Hogan School of Real Estate

THE HEWLETT-PACKARD CALCULATORS AS REAL ESTATE PROBLEM SOLVERS

by

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INTRODUCTION

Until recently, much time had to be spent in order to perform financial calculations of any sophistication. With the advent of the financial calculator, these complex calculations can be performed with ease.

The object of this course is to help the real estate practitioner learn to use the Hewlett-Packard calculators to solve everday problems encountered in both residential and investment real estate.

The workbook is not designed to replace the HP Owner's Handbook and it will be necessary to refer to the Owner's Handbook for information which is not discussed here.

I would like to thank and acknowledge the talents of Esther Johnson, Office Manager Extraordinaire, and Rachelle Pellissier, who spent many hours putting these numbers through the typewriter. I would also like to thank Ellen Hogan and everyone else at Hogan School of Real Estate for their help and patience.

I hope this course and the calculator helps you to become a little more professional and enables you to put more dollars and free time in you pocket.

Sincerely,

Jim Hogan

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UNDERSTANDING THE HP12C KEYS

For most real estate problems you need to know the following keys and information:

ON	Turns the calculator on and off. We have found that the calculator works best when turned on.

CLEARING THE CALCULATOR

Throughout this manual it is assumed that you clear your calculator of all information before beginning a new problem.

With the HP 12C: This is done by pressing the f CLX (CLEAR REG - clear registers) key. The HP 12C has a continuous memory and turning the calculator Off does not clear any information.

UNDERSTANDING THE HP38 KEYS

For this course and most real estate problems you need to know the following keys and information:

OFF - ON	Turns calculator on or off. We have found that the calculator works best when this switch is in the ON position.
D.MY - M.DY BEGIN END	Has two uses. D.MY (Day, Month, Year) and M.DY (Month, Day, Year) is used to index data for the calendar functions. BEGIN-END: Specifies if payments are made or received at the beginning or end of a period.

CLEARING THE CALCULATOR

Throughout this course it is assumed that you clear your calculator of all information before begining a new problem.

With the HP 38E: This can be done by pressing f CLX (CLEAR ALL) key or by turning the calculator Off and then On again.

With the HP 38C: This can only be done by pressing the f CLX (CLEAR ALL) key. Please do not clear the calculator between steps in a problem unless instructed to do so.

UNDERSTANDING THE HP 12C KEYS

Functions on the face of the ksys

n	Number of periods (days, months, years).
i	Interest rate.
PV	Present value.
PMT	Payment per period.
FV	Future Value.
STO	Store - used to store an entry in storage registers.
RCL	Recall - used to recall an entry.
%	Percent.
ENTER	Used to enter a number into the next register.
CHS	Change Sign - Change positive numbers to negative and visa versa.
x≷y	Exchange - used to determine the number (if one exists) in the Y register.
CLX	Clear entry - clears only the entry in the display (X Register).
R ↓	Roll Down; rolls the numbers in the stack down into the next lowest register.

Functions printed in GOLD above keys

To select a function printed in gold above a key, first press the gold prefix key, f, then press the function key.

AMORT	Amortize; used to amortize a loan.
NPV	Net Present Value; calculates the present value of a series of irregular cash flows.
IRR	Internal Rate of Return; calculates IRR when cash flows are irregular.
CLEAR REG	Clears all information in calculator except programs.
CLEAR PREFIX	Clears a prefix (f, g, RCL, STO) which was pressed incorrectly.
CLEAR FIN	Clears information in Financial Registers only.
CLEAR E	Clears data in Storage Registers 1 through 6.

Functions printed in BLUE on slanted face of keys

To select a function printed in blue on the slanted face of the key, first press the blue prefix key, g, then press the function key.

g n (12X) Used to convert annual periods to monthly periods. Automatically enters the monthly periods in n.

g i (12÷) Used to convert annual interest to monthly interest. Automatically enters the monthly interest in i.

CFo Cash flow at time of investment (Year 0) - used when cash flows are irregular.

CFj Periodic cash flow - used when cash flows are irregular.

Nj Number of periods for same cash flow. (Up to 99 periods per cash flow).

BEG Compounds interest from beginning of each period. Note BEGIN in display.

END Compounds interest from end of each period. Not indicated in display.

CLEAR PRGM
P/R
GTO
BST
SST
MEM
PSE

> Programming keys: discussed in Programming Section

UNDERSTANDING THE HP 38C & E KEYS

Functions on the face of the keys

n	Number of periods (days, months, years).
i	Interest rate.
PV	Present Value.
PMT	Payment per period.
FV	Future Value.
STO	Store - used to store an entry in storage registers.
RCL	Recall - used to recall an entry.
%	Percent.
ENTER	Used to enter a number into the next register.
CHS	Change Sign - Change positive numbers to negative and visa versa.
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CLEAR ALL	Clears all information in calculator except programs.
CLEAR PREFIX	Clears a prefix (f, g, RCL, STO) which was pressed incorrectly.
CLEAR FIN	Clears information in Financial Registers only.
CLEAR E	Clears data in Storage Registers 1 through 6.
△DAYS	Computes the number of days between two dates you have entered.

Functions printed in BLUE on slanted face of keys

To select a function printed in blue on the slanted face of the key, first press the blue prefix key, g, then press the function key.

g n (12X) Used to convert annual periods to monthly periods. Automatically enters the monthly periods in n.

g i (12÷) Used to convert annual periods to monthly periods. Automatically enters the monthly interest in i.

CFo Cash flow at time of investment (Year 0) - used when cash flows are irregular.

CFj Periodic cash flow - used when cash flows are irregular.

Nj Number of periods for same cash flow. (Up to 99 periods per cash flow).

R ↓ Roll Down; rolls the numbers in the stack down into the next lowest register.

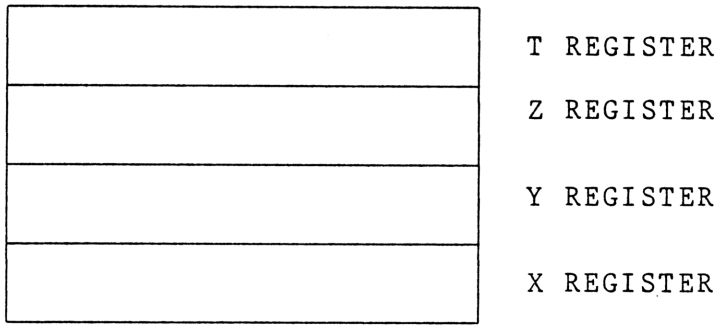
CLP
P/R
GTO
BST
SST
MEM
PSE

Programming keys: discussed in Programming Section

BASIC CALCULATIONS

The Operational Stack:

The calculator has four memory registers that are "stacked". The registers are designated as follows:



The content of the X register is always in the display, while contents of the Y, Z and T registers (if any) are remembered by the calculator.

The ENTER key copies the number in the X register and pushes it up into the Y register and pushes the values in the Z and T registers up one register.

Arithmetic functions are performed with the X and Y registers only. The Z and T registers hold values until needed.

The x⇄y key exchanges the contents of the X and Y registers.

The R↓ (Roll Down) function rotates the stack one quarter turn. Pressing R↓ four times will reveal all the values in the stack.

Basic Arithmetic:

To perform any arithmetic calculation the following basic keystrokes are used:

1. Number
2. ENTER
3. Number
4. Arithmetic Function (+, -, x, ÷)

Examples:

1. 27 x 14 = ?

Keystrokes	Display	Remarks
27	27	27 in X register
ENTER	27.00	27 in X & Y registers
14	14	27 in Y, 14 in X
x	378	Answer in X

The stack would appear as follows:

T						
Z						
Y		27	27			
X	27	27	14	378		
KEYSTROKES	27	ENTER	14	X		

2. 478 - 264 = ?

Keystrokes	Display	Remarks
478	478	478 in X
ENTER	478.00	478 in X & Y
264	264	478 in Y, 264 in X
-	214	Answer in X

The stack would appear as follows:

T						
Z						
Y		478	478			
X	478	478	264	214		

KEYSTROKES 478 ENTER 264 -

3. 46 + 119 = ?

Keystrokes	Display	Remarks
46	46	46 in X
ENTER	46.00	46 in X & Y
119	119	46 in Y, 119 in X
+	165	Answer in X

The stack would appear as follows:

T						
Z						
Y		46	46			
X	46	46	119	165		

KEYSTROKES 46 ENTER 119 +

4. 62 ÷ 427 = ?

Keystrokes	Display	Remarks
62	62	62 in X
ENTER	62.00	62 in X & Y
427	427	62 in Y, 427 in X
÷	.15	Answer in X (rounded)
f 9	.145199063	Answer to 9 decimal places

You can set your calculator to display up to 9 decimal places (or less) by pressing f and the number of decimal places you wish to display.

This manual is written with the calculator set to display two decimal places.

The stack would appear as follows:

T						
Z						
Y		62	62			
X	62	62	427	.15	.145199063	
KEYSTROKES	62	ENTER	427	+	f 9	

Reset to two decimal places by pressing f 2.

5. a) 27.2% of 643 = ?

b) What is the difference between them?

Keystrokes	Display	Remarks
643	643	643 in X
ENTER	643.00	643 in X & Y
27.2	27.2	643 in Y, 27.2 in X
%	174.90	174.90 in X, 643 in Y. Answer to part "a"
-	468.10	Difference in X

The stack would appear as follows:

T						
Z						
Y		643	643	643		
X	643	643	27.2	174.90	468.10	

KEYSTROKES 643 ENTER 27.2 % -

USING THE STACK TO COMPOUND WITH FACTORS

Example

A property is leased for 7 years. The first year's rent is $100,000 with the rent increasing by 5% per year. What are the rents for each of the 7 years.

Procedure

Load the stack with the compounding factor (1.05). Key in the base rent $100,000 into the display and press x (multiply).

Keystrokes		Display	Remarks
1.05	ENTER	1.05	1.05 in x & y
	ENTER	1.05	1.05 in z
	ENTER	1.05	1.05 in T
100,000			Rent Year 1
	x	105,000	Rent Year 2
	x	110,250	Rent Year 3
	x	115,762.50	Rent Year 4
	x	121,550.63	Rent Year 5
	x	127,628.16	Rent Year 6
	x	134,009.56	Rent Year 7

Problem

A property has a Net Operating Income of $40,000. You predict that the N.O.I. will increase by 4% per year over the next 10 years. What are the rents through year 11?

Keystrokes		Display	Remarks
1.04	ENTER	1.04	
	ENTER	1.04	
	ENTER	1.04	
40,000		40,000	N.O.I. Year 1
	x	41,600	N.O.I. Year 2
	x	43,264	N.O.I. Year 3
	x	44,994.56	N.O.I. Year 4
	x	46,794.34	N.O.I. Year 5
	x	48,666.12	N.O.I. Year 6
	x	50,612.76	N.O.I. Year 7
	x	52,637.27	N.O.I. Year 8
	x	54,742.76	N.O.I. Year 9
	x	56,932.47	N.O.I. Year 10
	x	59,209.77	N.O.I. Year 11

STORAGE REGISTERS

Your calculator contains 20 storage registers (0 through 9 & .0 through .9) which allow you to store a number and recall that number at a later time.

To store a number press the "store" key, STO, and the number key (0 through 9 or .0 through .9) for the storage register in which you wish to store that number.

To recall a stored entry press the 'recall' key, RCL, and the number key (0 through 9 or .0 through .9) for the storage register in which that number was stored.

Example:

Keystrokes		Display	Remarks
27	STO 0	27	
64	STO 1	64	
427	STO 2	427	
4854	STO 3	4854	
	RCL 0	27	
	RCL 1	64	
	RCL 2	427	
	RCL 3	4854	

Storage Registers 0 through 4 can be used for storage register arithmetic. Any number can be added to, subtracted from, multiplied or divided by any number which has been stored in registers 0 through 4.

To perform storage register arithmetic the following keystrokes are used:

1. Number (in Display)
2. STO
3. Arithmetic Function (+,-, x, ÷)
4. Storage Register Number (0, 1, 2, etc.)

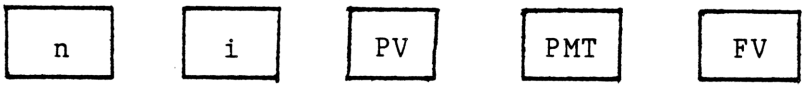
Examples:

Keystrokes		Display	Remarks
567	STO 0	567	Stores 567 in STO 0
42	STO + 0	42	Adds 42 to 567 and places result in STO 0
	RCL 0	609	

Keystrokes		Display	Remarks
89	STO 1	89	Stores 89 in STO 1
3	STO x 1	3	Multiplies 89 by 3 and places result in STO 1
	RCL 1	267	

FINANCIAL CALCULATIONS

Special memories, called financial storage registers, are reserved for financial calculations. Many complex financial functions can be performed with these registers which appear along the top row of keys.



n Number of periods

i Interest rate per period

PV Present Value (A lump sum paid or received today)

PMT Payment per period (A series of level payments paid or received)

FV Future Value (A lump sum paid or received in the future)

Once a value is stored in a particular register it remains in the register for future use until it is overwritten or cleared.

When performing any calculation with the financial registers the interest rate must be the rate per payment period. For example, if the payments are monthly, the interest rate must be a monthly rate. If the payments are quarterly, the interest rate must be a quarterly rate.

Because most real estate loans are paid monthly, your calculator is equipped to easily convert the annual interest to monthly interest and the number of years to months through the use of the g i (12÷) and g n (12×) keys.

When the g i key is used, the interest rate is automatically converted to a monthly figure and is stored in the i register.

When the g n key is used, the number of years is automatically converted to the number of months and is stored in the n register.

When performing most real estate calculations, the payments are made at the END of the payment period.

For the 12C: Press g END and all payments are assumed to be made at the end of the period. If the payments are made at the beginning of the period press g BEG and the payments will be assumed to be made at the beginning of the period.

For the 38C or 38E: The switch in the upper right corner of the calculator should be set to END so the payments are assumed to be made at the end of the period.

The sequence in which you index data in the financial registers does not matter.

If you know any four of the values in the financial registers you can solve for the fifth.

When entering cash flows into the financial registers it is important to use the proper sign convention.

Use Positive numbers for cash received.
Use Negative numbers for cash paid out.

If you wish to display a number which has been placed into any of the financial registers you may do so by pressing the RCL key and the financial register desired. Example: RCL n, RCL PV, etc.

SOLVING FOR FRACTIONAL PERIODS WITH 12C ONLY:
When solving for the number of periods, n, the HP 12C always rounds the result up to the next highest whole number. To calculate the exact value of n, a program is used and can be found on page 83.

When the number of periods indexed in n is not a whole number, the HP 12C will continuously compound or discount only for the whole number of periods and will calculate simple interest for the "odd period".

To have your calculator perform continuous compounding for the "odd period" press STO EEX. A "C" should appear in the display indicating continuous compounding. Press STO EEX to remove the continuous compounding.

When compounding or discounting or when calculating payments on loans, the frequency of the payments will determine the number by which you will multiply the number of years and divide the annual interest rate.

Frequency of Payments	n Multiply # of years by	i Divide annual rate by
Annual	1	1
Semi-annual	2	2
Quarterly	4	4
Monthly	12	12
Weekly	52	52
Daily	365 or 360	365 or 360

AMORTIZATION OF LOANS

A. Determining Loan Payments

Steps to calculating payments:

1. Key in financial data in n, i, PV
2. Solve for payment by pressing PMT

Examples:

1) What are the monthly payments on a $65,000 loan at 11 3/4% interest for 30 years:

n
i
PV
PMT
FV

Index		Display	Remarks
65000	PV	65,000	Loan amount
11.75	g i	.98	Monthly interest
30	g n	360	No. of actual monthly pmts.
	PMT	-656.12	Monthly payment

2) What are the annual payments on a $42,000 loan at 10.3% interest for 12 years:

n
i
PV
PMT
FV

Index		Display	Remarks
42000	PV	42,000	Loan amount
10.3	i	10.30	Annual interest
12	n	12	No. of actual annual pmts.
	PMT	-6,254.93	Annual payment

PROCEDURE FOR MONTHLY PAYMENTS

Loan Amount	PV
Annual Rate	g i
Annual Term	g n
	PMT

PAYMENT PROBLEMS

Find the monthly payments on the following loans:

1. $ 80,000 @ 12% for 30 years
2. $165,000 @ 14% for 20 years
3. $200,000 @ 9½% for 25 years
4. $ 50,000 @ 7% for 50 years
5. $120,000 @ 18% for 3 years
6. $ 20,000 @ 11% for 10 years
7. $ 79,200 @ 16½% for 13 years
8. $ 58,500 @ 11½% for 27 months
9. $827,000 @ 9% for 200 months
10. $140,000 @ 15% for 300 months

SOLUTIONS:

1. $822.89
2. $2,051.81
3. $1,747.39
4. $300.84
5. $4,338.29
6. $275.50
7. $1,235.80
8. $2,469.36
9. $7,996.85
10. $1,793.16

PITI PAYMENT

Example

An offer is made on a home for $275,000 with 20% down and the buyers to obtain a new loan at 11.5% interest with monthly payments for 30 years. Taxes on the proprety are $3,700 per year and insurance premiums will be $660 annually.

