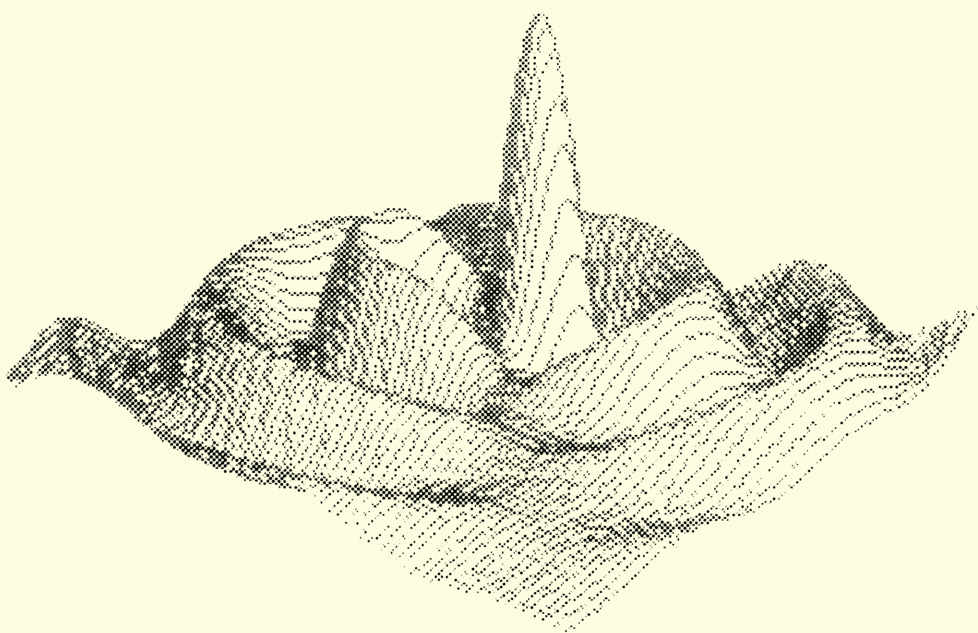
CURVE FITTING FOR PROGRAMMABLE CALCULATORS

THIRD EDITION



by

William M. Kolb

THIRD EDITION

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CURVE FITTING

FOR

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William M. Kolb

To Hiroko

PREFACE

This is a practical sourcebook of curve fitting formulas for users of programmable calculators and micro-computers. All of the essential information needed to fit data to the most common curves is provided in a form that minimizes calculations and makes programming easy. The curves selected for this volume consist primarily of models with one- and two-independent variables that do not require iterative solutions. A basic introduction is included for the novice user of statistical models while the more intrepid explorer may find the sections on transformation, derivation, decomposition and substitution valuable for developing custom curve fitting routines. Owners of the Hewlett-Packard HP-41C programmable pocket computer can make immediate use of many models presented with the program and barcode included. Similar programs for other popular handheld computers are also provided.

The figures used to illustrate the text were drawn on an HP-85 desktop computer with a modified version of the computer's Standard Pac. The cover design was also done on the HP-85 with a hidden-line plotting algorithm.

It is a pleasure to acknowledge the contributions of Maurice Swinnen and Rick Conner to this edition. Maurice prepared the multiple curve fitting routines for the TI-59 under severe running time and storage contraints without sacrificing user friendliness or versatility. Rick prepared an excellent BASIC language version of the HP-75 program that fits nineteen curves and uses many of the special features of the PC-1500. I am especially indebted to Robert, Jeanne, Michael and Hiroko for their continuing enthusiasm and support in revising and updating previous editions.

January 1984 William M. Kolb

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"If you know it to one significant place, then you know it!"

G. S. Shostak

INTRODUCTION

This book presents a wide variety of curve fitting formulas intended to help anyone with an interest in data analysis. It can be used to fit a specific curve to your data or as a guide in selecting appropriate models. The curves consist of equations with one-, two- and three-independent variables that can be solved simply and without complicated mathematics. A number of special-purpose curve fitting routines are included to help solve unusual problems.

The equations for these models are arranged to minimize the number of computations required and are therefore ideally suited to the programmable scientific calculator. Register numbers are used in a consistent manner for all formulas which makes it easy to experiment with different models once the data have been entered. If your calculator has less than 90 data registers available, however, you may find it necessary to use different register assignments for some equations. Graphs and general descriptions of the various equations are included to help with model selection and an example is worked out for each equation to assist in debugging programs.

The book is divided into three major sections. This first section is a general discussion of the curve fitting process intended as a primer for the beginner. The second section contains various statistical models and the formulas necessary for estimating the coefficients. The final section is a series of appendixes that will help you program these models and develop new ones.

The remainder of this introduction provides some insight into the techniques used to formulate a model and the basics of regression analysis. The uninitiated user of statistical models should review this section carefully before drawing conclusions about the significance of any model.

1. REGRESSION

If you were asked to estimate the heights of various trees in a forest, how would you go about it. One way is to develop a mathematical relationship between the height of the tree and its diameter. The height of a particular tree could then be estimated from a simple measurement of its diameter. In this case there would be one independent variable (diameter) and one dependent variable (height). Unfortunately, the relationship between diameter and height may not work particularly well in all instances. We might conclude that other independent variables are required to take into account such things as annual rainfall, forest density and type of tree.

One technique used to formulate a statistical relationship among such variables is regression (the term regression is applied for historical reasons). In our forestry problem, actual data consisting of diameters and heights for various trees would be regressed to produce the best fitting line through all of the points.

The usual meaning of best fit is the line that minimizes the sum of the squares of the vertical distances from each data point to the line of regression: the term least squares describes this fitting method. Other regressions, however, could be developed where best fit defines a line that minimizes the sum of all perpendicular distances between the regression line and the data points, or the line that minimizes the square of vertical distances to transformed data points, e.g., the log of height (more on this in the next section).

The most common form of regression is a straight-line fit. In this case, we are trying to find the coefficients A and B that result in a least squares vertical deviation from the straight line: $y = A + Bx$. Actually the values obtained for A and B are only estimates and are sometimes written A′ and B′ to remind us of this. When we calculate y from a value of x using these coefficients, it too may be written y′. Note that the coefficients derived in this regression process are specifically designed to predict y from a knowledge of x: the values obtained for A and B are generally not the best ones to use if we want to estimate x from a knowledge of y.

The more data points we have in our sample, the closer our estimate of A and B will be to the expected values. If the data sample is fairly small, as it frequently is, we should not be tempted to use more than two or three significant places for A and B. Even if A and B are determined with great precision from ample data, we should not conclude that x and y are actually related by the equation $y = A + Bx$. The values obtained for A and B merely reflect our assumption that y can be predicted from x using a straight-line approximation.

2. TRANSFORMATION

Often we can plot our data and see immediately that the assumption of a straight-line fit is not correct. Our intuition or experience may suggest a totally different relationship between y and x such as $(y-A)(x+B)=1$ or $y=kc^x$. Rather than invent a new regression for each case, it is common practice to transform the expression into one that has the properties of a straight line. Consider the expression $y=kc^x$, for example. By taking the natural logarithm of both sides, we obtain $\ln(y)=\ln(k)+ x\ln(c)$. Since the logarithm of a constant is a constant, we can rewrite the last expression as $\ln(y)=A+Bx$ where $A=\ln(k)$ and $B=\ln(c)$. Now we have a dependent variable on the left that is linearly related to an independent variable on the right which means we can apply a straight-line fit. The transform applied in this case is the natural logarithm, and instead of using y in the regression formula, we input $\ln(y)$. The values for coefficients A and B, however, must be transformed back using anti-logs to obtain estimates for k and c.

It should be noted that the transformation applied in this example produces a least squares regression of $\ln(y)$, rather than y. The coefficients k and c obtained in this manner are not necessarily the same ones that would be obtained by an actual least squares regression of y on x. The difference will be small, however, if the fit is good. Transforms thus provide a convenient and relatively uncomplicated method for estimating coefficients in more complex models without resorting to unwieldy iterative techniques.

3. MULTIPLE LINEAR REGRESSION

Many phenomena cannot be expressed in terms of a simple linear model. What happens, for example, when there are several variables? We would still like a regression technique that minimizes the sum of squared deviations between the regression line and the actual data. One such linear regression model for a dependent variable, w, and three independent variables, can be written as: $w = A + Bx + Cy + Dz$. Y and z can represent almost any independent variable we want including a function of x such as $1/x$ or x^2. Thus an easy way to fit data to a cubic equation would be to substitute x^2 for y, and x^3 for z. Where the input of our regression formula calls for x, we input x. Where it calls for y and z, we input x^2 and x^3, respectively. After the coefficients A, B, C and D are computed, we will have a best fit curve for the expression

$$y = A + Bx + Cx^2 + Dx^3$$

More complicated equations can be fitted by transforming the data to an expression that has the form of a multiple linear regression. This process can be used to develop models for probability distribution functions such as the familiar bell-shaped (normal) curve. Unfortunately, not all equations can be transformed to a linear model and iterative methods must often be employed to fit data to a curve.

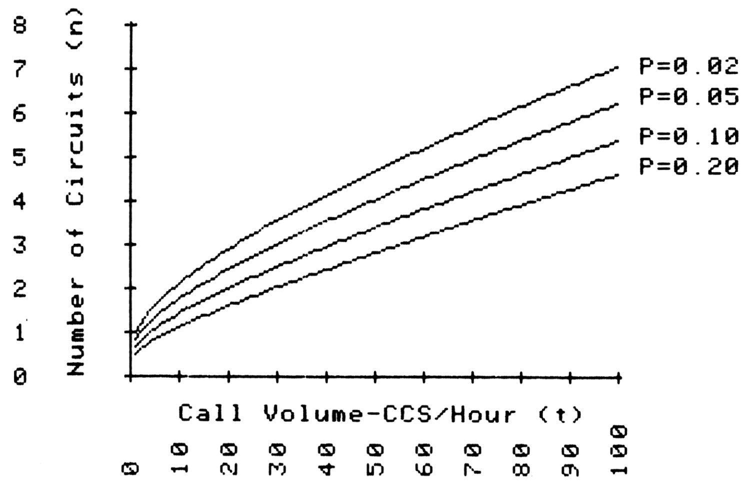
Sometimes an alternative to iterative methods is to divide the curve into two or more pieces, each of which can be approximated by a separate regression equation. If it is desirable to join these curves together precisely, we need to specify a particular point through which one or both curves must pass. Several regressions are included in this book to provide a least squares fit through such a point. Others can be developed as needed.

The general technique for joining two curves consists of moving the origin (0,0) to the desired point (h,k) and then forcing one or both regression curves through the origin. For a simple linear model, merely subtracting h from each x value and subtracting k from each y value as the data is entered will move the origin to (h,k). The Generalized 2nd Order and Generalized 3rd Order equations are then used to force a regression line through the origin, now located at (h,k). Once the coefficients for this regression model are computed, substitute the value of h for x and the value of k for y and solve for the constant term A in the regression equation.

The underlying assumptions to keep in mind whenever linear regression is used are 1) the independent variables are free of error, 2) the independent variables are not affected by the dependent variable, 3) the data consists of a random sample from the population and not the entire population, and 4) the model accurately describes the data, although it need not correctly define the underlying physical phenomena involved.

4. FITTING A FAMILY OF CURVES

Sometimes it is necessary to find a single equation that describes a family of curves such as those shown in the following figure. A useful technique for doing this involves fitting just one of the curves to determine the general equation that best represents the entire family.



When choosing the general equation, it is important to check the curves at either extreme for goodness of fit. Once a general equation has been selected, each individual curve in the family is fit to that model. In the example shown, the best general equation was found to be

$$n = A\ e^{(B\ +\ \ln\ t)^2/C}$$

If each curve in the family is fit to this model, we obtain the following set of equations for the different values of P:

$$n = 0.726\ e^{(2.79\ +\ \ln\ t)^2/23.8} \quad \text{for } P=0.02$$

$$n = 0.696\ e^{(1.99\ +\ \ln\ t)^2/19.8} \quad \text{for } P=0.05$$

$$n = 0.555\ e^{(1.93\ +\ \ln\ t)^2/18.6} \quad \text{for } P=0.10$$

$$n = 0.414\ e^{(1.78\ +\ \ln\ t)^2/16.9} \quad \text{for } P=0.20$$

The next step is to find a relationship for each of the coefficients appearing in the model. In other words, we are looking for a set of equations that estimate A, B and C based on P. When P=0.02 we need an equation for A that gives A=0.726, when P=.05 we want A=0.696, etc. We likewise need an equation to estimate B so that B=2.79 when P=0.02, and B=1.99 when P=0.05, etc. After curve fitting each coefficient, we might finally decide on the following equations:

$$A = 0.188\ (0.985^{1/P})\ (P^{-0.535})$$

$$B = 1.77\ P^{-0.0023/P}$$

$$C = 18.1 - 8.45P + 0.118/P$$

Given these equations, we can now estimate A, B and C for any value of P in the range from 0.02 to 0.20. We can then substitute these values in the original model along with a value for t in order to estimate n.

5. DECOMPOSITION AND SUBSTITUTION

It is possible to express a generalized regression equation in several different forms by decomposing its coefficients or by substituting other variables for the ones given (this was done earlier when x^2 was substituted for y and x^3 for z). These techniques should always be tried before developing a new regression formula. The following examples illustrate a few of the variations possible with models presented in this book.

General Equation	Alternate Form
$y = ax^b$	$y = 1/(rx^t)$
$y = ab^x$	$y = re^{(sx)}$
$y = ab^x x^c$	$y = rx^s e^{tx}$
$y = ab^x x^c$	$y = r(x/s)^t e^{(x/s)}$
$y = bx$	$y = b/x$
$y = a + bx$	$y = a + b*\arcsin(x)$
$y = a + bx$	$\ln y = a + bx$

The first three examples illustrate substitution of new coefficients for the originals. In the first case, the coefficients are related by $r=1/a$ and $t=-b$. In the second case, $r=a$ and $s=\ln b$. In the third example, $r=a$, $t=\ln b$, and $s=c$. The fourth example involves decomposing or partitioning a coefficient into factors such that $a=r/(s^t)$, $c=t$ and $\ln b=1/s$. The last three examples involve substituting a new variable in the general equation. In the fifth example, the reciprocal of x is substituted for x. In the sixth example, the inverse sin of x is substituted for x and in the last, lny is substituted for y. These techniques can also be applied simultaneously to greatly expand the range of equations that can be fit with just a few basic regression formulas.

6. SCALING

Most programmable calculators are capable of storing only ten digits in a register. This limitation presents a problem if the less significant digits are lost during calculation. The problem can be particularly acute when one variable is much larger than another or when one variable changes much more rapidly than another. There are two techniques that are often used in such cases to maintain maximum accuracy.

The first consists of subtracting a common value from each y value before entering the data. After the coefficients have been determined, this value is then added to the general equation for y. If our data consisted of X,Y pairs such as (10,994), (20,1015), (30,1033), and (40,1057), we could subtract 1000 from each y value and use (10,-6), (20,15), (30,33), and (40,57) to find the regression coefficients. Using a linear fit, the final result would be $(y-1000) = -27 + 2.07x$ which can be rewritten as $y = 973 + 2.07x$. The same result is obtained using the original data but with some loss of accuracy.

The second technique consists of multiplying or dividing by a constant factor before entering the data. This technique would be very useful with the following data: (10,994), (20,1930), (30,3160), and (40,4050). Dividing each y value by 100 gives us (10,9.94), (20,19.3), (30,31.6), and (40,40.5). The best linear fit would then be $(y/100) = -0.660 + 1.04x$ which can be rewritten as $y = 104x - 660$. While this technique could have been used with the data in the previous example, it would not have emphasized the absolute differences in y. On the other hand, if we had subtracted 1000 from each y in the second example, we would still have the problem of some y values being much larger than x. Both of these techniques can be applied to x as well as y. They may also be used simultaneously as long as you remember to rewrite the final equation accordingly.

7. GOODNESS OF FIT

For peace of mind, we would like some assurance that the straight line resulting from our regression analysis is a reasonable approximation or good fit. There are a number of ways to evaluate how good the fit is, each with its own advantages and limitations. Perhaps the most commonly used measure of goodness of fit is the coefficient of determination, RR. (The square-root of RR is called the correlation coefficient.) A useful property of the coefficient of determination is that it applies to any linear regression and may be used to determine which transform produces the best fit.

Another property of RR is that it ranges from 0 to 1: it is 1 when all of the data points (or transformed data points) fall exactly on a straight line, and it is 0 for values of x and y chosen at random. RR has a direct interpretation as well: it is the proportion of the total variation in y explained by the regression line. Thus an RR of 0.80 means that 80% of the observed variation in y can be attributed to variation in x: 20% of the variation in y is unexplained by the regression line. If the data are very noisy (i.e., contain significant random errors), an RR of 0.8 may represent a fairly good fit. It is possible to have a coefficient of determination near 1, however, and not have a good fit if the data have very little noise. Even with a high degree of correlation, we cannot infer that the data actually fit a particular model without assuming something about the distribution of errors in the measurements of y. Moreover, a high RR does not prove causality and does not guarantee that new data will necessarily fit the model.

In general, a model is only valid over the range of input data and should not be used to extrapolate values outside of this range. A linear relationship between age and weight in children, for example, could not be used to accurately predict adult weight. Furthermore, losing weight will not reduce your age no matter how good the fit.

Whenever RR is used to compare one model with another, it should be corrected to eliminate any bias due to the size of the data sample and the number of coefficients being estimated. Appropriate corrections are included for most models in the book. It should be noted that RR is purely a function of the original (or transformed) values of x and y and expresses the strength of the linear relationship between them regardless of the curve being fit. It is not useful for measuring the goodness of fit in special cases such as lines forced through a point.

8. SIGNIFICANCE

We should always select a model for our data on the basis of either theoretical or empirical knowledge. Whenever possible, we should even design the data-collection so that goodness of fit, lack of fit, and measurement errors can all be tested. Unfortunately, it is not always possible to control what data are collected or to completely understand the phenomena involved. In such cases, the model must always be suspect until tests of significance are applied to each of the variables.

For regressions involving several terms, it is possible to determine the correlation between each pair of variables. If two independent variables are highly correlated, we should consider redoing the regression and omitting one of them since it contributes little toward reducing the variance in the dependent variable. If an independent variable exhibits little or no correlation to the dependent variable, we should also consider omitting it from the model.

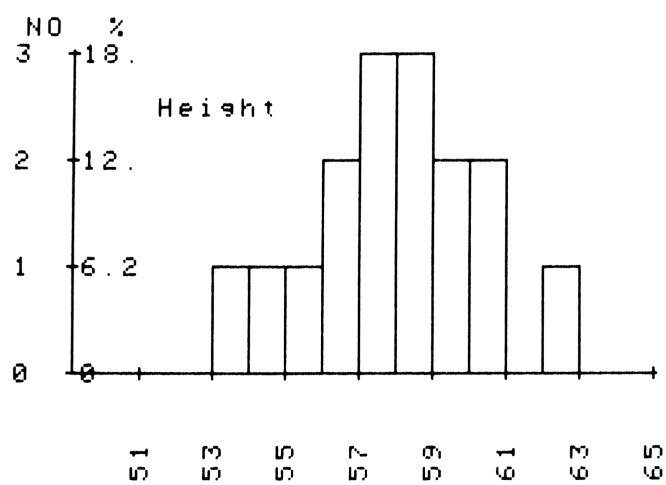
It is usually the case that higher order curves with more variables will produce a better coefficient of determination. It is therefore prudent to determine if the improvement in RR is truly significant before opting to use the higher order curve. There are a number of methods for testing the significance of the coefficient of determination, but one of the most commonly used is the F-test. This is essentially a test of the hypothesis that the dependent variable is linearly related to the independent variables. The F value can be computed from the sample size and RR using the equation F = (n - 2)RR/(1 - RR), where n is the number of data points. F may be adjusted for the number of degrees of freedom (m) as follows: F = (n-m-1)RR/m(1-RR). The interpretation of such tests is beyond the scope of this book, however, and the interested reader should consult one of the more advanced statistical texts listed under references.

8. GETTING STARTED

We shall conclude with an example that illustrates how to fit data to a curve using the information in this book. Suppose the heights of a small group of adults picked at random were measured as follows: 53, 54, 55, 56, 56, 57, 57, 57, 58, 58, 58, 59, 59, 60, 60, 62 and the frequency of each height summarized in a table.

Height (x)	Frequency (y)
53 in.	6.25%
54 in.	6.25%
55 in.	6.25%
56 in.	12.50%
57 in.	18.75%
58 in.	18.75%
59 in.	12.50%
60 in.	12.50%
62 in.	6.25%

We would like to find a curve that fits these data and use it to estimate what percentage of the adult population have a height of 65 inches. We have converted the data to percentages in this case only because we want the final answer to be a percentage. The first step is to plot the data. Since height is the independent (or given) variable, it is plotted along the x-axis while frequency (or the unknown) is plotted along the y-axis. The plot will look something like this



We have the option at this point of scaling the data, if it is desirable. Since all of the x values are clustered between 53 and 62, we could simply subtract 50 from each and remember to add 50 to x in the final regression equation. In this example, the differences between x and y are not quite large enough to be troublesome so we shall not bother to scale them.

The next step is to examine the curve and delete any points that appear to be extremes, or outliers. This should be done with discretion so that the results are not biased by preconceptions of what the curve should look like.

We should also consider how the curve might behave on either side of the data sample we have plotted: does it reach some limit or does it swing upward again at some point? Does the curve pass through the origin? Knowing or assuming these details will make the selection of a model easier. In some instances it may help to make up and use additional data. We may assume, for example, that there are no adults two feet tall or ten feet tall.

Now we look through the graphs for various equations and pick out ones that are similar to the plotted data. When it is difficult to find a curve that looks similar to our plot, we might replot the data with x values on the y-axis and y values on the x-axis to find a better match. In this particular case, it seems reasonable that the data belong to a normal distribution and replotting isn't necessary.

Turning to the normal distribution, the first thing we find is a list of summation terms that are required. The first sum (R16) is simply the total of all the x values. The second sum (R17) involves squaring each x value before adding them together. The fourth sum requires taking the natural logarithm of each y value and then adding the logarithms together. When each of these summations has been calculated, the results should look like this:

R16 $= \Sigma x_i$	514.000
R17 $= \Sigma x_i^2$	29,424.000
R21 $= n$	9.000
R30 $= \Sigma \ln y_i$	20.770
R31 $= \Sigma \ln y_i^2$	49.755
R40 $= \Sigma x_i^3$	1,688,344.000
R43 $= \Sigma x_i^4$	97,104,852.000
R46 $= \Sigma x_i * \ln y_i$	1,189.588
R54 $= \Sigma x_i^2 * \ln y_i$	68,268.885

If suggested register assignments are used, the formulas in the text can be applied directly to calculate all of the terms required. For this particular example, the terms become

R05 =	620.000
R06 =	3,286.447
R07 =	71,160.000
R08 =	30.564
R09 =	8,171,892.000
R11 =	-160.109
R12 =	5.625
R13 =	-0.049

The coefficients of the normal distribution equation are calculated from the last three registers.

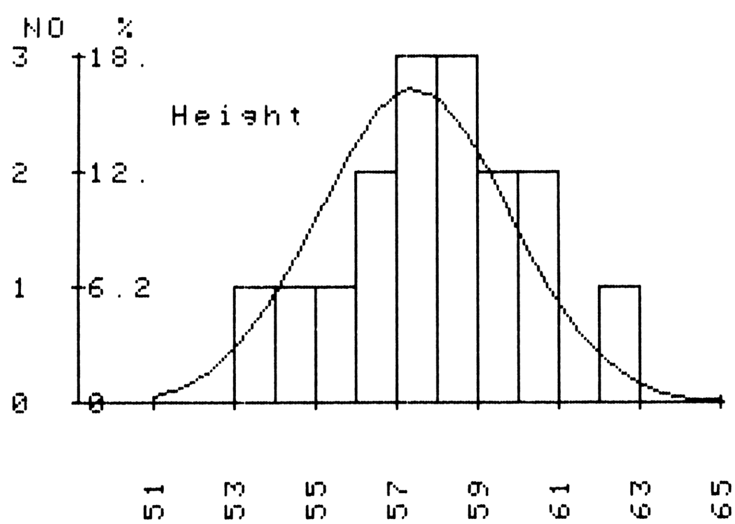
a =	15.020
b =	57.895
c =	-20.586

Substituting these coefficients into the general equation gives us the best fitting normal distribution for our data.

$$y = 15.0\ e^{(x-57.9)^2/(-20.6)}$$

The coefficient of determination is found to be 0.748, meaning that approximately 75% of the observed variance in y is explained by x. For the limited data sample used, this probably represents a fairly good fit. If we were to try another curve with the hope of getting a better fit, it would be necessary to calculate the corrected coefficient of determination (0.664) for comparison. The curve with the larger corrected value of RR would generally be considered the better fitting curve. In this case, there is at least one other curve with a higher coefficient of determination, namely the cauchy distribution. Unless there is some strong justification for believing the adult population follows a cauchy distribution, however, we should not select it over the normal distribution for extrapolating outside of the range of our data. The regression curve we finally settle on should be plotted along with our data as a visual check of the calculations and goodness of fit.

Assuming we are satisfied with the model, we can now use our regression coefficients to estimate any value of y. By substituting 65 for x in the last expression, we can estimate the expected proportion of the adult population that is 65 inches in height. Based on the data sample used, we would expect about 1.3% of the population to be 65 inches tall. (The cauchy distribution model would have predicted 3.1%.)



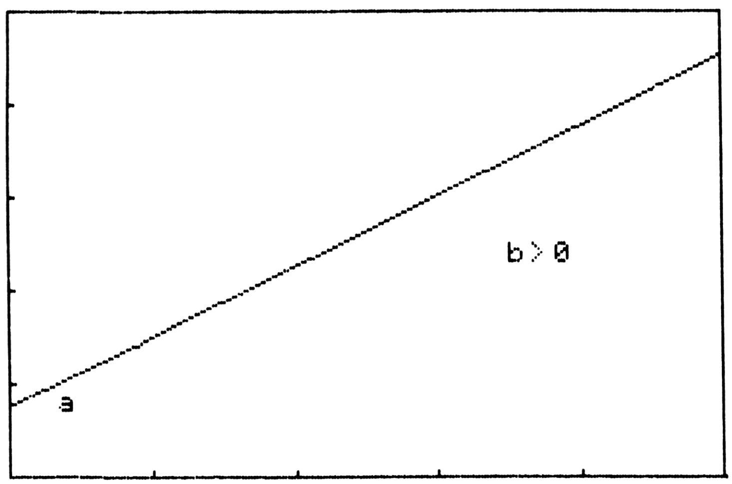
PART II

GENERAL CURVE FITTING EQUATIONS

STRAIGHT LINE

General Equation: $y = a + bx$

This is perhaps the most common equation used to fit data. It enjoys widespread use in general forecasting, biology, economics and engineering. It can be used any time y is proportional to x. The following formulas will estimate the coefficients of a linear equation that best fits the data when three or more points are given. X and y may be positive, negative or equal to zero.



$R16 = \Sigma x_i$ $R18 = \Sigma y_i$ $R20 = \Sigma x_i * y_i$

$R17 = \Sigma x_i^2$ $R19 = \Sigma y_i^2$ $R21 = n$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit straight line are computed as follows:

$$R05 = R17*R21 - (R16)^2$$

$$R11 = (R17*R18 - R16*R20)/R05$$

$$R12 = (R20*R21 - R16*R18)/R05$$

The coefficients of the best fit line are:

$$a = R11$$

$$b = R12$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R18 + R12*R20 - (R18)^2/R21}{R19 - (R18)^2/R21}$$

When RR is to be used for comparison with other regression curves, it should be corrected to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 2)}$$

Example:

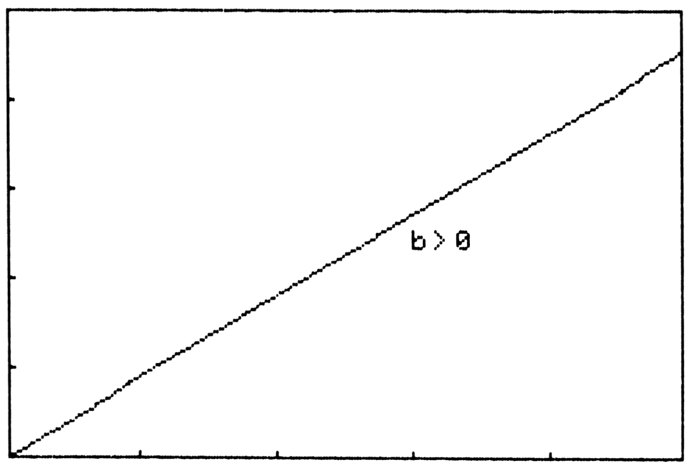
X =	10	20	30	40	50
Y =	28	32	46	59	72

R16 =	150.00	R21 =	5.00
R17 =	5,500.00	R05 =	5,000.00
R18 =	237.00	a =	12.90
R19 =	12,589.00	b =	1.15
R20 =	8,260.00	RR =	0.976

STRAIGHT LINE
THROUGH THE ORIGIN

General Equation: $y = bx$

There are many situations where the relationship between x and y is such that y must be zero when x is zero. If blood pressure is zero, for example, then the volume of blood flow is zero. Voltage and current in simple electrical circuits exhibits a similar relationship. This equation can be used to fit a straight line to these kinds of data and is used any time y is directly proportional to x. The following formulas will estimate the coefficient of a linear equation that best fits the data when two or more points are given. X and y may be positive, negative, or equal to zero.



R17 = x_i^2 R20 = x_i*y_i

where x and y are the values associated with each data point. The coefficient of the best fit straight line is computed as follows:

$$R12 = R20/R17$$

The coefficient of the best fit line through the origin is:

$$b = R12$$

Example:

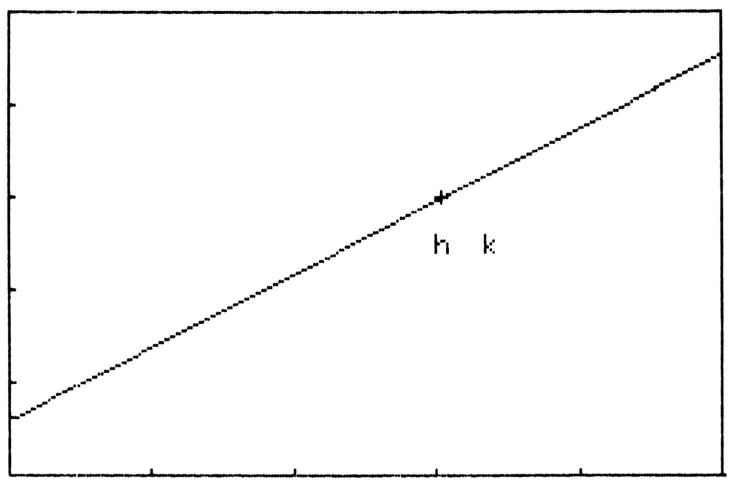
X =	11	17	23	29
Y =	15	23	31	39

R17 = 1,780.00
R20 = 2,400.00
b = 1.348

STRAIGHT LINE
THROUGH A GIVEN POINT

General Equation: $y = a + bx$

This is a variation of the linear equation. It is used to fit data to a straight line which passes through the point h,k. It can be used whenever the value of one point is known or assumed to be correct, e.g., surveying through a known tie point or benchmark. The following formulas will estimate the coefficients of a linear equation that best fits the data when three or more points are given. X and y may be positive, negative or equal to zero.



R16 = x_i R18 = y_i R20 = $x_i * y_i$

R17 = x_i^2 R21 = n

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit straight line are computed as follows:

$$R11 = \frac{h*k*R16 - k*R17 - h^2*R18 + h*R20}{2*h*R16 - R17 - h^2*R21}$$

The coefficients of the best fit line are:

$$a = R11$$

$$b = (k - R11)/h$$

Example:

X =	100	200	300	400	500
Y =	140	230	310	400	480

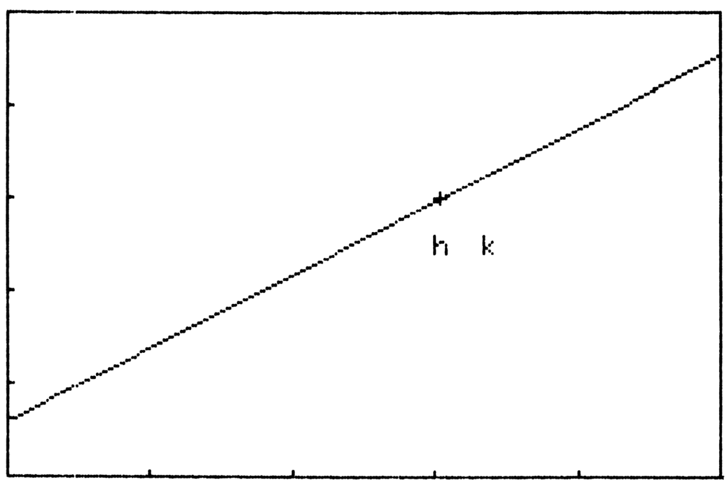
Find the best fitting straight line that passes through the point (300,310).

R16 =	1,500.00	R21 =	5.00
R17 =	550,000.00	a =	55.00
R18 =	1,560.00	b =	0.850
R20 =	553,000.00		

ALTERNATIVE STRAIGHT LINE
THROUGH A GIVEN POINT

General Equation: $y = a + bx$

These equations will find the best fitting straight line that passes through the point h,k. It produces the same coefficients as the previous method but the technique is somewhat different and is generally applicable to other regression curves. It may be used to join two curves at a common point, e.g., a straight line and an arc. The following formulas will estimate the coefficients of a linear equation that best fits the data when three or more points are given. Note that h is subtracted from each x value and k is subtracted from each y value as the sums are calculated. X and y may be positive, negative or equal to zero.



$R17 = (x_i - h)^2$ $\qquad$ $R20 = (x_i - h) * (y_i - k)$

where x and y are the values associated with each data point and h and k are the coordinates of the fixed points. The coefficients of the best fit straight line are computed as follows:

$$R12 = R20/R17$$

$$R11 = k - h*R12$$

The coefficients of the best fit line are:

a = R11

b = R12

Example:

X =	100	200	300	400	500
Y =	140	230	310	400	480

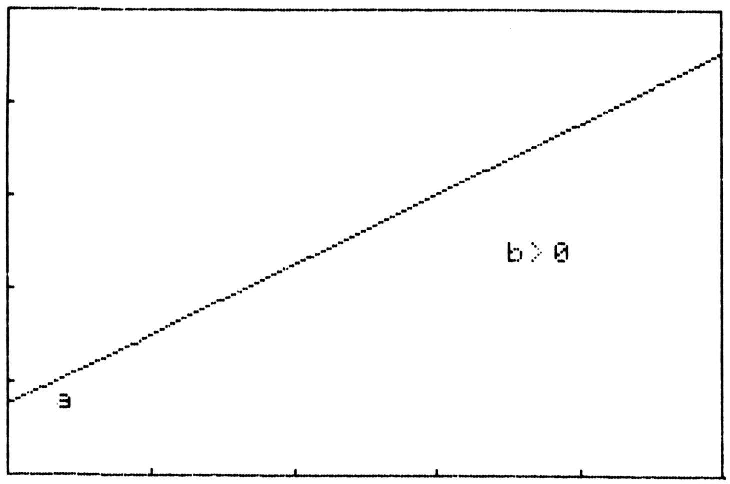
Find the best fitting straight line that passes through the point (300,310).

