Professional Real Estate Problem Solving Using the HP 12C

Second Edition

A Step By Step Approach

By John A. Tirone

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PREFACE

The second edition of this book, as was the case with its predecessor, addresses you, the reader, as a professional. The book is squarely designed and written around this premise. Your time is valuable and respected, and thus the materials zero-in as quickly and efficiently as possible.

The topical coverage and organization in this book is arranged primarily on the basis of a profile of what practicing real estate professionals (agents, brokers, appraisers, lenders, etc.) need most often. To achieve this end a profile was developed of the topics and depth of coverage most often requested by real estate professionals surveyed by this writer. The data was collected from the stated needs of my private seminar participants, participants in in-house firm-sponsored specialized classes held at various banks, real estate firms (commercial, appraisal, residential) and Fortune 500 Corporations. Also, data was collected from my real estate investor clients, college students, and from several instructors who also teach real estate problem solving on the HP 12C.

This book is clearly not one which flows from my assumptions about the real world of real estate financial calculations. With the sole exception of the modified internal rate of return (MIRR) example on page 142, each routine--each computation--in this book comes directly from solutions which I applied in my law practice, or taught--and have been asked to teach--practicing real estate people over the past 25 years. Each example is field tested. In fact, there are leases used by national companies which are structured around a number of the lease examples. Your fellow real estate practitioners from California to New York have caused each and every routine to be designed, perfected, proofed and taught by me for you, a participant in this, a most fascinating component of the real estate business.

The book works from the premise that all of us who perform real estate calculations must build our confidence in not only how to use the calculator, but more important we must build our confidence by understanding the techniques and principles underlying the keystrokes we must perform to solve the problem we are working on. Indeed, a significant flaw I see is that many practitioners look for "quick" or "easy" answers for developing practising level competence in this field. There are none! Thus they lose confidence in the field and never experience anywhere near the depth of what we can achieve using a high-level financial calculator such as the HP 12C.

Part of the approach to building confidence in the use of the HP 12C is squarely centered on learning the calculator's STACK. You will understand and have command of the STACK after your first few hours with this book. Once you are comfortable with the workings of the STACK you will find that it greatly enhances your confidence in performing calculations which may have been thought to be out of your realm of expertise or patience.

Competence, and hence confidence in the area of real estate computations can only come from study, some of it admittedly concentrated. However, once you master the underlying techniques of real estate problem solving--even 50% of the techniques in this book--you will not only amaze yourself, but more important, you will have the confidence necessary to "crunch numbers" in a way that will increase your efficiency and income, no matter what segment of the real estate industry you are in. As well, if you are a college student you will have a book which lays a solid foundation for any calculation you will perform in a real estate principles or investment course.

By providing the reader with a broad range of topics it is believed that you will be better able to expand your own solutions into areas which are not covered in this book. A treatment of the "what" and "whys" of many of the calculations covered in this book will indeed provide you with a much better foundation for understanding additional time value of money applications which may suit your specific business needs and activities.

The greatest number of pages are devoted to the areas of home buyer income qualification and real estate appraisal analysis. We approach the buyer income qualification topic by not only calculating the annual income needed to qualify a buyer, but we also determine the monthly PITI (or PITI plus PMI and "other" costs), maximum affordable price and required mortgage. As well, extensive coverage is given to a number of residential real estate routines which are novel on the HP 12C, such as taking a given amount of funds which the buyer has available for the downpayment and allocating them over points, prepaids, PITI reserves, and the like. Computer programs make these solutions effortless.

The appraisal section was extended through expansion of the coverage of the section dealing with limiting the maximum allowable mortgage loan with a debt service coverage ratio (DCSR) constraint. A program was added that enables the user to estimate the value of an income producing property where the financing is constrained by the lender's debt service coverage ratio (DSCR) requirements.

Coverage of various "creative financing" arrangements is retained, the belief being that the real estate practitioner should have a working familiarity with several of the more sophisticated--indeed, creative--financing techniques, such as the WRAPAROUND mortgage, GEM, ARM, and the GPM.

One of the hallmarks of this book is that it includes proofs of a number of routines and procedures. Why proofs? Why do I take precious space and your valuable time to show you that a number of the solutions--indeed a number of the results--we arrive at by punching certain keys or by running certain computer programs are in fact accurate? The reason for this is grounded on the fact that too many real estate and finance practitioners have simply put their calculators down and given up on performing calculations, particularly calculations which might be considered high-level real estate computations.

By seeing why an answer or a result is accurate you eliminate most, if not all of your real estate computations anxiety. Many times these proofs are carried out by calculating the internal rate of return (IRR) on the results obtained in a specific problem or from results obtained with one of the programs covered in the book. As you please, you can and will master not only the HP 12C, but you can also be an authority on the broad level of real estate problem solving addressed in this book.

Hundreds of problems are included along with the answers. By drill and repetition the reader builds a higher level of confidence in applying the techniques in his or her specific applications. The author has repeatedly been told by both practitioners and students of real estate that "more examples and more problems" are needed in real estate computations books. I hope I have hit a happy medium with the number given in this book.

A number of solution templates are provided for routines which are carried out with computer programs. However, not every routine based upon a program has a solution template included for it. Rather, templates were left out in a few cases where it was felt they were not needed or where the variety of inputs into a program were so broad that it was felt that working through a two page template might take more time than would otherwise be saved by using the template. These cases present no difficulty since the keystroke examples are very thorough and easily guide you through the necessary inputs.

Though I did all the math and programming that made this book (as well as its HP 17BII and HP 19BII counterparts) possible, still a book like this cannot be written without substantial inputs from others. I want to express my appreciation to all of my seminar participants as well the many sponsors who brought me in to present real estate computation seminars. In addition, I thank my university students who took me--by design or even by default--in the various programs I teach. It is the students and seminar participants (and admittedly one's clients!) who sharpen the edge of every practitioner/writer. They provide the needs and the real-world problems for the technicians--like myself--to solve. To them I owe my never ending gratitude.

I am most grateful to Dr. Edward J. Farragher, Oakland University, Rochester, Michigan, who read the manuscript for the first edition of this book.

In the appraisal area, I acknowledge that I turned a number of years ago to two excellent works, Income Property Appraisal, by William N. Kinnard, Jr., and Income Capitalization Techniques: A Study Guide, by Charles B. Akerson. I consider both of these works to be references for the appraisal section of this book. However, the presentation deficiencies, if any, are solely mine.

I acknowledge that several consultations with Dr. Elbert B. Greynolds, Jr., Southern Methodist University, Dallas, Texas, were most helpful in getting me on track with respect to several changes in this edition. Further, it should be acknowledged that I have turned to Bert's pioneering work, Financial Analysis Using Calculators: Time Value of Money, in order verify several of the mathematical derivations I did which were necessary to put a book like this one together. Clearly, the value of Doctor Greynolds' work to the serious student of time value of money cannot be overstated.

Finally, I acknowledge the excellent counsel I received from Janet M. Cryer at the time the first edition of this book was written.

I wish you a most pleasant trip through this book.

John A. Tirone

QUICK STARTING THE HP 12C

Turning the HP 12C On and Off

Pressing **[ON]** turns the calculator on. Pressing **[ON]** again turns it off. Since the calculator has "continuous memory" capability, you do not lose data stored in the device when it is turned off.

The Logic System

Most of us are familiar with the standard **algebraic logic system** which is found in most calculators. Basically, algebraic logic systems allow us to do calculations in accord with the following format:

"this" **times** "that" **equals** "something".

The **logic system** of the HP 12C, however, is what is called **RPN**--that is, "Reverse Polish Notation". (Note that Hewlett-Packard Company's "OmniGo Organizer Plus" contains an emulation of the HP 12C.) It is named in this manner since it was derived from a similar system developed by a mathematician at the time he was associated with the Hewlett-Packard Company.

With **RPN logic,** we generally use the following format:

enter "this"; type "that"; **press** an **operation symbol** (+, -, x, ÷).

Basically, **RPN logic** is exceptionally useful and likely the most user-friendly computation system available for those who regularly perform long computations, commonly called "chain calculations" or "mixed calculations". You will note that the **RPN** logic system does not require the use of an equal ("=") sign. This system is still the keystroke operating system of the HP 12C, but also is available as an election on the "menu driven" Hewlett-Packard Financial Calculators, such as the HP 17BII and the HP 19BII.

Note the large vertical **[ENTER]** key (shown horizontally here). When working with **RPN** logic, we use this key to **input**--to **enter**--the first number we are working with into the calculator's "STACK", assuming the number was not already in the display as a result of our having performed a previous calculation or having recalled [RCL] the number from a storage register.

The Keyboard

The HP 12C is capable of performing many more functions than would appear possible from the number of **keys** on the Keyboard. When you look at the keys you find many of them enable you to perform dual or triple functions. For example, take the **[n]** key. It appears in the first row, first column of keys. Note the word printed in GOLD above the key (AMORT), and further note the "12x" printed in BLUE at the bottom of the key.

