THE HP-12C MADE EASY

VOLUME II

Irregular Cash Flows, Statistics, Programming

By EDRIC CANE, Ph.D., Realtor

A STEP-BY-STEP, EASY-TO-FOLLOW SELF-STUDY COURSE:

- **FINANCIAL CONCEPTS**
- **CALCULATOR PROCEDURES**

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NOTICE

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My deepest gratitude goes to **BOB NEW** of **BOB NEW LEASING** and **BOB NEW GENERAL FINANCIAL SERVICES** in Glendale, California. He graciously accepted to check the answers to all the problems in this volume. In the process, he pointed out many passages that were not clear to him. If it wasn't clear to Bob, it wasn't good enough for me as my objective is to make it clear to the enlightened layman and practitioner, not just to the specialist in the various areas covered. Much improvement resulted from Bob New's corrections and suggestions.

INTRODUCTION

Response to Volume I has been most gratifying--especially when coming from readers who had picked up the book not really knowing what to expect and who found, it appears, more than they bargained for. Their expression of support has sustained me in the long hours of writing and re-writing required to achieve the clarity and simplicity that are my objective.

Volumes I and II are not so much a set of books as a course, and not so much a course on the HP-12C as a course on money, on exchanges of money in time. The calculator is just a tool. We need to master it only to the extent required by our purpose. Those who do not need a particular skill should skip or postpone and concentrate on those features that they need. The presentation of irregular cash flow functions has been divided into three levels to help with the selection process.

I hope readers of this second volume will select the degree of expertise they need and take the time to practice keying in and retrieving data until the process no longer represents a hurdle. Free from having to devote too much attention to the mechanical aspects, they may then concentrate on the financial situation at hand.

Despite the infinite variety of names under which applications choose to present themselves, the user is still dealing with the simple concepts and realities that underlie all time-value-of-money problems and the calculator procedures that reflect them.

With irregular cash flow data, we have, first of all, the reality of specific amounts of money changing hands at specific points in time and the need to submit that reality to the calculator whenever we seek to calculate a rate. We also have the option of imposing a rate and of asking the calculator to give the Present Value of all the cash flows we submit discounted at that rate, or the Present Value of those extra amounts that are needed at some point in time in order to achieve the desired rate. Knowledge of the Present Value of those missing amounts then leads to the actual amounts that are missing.

Time-value-of-money problems are examples and combinations of these simple themes. They will become clear to the reader as we introduce them gradually and apply them to a considerable variety of situations. Keeping the simplicity of the process in mind despite the occasional complexity of the data should allow those who have studied these pages to solve problems that may not even be included here.

Good luck.

C O N T E N T S

P R E L I M I N A R I E S

- With a basic understanding and experience of calculator procedures and regular cash flow functions, much of this second volume should be accessible to those who have not studied--or not yet studied-- Volume I. However, there is no doubt that Volume I would help.

- Units 1 to 10 are in Volume I. There are no Units 11 to 20. Volume II begins with Unit 21. Units 21 to 28 are devoted to Irregular Cash Flow functions. There are no Units 29 and 30. Statistics and programming begin with Unit 31.

- There is no need to clear previous answers from the display or previously used data from the display or from other data registers when keying in the models given in this book. Provided you do not have an unwanted letter indicator in the diplay (**PRGM, BEGIN**), the keystrokes suggested here should lead to the answer given in parenthesis. All the clearing that needs to be done, if any, is provided in the models.

- When a key is first introduced, its position is described for the reader's convenience by a code that gives its row and column number. Key 13 means row 1, column 3, or **PV**. Key 25 is in row 2 and in column 5: it is the percentage (**%**) key. This code is used by the calculator itself in programming mode.

KEY CODE BY ROW AND COLUMN.

11	12	13	14	15	16	7	8	9	10
21	22	23	24	25	26	4	5	6	20
31	32	33	34	35	36	1	2	3	30
ON	42	43	44	45		0	48	49	40

IRREGULAR CASH FLOWS

LEVEL I

Level I considers shorter term, mostly yearly data. It uses only the CFo and CFj functions that allow for a maximum of 21 different amounts.

UNIT 21

INTERNAL RATE OF RETURN (IRR) AND PRESENT VALUE

I R R E G U L A R C A S H F L O W
L E V E L O N E

MODEL

Let's consider a simple investment:
a savings account on which we make transactions once a year.

	Deposit (money I give)	Withdrawal (money I get)
Initial deposit:	-2,000.00	
Deposit, End of Year (EOY) 1:	-900.00	
Withdrawal, EOY 2:		1,200.00
Withdrawal, EOY 3:		500.00
Withdrawal, EOY 4:		1,500.00
Withdrawal, EOY 5 to close the account:		1,123.25

The deposits are money I GIVE, money I INVEST.
The withdrawals are money I GET, income I RECEIVE IN EXCHANGE.

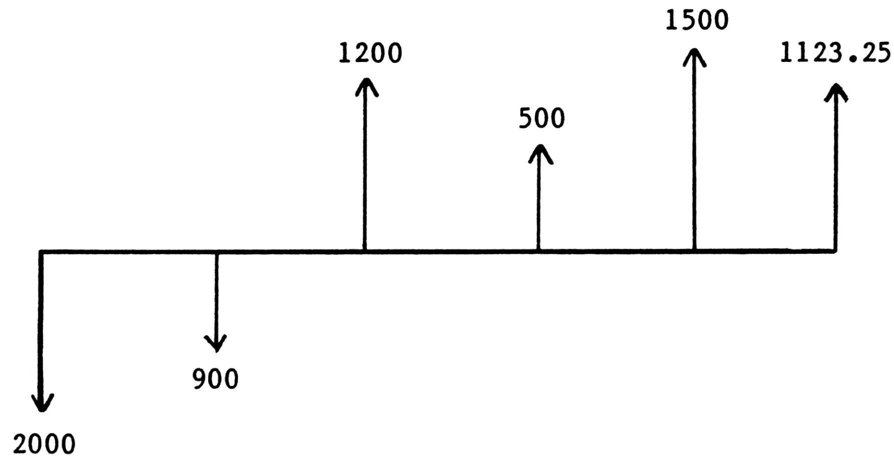
We know all the amounts that change hands, when they change hands, and whether they are given or received. We have a complete exchange of money in time. So we have all the information needed to calculate the rate of interest on the account. We just need to:

- Provide the **DATA**
- Ask the **QUESTION**

DATA		KEYSTROKES				
Time 0:	-2,000	2000	CHS	BLUE	CFo	Initial amount: **CFo**
EOY 1:	-900	900	CHS	BLUE	CFj	
EOY 2:	1,200	1200		BLUE	CFj	All other dollar amounts keyed in with **CFj**
EOY 3:	500	500		BLUE	CFj	
EOY 4:	1500	1500		BLUE	CFj	
EOY 5:	1,123.25	1123.25		BLUE	CFj	
QUESTION						
What is the rate?				GOLD	IRR	**ANSWER:** 13.25%

The word "**running**" in half-size characters flashes sometimes for quite a while as the calculation is being made.

THE CASH FLOW DIAGRAM:



Here, 13.25% is a rate of interest. If the same exchange of money was the result of another kind of investment, the rate would still be 13.25% though we might choose to call it rate of return, IRR, yield, etc.

Calculating a rate on uneven Cash Flows is a simple matter of communicating the correct Cash Flows to the calculator and letting the calculator do the rest by pressing **GOLD IRR.**

Rates are simply means of grading or 'rating' exchanges of money in time. Whenever we want to find a rate we should think "Cash Flow". Under normal conditions, if we can establish the Cash Flows and communicate them to the calculator, the calculator will compute the rate.

Except when we are considering a 'simple interest' agreement that specifies that interest is to be earned without being either distributed or allowed to earn interest--a situation that is unusual except for rather short term contracts and is very misleading over extended periods as it does not fully take into account the time value of money--, the rate considered under a variety of names is a compound interest rate, and the change in names (IRR, APR, interest, yield, rate of return, etc.) reflects the nature or origin of the Cash Flows or the frequency of the compounding process, not the calculation itself.

PRIORITY: To acquire the ability to key in, check, and modify irregular Cash Flows with ease and confidence.

THREE DATA KEYS

NPV		**IRR**
PV	PMT	FV
CFo	**CFj**	**Nj**

BLUE CFo CASH-FLOW-ZERO [BLUE and the PV key]

Cash Flow zero is used to key in the INITIAL AMOUNT, not because it is negative--it is not necessarily so--but because it occurs at time ZERO.

Its main function, apart from communicating the initial amount to the calculator, is to get a new irregular Cash Flow process going.

Using **BLUE CFo** is **THE ONLY CLEARING THAT NEEDS TO BE DONE** when beginning a new irregular cash flow calculation.

BLUE CFj CASH-FLOW-J [BLUE and the PMT key]

Cash Flow j is used to key in, in chronological order, all other dollar amounts, whether positive or negative.

The 'j' stands for '1', then '2', then '3', etc. to a maximum of 20.

We have:			
	CF0	Cash Flow Zero.	→ Keyed in with **BLUE CFo**
	CF1	Cash Flow One.	}
	CF2	Cash Flow Two.	}
	CF3	Cash Flow Three.	} Keyed in with **BLUE CFj**
	-		}
	-		}
	CF20	Cash Flow Twenty.	}

BLUE Nj Nj [BLUE and FV key]

With Nj, any CFo and CFj value may be repeated up to 99 times. For instance, we may key in 12 equal amounts of $500.00 as follows:

500 BLUE CFj 12 BLUE Nj

Some people, dealing mostly with short term annual data, may not need the Nj function. For others, it is indispensable. We will present it in detail in **IRREGULAR CASH FLOW: LEVEL II.**

TWO QUESTION KEYS

GOLD IRR INTERNAL RATE OF RETURN [GOLD and FV key]

This **QUESTION** tells the calculator to calculate a **RATE.**

In the case of our Savings Account, it is easy to understand why the rate is called an **INTERNAL** rate of return: it takes into consideration only the amounts that are in the account for the period of time that they remain in the account.

The Savings Institution is not concerned with the $900 deposit during the first year. Maybe we didn't have the $900 when we invested the original $2,000.00, or maybe we kept it in cash or had it invested at a different rate. The Savings Institution that is giving us 13.25% makes no assumption about the $900 until we deposit it.

Similarly, the 13.25% interest rate completely disregards what may happen to the $1,200 after we have withdrawn it. We may use it to pay off a debt, to buy food, or to invest at 5% interest or 25% interest. The Savings Institution takes into account only the amounts that are **in** the account for the period of time that they remain **in** the account.

Though the word we choose to use may change according to the nature of the Cash Flows, the Internal Rate of Return is the common expected yardstick used to measure and compare exchanges of money in time.

GOLD NPV NPV [GOLD and PV key]

This **QUESTION** calculates a **DOLLAR AMOUNT**. It is a **PRESENT VALUE:**

- The **PRESENT VALUE** of all irregular cash flows keyed in discounted at the chosen rate which we store in **i.**

- The **NET PRESENT VALUE:** if we submit both positive and negative Cash Flows, then GOLD NPV calculates the Present Value of the positive amounts **minus** the Present Value of the negative amounts, or **NET** PRESENT VALUE. This gives its name to the function.

- The **PRESENT VALUE** of any amount that is missing if one is to achieve the rate stored in **i.** (In this case, the Present Value has the **wrong sign**).

In brief, **GOLD NPV** is:
- The Present Value of what we have,
- The Present Value of what we do not have.

THE SIGN REQUIREMENT

In the current example (Model page 1), we use the **CHS** function [Key location 16] to key in some amounts as negative numbers.

In order to calculate a rate, the calculator needs to know which amounts are being exchanged for which. As with regular cash flow data, the **SIGN REQUIREMENT** provides that information. Amounts that are on opposite sides of a transaction must have a different sign.

One way of implementing the sign requirement is to apply the "MONEY IN - POSITIVE, MONEY OUT - NEGATIVE" approach. In investment situations, this means beginning with negative amounts (the investment) followed by positive amounts (the return), and this is what we we will do in such situations. We will also routinely continue to key in negative amounts first followed by positive amounts even when the problem is seen from the borrower's point of view.

So our approach is the opposite of what we did with regular cash flow data: (PV positive, PMT and FV negative).

If we key in all the dollar amounts as positive and then ask the calculator to calculate a rate (**GOLD IRR**), we get an **ERROR** sign: we have not provided the calculator with an exchange of money and there is no rate to calculate. We do, however, calculate a Present Value (**GOLD NPV**) with all Cash Flows keyed in with the same sign.

THE TIME REQUIREMENT

Irregular Cash Flows are irregular in amount only: the payments must still occur on a regular basis: every week, every month, every quarter, every year. If the payments do not occur on a regular basis, then we need to choose a regular period small enough to accomodate all the data that we have, and fill in with amounts of zero for those times when no money changes hand.

As with regular cash flow functions, the rate, the period at which the Cash Flows occur, and the compounding period must be on the same time scale: if the dollars keyed in are yearly amounts, then the rate is a yearly rate yearly compounded; if the amounts are monthly, then the rate is a monthly rate and needs to be multiplied by 12 to provide an annual, monthly compounded, rate.

The calculator requires consistency precisely because **TIME IS IN THE MIND OF THE USER**. The calculator does not know what the period is anymore than it knows that we are talking about dollars rather than yens or francs. We know, and this allows us to interpret the answer accordingly. The same numbers that, interpreted as yearly amounts, result in a yearly rate of 4%, result in a monthly rate of 4% if we interpret them as being monthly amounts. That 4% monthly rate, multiplied by 12, gives an annual rate of 48%, monthly compounded.

ILLUSTRATION AND COMMENTS

We have already acquired a powerful tool: the ability to calculate the rate of return on a considerable variety of investment and loan situations. Let's illustrate that variety and practice the procedure before we expand on the options available to us.

ILLUSTRATION: SEMI-ANNUAL PAYMENTS

You borrow $7,000:	7000 CHS BLUE CFo	
You pay back with semi-annual payments as follows:		
Year 1: 2 payments of $1,000:	1000 BLUE CFj BLUE CFj	
Year 2: 2 Payments of $1,500:	1500 BLUE CFJ BLUE CFj	
Year 3: 2 Payments of $2,000:	2000 BLUE CFj BLUE CFj	
What is the semi-annual rate?	GOLD IRR	(6.74%)
Annual rate of interest:	2 x	(13.47%)

Remarks:

- If we key in money received as positive and money paid out as negative, the loan amount is positive and all the yearly payments negative. We get the same answer if we reverse the signs--calculating the rate, as it were, from the lender's point of view. This eliminates the need to press **CHS** with each new payment amount. We will generally key in initial amounts as negative even when they do not correspond to money paid out.
- Here, we have semi-annual data, so the IRR provided by the calculator (6.74%) is a semi-annual rate that we multiply by 2 to get an annual rate. The calculator does not know the period of the cash flows anymore than it knows whether we are keying in dollars, francs, or yens. We do, and we interprete the answer accordingly.
- Pressing **BLUE CFj** twice is a convenient means of keying in 2 cash flows of equal value--no need to key the number in twice. We could, of course, use **BLUE Nj** instead, and would do so if we had more than 2 or 3 equal amounts or if we wanted to use fewer Register memories.

ILLUSTRATION: 5-YEAR PROJECTION

An investor is buying an income property and establishes the following net projection for the 5-year holding period:

Year	Cash flow	Keystrokes				
0	- 500,000	500000	CHS	BLUE	CFo	
1	-30,000	30000	CHS	BLUE	CFj	
2	0	0		BLUE	CFj	
3	70,000	70000		BLUE	CFj	
4	- 50,000	50000	CHS	BLUE	CFj	
5	1,000,000	1000000		BLUE	CFj	
What is the IRR?		GOLD	IRR			(14.48%)
		DO NOT CLEAR				

- Note the importance of the correct sign and the fact that only the initial cash flow ($500,000) is keyed in with CFo, not because it is negative, but because it occurs at time zero!

- We key in 0 for year 2 where no money changes hands. Having to wait these extra years before we get back $70,000 and $1,000,000 affects the rate of return on the investment. Once we have selected a time period, here the year, something has to be communicated for each period, whether it is a positive amount, a negative amount, or a dollar amount of zero.

- The calculator has no way of knowing what the cash flows actually represent: if the cash flows are the same, the answer is the same whether the cash flows represent deposits and withdrawals from a savings account, actual or projected investments and returns on a business venture, before-tax or after-tax cash flows, etc. We know what the cash flows represent, and we interpret and use the rate accordingly.

- It is typical of irregular cash flow problems that a lot of work goes into establishing the bottom line. Here, the net projection that we keyed in is the result of many assumptions and calculations by the investor: assumptions concerning appreciation and inflation, calculations to establish the balance on the loans, yearly depreciation, and other tax consequences.

There are courses, books, and forms that cover these topics for various kinds of investments, in the various ways required by different professions, that provide the guidelines and prudent ratios, and that are updated with each change in the tax laws. Our essential objective is not to duplicate these presentations, but to provide the tool and the understanding that can be applied at various levels of the process, in any of these circumstances.

See UNIT 26 for some of the calculations that might go into establishing this 5-year cash flow projection.

RECALLING THE DATA

With the previous data still in the calculator, let us explore Register memories 0 to 5 as well as regular cash flow functions **n** and **i**:

	Display	
RCL 0	-500,000.00	Cash Flow zero: CFo.
RCL 1	-30,000.00	Cash Flow one: CF1.
RCL 2	0.00	Cash Flow two: CF2.
RCL 3	70,000.00	Cash Flow three:CF3.
RCL 4	-50,000.00	Cash Flow four: CF4.
RCL 5	1,000,000.00	Cash Flow five: CF5.
RCL n	5.00	Number of CFj entries.
RCL i	14.48	The Internal Rate of Return

We have discovered three facts:

1) The dollar amounts keyed in with CFo and CFj are automatically stored in Register memories 0, 1, 2, 3, etc. (R0, R1, etc).

Using CFo and CFj automatically erases the content of the Register memories that are taken over by irregular cash flow data: keying in the data for the present problem automatically erases any number previously stored in Register memories 0 to 5.

2) **n** automatically registers the number of **CFj** values that have been keyed in. In the current problem, we have 6 Cash Flow values but only 5 **CFj** amounts: the value in **n** is 5, not 6.

Pressing BLUE CFo puts 0 in **n**.
Pressing BLUE CFj adds 1 to **n**.

So the number in **n** is automatically adjusted to the "j" value of the last Cash Flow keyed in:
0 in **n** for CF0 stored in R0,
1 in **n** for CF1 stored in R1,
2 in **n** for CF2 stored in R2, etc.

A 5 in **n** just informs the calculator that it should seek irregular Cash Flow data all the way up to Register memory 5, in the 6 memories from 0 to 5. Recalling **n** is a convenient way to check that we have the right number of Cash Flows before we question the calculator for the rate.

We shall see later that the ability to manipulate **n** gives us considerable control over irregular Cash Flow data.

3) The RATE given as the answer by the calculator is automatically stored in the **i** memory from which it can be recalled at any time.

We have 20 register memories, memories 0 to 19. In fact we can have one cash flow zero value, stored in Register 0, and 20 CFj values, stored in Register memories 1 to 19, and automatically in the FV memory for the 20th CFj amount.

With the use of the Nj function, each one of these amounts (including the CFo amount) can be repeated 99 times for a total of 2,079 cash flows of up to 21 different values. (See Level II).

NOTE ON REGISTER MEMORY ALLOCATION.

We do not always have 20 Register memories available for irregular Cash Flow data.

Programs keyed into the calculator also claim Register memories beginning with memory 19 and moving down all the way to memory 7 as required. Memories claimed by user-written programs are no longer available as Register memories as long as the program is retained. With a 99 step program in the calculator only memories 0 to 6 remain available for irregular Cash Flow data, or for any other purpose for that matter.

An **Error 6** signal appears in the display when an attempt is made to key in a CFj value when the required Register memory does not exists. You may then claim the Register memories back for irregular cash flow data by CLEARING THE PROGRAMS using functions that will be studied in UNIT 32:

Keystrokes	Key location	Function
CLx	35	Clears **Error** sign.
GOLD P/R	f 31	Switches into Program mode.
GOLD Clear-PRGM	f 33	Clears programs. (Press 2 keys only)
GOLD P/R	f 31	Switches out of Program mode.

NOTE: You have now lost all user-written programs.

You may now press **BLUE CFj** to key in the CFj value that prompted the error signal and that has now reappeared in the display.

CHANGING THE DATA

Dollar amounts that have been keyed in using CFo and CFj can be overwritten using the **STO** key directly in the Register memory where they were automatically stored. There is no need to Recall the original number first, as that does not take the number out of the Register memory.

Data can be changed for any reason: an error was made, we have changed our mind, or we want to test a different assumption. It can be changed before or after IRR or NPV have been calculated.

MODEL

Consider the following yearly investment projection:

0	-120,000	120000	CHS	BLUE	CFo	
1	20,000	20000		BLUE	CFj	
2	30,000	30000		BLUE	CFj	
3	40,000	40000		BLUE	CFj	
4	50,000	50000		BLUE	CFj	
Rate?		GOLD	IRR			(5.61%)

CHANGING THE REQUIREMENTS

The investor is not satisfied with a rate of 5.61% and decides to increase his return by requesting $75,000 in the 4th year instead of $50,000. How does that affect the rate?

75000	STO	4	
GOLD	IRR		(11.32%)

The investor now decides that he will go along with the transaction only if he can buy these cash flows (including the $75,000 for year 4) for $100,000 instead of $120,000. What is his rate of return now?

100000	CHS	STO	0	
GOLD	IRR			(18.63%)

We cannot correct this amount with **BLUE CFo** as this would initiate a new irregular cash flow sequence, invalidating in the process all the CFj values already keyed in. (They could be re-validated by pressing 4 back into **n**. See later for details).

CORRECTING A MISTAKE

Let's key the initial data back in and make a MISTAKE by omitting the **CHS** required for CFo:

120000	BLUE	CFo	
20000	BLUE	CFj	
30000	BLUE	CFj	
40000	BLUE	CFj	
50000	BLUE	CFj	
GOLD	IRR		**Error 7**

With no negative amount, there is no exchange of money in the calculator, so no rate can be calculated. The error sign tells us that we have requested of the calculator something that it cannot perform.

Before we do anything else, we must **CLEAR THE ERROR SIGN,** as the first key we press will clear the error sign anyway, and would not be operative for any other purpose. We may then explore the data and correct the mistake:

CLx		Clears **Error** sign. **IMPORTANT FIRST STEP.**
RCL	0	120,000.00: the mistake is discovered.
CHS		Mistake is corrected in display.
STO	0	-120,000.00 now in Register 0.
GOLD	IRR	(5.61%)

Other data errors can be corrected in the appropriate Register memory.

If one Cash Flow is omitted by error, then all subsequent Cash Flows are in the wrong memory and need to be corrected. A convenient procedure then is to key back in **n** the number of the **last correct Cash Flow J value,** and to key the missing amount and subsequent Cash Flows back in with BLUE CFj.

REMEMBER

The ability to check data is essential, as knowing that we have the correct data is the only assurance that we have the correct answer.

Once dollar amounts have been keyed in using CFo and CFj, they are most conveniently changed directly in the Register memories where they are stored.

PRACTICE

It is important to acquire considerable fluency in the mechanics of keying in, checking, and changing data. Calculating the rate will serve here as a check that the correct data has indeed been communicated to the calculator.

Most common mistakes:

- Forgetting to key in the initial investment as negative.
- Pressing CFo instead of CFj for a subsequent amount.
- Forgetting to press the BLUE key with CFo or CFj.
- Somehow changing the value in **n**.
- Keying in a wrong number, in amount or in sign.

Check the Register memories, check **n**. The answer you get is always the correct answer for the data that you have in the calculator.

1) Calculate the rate of return on the following yearly data:

0	-200,000
1	20,000
2	25,000
3	30,000
4	35,000
5	240,000

(14.34%)

- Recall the dollar amounts, the number in **n**, and the rate.

2) Calculate the rate of return on the following yearly data:

0	-300,000
1	-40,000
2	-10,000
3	0
4	20,000
5	50,000
6	700,000

(14.80%)

- Check the numbers in **n** and in Register memories 0 to 6.
- Rate of return if initial investment is reduced to $250,000. (18.03%)
- Rate of return if initial investment is reduced to $200,000. (22.03%)

3) A business investment offers the following yearly cash flows.
- What is the rate of return?

0	- 50,000
1	- 50,000
2	- 50,000
3	90,000
4	90,000
5	90,000

21.64%)

4) In looking back on a venture capital investment, we establish the following cash flows. What is the rate of return?

0	- 20,000
1	- 35,000
2	- 50,000
3	0
4	0
5	300,000

(31.84%)

- Rate of return if the amount for year 5 is cut in half? (10%)
- What if the return for year 5 is $200,000? (18.66%)

5) An investment opportunity suggests the following cash flows:

0	-350,000	Initial investment.
1	-50,000	
2	0	
3	30,000	
4	60,000	
5	90,000	
6	500,000	

- What is the rate of return to the investor? (10.32%)
- What is the rate of return if the initial outlay can be reduced from $350,000 to $300,000? (13.14%)

6) A $500,000 loan is paid back with yearly payments as follows:

End of Year 1: 40,000
End of Year 2: 50,000
EOY 3: 60,000
EOY 4: 70,000
EOY 5: 580,000 balloon payment.

- What interest is being charged? (11.56%)
- What is the rate of return to an investor who purchases the loan at a discount for $440,000? (15.03%)
- What is the rate if the loan is purchased for $400,000? (17.71%)

7) Consider the following investment projection.

0	-900,000
1	100,000
2	200,000
3	300,000
4	400,000
5	500,000
6	600,000

- What is the internal rate of return? (23.01%)
- What is the IRR if you invest $1,200,000 instead of $900,000? (14.33%)
- Check the dollar amounts, the value stored in **n**, the rate of return.

PRESENT VALUE GOLD NPV

Calculating the Present Value of irregular cash flow data is simple. We just:

1) Key in the cash flows as already practiced.
2) Key in the discount rate in **i.**
3) Press **GOLD NPV.**

MODEL

Present Value of the following yearly cash flows discounted at 11%:

0:	$300,000	300000	BLUE	CFo	
1:	90,000	90000	BLUE	CFj	
2:	120,000	120000	BLUE	CFj	
3:	150,000	150000	BLUE	CFj	
4:	200,000	200000	BLUE	CFj	
11% discount rate:		11	i		
Present Value:		GOLD	NPV		($719,900.68)

By simply changing the rate, we may calculate the Present Value of the same cash flows discounted at a different rate.

14% discount rate:	14	i	
Present Value:	GOLD	NPV	($690,945.25)
18% discount rate:	18	i	
Present Value:	GOLD	NPV	($656,905.72)

- Here all cash flows are positive. There is no exchange of money.

- If there are both positive and negative cash flows, pressing NPV calculates the **Net Present Value:** the Present Value of the positive amounts minus the Present Value of the negative amounts. This gives its name to the function. See later for comments on Net Present Value.

- The answer given by the calculator is the Present Value of <u>all</u> the cash flows submitted, <u>including</u> the initial $300,000 amount which is itself a Present Value and contributes $300,000 to the final answer. This is in contrast to what happens with REGULAR cash flow data where pressing PV calculates the Present Value of the PMT and FV data only, ignoring and overwriting any data that might be stored in PV.

- The NPV answer is stored in PV. If cleared from the display, NPV can be recalled directly from the PV memory:

Keys	Display
RCL 0	$300,000: initial cash flow unchanged in R0.
RCL PV	$656,905.72: last NPV value retrieved from PV.

- With a discount rate of zero, all the irregular cash flows are added up without being discounted. This is a convenient way of just finding the sum of all the cash flows:

Keys	Display
0 i	
GOLD NPV	$860,000: sum of all the cash flows.

- We may press **minus** to find the difference between two Present Values calculated in immediate succession on the same cash flows:

Keys	Display
11 i GOLD NPV	$719,900.68
14 i GOLD NPV	$690,945.25
-	$28,955.42

There is a $28,955.42 difference in the Present Values when the cash flows are discounted at 11% and 14%.

Because it is sometimes so difficult to decide on the discount rate one should apply, it is frequently convenient to calculate the present values at two possible discount rates, one on the low side, the other on the high side, and to find out how sensitive the cash flows are to a change in the discount rate. The more cash flows are postponed in time, the more sensitive they are to the rate at which they are being discounted: with cash flows that are 30 years in the future, a slight change in the discount rate has considerable impact on the Present Value.

ILLUSTRATION

What loan amount can be obtained with the following semi-annual payments if the interest rate is 11.25%?

Payments year 1:	$20,000
Payments year 2:	$25,000
Payments year 3:	$30,000
Payments year 4:	$35,000

```
    0  BLUE  CFo
20000  BLUE  CFj   BLUE  CFj
25000  BLUE  CFj   BLUE  CFj
30000  BLUE  CFj   BLUE  CFj
35000  BLUE  CFj   BLUE  CFj

11.25  ENTER  2  ÷    i

GOLD NPV
```

The answer is $169,035.84.

- Of course, the discount rate stored in **i** and the period separating the various cash flows must represent the same time span: with yearly cash flows, the rate must be a yearly amount, with semi-annual cash flows, as here, the rate in **i** must be semi-annual.

- It is important to begin with **0 BLUE CFo**. With payments on a loan made in arrears, the first payment occurs six months after the loan has been made. Pressing **0 BLUE CFo** informs the calculator that the first actual dollar amount keyed in does not occur at Time 0 but at the end of the first period. On being presented with cash flow data, we often need to analyze the circumstances to determine whether or not the initial amount occurs at Time 0.

Find the total amount paid back and the total amount of interest charged over the life of the loan:

Total amount paid:	0 i GOLD NPV	($220,000.00)
Total amount of interest:	-	($50,964.16)

- Discounting at a rate of 0% adds all the irregular cash flows keyed in without discounting them: we get the total amount of money paid out over the life of the loan ($220,000). Pressing MINUS (-) calculates the difference between that answer and the previous NPV value of $169,039.64, here the difference between the loan amount and the total payments on the loan: the total amount of interest.

- There are infinite numbers of applications that consist of a Present Value calculation. Some will be explored after multiple CFj values have been studied (Level II). Calculating a cash equivalent price, a cost equivalent price, or, as here, finding the loan amount that given cash flows can purchase, are all Present Value calculations.

PRACTICE: PRESENT VALUE

8) What is the Present Value of the following yearly income stream?

Time 0:	$50,000
End Of Year 1:	$10,000
End Of Year 2:	$20,000
End Of Year 3:	$30,000
End Of Year 4:	$40,000
End Of Year 5:	$50,000

(See bottom of page for answer)

9) Present Value of income stream at 8% and 12% discount:

Time 0:	$50,000
EOY 1:	$100,000
EOY 2:	$150,000
EOY 3:	$200,000
EOY 4:	$250,000
EOY 5:	$300,000
EOY 6:	$350,000

($1,038,451.66 at 8%, $907,649.31 at 12%)

10) Present Value at discount rates of 7% and 10%. What is the difference?

0	$100,000
1	$80,000
2	$60,000
3	$40,000
4	$20,000

($275,082.50 and $266,026.91, a $9,055.59 difference)
The difference is small because the largest amounts come in early and are less affected by the changing discount rate than the smaller amounts that come late. In fact, the largest amount of all occurs at Time 0 and is not discounted. It contributes a full $100,000 to each Present Value and nothing at all to the difference).

11) Present Value of same income stream as in (10) at the same discount rates, but with every cash flow postponed 1 year ($100,000 begins at End Of Year 1 instead of at Time 0). Give the difference as above.
($257,086.45 and $241,842.65 for a $15,243.80 difference)

12) Present Value at 9% discount and 0%. What is the difference?

EOY 1:	$2,000,000
EOY 2:	$3,000,000
EOY 3:	$4,000,000
EOY 4:	$5,000,000

($10,990,762.34 and $14,000,000 for a $3,009,237.66 difference)
Discounting at 0%, of course, just adds up all the cash flows. Finding the difference as above gives the total amount of the discounts.

(8) Answer. There is no answer as a Present Value question is meaningless unless a discount rate is specified. Using a 15% discount rate the answer is $141,127.98)

FUTURE VALUE

MODEL

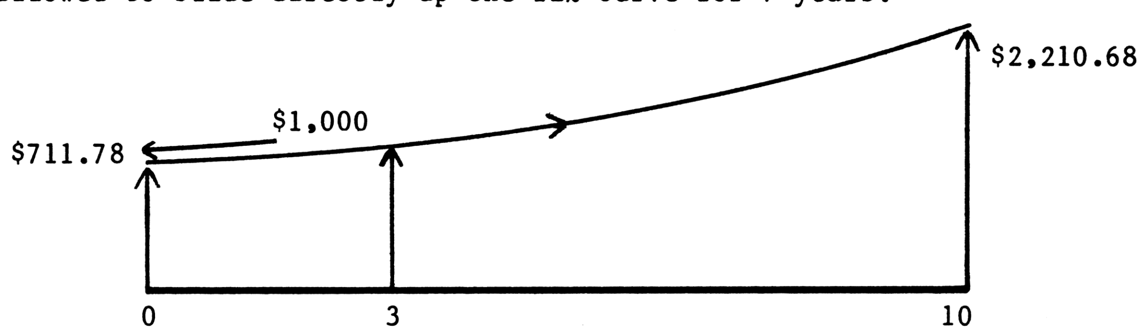
You invest the following yearly amounts in a Savings account earning 12% interest. Find the accumulated value of the investment after 10 years.

Time 0:	$8,000	8000	BLUE	CFo			
EOY 1:	3,000	3000	BLUE	CFj			
EOY 2:	5,000	5000	BLUE	CFJ			
EOY 3:	1,000	1000	BLUE	CFj			
EOY 4:	9,000	9000	BLUE	CFj			
EOY 5:	4,000	4000	BLUE	CFj			
12% rate:		12	i				
Present Value of amounts:			GOLD	NPV			($23,365.69)
Future Value in 10 years:		10	n	0	PMT	FV	($72,570.29)

In Volume I, Unit 6, we saw that keying in a rate was like communicating a curve to the calculator, a curve that automatically adjusts cash flows to changes made in the time at which they occur. Once we have agreed to the rate of exchange defined by the curve, the only thing Cash Flows can do is slide up and down that curve as the money shifts up or down in time, without ever changing in value. The change allowed by the curve automatically adjusts the amount to the change in time so as to preserve the constant value.

Here, in order to slide all our cash flows up the 12% curve to the end of year 10, we first have them all slide down the same curve to Time 0. This consolidates all the cash flows into one amount of $23,365.69. We then slide this amount along the same 12% curve all the way up to the end of the 10th year to find the Future Value.

Let's see what happens in the process to our $1,000 cash flow for year 3. It first slides down to Time 0 and contributes $711.78 to the total Present Value. The $711.78 amount is then made to slide back up the same curve to adjust for a 10 year shift in time. This changes the amount to $2,210.68 which is what the $1,000 cash flow contributes to the total Future Value. It is exactly the same amount as if $1,000 had been allowed to slide directly up the 12% curve for 7 years.



So, to calculate the Future Value of a series of irregular cash flows invested at a given rate, we:

1) Key in the cash flows and the rate.
   What else could we do?
   The calculator needs to know these amounts.

2) Press **GOLD NPV** to calculate their Present Value.
   What else could we do?
   This is the only question that calculates a dollar amount in reference to irregular cash flow data.

3) Press the desired term in **n**, and, with 0 in PMT, proceed with a REGULAR cash flow Present Value--Future Value transfer.

In proceeding with the calculation, it is important to remember that the NPV amount is automatically stored in the Present Value memory.

We have assumed yearly payments and a 12% yearly compounded rate. If the compounding period of the rate does not match the period defined by the payments, then the rate needs to be adjusted. See Index under "Compounding".

The same approach will be used in **UNIT 22** when we consider NPV as the Present Value of a missing amount. This is not surprising, as indeed the Future Value calculated here is the missing amount that we need to receive 10 years in the future in order to earn 12% on the investment made during the first 5 years.

(See **UNIT 22** for details and practice)

UNIT 22

CALCULATING A MISSING AMOUNT

NPV AS PRESENT VALUE OF MISSING AMOUNT

So far, we have solved the following problems:

1) Knowing the Cash Flows of a complete exchange of money in time we calculated the RATE (GOLD IRR).

2) Knowing Cash Flows and a discount rate, we calculated the PRESENT VALUE of the Cash Flows discounted at the given rate (GOLD NPV).

   One calculation gave us a rate, the other a dollar amount.

3) We also calculated a Future Value, the accumulated value of a series of cash flows invested for a given number of years at a given rate.

   We now want to expand on that last calculation by seeking to calculate any missing cash flows. By "missing" cash flows we mean the amount or amounts needed at some point in time if we are to get the desired rate of return.

There are any number of problems where we know some of the cash flows, and we know the rate that we want or that has been promised. But to get that rate we need to receive more money than provided by those cash flows that are known (or invest less than assumed by the cash flows that we are considering).

How do we calculate the amount (or amounts) that we need to add to those cash flows that we already have if we are to get the required rate?

The key is in our alternative interpretation of the NPV function:

NPV as
PRESENT VALUE OF THE MISSING AMOUNTS

FINDING A MISSING AMOUNT

MODEL

You are considering the purchase of an investment property. Net investment and income projection for the first 5 years are as follows:

Initial investment:	-350,000	
End of Year 1:	-95,000	
EOY 2:	30,000	
EOY 3:	40,000	
EOY 4:	50,000	
EOY 5:	60,000	
EOY 6:	?	(Amount unknown)

If you sell the property at the end of Year 6, what total net amount must you receive for that year (Net income and reversion from sale) in order to get a 25% rate of return on your investment?

SOLUTION

1) Key in the Cash Flows:	350000 CHS BLUE CFo 95000 CHS BLUE CFj 30000 BLUE CFj 40000 BLUE CFj 50000 BLUE CFj 60000 BLUE CFj	
2) Key in the Rate:	25 i	
3) Press **GOLD NPV** to calculate the PV of the missing amount:	GOLD NPV	($346,179.20)
4) Transfer that PV to its rightful place in time (here a simple PV-FV transfer)	6 n 0 PMT FV	($1,320,568.85)

The key to understanding this procedure is to remember that

NPV gives us the PRESENT VALUE OF THE MISSING AMOUNTS

Let's consider this procedure one step at a time. Initially, we are simply taking the obvious steps suggested by our understanding of the fundamental calculator procedure: **DATA** then **QUESTION**. We key in the data (**Cash Flows and rate**), and ask the only question that makes sense (**NPV**).

STEP ONE. We know some Cash Flows: let's key them in. Clearly the calculator needs to have that data before it can solve the problem.

350000	CHS	BLUE	CFo
95000	CHS	BLUE	CFj
30000		BLUE	CFj
40000		BLUE	CFj
50000		BLUE	CFj
60000		BLUE	CFj

STEP TWO. We know the rate, let's key it in. The calculatator also needs to know that we require 25%, not just 22% or 20%.

25	i

STEP THREE. We want to calculate a dollar amount: let's press **GOLD NPV** as this is the only question we may ask about irregular Cash Flow data that gives a dollar amount.

GOLD	NPV	($346,179.20)

$346,179.20 is not the answer. It is **THE PRESENT VALUE OF THE MISSING AMOUNT** discounted at the rate stored in **i** (25%).

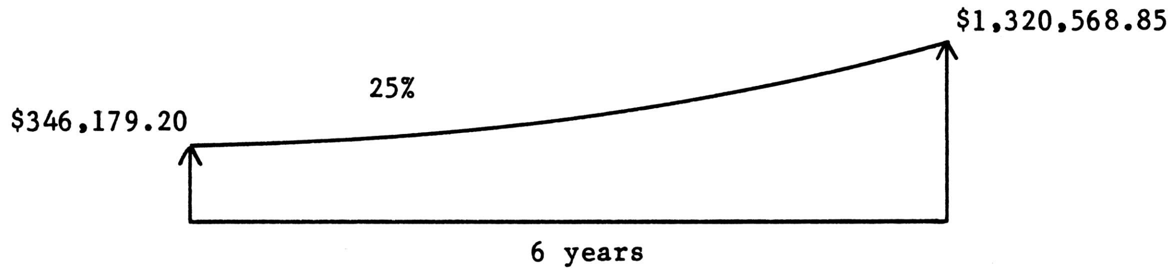
STEP FOUR. This is where the crucial transformation occurs. We no longer need the irregular cash flows initially communicated to the calculator: we could clear them if we so wanted, though we may just ignore them.

We are left with a very simple problem if we just accept to consider it on its own merits: we know the Present Value, discounted at 25%, of an amount that occurs 6 years in the future. Calculating that amount is a simple Present Value--Future Value calculation that we may solve with the **REGULAR CASH FLOW** functions of the calculator:

- 346,179.20 is the **PRESENT VALUE** and is **ALREADY STORED IN PV.**
- The 25% rate is already in **i.**
- We just put the 6 year time span in **n**, make sure there is no leftover data in the **PMT** memory, and press **FV** to get the answer.

6	n	
0	PMT	
FV		($1,320,568.85)

If we receive $1,320,568.85 for the 6th year we will get a 25% return on our investment.



COMMENTS:

- $1,320,568.85 is the total net amount for year 6: it includes net income for that year plus the net reversion on sale of the property (selling price minus selling costs, balance due on loans, and tax payments).

- Note the ease with which we switch from an irregular Cash Flow problem to a regular Cash Flow situation. With the correct data already in **i** and **PV**, we just adjust **n** to the regular Cash Flow requirement, make sure no leftover data in **PMT** interferes, and press **FV**.

- The number left in **PV** when we press **GOLD NPV** is the Present Value of the missing amount **WITH THE WRONG SIGN** (a negative value if the missing amount is positive, a positive one if the missing amount is negative). Having the wrong sign is most convenient here as calculating the Future Value automatically changes the sign back to the correct value - from negative to positive in our case. (We are not concerned with the routine "PV - positive" approach we have followed in most instances: this was for our convenience, and the opposite works just as well).

- This procedure--which we summarize by stating that **NPV** calculates the Present Value of the missing amounts--can be applied anytime we want to calculate a missing cash flow or a series of missing cash flows provided we have enough information about the missing amounts to allow us to calculate them from a knowledge of their Present Value. In particular, we need to know **WHEN** the missing cash flows occur, otherwise there are any number of possible answers.

- As we expand on this procedure and its applications we shall see what considerable power it puts at our disposal:

    - With **GOLD IRR** we are able to calculate a rate when we know the complete exchange of money in time.

    - With **GOLD NPV** we acquire the power to calculate missing amounts from a knowledge of the **RATE**, of **SOME CASH FLOWS**, and of **WHEN** the missing amounts occur.

We have now acquired the power to "balance the books" on irregular cash flow exchanges, whether the unknown number is a rate or a dollar amount somewhere during the transaction.

ILLUSTRATIONS:
NPV as PV of missing amounts

TESTING DIFFERENT RATES

Consider the following investment and income projection on a an appartment house purchase. What net reversions on sale at the end of year 5 must we add to the net income for that year to have a 20% and a 24% rate of return?

