The HP 10BII Pocket Guide

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The HP 10BII Pocket Guide

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Contents

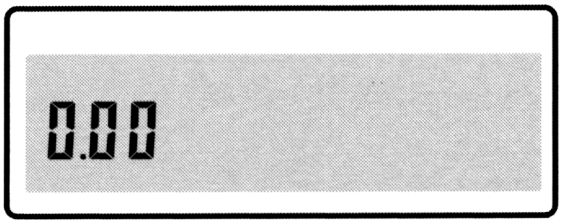
(For a much more detailed cross-reference,
see the **Index** on pages 63-64.)

Getting Started

The ON Button To turn the calculator on, press ON at the lower left of the keyboard. To turn it off, press shift OFF. In other words, press the *shift* key (shift), then ON, which has OFF written below it.

The Machine Settings To set the machine for most common financial calculations, do these keystrokes first. (You'll seldom need to do this in the future—it's mainly so you can follow along in this book.)

First, press 1 2 shift P/YR. That is, press the *shift* key (shift), then the PMT key, which has P/YR written below it. Next, press shift DISP 2. Then, if BEGIN appears in the display, press shift BEG/END to remove it. Finally, press shift C ALL.



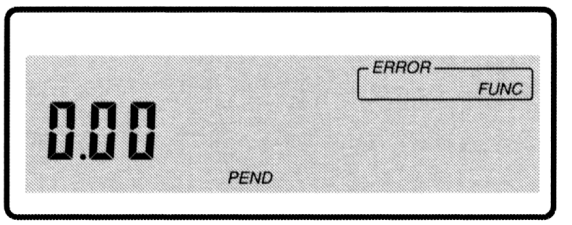
The Display

The Viewing Angle If you can't see and read the display comfortably, it may be at the wrong viewing angle. You can adjust that angle: With the machine on, press *and hold down* the ON key. Then press the + or the − key until you find a comfortable angle.

The Annunciators The areas above and below the number line in the display have symbols to indicate certain states. For example, whenever you press [f], the SHIFT annunciator will appear, indicating that the next key you press will perform its gold (shifted) function rather than its white (un-shifted) function. ([f] is a toggle key: Press it once to turn it on; press it again to turn it off.)

Similarly, whenever you press [g], the STATS annunciator appears, indicating that if the next key you press has a purple (statistical) function shown above it, that's what you'll get. ([g] is also a toggle key.)

But annunciators appear above the number line, too. Try this: Press [2] [÷] [0] [=]. Of course, you can't divide any number by zero, and the HP 10BII tells you that this is an error, via its message area in the upper right corner:



To recover from such errors, press [←] (or [C]). Then you'll still see a PEND annunciator. The PEND means a calculation is *pending*; you're part-way through it. (It appeared here when you pressed [÷].) But since you're not concerned with completing this division operation with a valid divisor, just press [←] (or [C]) again to cancel the calculation altogether.

Clearing the Display What ← clears depends on what you were doing just prior to pressing it. If you were keying in a number or operation on the number line, ← is a "backspace," clearing one digit. But if a complete result is already on the number line, ← clears the whole line. (And, as you've seen on page 5, if you get an error message, ← clears the message without cancelling the whole calculation.)

There are other clearing keys, too: C (like ←) will clear an error message, if there is one. But otherwise it clears the entire number line. And ▬ C ALL clears the number line and all the memory registers.

Setting Decimal Places To set a certain number of displayed decimal places, press ▬ DISP, then the desired number of decimal places. For example, press 1 1 ÷ 8 =. If your machine is set for 2 decimal places, this will display as 1.38. But internally, the actual value is there. To see it (1.375), just ask for them: ▬ DISP 3; or ▬ DISP 9; or to see it to 0 decimal places, press ▬ DISP 0; etc. (To see every digit except trailing zeros, use ▬ DISP •.) Internally, the HP 10BII always keeps a 12-digit value. To actually round that internal value to match the display's setting, you have to use ▬ RND.

Choosing the decimal point You can select either a period or a comma (i.e. European format) for the *radix*—the "decimal point"—in displayed numbers: Press ▬ .,. (This is a toggle key: Press it again to change to the other setting.)

Arithmetic and Math

Here are good examples to practice with. Remember that [←] or [C] will back out or clear any mistakes.

Example: Calculate 342 – 173 + 13
Solution: [3][4][2] [–] [1][7][3] [+] [1][3] [=]
Answer: 182.00

Example: Calculate $\frac{101.00 \times (47.50 - 2)}{64 + (25 \times 3/4)}$

Solution: [1][0][1] [×] [(] [4][7][•][5] [–] [2] [)]
[÷] [(] [6][4] [+] [(] [2][5] [×] [(]
[3] [÷] [4] [=]
Answer: 55.53

(Notice that you don't need to close the parentheses at the end of the problem.)

Changing the Sign of a Number

Example: Find 34 x (–19)
Solution: [3][4] [×] [1][9] [+/–] [=]
Answer: -646.00

Example: Change the -646.00 into 646.00.
Solution: Press the [+/–] key.
Answer: 646.00

[+/–] is the "change sign" key. It changes the sign of the number you're working on (and notice that it's a toggle key—with alternating effects).

Scientific Notation

Example: What's 2,000,000 x 2,000,000?
Solution: 2 0 0 0 0 0 0 × 2 0 0 0 0 0 0 =
Answer: 4.00E 12

You read this as "four-point-zero-zero times 10 to the 12th power." The E means "...times 10 to the..."

Example: Find "two million times two million" without using the 0 key.
Solution: 2 (-) E 6 × 2 (-) E 6 =
Answer: 4.00E 12

Logarithms and Exponentials

Example: What's LN(10)?
Solution: 1 0 (-) LN
Answer: 2.30

Example: Find e^{rt} if r = .06 and t = 0.5.
Solution: • 0 6 × • 5 = (-) e^x
Answer: 1.03

Factorials

Example: Calculate 5!
Solution: 5 (-) n!.
Answer: 120.00

Roots and Powers

Example: Calculate $(-19)^7$
Solution: 1 9 +/− y^x 7 =.
Answer: -893,871,739.00

Example: Calculate $\sqrt{3^2 + 4^2}$
Solution: 3 x^2 + 4 x^2 = $\sqrt{x}$
Answer: 5.00

Example: Calculate $\sqrt[8]{6561}$
Solution: 6561 y^x 8 $1/x$ =
Answer: 3.00

Simple Percentages

Unless it is preceded by a + or −, the % key simply divides a number by 100. Thus, to find a given percentage of any number, you just multiply the number by that percentage. But to increase or decrease a number by a known percentage, you add or subtract that percentage.

Example: What's 40% of $21.95?
Solution: 21.95 × 40 %
Answer: 8.78

Example: What's 25% more than 134?
Solution: 134 + 25 %
Answer: 167.50

Example: Last year, out of your gross income of $25,000, you paid $5,602.50 in taxes. What was your effective tax bracket?

Solution: First, find the fraction: 5602.5 ÷ 25000 =. (Result: 0.22)
Now convert that to a percentage: × 100 =.

Answer: 22.41

Example: What total tax bill in the above problem would have resulted in an 18% effective tax bracket?

Solution: 25000 × 18% =.

Answer: 4,500.00

Example: Last year, the retail trade per square foot in a shop was $64. What level of trade this year would be a 20% increase?

Solution: 64 + 20% =.

Answer: 76.80

Percentage Changes

Example: The retail trade per square foot in a shop was \$75 this year. Last year it was \$64. By what percentage did it change?

Solution: 64 INPUT 75 ▬ %CHG

Answer: 17.19 (Positive change is an *increase*.)

Example: The per-foot trade in another shop was \$75 last year and \$64 this year. By what percentage did it change?

Solution: 75 INPUT 64 ▬ %CHG

Answer: -14.6 (Negative change is a *decrease*.)

Example: If the appraised value of a property decreased by 36% this year and 25% last year, what was the total percentage of its change in value over the two years?

Solution: Start with a convenient basis for the original value: 100 INPUT.
Now find the current value: 100 − 25 % − 36 % =. (Result: 48.00)
Now calculate the percent change between the two values: ▬ %CHG

Answer: -52.00

Markups and Margins

Closely related to percentage-change calculations are the pricing calculations: markup and margin. Given the purchase cost of an item and the subsequent sale price, the markup is the percent change from cost to price. The margin is the percent change (without the minus sign) from price to cost.

Example: A broker buys a property for $\$12/ft^2$, then sells it later for $\$15/ft^2$. Find the markup and the margin for the sale.

Solution: 12 CST 15 PRC MU, then MAR.
(*Or* 12 INPUT 15 −%CHG,
then 15 INPUT 12 −%CHG +/−.)

Answer: 25.00 (markup) and 20.00 (margin).

The four keys, CST, PRC, MU and MAR let you play "What-If?" with pricing decisions. By specifying any two of these values (at least one being Cost or Price), you can calculate the other two values.

Example: A book costs $5.50 wholesale, and sells at retail for $9.95. What's the margin? The markup? What price would give a 60% margin? A 75% markup?

