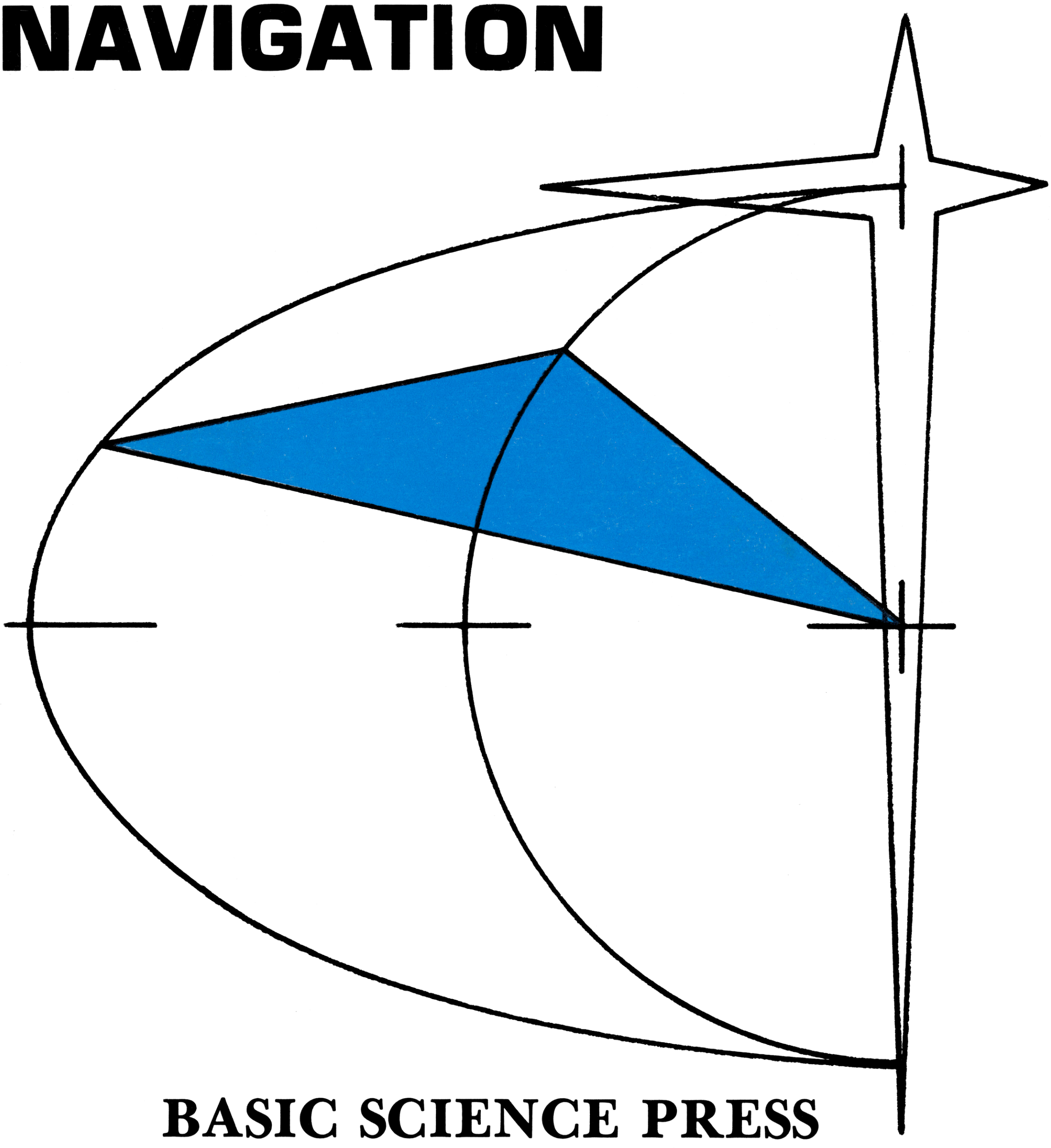
JONAH SLOCUM

CELESTIAL NAVIGATION



BASIC SCIENCE PRESS

CELESTIAL NAVIGATION

by

Jonah Slocum

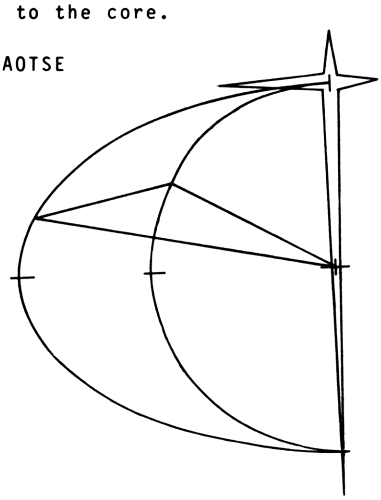
Basic Science Press
Palos Verdes Estates, California 90274

Third Edition

ISBN 0-917410-08-4

Before the Heaven and Earth existed
There was something nebulous:
      Silent, isolated,
      Standing alone, changing not,
      Eternally revolving without fail,
      Worthy to be Mother of All Things.
I know not its name
      And address it as Tao.
....
How the universe is like a bellows!
      Empty, yet it gives a supply that never fails;
      The more it is worked, the more it brings forth.
....
By many words is wit exhausted.
Rather, therefore, hold to the core.

      The Book of Tao, LAOTSE



"I find my zenith doth depend upon a most auspicious star"
The Tempest, Shakespeare

FOREWORD

Our zenith is directly over our head. The zenith of a star is its highest point of ascendancy as it climbs and descends the horizon. A line from our position, our zenith, through the zenith of the star marks the earth's polar axis. It is one of the simplest and most useful truths of celestial navigation.

The practicality of using the sun and the stars for navigation has been greatly advanced by development of portable calculators and computers which render feasible the tedious and lengthy calculations formerly made by use of logarithms aided by charts and tables.

A second advance is given by this book (for what is believed to be the first time) in using the Celestial Sphere as a framework for orthographic projections to present a unified picture of the location of the Observer, his position, and that of the Star. This method allows identifying the essential data needed for analysis and shows how they are related. Further, it indicates the correctness of the solution.

Celestial navigation penetrates the nature of the universe. Is the earth its center? What is the distance to the stars? These questions have occupied the thoughts of all thinking men, among them Aristotle, Galileo, Newton, Einstein and Kepler. They have built our concept of the universe. New data are being accumulated; new theories are being formulated. The study of celestial navigation prepares us for improved future understanding.

INTRODUCTION

Celestial Navigation combines the science of applied Mathematics (principally plane and solid geometry) with the science of Astronomy.

Geometry is, like Astronomy, an ancient science, dating at least from the Babylonian era of some 4000 years ago. Our basic texts on Geometry were devised by Euclid over 2000 years ago. Geometry deals with (1) the relation between the angles, lengths and surfaces of a triangle, and other **Plane** (that is, flat) shapes, and (2) the relation between the angles, lengths and surfaces of a sphere and other **Solid** shapes .

Astronomy deals with the materials and energies of the universe. It is subject to the universal laws of nature which govern the relations between the elements. These include the phenomena of Gravitation and Electromagnetic Waves (including Light), and the Conservation of Energy.

Astronomy is an Observational Science. We cannot employ the usual scientific approach to conducting experiments by varying parameters. Yet, the countless observations recorded since the dawn of history have supplied us with data to prove and disprove many profound theories of the Universe.

The study of any science has a value, not only in defining those laws which to it apply, but by pointing the way to central fundamental laws. Celestial Navigation calls our attention to (1) The Universal Totality of Energy and its conservation, and (2) The Law of Rhythmical or Cyclical Behaviour.

The Totality of Energy is shown through the immutability of the stars on the background of the Celestial Sphere. It is as if, as Omar Khayyam said, they might be mere perpetual pin points of light lit by a distant candle beyond the curtain of darkness. At the same time, there is a destiny in our far future, as the novelist H.G.Wells predicted, in which our Sun becomes a dull and dying star. This, too, conforms to another scientific truth, the Law of Entropy, in which all organized directed energy degrades inevitably into chaotic unavailable energy.

Cyclical behaviour is shown by the yearly movement of the sun to the north and south, by the rise and fall of the tides, and by the waxing and waning of the moon. As le Chatelier wrote, any change of a condition generates opposing forces tending to restore the original condition. Thus, does a cycle develop.

These factors play a part in the nature of our world and the universe. The study of Celestial Navigation can reveal many of their workings and evidences of their existance.

METHODOLOGY

What instruments and data are needed for celestial navigation?

First, a sextant, quadrant or astrolabe for finding the elevation of the celestial body, that is, its angle from the horizontal.

Second, a clock for determining the time of the observation.

Third, a method for finding the predicted location of this body at this time combined with data given by the Nautical Almanac. (One of the purposes of this book is to explain the origin and significance of the data given in the Almanac and to show how and why it changes from hour to hour; from day to day and from year to year.)

Finally, a trusted compass is needed at all times to maintain a continuity between position fixes, especially in times of heavy weather.

NAVIGATION TERMS

Location

CELESTIAL SPHERE - As used in this book, a shell of zero thickness having a radius of one Celestial Unit with the Observer of zero dimension at its center and upon which all celestial bodies are located. From this Celestial Sphere can be made orthographic (right angle) projections, or views, to show the Zenith Plane, the Orbit Plane, the Horizon Plane and the Latitude Plane, on which all necessary angles and directions are shown to scale.

ZENITH - (Spanish, cenit, from error in transcription of the Arabic semt ar-ra's, the place of the head.) The point on the Celestial Sphere directly above the Observer.

NADIR - Arabic, opposed, the point on the Celestial Sphere below the observer's vertical.

POLE - The axis through the Earth's center toward the portion of the Celestial Globe to which it always points. In the Northern Hemisphere, the pole points to Polaris, the North Star. In the Southern Hemisphere there is no prominent star to mark it, but it is found with the aid of the Southern Cross.

EQUATOR - The plane through the center of the Earth perpendicular to the Pole.

ECLIPTIC - The plane of the Earth's orbit about the Sun, and on which the eclipses of the Moon occur; hense, the name.

UPPER TRANSIT - The highest elevation of the observed body as it reaches its zenith in its orbit.

LOWER TRANSIT - The lowest elevation of the body as it reaches the nadir in its orbit.

LATITUDE - The angle from the Earth's center, north or south of the equator, from 0 to 90^{o}.

MERIDIAN - The arc from pole to pole through a designated position on the Earth's surface or on the Celestial Sphere.

GREENWICH - The original location of the Greenwich Royal Observatory, just outside London, used as the zero reference for positions east or west.

LONGITUDE - The angle from the Earth's center, east or west measured from the Greenwich Meridian to the observer.

DECLINATION - The angular distance of a celestial body North or South of the equator.

VERNAL EQUINOX - The spring passage of the Sun across the equator.

ZODIAC - A band on the Celestial Sphere 8^{o} to each side of the ecliptic and divided into twelve 30^{o} segments. During the year, as the Sun appears to move in its circuit about the Celestial Sphere, new portions of the Zodiac reach their ascendancy during the hours of darkness and become visible.

FIRST OF ARIES - The position of the sky at the time of the Vernal Equinox, originally at the start of the ascendancy of Aries, the Ram.

NAVIGATION TERMS

Directions

COURSE - The path of movement of the vessel.

BEARING - The direction of the object as viewed by the observer in the Horizon Plane; measured from the North and to the East or West according to the location of the object on the observer' meridian.

RHUMB LINE - (Spanish, straight) A course of constant direction.

GREAT CIRCLE ROUTE - A course of changing direction; the shortest distance along the Earth's spherical surface.

AZIMUTH - The same or a complement of the bearing but with the angle measured eastward only.

ELEVATION - The vertical angle from horizon to observer to the celestial body;

HOUR ANGLE - The angle between the designated meridian, the Earth's center and the body measured westward from upper transit.

Time

APPARENT SOLAR TIME - The time shown by the sundial, telling the arrival of the Sun at it's Zenith at the Observer's meridian.

MEAN TIME - The average time of a day given by a perfect clock.

EQUATION OF TIME - The difference between mean time and apparent time.

SIDEREAL TIME - Time as measured by the stars.

CONTENTS

BASIC CONSIDERATIONS

CELESTIAL BODIES

The Sun

The Sun's energy, like that of other stars, is the result of thermonuclear processes. Its core temperature is estimated to be 15 million degrees Kelvin (Celsius + 273). At the surface its temperature is 6,000 K.

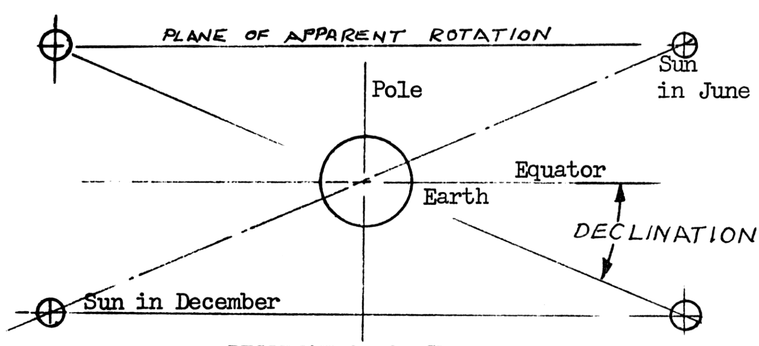
The surface of the sun is entirely gaseous and there can be seen dark patches of turbulence known as sunspots. At the edges there are discharges called flares, streamers and prominences. These produce electromagnetic effects which interfere with radio and TV reception and satellite navigation aids. Solar activity varies in intensity and some studies indicate a cycle of seven years.

The composition of the sun was first investigated by Joseph Von Fraunhofer, a German optician, in the early 1880's. He mapped the spectral separation of light into its wavelengths by passing it through a triangular glass prism. Light emmited by each element and viewed through such a prism creates a pattern of dark lines (shadows). The pattern identifies the element. Von Fraunhofer found an element unknown at the time which he named Helium (Gr. helios, the sun).

The sun appears to move through the sky so that its track is fartherest North in June and fartherest South in December. Its angle from the earth's equator during this travel is called the Declination. North Declination is signed positive; South Declination is signed negative.

Declination is a maximum at the time of the two Solstices (literally, the sun stands still) which occcurs on June 21 and December 21 for non-leap years and on June 20 and December 20 for leap years.

Declination is zero at the time of the two Equinoxes (literally equal day and night) which occur on March 21 and September 21 for non-leap years and on March 20 and September 20 for leap years.



DECLINATION OF SUN

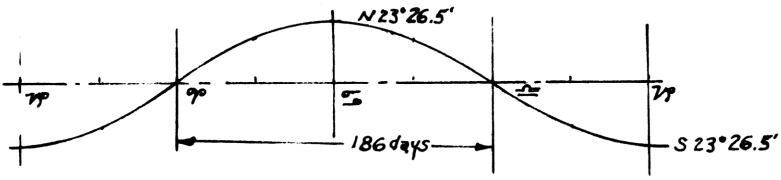
The usual reference for astronomical calculations is the Vernal (Spring) Equinox, or First of Aries, ♈, named originally for the constellation of stars visible at the time.

The time between successive events of solstices or of equinoxes, as measured by the apparent travel of the sun about the heavens, is approximately one-fourth year. However, as Hipparchus noted in -120, it takes 186 days for the sun to pass from the Vernal Equinox to the Autumnal but only 179 days to return.

As an approximation, the sun's declination can be expressed as a sine function:

Dec. = 23.44 sin θ

where Dec. = Declination, degrees
θ = Travel from Aries, ♈, deg.



Example DECLINATION OF SUN

For year 1984. Vernal Equinox March 20, 10h GMT
Find sun declination August 1, 10h GMT

Count days between dates:

MAR	APR	MAY	JUN	JUL	AUG	
20	20	20	20	20	1	
31+	30+	31+	30+	12		= 134 days

Convert to angle : (134/365.24) x360= 132.08°

Dec. = 23.44 sin 132.08 = 17.40°
(Compare to Almanac, N17° 54.8')

The Earth

The nearly spherical form of the earth was taught by the Greeks: Pythagoras in -600 and Aristotle in -400. The latter cited the shape of the earth's shadow on the moon during an eclipse. Other arguments for this shape are the progressive upward emergence of a distant body as we approach it on the horizon and the changing position of the stars as we move from north to south. Magellan's ship circumnavigated the globe in 1522 to prove this postulate and recent space explorations sent back visual evidence.

The earth's size was first determined by Eratosthenes of Alexandria in -250. He made two simultaneous observations of the sun's altitude in midsummer: one at Syene, near the equator, and one at Alexandria, 500 miles to the north. He found the sun at Syene to give a shadow at its zenith which was 7-1/4 degrees less than the one at Alexandria. He deduced that the angular difference corresponded to the difference in latitude.

By ratioing Eratosthenes data, we see that:

$$(360/7.25) \times 500 = 25{,}000 \text{ miles}$$

At the time of Columbus the earth was thought to be considerably smaller, which is the reason that Columbus thought he had found islands off the coast of China (hense, the West Indes).

The earth's rotation was demonstrated in 1851 by Jean Bernard Foucault (who invented the gyroscope) using a heroic experiment in which he swung an iron ball from the dome of the Pantheon in Paris. The motion was started by the burning of a cloth tether with the ball drawn to the side. The plane of motion was found to shift with each swing (marking a track along the sanded floor) and to rotate its plane once each 32 hours.

Considering that the ball swings in a constant plane and the earth rotates, the rotation should be 24 hours at the pole and increasing toward the equator.

Deviation= 15.00 sin L   in degrees/hour
L= Latitude, deg.

Example   LATITUDE BY FOUCAULT PENDULUM
Observed: 40 hours per revolution

Latitude= $\sin^{-1}((360/40)/15) = 36.87^{\circ}$

The Moon

The moon rotates about the earth and rotates about its own axis at a rate such that only one half of the moon's surface is ever visible from the earth.

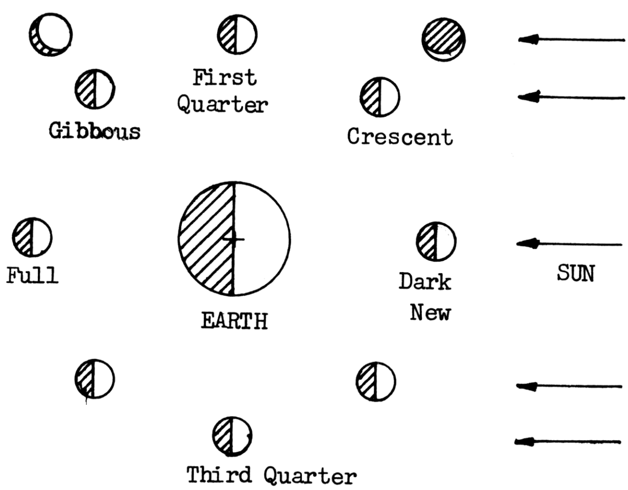
The distance of the moon from the earth can be determined by triangulation. That is, by measuring the angle of the moon's direction from two points separated by a known distance. The method is known as **Geocentric Parallax.** The mean distance from the earth to the moon is 238,000 miles.

The moon's diameter can be found by observing its angular width in a telescope and then computed from the known distance.

The moon, like the sun, moves through the sky in an east-to-west direction, but falls behind the motion of the stars by some 13 degrees per night. It makes a circuit of the stars, a lunar month, in approximately 27-1/3 days.

With reference to the sun, the lunar month is about 29-1/2 days (a month is a moon). This is known as a synodic month (a conjunction). The time varies more than half a day by effects of the earth and sun gravity.

The **phases** of the moon result from the changing angle of view from the earth:



PHASES OF THE MOON

The Stars

The Celestial Pole is the extension of the Earth's Pole and is marked by two points on the Celestial Sphere. In the northern hemisphere the pole is located within one degree of Polaris; in the southern hemisphere there is no prominent pole star but it can be found by the Southern Cross. The stars appear to rotate daily about the two poles as viewed from either hemisphere. It is the constancy of the rate of rotation and angle of rotation of the Earth, like a constant spinning top, that causes this effect.

The stars are visible during the daylight hours and during their elevation above the horizon of the observer. This precludes viewing those stars which reach our horizon only during daylight; they become visible when the Sun has moved to a new location in its apparent yearly circuit about the sky.

In the whole celestial sphere there are some six or seven thousand stars visible to the naked eye. The stars were studied by the Babylonians some four thousand years ago and who had vast libraries of clay tablets inscribed by cuneiform (wedgeshaped) writing equivalent to phonetic script on subjects which included astronomy and mathematics. The Babylonians were the first to use the structural arch and from them we derive our system of sets of 12 and 30; our circle of 360 divisions.

The first star catalog was made by Hipparchus of Bithynia in -125 and listed 1080 stars. This catalog was used by Ptolemy 250 years later. In 1450 a star catalog was made by Ulug Beigh at Samarcand. Tycho Brahe noted 1005 stars. The Nautical Almanac lists 57 stars as being of importance to navigators.

Hipparchus and Ptolemy graded stars into six classes of magnitude with the brightest as Class 1 and the barely visible as Class 6. Sir John Herschel, using photography, showed about 1830 that the first magnitude star is about 100 times as bright as the sixth. Negative values, as for Venus, indicate exceptional brilliance.

Star distance is so vast that until 1833 no estimates were made. In that year Henderson of th Cape of Good Hope estimated the distance of a star using the comparative angles as viewed in Winter and Summer, in effect using the Earth's orbit as a triangulation base. The nearest star is Alpha Centauri at 271,000 AU (Astronomical Units, the distance of the Earth to the Sun).

Our own sun is a star in the Milky Way Galaxy (Gr. Milky) which stretches across the sky as countless points of light. Telescopes resolve other apparent stars as other galaxies, each composed of millions of suns.

Change in Star Positions

The stars have changed little in observed declination in the last two thousand years but have changed some 30 degrees in Right Ascension, or Sidereal Hour Angle. The principal factor in changing the star's apparent position is the rotation of the intersection of the earth's pole on the Celestial Sphere. This rotation is at the rate of 25,800 years or about 50.2" per year.

The physical cause of precession was described by Newton as being the gyroscopic effect caused by the equatorial bulge and the earth's rotation. The attraction of the Moon has a disturbing effect on precession with the Sun as a lesser factor. The effect of the Sun and the Moon is zero at the time both are at the equator. The maximum disturbance is about 9.2" and runs over a period of 19 years, corresponding to the Lunar Cycle.

Another factor to change the apparent star positions is the aberration of light, as discovered by Bradley in 1725. This results by combining the effects of the speed of light and the earth's motion. The maximum aberration of any star is about 20.5", which is called the Constant of Aberration. Stars near the pole move in a circle of diameter 41"; stars at the ecliptic move in straight lines of length 41". Those between the pole and the ecliptic move in an ellipse of major axis parallel to the ecliptic and always 41" long.

Almanac data show the following change in location for the star Beta Gemmini, Pollux:

1925 Jan 1 R.A. = 7h 40m 44.3s (7.6789h)
SHA $= 360^{\circ} - (7.6789 \times (360/24)$
$= 244.8154^{\circ}$
$= 244^{\circ}\ 48.9'$
1983 Jan 1 SHA = 243 55.8

SOLAR SYSTEM DATA

Body		Mean Distance From Sun, AU	Sidereal Period *	Diameter Miles	Inclination to Ecliptic	Known Moons
Sun	☉	---	---	865,000.	---	---
Moon	☾	---	27.3 days	2,160.	$5^{\circ}\ 9'$	---
Mercury	☿	0.387	87.9 days	3,100.	$7^{\circ}\ 0'$	0
Venus	♀	0.723	224.7days	7,700.	$3^{\circ}\ 24'$	0
Earth	⊕	1.000	365.24 days	7,927.	---	1
Mars	♂	1.524	1.88 years	4,200.	$1^{\circ}\ 51'$	2
Jupiter	♃	5.203	11.86 years	88,700.	$1^{\circ}\ 18'$	12
Saturn	♄	9.539	29.46 years	75,100.	$2^{\circ}\ 29'$	9
Uranus	♅	19.182	84.01 years	32,000.	$0^{\circ}\ 46'$	5
Neptune	♆	30.058	164.79 years	27,700.	$1^{\circ}\ 46'$	2
Pluto		39.518	248.43 years	3,600.	$17^{\circ}\ 9'$	0

* Mean Solar Days

The Planets

Planet is a Greek term meaning "the wanderer". As such, they grouped the Sun, the Moon, and Mercury, Venus, Mars, Jupiter and Saturn.

Mercury is visible to the naked eye near the horizon before sunrise or after sunset. (The noted observer Tycho Brahe was never able to see Mercury.) Venus is brighter than any star and can be seen in the daytime if not too near the sun. The angle between Venus and the Sun is never more than 47 degrees. Jupiter, and sometimes Mars, are brighter than any star. Saturn appears as another star.

When two planets are close together the body with the greatest **Right Ascension** is most eastward. The term comes from the right angle which the body makes with the horizon as it first becomes visible in moving to its zenith.

The Constellations

The constellations are configurations of stars. The Greeks recognized 48 constellations with which they connected the forms of heroes and animals. Twelve of these comprise the Zodiac.

The stars in a constellation are subnamed generally by the method of Bayer in 1603. He preceeded each star in the constellation with a letter of the Greek alphabet according to its relative brightness.

The stars in some constellations are named with the Greek letter according to its order of appearance on the horizon (its Right Ascension).

THE GREEK ALPHABET

1	Alpha	Α α		13	Nu	Ν ν
2	Beta	Β β		14	Xi	Ξ ξ
3	Gamma	Γ γ		15	Omicron	Ο ο
4	Delta	Δ δ		16	Pi	Π π
5	Epsilon	Ε ε		17	Rho	Ρ ρ
6	Zeta	Ζ ζ		18	Sigma	Σ σ
7	Eta	Η η		19	Tau	Τ τ
8	Theta	Θ θ		20	Upsilon	Υ υ
9	Iota	Ι ι		21	Phi	Φ φ
10	Kappa	Κ κ		22	Chi	Χ χ
11	Lambda	Λ λ		23	Psi	Ψ ψ
12	Mu	Μ μ		24	Omega	Ω ω

The Zodiac

The Zodiac (Gr. Zoo) was described by Hipparchus in -200. It is a part of the celestial sphere through which the sun and planets move. The Ecliptic, the path of the sun (along which it may eclipe the moon), is at the center of the Zodiacal Band which has a width of 16 degrees.

The Zodiac is divided into twelve constellations or Signs or Houses. Half of these lie north of the equator.

The **Precession of the Equinoxes** causes the stars to advance in relation to the Vernal Equinox (the time of zero sun declination in the spring). This effect was first described by Hipparchus in -125 who cited its discovery by the Babylonians some 400 years earlier. The rate is one day in 70 years. The stars have shifted 30 degrees since the time of Hipparchus. While we say the Vernal Equinox to occur at the first point of Aries our observations show it to be in Pisces.

SIGNS OF THE ZODIAC

1	♈	ARIES, Ram	Mar 21	Vernal Equinox
2	♉	TAURUS, Bull	Apr 20	
3	♊	GEMINI, Twins	May 21	
4	♋	CANCER, Crab	Jun 22	Summer Solstice
5	♌	LEO, Lion	Jul 23	
6	♍	VIRGIO, Virgin	Aug 23	
7	♎	LIBRA, Scales	Sep 23	Autumn Equinox
8	♏	SCORPIO, Scorpion	Oct 23	
9	♐	SAGITTARIUS, Archer	Nov 22	
10	♑	CAPRICORNUS, Goat	Dec 22	Winter Solstice
11	♒	AQUARIUS, Water Bearer	Jan 20	
12	♓	PISCES, Fishes	Feb 19	

CELESTIAL MECHANICS

Historical

The Geocentric Theory- The Geocentric (Gr. Earth) theory holds that the earth is the center of the universe. It assumes the earth to be stationary and the sun, planets and stars to revolve about it. It is the theory most in accord with our senses. The theory was advanced by Hipparchus and is also known by the name of Claudius Ptolemaeus, a Greek-Egyptian mathematician and mapmaker.

The Geocentric Theory has one disturbing effect to account for. At times the planets appear to move backwards through the sky to describe elongated ellipses. The ancients explained that the planets both moved in circles about the earth as well as in circles about themselves, producing Epicycloidal Motions.

The Heliocentric Theory- This theory, that the sun is the center of the universe, was first attributed to Aristarco of Samos in -270. The theory was revived by Nicolaus Copernicus, a Polish astronomer, in 1543. The theory explains the apparent motion of the planets.

The Tychonic System- Tycho Brahe, a Danish astronomer, held that the earth was stationary but that the other planets moved about the sun. He made special instruments including a steel quadrant with which he could measure within a minute of arc.

Only the Copernican theory agrees with the test of the aberration of light (the apparent shifting of position of the stars caused by their motion) and the annual parallax of the stars (the shift in star position with the seasons).

Galileo's Law of the Pendulum

Galileo Galilei was the first to observe celestial bodies with a telescope. His work was published in 1610. He discovered the moons of Jupiter and noted its analogy with the theory of Copernicus. For denying the Geocentric Theory he was imprisoned by the authorities but allowed to continue his work. (The Holy Roman Empire ruled Central Europe from 962 to 1806). He began a scientific revolution and made findings of (1) the principle of inertia, (2) the law of the descent of bodies, (3) the principle of relativity, (4) the composition of velocities, and (5) the law of the pendulum.

The Law of the Pendulum:

$$T = 2\pi(L/g)^{1/2}$$

where T = Period, seconds (time for swing cycle)
L = Length of swing, feet
g = Gravity, feet/sec-sec

Kepler's Laws of Planetary Motion

Johannes Kepler worked with Tycho Brahe and used his data and observatory to devise fundamental laws of planetary motion. The first two were published in 1609.

1. The orbit of each planet is an ellipse, with the sun being at one foci.
2. The radius vector of each planet (the line joining its center with that of the sun) moves over equal areas in equal times.
3. The square of the period of each planet's revolution around the sun is proportional to the cube of its distance from the sun.

