# THE HP-12C MADE EASY

## Regular Compound Interest Functions


By Edric Cane, Ph.D., Realtor

A STEP-BY-STEP, EASY-TO-FOLLOW SELF-STUDY COURSE:

- FINANCIAL CONCEPTS
- CALCULATOR PROCEDURES

# THE HP-12C MADE EASY

## Regular Compound Interest Functions

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**NOTICE**

Use this material at your own risk. Laws change, applications are infinitely varied. Furthermore, even this author has been known to make mistakes. So Edric Cane makes no express or implied warranty as to the validity of this material for any specific application and shall not be liable for any incidental or consequential damages in connection with its use. The user (not Edric Cane, Edric Cane Seminars, or any other party) shall bear the entire cost of all necessary corrections and all incidental or consequential damages.

My thanks to Jesse Pratt of Woodland Hills, California, who was kind enough to double check all the solutions to the many problems. Please address any constructive suggestions for further corrections and improvements directly to me. All nasty complaints can go directly to Jesse.



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To all the friends made while teaching my **Keys to Creative Financing** seminars. With some, our paths cross regularly. Others are far away, and I haven't seen them for years. For their encouragement and support, their questions, their suggestions, their help, and above all their friendship, I am deeply grateful.

# INTRODUCTION

Teaching is not dumping a truckload of bricks on your front lawn. It's gradually showing how the bricks can fit together to build a structure.

The aim of this book is to teach. It seeks to provide:

- **an understanding of financial concepts,**
- **a practical understanding of calculator procedures,**
- **practice in putting financial concepts and calculator procedures together to solve actual problems.**

It uses the calculator to clarify financial concepts, and financial concepts to clarify calculator procedures.

**This presentation assumes no prior specialized knowledge of any kind, certainly none whatsoever of the calculator: open the box, open the book, and you're in business.**

The ideal is not so much to simplify as to make obvious. The obvious implies **understanding the simplicity.** As such the most successful passages are those where the reader is left with the impression that he already knew it, or at least that there is nothing that he needs to remember. If the reader cooperates in this approach, if he strives to understand the simplicity rather than attempt to memorize the simple, he will retain the creativity required to adjust to the infinite variety of circumstances encountered in the real world.

A majority of the examples used here are taken from real estate. That emphasis gives unity to the book. It allows the major financial concepts and calculator procedures to be expressed in terms of financial situations to which most people can relate, and that can yet be transferred to applications ranging from estate planning to business finance. In terms of financial functions, this volume limits itself to the regular compound interest features of the calculator, as opposed to irregular cash flow functions, statistical functions, and programming features. A continuation is being actively prepared.

Born of a seminar offered at UCLA-Extension, UC Berkeley, and to a variety of real estate, banking, and investment groups from New York to Hawaii, this written course will remain a text for my own seminars. There are so many courses, excellent in their own right, which could be made so much more productive if the student came equipped with the understanding and the practical abilities offered here! It is hoped that this volume and the one that will follow, will be recommended for other courses or seminars where the use of the HP-12C is advised: appraisal, property management, investment analysis, estate and financial planning, mortgage banking, as well as business courses and MBA programs.

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# UNIT 1

# BASIC CONCEPTS

# UNIT 1

# BASIC CONCEPTS

BASIC FINANCIAL CONCEPT: EXCHANGING MONEY IN TIME

BASIC CALCULATOR PROCEDURE: DATA THEN QUESTION

A FIRST LOOK AT THE CALCULATOR

# EXCHANGING MONEY IN TIME

- **BORROWING** is **EXCHANGING MONEY IN TIME.**

  We get money now, and we pay back more money later.

- **INVESTING** is **EXCHANGING MONEY IN TIME.**

  We give money, and we hope to get back more money later.

- **CHOOSING BETWEEN FINANCIAL ALTERNATIVES** is **EXCHANGING MONEY IN TIME.**

  Should we buy or rent, refinance or add a second, choose one offer with more cash but less paper to carry, or another offer with a higher selling price but less cash? <u>The choice itself</u> is between getting more money now but less later, or vice versa.

**BORROWING,**

**INVESTING,**

**CHOOSING** between financial options.

This covers a lot of ground. Much of it is simply

**EXCHANGING MONEY IN TIME.**

MAJOR OBJECTIVE

TO ANSWER MOST NUMBER QUESTIONS INVOLVING
**THE EXCHANGE OF MONEY IN TIME.**

# THE CASH FLOW DIAGRAM

The questions that can be asked concerning EXCHANGES OF MONEY IN TIME are exceedingly varied and may involve such words as

Interest rate
Annual Percentage Rate (APR)
Yield
Internal Rate of Return (IRR)
Rate of inflation
Payments,
Cash flow
Balance
Balloon payment
Reversion
Equity,
Future Value
Present Value
Discounted Value
Investment
Downpayment
Loan amount
Principal
Principal reduction
Interest payment
Pay down
Buy down
Term
Due date, etc.

Under the great variety of names, there is the simple reality of an **EXCHANGE OF MONEY IN TIME**: the reality of

**SPECIFIC SUMS OF MONEY** that we receive and that we give at

**SPECIFIC POINTS IN TIME**, and of the

**RATE OF INCREASE** that is required for the money received to balance off the money given out.

We may represent the sums that make up the exchange by arrows spread out on the line of time, arrows that point up for the money received and down for the money given, or vice-versa.

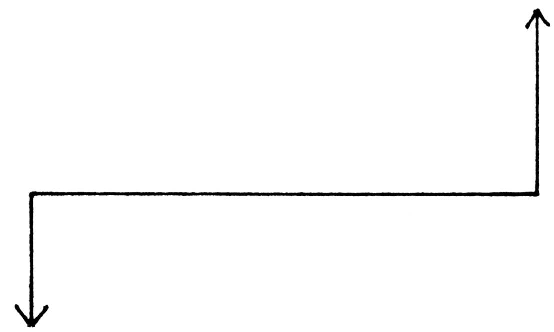
This visual representation is the

**CASH FLOW DIAGRAM.**

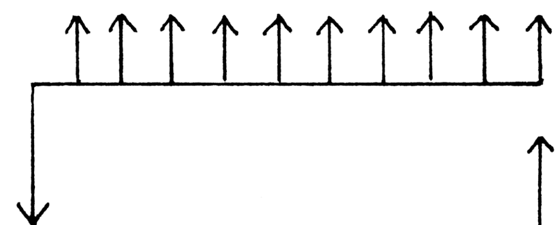
## SAMPLE CASH FLOW DIAGRAMS

### REGULAR CASH FLOW

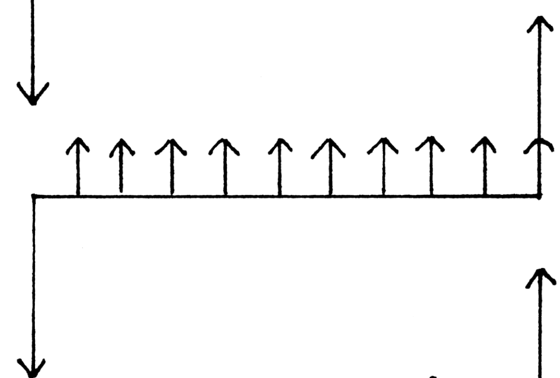
**STRAIGHT NOTE:** I make a lump deposit in a savings account, and later withdraw principle and interest in one lump sum. Or vice-versa: I borrow, and pay back in one lump sum. This is the simplest EXCHANGE OF MONEY IN TIME.



**FULLY AMORTIZED LOAN:** I borrow, and then pay back with regular equal payments.

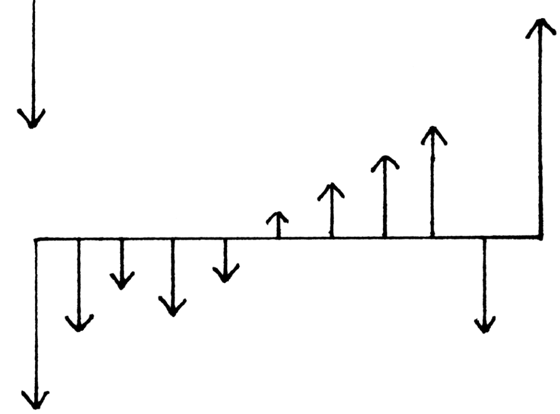


**PARTIALLY AMORTIZED LOAN:** I borrow, and then pay back with equal payments for a while, then pay off the remaining balance with one lump sum.



### IRREGULAR CASH FLOW

**AN INVESTMENT** requires me to spend various sums for a while, then I get more and more money back in return. After a final disbursement, I cash in on my investment.



The HP-12C divides exchanges of money in time into two broad categories:

- **REGULAR CASH FLOW**
- **IRREGULAR CASH FLOW**

**REGULAR CASH FLOW** situations: as in the first three cash flow diagrams, all the amounts except the initial and final amounts are equal.

**IRREGULAR CASH FLOW** situations: as in the final example, they accommodate considerable fluctuations in the cash flows.

| THE PRESENT VOLUME DEALS ONLY WITH REGULAR CASH FLOW SITUATIONS |
|---|

Throughout this presentation we will be moving back and forth between **TWO LEVELS OF UNDERSTANDING:**

- **UNDERSTANDING FINANCIAL CONCEPTS**
- **UNDERSTANDING CALCULATOR PROCEDURES.**

We have considered so far **the most important financial concept,** one that we will be constantly referring back to throughout this course, the notion of

| **EXCHANGING MONEY IN TIME** |
|---|

It is more dynamic than the more usual concept of the time value of money as it stresses the consequences of our own decisions, but basically **anything covered by the notion of the TIME VALUE OF MONEY is included in the notion of EXCHANGING MONEY IN TIME.** We will return to the phrase "Time Value of Money" when we consider the exchanges of money in time from a more theoretical point of view.

We now turn to **the most important calculator procedure:**

| **DATA then QUESTION** |
|---|

It simply means that to solve problems we need first to provide the **DATA,** then to ask the **QUESTION.** With our calculator, for all the financial and other functions that it possesses, the **ANSWER** automatically appears.

The following comparison makes the point.

(In keeping with our objective of avoiding technical terms whenever possible, we will be using the word "DATA" as a collective singular)

**DATA** then **QUESTION**

## A ROBOT IN YOUR KITCHEN!

How would you like to have a robot that could do the cooking for you? Well, one has just settled in your kitchen. It has rows of buttons on its chest labelled for all your favorite recipes: lemon chiffon pie, chocolate chip cookies, quiche lorraine, etc. When you press the quiche lorraine button, the robot spins towards the refrigerator and reaches for the eggs, the bacon, the milk and the butter. It swings towards the shelf and gets the flour and salt. It mixes the ingredients together according to its instructions, bakes the mixture for the right amount of time, and presents you with a piping hot dish.

Excellent! Provided of course you have the right ingredients on the shelves. You see, the robot doesn't really have a sense of taste or smell, it doesn't have eyes: it just has instructions to go, to grab, and to mix. If the salt on the shelf is where the flour is supposed to be, and the flour is where the salt should be, the quiche ends up with two cups of salt and a pinch of flour. We first need to have the right ingredients in their proper place, and then we can get the process going by pressing the right key.

The calculator is like this kitchen with the robot and the shelves.

The shelves and refrigerator are just places where ingredients are stored. In the calculator, they correspond to electronic memories where we can store our **DATA** in the form of numbers.

The robot itself embodies a set of instructions and the ability to carry them out: instructions to reach into specific places and to mix according to specific formulas. **QUESTION** keys on the calculator perform the same function: they set in motion pre-programmed instructions that tell the calculator where to reach for its data,--what memories to explore--, and what mathematical processing to apply to the numbers that it finds there. The result of the reaching and processing is the **ANSWER** to our question

Of course, we get the right answer only if we provide the correct data. As with the flour and the salt for our kitchen robot, if we have the wrong numbers, or if some number is in the wrong place or is missing, we get the wrong answer. Or rather:

**WE NEVER GET THE WRONG ANSWER,**
**WE JUST GET THE CORRECT ANSWER TO INCORRECT DATA.**

## ELECTRONIC MEMORIES and PRE-RECORDED PROGRAMS.

**DATA** then **QUESTION.** This is our fundamental procedure. This also corresponds to <u>two different parts of the calculator</u>. We have data keys and question keys, and a few keys that perform the two functions under different circumtances.

The **DATA KEYS** correspond to **ELECTRONIC MEMORIES** where we store **NUMBERS.** We may think of them as the shelves and refrigerator in our kitchen, or as boxes below the keys.

Data keys allow us to put the numbers we choose in specific places, to look into the content of a memory, to shift data from one memory to another, or to wipe a memory clean of any data. The data stored in the calculator is always available: if it is anywhere in the calculator, then it can be retrieved, checked, and changed if so desired.

Acquiring full control over the data we have in the calculator is an important objective we should set for ourselves.

The **QUESTION KEYS** correspond to **PERMANENT SETS OF INSTRUCTIONS** or programs remembered by the calculator and executed when the key is pressed--like the keys on the robot. When we refer to "questioning the calculator" or to "solving" for a specific piece of information, we just mean pressing the appropriate question key.

A financial calculator is a calculator that remembers programs that solve financial questions. A scientific calculator could be very similar, except that it would remember programs of interest to the scientist.

> **The instructions set in motion by pressing the question keys are instructions to take the numbers in specific memories and to process them according to specific mathematical formulas.**

Different question keys can explore different groups of memories, but any given question key always explores the same data memories. It is important to know what memories are going to be explored when we press a certain question key: that is where our correct data is supposed to be.

| DATA | QUESTION |
|---|---|
| **NUMBERS**<br>stored in<br>**ELECTRONIC MEMORIES**<br>by the user. | **PERMANENT INSTRUCTIONS**<br>**to take** the numbers in specific memories<br>**and to process** them according to specific<br>mathematical formulas. |

## LET THE CALCULATOR TAKE CARE OF THE MATH!

Because of the pre-recorded mathematical formulas and processing instructions, providing the **DATA** and asking the **QUESTION** takes care of the mathematics for us.

**This is not a course where we need to be good in mathematics.** We are not concerned with formulas and computations. We make full use of the calculator, and that means dispensing with the mathematics when the calculator knows it and does it for us. The course helps us understand financial problems so that we may express them in terms of data and question, and understand the calculator to the extent needed to communicate our data and ask the question.

## DO NOT TRY TO REMEMBER SERIES OF KEYSTROKES!

**DATA** then **QUESTION** is the basic procedure that we have to remember. It's easier than to remember 1500 different series of keystrokes! As our basic attitude towards learning how to use the calculator we should **deliberately avoid trying to remember long series of keystrokes!.**

Instead, we should ask ourselves:

- What is my **DATA**?
- In what memories should I store this data?
- What is my **QUESTION**?

The answer to these three questions should be relatively obvious. With the right data in the right place, asking the right question is a simple matter that provides the correct answer.

**I need your cooperation on this matter. If you try to memorize series of keystrokes, you are bound rapidly to reach a point of saturation and frustration. There is an infinite number of problems. Each problem, as it occurs in real life, may present itself from a variety of angles. How can we memorize so many variations? How can we adjust creatively to constant changes if we are relying on memory rather than understanding?**

Solving problems becomes simple if we remember the basic procedure:

$$\boxed{\textbf{DATA} \quad \text{then} \quad \textbf{QUESTION}}$$

## THE NAMING OF KEYS

Keys are referred to by the name or symbol of the function they perform.

On occasion it may be convenient to refer to the **position** of the key. We will do so by giving each key a number corresponding to its **COLUMN** and its **ROW** as follows:

| COLUMN > <br> ROW v | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 11 | 12 | 13 | 14 | 15 | 16 | **7** | **8** | **9** | 10 |
| 2 | 21 | 22 | 23 | 24 | 25 | 26 | **4** | **5** | **6** | 20 |
| 3 | 31 | 32 | 33 | 34 | 35 | 36 | **1** | **2** | **3** | 30 |
| 4 | **ON** | 42 | 43 | 44 | 45 | | **0** | 48 | 49 | 40 |

The keys for numbers 0 to 9 retain their own numeral as reference.

The first time we use some of these keys, we will refer to their function name and, between square brackets, to their position. For instance:

**PLUS** may be referred to as: + [40] (40: Row 4, column 10)

The **PERCENTAGE** key as: % [25] (25: Row 2, column 5)

The calculator itself refers to its own keys in this way in programs.

**f** **g** **GOLD** and **BLUE**

Now turn the calculator **ON.** [41]
The **ON** key also turns the calculator **OFF.**

The two most prominent keys, **f** and **g**, [42 and 43] have NO MEANING OF THEMSELVES. We will call them **GOLD** and **BLUE**.

When pressed and released before another key is pressed, they give a second and third meaning to that other key.

The second and third meanings are written in gold above the key for the functions that require the **GOLD** key as a prefix, and in blue on the front face of the key for the functions that require **BLUE** as a prefix.

The three meanings of the **Clx** key [35] may serve as an example:

| | | |
|---|---|---|
| **GOLD** | **CLEAR – REG** | Clears all data memories. |
| | **CLx** | Clears the display only. |
| **BLUE** | **x = 0** | A programming function. (of no interest to us at this stage) |

(The word "**CLEAR**" applies to the 5 gold functions, all on the same row, that have the gold line over them).

We may compare the **GOLD** and **BLUE** keys to the **SHIFT** key of a typewriter. It too can give an entirely different meaning to the same key--my "4" becomes a "$" sign! However, we release the **GOLD** and **BLUE** keys before we press the second key, we do not keep them pressed down as with the shift key of the typewriter.

When we press the **GOLD** key, a small **f** appears in the display. This status indicator warns that the next key pressed is going to have its gold function executed. A small **g** appears when the **BLUE** key is pressed.

## DISPLAY FORMAT

The normal format shows two decimals, dollars and cents. If a previous user selected a different format, select the normal format by pressing:

**GOLD** **2**

## MAJOR CLEARING FUNCTIONS

**CLx** **CLEAR x** [35]

**x** is the number shown on the display.

**CLx** stands for **CLEAR x.** Pressing **Clx** clears (erases) the number in the display.

The **CLx** key eliminates the number in the display without affecting any other data in the calculator. Use **CLx** in particular when the wrong number has been keyed in:

789 Oops! I really wanted 759. Press **CLx**, and then 759

**GOLD** **CLEAR-REG** **Clear Register**

- Press the **Gold** key.
- Release it.
- Then press the **CLx** key.

The instruction now given is **CLEAR-REGISTER.**

**GOLD CLEAR-REG** and **CLx** are two entirely different instructions though they both happen to be clearing functions: **GOLD CLEAR-REG** clears all the memories of the calculator, not just the display, everything, that is, except the program memories.

Some people use **GOLD CLEAR-REG** or other clearing functions systematically between problems. If this becomes a habit, it can prevent you from taking full advantage of the flexibility of the calculator. I propose an approach that clears only what needs to be cleared.

> **From here to the end of these instructions, all the clearing that needs to be done as you move from one illustration to the other is included in the keystrokes. No clearing needs to be done between practice problems. You do not need to clear when you get the wrong answer and want to start a problem again. The assumption is made that you will not clear anything between calculations unless directed to do so.**

However, do use **Clx** if you press the wrong digits, and **GOLD CLEAR-REG** does have a psychological effect: if a problem gets you all confused, it is sometimes good to **know** that you have been absolved of all your past keystrokes.

## TRIAL AND ERROR
## AND THE LEARNING PROCESS

You cannot harm the calculator by "pushing the wrong button". So test your ideas. If you are not sure of a procedure, try it. If it works, you have discovered something. If it doesn't, that too was worth learning.

As you move through procedures that you have already experienced, learn to trust your first instinct and to act upon it with speed and confidence....if only from the knowledge that no harm has been done if that was not the correct move.

Set yourself goals as you move through this course. Some things you really need to know. Work on them. Work on your dexterity and speed in achieving them. Other features are just applications: some you may need, others you don't. Be selective as you go along. Glance, skip, and study as the subject requires.

We may speak of four stages in the learning process:

1) **DISCOVERY AND AWARENESS:** "Hey, that's interesting!" "It makes sense. How come I didn't think of it before?" "I could use that!" You have just seen someone ride by on a bicycle.

2) **AWKWARDNESS AND FRUSTRATION** as you try to implement something that seemed simple at first sight. "I'll never learn how to keep my balance on that thing. I wonder that anybody could." You know you are at that stage when you become convinced that your machine isn't working properly.

3) **SKILLFULNESS:** with careful attention you get it right most of the time. But it doesn't take much to throw you back to the previous stage.

4) **INTEGRATION:** now it comes naturally and does not require much mental effort on your part. You've learned how to ride the bicycle, you can wave as you go by, you enjoy the mastery, and you take it for granted.

It pays to come pretty close to integration on some of the basic procedures. Skillfulness is not enough for such fundamental calculations as finding the payments or the balance on a loan. These are essential building blocs, and integration will allow you to shift your attention to what comes afterwards, to understanding the problems and applications that use these simple procedures or modify them for their own purpose.

This is why, along with the more systematic progression followed in these pages, relaxed, casual, fun-filled attempts to solve simple problems of your own making, with no pressure to get the answers right, can be an important part of the learning process.

# UNIT 2

# THE TRADITIONAL REAL ESTATE LOAN

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# THE TRADITIONAL REAL ESTATE LOAN

This unit provides essential routine information on the most common exchange of money in time: the amortized loan. Other units will bring us back to the traditional real estate loan with more details and specific applications, but this unit provides the basic building block both for the understanding of the logic and for the practical ability to compute answers.

# THE TRADITIONAL REAL ESTATE LOAN

We have considered a major financial concept:

**EXCHANGING MONEY IN TIME.**

We have seen a major calculator procedure:

**DATA then QUESTION.**

Let us put the two together and start solving problems on the traditional real estate loan.

The traditional real estate loan is

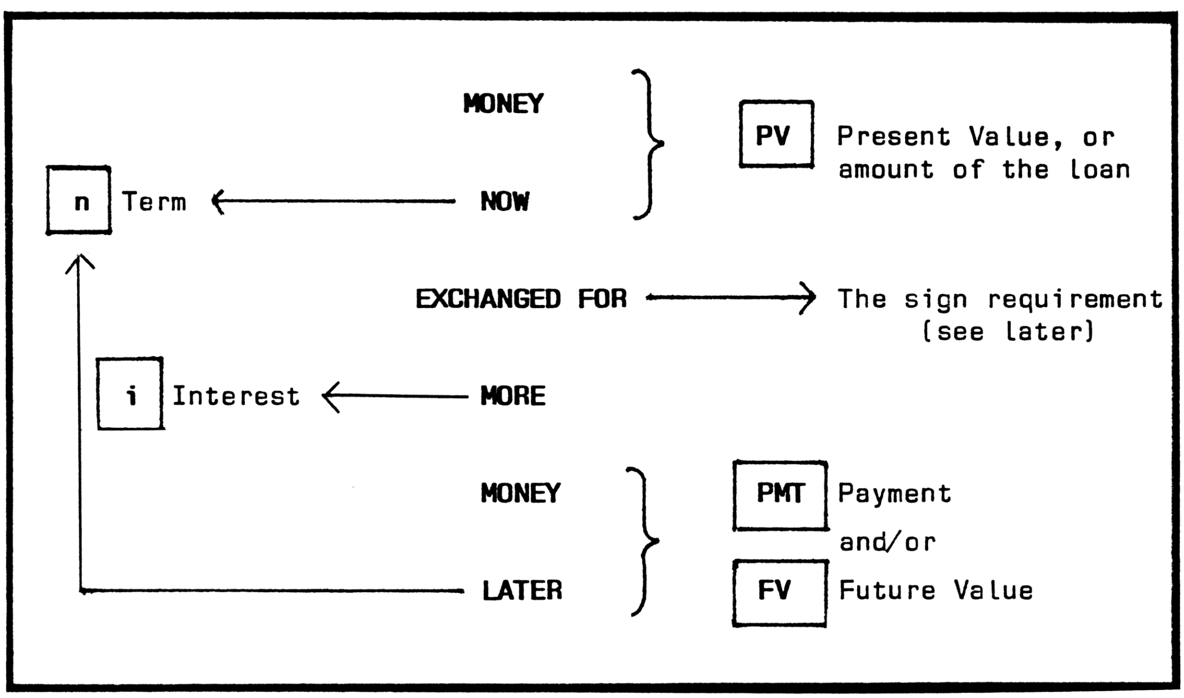
**MONEY NOW EXCHANGED FOR MORE MONEY LATER.**

The various words of this definition are represented by the

**FIVE TOP ROW FINANCIAL KEYS:**

| | | | | |
|---|---|---|---|---|
| **n** | **i** | **PV** | **PMT** | **FV** |

They are the five most important keys on our calculator.



## n i PV PMT FV

## AND THE TRADITIONAL REAL ESTATE LOAN

| n | **TERM** of the loan and **NUMBER OF PAYMENTS.** With a 30 year loan, 360 represents the <u>number of payments</u> and the <u>number of months</u>. |
|---|---|
| i | **INTEREST** as a percentage rate **PER PERIOD:** For a loan with monthly payments, an annual rate of 12% means an interest rate of 1% per month. |
| PV | **PRESENT VALUE** or the **AMOUNT OF THE LOAN.** |
| PMT | **PAYMENT** amount. |
| FV | **FUTURE VALUE** or the **BALANCE OF THE LOAN.** |

**THE TIME REQUIREMENT.**

These definitions bring out a fundamental requirement: **n**, **i**, and **PMT** must be on the same **TIME SCALE:** if we have <u>monthly</u> payments, the interest must be expressed <u>per month</u>, and **n** will measure a number of <u>months</u>.

The 5 regular cash flow keys allow us to answer a number of questions involving financial situations. We are applying them first to the traditional real estate loan paid off with monthly payments. We will later broaden our definitions and applications to include a greater variety of financial circumstances.

Let us begin with two basic problems: calculating the payments and the balance of a loan.

**IT IS ESSENTIAL TO ACQUIRE CONSIDERABLE FACILITY IN SOLVING FOR THE PAYMENTS AND THE BALANCE, AS THOSE BASIC PROCEDURES ARE THE FIRST STEP IN MANY OTHER CALCULATIONS.**

## PAYMENTS ON AN AMORTIZED LOAN.

### MODEL PROBLEM

Calculate the monthly payments on a $78,000 loan, 9% interest, amortized over 30 years.

### SOLUTION

| | | | | |
|---|---|---|---|---|
| DATA | 78000 | PV | | |
| | 9 | BLUE | i | Display shows 0.75% monthly rate |
| | 30 | BLUE | n | Display shows 360 months. |
| | 0 | FV | | 0 in FV = fully amortized. |
| QUESTION | | PMT | | The **ANSWER**: $627.61. |

**If your answer is $622.93**, you also have the word **BEGIN** in small characters in your display. This status indicator shows that your calculator has been told to interpret every payment as being made at the beginning of each period instead of at the end. The first payment is made the very day you get the loan! So of course your payments are a little lower.

To tell the calculator that your payments are made at the end of each month, just press: **BLUE** **END** [**BLUE 8**]

You may now press the **PMT** key again, and get the correct answer—no need to key the data in again.

## UNDERSTANDING

**78000 PV** Puts the number 78,000 in the Present Value memory.

**9 BLUE i**

**30 BLUE n** Used with **n** and **i**, the **BLUE** key transforms the monthly data (30 years, and 9% per annum) into the monthly equivalent (360 months and 0.75% per month), and stores that monthly data in **n** and **i**.

We could also key the monthly data directly into **n** and **i**, by pressing **360 n** and **0.75 i**. Of course, that option would not be open with a 10.25% interest loan amortized over 27 years!

**0 FV** Tells the calculator that we want to find the payments that FULLY AMORTIZE the loan: there is no outstanding balance at the end of the 360 payments.

**0 FV** allows us here not to clear between financial problems. Though we do not need to press **0 FV** if we already have 0 in **FV** we will at this stage train ourselves to press **0 FV** as a matter of routine when dealing with fully amortized situations. (More on the subject later).

**PMT** This is the **QUESTION.** The word "**running**" flashes briefly in half size characters in the display while the calculator calculates. Almost immediately, we get the **ANSWER.**

The answer is negative: I receive the positive **PV,** I pay back the negative **PMT.** The change in sign represents the **exchange** of money. More on the **SIGN REQUIREMENT** later.

The 5 regular cash flow keys are both **DATA** keys and **QUESTION** keys. Pressing **PMT** is interpreted as a question because no NEW data has been put in the display.

| n | i | PV | PMT | FV |
|---|---|---|---|---|
| 360 | 0.75 | 78,000 | ? | 0 |

First, we key in the **DATA.** Then we ask the **QUESTION.** This instructs the calculator to explore the other four financial memories, and to calculate the only **PMT** amount that is consistent with the data it finds: the only amount that is fair to the borrower and the lender on the basis of the terms represented by the data.

Calculating the payment **BALANCES THE BOOKS** on the transaction. The calculator now holds full information on the **EXCHANGE OF MONEY IN TIME.**

## PRACTICE

Practice is essential. Some need more practice than others. Most of us need to forget things a few times before they become second nature. So going over the same practice problems a number of times, with some time to forget in between, may be an important part of your strategy.

- Practice the model problem until the keystrokes come easily to your finger. You may want to take each line separately and repeat it 5 or 6 times or until your finger no longer has to search for the sequence. Acquiring a certain amount of speed and dexterity is important:

| | | |
|---|---|---|
| 9 | BLUE | i |
| 9 | BLUE | i |
| 9 | BLUE | i |
| 9 | BLUE | i |

| | |
|---|---|
| 78000 | PV |
| 78000 | PV |
| 78000 | PV |
| 78000 | PV |

| | | |
|---|---|---|
| 30 | BLUE | n |
| 30 | BLUE | n |
| 30 | BLUE | n |
| 30 | BLUE | n |

| | | |
|---|---|---|
| 0 | FV | PMT |
| 0 | FV | PMT |
| 0 | FV | PMT |
| 0 | FV | PMT |

- Use the same keystrokes with the following problems.
  Make sure you press **0 FV** even though with this particular sequence of problems omitting **0 FV** should still give you the correct answer.
  Key the data in the order in which it appears in the problem--the order is not important, so you don't have to stay with the order of the model.

- If you get an incorrect answer, just start the problem over again--and make sure you do not have **BEGIN** in the display (Unit 2, page 4)

- Press **GOLD 2** if your display does not show 2 decimals (Unit 1, page 11)

- There is no need to clear between problems or after an incorrect answer.

## PROBLEMS

Calculate the monthly payments:

1) $135,000 loan, 13.5% interest, amortized over 30 years. (1,546.31)

2) $25,000 loan, 9% interest, 20 years. (224.93)

3) 30 year loan at 12.75% interest in the amount of $75,000. (815.02)

4) $96,000 loan, 30 years, 14.625 interest. (1,185.13)

5) $235,000 loan, 15% interest, 25 years. (3,009.95)

6) 8.5% interest, 30 years, loan of $15,000. (115.34)

Practice these problems and problems of your own making until the keystrokes come naturally to your fingers.

## CHANGING THE DATA

Let's key our $78,000, 9%, 30 year loan back in:

| n | i | PV | PMT | FV |
|---|---|---|---|---|
| 360 | 0.75 | 78,000 | | 0 |

As we key in the data, the **ONLY** thing we are doing is putting numbers into those four boxes, those four electronic memories. As we do so we eliminate any other number that might have been there.

So now if I key in **81000 PV**, I just replace the $78,000 loan with an $81,000 loan. If I key in **10 BLUE i**, I replace my 9% interest with 10%.

As I press the **PMT** key, the only thing the calculator knows is the data that is now in the four other financial memories. It doesn't know when the data was keyed in, in what order, or what data was there before. It just calculates the payment corresponding to the data that it finds.

**CHANGING DATA is as simple as KEYING IN THE NEW DATA.**

**I can change data** because I keyed in the wrong number: I just overwrite with the correct number.

**I can change data** because I have changed my mind, or the banker has changed his rate: I just overwrite with the latest data.

**I can change data** before I calculate an answer or after: I just overwrite with the new data and solve for the new answer.

Any number that I do not change remains in the financial memory and is used by the next financial calculation.

For instance:

| INSTRUCTIONS | KEYSTROKES | |
|---|---|---|
| A $66,000 loan: | 66000 PV | |
| Amortized over 30 years: | 30 BLUE n | |
| At 11% interest: | 11 BLUE i | |
| Well, let's make it 12%: | 12 BLUE i | |
| And let's increase the loan to $69,000: | 69000 PV | |
| What are the monthly payments? | 0 FV PMT | ($709.74) |

## RCL RECALLING THE DATA

Let us key in the model problem once again:

| | | | | | |
|---|---|---|---|---|---|
| 30 | BLUE | n | 78000 | PV | |
| 9 | BLUE | i | 0 | FV | PMT |

We now have numbers in the 5 financial memories: the 4 numbers we keyed in and the number we calculated.

We can check what those numbers are with the **RECALL** key: **RCL** [45]

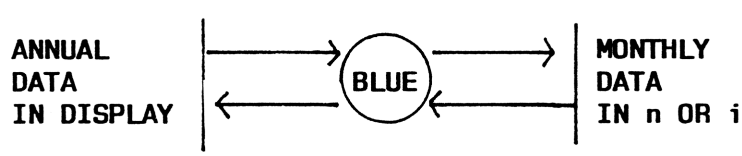
| | |
|---|---|
| RCL PV | $78,000 loan. |
| RCL i | 0.75%, the monthly interest rate. |
| RCL n | 360, the number of payments. |
| RCL FV | 0: the loan is fully amortized. |
| RCL PMT | $627.61: the payment amoount calculated. |

Recalling does not take a number out of the memory. It just duplicates the number in the display.

Now let us try:

| | |
|---|---|
| RCL BLUE i | 9%, our annual rate of interest. |
| RCL BLUE n | 30 years, the term of the loan in years. |

Used with **n** and **i** the **BLUE** key takes the yearly data in the display, and stores its monthly equivalent in **n** or **i**. It also takes the monthly data in **n** or **i**, and shows its annual equivalent in the display--however the monthly data remains unchanged in **n** and **i**.




Note: After you RECALL financial data, you need to press a financial key twice before it calculates an answer. The first time, the number you have in the display is merely stored in the financial memory.

**IF YOUR DATA IS CORRECT, YOU KNOW YOUR ANSWER IS CORRECT.**

In the field, with no answer between brackets at the end of the line, checking the data is the only way to confirm that the answer is correct.

However, you must explore the four data memories with the **RCL** key. Do not omit to check the **FV** memory, or any other financial memory where you had no specific data to store. When you ask the question, the calculator explores the four memories other than the one you select for your question. It takes into account all the data that it finds there, including any leftover data from a previous calculation!

**IF YOU HAVE THE WRONG ANSWER, YOU KNOW YOUR DATA IS INCORRECT.**

This is just looking at the previous statement from another angle. If you get the wrong answer, and we all do on occasion, our response should be: **"What data do I have wrong?"**

It should not be: "Where in the world did I punch the wrong key?" Of course you punched the wrong key sometime in the past, but the mistake is now in the present. You can look at the data in the calculator <u>now</u> and find the missing, misplaced, or incorrect piece of data. You can then correct just that piece of data by keying in the correct amount and question the calculator once again, this time for the correct answer.

Another way of putting it is to say that:

**THERE ARE NO INCORRECT ANSWERS....ONLY INCORRECT DATA.**

So let's learn from our mistakes and build confidence in ourselves and in the calculator by checking our financial data with the **RCL** key.

**REMINDER:**

The other possible cause of mistakes is signalled by the presence of the **BEGIN** indicator in the display when we are really dealing with the usual **END** situation. The calculator would then be instructed to give an incorrect interpretation to the correct data by assuming that the payments are made at the beginning of each period. Press **BLUE END** [BLUE 8] to select the normal **END** option and eliminate the unwanted **BEGIN** indicator.

## CHS CHANGE SIGN

We are going to need a new key: **CHS** [16] Next to the **FV** key.

It changes the sign of whatever number is in the display: a positive number becomes negative, a negative number becomes positive.

| Keystrokes | | Display |
|---|---|---|
| 789 | CHS | -789 |
| | CHS | 789 |
| | CHS | -789 |
| | CHS | 787 |
| | CHS | -789 |

We are going to need to key a negative number in the **PMT** memory:

789 CHS PMT

It is important to see the negative number in the display **before** we press the **PMT** key. The following keystroke is **not satisfactory:**

| | | |
|---|---|---|
| 789 | PMT | Puts positive 789 in **PMT**. |
| | CHS | Leaves -789 in the display. |

We would have -789 showing, but a positive 789 in **PMT**.

## BALANCE OF A LOAN
(Payment amount known)

$55,000 loan, 15% interest, monthly payments of $700.
Calculate the balance of the loan after 5, 10, and 4 years.

Here again it is quite obvious where the data needs to be stored and which of our 5 top row financial keys needs to be pressed to solve for the balance. There is just one requirement that might not be obvious:

### THE SIGN REQUIREMENT

We borrow $55,000 and we give back $700 amounts as partial repayment. If the $55,000 amount is keyed in as a **positive** number, the $700 amount must be keyed in as a **negative** number. The calculator would give us a negative amount if it calculated the **PMT**. We need to key in a negative amount when we impose the **PMT** amount. The difference in sign is what allows the calculator to know that the **PV** and the **PMT** are on opposite sides of the transaction. This is the SIGN REQUIREMENT.

With the **TIME** and **SIGN** requirements in mind, it just becomes a matter of keying in the **DATA** and asking the **QUESTION**.

### MODEL

$55,000 loan, 15% interest, monthly payments of $700.
Calculate the balance of the loan after 5, 10, and 4 years.

| | | | | |
|---|---|---|---|---|
| 55000 | | PV | | |
| 15 | BLUE | i | | |
| 700 | CHS | PMT | | |
| 5 | BLUE | n | FV | ($53,892.82, 5 years balance) |
| 10 | BLUE | n | FV | ($51,559.79, 10 year balance) |
| 4 | BLUE | n | FV | ($54,184.65, 4 year balance) |

The calculator gives **negative** amounts for **FV** also: the borrower gets the loan amount, and has to pay back the payments and the balance.

## PRACTICE

Practice the model until the keystrokes come easily, then turn to the practice problems. There is no need to clear as you move from one problem to another. There is no need to press **0 FV** except in problem 9 where you have a fully amortized situation.

7) $72,000 loan at 12%. What is the balance after 3 years if the monthly payments are $800? ($68,553.85)

8) $115,000 loan at 16% with monthly payments of $1,550.

| | |
|---|---|
| What is the balance after 5 years? | ($113,482.74) |
| 10 years ? | ($110,123.82) |
| 15 years ? | ($102,687.83) |
| 20 years ? | ($86,225.97) |
| 25 years ? | ($49,782.60) |

9) $195,000 loan amortized over 30 years. Key this information in only once, and check what the monthly payment would be at the following rates and at rates of your own choosing:

| | | | |
|---|---|---|---|
| 8% | ($1,430.84) | 12% | ($2,005.79) |
| 10% | ($1,711.26) | 15% | ($2,465.67) |

10) 11.75% interest loan of $225,000. Payments of $2,250 per month.

| | |
|---|---|
| Balance after 4 years. | ($222,145.18) |
| 5 years. | ($221,197.27) |
| 6 years. | ($220,131.77) |

Recall the data presently in the 5 financial memories.

11) What is the balance after 10 years on a 7% interest loan of $100,000 with monthly payments of $750 ? ($71,152.53)

| | |
|---|---|
| With payments of $650? | ($88,461.01) |
| With payments of $550? | ($105,769.49, negative amortization) |
| With payments of $850? | **($53,844.05)** |

12) $98,000 loan, 14.5% interest, payments of $1,250.

| | |
|---|---|
| Balance after 2 years. | (96,179.69) |
| Balance after 3 years. | (95,052.81) |
| Balance after **7 months**? | ($97,522.12) |
| Balance after 17 months? | ($96,765.83) |

What have we been keying into the **n** memory all along? The number of **months**! We used the **BLUE** key as a convenient way of transforming yearly data into the monthly equivalent.
Now we no longer need the **BLUE** key:
Balance after 7 months: **7 n FV**

## BALANCE OF A LOAN.
(Payment amount not known)

$100,000 loan, 14% interest, amortized over 30 years.
What is the balance after 5, 10, 15 years?

This problem needs to be solved in two stages:

1) Calculate the payments that **fully amortize (0 in FV)**.
2) Calculate the balance.

Clearly, the balance of a loan at a given time depends on the amount of the payments: the higher the payments, the lower the remaining balance. In our problem, we are not directly given the amount of the payments, but "amortized over 30 years" is an indirect way of telling us what the payments are: the payments are such that they would fully amortize the loan in 30 years. We are going to use this information to calculate the payments, <u>and</u> <u>then</u> use our knowledge of the payments to calculate the balance.

### MODEL

$100,000 loan, 14% interest, amortized over 30 years.
What is the balance after 5 years, 10, 15 years?

| | | | | |
|---|---|---|---|---|
| 100,000 | PV | | | |
| 14 | BLUE | i | | |
| 30 | BLUE | n | | |
| 0 | FV | | PMT | ($1,184.87: PMT.) |
| 5 | BLUE | n | FV | ($98,430.61: 5 year balance) |
| 10 | BLUE | n | FV | ($95,283.63: 10 year balance) |
| 15 | BLUE | n | FV | ($88,971.61: 15 year balance) |

Note: **The BALANCE is NOT the BALLOON PAYMENT.** (See Unit 3, page 16)
To calculate the balloon payment, add one payment as follows:

| | | | |
|---|---|---|---|
| RCL | PMT | + | ($90,156.48: balloon after 15 years) |

## ROUNDING THE PAYMENT AMOUNT

**GOLD | RND | PMT**

We have just calculated a payment amount of $1,184.87, but internally, inside the payment memory, the number still has additional hidden decimals that we will learn how to explored in the next Unit.

To eliminate those extra decimals and round the amount in the **PMT** memory to an exact number of dollars and cents, we may press the **ROUND** function: **GOLD RND** [GOLD and the PMT key]. This rounds the number in the display to what is actually showing in the display. We then need to press that rounded number back into the **PMT** memory by pressing the **PMT** key again.

We may think of the procedure as **GOLD PMT PMT.** Performed when the payment amount is showing, it assures that no hidden decimals will interfere with the accuracy of a Future Value calculation.

Previous problem with the rounding routine:

| | | | | |
|---|---|---|---|---|
| 100,000 | PV | | | |
| 14 | BLUE | i | | |
| 30 | BLUE | n | | |
| 0 | FV | | PMT | ($1,184.87: PMT) |
| GOLD | RND | PMT | | **PMT rounded to exactly $1,184.87** |
| 5 | BLUE | n | FV | ($98,430.96, or 16 cents more) |
| 10 | BLUE | n | FV | ($95,284.08, or 46 cents more) |
| 15 | BLUE | n | FV | ($88,972.67, or $1.06 more) |

Even in the worst of cases the difference is very small. We will not include this routine in our calculations on a regular basis, but users who feel the need for the extra accuracy should practice including it whenever payments are calculated and then used to calculate a balance.

## PRACTICE PROBLEMS

This essential calculation needs to be practiced until it becomes second nature. Make sure you are familiar with the model, then turn to the practice problems. There is no need to clear between calculations if you press **0 FV** before you calculate the payment amount that **fully amortizes** the loans.

For the first few problems, the answer provides the payment amount that needs to be calculated. For later problems, the answer is given for the balance only, though the correct payment must still be calculated first.

| | | |
|---|---|---|
| 13) | $50,000 loan, 12% interest, fully amortized in 30 years. | ($514.31) |
| | Balance after 5 years: | ($48,831.61) |
| | 15 years: | ($42,852.86) |
| | 25 years: | ($23,120.66) |
| 14) | $109,000 loan, 10%, 30 years. | ($956.55) |
| | Balance after 10 years. | ($99,122.44) |
| 15) | $88,500 loan, 14.25%, 25 years. | ($1,082.29) |
| | Balance after 5 years: | ($85,779.08) |
| | 10 years: | ($80,254.13) |
| | 15 years: | ($69,035.49) |
| 16) | $135,000 loan, 30 years, 8.75%. | ($1,062.05) |
| | Balance after 6 years. | ($127,679.53) |
| 17) | $25,000 loan, 9% interest, 30 years. | ($201.16) |
| | Balance after 12 years: | ($21,480.78) |
| | 18 years: | ($17,675.77) |
| | 3 years: | ($24,438.03) |
| 18) | $52,000 loan, 12% interest, amortized over 30 years, all due in 3 years. | |
| | What is the balance? | ($51,359.08) |
| | What is the balloon payment? | ($51,893.96) |
| 19) | $195,000 loan, 16% interest, amortized over 30 years. | |
| | What is the balance after 5 years? | ($192,972.08) |
| 20) | $15,000 loan, 9% interest, 25 years. | |
| | Balance after 10 years. | ($12,410.88) |

- Practice with loans of your own choosing.
- Use the **RCL** key to check the data in the 5 financial memories.
- Choose one problem and practice solving it 5 or 6 times in succession with less and less hesitation between keystrokes.
- Run through these problems again, this time including the rounding routine. Notice the size of the discrepancy.

Taking stock.

## **PAYMENT** and **BALANCE**

We have learned how to calculate the payments and the balance on the traditional real estate loan.

This represents absolutely vital information and is a frequent first step in more advanced applications. These calculations should be allowed to sink in and become second nature. See Unit 4 for further details and options.

## **SUMMARY**

- **TIME REQUIREMENT: n, i,** and **PMT** must be on the same time scale. With monthly payments, **n** and **i** must be expressed in months. Using the **BLUE** key with **n** and **i** automatically transforms yearly data into the monthly equivalent. It takes care of the time requirement under normal conditions.

- **SIGN REQUIREMENT:** with **PV** as a positive number, the calculator gives negative values for **PMT** and **FV** because these amounts are on the opposite side of the transaction--I get the loan and give back the payments and the balance; or vice-versa, I make the loan and get back the payments and the balance. Similarly, when we key in a value for **PMT** or for **FV,** we are going to key it in as a negative amount. Use the **CHANGE SIGN** key **CHS** for that purpose.

- **FULLY AMORTIZED** means that there is **nothing** left over after the payments are made. This is communicated to the calculator by having **0** in **FV.**

- To calculate the **BALANCE** of a loan, we need to know what the **PAYMENTS** are. If we do not know what the payments are, then we need to calculate those payments before we can press **FV** to solve for the balance.

- To find the **BALLOON PAYMENT,** add one payment to the balance.

- **IF THE DATA IS CORRECT, THE ANSWER IS CORRECT.** We can always check the data with the **RCL** key. (Of course, with payments made in arrears, we do not want the word **BEGIN** in the the display)

- **CHANGING DATA is as simple as KEYING IN THE NEW DATA**

# UNIT 3

## FORMAT

## MEMORY SYSTEM

## BASIC ARITHMETIC

# UNIT 3

# FORMAT

# MEMORY SYSTEM

# BASIC ARITHMETIC

The time has come to learn how to add!

This unit presents a number of basic arithmetic functions, including percentage calculations. It also gives a basic understanding of the memory system and how it is designed to make calculations easy and natural--with a little bit of practice.

The word 'basic' is important, as we do not seek at this stage to present a complete inventory of the features. We are concerned with immediate practical needs and with providing you with some time to become familiar with essential procedures before exploring the full power and convenience of the system.

See Unit 8 for a more systematic presentation.

## DISPLAY FORMAT

- In the normal setting, the number in the display shows 2 decimals (dollars and cents).
- The number actually in the calculator may have more than 2 decimals.
- The number visible in the display is rounded up or down to the closest approximation allowed by the number of decimals selected.
- To change the format, press the **GOLD** key, release it, then press the numeral for the number of decimals desired.

For instance, let's key into **i** a rate of 8.5%: **8.5 BLUE i**. We see **0.71** in the display. Then let's change the display format as follows:

| KEYSTROKES | | | DISPLAY |
|---|---|---|---|
| 8.5 | BLUE | i | **0.71** |
| | GOLD | 3 | **0.708** |
| | GOLD | 4 | **0.7083** |
| | GOLD | 9 | **0.708333333** |
| | GOLD | 0 | **1.** |
| | GOLD | 2 | **0.71** |
| | GOLD | ENTER | 70833333333 then 0.71 |

**GOLD ENTER** offers a convenient way to check on the internal number. The regular format reappears when **ENTER** is released.

Whatever the format of the display, **the full internal number of decimals is used in calculations.** So it is important not to key in **0.71** the second time you need the same 8.5% rate.

If you suddenly find yourself with too many or too few decimals and want to revert to the **NORMAL FORMAT**, press:

| **GOLD** | **2** |
|---|---|

## THE MEMORY SYSTEM

There are three major groups of memories:

- **5** regular cash flow **FINANCIAL MEMORIES:**
  **n, i, PV, PMT, FV** which we have already used extensively.

- **20 REGISTER MEMORIES:** a scratch pad and much more.

- **THE STACK:** the crossroad of all information, a powerful and flexible tool.

The content of all these memories is retained when the calculator is turned **OFF**--that's what "continuous memory" means.

Acquiring full control over the memories of the calculator is an objective we should set for ourselves. After a while, we should be able to check the content of any memory, change it, clear the memory, and shift data around from one memory to another.

The following explanations give the essentials only. A full exploration of the features of the REGISTER and of the STACK is reserved for later. (See Unit 8)

## THE REGISTER

**STO** **RCL**

The **REGISTER** is a group of 20 memories (Register memories 0 to 19) in which we can store numbers for future use. They are accessed with the **STO** (Store) and **RCL** (Recall) keys. [44 and 45]

| | | | |
|---|---|---|---|
| 789 | STO | 1 | Stores 789 in memory 1, |
| 456 | STO | 0 | 456 in memory 0, |
| 19 | STO | 7 | 19 in memory 7. |
| RCL | 1 | | 789 back in display. |
| RCL | 7 | | 19 back in display. |
| 35 | STO | 1 | Replaces 789 with 35 in memory 1. |
| RCL | 0 | | 456 in display. |
| RCL | 7 | | 19 again in display. |
| RCL | 1 | | 35 in display. |
| RCL | 0 | | 456 again in display. |

As with the financial memories, the last number to be keyed into the memory entirely replaces whatever number was there before and can be RECALLed any number of times.

Use the **REGISTER** as a scratch pad to store numbers you may need later in your calculations, telephone numbers, etc. Other uses of the Register will be introduced later.