Solution: 5 • 5 CST 9 • 9 5 PRC MAR; MU.
Then 60 MAR PRC; 75 MU PRC

Answer: The margin at a $9.95 price: 44.72; the markup: 80.91. For a 60% margin, the price is 13.75; for a 75% markup, 9.63.

Storing and Recalling Numbers

Besides just doing calculations on the number line in the display, you can store results or other useful values in a variety of memory *registers* (one value at a time per register).

The M Register The M register ("M" stands for "Memory") is the most accessible place to store a value temporarily until you need it again.

Example: Suppose you do many calculations with your effective income tax bracket, 0.21. Store this into the M register.

Solution: [•][2][1][→M].

Result: The display will still contain the original value, but now there's also a copy in the M register. To prove it, press [C], then [RM] ("recall memory"): 0.21

Note that you can also add (or subtract) directly into the M register.

Example: Suppose your effective income tax bracket has risen by 2 points. Make this change directly to the M register.

Solution: [•][0][2][M+].

Result: The display will still contain 0.02, but now there's the modified value in the M register. To prove this, press [RM]: 0.23

The Numbered Storage Registers You can also save values in any of ten numbered storage registers (0-9) using [STO] and [RCL] ("store" and "recall").

Example: Calculate 25.3 + 19.8, then store the result in register 1.

Solution: [2][5][•][3][+][1][9][•][8][=][STO][1]

Result: The display will still contain the original, but now there's a copy in register 1. To prove this, press [C], then [RCL][1].

Other Storage Registers [STO] and [RCL] work for other registers, too—such as the registers you use to solve compound interest problems.

Example: Suppose you've keyed in a loan situation and are trying to calculate payment ([PMT]), but the result is far too low to be reasonable. What can you do?

Solution: Use the [RCL] key to inspect other values you've entered: [RCL][N], [RCL][I/YR], [RCL][PV], [RCL][FV], [RCL][P/YR]. (Probably one of these is erroneous, and that's the reason for the strange PMT result.)

Clearing Registers Every memory register always contains a value. To "clear" a register, then, means simply to set that value to zero.

To clear only the M register, press [0] [→M].

To clear any numbered register, just store a 0 into it (e.g. [0] [STO] <*register #*>). Do likewise to clear a financial register (e.g. [0] [STO] [FV] or [0] [FV]).

To clear all registers at once, press [C ALL].

Storing an Operation Just as the M register lets you store a certain *value* for repeated use, the [K] key lets you store a specific *operation* (that is, operator and value together) for repeated use.

Example: Grading a test with 34 questions, you need to divide each raw score by 34 to find the fraction correct. Use [K] to help.

Solution: Do the first calculation normally (supposing that raw score is, say, 28), but note the operation "in passing" with [K]:

[2] [8] [÷] [3] [4] [K] [=]

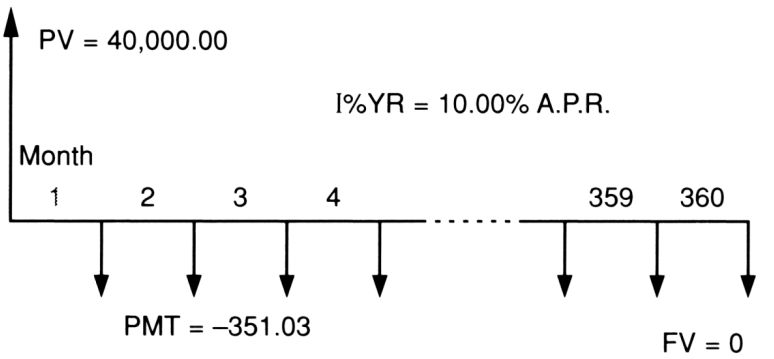
Now, for all subsequent raw scores, you just key in a score and press [=].

[K] stores percent operations, too: [1] [−] [2] [5] [%] [K] [=] then lets you deduct 25% from any value, using only [=]. And [K] works even with powers and roots: Try [1] [y^x] [2] [K] [=] or [1] [y^x] [3] [1/x] [K] [=].

Cash Flow Diagrams

Every loan, lease or investment is described by a certain periodic cash flow scenario. At specified regular intervals, you either receive or pay money.

You can represent this conveniently on a cash flow diagram. For example, here's a diagram of a typical mortgage:



This particular diagram is drawn from the perspective of the borrower. The initial cash flow is positive (with an upward arrow), to represent the borrowed money received to purchase the house. The subsequent payments are negative (downward arrows), to represent money paid back in monthly installments. (Remember that [+/–] is the key to use to change numbers from positive to negative—or vice versa—when you are describing a cash flow diagram to your calculator.)

Besides the sign convention, follow these three rules when drawing a cash flow diagram (because these are the rules and assumptions your calculator uses):

1. The periods must be regular. A month is a common period, but the period can be quarterly, annual, or any other defined length of time.

2. A cash flow can be any amount, including zero. If more than one cash flow occurs at any point in time, then these simultaneous flows may be netted together; only one net cash flow occurs each period. In addition, one initial cash flow occurs at the beginning of the timeline, and one final cash flow occurs at the end.

3. The compounding period of the interest must be the same as the payment or cash flow period described in rule 1 above.

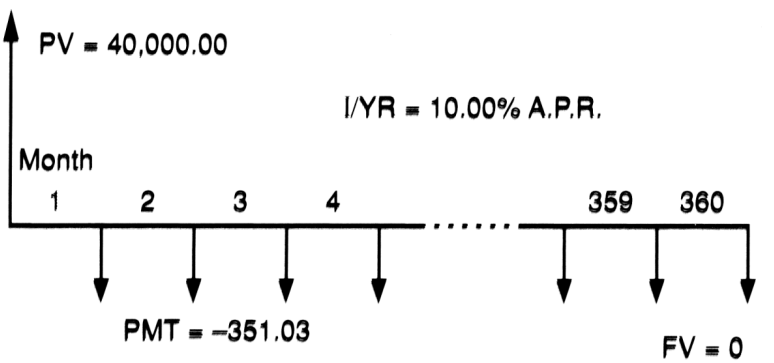
Any cash flow situation with an identical cash flow amount for each period (except the initial and final flows, which can be different) may be analyzed using the compound interest keys—also called the Time Value of Money (TVM) keys: [N], [I/YR], [PV], [PMT] and [FV] (along with [▬][P/YR] and [▬][BEG/END]).

When the periodic amounts are not identical, you can still analyze the situation, but you must use the discounted cash flow keys: [CFj], [▬][Nj], [▬][NPV] and [▬][IRR/YR], along with [▬][P/YR] and [I/YR]. (See page 42 for more on that topic.)

The TVM Keys

The TVM keys let you solve most types of financial calculations that involve steady periodic cash flows. The main five TVM keys are on the top row of the HP 10BII's keyboard. Along with [▬][BEG/END] and [▬][P/YR] (additional parameters) these keys let you draw a cash flow diagram for your calculator. Note how the keys correspond to the various parts in the diagram illustrated below:

[N] = the Number of Interest Periods.
[I/YR] = the Interest rate per YeaR (as a percent).
[PV] = the Present Value.*
[PMT] = the periodic PayMenT amount.
[FV] = the Future Value.*



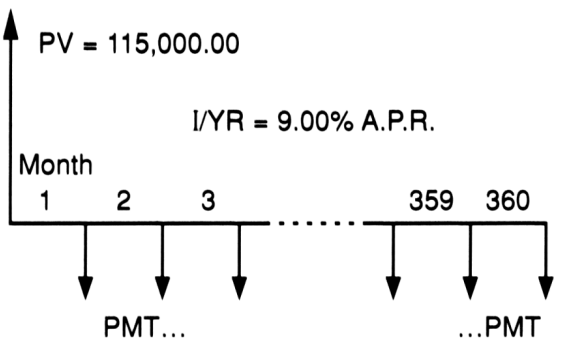
You can solve for any one of the above five values if you already know the other four. (The P/YR and BEG/END values must always be known.)

*Note that the Present Value (PV) is the cash flow that occurs *over and above* any PMT cash flow at the very *beginning* of the time line. Likewise, the Future Value (FV) is the cash flow that occurs *over and above* any PMT cash flow at the very *end* of the time line.

Solving for Payment (PMT)

Example: You're buying a $125,000 property with $10,000 down, financing the rest for 30 years at 9%. Find the monthly payment.

The situation, as the borrower sees it:



m	$
0	115,000
1	(PMT)
2	(PMT)
3	(PMT)
⋮	
360	(PMT)

FV (Future Value) is zero; the loan is completely paid in 360 months. And though you key in I/YR as an annual rate (9%), it's compounded monthly.

Keystrokes	*Comments*
[▬] [BEG/END]	(Do this to remove BEGIN from the screen, as nec.)
[1] [2] [▬] [P/YR]	12 payments per year.
[3] [0] [▬] [x P/YR]	Convert the loan term to months and store into N.
[9] [I/YR]	The annual interest rate.
[1] [1] [5] [0] [0] [0] [PV]	The amount financed.
[0] [FV]	Store zero into FV.
[PMT]	Calculate the payment.

Answer: -925.32

Example: In the previous example, if the monthly payment is at the BEGINning of the period (uncommon), find that payment.