1. What is the down payment and loan amount?
2. What is the monthly PI payment?
3. What is the monthly PITI payment?

Keystrokes		Display	Remarks
275,000	ENTER	275,000	
25	%	68,750	Down Payment
	-	206,250	Loan Amount
	PV	206,250	
11.5	g i	.96	
30	g n	360	
	PMT	-2,042.48	Monthly PI payment
CHS	STO o	2,042.48	
3,700	ENTER	3,700	Annual Taxes
660	+	4,360.00	Annual T & I
12	÷	363.33	Monthly T & I
STO	+ 0	363.33	
RCL	0	2,405.81	PITI Payment

BACK INTO FINANCING USING DEBT COVERAGE RATIOS

A property has a Net Operating Income of $820,000 annually. Lenders will finance the property using a 1.1 to 1 debt coverage ratio at 11.5% per annum for 30 years with a 7 year call.

What is the maximum loan the property can handle.

Step 1: Find the amount available for annual debt service and monthly payment.

Keystrokes		Display	Remarks
820,000	ENTER	820,000	N.O.I.
1.1	÷	745,454.55	Annual Debt Service
12	÷	62,121.21	Monthly PMT

Step 2: Find loan amount

Keystrokes		Display	Remarks
62,121.21	PMT	62,121.21	
11.5	g i	.96	
30	g n	360	
	PV	-6,273,023.06	Loan Amount

B. Determining Remaining Principal Balance at Future Date (Balloon Payments)

A balloon payment is the remaining balance (Future Value) of a loan after a certain number of payments have been made plus the payment to be made at that time. Throughout this manual we have calculated the remaining principal balance of the loan. Please keep in mind that the total due at the balloon payment date is the payment then due plus the remaining principal balance.

THE STEPS TO CALCULATE REMAINING BALANCE ARE:

1. Key in Financial Data (n, i, PV)
2. Solve for Payment
3. Key in # of periods to due date in n
4. Solve for FV (Remaining Balance)

Examples:

1) On a loan of $65,000 at 11 3/4% for 30 years with monthly payments, what is the remaining balance at the end of one year:

n
i
PV
PMT
FV

Keystrokes		Display	Remarks
65000	PV	65,000	
11.75	g i	.98	
30	g n	360	
	PMT	-656.12	Monthly payment for 30 years
1	g n	12	No. of pmts. made in 1 yr.
	FV	-64,750.98	Balance EOY 1 (EOM 12)

$64,750.98 is the future value or (remaining balance) of this loan after 1 year or 12 monthly payments have been made.

2) On a loan of $124,000 at 12 1/2% interest for 30 years with monthly payments, what will be the remaining balance after 5 years; 8 years; 15 years:

n
i
PV
PMT
FV

Keystrokes		Display	Remarks
124000	PV	124,000	
12.5	g i	1.04	
30	g n	360	
	PMT	-1,323.40	Monthly payment
5	g n	60	No. of payments made in 5 years
	FV	-121,373.38	Balance EOY 5 (EOM 60)
8	g n	96	
	FV	-118,808.21	Balance EOY 8
15	g n	180	
	FV	-107,373.30	Balance EOY 15

PROCEDURE FOR REMAINING BALANCE ON MONTHLY PAYMENT LOANS

Loan Amount	PV
Annual Rate	g i
Annual Term	g n
	PMT
Years to Call date	g n
	FV

REMAINING BALANCE PROBLEMS

Find the monthly payment and remaining balance on the following loans:

1. $ 68,500 @ 9%, 20 years after 5 years
2. $920,000 @ 12%, 30 years after 7 years
3. $ 6,000 @ 14%, 10 years after 8 years
4. $ 56,000 @ 8%, 15 years after 12 years
5. $100,000 @ 18%, 20 years after 6 years
6. $ 76,000 @ 10%, 5 years after 2 years
7. $120,000 @ 15%, 10 years after 4 years
8. $ 22,000 @ 11%, 15 years after 7.5 years
9. $ 34,000 @ 7%, 12 years after 9 years
10. $ 62,000 @ 9%, 18 years after 5 years

SOLUTIONS

	PAYMENT	REMAINING BALANCE
1.	$ 616.31	$ 60,764.33
2.	$9,463.24	$885,602.37
3.	$ 93.16	$ 1,940.31
4.	$ 535.17	$ 17,078.09
5.	$1,543.31	$ 94,452.78
6.	$1,614.78	$ 50,043.88
7.	$1,936.02	$ 91,559.15
8.	$ 250.05	$ 15,279.00
9.	$ 349.65	$ 11,323.91
10.	$ 580.60	$ 53,281.29

PAYMENT & REMAINING BALANCE PROBLEMS

1. Find the monthly payments on the following loans:

   a) $ 72,000 @ 10% for 25 years
   b) $145,000 @ 12½% for 30 years
   c) $ 22,600 @ 9% for 18 years
   d) $100,000 @ 18% for 30 years
   e) $420,000 @ 16% for 20 years

   n
   i
   PV
   PMT
   FV

2. Find the remaining balance on the following <u>monthly payment</u> loans:

   a) $ 65,000 @ 14% for 30 years after 6 years
   b) $127,000 @ 16½% for 25 years after 8 years
   c) $ 90,000 @ 12% for 20 years after 5 years
   d) $ 36,000 @ 22% for 8 years after 3 years

   n
   i
   PV
   PMT
   FV

3. On a loan of $87,000 at 16% interest for 30 years with monthly payments, what will be the remaining balance after 6, 12, & 18 years:

   n
   i
   PV
   PMT
   FV

4. The Skinflints purchased a $100,000 home with an $80,000 new 1st loan. If the loan had monthly payments for 30 years at 16½% interest:

   a) What is their monthly payment?
   b) What will the loan balance be in 10 years?

   n
   i
   PV
   PMT
   FV

5. On a loan of $180,000 at 14% interest with interest only monthly payments:

   a) Find the monthly payment
   b) Find the remaining balance at end of year 6.

   n
   i
   PV
   PMT
   FV

SOLUTIONS TO PAYMENT & REMAINING BALANCE PROBLEMS

1. Find the monthly payments on the following loans:

a) $ 72,000 @ 10% for 25 years.

Keystrokes		Display	Remarks
72000	PV	72,000	
10	g i	.83	
25	g n	300	
	PMT	-654.26	

b) $145,000 @ 12½% for 30 years.

Keystrokes		Display	Remarks
145000	PV	145,000	
12.5	g i	1.04	
30	g n	360	
	PMT	-1,547.52	

c) $22,600 @ 9% for 18 years.

Keystrokes		Display	Remarks
22600	PV	22,600	
9	g i	.75	
18	g n	216	
	PMT	-211.64	

d) $100,000 @ 18% for 30 years.

Keystrokes		Display	Remarks
100000	PV	100,000	
18	g i	1.50	
30	g n	360	
	PMT	-1,507.09	

e) $420,000 @ 16% for 20 years.

Keystrokes		Display	Remarks
420000	PV	420,000	
16	g i	1.33	
20	g n	240	
	PMT	-5,843.27	

2. Find the remaining balance on the following monthly payment loans:

a) $65,000 @ 14% for 30 years after 6 years

Keystrokes		Display	Remarks
65000	PV	65,000	
14	g i	1.17	
30	g n	360	
	PMT	-770.17	Monthly Payment
6	g n	72	Months to Stop Date in n
	FV	-63,676.23	Remaining balance

b) $127,000 @ 16 1/2% for 25 years after 8 years

Keystrokes		Display	Remarks
127000	PV	127,000	
16.5	g i	1.38	
25	g n	300	
	PMT	-1,775.77	
8	g n	96	
	FV	-121,181.93	Remaining balance

c) $90,000 @ 12% for 20 years after 5 years

Keystrokes		Display	Remarks
90000	PV	90,000	
12	g i	1.00	
20	g n	240	
	PMT	-990.98	
5	g n	60	
	FV	-82,569.90	Remaining balance

d) $36,000 @ 22% for 8 years after 3 years

Keystrokes		Display	Remarks
36000	PV	36,000	
22	g i	1.83	
8	g n	96	
	PMT	-799.81	
3	g n	36	
	FV	-28,958.96	Remaining balance

3. On a loan of $87,000 at 16% interest for 30 years with monthly payments, what will be the remaining balance after 6, 12, & 18 years:

Keystrokes		Display	Remarks
87000	PV	87,000	
16	g i	1.33	
30	g n	360	
	PMT	-1,169.94	Monthly payment
6	g n	72	
	FV	-85,810.96	Balance EOY 6
12	g n	144	
	FV	-82,725.19	Balance EOY 12
18	g n	216	
	FV	-74,717.06	Balance EOY 18

4. The Skinflints purchased a $100,000 home with an $80,000 new 1st loan. If the loan had monthly payments for 30 years at 16½% interest:

a) What is their monthly payment?
b) What will the loan balance be in 10 years?

Keystrokes		Display	Remarks
80000	PV	80,000	
30	g n	360	
16.5	g i	1.38	
	PMT	-1,108.12	Monthly Payment
10	g n	120	Months to Balloon PMT
	FV	-77,550.42	Balance EOY 10

5. On a loan of $180,000 at 14% interest with interest only monthly payments:

a) Find the monthly payment
b) Find the remaining balance at end of year 5

When solving for interest only payments, the financial registers need not be used as the problem is a simple arithmetic problem.

Keystrokes		Display	Remarks
180000	ENTER	180,000	
14	%	25,200	Annual interest
12	÷	2,100.00	Monthly Payment

The remaining balance at EOY 5 is $180,000 as no principal has been paid.

SOLVING FOR REMAINING BALANCE USING PV REGISTER

An alternative approach to finding a remaining balance on a loan is to calculate the Present Value (PV) of the payment stream for the remaining periods after the due date.

Example

A loan was placed for $150,000 at 13% interest with monthly payments for 30 years. What will be the balance in 5 years?

1. Calculate the monthly payment.
2. Change n to the number of periods remaining after the call date.
3. Solve for PV.

Keystrokes		Display	Remarks
150,000	PV	150,000	
13	g i	1.08	
30	g n	360	
	PMT	-1,659.30	Payment
25	g n	300	Periods after call date
	PV	147,122.48	Balance after 5 years

Compare this balance to the balance calculated using the Future Value register....

Keystrokes		Display	Remarks
150,000	PV	150,000	
13	g i	1.08	
30	g n	360	
	PMT	-1,659.30	Payment
5	g n	60	
	FV	-147,122.48	Balance in 5 years

LOAN AMORTIZATION

Your calculator is programmed to amortize loans to allow you to determine the interest and principal paid for any number of periods during the life of the loan. You can also determine the remaining balance of a loan at any point in time.

To amortize a loan the f AMORT key is used.

When amortizing loans:

1. The payment must be calculated before the loan can be amortized.
2. To keep track of the number of periods amortized, set n to zero before amortizing. (This step is optional & is like setting a trip meter to zero).
3. The first number to appear in the display (X register) is the interest paid for the number of periods amortized.
4. The principal for those preiods is automatically stored in the Y register.
5. The remaining balance of the loan is automatically stored in the PV register after the loan is amortized.

After entering the loan information into the financial registers, solving for the payment and setting n to zero, the keystrokes for a monthly amortization are:

Index		Remarks
1	f AMORT	Displays interest
	x ≷ y	Displays principal
	RCL PV	Displays remaining balance
	RCL n	Displays total number periods amortized

EXAMPLES:

1. Using the table below, prepare an amortization schedule for the first 6 months on a loan of $50,000 at 10% interest with monthly payments for 30 years.

Procedure for 1st month is as follows:

Keystrokes		Display	Remarks
50000	PV	50,000	
10	g i	.83	
30	g n	360	
	PMT	-438.79	Constant monthly payment
0	n	0	Sets trip meter to 0
1	f AMORT	-416.67	Int. for 1st mo.
	x⇄y	-22.12	Princ. for 1st mo.
	RCL PV	49,977.88	Bal. after 1st mo.
	RCL n	1	

Repeat keystrokes for last 4 steps for next 5 months.

SOLUTION TO LOAN AMORTIZATION PROBLEM

Month	Beginning Loan Balance	Total Pmt.	Int. Pd.	Princ. Pd.	Ending Loan Balance
1	$50,000.00	$438.79	$416.67	$22.12	$49,977.88
2	49,977.88	438.79	416.48	22.31	49,955.57
3	49,955.57	438.79	416.30	22.49	49,933.08
4	49,933.08	438.79	416.11	22.68	49,910.40
5	49,910.40	438.79	415.92	22.87	49,887.53
6	49,887.53	438.79	415.73	23.06	49,864.47

2 A loan of $100,000 at 9% interest with monthly payments is made for 30 years. At the end of one year, how much interest has been paid? How much principal? What is the remaining balance?

Keystrokes		Display	Remarks
100000	PV	100,000	
9	g i	.75	
30	g n	360	
	PMT	-804.62	
0	n	0	Sets calculator at beginning of loan.
12	f AMORT	-8,972.28	Interest paid over 12 periods.
	x≷y	-683.16	Principal paid over 12 periods.
	RCL PV	99,316.84	Remaining balance after 12 periods.
	RCL n	12	No. of periods amortized.

DO NOT CLEAR CALCULATOR

What is the interest and principal paid for the next month? Remaining balance?

Keystrokes		Display	Remarks
1	f AMORT	-744.88	Interest paid for next 1 period.
	x≷y	-59.74	Principal paid for next 1 period
	RCL PV	99,257.10	Principal Balance after next 1 period
	RCL n	13	Total no. of periods amortized

DO NOT CLEAR CALCULATOR

How much interest will have been paid over the next 11 months? Principal? Remaining Balance?

Keystrokes		Display	Remarks
11	f AMORT	-8,163.32	Interest for next 11 mos.
	x⇄y	- 687.50	Principal for next 11 mos.
	RCL PV	98,569.60	Balance after 2 years
	RCL n	24	Total no. of mos. amortized

To find the total inteest and principal paid for the first 2 years proceed as follows:

Keystrokes		Display	Remarks
100000	PV	100,000	Places original loan balance in PV
0	n	0	Sets calculator at beginning of loan
24	F AMORT	-17,880.48	Total interest paid over 1st 2 years
	x⇄y	- 1,430.40	Total principal paid over 1st 2 years
	RCL PV	98,569.60	Remaining Balance after 2 years

LOAN AMORTIZATION PROBLEM

1. On a loan of $65,000 at 12 % interest with monthly payments for 25 years, find:

   a) Monthly payments
   b) Total interest and principal paid for years 1, 2, and 3
   c) Principal balances at end of years 1, 2, and 3

Keystrokes		Display	Remarks
65000	PV	65,000	
12	g i	1.00	
25	g n	300	
	PMT	-684.60	Monthly payment
0	n	0	
12	f AMORT	-7,776.38	Int. in year 1
	x≷y	- 438.82	Princ. in year 1
	RCL PV	64,561.18	Bal. at EOY 1
	RCL n	12	No. of periods amortized
12	F AMORT	-7,720.72	Int. in year 2
	x≷y	- 494.48	Princ. in year 2
	RCL PV	64,066.70	Bal. at EOY 2
	RCL n	25	Total no. of periods amortized
12	f AMORT	-7,658.03	Int. inyear 3
	x≷y	- 557.17	Princ. in year 3
	RCL PV	63,509.53	Bal. at EOY 3
	RCL n	36	Total no. of periods amortized

Graduated Payment Loans:

In today's market, the payment on loans often increase over the life of the loan. This will cause the amortization schedule to change and affect the principal and interest paid as well as the remaining balance.

Example:

A loan of $50,000 at 14% interest is placed with monthly payments as follows:

Yr. 1	$600
Yr. 2	$650
Yr. 3	$700 till paid

How much interest and principal are paid each of the first 3 years? What are the remaining balances?

Keystrokes		Display	Remarks
14	g i	1.17	
50,000	CHS PV	-50,000	
600	PMT	600	
12	f AMORT	6,986.63	1st years Interest
x≷y		213.37	1st years Principal
RCL	PV	-49,786.63	Balance EOY 1
650	PMT	650	Changes Payment
12	f AMORT	6,914.74	2nd years Interest
x≷y		885.26	2nd years Principal
RCL	PV	-48,901.37	Balance EOY 2
700	PMT	700	Changes Payment
12	f AMORT	6,742.51	3rd years Interest
x≷y		1,657.49	3rd years Principal
RCL	PV	-47,243.88	Balance EOY 3

ADJUSTABLE RATE LOANS (ARM'S) OR VARIABLE INTEREST RATE LOANS (VRM'S)

A lender places a new loan of $130,000 with monthly payments for 30 years. The interest rate for the first year is 11%. For each of the next 2 years the interest rate will increase .5% and the payment will be recalculated over the remianing term. Find the monthly payment for the first 3 years.

Step 1: Find payment for Year 1

Keystrokes		Display	Remarks
130,000	PV	130,000	
30	g n	360.00	
11	g i	.92	
	PMT	-1,238.02	Payment Year 1

Step 2: Amortize the loan for 12 months, increase the interest rate, change the amortization term and calculate the new payment

Keystrokes		Display	Remarks
12	f AMORT	14,271.08	Interest for Year 1
11.5	g i	.96	Rate for Year 2
29	g n	348	Amortization Term
	PMT	-1,286.79	Payment for Year 2
12	f AMORT	14,852.28	Interest for Year 2
12	g i	1.00	Rate for Year 3
28	g n	336	Amortization Term
	PMT	-1,335.42	Payment for Year 3

ADDITIONAL PRINCIPAL PAYMENTS

Loans are often structured to have additional principal payments at certain times during the life of the loan. This changes the amortization schedule of the loan and will change the balance due at any given time.