R17 =	100,000.000	a =	55.000
R20 =	85,000.000	b =	0.850

ISOTONIC LINEAR REGRESSION

General Equation: $y = a + bx$

This is a variation of the linear equation based on minimizing the sum of squared deviations as measured perpendicular to the regression line. It corresponds most nearly to a free-hand line drawn through the points. Isotonic regression can be used when there are equal errors in both x and y, e.g., surveying through a number of points that lie on a straight line. The following formulas will estimate the coefficients of a linear equation that best fits the data when three or more points are given. X and y may be positive, negative or equal to zero.



$R16 = \Sigma x_i$ $R18 = \Sigma y_i$ $R20 = \Sigma x_i * y_i$

$R17 = \Sigma x_i^2$ $R19 = \Sigma y_i^2$ $R21 = n$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit straight line are computed as follows:

$$R05 = \frac{(R17 - R19)*R21 + (R18)^2 - (R16)^2}{2*(R20*R21 - R16*R18)}$$

$$R11 = -R05 \pm \sqrt{R05^2 + 1}$$

$$R12 = (R18 - R11*R16)/R21$$

There are two possible lines which satisfy the regression equation, each is perpendicular to the other. The correct solution for most applications is given by the value of R11 that minimizes the expression

$$|(R12*R16 - R20)*R11 - R12*R18|$$

The coefficients of the best fitting straight line are then given by

$$a = R12$$

$$b = R11$$

Example:

X =	100	200	300	400	500
Y =	140	230	310	400	480

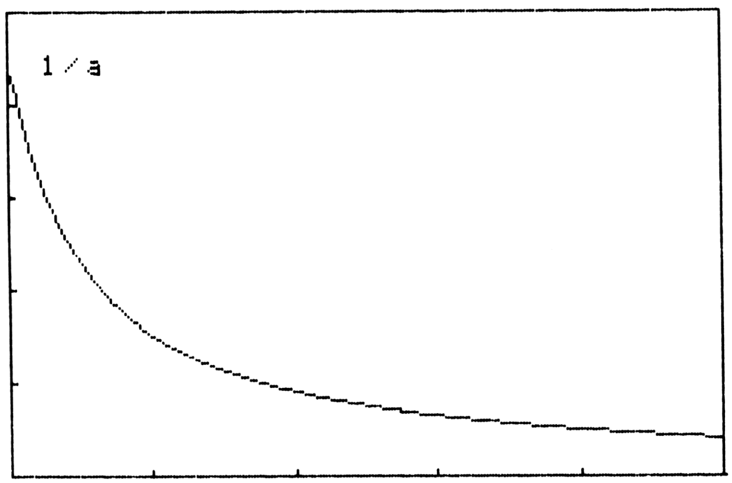
R16 =	1500	R21 =	5.000
R17 =	550,000	R05 =	0.163
R18 =	1,560	a =	664.88 or 56.96
R19 =	559,000	b =	-1.176 or 0.850
R20 =	553,000		

Since the expression $|(R12*R16 - R20)*R11 - R12*R18|$ is minimum (486,351 versus 1,559,850) when R11 equals 0.85, the coefficients of the best fit line are: a = 56.96 and b = 0.85.

RECIPROCAL OF STRAIGHT LINE

General Equation: $$y = \frac{1}{a + bx}$$

This equation is the reciprocal of a straight line. It is used when x is inversely proportional to y, e.g., exposure time versus brightness in photography. The following formulas will estimate the coefficients of a reciprocal equation that best fits the data when three or more points are given. Y must not be equal to zero (any small number may be substituted for y when it is).



$R16 = \Sigma x_i$ $R21 = n$ $R25 = \Sigma 1/y_i^2$

$R17 = \Sigma x_i^2$ $R24 = \Sigma 1/y_i$ $R34 = \Sigma x_i/y_i$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit curve are computed as follows:

$$R05 = R17*R21 - (R16)^2$$

$$R11 = (R17*R24 - R16*R34)/R05$$

$$R12 = (R21*R34 - R16*R24)/R05$$

The coefficients of the best fit curve are:

$$a = R11$$

$$b = R12$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R24 + R12*R34 - (R24)^2/R21}{R25 - (R24)^2/R21}$$

When RR is to be used for comparison with other regression curves, it should be corrected to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 2)}$$

Example:

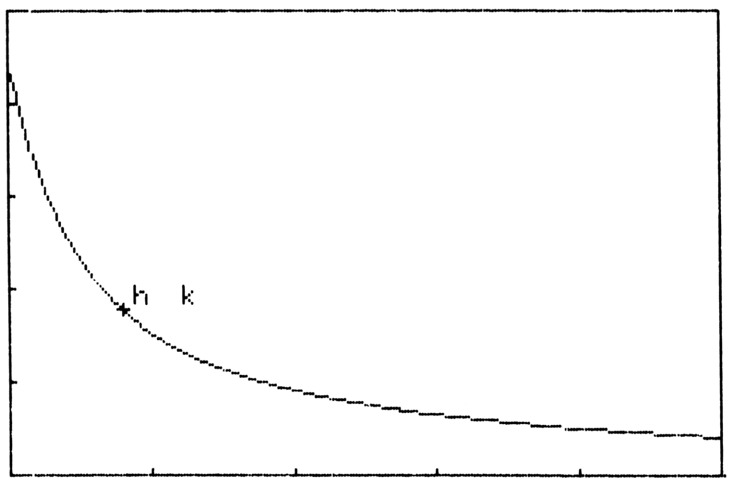
X =	1.0	2.0	3.0	4.0	5.0
Y =	5.1	3.1	2.2	1.7	1.4

R16 =	15.000	R34 =	8.129
R17 =	55.000	R05 =	50.000
R21 =	5.000	a =	0.065
R24 =	2.276	b =	0.130
R25 =	1.205	RR =	1.000

RECIPROCAL OF STRAIGHT LINE
THROUGH A GIVEN POINT

General Equation: $$y = \frac{1}{a + bx}$$

This equation is the reciprocal of a straight line forced to pass through a given point h,k and can be used to join two curves through a common point. The following formulas will estimate the coefficients of a reciprocal curve when three or more points are given. Subtract h from each x value and 1/k from 1/y when calculating the sums. Y must not be equal to zero (any small number may be substituted for y when it is).



$R17 = (x_i-h)^2$ $\qquad$ $R34 = (x_i-h)*(1/y_i-1/k)$

where x and y are the values associated with each data point and h and k are the coordinates of the given point. The coefficients of the best fit curve are computed as follows:

$$R12 = R34/R17$$

$$R11 = (1/k) - h*R12$$

The coefficients of the best fit curve are:

a = R11

b = R12

Example:

X =	1.0	2.0	3.0	4.0	5.0
Y =	5.1	3.1	2.2	1.7	1.4

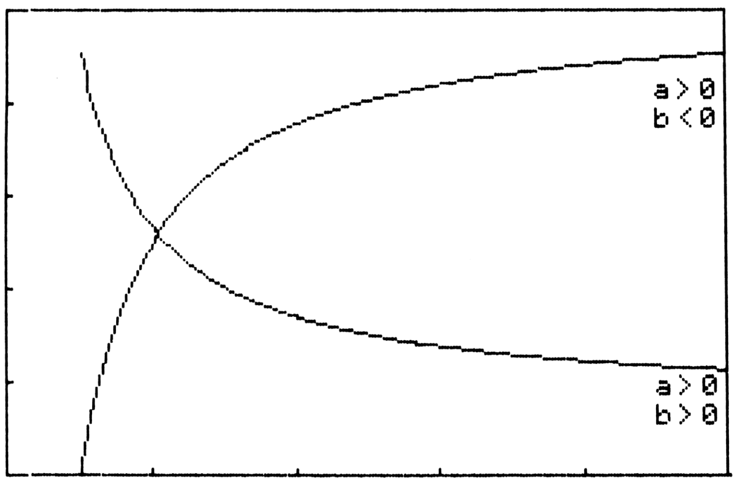
Find the best fitting reciprocal curve that passes through the point (3,2).

R17 =	10.000	a =	0.109
R34 =	1.302	b =	0.130

HYPERBOLA

General Equation: $y = a + b/x$

The following formulas can be used to estimate the coefficients of a hyperbolic equation that best fits the data when three or more points are given. X must not be equal to zero (any small number may be substituted for x when it is).



R18 = $\Sigma\, y_i$ R21 = n R23 = $\Sigma\, 1/x_i^2$

R19 = $\Sigma\, y_i^2$ R22 = $\Sigma\, 1/x_i$ R35 = $\Sigma\, y_i/x_i$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit hyperbola are computed as follows:

R05 = R21*R23 - (R22)²

R11 = (R18*R23 - R22*R35)/R05

R12 = (R21*R35 - R18*R22)/R05

The coefficients of the best fit hyperbola are:

$$a = R11$$

$$b = R12$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R18 + R12*R35 - (R18)^2/R21}{R19 - (R18)^2/R21}$$

When RR is to be used for comparison with other regression curves, it should be corrected to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 2)}$$

Example:

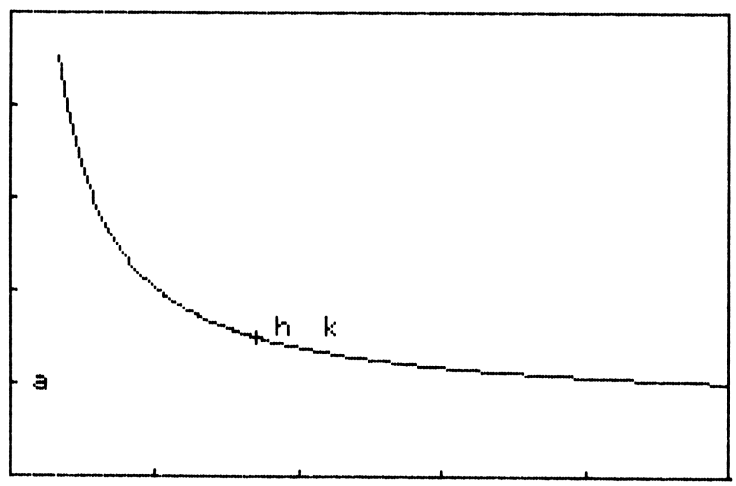
X =	1.0	2.0	3.0	4.0	5.0
Y =	5.1	3.1	2.2	1.7	1.4

R18 =	13.500	R35 =	8.088
R19 =	43.310	R05 =	2.104
R21 =	5.000	a =	0.613
R22 =	2.283	b =	4.570
R23 =	1.464	RR =	0.992

HYPERBOLA THROUGH A GIVEN POINT

General Equation: $y = a + b/x$

The following formulas can be used to estimate the coefficients of a hyperbolic equation that passes through the point h,k when three or more points are given. Subtract k from each y value and 1/h from 1/x when calculating the sums. X must not be equal to zero (any small number may be substituted for x when it is).



$$R23 = (1/x_i - 1/h)^2 \qquad R35 = (y_i - k) * (1/x_i - 1/h)$$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit hyperbola are computed as follows:

$$R12 = R35/R23$$

$$R11 = k - R12/h$$

The coefficients of the best fit hyperbola are:

a = R11

b = R12

Example:

Find the best fit hyperbola that passes through the point (3,2).

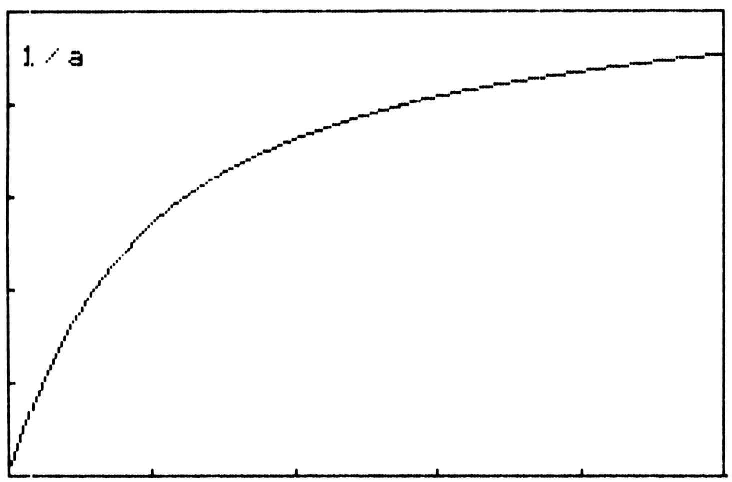
X =	1.0	2.0	3.0	4.0	5.0
Y =	5.1	3.1	2.2	1.7	1.4

R23 =	0.497	a =	0.420
R35 =	2.355	b =	4.739

RECIPROCAL OF A HYPERBOLA

General Equation: $$y = \frac{x}{ax + b}$$

This equation is the reciprocal of a hyperbola. The following formulas will estimate the coefficients of the equation that best fits the data when three or more points are given. Neither x nor y can be equal to zero (any small number may be substituted for y when they are).



R21 = n

R22 = $\Sigma\, 1/x_i$

R23 = $\Sigma\, 1/x_i^2$

R24 = $\Sigma\, 1/y_i$

R25 = $\Sigma\, 1/y_i^2$

R26 = $\Sigma\, 1/(x_i * y_i)$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit curve are computed as follows:

R05 = R21*R23 - $(R22)^2$

R11 = (R23*R24 - R22*R26)/R05

R12 = (R21*R26 - R22*R24)/R05

The coefficients of the best fit curve are:

$$a = R11$$

$$b = R12$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R24 + R12*R26 - (R24)^2/R21}{R25 - (R24)^2/R21}$$

When RR is to be used for comparison with other regression curves, it should be corrected to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 2)}$$

Example:

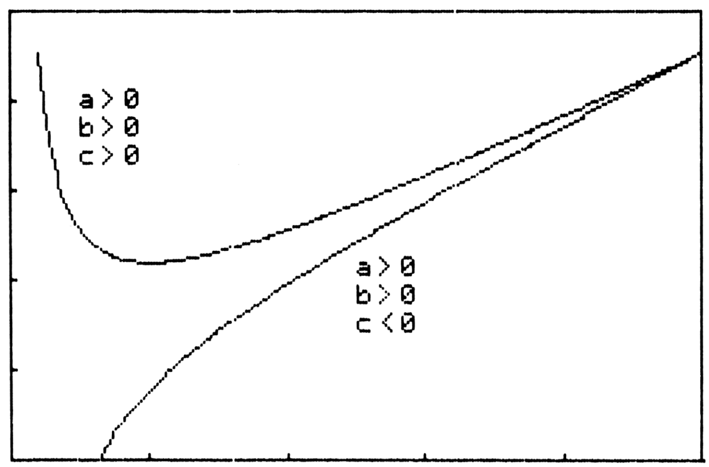
X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R21 =	5.000	R26 =	0.656
R22 =	2.283	R05 =	2.104
R23 =	1.464	a =	0.120
R24 =	1.195	b =	0.262
R25 =	0.320	RR =	0.830

COMBINED LINEAR AND HYPERBOLIC

General Equation: $y = a + bx + c/x$

This equation combines a straight line with a hyperbola and is especially useful for fitting many curves when the underlying phenomena is not well defined. The following formulas will estimate the coefficients of such a curve when four or more points are given. X must not be equal to zero (any small number may be substituted for x when it is).



$R16 = \Sigma x_i$ $\quad$ $R19 = \Sigma y_i^2$ $\quad$ $R22 = \Sigma 1/x_i$

$R17 = \Sigma x_i^2$ $\quad$ $R20 = \Sigma x_i * y_i$ $\quad$ $R23 = \Sigma 1/x_i^2$

$R18 = \Sigma y_i$ $\quad$ $R21 = n$ $\quad$ $R35 = \Sigma y_i/x_i$

where x and y are the values associated with each data point and n is the total number of points. The following terms must now be calculated in order to obtain the coefficients of the equation:

$R05 = R17*R21 - (R16)^2$ $\quad$ $R08 = R20*R21 - R16*R18$

$R06 = R21*R35 - R18*R22$ $\quad$ $R09 = R21*R23 - (R22)^2$

$R07 = (R21)^2 - R16*R22$

$$R13 = (R05*R06 - R07*R08)/(R05*R09 - R07^2)$$

$$R12 = (R08 - R07*R13)/R05$$

$$R11 = (R18 - R12*R16 - R13*R22)/R21$$

The coefficients of the best fit curve are:

$$a = R11$$

$$b = R12$$

$$c = R13$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R18 + R12*R20 + R13*R35 - (R18)^2/R21}{R19 - (R18)^2/R21}$$

When RR is to used for comparison with other regression curves, it should be corrected as follows to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 3)}$$

Example:

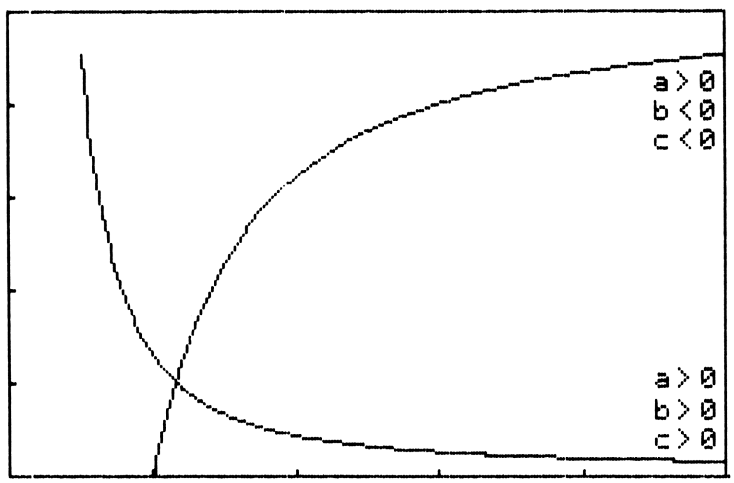
X =	5	10	15	20	25
Y =	21	12	15	21	28

R16 =	75.000	R05 =	1,250.000
R17 =	1,375.000	R06 =	-1.447
R18 =	97.000	R07 =	-9.250
R19 =	2,035.000	R08 =	575.000
R20 =	1,570.000	R09 =	0.084
R21 =	5.000	a =	-23.628
R22 =	0.457	b =	1.781
R23 =	0.0585	c =	178.559
R35 =	8.570	RR =	1.000

SECOND ORDER HYPERBOLA

General Equation: $y = a + b/x + c/x^2$

This is a second degree polynomial where 1/x has been substituted for x. It is similar to the hyperbola but generally exhibits a steeper descent along the y-axis. The following formulas will estimate the coefficients of such a curve when four or more points are given. X must not be equal to zero (any small number may be substituted for x when it is).



$R18 = \Sigma y_i$ $R22 = \Sigma 1/x_i$ $R38 = \Sigma y_i/x_i^2$

$R19 = \Sigma y_i^2$ $R23 = \Sigma 1/x_i^2$ $R41 = \Sigma 1/x_i^3$

$R21 = n$ $R35 = \Sigma y_i/x_i$ $R44 = \Sigma 1/x_i^4$

where x and y are the values associated with each data point and n is the total number of points. The following terms must now be calculated in order to obtain the coefficients of the equation:

R05 = R21*R23 - $(R22)^2$

R06 = R21*R38 - R18*R23

R07 = R21*R41 - R22*R23

R08 = R21*R35 - R18*R22

R09 = R21*R44 - $(R23)^2$

$$R13 = (R05*R06 - R07*R08)/(R05*R09 - R07^2)$$

$$R12 = (R08 - R07*R13)/R05$$

$$R11 = (R18 - R12*R22 - R13*R23)/R21$$

The coefficients of the best fit curve are:

$$a = R11$$

$$b = R12$$

$$c = R13$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R18 + R12*R35 + R13*R38 - (R18)^2/R21}{R19 - (R18)^2/R21}$$

When RR is to used for comparison with other regression curves, it should be corrected as follows to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 3)}$$

Example:

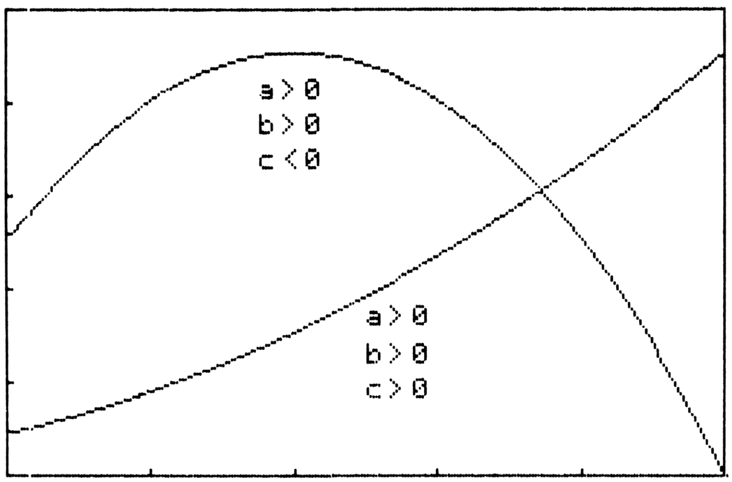
X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R18 =	23.700	R05 =	2.104
R19 =	125.890	R06 =	-10.848
R21 =	5.000	R07 =	2.586
R22 =	2.283	R08 =	-9.873
R23 =	1.464	R09 =	3.260
R35 =	8.848	a =	11.153
R38 =	4.768	b =	-24.238
R41 =	1.186	c =	15.904
R44 =	1.080	RR =	0.986

PARABOLA

General Equation: $y = a + bx + cx^2$

The parabolic equation belongs to a family of curves known as polynomials. This particular equation is a second degree polynomial with many applications in the physical sciences, e.g., describing the motion of an object under the influence of gravity or acceleration. The following formulas will estimate the coefficients of such a curve when four or more points are given. X and y may be positive, negative, or equal to zero.



$R16 = \Sigma x_i$

$R17 = \Sigma x_i^2$

$R18 = \Sigma y_i$

$R19 = \Sigma y_i^2$

$R20 = \Sigma x_i * y_i$

$R21 = n$

$R36 = \Sigma x_i^2 * y_i$

$R40 = \Sigma x_i^3$

$R43 = \Sigma x_i^4$

where x and y are the values associated with each data point and n is the total number of points. The following terms must now be calculated in order to obtain the coefficients of the equation:

$R05 = R17*R21 - (R16)^2$

$R06 = R21*R36 - R17*R18$

$R07 = R21*R40 - R16*R17$

$R08 = R20*R21 - R16*R18$

$R09 = R21*R43 - (R17)^2$

$$R13 = (R05*R06 - R07*R08)/(R05*R09 - R07^2)$$

$$R12 = (R08 - R07*R13)/R05$$

$$R11 = (R18 - R12*R16 - R13*R17)/R21$$

The coefficients of the best fit curve are:

$$a = R11$$

$$b = R12$$

$$c = R13$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R18 + R12*R20 + R13*R36 - (R18)^2/R21}{R19 - (R18)^2/R21}$$

When RR is to used for comparison with other regression curves, it should be corrected as follows to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 3)}$$

Example:

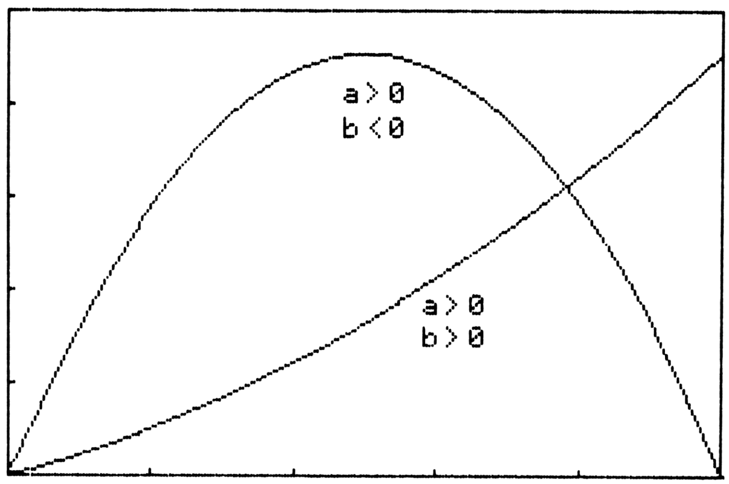
X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R16 =	15.000	R05 =	50.000
R17 =	55.000	R06 =	353.500
R18 =	23.700	R07 =	300.000
R19 =	125.890	R08 =	57.500
R20 =	82.600	R09 =	1,870.000
R21 =	5.000	a =	2.140
R36 =	331.400	b =	0.421
R40 =	225.000	c =	0.121
R43 =	979.000	RR =	0.991

PARABOLA THROUGH THE ORIGIN

General Equation: $y = ax + bx^2$

This equation represents a parabolic curve that is constrained to pass through the origin. It is especially useful for estimating the relationship between elapsed time and counter readings on tape recorders and video recorders. The following formulas will estimate the coefficients of such a curve when three or more points are given. X and y may be positive, negative, or equal to zero.



R17 = x_i^2

R20 = $x_i * y_i$

R36 = $x_i^2 * y_i$

R40 = x_i^3

R43 = x_i^4

where x and y are the values associated with each data point. The following terms must now be calculated in order to obtain the coefficients of the equation:

$$R05 = R17*R43 - (R40)^2$$

$$R11 = (R20*R43 - R36*R40)/R05$$

$$R12 = (R17*R36 - R20*R40)/R05$$

The coefficients of the best fit curve are:

$$a = R11$$

$$b = R12$$

Example:

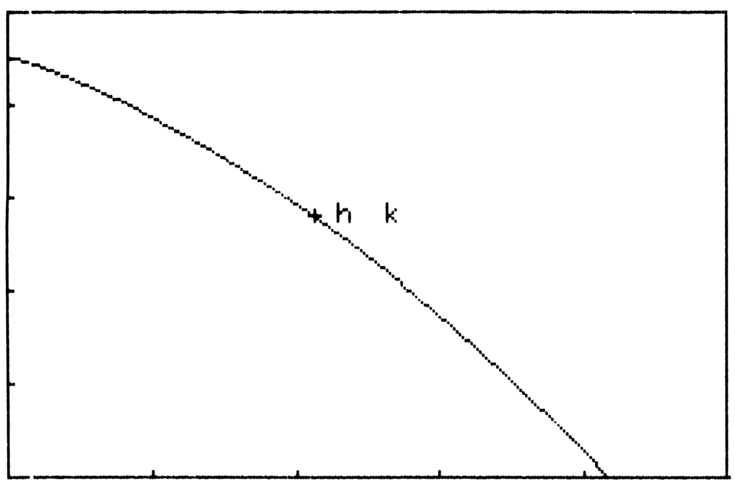
X =	1.0	2.0	3.0	4.0	5.0
Y =	49	84	113	138	161

R17 =	55.00	R43 =	979.00
R20 =	1,913.00	R05 =	3,220.00
R36 =	7,635.00	a =	48.12
R40 =	225.00	b =	-3.26

PARABOLA THROUGH A GIVEN POINT

General Equation: $y = a + bx + cx^2$

This equation represents a parabolic curve that is constrained to pass through the point h,k. The following formulas will estimate the coefficients of such a curve when four or more points are given. Subtract h from each x value and k from each y value when calculating the sums. X and y may be positive, negative, or equal to zero.



$R17 = \Sigma (x_i - h)^2$

$R20 = \Sigma (x_i - h)*(y_i - k)$

$R36 = \Sigma (y_i - k)*(x_i - h)^2$

$R40 = \Sigma (x_i - h)^3$

$R43 = \Sigma (x_i - h)^4$

where x and y are the values associated with each data point and h and k are the coordinates of the given point. The following terms must now be calculated in order to obtain the coefficients of the equation:

$$R05 = R17*R43 - (R40)^2$$

$$R11 = (R20*R43 - R36*R40)/R05$$

$$R12 = (R17*R36 - R20*R40)/R05$$

The coefficients of the best fit parabola are:

$$a = k - (R11 - R12*h)*h$$

$$b = R11 - 2*h*R12$$

$$c = R12$$

Example:

X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

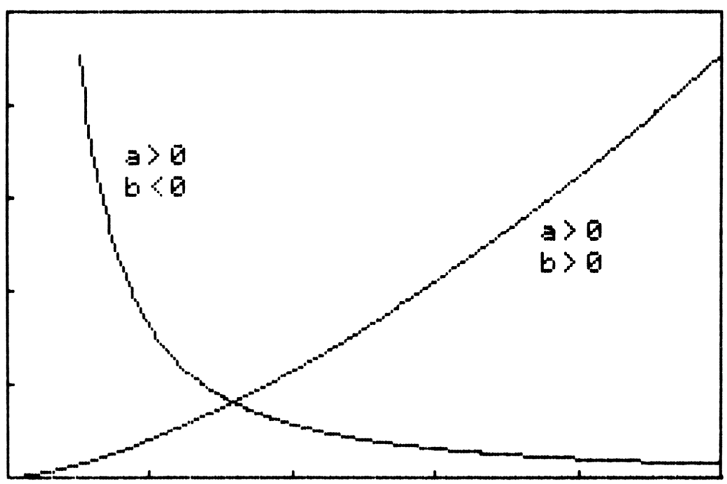
Find the best fitting parabola through the point (4.5,6.5).

R17 =	21.250	R05 =	300.625
R20 =	24.700	R11 =	1.516
R36 =	-70.200	a =	2.139
R40 =	-61.875	b =	0.422
R43 =	194.313	c =	0.122

POWER

General Equation: $y = a\ x^b$

This equation is commonly referred to as the learning curve. It describes trends which are geometric in nature and is often applied when y increases at a much faster (geometric) rate than x. The following formulas will estimate the coefficients of a power curve that best fits the data when three or more points are given. X and y must be positive numbers greater than zero.



R21 = n	R29 = $\Sigma\ (\ln x_i)^2$	R31 = $\Sigma\ (\ln y_i)^2$
R28 = $\Sigma \ln x_i$	R30 = $\Sigma \ln y_i$	R32 = $\Sigma\ (\ln x_i * \ln y_i)$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit curve are computed as follows:

$$R05 = R21*R29 - (R28)^2$$

$$R11 = (R29*R30 - R28*R32)/R05$$

$$R12 = (R21*R32 - R28*R30)/R05$$

The coefficients of the best fit power curve are:

$$a = e^{R11}$$

$$b = R12$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R30 + R12*R32 - (R30)^2/R21}{R31 - (R30)^2/R21}$$

When RR is to be used for comparison with other regression curves, it should be corrected to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 2)}$$

Example:

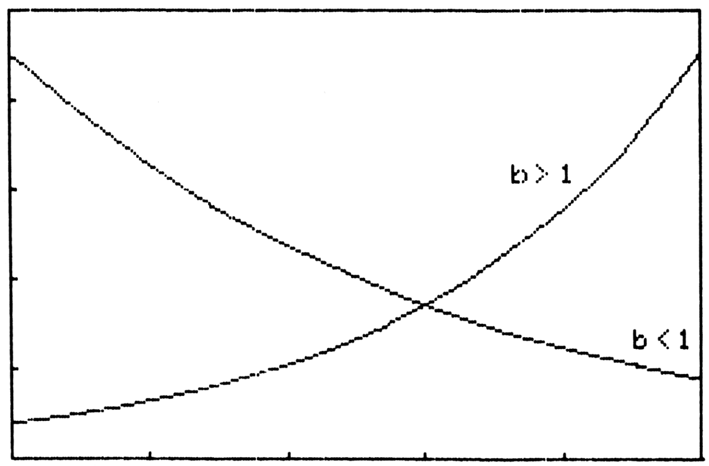
X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R21 =	5.000	R05 =	8.077
R28 =	4.787	R11 =	0.919
R29 =	6.200	a =	2.506
R30 =	7.468	b =	0.600
R31 =	11.789	RR =	0.917
R32 =	8.121		

MODIFIED POWER

General Equation: $y = a\,b^{x}$

This equation is a variation of the power curve. It also describes trends which are geometric in nature and is applied when the ratio between successive terms in a series is constant. The following formulas will estimate the coefficients of a modified power curve that best fits the data when three or more points are given. Y must be a positive number greater than zero.



$R16 = \Sigma\, x_i$ $R21 = n$ $R31 = \Sigma\,(\ln y_i)^2$

$R17 = \Sigma\, x_i^2$ $R30 = \Sigma \ln y_i$ $R46 = \Sigma\,(x_i * \ln y_i)$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit curve are computed as follows:

$$R05 = R17*R21 - (R16)^2$$

$$R11 = (R17*R30 - R16*R46)/R05$$

$$R12 = (R21*R46 - R16*R30)/R05$$

The coefficients of the best fit curve are:

$$a = e^{R11}$$

$$b = e^{R12}$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R30 + R12*R46 - (R30)^2/R21}{R31 - (R30)^2/R21}$$

When RR is to be used for comparison with other regression curves, it should be corrected to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 2)}$$

Example:

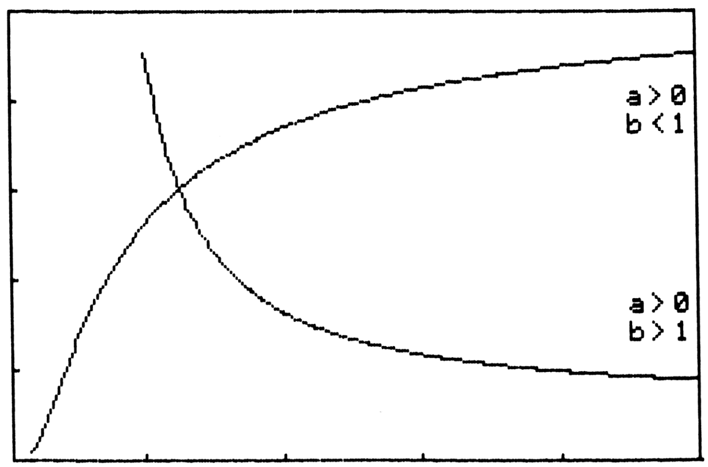
X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R16 =	15.000	R05 =	50.000
R17 =	55.000	R11 =	0.743
R21 =	5.000	R12 =	0.250
R30 =	7.468	a =	2.103
R31 =	11.789	b =	1.284
R46 =	24.904	RR =	0.984

ROOT

General Equation: $y = a\, b^{1/x}$

This equation is a variation of the modified power curve. It fits the xth root of a constant to the dependent variable y. The following formulas will estimate the coefficients of this equation when three or more points are given. Y must be a positive number greater than zero and x must not be equal to zero (any small number may be substituted for x when it is).