The five main TVM values are the same as in the previous example; having just worked that, you need not re-enter them. Just change the annuity mode.

Keystrokes	*Comments*
[BEG/END]	(Do this to put BEGIN onto the screen, as necessary.)
[PMT]	Re-calculate the payment.

Answer: -918.43

About $7/month (more than $2,400 over 30 years) is saved just by paying at the beginning of the month.

Example: Another lender offers the same terms as above, except the rate is 8.5%. What's the monthly (end of month) payment?

Continuing from above, three of the five TVM parameters are already correct; you can verify them via [RCL][N], [RCL][PV], etc. Just input the changed values:

Keystrokes	*Comments*
[BEG/END]	(Do this to remove BEGIN from the screen, as nec.)
[8][.][5][I/YR]	The new interest rate.
[PMT]	Find the new payment.

Answer: -884.25

Even small changes in the interest rate make a difference in the monthly payment.

Example: Find the payment amount necessary to fully pay off the loan in the previous example—in 25, 20, and 15 years.

Keystrokes		*Comments*
25 [-] x P/YR		Convert the loan term to months and store into N.*
PMT		Solve for the payment.
Answer:	-926.01	
20 [-] x P/YR		Convert the loan term to months and store into N.*
PMT		Solve for the payment.
Answer:	-998.00	
15 [-] x P/YR		Convert the loan term to months and store into N.*
PMT		Solve for the payment.
Answer:	-1,132.45	

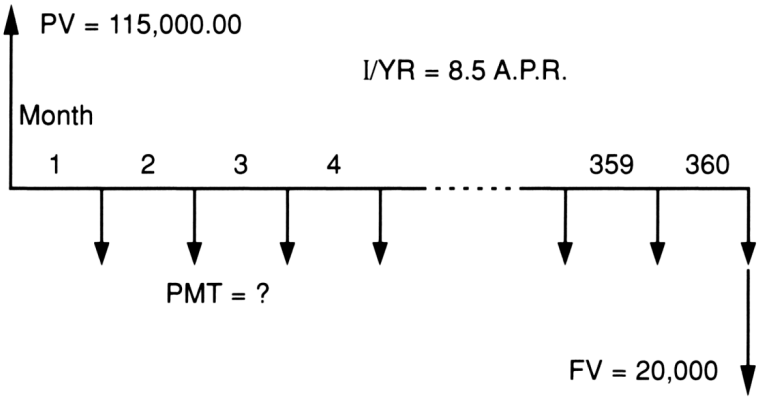
For less than $250 more per month, the loan term is cut in half!

*Keep in mind that by using the [-] x P/YR key, you're simply taking a shortcut to what you'd have to do otherwise: Convert the years to months by multiplying by 12, then pressing N to store this number of months. The [-] x P/YR key does this all at once, as its name implies. It multiplies the number of years you give by the number of payment periods per year (i.e. the value you've previously stored in P/YR), then stores the result into N.

For more explanation and practice with these concepts, see also "An Easy Course in Financial Calculations."

Example: A 30-year, $115,000 mortgage at 8.5% requires a final $20,000 balloon payment. Find the monthly payment.

Here's the situation from a borrower's perspective (so the payments and final balloon are negative):

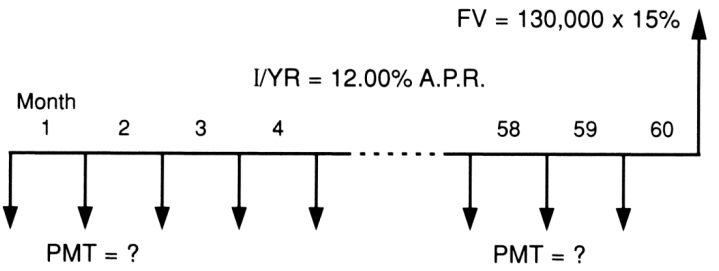


Keystrokes	*Comments*
[BEG/END]	(Do this to remove BEGIN from the screen, as nec.)
[1][2] [P/YR]	12 payments per year.
[3][0] [x P/YR]	Convert the loan term to months and store into N.
[8][•][5] [I/YR]	The annual interest rate.
[1][1][5][0][0][0] [PV]	The amount financed.
[2][0][0][0][0] [+/–] [FV]	Balloon amount is FV.
[PMT]	Solve for the payment.

Answer: -872.13

Example: How much must you invest each month, in an account paying 12% (compounded monthly), to save a 15% down-payment on a $130,000 home in 5 years?

From your perspective:



Keystrokes	*Comments*
[BEG/END]	Do this to put BEGIN onto the screen. (You make the first deposit right when you open the account.)
[1][2] [P/YR] [5] [x P/YR]	12 payments per year. Convert the loan term from 5 years to 60 mo.
[1][2] [I/YR] [0] [PV]	The annual interest rate. The account was empty before the first deposit.
[1][3][0][0][0][0] [×][1][5][%][=]	Compute amount needed. at the end of 5 years.
[FV]	Store result in FV.
[PMT]	Find the PMT (i.e. monthly savings deposit) required to reach the goal.

Answer: -236.40

Example: Interest rates for a 30-year, $120,000 Adjustable Rate Mortgage (ARM) are set at 7.5% for years 1-3, 8.5% for year 4, and 9.0% for year 5. Find the corresponding monthly payments.

Each time the rate changes, you find the remaining balance, then use it—along with the new rate and the remaining term—to compute the new payment.

Keystrokes	*Comments*
[▬][BEG/END]	(Do this to remove BEGIN from the screen, as nec.)
[1][2] [▬][P/YR]	12 payments per year.
[3][0] [▬][x P/YR]	Convert the loan term to months and store into N.
[7][•][5] [I/YR]	The annual interest rate.
[1][2][0][0][0][0] [PV]	The loan amount.
[0] [FV]	You want to calculate the payment necessary to pay off the loan entirely over the given term.
[PMT]	Calculate the payment.

Answer: -839.06

This is the monthly payment amount for the first 3 years. Now calculate the remaining balance after those 3 years. Just change N and solve for FV:

[3] [▬][x P/YR]	3 years = 36 months.
[FV]	Find the loan balance.

Result: -116,417.10

Use this loan amount for the next PMT calculation:

Keystrokes	*Comments*
[+/–] [PV]	Store the result as PV.
[2] [7] [x P/YR]	27 years remaining.
[8] [•] [5] [I/YR]	The new interest rate.
[0] [FV]	Again, figure the payment to completely pay off the loan.
[PMT]	Calculate that payment.

Answer: -917.86

This is the monthly payment amount for year 4. Now find the remaining balance after that one year:

[1] [x P/YR]	1 year = 12 months.
[FV]	Find the loan balance.

Result: -115,253.63

Use this loan amount for the next PMT calculation:

Keystrokes	*Comments*
[+/–] [PV]	Store the result as PV.
[2] [6] [x P/YR]	26 years remaining.
[9] [I/YR]	The new interest rate.
[0] [FV]	Again, figure the payment to completely pay off the loan.
[PMT]	Calculate that payment.

Answer: -957.44

This is the monthly payment amount for year 5.

Solving for Future Value (FV)

Example: After seven years of $1,115 monthly payments on a $145,000 mortgage at 8.25%, what's the remaining balance?

Keystrokes	*Comments*
[▬][BEG/END]	(Do this to remove BEGIN from the screen, as nec.)
[1][2] [▬][P/YR]	12 payments per year.
[7] [▬][x P/YR]	7 years is 84 months.
[8][•][2][5] [I/YR]	The annual interest rate.
[1][4][5][0][0][0] [PV]	The loan amount.
[1][1][1][5][+/–] [PMT]	Specify the payment.
[FV]	Calculate the balance.

Answer: -131,631.48

Example: A $160,000 home is increasing in value at 6%/year. What will it sell for in 7 years?

Keystrokes	*Comments*
[1] [▬][P/YR]	1 period per year.
[7] [N]	A 7-year appreciation.
[6] [I/YR]	The appreciation rate.
[1][6][0][0][0][0][+/–] [PV]	The starting value (negative; it's an investment).
[0] [PMT]	No periodic value added.
[FV]	The value after 7 years.

Answer: 240,580.84

Example: A $110,000 mortgage at 9.5% has 360 monthly payments (in arrears) of $850, plus a final payoff balloon. What's that payoff amount?

Keystrokes	*Comments*
[BEG/END]	(Do this to remove BEGIN from the screen, as nec.)
[1][2] [P/YR]	12 payments per year.
[1][1][0][0][0][0] [PV]	Loan amount.
[3][0] [x P/YR]	30 years is 360 months.
[9][•][5] [I/YR]	The annual interest rate.
[8][5][0][+/–] [PMT]	Monthly payment.
[FV]	Find remaining balance.

Answer: -152,354.90

The balloon payoff is more than the original mortgage because the $850 monthly payment didn't even cover the interest, let alone reduce the principal. So the principal remained, and extra interest accrued—for 30 years. This is called *negative amortization.*

For more explanation and practice with these concepts, see also "An Easy Course in Financial Calculations."

Example: What is the balance after 48 end-of-month payments of $188 on a loan of $7,500 at 12.5%, if the first payment is due 3 months from the date of the loan?