## BASIC ARITHMETIC

Preliminary exploration

The calculator does not have an equal sign. This is because even arithmetic operations follow the Golden Rule:

**DATA** then **QUESTION**

We first key in the **DATA**, then we ask the **QUESTION**. The **ANSWER** appears without any need to press an equal key.

If I want to add 21 and 17, these two numbers are my **DATA**, and I have to communicate them to the calculator. If I press the two numbers in succession, I get 2,117. So I have to tell the calculator where one number ends and the other one begins. This is done by pressing the long **ENTER** key. [36]

| 21 | ENTER | 17 |
|---|---|---|

The **DATA** is now in the calculator. I can now ask the **QUESTION**. When I press the + key the answer, 38, appears in the display.

I can execute in this way all two-number functions, and in particular any one of the four basic operations:

| **PROBLEM** | **KEYSTROKES** | | | | **ANSWER** |
|---|---|---|---|---|---|
| 21 + 17 | 21 | ENTER | 17 | + | [38] |
| 21 x 17 | 21 | ENTER | 17 | x | [357] |
| 21 − 17 | 21 | ENTER | 17 | − | [4] |
| 21 : 17 | 21 | ENTER | 17 | ÷ | [1.24] |

These arithmetic operations, like all other functions requiring two numbers, are performed on the content of the two lower memories of the **STACK**.

## THE STACK

**The Stack** is a group of four memories named **x**, **y**, **z**, and **T** stacked one on top of the other. The lowest memory, **x**, is what we see in the **DISPLAY**.

| | | |
|---|---|---|
| Top memory | T | |
| | z | |
| | y | |
| Display | x | |

Arithmetic calculation performed on content of **x** and **y** memories.

Let's look at the **STACK** in time as we add two numbers:

| | | | | | |
|---|---|---|---|---|---|
| The Stack | T | | | | |
| | z | | | | |
| | y | | 21 | **21** | |
| Display | x | 21 | 21 | **17** | **38** |
| Keystrokes: | | **21** | **ENTER** | **17** | **+** |

So before we tell the calculator to add, we need to have the two numbers—the **DATA**—stacked in the **x** and **y** memories one above the other as they would be if we were performing the operation on paper. This is what is achieved by pressing **21 ENTER 17**.

When we press the **+** key, the numbers in **x** and **y** are replaced by their sum visible in the **x** memory. **21** and **17** have disappeared from the Stack.

$$\begin{array}{r} 21 \\ +\ \underline{17} \\ 38 \end{array}$$

Operations are performed as we naturally do them, not as we write them down with the = sign.

The full power and flexibility of the Stack, its ability to become an automatic memory for partial answers that float up and down until they are needed, will be studied more systematically in Unit 8. At this stage, let us concentrate on the more fundamental aspects.

Let's practice a few operations:

There is **no need to clear** between calculations. Just ignore the previous answer if you do not need it.
(Answers are provided between brackets).

| | | | | | |
|---|---|---|---|---|---|
| 56.92 | 9,324 | 38.75 | 15.36 | 39 | 342 |
| + 37.27 | + 1,207.56 | + 61.25 | - 11.45 | x 74 | - 123 |
| (94.19) | (10,531.56) | (100.00) | (3.91) | (2,886) | (219) |

We now have 219 showing in the display. To add 150 to that number I do not need to press the **ENTER** key: I just key in my new piece of data (150), and press +. 219 was calculated by the calculator, and automatically moves up into the **y** memory when I key in my new data.

(With 219 still in display) 150 + (369)

WE PRESS THE **ENTER** KEY ONLY WHEN WE KEY IN TWO NUMBERS OURSELVES IN IMMEDIATE SUCCESSION.

With many chain calculations only the first two numbers are keyed in by us in immediate succession: they alone need to be separated by pressing the **ENTER** key:

| **DATA** | | **KEYSTROKES** | | **THE DISPLAY** |
|---|---|---|---|---|
| Add | 64 | 64 | ENTER | 64.00 |
| | 91 | 91 | + | 155.00 |
| | 32 | 32 | + | 187.00 |
| Now multiply by 12 | | 12 | x | 2,244.00 |
| and divide by 8 | | 8 | ÷ | 280.50 |

We now know how to key in an interest rate of 9 7/8.
We calculate 7/8 and we add 9:

| | |
|---|---|
| 7 ENTER 8 ÷ | (7/8 is 7 divided by 8) |
| 9 + | (9.88 in display, but check hidden decimals) |

## CHECKING THE Y DATA $\boxed{x \rightleftarrows y}$

With all two-number operations performed on the content of the **x** and **y** memories, it is important to be able to check the content of the **y** memory. This can be done by pressing the **x⇄y** key [34].

**x⇄y** puts the **y** data in the **x** memory where it can be seen, and the **x** data in the **y** memory.

Pressing **x⇄y** twice is a convenient way of checking what we have in the **y** memory, and putting the data back in its original order.

Pressing **x⇄y** once re-establishes the correct order if we have the correct data in **x** and **y**, but in the wrong order.

Example:

| 32 ENTER 789 |
|---|

Now press **x⇄y** a number of times to see the two numbers alternatively in the display. Stop when 32 is showing. You may now press ÷ to divide 789 by 32.

| | | | | | | |
|---|---|---|---|---|---|---|
| y | | 32 | 789 | 32 | 789 | |
| x | 32 | 789 | 32 | 789 | 32 | 24.66 |

**32 ENTER 789 x⇄y x⇄y x⇄y ÷**

## PRACTICE PROBLEMS

There is no need to clear between problems.

1) Add the following amounts and multiply the result by 12:

| | | | | | |
|---|---|---|---|---|---|
| 789 | 56 | 15.50 | 197 | 32.51 | 300 |
| 456 | 44 | 30.75 | 372 | 19.66 | 45 |
| 123 | 82 | 65.45 | 507 | 12.19 | 210 |
| 632 | 17 | 95.00 | 229 | 79.37 | 173 |
| (24,000) | (2,388) | (2,480.40) | (15,660) | (1,724.76) | (8,736) |

2) Add or subtract as indicated.

| | | | | | |
|---|---|---|---|---|---|
| 950 | 33 | 77.31 | 357 | 19.65 | 713 |
| -212 | 13 | -51.03 | -112 | 12.96 | -502 |
| -197 | -29 | 89.61 | -513 | -25.00 | -121 |
| 721 | 57 | -45.13 | -374 | -16.33 | 419 |
| (1,262) | (74) | (70.76) | (-642) | (-8.72) | (509) |

3) Add or subtract as indicated.

13,952 + 17,321 + 5,076 + 23,994 = (60,343)

532 - 64.25 + 145 - 278 - 56.77 = (277.98)

32.27 + 19.66 - 45.00 + 16.21 + 71.75 - 12.23 = (82.66)

4) Multiply as indicated and divide the answers by 17.

| | | | | | |
|---|---|---|---|---|---|
| 47 | 912 | 13,550 | 19.62 | 132 | 67.91 |
| x 21 | x 57 | x 113 | x 9.52 | x 75 | x 33.33 |
| (58.06) | (3,057.88) | (90,067.65) | (10.99) | (582.35) | (133.14) |

5) Calculate the following interest rates (annual). Check hidden decimals.

| | | | | |
|---|---|---|---|---|
| 11 7/8 % | 8 5/8 % | 13 1/8 % | 10 3/8 % | 9 5/8 % |
| (11.88) | (8.63) | (13.13) | (10.38) | (9.63) |
| (11.875) | (8.625) | (13.125) | (10.375) | (9.625) |

(Press **GOLD 2** to switch back to regular format)

## AUTOMATIC MEMORY FOR PARTIAL ANSWERS

The Stack makes it very convenient to perform an operation while another operation is in progress. The sub-total or partial answer automatically floats up into the **Z** memory while the second operation is being performed, and floats back down into the **y** memory when it is needed.

A Find the sum of the following numbers:

| DATA | KEYSTROKES | | | | |
|---|---|---|---|---|---|
| 252.75 | 252.75 | ENTER | | | |
| 31.96 | 31.96 | + | | | |
| 164.15 | 164.15 | + | | | Sub-total: 448.86 |
| 87.32 | | | | | |
| 87.32 | | | | | |
| 87.32 | 87.32 | ENTER | 5 | x | 87.32 x 5 = 436.60 |
| 87.32 | | + | | | Sub-total: 885.46 |
| 87.32 | | | | | |
| 359.26 | 359.26 | + | | | Grand total: 1,244.72 |

Notice the ease with which the addition is interrupted to perform a multiplication and resumed by adding the product back in.

B Find the sum of the following products:

| 72 | 94 | 61 | 37 |
|---|---|---|---|
| x15 | x66 | x51 | x69 |

Each multiplication can be performed and added to the sum of the previous products as follows:

| | | | | | | |
|---|---|---|---|---|---|---|
| 72 x 15 | 72 | ENTER | 15 | x | | |
| 94 x 66 | 94 | ENTER | 66 | x | + | |
| 61 x 51 | 61 | ENTER | 51 | x | + | |
| 37 x 69 | 37 | ENTER | 69 | x | + | 12,948 grand total. |

This feature is studied in greater detail in Unit 8

## PERCENTAGE

### MODEL PROBLEM

You buy a $789.00 refrigerator.
Calculate the 6% sales tax.
What total amount do you have to pay?

| | |
|---|---|
| 789 ENTER 6 | The **DATA.** |
| % | $47.34 sales tax. |
| + | $836.34 total cost. |

- We enter the two numbers in the **x** and **y** memories of the Stack.
- Pressing **%** calculates the tax.
- The base number (789) remains in the **y** memory. Pressing **+** adds it back to the sales tax to give the total cost.

The Stack.

| | | | | | |
|---|---|---|---|---|---|
| y | | 789 | 789 | 789 | |
| x | 789 | 789 | 6 | 47.34 | 836.34 |
| The keystrokes: | 789 | ENTER | 6 | % | + |

If the store offers a 15% discount, the keystrokes are as follows:

| | |
|---|---|
| 789 ENTER | Initial number ENTERED. |
| 15 % − | 15% discount calculated and deducted. |
| 6 % + | 6% tax calculated and added ($710.89) |

These examples should provide some feel for how natural and close to our normal thinking pattern are the keystrokes made possible by the Stack.

Study the following **percentage** problems:

A A Growing Equity Mortgage (GEM loan) has initial payments of $456.00. They are to increase by 4% each year. What are the payments after each of the first 9 increases?

| | | | | | | |
|---|---|---|---|---|---|---|
| 456 | ENTER | 4 | % | + | | (474.24) |
| | | 4 | % | + | | (493.21) |
| | | 4 | % | + | Etc. | (512.94) |

B A $117,000 property is financed with a loan and a 20% downpayment. Store the downpayment in Register memory 0. Put the amount of the loan in Present Value:

| | | | | |
|---|---|---|---|---|
| 117000 | ENTER | | | |
| 20 % | | STO | 0 | ($23,400 downpayment in memory 0) |
| − | | PV | | ($93,600 loan in Present Value) |

C Calculate the **interest-only** payment on a $77,000 loan at 9.75% interest.

| | | |
|---|---|---|
| 77000 | ENTER | |
| 9.75 | % | (Annual interest only payment: $7,507.50) |
| 12 | ÷ | (Monthly interest only payment: $625.63) |

## PERCENTAGE DIFFERENCE and PERCENTAGE OF TOTAL

The calculator has two more percentage functions. They are introduced here for the record only, and will be studied in detail in Unit 8.

### PERCENTAGE DIFFERENCE $\boxed{\Delta\%}$ [24]

What percentage difference (percentage discount in this case) is there as we go from a face value of $31,000 to a discounted price of $23,000?

| Keystrokes | |
|---|---|
| 31000 ENTER 23000 | (**DATA** in **x** and **y**) |
| Δ% | (A 25.81% decrease) |

### PERCENTAGE OF TOTAL $\boxed{\%T}$ [23]

A broker is promised a commission of $25,000 if he sells a $470,000 property. Express that commission as a percentage of the total price:

| Keystrokes | |
|---|---|
| 470000 ENTER 25000 | (**DATA** in **x** and **y**) |
| %T | (5.32% commission) |

## PRACTICE PROBLEMS

6) Find the 5% sales tax and total cost of a $34.25 video game cartridge. ($1.71 and $35.96)

7) 6.5% sales tax and total cost of a $450.00 computer software package. ($29.25 and $$479.25)

8) 6% sales tax and total cost of a $27.50 book. ($1.65 and $29.15)

9) Calculate the negociated 6% commission on a $235,000 real estate sale. ($14,100)

10) Total cost of a $92.85 case of wine if you get a 12% discount but have to pay 6.5% tax on the discounted price? ($87.02)

11) An investor purchases a house for $83,000, remodels it, and increases the price by 30% when he puts it back on the market.
What is his asking price? ($107,900)

12) A property is purchased for $377,000 with 25% down and a 30 year loan at 10.5% interest. Store the amount of the downpayment in memory 0 and calculate the monthly payment.
($94,250 downpayment, $2,586.43 payment)

13) Same question with $130,000 property, 20% down, 30 year loan at 12%. ($26,000 and $1,069.76)

14) First year payment on a Graduated Payment Mortgage is $732.25. The payments increase by 7.5% every year for the next 5 years: what are those payments? ($787.17, $846.21, $909.67, $977.90, $1,051.24)

15) Calculate the interest only payment on a $91,000 loan at 11%. ($834.17)

16) Interest only payment on a $500,000 loan at 9.5%. ($3,958.33)

17) Calculate the 2 5/8 % seller's commission on a $466,000 sale. ($12,232.50)

## ARITHMETIC PERFORMED ON LOAN DATA

Arithmetic operations can be performed at any time on numbers that are brought into the **x** and **y** memories of the Stack by recalling them from some other calculator memories, in particular from the financial memories.

Calculate the monthly payments and the balance on a $37,000 loan at 13% interest, amortized over 30 years, due in 5 years.

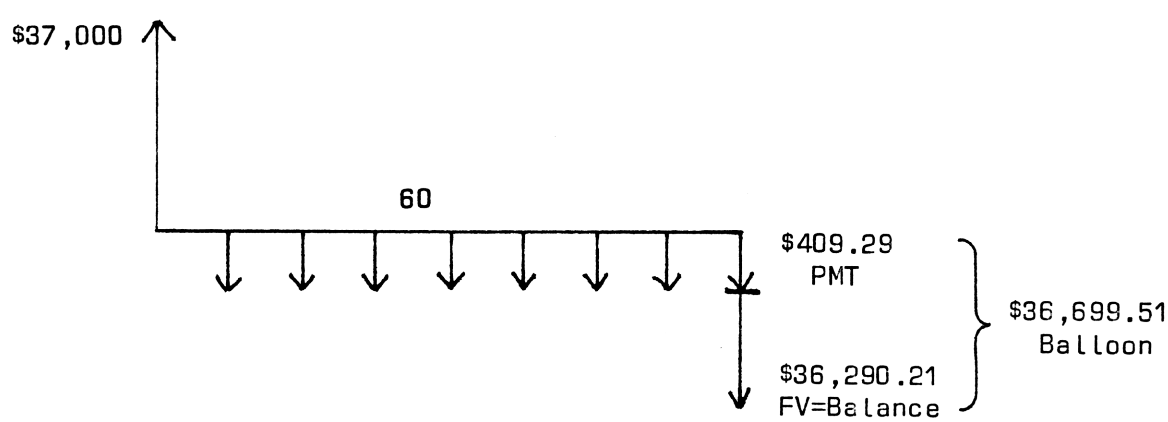
This leaves us with the following data in the financial keys:

| **n** | **i** | **PV** | **PMT** | **FV** |
|---|---|---|---|---|
| 60 | 1.08 | 37,000 | -409.29 | -36,290.21 |

A The **BALLOON PAYMENT** is **FV + PMT**:

RCL FV RCL PMT +

$36,699.51 balloon payment. Omit **RCL FV** if the balance is still showing in display.




Notice how the last payment and the balance are due at the same time when the loan is paid off. The total amount paid at that point in time is the legal definition of the balloon payment.

B **EQUITY BUILD-UP** or **PRINCIPAL REDUCTION**

This is the difference between the original balance and the balance at a later time, or **PV** - **FV**.

Because of different signs, we convert the negtive number into a positive one. (We could also press **+** instead of **CHS** -)

| | | | |
|---|---|---|---|
| RCL | PV | RCL | FV |
| CHS | - | | |

C **TOTAL AMOUNT** (**PRINCIPAL** and **INTEREST**) spent to pay off the loan:

The sum of all the checks is: (**PMT** x **n**) + **FV**

| | | | | |
|---|---|---|---|---|
| RCL | PMT | RCL | n | |
| x | | | | Brings **PMT** and **n** in Stack and Multiplies **PMT** by No of **PMT**: $24,557.63 spent on 60 payments. |
| RCL | FV | + | | Adds balance to previous total: $60,847.84 total pay-back. |

Because both numbers are negative, we can add without concern for the sign. Do not clear data!

D What is the **TOTAL AMOUNT OF INTEREST** paid on the loan?

The total amount paid ($60,847.84 still in the display) is both interest and principal reduction. Because the whole loan has been paid off, the loan amount in **PV** is the principal reduction. By deducting it from the total amount paid we get the total amount of interest.

The amount in the display is negative. It is convenient to change it back into a positive number (**CHS**) before deducting from it the positive value in **PV**.

| | | | |
|---|---|---|---|
| CHS | | | |
| RCL | PV | - | Deducts **PV** from total amount paid: $23,847.84 total interest over 5 years |

E **INTEREST PORTION** of payment and **INTEREST-ONLY** payment:

The interest portion of a payment is not affected by the amount of the payment. It is determined by the balance of the loan and the periodic rate. Any amount not claimed by the interest goes to reducing the principal balance.

1.08% stored in **i** (with hidden decimals) is the periodic rate.
1.08% of **PV** gives the interest portion of the first payment and the interest-only payment if the loan had been interest-only.
1.08% of **FV** (here the balance after 60 payments) gives the interest portion of the 61st payment.

| Keystrokes | Result | Keystrokes | Result |
|---|---|---|---|
| RCL PV<br>RCL i % | Interest-only payment and interest portion of 1st payment: $400.83. | RCL FV<br>RCL i % | Interest portion of 61st payment: $393.14. |

Despite the calculations we have done on the loan, the financial memories still hold all the original data. In fact, you may want to start over again with problem A, but now without looking at the keystrokes.

**ARITHMETIC** can also be performed **ON TWO SUCCESSIVE ANSWERS.**

This is because data keyed into financial memories is not pushed up into the **y** memory, but disappears from the Stack when a new number is put in the display. So the previous answer remains in **y**. Examples:

F Calculate the payments on a $92,000, 30 year loan at 13% interest. You should find $1,017.70. Now change the rate to 12.5%:

| Keystrokes | Result |
|---|---|
| 92000 PV 30 BLUE n | |
| 13 BLUE i 0 FV PMT | ($1,017.70) |
| 12.5 BLUE i PMT | (New payment: $981.88) |
| – | (Difference: $35.83) |

G There are three loans on a property, as follows:
Loan 1: $125,000, 11%, 30 years.
Loan 2: $55,000, 8%, 20 years.
Loan 3: 36,000, 14%, 30 years.
What is the total monthly debt service?

| | | |
|---|---|---|
| Loan 1: | 125000 PV 11 BLUE i | |
| | 30 BLUE n 0 FV PMT | ($1,190.40) |
| Loan 2: | 55000 PV 8 BLUE i | |
| | 20 BLUE n 0 FV PMT | ($460.04) |
| Loan 1 + 2: | + | ($1,650.45) |
| Loan 3: | 36000 PV 14 BLUE i | |
| | 30 BLUE n 0 FV PMT | ($426.55) |
| Loans 1 + 2 + 3: | + | ($2,077.00 total debt service) |

Arithmetic on loan data

## PRACTICE PROBLEMS

A $35,000 loan at 10% interest, amortized over 20 years, all due in 4 years.

18) Calculate the payment amount and the balance. ($337.76 and $32,293.43)

19) What is the balloon payment? ($32,631.18)

20) Total amount spent on paying off the loan? ($48,505.79)

21) Total amount spent on interest? ($13,505.79)

22) What would the payments be if the loan was interest only? ($291.67)

With all the loan data still unchanged in the financial memories, run through these calculations over again. It is important not to concentrate on the series of keystrokes, but on understanding where the data is--the financial memories--, and where it needs to be--**x** and **y**--in order to perform the arithmetic operation that you want.

23) What is the total interest paid on a $100,000 loan at 8.5% with monthly payments of $770.00 if the loan is kept for 15 years? ($116,289.84)

24) We have a $250,000 loan, 11.5% interest, amortized over 30 years.
Calculate the payment. ($2,475.73)
What is the payment if the rate goes down to 10.5%? ($2,286.85)
Calculate the difference between the two payments. ($188.88)

25) Calculate the payment and 10 year balance on a $45,000 30-year loan. The interest is 9.75%. ($386.62 and $40,760.42)
What is the balance if the payments are $450? ($27,961.51)
Give the difference between the two balances. ($12,798.91)

## RANDOM PRACTICE PROBLEMS

Here again it is essential to concentrate not on remembering a series of keystrokes but on controlling the data: shifting it to where you want it to be and performing on it the operations that you desire.

26) Add the following numbers. Store the answer in memory 1.

52 + 126 + 95 + 31 + 423 + 21 (748)

27) Add the result of these two multiplications and store in memory 2.

221 x 37 423 x 61 (33,980)

28) Divide 800 by 81. Explore the full range of hidden decimals, then switch back to normal format. (9.876543210)

29) A $185,000 property is financed with a 20% downpayment and a 13% loan with interest only payments. Store the downpayment in memory 3 and the monthly payments in memory 4. ($37,000 and $1,603.33)

30) A $15,000 expense is expected to increase by 4% per year. Give the amount for the next 4 years. ($15,600, $16,224, $16,872.96, $17,547.88.)

31) Using the data stored in memories 1 and 2, divide the answer to problem 27 by the answer to problem 26. (45.43)

32) Select a display format with no decimals.
Multiply 0.84 by 4. (3)
Multiply the answer by 3. (10)
Switch back to normal format.
If you are baffled, do the calculations again in normal format.

33) Calculate the **balloon payment** (not the balance!) on a $56,000 loan, 11% interest, monthly payments of $560 (**560 CHS PMT**) with a 4 year due date. ($53,762.05)
Calculate the total amount spent on the loan. ($80,082.05)
Calculate the total amount of interest paid during the four years. ($24,082.05)

Go over these last calculations with the data that is still undisturbed in the financial memories.

34) Calculate the 25% downpayment on a $235,000 property. ($58,750)
Check the number that you now have in the **y** memory and store it in memory 0. (The selling price goes into memory 0)

## THE MASTER CLEAR (MINUS---ON)

To clear EVERYTHING in the calculator turn the calculator **ON** while keeping the **MINUS** key pressed down--think of "minus" as "taking away":

- With the calculator turned **off**,
- Press the **MINUS** key--[30]--, and keep it pressed down as you
- Turn the calculator **ON** with a firm strike of the finger.
- As you release the **ON** key, the display should read "**Pr Error**".
- You may now release the **MINUS** key.
- Pressing any key--for instance **CLx**--eliminates the Error sign.

You now have **0.00** in the display and the calculator is exactly as it was when it left the factory: you have cleared all the memories, all programs, all status indicators, and brought the format back to its normal setting.

"**Pr Error**" stands for "Program error": it warns that any user-written programs have been lost.

> **USE THIS MASTER CLEAR PROCEDURE** whenever you find yourself with an unwanted feature that you do not know how to clear selectively--unwanted indicators, an unwanted format, a comma instead of the decimal point, etc.--provided that there is no user-written program or data in the calculator that you want to preserve.

To eliminate indicators selectively refer to **APPENDIX I**. At this stage you should be concerned with not having the word **BEGIN** in the display. Other indicators will be introduced before their presence could create a problem.

With a few HP-12Cs that perform well in other respects this master clear procedure—and the master check—do not turn the calculator on. If this is the case, try a few times, have somebody else try. If nothing works, just keep going: we will take care of any possible problem before it could become an inconvenience--or you may want to take a closer look at **Appendix I** or have the calculator checked and repaired.

# UNIT 4

# THE FIVE REGULAR CASH FLOW KEYS

## DISCOVERING FREEDOM

# UNIT 4

# THE FIVE REGULAR CASH FLOW KEYS

## DISCOVERING FREEDOM

So far, we have learned how to calculate the payments and the balance on a traditional real estate loan. This is only a small part of what the regular cash flow keys can do for us: they just about give us the full freedom to ask any question that has an answer concerning the five elements represented by **n**, **i**, **PV**, **PMT**, **FV**.

We are going to explore that freedom as we would an unknown land: by gradually expanding the area with which we are familiar. This implies acquiring much greater flexibility in answering questions concerning the traditional real estate loan, and will lead gradually beyond loan problems to other matters that can be analyzed with the same keys. As we do so we should remember that knowing that we have full freedom says it all: it is just a matter of discovering that freedom and claiming it as ours.

## DATA then QUESTION and the financial keys

### MODEL

I borrow **$50,000** and pay back with monthly payments of **$625** for **10** years, plus a balance of **$35,000** at the end of the term. **What interest rate** am I being charged?

| | | | |
|---|---|---|---|
| **50000** | PV | | |
| **625** | CHS | PMT | The **DATA** |
| **35000** | CHS | FV | |
| **10** | BLUE | n | |
| | i | | We ask the **QUESTION** and get the **ANSWER**: 1.13%, monthly rate. or 13.57% annual rate. |
| | 12 | x | |

Any number of problems can be solved by just keying in the **DATA** and asking the **QUESTION** as above.

All the **NUMBERS** that figured in the problem were keyed into the appropriate financial memory--and it was quite obvious where each one belonged. We wanted to know the **INTEREST RATE**, so **i** was the question.

The only precautions we took concerned two requirements:

The **TIME** requirement
The **SIGN** requirement

Under routine circumstances these two requirements can be met when keying in data by systematically using, as above:

**BLUE n**
**BLUE i** ⟶ the time requirement.

**CHS PMT**
**CHS FV** ⟶ the sign requirement.

## THE TIME REQUIREMENT

The calculator does not know about time any more than it knows that the payment it calculates is expressed in dollars, yens, pesos, or swiss francs. The answer is in dollars because I key in amounts that in my mind are dollars. The time is the year, the month, the week, because in my mind it is so. We could say that the calculator deals in **periods** and that we give a meaning to that period.

So the only requirement imposed by the logic of the 5 financial keys is **consistency:** the period can have one meaning, and one only:

**n**, **i**, and **PMT** must be on the same time scale.

With monthly payments, **n** and **i** must be expressed in months. If I key yearly data into **n** and **i**, the payments are yearly amounts. As we shall see later, the compounding period is also on the same time scale as **n**, **i**, and **PMT**.

In loan situations, under routine circumstances, the payments are monthly. But the data we get for the term and rate of the loan is traditionally expressed in years. So the routine keystrokes use **BLUE n** and **BLUE i** to transform yearly data into the monthly equivalent and meet the time requirement of consistency.

Of course we retain the freedom to key the rate and term directly into **n** and **i** without using the **BLUE** key provided that we express the data in terms of the appropriate period.

## THE SIGN REQUIREMENT

Amounts that occur on opposite sides of a transaction must be of different signs. This is the sign requirement.

One approach to meeting this requirement--this will not be ours--is to decide that money you get is positive and money you give negative. It makes sense, and it works, but it is easier to explain than to implement. Among the inconveniences:

- It forces us to make a decision each time we solve a problem.

- It means that we have to key in the data differently when we calculate the payments from the lenders point of view and when we are looking at the loan from the borrower's perspective, though the amount of the payments doesn't change.

- It leaves us in a quandary when our perspective changes in the middle of a problem, or when we are caught in the middle, with the borrower on one side and the lender on the other and we forget along the way which arbitrary point of view we decided to adopt, or again when the problem we are solving does not really lend itself to being expressed in terms of money in and money out.

Another approach--it will be ours here--is to realize that for most users, 99% of all problems have the Present Value on one side of the transaction, the Payments and Future value on the other. Why not systematically decide that we are going to have Present Value positive, and Payments and Future Value negative? There is no decision to make, just a routine to get accustomed to.

Of course, as with the **BLUE** key, we remain ready to modify our use of **CHS** with **PMT** and **FV** in those circumstances where Payments and Future Value are on opposite sides of the transaction. We can then switch back to the "Money in, positive--Money out, negative" approach. More on this later. [Unit 10]

AS A MATTER OF ROUTINE

When keying data in, we are systematically going to press **CHS PMT** and **CHS FV**, unless circumstances absolutely prevent us from doing so.

Keying in the data: routine procedure

**PRACTICE**

| n | i | PV | PMT | FV |
|---|---|---|---|---|

BLUE CHS

Just key in the data in the appropriate financial memory using **BLUE** with **n** and **i**, and **CHS** with **PMT** and **FV**. (No problems are being solved).

| **DATA** | **ROUTINE KEYSTROKES** | | |
|---|---|---|---|
| $500,000 loan. | 500000 | | PV |
| Payments of $450.00. | 450 | CHS | PMT |
| Balance of $95,000. | 95000 | CHS | FV |
| 16% interest. | 16 | BLUE | i |
| 14% interest. | 14 | BLUE | i |
| Payments of $2,000.00. | 2000 | CHS | PMT |
| A loan of $250,000. | 250000 | | PV |
| 30 year loan. | 30 | BLUE | n |
| 4 year due date. | 4 | BLUE | n |
| 10 year due date. | 10 | BLUE | n |
| Payable $1,500 a month. | 1500 | CHS | PMT |
| 9% interest. | 9 | BLUE | i |
| $17,000 loan. | 17000 | | PV |
| $185,000 balance. | 185000 | CHS | FV |
| $25,000 balance. | 25000 | CHS | FV |
| 2 year due date. | 2 | BLUE | n |

## CLEARING BETWEEN FINANCIAL CALCULATIONS.

There is at most one financial memory that needs to be cleared between financial calculations. The four others are either overwritten with new data or, for the QUESTION key, automatically overwritten with the new answer. We have chosen consciously to clear that one unused memory by keying **0** in it as needed. This approach offers maximum flexibility and control. It can be performed at any time before the question is asked, and even after if it was omitted and we have the wrong answer. When it is used there is no need to clear between financial calculations. But other options are available.

**GOLD CLEAR-FIN** [GOLD and 34] (2 keys only. Do not press **CLx**)

It puts **0** in each of the 5 financial memories, **n, i, PV, PMT, FV.**
**It does not clear the display.** In particular, if the last financial answer is still showing, it remains in the Stack and can be added to the next financial answer or subtracted as needed.

GOLD CLEAR-FINANCE can be selected as a **very convenient routine procedure** between financial calculations. It eliminates the need to press **0 FV** for a fully amortized calculation, **0 PMT** for a **PV-FV** calculation with no payments, or **0 PV** for a **PMT-FV** exchange with no **PV.**

The disadvantages of using GOLD CLEAR-FINANCE are:

- It clears 5 memories when at most only one needs to be cleared. In many circumstances some of the data that is cleared could have been used in the next calculation but now needs to be keyed back in again.

- It must be used before we begin keying in new data. If it is the only procedure available to the user, forgetting to use it means that all the data needs to be cleared and keyed in again to correct the omission.

- It does not clear the display. Some users find this disturbing and switch to pressing **GOLD CLEAR-REG** instead.

Provided the user retains the option of clearing a single financial memory where convenient, GOLD CLEAR-FIN can be used as an alternative to keying **0** in selected financial memories when strings of unrelated calculations are performed.

**GOLD CLEAR-REG** [GOLD and 35]

GOLD CLEAR-REGISTER clears all non-program memories in the calculator. It is a serious overkill when only one memory needs to be cleared. All numbers stored in the Register and the Stack are lost. But knowing that there can be no data to interfere with our calculations, and seeing **0** in the display, can have psychological advantages to a user running through a series of tedious unrelated financial calculations.

Beginning with our model for this Unit, let's review problems that can be solved by just keying in the data as explained in the previous pages, and asking the question.

## SOLVING FOR THE RATE

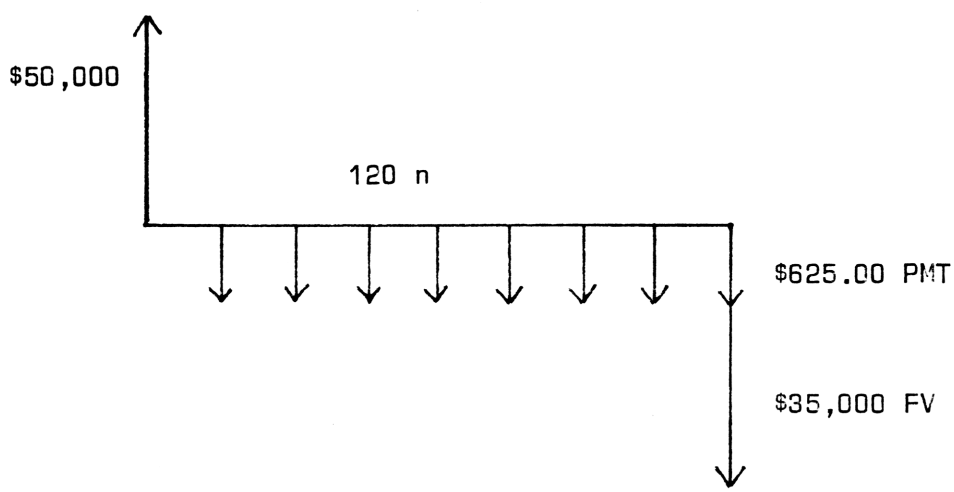
I borrow $50,000 and pay back with monthly payments of $625 for 10 years, plus a balance of $35,000 at the end of the term. What rate am I being charged?

| | | |
|---|---|---|
| 50000 | PV | |
| 625 | CHS | PMT |
| 35000 | CHS | FV |
| 10 | BLUE | n |
| | i | |
| | 12 | x |

The **DATA**

The **QUESTION** gives the **ANSWER**:
1.13%, monthly rate.
13.57% annual rate.

The data alone makes it possible to draw the full cash flow diagram and to put numbers on the various elements:




I am exchanging $50,000 for 120 payments of $625 and an extra lump sum payment of $35,000 at the same time as the last payment. The dollar amounts on opposite sides of the transaction are of opposite signs. The only number that remains to be calculated is the rate that makes sense of the exchange. Pressing **i** gives a monthly rate. We multiply it by 12 to get the annual rate: **12 x** or **RCL BLUE i**

## SOLVING FOR THE LOAN AMOUNT

> What traditional 30 year fully amortized loan can I afford to get with payments of $1,200 a month if the interest is 10% per year?

Let's put the numbers in the 5 financial memories.

| INFORMATION | KEYSTROKES | |
|---|---|---|
| The loan is fully amortized | 0 FV | |
| over 30 years | 30 BLUE n | |
| Payments of $1,200 | 1200 CHS PMT | |
| Interest is 10% | 10 BLUE i | |
| I want to know the loan amount | PV | (136,740.98) |

Here again, **DATA** then **QUESTION** fully solves my problem.

- I use the **BLUE** key with **n** and **i** because I want the monthly equivalent of the interest rate and amortization term.

- I press **CHS PMT** perhaps more as a matter of routine than of real necessity in this particular case.

- I press **0 FV** just in case there is left-over data in **FV** that needs to be erased. Before I press the question key, I want to be assured that I have the correct data in the four other financial memories. **0 FV** means **fully amortized** and is as much a part of the data as the amount of the payments.

- I solve for **PV** because that is what I want to know.

In the memories, the data appears as follows:

| n | i | PV | PMT | FV |
|---|---|---|---|---|
| 360 | 10/12 | | -1200 | 0 |

When I solve for PV, the calculator calculates the only loan amount that makes sense of my data: once again it BALANCES THE BOOKS on the rest of the data.

## SOLVING FOR THE TERM OF THE LOAN.

How long does it take to bring the balance of a $100,000 loan, 9% interest, with payments of $800, down to $60,000?

| | | | |
|---|---|---|---|
| 100000 | PV | | |
| 9 | BLUE | i | |
| 800 | CHS | PMT | |
| 60000 | CHS | FV | |
| | n | | (261 months) or |
| 12 | ÷ | | 21.75 years) |

Again, it is just a matter of keying in the data.

The calculator gives an exact number of months and stores that number in **n**. Let's now press **FV**.

| | |
|---|---|
| FV | ($59,799.57) |

We might expect to get back the $60,000 we keyed in initially. Instead we get $59,799.57. Why is that?

By making 261 payments we pushed the balance too far down. Let's see what balance we would have after just 260 payments:

| | | | |
|---|---|---|---|
| 260 | n | FV | ($60,148.46) |

The logic here is that until I make that 261st payment the balance has not been brought back down to $60,000. When I make that 261st payment, the balance drops below $60,000. The calculator gives the value for **n** that reaches the goal, and to know by how much we overshot the mark we have to recalculate the **FV**. Until we do this we have not fully balanced the books on the transaction.

## SOLVING FOR THE REMAINING TERM.

There is an old loan on a property with a balance of $46,322. The interest rate is 8% and the monthly payments are $385. What is the remaining term of the loan?

We have no way of reconstructing the past: it could have been a 30 year loan, or a 25 year loan, or an interest only loan where the borrowers exercised the "or more" clause for a while and brought the balance down. So let's concentrate on the present and the future, not the past.

The balance is the **indebtedness now,** and becomes our Present Value. We want to know how long it will take to fully amortize the loan, or how long it will take to bring the **FV** back to **0.** With these points in mind, we just key in the data and ask the question:

| | | | |
|---|---|---|---|
| 46322 | PV | | |
| 385 | CHS | PMT | |
| 8 | BLUE | i | |
| 0 | FV | n | 244 months. |

As with the previous problem, by making 244 full payments I may slightly overpay my obligation. Let's press **FV:** we should have **0** but we find a **positive** 70.01. That's how much the lender owes me back for having overpaid him!

I may now calculate the "balloon payment": **RCL PMT +.** The number in my display, $314.99, is the amount of the 244th payment. I can pay off the loan by making 243 payments of $385 and one final 244th payment of $314.99. So we completed the previous keystrokes as follows:

| | | | |
|---|---|---|---|
| | FV | | $70.01 positive amount. |
| RCL | PMT | + | $314.99, amount of last payment |

Keying in the data and asking the question

**PRACTICE**

The answer is given in parenthesis. Do not key it in!
Remember **CHS** with **PMT** and **FV** data.

1) A debt in the amount of $21,000 is to be paid off with monthly payments of $300. How many payments pay off the loan if the rate is 11%?

| n | i | PV | PMT | FV |
|---|---|---|---|---|
| (113) | 11/12 | 21,000 | -300 | 0 |

**RCL BLUE n** or **12 ÷** gives the answer in years: 9.42 years.

2) You agree to pay back a $100,000 loan with monthly payments of $1,500 for the next 10 years. What interest are you paying?

| n | i | PV | PMT | FV |
|---|---|---|---|---|
| 120 | (1.09) | 100,000 | -1,500 | 0 |

**RCL BLUE i**, or **12 x**, gives the annual rate of 13.12%.

3) You agree to pay back a $100,000 loan with monthly payments of $1,200 for the next 10 years, plus an extra amount of $70,000 at the end of the 10 years. What interest are you paying? (12.92% annual)

| n | i | PV | PMT | FV |
|---|---|---|---|---|
| 120 | (1.08) | 100,000 | -1,200 | -70,000 |

4) The buyer of a home needs to borrow $115,000 to complete the transaction. He qualifies for a traditional amortized loan at 13.5% interest. He wants to bring the balance down to $70,000 in 6 years in order to use the balloon payment on a second trust deed he owns to pay off the loan and own the property free and clear. What monthly payments does he need to make?

| n | i | PV | PMT | FV |
|---|---|---|---|---|
| 72 | 13.5/12 | 115,000 | (-1,702.75) | -70,000 |

5) How much can you borrow at 9% interest if you are willing to pay $800 a month for the next 7 years, plus an extra $15,000 at the end of the 7 years?

| n | i | PV | PMT | FV |
|---|---|---|---|---|
| 84 | 9/12 | (57,730.85) | -800 | -15,000 |

6) A homeowner has a loan on his property for which he is paying $532 a month. He knows that the rate is 7.75% and that the present balance is $31,200. As he inherited the property from distant relatives, he does not know when the loan was created or the original amount of the loan. How long will it take to pay off the loan?
(74 months, or 6 years and 2 months)

| n | i | PV | PMT | FV |
|---|---|---|---|---|
| (74) | 7.75/12 | 31,200 | -532 | 0 |

7) How much can you borrow with monthly payments of $2,000 if you can get a 30 year loan at 11%? ($210,012.69)

8) How long does it take to reach a $50,000 balance on a $100,000 loan at 16% interest with payments of $1,345? (307 months, or over 25 years)

9) There is a $44,000 balance on a 9.5% loan. The payments are $415 per month. What is the remaining term of the loan? (232 month, 19.33 years)

10) You are buying a discounted mortgage for $18,000. The note promises monthly payments of $350 for 6 years, plus a balance of $23,000 at the end of the term. What yield are you getting on your purchase?
(2.11% per month, 25.34% annually)

11) How much can you borrow at 8% interest with monthly payments of $1000 for 10 years, plus a balance of $100,000 at the end of the term?
($127,473.83)

12) You have a $37,000 loan at 9% interest. What monthly payments will leave you with a balance of $10,000 after 5 years? ($635.48)

Because of the ease with which these problems are solved, the calculator may become an extension of your hand, and you may find yourself keying in numbers as they are expressed by your broker, clients, or banker. You will be ready with the answer even before the question is formulated.

## INTEREST-ONLY LOAN

Calculating the monthly payments on an interest-only loan requires very simple arithmetic and we have already solved the problem in two different ways. We will review them here and add a third.

### A SIMPLE ARITHMETIC

Interest-only monthly payments on a $47,500 loan at 14%?

We may take 14% of $47,500, which gives us the annual interest, and divide by 12 to get the interest-only payment:

| Keystrokes | | Result |
|---|---|---|
| 47500 | ENTER | |
| 14 | % | (Annual interest: $6,650) |
| 12 | ÷ | (Monthly interest-only payment: $554.17) |

### B USING MONTHLY RATE STORED IN i

This approach is particularly useful when we already have an amortized loan in the financial memories, and we want to know what difference an interest-only situation would make. The rate already stored in **BLUE i** is the annual rate divided by 12, for instance 1.5% in the case of an 18% loan. Taking 1.5% of the amount of the loan gives the interest-only payment. (This is also the interest portion of the first amortized payment).

Calculate the monthly payment on a $47,500 loan
at 14% interest, amortized over 30 years.
What would the payments be if the loan was interest-only?

| Keystrokes | | | | | | Result |
|---|---|---|---|---|---|---|
| 14 | BLUE | i | 30 | BLUE | n | |
| 47500 | PV | 0 | FV | PMT | | (Amortized payment: $562.81) |
| RCL | PV | RCL | i | % | | (Interest-only payment: $554.17) |
| RCL | PMT | CHS | | - | | ($8.65 difference between amortized and interest-only payments. $8.65 is is also the principal reduction amount included in first payment of amortized loan). |

## C FINANCIAL KEYS APPROACH

With an interest-only loan there is no principal reduction: We borrow a certain amount, we faithfully pay the interest every month, and we are still left with the full loan amount to repay when the loan becomes due. So the Future Value is the same amount as the Present Value, except that it has to be of a different sign. Let's use this observation to calculate the interest-only payments:

A 14% interest-only loan of $47,500 is all due in 5 years. What are the monthly payments?

| Keystrokes | Comment |
|---|---|
| 47500 PV CHS FV | Puts loan amount in **PV AND FV**, with different sign. |
| 14 BLUE i | |
| 5 BLUE n | |
| PMT | Interest-only payment: $554.17. |

This approach leaves the loan fully "balanced" in the financial memories where it can be discounted or used for other calculations. It also has the advantage of using the routine financial key procedure. We just remember that we know what the **FV** is and can key it in in the usual way.

Note: - If we omit to press **CHS FV** when the **PV** amount is still showing in the display, we may at any time press **RCL PV CHS FV** instead.

- The interest-only payments are the same whatever the term of the loan. However, it is worth putting the correct term in **n** as the whole purpose of this approach is to discount the loan or ask other questions that require the correct value for **n**.

## NEGATIVE AMORTIZATION

This is when the payments on a loan do not even cover the interest. Whatever interest is earned but not paid is added to the principal, which therefore increases instead of decreasing--so the name **negative** amortization.

The interest added to the principal every month earns interest of its own: we have compounding. The calculator automatically takes care of the calculation if the data submitted implies that situation.

Calculate the 5 year balance on $200,000 loan at 10% interest with monthly payments of $1,200.

```
200000   PV
5   BLUE   n      10   BLUE   i
1200   CHS   PMT
FV
```

($236,137.30)

## STRAIGHT NOTE

A loan with no payment is an extreme case of negative amortization. We just need to key **zero** into **PMT** and proceed as usual in all respects.

Calculate the balance after 5 years on a $200,000 loan at 10% interest with no payments made on the loan. Assume monthly compounding.

```
200000   PV
5   BLUE   n      10   BLUE   i
0   PMT
FV
```

($329,061.79)

By keying in the values for **n** and **i** on a monthly basis we have imposed monthly compounding on the situation. Keying in yearly data would imply yearly compounding.

Loans that do not amortize

**PRACTICE**

Calculate the interest-only payments by the arithmetic approach:

13) $23,500 loan, 7.5% interest. ($146.88)

14) $79,000 loan, 11% interest. (724.17)

15) $19,000 loan, 17% interest. (269.17)

Calculate the payment amount on the following 30 year loans. Then use the rate in **i** to check what the interest-only payments would be and calculate the difference between the two payments:

16) $25,000 loan at 15%. ($316.11; $312.50; $3.61)

17) $186,000 loan at 8%. ($1,364.80; $1,240.00; $124.80)

18) $186,000 loan at 18%. ($2803.18; $2,790.00; $13.18)

19) Now, using a 5 year due date on all the loans, let the payment key of the calculator calculate the interest-only payments on the six loans defined by the previous problems.

20) Calculate the balance after 3 years on a $29,000 loan at 8% interest with payments of $10 per month. ($36,431.52)

21) Calculate the monthly payment that will allow the balance on a $45,000 loan at 14% interest to increase to $55,000 after 6 years. ($435.61)

22) What is the balance after 2 years on a $156,000 loan at 12% interest if no payments are made on the loan? (Monthly compounding) ($198,078.61)

## ANNUAL, SEMI-ANNUAL, QUARTERLY PAYMENTS, etc.

So far we have considered loans with monthly payments. Balancing the books on loans with payments that occur on a different schedule is simple: we just need to **express n, i, and PMT on the same time scale**, the time scale defined by the time that separates the payments.

With yearly payments, **n**, **i**, and **PMT** are expressed per annum.
With quarterly payments or payments twice a year, **n**, **i** and **PMT** are expressed on a quarterly or semi-annual basis.

Note: Yearly payments are not equal to 12 times the monthly payments. They are higher. Monthly payments speed up the amortization process.

Study these examples:

A Loan of $200,000, 13% interest, amortized with 10 **yearly** payments.
Calculate the yearly payments.
Calculate the balance and balloon payment after 5 years.

| | | |
|---|---|---|
| | 200000 PV | |
| Annual rate | 13 i | |
| Number of payments and number of years | 10 n | |
| Yearly payment: | 0 FV PMT | ($36,857.91) |
| Balance after 5 years | 5 n FV | ($129,637.80) |
| Balloon payment: | RCL PMT + | ($166,495.71) |

B Loan of $200,000, 13% interest, amortized over 10 years with payments every 6 months. What are those payments?

| | | |
|---|---|---|
| | 200000 PV | |
| Interest per period: | 13 ENTER 2 ÷ i | i = 13 ÷ 2 |
| Number of payments and number of periods | 10 ENTER 2 x n | n = 10 x 2 |
| Semi-annual payment: | 0 FV PMT | ($18,151.28) |

C  Same loan, same rate, same term, but quarterly payments:

| | | |
|---|---|---|
| | 200000 PV | |
| Interest per period: | 13 ENTER 4 ÷ i | i = 13 ÷ 4 |
| Number of payments and number of quarters | 10 ENTER 4 x n | n = 10 x 4 |
| Quarterly payment: | 0 FV PMT | ($9,005.59) |

Different payment periods

**PRACTICE**

23) Calculate the semi-annual payments and the balance after 3 years on a $45,000 loan, 9% interest, amortized over 20 years.
(Semi-annual payments: $2,445.44.)
(Balance after 3 years--or 6 payments--$42,175.94)

24) A $900,000 loan at 11% interest is amortized over 30 years with payments every 2 months. What are those payments? ($17,151.80)

25) A $100,000 loan at 14% requires quarterly payments of $4,000. What is the balance of the loan after 5 years? ($85,860.16)

26) $7,000 loan, 17% interest, amortized with annual payments over 5 years. What are the payments? ($2,187.95)

27) $59,000 loan, 13% interest, amortized over 15 years with payments every two months. What is the balance and the balloon payment at the end of the third year?
(PMT $1,495.60; balance $54,278.64; balloon $55,774.24)

28) What 30 year loan can a borrower get with payments of $3,600 every 2 months if he is charged 9% interest? ($223,544.02)
What is the balance of the loan after 10 years? ($199,794.43)

## CHANGING THE DATA

We can at any point change some of the data and ask a question based on the new requirements. Let's give one illustration.

A buyer needs a $123,000 loan to purchase a property. He can get the loan at 11.5% amortized over 30 years. He plans on retiring in 10 years and on paying off the loan at that time to own the property free and clear during retirement. What are his payments, and the 10 year balance?

| | | | | |
|---|---|---|---|---|
| 123000 | | PV | | |
| 11.5 | BLUE | i | | |
| 30 | BLUE | n | | |
| 0 | FV | PMT | | $1,218.06 to amortize in 30 years. |
| 10 | BLUE | n | FV | $114,218.36 balance after 10 years. |

On learning of the balance, the borrower asks: "What payments should I make to bring the balance down to $80,000?" What is the answer?

| | | | |
|---|---|---|---|
| RCL | PMT | | Brings **PMT** back into the display: $1,218.06 |
| 80000 | CHS | FV | Imposes the desired balance. |
| | PMT | | Calculates the required payment: $1,371.23 |
| | - | | Required increase in payment amount:$153.17 |

By recalling the initial payment before we begin to change the data we make it possible for that amount to be in the **y** memory of the Stack when the new payment amount is calculated. Pressing **minus** then gives the difference between the two payments. These two lines could be omitted.