Example:

A $31,000 loan is created @ 11% with monthly payments for 40 years, all due in 4 years. In addition to the payments the borrower agrees to make additional principal payments as follows:

EOY 1	$2,000
EOY 2	$3,000
EOY 3	$5,000

What is the balance due at the stop date?

Step 1: Find monthly payment

Keystrokes		Display	Remarks
31000	PV	31,000	
40	g n	480	
11	g i	.92	
	PMT	-287.77	Monthly PMT

Step 2: Find balance due EOY 1 (Use AMORT Function)

Keystrokes		Display	Remarks
12	f AMORT	-3407.75	Interest for Yr. 1
	RCL PV	30954.51	Balance EOY 1

Step 3: Reduce remaining balance by extra principal paid at EOY 1 and index new loan balance in PV

Keystrokes		Display	Remarks
		30954.51	In Display
2000		2,000	Additional principal paid EOY 1
-	PV	28954.51	New PV

Step 4: Amortize loan for Year 2 to find remaining balance at EOY 2

Keystrokes		Display	Remarks
12	f AMORT	-3171.05	Interest for Year 2
	RCL PV	28672.32	Balance EOY 2

Step 5: Repeat Steps 3 & 4 for next 2 years

Keystrokes		Display	Remarks
		28672.32	In display
3000		3000	Additional principal paid
-	PV	25672.32	New Loan Balance
12	f AMORT	-2791.23	Interest for Year 3
RCL	PV	25010.31	Balance EOY 3
5000		5000	Additional principal paid
-	PV	20010.31	New Loan Balance
12	f AMORT	-2136.06	Interest for Year 4
RCL	PV	18693.13	Balance EOY 4

COMPOUNDING

Definition: Finding the Future Value (FV) of amounts invested today when interest is calculated on the principal and accumulated interest.

Examples:

A. If you invested $100 @ 5% compounded annually, what would the balance be at the end of years 1, 2, and 3:

1) Without the calculator:

EOY 1 $100.00 x 1.05 = $105.00

EOY 2 $105.00 x 1.05 = $110.25

EOY 3 $110.25 x 1.05 = $115.76

2) Using the calculator:

n

i

PV

PMT

FV

Keystrokes			Display	Remarks
100	CHS	PV	-100	Negative for cash out of pocket
5	i		5	5% interest
1	n		1	1 year
	FV		105.00	Balance EOY 1
2	n		2	Overwrites value in n
	FV		110.25	Balance EOY 2
3	n		3	Overwrites value in n
	FV		115.76	Balance EOY 3

B. If you purchased a home today for $125,000 what would be the value of yourhome 10 years from now if appreciation occurred at 5% per annum:

n

i

PV

PMT

FV

Keystrokes		Display	Remarks
125000	PV	125,000	
5	i	5	
10	n	10	
	FV	-203,611.83	Value in 10 years

COMPOUNDING A UNIFORM PAYMENT

Example:

If you invest $50 at the end of each month in an account which earns 6% interest per annum, how much will your investment be worth at the end of 3 years:

n

i

PV

PMT

FV

Keystrokes		Display	Remarks
6	g i	.50	Monthly interest stored in i
3	g n	36	Number of months stored in n
50	CHS PMT	-50	Monthly payment
	FV	1966.81	

APPRECIATION & LEVERAGE

A home is purchased today for $250,000 with $50,000 down and the balance financed over 30 years with monthly payments at 10.5% interest. If the property appreciates at 5% per annum compounded annually:

1. What will the value be in 10 years?
2. What will be the seller's equity in 10 years?
3. If the property was sold for cash in 10 years and costs of sale are 8% of the sale price, what is the owner's rate of return on equity over the 10 year period?

Step 1: Find value in 10 years (Appreciation)

Keystrokes		Display	Remarks
250,000	PV	250,000	
10	n	10	
5	i	5	
	FV	-407,223.66	Value EOY 10

Step 2: Find payment, Balloon & Equity in 10 years

Keystrokes		Display	Remarks
f	CLX	0.00	Clears Calculator
200,000	PV	200,000	
10.5	g i	.88	
30	g n	360	
	PMT	-1,829.48	Monthly payment
10	g n	120	
	FV	-183,244.74	Balance EOY 10
407,223.66	ENTER	407,223.66	Value EOY 10
8	%	32,577.89	Costs of Sale
	-	374,645.77	Value - Costs
RCL	FV	-183,244.74	Balance on Loan
	+	191,401.03	Net Equity EOY 10

Step 3: Find Rate of Return on Equity

Keystrokes		Display	Remarks
f	CLX	0.00	
50,000	CHS PV	-50,000	Down payment
191,401.03	FV	191,401.03	Net Equity EOY 10
10	n	10	Term
	i	14.37	Rate of Return on Equity

DISCOUNTING

Definition:

Determining the present value (PV) of amounts to be received in the future.

Examples:

A. What is today's value of $4,650 to be received 3½ years from now discounted at 8%:

n
i
PV
PMT
FV

12C

Keystrokes		Display	Remarks
8	i	8	
3.5	n	3.5	
4650	FV	4650	
	PV	-3549.35	Value Today

38C/E

Keystrokes		Display	Remarks
8	i	8	
3.5	n	3.5	
4650	FV	4650	
	PV	-3551.97	Value Today

Note: FOR 12C CALCULATOR Because the number of periods is 3.5 years 12C uses continuous compounding only for 3 periods and calculates simple interest for the 1/2 period. To have the 12C use continuous compounding press STO EEX; Note the c in display for continuous compounding.

COMPOUNDING & DISCOUNTING PROBLEMS

1. The Skinflints purchase a home today for $100,000 with $20,000 down. If appreciation occurs at 8% per year and there is no principal reduction on the loans;

   a) What will their property be worth in 7 years?

   b) What will be their equity in 7 years?

   c) Their equity will represent what rate of return based upon their down payment?

   n
   i
   PV
   PMT
   FV

2. How much interest will you earn if you invest $2,000 today for 10 years at 7 1/2% compounded annually?

   n
   i
   PV
   PMT
   FV

3. If you had $6,300 to invest today how long would it take you to double your money at

   a) 5% per year
   b) 7% per year
   c) 12% per year

   n
   i
   PV
   PMT
   FV

4. If in 6 years you will need $14,200 to replace an air conditioning system, how much must you invest at the end of each year at 8% per annum?

n
i
PV
PMT
FV

5. Congratulations! You've just "Picked the Pick" and won $3,000,000 in the Arizona Lottery. If you are paid $150,000 per year over the next 20 years, what is the present day purchasing power of your winnings if inflation averages 8% per annum.

n
i
PV
PMT
FV

6. You sell your home and carry back a loan with monthly payments of $200 for 12 years. If inflation remains constant at 10%, what is the present value of those payments?

n
i
PV
PMT
FV

SOLUTIONS TO COMPOUNDING & DISCOUNTING PROBLEMS

1. The Skinflints purchase a home today for $100,000 with $20,000 down. If appreciation occurs at 8% per year and there is no principal reduction on the loans;
   a) What will their property be worth in 7 years?
   b) What will be their equity in 7 years?
   c) Their equity will represent what rate of return based upon their down payment?

   a) Step 1: Find Value of house in 7 years.

Keystrokes		Display	Remarks
100,000	CHS PV	-100,000	
7	n	7	
8	i	8	
	FV	171,382.43	Value in 7 years

   Do not Clear Calculator

   b) Step 2: Find equity in 7 years.

Keystrokes		Display	Remarks
		171,382.43	Already in display
80,000		80,000	Loan balance
	-	91,382.43	Equity in 7 years

   Do Not Clear Calculator

   c) Step 3: Find rate of return based on down payment.

Keystrokes		Display	Remarks
		91,382.43	Already in display
FV		91,382.43	Indexes equity in FV
20,000	CHS PV	-20,000	Indexes original equity in PV
i		24.24	Annual rate of return over 7 years.

2. How much interest will you earn if you invest $2,000 today for 10 years at 7 1/2% compounded annually:

Keystrokes		Display	Remarks
2000	CHS PV	-2,000	
7.5	i	7.5	
10	n	10	
	FV	4,122.06	Value in 10 years
2000	-	2,122.06	Interest earned

3. If you had $6,300 to invest today how long would it take you to double your money at
   a) 5% per year
   b) 7% per year
   c) 12% per year

12C

Keystrokes		Display	Remarks
6300	CHS PV	-6300	
5	i	5	
12600	FV	12,600	Dbl. Money
	n	15.00	Yrs. to dbl @ 5%
7	i	7	
	n	11.00	Yrs. to dbl @ 7%
12	i	12	
	n	7.00	Yrs. to dbl @ 12%

38C/E

Keystrokes		Display	Remarks
6300	CHS PV	-6300	
5	i	5	
12600	FV	12,600	Dbl. Money
	n	14.21	Yrs. to dbl @ 5%
7	i	7	
	n	10.24	Yrs. to dbl @ 7%
12	i	12	
	n	6.12	Yrs. to dbl @ 12%

Note: FOR 12C CALCULATOR When solving for n, your 12C always rounds up to the next highest whole period. To calculate the exact number of periods see page 83.

4. In 6 years you will need $14,200 to replace an air conditioning system. How much must you invest at the end of each year at 8% per annum?

Keystrokes		Display	Remarks
8	i	8	
6	n	6	
14,200	FV	14,200	
	PMT	-1,935.68	Yearly investment

5. Congratulations! Your've just "Picked the Pick" and won $3,000,000 in the Arizona Lottery. If you are paid $150,000 per year over the next 20 years what is the present day purchasing power of year winnings if inflation averages 8% per annum.

Keystrokes		Display	Remarks
150,000	PMT	150,000	
20	n	20	
8	i	8	
	PV	1,472,722.11	Today's Value

6. You sell your home and carry back a loan with monthly payments of $200 for 12 years. If inflation remains constant at 10%, what is the present value of those payments?

Keystrokes		Display	Remarks
10	g i	.83	
12	g n	144	
200	PMT	200	
	PV	-16,735.31	Today's value of all payments

YIELDS ON INVESTMENTS AND PRESENT VALUE ANALYSIS

EXAMPLE

You are considering investing $200,000 in a property which will pay you an annual income of $35,000 for the next 10 years. What will be the yield on your investment?

Keystrokes		Display	Remarks
200,000	CHS PV	-200,000	
35,000	PMT	35,000	
10	n	10	
	i	11.73	Annual Yield

Do not clear calculator.

How much should you invest to achieve yields of 14%, 15%, & 16%?

Keystrokes		Display	Remarks
		11.73	in Display
14	i	14	Desired Yield
	PV	-182,564.05	PV to yield 14%
15	i	15	
	PV	-175,656.90	PV to yield 15%
16	i	16	
	PV	-169,162.96	PV to yield 16%

PRESENT VALUE OF LOANS

Example

A seller carries back a note for $100,000 at 11.5% with monthly payments for 10 years. The seller wishes to sell this note at closing to cash out of the transaction.

To sell the note mortgage brokers tell you the seller will have to discount the note to allow an investor to yield 18%.

Based on this information, how much cash will the seller receive?

Step 1: Find payment on note

Keystrokes		Display	Remarks
100000	PV	100,000	
11.5	g i	.96	
10	g n	120	
	PMT	-1,405.95	Monthly Payment

Step 2: Key in desired yield in i and find PV

Keystrokes		Display	Remarks
18	g i	1.5	Desired Yield
	PV	78,028.30	Cash to Seller

Do not clear calculator.

If you find an investor who will buy the note to yield 16%, how much cash will the seller net?

Keystrokes		Display	Remarks
		78,028.30	In display
16	g i	1.33	Yield
	PV	83,931.00	Cash to Seller

Example

You sell a free & clear property for $325,000 with $100,000 down and the balance paid to the seller at 10% interest amortized over 20 years with the entire balance due in 10 years. If notes can be sold to yield 15% how much cash can the seller net if she sells the note?

Step 1: Find payment and remaining balance

Keystrokes		Display	Remarks
225,000	PV	225,000	
10	g i	.83	
20	g n	240	
	PMT	-2,171.30	Monthly Payment
10	g n	120	
	FV	-164,304.70	Balance EOY 10

Step 2: Input 15% yield into i and solve for PV

Keystrokes		Display	Remarks
15	g i	1.25	Desired yield
	PV	171,587.06	Cash to Seller

YIELDS ON LOANS
& PRESENT VALUE OF LOANS

LOAN YIELD DEFINITIONS:

In order to understand loan yields the following definitions must be understood:

Interest Rate:	Rate paid by borrower, based on Loan amount.
Yield:	Rate of Return to Lender (investor) based on lender's investment.
Discounted Value: (Present Value)	Lender's (investor's) actual investment to achieve a desired yield.
Discount:	Difference between Loan Amount and Discounted Value.
% Discount: (Points)	Discount divided by Loan Amount.

A. Existing Loans Purchased at a Discount

Steps for finding yield on existing loans purchased at a discount:

1. Find payment & remaining balance, if any.
2. Key in discounted value in PV.
3. Solve for i (yield).

To find the discounted value:

1. FInd payment & remaining balance, if any.
2. Key in desired yield in i.
3. Solve for PV (discounted value).

Example:

A loan has a remaining balance of $62,857 with monthly payments of $589.75 including 10% interest. The loan has 22 years remaining If you purchase this loan for $50,000, what would be your yield?

Step 1: Compare terms of note to buyer's investment.

	NOTE	BUYER'S INVESTMENT
n	22 g n	22 g n
i	10 g i	i = ?
PV	62,857	50,000
PMT	-589.75	-589.75
FV	0	0

Step 2: Find Yield

Keystrokes		Display	Remarks
50000	PV	50,000	Your Investment
589.75	CHS PMT	-589.75	Payment
22	g n	264	Months remaining
	i	1.12	Monthly Yield
12	X	13.40	Annual Yield

DO NOT CLEAR CALCULATOR

What can you pay to yield 20%? 24%?

Keystrokes		Display	Remarks
20	g i	1.67	Desired yield in i
	PV	34,934.72	PV to yield 20%
24	g i	2.00	Desired yield in i
	PV	29,329.47	PV to yield 24%

B. Loans Placed at a Discount

Steps for finding yields on loans placed at a discount:

1. Find Payment (& remaining balance if there is a balloon payment).
2. Find Discount and discounted value.
3. Key in discounted value in PV
4. Solve for i (yield).

To solve for discounted value & % discount (points):

1. Find payment (& remaining balance if there is a balloon Payment).
2. Key in desired yield in i.
3. Solve for PV (Discounted Value)

Example:

What is the lender's annual yield on a $75,000 loan at 11 1/2% interest with monthly payments for 30 years if 10 points discount are paid?

Step 1: Find monthly payment

		Display	Remarks
75000	PV	75,000	
11.5	g i	.96	
30	g n	360	
	PMT	-742.72	Monthly payment

Step 2: Find lender's discounted value.

		Display	Remarks
75000	ENTER	75,000	
10	%	7,500	Amt. of discount
	-	67,500	Discounted value

Step 3: Compare terms of note to Lender's Investment.

	NOTE	LENDER
n	360	360
i	11.5 g i	i = ?
PV	75,000	67,500
PMT	-742.72	-742.72
FV	0	0

Step 4: Find yield (rate of return) to lender

Index		Display	Remarks
67500	PV	67,500	Replaces 75,000 in PV
	i	1.08	Monthly yield
12	X	12.92	Annual yield

DO NOT CLEAR CALCULATOR

In order to achieve a yield of 18% per annum, how much can the lender invest? How much discount? How many points will be charged?

Index		Display	Remarks
18	g i	1.5	Desired yield in i
	PV	49,281.79	Discounted Value to yield 18%
75000		75,000	Loan Amount
x ⇄ y		49,281.79	Discounted Value in x Loan Amount in y
-		25,718.21	Discount
75000	÷	.34	34 Points Discount

C. Loans with Balloon Payments

Example:

A loan of $50,000 is made at 10% interest with monthly payments for 30 years, the entire balance due in 8 years. If the loan is sold 2 years from now for $40,000, what will be the buyer's annual yield?

Step 1: Determine monthly payment and balance at due date.

Keystrokes		Display	Remarks
50000	PV	50,000	
10	g i	.83	
30	g n	360	
	PMT	-438.79	Monthly payment
8	g n	96	
	FV	-46,766.60	Balance EOY 8

Step 2: Compare terms of note with 6 years remaining to buyer's investment.

	NOTE	BUYER'S INVESTMENT
n	72	72
i	10 g i	i = ?
PV	50,000	40,000
PMT	-438.79	-438.79
FV	-46,766.60	-46,766.60

Step 3: Determine yield to buyer for remaining term at time of purchase.

Keystrokes		Display	Remarks
40000	PV	40,000	Buyer's discounted value
6	g 12X	72	Remaining term
	i	1.24	Monthly yield
12	X	14.92	Annual yield

PRESENT VALUE & LOAN YIELD PROBLEMS

1. When the Skinflints purchased their home, they placed an $80,000 loan @ 16½% monthly payments for 30 years. If the lender charged 5 points discount, what was the lender's annual yield on the loan?