Newton's Laws of Motion and Gravity

Isaac Newton published in 1687 the Principia Mathematicia which proved the assumptions of Kepler and added new laws:

1. A body in motion tends to remain in motion unless acted upon by external force.
2. Force is equal to mass times acceleration.
3. The attraction between any two particles is proportional to the product of the mass of each particle divided by the square of the distance between them.

Bode's Law of Planetary Spacing

J.E. Bode, a German astronomer, attributed to Titus of Wittenburg the relation which was responsible for discovery of the asteroids. The distance from the planet to the sun is found (in A.U.) by the number 4 plus 3 doubled for each increment.

4	4	4	4	4	4	4	4	4
+ 0	+ 3	+ 6	+12	+24	+ 48	+ 96	+192	+384
4	7	10	16	28	52	100	196	388
0.4	0.7	1.0	1.6	2.8	5.2	10.0	19.6	38.8
Mercury	Venus	Earth	Mars	Asteroids	Jupiter	Saturn	Uranus	Neptune

RHYTHM

The realization of Newton's First Law of Motion implies the absence of any second particle of matter in the universe. This not being so, as the simplest case imagine a second particle of a different mass and with a velocity in another direction.

The two particles will be attracted to each other, toward a center of mass between the two particles. They will accelerate toward this point. The point is not fixed, however, but moving.

Considering one body alone, it will move in a closed curve of which one of the foci must coincide in position with the common center of gravity.

This condition is realized in every case of planetary revolution. This constitutes rhythmical behaviour; velocitys are rhythmically augmented and diminished.

Thus in all cases, whether molar or molecular, the rhythm of motion is necessitated by the fact that no portion of matter is uninfluenced by other forms of matter.

The theory of rhythm is eloquently put by John Fiske, circa 1890, in his Outlines of Cosmic Philosophy:

> "Periodicity, rise and fall, recurrence of maxima and minima,...this is the law of all motions whatever, whether exemplified by the star rushing through space, by the leaf that quivers in the breeze, by the stream of blood that courses through the arteries, or by the atom of oxygen that oscillates in harmony with its companion atoms of hydrogen in the raindrop...The phenomena which are represented to our consciousness as light, heat, electricity and magnetism, are the products of a perpetual trembling, or swaying to and fro of the invisible atoms of which visible bodies are composed. When we contemplate the heavens on a clear autumn evening, and marvel at the beauty of Sirius, that beauty is conveyed to our senses by the medium of atomic shivers, kept up unceasingly during the past twenty-two years, at the average rate of six hundred millions of millions per second."

TERRESTRIAL COORDINATES

The earth is usually assumed to be a sphere. The equatorial diameter is about 7,926 miles; the polar diameter about 27 miles less. This results in an equatorial circumference of 24,900 miles.

Navigation distances are given in nautical miles of 6080 feet (as compared to 5280 feet per statute mile, for a ratio of 1.1515 nm/sm). Accordingly, the equatorial diameter is 21624 nm, or 60 nm/degree or 1 nm/minute of arc.

Latitude- The earth is assumed to be circled by parallel lines extending east and west and reaching from the equator to the poles. The parallel of zero latitude is the equator. The parallel of 90 degree latitude is at the pole. The distance between any two parallels is constant.

Longitude- The earth is assumed to be covered by arcs or meridians extending from pole to pole. The **Prime Meridian** passes through the Royal Observatory at Greenwich, near London. Meridians of longitude extend 180 degrees to the west and to the east (signed positive for west and negative for east in this text). East and West Longitudes meet in the Pacific at the International Date Line, at which the new day begins.

MAPMAKING

The Greeks were the first to elevate mapmaking to a science. Erastostenes, head of the great library of Alexandria (which was later burned totally), determined the circumference and tilt of the world. His map showed known geographical features with considerable precision of latitude and longitude. Strabo (-63 to 21) is known for 27 volumes of geography.

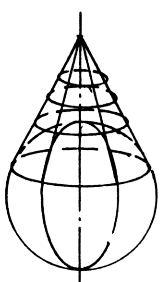
The New World is named for Amerigo Vespucci, famed Florentine navigator and cartographer, who discovered the Falklands in 1504. It was Vespucci who determined that Columbus had found a new land. Vespucci also discovered the mouths of the Amazon and devised a system for computing exact longitude by use of the moons of Jupiter.

In general, however, early maps appear distorted by (1) the general inability of the ancients to determine proper longitude, and (2) lack of a good method for showing the earth's sphere on a flat plane.

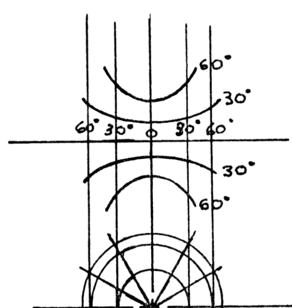
The most used projection for navigation was devised by Mercator a Flemish man named Gerhard Kremer (Kremer, German, merchant). His map appeared in 1569. This is a **Conformal Projection**, one in which the shape of a feature is the same as on the globe. The meridians appear as evenly spaced vertical lines. The latitudes are spaced proportional to meridian spacing on the globe. Areas are exaggerated near the poles. Compass directions appear as straight lines. A great circle is a curved line.

The **Gnomic Projection** was originated by Jean Dominique Cassini (1625-1712), member of a famous family of French Astronomers originally from Italy. Jean Dominique used simultaneous observations at the Paris observatory and at other localities to prepare maps of great accuracy. In the Gnomic projection, which is a projection upon a tangent plane from the earth's center, all great circles are shown as straight lines. Use is made of the Polar and the Equatorial case.

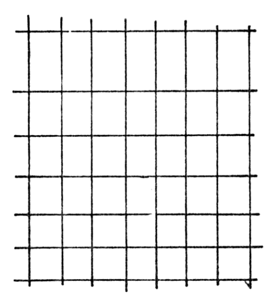
The **Lambert Conformal** projection shows the earth's surface upon a cone. Meridians and parallels intersects at right angles. Three cases are used: the Polar Hemisphere, the Equatorial Hemisphere, and the Oblique World Map.



CONIC



GNOMONIC



MERCATOR

NAVIGATION INSTRUMENTS

The Compass

A primitive compass can be made by floating on a cork a naturally magnetized lodestone. The stone will align its axis with the local magnetic field so that a particular end will always point to magnetic north.

The magnetic north is in the vicinity of Hudson's Bay in northern Canada. Magnetic north has had historical migrations as shown by ancient stones.

The compass direction of magnetic north can vary according to earth magnetic fields. The difference in compass direction and true north is called **variation** or **magnetic declination.** It can exceed 30 degrees.

An installed compass must be calibrated for **deviation,** the effect of the ship's magnetism. This is done be swinging the ship, that is, facing the ship in directions around the compass and noting the compass error at each point. Deviation can be minimized by use of "Flinders' Bars" (named for Captain Flinders) to counteract such local magnetism as the engine.

The modern compass has been rendered insensitive to most error by addition of the gyroscope in 1920 by Elmer Ambrose Sperry. The gyroscope is the nucleus of inertial guidance instrument which sums motions in three axes. Like all complex instruments it should have backup provisions.

The compass is graduated into 360 degrees of arc with each degree divided into 60 minutes and each minute into 60 seconds. The marine compass is divided into 32 points of 11-1/4 degrees each. The **Cardinal Points** are North, South, East and West. Boxing the Compass consists of naming the points: North, N by E, NNE, NE by N, NE, NE by E, ENE, E by N, East, etc.

The **Lubber's Line** is a mark on the compass bowl to indicate the direction of the ship's keel. The **Pelorus** is a dummy compass with sighting vanes and a lubber's line to take bearings of distant objects.

The Astrolabe and Quadrant

These are device for measuring angles, usually from the horizon to a star. The astrolabe (from the Greek, meaning to know the stars) is thought to have been invented in the latter part of the sixteenth century by the Arabs. It has a small circular disc with moveable sights and scribed markings. They were often carried by travelers on a lanyard about the neck. Quadrants served the same purpose for stationary studies and were often made in large sizes.

The Telescope

Galileo is credited with making the first telescope in 1609. He is said to have heard of such devices in Holland. His was a **refracting** telescope consisting of two lenses through which light passes with a change of direction, refracted by the glass boundaries.

The reflecting telescope consists of a curved mirror by which light is concentrated to an observation point. The 200 inch Hale telescope at Mount Palomar is of this type. Mount Palomar has also a Schmidt telescope which combines the reflecting mirror with a refractive lens for greater angle of field.

Archimedes of Syracuse is reported in the Roman siege of -212 to have set fire to the ships in the harbor by use of mirrors to concentrate the sun. The laws of optics were studied by the Islamic Ibn al-Haytham, or Alhazen (965-1038). He investigated reflection and developed equations describing use of curved mirrors. His work on lenses was unsurpassed for centuries.

Soon after Galileo's discovery, Christiaan Huygens published works on improved lenses and theory. He gave us the principle of the wave theory of light. The wave theory of light was developed by Augustin Jean Fresnel (1788-1827). He dealt with interference and polarization and invented the Fresnel Lens, flat transparent and ridged with circular groves on one side.

> Huygens Principle: Every point on a wave front of light is the source of new waves.

The classic work on Optics was written in 1704 by Isaac Newton who treated light as minute particles. Those of different size have different color.

The Sextant

The sextant (sixth of a circle) came into use about 1730. The framework is a circular arc marked from 0 to 120 degrees. A moveable arm intercepts the graduated arc and holds a mirror at its pivot. A second fixed mirror on the frame reflects light to the observer who also sees simultaneously the horizon through the fixed mirror which is half-silvered. When the sextant is set at zero angle the horizon viewed through the clear glass and the two mirrors should appear as a single line. If the reading is not zero an "index error" occurs, which can be adjusted to zero.

In using the sextant on a star, the angle is set to zero to view the star, then, holding the star in view, draw down the angle to the horizon.

Corrections must be made for the difference between true and observed angle caused by **refraction** and **parallax.** Refraction is caused by bending of the light ray as it enters the earth's atmosphere. Parallax is caused by the observer not being at the earth's center. Values for refraction and parallax are combined in Almanac "Altitude Tables".

The horizon appears to be lower as the observer's height is increased. This effect is known as **dip.** The value of dip is given by the Almanac.

Observations of the sun and moon are made at the upper or lower edge, spoken of as the **limb.** The semidiameter must be added or subtracted using Almanac values.

Example SEXTANT CORRECTIONS

Data: Observer's height, 45 feet
Observed star at altitude, 46° 00.5'

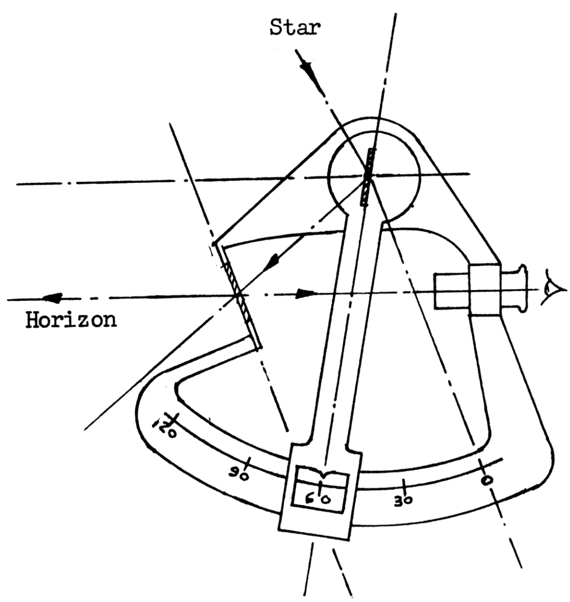
Procedure, with Values from Almanac:

Sextant Altitude,	H_s=	46° 00.5'
	Dip =	-11.2'
Sextant Apparent Altitude,	H_a=	45° 48.3'
	Altitude Correction=	-00.9'
Observed (corrected) Altitude,	H_o=	45° 47.4'

The Chronometer

Mechanical clocks date from the fourteenth century and were at the time so large they were placed in clock towers where everyone could see. Portable clocks were made in 1430, and in 1509 the first pocket watch was made. Christiaan Hugens in 1660 made his first chronometer. John Harrison made an accurate timekeeper in 1735 weighing 65 pounds and using pendulums.

The chronometer (Gr. **chronos**, time) is the name for a highly accurate clock used for navigation. Current technology has brought high accuracy to the inexpensive quartz watch, timed by the vibrations of the quartz crystal excited by electrical impulses. Quartz watch accuracy of 30 seconds in a year is typical, with a battery life of two years or more. A spare watch should be carried and set to Greenwich time.



SEXTANT

PLANE NAVIGATION

Analytical Considerations

Navigation consists of setting a compass course, or series of courses, from an origin described by a set of latitude and longitude to a destination described by a new set of latitude and longitude.

The essentials of navigation are increments of speed, time and direction. Each of these variables needs to be recorded in the log at the instant that the change occurs, or if constant, confirmed at intervals. Paths of constant speed and direction constitute legs of the journey and are used to compute the dead reckoned position.

Plotting should be used where feasible to indicate the chart position and heading. Plotting can reveal gross errors not seen when inputting data to computers.

A navigation position is most accurately determined by direct observation of known geographical features. Navigation aids, such as satellite data, bouys, radio direction signals, and navigation lights are the usual source of navigation data. It is always desirable to confirm their validity by celestial navigation.

Precision of Computation

The precision with which a position can be located depends on the accuracy of the data and its depth of subdivision. By design, one minute of arc equals one nautical mile. For a positional accuracy of one mile, then, we require data and computation accurate to one minute of arc.

> (Note that an accuracy of one nautical mile allows a latitude tolerance of plus or minus one mile as well as a longitude tolerance of one mile.)

An equivalent precision is required for time measurements. The global circumference in terms of time is 24h x 60m, while in terms of arc it is 360^{o} x 60'. We see then that one minute of arc is equivalent to 24m/360' or 1/15m = 4s. Accordingly, one nautical mile accuracy would require a time measurement within 4 seconds.

In fractional terms, one minute of arc requires a computational accuracy of

$$1'/\ 60' = 0.0167 \text{ degree}$$

$$4s/(60m \times 60s) = 0.0011 \text{ hour}$$

Plane Geometry

Geometry by Inspection - The Propositions of Euclid (of which G.A.Wentworth in Plane and Solid Geometry, Ginn & Co., 1902, describes over a hundred) form the basis of plane geometry and provide rules by which geometric figures can be rapidly categorized. Some of the most useful are:

1. Parallel lines are everywhere equidistant.
2. Parallel lines cut by a transversal form angles which are the same at each parallel line.
3. For two intersecting straight lines the opposite angles are equal.
4. A perpendicular is the shortest line which can be drawn to a straight line from an external point.
5. An **acute** angle is one of less than 90 degrees; an **obtuse** angle is one between 90 and 180 degrees.

The extention of one side of an acute angle past the apex forms an obtuse angle equal to 180 degrees minus the acute angle.

6. A right triangle is one which has one interior angle equal to 90 degrees.
7. The sum of the interior angles of a triangle is 180 degrees.

Pythagorean Theorem - The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

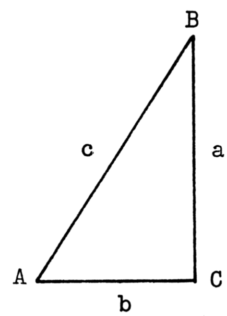
$$a^2 + b^2 = c^2$$

Geometric Identities - There are three fundamental geometric identities which relate to a right triangle:

SINE - or sin; opposite side divided by hypotenuse; sin A=a/c

COSINE - or cos; adjacent side divided by hypotenuse; cos A=b/c

TANGENT - or tan; opposite side divided by adjacent side; tan A=a/b



The Law of Sines - The sides of a triangle are proportional to the sines of the opposite angles:

$$a/\sin A = b/\sin B = c/\sin C$$

The Law of Cosines - The square of any side of a plane triangle is equal to the sum of the squares of the other two sides diminished by the product of the othe two sides and the cosine of their included angle:

$$a^2 = b^2 + c^2 - 2\ bc \cos A$$

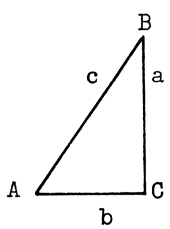
Vectors

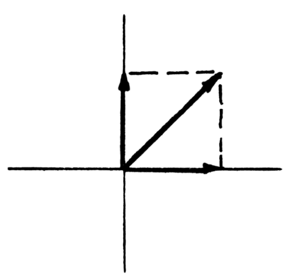
A vector is a straight line which represents both a magnitude and a direction. For example, a speed of ten knots and a course of 45 degrees could be represented by a line of a certain scale length and a particular angle to the chart base line. The top of the chart is usually North, and is so noted. The direction is represented by an arrow head, with the other end called the tail of the vector.

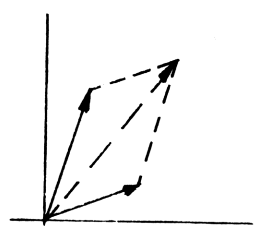
A vector can be resolved into components which are parallel to the vertical and horizontal axes. For navigation, the vertical dimension is the change of **latitude**; the horizontal dimension is the change of **departure.** (Considering that distances between longitudes are variable.)

Vectors can be combined on a chart by adding the tail of the second vector to the head of the first. The **resultant** vector is drawn from the origin to the end.

Alternately, vectors can be combined at the origin. Then, a parallelogram is constructed by adding two parallel sides. The diagonal is the resultant.







Earth locations on a map are determined by Cartesian Coordinates (devised by Rene' Descartes). The origin is our starting point and through it extend horizontal and vertical lines representing the four cardinal compass points, N,E,S, and W. These lines separate the the circle into four Quadrants, reading clockwise numbered I, II, III, and IV. Plus is to the North and East. Minus is to the South and West. Accordingly, any point on the map can be connected with the origin and the base lines to form a triangle. The trignometric functions of this triangle have signs corresponding to the quadrants.

The direction measured clockwise from North is called the **Azimuth.** By use of trignometric relations we can use this angle to find the vector components and resultant vector for, say, two legs of a traverse.

East-West distance (Departure):

$$dD = D_1 \sin A_1 + D_2 \sin A_2$$

North-South distance, Latitude:

$$dL = D_1 \cos A_1 + D_2 \cos {}_2$$

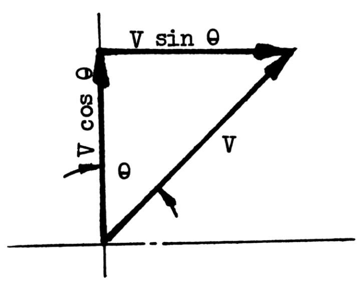
Resultant: $R = (dD^2 + dL^2)^{1/2}$

In the First Quadrant, the azimuth angle, 0, is:

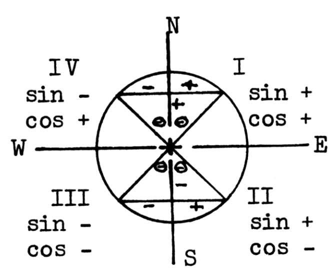
$\theta = \tan^{-1}(dD/dL)$.....which is read "the angle whose tangent is"

Angles in other Quadrants have equivalent values:

Second Quadrant, $\theta = 180 - \text{abs } \theta$
Third Quadrant, $\theta = 180 + \text{abs } \theta$
Fourth Quadrant, $\theta = 360 - \text{abs } \theta$



ADDITION OF VECTORS



SIGNS OF COMPASS QUADRANTS

The **Rhumb Line** and **Traverse:** - A course of constant direction is a Rhumb Line (Span. direction). It appears on a Mercator chart as a straight line. A series of rhumb lines constitute a Traverse.

Example TRAVERSE DIRECTION AND TRUE COURSE

Data: Leg 1 Distance, D_1 = 240 nautical miles
Azimuth, A_1 = 68. degrees

Leg 2 Distance, D_2 = 15 nm
Azimuth, A_2 = $165.^\circ$

Procedure:

(1) Find value for departure, dD

$$dD = D_1 \sin A_1 + D_2 \sin A_2$$

$$= 240 \sin 68. + 15 \sin 165. = 226.41 \text{ nm}$$

(2) Find value for latitude, dL

$$dL = D_1 \cos A_1 + D_2 \cos A_2$$

$$= 240 \cos 68. + 15 \cos 165 = 75.42 \text{ nm}$$

(3) Find Resultant, R

$$R = (dD^2 + dL^2)^{1/2}$$

$$= ((226.41)^2 + (75.42)^2)^{1/2} = 238.64\text{nm}$$

(4) Find azimuth of resultant, θ

$$\theta = \tan^{-1}(dD/dL)$$

$$= \tan^{-1}(226.41/75.42) = 71.58^\circ$$

The vector is in the First Quadrant

Example COURSE CORRECTION FOR DRIFT

Data: Azimuth, true heading, θ_H 68.°

Speed, True Heading, S_H 240. knots

Azimuth, Drift, θ_D 165.°

Speed, Drift, S_D 15. knots

(Currents are described in direction as **set** with speed in knots as **drift**)

(1) Departure component, C_H

$$C_H = S_H \sin \theta_H + S_D \sin \theta_D$$

$$= 240 \sin 68 + 15 \sin 165 = 226.41 \text{ knots}$$

(2) Latitude component, C_V

$$C_V = S_H \cos \theta_H + S_D \cos \theta_D$$

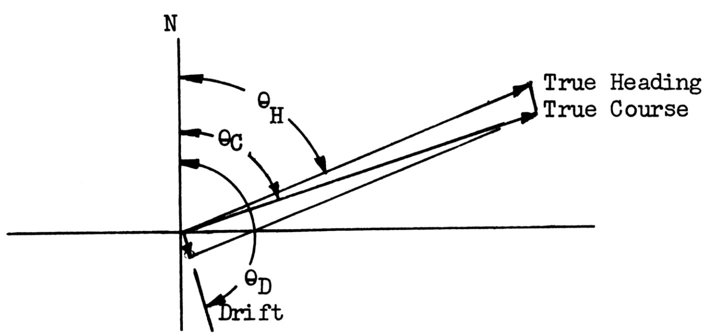
$$= 240 \cos 68 + 15 \cos 165 = 75.42 \text{ knots}$$

(3) Resultant, R

$$R = ((226.41)^2 + (75.42)^2)^{1/2} = 238.63 \text{ knots}$$

(4) True Course, θ

$$\theta = \tan^{-1}(226.41/75.42) = 71.57^\circ$$



Middle Latitude Sailing

"Sailing" is the term which describes a method of solving navigation problems. These problems are:

1. To find the course and distance between two known points.
2. To find the position after sailing a known distance on a given course.

Middle Latitude Sailing is a navigation analysis method which uses the latitude midway between the point of departure (origin) and point of destination to determine the departure (East-West distance). The root of the problem is that the distance between longitudes is not constant, being 60 nautical miles at the equator amd zero at the poles.

By definition the Longitude change at the Equator per degree is:

$$dm/d_e = 60 \text{ nm/deg} = 2 \text{ pi } r_e/360$$

The radius of the earth at a given Latitude is:

$$r_L = r_e \cos L$$

The Longitude change per degree at the Latitude is:

$$dm/d_L = 2 \text{ pi } (r_e \cos L)/360$$

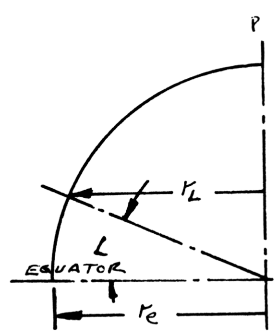
Transposing: $r_e = 60 \times 360/(2 \text{ pi})$

then $dm/d_L = 60 \times (2 \text{ pi}/360) \times (360/(2 \text{ pi})) \cos L$
$= 60 \cos L$ deg/nm

for instance, the longitude spacing at latitude 59 is

$$dm/d_{59} = 60 \cos 59.0000 = 30.9022 \text{ nm/deg}$$

further, the departure distance per minute of arc at 59^o is simply $\cos 59.0000 = .5150$nm/'



Dead Reckoning

Dead reckoning is the term used to describe estimating the location by use of a known point of origin and measured courses and distances. Middle latitude sailing can be used in this analysis.

The indicated direction needs to be corrected for (1) Compass Deviation, (2) Compass Variation, (3) Vessel Leeway, and (4) Vessel Drift.

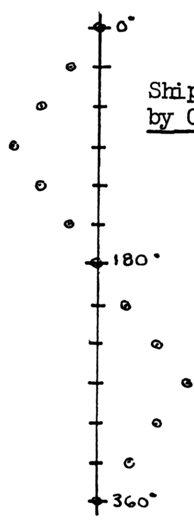
Deviation is the result of onboard magnetic disturbances. The deviation varies with the heading of the vessel. A "compass card" is made up to describe the magnitude and direction of the correction to be made as found by test.

Variation is read from a chart of the locality. It can change over a period of years.

Leeway describes the action of the wind and the waves which cause the vessel to deviate from the desired direction of travel. It may be influenced by unsymmetrical drag.

Drift refers to the currents which add secondary components of motion to the vessel.

These items are tabulated in columns with rows for each course, together with distance and the Latitude/Departure components; then summarized. The direction of the wind should be noted.



SKELETON DEVIATION CARD

Standard COMPASS: S.S *Alta Star*

Ship Heading by Compass	Deviation	Ship Heading by Compass	Deviation
000°	0°	180°	0°
030	3° W	210	3° E
060	6 W	240	6 E
090	9 W	270	9 E
120	6 W	300	6 E
150	3 W	330	3 E
180	0	360	0

Example
DEAD RECKONING FROM FARALLON ISLAND LIGHT

Location of Farallon Island Light: Latitude 37° 41' 51" N
Longitude 123° 00' 07" W

Bearing by compass to light: N 89° E, 12 miles distant

Current set (drift) S 15° E (correct magnetic) 9 miles per day

Leg	Compass Course	Chart Variation	Ship's Deviation	Wind	Leeway	Total Error	True Course		Time	Speed	Distance
D.P.	S 89°W	6°E	12°W	---	---	6°W	S83°W	(263.00)	2/14/68 06:00	---	12 nm
1	North	"	12°W	E	2°W	8°W	N08°W	(352.00)	06:00 12:00	10 knots	--- 60
2	S 15°W	"	14°W	W	4°W	12°W	S03°W	(183.00)	12 00 00:30'	10 kn (2/15)	--- 65
3	S 80°W	"	6°W	NW	5°W	5°W	S75°W	(255.00)	00:30' 06:54'	10 kn	--- 64
4	N 80°W	"	5°E	N	4°W	7°E	N73°W	(287.00)	06:54' 13:12'	10 kn	--- 63
5	N 50°W	"	4°E	N	3°W	7°E	N43°W	(317.00)	13:12' 19:24'	10 kn	--- 62
6	N 20°W	"	8°W	NE	4°W	6°W	N26°W	(334.00)	19:24' 01:30'	10 kn (2/16)	--- 61
Set	S15°E	"	--	---	---	6°E	S09°E	(171.00)	2 days	9 m/day	--- 18

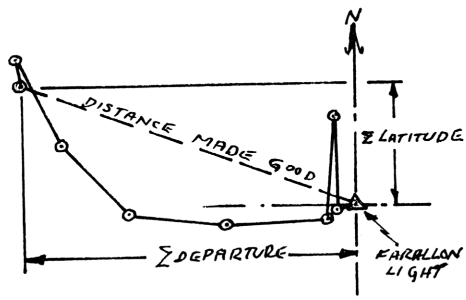
Example (continued)

Dead Reckoning From Farallon Island Light

Use Program TRAVERSE COMPONENT SUMMATION

ENTER: N = 8

1	Dist.	= 12
	Head.	= 263
2	Dist.	= 60
	Head	= 352
3	Dist	= 65
	Head	= 183
4	Dist	= 64
	Head	= 255
5	Dist	= 63
	Head	= 287
6	Dist	= 62
	Head	= 317
7	Dist	= 61
	Head	= 334
8	Dist	= 18
	Head	= 171



Find: Latitude = 77.29 nm
Departure = 211.94 nm
Distance made good = 225.59 nm

Find Middle Latitude:

Change of latitude: 77.29 nm/60 m/deg = 1.29 deg

Original latitude: 37° 41' 51" N = 37.70°N

Final Latitude : 37.70 + 1.29 = 38.99°N

Middle Latitude : 37.70 + 1/2 x 1.29 = 38.35°N

MIDDLE LATITUDE LONGITUDE SPACING
Spacing is 47.04 nm/degree

Final Longitude = 123° 00' 07" W + (211.94/47.04)
= 127.51°W

Dead Reckoned New Position: Latitude 38.35°N
Longitude 127.51°W

True Speed by Observation of Fixed Object

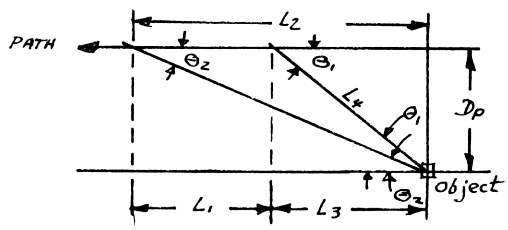
The navigator can determine his true speed by observing a fixed object while maintaining a constant speed and direction. The method consists of observing the angle between the true course and the object through an angle change of some 20 degrees and noting the time lapse between readings.

Note that the true course is the track across the earth's surface accounting for drift. The true heading is the corrected compass direction.