**RECALLING AHEAD** is the term we will use for this last procedure. It consists in bringing back into the display the content of one of the 5 financial elements that is going to be erased by a new calculation of that element—here the initial value for the payments was going to be erased by the calculation of the new value. For greater efficiency it should be done before we have changed all the data required by the new calculation—here we recalled before we keyed in the new Future Value. This procedure is an alternative to having to key the number back in later on. (See Unit 3, pages 18 & 19 for related material).

Changing the data

**PRACTICE**

29) $96,000 loan, 9% interest, 30 years.
Calculate the payments and the balance after 7 years.
What payment would bring the balance down to $50,000?
What dollar increase would that be over the initial payment?
(Initial amounts: $772.44 and $89,894.84)
(New payment: $1,115.10, a $342.66 difference)

30) $186,000 loan, 12.75% interest, 25 years.
What is the balance after 10 years?
What is the balance after 11 years?
What is the principal reduction during the 11th year?
(Calculate the payment: $2,062.84)
(Calculate the 10 year balance: $165,180.66)
(Calculate the 11 year balance: $161,263.48)
(Press – to get the difference: $3,917.18)

31) You want to make payments of $4,800 per month.
What loan can you afford to borrow at 11%, amortized over 20 years?
How much more money can you borrow with a 30 year loan at 11.25%?
(First loan: $465,031.39)
(Second loan:$494,202.70)
(Difference: $29,171.31)

32) $199,000 loan, 8% interest, 30 years.
What is the balance after 12 years?
What is the balance if you increase the payments by 10%?
What difference between the balances?
(Initial payment: $1,460.19. Initial balance: $166,886.16
Increased payment: $1606.21. New balance: $131,767.33.
Difference: $35,118.83).

(Here as elsewhere the answers are given without rounding the payment amount to an exact number of dollars and cents. In this last problem, rounding the internal values in **PMT** would give a final 12 year balance of $131,767.48, a 15 cents difference. See Unit 2, page 14).

**IF YOU GET AN INCORRECT ANSWER.......**

**........YOUR DATA IS INCORRECT.**

Check the data with the **RCL** key:

- Check the data in all 5 financial memories, in particular the memory that you have not used, if any.
- If you have monthly payments, check that you have monthly data in **n** and **i**.
- Check that the signs reflect the exchange of money in time: **PV** positive, **PMT** and **FV** negative unless you have good reason to have it otherwise.
- Check that you do not have the word **BEGIN** as an indicator in the display unless you have reason to select that option.

The numbers in the five memories must reflect the reality of the situation that you are analyzing: it is not a question of keystrokes or meaningless formulas but of **REALITY**.

> In the field, checking the accuracy of the data is the only means of ascertaining the accuracy of the answer.

When you RECALL data, the number that you have recalled is like fresh data in the calculator. That number is stored as data by the first financial key that you press. If you intended that key to solve for an answer, it has not done so. Press it a second time to have it respond as a question key. This is the small price we pay for having those 5 keys act both as data keys and question keys:

- Key in data for a loan: $30,000, 10%, $290/month, 5 years.
- Recall that data.
- Now solve for the **FV**: you have to press **FV** twice!

## BALANCING THE BOOKS

By "balancing the books" on a loan transaction we mean calculating those elements of the transaction that were not known, but were nevertheless implied by the known data. If I agree with a lender to borrow one hundred thousand dollars at 12% interest amortized with equal monthly payments over 30 years, even though no mention has been made of the amount of the payments, there is only one payment amount that fulfills the agreement, only one amount that is fair to both borrower and lender. Calculating that amount "balances the books" on the transaction.

Balancing the books may require that we define a dollar amount, either by calculating the amount (**PV, PMT, FV**) or the number of times that it occurs (**n**), or when it should occur (again **n**). Or if all the cash flow is known both in amounts and in timing, balancing the books means calculating the rate that makes sense of that exchange of money in time. It means, in short, making sure that we have the full picture under the conditions that we assume, and just as important, making sure that **the calculator** has the full picture as well.

Unit 4 has provided us with considerable flexibility in balancing the books on loan transactions under greatly varied circumstances. With our ability to balance the books we have a powerful tool that can be applied to a variety of problems. This is what we want to do now.

# UNIT 5

# LOOKING AT A LOAN FROM TWO POINTS OF VIEWS

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# LOOKING AT A LOAN FROM TWO POINTS OF VIEWS

**ANNUAL PERCENTAGE RATE (APR)**

**DISCOUNTED NOTES: YIELD AND PRICE**

**CASH EQUIVALENT PRICE**

**COST EQUIVALENT PRICE**

**BUYDOWN**

Now is when our newly acquired abilities pay off!

My effort will be to show that these problems are really very much the same thing under different names, and that the procedure to solve them is essentially the same.

## LOOKING AT A LOAN FROM TWO POINTS OF VIEWS

Annual Percentage Rate (APR)
Discounted note
Cash equivalent price
Cost equivalent price
Buydowns

All these situations imply looking at a loan from two points of views. As we balance the books from one perspective, we realize that this does not meet the requirements of the second point of view. So we change the Present Value to meet the new requirement, and we must then re-calculate the rate to balance the books once again on the data; or we change the rate to meet the requirement of the second perspective, and then we balance the books by re-adjusting the Present Value.

At the heart of each problem then we have the following steps:

(1) Balance the books on a loan.

(2) Change **PV** or (2) Change **i**

(3) Re-calculate **i**. (3) Re-calculate **PV**.

**REMINDER:** Though most examples will be on amortized loans with an early due date, exactly the same procedures apply to FULLY AMORTIZED, INTEREST-ONLY, NEGATIVE AMORTIZATION, and STRAIGHT (no payment) LOANS.

# A ANNUAL PERCENTAGE RATE (APR)

The two perspectives:

The lender: "Here is an $80,000 loan!"
The borrower: "Yes, but
you charged me $2,000 in points and costs
so I really only received $78,000!"

## APR
## MODEL PROBLEM

An $80,000 loan at 13% interest, payments of $890, 5 year due date.
Because of loan costs the borrower only receives $78,000.
What is the APR?

| | | |
|---|---|---|
| **Step 1** | Balance the books on the face value. | 80000 PV 890 CHS PMT<br>13 BLUE i 5 BLUE n |
| | ($78,042.46 balance) | FV |
| **Step 2:** | The borrower in effect only received $78,000: | 78000 PV |
| **Step 3:** | Real cost of the money to the borrower: (13.70% APR) | i (wait) 12 x |

**Inside the financial memories:**

| | n | i | PV | PMT | FV |
|---|---|---|---|---|---|
| **Step 1:** Exchange of money in time based on face value of loan: | 60 | 13/12 | 80,000 | -890 | -78,042.46 |
| **Step 2:** Exchange of money in time from borrower's perspective. | 60 | | 78,000 | -890 | -78,042.46 |
| **Step 3:** Calculates the rate corresponding to the exchange of money established in step 2. | | 13.7/12 | | | |

In most cases we have to calculate the net amount received by the borrower from knowledge of the costs charged to the borrower. This can be done while we are performing the calculations previously described.

Loan charges are frequently expressed in "points", which means percentage points: 1 point is 1% of the loan amount. It makes no difference whether the points and other costs are paid by the borrower in advance of getting the loan, or deducted by the lender from the loan amount: in both cases the net amount left for the borrower to use for his purpose is reduced by those costs.

## MODEL APR PROBLEM
including loan proceeds calculation

$80,000 loan, 13% interest, payments of $890, all due in 5 years. What is the APR if the lender charges 2 points (2% of the face value of the loan) plus $400.00?

| | |
|---|---|
| 80000 PV 890 CHS PMT<br>13 BLUE i 5 BLUE n<br>FV | Balances the books on the loan. |
| RCL PV<br>2 % −<br>400 − | Calculates net amount received by borrower. ($78,000) |
| PV | $78,000 in **PV** |
| i (wait) 12 x | (13.70% APR) |

Instead of just keying in the $78,000.00 net proceeds, we must here calculate the amount. We do so in parenthesis, as it were, as a prelude to keying the amount into **PV**.

Of course it is highly important to balance the books correctly, and this may be done in a variety of ways according to the nature of the loan and of the information we have on it. If there is a due date, it is essential to key it in and to calculate the **FV** based on that due date: it makes a significant difference if the initial costs of the loan are absorbed in 5 years or can be spread over 30 years.

Without the lender's obligation to disclose the APR, points and costs could be used to hide the real cost of the money actually received: the worst loan in town could advertise the lowest rate in town! **Regulation Z "Truth in Lending"** that requires disclosure of the APR has made it easier for borrowers to choose between loans that have different rates of interest and different loan costs.

For details on the points, costs, reconveyance fees, required loan insurance fees, etc. that must be taken into account in calculating the APR, see the "Regulation Z: Truth in Lending" publications of the Board of Governors of the Federal Reserve System.

The Board of Governors also provides tables and rule of thumb formulas for calculating the rate under various conditions: these tables and formulas are only meant to be of assistance to those who have no better means of calculating the APR, and they can be significantly inaccurate, especially at higher rates. Properly used, the calculator is a simpler, more accurate, and perfectly valid means of obtaining the APR.

## COSTS AND NET LOAN PROCEEDS

A lender advertises 10.75% interest and a 11.21% APR on fully amortized 30 year loans. What are the costs and the net loan proceeds on a $150,000 loan?

| | | | | | | |
|---|---|---|---|---|---|---|
| 150000 | PV | 30 | BLUE | n | | |
| 10.75 | BLUE | i | 0 | FV | PMT | ($1,400.22) |
| | RCL | PV | | | | Recall ahead procedure |
| 11.21 | BLUE | i | | PV | | (Loan proceeds: $144,617.42) |
| | - | | | | | (Costs of loan: $5,382.58) |

Because the APR is frequently only approximative, the answers here are likely to be approximations.

Notice how we just choose the second option: after balancing the books, we impose the APR, and calculate the net loan proceeds.

We also include here the "Recall ahead" procedure that brings the original loan amount back into the Stack before we change the data. This makes it possible to just press - at the end to get the difference between the two successive values for **PV**.

## PERSONALIZED APR

The APR is defined by laws and regulations. As such the legal APR cannot always fully take into account the specific costs and circumstances affecting a borrower. So we may want on occasion to substitute the notion of a **personalized APR** that implements the intent of the APR but more fully adapts to the circumstances of a specific borrower.

> This effort to improve on the APR points to a major assumption of all the calculations that follow (discounted loan, cash equivalent price, etc.). They are valid "all other things being equal", which is seldom exactly the case. If major distortions are introduced--by tax consequences for instance, or the possibility that the loan could be paid off early,--then, either the tax consequence or other distorting element should be taken into account, or we should be aware of the limitations of the answer that we obtain.

## MODEL
## PERSONALIZED APR

A borrower needs to choose between:

Loan A: $100,000 loan at 12% interest, amortized over 30 years.
The loan charges are 2 points and $200.
(We calculate the APR and find 12.30%)

Loan B: A similar loan at 13% interest, with no points, no costs, and no penalties of any kind made available by his credit union.
The APR of course is 13%.

On the basis of the APR alone, loan A seems cheaper than loan B. But the borrower is expecting to stay only two years in the area. Should that affect his choice between the two loans? Let's calculate the personalized APR on loan A based on a 2 year self-imposed due date.

| | |
|---|---|
| 100000 PV 12 BLUE i | |
| 30 BLUE n 0 FV PMT | |
| 2 BLUE n FV | ($99,228.22) |
| RCL PV 2 % - 200 - | (Arithmetic: $97,800) |
| PV i (wait) 12 x | (13.26%) |

With a personalized APR of 13.26% loan A is now more costly than loan B.

If we assume that loan A would also require a prepayment penalty of $5,000 if paid off after only two years, then we can add this penalty to the balance left in the **FV** memory, and calculate the personalized APR that takes the penalty into account:

| Keystrokes | |
|---|---|
| RCL FV CHS | |
| 5000 + | (Adds $5,000 to FV amount) |
| CHS FV | (Adjusted balance keyed back into FV as negative amount) |
| i (wait) 12 x | (Personalized APR: 15.45%, much higher than the 12.30% APR!) |

Other incidental costs, such as extensive termite work required by one lender as a condition for the loan but judged useless by the borrower, could be included in personalized APR calculations.

(There are shorter ways of adding $5,000 to the **FV**. But the safe approach used here consists in changing negative numbers back into positive numbers before performing additions or subtractions on numbers that have different signs)

> Any change in the amount or the timing of cash flows affects the APR or the personalized APR. The key is to have in the calculator the reality of the exchange of money as seen from the borrower's point of view. Whether the loan is **fully or partially amortized** or **interest-only**, whether it has **negative amortization** or **no payments** at all, whether the payments are **monthly**, **quarterly** or **annual** is of no importance provided the calculator faithfully reflects that reality and records the net amounts of money changing hands at each point in time.

With irregular payments, the same procedure is implemented with the use of the irregular cash flow keys (see separate volume).

**PRACTICE**

There is no difference in the actual calculation of the APR and of the personalized APR, except that the costs or the term, or both, are different.

When calculating first the APR and then the personalized APR the safest approach is to balance the books again from scratch, without trying to rescue some data from one calculation for the next one. The payments are the same, but just about everything else would probably need to be changed.

If you come up by mistake with the original nominal rate when calculating an APR this means that you forgot to key back into **PV** the net loan proceeds that you calculated.

Calculate the APR where appropriate.

1) $75,000 loan, 30 years, 9.25% interest. Loan costs are 2 points and $400.00. (9.54%)

2) $17,250 loan, 15% interest, payable $258.75 per month, due in 3 years. Loan costs are 10% plus $250.00. (20.37%)

3) $115,000 loan, 8.5% interest, amortized over 30 years. Loan costs are 2 points and $200.00. (8.74%)

4) $6,000 loan, 16% interest, payable $90 per month, all due in 1 year. Loan costs are 6% plus $200.00. (26.84%)

5) Nominal rate: 9.5%, APR: 10.10, fully amortized in 30 years. What are the net loan proceeds and costs on a $92,000 loan.
($87,413.78; $4,586.22)

6) $45,000 loan, 10%, 30 years. Cost: 3 points and $300.00. Give the APR. Calculate the personalized APR for a borrower who intends to keep the loan only 3 years. (APR: 10.45%. Personalized APR: 11.46%)

7) $150,000 loan, 9% interest, 30 years. The costs are 2.5 points and $400. Calculate the APR. Calculate the personalized APR on the assumption that the borrower will pay off the loan after 4 years and will have to pay a prepayment penalty of $6,600. (APR: 9.32%. Personalized APR: 10.76%)

8) $19,000 15% **interest-only** loan. 5 year due date.
Cost: 6 points and $700. What is the APR?
What is the personalized APR if the borrower uses that loan as a short term swing loan, and pays it off after 6 months?
(APR: 17.95%. Personalized APR: 36.48%)
(19000 PV CHS FV 15 BLUE i 5 BLUE n PMT
RCL PV 6 % – 700 – PV i 12 x (APR)
6 n i 12 x (personalized APR)

## B DISCOUNTED LOANS

The two perspectives:

| | |
|---|---|
| Original lender: | This is a $15,000 loan. |
| New owner of the loan: | Yes, but I am buying it from you at a discount for $13,000. |

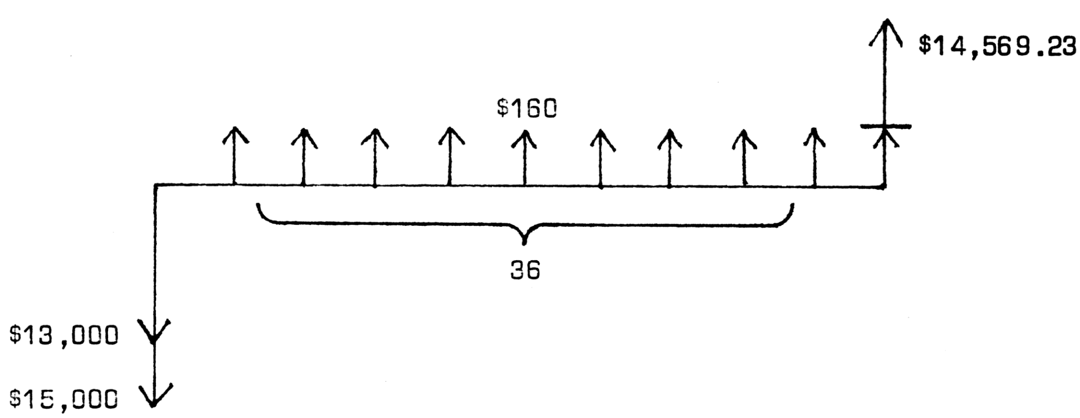
Ownership of a loan is the right to receive a certain number of payments and possibly a Future Value. That right can be sold. The sale does not generally occur at the face value of the loan, but at a discount.

We saw that the APR was the rate from the point of view of a borrower who does not get the full face value of the loan and yet has to pay it off in full. The yield on a discounted loan is the rate from the point of view of an investor who does not invest the full face value of the loan and yet receives the full benefit of the payments.

### PROBLEM

A $15,000 loan at 12% interest with monthly payments of $160 and a 3 year due date is sold at a discount for $13,000. What is the rate of return (yield) to the buyer?

We have all the information needed to know what amounts the buyer of the discounted note receives in return for his $13,000 investment. Once these numbers and the $13,000 investment are in the calculator, pressing **i** calculates the rate of return. If we knew all these amounts from the beginning, we could key them in and press **i**. We would then have no need to know the face value and nominal rate of the note. We require these numbers here only because they are needed to calculate the balance after 3 years ($14,569.32). Calculating this last number--or the payment amount if it too was missing--is the only but essential purpose achieved by "balancing the books" on the original loan.



## MODEL PROBLEM I

A $15,000 loan at 12% interest with monthly payments of $160 and a 3 year due date is sold at a discount for $13,000. What is the rate of return (yield) to the buyer?

| | |
|---|---|
| | 15000 PV 12 BLUE i |
| **Step 1:** Balancing the books ($14,569.23 balance) | 160 CHS PMT 3 BLUE n FV |
| **Step 2:** Changing **PV.** | 13000 PV |
| **Step 3:** Calculating **i.** (17.84% yield) | i (wait) 12 x |

## MODEL PROBLEM II

Same $15,000 loan at 12% interest with monthly payments of $160 and a 3 year due date.
What discounted selling price provides a 21% rate of return?

| | |
|---|---|
| | 15000 PV 12 BLUE i |
| **Step 1:** Balancing the books exactly as above. | 160 CHS PMT 3 BLUE n FV |
| **Step 2:** Changing **i.** | 21 BLUE i |
| **Step 3:** Discounted price. ($12,048.69) | PV |

With MODEL I still in the calculator there was really no need to balance the books again on the loan. When this has been done once, a whole sequence of questions can be asked:

| | |
|---|---|
| What is the yield if the note is sold for $11,000? (24.84%) | 11000 PV i 12 x |
| Yield if sold for $14,000? (14.80%) | 14000 PV i 12 x |
| Discounted price to yield 16%? ($13,594.85) | 16 BLUE i PV |
| Discounted price to yield 13%? ($14,633.54) | 13 BLUE i PV |

**PRACTICE**

9) $18,000 loan, 10% interest, payments of $180 per month, 6 year due date.
Yield if sold for $14,400? (15.47%)
Yield if sold for $13,500? (17.11%)
Price to obtain 12% return? ($16,562.19)
Price to obtain 19% return? ($12,558.57)
Price to obtain 13% return? ($15,897.85)
(When initially balancing the books, you found a FV of $15,056.66)

10) $25,000 loan, amortized over 20 years at 8% interest, due in 5 years.
Discounted price to yield 14%? ($19,897.02)
Discounted price to yield 16%? ($18,483.02)
Yield if sold for $21,000? (12.56%)
(You initially found PMT of $209.11 and FV of $21,881.40)

11) $400,000 loan at 7%, monthly payments of $3,500, 7 year due date.
Price to yield 10%? ($347,288.54)
Price to yield 12%? ($317,053.34)
Yield if sold for $300,000? (13.24%)
(When initially balancing the books, you found a FV of $274,001.19)

12) Interest-only loan of $6,000 at 13% interest with a 3 year due date.
Price to yield 19%? ($5,181.58)
Price to yield 15%? ($5,711.53)
Yield if sold for $5,000? (20.48%)
(Did you find payments of $65.00?)

## A LOAN, BY ANY OTHER NAME, IS STILL A LOAN

Real estate loans are frequently called by the nature of the security agreement that covers them. These may change in various parts of the country: mortgages in the East, Notes, Trust Deeds--First, Second, or Third Trust Deeds--in the West. We will generally refer to all of them simply as "loans". In the West, discounted loans most frequently apply to second Trust Deeds (2nd TDs) and are then frequently referred to as "discounted seconds".

## PERCENTAGE DISCOUNT

When a loan is sold at a discount, the discounted selling price is frequently expressed by specifying the amount of the discount as a percentage of the face value. A $30,000 loan sold for $24,000 is said to be sold "at a 20% discount": 24,000 is 30,000 minus 20% of 30,000.

If the discounted selling price is expressed in that fashion, the simple calculation that tells us the actual selling price can be done after the full loan has been balanced in the financial keys:

**MODEL PROBLEM**
**DISCOUNTED LOAN**
with discounted price expressed as a rate.

$30,000 loan, 10% interest, payments of $$290.00, 5 year due date.
What is the yield to the buyer if it is sold at a 20% discount?

| | |
|---|---|
| 30000 PV 10 BLUE i | |
| 290 CHS PMT 5 BLUE n | |
| FV | ($26,902.52) |
| RCL PV | Calculates the selling price: |
| 20 % − | ($24,000) |
| PV i 12 x | (16.09%) |

The rate at which the face value is discounted (here 20%) has very little to do with the yield (here 16.09%). A loan could have a yield of 16.09% without being discounted at all, if that was the nominal rate on the loan. A 20% discount means a one-time cut of 20%, from $30,000 to $24,000 in our example. The yield is the annual rate of return to the investor over the life of the investment.

**CALCULATING THE DISCOUNT AS A RATE.**

A $19,000 loan at 9% interest, with payments of $180.00 per month and a 4 year due date is discounted to yield 20%.
What is the discounted price?
What percentage discount is that?

Getting the answer to the last question is very simple: we need the face value in **y** and the discounted price in the display. We then press $\Delta$%.

This can be implemented in two ways, as illustrated below. In one case we use the "Recall ahead" procedure to avoid having to key in the loan amount a second time. In the other case we use the x≷y key to re-establish the correct order before we press $\Delta$%.

19000 PV 9 BLUE i

180 CHS PMT 4 BLUE n

FV

| | | |
|---|---|---|
| RCL PV | or | |
| 20 BLUE i PV | ($13,533.25 discount price.) | 20 BLUE i PV |
| | | 19000 x≷y |
| $\Delta$% | (A 28.77% discount) | $\Delta$% |

## DISCOUNTING A MATURE LOAN

So far we have assumed that the loan was being sold at a discount as soon as it was created. This need not be the case. Let's take the loan that we still have in the calculator. What if it was being sold to yield 20% 10 months after its creation? What numbers in the calculator no longer apply? Let's use the **RCL** key to look into the financial memories. We find the following numbers:

| n | i | PV | PMT | FV |
|---|---|---|---|---|
| 48 | 20/12 | 13,533.25 | -180 | -16,842.97 |

When I buy that note 10 months after its creation, I am still buying the right to receive payments of $180.00 a month and a Future Value of $16,842.97, and I still want a 20% return. But I will get only 38 payments, and will receive the balance in 38 months, not 48. Putting 38 in **n** is the only change that needs to be made before calculating **PV.**

| | | |
|---|---|---|
| 38 | n | PV |

($14,024.52)

**Note:** - Even though I will receive the balance in only 38 months, it is the balance of the loan after 4 years that I receive, and it is with the full 4 year term in **n** that the balance must be calculated.

- I have allowed the original owner to receive 10 payments, and yet the calculator tells me that I should be willing to pay more for the loan than if I had bought it at a discount when it still had 4 years to run. The reason is that I will not have to wait as long before I receive the $16,842.97 balance, or the $17,022.97 balloon payment. Think of it, how much would you be willing to pay if you had just one month to wait before you received the $17,022.97 balloon payment? All our calculations take the time value of money into account, and <u>when</u> money is received or paid out is as important as <u>how much</u>. The complete model follows:

$19,000 loan, 9% interest, payments of $180, 4 year due date. What discounted selling price will yield 20% return if the loan is sold when 38 months and 38 payments still remain?

| | | | | | | |
|---|---|---|---|---|---|---|
| 19000 | PV | | 9 | BLUE | i | |
| 180 | CHS | PMT | 4 | BLUE | n | FV |
| | | | 38 | | n | |
| 20 | BLUE | i | PV | | | |

($14,024.52)

## RENEGOTIATING THE TERMS OF A LOAN
**before selling it at a discount**

Loan A: $20,000, 12% interest,
payments of $210, 7 year due date.

Loan B: $20,000, 10% interest,
payments of $420, 5 year due date.

You are the lender.
Which loan do you prefer? The one at 12% or the one at 10%?
The tendency of investors is to prefer 12% interest over 10% interest.

Let's discount these two loans to yield 20%. To provide that yield

-- loan A must sell for $14,120.02, a 29.40% discount.

-- loan B must sell for $15,994.63, a 20.03% discount.

If you have to sell these loans under these conditions, the 10% loan is significantly more valuable than the 12% loan. The reason is that "money is better sooner": with loan B money comes back sooner, and therefore it does not have to be discounted as much to provide the same 20% yield.

It is also possible that the borrower would prefer loan B to loan A:

-- The interest is lower.
-- The term is shorter.
-- It is almost fully amortized during the shorter term instead of leaving an $18,903.28 balloon in 7 years.

Of course, the payments are double, but the trade-off might be worth the higher payments.

Is it possible to imagine that the owner of loan A might renegotiate it with his borrower into loan B before selling it at a discount? Is it possible to imagine that we could buy loan A at a discount to yield 20%, renegotiate it with the borrower into loan B, sell the new loan to yield 20%, and keep the difference?

The ability to discount loans to test any hypothesis puts a powerful tool at our disposal and can be used in a variety of creative ways.

## MORE PRACTICE

13) A $10,000 loan, 10% interest, payments of $100,
due in 7 years is sold 3 years after its creation.
What is the selling price to yield 14%? ($8,234.89)

14) A $56,000 loan at 8% interest with monthly payments of $440
and a 10 year due date is sold at a 30% discount.
What yield does that represent? (14.01%)
What is the yield if it is sold for the same price
23 months after its creation? (14.25%)

15) A $750,000 loan at 10% interest, with payments of $6,500
and a 5 year due date is offered for sale to yield 12%.
What is the selling price? ($694,388.66)
What percentage discount is that? (7.41%)
What is the yield if it is sold for $650,000? (13.74%)
What is the yield if it is sold for $610,000? (15.43%)

16) An **interest-only** loan of $140,000, 13% interest, with a
4 year call, is offered for sale 15 months after its creation.
What is the yield if it sells for $110,000? (23.68%)
What selling price results in a 21% yield? ($116,752.56)
What selling price results in a 19% yield? ($122,115.45)

17) $40,000 loan, 9.75% interest, amortized over 30 years,
all due in 7 years is offered for sale at a 20% discount.
What yield is that to the buyer? (14.39%)
What is the yield if it sells for $30,000? (15.79%)
What is the selling price that results in a 17% yield? ($28,401.70)
What percentage discount is that? (29%)

18) $21,000 loan, 7.5% interest, payments of $170,
all due in 4 year.
What selling price will double the yield to 15%? ($16,485.76)
What percentage discount is that? (21.5%)

## C CASH EQUIVALENT PRICE

The two perspectives:

Seller as seller: To help sell my house, I will carry a $100,000 mortgage at 9% interest!

Seller as investor: If I received cash instead of carrying the mortgage I could invest at 12%!

The property is selling for $125,000: the seller gets $25,000 in cash—or in cash and debt relief as the case may be—, and carries the balance at 9% interest. He is not really getting the $100,000 balance: that number is just a word on a piece of paper. What he is getting are the payments—$760.00 dollars a month shall we say—, and the balance of the loan 8 years from now when the loan becomes due. Let's balance the books on the loan and do our usual little routine. Let's then add back the down payments:

| | |
|---|---|
| 100000 PV 760 CHS PMT | |
| 9 BLUE i 8 BLUE n FV | (Balance of the loan: $98,601.44.) |
| 12 BLUE i PV | (Discounted value of loan: $84,695.29) |
| 25000 + | (Cash equivalent price: $109,695.29) |

As an investor who wants 12% return on his investments, the seller would be willing to pay $84,695.29 for the note that he now possesses. If that is what he would be willing to pay for that 9% loan, discounted to yield 12%, that is what the loan is really worth to him. If we add back the down payment received in cash, we get $109,695.29: that is the VALUE of what the seller receives for the property, the **CASH EQUIVALENT PRICE** of the property.

Any cash offer higher than the cash equivalent price is a better offer than a full $125,000 offer with the $100,000 owner-carried loan at 9%—we are not here of course taking into account the numerous tax ramifications of the various options.

If the seller was made a cash offer of $109,395.29 it would be as good an offer as the $125,000 selling price with the loan at 9%: it would be an offer that could, if desired, provide him with the same amounts of cash now and in the future as the full $125,000 price with owner financing.

The difference between the selling price and the cash equivalent price is merely the amount of the discount on the $100,000 loan at 9% when it is required to yield 12%. It is as if the seller, who accepted 9% interest in order to sell the property, was now, as an investor, buying that note from himself at a discount that will yield a 12% return. He decides to ignore the face value of the note and consider instead its real value to him now.

Let's give the full keystrokes leading from the selling price of the property to the cash equivalent price using register memory 0 (R0) to store the downpayment:

## MODEL PROBLEM
## CASH EQUIVALENT PRICE

A property sells for $125,000 with a 20% downpayment and the balance carried by the seller at a low 9% interest (payments of $760, balance due in 8 years). What is the cash equivalent price for an owner who generally expects 12% on his investments?

| | |
|---|---|
| 125000 ENTER 20 % STO 0 | (Downpayment in R0) |
| - PV | (Loan in PV) |
| 760 CHS PMT 8 BLUE n | |
| 9 BLUE i FV | ($98,601.44 balance) |
| 12 BLUE i PV | ($84,695.29: value of discounted loan) |
| RCL 0 + | ($109,695.29: cash equivalent price) |

The seller has many possible reasons to use the cash equivalent price. Having offered 9% financing, he might be tempted to refuse a cash offer of $115,000 that would be a much better offer than the $125,000. When debating how to offer the property for sale, he might decide that a $109,900 cash-to-new-loan listing would be easier to sell—and a better value—than the $125,000 price with owner financing at 9%.

Appraisers may use the cash equivalent price calculation in order to compare properties that are sold with very different financing on the assumption that the better the financing, the more people are willing to pay for a property. The financing obviously affects the selling price—to what degree may be subject to debate—and translating all the selling prices into their respective cash equivalent price can provide useful market data despite the differences in the terms. In this case, the rate projected into the calculations—equivalent to the 12% in our example—would be the standard rate for the financing of such properties at the time. A common term for all the loans, such as 5 years, would also be used on the basis that a property is not kept for 30 years even when there is no early due date.

So the CASH EQUIVALENT PRICE really helps the seller or appraiser compare various terms for the sale of property, and that is what it can be used for by a seller confronted with different offers at different selling prices and with different terms. For instance:

**Offer A:** $1,000,000, 20% cash, balance carried by seller at 9% interest-only for 5 years.
**Offer B:** $910,000, $500,000 cash, balance carried by seller at 11%, amortized over 30 years, due in 6 years.

Which is the best offer for a seller who can really expect 13% from his investments?

The cash equivalent prices are $882,799.71 and $876,421.95 respectively, a difference of about $6,400 in favor of the $1,000,000 offer.

Which is the best offer for a seller who really expects 15% on his investments?

The answers are $831,861.63 and $846,223.45: the $910,000 offer now appears to be a better deal.

Let's give the keystrokes for the very last of these calculations.

| | |
|---|---|
| 910000 ENTER 500000 STO 0 | (Downpayment in R0) |
| − PV 0 FV | |
| 30 BLUE n 11 BLUE i PMT | ($3,904.53) |
| 6 BLUE n FV | ($395,184.30 balance) |
| 15 BLUE i PV | ($346,223.45) |
| RCL 0 + | ($846,223.45 cash equivalent price) |

(For an alternative approach to these choices, see Unit 7, page 10.)

**PRACTICE**

19) $97,000 property. 20% downpayment. Balance is owner-carried at 11% (30 years, due in 7 years). What is the cash equivalent price if the buyer could expect 13% on his investments? ($90,004.79)

20) A seller has listed his property for $210,000. The seller is willing to carry $80,000 at 7%, amortized over 30 years, with a 5 year due date.

Knowing that the seller could really expect 11% on his investments, what is the cash equivalent price of his property from his perspective? ($198,035.78)

21) $159,000 property, $35,000 downpayment, the balance is an assumable loan at 8% interest with payments of $1,000.

What is the cash equivalent price of the property from the point of view of an appraiser who is bringing in line all properties financed with loans other than the current 10% rate on regular mortgages, and who assumes a 5 year due date on all financing? ($149,690.37)

22) A seller is presented with three offers for a property. Knowing that he can expect 11% return on his investments, use the cash equivalent approach to select the best offer.

Offer A: $220,000, cash to new loan.
Offer B: $235,000, $80,000 down, owner carries balance at 8% interest only for 7 years.
Offer C: $230,000, $120,000 down, balance carried by the owner at 9% interest, amortized over 30 years, due in 5.

(Cash equivalent prices: $220,000, $212,368.87, $221,710.22. The $230,000 offer seems best, and the $235,000 offer is significantly less satisfactory than the other two.)

## COST EQUIVALENT PRICE

The two perspectives:

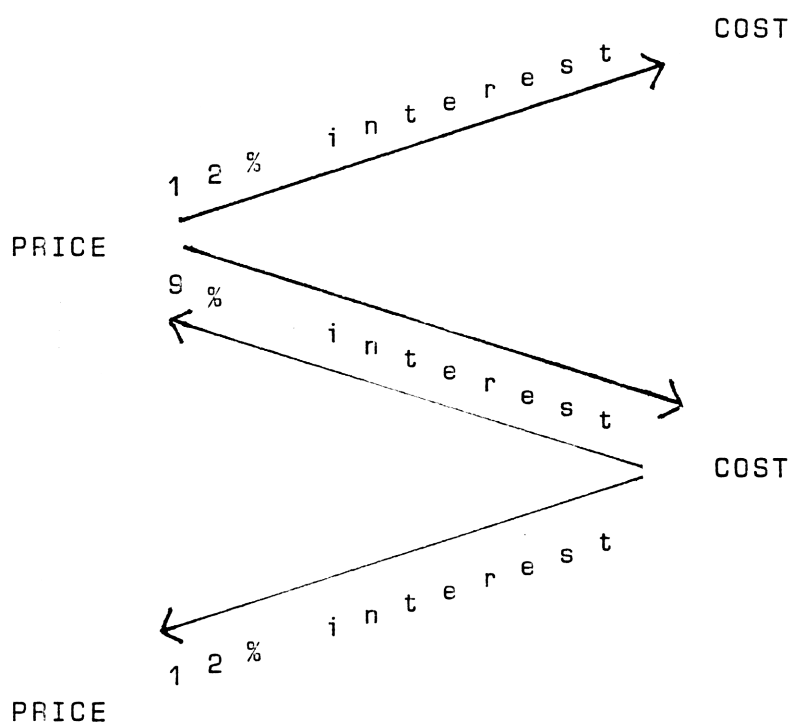
Buyer: - It's great, I can get 9% financing on this purchase.

- I would have to pay 12% for any other property.

buyers do not generally pay CASH for Real Estate, they finance their purchase with a loan. So they do not really pay for the PRICE of the property, they pay for the COST, which implies a downpayment and various payments stretched out in time.

The same PRICE can correspond to very different COSTS if one property is financed at 9% interest, for example, and the other selling for the same price is financed at 12%.

Similarly, the same COST can correspond to very different PRICES if one cost is the result of high rates and the other of lower rates.




The PRICE is just a number on a piece of paper. The reality of money changing hands is represented by the COST.

When all properties are financed at about the same rate, the price is an adequate reflection of the cost. But when properties are available at rates significantly different from the norm, then the question arises: how can we compare the costs rather than the prices?

The availability of a low interest loan on a property tends to make it a good buy. So the ads stress: "Owner will carry at 9%" or "Assume FHA loan at 8%" when the market rate on loans is 12%. We have even seen no-interest financing offers with substantial downpayments. But how good a deal are we looking at, at what point does an inflated selling price offset the advantages of lower financing?

A solution is to establish the cost of buying that property, and then to work backwards from the cost to the price that would require the same cost if the financing was at the standard rate: in other words the COST EQUIVALENT PRICE. The calculation is none other than a cash equivalent price calculation done from the point of view of the buyer.

## MODEL PROBLEM
## COST EQUIVALENT PRICE

A $141,000 property is offered with 9% financing (owner-carried, assumable, or wrap-around, whatever the case may be), provided that the buyer can come up with a 25% downpayment. The 9% loan is amortized over 30 years and has a 6 year due date. Calculate the cost equivalent price knowing that market rate for mortgages is presently 12%.

| | |
|---|---|
| 141000 ENTER 25 %<br>STO 0 − PV | Stores $35,250 downpayment in Register memory 0 and puts $105,750 loan in PV. |
| 9 BLUE i 30 BLUE n | |
| 0 FV PMT 6 BLUE n FV | ($100,262.07) |
| 12 BLUE i PV | Standard routine calculates the loan we could afford at 12% with same payments and FV |
| RCL 0 + | Adds downpayment to obtain COST EGUIVALENT PRICE of $127,750.90 |

We now know that if we had to pay the market rate of 12%, we would have to go all the way down to a price of $127,750.90 before we could buy a property with the same downpayment, the same payments for 6 years, the same balance at the end of 6 years as are required by the purchase of the $141,000 property.

Like the cash equivalent price, the cost equivalent price allows us to compare situations that, under realistic conditions, are not equivalent in all respects. The whole point is precisely to resolve a variety of differences into one single number.

However, there are also differences that are not taken into account, in particular the numerous tax consequences that are affected by the sales price--property tax, depreciation and investment tax credit, capital gains tax on resale, possibly capital gains tax on previous property--, and those affected by the loan amount and the interest on the loan. And of course if the property is not kept for the number of years that was assumed, then the calculation is no longer valid: what is the advantage of a low rate if circumstances force us to sell the property one month after it was bought?

So the cash equivalent and cost equivalent calculations--like the APR before them--should be seen as useful tools, not as an assertion that two situations would be perfectly equivalent. These numbers cannot make a decision for us, but they can make an important contribution to the decision-making process.

**PRACTICE**

23) A property is being offered for sale for $89,000. There is an assumable FHA loan at 7.5% interest with a balance of $61,000 and monthly payments of $480.00. What is the cost equivalent price of the property if current rates are 10% from the point of view of a buyer who expects to keep the property at least 6 years. ($82,549.45)

What is the cost equivalent price if current rates are 13%? ($75,874.19)

24) A $319,000 property can be financed with wrap-around loan at 11% interest (amortized over 30 years, balance in 7 years) for a buyer with a 25% downpayment. What is the cost equivalent price of the property for someone who would keep the property for the 7 years if regular financing is at 13.5%. ($292,436.45)

25) A property can be bought with no-interest financing! The selling price is $100,000. A $40,000 downpayment is required. The balance is paid off in equal monthly payments over 5 years. What is the cost equivalent price if mortgage rates are around 12.5% ($84,448.52)

26) At today's interest of 13.75% a buyer can purchase a $175,000 property with 20% down and the balance amortized over 30 years. He is assuming that he would keep the loan for 5 years. What price property could you purchase with the same downpayments, the same payments, and the same balance if rates were at 12%? ($184,119.74, a 5.21% increase)

27) A $70,000 commercial building can be financed with no downpayment and a 10% interest loan amortized with monthly payments over 5 years, or with a $10,000 downpayment, and an 8% loan amortized over 8 years. Calculate the cost equivalent price under each circumstance from the point of view of a 13% rate for loans. ($65,366.69 and $60,466.35)

## E BUYDOWN

The two perspectives:

Buyer: I'm only paying 10% interest for the first 3 years.

Lender: I'm getting 13% interest on my money for the length of the loan--and that means even during the first 3 years.

A buydown occurs when a third party bridges the gap between the two conflicting requirements by compensating the lender up front for his loss.

### SAMPLE PROBLEM

A developer has a commitment of 13%, amortized over 30 years, for the $110,000 loans his clients need to purchase his units. To attract buyers, he wants to offer 10% financing, amortized over 30 years, for the first three years of a loan. What must he pay up front to the lender to compensate him for receiving only 10% interest for 3 years?

Nothing in this scenario indicates that the lender is willing to charge less than 13% for his money. The buydown, therefore, is a means of reducing the amount of the loan so that the reduced payments provide the full 13% rate to the lender. The question becomes: "How much can the lender be expected to lend, given the payments we are willing to make, if he is to receive 13% return on his money?" The solution requires exactly the same keystrokes as for discounted loans because indeed we are discounting that 3 year 10% loan to yield 13%. Only here a third party steps in to pay for the amount of the discount.

The buydown itself is the amount of the discount. Because that is what we want to calculate, it is convenient to "recall ahead" the original loan amount after we have balanced the books. This retains the number in the Stack even after **PV** has been recalculated, and allows us to calculate the discount by just pressing the minus key. In summary:

1) 'Balance the books' on the bought down section of the loan.

2) | RCL | PV | (Keeps original loan amount in the Stack)

3) Proceed with the standard routine: key in the market rate and press **PV.**

4) Press – (minus) to get the cost of the buydown. This calculates the difference between the full face value ($110,000) and the new **PV.**

## BUYDOWN
## MODEL PROBLEM

A developer has a commitment of 13%, amortized over 30 years, for the $110,000 loans his clients need to purchase his units. To attract buyers, he wants to be able to offer 10% financing, amortized over 30 years, for the first three years of a loan. What must he pay up front to the lender to compensate him for receiving only 10% interest for 3 years?

| Keystrokes | Comment |
|---|---|
| 110000 PV 10 BLUE i | |
| 30 BLUE n 0 FV PMT | |
| 3 BLUE n FV | Balances the books on the 3 year, 10% paydown portion of the loan. ($107,966.81 FV). |
| RCL PV | (Puts 110,000 back in stack) |
| $\boxed{\text{13 BLUE i PV}}$ | ($101,903.06 net loan) |
| − | ($8,096.94 buydown cost $110,000 − $101,903.06) |

From the point of view of the lender this is a net loan of $101,903.96: he lends $110,000 to the buyer but receives $8,096.94 from the seller. This net loan amount allows the lender to get 13% on his investment.

Buydown

**PRACTICE**

28) A developer has a take-out loan commitment at 12%, amortized over 30 years. He wants to offer 9% interest for the first 2 years, also amortized over 30 years. What is the cost of the buydown on a home that requires an $80,000 loan? ($4,221.66)

29) $125,000 loan, 15% interest, amortized over 30 years. What is the cost of the buydown that will bring the interest down to 13% for the first 4 years? ($7,443.13)

30) $500,000 loan, 11% interest, 20 years. Cost of buying down the rate to 10% for the first 5 years. ($18,371.23)

**COMMENT:**

**BUYDOWN versus NEGATIVE AMORTIZATION.**

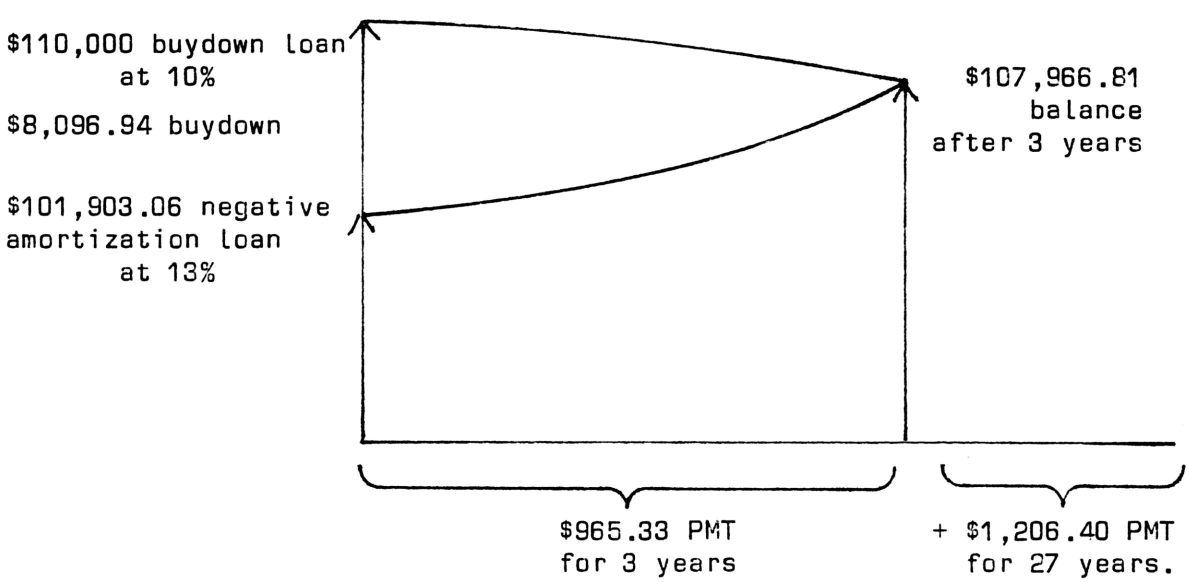
A buydown allows the buyer to make lower payments for a period of time. Apart from unavoidable tax ramifications, what is the difference between the buydown we have just analyzed and a sale where instead of giving the buydown to the lender, the seller would just reduce the price of the property by the same amount, and where the buyer would still be required to make the same downpayment and the same payments that increase in the same way after 3 years? This would allow the loan to be reduced by the amount of the buydown and the lender would be getting the 13% that he requires.

The $101,903.06 loan would then be a graduated payment loan with negative amortization for the first 3 years as the low payments would not cover the interest. The balance of the loan, after 3 years of negative amortization would be exactly $107,966.81, which is the 3 year balance on the $110,000 buydown loan. So the only difference for the buyer would be that during the first three years his indebtedness would be significantly lower with the graduated payment loan than with the buydown alternative: a significant advantage if circumstances force him to sell during that time.

A buydown is a disguised negative amortization loan where the lender is paid up front as full compensation for the lower payments whether or not the buyer keeps the loan for the whole buydown period.

**GRAPH**

showing loan balance (indebtedness)
for buydown loan and negative amortization loan
that both require the same payments from the buyer.



## IRREGULAR CASH FLOW FUNCTIONS FOR IRREGULAR PAYMENTS

Buydown loans frequently imply gradually increasing payments during the buydown period. With a 3-2-1 buydown, for example, the rate is reduced by 3 percentage points the first year, by 2 points the second year, and by 1 percentage point only the third year. In such cases the user needs the irregular cash flow functions of the calculator.

The same is true for the other applications in this unit--APR, discounted loan, cash and cost equivalent price--and elsewhere. If the payments are not equal throughout the life of the loan, we can still solve the problem in very much the same way as shown here except that the irregular cash flow functions of the calculator are required.

Looking at a loan from two points of views

### SUMMARY

We have given different meanings to the same calculations. Let's take a last look at these meanings by interpreting the same numbers and the same cash flow diagram in terms of the various applications.

- The loan:

$50,000, 10% interest, payments of $430, due in 3 years.
Balancing the books shows a balance of $49,442.

- Second point of view:

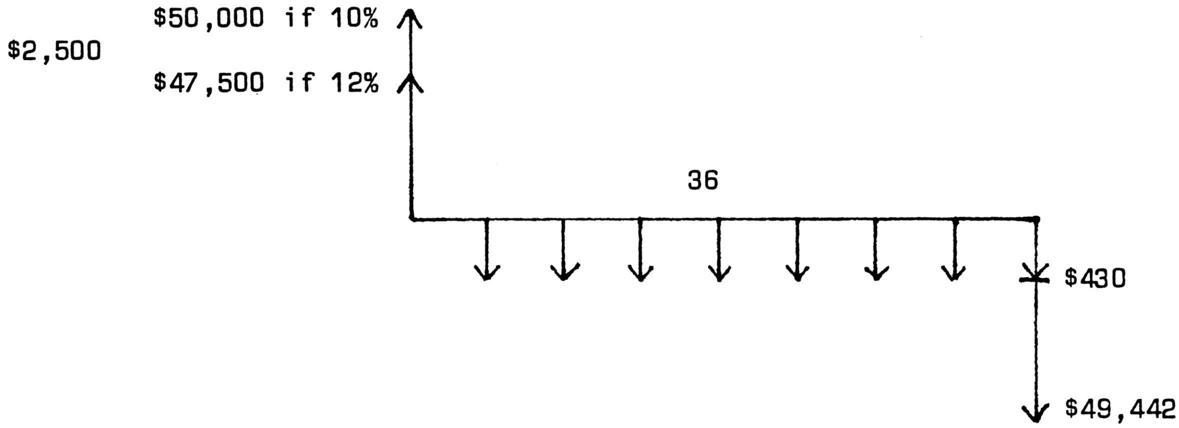
Increasing the yield to 12% decreases the Present Value to $47,500.
Decreasing the Present Value to $47,500 increases the yield to 12%.

- Keystrokes:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 50000 PV | 3 | BLUE | n | 430 | CHS | PMT | | |
| 10 BLUE | i | FV | | | | | | ($49,442) |
| 12 BLUE | i | PV | | | | | | ($47,500) |
| 47500 PV | | i | 12 | x | | | | (12%) |

For convenience, numbers are rounded up or down a few dollars. Where needed in the applications a $100,000 cash downpayment (or its equivalent in debt relief or financing at market rate) is included.

The same cash flow diagram and the calculations that it represents apply to anyone of these five interpretations:




(1) **ANNUAL PERCENTAGE RATE (APR).**

With $2,500 in points and other loan costs the APR is 12%.

(2) **DISCOUNTED LOAN.**

When sold at a discount for $47,500, the yield is 12%, or:
To yield 12%, the loan must be sold for $47,500.

(3) **CASH EQUIVALENT PRICE.**

A seller receives $100,000 cash or equivalent for his property and carries $50,000 for 3 years at 10% when he could really expect 12% on his money. The Cash equivalent price of the property is $147,500.

