PRACTICAL REAL ESTATE FINANCIAL ANALYSIS:

USING THE HP-12C CALCULATOR

Elbert B. Greynolds, Jr.
Julius S. Aronofsky

A Step-by-Step Approach

PRACTICAL REAL ESTATE FINANCIAL ANALYSIS:
USING THE HP-12C CALCULATOR

Elbert B. Greynolds, Jr.
Julius S. Aronofsky

A Step-by-Step Approach

Real Estate
Education Company
a division of Dearborn Financial Publishing, Inc.

While a great deal of care has been taken to provide accurate and current information, the ideas, suggestions, general principles, and conclusions presented in this book are subject to local, state, and federal laws and regulations, court cases, and any revisions of same. The reader is thus urged to consult legal counsel regarding any points of law—this publication should not be used as a substitute for competent legal advice.

Published by Real Estate Education Company/Chicago
a subsidiary company of Dearborn Financial Publishing, Inc.

10 9 8 7 6 5 4

Library of Congress Cataloging in Publication Data

Greynolds, Elbert B.
Practical real estate financial analysis.

Bibliography: p.
Includes index.
1. Business mathematics—Real estate business.
2. HP-12C (Calculating-machine) I. Aronofsky, Julius S.
II. Title.
HF5716.R4G73 1983 332.7'2'02854 83-563
ISBN 0-88462-378-5

Cover photo of HP-12C calculator courtesy of Hewlett-Packard Company, Corvallis, Oregon. Used with permission.

table of contents

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

hp-12c program index

preface

This book originated out of our activities in developing materials on the use of hand-held calculators by real estate executives. More specifically, over a period of four years, we have been involved in presenting a short course on real estate financial analysis for the Costa Institute of Real Estate, Edwin L. Cox School of Business, Southern Methodist University. For several years we have been interested in organizing the formulas and methods for solution in financial analysis into a form that would be directly usable on hand-held calculators. This effort resulted in publishing the book Financial Analysis Using Calculators: Time Value of Money. In this prior work, we attempted to reorganize and integrate the traditional topics into a form that is suitable for use by practitioners in modern financial applications. In this early volume, we also attempted to develop material that would be applicable on all types of hand-held calculators, from the simplest manual calculators to the most advanced programmable units. Such material has its proper place but we have something much more specific in mind for the current volume, which is described below.

What we hoped to develop for the Costa Institute was a very pragmatic short course that would spell out how to solve problems in real estate financial analysis by use of hand-held calculators. In order to present in a clear, concise manner how to solve real estate financial problems on a calculator, we organized the course around one and only one popular financial calculator, the Hewlett Packard HP-12C. The selection of such a calculator with its enriched financial function keys enabled us to eliminate the use of tables and extend the methods to problem areas that are usually considered too involved or complex using traditional methods and tables. As this course became popular with real estate executives, appraisers, consultants, bankers, accountants, lawyers, and students, we had repeated requests to develop material that could be used in the short course and for future reference.

This is the book that emerged and its description is given in Chapter 1. Typically, in a short course, we will work through essentially all of the material in this book in 16 to 18 classroom hours. We do believe that the same material can be covered on a self-study basis by individuals who have a background in traditional real estate financial analysis but no prior exposure to financial hand-held calculators.

We would like to express our gratitude to the late Professor Robert O. Harvey, Chairman of the Real Estate Activity and Director of the Costa Institute at the Edwin L. Cox School of Business. We shall miss him as a friend and colleague who was always encouraging and enthusiastic about this work to which he made a contribution.

We also would like to express our appreciation to Ms. Jean Long, Assistant Director of the Costa Institute, for her cooperation in organizing the short courses. We owe thanks to the Word Processing Department -- Mary Kesner, Edith Benham, Bonnie Campion, Madelon Gafford, Wanda Hanson, Betty Johnson, Patricia Shield, Nancy Thomas and Bess Vick -- who diligently worked on the manuscript through all the phases of its production.

Elbert B. Greynolds, Jr.
Julius S. Aronofsky

1

AN INTRODUCTION

This book deals with financial applications that will be useful to real estate professionals and real estate students. The emphasis is on the use of hand-held calculators as a tool to analyze the many financial problem situations that realtors encounter in the conduct of their practice. More specifically, this book deals with the use of a special class of hand-held calculators in real estate analysis: namely, financial calculators.

In recent years, changing market conditions and high interest rates have forced many changes on individuals who perform financial analysis. New types of mortgages and other "creative financing" arrangements require analysis that is either difficult or impossible with traditional tables. As a result, real estate professionals have found that financial calculators are devices uniquely designed to satisfy current financial analysis requirements. Examples of such problem situations are analysis of investments, regular mortgages, graduated payment mortgages, wrap-around mortgages, variable payment leases, and other financial arrangements where time value of money plays a critical role. Although several financial calculators are available in the marketplace, a particular financial calculator was selected for use in this book: the Hewlett Packard HP-12C.

PURPOSE OF THIS BOOK

The purpose of this book is to:

1. Present self-study material on how to use the HP-12C calculator to solve commonly occurring real estate problems in financial analysis.

2. Demonstrate how the fundamental concepts in time value of money can be used to extend the use of the HP-12C calculator to solve more advanced real estate problems.

3. Provide sufficient drill and exercise material so that this text can be used to help study for certification examinations.

4. Introduce a unique template-format approach that will enable a busy manager to solve advanced problems essentially by "filling in the blanks" in the template. The manager can also prepare a template and give it to a subordinate for solution. The template serves as a "hard copy" record of the analysis for later reference.

5. Aid managers who do not perform financial analysis on a day-to-day basis. All applications in this book are self-contained, so the manager fills in the information required by the template and solves the problem. For problems without templates, the user is shown how to develop the appropriate input values using either time-line diagrams or "canned" programs.

6. Show the manager how to enter and execute a canned (pre-written) program. A number of useful programs are included in the book. These programs include amortization schedules for regular, graduated payment, and constant principal payment mortgages, construction loans, graduated payment loan schedules, and financial management rate of return as well as other programs.

READING THE BOOK

The following chapters are based on discussions with attendees of the short course described in the Preface. In developing the format for the following chapters, considerable effort was given to a clear and concise presentation of the more complex problems that are emerging due to changing money markets and the decline in availability of fixed rates and/or fixed-term mortgages.

No attempt is made to present a complete story on the HP-12C calculator, nor on the fundamentals of time value of money. However, sufficient material has been included so that the following chapters are reasonably self-contained, assuming the reader has ready access to the HP-12C Owner's Handbook and Problem-Solving Guide, (2),* and will refer to it occasionally. For additional materials on the concepts of time value of money the reader is referred to the book by Greynolds, Aronofsky, and Frame (1).**

Based on our short course experience, we recommend two approaches for reading this book -- one for the reader who is well-grounded in time value of money theory and another for the reader who intends to use the book for a self-study program.

For the reader well-grounded in time value of money, the recommended sequence is:

1. Read Chapter 2 which covers the function keys and keystroke procedures necessary for performing the analysis techniques in the book.

2. Read Chapter 3 to become familiar with the terminology and financial function keys.

3. Read pages 163 and 164 of Chapter 8 to learn how canned programs are entered and executed with the HP-12C. Then perform the monthly mortgage amortization schedule application on page 165.

4. Go to the application of interest in the book.

For the reader intending to use the book for a self-study program, the recommended sequence is:

1. Read Chapter 2 and work all examples in the chapter. This provides practice with the function keys and keystroke procedures used for analysis in this book.

2. Read the sections of Chapter 3 entitled "The Equivalent Value of Money" and "Compound Interest." This provides the basic theory for the applications in this book.

3. Read the section of Chapter 3 entitled "Unequal Multiple Cash Flows." This explains present values and interest rates (IRR) and how they are calculated on the HP-12C.

---

*See Reference (2) in the Bibliography.

**See Reference (1) in the Bibliography.

4. Work the following applications to reinforce the material on unequal multiple cash flows.

   Chapter 7 a) Solving for the Price (Present Value of a Graduated Payment Mortgage)

   b) Solving for the Yield of a Graduated Payment Mortgage

   Chapter 5 a) Solving for the Present Value of a Lease with Variable or Grouped Payments

   b) Solving for the Interest Rate of a Lease with Variable or Grouped Payments

5. Read the sections of Chapter 3 entitled "Equal Multiple Cash Flows" through "Ordinary Annuity." This explains how to use the Financial or Annuity keys and ordinary annuities.

6. Work the following applications to reinforce the material on ordinary annuities.

   Chapter 4 a) Regular Mortgages
   Amount of Mortgage Payment
   APR of Regular Mortgage
   Price of a Regular Mortgage - Scheduled Payments Assumed
   Remaining Balance of a Regular Mortgage

   b) Balloon Mortgages
   Amount of Payment
   APR for Balloon Mortgages
   Price of Mortgage with Balloon

7. Read the section of Chapter 3 entitled "Annuity Due."

8. Work the following applications to reinforce the material on annuities due.

   Chapter 5 Ordinary Leases
   Solve for Lease Payment
   Solve for Yield on Lease
   Solve for Present Value of Lease

9. Read pages 163 and 164 of Chapter 8 to learn how canned programs are entered and executed with the HP-12C.

10. Work the following applications to reinforce the material on canned programs.

    Chapter 8 a) Monthly Mortgage Amortization Schedule
    b) Annual Mortgage Amortization Schedule
    c) Amortization of Constant Principal Payment Loans

11. Read Chapter 9, enter the Financial Management Rate of Return program and solve the example.

12. Read Chapter 10 and then solve the two wrap-around mortgage examples in Chapter 7.

After following the above schedule, the reader should be able to solve all applications in this book.

ORGANIZATION OF THE BOOK

Chapter 2 presents the KEYSTROKE FUNDAMENTALS for the HP-12C. The point is made in this chapter that careful study of the keystroke instructions is a prerequisite for applying the calculator to the applications outlined in subsequent chapters. No attempt is made to explain all the function keys on the HP-12C, but the function keys and procedures necessary for solving the applications in this book are explained.

Chapter 3 introduces some fundamental concepts on the TIME VALUE OF MONEY. It is strongly recommended that you review the material in this chapter on how to use the grouped cash-flow keys on the HP-12C to solve a broad class of problems. Also a distinction is made in this chapter as to when to use the so-called financial keys, or the annuity keys. Both the "grouped cash-flow keys" and the "financial keys" will be used in subsequent chapters with an indication when one set of keys is applicable or even preferable to the other.

Chapter 4 covers in some detail how to make calculations related to REGULAR MORTGAGES and BALLOON MORTGAGES. Most of the commonly-occurring problems for regular ordinary mortgages are presented here, almost in the form of a catalog, along with a reusable template for each problem situation. The "blank" template is reusable in the sense that the user first has to identify the correct template for a particular problem situation and then simply fill in the "blanks" with specific data in the order requested.

Chapter 5 deals with LEASES. The key difference between the material in Chapters 4 and Chapter 5 is that for ordinary mortgages the payments occur at the end of each payment period whereas for leases the payments occur at the beginning of each period. In many ways Chapter 5 is a companion to Chapter 4 and builds on the material already learned in the prior chapter.

Chapter 6 covers the three commonly used methods for DEPRECIATION -- straight-line, sum-of-years'-digits, and declining balance. Such material on depreciation is introduced here to set the stage for solving more advanced problems on an after-tax basis. Again the same blank-template approach is used to present the keystroke sequences.

Chapter 7 introduces you to a group of SPECIALIZED MORTGAGES. A manual keystroke template and a program are provided for computing the payment schedule for a graduated payment mortgage. Templates are also provided for determining the price and yield of a graduated payment mortgage. Deferred mortgages are treated in depth with templates provided for computing the payment, price, yield or balloon. Two wrap-around mortgage procedures are discussed. A simple wrap-around procedure and template are provided where both mortgages have equal terms. A second procedure for a complex wrap-around allows any combination of mortgage terms and payments. Two procedures with templates and programs are provided for determining the yield on construction loans. The interest on the loan can be included or omitted from the periodic draw.

Chapter 8 deals with AMORTIZATION of regular mortgages, variable payment mortgages, graduated payment mortgages, constant principal payment mortgages, and deferred mortgages. For each mortgage type, a manual template and/or program are provided. The procedures are designed for calculating a monthly or annual schedule. The explanation of how a canned HP-12C program is entered and executed is explained in this chapter.

Chapter 9 explains the concept and solution procedure involving VARIABLE CASH FLOWS for net present value and internal rate of return. The problem of multiple internal rates of return that can occur when multiple negative cash flows are present in the cash-flow stream is discussed, along with the calculator procedure necessary to find the rates. One alternative to IRR, Financial Management Rate of Return, is discussed and a program provided for computing the value.

Chapter 10, EVALUATING PROJECT USING CASH FLOWS, shows you how to combine the techniques contained in the previous chapters for evaluating projects using cash flows. Because such projects often differ, several examples are employed to demonstrate the underlying solution procedures. The chapter also shows you how to calculate annual effective interest rates and how to convert interest rates.

2

KEYSTROKE FUNDAMENTALS

The keystroke techniques needed to master the HP-12C for this book are introduced in this chapter. Careful study of the keystroke instructions is prerequisite to applying the applications outlined in subsequent chapters.

The limited keystrokes described here may be found, also, in the HP-12C Owner's Handbook and Problem-Solving Guide. For an in-depth discussion of the features of the HP-12C, refer to the Owner's Handbook.

A number of programs -- sequences of calculations placed in the calculator that can be activated as a unit -- and their applications are described in subsequent chapters. Instructions for programming are not included. For programming instructions, refer to the Owner's Handbook.

ARITHMETIC

HP calculators use a logic system for arithmetic called Reverse Polish Notation (RPN). For details of RPN, refer to the manual, Your HP Financial Calculator, which accompanied the user manual. Numbers are entered first, then an arithmetic operation (+, -, x, ÷) is performed. Adding two numbers, for example, requires entering the numbers and then pressing the [+] key.

The basic procedure is:

1. Enter the first number, then separate it from values entered subsequently by pressing [ENTER].
2. Enter a second number and press the appropriate symbol, +, -, ÷, or X, to perform a calculation.
3. To use the displayed result in a calculation, key in another number and press the arithmetic operation (+, -, ÷, or X).

Complete the following arithmetic calculations:

EXAMPLE 1:

1. 2 + 5 = ?
2. 7 - 5 = ?
3. 6 X 4 = ?
4. 20 ÷ 5 + 8 = ?

Keystroke Solution:

Procedure	Keystrokes	Display
1. a. enter 2	2, [ENTER]	2.00
b. key in 5	5	5.
c. add	+	7.00

Keystroke Solution: (continued)

Procedure		Keystrokes	Display
2.	a. enter 7	7, [ENTER]	7.00
	b. key in 5	5	5.
	c. subtract	-	2.00
3.	a. enter 6	6, [ENTER]	6.00
	b. key in 4	4	4.
	c. multiply	X	24.00
4.	a. enter 20	20, [ENTER]	20.00
	b. key in 5	5	5.
	c. divide	÷	4.00
	d. key in 8	8	8.00
	e. add	+	12.00

Always anchor the first number by pushing the [ENTER] key immediately after entering it.

Sometimes it is appropriate to use a negative number. Enter a negative number in two steps: 1) key in the number, and 2) press the [CHS] key. The [CHS] key changes the sign of the number shown in the display. Pressing [CHS] changes a positive (+) display number to a negative (-) number, and changes a negative number to positive.

Example 2 shows a series of arithmetic operations that use the same steps as in Example 1, but only the first number is followed by [ENTER].

EXAMPLE 2:

$$(-6 + 5) \times 3 \div (-1) = ?$$

Keystroke Solution:

Procedure	Keystrokes	Display
1. key in 6	6	6.
2. change sign	CHS	- 6.
3. Enter	[ENTER]	- 6.00
4. key in 5	5	5.
5. add (-6 + 5)	+	- 1.00
6. key in 3	3	3.
7. multiply (-6 + 5) by 3	X	- 3.00
8. key in 1	1	1.
9. change sign	CHS	- 1.
10. divide (-6 + 5) X 3 by -1	÷	3.00

The [CHS] key is pressed before pressing [ENTER] or an arithmetic operator (+, -, X, ÷) as indicated in steps 2 and 9.

Example 3, below, illustrates the difference between using a negative number and completing a subtraction operation, and also demonstrates the use of the decimal point key [.].

EXAMPLE 3:

(-7.5 - 5 + 2.5) ÷ 4 X 6 = ?

Keystroke Solution:

Procedure	Keystrokes	Display
1. key in 7	7	7.
2. key in decimal point	[.]	7.
3. key in 5	5	7.5
4. change sign	CHS	- 7.5
5. enter	[ENTER]	- 7.50
6. key in 5	5	5.
7. subtract 5 from - 7.5	-	- 12.50
8. key in 2	2	2.
9. key in decimal point	[.]	2.
10. key in 5	5	2.5
11. add 2.5 to -12.5	+	- 10.00
12. key in 4	4	4.
13. divide -10 by 4	÷	- 2.50
14. key in 6	6	6.
15. multiply -2.5 by 6	X	- 15.00

The calculator always shows a decimal when a number is entered. This decimal point is assumed to be to the right of the last number entered unless the decimal point key is pressed [.] as in steps 2 and 9 above. Pressing [.] fixes the decimal point location for the number in the display. The (-) in front of 7.5 means that 7.5 is negative and obtained by pressing [CHS], but the (-) preceding the 5 means subtract 5 so the [-] key is pressed.

MEMORY (STORAGE REGISTERS) OPERATIONS

Using Stored Values. The HP-12C calculator has the capacity for storing up to 20 individual items. Numeric values are stored and recalled using the [RCL] and [STO] keys. Each storage register has an address or locating number. For the first ten registers the addresses are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The second ten registers have a decimal point in front of the address number and the addresses are .0, .1, .2, .3, .4, .5, .6, .7, .8, and .9.

Store information by keying in the number and pressing [STO] and the desired address, recall numbers by pressing [RCL] and the storage register address. For example, to store 7 in register 2 and 8 in register .3 press:

7 STO 2

8 STO .3

To recall these values press:

RCL 2

RCL .3

When [STO] is pressed followed by a memory address, any previous value in the register selected is replaced by the new display value.

Arithmetic operations previously discussed may be performed by using values stored in memory as shown in Example 4.

EXAMPLE 4:

Store -5 in register 1, and 10 in register 2 and perform the following arithmetic operations.

1. -5 + 10 = ?
2. 10 ÷ (-5) = ?
3. -5 X 10 = ?

Keystroke Solution:

Procedure	Keystrokes	Display
1. key in -5	5 CHS	- 5.
2. store in register 1	STO 1	- 5.00
3. key in 10	10	10.
4. store in register 2	STO 2	10.00
5. add -5 + 10		
a. recall -5	RCL 1	- 5.00
b. recall 10	RCL 2	10.00
c. add	+	5.00
6. divide 10 by -5		
a. recall 10	RCL 2	10.00
b. recall -5	RCL 1	- 5.00
c. divide	÷	- 2.00
7. multiply -5 by 10		
a. recall -5	RCL 1	- 5.00
b. recall 10	RCL 2	10.00
c. multiply	X	- 50.00

A value stored can be recalled any number of times for use in calculations. Do not press [ENTER] after recalling an entry because the [RCL] command does it for you. A value remains in memory until changed or cleared. The clearing keys and strokes are discussed in the next section.

Performing Operations Without Using RCL. Arithmetic operations can be performed on the contents of storage registers 0, 1, 2, 3 or 4 without recalling the values stored in them. Let # REG stand for any one of the registers 0 through 4; to complete an arithmetic calculation with a number in the display and a stored value, simply enter a number into display and then press:

STO [+] # Reg to add

STO [-] # Reg to subtract

STO [X] # Reg to multiply

STO [÷] # Reg to divide

The sequence STO [+] # Reg adds the display value to the value stored in # Reg.

The sequence STO [-] # Reg subtracts the display value from the value stored in # Reg.

The sequence STO [X] # Reg multiplies the contents of # Reg by the display value.

The sequence STO [÷] # Reg divides the contents of # Reg by the display value.

The display value is not changed by the keystrokes described above. To see the contents of a register after a calculation, press the [RCL] key and the register number.

Arithmetic operations using a memory register can be inserted in the middle of a calculation without affecting the current calculation.

EXAMPLE 5:

Store 5 in register 3 and while calculating 7 X 6 = ? Add the 6 to the contents of register 3. Recall register 3 (RCL, 3); its value should now be 11.

Keystroke Solution:

Procedure	Keystrokes	Display
1. store 5 in register 3	5 STO 3	5.00
2. key in 7	7	7.
3. enter	[ENTER]	7.00
4. key in 6	6	6.
5. add the 6 to the contents of register 3	STO + 3	6.00
6. multiply 7 X 6	X	42.00
7. Recall the contents of register 3	RCL 3	11.00

The STO [+] # REG sequence is frequently used for summing a series of values during calculations. Remember to store the first value in the series or to store a zero in the register before starting the summing operation; otherwise, the process may start with an undesired value in the register.

SPECIAL KEYS

Keys with special functions frequently used in financial calculations are described below.

The key with the white [1/x] on its face is used to divide one by the display value. Above the [1/x] key is [YTM] in gold; it is used to calculate the yield to maturity for a bond. The [1/x] key becomes the [YTM] key whenever the gold key, [f], is pressed before pressing [1/x] and the calculator calculates the yield to maturity. The gold keys on the

HP-12C are used for internal programs. We will show you examples of these keys in the applications.

The blue key [g] activates the functions shown in blue on the front of certain keys. The front of the [n] key has [12X] in blue and the [i] key, [12÷] in blue. When [g], [12X] are pressed, a display value is multiplied by 12 and entered as [n] for annuity or compound interest calculations. The [12X] key is used most often to convert years to months for mortgage calculations. The [g], [12÷] key sequence divides the contents of a display value by 12 and enters it as the monthly component of an annual interest rate. These key sequences are employed in mortgage calculations and are labeled g, [12X] and g, [12÷] in subsequent keystroke instructions.

Other special keys are [%], [y^x] and [X≷Y].

The [%] key is used for percent calculations. The format is:

1. Enter value on which percent calculation is based.
2. Enter the percent to be calculated.
3. Press [%] to compute the percentage amount.

4a. Press [+] to add value in step 3 to amount in step 1.
4b. Press [-] to subtract value in step 3 from amount in step 1.

To compute 5% of 100 and add it to 100 requires the following keystrokes:

100 [ENTER] 5 [%], [+] answer = 105

To compute 10% of 200 and subtract it from 200 requires:

200 [ENTER] 10 [%], [-] answer = 180

The [y^x] key is used to raise a number to a power. "y" is the number to be raised and "x" is the exponent or power. The sequence is:

1. Enter the y value.
2. Enter the x value.
3. Press [y^x].

For example, raise two to the third power, (2^3). The keystrokes are:

2 [ENTER] 3, [y^x] answer = 8

Squaring 5 to obtain 25 requires:

5 [ENTER] 2, [y^x] answer = 25

The [X≷Y] is the symbol used in this book for the key above the [STO] key. An exact replica of the key symbol is not available. Pressing [X≷Y] exchanges a display value (x) with a value stored in the register y. The [X≷Y] key is used following the calculation of items such as mortgage amortization and days between dates. If an amortization schedule has been developed, pressing [X≷Y] will display the principal component of a payment.

Additional information on all keys on the HP-12C may be found in the Owner's Manual.

DISPLAY CONTROL

When the HP-12C is turned on, only two decimal places are shown in the display. The number of decimal places displayed may be changed by pressing f, and a digit 0 through 9. Regardless of the display setting, the calculator uses all nine decimal places in calculations. Setting the display to, say, three decimal places has no effect on the values calculated or stored in memory.

The calculator rounds displayed values using the "5/4" rule. When two decimal places are specified, a third value, not shown, if less than five does not alter the second decimal place, but if the third decimal value is 5 or greater the second decimal place value is increased by one. The rounding rule is illustrated in Example 6.

EXAMPLE 6:

Enter the number 1.548298339; observe the display when decimal settings of 0, 4, 6, 9, and 2 are employed.

Keystroke Solution:

Procedure	Keystroke	Display
1. turn calculator on		0.00
2. key in	1.548298339	1.548298339
3. enter	[ENTER]	1.55
4. zero digit display	f, 0	2.00
5. four digit display	f, 4	1.5483
6. six digit display	f, 6	1.548298
7. nine digit display	f, 9	1.548298339
8. two digit display	f, 2	1.55

Remember, rounding changes only displayed numbers; calculations within the instrument use all known digits to the right of the decimal point.

CLEARING THE CALCULATOR

The HP-12C has five clearing functions. Four are for manual calculations and one is for programs, where a program is a sequence of calculations placed in the calculator that can be activated as a unit. These functions are:

[CLX] -- This key clears the display register and does not affect any calculations in progress.

f, [FIN] -- clears the n, i, PV, PMT and FV registers when pressed but does not clear the display or other storage registers.

f, [Σ] -- clears the storage register 0, 1, 2, 3, 4, 5, and 6. Other storage registers, financial registers and the display are not cleared. The [Σ] key is normally used with statistical applications in which values are automatically calculated and stored in registers 0 through 6.

f, [REG] -- clears the display, financial registers and storage registers, but does not erase a program.

f, [PRGM] -- is used to erase programs that have been stored. If the calculator is set in the mode to recieve a program, pressing f, [PRGM] erases the program. In the regular keystroke mode (non-programming mode), pressing f, [PRGM] prepares the calculator to execute the program from its beginning. See the amortization program in Chapter 8 for more details.

ENTERING FRACTIONAL INTEREST RATES

A minor but cumbersome procedure is required to enter an interest rate that has a fraction. Entering 12 7/8% requires the following steps.

1. Key in and enter the numerator of the fraction (7).
2. Key in the denominator (8) and divide.
3. Add the integer value (12).

For example:

		Display
7	[ENTER]	7.00
8	÷	0.88
12	+	12.88

Reducing the fraction to decimals in the calculator will make available all digits to the right of the decimal point, displayed or not, when entering an interest rate using the key [i].

ENTERING YEARS AND MONTHS

Mortgage and lease terms may be expressed in years and months: 5 years, 3 months. But if the compounding period is monthly, the input for [n] must be in months. To convert years and months to months enter the number of years, multiply by 12, then add the months.

For example:

		Display
5	[ENTER]	5
12	X	60
3	+	63

3

TIME VALUE OF MONEY THEORY

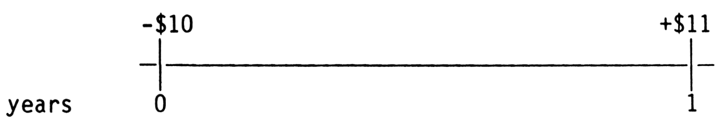
The HP-12C allows you to discard financial tables and to perform real estate analyses that are ordinarily feasible only on a computer. Using the concept of the time value of money will enable you to frame an approach to a problem and to determine an answer. This chapter reviews the basic concepts of the time value of money. The approach is non-mathematical; the focus is on concepts and not equations.

Many applications in this book can be solved omitting this chapter's material, but the advanced applications in Chapters 7 and 10 require a careful study of this chapter.

THE EQUIVALENT VALUE OF MONEY

"A dollar today is worth more than a dollar a year from now!" A dollar available today can be invested for a year and become worth more than one dollar at the end of the year. For example, \$10 invested for one year at 10 percent earns \$1 of interest by the end of the year. So, \$10 invested at 10 percent will grow to \$11 one year later.

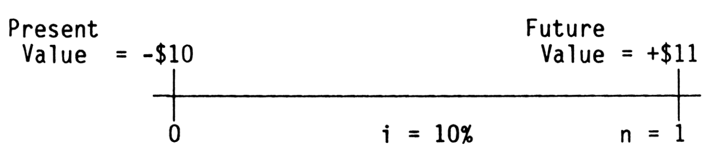
The Time Diagram, below, depicts the investment and the interest process.



Time Diagram 3.1

The 0 indicates the point in time when money was invested and the 1 indicates the end of one year. The -\$10 is shown as a negative value since the HP-12C treats money paid out as a negative number. The +\$11, on the other hand, represents the amount available at the end of the year and is a positive value.

The next step is to label the time values as shown in Time Diagram 3.2



Time Diagram 3.2

The present value, called PV, is the amount of money invested at the beginning of year one or at time period zero. The future value, labeled FV, is the amount of money accumulated at the end of year one. The difference between PV, \$10, and FV, \$11, is the interest earned on the investment. The annual interest rate is 10 percent represented by the symbol "i." The symbol "n" represents the number of time periods for compounding interest.

A PV of $10 is equivalent to a FV of $11 assuming an annual interest rate (i) of 10 percent. As a result, if money can be invested at 10 percent, an investor would be indifferent between the worth of $10 at time period 0 or $11 at time period 1.

To restate, the future value (FV) is the equivalent value after n time periods of the present value (PV) invested at i percent. The present value (PV) is the equivalent value that must be invested at time period zero for n periods at i percent in order to have a specified future value (FV).

When computing FV, the term compounding is used to indicate that interest is being computed on interest previously earned. An investment of $10 at 10 percent for one year will become $11 and interest will be earned on the $11 in year two.

If the future value and the interest rate are known, the term discounting is used when determining the present value. The future value of $11 is discounted one time period at 10 percent to determine the present value of $10. In both cases the value used for compounding and for discounting is the interest rate of 10 percent. The term compounding is employed when computing the future value of an investment, and discounting when determining the present value of an investment.

If $10 is invested at 10 percent, what is the investment amount at the end of one year? Time Diagram 3.3 shows, once again, that

```
PV = -$10                                FV = ? = $11
   |-------------------------------------------|
   0                 i = 10%                 n = 1
```

Time Diagram 3.3

the FV value is $11. The FV is often called the value of the reversion: the sum that will revert to the investor at the end of one year.

An investment opportunity will produce $11 at the end of one year. How much should be invested to earn 10 percent? Time Diagram 3.4 shows that the unknown value, PV, is $10.

```
PV = ? = $10                                  $11
   |-------------------------------------------|
   0                 i = 10%                 n = 1
```

Time Diagram 3.4

Depending on the problem situation, the PV of $10 may be called the price of the investment, the market value, or the cost of the investment.

An investment opportunity requiring a $10 outlay returns $11 at the end of one year. What interest rate will be earned on the investment? Time Diagram 3.5 shows the unknown value.

```
PV = -$10                                   FV = $11
   |-------------------------------------------|
   0                  i = ?                  n = 1
```

Time Diagram 3.5

It can be determined that an interest rate of 10 percent will either compound the PV of \$10 forward to the FV of \$11, or discount the FV of \$11 back to a PV of \$10. The interest rate of 10 percent is variously called the <u>internal rate of return</u>, the <u>yield</u>, or the <u>return</u>.

SUMMARY: This section has shown the relationship between FV and PV when the interest rate (i) is known. Time diagrams and other key items in solving time value of money problems have been introduced.

COMPOUND INTEREST

In this section, compounding over several periods is explained. Suppose \$10 is invested for 3 years at a 10 percent annual interest rate. The situation is shown in Time Diagram 3.6.

```
PV = -$10                                  FV = ?
   |--------------|--------------|--------------|
   0              1              2          n = 3
                      i = 10%
```

Time Diagram 3.6

The original investment, the PV, is -\$10, the i is 10 percent and the FV is the unknown, and there are three time periods. Compound interest means that interest previously earned is added to the investment and interest is earned both on the original investment and accumulated interest.

Compound interest calculations are based on several assumptions. First, interest is calculated at the end of each compounding period. The compounding period is a standard period of time such as a day, month, quarter or year. Second, the interest rate used for calculations is the effective interest rate <u>per compounding period</u>. Finally, the symbol n represents the number of <u>compounding periods</u>. In this example, years and compounding periods are the same because interest is compounded at the end of each year.

<u>Manual Calculations of FV and PV</u>. With an effective interest rate of 10 percent per year and a compounding period equal to one year, a future value is derived as follows:

Amount invested	\$10.00
Interest earned at the end of year one \$10 x .1 =	1.00
Value at end of year 1	\$11.00
Interest earned at the end of year two \$11 x .1 =	1.10
Value at end of year 2	12.10
Interest earned at the end of year three 12.1 x .1 =	1.21
Future value at end of year three	\$13.31

Ten dollars invested at 10 percent annual rate becomes \$13.31 after three years. That is, an investment amounting to \$13.31 at the end of three years is produced by an original investment of \$10 earning 10 percent annually, or, an investor paying \$10 for such an investment would earn 10 percent compounded annually.

Interest is usually compounded in periods of less than one year even though rates are quoted in annual terms. The effective interest rate per compounding period in the United States is normally determined by dividing the stated annual rate by the number of compounding periods per year. Accordingly, the compounding period is one year for the preceding example with an effective interest rate per compounding period of 10 percent. Furthermore, the interest is "discrete" which means the interest is calculated at a specified point in time -- at the end of each yearly compounding period.

Now change the example by assuming interest is compounded at the end of each six months (semi-annually). The $10 is still invested for three years at a 10 percent annual rate, but the number of compounding periods is six instead of three, as shown in Time Diagram 3.7.

i = 10%/2 = 5%

Time Diagram 3.7

The effective interest rate per compounding period is 5 percent because the annual rate of 10 percent is divided by the number of compounding periods per year (2). Notice that while the number of compounding periods has doubled to six, the time period for the investment has remained constant at three years.

The future value can be computed as follows:

Amount invested	$10.00
Interest at the end of first six months $10 x .05 =	.50
Value at end of first six months	10.50
Interest at the end of second six-month period $10.50 x .05 =	.53
Value at end of the first year	11.03
Interest at end of third six-month period	.55
Value at end of third six-month period	11.58
Interest at end of fourth six-month period	.58
Value at end of second year	12.16
Interest at end of fifth six-month period	.61
Value at end of fifth six-month period	12.76
Interest at end of sixth six-month period	.64
Value at the end of the third year	$13.40

The future value at the end of three years is $13.40 with semi-annual compounding as compared to the future value of $13.31 with annual compounding. The increase of $0.09 occurred because the number of compounding periods per year increased.

Equivalent value is an important concept to understand in conjunction with compound interest. Compound interest means determining the equivalent future value of the present value or vice versa while holding the interest rate constant. For example, $10 invested at 10 percent annual compounded semi-annually is equivalent to:

$10.50 at the end of 6 months, or

$11.03 at the end of 12 months, or

$11.58 at the end of 18 months, or

$12.16 at the end of 24 months, or

$12.76 at the end of 30 months, or

$13.40 at the end of 36 months.

The number of compounding periods determines the specific future value. So if the investment were for two years, the future value would be $12.16.

If $11.03 is invested at the end of year one rather than $10 at time period zero, the future value at the end of year three is still $13.40.

Furthermore, the future value of $13.40 at the end of year three has a present value at the end of year two of $12.16, or at the end of year one of $11.03, or at the beginning of year one (time period zero) of $10.

In summary, the future value is the equivalent of a series of payments compounded forward. The present value is the equivalent worth of a series of discounted cash flows. Thus, the principal sum of a mortgage, the price of leased property, or value of a bond, is actually the present value of a future cash flow or flows for a given interest rate. The yield or internal rate of return is the interest rate that makes the present value of future flows equal to an initial investment.

A simple but cumbersome method was used to calculate future values in the preceding example. The HP-12C calculator, however, computes present and future values directly and simply.

Calculator Solution Using "Financial" Keys. Your HP-12C has a row of keys we call the "Financial" or "Annuity" keys. These keys are [n], [i], [PV], [PMT], and [FV]. These keys are used to solve compound interest problems and annuities (explained later).

First, the HP-12C logic requires that all cash flows be positive or negative. We will normally show cash outflows as negative (minus) values and cash inflows as positive (plus) values. For compound interest, the keys* are defined as:

n = The number of compounding periods.

i = The effective interest rate per compounding period. This value is entered as a percent and is computed by dividing the annual interest rate by the number of compounding periods per year.

PV = The equivalent value of the discounted future cash flow (FV).

FV = The equivalent value of the compounded cash flow (PV).

PMT = A zero value for a single payment, or compound interest.

Either PV or FV must be entered as a negative number when computing n or i; otherwise you will see "Error 5" in the calculator display when you solve for an unknown value. So enter a positive value if you receive the cash and a negative value if you pay out the cash.