Investment:	-250,000
Net income EOY 1:	10,000
Net income EOY 2:	20,000
Net income EOY 3:	30,000
Net income EOY 4:	40,000
Net income EOY 5:	50,000 + unknown reversion on sale.

250000	CHS	BLUE	CJo	
10000		BLUE	CFj	
20000		BLUE	CFj	
30000		BLUE	CFj	
40000		BLUE	CFj	
50000		BLUE	CFj	
20	i	GOLD	NPV	($171,032.66)
0	PMT	FV		(Reversion for 20% return: $425,584.00)
24	i	GOLD	NPV	
FV	FV			(Reversion for 24% return: $525,403.65)
-				(Difference in reversions: $99,819.65)

- Notice the ease with which we may test various rate assumptions: there is no need to key the Cash Flows back in for each run.

- Here the missing amount is not a complete Cash Flow but only a part of what is received for year 5. GOLD NPV gives the Present Value of what is missing with no restrictions on what can be missing: it can be a single Cash Flow, a group of Cash Flows, or part of one or more Cash Flows.

- Exceptionally here, the value for **n** (5) is the same for the irregular Cash Flow part of the calculation and for the regular Cash Flow sequence. This is not generally the case.

- Notice the need to press FV twice whenever we calculate FV immediately after having calculated NPV. The first time, NPV is stored in FV. The second time around FV is interpreted as a question.

CASH FLOW MISSING IN MIDDLE OF KNOWN AMOUNTS

Consider the following Cash Flows. What amount must we have for End Of Year 3 if we want a 19% return?

Initial investment:	-500,000	500000 CHS BLUE CFo	
Investment EOY 1:	-150,000	150000 CHS BLUE CFj	
EOY 2:	100,000	100000 BLUE CFj	
EOY 3:	?	0 BLUE CFj	
EOY 4:	700,000	700000 BLUE CFj	
19% rate:		19 i	
PV of missing amount:		GOLD NPV	(206,365.81)
Missing amount EOY 3:		3 n 0 PMT FV	($347,759.21)

- The missing amount ($347,759.21) is in the middle of the known Cash Flows. Its place is initially reserved by keying in a value of zero.

INITIAL CASH FLOW MISSING

An investment promises the following returns. What should you be willing to invest in order to get a 17% rate of return?

Initial investment unknown:	0 BLUE CFo	
EOY 1: $55,000 return:	55000 BLUE CFj	
EOY 2: $77,000 return:	77000 BLUE CFj	
EOY 3: $99,000 return:	99000 BLUE CFj	
EOY 4:$400,000 return:	400000 BLUE CFj	
Required return:	17 i	
PV of missing amount: or PV of amounts submitted:	GOLD NPV	($378,530.79)

The required investment is $379,530.79.

- Here the missing amount is a Present Value. So the two interpretations of NPV (as PV of all the Cash Flows submitted and PV of those Cash Flows that are missing) are interchangeable:

  a) $378,530.79 is the Present Value of the missing amount which is also a Present Value. No transfer in time occurs, and NPV directly gives us the missing amount, <u>with the wrong sign</u>.

  b) If the Present Value of all the Cash Flows is $379,530.79 when discounted at 17%, then I will get a 17% return if I invest that amount in the transaction.

MISSING AMOUNTS SPREAD OVER TERM OF INVESTMENT

Consider the following projections:

Initial investment:	$35,000	35000 CHS BLUE CFo		
Income year 1:	6,000	6000 BLUE CFj		
Income year 2:	9,000	9000 BLUE CFj		
Income year 3:	15,000	15000 BLUE CFj		
Income year 4:	17,000	17000 BLUE CFj		
What is the rate of return?		GOLD IRR	(10.86%)	

By how much should each of the yearly income amounts be increased to get a 14.5% rate of return?

Present Value of missing amounts:	14.5 i GOLD NPV	($3,011.75)
Actual value of missing amounts:	0 FV PMT	($1,044.26)

Adding $1,044.26 to the income for each year results in a 14.5% rate of return. This means $7,044.26 for year 1, $10,044.26 for year 2, etc.

- Here the Present Value of the missing amounts (NPV) needs to be spread equally between the 4 years that follow while fully taking into account the time value of money--we are not dividing $3,044.26 by 4! This is easily achieved with a regular cash flow PV--PMT transfer.

- By changing the rate requirement we transform a valid exchange of money in time with a rate of return of 10.86% into an incomplete exchange. As in all other cases of an incomplete exchange for which we know the rate, we have no problem calculating the Present Value of the missing amounts. It is then just a question of transferring the Present Value of the missing amounts to their rightful place in time.

- The important is to keep each step of the procedure separate:

  1) We key in the known Cash Flows and the required rate.
  2) We calculate **NPV**, the Present Value of the missing amounts.
  3) We transfer that Present Value to the Cash Flows' rightful place in time, adjusting for the time value of money in the process.

The illustration that follows stresses the fact that the Present Value of the missing amounts (NPV) is independent of where the missing amounts are actually located.

NPV INDEPENDANT OF WHERE AMOUNTS ARE MISSING

An investor is considering the following Cash Flows. He keys in the data and calculates the rate:

Initial outlay:	-400,000	400000	CHS	BLUE	CFo	
Year 1:	-150,000	150000	CHS	BLUE	CFj	
Year 2:	100,000	100000		BLUE	CFj	
Year 3:	200,000	200000		BLUE	CFj	
Year 4:	300,000	300000		BLUE	CFj	
Year 5:	400,000	400000		BLUE	CFj	
Rate of Return:		GOLD	IRR			(17.75%)

The investor is not satisfied with a 17.75% return. He wants 20%.
To get that higher rate, he needs to invest less or get more money back.
What is the Present Value of the missing amounts?

20 i GOLD NPV (\$34,387.86)

We now know the Present Value of the missing amounts, but we have not yet decided **WHEN** they are supposed to occur. The Present Value of the missing amounts depends solely on the Cash Flows submitted and on the rate requirement, not on when the missing amounts actually occur. There are any number of possible ways of increasing the rate to 20%: they all have a Present Value of \$34,387.86. Let's review a few options:

1) 20% is achieved by reducing the initial investment. By how much?

NPV is the answer: \$34,387.86. We just need to change the sign (Present Value of missing amount, with the **wrong** sign. Remember?) and add the value to the amount already changing hands at that time:

- 400,000 + 34,387.86 = - 365,612.14.

An initial investment of \$365,612.14 meets the 20% requirement.

2) The rate is achieved by receiving more in Year 4. How much more?

4 n 0 PMT FV (\$71,306.67)

Adding \$71,306.67 to \$300,000 for year 4 meets our objective.

3) The rate is achieved by increasing the income or decreasing the disbursement by an equal amount each year for years 1 to 5:

5 n 0 FV PMT (\$11,498.60)

Adding \$11,498.60 to each of the 5 years also raises the rate to 20%. This means a negative \$138,501.40 for year 1 (-150,000 + 11,498.60), \$111,498.60 for year 2 (100<000 + 11,498.60), \$211,498.60 for year 3, etc.

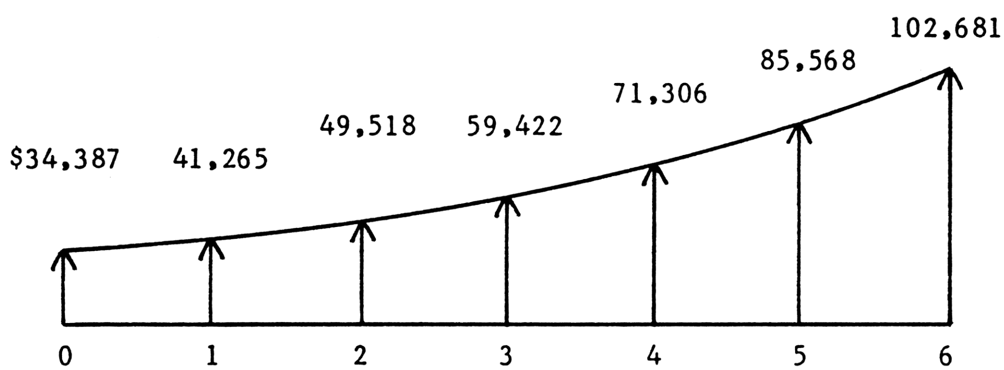
4) Let's examine a little more closely what happens when the missing amount is a single cash flow, as in options 1 and 2 above.

Of course, the missing amount changes according to WHEN it is missing, but its Present Value remains $34,387.86. The various alternative options for the missing amount--and the resulting net Cash Flow for that year--can be calculated as follows:

Year	Keystrokes	Known Cash Flows	+	Missing amount	=	Net Cash Flow
	0 PMT					
0	GOLD NPV	- 400,000.00	+	34,387.86	=	- 365,612.14
1	1 n FV	- 150,000.00	+	41,265.43	=	- 108,734.57
2	2 n FV	100,000.00	+	49,518.52	=	149,518.52
3	3 n FV	200,000.00	+	59,422.22	=	259,422.52
4	4 n FV	300,000.00	+	71,306.67	=	371,306.67
5	5 n FV	400,000.00	+	85,568.00	=	485,568.00
6	6 n FV	0.00	+	102,681.60	=	102,681.60
Etc.						
10	10 n FV	0.00	+	212,920.57	=	212,920.57

Adding any one of the missing amounts increases the rate to 20%.

We may graph these missing amounts as follows on the 20% rate curve:



Once the procedure illustrated in these models has been mastered, perhaps the most difficult for the practioner faced with an actual problem is to recognize and consciously state to himself that he is faced with the task of calculating a missing amount, and that he has all the information needed to find the Present Value of that missing amount. For that missing amount will present itself under an infinite variety of names, a few among which are future value, balance, reversion, balloon payment, return on investment, and others that do not necessarily represent one lump sum at the end of the transaction. They have one thing in common: they are all dollar amounts.

See Unit 28, page 7, for circumstances where multiple missing amounts are unevenly spread in the future.

Calculating a missing amount.

PRACTICE

1) Key in the following yearly cash flows:

Initial investment:	-35,000
End Of Year 1 return:	5,000
EOY 2:	8,000
EOY 3:	12,000
EOY 4:	15,000

- You want a 15% rate of return. What is the Present Value of the missing amounts? ($8,136.53)

- What amount must you receive in year 4, in addition to the $15,000 already scheduled, in order to get a 15% rate of return? ($14,230.84)

- If instead your rate is increased by receiving an additional cash flow at the End Of Year 5, what amount must you receive? ($16,365.47)

2) You have established the following 5 year net income projection on the purchase of a property:

Time 0:	-420,000	
EOY 1:	-50,000	(Negative!)
EOY 2:	0	
EOY 3:	30,000	
EOY 4	50,000	
EOY 5:	70,000	(+ unknown reversion on sale)

You sell the property at the end of year 5.
- What net reversion gives you a 17% rate of return? ($844,955.54)
- Net reversion to get a 20% return? ($975,574.40)
- Net reversion to get a 25% return? ($1,224,433.59)
- Rate of return if the net reversion is $1,100,000?) (22.60%)

3) Beginning right now, you invest $2,000 a year for 3 years and $3,000 a year for 2 years in an account that offers a 10% annual rate of return.

- What amount can you withdraw 10 years from now? ($25,351.39)
- What amount can you withdraw 20 years from now? ($65,754.98)
- Same questions with a 12% rate of return. ($29,263.29)
  ($90,887.34)

(Initial $2,000 in CFo. Key 4 back into **n** when switching to 12% rate).

4) You invest the following amounts at the beginning of each year in an account that offers a 7.5% rate of interest.
Watch your money grow during years 5 to 10 by calculating the balance at the end of each of these years.

Intial investment (Time 0):	$10,000
Investment beginning of year 2:	$12,000
Investment beginning of year 3:	$14,000
Investment beginning of year 4:	$16,000
Investment beginning of year 5:	$18,000

(Present Value of missing amount is $59,635.23.)
(Balance, EOY 5: $85,614.08)
(EOY 6: $92,035.14)
(EOY 7: $98,937.77)
(EOY 8: $106,358.10)
(EOY 9: $114,334.96)
(EOY 10: $122,910.08)

5) You are considering making the following loan with yearly payments:

Loan amount:	$650,000	
Payment, EOY 1:	50,000	
Payments, EOY 2 & 3:	100,000	
payments, EOY 4 & 5:	150,000	2 years each.
Payments, EOY 6 & 7:	200,000	

- What is the rate of interest? (8.61%)

You want to increase the rate to 10% and consider these alternatives:
- By how much should you decrease the original loan? ($35,652.72)
- By how much should you increase the balance at EOY 7? ($69,477.07)
- By how much should you increase the payment at EOY 4? ($52,199.15)
- By how much should you increase each yearly payment? ($7,323.26)
- What payment amount should you add at EOY 8? (76,424.77)

Clearly, of crucial importance is the ability to communicate the desired cash flows to the calculator. We may then let the calculator calculate the rate or, if we know the desired rate, find means of calculating the missing amounts required to achieve that rate.

The section that follows will increase our control over the cash flows we communicate to the calculator essentially through understanding the role of 'n' and how we can manipulate its value for our purpose.

THE ROLE OF 'n'
WITH IRREGULAR CASH FLOW DATA

MANIPULATING 'n' FOR OUR CONVENIENCE

So far, we have changed **n** only as we switched from irregular cash flow to regular cash flow problems, and vice versa. We now want to manipulate **n** as it affects irregular Cash Flow data.

ROLE OF n: A SUMMARY

When we key in a series of irregular Cash Flows using CFo and CFj, the value in **n** automatically adjusts to reflect the number of the Register memory where the Cash Flow is being stored. This is also the **'j'** value.

We have 0 in **'n'**	for CF0	stored in Register memory	0
1	CF1		1
2	CF2		2

- When we key in the initial amount with CFo, as soon as we press CFo
    - We automatically clear **n**. (We put zero in **n**).
    - We automatically store the initial amount in Register memory 0.
- When we key in subsequent amounts with CFj, as soon as we press CFj
    - We automatically increase **n** by 1.
    - We automatically store the new Cash Flow in the Register memory corresponding to the value that is being assumed by **n**.

In other words:

- We need 4 in **n** in order to key in Cash Flow 5.
- As soon as we key in Cash Flow 5, **n** is increased to 5, and Cash Flow 5 is stored in Register memory 5.
- With 5 in **n**, the calculator knows that it should explore all the way up to memory 5 when we ask one of the two irregular Cash Flow questions: IRR or NPV.

MANIPULATING n TO ISOLATE GROUPS OF CASH FLOWS

We may use these features to our advantage by changing the value that was initially assumed by **n**.

By keying in a lower number in **n**, we instruct the calculator to ignore all Cash Flows keyed into memories with higher address numbers. We may then proceed as if we had never keyed in the higher Cash Flows. However these Cash Flows remain in the calculator and may be re-instated by keying back the higher numbers in **n**.

For instance, let's key in the following amounts and calculate their Present Value at a 12% discount rate:

0	$75,000	75000	BLUE	CFo	
1	35,000	35000	BLUE	CFj	
2	45,000	45000	BLUE	CFj	
3	55,000	55000	BLUE	CFj	
4	65,000	65000	BLUE	CFj	
5	125,000	120000	BLUE	CFj	
Discount rate:		12	i		
Present Value:		GOLD	NPV		($293,508.67)

If we now want to calculate the Present Value of years 0, 1 and 2 only, we may simply isolate Cash Flows 0 to 2 by keying 2 in **n**:

2 n GOLD NPV	($142,123.72)

Similarly, subsequent years can be re-instated as desired:

PV of years 0 to 3:	3 n GOLD NPV	($181,271.64)
PV of years 0 to 4:	4 n GOLD NPV	($222,580.31)

If we want to calculate the Present Value of Cash Flows 2 and 3 only, we may press 3 in **n** and errase from the appropriate memory any unwonted Cash Flows that are still retained:

3 n	
0 STO 0	
STO 1	
GOLD NPV	($75,021.64)

The next two illustrations take further advantage of this procedure.

CHANGING THE PROJECTION TIME

An investment promises the following income, plus the reversion on sale of the business, real estate, or whatever that particular investment is:

Initial outlay:	$125,000	125000	CHS	BLUE	CFo
Income, year 1:	21,000	21000		BLUE	CFj
Income, year 2:	29,000	29000		BLUE	CFj
Income, year 3:	36,000	36000		BLUE	CFj
Income, year 4:	41,000	41000		BLUE	CFj
Income, year 5:	41,000	41000		BLUE	CFj

Calculate the reversion needed to obtain a 22% rate of return if the investment is sold in either one of the last 3 years:

	22 i	
Year 3 reversion:	3 n GOLD NPV 0 PMT FV	($124,344.60)
Year 4 reversion:	4 n GOLD NPV FV FV	($110,700.41)
Year 5 reversion:	5 n GOLD NPV FV FV	($94,054.50)

If we now want to consider the option of selling the investment at the end of the 6th year, we no longer need to re-calculate a new NPV value as NPV calculated with 5 in **n** already takes all known Cash Flows into account. So we may calculate the total required return for that year (income and reversion) as follows:

Year 6 return:	6 n FV	($114,746.49)

For year 6 we should not press **6 n GOLD NPV FV FV** as this would re-activate Register memory 6 and might allow a telephone number or other leftover data to interfere with the NPV calculation.

RUNNING BALANCE

Calculate the running balance on the following monthly Savings Account transactions. The account offers 8.35% interest compounded monthly.

	Deposit	Withdrawal	Keystrokes							
Time 0:	$2,500		2500	CHS	BLUE	CFo				
End Of Month 1:	1,500		1500	CHS	BLUE	CFj				
EOM 2:		$800	800		BLUE	CFj				
EOM 3:		500	500		BLUE	CFj				
EOM 4:	1,300		1300	CHS	BLUE	CFj				
EOM 5:		1,600	1600		BLUE	CFj				
EOM 6:	750		750	CHS	BLUE	CFj				
8.35% monthly compounded interest:			8.35	BLUE	i					
End Of Month 1 balance:	1	n	GOLD	NPV	0	PMT	FV	($4,017.40)		
EOM 2 balance:	2	n	GOLD	NPV		FV	FV	($3,245.35)		
EOM 3 balance:	3	n	GOLD	NPV		FV	FV	($2,767.93)		
EOM 4 balance:	4	n	GOLD	NPV		FV	FV	($4,087.19)		
EOM 5 balance:	5	n	GOLD	NPV		FV	FV	($2,515.63)		
EOM 6 balance:	6	n	GOLD	NPV		FV	FV	($3,283.14)		
And, assuming no new deposit or withdrawal:										
EOM 7 balance:	7	n	FV					($3,305.98)		
EOM 8 balance:	8	n	FV					($3,328.99)		
EOM 17 balance:	17	n	FV					($3,543.36)		

- Note how we key in all the data, and then at leisure isolate the number of periods that we want to consider.

- Here, Cash Flows are monthly amounts: the monthly rate is keyed in by pressing **8.35 BLUE i.**

- Some might find it easier to take the point of view of the account itself, and key deposits as positive and withdrawals as negative: the results, of course, are the same.

- The balance at any one time is the "missing amount", the amount that, if withdrawn, closes the account and balances the books on the transaction at that point in time. It is the amount the depositor is entitled to if he is to receive his 8.35% interest.

- The NPV value calculated for End of Month 6 ($3,149.34) remains the same for all following months as long as no new transaction occurs. We may just watch the 6 month balance of $3,283.14 grow as we allow it to remain in the account and earn interest for additional months.

- The procedure also applies if only deposits are made and may therefore be used to find the Future Value of irregular Cash Flow data.

KEYING A MISSING AMOUNT BACK IN

Once we have calculated a missing amount, we may want to add it to the irregular Cash Flows we already have. This "balances the books" on the transaction and allows us to check that we are indeed getting the expected rate or to proceed with other calculations on the adjusted data, such as calculating an APR on loan data or discounting the loan to raise the yield.

If the 'missing amount' that we calculated is to be added (or subtracted) from Cash Flows that have already been keyed in, we may modify the existing Cash Flow with simple arithmetic: we may recall the original Cash Flow, add the new amount, and store the adjusted Cash Flow back in, or we may use storage register arithmetic when appropriate (memories 0 to 4).

If the 'missing amount' represents an additional Cash Flow, then we need to restore the original irregular Cash Flow value for **n** and key the new amount in with BLUE CFj. (Using CFj instead of just storing the number in the approriate Register memory has the advantage of clearing any leftover Nj value that could be affecting the Register memory).

For instance, let us consider the following incomplete exchange of money. Let us calculate what additional CF5 amount increases the rate to 20%, and let's key that amount in as a 5th CFJ value:

0	- 100,000	100000 CHS BLUE CFo	
1	20,000	20000 BLUE CFj	
2	30,000	30000 BLUE CFj	
3	40,000	40000 BLUE CFj	
4	50,000	50000 BLUE CFj	
5	?		
20% rate:		20 i	
PV of missing amount:		GOLD NPV	($15,239.20)

We have 4 in **n** and could key in a 5th CFj amount by just keying the additional value in with BLUE CFj. But we do not yet know the missing amount. To calculate the missing amount, we need to change the number in **n** (from 4 to 5). When we finally know the missing amount and are ready to key it in as CF5, we simply need to press back 4 in **n**:

Missing amount:	5 n 0 PMT FV	($37,920.00)
Keying in the	4 n	
missing amount:	RCL FV BLUE CFj	

The logic is simple: we need 4 in **n** in order to key in a 5th Cfj amount. Normally, the correct value is automatically placed in **n**, but if it has been changed, then we just need to key the original value back in before we resume keying in irregular Cash Flow data.

CHECKING THE ANSWER

We now have the complete exchange in the calculator, and we may check that we indeed are getting the required 20% return:

GOLD IRR	(20%)

The IRR calculation takes a certain amount of time, especially with more complex data. A shorter way of checking that we have the rate that we expect is to make sure that we have 20 in **i** (we alredy do, but who knows?), and to check that there is **no missing amount**:

20 i GOLD NPV	($0.00)

The Present Value of what is missing is zero, so the missing amount is also zero: there is no missing amount, the Cash Flows submitted offer a 20% rate of return. (Sometimes, instead of 0.00, we find something like 0.00002. These few fractions of a cent are due to rounding discrepancies and should be disregarded)

CHANGING CASH FLOWS

In the previous example, pressing 4 in **n** allowed us to key in Cash Flow 5. This manipulation of **n** can be quite convenient if a whole sequence of Cash Flows need to be changed. Let's consider the following example where one Cash Flow is inadvertently omitted:

0	-25,000	25000 CHS BLUE CFo	
1	4,000	4000 BLUE CFj	
2	4,500	4500 BLUE CFj	
3	5,000	5000 BLUE CFj	
4	5,500	5500 BLUE CFj	
5	6,000	(omitted)	
6	6,500	6500 BLUE CFj	
7	7,000	7000 BLUE CFj	
8	7,500	7500 BLUE CFJ	
		RCL n	(7)

Recalling **n** shows that 1 Cash Flow value is missing. Recalling the various Register memories (RCL 0, RCL 1, etc) would then show that CF5 was inadvertently omitted. All subsequent Cash Flows are in the wrong memories. We may correct as follows:

		4 n	
5	6,000	6000 BLUE CFj	
6	6,500	6500 BLUE CFj	
7	7,000	7000 BLUE CFj	
8	7,500	7500 BLUE CFJ	
		GOLD IRR	(14.11%)

SUMMARY
CHANGING AND CHECKING DATA

To overwrite Cash Flows with new values, we normally choose to go directly to the Register memory, though manipulating **n** and using CFj is a convenient alternative when many Cash Flows are involved.

To add new Cash Flows to previously keyed in data, adjust **n** back to the original value if it has been disturbed, and key the additional Cash Flows with **BLUE CFj**.

To check dollar values, recall the appropriate Register memory.

To check that Cash Flows already in the calculator meet the expected rate requirement we may proceed in one of two ways:

- Let **GOLD IRR** calculate the rate.
- Key in the rate and let **GOLD NPV** verify that no amounts are missing.

IRREGULAR CASH FLOWS
LEVEL II

Level II introduces the Nj function, which allows each of 21 cash flow amounts to be repeated up to 99 times. This becomes indipensable with long term exchanges and the monthly data provided by many loans.

UNIT 23

MULTIPLE CASH FLOW VALUES

IRREGULAR CASH FLOWS: LEVEL II

MULTIPLE CFj VALUES: BLUE Nj

CFo and CFj values can be repeated up to 99 times with BLUE Nj

MODEL

$200,000 loan to be paid off over 10 years as follows:
  $2,000 a month for the first 2 years (24 payments).
  $3,000 a month for the following 3 years (36 payments).
  $4,000 a month for the 5 remaining years (60 payments).
What is the rate of interest?

We just need to communicate the data and solve for the Rate of Return:

DATA		KEYSTROKES	ANSWER
-200,000	(1)	200000 CHS BLUE CFo	
2,000	(24)	2000 BLUE CFj 24 BLUE Nj	
3,000	(36)	3000 BLUE CFj 36 BLUE Nj	
4,000	(60)	4000 BLUE CFj 60 BLUE Nj	
Monthly rate?		GOLD IRR	(1.11%)
Annual rate?		12 x	(13.31%)

- With REGULAR cash flows, a dollar amount stored in the PMT memory can be repeated a given number of times by specifying that number in **n**: **Nj performs the same function for each CFo or CFj amount:**
  2000 BLUE CFj 24 BLUE Nj means 24 successive cash flows of $2,000.

- Note the simple **T-CHART** that allows us to formulate the data. On one side we have the CFo and CFj values (dollar amounts), on the other the corresponding Nj values that expresses the number of times each Cash Flow must be repeated. The numbers offer themselves in the order in which they are keyed in.

- The calculator automatically attributes an Nj value of 1 when a cash flow is first entered with **BLUE CFo** or **BLUE CFj**. We may then overwrite the value (one) with the desired Nj value. Because we do not need to key in an Nj value of 1, we will often omit showing that number in the T-charts.

- With Nj, we greatly increase our ability to communicate complex irregular cash flow data to the calculator. All the problems that we learned to solve in situations requiring only the use of CFo and CFj (a maximum of 21 cash flows) can now be solved in situations where each one of these amounts is repeated up to 99 times.

CASH FLOW REPEATED MORE THAN 99 TIMES

A $91,000 loan is paid back with monthly payments of $900 for 10 years and payments of $1,500 for the following 15 years. What is the rate of interest on the loan?

The exchange of money in time can be expressed in such a way that no amount is repeated more than 99 times:

- 91,000	(1)	
900	(60)	120 months
900	(60)	
1500	(90)	180 months
1500	(90)	

We may calculate the rate as follows:

91000	CHS	BLUE	CFo				
900		BLUE	CFj	60	BLUE	Nj	
900		BLUE	CFj	60	BLUE	Nj	
1500		BLUE	CFj	90	BLUE	NJ	
1500		BLUE	CFj	90	BLUE	Nj	
		GOLD	IRR				(1.10%)
		12	x				(13.25%)

It seems a pity to have to key in some of these numbers twice. And indeed they can easily be retrieved from the **y** memory of the Stack as follows:

91000	CHS	BLUE	CFo				
900		BLUE	CFj	60	BLUE	Nj	
xy		BLUE	CFj	xy	BLUE	Nj	
1500		BLUE	CFj	90	BLUE	NJ	
xy		BLUE	CFj	xy	BLUE	Nj	
		GOLD	IRR				(1.10%)
		12	x				(13.25%)

(Keep data in calculator)

CHECKING AND CHANGING CASH FLOW AMOUNTS

With the previous data retained, we may check the numbers as follows:

RCL n	(4 CFj entries)

This provides a convenient check that the correct number of entries has been made. The 4 in **n** indicates merely that data has been put in Register memories 0 to 4 (5 entries including CFo, 4 CFj entries). Because it does not reflect the Nj values, there is no longer any relationship between the value in **n** and the time span of the transaction. Here, 4 in **n** corresponds to 25 years.

RCL BLUE i	(13.25% annual rate)

A rate, once it has been calculated by the **GOLD IRR** procedure, is stored in the i memory. If it happens to be a monthly rate, we may recall the yearly equivalent by using the BLUE function as above.

RCL 0	(-91,000.00)
RCL 1	(900.00)
RCL 2	(900.00)
RCL 3	(1,500.00)
RCL 4	(1,500.00)

Dollar amounts are stored in the corresponding Register memory and can be recalled and changed directly in those memories.

Changing the dollar amounts does not affect the Nj values already attributed to these cash flows. For instance, if the payments are doubled after the initial 10 years ($1,800.00 instead of $1,500.00), the data can be changed and the new rate calculated as follows:

1800 STO 3	
STO 4	
GOLD IRR 12 x	(14.08%)

The ability to check, change, and manipulate Nj values will be presented in the following Unit. With no explanations, let us just provide the following checking procedure at this stage:

RCL BLUE Nj RCL BLUE CFj	90 cash flows of $1,800.00.
RCL BLUE Nj RCL BLUE CFj	90 cash flows of $1,800.00.
RCL BLUE Nj RCL BLUE CFj	60 cash flows of $900.00.
RCL BLUE Nj RCL BLUE CFj	60 cash flows of $900.00.
RCL BLUE Nj RCL BLUE CFj	1 cash flow of -91,000.00.
4 n GOLD IRR 12 x	(14.08%)

Once data has been checked in this way, we need to press the original **n** value back in **n** before we can solve for a rate or a Present Value.

ILLUSTRATION PROBLEMS

At this stage you probably need some practice in just keying in data using CFo, CFj, and Nj. The mechanics needs to come easily to our fingers before you can free your mind to think exclusively of the problem at hand. The illustrations that follow provide opportunities for practice and introduce extra features and precautions in the process. You may want to see how you would solve the problem before studying the proposed keystrokes and comments.

SPAN OF TIME WITH NO CASH FLOWS

A $100,000 loan is to be repaid as follows:
no payments for the first 2 years, followed by
monthly payments of $1,400 for the next 20 years.
What is the rate of interest?

-100,000		100000	CHS	BLUE	CFo				
0	(24)	0		BLUE	CFj	24	BLUE	Nj	
1,400	(99)	1400		BLUE	CFj	99	BLUE	Nj	
1,400	(99)	xy		BLUE	CFj	xy	BLUE	Nj	
1,400	(42)	xy		BLUE	CFj	42	BLUE	Nj	
				GOLD	IRR	12	x		(12.02%)

- Once we have chosen the month as the regular period, data must be submitted for every month considered. Months where nothing happens are taken into account by keying in cash flows of **zero dollars.** With a yearly or quarterly period, data must be submitted for every year or every quarter.

- Note how a problem concerning a loan that is regular in most aspects but does not quite fit into the regular cash flow pattern can be solved using irregular cash flow functions.

Nj VALUE AFFECTING CFO.

You invest $2,000 now and every month that follows for 2 years (24 payments), then no money changes hands for the following 2 years, then you receive $3,000 a month for 2 years. What is the rate of return on your investment?

-2,000	(24)
0	(24)
3,000	(24)

2000	CHS	BLUE	CFo	24	BLUE	Nj
0		BLUE	CFj	xy	BLUE	Nj
3000		BLUE	CFj	xy	BLUE	Nj
		GOLD	IRR	12	x	

(10.18%)

- An Nj factor can be attributed to the dollar amount keyed in with CFo. Of course, only the first of the 24 $2,000 amounts actually occurs at time zero. The remain 23 are spread out over the following 23 months.
- To be consistent with the monthly period imposed by the data, the 2 years where no money changes hands are communicated to the calculator as 24 monthly periods of 0.00 dollars.

TIME PERIOD IS IN THE MIND OF THE USER

Find the rate of return on the following investment opportunity.
Initial investment: $45,000.
Returns of: $6,000 a year for the first 5 years and $10,000 a year for the next 10 years.

-45,000	
6,000	(5)
10,000	(10)

45000	CHS	BLUE	CFo			
6000		BLUE	CFj	5	BLUE	Nj
10000		BLUE	CFj	10	BLUE	Nj
		GOLD	IRR			

(15.03%)

- How does the calculator know that we have yearly data? It does not. Because I know that my cash flows are yearly amounts, I interpret the answer as being a yearly rate. If the same cash flows represented monthly data, the answer would be 15.03% per month, if semi-annual data, a semi-annual rate.
- We do not necessarily use Nj every time one cash flow occurs more than once. Here, we could theoretically solve the problem without using Nj, but keying in $6,000 five times and $10,000 ten times with BLUE CFj would be time-consuming and would use up a lot of memory space.
- Note the importance of keying in the initial amount as negative: a rate can be calculated only if there is an exchange of money in time: with no negative amount there is no exchange and no rate...just an **Error** sign as a convenient reminder.

PAYMENTS WITH DIFFERENT PERIODS

What is the rate of interest on a $350,000 loan paid back as follows:
$25,000 every 6 months for the first 5 years,
$100,000 a year for the following 4 years.

The period we need to select is decided by the smallest time period defined by the data, here the semi-annual payments. In order to express the yearly data on a semi-annual basis, we have to introduce semi-annual payments of ZERO dollars at the appropriate time.

- 325,000		325000	CHS	BLUE	CJo				
25,000	(10)	25000		BLUE	CFj	10	BLUE	Nj	
100,000	(1)	100000		BLUE	CFj				
0	(1)	0		BLUE	CFj				
100,000	(1)	xy		BLUE	CFj				
0	(1)	xy		BLUE	CFj				
100,000	(1)	xy		BLUE	CFj				
0	(1)	xy		BLUE	CFj				
100,000	(1)	xy		BLUE	CFj				
				RCL	n				(8)
				GOLD	IRR	2	x		(14.54%)

- This kind of data can easily use up the maximum 20 CFj values that can be keyed into the calculator and should be adjusted to more manageable data if possible.

- Note how **xy** retrieves alternatively the $100,000 and $0.00 amounts.

- Recalling **n** before the time-consuming IRR calculation provides an optional check that we have indeed the correct number of entries.

- The initial rate of 7.27% applies to semi-annual data and is a semi-annual rate. It is multiplied by 2 to give an annual rate (14.54%) which still implies semi-annual compounding. It can be converted into an annualized yield if desired: (See Index for more details on Compounding).

2	n	100	PV	
0	PMT		FV	(115.07 for 15.07% yield)
1	n	i		(15.07% now in i memory)

For a monthly-compounded equivalent:

12	n	i	12 x	(14.12%)

The loan provides the investor an annualized yield of 15.07% which is equivalent to a rate of 14.54 semi-annually compounded and to a monthly compounded rate of 14.25%.

CONTRAST WITH REGULAR CASH FLOW DATA

A $300,000 loan is paid off as follows:
No payments for 1 year,
$4,000 a month for 1 year,
$5,000 a month for another year,
$6,000 a month for a final year, with
$300,000 balance added to the last monthly payment.
What is the rate of interest?

-300,000	
0	(12)
4,000	(12)
5,000	(12)
6,000	**(11)**
306,000	(1)

300000	CHS	BLUE	CFo		
0	BLUE	CFj	12	BLUE	Nj
4000	BLUE	CFj	xy	BLUE	Nj
5000	BLUE	CFj	xy	BLUE	Nj
6000	BLUE	CFj	11	BLUE	Nj
306000	BLUE	CFj			
	GOLD	IRR	12	x	

(13.69%)

- With IRREGULAR cash flow data, every separate dollar amount that we key in occurs at a different time. The final dollar entry for a loan on which we have complete cash flow data is the **BALLOON PAYMENT,** not the balance which occurs at the same time as the last payment and is therefore not a complete cash flow amount. The initial data specifies 12 payments of $6,000 for year 4, plus a $300,000 balance added to the last payment: to key the data in we need to understand that we really have 11 payments of $6,000 and a balloon payment of $306,000 at the end of the 4th year.

- This represents one difference between irregular cash flow logic and regular cash flow logic. With regular cash flow data, the Future Value is an amount that occurs at the same time as the last Payment amount (in normal "END" mode). In the "BEGIN" mode the first Payment occurs at the same time as the Present Value amount.

- The **BEGIN - END option does not apply** to irregular cash flow data, as we are already assigning a specific point in time to each dollar amount that we key in.

- Provided that we take these points into account as we communicate data to the calculator, many regular cash flow problems can be solved with the irregular cash flow keys: when the same cash flows are communicated in the appopriate way, the same answer is obtained.

PRESENT VALUE WITH Nj DATA

What is the Present Value of the following monthly amounts discounted at the rate of 12%?

0	$25,000		25000	BLUE	CFo				
1	5,000	(12)	5000	BLUE	CFj	12	BLUE	Nj	
2	6,000	(12)	6000	BLUE	CFj	12	BLUE	Nj	
3	7,000	(18)	7000	BLUE	CFj	18	BLUE	Nj	
4	8,000	(6)	8000	BLUE	CFj	6	BLUE	Nj	
12% rate:			12	BLUE	i				
Present Value:				GOLD	NPV				($262,135.10)

- We have monthly data, so we key in the discount rate as a monthly amount (12 BLUE i). With yearly data, we would key in a yearly rate, with semi-annual cash flows, a semi-annual rate.

- With the data still in the calculator, we may of course discount at any other rate. For instance:

15% discount rate:	15	BLUE	i	
Present Value:		GOLD	NPV	($248,226.98)
Difference between 12% and 15% rates:		-		($13,908.12)

- Checking the Present Values of cash flows discounted at two different discount rates and calculating the difference as above is a convenient way of seeing how sensitive the Present Values are to the choice of the discount rate.

- 'Discounting' at a rate of 0% adds up all the cash flows without actually changing their value. This may be used to find the sum of all the cash flows.

Total amount received:	0	i	GOLD	NPV	($331,000)

331,000 is equal to:

25,000 + (5000 x 12) + (6000 x 12) + (7000 x 18) + (8000 x 6).

DISCOUNTED CASH FLOW ANALYSIS

With the ability to calculate the Present Value of complex uneven cash flows comes the power to apply discounted cash flow analysis procedures to any sequence of future income or expenses. Let's give a few examples.

Choose between the following offers on a property you are selling by discounting at 9% and 13%.

A $650,000 cash.
B $300,000 cash and $400,000 carried for 10 years at 10% interest, interest only. (Selling price is $700,000).
C $180,000 cash and $475,000 carried at 12% fully amortized over 8 years. (Selling price is $655,000).

We first establish the cash flows for options B and C. The results:

B	$300,000.00	
	3,333.33	(119)
	403,333.33	(1)

C	$180,000.00	
	7,720.10	(96)

We then discount each sequence at the selected rate or rates:

	9% discount rate.	13% discount rate.
Option A	$650,000.00	$650,000.00
Option B	726,313.63	633,025.36
Option C	706,961.97	639,331.40

At a 9% discount, option B is the best offer, at 13% it is the worst. The classification itself is affected by the rate we choose. So we have to make a choice of the rate that really makes sense. We need to ask ourselves: What will I do with the cash I get?

If I can invest at 9%, I choose option B: I get a higher selling price and amounts not received in cash remain invested at 10%, not just 9%.

If I can invest at 13%, on the other hand, choosing the cash offer is the best solution: I can always get anything that options B and C provide, and more, by investing the extra cash at 13%.

If I am selling the property to buy another property and will borrow large amounts at 9% interest, I am again better off choosing option B and using the payments I receive on my carryback loan to subsidize my new loan. If I borrow at 13% on the new purchase, then the cash offer allows me to reduce the new loan and realize savings on the payments that the other options could not subsidize.

Once the choice is made of a discount rate, then a Present Value approach summarizes many variables--here different selling prices, different downpayments, different carryback loans at different rates--into a set of single numbers that can be compared.

Let's provide the keystrokes for options B and C, and also discount each offer at the rate on the financing. (NPV answers are rounded).

```
400000   ENTER
10  %    12  ÷                (3,333.33)

  300000  BLUE CFo
  3333.33 BLUE CFj    99 BLUE Nj
  3333.33 BLUE CFj    20 BLUE Nj
403333.33 BLUE CFj

 9 BLUE i   GOLD NPV    (726,313)
10 BLUE i   GOLD NPV    (700,000)
13 BLUE i   GOLD NPV    (633,025)
```

```
475000  PV
12 BLUE i
8  BLUE n
0 FV    PMT                      (7,720.10)

 180000   BLUE CFo
7720.10 BLUE CFj  96 BLUE Nj

 9 BLUE i   GOLD NPV       (706,961)
12 BLUE i   GOLD NPV       (655,000)
13 BLUE i   GOLD NPV       (639,331)
```

When we discount at the rate of the carryback financing (10% for option B, 12% for C), the Present Value calculation gives us back the selling price. This is not surprising as the selling price is the Present Value when the rate of interest we get on the carryback loan is seen as a valid compensation for lending the money instead of receiving the full cash price. (The difference of a few cents is the result of rounding the payment amounts to an even dollars and cents figure).

Discounting at the carryback rate is a simple means of verifying that the cash flows keyed in are correct.

SECOND ILLUSTRATION: PRESENT VALUE OF LEASES

In leasing a commercial property for 5 years you have the choice between two payment methods:

A No payments for the first year.
$5,000.00 per month for the following 48 months.

B Monthly payments that increase every year as follows:

First year:	2,500	(12)
Second year:	3,500	(12)
Third year:	4,000	(12)
Fourth year:	4,500	(12)
Fifth year:	5,000	(12)

We may discount at 8% and 14% and find the following Present Values:

	Option A	Option B	
8% discount:.	190,374.01	188,854.95	Option A is more expensive.
13% discount.	161,055.11	162,278.45	Option B is more expensive.

DIFFERENTIAL CASH FLOW APPROACH

AS ALTERNATIVE TO DISCOUNTED CASH FLOW ANALYSIS

The two options of the previous problem lend themselves to a differential cash flow solution. (See Volume I, Unit 7, pages 8 and following), which requires that we establish the DIFFERENCE between the two options at each point in time. If I choose option B, I lose money the first year compared to option A but make it up during years 2, 3 and 4 as follows:

OPTION A - OPTION B =	DIFFERENCE	
0 - 2,500 =	- 2,500	(12)
5,000 - 3,500 =	1,500	(12)
5,000 - 4,000 =	1,000	(12)
5,000 - 4,500 =	500	(12)
5,000 - 5,000 =	0	(12)

```
2500 CHS BLUE CFo    12 BLUE Nj
1500     BLUE CFj    12 BLUE NJ
1000     BLUE CFj    12 BLUE Nj
 500     BLUE CFj    12 BLUE Nj

GOLD  IRR    12  x
```

The cash flows in the 'Difference' column represent the reality of the choice I have to make: do I want to save myself $2,500 the first year and pay for that benefit with payments that are $1,500, $1,000 and $500 higher in the years that follow (Option A), or do I prefer to spend $2,500 more the first year and keep my payments lower by $1,500, $1,000 and $500 later (Option B)?

We key in the exchange of money represented by the difference, and calculate a rate of 11.19%. That is the differential rate. Choosing option A is like borrowing $2,500 at that rate. Choosing option B is like investing $2,500 at that rate. At that rate, do I prefer to invest or to borrow? Answering that question enables us to choose between the two options.

11.19% also happens to be the discount rate for which the Present Values of the two options are equal. Discounting the cash flows at a lower rate (8% for instance) provides one classification between options A and B. Discounting at a rate higher than 11.19% (14% for instance) establishes the opposite order.