$R21 = n$ $\quad$ $R23 = \Sigma\, 1/x_i^2$ $\quad$ $R31 = \Sigma\, (\ln y_i)^2$

$R22 = \Sigma\, 1/x_i$ $\quad$ $R30 = \Sigma \ln y_i$ $\quad$ $R47 = \Sigma\, (\ln y_i)/x_i$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best curve are computed as follows:

$$R05 = R21*R23 - (R22)^2$$

$$R11 = (R23*R30 - R22*R47)/R05$$

$$R12 = (R21*R47 - R22*R30)/R05$$

The coefficients of the best fit curve are:

$$a = e^{R11}$$

$$b = e^{R12}$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R30 + R12*R47 - R(30)^2/R21}{R31 - R30^2/R21}$$

When RR is to be used for comparison with other regression curves, it should be corrected to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 2)}$$

Example:

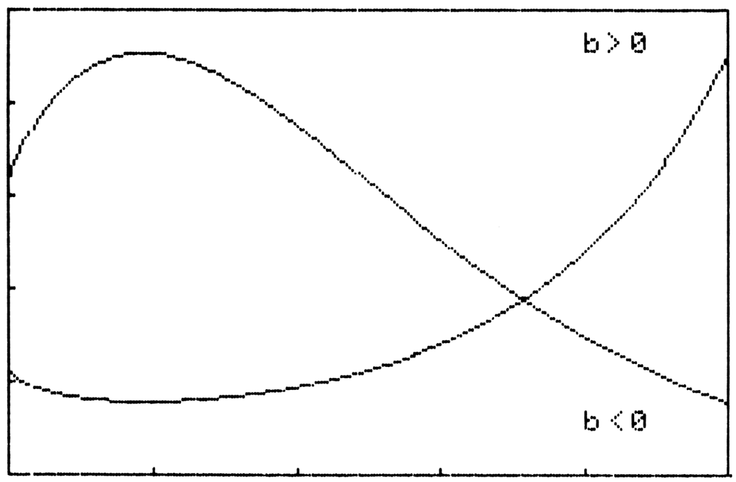
X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R21 =	5.000	R05 =	2.104
R22 =	2.283	R11 =	1.984
R23 =	1.464	R12 =	-1.074
R30 =	7.468	a =	7.271
R31 =	11.789	b =	0.342
R47 =	2.958	RR =	0.763

SUPER GEOMETRIC

General Equation: $y = a\, x^{bx}$

This equation is similar to the power curve but changes much more rapidly. The following formulas will estimate the coefficients of this curve when three or more points are given. X and y must be positive numbers greater than zero.



$R21 = n$ $R31 = \Sigma (\ln y_i)^2$ $R49 = \Sigma (x_i * \ln x_i)^2$

$R30 = \Sigma \ln y_i$ $R48 = \Sigma x_i * \ln x_i$ $R50 = \Sigma (x_i * \ln x_i * \ln y_i)$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit curve are computed as follows:

$$R05 = R21*R49 - (R48)^2$$

$$R11 = (R30*R49 - R48*R50)/R05$$

$$R12 = (R21*R50 - R30*R48)/R05$$

The coefficients of the best fit curve are:

$$a = e^{R11}$$

$$b = R12$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R30 + R12*R50 - (R30)^2/R21}{R31 - (R30)^2/R21}$$

When RR is to be used for comparison with other regression curves, it should be corrected to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 2)}$$

Example:

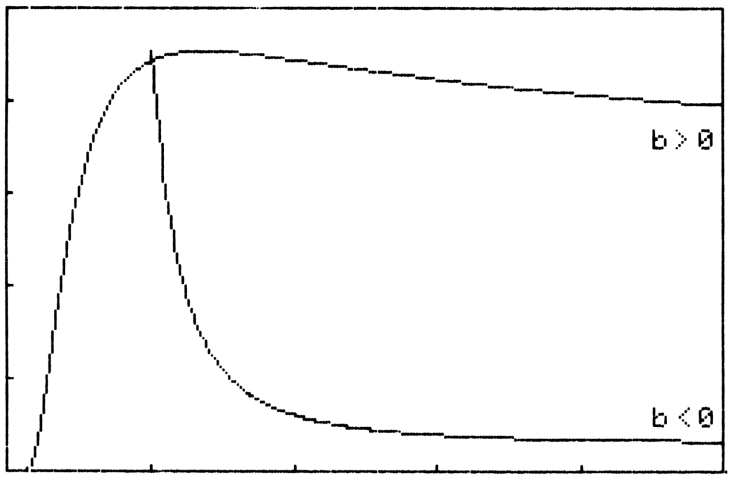
X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R21 =	5.000	R05 =	207.496
R30 =	7.468	R11 =	1.047
R31 =	11.789	a =	2.848
R48 =	18.274	b =	0.122
R49 =	108.291	RR =	0.977
R50 =	32.370		

MODIFIED GEOMETRIC

General Equation: $y = a\, x^{b/x}$

This equation is another variation of the modified power curve. The following formulas will estimate the coefficients of this curve when three or more points are given. X and y must be positive numbers greater than zero.



$R21 = n$

$R30 = \Sigma \ln y_i$

$R31 = \Sigma (\ln y_i)^2$

$R45 = \Sigma (\ln x_i)/x_i$

$R53 = \Sigma [(\ln x_i)/x_i]^2$

$R58 = \Sigma (\ln x_i * \ln y_i)/x_i$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit curve are computed as follows:

$$R05 = R21*R53 - (R45)^2$$

$$R11 = (R30*R53 - R45*R58)/R05$$

$$R12 = (R21*R58 - R30*R45)/R05$$

The coefficients of the best fit curve are:

$$a = e^{R11}$$

$$b = R12$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R30 + R12*R58 - (R30)^2/R21}{R31 - (R30)^2/R21}$$

When RR is to be used for comparison with other regression curves, it should be corrected to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 2)}$$

Example:

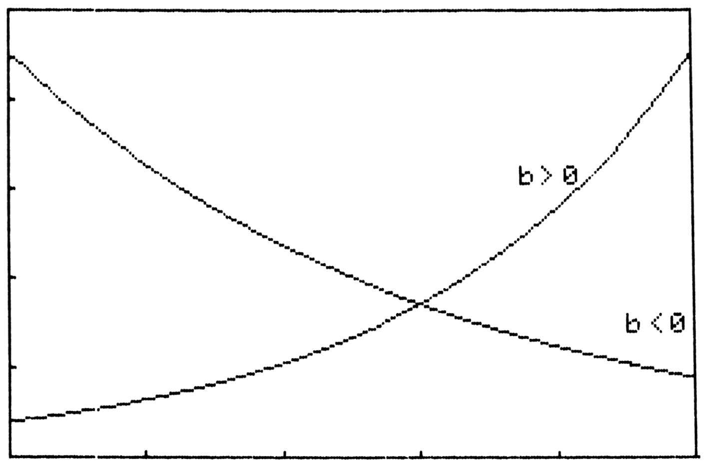
X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R21 =	5.000	R05 =	0.482
R30 =	7.468	R11 =	1.065
R31 =	11.789	a =	2.900
R45 =	1.381	b =	1.552
R53 =	0.478	RR =	0.365
R58 =	2.213		

EXPONENTIAL

General Equation: $y = a\,e^{bx}$

This equation is used to model many natural phenomena including: chance or random failures, the time between failures in systems with many independent components, radial errors in centering, the distribution of lifetimes, and radioactive decay. It is essentially the same as the modified power curve. The following formulas will estimate the coefficients of an exponential curve that best fits the data when three or more points are given. Y must be a positive number greater than zero.



$R16 = \Sigma x_i$ $R21 = n$ $R31 = \Sigma (\ln y_i)^2$

$R17 = \Sigma x_i^2$ $R30 = \Sigma \ln y_i$ $R46 = \Sigma (x_i * \ln y_i)$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit curve are computed as follows:

$$R05 = R17*R21 - (R16)^2$$

$$R11 = (R17*R30 - R16*R46)/R05$$

$$R12 = (R21*R46 - R16*R30)/R05$$

The coefficients of the best fit exponential curve are:

$$a = e^{R11}$$

$$b = R12$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R30 + R12*R46 - (R30)^2/R21}{R31 - (R30)^2/R21}$$

When RR is to be used for comparison with other regression curves, it should be corrected to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 2)}$$

Example:

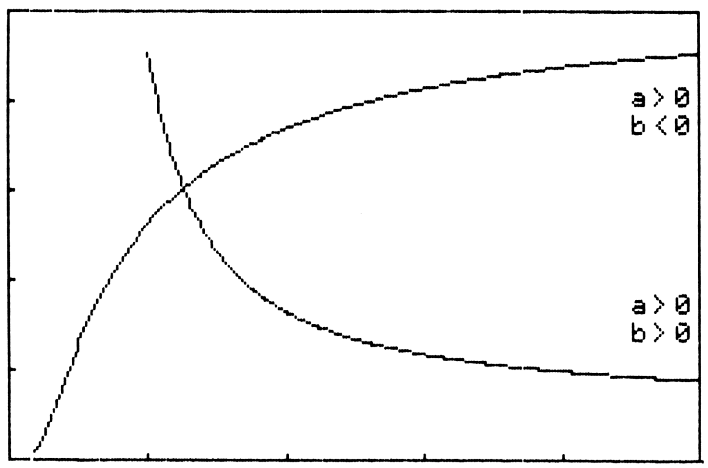
X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R16 =	15.000	R05 =	50.000
R17 =	55.000	R11 =	0.743
R21 =	5.000	a =	2.103
R30 =	7.468	b =	0.250
R31 =	11.789	RR =	0.984
R46 =	24.904		

MODIFIED EXPONENTIAL

General Equation: $y = a\ e^{b/x}$

This equation is a variation of the exponential curve and is essentially the same as the root curve. The following formulas will estimate the coefficients of this equation when three or more points are given. Y must be a positive number greater than zero and x must not be equal to zero (any small number may be substituted for x when it is).



R21 = n

R22 = $\Sigma\ 1/x_i$

R23 = $\Sigma\ 1/x_i^2$

R30 = $\Sigma\ \ln y_i$

R31 = $\Sigma\ (\ln y_i)^2$

R47 = $\Sigma\ (\ln y_i)/x_i$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best curve are computed as follows:

R05 = R21*R23 - (R22)²

R11 = (R23*R30 - R22*R47)/R05

R12 = (R21*R47 - R22*R30)/R05

The coefficients of the best fit curve are:

$$a = e^{R11}$$

$$b = R12$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R30 + R12*R47 - (R30)^2/R21}{R31 - (R30)^2/R21}$$

When RR is to be used for comparison with other regression curves, it should be corrected to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 2)}$$

Example:

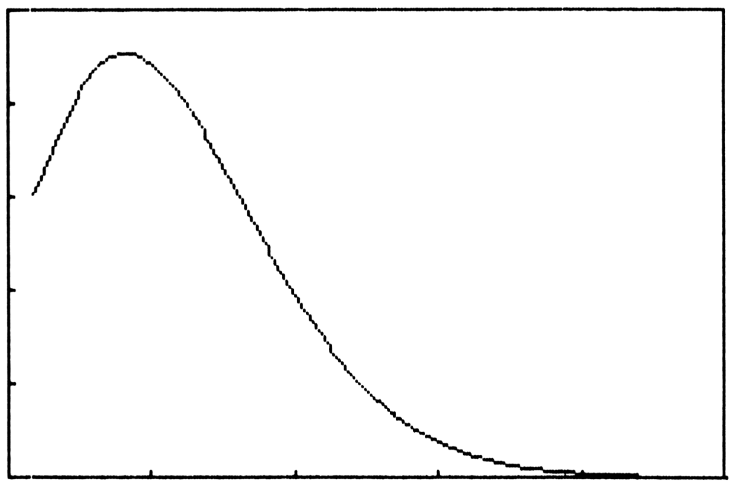
X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R21 =	5.000	R05 =	2.104
R22 =	2.283	R11 =	1.984
R23 =	1.464	a =	7.271
R30 =	7.468	b =	-1.074
R31 =	11.789	RR =	0.763
R47 =	2.958		

POISSON

General Equation: $y = ab^x/x!$

The Poisson distribution is commonly used to describe the space or time distribution of random events, e.g., the probability of exactly x arrivals in a given period of time. The following formulas will estimate the coefficients of this curve when three or more points are given. Y must be a positive number greater than zero and x must be a positive integer.



$R16 = \Sigma x_i$

$R17 = \Sigma x_i^2$

$R21 = n$

$R27 = \Sigma \ln(x_i!)$

$R30 = \Sigma \ln y_i$

$R33 = \Sigma x_i * \ln(x_i!)$

$R46 = \Sigma (x_i * \ln y_i)$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit curve are computed as follows:

$$R05 = R17*R21 - (R16)^2$$

$$R06 = R27 + R30$$

$$R07 = R33 + R46$$

$$R11 = (R06*R17 - R07*R16)/R05$$

$$R12 = (R07*R21 - R06*R16)/R05$$

The coefficients of the best fit curve are:

$$a = e^{R11}$$

$$b = e^{R12}$$

Example:

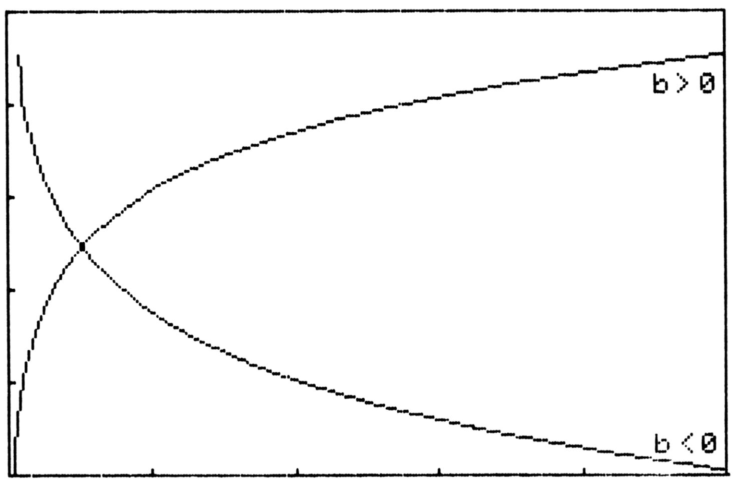
X =	0	1	2	3	4
Y =	1.000	0.800	0.320	0.085	0.017

R16 =	10.000	R05 =	50.000
R17 =	30.000	R06 =	-2.239
R21 =	5.000	R07 =	-6.722
R27 =	5.663	R11 =	0.001
R30 =	-7.902	R12 =	-0.224
R33 =	19.474	a =	1.001
R46 =	-26.195	b =	0.799

LOGARITHMIC

General Equation: $y = a + b*\ln x$

This equation represents a logarithmic curve. It describes trends where y increases at a much slower rate than x. The following formulas will estimate the coefficients of a logarithmic curve that best fits the data when three or more points are given. X must be a positive number greater than zero.



$R18 = \Sigma y_i$ $R21 = n$ $R29 = \Sigma (\ln x_i)^2$

$R19 = \Sigma y_i^2$ $R28 = \Sigma \ln x_i$ $R51 = \Sigma y_i*\ln x_i$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit curve are computed as follows:

$$R05 = R21*R29 - (R28)^2$$

$$R11 = (R18*R29 - R28*R51)/R05$$

$$R12 = (R21*R51 - R18*R28)/R05$$

The coefficients of the best fit logarithmic curve are:

$$a = R11$$

$$b = R12$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R18 + R12*R51 - (R18)^2/R21}{R19 - (R18)^2/R21}$$

When RR is to be used for comparison with other regression curves, it should be corrected to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 2)}$$

Example:

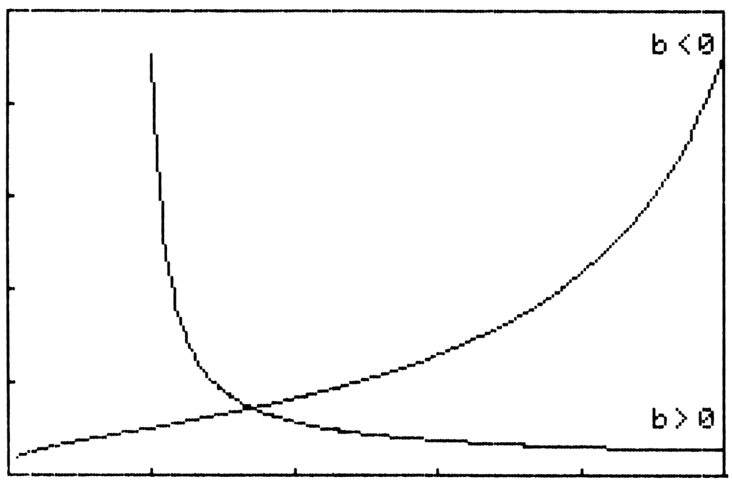
X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R18 =	23.700	R51 =	27.039
R19 =	125.890	R05 =	8.077
R21 =	5.000	a =	2.164
R28 =	4.787	b =	2.690
R29 =	6.200	RR =	0.863

RECIPROCAL OF LOGARITHMIC

General Equation: $$y = \frac{1}{a + b*\ln x}$$

This equation is the reciprocal of the logarithmic curve. The following formulas will estimate the coefficients of such a curve when three or more points are given. X must be a positive number greater than zero. Y must not be equal to zero.



R21 = n

R24 = $1/y_i$

R25 = $1/y_i^2$

R28 = $\ln x_i$

R29 = $(\ln x_i)^2$

R52 = $(\ln x_i)/y_i$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit curve are computed as follows:

R05 = R21*R29 - $(R28)^2$

R11 = (R24*R29 - R28*R52)/R05

R12 = (R21*R52 - R24*R28)/R05

The coefficients of the best fit curve are:

$$a = R11$$

$$b = R12$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R24 + R12*R52 - (R24)^2/R21}{R25 - (R24)^2/R21}$$

When RR is to be used for comparison with other regression curves, it should be corrected to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 2)}$$

Example:

X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

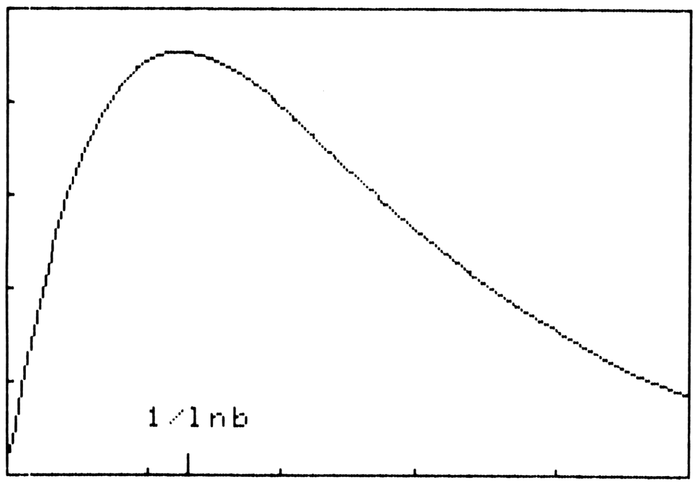
R21 = 5.000
R24 = 1.195
R25 = 0.320
R28 = 4.787
R29 = 6.200

R52 = 0.914
R05 = 8.077
a = 0.376
b = -0.143
RR = 0.950

LINEAR-EXPONENTIAL

General Equation: $y = ax/b^x$

This equation is used to model many biological phenomena including dose response and stimuli response times. The following formulas will estimate the coefficients of this curve when three or more points are given. Both x and y must be positive numbers greater than zero.



$R16 = \Sigma x_i$

$R17 = \Sigma x_i^2$

$R21 = n$

$R28 = \Sigma \ln x_i$

$R30 = \Sigma \ln y_i$

$R46 = \Sigma (x_i * \ln y_i)$

$R48 = \Sigma (x_i * \ln x_i)$

where x and y are the values associated with each data point and n is the total number of points. The coefficients of the best fit curve are computed as follows:

$R05 = R17*R21 - (R16)^2$

$R06 = R30 - R28$

$R07 = R48 - R46$

$R11 = (R06*R17 + R07*R16)/R05$

$R12 = (R07*R21 + R06*R16)/R05$

The coefficients of the best fit curve are:

$$a = e^{R11}$$

$$b = e^{R12}$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*(R30 - R28) + R12*(R48 - R46) - (R30 - R28)^2/R21}{R29 + R31 - 2*R32 - (R30 - R28)^2/R21}$$

When RR is to be used for comparison with other regression curves, it should be corrected to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 2)}$$

Example:

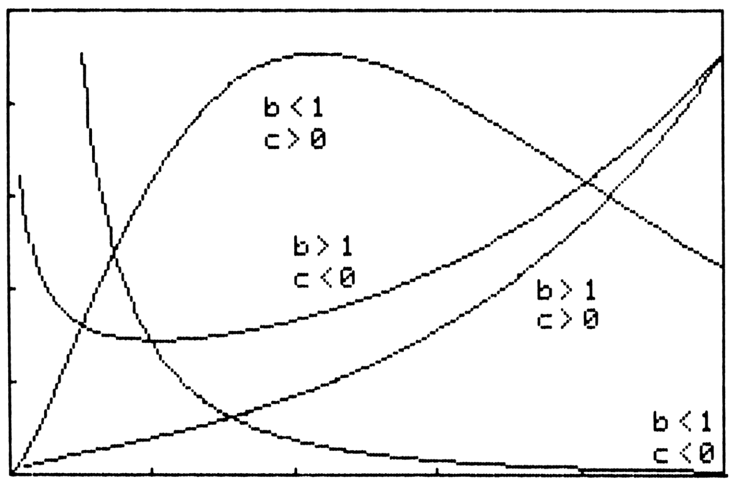
X =	1.0	2.0	3.0	4.0	5.0
Y =	0.667	0.889	0.889	0.790	0.658

R16 =	15.000	R05 =	50.000
R17 =	55.000	R06 =	-6.082
R21 =	5.000	R07 =	22.303
R28 =	4.787	R11 =	0.001
R30 =	-1.295	a =	1.001
R46 =	-4.029	b =	1.500
R48 =	18.274		

HOERL FUNCTION

General Equation: $y = a\, b^{x}\, x^{c}$

This is a generalized form of Hoerl's equation. The following formulas will estimate the coefficients of such a curve when four or more points are given. Both x and y must be positive numbers greater than zero (any small number may be substituted for 0).



$R16 = \Sigma\, x_i$

$R17 = \Sigma\, x_i^2$

$R21 = n$

$R28 = \Sigma\, \ln x_i$

$R29 = \Sigma\, (\ln x_i)^2$

$R30 = \Sigma\, \ln y_i$

$R31 = \Sigma\, (\ln y_i)^2$

$R32 = \Sigma\, \ln x_i * \ln y_i$

$R46 = \Sigma\, x_i * \ln y_i$

$R48 = \Sigma\, x_i * \ln x_i$

where x and y are the values associated with each data point and n is the total number of points. The following terms must now be calculated in order to obtain the coefficients of the equation:

$R05 = R17*R21 - (R16)^2$

$R06 = R21*R32 - R28*R30$

$R07 = R21*R48 - R16*R28$

$R08 = R21*R46 - R16*R30$

$R09 = R21*R29 - (R28)^2$

$$R13 = (R05*R06 - R07*R08)/(R05*R09 - R07^2)$$

$$R12 = (R08 - R07*R13)/R05$$

$$R11 = (R30 - R12*R16 - R13*R28)/R21$$

The coefficients of the best fit curve are:

$$a = e^{R11}$$

$$b = e^{R12}$$

$$c = R13$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R30 + R12*R46 + R13*R32 - (R30)^2/R21}{R31 - (R30)^2/R21}$$

When RR is to used for comparison with other regression curves, it should be corrected as follows to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 3)}$$

Example:

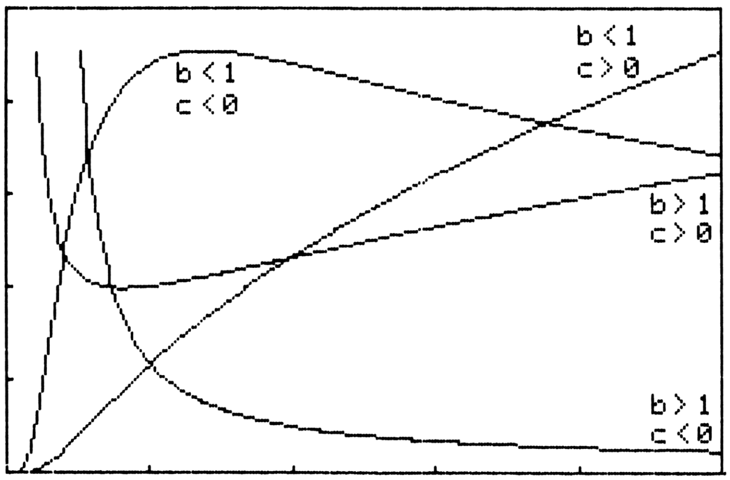
X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R16 =	15.000	R06 =	4.850
R17 =	55.000	R07 =	19.560
R21 =	5.000	R08 =	12.504
R28 =	4.787	R09 =	8.077
R29 =	6.200	R11 =	0.723
R30 =	7.468	R12 =	0.288
R31 =	11.789	a =	2.060
R32 =	8.121	b =	1.333
R46 =	24.904	c =	-0.096
R48 =	18.274	RR =	0.985
R05 =	50.000		

MODIFIED HOERL FUNCTION

General Equation: $y = a\, b^{1/x}\, x^c$

This is a modified form of Hoerl's equation. The following formulas will estimate the coefficients of this equation when four or more points are given. Both x and y must be positive numbers greater than zero (any small number may be substituted for 0).



$R21 = n$

$R22 = \Sigma\, 1/x_i$

$R23 = \Sigma\, 1/x_i^2$

$R28 = \Sigma \ln x_i$

$R29 = \Sigma (\ln x_i)^2$

$R30 = \Sigma \ln y_i$

$R31 = \Sigma (\ln y_i)^2$

$R32 = \Sigma \ln x_i * \ln y_i$

$R45 = \Sigma (\ln x_i)/x_i$

$R47 = \Sigma (\ln y_i)/x_i$

where x and y are the values associated with each data point and n is the total number of points. The following terms must now be calculated in order to obtain the coefficients of the equation:

$R05 = R21*R23 - (R22)^2$

$R06 = R21*R32 - R28*R30$

$R07 = R21*R45 - R22*R28$

$R08 = R21*R47 - R22*R30$

$R09 = R21*R29 - (R28)^2$

$$R13 = (R05*R06 - R07*R08)/(R05*R09 - R07^2)$$

$$R12 = (R08 - R07*R13)/R05$$

$$R11 = (R30 - R12*R22 - R13*R28)/R21$$

The coefficients of the best fit curve are:

$$a = e^{R11}$$

$$b = e^{R12}$$

$$c = R13$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R30 + R12*R47 + R13*R32 - (R30)^2/R21}{R31 - (R30)^2/R21}$$

When RR is to used for comparison with other regression curves, it should be corrected as follows to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 3)}$$

Example:

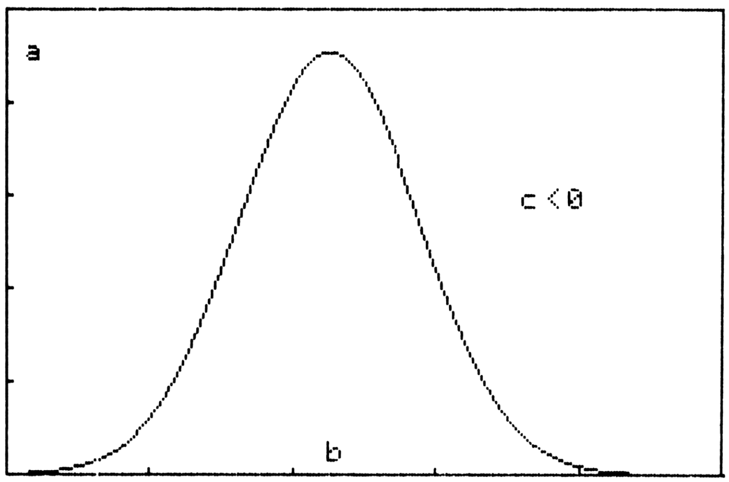
X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R21 =	5.000	R06 =	4.850
R22 =	2.283	R07 =	-4.025
R23 =	1.464	R08 =	-2.259
R28 =	4.787	R09 =	8.077
R29 =	6.200	R11 =	-0.576
R30 =	7.468	R12 =	1.600
R31 =	11.789	a =	0.562
R32 =	8.121	b =	4.954
R45 =	1.381	c =	1.398
R47 =	2.958	RR =	0.996
R05 =	2.104		

NORMAL DISTRIBUTION

General Equation: $y = a\ e^{(x-b)^2/c}$

The normal or Gaussian distribution is often used to describe the expected frequency of some characteristic in a large population, e.g., the height of adult males. It applies to many natural and biological phenomena and is particularly appropriate when the data results from numerous additive factors. The following formulas will estimate the coefficients of this curve when four or more points are given. Y must be a positive number greater than zero.



R16 = $\Sigma\ x_i$	R30 = $\Sigma \ln y_i$	R43 = $\Sigma\ x_i^4$
R17 = $\Sigma\ x_i^2$	R31 = $\Sigma (\ln y_i)^2$	R46 = $\Sigma\ x_i * \ln y_i$
R21 = n	R40 = Σx_i^3	R54 = $\Sigma\ x_i^2 * \ln y_i$

where x and y are the values associated with each data point and n is the total number of points. The following terms must now be calculated in order to obtain the coefficients of the equation:

R05 = R17*R21 - (R16)²

R06 = R21*R54 - R17*R30

R07 = R21*R40 - R16*R17

R08 = R21*R46 - R16*R30

R09 = R21*R43 - (R17)²

$$R13 = (R05*R06 - R07*R08)/(R05*R09 - R07^2)$$

$$R12 = (R08 - R07*R13)/R05$$

$$R11 = (R30 - R12*R16 - R13*R17)/R21$$

The coefficients of the best fit curve are:

$$a = e^{[R11 - (R12)^2/(4*R13)]}$$

$$b = -(R12)/(2*R13)$$

$$c = 1/R13$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R30 + R12*R46 + R13*R54 - (R30)^2/R21}{R31 - (R30)^2/R21}$$

When RR is to used for comparison with other regression curves, it should be corrected as follows to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 3)}$$

Example:

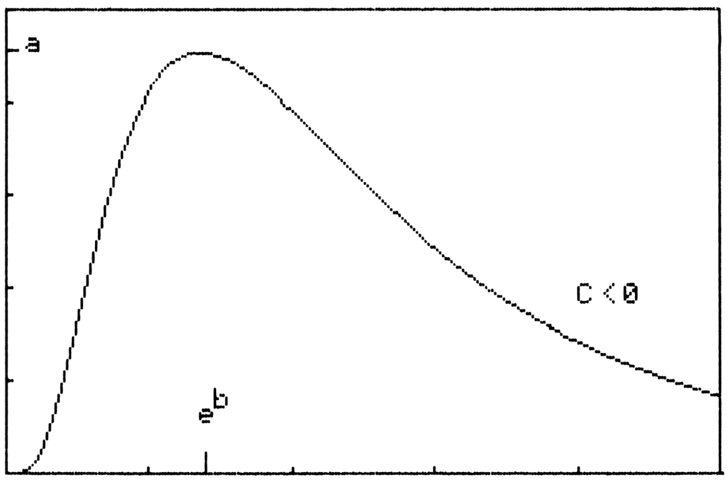
X =	1.0	2.0	3.0	4.0	5.0
Y =	0.9	3.7	4.7	1.8	0.2

R16 =	15.000	R07 =	300.000
R17 =	55.000	R08 =	-18.644
R21 =	5.000	R09 =	1,870.000
R30 =	1.729	R11 =	-2.746
R31 =	7.054	R12 =	3.236
R40 =	225.000	R13 =	-0.601
R43 =	979.000	a =	4.986
R46 =	1.458	b =	2.690
R54 =	-11.775	c =	-1.663
R05 =	50.000	RR =	1.000
R06 =	-153.965		

LOG-NORMAL DISTRIBUTION

General Equation: $y = a\, e^{(b - \ln x)^2/c}$

The log-normal distribution is often used when the range of data spans several orders of magnitude. It is frequently applied in economics, biology, and the physical sciences when the data results from numerous multiplicative factors. The following formulas will estimate the coefficients of this curve when four or more points are given. Both x and y must be positive numbers greater than zero.



$R21 = n$

$R28 = \Sigma \ln x_i$

$R29 = \Sigma (\ln x_i)^2$

$R30 = \Sigma \ln y_i$

$R31 = \Sigma (\ln y_i)^2$

$R32 = \Sigma \ln x_i * \ln y_i$

$R55 = \Sigma (\ln x_i)^3$

$R56 = \Sigma (\ln x_i)^4$

$R57 = \Sigma (\ln x_i)^2 * \ln y_i$

where x and y are the values associated with each data point and n is the total number of points. The following terms must now be calculated in order to obtain the coefficients of the equation:

$R05 = R21*R29 - (R28)^2$

$R06 = R21*R57 - R29*R30$

$R07 = R21*R55 - R28*R29$

$R08 = R21*R32 - R28*R30$

$R09 = R21*R56 - (R29)^2$

$$R13 = (R05*R06 - R07*R08)/(R05*R09 - R07^2)$$

$$R12 = (R08 - R07*R13)/R05$$

$$R11 = (R30 - R12*R28 - R13*R29)/R21$$

The coefficients of the best fit curve are:

$$a = e^{[R11 - (R12)^2/(4*R13)]}$$

$$b = -(R12)/(2*R13)$$

$$c = 1/R13$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R30 + R12*R32 + R13*R57 - (R30)^2/R21}{R31 - (R30)^2/R21}$$

When RR is to used for comparison with other regression curves, it should be corrected as follows to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 3)}$$

Example:

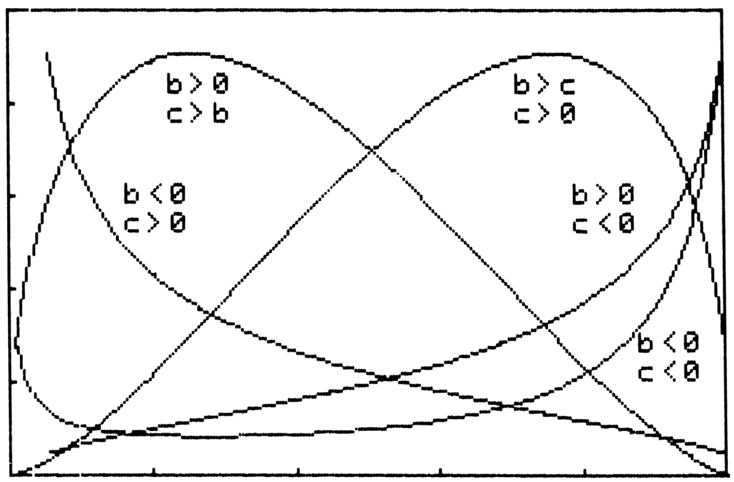
X =	100	200	300	400	500
Y =	330	300	270	240	220

R21 =	5.000	R07 =	87.176
R28 =	27.813	R08 =	-2.020
R29 =	156.332	R09 =	942.656
R30 =	27.976	R11 =	3.268
R31 =	156.634	R12 =	1.145
R32 =	155.215	R13 =	-0.129
R55 =	887.058	a =	331.304
R56 =	5,076.463	b =	4.429
R57 =	870.288	c =	-7.737
R05 =	8.077	RR =	0.999
R06 =	-22.034		

BETA DISTRIBUTION

General Equation: $y = a\, x^{b}\, (1 - x)^{c}$

The beta distribution is sometimes encountered in statistics when the independent variable ranges between zero and one. The following formulas will estimate the coefficients of this equation when four or more points are given. X must be between zero and one and y must be a positive number greater than zero (any small number may be substituted for 0).