Solving for an unknown value is easy because you enter three of the four values (n, i, PV, FV) and solve for the unknown by pressing its key.

---

*The five keys [n], [i], [PV], [FV], and [PMT] will be referred to as the financial keys or as the annuity keys (see the owner's handbook). Likewise, the five registers associated with these five keys will be referred to as the financial registers or sometimes as the annuity registers.

To demonstrate, let's solve two of the previous examples using the HP-12C.

EXAMPLE 1:

Compute the future value of $10 invested for one year at 10 percent compounded annually.

Solution:

First identify the known values, which are: n = 1 because there is one compounding period; i = 10, because the 10 percent interest is compounded annually; and PV = -10, because we invest $10. Remember, clear the calculator by selecting the option [f], [REG], or [f], [FIN], or with the [PMT] register by entering zero. This last option is used in the keystroke solution below.

Keystroke Solution:

Procedure	Action Step	Display
1. Clear the PMT register	0, PMT	0.00
2. Enter the number of compounding periods	1, n	1.00
3. Enter the effective interest rate per compounding period	10, i	10.00
4. Enter the cash outlay at time period zero as a negative number	10, CHS, PV	- 10.00
5. Compute the equivalent value at the end of year one	FV	11.00

Notice that after entering the known values, we found the unknown future value by simply pressing [FV]. Also, because the present value is entered as a negative number, the future value is shown as a positive number. Remember you can solve for any unknown if three other values are entered. Now solve the second example.

EXAMPLE 2:

Assuming an initial investment of $10 with annual interest of 10 percent compounded semiannually, what is the future value or amount you will receive at the end of three years?

Solution:

Because interest is not compounded annually, we cannot use the annual rate of 10 percent for [i] and the number of years for [n]. We must compute the number of compounding periods for [n] and the effective interest rate per compounding period for [i]. First, determine n by multiplying the number of years by the number of compounding periods per year -- so, n = 6 (3 years x 2 compounding periods per year). Second, determine the effective interest rate per compounding period by dividing the number of compounding periods per year into the annual interest rate and get 5 percent (10% ÷ 2 compounding periods per year). Finally, [PV] is -10 and [PMT] is 0. We compute the input values for [n] and [i] to avoid rounding errors.

In order to demonstrate the power of the calculator, we first solve for future value in the keystroke solution, and then solve for n, i, and PV, assuming the other values are known.

Keystroke Solution:

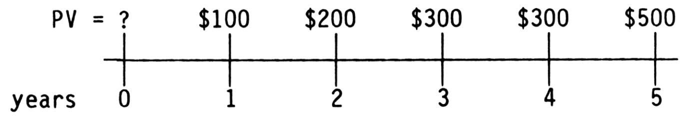
Procedure	Action Step	Display
1. Clear the PMT register	0, PMT	0.00
2. Enter the number of years	3 [ENTER]	3.00
3. Enter the number of compounding periods per year	2	2
4. Compute the total number of compounding periods	X, n	6.00
5. Enter the annual interest rate	10 [ENTER]	10.00
6. Enter the number of compounding periods per year	2	2
7. Compute the effective interest rate per compounding period	÷, i	5.00
8. Enter the cash outflow at time zero as a negative number	10, CHS, PV	- 10.00
9. Solve for the equivalent value after six compounding periods	FV	13.40
10. Assuming i, PV and FV are known, solve for n	n	6.00
11. Assuming n, PV and FV are known, solve for i	i	5.00
12. Assuming n, i, and FV are known, solve for PV	PV	- 10.00

Please notice that in the example above, the calculator computed the effective interest rate per compounding period of 5 percent, not the annual rate of 10 percent. You must multiply the computed i value by the number of compounding periods per year in order to determine the annual rate. Also, the computed n value is the total number of compounding periods. The number of years is found by dividing the computed n value by the number of compounding periods per year.

SUMMARY: The concepts discussed in this section on compound interest are the fundamental building blocks for all time-value-of-money applications. The key point is the equivalent value concept which we continue to discuss in the next section on multiple cash flows.

UNEQUAL MULTIPLE CASH FLOWS

Multiple cash flows can be converted to equivalent present or future values using the compound interest approach discussed in the previous section. The only difference is that more than one cash inflow or outflow exists. Consider, as an illustration, the cash flows shown in Time Diagram 3.8.

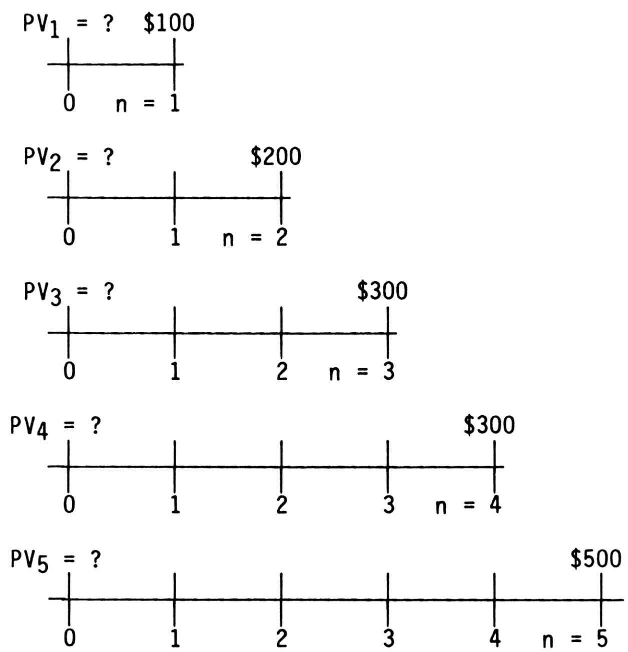


i = 10%

Time Diagram 3.8

The unknown value is PV, because we want to determine the equivalent present value of the five cash flows at time zero. Or you can treat the unknown as an investment returning the five cash flows on which you want to earn 10 percent compounded annually. How much should you pay now, in the present, for the future cash flows?

The equivalent present value at time zero (amount to pay) is determined by treating the future cash flows as five individual compound interest problems. As a result, Time Diagram 3.8 is separated into five individual diagrams as shown in Time Diagram 3.9.



i = 10%

Time Diagram 3.9

As the diagram illustrates, we now have five individual compound interest problems. The sum of the five present values represents the total equivalent present value of the five cash flows.

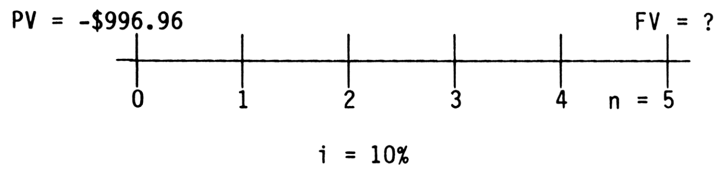
Using the calculator, you can easily compute the five individual present values. Rather than showing a detailed keystroke solution, only the inputs are given. The detailed keystrokes would follow the same format as in Example 3.

input values				unknown
n	i	PMT	FV	PV
1	10	0	$100	$ -90.909
2	10	0	$200	-165.289
3	10	0	$300	-225.394
4	10	0	$300	-204.904
5	10	0	$500	-310.461
Sum of present values				$-996.957

The equivalent value at time zero, present value, of the $100 received at the end of year 1 is $90.91. The other four present values are also the equivalent values at time zero of the individual future cash flows. The sum of the five present values is the total equivalent value of the future flows. The total present value of $996.96 is the equivalent cumulative value of $100 at the end of year 1, $200 at the end of year 2, $300 at the end of year 3, $300 at the end of year 4, and $500 at the end of year 5 assuming an interest rate of 10 percent. Thus assuming that 10 percent is the best rate you can receive on invested money, you would pay $996.96 for the five future payments.

As you see from this example, determining the equivalent present value for a stream of cash flows is accomplished simply by finding the present values for the individual cash flows and then summing these values to determine the total present value.

Once the equivalent present value of the future cash flows is known, we can move it to any point in time, such as five years later, assuming the interest rate remains constant. For example, the equivalent value at the end of five years for the five cash flows is $1,605.61. First draw Time Diagram 3.10.

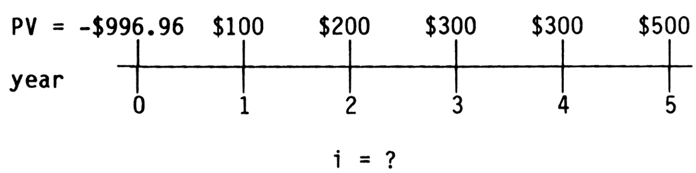


Time Diagram 3.10

The present value of -$996.96 is shown on the diagram rather than the five individual cash flows because we want to move one equivalent value to another point in time -- the end of five years. As a result, finding the future value of the five cash flows at the end of five years is a compound interest problem. Using values of n = 5, i = 10, PV = -996.96, PMT = 0, the future value is easily computed as $1,605.61.

So holding the ten-percent interest rate constant, we can say the five cash flows are equivalent to $996.96 at time zero or $1,605.61 at the end of year five. Putting this equivalency relationship into problem statements, we can say that the amount to invest, or the price to pay, in order to receive the five cash flows is $996.96. We can also say that if the five cash flows are deposited into a savings account at the end of each year when they occur, we could withdraw $1,605.61 at the end of five years.

Now let's change the example and assume that the interest rate is the unknown value while the present value is known. The revised Time Diagram 3.11 for the new problem is shown below.



Time Diagram 3.11

When solving for the interest rate that makes the five cash flows have an equivalent value at time zero equal to $996.96, we are also solving for the internal rate of return. Here we know the interest rate is 10 percent. Solving for the interest rate requires lengthy and complex mathematical techniques. Because your calculator solves for the interest rate directly by pressing the i key, we will not discuss the mathematics involved. Instead we will concentrate on teaching you the meaning of the calculated values. While we discuss the general concepts here, Chapter 9 has an in-depth discussion on rates of return.

Assumptions for Multiple Cash Flows. The preceding material presented an overview for understanding the meaning of the present value, the future value and the interest rate for multiple cash flows. Now we discuss three key assumptions underlying these values.

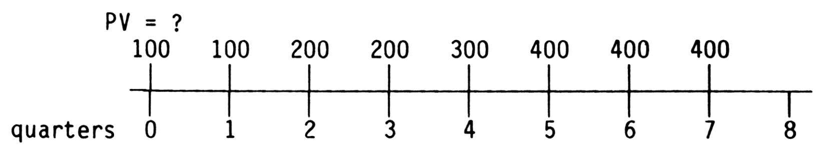
First, the length of the cash-flow period and the compounding period are equal. For example, if cash flows occur monthly, then compounding occurs monthly; if cash flows occur annually then compounding occurs annually.

Second, interest is compounded at the end of each cash-flow period. Thus, a cash flow occurring at time zero is not discounted. Third, unless otherwise specified, a constant interest rate is assumed when solving for the present value, the future value, or the interest rate.

Finally, the interest rate used for computations is the effective interest rate per compounding period. It is also the interest rate per cash-flow period because the cash-flow and compounding periods are equal. When you solve for the interest rate, it is the effective rate per cash-flow period and compounding period. As a result the interest rate used for calculations is the annual rate divided by the number of cash flows occurring per year.

The process of determining the present value or the future value of a series of cash flows is simply an extension of the compound interest concepts. The same concepts apply whether single or multiple cash flows occur.

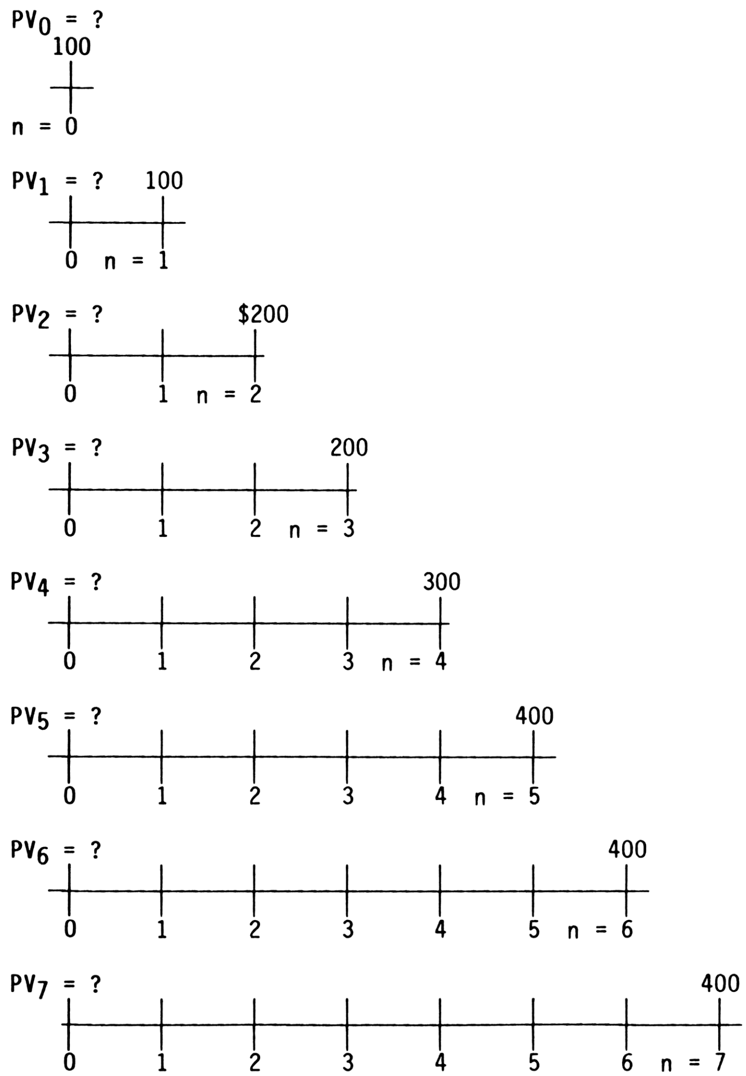
A more complex problem involving multiple cash flows is shown in Time Diagram 3.12.



i = 12%/4 = 3%

Time Diagram 3.12

This cash-flow pattern is representative of a lease because a cash flow occurs at time zero. The annual interest rate is 12 percent but the effective rate per cash-flow period is 3 percent (12 ÷ 4) because cash flows occur quarterly. As before, we can determine the present value by finding the equivalent value of each cash flow at time zero and summing them. Our first step is drawing Time Diagram 3.13.



i = 12%/4 = 3%

Time Diagram 3.13

Notice that the first cash flow occurs at time zero; thus no compounding occurs, and its equivalent value at time zero is $100. Also, while eight cash flows occur, the last cash flow is at the end of period 7 or the beginning of period 8.

Using the calculator the individual present values are found using the keystroke values shown below:

input values				unknown
n	i	PMT	FV	PV
0	3	0	$100	-100.0000
1	3	0	100	-97.0874
2	3	0	200	-188.5192
3	3	0	200	-183.0283
4	3	0	300	-266.5461
5	3	0	400	-345.0435
6	3	0	400	-334.9937
7	3	0	400	-325.2366
Sum of present values				$1,840.4548

The equivalent value at time zero (present value) of the eight cash flows, rounded to two places, is $1,840.45. That is, the outlay of $1,840.45 at time zero is equivalent to receiving the eight cash flows assuming an effective interest rate of 3 percent per quarter. We can also say that if you pay $1,840.45 in order to receive the eight cash flows, the interest rate that makes the cash flows equivalent to the outlay cost is 3 percent per quarter, or 12 percent per year (3% x 4 quarters per year).

Calculator Solution for Multiple Cash Flows Using "Grouped Cash Flow" Keys. In order to explain the meaning of the present value, future value and interest rate for multiple cash flows, we repeatedly used compound interest for single cash flows. But your HP-12C has special keys to use for computing the present value and the interest rate. The calculator has three cash-flow entry keys: g, [CF_0]; g, [CF_j]; and g, [N_j], and two unknown value keys f, [NPV]; and f, [IRR].

Cash outflows are entered as negative values and cash inflows are entered as positive values. The cash flow occurring at time zero is entered using the g [CF_0] key. If no cash flow at time zero occurs, then enter a zero. The g, [CF_j] key is used to enter the cash flows occurring at the end of each period starting with period 1. The "j" is an index indicating j = 1, 2, 3, ..., n with n being the last cash-flow period.

You can enter 20 different cash flows using the g, [CF_j] key. But if you have cash flows that are identical and grouped together, then you can use the g, [N_j] key to tell the calculator how many grouped cash flows occur together and enter the entire group at one time. For example, if you have 5 cash flows of $1,500, the key sequence 1500, g, [CF_j] enters the amount and 5, g, [Nj] enters the number of $1,500 cash flows. You are then ready to enter the next cash flow. You can enter up to 99 cash flows using the [N_j] key, but if you have, say, 150 cash flows, then two entries must be made, i.e., 1500, g, [CF_j], 75, g [N_j] enters the first 75 flows and 1500, g, [CF_j], 75, g [N_j] enters the second 75 flows. You can enter 20 grouped cash flows in your calculator.

After the cash flows are entered, you can solve for the present value by first keying the interest rate per cash-flow period in [i] and then pressing f, [NPV]. The f, [NPV] key stands for net present value but for now we will use the key to solve for present value as explained in this section. We will discuss the net present value in a later section.

Solving for the interest rate is accomplished after entering the cash flows by pressing f, [IRR]. The [IRR] stands for internal rate of return and is discussed in Chapter 9.

Now let's demonstrate the use of these keys by solving the two multiple cash-flow examples previously discussed.

EXAMPLE 3a:

Find the present value of the annual cash flows shown in Time Diagram 3.14.

PV = ?

	0	$100	$200	$300	$300	$500
years	0	1	2	3	4	5

i = 10%

Time Diagram 3.14

Solution:

If you set up your time diagram properly, the input values for the various keys are already labeled in the diagram. First, the input for [i] is 10 percent as shown. Second, the value to enter for g, [CF_0] is shown above the time period 0 mark. In this case the g, [CF_0] value is 0. Third, the values to enter using the g, [CF_j] key are shown above the periods marked 1, 2, 3, 4, and 5. The first value entered using the g, [CF_j] key is 100 because we start with period 1 when using the g, [CFj] key. We will use the g, [N_j]

key to enter the two $300 flows at the end of periods 3 and 4 because they are identical cash flows occurring in a group.

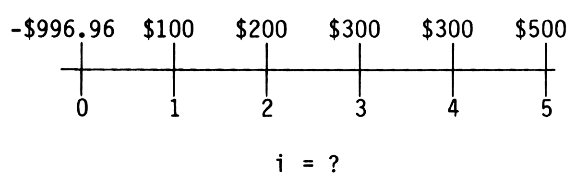
Keystroke Solution:

Procedure	Action Step	Display
1. Clear the calculator	f, REG	0.00
2. Enter the interest rate per cash-flow period	10 i	10.00
3. Enter the CF_0 value	0 g, CF_0	0.00
4. Enter the first cash flow	100 g, CF_j	100.00
5. Enter the second cash flow	200 g, CF_j	200.00
6a. Enter the amount of the third and fourth cash flows	300 g, CF_j	300.00
b. Enter the number of identical cash flows	2 g, N_j	2.00
7. Enter the fifth cash flow	500 g, CF_j	500.00
8. Solve for the present value	f, NPV	996.96

Notice that the sign of the answer is not opposite to the sign of the cash flows as it is when using the compound interest keys for single cash flows. The positive present value of the positive cash flows is $996.96. So, if you wanted to buy these cash flows and earn 10 percent, you would pay $996.96. The next example will clarify this sign difference.

EXAMPLE 3b:

Find the interest rate that makes the discounted cash inflows equal to the initial outlay for the values shown in Time Diagram 3.15.



Time Diagram 3.15

Solution:

In this example, the unknown value is the interest rate. Again, the time diagram shows the input values for g, [CF_0] and g, [CF_j]. Enter -$996.96 for the g, [$CF_0$] value and the other cash flows using the g, [CF_j] key as shown in the previous example.

Keystroke Solution:

Procedure	Action Step	Display
1. Clear the calculator	f, REG	0.00
2. Enter the cash flow at time zero	996.96 CHS, g, CF_0	- 996.96
3. Enter the end-of-period cash flows		
a. period 1	100 g, CF_j	100.00
b. period 2	200 g, CF_j	200.00
c. periods 3 and 4	300 g, CF_j 2 g, N_j	300.00 2.00
d. period 5	500 g, CF_j	500.00
4. Solve for the interest rate per payment period	f, IRR	10.00

The answer of 10 percent is the interest rate that makes the five end-of-period cash inflows equal to the zero period cash outflow. So the equivalent value for the five inflows is +\$996.96 while the outflow at period zero is -\$996.96.

The next example also demonstrates the usefulness of time diagrams when working with the cash-flow keys.

EXAMPLE 4a:

Find the present value of the quarterly cash flows shown in Time Diagram 3.16.

	PV = ?							
	100	100	200	200	300	400	400	400
quarters	0	1	2	3	4	5	6	7

$i = 12\%/4 = 3\%$

Time Diagram 3.16

Solution:

As in Examples 3a and 3b, the cash flows are entered as shown on the time diagram. But in this example, we have to deal with a non-zero cash flow at time zero in addition to the present value calculation at time zero. So, \$100 is entered for g, [CF_0], and the g [CF_j] keys are used to enter the other cash flows beginning with period 1. The g, [N_j] key is used a number of times to reduce the number of keystrokes. The keystroke solution shows the preliminary calculation of converting the annual rate to the effective interest rate per quarter. Remember, you should calculate the effective rate per cash-flow period on your calculator to avoid rounding errors.

Keystroke Solution:

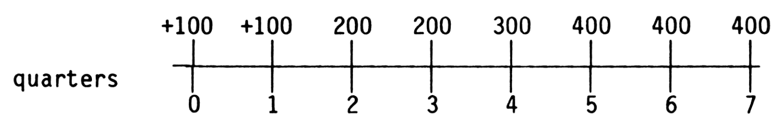
Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Enter the annual interest rate	12 [ENTER]	12.00
3. Enter the number of cash flows per year and calculate the effective interest rate	4, ÷, i	3.00
4. Enter the cash flow at time zero	100 g, CF_0	100.00
5. Enter the cash flow at the end of period 1	100 g, CF_j	100.00
6. Enter the cash flows for periods 2 and 3		
a. amount	200 g, CF_j	200.00
b. number of times the $200 occurs together	2 g, N_j	2.00
7. Enter cash flow for period 4	300 g, CF_j	300.00
8. Enter the cash flows for periods 5, 6, and 7		
a. amount	400, g, CF_j	400.00
b. number of times the $400 occurs together	3 g, N_j	3.00
9. Compute the equivalent value of the cash flows at time period zero - the present value	f, NPV	1840.45

Notice how the use of the g, [N_j] key not only reduced the keystrokes, but only used four registers to store the seven cash flows after the initial cash flow at time zero. The $1,840.45 is the equivalent value of the eight positive cash flows at time zero. Since the +$100 cash flow at time zero has an equivalent present value of $100, we can also say the equivalent value at time zero (present value) of the seven cash flows starting at the end of period 1 is $1,740.45 (1,840.45 -100). The next example will use this value as shown below.

EXAMPLE 4b:

Find the interest rate that makes the eight cash flows in Time Diagram 3.17 have an equivalent value equal to an outlay of $1,840.45.

PV = -$1,840.45

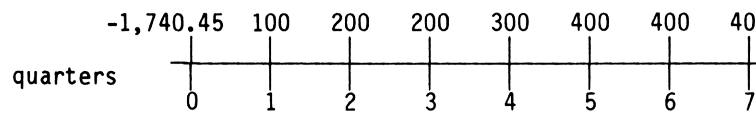


i = ?

Time Diagram 3.17

Solution:

Because two types of cash flows occur at time period zero, we must add them together to have one equivalent cash flow. We must do this because the calculator allows only one value to be entered for g, [CF_0]. As a result, the net value for time period zero is -$1,740.45 (-1840.45 + 100). In fact, when multiple cash flows occur at the end of any time period, they must be added together because the calculator allows only one value per time period to be entered using either the g, [CF_0] or g, [CF_j] key. The revised Time Diagram 3.18 is shown below.



i = ?

Time Diagram 3.18

Now that we have a time diagram with one cash flow per period, we can proceed to solve for the interest rate. Remember, the interest rate found by the calculator is the effective rate per cash-flow period and you must convert the effective rate to the annual rate. The annual rate is found by multiplying the effective rate per cash-flow period by the number of cash flows per year.

Keystroke Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Enter the cash flow at time period zero	1740.45 CHS, g, CF_0	- 1740.45
3. Enter the remaining cash flows		
a. period 1	100 g, CF_j	100.00
b. periods 2 and 3	200 g, CF_j 2 g, N_j	200.00 2.00
c. period 4	300 g, CF_j	300.00
d. periods 5, 6, and 7	400 g, CF_j 3 g, N_j	400.00 3.00
4. Solve for the effective interest rate per cash-flow period	f, IRR	3.00
5. Convert to the annual rate by entering the number of cash flows per year	4 X	12.00

As shown in Step 4 above, the calculator shows 3 percent as the effective interest rate per cash-flow period (quarter), and the annual rate is found by multiplying the effective rate by the number of cash flows per year. The material in Chapter 9 discusses the assumptions underlying the interpretation of the interest rate as a return rate. We will not repeat that discussion here.

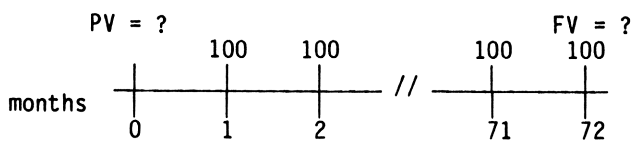
SUMMARY: In this section above we have shown the extension of compound interest concepts to situations involving multiple cash flows. We have stressed the understanding of the present value and the interest rate when multiple cash flows are involved, omitting the underlying mathematics. If you are interested in the mathematical theory, refer to Chapters 7 and 9 of Financial Analysis Using Calculator: Time Value of Money by Greynolds, Aronofsky and Frame.

Next, we discuss the situation where all multiple cash flows are equal.

EQUAL MULTIPLE CASH FLOWS (ANNUITIES)

Before the easy access to calculators and computers, tables were normally used when working with time value of money problems. One popular set of tables is called Annuity Tables. An annuity is simply a series of cash flows where all cash flows were equal. Because the tables are static, an elaborate system of notation was developed which perhaps confused some users as much as it helped them. Here we will show you the linkage between the Standard Annuity Tables and the grouped cash-flow keys on your calculator. In the following section we will solve more advanced annuity problems by use of the "financial" keys and then give a side-by-side comparison of the two keystroke approaches.

Using the Cash Flow Keys. Consider the series of equal cash flows in Time Diagram 3.19. Solve first for present value, PV, and then for the future value, FV.



i = 12%/12 = 1%

Time Diagram 3.19

In the diagram there are 72 cash flows, starting at the end of month 1. By use of the grouped cash-flow keys, we can determine the equivalent present value at time period 0. This is the first answer requested. In order to obtain the second answer for the FV, we will abandon the convenience of the grouped cash-flow keys. A simple approach is to use the equivalent value at time period zero (PV), and convert it to the equivalent future value at the end of month 72 (FV), using the financial keys previously discussed in the compound interest method.

Solution:

For the first answer, the grouped cash-flow keys with the input are shown below. We find that the present value is $5,115.04. The inputs are:

Grouped Cash Flow Keystrokes	Display
f, REG	0.00
1, i	1.00
0 g, CF_0	0.00
100 g, CF_j	100.00
72 g, N_j	72.00
f, NPV	5115.04

For the second answer we compute the equivalent future value at the end of month 72 (FV) where the present value and interest rate are known. The future value of $10,470.99 is found using the financial keys with the following inputs.

Annuity Keystrokes	Display
f, REG	0.00
5115.04 CHS, PV	-5115.04
72 n	72.00
1, i	1.00
0, PMT	0.00
FV	10,470.99

As we have seen, the grouped cash-flow keys were used to solve for the PV, but not for the FV. Another convenient use of the grouped cash-flow keys is to solve for the interest (i) when the cash flows and PV are known. Refer back to the problem worked out above after the first question was answered.

Monthly rate. We can also solve for the monthly rate of 1 percent with the grouped cash-flow keys using the inputs shown below.

Grouped Cash Flow Keystrokes	Display
f, REG	0.00
5115.04 CHS, g, CF_0	-5115.04
100 g, CF_j	100.00
72 g, N_j	72.00
f, IRR	1.00

In summary, the grouped cash-flow keys are very convenient to use when solving for either the present value (PV) or the interest rate (i). For more common annuity situations we will use the financial keys, as described in the next section.

Using the Financial Keys. Whenever the cash flows are equal there is an alternative to using the grouped cash-flow keys; namely, to use the financial keys for annuities.

Because the cash flows are all equal we can use equations to solve for the values, thus reducing the keystrokes involved. These equations are generally called "Annuity Equations" and utilize the financial keys. Furthermore, we can solve for the amount of each payment and the number of payments. These equations are widely used in real estate analysis because many mortgages and leases have equal payments. You, however, must learn some new terms in order to solve annuities on your calculator.

First, the cash flows are called payments and the [PMT] key is used to enter the amount of the equal payment. Payments received are positive and those paid out are negative.

Second, the number of equal payments is entered with the [n] key. Notice that, while the [n] key is used for the number of payments when solving annuities, the [n] key is also used for the number of compounding periods when solving single-payment, compound-interest problems. The calculator knows the difference by checking the value entered for [PMT]. If [PMT] is zero, then [n] is the number of compounding periods. If [PMT] is not equal to zero, then [n] is the number of equal payments. This is the reason we showed 0 being entered for [PMT] in the compound interest section described above.

Third, the [i] value is the effective interest rate per payment period. The interest rate for annuities has the same meaning as in the previous section on unequal multiple cash flows.

Fourth, the [PV] key represents the equivalent present value at time period 0 and is equivalent to the following cash flows; all of the equal multiple cash flows and, in addition, an extra final payment which is entered by use of the [FV] key and occurs at the end of the last period.

Finally, the [FV] key represents the equivalent future value at the end of the last payment period and is equivalent to the following cash flows: all of the equal multiple cash flows and, in addition, an extra initial payment which is introduced by use of the [PV] key and occurs at time 0.

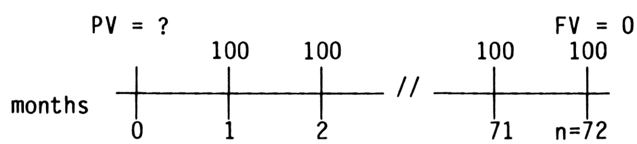
In summary, you have five values when working with annuities -- n, i, PV, PMT and FV. You can solve for any unknown as long as the other four values are known.

A point that often confuses calculator users is the fact that up to three cash flows are identified with annuities: an outflow at time zero (PV), the regular even cash flows (PMT), and an additional cash flow at the end of the final payment period (FV). In the following discussion we will clarify these three values for you.

A major assumption for annuities is the timing of the first of the even payments. When the first payment occurs at the end of the first period, it is called an "Ordinary Annuity." If the first payment occurs at the beginning of the first payment period (time 0), it is called an "Annuity Due." As a result, in order to solve annuities on your calculator you must identify the annuity type as an ordinary annuity or an annuity due.

You indicate the type of annuity you are solving by pressing the g, [END] or g, [BEG] keys. For cases where the first payment occurs at the end of the first payment period, press the g, [END] key. This tells the calculator you are solving an ordinary annuity. Press the g, [BEG] key if the first payment occurs at time period 0, or the beginning of the first payment period. Forgetting to set the payment timing key (switch) is a common error HP-12C users make. That is the reason we show the keys being set in all keystroke solutions in this book even when it has no effect. For instance, the calculator ignores the [END] and [BEG] setting when solving problems using the financial keys with a single payment or using the cash-flow keys.

Ordinary annuity. Now let's return to the example in Time Diagram 3.19 and solve for the PV and FV using the annuity keys. Refer to the new diagram below.



i = 12%/12 = 1%

Time Diagram 3.20

We have only one unknown value (PV) because a zero is shown for FV and this zero indicates that no additional payment occurs at the end of the last period. First, we set the payment timing to g, [END] because the first payment occurs at the end of period 1 and this indicates an ordinary annuity.

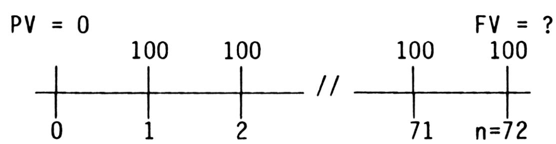
Second, we enter the values as shown on the time diagram, and compute the present value.

Annuity Keystroke Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Enter the number of payments	72 n	72.00
3. Enter the annual interest rate	12 [ENTER]	12.00
4. Enter the number of payments per year and calculate the interest rate per payment period	12, ÷, i	1.00
5. Enter the payment	100 PMT	100.00
6. Enter 0 for additional payment at the end of last period	0 FV	0.00
7. Solve for the present value	PV	- 5,115.04

We get an answer of -$5,115.04 which, except for the sign, is the same answer we calculated using the grouped cash-flow keys. The annuity keys assume that if the [PMT] amount is positive the [PV] entry is negative and vice versa. You can solve for any unknown as shown here by entering the known values and pressing the key for the unknown value. Notice that when either PV or FV is zero, you must make sure that there is a zero in the appropriate register.

Now let's solve for the future value of the cash flows. Again we will draw a new time diagram:



i = 12%/12 = 1%

Time Diagram 3.21

When solving for the equivalent future value (FV) for an ordinary annuity, it means solve for FV after the last payment is made. Because the keystrokes are so similar to the last example, only the inputs are shown without commentaries. The inputs are:

Annuity Keystrokes	Display
CLX, f, FIN	0.00
72, n	72.00
1, i	1.00
0, PV	0.00
100, PMT	100.00
FV	-10,470.99

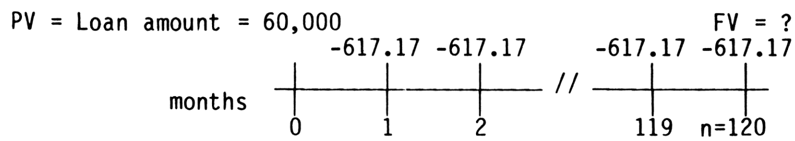
Again we see the answer is the same as that computed for time Diagram 3.19, except for the sign.

For real estate analysis, ordinary annuities are used when computing mortgage values. A common situation is to compute the balloon, or payoff amount, if the mortgage is paid off early. One point you must remember -- THE BALLOON CALCULATED BY THE CALCULATOR IS THE AMOUNT DUE AFTER THE LAST PAYMENT IS MADE. We will explain this statement with the example below.

EXAMPLE 5:

Let's assume a $60,000, thirty year mortgage at 12 percent annual, compounded monthly, with payments of $617.17 and where the mortgage is paid off at the end of ten years. What is the payoff amount or the balloon?

First, we draw Time Diagram 3.22.



i = 12%/12 = 1%

Time Diagram 3.22

Solution:

The time diagram is developed from the borrower's viewpoint. Thus, the amount received (PV = 60,000) is shown as a positive number with the monthly payments shown as negative values because the borrower makes the payments. The FV = ? is shown above the payment made at the end of month 120. As a result, the FV value we compute will be the amount owed on the mortgage after payment 120 is made and will be a negative value because it is the amount necessary to pay off the mortgage. Now that the values are identified, we can compute the payoff amount as shown in the keystroke diagram.

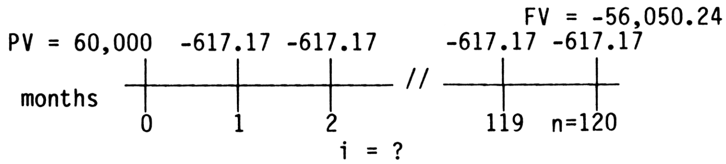
Annuity Keystroke Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set switch	g, END	0.00
3. Enter number of payments	120, n	120.00
4. Enter interest rate per payment period	1, i	1.00
5. Enter cash flow at time zero (PV)	60000 PV	60,000.00
6. Enter payment amount	617.17, CHS, PMT	- 617.17
7. Solve for equivalent value at end of month 120	FV	- 56,050.24

The future value of $56,050.24 is the equivalent future value of the combination of the +$60,000 and the 120 payments of -$617.17 all compounded to the end of month 120. The calculator shows a -$56,050.24 because that is the amount necessary to balance the relationship. Again, notice that the calculator treats the equivalent future value separately from the final payment amount. The total amount necessary to pay off the loan at the end of the 120th month is $56,667.41 (56,050.24 + 617.17).