The perpendicular distance from the true course to the object must be known. For an airplane this is the altitude. The direct distance from the ship to the object, as measured by a rangefinder, must be corrected to the perpendicular.

$$D_p = L_4 \sin \theta$$

where D_p = Perpendicular distance
L_4 = Range from ship to object
θ = Angle between path of ship and object



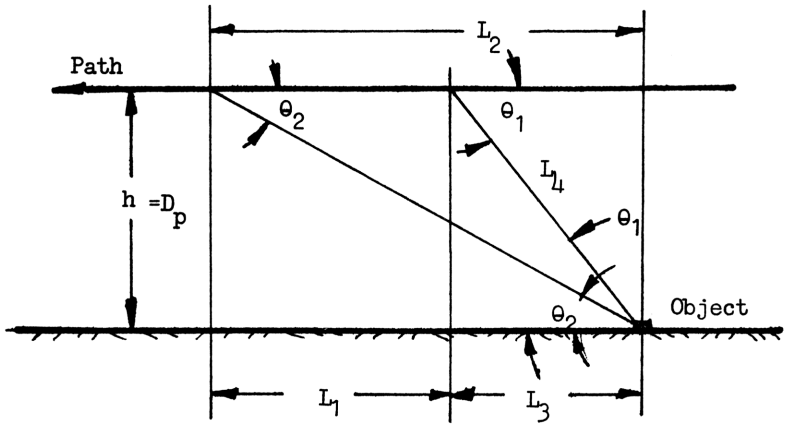
The distance from the object on a perpendicular through the object is

$$L_2 = D_p/\tan \theta_2$$

$$L_3 = D_p/\tan \theta_1$$

The true speed of the ship during the interval dT is:

$$S = (D_p/\tan \theta_2 - D_p/\tan \theta_1)/dT$$



TRUE SPEED BY FIXED OBJECT

Example TRUE SPEED BY OBSERVING FIXED OBJECT

DATA: Range to object, 5320 feet, L_4
First angle from path to object, θ_1 = 70. deg
Second angle from path to object, θ_2 = 40. deg
Time interval for observation, dT = 14. sec

PROCEDURE: TRUE SPEED BY FIXED OBJECT

(1) Find perpendicular distance to object,

$$D_p = L_4 \sin \theta_1$$
$$= 5320 \sin 70 = 4999.\ \text{ft}$$

(2) Find distance travelled

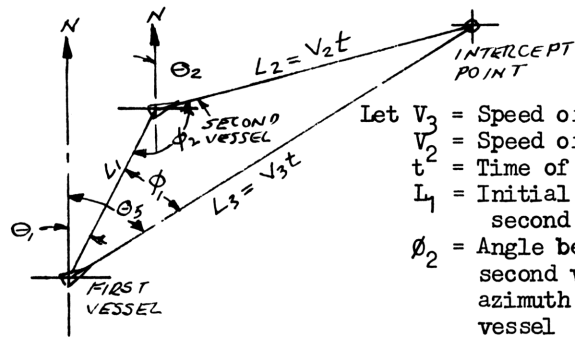
$$dL = L_1 = (4999/\tan 40) - (4999/\tan 70)$$
$$= 4138.\ \text{ft}$$

(3) Find true speed

$$S = 4138/14 = 295.6\ \text{ft/sec}$$
$$\times\ 3600/6080 = 175.\ \text{knots}$$

Course For Intercept Mission

A course for intercepting a second vessel can be calculated from observing its direction, path and speed.



Let V_3 = Speed of first vessel
V_2 = Speed of second vessel
t = Time of interception
L_1 = Initial range to second vessel
ϕ_2 = Angle between path of second vessel and azimuth to second vessel

These variables can be related by the above described Law of Cosines:

$$(V_3 t)^2 = L_1^2 + (V_2\ t)^2 - 2\ L_1\ (V_2\ t) \cos \phi_2$$

Also, $\phi_2 = \Theta_1 + (180 - \Theta_2)$ by inspection

Collecting terms of the first equation:

$$(V_3^2 - V_2^2)t^2 + (2\ L_1 V_2 \cos \phi_2)\ t\ - L_1^2 = 0$$

This is a quadratic equation and can be solved for t by the formula

$$t = -b \pm \sqrt{b^2 - 4\ a\ c}/(2a)$$

$$\text{where}\quad a = V_3^2 - V_2^2$$

$$b = 2\ L_1 V_2 \cos \phi_2$$

$$c = -L_1^2$$

Again, by the Law of Cosines, find the angle ϕ_1

$$L_2^2 = L_1^2 + L_3^2 - 2\ L_1\ L_3 \cos \phi_1$$

$$\phi_1 = \cos^{-1}((L_1^2 + L_3^2 - L_2^2)/(2\ L_1\ L_3))$$

The course for intercept is $\Theta_1 + \phi_1$

Example COURSE FOR INTERCEPT MISSION

DATA: Azimuth of second vessel, θ_1 = 15. degrees
Range to second vessel, L_1 = 12. nm
Speed of second vessel, V_2 = 10. knots
Path of second vessel, θ_2 = 75. degrees
Speed of intercepting ship, V_3 = 15. knots

PROCEDURE: COURSE FOR INTERCEPT MISSION

(1) Find angle ϕ_2, the angle between its path and azimuth from first vessel

$$\phi_2 = 15. + (180. - 75) = 120. \text{ deg}$$

(2) Find coefficients for quadratic equation:

$$a = (15)^2 - (10)^2 = 125.$$

$$b = 2 \times 12 \times 10 \cos 120 = -120.$$

$$c = -(12)^2 = -144.$$

(3) Find value for intercept time, t:

$$((-120)^2 - (4 \times 125 \times (-144)))^{1/2} = 293.94$$

$$t = (120 \pm 293.94)/(2 \times 125) = 1.66 \text{ hour}$$

(4) Determine distance travelled

$$L_3 = 15 \times 1.66 = 24.83 \text{ nm}$$

$$L_2 = 10 \times 1.66 = 16.6 \text{ nm}$$

(5) Find angle ϕ_1 between azimuth to second vessel

$$\phi_1 = \cos^{-1}((L_3^2 + L_1^2 - L_2^2)/(2\, L_3\, L_1))$$
$$= \cos^{-1}((24.83)^2 + (12)^2 - (16.6)^2)/(2 \times 24.83 \times 12)$$
$$= 35.48 \text{ deg}$$

(6) Find course for interception, θ_5:

$$\theta_5 = 35.48 + 15 = 50.48 \text{ deg}$$

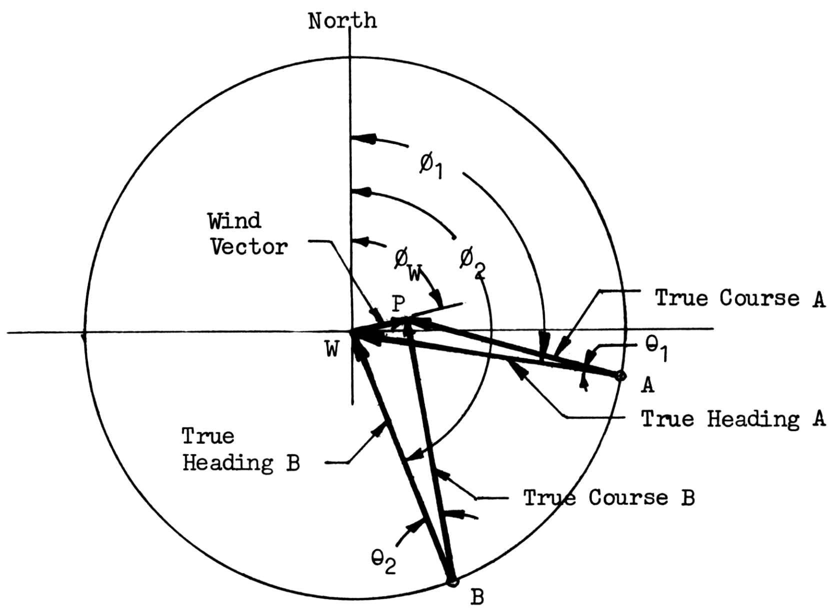
Wind-Star Wind Vector

The wind-star method for determining wind direction and speed requires two observations of the true course and the drift angle at a constant speed. A vector triangle can be constructed for each of the observations within a circle.

The three vectors are:

True Course
True Heading
Wind Vector

The true heading extends from the perimeter of the circle to the center, labelled BW and AW. The true course is drawn from the perimeter to the head of the wind vector whose origin is at the center of the circle. A vertical line through the center of the circle represents true north. The angles WAP and BWP are the observed drift angles.



WIND-STAR WIND VECTOR

Inspection of the diagram shows that the wind vector, PW, can be evaluated by summing the north-south and east-west values.

East-West components:

$$V_{AW} \sin \emptyset_1 - V_{AP} \sin (\emptyset_1 + \theta_1) = V_{BW} \sin \emptyset_2 - V_{BP} \sin(\emptyset_2 + \theta_2)$$

North-South components:

$$V_{AW} \cos \emptyset_1 - V_{AP} \cos (\emptyset_1 + \theta_1) = V_{BW} \cos \emptyset_2 - V_{BP} \cos (\emptyset_2 + \theta_2)$$

where $\emptyset$ = True Heading, degrees
θ = Drift Angle, degrees (positive for right drift; negative for left drift)

Solve the first equation for V_{AP}: $(V_{AW} = V_{BW})$

$$V_{AP} = \frac{V_{AW,BW} (\sin \emptyset_1 - \sin \emptyset_2) + V_{BP} \sin (\emptyset_2 + \theta_2)}{\sin (\emptyset_1 + \theta_1)}$$

now, let $M = V_{AW,BW} (\sin \emptyset_1 - \sin \emptyset_2)/\sin (\emptyset_1 + \theta_1)$

$N = \sin (\emptyset_2 + \theta_2)/\sin (\emptyset_1 + \theta_1)$

then, $V_{AP} = M + V_{BP} N$

Similarly, solve the second equation for V_{BP}:

$$V_{BP} = \frac{V_{AW,BW}(\cos \emptyset_2 - \cos \emptyset_1) + V_{AP} \cos (\emptyset_1 + \theta_1)}{\cos (\emptyset_2 + \theta_2)}$$

now, let $T = V_{AW,BW} (\cos \emptyset_2 - \cos \emptyset_1)/\cos (\emptyset_2 + \theta_2)$

$U = \cos (\emptyset_1 + \theta_1)/\cos (\emptyset_2 + \theta_2)$

then, $V_{BP} = T + V_{AP} U$

Now, substituting the first equation in the second:

$$V_{BP} = (T + U M)/(1 - U N)$$

With two sides of the triangle and the included angle WAP known, the third side WP can be found by the Law of Cosines:

$$V_{WP}^2 = V_{AP}^2 + V_{AW}^2 - 2 V_{AP} V_{AW} \cos \theta_1$$

The angle PWA, or j, can also be found by the Law of Cosines.

$$\cos j = (V_{AW}^2 + V_{WP}^2 - V_{AP}^2)/(2\ V_{AW}\ V_{WP})$$

$$j = \cos^{-1} j$$

Finally, the wind direction equals the angle j subtracted from the true course, ϕ_1:

$$\phi_W = \phi_1 - j$$

Example WIND-STAR DRIFT VELOCITY

Data: True Heading, ϕ_1, 100 degrees; Drift Angle, θ_1, 5. deg.
True Heading, ϕ_2, 158 degrees; Drift Angle, θ_2, 12. deg.
Ship Speed, $V_{AW,BW}$, 95 MPH

WIND STAR WIND VELOCITY

PROCEDURE:

(1) Find values for N,U, M and T

$N = \sin(158 + 12)/\sin(100 + 5) = 0.1798$

$U = \cos(100 + 5)/\cos(158 + 12) = 0.2628$

$M = 95(\sin 100 - \sin 158)/\sin(100 + 5) = 60.0140$

$T = (95 \cos 158 - 95 \cos 100)/\cos(158 + 12) = 72.6902$

(2) Find values for True Course BP and AP and wind velocity WP

$V_{BP} = (72.6902 + (0.2628 \times 60.0140))/(1 - (0.2628 \times 0.1798))$
$= 92.85$ MPH

$V_{AP} = 60.0140 + (0.1798 \times 92.85) = 76.71$ MPH

$V_{WP} = ((76.71)^2 + (95)^2 - (2 \times 76.71 \times 95 \cos 5))^{1/2} = 19.75$ MPH

(3) Find Wind Direction, ϕ_W

$j = \cos^{-1}(((95)^2 + (19.75)^2 - (76.71)^2)/(2 \times 95 \times 19.75))$
$= 19.80$ deg

$\phi_W = 100 - 19.80 = 80.20$ deg

TIME

The Calendar

The Year - By noting the seasons we can mark the year. The ancients observed the night sky and prepared maps of the visible stars. They inferred the yearly circuit of the Sun through the heavens; through the twelve houses of the Zodiac. They marked the yearly travel of the Sun from north to south and return; its extremities and its mean. Its midway point, in Spring and Autumn, we call the Equinox. The Spring, or Vernal Equinox, with the Sun moving north, is the reference point for astronomers and navigators and is commonly called the First Point of Aries, or simply Aries, the Ram, ♈, and marks the celestial longitude at which the house of Aries begins, and into which the Sun entered when the name was applied some two thousand years ago.

The length of time of passage from one Vernal Equinox to the next marks the Equinoctial Year. This time can be found from Almanac data by interpolating to find the time at which the declination is zero for the two years. Note that the time elapsed is the same for any two specific declinations. The true length of the Equinoctial Year is 365d 5h 48m 45.5s.

The Julian Year is the fundamental basis for our calendar and consists of 365 days and 1/4 solar years. The day is too long by 11m 14.5s. On October 4, 1582 Pope Gregory ordered ten days striken from the Julian calendar. Further, he ordered that leap years should be omitted in century years not divisible by 400 (as the year 2000 is a leap year, which will not occur again until 2400).

The time of the Vernal Equinox can be estimated by using reference Almanac values to describe the yearly increment:

The 1976 Almanac gives V.E. at Mar 20, 11:80h
diff.=-1:40h
The 1984 Almanac gives V.E. at Mar 20, 10:40h or .175h/yr

Example: Prediction of Time of Vernal Equinox for 1983
Use 1976 as reference year
Note that V.E. for 1983 falls on March 21
The years correction is 7yr x .175h/yr = 1.225h
Observe that 1983 is 3 years past leap year

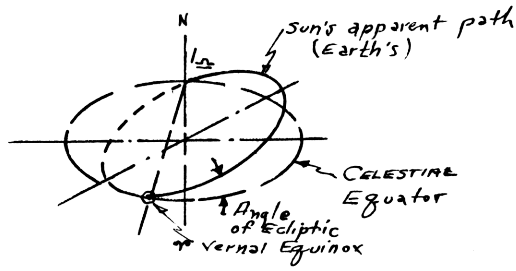
Add a correction of 3 yrs x (6hr/yr + .0375hr/yr)= 18.110h
New time of V.E. = Mar 20, 11.80h + 18.110 - 1.225
= Mar 20, 28.685h or Mar 21, 4.68h
(Almanac value: Mar 21, 4.6h)

The Months- There are 235 lunar orbits to correspond to the same position of the sun as found by Meton in -433. This is approximately nineteen solar years. The lunar month is then 29.53 days.

The early Roman calendar had ten months. Julius Caesar, on the advice of the Alexandrian astronomer Sosigenes, established in -45 the present calendar. He adopted a year of 365 days with each fourth year, when divisible evenly by four, to be a "leap year" with one day added. He also added a month, named for himself. His successor, Augustus, added a second month. Thus, we have the latter months improperly named: September (seven), October (eight), November (nine), and December (ten).

The Days- The days of the week have been in most languages named for the seven visible heavenly wanderers. Further, their order of naming is in sequence of their relative motions.

Planet	English	Spanish
Sun	Sunday	Domingo
Moon	Monday	Luna
Mars	Tuesday	Martes
Mercury	Wednesday	Miercoles
Jupiter	Thursday	Jueves
Venus	Friday	Viernes
Saturn	Saturday	Sabado



The Hours

The day is divided into 24 hours of which the 12 hours before noon are classed as A.M. (ante-meridian) and the 12 hours after noon as P.M. (post-meridian).

The **Civil Day** begins with the sun at its nadir at midnight, 00hours. Mid-day occurs at 12hours, at its zenith, high noon, and, in navigation terms, we continue counting to complete the cycle in 24hours.

Local Apparent Time is marked by the zenith of the sun, frequently called its **transit**, at our particular locality, that is at the exact longitude we happen to be in at that instant. Until the coming of the railroad in the last century, requiring coordination of time, it was customary for each locality to have its clocks set to agree with its local apparent time.

The Sun moves through each day in a slightly irregular fashion so that, as measured by a perfect timepiece, it sometimes is early to its zenith and sometimes late. This discrepancy as measured in minutes is called the **Equation of Time.**

The Equation of Time refers to the difference between the mean sun time and the true sun time (its position). The term "equation" is used in the sense of being a variation.

The Equation of Time is a yearly recurring phenomena and is zero at four times of the year. The value can be positive or negative and as much as 15 minutes. The values are given in the Almanac for each day at 00h and at 12h. They are also obtainable from the main table of the Sun's location by converting degrees to hours and comparing to the mean time.

Mean Time is given by a precision clock. The location of the Sun can be found by adding to the Mean Time the Equation of Time. Conversely, the local noon can be found by an observation of the Sun and application of the Equation of Time in the opposite manner.

As we move from East to West we find that the time of high noon is delayed by an amount corresponding to our distance from the reference longitude. The navigation reference meridian, the half circle of zero longitude, is at the original site of Greenwich Observatory, near London. Since there are 360 degrees in the Earth's circumference, and our Sun rotates once in 24 hours, it follows that a change of 15 degrees of longitude corresponds to a one hour delay in the Sun's zenith.

The **Hour Circle** is a circle drawn on the Celestial Sphere through the heavenly body. The **Hour Angle** is the angle between the Hour Circle of the body and the meridian of the observer.

<u>Civil Time</u> at any place equals the hour angle of the mean sun plus 12 hours, dropping 24 hours if the sun exceeds that amount. At high noon, the hour angle of the Sun is zero; thus the Civil Time is 12:00 o'clock.

Zone Time was established by the US Navy in 1920 for ships at sea. There is a Zero Zone of 7.5 degrees to each side of the zero meridian. From this Zero Zone are measured twelve Zones to the east and west. Zones One to Eleven contain 15 degrees of Longitude; the Twelfth Zone has 7.5 degrees to each side.

The Time Zone at a specified longitude can be found by the equation:

Zone= INT((abs(Longitude - 7.5)/15.)+1.)

(using negative values for East Longitude)

Longitude:	173^{o} 47'E.	Zone -12
	6^{o} 57'W.	0
	141^{o} 33'E.	-9
	128^{o} 02'W.	9

Continental Time Zones in the United States were introduced as Standard Railway Time on Nov. 18, 1883 and legalized by Congress in 1918. It provides four time zones: Eastern, Central, Mountain and Western with numerous adjustments for local preference to a particular zone. Daylight Saving Time was invented by Benjamin Franklin and advances the clock one hour in the spring and retards it one hour in the fall ("Spring ahead; Fall behind"). These time zones are irregularly applied according to local preference.

Example: <u>Conversion of Local Zone Time to GMT, LMT and LAT</u>

Data: Local Zone Time is 6:15PM   Date: Nov 10, 1984
Longitude is 76^{o} 30'W

Procedure for GMT:

1. Find Time Zone = INT(abs(76.5-7.5)/15. +1) = 5
2. GMT = 5h + 6h 25m + 12h = 23h 15m, Nov 10

Procedure for LMT:

LMT = GMT - (Longit./15.)
= 23.25h - (76.50/15.) = 18.15h or 18h 9m

Procedure for LAT:

1. By Almanac, Equation of Time at 12h Nov 10 = 16m 03s
at 00h Nov 11 = 16m 00s
Meridian Passage at 12h Nov 10 is 11h 44m
so, Sun preceeds Mean Time
By interpolation, EqTm = -(16m + (3s-((23.25-12)/12)x03))
= -16m 0.1s
2. LAT = LAT + EqTm
= 18h 9m + (-16m 0.1s) = 17h 52m 59.9s

Example   Find Meridian Angle and Local Apparent Time

DATA: Date is Dec. 3, 1976; GMT is 14h 36m
Position: 30° 25'N, 81° 251W
By Almanac, EqTm at Dec 3, 12h= 10m 04s
Dec 4, 00h= 09m 52s

1. By interpolation, EqTm= 10.06m - (.1083 x (10.06-9.86))
   = 10.04m
2. Find Greenwich Hour Angle (of Sun):

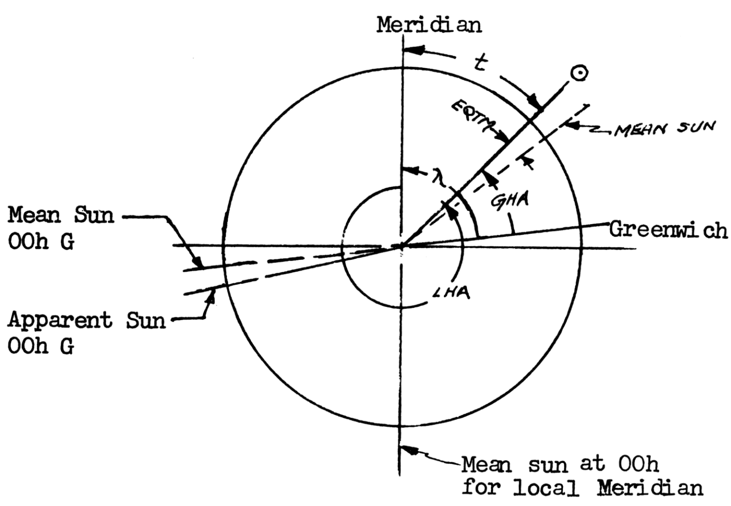
   GHA = (GMT + EqTm) - 12h
   = (14.600+.167) - 12 = 2.767h

3. Find Meridian Angle:

   t = Longit. - (GHA x (360/24))
   = 81.417 - (2.767 x (360/24)) = 39.912°E.

4. Find Local Apparent Time:

   LAT = 12h - (t x (24/360))
   = 12h - (39.912 x (24/360)) = 9.339h or 9 h 20.35m



Sidereal Time

Sidereal (si-de're-al, L. Star or constellation) time is measured by the stars. The sky appears to rotate about the pole daily and a sidereal day can be marked by two passages of a star across a north-south transit. The sidereal year is the consecutive passages of a star across such a transit at the end of the earth's circuit about its orbit.

The reference meridian of the sky is called the First Point of Aries and has its zenith at the moment of the Sun's Vernal Equinox (passing from South Declination to North Declination). Our navigation methods relate the position of the stars to that of Aries. Unfortunately, there are no significant stars along the path of this line and it must be found by mathematical analysis. There is accordingly a constant need in the practice of Celestial Navigation for the conversion of clock time and angles of rotation and spherical measurement.

The length of the solar day exceeds the length of the sidereal day by 3m 56.6s. This can be expressed as an angle:

$$3.94333 \times ((360 \times 60)/(24 \times 60) = 59.15'$$

By reference to the 1984 Almanac:

Mar 19, 00h GMT Aries	176°	43.3'
Mar 20, 00h	177	42.4'
diff:		59.1'

In a solar year this amounts to a full circle so that the sidereal year consists of an extra day. The long term consequence of this difference in time is that the position of the stars advances at the time of the Vernal Equinox (or any other similar base point). This is known as the Procession of the Equinoxes. It has moved some 30 degrees in two thousand years.

Consider the change in the location of Aries over a short time as given by Almanac data:

Mar 20, 1976 00h GMT GHA_{aries}	177°	39.1'
Mar 20, 1984 00h GMT	177	42.4
diff		3.3'/8yr
		or 0.0069°/yr

Example 13. <u>Find Meridian Angle of Star</u>

DATA: Date is Jan 1, 1976; Zone Time is 06h 15m
Position: 74^{o}W
Observed: Spica
Almanac data: Spica (#33);Magnitude 1.2;SHA 159.008^{o}
Declination -11.040^{o}

1. Determine Time Zone = Int((74 - 7.5)/15 + 1)= 5

2. Find GMT = Zone Time + Zone
= 6.25h + 5 = 11.25h

3. Find position of Aries from Almanac:

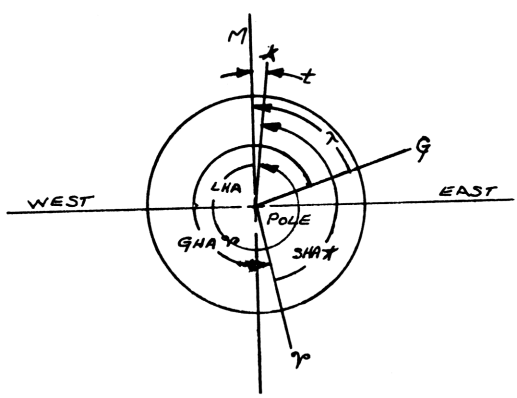
Aries at 11h 00m GMT 265^{o} 14.3' GHA
12h 00m 280^{o} 16.7'
by interpolation, increment =(15/60min) x 15^{o} 2.4'
increment = 3^{o} 45.6'
GHA, Aries = 265.238+3.760 = 268.998^{o}

4. Find Local Hour Angle, LHA, of Spica (angle measured westward from upper branch of meridian)

$$\begin{aligned} LHA &= -\text{Longitude} + GHA_{aries} + SHA_{star}) \\ &= -74 + 268.998 + 159.008 \\ &= 354.006^{o} \end{aligned}$$

Note: Correct value as necessary for excess of 360^{o}

5. Find Meridian angle, t: The Meridian Angle is measured east or west of the meridian according to the location.
In this case, t = 5.994^{o}E.



SPHERICAL NAVIGATION

SPHERICAL GEOMETRY

Propositions:

1. If two angles of a spherical triangle are equal, the sides opposite are equal; and conversely.

2. The sum of the angles of a spherical triangle is more than 180 degrees and less than 540 degrees.

3. The sum of two sides of a spherical triangle is greater than the third side.

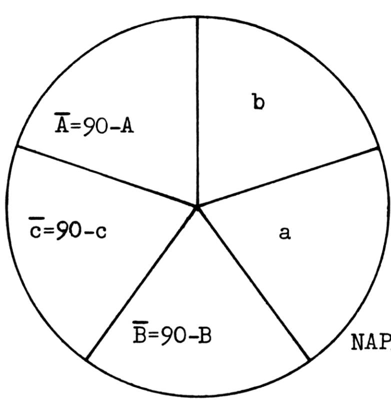
Right Spherical Triangles -John Napier, Scot. mathematican (1550-1617), the inventor of natural logarithms, and the decimal point for numbers, represented the two angles and three sides of a spherical triangle as being parts of a circle. Proceeding from the right angle, we have:

b = side of right angle opposite B, leg
$\bar{A} = 90-A$ = angle opposite side a, co-value of A
$\bar{c} = 90-c$ = side opposite right angle C, hypotenuse
$\bar{B} = 90-B$ = angle opposite side b, co-value of B
a = side opposite angle A, leg

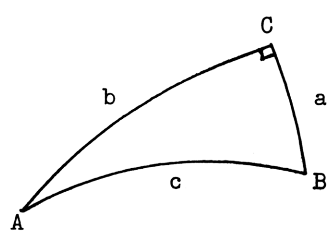
Napier's Rules:

I. The sine of any middle part is equal to the product of the cosines of the opposite part.
II. The sine of any middle part is equal to the product of the tangents of the adjacent parts.

Briefly: sin middle = cos opposite = tan adjacent



NAPIER'S CIRCLE



SPHERICAL RIGHT TRIANGLE

Radius of Inscribed Circle for Spherical Triangle

$$r = (\sin(s-a)\ \sin(s-b)\ \sin(s-c)/\sin(s))^{1/2}$$

where $s = 1/2(a+b+c)$

Angle of Spherical Triangle with Known Sides

$$\tan(1/2\ A) = r/(\sin(s-a))$$

Law of Sines for Spherical Triangle

$$\sin A/\sin a = \sin B/\sin b = \sin C/\sin c$$

Note: Since $\sin A = \sin(180-A)$ the part found may be in either the first or the second quadrant. It may help to know that:

(1) The sum of two sides is greater than the third side.

(2) The order of magnitude of the sides is the same as the order of magnitude of the respective opposite angles, as

if $a<b<c$ then $A<B<C$

Equivalence of Trigonometric Functions

$\cos(90-\phi) = \sin\phi$	$1/\sin\phi = \csc\phi =$ cosecant ϕ
$\sin(90-\phi) = \cos\phi$	$1/\cos\phi = \sec\phi =$ secant ϕ
$\tan(90-\phi) = 1/\tan\phi$	$1/\tan\phi = \cot\phi$ (cotan ϕ)

Law of Cosines for Sides

The cosine of any side of a spherical triangle is equal to the product of the cosines of the other two sides increased by the product of the sines of the other two sides and the cosine of the angle included between them, e.g.:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

Law of Cosines for Angles

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$

Note All values must appear as angles (degrees or radians)

THE ASTRONOMICAL TRIANGLE

The astronomical triangle is a spherical triangle drawn upon the celestial sphere with points at the pole, the observer and the celestial body. It relates the four parameters essential to celestial navigation:

t, the meridian angle of the observed body
d, the declination of the observed body
h, the altitude of the observed body
L, the latitude of the observer

It will be seen that these parameters consist of three sides and one angle. Each must be expressed as degrees of the arc subtended by a full circle.

Conventionally, the pole point is designated P; the observed body is M; and the observer's zenith is Z.

The observer's meridian is the arc PZ. The length of this arc is equal to his co-latitude, or 90 degrees minus the latitude.

The body's altitude is the extension of the arc ZM to the horizon. The length of the arc ZM is the co-altitude of the altitude, or 90 degrees minus the altitude.

The body's declination is the extension of the arc PM to the equator and may be either North or South. The length of the arc PM is the co-declination which equals 90 degrees plus or minus the declination to describe the distance from the pole.

The zenith angle is the angle between the pole, P, and the body, M, as measured at the observer, Z.

The meridian angle, t, is the angle between the observer's meridian, Z, and the body, M, as measured at the pole, P. The meridian angle is measured and designated East or West as required and cannot exceed 180 degrees.