(4) **COST EQUIVALENT PRICE.**

A buyer purchases a $150,000 condominium with $100,000 cash and loans at current 12% market rate, plus a $50,000 loan at 10% for 3 years. He could only have purchased a $147,500 property with the same downpayment, the same payments for 3 years and the same balance at the end of 3 years if all the financing had been at market rate.

(5) **BUYDOWN**

A lender requires a 12% rate of return on his loans. By making a buydown payment of $2,500 a seller is able to secure from him a 3-year $50,000 loan at 10% for the buyer of his property.

> **Reminder**: The same keystrokes apply to negative amortization, interest-only and no-payment loans except that we 'balance the books' initially in a way that reflects the realities of such loans.

# UNIT 6

# TIME AND MONEY

# UNIT 6

# TIME AND MONEY

**THE TIME VALUE OF MONEY**

**PRESENT VALUE AND DISCOUNT RATE**

**INFLATION AND APPRECIATION**

**COMPOUNDING**

We have acquired practical ability in using the 5 regular cash flow keys and have applied that ability to solving a number of concrete problems. We now need to turn to the more abstract notion of the TIME VALUE OF MONEY and some related topics. The time value of money is at the heart of everything we have done so far. In fact, as a <u>financial</u> calculator, the HP-12C is a tool that does one simple task well: take the time value of money into account.

## THE TIME VALUE OF MONEY

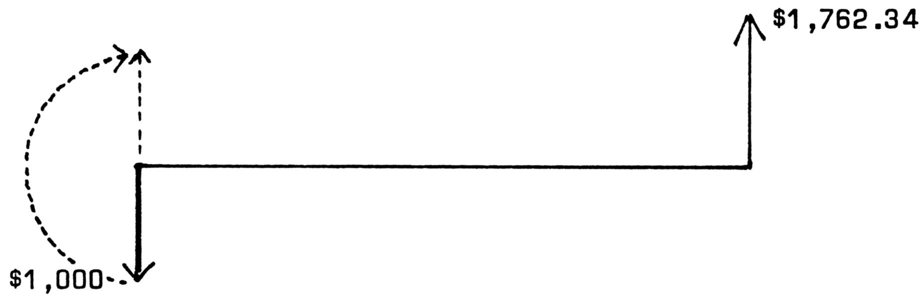
$1,000 is deposited in a 12% account, annual compounding.
How much is there in the account 5 years later?

| | | | |
|---|---|---|---|
| 1000 | PV | 0 | PMT |
| 12 | i | 5 | n |
| FV | | | |

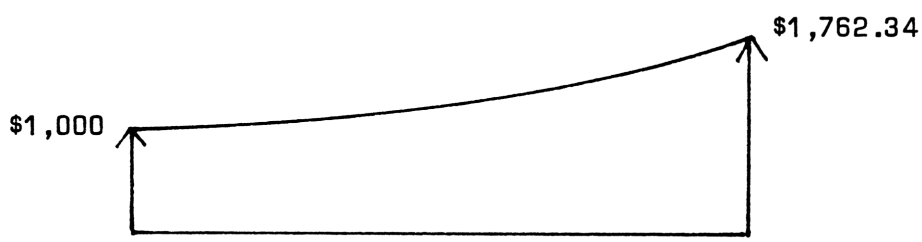
Annual compounding so annual data.

($1,762.34)

By investing I am exchanging $1,000 now for $1,762.34 in five years.

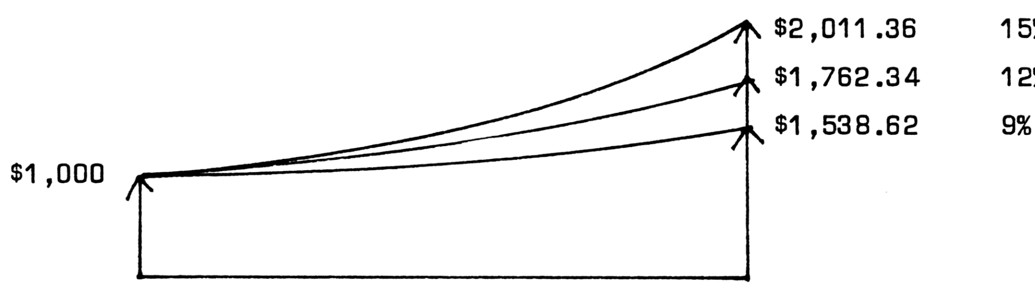



To visualize the growth of my money in the account, I can put the two amounts on the same side of the time line. The rate of growth (12%) can then be represented by a curve, as follows:




That curve is what the TIME VALUE OF MONEY is all about.

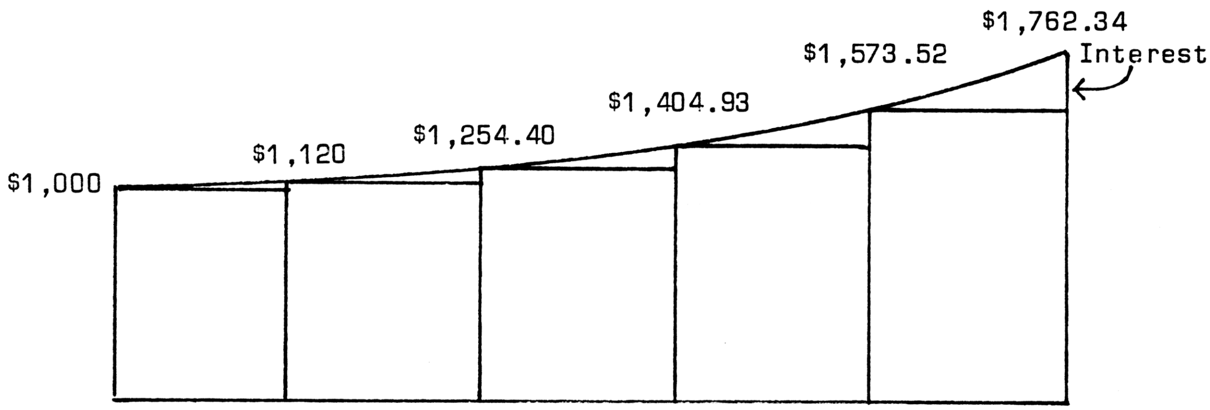
Different rates correspond to different curves.



Let's press **12 i** once again to key the 12% curve back into the calculator, and let's get a better feel for the growth of our investment by calculating how much we have in the account at the end of each year:

| | | | |
|---|---|---|---|
| 1 | n | FV | $1,120.00 |
| 2 | n | FV | $1,254.40 |
| 3 | n | FV | $1,404.93 |
| 4 | n | FV | $1,573.52 |
| 5 | n | FV | $1,762.34 |

Can you see the money grow? It is just sliding up that 12% curve in stepped up increments really for each of the successive periods:




The same numbers can be obtained by simple arithmetic:

| | | | | | Interest | Balance |
|---|---|---|---|---|---|---|
| 1000 | ENTER | 12 | % | + | $120.00 | $1,120.00 |
| | | 12 | % | + | $134.40 | $1,254.40 |
| | | 12 | % | + | $150.53 | $1,404.93 |
| | | 12 | % | + | $168.59 | $1,573.52 |
| | | 12 | % | + | $188.82 | $1,762.34 |

The effect of compounding can be seen in the increasing amount of interest earned. Each year I get in interest 12% of the balance of the previous year, which includes the interest earned in previous years. I am earning interest on interest, which is called compounding. The financial keys automatically take care of the compounding effect.

## THE RATE CURVE DEFINES NUMBERS THAT, IN TIME, ARE "EQUAL"

When I buy a piece of property for $120,000, I decide that the VALUE of the property is $120,000,-- the benefits of owning the property in the future are worth that amount of money now--, and therefore I am willing to exchange 120,000 dollars for that property. The seller also agrees on the VALUE of the property, and is willing to exchange the property for $120,000. We establish an equality of some sort between a certain amount of money now and certain benefits to be received over time from ownership of the property, and the word VALUE, equating as it does the dollar amount with the future benefits, is used to express that equality.

The same kind of equality is assumed when I decide to invest $1,000 at 12% for some years, exchanging $1,000 for $1,762.34 five years from now or for $1,254.40 in two years. I find benefits of my own in the exchange, the borrower finds benefits of his own in the reverse transaction. But from the common point of view we have agreed upon an equality of a sort has been established between different amounts at different points in time.

| | | |
|---|---|---|
| $1,000 | = | $1,762.34 |
| (now) | | (in 5 years) |

Agreeing to the 12% rate is to agree to a rate of exchange. From the point of view of that rate of exchange in time, all the amounts along that 12% curve can be said to be equal:

$1,000 now is equal to $1,404.93 three years from now,
$1,762.34 in five years is equal to $1,404.93 three years from now,
$1,573.52 in four years is equal to $1,254.40 two years from now.

By keying a rate into the calculator we are imposing a curve. The only thing that money can do then is slide up and down that curve WITHOUT EVER CHANGING IN VALUE from our particular point of view: all the amounts calculated along that curve are equal to the initial amount and equal to one another FROM THE POINT OF VIEW OF THE RATE OF EXCHANGE DEFINED BY THE CURVE.

Acquiring a feel for that curve,
for the growth or decrease that it represents,
for the equality in value between all the different amounts that money changes into along the way,
is what the TIME VALUE OF MONEY is all about.

## PRESENT VALUE and DISCOUNT RATE

We've seen money increase with time along the rate curve. Let's now look at money sliding back in time along the same curve, in fact sliding all the way back to the present.

As we have seen, given a rate of 12%, $1,000 now is "equal" to $1,762.34 five years from now, and $1,762.34 five years from now is "equal" to $1,000 now. Another way of expressing this equivalence is to say:

$1,762.34 is the **FUTURE VALUE** of $1,000 invested for 5 years at 12%.
$1,000.00 is the **PRESENT VALUE** of $1,762.34 five years from now at the **DISCOUNT RATE** of 12%.

The notion of **PRESENT VALUE** is an essential concept related to the time value of money. Well understood, it clarifies a number of problems and calculations. So let's look at a practical illustration by pretending that you have just receive the following note:

> Today's date
>
> I promise to pay you, (your name), or bearer, the sum of $10,000 exactly 10 years from today.
>
> Signed:
> The Richest Man in Texas

If you wait 10 years you will receive $10,000. But "money is better sooner" and the note gives your name "or bearer". So you decide to sell the note right now.

Without using the financial keys of the calculator, check the MINIMUM SELLING PRICE that you are willing to accept:

| | | | | |
|---|---|---|---|---|
| Lowest offer you will accept: | $10,000 | ( ) | $5,000 | ( ) |
| | $9,000 | ( ) | $4,000 | ( ) |
| | $8,000 | ( ) | $3,000 | ( ) |
| | $7,000 | ( ) | $2,000 | ( ) |
| | $6,000 | ( ) | $1,000 | ( ) |

Let's say that you have checked $7,000. At that price or at any other higher price you are willing to sell, at any lower price you will keep the note and wait 10 years.

Your choice means that, for you, $7,000 is the **PRESENT VALUE** of $10,000 in ten years. The Present Value is exactly what it means: the **value now** of an amount that is to be received only some time in the future. Because $10,000 in 10 years is worth $7,000 to you now, you are willing to exchange your right to receive $10,000 in 10 years for $7,000 now: you are exchanging amounts that have equal value.

The Present Value that you selected ($7,000 in our assumption) was just a guess. Let's see if the calculator can help us analyze that guess. We already know how to do it from the point of view of the buyer of the note. For him it is an investment. He invests $7,000 now and gets back $10,000 in ten years. What rate of return is he getting on his investment?

| | | | | | |
|---|---|---|---|---|---|
| 7000 | PV | 10000 | CHS | FV | |
| 10 n | | 0 | PMT | i | (3.63%) |

He gets a rate of return of 3.63% on his investment.

The same rate expresses the exchange of money from the investor's point of view and from your point of view as the seller of the note. But from your point of view it can no longer be called "rate of interest", "rate of return" or "rate of increase", because that implies money moving with time, not against it. This is why, from your point of view, the rate is called a **DISCOUNT RATE.**

By the way, 3.63% is not a very good rate of return for the investor, and you won't find a buyer when other safe investments provide 2 or 3 times that rate. **From your point of view also 3.63% is an unacceptable discount rate.** Refusing to sell for less than $7,000 means that you have little hope of investing at much more than 3.63%, and we have all heard of 5 1/2% passbook accounts.

It is difficult to guess at a Present Value--even more so if there were more than one cash flow. Our approach should be to decide on a discount rate and use it to calculate the Present Value. So let us choose a discount rate a couple of points lower than what we feel we can get as a rate of return on our secure investments over the next 10 years:

7% ( )     9% ( )     11% ( )     13% ( )     15% ( )     20% ( )

If we select 11% we can now calculate the Present Value:

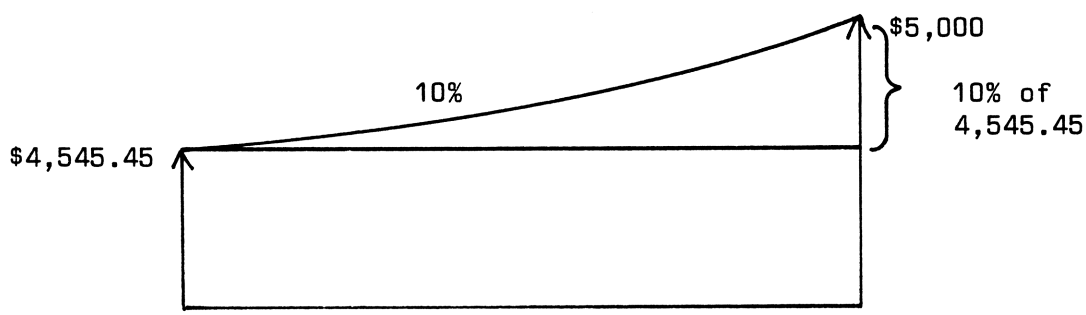
| | | | | | |
|---|---|---|---|---|---|
| 10000 | CHS | FV | 11 | i | |
| 10 n | 0 | PMT | | PV | ($3,521.84) |

We should be willing to sell the note for $3,521.84.

The discount rate that we select, and the resulting Present Value, need not be the same for everybody. Within the framework of the financial market at the time, it very much depends on the investment and borrowing options that are available to use.

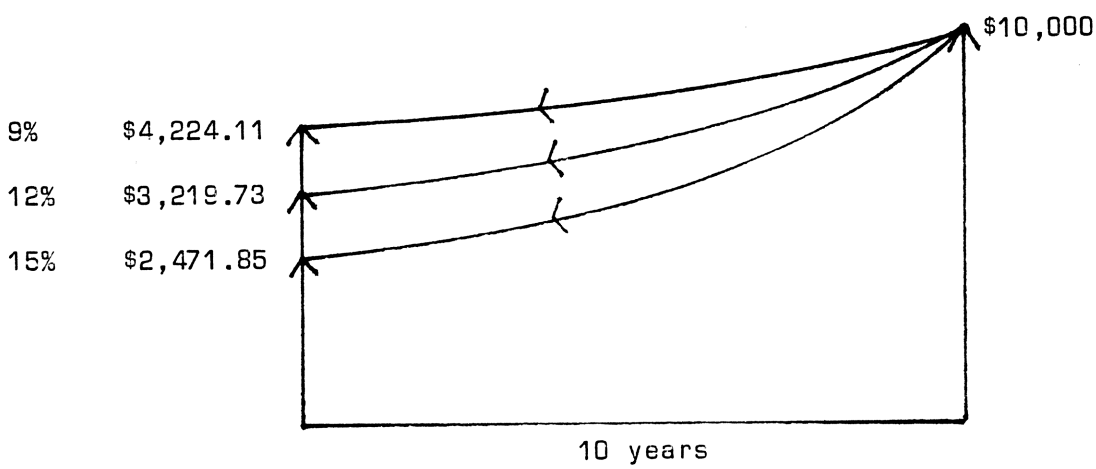
A discount rate of 10% does not mean that we discount the Future Value amount by 10% for every year that we bring that amount closer to us. It is the rate that would allow the Present Value to grow into the Future Value but seen from the point of view of the Future Value shrinking back to an earlier period of time, sliding back, in other words, along the very same curve that is defined by a rate of return of 10%.

So there is a major difference between a discount rate and the more common usage of the word discount, even when expressed as a rate. When a $5,000 object—whether a loan or an air conditioning unit—is sold at a 10% discount, it is sold for $4,500, which is $5,000 minus 10% of $5,000. When a 10% discount rate is applied to a $5,000 amount due in one year, the Present Value is $4,545.45 which is the amount that, increased by 10% of itself ($454.55), gives $5,000.




The difference is equal to 10% of $4,545.45, not 10% of $5,000. The 10% discount rate is the rate that allows $4,545.45 to grow into $5,000 in one year, but from the point of view of the $5,000 sliding back down to the present along the very same 10% curve.

The Present Value of a future sum depends on the rate that is applied to the calculation: the higher the rate, the smaller the Present Value. At 12%, the Present Value of $10,000 ten years from now is $3,219.73. At 15% it is only $2,471.85, but at 9% it increases to $4,224.11:



## SOME BASIC TIME VALUE OF MONEY CALCULATIONS

**A** What is the Present Value of $10,000 ten years from now? As we have seen, there is no answer unless we also submit a discount rate.

| | | | | | |
|---|---|---|---|---|---|
| | 10000 | CHS | FV | | |
| | 10 | n | | | |
| | 0 | PMT | | | |
| At a discount rate of 12%: | 12 | i | | PV | ($3,219.73) |
| At a discount rate of 15%: | 15 | i | | PV | ($2,471.85) |
| At a discount rate of 9%: | 9 | i | | PV | ($4,224.11) |

These numbers imply that I would have to invest $4,224.11 at 9% in order to accumulate $10,000 ten years from now, but that I would only have to invest $3,219.73 at 12% or $2,471.85 at 15% to get the same $10,000 after the same 10 year period: the Present Value calculation is just the flip side of the coin of the Future Value calculation.

We pressed **CHS FV** more as a matter of routine than of necessity. Omitting it would just give us a negative amount for the **PV** answer.

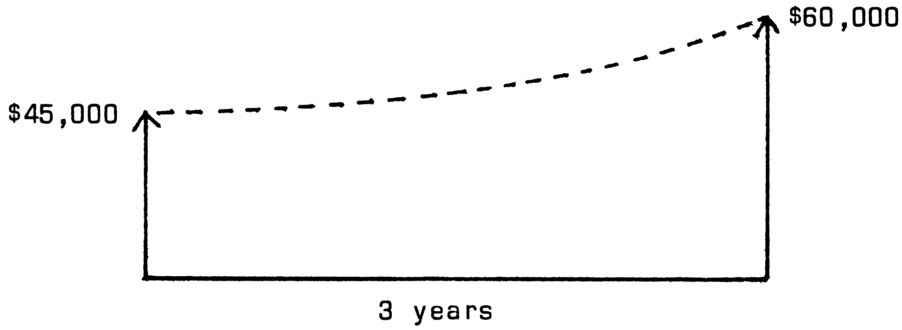
So the **PV** key really gives us the Present Value with the **wrong sign.** This is because the regular cash flow keys take the point of view of the exchange of money rather than just simply calculating the (positive) Present Value of a (positive) future amount, or the (positive) Future Value of a (positive) present amount. The calculator is telling us: if it is worth $3,219.73 cents, then you should be willing to pay $3,219.73 for it.

Similarly we can calculate the discount rate if we know the Present Value, the Future Value, and the time span involved:

**B** What discount rate have you established for yourself if you decide that $60,000 three years in the future is only worth $45,000 to you?

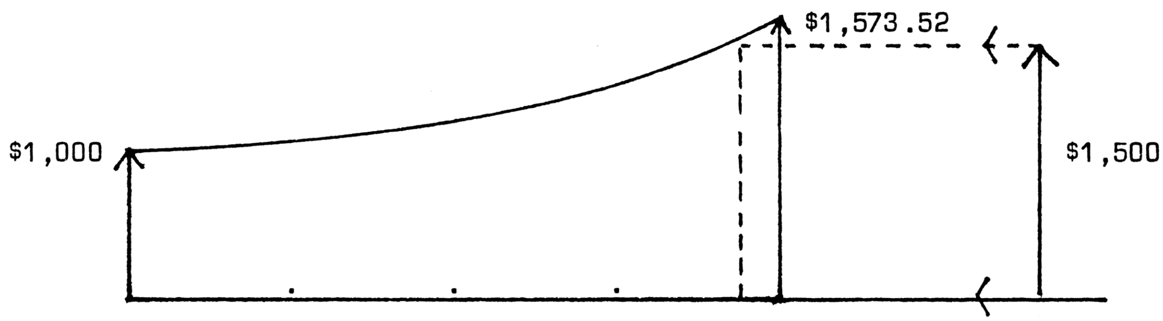
| | | | | | |
|---|---|---|---|---|---|
| 60000 | CHS | FV | 45000 | PV | |
| 3 n | 0 | PMT | i | | (10.06%) |

In this case we know the dollar amounts and we calculate the curve:




In the same way we can calculate **n** if we know the three other elements:

**C** How long does it take for $1,000 to grow into $1,500 if we invest it at 12%, annual compounding?




We know the Present Value, the curve, the amount of the Future Value but not its position in time. Graphically, we can imagine the $1,500 amount sliding until it touches the curve. That defines a value for **n**.

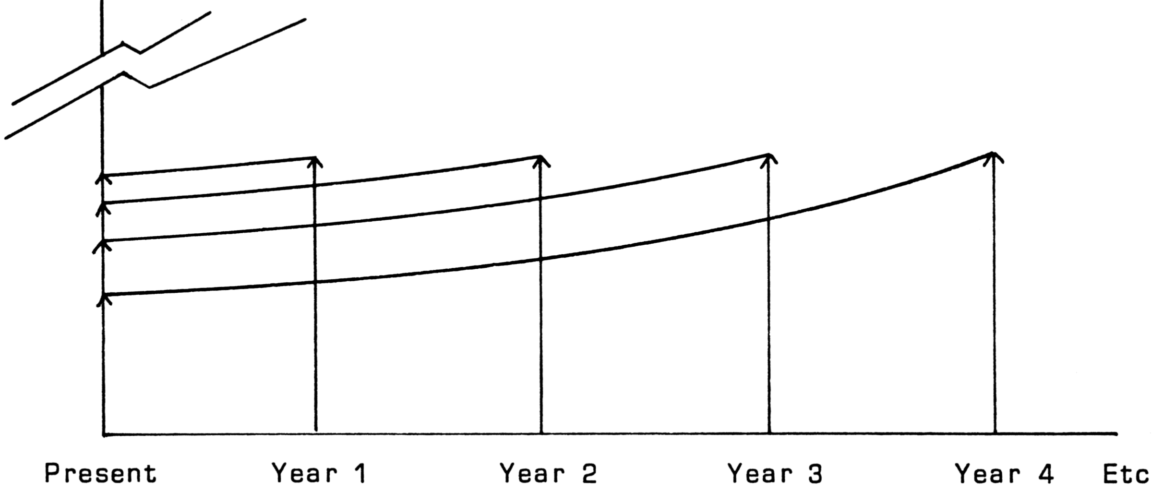
| | | | | |
|---|---|---|---|---|
| 1000 | PV | 1500 | CHS | FV |
| 12 | I | 0 | PMT | n |

(4)

The calculator gives 4 years as the answer, but this number has been rounded up to an exact number of years--on the HP-12C **n** does not calculate fractional numbers. By pressing **FV** now, we calculate $1,573.52 as the amount earned after 4 years, not just $1,500. The calculator gives 4 as the answer because it assumes that no interest is earned until a full compounding period is over.

(For more details on **n** see Unit 10 and Appendix II)

**D** What is the Present Value of $10,000 a year received at the end of each year for the next 10 years if we choose a discount rate of 12%?




We can discount each amount separately at the same 12% discount rate. The Present Value of the first $10,000 payment is higher than the Present Value of the second, because we do not have to wait as long before we receive the amount: "Money is better sooner". By adding up all the Present Values we get the Present Value of the income stream.

To obtain the result with the calculator, rather than repeat the process with each future amount, we use the **PMT** function:

| | | | | | | |
|---|---|---|---|---|---|---|
| 10000 | CHS | PMT | 10 | n | 12 | i |
| 0 | FV | PV | | | | |

($56,502.23)

As you press that **PV** key, imagine those 10 payments sliding down the curve and added up. Given the curve defined by the 12% rate, $56,502.23 now is "equal" to payments of $10,000 once a year for the next 10 years.

If we have an extra amount of $50,000 added to the last payment, that can be added to our cash flows as a Future Value. The increased Present Value can be calculated by pressing **PV**:

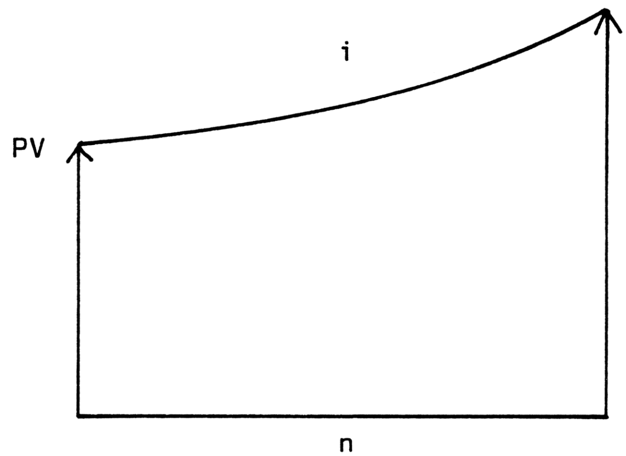
| | | | |
|---|---|---|---|
| 50000 | CHS | FV | PV |

($72,600.89)

We may now press **Minus** to get the difference between the two latest answers: that difference ($16,098.66) is the Present Value of the additional $50,000 ten years from now.

The **PMT** key, in combination with **n**, offers a convenient means of communicating a whole batch of future values to the calculator, all equal amounts equally spread out in time.

So we are back to the simple Present Value - Future Value transfer along the curve represented by **i**, except that now we can deal simultaneously with a whole group of future values at a time.




One or more future amounts defined by **FV** function or by **PMT** function in combination with **n.**

The missing element can be calculated if the others are known.

**That simple task is the only thing the 5 financial functions do for us.** But they do so with large numbers of future amounts, in any combination, and at considerable speed. The irregular cash flow functions perform the same task with even more complex data.

Beyond the complexity of the data, there remains the simple reality that is the **TIME VALUE OF MONEY.**

We spoke earlier of an equality of a kind between the various amounts $1,000 could grow into when invested at 12%. We establish the same kind of equality, by the same process, when we discount a loan to yield 20%. We calculate the Present Value of all the future benefits at the monthly compounded discount rate of 20%. If the answer comes out as $27,500 the calculator is telling us that, at that discount rate, all the future benefits are worth $27,500 now. If this is what they are worth, then it is what the investor should be willing to pay. With 20% as the equalizing rate, $27,500 now is "equal" to all those payments and the Future Value that are being purchased and that were put in the calculator when we balanced the books on the original loan.

Balancing the books, in fact, is merely establishing such an equality. When we are given a $78,000 loan at 9%, amortized over 30 years, calculating the $627.61 payment is equivalent to saying that $78,000 now is "equal" to 360 monthly payments of $627.61. Because the two sides of the transactions have been judged to have the same value now on the basis of the rate that was agreed upon, then borrower and lender are willing to exchange these amounts.

# THE TIME VALUE OF MONEY

## CONCLUSION

So at the heart of all the calculations of the financial keys, regular cash flow and irregular cash flow, there is the simple relationship that exists between **n**, **i**, **PV**, and **FV** that we were able to mimic by pressing 12 % +, 12 % +, etc.

The financial keys, of course, do not have to use this clumsy repetitive process: they use a formula to go directly to the answer. They are able to solve for any one of the four variables if the other three are known, and are able to do this even in situations where a considerable number of future amounts are submitted by means of the **PMT** key--thus introducing a fifth variable. So the mathematics may not be simple.

But at the center of it all there is the simple reality of the **TIME VALUE OF MONEY**, and of a tool that has the ability to handle that reality: that allows money to slide up and down the curve, or that calculates the curve if we submit all the dollar amounts and their place in time.

**We are the ones who give constantly changing meanings to that simple reality.**

## PRACTICE

Remember to either clear the financial memories between financial calculations (**GOLD CLEAR-FIN** or, if you must, **GOLD CLEAR-REG**), or preferably to clear the 5th financial memory in those cases where you do not use it by pressing **0 FV, 0 PMT** or **0 PV** at any point before you 'ask the question'.

1) You are to receive $79,300 four years from now. What is the Present Value of that amount if you apply a discount rate of 13% annually compounded? ($48,636.18)

2) You purchase a car by taking over the payments that remain to be made on the loan. Knowing that you could borrow money at 9.75%, what is the Present Value of the 33 monthly payments of $212.00 that you are agreeing to make beginning one month from today? ($6,114.96)

3) You are purchasing the right to receive 39 monthly payments of $666.00 plus an additional amount of $85,000 at the end of the 39 months. What is the Present Value of those amounts if you use a discount rate of 14%? ($74,842.58)

4) Leave the same cash flows in the calculator. Now key in a discount rate of 0% by just pressing **0 i**. What kind of a curve corresponds to a rate of 0%? What is the meaning of the Present Value that you calculate with **0** in **i**?

   0% corresponds to a flat horizontal curve--a straight line. The **PV** of $110,974.00 is the sum of all the Payments and the Future Value that are just brought back to time zero and added up without being discounted. Check by multiplying $666 by 39 and adding $85,000.

5) What discount rate is applied if a $5,000 amount due 5 years from now is given a Present Value of $3,000? (Annual compounding). (10.76%. Do not forget the sign requirement)

6) You invest $11,000 at 9% (annual compounding) for 5 years. Use the financial keys to watch the money grow year after year. Get the same numbers using the percentage key (**%**). ($11,990; $13,069.10; $14,245.32; $15,527.40; $16,924.86)

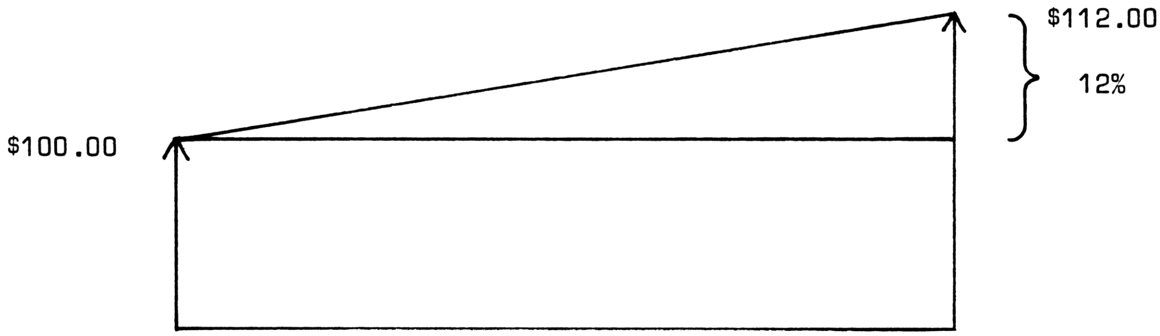
## COMPOUNDING

The same amount, invested at the same rate for the same period of time, can produce different increases if the compounding period is different:

$100 invested for 1 year at 12%, annual compounding.

| | | | | | |
|---|---|---|---|---|---|
| 100 | PV | 1 | n | 12 | i |
| 0 | PMT | FV | | | |

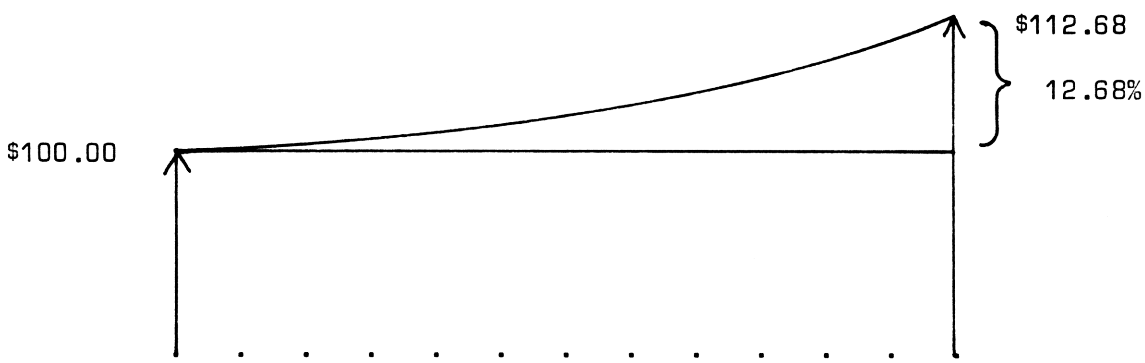
($112.00)




$100 invested for one year at 12%, monthly compounding.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 100 | PV | 1 | BLUE | n | 12 | BLUE | i |
| 0 | PMT | FV | | | | | |

($112.68)




The same $100 invested for one year at 12% grows into $112 with annual compounding (a 12% increase), and into $112.68 with monthly compounding (a 12.68% increase).

## THE EFFECT OF COMPOUNDING

Let's invest $100 at 12% with different compounding periods:

| Compounding period | 1 year | 10 years |
|---|---|---|
| Yearly | $112 | $310.58 |
| Monthly | $112.68 | $330.04 |
| Daily | $112.75 | $331.95 |
| Continuous | $112.75 | $332.01 |

The shorter the compounding period, the faster the investment grows.

There is a substantial difference between yearly and monthly compounding, a much smaller difference between monthly and daily, and only a marginal difference between daily and continuous compounding. Continuous compounding is impressive, but it does not mean you have hit the jackpot and the dollars come pouring out (See Appendix II, page 4)

The **PERIODIC RATE** is the percentage increase **per compounding period.** An annual rate of 12% compounded monthly has a periodic rate of 1%: every $100 earns $1.00 every month. That dollar is added to the principal and in turn earns additional interest. Because of interest earned by interest $100 earns more than $12 in 12 months. Without compounding, the increase would be along a straight line, not a curve.

## CALCULATING THE EFFECT OF COMPOUNDING

To solve problems, we key the periodic rate in **i** and the number of periods in **n**, which means that we key **n** and **i** expressed in terms of the compounding period. For instance:

You deposit $5,000 in a 12% savings account that offers daily compounding. How much do you have in the account after 3 years? (Use a 365 day year)

| | | | | | | |
|---|---|---|---|---|---|---|
| 5000 | PV | 0 | PMT | | | |
| 12 | ENTER | 365 | ÷ | i | Periodic rate: | 12/365 |
| 3 | ENTER | 365 | x | n | Number of periods: | 3 x 365 |
| FV | | | | | | ($7,166.22) |

## A CASE STUDY

Let us compare two $10,000 one-year loans.

> **Loan A:** a straight note (no payments)
> returning $12,400 at the end of the year.
> **Loan B:** 12 monthly payments of $100,
> plus $11,200 at the end of the year.

From the lender's point of view, which is the best loan? Quite obviously loan B, as they both return $2,400 in interest, but loan B returns some of that money sooner. Let's have the calculator confirm our judgment:

| | | |
|---|---|---|
| Rate of return on A: | 10000 PV 12400 CHS FV<br>1 n 0 PMT<br>i | (24% return) |
| Rate of return on B: | 10000 PV 100 CHS PMT<br>11200 CHS FV 1 BLUE n<br>i (wait) 12 x | (22.8% return) |

Contrary to expectation, loan B shows a lower rate of return than A. Were we wrong in our judgment, or is the calculator wrong? Neither really. The calculator has provided us with two rates that we cannot compare because they have different compounding periods: annual compounding for loan A where **n** and **i** are expressed in years, monthly compounding for loan B where **n**, **i**, and **PMT** are expressed in months.

The period selected for **n**, **i**, and **PMT**--or imposed by the data--is also the compounding period.

**Rates can be compared accurately only if they have the same compounding period.**

In the previous example, pressing **1 BLUE n** for loan A instead of **1 n** calculates a monthly rate of 1.81% that multiplied by 12 gives a monthly compounded rate of 21.71%. That rate can now be compared to the 22.8% monthly compounded rate for loan B and seen to be lower as expected.

## EFFECTIVE YIELD

You may have seen posters advertising rates on savings account:

**INVEST IN OUR ACCOUNT!**

**Interest Rate: 10.670%**

**Annual Yield: 11.425%**

The two rates are correct, and they both are a measure of the same exchange of money in time. But they imply different compounding periods.

The annual yield, or less ambiguously, annualized yield or effective yield, implies yearly compounding. It is the rate of increase that affects an investment that remains invested for one year at the nominal rate.

Whatever the compounding period, rates are generally expressed as annual rates. The compounding period either needs to be specified, or, if there are payments involved, is implied by the data.

When we speak of a rate in a situation where we have monthly payments, the implication is that we are referring to a monthly compounded rate. It would not be ethical to use the effective yield without making it clear that the compounding period has been changed. But we can do like the Savings and Loans and use the two rates if that serves our purpose.

For instance, if we discount a loan with monthly payments to yield 18.95%, that is a monthly compounded rate. We may refer to that number and also specify that it corresponds to a 20.69% effective yield, which may help us sell the note to a buyer who insists that he wants 20% on his money: for every dollar that remains invested for one year he will earn $20.69.

## CALCULATING AN EFFECTIVE YIELD

The easiest way to calculate an effective yield is to invest $100 for one year at the nominal rate and to see the effective increase:

We just saw that $100 invested for one year at 12%, compounded monthly, yielded $112.68. That is a 12.68% increase. So 12% with monthly compounding has an effective yield of 12.68%. For amounts that remain invested a set number of years, a monthly compounded rate of 12% or an annually compounded rate of 12.68% have the same effect.

What is the annualized yield corresponding to a rate of 8% compounded monthly?

| | | | |
|---|---|---|---|
| 100 | PV | | |
| 8 | BLUE | i | |
| 1 | BLUE | n | |
| 0 | PMT | FV | (108.30) |

The effective yield is 8.30%.

Converting rates to the corresponding effective yields is one way of comparing rates with different compounding periods:

Which produces more interest, a 14.75% loan with monthly payments
or a 15% loan with semi-annual payments?

| | | | | | | |
|---|---|---|---|---|---|---|
| 14.75 | BLUE | i | 1 | BLUE | n | |
| 100 | PV | 0 | PMT | FV | | ($115.79 or 15.79% annualized yield) |
| 15 | ENTER | 2 | n | ÷ | i | (2 in **n**, 15/2 in **i**) |
| FV | | | | | | ($115.56 or 15.56% annualized yield) |

14.75% compounded monthly is higher than 15% compounded semi-annually.

With the data from the case study involving two $10,000 investments, we can translate the 22.8% monthly compounded rate into an annual yield of 25.34%, which is better than 24% and confirms our initial judgement.

## CONVERTING RATES FROM ONE COMPOUNDING PERIOD TO ANOTHER

(Included here mostly for the record.
See Unit 10, page 8 for application)

When we have monthly payments we need monthly data in **n** and **i**. This imposes monthly compounding for the rate stored in **i**. If the actual nominal rate has a different compounding period--for instance daily compounding--then we need to replace that rate with an equivalent rate having the required compounding period--the month in our example.

The key to making such a transformation is to leave in the **PV** and **FV** memories an exchange of money in time corresponding to the known rate with its stated compounding period and to allow the calculator to express the rate on that exchange in terms of the other compounding period. It is convenient to choose 1 year and a $100 Present Value for the calculation.

What is the monthly compounded rate that corresponds to a 9% daily compounded rate?

| | |
|---|---|
| 9 ENTER 365 n ÷ i | Puts 365 in **n** and periodic rate 12/365 in **i**. |
| 100 PV 0 PMT FV | $109.42. Completes **PV-FV** exchange at 9% compounded daily. |
| 12 n | 12 monthly compounded periods. |
| i | The calculator expresses the rate on the initial exchange of money in terms of monthly compounding. |

The answer shows as 0.75% per month. The internal number is 0.75272663985 (**GOLD ENTER**). The annual rate appears as 9.03%.

In short:

(1) Balance the books on a **PV-FV** exchange at the known rate. ($100 **PV**, 1 year are convenient numbers to use)

(2) Re-express **n** in terms of the period of the unknown rate. (generally the period at which Payments occur)

(3) Calculate **i**.

**PRACTICE**

7) Calculate the annualized yield on the following rates:
9% compounded monthly. (9.38%)
12% compounded monthly. (12.68%)
12% with semi-annual compounding. (12.36%)
15% with quarterly compounding. (15.87%)
7.5% with daily compounding. (7.79%)

8) What is the difference in dollars between investing $10,000 for 5 years at 10% with daily compounding and with yearly compounding? ($380.98)

9) Which has the lowest rate?
Loan A at 10% interest, with annual payments.
Loan B at 9.5% interest with monthly payments.
(9.92% annualized yield on loan B is still lower than 10% effective yield of loan A)

10) Compare these two loans:
Loan C is at 17% with monthly payments.
Loan D is at 17.25% with semi-annual payments.
(Effective yield: loan C, 18.39%; loan D, 17.99%)

11) Calculate in succession the one year balance on these two amortized loans. Then press – to get the difference between the balances.
What conclusion do you draw?
Loan E: $85,000, 12% interest, payments of $950 per month. ($83,731.75)
Loan F: $85,100, 12% interest, payments of $950 per month. ($83,844.43)
(Difference: $112.68)
(The extra $100 invested in loan F has earned $12.68 in interest over the year. 12.68% is the effective yield corresponding to 12% compounded monthly. Even though interest is paid as soon as it is earned and is never allowed to be added to the principal and earn interest of its own, the very fact that interest is paid every month instead of every year is a benefit that automatically translates into a 12.68% effective yield. The nominal rate on regular real estate loans with monthly payments is a monthly compounded rate that corresponds to a higher effective annual yield)

12) Calculate and leave in **i** the quarterly compounded rate corresponding to 14% with daily compounding. (3.56% per quarter, 14.25% per annum)

# APPRECIATION AND INFLATION

Appreciation and inflation are important aspects of the interaction between time and money. They will provide reinforcement for our sense of money sliding up and down the curve defined by a rate of increase.

However, it is not only because of inflation that money is better sooner. Though inflation is a factor, often a major factor, in determining how much more valuable money is when it is not deferred, the underlying cause of the time value of money is productivity: the ability of capital investment (money now) to increase the amount of goods and services that can be produced and sold in the future (money later).

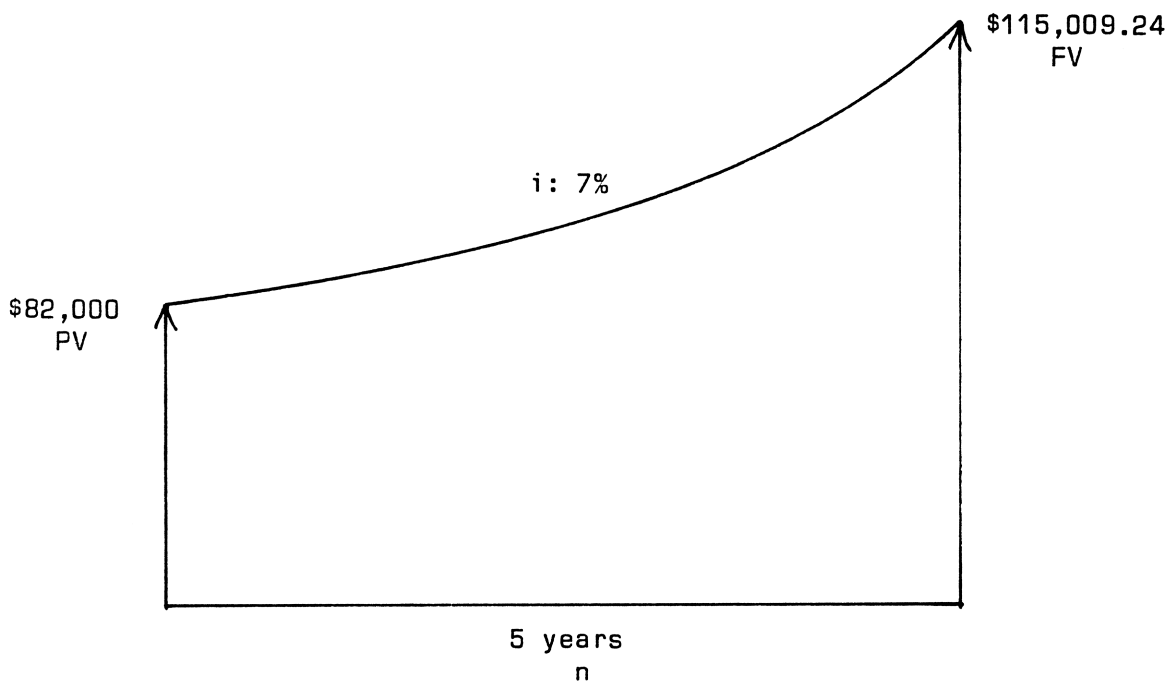
Every decision we make is based on assumptions. An important decision such as the purchase of a home is based on many assumptions: about one's health, one's family, one's job, and the general state of the economy. About the San Andreas Fault or the waters of the Mississippi river. None of the assumptions is certain to be realized, but they represent the best we have to go on. A major assumption affecting Real Estate decisions concerns inflation and appreciation. Our examples will be taken from Real Estate, but the calculations can be applied to any other product or service.

Given an assumption concerning a rate of inflation or appreciation, the calculator allows us to translate the effects of the assumption into important decision-making data. It does so with the simplest possible keystrokes.

**Appreciation** is inflation as it affects a specific product, one particular item. **Inflation** is the impact on the economy as a whole of the different rates of appreciation that affect the various goods and services: as a result it is the effect of appreciation on the value of money itself.

**Deflation** and **depreciation** measure the corresponding change in value when goods go down in price instead of up: they represent inflation and appreciation at a negative rate. Though we will talk mostly in terms of appreciation and inflation, all the calculations could apply to deflation and depreciation by just keying a negative rate into **i**.

# APPRECIATION




A 7% yearly increase has the same effect on the initial $82,000 amount whether the rate of increase is interest on an investment or appreciation of an asset.

## APPRECIATION

Assuming a 7% rate of appreciation, what is the value of an $82,000 property after 5 years?

| | | |
|---|---|---|
| 82000 | PV | Present value of property in **PV.** |
| 7 | i | Annual rate in **i.** |
| 5 | n | Number of years in **n.** |
| 0 | PMT | No payments to be taken into account. |
| | FV | Future Value expect to be $115,009.24. |

7% is an annual rate and implies no monthly compounding: **n** and **i** are keyed in directly without using the BLUE key.

Note the negative answer: the sign convention still applies. After all, how could the calculator know we are dealing with appreciation and not with interest on an investment?

Because of the high degree of uncertainty implied in the initial assumption it makes no sense not to round very generously when dealing with such calculations. We will not round when giving the answer, but will round on occasion as we refer back to the numbers in the text.

We may now change the initial data in any way we want:

| | | | |
|---|---|---|---|
| 10 | n | FV | $161,306.41 value after 10 years. |
| 4 | i | FV | $121,380.03 value after 10 years of appreciation at 4% per year. |
| 12 | i | FV | $254,679.55 value after 10 years of appreciation at 12% per year. |

What would be the value of the property after 4 years of <u>depreciation</u> at 7% per year? We just key a negative value into **i**:

| | | | | | | |
|---|---|---|---|---|---|---|
| 4 | n | 7 | CHS | i | FV | ($61,340.26) |

After 10 years of appreciation at 6% per year?:

| | | | | | |
|---|---|---|---|---|---|
| 10 | n | 6 | i | FV | (146,849.51) |

## CALCULATING A RATE OF APPRECIATION

Calculating a rate of appreciation when we know the value of the property at two points in time is as simple as keying in the numbers. Of course, we have to apply the sign convention.

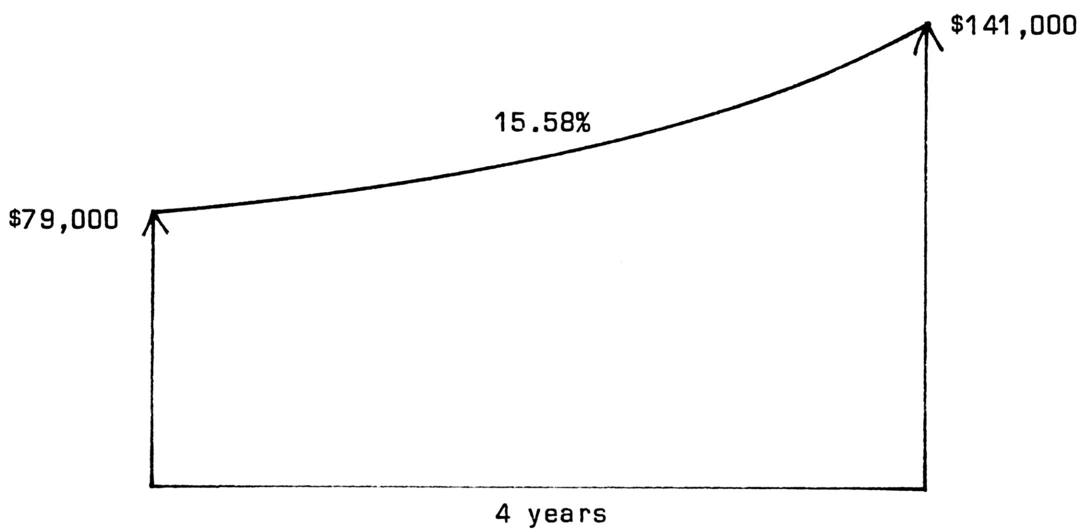
> A property was purchased for $79,000 and is being sold four years later for $141,000. What annual rate of appreciation does that represent?

| | | | | |
|---|---|---|---|---|
| 79000 | PV | 141000 | CHS | FV |
| 4 n | 0 | PMT | i | |

(15.58%)

## DIAGRAM

All these calculations simply involve values sliding up and down the curve defined by the rate of appreciation, or calculating the rate if we know the values:



## EQUITY INCREASE DUE TO APPRECIATION

Simple arithmetic with numbers that we have in the financial memories tells us by how much the value of a property has increased over time:

What is the equity increase due to a 7% rate of appreciation for 5 years on an $82,000 property?

| | |
|---|---|
| 82000 PV 7 i | |
| 5 n 0 PMT FV | ($115,009.24) |
| CHS RCL PV − | ($33,009.24 equity increase) |

Same $82,000 property, same 5 year time span. What is the decrease in equity due to a 7% rate of depreciation?

| | |
|---|---|
| 82000 PV 7 CHS i | |
| 5 n 0 PMT FV | ($57,046.45) |
| CHS RCL PV − | ($24,953.55) |

The same rate has a more dramatic effect on appreciation than on depreciation: the increase gets larger and larger every year as the property appreciates, the decrease due to depreciation gets smaller and smaller as the value decreases.