$R21 = n$

$R28 = \Sigma \ln x_i$

$R29 = \Sigma (\ln x_i)^2$

$R30 = \Sigma \ln y_i$

$R31 = \Sigma (\ln y_i)^2$

$R32 = \Sigma \ln x_i * \ln y_i$

$R59 = \Sigma \ln(1-x_i)$

$R60 = \Sigma [\ln(1-x_i)]^2$

$R61 = \Sigma \ln x_i * \ln(1-x_i)$

$R62 = \Sigma \ln y_i * \ln(1-x_i)$

where x and y are the values associated with each data point and n is the total number of points. The following terms must now be calculated in order to obtain the coefficients of the equation:

$R05 = R21*R29 - (R28)^2$

$R06 = R21*R62 - R30*R59$

$R07 = R21*R61 - R28*R59$

$R08 = R21*R32 - R28*R30$

$R09 = R21*R60 - (R59)^2$

$$R13 = (R05*R06 - R07*R08)/(R05*R09 - R07^2)$$

$$R12 = (R08 - R07*R13)/R05$$

$$R11 = (R30 - R12*R28 - R13*R59)/R21$$

The coefficients of the best fit curve are:

$$a = e^{R11}$$

$$b = R12$$

$$c = R13$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R30 + R12*R32 + R13*R62 - (R30)^2/R21}{R31 - (R30)^2/R21}$$

When RR is to used for comparison with other regression curves, it should be corrected as follows to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 3)}$$

Example:

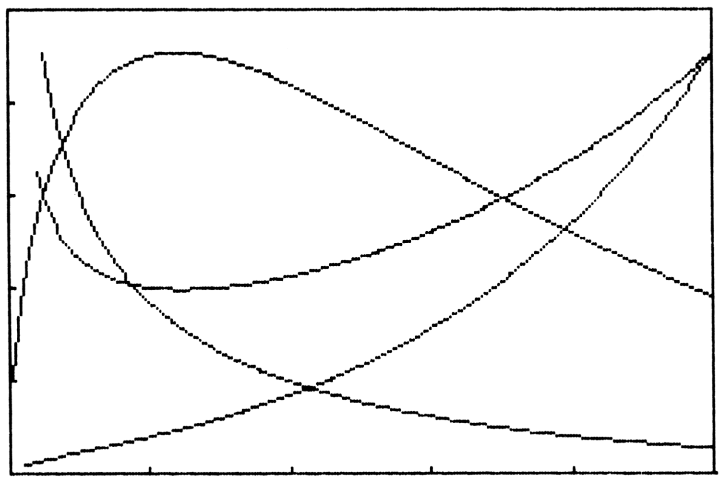
X =	0.1	0.2	0.3	0.4	0.5
Y =	1.0	1.6	2.2	2.6	3.1

R21 =	5.000	R05 =	8.077
R28 =	-6.725	R06 =	-1.972
R29 =	10.662	R07 =	-2.807
R30 =	3.345	R08 =	5.672
R31 =	3.036	R09 =	1.079
R32 =	-3.365	R11 =	1.603
R59 =	-1.889	a =	4.968
R60 =	0.930	b =	0.698
R61 =	1.980	c =	-0.012
R62 =	-1.658	RR =	0.999

GAMMA DISTRIBUTION

General Equation: $y = a\ (x/b)^{c}\ e^{x/b}$

The gamma distribution is commonly used in statistics to describe the sum of several identical exponential distributions. It is essentially the same as the Hoerl function. The following formulas will estimate the coefficients of such a curve when four or more points are given. Both x and y must be positive numbers greater than zero (any small number may be substituted for 0).



R16 = x_i

R17 = x_i^2

R21 = n

R28 = $\ln x_i$

R29 = $(\ln x_i)^2$

R30 = $\ln y_i$

R31 = $(\ln y_i)^2$

R32 = $\ln x_i * \ln y_i$

R46 = $x_i * \ln y_i$

R48 = $x_i * \ln x_i$

where x and y are the values associated with each data point and n is the total number of points. The following terms must now be calculated in order to obtain the coefficients of the equation:

R05 = R17*R21 - (R16)²

R06 = R21*R32 - R28*R30

R07 = R21*R48 - R16*R28

R08 = R21*R46 - R16*R30

R09 = R21*R29 - (R28)²

$$R13 = (R05*R06 - R07*R08)/(R05*R09 - R07^2)$$

$$R12 = (R08 - R07*R13)/R05$$

$$R11 = (R30 - R12*R16 - R13*R28)/R21$$

The coefficients of the best fit curve are:

$$a = e^{[R11 + R13*\ln(1/R12)]}$$

$$b = 1/R12$$

$$c = R13$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R30 + R12*R46 + R13*R32 - (R30)^2/R21}{R31 - (R30)^2/R21}$$

When RR is to used for comparison with other regression curves, it should be corrected as follows to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 3)}$$

Example:

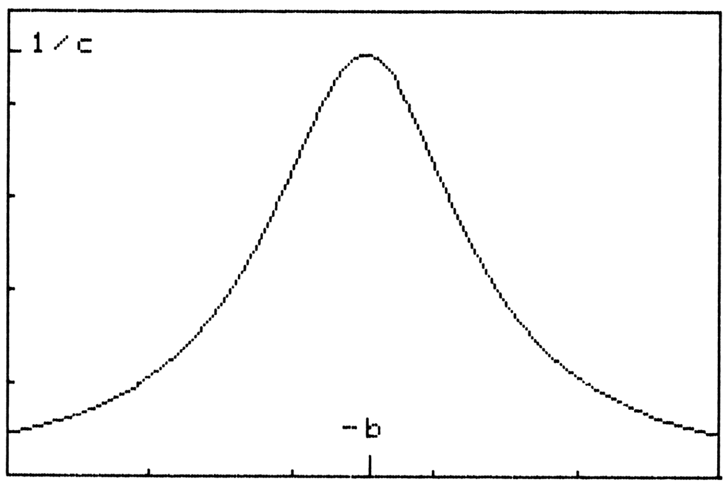
X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R16 =	15.000	R06 =	4.850
R17 =	55.000	R07 =	19.560
R21 =	5.000	R08 =	12.504
R28 =	4.787	R09 =	8.077
R29 =	6.200	R11 =	0.723
R30 =	7.468	R12 =	0.288
R31 =	11.789	a =	1.826
R32 =	8.121	b =	3.475
R46 =	24.904	c =	-0.096
R48 =	18.274	RR =	0.985
R05 =	50.000		

CAUCHY DISTRIBUTION

General Equation: $$y = \frac{1}{a(x+b)^2 + c}$$

The cauchy distribution is sometimes used in statistics to describe distributions that have a mean but no standard deviation. The following formulas will estimate the coefficients of this curve when four or more points are given. Y must not be equal to zero (any small number may be substituted for y when it is).



$R16 = \Sigma x_i$

$R17 = \Sigma x_i^2$

$R21 = n$

$R24 = \Sigma 1/y_i$

$R25 = \Sigma 1/y_i^2$

$R34 = \Sigma x_i/y_i$

$R37 = \Sigma x_i^2/y_i$

$R40 = \Sigma x_i^3$

$R43 = \Sigma x_i^4$

where x and y are the values associated with each data point and n is the total number of points. The following terms must now be calculated in order to obtain the coefficients of the equation:

$R05 = R17*R21 - (R16)^2$

$R06 = R21*R37 - R17*R24$

$R07 = R21*R40 - R16*R17$

$R08 = R21*R34 - R16*R24$

$R09 = R21*R43 - (R17)^2$

$$R13 = (R05*R06 - R07*R08)/(R05*R09 - R07^2)$$

$$R12 = (R08 - R07*R13)/R05$$

$$R11 = (R24 - R12*R16 - R13*R17)/R21$$

The coefficients of the best fit curve are:

$$a = R13$$

$$b = R12/(2*R13)$$

$$c = R11 - [(R12)^2/(4*R13)]$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R24 + R12*R34 + R13*R37 - (R24)^2/R21}{R25 - (R24)^2/R21}$$

When RR is to used for comparison with other regression curves, it should be corrected as follows to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 3)}$$

Example:

X =	1.0	2.0	3.0	4.0	5.0
Y =	2.8	3.2	4.6	5.9	7.2

R16 =	15.000	R06 =	-17.009
R17 =	55.000	R07 =	300.000
R21 =	5.000	R08 =	-2.898
R24 =	1.195	R09 =	1,870.000
R25 =	0.320	R11 =	0.451
R34 =	3.007	R12 =	-0.090
R37 =	9.748	a =	0.005
R40 =	225.000	b =	-8.388
R43 =	979.000	c =	0.072
R05 =	50.000	RR =	0.980

MULTIPLE LINEAR REGRESSION

TWO INDEPENDENT VARIABLES

General Equation: $z = a + bx + cy$

This equation can be used to fit any linear equation involving two independent variables. The following formulas will estimate the coefficients of such an equation when four or more points are given. X, y and z may be positive, negative, or equal to zero.

$R16 = \sum x_i$ $\quad R19 = \sum y_i^2$ $\quad R64 = \sum z_i^2$

$R17 = \sum x_i^2$ $\quad R20 = \sum x_i * y_i$ $\quad R65 = \sum x_i * z_i$

$R18 = \sum y_i$ $\quad R21 = n$ $\quad R66 = \sum y_i * z_i$

$R63 = \sum z_i$

where x, y and z are the values associated with each data point and n is the total number of points. The following terms must now be calculated in order to obtain the coefficients of the equation:

R05 = R17*R21 - (R16)²

R07 = R20*R21 - R16*R18

R06 = R21*R66 - R18*R63

R08 = R21*R65 - R16*R63

R09 = R19*R21 - (R18)²

R13 = (R05*R06 - R07*R08)/(R05*R09 - R07²)

R12 = (R08 - R07*R13)/R05

R11 = (R63 - R12*R16 - R13*R18)/R21

The coefficients of the best fit curve are:

a = R11

b = R12

c = R13

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R63 + R12*R65 + R13*R66 - (R63)^2/R21}{R64 - (R63)^2/R21}$$

When RR is to used for comparison with other regression curves, it should be corrected as follows to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 3)}$$

Example:

X =	1.1	2.3	3.2	4.5	5.1
Y =	1.7	3.0	5.2	7.1	9.2
Y =	2.8	4.6	3.8	4.9	3.3

R16 =	16.200	R05 =	52.560
R17 =	63.000	R06 =	9.070
R18 =	26.200	R07 =	96.960
R19 =	173.980	R08 =	9.220
R20 =	104.280	R09 =	183.460
R21 =	5.000	a =	2.038
R63 =	19.400	b =	3.364
R64 =	78.340	c =	-1.728
R65 =	64.700	RR =	1.000
R66 =	103.470		

MULTIPLE LINEAR REGRESSION

THREE INDEPENDENT VARIABLES

General Equation: $t = a + bx + cy + dz$

This regression can be used to fit any linear equation involving three independent variables. The following formulas will estimate the coefficients of such an equation when five or more points are given. X, y, z and t may be positive, negative, or equal to zero.

$R16 = \Sigma x_i$	$R21 = n$	$R67 = \Sigma t_i$
$R17 = \Sigma x_i^2$	$R63 = \Sigma z_i$	$R68 = \Sigma t_i^2$
$R18 = \Sigma y_i$	$R64 = \Sigma z_i^2$	$R69 = \Sigma x_i * t_i$
$R19 = \Sigma y_i^2$	$R65 = \Sigma x_i * z_i$	$R70 = \Sigma y_i * t_i$
$R20 = \Sigma x_i * y_i$	$R66 = \Sigma y_i * z_i$	$R71 = \Sigma z_i * t_i$

where x, y, z and t are the values associated with each data point and n is the total number of points. The following terms must now be calculated in order to obtain the coefficients of the equation:

$R81 = (R20^2 - R17*R19)*R63 + (R17*R66 - R20*R65)*R18 + (R19*R65 - R20*R66)*R16$

$R82 = (R20*R21 - R16*R18)*R66 + (R16*R19 - R18*R20)*R63 + (R18^2 - R19*R21)*R65$

$R83 = (R17*R63 - R16*R65)*R18 + (R21*R65 - R16*R63)*R20 + (R16^2 - R17*R21)*R66$

$R84 = R17*R21 - R16^2$

$R85 = R19*R84 + 2*(R16*R18*R20) - (R21*R20^2) - (R17*R18^2)$

$R86 = R21*R69 - R16*R67$

$R87 = R20*R21 - R16*R18$

$R88 = R21*R65 - R16*R63$

$R89 = (R21*R70 - R18*R67)*R84 - (R86*R87)$

$$R14 = \frac{(R67*R81 + R69*R82 + R70*R83 + R71*R85)}{(R63*R81 + R65*R82 + R66*R83 + R64*R85)}$$

$$R13 = \frac{[(R18*R63 - R21*R66)*R84 + (R87*R88)]*R14 + R89}{(R19*R21 - R18^2)*R84 - (R87)^2}$$

$$R12 = \frac{R86 - R13*R87 - R14*R88}{R84}$$

$$R11 = \frac{R67 - R12*R16 - R13*R18 - R14*R63}{R21}$$

The coefficients of the best fit curve are:

$$a = R11$$

$$b = R12$$

$$c = R13$$

$$d = R14$$

The goodness of fit (coefficient of determination) is calculated from the following expression:

$$RR = \frac{R11*R67 + R12*R69 + R13*R70 + R14*R71 - (R67)^2/R21}{R68 - (R67)^2/R21}$$

When RR is to used for comparison with other regression curves, it should be corrected as follows to obtain an unbiased estimate for the coefficient of determination.

$$RR_{corrected} = 1 - \frac{(1 - RR)*(R21 - 1)}{(R21 - 4)}$$

Example:

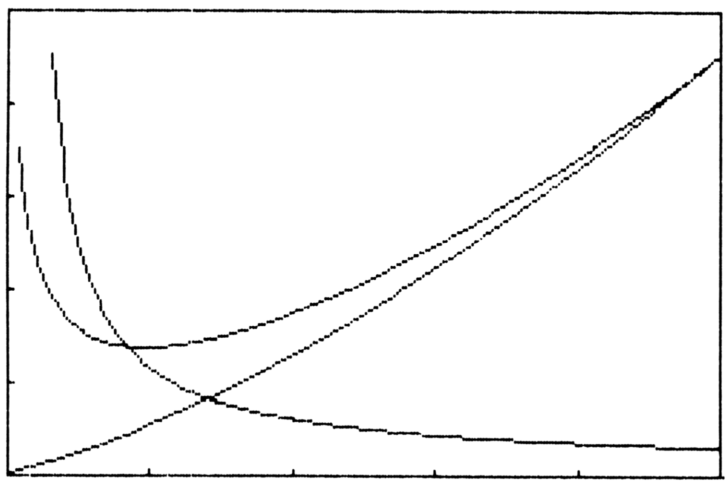
X =	1.0	2.0	3.0	4.0	5.0	6.0
Y =	2.8	3.2	4.6	5.8	7.2	8.4
Z =	4.8	2.6	9.4	6.4	2.4	8.0
T =	4.0	2.5	8.8	5.4	1.1	6.8

R16 =	21.000	R81 =	−153.768
R17 =	91.000	R82 =	−0.976
R18 =	32.000	R83 =	−10.680
R19 =	195.280	R84 =	105.000
R20 =	132.600	R85 =	38.240
R21 =	6.000	R86 =	19.200
R63 =	33.600	R87 =	123.600
R64 =	228.880	R88 =	37.200
R65 =	123.800	R89 =	−163.920
R66 =	186.600	a =	0.941
R67 =	28.600	b =	0.998
R68 =	176.300	c =	−0.996
R69 =	103.300	d =	1.008
R70 =	156.040	RR =	0.998
R71 =	200.020		

GENERALIZED 2ND ORDER POLYNOMIAL

General Equation: $y = ax^r + bx^s$

This is a generalized second order polynomial where r and s must both be specified to use the regression. When r equals zero and s equals one, it reduces to a straight line equation. When r and s have values other than zero, the curve will pass through the origin. The following formulas will estimate the coefficients of such an equation when three or more points are given. Y may be positive, negative, or equal to zero. If either r or s is negative, x must not be equal to zero. If either r or s is non-integer, x must be positive.



$R18 = \Sigma y_i$ $\quad$ $R72 = \Sigma x_i^{2r}$ $\quad$ $R78 = \Sigma y_i * x_i^r$

$R19 = \Sigma y_i^2$ $\quad$ $R73 = \Sigma x_i^{2s}$ $\quad$ $R79 = \Sigma y_i * x_i^s$

$R21 = n$ $\quad$ $R75 = \Sigma x_i^{r+s}$

where x and y are the values associated with each data point and n is the total number of points. The following terms must now be calculated in order to obtain the coefficients of the equation:

$$R12 = (R72*R79 - R75*R78)/(R72*R73 - R75^2)$$

$$R11 = (R78 - R12*R75)/R72$$

The coefficients of the best fit curve are:

$$a = R11$$

$$b = R12$$

Example:

X =	1.0	2.0	3.0	4.0	5.0
Y =	1.0	0.9	0.5	0.3	0.2

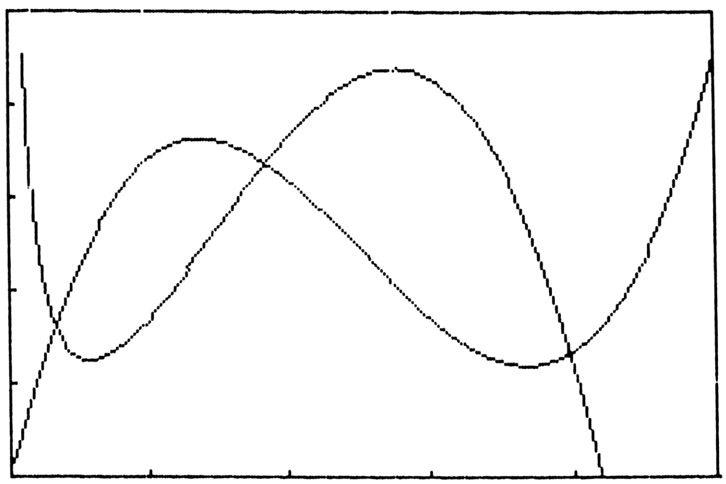
Given that r = −2 and s = −3, find the coefficients of the best fitting curve.

r =	−2.000	R73 =	1.017
s =	−3.000	R75 =	1.037
R18 =	2.900	R78 =	1.307
R19 =	2.190	R79 =	1.137
R21 =	5.000	a =	6.191
R72 =	1.080	b =	−5.191

GENERALIZED 3RD ORDER POLYNOMIAL

General Equation: $y = ax^r + bx^s + cx^t$

This is a generalized third order polynomial where the values of r, s and t must be specified to use the regression. When r equals zero, s equals one, and t equals 2, it reduces to a parabola. When r, s and t have values other than zero, the curve will pass through the origin. The following formulas will estimate the coefficients of such an equation when four or more points are given. Y may be positive, negative, or equal to zero. If r, s or t is negative, x must not be equal to zero. If r, s or t is non-integer, x must be positive.



$R18 = \Sigma\, y_i$

$R19 = \Sigma\, y_i^2$

$R21 = n$

$R72 = \Sigma\, x_i^{2r}$

$R73 = \Sigma\, x_i^{2s}$

$R74 = \Sigma\, x_i^{2t}$

$R75 = \Sigma\, x_i^{r+s}$

$R76 = \Sigma\, x_i^{r+t}$

$R77 = \Sigma\, x_i^{s+t}$

$R78 = \Sigma\, y_i * x_i^r$

$R79 = \Sigma\, y_i * x_i^s$

$R80 = \Sigma\, y_i * x_i^t$

where x and y are the values associated with each data point and n is the total number of points. The following terms must now be calculated in order to obtain the coefficients of the equation:

$$R81 = (R72*R73*R80) + (R75*R76*R79) + (R75*R77*R78) - R80*(R75)^2 - (R73*R76*R78) - (R72*R77*R79)$$

$$R82 = (R72*R73*R74) + 2*(R75*R76*R77) - R74*(R75)^2 - R73(R76)^2 - R72*(R77)^2$$

$$R13 = (R81/R82)$$

$$R12 = \frac{(R75*R76 - R72*R77)*R13 + R72*R79 - R75*R78}{R72*R73 - (R75)^2}$$

$$R11 = (R78 - R12*R75 - R13*R76)/R72$$

The coefficients of the best fit curve are:

a = R11

b = R12

c = R13

Example:

X =	1.0	2.0	3.0	4.0	5.0
Y =	0.5	1.4	2.8	4.6	6.6

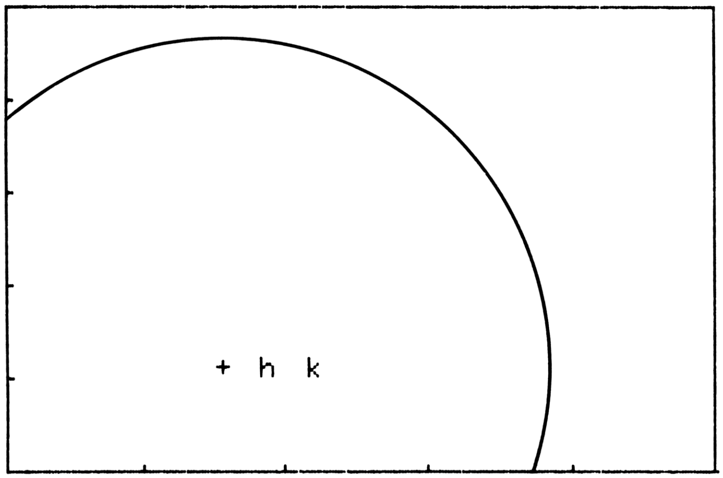
Given that the exponents of x are: r = 0.2, s = 0.8, and t = 1.2, calculate the coefficients of the best fitting curve.

r =	0.200	R76 =	24.777
s =	0.800	R77 =	55.000
t =	1.200	R78 =	20.772
R18 =	15.900	R79 =	47.543
R19 =	74.770	R80 =	83.991
R21 =	5.000	R81 =	4.552
R72 =	7.516	R82 =	1.914
R73 =	32.153	a =	1.328
R74 =	95.694	b =	-3.209
R75 =	15.000	c =	2.378

CIRCLE

General Equation: $r^2 = (x - h)^2 + (y - k)^2$

This is the general equation for a circle with a radius of r and center at h,k. It is used in surveying to determine the best fit circle for points which lie on an arc. The circle is best fit in the sense that the squared deviation of the area is minimized. The following formulas will determine the coordinates of a best fit circle when four or more points are given. X and y may be positive, negative, or equal to zero.



$R16 = \Sigma x_i$

$R17 = \Sigma x_i^2$

$R18 = \Sigma y_i$

$R19 = \Sigma y_i^2$

$R20 = \Sigma x_i * y_i$

$R21 = n$

$R36 = \Sigma y_i * x_i^2$

$R39 = \Sigma x_i * y_i^2$

$R40 = \Sigma x_i^3$

$R42 = \Sigma y_i^3$

where x and y are the values associated with each data point and n is the total number of points.

$$R05 = R20*R21 - R16*R18$$

$$R06 = (R17 + R19)*R18 - (R36 + R42)*R21$$

$$R07 = (R18)^2 - R19*R21$$

$$R08 = (R17 + R19)*R16 - (R39 + R40)*R21$$

$$R09 = (R16)^2 - R17*R21$$

$$R10 = R07*R09 - (R05)^2$$

The solution is undefined whenever R08 is equal to zero. Otherwise the parameters of the best fit circle are

$$h = (R05*R06 + R07*R08)/(2*R10)$$

$$k = (R05*R08 + R06*R09)/(2*R10)$$

After the values of h and k have been determined, the radius of the circle can be calculated as follows:

$$r = \sqrt{\frac{R17 + R19 - 2*(h*R16 + k*R18) + (h^2 + k^2)*R21}{R21}}$$

Example:

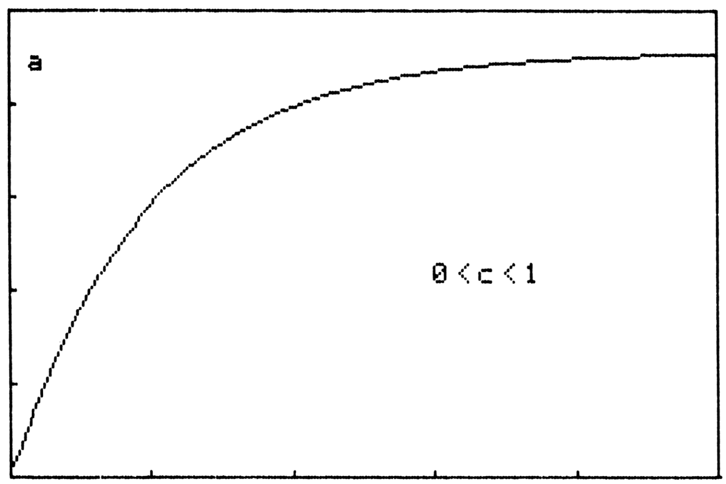
X =	6	10	13	13	14
Y =	12	11	8	0	4

R16 =	56	R05 =	−250
R17 =	670	R06 =	−990
R18 =	35	R07 =	−500
R19 =	345	R08 =	−580
R20 =	342	R09 =	−214
R21 =	5	R10 =	44,500
R36 =	3,668	h =	6.04
R39 =	3,130	k =	4.01
R40 =	8,354	r =	8.01
R42 =	3,635		

CORRECTION EXPONENTIAL

General Equation: $y = a + bc^x$

The correction exponential curve is commonly used in economic forecasting but it generally requires iterative techniques to find the coefficients. If the values of x are evenly spaced, however, the following non-iterative technique can be applied to estimate the coefficients.



The x,y pairs must first be arranged in ascending order starting with the lowest value of x and ending with the highest value of x. The data must now be divided into three even groups with the same number of points in each group. The first one or two data points can be discarded in order to make the groups come out even. Now replace each x value with the numbers zero through N-1 where N is the total number of points to be used. Sums for each of the groups are calculated as follows:

$$R05 = \sum_{i=0}^{(N-3)/3} y_i$$

$$R06 = \sum_{i=N/3}^{(2N-3)/3} y_i$$

$$R07 = \sum_{i=(2N/3)}^{(N-1)} y_i$$

The following terms must now be calculated in order to obtain the coefficients of the equation.

$$R08 = (R07 - R06)/(R06 - R05)$$

$$R13 = (R08)^{3/N}$$

$$R12 = (R06 - R05)*(R13 - 1)/(R08 - 1)^2$$

$$R11 = \frac{R05 - [(R08 - 1)/(R13 - 1)]*R12}{(N/3)}$$

The coefficients of the best fit correction exponential are

$$a = R11$$

$$b = R12$$

$$c = R13$$

Example:

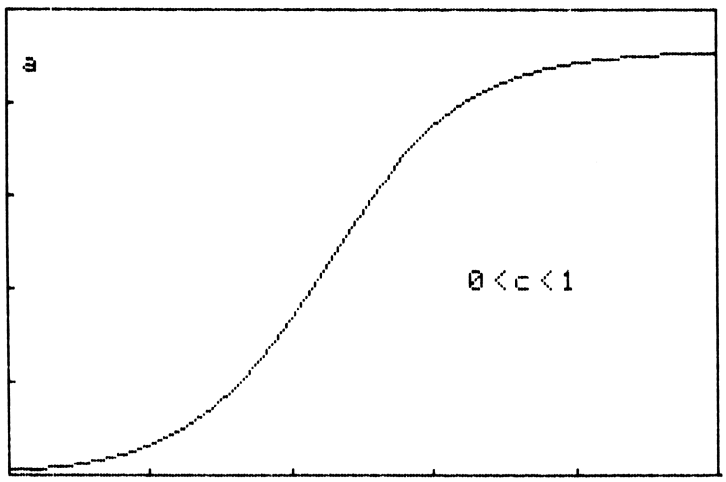
Year	i	Production
1922	0	33.8
1923	1	38.9
1924	2	37.7
1925	3	42.5
1926	4	46.3
R05	=	199.2
1927	5	50.6
1928	6	55.2
1929	7	58.9
1930	8	58.0
1931	9	60.5
R06	=	283.2
1932	10	62.8
1933	11	63.5
1934	12	60.4
1935	13	63.9
1936	14	68.2
R07	=	318.8

N	=	15.000
R08	=	0.424
R11	=	68.997
R12	=	-39.917
R13	=	0.842

LOGISTIC CURVE

General Equation: $$y = \frac{a}{1 + bc^x}$$

The logistic curve is commonly used in economic forecasting but it generally requires iterative techniques to find the coefficients. If the values of x are evenly spaced, however, the following non-iterative technique can be applied to estimate the coefficients.



The x,y pairs must first be arranged in ascending order starting with the lowest value of x and ending with the highest value of x. The data must now be divided into three even groups with the same number of points in each group. The first one or two data points can be discarded in order to make the groups come out even. Now replace each x value with the numbers zero through N-1 where N is the total number of points to be used. Sum for each of the groups are calculated as follows:

$$R05 = \sum_{i=0}^{(N-3)/3} 1/y_i$$

$$R06 = \sum_{i=N/3}^{(2N-3)/3} 1/y_i$$

$$R07 = \sum_{i=(2N/3)}^{(N-1)} 1/y_i$$

The following terms must now be calculated in order to obtain the coefficients of the equation.

$$R08 = (R07 - R06)/(R06 - R05)$$

$$R13 = (R08)^{3/N}$$

$$R12 = (R06 - R05)*(R13 - 1)/(R08 - 1)^2$$

$$R11 = \frac{R05 - [(R08 - 1)/(R13 - 1)]*R12}{(N/3)}$$

The coefficients of the best fit correction exponential are

$$a = 1/R11$$

$$b = R12/R11$$

$$c = R13$$

Example:

Year	i	Production	1/Production
1956	0	40	0.02500
1957	1	50	0.02000
1958	2	67	0.01493
1959	3	88	0.01136
1960	4	119	0.00840
1961	5	146	0.00685
R05		=	0.0865
1962	6	182	0.00549
1963	7	123	0.00813
1964	8	273	0.00336
1965	9	322	0.00311
1966	10	388	0.00258
1967	11	475	0.00211
R06		=	0.02508
1968	12	591	0.00169
1969	13	713	0.00140
1970	14	845	0.00118
1971	15	983	0.00102
1972	16	1143	0.00087
1973	17	1256	0.00080
R07		=	0.00697

N	=	18.000
R08	=	0.295
R11	=	-9.961 E-05
R12	=	0.023
R13	=	0.816
a	=	-10,039.033
b	=	-228.532
c	=	0.816

ABBREVIATIONS AND SYMBOLS

e	the base of the natural logarithm (2.71828....)
e^x	e raised to the x power
i	an index used to refer to the ith data point
ln x	the natural logarithm of x
n	the total number of data points
Rnn	the contents of calculator register nn
RR	the coefficient of determination (the square-root of RR is the correlation coefficient)
x_i	the value of x at the ith data point
y_i	the value of y at the ith data point
$\|X\|$	the absolute value of x
X!	the factorial of X, e.g., 5! = 5x4x3x2x1 = 120
Σ	the sum of all terms from the first data point to the last

REGISTER ASSIGNMENTS

R00 = curve number

R01 = a or last X

R02 = b or last Y

R03 = c

R04 = d

R05 = used

R06 = used

R07 = used

R08 = used

R09 = used

R10 = corrected RR

R11 = used

R12 = used

R13 = used

R14 = used

R15 = best curve

R16 = $\Sigma\ x_i$

R17 = $\Sigma\ x_i^2$

R18 = $\Sigma\ y_i$

R19 = $\Sigma\ y_i^2$

R20 = $\Sigma\ x_i * y_i$

R21 = n

R22 = $\Sigma\ 1/x_i$

R23 = $\Sigma\ 1/x_i^2$

R24 = $\Sigma\ 1/y_i$

R25 = $\Sigma\ 1/y_i^2$

R26 = $\Sigma\ 1/(x_i * y_i)$

R27 = $\Sigma\ \ln(x_i!)$

R28 = $\Sigma\ \ln\ x_i$

R29 = $\Sigma\ (\ln\ x_i)^2$

R30 = $\Sigma\ \ln\ y_i$

R31 = $\Sigma\ (\ln\ y_i)^2$

R32 = $\Sigma\ (\ln\ x_i)*(\ln\ y_i)$

R33 = $\Sigma\ x_i * \ln(x_i!)$

R34 = $\Sigma\ x_i/y_i$

R35 = $\Sigma\ y_i/x_i$

R36 = $\Sigma\ x_i^2 * y_i$

R37 = $\Sigma\ x_i^2/y_i$

R38 = $\Sigma\ y_i/x_i^2$

R39 = $\Sigma\ x_i * y_i^2$

R40 = $\Sigma\ x_i^3$

R41 = $\Sigma\ 1/x_i^3$

R42 = $\Sigma\ y_i^3$

R43 = $\Sigma\ x_i^4$

R44 = $\Sigma\ 1/x_i^4$

R45 = $\Sigma\ (\ln\ x_i)/x_i$

REGISTER ASSIGNMENTS

R46 = $\Sigma\ x_i * \ln y_i$

R47 = $\Sigma\ (\ln y_i)/x_i$

R48 = $\Sigma\ x_i * \ln x_i$

R49 = $\Sigma\ (x_i * \ln x_i)^2$

R50 = $\Sigma\ x_i * (\ln x_i) * (\ln y_i)$

R51 = $\Sigma\ y_i * \ln x_i$

R52 = $\Sigma\ (\ln x_i)/y_i$

R53 = $\Sigma\ [(\ln x_i)/x_i]^2$

R54 = $\Sigma\ x_i^2 * \ln y_i$

R55 = $\Sigma\ (\ln x_i)^3$

R56 = $\Sigma\ (\ln x_i)^4$

R57 = $\Sigma\ (\ln x_i)^2 * \ln y_i$

R58 = $\Sigma\ (\ln x_i * \ln y_i)/x_i$

R59 = $\Sigma\ \ln(1 - x_i)$

R60 = $\Sigma\ [\ln(1 - x_i)]^2$

R61 = $\Sigma\ (\ln x_i) * \ln(1 - x_i)$

R62 = $\Sigma\ (\ln y_i) * \ln(1 - x_i)$

R63 = $\Sigma\ z_i$

R64 = $\Sigma\ z_i^2$

R65 = $\Sigma\ x_i * z_i$

R66 = $\Sigma\ y_i * z_i$

R67 = $\Sigma\ t_i$

R68 = $\Sigma\ t_i^2$

R69 = $\Sigma\ x_i * t_i$

R70 = $\Sigma\ y_i * t_i$

R71 = $\Sigma\ z_i * t_i$

R72 = $\Sigma\ x_i^{2r}$

R73 = $\Sigma\ x_i^{2s}$

R74 = $\Sigma\ x_i^{2t}$

R75 = $\Sigma\ x_i^{r+s}$

R76 = $\Sigma\ x_i^{r+t}$

R77 = $\Sigma\ x_i^{s+t}$

R78 = $\Sigma\ y_i * x_i^r$

R79 = $\Sigma\ y_i * x_i^s$

R80 = $\Sigma\ y_i * x_i^t$

R81 = used

R82 = used

R83 = used

R84 = used

R85 = used

R86 = used

R87 = used

R88 = used

R89 = used

DERIVATION OF A REGRESSION FORMULA

The following example illustrates how a regression formula can be derived for the equation

$$y = ax + bx^2$$

The estimated values of Y for this equation that correspond to given values of X, i.e., X_1, X_2, X_3, ..., X_n are

$$aX_1+bX_1^2,\ aX_2+bX_2^2,\ aX_3+bX_3^2,\ \ldots\ldots\ldots\ldots,\ aX_n+bX_n^2$$

The actual values of Y, however, are Y_1, Y_2, Y_3, ..., Y_n.