The GOLD "AMORT" printed above the **[n]** key enables you to perform loan amortization calculations, while the BLUE "12x" function enables you to multiply the number of years in a loan times "12" and at the same time enter the number of months in the loan directly into the calculator's **[n]** register.

Your calculator contains:

1. **White faced Keys.**
2. **Gold functions.** You press the GOLD **[f]** key to access these.
3. **Blue functions.** You press the BLUE **[g]** key to access these.

You should think of the BLUE and GOLD functions on your HP 12C as "shifted" or "second" functions. That is, they are functions which are **secondary** to the primary function printed on the face of the key. The BLUE and GOLD keys perform the same function as the "Shift" key on a typewriter. Just as you would press the "Shift" key, followed by the "H" key to type a capital "H" on a typewriter, you would type the HP 12C's BLUE key, followed by the white-faced "8" key to set your calculator to "END" mode.

Locating the Keys by their Program Mode Designations

The keys are identified by their row and column positions, as per the following straight line schematic of the HP 12C:

Column # → **Row #** ↓	1	2	3	4	5	6	7	8	9	0
1	11	12	13	14	15	16	**7**	**8**	**9**	10
2	21	22	23	24	25	26	**4**	**5**	**6**	20
3	31	32	33	34	35	36	**1**	**2**	**3**	30
4	ON	42	43	44	45		**0**	48	49	40

Keys 0 through 9 retain their own values for programming identification.

For example:

a) The [n] (number of compounding periods) key is key number "11".
b) The [i] (interest rate per compounding period) key is key "12".
c) The Store [STO] key is key number "44".
d) The Recall [RCL] key is key number "45".

Setting the Display Format

Your calculator can show numbers in its display under both the **American convention** and the **European convention.** With the American convention you will have a decimal point [.] separating the decimal portion of a number from the whole number (or "integer") portion of the complete number. For example, the number **$1,028.61** is displayed in its American format. On the other hand, the European convention for displaying this number would be: **$1.028,61.** Thus, the only difference between the two conventions is that the decimal point and the comma are interchanged.

To set your HP 12C to the European convention--or to get it **out** of the European convention if it is already set this way--you must (1) turn the calculator **off** by pressing the **[ON]** key; (2) then press and **hold down** the decimal point **[.]**; (3) while **still holding down the decimal point [.]** you press the **[ON]** key; (4) release the **[ON]** key; (5) then release the decimal point **[.]**. In keystroke form, it looks like this:

KEYSTROKES:	[ON]	Turns your calculator **off**
	[.]	Press **and hold down the decimal point**
	[ON]	Press the [ON] key **while still holding down the decimal point**
	LET GO OF [ON]	**Release** the [ON] key
	LET GO OF [.]	**Release** the decimal point

For example, if the number in your display is **1.028,610000** and you wanted to get back to the American convention, you would turn the calculator **off** by pressing the **[ON]** key; then hold down the decimal point **[.]**; then press the **[ON]** key again; let it go; and then release the decimal point **[.]**. Your display will show **1,028.610000.**

To set the number of decimal places your calculator is capable of showing in its display window--the **"displayed X register"**--you will use the following procedure:

> Press the GOLD **[f]** key followed by a number key (0 through 9). For example, if you wanted to show two decimal place accuracy in your **displayed X register**, you will press the GOLD [f] followed by the "2".
>
> To show all decimal places your calculator is capable of displaying, you will press the GOLD **[f]** followed by the "9".

For example, if you typed the number **1,028.612597** and immediately afterwards pressed **[f] [2]**, your display window will show **1,028.61**. Now, if you wanted to show, say, five decimal places, press the GOLD **[f]** key followed by the **[5]**. Your display window will show **1,028.61260**.

Note that your calculator **rounds** the displayed amount to the number of decimal places which you cause it to display. For example, if you are set to 4 decimal places--because you previously typed **[f] 4**--and you **enter** the number "1.123456789" into your calculator's **X** and **Y** registers by pressing the (vertical) **[ENTER]** key, your display will show "1.1235". This is so because the typed digit "5" appears immediately to the right of the typed "4" in the sequence "1.123456789". Therefore, your calculator's programming "borrows" a "1" from the typed "5", thus causing the **internally stored** "4" to **display** as a "5" in your window. Let's perform the keystrokes.

KEYSTROKES:	1.123456789	Typed the number "1.123456789"
	f 4	Sets four decimal places
	DISPLAYS:	
	1.1235	

At this time, let's set the display format to the **European convention.** This will cause the displayed number **(1.1235)** to appear as **"1,1235".**

KEYSTROKES:	[ON]	Turns your calculator off
	[.]	**Press and hold down the decimal point** while you press the **[ON] key** in the next step
	[ON]	Turns calculator back on
	LET GO OF [ON]	**Release** the [ON] key
	LET GO OF [.]	**Release** the decimal point
	DISPLAYS:	(You are in the European convention)
	1,1235	

Let's leave the European convention and go back to the American convention.

KEYSTROKES:	[ON]	Turns your calculator off
	[.]	**Press and hold down the decimal point** while you press the **[ON] key** in the next step
	[ON]	Turns calculator back on
	LET GO OF [ON]	**Release** the [ON] key
	LET GO OF [.]	**Release** the decimal point
	DISPLAYS:	(You are in the American convention)
	1.1235	

Clearing Erroneously Typed Numbers

To clear only erroneously typed numbers, you will use the **[CLx]** key. That is, if you type an incorrect number, which is still in your display, and if you want it out of the calculator--without disturbing other data that may already be entered in the stack, storage or financial registers--just press **[CLx]**. For example, if you typed a "5" instead of, say, a "4", you could remove the **displayed** "5" by pressing the **[CLx]** key. You would then type the "4". In keystroke form, it looks like this:

KEYSTROKES:	f 2	We set 2 decimal places
	5	We typed a "5" instead of typing the "4"
	CLX	Clears the displayed "x" register
	DISPLAYS: 0.00	(the "x" register is clear)
	4	Retyped the correct number

You should note that pressing the [CLx] key to clear an erroneously typed number **does not clear the complete stack,** nor does it clear any other storage or financial register on your HP 12C. It **only** clears the **displayed X register.** Note further the face of the [CLx] key: It has capitals for "CL" and a small "x". This is totally consistent with the kind of detail we have come to expect from Hewlett-Packard Company. The "CL" stands for "Clear", while the "x" designates the **displayed X register.**

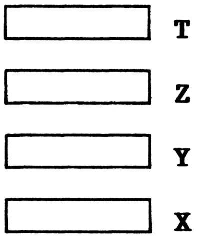
Read together, **CLx** stands for: **Clear** the displayed **"X" register.**

THE STACK

The operating system of the HP 12C--at least from our perspective--is built around its STACK. The stack has four storage areas, which areas you might think of as "boxes" or storage locations. There are four such boxes, namely: **X, Y, Z,** and **T.** Numbers can be stored in each of the stack-registers; that is, you can have a different number stored in each stack-register, and you can tie each of them together mathematically by simply pressing the appropriate operation key, like [+], [-], [x], and [÷].

Think of the STACK this way:

1. **T** is at the top of the stack.
2. **Z** holds interim data.
3. Math calculations are carried out in the **X** and **Y** registers.



In addition to the displayed **X** register, the HP 12C has a "**Last X**" function. Though we will not cover this function in detail, basically what it does is allow the user to bring back the **last number** which was in the calculator's displayed **X register.** The **LSTx** function appears in BLUE at the bottom of the vertical **[ENTER]** key.

For example, let's multiply 10 times 5. The result will be "50". However, to assure ourselves that we in fact used a "5" as the **last** number we typed, we can recall it by using the **"Last X"** function. The keystrokes follow:

KEYSTROKES:	f 2	We set 2 decimal places
	10 ENTER	We entered "10" into the STACK
	5 x	Multiplied "5" times "10"
	DISPLAYS: 50.00	
	[g] (LSTX)	Brings up the last number we had in the displayed "x" register
	DISPLAYS: **5.00**	

Now multiply the displayed "5.00" times "8". Your result will be "40". Then, bring up the **last** number we typed. The keystrokes follow:

KEYSTROKES:	8 x	Multiplied "8" times "5"
	DISPLAYS: 40.00	
	[g] (LSTX)	Brings up the last number we had in the displayed "x" register
	DISPLAYS: **8.00**	

CLEARING THE CALCULATOR

There are a number of different ways we can clear the registers on the HP 12C, depending upon what we are trying to achieve. A summary of the most common clearing methods follows:

1.	[CLX]	Clears only the display window (**X register**)
2.	0 [ENTER] [ENTER][ENTER]	Clears only the STACK (**X, Y, Z,** and **T** registers)
3.	[f][FIN]	Clears the Financial Registers (n, i, PV, PMT, and FV)
4.	[f][CLX]	Clears the STACK, Financial Registers, the 20 storage registers, and the "last x" function. It does not clear stored programs.
5.	[f][R↓]	Clears all programs, assuming the keys are pressed while you are in the programming mode. (You get into the programming mode by pressing [f] [R/S].)