First calculate how much interest the $7,500 accrues over the 2 extra months. Then use that result as the adjusted "starting loan amount."

Keystrokes	*Comments*
[BEG/END]	(Do this to remove BEGIN from the screen, as nec.)
[1][2] [P/YR]	12 payments per year.
[2] [N]	2 months accrual.
[1][2][•][5] [I/YR]	Annual interest rate.
[7][5][0][0] [PV]	Original loan amount.
[0] [PMT]	No PMTs for 2 months.
[FV]	Solve for new balance.
Result: -7,657.06	
[+/–] [PV]	Store that as the adjusted "starting principal" amount in PV.
[4][8] [N] (or [4] [x P/YR])	48 end-of-month pmts.
[1][8][8][+/–] [PMT]	The payment amount.
[FV]	Balance after 48 pmts.
Answer: -960.49	

Example: If you invest $10,000 today into a fund that earns 11% per year, what buying power (i.e. value in today's dollars) will be in this account after 30 years, if inflation is a steady 3% during that time?

Use a rate that reflects both the fund growth rate and the inflation rate.

Keystrokes	*Comments*
1 • 1 1 ÷ 1 • 0 3 − 1 × 1 0 0 =	Find the adjusted rate.
Result: 7.77	
I/YR	Store this adjusted rate.
1 2nd P/YR	Set payments/year to 1.
3 0 N	30 years.
1 0 0 0 0 +/− PV	The initial investment.
0 PMT	No periodic value added.
FV	Solve for FV.
Answer: 94,313.23	

For more explanation and practice with these concepts, see also "An Easy Course in Financial Calculations."

Solving for Present Value (PV)

Example: How much can you finance over 30 years at 8.50%, paying $850 (principal and interest) at the end of each month?

Keystrokes	*Comments*
[BEG/END]	(Do this to remove BEGIN from the screen, as nec.)
[1][2] [P/YR]	Set payments/year to 12.
[3][0] [x P/YR]	30 years = 360 months.
[8][.][5] [I/YR]	The annual interest rate.
[8][5][0] [+/−] [PMT]	Specify the payment.
[0] [FV]	No balance left in 30 yrs.
[PV]	Solve for PV.

Answer: 110,545.60

Example: A 30-year mortgage at 9.5% has monthly payments (in arrears) of $924.94. What was the loan amount?

Keystrokes	*Comments*
[BEG/END]	(Do this to remove BEGIN from the screen, as nec.)
[1][2] [P/YR]	Set payments/year to 12.
[3][0] [x P/YR]	30 years = 360 months.
[9][.][5] [I/YR]	The annual interest rate.
[9][2][4][.][9][4] [PMT]	The monthly payment.
[0] [FV]	No balance left in 30 yrs.
[PV]	Find the loan amount.

Answer: -110,000.04

Example: If your money can earn 11% annually in a mutual fund, and you win a $1 million lottery jackpot, would you rather receive it tax-free, paid out at $100,000 a year over 10 years, or in a taxable lump-sum now? Your tax bracket is 38%.

First, calculate the taxable lump-sum option:

1 0 0 0 0 0 0 − 3 8 % =

Result: 620,000.00

Then, for the 10-year, tax-free payout option:

Keystrokes	*Comments*
BEG/END	Do this to put BEGIN onto the screen. (You get the first tax-free pay-out right away.)
1 P/YR	Just 1 payment per year.
1 0 N	Years of the payout.
1 1 I/YR	Store annual interest rate.
1 0 0 0 0 0 PMT	Yearly payment amount.
0 FV	All paid out in 10 years.
PV	Calculate the PV.

Answer: -653,704.75

Take the 10-year payout—it's worth about $33,700 more to you.

For more explanation and practice with these concepts, see also "An Easy Course in Financial Calculations."

Example: To qualify for a mortgage, a prospective borrower's projected PITI (principal, interest, taxes and insurance) plus other current debt payments must not exceed 40% of her gross income. If she grosses $4,500/mo. and makes $350 in other debt payments, how much can she borrow on a 30-year fixed-rate mortgage at 8%? Assume that taxes and insurance would total about $200/mo.

Keystrokes	*Comments*
[↓] [BEG/END]	(Do this to remove BEGIN from the screen, as nec.)
[1] [2] [↓] [P/YR]	Set payments/year to 12.
[3] [0] [↓] [x P/YR]	30 years = 360 months.
[8] [I/YR]	The annual interest rate.
[4] [5] [0] [0] [×] [4] [0] [%] [−]	Compute the maximum
[3] [5] [0] [−] [2] [0] [0] [=]	P&I payment, then store
[+/−] [PMT]	this in PMT.
[0] [FV]	No balance left in 30 yrs.
[PV]	Solve for Present Value.

Answer: 170,354.37

So, with a 10% down-payment, she can buy a house priced at about $189,000; with 20% down, she can buy a $213,000 home. (To find the exact numbers, divide the above PV result by .9 or .8, respectively.)

Solving for Number of Periods (N)

Example: A homeowner has a 30-year, $85,000 mortgage at 8.5%, with month-end payments of $653.58. If she pays $20 more per month, when will the loan be paid?

Keystrokes	*Comments*
[gold] BEG/END	(Do this to remove BEGIN from the screen, as nec.)
12 [gold] P/YR	Set payments/year to 12.
8.5 I/YR	Annual interest rate.
85000 PV	Loan amount.
0 FV	Zero FV (no balloon).
673.58 +/– PMT	The total actual payment, including the extra $20.
N	Solve for N—in months.

Answer: 311.11 (That's about 26.5 yrs.)

Example: How long will it take you to pay off a 30-year, $145,000 mortgage at 8.25%, if you make biweekly payments (i.e. 26 payments/year), each being half the normal monthly payment ($1,089.34)?

Keystrokes	*Comments*
[gold] BEG/END	(Do this to remove BEGIN from the screen, as nec.)
12 [gold] P/YR	Set payments/year to 12.
30 [gold] x P/YR	30 years is 360 months.
8.25 I/YR	The annual interest rate.
145000 PV	The loan amount.

0 FV	Totally paid in 30 years.
PMT	Solve for monthly pay-
÷ 2 =	ment, then divide it by 2.
PMT	Store it as the new PMT.
2 6 P/YR	Set payments/year to 26.
N	Solve for N.

Result: 587.92

Divide this by 26 payments per year, and you'll find it will take just under 23 years to pay off the loan—7 years faster than a conventional monthly payment.

Example: A 30-year, $100,000 mortgage at 8.75% has month-end payments of $786.70. How long until the loan is half paid?

Keystrokes	*Comments*
BEG/END	(Do this to remove BEGIN from the screen, as nec.)
1 2 P/YR	Set payments/year to 12.
8 • 7 5 I/YR	Annual percent rate.
1 0 0 0 0 0 PV	Initial mortgage amount.
5 0 0 0 0 +/– FV	Desired remaining bal.
7 8 6 • 7 +/– PMT	The monthly payment.
N	Solve for N.

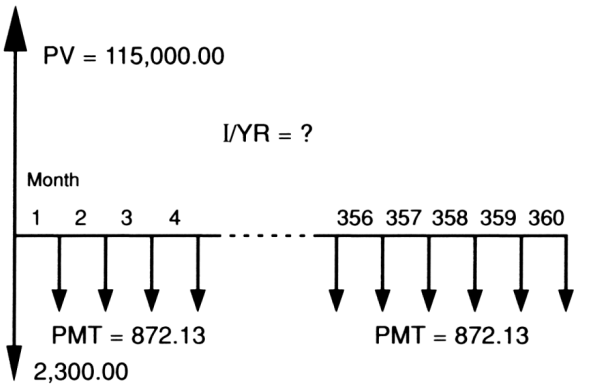
Answer: 274.31 (months—about 23 yrs.)

For more explanation and practice with these concepts, see also "An Easy Course in Financial Calculations."

Solving for Interest Rate (I/YR)

Example: A $115,000, 30-year mortgage specifies 360 month-end payments of $872.13, and a loan fee of 2% of the amount financed ($2,300). What's the A.P.R.?

The A.P.R. must include the loan fee, which effectively reduces the loan amount.



m	$
0	115,000
	–(2,300)
1	(PMT)
2	(PMT)
3	(PMT)
⋮	
360	(PMT)

Keystrokes	*Comments*
[▬] [BEG/END]	(Do this to remove BEGIN from the screen, as nec.)
[1] [2] [▬] [P/YR]	Set payments/year to 12.
[3] [0] [▬] [x P/YR]	The loan term.
[8] [7] [2] [•] [1] [3] [+/–] [PMT]	The payment amount.
[1] [1] [5] [0] [0] [0] [–] [2] [%]	Calculate the net amount
[=]	financed.
[PV]	Store that amount in PV.
[0] [FV]	No balloon payment.
[I/YR]	Find the real A.P.R.

Answer: 8.57

Example: 25 years ago, a burger cost $0.89. Now it's $2.59. What was the inflation rate?

Keystrokes	*Comments*
1 ▬ P/YR	Set payments/year to 1.
2 5 N	Analyze over 25 years.
• 8 9 +/– PV	The price 25 years ago.
0 PMT	No periodic value added.
2 • 5 9 FV	The current price.
I/YR	The annual inflation rate.