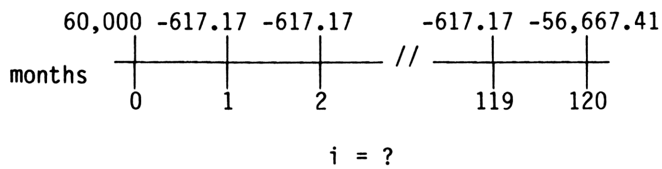
We can also say the $60,000 at time 0 is equivalent to the 120 payments of $617.17 and the balloon or payoff amount of $56,050.24.

Now let's reverse the problem and solve for the monthly interest rate. Two diagrams are shown because the inputs vary depending on whether you are using the grouped cash-flow keys or the annuity keys. We will indicate both keystroke types, side-by-side.



CASH FLOWS USING FINANCIAL KEYS

Time Diagram 3.23



CASH FLOWS USING GROUPED CASH-FLOW KEYS

Time Diagram 3.24

Time Diagram 3.23 shows the cash flows in a form suitable for use with the financial keys. Notice that the cash flow at time 0 is labeled as PV. The FV amount is shown separately from the final payment because the calculator treats it as an individual value. On the other hand, Time Diagram 3.24 does not label a PV or FV value because you enter the total cash flow occurring in period 0 in g, CF_0, and 119 payments of -617.17 in CF_j and N_j with a final total payment of -56,667.41 in CF_j. Both methods will give the same answer with the HP-12C, as the following inputs show.

<u>inputs for financial keys</u>

CLX, f, FIN

120, n

60000 PV

617.17 CHS, PMT

56050.24, CHS, FV

i, which displays 1.00

<u>inputs for grouped cash flow keys</u>

f, REG

60000 g, CF_0

617.17 CHS, g, CF_j

90 g, N_j

617.17 CHS, g CF_j

29 g, N_j

56667.41 CHS,

g, CF_j

f, IRR, which displays 1.00

As you see, both methods give the same answers for i, but each method requires slightly different inputs.

In either case, the 1 percent interest rate is necessary to make the 120 payments of \$617.17 and the balloon of 56,050.24 equivalent to the initial cash flow of \$60,000. It is also the rate that makes the initial cash inflow of +\$60,000 and the 120 payments of -\$617.17 equivalent to -\$56,050.24. This interpretation of the interest rate is consistent with the earlier definitions because it is the rate that makes a series of cash flows equivalent to a single flow at some point in time. The assumptions necessary to call the interest rate a return on earnings rate are discussed in Chapter 9.

The annuity keys are used to compute the n, PMT, or FV values, but either the annuity keys or grouped cash-flow keys may be used to calculate the present value or interest rate. The keystroke solutions in this book use the more convenient option for the application.

Annuity due. We defined ordinary annuity by the fact that the first payment occurred at the end of the first payment period. If the first payment occurs at the beginning of the first payment period (time period 0), we have an annuity due. We will illustrate an annuity due situation with a lease example, because most leases require a payment at the start of the lease (time period 0).

EXAMPLE 6:

You have considered leasing an asset that has 24 quarterly payments of \$2,767.60 with a residual value, or market value, of \$20,000 which occurs at the end of the final period. However, you can purchase the asset outright. How much should you pay if the money used for the purchase would otherwise earn 16 percent annual compounded quarterly? The first step in determining the purchase price is to draw Time Diagram 3.25.

```
             PV = ?                                    FV = 20,000.00
             2,767.60   2,767.60   2,767.60   2,767.60
                |          |          |          |         |
quarters     ---+----------+----------+--- // ---+---------+---
                0          1          2          23       n=24

                       i = 16%/4 = 4%
```

Time Diagram 3.25

Solution:

First, notice that the regular lease payments start at time 0, thus indicating an Annuity Due. While 24 payments are scheduled, the last payment occurs at the end of quarter 23 or beginning of quarter 24. From the seller's point of view, if he sells the asset he will forego the 24 payments and the residual value of \$20,000. You want to determine the amount at time period 0 that is equivalent to these cash flows with a quarterly interest rate of 4 percent. As a result, the purchase price you will pay is the present value of these cash flows, using interest of 4 percent per quarter.

Second, observe that the future value of \$20,000 is also the total of all cash-flow amounts compounded to the end of period 24. As a result, the FV for an Annuity Due is also the residual and the total final cash flow, as contrasted to an Ordinary Annuity. The difference is due to the timing of the regular payments.

Grouped Cash Flow Keystroke Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Enter the interest rate period	4 i	4.00
3. Enter first payment	2767.60 g, CF_0	2767.60
4. Enter next 23 payments	2767.60 g, CF_j 23 g, N_j	2767.60 23.00
5. Enter final total payment	20000 g, CF_j	20000.00
6. Compute present value	f, NPV	51687.82

For grouped cash flows we enter 25 cash flows -- 24 payments of $2,767.60 and one for $20,000. But if we use the annuity keys to solve for the present value, then the 24 payments are treated separately from the final flow of $20,000.

Annuity Keystroke Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set switch	g, BEG	0.00
3. Enter number of even payments	24, n	24.00
4. Enter the interest rate per payment period	4 i	4.00
5. Enter the payment	2767.60 PMT	2,767.60
6. Enter the cash flow at the end of the last payment period	20000 FV	20,000.00
7. Solve for the equivalent value at time zero, (PV)	PV	- 51,687.82

The solution using the financial keys, except for the sign, is the same as that using the grouped cash-flow keys. In either case, the 51,687.82 represents the equivalent value of the future cash flows at time period 0 assuming an interest rate of 4 percent per quarter.

In Step 2, we set the payment switch to g, [BEG] because the first regular (equal) payment occurs at time period zero. This tells the calculator you are solving an Annuity Due. Setting the payment switch is critical when you use the financial keys because it determines the type of annuity you are solving.

Now let's assume next that the purchase price is known and we are solving for the interest.

EXAMPLE 7:

You have the option of purchasing an asset for $51,687.82 or leasing it for 24 quarterly payments of $2,767.60. The asset has a residual (market) value of $20,000 at the end of the 24th quarter. What interest rate makes the cash flows equivalent to the purchase price? As usual, our first step is drawing a time diagram.

PV = Purchase price = -51,687.82   FV = Residual value = 20,000

2,767.60 2,767.60   2,767.60

quarters 0 1 2 // 23 n=24

i = ?

Time Diagram 3.26

Diagram 3.26 is in the form suitable for using the financial keys. The present value is shown separately from the first regular payment, and the future value is shown separately from the last regular payment. The inputs, using the annuity keys, are:

Annuity Keystrokes	Display
f, REG	0.00
g, BEG	0.00
24 n	24.00
51,687.82, CHS, PV	-51,687.82
2767.60 PMT	2767.60
20000, FV	20,000.00
i	4.00

The answer of 4 percent agrees with the interest rate used in Example 6.

Using the grouped cash-flow keys to solve for the interest rate requires adding together the cash flows for each period so that only one occurs at each period.

-48,920.22 2,767.60 2,747.60   2,767.60 20,000.00

quarters 0 1 2 // 23 24

i = ?

Time Diagram 3.27

Because we can enter only one cash flow in g, CF_0, the -51,687.82 and the 2,767.60 were added together to get -48,920.22. Next the interest rate is found using these inputs.

Grouped Cash-Flow Key Strokes	Display
f, REG	0.00
48920.22 CHS, g, CF_0	-48,920.22
2767.60 g, CF_j	2767.60
23 g, N_j	23.00
20000 g, CF_j	20,000.00
f, IRR	4.00

Again we get an interest rate of 4 percent. Notice that we did not set the payment switch because it is ignored when you use the grouped cash-flow keys. While you can use either approach to solve for the present value or interest rate, the financial keys must be used to solve for n, PMT or FV when you have equal payments. You cannot solve directly for n or the individual cash flows when the cash flows are uneven.

SUMMARY: In summary, you can use either the grouped cash-flow keys or the financial keys to solve for the interest rate or the present value. But if you use the financial keys, then remember the timing assumptions for the cash flows. For your convenience, four common annuity cash-flow patterns are shown below. If your application matches one of the patterns, you can solve for n, i, PV, PMT, or FV. Remember, one of the three values, PV, PMT or FV, must be negative when solving for i and n or the calculator will give you an error message.

ORDINARY ANNUITY - USED FOR MORTGAGE CALCULATIONS

```
                                        -FV
+PV    -PMT    -PMT            -PMT     -PMT
 |       |       |               |        |
-+-------+-------+----- // ------+--------+-
 |       |       |               |        |
 0       1       2              n-1       n
```

Switch = g, END

Time Diagram 3.28

ANNUITY DUE - USED FOR LEASE CALCULATIONS

```
+PV                                     -FV
-PMT   -PMT    -PMT            -PMT
 |       |       |               |        |
-+-------+-------+----- // ------+--------+-
 |       |       |               |        |
 0       1       2              n-1       n
```

Switch = g, BEG

Time Diagram 3.29

ORDINARY ANNUITY - USED FOR DETERMINING FUTURE VALUE OF AN INITIAL DEPOSIT AND A SERIES OF END-OF-PERIOD PAYMENTS

```
                                        +FV
-PV    -PMT    -PMT            -PMT     -PMT
 |       |       |               |        |
-+-------+-------+----- // ------+--------+-
 |       |       |               |        |
 0       1       2              n-1       n
```

Switch = g, END

Time Diagram 3.30

ANNUITY DUE - USED FOR DETERMINING FUTURE VALUE OF AN INITIAL DEPOSIT AND A SERIES OF BEGINNING OF PERIOD PAYMENTS

```
-PV
-PMT   -PMT    -PMT            -PMT     +FV
 |       |       |               |        |
-+-------+-------+----- // ------+--------+-
 |       |       |               |        |
 0       1       2              n-1       n
```

Switch = g, BEG

Time Diagram 3.31

Special Annuity and Compound Interest Features of the HP-12C. The HP-12C has a special feature built into the annuity and compound interest routine. This feature allows "odd-period" calculations, but does cause problems with certain estate calculations.

First, the calculator will always show an integer value when solving for n, even though the value may be a fractional amount. The calculator rounds the answer to the next whole number. You will be shown how to compute the number of whole payments and the amount of the final partial payment when computing n values. You can, however, enter a fractional n value and when the payment register has a zero value compute the correct PV and FV values for compound interest.

Second, when computing annuity or compound interest values where n is a fractional amount of a whole number plus a fractional amount, the calculator does not use the same annuity equation as when n is a whole number. Furthermore, depending on the type of compounding selected for the partial n period, you can get two different answers. As a result, in this book, you will always have the compound interest option selected for all calculations. This option is selected by pressing [STO] [EEX] until a "c" appears in the display. Once you have made this selection, the calculator will not change it even when the machine is turned off and back on. So at this time, press [STO] [EEX] until the "c" appears. All following applications assume the "c" is in the display.

For a detailed discussion of these two special features see the HP-12C Owner's Handbook and Problem-Solving Guide, pages 44-49 and 57-61.

SUMMARY

The concepts discussed in this chapter should help you to understand the applications described in this book, especially those in Chapter 7. Whether you use the annuity keys or the grouped cash-flow keys, the first step is to draw a time diagram and then to decide on which set of keys to use -- the financial keys or the grouped cash-flow keys. The financial keys require that the present value and future value be entered separately from the regular equal payments. You must also identify the timing of the first payment and set the switch to g, [END] or g, [BEG]. You can solve for n, i, PV, PMT or FV.

The grouped cash-flow keys allow only one cash flow per period to be entered using g, CF_0 or g, CF_j, so the present value and future value are treated as part of the regular cash flows. For practical purposes, you can use these keys to solve for the present value or interest rate.

The key to understanding the values you compute, using either the financial or grouped cash-flow keys, is the discussion on compound interest and the equivalent value of money. We explained the concepts without showing the actual equations used. If you are interested in the mathematics, consult Financial Analysis Using Calculators: Time Value of Money by Greynolds, Aronofsky and Frame.

4

REGULAR MORTGAGES

In this chapter, you are shown the solution procedure for regular mortgages with and without balloon payments. All examples assume even or level mortgage payments. The calculator solution procedures for the traditional Six Functions of One Dollar conclude the chapter. Other mortgage types such as Graduated Payment Mortgages or Wraps are discussed in Chapter 7.

To avoid confusion on the signs (+, -) for the PV, PMT, and FV values, we always compute the mortgage from the borrower's point of view. The money borrowed is an inflow and entered as a positive value. The monthly payments and final balloon amount are outflows and entered as negative values.

AMOUNT OF MORTGAGE PAYMENT:

Objective: To calculate the monthly payment for an ordinary mortgage.

Information Required:

1. Annual interest rate as a percent.
2. Number of payments.
3. Amount borrowed.

Example:

What is the monthly payment for a 30 year, $75,000 mortgage with a 12 percent annual interest rate compounded monthly?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter years*	30 g, [12x]	360.00
4. Enter annual interest rate*	12 g, [12÷]	1.00
5. Enter amount of mortgage	75000 PV	75,000.00
6. Solve for monthly payment	PMT	- 771.46

*See Chapter 2 for additional information on g, [12X] and g, [12÷].

HP-12C Keystroke Template: Amount of Mortgage Payment:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter years	☐ g, [12x],	☐
4. Enter months*	[ENTER] ☐ + n	☐
5. Enter annual interest rate	☐ g, [12÷]	☐
6. Enter amount of mortgage	☐ PV	☐
7. Solve for monthly payment	PMT	☐

*This keystroke template allows for loans with years and odd months, but for loans with whole year terms omit step 4.

APR OF REGULAR MORTGAGE:

Objective: To compute the Annual Percent Rate (APR) on a regular mortgage.

Information Required:

1. Number of payments.
2. Amount of payment.
3. Amount borrowed.
4. Any fees that must be included in the APR calculation.

Comments:

The APR occasionally differs from the yield of a mortgage because some fees are omitted from the calculation. Fees and points included in the calculation should be deducted from the amount borrowed before computing the interest rate.

Example:

John Doe borrows $60,000 for thirty years from the Any City bank. His monthly payments are $617.17. Assuming no fees or points are paid, what is the APR?

HP-12C Solution

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter years	30 g, [12x]	360.00
4. Enter amount borrowed	60000 PV	60,000
5. Enter monthly payment	617.17 CHS PMT	- 617.17
6. Solve for monthly percent rate	i	1.00
7. Compute APR	12 X	12.00

HP-12C Keystroke Template: Solving for APR of a Regular Mortgage:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter years	☐ g, [12x]	☐
4. Enter amount borrowed	☐ [ENTER]	☐
5. Less Fees if any, otherwise enter a zero.	☐ - PV	☐
6. Enter monthly payment	☐ CHS PMT	☐
7. Solve for monthly percent rate	i	☐
8. Compute APR	12 X	☐

YIELD OF REGULAR MORTGAGE WITH FEES AND POINTS:

Objective: To calculate the yield (annual nominal interest rate) on a regular mortgage.

Information Required:

1. Number of payments.
2. Amount of payment.
3. Amount borrowed.
4. Points as a percent.
5. Amount of fees.

Example:

John Doe borrows $60,000 for thirty years from the Any City Bank. His monthly payments are $617.17. He pays 3 points and a $150 fee to obtain the 12 percent mortgage. What is the yield per month and the annual yield?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter years	30 g, [12x]	360.00
4. Enter amount of monthly payment	617.17 CHS PMT	- 617.17
5. Enter amount borrowed	60000	60,000
6. Enter points	[ENTER] 3%	1,800.00
7. Net cash less points	-	58,200.00
8. Enter fees	150	150.00
9. Net cash received	- PV	58,050.00
10. Solve for monthly yield	i	1.04
11. Annual yield	12 X	12.45

HP-12C Keystroke Template: Yield of Regular Mortgage with Fees and Points:

Procedure	Action Step	Display
1. Clear Calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter years	☐ g, [12x],	☐
4. Enter months*	[ENTER] ☐ + n	☐
5. Enter monthly payments	☐ CHS PMT	☐
6. Enter amount borrowed	☐ [ENTER]	☐
7. Enter points; if none, key in zero	☐ %	☐
8. Net cash less points	-	☐
9. Enter fees; if none, key in zero	☐	☐
10. Net cash received	- PV	☐
11. Solve for monthly yield	i	☐
12. Annual yield	12 X	☐

*This keystroke template allows for loans with years and odd months, but for loans with whole year terms omit step 4.

PRICE OF A REGULAR MORTGAGE - SCHEDULED PAYMENTS ASSUMED:

Objective: To determine the price to pay for a mortgage without a balloon payment to earn a specified annual yield.

Information Required:

1. Annual yield compounded monthly.
2. Number of payments remaining.
3. Current loan balance.
4. Amount of monthly payment.

Comments:

All payments made on schedule without extra payments made on principal.

Example:

You are buying a mortgage and you require a 12.5 percent annual yield compounded monthly. The mortgage has a balance of $37,825.02 with 295 monthly payments of $293.51 remaining. What is the maximum price you can pay and earn the specified yield? The loan has an 8 percent annual rate.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter number of remaining payments	295 n	295.00
4. Enter desired annual yield	12.5 g, [12÷]	1.04
5. Enter amount of monthly payment	293.51 [CHS][PMT]	- 293.51
6. Solve for price of mortgage	PV	26,851.87

HP-12C Keystroke Template: Price of a Regular Mortgage - Scheduled Payments Assumed:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter number of remaining payments	☐ n	☐
4. Enter desired annual yield	☐ g, [12÷]	☐
5. Enter amount of monthly payment	☐ CHS PMT	☐
6. Solve for price of mortgage	PV	☐

PRICE OF A REGULAR MORTGAGE - NONSCHEDULED PAYMENTS MADE:

Objective: To determine the price to pay for a mortgage to earn a specified annual yield when nonscheduled payments have been made.

Information Required:

1. Annual yield compounded monthly.
2. Current loan balance.
3. Amount of monthly payments.
4. Number of scheduled payments remaining.
5. Original mortgage interest rate.

Comments:

Extra payments on principal have been made. This will require computing the number of actual scheduled payments and the partial final payment.

Example:

You are buying an 8 percent mortgage to yield 12.5 percent. The mortgage has a current balance of $35,547.83 with 295 scheduled monthly payments of $293.51. Extra payments to principal have been made. What price should you pay?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter original loan annual rate	8 g, [12÷]	0.67
4. Enter amount of monthly payment	293.51 CHS PMT	- 293.51
5. Enter current balance	35547.83 PV	35,547.83
6. Compute number of remaining payments	n	248.00
7. Recompute remaining balance	FV	26.80
8. Enter desired yield	12.5 g, [12÷]	1.04
9. Solve for price	PV	26,018.31

HP-12C Keystroke Template: Price of a Regular Mortgage - Nonscheduled Payments Made:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter original loan annual rate	☐ g, [12÷]	☐
4. Enter amount of monthly payment	☐ CHS PMT	☐
5. Enter current balance	☐ PV	☐
6. Compute number of remaining payments	n	☐
7. Recompute remaining balance	FV	☐
8. Enter desired yield	☐ g, [12÷]	☐
9. Solve for price	PV	☐

REMAINING BALANCE OF A REGULAR MORTGAGE:

Objective: To determine the remaining balance of a mortgage immediately after a scheduled payment.

Information Required:

1. Annual interest rate compounded monthly.
2. Amount borrowed.
3. Monthly payment.
4. Number of payments.

Assumptions:

This analysis assumes that all payments are made as scheduled with no extra principal payments.

Example:

Debbie Doe borrows $105,000 at 16 percent annually. The payment amount of $1,411.99 is based on a 30-year amortization schedule. The loan, however, must be paid off at the end of five years. What is the remaining balance after the 60th payment?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual rate	16 g [12÷]	1.33
4. Enter amount borrowed	105000 PV	105,000.00
5. Enter amount of payment	1411.99 CHS PMT	- 1,411.99
6. Enter number of payments made	60 n	60.00
7. Compute remaining balance	FV	- 103,908.48

The remaining balance after 60 payments is $103,908.48.

HP-12C Keystroke Template: Remaining Balance of a Regular Mortgage:

Procedure	Action Step		Display
1. Clear calculator	CLX, f, FIN		0.00
2. Set payment switch	g, END		0.00
3. Enter annual rate	☐	g [12÷]	☐
4. Enter amount borrowed	☐	PV	☐
5. Enter payment amount	☐	CHS PMT	☐
6. Enter number of payments made	☐	n	☐
7. Compute remaining balance	FV		☐

TOTAL FINAL PAYMENT FOR A REGULAR MORTGAGE:

Objective: To determine the final total payment required to pay off a regular mortgage.

Information Required:

1. Annual interest rate compounded monthly.
2. Amount borrowed.
3. Number of years for computing payment.
4. Number of years used as basis for monthly payments.

Assumptions:

The payment amount computed in this example is rounded to two decimal places. The calculator, however, will use all the decimal places in the calculated payment amount unless you change the value. So you should reenter the payment amount to the nearest cent before computing the balloon payment, or use the f, RND key to set the internal digits equal to the displayed value.

Example:

Sue Doe borrows $180,000 at 13.4 percent annually. The payments are computed based on a term of 30 years. Sue, however, will pay off the mortgage at the end of 10 years. What is the monthly payment and the final total payment?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual rate	13.4 g [12÷]	1.12
4. Enter number of years for calculating payment	30 g, [12x]	360.00
5. Enter amount	180000 PV	180,000.00
6. Compute payment	PMT	- 2047.59
7. Enter payment to nearest cent	f, RND PMT	- 2047.59
8. Enter number of years for balloon	10 g, [12x]	120.00
9. Compute remaining balance	FV	- 170,605.46
10. Total final payment	RCL PMT +	- 172,653.05

Note: 119 payments of $2047.59 and a final payment of $172,653.05 are required.

HP-12C Keystroke Template: Total Final Payment for a Regular Mortgage:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual rate	☐ g, [12÷]	☐
4. Enter number of years for calculating payment	☐ g, [12x]	☐
5. Enter amount	☐ PV	☐
6. Compute payment	PMT	☐
7. Enter payment rounded to nearest penny	☐ f, RND PMT	☐
8. Enter number of years for balloon	☐ g, [12x]	☐
9. Compute remaining balance	FV	☐
10. Total final payment	RCL PMT +	☐

NUMBER OF PAYMENTS:

Objective: To calculate the number of payments necessary to pay off a mortgage.

Information Required:

1. Amount borrowed.
2. Amount of anticipated payment.
3. Annual interest rate.

Example:

You borrow $50,000 at 11.4 percent with monthly payments of 491.33. The loan was scheduled for 30 years, but you plan to pay $525 per month. How many payments are necessary to pay off the loan?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual rate	11.4 g, [12÷]	0.95
4. Enter amount borrowed	50000 PV	50,000.00
5. Enter payment	525 CHS PMT	- 525.00
6. Compute number of payments	n	249.00
7. Compute number of years	12	20.75
8. Calculate over- or underpayment if 249 payments are made.	FV, FV	163.78
9. Calculate final fractional payment (regular + fractional)	RCL PMT +	- 361.22

There are 248 payments of $525 and a final payment of $361.22.

HP-12C Keystroke Template: Number of Payments:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual rate	[ ] g, [12÷]	[ ]
4. Enter amount borrowed	[ ] PV	[ ]
5. Enter payment	[ ] CHS PMT	[ ]
6. Compute number of payments	n	[ ]
7. Compute number of years	12	[ ]
8. Calculate over- or underpayment for final period	FV, FV	[ ]
9. Calculate final fractional payment (regular + fractional)	RCL PMT +	[ ]

BALLOON MORTGAGES:

In this book we define a "balloon payment" as the amount necessary to pay off a loan immediately after a scheduled regular payment. By using this definition of a balloon payment, we can solve directly for the unknown values using the HP-12C calculator.

The final payment for a mortgage consists of two parts -- the regular payment and the balloon (unpaid balance).

AMOUNT OF PAYMENT:

Objective: To calculate the monthly payment for a mortgage when the balloon is a specified amount.

Information Required:

1. Annual interest rate.
2. Amount borrowed.
3. Number of payments.
4. Specified balloon.

Example:

The ROH Land Company is borrowing $1,500,000 at 13.3 percent. They plan to pay $750,000 as a final balloon payment after making 120 monthly payments (10 years). What is the amount of the regular end-of-month payment?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter number of years	10g, [12x]	120.00
4. Enter annual interest	13.3 g, [12÷]	1.11
5. Enter amount borrowed	1500000 PV	1,500,000.00
6. Enter balloon payment	750000 CHS FV	- 750,000.00
7. Solve for regular payment	PMT	- 19,643.91

HP-12C Keystroke Template: Amount of Payment:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter number of years	☐ g, [12x]	☐
4. Enter annual interest	☐ g, [12÷]	☐
5. Enter amount borrowed	☐ PV	☐
6. Enter balloon payment	☐ CHS FV	☐
7. Solve for regular payment	PMT	☐

AMOUNT OF BALLOON:

Objective: To calculate the balloon payment after a specified number of payments.

Information Required:

1. Number of regular payments.
2. Annual interest rate.
3. Amount borrowed.
4. Amount of regular payment.

Example:

BG is borrowing $840,000 to finance an apartment complex. The annual interest rate is 11.5 percent. He can make monthly payments of $8,800 and he can pay off the loan after 15 years. What is the final balloon amount (remaining balance)?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter number of payment years	15 g, [12x]	180.00
4. Enter annual interest rate	11.5 g, [12÷]	0.96
5. Enter amount borrowed	840000 PV	840,000.00
6. Enter monthly payment	8800 CHS PMT	- 8,800.00
7. Compute balloon	FV	- 482,612.89

HP-12C Keystroke Template: Amount of Balloon:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter number of years	[ ] g, [12x]	[ ]
4. Enter annual interest rate	[ ] g, [12÷]	[ ]
5. Enter amount borrowed	[ ] PV	[ ]
6. Enter monthly payment	[ ] CHS PMT	[ ]
7. Compute balloon	FV	[ ]

APR FOR BALLOON MORTGAGE:

Objective: To calculate the APR for a balloon mortgage.

Information Required:

1. Number of payments.
2. Amount borrowed.
3. Amount of regular and a balloon payment.
4. Any fees that must be included in the APR calculation.

Comments:

The APR occasionally differs from the yield of a mortgage because some fees are omitted from the calculation. Fees and points included in the calculation should be deducted from the amount borrowed before computing the interest rate.

Example:

Ann borrowed $60,000 from the Any City Bank. Her monthly payments are $617.17 and a balloon payment of $43,014.49 is due at the end of 20 years. Assuming no fees or points are paid, what is the APR?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter years	20 g, [12x]	240.00
4. Enter amount borrowed	60000 PV	60,000.00
5. Enter monthly payment	617.17 CHS PMT	- 617.17
6. Enter balloon	43014.49 CHS FV	- 43,014.49
7. Solve for monthly percent rate	i	1.00
8. Compute APR	12 X	12.00

HP-12C Keystroke Template: APR for Balloon Mortgage:

Procedure	Action Step		Display
1. Clear calculator	CLX, f, FIN		0.00
2. Set payment switch	g, END		0.00
3. Enter years		g, [12x]	
4. Enter amount borrowed		[ENTER]	
5. Enter any fees; if none enter zero		- PV	
6. Enter monthly payment		CHS PMT	
7. Enter balloon		CHS FV	
8. Solve for monthly percent rate	i		
9. Compute APR	12 X		

YIELD OF MORTGAGE WITH BALLOON, FEES, AND POINTS:

Objective: To calculate the annual nominal interest rate (yield) on a balloon mortgage.

Information Required:

1. Number of payments.
2. Amount borrowed.
3. Amount of regular payment.
4. Amount of balloon.
5. Points as a percent.
6. Amount of fees.

Example:

Ann borrowed $60,000 from the Any City Bank. She paid 3 points and $150 in fees. Her monthly payments are $617.17 and a balloon payment of $43,014.49 is due at the end of 20 years. The APR is 12 percent. What is the yield on the loan?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter years	20 g, [12x]	240.00
4. Enter amount borrowed	60000 [ENTER]	60,000.00
5. Enter points	3%	1,800.00
6. Amount borrowed less points	-	58,200.00
7. Enter fees	150	150.
8. Net cash	- PV	58,050.00
9. Enter monthly payment	617.17 CHS PMT	- 617.17
10. Enter balloon	43014.49 CHS FV	- 43,014.49
11. Solve for monthly yield	i	1.04
12. Compute annual yield	12 X	12.46

HP-12C Keystroke Template: Yield of Mortgage with Balloon, Fees and Points:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter years	☐ g, [12x]	☐
4. Enter amount borrowed	☐ [ENTER]	☐
5. Enter points; if none enter zero	☐ %	☐
6. Amount borrowed less points	-	☐
7. Enter fees; if none enter zero	☐	☐
8. Net cash	- PV	☐
9. Enter monthly payment	☐ CHS PMT	☐
10. Enter balloon	☐ CHS FV	☐
11. Solve for monthly yield	i	☐
12. Compute annual yield	12 X	☐

YIELD ON LOAN PAID OFF EARLY WITH PENALTIES:

Objective: To calculate the yield on a loan with an early pay off and penalties.

Information Required:

1. Number of payments made before payoff.
2. Amount borrowed.
3. Amount of regular payment.
4. Remaining balance (computed and then used).
5. Amount of points and fees to obtain loan.
6. Amount of penalty for early payoff.

Comments:

The methods for computing early payoff penalties vary. So in this example we assume the penalty is six months interest using simple interest. For other methods, determine the amount of penalty and add it to the remaining balance.

Example:

Ann borrowed $60,000 from the Any City Bank at 12 percent. She paid 3 points and $150 in fees to obtain the loan. She has monthly payments of $617.17 for 20 years and a balloon payment of $43,014.49 in addition to the last payment. For early retirement, six months interest must be paid as a penalty.

What is the yield if Ann pays off the loan after 98 payments?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
A. Compute Remaining Balance		
3. Enter number of payments	98 n	98.00
4. Enter amount borrowed	60000 PV	60,000.00
5. Enter payment amount	617.17 CHS PMT	- 617.17
6. Enter annual loan interest rate	12 g, [12÷]	1.00
7. Compute remaining balance	FV	- 57,164.34
B. Compute Penalty		
8. Compute penalty	RCL i, %, 6 X	- 3,429.86
9. Add to remaining balance	+ FV	- 60,594.20
C. Compute Net Cash Received from Loan and the Yield		
10. Recall Loan amount	RCL PV	60,000.00
11. Enter points	3 %	1,800.00
12. Enter fees	- 150	150.

HP-12C Solution: (continued)

Procedure	Action Step	Display
13. Net cash	- PV	58,050.00
14. Solve for monthly yield	i	1.09
15. Annual yield	12 X	13.06

HP-12C Keystroke Template: Yield on Loan Paid Off Early with Penalties:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
A. Compute Remaining Balance		
3. Enter number of payments	n	
4. Enter amount borrowed	PV	
5. Enter payment amount	CHS PMT	
6. Enter annual loan interest rate	g, [12÷]	
7. Compute remaining balance	FV	
B. Compute Penalty		
8. Enter amount of penalty; if none enter zero		
9. Add to remaining balance	+ FV	
C. Compute Net Cash Received from Loan and the Yield		
10. Recall loan amount	RCL PV	
11. Enter points as a percent; if none enter zero	%	
12. Enter fees; if none enter zero	-	
13. Net cash	- PV	
14. Solve for monthly yield	i	
15. Annual yield	12 X	

NUMBER OF PAYMENTS FOR LOAN WITH BALLOON:

Objective: To compute the number of payments when the balloon amount is specified.

Information Required:

1. Annual interest rate.
2. Amount borrowed.
3. Amount of regular payment.
4. Amount of balloon.

Comment:

The number of payments when computed is normally a fraction. Because the HP-12C rounds the computed number of payments, the amount of the final payment should be computed. This example shows the procedure for determining the final payment.

Example:

Jean is longing for a lake home. So she borrows $70,000 at 12.5 percent. She can afford to pay $800 a month and can make a balloon payment of $25,000 at the end of the loan. How many regular payments are necessary and what is the final total payment?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	12.5 g, [12÷]	1.04
4. Enter amount borrowed	70000 PV	70,000.00
5. Enter monthly payment	800 CHS PMT	- 800.00
6. Enter balloon	25000 CHS FV	- 25,000.00
7. Compute number of monthly payments	n	196.00
8. Number of payments at $800 per month	1 - n	195.00
9. Compute balance at beginning of final month	FV	- 25,501.27
10. Compute amount of Total final payment	RCL FV, CHS, PV 0 PMT, 1 n, FV	- 25,766.90
11. Enter balloon	25000	25,000.
12. Final payment at end of month 196, net of balloon	+	- 766.90

Note: Payments of $800 will be made for 195 months with a revised $766.90 payment at the end of month 196 along with the balloon at $25,000. The final total payment at the end of month 196 is $25,766.90.

HP-12C Keystroke Template: Number of Payments for Loan with Balloon:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	g, [12÷]	
4. Enter amount borrowed	PV	
5. Enter monthly payment	CHS PMT	
6. Enter balloon	CHS FV	
7. Compute number of payments	n	
8. Number of payments at amount entered in Step 5	1 - n	
9. Compute balance at beginning of final month	FV	
10. Compute amount of Total final payment	RCL, FV, CHS, PV 0 PMT, 1 n, FV	
11. Enter balloon		
12. Final payment net of balloon	+	

PRICE OF MORTGAGE WITH BALLOON:

Objective: To determine the price of a mortgage with a balloon payment and a specified yield.

Information Required:

1. Number of remaining payments.
2. Required annual yield.
3. Monthly payment.
4. Balloon.

Comment:

This analysis assumes all payments were made as scheduled.

Example:

Your company is purchasing a mortgage from the Any City Bank. The mortgage has a current balance of $58,597.73 with 180 monthly payments of $617.17 and a balloon of $43,014.49. How much should be paid for the mortgage to earn an annual yield of 13.5 percent?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter number of remaining payments	180 n	180.00
4. Enter annual yield	13.5 g, [12÷]	1.13
5. Enter monthly payment	617.17 CHS PMT	- 617.17
6. Enter balloon	43014.49 CHS FV	- 43,014.49
7. Solve for price	PV	53,278.30

HP-12C Keystroke Template: Price of Mortgage with Balloon:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter number of remaining payments	☐ n	☐
4. Enter annual yield	☐ g, [12÷]	☐
5. Enter monthly payment	☐ CHS PMT	☐
6. Enter balloon	☐ CHS FV	☐
7. Solve for price	PV	☐

THE SIX FUNCTIONS OF ONE DOLLAR:

In this section of Chapter 4, we show you how to calculate the six functions of one dollar. The six functions of one dollar are the column heading of "Compound Interest and Annuity" tables. The columns are often labeled as:

Column 1: Future value of one dollar

Column 2: Accumulation of one dollar per period

Column 3: Sinking fund factor

Column 4: Present value of one dollar

Column 5: Present value of one dollar per period

Column 6: Installment to amortize one dollar per period

The following four examples are included in this chapter for the convenience of readers who are familiar with "Six Functions of One Dollar" tables and want to analyze problems in this way or want to reconcile the calculator procedures with the tables.

COLUMN NO. 1: FUTURE VALUE OF ONE DOLLAR:

Objective: To calculate the future value of one dollar.