So using a differential cash flow approach instead of a Present Value approach should ultimately lead to the same conclusion. The advantage of the differential cash flow approach is that we do not have to choose a somewhat arbitrary discount rate with little knowledge of whether a slight change in the rate would reverse the order. We go directly to the number that represents the turning point and ask ourselves whether, at that rate, we prefer to invest or to borrow.

PRESENT VALUE

WITH MORE THAN 21 DIFFERENT AMOUNTS

When calculating the Present Value of a series of cash flows we are really having the cash flows slide all the way back to the present along the curve defined by the rate. In sliding along the discount rate curve, it is always possible to make one or more stops on the way. This can be useful if we need to calculate the Present Value of more than 21 different amounts, as 21 different values is the maximum number of cash flows that the calculator will hold.

Let's find the Present Value discounted at 10% of a set of 30 different yearly amounts (0 to 29), with cash flow 0 occurring right now:

0	2,000	6	2,600	12	3,200	18	3,800	24	4,400
1	2,100	7	2,700	13	3,300	19	3,900	25	4,500
2	2,200	8	2,800	14	3,400	20	4,000	26	4,600
3	2,300	9	2,900	15	3,500	21	4,100	27	4,700
4	2,400	10	3,000	16	3,600	22	4,200	28	4,800
5	2,500	11	3,100	17	3,700	23.	4,300	29	4,900

Let's calculate the 'Present' Value at time 15 of cash flows 15 to 29 (with $3,500 keyed into CFo, $3,600 into CF1, etc). At the chosen discount rate of 10% we find a Present Value of $33,700.12. That is the Value of cash flows 15 to 29 brought back to time 15. We may now replace cash flow 15 with that value, and key in cash flows 0 to 15, with 15 as modified:

0	2,000	6	2,600	12	3,200
1	2,100	7	2,700	13	3,300
2	2,200	8	2,800	14	3,400
3	2,300	9	2,900	15	33,700.12.
4	2,400	10	3,000		
5	2,500	11	3,100		

We now find a Present Value of $29,217.63 which is the Present Value of all 30 amounts discounted at 10%.

An alternative approach consists in dividing the cash flows into two groups (0 to 14 and 15 to 29 for instance), discounting each group separately directly to the present (time 0), and adding the two Present Values. Calculating the Present Value of cash flows 15 to 29 is possible by keying in an initial CFo amount of $0.00 with an Nj value of 15 to represent the 15 years where no cash flows are being considered.

The first of these two procedures is more convenient if the cash flows to be discounted already require the use of the Nj function.

MISSING AMOUNT WITH Nj DATA

With or without Nj data, NPV can be interpreted as the Present Value of any amount that needs to be received if we are to achieve the desired rate. We will just give two example here, with more being presented as we address specific applications.

Consider a $50,000 investment and yearly returns of $12,000 for 5 years and $9,000 for the following 5 years.

What amount (reversion on sale of the investment) should you receive over and above the final $9,000 for year 10 if you want a 24% return?

Investment, $50,000:	50000 CHS BLUE CFo	
Returns:		
Years 1 - 5, $12,000:	12000 BLUE CFj 5 BLUE Nj	
Years 6 - 10, $9,000:	9000 BLUE CFj 5 BLUE Nj	
Required rate, 24%:	24 i	
PV of missing amount:	GOLD NPV	($8,627.14)
Missing amount 10 years in future:	10 n 0 PMT FV	($74,145.31)

- As previously stated, NPV provides the Present Value of the missing amount. We find the missing amount by transferring its Present Value to the actual place in time of the missing amount, allowing the calculator to adjust for the time transfer as we do so.

- We get the Present Value of the missing amount **with the wrong sign.** Here, the PV-FV transfer automatically re-establishes the correct sign for the missing amount.

- The initial 10 year projection is communicated to the calculator as IRREGULAR CASH FLOW data with only 2 CFj values: the number in **n** is 2 and does not bear any direct relation to the actual length of time.

  However, when we return to REGULAR CASH FLOW functions (**n, i, PV, PMT, FV**), **n** needs to express the actual number of periods and the time span considered. So we press 10 in **n** to allow a regular cash flow PV--FV transfer to shift the Present Value of the missing amount 10 years into the future.

  Switching the value of **n** around as we go back and forth from irregular cash flow to regular cash flow calculations is a crucial step of all such procedures. Omitting that step may result in an **Error** sign or in substantially erroneous answers.

MISSING AMOUNT: ALTERNATIVE ANSWERS

(See also Unit 22, pages 8 & 9)

Let's consider the following monthly cash flows spread over 5 years:

0	-115,000		115000 CHS BLUE CFo	
1	2,000	(20)	2000 BLUE CFj 20 BLUE Nj	
2	3,000	(18)	3000 BLUE CFj 18 BLUE Nj	
3	4,000	(22)	4000 BLUE CFj 22 BLUE Nj	
What is the rate?			GOLD IRR 12 x	(16.80%)

16.80% is not enough. How can I get the 20% return that I require? By changing the cash flows that we already have: we may decrease the initial investment or add more money at any point in time. The dollar amount of the change depends on where the change occurs, but all the infinitely varied possible amounts and combinations have one thing in common: they all have the same Present Value. Let's calculate it:

PV of missing amounts:	20 BLUE i GOLD NPV	($8,895.76)

This means that to get a 20% return, we need to increase the value of what we are getting by $8,895.76. The actual amount that we are to receive depends on when we receive the amount.

Let's consider four changes, any one of which increases the rate to 20%.

1) I can receive the extra value now. This corresponds to decreasing my initial investment by $8,895.76. When the missing amount is a Present Value, we adjust the initial cash flow by the NPV value calculated.

2) I can increase the amount received at the very end of the transaction: the missing amount is 60 months in the future:

60 n 0 PMT FV	($23,982.69)

3) I can receive an extra amount 10 years in the future:

10 BLUE n FV	($64,656.62)

4) I can increase each of the 60 positive payments by an equal amount.

60 n 0 FV PMT	($235.68)

Adding $235.68 to each of the 60 payments increases the yield to 20%.

So $8,895.76 now, $23,982.69 in 5 years, $64,656.62 in 10 years, and $235.68 every month for 60 months all have a Present Value of $8,895.76 at the discount rate of 20%. Any one of these options, added to those cash flows that we already have, increases the rate of return to 20%.

INITIAL INVESTMENT IS THE MISSING AMOUNT

There are any number of problems where the missing amount is the initial Cash Flow, the value that, if known, would be communicated to the calculator with CFo. When such is the case, it may be easier to analyze and solve the problem if we think of that amount as the missing amount, rather than having to interpret it as being the Present Value or Net Present Value of the known amounts. Let us give two examples.

I An investment promises the following returns. What should we be willing to invest in order to get a 14% rate of return?

Initial investment unknown:	0 BLUE CFo
Cost, End Of Year 1, $30,000:	30000 CHS BLUE CFj
Years 2, 3, 4, & 5, no return:	0 BLUE CFj 4 BLUE Nj
Years 6 to 10 return, $40,000:	40000 BLUE CFj 5 BLUE Nj
Required rate, 14%:	14 i
Missing amount:	GOLD NPV

The investor can afford an initial investment of $45,005.60.

II Calculate the rate of return on the following investment projection. Then calculate the initial investment that will result in a 12% rate of return.

Initial investment $100,000:	100000 CHS BLUE CFo
Return years 1 to 5, $15,000:	15000 BLUE CFj 5 BLUE Nj
Return years 6 to 15, $10,000:	10000 BLUE CFj 10 BLUE Nj
Rate of return:	GOLD IRR

We find 9.01% but want to increase the yield to 12% by investing less than the $100,000 presently considered. Let's take out from the data the $100,000 amount that is no longer valid, leaving a blanc space for the missing amount which we now know how to calculate:

Delete $100,000 from memory 0:	0 STO 0
Impose required 12% rate:	12 i
Calculate missing amount:	GOLD NPV
(Answer: $86,132.53)	

When the missing amount is a Present Value, Gold NPV immediately provides the amount (with the wrong sign which we ignore). Other useful applications will be considered as we examine loans.

PRACTICE PROBLEMS

1) Rate of return on the following investment projection:

Initial investment:	$36,000	
Return, years 1 to 10:	$3,000	
Return, years 11 to 20:	$4,000	(6.78%)

2) Rate of return on the following monthly exchange of money:

-66,000	(1)	
400	(12)	
500	(12)	
600	(36)	
700	(60)	
800	(120)	(10.26%)

3) Interest rate on the following loan with monthly payments:

Loan amount:	$2,500,000		
Monthly payments first 3 years:	$20,000	(36)	
Monthly payments years 4 to 10:	$25,000	(84)	
Monthly payments years 11 to 15:	$30,000	(60)	(8.41% annual)

4) You invest $10,000, get nothing back for 5 years and then $200 a month for the next 5 years and $400 a month for the following 5 years. What is the rate of return?

-10,000		
0	(60)	
200	(60)	
400	(60)	(12.40)

5) What is the Present Value of the following monthly cash flows (all positive) discounted at 10%?

CFo:	$100,000	(1)	
CF1:	1,900	(35)	
CF2:	51,200	(1)	
CF3:	1 300	(83)	
CF4:	115,300	(1)	($295,647.42)

Recall **n**, **i**, **PV** and check the dollar amounts keyed in.

6) Present Value of the following monthly payments, paid in arrears, at 10% and 12% discount rates. What is the difference between these amounts?

No initial amount.

Monthly payments, first 2 years:	$620.00.	
Monthly payments years 3, 4 & 5:	$770.00.	
Monthly payments years 6 to 10:	$920.00.	(59 payments only!)
Balloon payment:	$77,500.00.	

($87,596.22 at 10%, $77,398.06 at 12%, $10,198.17 difference)

7) A $300,000 investment promises the following returns:

$50,000 a year for 5 years,
$70,000 a year for the next 7 years.

- What is the rate of return? (15.88%)
- Put 0 in Register 0 and calculate the initial investment that results in a 20% return. ($250,932.93)
- What is the IRR if the initial investment is $230,000? (22.18%)

8) Consider the following yearly investment projection:

Initial investment:	-125,000
End of Year 1:	-35,000
Years 2 to 5:	0
Years 6 to 10:	70,000

- What is the rate of return? (10.72%)
- By how much should you decrease the initial investment to increase the rate of return to 15%? ($38,771.84)
- By how much should you increase the cash flow for year 10 to increase the rate of return to 15%? ($156,853.70)

9) You make the following investments at the beginning of each year:

Initial investment:	-78,000
Years 1 to 7:	-15,000
Years 8 to 14:	-23,000

- What amount should you receive at the end of the 14th year to get a 10% return on your investment? ($791,726.99)
- If this is a Savings Account offering 10% interest (compounded yearly), what is the balance in the account at the end of year 14? (Same as above: $791,726.99)

10) Consider the following 20 yearly cash flows:

$50,000	(3)
70,000	(7)
100,000	(5)
140,000	(5)

Find the Present Value at a 10% discount under these two assumption:

- First $50,000 amount occurs now. ($718,939.86)
- First $50,000 amount occurs 1 year from now. ($653,581.69)

UNIT 24

CHECKING AND EDITING MULTIPLE CASH FLOW DATA

This short unit seeks to convey the ability to manipulate Nj values.

Some may want to postpone the material on first studying this course as most irregular Cash Flow calculations and applications are possible without manipulating Nj. It just means we should key in the material correctly from the start or key it in afresh if we discover a mistake or want to make some changes. However, mastering the material will provide great flexibility in the handling of irregular Cash Flow data.

RECALLING AND CHANGING Nj VALUES

"j", as in Nj and CFj, stands for 0, 1, 2, 3, etc. to a maximum of 20. So Nj stands for N0, N1, N2, etc. and CFj stands for CF1, CF2, CF3, etc.

When a line of data has been keyed in, the "j" value in CFj and Nj, the value stored in "n", and the number of the Register memory where the Cash Flow is stored have the same value.

To recall or overwrite an Nj value, we key the "j" value in "n".

Let us key in the following monthly cash flows:

0	-100,000	
1	1,000	(24)
2	2,000	(36)
3	3,000	(59)
4	43,000	(1)

```
100000  CHS   BLUE   CFO
  1000  BLUE  CFj     24  BLUE  Nj
  2000  BLUE  CFj     36  BLUE  Nj
  3000  BLUE  CFj     59  BLUE  Nj
 43000  BLUE  CFj
```

1) I may check the value for N2 as follows:

```
2  n   RCL  BLUE  Nj                (36)
```

2) I may change N1 so as to have only 12 CF1 values instead of 24:

```
1  n   12  BLUE  Nj
```

3) **n** is not affected when I key in an Nj value. So if I key in the wrong Nj value, I may immediately correct it by keying in another value:

```
3  n   23  BLUE  Nj
       66  BLUE  Nj
       59  BLUE  Nj
```

Here N3 is changed to 23, 66, and back to 59.

4) I may completely eliminate a CFj (or CFo) value, in amount **and in the time delay that it represents** by affecting it with an Nj value of 0 (zero). This cannot be achieved by just keying 0 in the appropriate Register memory, and does not require that we clear the Register memory.

Let's completely eliminate from our data the 36 CF2 payments of $2,000:

```
2  n    0  BLUE  Nj
```

VERY IMPORTANT: exploring or changing data in this way disrupts the value stored in **n.** The correct value for **n** must be keyed back in (here by pressing **4 n**) before we can solve for NPV or IRR.

SYSTEMATIC EXPLORATION OF CFj AND Nj DATA

With cash flows and Nj values keyed in, and the full **n** value undisturbed in **n**, we may explore the whole exchange of money (dollar amounts and Nj values) in the opposite order from which the numbers were keyed in.

To recall the data still in the calculator, for instance:

Keystrokes	Result	
RCL BLUE Nj RCL BLUE CFj	1 cash flow of $43,000.	
RCL BLUE Nj RCL BLUE CFj	59 cash flows Of $3,000.	
RCL BLUE Nj RCL BLUE CFj	0 cash flow of $2,000.	
RCL BLUE Nj RCL BLUE CFj	12 cash flows of $1,000.	
RCL BLUE Nj RCL BLUE CFj	1 initial cash flow of -100,000 dollars.	
4 n GOLD IRR 12 x		(2.07%) (24.80%)

- Note that CFo is recalled by using RCL BLUE CFj, not CFo.

- Any Nj value may be overwritten as soon as it has been recalled.

- A CFo or CFj value can also be changed with **BLUE CFj** as soon as it has been recalled, provided that we also key back in the appropriate Nj value if other that 1.

- This procedure is made possible because pressing **RCL BLUE CFj** not only recalls the appropriate CFj value, but also decreases "n" by 1: we are ready to explore the previous line or overwrite the line just examined.

- Here again, we need to re-establish the correct value in **n** before we can solve for IRR or NPV.

UNDERSTANDING

To see why we may explore and modify data as above, it is important to understand the crucial role played by the value in **n** as the cash flows are initially keyed in. So let's go back to our original data and follow what happens as we key the data in:

As we key in **-100,000 CHS BLUE CFo**, we **AUTOMATICALLY** accomplish the following:

- We put 0 in **n**: **n** is cleared every time we press **CFo**. This simple feature is what automatically initiates a new irregular cash flow procedure.
- We automatically store -100,000 in Register 0. (That value can later be recalled by pressing **RCL 0**)
- We automatically put 1 in N0, the Nj memory corresponding to CFo. (N0 is "cleared" of any value previously attributed to it).

As we key **1000 BLUE CFj**:

- We increase **n** by 1: 0 becomes 1.
- We put 1000 in Register memory 1. (That value can later be recalled by pressing **RCL 1**)
- We bring N1 back to 1.

As we key **24 BLUE Nj**, with 1 in **n**:

- We store 24 in N1. Instead of one $1,000 value, the calculator now considers 24 such amounts.

As we key **2000 BLUE CFj**:

- We increase **n** by 1: the value in **n** changes from 1 to 2.
- We store 2000 in Register memory 2.
- We bring N2 back to 1.

We now have 2 in **n**, and when we key in **36 BLUE Nj**:

- We store 36 in N2, which tells the calculator that the value stored in memory Register 2 is repeated 36 times.
- We are also ready to key in CF3.

To explore and change Nj values, we simply manipulate **n** to put ourselves back in the position we were in when the data was originally keyed in.

By pressing 2 in **n**, we put ourselves back in the situation we were in after having keyed in CF2 of our original data. This is the right position to be in if we want to key in or overwrite a value for N2.

2 is also what we want in **n** if we are to key in the next CFj value, CF3.

IN SUMMARY:

The value in **n** is modified by the calculator itself in three ways:

1) Keying in data with BLUE CFo	**CLEARS n.**
2) Keying in data with BLUE CFj	**ADDS 1 to n.**
3) Recalling data with BLUE CFj	**DECREASES n BY 1.**

Values for CFj and Nj are keyed in OR OVERWRITTEN as follows:

1) It is with 2 in **n** that we key in CF3.	2	3
2) It is with 3 in **n** that we key in (and Recall) N3.	3	3

Keying in a dollar value with CFo or CFj puts 1 in the corresponding Nj memory: there is no need to press 1 BLUE Nj when the cash flow keyed in with CFo or CFj occurs only once. Keying in a dollar amount directly into the appropriate Register memory (900 STO 2) does not clear the corresponding Nj memory. This is why it makes sense to key in a CFj value first with BLUE CFj: we may change it later directly in the appropriate register memory if the Nj value remains unchanged.

When we solve for IRR or NPV, the value in **n** determines the Register memories, and therefore the CFj and NJ values, that are taken into account in the calculation. With 4 in **n**, only Register memories 0 to 4 are considered. So we may tell the calculator to ignore any cash flow data above a certain j value by storing that value in **n**.

EXPANDED T-CHART

n	CF0 & CFj	Nj	
0	-100,000		\$100,000 amount exchanged for
1	1,000	(24)	24 cash flows of \$1,000, followed by
2	2,000	(36)	36 cash flows of \$2,000, and
3	3,000	(59)	59 cash flows of \$3,000, and
4	43,000	(1)	1 last cash flow of \$43,000.

		120	120 periods (exclude initial amount).

In this expanded T-chart the left column expresses the "j" value--CF0, CF1, CF2--and also the number automatically put in **n** as the dollar value is keyed in with CF0 and CFj, and the Register memory where the dollar amount is stored and from which it can be recalled. It is also the value we need to have in **n** to recall or overwrite an Nj value.

A clear visual presentation of the data is frequently an important first step towards a correct solution. It helps identify any ambiguity or uncertainty and more clearly define the problem. In particular, it is often necessary to determine whether or not there is money changing hands at time 0, and to express the data accordingly.

CHANGING THE DATA: PRACTICE PROBLEM

- Key in the following monthly exchange of money in time (8 years):

CF0	-500,000	(1)	Key in as usual.
CF1	2,000	(12)	
CF2	4,000	(18)	
CF3	5,000	(6)	
CF4	6,000	(24)	
CF5	8,000	(36)	

- Change the data so that CF2 and CF3 occur both 12 times:

```
2   n   12   BLUE   Nj
3   n   12   BLUE   Nj
```

- Add a $456,000 balloon payment as CF6:

```
5   n   456000   BLUE   CFj
```

- This is really a balloon payment that occurs at the end of year 8. It includes the last $8,000 payment, so you have one payment too many affecting CF5. Change N5 to reflect the correct data:

```
5   n   35   BLUE   Nj
```

Why do we need to press **5 n** once again? Is it because we had 6 in **n**?

- Change CF1 from $2,000 to $2,500 using CFj:

```
0   n   2500   BLUE   CFj
          12   BLUE   Nj
```

Here, keying in new CFj data clears the old Nj value: we have to re-write the whole line. An easy alternative would be: **2500 STO 1.**

- CF3, the $5,000 amount for 12 months, was keyed in by mistake. Eliminate that entry.

```
3   n   0   BLUE   Nj
```

There is no need to clear the $5,000. With 0 in N3 the whole line is ignored).

- Now calculate the rate on the data, as modified:

```
6   n   GOLD   IRR
        12     x          (12.49%)
```

EDITING CASH FLOW VALUES

We have seen how to eliminate a line of data (CFj and Nj) by changing the Nj value to 0. This is most convenient if a line of data has been entered twice by error.

What if a line of data has been omitted? We certainly do not have to start again from scratch. We may retain all the correct data by pressing in **n** the **j** value of the last correct entry and then keying back in all subsequent data.

Let's consider the following annual amounts spread over 30 years. We want to calculate their present value at a 10% discount, but in keying in the data, we omit the $30,000 amount that should have been CF5.

0	0	
1	10,000	(2)
2	15,000	(2)
3	20,000	(3)
4	25,000	(3)
5	30,000	(5)
6	35,000	(5)
7	40,000	(10)

0	BLUE	CFo			
10000	BLUE	CFj	2	BLUE	Nj
15000	BLUE	CFj	2	BLUE	Nj
20000	BLUE	CFj	3	BLUE	Nj
25000	BLUE	CFj	3	BLUE	Nj
(Data omitted by error)					
35000	BLUE	CFj	5	BLUE	Nj
40000	BLUE	CFj	10	BLUE	Nj

The data keyed in is correct until CF4. We may press 4 in **n** to retain all the correct data, and key back in all subsequent cash flows:

CF5	30,000	(5)
CF6	35,000	(5)
CF7	40,000	(10)

NPV at 10% discount:
(Answer: $216,886.42)

4	n				
30000	BLUE	CFj	5	BLUE	Nj
35000	BLUE	CFj	5	BLUE	Nj
40000	BLUE	CFj	10	BLUE	Nj
10	i	GOLD	NPV		

BALANCING THE BOOKS ON IRREGULAR CASH FLOW DATA

After a missing amount has been calculated, there are many circumstances where we need to add it back to the original cash flows in order to have a fully balanced exchange of money in the calculator. We may do this without having to key back in the complete exchange of money.

1) **IF THE AMOUNT TO BE ADDED IS NOT A NEW CASH FLOW,** but an amount that will increase cash flows already keyed in, with no discrepancy between Nj values, we may do so directly in the appropriate Register memories. To add $3,000 to the amount already stored in Register 5:

```
RCL 5    3000 +    STO 5
```

With memories 0 to 4 only, we may use storage register arithmetic. For instance, we may add $260.00 to the amounts already stored in registers 1, 2, and 3 as follows:

```
260 STO + 1
    STO + 2
    STO + 3
```

2) **IF THE MISSING AMOUNT IS A NEW CASH FLOW,** a final reversion or balloon payment for instance, then we should key it in with **BLUE CFj.** This means restoring **n** to its original irregular cash flow value.

Let's "balance the books" on this 13.5% interest, 10 year loan.

Loan amount:	-220,000		220000 CHS BLUE CFo
CF1 payments:	2,000	(24)	2000 BLUE CFj 24 BLUE Nj
CF2 payments:	2,500	(36)	2500 BLUE CFj 36 BLUE Nj
CF3 payments:	3,000	(59)	3000 BLUE CFj 59 BLUE Nj
CF4 balloon:	?		
Present Value of missing amount:			13.5 BLUE i GOLD NPV
Balloon payment ($214,262.89):			120 n 0 PMT FV
Balloon payment keyed back in:			3 n RCL FV BLUE CFj

- Pressing 3 back in **n** allows us to key in the balloon payment as CF4.

- We keyed in 59 payments of $3,000, not 60. This conveniently establishes **the whole balloon payment as the missing amount.** If we had keyed in 60 payments of $3,000 then the balance (not the balloon payment) would be the missing amount and it would be more difficult to add that partial cash flow back in in order to balance the books on the transaction.

PRACTICE

The following exercises provide sets of cash flows on which questions are asked. You should go from one set to the next just by editing the previous data, without keying the whole material back in.

1) Yearly data. Calculate the rate of return.

- 45,000	
5,000	(5)
7,000	(5)

- 45,000	
5,000	(5)
7,000	(5)
8,000	(2)

- 45,000	
5,000	(2)
7,000	(8)
8,000	(2)

Ans.: (5.17%) (8.23%) (9.96%)

2) Monthly data. Find the Present Value at a 10% discount.

0	(1)
15,000	(12)
25,000	(36)
35,000	(72)

15,000	(12)
25,000	(36)
35,000	(72)

15,000	(24)
25,000	(24)
35,000	(72)

Ans.: ($2,140,464.03) ($2,158,301.23) ($2,054,479.74)

3) Monthly data. Check the data then find the rate of return.

- 20,000	
100	(36)
200	(24)
400	(60)

- 25,000	
100	(36)
200	(24)
400	(60)

- 25,492.57	
200	(36)
300	(24)
400	(60)

Annual ans.: (7.93%) (4.17%) (7.50%)

4) Monthly data. Find the rate.

0	(60)
- 100,000	(1)
2,000	(60)
3,000	(60)

- 100,000	(1)
2,000	(60)
3,000	(60)

(Same 24.37% annual rate for both problems. The rate is not affected by when an exchange occurs provided the exchange itself is unchanged. This does not apply to a Present Value calculations, where the choice of what constitutes 'the present' is crucial.)

5) Monthly data. Calculate the Present Value at a 15% discount rate.

1,000	(12)
2,000	(48)
3,000	(36)
4,000	(24)
5,000	(60)

1,000	(36)
2,000	(36)
3,000	(36)
4,000	(24)
5,000	(60)

0	(36)
0	(36)
0	(36)
4000	(24)
5000	(60)

Ans.: ($188,757.23) ($165,.507.96) ($63,124.18)

Check Nj value for cash flows 0 and 2.

6) Monthly data. Calculate the Present Value at a 10% discount.

0	(1)
500	(24)
600	(36)
700	(36)
800	(24)
900	(60)

0	(1)
500	(24)
600	(36)
700	(36)

0	(1)
500	(24)
600	(36)
700	(36)
800	(24)

0	(1)
500	(24)
600	(36)
700	(36)
800	(24)
900	(60)
1,000	(60)

Ans.: ($62,720.81) ($39,257.42) ($47,073.16) ($73,287.99)

Check the Nj value for cash flow 3 and 4.

7) Monthly data. Check the complete exchange, then calculate the rate.

- 50,000	(1)
500	(12)
550	(18)
600	(24)
700	(30)
750	(36)
800	(42)

- 50,000	
500	(12)
550	(18)
600	(24)
650	(30)
700	(36)
750	(42)
800	(48)

(12.61%) (13.75%)

UNIT 25

SPECIAL APPLICATION: REAL ESTATE LOANS

The skills acquired so far in solving time value of money problems--both irregular cash flow exchanges studied in this volume and regular cash flow situations considered in Volume I--, lead to an infinite variety of applications in various fields of finance and financial analysis. Let us explore some here involving real estate loans.

These applications do not all require full mastery over the manipulation of Nj data, though on occasion those manipulative skills will be called upon. They do require the ability to key in data using **CFo**, **CFj** and **Nj**, and the ability to change dollar amounts directly in the Register memories. Many, that rely only on the regular cash flow functions, may later be transferred to a new edition of Volume I.

IRREGULAR CASH FLOW LOANS

Let's consider a number of problems that all require as a first step that we key in complete, balanced loan data into the calculator. Complete, that is, except for the face value of the loan (the loan amount) that in some cases is not required.

We will key the balanced loan data in once, and move from problem to problem without having to key the original loan back in. The user selecting a single application must begin by keying in the original loan data.

BALANCED LOAN DATA

Let's key in the following complete data on a 10 year loan with monthly payments:

Loan:	-220,000		220000	CHS	BLUE	CFo		
CF1:	2,000	(24)	2000	BLUE	CFj	24	BLUE	Nj
CF2:	2,500	(36)	2500	BLUE	CFj	36	BLUE	Nj
CF3:	3,000	(59)	3000	BLUE	CFj	59	BLUE	Nj
CF4:	214,262.89	(1)	214262.89	BLUE	CFj			

This represents a complete exchange of money in time. We have the loan amount (as a negative), the changing payments, and the balloon payment. If the balloon payment was not known, it should be established and entered as irregular cash flow data (See above). We have called establishing that data and keying it into the calculator 'balancing the books' on the exchange of money in time.

I CHECKING THE RATE

The interest is 13.5%. Let us check that we indeed have a 13.5% rate:

GOLD IRR 12 x	(13.5%)

An alternative procedure for checking that we have the correct rate is to key that rate in and question for NPV: the answer should be 0 (zero).

13.5 BLUE i GOLD NPV	(Close to 0)

The expected rate is achieved if we find that there is no missing amount preventing us from reaching the expected rate (NPV = 0). Instead of zero, we generally find some small fraction of a cent, such as 0.000004, which is close enough to 0 by any standard.

This last procedure results in a shorter calculation than actually solving for the IRR.

II ANNUAL PERCENTAGE RATE (APR)

Same loan. What is the Annual Percentage Rate (APR) if loan costs are 3 points and $500.00?

The Annual Percentage Rate (APR) is the rate from the point of view of a borrower who makes all the scheduled payments but does not receive the full face value of the loan because he has to pay points and other loan costs.

By adjusting in Register 0 the amount of the loan to reflect the net amount actually received by the borrower, we are able to calculate the real cost of the loan, the APR. This procedure remains valid whether or not the points and costs are paid up front by the borrower, or whether they are deducted by the lender from the loan proceeds.

	(With loan data 'balanced' in the calculator)	
Recall loan amount:	RCL 0 CHS	($220,000)
Calculate net loan amount:	3 % - 500 -	($212,900)
Key net loan back in (negative):	CHS STO 0	(-212,900)
Calculate APR:	GOLD IRR 12 x	(14.08%)

- The keystrokes given above assume that the loan amount was initially keyed in as a negative, and all subsequent payments as positive amounts. If the opposite is the case, the two **CHS** keystrokes should be omitted.

- With the signs as we have them, we need to perform some arithmetic calculations on the $220,000 loan amount that we have in Register 0 as a negative number. The safe approach is to change that negative number into a positive amount, perform the arithmetic, and change the answer back into a negative. An alternative to changing the signs is to ADD the $500 we want to subtract:

```
RCL  0
3  %     -     500   +
STO  0
GOLD  IRR     12    x
```

- The user should refer back to the appropriate rules and regulations to establish which costs are to be taken into account in the calculation. With variable rate loans, the APR is presently calculated on the scheduled repayment amounts that would occur if there were to be no changes in the underlying index.

III DISCOUNTED NOTE CALCULATIONS

YIELD ON DISCOUNTED LOAN:

Same loan. What is the yield if the loan is bought for $190,000?

(With loan data 'balanced' in the calculator) 190000 CHS STO 0 GOLD IRR 12 x	(16.14%)

By changing CFO to reflect the discounted purchase price, we leave in the calculator the exchange of money in time from the point of view of the buyer of the discounted note, a buyer who receives the scheduled payments but does not have to invest the full face value of the loan. Once we have the reality of that exchange of money in the calculator, pressing **GOLD IRR** gives the rate of return (yield) on the buyer's investment.

PRICE OF DISCOUNTED LOAN:

Same loan. What discounted selling price gives a 20% yield or a 25% yield?

	(With loan data 'balanced' in the calculator)	
Eliminate loan amount that no longer applies:	0 STO 0	
Price for 20% yield:	20 BLUE i GOLD NPV	($155,605.02)
Price for 25% yield:	25 BLUE i GOLD NPV	($123,261.63)

By pressing **0 STO 0** we eliminate from our irregular cash flow data the initial loan amount (or whatever amount has replaced it in Register 0). The initial cash flow then becomes the missing amount which we calculate by imposing the desired rate of return and pressing **GOLD NPV.**

- o 0 o -

To prepare for the next problem, let's re-establish the original loan data in the calculator by keying the loan amount back in memory 0:

```
220000 CHS STO 0
```

DISCOUNT TO ACHIEVE GIVEN YIELD

We still want a 20% and a 25% yield. But we want to know the amount of the discount, not the discounted value. The same keystrokes give the answer provided we retain the original loan amount in Register 0:

	With loan data 'balanced' in the calculator	
Discount for 20% yield:	20 BLUE i GOLD NPV	($64,394.98)
Discount for 25% yield:	25 BLUE i GOLD NPV	($96,738.37)

Of course, Discount + Discounted price = Face value.
64,394.98 + 155,605.02 = 220,000.00
96,738.37 + 123,261.63 = 220,000.00

YIELD ON MATURE DISCOUNTED LOAN.

What is the yield if the same loan is bought for $190,000 thirty (30) months after its creation? (No CF1 and only 30 CF2 payments are received).

	(With loan data 'balanced' in the calculator)	
Eliminate CF1:	1 n 0 BLUE Nj	
Adjust CF2:	2 n 30 BLUE Nj	
Restore **n** value:	4 n	
	190000 CHS STO 0	
	GOLD IRR 12 x	(18.11%)

The key, here again, is to leave in the calculator the REALITY of the exchange of money from the point of view of the buyer of the discounted note: what is he investing, what is he getting back in return at each point in time? Here, we adjust the original loan directly in the calculator to reflect what is left of it. It might be easier to establish those numbers on paper, and to key in the new values from scratch:

Investment:	$190,000	
Cash Flows Purchased:	2,500	(30)
	3,000	(59)
	214,262.89	

190000	CHS	BLUE	CFo			
2500	BLUE	CFj	30	BLUE	Nj	
3000	BLUE	CFj	59	BLUE	Nj	
214262.89	BLUE	CFj				
GOLD	IRR	12	x			

There is no shortcut to first establishing the reality of the exchange in our minds and then to communicating it faithfully to the calculator.

IV BALANCE AND INTEREST ON IRREGULAR CASH FLOW LOAN

We may also use the loan data 'balanced' in the calculator to find the remaining balance of a loan at any point in time. The key is to manipulate **n** and **Nj** so as to have the calculator consider only the payments that have been made during the time period considered. THE BALANCE OF THE LOAN IS THE AMOUNT THAT, IF PAID, CLOSES THE ACCOUNT ON THE EXCHANGE OF MONEY. IT IS THE MISSING AMOUNT THAT BALANCES THE BOOKS ON THE EXCHANGE OF MONEY UP TO THAT POINT IN TIME.

Let's key back in the $220,000 as on page 1, and the 13.5% rate:

Loan:	-220,000	
CF1:	2,000	(24)
CF2:	2,500	(36)
CF3:	3,000	(59)
CF4:	214,262.89	(1)
Interest rate:		

220000	CHS	BLUE	CFo			
2000	BLUE	CFj	24	BLUE	Nj	
2500	BLUE	CFj	36	BLUE	Nj	
3000	BLUE	CFj	59	BLUE	Nj	
214262.89	BLUE	CFj				
13.5	BLUE	i				

- What is the balance of the loan at the end of 5 years and 7 months (68 months) and the interest earned to date?

Isolate relevant CFs:	3 n	
Retain 7 payments for CF3:	7 BLUE Nj	
PV of missing amount:	GOLD NPV	
Calculate missing amount	68 n	
68 months in the future:	0 PMT FV	($238,699.92)

- The interest to date is the total amount paid out less the principal reduction or plus the principal increase and, with the balance still in the display, may be calculated as follows:

	3 n	
	0 i GOLD NPV	
Interest 1st 68 months:	+	($177,699.92)

3 n isolates once again the relevant 68 cash flows. **0 i GOLD NPV** calculates the total amount of money paid out minus the loan amount. Pressing **plus** adds back the balance that remains in the Stack. The reader may check that this is equivalent to total amount paid out minus principal reduction or plus principal increase.

Similar calculations made for the beginning and end of a span of time allow us to take the difference and find the principal increase or decrease, and the amount of interest, that occurs during that period of time.

V CASH EQUIVALENT CALCULATION

A seller receives a $375,000 downpayment for his property plus the following monthly payments on a 10 year loan that he carries at 9% interest:

First 3 years:	$4,500.00.	(36 payments).
Years 4 to 6:	$5,500.00.	(36 payments).
Last 4 years:	$6,500.00.	(47 payments).
Balloon payment:	$493.836.29.	(Includes last payment).

What is the cash equivalent price of the sale if money is worth 13% to the seller?

The seller has agreed to finance part of his property at 9% interest even though alternative investments could have yielded 13%. He has done so in order to sell the property. However, that comparatively low rate of interest represents a loss, and the actual selling price misrepresents the actual value of what the seller is receiving for the transaction. To find the real value of what the seller is getting for his property, we may discount all the amounts he receives at the 13% rate that other investments could provide. The Cash Equivalent calculation is a simple Present Value calculation based on the those amounts actually received (all positive amounts), and 13% used as the discount rate.

```
  375000  BLUE  CF0
    4500  BLUE  CFj   36  BLUE  Nj
    5500  BLUE  CFj   36  BLUE  Nj
    6500  BLUE  CFj   47  BLUE  Nj
493836.29 BLUE  CFj

13  BLUE  i     GOLD  NPV                 ($864,585.81)
```

A Cash Equivalent price of $864,585.81 means that a cash offer $864,585.81, with part of that amount invested at 13%, could provide the seller with the same cash flow benefits as the current sale with owner financing.

We have not used the 9% interest--the same payments could easily have been provided by other loans at different rates without affecting the Cash Equivalent price--, nor have we been told the actual selling price or the amount of the seller-carried loan. However, because we know the rate on the carry back financing, we may establish both amounts:

```
Selling price:  9  BLUE  i     GOLD  NPV      ($1,000,000.00)
Loan amount:    375000   -                      ($625,000.00)
```

When we key in the actual rate of interest on the carryback financing, the Cash Equivalent price calculation provides the actual selling price.

VI BLENDED RATE

Any time we think of "rate", we should think "cash flow", and more precisely "exchange of money in time", as a rate is nothing more than a means of rating such an exchange. When we ask what the blended rate is on a package of two or more loans, we simply mean the rate that corresponds to the total amount borrowed exchanged for the total amount paid back in exchange. It is the rate corresponding to an exchange of money represented by the total amount of money changing hands at each point in time. We may calculate the blended rate by establishing that exchange, keying it in, and solving for the rate.

DIFFERENT DUE DATES: IRREGULAR CASH FLOW DATA:

When the loans have different due dates, the total exchange leaves us with irregular cash flow data. Let's take a simple example and carefully establish the total cash flows.

The purchase of a property is financed with two loans:

Loan A: $95,000 loan, 10.5% interest, monthly payments of $900.00, balance due in 10 years.

Loan B: $57,000 loan, 14% interest, amortized over 20 years, balance due in 4.
What is the blended rate?

The first step is to get all the information we can on each of the loans, which means calculating the payments and the balances that are not yet known:

```
95000 PV   10.5 BLUE i
900 CHS PMT   10 BLUE n
FV                                   ($80,506.48: 10 year balance)

57000 PV   14 BLUE i   20 BLUE n
0 FV   PMT                           ($708.81: monthly payment).
4 BLUE n   FV                        ($54,202.59: 4 year balance)
```

We now carefully establish the net amount of money changing hands at each point in time. In keying in the numbers as irregular cash flow data, we may want to retain the "money in - positive, money out - negative" approach to the signs if there is any danger of confusion on the subject. Here, we will simply key the total amount of the loans in as a negative and save ourselves the trouble of changing the sign on all subsequent payments.

	LOAN 1	+	LOAN 2	=	TOTAL CASHFLOW	
1	95,000	+	57,000	=	152,000	(negative)
2	900	+	708.81	=	1,608.81	(47)
3	900 + 80,506.48	+	708.81	=	82,115.29	(1)
4			708.81	=	708.81	(71)
5			708.81 + 54,202.59	=	54,911.40	(1)

We now key in the cash flows represented in the T-chart with the usual irregular cash flow procedure (making sure to key $152,000 in as a negative). Pressing **GOLD IRR** gives the monthly rate, which we mupltiply by 12 to get the blended annual rate (11.67%).

In the process, we have acquired a clear picture of the payments that will be rquired of the borrower, which is useful information even apart from the rate calculation it allows.

SAME DUE DATE: REGULAR CASH FLOW DATA.

For good measure, let us perform the equivalent calculation on the assumption that the loans are to be paid off at the end of year 4. If all the loans have the same due date, the sum of all the loans is a regular cash flow exchange. We may balance the books on each of the loans, accumulate the PV, PMT, and FV data in memories 1, 2, and 3 using storage register arithmetic, transfer the total of the loans, the payments, and the balances back into PV, PMT, and FV, and solve for **i**:

```
95000  PV      10.5  BLUE  i
900  CHS  PMT     4  BLUE  n
FV                                  ($90,920.70: 4 year balance)

          STO 3                  }
RCL  PMT  STO 2                  }  LOAN 1 IN REGISTERS 1, 2 & 3.
RCL  PV   STO 1                  }

57000  PV    14  BLUE  i
20  BLUE  n   0  FV    PMT          (-708.81: monthly payment)
4  BLUE  n      FV                  (-54,202.59:  4 year balance)

          STO + 3                }
RCL  PMT  STO + 2                }  LOAN 2 ADDED TO REG. 1, 2 & 3.
RCL  PV   STO + 1                }

RCL  1   PV           ($152,000)      }
RCL  2   PMT          (- 1608.81)     }  SUM OF
RCL  3   FV           (- 145,123.29)  }  2 LOANS
  i                                     (0.98%: monthly rate)
 12  x                                  (11.81%: blended rate)
```

VII ALL INCLUSIVE TRUST DEED (AITD)

A house is selling for $200,000. There is a loan at 10% interest on it with a balance of $110,000. The current rate on new loans is 13%. The traditional way to finance the purchase is to get the buyer to put 20% down ($40,000) and get a new loan for the balance ($160,000). The proceeds on the new loan are then used to pay off the old loan, and the $50,000 that remain are given to the seller along with the $40,000 downpayment. The seller has paid off his obligations on the property and has received his equity in cash.

Nothing needs to change from the buyer's point of view if the seller agrees to carry an AITD (or wraparound loan). He still puts $40,000 down and borrows $160,000. But instead of borrowing from an outside lender, he borrows from the seller.

The seller, in turn, is responsible for paying off the existing loan, but instead of paying it off right away, he keeps the loan and pays it off as it was meant to be paid: in monthly instalments spread over the term of the loan. The seller's situation is now as follows:

- His investment in the AITD is not the full $160,000, but only the $50,000 that he would otherwise have received and agrees not to receive.

- The $110,000 which he is borrowing at 10% is re-invested in the AITD at 13%, which results in a significant profit.

- Each month, he receives the payments on the AITD but must make the payments on the existing loan. The difference between what he receives and what he must pay out is what he gets in exchange for his $50,000 investment.

The buyer normally also shares in the benefits. His rate is likely to be lower than market rate, it will be easier for him to qualify, the transaction can close faster.

From the seller's point of view, an important consideration is the yield that the AITD offers on his net investment (the $50,000 AITD equity in our example). That yield can be calculated exactly as in the blended rate calculations above, except that now we should take the difference between the AITD and the underlying loans, not the sum. Here again, if we have multiple underlying loans with different due dates, establishing the bottom line requires some careful calculations and results in irregular cash flow data. Let us just take the previous example and specify an 8 year due date on the AITD--which means that the underlying loan will be paid off at that time. The loan could also be assumed by the buyer, which is the the same as paying it off from the seller's point of view as in both cases he has to pay someone for relieving him of the debt.