Answer: 4.37

Example: A $27,000 car lease has monthly payments of $475 and a buyout of $6,000 in 7 years. Should you take this lease or put the $27,000 on a credit card, at an annual rate of 14.9%?

Keystrokes	*Comments*
▬ BEG/END	Do this to put BEGIN onto the screen. (Lease payments are generally at the beginning of each month.)
1 2 ▬ P/YR	Set payments/year to 12.
7 ▬ x P/YR	7 years is 84 months.
2 7 0 0 0 PV	The car value.
4 7 5 +/– PMT	The monthly payment.
6 0 0 0 +/– FV	The buyout amount.
I/YR	The annual interest rate.

Answer: 15.04

The credit card purchase looks slightly better.

Interest Conversions

with the TVM keys, the interest compounding period must always match the payment period, but many real-life situations don't reflect this. To allow for this, you find a rate that is *equivalent* to the given rate but which compounds with the payment period:

Key in the quoted rate; press [NOM%].

Key in compounding periods/year; press [P/YR].

Press [EFF%].

Key in payment periods/year; press [P/YR].

Press [NOM%] for I/YR to use in TVM calculations.

Example: You're converting a credit card balance into a monthly installment loan. The card's rate is 14.9%, compounded daily. What I/YR do you use in TVM to get a monthly payment (i.e. P/YR is 12)?

Keystrokes		*Comments*
14.9 [NOM%]		The card's NOMinal rate.
365 [P/YR]		Its compounding per./yr.
[EFF%]		Calculate…
Result:	16.06	the card's EFFective rate.
12 [P/YR]		Now set monthly compounding to match the payment period.
[NOM%]		Find the I/YR you need to use in TVM calculations.
Answer:	14.99	

Example: Your Canadian friend tells you that he just got a $100,000 mortgage, with 360 $724.71 month-end payments, at 8%. Find the equivalent U.S. mortgage rate.

In Canada, mortgage payments are usually monthly, but interest often compounds semi-annually.

Keystrokes	*Comments*
8 NOM%	Key in the NOMinal rate.
2 P/YR	The compounding period is semi-annual.
EFF%	Find the EFFective rate.
Result: 8.16	
12 P/YR	Key in the payments/year.
NOM%	Solve for the U.S. rate.
Answer: 7.87	

Notice, by the way, that whatever goes into NOM% also goes into the I/YR register—use RCL I/YR to verify this, if you wish—so that you're all ready to proceed with a TVM calculation after you have calculated the equivalent nominal rate. (It's no accident that HP chose to locate NOM% on the I/YR key.)

For more explanation and practice with these concepts, see also "An Easy Course in Financial Calculations."

Amortization Schedules (AMORT)

After you know the terms of a mortgage, the next questions you usually have are: "How much interest and principal will I be paying each year?" And: "What's the remaining balance?" The HP 10BII's amortization feature helps answer such questions. An amortization schedule is an itemized computation of the total principal and interest paid during a given set of periods within the term of a mortgage.

The HP 10BII's ↓ AMORT key uses the values in the TVM registers.

You have a straightforward 30-year, fully-amortized mortgage for $130,000 at 8.75%, with monthly payments (in arrears). Find the principal and interest you'd pay in each of the first 2 years, and the remaining balance still due after the 2nd year.

First, you find the payment, with all the usual TVM parameters:

Keystrokes	*Comments*
↓ BEG/END	(Do this to remove BEGIN from the screen, as nec.)
12 ↓ P/YR	Set payments/year to 12.
30 ↓ x P/YR	Key in all the other
8.75 I/YR	known values....
130000 PV	
0 FV	
PMT	Solve for payment.

Result: -1,022.71

Now that you know the payment amount, you can proceed with the actual amortization:

Keystrokes		*Comments*
[AMORT]		Begin the amortization.
Result:	1-12	The loan periods amortized.
[=]		(View the amortization.)
Result:	-934.42	Amount paid to principal.
[=]		(View the amortization.)
Result:	-11,338.10	The amount paid to interest.
[=]		(View the amortization.)
Result:	129,065.58	The balance remaining.*
[AMORT]		Continue the amortization.
Result:	13-24	The loan periods amortized.
[=]		(View the amortization.)
Result:	-1,019.53	Amount paid to principal.
[=]		(View the amortization.)
Result:	-11,252.99	The amount paid to interest.
[=]		(View the amortization.)
Result:	128,046.05	The balance remaining.*

You can continue this process for every year, as you wish. Notice, too, that you can amortize more than one year at a time, by keying in the desired periods before pressing [AMORT]. For example, to amortize all 24 periods above, you'd press [1] [INPUT] [2] [4] [AMORT], then [=], etc.

**Note:* Balances given by [AMORT] will be slightly different than that calculated directly via [FV], because [AMORT] rounds each period's balance to the decimal places currently displayed by the calculator (usually two), to reflect real-life payments and balances, whereas the TVM keys use 12 digits in all their calculations.

Example: If a new homeowner's mortgage interest and property taxes are both fully tax deductible, and his effective income tax bracket is 33%, what is his after-tax monthly PITI (principal, interest, taxes and insurance) payment in the first year of a $170,000 30-year fixed-rate mortgage of 8%? His property taxes are $150/mo. His insurance is $50/mo.

Keystrokes		*Comments*
[▬][BEG/END]		(Do this to remove BEGIN from the screen, as nec.)
[1][2] [▬][P/YR]		Set payments/year to 12.
[3][0] [▬][x P/YR]		30 years = 360 months.
[8] [I/YR]		The annual interest rate.
[1][7][0][0][0][0] [PV]		
[0] [FV]		No balance left in 30 yrs.
[PMT]		Solve for PMT.
Result:	- 1,247.40	

So pre-tax PITI is $1,247.40+150+50 = $1,447.40.

[▬][AMORT]		Begin the amortization.
Result:	1 - 12	The loan periods amortized.
[▬] [▬]		(View the amortization.)
Result:	- 13,548.67	The amount paid to interest.

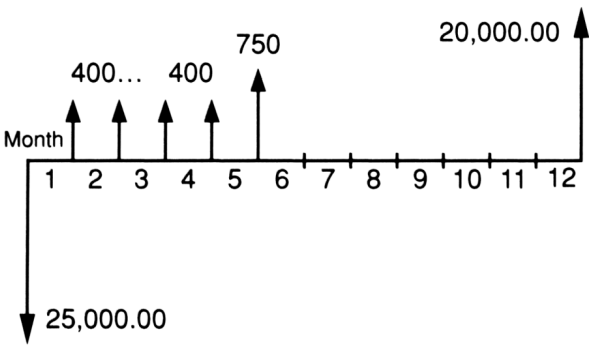
A tax-deductible payment is effectively reduced by by the tax bracket percentage. So the monthly PITI for the first year, after taxes, would effectively be

$$\$1{,}447.40 - .33[(13{,}548.67 \div 12) + 150].$$

Doing the arithmetic gives $1,025.31.

Discounted Cash Flow Analysis

The TVM keys work only when there is a uniform PMT every period. When periodic cash flows are not the same amount, you must use discounted cash flow analysis. (The *timing* of the cash flows must still be regular, but the *amounts* may be uneven.)



m	$
0	(25,000)
1	400
2	400
3	400
4	400
5	750
6	0
7	0
8	0
9	0
10	0
11	0
12	20,000

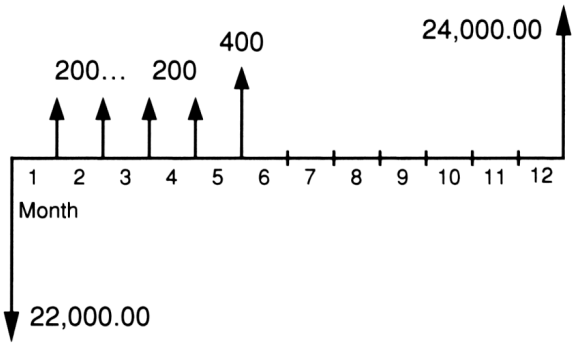
Example: Key in the above cash-flow situation.

First, press [▬] [C ALL]. Then, to "draw the picture" of a situation such as that above, enter the information in cash flow *groups*. Watch the display as you go:

Keystrokes	*Comments*
[2] [5] [0] [0] [0] [+/−] [CFj]	The initial cash flow (group 0) is –$25,000.
[4] [0] [0] [CFj]	Group 1's cash flows are $400.
[4] [▬] [Nj]	Group 1 has 4 flows.
[7] [5] [0] [CFj]	Group 2's flows are $750.
[1] [▬] [Nj]	Group 2 has 1 flow.

0 CFj	Group 3's flows are $0.
6 −Nj	Group 3 has 6 flows.
20000 CFj	Group 4's flows are $20,000.
1 −Nj	Group 4 has 1 flow.