A $125,000 property. 5% projected rate of appreciation. What is the equity increase during the 7th year?

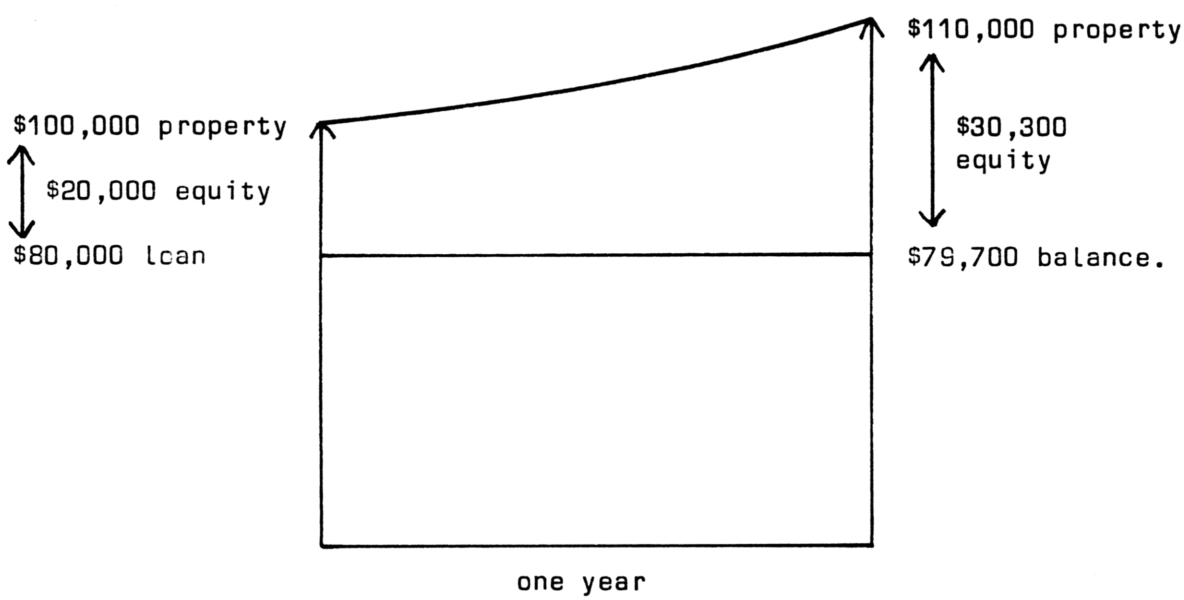
| | |
|---|---|
| 125000 PV 5 i 0 PMT | |
| 6 n FV | ($167,511.96) |
| 7 n FV | ($175,887.55) |
| − | (Difference: $8,375.60) |

Just calculate the increased value after 6 year, then after 7 years, and press − to get the difference.

**LEVERAGE AND APPRECIATION**

People do not generally own real estate free and clear. A 20% downpayment is often more than enough to purchase a property. The owner has an initial 20% equity in the property, an equity that increases as the loan is paid down and as the value of the property is affected by appreciation. The smaller the owner's equity in relation to the value of the property, the greater the leverage.

It is because of leverage that appreciation has such an impact on real estate ownership: I may only own 20% of the value of the property, but I benefit from appreciation on 100% of its value. With a 20% equity, a 10% appreciation in the first year increases the equity by 50%. Let us illustrate this in the case of a $100,000 property.




The $300 equity buildup we can expect from principal reduction is negligible compared to the $10,000 equity increase due to appreciation. The 50% increase in equity due to appreciation is not a rate of return: there are too many other factors involved, not least of which the payments on the loans and the costs involved in buying and selling property.

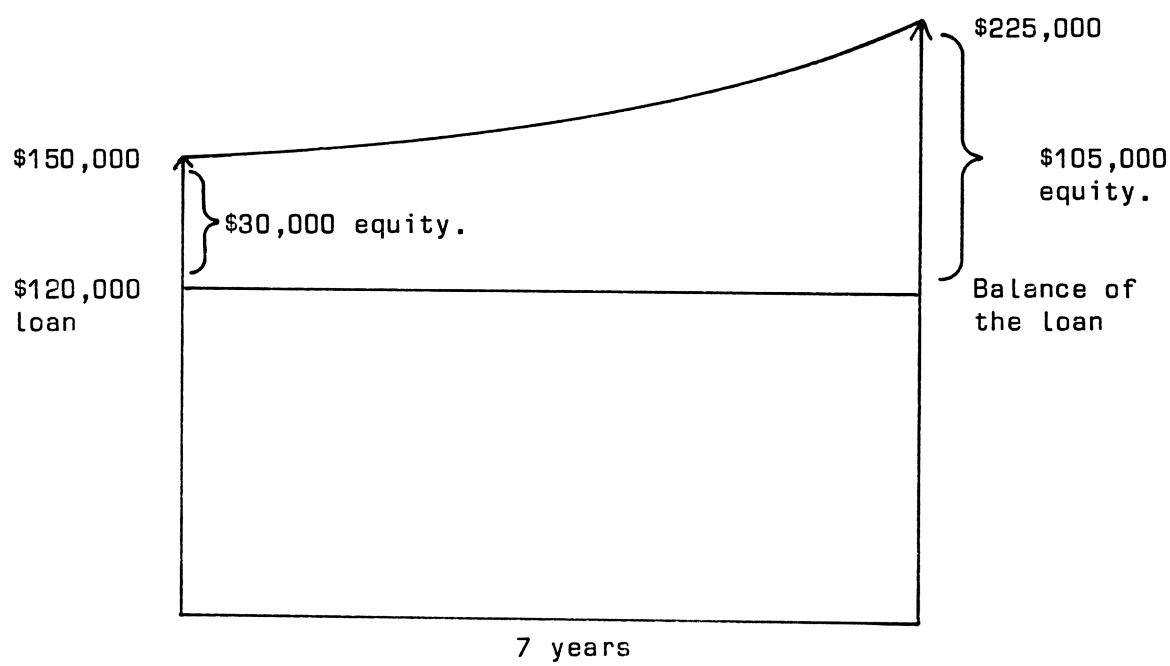
If the property had been bought with a 10% downpayment, the same 10% appreciation the first year would double the equity, plus any additional equity buildup due to principal reduction on the loan.

Let's look at a similar transformation taking place over several years.

A $150,000 property appreciates 6% per year for 7 years. It's value increases to $225,000 in round figures. The property was purchased with a 20% downpayment and a $120,000 loan--let's make it interest-only so as to ignore the $5,000 or $6,000 principal reduction that might have occurred.

The 6% rate of appreciation was also the rate of inflation: the property has just kept up with inflation and in constant dollars is not worth a penny more than when it was purchased 7 years before.

There is, however, a major difference. Without appreciation the owner would still have a $30,000 or 20% equity in the property (24% maximum with some principal reduction). With appreciation, he now has a $105,000 equity in the property, or 46.67% of the $225,000 value. This is a substantial and very real difference. He could use the equity to move into a property worth about half as much and own the property free and clear. Without appreciation, he could exchange his equity for a property worth only one fifth. (See Unit 8 for percentage calculations)



**PRACTICE**

13) What is the value of a $136,000 property
after 5 years of appreciation at 5% per year? ($173,574.29)
after 5 years at 9% per year? ($209,252.86)
after 10 years at 9% per year? ($321,961.46)
after 6 years at 4% per year? ($172,083.39)

14) A $92,000 property is affected by a 6% rate of appreciation.
Calculate
- The value of the property after 5 years. ($123,116.75)
- The equity increase due to appreciation. ($31,116.75)
- The total equity at the end of the 5th year if the property was bought with an 80% loan at 10% interest, amortized over 30 years. ($52,038.05)

15) A property was purchased for $185,000 and is sold for $300,000 three years later. What is the rate of appreciation? (17.49%)

16) A property is purchased for $325,000.
What is the equity increase due to appreciation during the 10th year of ownership (from the end of the 9th year to the end of the 10th year) if appreciation is 4% per year? ($18,503.05)
Same question if the rate of appreciation is 6% per year. ($32,944.84)
Compare that last amount to the total annual cost of a 12% 30 year mortgage on the property in the amount of 80% of the purchase price.
($32,092.71 annual payments on $260,000 loan. Even before tax advantages the 6% appreciation more than pays for the 12% mortgage payments by that time.)

## INFLATION

So far we have considered the impact of appreciation on real estate ownership. Almost as important is the impact of inflation itself, whether or not there is any appreciation on the property.

Leveraged real estate is purchased with a downpayment and with a loan on which payments must be made. Inflation means that the payments become easier and easier to make.

> A buyer purchases a home for $165,000. He makes a 20% downpayment and finances the rest with a 30 year loan at 12.5%. The payments of $1,408.78 represent 1/3 of his income, and it hurts. How painful is it going to be to make that same payment after 7 years of inflation at 6% per year?

| | | | | | |
|---|---|---|---|---|---|
| | 1408.78 | CHS | | FV | Why in **FV?** |
| After 7 years at 6%: | 7 | n | 6 | i | |
| | 0 | PMT | | PV | ($936.92) |

The calculation considers only <u>one</u> payment that changes hand 7 years from now, and therefore treats that amount as a **FUTURE VALUE.**

The $1,408.78 check that will be written 7 years from now will be worth only $936.92 in terms of today's dollars because it is paid in dollars that have lost some of their value. Wouldn't it be great to be able to buy that property with payments of $937? If the buyer's income keeps up with inflation--and that is a basic assumption made by the purchaser and by the lender--, then the payment will no longer be 1/3 of his income and will be as easy to make as a $937 payment would be today.

| | | | | |
|---|---|---|---|---|
| What about after 10 years? | 10 | n | PV | ($786.66) |
| After just 4 years? | 4 | n | PV | ($1,115.89) |
| After 4 years at 4% only? | 4 | i | PV | ($1,204.23) |

Whatever the assumptions--they should be conservative, and it does not make much sense to go too far into the future--the impact is very real.

The benefits of inflation for the property owner are independent of any appreciation on the property. The property could decrease in value: if there still is inflation for the cost of living in general, the payments are going to be easier and easier to make. If there is both inflation and appreciation, then the property owner benefits from the two.

## YIELD AFTER INFLATION

Our response to rates is very relative. 21% as a rate of return may be reasonable at a time when inflation is 17%, it may seem outrageous when inflation is 2% or 3%, and it would be outrageously low in a country such as Argentina or Israel where inflation is 100% or even 200%. Rates are used as means of comparing the cost of money or the return on one's investment to the cost of other loans and the return on other investments that are all living in the same inflationary environment—or deflationary as the case may be.

So it wouldn't make sense to take the market rate of return we can expect on an investment—let's say 13%— and to say: "Hey, I want my 13% over and above inflation!" 13% is the market rate precisely because inflation is around 8%. With inflation at 2%, the rate we could expect might be just 5% or 6%. So the correction for inflation is already built into our expectation of a 13% return.

As already pointed out, inflation is not the primary reason why we can expect to earn interest on the money we agree to invest. It may be an important ingredient of that rate, but we could still expect some interest even without inflation. The increased productivity due to capital investment is the primary cause of interest: by depositing my money in a savings account, I agree not to spend it. This allows somebody else to spend it on tools that will increase his productivity: for the privilege of buying the tools with my money, he is willing to share the benefits of that increased productivity with me in the form of interest. This, at least, is the theory.

How can we calculate that underlying rate which is the real return adjusted for inflation? Just taking the difference between the rate of return and the rate of inflation is not an accurate measurement. The easiest approach is to take $100.00 and 1 year:

What is the real rate of return on a 15% investment, adjusted for an 8% rate of inflation?

| | | | | | | |
|---|---|---|---|---|---|---|
| 100 | PV | 15 | i | 1 | n | |
| 0 | PMT | FV | | | | ($115) |
| 8 | i | PV | | | | ($106.48) |

In 1 year $100 grows into $115. But $115 one year from now is only worth $106.48 after adjustment for inflation. The rate of return adjusted for inflation is 6.48%.

## APPRECIATION AND INFLATION
## A COMBINED PROBLEM

A property selling for $225,000 requires monthly payments of $1,750. What can you say concerning the price of the property and the value of the monthly payments 10 years from now if you assume a 6% rate of inflation and that the property just keeps up its value by appeciating at the same 6% rate as inflation?

| Keystrokes | Result |
|---|---|
| 10 n 6 i | |
| 225000 PV 0 PMT FV | ($402,940.73: price after 10 years of appreciation) |
| ( CHS RCL PV – ) | (177,940.73: Optional equity build-up calculation) |
| 1750 CHS FV PV | ($977.19: payment adjusted for 10 years of inflation) |

In 10 years, under the assumptions made, the property would be priced at over $400,000 (a $178,000 build-up in equity), and the monthly payments would be as easy to make as payments of about $1,000 would be now.

Inflation

**PRACTICE**

17) Same problem as above using 5% inflation for 5 years.
($287,163.35 price, $62,163.35 equity build-up due to appreciation, $1,371.17 value of $1,750 payment adjusted for inflation.)

18) Express in current dollars the value of a $250,000 balloon payment due in 5 years if the rate of inflation is 5% per year? ($195,881.54)

19) You make the assumption that inflation will average 8% per year over the next 35 years, and that you will be able to invest $2,500 at the end of each year at an interest rate of 12%, a few percentage points higher than the rate of inflation.
What amount of money will be yours when you retire in 35 years?
(Congratulations, you will be a millionaire: $1,079,158.74)
What is the value of that amount adjusted for inflation? ($72,988.41)
What is the total amount of money invested? ($87,500)
It seems that you are getting back even less than the total amount invested. This is because you are not investing a $2,500 value every year. Because of inflation, the $2,500 amount is worth less and less each year.

20) Assuming an annual rate of inflation of 4%, what is the value in current dollars of a $1,000 payment to be made 10 years from now? ($675.56)
What if inflation averages 6%, or only 3%? ($558.39, $744.09)

# UNIT 7

# CHOOSING BETWEEN FINANCIAL ALTERNATIVES

# UNIT 7

# CHOOSING

# BETWEEN FINANCIAL ALTERNATIVES

This is a short unit that allows the concept of Present Value and the feel for money sliding up and down the curve defined by a rate to bear on problems we have already studied: in particular cash equivalent calculations.

It also proposes an alternative approach to choosing between two financial options: establishing the difference between the two cash flows, and calculating the rate on that difference.

## CHOOSING BETWEEN FINANCIAL ALTERNATIVES AND THE TIME VALUE OF MONEY

Numbers are constantly used to evaluate opportunities. This is most frequently done either by calculating a rate, which we then compare to other rates, or by calculating a Present Value, which we then compare to the value of other assets, now, in the present.

Calculating a rate is possible only when there is an exchange of money in time and when we know what that exchange is or are able to project what we expect it to be. Calculating a Present Value is most useful when we do not have an exchange of money on which to calculate a rate, either because there is no exchange, or because we have incomplete data on the exchange.

Money changing hands in time is the reality we are considering. The rate and the Present Value are measures that allow us to characterize that reality and compare it to other realities of a similar nature. We have already used the two approaches under a variety of names.

### COMPARING RATES

Calculating the **APR** or the **PERSONALIZED APR**, for instance, is a means for the borrower to compare different loans by establishing the real cost of the money he borrows. Calculating the rate of return on an investment--the yield on a discounted loan we want to purchase, for instance--even if we are not specifically comparing that rate to other investments, has little meaning unless it is used to compare with investments in general or with rates of return readily available in the market for similar investments.

It helps to think of rates as being essentially a means of comparing exchanges of money in time. As a result, in many circumstances, consistency is more important than accuracy. Though a rate calculated on after-tax cash flows may be more accurate than a before-tax rate, it becomes a meaningless tool if used to make a comparison with other before-tax rates. It is customary in investment analysis on income property to lump all the expenses as if they occurred at the end of the year. Taking into account the timing of major expenses during a year would affect the rate. But the adjusted rate, though more accurate, would be a less valuable tool when used to compare to the rate on other investments calculated in the traditional way. See also Unit 6, page 17, for the importance of consistency in the choice of the compounding period.

Calculating a rate used to be a time consuming procedure. With a financial calculator it is as simple as communicating the data and asking the question. We have learnt how to do it in regular cash flow situations which has opened up to us a variety of applications. The reader may want to acquire the same ability when dealing with exchanges of money in time involving irregular cash flows.

## COMPARING PRESENT VALUES

The **CASH EQUIVALENT PRICE** and **COST EQUIVALENT PRICE** calculations are simply Present Value calculations: we discount future amounts back to the present and add any amount received in the present. Calculating the cash equivalent price or the cost equivalent price has little purpose unless these numbers are used to compare different offers or the price of different properties, either specifically or in terms of the market as a whole. If, after having calculated a cost equivalent price we are able to say: "I didn't realize how good a deal it was", that implies a comparison with the cost of similar properties.

Calculating, as we did in Unit 5, the selling price of a discounted note to yield a required rate of return is also a Present Value calculation. We submit to the calculator all the future benefits that will result from purchasing the loan--this is done by 'balancing the books' on the initial transaction but could be done by just keying in the correct values for **PMT**, **FV**, and **n**, with no concern for the nominal rate and the amount of the original loan. We then discount these amounts at the required rate. Having established the Present Value, we are able to say: "If that is their present worth, that is what I should be willing to pay for them".

In using Present Value calculations to establish the relative worth of what we are buying or selling it is important to remember the obvious: future cash flows do not of themselves have one single permanent Present Value. The Present Value depends on the discount rate that we select. Furthermore, the relative present worth of various cash flows is also affected by the choice of the discount rate. This is because more distant cash flows are more sensitive to a change in rate than those that are closer. Let's take a typical example:

### MODEL PROBLEM

Which of these after-tax cash flows has the highest value?

Option A: $750,000 in 5 years.
Option B: $75,000 at the end of each year for 10 years.
Option C: $50,000 each year for 15 years, plus
$400,000 at the end of the 15th year.

Compare the alternatives by calculating the Present Value of the cash flows at after-tax discount rates of 10% and 15%:

At 10% we have the following answers:

| | | |
|---|---|---|
| Option A: | $465,690.99. | (2) |
| Option B: | $460,842.53. | (3) |
| Option C: | $476,060.80. | (1) |

At 15% we have the following:

| | | |
|---|---|---|
| Option A: | $372,882.55 | (2) |
| Option B: | $376,407.65 | (1) |
| Option C: | $341,526.30 | (3) |

Option C has the highest value for someone with alternative investments that could yield an after-tax rate of 10%. It becomes the worst choice for someone who could expect 15% on his investments. This is because more money is postponed for a longer period of time with option C, and the higher discount rate has more impact on it than on the others.

We know how to calculate these answers. For good measure, let us give the keystrokes that would calculate the Present Value under the two assumptions:

> (We will retain our routine use of **CHS** when keying data into **PMT** and **FV** though omitting the negative here would certainly make sense for anyone who has more than just a few to key in).

| | |
|---|---|
| 750000 CHS FV 0 PMT 5 n | |
| 10 i PV | ($465,690.99) |
| 15 i PV | ($372,882.55) |

| | |
|---|---|
| 75000 CHS PMT 0 FV 10 n | |
| 10 i PV | ($460,842.53) |
| 15 i PV | ($376,407.65) |

| | |
|---|---|
| 50000 CHS PMT 400000 CHS FV 15 n | |
| 10 i PV | ($476,060.80) |
| 15 i PV | ($341,526.30) |

This, in essence, broadened to include irregular cash flows, is what discounted cash flow analysis is all about.

Of course, if each of the options could be purchased for a specific amount, and that amount was known, we would then have no need for a Present Value calculation. We would simply calculate and compare the rates of return.

## PRESENT VALUE: WHAT TO EXPECT

In many cases, the amounts to be received in the future are the result of a loan that has a face value and a nominal rate of interest. We then discount the Payments and the Future Value--any amount that changes hand in the future--at a specific discount rate. This process occurs in discounted loan, cash equivalent, and cost equivalent situations.

(1) If the discount rate is higher than the nominal rate, the Present Value is lower than the face value.

(2) If the discount rate is the same as the nominal rate, the Present Value and the face value are the same. We are getting from the interest what we expect of our investments. The Present Value is not affected by the length of the loan. The cash equivalent price is equal to the selling price whatever the amount and terms of the loan. It is only when a second rate is thrown into the calculation, different from the nominal rate on the loan, that the Present Value is affected.

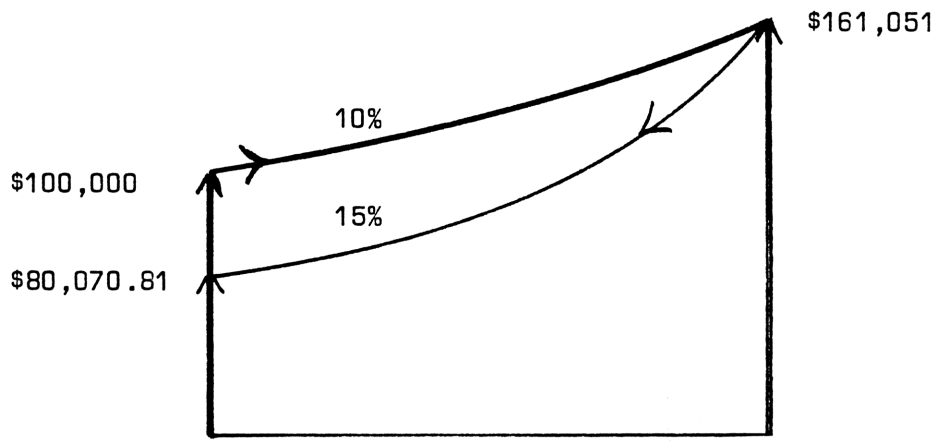
(3) If the discount rate is lower than the nominal rate, the Present Value is higher than the face value. The lender is getting more from the loan than he could expect from the same amount of cash.

This applies to any kind of loan, whatever the due date. But to illustrate the point, let us look at one single amount in the future, at a loan that is fully paid off with one lump sum:

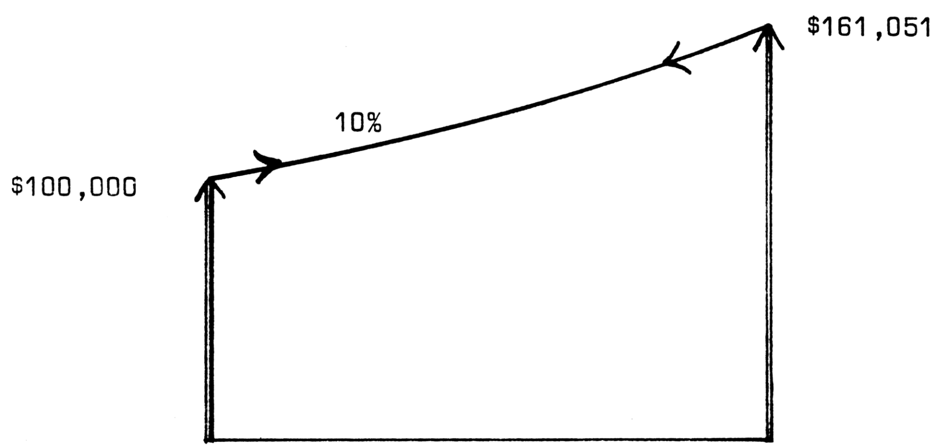
A straight note (no payments) with a face value of $100,000 and a nominal rate of 10%, annual compounding, is due 5 years from now. What is the value of the note discounted to yield 15%, 10%, and 7%? Or, which is the same thing, what is the Present Value of the $161,051.00 that must be paid back in 5 years if we discount that amount at 15%, 10%, and 7%?

| | | | | | | |
|---|---|---|---|---|---|---|
| 100000 | PV | 0 | PMT | 5 | n | |
| 10 | i | FV | | | | ($161,051.00) |
| 15 | i | PV | | | | ($80,070.81) |
| 10 | i | PV | | | | ($100,000) |
| 7 | i | PV | | | | ($114,827.14) |

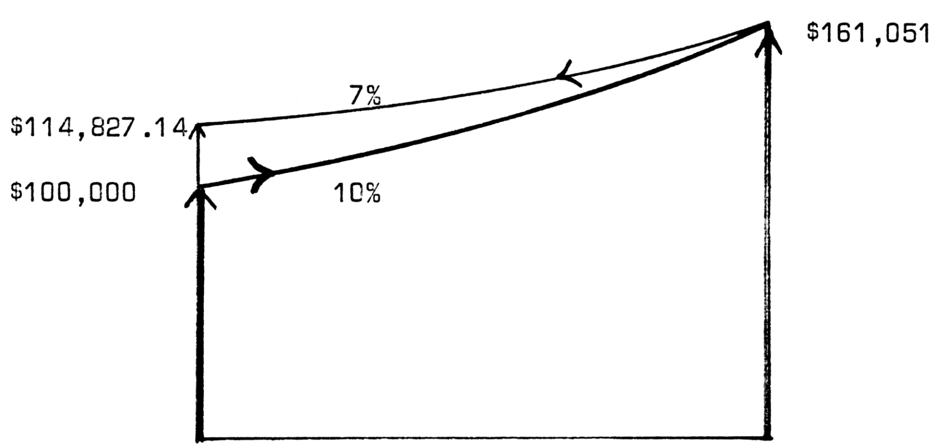
Apart from knowing how to calculate the answers, it is important to have a feel for what kind of answer to expect. Seeing amounts slide up and down the rate curve may help us do so.




(1) 10% interest, 15% discount rate. The $100,000 note is worth $80,070.81




(2) 10% interest, 10% discount rate. Present Value and face value are the same. The interest earned on the $100,000 fully compensates the purchaser of the note for the 10% return that he requires.




(3) 10% interest, 7% discount rate. The $100,000 note is worth $114,827.14 which is the Present Value of $161,051 in 5 years discounted at 7%.

## CHOOSING BETWEEN FINANCIAL ALTERNATIVES

"It's the difference that makes the difference"

### MODEL PROBLEM I

Homes are being offered for sale for $100,000 in a development. The available financing is as follows:

--12.75% 30-year fixed rate with 15% down

--13.75% 30-year fixed rate with 5% down.

How can the buyer choose between these options?

From one point of view the choice is very clear: 12.75% is better than 13.75%. Everything else being equal, the first loan has to be a better deal. But everything else is not equal, otherwise why even bother offering the choice? So let's take a different approach.

Let's: 1) Establish the cash flow required by each option.

2) Calculate the difference between these amounts at each point in time: that difference represents an exchange of money in time.

3) Calculate the rate on that exchange of money in time.

4) Ask ourselves: "At that rate, do I want to be the investor or the borrower?"

| | Downpayment | Loan amount | Rate | Monthly payment |
|---|---|---|---|---|
| Option I | 15,000 | 85,000 | 12.75 | 923.69 |
| Option II | 5,000 | 95,000 | 13.75 | 1,106.86 |
| Difference | 10,000 | | | -183.17 |

Shifting from option I to option II is equivalent to borrowing $10,000 more and paying for that extra amount with monthly payments of $183.17 for 30 years. We may key that exchange of money in the calculator and find that it corresponds to a rate of 21.95%. At that rate, do I really want to borrow the extra $10,000? If I need the extra amount or insist on the more leveraged situation, are there cheaper ways of borrowing the $10,000?

Alternatively, shifting from option II to option I implies by comparison an extra investment of $10,000 which is rewarded by an extra amount of $183.17 retained each month for the next 30 years. This time we are considering a rate of return of 21.95% on the $10,000 investment. So the choice is whether, at 21.95%, we want to borrow or to invest $10,000.

The simple keystrokes on the financial calculator are as follows:

| | |
|---|---|
| 85000 PV 30 BLUE n | |
| 12.75 BLUE i 0 FV PMT | ($923.69) |
| 95000 PV | |
| 13.75 BLUE i PMT | ($1,106.86) |
| - | (Difference: $183.17) |
| CHS PMT 10000 PV | |
| i (wait) 12 x | (Annual rate: 21.95%) |

The answer remains valid whatever the price of the property.

If we assume that the two loans are going to be kept for 5 years only, then we can calculate the difference between the two 5 year loans, including the difference between the Future Values, and calculate the rate on the difference. Under those conditions, borrowing the extra $10,000 costs us 22.14%.

- o 0 o -

We have here a procedure that can be applied just about any time there is a real choice between two options. If there are financial benefits and inconveniences for each one of the options, then **IT'S THE DIFFERENCE THAT MAKES THE DIFFERENCE.** That difference is what the choice is all about, and because it implies more money here and less there, we can say that the choice itself is an exchange of money in time and can be analyzed as such.

As in the previous example, we can calculate the rate of return on that exchange and ask ourselves: **"AT THAT RATE, DO I WANT TO BE THE BORROWER OR THE INVESTOR?"** If we decide that we want to be the borrower, we choose the option that provides us with more money now and less later. If the rate is one at which we are quite happy to invest, then we choose the option that provides less money now and more later.

Additional examples will show the variety of situations where that kind of approach can help. Of course, many circumstances require the use of the irregular cash flow functions to calculate the rate on a difference that does not fit into the neat regular pattern we have been analyzing so far.

## PROBLEM II

A seller has to choose between two offers on a property.

--Offer I: $450,000 selling price with $400,000 cash or equivalent (from downpayment, new loan, debt relief), and $50,000 to be carried by the owner at 13% interest, interest-only for 5 years.

--Offer II: $490,000 selling price, with only $190,000 cash or equivalent, and $300,000 carried by the owner at 10% interest, interest-only for 5 years.

Comparing the offers:

| | Cash | Owner carries | rate | monthly pmt | 5 year balance |
|---|---|---|---|---|---|
| Offer I | 400,000 | 50,000 | 13% | 541.67 | 50,000 |
| Offer II | 190,000 | 300,000 | 10% | 2,500.00 | 300,000 |
| Difference | 210,000 | | | -1,958.33 | -250,000 |

We have now established the difference between the cash flows. **That DIFFERENCE embodies the CHOICE** we are asked to make between the two options. If one offer provides me with $210,000 more now than the other one, why would I not immediately accept that offer? Because taking the extra $210,000 means giving up $1,958.33 a month for the next five years and the additional $250,000 at the end of the 5 years.

Calculating the rate on the exchange of money helps us decide:

| | | | | | |
|---|---|---|---|---|---|
| 210000 | PV | 1958.33 | CHS | PMT | |
| 250000 | CHS | FV | 5 | BLUE | n |
| i | (wait) | 12 | x | | |

(Answer: 13.85%)

The CHOICE between the two options implies a rate of 13.85%. At that rate, do I want to borrow $210,000, or do I prefer to invest $210,000? If I prefer to borrow, I choose the $450,000 offer that provides me with the extra cash. If I choose to invest, I select the $490,000 offer. If I am still hesitant, then I know that the two offers are about equivalent from a strictly financial point of view, and that too is useful knowledge.

With this example we could also take the cash equivalent (Present Value) approach. Let us do so at discount rates of 12% and 15%.

At a discount rate of 12%, the cash equivalent price is as follows:

| | | |
|---|---|---|
| Offer I: | $451,873.13 | (2) |
| Offer II: | $467,522.48 | (1) |

Offer II is superior. This is also the conclusion we would draw from knowing the 13.85% rate: if money is worth 12% to us, the opportunity to make 13.85% is too good to reject. We choose the option where the $210,000 remains invested.

At a discount of 15%, the cash equivalent prices are as follows:

| | | |
|---|---|---|
| Offer I: | $446,497.12 | (1) |
| Offer II: | $437,456.76 | (2) |

Now offer I is superior. This is also the choice which knowledge of the 13.85% rate would imply: if money is worth 15% to us, we do not want to invest at 13.85% but will choose to borrow at that rate.

By calculating the rate on the <u>difference</u> I have in fact calculated the rate which would give <u>the same Present Value</u> (about $448,500) to the two offers.

Which of the two approaches provides the most useful information to the seller: (1) Deciding on a somewhat arbitrary rate such as 15%, and calculating a difference of $9,040.36 between the two offers discounted at that rate; (2) Knowing that the choice amounts to either borrowing or investing $210,000 at 13.85%? With the first approach we do not know whether discounting at a rate of 14.5%, for instance, would lead to the opposite conclusion. Calculating the rate on the difference, when available, is a much more precise gage of the choice we have to make.

In the past, the technique of comparing Present Values was most frequently used because calculating a rate was a much more time consuming process. With financial calculators now offering a fast and simple means of finding the rate the second approach becomes readily available.

With more than two offers to choose from, it is possible to choose the better of two options using the rate approach, and then use the same approach again to compare it to a third option. With too many options the Present Value approach becomes indispensable.

There would be different tax consequences, of course, on two such offers. These can be taken into account by submitting after-tax cash flows to one or the other calculation. The rate that we calculate on the difference, or the discount rate that we select, is then an after-tax rate that should be compared to other after-tax rates.

## PROBLEM III

I receive my home insurance bill for the year and am offered a choice: I can pay $700 now for the whole year, or I can pay on an installment basis, $72 now followed by 9 more monthly payments of $72 (for a total of 10 payments).

If I choose to pay now, I am spending $628 more now than I really have to, but this spares me the cost of 9 additional payments of $72. Or again I can save $628 now, and pay for this saving with 9 monthly payments of $72.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Option I: | 700 | | | | | | | | | |
| Option II: | 72 | 72 | 72 | 72 | 72 | 72 | 72 | 72 | 72 | 72 |
| Difference: | 628 | -72 | -72 | -72 | -72 | -72 | -72 | -72 | -72 | -72 |

Having established the difference between the two options, I calculate the rate of return on that exchange of money in time:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 628 | PV | 72 | CHS | PMT | | | |
| 9 | n | 0 | FV | i | 12 | x | (7.58%) |

At 7.58% I prefer being the borrower than the investor. So I choose to write a check for $72 instead of $700.

- o 0 o -

We have chosen simple examples to illustrate this approach: by considering the **DIFFERENCE** between the cash flows provided by two alternative financial options, and analyzing that difference as one would any other **EXCHANGE OF MONEY IN TIME**, we have a very accurate means of focusing on what precisely makes the difference.

It is a very versatile approach that can apply to situations where comparing rates or Present Values would not be possible or appropriate--as in Model Problem I.

The major limitation to this approach is that it forces us to pair options one on one. We also very often end up with an irregular cash flow differential on which we need to calculate the rate. In such cases we need to resort to the irregular cash flow functions of the calculator.

## PRACTICE PROBLEMS

1) You are choosing between buying a computer system with a bank loan or leasing it with option to buy. You establish the following after-tax cash flow for each option:
- Buying: $2,500 down, 60 monthly payments of $300.
- Leasing: $1,000 down, 60 monthly payments of $335, and $1,000 five years from now as you exercise the option (which you intend to do).

Calculate the rate on the difference to help you choose.

(2.05% per month, 24.55% annual after-tax rate. You decide that you prefer investing the extra $1,500 and buy the system outright.)

2) Same choice except that the leasing now requires:
- Leasing: $1,000 down, 60 monthly payments of $275, and $4,000 in five years when you exercise the option to buy.

(7.54% after-tax rate: do you wish to invest or to borrow? Here leasing saves money at the outset, and also saves $25 each month for 5 years. The $25 payments need to be keyed in with the same sign as the $1,500 Present Value)

3) You are choosing between two 30-year loans and comparing the maximum net cash each one will provide.
- Loan A: $72,300 net cash, 11.31% APR.
- Loan B: $68,900 net cash, 11% APR.

You would prefer as much cash as possible, but how much is the extra amount going to cost?

(17.32% rate on extra $3,400.
Note that we may deal with net cash (loan proceeds) and APR as we would if these were the loan amount and the nominal rate on the loan)

4) The seller of a property has an offer of $350,000 which requires him to carry $150,000 for 5 years at 10% interest with monthly payments of $1,350.
What is the Present Value (Cash equivalent price) of that offer if money is worth 12% to the seller? ($338,994.22)

5) You receive a bill for one year of home insurance and are offered the choice of paying $655, or the first of 10 monthly payments of $67. At what rate are you borrowing if you decide to choose the installment plan? (6.08%)

6) You are choosing between two offers:
- Offer C: $185,000 cash to new loan.
- Offer D: $190,000 with a $75,000 carryback, 8% interest-only, 6 years.

Money is worth 10% to you.
Use a cash equivalent approach, ($183,252.67 is PV of offer D)
and calculate the rate on the difference. (9.46%)

(Both calculations point to choosing the all-cash offer. With knowledge of the rate you might fine-tune your decision: "9.46% is not that bad after all. It's on a property I know. I threw in 10% in reaction to the 8% interest that is decidedly too low, but it seems the extra $5,000 on the offer just about makes up for it."

7) You want to calculate the Present Value of the following cash flow discounted at 13.5%:
- $900 a month for 5 years and $600 a month for the following 5 years, with the first $900 payment to be received in one month.

Not having the irregular cash flow functions at your disposal, you decide to find the answer by adding the Present Values of $600 a month for 10 years and $300 a month for 5 years. Allow the Stack to automatically store the partial answer for you. ($52,440.48)
- How does that compare to receiving $700 a month for 15 years? ($53,915.89)
- Now compare the two cash flows on the basis of a 10% discount rate. ($59,522.31 and $65,140.21)

8) Using a discount rate of 10% (monthly compounding), rate these amounts in terms of their Present Value: (Payments begin in one month)
- $3,000 a month for 5 years. ($141,196.11)
- $2,000 a month for 9 years. ($142,058.71)
- $375,000 in 10 years. ($138,527.61)
- $1,500 a month for 5 years, plus $125,000 at the end of year 5. ($146,571.63)

# UNIT 8

# ARITHMETIC

# AND NUMBER MANIPULATION

# UNIT 8

# ARITHMETIC

# AND NUMBER MANIPULATION

## The Register and the Stack

We took a brief look in Unit 3 at the 20 Register memories and at the Stack, and we have been using their basic features ever since. We now need to **review** and **expand** our undertanding of those memories and of the functions that use them, in particular the various arithmetic functions.

We have reached the point where a more systematic presentation is in order. This calls for some repetition and also for the inclusion of material that not all users may necessarily require—some one-number functions are prominent among the latter. The reader may well want to skim and skip where appropriate.

The unit ends with a detailed study of percentage functions.

## THE REGISTER

**STO** **RCL**

| | | |
|---|---|---|
| 78 | STO | 1 |
| 59 | STO | 4 |

If I now want to store 25 in memory 12, can I really press...

25 STO 1 2 ? No! 25 would end up in memory 1 before I had time to press the 2.

Solution: the decimal point replaces 1 in 10, 11, 12, etc:

| | | | |
|---|---|---|---|
| 25 | STO | .2 | |
| RCL | 4 | | (Brings 59 back in the display) |
| RCL | .2 | | (Brings 25 back in the display) |

The 20 Register memories--memories 0 to 19--are referred to as:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **R0** | **R1** | **R2** | **R3** | **R4** | **R5** | **R6** | **R7** | **R8** | **R9** |
| **R.0** | **R.1** | **R.2** | **R.3** | **R.4** | **R.5** | **R.6** | **R.7** | **R.8** | **R.9** |

### POINTS TO REMEMBER

- Storing a number in a Register memory eliminates any number that might have been there.

- Recalling a number does not take it out of the Register memory. It just duplicates it in the display. Any Register memory can be used as a constant from which we may recall over and over again a number needed in a calculation.

- To clear one specific Register memory, we just store 0 in it.

- A number can be stored in more than one memory:

| | | | |
|---|---|---|---|
| 0 | STO | 0 | |
| | STO | 1 | |
| | STO | 2 | Clears memories 0, 1, and 2. |

– Storing a number preserves all the hidden decimals. So storing and recalling can provide greater accuracy than keying back in the visible part of a previous calculation.

– A number can be stored while it is being used in a calculation in progress:

| Keystrokes | Explanation |
|---|---|
| 789 ENTER 6 % | Calculates 6% sales tax. |
| STO 0 | Stores sales tax in memory 0 |
| + | Adds price to sales tax. |

| Keystrokes | Explanation |
|---|---|
| 197000 ENTER 20 % | Calculates 20% downpayment on $197,000 property. |
| STO 1 | Stores downpayment in R1. |
| – PV | Calculates loan and puts amount in Present Value. |

– Storing a number in no way disturbs the stack, as the previous example shows. But it does transform the number in the display into a complete number, one that does not need to be followed by **ENTER** before we key in another number. Let's add 31 and 22 in the following ways:

| | |
|---|---|
| 31 ENTER | 31 STO 2 |
| 22 + | 22 + |

– We may recall a number and use it in a calculation in progress. It is as if we keyed in the number except that we do not need to press **ENTER** either before or after we recall.

We still have 59 in memory 4. Let's perform: 19 + (59 x 6) with and without the **RCL** key:

| Without **R4** | Using data in **R4** |
|---|---|
| 19 ENTER | 19 |
| 59 ENTER | RCL 4 |
| 6 x | 6 x |
| + | + |

## STORAGE REGISTER ARITHMETIC

The four basic arithmetic operations can be performed directly on the content of Register memories 0 to 4: **R0, R1, R2, R3, R4.**

| | | | | |
|---|---|---|---|---|
| 789 | STO | 1 | | |
| 456 | STO | + | 1 | Adds 456 to content of **R1** |
| 100 | STO | x | 1 | Multiplies **R1** by 100) |
| RCL | 1 | | | 124,500, or (789 + 456) x 100 |

This feature is particularly useful when we want to accumulate successive answers in a memory, like dropping money in a piggy bank. For instance:

You have 3 loans with a **five year due date** on a property. Establish the total cash flow picture and calculate the rate on the total financial package.

**Loan A:** $100,000, 30 years, 10%.
**Loan B:** $39,000, 15%, 20 year amortization.
**Loan C:** $51,000, 12%, payments of $550.

| | |
|---|---|
| **Loan A:** | 100000 PV 30 BLUE n 10 BLUE i |
| | 0 FV PMT 5 BLUE n FV |
| | STO 3 RCL PMT STO 2 RCL PV STO 1 |
| **Loan B:** | 39000 PV 20 BLUE n 15 BLUE i |
| | 0 FV PMT 5 BLUE n FV |
| | STO + 3 RCL PMT STO + 2 RCL PV STO + 1 |
| **Loan C:** | 51000 PV 12 BLUE i 550 CHS PMT FV |
| | STO + 3 RCL PMT STO + 2 RCL PV STO + 1 |
| **Total:** | RCL 1 PV RCL 2 PMT RCL 3 FV |
| | i (wait) 12 x (11.56%: blended rate) |

Storing numbers as they first appear would lead to greater efficiency but would require more attention and offer greater risk of omission.

**REMARKS on Storage Register Arithmetic:**

- The first time around we did not press **STO + 3**, etc. This avoids the danger of adding the first amount to leftover data from a previous calculation.

- Afterwards, omitting + would clear the previous numbers and force us to start over again.

- When dealing with loan data, I like to use **R1** for **PV**, **R2** for **PMT**, and **R3** for **FV**. Notice how we accumulate these values in the appropriate memories.

## SHARING THE REGISTER MEMORIES

The calculator needs memories to automatically store data that we submit for various kinds of operations: irregular cash flow data, statistical functions, programming. Rather than permanently share the total number of memory space available among these functions and the user, those who designed the calculator made just about all the memories directly available to the user for storing his own data. But the calculator is able to claim back some of these memories for other purposes as the need arises.

**Irregular cash flow** data can use all Register memories, beginning with R0, R1, etc., using only as many as are needed. Data originally in the Register memories is automatically erased from the memories that are taken over by irregular cash flow data.

**Statistical functions** use all 6 memories from R1 to R6 automatically as the first statistical piece of data is keyed in, and these memories must be cleared before the statistical data is entered. (**GOLD CLEAR Σ** clears memories R1 to R6).

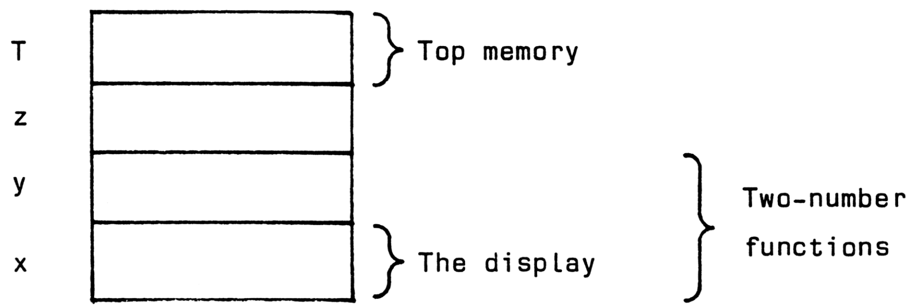
**Programming** can claim all the memories from R.9 to R7, beginning with R.9 and moving backwards as the need is felt.

Register memories claimed by irregular cash flow and statistical data retain their full identity as register memories. The data can be recalled with the **RCL** function and overwritten with the **STO** key.

With programming, on the other hand, each Register memory is transformed into 7 program memories, and once transformed, the Register memories are no longer available for storing and recalling as long as the program is retained. **ERROR 6** appears in the display if attempts are made to store or recall from memories that have been allocated to programming. Only memories R0 to R6 cannot be taken over by programs. There are special procedures to explore and change program memories.

## THE STACK

A stack of 4 memories:




Data floats up and down in the Stack forming an automatic memory bank for partial answers. (See "The Stack at work")

The bottom memory, x, is visible in the display though the format we select shows us a smaller number of decimals than actually present in **x.** (See "Format")

It is through memory x and the display that all communication occurs between the calculator and the user.

**ONE-NUMBER FUNCTIONS** are performed on the content of memory x.

**TWO-NUMBER FUNCTIONS** are performed on the content of memories x and y.

For these one and two-number functions, it is a simple matter of putting the **DATA** in memories y and/or x, and asking the **QUESTION.** The **ANSWER** appears in the display:

**DATA** then **QUESTION**

just about says it all

Following is a list of one-number and two-number functions.
You may want to look them over and select those you want to practice.

## ONE-NUMBER FUNCTIONS

| BLUE | $\sqrt{x}$ | **SQUARE ROOT** [BLUE 21]

The square root of x is the number that multiplied by itself gives x

- What is the square root of 789? | 789 BLUE $\sqrt{x}$ | (28.09)

| 1/x | **RECIPROCAL** [22]

The reciprocal of a number is 1 divided by that number. Multiplying by a reciprocal is equivalent to dividing by the original number.

- What is the reciprocal of 5? | 5 1/x | (0.20)

| BLUE | n! | **FACTORIAL** [BLUE 3]

The factorial of 5 is 5 x 4 x 3 x 2 x 1.

- What is the factorial of 5? | 5 BLUE n! | (120)

| BLUE | LN | **NATURAL LOGARITHM** or **LOG.** [BLUE 23]

5 x 7 = 35. Logarithms are numbers such that the log of 5 **plus** the log of 7 is equal to the log of 35. They can be used to downgrade multiplications into additions, root calculations into divisions, etc.

- What is the logarithm of 5? | 5 BLUE LN | (1.61)

(Check hidden decimals for full internal number: 1.609437912)

| BLUE | $e^x$ | **ANTILOGARITHM** [BLUE 22]

The antilog of 1.609437912 is 5.

- What is the antilog of 10? | 10 BLUE $e^x$ | (22,026.47)

## TWO-NUMBER FUNCTIONS

They are performed on the content of the **x** and **y** memories of the Stack.

They include the 4 basic operations and the 3 percentage functions that have already been studied (Unit 3) and will be explored from different angles in the pages that follow:

[+] [−] [x] [÷] [%] [Δ%] [%T]

The 3 percentage functions have one characteristic that sets them apart. When the question is asked, the **x** data is replaced by the answer, but the **y** data remains undisturbed in the **y** memory. So the answer can easily be added or subtracted from the base number in **y** when needed. See later in this unit for a systematic presentation of the percentage functions.

Two-number functions also include $\mathbf{y^x}$, and two functions that calculate dates and the number of days between two dates.

### $y^x$ Y TO THE POWER OF X [21]

3 to the power of 8, or $3^8$, is 3 multiplied 8 times by itself.

- Calculate $3^8$:

| | | |
|---|---|---|
| 3 ENTER 8 $y^x$ | | (6,561) |

The same function also calculates roots:

The cube root of 15 is the number that multiplied 3 times by itself gives 15. It is calculated by putting 15 to the power of 1/3:

- What is $\sqrt[3]{15}$, or the cube root of 15?

| | | |
|---|---|---|
| 15 ENTER 3 1/x $y^x$ | | (2.47) |

We used the reciprocal here as a short cut to calculate 1 divided by 3.

## DATES AND DAYS

### FORMAT FOR DATES

Numbers that will be interpreted as dates need to be keyed in as follows:

| DECEMBER 16, 1985 | MARCH 6, 1953 |
|---|---|
| 1 2 . 1 6 1 9 8 5 | 3 . 0 6 1 9 5 3 |

The format here is **M.DY**, (Month. Days Year). It requires a decimal point after the number of the month, and two numbers for the day of the month.

Pressing **BLUE D.MY** (Day, Month Year) [BLUE 4] selects the European format for dates. It puts the indicator **D.MY** in the display. To switch back to the traditional American format, press **BLUE M.DY** [BLUE 5].

### NUMBER OF DAYS BETWEEN TWO DATES

| BLUE | ΔDYS | **DELTA DAYS** [BLUE 26]

- How many days between February 7, 1985 and March 25, 1986?

```
2.071985   ENTER   3.251986

BLUE   ΔDYS
```

(411 days)

Press **x≷y** to get the answer with a banker's year of 360 days. (408 days)

### DATE

| BLUE | DATE | [BLUE 16]

This function calculates a date that is **x** days away from another date.

- A 120 day listing agreement is signed on June 10, 1984. When does the listing expire?

```
6.101984   ENTER   120

BLUE   DATE
```

(10,08,1984 1:
Monday, October 8th, 1984)

The **1** stands for **Monday**, the first day of the working week. Similarly, 5 means Friday, 7 Sunday, etc. (Yes, the week-end is at the end of the week, not the beginning).