AITD:	$160,000, 13% interest, amortized over 30 years.
Underlying loan:	$110,000, 10% interest, $952.00 per month.
	Both loans are paid off in 8 years.

Keystrokes	Result / Comment
160000 PV 13 BLUE i	
30 BLUE n 0 FV PMT	($1,769.92)
8 BLUE n FV	($153,875.73)
STO 3	
RCL PMT STO 2	Transfer AITD to R1, R2, R3.
RCL PV STO 1	
110000 PV 10 BLUE i	
952 CHS PMT	
8 BLUE n FV	($104,834.94)
STO - 3	
RCL PMT STO - 2	Deduct underlying loan from AITD in memories R1, R2, R3.
RCL PV STO - 1	
RCL 1 PV	($50,000) AITD equity.
RCL 2 PMT	($817.92) Net payment.
RCL 3 FV	($49,040.80) Net balance.
i 12 x	(19.53%: yield on AITD equity.

The seller receives a 19.53% rate of return on his $50,000 investment.

Multiple underlying loans with different due dates require the careful elaboration of the net amounts to change hands at each point in time, and the use of irregular cash flow functions to calculate the rate.

See statistical functions (Unit 31, page 8) for a weighted average approach to blended rate and AITD equity rate calculations.

VIII **BUYDOWN**

A buydown occurs when a lender is willing to lend (at 13% for instance), and a borrower is willing to borrow at a lower rate. For a price, the lender agrees to lend at the lower rate. That price a cash payment to him that, in effect, lowers his net investment in the loan in such a way that his yield is increased to the desired (13%) rate. The cash payment to the lender can be made either by a third party--for instance the seller of the property--, or by the borrower himself.

Let us give here an example of the typical 3-2-1 buydown generally paid by the seller, a buydown that requires irregular cash flow functions. We will return to buydowns that apply to the life of the loan in the section that follows on choices and options open to borrowers.

> A lender is offering 30 year loans at 13% interest to finance the homes a developer is trying to sell. The seller wants to offer a lower rate to his buyers: 10% interest the 1st year, 11% the second year, 12% the third year, and the full 13% thereafter-- 3, 2, and 1 percentage points lower than the commitment rate for the first three years of the loan. What is the cost of the buydown that compensates the lender for the lower payments?

By performing the calculations on a $100,000 loan, we will reach results that can easily be expressed as a percentge of the loan and can then be used with loans of other amounts.

STEP 1: Let's establish the payments that the buyer is required to make as the rate increases from 10% to 11%, 12%, and finally 13%.

	100,000	PV	0	FV			
10	BLUE	i	30	BLUE	n	PMT	($877.57)
	12	GOLD	AMORT				
11	BLUE	i	29	BLUE	n	PMT	($951.31)
	12	GOLD	AMORT				
12	BLUE	i	28	BLUE	n	PMT	($1,025.65)
	12	GOLD	AMORT				
	RCL	PV	RCL	PMT	-		(Balloon: $99,508.69)
13	BLUE	i	27	BLUE	n	PMT	($1,100.43)

The **12 GOLD AMORT** procedure is used here simply as a means of automatically adjusting the loan amount in **PV** to the remaining balance at the end of the current year. **The balance is then amortized at the new rate over the remaining term of the loan.** This procedure allows us to establish the payments that will be made, and the $99,508.69 balloon payment at the end of the three year buydown period.

- **STEP 2:** We may now key in the actual exchange of money for the three year buydown period as if the loan was being paid off at the end of year 3. After year 3 the lender gets the required 13% yield directly from the buyer and we may ignore those later years, though keying them in would not change the final answer.

```
100000  CHS  BLUE  CFo
  877.57  BLUE  CFj   12  BLUE  Nj
  951.31  BLUE  CFj   12  BLUE  Nj
 1025.65  BLUE  CFj   11  BLUE  Nj
99508.69  BLUE  CFj
    13  BLUE  i
    GOLD  NPV
```

($5,130.48, or 5.13% of loan)

- Pressing **GOLD NPV** calculates the missing amount: the amount that allows the lender to get a 13% return. $5,130.48 for a $100,000 loan means 5.13048% of the loan for loans of different amounts. Charging 6 points on the basis that 3 + 2 + 1 = 6 makes no mathematical sense.

- Even if 5.13% were charged for buydown costs, a buyer would be better off if the seller used the amount of the buydown to reduce the selling price and the loan amount, while keeping the downpayment and all the payments the same and retaining the full (13%) rate on the loan. The resulting negative amortization loan would then slowly increase to the End Of Year 3 balance established above. This would provide the buyer with the important advantage of a lower loan balance if he happended to sell the property and pay off the loan before the end of the buydown period. It might also offer him higher interest deductions. (See Volume I, Unit 5, p. 27 for details).

OPTIONS AND STRATEGIES FOR HOME LOAN BORROWERS

Beginning with the option that borrowers sometimes have of buying down the rate of interest for the life of the loan, we are going to consider now a number of choices that borrowers have to make, and some procedures and strategies to help them in their choice.

Most calculations require only the regular cash flow functions of the calculator, and this section may well be transferred to volume I in a later edition of this course.

BUYDOWN ON FIXED RATE LOAN

A different kind of buydown occurs when a borrower is offered a slightly lower rate on a mortgage if he accepts to pay slightly higher points 'up front'. Let's take an example of what is currently available.

> Compare a 30 year, 12% loan to a 30 year, 11.75% loan that requires 1/2 point more in loan costs. Let's take a $100,000 loan with an extra cost of $500.00 for the lower rate.

We may calculate the APR and find that the 11.75% loan offers a slightly lower rate. But let's proceed with a differential cash flow analysis approach by first establishing the cash flow for both options:

```
100000  PV   30  BLUE  n     12  BLUE  i  0  FV   PMT      ($1,028.61)
                          11.75  BLUE  i          PMT      ($1,009.41)
-                                                            ($19.20)
PMT    500  PV    i   12  x                                  (46.09%)
5  BLUE  n        i   12  x                                  (39.48%)
3  BLUE  n        1   12  x                                  (22.42%)
```

Here we calculate the monthly payments on the two loans, and the $19.20 difference between those payments. The cost of saving $19.20 for the next 30 years is the extra 1/2 point on the loan costs, or $500.00. So we calculate the rate of return on a $500.00 investment that produces a $19.20 return each month for the next 30 years: we find 46.09%. Because we are not sure of keeping the loan 30 years, we assume that we are going to enjoy the savings for only 5 years: the rate of return is then 39.48%. It falls down to 22.42% if we keep the loan only 3 years. (We ignore the slight discrepancy in the balance that would marginally increase these rates).

We may divide the $500 investment by the monthly savings to calculate the break even point:

```
RCL  PV   RCL  PMT   ÷                                   (26.04 months)
```

How long does it take to get a given rate of return, for instance 10%, on the $500.00 investment?

```
10  BLUE  i    n                                         (30 months)
```

The key to this analysis is to recognize that we are focussing upon the reality of what is being requested of the borrower (come up with an additional $500.00), and the reality of what the resulting consequence is for him (save $19.20 a month for 30 years, 5 years, or just 3 years).

Whether similar buydowns are as good an investment as suggested by these figures depends on the numbers themselves.

30 YEAR LOAN VERSUS 15 YEAR LOAN

The advantage of a 15 year loan over a 30 year loan is not that the first is paid off sooner--the 30 year loan can also be paid off sooner if we make higher payments. The benefit has to be a lower rate.

We have to choose between a 30 year loan at 9.5% interest, and a 15 year loan at 9%. Let's take an arbitrary $100,000 amount for the loan.

100000 PV 0 FV	
15 BLUE n 9 BLUE i PMT	($1,014.27)
30 BLUE n 9.5 BLUE i PMT	($840.85)
-	($173.41)
15 BLUE n FV	($80,524.26)
173.41 PMT 0 PV i 12 x	(11.22%)

Do we want to invest $173.41 every month for 15 years and be rewarded with not owing what would otherwise be a $80,524.26 balance? In other words, do I want to invest $173.41 a month for 15 years at 11.22% interest? Such is the choice we are asked to make.

If we assume that we do not keep the loan 15 years, then we need to calculate the difference between the balances at the chosen term. The rate of return on the investment increases if we pay off the loan early.

LOWER RATE FOR HIGHER PAYMENTS

A borrower has a 7% interest loan with a balance of $44,000. He makes payments of $385.00. Because the 7% rate is low, the bank offers to lower the rate to 5% if he agrees to double his payments. The advantge to the bank is that the loan will be paid off sooner.

Let's calculate the remaining terms on the existing and recast loans:

44000 PV	
385 CHS PMT 0 FV 7 BLUE i n	(189 months)
RCL PMT 2 x PMT 5 BLUE i n	(66 months)
-	(123 months)

The borrower is asked to make extra payments of $385.00 for 66 months which will result in savings of $385.00 a month for the following 123 months. What is the rate of return on the investment? The 8.20% answer allows the borrower to see if he has better things to do with his money.

385 CHS BLUE CFj 66 BLUE Nj	
385 BLUE CFj 99 BLUE Nj	
385 BLUE CFj 24 BLUE Nj	
GOLD IRR 12 x	(8.20%)

PAYING MORE THAN THE MINIMUM ON A LOAN

Some lenders make it difficult for the borrower to pay more than the required payments. There may be lock-in clauses that prevent the prepayment of a loan before a certain date, prepayment penalties that penalize prepayment over a certain percentage (20% of the original loan balance per year, for instance), or requirements that any payment over the required amount be in increments of $100.00. But most home loan lenders fix a minimum payment amount and allow the borrower to pay that amount 'or more' with no or few limitations.

In most cases it is not even necessary to write a separate check or to assign the additional amount to principal reduction. The amount of interest that is due depends on the rate and on the outstanding balance of the loan. Any extra amount automatically goes to principal reduction.

We will assume here that borrowers have checked with their lenders and have assured themselves that they can increase their payments at will.

UNVESTING IN YOUR OWN LOAN

> Let's consider a 10.25% interest loan with monthly payments. What happens if one month the borrower writes a check for $100.00 more than the required payment?

It immediately reduces the outstanding balance by $100.00 and it reduces the balance on the loan at a later point in time by an amount equal to what $100.00 would grow into if it was invested at 10.25% interest, compounded monthly, for the same period of time.

So making an extra payment is like investing that amount at the rate that is being charged on the loan. The investment is redeemed in the form of a reduced balance and increased equity when the loan is paid off. Most borrowers have the option of investing at the rate that they are paying on their home mortgage. That investment cannot be redeemed until they pay off the loan, but it may offer a substantially higher rate of return than other investments available to them.

The investment we make in our loan is not tax free. By reducing the loan balance, the extra payment reduces the amount of interest we pay, and therefore the tax savings that would result from higher interest payments. The loss in tax savings is exactly equal to the tax liability that would result from investing the same amount in a savings account at the same rate as the mortgage and for the same period of time.

Making higher than required payments has the same effect on adjustable rate mortgages, except that the rate of return on the investment varies with the rate of interest on the loan, and that such investments also affect the payment amount that is later required. (See Unit 25, p. 18).

EFFECT OF INCREASED PAYMENTS ON THE BALANCE OF A LOAN

The effect of increased payments on the future balance of a loan does not depend on the amount of the loan or term over which it is amortized. It depends only on the amount of the increase and the rate of interest.

If I increase my payments by $100.00 every month for 5 years on a 10.25% interest loan, the balance will be reduced by $7,795.08:

Keystrokes	Result
100 CHS PMT	
5 BLUE n	
10.25 BLUE i	
0 PV FV	($7,795.08)

To verify this, let's calculate the payments and the 5 year balance on a $75,000 loan at 10.25% interest amortized over 15 years. We may then increase the payments by $100.00 and see how that affects the balance.

Keystrokes	Result
75000 PV	
15 BLUE n	
10.25 BLUE i	
0 FV PMT	($817.45: monthly payment)
5 BLUE n FV	($61,215.31: 5 year balance)
RCL PMT 100 -	($917.45: new payment amount)
PMT FV	($53,420.23: new 5 year balance)
-	($7,795.08: difference between balances)

EFFECT OF INCREASED PAYMENTS ON THE TERM OF A LOAN

Let's take a 30 years fixed rate loan at 12.5% interest and calculate how a 15% increase in the payments reduces the term of the loan:

Keystrokes	Result
100000 PV	
30 BLUE n	
12.5 BLUE i	
0 FV PMT	($1,067.26)
15 % +	($166.09 increase, $1,227.35 payment)
PMT n	(Term in months: 183 or 15.25 years)

If we express the increase as a percentage of the payments, the new term is the same whatever the amount of the loan. But the new term is affected by the rate on the loan: it becomes more dramatic on loans with high rates, and less so on loans with lower rates. At 10% interest, a 15% increase in the payments reduces the term to 17 years and 7 months.

STRATEGIES FOR PAYING A LOAN OFF EARLY

The best strategy for reducing the term of a loan is for the borrower to increase the monthly payment by an amount that he can afford, and to keep making that higher payment as if it were the required amount. Every few years he may want to reassess whether he can afford to increase the amount.

Other strategies suggest doubling the principal reduction portion of each payment (and writing a separate check for that amount), or making payments every two weeks of half of the required amount. Let's discuss these options.

The inconvenience of doubling the principal reduction portion is that the amount changes every month. Do I really have to increase my payment by $35.65 the first month and by $36.31 the next as I would with a $100,000 loan at 11% interest? Why not instead round the $952.32 payment to $1,000.00 for a year or two, and later increase it by an extra 50 or 100 dollars? Of course, most borrowers would have no need to write a separate check for the extra amount.

Making payments every two weeks can have a dramatic effect on the term of the loan. However, this is not because we are making one half of the payment half a month early--few lenders would actually take this into account and credit the borrower for the lower loan balance. The advantage comes from the fact that there are 52 weeks in a year (in fact, slightly more). So making half a payment every two weeks means that we make 26 half payments every year, or the equivalent of 13 full payments, one more than otherwise. Increasing the monthly payment by 1/12 would have just about the same effect.

There is an easy way of checking what effect making a full payment one full month ahead of schedule has on the term of a loan: that is to switch to the **BEGIN** option of the calculator:

100000	PV		
11	BLUE	i	
30	BLUE	n	
0	FV	PMT	($952.32)
BLUE	BEG	n	(337 months or 28 years and 1 month)
BLUE	END		(Back to normal setting)

Paying only half the payment half a month ahead of schedule offers at best 1/4 of the advantage of the previous assumption, and could reduce the term only marginally. Without the two extra half payments every year, the suggested strategy would have little effect.

EFFECT OF INCREASED PAYMENTS ON ADJUSTABLE RATE MORTGAGES

With a fixed rate loan, making payments higher than required reduces the balance of the loan and the remaining term of the loan, but the minimum payment required by the lender is not affected.

Not so with the standard Adjustable Rate Mortgage: the term of the loan is not reduced; instead, the required payments are. This is because on every change date the payments are recalculated on the basis of the new rate. When that happens, the outstanding balance of the loan is amortized over the remaining term of the loan. By making higher than required payments, the borrower reduces the balance of the loan, and so reduces the payments that are required of him compared to what they would otherwise have been after the change.

This can be used as a strategy to protect oneself against the effect of increasing rates and as an analytical tool in choosing between a fixed rate and an adjustable rate mortgage. So let's take a closer look at Adjustable Rate Mortgages.

ADJUSTABLE RATE MORTGAGES (ARM)

There is a considerable variety of adjustable rate mortgages. Let's consider some calculations involving them: procedures that allow us to follow such loans through various rate change scenarios, approaches that may help in choosing between fixed rate and ARM loans, and strategies that may protect an ARM borrower from sudden increases in his payments.

TYPICAL ARM LOANS

PERIOD AND CHANGE DATE

ARMs define a period at the end of which rates are adjusted. Rates are typically adjusted every month, every 6 months, or every 1, 2, 3 or 5 years. The end of each period marks the 'change date'.

INDEX

ARMs also provide for an index frequently tied to the adjustment period (1 year Treasury Bill rate for 1 year ARMs, for instance), or to the average cost of funds for a particular region. The change in the index determines the change in the rate on the change date.

An index based on the average cost of funds has traditionally been more stable than those based on Treasury Bills or other financial instruments. Longer terms also tend to provide greater stability in the index: the 6 month T-Bill being more volatile than the 3-year equivalent. Stability slows the increase and also the decrease in the rate. Before choosing stability borrowers should consider carefully the initial rates that each index offers.

MARGIN

The new rate is determined by adding a fixed MARGIN to the current value of the INDEX. If the index rate is 7% for a loan that was created with a 2.65% margin, the rate on the loan will increase or decrease to 9.65%.

TEASER RATE

Typically, with ARM loans, the rate for the first period is not defined by the current index and margin, but is 'arbitrarily' chosen by the lender. This initial rate is referred to as the 'teaser' rate, and is normally lower than what index and margin would define.

The teaser rate is in effect only during the first period. In choosing between loans, borrowers should look beyond the teaser rate, and should request solid information on the rate that would prevail if index and margin were immediately in effect.

However, as a result of the teaser rate, what the index is on the day you lock in your initial rate generally has no direct effect on the loan itself. What the index is on the first change date is important.

CAPS

ARM loans also traditionnally offer some protection to the borrower against sudden increases in the payments. This is done by means of a limit or cap on the increase (or decrease) that can be imposed, either on the rate or on the payments themselves.

In this respect there are two important categories of ARM loans: those that allow for a possible negative amortization on the loan, and those that do not.

NEGATIVE AMORTIZATION LOANS

Those that allow negative amortization normally put a cap on the increase in the amount of the PAYMENTS, but not on the rate. For instance, payments may not increase by more than 7.5% with each adjustment. If the increase in the rate calls for a higher increase in the payments than allowed by the cap, this does not result in a loss by the lender: instead, the balance of the loan is increased each month by the amount of interest owed but not paid: we have negative amortization.

NO NEGATIVE LOANS

ARMs that do not allow negative amortization generally put a cap on the amount by which the RATE may increase or decrease at each change date. The outstanding loan balance is then amortized at the new rate over the remaining term of the loan. For instance, a 1-year ARM may call for a 2% cap on the increase in any single year.

LIFETIME CAP

In addition to the periodic cap, no-negative ARMs often have a maximum lifetime cap. For instance, a 1-year ARM with an initial rate of 9.75% and a 2% yearly cap, may also have a 5% lifetime cap. The worst case increase would result in the following yearly rates:

9.75% for year 1.
11.75% for year 2.
13.75% for year 3.
14.75% for the remaining 27 years.

It is important for the borrower to assure himself that the cap applies to the initial teaser rate and does not just go into effect after the first adjustment. Otherwise the borrower of our previous loan could see his rate jump to 13% after the first year, followed by a possible increase to 18% at some point during the life of the loan.

Borrowers can not be told what the future of the index will be, but they have a right, not fully guaranteed by Truth in Lending regulations, to be informed in no uncertain terms of the ground rules. In particular, they should have a clear, practical understanding of what would happen and what rates would apply if no change in the index occurred and if the worst case scenario unfolded.

FIXED RATE VERSUS VARIABLE RATE MORTGAGE

This section addresses the fear of variable rate mortgages by showing what numbers should really be compared as we choose between fixed rate loans and Adjustable Rate Mortgages. It also offers strategies that can be actually implemented to mitigate the dangers of a changing rate. As of this writing, rates are changing fast, and the difference between the rates offered on fixed rate loans and alternative Adjustable Rate Mortgages is also decreasing rapidly. Different numbers will lead to different results and different conclusions. The analytical process, however, remains the same and can be applied whatever rates are current at the time of reading.

NO-NEGATIVE ARM: WORST CASE SCENARIO

Let's take a 30 year, 9% interest variable rate loan in the amount of $124,281.87. The monthly payments are exactly $1,000.00. The rate is adjusted every year. There is a 2% cap on the yearly rate increase and a 5% lifetime cap. Under worst case conditions, the rate would increase to 11%, 13%, and finally 14% for the remaining term of the loan.

Let's follow that loan through the worst case scenario.

Keystrokes				Comment
124,281.87	PV			
9	BLUE	i		
30	BLUE	n		
0	FV	PMT		($1000.00 monthly payments)
1	BLUE	n	FV	($123,432.78: 1 year balance)
CHS	PV			(Balance becomes new Present Value)
11	BLUE	i		
29	BLUE	n		
0	FV	PMT		(Second year payment: $1,180.79)
1	BLUE	n	FV	(Balance, end of second year: $122,810.08)
CHS	PV			
13	BLUE	i		
28	BLUE	n		
0	FV	PMT		(Third year payment: 1,367.04)
1	BLUE	n	FV	(Balance, end of 3rd year: $122,343.78)
CHS	PV			
14	BLUE	i		
27	BLUE	n		
0	FV	PMT		(Payment after 3 years: $1,461.44)

Notice how, after each year, we amortize the remaining balance at the new rate over the remaining term of the loan--29 years, 28 years, 27 years. We do so by taking the remaining balance and keying it back in as the new Present Value.

Because we chose initial monthly payments of $1,000.00, the increase can immediately be seen as a percentage of the original loan. After 3 years, the monthly payments have increased by 46.14%. If the borrower barely qualified for the variable rate loan, this can be quite a traumatic increase.

Just looking at the possible 46.14% increase, however, could be somewhat misleading, especially from the point of view of a borrower who has the choice between the variable rate loan and a fixed rate loan. For fear of a possible 46% increase should he get a fixed rate loan at 12%? His payments would then be $1,278.38 or almost 28% more than the initial variable rate payments. That amount would furthermore be guaranteed for the term of the loan and would be charged immediately. Does it make sense to condemn oneself to a guaranteed 28% penalty just to avoid a possible 46% increase?

We should really compare the fixed rate payment of $1,278.38 to the worst case maximum payment in 3 years of $1,461.44, and conclude that what we are avoiding by choosing the fixed rate loan is a payment that could, three years later, under unlikely worst case conditions, be 14.32% higher than the fixed rate payments. What's more, if we choose the risk of the variable rate loan, we also benefit from the guaranteed lower initial payments.

A STRATEGY FOR ARM LOANS: CHOOSE ARM RATE, MAKE FIXED RATE PAYMENTS

We may adjust for the benefits of initially lower payments by deciding to choose the variable rate loan but still make the higher fixed rate payments as long as they meet the minimum requirement. Let's do this with the previous data, checking the required payment on the rapidly declining balance, but still keying back in the higher fixed rate payment as long as it is higher than the required payment.

YEAR	RATE	TERM	BEG. BALANCE	MINIMUN PMT	PMT MADE	ENDING BALANCE
1	9%	30	124,281.87	1,000	1,278.38	119,950.93
2	11%	29	119,950.93	1,147.49	1,278.38	117,693.42
3	13%	28	117,693.42	1,310.08	1,310.08	117,246.54
4	14%	27	117,246.54	1,400.55	1,400.55	

Here, where we choose no longer to benefit from the lower variable rate payments for the first years, the maximum possible increase over the fixed rate payment under the worst case scenario is from $1,278.38 to $1,400.55, or 9.56%.

This strategy can have an even more dramatic effect if the ARM rates are adjusted every 2 or 3 years instead of every year. I have seen the same lender offer 30 year loans at a fixed rate of 12.5% and 3-year ARMs at an initial rate of 10.125%, a loan that adjusts only every three years by a maximum of 2 percentage points, with a 5% maximum lifetime interest cap. By choosing the variable rate loan but making the fixed rate payment for the first 9 years, a worst case analysis similar to the one above shows that the payment for the remaining 21 years would be lower than the fixed rate payment!

In that particular case, apart from any difference in the initial points, even if the worst case scenario is realized one would be better off with the variable rate loan. If one chose the variable rate loan but kept on making the fixed rate payments, still under worst case assumptions, the loan would be paid off in less than 25 years instead of 30. It would off course be paid off much faster under the very likely circumstance that rates would not increase by the maximum allowable amount at each change date.

Results will not normally be quite so dramatic. But the question may still be asked of other no-negative ARM loans: what monthly payment amount GUARANTEES that the payments can never increase?

ARM LOAN WITH LEVEL PAYMENTS!

Let's take a 1-year, no-negative Adjustable Rate Mortgage (9% interest, 30 years, 2/5 caps as above), and find the level payments that bring the balance down in such a way that the payments need never increase.

To solve this problem, we take an arbitrary payment amount of $1.00 and work our way backwards to establishing the ARM loan such a payment allows under worst case conditions. We may then find the desired payment by dividing the actual loan amount by the loan amount made possible by $1.00 payments.

27 BLUE n	
1 CHS PMT	
0 FV	
14 BLUE i	
PV CHS FV	($83.71: 3 year balance)
1 BLUE n	
13 BLUE i	
PV CHS FV	($84.76: 2 year balance)
11 BLUE i	
PV CHS FV	($87.28: 1 year balance)
9 BLUE i	
PV	($91.23: initial loan amount)

91.23 becomes a factor that can be used to calculate the level payments on various loan amounts. Level payments on the $124,281.87 ARM loan would be $1,362.29 ($124,281.87 divided by 91.23).

A potential borrower concerned about variable rates and seriously considering the fixed rate alternative could be told:

> You can choose the fixed rate at 12% interest with payments of $1,278.38 for the next 30 years. Or you can choose the variable rate loan at a significantly lower rate of 9% and decide to make payments of $1,362.29, just 6.56% higher than on the fixed rate.
>
> The extra $83.91 is a cushion that fully absorbs the risk of higher rates. Paying that insurance premium guarantees that the payments will never increase while preserving all the advantages of the variable rate: a substantially lower initial rate and the possibility that the rate will remain low or even decrease. This means, in particular:
>
> - Significantly lower loan balance, even under the worst case scenario, especially during the crucial years (2 to 12, let us say), when properties are often sold and loans paid off.
> - Loan paid off in less than 15 years if the rate stabilizes around 10%, much faster if it remains at 9% or even declines.
> - No need to incur the cost of refinancing if rates go down: the variable rate feature will automatically adjust to lower rates.
> - The option of making lower payments if the need arises during the early years. Using this option, of course, cancels out some of the benefits of the higher payments.

WHAT SHOULD THE ARM BORROWER HOPE FOR?

Having chosen an ARM loan, the borrower may be tempted to hope for a decline in the index and a decline in the required payments. It may help further ease the fear of seeing rates go up if he reflects that it might well be to his advantage if the opposite happens.

If the index goes up and the rate goes up, this almost certainly means that inflation has flared up. So future payments, though higher, are made with cheaper dollars which, in part, mitigates the pain of increased payments. Furthermore, owning leveraged property in an inflationary environment has such a dramatic effect on equity that it would almost certainly more than compensates the owner for the higher payments. (See Volume I, Unit 6, Page 22).

Under the right circumstances, choosing an ARM loan is like hedging your bets: if rates stay low, you win because of the lower payments. If inflation and rates flare up, you still win more from inflation than you lose from the higher payments.

NEGATIVE AMORTIZATION ARM

Because of the teaser rate, payments are likely to go up initially on a variable rate loan even if the index has not changed. This increase can be continued through more than one change date on negative amortization ARM loans to the dismay of the borrower who learns through the media of the general decline in interest rates. Let's follow a negative amortization ARM loan under the assumption that the index does not change:

> A 30 year $100,000 Adjustable Rate Mortgage (ARM) offers a rate of 10.5% (qualifying rate) for the first year of the loan. The rate is then pegged at 2.5 percentage points over the underlying variable index rate, presently 10.27%, which implies a 12.77% rate beginning in year 2 under the assumption that the index does not change. Payments are adjusted yearly and cannot increase by more than 7.5% on any change date.

The payments that result from this contract--under the assumption that the index remains unchanged--can be calculated using the regular cash flow keys to calculate the payments required to pay off the loan in the remaining term, and **GOLD AMORT** to update the loan each year until stable payments are achieved.

We first calculate the payment amount for the first year, and the resulting maximum payments possible under the 7.5% cap for the 4 or 5 years that follow. We will write the answers in the right-hand column.

We then proceed to establish in PV the balance of the loan at the end of each year (**12 GOLD AMORT**), and find the monthly payments required to pay off that balance in the remaining term of the loan. For each year we select the lower of the two payments provided by the cap or the amortization process. For our own information, we will also check the interest earned by the lender, the principal reduction or principal increase, and the balance of the loan at the end of the year.

PAYMENT CALCULATION, NEGATIVE AMORTIZATION ARM

ASSUMPTION: no change in the index.

YEAR		INTEREST PRINCIPAL CHANGE (Increase/ Reduction) END OF YEAR BALANCE	PAYMENTS: CHOOSE LOWEST FOR EACH YEAR	
1	100000 PV 10.5 BLUE i 30 BLUE n 0 FV PMT 7.5 % + 7.5 % + 7.5 % +			$914.74
	12 GOLD AMORT xy RCL PV	(INT: $10,476.36) (P.REDUC: -500.52) (BAL: 99,499.48)		
2	12.77 BLUE i 29 BLUE n PMT		~~1,086.13~~	983.35
	983.35 CHS PMT 12 GOLD AMORT xy RCL PV	(INT: $12,761.04) (P.INCR: 960.84) (BAL: 100,460.32)		
3	28 BLUE n PMT		~~$1,100.46~~	1057.10
	1057.10 CHS PMT 12 GOLD AMORT xy RCL PV	(INT: $12,837.49) (P.INCR: 152.29) (BAL: 100,612.61)		
4	27 BLUE n PMT		1,106.53	~~1136.38~~
	12 GOLD AMORT xy RCL PV	(INT: $12,822.15) (P.REDUC: -456.21) (BAL: 100,156.40)		

By year 4, the amortized payments are lower than the limit imposed by the cap, and, with no change in the index, the $1,106.53 amount would pay off the loan over the remaining 27 years.

APR ON ADJUSTABLE RATE MORTGAGE

Now that we have established the payments (under the incredible but necessary assumption of an index that does not change), we may calculate the Annual Percentage Rate (APR) if loan costs are 3 points and $300.00.

Net proceeds to borrower:

```
100000  ENTER  3  %   -   300   -
```

The borrower nets $96,700 after loan costs. We have now established the complete net exchange of money in time from the point of view of the borrower. Keying that data in allows us to calculate the APR:

-96,700			96700 CHS BLUE CF0.
914.74	(12)		914.74 BLUE CFj 12 BLUE Nj
983.35	(12)		983.35 BLUE CFj 12 BLUE Nj
1057.10	(12)		1057.10 BLUE CFj 12 BLUE Nj
1106.53	(99)	(324)	1106.53 BLUE CFj 99 BLUE Nj
1106.53	(99)		xy BLUE CFj xy BLUE Nj
1106.53	(99)		xy BLUE CFj xy BLUE Nj
1106.53	(27)		xy BLUE CFj 27 BLUE Nj
(Answer: 12.94%)			GOLD IRR 12 x

- The amounts thus established are of interest to the borrower even apart from providing the APR: they show him what the payments and balances would be if there were no change in the index. Borrowers of such negative amortization loans are frequently disturbed to see their payments increase for some years even though rates in general may have remained stable or have slightly decreased.

- Should the underlying index rate increase, the payments would not change for the first 3 years as they already increase by the maximum amount allowed, but the negative amortization balance would be more substantial, and the increase in the payments under the allowable cap would continue for a longer period of time. A decrease in the index would reduce or eliminate those increases.

- Negative amortization is when interest owed is not paid but added to the loan amount instead, as occurs here in years 2 and 3. Check with a tax accountant on how to handle such deferred interest for tax purposes.

- Because of the 99 limit on Nj data, the 324 payments of $1,106.53 are keyed in as 3 groups of 99 payments and one group of 3 + 24 = 27. Similarly 263 payments could be keyed in as 2 groups of 99 plus one group of 2 + 63 = 65.

UNIT 26

SPECIAL APPLICATION:
PROJECTION
ON INCOME PROPERTY INVESTMENT

INCOME PROJECTION

Any number of problems require preliminary calculations before we reach the bottom line on which we may finally calculate a rate. This was the case with the blended rate and All Inclusive Mortgage yield calculations in Unit 25. Let's take one example involving income: a 5 year projection on the purchase and resale of an income property.

It is not the purpose of this book to make a systematic presentation of the various options, tax requirements, prudent ratios and expectations that go into establishing a cash flow forecast on investment real estate any more than on other forms of investments. The purpose is merely to SUGGEST SOME KEYSTROKES for those who already know the applicable tax laws and prudent business practices, and who have selected the printed form appropriate for their purpose.

In similar circumstances, I have seen practitioners trying to develop a super-efficient program that does it all for them--except that it allows for no mistakes and in practice ends up in a nightmare of unverifiable numbers. I prefer to accept the limitations of the HP-12C for handling simultaneously a considerable amount of data, and to take into account the human propensity to make mistakes--and therefore the need to allow for some flexibility in checking and correcting.

This is can be achieved by writing down intermediate numbers on a form as we go along. The intermediate data becomes an important part of the knowledge we acquire on the transaction and it provides a clearer picture of the importance of the various assumptions made in the process of calculating the rate. It also makes it possible to make a mistake and not have to start the whole process from the beginning.

We present here first the very basic keystrokes required to fill in the form. We will need to key numbers back in and total some columns in order to establish the bottom line. We may then key the bottom line in the calculator and calculate the rate of return.

We will then go through the same process again and integrate it with some storing of intermediate data that gradually builds up to the bottom line directly in the calculator. This may suggest some procedures and shortcuts of use to those who frequently do such calculations.

SAMPLE 5-YEAR INCOME PROPERTY INCOME PROJECTION FORM

	Year 1	Year 2	Year 3	Year 4	Year 5
1) Gross Scheduled Income (Increase %/year)					
2) - Vacancy & credit loss (% of GSI)					
3) = Gross Operating Income					
4) - Operating Expenses (Increase %/year)					
5) = Net Operating Income					
6) - Interest on loans					
7) - Depreciation					
8) = Taxable income / Loss) (Marginal rate: %)					

9) NOI (From line 5):					
10) - Annual Debt Service					
11) - Funded reserves & capital improvements					
12) = Cash Flow before Taxes					
13) - Tax owed / + Rebate (% of 8)					
14) = Net After-Tax Income					

15) Reversion on sale:

a) Selling Price:	
b) - Selling costs:	
c) - Balance on loans:	
d) - Tax liability:	
e) = Net reversion:	
f) + Net income Year 5 (Line 14):	
g) = Net Cash Flow End-of-Year 5:	

16) The bottom line: Net investment and returns:

Initial investment	Year 1	Year 2	Year 3	Year 4	Year 5

17) Internal Rate of Return (IRR): ____

5 YEAR INCOME PROJECTION: SAMPLE PROCEDURE

BASIC INCOME PROPERTY DATA

Purchase price:	$450,000, 25% land, 75% improvements.
Down payment:	20% or $90,000 + $5,000 buying costs.
Loan:	$360,000, 11% interest, 30 years.
Gross Scheduled Income (GSI):	$52,000 for year 1.
Vacancy and credit loss:	6% of GSI. (Assumption).
Operating expense:	$12,000 for year 1.
GSI and expenses:	4% increase per year. (Assumption).

Depreciation based on 19 year life, 175% declining balance approach.
Marginal tax bracket of owner: 40%

Selling price:	$547,500 (Assumes 4% appreciation).
Selling costs:	7% of selling price.
Capital gains Tax on sale:	$15,000 (as provided by accountant).

The form itself may not fill the needs of all users and the numbers used for depreciation and tax consequences need to be checked with a tax attorney or accountant.

NET OPERATING INCOME: Line 5

Because Gross Scheduled Income, Vacancy and Credit Loss, and Operating Expenses are all affected by the same 4% yearly increase, the Net Operating Income (NOI) also increases by the same ratio. So we may dispense with projecting lines 1 to 4 over the full 5 year holding period, and project only the NOI figure (line 5). This would not be the case, for instance, if Gross Income was scheduled to increase by 6%, while the Operating Expenses figure was expected to rise only 4% each year.

Keystrokes		Lines
52000 ENTER		(1)
6 %	(Vacancy: $3,120)	(2)
-	(GOI: $48,880)	(3)
12000		(4)
-	(NOI: $36,880)	(5)
4 % +	($38,355)	(5)
4 % +	($39,889)	(5)
4 % +	($41,484)	(5)
4 % +	($43,144)	(5)

INTEREST: Line 6

As we calculate the interest (line 6), we also record the total debt service (line 10), and the balance of the loan at the end of the 5 year holding period (line 15c).

Keystrokes	Result	Line
30 BLUE n 11 BLUE i		
360000 PV 0 FV PMT	($3,428.36)	
12 x	($41,140)	(10)
12 GOLD AMORT	($39,519)	(6)
12 GOLD AMORT	($39,332)	(6)
12 GOLD AMORT	($39,123)	(6)
12 GOLD AMORT	($38,889)	(6)
12 GOLD AMORT	($38,629)	(6)
RCL PV	($349,793)	(15c)

DEPRECIATION: Line 7

Tax laws have been changing fast on this matter. The example given here uses the depreciation functions of the HP-12C calculator, a 175% declining balance approach with a 19 year life on depreciable improvement appraised at 75% of the purchase price. Readers are urged to refer back to tax laws and tax tables.

We do not take into account here at what point during the year the investment was put into service, a factor that would affect the depreciation allowed for each tax year. This is justified if the projection is made before we know exactly when the property will sell.

The residual depreciable value at the end of the 5-year holding period is important for capital gains tax calculations. It is shown here though we do not later use the figure.

Keystrokes	Result	Line
450000 ENTER 75 % PV	($337,500)	
175 i 19 n 0 FV		
1 GOLD DB	($31,085)	(7)
2 GOLD DB	($28,222)	(7)
3 GOLD DB	($25,622)	(7)
4 GOLD DB	($23,262)	(7)
5 GOLD DB	($21,120)	(7)
xy	(Residual value: $208,185)	

TAXABLE INCOME (OR LOSS): Line 8
TAX OWED (OR TAX REBATE): Line 13

We may now perform the arithmetic that establishes taxable income or loss and write the answer on line 8.

This is a convenient time to calculate the tax owed or the tax rebate resulting from that income or loss, and fill in line 13.

We assumed a 40% marginal tax bracket or that 40% of any income is paid out in taxes, and that 40% of any loss offsets taxes that would otherwise be paid on other income. Calculations for year 1:

Line 5:	36880	ENTER		
Minus line 6:	39519	-		
Minus line 7:	31085	-	(- 33,724)	(8)
40% of line 8:	40	%	(- 13,489)	(13)

We have a loss for tax purposes of $33,724 and a tax rebate of $13,489. We proceed with the same calculations for the following years:

	Year 1	Year 2	Year 3	Year 4	Year 5
Line 8:	-33,724	-29,199	-24,856	-20.667	-16,605
Line 13:	-13,489	-11,679	-9,942	-8,266	-6,642

CASH FLOW BEFORE TAX: Line 12
NET AFTER-TAX CASH FLOW: Line 14

We are now ready to calculate the Net After-Tax Income projection (line 14) by again performing simple arithmetic. With no amounts projected for reserves and capital improvement, we skip line 11.

Line 9:	36880	ENTER		
Minus line 10:	41140	-	- 4,260	(12)
Plus rebate, line 13:	13489	+	9,229	(14)

A before-tax loss of $4,260 is transformed into a Net after-tax income of $9,229.

The same calculations for each year complete lines 12 and 14.

	Year 1	Year 2	Year 3	Year 4	Year 5
Line 12:	-4,260	-2,785	-1,251	344	2,004
Line 14:	9,229	8,894	8,691	8,610	8,646

REVERSION ON SALE: Line 15

We have established for year 5 the net income that results from HOLDING the property. We now need to calculate and add back in the net income that results from SELLING the property, the reversion on sale:

	Keystrokes		Result	Line
Selling price:	547500	ENTER		
Minus selling costs:	7	%	(38,325)	(15b)
		-	(509,175)	
Minus loan balance:	349793	-	(159,382)	
Minus tax liability:	15000	-	(144,382)	(15e)
Plus Net Income Year 5:	8646	+	(153,028)	(15g)

THE BOTTOM LINE: NET INVESTMENT AND RETURNS: Line 16

We now fill in the bottom line: the initial investment of $95,000 followed by the net after-tax income (or loss) for each of the 5 years, with year 5 adjusted for the net reversion on sale as in line 15g.

Initial investment	Year 1	Year 2	Year 3	Year 4	Year 5
- 95,000	9,229	8,894	8,691	8,610	153,028

INTERNAL RATE OF RETURN: Line 17

Calculating the Internal Rate of Return is just a matter of keying in the data and asking the question:

		Keystrokes				Result
0	- 95,000	95000	CHS	BLUE	CFo	
1	9,229	9229		BLUE	CFj	
2	8,894	8894		BLUE	CFj	
3	8,691	8691		BLUE	CFj	
4	8,610	8610		BLUE	CFj	
5	153,028	153028		BLUE	CFj	
		GOLD	IRR			(16.77%)

16.77% is the PROJECTED rate of return. That rate fully embodies all the assumptions that went into establishing the bottom line, whether justified or unjustified.

Calculating the rate of return is a means of establishing a rating for that crucial bottom line. That rating or rate makes it easier to compare the exchange of money in time that the bottom line represents to other exchanges of money offered by other properties or to entirely different investments.

SAMPLE 5-YEAR INCOME PROPERTY INCOME PROJECTION FORM

	Year 1	Year 2	Year 3	Year 4	Year 5
1) Gross Scheduled Income (Increase 4 %/year)	52,000				
2) - Vacancy & credit loss (6 % of GSI)	3,120				
3) = Gross Operating Income	48,880				
4) - Operating Expenses (Increase 4 %/year)	12,000				
5) = Net Operating Income	36,880	38,355	39,889	41,484	43,144
6) - Interest on loans	39,519	39,332	39,123	38,889	38,629
7) - Depreciation	31,085	28,222	25,622	23,262	21,120
8) = Taxable income / Loss) (Marginal rate: 40 %)	<33,724>	<29,199>	<24,856>	<20,667>	<16,605>

	Year 1	Year 2	Year 3	Year 4	Year 5
9) NOI (From line 5):	36,880	38,355	39,889	41,484	43,144
10) - Annual Debt Service	41,140	41,140	41,140	41,140	41,140
11) - Funded reserves & capital improvements	—	—	—	—	—
12) = Cash Flow before Taxes	<4,260>	<2,785>	<1,251>	344	2,004
13) - Tax owed / + Rebate (40 % of 8)	<13,489>	<11,679>	<9,942>	<8,266>	<6,642>
14) = Net After-Tax Income	9,229	8,894	8,691	8,610	8,646

15) Reversion on sale:

a) Selling Price:	547,500
b) - Selling costs:	38,325
c) - Balance on loans:	349,793
d) - Tax liability:	15,000
e) = Net reversion:	144,382
f) + Net income Year 5 (Line 14):	8,646
g) = Net Cash Flow End-of-Year 5:	153,028

16) The bottom line: Net investment and returns:

Initial investment	Year 1	Year 2	Year 3	Year 4	Year 5
— 95,000	9,229	8,894	8,691	8,610	153028

17) Internal Rate of Return (IRR): 16.77%

INCOME PROJECTION USING STORAGE REGISTERS

In the previous calculations we had to key back into the calculator a considerable amount of numbers that had been previously calculated. This gives us great control over numbers and makes it easy to correct a mistake. But clearly some of these numbers could be stored and retrieved for our convenience rather than punched in all over again. The NOI (lines 5 and 9) could be stored in Registers 1 to 5 when first established and recalled later as needed. The $41,140 debt service should be stored (Register 0, for instance), and later retrieved 5 times instead of being keyed in 5 times.