Example: Assuming the previous cash flow situation is still stored, edit that data into the situation below.



m	$
0	(22,000)
1	200
2	200
3	200
4	200
5	400
6	0
7	0
8	0
9	0
10	0
11	0
12	24,000

Keystrokes	*Comments*
22000 +/− −STO CFj 0	New amount of group 0.
200 −STO CFj 1	New amount of group 1.
400 −STO CFj 2	New amount of group 2.
24000 −STO CFj 4	New amount of group 4.*

*In this case, you didn't need to replace any of the Nj values, but note that when you do, you can't just press −STO−Nj. First, *recall the corresponding CFj value*—to indicate the group. For example, to replace the Nj value for group 2, first recall that CFj value (RCL CFj 2), *then* key in its new Nj value and −STO−Nj.

Net Present Value (NPV)

A dollar today is worth more than a dollar tomorrow because you can put it to work earning interest in the meantime. That potential interest rate is also called the *discount* rate, because it discounts a future dollar's value—calculates its lesser, equivalent *Present Value* today. When you apply such a discount rate to an entire *set* of cash flows and sum all their resulting Present Values, this is the Net Present Value of the cash flow stream. PV can't do this unless all the cash flows are identical, but NPV can.

Example: If you can earn 15% on your money, what's the Net Present Value to you of the situation described by the cash flow situation on the previous page)? What if you can earn only 10%? Or 0%?

First, if you haven't done so already, key in the situation on the previous page. Next, in order to calculate NPV, you must specify a discount rate (I/YR) and the frequency at which it compounds (P/YR). So press 12 P/YR, then 15 I/YR. Now calculate: NPV.... *Answer:* -172.27

For a 10% discount rate: 10 I/YR, then NPV.... *Answer:* 892.45

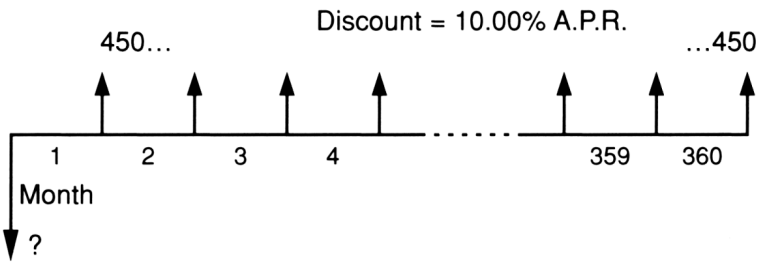
For a 0% discount rate: 0 I/YR, then NPV.... *Answer:* 3,200.00

(Using a 0% discount rate is not really discounting at all, of course; it's just summing the cash flows.)

These NPV values indicate the relative value of this cash flow situation compared with a similar investment that earns exactly the given discount rate. If you can expect to earn 15% on your money in a similar investment, you'd *lose* $172.27 (in present value—today's dollars) by choosing this deal over that other investment. But if can you expect to earn only 10% elsewhere, this deal is better by $892.45.

Example: A loan was written at 10% A.P.R., with $450 monthly payments (in arrears), for 30 years. You could use the TVM keys to find the loan amount, but as an exercise, use NPV instead. Repeat for annuity in advance (i.e. with BEGIN on).

Here's the annuity in arrears case (lender's view):



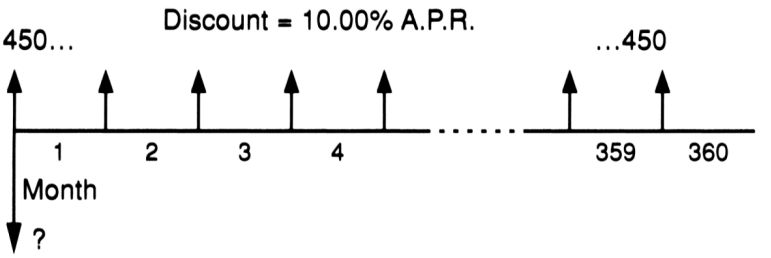
The unknown is an initial cash flow, so if you omit it (make it $0) and take the NPV of the rest of the picture—with the loan rate as the discount rate—the result will exactly balance the unknown initial flow.

Keystrokes	*Comments*
C ALL	Start a new list.
0 CFj	The initial cash flow (group 0) is $0.

4 5 0 CFj	Group 1's flows are $450.
9 0 − Nj	Group 1 has 90 flows.*
4 5 0 CFj	Group 2's flows are $450.
9 0 − Nj	Group 2 has 90 flows.
4 5 0 CFj	Group 3's flows are $450.
9 0 − Nj	Group 3 has 90 flows.
4 5 0 CFj	Group 4's flows are $450.
9 0 − Nj	Group 4 has 90 flows.
1 2 − P/YR	Key in the periods/year.
1 0 I/YR	Key in the discount rate.
− NPV	Solve for NPV.

Answer: 51,277.87

Now solve the case of annuity in advance:



4 5 0 − STO CFj 0	There's a known flow amount now in group 0.
RCL CFj 1	
8 9 − STO − Nj	Change the number of flows in group 1 to 89.
− NPV	Re-solve for NPV.

Answer: 51,705.18

*Actually, of course, there's just one big group of 360 flows, but since the HP 10BII is limited to just 99 flows per group, you need to use several consecutive groups to build the whole picture. (You could have used groups of 99, 99, 99 and 63—or some other such combination that adds to 360—but four 90's seemed easiest.)

Example: How much should you deposit now in a bank account earning 8% annually, compounded monthly, so you can draw $5,000 at the 1st of each month for 20 years, starting 20 years from now?

Keystrokes	*Comments*
	Start a new list.
0 CFj	The initial flow (group 0) is unknown, so use $0.
0 CFj	Group 1's flows are $0.
80	Group 1 has 80 flows.*
0 CFj	Group 2's flows are $0.
80	Group 2 has 80 flows.
0 CFj	Group 3's flows are $0.
79	Group 3 has 79 flows.
5000 CFj	Grp. 4's flows are $5000.
80	Group 4 has 80 flows.*
5000 CFj	Grp. 5's flows are $5000.
80	Group 5 has 80 flows.
5000 CFj	Grp. 6's flows are $5000.
80	Group 6 has 80 flows.
12	Key in the periods/year.
8 I/YR	Key in the discount rate.
	Solve for NPV.

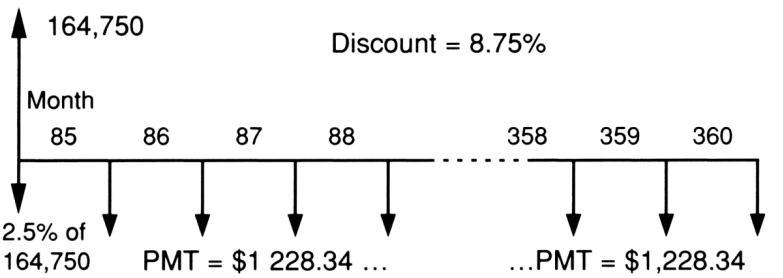
Answer: 122,139.37

*Again, you must split up groups in excess of 99 flows.

For more explanation and practice with these concepts, see also "An Easy Course in Financial Calculations."

Example: You have just made the 84th end-of-month payment on a 30-year mortgage, whose rate is 8.75% and balance is now $164,750. You can refinance at 7.25% (new payments of $1,228.34 for the remaining 276 months), with a 2.5% fee. If you plan to stay in the house for the entire term, should you refinance?

Take the NPV of the proposed refinancing, using the original mortgage rate as the discount rate. You should refinance only if the result is positive.



Keystrokes	*Comments*
[C ALL]	Start a new list.
[1][6][4][7][5][0]	Compute the initial flow
[–][2][.][5][%][=] [CFj]	(group 0).
[1][2][2][8][.][3][4][+/–] [CFj]	Grp 1 flows: –$1,228.34.
[9][0] [Nj]	Group 1 has 90 flows.*
[1][2][2][8][.][3][4][+/–] [CFj]	Grp 2 flows: –$1,228.34.
[9][0] [Nj]	Group 2 has 90 flows.*
[1][2][2][8][.][3][4][+/–] [CFj]	Grp 3 flows: –$1,228.34.
[9][6] [Nj]	Group 3 has 96 flows.*
[1][2] [P/YR]	Key in the periods/year.
[8][.][7][5] [I/YR]	Key in the discount rate.
[NPV]	Solve for NPV.

Answer: 14,853.31

Example: On your 26th birthday, you begin to invest $2,000/year for 7 years into a (tax-deferred) IRA yielding 10% annually. At that point, you stop paying into the IRA, but your spouse begins to invest $2,000/year into another IRA with the same yield. On your 60th birthday, whose IRA has a higher balance—yours with just 7 early years of investing, or your spouse's with 28 later years?