Leaving a negative number in **x** calculates the date that is so many days before the date keyed into **y**:

- A lease needs to be renewed 90 days before its December 3rd, 1985 expiration. By what date must it be renewed?

| | | | |
|---|---|---|---|
| 12.031985 | ENTER | 90 | CHS |
| BLUE | DATE | | |

(9,04,1985 3,or Wednesday,September 4th, 1985)

**BIRTHDAY**

Putting **0** in **x** is a convenient way of calculating the day of the week for a given date, for instance the birthday of this nation: we ask the calculator to calculate the date that is **zero** days away from the date we have in **y**.

- What day of the week was July 4th, 1776?

| | | |
|---|---|---|
| 7.041776 | ENTER | 0 |
| BLUE | DATE | |

(7,04,1776 4: a Thursday)

**REMINDER:** You must not have the **D.MY** status indicator in the display if you key in dates in the traditional American format.

One and two number functions

**PRACTICE**

1) Find the day of the week for:

| | |
|---|---|
| June 25, 1935? | (Tuesday) |
| January 1, 2000? | (Saturday) |
| October 14, 1987? | (Wednesday) |
| August 24, 1985? | (Saturday) |
| Your date of birth. | |

2) How many days are there between:

| | |
|---|---|
| December 23, 1984 and September 1, 1985? | (252) |
| April 9, 1931 and June 19, 1952? | (7,742) |
| Same question with banker's year of 360 days. | (7,630) |
| February 28, 1985 and March 1, 1985? | (1) |
| February 28, 1984 and March 1, 1984? | (2) |
| Same question with banker's year. | (3) |
| January 3, 1985 and March 15, 1985? | (71) |

3) Find the date that is 120 days after:

| | |
|---|---|
| July 12, 1988? | (Wednesday, November 9, 1988) |
| May 4, 1984? | (Saturday, September 1st, 1984) |
| September 3, 1991? | (Wednesday, January 1, 1992) |

4) What is the square root of 75? (8.66)

5) What is 7 to the 10th power, or $7^{10}$ ? (282,475,249)

6) You have to renew a lease 90 days before its October 1st, 1984 expiration. By what date must you renew? (Tuesday, July 3rd, 1984)

7) With the Log of 5 as an excellent approximation of the factor that transforms miles into kilometers and vice versa, give the number of kilometers in

| | |
|---|---|
| 1,000 miles. | (1,609.44) |
| 250 miles. | (402.36) |
| 55 miles. | (88.52) |

8) Using the Log of 5 as factor, calculate the number of miles in

| | |
|---|---|
| 1,000 kilometers. | (621.33) |
| a 6 K run. | (3.73) |

9) You borrow money on June 6, 1984 and pay back on April 15, 1985. How many days have you kept the money? (313)

10) What is the square root of 600? (24.49)

11) What is the expiration data of a 180 day listing that originated on April 6, 1986? (Friday, October 3rd, 1986)

12) By trial and error find out the earliest and the latest dates that the calculator can accommodate.

## TIPS ON ARITHMETIC

- To multiply by two you can press **ENTER** **+**

What is 136 x 2?

| | | |
|---|---|---|
| 136 ENTER<br>+ | instead of | 136 ENTER<br>2 x |

- To square a number you can press **ENTER** **x**

What is $37^2$ ?

| | | |
|---|---|---|
| 37 ENTER<br>x | instead of | 37 ENTER<br>2 $y^x$ |

- Never use **BLUE n** just to multiply by 12 or **BLUE i** just to divide by 12. It puts unwanted numbers in **n** and **i** and can be the source of many frustrations. There are keystrokes that work, and that may even save a step or two but that can become inconvenient habits in many circumstances.

  Use **BLUE n** and **BLUE i** only when you really want to key the data in **n** and **i**. Certainly use **RCL BLUE i** to mutliply by 12 monthly data that you already have in **i**, and **RCL BLUE n** to divide data that you have in **n**, especially when the monthly numbers are not showing in the display.

- It is a good habit to RECALL data that is already stored in a memory instead of keying it in again or of calculating it again.

# THE STACK AT WORK

| ACTION (Answers between brackets) | KEYSTROKES |
|---|---|
| 1) I go Christmas shopping with $99.00 in my pocket. | 99 ENTER |
| 2) I see a gift costing $12.00. | 12 ENTER |
| 3) I buy 2 for a total of ($24.00). | 2 x |
| 4) I see another item costing $17.00. | 17 ENTER |
| 5) I buy 3 for a total of ($51.00). | 3 x |
| 6) My total purchase is ($75.00). | + |
| 7) The 6% sales tax is ($4.50). | 6 % |
| 8) For a total cost of ($79.50). | + |
| 9) Which leaves me with ($28.50). | − |

Let's see how this series of keystrokes is implemented in the Stack:

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | | | | | | | | 99 | 99 | | | | | | |
| z | | | | 99 | 99 | | 99 | 24 | 24 | 99 | | 99 | 99 | | |
| y | | 99 | 99 | 12 | 12 | 99 | 24 | 17 | 17 | 24 | 99 | 75 | 75 | 99 | |
| x | 99 | 99 | 12 | 12 | 2 | 24 | 17 | 17 | 3 | 51 | 75 | 6 | 4.50 | 79.50 | 20.50 |
| Keys | 99 ENTER | | 12 ENTER | | 2 | x | 17 ENTER | | 3 | x | + | 6 % | | + | − |
| | (1) | | (2) | | (3) | | (4) | | (5) | | (6) | (7) | | (8) | (9) |

Notice how:

- The $99.00 amount keyed in at the beginning floats up and down the Stack. It becomes available when needed because all intermediate numbers have been used up. See also 24 in lines 3, 4, 5.

- When the initial 99 is ENTERed, it is pushed into the **y** memory, but it remains visible in the display for our convenience until we key in the next number.

- When a number appearing in the display (**x**) is the result of a calculation there is no need to press **ENTER** before keying in a new number. The initial number is automatically pushed up into the **y** memory (24 pushed up by 17 in line 4). Press **ENTER** only when you key in two numbers yourself in immediate succession.

- When a two number arithmetic operation is performed on the content of **x** and **y**, the two numbers are replaced by the result of the operation (Lines 3, 5, 6, 8, 9). However this is not true of percentage calculations (Line 7): here the base number remains in the **y** memory where it can be used to calculate a percentage increase or decrease (line 8).

- In all cases--including one-number functions--the **x** data is replaced by the answer. We shall see that the last value for **x** eliminated from the Stack in this manner is temporarily retained in a special **LAST x** memory.

- Also not illustrated in our representation of the Stack is the fact that 99, once it has reached the Top memory, not only floats back down to **z**, but also duplicates itself in the Top memory. The applications of these last two features will be studied later in this section.

## EXPLORING THE STACK

**x≷y** [34] Exchanges the content of memories **x** and **y**.

**x≷y** can be used to put the data in the correct position for a two number operation.

Pressing **x≷y** twice allows us to check what number we have in **y** and to put the numbers back in their original order.

| 5 ENTER 789 |
|---|

Now press **x≷y** a number of times:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| y | | 5 | 5 | 789 | 5 | 789 | 5 |
| x | 5 | 5 | 789 | 5 | 789 | 5 | 789 |

**R↓** [33] **Roll down.**

The content of each memory of the Stack falls down to the memory below, while **x** moves up to the Top memory. By pressing **R↓** four times, we may review the whole content of the Stack and leave it in its original order.

| 3 ENTER 78 ENTER 6 ENTER 10 |
|---|

This leaves the Stack as in the first column of numbers.
Pressing **R↓** then moves the data as illustrated:

| | | |
|---|---|---|
| T | 3 | 10 |
| z | 78 | 3 |
| y | 6 | 78 |
| x | 10 | 6 |

**ENTER**

**ENTER** also manipulates the Stack. Pressing **ENTER** 3 times loads the Stack with the number in the display. (See "The Stack as a constant")

**APPLICATIONS**

**A** Let's add the following column of numbers:

| Numbers | Keystrokes | Display |
|---|---|---|
| 37.50 | 37.5 ENTER | |
| 13.02 | 13.02 + | (50.52) |
| 9.67 | 9.67 + | (60.19 sub-total) |
| 52.95 | | |
| 52.95 | | |
| 52.95 | | |
| 52.95 | 52.95 ENTER 7 x | (370.65 partial answer) |
| 52.95 | + | (430.84 sub-total) |
| 52.95 | | |
| 52.95 | | |
| 7.75 | 7.75 + | (438.59) |
| 13.21 | 13.21 + | (451.80, grand total) |

Note how we are able to stop in the middle of the additions to perform a multiplication and then simply add the result of the multiplication to the initial sub-total (60.19) that is temporarily stored in the **z** memory and comes floating back down to **y** when we need it:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| z | | | | | **60.19** | **60.19** | | |
| y | | 50.52 | | **60.19** | 52.95 | 52.95 | **60.19** | |
| x | 50.52 | 9.67 | **60.19** | 52.95 | 52.95 | 7 | 370.65 | 430.84 |
| | + | 9.67 | + | 52.95 | ENTER | 7 | x | + |

**B** Key in a rate of interest of 11 7/8 %.

11 7/8 (eleven and seven eighth) is indeed 11 <u>and</u> 7/8. So we need to add these two values.

7/8 is seven divided by eight which we know how to calculate.

Keystrokes:

| Keystrokes | |
|---|---|
| 7 ENTER 8 ÷ | (Calculates 7/8) |
| 11 + | (Display shows 11.88 though internal number is 11.875) |

Alternative keystrokes:

| |
|---|
| 11 ENTER 7 ENTER 8 |
| ÷ + |

**C** Though not really our concern here, the Stack is powerful enough to perform the most complex calculations that could result from algebraic formulations. We only need to start with the central parenthesis (for the more complex calculations), and immediately perform operations on numbers that are in the calculator when that is possible.

Note in the following example how we may leave partial answers in the Stack and then automatically return to them as the need arises:

$$\frac{(4 \times [16 + \sqrt{3}])^2 + (107 - 31)}{(21 + 51)(17 - 5)}$$

| Keystrokes | |
|---|---|
| 16 ENTER 3 BLUE $\sqrt{x}$ + 4 x 2 $y^x$ | |
| 107 ENTER 31 − + | |
| 21 ENTER 51 + 17 ENTER 5 − x ÷ | (5.91) |

## **LAST x**

| BLUE | LSTx |
|---|---|

When an operation is performed in the Stack the original **x** number is always discarded and replaced by the result of the operation. However the value of x that has been discarded is automatically and temporarily retained in a 5th memory associated with the Stack, the **LAST x** memory.

It can be retrieved by pressing **BLUE LSTx.**     [BLUE and the ENTER key]

Example:   78 + 22

| | **78** | **ENTER** | **22** | **+** | **BLUE LSTx** |
|---|---|---|---|---|---|
| T | | | | | |
| z | | | | | |
| y | | 78 | 78 | | 100 |
| x | 78 | 78 | 22 | 100 | 22 |
| LAST x | | | | 22 | 22 |

**LAST x** is a fail safe feature. Using it is like retrieving a document from the waste basket. It is possible only until a new calculation performed with the Stack stores a new value in the **LAST x** memory.

Pushing 78 up into the **y** memory by keying in 22 does not put 78 in the **LAST x** memory. To go into **Last x**, the **x** data must be eliminated from the Stack by a calculation or a number altering function as 22 was eliminated from the Stack in the example.

**LAST x** can be used anytime we want to check or to use over again our Last x value as previously defined. It can also be used to recreate the previous **y** value in two-number operations: with the previous data still in, pressing **minus** retrieves 78 and sends 22 back into **Last x.**

The applications of the **LAST x** function are many. Let's review a few.

**A Which number did I just key in? Ask LAST x.**

Have you ever been interrupted in the middle of a long series of additions by a telephone call or a spouse, or just allowed your finger to slip and lost track of where you were? Let's put ourselves in that position.

Add these numbers: 789 + 456 + 123 + 632 + 75 + 97

| | |
|---|---|
| 789 ENTER 456 + | |
| 123 + | |
| 632 + | (2000) |

At this stage we are interrupted. We come back to the calculator to find 2000 in the display, but we do not remember where we stopped.

| | | |
|---|---|---|
| BLUE LSTx | (632) | Now we know where we stopped. |
| R↓ | (or $x \gtrless y$) | Brings back 2000. We may resume the calculations: |
| 75 + | | |
| 97 + | (2,172) | The grand total. |

## B Did I key in the correct number? Ask LAST x

Checking the last number that was added (or subtracted, etc) may be desirable when we are not quite sure that we keyed in the correct number.

Let's add the same numbers but make a mistake as we key in 123.

| | |
|---|---|
| 789 ENTER 456 + | (1,245) |
| 113 + | (1,358 grand total) |

At this stage we have the strong suspicion that our finger slipped or that we may somehow have added the wrong number.

| | |
|---|---|
| BLUE LSTx | (Shows 113) |

Yes, we made a mistake. 113 was added by mistake: pressing **minus** subtracts it from the total that has been pushed up into the **y** memory. We may then carefully add back the correct amount:

| | |
|---|---|
| − | (1,245: the number in the display before we keyed in the wrong number) |
| 123 + | Mistake corrected. |
| 632 + | |
| Etc. | |

If no mistake was made, then **x≷y** or **R↓** brings back the correct sub-total.

## C Wrong operation was performed: correct with LAST x

You have 789.55 in **y** and 456.32 in **x** and you want to add them. But you make a mistake and press the multiplication key instead:

| | | |
|---|---|---|
| 789.55 | ENTER | |
| 56.32 | x | (44,467.46) You multiplied by mistake. |
| BLUE | LSTx | Brings 56.32 back into the display. |
| ÷ | | The <u>opposite</u> of the erroneous multiplication: divides 44,467.46 by 56.32 to give 789.55. 56.32 is lost once again to the Last x memory. |
| BLUE | LSTx | Puts 56.32 back into **x** and 789.55 in **y**. We are back to where we were before the mistake. |
| + | | 845.87: the correct sum. |

So:

| | | |
|---|---|---|
| BLUE | LSTx | |
| ( ) | | (Opposite of wrong operation) |
| BLUE | LSTx | |

This simple keystroke restores the Stack to the situation that existed <u>before</u> the mistake was made. The correct operation can now be performed.

**D LAST x as a constant.**

**BLUE LSTx** offers a convenient way of recalling the previous value of **x** when we need that number for another operation.

We need to divide $2,000.00, $15,800.00, $9,000, and $7,500 by 16.37:

| | | | | |
|---|---|---|---|---|
| 2000 | ENTER | 16.37 | ÷ | (122.17) |
| 15800 | BLUE | LSTx | ÷ | (965.18) |
| 9000 | BLUE | LSTx | ÷ | (549.79) |
| 7500 | BLUE | LSTx | ÷ | (458.16) |

Using **BLUE LSTx** has saved us the trouble of keying in the number four times, or of storing and recalling it from a Register memory.

## THE STACK AS A CONSTANT

As seen earlier, any Register memory can be used to store a constant--**STO 1**, **STO 7**, etc--, which can then be recalled at will over an over again when needed--**RCL 1**, **RCL 7**, **RCL 1**, etc.

We have just seen that the **LAST x** memory could also used to provide a constant.

A third option is to use the Stack itself to provide an unlimited supply of a number that we need to use over and over again.

This use of the Stack results from a feature that was not illustrated in our recent depiction of the Stack at work: when a number reaches the Top memory and is then made to float back down again, it not only floats back down but also duplicates itself in the Top memory thus providing the user with an unlimited supply of that number.

Example:

A Growing Equity Mortgage (GEM) has an initial monthly payment of $456.00. That payment is scheduled to increase by 4% every year for the next 14 years. What are those growing payments?

We could use the percentage key:

| | | | | | |
|---|---|---|---|---|---|
| 456 | ENTER | 4 | % | + | (474.24) |
| | | 4 | % | + | (493.21) |
| | | 4 | % | + | (512.94) |
| | | Etc. | | | |

We could use the top row financial keys:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 456 | PV | 0 | PMT | 4 | i | 1 | n | FV | (474.24) |
| | | | | | | 2 | n | FV | (493.21) |
| | | | | | | 3 | n | FV | (512.94) |
| | | | | | | Etc. | | | |

But let us use a constant, and more precisely the Stack as a constant:

| | | |
|---|---|---|
| 1.04 ENTER ENTER ENTER 456 | x | (474.24) |
| | x | (493.21) |
| | x | (512.94) |
| | x | (533.46) |
| | x | (554.79) |
| | etc. | |

Calculating 4% is the same as multiplying by 0.04.
Increasing by 4% is the same as multiplying by 1.04--the 1 gives back the original number, the .04 adds 4%. So we use 1.04 as a constant.

To establish 1.04 as a constant in the Stack we need to push 1.04 all the way up to the Top memory by pressing the **ENTER** key at least three times:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| T | | | | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 |
| z | | | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 |
| y | | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 |
| x | 1.04 | 1.04 | 1.04 | 1.04 | 456 | 474.24 | 493.21 | 512.94 | 533.46 | 554.79 |
| Key in: | 1.04 | ENTER | ENTER | ENTER | 456 | x | x | x | x | x |

Establish constant — Calculate 14 successive increases

Note how the 1.04 constant is always positioned in the **y** memory just above the last payment amount: pressing **x** multiplies the two numbers to provide the next payment. The process can be repeated indefinitely.

Manipulating data:

**PRACTICE**

13) Perform the following multiplications. As you do so add the top numbers in memory 1, the bottom ones in memory 2, and the answers in memory 3.

| | | | | | |
|---|---|---|---|---|---|
| 57 | 92 | 195 | 61 | 45 | (450) |
| x 21 | x 17 | x 39 | x 14 | x 25 | (116) |
| | | | | | (12,345) |

14) Use the **LAST x** memory to divide the following numbers by 7.555:

| | |
|---|---|
| 778 | (102.98) |
| 3,023 | (400.13) |
| 942 | (124.69) |
| 75 | (9.93) |

15) Use the Stack as a constant to calculate the 10 yearly increases on a Graduated Payment Loan (GPM) with initial payment of $885 and annual increases of 7.5%. ($951.38; 1022.73; 1099.43 etc.)

16) When you get the answer to each of the following multiplications, retrieve the bottom number from the **LAST x** memory and divide to find again the top number.

| | | | | |
|---|---|---|---|---|
| 618 | 309 | 562 | 441 | 94 |
| x 75.2 | x 37 | x 156 | x 529 | x 36 |

17) Calculate the monthly payment on a $100,000 30-year loan at the following interest rates:

| | |
|---|---|
| 8 5/6% | ($777.79) |
| 11 3/8% | ($980.77) |
| 12 7/8% | ($1,096.44) |
| 10 1/8% | ($886.82) |

18) Find the product of these sums:

(59 + 17)(92 + 119)(36 + 38)(15 +31) = (54,586,544)

19) You calculated the following products using the middle line numbers by mistake instead of the numbers just below. First find the answer using the middle line number. Then divide your incorrect answer by the **LAST x** data and correct your error.

| | | | | |
|---|---|---|---|---|
| | 888 | 523 | 3.75 | 78,456 |
| By error> | x 5.6 | x 44.91 | x 65.5 | x 41.23 |
| Correct > | x 56 | x 144.91 | x 66.5 | x 51.23 |
| | (49,728) | (75,787.93) | (249.38) | (4,019,300.88) |

## FORMAT AND HIDDEN DECIMALS

(Review page)

| | Keys | Display |
|---|---|---|
| Divide 80 by 81: | | |
| | 80 ENTER 81 ÷ | 0.99 |
| Now press: | GOLD 4 | 0.9877 |
| | GOLD 9 | 0.987654321 |
| | GOLD 0 | 1 |
| | GOLD ENTER | 9876543210 then 1. |
| Multiply 2.51 by 2: | | |
| | 2.51 ENTER | 3 |
| | 2 x | 5 (3 x 2 = 5?) |
| | GOLD 2 | 5.02(Normal format) |

**BASIC RULES AFFECTING FORMAT:**

- The normal format shows numbers with 2 decimals (dollars and cents).

- To change the format press **GOLD** and the number of decimals selected. The change is permanent until a different format is selected.

- **PRESSING GOLD 2 RE-ESTABLISHES THE NORMAL 2 DECIMAL FORMAT.**

- Numbers are rounded up or down in the display to the closest approximation allowed by the number of decimals selected.

- Calculations take into account the full range of decimals even when they do not appear in the display: the answer--including hidden decimals--is not affected by the chosen format.

  In the last calculation 2.51 appeared as '3' in the display. However, internally, the number remained 2.51. When multiplied by 2, the answer appeared as '5', which is a better approximation of the real internal answer (5.02) than the '6' one might expect.

- For a **temporary glance** at the full sequence of internal numbers, press and hold down **GOLD ENTER**.

**CONSEQUENCES.**

**A TO PRESERVE THE HIDDEN DECIMALS:**

Leave the numbers in the calculator whenever possible: Storing and Recalling preserves all hidden decimals.

A loan with monthly payments has a rate of 13%. You key it in as:

| 13 BLUE i | The display shows 1.08 |
|---|---|

Do not later key in 1.08 instead of **13 BLUE i.** You would fail to take the important string of hidden decimals into account in your loan calculations. The real monthly rate in the calculator is 1.083333333

**B TO ELIMINATE THE HIDDEN DECIMALS:**

Press **GOLD RND** [GOLD 14]

**GOLD ROUND,** the gold function corresponding to the **PMT** key, rounds the internal number in **x** (not in the **PMT** memory!) to the digits showing in the display:

The INTERNAL number in **x** now conforms to the display.

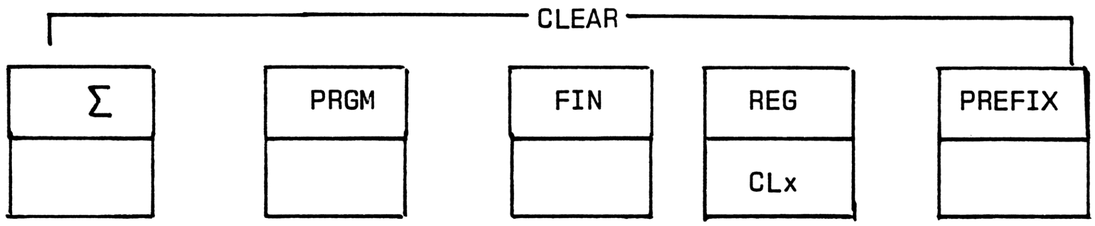
**Application:**

When we calculate a payment amount with the financial keys and want to calculate the balance with some precision based on that payment, pressing **GOLD RND PMT** when the payment is showing rounds the internal number in **x** to the dollar-and-cents amount shown in the display, and presses the rounded number back into the **PMT** memory. At 15% interest, failure to do this might result in a discrepancy of $7.50 after 20 years.

Should you decide that you need this routine, you may want to think of it as **GOLD PMT PMT.**

(Answers given here to financial practice problems do not assume that the payment amount has been rounded in this way)

## CLEARING FUNCTIONS
(Summary)




A row of clearing functions, all requiring the **GOLD** prefix, are marked by a gold line labelled **CLEAR**. [Keys 32, 33, 34, 35, 36]

| **GOLD** | **CLEAR Σ** |
|---|---|

Clear Sigma.
[Press **GOLD** key, release, then press key 32].

Clears Register memories 1 to 6 (**R1** to **R6**), the 6 Statistical memories.

Unfortunately, this also clears the Stack, which serves no purpose and is indeed very inconvenient when dealing with statistical problems.

| **GOLD** | **CLEAR-PRGM** |
|---|---|

Clear Program. [GOLD and 33]

Clears user-written programs. As a fail safe precaution, the function is operational only when the calculator is switched into programming mode.

| **GOLD** | **CLEAR-FIN** |
|---|---|

Clear financial memories. [GOLD and 34]

Clears the 5 regular cash flow financial memories: **n** **i** **PV** **PMT** **FV**

| **GOLD** | **CLEAR-REG** |
|---|---|

Clear register. [GOLD and 35]

Clears all user memories except user-written programs.

Though this clearing function does not clear programs, it does set programs back to step 00.

| **GOLD** | **CLEAR-PREFIX** |
|---|---|

[GOLD and ENTER]

Clears the four prefixes: **GOLD** (**f**), **BLUE** (**g**), **STO**, **RCL**, (and **BLUE GTO**).

These are keys that operate only when a second key follows. If we have pressed one of them by mistake, we should undo that mistake before we select the second key that will complete the function. We may do so by pressing **GOLD CLEAR PREFIX**

With few exceptions, pressing a second prefix automatically clears the previous one--no need to press **GOLD CLEAR PREFIX.**

Pressing **GOLD CLEAR PREFIX** also shows the whole mantissa, all the digits in a number including hidden decimals, for as long as **ENTER** is kept pressed down.

| CLx |
|---|

[35]

Clears the **x** memory of the Stack (the display). This function preserves all the data that may be stored in other memories of the calculator.

**MASTER CLEAR**

Turning the calculator **ON** while the **minus** key [30] is being pressed down clears all the user memories of the calculator, including programs, and brings all indicators and formats back to their standard setting. **Pr Error** in the display is a reminder that programs, if any, have been cleared. Pressing any key (for instance **CLx**) clears this and any other error sign.

You may use the **MASTER CLEAR** whenever an inconvenient format or unwanted indicator appears and you do not know how to clear it selectively, provided there are no data and programs in the calculator that you need to retain.

See also **APPENDIX I** for details on MASTER CLEAR and selective clearing of status indicators.

# PERCENTAGE CALCULATIONS

A property is purchased for **$239,000** with a **20%** downpayment of **$47,800.**

We have here the **TOTAL,** $239,000
the **PART,** $47,800
the **RATE,** 20%: the part as a **percentage** of the total.

Speaking in terms of **total** and **part** is convenient in examining the three percentage calculations which consist in calculating the third number from knowledge of the other two elements. The calculations could be performed arithmetically, but two special calculator functions-- **%** and **%T**--make the task even easier.

A third percentage function **△%** will help us calculate as a percentage the difference that exists between two numbers.

With all three question keys **the data must be stored in the x and y memories of the stack.**

## A PERCENTAGE [%] [25]

I know the **TOTAL** and the **RATE.** What is the **PART**?

A $239,000 property is purchased with a 20% downpayment.
What is the amount of the downpayment?

| | |
|---|---|
| 239000 ENTER 20 % | ($47,800) |

| | Data | | Question | Answer | |
|---|---|---|---|---|---|
| y | TOTAL | 239,000 | | 239,000 | TOTAL |
| x | RATE | 20 | % | 47,800 | PART |

Because the total remains in the **y** memory, it is now possible to press **minus** and get the loan amount ($191,200). Other problems require that we press **plus** instead:

How much do you have to pay for a $456.00 microwave oven if you are given a 15% discount and have to pay 6% tax on the discounted price?

| | |
|---|---|
| 456 ENTER 15 % − | ($387.60 discounted price) |
| 6 % + | ($410.86 cost including $23.26 tax) |

## B PERCENTAGE OF TOTAL $\boxed{\%T}$ [23]

I know the **TOTAL** and the **PART**. What is the **RATE**?

> A $239,000 property is purchased with a downpayment of $47,800. Express the downpayment as a **percentage of the total** price?

| | | | |
|---|---|---|---|
| 239000 | ENTER | 47800 | %T |

| | Before | | | | After | |
|---|---|---|---|---|---|---|
| y | TOTAL | 239,000 | | | 239,000 | TOTAL |
| x | PART | 47,800 | ⟷ %T ⟶ | | 20 | RATE |

We are performing here the exact opposite of the **%** calculation: we know the part expressed as an amount, and we re-express it as a rate. Press alternatively **%** and **%T** a few times: the PART is tranformed into the RATE and then the RATE into the PART.

## C FROM THE PERCENTAGE RATE TO THE TOTAL

We know the **RATE** and the **PART**, what is the **TOTAL**?

This is the last of the three obvious combinations to which we can subject our initial data. We are going to solve the problem here with an <u>unconventional use</u> of the same **%T** function used in the previous Percentage of Total calculation. If you find this different usage of the same function confusing, forget it and study the more conventional presentation entitled "The three step solution".

> I have $47,500 to meet the 20% downpayment requirement. What price property can I afford?

| | | | |
|---|---|---|---|
| 20 | ENTER | 47800 | %T |

| | Before | | | | After | |
|---|---|---|---|---|---|---|
| y | RATE | 20 | | | 20 | RATE |
| x | PART | 47800 | ⟶ %T ⟶ | | 239,000 | TOTAL |

This is an unconventional use of the **%T** function. If you decide that you want it as part of your arsenal of percentage functions, think of it as leading from the RATE (which you key in first) to the TOTAL that you want to calculate.

**THE 3 STEP SOLUTION.**

This simple arithmetic reasoning can be put to many uses and is **an alternative to the unconventional use of %T.**

The reasoning

A broker is setting up a new office. He is told by a furniture store that 7 desks will cost him $903.00. He then decides to order 9 desks. How much will that cost?

STEP ONE states the fact: 7 desks cost $903.00.

STEP TWO calculates the cost of ONE desk: 1 desk costs $903 ÷ 7.

STEP THREE draws the conclusion: 9 desks cost 9 times more.

The same reasoning can be done with 1 percent instead of 1 desk.

A buyer can afford a $92,000 loan. If the loan is 80% of the value of the property, what price property can the buyer purchase?

STEP ONE: 80% is $92,000.

STEP TWO: 1% is $92,000 ÷ 80

STEP THREE: 100% is ($92,000 ÷ 80) x 100

| | | | |
|---|---|---|---|
| 92000 | ENTER | 80 | ÷ |
| 100 | x | | |

($115,000)

Some will want to remember that dividing by 80 and multiplying by 100 is the same as dividing by .8. So the keystrokes could be:

| | | | |
|---|---|---|---|
| 92000 | ENTER | .8 | ÷ |

And as we have just seen, we could also use the **%T** function:

| | | | |
|---|---|---|---|
| 80 | ENTER | 92000 | %T |

But there are times when reasoning things out is a safer alternative.

## D PERCENTAGE DIFFERENCE $\boxed{\Delta\%}$ [24]

There is a frequent need to calculate the difference between two numbers expressed as a percentage. A third percentage key, $\Delta$% DELTA PERCENTAGE or PERCENTAGE DIFFERENCE does that for us. (Delta in mathematics frequently stands for the difference).

A house was purchased for $175,000 and resold for $222,000. What percentage increase is that?

| | |
|---|---|
| 175000 ENTER 222000 $\Delta$% | (26.86%) |

| | | | | | |
|---|---|---|---|---|---|
| y | First number | 175000 | | 175000 | First number |
| x | Second number | 222000 | $\Delta$% | 26.86 | Difference as % |

Now if you want to know the actual amount of the increase, press **%**.
($47,000)

We are frequently interested in measuring as a percentage the change that occurrs over time from an original state to a later state. It may help to remember that we just key in the data in chronological order.

The rate calculated measures the increase (positive answer) or decrease (the discount, a negative answer) that has occurred.

**Calculating the discount on a loan as a rate.**

When a loan is sold at a discount the discount is frequently expressed as a percentage of the face value. That rate of discount should not be confused with the yield on the discounted note: it is not a discount rate, just the one time percentage difference between the face value and the discounted selling price.

A loan with a face value of $46,000 is sold for $37,700. What discount is that, as a rate and in dollars?

| | |
|---|---|
| 46000 ENTER 37700 $\Delta$% | (-18.04) |
| % | (-8,300) |

An 18.04% discount. A discount of $8,300. That discount, whether expressed as a dollar amount or as a rate, is not the yield (rate of return) on the discounted note. That yield here could be any number, say 13% or 32%.

## ILLUSTRATION and PRACTICE

### A PERCENTAGE. $\boxed{\%}$

21) A property is selling for 176,000. Calculate the amount of the downpayment when the buyer puts 15%, 20%, and 25% down.

| | | | | |
|---|---|---|---|---|
| 176000 | ENTER | 15 | % | ($26,400) |
| | x≷y | 20 | % | ($35,200) |
| | x≷y | 25 | % | ($44,000) |

Notice how the **x≷y** key is used to bring back the total selling price that remained in the **y** memory. The **CLx** key could also be used.

22) A property is sold for $223,000. Calculate the commission breakdown between three brokers who receive the following percentages:

- A: 1.25% ($2,787.50)
- B: 1.75% ($3,902.50)
- C: 3.00% ($6,690.00)

23) A $62,000 cash return from a partnership is divided between three partners. Calculate each partner's share:

- Partner A receives 33%. ($20,460)
- Partner B receives 45%. ($27,900)
- Partner C receives 22%. ($13,640)

24) A $137,000 home is purchased with a 20% downpayment. Put the amount of the downpayment in Register memory 1, and the loan amount in **PV**.

| | | | |
|---|---|---|---|
| 137000 | ENTER | 20 | (Puts data in **x** and **y** as required) |
| % | STO | 1 | (Calculates $27,400 downpayment and stores it in memory 1) |
| − | PV | | (Calculates $109,600 loan and stores it in PV) |

Note how each intermediate answer can be stored in a memory without disturbing the ongoing calculations.

25) For each property, store the loan in **PV** and the downpayment in memory 1 and calculate the monthly payment assuming 12% interest, 30 year loans.
   $83,000 property with 25% downpayment.($20,750, $62,250 and $640.31)
   $750,000 property with 80% loan. ($600,000, $150,000 and $6,171.68)
   $183,000 property with 35% down. ($64,050, $118,950 and $1,223.53)

26) A computer store is discounting list prices by 17%. Calculate the discounted price, the 6% sales tax, and the total cost to the buyer for the items that follow:
   List price $1,250. ($1,037.50; $62.25; 1,099.75)
   List price $888. ($737.04; $44.22; $781.26)
   List price $3,000. ($2,490; $149.40; $2,639.40)

27) Calculate the net proceed to the borrower on the following loans:
   $300,000 loan, 1.5 points and $800 in costs. ($294,700)
   $35,000 loan, 3.00 points and $300 in costs. (33,650)

## B PERCENTAGE OF TOTAL

%T

28) In negociating a commission a broker is offered $20,000 to sell a $375,000 property. Express the commission as a percentage of the selling price.

| 375000 | ENTER | 20000 | %T |
|---|---|---|---|

(5.33%)

29) Express as a percentage a participating broker's $5,000 share in a $385,000 transaction. (1.30%)

30) A seller offers a $12,000 commission for the sale of a $235,000 property. What percentage is that? (5.11%)

31) Out of 43 telephone inquiries, 15 wanted product A, 20 product B, and 8 product C. Express the request for each product as a percentage. (Key 43 in only once: use **x⇄y** or **CLx** between calculations).
   (A: 34.88%. B: 46.51%. C: 18.60%.

32) Express as a percentage of the total $550,000 budget the share assigned to these expenses--key $550,000 in only once:
   $37,000 for printing (6.73%)
   $92,000 for advertising (16.73%)
   $265,000 for salaries (48.18%)
   $24,000 for office supplies (4.36%)

## C FROM PERCENTAGE TO TOTAL (Unconventional use of %T function)

34) A partner entitled to 16.66% of the income receives $17,500. What is the total income?

| 16.66 | ENTER | 17500 | %T |
|---|---|---|---|

($105,042.02)

35) A broker entitled to a 2.75% commission receives $8,250. What is the selling price of the property? ($300,000).

36) The following loans are 80% of the property value. Calculate the price of the property. Key 80 in only once.

| | |
|---|---|
| $72,000 | ($90,000) |
| $126,000 | ($157,500) |
| $232,000 | ($290,000) |
| $88,000 | ($110,000) |

37) A $119,500 Net Operating Income represents 9.56% of the price of a property. What is the price of the property? ($1,250,000)

38) You read in the paper:"The city spends $2,700,000 for the fire and police departments, or 38% of the city budget". What is the total city budget? ($7,105,263.16 is the mathematical answer)

39) A borrower wants to net $15,000 from his loan. How much does he need to borrow if the lender charges 4 (percentage) points.

If the lender charges 4 points, then $15,000 is equal to 96% of the total loan (100 - 4 = 96). The total loan must be:

| 96 | ENTER | 15000 | %T |
|---|---|---|---|

($15,625)

40) A borrower wants to net $36,000 from his loan. How much does he need to borrow if the lender charges 3 points and $500.00?

We now need to add the $500 to the net loan proceeds in order to have 97% of the loan.

| 97 | ENTER | 36500 | %T |
|---|---|---|---|

($37,628.87)

or

| 97 | ENTER | 36000 | ENTER | 500 | + | %T |
|---|---|---|---|---|---|---|

Check that you can pay the points and costs and still net $36,000.

41) Calculate the total amount that needs to be borrowed in order to net the following amounts:

Net $32,000. Loan charges: 10 points ($35,555.56)
Net $32,000. Loan charges: 10 pts and $400. ($36,000)
Net $50,000. Loan charges: 2 points and $150. ($51,173.47)
Net $12,000. Loan charges: 5 points and $500. ($13,157.89)

42) A seller wants to net $112,000 from the sale of a property he owns free and clear. Commission and other selling costs will approximate 8% of the selling price. What should be the selling price of the property? ($121,739.13)

43) An owner wants to sell if he can net $56,000 after paying 8% of the price of the property for selling costs and $42,000 to pay off existing loans. What selling price will meet his requirement? (106,521.74)

## PERCENTAGE DIFFERENCE $\boxed{\Delta\%}$

44) Purchase price: $111,000. Resale price $170,000.
What percentage increase is that? (53.15%)

45) A note with a face value of $23,000 is sold for $17,000.
What percentage discount is that? (26.09%)

46) Calculate the discount on each of the following loans:

| Face value | Discount price | |
|---|---|---|
| $45,000 | $32,200 | (28.44%) |
| $21,000 | $14,300 | (31.90%) |
| $92,000 | $59,666 | (35.15%) |
| $19,700 | $17,700 | (10.15%) |

47) A year ago, a Board of Realtors sold $6,532,900 worth of properties for the month. The figure for this year is $7,880,000.
What percentage increase is that? (20.62%)

48) An item bought for $85.00 is resold for $111.00.
What mark-up is that?
(30.59%)

49) Inventory of unsold properties goes down from 325 to 266.
What percentage decrease is that? (18.15%)

# UNIT 9

# SUBORDINATE FINANCIAL FUNCTIONS

# UNIT 9

# SUBORDINATE FINANCIAL FUNCTIONS

The amortization function (**GOLD AMORT**) and the short term simple interest function (**GOLD INT**) are considered in this section, with emphasis on the memory requirements of each and on the flexible uses of the amortization function.

Subordinate financial functions:

## UNDERSTANDING THE MEMORY REQUIREMENTS

The 5 top row regular cash flow keys--**n, i, PV, PMT, FV**--are both DATA keys and QUESTION keys. To use one of them as a question key we need to have already used the others as data keys to store the information about the problem we want to solve. As we do this specific rules need to be followed, in particular the TIME requirement and the SIGN requirement.

But if we have no intention of SOLVING for **n, i, PV, PMT**, or **FV**, then we can still use those five memories to store numbers--a telephone number, someone's birthday, the distance from the earth to the moon.

Similarly, the calculator uses some of these memories for the purpose of storing data that will be used by OTHER QUESTION KEYS that sometimes have specifications of their own concerning the format required for the data. We should not be surprised if the 'logic' we have learned to use and to respect no longer fully applies when these other questions are being asked.

There are four groups of QUESTION KEYS that use the top row memories with somewhat different requirements. We will study two of them in detail. The other two (Bond and Depreciation calculations) are mentioned only for the record here as there is not much we would want to add to the **OWNER'S MANUAL** presentation.

> Using these subordinate financial functions is simple if we keep in mind the basic logic of the calculator--**DATA** then **QUESTION**--and focus on understanding the memory requirements of each question as summarized on the back of the HP-12C calculator. We may also notice that most of these functions immediately provide answers to more than one question, with one answer visible in the display and the others stored in other memories from which they can be retrieved if needed.

**GOLD AMORT** [GOLD 11]

This function helps us write AMORTIZATION SCHEDULES for loans. It is also a convenient tool to analyze loans where the payments or the rate change, or where a paydown occurs. Our detailed presentation stresses the flexible uses of the function.

Pressing **GOLD AMORT** tells the calculator to explore only **i**, **PV**, and **PMT**, and the regular TIME and SIGN requirements apply to that data. **n** and **FV** do not intervene in the calculation, though **n** becomes a convenient counter if we take the trouble to clear it at the start (**0 n**).

**GOLD AMORT** also changes the value in **PV**, adjusting it to what is left of the loan--the balance of the loan! (See below for details).

**GOLD INT** [GOLD 12]

It calculates the interest on short term **SIMPLE INTEREST** bank loans.

Pressing **GOLD INT** explores only **n**, **i**, and **PV**. The term in **n** must be expressed **per day**, and the **annual** rate is stored in **i**.

The answers are stored in the **x**, **y**, and **z** memories of the Stack. (See below for details).

**BONDS** [GOLD 21 and GOLD 22]

Two functions calculate the price (**GOLD PRICE**) if we know the yield, or the yield to maturity (**GOLD YTM**) if we know the price.

**PMT** is used to store the coupon (as a rate), and either **PV** for the price or **i** for the yield. The purchase date is stored in **y** and the date of maturity in **x**.

**DEPRECIATION** [GOLD 23, GOLD 24, GOLD 25]

Three functions allow us to depreciate assets on a Straight Line (**GOLD SL**), Sum-Of-Year-Digits (**GOLD SOYD**), or Declining Balance (**GOLD DB**) approach.

These functions use **PV** to store the cost of the depreciable asset and **FV** for the salvage value--with no sign change. For real estate, only improvements are depreciable, not the land value. It is **not possible** to put the total selling price in **PV** and the value of the land in **FV** as this would still allow the land to be depreciated. The number of years (life) of the asset is stored in **n**. For Declining Balance, store in **i** the appropriate percentage over straight line (125, 150, 175, or 200).

The Declining Balance function is less valuable now that recent tax law changes take into account the month in which the asset is put into productive use and force us to go back to tax tables for the precise numbers.

## AMORTIZATION with GOLD AMORT

Write a monthly amortization schedule for a $45,000 loan at 13% interest with monthly payments of $510.00.

A special function of the calculator helps us do this. Though we haven't seen that function yet, we already know how to use it: **DATA** then **QUESTION** tells us just about all that we need to know.

**DATA:** Keyed in in the usual way:

| | | |
|---|---|---|
| 45000 | PV | |
| 13 | BLUE | i |
| 510 | CHS | PMT |

There is no need to know how long it would take to amortize the loan. We do not need to clear **FV** but there is some advantage in clearing **n:**

0 n (Optional)

**QUESTION:** The word **AMORT** written in gold over the **n** key is a clear indication that **GOLD AMORT** is the question.

However, before we press **GOLD AMORT** we need to tell the calculator that we want to amortize the loan one payment at a time, not 5, 8 or 12 payments at a time. So we press:

1 GOLD AMORT

**ANSWERS:**

In so doing we have already calculate three numbers concerning the first month of the loan. They are stored in the display (**x**), in the **y** memory of the Stack, and in **PV**.

In **x**: the **INTEREST** portion of the first payment: $487.50 (negative).
In **y**: the **PRINCIPAL REDUCTION** portion of that payment.
In **PV**: the **REMAINING BALANCE** of the loan at the end of the period.

To explore the last two numbers we press:

| | |
|---|---|
| x⇄y | ($22.50, also negative: the principal reduction) |
| RCL PV | ($44,977.50: the balance) |

Because we now have in **PV** what remains of the indebtedness, we may get the answers for the second month by pressing once again:

| | |
|---|---|
| 1 GOLD AMORT | ($487.26: interest, 2nd month) |
| x⇄y | ($22.74: principal reduction, 2nd month) |
| RCL PV | ($44,954.76: balance, end of 2nd month) |

Repeating the procedure gives the answers for each succeeding month. Let's do this 3 more times with the following results:

| | Interest | Principal reduction | Balance of the loan |
|---|---|---|---|
| 3rd month: | 487.01 | 22.99 | 44,931.77 |
| 4th month: | 486.76 | 23.24 | 44,908.53 |
| 5th month: | 486.51 | 23.49 | 44,885.04 |

To check which month we have reached we may press:

| | |
|---|---|
| RCL n | (5 periods) |

We find 5 in **n** only because we started with 0 in **n**. If we had left 60 in **n** from a previous problem, we would now find 65. **n** here is like a trip mileage indicator: if we do not set it back to zero we won't know the mileage but it doesn't prevent the car from running.

To amortize more than one period at a time, we just need to press the chosen number of payments instead of 1 before **GOLD AMORT**. For instance, having amortized the loan for 5 months, let's assume that we have brought it up to date to the end of a calendar year and that we now want to amortize it one year at a time:

| | |
|---|---|
| 12 GOLD AMORT | ($5,817.45: interest for first full calendar year) |
| x⇄y | ($302.55: principal reduction first full calendar year) |
| RCL PV | ($44,582.49: balance of the loan end of 17th month) |

We may repeat this last group of keystrokes for each successive full calendar year. Because we started with zero in **n**, at any point **RCL n** will show the number of months amortized.

## AMORTIZATION SCHEDULE
## MODEL

Write a **MONTHLY** amortization schedule for a
$45,000 loan at 13% interest with monthly payments of $510.

| | | | | |
|---|---|---|---|---|
| | 45000 | PV | | |
| | 13 | BLUE | i | |
| | 510 | CHS | PMT | |
| First month: | 1 | GOLD | AMORT | (Interest: $487.50) |
| | | x≷y | | (Principal reduction: $22.50) |
| | | RCL | PV | (Balance: $44,977.50) |
| Second month: | 1 | GOLD | AMORT | (Interest: $487.26) |
| | | x≷y | | (Principal reduction: $22.74) |
| | | RCL | PV | (Balance: $44,954.76) |

Write an **ANNUAL** amortization schedule for a
$45,000 loan, at 13% interest, monthly payments of $510.

| | | | | |
|---|---|---|---|---|
| | 45000 | PV | | |
| | 13 | BLUE | i | |
| | 510 | CHS | PMT | |
| First year: | 12 | GOLD | AMORT | (Interest: $5,833.32) |
| | | x≷y | | (Principal reduction: $286.68) |
| | | RCL | PV | (Balance: $44,713.32) |
| Second year: | 12 | GOLD | AMORT | ($5,793.75) |
| | | x≷y | | ($326.25) |
| | | RCL PV | | ($44,387.07) |

There is no need to press **x$\gtrless$y** or **RCL PV** unless we want to see the principal reduction and remaining balance: the amount of the loan is adjusted in **PV** whether we look at it or not. For instance:

> A $250,000 loan, 10% interest, payments of $2,350. It is a four year loan, with 7 payments made during the first year (1984). Establish the schedule of interest paid during each calendar year and the balloon payment.

| | | | | | |
|---|---|---|---|---|---|
| | 250000 | PV | | | |
| | 10 | BLUE | i | | |
| | 2350 | CHS | PMT | | |
| 1984 (7 months): | 7 | GOLD | AMORT | | ($14,536.01) |
| 1985: | 12 | GOLD | AMORT | | ($24,648.75) |
| 1986: | 12 | GOLD | AMORT | | ($24,276.90) |
| 1987: | 12 | GOLD | AMORT | | ($23,866.09) |
| 1988 (5 months): | 5 | GOLD | AMORT | | ($9,812.89) |
| 4 year balance: | RCL | PV | | | ($234,340.64) |
| Balloon payment: | RCL | PMT | CHS | + | ($236,690.64) |

## NEGATIVE AMORTIZATION

The **GOLD AMORT** procedure is perfectly adapted to handle negative amortization loans:

- The interest that shows in the display is the total amount of **interest earned**, not the total amount of interest paid which in negative amortization loans is just the sum of all the payments made.

- The amount left in **y** is now positive: it is the **PRINCIPAL INCREASE.**

- The Principal increase is added to the amount in **PV** and immediately starts earning interest. So **PV** is still automatically adjusted to changes in the loan amount, but now we have negative amortization: an increase in the loan balance.

- The **GOLD AMORT** procedure does not handle loans where the interest that is not paid is just deferred without earning interest: simply multiplying the amount of the unpaid interest each month by the number of months provides us with all the information we want on such loans.

## CHANGING DATA with GOLD AMORT

The **GOLD AMORT** function is most useful when we want to calculate a whole series of intermediate data concerning a loan that is being amortized. There is no point in using the function just to calculate the balance 5 years in the future—**5 BLUE n FV** is a much faster procedure.

However, if strange things happen to the loan at various points in time—a change in the interest rate, a paydown, a change in the payment amount—then the **GOLD AMORT** function may provide the most efficient way of calculating the final balance, even if we have no need for intermediate numbers. It is a simple matter of **implementing the changes as they occur:**

### PROBLEM

Calculate the balance and balloon payment on a $60,000 five year loan with monthly payments of: $500 for the first year
$600 for the second year
$700 for years 3 TO 5.
The interest is 11% for years 1 and 2 and 13% for years 3 to 5.

| | |
|---|---|
| 60000 PV | |
| 500 CHS PMT | |
| 11 BLUE i | |
| 0 n | (Optional) |
| 12 GOLD AMORT | (Year 1) |
| 600 CHS PMT | |
| 12 GOLD AMORT | (Year 2) |
| 700 CHS PMT | |
| 13 BLUE i | |
| 36 GOLD AMORT | (Years 3, 4, and 5) |
| RCL PV | ($57,920.50: balance after 5 years) |
| RCL PMT CHS + | ($58,620.50: balloon payment) |
| RCL n | (60. Optional check that we have amortized the loan for a total of 60 months) |

The **GOLD AMORT** function can be used in conjunction with regular cash flow key calculations as in the following example:

> Loan of $200,000, amortized over 30 years. Interest of 10% the first year, 11% the second year, 12% thereafter. Find the balance after 10 years.

| | | |
|---|---|---|
| | 200000 PV 0 FV | |
| 10 | BLUE i 30 BLUE n | |
| | PMT | ($1,755.14) |
| | 12 GOLD AMORT | |
| 11 | BLUE i 29 BLUE n | |
| | PMT | ($1,902.62) |
| | 12 GOLD AMORT | |
| 12 | BLUE i 28 BLUE n | |
| | PMT | ($2,051.30) |
| | 8 BLUE n FV | ($186,297.99) |

The last line could also have been—and this can be keyed in right now:

| | |
|---|---|
| 8 ENTER 12 x | (96 remaing payments) |
| GOLD AMORT | |
| RCL PV | ($186,298.21) |

The 22 cents difference is due to the hidden decimals retained in the **PMT** memory during the **FV** calculation. The **GOLD AMORT** procedure automatically eliminates hidden decimals from the amount in the **PMT** memory. To achieve the same result, we would need to press **GOLD RND PMT** before pressing **8 BLUE n FV.**

(In fact this would still leave us with a few cents difference as the **GOLD AMORT** procedure also rounds internally to the display format decimals—dollars and cents in normal format—the interest and principal reduction amounts calculated for each month.