Equation for the Astronomical Triangle

By the law of cosines for sides:

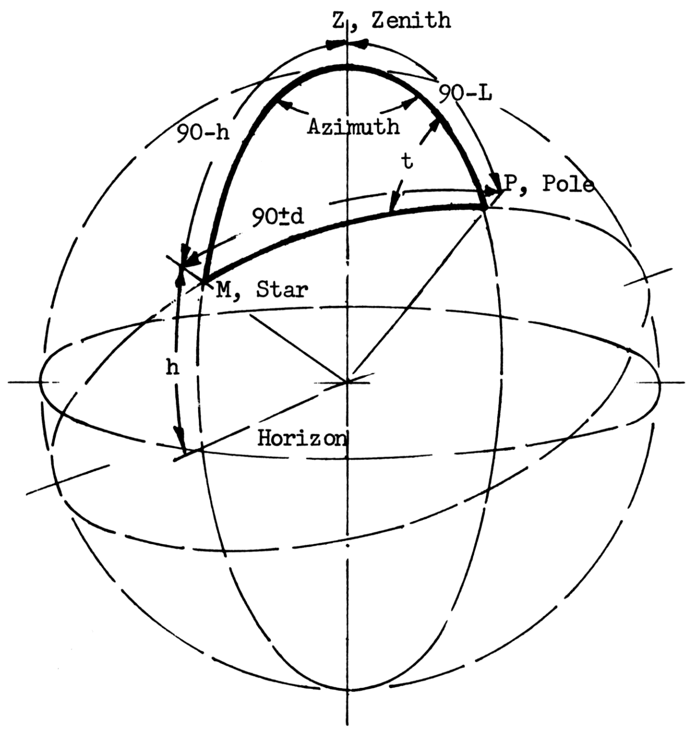
$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

here:
$$a = 90 - h$$
$$b = 90 - L$$
$$c = 90 - d$$
$$A = t$$

then:
$$\cos(90-h) = \cos(90-L)\cos(90-d) + \sin(90-L)\sin(90-d)\cos t$$

by trigonometric equivalents:

$$\sin h = \sin L \sin d + \cos L \cos d \cos t$$



THE ASTRONOMICAL TRIANGLE

THE CELESTIAL SPHERE

Celestial Sphere: Omar Khayyam spoke of "this inverted bowl we call the sky." It is a useful similie. Consider the heavens to be a thin bowl with a radius of One Celestial Unit upon which the stars and the sun are placed. In effect, we have conceived a "Celestial Sphere". Our position is at the center of the sphere, at the location usually designated "O" for Observer.

The spot above the Observer's head is designated the Zenith; the spot on which he stands is at a certain Meridian (which goes from pole to pole at some particular angle of longitude from the Greenwich Prime Meridian.)

Zenith Plane: The key diagram for understanding is the Zenith Plane. Given a Horizontal Axis and a Vertical Zenith Line at the intersection of which the Observer (of zero size) is located. To the left is North; to the right is South. Draw then a circle of radius=1.0 which represents the Celestial Sphere.

Now, lay off a line at an angle to the Zenith which is equal to the Latitude of the Observer, To the right for North Latitudes; to the left for South Latitudes. (The angle is located by measuring off a distance from the Z axis equal to the tangent of the angle, as, for Latitude 34degN, measure .6745 to the right from the periphery of the circle.) The line thus located is the Equator. At 90 degrees to it draw the Pole.

From the Almanac value of the Declination of the Star (or the Sun) measure angles from the Equator equal to the tangent of this value according to whether it be South or North. The intersections of these radii from the center with the Celestial Globe mark the Upper Transit and the Lower Transit. The connecting line between them locates the Star Plane, or the Orbit Plane. The length of this line determines the diameter of the Orbit.

Orbit Plane: The Orbit Plane or the Star Plane corresponds to the familiar Equatorial Diagram, but in this case with the added convenience of having a definite diameter and orientation. An orthographic projection of the Orbit Plane (at right angle to the Polar axis) turns the Star Plane into a circle. We can identify the Upper Transit and the Lower Transit (which fall along the Meridian Line) and directions East and West. The day begins at the Lower Transit and moves Eastward. The position of the star can be located on this circle according to its Local Hour Angle, LHA, which is measured westward from upper transit. Meridian Angle, t, is the smallest angle between the star and the Meridian.

A parallel projection from the star in the Orbit Plane to intersect the Star Plane in the Zenith Plane locates the Star in the Zenith Plane. Note, particularly, that the angle of a line drawn from the Observer origin to the star in the Zenith Plane does not give the observed altitude of the star unless the star happens to be at Upper or Lower Transit. This is because the star is not really in the Zenith Plane.

Horizon Plane: The orthographic projection of the Star Plane parallel to the Z axis gives an ellipse in the Horizon Plane. The vertical centerline of the ellipse is the projection of the intersection of the Pole and the Star Plane in the Zenith Plane. Extentions of the Upper and Lower Transits give the semidiameter in the North-South axis. The semidiameter along the Pole axis is the same as that of the Star Plane.

Positions along the ellipse can be calculated by use of the equation of the ellipse:

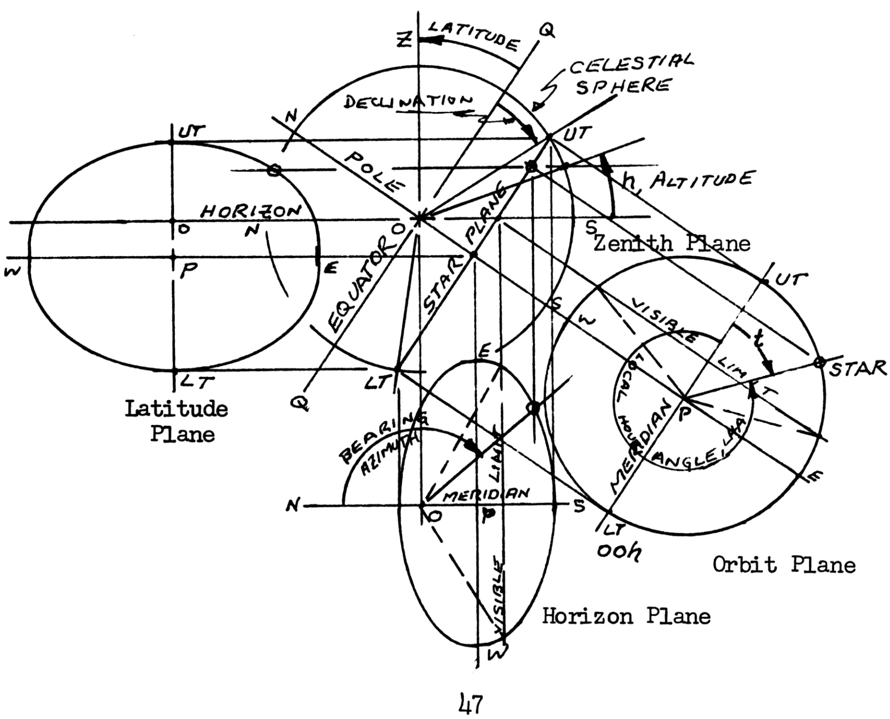
$$(x^2/\mathrm{sdiax}^2) + (y^2/\mathrm{sdiay}^2) = 1.0$$

Then, for a given value of y,

$$x = \mathrm{sqrt}(\mathrm{sdiax}^2 \text{ x } (1.0 - (y^2/\mathrm{sdiay}^2)))$$

The bearing is the angle measured from the North from the Observer to the star in this Horizon Plane measured eastward or westward according to the direction of the star from the meridian. The azimuth is the same angle measured from the North in an easterly direction.

Latitude Plane: An orthographic projection of the Star Plane parallel to the Horizon gives an ellipse in the Latitude Plane. This view shows the location of the Observer relative to the Pole about which the star rotates and indicates the portion of the Orbit visible to the Observer as well as the elevation of the Pole and the apparent shape and size of the Orbit.



Path of Sun

The methods already discussed can be used to determine the change in elevation of the Sun during the course of the day as well as its azimuth and the length of shadow of a gnomen.

Example: Given Latitude 33.4°N
Date Dec 18, 1984

1. Zone = Int(117.4 - 7.5)/15. + 1) = 8
2. At 12 noon, Local Mean Time, GMT = 18h, Dec 18
3. By Almanac, Declination of Sun is S23° 24.5'

Find Elevation, h, and Zenith Angle, Z

3 hours before noon

$$t = (3/12) \times 180^\circ = 45^\circ$$
$$h = \sin^{-1}(\sin L \sin d + \cos L \cos d \cos t)$$
$$= \sin^{-1}(\sin(33.4)\ \sin(-23.4083) + (\cos(33.4)\ \cos(-23.4083)\ \cos(45.))$$
$$= 18.8472^\circ$$
$$Z = \cos^{-1}((\sin d - \sin L \sin h)/(\cos L \cos h))$$
$$= \cos^{-1}((\sin(-23.4083) - \sin(33.4)\ \sin(18.8472))/\cos(33.4)\ \cos(18.8472))$$
$$= 136.7114^\circ$$

4 hours before noon

$$t = (4/12) \times 180^\circ = 60^\circ$$
$$h = \sin^{-1}(\sin(33.4)\ \sin(-23.4083) + (\cos(33.4)\ \cos(-23.4083)\ \cos(60.))$$
$$= 9.4608^\circ$$
$$Z = \cos^{-1}((\sin(-23.4083) - \sin(33.4)\ \sin(9.4608))/(\cos(33.4)\ \cos(9.4608))$$
$$= 126.3213^\circ$$

1 hour before noon

$$t = (1/12) \times 180 = 15^\circ$$
$$h = \sin^{-1}(\sin(33.4)\ \sin(-23.4083) + (\cos(33.4)\ \sin(-23.4083)\ \cos(15.))$$
$$= 31.4219^\circ$$
$$Z = \cos^{-1}((\sin(-23.4083) - \sin(33.4)\ \sin(31.4219))/(\cos(33.4)\ \cos(31.4219))$$
$$= 81.0937^\circ$$

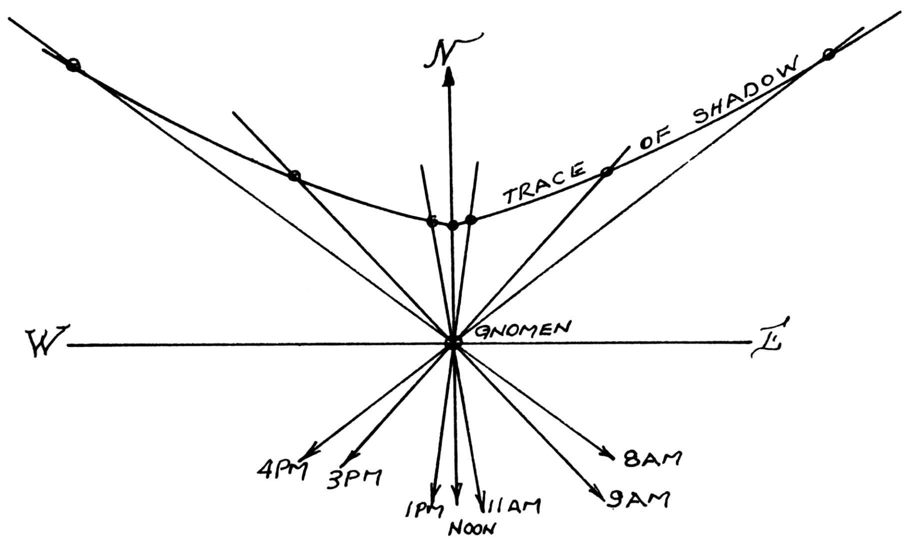
Noon

$$t = 0$$
$$h = \sin^{-1}(\sin(33.4)\ \sin(-23.4083) + (\cos(33.4)\ \cos(-23.4083)\ \cos(0.))$$
$$= 33.1917^\circ$$
$$Z = 180.^\circ$$

Path of Sun (Continued

Find Length of Shadow

Height of gnomen is 2.0"
Noon: Shad = 2.0/tan(h)
= 2.0/tan(33.1917) = 3.0573"
8 AM, Shad = 2.0/tan(09.4608) =12.0020"
9 AM, Shad = 2.0/tan(18.8472) = 5.8591"
11AM, Shad = 2.0/tan(31.4219) = 3.2737"



SUNSET

Night comes on gradually because the upper air reflects the sunlight after the sun is no longer visible. As Professor Baker, in his Manual of Astronomy describes it poetically:

> "...as the Sun sinks farther below the horizon;
> as the dull blue twilight arch of the Earth's shadow
> rises in the East and overspreads the sky..."

Sunrise and Sunset occurs with the Sun at the Horizon, 90° from the Zenith, plus the Sun semidiameter of 16' and refraction of 35', giving a total angle of 0.850° below the Horizon.

Civil twilight occurs with the Sun's center 6° below the horizon. It is the state of darkness that precludes normal daylight activity.

Nautical twilight occurs with the Sun at 102° from the Zenith (12° below the Horizon). At this time, the Horizon is generally not visible.

Astronomical twilight occurs with the Sun at 18° below the Horizon. At this time, the faintest stars are visible.

Example: Find the time of Sunset
at 35deg N on April 25, 1984
by the Almanac, the Sun declination is N13° 26.0'

1. The elevation, $h = -0.833^{\circ}$
2. Meridian Angle,
$t = \cos^{-1}((\sin h - \sin L \sin d)/(\cos L \cos d))$
$= \cos^{-1}((\sin(-0.833) - \sin(35.) \sin(13.4333)/(\cos(35.) \cos(13.4333))$
$= 100.6897^{\circ}$
3. Equation of Time = 2m 07s with sun leading clock
5. Local Mean Time of Sunset
= 12h + 6h + (4m/deg x 100.6897-90)) -(2m 07s)
= 18h + 42.7588m - (2m 07s)
= 18h 40m 45s

Example: Find the time of Nautical Twilight
at 35deg N on April 25, 1984
by the Almanac, the Sun declination is N13° 26.0'

1. The elevation, $h = -12.^{\circ}$
2. Meridian Angle,

$$t = \cos^{-1}((\sin h - \sin L \sin d)/(\cos L \cos d))$$

$$= \cos^{-1}((\sin(-12.) - \sin(35.) \sin(13.4333))/(\cos(35.) \cos(13.4333))$$

$$= 115.3531^{\circ}$$

3. Equation of Time = 2m 07s with sun leading clock
4. Local Mean Time of Twilight

= 12h + 6h + (4m/deg x (115.3531-90)) - (2m 07s)
= 18h + 101.41254m - (2m 07s)
= 19h 39m 18s

Example: Find the time of Civil Twilight
at 64deg N on April 25, 1984

1. The elevation, $h = -6.^{\circ}$
2. Meridian Angle,

$$t = \cos^{-1}((\sin(-6.) - \sin(64.) \sin(13.4333))/(\cos(64.) \cos(13.4333))$$

$$= 137.2958^{\circ}$$

3. Local Mean Time of Civil Twilight

= 12h + 6h + (4m/deg x (137.2958-90)) - (2m 07s)
= 18h + 189.1833m - (2m 07s)
= 21h 07m 04s

AZIMUTH AND BEARING

The navigator's work at sea consists largely of computing lines of position which have only two essentials: the elevation of the observed celestial body and its direction, as measured by its azimuth.

The calculation of altitude, with which the observed reading must be compared, is relatively simple and straightforward.

The calculation of azimuth is complicated by the fact that the angle is measured in the Horizon Plane with the Observer either North or South of the center of the apparent ellipse along which which the body moves. Accordingly, a large number of methods have been developed for computing the azimuth. In the past, and even today, these methods relied on voluminous tables for selected latitudes and declinations to simplify the work of the navigator. Such tables, or the specific case desired, can now be generated effortlessly for normal usage by the modern calculator and portable computer.

Keep in mind that azimuth is the direction of the star from the observer's north, measured in an easterly direction. Bearing is the direction of the star from the observer's north, measured to either the east or west according to the side of the meridian on which the body is located at the time. Bearing corresponds to the Zenith Angle of the Astronomical Triangle. (Accordingly, we find four cases for determination of azimuth: Star east or west; observer north or south of center.) Bearing equals azimuth when the bearing is toward the East. When bearing is toward the west, the azimuth is 360 degrees minus the bearing.

Bearing can be observed by use of an azimuth prism attachment to the compass. It consists of a system of mirrors and prisms mounted in a fitting which turns about the center of the compass. Bearings can be taken by: 1. reflecting light from the body directly onto the compass card, 2. by reflecting part of the compass card onto the field of view, or 3. by observing the reflection of the body on the prism directly in line with the compass card.

This section gives several methods for calculating azimuth and bearing.

The first method finds the Zenith Angle by direct use of formulas from the Astronomical Triangle. A sketch of the Celestial Sphere helps to locate the azimuth. Two examples show the contrast between the cases of having the observer positioned north or south of the star's pole.

The second method finds the Zenith Angle by a mathematical relation between the sides of the Astronomical Triangle and its inscribed circle.

The third method finds the Zenith Angle by the Law of Sines and the Astronomical Triangle.

Locating Stars from the Astronomical Triangle

Given data: Position 30° 24'N Latitude, 134° 40'W Longitude
Date: July 3, 1984, 21h 00m ship time
Locate: Denebola and Antares

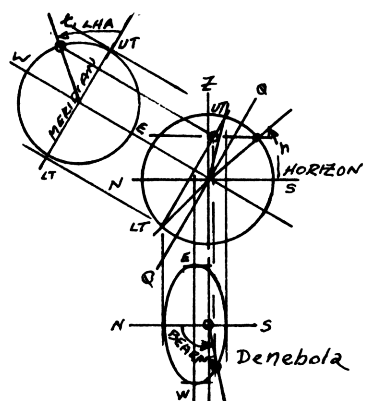
Time Zone= Int(134.667−7.5)/15 + 1) = 9; GMT 06h 00m July 4
By Almanac: $GHA_{Aries} = 12^{\circ}$ 25.9'
$SHA_{Denebola} = 182^{\circ}$ 56.3'; dec=N14° 39.7'
$SHA_{Antares} = 112^{\circ}$ 52.3'; dec=S26° 24.0'

Find LHA's: $LHA_D = SHA_{star} + GHA_{Aries} - \text{Longitude}$
$= 182.938 + 12.432 - 134.667 = 60.703^{\circ}$
LHA is less than 90; Denebola $t=60.703^{\circ}$
$LHA_A = 112.887 + 12.432 - 134.667 = 9.348^{\circ}$
LHA is less than 90; Antares $t=9.348^{\circ}$

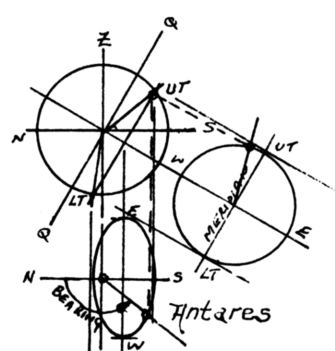
Find Altitudes: $h_D = \sin^{-1}(\sin L \sin d + \cos L \cos d \cos t)$
$= \sin^{-1}(\sin(30.400)\ \sin(14.662) + (\cos(30.400)\ \cos(14.662)\ \cos(60.703))$
$= 32.439^{\circ}$
$h_A = \sin^{-1}(\sin(30.400)\ \sin(-26.400) + (\cos(30.400)\ \cos(-26.400)\ \cos(9.348))$
$= 32.500^{\circ}$

Find Zenith Angles:
$Z = \cos^{-1}((\sin d - \sin L \sin h)/(\cos L \cos h))$
$Z_D = \cos^{-1}((\sin(14.662) - \sin(30.400)\ \sin(32.439))/(\cos(30.400)\ \cos(32.439))$
$= 91.442^{\circ}$
since bearing is W, $\text{Azimuth}_D = 360-91.442 = 268.558^{\circ}$

$Z_A = \cos^{-1}((\sin(-26.400) - \sin(30.400)\ \sin(32.500))/(\cos(30.400)\ \cos(32.500))$
$= 169.845^{\circ}$
since bearing is W, $\text{Azimuth}_A = 360-169.845 = 190.155^{\circ}$



Observer South of star's pole



Observer North of star's pole

True Azimuth by Radius of Inscribed Circle and Opposite Angle

The true azimuth can be computed by finding the radius of the inscribed circle and from that the angle of the azimuth. For example, recreate Captain Sumner's observation of the Sun at 10AM, Dec 18, 1837 at true position of Latitude 51° 30'N, Longitude 6° 35'W.

Given:

Time Zone = 0; Ship time = 10.00h
Sun GHA = 330° 54.6'
Sun Declination, d = S23° 22.7'
Meridian angle, t = 35.6733°E.
Elevation, true = 8.8384°

Find the parameter s, where $a=90-d=90-(-23.3783)=113.3783^\circ$
$b=90-h=90-8.8384 = 81.1616^\circ$
$c=90-L=90-51.5000 = 38.5000^\circ$

$$s=1/2(a+b+c)=1/2(113.3783+81.1616+38.5000)=116.5199^\circ$$

Find the radius of the inscribed circle,

$$r=(\sin(s-a)\ \sin(s-b)\ \sin(s-c)/\sin s))^{1/2}$$

$$r=(\sin(116.5199-113.3783)\ \sin(116.3783-81.1616)\ \sin(116.5199-38.5000)/\sin(116.5199))^{1/2}$$

$$= (\sin(3.1416)\ \sin(35.2167)\ \sin(78.0199)/\sin(116.5199)^\circ$$

$$= .1859^\circ$$

Find the Zenith Angle,

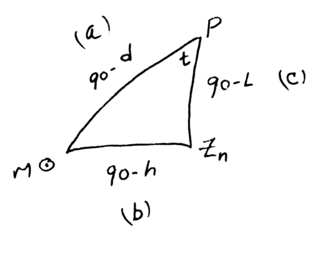
which is the angle opposite the side a:

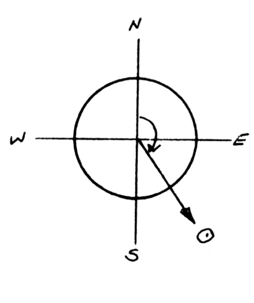
$$Z_n = 2\tan^{-1}(r/\sin(s-a))$$

$$= 2\tan^{-1}(.1859/\sin(116.5199-113.3783))$$

$$= 147.1487^\circ$$

Since the time is A.M., the Sun has an Easterly bearing and the bearing and azimuth both equal the Zenith Angle.





True Azimuth by Law of Sines

The true azimuth can be found by the Law of Sines with consideration to locating the vector in the proper quadrant.

Consider again the observation of Capt. Sumner at his true position of Latitude 51° 30'N, Longitude 6° 35'W at 10AM, Dec 18,1837:

Time Zone = 0 Ship time = 10.00h GMT=10.00h
Sun GHA = 330° 54.6'
Sun declination, d = S23° 22.7'
Meridian angle, t = 35.6733°
Elevation, true, h= 8.8384°

$$
\begin{aligned}
Z &= \sin^{-1}(\sin t \cos d/\cos h)\\
&= \sin^{-1}(\sin(35.6733) \cos(-23.3783)/\cos(8.8384))\\
&= 32.8008^\circ
\end{aligned}
$$

Find angle at star from zenith to pole, PMZ:

$$
\begin{aligned}
PMZ &= \cos^{-1}((\sin L - \sin d \sin h)/(\cos d \cos h))\\
&= \cos^{-1}((\sin(51.5000)-(\sin(-23.3783) \sin(8.8384)))/\\
&\qquad (\cos(-23.3783) \cos(8.8384)))\\
&= 21.5547^\circ
\end{aligned}
$$

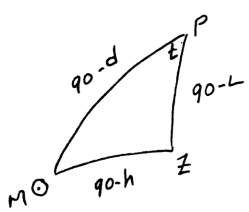
Compute sum of angles, S= 21.5547+32.8008+35.6733= 90.0288°

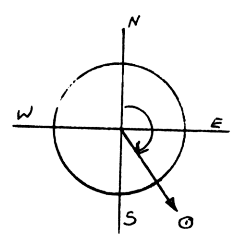
Sum of angles must be more than 180° and less than 540°

then, Z = 180−32.8008 = 147.1992°

now S=147.1992+21.5547+35.6733= 204.4272°

The test is satisfied and Z_n = 147.1992°

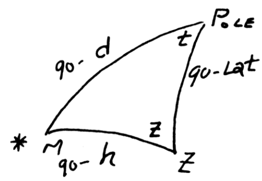




Identifying a Star

It is good practice when sighting a star to note its bearing as well as its altitude. Calculations can then be made to confirm its identity.

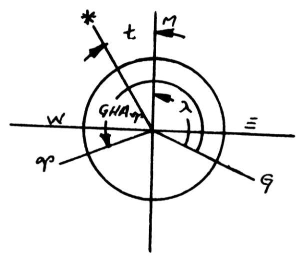
The Astronomical Triangle can tell us the Declination, d, with the bearing giving the Zenith Angle, Z, between the star and the Pole at the Observer's head. Using the Law of Cosines for Sides:



cos a = cos b cos c + sin b sin c cos A

then, cos(90–d)=cos(90–L) cos(90–h)
+sin(90–L) sin(90–h) cos Z
by equivalents,

sin d = sin L sin h + cos L cosh cos Z



by the Law of Sines:

sin t/sin(90–h) = sin Z/sin(90–d)

then, t = $\sin^{-1}$(sin Z cos h/cos d)

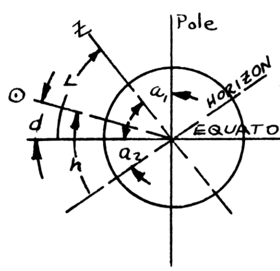
<u>Example: Identifying a Star</u>
Data: Observed unknown star Apr 13, 1984
Local Mean Time 06h 43m P.M. Local Mean Time
Estimated Position, 33° 45'N, 118° 25'W
Altitude 34° 18', Bearing N150W
Calculation:
Zone = Int(((abs(118.416)–7.5)/15)+1) = 8
GMT = 8 + 12 + 6.716 = 26.716h or 2.716h Apr 14
By Almanac, Aries = 243.183°
Find declination, d:
d = $\sin^{-1}$(sin L sin h + cos L cos h cos Z)
= $\sin^{-1}$(sin(33.750) sin(34.300) +
(cos(33.750) cos(34.300) cos(150.))
= –16.366° (south)
Find meridian angle, t= $\sin^{-1}$(sin Z cos h/cos d)
t = $\sin^{-1}$(sin(210) cos(34.300)/cos(–16.366))
= –25.499° (West)
Find SHA of star:
LHA_{Aries} = – Long. + GHA_{Aries}
= – 118.417 + 243.183
= 124.766°
LHA_{star} = 25.499^{op}
SHA_{star} = (180–124.766) + 180 + 25.499 = 260.733
By Almanac, star is Sirius, # 18
SHA 258° 53.5', Decl. S 16° 41.8'

THE NOON SIGHT

The noon sight (of the sun) can be considered the single most important job of the navigator. It tells at once with a minimum of calculation the position in latitude and longitude.

Determination of Latitude

As Mixter put it in his Primer of Navigation, "When the sun, on the meridian, bears true north and south at the time of the local apparent noon, the momentary collapse of the astronomical triangle makes possible the ages old solution used by all mariners" for finding latitude.



Consider the illustration of a noon sight in north latitude with observed sun at 60 deg and declination at 20 deg.

By inspection, $a_1 = a_2$

$$a_1 = 90 - L$$
$$a_2 = h - d$$

then, $h - d = 90 - L$

or, $L = 90 - (h - d)$

in this case, $L = 90 - (60 - 20)$
$= 50^{\circ}N.$

Since time is not a factor, a watch is not necessary, however, as a practical matter, the time of local apparent noon, L.A.N., is needed to know when to take the observation. Further, since there may be error in the reading, particularly in heavy seas and in a small vessel, it is wise to take several readings before and after the sun's zenith to obtain an mean value at noon.

A reading of the sun made near the zenith can be "reduced to the meridian" by use of tables developed by Bowditch. These are rather extensive since the sun elevation and its rate of rise depends on the latitude.

Determination of Longitude

At the time of the noon sight, the local meridian angle is zero. The Civil Mean Time at the Greenwich meridian is known. The Almanac gives the value of the GHA (Greenwich Hour Angle) of the sun at this instant. Since the sun is at this instance directly in line with the overhead meridian the local longitude equals the value of the GHA if less than 180 degrees or 360-GHA if the GHA exceeds 180 degrees.

Longitude based on noon sight

Consider the case for finding longitude with a noon sight observed at 10h GMT on August 9, 1983.

By Almanac: GHA Sun is 328^{o} 37.0'

The meridian angle of the sun at Greenwich is

$$\begin{array}{r} 359^{o}\ 60.0' \\ \underline{328^{o}\ 37.0'} \\ t = 31^{o}\ 23'\ E. \end{array}$$

Since it is noon at the local meridian but only 10 o'clock AM at Greenwich, then the position must be East of Greenwich. It is east of Greenwich by the value just determined. Observe that this value when divided by 15 degrees per hour gives two plus hours which agrees with the difference in local and Greenwich time of two hours.

Consider now the Equation of Time. By the Almanac, the value is 05m 34s with the sun lagging the hour. This corresponds to an angle of:

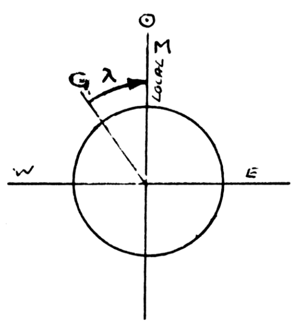
$$((5.567m/60)/12.) \times 180. = 1.392^{o}$$

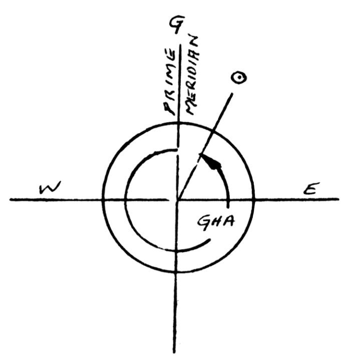
The GHA for 10.00h for Civil Mean Time at EqTm=0 is

$$180^{o} + ((10/12) \times 180 = 330.00^{o}$$

The indicated Sun GHA is then: $330.000-1.392 = 328.608^{o}$ or 328^{o} 36'

which is in agreement with the Almanac value





LATITUDE BY PHI-1/PHI-2 METHOD

A review of the problem of locating one's position shows that Longitude can be readily established having the essentials of a watch to show Greenwich time and an Almanac to find Greenwich Hour Angle of the observed celestial body.