NOTE	LENDER (INVESTOR)
n	
i	
PV	
PMT	
FV	

2. A lender places a loan of $125,000 at 11% interest with annual payments for 30 years and 4 points discount.
   a) What is borrower's annual payment?
   b) What is lender's yield?

NOTE	LENDER (INVESTOR)
n	
i	
PV	
PMT	
FV	

3. An existing loan of $62,575 is purchased today for $46,250. The loan has monthly payments of $602.03 P.I. @ 10.75% and has a balloon payments in 6¼ years. What will be the purchaser's yield?

NOTE	LENDER (INVESTOR)
n	
i	
PV	
PMT	
FV	

4. A $13,500 note payable at $135 per month interest only at 12% has 48 more monthly payments to be made. What is the discounted value of this note to yield 25%, 30%, 35%?

NOTE	LENDER (INVESTOR)
n	
i	
PV	
PMT	
FV	

SOLUTIONS TO PRESENT VALUE & LOAN YIELD PROBLEMS

1. When the Skinflints purchased their home, they placed an $80,000 loan @ 16 1/2% monthly payments for 30 years. If the lender charged 5 points discount, what was the lender's annual yield on the loan?

Step 1: Find monthly payment.

Keystrokes		Display	Remarks
80000	CHS PV	-80,000	
30	g n	360	
16.5	g i	1.38	
	PMT	1,108.12	Monthly Payment

Step 2: Calculate discount & discounted value.

Keystrokes		Display	Remarks
80000	ENTER	80,000	Loan
5	%	4,000	Discount
-		76,000	Discounted Value

Step 3: Compare the Note with the Lender's Investment.

NOTE		LENDER (INVESTOR)
n	360	360
i	16.5 g i	i = ?
PV	-80,000	-76,000
PMT	1,108.12	1,108.12
FV	0	0

Step 4: Find lender's yield.

Keystrokes		Display	Remarks
		76,000	Already in display
CHS	PV	-76,000	Discounted value in PV
i		1.45	Monthly yield
12	X	17.40	Annual yield

2. A lender places a loan of $125,000 at 11% interest with annual payments for 30 years and 4 points discount.

a) What is borrower's annual payment?
b) What is lender's yield?

Step 1: Find Annual Payment.

Keystrokes		Display	Remarks
125000	CHS PV	-125,000	
11	i	11	
30	n	30	
	PMT	14,378.07	Annual payment

Step 2: Find lender's actual investment.

Keystrokes		Display	Remarks
125000	ENTER	125,000	
4	%	5,000	
	-	120,000	Lender's investment

Step 3: Compare the Note with the Lender's Investment.

	NOTE	LENDER (INVESTOR)
n	30	30
i	11	i = ?
PV	-125,000	-120,000
PMT	14,378.07	14,378.07
FV	0	0

Step 4: Find lender's yield.

Keystrokes		Display	Remarks
120000	CHS PV	-120,000	Replaces 125,000 in PV
	i	11.53	Annual yield

3. An existing loan of $62,575 is purchased today for $46,250. The loan has monthly payments of $602.03 P.I. @ 10.75% and has a balloon payment in 6 1/4 years. What will be the buyer's yield:

Step 1: Find balloon payment.

Keystrokes		Display	Remarks
62575	CHS PV	-62,575	Present balance
10.75	g i	.90	
6.25	g n	75	Remaining term
602.03	PMT	602.03	Monthly payment
	FV	58,168.55	Balloon payment

Step 2: Compare the Note to the Buyer's Investment.

NOTE	BUYER'S INVESTMENT
n 6.25 g n	6.25 g n
i 10.75g i	i = ?
PV -62,575	PV - 46,250
PMT 602.03	PMT 602.03
FV 58,168.55	FV 58,168.55

Step 3: Determine yield to investor (buyer).

Keystrokes		Display	Remarks
46250	CHS PV	-46,250	Buyer's investment
	i	1.49	Monthly yield
12	X	17.89	Annual yield

4. A $13,500 note payable at $135 per month interest only at 12% has 48 more monthly payments to be made. What is the discounted value of this note to yield 25%, 30%, 35%?

Step 1: Compare the Note to the Buyer's Investment.

NOTE	BUYER'S INVESTMENT
n 48	48
i 1	i = 25 g i, 30 g i, 35 g i
PV -13,500	PV = ?
PMT 135	PMT 135
FV 13,500	FV 13,500

Step 2: Find Discounted Value to yield 25%, 30% and 35%

Keystrokes		Display	Remarks
135	PMT	135	Monthly payment
13500	FV	13,500	Balloon payment
48	n	48	No. of payments
25	g i	2.08	Desired yield
	PV	-9,089.18	Discounted value to yield 25%
30	g i	2.50	Overwrites desired yield
	PV	-7,875.94	Discounted value to yield 30%
35	g i	2.92	Overwrites desired yield
	PV	-6,860.50	Discounted value to yield 35%

CASH FLOW DIAGRAMS

Knowing how to utilize the basic financial functions of the calculator we can now utilize this knowledge in a number of more interesting ways.

In order to make any problem easier to solve, cash flow diagrams can be used to analyze the problem.

For example: If a lender makes a $75,000 loan for 30 years at 12% interest with monthly payments of $771.46, the cash flow diagram would appear as follows:

n	$	
0	(75,000)	
1	771.46	i = 12 g i
↓	↓	
360	771.46	

In month 0, the lender invests $75,000 (negative for cash out) and months 1 through 360 will receive $771.46.

INTERNAL RATE OF RETURN (IRR)

Definition: IRR is the rate at which all cash flows received in the future would be discounted to equal the initial investment.

IRR USING EQUAL CASH FLOWS

When all cash flows to be received are equal, the IRR is the same as the yield to a lender or the Rate of Return on a level payment annuity.

Examples:

1. You purchase a property for $475,000 with $150,000 down and you anticipate receiving annual cash flows of $6,750. At time of sale 10 years from now, you also expect to net $225,000 from the sale. What is your IRR?

   Step 1: Prepare a cash flow diagram.

n	$	
0	(150,000)	
1	6,750	
↓	↓	IRR = ?
10	6,750 + 225,000	

   Step 2: Find the IRR.

Keystrokes		Display	Remarks
150000	CHS PV	-150,000	Initial investment
6750	PMT	6,750	Annual cash flow
225000	FV	225,000	Cash flow at sale
10	n	10	Number of annual cash flows
	i	7.96	Annual IRR

IRR USING UNEVEN (IRREGULAR) CASH FLOWS

Many times the cash flows from an investment are not even. Some common examples would be:

1. Cash flow analysis projection of after tax cash flows
2. Wraparound loan when the underlying loan is paid off before the wraparound
3. Graduated payment loans
4. Graduated lease payments
5. Periodic payments of additional principal on a loan

To calculate the yield or IRR on investments like these, the financial registers as we have used them are insufficient. We can, however, use the CFo, CFj and Nj keys to enter the irregular cash flows and the IRR key to calculate the IRR.

CFo — Indexes initial cash flow (stores initial cash flow in STO 0). Must be preceeded by g key.

CFj — Indexes periodic cash flows (up to 20) and stores them in STO 1 through STO .9 with the 20th cash flow stored in FV. Must be preceeded by g key.

Nj — Indexes the number of times the same periodic cash flow is repeated. (up to 99 times). Must be preceeded by g key.

IRR — Calculates the internal rate of return for the cash flows entered. Must be preceeded by f key.

Note: There must be at least one change of sign in order for this calculation to be completed.

Examples:

1. You invest $86,000 cash today and expect to receive the following annual cash flows:

Year 1:	$2,400
Year 2:	$3,000
Year 3:	$3,925
Year 4:	$6,250

At the end of the fourth year you sell the property and net $176,000 cash. What is your IRR:

Step 1: Prepare cash flow diagram

Year	Nj	$	
0		(86,000)	
1	1	2,400	
2	1	3,000	IRR = ?
3	1	3,925	
4	1	6,250	+ 176,000

Step 2: Calculate IRR

Keystrokes	Display	Remarks
86000 CHS g CFo	-86,000	Initial investment
2400 g CFj	2,400	Cash flow Yr. 1
3000 g CFj	3,000	Cash flow Yr. 2
3925 g CFj	3,925	Cash flow Yr. 3
6250 ENTER	6,250	Annual cash flow Yr. 4
176000 +	182,250	Total cash flow Yr. 4
g CFj	182,250	
f IRR	22.88	Annual IRR

2. With a $50,000 initial cash investment you expect to receive the following cash flows: Year 1 ($2,100); Year 2 ($850); Years 3, 4, & 5 $120; Years 6 & 7 $2,000 and Years 8, 9, & 10, $5,000. At the end of year 10 you sell the property and will net $60,000 from the sale. What is your IRR:

Step 1: Prepare cash flow diagram

Year	Nj	$
0		(50,000)
1	1	(2,100)
2	1	(850)
3 - 5	3	120
6 - 7	2	2,000
8 - 9	2	5,000
10	1	5,000 + 60,000

Step 2: Calculate IRR

Keystrokes		Display	Remarks
50000	CHS g CFo	-50,000	Initial investment
2100	CHS g CFj	-2,100	Cash flow Yr. 1
850	CHS g CFj	-850	Cash flow Yr. 2
120	g CFj	120	Next Cash flow
3	g Nj	3.00	Same cash flow for 3 years
2000	g CFj	2,000	Next cash flow
2	g Nj	2.00	Same cash flow for 2 years
5000	g CFj	5,000	Next cash flow
2	g Nj	2.00	Same cash flow for 2 years
5000	ENTER	5,000	
60000	+ g CFj	65,000	Cash flow Yr. 10
	f IRR	4.34	IRR

IRR PROBLEMS

1. If you buy a loan of $42,500 for $30,000 and will receive $394 per month for the next 15 years plus a final payment of $25,614.00, what is your annual yield (IRR)?

2. The Skinflints buy a property today for $600,000 with $225,000 down payment and sell it seven years from now and net $560,000 cash. What will their IRR be if they also receive the following cash flows:

Year 1:	($4,050)
Year 2:	$50
Year 3:	$4,800
Year 4:	$4,800
Year 5:	($100)
Year 6:	$9,000
Year 7:	$9,500

3. You invest $125,000 cash today and expect to receive the following cash flows:

Year 1:	($650)
Year 2:	0
Year 3:	$1,250
Year 4:	($200)
Year 5:	$2,050

At the end of the 5th year you expect to sell the property and net $317,000. Calculate the IRR.

SOLUTIONS TO IRR PROBLEMS

1. If you buy a loan of $42,500 for $30,000 and will receive $394.00 per month for the next 15 years plus a final payment of $25,614.00, what is your annual yield (IRR)?

Step 1: Prepare cash flow diagram

n	$
0	(30,000)
1	394
↓	↓
180	394 + 25,614

Step 2: Calculate IRR

Keystrokes		Display	Remarks
30000	CHS PV	-30,000	
394	PMT	394	
25614	FV	25,614	
15	g 12X	180	
	i	1.29	Monthly IRR
12	X	15.51	Annual IRR

2. The Skinflints buy a property today for $600,000 with $225,000 down payment and sell it seven years from now and net $560,000 cash. What will be their IRR if they also receive the following cash flows:

Yr. 1 ($4,050): Yr. 2 $50 Yr. 3 & 4, $4,800;
Yr. 5 ($100); Yr. 6 $9,000; Yr. 7 $9,500.

Step 1: Prepare cash flow diagram

Year	Nj	$
0		(225,000)
1	1	(4,050)
2	1	50
3 - 4	2	4,800
5	1	(100)
6	1	9,000
7	1	9,500 + 560,000

Step 2: Calculate IRR

Keystrokes		Display	Remarks
225000	CHS g CFo	-225,000	Initial investment
4050	CHS g CFj	-4,050	Cash flow Yr. 1
50	g CFj	50	Cash flow Yr. 2
4800	g CFj	4,800	Next cash flow
2	g Nj	2	For next 2 years
100	CHS g CFj	-100	Cash flow Yr. 5
9000	g CFj	9,000	Cash flow Yr. 6
9500	ENTER	9,500	
560000	+ g CFj	569,500	Cash flow Yr. 7
	f IRR	14.66	Annual IRR

3. You invest $125,000 cash today and expect to receive the following cash flows:

Yr. 1 ($650); Yr. 2 0; Yr. 3 $1,250;
Yr. 4 ($200); Yr. 5 $2,050

At the end of the 5th year you expect to sell the property and net $317,000. Calculate the IRR;

Year	Nj	$
0		(125,000)
1	1	(650)
2	1	0
3	1	1,250
4	1	(200)
5	1	2,050 + 317,000

Keystrokes		Display	Remarks
125000	CHS g CFo	-125,000	Initial investment
650	CHS g CFj	-650	Cash flow Yr. 1
0	g CFj	0	Cash flow Yr. 2
1250	g CFj	1,250	Cash flow Yr. 3
200	CHS g CFj	-200	Cash flow Yr. 4
2050	ENTER	2,050	Cash flow Yr. 5
317000	+ g CFj	319,050	Total cash flow Yr. 5
	f IRR	20.63	Annual IRR

PROOF OF IRR

In calculating IRR does the calculator assume that the Cash Flows must be reinvested at the IRR?

Does It?

or

Doesn't It?

Let's look at the following example.

n	$
0	(100,000)
1	5,000
2	10,000
3	15,000
4	20,000
5	165,000

Calculate IRR.

Keystrokes			Display	Remarks
100000	CHS	g CFo	-100,000	
5000		g CFj	5,000	
10000		g CFj	10,000	
15000		g CFj	15,000	
20000		g CFj	20,000	
165000		g CFj	165,000	
		f IRR	18.80	IRR

DO NOT CLEAR CALCULATOR

The IRR of 18.80% (rounded) is the rate of return on the amount of dollars remaining, <u>within</u> (Internal to) the investment. The annual Cash Flows can be spent and are <u>not</u> reinvested.

The following calculations should help.

Year	$ Remaining Within Investment		IRR		Total Interest		Cash Flow		$ Reinvested
1	100,000	x	18.80%	=	18,800	-	5,000	=	13,800
	+ 13,800								
2	113,800	x	18.80%	=	21,394	-	10,000	=	11,394
	+ 11,394								
3	125,194	x	18.80%	=	23,536	-	15,000	=	8,536
	+ 8,536								
4	133,730	x	18.80%		25,141	-	20,000	=	5,141
	+ 5,141								
5	138,872	x	18.80%	=	26,108	+	138,872	=	164,980

$165,000 ($164,980 using rounded 18.80%) is the amount remaining after 5 years.

The annual cash flows <u>are not</u> reinvested but taken out of the investment and the 18.80% is applied only to the amount remaining internal to the investment.

Therefore:

IRR <u>does not</u> assume that cash flows must be reinvested at the same rate as the IRR.

FINANCIAL MANAGEMENT RATE OF RETURN

One of the shortcomings of the IRR is that it does not take into account how negative cash flows which occur in the future will be raised.

When future cash flows are negative, the investor may have to lay some money aside at the beginning of the investment period. This money is invested at a "Safe Rate" so that it will be available when needed. After all negative cash flows are taken care of remaining cash flows are reinvested at a higher "Reinvestment Rate".

The Financial Management Rate of Return (FMRR) is an alternative approach to estimating returns on investments.

Example: A client invests $240,000 cash today and expects to receive the following cash flows:

Year 1:	$24,000	
Year 2:	-$20,000	
Year 3:	-$27,000	IRR = 12.34%
Year 4:	$30,000	
Year 5:	$420,000	

Using a "Safe Rate" of 6% and a reinvestment rate of 10%, calculate the FMRR:

Step 1: Prepare a cash flow diagram.

n	$
0	(240,000)
1	24,000
2	(20,000)
3	(27,000)
4	30,000
5	420,000

Step 2: Reduce all negative cash flows, where possible, by compounding positive cash flows forward at the safe rate of 6%.

n	$
0	(240,000)
1	24,000 —6%→
2	(20,000) + 25,440 = 5,440 —6%→
3	(27,000) + 5,766.40 = (21,233.60)
4	30,000
5	420,000

Keystrokes		Display	Remarks
24000	CHS PV	-24,000	24,000 received in Yr. 1
6	i	6	Safe Rate
1	n	1	1 Year
	FV	25,440	Value of 24,000 after 1 Yr.
20000		20,000	Negative cash flow Yr. 2
	-	5,440	Total cash flow Yr. 2
	CHS PV	- 5,440	5,440 invested at safe rate
	FV	5,766.40	Value of 15,440 after 1 Yr.
27000			Negative cash flow Yr. 3
	-	-21,233.60	Total cash flow Yr. 3

DO NOT CLEAR CALCULATOR

The cash flows now appear as follows:

n	$
0	(240,000)
1	0
2	0
3	(21,233.60)
4	30,000
5	420,000

Step 3: Discount any remaining negative cash flows back to Year 0 using the safe rate.

n	$			
0	(240,000)	+	(17,828.14)	= (257,828.14)
1	0			
2	0			
3	(21,233.60)		6%	
4	30,000			
5	320,000			

Keystrokes		Display	Remarks
		-21,233.60	Already in display
	CHS FV	21,233.60	Need 21,233.60 in Yr. 3
3	n	3	
6	i	6	Safe rate
	PV	-17,828.14	Amt. needed to invest in Yr. 0 to have 21,233.60 in Yr. 3
240000	CHS		Original cash flow Yr. 0
	+	-257,828.14	New investment, Yr.0

The cash flows now appear as follows:

n	$
0	(257,828.14)
1	0
2	0
3	0
4	30,000
5	420,000

Step 4: Compound the remaining cash flows forward at the reinvestment rate of 10%.

n	$
0	(257,828.14)
1	0
2	0
3	0
4	30,000 — 10% →
5	420,000 + 33,000 = 453,000

Keystrokes		Display	Remarks
f	CLEAR FIN	-257,828.14	Clears financial registers
30000	CHS PV	-30,000	Received in Yr. 4
10	i	10	Reinvestment rate
1	n	1	1 year
	FV	33,000	Value of 30,000 after 1 Yr.
420000		420,000	Cash flow Yr. 5
	+	453,000	Total cash flow Yr.5

DO NOT CLEAR CALCULATOR

The cash flows now appear as follows:

n	$
0	(257,828.14)
1	0
2	0
3	0
4	0
5	453,000

Step 5: Calculate the FMRR. The FMRR is an IRR calculation without intermediate cash flows.