## GROWING EQUITY MORTGAGE (GEM).

Growing Equity Mortgages are another example of loans that can be followed with ease thanks to the **GOLD AMORT** procedure:

> A lender is offering Growing Equity Mortgages at a rate of 11.75% with initial payments based on a 30 year amortization schedule. The payment amount increases by 4% each year. Establish a schedule showing the balance of the loan as a percentage of the loan amount.

| | | | | | | |
|---|---|---|---|---|---|---|
| 100 | PV | 11.75 | BLUE | i | | (Or actual loan amount in **PV**) |
| 30 | BLUE | n | 0 | FV | PMT | |
| 12 | GOLD | AMORT | RCL | PV | | (99.64%: 1st year balance) |
| RCL | PMT | 4 | % | + | PMT | |
| 12 | GOLD | AMORT | RCL | PV | | (98.69%: 2nd year) |
| RCL | PMT | 4 | % | + | PMT | |
| 12 | GOLD | AMORT | RCL | PV | | (97.13%: 3rd year) |

Balance for the following years:

| Year | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|
| Bal: | 94.86 | 91.68 | 87.46 | 82.10 | 75.43 | 67.31 | 57.42 | 45.55 | 31.44 | 14.83 |

The loan is clearly going to be fully amortized sometime during the 14th year. We may now switch to a month by month approach, or adjust the payment and switch to the logic of the regular cash flow keys to calculate the number of months that remain:

| | | | | | | |
|---|---|---|---|---|---|---|
| RCL | PMT | 4 | % | + | PMT | |
| 0 | FV | n | | | | (10. The loan is paid off in 13 years and 10 months) |

Because we wanted all the answers as a percentage of the loan amount, we did the calculations with $100 as that loan amount: the answers are all immediately expressed as a percentage of the loan.

The repetitive procedures often used with **GOLD AMORT** are prime candidates for the programmming features of the calculator.

## PRACTICE

Find the keystrokes using **GOLD AMORT** that duplicate the answers provided for each loan.

1) Monthly amortization schedule on a $19,000 loan at 9% interest with payments of $160.

| Month. | Interest. | Principal reduction. | Balance at end of month. |
|---|---|---|---|
| 1 | 142.50 | 17.50 | 18,982.50 |
| 2 | 142.37 | 17.63 | 18,964.87 |
| 3 | 142.24 | 17.76 | 18,947.11 |
| 4 | 142.10 | 17.90 | 18,929.21 |
| 5 | 141.97 | 18.03 | 18,911.18 |

2) Calendar year schedule for $250,000 loan at 11% interest with payments of $2,500. Only 4 payments are made during first calendar year (1984). This is a five-year loan.

| Year | Interest. | Principal reduction. | Balance at end of year. |
|---|---|---|---|
| 1984 | 9,155.14 | 844.86 | 249,155.14 |
| 1985 | 27,272.27 | 2,727.73 | 246,427.41 |
| 1986 | 26,956.61 | 3,043.39 | 243,384.02 |
| 1987 | 26,604.43 | 3,395.57 | 239,988.45 |
| 1988 | 26,211.51 | 3,788.49 | 236,199.96 |
| 1989 | 17,233.80 | 2,766.20 | 233,433.76 |

(Note: 4 months only in 1984, 8 months only in 1989)

3) Annual amortization schedule for a $100,000 loan at 13% interest with payments as follows:

Year 1: $800
Year 2: $950
Year 3: $1,100
Year 4: $1,250
Year 5: $1,400

| Year | Interest earned | Principal increase or <principal reduction> | Balance at end of 12 month period. |
|---|---|---|---|
| 1 | 13,210.09 | 3,610.09 | 103,610.09 |
| 2 | 13,597.17 | 2,197.17 | 105,807.26 |
| 3 | 13,789.23 | 589.23 | 106,396.49 |
| 4 | 13,759.35 | <1,240.65> | 105,155.84 |
| 5 | 13,476.89 | <3,323.11> | 101,832.73 |

4) Find the interest for each calendar year and the balloon payment on a $500,000 loan at 9% interest with payments of $4,000 per month. This is a 5-year loan with 10 payments made during the first calendar year.

($37,413.93; $44,630.51; $44,314.42
$43,968.68; $43,590.54; $7,225.88
Balloon payment: $485,143.96)

5) Find the 4-year balance on a $100,000 loan, payments of $1000, interest of 11%. There is a $10,000 paydown at the end of years 2 and 3.

($71,398.17)

6) $150,000 loan, amortized over 30 years, interest of 9% the first year, 10% the second year, 11% thereafter. What is the balance at the end of 40 months? ($146,917.58)

7) Write an amortization schedule for a $500,000 loan at 8% interest fully amortized over 10 years with **annual payments.** (PMT is $74,514.74)

| Year: | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Interest: | 40,000.00 | 37,238.82 | 34,256.75 | 31,036.11 |
| Prin. reduc.: | 34,514.74 | 37,275.92 | 40,257.99 | 43,478.63 |
| Balance: | 465,485.26 | 428,209.34 | 387,951.35 | 344,472.72 |

8) Yearly amortization schedule for a Growing Equity Mortgage of $81,000. The rate is 10.5%. Monthly payments for first year based on 30 year amortization. Payments increase by 4% each year.

| Year: | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Payment: | 740.94 | 770.58 | 801.40 | 833.46 |
| Interest: | 8,485.86 | 8,423.53 | 8,314.48 | 8,151.87 |
| Princ. reduc.: | 405.42 | 823.43 | 1,302.32 | 1,849.65 |
| Balance: | 80,594.58 | 79,771.15 | 78,468.83 | 76,619.18 |

## CHANGING THE DATA WITH THE 5 FINANCIAL FUNCTIONS

Because the **GOLD AMORT** procedure automatically adjusts the loan amount in **PV** with what is left of the indebtedness, it provides, as we have just seen, a convenient way of following a loan where payments and rates change, or where a paydown suddenly affects the **PV.**

But **GOLD AMORT** is not the only way of solving these problems: the full logic of the regular cash flow keys in particular offers a convenient alternative in circumstances where only a few changes occur.

It is always possible to bring a loan up-to-date by calculating **FV** and then use that Future Value as the **PV** of a new loan that implements the changes in the payment or the rate, or that can be modified by a paydown. Pressing **CHS** as we transfer the Future Value into **PV** here serves a dual purpose: it allows us to stay with our decision to have positive numbers in **PV** whenever possible; it also makes it possible to store the amount into **PV** as you may see if you try to omit that step in the first problem.

A $210,000 loan at 9% interest has payments of $1,350 for the first 2 years and of $1,750 thereafter. How long does it take to fully payoff the loan?

```
210000   PV        1350   CHS   PMT
2   BLUE   n       9   BLUE   i
FV                            (2 year balance: $215,892.41)
CHS   PV           1750   CHS   PMT
0   FV   n         24   +                   (372, or 31 years)
```

Notice the negative amortization during the first two years and how we simply add 24 months to the number of months it takes to amortize the 2-year balance with payments of $1,750.

Calculate the 10 year balance on a $15,000 loan at 16% interest with payments of $210 if there is a $4,000 paydown at the end of the 4th year.

```
15000   PV          16   BLUE   i
210   CHS   PMT      4   BLUE   n
FV   CHS   4000      -   PV
6   BLUE   n        FV                       ($1,693.57)
```

## SIMPLE INTEREST

| GOLD | INT |
|---|---|

**GOLD INT** is a special function that allows us to calculate the interest on the traditional simple interest short term bank loan, the kind frequently made to businesses to carry inventory, that can be used in Real Estate as a swing loan to bridge the gap between two transactions, and that is sometimes used when drawing on a line of credit. The rate is frequently expressed as being a few percentage points over the prime rate and should not change for the duration of the loan.

**GOLD INT** , of course, is the **QUESTION.**

The **DATA** that is required is as simple to key in as can be imagined, with **n, i,** and **PV** used to store the term in days, the annual interest, and the amount of the loan. We just need to remember that these memories are used only as memories, not as part of the logical system required by the five regular cash flow functions: **n** and **i** are **no longer on the same time scale.**

### MODEL PROBLEM

I borrow $27,500 at 11.5% for 100 days. What is the amount of the interest and the total amount that I must pay back.

| | | | |
|---|---|---|---|
| | 27500 | PV | |
| DATA | 11.5 | i | |
| | 100 | n | |
| QUESTION | GOLD | INT | ($878.47 in interest, 360 day year) |
| | + | | ($28,378.47 total to pay back) |

The answer in the display is calculated on the basis of a banker's year of **360 days**--12 months of 30 days.

If the fine print shows that the loan implies a year of **365 days** (a better deal for the borrower), the amount of interest is already in the calculator. Initially in the **z** memory of the Stack, it has now just dropped down from the **z** to the **y** memory of the Stack and can be seen by pressing **x≷y:** ($866.44).

When we press **GOLD INT** with all the original data in the calculator--as we may do right now once again because the data has not changed--the calculator puts answers in the Stack as follows:

| | | |
|---|---|---|
| T | | |
| z | - 866.44 | Interest if 365 day year. |
| y | - 27,500.00 | Original loan amount. |
| x | - 878.47 | Interest if 360 day year. |

With a note based on a 365 day year, we may get the answers by pressing:

| | |
|---|---|
| GOLD INT | |
| R↓ x≷y | ($866.44 interest if 365 day year) |
| + | ($28,366.44 total to be paid back) |

Simple interest

**PRACTICE**

Calculate the interest and the total amount to be paid back.

1) $18,750 loan, 14% simple interest, for 45 days. (Banker's year)
(Interest is $328.13, for a total payback of $19,078.13)

2) $49,000 loan, 10% simple interest for 95 days. (365 day year).
($1,275.34 in interest, $50,275.34 payback)

3) $7,500 loan, interest is 2 points over 9.5% prime, for 90 days. (360 day year). ($215.63 in interest, for a total of $7,715.63)

4) $15,900 loan borrowed on December 10, 1984 and paid back on September 1st, 1985. Interest is 10.75%. (360 day year).
(265 day loan, $1,258.20 in interest, $17,158.20 payback)
Reminder: Keystrokes for number of days:
12.101984 ENTER 9.011985 BLUE △DYS

5) On January 5, 1984 you borrow $81,000 at 13.5% simple interest (calculated on 365 day year). You borrow that amount for 120 days.
What interest are you charged? ($3,595.07)
What amount do you have to pay back? ($84,595.07)
When do you have to pay the loan back? (Friday, May 4th, 1984)
Reminder. At end of financial calculation calculate the date as follows: 1.051984 RCL n BLUE DATE

# UNIT 10

## THE FIVE REGULAR CASH FLOW KEYS

## DIFFERENT COMBINATIONS

## DIFFERENT OPTIONS

# UNIT 10

# THE FIVE REGULAR CASH FLOW KEYS

## DIFFERENT COMBINATIONS

## DIFFERENT OPTIONS

We are now going beyond two limitations that we imposed upon ourselves in the use of the the 5 regular cash flow keys:

- So far **PMT** and **FV** were always on the same side of the transaction. We are going to consider situations where they are on opposite sides of the transaction, thus forcing us on occasion to change our routine way of meeting the sign requirement.

- So far Payments were always made in arrears, at the end of the period. We are going to consider situations where they are made at the beginning of the period, where the first payment is made at the same time as the Present Value and the last payment one full period before the Future Value.

We will also take a look at some idiosyncrasies of the **n** function.

**WARNING**

Some readers may need many of the features presented in this unit. For others quite a few features may be of limited use and should be given much lower priority than acquiring considerable fluency with previous material or exploring irregular cash flow functions and basic programming.

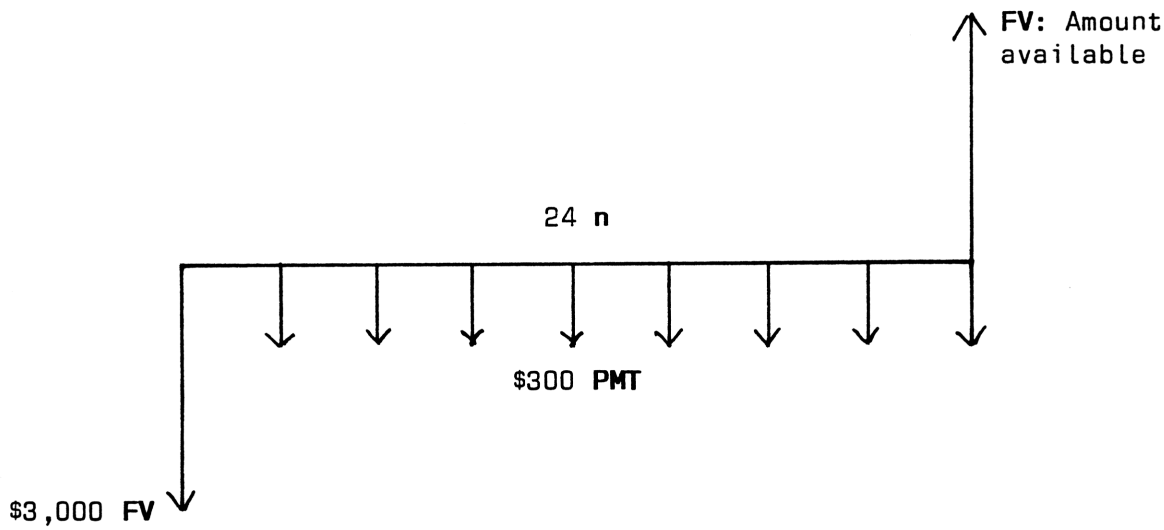
## THE SIGN REQUIREMENT REVISITED

### PROBLEM

How much money will I have available 2 years from now if I deposit $3,000 immediately in a savings account offering 8% interest (compounded monthly), and keep depositing $300 a month in the account for 24 months?

### UNDERSTANDING

First, it is important to agree on the facts: there will be a $3,000 deposit now, a $300 deposit one month from now and every month thereafter for a total of 24 months at which time the total amount is withdrawn. The last deposit occurs on the same day as the withdrawal and has no time to earn interest. The total amount of money available is the same whether the last $300 amount is actually deposited and withdrawn with the rest of the money or is just added to the money that has been previously deposited and is now being withdrawn.




Contrary to all the problems we have dealt with so far, the Payments and the Future Value are on opposite sides of the transaction. Previously, in regular loan situations, increasing the payments would lower the Future Value as the loan balance was being offset by the Payments. Here, if we increased the amount we deposit each month we would increase the amount we may finally withdraw. The sign requirement is what enables the calculator to tell the difference between one and the other situation: for the first time, **the Payment amount must have the same sign as the the Present Value because they are both on the same side of the transaction.**

| | | | | | |
|---|---|---|---|---|---|
| 3000 | PV | 300 | PMT | | |
| 24 | n | 8 | BLUE | i | (or 2 BLUE n) |
| FV | | | | | ($11,298.62) |

The need to deviate from our routine pattern (Present Value positive, Payment and Future Value negative) occurs when two conditions are met:

- We are dealing with a situation where **PMT** and **FV** are on opposite sides of the transaction--most often regular investments followed by a lump sum withdrawal as in the previous example.

- We are keying in more than one dollar amount ourselves. If we were keying in just one dollar amount--**PV**, **PMT** or **FV**--and allowed the calculator to give us the second amount, then we could still follow our regular pattern and allow the calculator to give us the unusual sign for the second amount.

So we can always follow our routine pattern for the first dollar amount we key in--as we did in the previous example. But when a second dollar amount is being keyed in, **then we have to be consistent with the amount already keyed in:** if the second amount is on the same side of the transaction, then the signs are the same, if the second amount is on the opposite side of the transaction, then the signs must be different. The same consistency is required when a third dollar amount is keyed in.

Another option is to go back to the "Money in, money out" approach--money coming in is positive, money going out negative--in which case both **PV** and **PMT** in the previous example would be keyed in as negative amounts. This is a very valid option that should be available to us if we have any hesitation concerning the signs.

Users should select as their preferred option the approach that minimizes the need for **CHS** in solving those problems that they deal with most often. Someone who constantly calculates Present Values on regular loan data, and constantly keys in Payments and Future Values for those loans would reverse our routine procedure and decide to have **PMT** and **FV** as positive amounts, and **PV** as a negative. He would still have to change his routine in dealing with the problem considered here.

There should be little difficulty in recognizing those situations where **PMT** and **FV** are on opposite sides of the transaction and being a little bit more on the alert when dealing with them. If we fail on that matter, the answer given by the calculator would generally be significantly different from what we would expect, thus drawing our attention to the error: pressing **CHS PMT** in the previous example would give us a balance of $4,261.29, significantly less than the total amount invested, an obvious inconsistency that any legitimate investor would notice.

The important is to be consistent within our solution to each problem.

## BEGIN – END OPTION

So far we have looked at situations where the amounts stored in the **PMT** memory are made at the END of each period--whether the month, the quarter, the year, etc. Payments can also be made at the BEGINNING of the period and, like any other change in the timing of the amounts, this affects the answer to our questions.

| BLUE | BEG |
|---|---|

**BEGIN OPTION** [BLUE and 7]

Pressing **BLUE BEGIN** tells the calculator that the payment amount stored in the **PMT** memory is made at the beginning of each period.

At the same time the word **BEGIN** appears in the display. This indicator reminds us of the choice we made.

| BLUE | END |
|---|---|

[BLUE and 8]

Pressing **BLUE END** shifts back to the standard setting for the option: payments are made at the end of each period. The **BEGIN** indicator disappears from the display.

**The BEGIN or END setting is permanent until the opposite setting is selected.**

Selecting the **BEGIN** option affects only calculations involving the logic of the 5 regular cash flow keys, and only when the **PMT** key is involved in the calculation:

- It does not affect a **PV** – **FV** exchange where the data in **PMT** is 0.

- It does not affect the **GOLD AMORT** procedure. With **GOLD AMORT**, payments are assumed to be in arrears, even when the **BEGIN** indicator is showing.

- It does not affect irregular cash flow data and calculations.

In all regular cash flow problems that involve **PMT** we are assuming that the **BEGIN** option and its indicator in the display are not switched on unless specifically selected. Model solutions show **BLUE BEG** where required, but do not show the alternative **BLUE END** keystrokes that switch the calculator back to the regular setting.

With regular cash flow problems that include payments:

> YOU SHOULD NEVER HAVE **BEGIN** IN THE DISPLAY UNLESS YOU SPECIFICALLY SELECT IT FOR THE PROBLEM THAT YOU ARE SOLVING.

**TESTING THE BEGIN - END OPTION**

**A** A $60,000 loan, 12% interest, is amortized over 30 years.
- Calculate the monthly payments. ($617.17)
- Press **BLUE BEG**, and press the **PMT** key again. ($611.06)
- Now press −. ($6.11)

If we make all the payments one month earlier, as implied by switching to the **BEGIN** option, we can afford to lower the payments by $6.11.

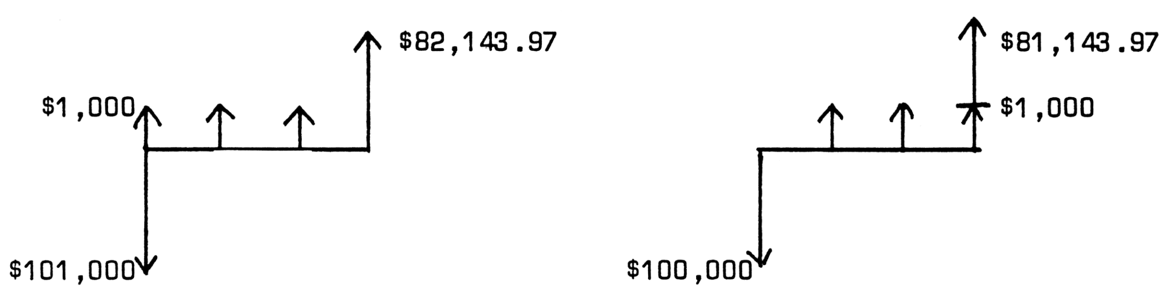
- The difference is significant, but too small to draw attention to the discrepancy if we just left the **BEGIN** option on by error.

- We do not need to key the data in again if we change the option. Just asking the question again interprets the same data differently. You may switch back to **END** and press **PMT** again if you wish.

**B** Key in a $100,000 loan, 9% interest, monthly payments of $1000.
What is the balance after 5 years? ($81,143.97)
What is the balloon payment? ($82,143.97)

Now key in a $101,000 loan, and keep the same interest, the same due date, and the same payment amount, except that the payments are made in advance (**BLUE BEG**). What is the balance? ($82,143.97)

In the second transaction, the borrower gets $101,000 at the very same time as he is required to make his first $1,000 payment, which leaves him with a net proceed of $100,000. He then makes 59 payments of $1,000, and a final disbursement equal to the balloon payment on the first transaction. These two transactions represent exactly the same net amounts changing hands at each point in time and are interchangeable.




The **BEGIN** and **END** options are just means of submitting to the calculator various cash flow patterns. Whether a situation requires one or the other option does not depend so much on the nature of the payments, or whether in fact those payments are due on the first day of the month or the last day of the previous month. The option we choose depends on the convenience with which we can submit to the calculator net amounts changing hands at specific points in time. That is the reality we and the calculator are concerned about.

# BEGIN OPTION
# MODEL PROBLEM

I am opening a Savings account that provides a rate of interest of 8% compounded monthly. My initial deposit consists of the first of 24 monthly payments of $300. How much will there be in the account when I withdraw it all 24 months from now—1 month after my last deposit?

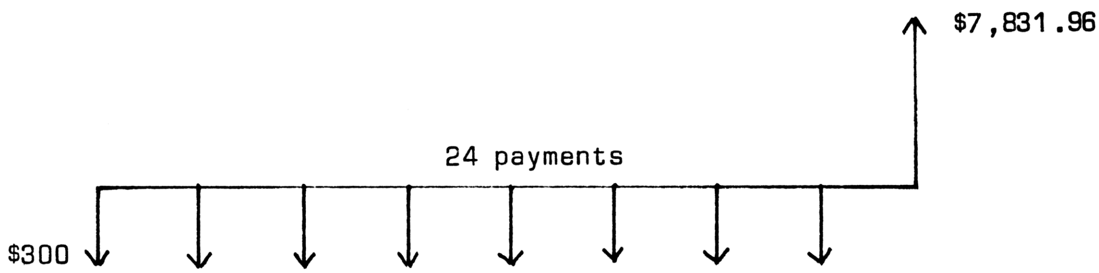
8 BLUE i 2 BLUE n (Or **24 n**)

300 CHS PMT

BLUE BEG

0 PV FV ($7,831.96)

Let's look at the cash flow diagram that corresponds to these numbers:




It is not only to accommodate the first payment that we switch to the **BEGIN** option here—we could key that first payment in as a Present Value of $300. We also want to let the money earn interest for one full month after the last deposit before we withdraw the balance.

There are many situations where the first of a series of equal Payments is made—or can be construed to be made—at the very beginning of the transaction, and where the Future Value, if any, occurs one full period after the last of these Payments.

When solving a **BEGIN** option problem, there is no need to press **BLUE BEGIN** if the **BEGIN** indicator is already showing in the display.

## TYPICAL "BEGIN" SITUATIONS

### ANNUITY

You have $47,000 in a savings account that pays 9.5% interest compounded monthly. How much can you afford to withdraw once a month, beginning today, if you want the income to last for 5 years?

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 47000 | PV | 9.5 | BLUE | i | 5 | BLUE | n | |
| BLUE | BEG | 0 | FV | PMT | | | | |

($979.33)

### LEASE PROBLEMS

The owner of a commercial property is setting up a Net Net Net lease with option to buy. He agrees that the lease should be $12,500 per month, payable in advance, and he wants to establish the price at which the option could be exercised 4 years from now such as the contract (lease plus selling price in 4 years) would have a Present Value of $1,300,000 when discounted at 14%.
What selling price 4 years from now should the option specify?

| | | | | |
|---|---|---|---|---|
| 12500 | CHS | PMT | 1300000 | PV |
| 4 | BLUE | n | 14 | BLUE i |
| BLUE | BEG | FV | | |

($1,460,974.71)

Here, as with most lease and rent problems, the **BEGIN** option is required. The lease payments are made at the beginning of each period, which means that the first lease payment is made as soon as the lease is implemented and the last payment is made one full month before the lease terminates.

**BEGIN** option problems are as varied as **END** option ones, and the flexibility offered by the calculator is as great under one option as under the other. We have just solved a **FV** problem involving a lease. We could similarly question for the rate of return, or the amount of the lease payments, or the Present Value of a lease, etc. Many lease situations with escalation clauses or other features that disturb the regularity of the **PMT** data require the use of the irregular cash flow functions of the calculator.

## COMPOUNDING PERIOD AND PAYMENT PERIOD ARE DIFFERENT

> You decide to invest $150 a month now and every month for the next 5 years (a total of 60 deposits) in a savings account that offers 8.75% interest compounded daily. What will you have in the account at the end of the 5th year?

This BEGIN option problem is very similar to the model problem except that the compounding period on the interest and the period defined by the Payments are different. A discrepancy between the compounding period and the payment period is a frequent occurance in such situations as there is no longer any reason why the periods for both should be the same.

Before we proceed with the calculation as in the model problem, we need to find the monthly compounded rate that is equivalent to a daily compounded rate of 8.75%. The approach is to put in the calculator a **PV–FV** exchange based on the rate of interest (8.75% compounded daily), and to allow the calculator to re-express the rate on that same exchange of money in terms of the period defined by the payments (here the month). That equivalent rate is now compatible with Payment data and we may use it instead of the actual interest rate. It is convenient to use a $100 **PV** investment and a 1 year term in these calculations. (See Unit 6, Page 20)

| Keystrokes | Comment |
|---|---|
| 8.75 ENTER 365 n ÷ i | (Puts 365 in **n**, calculates and stores daily rate in **i**) |
| 100 PV 0 PMT FV | (109.14: a 9.14% annual yield) |
| 12 n i | (0.73% monthly, 8.78% annual monthly compounded rate in **i**) |

With the adjusted rate in **i**, we may now proceed with the problem.

| Keystrokes | Result |
|---|---|
| 150 CHS PMT 5 BLUE n | |
| BLUE BEG 0 PV FV | ($11,331.11) |

Both the 8% daily compounded rate and the 8.78% monthly compounded rate have the same annualized yield of 9.14%. In other words, over an equal number of months they both result in the same increase.

## ADJUSTING NUMBERS TO FIT BEGIN OR END PATTERN

The regular cash flow keys of the calculator handle only problems where one of the payments (**PMT**) occurs either at the very beginning or at the very end of the time span considered. We need to submit to that requirement without changing the actual net amount changing hands at each point in time even in cases where the data is expressed in ways that do not directly fit the pattern.

We may do so either by interpreting an initial amount as being made up of a first payment in **BEGIN** mode plus a lower initial **PV** (see the next problem), or by adjusting the **FV** in a similar way while in the **END** mode as in the second illustration.

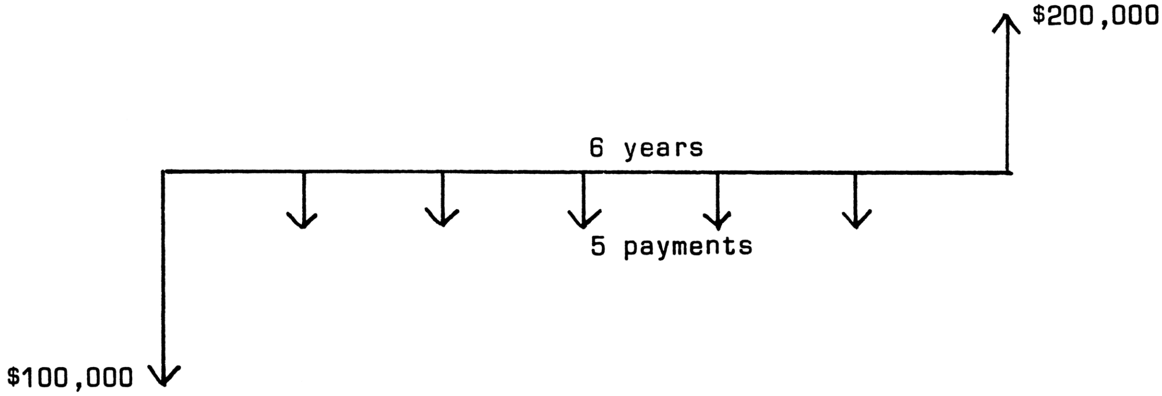
The key to solving such problems correctly is to be very clear in one's mind what the cash flow data implies. Drawing the cash flow diagram can be helpful in that respect. If we are not sure what is taking place then there is little likelihood that we will communicate the data accurately to the calculator.

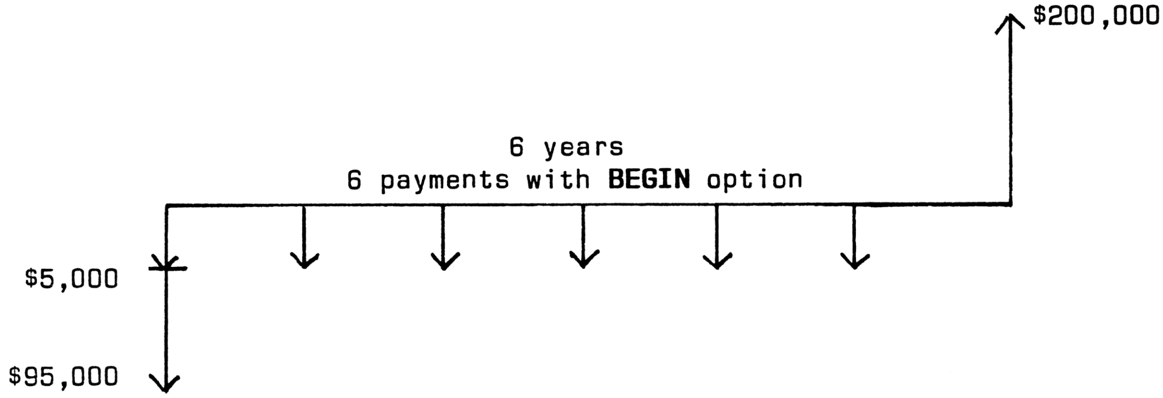
> You invest $100,000 now, plus $5,000 at the end of each year for 5 years. You then receive $200,000 at the end of the 6th year. What rate of return are you getting?

As the problem is expressed, none of the regular payments occur at the same time as either the initial investment or the final return. As a result we have 5 payments and an investment that stretches over 6 years. An obvious out is to interpret the initial $100,000 investment as made up of a **PV** of $95,000 plus the first of 6 payments of $5,000 made at the beginning of each of the 6 years. (Check the cash flow diagrams on the next page)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 95000 | PV | 5000 | PMT | 200000 | CHS | FV | |
| 6 | n | BLUE | BEG | i | | | (8.97%) |

The irregular cash flow functions of the calculator would solve this problem with no need to modify the numbers as we do here. They also offer the only really convenient and accurate way of solving for the payment when we know the initial investment and the **balloon** payment, as opposed to knowing the initial investment and the balance.




The two cash flow diagrams offer the same net amounts changing hands at each point in time. They are interchangeable. Using the second interpretation makes it possible to solve the problem with the regular cash flow keys.

> You invest $100,000, which allows you to **receive** $5,000 at the end of each year for 5 years, plus an amount of $200,000 at the end of the sixth year. What rate of return are you getting?

Same kind of problem as above, except that now it makes more sense to identify the $200,000 as a balloon payment made up of a 6th payments of $5,000 paid in arrears, plus a balance of $195,000. We should have no problem finding a rate of 15.67%.

```
BLUE END
100000   PV    5000   CHS   PMT
195000   CHS   FV     6     n     i
```

**ILLUSTRATION and PRACTICE**

The key to solving problems correctly, here as elsewhere, is to understand the problems correctly. What cash flows does the data provide? Precisely when do they occur? How do I communicate that information to the calculator?

1) You deposit $2,000 a year in an IRA account beginning now. You assume that the annual yield remains at 10%.
What can you withdraw at the end of years 10, 15, and 20?
($35,062.33; $69,899.46; $126,005.00)

2) You open a Christmas Club account with an initial deposit of $200. You intend to deposit $30 at the end of every week for 42 weeks. What amount can you withdraw at the end of the 43rd week if you are earning interest at a rate of 7.26% with weekly compounding? (Assume 52 weeks each year)
(1,510.92)
43 in **n**, 170 in **PV**
and the BEGIN option.

3) A city has issued bonds and is committed to redeeming them 15 years from now. How much must they invest at 9% interest now and at the beginning of each of the years that follow in order to accummulate $12,000,000 in the sinking fund by the end of the 15th year? ($374,960.18)
15 n 12000000 CHS FV 9 i
BLUE BEG 0 PV PMT

4) Same problem, but the city decides to add $1,000,000 to the initial amount invested in the sinking fund. ($261,144.69)

5) You are leasing a property at $2,000 a month below current market rates. The owner wants to buy you out by giving you the present value of those $2,000 discounted at 9% interest. What amount should you receive if you leave when the lease has 2 1/2 years to go? ($53,951.79)

6) You agree to purchase a computer for a small business for a total of $13,000 and lease it to them for 5 years for $300 per month, payable at the beginning of each month. You expect a salvage value of $3,000, or specify an option to buy for that amount at the end of the lease. What is the expected rate of return on your investment? (18.72%)

7) A private party lends $45,000 to be paid back with 40 monthly payments of $1,000 and a balloon payment of $30,000 at the end of the 41st month. What interest is he getting on his loan? (19.23%)
END option, 41 in **n**, $29,000 in **FV**

## THE LOGIC OF THE n FUNCTION

When we calculate **n** the answer is always a whole number, rounded up from the fractional number that might be a more accurate answer. The justification for this feature is that **n** represents a number of periods or payments, and that no interest is earned and no payment made in between periods.

But waiting out the whole last period may overpay our loan, or leave us with a balance other than we requested. To calculate by how much we overpaid, or what balance we have really reached, we should finish the calculation off by pressing **FV**: this correctly balances the books on the numbers in the calculator.

At 10% interest compounded yearly, how long does it take for $1,000 to grow into $3,000?

| | | | | | |
|---|---|---|---|---|---|
| 1000 | PV | 3000 | CHS | FV | |
| 10 | i | 0 | PMT | n | (12 years) |
| | FV | | | | ($3,138.43) |

It takes 12 years, though of course waiting 12 years really makes $1,000 grow into $3,138.43.

A 9% loan has a current balance (**PV**) of $64,500, the payments are $700 per month. How long does it take to fully amortize the loan?

| | | | | | | |
|---|---|---|---|---|---|---|
| 64500 | PV | | 700 | CHS | PMT | |
| 9 | BLUE | i | 0 | FV | n | (158 months) |
| FV | | | | | | ($555.41) |
| RCL | PMT | + | | | | (144.59) |

It takes 158 months, at which time we have overpaid the loan by $555.59. We are justified in making a "balloon" payment of $144.59 instead of a final payment of $700: the loan can be paid off with 157 payments of $700 and a 158th payment of $144.59.

## THE ODD PERIOD

Keying in fractional data for **n.**

We have seen that **n** calculates only whole numbers. It is however possible to key a fractional number into **n**, such as **36.5 n.**

When we do so, the fractional part is interpreted as a fraction of a period squeezed in before the first full period when the **END** option is set, or as a fraction of a period squeezed in before the first payment when the **BEGIN** option is selected.

In a situation with monthly payments and with the normal **END** option, **36.5 n** requires 36 monthly payments, with the first payment being made one and a half month after loan disbursement.

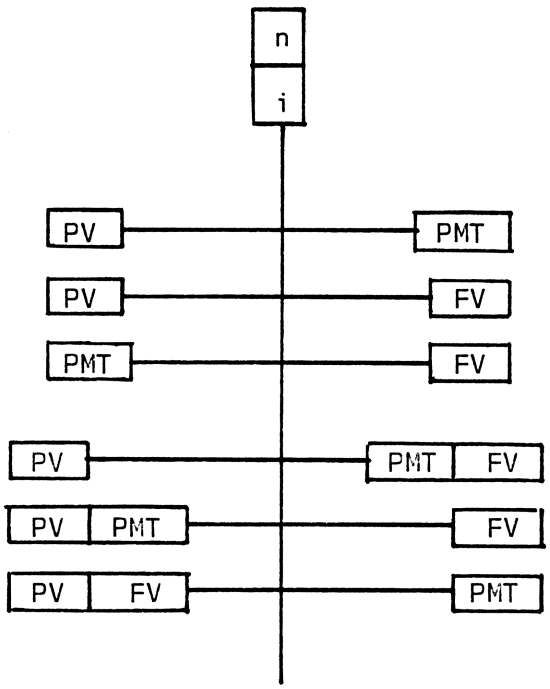
With the **BEGIN** option, the same **36.5 n** also requires 36 payments, but the first payment is made half a month after the **PV.**

The odd period feature of the calculator may be of use in a number of circumstances, such as when the first payment is postponed for more than one full period (but less than two periods) and the interest for that fractional period is amortized along with the loan amount, or when a discounted note is purchased with less than one full period remaining before the first payment is received, or when we want to project appreciation or inflation for a fractional number of years.

> Though very convenient in rather specialized circumstances, this odd period feature may not be of general interest.
> See **Appendix II** for a more systematic presentation.

## n i PV PMT FV
## THE VARIOUS COMBINATIONS: A FINAL LOOK

A fully balanced transaction includes data for **n** and **i** and at least two amounts of money representing the two sides of the exchange of money in time. If we consider the three memories that store amounts of money--**PV PMT FV**--we see that we can have the content of one of these memories balanced off by the content of another, or any one of these amounts balanced off by the remaining two. This leads to the following chart:




(1) Fully amortized loan. (0 FV)

(2) Lump deposit, lump withdrawal (0 PMT)

(3) Sinking fund. (0 PV)

(4) Loan with due date.

(5) Sinking fund with seed money.

(6) Not standard transaction.

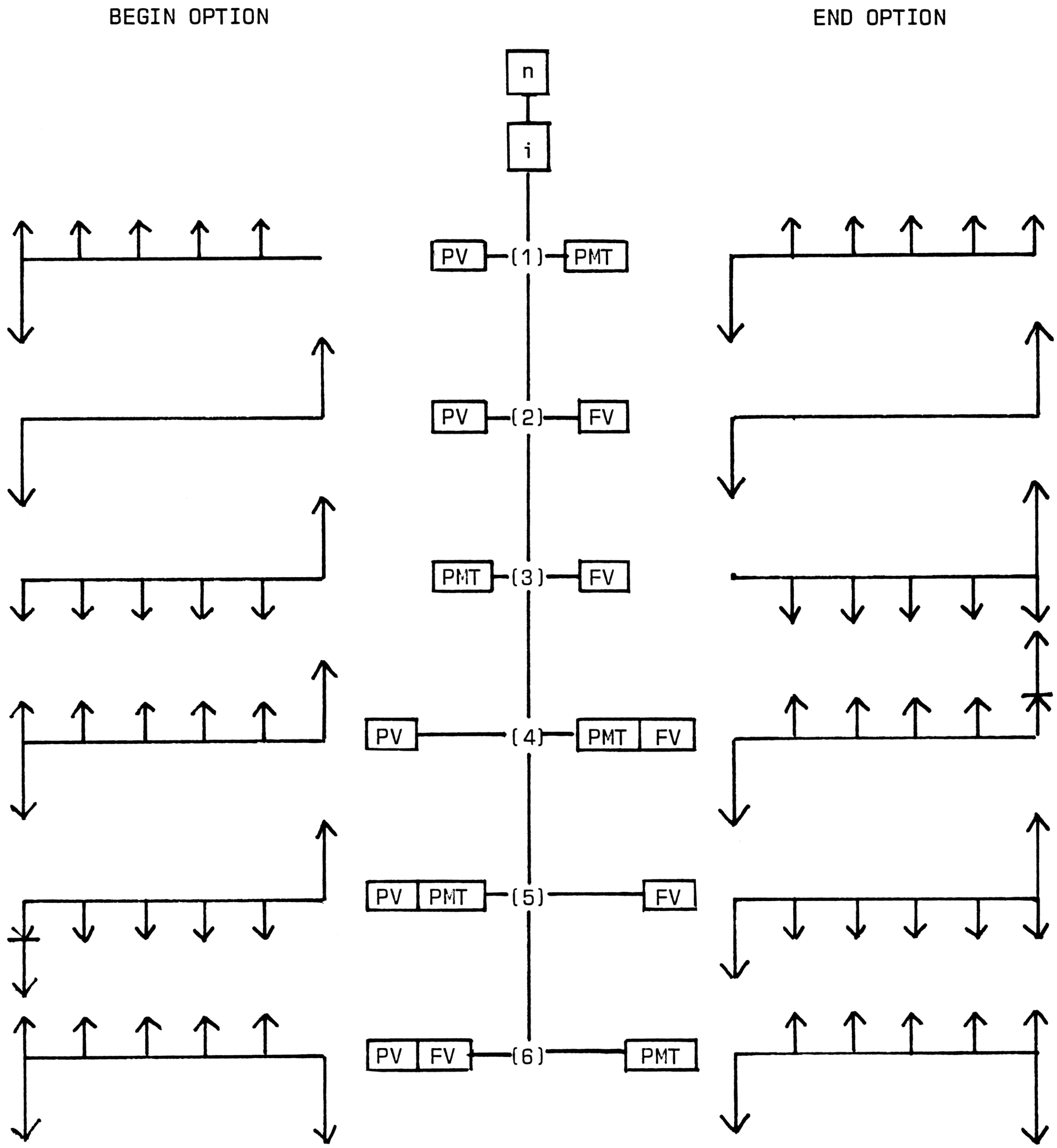
Every financial transaction must include the vertical line with **n** and **i** and one of the horizontal lines with the money keys either paired one on one with zero in the third memory, or combined in various ways with numbers in the three.

The actual situations given as an example for each combination are not exclusive of other interpretations: line (2) also corresponds to inflation, lines (1), (2), and (4) can represent Present Value calculations independently of any loan, etc.

The sign requirement states that one side of the chart must be of one sign, the other side of the other sign. That's what enables the calculator to differentiate between lines (4) and (5) in particular: it makes a difference in the balance if the payments are paying off the loan or are extra amounts borrowed and added to the amount of the loan!

Our practical approach to the sign requirement--**PV** positive, **PMT** and **FV** negative--is valid with combinations (1) (2) and (4), the three most frequently used combinations.

Each horizontal line corresponds to a different cash flow diagram pattern, or rather to two separate patterns according to whether we have selected the **BEGIN** or **END** option. Let's take another look with this in mind:




With data in four of the memories--including a zero if need be--pressing the fifth key balances the books on the transaction.

## THE SIX FUNCTIONS OF THE DOLLAR

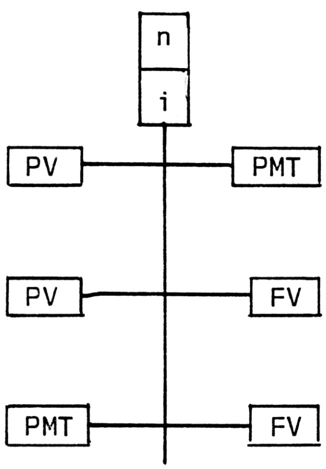
Calculate the monthly payment on a $1.00 (one dollar) loan at 8% interest amortized over 30 years.

| | | |
|---|---|---|
| 8 | BLUE | i |
| 30 | BLUE | n |
| 1 | PV | |
| 0 | FV | PMT |
| GOLD | ENTER | |

($0.007337645739)

Nobody is going to write a check for that amount. But then nobody is going to borrow one dollar amortized over 30 years. The number we calculated is a factor that, multiplied by the actual amount of the loan, gives the monthly payment on that loan.

Before the advent of the financial calculator thick volumes of tables provided such factors for this and similar questions all involving ONE DOLLAR. They did so for the six different questions that can be asked concerning dollar amounts when the Present Value, the Payment, and the Future Value are exchanged as pairs. This corresponds to the first 3 lines of the chart showing the possible combinations of **PV**, **PMT**, and **FV**. So six factors were given for each of the most likely combinations of rate and term.




| PV | PMT | FV | Name of factor. |
|---|---|---|---|
| 1 | ? | 0 | Partial payment. |
| ? | 1 | 0 | Present worth of 1 per period. |
| 1 | 0 | ? | Amount of one. |
| ? | 0 | 1 | Present worth of one. |
| 0 | 1 | ? | Amount of 1 per period. |
| 0 | ? | 1 | Sinking fund factor. |

The easiest way to visualize the six functions of the dollar is to interpret them in terms of the possible combinations of **PV**, **PMT**, and **FV** as pairs: a sinking fund situation, for instance, is when we know the **FV** and we want to know what **PMT** will build up to that **FV**.

It is clear that we can duplicate with the five regular cash flow keys of the calculator any one of the Six Functions of the Dollar factors, with the following differences, all to the advantage of the calculator:

1) We do not need the factors of ONE dollar. By keying in directly the correct amount for **PV**, **PMT**, or **FV** instead of just ONE, we directly get the actual answer, instead of a factor that still needs to be multiplied by the actual amount.

2) With ten significant digits--and possibly many more decimals as in our initial example--we get much greater precision than with the factors.

3) We can get the answer for **any** combination of **n** and **i**, not just for the most likely ones--which sometimes meant not going higher than a rate of 12%.

4) We can find the answer for all the various combinations of **PV**, **PMT**, and **FV**, including those involving those three elements, not just the combinations that pair them off. (Not all financial calculators that have the 5 regular cash flow keys offer that flexibility, or the ones that follow)

5) For all these combinations we can directly solve for **n** and **i**.

6) We are provided with the **BEGIN - END** option, and some options concerning odd initial periods.

And of course, the irregular cash flow functions of the calculator provide us with similar flexibilty in situations where the payments are subjected to considerable irregularities instead of all being equal. This can be very convenient with loans having irregular payment schedules and becomes indispensable with income projections and investment analysis.

# APPENDIX

# APPENDIX I

## EMERGENCY PROCEDURES
## INDICATORS
## FORMAT

## THREE EMERGENCY FUNCTIONS

**KEYS DO NOT RESPOND: PMT ON**

When the calculator does not turn on, or when the display freezes and does not respond to keystrokes, pressing the **ON** key while the **PMT** key is being held down may correct the problem.

If the calculator still does not operate, take the batteries out, wipe them on cloth or paper, and put them back in again. If you still have no succes, try the whole process over again with **PMT ON** and then with the batteries, but this time leave the batteries out for a few minutes before you put them back in. (You will of course lose any user-written program and should see a **Pr Error** sign in the display when it is restored to proper operation). I do not recall a time when a little perseverance does not get the calculator back in business.

**MASTER CLEAR: MINUS ON**

Clears all user memories **including programs** and sets all status indicators and the display format back to their normal setting. Use when you don't know how to clear a disturbing setting or format selectively (see next page).

- Make sure the calculator is turned off (**ON** key, bottom left).
- Press the **minus** key (key [30], 3rd row, 10th column) and **keep it pressed down.**
- Turn the calculator ON.
- Release the **minus** key.

**Pr Error** appears in the display to advise you that any program you may have had has been cleared. Press any key to get rid of the error sign

For a small percentage of HP-12Cs that otherwise function properly this function and the one that follows do not operate. If so, you may want to have it repaired. For others, it takes more than one attempt.

**MASTER CHECK: MULTIPLY ON**

This checks the electronic circuitry and the calculator's own programs. Use after major shock to the calculator or when you need to be reassured that an incorrect answer cannot be blamed on the calculator.

Same procedure as with the **MASTER CLEAR** above except that you keep the multiplication key pressed down (**x**) while you turn the calculator on.

After you release **x**, the calculator shows the word **running** blinking in the display, then the display shows **-8,8,8,8,8,8,8,8,8,8,**, and all the status indicators including some not used on the HP-12C. **Error 9** indicates that the electronic circuitry is not functioning properly. This procedure does not clear calculator memories.

## STATUS INDICATORS AND HOW TO CLEAR THEM SELECTIVELY

Small letters sometimes appear in the display. They are status indicators drawing our attention to unusual options we have selected. These options affect various calculations. **We do not want an indicator in the display unless we specifically choose that option for the problem we are solving.** The keystrokes provided in this course assume that no indicator is showing in the display except where the keystrokes themselves call one up. It is important to know how to select or turn off (clear) the various indicators.

### MASTER CLEAR

All indicators and the corresponding settings can be cleared with the **MASTER CLEAR**, which consists in turning the calculator **ON** while keeping the **MINUS** key pressed down. This also clears everything in the calculator (See previous page). Indicators can also be cleared or selected as follows:

**c** (Continuous compounding during odd period)

Cleared by pressing **STO EEX.** [44 26]

Also set by pressing **STO EEX.**

When set odd period interest is calculated at a continuously compounded rate. Has an effect only if we have a fractional value keyed into **n**, such as 36.25, and only for the duration of the odd period.

When cleared, odd period interest is prorated on a straight line, simple interest basis. See APPENDIX II.

### BEGIN

Cleeared by pressing **BLUE END.** [BLUE 8]

Set by pressing **BLUE BEG.** [BLUE 7]

When set, data in the **PMT** memory is interpreted as being received or given at the beginning of each period. When cleared, Payments are at the end of each period. See Unit 10, page 4.

**D.MY** (Month. Day Year)

Cleared by pressing **BLUE M.DY.** [BLUE 5] (Month.Day Year)

Set by pressing **BLUE D.MY.** [BLUE 4]

When set, dates must be entered in the European format, Days first, then the decimal point, then two digits for the Month and then the Year. When cleared the calculator requires the usual American order of **Month. Day Year**, with a decimal point after the month, and two digits for the days. See Unit 8, p.10.

**f** or **g**

They are a reminder that the **GOLD** key (**f**) or the **BLUE** key (**g**) has been pressed, and that the next key you press will see its gold function or its blue function executed.

They clear automatically when that second key is pressed, or when one of the other prefixes is selected (**BLUE** or **GOLD**, **STO**, **RCL**), or when the calculator is turned off.

Both can be cleared by pressing **GOLD CLEAR PREFIX** [GOLD ENTER].

**PRGM**

The calculator is set in programming mode: keys you press will not be executed but remembered. You also have two digits on the right of the display followed by a dash: 00- probably, or 46- for instance.