What we propose here goes one step further and gradully establishes the bottom line in the appropriate Register memories (Registers 0 to 5). Some amount of keying back in will be required, but considerably less than in the previous procedure. Those who frequently need to establish 5-year income projections or other similar projections may find some procedures convenient in working out their own strategy.

At the heart of the process is our ability to manipulate data directly in the Register memories through storage register arithmetic and to calculate an Internal Rate of Return on data that has been keyed and adjusted directly in the appropriate Register memories. This is possible provided that we do not need to use the **Nj** function and have cleared the calculator of any previous Nj values by pressing **GOLD CLEAR-REG**. This clears all Register memories and associated Nj values.

1) STORAGE REGISTER ARITHMETIC

We may modify data stored in the Register memories 0 to 4 only with storage register arithmetic. For instance, to subtract $41,140 annual loan payments from the income of years 1 to 5 previously stored in memories 1 to 5, we may proceed as follows:

```
41140  STO - 1
       STO - 2
       STO - 3
       STO - 4
RCL 5  xy  -  STO 5
```

OR

```
41140 CHS  STO + 1
           STO + 2
           STO + 3
           STO + 4
       RCL 5  +   STO 5
```

The order makes a difference when you subtract, it doesn't when you add. The **xy** in the first approach re-establishes the correct order.

If we press **STO + 5** or **STO - 5** by mistake, then an **ERROR 4** indication shows up. We should then clear the error sign (**CLx**) and proceed with the correct keystrokes. There is no need to start the calculations again.

2) PERCENTAGE INCREASE

Income projections traditionally project a given rate of increase over the years for various income and expense items. A 4% increase per year affecting a $15,000 expense item may be calculated in one of two ways:

	OR		
15000 ENTER		1.04 ENTER ENTER ENTER	
4 % +		15000 x	($15,600.00)
4 % +		x	($16,224.00)
4 % +		x	($16,872.96)
4 % +		x	($17,547.88)
4 % +		x	($18,249.79)
4 % +		x	($18,979.79)

The greater the number of increases, the more time-saving the second procedure where **x** (multiplication key) can be repeated any number of times.

We may combine percentage increase and storage register arithmetic. For instance, a $15,000 expense in year 1 increases by 4% per year. We may subtract this expense item from data already keyed into Register memories 1 to 5 as follows:

1.04 ENTER ENTER ENTER	
15000 CHS STO + 1	($15,000.00)
x STO + 2	($15,600.00)
x STO + 3	($16,224.00)
x STO + 4	($16,872.96)
x RCL 5 + STO 5	($17,574.88)

This reduces the content of Register memories 1 to 5 by an amount of $15,000 for year 1 that increases by 4% for each of the following years.

- o 0 o -

We may now use these building blocs to go over the previous 5-year income projection calculations in a way that will save quite a few steps. The penalty is that mistakes are not as easy to correct. We will make some provisions for filling in the form when practical as a means of not having to start from scratch every time a mistake is made.

Because numbers remain in the calculator instead of being rounded and keyed back in, there may be slight discrepancies of no significance with the previous procedure.

NET OPERATING INCOME: Line 5

NOI amounts are stored in memories 1 to 5 and go on lines 5 and 9 of the form. For practical considerations, we will store the numbers first and then recall them all in succession to write them down on lines 5 and 9.

Keystrokes	Display	Lines
GOLD CLEAR-REG	(To clear Nj memories)	
52000 ENTER 6 %	(Vacancy: $3,120)	(2)
-	(GOI: $48,880)	(3)
12000 - STO 1	(NOI: $36,880)	
4 % + STO 2	($38,355)	
4 % + STO 3	($39,889)	
4 % + STO 4	($41,484)	
4 % + STO 5	($43,144)	
RCL 1 RCL 2 RCL 3		
RCL 4 RCL 5		(5 & 9)

INTEREST: Line 6

Because the interest amount appears as a negative, we may just add that negative number into memories 1 to 5:

30 BLUE n 11 BLUE i		
360000 PV 0 FV PMT	($3,428.36)	
12 x	($41,140)	(10)
12 GOLD AMORT STO + 1	($39,519)	(6)
12 GOLD AMORT STO + 2	($39,332)	(6)
12 GOLD AMORT STO + 3	($39,123)	(6)
12 GOLD AMORT STO + 4	($38,889)	(6)
12 GOLD AMORT	($38,629)	(6)
RCL 5 + STO 5	(4,515)	
RCL PV	($349,793)	(15c)

DEPRECIATION: Line 7

450000 ENTER 75 % PV	($337,500)	
175 i 19 n 0 FV		
1 GOLD DB STO - 1	($31,085)	(7)
2 GOLD DB STO - 2	($28,222)	(7)
3 GOLD DB STO - 3	($25,622)	(7)
4 GOLD DB STO - 4	($23,262)	(7)
5 GOLD DB	($21,120)	(7)
RCL 5 xy - STO 5	(-16,605)	
xy	(Residual value: $208,185)	

TAXABLE INCOME (OR LOSS): Line 8

We may now recall Taxable Income from memories 1 to 5 and fill in line 8. That data will be erased during the next step.

TAX OWED OR TAX REBATE: Line 13

During the same run or, more conveniently, through a second recalling process, we may calculate 40% of the taxable income (or loss), reverse the sign, and store the result back in memories 1 to 5:

Note that the keystrokes apply whether the amount is a tax owed or a tax rebate. This is because tax owed would appear as positive (a set percentage of positive income) and a tax rebate as a negative (a percentage of negative loss). Changing the sign and storing in the memories keys in tax owed as a negative and a tax rebate as a positive amount.

RCL 1	40	%	CHS	STO 1	(13,490)	(13)
RCL 2	40	%	CHS	STO 2	(11,679)	(13)
RCL 3	40	%	CHS	STO 3	(9,942)	(13)
RCL 4	40	%	CHS	STO 4	(8,267)	(13)
RCL 5	40	%	CHS	STO 5	(6,642)	(13)

NET AFTER-TAX CASH FLOW: Line 14

We may now manipulate memories 1 to 5 by adding back the NOI (from lines 5 or 9) and deducting the $41,140 annual debt service. This leaves the Net After-Tax Income in memories 1 to 5:

```
36880  STO + 1                 or    1.04  ENTER  ENTER  ENTER
38355  STO + 2                       36880  STO + 1
39889  STO + 3                           x  STO + 2
41484  STO + 4                           x  STO + 3
43144  RCL 5  +  STO 5                   x  STO + 4
                                         x  RCL 5  +  STO 5

41140  STO - 1
       STO - 2
       STO - 3
       STO - 4
RCL 5  xy  -  STO 5
```

We may now recall Net After-Tax Income from memories 1 to 5 and fill in line 14. (See Form for numbers).

REVERSION ON SALE

We now calculate the new reversion (net proceeds) on the sale of the property and add the amount to the after-tax cash flow for year 5 already stored in memory 5, filling in line 15 as required.

547500 ENTER 7 % -	($38,325 & $509,175)	
349793 -	(Adjust for loan balance)	
15000 -	(Net reversion: $144,382)	(15e)
RCL 5 +	(Net CF, year 5: $153,028)	(15g)
STO 5		

INTERNAL RATE OF RETURN

We now have in memories 1 to 5 the cash flows for years 1 to 5. We may key into memory 0, as a negative amount, the initial investment (downpayment and buying costs if any), key in 5 in **n** to instruct the calculator to search memories 0 to 5 for irregular cash flow data, and calculate the Internal Rate of Return. The numbers in memories 0 to 5 may now be recalled to fill in line 16.

95000 CHS STO 0		
5 n		
GOLD IRR	(IRR: 16.77%)	(17)

IN SUMMARY

We gradually adjust the Net Operating Income initially stored in Registers 1 to 5 to establish the after-tax cash flows. We further adjust End-Of-Year 5 income by adding the net proceeds from the sale. We store the initial investment as a negative amount in Register 0. We then allow the calculator to calculate the Rate of Return.

Though not quite as easily as in the line-by-line procedure, we may at this stage modify Register 5 to reflect a different assumption concerning the sale of the property, and calculate the new Rate of Return.

IRREGULAR CASH FLOWS

LEVEL III

Level III provides an understanding of the Internal Rate of Return (IRR) and Net Present Value (NPV) functions. It addresses some of the limitations of the IRR and presents modified rate approaches that seek to overcome these limitations.

UNIT 27

UNDERSTANDING IRR AND NPV

UNDERSTANDING IRR AND NPV CONCEPTS

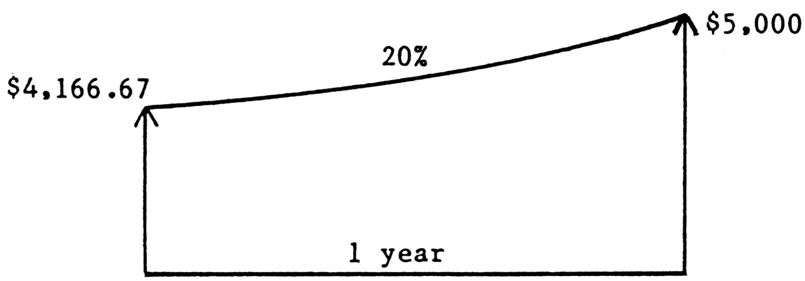
This section does not provide new practical applications. It seeks to give a better understanding of what happens and why it happens.

AN EXPERIMENT

You are offered the opportunity of receiving $5,000 one year from now. How much should you be willing to invest now if you want a 20% return?

This is a simple regular cash flow Future Value - Present Value calculation. $5,000 is the Future Value, i is 20%, n is 1 year, and we have 0 in PMT:

```
  20  i     0  PMT
5000  FV    1  n     PV    STO  0            ($4,166.67)
```



We have stored the answer in Register 0. Let us now proceed in the same way with additional cash flows spread over 5 years, and accumulate their Present Values in memory 0 as follows:

```
 8000  FV    2  n     PV    STO  +  0      ($5,555.56)
 9500  FV    3  n     PV    STO  +  0      ($5,497.69)
12000  FV    4  n     PV    STO  +  0      ($5,787.04)
15000  FV    5  n     PV    STO  +  0      ($6,028.16)
  RCL  0                                  ($27,035.11)
```

We may summarize the procedure as follows:

	FV	PV
EOY 1:	5,000	4,166.67
EOY 2:	8,000	5,555.56
EOY 3:	9,500	5,497.69
EOY 4:	12,000	5,787.04
EOY 5:	15,000	6,028.16
Total:		27,035.11

27,035.11 is the sum of all the Present Values. It is also the Present Value of the 5 cash flows taken as a whole and discounted at 20%.

Discounted at 20%, these 5 cash flows are worth $27,035.11. If they are worth $27,035.11, that is what I should pay for the benefit of receiving them: in doing so I am getting a Rate of Return or IRR of 20%. Let's adjust the chart to reflect the $27,035.11 investment:

	Cash Flows	PV
Time 0:	-27,035.11	-27,035.11
EOY 1:	5,000	4,166.67
EOY 2:	8,000	5,555.56
EOY 3:	9,500	5,497.69
EOY 4:	12,000	5,787.04
EOY 5:	15,000	6,028.16
Total (NPV):		**0.00**

The Future Value column now shows a complete exchange of money in time and is labelled Cash Flows. The Present Value column has the Present Value of all the cash flows, including the initial negative $27,035.11 investment which is already a Present Value. As a result, the total of all the Present Values is now equal to **ZERO**.

Let's consider that **$0.00** for a moment.

- It is the **NET PRESENT VALUE (NPV)** of all the Cash Flows in the central column discounted at 20%., the Present Value of the positive amounts, minus the Present Value of the negative amounts.

- A **NET PRESENT VALUE** of $0.00 indicates that the Present Value of all the positive amounts is equal to the Present Value of the negative amounts. In other words, at the selected discount rate, what I get is equal in value to what I give. Given that discount rate, we have a fair exchange: we are getting a 20% rate of return on the exchange as required.

- A possible definition of the IRR is as follows: it is the rate that results in a Net Present Value of $0.00. This should be understood in terms of the previous comment: the IRR is the rate that balances the books on the exchange because at that rate what we give is equal in value to what we get--the difference between the value of what we give and the value of what we get is $0.00.

- If the difference between the Present Values of what we get and what we give is $0.00, then the difference between the Future Values of what we get and of what we give is also $0.00, as would the difference between the values of the cash flows brought back to any point in time that we may choose. Comparing the Present Values is a matter of convenience, any other point in time would do just as well as a means of defining the same Internal Rate of Return.

NET PRESENT VALUE: AN ALTERNATIVE TO KNOWING THE IRR

Let's assume that the same returns can be purchased for $29,035.11 instead of $27,035.11. The Net Present Value of all the Cash Flows discounted at 20% is now -2,000. Knowing that number tells us that we should pay $2,000 less if we are to get a 20% return. We do not know the rate of return on the $29,035.11 investment, but we know that we are **not** getting 20%. If we require a 20% return, this investment is not for us.

	Cash Flows	PV
Time 0:	-29,035.11	-29,035.11
EOY 1:	5,000	4,166.67
EOY 2:	8,000	5,555.56
EOY 3:	9,500	5,497.69
EOY 4:	12,000	5,787.04
EOY 5:	15,000	6,028.16
Total (NPV):		-2,000.00

On the other hand, if the same returns can be purchased for $24,035.11, then we have a positive $3,000 Net Present Value. This means we can afford to invest $3,000 more (or $27,035.11), and still get a 20% return. We do not know what the rate of return really is. But as it is more than our 20% requirement, we may want to make the investment.

	Cash Flows	PV
Time 0:	-24,035.11	-24,035.11
EOY 1:	5,000	4,166.67
EOY 2:	8,000	5,555.56
EOY 3:	9,500	5,497.69
EOY 4:	12,000	5,787.04
EOY 5:	15,000	6,028.16
Total (NPV):		3,000.00

Before financial calculators provided a simple means of calculating the actual rate of return, it was possible to rate an investment opportunity on the basis of its Net Present Value. We could:

- Select a rate that represented our requirement for that exchange.
- Use the FV-PV column of Time Value of Money tables to discount each cash flow separately at the selected rate.
- Add up all the Present Values to calculate a Net Present Value.
- Judge on the basis of the Net Present Value whether the required rate was being achieved.

NPV AS A MEANS OF CALCULATING THE IRR

We may even move one step further. If we insist on knowing the rate of return on our $24,035.11 investment, we may make another guess--higher if NPV is positive, lower if it is negative--, and once again discount all the cash flows at that new rate. Clearly, an iterative process of TRIAL AND ERROR would allow us to find a rate that resulted in a Net Present Value equal to ZERO, ot close enough to zero to meet our purpose.

This time-consuming procedure was the only way of calculating a rate of return (IRR) before financial calculators came along. In fact, our calculator itself uses this procedure: it makes a guess about the rate, discounts all the cash flows at that rate. If the Net Present Value is positive, it discounts again at a higher rate. If it is negative, it tries again at a lower rate. It repeats the process until it finds a rate, accurate to 10 significant digits.

NPV AS PRESENT VALUE OF MISSING AMOUNT

Let's go back to our initial $27,035.11 investment and the cash flows that it buys. At 20% the Net Present Value is $0.00. But now, let's pretend that End Of Year 3 cash flow is missing. What amount should we receive for Year 3 if we are to get our 20% return?

	Cash Flows	PV
Time 0:	-27,035.11	-27,035.11
EOY 1:	5,000	4,166.67
EOY 2:	8,000	5,555.56
EOY 3:	~~9,500~~ ?	~~5,497.69~~
EOY 4:	12,000	5,787.04
EOY 5:	15,000	6,028.16
Total (NPV):		-5,497.69

If we calculate the Net Present Value of all the cash flows that we know, discounted at 20%, we find -5,497.69: the Present Value of the missing amount discounted at 20% with the wrong sign: $9,500 is missing from the FV column, so $5,497.69 is missing from the PV column and the total Net Present Value, instead of being $0.00 is -5,497.69.

A simple PV-FV calculation with 3 in **n** then allows us to calculate the missing amount: $9,500. This PV-FV calculation is the exact opposite of the FV-PV calculation that initially established the $5,497.69 Present Value. It also automatically gives back the correct sign for the missing Cash Flow.

This may show why the same function is able to provide us with the Present Value of all the cash flows submitted, the Net Present Value if some are positive and others negative, and also the Present Value (with the wrong sign) of any missing amount.

REVIEWING THE EXPERIMENT

Let's use irregular cash flow keys to go over the procedures examined in this experiment. The keystrokes, with few comments, should suffice.

- **PRESENT VALUE** of returns discounted at 20%.

Keystrokes	Result
0 BLUE CF0	
5000 BLUE CFj	
8000 BLUE CFj	
9500 BLUE CFj	
12000 BLUE CFj	
15000 BLUE CFj	
20 i GOLD NPV	($27,035.11)

- **NET PRESENT VALUE** of investment for different purchase prices.

CHS STO 0	Key in $27,035.11 initial investment.
GOLD NPV	($0.000005, $0.00 for practical purposes)
29035.11 CHS STO 0	Key in $29,035.11 initial investment.
GOLD NPV	(Net Present Value: -2,000)
24035.11 CHS STO 0	(Key in $24,035.11 initial investment.
GOLD NPV	(Net Present Value: $3,000)

- **TRIAL AND ERROR** procedure to find rate.

25 i GOLD NPV	(NPV = -220.71. Rate is too high)
24 i GOLD NPV	(NPV = 374.99. Rate now too low)
24.50 i GOLD NPV	(NPV = 74.33. Rate still too low)
24.60 i GOLD NPV	(NPV = 14.88. Rate still too low)
24.65 i GOLD NPV	(NPV = -14.77. Rate now too high)
24.625 i Gold NPV	(NPV = .05, close enough to zero)

The rate is 24.625%. Let's check with the IRR function:

Keystrokes	Result
GOLD IRR	(24.63%)
GOLD ENTER	(24.62508187% with hidden decimals)

- **PRESENT VALUE OF MISSING AMOUNT.**

Let's first key back in our initial $27,035.11 investment and the 20% requirement, and delete CF3 from the calculator. We may now calculate NPV--do you recognize the amount?--, and calculate the missing amount!

27035.11 CHS STO 0	(Original data back in and
20 i 0 STO 3	$9,500 erased from memory 3)
GOLD NPV	(-5,497.69, PV of missing amount)
3 n 0 PMT FV	($9,500: amount of missing cash flow)

LIMITATIONS OF INTERNAL RATE OF RETURN

The calculator invites us to identify the Internal Rate of Return (IRR) with the mathematical procedure implemented by the **GOLD IRR** function, and also by the regular cash flow **i** function. That unique mathematical procedure is given a variety of names according to the nature of the cash flows considered or the point of view we choose to select. We may then speak of a yield, an Annual Percentage Rate (APR), a discount rate, a rate of interest, even a rate of inflation.

In a narrow sense of the word, Internal Rate of Return does not really apply to some of these other rates because the word 'RATE OF RETURN' would seem to imply that we have taken the point of view of the INVESTOR. But given the same cash flows, the rate is the same whether we are considering the exchange from the point of view of the investor, of the borrower, or any other we choose to imagine. (See Volume I, Unit 6).

Under the various names we choose to disguise that unique procedure, and whether we use **i** or **GOLD IRR** to calculate it, that procedure remains the best and most universal yardstick for measuring and comparing exchanges of money over time. But is has limitations that we need to consider.

We calculate a rate in order to make a value judgment, to **COMPARE** exchanges of money in time. That's what the word itself says: it allows us to establish a **RATING** which implies a comparison. Even when no formal comparison is made, the rate is still being used in reference to "the market" in general at this particular time, and loses much of its meaning outside that context.

But the reality is not the rate, it is money changing hands at various points in time. The rate is here to help us rate that exchange. What are the limitations of the rating system provided by the rate?

1) Rates can be compared with precision only if the calculation is consistent in the assumptions it makes concerning compounding: 14% compounded daily provides a better return than 15% compounded yearly. (For the concepts and the calculations that allow us to convert rates from one compounding period into another, see Volume I, Unit 6).

2) A rate is as good as the cash flows submitted. If we calculate a rate based on normal expectations for returns, the rate is valid only if those expectations are met. When rates are calculated on cash flows that are the results of assumptions, the rate is only as certain as the assumptions themselves. We should not allow the precision of the calculation to blind us to the imprecision of the assumptions.

Needless to say, rates can be compared only if the nature of the cash flows that provide them are reasonably consistent. We may calculate before-tax rates on before-tax cash flows, and after-tax rates on cash flows adjusted for taxes, but it doesn't make sense to make a direct comparison between before-tax rates and after-tax rates without adjusting significantly for the discrepancy.

3) Because a rate considers only cash flows, any factor that has bearing on the transaction but is not reflected by the cash flows is being disregarded if we look only at the rate. Among these are convenience and liquidity as well as risks and uncertainties.

4) The same rate is a measure of exchanges that can involve vastly different amounts of money, over vastly different periods of time. It is not possible, in comparing investment options, to look just at the rate and close our eyes to size and duration.

5) Finally, as a tool for making value judgments, a rate forces us to take a point of view: are we an investor or a borrower? Clearly, if we compare 10% and 20%, our choice of which is best is reversed if we are the borrower rather than the investor. Some exchanges of money in time, by their very nature, do not allow us to decide whether we are borrowers or investors. We really are both at different times. When this occurs, not only is the rate of no great help, but the mathematics itself might allow for more than one rate to fit the exchange!

Let's examine some of these features.

RISK

What is the degree of certainty that the predicted cash flows will be realized? Two investments offering the same 12% return are not equal if the risk factors are different. This is why bond issues are 'rated' according to degree of risk, and the less secure offer a higher rate of return than those rated higher. This is why high rates of return are required on risky venture capital investments. This is why, before deregulation, when Savings institutions were prevented by law from offering competitive rates of interest on savings, they were compelled to sell "Peace of mind", the almost perfect security they were offering, in contrast to the higher yields available from more risky investments.

SPECULATION

The flip side of the "risk" factor is the potential for unusually high returns. The expected rate of return on an investment may be rather low, but there is the chance, if ever so small, of making a killing, and the investor is willing to speculate on that chance. The Stock market is a case in point. Some stocks offer mostly stable earnings expectations and the price is such that one earns a respectable rate of return from the earnings. Others offer potential for capital gains, and are bought on the strength of that hope at much higher price/earnings ratios. Again, an apartment house purchase may offer a rather low rate of return under conservative assumptions--the investor may want to ask himself "Could I live with that?"--, but would offer a very high return if inflation flares up again, if that airport is finally built, if that law is passed or repealed.

INVOLVEMENT AND CONVENIENCE

"Peace of mind" also stresses the convenience with which the investment can be acquired or redeemed, and the complete absence of involvement in the investment. Clearly, buying an apartment house is more demanding of the investor's time and involvement than opening a savings account. Even if he does not choose to contribute any "sweat equity" and pays others to take care of the yard, unplug the sink, collect the rent, and select the tenant, he is likely to be involved in decisions concerning the property, and to be affected by problems that may arise. It may not be possible to factor these concerns into the cash flows themselves (as can be done with actual "sweat equity", where the costs of maintenance and management can be taken into account even if the investor himself takes on the task and does not actually pay anybody to do the job), but then the rate requirement should be increased.

LIQUIDITY

Another advantage of the passbook savings account and some money market funds over other accounts and investments is their complete liquidity: the money is available at a moment's notice. It is also possible to add to the investment: any amount, at any time. Other accounts offer less flexibility. If the investor has to commit himself for 1, 2, or 10 years, he is entitled to a higher return because he gives up liquidity. Buying an appartment house offers much less flexibility: you can't just sell it overnight if the need arises (though you might possibly borrow on the equity), you need to meet the mortgage payments and other expenses when they arise instead of retaining the option to invest into the transaction only at your own discretion, you must receive the income when it is earned whether or not you need it or know what to do with it. The absence of flexibility in the income stream is a liability that should be compensated by an expectation of higher returns.

TERM AND TIMING

Even with investments that do not offer much flexibility, some may be for the long term, others for shorter terms. Do you prefer to invest $10,000 at 35% or at 25% return? The question has little meaning unless we consider for how long. If the 35% return is available only for one month, we have just made ourselves $291.67. If the 25% return implies 5 year amortization, we will be receiving $293.51 a month for the next 5 years, though part of these payments are a return of the original investment. We cannot choose between these two investments just by looking at the rate.

Even when two investments have the same overall term, if one spins off some returns early while the other pays back in one lump sum at the end of the term (regular bonds versus zero-coupon bonds), then, with the first, some parts of the investment are not invested for the whole term.

AMOUNT

The amount required to go into an investment is related to the previous concern. We may not want to give up the opportunity of investing $2,000,000 at 20% for 10 years just for the sake of a $300,000 investment at 25% for 3 years.

In the next unit, we will consider adjusted rates of return that attempt to compensate for these differences.

ASSUMPTIONS

Man lives by assumptions. Without making some assumptions he wouldn't get up in the morning, and he certainly would not invest. There may be different degrees of likelihood between assumptions, and there is no guarantee that they will be realized. We should not ask for that impossible guarantee, but we can at least take two precautions:

BE CONSERVATIVE. That 5 year projection on an appartment house seems to promise such a good rate of return on the basis of the 8% rate of appreciation thrown into the calculations! How does it look if we project a more modest, but still significant, 4 or 5% rate of appreciation? How dependant is the rate of return on the assumption that is being made about appreciation?

BE CONSISTENT. You are looking at the Internal Rate of Return on two 5 year projections on appartment house investments. Comparing the rate is meaningless if one calculation projects 4% inflation on maintenance costs while the other uses 8%. Whether inflation turns out to be 4% or 8%, or 2% or 12%, may be less important than the fact that it will be the same for both properties! Similarly for the ratio of expenses to income, the vacancy factor, the reserve for maintenance, the rate of increase for rental income, and the rate of appreciation for the property as a whole: it is not so important that these numbers be proved accurate as that they be consistent, that any discrepancy be justified by particular circumstances. When deciding to invest in an appartment house, we made the basic decision to accept some elements of risk and speculation. In choosing between alternative appartment houses, we should at least insist on consistency between the assumptions used in establishing the cash flow projection.

- o 0 o -

Clearly, rates allow us to rate investments only to the extent that all other things are equal, or at least that they are not so significantly unequal as to make comparisons impossible. A rate, once calculated, does not allow us to close our eyes to all the assumptions that went into the calculation of the rate, or the various circumstances that affect the investment but are not fully reflected in the rate. It does not overrule common sense and a sense of proportions.

INVESTOR OR BORROWER ?

> Two friends have an agreement that they can borrow money from one another. They borrow, and pay back, and lend on a regular basis in such a way that both borrow about as much, and for as long, as they lend. Clearly, once the slight advantage that the first borrower might gain has dimmed into the past, it wouldn't make much difference if they pretended that interest indeed had been charged, or at what rate, as, whatever the rate, each would have paid as much in interest as he would have received.

The problem that we feel intuitively to exist--that 0%, 10%, and 40% could just as well balance the books on the account between the two friends--is reflected in the mathematics of the rate calculation. If the parties switched an infinite number of times from being lender to becoming borrowers, there would be an infinite number of mathematically correct rates! If they just switched 50 times, there would then be only 50 correct rates!

A rate then becomes meaningless. First, because we wouldn't know whether to prefer a low rate as a borrower or a high rate as an investor. Secondly, because other, equally valid rates also fit the exchange.

Let's contrast two exchanges, one where the IRR presents no problem, and one where the rate loses much of its meaning. (Numbers are rounded for convenience).

A) Yearly Cash flows on income property investment with large capital improvement expenses in the year prior to the sale of the property:

Time 0:	-150,000
EOY 1:	20,000
EOY 2:	30,000
EOY 3:	40,000
EOY 4:	-100,000
EOY 5:	400,000

The calculator shows a rate of return of 24.15%.

The negative cash flow for year 4 presents no problem as can be seen if we consider the previous investment as the sum of two investments, each one of which offers the same same 24.15% annual rate of return:

Time 0:	-150,000	0
EOY 1:	20,000	0
EOY 2:	30,000	0
EOY 3:	40,000	0
EOY 4:	0	-100,000
EOY 5:	275,850	124,150

Each exchange is a perfectly valid investment that produces a 24.15% rate of return, and the sum, of course also provides a 24.15% return.

B) Cash flows provided by a business venture that turned sour:

Time 0:	-11,000
EOY 1:	30,000
EOY 2:	-20,000

A first problem occurs when we press **Gold IRR:** we get an **ERROR 3** signal. However, by using **GOLD NPV** and a trial and error procedure, we may establish two rates, 16.03% and 56.69%, that both meet the requirement of providing a Net Present Value of zero. Both rates seem high when we are getting back $1,000 less than we invested! What is the problem?

At 16.03%, the exchange of money can be seen as the sum of the two following transactions:

Time 0:	-11,000	0
EOY 1:	12,763	17,237
EOY 2:	0	-20,000

Both are valid transactions, both correspond to a rate of 16.03%. Unfortunately, with the first transaction we invests $11,000 for one year and with the second we borrow $17,237 for one year.

At 56.69%, the numbers are different, but the result is similar:

Time 0:	-11,000	0
EOY 1:	17,237	12,763
EOY 2:	0	-20,000

Whichever rate we choose, it is a rate at which we are both investing and borrowing. Though both rates are mathematically correct, they are also both meaningless as a means of rating the exchange of money. This becomes clear if we select one of the rates and ask ourselves: "Would I prefer a higher rate or a lower rate?" Being both investor and borrower, we are pulled in two different directions and are unable to answer the question.

How to deal with the problem in concrete terms? These points may help:

1) Exchanges where there is only one change of sign as we go down the series of cash flows present no problem, nor should most usual situations where the initial investment and the final return are by far the largest amounts in the transaction. There has to be 2 or more changes in the sign for there to be the possibility of a problem.

2) Even the presence of 2 or more changes in the signs does not necessarily mean that we have a problem. Transactions on a savings account, with constant deposits (investments) and withdrawals (returns) display innumerable changes in the sign, yet there is no problem because we never withdraw more than what we have in the account. If we withdraw more than the available balance, then from investors we become borrowers which introduces a measure of discrepancy in the rate.

3) It is possible to have an ambiguous exchange of money and a meaningless rate even though the IRR function provides an answer, as we may find by testing the previous problem with the final -20,000 replaced by -5,000. (IRR then shows 154.9% and a trial and error procedure flushes out a second rate of -82.17).

4) When there is the possibility of a problem, then, with the calculated rate in **i**, we should calculate **GOLD NPV** for successively larger numbers of cash flows (1 n GOLD NPV, 2 n GOLD NPV, etc). If the Net Present Value remains of the same sign as CFo (negative, shall we say) until all cash flows are included, then there is no problem as we never put ourselves in a situation where the roles are changed (a negative balance for a savings account). If the Net Present Value changes sign, then we have just calculated the Present Value of the amount that is being borrowed within that investment transaction. The larger that amount compared to other cash flows, the more our rate is distorted by the ambiguity of the data.

5) Where there is a problem, it is good to remember that the problem is with the cash flows themselves. They are such that they could not be provided by the transactions on a savings account without the lending institution agreeing to lend us money at the same rate as it is paying us when we have a positive balance. We may also want to remember that a rate is just here to help us rate an exchange. If the rate is unable to help, we may still look at the exchange of money. For instance: do I want to invest $11,000 now, get $30,000 back in one year, and then have to pay $20,000 a year later?

6) To better answer this last question we should really make an assumption about what we can do with the cash flows: we could certainly re-invest the $30,000 for one year at a readily available 10% rate of interest. This would provide us with $33,000 at the end of the second year, allowing us to pay off the $20,000 and retain $13,000. The initial $11,000 investment would then result in a $13,000 net return two years later, which corresponds to a rate of return of 8.71%. That rate, which is here a blended rate on 2 investments, is called a Financial Manager's Rate of Return, or FMRR. It will be discussed more fully in the next Unit. 8.71% certainly gives a better feel for the desirability of the transaction than either 16.03% or 56.69%!

CHOOSING THE INITIAL TRIAL RATE IN AN IRR CALCULATION

The Internal Rate of Return calculation is a trial and error process and the calculator needs to make an initial guess. Sometimes that initial guess is far off the mark and the calculator runs for a long time in pursuit of the correct rate. At other times, the nature of the data makes it difficult for the calculator to make that initial determination, and we get an **Error 3** signal.

These problems can be avoided if we choose ourselves the initial trial rate and key it into **i**. We may then start the IRR calculation by pressing **RCL BLUE R/S**.

SHORTENING AN IRR CALCULATION

We may interrupt an IRR calculation that seems to be going on for ever by pressing and holding down **R/S**. When we do so, the calculator finishes the trial run that is currently in progress but does not pursue the calculation any further. The number that is then displayed is the rate currently being tested. Because that rate is already stored in the **i** memory, we may press **GOLD NPV** to find how close we are to the desired ZERO value that would show a perfectly accurate IRR. If we are a few dollars off on cash flows of thousands of dollars, the rate achieved so far may provide more than enough precision for our purpose. If NPV is still too large, we may resume the IRR calculation where we left off by pressing **RCL BLUE R/S**. This procedure allows us not to pursue an IRR calculation all the way to the tenth decimal which we will not bother to consider anyway.

PRACTICE

Key in the following exchange of money in time:

-150,000
-40,000
-20,000
70,000
80,000
90,000
100,000
110,000

1) Find the IRR by the trial and error approach and **GOLD NPV.**

2) Find the IRR with **GOLD IRR** and check hidden decimals.

3) Press an initial guess of 16 into **i** and calculate the IRR with **RCL BLUE R/S.**

4) Press **GOLD IRR** and interrupt the calculation after a few seconds by pressing and holding down **R/S.** Then:

   - Press **GOLD NPV** to see how far you are from the correct rate.

   - Press **RCL BLUE R/S** to pursue the calculation to the correct answer.

(To check your final answer, divide 122.81 by 7)

UNIT 28

MODIFIED RATES AND FINANCIAL MANAGEMENT RATE OF RETURN (FMRR)

FUTURE VALUE CALCULATION AND MODIFIED RATE

> I have $100,000 to invest for 5 years. I am offered the choice of investing the full $100,000 for 5 years at 16% compounded monthly, with no payments received until the end, or of investing $70,000 for 5 years at 20% with monthly interest-only payments of $1,166.67.

The choice cannot be made unless we know a little more about circumstances and alternatives. So let's add the following:

> I do not need the interest-only income: what I really want is to maximize the total amount of money I will end up with 5 years from now. Furthermore, I am willing to assume that any amount not invested in one or the other investment, can be deposited at 11% interest, compounded monthly.

Solving the problem is easy if we reflect that what we want here is as much cash as possible at the end of the 5th year. So let's find out how much cash we get at the end of 5 years under each option:

OPTION A: $100,000 for 5 years at 16%, with no payment.

```
100000  PV     5  BLUE  n     16  BLUE  i
0  PMT    FV                                    ($221,380.69)
```

OPTION B: After 5 years we get the $70,000 balance on the 20% interest-only loan, plus the Future Value of all amounts invested at 11% until the end of the 5th year: the initial $30,000 not needed for the 20% loan and the $1,166.67 monthly interest payments.

```
30000  PV    1166.67  PMT
5  BLUE  n     11  BLUE  i      FV              (-144,638.83)
CHS    70000  +                                 ($214,638.83)
```

Option B allows for the exchange of $100,000 for $214,638.83 in 5 years, which less than the $221,380.69 provided by the 16% investment. We may now calculate the overall rate provided by option B, the rate that allows $100,000 to grow into $214,638.83 in 5 years:

```
100000  PV     214638.83  CHS  FV
5  BLUE  n
0  PMT      i     12  x                         (15.37%)
```

Far from providing a 20% rate of return, option B, as modified by the re-investment assumption, offers only 15.37%. 15.37% is the blended rate on the two investments that make up option B. It is an adjusted rate of return that more adequately allows us to compare the full consequences of choosing option B to the 16% rate promised by option A. Just looking at the 20% rate of return ignores the fact that option B does not invest all the money all the time at that rate.

In relation to the cash flows that support them, 11%, 16%, 20%, and the modified rate of 15.37%, are all Internal Rates of Return.

MODIFIED RATE: IRREGULAR CASH FLOW EXAMPLE

Initial investment:	$250,000	250000	CHS	BLUE	CFo
EOY 1 return:	50,000	50000	BLUE	CFj	
EOY 2:	60,000	60000	BLUE	CFj	
EOY 3:	70,000	70000	BLUE	CFj	
EOY 4:	80,000	80000	BLUE	CFj	
EOY 5:	90,000	90000	BLUE	CFj	
EOY 6:	250,000	250000	BLUE	CFj	
The rate of return is 23.92% rate:		GOLD	IRR		

If the investor uses the yearly returns as income which he needs to live on, the rate of 23.92% is a good measure of his return. If the investor is a very active investor, for whom this is only a small portion of his constant dealings, then the yearly amounts received may be expected to join a larger pool of money and find efficient use at equally high rates of return, and here again the 23.92% rate is a good measure of that particular investment's contribution.

But let us take an investor who is somewhere in between: he doesn't need the yearly amounts as income, but, for lack of capital or other reasons, he cannot take on two such investments at the same time. The yearly amounts will be allowed to accumulate in some form of certificate of deposit and can be expected to earn 11%.

In this case we cannot use the 23.92% to compare this investment to other investments that may well have lower rates but that would allow all the money to remain invested all the time at that rate. We need once again to calculate a modified rate of return that is a blended rate on the two investments (at 23.92% and 11%) allowed by the circumstances.

Let's use the data already in the calculator to calculate the blended rate, or modified rate of return, as follows:

1) Calculate the Future Value of all the positive returns invested at 11%. To do so, we eliminate the $250,000 initial investment.

2) Calculate the rate of return on a $250,000 investment that can be exchanged for the stated Future Value.

(With previous data)	
0 STO 0	Leaves only positive returns in calculator.
11 i GOLD NPV	Present Value of returns discounted at 11%.
0 PMT FV	\$719,539.30: Future Value of all positive returns if invested at 11%.
250000 PV i	(19.27%, modified rate of return).

19.27% is the rate of return that allows 250,000 to grow into \$719,539.30 in 6 years.

We now know that if the investor finds an opportunity to invest all his \$250,000 for a full 6 years at 20%, he is better off than investing some of it part of the time at 23.92% as above, and the rest for part of the time at only 11%.

FINANCIAL MANAGEMENT RATE OF RETURN (FMRR)

A financial manager is telling a client that he has a beautiful 5 year investment for him: it offers a 30% rate of return! The client asks how much he must invest. He is told \$681,237.30. He gives the money and leaves for a 5 year cruise.

The financial manager immediately invests as follows:

CFo	-500,000
CF1	-150,000
CF2	-50,000
CF3	400,000
CF4	600,000
CF5	938,730

We may verify that this indeed provides a 30% rate of return.

But there are a few problems:

1) The investor only provided \$681,237.30 when the investment requires a total of \$700,00 over the first 2 years. That is easily solved by investing the money that was not immediately required at 8.25%, a safe and liquid rate readily available that will provide for the needed cash flows. The manager knew he would invest some amounts at that rate, and therefore did not request the full \$700,000.

2) What should the money manager do with the \$400,000 and 600,000 that revert to him before the 5 year deadline? He has no way of investing them at 30%, but knows he can invest the amount at 12%, still a safe rate, but not as liquid as the 8.25% investment. This allows the \$1,000,000 received early to grow into \$1,173,760.00.

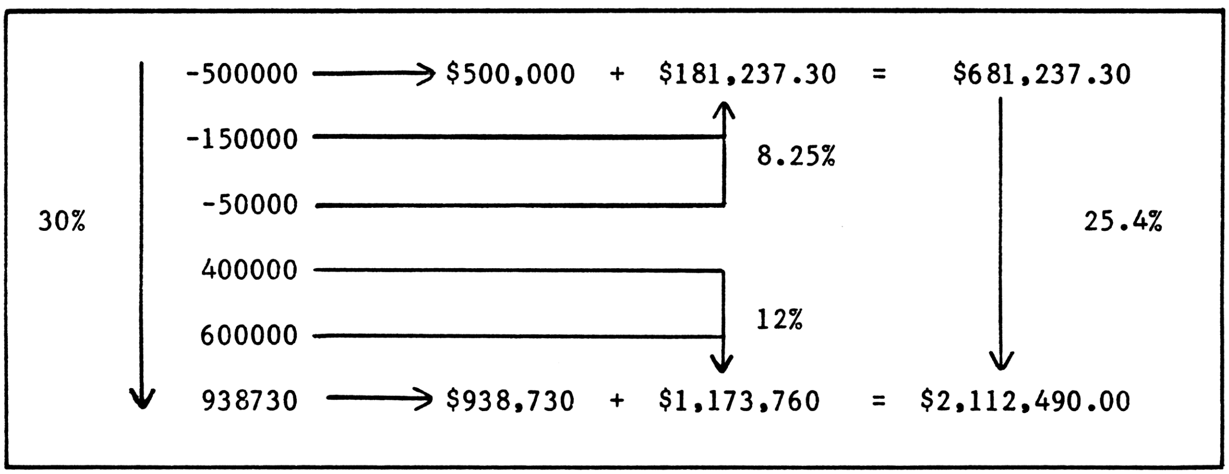
When the investor comes back to claim his money 5 years later he is given $938,730.00 plus $1,173,760.00, a total of $2,112,490.00. The investor takes out his calculator and says: "This is not a 30% return. It's only 25.4%".

That 25.4% rate is what is frequently called a Financial Management Rate of Return or FMRR. It is a rate that makes more sense than the higher 30% in situations where the ideal would be to keep all the money invested all the time at the higher rate, but where the nature of the high rate investment does not allow that to happen. It is indeed the rate that the finanacial manager should have quoted.

The FMRR modifies the original IRR in a procedure that simulates what happened above. Whether or not this is indeed the case, it assumes that all the money that will be needed is invested at a safe and liquid rate (here 8.25%) until it is needed. It also assumes that early returns are re-invested at a somewhat higher reinvestment rate (here 12%). The FMRR is the blended rate on the combination of these 3 investments that, together, keep all the money invested all the time at the highest rate available under the circumstances.

The four rates--8.25%, 12%, 25.4%, and 30%--are all Internal Rates of Return in reference to the cash flows that provide them. The FMRR of 25.4% is the blended rate of three investments. It is 'internal' to the exchange of money that best represents the transaction from the point of view of the mythical investor.

We may illustrate this FMRR procedure as follows:



From the point of view of the original investment opportunity, 30% is the Internal Rate of Return (IRR), and 25.4% is the Financial Management Rate of Return (FMRR). Mathematically, from the point of view of the cash flows that support each of them, the four different rates are all Internal Rates of Return.