The least square regression curve is the one that minimizes the sum of the squared differences between the estimated Y values and the actual Y values.

$$S = (aX_1 + bX_1^2 - Y_1)^2 + (aX_2 + bX_2^2 - Y_2)^2 + \ldots\ldots + (aX_n + bX_n^2 - Y_n)^2$$

The minimum value of S can be determined from the calculus by taking the partial derivatives with respect to each coefficient (a and b in this case) and equating them to zero.

$$\frac{\partial S}{\partial a} = 0 = 2(aX_1 + bX_1^2 - Y_1)X_1 + 2(aX_2 + bX_2^2 - Y_2)X_2 + \ldots$$
$$\ldots + 2(aX_n + bX_n^2 - Y_1)X_n$$

$$\frac{\partial S}{\partial b} = 0 = 2(aX_1 + bX_1^2 - Y_1)X_1^2 + 2(aX_2 + bX_2^2 - Y_2)X_2^2 + \ldots$$
$$\ldots + 2(aX_n + bX_n^2 - Y_1)X_n^2$$

These equations can be reduced and expressed in summation form as

$$a \Sigma X^2 + b \Sigma X^3 - \Sigma XY = 0 \qquad (1)$$

$$b \Sigma X^3 + b \Sigma X^4 - \Sigma X^2 Y = 0 \qquad (2)$$

Rewriting (1) and (2) to solve for the coefficients a and b gives:

$$a = \frac{\Sigma XY - b \Sigma X^3}{\Sigma X^2} \qquad (3)$$

$$b = \frac{\Sigma X^2 Y - a \Sigma X^3}{\Sigma X^4} \qquad (4)$$

Solve for the coefficients a and b by first substituting b from equation (4) into equation (3), and then substituting a from equation (3) into equation (4).

$$a = \frac{\Sigma XY * \Sigma X^4 - \Sigma X^2 Y * \Sigma X^3}{\Sigma X^2 * \Sigma X^4 - \Sigma X^3 Y * \Sigma X^3}$$

$$b = \frac{\Sigma X^2 * \Sigma X^2 Y - \Sigma XY * \Sigma X^3}{\Sigma X^2 * \Sigma X^4 - \Sigma X^3 Y * \Sigma X^3}$$

DERIVATION OF MODELS BY TRANSFORMATION

The general linear regression and multiple linear regression models can be used to fit many other curves. The following tables provide a framework for deriving many of the equations presented in the text and illustrate how new models can be developed by transforming X and Y. These tables are especially useful for developing your own curve fitting programs. Remember that a least squares fit based on transforms is not identical to a least squares fit of the original X and Y (see p. 2).

<u>One Independent Variable</u>

A general linear regression model for one independent variable can be written as:

$$R11 = (A*D - B*E)/(A*N - B^2)$$

$$R12 = (E*N - B*D)/(A*N - B^2)$$

These two equations can be used to fit many curves by simply substituting the appropriate transformations from Table 1.

TABLE 1. TRANSFORMATIONS FOR ONE INDEPENDENT VARIABLE

General Equation	Page	B	A	D	F	E	N	Coefficients a	b
$Y = a + bX$	12	ΣX	ΣX^2	ΣY	ΣY^2	ΣXY	n	R11	R12
$Y = a + b/X$	26	$\Sigma 1/X$	$\Sigma 1/X^2$	ΣY	ΣY^2	$\Sigma Y/X$	n	R11	R12
$Y = X/(aX + b)$	30	$\Sigma 1/X$	$\Sigma 1/X^2$	$\Sigma 1/Y$	$\Sigma 1/Y^2$	$\Sigma 1/(XY)$	n	R11	R12
$Y = 1/(a + bX)$	22	ΣX	ΣX^2	$\Sigma 1/Y$	$\Sigma 1/Y^2$	$\Sigma X/Y$	n	R11	R12
$Y = a + b*\ln X$	58	$\Sigma \ln X$	$\Sigma (\ln X)^2$	ΣY	ΣY^2	$\Sigma Y*\ln X$	n	R11	R12
$Y = 1/(a + b*\ln X)$	60	$\Sigma \ln X$	$\Sigma (\ln X)^2$	$\Sigma 1/Y$	$\Sigma 1/Y^2$	$\Sigma (\ln X)/Y$	n	R11	R12
$Y = aX^b$	42	$\Sigma \ln X$	$\Sigma (\ln X)^2$	$\Sigma \ln Y$	$\Sigma (\ln Y)^2$	$\Sigma \ln X*\ln Y$	n	e^{R11}	R12
$Y = ab^X$	44	ΣX	ΣX^2	$\Sigma \ln Y$	$\Sigma (\ln Y)^2$	$\Sigma X*\ln Y$	n	e^{R11}	R12
$Y = ab^{(1/X)}$	46	$\Sigma 1/X$	$\Sigma 1/X^2$	$\Sigma \ln Y$	$\Sigma (\ln Y)^2$	$\Sigma (\ln Y)/X$	n	e^{R11}	R12
$Y = aX^{bX}$	48	$\Sigma X*\ln X$	$\Sigma (X*\ln X)^2$	$\Sigma \ln Y$	$\Sigma (\ln Y)^2$	$\Sigma X*\ln X*\ln Y$	n	e^{R11}	R12
$Y = aX^{b/X}$	50	$\Sigma (\ln X)/X$	$\Sigma [(\ln X)/X]^2$	$\Sigma \ln Y$	$\Sigma (\ln Y)^2$	$\Sigma (\ln X*\ln Y)/X$	n	e^{R11}	R12

Two Independent Variables

A general multiple linear regression model for two independent variables can be solved as follows:

$$R05 = B*N - A^2$$

$$R06 = H*N - D*E$$

$$R07 = J*N - A*D$$

$$R08 = G*N - A*E$$

$$R09 = C*N - D^2$$

where N is the total number of data points. The different curves are fit by substituting appropriate transformations from Table 2 in these equations and then solving for the following

$$R13 = (R05*R06 - R07*R08)/(R05*R09 - R07^2)$$

$$R12 = (R08 - R07*R13)/R05$$

$$R11 = (E - A*R12 - D*R13)/N$$

The coefficients a, b and c are derived from R11, R12 and R13 as shown in the last three columns of the table.

TABLE 2. TRANSFORMATIONS FOR TWO INDEPENDENT VARIABLES

General Equation	Page	A	B	D	C	J	E	F	G	H	Coefficients a	b	c
$Y = a + bX + c/X$	32	ΣX	ΣX^2	$\Sigma 1/X$	$\Sigma 1/X^2$	n	ΣY	ΣY^2	ΣXY	$\Sigma Y/X$	R11	R12	R13
$Y = a + b/X + c/X^2$	34	$\Sigma 1/X$	$\Sigma 1/X^2$	$\Sigma 1/X^2$	$\Sigma 1/X^4$	$\Sigma 1/X^3$	ΣY	ΣY^2	$\Sigma Y/X$	$\Sigma Y/X^2$	R11	R12	R13
$Y = a + bX + cX^2$	36	ΣX	ΣX^2	ΣX^2	ΣX^4	ΣX^3	ΣY	ΣY^2	ΣXY	ΣX^2Y	R11	R12	R13
$Y = ab^X X^c$	64	ΣX	ΣX^2	$\Sigma \ln X$	$\Sigma (\ln X)^2$	$\Sigma X*\ln X$	$\Sigma \ln Y$	$\Sigma (\ln Y)^2$	$\Sigma X*\ln Y$	$\Sigma \ln X*\ln Y$	e^{R11}	e^{R12}	R13
$Y = ab^{1/X} X^c$	66	$\Sigma 1/X$	$\Sigma 1/X^2$	$\Sigma \ln X$	$\Sigma (\ln X)^2$	$\Sigma (\ln X)/X$	$\Sigma \ln Y$	$\Sigma (\ln Y)^2$	$\Sigma (\ln Y)/X$	$\Sigma \ln X*\ln Y$	e^{R11}	e^{R12}	R13
$Y = ae^{(b-X)^2/c}$	68	ΣX	ΣX^2	ΣX^2	ΣX^4	ΣX^3	$\Sigma \ln Y$	$\Sigma (\ln Y)^2$	$\Sigma X*\ln Y$	$\Sigma X^2\ln Y$	e^*	-R12/2R13	1/R13
$Y = ae^{(b-\ln X)^2/c}$	70	$\Sigma \ln X$	$\Sigma (\ln X)^2$	$\Sigma (\ln X)^2$	$\Sigma (\ln X)^4$	$\Sigma (\ln X)^3$	$\Sigma \ln Y$	$\Sigma (\ln Y)^2$	$\Sigma \ln X*\ln Y$	$\Sigma (\ln X)^2\ln Y$	e^*	-R12/2R13	1/R13
$Y = 1/[a(X+b)^2 + c]$	76	ΣX	ΣX^2	ΣX^2	ΣX^4	ΣX^3	$\Sigma 1/Y$	$\Sigma 1/Y^2$	$\Sigma X/Y$	$\Sigma X^2/Y$	R13	R12/2R13	*

* R11-[(R12)²/(4R13)]

MULTIPLE CURVE FITTING PROGRAM

HP-41C/V

This program fits up to 19 curves to an unlimited number of X,Y data points. Any curve can be selected by entering the appropriate curve number from the table below. The best fitting curve can be determined automatically based on the adjusted coefficient of determination, RR. Once a curve has been selected either manually or automatically, values of Y can be estimated for any given value of X.

Curve Number	Type	General Equation	Page
1	Linear	$Y = a + bX$	12
2	Reciprocal	$Y = 1/(a + bX)$	22
3	Linear-Hyperbolic	$Y = a + bX + c/X$	32
4	Hyperbola	$Y = a + b/X$	26
5	Reciprocal Hyperbola	$Y = X/(aX + b)$	30
6	2nd Order Hyperbola	$Y = a + b/X + c/X^2$	34
7	Parabola	$Y = a + bX + cX^2$	36
8	Cauchy Distribution	$Y = 1/[a(X+b)^2 + c]$	76
9	Power	$Y = aX^b$	42
10	Super Geometric	$Y = aX^{bX}$	48
11	Modified Geometric	$Y = aX^{b/X}$	50
12	Hoerl Function	$Y = ab^X X^c$	64
13	Modified Hoerl	$Y = ab^{1/X} X^c$	66
14	Log-Normal	$Y = ae^{(b-\ln X)^2/c}$	70
15	Logarithmic	$Y = a + b\ln X$	58
16	Reciprocal Log	$Y = 1/(a + b\ln X)$	60
17	Modified Power	$Y = ab^X$	44
18	Root	$Y = ab^{1/X}$	46
19	Normal Distribution	$Y = ae^{(X-b)^2/c}$	68

Program Operation

The program displays the equation for any selected curve as well as the coefficients and the adjusted coefficient of determination, RR. Errors are easily corrected and accidentally pressing R/S in most cases simply repeats the last function executed. The program requires a QUAD Memory Module when used with the HP-41C. A printer is optional. After running the program, remember to save the data on magnetic cards, cassette tape or in Extended Memory. You may then easily reenter this data at any time to add more points, delete points or try a different equation.

Early HP-41C Calculators

Very early HP-41C calculators did not store X in the LSTX register when executing Σ+ or Σ−. To see if you have one of these machines, enter 169, press Σ+, and then press LSTX. If the display shows anything other than 169, you have an early HP-41C and the Multiple Curve Fitting (MCF) program must be changed as follows to work on your calculator.

(1) Switch to PRGM mode, (2) Press GTO.150 and then enter STO.L, (3) Press GTO.117 and enter STO.L, (4) Press GTO.051 and enter STO.L, (5) Press GTO.018 and enter STO.L, (6) Switch out of PRGM mode and execute PACK.

Be sure to make these changes in the order shown. Use the example on page 33 for the combined linear-hyperbolic equation to verify that the program now works correctly.

Compiling Your Program

If you enter the MCF program manually or with the wand, several minutes will be required to execute the automatic curve fit routine (LBL E) the first time. This is because numerous XEQ and GTO instructions in the program are being compiled. The next time you execute LBL E the program will run in about 90 seconds. Remember that anytime you modify a program, it automatically reverts to the decompiled state.

Limits and Warnings

Enter at least four data points when using the automatic curve selection feature (LBL E). This is necessary because the corrected RR calculation uses a divisor of N-3 for curves 3, 6, 7, 8, 12, 13, 14 and 19, where N is the number of data points. All other curves require a minimum of three data points. It is a good practice to run the MCF program a second time with the values of X and Y exchanged to determine if a higher RR can be obtained.

The corrected coefficient of determination is displayed for RR and used in all comparisons to find the best fitting curve. If the corrected RR is negative, it is set to zero. Because of round-off errors, it is also possible for RR to be slightly greater than one (refer to the section on scaling, p. 6, in such cases).

Whenever X or Y is zero, it is replaced with 9E-09 by the data entry routine. This technique for dealing with zero may sometimes cause curves with 1/X or 1/Y terms to halt the program and display DATA ERROR. If this happens, either eliminate any point with a zero or avoid fitting curves that would involve the reciprocal of zero. If any value of X is negative, curves 9 through 16 should not be selected. If any value of Y is negative, curves 9 through 14 and curves 17 through 19 should not be selected.

The Σ- (SHIFT A) and DELETE LST X,Y (SHIFT B) functions are limited by the internal accuracy of the HP-41. The last few digits in certain summations may be in error as a result of using these two functions.

Registers

The MCF program requires a total of 241 program registers and 70 data registers. Data register assignments are consistent with those used in the text except for R27, R33, R39 and R42 which are not computed, and R59 through R69 which are used for computations.

Flags

F01 is set when the program is searching for the best fit curve.

F02 is set if any value of X is negative.

F03 is set if any value of Y is negative.

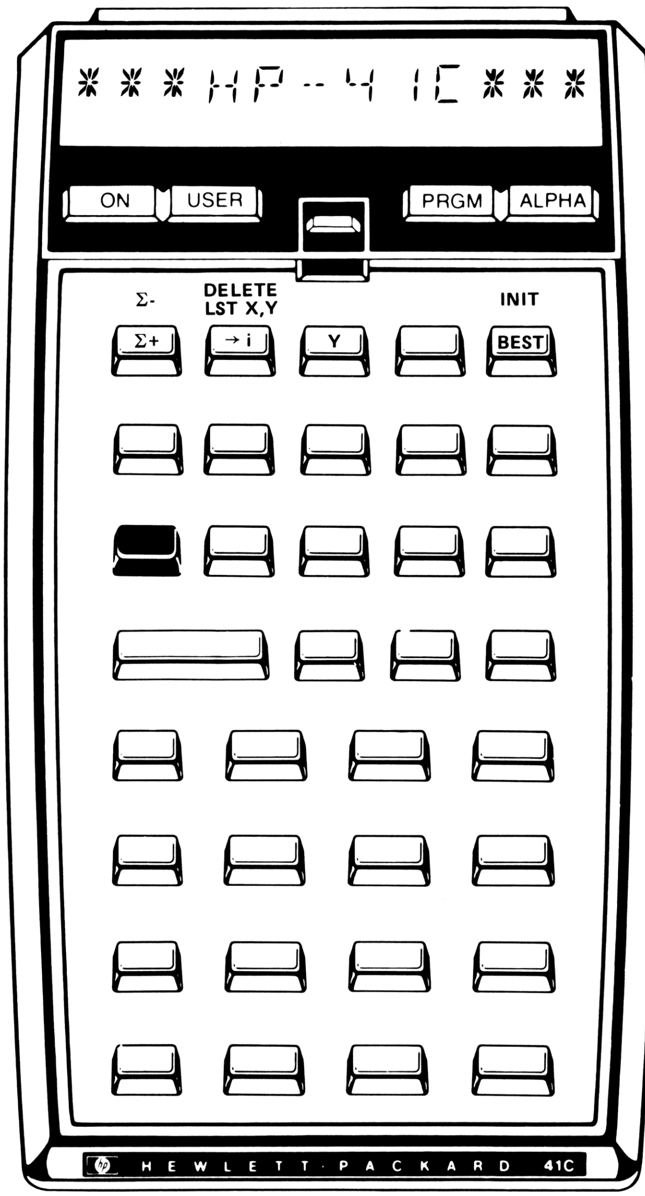
F21 is used to automatically control the printer if attached.

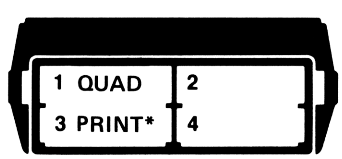
Key Assignments

When the calculator is in USER mode, the upper row of keys are assigned alternate functions as follows

A	Used to input values of X and Y.
SHIFT A	Used to delete values of X and Y once entered.
B	Fits the curve designated in the X-register.
SHIFT B	Deletes the last value of X and Y (use only during data entry).
C	Predicts Y for a given value of X using the curve selected.
E	Finds the best fitting curve automatically.
SHIFT E	Clears registers and initializes the data entry routine.

HP-41C/V KEY ASSIGNMENTS





*Printer is optional

HP-41C/V USER INSTRUCTIONS

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Set calculator to USER mode.			
1	Set SIZE to 070.		XEQ SIZE 070	
2	Load program "MCF" and execute GTO · · in order to pack memory. Now execute GTO "MCF" to begin.		GTO · · GTO "MCF"	
3	Initialize program memory and flags.		SHIFT E	X, ENTER, Y, Σ+
4	Key in the X value for the first point.	X	ENTER	X
5	Key in the Y value for the first point and press Σ+ or R/S. Each entry takes about 5 seconds. The total number of points is displayed after each entry.	Y	A or R/S	n
6	Repeat steps 4 and 5 for all points.			
7a	Correct the last entry by pressing Shift B and going to steps 4 and 5.		SHIFT B	n - 1
7b	Delete any point by entering X and Y, then pressing the Σ- key.	X Y	ENTER SHIFT A	X n - 1
8a	Find the best fitting curve. The calculator displays the number for each curve and the corrected value of RR (coefficient of determination). The equation of the best fitting curve will be displayed after approximately 90 seconds.		E	i.RR equation
8b	Press R/S to obtain the coefficients of the best fitting curve and the corrected coefficient of determination (RR).*		R/S R/S R/S R/S	a b c RR
9	Determine the coefficients for any selected curve at any time by entering the curve number (i) and pressing 'B.' Press R/S to get each of the coefficients.*	i	B R/S R/S R/S R/S	equation a b c RR
10a	Calculate the value of Y corresponding to any given value of X by entering X and pressing 'C.' (This routine uses the coefficients of the last equation displayed.)	X	C	Y
10b	Calculate additional values of Y by entering values of X and pressing 'C' or R/S.	X	C or R/S	Y
	*If the printer is attached, the coefficients are automatically printed out.			

```
01♦LBL e
02 CF 02
03 CF 03
04 CF 21
05 CLRG
06 "X, ENTE
R, Y, Σ+"
07 AVIEW
08 RTN
09♦LBL "MCF
"
10♦LBL A
11 X=0?
12 XEQ 89
13 STO 02
14 X<>Y
15 X=0?
16 XEQ 89
17 STO 01
18 ΣREG 16
19 Σ+
20 LASTX
21 /
22 ST+ 35
23 1/X
24 ST+ 34
25 RCL 01
26 *
27 ST+ 37
28 1/X
29 ST+ 38
30 RCL 02
31 RCL 01
32 X↑2
33 *
34 ST+ 36
35 LASTX
36 X↑2
37 ST+ 43
38 1/X
39 ST+ 44
40 RCL 01
41 *
42 ST+ 41
43 1/X
44 ST+ 40
45 RCL 02
46 1/X
47 RCL 01
48 1/X
49 ΣREG 22
50 Σ+
51 XEQ 88
52 Σ+
53 LASTX
54 RCL 01
55 *
56 ST+ 48
57 *
58 ST+ 50
59 LASTX
60 X↑2
61 ST+ 49
62 RCL 03
63 RCL 01
64 /
65 ST+ 45
66 X↑2
67 ST+ 53
68 LASTX
69 RCL 04
70 *
71 ST+ 58
72 LASTX
73 RCL 01
74 *
75 ST+ 46
76 LASTX
77 *
78 ST+ 54
79 RCL 04
80 LASTX
81 /
82 ST+ 47
83 RCL 02
84 RCL 03
85 *
86 ST+ 51
87 LASTX
88 RCL 02
89 /
90 ST+ 52
91 RCL 04
92 RCL 03
93 X↑2
94 *
95 ST+ 57
96 LASTX
97 X↑2
98 ST+ 56
99 LASTX
100 RCL 03
101 *
102 ST+ 55
103 RCL 21
104 RTN
105 GTO "MCF
"
106♦LBL b
107 RCL 01
108 RCL 02
109♦LBL a
110 X=0?
111 XEQ 89
112 STO 02
113 X<>Y
114 X=0?
115 XEQ 89
116 STO 01
117 ΣREG 16
118 Σ-
119 LASTX
120 /
121 ST- 35
122 1/X
123 ST- 34
124 RCL 01
125 *
126 ST- 37
127 1/X
128 ST- 38
129 RCL 02
130 RCL 01
131 X↑2
132 *
133 ST- 36
134 LASTX
135 X↑2
136 ST- 43
137 1/X
138 ST- 44
139 RCL 01
140 *
141 ST- 41
142 1/X
143 ST- 40
144 RCL 02
145 1/X
146 RCL 01
147 1/X
148 ΣREG 22
149 Σ-
150 XEQ 88
151 Σ-
152 LASTX
153 RCL 01
154 *
155 ST- 48
156 *
157 ST- 50
158 LASTX
159 X↑2
160 ST- 49
161 RCL 03
162 RCL 01
163 /
164 ST- 45
165 X↑2
166 ST- 53
167 LASTX
168 RCL 04
169 *
170 ST- 58
171 LASTX
172 RCL 01
173 *
174 ST- 46
175 LASTX
176 *
177 ST- 54
178 RCL 04
179 LASTX
180 /
181 ST- 47
182 RCL 02
183 RCL 03
184 *
185 ST- 51
186 LASTX
187 RCL 02
188 /
189 ST- 52
190 RCL 04
191 RCL 03
192 X↑2
193 *
194 ST- 57
195 LASTX
196 X↑2
197 ST- 56
198 LASTX
199 RCL 03
200 *
201 ST- 55
202 RCL 21
203 RTN
204 GTO "MCF
"
205♦LBL 88
206 RCL 02
207 X<=0?
208 SF 03
209 X<=0?
210 XEQ 89
211 LN
212 STO 04
213 RCL 01
214 X<=0?
215 SF 02
216 X<=0?
217 XEQ 89
218 LN
219 STO 03
220 ΣREG 28
221 RTN
222♦LBL 89
223 CLX
224 9 E-9
225 RTN
226♦LBL E
227 CLX
228 STO 10
229♦LBL 01
230 XEQ 23
231 XEQ 24
232 RCL 20
233 1
234 XEQ 20
235 FC?C 01
236 GTO 02
237 "Y=a+bX"
238 RTN
239♦LBL 02
240 XEQ 23
241 XEQ 25
242 RCL 34
243 2
244 XEQ 20
245 FC?C 01
246 GTO 03
247 "Y=1/(a+
bX)"
248 RTN
249♦LBL 03
250 XEQ 23
251 XEQ 24
252 RCL 22
253 STO 66
254 RCL 23
255 STO 67
256 RCL 20
257 RCL 21
258 RCL 35
259 3
260 XEQ 21
261 FC?C 01
262 GTO 04
263 "Y=a+bX+
c/X"
264 RTN
265♦LBL 04
266 XEQ 26
267 XEQ 24
268 RCL 35
269 4
270 XEQ 20
271 FC?C 01
272 GTO 05
273 "Y=a+b/X
"
274 RTN
275♦LBL 05
276 XEQ 26
277 XEQ 25
278 RCL 26
279 5
280 XEQ 20
281 FC?C 01
282 GTO 06
283 "Y=X/(aX
+b)"
284 RTN
285♦LBL 06
286 XEQ 26
287 XEQ 24
288 RCL 23
289 STO 66
290 RCL 44
291 STO 67
292 RCL 35
293 RCL 41
294 RCL 38
295 6
296 XEQ 21
297 FC?C 01
298 GTO 07
299 "Y=a+b/X
+c/X↑2"
300 RTN
301♦LBL 07
302 XEQ 23
303 XEQ 24
304 XEQ 27
305 RCL 20
306 RCL 40
307 RCL 36
308 7
309 XEQ 21
310 FC?C 01
311 GTO 08
312 "Y=a+bX+
cX↑2"
313 RTN
314♦LBL 08
315 XEQ 23
316 XEQ 25
317 XEQ 27
318 RCL 34
319 RCL 40
320 RCL 37
321 8
322 XEQ 21
323 FC?C 01
324 GTO 09
325 RCL 11
326 RCL 12
327 X↑2
328 RCL 13
329 STO 01
330 ST+ X
331 ST/ 02
332 ST+ X
333 /
334 -
335 STO 03
336 "Y=1/[c+
a(X+b)↑2"
337 "⊢]"
338 RTN
339♦LBL 09
340 FS? 02
341 GTO 17
342 FS? 03
343 GTO 15
344 XEQ 28
345 XEQ 29
346 RCL 32
347 9
348 XEQ 20
349 FC?C 01
350 GTO 10
351 RCL 11
352 E↑X
353 STO 01
354 "Y=aX↑b"
355 RTN
356♦LBL 10
357 RCL 48
358 STO 61
359 RCL 49
360 STO 62
361 XEQ 29
362 RCL 50
363 10
364 XEQ 20
365 FC?C 01
366 GTO 11
367 RCL 11
368 E↑X
369 STO 01
370 "Y=aX↑(b
X)"
371 RTN
372♦LBL 11
373 RCL 45
374 STO 61
375 RCL 53
376 STO 62
377 XEQ 29
378 RCL 58
379 11
380 XEQ 20
381 FC?C 01
382 GTO 12
383 RCL 11
384 E↑X
385 STO 01
386 "Y=aX↑(b
/X)"
387 RTN
388♦LBL 12
389 XEQ 23
390 XEQ 29
391 XEQ 30
392 RCL 46
393 RCL 48
394 RCL 32
395 12
396 XEQ 21
397 FC?C 01
398 GTO 13
399 RCL 11
400 E↑X
401 STO 01
402 RCL 12
403 E↑X
404 STO 02
405 "Y=a(b↑X
)(X↑c)"
406 RTN
407♦LBL 13
408 XEQ 26
409 XEQ 29
410 XEQ 30
411 RCL 47
412 RCL 45
413 RCL 32
414 13
415 XEQ 21
416 FC?C 01
417 GTO 14
418 RCL 11
419 E↑X
420 STO 01
421 RCL 12
422 E↑X
423 STO 02
424 "Y=a(b↑1
/X)(X↑c)"
425 RTN
426♦LBL 14
427 XEQ 28
428 XEQ 29
429 RCL 29
430 STO 66
431 RCL 56
432 STO 67
433 RCL 32
434 RCL 55
435 RCL 57
436 14
437 XEQ 21
438 FC?C 01
439 GTO 15
440 RCL 11
441 RCL 12
442 X↑2
443 RCL 13
444 ST+ X
445 CHS
446 ST/ 02
447 ST+ X
448 /
449 +
450 E↑X
451 STO 01
452 RCL 13
453 1/X
454 STO 03
455 "Y=aEXP[
(1/c)(LN"
456 "⊢X-b)↑2
]"
457 RTN
458♦LBL 15
459 XEQ 28
460 XEQ 24
461 RCL 51
462 15
463 XEQ 20
464 FC?C 01
465 GTO 16
466 "Y=a+bLN
X"
467 RTN
468♦LBL 16
469 XEQ 28
470 XEQ 25
471 RCL 52
472 16
473 XEQ 20
474 FC?C 01
475 GTO 17
476 "Y=1/(a+
bLNX)"
477 RTN
478♦LBL 17
479 FS? 03
480 GTO 31
481 XEQ 23
482 XEQ 29
483 RCL 46
484 17
485 XEQ 20
486 FC?C 01
487 GTO 18
488 RCL 11
489 E↑X
490 STO 01
491 RCL 12
492 E↑X
493 STO 02
494 "Y=ab↑X"
495 RTN
496♦LBL 18
497 XEQ 26
498 XEQ 29
499 RCL 47
500 18
```

```
501 XEQ 20
502 FC?C 01
503 GTO 19
504 RCL 11
505 E↑X
506 STO 01
507 RCL 12
508 E↑X
509 STO 02
510 "Y=ab↑(1
/X)"
511 RTN
512♦LBL 19
513 XEQ 23
514 XEQ 29
515 XEQ 27
516 RCL 46
517 RCL 40
518 RCL 54
519 19
520 XEQ 21
521 FC?C 01
522 GTO 31
523 RCL 11
524 RCL 12
525 X↑2
526 RCL 13
527 ST+ X
528 CHS
529 ST/ 02
530 ST+ X
531 /
532 +
533 E↑X
534 STO 01
535 RCL 13
536 1/X
537 STO 03
538 "Y=aEXP[
(1/c)(X-"
539 "⊢b)↑2]"
540 RTN
541♦LBL 20
542 STO 00
543 RDN
544 STO 65
545 RCL 62
546 RCL 21
547 *
548 RCL 61
549 X↑2
550 -
551 STO 05
552 RCL 62
553 RCL 63
554 *
555 RCL 61
556 RCL 65
557 *
558 -
559 RCL 05
560 /
561 STO 11
562 STO 01
563 RCL 65
564 RCL 21
565 *
566 RCL 61
567 RCL 63
568 *
569 -
570 RCL 05
571 /
572 STO 12
573 STO 02
574 CLX
575 STO 03
576 STO 13
577 2
578 GTO 22
579♦LBL 21
580 STO 00
581 RDN
582 STO 69
583 RDN
584 STO 68
585 RDN
586 STO 65
587 RCL 62
588 RCL 21
589 *
590 RCL 61
591 X↑2
592 -
593 STO 05
594 RCL 21
595 RCL 69
596 *
597 RCL 63
598 RCL 66
599 *
600 -
601 STO 06
602 RCL 21
603 RCL 68
604 *
605 RCL 61
606 RCL 66
607 *
608 -
609 STO 07
610 RCL 65
611 RCL 21
612 *
613 RCL 61
614 RCL 63
615 *
616 -
617 STO 08
618 RCL 21
619 RCL 67
620 *
621 RCL 66
622 X↑2
623 -
624 STO 09
625 RCL 05
626 RCL 06
627 *
628 RCL 07
629 RCL 08
630 *
631 -
632 RCL 05
633 RCL 09
634 *
635 RCL 07
636 X↑2
637 -
638 /
639 STO 13
640 STO 03
641 RCL 08
642 RCL 07
643 RCL 13
644 *
645 -
646 RCL 05
647 /
648 STO 12
649 STO 02
650 RCL 63
651 RCL 12
652 RCL 61
653 *
654 -
655 RCL 13
656 RCL 66
657 *
658 -
659 RCL 21
660 /
661 STO 11
662 STO 01
663 3
664♦LBL 22
665 RCL 11
666 RCL 63
667 *
668 RCL 12
669 RCL 65
670 *
671 +
672 RCL 13
673 RCL 69
674 *
675 +
676 RCL 63
677 X↑2
678 RCL 21
679 /
680 -
681 RCL 64
682 LASTX
683 -
684 /
685 1
686 -
687 LASTX
688 RCL 21
689 -
690 *
691 X<>Y
692 RCL 21
693 -
694 /
695 1
696 +
697 X<0?
698 CLX
699 FS? 01
700 GTO 00
701 CLA
702 FIX 0
703 ARCL 00
704 "⊢ "
705 FIX 3
706 ARCL X
707 CF 21
708 AVIEW
709 SF 21
710 RCL 10
711 X<>Y
712 X<=Y?
713 RTN
714♦LBL 00
715 STO 10
716 RCL 00
717 STO 15
718 RTN
719♦LBL 23
720 RCL 16
721 STO 61
722 RCL 17
723 STO 62
724 RTN
725♦LBL 24
726 RCL 18
727 STO 63
728 RCL 19
729 STO 64
730 RTN
731♦LBL 25
732 RCL 24
733 STO 63
734 RCL 25
735 STO 64
736 RTN
737♦LBL 26
738 RCL 22
739 STO 61
740 RCL 23
741 STO 62
742 RTN
743♦LBL 27
744 RCL 17
745 STO 66
746 RCL 43
747 STO 67
748 RTN
749♦LBL 28
750 RCL 28
751 STO 61
752 RCL 29
753 STO 62
754 RTN
755♦LBL 29
756 RCL 30
757 STO 63
758 RCL 31
759 STO 64
760 RTN
761♦LBL 30
762 RCL 28
763 STO 66
764 RCL 29
765 STO 67
766 RTN
767♦LBL 31
768 RCL 15
769♦LBL B
770 STO 00
771 SF 01
772 SF 21
773 XEQ IND
00
774 TONE 7
775 AVIEW
776 "a= "
777 ARCL 01
778 AVIEW
779 "b= "
780 ARCL 02
781 AVIEW
782 RCL 03
783 X=0?
784 GTO 00
785 "c= "
786 ARCL X
787 AVIEW
788♦LBL 00
789 "RR= "
790 ARCL 10
791 AVIEW
792 ADV
793 RTN
794 GTO 31
795♦LBL C
796 CF 21
797 RCL 15
798 60
799 +
800 X<>Y
801 ENTER↑
802 XEQ IND
Z
803 "Y= "
804 ARCL X
805 AVIEW
806 RTN
807 GTO C
808♦LBL 75
809 LN
810♦LBL 61
811 RCL 02
812 *
813 RCL 01
814 +
815 RTN
816♦LBL 62
817 XEQ 61
818 1/X
819 RTN
820♦LBL 63
821 RCL 03
822 X<>Y
823 /
824 RCL 02
825 LASTX
826 *
827 +
828 RCL 01
829 +
830 RTN
831♦LBL 64
832 RCL 02
833 X<>Y
834 /
835 RCL 01
836 +
837 RTN
838♦LBL 65
839 XEQ 64
840 1/X
841 RTN
842♦LBL 66
843 1/X
844 ENTER↑
845 ENTER↑
846♦LBL 67
847 RCL 03
848 *
849 RCL 02
850 +
851 *
852 RCL 01
853 +
854 RTN
855♦LBL 68
856 RCL 02
857 +
858 X↑2
859 RCL 01
860 *
861 RCL 03
862 +
863 1/X
864 RTN
865♦LBL 69
866 RCL 02
867 Y↑X
868 RCL 01
869 *
870 RTN
871♦LBL 78
872 1/X
873♦LBL 77
874 RCL 02
875 X<>Y
876 Y↑X
877 RCL 01
878 *
879 RTN
880♦LBL 71
881 1/X
882♦LBL 70
883 RCL 02
884 *
885 Y↑X
886 RCL 01
887 *
888 RTN
889♦LBL 72
890 RCL 02
891 X<>Y
892 Y↑X
893 X<>Y
894 RCL 03
895 Y↑X
896 *
897 RCL 01
898 *
899 RTN
900♦LBL 73
901 RCL 03
902 Y↑X
903 RCL 02
904 RCL Z
905 1/X
906 Y↑X
907 *
908 RCL 01
909 *
910 RTN
911♦LBL 74
912 LN
913♦LBL 79
914 RCL 02
915 -
916 X↑2
917 RCL 03
918 /
919 E↑X
920 RCL 01
921 *
922 RTN
923♦LBL 76
924 XEQ 75
925 1/X
926 RTN
927 .END.

WM KOLB
```

See page E-8 for notes regarding the use of special characters ( ) [ ] in programs.