In addition, there is a **master clear** function which clears everything, including stored programs. To activate the master clear you should do the following:

1. Turn your HP 12C off and release the [ON] key.
2. Press and hold down the minus key **[-]**.
3. Press and release the **[ON]** key.

After performing the above keystroke sequence, your display will show:

Pr Error

Note: "**Pr Error**" means all computer programs have been cleared.

As a result of using this clearing procedure, you have:

1. Set the display format to 2 decimal places.
2. Set the payment mode to END mode.
3. Cleared all data stored in the calculator, including all programs.
4. Set the other functions back to the factory setting, including the DAY/DATE function.

THE STORAGE REGISTERS

There are 20 general storage registers on your HP 12C, plus an additional location where 8 lines of program can be stored. If we counted all storage registers on the calculator, they total 30. In addition to the 20 general storage registers, we have five Financial Registers; the four column STACK; plus the **"last x register"**.

The 20 storage registers are consistent with numbered keys **0** through **9**, and keys **.0** ("point zero") through **.9** ("point nine"). That is, we can store data in memory registers **0** through and including **.9**, assuming we did not have more than an eight line program entered in the calculator.

The memory registers are typically designated in the following manner:

R_0 represents storage location accessed through the "0" key.
R_1 " " " " " " "1" key.
R_2 " " " " " " "2" key.
.
.
.
$R_{.7}$ represents storage location accessed through the ".7" key.
$R_{.8}$ " " " " " " ".8" key.
$R_{.9}$ " " " " " " ".9" key.

Assuming your calculator contains nothing longer than an 8 line program, you will have 20 registers available for storage. If, however, you go beyond an 8 line program, your calculator will begin to allocate one or more of the "open" storage registers in accord with the length of the program input.

Each storage register (starting with register **R.9** and working backwards through register **R7**) may be utilized for up to 7 lines of computer programming. Registers **R.9** through and including **R7** represent a total of 13 registers. Since each is capable of holding 7 lines of programming we can hold a maximum of 99 lines of programming in the HP 12C. This is arrived at by adding the extra 8 lines of program to the 91 lines (13 registers x 7 lines per register) available from the storage registers.

For example, if you had a 9 line computer program entered into your HP 12C, you would have 19 storage registers (**R.8** through and including **R0**) available for storage. As well, since you used one storage register, you could hold a maximum 15 line program in your calculator and still maintain the 19 storage registers free.

To determine the number of storage registers available, and as well determine the maximum program length consistent with the number of registers currently allocated for existing programs, you should press the BLUE [g] key, let it go, and then **hold down** the [9] key. If, for example, your calculator was free of programs--or if you had a maximum of an 8 line program entered--your display would read: **P - 08 r - 20.** On the other hand, assume you had 99 lines of programming entered. Checking again for available memory, you press and release the BLUE [g] key and hold the [9] down: Your display would read: **P - 99 r - 07.**

Storing and Recalling Numbers

To **save** a number in one of the 20 data storage registers, you should:

1. Type the number to be stored, assuming it is not already in your display.
2. Press the store **[STO]** key.
3. Type the register number where you desire to store the number: **0** through **9** for storage registers **R0** through **R9**, or **[.] 0** ("point zero") through **[.] 9** ("point nine") for registers **R.0** through **R.9.**

Likewise, to **recall** a number from a data storage register, you should:

1. Press the recall **[RCL]** key.
2. Type the register number where the desired data is stored.

Example: Perform the following storing [STO] and recall [RCL] keystrokes:

Keystroke:	**Display:**	**Comments:**
f CLX f 2	0.00	Clears all registers; sets 2 decimal places
100 STO 0	100.00	Stores 100 in memory register 0
500 STO [.] 9	500.00	Stores 500 in memory register "point nine"
43,560 STO 1	43,560.00	Stores number of square feet in an acre in memory register 1
STO 2	43,560.00	Stores number of square feet in an acre in memory register 2
CLX	0.00	Clears displayed "x" register
ON	(SCREEN BLANK)	Turned calculator off
ON RCL 0	100.00	Calculator ON and recalls data stored in memory register 0
RCL [.] 9	500.00	Recalls data stored in memory register "point nine"
STO 2 RCL 2	500.00	Stores and recalls 500 from memory register 2
RCL 1	43,560.00	Recalls data stored in memory register 1
f CLX	0.00	Clears all 20 memory storage registers (plus financial, etc.)
RCL 0	0.00	Memory register cleared
RCL 1	0.00	Memory register cleared

Storage Register Arithmetic

The HP 12C has a built-in function which enables you to perform **storage register arithmetic** on a number stored in memory registers **R0, R1, R2, R3,** and **R4.** That is, you can perform an **arithmetic operation** (+, -, x, and ÷) on a number stored in these data storage registers **without ever recalling it.**

This feature seems to be most in favor with real estate practitioners and accountants who need to add successive numbers--answers or results--to a number previously stored in a memory register. For example, the percentage of a property's cost represented by a given number of its components, such as the land, roof, personal property, landscaping, garage, and so forth.

To use storage register arithmetic, you should:

1. Type the number which you want to use to perform an arithmetic operation on another number already stored in memory register **R0, R1, R2, R3,** or **R4.**
2. Press the appropriate operation desired [+], [-], [x], or [÷].
3. Type the register number (0, 1, 2, 3, or 4) in which the base data is stored which you want to manipulate arithmetically through the use of the number you typed in step 1 above.

An example follows:

Keystroke:	**Display:**	**Comments:**
f CLX	0.00	Clears all registers
100 STO 0	100.00	Stores 100 in memory register 0
43,560 STO 1	43,560.00	Stores number in memory register 1
200 STO + 0	200.00	Adds 200 to memory register 0
3 STO x 1	3.00	Multiplies number in memory register 1 times three
CLX	0.00	Clears displayed "x" register
RCL 0	300.00	Recalls the number in memory register 0 to check the new total
RCL 1	130,680.00	Recalls the number in memory register 1 to check its total
STO - 4	130,680.00	Subtracts from memory register 4
RCL 4	-130,680.00	Recalls data in memory register 4
43,560 STO ÷ 1	43,560.00	Divides 43,560 into data stored in memory register 1
RCL 1	3.00	Recalls data in memory register 1
STO + 5	**Error 4**	**R4** is as far as you can go
f CLX	0.00	Clears all registers

WORKING WITH RPN LOGIC

The **[ENTER]** key is an important part of the HP 12C's RPN logic system. When we key a number in--such as typing the number "8" in the first example--and immediately press the **[ENTER]** key, we cause the typed number to be copied ("replicated") from the calculator's **X** register directly into the **Y** register. The number typed prior to pressing the **[ENTER]** key remains in the calculator's "displayed **X** register", though it is copied into the **Y** register.

Let's clear the four column STACK (X, Y, Z, and T registers) of your calculator:

Press: 0 [ENTER] [ENTER] [ENTER] (Clears X, Y, Z, and T registers.)

Addition Examples

8 + 5	8 [ENTER] 5 [+] **DISPLAYS: 13**
7 + 10 + 2 + 5	7 [ENTER] 10 [+] 2 [+] 5 [+] **DISPLAYS: 24**
24 + 26	[**Note:** You do not need to enter the "24" here.] 26 [+] **DISPLAYS: 50**
50 + 100	[**Note:** You do not need to enter the "50" here.] 100 [+] **DISPLAYS: 150**

Subtraction Examples

23 - 5	23 [ENTER] 5 [-] **DISPLAYS: 18**
19 - 25	19 [ENTER] 25 [-] **DISPLAYS: -6**
50 - 20 - 16	50 [ENTER] 20 [-] 16 [-] **DISPLAYS: 14**
40 - **-40**	40 [ENTER] 40 [CHS] [-] **DISPLAYS: 80**

Multiplication Examples

9 x 5 x 2	9 [ENTER] 5 [x] 2 [x] **DISPLAYS: 90**
90 x 3	[**Note:** You do not need to enter the "90" here.] 3 [x] **DISPLAYS: 270**
270 x 3/5	[**Note:** You do not need to enter the "270" here.] 3 [ENTER] 5 [÷] [x] **DISPLAYS: 162**
5 x -40	5 [ENTER] 40 [CHS] [x] **DISPLAYS: -200**
-10 x -15	10 [CHS] [ENTER] 15 [CHS] [x] **DISPLAYS: 150**

Division Examples

30/6	30 [ENTER] 6 [÷] **DISPLAYS: 5**
125/5	125 [ENTER] 5 [÷] **DISPLAYS: 25**
25/12.5	[**Note:** You do not need to enter the "25" here.] 12.5 [÷] **DISPLAYS: 2**
18/(72/2)	18 [ENTER] 72 [ENTER] 2 [÷] [÷] **DISPLAYS: 0.50**

Mixed (Chain) Calculations

5 x (15 - 10)	15 [ENTER] 10 [-] 5 [x] **DISPLAYS: 25**
(12 x 13) - 15	12 [ENTER] 13 [x] 15 [-] **DISPLAYS: 141**
5 x [(18 - 16)/3]	18 [ENTER] 16 [-] 3 [÷] 5 [x] ________?
$\frac{4\ [(25\ x\ 10) - 100]}{[(30 - 15)\ x\ 40]}$	[**Note:** Let's clear the STACK. A good practice.] 0 [ENTER] [ENTER] [ENTER] 25 [ENTER] 10 [x] 100 [-] 4 [x] [**Note:** Above keystrokes solved the numerator.] 30 [ENTER] 15 [-] 40 [x] [**Note:** Above keystrokes solved the denominator.] [÷] **DISPLAYS: 1**

Word Problem

A **Section** of land contains one square mile. We also know that a mile is 5,280 feet. By definition it was decided that a **section** of land would contain 640 acres. How many square feet are there in one acre of land?