Finding Latitude by observation of the Sun at its zenith or of a star at either its zenith or nadir (upper transit or lower transit) is similarly readily accomplished. Observation of Polaris, which is less than a degree from the Celestial Pole, is particularly suitable for Latitude observation.

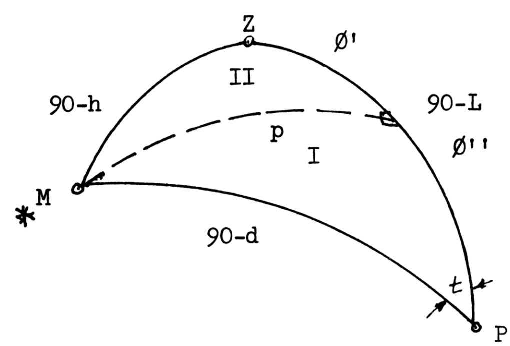
Finding Latitude by observation of celestial bodies at positions in which their meridian angle from the observer is not zero is more complex when done entirely by computation. (A later chapter shows how to find position by a combination of calculation and "line of position" charting.)

Latitude can be found by use of the Astronomical Triangle, separating it into two right angles by extending a perpendicular from the star to the observer's meridian; that it, from the star to a line from the Pole to the Zenith. The distance from the Pole to the intersection is called $\emptyset_1$; the distance from the Zenith to the intersection is called $\emptyset_2$. This is known accordingly as the Phi-1/Phi-2 Method as described in Bodwitch's 1936 American Practical Navigator.

Triangle 1 encompasses M (star), Pole (with angle t), and Phi-2. M-P is 90-d.

Triangle 2 encompasses M (star), Zenith, and Phi-1. The length of the perpendicular is p. M-Z is 90-h.

The sum of Phi-1 and Phi-2 equals 90-L.



LATITUDE BY ASTRONOMICAL TRIANGLE

The Phi-1/Phi-2 Method

By Triangle 1, the length of the perpendicular, p, can be found by Napier's rule for opposite parts:

$$\sin p = \cos \overline{(90-d)} \cos \overline{P}$$

now, the co-value of $\overline{90-d}$ is d
P = t and its co-value is 90-t
and by equivalence, $\cos \overline{P} = \sin t$

then, $\sin p = \cos d \sin t$

The value of Phi-2 in Triangle 1 can now be found by Napier's rule for opposite parts:

$$\sin \overline{(90-d)} = \cos p \cos \phi''$$

since $\overline{(90-d)} = d$

$$\cos \phi'' = \sin d/\cos p$$

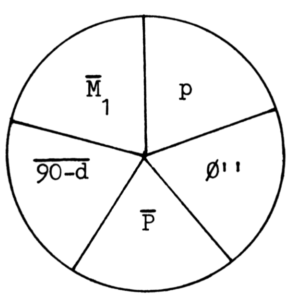
The value of Phi-1 in Triangle 2 is found similarly:

$$\sin \overline{(90-h)} = \cos \phi' \cos p$$

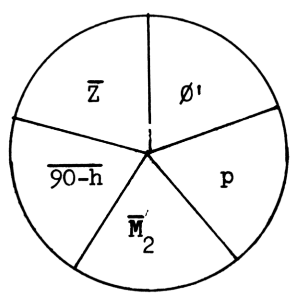
since $\overline{90-h} = h$

$$\cos \phi' = \sin h/\cos p$$

Mark Ø" North or South according to the name of the declination; mark Ø' North or South according to the Zenith distance, it being North if the body bears South and East or South and West, and South if it bears North and East or North and West. Then, combine Phi-1 and Phi-2 according to their names (that is, using plus and minus for North and South), except that in the case of bodies of lower transit, when 180-Ø" must be substituted for Ø" to obtain the Latitude.



NAPIER'S CIRCLE FOR LATITUDE
TRIANGLE I



NAPIER'S CIRCLE FOR LATITUDE
TRIANGLE II

Latitude by Observation of Star

Data: (Case Number 2, Achernar, Bowditch(1936,p.145))
Estimated Latitude, -53.0856deg; Longitude, -146.5330deg
Date: Aug 6,1925   Zone Time 19.0617hr
Star Number 5, Elevation 23.9300; Near Lower Transit

Reference Data: Achernar; Magn. 0.6; SSHA 335.8000
Declin. -57.3600

Calculations:

Zone =- Int(((abs(Longit.)-7.5)/15)+1)
=- Int(((abs(-146.5330)-7.5)/15)+1) = -10

GMT = Zone + Zone Time = 19.0617 - 10 = 9.0617hr

Greenwich date is Aug 6,
GHA Aries by Almanac (estimated) 89.6165deg
SHA Star " " 336.5036 "

Equatorial Direction star,
eqtdr = 360+(Longit. -SHAA - SHA)
= 360 +(-146.5330-89.6165-336.5036)
= -212.6522 (+360)= 147.3478 deg
Meridian Angle, t = 147.3478deg

Length of Perpendicular, $p = \sin^{-1}(\cos(d)\ \sin(t))$
$p=\sin^{-1}(\cos(-57.3600)\ \sin(147.3478))$
= 16.9181deg

Declination Distance,
$\text{Phi-2} = \text{abs}(\sin^{-1}(\sin(d)/\cos(p)))$
$\text{Phi-2}=\text{abs}(\sin^{-1}(\sin(-57.3600)/\cos(16.9181)))$
= 61.6626deg
"...except that for bodies of lower transit, use 180-(Phi-2)
then, Phi-2=180-61.6626= 118.3374deg
"Mark Phi-2 according to the declination, being North for a South Body..."
then, Phi-2=-118.3374deg
Zenith distance, $\text{Phi-1} = \cos^{-1}(\sin(h)/\cos(p))$
$\text{Phi-1} = \cos^{-1}(\sin(23.9300)/\cos(16.9181))$
= 64.914deg
"Mark Phi-1 according to Zenith distance, minus if North..."
then, Phi-1 = 64.914deg

Latitude = Phi-1 + Phi-2
= 64.914 - 118.3374 = -53.4234deg

Latitude by Observation of Sun

Data: (Case Number 12, Achernar, Bowditch(1936,p.144))
Estimated Latitude, 30.4160deg; Longitude, 81.4250deg
Date: Jun 7,1925 Zone Time 13.3378hr
Star Number 59, Elevation 75.7580; Bearing South and West

Reference Data: Sun; Magn. ***; SSHA ---
Declin. --

Calculations:

Zone = Int(((abs(Longit.)-7.5)/15)+1)
= Int(((abs(81.4250)-7.5)/15)+1) = 5

GMT = Zone + Zone Time = 5+ 13.3378= 18.3378hr

Greenwich date is Jun 7,
Equation of Time by Almanac(estimated) = -1.1463min
Declination by Almanac(estimated) = 22.8282deg
Sun at Greenwich, GHAS=((GMT-(EQTM/60)+12)/24) x 360
GHAS=((18.3378-(-1.1463/60)+12)/24)x360
= 455.3535 (-360)=95.3536deg
Equatorial Direction Sun,
eqtdr = 360+(Longit. -GHAS)
= 360 +(81.4250-95.3536)
= 346.0714deg
Meridian Angle, t =360-346.0714 = 13.9286deg

Length of Perpendicular, p = $\sin^{-1}$(cos(d) sin(t))
p=$\sin^{-1}$(cos(22.8282) sin(13.9286))
= 12.8182deg

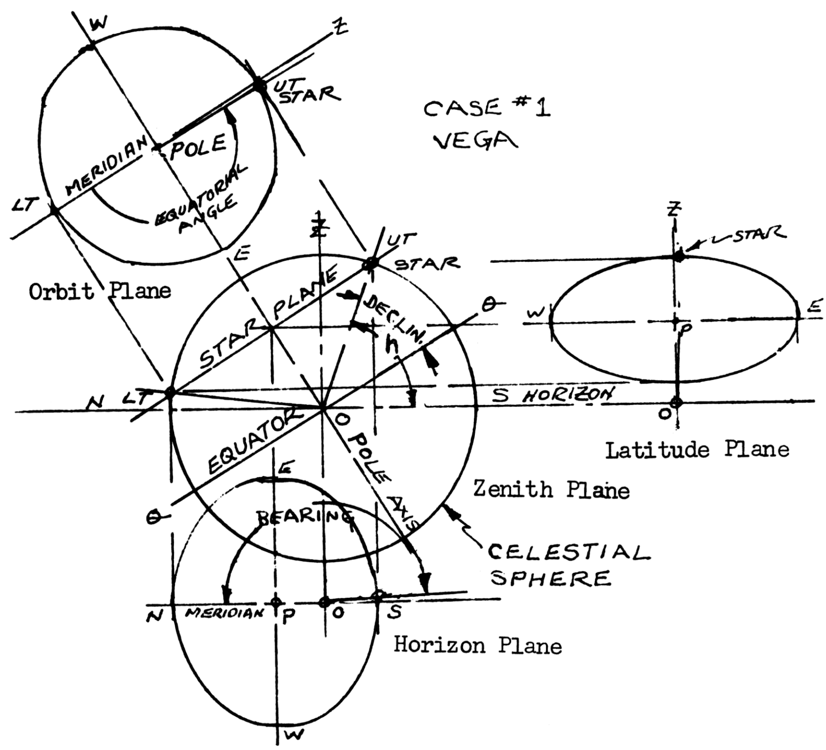
Declination Distance,
Phi-2 = abs($\sin^{-1}$(sin(d)/cos(p)))
Phi-2=abs($\sin^{-1}$(sin(22.8282)/cos(12.8182)))
= 23.4460deg
"...except that for bodies of lower transit, use 180-(Phi-2)
then, Phi-2=23.4460deg
"Mark Phi-2 according to the declination, being North for a South Body..."
then, Phi-2= 23.4460deg
Zenith distance, Phi-1 = $\cos^{-1}$(sin(h)/cos(p))
Phi-1 = $\cos^{-1}$(sin(75.7580)/cos(12.8182))
= 6.2598deg
"Mark Phi-1 according to Zenith distance, minus if North..."
then, Phi-1 = 6.2598deg

Latitude = Phi-1 + Phi-2
= 6.2598 + 23.4460 = 29.7058deg

LATITUDE BY PHI-1/PHI-2 METHOD

Case Number 1 Vega, Dutton(1942,p.320)

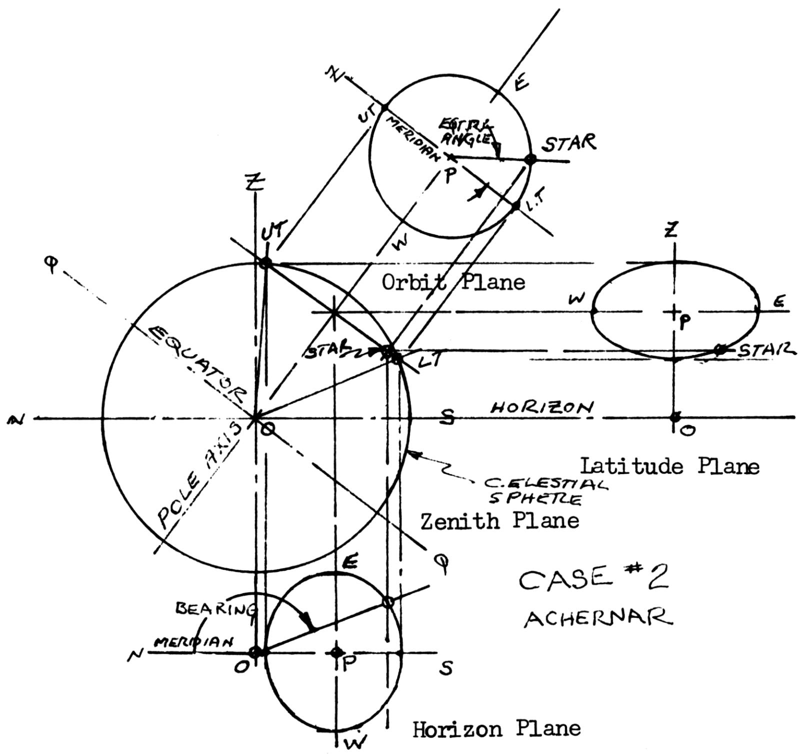
Est. Latitude, deg.	59.5000	Est. Longitude, deg.	22.0167
Zone Time, hr.	4.4969	Month/Day/Year	5/ 1/1939
Star Number/Elev.	49/69.2500	At upper transit	
Reference Data:		Vega	
Magnitude	.1	SHA/Declin.	80.9670/ 38.7590
Calculated Results			
TimeZone/Grnwchdate	1// 5/ 1	Ref.day/days to Ref.day	21/ 41
Hr. Sun Zero Decl.	12.9164	GHA Sun	263.1721
Eqtrl direct. star	179.9506	GHA Aries	300.4875
Meridian Angle, t	.0494	SHA Star	81.4797
Ht.UT/LT(rads=1.)	.9352/ .1436	Ht. Star(radius=1.)	.9351
AngleEqtr(frmSouth)	30.5000	Angle UT/LT	69.2590/171.7410
Bearing	178.0977	Phi-1	20.7515
Dist. Perpendicular	.0386	Phi-2	38.7590
GMT, hr.	5.4969	Latitude	59.5105



LATITUDE BY PHI-1/PHI-2 METHOD

Case Number 2 Achernar, Bowditch(1936,p.145)

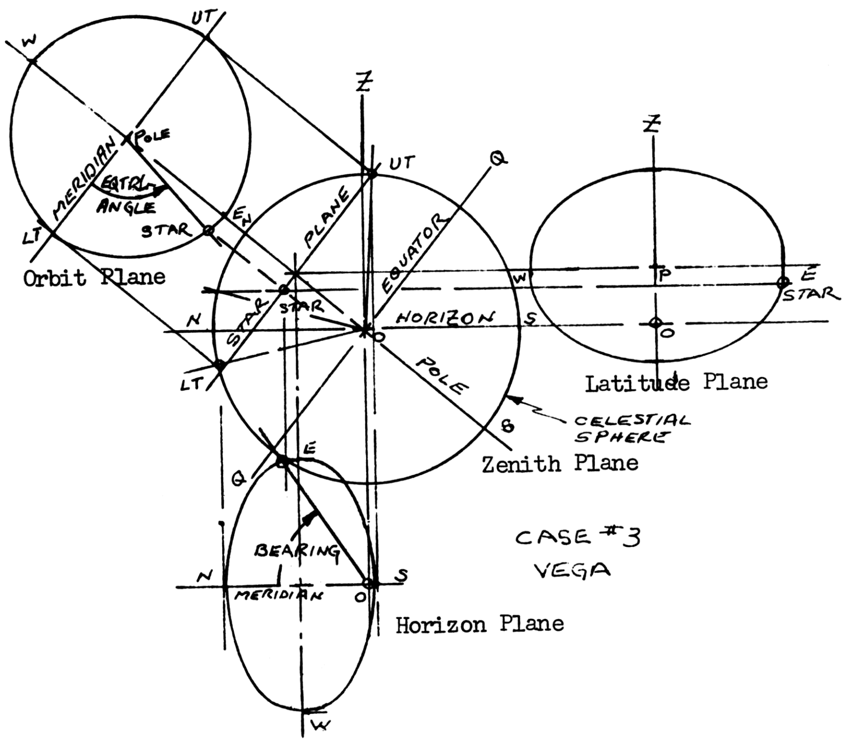
Est. Latitude, deg.	-53.0856	Est. Longitude, deg.	-146.5330
Zone Time, hr.	19.0610	Month/Day/Year	8/ 6/1925
Star Number/Elev.	5/23.9000		
Reference Data:		Achernar	
Magnitude	.6	SHA/Declin.	335.8000/-57.3600
Calculated Results			
TimeZone/Grnwchdate	-10// 8/ 6	Ref.day/days to Ref.day	21/138
Hr. Sun Zero Decl.	3.5511	GHA Sun	314.4931
Eqtrl direct. star	32.6426	GHA Aries	89.6059
Meridian Angle, t	32.6426	SHA Star	336.5036
Ht.UT/LT(rads=1.)	.9972/ .3494	Ht. Star(radius=1.)	.4051
AngleEqtr(frmSouth)	143.0856	Angle UT/LT	85.7256/ 20.4456
Bearing	160.6598	Phi-1	64.9484
Dist. Perpendicular	16.9140	Phi-2	-118.3399
GMT, hr.	9.0610	Latitude	-53.3915



LATITUDE BY PHI-1/PHI-2 METHOD

Case Number 3 Vega, Bowditch(1936,p.178m)

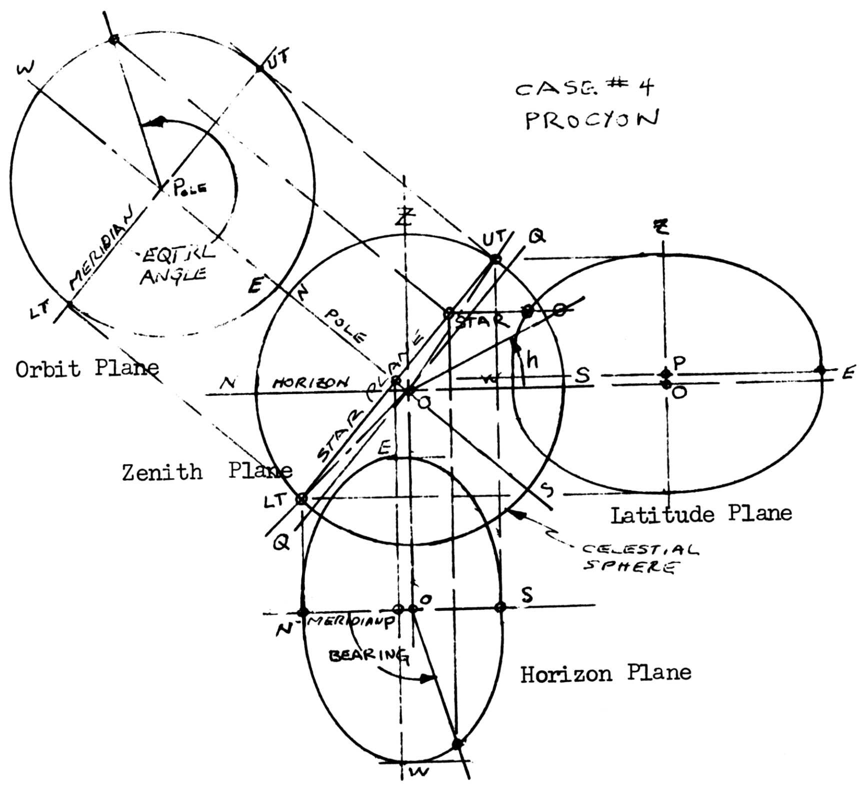
Est. Latitude, deg.	40.7200	Est. Longitude, deg.	68.5000
Zone Time, hr.	19.7542	Month/Day/Year	5/15/1934
Star Number/Elev.	49/14.5000	Bearing N51degE	
Reference Data:		Vega	
Magnitude	.1	SHA/Declin.	80.9670/ 38.7590
Calculated Results			
TimeZone/Grnwchdate	5// 5/16	Ref.day/days to Ref.day	21/ 55
Hr. Sun Zero Decl.	7.8471	GHA Sun	192.2647
Eqtrl direct. star	75.9655	GHA Aries	242.9168
Meridian Angle, t	75.9655	SHA Star	81.5487
Ht.UT/LT(rads=1.)	.9994/-.1826	Ht. Star(radius=1.)	.2504
AngleEqtr(frmSouth)	49.2800	Angle UT/LT	88.0390/190.5210
Bearing	50.9069	Phi-1	-67.4908
Dist. Perpendicular	49.1585	Phi-2	106.8080
GMT, hr.	.7542	Latitude	39.3172



LATITUDE BY PHI-1/PHI-2 METHOD

Case Number 4 Procyon, Bowditch(1936,p.178m)

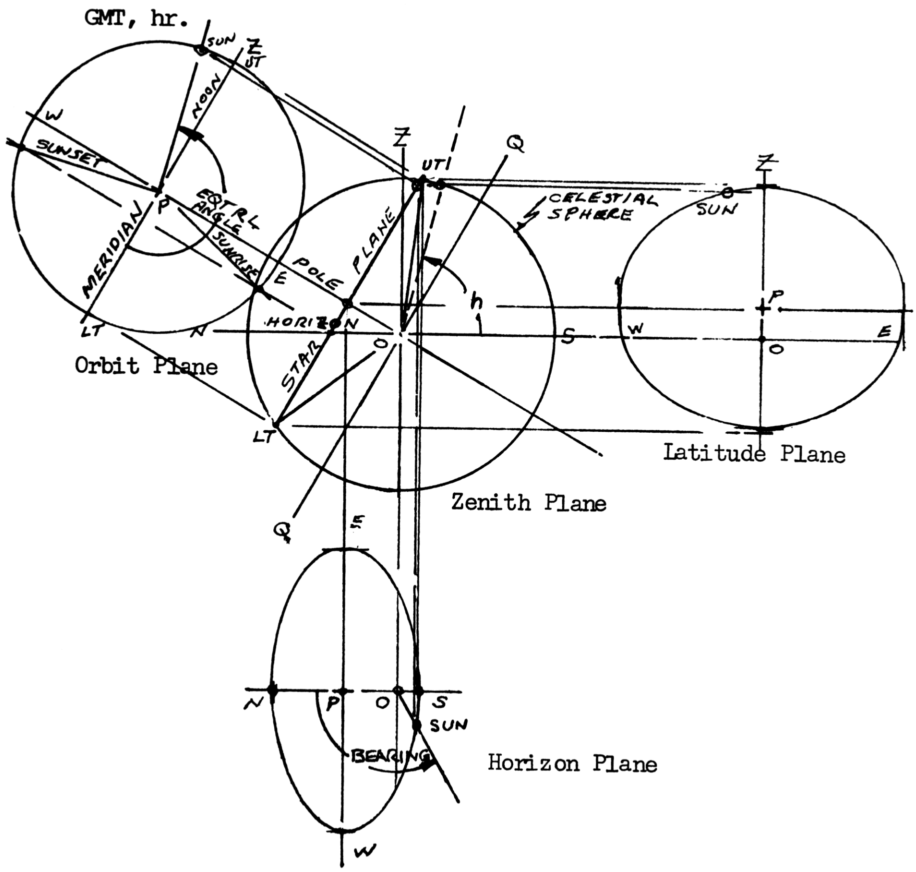
Est. Latitude, deg.	40.7200	Est. Longitude, deg.	68.5000
Zone Time, hr.	19.7542	Month/Day/Year	5/15/1934
Star Number/Elev.	20/26.5500	Bearing N107degW	
Reference Data:		Procyon	
Magnitude	.5	SHA/Declin.	245.4800/ 5.2830
Calculated Results			
TimeZone/Grnwchdate	5// 5/16	Ref.day/days to Ref.day	21/ 55
Hr. Sun Zero Decl.	7.8471	GHA Sun	192.2647
Eqtrl direct. star	240.4785	GHA Aries	242.9168
Meridian Angle, t	60.4785	SHA Star	246.0617
Ht.UT/LT(rads=1.)	.8147/-.6946	Ht. Star(radius=1.)	.4470
AngleEqtr(frmSouth)	49.2800	Angle UT/LT	54.5630/223.9970
Bearing	107.1134	Phi-1	26.4520
Dist. Perpendicular	60.0515	Phi-2	10.6280
GMT, hr.	.7542	Latitude	37.0800



LATITUDE BY PHI-1/PHI-2 METHOD

Case Number 12 Sun, Bowditch(1936,p.144)

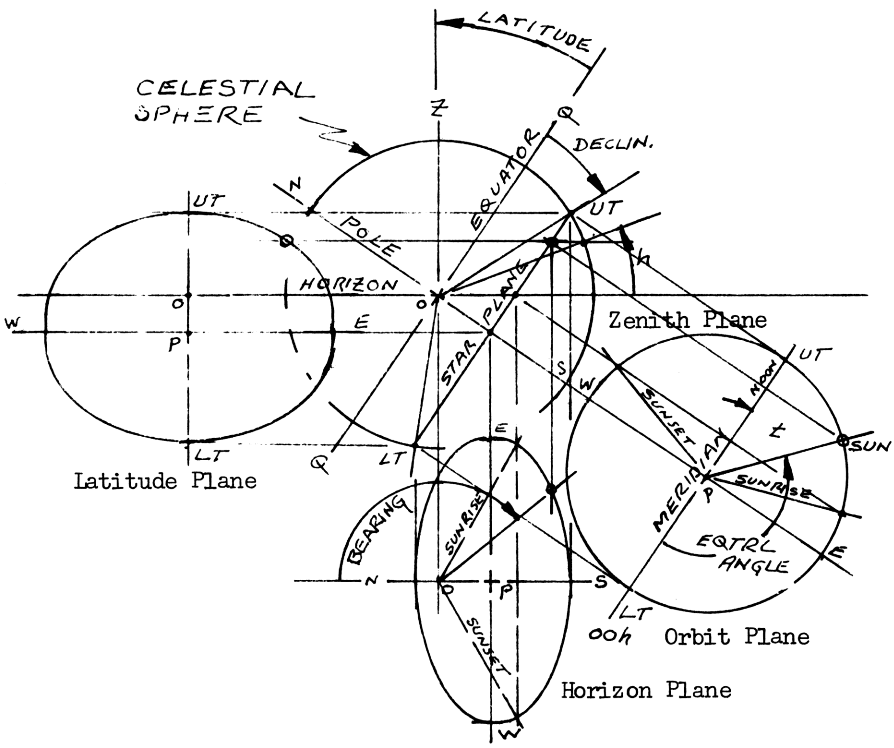
Est. Latitude, deg.	30.4160	Est. Longitude, deg.	81.4250
Zone Time, hr.	13.3378	Month/Day/Year	6/ 7/1925
Star Number/Elev.	59/75.7580	Bearing South and West	
Reference Data:		Sun	
Magnitude	-9.0	SHA/Declin.	.0000/ 22.8282
Calculated Results			
TimeZone/Grnwchdate	5// 6/ 7	Ref.day/days to Ref.day	21/ 78
Hr. Sun Zero Decl.	3.5511	GHA Sun	95.3536
Eqtrl direct. star	193.9286	GHA Aries	170.0005
Meridian Angle, t	13.9286	SHA Star	.7059
Ht.UT/LT(rads=1.)	.9912/-.5984	Ht. Star(radius=1.)	.9693
AngleEqtr(frmSouth)	59.5840	Angle UT/LT	82.4122/216.7558
dT Ellipt. Path	3.1962		
dT Eclipt. Path	-4.3426	Equation of Time	-1.1463
Bearing	118.9584	Phi-1	6.2632
Dist. Perpendicular	12.8182	Phi-2	23.4460
GMT, hr.	18.3378	Latitude	29.7093



LATITUDE BY PHI-1/PHI-2 METHOD

Case Number 14 Data to Calculate Sun Path

Est. Latitude, deg.	33.4000	Est. Longitude, deg.	117.4000
Zone Time, hr.	9.0000	Month/Day/Year	12/18/1984
Star Number/Elev.	59/20.8070	Bearing South and East	
Reference Data:		Sun	
Magnitude	-9.0	SHA/Declin.	.0000/-23.4370
Calculated Results			
TimeZone/Grnwchdate	8//12/18	Ref.day/days to Ref.day	20/273
Hr. Sun Zero Decl.	10.2536	GHA Sun	75.8446
Eqtrl direct. star	138.4446	GHA Aries	342.4855
Meridian Angle, t	41.5554	SHA Star	-.1216
Ht.UT/LT(rads=1.)	.5470/-.9849	Ht. Star(radius=1.)	.3552
AngleEqtr(frmSouth)	56.6000	Angle UT/LT	33.1630/260.0370
dT Ellipt. Path	-1.8077		
dT Eclipt. Path	-1.5708	Equation of Time	-3.3785
Bearing	139.4810	Phi-1	63.4065
Dist. Perpendicular	37.4897	Phi-2	-30.0839
GMT, hr.	17.0000	Latitude	33.3227



Sight Reduction

Sight Reduction is the process of determining a Line of Position (or simply a Position) from observation of celestial bodies. The process can be separated into six steps for finding a Line of Position: (1) Correction of sextant altitude, (2) Determination of GHA and declination, (3) Selection of assumed position and finding meridian angle, and (4) Computation of altitude and azimuth, (5) Comparison of computed and observed altitudes, and (6) Plot of the Line of Position.

Tables and methods for facilitating sight reduction are based on the Astronomical Triangle. The first to divide this triangle into two parts by dropping a perpendicular from the Zenith was Souillagouet in 1891. The 1962 edition of Bowditch lists more than twenty variations of the method. The 1936 edition of Bowditch cites the Phi-1/Phi-2 Method (named for the two triangles) and cautions that "the solution is impractical when the declination is 0 degrees as well as when the hour angle is 6h; and in fact, it is commonly advised that this observation should be limited to conditions where the celestial body is within three hours of meridian passage and where it is not more than 45 degrees from the meridian; also where the declination is at least 3 degrees". (The solution adds or subtracts two legs of the triangles according to whether the bearing is North or South; accordingly, a small error in bearing when at nearly 90 degrees can change the result drastically.)

A small error leading to a determination of excessive altitude can prevent solution by trigonometric analysis where inverse sine and cosine functions are unacceptably found to exceed a value of 1.0.

Terminology of Positions

A <u>fix</u> is an accurate determination of latitude and longitude, most reliably by two lines of position. <u>Dead Reckoning</u> is a method of determining a ship's position by applying the to latest well determined position fix the course changes using only directions and speeds. A <u>Running Fix</u> is a position determined by the intersection of two or more lines of position established at different times, but adjusted to the same time by consideration of speed and course. <u>Estimated Position</u> is the best position obtainable short of a fix or running fix.