Keystrokes		Display	Remarks
453000		453,000	Already in display
	FV	453,000	Cash flow Yr. 5
257.828.14	CHS PV	-257,828.14	Cash flow Yr. 0
0	PMT	0.00	Intermediate cash flows
5	n	5.00	
	i	11.93	FMRR

FMRR PROBLEM

A property is purchased with $80,000 down and you expect to receeve the following cash flows.

Yr. 1	(2,000)
Yr. 2	6,000
Yr. 3	(3,950)
Yr. 4	16,800
Yr. 5	9,850
Yr. 6	3,200
Yr. 7	155,000

Problem:

1. Find the IRR

2. Using 10% as both the "safe rate" and "reinvestment rate" calculate the FMRR.

Solution:

1. Find the IRR.

   Step 1: Prepare Cash Flow Diagram

n	$
0	(80,000)
1	(2,000)
2	6,000
3	(3,950)
4	16,800
5	9,850
6	3,200
7	155,000

   Step 2 : Solve for IRR

Keystrokes			Display	Remarks
80,000	CHS	g CFo	-80,000	
2,000	CHS	g CFj	-2,000	
6,000		g CFj	6,000	
3,950	CHS	g CFj	-3,950	
16,800		g CFj	16,800	
9,850		g CFj	9,850	
3,200		g CFj	3,200	
155,000		g CFj	155,000	
		f IRR	13.70	IRR

2. Using 10% as both the 'safe rate' and 'reinvestment rate' calculate the FMRR.

Step 3: Reduce the negative cash flow in year 3 by compounding year 2 cash flow forward.

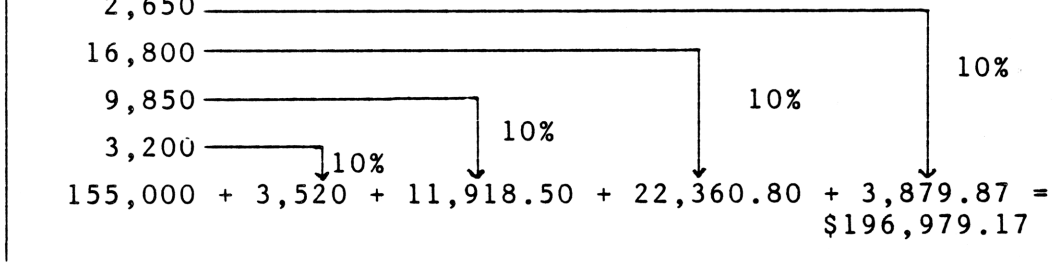
n	$
0	(80,000)
1	(2,000)
2	6,000 ——10%→
3	(3,950) + 6,600 = 2,650
4	16,800
5	9,850
6	3,200
7	155,000

Step 4: Discount the negative cash flow in year 1 back to year 0.

n	$
0	(80,000) + (1,818.18) = 81,818.18
1	(2,000) ←10%——
2	0
3	2,650
4	16,800
5	9,850
6	3,200
7	155,000

Step 5: Compound the cash flow in year 3,4,5, & 6 forward to year 7.

n	$
0	(81,818.18)
1	0
2	0
3	2,650
4	16,800
5	9,850
6	3,200
7	155,000 + 3,520 + 11,918.50 + 22,360.80 + 3,879.87 = $196,979.17



Step 6: Calculate FMRR

n	$
0	(81,818.18)
1	0
2	0
3	0
4	0
5	0
6	0
7	196,679.17

Keystrokes			Details	Remarks
81,818.18	CHS	PV	-81,818.18	
196,679.17		FV	196,679.17	
7		n	7.00	
		i	13.35	FMRR

NET PRESENT VALUE

Many times you may anticipate certain uneven cash flows in future years and wish to achieve a specific rate of return. In order to determine the amount that your initial investment should be to achieve that return the Net Present Value calculation can be used.

Steps to Solve for Net Present Value:

1. Index cash flows using g CFj registers & Nj registers (Do not use g CFo).
2. Index desired yield in i.
3. Solve for Net Present Value using f NPV.

Example:

Over the next 4 years you expect to receive the following cash flows:

Year 1:	($28,000)
Year 2:	$31,500
Year 3:	$42,000
Year 4:	$51,000

How much can you invest in order to receive a return of 14.5%?

Step 1: Prepare cash flow diagram

Year	Nj	$	
0		?	
1	1	(28,000)	
2	1	31,500	i = 14.5%
3	1	42,000	
4	1	51,000	

Keystrokes		Display	Remarks
28000	CHS g CFj	-28,000	Cash flow Yr. 1
31500	g CFj	31,500	Cash flow Yr. 2
42000	g CFj	42,000	Cash flow Yr. 3
51000	g CFj	51,000	Cash flow Yr. 4
14.5	i	14.50	Desired rate of return
	f NPV	57,223.99	Net Present Value

You can afford to invest $57,223.99 cash to earn a 14.5% return with those cash flows.

Note: The positive value of the Net Present Value indicates that the investment does meet the desired rate of return.

NET PRESENT VALUE PROBLEMS

1. An existing loan has the following remaining payment schedule:

Next	8 months	$75
Next	24 months	$125
Next	24 months	$175
Next	36 months	$250 plus a $16,000 balloon payment at the time the last payment is due.

   At what price can you purchase this loan to achieve a desired yield of 23% per annum.

2. A property is triple net leased according to the following schedule:

Years 1 - 4	$20,000 net to lessor
Years 5 - 8	$25,000 net to lessor
Years 9 - 12	$30,000 net to lessor
Years 13 - 20	$40,000 net to lessor

   What is the present value of the lessor's income stream if we assume inflation to average 10% over the life of the lease?

3. In Problem #2, if the lessor wishes to generate additional cash at the beginning of the lease, how much could a partner pay today for the right to receive the cash flows from years 4 thru 6 and yield that partner a 18% return on his investment?

4. The Skinflints sell their home for $140,000 and carry back a $40,000 2nd with no interest and monthly payments as follows:

Year 1:	$1,000 per month
Year 2 & After:	$2,000 per month till paid

   If they sell the 2nd at a discount to yield 18%, how much cash will they receive?

SOLUTIONS TO NET PRESENT VALUE PROBLEMS

1. An existing loan has the following remaining payment schedule:

Next 8 months	$75
Next 24 months	$125
Next 24 months	$175
Next 36 months	$250 plus a $16,000 balloon payment at the time the last payment is due.

At what price can you purchase this loan to achieve a desired yield of 23% per annum.

Step 1: Prepare cash flow diagram

Month	Nj	$	
0		?	
1 - 8	8	75	
9 - 32	24	125	
33 - 56	24	175	i = 23 g i
57 - 91	35	250	
92	1	16,250	

Keystrokes		Display	Remarks
75	g CFj	75	1st cash flow ...
8	g Nj	8	... for 8 periods
125	g CFj	125	2nd cash flow ...
24	g Nj	24	... for 24 periods
175	g CFj	175	3rd cash flow ...
24	g Nj	24	... for 24 periods
250	g CFj	250	4th cash flow ...
35	g Nj	35	... for 35 periods
16250	g CFj	16,250	Last cash flow (monthly pmt. plus balloon)
23	g i	1.92	Desired monthly yield
	f NPV	9,441.98	Discounted value to yield 23%

2. A property is triple net leased according to the following schedule:

Years 1 - 4	$20,000 net to lessor
Years 5 - 8	$25,000 net to lessor
Years 9 - 12	$30,000 net to lessor
Years 13 - 20	$40,000 net to lessor

What is the present value of the lessor's income stream if we assume inflation to average 10% over the life of the lease?

Step 1: Prepare cash flow diagram

Year	Nj	$
0		?
1 - 4	4	20,000
5 - 8	4	25,000
9 - 12	4	30,000
13 - 20	8	40,000

Keystrokes		Display	Remarks
20000	g CFj	20,000	1st cash flow ...
4	g Nj	4	... for 1st 4 years
25000	g CFj	25,000	2nd cash flow ...
4	g Nj	4	... for next 4 years
30000	g CFj	30,000	3rd cash flow ...
4	g Nj	4	... for next 4 years
40000	g CFj	40,000	4th cash flow ...
8	g Nj	8	... for last 8 years
10	i	10	Annual inflation
	f NPV	229,881.67	Net present value

3. In Problem #2, if the lessor wishes to generate additional cash at the beginning of the lease, how much could a partner pay today for the right to receive the cash flows from years 4 thru 6 and yield that partner a 18% return on his investment?

Step 1: Prepare a cash flow diagram

Year	Nj	$
0		?
1 - 3	3	0 i = 18
4	1	20,000
5 - 6	2	25,000

Keystrokes		Display	Remarks
0	g CFj	0	Cash flow for ...
3	g Nj	3	... 1st 3 years
20000	g CFj	20,000	Cash flow for 4th Yr.
25000	g CFj	25,000	Cash flow for ...
2	g Nj	2	... years 5 & 6
18	i	18	Desired yield
	f NPV	30,504.30	Partner's Investment

4. The Skinflints sell their home for $140,000 and carry back a $40,000 2nd with no interest and monthly payments as follows:

Year 1: $1,000 per month
Year 2 & After: $2,000 per month till paid.

If they sell the 2nd at a discount to yield 18%, how much cash will they receive?

Step 1: Prepare cash flow diagram

Month	Nj	$	
0		NPV = ?	
1 - 12	12	1,000	i = 18 g i
13 - 26	14	2,000	

Keystrokes		Display	Remarks
1000	g CFj	1,000	Payment for ...
12	g Nj	12	... 1st 12 mos.
2000	g CFj	2,000	Payment for ...
14	g Nj	14	... next 14 mos.
18	g i	1.5	Desired Yield
	f NPV	31,889.76	Cash to yield 18%

COMPOUNDING UNEVEN CASH FLOWS

In compounding level payment cash flows, the PMT and FV registers can be used. However, when you want to compound a series of uneven cash flows the PMT register can not be used.

In order to compound uneven cash flows, we must first find the Present Value of those cash flows using NPV and then find the Future Value of the Present Value.

Example:

You receive the following cash flows:

Yr. 1	10,000
Yr. 2	15,000
Yr. 3	20,000
Yr. 4	25,000
Yr. 5	30,000
Yr. 6	35,000
Yr. 7	100,000

If you invest all these cash flows at 10% per annum, what will be the total value of your investment at the end of year 7?

Step 1: Find the PV of the cash flows discounted at 10%.

Keystrokes		Display	Remarks
10000	g CFj	10,000	
15000	g CFj	15,000	
20000	g CFj	20,000	
25000	g CFj	25,000	
30000	g CFj	30,000	
35000	g CFj	35,000	
100000	g CFj	100,000	
10	i	10	
	f NPV	143,289.27	PV of Cash Flows Stored in PV

DO NOT CLEAR CALCULATOR

Step 2: Find Future Value (Remember n = 7, i = 10, PV = 143,289,27) Just solve for FV.

Keystrokes	Display	Remarks
FV	143,289.27	Inputs value into FV
FV	-279,230.26	Solves for FV compounded at 10%

COMPOUNDING UNEVEN CASH FLOWS TO FIND SALES PRICE

TO ACHIEVE DESIRED YIELD

Example:

Over the last 8 years, a property has produces the following before tax cash flows:

Initial Investment	(3,250,000)
Year 1	300,000
Year 2	325,000
Year 3	400,000
Year 4	200,000
Year 5	100,000
Year 6	250,000
Year 7	300,000
Year 8	275,000

How much must you sell the property for at EOY 8 in order to achieve an annual yield of 15% on your investment?

Step 1: Prepare a Cash Flow Diagram.

n	$
0	(3,250,000)
1	300,000
2	325,000
3	400,000
4	200,000
5	100,000
6	250,000
7	300,000
8	275,000 + ? Sale Price

Step 2: Find the Net Present Value of the Cash Flows discounted at 15%.

Keystrokes		Display	Remarks
3250000 CHS	g CFo	-3,250,000	
300000	g CFj	300,000	
325000	g CFj	325,000	
400000	g CFj	400,000	
200000	g CFj	200,000	
100000	g CFj	100,000	
250000	g CFj	250,000	
300000	g CFj	300,000	
275000	g CFj	275,000	
15	i	15	
	f NPV	-2,005,547.93	NPV of all Cash Flows discounted at 15%

DO NOT CLEAR CALCULATOR

Step 3: The FV of the NPV compounded at 15% is the desired sale price.

Keystrokes	Display	Remarks
FV	-2,005,547.93	Inputs NPV into FV
FV	6,135,016.97	Solves for FV compounded at 15%. Result is desired Sale Price.

DO NOT CLEAR CALCULATOR

Step 4: Check your answer by adding sale price to year 8 cash flow, input the sum into STO 8 and solve for IRR (should be 15%).

Keystrokes		Display	Remarks
RCL	8	275,000	Year 8 Cash Flow
	+	6,410,016.97	Total EOY 8
STO	8	6,410,016.97	Inputs total in STO 8
	f IRR	15	IRR

PROGRAMMING

A program is a swquence of manual keystrokes that is remembered by the calculator. This enables you to perform repetitive calculations of up to 99 steps with just one keystroke.

The most important keys used in programming are:

P/R (Program/run) has two uses:

1. Places the calculator in Program Mode allowing you to index the program.

2. Takes the calculator out of Program Mode and into Automatic Run Mode Allowing you to run the program you have indexed.

R/S (Run/Stop) has two uses:

1. When in Automatic Run Mode, R/S is pressed to actually run the program.

2. When in Program Mode, R/S can be used as a step in the Program which will stop the program execution and allow you to write down an intermediate result.

CLEAR PRGM (12C)

CLP (38)

Clear Program has two uses:

1. When in Program Mode, used to clear an existing program.

2. When in Automatic Run Mode, resets calculator so operations begin at beginning of program.

WRITING A PROGRAM

To illustrate how a program is written and entered into the calculator, let's use the problem on page 34 .

The problem asked us to generate an amortization schedule for the first 6 months on a loan of $50,000 loaned at 10% with montly payments for 30 years.

Here are the keystrokes we used in solving that problem.

Keystrokes		Display	Remarks
50000	PV	50,000	
10	g i	.83	
30	g n	360	
	PMT	-438.79	Monthly Payment
0	n	0	sets trip meter to 0
1	f AMORT	-416.67	Interest Mo. 1
	x y	-22.12	Principal Mo. 1
	RCL PV	49,977.88	Balance EOM 1

To amortize the loan for the remainder of the loan we would repeat the keystrokes:

1	f AMORT
	x y
	RCL PV
	RCL n

Because we have a set of keystrokes which will be repeated, a program can be used to solve the problem.

The repetitive keystrokes to solve the problem are as follows:

1	f AMORT	Calculates Interest
	x y	Displays Principal
	RCL PV	Displays Balance
	RCL n	Displays Periods Amortized

We can program the calculator to perform these steps by pressing only one key.

INDEXING THE PROGRAM

To index the program, press P/R to place the calculator in Program Mode. The display will read 00- (with the 12C letters PRGM will appear in the display). This tells you that you are at the beginning of program memory and that you are ready to key in your program. Press CLEAR PRGM (12C), CLP (38) to clear any programs.

Press the first key of the program, 1, and the display with change to:

01- 1

The two digits displayed on the left designate the line number of the program memory (01 through 99), while the digits displayed on the right designate the key stored in that line.

Each key (except digit keys 0 through 9) on the keyboard has a two-digit keycode based on its position on the keyboard. The first digit denotes the row of the keys and the second digit denotes the number of the keys in that row.

For convenience, the digit keys (0 through 9) are coded with a single digit key coade 0 through 9.

Let's index the remaining keystrokes in our program and verify the keycode in the display.