$\text{Sq.ft/acre} = \frac{5,280\text{ ft x }5,280\text{ ft}}{640\text{ acres}}$	
$= \frac{27,878,400\text{ sq.ft.}}{640\text{ acres}}$	5,280 [ENTER] [x]
Sq.ft/acre = 43,560	640 [÷] **DISPLAYS: 43,560**

You will note that in the above keystroke sequence we did not retype 5,280 after we **entered** it into the calculator's "Y" register. Since we were multiplying 5,280 times itself, all that was required was to press the multiplication key [x] after we entered the number into the "Y" register of your calculator's STACK. Pressing [x] caused the number in "Y" (5,280) to be multiplied times the number in the displayed "X" register (5,280).

WATCHING THE STACK IN OPERATION

It is suggested that you work mixed calculations on your HP 12C by working from the inside of a parenthesis to the outside, and from the numerator to the denominator.

Problem: $$\frac{40\ [(25 \times 10) - 100]}{[(30 - 15) \times 40]} = 10$$

Let's clear the STACK: 0 **[ENTER][ENTER][ENTER]** (This is a good practice.)

You can also clear the STACK by pressing the GOLD [f] key followed by [CLX]. As pointed out earlier, this clearing procedure also clears the 20 storage registers as well as the five financial registers.

STACK								Numerator solved →						Denominator solved →	
T															
z										6000	6000		6000		
y		25	25		250		150		6000	30	30	6000	15	6000	
x	25	25	10	250	100	150	40	6000	30	30	15	15	40	600	10

Key-stroke	25	ENTER	10	x	100	-	40	x	30	ENTER	15	-	40	x	÷

Problem: $$\frac{4\ [(8 \times (6/3) - 10)]}{(12 \times 2)} = ?$$

Note: Multiplication has priority over subtraction.

STACK									Numerator solved →					Denominator solved →	
T															
z												24	24		
y		6	6		2		16		6		24	12	12	24	
x	6	6	3	2	8	16	10	6	4	24	12	12	2	24	1

Key-stroke	6	ENTER	3	÷	8	x	10	-	4	x	12	ENTER	2	x	÷

REVIEWING DATA STORED IN THE STACK

1. The **Roll Down Key [R↓]** rolls the numbers in your STACK so that you can review the data previously stored in it. You will note that the Roll Down Key is located in the third row, third column of keys; hence its designation as key number "33".

2. **Perform the following Keystrokes:** 4 ENTER 3 ENTER 2 ENTER 1, but **do not** ENTER the 1.

   Next, press the **Roll Down Key** [R↓]. Continue pressing [R↓] until you have reviewed all the numbers as shown below.

STACK												
T						4	4	1	2	3	4	1
z				4	4	3	3	4	1	2	3	4
y		4	4	3	3	2	2	3	4	1	2	3
x	4	4	3	3	2	2	1	2	3	4	1	2

Key-stroke	4	ENTER	3	ENTER	2	ENTER	1	R↓	R↓	R↓	R↓	R↓

3. **Watch** for this potential **trap** in using your calculator: If you do not clear the STACK before working on a problem you could inadvertently use data from a previous problem if you did not pay close attention to your keystrokes.

STACK										* →				
T						4	4	4	4	4	4	14	14	14
z				4	4	3	3	4	4	4	4	4	14	14
y		4	4	3	3	2	2	3	4	4	4	4	4	14
x	4	4	3	3	2	2	1	3	6	10	14	4	8	12

Key-stroke	4	ENTER	3	ENTER	2	ENTER	1	+	+	+	+	R↓	+	+

* You will note that we entered **only** one "4" in the above keystroke sequence. However, by **design,** your calculator will cause a number entered into the **"T"** register to drop down, unless cleared.

LOSING NUMBERS THROUGH THE TOP ("T" REGISTER) OF THE STACK

The Problem: As stated earlier, the HP 12C's STACK consists of four separate storage areas in which numbers can be stored or manipulated mathematically. Always remember that the STACK is indeed **four rows high;** nothing more, and nothing less. This feature must be dealt with when you are attempting a long chain calculation since it is very easy to **lose** data through the top ("T" Register) of the STACK.

Let's see how this can happen. Here we duplicate the chain calculation first covered on page 12 of this book. Instead of working the problem from the "inside-out", let's cause more data to be input than the STACK can handle.

We will assume that the user intends to start the problem from the left-side of the equation and somehow work his/her way through the numerator. As we go through the keystrokes below, keep in mind that a four row STACK enables you to make a **maximum of three entry [ENTER] operations** into it, at which time you must conclude with a mathematical operation performed on two of the storage locations (typically data stored in the **X** and **Y** registers).

If you exceed three [ENTER] keystrokes, you will lose the number first put into the STACK. Watch this in operation as we try to solve the problem.

Problem:

$$\frac{4\ [(8 \text{ x } (6/3) - 10)]}{(12 \text{ x } 2)} = ?$$

STACK **Lost→ 4**

Solution obviously going nowhere!

T						4	**4**	8	8	8	8	8
z				4	4	8	8	6	8	8	8	8
y		4	4	8	8	6	6	3	6	8	6	8
x	4	4	8	8	6	6	3	3	1	6	10	-4
Key-stroke	4	ENTER	8	ENTER	6	ENTER	3	**ENTER**	÷	x	10	-

Maximum number of ENTER keystrokes made →

Note: The **bold "4"** and the **bold "ENTER"** are used for emphasis.

It is doubtful there has ever been a user of the HP 12C who has not lost data by making more than three ENTER operations into the STACK, this author included. Though your expertise can only build as you use the calculator, **never** get "brave", and always be on guard for losing data through the STACK.

CALENDAR FUNCTION

One of the most useful functions on your HP 12C is the calendar function. Your calculator is programmed to work with dates from October 15, 1582 through November 25, 4046.

The Date Format: We concern ourselves only with the "month-day-year" format. To make calendar calculations on the basis of the month-day-year format, you must do the following:

(1) Set the **date format** to **month-date-year** by pressing the blue [g] key followed by the [5]. (Please note the **blue "M.DY"** designation printed at the bottom of the [5] key.)

(2) Type the **initial date** in accord with the following format: **MM.DDYYYY.**

- (a) That is, you type one or two digits for the **month.**
- (b) Type the **decimal point** [.].
- (c) Then type two digits for the **day of the month.**
- (d) Then type the four digits of the year.

(3) Press the **ENTER** key to input the initial date into the calculator.

(4) Type the ending date using the **MM.DDYYYY (MONTH.**DATE**YEAR)** format, except you do not use the ENTER key.

(5) Type the blue **[g] key** followed by the **[EEX] key.** This operates the **(ΔDYS)** (change in days) function. The answer displayed will be the number of days **between** the two dates, taking into account leap days, if any.

(6) If you prefer the answer on the basis of a 30 day month, press the [x≷y] ("x-to-y") key. The answer displayed will be the number of days **between** the two dates on the basis of a 360 day year (30 days/month).

Example: Let's calculate the number of days between November 25, 1996, and July 1, 1997. Calculate the actual number of days, and then determine the number of days on the basis of a 30 day month. The keystrokes follow below:

KEYSTROKES:	f CLX f 2	Clears all registers; set 2 places
	g [M.DY]	Make sure you are set to month-day-year format
	11.251996 [ENTER]	Initial date entered into STACK
	7.011997	Type the ending date
	g [ΔDYS]	Activated days-between-date function
	DISPLAYS: **218.00**	Number of actual days
	[x≷y]	Brings up number of days on the basis of a 30 day month

DISPLAYS:
216.00

The calendar function also enables us to determine the **actual** day of the week on the basis of going forward or backwards in time by a given number of days from a stated date. For example, we can determine what day would fall 180 days from October 3, 1996. Let's perform the keystrokes below:

KEYSTROKES:	10.031996 [ENTER]	Initial date entered into STACK
	180	The number of days between dates
	g DATE	Operates the DATE function
	DISPLAYS:	The ending date is April 1, 1997, a Tuesday
	4,01,1997 2	

Comment: Note the format displayed in the above answer: **4,01,1997 2.** This tells us the month is April; the date is the 1st of the month; the year is 1997; and the day of the week is a Tuesday. The format used on the HP 12C to designate the **day of the week** is as follows: Monday = 1; Tuesday = 2; Wednesday = 3; Thursday = 4; Friday = 5; Saturday = 6; and Sunday = 7.