Cleared and set by pressing **GOLD P/R** [GOLD 31]

**Error**

**Pr Error**, or **Error 0**, **Error 5**, etc.

Cleared by pressing any key--for instance the **CLx** key. That first key is not operational: it just clears the error sign and normally sets the calculator back to just before the error was made.

**Pr Error** is a warning that user-written programs have been erased.

**Error 0, Error 1,....Error 9** indicate that the last key pressed is requesting of the calculator something that it cannot perform--insufficient data, incorrect format, mathematical impossibility. The number indicates the kind of error that occurred (See Hewlett-Packard manual). In most cases just knowing what key we pressed last is enough of a hint. For instance, you press **n** to solve for the term of a loan and get an **ERROR 5** signal. Check your data: either the numbers are incomplete or incorrect (all positive numbers for instance), or the amount of the **PMT** is just interest-only, or less, and the loan will never be amortized.

## DISPLAY FORMAT

(See Unit 8, p. 27 and APPENDIX II for details)

You want to come back to the regular two decimal display format. Press:

| **GOLD 2** |
|---|

Pressing the **GOLD** key, and then the numberal **2** selects the regular two decimal format for the display. It brings back the regular format in all cases except the following:

- **Too many decimals** because the data in the calculator is too small to be expressed with only 2 decimals. When you change your data the 2 decimal format reappears.

- **Scientific notation format** such as:

  **7.892356 19** **3.589600 -25** **9.999999 99**

  The number in the display is too large or too small to be expressed with the regular format. The last example with the 9.999999 99 indicates that the calculator has reached the maximum of its capacity for any one number. When you clear x, **CLx** [35], or key in a new number, the regular format reappears.

- Programming format such as:

  **00-**
  **01- 42 2**
  **02- 35**

  You also have **PRGM** on the bottom right of the display. You are in programming mode. Switch out by pressing **GOLD P/R** [GOLD 31].

- **COMMA** instead of **DECIMAL** and vice versa.

  You have selected the European way of writing numbers: FF 3.524,65 for 3,524.65 French Francs.

  Turn the calculator off, and turn it back on **while you keep the decimal point pressed down.** This brings you back to the regular american format--or switches you to the European format if you are already in the American format.

(To clear letter indicators, see previous pages)

The **MASTER CLEAR** (Appendix I, page 1) takes care of all possible problems and erases all status indicators from the display, but it also clears all your data and all user-written programs.

# APPENDIX II

# ADDITIONAL DETAILS

This appendix gathers a number of features and applications judged too technical or too limited in their usefulness, or both, to be included with the bulk of this course. I am thinking of those compulsive learners who would want to master the technique "because it is there". The inclusion of these matters in the Appendix should give them an excuse to pass unless they really have the need for such topics.

## NUMBER MODIFICATION FUNCTIONS

**BLUE INTG** INTEGER [BLUE 25]

Cuts off the decimals from the number in the display (the **x** memory).

This does not affect just the format: the internal number is changed.

**This does not round to the closest integer: it rounds down.**

75.92 BLUE INTG — Retains only 75.00

**BLUE FRAC** FRACTION [BLUE 24]

Retains only the fractional part of a number.

Example of use:

You have calculated a distance of 14.62 feet and want to express it in feet, inches, and 16th of an inch:

| | | | |
|---|---|---|---|
| 14.62 | BLUE | FRAC | (0.62) |
| | 12 | x | (7.44 inches) |
| | BLUE | FRAC | (0.44) |
| | 16 | x | (7.04 sixteenth) |

Answer: 14' 7" 7/16 (plus 4/100 of 1/16)

**GOLD RND** ROUND [GOLD and 14]

Rounds the display number internally to the format showing in the display.

(See Unit 2, page 14 for application with loan payments)

When **BLUE INTG, BLUE FRAC** and **GOLD RND** are used, the complete original number with the hidden decimals is retained in the **LAST x** memory.
Press **BLUE LSTx** (BLUE and ENTER) to retrieve the original number.

## SCIENTIFIC NOTATION FORMAT

In SCIENTIFIC NOTATION numbers are expressed as a single digit from 1 to 9--followed, if need be, by decimals--multiplied by a power of 10:

| | | |
|---|---|---|
| 300 | (3 x 100) becomes: | $3 \times 10^2$ |
| 90,000 | (9 x 10,000) becomes: | $9 \times 10^4$ |
| 1723 | (1.723 x 1000) becomes: | $1.723 \times 10^3$ |

Pressing **GOLD** and the **DECIMAL POINT** selects scientific notation as the display format. However the display expresses only the digits and the power of 10 (the exponent). The 10 itself is left as a blank space.

456 $4.56 \times 10^2$ is seen as **4.560000 02**

Switch to scientific notation format: **GOLD** and **DECIMAL POINT.**
Key in the following numbers and press **ENTER** to see them in scientific notation format:

| | | | |
|---|---|---|---|
| 60,000 | ENTER | 6.000000 04 | |
| 14 | ENTER | 1.400000 01 | |
| 5.23 | ENTER | 5.230000 00 | |
| 0.0002 | ENTER | 2.000000-04 | (Negative exponent moves |
| 0.5 | ENTER | 5.000000-01 | decimal to left) |
| 789,456.32 | ENTER | 7.894563 05 | |

Switch back to regular format: **GOLD 2**

The exponent of 10 can be keyed in using **EEX** (Enter **EX**ponent) [26]

789,000,000,000,000,000,000,000,000,000,000 can be entered as:

789 EEX 30 ENTER (7.89000 32)

- The display automatically switches to scientific notation if the number calculated is too large or too small to appear in regular format.

- The maximum exponent allowed by the display is 99.

- Using **EEX** can help save keystrokes when keying in numbers--especially important in programming:

| | | | |
|---|---|---|---|
| 5,000,000 keyed in as: | 5 EEX 6 | |
| 100,000 keyed in as: | EEX 5 | |
| 100 keyed in as: | EEX 2 | (Saves 1 step) |

## CONTINUOUS COMPOUNDING

Continuous compounding is the effect of a compounding period that becomes infinitely small--there is an infinite number of compounding periods within any given period of time.

There is a formula using logarithms to calculate the effective yield of a continuously compounded rate.

> What is the effective yield corresponding to 12% interest with continuous compounding?

| | | | | | | |
|---|---|---|---|---|---|---|
| 0.12 | BLUE | $e^x$ | 1 | − | 100 | x |

(12.7496852 with hidden decimals)

Another approach (easier to remember) uses the regular calculation for an annualized yield but with a very high number of periods per year. Daily compounding has 365 periods per year. Let's take 99,999,999 periods per year to approximate continuous compounding:

| | | | | | | |
|---|---|---|---|---|---|---|
| 100 | PV | 0 | PMT | | | |
| 12 | ENTER | 99999999 | n | : | i | (Puts 99999999 in **n** and 12/99999999 in **i**) |
| FV | | | | | | (112.7496851 with hidden decimals) |

Using $100 immediately shows the effective yield as 12.7496851%.
To put that rate in **i** we may allow the calculator to calculate it:

| | | |
|---|---|---|
| 1 | n | i |

(12.75; or 12.7496851)

## KEYING IN FRACTIONAL DATA FOR n: THE ODD PERIOD

When a value for **n** is keyed in with decimals, the fractional part is interpreted as extra time added to the first full period (**END** option), or, with the **BEGIN** option, as time squeezed in before the first payment.

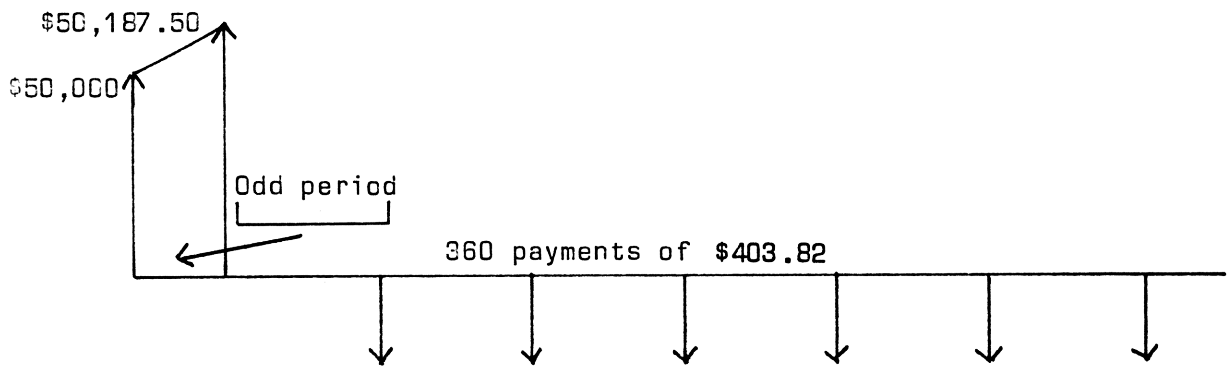
**The non-decimal (integer) portion** of the number keyed into **n** still defines **the number of payments or of full compounding periods.**

### ODD PERIOD INTEREST ADDED TO PRINCIPAL AND AMORTIZED

A $50,000 loan, 9% interest, is amortized with 360 monthly payments. What is the amount of the payments if the first payment is due 1 1/2 month after disbursement of the loan?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 50000 | PV | 9 | BLUE | i | 0 | FV | |
| 360.5 | n | PMT | | | | | (**$403.82**) |

Regular payments would be $402.31. The difference is due to the interest earned during the extra half month. It is as if the Present Value was increased by the interest earned during that odd period, and then amortized in 360 payments along with the rest of the loan.




You may want to calculate that $403.31 is the payment amount on a $50,187.50 loan at 9% interest, amortized over exactly 30 years, and that the extra $187.50 is 9% interest on $50,000 loan prorated for half a month.

What is the balance after 60 payments? (60.5 months after loan disbursement)

| | | | |
|---|---|---|---|
| 60.5 | n | FV | ($48,119.84) |

## THE ODD PERIOD WITH THE BEGIN OPTION

With the **BEGIN** status indicator turned on (**BLUE BEG**), and the word BEGIN showing in the display, a fractional amount added to the number keyed into **n** squeezes that fraction of a period before the first payment is made. So the first payment no longer occurs at the same time as the **PV**.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 150000 | PV | 20000 | CHS | PMT | 90000 | CHS | FV |
| BLUE | BEGIN | 6.5 | n | i | | | |

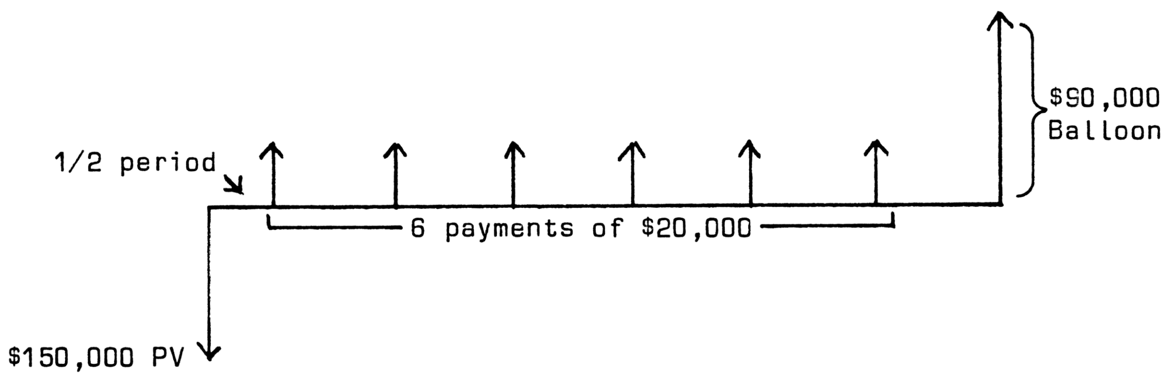
With the previous yearly cash flows keyed in, pressing **BLUE BEGIN 6.5 n** means the following:

- There are 6.5 years between Present Value and Future Value,

- There are 6 yearly payments of $20,000,

- The first payment begins 1/2 year after the Present Value.

Pressing **i** calculates the rate of return on that exchange.




The **BEGIN** option in combination with the odd period fractional data for **n** can be used in particular in loan situations normally associated with the **END** option when for some reason the first payment is delayed by less than one full period.

With the **BEGIN** option, the **FV** is the balloon payment--the last payment is made at the **beginning** of the last period, one full month before the **FV** is due. So in transforming an **END** situation into an equivalent **BEGIN** exchange we must key in the **BALLOON PAYMENT** as **FV**, and as the balloon payment already includes one payment, we must reduce the number of payments keyed into **n** by one.

As far as the **PMT** and **FV** parts of the exchange are concerned, 6 yearly payments of $20,000 made in advance and a balance of $90,000 one year later is the same thing as 7 payments of $20,000 made in arrears and a **FV** of only $70,000 at the same time as the last payment.

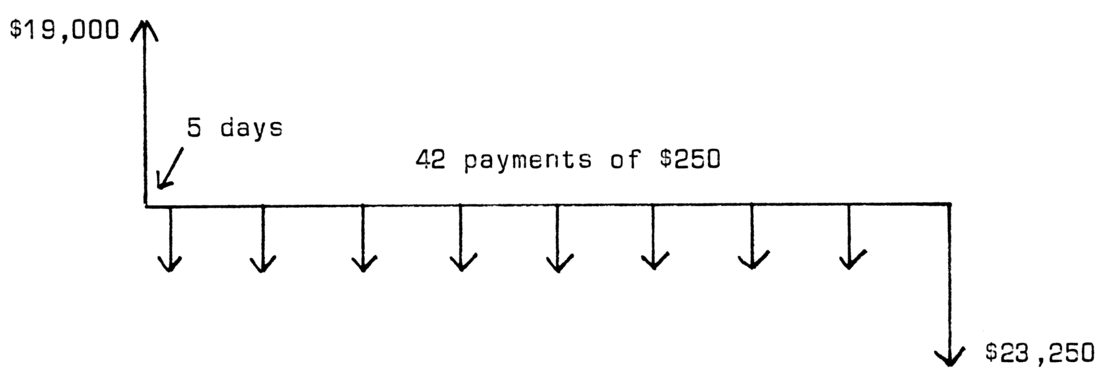
## DISCOUNTED LOAN WHEN FIRST PAYMENT COMES IN EARLY

You buy a discounted loan for $19,000. You are going to receive 43 payments of $250, plus a balance of $23,000--which is the same thing as 42 payments of $250, followed by a balloon payment of $23,250.
What yield are you getting if you are to receive the first payment 5 days (5/30 of a month) after you purchase the loan, and all subsequent payments 25 days earlier than in usual situations?
What selling price gives a 16% rate of return?

| Keystrokes | |
|---|---|
| 19000 PV 250 CHS PMT | We balance the books on the adjusted "BEGIN" data. |
| 23250 CHS FV $\boxed{\text{BLUE BEG}}$ | |
| $\boxed{\text{5 ENTER 30 ÷ 42 + n}}$ | 42 $^{5}/30$ in **n**. |
| i (wait) 12 x | (Yield for $19,000 price: 20.38%) |
| 16 BLUE i PV | (Price for 16% yield: $21,389.11) |

Ignoring the fact that the first payment is due in 5 days and, more importantly, that all subsequent payments and the balloon payment are received 25 days earlier then could be expected, we would find a rate of 19.87% and a selling price of $21,154.58.

The cash flow and timing are submitted to the calculator by switching to **BEGIN** and keying a fractional number into **n**: the first payment occurs that fraction of a month after the Present Value. Because we have selected the **BEGIN** option, the **PMT** and **FV** amounts must be keyed in as 42 payments plus the balloon payment one month later, rather than as 43 payments with the balance added to the last payment.



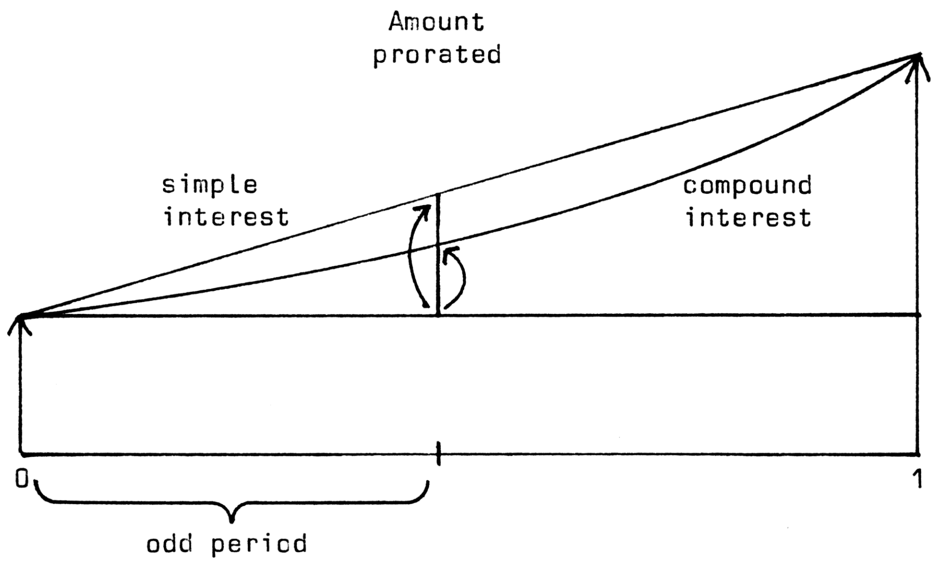
## COMPOUNDING DURING THE ODD PERIOD **STO EEX**

The interest earned during the odd period can be calculated in two ways:

In the normal setting--the one used so far--the interest is just prorated on a straight line basis for the partial period. This is a simple interest procedure.

By pressing **STO EEX** we select a lower, continuously compounded rate for that partial period--and for that partial period only--such as, if the partial period extended to a full period, the total increase would equal the regular periodic rate increase. That lower, continuously compounded rate has the same yield as the regular periodic rate. Choosing the continuous compounding option provides less interest than the straight line approach: it would provide the same amount of interest only if the continuous compounding was pursued for a full period.

Choosing the compounded option puts a small **c** as an indicator in the display. Pressing **STO EEX** again returns the calculator to the straight line option and eliminates the **c** from the display.




The compound interest option affects only the odd period: selecting the simple interest option--the normal setting--does not prevent the regular periodic compounding that occurs after the odd period. Even when theoretically justified, the compound interest option may often be somewhat of an overkill.

## ODD PERIOD WITH PV-FV EXCHANGES

You lend $30,000 and agree on an interest rate of 8% to be added annually to the principal. There are no payments. You are paid back after 2 1/4 years and agree to prorate the interest for the partial year. How much should you receive?

| | | | | | |
|---|---|---|---|---|---|
| 30000 | PV | 0 | PMT | | |
| 2.25 | n | 8 | i | FV | ($35,691.84) |

If the loan is paid off after only 9 months (.75 of a year) the Future Value is:

| | | | |
|---|---|---|---|
| .75 | n | FV | ($31,800) |

The compound interest option should not be used in this particular case: straight line proration was agreed upon and is perfectly justified. But using the compound interest option (**STO EEX** if **c** indicator is not already showing) is an obvious choice when analyzing inflation or appreciation as the annual rate is the result of a continuously compounded increase--at least if such precision is justified when working with rates that are often nothing more than educated guesses.

Calculate the rate of appreciation that has affected a property that sold for $62,000 three and one half years ago, and that is now selling for $95,000.

| | | | | | | |
|---|---|---|---|---|---|---|
| STO | EEX | | | | | (Press only if **c** indicator is not already on) |
| 62000 | PV | | 95000 | CHS | FV | |
| 0 | PMT | 3.5 | n | i | | (12.97%) |

Calculate the value of a $230,000 property after 5 years and 10 months of appreciation at 6% per year.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (Retain **c** indicator) | | | | | | | |
| 230000 | PV | 0 | PMT | | | | |
| 10 | ENTER | 12 | ÷ | 5 | + | n | (5.83 in **n**) |
| 6 | i | FV | | | | | ($323,106.27, only $75.21 less than without the **c** option). |

Fractional data in **n**

**PRACTICE**

Use 30-day months and no compounding during odd period.

1) $90,000 loan at 13% interest. What is the balance after 60 monthly payments of $1,000? What is the balance if the first payment is due 1 month and 25 days after origination of the loan?
($87,902.64 and $89,453.58)

2) $300,000 loan at 10% interest, semi-annual payments of $17,000 beginning 10 months after loan disbursement. What is the balance after the 8th payment is made (4 years and 4 months)? ($295,676.34)

3) A $20,000 loan is amortized with 90 monthly payments. The interest is 15%. What are the payments? ($371.43)
What are the payments if the first payment begins 1 month and 21 days after the loan is made (21 days later than in the previous question)
($374.68)

4) $175,000 loan, the interest is 11%, the payments are $1,650. What is the balance at the end of 5 years under normal conditions. What is it if the first payment is made one and a half month after disbursement of the loan? ($171,355.42 and $172,742.16)

5) You borrow $30,000 for 3 1/2 years at 13.5% annual yield with interest prorated for the half year. There are no payments. What amount must you pay back? ($46,224.89)

6) A property was bought for $92,000 and resold for $137,900 three and a half years later. What rate of appreciation is that? (Use compounding option)
(12.26%)

7) You agree to sell a loan at a discount at a price that gives a 16% monthly compounded yield to the buyer. There are 63 monthly payments of $1,120 left to be received, plus a balance of $93,444 at the same time as the last payment. What should be the discounted price <u>if the first payment is due 6 days from today</u>? ($89,037.00)

Remember that **STO EEX** is a toggle switch: it both selects and clears the compounding option for odd period calculations.

## LONGER DELAY IN FIRST PAYMENT

With monthly payments, and in normal **END** mode, the odd period procedure automatically adds a fraction of a month before the first full month--a full month that ends with the first payment.

If the first payment is delayed by two months or more, then we cannot use the odd period approach because only the fractional amount is interpreted as adding time without adding payments.

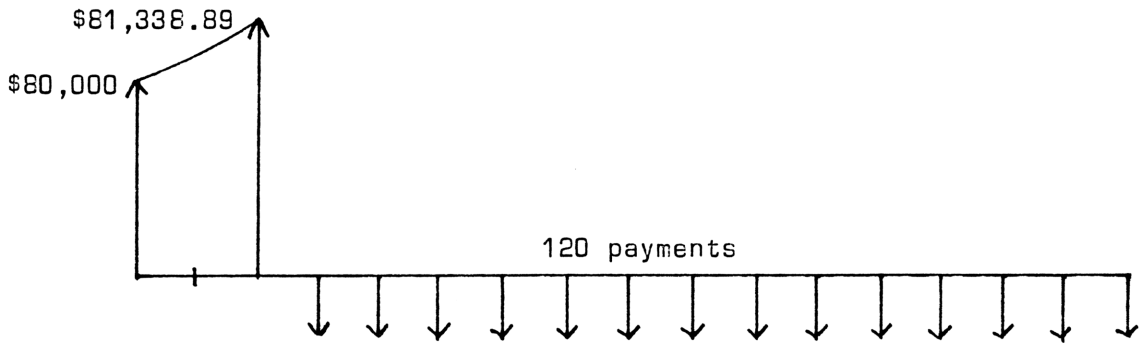
So when payments are delayed by more than a fraction of a period, we need to take over the adjustment ourselves, and execute by stages what the odd period approach does for us automatically.

### MODEL PROBLEM

> An $80,000 loan, 10% interest is amortized with 120 equal monthly payments beginning 3 months after the loan is made. Calculate the payment amount.

Here we allow the $80,000 loan to grow at its regular 10% monthly compounded rate for two months, during which time no payments are made. The $81,338.89 balance occurs one month before the first payment is made. We then use that amount as the Present Value of a loan amortized with 120 payments beginning one month later--the traditional payment calculation:

| | |
|---|---|
| 80000 PV | |
| 10 BLUE i 2 n | |
| 0 PMT FV | ($81,338.89, negative) |
| CHS PV | $81,338.89 in **PV** |
| 120 n 0 FV PMT | ($1,074.90) |



We may now use the adjusted $81,338.89 loan to ask other questions concerning the original loan. For instance:

What is the balance of the loan after 4 years?

| | | | |
|---|---|---|---|
| 46 | n | FV | ($59,189.79) |

The balance on the actual loan after 48 months corresponds to a balance on the ajusted loan after 46 months.

A $500,000 loan, 13% interest, 72 payments of $6,000 beginning 5.5 months after the loan is made. What is the **balloon payment**?

**INTEREST COMPOUNDED**

| | |
|---|---|
| 500000 PV 13 BLUE i | |
| 4.5 n 0 PMT FV | ($524,848.92) |
| CHS PV 6000 CHS PMT | |
| 72 n FV RCL PMT + | ($496,854.27) |

Again we adjust the Present Value to one month before the first payment--we use the odd period procedure here for that purpose--, and we treat the adjusted loan as we would any other traditional loan.

The interest that is not paid is added to the principal and compounded. If the interest is just DEFERRED without earning interest a simple arithmetic calculation would be required to adjust the Present Value to the actual indebtedness 4.5 months after loan disbursement:

**INTEREST DEFERRED, NOT COMPOUNDED**

| | |
|---|---|
| 500000 ENTER 13 BLUE i | (Arithmetic adds 4.5 months of interest to original loan for an adjusted loan amount of $524,375) |
| % 4.5 x + | which is stored in **PV.** |
| PV 6000 CHS PMT | |
| 72 n FV RCL PMT + | ($495,824.77 balloon) |

Questions involving the calculation of **i** for such loans would require the use of the irregular cash flow functions.

## PRACTICE

(Assume that unpaid interest earns interest compounded at the regular rate)

8) A $230,000 loan at 13.5% interest is amortized over 10 years with no payments for 2 years followed by 8 years of monthly payments. (With payments in arrears, the first payment occurs at the end of the 24th month). What are the monthly payments? When does the balance go back down below $230,000?

($5,140.77; 58 months: until then a tax-payer filing on a cash basis may consider all payments as deferred interest)

9) $37,000 loan, 8% interest,payments of $300 beginning 6 months and 21 days after the creation of the loan. What is the balance after 4 years (and 21 days)? ($36,255.07)

(Balance after 5 months: $38,249.89. This becomes PV of loan that has 43 months and 21 days to go)

10) $95,000 loan, 11% interest, no payments for 3 and 1/2 months. What payment amount then amortizes the loan in 298 payments (25 year loan)?

($953.82. Key 2.5 into **n** initially)

## LOAN CONSTANT

The annual loan constant is the total annual payments expressed as a percentage of the loan amount. (It can also be expressed as a decimal).

On a loan with monthly payments, the loan constant is identical to the monthly payment on a $1,200 loan.

Let's find the loan constant on a 10% interest loan with monthly payments for different amortization periods:

| | | | | | |
|---|---|---|---|---|---|
| | 1200 | PV | 0 | FV | |
| | 10 | BLUE | i | | |
| 30 year loan: | 30 | BLUE | n | PMT | (10.53) |
| 25 year loan: | 25 | BLUE | n | PMT | (10.90) |
| 20 year loan: | 20 | BLUE | n | PMT | (11.58) |
| 15 year loan: | 15 | BLUE | n | PMT | (12.90) |

With an interest-only loan the rate of interest itself is the loan constant: with a 10% interest-only loan the borrower must pay 10% of the original loan each year to satisfy his obligations. To amortize the loan, the borrower needs to increase his payments to pay for the principal reduction, and the faster the loan is to be paid off, the higher the payments become. Hence the increase of the loan constant in our example beyond 10% to 10.53%, 10.90%, 11.58%, and 12.90%.

To keep the payments as low as possible, borrowers are sometimes more interested in the loan constant than in the interest rate: a 12% interest-only loan requires lower payments than a 10% loan amortized over 15 years (loan constant of 12% and 12.90% respectively).

Loan constants are also used in a variety of calculations affecting income property valuation.

**Why is the loan constant equal to the payment on a $1,200 loan?**

With only $100 in **PV**, the monthly payment would measure the monthly obligation as a percentage of the loan. To get the annual loan constant we would need to multiply that monthly payment by 12. The short cut proposed here is to multiply the $100 loan amount by 12 instead.

# APPENDIX III

# USING THIS COURSE WITH OTHER CALCULATORS

## HP-37 HP-38C & E HP-92

## HP-41C

With very few modifications, most of the material in this course can be applied to these other Hewlett-Packard calculators: the older model financial calculators, HP-37, HP-38E, HP-38C, and HP-92, and the more powerful HP-41 series, HP-41C, HP-41CV, HP-41CX when equipped with the appropriate application module. The key to making the adjustment is common sense and trial and error. If you try to follow this course with one of these calculators you will be surprised how many Units you can glide through with only cosmetic changes. The following remarks should help in making what adjustments are required.

With financial calculators other than the Hewlett-Packard line and with computer programs providing the 5 financial functions **n**, **i**, **PV**, **PMT**, and **FV**, this course could still be of some use in providing the financial concepts and the general approach to using those 5 variables. The actual implementation would requiring adjusting to the particular logic of the instrument, and all peripheral functions and arithmetic calculations could be significantly different.

**HP-37, HP-38C, HP-38E, HP-92**

Written for the the HP-12C, the present course also applies with very few modifications to the older model HP-37, HP-38 and HP-92 financial calculators. Adjustments may need to be made in the following categories:

1) **The name of the function.** The same function, operating in exactly the same way, may be listed under a different name, or may require a different prefix (**GOLD** or **BLUE**). For instance, the **GOLD CLEAR-REG** function of the HP-12C is labeled **GOLD CLEAR-ALL** on the HP-38 series, and the **%T** function, a direct function with the HP-12C, requires the **GOLD** prefix on the HP-38 series. Of course, the position of the keys most often differs.

2) **The 5 top-row financial functions (n, i, PV, PMT, FV)**, operate in exactly the same way, except that the more abstruse characteristics of the **n** function introduced in UNIT 10 and APPENDIX II apply only to the HP-12C. **n** is easier to manage with these other calculators though some marginal flexibility may be lost in the process. In summary:

   Except for the HP-12C, **n** pressed as a question gives a fractional answer if required by the data. With these other calculators, a fractional value given by **n** as an answer or keyed into **n** as data implies a partial payment at the end of the full time span, not the 'odd period' squeazed in before the first full period as with the HP-12C. For some, this simpler logic may come as a relief.

3) The **BEGIN** and **END** options are selected by a toggle switch, as are the **ON** and **OFF** functions. This is particularly important as no indicator in the display draws user's attention to the option that has been selected. These calculators can be switched to the **BEGIN** option without the user being aware of the change, thus throwing all **PMT** calculations off by a small amount. Some users may want to tape the **BEGIN-END** switch in the **END** position. The same switch also selects the **D.MY** or **M.DY** format for the dates on the HP-38 and HP-92.

4) **The choice of arithmetic functions** differs from one model to another as does the **number of Register memories.** In particular the HP-37 has only 7 Register memories and the smallest number of mathematical functions. The 38 is most similar to the 12C, though storage register arithmetic can be performed on 7 of its 20 Register memories instead of only 5 for the 12C, and the **BOND** and **DEPRECIATION** functions do not exist on the 37 and the 38.

5) **The Stack** performs in exactly the same way with all these models, except that the HP-37 does not have a **LASTx** function.

6) The **AMORTIZATION** and **SIMPLE INTEREST** functions perform in the same way.

   With these points in mind, try it. The calculator itself will point out those very few instances where HP-12C procedures may not apply.

## HP-41C, HP-41CV, HP-41CX

This course can be applied with very few modifications to the HP-41 series calculators when equiped with the **"Compound Interest and Loan Amortization"** program found in the **REAL ESTATE PAC.** I have not found an alternative **FINANCIAL DECISIONS PAC** as satisfactory as the Real Estate Pac. There is also a **PPC ROM** module with a compound interest program that offers the extra option of choosing a different time span for the compounding period and the payment period. The specifics of the instructions that follow apply to the HP-41C and HP-41CV equipped with the **REAL ESTATE** pac.

As with the HP-37 and HP-38 calculators:

- the name and position of the function may differ.
- The Stack is essentially the same.
- The 5 regular compound interest functions, once they have been loaded from the application pac, are the same except for some particularities of the **n** function where the HP-41 retains the simpler logic of the HP-37 and HP-38.

However the HP-41 calculators are immensely more powerful than the HP-12C. With the extra power come extra demands on the user. Switching from the HP-12C to the HP-41 is like going from driving a car to flying a Boing 747. There are advantages to the 747, but also constraints.

Among the features of the HP-41 not found on the HP-12C are:

- The immensely superior memory capacity for storing data and, even more so, programs.
- The vast array of mathematical functions.
- The customized keyboard capabilities when in "USER" mode.
- Alphanumeric capability which allows letters of the alphabet to prompt the user or permits him to spell out the name of the function he wants to execute.
- The availability of plug-in peripherals allowing the reader to load and/or store data and programs using printer, magnetic card, screen, tape-recorder, bar-code, sensors.
- The availability of sophisticated programs on application modules, in bar-codes, etc. to fit an ever increasing number of specialized applications, from navigation to agriculture.

The explanations that follow do not attempt to open up the whole power available to the user of an HP-41 calculator. Those that have that power at their command will have little difficulty in learning from the financial material included in this course and in combining it to the extra features of the HP-41 though the notes that follow may give them a few pointers on what to expect. What follows is addressed essentially to the first time user of the HP-41, who, while he gradually opens up new areas of competence with the HP-41, would at least like to do with the 41 what he could previously do with the 12C.

## MAJOR DIFFERENCES IN USING THIS COURSE WITH AN HP-41.

Beyond the need to adjust to such matters as a different position for most keys and on occasion a different name for a function, the user of an HP-41 will find the following three differences in using this course with an HP-41:

- The need to load the appropriate "Compound Interest" program in the calculator (See below).
- The Stack and the 5 financial functions, essentially the same as in the HP-12C when taken separately, do not work together in the same convenient way as with the 12C. This also applies to the regular cash flow and irregular cash flow programs that can be loaded from application pacs: the cooperation between the two is not as automatic as with the HP-12C.
- **DATE** and **DAYS, SIMPLE INTEREST, BOND** and **DEPRECIATION** functions also require loading separate programs.

## ACTIVATING THE FINANCIAL FUNCTIONS ON THE HP-41C

Brief instructions for first-time HP-41 user
to allow him to perform most of the **HP-12C MADE EASY** procedures.

1) With calculator turned OFF, plug in the **REAL ESTATE** application module (Pac) into lowest numbered port not used for extra memory modules (See Hewlett-Packard Pac instructions for precautions).

2) Load the "Compound interest and loan amortization" program from the module into the calculator as follows:

| Press | Meaning and position. |
|---|---|
| XEQ | Letter K. 'Execute'. |
| ALPHA | This is the top right hand switch. |
| GOLD<br>P | Used with GOLD (Shift) as a prefix, the 'P' key really calls up the '$' character, which is the name assigned to the Compound Interest and Loan Amortization program of the Real Estate module. |
| ALPHA | |

3) You have now transformed the 5 top row keys (A to E) into the five regular cash flow functions **n, i, PV, PMT, FV.** Most of the material presented in this course can now be performed on your HP-41.

In addition, keys H, I, J, and the GOLD functions corresponding to keys A to E, have been assigned new meanings as follows:

| | | | | |
|---|---|---|---|---|
| 12x | 12: | BEG/END | LIST | CLFIN |
| n | i | PV | PMT | FV |
| (A) | (B) | (C) | (D) | (E) |

| | | | | |
|---|---|---|---|---|
| x y | R | % | %CH | AMORT |
| (F) | (G) | (H) | (I) | (J) |

The word 'USER' appears in the display, indicating that the keyboard has been switched to the user-defined setting. Pressing the **USER** switch (next to the **ON** switch) clears the 'USER' setting and gives back their original meaning to keys A to J. Pressing the **USER** switch again re-activates the user-assigned meanings as above.

The new key assignment that transforms the 5 top keys of the HP-41 into the 5 essential financial functions similar to those of the HP-12C is permanent unless specific action is taken to clear or overwrite the assignment. It is in operation whenever the **USER** indicator is set. It is not affected by turning the calculator off. Should it be cleared by error, loading the program in again as above re-establishes the assignment.

| |
|---|
| WITH THE HP-41, YOU MUST BE IN **USER** MODE TO ACTIVATE THE FINANCIAL FUNCTIONS |

An overlay may help you remember the meaning assigned to these keys. You may also press these keys and hold them down until the word "NULL" appears. In the interval, the user-assigned meaning appears in the display. If you release the key before the word "NULL" appears, then you have asked the calculator to perform the function, which may or may not be what you had in mind. If the display shows **DATA ERROR,** wait a second until the indicator **PRGM** (Program) has disappeared from the display before you press the next key. That function clears the error sign and is executed. (It just clears the error signal in the HP-12C)

## USING THE 5 FINANCIAL FUNCTIONS WITH AN HP-41C

You should have no problem running through UNITS 1, 2, 4, 5, 6, 7 of this course and may want to start even before you read any further. The slight discrepancies you may encounter are listed below:

- Previous users of the HP-12C will find that keying in financial data with the HP-41 takes more time. Data takes a fraction of a second to be recorded and the user must wait until that has taken place before keying in the next number or solving for an answer. Answers also may be a little longer in coming, and a small duck will fly haltingly across the screen replacing the "**running**" display of the HP-12C.

- **THE BEGIN/END OPTION** is selected by pressing **GOLD C.** This is a toggle switch that sets or clears the **BEGIN** option. A small **o** indicator (called a flag) is present in the display when the BEGIN option is selected. **YOU DO NOT WANT THAT FLAG IN THE DISPLAY** unless the BEGIN option is desired. There is no need to set the selected option for each problem: once set, the option remains valid until the other option is selected.

- Of course, with only one shift key, **30 BLUE n 11 BLUE i** now become **30 GOLD n 11 GOLD i** which puts the monthly equivalent of 30 years and 11% interest in **n** and **i**. It is no longer possible to recall the annual values by pressing RCL GOLD n or RCL GOLD i. Instead, press "**RCL n 12 :**" and "**RCL i 12 x**".

- The interaction between financial calculations and the Stack, so convenient in the HP-12C, is no longer present in the HP-41. We may still calculate the payment on a loan, change the interest and calculate the new payment. But we can no longer press **minus** and automatically get the difference between the two payments. The "Recall ahead" procedure no longer applies. Instead specific action must be taken to store previous data and recall it at the appropriate time.

- **n.** The logic of the **n** function is that of the HP-37 and HP-38 series, not of the HP-12C. This makes **n** easier to use than with a 12C calculator, but invalidates the details on **n** and on the 'odd period' presented in UNIT 10 and in APPENDIX II:

  With the "$" program of the HP-41 Real Estate Module, **n** provides a fractional answer if the data so requires instead of rounding up to a whole number as with the 12C. Also, when fractional data is keyed into **n** the fractional amount is interpreted as indicating a lower fractional payment at the end of the transaction, not an extended 'odd period' squeazed in before the first payment. The user should have little difficulty adjusting to this simpler logic, but should also understand that applications such as the one offered in Appendix II, page 7 (Discounted loans when first payments comes in early) are no longer possible.

**HP-41** (Continued)

In **PV - FV** exchanges, where no Payments are involved, keying fractional data into **n** provides the same logic as on the 12C with the 'Compound Interest" option turned on (small **c** indicator in HP-12C display, set or cleared by pressing **STO EEX**. See Appendix II).

- There is no need to clear all the financial memories between financial calculations any more than with the HP-12C provided we keep control over the data in the four data memories as explained in the body of the manual. If we select to clear the 5 memories between financial problems, then we do so by pressing **GOLD E** which has been reassigned the CLEAR FINANCE function (**GOLD CLFIN**).

## SOME ESSENTIAL HP-41 FUNCTIONS

The intention is not to open up the whole power of the HP-41 calculators, just to provide some very essential procedures that differ from those of the HP-12C.

### CLEARING FUNCTIONS

**GOLD CLx/A** [GOLD and key just left of **P**]

Clears display of numbers or characters.

Pressing the same delete key without the GOLD prefix edits a number or string of characters that are being keyed in by deleting the right-most figure or character, a convenience that is not offered on the HP-12C. The whole display is cleared if its content was not in the process of being keyed in.

### MASTER CLEAR

Turning the calculator on while the DELETE key is being pressed down erases all data and all programs from the calculator. The display shows **MEMORY LOST** which disappears as the first key is pressed. That first key is operational, which is not the case with the 12C.

When there are no programs or data in the calculator that need to be retained, the MASTER CLEAR is a convenient way for a beginner to clear unwanted data or programs, flags, indicators and unusual display formats that he does not know how to clear selectively.

## DISPLAY FORMAT WITH THE HP-41

The normal format shows 4 decimals. Loading the "**$**" program switches to the two-decimal "dollars and cents" format. This can also be achieved by pressing:

| | |
|---|---|
| GOLD | |
| 1 | "**FIX**" function selected by pressing "GOLD 1". |
| 2 | Number of decimals selected. |

## STORING and RECALLING

Directly accessible register memories are numbered from 00 to 99 though 99 memories may not necessarily be available for data storage. The storage register capacity differs according to the HP-41 model being used, the presence of additional memory modules, and the claim on memory space made by programs. The number of memories reserved for data storage can be specified by a "SIZE" instruction (see below).

To store 789 in memory 21 and 110 in memory 08 press:

| | | | | | |
|---|---|---|---|---|---|
| 789 | STO | 21 | 110 | STO | 08 |

To recall these numbers press:

| | | |
|---|---|---|
| RCL | 21 | (789) |
| RCL | 08 | (110) |

Note the need for two digits for the address of the memory.

Storage register arithmetic can be performed on all register memories.

Keys A to J (the two top rows) substitute for 01 to 10 for storing and recalling purposes. For instance 110 in memory 8 could be recalled by just pressing **RCL H**. This identification of memories 1 to 10 with the two top rows remains valid in **USER** mode and when the "$" program is loaded.

So with the "$" program activated, the five financial memories are memories 01, 02, 03, 04, and 05. It is possible to store and recall financial data with the **STO** and **RCL** keys using these addresses. It is also possible to use storage register arithmetic with the 5 financial keys. For instance, to incease a negative Payment amount stored in D by $100.00, the following keystrokes are possible: **100 CHS STO + D**, **100 STO - D**, or **100 STO - 04.**

Because the "**$**" program uses Register memories 00 to 12 for its own data purposes, we should not use these memories when also using the "**$**" program. I like to use memories 14, 15, 16 to store **PV**, **PMT**, **FV** respectively when needed, and memory 13 instead of memory 0 in the 12C.

Instead of recalling the data from the 5 memories, we may tell the calculator to list the financial values by pressing **GOLD D**, the **LIST** function. The display then shows the **BEGIN** or **END** option selected, and as we press **R/S** displays in succession the values for **n**, **i**, **PV**, **PMT**, **FV**.

Register memories can be transformed into programming memories, and programs themselves may need Register memories to operate. The "**$**" program uses a minimum of 13 Register memories (memories 00 to 12). To reserve 20 register memories for one's own and the program's register needs, execute the **SIZE** instruction as follows:

| | |
|---|---|
| XEQ | Execute |
| ALPHA | Alpha switch. |
| S | Spell out "SIZE" |
| I | one character at a time. |
| Z | |
| E | |
| ALPHA | Switch out of Alpha mode. |
| 020 | 3 digits required as suggested by prompt. |

We now have only 20 Register memories (0 to 19), and attempting to store data in Register memory 20 (the 21st memory) will result in a **NONEXISTENT** signal. We should then change to a lower Register memory or increase the number of available Register memories with a new SIZE instruction. If there are too many programs for this to be possible, we should clear a program or add a memory module (HP-41C only).

## PERCENTAGES

When loading the "**$**" program,

- **H** is automatically assigned the **%** function.
- **I** is automatically assigned the **%CH** function (percentage change), which is none other than the % function of the HP-12C. Both these functions are used exactly like their HP-12C counterparts.
- There is no **%T** function. A substitute for the regular use of that function is to use the **%CH** function followed by **100 +**.

## AMORTIZATION WITH THE HP-41C

The **AMORT** function (**J** with "**$**" program loaded and the calculator in USER mode) is similar to the equivalent HP-12C function except that prompts are used to select the various options and Register memories are put to a different use.

The initial data is still stored in the usual way in **i**, **PV**, and **PMT**. (negative amount in **PMT**). Pressing **AMORT** (the **J** key) leads the calculator to ask a series of questions to which we respond by a number followed by the **R/S** key. When the prompts have all been answered, pressing **R/S** over and over provides the various answers, each labelled as interest, principal reduction (**PRN**) and balance. At the end, the calculator provides the cumulative amounts for each value.

We retain the flexibility to change the interest and the payment amount in mid-stream. But the value in **PV** is not adjusted as in the HP-12C and cannot just be modified to account for a paydown. Also it is more difficult to amortize for 7 months corresponding to the first partial calendar year, then one calendar year at a time for some years, and finally for the remaining 5 months of a 5-year loan, for instance. We encounter here a frequent occurence with more powerful machines. The extra power is put to good use to free the user of some of the chores--here the prompts makes it easier to remember what data should be provided. However, because the computer takes over, the user loses some of his freedom or at least some flexibility.

The answers, calculated as the program prepares to announce the new period number, are stored in the following memories:

| | | |
|---|---|---|
| Accumulated interest: | memory 06 | (**F**) |
| Total principal reduction: | memory 07 | (**G**) |
| Interest for the period: | memory 08 | (**H**) |
| Increment selected: | memory 09 | (**I**) |
| Balance of the loan: | memory 10 | (**J**) |
| Principal reduction for the period: | memory 11 | |
| Number of periods amortized: | memory 12 | |

We may regain some of the flexibility provided by the 12C by manipulating these amounts directly in those memories. For a $20,000 paydown occuring at the end of the first year, for instance, we would bring the loan up to date to the end of the 12th month and just before the calculator shows **PERIOD 13** we would press **20000 STO − J** and then resume pressing **R/S**. (Of course, the final cumulative principal reduction amount would not take the paydown into account).

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# INDEX

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## THE SEMINAR

This written course course was born of a seminar on the HP-12C that also included irregular cash flow procedures and applications and programming. Following are some comments by participants in the seminar.

I have spent 18+ years evaluating teaching and teaching effective teaching methods. I have attended a myriad of graduate courses from California to Colorado...Edric Cane rates at the very top.

Richard C. Leyva, San Jose, California.

Thank you for such a magnificent course! You present the material in such an efficient, organized manner that in three days you are able to condense what should be a year's course in finance. Your seminar is the only one I know in which the student gets his money's worth, ten times over, at the very first lesson. Your course is the best investment that a practitioner could ever make in his or her career. It is a must for the truly professional.

Maritza Bodine, Sherman Oaks, California.

High-powered fun. Cynthia Burke, Richmond, California.

Whether he knows it or not, anyone who owns an HP-12C needs your course.

Jim Brondino, Claremont, California.

Superbly done! The most stimulating and well thought out seminar I have yet attended. The quality of the information was only exceeded by the raw teaching ability of the instructor.

From anonymous evaluation form, UC Berkeley.

I attended Dr. Cane's three-day course <u>after</u> I had used (the calculator) for 6 months, read three instruction manuals, and sat through a one-day training course by another instructor. Edric Cane's techniques and ease of delivery taught me more the first morning than I had learned up to that time. The remaining 2 1/2 days provided an exciting and rewarding learning experience.

A.J.(Tony) Coco, Education V-P,
Realty Investment Association of Orange County.

This seminar is one of the rare 100% VALUE PROGRAMS I have attended anywhere.

Len Burbine, President, Education Committee
Contra Costa Board of Realtors.

## A WORD FROM THE COMPUTER

As a word processor, I have helped write this course on the use of the HP-12C calculator. Now I would like to have you, it's reader, on my floppy disks. You would then be able to receive more material and information on new material that is being prepared. The care that has been put into writing this manual, in making it clear and easy to understand, could then be continued even after you have received it.

For instance you may be interested in the following, some of which are already being prepared:

- Correction sheet. With so many numbers, even a computer can slip.
- Additional applications in specialized areas.
- Brochure on suggested keystrokes for all practice problems.
- Model problems in card form to carry with calculator.
- Tapes and video-tapes related to this book.
- Further volume on irregular cash flows and programming.
- Information on Continuing Education requirements met by this course for Departments of Real Estate and Boards of Accountancy.
- Information on seminars in your area....or in Hawaii.

In the changing world of finances and calculators, this course is printed in rather small runs. Additions and corrections can be made with each printing. Your suggestions, corrections and comments will be most welcome.

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## OPEN THE BOX. OPEN THE BOOK. AND YOU'RE IN BUSINESS.

### THE BOOK

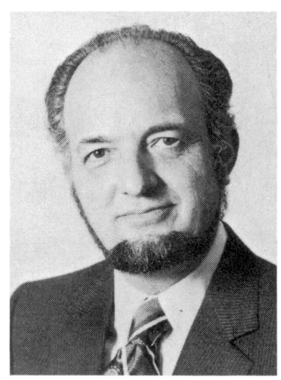
No previous knowledge required with this course to unravel calculator procedures, time-value-of-money concepts, and financial applications. Born of a seminar, it anticipates the questions and concerns of first-time HP-12C users and provides long-time users with a whole range of decision-making and money-making applications and perspectives on financial analysis. A second volume on irregular cash-flows, programming, and statistics is ready to go to press.

This book is for anyone with a serious interest in money at work:

- Real Estate professionals
- Appraisers
- Bankers and Mortgage Brokers
- Business and MBA students
- Investors
- Financial and Estate Planners
- CPAs and Lawyers
- Business executives

"THIS BOOK IS FANTASTIC! YOU ARE A GIFTED TEACHER AND WRITER. THE TEXT IS SO LUCID AND LOGICAL... A TRUE GOLD MINE."
Unsolicited comment, Paul Reilly, Thousand Oaks, California

### THE AUTHOR



Edric Cane was born in France. He has a B.A., an M.A. and a teaching degree from the Sorbonne. He spent two years as a Besse Scholar at Oxford University. After some teaching in France, he came to the United States where he obtained a Ph.D. from the University of Michigan—he wrote his dissertation on a French mathematician of the 18th Century. He has been on the faculty at Oberlin College, Ohio, and Occidental College, Los Angeles.

He then began a new career in Real Estate and is presently a licensed broker and Realtor. His old love of teaching has led him to join his two careers in a series of seminars on the Hewlett-Packard HP-12C financial calculator and its applications to financial analysis. He has given these seminars for UCLA-Extension, UC Berkeley, and to Boards of Realtors, Business Executives and Investors from New York to Hawaii. He lives with his wife, Wyn, and his children, ages 2 to 12, in La Canada Flintridge near Los Angeles.

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