HP-41C/V MULTIPLE CURVE FITTING PROGRAM

(Program Registers Required: 241)

ROW 1 (1 : 6)

ROW 2 (6 : 7)

ROW 3 (8 : 12)

ROW 4 (12 : 21)

ROW 5 (22 : 30)

ROW 6 (31 : 40)

ROW 7 (41 : 50)

ROW 8 (51 : 59)

ROW 9 (60 : 69)

ROW 10 (70 : 79)

ROW 11 (80 : 90)

ROW 12 (90 : 100)

ROW 13 (101 : 106)

ROW 14 (107 : 115)

ROW 15 (115 : 124)

ROW 16 (125 : 134)

ROW 17 (135 : 143)

ROW 18 (144 : 153)

HP-41C/V MULTIPLE CURVE FITTING PROGRAM

ROW 19 (154 : 163)

ROW 20 (164 : 173)

ROW 21 (174 : 183)

ROW 22 (184 : 194)

ROW 23 (194 : 203)

ROW 24 (204 : 210)

ROW 25 (210 : 218)

ROW 26 (219 : 226)

ROW 27 (226 : 233)

ROW 28 (234 : 237)

ROW 29 (237 : 244)

ROW 30 (244 : 247)

ROW 31 (247 : 252)

ROW 32 (252 : 258)

ROW 33 (259 : 263)

ROW 34 (263 : 267)

ROW 35 (267 : 273)

ROW 36 (273 : 277)

HP-41C/V MULTIPLE CURVE FITTING PROGRAM

ROW 37 (277 : 283)

ROW 38 (283 : 286)

ROW 39 (286 : 292)

ROW 40 (292 : 298)

ROW 41 (299 : 299)

ROW 42 (299 : 305)

ROW 43 (305 : 311)

ROW 44 (312 : 313)

ROW 45 (314 : 319)

ROW 46 (319 : 326)

ROW 47 (327 : 336)

ROW 48 (336 : 336)

ROW 49 (336 : 342)

ROW 50 (342 : 347)

ROW 51 (348 : 354)

ROW 52 (354 : 360)

ROW 53 (360 : 365)

ROW 54 (366 : 370)

ROW 55 (370 : 377)

ROW 56 (377 : 382)

ROW 57 (383 : 386)

ROW 58 (386 : 392)

ROW 59 (392 : 398)

ROW 60 (398 : 405)

ROW 61 (405 : 408)

ROW 62 (409 : 414)

ROW 63 (414 : 422)

ROW 64 (423 : 424)

ROW 65 (424 : 429)

ROW 66 (429 : 435)

ROW 67 (436 : 442)

ROW 68 (443 : 452)

ROW 69 (453 : 455)

ROW 70 (455 : 456)

ROW 71 (456 : 462)

ROW 72 (462 : 466)

ROW 73 (466 : 470)

ROW 74 (470 : 475)

ROW 75 (476 : 476)

ROW 76 (477 : 482)

ROW 77 (482 : 487)

ROW 78 (488 : 494)

ROW 79 (495 : 500)

ROW 80 (501 : 508)

ROW 81 (509 : 511)

ROW 82 (512 : 516)

ROW 83 (517 : 522)

ROW 84 (522 : 531)

ROW 85 (532 : 538)

ROW 86 (538 : 539)

ROW 87 (539 : 546)

ROW 88 (546 : 555)

ROW 89 (555 : 564)

ROW 90 (565 : 575)

ROW 91 (576 : 584)

ROW 92 (584 : 592)

ROW 93 (593 : 601)

ROW 94 (602 : 610)

ROW 95 (610 : 618)

ROW 96 (619 : 629)

ROW 97 (630 : 642)

ROW 98 (643 : 653)

ROW 99 (654 : 664)

ROW 100 (664 : 673)

ROW 101 (674 : 683)

ROW 102 (684 : 694)

ROW 103 (695 : 703)

ROW 104 (704 : 709)

ROW 105 (710 : 720)

ROW 106 (721 : 727)

ROW 107 (728 : 734)

ROW 108 (735 : 741)

ROW 109 (742 : 749)

ROW 110 (749 : 756)

ROW 111 (756 : 763)

ROW 112 (763 : 771)

ROW 113 (771 : 777)

ROW 114 (777 : 784)

ROW 115 (785 : 789)

ROW 116 (790 : 797)

ROW 117 (798 : 804)

ROW 118 (805 : 813)

ROW 119 (814 : 822)

ROW 120 (823 : 834)

ROW 121 (835 : 843)

ROW 122 (844 : 855)

ROW 123 (855 : 866)

ROW 124 (867 : 877)

ROW 125 (878 : 888)

ROW 126 (889 : 900)

ROW 127 (900 : 911)

ROW 128 (911 : 922)

ROW 129 (923 : 927)

(

)

[

]

Special characters such as ( ) [ ] cannot be keyed into a program directly. Custom barcodes are available, however, to put these characters in any alpha string with the aid of the wand. Users interested in advanced programming techniques such as these should contact the PPC, 2545 West Camden Place, Santa Ana, California 92704, U.S.A. The PPC is the oldest and largest member supported personal computing users group.

NOTE: If you are unable to read a line of barcode, press the SST key and continue reading on the next line. Note the program steps in parentheses above the missing line. If you are unable to read the last line of barcode, press the BACK ARROW key. After the program is entered, manually key in the missing steps from the program listing on pages D-1 and D-2.

MULTIPLE CURVE FITTING PROGRAM FOR THE TI-59

by Maurice Swinnen

This set of programs fits as many as 9 curves to an unlimited number of X,Y data points. The best fitting curve can be determined automatically based on the coefficient of determination, RR. Once a curve has been selected, values of Y can be estimated for any given value of X, and X may calculated for any given Y. A specific curve may also be selected manually by entering the appropriate curve number. The programs can be used with or without the PC-100 printer. For hand-held use only, the program will stop at the appropriate places to allow you to copy the results by hand. With the printer attached, print out is automatic.

Curve Number	Type	General Equation	Page
1	Linear	$Y = a + bX$	12
2	Logarithmic	$Y = a + b\ln X$	58
3	Quadratic	$Y = a + bX^2$	83
4	Hyperbola	$Y = a + b/X$	26
5	Reciprocal	$Y = 1/(a + bX)$	22
6	Exponential	$Y = ae^{bX}$	52
7	Power	$Y = aX^b$	42
8	Parabola	$Y = aX^2 + bX + c$	36
9	Linear-Hyperbolic	$Y = a + bX + c/X$	32

Program Operation

The program consists of four magnetic cards. The first magnetic card contains a data entry routine that creates the data base used by all curve fitting routines. It allows an unlimited number of data points to be entered in the format: X, R/S, Y, R/S. Each data point requires about 12 seconds to enter. An error correction routine resides in the same program and allows you to delete a data point at any time. With the printer attached, all data points are printed.

The second magnetic card contains a program that will compute the coefficient of determination (RR) and the coefficients a and b for the first seven curves. This program also has provisions for automatically selecting the best fitting curve based on RR. The automatic curve fitting routine requires about 40 seconds to execute. Two additional routines allow you to estimate Y for a given X, and to calculate X for a given Y.

The third magnetic card contains a polynomial (parabolic) curve fit routine, while the fourth magnetic card has a program that fits a combined linear and hyperbolic curve. Both of these programs, although of great practical value in themselves, are also intended to serve as examples of how you can expand your library of curve fitting programs for use with the data entry routine. Even a novice programmer will have no difficulty in deciphering the code in these rather simple routines. Programs such as the multiple curve fit on the second magnetic card, however, are quite complex and should be left to experienced programmers.

It should be noted that comparing the coefficient of determination (RR) of the first seven curves to each other is valid, since each equation has the same number of coefficients. Comparing RR from these seven curves to RR for curves 8 and 9 is not valid because the extra coefficient removes one degree of freedom. The corrected coefficient of determination (see page 7) should be calculated and used in all comparisons between curves with an unequal number of coefficients. (Note that there is enough room on the third and fourth magnetic cards to add these routines, if so desired.) It is also a good practice to run the curve fitting routines a second time with the values of X and Y exchanged to determine if a higher value of RR can be obtained.

This program could not have been written without heavy borrowing from two prior efforts, one by Bill Skillman and another by Frank Blachly. I thank both of them for allowing me to use some of their superbly written code.

Programming Aides

Key in each program according to the program listing. After each program is entered, record it on a magnetic card by entering the bank number (1 in this case), pressing 2nd WRITE, and then inserting a magnetic card. After the first bank is recorded, enter 2, press 2nd WRITE, turn the magnetic card end for end and insert it again. Label the card so that you can identify the program. Do this for each program to make a complete set of program cards. The data entry card is a special case and should be recorded using bank 1 and bank 3. This is because printer formatting data is stored registers 30 through 51 of bank 3 in addition to the program in bank 1. Note that banks 1 and 2 are always reserved for program storage while banks 3 and 4 are used exclusively for data. Once entered, data may also be stored on magnetic cards by following the above procedure for banks 3 and 4.

In order to get RTN (INV SBR) into place at the end of the multiple curve fitting program, temporarily partition to 5 OP 17, thus making more memory available. Then, when you reach the end of your listing, repartition to 6 OP 17 before recording the program on a magnetic card.

Users interested in further information on advanced programming techniques such as those used in these programs should contact TI PPC Notes, P.O. Box 1421, Largo, Florida 34294-1421, U.S.A.

MULTIPLE CURVE FITTING PROGRAM

TI-59

Program Registers

The limited amount of memory available in the TI-59 requires that registers be assigned differently from those in the text. The following table provides a cross-reference between the assignments used in the text and those used in the TI-59. Note that data is stored exclusively in memory banks 3 and 4 while the various curve fitting routines are stored in banks 1 and 2 so that either can be changed without affecting the other.

TI-59	CONTENTS	TEXT	TEXT	CONTENTS	TI-59
R01	Σ Y	R18	R01	X	R14
R02	Σ Y^2	R19	R02	Y	R07
R03	n	R21	R16	Σ X	R04
R04	Σ X	R16	R17	Σ X^2	R05
R05	Σ X^2	R17	R18	Σ Y	R01
R06	Σ X*Y	R20	R19	Σ Y^2	R02
R07	Y	R02	R20	Σ X*Y	R06
R10	Σ lnY	R30	R21	n	R03
R11	Σ $(\ln Y)^2$	R31	R22	Σ 1/X	R19
R12	Σ 1/Y	R24	R23	Σ $1/X^2$	R20
R13	Σ $1/Y^2$	R25	R24	Σ 1/Y	R12
R14	X	R01	R25	Σ $1/Y^2$	R13
R17	Σ lnX	R28	R28	Σ lnX	R17
R18	Σ $(\ln X)^2$	R29	R29	Σ $(\ln X)^2$	R18
R19	Σ 1/X	R22	R30	Σ lnY	R10
R20	Σ $1/X^2$	R23	R31	Σ $(\ln Y)^2$	R11
R21	Σ X^3	R40	R32	Σ (lnX*lnY)	R29
R22	Σ X^4	R43	R34	Σ X/Y	R27
R24	Σ Y*lnX	R51	R35	Σ Y/X	R26
R25	Σ $Y*X^2$	R36	R36	Σ $Y*X^2$	R25
R26	Σ Y/X	R35	R40	Σ X^3	R21
R27	Σ X/Y	R34	R43	Σ X^4	R22
R28	Σ X*lnY	R46	R46	Σ X*lnY	R28
R29	Σ (lnX*lnY)	R32	R51	Σ Y*lnX	R24

TI-59 USER INSTRUCTIONS

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Load side 1 of the data entry card.		CLR	1
	Load side 2 of the data entry card.		CLR	3
1	Initialize program and registers.		2nd E'	
2	Enter the X value and press R/S.	X	R/S	X
3	Enter the Y value and press R/S.	Y	R/S	Y
	The point number is displayed in approximately 12 seconds.			n
4	Repeat Steps 2 and 3 for all data.			
	To recover from an error, use one of the following procedures.			
5a	After X has been entered but not Y, enter the correct X and press A.	X	A	
5b	Enter Y and press R/S.	Y	R/S	n
5c	Continue entering data (Step 2).			
6a	Delete the last X and Y entered.		2nd A'	-(n-1)
6b	Enter the correct X and press A.	X	A	
6c	Enter the correct Y and press R/S.	Y	R/S	n
6d	Continue entering data (Step 2).			
7a	Delete any data point.			
7b	Enter the X value to be deleted.	X	STO 14	X
7c	Enter the Y value to be deleted.	Y	STO 07	Y
7d	Press 2nd A' to delete this point.		2nd A'	-(n-1)
7e	Go to Step 6b.			
	Load sides 1 and 2 of the multiple curve fit program.		CLR CLR	1 2
8a	Find the best fitting curve and display the coefficient of determination.		C	RR
8b	Display the A coefficient.		R/S	A
8c	Display the B coefficient.		R/S	B
9a	Find the coefficients of any curve.			
9b	Enter the curve number and press A.	i	A	
9c	Display the coefficient of determination.			RR
9d	Display the A coefficient.		B	A
9e	Display the B coefficient.		R/S	B

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
10a	Estimate Y for a given X.			
10b	Enter the X value and press D.	X	D	Y'
11a	Calculate X for a given Y.			
11b	Enter the Y value and press E.	Y	E	X
	Load sides 1 and 2 of the polynomial curve fit program.		CLR CLR	1 2
12	Press E to fit the curve and display the coefficient of determination.		E	RR
13	Display the A coefficient.		A	A
14	Display the B coefficient.		B	B
15	Display the C coefficient.		C	C
16a	Estimate Y for a given X.			
16b	Enter the X value and press D.	X	D	Y'
	Load sides 1 and 2 of the linear-hyperbolic curve fit program.		CLR CLR	1 2
17	Press E to fit the curve and display the coefficient of determination.		E	RR
18	Display the A coefficient.		A	A
19	Display the B coefficient.		B	B
20	Display the C coefficient.		C	C
21a	Estimate Y for a given X.			
21b	Enter the X value and press D.	X	D	Y'

TI-59 DATA ENTRY ROUTINE

```
000  76 LBL      067  44 SUM      134  42 STO      201  01  1
001  18 C'       068  17  17      135  15  15      202  06  6
002  33 X^2      069  18 C'       136  43 RCL      203  01  1
003  76 LBL      070  44 SUM      137  05  05      204  03  3
004  19 D'       071  18  18      138  42 STO      205  03  3
005  95  =       072  43 RCL      139  16  16      206  07  7
006  22 INV      073  14  14      140  43 RCL      207  01  1
007  87 IFF      074  35 1/X      141  06  06      208  03  3
008  00  00      075  19 D'       142  42 STO      209  69 OP
009  00  00      076  44 SUM      143  23  23      210  02  02
010  12  12      077  19  19      144  43 RCL      211  69 OP
011  94 +/-      078  18 C'       145  03  03      212  05  05
012  92 RTN      079  44 SUM      146  19 D'       213  69 OP
013  76 LBL      080  20  20      147  98 ADV      214  00  00
014  11  A       081  35 1/X      148  99 PRT      215  04  4
015  42 STO      082  18 C'       149  32 X:T      216  04  4
016  14  14      083  44 SUM      150  04  4       217  69 OP
017  91 R/S      084  22  22      151  04  4       218  01  01
018  42 STO      085  43 RCL      152  69 OP       219  43 RCL
019  07  07      086  07  07      153  04  04      220  51  51
020  22 INV      087  65  ×       154  43 RCL      221  69 OP
021  86 STF      088  43 RCL      155  14  14      222  02  02
022  00  00      089  09  09      156  69 OP       223  04  4
023  32 X:T      090  19 D'       157  06  06      224  05  5
024  43 RCL      091  44 SUM      158  04  4       225  69 OP
025  14  14      092  24  24      159  05  5       226  03  03
026  32 X:T      093  43 RCL      160  69 OP       227  43 RCL
027  22 INV      094  07  07      161  04  04      228  51  51
028  87 IFF      095  65  ×       162  43 RCL      229  69 OP
029  00  00      096  43 RCL      163  07  07      230  04  04
030  00  00      097  14  14      164  69 OP       231  69 OP
031  33  33      098  18 C'       165  06  06      232  05  05
032  22 INV      099  44 SUM      166  32 X:T      233  69 OP
033  78 Σ+       100  25  25      167  91 R/S      234  00  00
034  43 RCL      101  43 RCL      168  61 GTO      235  98 ADV
035  14  14      102  07  07      169  11  A       236  25 CLR
036  45 Y^X      103  55  ÷       170  76 LBL      237  91 R/S
037  03  3       104  43 RCL      171  16 A'       238  61 GTO
038  19 D'       105  14  14      172  43 RCL      239  11  A
039  44 SUM      106  19 D'       173  07  07
040  21  21      107  44 SUM      174  61 GTO
041  43 RCL      108  26  26      175  00  00
042  07  07      109  35 1/X      176  21  21
043  23 LNX      110  44 SUM      177  76 LBL
044  42 STO      111  27  27      178  10 E'
045  08  08      112  43 RCL      179  03  3
046  19 D'       113  14  14      180  69 OP
047  44 SUM      114  65  ×       181  17  17
048  10  10      115  43 RCL      182  47 CMS
049  18 C'       116  08  08      183  06  6
050  44 SUM      117  19 D'       184  69 OP
051  11  11      118  44 SUM      185  17  17
052  43 RCL      119  28  28      186  25 CLR
053  07  07      120  43 RCL      187  69 OP
054  35 1/X      121  01  01      188  00  00
055  19 D'       122  48 EXC      189  01  1
056  44 SUM      123  08  08      190  07  7
057  12  12      124  65  ×       191  02  2
058  18 C'       125  43 RCL      192  09  9
059  44 SUM      126  02  02      193  03  3
060  13  13      127  48 EXC      194  07  7
061  43 RCL      128  09  09      195  01  1
062  14  14      129  19 D'       196  07  7
063  23 LNX      130  44 SUM      197  03  3
064  42 STO      131  29  29      198  05  5
065  09  09      132  43 RCL      199  69 OP
066  19 D'       133  04  04      200  01  01
```

DATA	REGISTER
5114173637.	30
315.454432	31
324.466431	32
339.461435	33
354.457439	34
369.453443	35
391.311448	36
408.465447	37
471444.	38
4714273144.	39
47144470.	40
47146344.	41
21243751.	42
5460144400.	43
4460140000.	44
4500640013.	45
465.447	46
447.	47
4470004700.	48
1444004715.	49
47156344.	50
35633600.	51

TI-59 MULTIPLE CURVE FITTING PROGRAM

```
000  76  LBL
001  43  RCL
002  43  RCL
003  15   15
004  42  STO
005  04   04
006  43  RCL
007  16   16
008  42  STO
009  05   05
010  92  RTN
011  76  LBL
012  33  X²
013  42  STO
014  00   00
015  43  RCL
016  08   08
017  42  STO
018  01   01
019  43  RCL
020  09   09
021  42  STO
022  02   02
023  03   3
024  00   0
025  44  SUM
026  00   00
027  73  RC*
028  00   00
029  42  STO
030  14   14
031  22  INV
032  59  INT
033  52  EE
034  03   3
035  22  INV
036  52  EE
037  42  STO
038  46   46
039  22  INV
040  59  INT
041  52  EE
042  03   3
043  22  INV
044  52  EE
045  42  STO
046  47   47
047  71  SBR
048  40  IND
049  14   14
050  69  OP
051  03   03
052  69  OP
053  13   13
054  33  X²
055  92  RTN
056  76  LBL
057  11   A
058  98  ADV
059  99  PRT
060  69  OP
061  00   00
062  71  SBR
063  33  X²
064  32  X:T
065  43  RCL
066  45   45
067  69  OP
068  02   02
069  69  OP
070  05   05
071  03   3
072  05   5
073  07   7
074  00   0
075  61  GTO
076  04   04
077  71   71
078  76  LBL
079  12   B
080  43  RCL
081  00   00
082  32  X:T
083  03   3
084  05   5
085  77   GE
086  00   00
087  89   89
088  22  INV
089  86  STF
090  01   01
091  69  OP
092  12   12
093  65   ×
094  01   1
095  03   3
096  69  OP
097  04   04
098  01   1
099  95   =
100  87  IFF
101  01   01
102  01   01
103  06   06
104  22  INV
105  23  LNX
106  71  SBR
107  04   04
108  74   74
109  01   1
110  04   4
111  61  GTO
112  04   04
113  71   71
114  76  LBL
115  13   C
116  07   7
117  42  STO
118  07   07
119  29  CP
120  98  ADV
121  69  OP
122  00   00
123  43  RCL
124  30   30
125  69  OP
126  02   02
127  43  RCL
128  42   42
129  69  OP
130  03   03
131  69  OP
132  05   05
133  43  RCL
134  07   07
135  71  SBR
136  33  X²
137  22  INV
138  77   GE
139  01   01
140  46   46
141  32  X:T
142  43  RCL
143  07   07
144  42  STO
145  59   59
146  97  DSZ
147  07   07
148  01   01
149  33   33
150  43  RCL
151  59   59
152  71  SBR
153  04   04
154  76   76
155  11   A
156  12   B
157  92  RTN
158  76  LBL
159  14   D
160  98  ADV
161  32  X:T
162  04   4
163  04   4
164  69  OP
165  04   04
166  32  X:T
167  69  OP
168  06   06
169  71  SBR
170  40  IND
171  47   47
172  32  X:T
173  04   4
174  05   5
175  06   6
176  05   5
177  61  GTO
178  04   04
179  71   71
180  76  LBL
181  15   E
182  98  ADV
183  32  X:T
184  04   4
185  05   5
186  69  OP
187  04   04
188  32  X:T
189  69  OP
190  06   06
191  71  SBR
192  40  IND
193  46   46
194  32  X:T
195  04   4
196  04   4
197  06   6
198  05   5
199  61  GTO
200  04   04
201  71   71
```

NOTE: Steps 202 through 310 should be left blank (00).

TI-59 MULTIPLE CURVE FITTING PROGRAM

```
311  23 LNX
312  69 OP
313  15  15
314  92 RTN
315  71 SBR
316  43 RCL
317  43 RCL
318  23  23
319  42 STO
320  06  06
321  43 RCL
322  38  38
323  92 RTN
324  43 RCL
325  17  17
326  42 STO
327  04  04
328  43 RCL
329  18  18
330  42 STO
331  05  05
332  43 RCL
333  24  24
334  42 STO
335  06  06
336  43 RCL
337  39  39
338  92 RTN
339  43 RCL
340  16  16
341  42 STO
342  04  04
343  43 RCL
344  22  22
345  42 STO
346  05  05
347  43 RCL
348  25  25
349  42 STO
350  06  06
351  43 RCL
352  40  40
353  92 RTN
354  43 RCL
355  19  19
356  42 STO
357  04  04
358  43 RCL
359  20  20
360  42 STO
361  05  05
362  43 RCL
363  26  26
364  42 STO
365  06  06
366  43 RCL
367  41  41
368  92 RTN
369  43 RCL
370  12  12
371  42 STO
372  01  01
373  43 RCL
374  13  13
375  42 STO
376  02  02
377  71 SBR
378  43 RCL
379  43 RCL
380  27  27
381  42 STO
382  06  06
383  02  2
384  06  6
385  03  3
386  69 OP
387  01  01
388  43 RCL
389  38  38
390  92 RTN
391  43 RCL
392  10  10
393  42 STO
394  01  01
395  43 RCL
396  11  11
397  42 STO
398  02  02
399  71 SBR
400  43 RCL
401  43 RCL
402  28  28
403  42 STO
404  06  06
405  43 RCL
406  43  43
407  92 RTN
408  43 RCL
409  10  10
410  42 STO
411  01  01
412  43 RCL
413  11  11
414  42 STO
415  02  02
416  43 RCL
417  17  17
418  42 STO
419  04  04
420  43 RCL
421  18  18
422  42 STO
423  05  05
424  43 RCL
425  29  29
426  42 STO
427  06  06
428  43 RCL
429  44  44
430  92 RTN
431  23 LNX
432  69 OP
433  14  14
434  92 RTN
435  33 X²
436  69 OP
437  14  14
438  92 RTN
439  35 1/X
440  69 OP
441  14  14
442  92 RTN
443  69 OP
444  14  14
445  35 1/X
446  92 RTN
447  23 LNX
448  69 OP
449  14  14
450  22 INV
451  23 LNX
452  92 RTN
453  35 1/X
454  69 OP
455  15  15
456  92 RTN
457  69 OP
458  15  15
459  35 1/X
460  92 RTN
461  69 OP
462  15  15
463  34 ΓX
464  92 RTN
465  23 LNX
466  69 OP
467  15  15
468  22 INV
469  23 LNX
470  92 RTN
471  69 OP
472  04  04
473  32 X:T
474  69 OP
475  06  06
476  69 OP
477  08  08
478  91 R/S
479  92 RTN
```

TI-59 POLYNOMIAL CURVE FITTING PROGRAM

000	76	LBL	061	65	×	122	43	RCL	183	01	01
001	69	OP	062	43	RCL	123	05	05	184	32	X⇌T
002	69	OP	063	23	23	124	95	=	185	01	1
003	00	00	064	75	-	125	42	STO	186	03	3
004	69	OP	065	43	RCL	126	00	00	187	71	SBR
005	04	04	066	15	15	127	43	RCL	188	69	OP
006	32	X⇌T	067	65	×	128	08	08	189	91	R/S
007	69	OP	068	43	RCL	129	75	-	190	76	LBL
008	06	06	069	08	08	130	43	RCL	191	12	B
009	92	RTN	070	54	)	131	01	01	192	43	RCL
010	76	LBL	071	42	STO	132	65	×	193	00	00
011	15	E	072	04	04	133	43	RCL	194	32	X⇌T
012	98	ADV	073	65	×	134	16	16	195	01	1
013	69	OP	074	53	(	135	75	-	196	04	4
014	00	00	075	43	RCL	136	43	RCL	197	71	SBR
015	43	RCL	076	03	03	137	00	00	198	69	OP
016	45	45	077	65	×	138	65	×	199	91	R/S
017	69	OP	078	43	RCL	139	43	RCL	200	76	LBL
018	02	02	079	21	21	140	15	15	201	13	C
019	43	RCL	080	75	-	141	95	=	202	43	RCL
020	48	48	081	43	RCL	142	55	÷	203	14	14
021	69	OP	082	16	16	143	43	RCL	204	32	X⇌T
022	03	03	083	65	×	144	03	03	205	01	1
023	43	RCL	084	43	RCL	145	95	=	206	05	5
024	49	49	085	15	15	146	42	STO	207	71	SBR
025	69	OP	086	54	)	147	14	14	208	69	OP
026	04	04	087	42	STO	148	43	RCL	209	98	ADV
027	69	OP	088	02	02	149	01	01	210	91	R/S
028	05	05	089	95	=	150	65	×	211	76	LBL
029	43	RCL	090	55	÷	151	43	RCL	212	14	D
030	03	03	091	53	(	152	06	06	213	42	STO
031	65	×	092	53	(	153	85	+	214	07	07
032	43	RCL	093	43	RCL	154	43	RCL	215	32	X⇌T
033	25	25	094	03	03	155	00	00	216	04	4
034	75	-	095	65	×	156	65	×	217	04	4
035	43	RCL	096	43	RCL	157	43	RCL	218	71	SBR
036	16	16	097	22	22	158	04	04	219	69	OP
037	65	×	098	75	-	159	95	=	220	33	X²
038	43	RCL	099	43	RCL	160	55	÷	221	65	×
039	08	08	100	16	16	161	53	(	222	43	RCL
040	95	=	101	33	X²	162	43	RCL	223	01	01
041	42	STO	102	54	)	163	03	03	224	95	=
042	06	06	103	65	×	164	65	×	225	85	+
043	65	×	104	43	RCL	165	43	RCL	226	43	RCL
044	53	(	105	05	05	166	09	09	227	07	07
045	43	RCL	106	75	-	167	75	-	228	65	×
046	03	03	107	43	RCL	168	43	RCL	229	43	RCL
047	65	×	108	02	02	169	08	08	230	00	00
048	43	RCL	109	33	X²	170	33	X²	231	85	+
049	16	16	110	95	=	171	95	=	232	43	RCL
050	75	-	111	42	STO	172	32	X⇌T	233	14	14
051	43	RCL	112	01	01	173	03	3	234	95	=
052	15	15	113	94	+/-	174	05	5	235	32	X⇌T
053	33	X²	114	65	×	175	06	6	236	04	4
054	54	)	115	43	RCL	176	08	8	237	05	5
055	42	STO	116	02	02	177	71	SBR	238	06	6
056	05	05	117	85	+	178	69	OP	239	05	5
057	75	-	118	43	RCL	179	91	R/S	240	71	SBR
058	53	(	119	04	04	180	76	LBL	241	69	OP
059	43	RCL	120	95	=	181	11	A	242	98	ADV
060	03	03	121	55	÷	182	43	RCL	243	91	R/S

TI-59 LINEAR-HYPERBOLIC CURVE FITTING PROGRAM

000	76	LBL
001	69	OP
002	69	OP
003	00	00
004	69	OP
005	04	04
006	32	X:T
007	69	OP
008	06	06
009	92	RTN
010	76	LBL
011	15	E
012	98	ADV
013	69	OP
014	00	00
015	43	RCL
016	45	45
017	69	OP
018	02	02
019	43	RCL
020	38	38
021	69	OP
022	03	03
023	43	RCL
024	50	50
025	69	OP
026	04	04
027	69	OP
028	05	05
029	43	RCL
030	05	05
031	65	×
032	43	RCL
033	03	03
034	54	)
035	75	-
036	43	RCL
037	04	04
038	33	X²
039	95	=
040	42	STO
041	52	52
042	43	RCL
043	03	03
044	65	×
045	43	RCL
046	26	26
047	54	)
048	75	-
049	43	RCL
050	01	01
051	65	×
052	43	RCL
053	19	19
054	95	=
055	42	STO
056	53	53
057	43	RCL
058	03	03
059	33	X²
060	75	-
061	43	RCL
062	04	04
063	65	×
064	43	RCL
065	19	19
066	95	=
067	42	STO
068	54	54
069	43	RCL
070	06	06
071	65	×
072	43	RCL
073	03	03
074	54	)
075	75	-
076	43	RCL
077	04	04
078	65	×
079	43	RCL
080	01	01
081	95	=
082	42	STO
083	55	55
084	43	RCL
085	03	03
086	65	×
087	43	RCL
088	20	20
089	54	)
090	75	-
091	43	RCL
092	19	19
093	33	X²
094	95	=
095	42	STO
096	56	56
097	43	RCL
098	52	52
099	65	×
100	43	RCL
101	53	53
102	75	-
103	43	RCL
104	54	54
105	65	×
106	43	RCL
107	55	55
108	95	=
109	55	÷
110	53	(
111	53	(
112	43	RCL
113	52	52
114	65	×
115	43	RCL
116	56	56
117	54	)
118	75	-
119	43	RCL
120	54	54
121	33	X²
122	95	=
123	42	STO
124	57	57
125	43	RCL
126	55	55
127	75	-
128	53	(
129	43	RCL
130	54	54
131	65	×
132	43	RCL
133	57	57
134	95	=
135	55	÷
136	43	RCL
137	52	52
138	95	=
139	42	STO
140	58	58
141	43	RCL
142	01	01
143	75	-
144	43	RCL
145	58	58
146	65	×
147	43	RCL
148	04	04
149	95	=
150	75	-
151	43	RCL
152	57	57
153	65	×
154	43	RCL
155	19	19
156	95	=
157	55	÷
158	43	RCL
159	03	03
160	95	=
161	42	STO
162	59	59
163	43	RCL
164	01	01
165	33	X²
166	55	÷
167	43	RCL
168	03	03
169	95	=
170	42	STO
171	14	14
172	43	RCL
173	59	59
174	65	×
175	43	RCL
176	01	01
177	54	)
178	85	+
179	43	RCL
180	58	58
181	65	×
182	43	RCL
183	06	06
184	54	)
185	85	+
186	43	RCL
187	57	57
188	65	×
189	43	RCL
190	26	26
191	54	)
192	75	-
193	43	RCL
194	14	14
195	95	=
196	55	÷
197	53	(
198	43	RCL
199	02	02
200	75	-
201	43	RCL
202	14	14
203	95	=
204	32	X:T
205	03	3
206	05	5
207	07	7
208	08	8
209	71	SBR
210	69	OP
211	91	R/S
212	76	LBL
213	11	A
214	43	RCL
215	59	59
216	32	X:T
217	01	1
218	03	3
219	71	SBR
220	69	OP
221	91	R/S
222	76	LBL
223	12	B
224	43	RCL
225	58	58
226	32	X:T
227	01	1
228	04	4
229	71	SBR
230	69	OP
231	91	R/S
232	76	LBL
233	13	C
234	43	RCL
235	57	57
236	32	X:T
237	01	1
238	05	5
239	71	SBR
240	69	OP
241	98	ADV
242	91	R/S
243	76	LBL
244	14	D
245	42	STO
246	07	07
247	32	X:T
248	04	4
249	04	4
250	71	SBR
251	69	OP
252	65	×
253	43	RCL
254	58	58
255	54	)
256	85	+
257	43	RCL
258	59	59
259	54	)
260	85	+
261	43	RCL
262	57	57
263	55	÷
264	43	RCL
265	07	07
266	95	=
267	32	X:T
268	04	4
269	05	5
270	06	6
271	05	5
272	71	SBR
273	69	OP
274	98	ADV
275	91	R/S

MULTIPLE CURVE FITTING PROGRAM

SHARP PC-1211/TRS-80 PC-1

This program fits 8 curves to a set of X,Y data points. Data may be added or deleted from the calculator at any time. The best fitting curve can be determined automatically in about 45 seconds based on the coefficient of determination, RR. A curve can also be selected manually by entering the appropriate curve number from the table below. An estimate of Y can be computed for any value of X once a curve has been selected.