Example: Let's go **backwards** in time 180 days from April 1, 1997. It will bring us back to October 3, 1996, a Thursday. Note that since we are going **backwards** in time we must change the sign of the 180 day input by making it a negative number. We do this by typing 180 followed by the **CHS** (change sign) **key.** The keystrokes follow:

KEYSTROKES:	4.011997 [ENTER]	Enters later date into STACK
	180 **CHS**	Number of days we are going backwards in time
	g DATE	Operates the DATE function
	DISPLAYS:	Determined initial date as October 3, 1996, a Thursday
	10,03,1996 4	

Example: Let's determine which day of the week November 6, 1996 falls on. To do this we will ask the calculator to tell us what date is zero days from November 6 1996. The keystrokes follow:

KEYSTROKES:	11.061996 [ENTER]	Enters date into STACK
	0	Number of "days" we are counting from the initial date
	g DATE	Operates the DATE function
	DISPLAYS:	Determined date falls on a Wednesday
	11,06,1996 3	

Note: To work calendar problems using a **day-month-year** format, you can do so by activating the **D.MY** (day-month-year) function on your HP 12C. To do this just press the blue **[g] key** followed by the **[4] key.** This activates the **D.MY** function. Your input pattern would then be to type the day of the week; type the decimal point; then type the month (use 2 digits) and year.

PERCENT FUNCTIONS

I. STRAIGHT PERCENT CALCULATIONS

1. Key-in the base number and press [ENTER].
2. Key-in the applicable percentage and press [%].

Example: You are a cooperating agent on a $100,000 real estate sale. The total commission is 7% of the sales price. Your participation entitles you to 25% of the total commission. Assuming your overall income tax bracket is 32% (28% + 4%), calculate your gross commission and the commission net of income taxes.

PROCEDURE USING [%] KEY	KEYSTROKE/INPUT	DISPLAY
Enter sales price; set 2 decimal places	100,000 ENTER f 2	100,000.00
Key-in applicable commission rate; calculate total commission	7 %	7,000.00
Calculate your commission	25 %	1,750.00
Calculate estimated income taxes	32 %	560.00
Calculate commission net of taxes	-	1,190.00

What Happened Mathematically?

To find the amount corresponding to a percentage of a number, multiply the base number times the decimal equivalent of the given percentage. To convert a percent to a decimal you divide by 100 (or multiply times ".01").

MATHEMATICAL PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter applicable percent; divide by 100	7 ENTER 100 ÷	0.07
Multiply times base amount	100,000 x	7,000.00
Enter participation commission; convert to a decimal; multiply times gross commission; ENTER twice into the STACK	25 ENTER .01 x x ENTER ENTER	0.25 1,750.00
Convert tax bracket to decimal; calculate income tax	32 ENTER .01 x x	.32 560.00
Calculate commission net of income taxes	-	1,190.00

II. PERCENT OF TOTAL

To calculate what percentage of one number is another number, you should:

1. Calculate the **total** amount just as you would in any chain calculation.
2. Key in the number whose percentage of total you wish to find.
3. Press the percent of total key **[%T]**.

Example: You are analyzing a real estate transaction in which the cost (basis) of the individual components are as follows:

a) Land	$45,000	d) Improvements to home	$15,000
b) Home	$125,000	e) Land improvements	$4,500
c) Garage	$18,000		

Solution: Using your HP 12C's **percent of total key [%T]**, calculate the percentage of the total price represented by each component. Verify the total of all computed percentages as equalling 100%. To do this, you will store [STO] the first percentage you calculate in memory register 0. Then, use register arithmetic to add successive component percentages of total into memory register 0.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Calculate basis of property	45,000 ENTER 125,000 + 18,000 + 15,000 + 4,500 +	207,500.00
Calculate land percent of total	45,000 [%T] STO 0	21.69
Roll down to call-up basis again	R↓	207,500.00
Calculate home percent of total	125,000 [%T] STO + 0	60.24
Roll down to call-up basis again	R↓	207,500.00
Calculate garage percent of total	18,000 [%T] STO + 0	8.67
Roll down to call-up basis again	R↓	207,500.00
Calculate improvements % of total	15,000 [%T] STO + 0	7.23
Roll down to call-up basis again	R↓	207,500.00
Calculate land improvements %	4,500 [%T] STO + 0	2.17
Verify summation of component percentages of total equals 100	RCL 0	100.00
Display all digits	f 9	100.0000000
Clear all registers; set 2 decimal places	f CLX f 2	0.00

III. PERCENTAGE DIFFERENCE (OR PERCENT CHANGE)

To find the percent difference [Δ%] between two numbers you should:

1. Key-in and enter [ENTER] the base number into your HP 12C. The base number will generally be the first (earliest) number in a two-number sequence.
2. Key-in the second number. This number represents the second (later) number in your two-number sequence.
3. Press the [Δ%] key to calculate the **change in percent** represented by the difference between the second number and the first number.

Note: Typically your first number will be the original sales price on a real estate transaction. The second number will be the current sales price (terminal value) at a later date. The percent change [Δ%] represents the difference between the current price and the original price expressed as a percentage of the original price.

Example: A parcel of land sold for $50,000 in 1986. In 1996 it sold for $100,000. What is the percent change in sales price for this parcel? Next, assume the numbers were reversed; that is, the original price was $100,000 and the later price in 1996 was $50,000. What is the percent change in price?

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter original sales price (1986)	50,000 ENTER	50,000.00
Key-in 1996 sales price	100,000	100,000.
Calculate percent change between second number and first number (First part of problem solved)	[Δ%]	100.00
Enter original (1986) sales price	100,000 ENTER	100,000.00
Key-in new (1996) sales price	50,000	50,000.
Calculate percent change between second number and first number (Second part of problem solved)	[Δ%]	-50.00

Interpretation

The first computed result (100.00) represents a 100% **change in price** going from 1986 to 1996. The result is quite obvious since a 100% increase in the original price represents an increase in value of $50,000 ($50,000 x 100% = $50,000). Adding the $50,000 **increase** in value to the original price of $50,000, we have a final price of $100,000.

In the second part of the problem we went from a price of $100,000 down to a price of $50,000. This represents a loss in value of 50%, or -50%.

WHAT DOES PERCENT DIFFERENCE REPRESENT MATHEMATICALLY?

To find the percent difference--or percent change--between two numbers, the calculator is programmed to subtract the first number (this is the number entered [ENTER]), such as the original sales price, from the second number (typically the final sale price or "terminal value"), divide this difference by the amount of the first number, and multiply the result by 100. It looks like this in formula-form:

$$\textbf{PERCENT DIFFERENCE} = \frac{\textbf{NEW PRICE - OLD PRICE}}{\textbf{OLD PRICE}} \times 100$$

EXAMPLE USING FORMULA METHOD	HP 12C KEYSTROKE	DISPLAY
% DIFFERENCE = $\frac{\text{NEW PRICE - OLD PRICE}}{\text{OLD PRICE}}$		
x 100		
$= \frac{100,000 - 50,000}{50,000} \times 100$	100,000 ENTER	100,000.00
	50,000 -	50,000.00
$= \frac{50,000}{50,000} \times 100$	50,000 ÷	1.00
= 1 x 100	100 x	100.00
% DIFFERENCE = 100%		
% DIFFERENCE = $\frac{\text{NEW PRICE - OLD PRICE}}{\text{OLD PRICE}}$		
x 100		
$= \frac{50,000 - 100,000}{100,000}$	50,000 ENTER	50,000.00
x 100	100,000 -	-50,000.00
	100,000 ÷	-0.50
$= \frac{-50,000}{100,000} \times 100$	100 x	-50.00
= -0.50 x 100		
% DIFFERENCE = -50%		

SOLVING FOR FUTURE VALUE

Introduction

Real estate, finance, and accounting professionals are often called upon to compute the **projected value** of a real estate investment based upon historical growth rates for the locale in which the property is situated. For example, **Buyer A** may seek to acquire a property selling for $100,000 in an area which has experienced yearly **compound** growth of five percent. What is the projected value of the investment after, say, five years?

Alternatively, assume **Buyer B** seeks to list his property at a price sufficient to cover an average **compound** inflation rate of 4.75% per year for the past six years. Assuming the property was purchased for $125,000, what should it sell for today to keep pace with inflation?