12C

Keystrokes	Display	Remarks
1	01- 1	One Period
f AMORT	02-42 11	Amortize Loan
R/S	03- 31	Display interest calculated
x≷y	04- 34	Calculate Principal
R/S	05- 31	Display Principal
RCL PV	06-45 13	Calculate Balance
R/S	07- 31	Display Balance
RCL n	08-45 11	Display periods amortized
f P/R	0.00	Run Mode

38C & E

Keystrokes	Display
1	01- 1
f AMORT	02-24 11
R/S	03- 74
x≷y	04- 33
R/S	05- 74
RCL PV	06-22 13
R/S	07- 74
RCL n	08-22 11
g P/R	0.00

RUNNING A PROGRAM

At this point you are ready to run the program. To take the calculator out of Program Mode and put it into Automatic Run Mode. press P/R.

Now index the financial data and set calculator at beginning of loan.

Keystrokes		Display	Remarks
50000	PV	50,000	Original loan Balance
10	g i	.83	
30	g n	360	
	PMT	-438.79	
0	n	0.00	Sets calculator at beginning of loan

Now the program is ready to be run.

Keystrokes	Display	Remarks
R/S	-416.67	1st month's interest
R/S	-22.12	1st month's princ.
R/S	49,988.88	Balance after 1st month
R/S	1	Number of periods amortized
R/S	-416.48	2nd month's interest
R/S	-22.31	2nd month's princ.
R/S	49,955.57	Balance after 2nd month
R/S	2	Number of periods amortized
R/S	-416.30	3rd month's interest
R/S	-22.49	3rd month's princ.
R/S	49,933.08	Balance after 3rd month
R/S	3	Number of periods amortized

These four steps can be repeated for 360 months until the loan is repaid.

The 6 month table follows:

SOLUTION TO LOAN AMORTIZATION PROBLEM

Month	Beginning Loan Balance	Total Pmt.	Int. Pd.	Princ. Pd.	Ending Loan Balance
1	$50,000.00	$438.79	$416.67	$22.12	$49,977.88
2	49,977.88	438.79	416.48	22.31	49,955.57
3	49,955.57	438.79	416.30	22.49	49,933.08
4	49,933.08	438.79	416.11	22.68	49,910.40
5	49,910.40	438.79	415.92	22.87	49,887.53
6	49,887.53	438.79	415.73	23.06	49,864.47

PROGRAM FOR ANNUAL AMORTIZATION SCHEDULES

Problem:

You sell your client some apartments upon which a loan is made for $125,000 at 14% with monthly payments for 25 years. To prepare a cash flow analysis for your client you need to know the interest, principal, and loan balances for each of the next 5 years.

1. Write a program to perform these calculations and display each of them.

2. Program the calculator, enter the financial data, and run the program to determine these values.

Program:

12C			38C & E		
Keystrokes	Display	Remarks	Keystrokes	Display	Remarks
f P/R	00-	Program Mode	g P/R	00-	Program Mode
f CLEAR PRGM	00-	Clears Programs	g CLP	00-	Clears Programs
1	01- 1		1	01- 1	
2	02- 2		2	02- 2	
f AMORT	03-42 11		f AMORT	03-24 11	
R/S	04- 31		R/S	04- 74	
x≷y	05- 34		x≷y	05- 33	
R/S	06- 31		R/S	06- 74	
RCL PV	07-45 13		RCL PV	07-22 13	
R/S	08- 31		R/S	08- 74	
RCL n	09- 4511		RCL n	09-22 11	
f P/R	0.00	Run Mode	g P/R	0.00	Run Mode

Enter Financial Data:

Keystrokes		Display	Remarks
125000	PV	125,000	Original Loan balance
14	g i	1.17	Monthly interest
25	g n	300	Number of monthly payments
	PMT	-1,504.70	Monthly payment
0	n	0.00	Sets calculator at beginning of loan

Run Program:

Keystrokes	Display	Remarks
R/S	-17,462.85	1st year's interest
R/S	-593.55	1st year's princ.
R/S	124,406.45	Balance EOY 1
R/S	12	Number of periods amortized.

These steps can be repeated for each year.

OTHER KEYS USED IN PROGRAMMING

SST (Single Step) used in Program Mode to display the contents of the next line of program memory.

BST (Back Step) used in Program Mode to display the contents of the previous line of program memory.

GTO (Go To) has three uses:

1. When in Program Mode GTO is used to display any line of program memory. To do this the GTO instruction must be followed by a decimal point and the desired two digot code.

2. When in Program Mode GTO can be used as an instruction in the program. To do this the GTO instruction should not be followed by a decimal point but only by the desired two digit code.

3. In Run Mode, a GTO instruction sets the program memory to the line number specified by the two digit code. This instruction can be made with or without a decimal point.

PSE (Pause) used in Program Mode as an instruction will momentarily interrupt program execution to display intermediate results.

DISCOUNTED CASH FLOW ANALYSIS

1. The cash flow analysis on Page 102 has been prepared on a rental house that an investor has purchased.

$66,000	Price
20,000	Down Payment
$46,000	Assume existing 1st loan @ 11.75%, $470 PI monthly

The property is depreciated on a straight line basis assuming a 15 year economic life. The investor is assumed to be in a 40% marginal tax bracket. The property appreciates at an 8% annual rate and is sold at the end of the fifth year.

Problem: Find the Internal Rate of Return over the five year period:

1. On a Before Tax Basis
2. On an After Tax Basis

Assume all cash flows occur at the end of each year.

Cash Flow Analysis

For Big Swinger #1

Purpose

Date

Purchase Price $66,000

Encumbrances $46,000

Investment $20,000

Mortgage Data

	Encumbrances	Beginning Balance	Remaining Term	Number of Payments Per Year	Interest Rate	Payment	Annual Debt Service	Remarks
1	1st Mortgage	46,000	27 yr.	12	11.75	470.00	5,640	
2	2nd Mortgage							
3	3rd Mortgage							

		Year: 1	Year: 2	Year: 3	Year: 4	Year: 5
	Ownership Analysis of Property Income:	**Taxable Income**				
4	Gross Scheduled Income					
5	Less: Vcy. & Credit Losses					
6	Gross Operating Income	4,800	5,100	5,400	5,700	6,000
7	– Operating Expenses	1,920	2,040	2,160	2,280	2,400
8	Net Operating Income	2,880	3,060	3,240	3,420	3,600
9	– Non-Operating Expense					
10	– Interest – 1st Mortgage	5,391	5,360	5,324	5,285	5,240
11	– Interest – 2nd Mortgage					
12	– Interest – 3rd Mortgage					
13	– Cost Recovery	3,733	3,733	3,733	3,733	3,733
14	Real Est. Taxable Income	(6,244)	(6,033)	(5,817)	(5,598)	(5,373)
		Cash Flows				
15	Net Operating Income	2,880	3,060	3,240	3,420	3,600
16	– Annual Debt Service	5,640	5,640	5,640	5,640	5,640
17	– Funded Reserves					
18	– Capital Additions					
19	Cash Flow before Taxes	(2,760)	(2,580)	(2,400)	(2,220)	(2,040)
20	– Minimum Tax					
21	– Tax Liability on Real Est.	(2,498)	(2,413)	(2,326)	(2,239)	(2,149)
22	Cash Flow after Taxes	(262)	(167)	(74)	19	109

EOY	Mortgage Balance
	1st Mortgage
1	45,746
2	45,461
3	45,141
4	44,781
5	44,376
	2nd Mortgage
	3rd Mortgage

Analysis of Sale Proceeds

Year:

	Adjusted Basis	
23	Original Basis	66,000
24	+ Capital Improvements	----
25	– Cost Recovery	18,665
26	– Partial Sales	----
27	AB	47,335
28		

Excess Cost Recovery (CR)	
Total CR	18,665
S/L CR	18,665
Excess CR*	-0-
Exc. CR Carryover	

Tax Liability on Sale	
Excess Recapture Tax	
Capital Gain Tax	6,394
Tax Liability on Sale	6,394

Gain	
Sale Price	97,000
– Cost of Sale	9,700
– AB	47,335
Gain	39,965
– Excess CR	-0-
Capital Gain	39,965

Sale Proceeds	
Sale Price	97,000
– Costs of Sale	9,700
– Mortgage	44,376
Proceeds before Taxes	42,924
– Tax Liability on Sale	6,394
Proceeds after Taxes	36,530

*NOTE: On non-residential property, if accelerated CR system is elected, all CR claimed is recaptured as ordinary income in the year of sale.

The statements and figures presented herein, while not guaranteed, are secured from sources we believe authoritative.

Prepared by

Step 1: Prepare Cash Flow Diagram for Before Tax Cash Flows.

n	$
0	(20,000)
1	(2,760)
2	(2,580)
3	(2,400)
4	(2,220)
5	(2,040) + 42,924

Step 2: Solve for tax IRR.

Keystrokes			Display	Remarks
20000	CHS	g CFo	-20,000	
2760	CHS	g CFj	-2,760	
2580	CHS	g CFj	-2,580	
2400	CHS	g CFj	-2,400	
2220	CHS	g CFj	-2,220	
2040	CHS	ENTER	-2,040	
42924	+	g CFj	40,884	
		f IRR	7.57	Before Tax IRR

Step 3: Prepare Cash Flow Diagram for After Tax Cash Flows.

n	$
0	(20,000)
1	(262)
2	(167)
3	(74)
4	19
5	109 + 36,530

Step 4: Solve for after tax IRR

Keystrokes			Display	Remarks
20000	CHS	g CFo	-20,000	
262	CHS	g CFj	-262	
167	CHS	g CFj	-167	
74	CHS	g CFj	-74	
19		g CFj	19	
109		ENTER	109	
36530	+	g CFj	36,639	
		f IRR	12.42	After Tax IRR

DISCOUNTED CASH FLOW ANALYSIS

The cash flow on page 107 has been prepared on a 10 unit apartment building.

A. Calculate the Before Tax IRR.

B. Calculate the After Tax IRR.

C. What is the Present Value of the after tax cash flows to achieve a 15% yield.

Before Tax IRR

Step 1: Prepare a Cash Flow Diagram.

n	$
0	(77,200)
1	2,168
2	3,168
3	4,168
4	5,168
5	6,168 + 92,332

Step 2: Calculate IRR

Keystrokes			Display	Remarks
77200	CHS	g CFo	-77,200	
2168		g CFj	2,168	
3168		g CFj	3,168	
4168		g CFj	4,168	
5168		g CFj	5,168	
6168		ENTER	6,168	
92332	+	g CFj	98,500	
		f IRR	8.50	Before Tax IRR

After Tax IRR

Step 3: Prepare Cash Flow Diagram

n	$
0	(77,200)
1	9,846
2	10,313
3	9,776
4	9,233
5	9,685 + 66,598

Step 4: Find IRR

Keystrokes			Display	Remarks
77200	CHS	g CFo	-77,200	
9846		g CFj	9,846	
10313		g CFj	10,313	
9776		g CFj	9,776	
9233		g CFj	9,233	
9685		ENTER	9,685	
66598	+	g CFj	76,283	
		f IRR	10.46	

DO NOT CLEAR CALCULATOR

Step 5: Find the PV to Achieve 15% yield.

Keystrokes		Display	Remarks
15	i	15	Changes yield to 15
0	STO 0	0	Changes Initial Investment to 0
	f NPV	65,992.86	PV to Yield to 15%

Cash Flow Analysis

For Big Swinger #2

Purpose

Date

Purchase Price $240,000

Encumbrances $162,800

Investment $ 77,200

Depreciable Basis = $200,000

Mortgage Data

	Encumbrances	Beginning Balance	Remaining Term	Number of Payments Per Year	Interest Rate	Payment	Annual Debt Service	Remarks
1	1st Mortgage	$162,800	30 yrs	12	13	$1,801	$21,612	
2	2nd Mortgage							
3	3rd Mortgage							

		Year: 1	Year: 2	Year: 3	Year: 4	Year: 5
	Ownership Analysis of Property Income:	**Taxable Income**				
4	Gross Scheduled Income	42 000				
5	Less: Vcy. & Credit Losses	2 940				
6	Gross Operating Income	39 060				
7	– Operating Expenses	15 280				
8	Net Operating Income	23 780	24 780	25 780	26 780	27 780
9	– Non-Operating Expense					
10	– Interest – 1st Mortgage	21 136	21 070	20 996	20 911	20 814
11	– Interest – 2nd Mortgage					
12	– Interest – 3rd Mortgage					
13	– Cost Recovery	18 000	18 000	16 000	14 000	14 000
14	Real Est. Taxable Income	(15 356)	(14 290)	(11 216)	(8 131)	(7 034)
		Cash Flows				
15	Net Operating Income	23 780	24 780	25 780	26 780	27 780
16	– Annual Debt Service	21 612	21 612	21 612	21 612	21 612
17	– Funded Reserves					
18	– Capital Additions					
19	Cash Flow before Taxes	2 168	3 168	4 168	5 168	6 168
20	– Minimum Tax					
21	– Tax Liability on Real Est.	(7 678)	(7 145)	(5 608)	(4 065)	(7 517)
22	Cash Flow after Taxes	9 846	10 313	9 776	9 233	9 685

EOY	Mortgage Balance
	1st Mortgage
1	162324
2	161783
3	161167
4	160466
5	159668
	2nd Mortgage
	3rd Mortgage

Analysis of Sale Proceeds

Year:

	Adjusted Basis		Excess Cost Recovery (CR)		Tax Liability on Sale	
23	Original Basis	240 000	Total CR	80 000	Excess Recapture Tax	12 223
24	+ Capital Improvements		S/L CR	55 555	Capital Gain Tax	13 511
25	– Cost Recovery	80 000	Excess CR*	24 445	Tax Liability on Sale	25 734
26	– Partial Sales		Exc. CR Carryover			
27	AB	160 000	Gain		Sale Proceeds	
28			Sale Price	280 000	Sale Price	280 000
			– Cost of Sale	28 000	– Costs of Sale	28 000
			– AB	160 000	– Mortgage	159 668
			Gain	92 000	Proceeds before Taxes	92 332
			– Excess CR	24 445	– Tax Liability on Sale	25 734
			Capital Gain	67 555	Proceeds after Taxes	66 598

*NOTE: On non-residential property, if accelerated CR system is elected, all CR claimed is recaptured as ordinary income in the year of sale.

The statements and figures presented herein, while not guaranteed, are secured from sources we believe authoritative.

Prepared by

Negative Amortizations: (Less than interest only payments)

Many times it is necessary to structure a loan with payments that do not even cover the interest due each period. There are three common ways of handling the interest which accrues but is not paid.

1. Unpaid interest to accrue but not compound
2. Unpaid interest to compound at the rate in the note
3. Unpaid interest to accrue and compound at a rate different from the rate in the note

Whenever a loan has payments that are less than interest only, the actual payment must be compared to the interest only payment. The difference is the unpaid interest.

Example:

A loan of $20,000 is created with monthly payments of $150 with interest at 12%, the entire balance due in 5 years. What is the remaining balance at EOY 5 if the unpaid interest.

1. Accrues but does not compound
2. Compounds at 12%
3. Compounds at 16%

At 12% interest only, the monthly payment on a $20,000 loan would be $200. Therefore $50 interest is not being paid.

1. Interest to accrue but not compound.

	$50	mo unpaid
x	60	months
	$3,000	Interest Accrued
+	$20,000	Principal Balance
	$23,000	Remaining Balance

2. Interest to compound at 12%.

When the npaid interest is to compound at the rate in the note the original loan balance and the actual payment can be used in the calculation. This calculation can also be performed as if the unpaid interest was compounding separately (see #3).

n	60	(5 years)
i	1	(12% per annum)
PV	20,000	
PMT	-150	
FV	?	

Keystrokes		Display	Remarks
60	n	60	
1	i	1	
20000	PV	20000	
150	CHS PMT	-150	
	FV	-24,083.48	Balance at EOY 5

3. Interest to compound @ 16%.

When the unpaid interest compounds at a rate different from the rate in the note you must compound the unpaid interest separately and then add the computed value to the original loan amount.

n	60	
i	26 g i	
PV	0	(Only unpaid interest is compounding at 16%)
PMT	-50	(Unpaid interest)
FV	?	

Keystrokes		Display	Remarks
60	n	60	
16	g i	1.33	
50	CHS PMT	-50	
	FV	4551.78	FV of unpaid interest
20000			Original loan AMT.
	+	24,551.78	Balance at EOY 5

NO PAYMENT NOTES

A seller carries back a $55,000 note with no payments and all principal and 10% interest due in 7 years. What will be the balloon payment if the interest....

1. Does not compound (simple interest)

2. Compounds annually

3. Compounds monthly

Simple Interest

Keystrokes		Display	Remarks
550,000	ENTER	55,000	
10	%	5,500	Annual Interest
7	x	38,500	Total Interest
	+	93,500	Total Balloon

Interest Compounding Annually

Keystrokes		Display	Remarks
55,000	PV	55,000	
10	i	10	
7	n	7	
	FV	107,179.44	Total balloon

Interest Compounding Monthly

Keystrokes		Display	Remarks
55,000	PV	55,000	
10	g i	.83	
7	g n	84	
	FV	-110,435.61	Total Balloon

BUILDER OR SELLER BUY DOWNS

When interest rates are high it is common for builders or sellers to "Buy Down" the interest rate on the new loan the purchaser will qualify for. In effect, the builder subsidizes the monthly payments of the buyer for a certainamount of time. The builder usually does this by paying an extra amount of cash into the transaction which allows the lender to place the loan at a very large discount. The buyer benefits from a lower interest rate and payment for a few years, the builder benefits by selling his product and the lender benefitsby achieving his desired yield.