As we run through the keystrokes for a FMRR calculation, note how:

1) We key in the data for the major investment and calculate the IRR.

2) Using the same data bank, we isolate the negative cash flows (2 n) and calculate the initial investment required to provide those negative amounts.

3) We eliminate the negative cash flows, re-instate the positive amounts (5 n), and calculate the Future Value of the positive cash flows invested at the 12% reinvestment rate.

4) We calculate the rate that transforms our initial investment into the final total return: the FMRR.

0	500000	CHS	BLUE	CFo	
1	150000	CHS	BLUE	CFj	
2	50000	CHS	BLUE	CFj	
3	400000		BLUE	CFj	
4	600000		BLUE	CFj	
5	938730		BLUE	CFj	
			GOLD	IRR	(30% IRR)
	2	n	8.25	i	
	GOLD	NPV			($681,237.30: initial investment)
	0	STO	0		
		STO	1		
		STO	2		(Clears 3 negative cash flows)
	5	n	12	i	
	GOLD	NPV			
	0	PMT	FV		(-2,112,490.00: FV of returns)
	681,237.30		PV	i	(25.40% FMRR)

- o 0 o -

When comparing FMRRs, it is important to make sure that they are based on the same assumptions--same safe rate, same reinvestment rate--, and to remain as close as possible to reality in the assumptions we make.

Those two additional rates, we should remember, affect only cash flows that occur between the initial and final disbursement.The smaller these intermediate cash flows compared to the initial investment and the final reversion, the smaller the discrepancy between the FMRR and the initial IRR and the less important it becomes to introduce one or two more assumptions into the equation.

IRR AND FMRR

We frequently hear claims that the IRR makes the unwarranted assumption that all negative cash flows are discounted to Time 0 at the high Internal Rate of Return, and that all early returns are re-invested at the high rate until the transaction is concluded.

This could be true from the point of view of the investor in our last example if we tried to use 30% (the IRR) as an answer to the very different question that he is addressing. But the problem then is that we are taking the rate that applies to one exchange of money (the 30% investment with varying amounts changing hands at various points in time) and using it as an answer to the question asked by someone who is considering a different exchange (the 25.4% Present Value - Future Value exchange). If we want a 30% return on all the money, invested all the time, then indeed we need to invest all the money all the time which is not the case with the data submitted to the IRR calculation.

The fact that, in our calculations, we may actually be discounting all cash flows to time zero is simply a mathematical convenience: selecting any other point in time to equate the value of the amounts invested and the value of those received would work just as well and provide the same answer.

The IRR itself makes no assumption. It just considers the amounts submitted and the timing of these amounts. It is the rate that precisely does not make any assumption about amounts that are not yet invested or about what happens to amounts that have been returned to the investor. After all, the interest rate on a savings account is an IRR, and certainly the savings institution makes no assumption about amounts that are not yet or no longer entrusted to its care.

But a savings account provides a flexibility that most other investments do not have. In circumstances where this lack of flexibility presents a major inconvenience, as in the case of our traveling investor who wants all his money invested all the time, then we are justified in adjusting the rate to best reflect the investor's requirements.

The whole point of a rate of return is to compare investments that are dissimilar in more than one feature--otherwise, just looking at the cash flows allows us to compare the exchanges. When the discrepancies are too great--such as when the cash flows are not invested all the time and yet we want to compare with investments where the money is invested all the time--then we may feel justified in using a Financial Management Rate of Return or some other form of modified rate. With the understanding that it is not so much the rate that is modified as the cash flows submitted to a rate calculation.

CALCULATING IRREGULAR MISSING AMOUNTS

We have seen many instances where calculating the missing amounts was a simple matter of finding the Present Value of those missing amounts (**GOLD NPV**), and then shifting that value to its rightful place in time through a simple **PV - FV** or **PV - PMT** transfer.

A PV - FV transfer is possible when there is only one missing cash flow. A PV - PMT transfer is possible when a number of missing cash flows are all equal and spread out over a period of time that immediately follows the time one considers as the present.

Missing cash flows, however, are not always so obliging as to fit into one of these two categories. Yet, if they are defined, they can generally be calculated with relative ease.

We will consider here three different procedures:

1) A switch in the time that one selects as being the 'Present' often allows us to bridge the gap in an otherwise regular series of missing amounts.

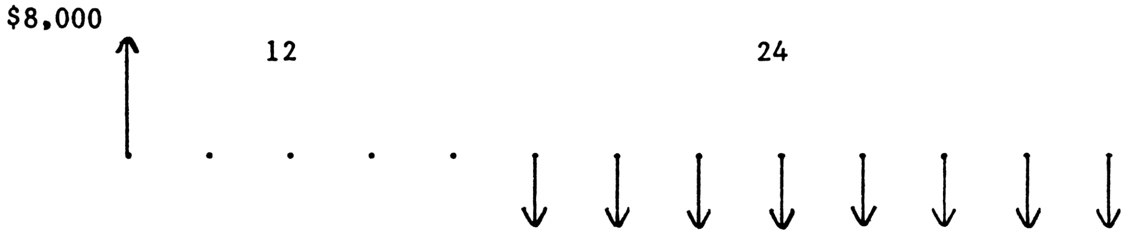
2) A procedure that consists in giving an arbitrary value to the missing amounts such that the known ratios are preserved allows us to calculate missing amounts of unequal value.

3) A procedure by which a single investment is considered as the sum of two (or more) separate investments, each of which can be analyzed separately by different methods.

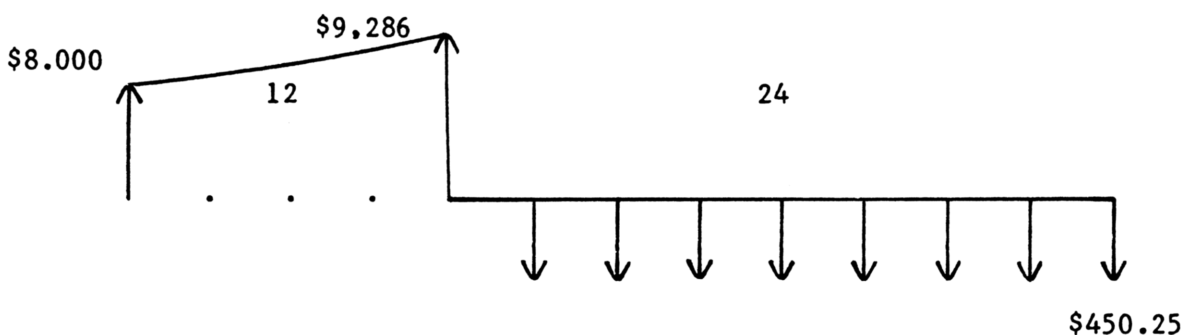
GAP IN EQUAL PAYMENTS

A frequent situation presents only a slight discrepancy from the equal payment situation: the payments are still equal and equally spread out in time, but they do not begin immediately. There is a gap that separates them from the Present Value. The solution here is simply to transfer the Present Value to the end of the last period of that gap, thus bridging the gap and allowing a PV - FV or PV - PMT transfer.

> The missing cash flows are 24 monthly amounts occurring at the end of months 13 to 36. Their NPV, discounted at 15%, is $8,000.



To calculate the 24 equal payments, we first transfer the $8,000 Present value to the end of the first year (the last empty period in the gap). As adjusted by the 15% rate, $8,000 at time 0 becomes $9,286.04 after 12 months. That new value becomes the Present Value in a PV - PMT exchange that allows us to find the value of the missing amounts ($450.25).



8000	PV	
15	BLUE i	
12	n	(or 1 BLUE n)
0	PMT FV	($9,286.04)
CHS	PV	(FV becomes the new PV)
24	n	
0	FV PMT	($450.25)

When considering a 'balanced' exhange of money in time, it is possible to take any cash flow and transfer it to any other point in time without disturbing the balance of the exchange, provided that we adjust the cash flow that we transfer as required by the rate. This is what we do here in order to create a situation that we know how to solve.

BRIDGING THE GAP: FURTHER ILLUSTRATION

Consider the following cash flows. What 2 equal payments do you need to receive for years 5 and 6 in order to obtain a 13% return?

Initial investment:	$350,000		350000 CHS BLUE CFo
EOY 1 & 2:	60,000	(2)	60000 BLUE CFj 2 BLUE Nj
EOY 3 & 4:	80,000	(2)	80000 BLUE CFj 2 BLUE Nj

PV of missing amounts:	13 i GOLD NPV	($145,404.34)
PV transferred 4 years into future:	4 n 0 PMT FV	($237<077.94)
FV becomes new PV:	CHS PV	
2 mising amounts calculated:	2 n 0 FV PMT	($142,124.33)

- o 0 o -

The procedure can also be used with regular cash flows that do not fully meet the regular cash flow requirements:

Calculate the monthly payments that fully amortize a $100,000, 30 year, 12.25% loan, knowing that no payments are made during the first 5 years of the loan. (12.25% is compounded monthly, even when no payments are received).

Here 'bridging the gap' means that we calculate the $183,931.79 outstanding balance on the loan at the end of the 5th year. That amount becomes the Present Value that we amortize over the remaining 25 years:

100000 PV 12.25 BLUE i	
5 BLUE n 0 PMT FV	($183,931.79)
CHS PV	
25 BLUE n 0 FV PMT	($1,971.28)

UNEVEN MISSING AMOUNTS: THE ARBITRARY VALUE APPROACH

The missing amounts need not be equal. When they are not, clearly some kind of ratio needs to be known establishing their relative value. Otherwise, there would be an infinite number of possible solutions. The model solution that follows considers 3 missing amounts representing respectively 20%, 50% and 30% of the total missing dollars.

The solution consists in solving for the Present Value of the missing amounts twice: once with the known cash flows as we have always done, another time by giving an arbitrary value to the missing amounts that respects their known ratio (here 20, 50 and 30 will be used). The ratio between the two Present Values gives the value of each arbitrary unit used to express the missing amounts. For instance, if the ratio between the two Present Values is 1,000, this means that the arbitrary values chosen are 1,000 times too small. The real value of each missing amount is obtained by multiplying its arbitrary value by 1,000.

MODEL PROBLEM

An investor-developer intends to buy a piece of land, make plans and obtain permits, proceed with on-site and off-site improvements, and finally build and sell a number of homes. His projected costs are as follows:

Year	Expense	Nature of the expense.
0	200,000	Downpayment on land and buying costs.
1	300,000	Financing, plans, permits.
2	700,000	Site improvement and financing costs.
3	1,200,000	Construction and financing costs.
4	1,500,000	Construction, financing, selling costs.
5	1,500,000	Construction, financing, selling costs.
6	400,000	Financing and selling costs.

These amounts represent expenses. The developer expects to make some money during the last three years of the project as completed properties are being sold. He expects to make 20% of his sales volume during year 4, 50% during year 5, and 30% during year 6.

What amount needs to be made during each of the last 3 years if the developer is to earn a 20% return on his investment?

SOLUTION

Let's first calculate the Present Value (discounted at 20%) of the missing returns by assigning them an arbitrary value of \$20.00, \$50.00, and \$30.00 respectively. The numbers are incorrect in their amount but correct in terms of their relative value.

```
  0  BLUE  CFo   4  BLUE  Nj          (Allows for years 0 to 3)
 20  BLUE  CFj
 50  BLUE  CFj
 30  BLUE  CFj
 20  i   GOLD  NPV                    ($39.79)
 STO 9                                (Store in memory not used
                                       by irregular cash flows)
```

Let's now calculate the Present Value of the missing amounts on the basis of the known cash flows:

```
  200000  BLUE  CFo
  300000  BLUE  CFj
  700000  BLUE  CFj
 1200000  BLUE  CFj
 1500000  BLUE  CFj
          BLUE  CFj
  400000  BLUE  CFj
    GOLD  NPV                          ($3,090,710.73)
     RCL  9   ÷                        (77,683.61)

 ENTER  ENTER  ENTER
        20  x                          ($1,553,672.19: return year 4)
   Clx  50  x                          ($3,884,180.47: return year 5)
   Clx  30  x                          ($2,330,508.28: return year 6)
```

Net returns for years 4 to 6 are projected as follows:

Year 4:	1,553,672.19	- 1,500,000	=	\$53,672.19
Year 5:	3,884,180.47	- 1,500,000	=	\$2,384,180.47
Year 6:	2,330,508.28	- 400,000	=	\$1,930,508.28

By storing these values in memories 4, 5 and 6, we may verify that we indeed have a 20% return. (Watch for sign consistency and for correct value in **n**).

The reasoning behind this approach is as follows:
- If the missing amounts were \$20, \$50, and \$30, their PV would be \$39.79.
- In reality their PV is \$3,090,710.73, or 77,683.61 times higher.
- If we multiply each arbitrary value by 77,683.61 we find cash flows that meet the double requirement: correct ratio and required Present Value for a 20% return.

APPLICATION TO LEASIMG PROBLEMS

Calculate the monthly payment on an equipment lease that meets the following requirements:

- Cost of equipement: $20,000.00.
- Required return: 18%.
- 4 payments paid in advance on receiving the equipement, remaining 32 payents paid at the beginning of each of the months that follow.
- Diregard residual value of $1.00.

Let's give an arbitrary value of $1.00 to the monthly payments and find their Present Value based on an 18% discount rate:

```
 4  BLUE  CF0
 1  BLUE  CFj    32  BLUE  Nj
18  BLUE  i

    GOLD  NPV                              ($29.27)
```

The Present Value of the payments, discounted at 18%, should really be $20,000. If we divide $20,000 by 29.27, we get the value of each payment:

```
20000  xy  ÷                     (Answer: $683.36)
```

CHECKING THE ANSWER

To verify that the projected exchange of money does result in the desired rate of return, we may now establish the net cash flow at each point in time--a net investment of $17,266.56 exchanged for 32 payments of $683.36--, and calculate the yield:

```
PMT                     ($683.36 stored in PMT memory)
ENTER  4  x             ($2,733.44: payents made up front)
20000  -   PV           (-17,266.56, net investment stored in PV)
32  n   0  FV   i       (1.50%, monthly rate)
12  x                   (18.00%, annual rate)
```

(Note the need to key either PV or PMT in as a negative. Also note that we need to press **ENTER** before we can multiply 683.36 by 4 once that number has been pressed into the PMT memory: the display image of a number pressed into a regular cash flow financial memory is inoperative unless re-activated in one way or another).

CLONE PROBLEMS

A firm constantly leasing equipement on the same basis--same rate requirement, same term, same number of payments up front, same percentage of cost as residual value--needs to calculate the Present Value of the arbitrary payments only once: that factor remains the same whatever the cost of the equipement. (Check the number of decimals you retain for the factor). Under the conditions in the previous example, the payments on a piece of equipement costing $111,000 would be:

```
111000  ENTER  29.27   ÷                    ($3,792.28)
```

LEASING PROBLEM WITH RESIDUAL VALUE

Same problem as above, except that now we have a 10% ($2,000) residual value at the end of the 3 year lease. The problem might be expressed as follows:

- Cost of equipement: $20,000.
- Required return: 18%.
- 36 month lease, with payments made at the beginning of each month, and the last three payments paid up-front on receiving the equipement, at the same time as the first regular payment. (So we still have 4 payments up-front).
- Residual value of equipement: 10% of initial cost.

Here, the $2,000 residual value received at the end of the lease prevents us from immediately giving an arbitrary value to all the payments received. The solution consists in reducing the $20,000 by the Present Value of the $2,000 residual discounted at 18%:

```
 2000  CHS   FV
   18  BLUE  i
   36  n                                (or 3 BLUE n)
    0  PMT   PV                          ($1,170.18)
20000  -                                ($18,829.82)
```

The monthly payments must offer an 18% return on $18,829.82, with the value of the residual presenting an 18% return on the $1,170.18 difference. We may now solve as in the first problem on the $18,829.82 value, or directly use the 29.27 factor if already known (slight discepancy due to rounding will result).

```
 4  BLUE  CF0
 1  BLUE  CFj     32  BLUE  Nj
18  BLUE  i       GOLD  NPV                 (29.27)

18829.82  xy  ÷                   (Answer: $643.38)
```

Whenever the residual value is expressed as a set percentage of the cost, we may establish a ratio between the cost and the monthly lease payments and use it as a factor for similar calculations--same rate requirement, same term, same number of payments up front, and residual value expressed as the same percentage of the cost.

With the previous problem, the factor is calculated by dividing $20,000 by $643.38:

20000 ENTER 643.38 ÷	(31.09)

We may now use that factor to calculate the required monthly payments on equipement costing $111,000.00, $273,000.00 and $51,000.00:

111000 ENTER 31.09 ÷	($3,570.28)
273000 ENTER 31.09 ÷	($8,780.96)
51000 ENTER 31.09 ÷	($1,640.40)

CHECKING THE FACTOR

Before using a factor extensively it is wise to check it on numbers other than those used to calculate it in the first place. Let us check whether monthly payments of $1,640.40 indeed result in an 18% rate of return on a $51,000 piece of equipement (last three payments made in advance, 10% residual value). We do so by establishing the net cash flow at each point in time and calculating the rate of return.

4 payments are received when the equipement is leased: the last three payments, plus the first of the regular payments. The net investment is $51,000 - (4 x 1,640.40) = 44,438.40. The reversion is 10% of $51,000 or $5,100. We establish the cash flows as follows:

- 44,438.40		44438.40 CHS BLUE CFo	
1,640.40	(32)	1640.40 BLUE CFj 32 BLUE Nj	
0.00	(3)	0 BLUE CFj 3 BLUE Nj	
5,100.00	(1)	5150 BLUE CFj	
		GOLD IRR 12 x	(17.99%)

Because the factor is rounded to 2 decimals, and also the payments, a slight rounding effect reduces the rate to 17.99%.

THERE ARE NO UNITS 29 & 30

UNIT 31

STATISTICAL FUNCTIONS

STATISTICAL FUNCTIONS

This section on STATISTICAL FUNCTIONS is meant as a practical introduction to procedures, concepts, and applications. It is hoped that it will open a door to some who never thought they could use statistical calculations for their purpose. We will avoid technicalities as much as possible.

Statistics is the art of drawing a general conclusion from the consideration of divers individual instances, a conclusion that can then be applied to making predictions about the likelihood of other individual occurences. By measuring a number of American men, I may calculate that their average height is 5' 11". Maybe none measured exactly 5' 11", and yet that single number summarizes all the measurements for me, and may allow me to feel some confidence that the man whom I know only from his voice on the phone is more likely to be 5' 11" than 6' 10" or 5' 2". But there is no guarantee that he is not 7' 2" or 4' 11".

A statistical conclusion such as the average height of American men is drawn from a consideration of the numbers (here the height) for each individual of a population, or, as we will assume here, for a representative sampling of that population. (The word "population" is used in statistics for a collection of individual subjects whether they are men, beasts, things, or occurences).

The most simple of statistical information is the average or mean. The HP-12C provides us with the ability to calculate averages, but also offers a number of statistical functions that are all, to some extent, significant improvements on the information provided by a simple knowledge of the average.

As with all other HP-12C calculations, statistical functions require that we provide the **DATA** and then ask the **QUESTION.** The Data that we key in is made up of those numbers that represent individual occurences--the height of each individual considered, for instance. However, the calculator does not hold in its memories each individual measurement. It builds up a sophisticated data-bank by adding values up in 6 memories: Register memories 1 to 6. The mathematical processing of the data-bank, which the statistical question-keys initiate, retrieves precious statistical information concerning the entries.

Let's explain the process for keying in data and asking questions with a simple average calculation, and--our first improvements--with standard deviation and weighted average calculations.

THREE STATISTICAL DATA KEYS

The three keys used to manipulate statistical data are marked with the greek letter SIGMA that resembles an awkward capital E. The greek letter SIGMA (here represented by **E**) stands for the summation process, the process of accumulating data, that preceeds statistical calculations.

E+ **SIGMA-PLUS** Key position 49.

This function enters numbers stored in **x** (the display) or in **x** and **y** into the statistical data bank (Register memories 1 to 6). SIGMA-PLUS is a specific key labeled **E+**. Do not press the + key.

BLUE E- **BLUE SIGMA-MINUS** Blue key and key 49.

SIGMA MINUS takes out of the statistical data bank the numbers that we have in the **x**, or **x** and **y** memories of the Stack. It can be used to correct an erroneous statistical entry or otherwise manipulate statistical data. BLUE SIGMA-MINUS is the BLUE function corresponding to the **E+** key. Do not press the - (minus) key.

GOLD CLEAR-E **GOLD CLEAR-SIGMA** Gold and key 32 (above Gold key).

This function CLEARS the statistical data memories (Register memories 1 to 6) before we begin keying in statistical data.

GOLD CLEAR-E also clears the Stack, which is very inconvenient and serves no useful purpose. It forces us to clear the statistical memories BEFORE we key the initial pair of data into the Stack.

GOLD CLEAR-REG can be used as an alternative to **GOLD CLEAR-E.**

THREE STATISTICAL QUESTIONS

Statistical questions that address data keyed into the statistical data bank are all BLUE functions. Let's consider three such questions:

BLUE x **AVERAGE OF x** BLUE and 0

Gives the average of the x entries. It also calculates and stores in the **y** memory the average of the y entries. (Retrieve by pressing **xy**).

BLUE s STANDARD DEVIATION BLUE and decimal point

Gives a measure of how spread out on each side of the average the population considered can be expected to be. (See later for details).

BLUE xw **WEIGHTED AVERAGE** BLUE and 6

This key is somewhat misnamed. It calculates the average of the **y** data weighted by the corresponding **x** values.

AVERAGE AND STANDARD DEVIATION

In order to evaluate the potential income on an appartment house for a client, a broker inquires about the rents for similar appartments in the neighborhood and comes up with the following numbers. He keys them in as statistical data and calculates the average rent:

$450.00, $525.00, $410.00, $535.00, $490.00, $490.00, $420.00.

```
GOLD  CLEAR-E
450  E+
525  E+
410  E+
535  E+
490  E+
490  E+
420  E+
```

Average rent:

```
BLUE  x
```

($474.29. We will round to $475.00).

If the calculation is aimed at making a judgment on a projected increase that would raise rents to $510.00, just to report an average rent of $475.00 for the area may not provide enough information: $475.00 could be the average of rentals ranging from $275.00 to $675.00--and a $510.00 rental is well within range--, or of rentals tightly grouped around the average--all between $450.00 and $500.00 for instance--, in which case a $510.00 rent might find very few takers.

A second question can be asked concerning the data already in the calculator. It will give a better feel for the spread that went into the calculation of the average: it is the STANDARD DEVIATION: **BLUE s.**

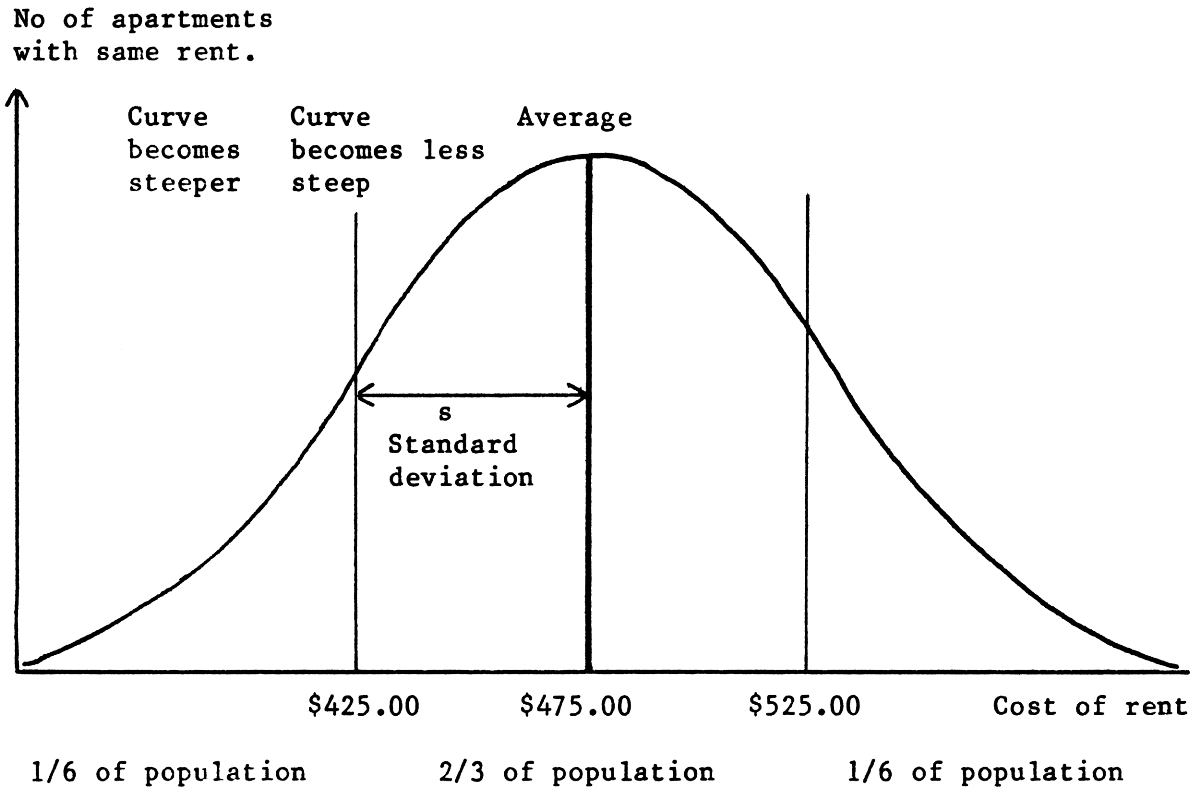
```
BLUE  s
```

(Standard deviation: $49.03. We round to $50.00 for discussion purposes)

So now we have an average of $475.00 with a standard deviation of $50.00. This tells us that, on the basis of the survey, about 2/3 of all appartments are likely to be rented for **$475.00 plus or minus $50.00**, or between $425 and $525. Only one appartment in six is likely to be priced lower than $425.00, and only one in six higher than $525.00. We may conclude that a $510.00 rental price is somewhat on the high side, but well within the bulk of existing rental costs.

The average, supplemented by the corresponding standard deviation, gives a much less ambiguous picture of the situation than would knowledge of the average alone.

We may think of STANDARD DEVIATION as being some kind of AVERAGE DEVIATION FROM THE AVERAGE.



On the bell-shaped curve that plots the distribution of a statistical population under ideal circumstances, the average corresponds to the highest point on the curve: we can expect to find the greatest number of rentals at the average price of $475.00, and find fewer and fewer rentals as we move further away fron the average.

The standard deviation is the distance between the average rent and the rental price that corresponds to the point on the curve where the slope of the curve changes from becoming steeper to becoming less steep. The statistical probability is that just about two thirds of the population lies within the limits marked by the standard deviation on either side of the average. Only 1/3 of the population is more than one standard deviation removed from the average, with 1/6 below the average, 1/6 above it. These numbers are independent of the number of entries used in the computation.

When both x and y data are keyed in, pressing **BLUE s** calculates the standard deviation of both x and y values. The standard deviation of the x values is stored in the display (x memory of the Stack) as above. The standard deviation of the y values is stored in the y memory of the Stack and can be retrieved by pressing **xy**.

AVERAGE AND WEIGHTED AVERAGE

MODEL PROBLEM

I have been buying potatoes for my restaurant as follows:

1st week:	100 lb at 12 cents a pound.
2nd week:	50 lb at 17 cents a pound.
3rd week:	175 lb at 8 cents a pound.
4th week:	150 lb at 10 cents a pound.
5th week:	50 lb at 18 cents a pound.

Let's clear the statistical memories and key the data in as pairs of x and y entries:

		DISPLAY
CLEARING:	GOLD CLEAR-E	0.00
DATA:	12 ENTER 100 E+	1.00
	17 ENTER 50 E+	2.00
	8 ENTER 175 E+	3.00
	10 ENTER 150 E+	4.00
	18 ENTER 50 E+	5.00
QUESTIONS:		
Average lbs purchased/week:	BLUE $\bar{x}$	105.00
Average price/lb over time:	xy	13.00
Weighted average (cost/lb):	BLUE $\bar{x}w$	11.14

So on the average, I purchased 105 pounds of potatoes each time and the average price of potatoes over those 5 weeks was 13 cents a pound. That average price is not affected by the amount of potatoes purchased.

But I bought more potatoes when they were cheaper and fewer when they were more expensive. This buying strategy allowed me to pay less per pound on the average than the 13 cents average price. My average cost per pound is 11.14 cents: the **WEIGHTED AVERAGE.** It is calculated by pressing BLUE xw.

COMMENTS

DATA

- Statistical calculations are not compatible with any other use of Register memories 1 to 6.

- Here, we key a pair of **x** and **y** values into the statistical memory bank. Which is **x** and which is **y** does make a difference, but the order in which the various pairs are entered is indifferent: we could begin with the 5th purchase, then the data pair for the first week, then jump to the third week, etc. This is because the values remembered by the calculator are added up, and when you have a sum, the order in which the numbers are added is indifferent.

- Because the values are added up, we could enter a thousand pairs and still not need more than the 6 Register memories used for the purpose: there is no practical limit to the number of entries other then the 9 to the power of 99 maximum value that can be stored in a register memory.

- As soon as an entry is made with **E+** the display shows the total number of entries already made.

- If an entry is made by error, it can be corrected by keying the same data in again with the **BLUE E-** function. (See **CHANGING THE DATA** section for details).

QUESTIONS

- The arithmetic average question, **BLUE x**, gives two answers: the average of all the **x** values, and the average of all the **y** values (if any), and each is stored in the corresponding memory of the Stack. Pressing **xy** is not a question, it just retrieves an answer that was calculated when we pressed **BLUE x**.

- **BLUE xw**, the key that questions for the weighted average, should really be labelled yw as it gives the average of the **y** values weighted by the **x** values. We wanted to calculate a price, not a weight, so all the dollar amounts had to be keyed into the **y** memory.

EXLORING THE STATISTICAL DATA-BANK

Statistical data accumulates in Register memories 1 to 6. These memories can be explored in the usual way by pressing **RCL 1**, **RCL 2**, etc.:

Remember: In mathematics, The greek letter SIGMA (E) means SUM.
SIGMA x (here Ex) means the sum of the various **x** entries.

MEMORY	CONTENT	MEANING
1		Counts the number of entries.
2	Ex	The SUM of each **x** value keyed in.
3	Ex^2	The SUM of the square of each **x**.
4	Ey	The SUM of each **y** entry.
5	Ey^2	The SUM of the square of each **y**.
6	Exy	The SUM of the product of each **x** and **y** pair.

The calculator uses these numbers to answer the various statistical questions. For instance, we may duplicate the weighted average answer by dividing the value in memory 6 (total price paid) by the content of memory 2 (total number of pounds purchased):

| RCL 6 RCL 2 ÷ | (11.14)

CHANGING THE DATA BLUE E-

The data bank is not affected by the questions that we ask, so we may ask a series of questions on the same data.

We may also delete an entry, and replace it with different data if we so choose, either to correct a mistake, or to test different assumptions.

For instance, on reviewing our data we realize that the second entry--50 pounds at 17 cents a pound--really refers to a purchase of sweet potatoes that should not have been included here. We may delete this entry by keying it in again with the **BLUE E-** function:

| 17 ENTER 50 BLUE E- |

The display shows 4 remaining entries. We may ask the questions again:

BLUE $\bar{x}$	118.75 pounds, average purchase.
xy	12 cents, average price per pound.
BLUE $\bar{x}w$	10.53 cents average cost per pound.

BLUE E- subtracts from memories 1 to 6 all the numbers that had been added when that entry was originally made. The order in which entries are made is of no importance, so we can change any entry at any time.

BLENDED RATE

There are three loans on a property. As they have the same due date (or you expect to pay them all off at the same time), you decide to use the weighted average approach to calculate the blended rate.

Loan 1: $520,000 at 13.75% interest.
Loan 2: $173,000 at 15.25% interest.
Loan 3: $370,000 at 10.00% interest.

SOLUTION

```
GOLD   CLEAR-E

13.75  ENTER  520000    E+
15.25  ENTER  173000    E+
   10  ENTER  370000    E+

BLUE   x̄w                                   (12.69%)
```

Because we want to calculate a rate, we key in the rate first (in the **y** memory).

The answer is a good approximation of the blended rate provided that the various loans have the same due date and are submitted to a reasonably similar rate of amortization.

```
RCL 2                          ($1,063,000: total financing)
```

Here, the sum of the **x** values in Register 2 provides the total financing.

For a perfectly accurate financial function approach, see Unit 25, pages 7 and 8, and Volume I, Unit 8, page 5.

YIELD ON AITD (WRAPAROUND LOAN) EQUITY

An All-Inclusive Trust Deed loan (AITD or wraparound) is being considered. Calculate the yield to the AITD investor assuming a 12% rate on the AITD loan. As all the loans are paid off at the same time, you decide to use the weighted average approach:

AITD:	$500,000 at 12%
Underlying loan balances:	$240,000 at 8.75%.
	$105,000 at 10.50%

SOLUTION

The AITD investor lends $500,000 and borrows the underlying amounts. His net investment is the AITD minus the underlying loans: we may enter the AITD with **E+** and take out the underlying loans with **BLUE E-** as follows:

GOLD	CLEAR-E				
12	ENTER	500000	E+		(1)
8.75	ENTER	240000	BLUE	E-	(0)
10.50	ENTER	105000	BLUE	E-	(-1)
	BLUE	$\bar{x}$w			(18.05%: yield on AITD equity)
	RCL	2			($155,000: AITD equity)

WHAT RATE WILL ACHIEVE DESIRED YIELD ON AITD EQUITY?

Same $500,000 AITD, same underlying loans, but now we want to find the rate that needs to be charged on the AITD loan (instead of 12%) in order to achieve an AITD equity yield of 21%.

The AITD loan is really made up of three investments: the two underlying loans, and the $155,000 AITD equity on which we require a 21% return. The rate we need to charge on the AITD loan is the BLENDED RATE on these three investments.

	GOLD	CLEAR-E			
8.75% on $240,000 loan:	8.75	ENTER	240000	E+	
10.50% on $105,000 loan:	10.50	ENTER	105000	E+	
21% on $155,000 loan:	21	ENTER	155000	E+	
Required AITD rate:		BLUE	$\bar{x}$w		(12.92%)
Total financing (AITD):		RCL	2		($500,000)

(A cash flow procedure to solving this problem might require a time-consuming trial and error approach)

RUNNING AVERAGE

A businessman wants to establish a monthly chart of gross sales. In order to smooth out the discrepancies due to chance fluctuations, he decides to calculate the running three-month average: the chart will always show the average sales for the past three months.

Gross sales data:		
	Jan.:	$82,000.00
	Feb.:	73,000.00
	Mar.:	87,000.00
	Apr.:	99,000.00
	May.:	105,000.00
	Jun.:	101,000.00
	Jul.:	110,000.00

SOLUTION

	Keystrokes	Three months average.
	GOLD CLEAR-E	
Jan. data:	82000 E+	
Feb. data:	73000 E+	
Mar. data:	87000 E+	
Jan. - Mar. average:	BLUE x	$80,666.67
Jan. data deleted:	82000 BLUE E-	
Apr. data added:	99000 E+	
Feb. - Apr. average:	BLUE x	$86,333.33
Feb. deleted:	73000 BLUE E-	
May added:	105000 E+	
Mar. - May average:	BLUE x	$97,000.00
March deleted:	87000 BLUE E-	
June added:	101000 E+	
Apr. - June average:	BLUE x	$101,666.67
April deleted:	99000 BLUE E-	
July added:	110000 E+	
May - July average:	BLUE x	$105,333.33

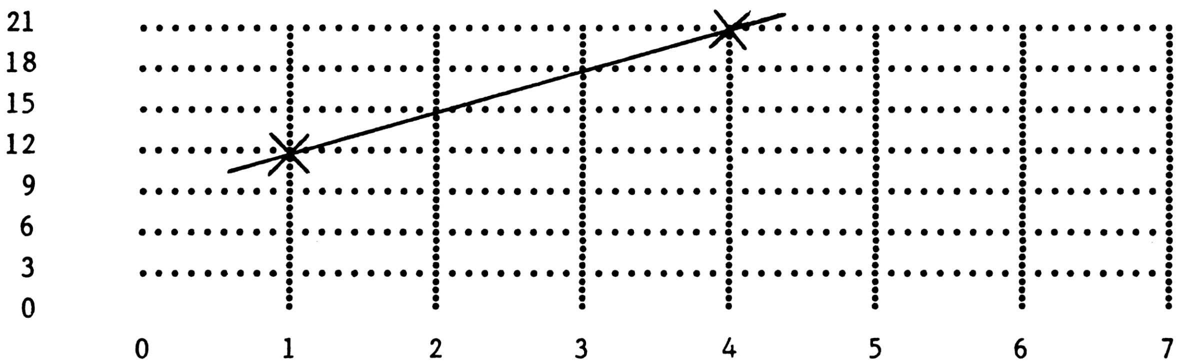
Smoothing out the curve is easily achieved by taking out the monthly data from 4 months back with **BLUE E-** and adding the latest gross sales figure with **E+**. In the present case, the three months running average provides a better picture of the regular improvement that is ocurring.

LINEAR PROJECTIONS

When there is a linear relationship between two factors, knowledge of two pairs of factors fully defines the relationship. The HP-12C allows us to key in two such pairs as statistical data, and easily solve for the missing factor in any other pair.

Let us imagine that a real estate broker gets 9 telephone calls a day without any advertising and 3 extra calls a day for every ad that he puts in the paper. The relationship between the number of ads and the number of calls can be plotted on a graph. It is represented by a straight line: there is a linear relationship.

Y axis: Calls/day.



X axis: number of ads in paper.

Each point on the line can be defined by the number of ads shown on the **x** axis and the corresponding number of calls on the **y** axis. With this line, we may find the number of ads required to generate a given number of calls, or the number of calls that result from a given number of ads.

Let us select two points: - With 1 ad (x=1) we have 12 calls (y=12)
- With 4 ads (x=4) we have 21 calls (y=21)

We may key in these two points as two pairs of **x** and **y** values in the Stack that we transform into 'statistical' data with **E+**. This defines the line within the calculator. We may then ask a number of questions:

- How many calls will we get with 10 ads?
  (We are given 10 as a value for **x**, and we solve for **y**).
- How many ads do we need to generate 45 calls?
  (Now we know **y** and we solve for **x**).

Two functions answer these questions:

BLUE x,r Calculates **x** if we submit a value for **y**. (BLUE and 1)

BLUE y,r Calculates **y** if we submit a value for **x**. (BLUE and 2)

So we communicate to the calculator a line defined by two of its points:

When x = 1, y = 12.

When x = 4, y = 21.

We may then calculate y for x = 10, and x for y = 45, as well as any other value for **x** or for **y** given the other element of the pair:

	GOLD	CLEAR-E			
DATA	12	ENTER	1	E+	(**y** value keyed in first)
	21	ENTER	4	E+	
QUESTIONS	10	BLUE	y,r		(39 calls for 10 ads)
	45	BLUE	x,r		(12 ads for 45 calls)

As we define the line, we need to key in the **y** values first, as these are pushed up into the **y** memory of the Stack when the second element of the pair (the **x** value) is keyed in. The numbers keyed into the **y** memory of the Stack represent data on the **y** axis. Those initially keyed in the **x** memory of the Stack (the display), represent data along the **x** axis. Which element of the relationship we select for the **x** and which for the **y** is arbitrary, though we need to remain consistent.

Let us now improve the procedure

THE BEST-FIT LINE

It is unlikely that the broker in our last example would really know that he gets 3 extra calls for each additional ad. It is more likely that he would consult his record of past performances and discover a less nicely organized set of relationships, for instance as follows. Let's still key in the whole data as in our previous example:

D A T A

Number of calls (y)	Number of ads (x)
30	6
30	8
38	10
32	10
12	2
20	6
37	11
21	3
8	0

K E Y S T R O K E S

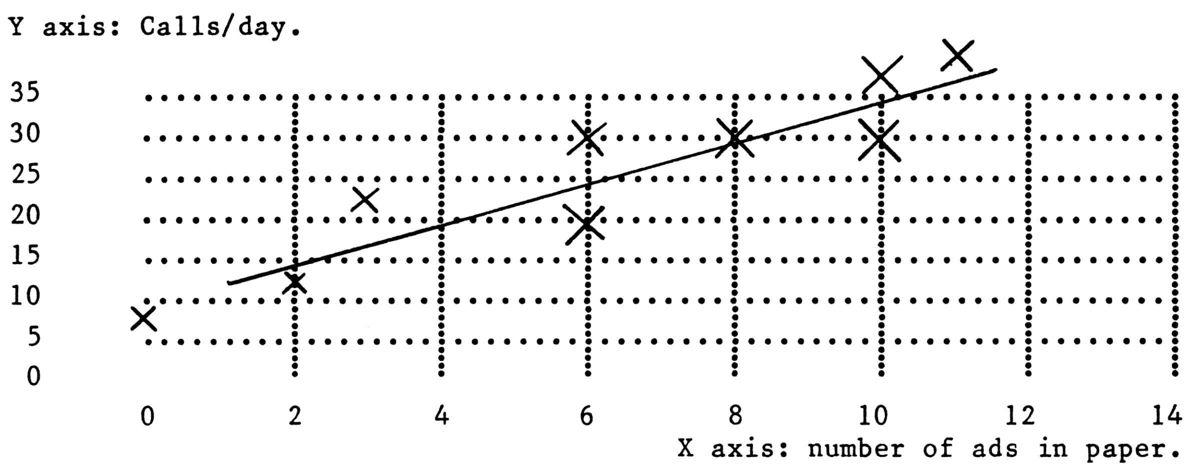
GOLD CLEAR-E

30	ENTER	6	E+
30	ENTER	8	E+
36	ENTER	10	E+
32	ENTER	10	E+
12	ENTER	2	E+
20	ENTER	6	E+
38	ENTER	11	E+
21	ENTER	3	E+
8	ENTER	0	E+

Obviously, these numbers are not on the same line: we have the same number of calls with a different number of ads, and also a different number of calls with the same number of ads. Yet, there does seem to be a relationship between the number of ads and the number of calls.

With the data keyed in, it is as if we had keyed into the calculator the line that BEST FITS all this somewhat contradictory record. We may use that line to make predictions as in the previous instance: predictions that take all the conflicting information into account.

How many ads (**x**) to get 45 calls (**y**)?	45 BLUE x,r	(13.91)
How many ads to get 25 calls?	25 BLUE x,r	(6.19)
How many calls likely with 10 ads?	10 BLUE y,r	(34.94)



BEST FIT LINE: APPLICATION TO APPRAISAL

There are innumerable situations in appraisals where a relationship is established between the price and some other feature of comparable properties: for instance the price in relation to the square footage or acreage. Sometimes a simple average calculation is appropriate in order to generalize to other properties: we calculate the average price per acre and multiply by the number of acres.

This is no longer possible when the very size of the property affects the price per unit of area. For instance when large commercial properties cost more per square foot than smaller properties, or when large agricultural properties cost less per acre than the smaller ones.

This is where a BEST FIT LINE calculation may become indispensable.