A second level future value problem concerns the process of inflation adjusting a property's projected value based upon separate rates for inflation and 'natural appreciation' in value. For example, if we assume a compound annual growth rate of 7% and a compound annual inflation rate of 4.75%, what is the projected inflation adjusted value of a subject-property a given number of years into the future? This problem, like the basic single component future value problem, has its solution-roots in the concept of the Future Value of One ($1.00) Dollar.

We start the study of this topic with an exploration of the underpinnings of the concept of the Future Value of One ($1.00) Dollar. Once the reader has a thorough understanding of the principles underlying this useful concept, it can be used beneficially and with total confidence.

FUTURE VALUE OF $1.00 -- PRESENT VALUE OF $1.00

Future Value of One ($1.00) Dollar (also sometimes called the Future Worth of One ($1.00) Dollar), in its simplest terms, is the final cash-flow that you will have as a result of making a "one-time deposit" of one ($1.00) dollar into an interest earning account and allowing the money to remain on deposit for a given length of time (term or number of time periods).

FUTURE VALUE -- The [FV] Key

On the Hewlett-Packard HP 12C, Future Value [FV] is not only represented by the [FV] key, it is also the key in position "15" on the keyboard of this programmable hand-held computer. This means that Future Value is computed by pressing the key in the calculator's first row, fifth column. Looking at the HP 12C's keyboard it is clear that the [FV] key appears in the first row, fifth column of keys; hence the designation as key number "15".

The Future Value [FV] key is part of your calculator's Financial Registers and appears in the top row of keys in the following sequence:

[n][i][PV][PMT][FV][CHS]

In order to solve for the Future Value of One ($1.00) Dollar, you must know the following:

(1) number of time periods in the projection period [n];
(2) interest rate per annual compounding period [i]; and
(3) present value [PV], which in this case is $1.00.

NUMBER OF TIME PERIODS -- The [n] Key

The length of time that the "one-time deposit" remains in an interest bearing account is known as the total number of "compounding periods". Compounding or time periods can be measured in days, weeks, months, quarters, years, or just about any other unit of time, though annual compounding is most common in real estate Future Value and Present Value projections. On the HP 12C, the number of time or compounding periods is represented by the [n] key.

For example, if interest is "paid" or "compounded" once per year and you are projecting the Future Value of $1.00 after being on deposit for two years, you would input the number "2" into your calculator's [n] register. Note, as well, that since interest is being "paid" or "compounded" once per year, you will enter the annual interest rate into your calculator's [i] register.

Let's assume, however, that interest is paid or compounded daily and that you are projecting a deposit of one ($1.00) dollar three years into the future. Here you have 1,095 **total compounding periods** (3 x 365) and therefore you must input "1,095" into your calculator's [n] register. You must also enter "1/365th" of the annual interest rate into the [i] register to permit daily compounding.

INTEREST -- The [i] Key

Interest is the amount paid or charged for the use of borrowed money. It is also defined as the compensation allowed by the law or fixed by the parties for the use or forbearance of money. Thus, the longer you use someone's money or "forbear" in the withdrawal of your money from an interest earning account, the greater will be the interest paid or earned, as the case may be. Clearly, **money has a time value.**

So ingrained in our economic system is the concept of a "time value of money" that if specific requirements are met, the Internal Revenue Code (IRC Sec. 483) "imputes"--that is, it "reads-in"--interest into certain sales or exchanges of property where the stated interest rate is "unreasonably low".

In finance calculations **interest is applied as a rate per compounding period.** That is, a year is generally divided into a number of compounding periods, with the annual interest rate then prorated over the number of annual compounding periods. For example, if the applicable interest rate is twelve percent (12%) per year, compounded (or paid) monthly, we would have one percent interest (12%/12 = 1%) compounded (or paid) monthly. The implication of this is that each month's interest will in turn earn or produce interest in successive months at the applicable interest rate of one percent (1%) per month.

On the HP 12C, interest rate is always entered as a rate **per compounding period.** Using the above example, if we have an annual interest of 12%, paid or compounded **monthly,** you therefore have one percent (1%) interest per month and will enter the number "1" into your calculator's [i] register. As well, you could use the HP 12C's built-in interest conversion function by performing the following keystrokes:

12 [g] [i] **Note: Some readers prefer to call the [g] key "[BLUE]"!**

The result is that we input an interest rate of one percent (1%) **per month** into the [i] register. (The calculator's programming further converts the interest rate per compounding period into its decimal equivalent by dividing by 100.)

Let's further assume ten percent (10%) interest per year with **daily** compounding over a holding period of three years. Therefore, interest will be "paid" or "compounded" daily, and thus must be prorated over three hundred and sixty-five (365) interest compounding periods per year. Your keystrokes follow:

[f] (FIN)	Clears financial registers
3 [ENTER] 365 [x] [n]	Enters 1,095 compounding periods
10 [ENTER] 365 [÷] [i]	Enters periodic interest rate

PRESENT VALUE — The [PV] Key

Present Value should be thought of as the "current" or "existing" or "today's" value of something given or exchanged for something else. It is the amount of money invested or deposited into an interest bearing account **at the beginning of the time period,** typically the beginning of the year. We can also say that it is the **one-time deposit** which is expected to produce a numerically greater, though financially equivalent future amount (Future Value) at a later date.

For example, if I gave you eighty dollars for your calculator, we are, in effect, setting or equating the value of the calculator to the amount of money exchanged for it. Thus, the Present Value--the "current" or "today's" value--of the calculator is eighty dollars. So too, if a financial institution lends $100,000 today at 10% annual interest, compounded monthly, for a term of 30 years, and receives in "exchange" the borrower's promise to pay $877.57 per month for 359 months plus a final (360th) payment of $881.12, we can say that the Present Value (or existing or current value) of the $100,000 given up by the lender today is **equivalent** to the series of three hundred and sixty payments for which the borrower is obligated.

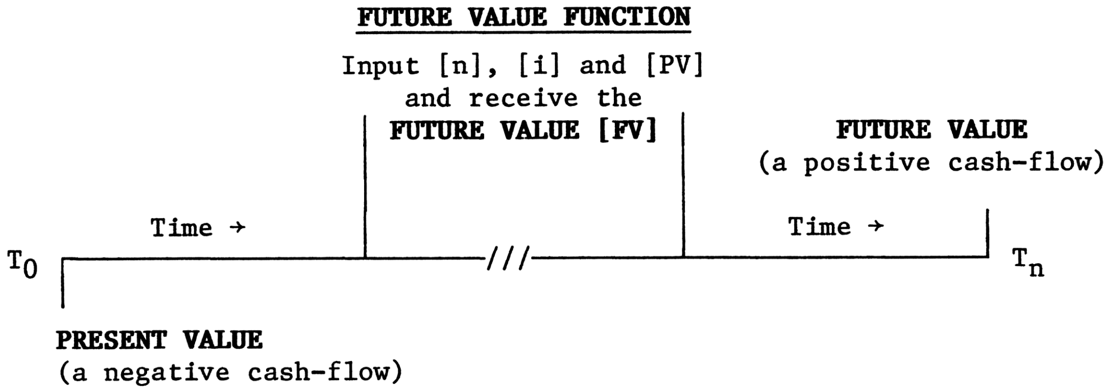
Present Value must be an equivalent value. This holds true whether the transfer takes place immediately--such as a hand-to-hand transfer of $80 for a calculator--or where a time interval separates the exchange of money and receipt of value in the future. The value received in the future--be it money, property, services, or what have you--has an equivalent value today. That "equivalent value" is its Present Value.

In our introduction to the concept of Future Value of One ($1.00) Dollar, reference is made to the making of a "one-time deposit" of a sum of money into an interest earning account. That "one-time deposit" is the Present Value (equivalent value) of the amount we are to receive at some future date. The amount received in the future--called the Future Value--depends upon the amount we charge or receive for the use of our money [i], the length of time the money is used [n], and the amount deposited [PV]. Of course, in a Future Value of One ($1.00) Dollar problem the amount "deposited" is always One ($1.00) Dollar.

We should **always think of Future Value [FV] and Present Value [PV] as equivalents of each other.** Though the two amounts will differ numerically (unless the interest or discount rate is zero), they are still **equivalent** to each other. This is so because the two concepts are bound together mathematically. Your calculator enables you to readily calculate the future value of a known amount (present value), or vice versa, without having to manipulate the mathematical formula which "generates" one concept from the other.

Both functionally and schematically, Future Value and Present Value occupy their respective positions on a **Timeline** or **Time Diagram.** The two concepts are interrelated by both the number of time periods [n] and the applicable periodic interest rate [i]. Therefore, a typical Future Value [FV] problem involves the input of three variables before it may be solved: interest rate per compounding period [i]; number of compounding periods [n]; and the Present Value [PV]. Alternatively, knowing any three of the variables enables us to readily solve for the fourth variable.