Example:

A purchaser places a $100,000 loan with a lender for 10 years. To sell the property the seller agrees to "buy down" the interest rate from 16% to 12% for the first 4 years ofthe loan. After the fourth year the buyer's monthly payment will increase and be based on 16%. Howmuch will it cost the builder to "buy down" this loan to yield the lender 16%?

Step 1: Prepare Cash Flow for Lender

n	$
0	(Lenders Investment)
1 - 48	PMTS @ 12%
49 - 120	PMTS @ 16%

Step 2: Find monthly payment at 12% and balance at EOY 4

Keystrokes		Display	Remarks
100000	CHS PV	-100,000	
10	g n	120	
12	g i	1	
	PMT	1434.71	Monthly payment @ 12%
4	g n	48	
	FV	73385.95	Balance EOY 4

Step 3: Find discounted value of 1st 48 months payments & balloon to yield 16%

n	$	
0	PV = ?	
1	1434.71	i = 16%
48	1434.71 + 73,385.95	

Keystrokes		Display	Remarks
16	g i	1.33	Desired Yield
	PV	-89484.24	Discounted value to yield 16%

Step 4: Find Buy Down

Keystrokes		Display	Remarks
		-89484.24	In display
100000	+	10515.76	Buy Down

NOTE: In this example, another approach to "Buy Downs" is for the seller to place the difference between the total of the monthly payments for 4 years at 12% and 16% in escrow at loan origination. The 12% monthly payment would then be subsidized for the 4 year period.

Keystrokes		Display	Remarks
100000	CHS PV	-1000000	
10	g n	120	
16	g i	1.33	
	PMT	1675.13	Monthly Pmt @ 16%
1434.71		1434.71	Monthly Pmt @ 12% (Previously calculated)
-		240.42	
48	X	11540.22	48 month buy down to be placed in escrow

OFFER-COUNTER OFFER

You have listed a home for $425,000. The house has an assumable 1st loan of $210,000 payable at $2,689 PITI including interest at 12.5%. An offer is presented with a price of $375,000 with $85,000 down with the buyer's assuming the existing 1st loan and giving the sellers a 2nd note and trust deed for the balance with monthly payments at 9% interest for 30 years with a call date in 15 years. Costs of sale include the following:

7% Brokerage Fee
1% Assumption Fee
$1,000 Closing Costs

1. What will be the net cash to the seller?
2. What will the monthly payment and balloon payment be on the 2nd note?
3. What will be the buyer's total monthly payment?

After discussing the offer and its terms the seller decides to make a counter offer as follows:

$410,000 Price
$110,000 Down
$210,000 Assume 1st

The balance is to be paid on a 2nd note in monthly payments at 11% interest for 30 years all due in 10 years. Based on the counter offer:

4. What will be the net cash to seller?
5. What will be the monthly payment and balloon on the 2nd note?
6. What will be the buyer's total monthly payment?

OFFER

Step 1: Find Net to Seller

Keystrokes		Display	Remarks
375,000	ENTER	375,000	Sale Price
7	%	26,250	Brokerage Fee
STO	o	26,250	
210,000	ENTER	210,000	1st Loan
1	%	2,100	Assumption Fee
STO	+ 0	2,100	
1,000		1,000	Closing Costs
STO	+ 0	1,000	
85,000	ENTER	85,000	Down Payment
RCL	0	29,350	Total Costs
	-	55,650	Net to Seller

Step 2: Find 2nd loan payment and balloon

Keystrokes		Display	Remarks
375,000	ENTER	375,000	Price
210,000	-	165,000	
85,000	-	80,000	2nd Loan Amount
	PV	80,000	
9	g i	.75	
30	g n	360	
	PMT	-643.70	Monthly Payment
15	g n	180	
	FV	-63,464.39	Balance EOY 15

Do not clear calculator.

Step 3: Find total monthly payment for buyers

Keystrokes		Display	Remarks
RCL	PMT CHS	643.70	
2,689		2,689.00	PITI on 1st
+		3,332.70	Total Payment

COUNTER-OFFER

Step 4: Find Net to Seller

Keystrokes		Display	Remarks
401,000	ENTER	410,000	
7	%	28,700	Brokerage Fee
STO	0	28,700	
2,100	STO + 0	2,100	Assumption Fee
1,000	STO + 0	1,000	Closing Costs
110,000	ENTER	110,000	Down Payment
RCL	0	31,800	Total Costs
	-	78,200	Net to Seller

Step 5: Find Payment and Balloon on 2nd

Keystrokes		Display	Remarks
410,000	ENTER	410,000	
110,000	-	300,000	
210,000	-	90,000	2nd Note
	PV	90,000	
11	g i	.92	
30	g n	360	
	PMT	-857.09	Payment on 2nd
10	g n	120	
	FV	-83,036.30	Balance EOY 10

Step 6: Find total payment for buyer

Keystrokes		Display	Remarks
RCL	PMT CHS	857.09	
2,689		2,689.00	PITI on 1st
	+	3,546.09	Total Payment

CASH EQUIVALENCY OF TERMS OFFER

A home was listed for $600,000 and has been on the market for 9 months. An all cash offer is made for $450,000 which the seller feels is well below what the home is worth. Comparable sales in the area indicate a price of $600,000 is reasonable with terms of 20% down and the seller taking back a note and trust deed for the balance at 10% interest only monthly payments and a balloon in 10 years. What would be the terms equivelant of the $450,000 cash offer?

The best approach to take to answer this question is to find the present value of the note the seller would carry discounted to achieve a market yield if sold. You can then add the PV of the note plus the down payment to determine the cash equivelancy of the terms offer. If the offers have a similar cash equivelancy the seller could accept the cash offer and then go out and purchase a note or notes which would yield the same cash flow as the carry-back. Assume the note can be sold to yield between 16% and 18%.

Step 1: Find the payment and balloon of the interest only note.

Keystrokes		Display	Remarks
600,000	ENTER	600,000	Sale Price
20	%	120,000	Down payment
	-	480,000	Carry back
10	%	48,000	Annual Interest
12	÷	4,000	Monthly Payment

Balloon in 10 years is $480,000 because payments are interest only.

Step 2: Find the PV of the note to yield 16% & 18% and add down payment

Keystrokes		Display	Remarks
4,000	PMT	4,000	Payment
480,000	FV	480,000	Balloon
10	g n	120	Term
16	g i	1.33	16% Yield
	PV	-336,727.64	PV to yield 16%
CHS		336,727.64	
120		120,000	Down Payment
	+	456,727.64	Cash equivalency if sold for 16%

Do not clear calculator.

Keystrokes		Display	Remarks
18	g i	1.5	18% yield
	PV	-302,404.95	PV to yield 18%
CHS		302,404.95	
120,000		120,000	Down payment
	+	422,404.95	Cash equivelency if sold for 18%

If the seller accepted a terms offer and sold the note to cash out she would net between $422,404 and $456,727. Obviously the $450,000 cash offer is equivelent to other terms offers in the area.

CASH SALE VS. CARRY BACK

Example

Two offers are made to you on a property you own.

Offer #1	$1,500,000	Cash
Offer #2	$1,800,000	Price
	600,000	Down
	1,200,000	Note payable at 11% with monthly payments for 30 years all due in 7 years

Which offer is better?

The answer may be different depending on the seller's needs.

1. If the seller needs to cash out he would have to sell the note at a discount. The cash to the seller would depend upon the yield investor's would demand. The higher the desired yield the less cash to seller.

   What would be the total cash to seller if the note in Offer #2 was purchased to yield 15%, 17%, 18%?

Step 1: Find the pyament and balloon in 7 years.

Keystrokes		Display	Remarks
1,200,000	PV	1,200,000	
11	g i	.92	
30	g n	360	
	PMT	-11,427.88	Payment
7	g n	84	
	FV	-1,146,217.56	Balloon EOY 7

Step 2: Key in desired yield in i and solve for PV

Keystrokes		Display	Remarks
15	g i	1.25	15% Yield
	PV	995,941.58	PV of note to yield 15%
600,000		600,000	Down payment
	+	1,595,941.58	Total cash to seller if sold for 15%
17	g i	1.42	17% Yield
	PV	910,836.50	PV of note to yield 17%
600,000		600,000	Down payment
	+	1,510,836.50	Total cash to seller if sold for 17%

Keystrokes		Display	Remarks
18	g i	1.50	18% Yield
	PV	871,908.53	
600,000		600,000.00	
	+	1,471,908.53	Total cash to seller if sold for 18%

2. If the seller only needs $600,000 cash from the sale and will invest all remaining cash or income in an account at 12% per annum compounded monthly, which offer will give the seller the greatest wealth in 7 years?

Cash offer: $1,500,000 minus $600,000 cash leaves $900,000 for investing. Find FV of $900,000 after 7 years at 12%.

Keystrokes		Display	Remarks
900,000	PV	900,000	
12	g i	1	
7	g n	84	
	FV	2,076,050.47	Total wealth EOY7

Terms offer: None of the $600,000 down payment is available for investment. The monthly payment of $11,427.88 recieved on the note is available for investment and at EOY 7 the note balloons at $1,146,217.56. Find the FV of the payments over 7 years at 12% and add the balloon to determine total wealth.

Keystrokes		Display	Remarks
11,427.88	PMT	11,427.88	
12	g i	1	
7	g n	84	
	FV	-1,493,307.07	FV of Pmts
	CHS	1.493,307.07	
1,146,217.56		1,146,217.56	Balloon EOY 7
	+	2,639,524.63	Total wealth EOY 7

CREATING TWO NOTES FROM ONE TO INCREASE CASH TO SELLER

An offer of $300,000 is made to your client. The terms are as follows.

$300,000	Sale Price
50,000	Down payment
100,000	Buyers to assume existing 1st
150,000	2nd Note to seller at 11% with monthly payments of $2,000 per month all due in 8 years

Problem:

The terms are acceptable to the seller except the seller needs another $50,000 cash from the sale.

Solution:

Create two notes and have the seller sell the 2nd for cash and keep the 3rd. Structure the terms of the 2nd to net the seller $50,000 cash when sold. When creating two notes from one, the buyer will usually insist that the monthly payment, interest rate and balloon which was offered remain the same. Therefore.....

Step 1: Analyze terms of note offered

Keystrokes		Display	Remarks
150,000	PV	150,000	Balance
2,000	CHS PMT	-2,000	Payment
11	g i	.92	Interest rate
8	g n	96	Term to call date
	FV	-54,459.95	Balance EOY 8

We know the buyer is willing to pay $2,000 per month with a balloon of $54,459.95 in 8 years. With this in mind lets structure the terms of the two notes. The 2nd will probably have "harder" terms to make it saleable and the 3rd will have "softer" terms.

Step 2: Let's assume that 2nd position notes can be sold to yield 18% and try the following terms:

$65,000 2nd at 11% fully amortized for 8 years.

Keystrokes		Display	Remarks
65,000	PV	65,000	
11	g i	.92	
8	g n	96	
	PMT	-1.021.05	Payment on 2nd
18	g i	1.5	18% yield
	PV	51,768.83	PV to yield 18%

With this 2nd the seller could net $51,768 more cash. But what would the terms of the 3rd note be?

Step 3: Structure terms of 3rd

Assuming the buyer's total monthly payment on the carryback cannot exceed $2,000 per month. We have $978.95 ($2,000.00 - $1,021.05) remaining for the payment on the 3rd note of $85,000. Based on our previous calculation in Step 1, we also know that the buyer is willing to have a $54,459.95 balloon in 8 years. Because the $65,000 2nd is fully amortized we can include this balloon in the 3rd.

Step 3: Structure terms of 3rd by

	$150,000	Offered 2nd
-	65,000	Restructured 2nd to be sold
	$ 85,000	Restructured 3rd
	$ 2,000.00	Offered payment
-	1,021.05	Restructured payment on 2nd to be sold
	$ 978.95	Restructured payment on 3rd
	$ 54,459.95	Balloon on ffered 2nd EOY 8
	-0-	Balloon on restructured 2nd to be sold
	$ 54,459.95	Balloon on restructured 3rd EOY 8

Step 4: Find interest rate on 3rd

Keystrokes		Display	Remarks
85,000	CHS PV	-85,000	
978.95	PMT	078.95	
54,459.95	FV	54,459.95	
8	g n	96	
	i	.92	Monthly rate
12	x	11.00	Annual Rate

Since we kept the same total payment, balloon and term as offered the interest rate must be the same.

Let's recap:

The offer gave the seller:

$50,000	Cash from down payment
$150,000	2nd note at 11%, $2,000 monthly with $54,459.95 balloon in 8 years.

The restructuring gives buyer the exact same terms and the seller

$50,000	Cash from down payment
+ 51,768	Cash from sale of $65,000 2nd
$85,000	3rd note at 11%, $978.95 monthly with $54,459.95 balloon in 8 years.

DETERMINING YIELDS ON WRAPAROUND LOANS

Wraparound (All inclusive) loans are a method of secondary financing Which will normally allow the seller to obtain a higher yield as compared to carrying a simple second loan. The following comparison may help to illustrate the difference between a simple second loan and wraparound financing.

Situation:		
	$100,000	Sales Price
	60,000	Existing 1st, 9% interest, $500 P.I. per mo.
	$ 40,000	Equity

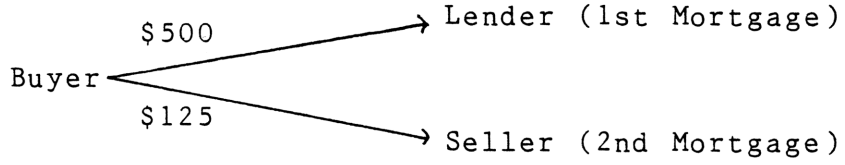
Buyer has $25,000 cash down payment.

Possible Solutins:

A. Seller carries 2nd mortgage

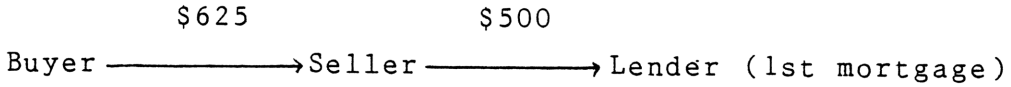
$25,000 down, buyer assumes existing 1st loan and given seller note and 2nd mortgage for $15,000. Terms of 2nd are interest only at 10% for 10 years.

Buyer is liable to the lender on the 1st and to the seller on the 2nd.



B. Wraparound Financing

$25,000 down and buyer executes a $75,000 note and mortgage to the seller, interest only at 10% for 10 years. The seller agrees to remain solely liable to the lender on the existing 1st.



In wraparound financing, if the wraparound loan is written at a higher interest rate than the 1st loan, the seller not only earns interest on the equity loaned to the buyer but also on the existing 1st owed to the lender.

To illustrate this the "Wraparound Worksheet" on the following page is very useful.

Example A

You sell a property today for $100,000 with $25,000 down and the buyer gives you a note and trust deed for $75,000 payable at $625 per month, interest only at 10% with a 10 year stop. The seller agrees to remain liable for an existing $60,000 1st loan payable at $500 per month P.I. @ 9%. What will be the seller's annual yield over the term of the wraparound loan.

Step 1: Analyze loan using Wraparound Worksheet

a) The wraparound loan amount ($75,000) and payment ($625) are given. Since the loan is paid interest only, the balance at payoff (EOY 10) will be $75,000.

b) The 1st loan amount ($60,000) and Payment ($500) are also given. The balance on the 1st loan at payoff (EOY 10) must be calculated.

Keystrokes		Display	Remarks
60000	CHS PV	-60,000	
500	PMT	500	
9	g i	.75	
10	g n	120	
	FV	50,324.29	Balance at 1st EOY 10

c) Find wrap equity now

Keystrokes		Display	Remarks
75000	ENTER	75,000	Wrap loan
60000		60,000	1st loan
	-	15,000	Wrap equity now
	STO 0	15,000	

WRAPAROUND WORKSHEET

PROPERTY LOCATION: _____ DATE: _____

SALE PRICE: $100,000

DOWN PAYMENT: 25,000

WRAP LOAN: 75,000 10 % $625.00 PI 10 Yr. Stop Term

1st LOAN: 60,000 9 % $500.00 PI _____ Orig. Bal. _____ Orig. Term _____ Old Now

2nd LOAN: _____ % _____ PI _____ Orig. Bal. _____ Orig. Term _____ Old Now

	Balance Now	Pmt. from mo/yr 1 to 120	Pmt. from mo/yr ____ to ____	Balance at time of wrap payoff EOY _____
Wrap Loan	$75,000.00	$625.00		$75,000.00
-1st Loan	$60,000.00	$500.00		$50,324.29
-2nd Loan				
Sellers Position	$15,000.00 Wrap Equity	$125.00 Net Payment	Net Payment	$24,675.71 Wrap Equity at Payoff

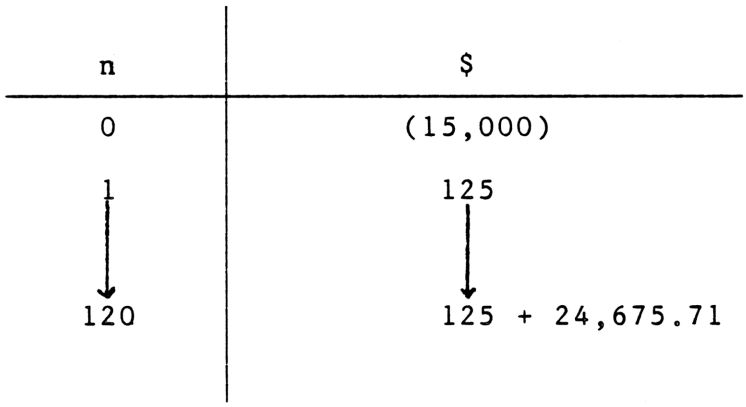
d) Find monthly net to seller

Keystrokes		Display	Remarks
625	ENTER	625	Pmt. on wrap
500		500	Pmt. on 1st
	-	125	Mo. net to seller
	STO 1	125	

e) Find wrap equity at payoff (EOY 10)

Keystrokes		Display	Remarks
75000	ENTER	75,000	Wrap bal. EOY 10
50324.29		50,324.29	1st bal. EOY 10
	-	24,675.71	Wrap equity at payoff
	STO 2	24,675.71	

Step 2: Prepare cash flow diagram



Step 3: Find yield to seller

Keystrokes		Display	Remarks
RCL 0	CHS PV	-15,000	Wrap equity now
RCL 1	PMT	125	Mo. net
RCL 2	FV	24,675.71	Wrap Equity
10	g n	120	No. of payments
	i	1.10	Monthly yield
12	X	13.14	Annual yield

Example B

An existing loan for $22,000 @ 8% with monthly payments of $446.08 will pay out in 5 years. The owner of the property sells the property on an all inclusive trust deed of $42,000 @ 15%, 30 year amortization all due in 7 years. What is the seller's annual yield over the 7 year period? If the seller sold the loan discounted to yield 20%, how much cash would she receive?