Information Required:

1. Annual interest rate as a percent.
2. Number of years and months for compounding.

Example:

What is the future value of one dollar invested for five years and four months with an 18 percent annual interest rate compounded monthly?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set switch	g, END	0.00
3. Enter annual interest rate	18 g, [12÷]	1.50
4. Enter 1 for present value	1 [CHS] PV	- 1.00
5. Enter number of years	5 g, [12x]	60.00
6. Add months	[ENTER] 4 + n	64.00
7. Solve for future value of $1: Col. 1	FV	2.59
8. To see ALL decimals	f, 9	2.593144416
	f, 2	2.59

HP-12C Keystroke Template: Column No. 1: Future Value of One Dollar:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	[ ] g, [12÷]	[ ]
4. Enter 1 for present value	1 [CHS] PV	- 1.00
5. Enter number of years	[ ] g, [12x]	[ ]
6. Add months*	[ENTER] [ ] + n	[ ]
7. Solve for future value of $1: Col. 1	FV	[ ]

*This keystroke template allows for loans with years and odd months, but for loans with whole year terms omit Step 6.

COLUMN NO. 4: PRESENT VALUE OF ONE DOLLAR:

Objective: To calculate present value of one dollar.

Information Required:

1. Annual interest rate as a percent.
2. Number of years and months for compounding.

Example:

What is the present value of one dollar received after 7 years and 2 months using an interest rate of 8.4 percent annual discounted monthly?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	8.4, g, [12÷]	0.70
4. Enter 1 for future value	1 [CHS] FV	- 1.00
5. Enter number of years	7 g, [12x]	84.00
6. Add months	[ENTER] 2 + n	86.00
7. Solve for present value of $1: Col. 4	PV	0.55
8. To see ALL decimals	f, 9	0.548864993
9. Set to 2 places	f, 2	0.55

HP-12C Keystroke Template: Column No. 4: Present Value of One Dollar:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	☐ g, [12÷]	☐
4. Enter 1 for future value	1 [CHS] FV	- 1.00
5. Enter number of years	☐ g, [12x]	☐
6. Add months*	[ENTER] ☐ + n	☐
7. Solve for present value of $1: Col. 4	PV	☐
8. To see all decimals	f, 9	☐

*This keystroke template allows for loans with years and odd months, but for loans with whole year terms omit Step 6.

COLUMN NO. 2: ACCUMULATION OF ONE DOLLAR PER PERIOD
COLUMN NO. 3: SINKING FUND FACTOR:

Objective: To calculate the accumulation of one dollar per month and the sinking fund factor.

Information Required:

1. Annual interest rate as a percent.
2. Number of years and months.

Example:

What is the future value and sinking fund factor of one dollar invested at the end of each month after 5 1/2 years if the annual interest rate is 11 percent compounded monthly?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	11 g, [12÷]	0.92
4. Enter 1 for payment	1 [CHS] PMT	- 1.00
5. Enter number of years	5 g [12x],	60.00
6. Add months*	[ENTER] 6 + n	66.00
7. Solve for accumulation of $1 per period: Col. 2	FV	90.13
8. To see all decimals	f, 9	90.13222557
9. Solve for sinking fund factor: Col. 3	1/x	0.011094811

*This keystroke template allows for loans with years and odd months, but for loans with whole year terms omit Step 6.

HP-12C Keystroke Template: Column No. 2: Accumulation of One Dollar Per Period
Column No. 3: Sinking Fund Factor:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	☐ g, [12÷]	☐
4. Enter 1 for payment	1 [CHS] PMT	- 1.00
5. Enter number of years	☐ g, [12x]	☐
6. Add months	[ENTER] ☐ + n	☐
7. Solve for accumulation of $1 per period: Col. 2	FV	☐
8. To see all decimals	f, 9	☐
9. Solve for sinking fund factor: Col. 3	1/x	☐

COLUMN NO. 5: PRESENT VALUE OF ONE DOLLAR PER PERIOD
COLUMN NO. 6: INSTALLMENT TO AMORTIZE ONE DOLLAR PER PERIOD:

Objective: To calculate the present value of one dollar per period and the installment to amortize one dollar per period.

Information Required:

1. Annual interest rate as a percent.
2. Number of years and months.

Example:

What is the present value and sinking fund factor of one dollar received at the end of each month for 7 years and 8 months if the annual interest rate is 10.75 percent compounded monthly?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	10.75 g, [12÷]	0.90
4. Enter 1 for payment	1 [CHS] PMT	- 1.00
5. Enter number of years	7 g, [12x]	84.00
6. Add months*	[ENTER] 8 + n	92.00
7. Solve for present value of $1 per period: Col. 5	PV	62.49
8. To see all decimals	f, 9	62.48/8/123
9. Solve for installment to amortize $1 per period: Col. 6	1/x	0.016003106

*This keystroke template allows for loans with years and odd months, but for loans with whole year terms omit Step 6.

HP-12C Keystroke Template: Column No. 5: Present Value of One Dollar Per Period
Column No. 6: Installment to Amortize One Dollar Per Period:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	[ ] g, [12÷]	[ ]
4. Enter 1 for payment	1 [CHS] PMT	- 1.00
5. Enter number of years	[ ] g, [12x]	[ ]
6. Add months	[ENTER] [ ] + n	[ ]
7. Solve for present value of $1 per period: Col. 5	PV	[ ]
8. To see all decimals	f, 9	[ ]
9. Solve for installment to amortize $1 per period: Col. 6	1/x	[ ]

5

LEASES

In this chapter we show you how to compute lease values. We assume that all payments are made at the _beginning_ of each payment period. We also assume that the residual value is the value of the asset (market or stated value) at the _end_ of the final payment period (after the last payment). For situations where lease payments occur at the end of each period, use the mortgage applications. The examples in this chapter will treat the current market value as a cash outflow, while the payments and the residual value will be cash inflows. The keystrokes and templates are designed to allow different payment periods, i.e., monthly, quarterly, etc. But the number of compounding periods per year _must_ equal the number of payments per year.

ORDINARY LEASES

The first section deals with leases requiring regular payments at the beginning of each compounding period, with no advance payments.

SOLVE FOR LEASE PAYMENT:

Objective: To compute the lease payment.

Information Required:

1. Annual interest rate.
2. Current market value of lease.
3. Number of payments.
4. Residual value of lease.
5. Number of payments per year.

Example:

The Land Ho Company is leasing an asset from the Big Company. The Big Company wants to earn 15 percent annually for ten years. At the end of ten years the residual value should be $25,000. The asset's current market value is $105,000. What is the monthly payment?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, BEG	0.00
3. Enter total number of payments	120 n	120.00
4. Enter annual interest rate as percent	15 [ENTER]	15.00
5. Enter number of payments per year	12 ÷ i	1.25
6. Enter current market value	105000 CHS PV	- 105,000.00
7. Enter residual value	25000 FV	25,000.00
8. Solve for payment	PMT	1,583.39

HP-12C Keystroke Template: Solve for Lease Payment:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, BEG	0.00
3. Enter total number of payments	☐ n	☐
4. Enter annual interest rate as percent	☐ [ENTER]	☐
5. Enter number of payments per year	☐ ÷ i	☐
6. Enter current market value	☐ CHS PV	☐
7. Enter residual value	☐ FV	☐
8. Solve for payment	PMT	☐

SOLVE FOR YIELD ON LEASE:

Objective: To compute the yield on a lease.

Information Required:

1. Number of payments.
2. Current market value of asset.
3. Payment amount.
4. Residual value.
5. Number of payments per year.

Example:

An asset is leased for 84 months with beginning-of-month payments of $3,312. The asset can be currently purchased for $240,000. The residual value at the end of the lease should be $150,000. What is the annual yield compounded monthly?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, BEG	0.00
3. Enter total number of payments	84 n	84.00
4. Enter current value	240000 CHS PV	- 240,000.00
5. Enter payment	3312 PMT	3,312.00
6. Enter residual value	150000 FV	150,000.00
7. Solve for yield per payment period	i	1.12
8. Enter number of payments per year	12	12.
9. Compute annual yield	X	13.50

HP-12C Keystroke Template: Solve for Yield on Lease:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, BEG	0.00
3. Enter total number of payments	☐ n	☐
4. Enter current value	☐ CHS PV	☐
5. Enter payment	☐ PMT	☐
6. Enter residual value	☐ FV	☐
7. Solve for yield per payment period	i	☐
8. Enter number of payments per year	☐	☐
9. Compute annual yield	X	☐

SOLVE FOR PRESENT VALUE OF LEASE:

Objective: To compute the present value of a lease.

Information Required:

1. Number of payments.
2. Annual interest rate.
3. Amount of payment.
4. Amount of residual value.
5. Number of payments per year.

Example:

An asset is leased for six years at 18 percent compounded quarterly. The quarterly payments are $5,711.93 and the residual value is $10,000. What is the present value of the lease?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, BEG	0.00
3. Enter total number of payments	24 n	24.00
4. Enter annual interest rate as a percent	18 [ENTER]	18.00
5. Enter number of payments per year	4 ÷ i	4.50
6. Enter payment	5711.93 PMT	5,711.93
7. Enter residual value	10000 FV	10,000.00
8. Solve for present value	PV	- 90,000.06

HP-12C Keystroke Template: Solve for Present Value of Lease:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, BEG	0.00
3. Enter total number of payments	☐ n	☐
4. Enter annual interest rate as a percent	☐ [ENTER]	☐
5. Enter number of payments per year	☐ ÷ i	☐
6. Enter payment	☐ PMT	☐
7. Enter residual value	☐ FV	☐
8. Solve for present value	PV	☐

SOLVE FOR RESIDUAL VALUE:

Objective: To compute the residual value of a lease.

Information Required:

1. Number of payments.
2. Annual interest rate.
3. Amount of payment.
4. Current value of asset.
5. Number of payments per year.

Example:

The Big Bert Company is leasing an asset costing $65,000 to the Little Joe Company for five years. The monthly payments are $1,508.56 and Big Bert expects to earn 17 percent annually compounded monthly from the lease. What residual value was used to compute the monthly payment amount?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, BEG	0.00
3. Enter total number of payments	60 n	60.00
4. Enter annual interest rate as a percent	17 [ENTER]	17.00
5. Enter number of payments per year	12 ÷ i	1.42
6. Enter current asset value	65000 CHS PV	- 65,000.00
7. Enter payment	1508.56 PMT	1,508.56
8. Solve for residual value	FV	7,999.90

HP-12C Keystroke Template: Solve for Residual Value:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, BEG	0.00
3. Enter total number of payments	☐ n	☐
4. Enter annual interest rate as a percent	☐ [ENTER]	☐
5. Enter number of payments per year	☐ ÷ i	☐
6. Enter current asset value	☐ CHS PV	☐
7. Enter payment	☐ PMT	☐
8. Solve for residual values	FV	☐

SOLVE FOR NUMBER OF LEASE PAYMENTS:

Objective: To compute the number of lease payments when other values are known and to compute the amount of the last regular payment.

Information Required:

1. Annual yield.
2. Number of payments per year.
3. Amount of payment.
4. Current value of asset.
5. Residual value.

Example:

An asset having a current value of $65,000 is leased with monthly payments of $1,349.85. At the end of the lease the residual value will be $5,000. Assuming the yield on the lease is 19 percent annually, compounded monthly, how many payments will be made? How much is the last regular payment?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, BEG	0.00
3. Enter annual interest rate as a percent	19 [ENTER]	19.00
4. Enter number of payments per year	12 ÷ i	1.58
5. Enter current value	65000 CHS PV	- 65,000.00
6. Enter payment	1349.85 PMT	1,349.85
7. Enter residual value	5000 FV STO 1	5,000.00
8. Solve for number of payments	n, n	85.00
9. Enter number of whole payments	1 - n	84.00
10. Compute	FV STO 2	5,766.31
11. Compute partial lease payment	RCL 1 FV 0 PMT 1 n PV RCL 2 +	844.24

This lease has 85 regular payments, but 84 are $1,349.85; the 85th is $844.24. The 85th payment occurs at the beginning of the period, while the residual occurs at the end of the period.

HP-12C Keystroke Template: Solve for Number of Lease Payments:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, BEG	0.00
3. Enter annual interest rate as a percent	[ ] [ENTER]	[ ]
4. Enter number of payments per year	[ ] ÷ i	[ ]
5. Enter current value	[ ] CHS PV	[ ]
6. Enter payment	[ ] PMT	[ ]
7. Enter residual value	[ ] FV STO 1	[ ]
8. Solve for number of payments	n, n	[ ]
9. Enter number of whole payments	1 - n	[ ]
10. Compute	FV STO 2	[ ]
11. Compute partial lease payment	RCL 1 FV 0 PMT 1 n PV RCL 2 +	[ ]

LEASES WITH ADVANCE PAYMENTS

In this section we compute the present value, payment amount, and interest rate for leases with advance payments.

The grouped cash-flow keys are used to solve for the present value and interest rate. Because the grouped cash-flow keys assume payments occur at the end of each period, the payment switch is set to g, [END]. This setting is used as a reminder.

A special keystroke routine is used to solve for the payment.

For this type of lease, we ask you to enter the total number of payments made at the start of the lease rather than advance payments so as to avoid confusion. We also assume that a regular payment is due at the start of the lease.

SOLVING FOR THE PRESENT VALUE OF A LEASE WITH ADVANCE PAYMENTS:

Objective: To compute the present value of a lease with advance payments.

Information Required:

1. Annual interest rate.
2. Number of payments.
3. Number of payment periods per year.
4. Number of payments made at start of lease.
5. Payment amount.
6. Residual value.

Example:

An asset leased for 3 years has a residual value of $25,000. The monthly payments are $3,650 and the lessee agrees to make three advance payments in addition to the first regular payment. These advance payments replace the last three regular payments. Assuming the lessor earns 16 percent annually, compounded monthly, what is the present value of the lease?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate as a percent	16 [ENTER]	16.00
4. Enter number of payments per year	12 ÷ i	1.33
5. Enter number of payments made at start of lease	4 [ENTER] STO 5	4.00
6. Enter payment	3650 STO 6, X g, CF_0	14,600.00
7. Enter payment	RCL 6 g, CF_j	3,650.00
8. Enter total number of payments	36	36.
9. Less payments at start of lease	RCL 5 - g, N_j	32.00
10. Enter zero	0 g, CF_j	0.00
11. Enter number of payments made at start, less 1	RCL 5, 1 - g, N_j	3.00
12. Enter residual	25000 g, CF_j	25,000.00
3. Solve for present value	f, NPV	124,692.83

HP-12C Keystroke Template: Solving for the Present Value of a Lease with Advance Payments:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate as a percent	[ ] [ENTER]	[ ]
4. Enter number of payments per year	[ ] ÷ i	[ ]
5. Enter number of payments made at start of lease	[ ] [ENTER] STO 5	[ ]
6. Enter payment	[ ] STO 6, X g, CF_0	[ ]
7. Enter payment	[ ] RCL 6 g, CF_j	[ ]
8. Enter total number of payments	[ ]	[ ]
9. Less payments at start	RCL 5 - g, N_j	[ ]
10. Enter zero	0 g, CF_j	0.00
11. Enter number of payments made at start, less 1	RCL 5, 1 - g, N_j	[ ]
12. Enter residual	[ ] g, CF_j	[ ]
13. Solve for present value	f, NPV	[ ]

SOLVING FOR THE INTEREST RATE OF A LEASE WITH ADVANCE PAYMENTS:

Objective: To compute the interest rate on a lease with advance payments.

Information Required:

1. Total number of payments.
2. Number of payments made at start of lease.
3. Number of payment periods per year.
4. Present value of lease or current value of asset.
5. Payment amount.
6. Residual value.

Example:

An asset leased for four years has a residual value of $26,000 and a current value of $133,000. Assuming the lessee agrees to make two advance payments (replacing the last two payments) in addition to the first regular monthly payment of $3,700, what is the annual yield on the lease?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter current value of asset	133000 [ENTER]	133,000.00
4. Enter number of payments made at start of lease	3 STO 5	3.00
5. Enter payment	3700 STO 6	3,700.00
6. Calculate adjusted present value	X, -, CHS, g, CF_0	- 121,900.00
7. Enter payment	RCL 6 g, CF_j	3,700.00
8. Enter total number of payments	48	48.
9. Less payments at start of lease	RCL 5, -, g, N_j	45.00
10. Enter zero	0 g, CF_j	0.00
11. Enter number of payments made at start less 1	RCL 5, 1 - g, N_j	2.00
12. Enter residual	26000 g, CF_j	26,000.00
13. Solve for interest rate per payment period	f, IRR	1.90
14. Enter payments per year	12	12.
15. Compute annual interest rate	X	22.75

HP-12C Keystroke Template: Solving for the Interest Rate of a Lease with Advance Payments:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter current value of asset	[ ] [ENTER]	[ ]
4. Enter number of payments made at start of lease	[ ] STO 5	[ ]
5. Enter payment	[ ] STO 6	[ ]
6. Calculate adjusted present value	[ ] X, -, CHS, g, CF_0	[ ]
7. Enter payment	RCL 6 g, CF_j	[ ]
8. Enter total number of payments	[ ]	[ ]
9. Less payments at start of lease	RCL5, -, g, N_j	[ ]
10. Enter zero	0 g, CFj	0.00
11. Enter number of payments made at start less 1	RCL 5, 1 - g, N_j	[ ]
12. Enter residual	[ ] g, CF_j	[ ]
13. Solve for interest rate per payment period	f, IRR	[ ]
14. Enter payments per year	[ ]	[ ]
15. Compute annual interest rate	X	[ ]

SOLVING FOR THE PAYMENT AMOUNT OF A LEASE WITH ADVANCE PAYMENTS

Objective: To compute the payment for a lease with advance payment and residual value.

Information Required:

1. Total number of payments.
2. Number of payments made at start of lease.
3. Number of payments per year.
4. Present value or current value of asset.
5. Annual interest rate.
6. Residual value.

Example:

An asset leased for 5 years has a residual value of $23,000 and a current value of $140,000. Assuming that the lessee agrees to make 3 advance payments (replacing the last three) in addition to the first regular payment, and the lessor earns 18 percent annually compounded monthly, what is the monthly payment?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	18 [ENTER]	18.00
4. Enter number of payments per year	12 ÷ i	1.50
5. Enter a negative one	1 CHS PMT	- 1.00
6. Enter total number of payments	60 STO 1	60.00
7. Less payments made at start of lease	4 STO 2 - n	56.00
8. Compute	PV STO + 2	37.71
9. Enter residual value	23000 FV	23,000.00
10 Compute present value of residual	0 PMT, RCL 1, n PV	- 9,413.81
11. Enter current value	140000	140,000.
12. Compute payment	+ RCL 2 ÷	3,131.12

HP-12C Keystroke Template: Solving for the Payment Amount of a Lease with Advance Payments:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	[ ] [ENTER]	[ ]
4. Enter number of payments per year	[ ] ÷ i	[ ]
5. Enter a negative one	1 CHS PMT	- 1.00
6. Enter total number of payments	[ ] STO 1	[ ]
7. Less payments made at start of lease	[ ] STO 2 - n	[ ]
8. Compute	PV STO + 2	[ ]
9. Enter residual value	[ ] FV	[ ]
10. Compute present value of residual	0 PMT, RCL 1, n PV	[ ]
11. Enter current value	[ ]	[ ]
12. Compute payment	+ RCL 2 ÷	[ ]

LEASES WITH VARIABLE OR GROUPED PAYMENTS

Some leases have variable payments; others require equal payments within certain portions of the lease's life -- $500 a month for the first year, $600 for the second and so on. We identify these payment patterns as variable or grouped. The HP-12C can solve for the net present value or the interest rate when these payment patterns exist.

The keys [CF_0], [CF_j] and [N_j] are used to enter the payments. The net cash flow at the beginning of the first or zero period is entered using the [CF_0] key. The payment at the end of each payment period is entered using the [CF_j] key. If the same payment amount occurs in sequence, enter the number of payments using the [N_j] key. If the payment amount is different each period, omit the [N_j] key entry. The calculator automatically enters a 1 for [N_j] if it is not pressed.

When more than 99 payments have the same value, then you must enter the payments in groups of 99 or less. For example, if 140 payments of $1,500 are made, then the entry sequence is:

1500 g, CF_j

99 g, N_j

1500 g, CF_j

41 g, N_j

In this section, we assume all actual payments occur at the beginning of each payment period. Thus, you must adjust the [CF_j] entry sequence because the calculator assumes payments occur at the end of each period.

When computing the present value of a lease, the first payment is entered using the [CF_0] key. The other payments are entered using the [CF_j] and [N_j] keys. Remember, we assume the first lease payment occurs at the beginning of the first period.

When computing the interest rate, the payment occurring at the beginning of the first payment period must be subtracted from the current asset value. This step is necessary because both the current value (usually an outlay) and the first payment (an inflow) occur at the beginning of the first period. For example, assume a current value of $85,000 and a first payment of $3,000. The amount entered using [CF_0] is -82000 (-85000 + 3000).

You can enter 20 variable or grouped cash flows. The keystroke templates show entries for 20 payments or groups, but if you have fewer payments then go to the next step.

We show the payment switch set to g, [END] in the examples, to remind you that the calculator assumes all payments occur at the end of each payment period. Setting the switch to g, [BEG] does not change the calculation procedure for f, [NPU] or f, [IRR].

SOLVING FOR THE PRESENT VALUE OF A LEASE WITH VARIABLE OR GROUPED PAYMENTS:

Objective: To compute the present value of a lease with variable or grouped payments and a residual value.

Information Required:

1. Payment amounts.
2. Annual interest rate.
3. Number of payments per year.
4. Residual value.

Comments:

We assume the lease payments occur at the beginning of each payment period. So the first payment must be stored in CF_0 since it occurs at the beginning of the first period.

Example:

What is the present value of this lease (payments made at the beginning of each period) using an 18 percent discount rate? Payments are made quarterly.

Quarters	Number of Grouped Payments	Quarterly Payment
1	1	$ 5,000
2 - 4	3	5,000
5 - 8	4	6,000
9	1	7,000
10	1	8,500
11	1	10,000
12	1	11,000

The residual value at the end of the three years will be $95,000.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter annual discount rate	18 [ENTER]	18.00
4. Enter number of payments per year	4 ÷ i	4.50
5. Enter 1st payment	5000 g, CF_0	5,000.00
6. a) Enter payment for quarters 2, 3, and 4	5000 g, CF_j	5,000.00
b) Enter number	3 g, N_j	3.00
7. a) Enter payment for quarters 5, 6, 7, and 8	6000 g, CF_j	6,000.00
b) Enter number	4 g, N_j	4.00
8. Enter payment for quarter 9	7000 g, CF_j	7,000.00
9. Enter payment for quarter 10	8500 g, CF_j	8,500.00
10. Enter payment for quarter 11	10000 g, CF_j	10,000.00
11. Enter payment for quarter 12	11000 g, CF_j	11,000.00
12. Enter residual value	95000 g, CF_j	95,000.00
13. Solve for present value	f, NPV	117,484.75

HP-12C Keystroke Template: Solving for the Present Value of a Lease with Variable or Grouped Payments:

Procedure	Action Step		Display
1. Clear calculator	f, REG		0.00
2. Set payment switch	g, END		0.00
3. Enter annual discount rate	[ ]	[ENTER]	[ ]
4. Enter number of payments per year	[ ]	÷ i	[ ]
5. Enter amount of first payment	[ ]	g, CF_0	[ ]

6. Enter remaining payments or groups of payments. Enter payment using [CF_j] and number of payments using [N_j] (omit N_j entry if n = 1). After entering last payment, go to step #7.

a)	[ ]	g, CF_j	[ ]	g, N_j	[ ]
b)	[ ]	g, CF_j	[ ]	g, N_j	[ ]
c)	[ ]	g, CF_j	[ ]	g, N_j	[ ]
d)	[ ]	g, CF_j	[ ]	g, N_j	[ ]
e)	[ ]	g, CF_j	[ ]	g, N_j	[ ]
f)	[ ]	g, CF_j	[ ]	g, N_j	[ ]
g)	[ ]	g, CF_j	[ ]	g, N_j	[ ]
h)	[ ]	g, CF_j	[ ]	g, N_j	[ ]
i)	[ ]	g, CF_j	[ ]	g, N_j	[ ]
j)	[ ]	g, CF_j	[ ]	g, N_j	[ ]
k)	[ ]	g, CF_j	[ ]	g, N_j	[ ]
l)	[ ]	g, CF_j	[ ]	g, N_j	[ ]
m)	[ ]	g, CF_j	[ ]	g, N_j	[ ]

HP-12C Keystroke Template: (continued)

Procedure		Action Step		Display
n)	[ ] g, CF_j	[ ]	g, N_j	[ ]
o)	[ ] g, CF_j	[ ]	g, N_j	[ ]
p)	[ ] g, CF_j	[ ]	g, N_j	[ ]
q)	[ ] g, CF_j	[ ]	g, N_j	[ ]
r)	[ ] g, CF_j	[ ]	g, N_j	[ ]
s)	[ ] g, CF_j	[ ]	g, N_j	[ ]
7.	Enter residual value	[ ]	g, CF_j	[ ]
8.	Solve for present value	f, NPV		[ ]

SOLVING FOR THE INTEREST RATE OF A LEASE WITH VARIABLE OR GROUPED PAYMENTS:

Objective: To compute the interest rate of a lease with variable or grouped payments and a residual value.

Information Required:

1. Current value of asset.
2. Payment amounts.
3. Number of payments per year.
4. Residual value.

Comments:

We assume the lease payments occur at the beginning of each period. So the first payment must be subtracted from the asset's current value because both occur at the beginning of the first period.

Example:

You lease an asset having a current value of $61,000 to the SPAD Co. The payments are due on the first of each month for three years. The residual value at the end of three years should be $43,000. What return (interest rate) are you earning on this leased asset? The scheduled payments are:

Months	Number of Grouped Payments	Monthly Payments
1	1	$1,000
2 - 12	11	1,000
13 - 24	12	1,500
25 - 30	6	2,000
31 - 36	6	2,500

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter current value	61000 [ENTER]	61,000.00
4. Enter amount of first payment	1000	1,000.
5. Net cash flow at start of first period	- CHS g, CF_0	- 60,000.00
6. Enter rest of payments for year 1		
a) payment	1000 g, CF_j	1,000.00
b) number	11 g, N_j	11.00
7. Enter payments for year 2		
a) payment	1500 g, CF_j	1,500.00
b) number	12 g, N_j	12.00
8. Enter payments for first half of year 3		
a) payment	2000 g, CF_j	2,000.00
b) number	6 g, N_j	6.00
9. Enter payment for second half of year 3		
a) payment	2500 g, CF_j	2,500.00
b) number	6 g, N_j	6.00
10. Enter residual value	43000 g, CF_j	43,000.00
11. Solve for monthly interest rate	f, IRR	1.91
12. Enter number of payments per year	12	12
13. Annual interest rate	X	22.87

HP-12C Keystroke Template: Solving for the Interest Rate of a Lease with Variable or Grouped Payments:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter current value	[ ] [ENTER]	[ ]
4. Enter amount of first payment	[ ]	[ ]
5. Net cash flow at start of first period	- CHS g, CF_0	[ ]

6. Enter remaining payments or groups of payments. Enter payment using [CF_j] and number of payments using [N_j]. (Omit N_j entry if n = 1.) After entering last payment, go to Step #7.

a)	[ ] g, CF_j	[ ] g, N_j	[ ]
b)	[ ] g, CF_j	[ ] g, N_j	[ ]
c)	[ ] g, CF_j	[ ] g, N_j	[ ]
d)	[ ] g, CF_j	[ ] g, N_j	[ ]
e)	[ ] g, CF_j	[ ] g, N_j	[ ]
f)	[ ] g, CF_j	[ ] g, N_j	[ ]
g)	[ ] g, CF_j	[ ] g, N_j	[ ]
h)	[ ] g, CF_j	[ ] g, N_j	[ ]
i)	[ ] g, CF_j	[ ] g, N_j	[ ]
j)	[ ] g, CF_j	[ ] g, N_j	[ ]
k)	[ ] g, CF_j	[ ] g, N_j	[ ]
l)	[ ] g, CF_j	[ ] g, N_j	[ ]
m)	[ ] g, CF_j	[ ] g, N_j	[ ]

Keystroke Template: (continued)

Procedure	Action Step	Display
n) [ ] g, CF_j	[ ] g, N_j	[ ]
o) [ ] g, CF_j	[ ] g, N_j	[ ]
p) [ ] g, CF_j	[ ] g, N_j	[ ]
q) [ ] g, CF_j	[ ] g, N_j	[ ]
r) [ ] g, CF_j	[ ] g, N_j	[ ]
s) [ ] g, CF_j	[ ] g, N_j	[ ]
7. Enter residual value	[ ] g, CF_j	[ ]
8. Solve for interest rate per payment period	f, IRR	[ ]
9. Enter number of payments per year	[ ]	[ ]
10. Annual interest rate	X	[ ]

6

DEPRECIATION

The applications in this chapter show you how to use the HP-12C's built-in depreciation programs, and how to solve for partial-year depreciation using manual procedures. The calculator will compute the three common methods of depreciation -- straight line, sum-of-years'-digits, and declining balance. Since the internal routines compute whole year depreciation, however, an alternative procedure must be employed for partial years. You have two choices for partial-year calculations: you can use the programs described on pages 153-166 of the HP-12C Owner's Handbook and Problem-Solving Guide, or you can use the manual procedures described in this chapter. Since the internal depreciation routines are easy to execute, solution forms are not provided.

STRAIGHT LINE DEPRECIATION: CALCULATOR INTERNAL ROUTINE:

Objective: To compute the annual depreciation expense using the S.L. depreciation method.

Information Required:

1. Asset cost.
2. Salvage.
3. Life in years.

Example:

Compute the annual depreciation expense for an asset costing $75,000 with a salvage value of $5,000, and a five year life.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Enter cost	75000 PV	75,000.00
3. Enter salvage	5000 FV	5,000.00
4. Enter life in years	5 n	5.00
5. Enter year for depreciation expense calculation	1	1.
6. Compute depreciation expense	f, SL	14,000.00

HP-12C Solution: (continued)

Procedure	Action Step	Display
7. Compute remaining balance for year 1	[X≷Y]	56,000.00
8. Repeat steps 5, 6 and 7 for each year		
a. year 2 depreciation expense	2, f, SL	14,000.00
b. remaining balance	[X≷Y]	42,000.00

STRAIGHT LINE DEPRECIATION: MANUAL SOLUTION:

Objective: To compute the periodic depreciation expense for partial and whole years using the S.L. depreciation method.

Information Required:

1. Asset cost.
2. Salvage.
3. Life.
4. Number of months depreciated in the first year.

Example:

An asset costing $80,000 with a $9,000 salvage value is purchased on May 1, 1980. Eight months of depreciation will be taken the first year. The asset has a seven year life.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Enter cost	80000 [ENTER]	80,000.00
3. Enter salvage	9000 -	71,000.00
4. Enter life	7 ÷ STO 0	10,142.86
5. Enter months of depreciation in first year	8 STO 1	8.00
6. Compute first year depreciation	RCL 0 RCL 1 X 12 ÷	6,761.90
7. Annual depreciation years 2 through 7	RCL 0	10,142.86
8. Partial depreciation year 8	RCL 0, 12, RCL 1 - X 12 ÷	3,380.95

HP-12C Keystroke Template: Computing Straight Line Depreciation:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Enter cost	☐ [ENTER]	☐
3. Enter salvage	☐ -	☐
4. Enter life	☐ ÷ STO 0	☐
5. Enter months of depreciation in first year	☐ STO 1	☐
6. Compute first year depreciation	RCL 0 RCL 1 X 12 ÷	☐
7. Annual whole year depreciation	RCL 0	☐
8. Partial depreciation for last year	RCL 0, 12, RCL 1 - X 12 ÷	☐

SUM-OF-THE-YEARS'-DIGITS DEPRECIATION: CALCULATOR INTERNAL ROUTINE:

Objective: To compute the annual depreciation expense using the S.Y.D. depreciation method.

Information Required:

1. Asset cost.
2. Salvage.
3. Life in years.

Example:

Compute the annual depreciation expense for an asset costing $75,000 with a salvage value of $5,000, and a five year life.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Enter cost	75000 PV	75,000.00
3. Enter salvage	5000 FV	5,000.00
4. Enter life in years	5 n	5.00
5. Enter year for depreciation expense calculation	1	1.
6. Compute depreciation expense	f, SOYD	23,333.33
7. Compute remaining balance for year 1	[X≷Y]	46,666.67
8. Repeat steps 5, 6 and 7 for each year		
a. year 2 depreciation expense	2, f, SOYD	18,666.67
b. remaining balance	[X≷Y]	28,000.00

SUM-OF-THE-YEARS'-DIGITS DEPRECIATION: MANUAL SOLUTION:

Objective: To compute the periodic depreciation expense for partial and whole years using the S.Y.D. depreciation method.

Information Required:

1. Asset cost.
2. Salvage.
3. Life.
4. Number of months depreciated in the first year.

Comments:

This application is designed for rapid calculation of the periodic depreciation expense. Enter a 12 for months taken in first year for a whole year of depreciation. The annual S.Y.D. depreciation is computed and prorated on a monthly basis in the keystroke solution.

Example:

An asset costing $80,000 with a $9,000 salvage value is purchased on May 1, 1980. Eight months of depreciation will be taken the first year. The asset has a seven year life.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Enter cost	80000 [ENTER]	80,000.00
3. Enter salvage	9000 -	71,000.00
4. Enter life in years	7 STO 3 [ENTER] [ENTER] 1 + 6 X X ÷ STO 0	211.31
5. Multiply by 12	12 X STO 1	2,535.71
6. First year depreciation:		
a) Enter months of depreciation in first year	8 STO 4	8.00
b) Compute depreciation expense	RCL 3 X RCL 0 X	11,833.33
7. Compute depreciation for year 2:	RCL 3, 12 X RCL 4 - RCL 0 X	16,059.52
8. Year 3 depreciation	RCL 1 -	13,523.81
9. Year 4 depreciation	RCL 1 -	10,988.10
10. Year 5 depreciation	RCL 1 -	8,452.38
11. Year 6 depreciation	RCL 1 -	5,916.67
12. Year 7 depreciation	RCL 1 -	3,380.95
13. Partial depreciation year 8	RCL 1 -	845.24

HP-12C Keystroke Template: Computing Sum-of-the-Years' Digits Depreciation:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Enter cost	[ENTER]	
3. Enter salvage	-	
4. Enter life in years	STO 3 [ENTER] [ENTER] 1 + 6 X X ÷ STO 0	
5. Multiply by 12	12 X STO 1	
6. First year depreciation:		
a) Enter months of depreciation in first year	STO 4	
b) Compute depreciation expense	RCL 3 X RCL 0 X	
7. Compute depreciation for year 2	RCL 3, 12 X RCL 4 - RCL 0 X	
8. Year 3 depreciation	RCL 1 -	
9. Year 4 depreciation	RCL 1 -	
10. Year 5 depreciation	RCL 1 -	
11. Year 6 depreciation	RCL 1 -	
12. Repeat Step 11 until the asset is fully depreciated		

DECLINING BALANCE DEPRECIATION: CALCULATOR INTERNAL ROUTINE:

Objective: To compute the annual depreciation expense using the D.B. depreciation method.

Information Required:

1. Asset cost.
2. Salvage.
3. Life in years.
4. Declining balance factor.

Example:

Compute the annual depreciation expense for an asset costing $75,000 with a salvage value of $5,000, and a five-year life. Use a Double Declining Balance factor (200 percent).

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Enter cost	75000 PV	75,000.00
3. Enter salvage	5000 FV	5,000.00
4. Enter life in years	5 n	5.00
5. Enter Declining Balance factor as a percent	200 i	200.00
6. Enter year for depreciation expense calculation	1	1.
7. Compute depreciation expense	f, DB	30,000.00
8. Compute remaining balance for year 1	[X⋛Y]	40,000.00
9. Repeat steps 5, 6 and 7 for each year		
a) year 2 depreciation expense	2, f, DB	18,000.00
b) remaining balance	[X⋛Y]	22,000.00

DECLINING BALANCE DEPRECIATION: MANUAL SOLUTION:

Objective: To compute the periodic depreciation expense for partial and whole years using the Declining Balance (D.B.) depreciation method.

Information Required:

1. Asset cost.
2. Salvage.
3. Life.
4. Number of months depreciated in the first year.
5. Declining balance factor.

Comments:

This application is designed for rapid calculation of the yearly depreciation expense. Enter a 12 for months taken in first year for a whole year of depreciation. The declining balance factors are entered as 200, 150 or 125. Remember, you stop taking depreciation when the net book value equals the salvage value.