A **Timeline Diagram** interrelating these variables--with Present Value placed **below** the line and Future Value **above** the line--looks like this:



The **"Future Value Function"** is nothing more than the mathematics which relates the four variables: [n], [i], [PV], and [FV]. Your financial calculator is programmed to run this computation as well as many other financial problems which are much more complex than the straightforward calculation necessary to solve for any of the four variables comprising the **Future Value**

Function.

A FUTURE VALUE OF ONE ($1.00) DOLLAR PROBLEM

Let's assume a deposit of one ($1.00) dollar into a savings account earning interest at the rate of seven percent (7%) per year, compounded monthly. Assume further that the interest earned is allowed to remain on deposit and is not taxed. How much will be in the account after: (a) 12 months; and (b) after 120 months?

Solution:

Since the one ($1.00) dollar deposited is a reduction in the amount of money the depositor has, it is considered a negative cash-flow under the "cash-flow sign convention" and therefore is entered into the HP 12C as a negative number. This is accomplished by typing: "1" [CHS] [PV]. Note, again, that in the **FUTURE VALUE FUNCTION** Timeline Diagram given on the previous page, Present Value is placed **below** the horizontal Timeline, while Future Value is placed **above** the line. This placement is consistent with the concept of monies paid out being treated as a cash outflow (negative cash-flow), while monies received or taken in are treated as a positive cash-flow.

To solve the problem, let's perform the following keystrokes:

[f] (FIN)	Clears Financial Registers
12 [n]	Inputs 12 compounding periods
7 [g] [i]	Inputs interest rate per monthly compounding period
1 [CHS] [PV]	Sets the Present Value to negative $1.00. Note: It makes no difference in this problem if you are in the **beginning** or **end mode.** You can leave the mode set as you find it.
[FV]	Computes Future Value of the $1.00 deposit
[f] 9	Sets all decimal places

DISPLAYS: 1.072290081

120 [n]	Inputs 120 interest compounding periods
[FV]	Computes Future Value of the $1.00 deposit after being "on deposit" for 120 months

DISPLAYS: 2.009661377

We read the above results to mean that were you to deposit one ($1.00) dollar into a tax-free account earning interest at the rate of 7% per year, compounded monthly, your deposit would be worth **$1.072290081** at the end of twelve months, and would be worth **$2.009661377** at the end of one hundred and twenty months. Of course, you cannot withdraw approximately twenty three-hundredth's of a penny ($.00229...) at the expiration of twelve months, nor could you make a withdrawal of approximately ninety seven-hundredths of a penny ($.00966...) at the expiration of one hundred and twenty months. However, those extra digits are most important--most significant--were we to have made a more substantial deposit.

For example, if we deposited $1,000,000 into our hypothetical tax-free account and allowed it to remain on deposit for (a) 12 months, or (b) 120 months, you will find that the **future values** of the deposit will be <u>exactly</u> one million times greater than the future value computed for a one ($1.00) dollar deposit over the given holding periods. That is, after making a deposit of $1,000,000, at 7% interest, compounded monthly, you will have $1,072,290.081 after twelve months, or the sum of $2,009,661.377 after one hundred and twenty months, assuming the account is free of tax and no withdrawals are made.

A TRADITIONAL REAL ESTATE FUTURE VALUE PROBLEM

Let's go back to our hypothetical **Buyer A** mentioned in the introduction to this topic. If **Buyer A** acquires a property for $100,000, and if we project a compound annual growth in value of five percent (5%) per year over a five year holding period, what will be the sales price the buyer (now seller) can expect? Also, what must **Buyer B's** property sell for (disregarding commissions, if any) to keep up with a <u>compound</u> inflation rate of 4.75% per year over the past six years, assuming the property was acquired for $125,000? How do we solve these kinds of problems?

First, we should think of the purchase price of **Buyer A's** property as an **out-of-pocket cash-flow** incurred by the buyer **at the beginning of the time period.** Financially, the purchase price (out-of-pocket cash-flow) parallels the **one-time deposit** you would make into an interest bearing account **at the beginning of the time period.** The purchase price of $100,000 is clearly <u>the</u> Present Value of the property.

The fact that the Present Value (in this part of the problem) is one hundred thousand times greater than the one ($1.00) dollar amount we worked with when projecting the Future Value of One (1.00) Dollar simply tells us that <u>the</u> **Future Value** we now compute <u>will be one hundred thousand times greater</u> than the Future Value of One ($1.00) Dollar projected at the same interest rate (5%) over the same holding period (5 years). In effect, the Future Value of <u>any</u> amount is nothing more than **a multiple** of the Future Value of One ($1.00) Dollar computed over <u>the same holding period</u>, at <u>the same interest rate</u>, and using <u>the same interest compounding periods</u> per year. <u>The</u> **multiple** is, of course, <u>the purchase price</u> of the property whose value we are projecting into the future.

The holding (or projection) period in real estate future value calculations is traditionally taken as a number of full years. The appreciation or growth rate is also applied as an annual rate, with compounding of the rate applied once per year. Accordingly, you input the projection period into your calculator's [n] register and enter the annual interest rate into the [i] register.

Let's project the Future Value of **Buyer A's** $100,000 property using the keystroke procedure setforth below.

Please note that we are not setting the payment mode to beginning (BEGIN) or end (END) mode. As long as your Future Value (or Present Value) calculation does not contain a series of payments (PMT), it makes no difference if you are in BEG or END mode.

[f] (FIN)	Clears financial registers
[f] 2	Sets display to 2 decimal places
5 [n]	Inputs 5 years into the [n] register
5 [i]	Sets annual growth rate to 5%
100,000 [CHS][PV]	Sets the Present Value to negative $100,000
[FV]	Computes the Future Value

DISPLAYS: 127,628.16

Note: Here is how the Financial Registers in your calculator are now loaded:

n	i	PV	PMT	FV	Payment Mode
5	5.00	-100,000.00	0.00	127,628.16	END or BEGIN

We read the displayed result to mean that **Buyer A's** $100,000 property will appreciate in value to approximately $127,628 in five years, assuming the growth rate remains at five (5%) percent, compounded once a year. **As a practice exercise** you should compute the Future Value of One ($1.00) Dollar using a growth rate of five (5%) percent and a holding period of five years. However, set the display format on your calculator to seven (7) decimal places. You will readily verify that the computed future value equals **1.2762816.** Clearly, the Future Value of $100,000 is exactly one hundred thousand times greater than the Future Value of $1.00.

We next project the minimum selling price (commissions, if any, excluded) of **Buyer B's** $125,000 property, held for six years, subject to a compound annual inflation rate of 4.75%. The keystrokes follow below:

[f] (FIN)	Clears financial registers (a good practice)
6 [n]	Inputs 6 years into the [n] register
4.75 [i]	Sets annual inflation rate to 4.75%
125,000 [CHS][PV]	Sets the Present Value to negative $125,000
[FV]	Computes the Future Value

DISPLAYS: 165,133.13

Note: Here is how the Financial Registers in your calculator are now loaded:

n	i	PV	PMT	FV	Payment Mode
6	4.75	-125,000.00	0.00	165,133.13	END or BEGIN

We read the displayed result to mean that the minimum selling price of the property--exclusive of sales commissions, if any--must be approximately $165,133 in order to keep pace with inflation.

Again, please note that in both future value projection problems we did not set the calculator's payment mode for either beginning [BEG] (g, 7) or end [END] (g, 8) mode. Though you will typically be set to [END] mode--to allow for **payments** occurring at the end of the month or end of the time period--, this is not a concern where the computation does not involve a periodic payment. That is, as long as the Future Value computation solely involves the input of a present value (out-of-pocket cash-flow) as the only cash-flow, it is not necessary to concern yourself with setting the payment mode to [END] or [BEG]; just leave it as you find it.

As a **practice exercise**, you should reset the payment mode on your calculator to reflect a payment mode different from what you were set at when performing the above future value calculations. (You will press **[g] [7]** to set the **beginning mode**, if you were set to the **end mode**; your "beginning status indicator" ("**BEGIN**") becomes illuminated. Otherwise, press **[g] [8]** to set your calculator to the **end mode.**) Now press the [FV] key. Your display will again show: **165,133.13.**

We will show the keystrokes for this problem both ways. Let's first solve for the Future Value with the payment mode set to beginning [BEG].

[f] (FIN)	Clears financial registers
[g] 7	**Sets beginning mode**
6 [n]	Inputs 6 years into the [n] register
4.75 [i]	Sets annual growth rate to 4.75%
125,000 [CHS][PV]	Sets the Present Value to negative $125,000
[FV]	Computes the Future Value

DISPLAYS: 165,133.13

Note: Here is how the Financial Registers in your calculator are now loaded:

n	i	PV	PMT	FV	Payment Mode
6	4.75	-125,000.00	0.00	165,133.13	**BEG**

Now let's set the payment mode to **END** and again solve for the future value.

[g] 8	**Sets end mode**
[FV]	Computes Future Value. (Same results.)