Your own scenario:

[▬] [C ALL]	Start a new list.
[2] [0] [0] [0] [CFj]	First year contribution.
[2] [0] [0] [0] [CFj] [6] [▬] [Nj]	Next 6 years of contrib.
[0] [CFj] [2] [8] [▬] [Nj]	No contrib. for 28 years.
[1] [▬] [P/YR]	Key in the periods/year.
[1] [0] [I/YR]	Key in the discount rate.
[▬] [NPV]	Solve for NPV.*

Answer: 10,710.52

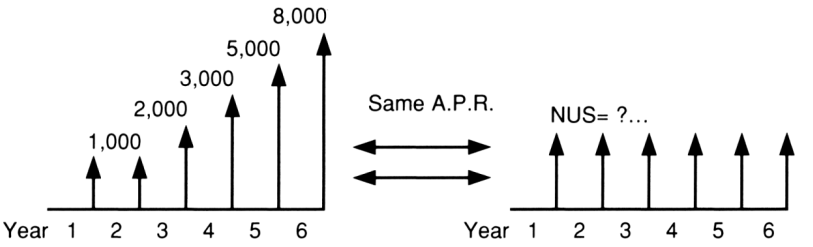
Your spouse's scenario:

[▬] [C ALL]	Start a new list.
[0] [CFj]	First year contribution.
[0] [CFj] [6] [▬] [Nj]	Next 6 years of contrib.
[2] [0] [0] [0] [CFj] [2] [8] [▬] [Nj]	Next 28 years of contrib.
[1] [0] [I/YR]	P/YR and discount rate.
[▬] [NPV]	Solve for NPV.*

Answer: 10,506.63

*This is Net *Present* Value—not the Net *Future* Value (i.e. not the balance on your 60th). But at the same discount rate, a larger NPV gives a larger NFV. To find the NFV, do a TVM calculation now N=34, don't touch I/YR or PV (NPV is already in PV!), PMT=0; find FV. Your NFV: 273,628.86; your spouse's: 268,419.87.

Example: If comparable investments yield you 16% annually, what level periodic payment stream (the "Net Uniform Series," N.U.S.) is equivalent to this scenario?



First find the NPV. Then use that as a "loan amount" for a TVM calculation to find the level PMT amount that "pays it off" over the 6-year time period .

Keystrokes	*Comments*
[↓] [C ALL]	Start a new list.
[0] [CFj]	The initial flow (group 0) is unknown, so use $0.
[1] [0] [0] [0] [CFj]	Grp. 1's flows are $1,000.
[2] [↓] [Nj]	Group 1 has 2 flows.
[2] [0] [0] [0] [CFj]	Grp. 2's flow is $2,000.*
[3] [0] [0] [0] [CFj]	Grp. 3's flow is $3,000.*
[5] [0] [0] [0] [CFj]	Grp. 4's flow is $5,000.*
[8] [0] [0] [0] [CFj]	Grp. 5's flow is $8,000.*
[1] [↓] [P/YR] [1] [6] [I/YR]	Per./yr. and discount rate.
[↓] [NPV]	Find NPV. (10,207.52)
[6] [N] [PMT] [+/−]	Equivalent level PMT.
Answer: 2,770.22	

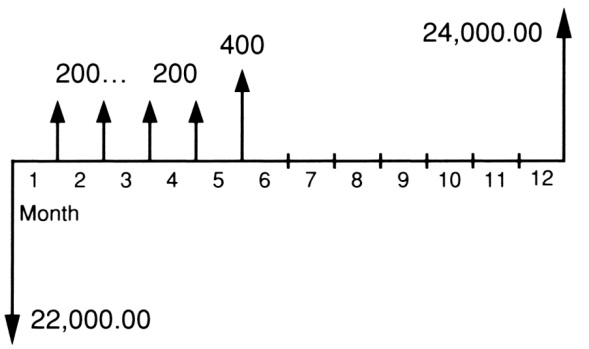
At a discount rate of 16%, the uneven cash flows are equivalent to 6 annual payments of $2,770.22.

*When a group contains just 1 fllow, you needn't use [↓] [Nj] at all.

Internal Rate of Return (IRR/YR)

Internal Rate of Return (IRR%) is the discount rate that produces a Net Present Value (NPV) of zero in a given cash flow situation.

Example: Find the internal rate of return of the following cash flow scenario.



m	$
0	(22,000)
1	200
2	200
3	200
4	200
5	400
6	0
7	0
8	0
9	0
10	0
11	0
12	24,000

Keystrokes	*Comments*
[↓] [C ALL]	Start a new list.*
[22000] [+/−] [CFj]	The initial cash flow.
[200] [CFj] [4] [↓] [Nj]	Group 1.
[400] [CFj]	Group 2.
[0] [CFj] [6] [↓] [Nj]	Group 3.
[24000] [CFj]	Group 4.
[12] [↓] [P/YR]	Key in the periods/year.
[↓] [IRR/YR]	Solve for IRR/YR.

Answer: 14.17

*This cash fllow situation is the same as that shown on page 43 See pages 42-43 for more on the details of keying it in.

Example: An adjustable-rate mortgage in the amount of $170,000 has monthly payments of $1,720.03 for the first year; $1,773.78 for years 2-5; and $1,878.75 for years 6-30. What is the overall rate blended rate over the 30 years?

Keystrokes	*Comments*
[gold] [C ALL]	Start a new list.
1 7 0 0 0 0 +/– CFj	The initial cash flow.
1 7 2 0 • 0 3 CFj	Year 1 monthly pmt.
1 2 [gold] Nj	12 months of that.
1 7 7 3 • 7 8 CFj	Years 2-5 monthly pmt.
4 8 [gold] Nj	48 months of that.
1 8 7 8 • 7 5 CFj	Years 6-30 monthly pmt.
7 5 [gold] Nj	300 months of that (split
1 8 7 8 • 7 5 CFj	into 4 consecutive groups
7 5 [gold] Nj	of 75 flows each).
1 8 7 8 • 7 5 CFj	
7 5 [gold] Nj	
1 8 7 8 • 7 5 CFj	
7 5 [gold] Nj	
1 2 [gold] P/YR	Key in the periods/year.
[gold] IRR/YR	Solve for IRR/YR.

Answer: 12.56

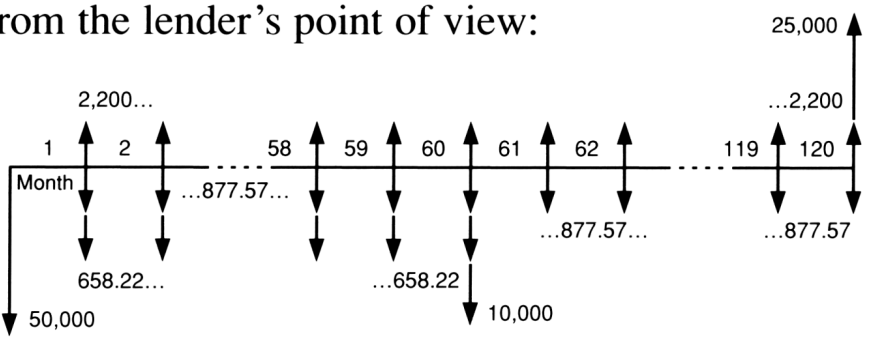
For more explanation and practice with these concepts, see also "An Easy Course in Financial Calculations."

Example: A home has two mortgages. The first is for $100,000 for 30 years at $877.57 per month, with no balloon. The second is for $50,000 starting 15 years later, with 120 PMTs of $658.22 and a $10,000 balloon. Now, after the 20th year, the owner wants to consolidate both loans and get financing for remodeling. You offer to "wrap" his current mortgages (cover all their obligations) and lend him $50,000 in new money, for 120 monthly payments of $2,200 and a $25,000 balloon. What's your yield? What's the borrower's overall A.P.R.? He paid 1 point on the first mortgage, 2 points on the second.

You'll need cash flow diagrams from two perspectives: the lender (that's you) and the borrower.

As the lender, you will make the monthly payments on both existing mortgages, as well as the 2nd mortgage's $10,000 balloon payment (in 5 more years).

From the lender's point of view:

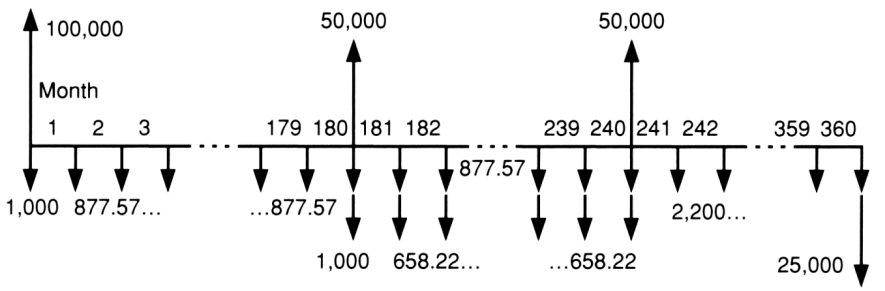


Keystrokes	*Comments*
[C ALL]	Start a new list.
[5][0][0][0][0][+/−] [CFj]	Initial cash flow.
[6][6][4][•][2][1] [CFj]	1st cash flow group is 2,200−877.57−658.22.
[5][9] [Nj]	59 flows in the group.
[9][3][3][5][•][7][9][+/−] [CFj]	2nd group (2,200–877.57 −658.22–10,000) has one flow.
[1][3][2][2][•][4][3] [CFj]	3rd group (2,200− 877.57) has 59 flows.
[5][9] [Nj]	
[2][6][3][2][2][•][4][3] [CFj]	4th group (2,200+ 25,000 −877.57) has one flow.
[1][2] [P/YR]	Key in the periods/year.
[IRR/YR]	Solve for IRR/YR.

Answer: 16.97

That's your yield as the lender.