Example:

An asset costing $80,000 with a $9,000 salvage value is purchased on May 1, 1980. Eight months of depreciation will be taken the first year. The asset has a seven-year life. The declining balance method with a 200 percent factor will be used to calculate the depreciation expense.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Enter declining balance factors	200 [ENTER]	200.00
3. Enter life in years	7 ÷ STO 0	28.57
4. Enter cost	80000 STO 1	80,000.00
5. Enter months of depreciation in year 1	8 STO 2	8.00
6. Compute depreciation for year 1	RCL 1, RCL 0, %, RCL 2 X 12 ÷	15,238.10
7. Net book value at end of year 1	-	64,761.90
8. Year 2		
a) Depreciation	RCL 0 %	18,503.40
b) Net book value	-	46,258.50
9. Year 3		
a) Depreciation	RCL 0 %	13,216.72
b) Net book value	-	33,041.79

HP-12C Solution: (continued)

Procedure	Action Step	Display
10. Year 4		
a) Depreciation	RCL 0 %	9,440.51
b) Net book value	-	23,601.28
11. Year 5		
a) Depreciation	RCL 0 %	6,743.22
b) Net book value	-	16,858.06
12. Year 6		
a) Depreciation	RCL 0 %	4,816.59
b) Net book value	-	12,041.47
13. Year 7		
a) Depreciation	RCL 0 %	3,440.42
b) Net book value	-	8,601.05

NOTE: Since the NBV is less than the salvage value, the depreciation for year 7 is year 6 NBV minus salvage.

14. Correct year 7 depreciation	12041.47 [ENTER] 9000 -	3,041.47

HP-12C Keystroke Template: Computing Declining Balance Depreciation:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Enter declining balance factor as a percent	[ ] [ENTER]	[ ]
3. Enter life in years	[ ] ÷ STO 0	[ ]
4. Enter cost	[ ] STO 1	[ ]
5. Enter months of depreciation in year 1	[ ] STO 2	[ ]
6. Compute depreciation expense for year 1	RCL 1, RCL 0, %, RCL 2 X 12 ÷	[ ]
7. Net book value at end of year 1	-	[ ]
8. Year 2		
a) Depreciation	RCL 0 %	[ ]
b) NBV	-	[ ]
9. Year 3		
a) Depreciation	RCL 0 %	[ ]
b) NBV	-	[ ]
10. Repeat Step 9 until the salvage value is reached		

7

SPECIAL MORTGAGES

This chapter shows you solution procedures for special mortgages: graduated payment mortgages, deferred mortgages, wrap-around mortgages, and construction loans.

GRADUATED PAYMENT MORTGAGES

This section shows you how to solve for the payment schedule, yield, and price or present value of a graduated payment mortgage. Both a manual procedure and a program are given for determining the payment schedule. Two key inputs for determining the payment amount are: 1) the number of years payments will increase after the first year; 2) the percentage increase each year. Remember, the number of payment increases means the ones after the first year.

SOLVING FOR THE PAYMENT SCHEDULE FOR A GRADUATED PAYMENT MORTGAGE - MANUAL PROCEDURE:

Objective: To compute the payment schedule for a graduated payment mortgage -- i.e., Section 245 mortgage.

Information Required:

1. Amount borrowed.
2. Total number of payments.
3. Percent amount payment will increase.
4. Number of years payments will increase.
5. Annual percent interest rate.
6. Number of payments per year.

Comments:

For a detailed explanation of the underlying mathematics see Financial Analysis Using Calculators, pp. 417 to 420. This solution procedure assumes a 30-year mortgage with monthly payments. The payments will increase for five years after the first year and will then remain constant. Within any given year all payments are the same amount and occur at the end of each month.

Example:

A $60,000, 30-year graduated payment mortgage at 13 percent annual, compounded monthly, has payments increasing 5 percent each year for 5 years after the first year, and then remaining constant from the beginning of the sixth year to the end of the thirtieth year. What is the payment schedule for this loan?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear all registers	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter percent increase in annual payments	5 i	5.00
4. Enter	1 CHS, PV	- 1.00
5. Enter	1 g CF_j, 12 g, N_j	12.00
6. Compute values for years 1 through 4	FV, FV, g, CF_j, 12, g, N_j	12.00
	FV, FV, g, CF_j, 12, g, N_j	12.00
	FV, FV, g, CF_j, 12, g, N_j	12.00
	FV, FV, g, CF_j, 12, g, N_j	12.00
7. Compute values for years 5 through 30	FV, FV, g, CF_j, 99, g, N_j	99.00
8. Enter remaining values	RCL FV, g, CF_j 99, g, N_j	99.00
	RCL FV, g, CF_j 99, g, N_j	99.00
	RCL FV, g, CF_j 3, g, N_j	3.00
9. Enter monthly mortgage rate	13 g, [12 ÷]	1.08
10. Compute	f, NPV	107.25
11. Enter amount borrowed and compute first year's payment	1/x, 60000, X	559.46
12. Enter first year's payment in PV	CHS, PV	- 559.46
13. Enter annual percent increase for payments	5 i	5.00
14. Compute second year's payment	1, n, FV	587.43
15. Third year's payment	2, n, FV	616.80
16. Fourth year's payment	3, n, FV	647.64
17. Fifth year's payment	4, n, FV	680.03
18. Payment for years 6 to 30	5, n, FV	714.03

HP-12C Keystroke Template: Solving for the Payment Schedule for a Graduated Payment Mortgage - Manual Procedure:

Procedure	Action Step	Display
1. Clear all registers	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter percent increase in annual payments	[ ] i	[ ]
4. Enter	1 CHS, PV	- 1.00
5. Enter	1 g, CF_j, 12, g, N_j	12.00
6. Compute values for years 1 through 4	FV FV g CF_j 12 g N_j	12.00
	FV FV g CF_j 12 g N_j	12.00
	FV FV g CF_j 12 g N_j	12.00
	FV FV g CF_j 12 g N_j	12.00
7. Compute values for years 5 through 30	FV FV g CF_j 99 g N_j	99.00
8. Enter remaining values	RCL FV g CF_j 99 g N_j	99.00
	RCL FV g CF_j 99 g N_j	99.00
	RCL FV g CF_j 3 g,N_j	3.00
9. Enter monthly mortgage rate	[ ] g, [12 ÷]	[ ]
10. Compute	f, NPV	[ ]
11. Enter amount borrowed and compute first year's payment	1/x [ ] X	[ ]
12. Enter first year's payment in PV	CHS, PV	[ ]
13. Enter annual percent increase for payments	[ ] i	[ ]
14. Compute second year's payment	1, n, FV	[ ]
15. Compute third year's payment	2, n, FV	[ ]
16. Compute fourth year's payment	3, n, FV	[ ]
17. Compute fifth year's payment	4, n, FV	[ ]
18. Compute payment for years 6 to 30	5, n, FV	[ ]

USING A PROGRAM TO SOLVE FOR A GRADUATED PAYMENT MORTGAGE SCHEDULE:

The previous manual solution for a graduated payment mortgage schedule is adequate for an occasional application. But solving for GPM schedules on a regular basis makes the manual procedure time consuming. As a result, the following GPM program should be used for frequent GPM calculations. The program is designed to calculate the mortgage payment schedule and display a 0.00 after the last payment increase. The example is the same problem solved using the manual procedure. Refer to the mortgage amortization program example in Chapter 8 for additional information on entering programs.

PROGRAM FOR PAYMENT SCHEDULE FOR A GPM:

Action Step	Press	Display
1	CLX	0.00
2	f, P/R, f, PRGM	00 -
3	f, FIN	01 - 42 34
4	1	02 - 1
5	RCL 6	03 - 45 6
6	%	04 - 25
7	+	05 - 40
8	1	06 - 1
9	RCL 5	07 - 45 5
10	%	08 - 25
11	+	09 - 40
12	1	10 - 1
13	2	11 - 2
14	y^x	12 - 21
15	÷	13 - 10
16	1	14 - 1
17	-	15 - 30
18	1	16 - 1
19	0	17 - 0
20	0	18 - 0
21	X	19 - 20
22	i	20 - 12
23	1	21 - 1
24	CHS	22 - 16
25	PMT	23 - 14
26	RCL 1	24 - 45 1
27	n	25 - 11
28	FV	26 - 15
29	STO 2	27 - 44 6
30	RCL 5	28 - 45 5
31	i	29 - 12
32	1	30 - 1
33	2	31 - 2
34	n	32 - 11
35	0	33 - 0
36	FV	34 - 15
37	PV	35 - 13
38	STO X 2	36 - 44 20 2
39	RCL 3	37 - 45 3
40	RCL 1	38 - 45 1
41	1	39 - 1
42	2	40 - 2
43	X	41 - 20
44	-	42 - 30
45	n	43 - 11
46	PV	44 - 13
47	1	45 - 1
48	RCL 6	46 - 45 6

PROGRAM FOR PAYMENT SCHEDULE FOR A GPM: (continued)

Action Step	Press	Display
49	%	47 - 25
50	+	48 - 40
51	RCL 1	49 - 45 1
52	y^x	50 - 21
53	X	51 - 20
54	1	52 - 1
55	RCL 5	53 - 45 5
56	%	54 - 25
57	+	55 - 40
58	RCL 1	56 - 45 1
59	1	57 - 1
60	2	58 - 2
61	X	59 - 20
62	y^x	60 - 21
63	÷	61 - 10
64	STO + 2	62 - 44 40 2
65	RCL 4	63 - 45 4
66	RCL 2	64 - 45 2
67	÷	65 - 10
68	STO 7	66 - 44 7
69	R/S	67 - 31
70	STO 2	68 - 44 2
71	1	69 - 1
72	STO 0	70 - 44 0
73	1	71 - 1
74	STO + 0	72 - 44 40 0
75	RCL 2	73 - 45 2
76	RCL 6	74 - 45 6
77	%	75 - 25
78	+	76 - 40
79	STO 2	77 - 44 2
80	R/S	78 - 31
81	RCL 1	79 - 45 1
82	RCL 0	80 - 45 0
83	g, $x < y$	81 - 43 34
84	g, GTO 71	82 - 43 33 71
85	CLX	83 - 35
86	g, GTO 00	84 - 43 33 00
87	f, P/R	0.00

HP-12C Solution:

Procedure	Action Step	Display
1. Key in GPM program		
2. Clear financial registers	CLX, f, FIN	0.00
3. Set payment switch	g, END	0.00
4. Enter total number of payments	360 STO 3	360.00
5. Enter amount borrowed	60000 STO 4	60,000.00
6. Enter annual interest rate	13 g, [12÷], STO 5	1.08
7. Enter percent increase in annual payment	5 STO 6	5.00
8. Enter number of years for increase entered in Step 7	5 STO 1	5.00
9. Set program to start	f, PRGM	5.00
10. Compute payment for year 1	R/S	559.46
11. Payment for year 2	R/S	587.43
12. Payment for year 3	R/S	616.80
13. Payment for year 4	R/S	647.64
14. Payment for year 5	R/S	680.03
15. Payment for year 6	R/S	714.03
16. Signal that payments will remain constant at last payment value	R/S	0.00

HP-12C Keystroke Template: Use of GPM Program Solving for the Payment Schedule of a Graduated Payment Mortgage:

Procedure	Action Step	Display
1. Key in GPM program		
2. Clear financial registers	CLX, f, FIN	0.00
3. Set payment switch	g, END	0.00
4. Enter total number of payments	☐ STO 3	☐
5. Enter amount borrowed	☐ STO 4	☐
6. Enter annual interest rate	☐ g, [12÷], STO 5	☐
7. Enter percent increase in annual payment	☐ STO 2	☐
8. Enter number of years for increase entered in Step 7	☐ STO 1	☐
9. Set program to start	f, PRGM	☐
10. Compute payment for year 1	R/S	☐
11. Payment for year 2	R/S	☐
12. Payment for year 3	R/S	☐
13. Payment for year 4	R/S	☐
14. Payment for year 5	R/S	☐
15. Payment for year 6	R/S	☐

Note: A 0.00 will appear in the display as a signal that the payments have stopped increasing and will remain consistent. The number of increased payments is 1 plus the value entered in step 8.

SOLVING FOR THE PRICE (PRESENT VALUE) OF A GRADUATED PAYMENT MORTGAGE:

Objective: To compute the price or present value of a graduated payment mortgage.

Information Required:

1. Payment schedule.
2. Number of remaining payments.
3. Required annual yield.

Comments:

A mortgage having the payment schedule shown is offered to you as an investment. How much should you pay in order to earn an annual return of 13.5 percent compounded monthly?

Number of Payments	Monthly Payment
7	$559.46
12	587.43
12	616.80
12	647.64
12	680.03
300	714.03

HP-12C Solution:

Procedure	Action Step	Display
1. Clear Calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter annual required yield	13.5 g, [12÷]	1.13
4. Enter payment groups		
year 1 - amount	559.46 g, CF_j	559.46
number	7 g, N_j	7.00
year 2 - amount	587.43 g, CF_j	587.43
number	12 g, N_j	12.00
year 3 - amount	616.80 g, CF_j	616.80
number	12 g, N_j	12.00
year 4 - amount	647.64 g, CF_j	647.64
number	12 g, N_j	12.00
year 5 - amount	680.03 g, CF_j	680.03
number	12 g, N_j	12.00
year 6 - break up into groups of 99 payments	714.03 g, CF_j	714.03
	99 g, N_j	99.00
	714.03 g, CF_j	714.03
	99 g, N_j	99.00
	714.03 g, CF_j	714.03
	99 g, N_j	99.00
	714.03 g, CF_j	714.03
	3 g, N_j	3.00
5. Compute present value (price)	f, NPV	58,294.45

HP-12C Keystroke Template: Solving for the Price (Present Value) of a Graduated Payment Mortgage:

Procedure		Action Step		Display
1. Clear calculator		f, REG		0.00
2. Set payment switch		g, END		0.00
3. Enter annual required yield		[ ]	g, [12÷]	[ ]
4. Enter payment groups. After last group entered, go to Step No. 5				
Group 1	Amt.	[ ]	g, CF_j	[ ]
	No.	[ ]	g, N_j	[ ]
Group 2	Amt.	[ ]	g, CF_j	[ ]
	No.	[ ]	g, N_j	[ ]
Group 3	Amt.	[ ]	g, CF_j	[ ]
	No.	[ ]	g, N_j	[ ]
Group 4	Amt.	[ ]	g, CF_j	[ ]
	No.	[ ]	g, N_j	[ ]
Group 5	Amt.	[ ]	g, CF_j	[ ]
	No.	[ ]	g, N_j	[ ]
Group 6	Amt.	[ ]	g, CF_j	[ ]
	No.	[ ]	g, N_j	[ ]
Group 7	Amt.	[ ]	g, CF_j	[ ]
	No.	[ ]	g, N_j	[ ]
Group 8	Amt.	[ ]	g, CF_j	[ ]
	No.	[ ]	g, N_j	[ ]

HP-12C Keystroke Template: (continued)

Procedure		Action Step		Display
Group 9	Amt.	$\boxed{}$	g, CF_j	$\boxed{}$
	No.	$\boxed{}$	g, N_j	$\boxed{}$
5. Compute present value (price)		f, NPV		$\boxed{}$

SOLVING FOR THE YIELD OF A GRADUATED PAYMENT MORTGAGE:

Objective: To compute the yield on a graduated payment mortgage when the borrower pays points.

Information Required:

1. Amount borrowed.
2. Graduated payment schedule.
3. Points paid.

Example:

A $60,000, 30-year 13 percent loan requiring the borrower to pay 2 points has the payment schedule shown below. What is the annual yield?

Number of Monthly Payments	Payment
12	$559.46
12	587.43
12	616.80
12	647.64
12	680.03
300	714.03

HP-12C Solution:

Procedure		Action Step	Display
1. Clear calculator		f, REG	0.00
2. Set payment switch		g, END	0.00
3. Enter amount borrowed		60000 [ENTER]	60,000.00
4. Enter points as a percent and compute dollar amount		2 %	1,200.00
5. Compute net cash		- CHS g, CF_0	- 58,800.00
6. Enter graduated payments			
Group 1	Pmt.	559.46 g, CF_j	559.46
	No.	12 g, N_j	12.00
Group 2	Pmt.	587.43 g, CF_j	587.43
	No.	12 g, N_j	12.00
Group 3	Pmt.	616.80 g, CF_j	616.80
	No.	12 g, N_j	12.00
Group 4	Pmt.	647.64 g, CF_j	647.64
	No.	12 g, N_j	12.00
Group 5	Pmt.	680.03 g, CF_j	680.03
	No.	12 g, N_j	12.00
Group 6	Pmt.	714.03 g, CF_j	714.03
	No.	99 g, N_j	99.00
	Pmt.	714.03 g, CF_j	714.03
	No.	99 g, N_j	99.00
	Pmt.	714.03 g, CF_j	714.03
	No.	99 g, N_j	99.00
	Pmt.	714.03 g, CF_j	714.03
	No.	3 g, N_j	3.00
7. Solve for yield per month		f, IRR	1.11
8. Compute annual yield		12 X	13.27

HP-12C Keystroke Template: Solving for the Yield of a Graduated Payment Mortgage:

Procedure		Action Step		Display
1. Clear calculator		f, REG		0.00
2. Set payment switch		g, END		0.00
3. Enter amount borrowed			[ENTER]	
4. Enter points as a percent and compute dollar amount			%	
5. Compute net cash		- CHS g, CF_0		
6. Enter graduated payments				
Group 1	Pmt.		g, CF_j	
	No.		g, N_j	
Group 2	Pmt.		g, CF_j	
	No.		g, N_j	
Group 3	Pmt.		g, CF_j	
	No.		g, N_j	
Group 4	Pmt.		g, CF_j	
	No.		g, N_j	
Group 5	Pmt.		g, CF_j	
	No.		g, N_j	
Group 6	Pmt.		g, CF_j	
	No.		g, N_j	
	Pmt.		g, CF_j	
	No.		g, N_j	
	Pmt.		g, CF_j	

HP-12C Keystroke Template: (continued)

Procedure	Action Step	Display
No.	[ ] g, N_j	[ ]
Pmt.	[ ] g, CF_j	[ ]
No.	[ ] g, N_j	[ ]
7. Solve for yield per month	f, IRR	[ ]
8. Compute annual yield	12 X	[ ]

DEFERRED MORTGAGES

In this section, mortgages that have the first payment made after the first normal payment date are discussed. This mortgage type is called a deferred mortgage. The applications in this section assume compound interest is used for the deferred period.

SOLVING FOR THE PAYMENT OF A DEFERRED MORTGAGE:

Objective: To compute the payment for a deferred mortgage when interest is added to the amount borrowed during the deferred period.

Information Required:

1. Amount borrowed.
2. Annual interest rate.
3. Number of payments.
4. Number of deferred payment periods.
5. Balloon, if any.

Comments:

Monthly payments are assumed. For a detailed explanation of the underlying mathematics see Financial Analysis Using Calculators, pp. 386 to 390.

Example:

A $50,000 loan at 11.4 percent annual compounded monthly requires 120 monthly payments with a balloon of $20,000 due in addition to the final payment. The first payment will be made 8 months after the origination date of the loan. Note: the payment is made at the end of the eighth month, so there are 8 deferred payment periods.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	11.4 g, [12÷]	0.95
4. Enter number of deferred payment periods	8 [ENTER] 1 - n	7.00
5. Enter amount borrowed	50000 CHS PV	- 50,000.00
6. Compute future value of amount borrowed one month before first payment	FV	53,421.28
7. Enter	STO PV	53,421.28
8. Enter number of payments	120 n	120.00
9. Enter balloon	20000 CHS FV	- 20,000.00
10. Compute monthly payment	PMT	- 657.98

HP-12C Keystroke Template: Solving for the Payment of a Deferred Mortgage:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	☐ g, [12 ÷]	☐
4. Enter number of deferred payment periods	☐ [ENTER] 1 - n	☐
5. Enter amount borrowed	☐ CHS PV	☐
6. Compute future value of amount borrowed one month before first payment	FV	☐
7. Enter	STO PV	☐
8. Enter number of payments	☐ n	☐
9. Enter balloon	☐ CHS FV	☐
10. Compute monthly payment	PMT	☐

SOLVING FOR THE PRICE (PRESENT VALUE) OF A DEFERRED MORTGAGE:

Objective: To compute the price or present value of a deferred mortgage.

Information Required:

1. Annual interest rate.
2. Payment.
3. Balloon if any.
4. Number of payments.
5. Number of deferred payment periods.

Example:

You are offered a $50,000 mortgage as an investment. The loan has 120 monthly payments of $657.98 and a $20,000 balloon due with the final payment. The first payment will be made at the end of 5 months. What price should you pay in order to earn 11.75 percent annually, compounded monthly?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	11.75 g, [12÷]	0.98
4. Enter number of payments	120 n	120.00
5. Enter payment amount	657.98 PMT	657.98
6. Enter balloon	20000 FV	20,000.00
7. Compute	PV	- 52,538.79
8. Enter	STO FV	- 52,538.79
	0 PMT	0.00
9. Enter number of deferred payment periods	5 [ENTER] 1- n	4.00
10. Compute price	PV	50,530.42

HP-12C Keystroke Template: Solving for the Price (Present Value) of a Deferred Mortgage:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	☐ g, [12÷]	☐
4. Enter number of payments	☐ n	☐
5. Enter payment amount	☐ PMT	☐
6. Enter balloon	☐ FV	☐
7. Compute	PV	☐
8. Enter	STO FV	☐
	0 PMT	0.00
9. Enter number of deferred payment periods	☐ [ENTER] 1- n	☐
10. Compute price	PV	☐

SOLVING FOR THE YIELD OF A DEFERRED MORTGAGE:

Objective: To compute the yield on a deferred payment mortgage.

Information Required:

1. Price of mortgage.
2. Monthly payment.
3. Balloon (if any).
4. Number of payments.
5. Number of deferred payment periods.

Example:

You are offered a mortgage for $50,530.42. The mortgage payments are $657.98 a month for ten years with a balloon of $20,000 due with the final payment. The first payment is due at the end of 5 months. What is the yield on the mortgage?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter price of mortgage	50530.42 CHS g, CF_0	- 50,530.42
4. Enter	0 g, CF_j	0.00
5. Enter number of deferred payment periods	5 [ENTER] 1 - g, N_j	4.00
6. Enter all payments but the final payment using the CF_j and N_j keys		
a) Pmt.	657.98 g, CF_j	657.98
No.	99 g, N_j	99.00
b) Pmt.	657.98 g, CF_j	657.98
No.	20 g, N_j	20.00
7. Enter final payment	657.98 [ENTER]	657.98
8. Enter balloon	20000	20,000.00
9. Enter final total payment	+ g, CF_j	20,657.98
	1, N_j	1.00
10. Solve for monthly interest rate	f, IRR	0.98
11. Solve for annual yield	12 X	11.75

HP-12C Keystroke Template: Solving for the Yield of a Deferred Mortgage:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter price of mortgage	☐ CHS g, CF_0	☐
4. Enter	0 g, CF_j	0.00
5. Enter number of deferred payment periods	☐ [ENTER] 1 - g, N_j	☐
6. Enter all payments <u>but</u> the <u>final</u> payment using the CF_j and N_j keys		
a) Pmt.	☐ g, CF_j	☐
No.	☐ g, N_j	☐
b) Pmt.	☐ g, CF_j	☐
No.	☐ g, N_j	☐
c) Pmt.	☐ g, CF_j	☐
No.	☐ g, N_j	☐
d) Pmt.	☐ g, CF_j	☐
No.	☐ g, N_j	☐
7. Enter final payment	☐ [ENTER]	☐
8. Enter balloon	☐	☐
9. Enter final total payment	+ g, CF_j	☐
	1, N_j	1.00
10. Solve for monthly interest rate	f, IRR	☐
11. Solve for annual yield	12 X	☐

SOLVING FOR THE BALLOON PAYMENT OF A DEFERRED MORTGAGE:

Objective: To compute the balloon payment for a deferred mortgage.

Information Required:

1. Amount borrowed.
2. Annual interest rate.
3. Number of payments.
4. Monthly payments.
5. Number of deferred payment periods.

Example:

A 30 year, $50,000 loan at 11.4 percent annual interest compounded monthly has monthly payments of $657.98. The first payment will be made at the end of 8 months. What balloon is required to pay off the loan after 120 payments?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	11.4 g, [12÷]	0.95
4. Enter number of deferred payments	8 [ENTER] 1 - n	7.00
5. Enter amount borrowed	50000 CHS PV	- 50,000.00
6. Compute future value of amount borrowed one month before first payment	FV	53,421.28
7. Enter	STO PV	53,421.28
8. Enter number of payments	120 n	120.00
9. Enter monthly payment	657.98 CHS, PMT	- 657.98
10. Compute balloon or amount to pay off loan	FV	- 19,999.53

HP-12C Keystroke Template: Solving for the Balloon Payment of a Deferred Mortgage:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	[ ] g, [12÷]	[ ]
4. Enter number of deferred payments	[ ] [ENTER] 1 - n	[ ]
5. Enter amount borrowed	[ ] CHS PV	[ ]
6. Compute future value of amount borrowed one month before first payment	FV	[ ]
7. Enter	STO PV	[ ]
8. Enter number of payments	[ ] n	[ ]
9. Enter monthly payment	[ ] CHS PMT	[ ]
10. Compute balloon or amount to pay off loan	FV	[ ]

WRAP-AROUND MORTGAGES

In this section we show you how to calculate the yield to the lender and the payment amount necessary to earn a specific yield on wrap-around mortgages. We assume that the lender makes the payments on the existing mortgage and advances the difference between the new mortgage amount and the old mortgage balance to the borrower. Examples for simple and complex wrap-around mortgages are shown.

SOLVING FOR THE YIELD TO THE LENDER FOR A SIMPLE WRAP-AROUND MORTGAGE:

Objective: To compute the lender's yield on a simple wrap-around mortgage.

Information Required:

1. Current balance on existing mortgage.
2. Payment amount for existing mortgage.
3. Number of payments remaining on existing mortgage.
4. Amount borrowed on new mortgage.
5. Payment amount for new mortgage.
6. Number of payments for new mortgage.
7. Balloon, if any, on new mortgage.
8. Annual interest rate for both mortgages.

Comments:

We assume that the number of payments for the new mortgage are equal to the number remaining on the old mortgage. We assume that all payments on the existing mortgage were made as scheduled and that the existing mortgage does not have a balloon payment.

See the complex wrap-around application for the case where the mortgages have unequal lives and where the old mortgage has a balloon.

Example:

Jane Doe has a 12 percent mortgage with 200 remaining payments of $1,028.62. The mortgage has a current balance of $88,802.54.

She has applied for a wrap-around loan from your firm. The new loan is for $120,000 with 200 payments of $1,449.12 at 13 percent annual interest. A balloon of $15,000 is due with the final payment.

What is the yield to your firm?

CALCULATION OF INPUT VALUE FOR A SIMPLE WRAP-AROUND MORTGAGE:

I. Net amount advanced by lender:

A.	New Mortgage	$ -120,000.00
B.	Existing mortgage balance	88,802.54
C.	Net amount advanced	$ - 31,197.46

II. Net payment:

A.	New mortgage	$ 1,449.12
B.	Existing mortgage	- 1,028.62
C.	Net payment	$ 420.50

III. Summary of cash flows:

A.	Net amount advanced from I, C	$ - 31,197.46
B.	Number of payments	200
C.	Net payment amount from II, C	$420.50
D.	Balloon on new mortgage	$15,000.00

IV. An alternative time diagram method for determining the inputs is:

Wrap PV	-$120,000	+$1,449.12	+$1,449.12	FV = $15,000.00 +$1,449.12
Old Mortgage PV	88,802.54	-1,028.62	-1,028.62	-1,028.62
Net Cash Flows PV	-$31,197.46	$ 420.50	$ 420.50	FV = $15,000.00 $ 420.50
	0	1	199	n = 200

i = ?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter number of payments from III, B	200 n	200.00
4. Enter net amount advanced from III, A	31197.46 CHS PV	- 31,197.46
5. Enter net payment from Step III, C	420.50 PMT	420.50
6. Enter balloon from Step III, D	15000 FV	15,000.00
7. Solve for interest rate per payment period	i	1.29
8. Enter number of payments per year	12	12.
9. Compute annual yield	X	15.51

HP-12C Keystroke Template: Solving for the Yield to the Lender for a Simple Wrap-Around Mortgage:

Calculation of Input Values for a Simple Wrap-Around

I. Net amount advanced by lender:

A. New mortgage $ - [ ]

B. Existing mortgage balance [ ]

C. Net amount advanced $ - [ ]

II. Net payment

A. New mortgage $ [ ]

B. Existing mortgage - [ ]

C. Net payment $ [ ]

III. Summary of cash flows

A. Net amount advanced from I, C $ - [ ]

B. Number of payments [ ]

C. Net payment amount from II, C [ ]

D. Balloon on a new mortgage [ ]

HP-12C Keystroke Template: (continued)

Yield to the Lender on a Simple Wrap-Around Mortgage

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter number of payments from III, B	☐ n	☐
4. Enter net amount advanced from III, A	☐ CHS PV	☐
5. Enter net payment from III, C	☐ PMT	☐
6. Enter balloon from III, D	☐ FV	☐
7. Solve for interest rate per payment period	☐ i	☐
8. Enter number of payments per year	☐	☐
9. Compute annual yield	X	☐

SOLVING FOR THE YIELD TO THE LENDER FOR A COMPLEX WRAP-AROUND MORTGAGE:

Objective: To compute the lender's yield on a complex wrap-around mortgage.

Information Required:

1. Current balance on existing or old mortgage.
2. Payment amount for existing mortgage.
3. Number of payments remaining on existing mortgage.
4. Balloon, if any, on existing mortgage.
5. Amount borrowed on new mortgage.
6. Payment amount for new mortgage.
7. Number of payments for new mortgage.
8. Balloon, if any, on new mortgage.
9. Annual interest rate for both mortgages.

Comments:

We assume that the existing mortgage payments have been made as scheduled. No extra payments to principal have been made. If extra principal payments have been made, then you must recalculate the number of remaining payments on the existing mortgage. We assume that payments for both mortgages are made at the same time. For example, both mortgages have monthly, quarterly, or annual payments. All payments are assumed to occur at the end of each payment period.

Example:

John Doe has a 9 percent mortgage with a current balance of $93,005.44. The mortgage has 240 remaining monthly payments of $784.39 including a balloon payment of $35,000 due with the last payment.

He has requested a loan from your firm. The new loan has a 13 percent annual rate with 300 monthly payments of $1,669.50. A balloon of $50,000 is due along with the final payment. The amount borrowed is $150,000.

What is the yield to your firm on this wrap-around?

CALCULATION OF INPUT VALUES FOR A COMPLEX WRAP-AROUND MORTGAGE:

I. Net amount advanced by lender:

A.	New mortgage	$ - 150,000.00
B.	Existing mortgage balance	93,005.44
C.	Net amount advanced	$ - 56,994.56

II. Net payments for life of existing mortgage:

A.	New mortgage	$ 1,669.50
B.	Existing mortgage	- 784.39
C.	Net payment	$ 885.11

III. Payment patterns for lender:

A.	Net amount advanced	$ - 56,994.56

B. Payments during term of existing mortgage

1.	number of payments excluding final payment	239
2.	net payment from II, C	$ 885.11
3.	balloon on existing mortgage	$ - 35,000.00
4.	final net payment	885.11
5.	cash flow at time of final payment on existing mortgage	$ - 34,114.89

C. Payments after existing mortgage is paid off

1.	number less final payment	59
2.	amount	$ 1,669.50
3.	balloon on new mortgage	50,000.00
4.	final payment	1,669.50
5.	cash flow at time of final payment on new mortgage	$ 51,669.50

IV. Summary of cash flows:

		Number of Paymemts	Amount
A.	Net amount advanced from I, C		$ - 56,994.56
B.	Payments during term of existing loan:		
1.	from III, B, 1 and III, B, 2	239	885.11
2.	from III, B, 3	1	- 34,114.89
C.	Payments after existing mortgage paid off:		
1.	from III, C, 1 and III, C, 2	59	1,669.50
2.	from III, C, 3	1	51,669.50

V. An alternative time diagram method for determining the inputs is:

Wrap	-$150,000.00	+$1,669.50	+$1,669.50	+$1,669.50	+$50,000.00 +$1,669.50
Old Mortgage	+ 93,005.44	-784.39	-35,000.00 -784.39		
Net Grouped Cash Flows	-$56,994.56	$885.11	-$34,114.89	$1,669.50	$51,669.50
Number of Grouped Cash Flows	0	239	1	59	1

i = ?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter net amount advanced from IV, A	56994.56 CHS g, CF_0	- 56,994.56
4. Enter net payments during term of existing loan:		
a. from IV, B, 1	885.11 g, CF_j	885.11
	99 g, N_j	99.00
	885.11 g, CF_j	885.11
	99 g, N_j	99.00
	885.11 g, CF_j	885.11
	41 g, N_j	41.00
b. from IV, B, 2	34114.89 CHS	- 34,114.89
	g, CF_j	- 34,114.89
5. Enter payments after existing mortgage is paid off:		
a. from IV, C, 1	1669.50 g, CF_j	1,669.50
	59 g, N_j	59.00
b. from IV, C, 2	51669.50 g, CF_j	51,669.50
6. Solve for interest rate per payment period	f, IRR	1.55
7. Enter number of payments per year	12	12.
8. Compute annual yield	X	18.58

HP-12C Keystroke Template: Solving for the Yield to the Lender for a Complex Wrap-Around Mortgage:

Calculation of Input Values for Complex Wrap-Around

I. Net amount advanced by lender:

A. New mortgage - $ [ ]

B. Existing mortgage balance + [ ]

C. Net amount advanced $ [ ]

II. Net payments for life of existing mortgage

A. New mortgage $ + [ ]

B. Existing mortgage - [ ]

C. Net payment $ [ ]

III. Payment patterns for lender:

A. Net amount advanced from I, C $ - [ ]

B. Payments during term of existing mortgage

1. number of payments excluding final payment [ ]

2. net payment from II, C $ [ ]

3. balloon on existing mortgage - $ [ ]

4. final net payment + [ ]

5. cash flow at time of final payment on existing mortgage (-3 + 4) $ [ ]

C. Payments after existing mortgage is paid off

1. number less final payment [ ]

2. amount $ [ ]

3. balloon on new mortgage $ [ ]

HP-12C Keystroke Template: (continued)

4. final payment $ [ ]

5. cash flow at time of final payment on new mortgage (3 + 4) $ [ ]

IV. Summary of cash flows:

	Number of Payments	Amount
A. Net amount advanced from I, C		$ - [ ]
B. Payments during term of existing loan:		
1. from III, B, 1 and III, B, 2	[ ]	[ ]
2. from III, B, 3	1	[ ]
C. Payments after existing mortgage paid off:		
1. from III, C, 1 and III, C, 2	[ ]	[ ]
2. from III, C, 3	1	[ ]

Procedure		Action Step		Display
1. Clear calculator		f, REG		0.00
2. Set payment switch		g, END		0.00
3. Enter net amount advanced from IV, A		[ ]	CHS g, CF_0	[ ]
4. Enter net payments during term of existing loan:				
a. from IV, B, 1	PMT	[ ]	g, CF_j	[ ]
NOTE:	No.	[ ]	g, N_j	[ ]
Each group entered using the N_j key cannot exceed 99.	PMT	[ ]	g, CF_j	[ ]
So break payments into groups of 99 or less.	No.	[ ]	g, N_j	[ ]
	PMT	[ ]	g, CF_j	[ ]

HP-12C Keystroke Template: (continued)

Procedure		Action Step		Display
	No.		g, N_j	
	PMT		g, CF_j	
	No.		g, N_j	
b. from IV, B, 2	PMT		CHS g, CF_j	
5. Enter payments after existing mortgage is paid off:				
a. from IV, C, 1	PMT		g, CF_j	
	No.		g, N_j	
	PMT		g, CF_j	
	No.		g, N_j	
	PMT		g, CF_j	
	No.		g, N_j	
	PMT		g, CF_j	
	No.		g, N_j	
b. from IV, C, 2	PMT		CHS g, CF_j	
6. Solve for interest rate per payment period		f, IRR		
7. Enter number of payments per year				
8. Compute annual yield		X		

CONSTRUCTION LOANS

In this section we discuss how to compute the yield on construction loans. We assume the loan is repaid either with the last payment or one period after the last payment. But by extending the procedure shown, the yield can be computed when payment occurs more than one period after the final draw.