Curve Number	Type	General Equation	Page
1	Linear	$Y = a + bX$	12
2	Reciprocal	$Y = 1/(a + bX)$	22
3	Hyperbola	$Y = a + b/X$	26
4	Reciprocal Hyperbola	$Y = X/(aX + b)$	30
5	Logarithmic	$Y = a + b\ln X$	58
6	Reciprocal Log	$Y = 1/(a + b\ln X)$	60
7	Modified Power	$Y = ab^X$	44
8	Power	$Y = aX^b$	42

Program Operation

The MCF program requires all of memory and is designed to work with the optional printer when attached. The program displays the equation of any selected curve as well as its coefficients and the coefficient of determination, RR. Each X,Y data point requires about 7 seconds to input and may be deleted or corrected at any time.

Limits and Warnings

The coefficient of determination is displayed for RR and is used in all comparisons for the best fitting curve. A corrected value of RR should be used for all comparisons with higher order curves as shown on page 13.

Whenever X or Y is zero, it is replaced with 1E-09 by the data entry routine. This technique for dealing with zero may sometimes cause curves with 1/X or 1/Y terms to halt the program and display an error. If this happens, either eliminate points with a zero or avoid fitting curves that would involve the reciprocal of zero. If any value of X is negative, curves 5, 6 and 8 should not selected. If any value of Y is negative, curves 7 and 8 should not be selected.

REGISTER ASSIGNMENTS

SHARP PC-1211/TRS-80 PC-1

The MCF program requires registers A(1) through A(32) for data storage and uses the remainder of memory for the program. The limited amount of memory available requires that registers be assigned differently from those in the text and that temporary registers be reused as much as possible.

A = X value, coefficient A

B = Y value, coefficient B

C = 1 (add data), -1 (delete data)

D = ln X, temporary

E = ln Y, temporary

F = RR, temporary

G = curve number

H = best RR, solve or print if 9

I = Σ X

J = ΣX^2

K = Σ Y

L = ΣY^2

M = Σ X*Y

N = number of data points

O = Σ 1/X

P = $\Sigma 1/X^2$

Q = Σ 1/Y

R = $\Sigma 1/Y^2$

S = Σ 1/(X*Y)

T = Σ X/Y

U = Σ Y/X

V = Σ ln X

W = $\Sigma (\ln X)^2$

X = Σ ln Y

Y = $\Sigma (\ln Y)^2$

Z = Σ (ln X)*(ln Y)

A(27) = Σ Y*ln X

A(28) = Σ X*ln Y

A(29) = Σ (ln X)/Y

A(30) = 0 if X<0, 1 otherwise

A(31) = 0 if Y<0, 1 otherwise

A(32) = best curve

KEY ASSIGNMENTS

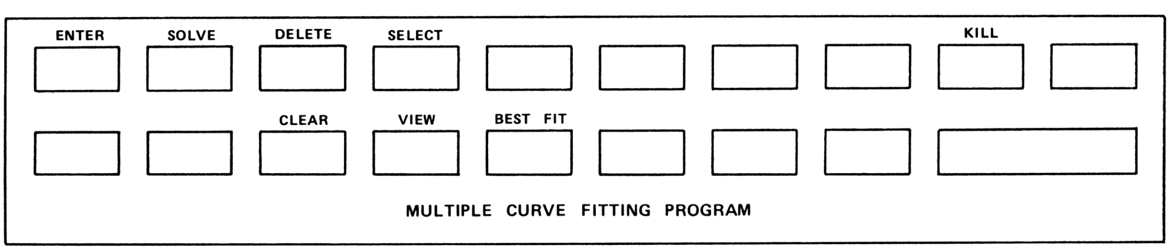
SHARP PC-1211/TRS-80 PC-1

When the calculator is in DEF mode, the following functions are assigned to the keyboard:

SHIFT A	Enter values of X and Y.
SHIFT B	Find the best curve.
SHIFT C	Clear all registers and initialize program.
SHIFT D	Delete any value of X and Y.
SHIFT F	Find coefficients for a specific curve.
SHIFT K	Delete the last values of X and Y entered. (Use before B or F.)
SHIFT S	Solve for Y using the selected curve and a value of X.
SHIFT V	View the last values of X and Y entered.

If the optional printer is attached, the calculator will automatically record all input and output data. If the printer is not attached, the calculator will stop after each output so that you may record the data.

It is a good practice to run the MCF program a second time with the X and Y values exchanged when you are trying to obtain the best possible fit.



STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Enter the program and set the calculator in DEF mode using the MODE key.			
1	Initialize program and registers.		SHIFT C	ENTER X:
2	Enter the X value.	X	ENTER	ENTER Y:
3	Enter the Y value.	Y	ENTER	
4	Repeat steps 2 and 3 for each data point.			ENTER X:
5a	View the last X value entered.		SHIFT V	X
5b	View the last Y value entered.		ENTER	Y
5c	Continue entering data (Step 2)		ENTER	ENTER X:
6a	Delete the last X and Y entered prior to Steps 7 through 11.		SHIFT K	
6b	Continue entering data (Step 2).			ENTER X:
7a	Delete any value of X and Y.		SHIFT D	DELETE DATA
7b	Press ENTER after the prompt.		ENTER	ENTER X:
7c	Enter the X value to be deleted.	X	ENTER	ENTER Y:
7d	Enter the Y value to be deleted.	Y	ENTER	
7e	Continue entering data (Step 2)			ENTER X:
8a	Find the best fitting curve.		SHIFT B	
8b	Display the A coefficient.		ENTER	A
8c	Display the B coefficient.		ENTER	B
8d	Display the coefficient of determination.		ENTER	RR
9a	Find the coefficients of any curve.		SHIFT F	ENTER CURVE NUMBER:
9b	Enter the curve number.	i	ENTER	
9c	Display the A coefficient.			A
9d	Display the B coefficient.		ENTER	B
9e	Display the coefficient of determination.		ENTER	RR
10a	Estimate Y for a given X.		SHIFT S	SOLVE FOR Y WHEN X=
10b	Enter the X value.	X	ENTER	X
10c	Estimate the value of Y.		ENTER	Y
10d	Solve for another Y (Step 10a).		ENTER ENTER ENTER	
11a	Enter additional X,Y values.		SHIFT A	ENTER X:
11b	Repeat steps 2 and 3.			

MULTIPLE CURVE FITTING PROGRAM

SHARP PC-1211/TRS 80 PC-1

```
10:"A":BEEP 1:C=1
11:INPUT "ENTER X: ";A,"ENTER Y: ";B:IF A=0LET A=E-9
12:IF B=0LET B=E-9
13:I=I+AC:J=J+AAC:K=K+BC:L=L+BBC:M=M+ABC:N=N+C:
   O=O+C/A:P=P+C/(AA):Q=Q+C/B
14:R=R+C/(BB):S=S+C/(AB):T=T+CA/B:U=U+CB/A
15:IF A>0LET D=LN (A):A(29)=A(29)+CD/B:GOTO 17
16:D=0:A(30)=0
17:IF B>0LET E=LN (B):GOTO 19
18:E=0:A(31)=0
19:V=V+CD:W=W+CDD:X=X+CE:Y=Y+CEE:Z=Z+CED:A(27)=A(27)+BCD:
   A(28)=A(28)+ACE:GOTO 10
20:"C":CLEAR :A(30)=1:A(31)=1:GOTO 10
21:"D":PRINT "DELETE DATA:":C=-1:GOTO 11
22:"V":PRINT "X=";A:PRINT "Y=";B:GOT0 10
23:"F":INPUT "ENTER CURVE NUMBER: ";G
24:H=9:GOTO G*6+21
25:"S":INPUT "SOLVE FOR Y WHEN X= ";D:GOTO G*6+25
26:"B":H=0
27:G=1:A=J:B=I:D=K:E=M:F=L:GOTO 78
29:PRINT "Y=A+BX":GOTO 81
31:E=A+BD:GOTO 82
33:G=2:A=J:B=I:D=Q:E=T:F=R:GOTO 78
35:PRINT "Y=1/(A+BX)":GOTO 81
37:E=1/(A+BD):GOTO 82
39:G=3:A=P:B=O:D=K:E=U:F=L:GOTO 78
41:PRINT "Y=A+B/X":GOTO 81
43:E=A+B/D:GOTO 82
45:G=4:A=P:B=O:D=Q:E=S:F=R:GOTO 78
47:PRINT "Y=X/(AX+B)":GOTO 81
49:E=D/(AD+B):GOTO 82
51:G=5:IF A(30)LET A=W:B=V:D=K:E=A(27):F=L:GOTO 78
52:GOTO 83
53:PRINT "Y=A+B*LN(X)":GOTO 81
55:E=A+B*LN (D):GOTO 82
57:G=6:IF A(30)LET A=W:B=V:D=Q:E=A(29):F=R:GOTO 78
58:GOTO 83
59:PRINT "Y=1/(A+B*LN(X))":GOTO 81
61:E=1/(A+B*LN (D)):GOTO 82
63:G=7:IF A(31)LET A=J:B=I:D=X:E=A(28):F=Y:GOTO 78
64:GOTO 83
65:PRINT "Y=A*B^X":A=EXP (A):B=EXP (B):GOTO 81
67:E=AB^D:GOTO 82
69:G=8:IF A(30)A(31)LET A=W:B=V:D=X:E=Z:F=Y:GOTO 78
70:GOTO 83
71:PRINT "Y=A*X^B":A=EXP (A):GOTO 81
73:E=AD^B:GOTO 82
75:G=A(32):GOTO 24
78:C=AN-BB:A=(AD-BE)/C:B=(EN-BD)/C:F=(AD+BE-DD/N)/(F-DD/N):
   IF F>HLET H=F:A(32)=G
79:IF H<>9GOTO G*6+27
80:BEEP 1:GOTO G*6+23
81:USING :PRINT "A= ";A:PRINT "B= ";B:PRINT USING "##.####";
 "RR= ";F:PRINT " ":END
82:USING :PRINT "X= ";D:PRINT "Y ";E:PRINT " ":GOTO 25
83:IF H<>9GOTO 79
84:PRINT USING ;"CANNOT FIT`NO. ";G
85:"K":C=-1:GOTO 13
```

NOTE: Lines 11 and 12 contain the exponent E-9.

MULTIPLE CURVE FITTING PROGRAM

HP-75C

This program fits up to 19 curves to an unlimited number of X,Y data points. Any curve can be selected by entering the appropriate curve number from the table below. The best fitting curve can be determined automatically based on the adjusted coefficient of determination, RR. Once a curve has been selected either manually or automatically, values of Y can be estimated for any given value of X.

Curve Number	Type	General Equation	Page
1	Linear	$Y = a + bX$	12
2	Reciprocal	$Y = 1/(a + bX)$	22
3	Linear-Hyperbolic	$Y = a + bX + c/X$	32
4	Hyperbola	$Y = a + b/X$	26
5	Reciprocal Hyperbola	$Y = X/(aX + b)$	30
6	2nd Order Hyperbola	$Y = a + b/X + c/X^2$	34
7	Parabola	$Y = a + bX + cX^2$	36
8	Cauchy Distribution	$Y = 1/[a(X+b)^2 + c]$	76
9	Logarithmic	$Y = a + b\ln X$	58
10	Reciprocal Log	$Y = 1/(a + b\ln X)$	60
11	Power	$Y = aX^b$	42
12	Super Geometric	$Y = aX^{bX}$	48
13	Modified Geometric	$Y = aX^{b/X}$	50
14	Hoerl Function	$Y = ab^X X^c$	64
15	Modified Hoerl	$Y = ab^{1/X} X^c$	66
16	Log-Normal	$Y = ae^{(b-\ln X)^2/c}$	70
17	Modified Power	$Y = ab^X$	44
18	Root	$Y = ab^{1/X}$	46
19	Normal Distribution	$Y = ae^{(X-b)^2/c}$	68

Program Operation

The program is designed so that the user need only step through a menu using the space bar until the desired selection is found. Pressing the RTN key causes the current menu item to be executed. Menu items are preceded by an * to distinguish them from commands requiring user input. When first executed, the MCF program also allows a print option to be selected.

The HP-75C is somewhat unique in that multiple data files may be stored in memory and randomly accessed. The program allows such files to be created and selected by the user at any time.

Most programmers will have little difficulty in adding new curves to the MCF program using the information on pages C-3 through C-5.

Limits and Warnings

Enter at least four data points when using the FIND BEST FIT option. This is necessary because the corrected RR calculation uses a divisor of N-3 for curves 3, 6, 7, 8, 12, 13, 14 and 19, where N is the number of data points. All other curves require a minimum of three data points.

Data may be deleted at any time by exercising the DELETE LAST X,Y or DELETE DATA options. When using the DELETE DATA option, be sure that you delete a valid data point or the data base will be meaningless. You can determine the last X and Y entered or deleted by halting the program and pressing X RTN or Y RTN. Resume program execution by keying CONT and pressing RTN.

Whenever X or Y is zero, it is replaced with 1E-10 by the data entry routine. This technique for dealing with zero may sometimes cause curves with 1/X or 1/Y terms to display a data error or warning. If this happens, either eliminate any point with a zero or avoid fitting curves that would involve the reciprocal of zero.

The logarithm of a negative data point is automatically set to zero during input. The results are therefore meaningless if you try to fit certain models to negative data. If any value of X is negative, curves 9 through 16 should not be selected. If any value of Y is negative, curves 9 through 14 and curves 17 through 19 should not be selected.

The corrected coefficient of determination is displayed for RR and used in all comparisons to find the best fitting curve. If the corrected RR is negative, it is set to zero.

When seeking the best possible fit, it is a good practice to run the MCF program a second time with the values of X and Y exchanged during input.

HP-75 REGISTER ASSIGNMENTS

The MCF program requires approximately 8k bytes of memory plus 387 bytes for each data file created by the curve fitting program. The correspondence between HP-75C registers and those used in the text is shown in the following table.

HP-75	REGISTER CONTENTS	TEXT	TEXT	REGISTER CONTENTS	HP-75
R(01)	Σ X	R16	R16	Σ X	R(01)
R(02)	Σ X^2	R17	R17	Σ X^2	R(02)
R(03)	Σ Y	R18	R18	Σ Y	R(03)
R(04)	Σ Y^2	R19	R19	Σ Y^2	R(04)
R(05)	Σ X*Y	R20	R20	Σ X*Y	R(05)
R(06)	n	R21	R21	n	R(06)
R(07)	Σ 1/X	R22	R22	Σ 1/X	R(07)
R(08)	Σ $1/X^2$	R23	R23	Σ $1/X^2$	R(08)
R(09)	Σ 1/Y	R24	R24	Σ 1/Y	R(09)
R(10)	Σ $1/Y^2$	R25	R25	Σ $1/Y^2$	R(10)
R(11)	Σ 1/(X*Y)	R26	R26	Σ 1/(X*Y)	R(11)
R(12)	Σ X/Y	R34	R28	Σ lnX	R(23)
R(13)	Σ Y/X	R35	R29	Σ $(\ln X)^2$	R(24)
R(14)	Σ X^2*Y	R36	R30	Σ lnY	R(34)
R(15)	Σ X^2/Y	R37	R31	Σ $(\ln Y)^2$	R(35)
R(16)	Σ Y/X^2	R38	R32	Σ (lnX)*(lnY)	R(39)
R(17)	Σ X*Y^2	R39	R34	Σ X/Y	R(12)
R(18)	Σ X^3	R40	R35	Σ Y/X	R(13)
R(19)	Σ $1/X^3$	R41	R36	Σ X^2*Y	R(14)
R(20)	Σ Y^3	R42	R37	Σ X^2/Y	R(15)
R(21)	Σ X^4	R43	R38	Σ Y/X^2	R(16)
R(22)	Σ $1/X^4$	R44	R39	Σ X*Y^2	R(17)
R(23)	Σ lnX	R28	R40	Σ X^3	R(18)
R(24)	Σ $(\ln X)^2$	R29	R41	Σ $1/X^3$	R(19)
R(25)	Σ (lnX)/X	R45	R42	Σ Y^3	R(20)
R(26)	Σ X*lnX	R48	R43	Σ X^4	R(21)
R(27)	Σ $(X*\ln X)^2$	R49	R44	Σ $1/X^4$	R(22)
R(28)	Σ Y*(lnX)	R51	R45	Σ (lnX)/X	R(25)
R(29)	Σ (lnX)/Y	R52	R46	Σ X*(lnY)	R(36)
R(30)	Σ $[(\ln X)/X]^2$	R53	R47	Σ (lnY)/X	R(37)
R(31)	Σ $(\ln X)^3$	R55	R48	Σ X*lnX	R(26)
R(32)	Σ $(\ln X)^4$	R56	R49	Σ $(X*\ln X)^2$	R(27)
R(33)	Σ Y*(lnX)/X	R58	R50	Σ X*(lnX)*(lnY)	R(40)
R(34)	Σ lnY	R30	R51	Σ Y*lnX	R(28)
R(35)	Σ $(\ln Y)^2$	R31	R52	Σ (lnX)/Y	R(29)
R(36)	Σ X*lnY	R46	R53	Σ $[(\ln X)/X]^2$	R(30)
R(37)	Σ (lnY)/X	R47	R54	Σ (lnY)*X^2	R(38)
R(38)	Σ (lnY)*X^2	R54	R55	Σ $(\ln X)^3$	R(31)
R(39)	Σ (lnX)*(lnY)	R32	R56	Σ $(\ln X)^4$	R(32)
R(40)	Σ X*(lnX)*(lnY)	R50	R57	Σ (lnY)*$(\ln X)^2$	R(41)
R(41)	Σ (lnY)*$(\ln X)^2$	R57	R58	Σ Y*(lnX)/X	R(33)

HP-75C USER INSTRUCTIONS

The following is a list of menu options in the MCF program. All menu items are preceded by an * to distinguish them from commands that require user input. Whenever the program prompts for an input, the RTN key may be pressed instead to take you back into the menu. Use the space bar whenever you want to step to the next menu item.

```
                                  * START NEW DATA FILE
                                  * USE OLD DATA FILE
IS HP-IL CONNECTED (Y/N)?         * SAVE DATA FILE        ** ADD DATA
PRINT OUTPUT (Y/N)?               * MODIFY DATA           ** DELETE LAST X,Y
                                  * SELECT A CURVE        ** DELETE DATA
                                  * FIND BEST FIT
                                  * SOLVE FOR Y
```

RUN "MCF". Start the multiple curve fitting program by entering RUN "MCF" and pressing the RTN key.

IS HP-IL CONNECTED (Y/N)? This option appears when the program is first started. Press Y and RTN if the HP-IL loop is connected to the HP-75 (make sure each device on the loop is first powered up). If you do not wish to use the HP-IL loop, press N (or any key other than Y) and RTN. (Be sure to execute the command 'OFFIO' prior to 1) turning off a device on the loop, 2) disconnecting a device on the loop, or 3) disconnecting the loop from the computer.)

PRINT OUTPUT (Y/N)? This option appears if the response to the previous question was yes. Press Y and RTN if a printer is attached and you desire to print all input and output data. The program will direct output to any peripheral assigned a device code of P. (Device codes are declared with the ASSIGNIO command. Output may be directed to both a printer and a tv monitor using the command PRINTER IS ':P,:TV'. You may also want to direct display information to a tv monitor using the command DISPLAY IS ':TV'). If you do not wish to direct output to a printer, respond to PRINT OUTPUT (Y/N) by pressing N (or any key other than Y) and RTN.

START NEW DATA FILE. Press RTN if you want to enter data that has not been previously stored in a file. The program will now prompt you for data entry as described under ADD DATA. This option can also be used to clear out the current data file without storing it.

USE OLD DATA FILE. Press RTN if you want to call an existing data file. The program will prompt you for the file name and load this file into the data registers. If the file does not exist, a tone will sound for 0.5 seconds before returning to the same menu item.

SAVE DATA. Press RTN if you want to save the current file. The program will prompt for the file name you want the data stored under and return to the main menu. If a name is not entered, a tone sounds for 0.5 seconds to warn you that no file has been saved. It is a good practice to save newly created files before modifying the data or fitting any curves.

MODIFY DATA. Press RTN if you want to add or delete data from the current file. A secondary menu will appear.

ADD DATA. Press RTN if you want to add data to the current file. The program will now prompt you for each data point by displaying ENTER X(n),Y(n). Enter the value of X followed by a comma and the value of Y. Press RTN to store the data. If you press RTN without entering data, the program will return to the main menu.

DELETE LAST X,Y. Press RTN if you want to delete the last data point entered. This option should only be used if you have been entering data. It should never be used once the curve fitting routines have been exercised since last X and Y are lost. If this option is selected, the display will momentarily show DELETED: and return to the main menu.

DELETE DATA. Press RTN if you want to delete a point from the current file. The program will prompt you for the value of X and Y and signify that the data are to be deleted by displaying ENTER X(-1),Y(-1). Enter the value of X followed by a comma and the value of Y. When you press RTN, the display will momentarily show DELETED: and return to the main menu. The program will also return to the main menu if no data are entered.

SELECT CURVE. Press RTN if you want to fit a specific curve. The program will prompt you for a curve number (see p. H-1). The curve corresponding to this number will be displayed along with the coefficients and the corrected coefficient of determination (RR). If a printer is not being used, press any key to display the next output. Note that the coefficient c is not displayed if it is equal to zero. When an invalid curve number is used, a tone sounds for 0.5 seconds before returning to the main menu.

FIND BEST FIT. Press RTN if you want the program to find the best fitting curve automatically. The display will show the corrected coefficient of determination (RR) for each curve before displaying the best fit curve and its coefficients. (Note that the output can be slowed down by changing the delay constant in line 15 of the program.) If a printer is not being used, press any key to display successive coefficients.

SOLVE FOR Y. Press RTN if you want to use the last curve displayed to calculate Y for a given X. The program will prompt you for a value of X. Press RTN to find the corresponding value of Y. The program will continue to prompt for X until you press RTN without entering a value. If you try to evaluate a curve such as Y=a+b/X for X=0, the program will prompt you to select another curve.

Two additional commands that will be useful are CAT ALL and PURGE. CAT ALL displays one file each time the ↓ key is pressed. Execute PURGE "filename" to delete any unwanted files.

HP-75C PROGRAM LISTING

```
10 OPTION BASE 1 @ DIM R(41)
15 F3=0 @ DELAY 1 @ REM DISPLAY PAUSE IS 1 SECOND
20 INPUT "IS HPIL CONNECTED (Y/N)? ";R$
25 IF R$#"Y" AND R$#"y" THEN 50
30 DISP "PRINT OUTPUT (Y/N)"; @ INPUT R$
35 IF R$#"Y" AND R$#"y" THEN PRINTER IS * @ GOTO 50
40 F3=1 @ PRINTER IS ':P' @ PRINT "MULTIPLE CURVE FIT" @ DELAY 0
50 DISP "* START NEW DATA FILE"
55 K=NUM(KEY$) @ IF K#13 AND K#32 THEN GOTO 55
60 IF K=13 THEN GOTO 250
65 DISP "* USE OLD DATA FILE"
70 K=NUM(KEY$) @ IF K#13 AND K#32 THEN GOTO 70
75 IF K=13 THEN GOTO 160
80 DISP "* MODIFY DATA"
85 K=NUM(KEY$) @ IF K#13 AND K#32 THEN GOTO 85
90 IF K=13 THEN GOTO 195
95 DISP "* SAVE DATA FILE"
100 K=NUM(KEY$) @ IF K#13 AND K#32 THEN GOTO 100
105 IF K=13 THEN W=1 @ R1=0 @ GOTO 180
110 DISP "* SELECT A CURVE"
115 K=NUM(KEY$) @ IF K#13 AND K#32 THEN GOTO 115
120 IF K=13 THEN GOTO 400
125 DISP "* FIND BEST FIT"
130 K=NUM(KEY$) @ IF K#13 AND K#32 THEN GOTO 130
135 IF K=13 THEN GOTO 415
140 DISP "* SOLVE FOR Y"
145 K=NUM(KEY$) @ IF K#13 AND K#32 THEN GOTO 145
150 IF K=13 THEN GOTO 675
155 GOTO 50
160 ON ERROR BEEP 2000,.5 @ GOTO 175
165 INPUT "OLD FILE NAME? "; R$ @ ASSIGN # 1 TO R$ @ READ # 1 ; R(),F1,F2
170 ASSIGN # 1 TO * @ OFF ERROR @ GOTO 80
175 DISP "NO SUCH FILE" @ GOTO 65
180 ON ERROR BEEP 2000,.5 @ GOTO 110
185 INPUT "FILE NAME? "; R$ @ ASSIGN # 1 TO R$ @ PRINT # 1 ; R(),F1,F2
190 ASSIGN # 1 TO * @ GOTO 80
195 DISP "** ADD DATA"
200 K=NUM(KEY$) @ IF K#13 AND K#32 THEN GOTO 200
205 IF K=13 THEN GOTO 255
210 ON ERROR GOTO 80
215 DISP "** DELETE LAST X,Y"
220 K=NUM(KEY$) @ IF K#13 AND K#32 THEN GOTO 220
225 IF K=13 THEN S=-1 @ GOTO 290
230 DISP "** DELETE DATA"
235 K=NUM(KEY$) @ IF K#13 AND K#32 THEN GOTO 235
240 IF K=13 THEN S=-1 @ I=S @ GOTO 270
245 GOTO 95
250 FOR I=1 TO 41 @ R(I)=0 @ NEXT I @ F1=0 @ F2=0 @ REM INITIALIZE REGISTERS
255 S=1 @ REM DATA ENTRY ROUTINE
260 ON ERROR GOTO 95
265 I=R(6)+S
270 DISP "ENTER X(";I;"),Y(";I;"):"; @ INPUT X,Y
275 IF S>0 AND F3 THEN GOSUB 910
280 IF X=0 THEN X=.0000000001 @ REM MAKE X NON-ZERO
285 IF Y=0 THEN Y=.0000000001 @ REM MAKE Y NON-ZERO
290 R(1)=R(1)+S*X @ R(2)=R(2)+S*X*X @ R(3)=R(3)+S*Y @ R(4)=R(4)+S*Y*Y
295 R(5)=R(5)+S*X*Y @ R(6)=R(6)+S @ R(7)=R(7)+S/X @ R(8)=R(8)+S/(X*X)
300 R(9)=R(9)+S/Y @ R(10)=R(10)+S/(Y*Y) @ R(11)=R(11)+S/(X*Y)
```

```
305 R(12)=R(12)+S*X/Y @ R(13)=R(13)+S*Y/X @ R(14)=R(14)+S*X*X*Y
310 R(15)=R(15)+S*X*X/Y @ R(16)=R(16)+S*Y/(X*X) @ R(17)=R(17)+S*X*Y*Y
315 T=X*X*X @ R(18)=R(18)+S*T @ R(19)=R(19)+S/T @ R(20)=R(20)+S*Y*Y*Y
320 T=X*T @ R(21)=R(21)+S*T @ R(22)=R(22)+S/T
325 IF X<0 THEN K=0 @ F1=1 @ GOTO 335
330 K=LOG(X)
335 R(23)=R(23)+S*K @ R(24)=R(24)+S*K*K @ R(25)=R(25)+S*K/X
340 T=K*X @ R(26)=R(26)+S*T @ R(27)=R(27)+S*T*T @ R(28)=R(28)+S*K*Y
345 R(29)=R(29)+S*K/Y @ R(30)=R(30)+S*K*K/(X*X)
350 T=K*K*K @ R(31)=R(31)+S*T @ R(32)=R(32)+S*K*T
355 IF Y<0 THEN L=0 @ F2=1 @ GOTO 365
360 L=LOG(Y)
365 R(34)=R(34)+S*L @ R(35)=R(35)+S*L*L @ R(36)=R(36)+S*X*L
370 R(37)=R(37)+S*L/X @ R(38)=R(38)+S*X*X*L @ R(33)=R(33)+S*K*L/X
375 R(39)=R(39)+S*K*L @ R(40)=R(40)+S*X*K*L @ R(41)=R(41)+S*K*K*L
380 IF S>0 THEN GOTO 265
385 PRINT "DELETED: "; @ IF F3=0 THEN PRINT @ GOTO 80
390 GOSUB 910 @ GOTO 80
395 GOTO 265
400 ON ERROR BEEP 2000,.5 @ GOTO 110 @ REM SELECT CURVE
405 DISP "INPUT CURVE NUMBER: "; @ INPUT I @ W=0 @ PRINT
410 ON I GOTO 420,430,440,455,465,475,490,505,530,540,555,565,575,585,600,6
15,635,645,655
415 R1=0 @ W=1 @ REM FIND BEST FIT
420 PRINT " 1:Y=a+bX              "; @ A=R(2) @ B=R(1) @ D=R(3) @ E=R(5) @ F=
R(4)
425 I=1 @ GOSUB 800 @ IF W=0 THEN GOTO 885
430 PRINT " 2:Y=1/(a+bX)          "; @ A=R(2) @ B=R(1) @ D=R(9) @ E=R(12) @ F
=R(10)
435 I=2 @ GOSUB 800 @ IF W=0 THEN GOTO 885
440 PRINT " 3:Y=a+bX+c/X          "; @ A=R(1) @ B=R(2) @ C=R(8) @ D=R(7)
445 E=R(3) @ F=R(4) @ G=R(5) @ H=R(13) @ J=R(6)
450 I=3 @ GOSUB 815 @ IF W=0 THEN GOTO 885
455 PRINT " 4:Y=a+b/X             "; @ A=R(8) @ B=R(7) @ D=R(3) @ E=R(13) @ F
=R(4)
460 I=4 @ GOSUB 800 @ IF W=0 THEN GOTO 885
465 PRINT " 5:Y=X/(aX+b)          "; @ A=R(8) @ B=R(7) @ D=R(9) @ E=R(11) @ F
=R(10)
470 I=5 @ GOSUB 800 @ IF W=0 THEN GOTO 885
475 PRINT " 6:Y=a+b/X+c/X^2       "; @ A=R(7) @ B=R(8) @ C=R(22) @ D=R(8)
480 E=R(3) @ F=R(4) @ G=R(13) @ H=R(16) @ J=R(19)
485 I=6 @ GOSUB 815 @ IF W=0 THEN GOTO 885
490 PRINT " 7:Y=a+bX+cX^2         "; @ A=R(1) @ B=R(2) @ C=R(21) @ D=R(2)
495 E=R(3) @ F=R(4) @ G=R(5) @ H=R(14) @ J=R(18)
500 I=7 @ GOSUB 815 @ IF W=0 THEN GOTO 885
505 PRINT " 8:Y=1/[c+a(X+b)^2]    "; @ A=R(1) @ B=R(2) @ C=R(21) @ D=R(2)
510 E=R(9) @ F=R(10) @ G=R(12) @ H=R(15) @ J=R(18)
515 I=8 @ GOSUB 815 @ IF W THEN GOTO 525
520 A=A-B*B/(4*C) @ B=B/(C+C) @ T=A @ A=C @ C=T @ GOTO 885
525 IF F1 THEN GOTO 630 @ REM LOG OF NEGATIVE X
530 PRINT " 9:Y=a+b*lnX           "; @ A=R(24) @ B=R(23) @ D=R(3) @ E=R(28) @
 F=R(4)
535 I=9 @ GOSUB 800 @ IF W=0 THEN GOTO 885
540 PRINT "10:Y=1/(a+b*lnX)       "; @ A=R(24) @ B=R(23) @ D=R(9) @ E=R(29) @
 F=R(10)
545 I=10 @ GOSUB 800 @ IF W=0 THEN GOTO 885
550 IF F1 THEN GOTO 905 @ REM LOG OF NEGATIVE Y
555 PRINT "11:Y=aX^b              "; @ A=R(24) @ B=R(23) @ D=R(34) @ E=R(39)
@ F=R(35)
560 I=11 @ GOSUB 800 @ IF W=0 THEN GOTO 875
565 PRINT "12:Y=aX^(bX)           "; @ A=R(27) @ B=R(26) @ D=R(34) @ E=R(40)
@ F=R(35)
```

```
570 I=12 @ GOSUB 800 @ IF W=0 THEN GOTO 875
575 PRINT "13:Y=aX^(b/X)          "; @ A=R(30) @ B=R(25) @ D=R(34) @ E=R(33)
@ F=R(35)
580 I=13 @ GOSUB 800 @ IF W=0 THEN GOTO 875
585 PRINT "14:Y=a(b^X)(X^c)      "; @ A=R(1) @ B=R(2) @ C=R(24) @ D=R(23)
590 E=R(34) @ F=R(35) @ G=R(36) @ H=R(39) @ J=R(26)
595 I=14 @ GOSUB 815 @ IF W=0 THEN GOTO 870
600 PRINT "15:Y=a(b^1/X)(X^c)    "; @ A=R(7) @ B=R(8) @ C=R(24) @ D=R(23)
605 E=R(34) @ F=R(35) @ G=R(37) @ H=R(39) @ J=R(25)
610 I=15 @ GOSUB 815 @ IF W=0 THEN GOTO 870
615 PRINT "16:Y=ae^[(b-lnX)^2/c]"; @ A=R(23) @ B=R(24) @ C=R(32) @ D=R(24)
620 E=R(34) @ F=R(35) @ G=R(39) @ H=R(41) @ J=R(31)
625 I=16 @ GOSUB 815 @ IF W=0 THEN GOTO 670
630 IF F2 THEN GOTO 905 @ REM LOG OF NEGATIVE Y
635 PRINT "17:Y=ab^X              "; @ A=R(2) @ B=R(1) @ D=R(34) @ E=R(36) @
F=R(35)
640 I=17 @ GOSUB 800 @ IF W=0 THEN GOTO 870
645 PRINT "18:Y=ab^(1/X)          "; @ A=R(8) @ B=R(7) @ D=R(34) @ E=R(37) @
F=R(35)
650 I=18 @ GOSUB 800 @ IF W=0 THEN GOTO 870
655 PRINT "19:Y=ae^[(b-X)^2/c]  "; @ A=R(1) @ B=R(2) @ C=R(21) @ D=R(2)
660 E=R(34) @ F=R(35) @ G=R(36) @ H=R(38) @ J=R(18)
665 I=19 @ GOSUB 815 @ IF W THEN W=0 @ I=I1 @ PRINT @ GOTO 410
670 A=EXP(A-B*B/(4*C)) @ B=-B/(C+C) @ C=1/C @ GOTO 885
675 ON ERROR GOTO 110 @ REM SOLVE FOR Y
680 DISP "ENTER X: "; @ INPUT X
685 ON I GOSUB 700,705,710,715,720,725,730,735,740,745,750,755,760,765,770,
775,780,785,790
690 GOSUB 910
695 GOTO 680
700 Y=A+B*X @ RETURN
705 Y=1/(A+B*X) @ RETURN
710 Y=A+B*X+C/X @ RETURN
715 Y=A+B/X @ RETURN
720 Y=X/(A*X+B) @ RETURN
725 Y=A+B/X+C/(X*X) @ RETURN
730 Y=A+B*X+C*X*X @ RETURN
735 Y=1/(C+A*(B+X)^2) @ RETURN
740 Y=A+B*LOG(X) @ RETURN
745 Y=1/(A+B*LOG(X)) @ RETURN
750 Y=A*X^B @ RETURN
755 Y=A*X^(B/X) @ RETURN
760 Y=A*X^B @ RETURN
765 Y=A*B^X*X^C @ RETURN
770 Y=A*B^(1/X)*X^C @ RETURN
775 Y=A*EXP((B-LOG(X))^2/C) @ RETURN
780 Y=A*B^X @ RETURN
785 Y=A*B^(1/X) @ RETURN
790 Y=A*EXP((B-X)^2/C) @ RETURN
795 REM FIND COEFFICIENTS A & B AND RR
800 C=A*R(6)-B*B @ A=(A*D-B*E)/C @ B=(E*R(6)-B*D)/C @ C=0
805 K=D*D/R(6) @ E=(A*D+B*E-K)/(F-K) @ F=2 @ GOTO 840
810 REM FIND COEFFICIENTS A, B & C AND RR
815 B=B*R(6)-A*A @ C=C*R(6)-D*D @ J=J*R(6)-A*D @ L=G*R(6)-A*E
820 C=(B*(H*R(6)-D*E)-J*L)/(B*C-J*J)
825 B=(L-C*J)/B @ A=(E-A*B-C*D)/R(6)
830 K=E*E/R(6) @ E=(A*E+B*G+C*H-K)/(F-K) @ F=3
835 REM COMPUTE CORRECTED RR
840 E=1-(1-E)*(R(6)-1)/(R(6)-F) @ IF E<0 THEN E=0
845 IF E>1 THEN E=1
850 IF W=0 THEN PRINT @ RETURN
855 PRINT USING "'   RR=',d.dddd" ; E
```

HP-75C PROGRAM LISTING

```
860 IF E>R1 THEN R1=E @ I1=I @ REM STORE BETTER CURVE
865 RETURN
870 B=EXP(B)
875 A=EXP(A)
880 REM DISPLAY COEFFICIENTS
885 GOSUB 915 @ PRINT " a= ";A @ GOSUB 915
890 PRINT " b= ";B @ GOSUB 915
895 IF C#0 THEN PRINT " c= ";C @ GOSUB 915
900 PRINT " RR= ";E @ GOSUB 915
905 PRINT @ GOTO 140
910 PRINT "X= ";X; @ PRINT "  Y= ";Y
915 IF F3 THEN RETURN
920 IF KEY$="" THEN GOTO 920 ELSE RETURN @ REM HALT UNTIL A KEY IS PRESSED
```

TRANSLATING HP BASIC

The following comments will be useful to anyone unfamiliar with HP BASIC when translating to another BASIC.