Now here's the borrower's picture:



Keystrokes	*Comments*
[orange] C ALL	Start a new list.
9 9 0 0 0 CFj	Initial flow amount is 100,000 – 1%.
8 7 7 • 5 7 +/– CFj	1st group is –877.57 and has 179 flows (so split into 2 groups).
9 9 [orange] Nj	
8 7 7 • 5 7 +/– CFj	
8 0 [orange] Nj	
4 8 1 2 2 • 4 3 CFj	Next grp. [(50,000 – 2%) – 877.57] has one flow.
1 5 3 5 • 7 9 +/–	Next group (–877.57 – 658.22) has 59 flows.
CFj 5 9 [orange] Nj	
4 8 4 6 4 • 2 1 CFj	Next group is 50,000 – 877.57 – 658.22 and has only one flow.
2 2 0 0 +/– CFj	Next group is –2,200 and has 119 flows (so split into 2 groups).
9 9 [orange] Nj	
2 2 0 0 +/– CFj	
2 0 [orange] Nj	
2 7 2 0 0 +/– CFj	Last group is –2,200 – 25,000 and has only one flow.
[orange] IRR/YR	Solve for IRR/YR.

Answer: 10.51

This is the borrower's overall interest rate—for the entire term of both of his mortgages and your new loan. That rate is different than yours as the new lender, of course, because you're not the only lender the borrower has dealt with.

Modified Internal Rate of Return (MIRR)

In some cash flow scenarios (typically where positive and negative flows alternate a lot along the time line), there may be more than one IRR%—more than one discount rate that will give an NPV of zero. When this happens, the calculator will ask you to guess at a rate, and then it finds the solution closest to your guess, leaving you to decide what it means.

To avoid this ambiguity, you can compute the yield differently. One easy method is Modified Internal Rate of Return: First you find the NPV of all negative cash flows, discounting at a "safe rate"—that of a liquid cash account. Next you find the NFV of all positive cash flows, using a "risk rate"—a rate reflecting the overall risk of the investment you're analyzing. With a single cash flow at each end of the timeline, it's then a simple I/YR calculation (via the TVM keys) to relate them. The result is the MIRR.

Example: You invest $50,000 in a low-income housing project with these cash flows:

Year	Cash Flow
0 (initial flow)	$ – 50,000
1	25,000
2	50,000
3	–25,000
4	50,000
5	–25,000
6	–10,000
7	–10,000

Find your yield via MIRR, assuming a safe rate of 5.5% and a risk rate of 18%.

First enter your positive cash flow list.

Keystrokes	*Comments*
[gold] [C ALL]	Start a new list.
0 [CFj]	Initial cash flow is 0.
2 5 0 0 0 [CFj]	1st grp.: 25,000 (1 flow).
5 0 0 0 0 [CFj]	2nd grp.: 50,000 (1 flow).
0 [CFj]	3rd group: 0 (1 flow).
5 0 0 0 0 [CFj]	4th grp.: 50,000 (1 flow).
0 [CFj] 3 [gold] [Nj]	5th group: 0 (3 flows).
1 [gold] [P/YR]	Key in the periods/year.
1 8 [I/YR]	Key in the discount rate.
[gold] [NPV]	Solve for NPV.

Result: 82,885.11

This is the Net Present Value (NPV) of all the positive cash flows—discounted at the risk rate. To find the Net Future Value, NFV, from this (see page 49 for more on NFV), press 7 [N], [FV].

Result: -264,028.34 (Ignore the minus sign.)

Now find the Net Present Value, NPV, of all the negative cash flows—discounted at the safe rate:

[gold] [C ALL]	Start a new list.
5 0 0 0 0 [+/–] [CFj]	Initial flow is –50,000.
0 [CFj] 2 [gold] [Nj]	Group 1: 0 (2 flows).
2 5 0 0 0 [+/–] [CFj]	Grp. 2: –25,000 (1 flow).
0 [CFj]	Group 3: 0 (1 flow).
2 5 0 0 0 [+/–] [CFj]	Grp. 4: –25,000 (1 flow).
1 0 0 0 0 [+/–] [CFj]	Grp. 5: –10,000 (2 flows).
2 [gold] [Nj]	

5 • 5 I/YR	Key in the discount rate.
NPV	Solve for NPV.

Result: -104,545.53

Now use the TVM keys to solve for MIRR (and keep in mind that the result of your NPV calculation has automatically been copied into the PV register already —very convenient):

Press 7 N, 2 6 4 0 2 8 • 3 4 FV, then solve for I/YR....

Answer: 14.15

MIRR is just one of several different alternatives to IRR%—probably the easiest, but not necessarily the most accurate.

In another method, called Financial Management Rate of Return (FMRR), rather than funding *all* of the scheduled negative flows with the up-front, safe-rate account investment, you seek to minimize that necessary starting cash by augmenting it with positive flows as they happen. Only excess cash inflows above those needs are reinvested at the risk rate.

This is a better reflection of the realities of investment, but it's not as easily calculated, since you must work backwards from the end of the timeline, one flow at a time.

For more explanation and practice with these concepts, see also "An Easy Course in Financial Calculations."

Statistics

The HP 10BII allows you to accumulate and analyze statistical data in six statistical registers specially reserved for that purpose.

Example: The list below shows the annual rental incomes (per ft^2) and floor areas for 11 comparable downtown office spaces.

Office	Annual Rental Income (per ft^2)	Office Floor Area (ft^2)
1	$ 8.83	3,150
2	9.02	2,500
3	9.19	1,900
4	9.67	1,500
5	9.03	2,100
6	8.97	2,600
7	8.96	2,650
8	8.95	2,700
9	9.10	2,000
10	8.91	2,900
11	8.57	3,750

Key in these figures as statistical data. Then:

(a) Find the total area for all eleven offices.

(b) Find the total annual income for the eleven offices.

(c) Find the average floor area per office.

(d) Find the standard deviation of the areas.

(e) Find the average office rental rate (per ft^2).

Keystrokes	*Comments*
[▬][CLΣ]	Start a new set of statistics.
[8][•][8][3] [INPUT] [3][1][5][0] [Σ+]	Key in the eleven pairs of values, separating the 2 values in each pair with [INPUT], then pressing [Σ+] after each pair. Notice that the display shows a running total of the number of pairs entered.
[9][•][0][2] [INPUT] [2][5][0][0] [Σ+]	
[9][•][1][9] [INPUT] [1][9][0][0] [Σ+]	
[9][•][6][7] [INPUT] [1][5][0][0] [Σ+]	
[9][•][0][3] [INPUT] [2][1][0][0] [Σ+]	
[8][•][9][7] [INPUT] [2][6][0][0] [Σ+]	
[8][•][9][6] [INPUT] [2][6][5][0] [Σ+]	
[8][•][9][5] [INPUT] [2][7][0][0] [Σ+]	
[9][•][1][0] [INPUT] [2][0][0][0] [Σ+]	
[8][•][9][1] [INPUT] [2][9][0][0] [Σ+]	
[8][•][5][7] [INPUT] [3][7][5][0] [Σ+]	

(a) The total area for all offices is the sum of the *y*-data. Press [▬][Σy]. *Answer:* 27,750.00

(b) The total annual income for all offices is the sum of the *products* of each *x*- and *y*-value. Press [▬][Σxy]. *Answer:* 248,701.00

(c) The average office area is the *y*-data mean. Press [▬][x̄,ȳ] [▬][SWAP]. *Answer:* 2,522.73

(d) The standard deviation you want is for the entire population (not just a sample) of the *y*-data. Press [▬][σx,σy] (not [▬][Sx,Sy]) [▬][SWAP]. *Answer:* 601.65

(e) The average annual rental rate per ft^2 is the *weighted mean* of the *x*-data. Press [▬][x̄w] (or [▬][Σxy] [÷] [▬][Σy] [=]). *Answer:* 8.96

Forecasting

Often you can use known data to help you forecast future results or study hypothetical scenarios.

Example: Continuing from the previous exercise, suppose that you're considering the purchase of another office with a floor area of 5,000 ft^2. Estimate the annual per-ft^2 rental for the new space. But before you do that, suppose you're told that office #5 was mismeasured—it's really 2,200 ft^2. (Alter the data accordingly.)

To make a change in a current statistical accumulation, simply use [g][Σ−] to remove the incorrect data pair values, then enter the corrected pair values.

Keystrokes	*Comments*
(DO NOT PRESS [g][CLΣ]!)	Keep the current data!
[9][•][0][3] [INPUT]	Remove the incorrect
[2][1][0][0] [g][Σ−]	data pair; then enter
[9][•][0][3] [INPUT] [2][2][0][0] [Σ+]	the correct data.

Now estimate an x-value (the new office's annual per-ft^2 rental rate) given a y-value (that office's floor area). Press [5][0][0][0] [g][$\hat{x}$,r]. *Answer:* 7.87

How good is this estimate? Press [g][SWAP] to see the correlation coefficient, r. *Answer:* -0.93

Any r-value near 1 or –1 is a good linear fit. (The minus sign indicates an inverse correlation here: The rental rate goes *down* as the floor area goes *up*.)

We hope this book is both convenient and informative for you. Please send your comments!

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