Let's take the example of commercial land where a relation exists between the front footage and the price of properties that have sold over the past year, and let's immediately key the data in:

Price (y)	Front foot (x)				
		GOLD	CLEAR-E		
$180,000	90'	180000	ENTER	90	E+
40,000	30'	40000	ENTER	30	E+
150,000	80'	150000	ENTER	80	E+
70,000	50'	70000	ENTER	50	E+
30,000	25'	30000	ENTER	25	E+
135,000	80'	135000	ENTER	80	E+

1) Average front foot size:	BLUE x	(59.17)
2) Average price per lot:	xy	($100.833.33)
3) Average price per front foot:	xy ÷	($1,704.23)
or directly:	BLUE x ÷	($1,704.23)
4) Likely price of a 90' lot:	90 BLUE y,r	($168,889.48)
5) Likely price of a 35' lot:	35 BLUE y,r	($47,492.03)

The data shows that larger lots are more expensive **PER FRONT FOOT** than the smaller ones. So simply using the average price per front foot would undervalue the larger lot ($1,704.23 x 90 = $153,380.28 instead of $168,889.48) and overvalue the smaller one ($1,704.23 x 35 = $59,647.89 instead of $47,492.03). The bias in the data is taken into account by the best fit line approach.

CORRELATION COEFFICIENT

The accuracy of the estimate is all the greater as the relationship between the two variables is more strictly linear. As we question for an **x** or **y** projection, the calculator also provides a measurement of the degree to which the data fits the linear relationship expressed by the best fit line. That measurement is called the correlation coefficient, or **'r'**. It is the **'r'** that we see as **x,r** and **y,r** on the two question keys that solve for **x** and **y**.

The correlation coefficient (r) is automatically stored in the **y** memory of the Stack when we solve for an **x** or **y** projection. It is retrieved by pressing **xy**. It is a value between 1 and -1. It is positive when **y** increases as **x** increases (positive slope), and negative when the slope is negative. The closer the coefficient is to 1 or to -1, the greater the correlation between the data and the best fit line.

In our very first example of linear projection, when the line was defined by two points only, then the correlation was perfect and the coefficient was 1. In the latest example it is 0.99 which shows that we still have excellent correlation.

EQUATION OF THE LINE

The equation of the line is of the model y = Ax + B

Clearly, if x = 0 then y = B. Solving for **y** with x = 0 calculates the value of **B**. **B** is the **y** intercept: the point where the line intercepts the **y** axis.

It is clear also that **y** increases by **A** when **x** increases by 1. **A** is the slope of the line. It is positive if the line slopes upwards, negative if the line slopes downwards. When the difference between two **x** values is one, the difference between the corresponding **y** values is A.

Keystrokes	Result
0 BLUE y,r	(B = -29,760.89. The y intercept)
STO 0	
1 BLUE y,r	
RCL 0 -	(A = 2,207.23. The slope)

The 2,207.23 value for the slope is a measure of the increase in the price of a lot per additional front foot. When compared to the $1,704.23 average price per front foot, it characterizes a situation where the larger the lot, the more valuable each front foot.

UNIT 32

PROGRAMMING

PROGRAMMING

TELLING THE CALCULATOR TO REMEMBER KEYS

When giving instructions to someone on how to prepare a gourmet dish, we may give an instruction, see it executed, and continue in this manner one direction at a time until the process is completed. Or we can say "Now listen and remember!", give all the instructions, and then say: "Now do it!". In this second case, we may even be tempted to say: "That was good. Do it again!".

The first situation is what we have been doing all along with the calculator: we press keys, and the calculator performs each instruction as we give it.

The second situation is what happens when we program the calculator: we first tell the calculator to remember a whole set of instructions, and then, whenever we want, one single instruction tells the calculator to perform these instructions, and to run them again and again, day after day or month after month, if we so choose.

A set of instructions remembered by the calculator is called a **PROGRAM.**

Keying in that set of instructions is called **PROGRAMMING.**

Of course, the calculator comes from the factory equipped with many permanent programs that cannot be erased or modified. We will not refer to them here when we speak of programs. We will reserve the word 'program' for user-written instructions remembered by the calculator.

The normal mode of operation which we have used so far, where the calculator executes each instruction as soon as it is given, is called the **RUN** mode, or calculator mode. We sometimes see the word **'running'** in the display as the calculator is executing an instruction. The mode of operation where the calculator remembers instead of performing is called the **PROGRAM** mode, or programming mode.

A program is keyed in in programming mode, but it is performed in the normal run mode.

THREE BASIC PROGRAMMING FUNCTIONS

GOLD P/R **PROGRAM/RUN** [f and key 31]

Switches the calculator from **RUN** mode to **PROGRAM** mode, or out of **PROGRAM** mode back into **RUN** mode.

To key in a program:
- We first switch into programming mode by pressing **GOLD P/R.**
- We key in the program.
- We switch back out of programming mode by pressing **GOLD P/R.**

While we are in programming mode, we have **PRGM** (Program) in small characters on the right side of the display. Instructions are remembered instead of being performed.

When we switch back out of programming (**GOLD P/R** again), we are back into the normal RUN mode, the calculator mode, where all instructions, **including the program that we have just recorded,** can be performed.

Observe the display as you Press **GOLD P/R** a number of times.

The very first stage in programming is
to remember how to switch in and out of programming
and to know when you are in and when you are out.

R/S **RUN/STOP** [Key 31]

Instruction to execute (run) a program or to stop a running program.

To perform or RUN a program, the program must have been keyed in and you must be OUT of programming mode. You must be in the normal RUN mode where functions are performed. If you then press **R/S** all the instructions recorded by the program are executed. Pressing **R/S** again while the program is running stops its execution.

Like the **ON** key that turns the calculator on but also turns it off, **GOLD P/R** and **R/S** are toggle keys.

GOLD CLEAR-PRGM **CLEAR-PROGRAM** [f and key 33]

Clears all user-written programs. The display then shows: **00-**

GOLD CLEAR-PRGM requires two keys only: the **GOLD** key (f), and the key just above the **BLUE** key. **DO NOT PRESS Clx.**

This clearing function operates **ONLY WHEN WE ARE IN PROGRAMMING MODE.**

A SIMPLE PROGRAM

You are a real estate broker and use the 5 regular cash flow functions of your calculator in a variety of ways. In particular, you are constantly coming back to 9.5% interest, 30 year loans that finance properties purchased with a 20% downpayment. This is the financing made available for homes on a development project for which you are exclusive agent. With these loans, you key in a selling price (let's take $136,000), and calculate as follows:

Keystrokes	Result
136000 ENTER	
20 % STO 0	($27,200 downpayment in memory 0)
- PV	($108,800 loan amount in PV)
9.5 BLUE i 30 BLUE n	(Rate and term keyed in)
0 FV PMT	(Monthly payment of $914.85)

You notice that, as the selling price changes from client to client, all the other keystrokes remain the same. Why not tell the calculator to remember them?

Keystrokes	Result
GOLD P/R	Switches into programming mode.
GOLD CLEAR-PRGM	Clears previous programs.
20 % STO 0 - PV 9.5 BLUE i 30 BLUE n 0 FV PMT	THE PROGRAM
GOLD P/R	Switches out of programming.

We may now key in a selling price and press **R/S**:

Selling price.		Monthly payment.
136000	R/S	$914.85
110000	R/S	$739.95
189000	R/S	$1,271.37
155000	R/S	$1,042.66

Each time, downpayment, loan amount, and payment can be recalled:

Keystrokes	Result
RCL 0	$31,000 downpayment.
RCL PV	$124,000 loan.
+	$155,000 selling price.
RCL PMT	$1,042.66 monthly payment.

WHAT MAY GO WRONG AS YOU KEY IN YOUR FIRST PROGRAM?

- The instructions, as given in the program frame above, assume that you are initially in the normal run mode. The initial **GOLD P/R** then switches you into program mode. Of course, if you were already in program mode, that instruction would switch you back into run mode and you would not be able to record the program.

- **GOLD CLEAR-PRGM** uses only two keys, leaving you with **00-** in the display. You do NOT press the **CLx** key at this stage.

- We will learn later how to check and correct a program. At this stage, if you press the wrong key while programming, just clear the whole program (GOLD CLEAR-PRGM) and start again.

REMARKS ON THE PROGRAM

- This program is an exact replica of the keystrokes that we used without a program. Some of the most useful programs are very simple, invented by the user on the spot to avoid having to repeat the same keystrokes over and over again. We will later discover specific functions used only in programming that allow us to devise more complex programs.

- **ENTER**, that separates the selling price from 20 in the initial keystrokes, is not needed here. The selling price remains outside the program and 20 is part of the program, so the calculator automatically assumes that they are different numbers.

IMPORTANT REMARKS THAT APPLY TO ALL PROGRAMS:

- As we key in a program, the program itself is sandwiched between two **GOLD P/R** instructions.

- We key in the program in **PROGRAM** mode. But we use the program, like any other question function, in the normal **RUN** mode of the calculator.

- Like the permanent instructions that are set in action by every question key, a user-written program is a set of instructions to reach for data and to submit that data to some form of mathematical processing. **R/S** is a normal question key, except that we have the power to define the question that it answers.

- Inside the program we have all the numbers and instructions that remain the same for all calculations--here, the downpayment ratio, the rate, and the term of the loan--, along with the mathematical processing.

- We leave outside the program all the numbers that change with each calculation (the variables),--here the selling price.

- When we switch out of programming, any number that was in the display before we switched into programming jumps back into the display. We may use that number or ignore it.

- When in programming mode, normal functions are remembered, not performed. So, when keying in a program, we cannot just clear a wrong number by pressing **CLx** as **Clx** would be remembered, not performed.

- Turning the calculator off does not clear the programs, nor can they be cleared while we are in the normal run mode. Pressing the major clearing function **GOLD CLEAR-REG**, or even **GOLD CLEAR-PRGM** if we are in the normal run mode does not clear programs. So once keyed in, a program can be used over and over for years.

- Programs are cleared when we use the MASTER CLEAR function (Minus ON), and may be lost if we take too much time to change the batteries. When programs are erased by such procedures, a **Pr Error** display warns us of the loss. Progams are also lost when we press **GOLD CLEAR-PRGM** while in programming mode, or if we overwrite an existing program with another program.

PROGRAMMING STEPS

As we key in a program, the first two digits in the display followed by a dash indicate the programming step that has been reached.

00- 01- 02- 03- . . 99-	Step 00, the initial "launching pad". Step 01, the first programming step. Step 02, the second step. Step 99. 99 steps: the most that can be keyed in.

A programming step is an instruction that can be executed: "789" is recorded as 3 steps, because something happens with each number we press. **RCL BLUE n** is recorded as only one programming step, because **RCL** and **BLUE** are only prefixes, and nothing happens until we have pressed **n.**

THE DISPLAY CODE

As we key in a program, the instruction that is recorded is projected in the display according to a simple code.

The digits that follow the programming step encode the function itself by describing it according to row and column as we have been doing to help locate new functions. Numerals from 0 to 9 are their own code.

	STEP	ROW	COLUMN	MEANING
06- 13	06	1	3	**PV**
05- 30	05	3	10	**Minus** (-).
11- 45 12	11	4 1	5 2	**RCL** **i**

KEY CODE BY ROW AND COLUMN.

11	12	13	14	15	16	**7**	**8**	**9**	10
21	22	23	24	25	26	**4**	**5**	**6**	20
31	32	33	34	35	36	**1**	**2**	**3**	30
ON	42	43	44	45		**0**	48	49	40

ORDERING A PROGRAM TO STOP OR PAUSE

The previous program calculates the downpayment and the loan amount, but does not show these amounts. It just stores them, and they can be recalled later. We have two ways of instructing the program itself to show these amounts: we may include in the program an instruction to pause, or an order to stop and wait for further instruction.

BLUE PSE Pause [g and key 31]

Instructs the calculator to pause for about a second.

R/S Run/Stop [Key 31]

When **R/S** is used as a programming step inside a program, execution of the program stops when that instruction is reached. The purpose is to display an answer or allow additional input to be keyed in. We resume execution by pressing **R/S**.

So **BLUE PSE** flashes the answer briefly, **R/S** interrupts the program and displays the answer for as long as we care to wait.

You may want to add two PAUSE instructions to the previous program:

```
GOLD  P/R
                  20  %   STO  0    BLUE  PSE
                  -   PV  BLUE  PSE
                  11.5 BLUE  i   30  BLUE  n
                  0  FV   PMT
GOLD  P/R
```

MODIFYING PROGRAMMING STEP SEQUENCE

A program executes orders in the sequence in which they are written: step 1, step 2, step 3, etc. That is, unless the instruction that must be obeyed is an order to jump to some other programming step. This is done through the use of a **BLUE GTO** function inside the program:

BLUE GTO 01 [g and key 33, then press ZERO and 1]

BLUE GTO must be followed by two digits, from 00 to 99 before it is recorded as a full programming step.

When the calculator reaches a GTO instruction it immediately jumps to the step specified in the instruction. For instance, **BLUE GTO 01** tells the calculator to run the whole program over again in an endless loop. **BLUE GTO 00** sets the program back to step 00, where it stops, ready to be used again.

SUMMARY

So far, we have seen the following program-related functions, and need to become familiar with them:

GOLD P/R		Switches IN and OUT of programming mode.
GOLD CLEAR-PRGM	PROGRAM mode:	Clears existing programs before writing a new one.
	RUN mode:	Does NOT clear programs. Sets program back to step 00. (See later).
R/S	PROGRAM mode:	Stops program execution.
	RUN mode:	Runs a program or stops a running program.
BLUE PSE	PROGRAM mode:	instruction to briefly show an intermediate answer.
BLUE GTO 01		(Or any 2 digits, from step 00 to 99)
	PROGRAM mode:	Records instruction to jump to step 01.
	RUN mode:	Sets program internally to step 01. (See later).

There is a lot more to learn about programming, but what we have already seen is sufficient for powerful and time-saving applications. Let us illustrate the latest functions before we expand to other features.

AMORTIZATION SCHEDULE PROGRAM

Here is a program that illustrates the use of the **PSE**, **R/S**, and **GTO** instructions. It has at its heart the routine procedure that would be used to obtain a monthly amortization schedule on a loan.

With basic loan data keyed in		
1 GOLD AMORT	xy	RCL PV

We then add two other short routines: a procedure at the beginning to keep track of the month for which data is presented, and a **GTO** instruction at the end to create a loop and continue the amortization process. We also add a **BLUE PSE** or a **R/S** instruction every time we need to see the number that is calculated.

	KEYSTROKES	DISPLAY		
Switch into programming:	GOLD P/R			
Clear previous programs:	GOLD CLEAR-PRGM	00-		
	1	01-		1
Counting routine:	n	02-		11
	RCL n	03-	45	11
	R/S	04-		31
	1	05-		1
	GOLD AMORT	06-	42	1
	R/S	07-		31
Heart of the program:	xy	08-		34
	R/S	09-		31
	RCL PV	10-	45	13
	R/S	11-		31
Creates loop:	BLUE GTO 03	12-	43,33	03
Switch out of program.:	GOLD P/R			

- Pressing 1 **GOLD AMORT** automatically increases **n** by 1. By instructing the calculator to start over at step 3 instead of step 1, we allow the calculator to recall from **n** a value that increases by 1 with each run.

- We illustrate on the right what the display shows as we key in the program. This is particularly useful with long programs as it can be used to check that the program has been keyed in correctly.

- We give here the version with **R/S**. You may want to test the program with **BLUE PSE** substituted for each **R/S** instruction within the program.

RUNNING THE PROGRAM

We may now key in the variables and run the program. For instance: write a monthly amortization schedule for a $45,000 loan at 9% interest with monthly payments of $400:

Keystrokes	Display	Description
45000 PV		
9 BLUE i		
400 CHS PMT		
R/S	1:	First month.
R/S	$337.50:	Interest portion for 1st month.
R/S	62.50:	Principal reduction, 1st month.
R/S	$44,937.50:	Balance, end of month 1.
R/S	2:	Second month.
R/S	$337.03:	Interest portion for 2nd month.
R/S	$62.97:	Principal reduction, 2nd month.
R/S	3:	Third month. Etc.

- This is not a linear program that runs its course, gives the answers, and sets itself back to step 00 ready to be used again. To use the program over we need to make sure that we start with the beginning of the program, not in the middle where we may have stopped. This may be done by pressing **BLUE GTO 00** while in NORMAL RUN MODE. We may now key in new data and amortize a new loan.

Any one of the following procedures, performed with the calculator in normal RUN mode, switches the program **INTERNALLY** back to step 00. The display is not affected except with **GOLD CLEAR-REG**.

BLUE GTO 00	An obvious procedure. Use in RUN mode only.
GOLD CLEAR-PRGM	Performed when in normal RUN mode, this procedure does not clear the program, but sets it back to step 00.
GOLD CLEAR-REG	Clears all the data memories, including the display, but not the program itself.
OFF / ON	The program is automatically set to Step 00 when the calculator is turned ON.
GOLD P/R GOLD P/R	The program is automatically set to Step 00 when the calculator is switched out of PROGRAMMING mode back into RUN mode.

ANNUALIZED YIELD PROGRAM

Converts annual rates into equivalent annualized yields.

REQUIRED INPUT: - In **y**: Annual rate compounded **x** times/year.
- In **x**: Number of compounding periods/year.

THE PROGRAM:

A 12 step program.
BLUE ENTER stands for for **Last x** function.

```
GOLD  P/R
GOLD  CLEAR-PRGM

÷  i
BLUE  ENTER
n
1  PV
0  PMT  FV
1  n    i

GOLD  P/R
```

RUNNING THE PROGRAM.

Find the annualized yield:

- 15% rate, monthly compounded:	15 ENTER 12 R/S	(16.08%)
- 13% rate, compounded daily:	13 ENTER 365 R/S	(13.88%)
- 10%, semi-annual compounding:	10 ENTER 2 R/S	(10.25%)
- 8%, continuous compounding:	8 ENTER 9 EEX 9 R/S	(8.33%)

(In this last example we use a periodicity of 9 followed by 9 zeros. This should be a more than adequate approximation for an infinite number of periods.)

- A loan of $40,000 is amortized over 10 years with monthly payments of $550.00. What is the annual rate of return (monthly compounding) and the annualized yield.

	40000 PV 550 CHS PMT 10 BLUE n
Annual rate:	0 FV i 12 x
Annualized yield:	12 R/S

So, when investing money for a given number of years, the following rates have the same effect: 10.96% compounded monthly.
11.52% compounded annually.

- You invest \$200.00 a month, beginning now, in a savings account that offers a 7% rate of interest compounded daily (365 times a year). What do you have in the account at the end of 5 years? (You have made 60 payments).

With program keyed in

Keystrokes	Comment
7 ENTER 365 R/S	(7.25%, annualized yield)
12 n i	(0.58%, monthly compounded rate)
200 CHS PMT	(**CHS** is optional)
5 BLUE n	
0 PV	(We have a PMT-FV exchange: 0 in PV)
BLUE BEG	(Switch to 'BEGIN' option)
FV	(\$14,409.72)
BLUE END	(Switch back to normal 'END' setting)

Here the program first calculates the annualized yield. The second line then calculates the monthly compounded rate (12 periods per year) that is equivalent to the annualized yield and to the daily compounded rate. We need to have that monthly compounded rate in **i** in order to handle the monthly payments.

We then key in the rest of the data (the 60 monthly payments of \$200.00). We switch to the 'BEGIN' option, and allow the calculator to calculate the Future Value.

0 PV clears the Present Value memory to allow the simple PMT-FV exchange required by the data. Note how inconvenient it would be at this stage to clear the financial memories with **GOLD CLEAR-FIN** as the correct rate would be cleared from the **i** memory.

Choosing the 'BEGIN' setting informs the calculator that the first of the 60 payments is made right now, not a month from now. Also, the last deposit is made a full month before the account is closed, not on the day it closes.

Immediately switching back to the 'END' setting is a good precaution for all who normally use that setting for their calculations.

- o 0 o -

The last two examples of the use of this program show how programs can be used in conjunction with the other functions of the calculator. A program, once keyed in, is no different then any other question function of the calculator. It is a question function where we have been allowed to define the question and which we asigned to the **R/S** key.

SQUARE FOOTAGE PROGRAM

This program allows measurements expressed in feet and inches to be keyed in as mock decimals: 12' 9" as 12.09, 3' 11" as 3.11.

THE PROGRAM

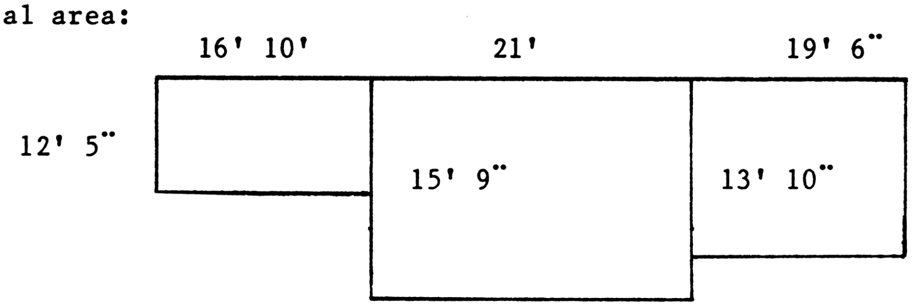
Keystrokes		Step	Code	
GOLD P/R	GOLD CLEAR-PRGM	00-		
	STO 0	01-	44	0
	BLUE FRAC	02-	43	24
	.	03-		48
	1	04-		1
	2	05-		2
	÷	06-		10
	RCL 0	07-	45	0
	BLUE INTGR	08-	43	25
	+	09-		40
GOLD P/R				

RUNNING THE PROGRAM

- Add the following measurements: 8' 11" + 12' 5" + 17' 9" + 6' 4"

Keystrokes	Display
8.11 R/S	(8.92)
12.05 R/S +	(12.42 / 21.33)
17.09 R/S +	(17.75 / 39.08)
6.04 R/S +	(6.33 / 45.42)
BLUE FRAC 12 x	(.42 ft = 5 in. for 42' 5" answer)

- Find the total area:



Keystrokes	Display
12.05 R/S 16.10 R/S x	(209.01 square feet)
15.09 R/S 21 R/S x +	(539.76 square feet)
13.10 R/S 19.06 R/S x +	(809.51 square feet)
BLUE FRAC 144 x	(Answer: 809 sq. ft and 74 sq. in.)

Note how decimal portion of final answer is transformed back into inches or square inches. Calculations may result in a slight rounding and in a final answer that does not provide an exact number of square inches.

PERCENTAGE OF TOTAL: A PROGRAM FOR TEACHERS

This two step program saves 1 step when calculating the number of correct answers on a test as a percentage of the total number of questions, knowing the total number of questions on the test and the number of mistakes a student makes.

THE PROGRAM

```
GOLD  P/R
GOLD  CLEAR-PRGM

Δ%
PV

GOLD  P/R
```

- A test has 77 questions. Calculate the grade as a percentage of the number of questions for the following students:

Number of wrong answers	Keystrokes	Grade (as %)
	77 ENTER	
10	10 R/S	87.01%
31	31 R/S	59.74%
14	14 R/S	81.82%
6	6 R/S	92.21%

- Now move on to another test with 115 questions:

	115 ENTER	
7	7 R/S	93.91%
22	22 R/S	80.87%
17	17 R/S	85.22%

The purpose of PV in the program is to leave in the display a number that disappears from the Stack when the next number is keyed in.

To use this program without clearing a long program already in the calculator, omit **GOLD CLEAR-PRGM** and press **BLUE GTO 00** after **PV**. To use the long program again, you will only need to key back in the 1st 3 steps. (See Unit 32, page 21 for details).

PROGRAMS OF CONVENIENCE / CANNED PROGRAMS

So far we have keyed in convenient little programs that can save us a few steps on calculations we could still manage to perform without a program. It is relatively easy to understand what each step of the program accomplishes and with a little effort and experience, we might write a similar program on the spur of the moment.

There are other programs that implement a considerably greater degree of complexity. They are generally programs that some person with experience in programming has taken days to write and to perfect. And yet, we can key the program in and be ready to use it over and over again in just a few minutes.

Using such programs is like buying a "canned" program for a computer: word processing, spread sheet, data management, or galactic game. We buy the floppy disk, load the program, and learn how to use it, but we would never dream of writing it. Sometimes such a program may be worth keying in even if we use it only once, as we have no other convenient way of soving the problem.

Such programs are available to meet a considerable number of situations. You will find some in the Hewlett-Packard applications books and from various other sources. **Edric Cane Seminars** intends to make some available as circumstances warrant.

These programs are good only in so far as they meet your particular requirements. Some, that require a considerable amount of input and provide answers to a variety of questions use an overlay to help remind the user where data must be keyed in and from what memories answers can be retrieved. That overlay in no way prevents us from using the regular functions in the usual way: it is simply of assistance when the program itself is being used. With others, the data is such that a little bit of practice allows the frequent user to remember order and positions.

- o 0 o -

The user may now want to try his hand at writing his own programs or at keying in and using complex "canned" programs such as those offered in **UNIT 33**. However, both of these activities will be greatly helped by an understanding of the functions and procedures discussed in the rest of this Unit.

EXPLORING A PROGRAM WHILE IN PROGRAM MODE

When in **PROGRAM MODE**, we have three functions that allow us to move around within a program, explore it, and if need be, change some lines.

SST SINGLE STEP. (Key 32)

This function allows us to move forward one step at a time.

BLUE BST BACK STEP. (g and key 32)

This function allows us to move backwards one step at a time.

To go back two steps we press **BLUE BST BLUE BST**, making sure not to omit the **BLUE** key for each instruction.

BLUE GTO . 00 GO TO step 00 (or any other step from 00 to 99).

To travel greater distances within a program we may press **BLUE GTO** followed by a **DECIMAL POINT**, and the **NUMBER OF THE STEP** we want to land on. The decimal point before the instruction is finalized is important as otherwise the GTO instruction is recorded as a program instruction instead of being interpreted as an order to go there without affecting the program.

When in programming mode, we may go back to step 00- by pressing **BLUE GTO.00** (watch for that decimal point!), or simply by switching out of programming and back in again.

We may not switch to a programming step that has not yet been created or at least claimed for programming. With a 10 step program in the calculator we have only 15 programming memories allocated to programming: we get an error sign if we press **BLUE GTO.23**.

CORRECTING A PROGRAM STEP.

To correct a program step, we position ourselves on the step immediately preceeding and, while in program mode, overwrite the step that follows by just keying in the correct instruction.

BLUE BST is particuliarly useful for this purpose when we record the wrong instruction while keying in a program:

For instance, in keying in a program we realize that we have just pressed **FV** for step 29 instead of the required **PV** instruction. We have now recorded the wrong step 29 instruction which we can see encoded in the display: **29- 15**. We may go back to step 28 by pressing **BLUE BST** and correct step 29 by just keying in the appropriate **PV** function. We have overwritten the wrong instruction with the correct one.

If, while keying in a program, we press the wrong prefix key (BLUE, GOLD, STO, RCL, RCL BLUE, BLUE GTO) which does not record as a program step until the instruction is completed, the easiest is to complete a wrong instruction that registers in the display, and then to overwrite it as above.

If one step has been omitted in the middle of a program, then all the steps that follow are keyed in in the wrong position. We may position ourselves on the last fully correct instruction and key back in all the steps that follow.

If on the other hand we realize that we have keyed in one step twice, or simply have one step that shouldn't be there, we may eliminate it by replacing it with an instruction to go to the next step. For instance:

1	STO 1
2	STO 2
3	xy
4	xy
5	1

In keying in a program we pressed the **xy** key twice by mistake. One of these steps needs to be deleted. This can be done by overwriting step 4 with the instruction **BLUE GTO 05**.

Of course, all subsequent steps are delayed by one step, and therefore all **BLUE GTO** instructions that refer back to a later step must be adjusted accordingly.

CHECKING A COMPLETE PROGRAM

This is done by switching into PROGRAM MODE and pressing **SST** over and over.

Let us imagine that we have just keyed in a program and have switched back into the normal run mode. We may:

- Make sure that the program is set internally to step 00. Switching the calculator OFF and ON is one safe way to do this.

- Switch back into programming mode: **GOLD P/R.**

- Press **SST** over and over again as we read the program line codes in the display and check them against those provided with the program. If the display code is not provided, then we can interprete the code to check that it indeed corresponds to the desired programming instruction.

With long programs, the procedure is best done with one person reading the display and a second checking the display code provided with the program.

If we find a program code in the display that is not as it should be, we should attempt to understand what went wrong. Did we skip a step, key one in twice, key the wrong step in because we misinterpreted the instruction? If we just have the wrong step, we need to go back to the previous step (**BLUE BST**) and overwrite with the correct instruction. We may then continue to check that no other mistake has been made. In other circumstances, we may need to position ourselves on the last correct program step and key back in all subsequent programming instructions.

AFTER THE LAST STEP

As we display the last step of the program, pressing **SST** one more time should display one of two things. Most likely, we will see the next step (for instance, step 57 for a 56 step program) with the following code:

	56- xx	Last program instruction.
SST	57- 43,33 00	This stands for **BLUE GTO 00**

Any programming memory that is available but has not been used is automatically encoded with the instruction to go back to step 00. That instruction is then overwritten by the actual step that we key in.

If the program ends on step 08, 15, 22, or another such number reached by adding 1 to multiples of 7, then pressing **SST** will show step **00-**. This is because there are no available memories left to be explored, and we have reached an automatic instruction to go back to step 00 that always ends an available programming sequence.

If instead we find an entirely different instruction, it is probably left over from a previous program that should have been cleared at the outset. If we attempt to clear that old program now, we will also clear all the correct instructions that have been recorded. An alternative is to Blue-backstep back to the last instruction of our program and to key in **BLUE GTO 00**. This creates a buffer between the new program and the remains of the old one.

It makes sense to think of every program as ending with a **BLUE GTO** instruction. Either it is the specific **BLUE GTO xx** instruction that we keyed in as the last step of our program in order to create a loop. Or, with linear programs, it is the **BLUE GTO 00** instruction provided by the calculator itself. With such linear programs, after executing the last step of a program the calculator explores the next instruction, which directs it back to step 00 where it stops. The program can now be used again.

An understanding of that feature provides us with an alternative to clearing programs at the outset.

ENDING A PROGRAM WITH 'BLUE GTO 00' AS AN ALTERNATIVE TO CLEARING PREVIOUS PROGRAMS

So far we have cleared previous programs before keying in a new program. We have done so by pressing the **GOLD** key and the **CLEAR-PRGM** key while in Programming mode. There are alternative procedures.

We are on step 00 ready to key in a new program. Why do we need to clear previous programs?

As we key in the program we overwrite steps of the previous program. If we have not cleared previous programs when the last step of our new program is reached the calculator continues with what is left of the old program and gives a meaningless answer.

Of course, that problem does not occur, and there is no need to clear, if the new program is longer than the steps previously keyed into the calculator, as in this case all of the previous steps are overwritten with new instructions.

There is also no reason to clear previous programs (other than to redeem Register memories) if the new program ends with a **GTO** instruction, as this instruction refers back to a step of the new program and we never reach whatever is left of the old programs.

When we have a linear program that is shorter than steps already keyed in, then we need to clear previous programs. Except that, if we forget, or choose not to do so, we may instead key in a **BLUE GTO 00** instruction at the end of the new program. This creates a perfectly adequate buffer between the new and what is left of the old.

There is however a penalty for keying in **BLUE GTO 00** at the end of a program instead of clearing other programs at the beginning: we use one more programming step, and once in every 7 programs this results in one more Register memory taken over by programming steps.

We may use this approach in the following circumstances:

- When we forget to clear and do not want to key back in the new program from scratch. Positioning ourselves on the last step of the new program and pressing **BLUE GTO 00** solves our problem.

- When we have a long program in the calculator that we would like to retain but we would also like to use a short program for a while. We may key in the short program with the **BLUE GTO 00** ending. When we are over with it, we just need to key back in the few initial programming steps of the first program that were overwritten by the new material. The rest of the long program is still there ready to be used.

- Understanding the procedure is useful in dealing with multiple programs.

EXPLORING A PROGRAM IN NORMAL "RUN" MODE

THE DEBUGGING PROCESS

SST SINGLE STEP

When in normal RUN mode, with the calculator internally on step 00 and all the variables required to run a program keyed in, pressing **SST** over and over displays each step and performs that step, thus running the program one step at a time. This allows us to see where a carefully written program doesn't work. "Debugging" a program is part and parcel of writing programs, and a constant and indispensable procedure used even by the best programmers. **SST** offers a convenient way of doing this. A few words of caution in this respect:

- **SST** used in normal RUN mode performs each instruction, including all **GTO** instruction. Complex programs may have internal loops that allow one part of the program to be run over and over again before the program runs its course and gives the final answer. **SST** used in the normal RUN mode makes the program go through all the loops. We may need to press **SST** a thousand times before the whole program enfolds. We should use **SST** in PROGRAMMING mode if we just want to check the steps encoded, not what they actually perform.

- We should allow every calculation to be fully performed before pressing **SST** again.

- A **R/S** instruction within the program is obeyed. We need to press **SST** a second time before we reach the next programming step.

- In one respect, running the program step by step in this manner differs from running it with one single push of **R/S**. This is one of the few obvious and very inconvenient mistakes made by the designers of the calculator. When we run a program in the usual way, a number keyed into the display does not blend in with a digit found inside the program. So there is generally no need to begin a program with **ENTER** as the calculator knows that digits inside the program and outside the program cannot be part of the same number. The calculator forgets that feature with the **SST** function. If a program requires a variable in the display and begins with a number, we should press **xy** twice when keying in the variable before we debug the program with the **SST** procedure. This transforms the variable in the display into a complete number without distorting other numbers in the Stack. In most circumstances though, just pressing **ENTER** before running the program should do just as well.

BLUE GTO **GO TO**

When in normal RUN mode, pressing **BLUE GTO** and an available programming step, such as 08 or 23, sets the calculator internally on that step. Nothing changes in the display itself, but if we switch into programming (**GOLD P/R**), we find ourselves on that step. This may be used as follows:

- To switch back to step 00 (**BLUE GTO 00**) before running a program. The calculator switches back to step 00 automatically under many circumstances, but needs to be set back by the user in some cases.

- With multiple programs, we switch the calculator to a program that begins on step 17, for instance, by pressing **BLUE GTO 17**. We may then still use the calculator's other functions and key in the variables. When we are ready to run the program beginning on step 17, we press **R/S**.

- To check a specific programming line--for instance step 23--, we may press **BLUE GTO 23** and then **SST**. As long as we hold down **SST**, we may see step 23 encoded in the display. (When we release **SST** that step is performed).

- If we want to be on a specific step when we switch back into programming mode, we may key that step in (**BLUE GTO 23** for instance) before switching back into programming. When we switch into programming mode, we find ourselves on that step and may, for instance, overwrite the next step with a new instruction.

BLUE BST **BACK STEP**

Used in normal RUN mode, this function displays programming steps in reverse order. Of course, it does not execute those steps.

GOLD CLEAR-PRGM

Used in normal RUN mode, this function does not clear the program but sets it back internally to step 00.

MEMORY ALLOCATION

BLUE MEM

Whether in PROGRAM or in RUN mode, **BLUE MEM** informs us of the allocation of memories between program and data registers.

With no program in the calculator, pressing **BLUE** and holding down key 9 (**MEM**, for **MEMORY**) displays the following:

P - 08	r - 20

This shows that there are 8 memories available for programming (8 program steps), and 20 Register memories available for regular storage or irregular cash flow data (memories 0 to 19).

If we key in a 9 step program and press **BLUE MEM**, we see the following:

P - 15	r - 19

This shows that we now have 15 memories available for programming steps, and only 19 Register memories left for other purposes (memories 0 to 18). Register memory 19 has been treansformed into 7 programming memories.

As we write longer programs, as soon as available programming memories are used up, one more Register memory is transformed into 7 new programming memories until we have reached the maximum of 99 programming memories. At that stage, pressing **BLUE MEM** shows the following:

P - 99	r - 7

We now have only Register memories 0 to 6 available for irregular cash flow data and other uses. If we try to store a number in Register 7, we get an **ERROR 6** sign: there no longer is a Register memory 7. When we clear the programs, the full 20 Register memories.

For irregular cash flow calculations, we would still be able to key in **CFo** in Register memory 0, and 7 **CFj** values, as the 7th **CFj** value would automatically be stored in the **FV** memory. We would get an **ERROR 6** sign as we tried to key in the 8th CFj value.

Register memories 0 to 6 can never be taken over by programming.

BRANCHING

Two programming functions allow a program to go into two different directions according to whether or not a given condition is met. These two functions are essential to most complex programs.

BLUE x=0

Here we test whether or not **x** is equal to zero.

If the condition is met and **x** is equal to zero, the program continues with the next programming step as in normal conditions. However, if **x** is not equal to zero, then the program skips two steps ahead instead of one.

For instance, with **BLUE x=0** on program line 17:

```
17-   BLUE x=0            If 'Yes'      If 'No'
18-   BLUE  GTO 00 <---- (If 'Yes')
19-   BLUE  GTO 05 <---- (If 'No')
```

If **x** is not equal to zero, the program jumps directly from line 17 to line 19 where it is instructed to resume the loop on line 5.

When finally **x** is equal to zero, then the programs goes from line 17 to line 18 where it is instructed to jump back to line 00: the program is over. It could also be instructed to jump to another programming step where a different calculation could begin.

Such a programming routine normally includes a counting routine that modifies the value of **x** with each run of the loop. For instance, before the branching routine we might have:

```
14-   1
15-   STO - 0
16-   RCL 0
```

With 10 in memory zero, this allows 10 loops before the content of Register 0, and therefore the value of **x** after **RCL 0**, is brought back down to zero at which time the program automatically stops.

BLUE x≤y   **x** inferior or equal to **y**.

This branching instruction compares the **x** and **y** memories of the Stack and skips a step if **x** is larger than **y**.

MULTIPLE PROGRAMS

It is a pity to select a few programs, key them in, and then use only these at the exclusion of others. Programs are not difficult to key in, and if we erase one to key another one in we may always key the first one back in in a minute or two. But there are circumstances where a few programs are constantly needed and might as well be permenantly recorded.

To key in multiple programs we should think of programs as having a **BEGINNING**, a **MIDDLE**, and an **END**, and each of these parts needs to be adjusted when a program is keyed in in second or third position. Programs normally begin with a launching pad and always end with a **BLUE GTO** instruction. In the middle, we may have other GTO instructions to adjust.

THE BEGINNING: A launching pad.

With a first program, the launching pad is step 00 where the calculator rests ready to run the program when we press **R/S**, and where it returns after it has been run, ready to run again.

With a second or third program, the launching pad is a **R/S** instruction where the calculator can be returned after the program is run. This makes it possible to select the program once and then run it over and over without having to select it at each run. We may dispense with this launching pad when the program terminates with an endless loop instruction, as the program in any case needs to be reset to the beginning before we use it again.

THE END: A **GTO** instruction.

With a first program, the final **GTO** instruction is either the one embodied in a loop instruction which refers back to some step within the program, or an automatic **BLUE GTO 00** instruction that sets the program back to the launching pad where it stops.

With a second or third program, the final **GTO** instructions must perform the same functions. If it is meant to create a loop, it must refer back to the correct programming step within the program. If it is meant to interrupt the program, it should refer back to the **R/S** instruction recorded as the launching pad for that program.

THE MIDDLE: Adjust any **GTO** instructions.

All **GTO** instructions within a program must be adjusted to refer back to the correct step when a program written as a first program is keyed in as a later program.

Let's consider the following programs, written as if they were first programs in a first column, written as one programming sequence in the second column.

The first program calculates a 6.5% sales tax. The second is our square footage program (Page 14). The third counts for us in case we should forget.

SEPARATE PROGRAMS

```
00-
01-   6
02-   .
03-   5
04-   %
05-   BLUE  GTO 00
```

```
00-
01-   STO 0
02-   BLUE  FRAC
03-   .
04-   1
05-   2
06-   ÷
07-   RCL 0
08-   BLUE  INTGR
09-   +
10-   BLUE  GTO 00
```

```
00-
01-   1
02-   +
03-   BLUE  PSE
04-   BLUE  GTO 01
```

PROGRAM SEQUENCE

```
00-
01-   6
02-   .
03-   5
04-   %
05-   BLUE  GTO 00

06-   R/S
07-   STO 0
08-   BLUE  FRAC
09-   .
10-   1
11-   2
12-   ÷
13-   RCL  0
14-   BLUE  INTGR
15-   +
16-   BLUE  GTO 06

17-   1
18-   +
19-   BLUE  PSE
20-   BLUE  GTO 17
```

In writing down the separate programs we have included the GTO instruction that, whether we know it or not, always exists at the conclusion of a program. This GTO instruction is adjusted in the sequential version to refer back to the corresponding line: the launching pad (lines 00 and 06) for programs 1 and 2; the first step of the program for the third program that ends with a looping instruction.

We should adjust similarly any other GTO instruction within a program.

Note that all programs end with a GTO instruction that serves, among other possible uses, as a buffer with subsequent programs.

RUNNING MULTIPLE PROGRAMS

To run the first program, press:	**BLUE GTO 00**	**R/S**
To run the second program, press:	**BLUE GTO 07**	**R/S**
To run the second program, press:	**BLUE GTO 17**	**R/S**

If no program has been used since the calculator was last turned ON, the calculator is already set on programming step 00, and the first program may be run by just pressing **R/S**.

Of course, the variables required by each program must also be keyed in. This can be done before or after we have set the program internally to the first active step of the program.

Once either one of the two linear programs has been selected, it can be run over and over again by just pressing **R/S**. Being an endless loop, the third program needs to be reset to its beginning if we want to use it again with different variables. This is true of endless loop programs even if they are keyed in as a first and only program.

PROGRAMS 1, 2 & 3 ON STEPS 1, 2 & 3

We may also consider the following option which is probably an overkill.

Let's push each program 3 steps further down so that the first begins on step 4, the second on step 10, and the third on step 20, making all necessary adjustments with the various **GTO** instructions. As the first three steps of the sequence, let's write the following:

01-	BLUE GTO 04
02-	BLUE GTO 10
03-	BLUE GTO 20

WITH THIS INITIAL ROUTINE:

To run the first program, press:	**BLUE GTO 00**	**R/S**
or:	**BLUE GTO 01**	**R/S**
To run the second program, press:	**BLUE GTO 02**	**R/S**
To run the second program, press:	**BLUE GTO 03**	**R/S**

KEEPING PROGRAMS SHORT

There is a penalty for having long programs or multiple programs in the calculator. We have fewer Register memories available for other purposes, in particular for irregular cash flow data.

WRITING A PROGRAM

Except for the very simplest of programs, writing a program means just that: writing the steps on paper in order to devise and implement a strategy that will provide the correct answers and make the program easy to use (user friendly).

We will usually follow the following stages:

1) SOLVE THE PROBLEM WITHOUT A PROGRAM. Except for exceptional iterative procedures, we clearly will not be able to write a program unless we can solve the problem itself without programming. In fact, the steps followed in solving the problem without a program are likely to offer us the best outline of the program itself, which can then be improved and modified for our greatest convenience. So writing down the keystrokes used to solve the problem without a program provides a good starting point.