Step 1: Analyze each loan separately using the wraparound worksheet.

a) Find PMT & balloon on all inclusive loan

Keystrokes		Display	Remarks
30	g n	360	
15	g i	1.25	
42000	PV	42,000	
	PMT	-531.07	PMT on wrap
7	g n	84	
	FV	-41,107.44	Balloon on Wrap EOY 7

b) The existing 1st loan has monthly payments of $446.08 for 5 years and no balloon payment.

Step 2: Find the seller's position in the wraparound

WRAPAROUND WORKSHEET

PROPERTY LOCATION: EXAMPLE B DATE: _____

SALE PRICE: _____

DOWN PAYMENT: _____

WRAP LOAN: $42,000 15% $531.07PI 7 years Term

1st LOAN: 22,000 8% 446.08PI _____Orig. Bal. _____Orig. Term _____ Old Now

2nd LOAN: _____ ____% _____PI _____Orig. Bal. _____Orig. Term _____ Old Now

	Balance Now	Pmt. from mo/yr 1 to 60	Pmt. from mo/yr 61 to 84	Balance at time of wrap payoff
Wrap Loan	$42,000	$531.07	$531.07	$41,107.44
-1st Loan	22,000	446.08	-0-	-0-
-2nd Loan	-0-	-0-	-0-	-0-
Sellers Position in	20,000 Wrap Equity	84.99 Net Payment	531.07 Net Payment	41,107.44 Net at Payoff

Step 3: Prepare cash flow diagram and calculate yield (IRR)

Months	Nj	$
	0	(20,000)
1 - 60	60	84.99
61 - 83	23	531.07
84	1	531.07 + 41,107.44

Keystrokes		Display	Remarks
20000	CHS g CFo	-20,000	
84.99	g CFj	84.99	
60	g Nj	60	
531.07	g CFj	531.07	
23	g Nj	23	
41638.51	g CFj	41,638.51	
	f IRR	1.46	
12	X	17.54	Annual yield to seller

DO NOT CLEAR CALCULATOR

Step 4: Find Net Present Value to yield 20%

Keystrokes	Display	Remarks
0 STO 0	0	Clears $20,000 in STO 0
20 g i	1.67	
f NPV	17,332.91	Present Value to yield 20%

Example C

A property has an existing $80,000 loan against it with monthly payments of $994.82 including 14% interest. You are asked to wrap the $80,000 loan with a loan which will net the borrower $50,000. At what rate of interest must the wraparound loan be written in order to yield you 19% on your investment?

Yields on wraparound loans are based on the lender's net position (wrap loan minus all underlying loans). To structure the wraparound loan for the lender to receive a 19% yield we must work from the net position and "back into" the terms of the wraparound loan.

Step 1: Use wraparound worksheet to analyze known information.

	Balance Now	Pmt. from mo/yr ____ to ____	Pmt. from mo/yr ____ to ____	Balance at time of wrap payoff
Wrap Loan	$130,000			
-1st Loan	80,000	$994.82 @ 14%		
-2nd Loan				
Sellers Position in	$50,000 Wrap Equity	Net Payment	Net Payment	Net at Payoff

Step 2: With this information find the term of the 1st loan

Keystrokes		Display	Remarks
14	g i	1.17	
80000	PV	80,000	
994.82	CHS PMT	-994.82	
	n	240	1st loan has 20 years remaining

Step 3: Assuming there is no balloon payment on the wrap-around loan we can now calculate the net payment to yield 19% to lender on $50,000 over 240 months.

Keystrokes		Display	Remarks
240	n	240	
19	g i	1.58	
50000	PV	50,000	
	PMT	-810.34	Net PMT to yield 19%

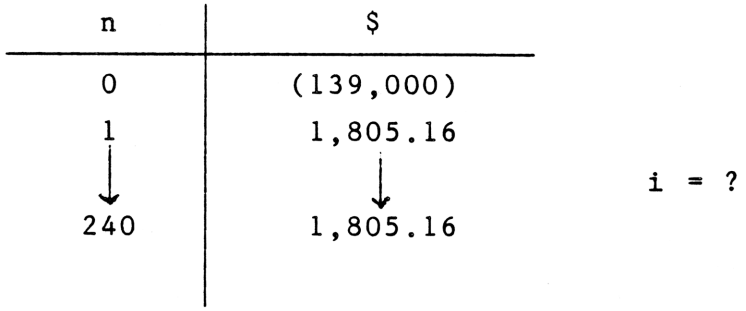
The wraparound worksheet now appears as follows:

	Balance Now	Pmt. from mo/yr 1 to 240	Pmt. from mo/yr ____ to ____	Balance at time of wrap payoff
Wrap Loan	$130,000			
-1st Loan	80,000	$994.82		
-2nd Loan	-0-	-0-	-0-	-0-
Sellers Position in	$50,000 Wrap Equity	$810.34 Net Payment	Net Payment	Net at Payoff

Step 4: Add the payment on the 1st loan to the net payment to find the required payment on the wraparound loan.

$ 994.82	Payment on 1st
+ 810.34	Net Payment
$1,805.16	Required wraparound payment

Step 5: Prepare cash flow diagram for wraparound loan and solve for interest rate.



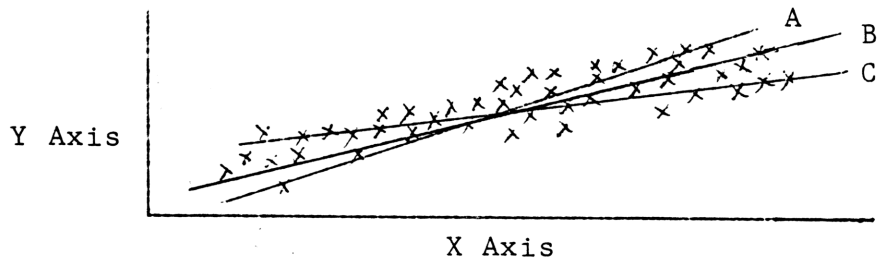
Keystrokes		Display	Remarks
240	n	240	
130000	CHS PV	-130000	
1805.16	PMT	1805.16	
	i	1.33	Monthly interest rate
12	X	15.96	Annual rate

TREND ANALYSIS & LINEAR REGRESSION

When performing market analyses, it is often necessary to be able to make projections or estimates based on known information. If the data can be plotted on a graph and a straight line can be fitted, either exactly or closely to the data points, that graph can be used to make the necessary estimates.

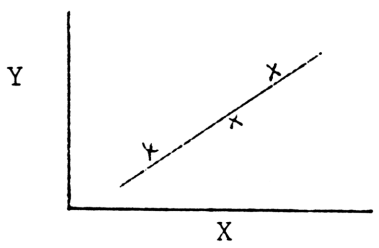
Data may show a linear (straight) trend and yet may be difficult to analyze. For example, in the diagram below, lines A, B & C each represent an attempt to visually fit a straight line to a set of data. Obviously, they are somewhat subjective in nature and prone to error.

Linear Regression is a statistical technique for defining the trend or projection line which provides the best mathematical fit to a set of data points.

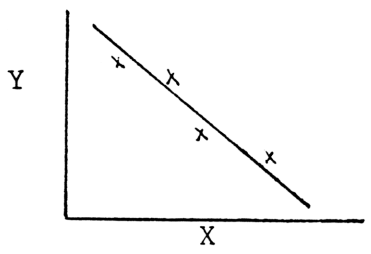


The Hp12C calculator is programmed to perform regression analyses. In essence, the calculator plots the data on a graph and then fits the best straight line to the data. The calculator then retains the line allowing you to make estimates or projections.

Once the known data has been entered and the calculator has plotted its straight line to the data, it is important to determine how well the line "fits" the data. The correlation coefficient (abbreviated "r" in statistics) will tell you how close to a straight line the data points lie. The correlation coefficient, r, is always a value between -1 and +1. If r = +1, then the line has a positive (upward) slope and the data fits perfectly. If r = -1, the data is still a perfect fit but the line has a negative (downward) slope.



Positive Slope



Negative Slope

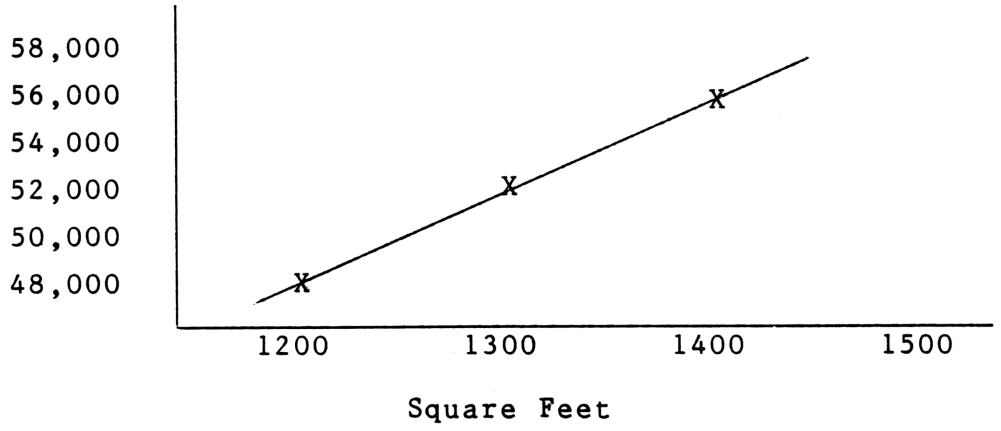
To enter the data and determine the correlation coefficient, the following keys are important.

ENTER	Enters the Y variable into Y register
E+	Summation Key: matches an entered Y variable with the X variable and plots it on the calculators graph.
$\hat{x}$,r	Predicts X along the plotted line
$\hat{y}$,r	Predicts Y along the plotted line
x⇄y	Displays correlation coefficient, r, when pressed after either $\hat{x}$,r or $\hat{y}$,r
Note:	When entering data, the Y variable must be entered first.

Example:

You wish to estimate the sale price for a 1,350 sq. foot home. You have found three properties comparable to the subject in every aspect except size.

Comparable	Size	Sale Price
A	1,200 sq. ft.	$47,000
B	1,300 sq. ft.	$52,000
C	1,400 sq. ft.	$55,000



Step 1: Enter data and have calculator pair the entries and plot the line. (Remember: Y variable first.)

Keystrokes		Display	Remarks
47000	ENTER	47,000	1st Y variable
1200	E+	1	X variable entered & paired (plotted) #1 in display indicated that this is 1st paired entry
52000	ENTER	52,000	2nd Y variable
1300	E+	2	2nd paired entry
55000	ENTER	55,000	3rd Y variable
1400	E+	3	3rd paired entry
g	$\hat{x}$,r x≷y	.99	Correlation coefficient

DO NOT CLEAR CALCULATOR

The calculator has plotted the best possible straight line and the .99 correlation coefficient, r, indicates a positive slope and an almost perfect fit. Knowing this, our estimate of the sale price for the 1,350 sq. foot home will be very accurate.

Step 2: Estimate the sale price of 1,350 sq. ft. home

Keystrokes		Display	Remarks
1350			X variable
g	$\hat{y}$,r	53,333.33	Estimated sale price

LINEAR REGRESSION PROBLEMS

1. The following data on warehouse construction was obtained for your area:

Warehouse	Size (sq. ft.)	Cost ($)
A	8,000	120,000
B	12,000	193,000
C	16,000	231,000
D	20,000	267,000
E	17,000	242,000

Using linear regression analysis:

a) determine the correlation coefficient

b) determine the cost of an 18,000 sq. ft. and a 10,000 sq. ft. warehouse

Step 1: Enter data (entering Y variable first)

Keystrokes		Display	Remarks
120000	ENTER	120,000	
8000	E+	1	
193000	ENTER	192,000	
12000	E+	2	
231000	ENTER	231,000	
16000	E+	3	
267000	ENTER	267,000	
20000	E+	4	
242000	ENTER	242,000	
17000	E+	5	

Step 2: Find correlation coefficient

Keystrokes		Display	Remarks
g	$\hat{x}$,r x≷y	.98	Good fit

Step 3: Find cost of properties

Keystrokes		Display	Remarks
18000			x variable
g	$\hat{y}$,r	251,626.15	Cost of 18,000 sq. ft. warehouse
10000			X variable
g	$\hat{y}$,r	155,094.04	Cost of 10,000 sq. ft. warehouse

2. You are given the following data on land prices.

Parcel	Size (acres)	Price ($)
A	5	50,000
B	8.4	78,000
C	9.6	70,000
D	22.4	120,000
E	50	300,000
F	32	240,000

Using linear regression, estimate the price of a 20, 30 and 40 acre parcel. How reliable are your estimates?

Step 1: Enter data & determine r.

Keystrokes		Display	Remarks
50000	ENTER	50,000	1st Y variable
5	E+	1.00	1st paired entry
78000	ENTER	78,000	
8.4	E+	2.00	
70000	ENTER	70,000	
9.6	E+	3.00	
120000	ENTER	120,000	
22.4	E+	4.00	
300000	ENTER	300,000	
50	E+	5.00	
240000	ENTER	240,000	
32	E+	6.00	
g	$\hat{x}$,r x≷y	.98	r (Good fit)

Step 2: Find price for 20, 30, and 40 acres.

Keystrokes		Display	Remarks
20			X variable
g	$\hat{y}$,r	135,864.21	Price of 20 acres
30			X variable
g	$\hat{y}$,r	193,722.00	Price of 30 acres
40			X variable
g	$\hat{y}$,r	251,579.80	Price of 40 acres

3. Through a market analysis you determine the following information on apartment units comparable to units you plan to purchase. Using regression analysis, estimate the rent for a 600, 750, and 850 sq. foot apartment.

Size (sq. ft.)	Rent ($)
650	200
725	215
800	225
900	245
875	240
775	222

Sept 1: Enter data and determine r.

Keystrokes		Display	Remarks
200	ENTER	200	1st Y variable
650	E+	1.00	1st paried entry
215	ENTER	215	
725	E+	2.00	
225	ENTER	225	
800	E+	3.00	
245	ENTER	245	
900	E+	4.00	
240	ENTER	240	
875	E+	5.00	
222	ENTER	222	
775	E+	6.00	
g	$\hat{x}$,r x≷y	1.00	r (good fit)

Step 2: Estimate rents for 600, 750 and 850 sq. ft.

Keystrokes		Display	Remarks
600			X variable
g	$\hat{y}$,r	191.42	Rent for 600 sq. ft.
750			X variable
g	$\hat{y}$,r	217.88	Rent for 750 sq. ft.
850			X variable
g	$\hat{y}$,r	235.53	Rent for 850 sq. ft.

COMPUTATION OF "n"

The 12C returns only integer values.

The 38C returns integer or non-integer values.

To make your 12C return the same answers when computing "n" as a 38C, use the following programs.

1. With any payment amount, including PMT = 0.

A. 38C Payment mode = BEG

```
f   P/R
RCL  FV
CHS
RCL PMT
+
RCL i
100
÷
X
RCL PMT
+
RCL i
100
÷
RCL PV
RCL PMT
+
X
RCL PMT
+
÷
```

```
g   LN
RCL  i
100
÷
1
+
g   LN
÷
f   P/R
```

B. 38C Payment mode = END

```
f   P/R
RCL  FV
RCL  i
100
÷
X
CHS
RCL PMT
+
RCL i
100
÷
RCL PV
X
RCL PMT
+
÷
g   LN
RCL i
100
÷
```

```
1
+
g LN
÷
f P/R
```

2. Enter Financial Data and Solve for n

3. Press R/S and calculator will solve for non-interger value of n.