Two types of draws are assumed. The first example assumes the interest is deducted from the gross draw. So the borrower actually receives a net cash payment less than the gross draw. The second example assumes the gross draw is paid to the borrower. A program is given for each type of draw.

Solving for the yield is simply a matter of identifying the cash flows, i.e., how much cash the borrower receives and pays back.

YIELD ON CONSTRUCTION LOAN - INTEREST DEDUCTED FROM DRAW:

Objective: To compute the yield on a construction loan when the interest is deducted from the draw amount.

Information Required:

1. Annual interest rate.
2. Number of draws per year.
3. Estimated pattern of draws after closing.
4. Amount borrowed at closing.
5. Net cash advanced at closing.

Comments:

This application has three basic solution steps. First, determine the net cash advanced to the borrower for each draw and the balance due at the end of the loan. Second, enter the net cash flows using the [CF_0], [CF_j], and [N_j] keys. Third, solve for the yield using the IRR key.

A keystroke template is not provided for this example because of the many ways the draw pattern can vary.

The program assumes draws occur at equal intervals each year.

Example:

The Big D Co. has made an $866,000 construction loan to the Little E Company. Annual interest is 12 percent. The initial amount advanced to Little E is $51,000, but Big D will keep $33,000 as a fee for making the loan. Thus, Little E will receive $18,000 at closing. The monthly draws will include the interest for the previous periods. As a result, the borrower will actually receive a cash amount that is less than the draw.

The estimated monthly draws are:

End of month	Draw
1	$ 0
2	175,000
3	175,000
4	150,000
5	125,000
6	100,000
7	90,000

What is the cost to Little E (yield to Big D) if the loan is paid off at the end of a) the seventh month, and b) the eighth month?

PROGRAM: NET DRAW AFTER INTEREST:

Action Step	Press	Display		
1	CLX	0.00		
2	f, P/R, f, PRGM	00 -		
3	STO 4	01 -	44	4
4	RCL 1	02 -	45	1
5	RCL 2	03 -	45	2
6	%	04 -		25
7	STO + 3	05 - 44	40	3
8	+	06 -		40
9	STO 1	07 -	44	1
10	RCL 4	08 -	45	4
11	g, [X = 0]	09 -	43	35
12	g, GTO 20	10 - 43	33	20
13	RCL 3	11 -	45	3
14	-	12 -		30
15	STO 4	13 -	44	4
16	STO + 1	14 - 44	40	1
17	CLX	15 -		35
18	STO 3	16 -	44	3
19	RCL 4	17 -	45	4
20	R/S	18 -		31
21	g, GTO 01	19 - 43	33	01
22	CLX	20 -		35
23	R/S	21 -		31
24	g, GTO 01	22 - 43	33	01
25	f, P/R	0.00		

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter net draw after interest program		
4. Set program to start	f, PRGM	0.00
5. Enter	0 STO 3, STO 4	0.00
6. Enter amount borrowed at closing	51000 STO 1	51,000.00
7. Enter annual interest rate	12 [ENTER]	12.00
8. Enter number of draws per year	12 ÷ STO 2	1.00
9. Compute net cash paid per draw and ending loan balance. Enter gross draw per period.		
a) net draw period 1	0, R/S	0.00
b) net draw period 2	175000, R/S	173,974.90
c) net draw period 3	175000, R/S	172,740.00
d) net draw period 4	150000, R/S	145,990.00
e) net draw period 5	125000, R/S	119,490.00
f) net draw period 6	100000, R/S	93,240.00
g) net draw period 7	90000, R/S	82,240.00
10. Balance end of period 7	RCL 1	866,000.00
11. Enter	0 R/S	0.00
12. Balance end of period 8	RCL 1	874,660.00
13. Solve for yield if loan repaid at end of period 7		
a) enter net cash advanced at closing	18000 g, CF_0	18,000.00
b) Enter all but last net draw from step 9		
1) net draw period 1	0 g, CF_j	0.00
2) net draw period 2	173,974.90 g, CF_j	173,974.90
3) net draw period 3	172,740.00 g, CF_j	172,740.00

HP-12C Solution: (continued)

Procedure	Action Step	Display
4) net draw period 4	145,990.00 g, CF_j	145,990.00
5) net draw period 5	119,490.00 g, CF_j	119,490.00
6) net draw period 6	93,240.00 g, CF_j	93,240.00
c) enter final net draw	82,240.00 [ENTER]	82,240.00
d) enter balance end of period 7	866000, -, g, CF_j	- 783,760.00
e) solve for periodic yield	f, IRR	2.37
f) enter number of draws per year	12	12.
g) annual yield	X	28.42
14. Solve for yield of loan repaid at end of period 8		
a) enter net cash advanced at closing	18000 g, CF_0	18,000.00
b) enter all net draws		
1) net draw period 1	0 g, CF_j	0.00
2) net draw period 2	173,974.90, g, CF_j	173,974.90
3) net draw period 3	172,740.00, g, CF_j	172,740.00
4) net draw period 4	145,990.00, g, CF_j	145,990.00
5) net draw period 5	119,490.00, g, CF_j	119,490.00
6) net draw period 6	93,240.00, g, CF_j	93,240.00
7) net draw period 7	82,240.00, g, CF_j	82,240.00
c) enter loan balance at end of period 8	874660, CHS, g, CF_j	- 874,660.00
d) solve for periodic yield	f, IRR	2.03
e) enter number of draws per year	12	12.
f) annual yield	X	24.41

YIELD ON CONSTRUCTION LOANS - INTEREST NOT DEDUCTED FROM DRAW:

Objective: To compute the yield on a construction loan when the interest is not deducted from the draw amount.

Information Required:

1. Annual interest rate.
2. Number of draws per year.
3. Estimated pattern of draws after closing.
4. Amount borrowed at closing.
5. Net cash advanced at closing.

Comments:

This application has three basic solution steps. First, determine the amount due at the time the loan is repaid using the program described below. Second, enter the net cash flows using the [CF_0], [CF_j], and [N_j] keys. Third, solve for the yield using the IRR key.

A keystroke template is not provided for this example because of the many ways the draw pattern can vary.

The program assumes draws occur at equal intervals each year.

Example:

The Big D Co. has made a $866,000 construction loan to the Little E Co. Annual interest is 12 percent. The initial amount advanced to Little E is $51,000, but Big D will keep $33,000 as a fee for making the loan. Thus, Little E will receive $18,000 at closing.

The estimated monthly draws are:

End of month	Draw
1	0
2	$175,000
3	175,000
4	150,000
5	125,000
6	100,000
7	90,000

What is the cost to Little E (yield to Big D) if the loan is paid off at the end of a) the seventh month, and b) the eighth month.

PROGRAM: FINAL BALANCE ON CONSTRUCTION LOAN:

Action Step	Press	Display
1	CLX	0.00
2	f, P/R, f PRGM	00 -
3	RCL 1	01 - 45 1
4	RCL 2	02 - 45 2
5	%	03 - 25
6	+	04 - 40
7	+	05 - 40
8	STO 1	06 - 44 1
9	R/S	07 - 31
10	g, GTO 01	08 - 44 33 01
11	f, P/R	0.00

Note: A new interest rate can be inserted by repeating steps 5 & 6 in the keystroke solution when the rate changes.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. a. Enter program		
b. Set program to start	f, PRGM	
4. Enter amount borrowed at closing	51000 STO 1	51,000.00
5. Enter annual interest rate	12 [ENTER]	12.00
6. Enter number of draws per year	12 ÷ STO 2	1.00
7. Compute balance of loan at end of periods 7 & 8. Enter draw for:		
a. Period 1	0	0.
balance end of period 1	R/S	51,510.00
b. Period 2	175000 R/S	227,025.10
c. Period 3	175000, R/S	404,295.35
d. Period 4	150000, R/S	558,338.30
e. Period 5	125000, R/S	688,921.69
f. Period 6	100000, R/S	795,810.90
g. Period 7	90000	90,000.
balance end of period 7	R/S	893,769.01
h. Period 8	0	0.
balance end of period 8	R/S	902,706.70
8. Solve for yield if loan repaid at end of period 7		
a) enter net cash advanced at closing	18000 g, CF_0	18,000.00
b) enter all but last draw		
Period 1	0 g, CF_j	0.00
Periods 2 & 3	175000 g, CF_j	175,000.00
	2 g, N_j	2.00
Period 4	150000 g, CF_j	150,000.00
Period 5	125000 g, CF_j	125,000.00
Period 6	100000 g, CF_j	100,000.00

HP-12C Solution: (continued)

Procedure	Action Step	Display
c) enter final draw	90000 [ENTER]	90,000.00
d) enter balance end of period 7 from step 7, g	893769.01 -	- 803,769.01
enter net cash flow	g, CF_j	- 803,769.01
e) solve for periodic yield	f, IRR	2.35
f) enter number of draws per year	12	12.
g) annual yield	X	28.15
9. Solve for yield if loan repaid at end of period 8		
a) enter net cash advanced at closing	18000 g, CF_0	18,000.00
b) enter draws		
Period 1	0 g, CF_j	0.00
Period 2 & 3	175000 g, CF_j	175,000.00
	2 g, N_j	2.00
Period 4	150000 g, CF_j	150,000.00
Period 5	125000 g, CF_j	125,000.00
Period 6	100000 g, CF_j	100,000.00
Period 7	90000 g, CF_j	90,000.00
c) enter amount repaid from step 7, h.	902706.70 CHS, G, CF_j	- 902,706.70
d) solve for periodic yield	f, IRR	2.01
e) enter number of draws per year	12	12.
f) annual yield	X	24.16

8

AMORTIZATION SCHEDULES

The examples and programs shown in this chapter will enable you to amortize mortgages. First a discussion of several important features of the amortization routine:

Payment Switch Setting - Mortgages with the first payment occurring at the end of the first month require a switch setting of g, [END]. Leases with the first payment occurring at the beginning of the first month require a switch setting of g, [BEG].

Rounding of Calculations - When you amortize using the f, [AMORT] key, the display setting determines the number of internal digits used in the calculation. A display setting of f, 2 (two decimals) will round the interest and principal portions of each payment to two places for internal calculations. So, you must decide in advance on the number of digits (i.e., 2, 3, 4, ..., 9) used for amortization.

Sign Notation - The amortization procedures in this book always enter the amount borrowed or the current lease value as a positive number and the payment as a negative number. As a result, they are calculated from the borrower's viewpoint. When the PV is positive and the PMT is negative, the interest and principal values will be negative if the payment amount exceeds the interest. For graduated payment mortgages, however, the early payments do not cover interest, and the difference between the interest due and the payment is added to the initial loan amount. In such cases, if the PV amount is positive, the principal paid will be positive instead of negative, until the payment amount exceeds the interest due. To avoid confusion, we suggest you adopt a standard sign sequence for amortization schedules. (PV is positive; PMT is negative.)

Rounding of Payment Value - In some situations you will compute a payment and then calculate an amortization schedule. But you should remember that the calculated payment is carried to nine decimal places internally. The actual cash payment, however, will be to the nearest penny. This actual payment is the amount to enter using the [PMT] key for amortization. For example, a payment of $-514.31 is calculated using n = 360, PV = 50,000, and i = 1%. When f, 9 is pressed the payment becomes $-514.3062985. The borrower can pay either $514.30 or $514.31, but not $514.3062985. Because the actual payment is to the penny, the loan will have a small positive or negative amount due after the last payment.

MANUAL PROCEDURE

The HP-12C mortgage amortization keystroke procedure is powerful and flexible. As a result, you should review the following material before continuing in this chapter. The amortization procedure is divided into two basic steps: (1) enter the mortgage values, and (2) compute the amortization schedule.

1. Enter the mortgage values.

   A. Set payment switch to g, END for mortgages with end of period payments, or to g, BEG for beginning of period payments.

B. Store 0 by pressing n. Since the cumulative number of payments amortized are contained in the n register, you should make sure it is cleared by storing a 0. When repeating a schedule always enter a zero in the n register.

C. Enter the mortgage amount in PV as a positive number. Because the calculator routine changes the value stored in the PV register, you must reenter the original loan amount when repeating the schedule.

D. Enter the payment in PMT as a negative value. This value remains constant for all calculations unless changed.

2. Compute the amortization schedule.

A. Key into the display the number of periods to amortize.

B. Press f, [AMORT]. The calculator amortizes the NUMBER of payments entered in the display before pressing f, [AMORT]. This feature confuses some users, because for a monthly schedule, a 1 is always entered. For an annual amortization schedule, a 12 is entered each time before pressing f, [AMORT]. The number of cumulative periods amortized is found by RCL n.

C. After pressing f, [AMORT], the interest for the number of periods entered is displayed.

D. Press [X $\gtrless$ Y] for the principal portion of the number of payments entered.

E. Press RCL PV for the remaining balance.

F. Press RCL n for the cumulative number of payments amortized.

G. Go to Step 2 A and repeat procedure for next period or periods.

You can change the frequency of payments entered in Step 2 A at any time, as well as the payment amount or interest rate.

However, you must make sure the interest rate and payment amount remain constant for the number of periods amortized. This procedure is employed in later applications.

MANUAL AMORTIZATION PROCEDURE

Objective: To manually amortize mortgage payments.

Information Required:

1. Annual interest rate.
2. Amount borrowed.
3. Periodic payment.
4. Number of payments per year.

Example:

Sue has a $75,000 mortgage with 360 monthly payments of $728.45. The annual interest rate is 11.25 percent and she made four payments in 1980. Prepare a monthly amortization schedule for the first four payments and an annual schedule for the next two years.

Comments:

The number of payments entered is 1 for Steps 8a, 9a, 10a, and 11a on the solution because monthly amortization is required. When annual amortization is desired, a 12 is entered as shown in solution Step 12a.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Set display for rounding accuracy	f, 2	0.00
4. Enter annual interest rate	11.25 g [12÷]	0.94
5. Enter amount of mortgage as a positive value	75000 PV	75,000.00
6. Enter monthly payment as a negative value	728.45 CHS PMT	- 728.45
7. Store 0	0 n	0.00
8. Amortize first month		
a. enter number of periods to amortize	1	1.
b. compute interest	f [AMORT]	- 703.13
c. compute principal	[X ≷ Y]	- 25.32
d. remaining balance	RCL PV	74,974.68
e. total number of payments amortized	RCL N	1.00
Repeat Step 8 for each monthly amortization.		
9. Second month		
a. number of periods to amortize	1	1.
b. interest	f, [AMORT]	- 702.89
c. compute principal	[X ≷ Y]	- 25.56
d. remaining balance	RCL PV	74,949.12
e. total number of payments amortized	RCL N	2.00
10. Third month		
a. number of periods to amortize	1	1.
b. interest	f, [AMORT]	- 702.65

HP-12C Solution: (continued)

Procedure	Action Step	Display
c. compute principal	[X ≷ Y]	- 25.80
d. remaining balance	RCL PV	74,923.32
e. total number of payments amortized	RCL N	3.00
11. Fourth month		
a. number of periods to amortize	1	1.
b. interest	f, [AMORT]	- 702.41
c. compute principal	[X ≷ Y]	- 26.04
d. remaining balance	RCL PV	74,897.28
e. total number of payments amortized	RCL N	4.00
12. Amortization for second year		
a. enter number of payments to amortize	12	12.
b. compute interest for 12 months	f, [AMORT]	- 8,409.17
c. compute principal for twelve months	[X ≷ Y]	- 332.23
d. balance after 12 payments	RCL PV	74,565.05
e. total number of payments amortized	RCL N	16.00
Repeat Step 12 for each year.		
13. Amortization for third year		
a. enter number of payments to amortize	12	12.
b. compute interest for 12 months	f, [AMORT]	- 8,369.78
c. compute principal for twelve months	[X ≷ Y]	- 371.62
d. balance after 12 payments	RCL PV	74,193.43
e. total number of payments amortized	RCL N	28.00

LEVEL PAYMENT AMORTIZATION PROGRAM:

The following program is a good example of simply programming the manual keystrokes. If you amortized a mortgage manually, the following keystrokes would be made.

1. Set switch to g, END.
2. Store 0 in n.
3. Enter amount borrowed in PV as a positive number.
4. Enter payment in PMT as a negative number.
5. a. Enter number of periods to amortize.
   b. Press f [AMORT] to see the interest paid.
   c. Press [X ⪌ Y] to see the principal paid.
   d. Press RCL PV to see the remaining balance.
   e. Press RCL n to see the total number of periods amortized.
6. Repeat step 5 for other periods.

Notice that after manual steps 1 through 4, step 5 is repeated over and over. So we need to tell the calculator to remember steps 5a through 5e and to stop between each step.

We will place the number of periods amortized in Register 1 to avoid keying them in each time. This step also gives us flexibility as demonstrated later.

When you press the [R/S] key one of two actions occurs. If a program is <u>not</u> executing, pressing [R/S] starts or runs the program. If a program is executing (running) and the key [R/S] is either pressed or occurs in a program step, the program stops executing.

The program for amortizing is simply the manual keystrokes with a [R/S] between each manual command. The program will stop and display the calculated value and you press [R/S] to calculate the next value.

The payment amount is entered using the PMT key. When the payment changes, enter the new amount and press PMT. The number of periods amortized each time is stored in Register 1. For a period-by-period schedule, store a 1 in Register 1. For annual amortization store a 12 in Register 1.

The short program shown below will be used for all remaining examples in this chapter except for graduated payment mortgages and deferred mortgages.

Action Step	Press	Display	
1	CLX, f, P/R, f, PRGM	00 -	
2	RCL 1	01 -	45 1
3	f, [AMORT]	02 -	42 11
4	R/S	03 -	31
5	[X ⪌ Y]	04 -	34
6	R/S	05 -	31
7	RCL PV	06 -	45 13
8	R/S	07 -	31
9	RCL n	08 -	45 11
10	R/S	09 -	31
11	g, GTO 01	10 - 43	33 01
12	f, P/R	0.00	

Program Step	Comment
Step 1 -	We place the calculator in the learn mode using f, P/R. The calculator is in the learn mode when "PRGM" appears in the display. The sequence f, PRGM clears any program currently in the calculator and moves the pointer to program line 00.
Step 2 -	The number of periods to amortize is stored in Register 1. The recall command places this value in the display.
Step 3 -	The interest is calculated using f, [AMORT].
Step 4 -	The [R/S] tells the calculator to stop and display the value calculated in step 3.
Step 5 -	The key [X $\gtrless$ Y] recalls the principal paid.
Step 6 -	The [R/S] tells the calculator to stop and display the value calculated in step 5.
Step 7 -	The RCL PV recalls the remaining balance.
Step 8 -	The [R/S] tells the calculator to stop and display the value calculated in step 7.
Step 9 -	The RCL n recalls the number of periods amortized.
Step 10 -	The [R/S] tells the calculator to stop and display the value calculated in step 9.
Step 11 -	The GTO 01 tells the calculator to go back to line 01 or step 2 the next time the [R/S] key is pressed.
Step 12 -	Pressing f, P/R takes the calculator out of the learn mode. The word "PRGM" disappears from the display.

MONTHLY MORTGAGE AMORTIZATION SCHEDULE:

Objective: To calculate a monthly amortization schedule.

Information Required:

1. Amount borrowed.
2. Annual interest rate.
3. Amount of payment.

Example:

Prepare a monthly amortization schedule for the Hardrock Co. They have a 30 year, 12 percent mortgage with monthly payments of $514.31. The amount borrowed is $50,000.

HP-12C Solution:

Procedure	Action Step	Display
1. Key in level payment amortization program		
2. Clear financial register and reset program	CLX, f, FIN, f, PRGM	0.00
3. Set payment switch	g, END	
4. Set display	f, 2	
5. Enter annual interest rate	12, g, [12÷]	1.00
6. Enter amount borrowed	50000 PV	50,000.00
7. Enter monthly payment	514.31 CHS, PMT	- 514.31
8. Enter 0 in n	0 n	0.00
9. Enter 1	1 STO 1	1.00
10. Calculate month 1 values		
a. interest	R/S	- 500.00
b. principal	R/S	- 14.31
c. remaining balance	R/S	49,985.69
d. months amortized	R/S	1.00
11. Calculate month 2 values		
a. interest	R/S	- 499.86
b. principal	R/S	- 14.45
c. remaining balance	R/S	49,971.24
d. months amortized	R/S	2.00
12. Repeat step 11 for each month		

HP-12C Keystroke Template: Monthly Amortization Schedule Using Program:

Procedure	Action Step	Display
1. Key in level payment amortization program		
2. Clear financial registers and reset program	CLX, f, FIN, f, PRGM	0.00
3. Set payment switch	g, END	0.00
4. Enter number of digits for rounding	f ☐	☐
5. Enter annual interest rate	☐ g, [12÷]	☐
6. Enter amount borrowed	☐ PV	☐
7. Enter monthly payment	☐ CHS, PMT	☐
8. Enter 0 in n	0 n	0.00
9. Enter 1	1 STO 1	1.00
10. Calculate month 1 values		
a. interest	R/S	☐
b. principal	R/S	☐
c. remaining balance	R/S	☐
d. months amortized	R/S	☐
11. Calculate month 2 values		
a. interest	R/S	☐
b. principal	R/S	☐
c. remaining balance	R/S	☐
d. months amortized	R/S	☐
12. Repeat step 11 for each month		

ANNUAL MORTGAGE AMORTIZATION SCHEDULE:

Objective: To calculate annual amortization values.

Information Required:

1. Amount borrowed.
2. Annual interest rate.
3. Amount of payment.
4. Number of payments made in year 1.

Example:

Sue has a $75,000 mortgage with 360 monthly payments of $728.45. The annual interest rate is 11.25 percent and she made four payments in 1980. Prepare an annual amortization schedule for the first two years.

HP-12C Solution:

Procedure	Action Step	Display
1. Key in level payment amortization program		
2. Clear financial registers and reset program	CLX, f, FIN, f, PRGM	0.00
3. Set payment switch	g, END	0.00
4. Set display	f, 2	0.00
5. Enter annual interest rate	11.25, g, [12÷]	0.94
6. Enter amount borrowed	75000 PV	75,000.00
7. Enter monthly payment	728.45 CHS PMT	- 728.45
8. Enter 0 in n	0 n	0.00
9. Enter number of payments made in year 1	4 STO 1	4.00
10. Calculate year 1 values		
a. interest	R/S	- 2,811.08
b. principal	R/S	- 102.72
c. remaining balance	R/S	74,897.28
d. months amortized	R/S	4.00
11. Enter 12 - number of payments per year	12 STO 1	12.00
12. Calculate year 2 values		
a. interest	R/S	- 8,409.17
b. principal	R/S	- 332.23

HP-12C Solution: (continued)

Procedure	Action Step	Display
c. remaining balance	R/S	74,565.05
d. months amortized	R/S	16.00

13. Repeat step 12 for each year until final year when 8 is stored in Register 1 and step 12 repeated.

HP-12C Keystroke Template: Annual Mortgage Amortization Schedule Using Program:

Procedure	Action Step	Display
1. Key in level payment amortization program		
2. Clear financial registers and reset program	CLX, f, FIN, f, PRGM	0.00
3. Set payment switch	g, END	0.00
4. Enter number of digits for rounding	f ☐	☐
5. Enter annual interest rate	☐ g, [12÷]	☐
6. Enter amount borrowed	☐ PV	☐
7. Enter monthly payment	☐ CHS PMT	☐
8. Enter 0	0 n	0.00
9. Enter number of payments made in year 1	☐ STO 1	☐
10. Calculate year 1 values		
a. interest	R/S	☐
b. principal	R/S	☐
c. remaining balance	R/S	☐
d. months amortized	R/S	☐
11. Enter 12 payments made per year	12 STO 1	12.00
12. Calculate year 2 values		
a. interest	R/S	☐
b. principal	R/S	☐
c. remaining balance	R/S	☐
d. months amortized	R/S	☐

HP-12C Keystroke Template: (continued)

Procedure	Action Step	Display
13. Calculate year 3 values		
a. interest	R/S	
b. principal	R/S	
c. remaining balance	R/S	
d. months amortized	R/S	

14. Repeat step 13 for each year except the final year. For the final year store the number of months in Register 1 and repeat step 13.

COMPUTING AN AMORTIZATION SCHEDULE WHEN PAYMENTS VARY:

Objective: To calculate an annual amortization schedule with different payments in each period.

Information Required:

1. Amount borrowed.
2. Annual interest rate.
3. Payment schedule.

Comments:

The example below was designed to show how to use the amortization program when payments are different for each period. Remember the interest rate and payment amount must be constant for the number of periods amortized each time.

Example:

You have made an unusual mortgage with five annual payments of differing amounts. The payments are:

Year	Payment
1	$10,000
2	15,000
3	20,000
4	25,000
5	30,000

The loan has an annual rate of 13 percent. The amount borrowed is $66,073.53.

HP-12C Solution:

Procedure	Action Step	Display
1. Key in level payment amortization program		
2. Clear financial registers and reset program	CLX, f, FIN, f, PRGM	0.00
3. Set payment switch	g, END	0.00
4. Set display	f, 2	0.00
5. Enter annual interest rate	13 i	13.00
6. Enter amount borrowed	66073.53 PV	66,073.53
7. Enter 0 in n	0 n	0.00
8. Enter 1	1 STO 1	1.00
9. Enter first payment	10000 CHS PMT	- 10,000.00
10. Calculate year 1 values		
a. interest	R/S	- 8,589.56

HP-12C Solution: (continued)

Procedure	Action Step	Display
b. principal	R/S	- 1,410.44
c. remaining balance	R/S	64,663.09
d. years amortized	R/S	1.00
11. Enter second payment	15000 CHS PMT	- 15,000.00
12. Calculate year 2 values		
a. interest	R/S	- 8,406.20
b. principal	R/S	- 6,593.80
c. remaining balance	R/S	58,069.29
d. years amortized	R/S	2.00
13. Enter third payment	20000 CHS PMT	- 20,000.00
14. Calculate year 3 values		
a. interest	R/S	- 7,549.01
b. principal	R/S	- 12,450.99
c. remaining balance	R/S	45,618.30
d. years amortized	R/S	3.00
15. Enter fourth payment	25000 CHS PMT	- 25,000.00
16. Calculate year 4 values		
a. interest	R/S	- 5,930.38
b. principal	R/S	- 19,069.62
c. remaining balance	R/S	26,548.68
d. years amortized	R/S	4.00
17. Enter fifth year payment	30000 CHS PMT	- 30,000.00
18. Calculate year 5 values		
a. interest	R/S	- 3,451.33
b. principal	R/S	- 26,548.67
c. remaining balance	R/S	0.01
d. years amortized	R/S	5.00

ANNUAL AMORTIZATION SCHEDULE FOR GRADUATED PAYMENT MORTGAGE - MANUAL PROCEDURE:

Objective: To compute the yearly interest and principal payments for a graduated payment mortgage using the manual keystroke procedure.

Information Required:

1. Annual interest rate.
2. Amount borrowed.
3. Payment schedule.

Comments:

This schedule assumes payments will increase for five years after the first year and then remain constant. This solution procedure also shows you how to amortize most types of mortgages with varying payments.

Example:

Compute a yearly amortization schedule for the first 7 years for a $60,000, 13 percent mortgage. The payment schedule is:

	Number of Payments	Amount
year 1	12	$559.46
2	12	587.43
3	12	616.80
4	12	647.64
5	12	680.03
6 to 30	300	714.03

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter amount borrowed	60000 PV	60,000.00
4. Enter annual interest rate	13 g, [12÷]	1.08
5. Enter 0 in n	0 n	0.00
6. a. enter payment for year 1	559.46 CHS PMT	- 559.46
b. compute interest	12 f, AMORT	- 7,867.12
c. compute principal paid	[X ≷ Y]	1,153.60
d. balance end of year 1	RCL PV	61,153.60
7. a. enter payment for year 2	587.43 CHS PMT	- 587.43
b. compute interest	12 f, AMORT	- 8,005.62
c. compute principal	[X ≷ Y]	956.46
d. balance end of year 2	RCL PV	62,110.06

HP-12C Solution: (continued)

Procedure	Action Step	Display
8. a. enter payment for year 3	616.80 CHS PMT	- 616.80
b. compute interest	12 f, AMORT	- 8,115.88
c. compute principal	[X ≷ Y]	714.28
d. balance end of year 3	RCL PV	62,824.34
9. a. enter payment for year 4	647.64 CHS PMT	- 647.64
b. compute interest	12 f, AMORT	- 8,191.59
c. compute principal	[X ≷ Y]	419.91
d. balance end of year 4	RCL PV	63,244.25
10. a. enter payment for year 5	680.03 CHS PMT	- 680.03
b. compute interest	12 f, AMORT	- 8,225.56
c. compute principal	[X ≷ Y]	65.20
d. balance end of year 5	RCL PV	63,309.45
11. a. enter payment for year 6	714.03 CHS PMT	- 714.03
b. compute interest	12 f, AMORT	- 8,209.34
c. compute principal	[X ≷ Y]	- 359.02
d. balance end of year 6	RCL PV	62,950.43
12. Year 7		
a. compute interest	12 f, AMORT	- 8,159.78
b. compute principal	[X ≷ Y]	- 408.58
c. balance end of year 7	RCL PV	62,541.85

HP-12C Keystroke Template: Annual Amortization Schedule for Graduated Payment Mortgage - Manual Procedure:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter amount borrowed	[ ] PV	[ ]
4. Enter annual interest rate	[ ] g, [12÷]	[ ]
5. Enter 0 in n	0 n	0.00
6. a. enter payment for year 1	[ ] CHS PMT	[ ]
b. interest	12 f, AMORT	[ ]
c. principal	[X ⪌ Y]	[ ]
d. balance	RCL PV	[ ]
7. a. enter payment for year 2	[ ] CHS PMT	[ ]
b. interest	12 f, AMORT	[ ]
c. principal	[X ⪌ Y]	[ ]
d. balance	RCL PV	[ ]
8. a. enter payment for year 3	[ ] CHS PMT	[ ]
b. interest	12 f, AMORT	[ ]
c. principal	[X ⪌ Y]	[ ]
d. balance	RCL P V	[ ]
9. a. enter payment for year 4	[ ] CHS PMT	[ ]
b. interest	12 f, AMORT	[ ]
c. principal	[X ⪌ Y]	[ ]

HP-12C Keystroke Template: (continued)

Procedure	Action Step	Display
d. balance	RCL PV	
10. a. enter payment for year 5	☐ CHS PMT	
b. interest	12 f, AMORT	
c. principal	[X ≷ Y]	
d. balance	RCL PV	
11. a. enter payment for year 6	☐ CHS PMT	
b. interest	12 f, AMORT	
c. principal	[X ≷ Y]	
d. balance	RCL PV	
12. Year 7		
a. interest	12 f, AMORT	
b. principal	[X ≷ Y]	
c. balance	RCL PV	

Repeat step 12 for remaining years.

ANNUAL AMORTIZATION PROGRAM FOR A GRADUATED PAYMENT MORTGAGE:

Objective: To calculate an annual amortization schedule for a graduated payment mortgage using a program.

Information Required:

1. Amount borrowed.
2. Annual interest rate.
3. Payment schedule.
4. Number of payments in the first year.

Example:

Prepare a yearly amortization program for a 30 year, $40,000 graduated payment mortgage with the payments shown below. The annual interest rate is 8.4 percent and four payments are made in the first year.

Number of Payments	Amount
12	252.01
12	264.61
12	277.84
12	291.73
12	306.32
300	321.64

Comments:

This program is valid for all periods except the final partial year when less than 12 payments are made the first year. Use the manual keystrokes to compute the final year's schedule. The procedure is:

RCL 2
f, AMORT
[X $\gtrless$ Y]
RCL PV
RCL n

If 12 payments are made the first year, then enter 12 in step 8 of the example.

GRADUATED PAYMENT MORTGAGE AMORTIZATION PROGRAM:

Action Step	Press	Display	Action Step	Press	Display
1	CLX, f, P/R, f, PRGM	00 -	19	RCL 3	18 - 45 3
2	RCL 6	01 - 45 6	20	R/S	19 - 31
3	g, [x = 0]	02 - 43 35	21	RCL 4	20 - 45 4
4	g, [GTO] 08	03 - 43 33 08	22	R/S	21 - 31
5	f, AMORT	04 - 42 11	23	RCL PV	22 - 45 13
6	STO + 3	05 - 44 40 3	24	R/S	23 - 31
7	[X ≷ Y]	06 - 34	25	RCL n	24 - 45 11
8	STO + 4	07 - 44 40 4	26	R/S	25 - 31
9	1	08 - 1	27	2	26 - 2
10	STO - 0	09 - 44 30 0	28	STO 0	27 - 44 0
11	RCL 0	10 - 45 0	29	0	28 - 0
12	g, [x = 0]	11 - 43 35	30	STO 3	29 - 44 3
13	g, GTO 18	12 - 43 33 18	31	STO 4	30 - 44 4
14	RCL 1	13 - 45 1	32	RCL 2	31 - 45 2
15	STO 6	14 - 44 6	33	STO 6	32 - 44 6
16	RCL 5	15 - 45 5	34	g, GTO 01	33 - 44 33 01
17	STO PMT	16 - 44 14	35	f, P/R	0.00
18	g, GTO 01	17 - 43 33 01			

HP-12C Solution:

Procedure	Action Step	Display
1. Key in Graduated Payment Mortgage amortization program		
2. Clear financial registers and reset program	CLX, f, FIN, f, PRGM	0.00
3. Set payment switch	g, END	0.00
4. Set display	f, 2	0.00
5. Enter annual interest rate	8.4 g, [12÷]	0.70
6. Enter amount borrowed	40000 PV	40,000.00
7. Enter	0 n 1 STO 0 0 STO 3, STO 4	0.00 1.00 0.00
8. Enter number of payments for year 1	4 STO 1 STO 6 12 - CHS STO 2	4.00 8.00
9. Enter first payment amount	252.01 CHS PMT	- 252.01
10. Compute schedule for year 1		
a. interest	R/S	- 1,121.18
b. principal	R/S	113.14
c. remaining balance	R/S	40,113.14
d. number of months amortized	R/S	4.00
11. Enter second payment amount	264.61 CHS STO 5	- 264.61
12. Compute schedule for year 2		
a. interest	R/S	- 3,382.57
b. principal	R/S	308.05
c. remaining balance	R/S	40,421.19
d. number of months amortized	R/S	16.00
13. Enter third payment amount	277.84 CHS STO 5	- 277.84
14. Compute schedule for year 3		
a. interest	R/S	- 3,403.51
b. principal	R/S	175.27
c. remaining balance	R/S	40,596.46
d. number of months amortized	R/S	28.00

HP-12C Solution: (continued)

Procedure	Action Step	Display
15. Enter fourth payment amount	291.73 CHS STO 5	- 291.73
16. Compute schedule for year 4		
a. interest	R/S	- 3,412.52
b. principal	R/S	22.88
c. remaining balance	R/S	40,619.34
d. number of months amortized	R/S	40.00
17. Enter fifth payment amount	306.32 CHS STO 5	- 306.32
18. Compute schedule for year 5		
a. interest	R/S	- 3,407.92
b. principal	R/S	- 151.20
c. remaining balance	R/S	40,468.14
d. number of months amortized	R/S	52.00
19. Enter sixth payment amount	321.64 CHS STO 5	- 321.64
20. Compute schedule for year 6		
a. interest	R/S	- 3,387.79
b. principal	R/S	- 349.33
c. remaining balance	R/S	40,118.81
d. number of months amortized	R/S	64.00
21. Compute schedule for year 7		
a. interest	R/S	- 3,350.69
b. principal	R/S	- 508.99
c. remaining balance	R/S	39,609.82
d. number of months amortized	R/S	76.00

22. Repeat step 21 for all but the final year on which you use the manual key sequence. After the last payment change, no further entries for the payment are necessary.