Three simultaneous lines of positions usually intersect to form a triangle. For any triangle, there are four points equidistant from the points; one inside and three outside. The most probable location is found by:

(1) Identify the vertexes as A-B, B-C and A-C according to the lines of position A, B and C.

(2) At Vertex A-B draw the azimuths of A and B. Similarly, draw in the azimuths at B and C.

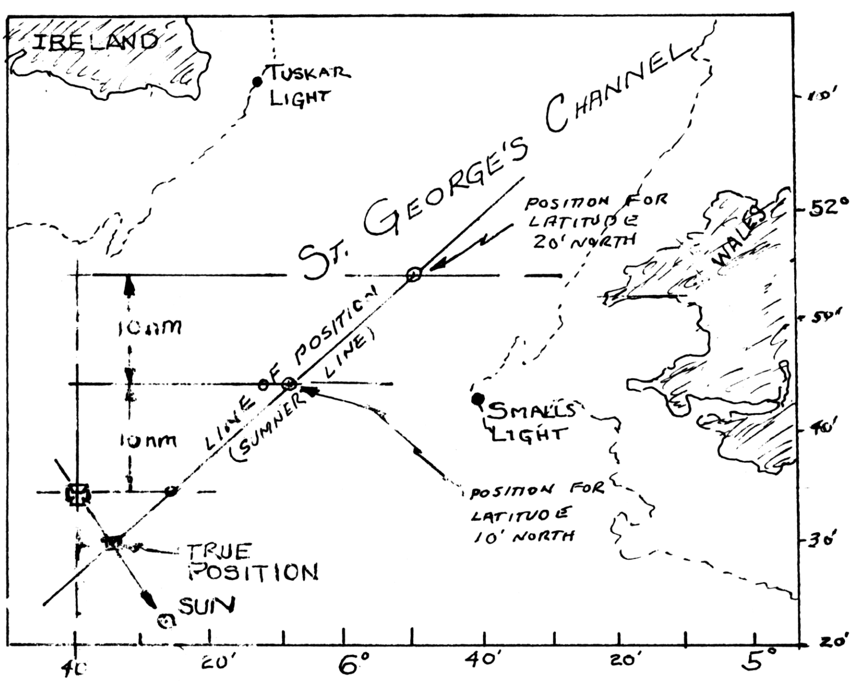
(3) Bisect the azimuths at each vertex. The bisection lines will intersect at the most probable position.

The Sumner Line

In Winter, 1837, Capt. Thomas A. Sumner, an American Master, was bound for Greenock (the port of Glascow) and making for the passage into the Irish Sea through St. George's Channel from the South mid Tuscar Light off Ireland's shore and Small's Light on the shoals of Wales. Having passed the Azores and Longitude 21°W, with winds from the southward and thick weather allowing no observation, soundings were made not far, it was supposed, from the shore.

In thick and heavy weather, the winds boisterous and toward the shore, he arrived about midnight, 17th December, within 40 miles by dead reckoning from Small's Light going ENE under short sails. About 10 A.M. an observation of the Sun was made and time noted at a dead reckoned position (as reconstructed) of 51°35'N, 6°40'W.

Using his estimated Latitude, he computed his Longitude. It was 15 minutes of arc East of his dead reckoned position and nearer the danger. A second calculation assumed 10' farther north gave a position 27nm ENE; a third latitude 10' farther north gave a position again ENE. Plotting these on his chart, he found them to fall in a straight line which passed through Small's Light. Maintaining this course of ENE with his compass, Small's Light was soon seen bearing ENE 1/2E and close aboard. (It has been calculated that the dead reckoned position was 8' north of the true position.)



Reconstructing the Sumner Line

Given the true position as Latitude 51^{o} 30'N, Longitude 6^{o} 35'W at 10.00 AM, Dec. 18, 1837.

Using the 1985 Almanac (similarly one year past the Leap Year):

Time Zone = 0; Ship time = GMT
Sun GHA = 330^{o} 54.6'
Sun declination, d = S23^{o} 22.7'

$$\text{Meridian angle, } t = \text{Longit} + 360 - GHA_{sun}$$
$$= 6.5833 + 360 - 330.9100 = 35.6733^{o}E.$$

Elevation (true),

$$h_{true} = \sin^{-1}(\sin L \sin d + \cos L \cos d \cos t)$$
$$= \sin^{-1}(\sin(51.5000)\ \sin(-23.3783) + (\cos(51.5000)\ \cos(-23.3783)\ \cos(35.6733))$$
$$= 8.8384^{o}$$

Now, presuming that the sighting was made from a location 10' farther north, find the angle t and the longitude:

$$t = \cos^{-1}((\sin h - \sin L \sin d)/(\cos L \cos d))$$
$$= \cos^{-1}((\sin(8.8384) - \sin(51.6667)\ \sin(-23.3783))/(\cos(51.6667)\ \cos(-23.3783)))$$
$$= 35.2540^{o}$$

$$\text{Long.} = t - 360 + GHA_{sun}$$
$$= 35.2540 - 360 + 330.9100 = 6.1650^{o}$$
$$\text{or } 6^{o}\ 9.8'\ W.$$

These two points mark the Sumner Line.

The East/West departure at Latitude 51.5 is
60 x cos(51.5) = 37.35 nm/degree
The North-South distance is 10 degrees or 10nm
The East-West departure is (6.5833-6.1650) x 37.35
= 15.6235nm

The slope of the Sumner Line is then
$\tan^{-1}(10.0000/15.6235) = 32.6217^{o}$

The Azimuth of the Sun is 180 - 32.6217 = 147.3782^{o}

THE LINE OF POSITION

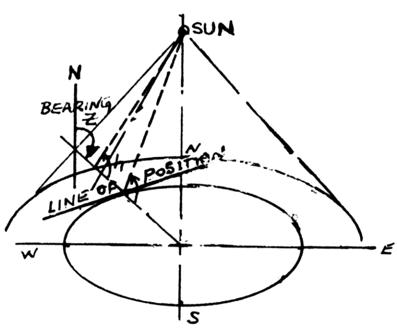
The Sumner Line was a monumental advance in the art of Navigation. It constitutes a method by which from an assumed Latitude and a single sighting of a Celestial Body at a known time a Line can be charted on which the probable position lies. The Sumner Line is at right angle from the observed body as shown by the forgoing analysis and charting.

In 1875 the French Commander Marq Saint-Hilare found that the altitude of a star represented a constant distance from the projection of that star's radius from the center of the earth. In other words, the altitude observation is constant at any position on a circle drawn with the focus at which the altitude is 90 degrees. The Sumner Line is a tangent to this "circle of equal latitudes".

It followed from St.-Hilare's insight that the Sumner Line must lie somewhere perpendicular to the line of bearing of the observed body and at a distance which differed from the assumed location by an amount proportional to the difference in the true (observed) elevation and the calculated elevation. This direct application is called St.-Hilare's Line of Position.

The true position can be nearer or farther toward the bearing of the observed body than the assumed position. The correct direction can be found by assuming an exaggerated condition. Suppose the true distance were several hundred miles farther away: the star would appear lower; if closer, the star would appear higher. Thus, if the star actual elevation is lower than the elevation computed by the assumed position, move away from the star to find the true position. The amount of movement is equal to the difference; 60nm/$^{\circ}$.

Suppose that Capt. Sumner had used the Method of St.-Hilare:
Estimated position: 51° 35'N, 6° 40'W; Dec 18, 1837, 10AM



By Almanac(1984), GMT 10h 00m 00s
GHA Sun $330^{\circ}54.6'$
Decl. Sun $S23^{\circ}22.7'$
Bearing, Z, of Sun 147.3782°
Altitude for true location on Line of Position (previous calc.),
Observed altitude:
$h_{true} = 8.8384^{\circ}$
Meridian angle, $t = 6.5833+360-330.9100$
$= 35.6733^{\circ}$
Altitudecalculated for assumed position (as recreated):

$$h_{calc} = \sin^{-1}(\sin(51.5833)\sin(-23.3783) + \cos(51.5833)\cos(-23.3783)\cos(35.8384))$$
$$= 8.7127^{\circ}$$

Altitude difference = $8.8384-8.7127 = 0.1257^{\circ}$ (calc h<observ h)
Make correction toward Sun along line of bearing a distance of
60nm/deg x $.1257^{\circ}$ = 7.54nm

Example. Running Fix

Position 1:

Dead reckoned position, 51° 30'N, 6° 10'W
Ship time 08h 46m 00s, Dec 18, 1983
Time zone Int((6.167-7.5)/15 +1)=0 GMT=08h 46m 00s
$GHA_{sun}=312^{\circ}$ 25.3';decl=S23° 22.6'
Sun meridian angle, t=360-212.422+6.667=53.745°E.
Observed elevation, $h_o=1^{\circ}32.5'$;bearing Z=132.377°
Calculated $h=\sin^{-1}(\sin L \sin d + \cos L \cos d \cos t)$
$=\sin^{-1}(\sin(51.500)\ \sin(-23.377)$
$+(\cos(51.500)\ \cos(-23.377)\ \cos(53.745))$
$= 1.750^{\circ}$ (1° 34.2')
minus observed h = 34.2'-32.4'=1.8nm
observed elevation is lower; move position away

Course and Speed: 238.93°, 15.71kn

Position 2:

Dead reckoned position, 51° 11.9'N, 6° 58.1W
Ship time 11h 00m 00s, Dec 18 GMT=same
$GHA_{sun}=345^{\circ}$ 54.2';decl=S23° 22.8'
Sun meridian angle, t=360-345.903+6.969=21.066E.
Observed elevation $h_o=13^{\circ}$ 20.0';bearing Z=160.697°
Calculated $h=\sin^{-1}(\sin(51.198)\ \sin(-23.380)$
$+\cos(51.198)\ \cos(-23.380)\ \cos(21.066))$
$= 13.149^{\circ}$ (13° 8.9')
minus observed h = 13° 8.9'-13° 20.0'=11.1nm
observed elevation is higher: move position away

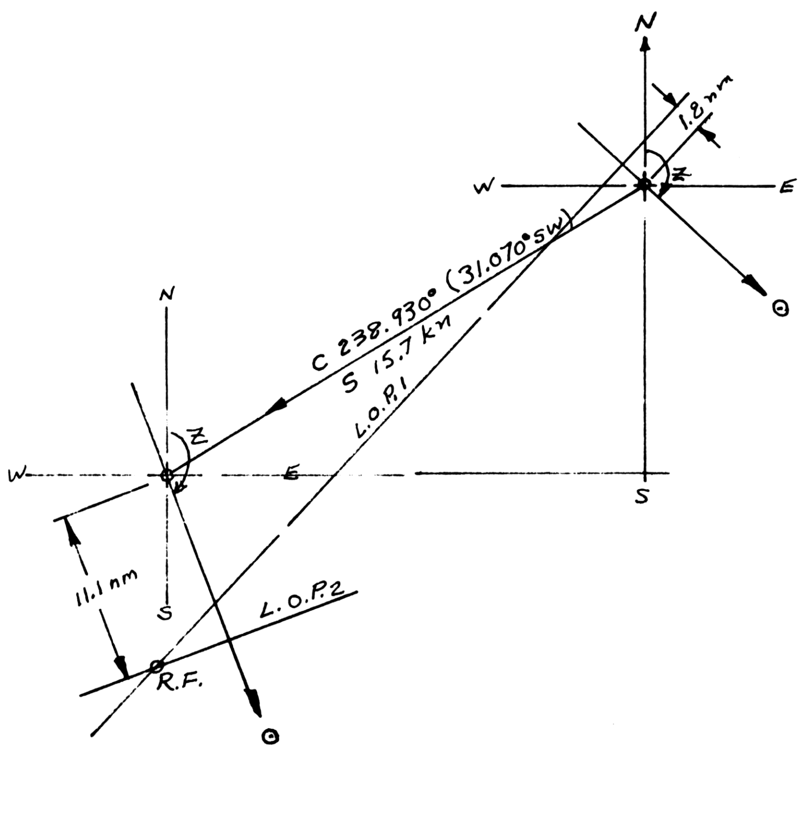
Latitude diff: 51° 30'N - 51° 11.9'N = 18.1'S=18.1nmS.
middle latitude= 51.349°;
longitude spacing=cos(51.349) = .6245nm/'
Longitude diff:6° 58.1'W - 6° 10'W = 48.1'W
departure = 48.1' x .6245nm/' = 30.04nmW.
Course: $\tan^{-1}$(dLat/departure)=$\tan^{-1}(-18.1/-30.04)$
= 31.070°S.W.
or, = 360 -(90+31.070) = 238.93°

Running Fix (continued)

The charting procedure is to start with the dead reckoned position (D.R.) at Position 1 (08h 40m) and the dead reckoned position at Position 2 (11h 00m) charted at the relative locations as connected by the course (C) and speed (S)

At $D.R._1$ the Sun azimuth vector is drawn and the Line of Position 1 is measured from the the origin in the proper direction and drawn through at 90 degrees.

At $D.R._2$ a parallel to the line of position at $D.R._1$ is drawn the same distance away as for $D.R._1$. The second Sun azimuth vector is drawn through $D.R._2$ and the Line of Position 2 is measured off and drawn at 90 degrees and extended to intersect Line of Position 1 which gives the Running Fix.



Navigation Fix, Two Stars with Azimuths Unknown

Dead reckoned position: 135°, 30°W
July 3, 1984, Zone Time 21.00h
Observe: Denebola, h = 32.439° (true)
Antares, h = 32.500° (true)

by Almanac, Denebola, SHA = 182° 56.3', decl. = N14° 39.7'
Antares, SHA = 112° 53.2', decl. = S26° 24.0'
Aries, GHAA = 12° 25.9'

Calculate meridian angle, t, and longitude for 30°N.

Denebola:

$$t=\cos^{-1}((\sin h - \sin L \sin d)/(\cos L \cos d))$$
$$=\cos^{-1}((\sin(32.439)-\sin(30.)\ \sin(14.662))/(\cos(30.)\ \cos(14.662))$$
$$= 60.714^{\circ}$$

Long. = SHA + GHAA − t
= 182.938 + 12.432 − 60.714 = 134.656°

Antares:

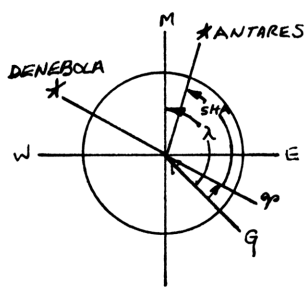
$$t=\cos^{-1}((\sin(32.500)-\sin(30.)\ \sin(-26.400))/(\cos(30.)\ \cos(-26.400))$$
$$= 11.691^{\circ}$$

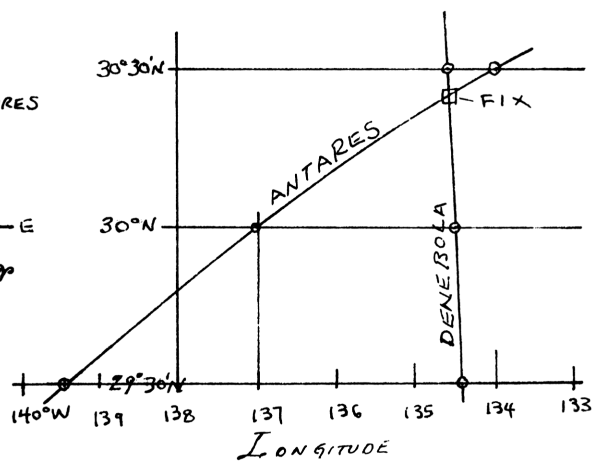
Long. = 112.887 + 12.432 − 11.691 = 137.010°

Similarly, find values for Latitude= 29° 30'N, and 30° 30'N

	Denebola Longit.	Antares Longit.
Assumed Latitude 29° 30'N	134.646	139.374
30° 00'N	134.656	137.010
30° 30'N	134.670	133.981

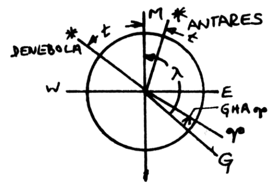
Plot and locate fix:





NAVIGATION FIX BY TWO STARS WITH KNOWN AZIMUTHS

Dead reckoned position 135°W, 30°N
July 3, 1984 21h 00m 00s ship time
Observe Denebola, $h = 32.439^{\circ}$(true)
Azimuth, $Z = 268.558^{\circ}$
Antares, $h = 32.500^{\circ}$(true)
$Z = 169.845^{\circ}$



Time Zone = INT((135-7.5)/15 +1)=9

GMT= 06h 00m 00s July 4

Almanac data: GHA♈ Aries=12°25.9'
Denebola, SHA =182°56.3' dec. N14°39.7'
Antares, SHA =112°53.2' dec. S26°24.0'

Calculate meridian angles:

Denebola, $t = SHA_{*} + GHA_{♈} + \lambda$

$= 182.938 + 12.432 - 135 = 60.370^{\circ}W$

Antares, $t = 112.887 + 12.432 - 135 = -9.681^{\circ}$ or $9.681^{\circ}E$

Calculate altitudes:

Denebola, $h = \sin^{-1}(\sin L \sin d + \cos L \cos d \cos t)$

$= \sin^{-1}(\sin 30.000 \sin 14.662 + \cos 30.000 \cos 14.662 \cos 60.370)$
$= 32.736^{\circ}$

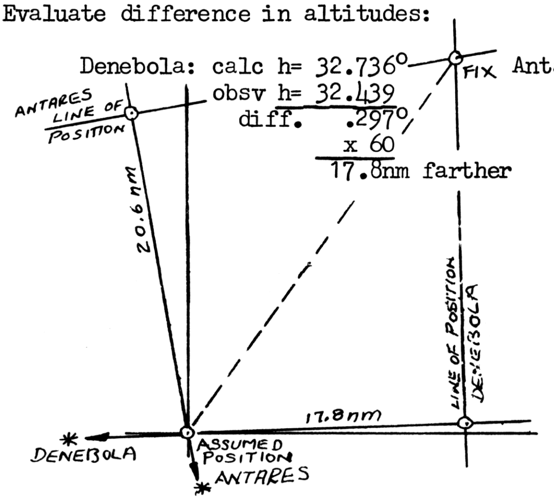
Antares, $h = \sin^{-1}(\sin 30.000 \sin (-26.400) + \cos 30.000 \cos (-26.400) \cos 9.681)$
$= 32.843^{\circ}$

Evaluate difference in altitudes:

Denebola: calc h= 32.736°
obsv h= 32.439
diff. .297°
x 60
17.8nm farther

Antares: calc h = 32.843°
obsv h = 32.500
diff. .343
x 60
20.6nm farther

Longitude nm/deg
60 sin(90-30)= 51.9



135°W- $\frac{17.2}{51.9}$ =134° 40'W
30° 24'N

A Classical Navigational Fix

There is an interesting fictional account of a navigational fix made under emergency conditions in Jules Verne's The Mysterious Island. Several ballonists, including Cyrus Harding, an engineer, are blown out to sea in a violent storm to become wrecked on an uninhabited island. The engineer has a watch set to the time of Washington, D.C. He proceeds to determine their location by use of his watch, the sun and the stars.

Longitude:

Fortitutiously, the date of their observation is April 16, one of the four times of the year at which the Equation of Time is zero (when mean time corresponds to sidereal time).

The engineer makes a gnomen: He sets a pole in the sand and observes the solar shadow. Its traced path locates the Sun's zenith, local high noon at his location. His watch shows 5 o'clock Washington time; indicating their longitude to be 5 hours West of Washington. At 15 degrees per hour, this is 75 degrees west of Washington which he knew to be at 77 degrees West longitude, giving their longitude as 152 degrees West.

Latitude:

The ballonists have determined that they are in the Southern hemisphere by noting the absence of the Northern stars and the presence of the Southern Cross, the star Fomalhaut, Antares in the constellation Scorpio, and the Centaur.

Harding decided to make a fix on the star Alpha of the Southern Cross, which he knew to be 27 degrees from the pole. He prepared a crude compass (astrolabe) by joining two flat boards flexibly at one end with an arrangement for fixing the angle of the two.

He had already determined North by his solar observation. Its direction was marked by the position of the shadow tip at noon from the gnomen.

He waited for the star to pass to the lowest point of its orbit at which time he took a fix of the angle to the horizon with his compass, holding the lower board level and sighting along the star with the upper board.

Harding improvised a protractor by scribing a circle which he divided into quadrants of 90 degrees. One of these he further divided. He found his fix to be 26 degrees. To this value he added the star polar distance of 27 degrees giving a distance of his horizon of 53 degrees. He then knew his latitude to be 90-53 or 37 degrees South.

Great Circle Route

A great circle is the intersection of a plane passing through the center of a sphere, in our case, the Earth. Consider a First location at Meridian M_1 Latitude A, and a Second location at Meridian M_2 Latitude B. The angle from the Pole between the two Meridians is P. Three great circles can then be formed: two from the Pole through A and B and the third through A and B, the latter being the required great circle route. Designate the sides by the angles opposite, as: side P-A is b, side P-B is a, and side A-B is D.

By the Law of Cosines for sides:

$$\cos D = \cos(90-Lat_A)\ \cos(90-Lat_B) + \sin(90-Lat_A)\ \sin(90-Lat_B)\ \cos P$$

by equivalences:

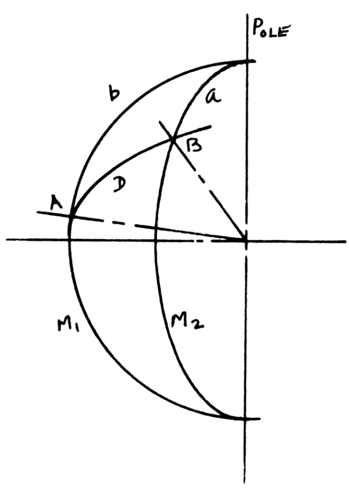
$$\cos D = \sin(Lat_A)\ \sin(Lat_B) + \cos(Lat_A)\ \cos(Lat_B)\ \cos P$$

Example: Great Circle Distance

Origin,	40° 45'N,	73° 37'W
Destination,	48° 50'N,	2° 20'E

Find angle P at Pole: $P = 73.6167 - (-2.3333) = 75.9500^\circ$

$$D = \cos^{-1}(\sin(40.7500)\ \sin(48.8333) + (\cos(40.7500)\ \cos(48.8333)\ \cos(75.9500))$$
$$= 52.2326 \text{ degrees}$$
$$\times 60 = 3133.9 \text{ nm}$$



Great Circle Course

Layout a great circle course from Los Angeles to Hawaii.

Origin: Pt. Fermin Light, 33° 42'N, 118° 18'W
Destination: Diamond Head Light
21° 16'N, 157° 49'W

Cruising speed, 8 knots, 192nm/day
Time of Departure, 06h 00m Zone Time, March 10, 1984

Time Zone = Int((118.3000 -7.5)/15) + 1) = 8
GMT at departure, 06 + 8 = 14h, Mar 10
Polar angle, P = 157.8167-118.3000 = 39.5167°
Great circle distance,

$$D = \cos^{-1}(\sin(33.7)\ \sin(21.2667) + \cos(33.7)\ \cos(21.2667)\ \cos(39.5167))$$
$$= 36.9325^{\circ}$$
$$\times 60 = 2215.9\text{nm}$$

<u>Leg 1</u>:

Find M_1, angle of spherical triangle at origin

$$\cos(90-Lat_B) = \cos(90-Lat_A)\ \cos(Dist) + \sin(90-Lat_A)\ \sin(Dist)\ \cos A$$

then $A = \cos^{-1}((\sin(Lat_B) - \sin(Lat_A)\ \cos(Dist))/(\cos(Lat_A)\ \sin(Dist)))$

$$A = \cos^{-1}((\sin(21.2667)-\sin(33.7)\ \cos(36.9325))/(\cos(33.7)\ \sin(36.9325)))$$
$$= 99.301^{\circ}\ (W9.301^{\circ}S)$$

At end of day: $Lat_1 = 33.7 - (192\text{nm}/60\text{nm}/\text{deg})\sin(9.301)$
$=33.183^{\circ}N$
$Long_1 = 118.3 + (192/60)\ \cos(9.301)$
$=122.084^{\circ}W$

Polar Angle, P = 157.8167-122.084 = 35.7327°

$$Dist_1 = \cos^{-1}(\sin(33.183)\ \sin(21.2667) + \cos(33.183)\ \cos(21.2667)\ \cos(35.7327))$$
$$= 33.7337^{\circ} \times 60 = 2024.0 \text{ nm}$$

GREAT CIRCLE COURSE-----LOS ANGELES TO HAWAII

```
Departure
Local: 3/10/84  Zone 8, 06:00h    Location:  33.700N, 118.300W
  GMT: 3/10/84          16:00h    Distance: 2215.9nm (36.933°)
    Course      260.699 (W9.301S)  Speed  8kn  TimeonCourse: 24h
    DistMadeGood        192nm (50.067nm/deg)
Local: 3/11/84  Zone 8  06:00h    Location:  33.183N, 122.084W
  GMT: 3/11/84          16:00h    Distance: 2024.0nm (33.734)
    Course      258.527 (W11.473S) Speed  8kn  TimeonCourse: 24h
    DistMadeGood        192nm (50.397nm/deg)
Local: 3/12/84  Zone 8  06:00h    Location:  32.547N, 125.818W
  GMT: 3/12/84          16:00h    Distance: 1832.1nm (30.535)
    Course      256.435 (W13.595S) Speed  8kn  TimeonCourse: 24h
    DistMadeGood        192nm (50.788nm/deg)
Local: 3/13/84  Zone 9  05:00h    Location:  31.795N, 129.493W
  GMT: 3/13/84          16:00h    Distance: 1640.1nm (27.335)
    Course      254.342 (W15.568S) Speed  8kn  TimeonCourse: 24h
    DistMadeGood        192nm (51.233nm/deg)
Local: 3/14/85  Zone 9  05:00h    Location:  30.931N, 133.102W
  GMT: 3/14/85          16:00h    Distance: 1448.1nm (24.135)
    Course      252.375 (W17.625S) Speed  8kn  TimeonCourse: 24h
    DistMadeGood        192nm (51.727nm/deg)
Local: 3/15/84  Zone 9  05:00h    Location:  29.961N, 136.639W
  GMT: 3/15/84          16:00h    Distance: 1256.1nm (20.936)
    Course      250.424 (W19.576S) Speed  8kn  TimeonCourse: 24h
    DistMadeGood        192nm (52.260nm/deg)
Local: 3/16/84  Zone 9  05:00h    Location:  28.889N, 140.101W
  GMT: 3/16/84          16:00h    Distance: 1064.2nm  (17.736)
    Course      248.533 (W21.427S) Speed  8kn  TimeonCourse: 24h
    DistMadeGood        192nm (52.826nm/deg)
Local: 3/17/84 Zone 10  04:00h    Location:  27.720N, 143.484W
  GMT: 3/17/84          16:00h    Distance:  872.2nm (14.537)
    Course      246.797 (W23.203S) Speed  8kn  TimeonCourse: 24h
    DistMadeGood        192nm (53.418nm/deg)
Local: 3/18/84 Zone 10  04:00h    Location:  26.459N, 146.788W
  GMT: 3/18/84          16:00h    Distance:  680.2nm (11.337)
        Departure nm/deg = 60 x cos(0.5 x (26.459+21.267))=54.87
     Rhumb Line Course:
     RLC = tan^-1(60x(21.267-26.459)/(54.87x(157.817-146.788)))
         = -27.238°, or W27.238S
     Speed  8kn  TimeonCourse: 680.2nm/8kn = 85:00h or 3d, 13h
Destination
Local: 3/21/84 Zone 11  16:00h    Location:  21.267N, 157.8167W
  GMT: 3/22/84          03:00h
```

Star Altitude Curves for a Flight Plan

The flight is from New York to London, Takeoff at 18h 30m, on Feb 07, 1984. Select two stars for viewing at three intermediate positions.