Line 10 OPTION BASE 1 starts array R at R(01) rather than R(00).

Line 15 DELAY n pauses the program n seconds each time output is displayed.

Line 25 # represents NOT EQUAL, e.g., <> in some BASICs.

Line 30 DISP directs program output to the LCD display.

Line 55 KEY returns the last character entered. NUM(KEY$) returns the value of the last key pressed (RTN=13 and SPACE=32).

Line 160 ON ERROR redirects program flow when an error is encountered. It is used frequently in this program to branch back to the main menu if numeric input does not follow an INPUT command. BEEP f,d causes a tone of frequency f to sound for d seconds.

Line 165 This statement reads the file named into array R() along with values for F1 and F2.

Line 180 This statement stores the contents of array R() along with F1 and F2 into the file named.

Line 385 PRINT directs output to the display if a printer is not used.

MULTIPLE CURVE FITTING PROGRAM

Sharp PC-1500 and TRS-80 PC-2

By Rick Conner

This program fits any of nineteen curves to an unlimited number of X,Y data points. The curve selection can be made manually or the program can be instructed to find the best fit curve automatically on the basis of the highest adjusted coefficient of determination. Once fitted, the equation may be solved for Y with any given value of X. Data may be printed on the PC's plotter/cassette interface if it is available.

Curve Number	Type	General Equation	Page
1	Linear	$Y = a + bX$	12
2	Reciprocal	$Y = 1/(a + bX)$	22
3	Linear-Hyperbolic	$Y = a + bX + c/X$	32
4	Hyperbola	$Y = a + b/X$	26
5	Reciprocal Hyperbola	$Y = X/(aX + b)$	30
6	2nd Order Hyperbola	$Y = a + b/X + c/X^2$	34
7	Parabola	$Y = a + bX + cX^2$	36
8	Cauchy Distribution	$Y = 1/[a(X+b)^2 + c]$	76
9	Power	$Y = aX^b$	42
10	Super Geometric	$Y = aX^{bX}$	48
11	Modified Geometric	$Y = aX^{b/X}$	50
12	Hoerl Function	$Y = ab^X X^c$	64
13	Modified Hoerl	$Y = ab^{1/X} X^c$	66
14	Log-Normal	$Y = ae^{(\ln X - b)^2/c}$	70
15	Logarithmic	$Y = a + b\ln X$	58
16	Reciprocal Log	$Y = 1/(a + b\ln X)$	60
17	Modified Power	$Y = ab^X$	44
18	Root	$Y = ab^{1/X}$	46
19	Normal Distribution	$Y = ae^{(X-b)^2/c}$	68

Entering the Program

Press MODE to put the PC-1500 into PRO mode. Type the command NEWØ to clear the program memory. Enter the program lines as shown in the program listing, remembering to switch back to upper case since the PC does not accept BASIC statements typed in lower case. The following hints will help save time when entering this very long program:

Assign repetitive keystroke sequences such as R(,)=R, and)+C* to the reserve function keys, i.e., keys F1 through F6.

The curve fitting routines are all similar in structure. Enter one of them, e.g., lines 311 to 314, and copy it 18 times by changing the line numbers (i.e., change line 311 to 321, 321 to 331,..., and 481 to 491, then change 312 to 322, 322 to 332,..., and 482 to 492, etc.). Now go back and edit each line to make the necessary individual changes.

When you have finished entering the program and whenever you load it into memory, you should first type DEF and then C to initialize it.

Limits and Warnings

Enter at least four data points when fitting curves 3, 6, 7, 8, 12, 13, 14, or 19 and whenever you are using the DEF B function. At least three data points are required in all other cases. (You may satisfy these requirements by entering each of the data points twice.)

Whenever an X or Y value of zero is entered, it is automatically changed to 1E-9 to avoid division by zero during certain calculations. This technique for dealing with zero may sometimes cause curves with 1/X or 1/Y terms to display a data error or warning when the program is running. If this happens, either eliminate any point with a zero or avoid fitting curves that would involve the reciprocal of zero.

Whenever a negative or zero value of X or Y is entered, the logarithm of the value is also set to zero to avoid a data error. Fitting such data to certain curves will therefore be meaningless:

If any X is less than or equal to zero, disregard curve numbers 9 through 16

If any Y is less than or equal to zero, disregard curve numbers 9 through 14 and curves 17 through 19.

The program uses the corrected coefficient of determination whenever RR is displayed and in all comparisons to find the best fitting curve. When seeking the best possible fit, it is a good practice to run the program a second time with the values of X and Y exchanged during input.

PC-1500 USER INSTRUCTIONS

This program consists of several carefully interwoven routines. You may start these routines simply by pressing the DEF key followed by the appropriate letter. You may also go directly to any one of these routines whenever the BUSY indicator is not on.

DEF A	ADD one or more X,Y data points.
DEF B	Find the BEST fitting curve.
DEF C	CLEAR registers and go to the data entry routine.
DEF D	DELETE one or more X and Y data points.
DEF F	FIT any curve.
DEF L	LIST functions on the display or plotter.
DEF M	Display MENU of curves.
DEF S	SOLVE for Y when X and the curve number are given.
DEF V	VIEW the last value of X and Y.

The user activates any of these predefined functions by pressing the DEF key followed by the appropriate letter. The following is a detailed account of what each of these functions does.

CLEAR. DEF C should always be the <u>first</u> thing you type whenever you load the program into memory. Executing the DEF C command will cause the program to clear all variables and registers (you will lose any data stored), and ask whether the plotter is to be used before branching to the ADD data function.

ADD. In the DEF A mode of operation, you will be prompted for X and Y values to add to the data base. A special 'sigma-plus' flag will appear on the right side of the display to remind you that you are entering data. If you are using the plotter, the X and Y values as well as the index (the number of points entered so far) will be printed in black. About five to ten seconds is required to enter each data pair, after which you will be prompted for the next pair.

DELETE. DEF D allows you to delete points from the data base at any time. The program does not store the X and Y values themselves but only the register totals derived from them. You should therefore make sure that the points you want to delete have actually been entered before using this function. The DELETE function will show a 'sigma-minus' sign in the display and print the X and Y values in red along with the index. Note that you must type DEF A in order to return to the ADD mode.

VIEW. The DEF V command will display or print the last data pair entered along with a flag indicating whether or not the point was added or deleted. If the point was added, the displayed index value includes that point. If the point was deleted, the index does not include that point.

FIT. The DEF F function allows you to fit any of the nineteen curves to your data, subject to the limitations previously discussed. About two seconds after you enter a curve number, the value of coefficient A will be displayed: press the ENTER key repetitively to see the values of B, C and RR. If you are using the plotter, these values will be printed in green.

BEST. Typing DEF B will instruct the program to fit each of the nineteen curves to your data in search of the one that results in the highest corrected coefficient of correlation (RR). This curve and its coefficients will be displayed after 20 seconds.

SOLVE. Use DEF S to fit any of the curves to your data and solve for Y. Enter the curve number of the desired equation as in the DEF F function. You will now be prompted for a value of X: the corresponding Y value will be calculated and displayed immediately. You may press ENTER to clear the display and get the next X prompt without entering the curve number again. Like the ADD and DELETE functions, you must either go to another routine or press BREAK to exit this function.

LIST. The DEF L command will cause the DEF key functions to be listed on the display or on the plotter.

MENU. The DEF M function will instruct the program to print the numbers, names, and formulas of the nineteen curve types in the display or on the plotter.

PC-1500 MULTIPLE CURVE FITTING PROGRAM

```
  1:REM  Multi cur
     ve fitting for
     PC-1500, PC-2
  2:REM  Adapted b
     y Rick Conner
  9:REM  CLEAR & R
     ESET PROGRAM
 10:"C":CLS :BEEP
     5:WAIT 100:
     PRINT "***** C
     LEAR PROGRAM *
     ****":CLEAR :
     DIM R(41):WA=1
     00
 11:INPUT "Use plo
     tter? (y/n)...
     ";P$
 12:IF P$="Y"GOSUB
     1000
 13:DIM NA$(1)*32,
     BD$(1)*36:BD$(
     1)="----------
     --------------
     ------------"
 14:GOTO 90
 19:REM  VIEW LAST
     POINT
 20:"V":IF P$="Y"
     GOSUB 1005:
     GOTO 100
 21:WAIT WA:PRINT
     "View last poi
     nt...":GOSUB 6
     00:WAIT 0:
     CURSOR 0:PRINT
     "X=";LX:CURSOR
     10:WAIT :PRINT
     "Y=";LY
 22:GOSUB 600:WAIT
     :PRINT "n = ";
     R(6):GOTO 100
 29:REM FIND BEST
     FIT
 30:"B":IF P$="Y"
     GOSUB 1010
 31:WAIT WA:PRINT
     "Finding best
     fit...":MR=0:M
     I=0:K=0
 32:ON ERROR GOTO
     750:GOTO 311
 39:REM  FIT A CUR
     VE
 40:"F":WAIT WA:
     PRINT "Fit any
     curve...":
     GOSUB 610:
     INPUT "Enter c
     urve...";I:
     GOSUB 630+I
 41:IF P$="Y"GOSUB
     1015:GOSUB 104
     0
 42:GOSUB 610:WAIT
     0:PRINT NA$(1)
 43:K=1:L=1:GOTO I
     *10+300
 49:REM  FIND Y-VA
     LUES
 50:"S":WAIT WA:
     PRINT "Solve f
     or Y...":GOSUB
     620:INPUR "Ent
     er curve numbe
     r...";I:L=1:K=
     2
 51:GOSUB 630+I:
     GOSUB 620:WAIT
     WA:PRINT NA$:
     IF P$="Y"GOSUB
     1020:GOSUB 104
     0
 52:GOTO 301+10*I
 53:GOSUB 620:
     INPUT "X = ";X
     :GOTO 303+I*10
 55:L=0:IF P$="Y"
     GOSUB 1050:
     GOTO 53
 56:GOSUB 620:WAIT
     :PRINT "Y = ";
     Y:GOTO 53
 59:REM  DISPLAY F
     UNCTIONS
 60:"L":IF P$="Y"
     GOSUB 1030:
     GOTO 65
 61:BEEP 1:WAIT WA
     :PRINT "List f
     unctions..."
 62:FOR L=1TO 8:
     GOSUB 660+L:
     WAIT :PRINT NA
     $(1)
 63:NEXT L
 64:GOTO 999
 65:FOR L=1TO 8:
     GOSUB 660+L:LF
     1:GOSUB 1040
 66:NEXT L
 67:LF 1:LPRINT BD
     $(1):LF 2:GOTO
     999
 69:REM DISPLAY C
     URVES
 70:"M":IF P$="Y"
     GOSUB 1035:
     GOTO 75
 71:FOR L=1TO 19:
     GOSUB 630+L:
     WAIT :PRINT NA
     $(1)
 72:NEXT L
 73:GOTO 999
 75:FOR L=1TO 19:
     GOSUB 630+L:
     GOSUB 1040
 76:NEXT L
 77:LPRINT BD$(1):
     LF 2
 78:GOTO 999
 79:REM  DELETE PO
     INTS
 80:"D":C=-1:WAIT
     WA:PRINT "Dele
     te points...":
     GOSUB 601:GOTO
     100
 89:REM  ADD POINT
     S
 90:"A":C=1:WAIT W
     A:PRINT "Add p
     oints...":
     GOSUB 601
 99:REM  REGISTERS
100:INPUT "X = ";X
     :CURSOR 11:
     INPUT "Y = ";Y
101:IF X=0LET X=1E
     -9
102:IF Y=0LET Y=1E
     -9
103:LX=0:IF X>0LET
     LX=LN X
104:LY=0:IF Y>0LET
     LY=LN Y
105:R(1)=R(1)+C*X:
     R(2)=R(2)+C*X*
     X:R(3)=R(3)+C*
     Y:R(4)=R(4)+C*
     Y*Y:R(5)=R(5)+
     C*X*Y
106:R(6)=R(6)+C:R(
     7)=R(7)+C/X:R(
     8)=R(8)+C/X/X:
     R(9)=R(9)+C/Y:
     R(10)=R(10)+C/
     Y/Y
107:R(11)=R(11)+C/
     X/Y:R(12)=R(12
     )+C*X/Y:R(13)=
     R(13)+C*Y/X:R(
     14)=R(14)+C*X*
     X*Y
108:R(15)=R(15)+C*
     X*X/Y:R(16)=R(
     16)+C*Y/X/X:R(
     17)=R(17)+C*X*
     Y*Y
109:R(18)=R(18)+C*
     X*X*X:R(19)=R(
     19)+C/X/X/X:R(
     20)=R(20)+C*Y*
     Y*Y
110:R(21)=R(21)+C*
     X*X*X*X:R(22)=
     R(22)+C/X/X/X/
     X:R(23)=R(23)+
     C*LX
111:R(24)=R(24)+C*
     LX*LX:R(25)=R(
     25)+C*LX/X:R(2
     6)=R(26)+C*LX*
     X
112:R(27)=R(27)+C*
     X*X*LX*LX:R(28
     )=R(28)+C*Y*LX
     :R(29)=R(29)+C
     *LX/Y
113:R(30)=R(30)+C*
     LX*LX/X/X:R(31
     )=R(31)+LX*LX*
     LX:R(32)=R(32)
     +C*LX^4
114:R(33)=R(33)+C*
     LY*LX/X:R(34)=
     R(34)+C*LY:R(3
     5)=R(35)+C*LY^
     2:R(36)=R(36)+
     C*X*LY
115:R(37)=R(37)+C*
     LY/X:R(38)=R(3
     8)+C*LY*X*X:R(
     39)=R(39)+C*LX
     *LY
116:R(40)=R(40)+C*
     X*LX*LY:R(41)=
     R(41)+C*LY*LX*
     LX:Q=C:LX=X:LY
     =Y
120:IF P$="Y"GOSUB
     1025
121:GOSUB 601:GOTO
     100
299:REM  CURVE-FIT
     ROUTINES
310:GOSUB 631:REM
     LINEAR
311:A=R(2):B=R(1):
     C=R(3):D=R(5):
     E=R(4):I=1
312:GOSUB 500:IF K
     =1OR K=2GOTO 5
     55
313:IF K=2LET Y=A+
     B*X:GOTO 55
314:IF RR>MRLET MR
     =RR:MI=I
320:GOSUB 632:REM
     RECIPROCAL
321:A=R(2):B=R(1):
     C=R(9):D=R(12)
     :E=R(10):I=2
322:GOSUB 500:IF K
     =1OR K=2GOTO 5
     55
323:IF K=2LET Y=1/
     (A+B*X):GOTO 5
     5
324:IF RR>MRLET MR
     =RR:MI=I
330:GOSUB 633:REM
     LIN-HYPERBOLI
     C
331:A=R(1):B=R(7):
     C=R(6):D=R(3):
     E=R(4):F=R(5):
     G=R(13):H=R(8)
     :J=R(2):I=3
332:GOSUB 520:IF K
     =1OR K=2GOTO 5
     50
333:IF K=2LET Y=A+
     B*X+C/X:GOTO 5
     5
334:IF RR>MRLET MR
     =RR:MI=I
340:GOSUB 634:REM
     HYPERBOLA
341:A=R(8):B=R(7):
     C=R(3):D=R(13)
     :E=R(4):I=4
342:GOSUB 500:IF K
     =1OR K=2GOTO 5
     55
343:IF K=2LET Y=A+
     B/X:GOTO 55
344:IF RR>MRLET MR
     =RR:MI=I
350:GOSUB 635:REM
     1/HYPERBOLA
```

PC-1500 MULTIPLE CURVE FITTING PROGRAM

```
351:A=R(8):B=R(7):
    C=R(9):D=R(11)
    :E=R(10):I=5
352:GOSUB 500:IF K
    =1OR K=2GOTO 5
    55
353:IF K=2LET Y=X/
    (A*X+B):GOTO 5
    5
354:IF RR>MRLET MR
    =RR:MI=I
360:GOSUB 636:REM
     2ND HYP.
361:A=R(7):B=R(8):
    C=R(19):D=R(3)
    :E=R(4):F=R(13
    ):G=R(16):H=R(
    22):J=R(8):I=6
362:GOSUB 520:IF K
    =1OR K=2GOTO 5
    50
363:IF K=2LET Y=A+
    B/X+C/X/X:GOTO
    55
364:IF RR>MRLET MR
    =RR:MI=I
370:GOSUB 637:REM
     PARABOLA
371:A=R(1):B=R(2):
    C=R(18):D=R(3)
    :E=R(4):F=R(5)
    :G=R(14):H=R(2
    1):J=R(2):I=7
372:GOSUB 520:IF K
    =1OR K=2GOTO 5
    50
373:IF K=2LET Y=A+
    B*X+C*X*X:GOTO
    55
374:IF RR>MRLET MR
    =RR:MI=I
380:GOSUB 638:REM
     CAUCHY
381:A=R(1):B=R(2):
    C=R(18):D=R(9)
    :E=R(10):F=R(1
    2):G=R(15):H=R
    (21):J=R(2):I=
    8
382:GOSUB 520:A=T6
    :B=T7/T6/2:C=T
    8-T7*T7/T6/4:
    IF K=1OR K=2
    GOTO 550
383:IF K=2LET Y=1/
    (A*(X+B)^2+C):
    GOTO 55
384:IF RR>MRLET MR
    =RR:MI=I
390:GOSUB 639:REM
     POWER
391:A=R(24):B=R(23
    ):C=R(34):D=R(
    39):E=R(35):I=
    9
392:GOSUB 500:A=
    EXP A:IF K=1OR
    K=2GOTO 555
393:IF K=2LET Y=A*
    X^B:GOTO 55
394:IF RR>MRLET MR
    =RR:MI=I
400:GOSUB 640:REM
     SUP. GEOM.
401:A=R(27):B=R(26
    ):C=R(34):D=R(
    40):E=R(35):I=
    10
402:GOSUB 500:A=
    EXP A:IF K=1OR
    K=2GOTO 555
403:IF K=2LET Y=A*
    X^(B*X)GOTO 5
    5
404:IF RR>MRLET MR
    =RR:MI=I
410:GOSUB 641:REM
     MOD. GEOM.
411:A=R(30):B=R(25
    ):C=R(34):D=R(
    33):E=R(35):I=
    11
412:GOSUB 500:A=
    EXP A:IF K=1OR
    K=2GOTO 555
413:IF K=2LET Y=A*
    X^(B/X):GOTO 5
    5
414:IF RR>MRLET MR
    =RR:MI=I
420:GOSUB 642:REM
     HOERL
421:A=R(1):B=R(23)
    :C=R(26):D=R(3
    4):E=R(35):F=R
    (36):G=R(39):H
    =R(24):J=R(2):
    I=12
422:GOSUB 520:A=
    EXP A:B=EXP B:
    IF K=1OR K=2
    GOTO 550
423:IF K=2LET Y=A*
    B^X*X^C:GOTO 5
    5
424:IF RR>MRLET MR
    =RR:MI=I
430:GOSUB 643:REM
     MOD. HOERL
431:A=R(7):B=R(23)
    :C=R(25):D=R(3
    4):E=R(35):F=R
    (37):G=R(39):H
    =R(24):J=R(8):
    I=13
432:GOSUB 520:A=
    EXP A:B=EXP B:
    IF K=1OR K=2
    GOTO 550
433:IF K=2LET Y=A*
    B^(1/X)*X^C:
    GOTO 55
434:IF RR>MR LET MR
    =RR:MI=I
440:GOSUB 644:REM
     LOG NORMAL
441:A=R(23):B=R(24
    ):C=R(31):D=R(
    34):E=R(35):F=
    R(39):G=R(41):
    H=R(32):J=R(24
    ):I=14
442:GOSUB 520:A=
    EXP (T8-T7*T7/
    T6/4):B=-1*T7/
    T6/2:C=1/T6:IF
    K=1OR K=2GOTO
    550
443:IF K=2LET Y=A*
    EXP ((B-LN X)^
    2/C):GOTO 55
444:IF RR>MRLET MR
    =RR:MI=I
450:GOSUB 645:REM
     LOGARITHMIC
451:A=R(24):B=R(23
    ):C=R(3):D=R(2
    8):E=R(4):I=15
452:GOSUB 500:IF K
    =1OR K=2GOTO 5
    55
453:IF K=2LET Y=A+
    B*LN X:GOTO 55
454:IF RR>MRLET MR
    =RR:MI=I
460:GOSUB 646:REM
     1/LOGARITHMIC
461:A=R(24):B=R(23
    ):C=R(9):D=R(2
    9):E=R(10):I=1
    6
462:GOSUB 500:IF K
    =1OR K=2GOTO 5
    55
463:IF K=2LET Y=1/
    (A+B*LN X):
    GOTO 55
464:IF RR>MRLET MR
    =RR:MI=I
470:GOSUB 647:REM
     MOD. POWER
471:A=R(2):B=R(1):
    C=R(34):D=R(36
    ):E=R(35):I=17
472:GOSUB 500:A=
    EXP A:B=EXP B:
    IF K=1OR K=2
    GOTO 555
473:IF K=2LET Y=A*
    B^X:GOTO 55
474:IF RR>MRLET MR
    =RR:MI=I
480:GOSUB 648:REM
     ROOT
481:A=R(8):B=R(7):
    C=R(34):D=R(37
    ):E=R(35):I=18
482:GOSUB 500:A=
    EXP A:B=EXP B:
    IF K=1OR K=2
    GOTO 555
483:IF K=2LET Y=A*
    B^(1/X):GOTO 5
    5
484:IF RR>MRLET MR
    =RR:MI=I
490:GOSUB 649:REM
     NORMAL
491:A=R(1):B=R(2):
    C=R(18):D=R(34
    ):E=R(35):F=R(
    36):G=R(38):H=
    R(21):J=R(2):I
    =19
492:GOSUB 520:A=
    EXP (T8-T7*T7/
    T6/4):B=-1*T7/
    T6/2:C=1/T6:IF
    K=1OR K=2GOTO
    550
493:IF K=2LET Y=A*
    EXP ((X-B)^2/C
    ):GOTO 55
494:IF RR>MRLET MR
    =RR:MI=I
495:REM  GO BACK T
    O BEST FIT
496:L=1:K=1:I=MI:
    GOSUB 630+I:IF
    P$="Y"GOSUB 10
    40:GOTO 301+10
    *I
497:WAIT :BEEP 3:
    PRINT NA$(1):
    GOTO 301+10*I
499:REM  CALC A,B,
    RR
500:GOSUB 610:T1=A
    *R(6)-B*B:T2=(
    A*C-B*D)/T1:T3
    =(D*R(6)-B*C)/
    T1
501:A=C*C/R(6):RR=
    (T2*C+T3*D-A)/
    (E-A):F=2:A=T2
    :B=T3:D=1:GOTO
    525
519:REM  CALC A,B,
    C,RR
520:GOSUB 610:T1=J
    *R(6)-A*A:T2=G
    *R(6)-D*B:T3=C
    *R(6)-A*B:T4=F
    *R(6)-A*D
521:T5=H*R(6)-B*B:
    T6=(T1*T2-T3*T
    4)/(T1*T5-T3*T
    3):T7=(T4-T3*T
    6)/T1
522:T8=(D-T7*A-T6*
    B)/R(6):A=T8:B
    =T7:C=T6:H=D*D
    /R(6)
523:RR=(A*D+B*F+C*
    G-H)/(E-H):F=3
    :D=0
524:REM  CALC CORR
    ECTED RR
525:RR=1-((1-RR)*(
    R(6)-1)/(R(6)-
    F)):IF RR<0LET
    RR=0
527:RETURN
549:REM  DISPLAY A
    ,B,C,RR
550:D=0:GOTO 557
555:D=1
557:IF P$="Y"GOSUB
     1045:GOTO 561
558:BEEP 2:GOSUB 6
     10:WAIT :PRINT
     "a = ";A:GOSUB
     610:WAIT :
     PRINT "b = ";B
559:IF D=0GOSUB 61
     0:WAIT :PRINT
     "c = ";C
```

PC-1500 MULTIPLE CURVE FITTING PROGRAM

```
560:GOSUB 610;WAIT
    :PRINT "RR = "
    ;RR
561:IF K=2GOTO 53
562:GOTO 999
599:REM  DISPLAY P
    ROMPTS
600:C=Q
601:CLS :GCURSOR 1
    38:WAIT 0
602:IF C=1GPRINT "
    0814224101456D
    55450111391101
    49556341"
603:IF C=-1GPRINT
    "0814224101456
    D5545011111110
    149556341"
604:CURSOR 0:
    RETURN
610:CLS :GCURSOR 1
    17:WAIT 0
611:GPRINT "416355
    49017D15150501
    7D0105057D0505
    41221408"
612:CURSOR 23:WAIT
    0:PRINT I:
    CURSOR 0:
    RETURN
620:CLS :GCURSOR 1
    11:WAIT 0
621:GPRINT "416355
    49017D25251901
    7D414101394545
    3901057D050141
    221408"
622:CURSOR 23:WAIT
    0:PRINT I:
    CURSOR 0:
    RETURN
630:REM  CURVE NAM
    ES
631:NA$(1)="1. Lin
    ear y=a+bx":
    RETURN
632:NA$(1)="2. 1/L
    inear y=1/(a+b
    x)":RETURN
633:NA$(1)="3. Lin
    -Hyp y=a+bx+c/
    x":RETURN
634:NA$(1)="4. Hyp
    erbola y=a+b/x
    ":RETURN
635:NA$(1)="5. 1/H
    yp y=x/(ax+b)"
    :RETURN
636:NA$(1)="6. 2nd
     Hyp y=a+b/x+c
    *x^2":RETURN
637:NA$(1)="7. Par
    abola y=a+bx+c
    x^2":RETURN
638:NA$(1)="8. Cau
    chy y=1/(a(x+b
    )^2+c)":RETURN
639:NA$(1)="9. Pow
    er y=ax^b":
    RETURN
640:NA$(1)="10. Su
    p.Geom. y=ax^(
    bx)":RETURN
641:NA$(1)="11. Mo
    d.Geom. y=ax^(
    b/x)":RETURN
642:NA$(1)="12. Ho
    erl y=a b^x x^
    c":RETURN
643:NA$(1)="13. Mo
    d.Hoerl y=ab^(
    1/x)x^c":
    RETURN
644:NA$(1)="14. LN
     y=ae^((b-lnx)
    ^2/c)":RETURN
645:NA$(1)="15. Lo
    garithm y=a+bl
    nx":RETURN
646:NA$(1)="16. 1/
    Log y=1/(a+bln
    x)":RETURN
647:NA$(1)="17. Mo
    d. Power y=ab^
    x":RETURN
648:NA$(1)="18. Ro
    ot y=ab^(1/x)"
    :RETURN
649:NA$(1)="19. No
    rmal y=ae^((x-
    b)^2/c)":
    RETURN
660:REM  FUNCTION
    NAMES
661:NA$(1)="<DEF>
    C - Clear & re
    set   ":RETURN
662:NA$(1)="<DEF>
    A - Add data p
    oints ":RETURN
663:NA$(1)="<DEF>
    D - Delete poi
    nts   ":RETURN
664:NA$(1)="<DEF>
    F - Fit a curv
    e     ":RETURN
665:NA$(1)="<DEF>
    B - Find best
    fit   ":RETURN
666:NA$(1)="<DEF>
    S - Solve for
    Y":RETURN
667:NA$(1)="<DEF>
    M - Curve menu
          ":RETURN
668:NA$(1)="<DEF>
    L - Function m
    enu   ":RETURN
750:BEEP 5:WAIT WA
    :PRINT "CALCUL
    ATION ERROR!!!
    "
751:GOSUB 610:
    INPUT "Restart
     @ curve#...";
    I:GOTO 300+I*1
    0
999:END
1000:TEXT :COLOR
    1:CSIZE 1:LF
    2:LPRINT "A/
    D";TAB 10;"
    X";TAB 20;"
    Y";TAB 32;"
    n"
1001:LPRINT "==="
    ;TAB 10;"===
    ";TAB 20;"==
    =";TAB 32;"=
    ==":RETURN
1005:LF 2:LPRINT
    "Last point
    entered/dele
    ted...":
    GOSUB 1025:
    RETURN
1010:LF 2:COLOR 2
    :LPRINT "Bes
    t-fit curve
    is...":
    RETURN
1015:LF 2:COLOR 2
    :LPRINT "Fit
    ting...":
    RETURN
1020:LF 2:COLOR 2
    :LPRINT "Fin
    ding y-value
    s for...":
    RETURN
1025:C=Q
1026:IF C=1COLOR
    0:LPRINT "(+
    )";TAB 10;LX
    ;TAB 20;LY;
    TAB 32;R(6):
    RETURN
1027:IF C=-1COLOR
    3:LPRINT "(-
    )";TAB 10;LX
    ;TAB 20;LY;
    TAB 32;R(6):
    RETURN
1030:COLOR 2:LF 2
    :LPRINT BD$(
    1):LPRINT "
      MULTIPLE C
    URVE FITTING
     PROGRAM"
1031:LPRINT TAB 1
    2;"Function
    menu":LPRINT
    BD$(1):
    RETURN
1035:COLOR 2:LF 2
    :LPRINT BD$(
    1):LPRINT "
      MULTIPLE C
    URVE FITTING
    PROGRAM"
1036:LPRINT TAB 1
    3;"Curve men
    u":LPRINT BD
    $(1):RETURN
1040:LPRINT NA$(1
    ):RETURN
1045:IF L=0RETURN
1046:LPRINT BD$(1
    ):LPRINT "
    a = ";A:
    LPRINT "  b
    = ";B
1047:IF D=0LPRINT
    "  c = ";C
1048:LF 1:LPRINT
    " RR = ";RR:
    LF 1:LPRINT
    BD$(1):LF 2:
    RETURN
1050:LPRINT "X =
    ";X;TAB 15;"
    Y = ";Y:
    RETURN
```

Note the difference between the letters "I" and "O" and the numbers "1" and "0" when entering this program.

Modifying the Program

This program requires approximately 8,500 bytes of memory plus an additional 400 bytes for the date file created by the program. Thus the 8k RAM extension module is a necessity. The register numbers used are identical to those for the HP-75C on page H-3.

All plotter commands are located in subroutines beginning at line 1000. If you don't have the plotter or don't wish to use it, you can either answer N when prompted by the CLEAR function, or you can eliminate these routines and their calls from the program altogether for a savings of about 1,100 bytes. In any case, these statements are disabled whenever the PC is disconnected from the plotter.

The PC preserves all programs and data in memory even while the power is off. In fact, the PC can even shut itself off while waiting for an input and will resume waiting at the same step when you turn it back on. You can also interrogate the PC for the value of any variable while in RUN mode simply by typing its name and pressing ENTER. This is a useful feature for debugging purposes.

Program statements unique to extended Pocket Computer BASIC include:

WAIT n - sets the display hold time to 'n' ticks. Without an argument, it indicates that the display is to be held until ENTER is pressed.

CURSOR n - positions the cursor to the nth (0 to 25) position in the LCD display.

GPRINT (string), GCURSOR n - any of the 155 columns of the LCD dot-matrix display may be independently turned on. GCURSOR n positions the display graphic cursor at one of the columns, and GPRINT (string) turns on the dot pattern specified by the string.

BEEP n - produces n beeps from the internal beeper.

COLOR n - changes the pen color of the plotter (0-black, 1-blue, 2-green, 3-red).

CSIZE n - selects one of the nine plotter character sizes.

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Reciprocal Through Given Point	$y = 1/(a + bx)$	24
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Hyperbola Through Given Point	$y = a + b/x$	28
Reciprocal of Hyperbola	$y = x/(ax + b)$	30
Linear-Hyperbolic	$y = a + bx + c/x$	32
Second Order Hyperbola	$y = a + b/x + c/x^2$	34
Parabola	$y = a + bx + cx^2$	36
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Parabola Through Given Point	$y = a + bx + cx^2$	40
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Super Geometric	$y = ax^{bx}$	48
Modified Geometric	$y = ax^{b/x}$	50
Exponential	$y = ae^{bx}$	52
Modified Exponential	$y = ae^{b/x}$	54
Poisson	$y = ab^x/x!$	56
Logarithmic	$y = a + b\ln x$	58
Reciprocal of Logarithmic	$y = 1/(a + b\ln x)$	60
Linear-Exponential	$y = ax/b^x$	62
Hoerl Function	$y = ab^x x^c$	64
Modified Hoerl Function	$y = ab^{1/x} x^c$	66
Normal Distribution	$y = ae^{(x-b)^2/c}$	68
Log-Normal Distribution	$y = ae^{(b-\ln x)^2/c}$	70
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--How much time is left on your video (or cassette) tape when the counter reads 790? (See page 38.)

--How much antifreeze should you add to your radiator for protection down to -20 degrees F? (See page 36.)

--How much aspirin remains in your blood if you consumed one gram 12 hours ago? (See page 62.)

--What is the distribution of insects per plant after treatment with a new insecticide? (See page 56.)

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* Subtract your average daily caloric intake divided by 15 from all weight measurements then add it back after fitting the curve.

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