HP-12C Keystroke Template: Annual Amortization Program for a Graduated Payment Mortgage:

Procedure	Action Step	Display
1. Key in G.P.M. amortization program		
2. Clear financial registers and reset program	CLX, f, FIN	0.00
3. Set payment switch	g, END	0.00
4. Enter number of digits for rounding	f [ ]	[ ]
5. Enter annual interest rate	[ ] g, [12÷]	[ ]
6. Enter amount borrowed	[ ] PV	[ ]
7. Enter	0 n, 1 STO 0, 0 STO 3, STO 4	0.00
8. Enter number of payments for year 1	[ ] STO 1	
	STO 6, 12, -, CHS, STO 2	[ ]
9. Enter first payment amount	[ ] CHS PMT	[ ]
10. Compute year 1 values		
a. interest	R/S	[ ]
b. principal	R/S	[ ]
c. remaining balance	R/S	[ ]
d. number of months amortized	R/S	[ ]
11. Enter second payment amount	[ ] CHS, STO 5	[ ]
12. Compute year 2 values		
a. interest	R/S	[ ]
b. principal	R/S	[ ]
c. remaining balance	R/S	[ ]
d. number of months amortized	R/S	[ ]

HP-12C Keystroke Template: (continued)

Procedure	Action Step	Display
13. Enter third payment amount	[ ] CHS, STO 5	[ ]
14. Compute year 3 values		
a. interest	R/S	[ ]
b. principal	R/S	[ ]
c. remaining balance	R/S	[ ]
d. number of months amortized	R/S	[ ]
15. Enter fourth payment amount	[ ] CHS, STO 5	[ ]
16. Compute year 4 values		
a. interest	R/S	[ ]
b. principal	R/S	[ ]
c. remaining balance	R/S	[ ]
d. number of months amortized	R/S	[ ]
17. Enter fifth payment amount	[ ] CHS, STO 5	[ ]
18. Compute year 5 values		
a. interest	R/S	[ ]
b. principal	R/S	[ ]
c. remaining balance	R/S	[ ]
d. number of months amortized	R/S	[ ]
19. Enter sixth payment amount	[ ] CHS, STO 5	[ ]
20. Compute year 6 values		
a. interest	R/S	[ ]
b. principal	R/S	[ ]

HP-12C Keystroke Template: (continued)

Procedure	Action Step	Display
c. remaining balance	R/S	
d. number of months amortized	R/S	
21. Compute year 7 values		
a. interest	R/S	
b. principal	R/S	
c. remaining balance	R/S	
d. number of months amortized	R/S	

22. Repeat step 21 for all but the final year on which you use the manual key sequence.

MONTHLY MORTGAGE AMORTIZATION SCHEDULE AND PROGRAM: CONSTANT PRINCIPAL PAYMENTS:

Objective: To calculate a monthly amortization schedule.

Information Required:

1. Amount borrowed.
2. Amount interest rate.
3. Amount of payment.

Comments:

Some loans have constant principal payments. This means each payment varies because the amount of interest paid changes as each payment is made.

Example:

Prepare a monthly amortization schedule for the Softrack Co. They have a 10 year 12 percent mortgage for $120,000. The monthly principal payment is $1,000 plus interest.

AMORTIZATION OF CONSTANT PRINCIPAL PAYMENT LOANS

The program shown below is used for amortization of constant principal payment loans. See pages 405-407 of Financial Analysis Using Calculators: Time Value of Money for the specific equations employed in the program. This program computes the accumulated interest and principal for the number of periods specified.

CONSTANT PRINCIPAL PAYMENT LOAN AMORTIZATION PROGRAM:

Action Step	Press	Display
1	CLX, f, P/R, f, PRGM	00 -
2	RCL 1	01 - 45 1
3	RCL n	02 - 45 11
4	+	03 - 40
5	STO n	04 - 44 11
6	[ENTER]	05 - 36
7	RCL PMT	06 - 45 14
8	X	07 - 20
9	CHS	08 - 16
10	RCL PV	09 - 45 13
11	+	10 - 40
12	STO 2	11 - 44 2
13	RCL PMT	12 - 45 14
14	RCL 1	13 - 45 1
15	X	14 - 20
16	+	15 - 40
17	RCL 1	16 - 45 1
18	X	17 - 20
19	STO 3	18 - 44 3
20	RCL 1	19 - 45 1
21	[ENTER]	20 - 36
22	[ENTER]	21 - 36
23	1	22 - 1
24	-	23 - 30
25	X	24 - 20
26	2	25 - 2
27	÷	26 - 10
28	RCL PMT	27 - 45 14
29	X	28 - 20
30	CHS	29 - 16
31	RCL 3	30 - 45 3
32	+	31 - 40
33	RCL i	32 - 45 12
34	1	33 - 1
35	0	34 - 0
36	0	35 - 0
37	÷	36 - 10
38	STO 4	37 - 44 4
39	X	38 - 20
40	R/S	39 - 31
41	RCL PMT	40 - 45 14
42	RCL 1	41 - 45 1
43	X	42 - 20
44	R/S	43 - 31
45	RCL 2	44 - 45 2
46	R/S	45 - 31
47	RCL 2	46 - 45 2
48	RCL PMT	47 - 45 14
49	+	48 - 40
50	RCL 4	49 - 45 4
51	X	50 - 20
52	RCL PMT	51 - 45 14
53	+	52 - 40
54	R/S	53 - 31
55	RCL n	54 - 45 11
56	R/S	55 - 31
57	g, GTO 01	56 - 44 33 01
58	f, P/R	0.00

HP-12C Solution:

Procedure	Action Step	Display
1. Key in constant principal payment program		
2. Clear registers	f, REG	0.00
3. Reset program	f, PRGM	0.00
4. Enter annual interest rate	12 g, [12÷]	1.00
5. Enter amount borrowed	120000 PV	120,000.00
6. Enter amount of constant principal payment as a positive value	1000 PMT	1,000.00
7. Enter 0 in n	0 n	0.00
8. Enter 1 for monthly payments	1 STO 1	1.00
9. Calculate month 1 values		
a. interest for month 1	R/S	1,200.00
b. principal for month 1	R/S	1,000.00
c. remaining balance	R/S	119,000.00
d. total payment	R/S	2,200.00
e. number of the amortized payment	R/S	1.00
10. Calculate month 2 values		
a. interest for month 2	R/S	1,190.00
b. principal for month 2	R/S	1,000.00
c. remaining balance at end of month 2	R/S	118,000.00
d. total payment for month 2	R/S	2,190.00
e. number of the amortized payment	R/S	2.00
11. Repeat step 10 for each month.		

HP-12C Keystroke Template: Monthly Amortization Schedule and Program: Constant Principal Payments:

Procedure	Action Step	Display
1. Key in constant principal payment program		
2. Clear registers	f, REG	0.00
3. Reset program	f, PRGM	0.00
4. Enter annual interest rate	☐ g, [12÷]	1.00
5. Enter amount borrowed	☐ PV	☐
6. Enter amount of constant principal payment as a positive value	☐ PMT	☐
7. Enter 0 in n	0 n	0.00
8. Enter 1 for monthly payments	1 STO 1	1.00
9. Calculate month 1 values		
a. interest for month 1	R/S	☐
b. principal for month 1	R/S	☐
c. remaining balance	R/S	☐
d. total payment	R/S	☐
e. number of the amortized payment	R/S	1.00
10. Calculate month 2 values		
a. interest for month 2	R/S	☐
b. principal for month 2	R/S	☐
c. remaining balance at end of month 2	R/S	☐
d. total payment for month 2	R/S	☐
e. number of the amortized payment	R/S	2.00
11. Repeat step 10 for each month.		

ANNUAL MORTGAGE AMORTIZATION SCHEDULE: CONSTANT PRINCIPAL PAYMENTS:

Objective: To calculate annual amortization value for a mortgage with constant principal payments.

Information required:

1. Amount borrowed.
2. Annual interest rate.
3. Amount of payment.
4. Number of payments made in year 1.

Example:

Bert has a $60,000 mortgage with 36 monthly payments. The annual interest rate is 18 percent with monthly compounding. He pays a constant principal payment of $1,666.67 ($60,000 ÷ 36) plus interest. He made 5 payments in 1981. Prepare an annual amortization schedule for this loan.

HP-12C Solution:

Procedure	Action Step	Display
1. Key in constant principal payment program		
2. Clear registers	f, REG	0.00
3. Reset program	f, PRGM	0.00
4. Enter annual interest rate	18 g, [12÷]	1.50
5. Enter amount borrowed	60000 PV	60,000.00
6. Enter amount of constant principal payment as a positive value	1666.67 PMT	1,666.67
7. Enter 0 in n	0 n	0.00
8. Enter number of payments made in year 1	5 STO 1	5.00
9. Calculate year 1 values		
a. interest for 5 payments	R/S	4,250.00
b. principal for 5 payments	R/S	8,333.35
c. balance after 5 payments	R/S	51,666.65
d. total payment value for 5th payment	R/S	2,466.67
e. number of payments amortized	R/S	5.00
10. Enter number of payments made in year 2	12 STO 1	12.00

HP-12C Solution: (continued)

Procedure	Action Step	Display
11. Calculate year 2 values (payments 6 through 17)		
a. interest for 12 payments	R/S	7,649.99
b. principal for 12 payments	R/S	20,000.04
c. balance after 17 payments	R/S	31,666.61
d. total payment value for 17th payment	R/S	2,166.67
e. total number of payments amortized	R/S	17.00
12. Calculate year 3 values (payments 18 through 29)		
a. interest	R/S	4,049.99
b. principal	R/S	20,000.04
c. balance after 29th payment	R/S	11,666.57
d. total amount of 29th payment	R/S	1,866.67
e. total number of payments amortized	R/S	29.00
13. Enter number of payments made in last year of the mortgage	7 STO 1	7.00
14. Calculate year 4 values (payments 30 through 36)		
a. interest for 7 months	R/S	699.99
b. principal for 7 months	R/S	11,666.69
c. balance after 36th payment	R/S	- 0.12
d. total amount of 36th payment	R/S	1,691.67
e. total number of payments amortized	R/S	36.00

The -0.12 in Step 14 c. is due to the rounding of the constant principal payment to the nearest penny.

HP-12C Keystroke Template: Annual Mortgage Amortization Schedule: Constant Principal Payments:

Procedure	Action Step	Display
1. Key in constant principal payment program		
2. Clear registers	f, REG	0.00
3. Reset program	f, PRGM	0.00
4. Enter annual interest rate	☐ g, [12÷]	☐
5. Enter amount borrowed	☐ PV	☐
6. Enter amount of constant principal payment as a positive value	☐ PMT	☐
7. Enter 0 in n	0 n	0.00
8. Enter number of payments made in year 1	☐ STO 1	☐
9. Calculate year 1 values		
a. interest	R/S	☐
b. principal	R/S	☐
c. balance	R/S	☐
d. total payment value	R/S	☐
e. number of payments amortized	R/S	☐
10. Enter number of payments made in year 2	☐ STO 1	☐
11. Calculate year 2 values		
a. interest for 12 payments	R/S	☐
b. principal for 12 payments	R/S	☐
c. balance	R/S	☐
d. total payment value	R/S	☐
e. total number of payments amortized	R/S	☐

HP-12C Template: (continued)

Procedure	Action Step	Display
12. Calculate year 3 values		
a. interest	R/S	
b. principal	R/S	
c. balance	R/S	
d. total amount of payment	R/S	
e. total number of payments amortized	R/S	

Repeat step 12 for each year until final year.

AMORTIZATION SCHEDULE FOR DEFERRED MORTGAGE:

Objective: To compute an amortization schedule for a deferred mortgage.

Information Required:

1. Annual interest rate.
2. Amount borrowed.
3. Payment.
4. Number of payments.
5. Number of deferred payments.

Example:

For a $50,000 mortgage with an 11.4 percent annual interest rate and monthly payments of $657.98, compute an amortization schedule for the first three years. The mortgage was closed on January 1, 1980, and the first payment is due on September 1, 1980.

Comments:

The number of deferred periods entered in step 7 is 8 because there are eight whole months between January 1 and September 1.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	11.4 g, [12÷]	0.95
4. Enter amount borrowed	50000 PV	50,000.00
5. Enter 0 in n	0 n	0.00
6. Enter 0 for deferred payment	0 PMT	0.00
7. Enter number of deferred periods	8 [ENTER] 1 -	7.00
a. compute interest	f, AMORT	3,421.29
b. compute principal	[X ≷ Y]	3,421.29
c. balance end of deferred period	RCL PV	53,421.29
8. Enter payment	657.98 CHS PMT	- 657.98
9. Enter number of payments made in year 1	4	4.
a. compute interest	f, AMORT	- 2,021.37
b. compute principal	[X ≷ Y]	- 610.55
c. balance end of year 1	RCL PV	52,810.74
10. Year 2		
a. number of payments	12	12.
b. compute interest	f, AMORT	- 5,919.27

HP-12C Solution:

Procedure	Action Step	Display
c. compute principal	[X $\gtrless$ Y]	- 1,976.49
d. balance end of year 2	RCL PV	50,834.25
11. Year 3		
a. number of payments	12	12.
b. compute interest	f, AMORT	- 5,681.81
c. compute principal	[X $\gtrless$ Y]	- 2,213.95
d. balance end of year 3	RCL PV	48,620.30

HP-12C Keystroke Template: Amortization Schedule for Deferred Mortgage:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter annual interest rate	☐ g, [12÷]	☐
4. Enter amount borrowed	☐ PV	☐
5. Enter 0 in n	0 n	0.00
6. Enter 0 for deferred payment	0 PMT	0.00
7. Enter number of deferred periods	☐ [ENTER] 1 -	☐
a. interest	f, AMORT	☐
b. principal	[X ≷ Y]	☐
c. balance	RCL PV	☐
8. Enter payment	☐ CHS PMT	☐
9. Enter number of payments made in year 1	☐	☐
a. interest	f, AMORT	☐
b. principal	[X ≷ Y]	☐
c. balance	RCL PV	☐
10. Year 2		
a. number of payments	☐	☐
b. interest	f, AMORT	☐
c. principal	[X ≷ Y]	☐
d. balance	RCL PV	☐

Repeat Step 10 for remaining years.

9

NET PRESENT VALUE AND RATES OF RETURN FOR VARIABLE CASH FLOWS

In this chapter we show you how to solve for Net Present Value (NPV), Internal Rate of Return (IRR), and Financial Management Rate of Return (FMRR). We assume in all cases that the after-tax cash flow per period is known.

NET PRESENT VALUE AND INTERNAL RATE OF RETURN

Net Present Value is the difference between the cash flow at time zero and the present value of a series of cash flows discounted at a specified interest rate. This "discount" or "investment" rate is the interest you can earn in the future on an investment made now. This rate differs from the interest rate that makes the cash flows equivalent to the present value.

Internal Rate of Return is the interest that makes the present value of a series of cash flows equal to the initial cash outflow at time zero. The IRR rate assumes that the cash flows are reinvested at that rate. Thus, if the cash flows are actually reinvested at a rate different than the IRR, the actual rate earned will also be different than the IRR. When only one negative cash flow occurs in a stream of positive cash flows, there is only one IRR rate. But if multiple negative cash flows occur among positive cash flows, then more than one IRR rate may exist. (For a detailed discussion see Financial Analysis Using Calculators, pp. 263-267.) The HP-12C has a special internal routine that normally will identify such cases as described later.

Solving for NPV and IRR. Solving for NPV and IRR on the HP-12C requires using the grouped or variable cash-flow keys: [CF_0], [CF_j], [N_j], [IRR], and [NPV]. Using these keys to solve for NPV or IRR requires two basic steps:

Step 1. Enter the cash flows.
Step 2. Solve for NPV or IRR.

Entering the cash flows is simple once you have identified them. The [CF_0] key is used to enter the net cash flow occurring at time zero or the beginning of the first payment period. Regular cash flows are entered using the [CF_j] and [N_j] keys. The calculator logic assumes that cash flows occur at equal intervals, i.e., monthly, quarterly, yearly, etc. The calculator logic also assumes that each cash flow occurs at the end of each period. This is true even if the switch is set to g, [BEG]. The applications using the grouped cash-flow keys in this book always show the switch set to g, [END] as a reminder of this assumption.

The calculator is designed so that 20 different (variable) cash flows, or 20 grouped cash flows, or a combination of 20 variable or grouped cash flows can be entered. You must indicate which flows are positive and negative.

To enter a variable cash flow, key in the amount and press [g][CF_j]. If another cash flow is entered using this sequence, the calculator will automatically enter a one for the [N_j] value.

But if you have a group of cash flows that are constant for several consecutive periods, say $2,500 for 3 periods and $5,000 for 6 periods, then you can enter them as follows:

2500 g, CF_j

3 g, N_j

5000 g, CF_j

6 g, N_j

Entering 3 using [g][N_j] for the $2,500 cash flow tells the calculator that this cash flow occurs three times. The next entry on those keys shows that the $5,000 cash flows occur six times. Since we only use two of our 20 registers available for cash flows, we can add up to 18 more grouped cash flows. Due to the calculator design, only two digit values can be entered using the [N_j] key, so the largest group you can enter is 99. A group of, say, 150 cash flows of $6,000 must be entered in two groups of less than 100; for example:

6000 g, CF_j

99 g, N_j

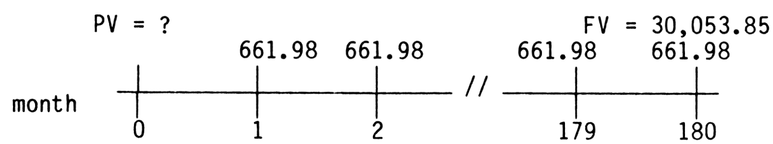
6000 g, CF_j

51 g, N_j

After entering the cash flows, you are ready to solve for either or both the NPV or IRR. The NPV is obtained by entering the required earnings rate per payment period in [i] and pressing f, NPV, and the IRR by pressing f, IRR. The mortgage problem in Example 1 illustrates the meaning of the NPV. The analysis following Example 1 involves the use of both the annuity keys and the cash-flow keys.

EXAMPLE 1: NET PRESENT VALUE:

Let's assume you are interested in purchasing a mortgage with a current balance of $60,721.33 and 180 remaining payments of $661.98 with a balloon of 30,053.85 due in addition to the final payment. The mortgage has an annual rate of 12 percent compounded monthly. If you don't purchase the mortgage, you can invest the money in your firm or other investments and earn 15 percent annual interest compounded monthly. You can purchase the mortgage for $43,783.30. Should you purchase the mortgage? As usual, our first step is to draw a time diagram:



i = 15%/12

Time Diagram 9.1

Time Diagram 9.1 has the cash flows set up for use with the annuity keys. The unknown value is the present value. The first step is to determine the equivalent value of the cash flows at time zero using an interest rate of 15 percent annual compounded monthly, or a rate of 1.25 percent per month (15/12). We need to compare the value of the money we pay out at time zero ($43,783.30) to the equivalent value of the future cash flows using our discount rate of 1.25 percent per month. If the equivalent value of the cash flows exceeds the cash outflow at time zero, then it is a good investment because we are receiving a greater equivalent amount of money at time zero than we are paying out. The -$50,510.37 equivalent value of the cash flows for Example 1 is found using the annuity keys.

Action Step	Display
f, FIN	0.00
g, END	0.00
180 n	180.00
1.25 i	1.25
661.98 PMT	661.98
30,053.85 FV	30,053.85
PV	- 50,510.37

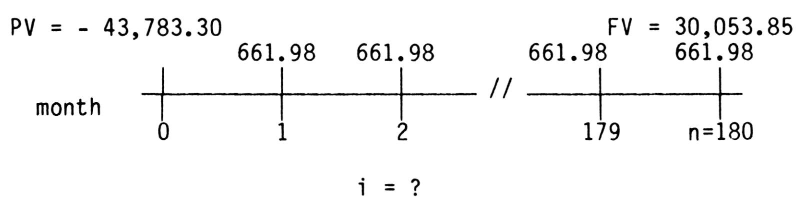
The payment switch is set to g, [END] because the first regular cash flow occurs at the end of the first payment period. The present value of $50,510.37 represents the equivalent value at time zero of the future cash flows using an interest rate of 1.25 percent per month. In other words, if you put $50,510.37 in a savings account paying 1.25 percent interest compounded monthly, you could withdraw $661.98 at the end of each month for 180 months. Furthermore, after you withdraw the 180th amount, you will have a balance of $30,053.85 that you can withdraw. But in the example, you only have to pay $43,783.30 to obtain this series of cash flows. You can think of the difference as the amount of profit you make on the investment. We also call the difference the net present value or NPV. In this book, the NPV for annuities is calculated as follows:

Step 1.	Compute the equivalent value of future cash flows using your investment rate (discount rate).	$50,510.37
Step 2.	Subtract the cash flow at time zero that is necessary to purchase the future cash flows.	43,783.30
Step 3.	Net present value.	$ 6,727.07

The $6,727.07 represents the excess value of the future cash flows (180 payments of 661.98 and 30,053.85) over the outlay cost of 43,783.30 at time zero assuming a monthly rate of 1.25 percent; or, you get more money back than you pay out if you purchase the mortgage. Remember that the NPV value is only as accurate as your estimate of the long run investment or discount rate. If the NPV had been negative, then you could earn more money in other investments.

We can also describe the present value of the future cash flows using the 1.25 percent rate as the amount you would pay if you wanted to earn 1.25 percent compounded monthly on the mortgage. In other words, you would pay $50,510.37 for the mortgage if you wanted to earn 1.25 percent compounded monthly.

We can also solve for the interest rate that makes the future cash flows equivalent to the price of 43,783.30. Time Diagram 9.2 shows the cash-flow pattern.



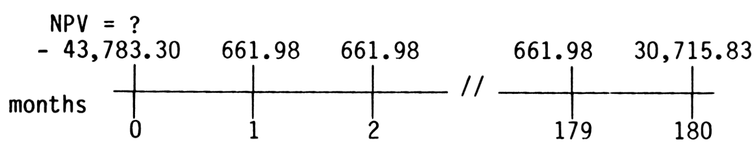
Time Diagram 9.2

The interest rate is found using the following annuity inputs.

Action Step	Display
CLX, f, FIN	0.00
g, END	0.00
180 n	180.00
43783.30, CHS, PV	- 43,783.30
661.98, PMT	661.98
30053.85 FV	30,053.85
i,	1.48

The rate of 1.48 percent per month or 17.72 percent per year (1.48% X 12) is the interest rate that makes the future cash flows equivalent to the purchase price of $43,783.30.

The g, [NPV] key. Now we are going to resolve Example 1 and explain why the key used to compute the equivalent value of variable (grouped) cash flows at time zero is called g, [NPV]. Because we are using the grouped cash-flow keys, we must draw a new time diagram with only one cash flow occurring per period.



i = 1.25%

Time Diagram 9.3

The balloon of $30,053.85 and the final regular payment of $661.98 have been added together to show one cash flow of $30,715.83 at the end of month 180. The unknown value is the net present value at time zero or the equivalent value of all the cash flows. The outlay cost of -$43,783.30 is shown at time zero. We solve for the equivalent value of these cash flows using these grouped cash-flow key inputs.

Action Step	Display
f, REG	0.00
1.25 i	1.25
43,783.30 CHS, g, CF_0	- 43,783.30
661.98 g, CF_j	661.98
99 g, N_j	99.00
661.98 g, CF_j	661.98
80 g, N_j	80.00
30,715.83 g, CF_j	30,715.83
f, NPV	6,727.07

The answer of 6,727.07 is the same answer we found earlier using the annuity keys. When you use the grouped cash-flow keys and store a value in CF_0 that has a sign opposite to the values stored in the CF_j registers, the calculator adds the negative CF_0 amount to the present value of the future cash flows. So HP called the equivalent value of all the cash flows the NPV. But as we demonstrated earlier, the key can be used for other purposes such as simply finding the equivalent value of a future series of cash flows with or without an amount stored in CF_0.

Finding the interest rate. Now let's solve for the interest rate which makes the cash flows equivalent to the outlay cost. The rate is found using the grouped cash-flow key inputs shown below.

Action Step	Display
f, REG	0.00
43,783.30 CHS, g, CF_0	- 43,783.30
661.98 g, CF_j	661.98
99 g, N_j	99.00
661.98 g, CF_j	661.98
80 g, N_j	80.00
30,715.83 g, CF_j	30,715.83
f, IRR	1.48

While the key is called IRR, the answer of 1.48 is technically the interest rate which makes the 179 payments of 661.98 and one payment of 30,715.83 equivalent to the outlay cost of 43,783.30. The net present value using the 1.48 percent interest rate is zero.

The IRR Assumption. The IRR is often called an earnings rate. Because of the underlying IRR assumptions, however, IRR is an earnings rate only when the future cash flows are reinvested at the IRR. The following example illustrates the reinvestment assumption.

EXAMPLE 2:

The AB Land Co. is evaluating an investment that has the following quarterly after-tax returns:

Quarter	Cash Flow
0	- 1,225,400
1	10,000
2	15,000
3	20,000
4	30,000
5	30,000
6	30,000
7	40,000
8	1,540,000

The company requires a 14 percent return on its investments. What is the NPV and IRR on the project?

Keystroke Solution: Solving for NPV and IRR:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set switch	g, END	0.00
3. Enter required annual return	14 [ENTER]	14.00
4. Enter number of payments per year	4	4.00
5. Compute and enter required return per payment period	÷ i	3.50
6. Enter initial cash outflow	1,225,400 CHS, g, CF_0	- 1,225,400.00
7. Enter end of period cash flows	10000 g, CF_j	10,000.00
	15000 g, CF_j	15,000.00
	20000 g, CF_j	20,000.00
	30000 g, CF_j	30,000.00
	3 g, N_j	3.00
	40000 g, CF_j	40,000.00
	1540000 g, CF_j	1,540,000.00
8. Solve for NPV	f, NPV	93,044.27
9. Solve for IRR per payment period	f, IRR	4.50
10. Enter number of payments per year	4	4.00
11. Compute annual IRR	X	18.00
12. Full decimals	f, 9	17.99985362

The NPV of $93,044.27 means that the present value of the cash flows for quarters 1 through 8 exceeds the outlay cost by that amount when discounted at 3.50 percent quarterly.

The IRR of 18.00 percent is the interest rate that discounts the quarterly cash flows back to an amount equal to the initial cash flow of $1,225,400.00. In other words, the IRR is the interest rate that makes the NPV equal to zero. When you use the IRR, remember that the rate is accurate only if the cash flows are reinvested at the same IRR.

EXAMPLE 3:

To illustrate the concept of reinvesting and its meaning for IRR, we will assume that as the cash is received it is invested in a savings account paying 17.99985362 percent annual interest compounded quarterly. After the eighth payment, the lump sum amount in the account will represent the future value of the initial cash outflow invested at the IRR. Once the present value (initial cash outflow) and the future value are known, the interest rate can be computed. This rate will be the same as the IRR only if the funds are reinvested at the IRR.

Keystroke Solution - with Reinvestment at IRR:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Enter interest rate per quarter as a decimal	17.99985362 [ENTER] 100 ÷ 4 ÷	0.04
3. Store 1 + i	1 + STO 0	1.04
4. Enter 1st cash flow	10000	10,000.00
	RCL 0	1.04
5. Balance end of year 1	X	10,450.00
6. Enter remaining cash flows and compound forward one period at a time	15000 + RCL 0 X	26,595.24
	20000 + RCL 0 X	48,692.01
	30000 + RCL 0 X	82,233.12
	30000 + RCL 0 X	117,283.57
	30000 + RCL 0 X	153,911.27
	40000 + RCL 0 X	202,637.21
7. Balance at end of quarter 8	1540000 +	1,742,637.21
8. Store as FV	FV	1,742,637.21
9. Enter outlay	1225400, CHS, PV	- 1,225,400.00
10. Enter 0 for PMT	0 PMT	0.00
11. Enter number of quarters	8 n	8.00
12. Compute quarterly return	i	4.50
13. Compute annual return	4 X	18.00
14. Full decimals	f, 9	17.99985362

As shown above, when the quarterly cash flows are compounded forward at the IRR we have two equivalent cash flows -- the initial outflow (PV) and the compounded value of the returns (PV). Solving for the interest rate that makes the FV equal to the PV, we see that the computed rate is the same as the IRR. So if the cash flows are reinvested at an interest rate equal to the IRR, then the earnings rate and IRR are identical. However, if the reinvestment rate is different from the IRR the earnings rate is also different.

For example, let's assume that the quarterly cash flows cannot be reinvested at the IRR, but at the firm's required rate of 14 percent.

Keystroke Solution - with Reinvestment at 14 Percent:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Enter reinvestment interest rate as a decimal	14 [ENTER] 100 ÷ 4 ÷	0.04
3. Store 1 + i	1 + STO 0	1.04
4. Enter 1st cash flow	10000	10,000.00
	RCL 0	1.04
5. Balance end of year 1	X	10,350.00
6. Enter remaining cash flows and compound forward one period at a time	15000 + RCL 0 X	26,237.25
	20000 + RCL 0 X	47,855.55
	30000 + RCL 0 X	80,580.50
	30000 + RCL 0 X	114,450.82
	30000 + RCL 0 X	149,506.59
	40000 + RCL 0 X	196,139.32
7. Balance at end of quarter 8	1540000 +	1,736,139.33
8. Store as FV	FV	1,736,139.33
9. Enter outlay	1225400, CHS, PV	- 1,225,400.00
10. Enter 0 for PMT	0 PMT	0.00
11. Enter number of quarters	8 n	8.00
12. Compute quarterly return	i	4.45
13. Compute annual return	4 X	17.80
14. Full decimals	f, 9	17.80470717

As shown above, the overall return, assuming reinvestment at 14 percent, drops to 17.80 percent, less than the IRR. For situations where there are more cash-flow periods, the difference in rates may be larger. Of course, if the reinvestment rate is higher than the IRR, the return will also be higher than the IRR. One problem in computing this adjusted return is accurately estimating the reinvestment rate for future projects.

Multiple IRRs. As previously mentioned, a number of IRRs *may* exist if multiple negative cash flows occur among the positive values. Your calculator usually indicates when such cases exist, and displays "Error 3." When this occurs you must enter a guess for IRR as a percent and press [RCL] [g] [R/S].

To illustrate this point consider the following cash flows (from Financial Analysis Using Calculators, p. 266).

cash outflow	- 500
cash inflows	
1	+ 1000
2	+ 6000
3	- 7000

Two interest rates will make the NPV equal zero, 7.741983634 percent and 309.5405477 percent.

If you enter the cash flows and press f, IRR, the calculator displays "Error 3." So press [CLX], key in a guess of 0 and press [RCL] [g] [R/S]. The answer of 7.74 percent appears. If you repeat the procedure with an initial guess of 900 percent the answer 309.54% appears.

The IRR in such cases is misleading and alternative methods such as FMRR should be considered when evaluating the yield. But remember, your HP-12C normally indicates when multiple rates may exist.

FINANCIAL MANAGEMENT RATE OF RETURN

An alternative to IRR, especially when multiple rates exist, is the Financial Management Rate of Return (FMRR).

The difference between IRR and FMRR is the reinvestment assumption. IRR assumes that all cash flows (positive or negative) are reinvested at the same rate. FMRR, however, uses two reinvestment rates. First, positive cash flows preceding negative flows are invested at a sure or low rate to fund the negative cash flows. Second, any positive cash flows not needed to fund the negative flows are invested at the high rate or the firm's minimum cost of capital. Any negative cash flows not funded by the positive flows are discounted back to the beginning of time zero. As a result, the original cash flows are converted to two values: a negative present value and a positve future value. The negative present value is the amount necessary to cover the initial cash outlay and any unfunded negative cash flows. The positive future value represents the positive flows value at the end of the analysis period. The FMRR is the interest rate that discounts the future value back to an amount equal to the present value. Remember, the answer is only as accurate as the estimate of the two investment rates.

Computing the FMRR requires using the following program.

FINANCIAL MANAGEMENT RATE OF RETURN PROGRAM:

Action Step	Press	Display	Action Step	Press	Display
1	CLX, f, P/R, f, PRGM	00 -	22	RCL 2	21 - 45 2
2	RCL 1	01 - 45 1	23	g, [x = 0]	22 - 43 35
3	+	02 - 40	24	g, GTO 35	23 - 43 33 35
4	0	03 - 0	25	RCL 6	24 - 45 6
5	g, [x $\leq$ y]	04 - 43 34	26	RCL 4	25 - 45 4
6	g, GTO 10	05 - 43 33 10	27	÷	26 - 10
7	STO 7	06 - 44 7	28	STO 1	27 - 44 1
8	[X $\gtrless$ Y]	07 - 34	29	RCL 2	28 - 45 2
9	STO 6	08 - 44 6	30	R/S	29 - 31
10	g, GTO 13	09 - 43 33 13	31	STO 9	30 - 44 9
11	STO 6	10 - 44 6	32	1	31 - 1
12	[X $\gtrless$ Y]	11 - 34	33	STO - 2	32 - 44 30 2
13	STO 7	12 - 44 7	34	RCL 9	33 - 45 9
14	RCL 3	13 - 45 3	35	g, GTO 01	34 - 43 33 01
15	RCL n	14 - 45 11	36	RCL 0	35 - 45 0
16	RCL 2	15 - 45 2	37	FV	36 - 15
17	-	16 - 30	38	RCL 6	37 - 45 6
18	y^x	17 - 21	39	PV	38 - 13
19	RCL 7	18 - 45 7	40	i	39 - 12
20	X	19 - 20	41	R/S	40 - 31
21	STO + 0	20 - 44 40 0	42	f, P/R	0.00

The program requires that you enter the cash flows in reverse order. The FMRR is computed after the cash flows are entered. The following example demonstrates how to use the program.

EXAMPLE 3:

The Land Ho Company is evaluating a project with the following cash flows occurring at the end of each year. The initial investment is -$18,500.00. Compute the FMRR using a safe investment rate of 5.5 percent and a high investment rate of 14.5 percent.

End of Year	Cash Flow
1	$ - 10,000
2	- 15,000
3	+ 21,000
4	- 22,500
5	+ 25,400
6	- 8,000
7	- 6,000
8	+ 9,100
9	+ 11,200
10	+ 184,500

Keystroke Solution: Financial Management Rate of Return:

Procedure	Action Step	Display
1. Enter FMRR program		
2. Clear calculator	f, REG	0.00
3. Set switch	g, END	0.00
4. Set program to start	f, PRGM	0.00
5. Enter number of end of period cash flows	10, n, STO 2	10.00
6. Enter high investment rate per period	14.5 [ENTER] 100 ÷ 1 + STO 3	1.15
7. Enter low investment rate per period	5.5 [ENTER] 100 ÷ 1 + STO 4	1.06
8. Enter cash flows in reverse order: Period 10	184,500 R/S	10.00
9	11,200 R/S	9.00
8	9,100 R/S	8.00
7	6,000 CHS, R/S	7.00
6	8,000 CHS, R/S	6.00
5	25,400 R/S	5.00
4	22,500 CHS, R/S	4.00
3	21,000 R/S	3.00
2	15,000 CHS, R/S	2.00
1	10,000 CHS, R/S	1.00
9. Enter initial cash outlay	18500, CHS	- 18,500.00
10. FMRR	R/S	18.80
11. Present value of outflows	RCL PV	- 41,733.95
12. Future value of inflows	RCL FV	233,709.51

The FMRR is 18.80 percent for the example problem as compared to an IRR of 21.60 percent. The present value of $41,733.95 represents the total investment in the project at the beginning of the first period. The amount includes the initial outlay and the negative cash flows not funded by the positive cash flows. The future value represents the total value of the net positive cash flows at the end of the 10th period.

To use the program for different values, start with step 2 and enter the new values. Be sure that you press f, REG to clear the data register before entering the new values and f, PRGM to reset the program.