Flt Plan 00h 30m: GMT=2/08/84, 00h 00m, 44.220°N,71.238°W.
" " 02h 30m: 2/08/84 00h 00m, 52.938°N,49.637°W.
" " 05h 10m: 2/08/84 00h,40m, 55.007°N,11.655°

GHA Aries at Flt Plan 00h 30m: 144° 48.9'
02h 30m: 174° 53.8'
05h 10m: 215° 00.5'

Local Hour Angle, LHA, of Aries = -Longit. +GHA_A
at Flt Plan 00h 30m: -71.238+144.815= 73.577°
02h 30m: -49.637+174.987=125.350°
05h 10m: -11.655+215.008=203.353°

Select Pollux(#21, Magn. 1.2, Gemini)
SHA 243° 54.8', decl. N28° 04.'
Alphard(#25, Magn. 2.2, Hydrae)
SHA 218° 17.8', decl. S08° 35.4'

Pollux	Alphard
Flt Plan 00h 30m:	
LHA=-71.238+144.815+243.913	LHA=-71.238+144.815+218.297
=317.490; t=42.51East	=291.874; t=68.126East
Flt Plan 02h 30m:	
LHA=-49.637+174.987+243.913	LHA=-49.637+174.987+218.297
=369.263 or 9.263;t=9.263West	=343.647; t=16.353East
Flt Plan 05h 10m:	
LHA=-11.655+215.008+243.913	LHA=-11.655+215.008+218.297
=447.266 or 87.266;t=87.266W	=421.650 or 61.650;t=61.65W

Altitudes

Altitude, $h = \sin^{-1}(\sin L \sin d + \cos L \cos d \cos t)$

00h 30m $h_P=\sin^{-1}(\sin(44.220)\ \sin(28.067) + \cos(44.220)\ \cos(28.067)\ \cos(42.510))=52.590^\circ$

02h 30m $h_P=\sin^{-1}(\sin(52.938)\ \sin(28.067) + \cos(52.938)\ \cos(28.067)\ \cos(-9.263))=64.200$

05h 10m $h_P=\sin^{-1}(\sin(55.007)\ \sin(28.067) + \cos(55.007)\ \cos(28.067)\ \cos(-87.266))=24.178$

00h 30m $h_A=\sin^{-1}(\sin(44.220)\ \sin(-8.590) + \cos(44.220)\ \cos(-8.590)\ \cos(68.126)=9.198$

02h 30m $h_A=\sin^{-1}(\sin(52.938)\ \sin(-8.590) + \cos(52.938)\ \cos(-8.590)\ \cos(16.353)=26.912$

05h 10m $h_A=\sin^{-1}(\sin(55.007)\ \sin(-8.590) + \cos(55.007)\ \cos(-8.590)\ \cos(-61.650)=8.447$

Star Altitude Curves (Continued)

Zenith Angle

$Z = \cos^{-1}((\sin d - \sin L \sin h)/(\cos L \cos h)$

Pollux

00h 30m $Z_P=\cos^{-1}((\sin(28.067)-\sin(44.220)\ \sin(52.590))$
$/(\cos(44.220)\ \cos(52.590)))= 101.051$

02h 30m $Z_P=\cos^{-1}((\sin(28.067)-\sin(52.938)\ \sin(64.200))$
$/(\cos(52.938)\ \cos(64.200)))=160.947$
since t is West, Azimuth =180+(180-160.947)=199.053

05h 10m $Z_P=\cos^{-1}((\sin(28.067)-\sin(55.007)\ \sin(24.178))$
$/(\cos(55.007)\ \cos(24.178)))=75.049$
since t is West, Azimuth =180+(180-75.049) =284.951

Alphard

00h 30m $Z_A=\cos^{-1}((\sin(-8.590)-\sin(44.220)\ \sin(9.198)$
$/(\cos(44.220)\ \cos(9.198)))=111.636$

02h 30m $Z_A=\cos^{-1}((\sin(-8.590)-\sin(52.938)\ \sin(26.912)$
$/(\cos(52.938)\ \cos(26.912)))=161.807$

05h 10m $Z_A=\cos^{-1}((\sin(-8.590)-\sin(55.007)\ \sin(8.447)$
$/(\cos(55.007)\ \cos(8.447)))=118.389$
since t is West, Azimuth =180+(180-118.389)=241.611

Test Z for Proper Quadrant

Angle at Star, M

$M=\cos^{-1}((\sin L - \sin d \sin h)/(\cos d \cos h))$

00h 30m $M_P=\cos^{-1}((\sin(44.220)-\sin(28.067)\ \sin(52.590))$
$/(\cos(28.067)\ \cos(52.590)))=52.856$

02h 30m $M_P=\cos^{-1}((\sin(52.938)-\sin(28.067)\ \sin(64.200))$
$/(\cos(28.067)\ \cos(64.200)))=12.882$

05h 10m $M_P=\cos^{-1}((\sin(55.007)-\sin(28.067)\ \sin(24.178))$
$/(\cos(28.067)\ \cos(24.178)))=38.896$

00h 30m $M_A=\cos^{-1}((\sin(44.220)-\sin(-8.590)\ \sin(9.198))$
$/(\cos(-8.590)\ \cos(9.198)))\ =42.356$

02h 30m $M_A=\cos^{-1}((\sin(52.938)-\sin(-8.590)\ \sin(26.912))$
$/(\cos(-8.590)\ \cos(26.912)))=10.970$

05h 10m $M_A=\cos^{-1}((\sin(55.007)-\sin(-8.590)\ \sin(8.447))$
$/(\cos(-8.590)\ \cos(8.447)))\ =35.396$

Find Sum of angles M+Z+t; must be more than 180° and less than 540° (use absolute value of t)

00h 30m S_P=52.590+101.051+42.510 =196
02h 30m S_P=12.882+199.053+9.263 =221
05h 10m S_P=38.896+284.951+87.26 =411

00h 30m S_A=42.356+111.636+68.126 =222
02h 30m S_A=10.970+161.807+16.351 =188
05h 10m S_A=35.396+241.611+61.650 =338 Z values are correct

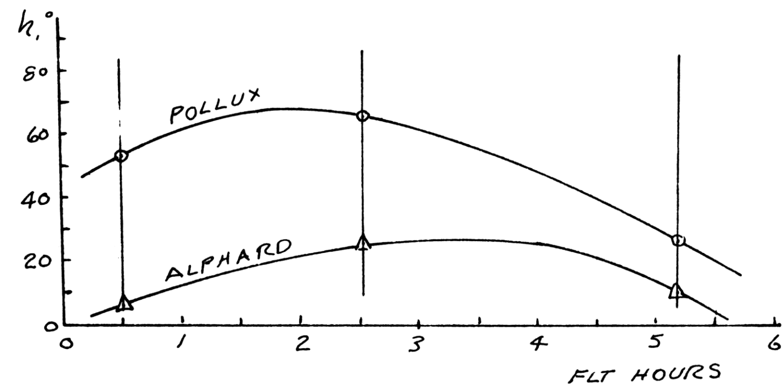
Star Altitude Curves (Continued)

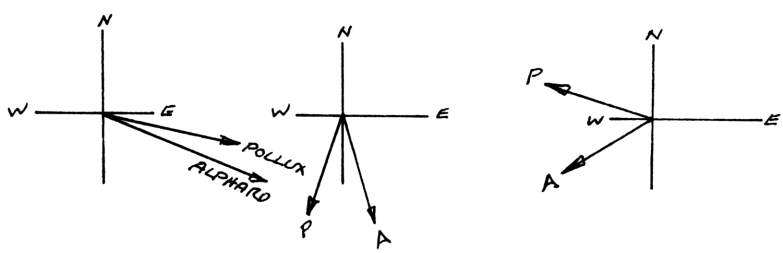
Summary of Results

00h 30m 44°13.2'N, 71°14.3'W
Pollux h=52°35.4' Azimuth =101° 3.1'
Alphard h= 9°11.9' Azimuth =111°38.2'

02h 30m 52°56.3'N, 49°38.2'W
Pollux h=64°12' Azimuth =199°03.2'
Alphard h=26°54.7' Azimuth =161°48.4'

05h 10m 55°00.4'N, 11°39.3'W
Pollux h=24°10.7' Azimuth =284°57.1'
Alphard h= 8°26.8' Azimuth =241°36.7'





TRUE SUN ELEVATION AND AZIMUTH FOR A SET COURSE

<u>Position 1</u>: Ship time 08h 46m 00s, Dec 18, 1983
$5^\circ 59'$W, $51^\circ 40'$N

Time Zone = Int(5.983-7.5/15 + 1)=0

GMT = 08h 46m 00s

GHA sun, by Almanac, $312^\circ 25.3'$, dec.=S$23^\circ 22.6'$

Meridian angle, t = 360-312.422+ 5.983 = 53.561°E

$$h = \sin^{-1}(\sin L \sin d + \cos L \cos d \cos t)$$
$$= \sin^{-1}(\sin 51.667 \sin(-23.377) + \cos 51.667 \cos(-23.377) \cos 53.377)$$
$$= 1.542^\circ \; (1^\circ 32.5')$$

$$\text{Azimuth, } Z = \cos^{-1}((\sin d - \sin L \sin h)/(\cos L \cos h))$$
$$= \cos^{-1}((\sin(-23.377) - \sin 51.667 \sin 1.542)/(\cos 51.667 \cos 1.542))$$
$$= 132.377^\circ$$

Course: 260° true (W10°S)
8 knots for 2h 14m
Distance made good 17.867nm

Longitude nm/deg = 60 sin (90 - 51.667)= 37.214

<u>Position 2</u>:

Latitude = 51.667 -(17.867/60)sin 10 = 51.615°N($51^\circ 36.9'$N)

Longitude = 5.983 +(17.867/37.214) cos 10 = 6.456°W($6^\circ 27.3'$W)

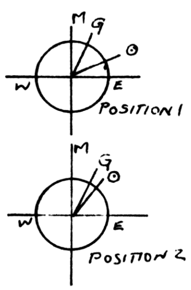
Time Zone = 0; GMT = 08h 46m + 2h 14m = 11h 00m 00s

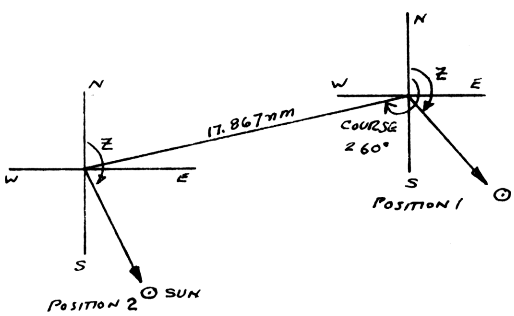
GHA sun by Almanac = $345^\circ 54.2'$, dec. = S$23^\circ 22.8'$

t = 360 - 345.903 + 6.456 = 20.553°E

$$h = \sin^{-1}(\sin 51.615 \sin(-23.380) + \cos 51.615 \cos(-23.380) \cos 20.553)$$
$$= 12.863^\circ \; (12^\circ 51.8')$$

$$Z = \cos^{-1}((\sin(-23.380) - \sin 51.615 \sin 12.863)/(\cos 51.615 \cos 12.863))$$
$$= 160.697^\circ$$





ORBITAL MECHANICS

Nomenclature

Planetary orbits necessarily have the shape of an ellipse, as proved by Newton. The ellipse is characterized by two axes at right angles. The long axis is called the major, and its distance from the center ,a, the major semidiameter; the short axis is the minor and its distance from the center ,b,the minor semidiameter. The ellipse has two foci, placed along the major axis such that the sum of the distances from each focus to any point of the ellipse is a constant, equal to 2a, with the distance between foci being equal to 2c (c being the distance between the center of the ellipse and one foci).

then, the ratio c/a is called the eccentricity, e

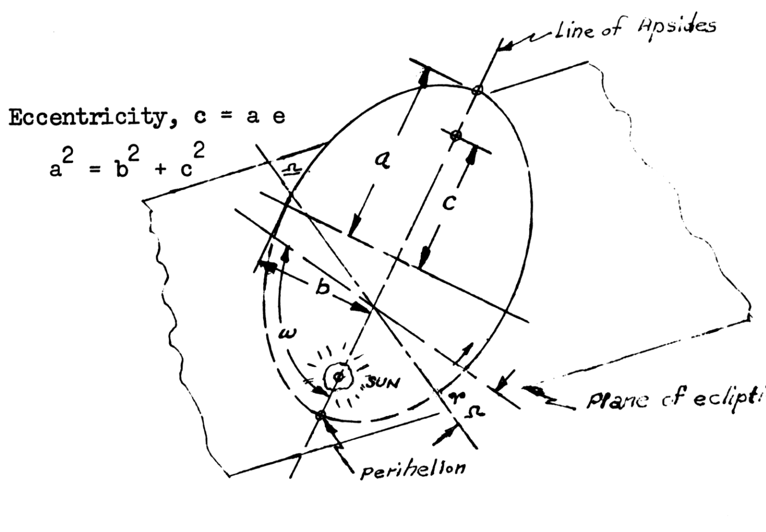
or $c = a\,e$

The Sun is at one focus of the planetary ellipse. The condition of <u>perihelion</u> occurs when the planet is nearest the Sun which is then along the major axis. The condition of <u>aphelion</u> occurs when the planet is fartherest from the Sun which is then also along the major axis. The <u>Line of Apsides</u> is the major axis projected in both directions.

A line drawn from the Sun to the planet at any point in its path is called a <u>radius vector</u>. The angle between the radius vector and the point of perihelion is called the <u>anomaly</u>.

The seven characteristics needed to define an orbit are:

- The semi-major axis, a
- The eccentricity, e
- The inclination to the ecliptic, i (0 for Earth)
- The longitude of the ascending node, Ω (0 for Earth)
- The longitude of perihelion, $\pi = \omega + \Omega$
- The period, P, or else the daily motion, μ
- The epoch, E, (the date)



Changes in Orbit of the Earth

Revolution of the Line of Apsides - This line now extends toward the opposite constellations of Gemini and Satittarius. It moves Eastward and at the current rate would rotate once in about one hundred and eight thousand years.

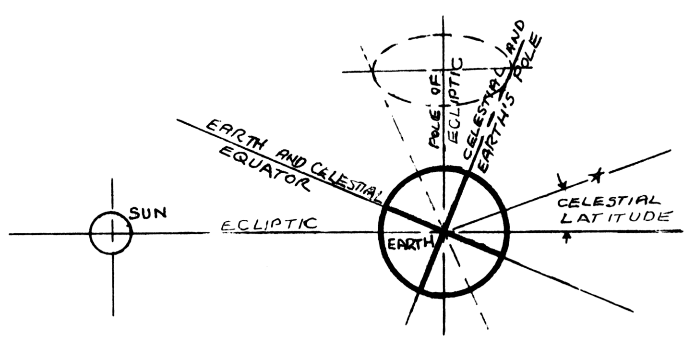
Change of Eccentricity - The eccentricity of the orbit is now 0.016 and is slowly diminishing with a predicted value of 0.003 in twenty-four thousand years.

Change in Obliquity of the Ecliptic - The plane of the Earth's orbit, which defines the ecliptic, is at present decreasing about 0.5" per year. The obliquity is now 24' less than it was two thousand years ago.

Periodic Disturbances in the Earth's Orbit - There is a monthly movement of the Earth's center above and below the true plane of the ecliptic by reason of the influence of the Moon. There are smaller effects caused by the planets which move the Earth forward, backward or sideways. These effects move the apparent location of the Sun, but do not affect the stars because of their great distance.

Precession of the Equinoxes

Hipparchus in -125 found that the time from Equinox to Equinox was less than that of repeated sequential pattern of the stars, that is the "Equinox preceded the stars". To put it another way, the year of the seasons is about 20 minutes shorter than the sidereal year. It is the motion of the equator and not of the ecliptic which causes the precession. That is, the plane of the Earth's orbit remains constant while the pole marks a circle upon the Celestial Sphere in the manner of a wobbly spinning top, making a circle in 25,800 years.



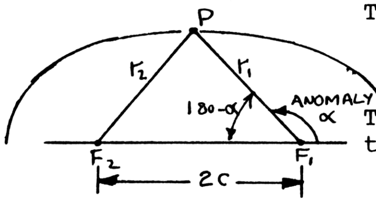
ELLIPTICAL MOTION

The Earth's motion is along an elliptical path and by Kepler's Law its speed is such that the area of the ellipse swept each day is a constant. The area of an ellipse is

A = pi a b where A = Area, a = major semidiameter
b = minor semidiameter

The mean distance of the Sun, a, is One Astronomical Unit, approximately 92,900,000 miles. The eccentricity of the earth, e, is 0.01667, and the distance of the two focal points of the ellipse (with the Sun occupying one) is

c = 1.0 x 0.01667 = 0.01667 AU



The minor semidiameter,

$b = sqrt(a^2 - c^2)$
$= sqrt(1.^2 - 0.01667^2)$
$= 0.999859$ AU

Then, by Kepler's Law, the area swept per day is

dA = 3.14159 x 1.0 x 0.999859/365.25
= 0.0086 sqAU/day

The daily angular change depends on the radius, PF_1. Observe that a triangle can be constructed with sides PF_1, 2c, and PF_2. The angle at F_1, as measured from the major axis at perihelion is known by the calendar. The angle F_1 of the triangle is 180-anom.

By the Law of Cosines:

$r_2^2 = r_1^2 + (2c)^2 - 2(r_1 \times 2c \cos(180-\alpha)$
$= r_1^2 + 4c^2 - 4r_1 c \cos(180-\alpha)$

Also, by ellipse geometry, $r_1 + r_2 = 2a$ where a =1.0

then, $r_2 = 2. - r_1$
and $r_2^2 = 4 - 4r_1 + r_1^2$

so: $4 - 4r_1 + r_1^2 = r_1^2 + 4c^2 - 4r_1 \cos(180-\alpha)$

which reduces to: $r_1 = (1.-c^2)/(1.- c \cos(180-\alpha)$

The <u>mean</u> distance along the orbit path we can call K, the mean anomoly. Then, K = days x (360deg/365.25days), degrees

The <u>true</u> distance along the orbit path is

$dA = 1/2 \times r_1 \times (r_1 \tan(\alpha)$
$= 1/2 \times r^2 \, d\alpha$

where $d\alpha$ is the daily anomoly, given in radians

or, $d\alpha_{true} = (2\ dA/r^2) \times (360/2\ pi)$, degrees

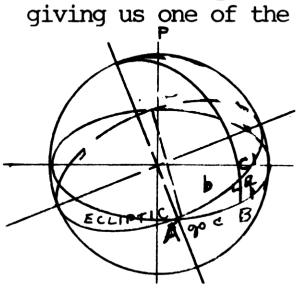
The daily correction is the cummulative difference between the true anomaly and the mean anomaly.

Days	True Anomaly	Mean Anomaly	Radius,AU	dt,minutes
1	1.0194	0.9856	0.9833	0.1350
10	10.1906	9.8562	0.9835	1.3374
20	20.3711	19.7124	0.9842	2.6350
60	60.8044	59.1372	0.9914	6.6687
100	100.4664	98.5620	1.0025	7.6177
140	139.2773	137.9968	1.0123	5.1619
180	177.4116	177.4116	1.0166	0.4828
280	274.0924	275.9737	0.9988	-7.5235
360	354.6568	354.8233	0.9834	-0.6660

THE ECLIPTIC

Consider the ecliptic as the surface of a bowl of water in which the Earth floats half submerged. Then, the surface and the ecliptic are equivalent to the plane in which the Earth revolves about the Sun.

Moreover, the Earth's pole is not perpendicular to the plane of motion but points constantly (at the north) to the star Polaris which is at an angle of 23-1/2 degrees from the perpendicular to the plane of motion. This causes the Sun to reach its daily zenith at a time different from the mean value, giving us one of the two components of the Equation of Time.

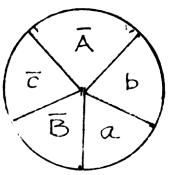


The illustration shows the mean path AB and the true path AC. These coincide at four times (solstices and equinoxes).

Note that the angle C is a right angle, being drawn from the pole through a diameter at right angles to that pole.

The value of c, the side perpendicular to the pole, with its origin at the Vernal Equinox, ♈, is and its length determined by the calendar is a known value.

The value of b, the slanted side which represents the Sun's path, can be found. We draw Napier's circle and see that we know c, and A, the angle of the ecliptic. We then have three adjacent parts. "The sine of any middle part is equal to the product of the tangents of the adjacent parts".



$$\sin \bar{A} = \tan \bar{c} \tan b$$

by equivalents becomes $b=\tan^{-1}(\cos A \tan c)$

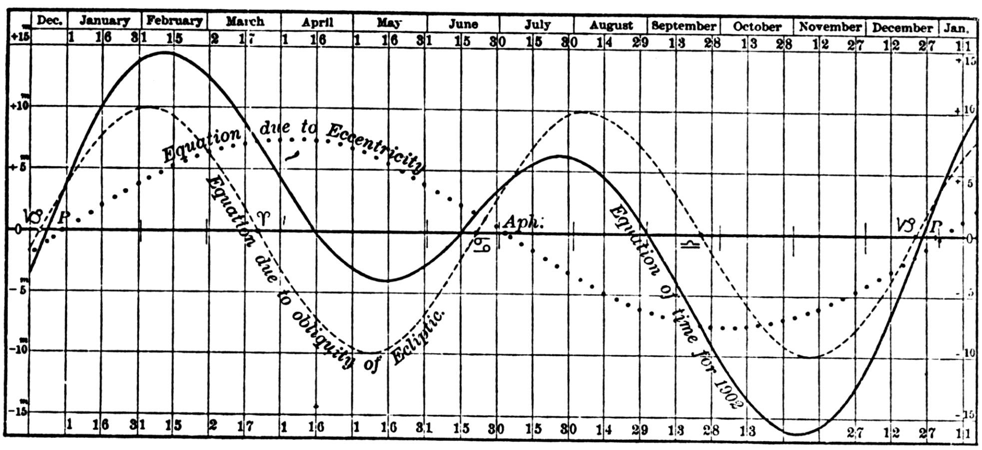
There are (24h x 60m) per day which correspond to a sun rotation of 360 degrees. Thus we convert the angle to minutes by multiplying degrees by 4.

$$Eqtm_{ecl} = b \times 4, \text{ minutes}$$

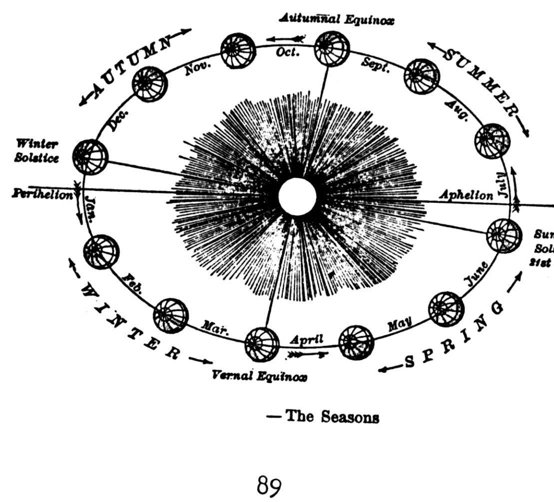
Angle from Aries	$Eqtm_{ecl}$	Angle from Aries	$Eqtm_{ecl}$
0 deg	0 min	205	7.3498
25	7.3490	225	9.8573
45	9.8573	245	7.7677
65	7.7679	270	0.
90	0.	295	-7.3498
115	-7.3498	315	-9.8573
135	-9.8573	335	-7.7679
155	-7.7679	360	0.
180	0.		

THE EQUATION OF TIME

The components of the effects of elliptical motion and ecliptical motion are summed as the Equation of Time. This is clearly shown in the illustration taken from Manual of Astronomy by Charles A. Young, Ginn and Company 1902. Professor Young stated that the "heavy-line curve is carefully laid out from the Nautical Almanac for 1902 (a mean year in the "leap-year cycle") and will give the equation of time for any date during the next fifty years within less than half a minute; not exactly because from year to year the equation of time for any day of the month varies a few seconds."



— The Equation of Time



— The Seasons

THE MOON

The Sidereal Month of the Moon is the time for it to revolve in the sky from a given star to the same star again. The time averages 27d 7h 43m (27.32166d), but it varies some three hours because of perturbations. The mean daily motion is 360/27.32166= 13^{o} 11'.

The Synodic Month of the Moon is the time for it to move between two successive conjunctions or oppositions. Its average value is 29d 12h 44m but it varies nearly thirteen hours, mainly because of the eccentricity of the lunar orbit.

The Moon's apparent diameter ranges from 33' 33" when nearest and 29' 24" when most remote. The orbit has an eccentricity of about 1/18, varying from 1/15 to 1/21. In her motion the Moon very nearly follows Kepler's Law of Equal Areas. Its path is deflected by the gravity of the Sun, always concave toward it.

The inclination of the Moon's orbit to the Ecliptic is 5^{o} 08' 40".

The Sidereal Month, the Synodic Month and the Sidereal Year are related by the simple equation:

$$1/M - 1/E = 1/S$$

where M = Moon's sidereal period, 27.32166 days
E = Length of sidereal year, 365.25635 days
S = Synodic month, 29.53058 days

The difference between the value of 1/M and 1/E represents the amount by which the Moon gains on the Sun each day. That is, the value of 1/S equals the fraction of a revolution per day.

$$1/S = (1/29.53058) \times 360 = 12.190^{o}/\text{day}$$

It is useful to remember that the Moon arrives each day on the Eastern horizon some 10 to 14 degrees behind its position on the previous day as shown by Almanac data: 1984 Almanac, 00h GMT

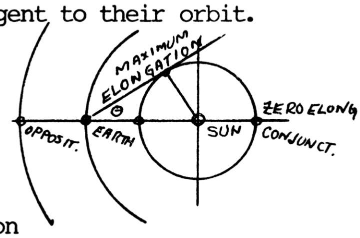
	MoonGHA-SunGHA	MoonGHA-AriesGHA
Jan	12.592^{o}	13.695^{o}
Mar	10.741	11.678
May	11.110	12.066
Jul	13.973	15.007
Sep	12.962	13.867
Nov	11.265	12.245

THE INNER PLANETS

The planets closer to the Sun than the Earth, of which there are only two, Venus and Mercury, are called the inner or inferior planets (as Mars would be an outer or superior planet). As seen from the Earth their maximum distance from the Sun is the angle, or elongation, between the Sun and a tangent to their orbit.

Venus: $\theta = \sin^{-1}(0.732)$
$= 47^{\circ}$

Mercury: $\theta = \sin^{-1}(0.387)$
$= 22.7^{\circ}$

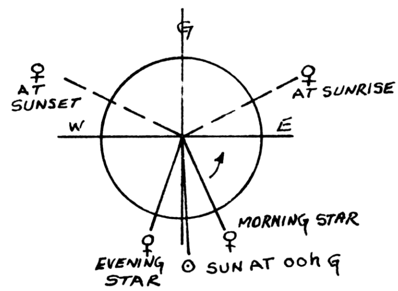


Conjunction occurs when the elongation is zero. For an inner planet there is inferior and superior conjunction. For the Moon and outer planets Opposition occurs when conjunction is 180 degrees and the planet rises at sunset. The time between two successive conjunctions is the synodic period.

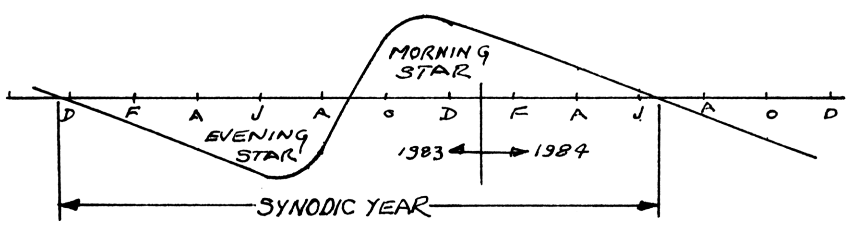
Mercury	☿	0.387AU	0.2408 sdrl.yr	0.317synd.yr
Venus	♀	0.723	0.6152	1.598

From Almanac at 00h GMT, 1st of Month:

1983	VenusGHA		SunGHA		diff.
Jan	163°	54.5'	179°	12.5'	-15.3°
Feb	155	17.3	176	37.5	-21.3
Apr	146	17.3	178	57.3	-32.7
Jun	131	55.4	180	35.9	-48.7
Aug	149	38.0	178	25.1	-28.8
Oct	221	04.3	182	30.6	38.6
Dec	226	43.8	182	49.6	43.9
1984					
Feb	212	09.5	176	38.7	35.5
Apr	196	42.9	179	00.4	17.7
Jun	184	49.2	180	34.1	04.3
Aug	165	18.9	178	25.7	-13.1
Oct	155	43.5	182	34.2	-26.8
Dec	137	37.6	182	45.5	-45.1



"Farewell, Morning Star, herald of the dawn and quickly come as the Evening Star bringing again in secret her whom thou takest away"
—MELEAGER



THE OUTER PLANETS

The three outer planets used for navigation are Mars, Jupiter and Saturn. The longitudes are presented in the Almanac as GHA values. This obscures the relative constancy of their locations as measured by the SHA value from Aries or as given by the more ancient method of designating geocentric longitudes by use of the Zodiac.

The twelve Houses of the Zodiac, of which Aries is the first, are divided each into 30 degrees. To say that "Saturn is in the twelvth degree of the Eight House" has a definite angular meaning as well as an elegance of expression and, when the symbols are used, brevity.

The data below were taken from a compilation by Raphael for Geocentric Longitudes and Declinations of Neptune, Herschel, Saturn, Jupiter & Mars, from 1900 to 2001, W. Foulsham & Co., London, circa 1900.

A study of such data reveals apparent retrograde movements which confounded some of the ancients and led to the epicyclic movements by which they described the planets. The effect is the result of the sometime nearly parallel paths of the earth and the planets.

MARS Sidereal Period 1.88 years

1984,	Jan	26° of Libra	(7th House),	decl. = 08°S	SHA=154°
	Mar	23° of Scorpio	(8th House),	decl. = 17°S	=127
	May	24° of Scorpio	(8th House),	decl. = 18°S	=126
	Jul	13° of Scorpio	(8th House),	decl. = 17°S	=137
	Sep	08° of Sagit.	(9th House),	decl. = 24°S	=112
	Nov	19° of Capr.	(10th House),	dec. = 24°S	=101
1985,	Jan	05° of Pisces	(12th House),	decl. = 10°S	= 25
	Mar	20° of Aries	(1st House),	decl. = 8°N	=340
	May	04° of Gemini	(3rd House),	decl. = 22°N	=278
	Jul	15° of Cancer	(4th House),	decl. = 24°N	=255
	Sep	25° of Leo	(5th House),	decl. = 14°N	=215
	Nov	03° of Libra	(7th House),	decl. = 00°S	=177

Jupiter Sidereal Period 11.86 years

1984	Jan	26° of Sagit.	(9th House),	decl. = 23°S	SHA= 94°
1985	Jan	22° of Capr.	(10th House),	decl. = 10°S	= 68
1986	Jan	18° of Aquar.	(11th House),	decl. = 16°S	= 42
1987	Jan	18° of Pisces	(12th House),	decl. = 06°S	= 12
1988	Jan	21° of Aries	(1st House),	decl. = 07°N	=339

Saturn Sidereal Period 29.46 years

1984	Jan	14° of Scorpio	(8th House),	decl. = 14°S	SHA=136°
1988	Jan	26° of Sagit.	(9th House),	decl. = 22°S	= 94
1992	Jan	06° of Aquar.	(11th House),	decl. = 19°S	= 54
1996	Jan	19° of Capr.	(10th House),	decl. = 06°S	= 71

WIND, WEATHER AND STORMS

The navigator is perpetually at the mercy of the weather which when favorable accelerates and pleasures his journey and when unfavorable perils his life and vessel. Consequently, there have developed over the ages a store of methods by which it is attempted to predict the weather. These are not always successful, as evidenced by the recent case of the tall sailing ship lost near Bermuda while attempting a sailing record. It is thought she went down in a sudden gust with full sails.