2) DEFINE THE PROBLEM. What data do I need in order to solve the problem? What answers do I want to obtain at the end of the process? Though it seems that this step should come before we solve the problem, in practice it may well come as an analytical sequence to our solving the problem the way we always have, without a program.

3) IDENTIFY THE VARIABLES AND THE CONSTANTS. Of the data needed to solve the problem, some numbers that remain constant should be included within the program, others that vary from one example to another, should remain outside the program itself. The choice of which numbers should be variables and which constants depends on the purpose of the program and is often a matter of strategy (see next stage).

4) DEVISE A STRATEGY to implement the purpose. This is the creative part of the process. We need to decide the most convenient (user friendly) way of communicating the variables to the program, and how we want the program to provide the answers. We then need to devise the calculations that will lead from the variables to the answers. Flow charts of the decision-making and computational stages will help in complex cases, and choices and compromises and a certain amount of trial and error are generally required.

5) TEST AND DEBUG THE PROGRAM. Now we turn to the calculator and key in our tentative program. If it works, congratulations. More often than not, something did not take place the way we expected it to: we have to identify the problem and once again devise a strategy to correct it.

CHOICES

Among the choices that have to be made in the process of writing a program are the following:

- Simplicity or complexity. I think we should accept the limitations of the HP-12C in terms of programming ability. I sometimes see programs that indeed provide the correct answer, but require such a variety of input, and such complexity in the presentation of these variables that it seems easier to perform all the calculations oneself or to limit the program to some essential parts of the total calculations. With no alphnumeric capability (words that can prompt you on what the next step or input should be), a problem that requires a considerable amount of input and only small arithmetic calculations in between is more often than not best solved without a program.

- Comprehensiveness versus length of the program. It is always tempting to ask a program to perform a few more tasks. But this must be weighed against the advantages of keeping a program short and simple. A shorter program is easier to key in, takes less time to run, and uses fewer Register memories.

- Convenience versus length of the program. The easiest way to key in the variables and present the answers may require more steps and a longer program than a less user-friendly procedure. Again, choices must be made.

- Time or space. Variables can be communicated to the program by keying them into specific memories, or by keying them at various times during the running of the program, when the program is interrupted for that purpose by a **R/S** instruction included in the program.

  The same is true for the answers that can be flashed during the running of the program or stored in chosen memories. I tend to prefer space over time for the variables, but the choice depends on the nature of the program.

- Essentially, the programs we write reflect an attitude. Simplicity of use and brevity, though sometimes in conflict, are both qualities in a program that are worth some trade-offs, some sacrifices in terms of what the program will do for us.

HOW TO ACHIEVE BREVITY IN PROGRAMS

There is a penalty in having long programs: we use up more data register memories. Some techniques can be used to shorten programs.

MATHEMATICAL SHORT CUTS

There are often more than one way of proceeding with a calculation. The most obvious mathematically may not be the shortest in terms of programming steps. For instance:

To divide by 25:	Take 4%.	
To calculate 25%:	Divide by 4.	
To calculate 20%:	Divide by 5.	
6% of 45000:	45000% of 6	(Order may be irrelevant).
0.125:	8 1/x	(Reciprocal saves 2 steps).
100,000:	EEX 5	(Scientific notation).
100:	EEX 2	(Saves 1 step).
Multiply by 12:	BLUE n	(Banned in Run mode, this is
Divide by 12:	BLUE i	justified in Program mode if
		there are no contradictions).

STRATEGIC AND PROCEDURAL SHORT CUTS

Keying in numbers is costly in terms of programming steps. A factor that is used more than once should be stored the first time around and recalled for later uses. It might even be possible to leave that number outside the program, as a variable.

Careful management of the Stack, of **BLUE LSTx**, of storage register arithmetic may save precious steps.

Within a program, memories and functions can be used creatively. Storing in a financial memory instead of a Register memory allows the number to automatically disappear from the Stack when a new number is keyed in. **E+** may be used, not for its statistical values, but just to bring up 1, 2, 3, etc. in the display and in Register 1.

A procedure that interrupts a calculation after a loan has been fully amortized is possible but costly. Why not just stop looking when we are no longer interested in the answers? The greatest savings are made by carefully defining what really needs to be included and what can, at minimum cost, be left out of a program.

With Register memories the most precious commodities, it is important to realize that programming claims one more Register memory only once every seven steps. If we have 6 programming steps available for programming before a new Register memory is claimed, let's splurge. Or rather, let's see if there is no way of saving 1 program step and thus redeem a full Register memory.

UNIT 33

READY-MADE PROGRAMS

READY-MADE PROGRAMS

An **AMORTIZATION SCHEDULE PRGRAM** and other shorter programs are included in UNIT 32.

The programs offered here meet a dual purpose.

They are directly related to financing of one kind or another and readers of this course may find one or more that meets their needs.

They are used as a tool to show what programs can do and how to use long programs.

USING READY-MADE PROGRAMS

Ready-made or 'canned' programs can be used to solve difficult problems, even if we need to do so only once. They implement mathematical calculations that most would not care to duplicate without a program. We may compare such programs to the software than can be purchased on discs for personal computers. No one would think of writing his own wordprocessing or spread sheet program. And yet, we may buy the disc, load the program into the computer, and use it. Such programs are available for the HP-12C from various sources, including Hewlet-Packard application handbooks and Edric Cane Seminars.

Such programs must be keyed in and tested before we use them. With a little experience, this can be done in a couple of minutes. It should never be done in front of a client, as it is easy to make mistakes and hunting for a mistake is a task better done without spectators.

To simplify the search for a mistake, these programs are generally presented 1 step per line, with a complete illustration of what appears in the display for each programming step. We will do so here.

To use a ready-made program, you do not need to understand the meaning of the keys pressed when keying in the program or the mathematical processing that the program implements. You just need to:

1) **KEY IN THE PROGRAM.**

2) **TEST THE PROGRAM TO CHECK ITS ACCURACY.**

These two steps are performed once and for all and need not be repeated as long as the program is retained in the calculator--years, if need be.

3) **USE THE PROGRAM.**

Like any other functions of the calculator, using the program implies:

- Providing the **DATA** which varies with each application. (The variables).

- Asking the **QUESTION:** here the **R/S** key.

Keying in the program transforms the **R/S** key into a question key like any other question key of the calculator, except that **WE** have defined the problem that it answers.

Though we do not need to know the mathematics of a program, we need to understand what the program does for us, what variables it requires, and how these variables should be communicated to the calculator.

We key in the program in **PROGRAMMING** mode.
We use the program in the normal **RUN** mode.

TESTING A PROGRAM

After a program has been keyed in, it must be tested by comparing the results it provides to results obtained through other means. So, along with the text of the program itself, you should retain sample answers that test the various parts of the program.

IF YOU DO NOT GET THE CORRECT ANSWERS consider these possibilities:

- The program may not be set internally to its beginning.
- You may not communicate the proper variables correctly.
- The correct program may lead into another program that should have been cleared at the outset.
- You may have an incorrect program in the calculator.

SETTING THE PROGRAM BACK TO ITS BEGINNING.

When using a program, we need to begin with the first step. The calculator in normal RUN mode is set on step 00, ready to run the first program, after we do any one of the following:

- Turn the calculator **ON**.
- Clear the data memories with **GOLD CLEAR-REG.**
- Press **BLUE GTO 00**
- Press **BLUE CLEAR-PRGM** (RUN mode only, please!)
- Satisfactorily run a linear program. (One that ends with an automatic **BLUE GTO 00** instruction).

COMMUNICATING THE PROPER VARIABLES CORRECTLY.

- You must have the PROPER VARIABLES in the PROPER MEMORY.
- You should have NO LETTER INDICATOR in the display unless specifically selected for the circumstance.
- Some programs require that you clear all or part of the calculator before you key in your variables, other programs take care of whatever clearing needs to be performed.

Press **GOLD CLEAR-REG** to clear all data memories and to set the program back to step 00. Key back in the required data and run the program.

PROGRAM LEADS INTO ANOTHER PROGRAM

If a program gives an incorrect answer because previous programs have not been cleared, there is no need to clear and start again. Position the calculator on the last correct step of the program, and, in programming mode, key in **BLUE GTO 00.** Switch back into RUN mode and test the program again (See UNIT 32, pages 17 and following).

INCORRECT PROGRAM

If the program was keyed in incorrectly, we may check and correct the program as indicated in Unit 32, pages 18 & 19. Or we may simply key the program back in without trying to identify where the error was made.

QUALIFYING A BUYER
FOR A REAL ESTATE LOAN

A PROGRAM

OBJECT: To allow real estate brokers and loan officers to calculate in seconds the selling price of a property for which a buyer qualifies using accepted ratios (See details).

REQUIRED INPUT:
- TERM and RATE on the loan.
- DOWNPAYMENT RATIO: 10% or 20% option.
- Monthly GROSS INCOME.
- Monthly DEBT PAYMENTS.

USING THE PROGRAM:

EXAMPLE: A buyer is considering an **11%, 30** year loan.
He expects to make a **20%** downpayment.
His gross monthly income is **$4,000**
He has **$300.00** in monthly liabilities.
What price property does he qualify for?

WITH THE PROGRAM PREVIOUSLY KEYED IN (See page 6) and the calculator in the usual **RUN** mode (NOT programming mode):

With program keyed in	
11 BLUE i	
30 BLUE n	
20 R/S	
4000 R/S	
300 R/S	$137,719.48 selling price.
	ADDITIONAL INFORMATION
RCL PV	$110,175.58: loan amount.
RCL PMT	$1,049.23 monthly payment.
RCL 1	1,220 The smallest of the 2 is
RCL 2	1,320 PITI (+ PMI if 90% loan)
-	-100 To qualify for a larger loan, reduce liability by $100.00. Reducing liability by more than $100 will not improve ratios.
RCL 3	$27,543.90: downpayment.
RCL 5	20%: downpayment ratio.
RCL 0	$137,719.48 selling price again.

ASSUMPTIONS MADE BY THE PROGRAM

The program makes allowance for the following ratios:

(PITI = Principal, Interest, Tax, Insurance on home)
(PMI = Private Mortgage Insurance, with 10% downpayment only)

MAXIMUM EXPENSE RATIOS ALLOWED

With 20% downpayment:
33% of gross income for PITI,
38% of gross income for PITI + debt payments.

With 10% downpayment:
28% of gross income for PITI + PMI,
36% of gross income for PITI + PMI + debt payments.

These ratios can be changed by just keying in a different number where they appear in the program (lines marked "A").

TAX, INSURANCE, AND PMI RATIOS INCLUDED IN THE CALCULATION:

Tax: 1.25% of selling price per year.
Insurance (as in PITI) .3% of loan per year.
PMI (on 90% loan only): .03% of loan per month.

These numbers are implemented in the program through two factors (lines marked "B"), and can be changed by modifying those factors.

The assumptions and ratios implemented in the program are typical but they may change with lenders, with time, and with geographic area. They should be adjusted within the program to meet current requirements.

The ratios implemented by the program, even when current, are only guidelines: most lenders consider other factors, such as credit record, stability of employment, and existence of savings, to adjust the maximum loan amount and resulting selling price one way or the other. The answers given by the program must be interpreted accordingly.

Qualifying a buyer. (Continued)

KEYING IN THE PROGRAM

Turn the calculator **ON**, or **OFF** and **ON**. Then:

KEY IN	DISPLAY SHOWS		COMMENTS
GOLD P/R	00-		Switches into programming mode. You have a small **PRGM** indicator in the display.
GOLD CLEAR-PRGM	00-		(**f** and key above the **g** key) Clears any other programs.
STO 5	01-	44 5	
2	02-	2	
0	03-	0	
BLUE x≤y	04-	43 34	(**g**, then **xy**: key 34)
BLUE GTO 24	05-	43,33 24	
R/S	06-	31	Begins 10% downpayment section.
2	07-	2	} A Change 28% ratio here.
8	08-	8	
%	09-	25	
STO 2	10-	44 2	
xy	11-	34	
3	12-	3	} A Change 36% ratio here.
6	13-	6	
%	14-	25	
STO 1	15-	44 1	
.	16-	48	(Decimal point)
1	17-	1	} B Change .154 ratio here
5	18-	5	
4	19-	4	
STO 4	20-	44 4	
9	21-	9	
0	22-	0	
BLUE GTO 41	23-	43,33 41	
R/S	24-	31	Begins 20% dowmpayment section.
3	25-	3	} A Change 33% ratio here.
3	26-	3	
%	27-	25	
STO 2	28-	44 2	
xy	29-	34	
3	30-	3	} A Change 38% ratio here.
8	31-	8	
%	32-	25	
STO 1	33-	44 1	

Keystrokes	Line	Code	Comments
.	34-	48	(Decimal point)
1	35-	1	B Change .124 ratio here.
2	36-	2	
4	37-	4	
STO 4	38-	44 4	
8	39-	8	
0	40-	0	
CHS	41-	16	Begins common section.
PV	42-	13	
R/S	43-	31	
STO - 1	44-	44 30 1	(STO, minus key, 1)
RCL 1	45-	45 1	
RCL 2	46-	45 2	
BLUE x≤y	47-	43 34	
BLUE GTO 50	48-	43,33 50	
xy	49-	34	
0	50-	0	
FV	51-	15	
PMT	52-	14	
RCL 4	53-	45 4	
+	54-	40	
xy	55-	34	
%T	56-	23	Program may terminate here.
STO 0	57-	44 0	
RCL 5	58-	45 5	
%	59-	25	
STO 3	60-	44 3	
-	61-	30	
PV	62-	13	
PMT	63-	14	
RCL 0	64-	45 0	
SST	00-		
GOLD P/R			Switches out of programming.

Qualifying a buyer (Continued).

TESTING THE PROGRAM

Key in the variables as below and check that you get the answers given in parenthesis. Here we retain the same rate and term for all four calculations. It is important to test all four columns.

Rate:	11 BLUE i			
Term:	30 BLUE n			
Down payment:	20 R/S	20 R/S	10 R/S	10 R/S
Gross income:	6000 R/S	5000 R/S	4000 R/S	3000 R/S
Liabilities:	600 R/S	100 R/S	100 R/S	300 R/S
Price:	(189,646.49)	(186,259.95)	(110,771.43)	(77,144.39)

You may then find:

Loan amount in **PV**:	($69,429.95)	RCL PV
Monthly payment in **PMT**:	($661.20)	RCL PMT
Purchase price in Register 0:	($77,144.39)	RCL 0
Lowest of Registers 1 & 2 is	(780.00)	RCL 1
PITI (+ PMI for 90% loan):	(840.00)	RCL 2
Reduce liability to maximize loan:	(-60.00)	-
Downpayment in Register 3:	($7,714.44)	RCL 3
Downpayment ratio in Register 5:	(10.00%)	RCL 5

IF YOU GET THE CORRECT ANSWERS, you may use the program in confidence and without checking it again for years if need be.

However, please remember that the program implements ratios and factors that may not permanently remain those you need to use, and that you might need to adjust them for your purpose.(See later for details).

IF YOU DO NOT GET THESE ANSWERS, the following matters could be wrong. Correct as suggested and test again as above.

PROBLEM	**CORRECTION**
- Data keyed in incorrectly. - Incorrect data keyed in. - Program not set at its beginning.	Press **GOLD CLEAR-REG**
- Incorrect **BEGIN** setting indicator.	Press **BLUE END** (g & 8).
- Program incorrectly keyed in.	Check and correct as in UNIT 32, Pages 18 & 19.

USING THE PROGRAM: DETAILED PROCEDURES AND EXPLANATIONS

The following procedures assume that the program has been keyed in and that the calculator is in the normal calculator (RUN) mode. The programming mode is for keying in a program, NOT using it.

STEP 1 Not needed under normal circumstances:
Turn the calculator **OFF** and **ON**,
or press **GOLD CLEAR-PRGM** (f and key above g)
or **BLUE GTO 00** (g, GTO, and two zeros)
or **GOLD CLEAR-REG**
Any one of these keystrokes assures that the program has not been interrupted in mid-stream and starts at the beginning.

STEP 1 is not required if you did not use a program since you last turned the calculator ON, or if you just used the present program satisfactorily.

STEP 2 Loan data: key in the term and rate in **n** and **i** in any order as you would to calculate the payments on the loan: Using the data from our previous example:

Keystrokes	Comment
30 BLUE n	For 30 year loan.
11 BLUE i	For 11% interest.

STEP 3 Punch in 10 or 20 for your choice of 10% or 20% downpayment and press R/S:

Keystrokes	Comment
10 R/S	For 10% downpayment.

The display then shows 20, even with a 10% loan: ignore it. The program accommodates only 10% or 20% options, though you may qualify for a 90% or 80% loan and add a higher downpayment to the loan amount found in **PV**.

STEP 4 Key in gross monthly income and press R/S:

Keystrokes	Comment
3000 R/S	$3,000 gross monthly income.

The display shows -80 with 80% loan, -90 with 90% loan.

STEP 5 Key in monthly liability as defined by lender. (Payments on outstanding debts, homeowner fees. Some lenders also include an estimate for utilities). Then press R/S:

Keystrokes	Comment
300 R/S	$300.00 monthly liability.

After a few seconds, the maximum selling price that meets the most stringent of the two ratio requirements appears in the display: here $77,144.39. That amount is also stored in Register memory 0.

ADDITIONAL DATA

Downpayment, loan amount, and monthly payments have already been calculated and stored. You may recall them in any order and as many times as you choose.

- Loan data in n, i, PV and PMT:	RCL BLUE n	(30 year loan)
	RCL BLUE i	(11% rate)
	RCL PV	($69,429.95 loan)
	RCL PMT	($661.20 payment)
- Downpayment in Register 3:	RCL 3	($7,714.44)
- Downpayment ratio in Register 5:	RCL 5	(10%)
- Selling price in Register 0:	RCL 0	($77,144.39)

PITI, plus PMI if applicable, is the smaller of the two dollar amounts stored in Registers 1 and 2. Recall them in that order:

RCL 1	($780.00)
RCL 2	($840.00)
-	(-60.00)

Here, PITI + PMI is $780.00.

By pressing minus (-), we calculate the difference between the numbers in memories 1 and 2. That difference, and the sign it has, is important:

- IF THE NUMBER IS NEGATIVE, as above, then a maximum qualifying value can be found by reducing the debt payments by that amount. Reducing the debt payments by a larger amount will not allow the buyer to qualify for a higher-priced property, as the lower ratio (here 28%) would then become more stringent.

- IF THE NUMBER IS POSITIVE, the monthly debt burden could be increased by that amount without reducing the price for which the buyer qualifyies under the guidelines. This is because the lower ratio is already limiting the selling price independently of the liability payments.

- IF THE NUMBER IS ZERO, the buyer has just the right amount of debts: more debts would reduce the qualifying price, but less debt would not qualify him for a higher priced property. It is important to realize than any liability over this ideal amount reduces allowable monthly payments on the loan almost dollar for dollar.

RUNNING THE PROGRAM WITH NEW DATA

We may check the impact of different data by returning to STEP 2 above, or directly to STEP 3 if the rate and term of the loan have not changed.

For instance: what if the same buyer chooses to reduce his liability by $100.00? (From $300.00 to $200.00).

STEP 3	10 R/S 3000 R/S 200 R/S	 ($83,078.57)

Lowering his liabilities by more than $60.00 did not increase the purchase price beyond the answer given here.

What if the buyer retains his original $300.00 in liabilities but selects a variable rate loan which allows him to qualify on the basis of an 8.25% interest 30 year loan:

STEP 2	8.25 BLUE i	(30 year term has not changed)
STEP 3	10 R/S 3000 R/S 300 R/S	 ($93,960.06)

We should go back to STEP 1 if the previous run was not satisfactorily completed.

Notes:

- There is no need to clear any part of the calculator before using this program.
- Running this program uses the 5 financial memories and register memories 0 to 5.
- The program itself uses up register memories 12 to 19. It leaves only 12 register memories (memories 0 to 11). As long as the program is retained in the calculator, memories 12 to 19 are not available for storage of data or irregular cash flow calculations.

CALCULATING THE RATIOS ACHIEVED

If you need to verify the ratios achieved with and without liabilities, you may calculate them as follows:

Of memory 1 or 2, select the one with the lowest number: that number is the only one to consider.

	RCL 1	($780.00)
	RCL 2	($840.00)

The lowest number is $780.00 in memory 1.

We may now calculate the ratios as follows:

- Key in gross monthly income:	3000	
- Recall memory 1:	RCL 1	
- Press %T:	%T	(26.00%)

The result (26%) is the ratio achieved instead of the low ratio requirement. It is lower than 28%, and therefore satisfactory.

To verify that the high ratio is acheived, just add monthly liabilities to the content of memory 1 in the same procedure:

	3000	
	RCL 1 300 +	
	%T	(36%)

If the lowest number is in memory 2, we just need to use RCL 2 instead of RCL 1 to find the two ratios, though here the unknown ratio would be obtained when we add the liabilities as immediately above.

ADJUSTING THE PROGRAM TO YOUR NEEDS

Before using the program, you need to make sure that the assumptions embodied in the program are those commonly made by lenders in your area. If they use different ratios, you should adjust the program accordingly.

The ratios used in computing the answer (28% and 36%, 33% and 38%) may be changed directly in the program (lines marked "A"). Just change the numbers on the written program and key the program in as modified. Do not change the number of steps. With some experience of programming, the ratios can be changed without having to re-write the whole program. For instance, to change the 28% ratio to 31%:

1) Look at the program listing and identify the 28% ratio as being on lines 7 and 8.

2) With the calculator still in normal calculator (run) mode:

Press: This positions the program internally one step before the steps you want to modify.	BLUE GTO 06
3) Now switch into programming: You are positioned on step 06-.	GOLD P/R
4) Press the digits for the new ratio:	31
5) You may now switch out of programming: The correction has been made.	GOLD P/R

The factors used in lines marked "B" may also be adjusted in the program. These factors express TI (monthly payments for Tax and Insurance), or TI + PMI with 10% downpayment, as a percentage of the selling price. They should be re-calculated and adjusted if need be on the basis of a $100.00 selling price and a $90.00 or $80.00 loan. The 20% downpayment factor (lines 34 to 37) is the sum of the monthly payments for Tax and Insurance that would be required on a property selling for $100.00 and financed with an $80.00 loan. The corresponding 10% factor (lines 16 to 19) is calculated on the basis of a $100.00 property financed with a $90.00 loan and includes monthly PMI charges.

If, in the process, you change the number of decimals, all GTO instructions in the program need to be readjusted. So if you insist on a factor of .16 instead of .154, key it in as .160 to retain the same number of decimals.

The program can be shortened slightly by omitting lines 58 to 64: it would then no longer calculate and store downpayment, loan amount, and monthly payment all of which can easily be calculated from knowledge of the selling price. The program can also be lengthened, for instance by making it calculate and store the ratios that have been met.

DOWNPAYMENTS OTHER THAN 10% AND 20%

Though the program calculates a purchase price, what the buyer really qualifies for is a loan amount. The actual purchase price affects the loan amount only insofar as property taxes enter into the calculation. However, this has only a marginal effect that can generally be disregarded. So we should go back to the loan amount and add the desired downpayment.

If a buyer is willing to make a downpayment that is less than 20% but more than 10%, we should qualify on the basis of a 10% downpayment, then recall the loan from **PV** and add the dollar amount of the downpayment that can be made.

If a buyer with a $6,600 gross income and $350 in liabilities gets a 9.25% interest, 30 year loan, and is also willing to put $80,000 down, more than the required 20%, we should qualify on the basis of 20% as follows:

9.25	BLUE	i		
30	BLUE	n		
20	R/S			
6600	R/S			
350	R/S			(Ignore $275,909.56)
RCL	PV	80000	+	($300,727.65)

We simply add the downpayment to the loan amount. The buyer in effect is putting down $25,000 more than otherwise required and can purchase a property that is more expensive by $25,000.

CREATING A GRADUATED PAYMENT LOAN

A PROGRAM

This program calculates the graduated payments on a loan so that it conforms to a variety of requirements: loan amount, rate, and amortized term (keyed in in the usual way). Ratio of the increase as a percentage of the initial payment and number of required increases (keyed into the **y** and **x** memories of the stack).

Credit for this program must go to HEWLETT-PACKARD. As offered here, it was slightly shortened and revised by Edric Cane.

EXAMPLE

A $100,000 loan at 13% interest is to be fully paid off in 20 years. The monthly payments increase by 4% every year for the first 10 years (10 different payments, but ONLY 9 INCREASES) and then remain fixed until the loan is fully paid off. What are these payments?

WITH THE PROGRAM KEYED IN:

1) Key in the term, interest, and loan amount in the usual way.

```
100000  PV
    13  BLUE   i
    20  BLUE   n
```

2) Key in yearly increase as a percentage, **ENTER**, and the number of increases.

```
     4  ENTER  9
```

3) Press **R/S** to find the monthly payment for year 1. (30 seconds).
4) Keep pressing **R/S** to get monthly payments for each succeeding year. After the appropriate number of increases, payments will remain level.

R/S	$967.74: year 1.
R/S	$1,006.45: year 2.
R/S	$1,046.71: year 3.
R/S	$1,088.58: year 4.
R/S	$1,132.12: year 5.
R/S	$1,177.51: year 6.
R/S	$1,224.51: year 7.
R/S	$1,273.49: year 8.
R/S	$1,324.43: year 9.
R/S	$1,377.40: year 10.
R/S	$1,377.40: level to end of term.

GRADUATED PAYMENT LOAN: THE PROGRAM.

STEP	KEYSTROKES	DISPLAY			
	GOLD P/R				Switches into programming
	GOLD CLEAR-PRGM	00-			Clears previous programs
1	STO 1	01-	44	1	
2	STO 2	02-	44	2	
3	xy	03-		34	
4	1	04-		1	
5	%	05-		25	
6	1	06-		1	
7	+	07-		40	
8	STO 0	08-	44	0	
9	RCL BLUE n	09-	45,43	11	
10	RCL 2	10-	45	2	
11	-	11-		30	
12	BLUE n	12-	43	11	
13	RCL PV	13-	45	13	
14	STO 3	14-	44	3	
15	1	15-		1	
16	CHS	16-		16	
17	PMT	17-		14	
18	0	18-		0	
19	FV	19-		15	
20	PV	20-		13	
21	CHS	21-		16	
22	FV	22-		15	
23	1	23-		1	
24	BLUE n	24-	43	11	
25	RCL PMT	25-	45	14	
26	RCL 0	26-	45	0	
27	÷	27-		10	
28	PMT	28-		14	
29	PV	29-		13	
30	CHS	30-		16	
31	FV	31-		15	
32	1	32-		1	
33	STO - 1	33-	44 30	1	
34	RCL 1	34-	45	1	
35	BLUE x=0	35-	43	35	
36	BLUE GTO 38	36-	43,33	38	
37	BLUE GTO 25	37-	43,33	25	
38	RCL 3	38-	45	3	
39	RCL PV	39-	45	13	
40	÷	40-		10	
41	STO 4	41-	44	4	
42	RCL 4	42-	45	4	
43	RCL 0	43-	45	0	
44	RCL 2	44-	45	2	

45	y^x	45-	21
46	÷	46-	10
47	R/S	47-	31
48	1	48-	1
49	STO - 2	49- 44 30	2
50	RCL 2	50- 45	2
51	BLUE x=0	51- 43	35
52	BLUE GTO 54	52- 43,33	54
53	BLUE GTO 42	53- 43,33	42
54	RCL 4	54- 45	4
55	R/S	55-	31
56	BLUE GTO 55	56- 43,33	55
	GOLD P/R		Ends programming

TESTING THE PROGRAM

Before using the program to find answers that meet your own requirements, you must make sure that is works properly. This is done by verifying that it provides the same answers as those shown in these pages. Go back to the $100,000 loan given as our initial example. Also check that you find the answers provided in the following section.

RUNNING THE PROGRAM AGAIN

Because this program is an endless loop, you need to set it back to step 00 before you can use it over again. This may be done by pressing **BLUE GTO 00** while in the normal run mode at any time before you press the initial **R/S**.

Create a **$125,000** loan at **11%** interest that fully amortizes in **15** years with payments that increase every year by **5%**.

With program keyed in	
BLUE GTO 00	
125000 PV	
11 BLUE i	
15 BLUE n	
0 FV PMT	(Level payment: $1,420.75. Optional.)
5 ENTER 14	(14, not 15)
R/S	($1,088.69)
R/S	($1,143.12)
R/S	($1,200.28)
R/S	($1,260.29)
R/S	($1,323.30)
R/S	($1,389.47)
R/S	($1,458.94)
R/S	($1,531.89)
R/S	($1,608.48)
R/S	($1,688.91)
R/S	($1,773.35)
R/S	($1,862.02)
R/S	($1,955.12)
R/S	($2,052.88)
R/S	($2,155.52)

- Note how we use the loan data keyed into **n**, **i**, and **PV** to calculate the equivalent level payments that would amortize the loan in 15 years. With the graduated payments, it will take 7 years before the borrower pays more than the level payments would require.

- Note that a yearly increase on a 15 year loan means 14 increases, not 15. The graduated requirements are keyed in as **5 ENTER 14.**

- This program may be used to calculate set FHA Graduated Payment Mortgage payments, except that the greater accuracy of the calculator will lead to discrepancies of a few cents with the numbers provided by FHA tables.

BAND OF INVESTMENT APPRAISAL
AN APPROACH TO INCOME PROPERTY VALUATION

A PROGRAM

Let's first explain the problem and solve it without using programming. We will then propose a program that solves the problem for us. This will help illustrate the much greater flexibility provided by programming.

THE PROBLEM

An income property has a Net Operating Income (NOI) of $135,000. (NOI is the gross income minus expenses but not including debt service). The property is being offered for sale on the following terms:
- 70% loan-to-value 1st mortgage at 13% amortized over 30 years.
- 10% loan-to-value owner-carried 2nd mortgage at 12%, interest only.
- 20% downpayment from a buyer who wants a 5% cash-on-cash return.

Only one selling price meets those requirements. Let's find it.

DEFINITIONS

NET OPERATING INCOME (NOI): what the owner would retain as income if he owned the property free and clear. It is the gross income, minus vacancy and credit losses, expenses for maintenance, reserves, and taxes on the property, but NOT expenses for servicing of the debt that still need to be paid out from NOI proceeds.

CASH-ON-CASH RETURN: net yearly income (after debt service) as a percentage of the buyer's cash investment here construed as equivalent to the downpayment. The cash-on-cash return does not take into account personal income tax liabilities or benefits and the tax benefits of depreciation.

When the NOI is all spent on loan payments, there is a cash-on-cash return of 0, or break-even. When the payments on the loans are higher than the NOI, the owner must put more money into the property: their is a negative cash-on-cash return.

The cash-on-cash is really a first year net balance for the buyer, before the tax consequences of depreciation. Most buyers expect the ratio to change in subsequent years.

LOAN CONSTANT: Yearly payments on a loan as a percentage of the original loan amount. The yearly loan constant is equivalent to the monthly payments on a $1,200.00 loan of the same rate and term. (See Index). For a 30 year loan at 13% interest the annual loan constant is 13.27%:

```
1200  PV    13  BLUE  i    30  BLUE  n
   0  FV    PMT
```

(13.27%)

UNDERSTANDING -- AND SOLVING -- THE PROBLEM

There are three investors and we know what proportion of the sales price each one is going to finance:

- The institutional lender: 70% loan.
- The seller: 10% 2nd mortgage.
- The buyer: 20% downpayment.

We also know what proportion of his investment each one wants as a yearly cash return.

- The 1st mortgage lender wants interest and principal on his loan, which, as a ratio, is expressed by the loan constant (13.27%).
- The seller also wants the loan constant back on his 2nd mortgage. Here, because the loan is interest-only, loan constant and interest are expressed by the same number, 12% per year.
- The buyer wants 5% of his investment back that first year.

Clearly, if the seller's loan is equal to 10% of the selling price, we may satisfy the seller by giving him either 12% of his loan or 1.2% of the selling price. These two ratios represent the same amount of money. So with these sets of ratios (70%, 10%, 20%, and 13.27%, 12%, 5%) we may calculate what each investor wants <u>as a percentage of the selling price</u>:

The lender wants 13.27% of his 70% loan, or 9.29% of the selling price.
The seller wants 12% of his 10% 2nd loan, or 1.2% of the selling price.
The buyer wants 5% of his 20% investment, or 1% of the selling price.

The calculations are very simple: we take 20% of 5, 10% of 12, and 70% of 13.27. We also get the correct numbers if instead we take 5% of 20, 12% of 10, and 13.27% of 70.

By adding each investor's requirements, we find that together they require 11.49% of the selling price. This requirement must be met by the NOI on which nobody else has a claim except our three investors (the two lenders and the buyer). So we now know that $135,000 is equal to 11.49% of the selling price of the property. A simple capitalization rate calculation gives us the selling price. Let's propose two options for the keystrokes:

Keystrokes	Result
135000 ENTER .1149 ÷	($1,174,934.73)
11.49 ENTER 135000 %T	($1,174,939.73)

We may arrange these numbers in columns as follows, with the crucial calculation being the passage from the Return on Investment expressed as a percentage of each investor's investment to the same amount expressed as a percentage of the sales price.

INVESTOR	REQUIREMENT	INVESTMENT as % of sales price	RETURN ON INVESTMENT as % of investment	as % of sales price
1st LENDER	13%, 30 yrs.	70%	13.27%	9.29%
SELLER	12% int.-only	10%	12%	1.20%
BUYER	5% cash-on-cash	20%	5%	1.00%
TOTALS		100%		11.49%

The problem may be solved as follows:

```
30  BLUE  n    13  BLUE  i     1200  PV
 0  FV   PMT                                (13.27%: loan constant)
      CHS  70  %   STO  0
12  ENTER  10  %   STO  +  0
 5  ENTER  20  %   STO  +  0    RCL  0      (11.49% cap rate)
135000  %T                                  ($1,174,722.48)
```

- The small discrepancy with the previous answers is caused by rounding off of internal decimals avoided in this sequential calculation.

- If the 2nd loan also had amortized payments, we should calculate the loan constant on it as with the 1st loan.

Now let us see how a program can solve the same problem for us.

BAND OF INVESTMENT PROGRAM

As shown earlier, the stages now are as follows:

1) Keying in the program.
2) Testing the program.
3) Using the program.

Stages 1 and 2 can be performed once and for all, allowing the program to be used over and over again for months or years. The extra flexibility provided may make it worth it to key in and test the program just to handle a single listing, packaging, or selling negociation.

KEYING IN THE PROGRAM

KEY IN	DISPLAY SHOWS	COMMENTS
GOLD P/R	00- (generally)	Switches into program mode.
GOLD CLEAR-PRGM	00-	(f and key above the g key)
1	01- 1	
2	02- 2	
0	03- 0	
0	04- 0	
CHS	05- 16	
PV	06- 13	
0	07- 0	
FV	08- 15	
PMT	09- 14	
EEX	10- 26	
2	11- 2	
RCL 4	12- 45 4	
RCL 2	13- 45 2	
+	14- 40	
-	15- 30	
BLUE PSE	16- 43 31	
%	17- 25	
STO 0	18- 44 0	
RCL 5	19- 45 5	
1	20- 1	
2	21- 2	
x	22- 20	
RCL 4	23- 45 4	
%	24- 25	
STO + 0	25- 44 40 0	
RCL 6	26- 45 6	
BLUE x=0	27- 43 35	
BLUE GTO 37	28- 43,33 37	
RCL 1	29- 45 1	
%T	30- 23	
RCL 0	31- 45 0	
-	32- 30	
RCL 2	33- 45 2	
xy	34- 34	
%T	35- 23	
BLUE GTO 0 0	36- 43,33 00	
RCL 3	37- 45 3	
RCL 2	38- 45 2	
%	39- 25	
STO + 0	40- 44,40 0	
RCL 0	41- 45 0	
RCL 1	42- 45 1	
%T	43- 23	
GOLD P/R	Ignore display.	Switches out of programming.

TESTING THE PROGRAM

With the program keyed in, make sure that you get the same answers as provided in parenthesis for questions 1 to 5 below.

ENTERING THE VARIABLES

Variables are entered in Register memories 1 to 6 and in **n** and **i**. They may be entered in any order we choose.

The major advantage of the program over the regular procedure is that now we may change any variable and see how it affects the selling price, or impose a selling price (in memrory 6) and calculate the cash-on-cash return. Changes are **CUMULATIVE** unless we key back in the original data.

Data from previous illustration:
- Net Operating Income (NOI): **$135,000.**
- **70%** loan-to-value 1st mortgage at **13%**, amortized over **30** years.
- **10%** loan-to-value owner-carried 2nd mortgage at **12%**, interest-only.
- **20%** downpayment from a buyer who wants a **5%** cash-on-cash return.

INITIAL DATA

		With program in
Register 1:	$135,000 Net Operating Income:	135000 STO 1
Register 2:	20% down payment:	20 STO 2
Register 3:	5% Cash-on-cash requirement:	5 STO 3
Register 4:	10% loan-to-value ratio on 2nd loan:	10 STO 4
Register 5:	**Monthly** payment on loan as % of loan amount (1% per month):	1 STO 5
Register 6:	Selling price if known, 0 if price unknown:	0 STO 6
n and **i**:	Term of 1st mortgage (30 years):	30 BLUE n
	Rate on 1st mortgage (13%):	13 BLUE i

CHANGING THE DATA AND ASKING QUESTIONS

QUESTIONS AND NEW DATA	ANSWERS		
1) Selling price for initial data:	($1,174,722.48)		R/S
2) Price for break-even cash-on-cash return:	($1,286,685.31)	0 STO 3	R/S
3) Cash-on-cash for $1,400,000 price?	(-4.25%)	1400000 STO 6	R/S
4) Cash-on-cash with a 30% downpayment?	(1.59)	30 STO 2	R/S
5) Price to get a 2% cash-on-cash?	($1,382,539.94)	0 STO 6 2 STO 3	R/S
6) Price if 2nd loan is at 9% (int. only):	($1,426,362.22)	9 ENTER 12 ÷ STO 5	R/S

- Changes are cumulative. Our last calculation considers a 30% downpayment, a 2% cash-on-cash return, a 9% interest-only 2nd T.D. (.75% per month).

- Any of the variables can be changed, including the NOI and the term and rate of the first loan.

- The loan-to-value ratio of the first loan (here 70% then 60%) is briefly flashed on the display for our convenience.

- To calculate a selling price, we need 0 in Register 6. Any other number stored in memory 6 is interpreted as the selling price for which we want to calculate the cash-on-cash ratio. Memory 6 is the switch that tells the calculator whether we are seeking a selling price or a cash-on-cash ratio.

- The program itself clears the financial memories as needed (it puts 0 in FV). It does not clear Register 6 where the required 0 if we are seeking to calculate a selling price must be considered as part of the data.

- The capitalization rate is automatically calculated and stored in Register 0, from which it may be recalled if needed.

- When a cash-on-cash value is calculated, it is not automatically stored in Register 3. Any previous value remains, though inoperative, as long as a selling price is stored in Register 6.

- If the 2nd loan is amortized, we should use the regular cash flow functions to calculate the monthly payments on a $100.00 loan and key the amount into Register 5 as a positive number.

- If there is no second loan, we just key 0 in register 4. If there are a 2nd and 3rd loan, it is possible to combine their ratios into a single set of data.

- Arithmetic calculations can be performed on the numbers stored in the various meories. For instance, with the last answer still showing in the display ($1,669,526.93), we may calculate as follows:

Downpayment:	RCL 2 %	($500,858.08)
Cash-on-cash:	RCL 3 %	($10,017.16))

- This program is a flexible tool that allows us to change any one of the variables and to see the effect on the selling price or on the cash-on-cash return, given the other element as data. Its speed also allows us to calculate by trial and error the value of another of the variables (loan rate or ratio, downpayment ratio, etc.) that would meet a given set of values for the selling price and the cash-on-cash return.

- The program can be used in a variety of circumstnces. It may be used to reach a realistic selling price when listing or preparing to sell the property--a seller may agree that the various requirements are reasonable, though he might not initially have agreed to the resulting selling price. It may be used to package an offer to sell--the seller may realize that a high rate of return on the second loan has a negative effect on the selling price, given a buyer's expected cash-on-cash requirement. It may be used to package an offer by a potential buyer who wants to find the best compromises between his needs and the seller's requirements.

- A major limitation of this approach is that it only allows us to handle loans expressed as a percentage of the selling price, not as a dollar amount as would be the case if there were large assumable loans at lower than market rates.

- The price of properties is affected by the market, not just by the financing. So every approach to appraisal must have under some form or another the input of market forces. Here the lever of market realities is introduced by the cash-on-cash return that buyers expect. Under different market conditions, they would accept a negative cash-on-cash, or a breakeven situation, or require a higher cash-on-cash ratio then those selected here. There is, of course, an interdependence between various ratios. For instance, a buyer providing a 30% or 40% downpayment may expect a higher cash-on-cash return than one offering 15% or 20%. The program invites us to manipulate data within the margins allowed by realistic considerations.

INDEX

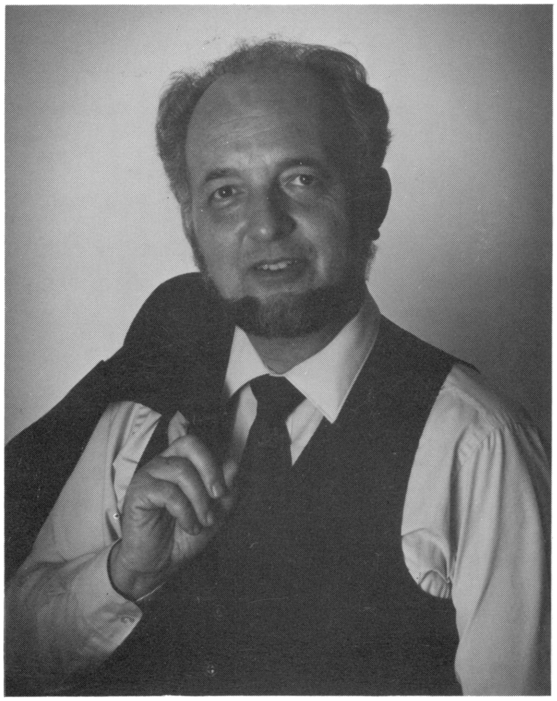
(For more information, see also Index to Volume I)

I N D E X

21-7 stands for Unit 21, page 7.
33-5/9 stands for Unit 33, pages 5 to 9.

UNDERSTANDING IS THE KEY TO CREATIVE APPLICATIONS

To provide the reader with the joy and power, and the professional edge, of almost complete control over exchanges of money in time. Such is the primary objective of **THE HP-12C MADE EASY, Volumes I & II.**



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Leslie Brown, Senior Loan Underwriter, Columbia Savings, Irvine, California.

"You sold me at the beginning with your advice to **understand the simplicity rather than memorize the simple.** Your book is a model of precise logic, concise expression and lapidary clarity."
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