HP-12C Keystroke Template: Financial Management Rate of Return:

Procedure	Action Step	Display
1. Enter FMRR program		
2. Clear calculator	f, REG	0.00
3. Set switch	g, END	0.00
4. Set program to start	f, PRGM	0.00
5. Enter number of end of period cash flows	[ ] n, STO 2	[ ]
6. Enter high investment rate per period	[ ] [ENTER] 100÷1+STO 3	[ ]
7. Enter low investment rate per period	[ ] [ENTER] 100÷1+STO 4	[ ]
8. Enter cash flows in reverse order. Remember to press CHS before R/S if the cash flow is negative.		
a. Last period	[ ] R/S	[ ]
b.	[ ] R/S	[ ]
c.	[ ] R/S	[ ]
d.	[ ] R/S	[ ]
e.	[ ] R/S	[ ]
f.	[ ] R/S	[ ]
g.	[ ] R/S	[ ]
h.	[ ] R/S	[ ]
i.	[ ] R/S	[ ]
j. Repeat step i until cash flow for period 1 is entered	[ ] CHS	1.00
9. Enter initial cash outlay	[ ] CHS	[ ]
10. FMRR	R/S	[ ]

HP-12C Keystroke Template: Financial Management Rate of Return: (continued)

Procedure	Action Step	Display
11. Present value of outflows	RCL PV	
12. Future value of inflows	RCL FV	

10

PROJECT ANALYSIS USING CASH FLOWS AND CONVERTING INTEREST RATES

In this chapter, we show you how to combine the techniques contained in the previous chapters for project analysis, and how to convert interest rates.

Because project analysis varies from project to project, three examples are selected to show you how to combine the various techniques discussed in this book for your project. The first is analyzing an interest-only loan while the second shows you how to analyze an interest-only loan with monthly payments made after a period of time. This second example also illustrates how to solve problems with odd interval cash flows. The third example is the traditional project analysis for analyzing after-tax cash flows.

The second topic is computing a buy-down mortgage payment schedule using a short program.

The third topic in this chapter concerns converting interest rates. You will learn how to convert annual nominal rates (APR) to an annual effective rate and vice versa. You will also learn how to convert a nominal rate to the equivalent nominal rate with a different number of compounding periods per year.

PROJECT WITH INTEREST-ONLY PAYMENTS

The Land Ho Company is evaluating 100 acres of land that sells for $3,000 per acre. The company can purchase the land by paying 20 percent and taking out an interest-only loan at 9 percent for the balance. At the end of seven years the company estimates the land can be sold for $6,000 per acre. The property will generate $3,000 in rent each six months. Assuming the interest is paid each six months, what is the internal rate of return, and the net present value using a 14 percent investment rate?

The first step is to identify the net cash flows as inflows or outflows.

A. Initial cash flow: Down payment 20% of 100 X $3,000 = - $60,000
B. Interest payments.

   1. Amount borrowed 80% of 100 X $3,000 = $240,000.
   2. Semiannual interest payment .09 X 1/2 X $240,000 = - $10,800.

C. Semiannual rents = $3,000.
D. Cash flow at end of seven years when the property is sold.

Sale Price 100 X $6,000 =	$600,000
Balance of mortgage	240,000
Net cash	+ $360,000

Now that we have identified the net cash flows, the next step is to draw a time diagram:

```
                                                          + $360,000
cash                - 10,800     - 10,800      - 10,800    - 10,800
flow     - 60,000   +  3,000     +  3,000      +  3,000    +  3,000

             |------------|------------|---- // ----|------------|
semi-        |            |            |             |            |
annual
periods      0            1            2             13           14
```

When the semiannual rents and interest payments are netted together, the following cash-flow diagram results:

```
                                                                + $360,000
cash      - 60,000      - 7,800      - 7,800        - 7,800      - 7,800
flow
               |------------|------------|---- // ----|------------|
semi-          |            |            |            |            |
annual
periods        0            1            2            13           14
```

Because the net semiannual cash flow is constant, we can treat this situation as an ordinary annuity. Remember the FV key is the amount after the final period's payment. The input values then are:

PV = - 60,000

PMT = - 7,800

FV = + 360,000

n = 14

The payment switch is set to END because the payments occur at the end of each period.

To compute the IRR, enter the above values and press [i]. The return per period is 7.86 percent or 15.72 percent per year. This is greater than the required return of 14 percent.

To compute the present value, key in the minimum return of 14 percent ÷ 2 or 7 percent and press PV CHS to get 71,399.56 which is the present value of the cash flows. Next add the initial outlay of -60,000 to determine the NPV of $11,399.56.

The formal keystroke solution for the above steps is on the next page.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter periods	14 n	14.00
4. Enter outlay cost	60000, CHS, PV	- 60,000.00
5. Enter semiannual cash flow	7800, CHS, PMT	- 7,800.00
6. Enter cash flow at end of period after last semiannual cash flow	360000 FV	360,000.00
7. Solve for semiannual return	i	7.86
8. Annual return	2 X	15.72
9. Solve for NPV		
a. Enter minimum return	14 [ENTER] 2 ÷ i	7.00
b. Compute present value of cash flows	PV	- 71,399.56
c. Change sign	CHS	71,399.56
d. Enter outlay	60000 CHS	- 60,000.00
e. NPV	+	11,399.56

PROJECTS WITH INTEREST-ONLY PAYMENTS AND LOAN AMORTIZATION

This example illustrates the solution procedure necessary for solving cash-flow problems with odd-interval payment periods. Assume the Land Ho Company purchases 100 acres of land for $300,000. At the end of seven years, the company expects to sell the land for $1,300,000 since the area should have a regional shopping center built near the land.

The loan has two components. Interest-only payments are made at the end of each year for the first two years with annual interest of 12 percent the first year and 14 percent the second year. After two years, Land Ho will make monthly payments of $3,793.33 at the end of each month. The loan will be repaid with a balloon payment of $296,162.07 immediately after the 60th payment. The loan payment and balloon were computed based on an annual interest rate of 15 percent with monthly compounding, thirty year amortization, and pay-off after five years.

Land Ho anticipates receiving rents from the land equal to $8,000 for the seven-year period.

Assuming a required annual interest rate of 18 percent compounded monthly, compute the net present value for this project, and the internal rate of return.

The first step is identifying the net cash flows as inflows or outflows for each period. Since this application has both monthly and annual cash flows, you must select the smallest payment period for the cash flows which is monthly.

A. The initial cash outflow at time zero is -$300,000.

B. Interest-only payments as outflows.

Year 1: $300,000 x 12% = -$36,000
Year 2: $300,000 x 14% = -$42,000

C. Annual rents as inflows are $8,000.

D. Monthly loan payments (outflows) starting at the end of the first month in year three are -$3,793.33.

E. Cash flow at the end of seven years when property is sold.

Sale price	$1,300,000.00
Balance of mortgage	296,162.07
Net Cash	$1,003,837.93

After identifying the cash-flow components, next draw a time diagram with cash inflows shown as positive values and cash outflows shown as negative values. Remember, the net cash flow occurring in step E is in addition to the final monthly payment. The time diagram shown below shows the groups of cash flows between the vertical lines with the number of cash flows in the group below the horizontal line. Because the diagram is long, it is broken into the three segments shown below.

Years 1 and 2

Interest			-36,000		-42,000
Rent			8,000		8,000
Net Cash Flow	-300,000	0	-28,000	0	-34,000
Number	1	11	1	11	1
Year	0		1		2

Years 3 and 4

Payment	-3,793.33	-3,793.33	-3,793.33	-3,793.33
Rent		8,000.00		8,000.00
Net Cash Flow	-3,793.33	4,206.67	-3,793.33	4,206.67
Number	11	1	11	1
Year		3		4

Year 5

Net cash from sale		1,003,837.93
Payment	-3,793.33	-3,793.33
Rent		8,000.00
Net Cash Flow	-3,793.33	1,008,044.60
Number	11	1
Year		5

As illustrated on the time diagram, months without a cash flow have a zero entered with the number of months shown below the horizontal line. For months having multiple cash flows, the net amount is entered with the number of months shown below the line.

Because the HP-12C has only twenty data registers, you may exceed its capacity when using this technique for examples having payments over a long period of time. In such cases, the cash flows can be combined, or a larger machine such as a HP-41C can be used.

After identifying the net amounts for each group of payments, the next step is entering them into the calculator using the grouped cash-flow keys and computing the net present value and internal rate of return.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set payment switch	g, END	0.00
3. Enter outlay cost	300000 CHS, g, CF_0	- 300,000.00
4. Enter cash flows for year 1	0 g, CF_j	0.00
	11 g, N_j	11.00
	28000 CHS, g, CF_j	- 28,000.00
	1 g, N_j	1.00
5. Enter cash flows for year 2	0 g, CF_j	0.00
	11 g, N_j	11.00
	34000 CHS, g, CF_j	- 32,000.00
	1 g, N_j	1.00
6. Enter cash flows for year 3	3793.33 CHS, g, CF_j	- 3,793.33
	11 g, N_j	11.00
	4206.67 g, CF_j	4,206.67
	1 g, N_j	1.00
7. Enter cash flows for year 4	3793.33 CHS, g, CF_j	- 3,793.33
	11 g, N_j	11.00
	4206.67 g, CF_j	4,206.67
	1 g, N_j	1.00
8. Enter cash flows for year 5	3793.33 CHS, g, CF_j	- 3,793.33
	11 g, N_j	11.00
	1008044.60 g, CF_j	1,008,044.60
	1 g, N_j	1.00
9. Enter required interest rate per month	15 g, [12÷]	1.25
10. Compute the net present value	f, NPV	59,134.13
11. Compute the interest rate per month	f, IRR	1.51
12. Multiply by number of payments per month for annual rate of return	12 X	18.12

The net present value is $59,134.13 and the annual rate of return (IRR) is 18.12 percent. The cash flows occurring one time had the number entered using the N_j key so that the keystroke entries would match the time line diagram. The N_j entry, however, can be omitted when the cash flow occurs one time.

PROJECT WITH UNEVEN CASH FLOWS AND TAXES

Here we consider a situation where the cash flows are uneven and where taxes are considered.

Determining either the net present value of a project or its internal rate of return requires your computing the after-tax cash flows. While no two projects are alike, this example illustrates the underlying solution procedure.

The solution procedure has 6 basic steps.

1. Compute taxable income.
2. Compute income taxes on income.
3. Compute after-tax cash flows.
4. Compute after-tax cash flow at end of the project.
5. Identify cash-flow pattern: annuity or variable.
6. Solve for NPV and IRR.

Because tax laws frequently change, the procedures employed here are illustrative and not intended to reflect exact tax treatment.

The Land Ho Company is considering the purchase of a new 5 unit apartment building for $120,000. They can obtain a $100,000 mortgage at 9 percent interest for 30 years. The building, valued at $90,000, will be depreciated over 40 years using the double declining balance method. Ordinary income taxes are 40 percent and capital gains taxes are 20 percent. The company expects a return of at least 16 percent after taxes on this investment. They expect the project to increase in value by 5 percent a year for five years at which time it will be sold. What is the IRR and NPV if estimated rents and operating expenses are:

Years	Rent	Operating Expense	Net Operating Income (NOI)
1	$12,000	$4,000	$ 8,000
2	15,000	5,000	10,000
3	15,000	5,000	10,000
4	15,000	5,000	10,000
5	19,000	6,000	13,000

Solving this problem requires a number of steps.

A. Compute the monthly mortgage payment using the Amount of Mortgage Payment Template in Chapter 4. The payment is $804.62, with annual payments of $9,655.44, the annual debt service.

B. Compute the annual interest and principal payments using the Level Payment Amortization Program and Annual Interest Amortization Template in Chapter 8. These values are:

Year	Interest	Principal	Remaining Balance
0			100,000.00
1	- 8,972.28	- 683.16	99,316.84
2	- 8,908.20	- 747.24	98,569.60
3	- 8,838.09	- 817.35	97,752.25
4	- 8,761.42	- 894.02	96,858.23
5	- 8,677.55	- 977.89	95,880.34

C. Compute the depreciation expenses using the procedure described for declining balance depreciation in Chapter 6. The annual expenses and net book values are:

Year	Depreciation	Net Book Value
1	$4,500.00	$85,500.00
2	4,275.00	81,225.00
3	4,061.25	77,163.75
4	3,858.19	73,305.56
5	3,665.28	69,640.28

D. Compute amount of excess depreciation as follows:

1. Annual Straight Line Depreciation.

$$\frac{\$90{,}000 - 0}{40} = \$2{,}250$$

2. Net Book value at end of 5 years using Straight Line Depreciation.

$90,000 - $2,250 X 5 = $78,750.00

3. Excess Depreciation.

S. L. net book value	$78,750.00
D.B. net book value	69,640.28
Excess Depreciation	$ 9,109.72

E. Compute annual net income and tax.

Year	NOI	-	Interest	-	Depreciation	=	Income	Tax at 40%
1	8,000	-	8,972.28	-	4,500.00	=	- 5,472.28	+ 2,188.91
2	10,000	-	8.908.20	-	4,275.00	=	- 3,183.20	+ 1,273.28
3	10,000	-	8,838.09	-	4,061.25	=	- 2,899.34	+ 1,159.74
4	10,000	-	8,761.42	-	3,858.19	=	- 2,619.61	+ 1,047.84
5	13,000	-	8,677.55	-	3,665.28	=	+ 657.17	- 262.87

Please note that the tax is shown as a positive amount because the net income is negative for years 1 to 4. We assume in this analysis that the company has other taxable income that will benefit from the tax loss on this project. In year 5 positive income of $657.17 is generated resulting in $262.87 of tax to pay and it is shown as a negative value.

F. Annual net cash flow from operations.

Year	NOI		Annual Debt Service		Tax		After Tax Cash Flow
1	8,000	-	9,655.44	+	2,188.91	=	533.47
2	10,000	-	9,655.44	+	1,273.28	=	1,617.84
3	10,000	-	9,655.44	+	1,159.74	=	1,504.30
4	10,000	-	9,655.44	+	1,047.84	=	1,392.40
5	13,000	-	9,655.44	-	262.87	=	3,081.69

Notice the effect of the tax benefit resulting from operating income losses in years 1 to 4. The tax is added while in year 5 a positive income occurs and results in a tax payment.

G. Compute net cash flow from sale of project after five years.

1. Sale price of project after appreciating at 5 percent for 5 years. The inputs are:

Action Step	Display
5 n	5.00
5 i	5.00
120000 CHS PV	- 120,000.00
0 PMT	0.00
FV	153,153.79

The sale price is $153,153.79.

2. Compute taxable gain.

Sale Price	$153,153.79
Less: Land	30,000.00
Net book value of building	69,640.28
Taxable Gain	$ 53,513.51

3. Compute tax on gain.

Taxable gain	$ 53,513.51	
Excess depreciation	9,109.72	
Amount taxed at captial gain rate	44,403.79	
Tax rate	.20	
Capital gains tax	$ 8,880.76	$ 8,880.76
Amount taxed at ordinary rate - Excess depreciation	$ 9,109.72	
Ordinary tax rate	.4	
Ordinary tax	$ 3,643.89	$ 3,643.89
Total tax ($8,880.76 + 3,643.89)		$12,524.65

4. Compute net cash flow.

Sale price	$153,153.79
Mortgage	- 95,880.34
Tax	- 12,524.65
Net cash flow from sale	$ 44,748.80

H. Draw diagram of net cash flows.

The initial outlay is $40,000, the difference between the price and the mortgage, while the operating cash flows are from step F. The cash flow from selling the asset is from step G.

cash						$44,748.80
flow	- 40,000	533.47	1,617.84	1,504.30	1,392.40	3,081.69
years	0	1	2	3	4	5

After adding the cash flows in period 5 together, the diagram is:

cash flow	- 40,000	533.47	1,617.84	1,504.30	1,392.40	47,830.49
years	0	1	2	3	4	5

Now that the analysis has been reduced to a series of after-tax cash flows, the next step is solving for the IRR and NPV.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	f, REG	0.00
2. Set switch	g, END	0.00
3. Enter outlay cost (down payment)	40000 CHS, g, CF_0	- 40,000.00
4. Enter cash flows	533.47 g, CH_j	533.47
	1617.84 g, CH_j	1,617.84
	1504.30 g, CF_j	1,504.30
	1392.40 g, CF_j	1,392.40
	47,830.49 g, CF_j	47,830.49
5. Solve for IRR	f, IRR	6.03
6. Enter annual required return rate	16 i	16.00
7. Compute NPV	f, NPV	- 13,832.32

Since the IRR is below the required annual return, and the net present value is negative, this project is a poor investment opportunity.

SOLVING FOR A BUY-DOWN MORTGAGE PAYMENT SCHEDULE:

Objective: To compute the payment schedule for a buy-down mortgage when the borrower's interest rate is subsidized.

Information Required:

1. Amount borrowed.
2. Total number of payments for loan.
3. Annual percent interest rate.
4. Number of payments per year.
5. Subsidized interest rates.

Comments:

This application assumes that the borrower receives a subsidy in the form of interest rates below the mortgage's stated rate. As a result, from the borrower's point of view, the loan payments increase each year for a number of years, and then remain constant. Someone, the builder selling the house for example, pays for the difference in the interest rates because from the lender's point of view the loan is a level payment mortgage as discussed in Chapter 4. The amount paid for the subsidy is called the "buy-down."

This application assumes that the payments change in yearly intervals, and that the amount necessary to pay the difference between the payment the borrower makes and the actual payment is deposited in an escrow account.

The payments the borrower makes are computed using the actual remaining balance, the number of payments remaining, and the subsidized rate for that period. The difference between this subsidized payment and the actual payment is covered by a party other than the borrower.

While this application assumes that the amount necessary to subsidize the loan (the buy-down amount) is the amount necessary for deposit in an account paying interest, this may not be a standard method. This type of loan is still new, and exact procedures vary in practice.

The amount of the full loan payment is calculated using the method discussed in Chapter 4. You also compute the remaining balance on the loan using the procedure in Chapter 4.

Once the payments made by the borrower are known, the price to pay for such a mortgage is calculated in the same exact fashion as in determining the price to pay for a graduated payment mortgage as discussed in Chapter 7.

Example:

Super-Builder Inc. has sold a house to John Doe who must borrow $100,000. In order to make the sale, Super-Builder has agreed to subsidize John's mortgage interest rate. John borrows the $100,000 from the Big City Bank with a thirty-year term and an annual interest rate of 14 percent with monthly compounding.

Super-Builder has agreed to adjust the payments so that John pays an 11 percent rate the first year, a 12 percent rate the second year, a 13 percent rate the third, with the full 14 percent rate taking effect in the fourth year.

What is the payment schedule for John? How much should Super-Builder pay to buy-down this mortgage assuming no interest is paid on the buy-down amount, and how much should they pay assuming the difference in payments is withdrawn from a savings account paying 6 percent annual interest with monthly compounding?

MORTGAGE BUY-DOWN PAYMENT SCHEDULE PROGRAM:

The following program computes the monthly payments for a buy-down mortgage situation where the borrower's annual rate changes each year. The procedure for using the program is given in the keystroke solution.

Action Step	Press	Display
1	CLX, f, P/R, f, PRGM	00 -
2	RCL 2	01 - 45 2
3	÷	02 - 10
4	STO i	03 - 44 12
5	RCL n	04 - 45 11
6	RCL 3	05 - 45 3
7	-	06 - 30
8	STO n	07 - 44 11
9	RCL 2	08 - 45 2
10	STO 3	09 - 44 3
11	PMT	10 - 14
12	PMT	11 - 14
13	f, RND	12 - 42 14
14	R/S	13 - 31
15	RCL 1	14 - 45 1
16	-	15 - 30
17	STO 5	16 - 44 5
18	RCL n	17 - 45 11
19	STO 7	18 - 44 7
20	RCL 3	19 - 45 3
21	STO n	20 - 44 11
22	RCL 1	21 - 45 1
23	STO PMT	22 - 44 14
24	RCL 0	23 - 45 0
25	STO i	24 - 44 12
26	FV	25 - 15
27	CHS	26 - 16
28	STO PV	27 - 44 13
29	0	28 - 0
30	STO FV	29 - 44 15
31	RCL 7	30 - 45 7
32	STO n	31 - 44 11
33	RCL 5	32 - 45 5
34	R/S	33 - 31
35	g, GTO 01	34 - 43 33 01
36	f, P/R	0.00

HP-12C Solution:

Procedure	Action Step	Display
1. Key in buy-down program		
2. Clear all registers	f, REG	0.00
3. Set payment switch	g, END	0.00
4. Enter total number of payments	360 n	360.00
5. Enter annual interest rate on loan and store	14 g, [12÷] STO 0	1.17
6. Enter amount of loan	100000 PV	100,000.00
7. Compute full payment, round to two decimals, and store	PMT f, RND, PMT STO 1	- 1,184.87
8. Enter number of payments per year and store	12 STO 2	12.00
9. Reset program	f, [PRGM]	12.00
10. First year's subsidized payment		
a. Enter subsidized rate for first year	11	11.
b. Compute monthly payment	R/S	- 952.32
c. Compute amount of monthly subsidy and record	R/S	232.55
11. Second year's subsidized payment		
a. Enter subsidized rate for second year	12	12.
b. Compute monthly payment	R/S	- 1,029.95
c. Compute amount of monthly subsidy and record	R/S	154.92
12. Third year's subsidized payment		
a. Enter subsidized rate for third year	13	13.
b. Compute monthly payment	R/S	- 1,107.56
c. Compute amount of monthly subsidy and record	R/S	77.31

HP-12C Solution: (continued)

Procedure	Action Step	Display
13. Check fourth year's payment		
a. Enter actual loan rate	14	14.
b. Compute monthly payment	R/S	- 1,184.87
c. Compute amount of difference	R/S	0.00
14. Enter data for buy-down calculation		
a. Clear calculator	f, REG	0.00
b. Enter the monthly difference in payments		
Year 1	232.55 g, CF_j 12 g, N_j	232.55 12.00
Year 2	154.92 g, CF_j 12 g, N_j	154.92 12.00
Year 3	77.31 g, CJ_j 12 g, N_j	77.31 12.00
15. Enter zero and compute buy-down amount without interest	0 i, f, NPV	5,194.34
16. Enter savings account rate per month and compute buy-down amount with interest	6 g, [12 ÷] f, NPV	0.50 5,186.30

The borrower, John, will make payments of $952.32 in year one, payments of $1,029.95 in year two, and payments of $1,107.56 in year three. Then in year four John will assume the full payment amount of $1,184.87. The company purchasing the buy-down will pay $5,568 if interest is not earned on the amount, or $5,194.34 if the amount is placed in a savings account paying 6 percent interest. In either situation, the amounts necessary to subsidize the three years of payments are $232.55 in year one, $154.92 in year two, and $77.31 in year three.

HP-12C Keystroke Template: Solving for a Buy-Down Mortgage Payment Schedule:

Procedure	Action Step	Display
1. Key in buy-down program		
2. Clear all registers	f, REG	0.00
3. Set payment switch	g, END	0.00
4. Enter total number of payments	☐ n	☐
5. Enter annual interest rate on loan and store	☐ g, [12÷] STO 0	☐
6. Enter amount of loan	☐ PV	☐
7. Compute full payment, round to two decimals, and store	PMT f, RND, PMT STO 1	☐
8. Enter number of payments per year and store	12 STO 2	12.00
9. Reset program	f, PRGM	☐
10. First year's subsidized payment		
a. Enter subsidized rate for first year	☐	☐
b. Compute monthly payment	R/S	☐
c. Compute amount of monthly subsidy and record	R/S	☐
11. Second year's subsidized payment		
a. Enter subsidized rate for second year	☐	☐
b. Compute monthly payment	R/S	☐
c. Compute amount of monthly subsidy and record	R/S	☐
12. Third year's subsidized payment		
a. Enter subsidized rate for third year	☐	☐
b. Compute monthly payment	R/S	☐
c. Compute amount of monthly subsidy and record	R/S	☐

HP-12C Keystroke Template: Solving for a Buy-Down Mortgage Payment Schedule: (continued)

Procedure	Action Step	Display
13. Check fourth year's payment		
a. Enter actual loan rate		
b. Compute monthly payment	R/S	
c. Compute amount of monthly difference	R/S	
14. Enter data for buy-down calculation		
a. Clear calculator	f, REG	0.00
b. Enter the monthly difference in payments		
Year 1	g, CF_j	
	g, N_j	
Year 2	g, CF_j	
	g, N_j	
Year 3	g, CF_j	
	g, N_j	
15. Enter zero and compute buy-down amount without interest	0 i, f, NPV	
16. Enter savings account rate per month and compute buy-down amount with interest	g, [12÷]	
	f, NPV	

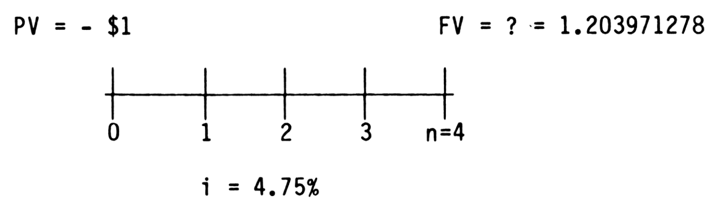
CONVERTING INTEREST RATES

You can easily convert interest rates using the HP-12C calculator. The concept is simple: first convert the nominal rate to the annual effective rate; second, convert the annual effective rate to the nominal rate with the desired number of compounding periods per year.

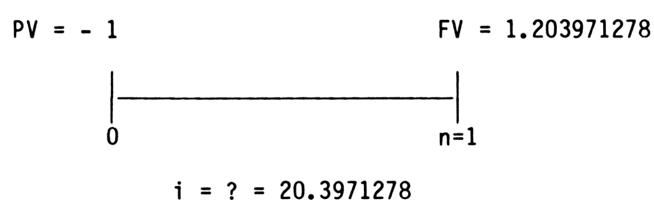
In the United States, annual interest rates are often stated as nominal rates. An annual nominal rate is equal to the interest rate per compounding period multiplied by the number of compounding periods per year.

<u>Converting Nominal Rates to Annual Effective Rates</u>. Nominal interest rates and annual effective rates may be compared by means of an example:

An investor wants to know the annual effective rate that is equivalent to an annual nominal rate of 19 percent annual compounded quarterly. The effective interest rate per period for the 19-percent annual nominal rate is 19/4 or 4.75 percent. During a year this means that the 4.75 percent rate will compound four times. So if one dollar were invested, we can solve for its value at the end of the year as shown on the time diagram below.



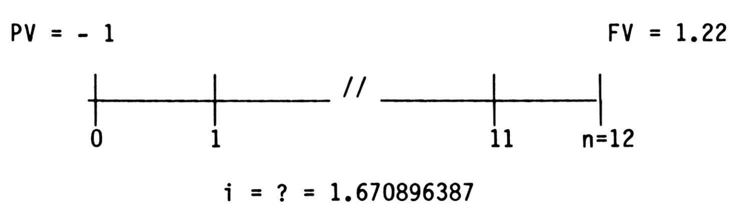
Using the annuity keys, the future value is 1.203971278. This means that the interest earned in dollars is $0.203971278 and as a percent of the investment is 20.3971278 percent. By finding the future value at the end of one year for the one dollar invested, you also determine the annual effective rate. In other words, the 19 percent annual nominal interest rate, compounded quarterly, increases the initial investment by 20.3971278 percent at the year end. This annual percentage increase is also the annual effective rate per year with one compounding period. We call it the annual effective rate because it is the effective compounding rate. Using your calculator, the rate is found as shown on the time diagram.



By entering the PV, FV and entering a 1 for n, the i value is computed.

<u>Converting AER to ANR</u>. Converting an annual effective rate to an annual nominal rate is the reverse of the above procedure.

Assume you want to convert an annual effective rate of 22 percent to the equivalent annual nominal rate with monthly compounding. One dollar invested at the annual effective rate is worth $1.22 at the end of one year. The question to answer is, "What monthly rate will cause $1 to equal $1.22 after 12 months?"



Using the annuity keys, we find that i is 1.670896387 percent per month or 20.05075664 percent when stated as an annual nominal rate with monthly compounding. Both the 22 percent annual effective rate and the 20.05075664 percent annual nominal rate cause $1 to equal $1.22 at the end of twelve months.

The next three keystroke applications illustrate interest rate conversion using the concepts discussed above. The 1 is entered as a positive value for PV in the keystroke solutions to save unnecessary keystrokes. Otherwise, the keystroke procedure is the same as explained above.

DETERMINING THE ANNUAL EFFECTIVE INTEREST RATE:

Objective: To determine the annual effective interest rate for a given annual nominal rate (ANR).

Information Required:

1. Annual nominal interest rate or ANR.
2. Number of payments per year.

Comments:

This procedure will convert any discrete interest rate to the equivalent annual effective rate. But always enter all the decimal values for the annual nominal rate (APR).

Example:

Project A has an annual nominal rate of 19 percent with quarterly cash flows. Project B has an annual nominal rate of 18.8 percent with monthly cash flows. What are the annual effective rates for these two projects?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
Project A		
3. Enter annual nominal rate	19 [ENTER]	19.00
4. Enter number of payments per year	4 n	4.
5. Divide and enter interest rate	÷, i	4.75
6. Enter zero for payment	0 PMT	0.00
7. Enter 1 for PV	1 PV	1.00
8. Compute future value	FV	- 1.20
9. Enter	1 n	1.00
10. a. Compute annual effective rate	i	20.40
b. Display all decimals	f, 9	20.3912780
c. Display two decimals	f, 2	20.40
Project B		
11. Enter annual nominal rate	18.8 [ENTER]	18.80
12. Enter number of payments per year	12 n	12.00
13. Divide and enter interest rate	÷, i	1.57
14. Enter zero for payment	0 PMT	0.00

HP-12C Solution: (continued)

Procedure	Action Step	Display
15. Enter 1 for PV	1 PV	1.00
16. Compute future value	FV	- 1.21
17. Enter	1 n	1.00
18. a. Compute annual effective rate	i	20.51
b. Display all decimals	f, 9	20.50758800
c. Display two decimals	f, 2	20.51

Project B has the higher annual effective rate of 20.51 percent as compared to Project A's annual effective rate of 20.40 percent.

HP-12C Keystroke Template: Determining the Annual Effective Interest Rate:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual nominal rate	[ENTER]	
4. Enter number of payments per year	n	
5. Divide and enter interest rate	÷, i	
6. Enter zero for payment	0 PMT	0.00
7. Enter 1 for PV	1 PV	1.00
8. Compute future value	FV	
9. Enter	1 n	1.00
10. Compute annual effective interest rate	i	
11. Display full decimals	f, 9	

DETERMINING THE ANNUAL NOMINAL RATE:

Objectve: To determine the annual nominal rate for a given annual effective interest rate.

Information Required:

1. Annual effective interest rate.
2. Number of payments per year.

Comments:

This procedure will convert the annual effective rate to the equivalent annual nominal rate for a specified number of payments per year. Remember, always enter all the decimal values for the annual effective rate.

Example:

The Land Ho Company requires all projects to earn an annual effective rate of 22 percent. A project with monthly cash flows is being evaluated. What equivalent annual nominal interest rate with monthly compounding should be used to calculate the net present value?

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual effective rate	22 i	22.00
4. Enter 1 for PV	1 PV	1.00
5. Enter zero for payment	0 PMT	0.00
6. Enter 1 for n	1 n	1.00
7. Compute FV	FV	- 1.22
8. Enter number of cash flows per year	12 n	12.00
9. Compute interest rate per payment period	i	1.67
10. Compute annual nominal rate	RCL n X	20.05
11. Display all decimals	f, 9	20.05075664

An annual nominal rate of 20.05075664 percent with monthly payments should be used for the net present value calculation.

HP-12C Keystroke Template: Determining the Annual Nominal Rate:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual effective rate	☐ i	☐
4. Enter 1 for PV	1 PV	1.00
5. Enter zero for payment	0 PMT	0.00
6. Enter 1 for n	1 n	1.00
7. Compute FV	FV	☐
8. Enter number of cash flows per year	☐ n	☐
9. Compute interest rate per payment period	i	☐
10. Compute annual nominal rate	RCL n X	☐
11. Display all decimals	f, 9	☐

CONVERTING ANNUAL NOMINAL RATES:

Objective: To convert an annual nominal rate compounded X times per year to an another annual nominal rate compounded Y times per year.

Information Required:

1. Annual nominal rate compounded X times per year.
2. Number of compounding periods occurring Y times per year.

Example:

Sam wants to earn 16 percent annual compounded monthly on a $1,000,000 sale. The buyer, however, wants to make quarterly payments for 5 years. What annual interest rate should Sam use to compute the quarterly payments?

Comments:

Since interest is compounded monthly and payments occur quarterly, the corresponding interest rate compounded quarterly must be determined. After converting the interest rate, the problem is solved as discussed in Chapter 4.

HP-12C Solution:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual nominal rate compounded X times per year	16 [ENTER]	16.00
4. Enter X compounding periods per year	12 n	12.00
5. Divide and enter	÷, i	1.33
6. Enter zero for payment	0 PMT	0.00
7. Enter 1 for PV	1 PV	1.00
8. Compute future value	FV	- 1.17
9. Enter Y compounding periods per year	4 n	4.00
10. Compute interest rate per Y period	i	4.05
11. Compute annual nominal rate with Y compounding periods per year	RCL n X	16.21
12. Display all decimals	f, 9	16.21428146

Sam should use an annual rate of 16.21428146 percent with quarterly compounding to compute the quarterly payment of $73,931.22.

HP-12C Keystroke Template: Converting Annual Nominal Rates:

Procedure	Action Step	Display
1. Clear calculator	CLX, f, FIN	0.00
2. Set payment switch	g, END	0.00
3. Enter annual nominal rate compounded X times per year	[ ] [ENTER]	[ ]
4. Enter X compounding periods per year	[ ] n	[ ]
5. Divide and enter	÷, i	[ ]
6. Enter zero for payment	0 PMT	0.00
7. Enter 1 for PV	1 PV	1.00
8. Compute FV	FV	[ ]
9. Enter Y compounding periods per year	[ ] n	[ ]
10. Compute interest rate per Y period	i	[ ]
11. Compute annual nominal rate with Y compounding periods per year	RCL n X	[ ]
12. Display all decimals	f, 9	[ ]

bibliography

1. Aronofsky, Julius, Robert Frame, and Elbert Greynolds. Programmable Calculators. New York: McGraw-Hill, 1978.

2. Benedict, Howard, George Herman, and William Kinnard. Calculator Techniques for Real Estate. Chicago: Realtors National Marketing Institute, 1977.

3. Greynolds, Elbert B., Julius Aronofsky, and Robert Frame. Financial Analysis Using Calculators: Time Value of Money. New York: McGraw-Hill, 1980.

4. Hewlett-Packard: HP-12C Owner's Handbook and Problem-Solving Guide. Corvallis, OR: Hewlett-Packard Company, 1981.

PRACTICAL REAL ESTATE FINANCIAL ANALYSIS: USING THE HP-12C CALCULATOR

has been classroom tested for over two years...and has been reviewed nationwide.

"The step-by-step instructions on how to key in the data are excellent... This text covers a broad subject in detail."

—Gary P. Tenos
Bernhard Realty
Pittsburgh, PA

Other books of interest from Real Estate Education Company:

- **By David Sirota—**

  ESSENTIALS OF REAL ESTATE FINANCE 3RD EDITION

  ESSENTIALS OF REAL ESTATE INVESTMENT

- **Developed By Real Estate Education Company in conjunction with Grubb & Ellis Commercial Brokerage—**

  SUCCESSFUL INDUSTRIAL REAL ESTATE BROKERAGE 2ND EDITION

  SUCCESSFUL LEASING AND SELLING OF OFFICE PROPERTY 2ND EDITION

  SUCCESSFUL LEASING AND SELLING OF RETAIL PROPERTY

- **By Ventolo & Allaway—**

  MASTERING REAL ESTATE MATHEMATICS, 3RD EDITION

- **By Ventolo & Williams—**

  FUNDAMENTALS OF REAL ESTATE APPRAISAL, 3RD EDITION

Real Estate Education Company®
a division of Dearborn Financial Publishing, Inc.
520 North Dearborn Street
Chicago, Illinois 60610-4354

REORDER NUMBER:1512-15
ISBN:0-88462-497-8