The normal background for wind and weather is presented in an exposition in the American Merchant Seaman's Manual, by F. Cornell and A. Hoffman, Cornell Maritime Press, 1942. The Earth's surface receives the greatest heat just north of the equator in a narrow band of latitudes called the Doldrums or Equatorial Belt of Calms which lies between two tradewind belts. The Doldrums is an area of low pressure, light variable rains, squalls and thunderstorms. As Cornell puts it, "the air, so warmed, rises and a low barometric pressure is continuous. Colder air from the higher pressure air to the North and South flows in and the trade winds are the result. If it were not for the Earth's rotation the trades would be north and south, but the Earth at latitudes away from the equator has a lower linear velocity. Consequently, the air in the higher latitudes takes the lower velocity of the Earth. The air tends to lag as it moves toward the equator where there is higher velocity. The rotation of the Earth being West to East, an easterly component is given to the air movements and the Trade Winds become Northeast in the Northern Hemisphere and Southeast in the Southern Hemisphere."

A further visualization can be made of the phemomena along the lines of fluid dynamics. There is a narrow band of warm, stagnant air rotating West to East. Along each side from the poles there is cold air flow toward this band; frictionally dragged along with it while dissipating its velocity and cooling which again carries it toward the poles. Such a movement is basically circular, but also three dimensioned and layered because of the perpetual thinness and coldness of the upper air. There are added turbulences and vortices caused by unequal heating over land and sea masses and by cloud cover which, when barometric differences become excessive, develop into cyclonic storms. In the Northern Hemisphere these cyclones rotate counterclockwise and move in a northern arc initially toward the west. In the Southern Hemisphere the cyclones rotate clockwise and move in a southern arc initially toward the west. At high altitudes toward the poles high velocity jet streams develop which affect aircraft true speed.

EPILOGUE

A final word of caution: Observations, data, charts and position determinations are fallible. The navigator carries the fate of the vessel in his hands.

Two things are critical: position and course. The navigator should always refer back to the last confirmed position and to the probable excursion from that base in time, distance and direction before judging his present position to be correct.

Choice of the course should consider prevailing winds, tides and currents as well as hazardous shoals, reefs, islets and, in the case of aircraft, peaks and towers. The chart furnished the navigator may lack notation of important features. There is the recent case of an aircraft bound from Madrid to Bilbao which crashed on a "hill" of some 4,000 feet on which was a TV antenna, neither of which was on his chart.

Especially, be wary of charts in less well traveled lanes. Topography does change; sometimes with startling suddenness as the result of volcanic elevation of land from the bottom of the ocean, or with equal suddenness their sinking and disappearance.

Further, there are areas in which it is hazardous to operate a vessel because of possible piracy or hostile action by foreign nationals. Unfortunately, this aspect covers such large areas as the Carribean, the West Coast of Africa and the South Pacific.

Regrettably, there is a history of incorrect positioning by responsible mariners. For example, in 1739 the French explorer Bouvet sighted an island 1500 miles southwest of the Cape of Good Hope, some five miles in diameter and partly covered with a glacier. Unsucessful searches were made for it by the noted explorer Captain James Cook in 1772 and 1775 as well as by Captain Furneaux in 1774. In 1808 two sealers reported sighting the Island and in 1822 Captain Benjamin Morell claimed to have landed on the island. Two British expeditions in 1843 and 1845 failed to find the island. Other expeditions were made with varying confirmation or denials. It is now definitely known to exist.

The prudent navigator takes advantage of every opportunity to confirm the accuracy of his position and to accertain that his course, as indicated by charts, radar, soundings or whatever means are at his disposal, will bring his vessel safely home.

BON VOYAGE

J. Slocum

REFERENCES

1 The Experts' Book of Boating, Edited by Ruth Brindze, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1960

2 The Nautical Almanac for the Year 1976, Nautical Almanac Office, U. S. Naval Observatory, Washington, D. C.

3 American Practical Navigator, Originally by Nathaniel Bowditch, United States Hydrographic Office, No. 9, U. S. Government Printing Office, Washington, 1936.

4 Navigation and Nautical Astronomy, Originally by Cmdr. Benjamin Dutton, United States Naval Institute, Annapolis, Md., 1942

5 Plane and Spherical Trigonometry, L. M. Kells, W. F. Kern, and J. R. Bland, McGraw-Hill Book Company, Inc., New York, 1940

6 Air Navigation, Technical Manual No. 1-205, War Department, Washington, November, 1940

7 Astronomy, Robert H. Baker, D. Van Nostrand Company, Inc., New York, 1938

8 Air Transport Navigation, P. Redpath, J. Coburn, and E. Schuett, Pitman Publishing Company, New York, 1943

9 Elementary Avigation, L. E. Moore, D. C. Heath and Company, Boston, 1943

10 Manual of Astronomy, Charles A. Young, Ginn and Company, Boston, 1902

11 American Merchant Seaman's Manual, F. Cornell and A. Hoffman, Cornell Maritime Press, New York, 1942

12 New Handbook of the Heavens, H. J. Bernard, D. Bennet and H. S. Rice, McGraw-Hill Book Company, New York, 1948

13 The Yachtsman's Modern Navigation and Practical Pilotage, L. Luard, Imray, Laurie Norie and Wilson, Ltd., London, 1932

14 Introduction to Astronomy, Dean B. McLaughlin, Houghton Mifflin Company, Boston, 1961

15 The Yachtsman's Vade Mecum, Peter Heaton, Adam & Charles Black, London, 1969

MARITIME POSITIONS

East Coast of North America		
Novia Scotia, Cape George Lighthouse	45°52'42"N	61°53'30"W
Massachusetts, Nantucket, Great Point	41 23 24	70 02 46
Florida, Cape Canaveral Lighthouse	28 27 37	80 32 37
Mexico, Vera Cruz, Blanquilla Reef	19 13 40	96 06 05
Panama, Cape Toro Lighthouse	09 21 52	82 12 42
West Coast of North America		
Alaska, Port Barrow	71 23 00	156 18 00
Washington, Grays Harbor Lighthouse	46 53 19	124 06 57
California, San Francisco, Goat Island Lth.	37 48 28	122 21 41
California, Los Angeles, Pt. Fermin Lighthouse	33 42 20	118 17 35
South America		
Brazil, Pernambuco, Recife Lighthouse	08°03'22"S	34°52'05"W
Argentina, Buenos Aires, Customhouse	34 36 30	58 22 14
" Arenas Point, Lighthouse	53 08 30	68 13 00
Chile, Valparaiso, Playa Ancha Lths.	33 01 08	71 38 52
Equador, Guayaquil Gulf, Arena Pt., Lths	03 01 45	80 06 50
Islands in Atlantic Ocean		
Canary Islands, Teneriffe, Teno Pt., Lths	28 20 40N	16 55 40
Cape Verde Island, Brava I., Ponta Jalunga L.	14 53 00	24 31 00
Atlantic Coast of Europe		
Great Britian, Lands End, Longships Light	50 04 10	05 44 45
Spain, Gibralter, Europa Pt. Lths	36 06 25	05 20 42

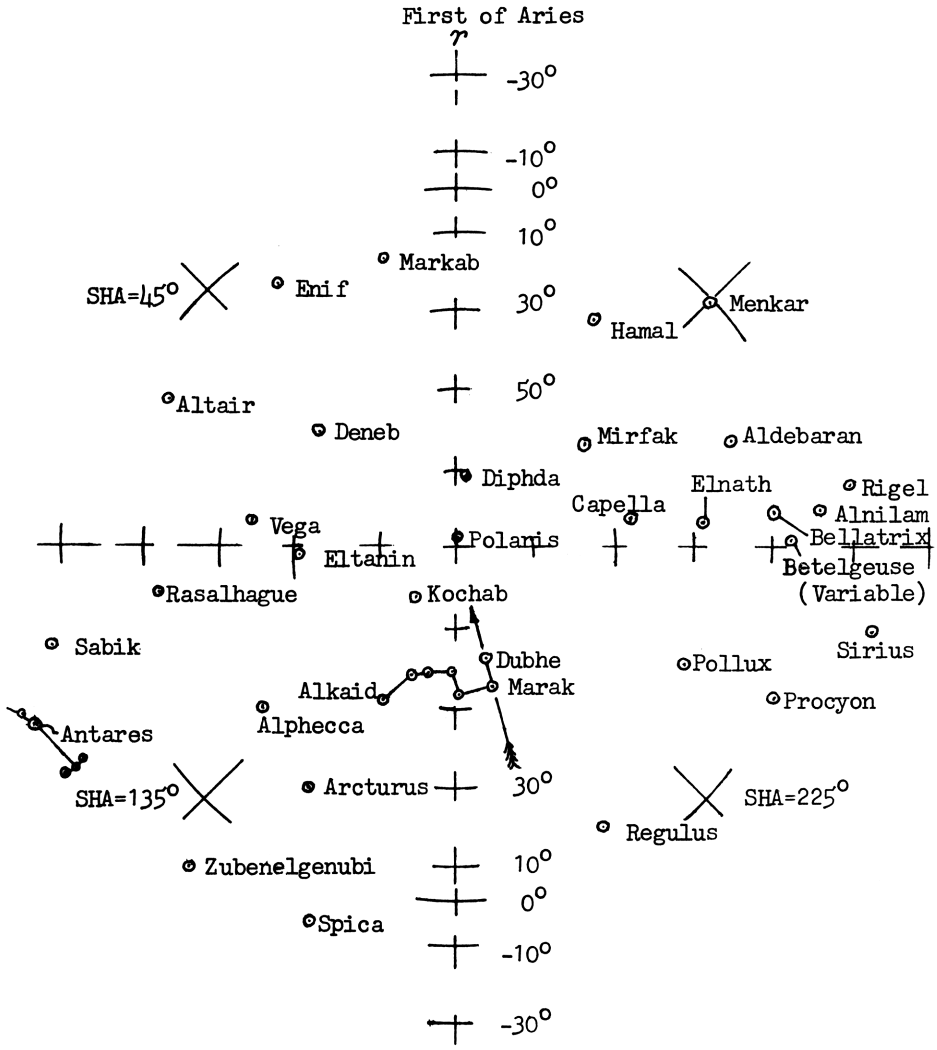
NAVIGATION STARS

July 1976

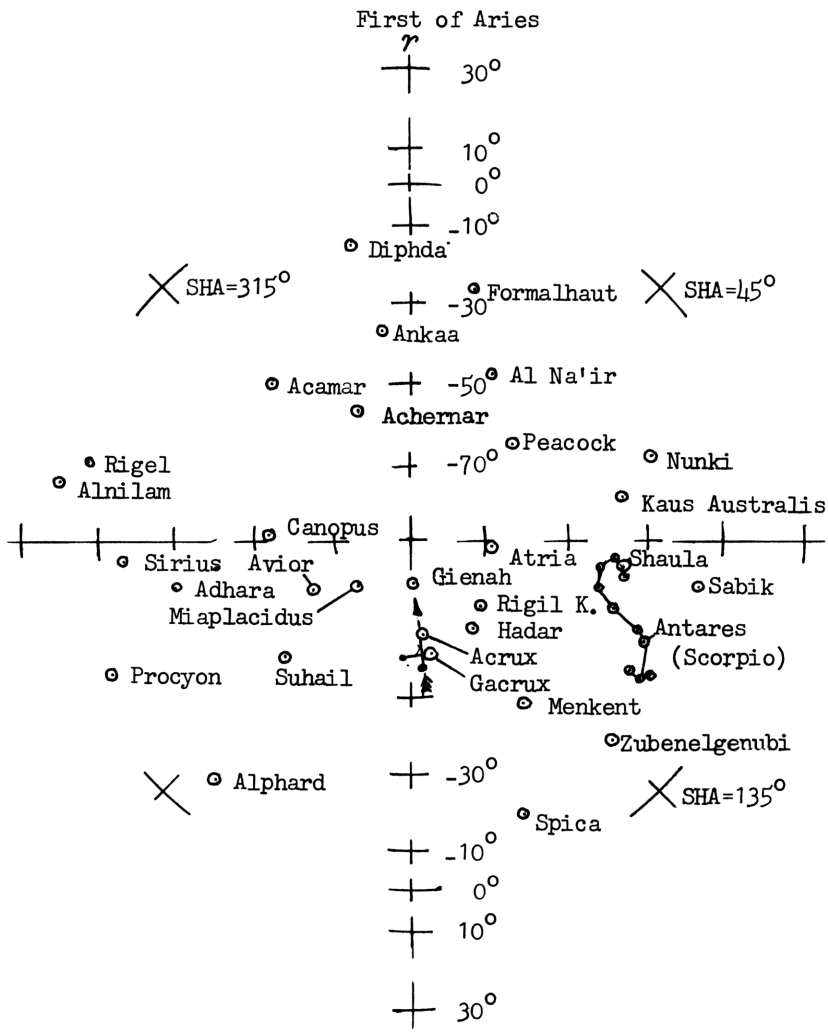
No.	Constellation	Name of Star	Magnitude	Sidereal Hour Angle	Declination
		Polaris		327° 39'	N 89° 09'.5
57	Pegasi	Markab	2.6	14° 05'	N 15° 05'
56	Piscis Aust	Fomalhaut	1.3	15 53	S 29 45
55	Gruis	Al Na'ir	2.2	28 18	S 47 04
54	Pegasi	Enif	2.5	34 14	N 09 46
53	Cygni	Deneb	1.3	49 50	N 45 12
52	Pavonis	Peacock	2.1	54 02	S 56 49
51	Aquilae	Altair	0.9	62 35	N 08 49
50	Sagittarii	Nunki	2.1	76 32	S 26 20
49	Lyrae	Vega	0.1	80 58	N 38 46
48	Sagittarii	Kaus Australis	2.0	84 20	S 34 24
47	Draconis	Eltanin	2.4	90 60	N 51 30
46	Ophiuchi	Rasalhague	2.1	96 32	N 12 35
45	Scorpii	Shaula	1.7	96 59	S 37 05
44	Ophiuchi	Sabik	2.6	102 44	S 15 42
43	Triang. Aust.	Atria	1.9	108 27	S 68 59
42	Scorpii	Antares	1.2	113 00	S 26 23
41	Coronae Bor.	Alphecca	2.3	126 34	N 26 48
40	Ursae Minoris	Kochab	2.2	137 20	N 74 15
39	Librae	Zubenelgenubi	2.9	137 36	S 15 56
38	Centauri	Rigil Kent	0.1	140 30	S 60 44
37	Bootis	Arcturus	0.2	146 21	N 19 18
36	Centauri	Menkent	2.3	148 40	S 36 15
35	Centauri	Hadar	0.9	149 27	S 60 16
34	Ursae Majoris	Alkaid	1.9	153 21	N 49 26
33	Virginis	Spica	1.2	159 00	S 11 02
32	Ursae Majoris	Alioth	1.7	166 45	N 56 05
31	Crucis	Gacrux	1.6	172 32	S 56 59
30	Crucis	Acrux	1.1	173 41	S 62 58

NAVIGATION STARS

No.	Constellation	Name of Star	Magnitude	July 1976 Sidereal Hour Angle	Declination
29	Corvi	Gienah	2.8	176° 21'	S 17° 25'
28	Leonis	Denebola	2.2	183 02	N 14 42
27	Ursae Majoris	Dubhe	2.0	194 26	N 61 52
26	Leonis	Regulus	1.3	208 13	N 12 05
25	Hydrae	Alphard	2.2	218 23	S 08 33
24	Carinae	Miaplacidus	1.8	221 46	S 69 37
23	Velorum	Suhail	2.2	223 13	S 43 20
22	Carinae	Avior	1.7	234 30	S 59 26
21	Geminorum	Pollux	1.2	244 01	N 28 05
20	Canis Minoris	Procyon	0.5	245 28	N 05 17
19	Canis Majoris	Adhara	1.6	255 34	S 28 56
18	Canis Majoris	Sirius	-1.6	258 58	S 16 41
17	Carinae	Canopus	-0.9	264 08	S 52 40
16	Orionis	Betelgeuse	0.1 to 1.2	271 31	N 07 24
15	Orionis	Alnilam	1.8	276 14	S 01 13
14	Tauri	Elnath	1.8	278 47	N 28 35
13	Orionis	Bellatrix	1.7	279 01	N 06 20
12	Aurigae	Capella	0.2	281 15	N 45 58
11	Orionis	Rigel	0.3	281 38	S 08 14
10	Tauri	Aldegaran	1.1	291 20	N 16 28
9	Persei	Mirfak	1.9	309 19	N 49 47
8	Ceti	Menkar	2.8	314 44	N 03 60
7	Eridani	Acamar	3.1	315 39	S 40 24
6	Arietis	Hamal	2.2	328 32	N 23 21
5	Eridani	Achernar	0.6	335 47	S 57 21
4	Ceti	Diphda	2.2	349 23	S 18 07
3	Cassiopeiae	Schedar	2.5	350 12	N 56 25
2	Phoenicis	Ankaa	2.4	353 43	S 42 26
1	Andromedae	Alpheratz	2.2	358 12	N 28 58



PRINCIPAL STARS OF NORTHERN HEMISPHERE



PRINCIPAL STARS OF SOUTHERN HEMISPHERE

APPENDIX I

ALGEBRAIC CALCULATOR PROGRAMS

Algebraic Program 1. **VECTOR COMPONENTS AND SUM**

Loc.	Name	Value	Dimension
00	Azimuth 1	68.	Degrees
01	Speed 1	240.	
02	Azimuth 2	165.	
03	Speed 2	15.	

Latitude change, $V_{NS} = V_1 \cos A_1 + V_2 \cos A_2$

Departure change, $V_{EW} = V_1 \sin A_1 + V_2 \sin A_2$

Combined Velocity = $\mathrm{sqrt}(V_{NS} + V_{EW})$

00	---	
01	(	
02	(	
03	RCL 01	
04	X	
05	RCL 00	
06	COS	
07	)	
08	+	
09	(	
10	RCL 03	
11	X	
12	RCL 02	
13	COS	
14	)	
15	)	
16	STOP	V_{EW}, 226.406
17	STO 04	
18	(	
19	(	
20	RCL 01	
21	X	
22	RCL 00	
23	SIN	
24	)	
25	+	
26	(	
27	RCL 03	
28	X	
29	RCL 02	
30	SIN	
31	)	
32	)	
33	STOP	V_{NS}, 75.417
34	SQUARE	
35	+	
36	RCL 04	
37	SQUARE	
38	EQUAL	
39	SQRT	
40	STOP	V, 238.63

Algebraic Program 2. **VECTOR HEADING**

Loc.	Name	Value	Dimension
03	scratch	----	
04	Latitude, V_{NS}	77.417	Degrees
05	Departure, V_{EW}	226.406	
06	C_1	90.	
07	Test	0.	

First Quadrant: $A_1 = \tan^{-1}(\mathrm{abs}(V_{EW}/V_{NS}))$

Second Quadrant: $A_2 = 180 - A_1$

Third Quadrant: $A_3 = 180 + A_1$

Fourth Quadrant: $A_4 = 360 - A_1$

00	---		22	RCL 03	
01	RCL 05		23	EQUAL	
02	DIVIDE		24	STOP	A_4
03	RCL 04		25	LBL 1	
04	EQUAL		26	RCL 05	
05	ABS		27	IF(x>t)	
06	TAN^{-1}		28	GOTO 03	
07	STO 03		29	RCL 06	
08	RCL 04		30	X	
09	IF(x<t)		31	2	
10	GOTO 1		32	EQUAL	
11	RCL 05		33	-	
12	IF(x<t)		34	RCL 03	
13	GOTO 2		35	EQUAL	
14	RCL 03		36	STOP	A_2
15	STOP	A_1,	37	LBL 2	
16	LBL 2	71.57deg	38	RCL 06	
17	RCL 06		39	X	
18	X		40	2	
19	4		41	EQUAL	
20	EQUAL		42	+	
21	-		43	RCL 03	
			44	EQUAL	
			45	STOP	A_3

Algebraic Program 3. **TRAVERSE COMPONENT SUMMATION**

Loc.	Name	Value	Dimension
00	scratch	---	
01	"		
02	"		
03	"		
04	"		
05	"		
Test	t (scratch)		

Latitude component = Distance cos(Heading)

Departure component = Distance sin(Heading)

```
00  ---
01  0
02  STO 00
03  STO 01
04  STO 05
05  STOP   Enter n=2 legs
06  STO 04
07  RCL 05
08  GOTO 1
09  LBL 1
10  STOP   Enter dist
11  STO 02      n=1,dist=240
12  X           n=2,dist= 15
13  STOP   Enter heading
14  STO 03      n=1,head=68°
15  COS         n=2,head=165°
16  EQUAL
17  SUM 00
18  RCL 03
19  SIN
20  X
21  RCL 02
22  EQUAL
23  SUM 01
24  1
25  SUM 05
26  RCL 05   (=t)
27  x (transposes) t
28  RCL 04   (=x)
29  IF (x ≠ t)
30  GOTO 1
31  RCL 00
32  STOP       Lat. = 75.42nm
33  SQUARE
34  +
35  RCL 01
36  STOP       Departure=226.41nm
37  SQUARE
38  EQUAL
39  SQRT
40  STOP       Dist.=238.64nm
```

Algebraic Program 4 **TRUE SPEED FROM SIGHTED OBJECT**

Loc.	Name	Value	Dimension
00	Initial Angle, A1	70.	Degrees
01	Final Angle, A2	40.	"
02	Normal Distance, DN	5000.	Feet
03	Time lapse, dt	14.	seconds
04	C1 (=3600/6080)	0.5921	knots(ft/sec)

True distance travelled, L1 = (DN/tan(A2)) - (DN/tan(A1))

True speed, VT = C1 x (L1/dt)

```
00  ---
01  RCL 02
02  DIVIDE
03  RCL 01
04  TAN
05  EQUAL
06  STO 05
07  RCL 02
08  DIVIDE
09  RCL 00
10  TAN
11  EQUAL
12  CHG SGN
13  +
14  RCL 05
15  EQUAL
16  DIVIDE
17  RCL 03
18  EQUAL
19  X
20  RCL 04
21  EQUAL
22  STOP        VT, 175kn
```

Algebraic Program 5. **COURSE FOR INTERCEPT MISSION**
Part 1 of 2

Loc.	Name	Value	Dimension
00	Distance to object, L1	12.	nm
01	Speed of object, V2	10.	knots
02	Speed of vessel, V3	15.	"
03	Azimuth of object, A1	15.	degrees
04	Path of object, A2	75.	"
05	C1	180.	"

Angle of object from vessel to point of interception,

$$A3 = A1 + (180-A2)$$

Elements of quadratic equation: $a = V3^2 - V2^2$

$c = -L1^2$ $\qquad b = 2\ L1\ V2\ \cos(A3)$

Time for intercept, $t = a\ t_2 + b\ t + c = 0$

$$t = (-b \pm \mathrm{sqrt}(b^2 - 4\ a\ c))/(2\ a)$$

```
00  ---                          16  STOP      A3, 120. deg.
01  (                            17  COS
02  RCL 02                       18  X
03  SQUARE                       19  (
04  -                            20  2
05  RCL 01                       21  X
06  SQUARE                       22  RCL 01
07  )                            23  X
08  STOP        a, 125.          24  RCL 00
09  (                            25  )
10  RCL 05                       26  EQUAL
11  -                            27  STOP      b, -120.
12  RCL 04                       28  RCL 00
13  +                            29  SQUARE
14  RCL 03                       30  CHG SGN
15  )                            31  STOP      c, -144.
```

Solve for t: $t = (120. \pm \mathrm{SQRT}((-120.)^2 - (4 \times 125. \times (-144))))/(2 \times 125.)$

$= 1.66$hours

Algebraic Program 5. COURSE FOR INTERCEPT MISSION
Part 2 of 2

Loc.	Name	Value	Dimension
00	Time to intercept, t	1.66	hours
01	Distance to object, L1	12.	nm
02	Speed of object, V2	10.	knots
03	Speed of vessel, V3	15	"
04	Azimuth of object, A1	15.	degrees
05	scratch	---	
06	scratch	---	
07	scratch	---	

Object distance to intercept point, L2 = V2 t

Vessel distance to intercept point, L3 = V3 t

Angle of vessel from object to intercept point,

$$A4 = \cos^{-1}((L3^2 + L1^2 - L2^2)/(2\ L3\ L1))$$

Vessel course for intercept, A5 = A4 + A1

00	---	
01	RCL 02	
02	X	
03	RCL 00	
04	EQUAL	
05	STO 05	L2, 16.60nm
06	RCL 03	
07	X	
08	RCL 00	
09	EQUAL	
10	STO 06	L3, 24.90nm
11	SQUARE	
12	STO 07	
13	RCL 01	
14	SQUARE	
15	SUM 07	
16	RCL 05	
17	SQUARE	
18	CHG SGN	
19	SUM 07	
20	RCL 07	
21	DIVIDE	
22	(	
23	2	
24	X	
25	RCL 06	
26	X	
27	RCL 01	
28	)	
29	EQUAL	
30	COS^{-1}	A4, 35.18deg
31	+	
32	RCL 04	
33	EQUAL	
34	STOP	A5, 50.18deg

Algebraic Program 6. **GREAT CIRCLE DISTANCE**

Loc.	Name	Value	Dimension
00	Latitude A	33.70	degrees
01	Longitude A	118.30	"
02	Latitude B	21.27	"
03	Longitude B	157.82	"

Dist. = $\cos^{-1}$((sin LatA sin LatB) +
(cos LatA cos LatB cos(LongA - LongB))

```
00  ---
01  RCL
02  01
03  -
04  RCL
05  03
06  EQUAL
07  COS
08  X
09  (
10  RCL
11  00
12  COS
13  X
14  RCL
15  02
16  COS
17  )
18  EQUAL
19  +
20  (
21  RCL
22  00
23  SIN
24  X
25  RCL
26  02
27  SIN
28  )
29  EQUAL
30  INVERT
31  COS
32  X
33  6
34  0
35  EQUAL
36  STOP      Dist.,2215.9nm
```

Algebraic Program 7. **ALTITUDE BY ASTRONOMICAL TRIANGLE**

Loc.	Name	Value	Dimension
00	Latitude, L	45.	degrees
01	Declination, d	22.76	degrees (North)
02	Meridian Angle, t	13.99	degrees

Altitude, $h = \sin^{-1}((\sin L \sin d) + (\cos L \cos d \cos t))$

```
00  ---
01  RCL 00
02  SIN
03  X
04  RCL 01
05  SIN
06  EQUAL
07  STO 03
08  (
09  RCL 00
10  COS
11  X
12  RCL 01
13  COS
14  X
15  RCL 02
16  COS
17  )
18  +
19  RCL 03
20  EQUAL
21  SIN-1
22  STOP        h, 64.99°
```

Example 8. **LATITUDE BY PHI-1/PHI-2 METHOD**
Part 1 of 2   Case Number 3, Vega

Loc.	Name	Value	Dimension
00	Altitude, h	14.5000	degrees
01	Declination, d	38.7590	degrees (North)
02	Meridian Angle, t	75.9655	degrees
03	C1	180	
04	C2	90	
05	Bearing	50.9069	degrees (North&East)
06	Equatorial Direct.	75.9655	degrees
07	Scratch (Phi-1)	---	
08	Scratch (Phi-2)	---	
09	Scratch (AP)	---	
10	C1	270	

AP = $\sin^{-1}$(cos d sin t)   Phi-1=$\cos^{-1}$(sin h/cos AP)
Phi-2=$\sin^{-1}$(sin d/cos AP)
but if Bearing<90 then Phi-1 = -Phi-1
and if Eqtdr<90 and Eqtdr<270 then Phi-2=180-Phi-2
and if d<0 then Phi-2=-Phi-2

Find AP:
00 ---
01 (
02 RCL
03 02
04 SIN
05 X
06 RCL
07 01
08 COS
09)
10 INVERSE
11 SIN
12 STO
13 09   AP, 49.1585deg
Find Phi-2:
14 (
15 RCL
16 01
17 SIN
18 DIVIDE
19 RCL
20 09
21 COS
22)
23 INVERSE
24 SIN
25 STO
26 08   Phi-2, 73.1926deg

Find Phi-1:
27 (
28 RCL
29 00
30 SIN
31 DIVIDE
32 RCL
33 09
34 COS
35)
36 INVERSE
37 COS
38 STO
39 07   Phi-1, 67.4894deg
Test for Phi-1 correction:
40 A
Test for Phi-2 correction:
41 B
Find Latitude:
42 RCL
43 07
44 +
45 RCL
46 08
47 EQUAL
48 STOP Lat., 39.3172°

Example 8. **LATITUDE BY PHI-1/PHI-2 METHOD**
Part 2 of 2

```
60  Label
61  A
62  RCL
63  04
64  exchange x for test
65  RCL
66  05
67  INVERSE
68  If(x>t)
69  C
70  Inverse Subroutine

80  Label
81  C
82  RCL
83  07
84  CHG SGN
85  STO
86  07
87  Inverse Subroutine

130 Label
131 D
132 RCL
133 10
134 exchange x for test
135 INVERSE
136 If(x>t)
137 E
138 Inverse Subroutine

90  Label
91  B
92  RCL
93  04
94  exchange x for test
95  RCL
96  06
97  INVERSE
98  If(x>t)
99  D
100 0
101 exchange x for test
102 INVERSE
103 If(x>test)
104 A'
105 Inverse Subroutine

110 Label
111 E
112 RCL
113 03
114 -
115 RCL
116 08
117 EQUAL
118 STO
119 08
120 Inverse Subroutine

150 Label
151 A'
152 RCL
153 08
154 CHG SGN
155 STO
156 08
157 Inverse Subroutine
```

CELESTIAL NAVIGATION