DISPLAYS: 165,133.13

Note: Here is how the Financial Registers in your calculator are now set:

n	i	PV	PMT	FV	Payment Mode
6	4.75	-125,000.00	0.00	165,133.13	**END**

MATHEMATICS SIDE-BAR

When calculating the Future Value of a given one-time deposit, your calculator is programmed to use the following equation:

$$\mathbf{FV = PV \times (1 + i)^{n}} .$$

To use this equation we must first convert the variable "i" to its decimal equivalent, and then convert the decimal rate into a rate per compounding period by dividing by the number of times per year the interest ("i") is paid or compounded. For example, an annual interest rate of 12%, paid or compounded monthly, yields ".01" as the decimal equivalent to be used in the equation. This results from the following division sequence:

$$12 / 100 / 12 = .01$$

Using the equation to verify the result obtained in the future value computation immediately above, we have:

$$\begin{aligned} \mathbf{FV} &= \mathbf{(1 + i)^{n}} \\ &= \$125{,}000 \times (1 + .0475)^{6} \\ &= \$125{,}000 \times 1.321065010 && \text{(to 9 places)} \\ &= \$165{,}133.1263 \\ FV &= \$165{,}133.13 && \text{(rounded to 2 places)} \end{aligned}$$

By keystroke you would do: 1.0475 [ENTER]
6 [y^{x}]
125,000 [x].

Your display will show: **165,133.1263** (The result is verified.)

AN INFLATION-ADJUSTED, GROWTH RATE FUTURE VALUE PROBLEM

Let's project the inflation adjusted future value of a $100,000 property six years into the future. Assume we expect an annual compound growth rate of 7%, and project an inflation rate of 4.75%, also compounded annually.

Solution:

We must first adjust the growth rate (7%) to reflect the effect of inflation (4.75%). In effect, we are "discounting" the growth rate on the basis of 4.75% per year. The equation for this adjustment follows:

$$d = [((1 + g/100)/(1 + r/100)) - 1] \times 100,$$

which is equivalent to: $$d = \left[\frac{1 + g}{1 + r} - 1 \right] \times 100$$

where: d = Inflation adjusted growth rate;
g = Annual compound growth rate; and
r = Annual compound inflation rate.

Your complete solution keystroke procedure follows:

[f] (FIN)	Clears financial registers
1.07 [ENTER]	Enters "1" plus decimal growth rate
1.0475 [÷]	Divides by "1" plus decimal inflation rate
1 [-] 100 [x]	Completes calculation of inflation adjusted growth rate. Displays "2.15" at 2 places.
[i]	Inputs into [i] register
6 [n]	Enters projection period
100,000 [CHS][PV]	Enters price, negative-signed
[FV]	Computes the inflation adjusted Future Value

DISPLAYS: 113,600.04

Note: The above solution method assumes that both inflation and annual growth in value are applied as **compound** rates. That is, we assume that each and every year the property's value increases by a **constant ratio,** and as well inflation is also assumed to grow by a **constant ratio.** If, however, we project inflation as a fixed rate every year--that is, it is assumed the inflation rate does not compound each year--, then a simple solution for computing the inflation adjusted growth rate would be to subtract the inflation rate from the projected growth rate. You will then use the adjusted rate as your input into the HP 12C's [i] register. (The difference between the two methods is not significant, unless the present value being projected is a substantial amount.)

FUTURE VALUE PRACTICE PROBLEMS

Problem A:

An investor has an opportunity to invest $15,000 for five years. Assuming the investment is not taxed during the holding period, what is the value of the investment under the following conditions:

Interest rate of 7%	Growth rate of 9%	**Decay** rate of 3%
Compounding:	Compounding:	Compounding:
(1) Annually;	(1) Semi-annual;	(1) Annually;
Ans: $21,038.28	**Ans: $23,294.54**	**Ans: $12,881.01**
(2) Quarterly;	(2) Daily:	
Ans: $21,221.67	**Ans: $23,523.38**	
(3) Daily;	(3) Annually;	
Ans: $21,285.30	**Ans: $23,079.36**	

Problem B:

Solve for the unknown financial variable in the following problems:

PV of $200,000	FV of $162,046.95	PV of $150,000.00
FV of $322,102	Monthly compounding	FV of $162,086.00
Annual compounding	1 year deposit term	1 year deposit term
5 year deposit term	7.75% interest rate	7.75% interest rate
Annual Interest Rate?	Amount of deposit?	Compounding periods?
Ans: 10%	**Ans: PV of $150,000**	**Ans: See below.**

The problem above in which we seek a solution for the number of compounding periods per year does not have a direct solution in finance mathematics, nor has a solution method for this type of problem been preprogrammed into your HP 12C calculator. Though we could write a program to solve this on the HP 12C, it is best to utilize the substantial computing power programmed into the equation SOLVER of the "OmniGo Organizer Plus", HP 17BII and HP 19BII (including the HP 17B, HP 19B, and the HP 18C) Financial Calculators--when properly programmed--to solve this type of problem.

The answer to the problem is that the 7.75% annual interest rate is being compounded daily. Therefore, we have 365 compounding periods. To solve this on the above noted Hewlett-Packard products, you would enter the following equation into the device's equation SOLVER:

FV = PV × (1 + I%YR ÷ P/YR ÷ 100)^(#YRS × P/YR)

After inputting the equation, the keystroke procedure would be:

162,086 |FV| 150,000 |PV| 7.75 |I%YR| 1 |#YRS| |P/YR|

Set to zero (0) decimal places, the display will show: **P/YR = 365.**

BASIC FINANCIAL CALCULATIONS

THE FINANCIAL REGISTERS IN A NUTSHELL

The Financial Registers of your HP 12C appear in the top row of keys in the following sequence:

[n][i][PV][PMT][FV][CHS]

Let's run through the keys:

Key	Description
[n]	Use to store or calculate the total number of payments or compounding periods. For example, a 30 year loan with monthly payments requires input of "360" into the [n] register.
[i]	Use to store or calculate the periodic interest rate expressed as a **percentage** (and not as a decimal).
[PV]	The Present Value register is used to store or calculate the current or existing value of something given or exchanged for something else. It is an initial out-of-pocket cash-flow, and, as well, it can be described as the discounted value of a stream of projected cash-flows (such as the discounted present value of a stream of mortgage payments you or your client intend to acquire).
[PMT]	Use the Payment register to calculate or store the amount of the periodic payment needed to amortize a mortgage loan or to achieve a required future value (FV). You are restricted to working with payments which are all equal. If your problem requires analysis of a stream of unequal or skipped payments, use the Unequal Cash-Flow registers (CFo, CFj, and Nj) of your calculator. **Caution:** Always be aware of the payment mode message that might appear in your calculator's screen. For example, if payments (or, for that matter, deposits) are made at the end of the month--indeed, at the end of the time period--, make sure that your "BEGIN" status indicator is not illuminated. We place the calculator in its end (END) mode by pressing the blue [g] key followed the [8] key. In the alternative, if payments (or deposits) occur at the beginning of the time period, make sure that your **BEGIN** status indicator is illuminated before you start a problem.
[FV]	Use the Future Value register to store or calculate the final cash-flow produced by a one-time deposit into an interest bearing account, or use this register to calculate or store the final value of an asset at the end of a known holding or projection period. You also use this register to calculate the final value of a series of deposits (PMTs) into an interest bearing account, and as well use it to calculate the final cash-flow produced by a one-time deposit [PV] plus a series of equal--and equally timed--deposits (PMTs) into the same account.
[CHS]	Use the "change sign key" to **change** the sign of the displayed number. For example, if a number displays as a positive number,

its sign can be changed by pressing the [CHS] key, and vice versa.

PRIMER ON THE "CASH-FLOW SIGN CONVENTION"

The "Cash-Flow Sign Convention" is a set of rules which determine the **sign** for the answers your calculator produces when performing financial calculations. This applies to calculations performed with both the Financial Registers and the Unequal Cash-Flow Registers. Though this convention is prominent and found across the product line with the Hewlett-Packard Financial Calculators, it still gives even the most experienced calculator user some trouble at times.

In a nutshell, this conventions tells us:

Input	**Calculation results produced**
Present Value [PV] positive	Future Value [FV] displays negative Payment [PMT] displays negative
Present Value [PV] negative	Future Value [FV] displays positive Payment [PMT] displays positive
Payment [PMT] negative	Future Value [FV] of an annuity is positive Present Value [PV] displays positive
Payment [PMT] positive	Present Value [PV] displays negative Future Value [FV] displays negative
Future Value [FV] positive	Present Value [PV] displays negative Payment [PMT] displays negative
Future Value [FV] negative	Present Value [PV] displays positive Payment [PMT] displays positive
All Cash-flows into Unequal Cash-Flow registers positive	Net Present Value [NPV] displays positive
All Cash-flows into Unequal Cash-Flow registers negative	Net Present Value [NPV] displays negative

Comment: There are no hard and fast rules on whether or not you should input data into, for example, Present Value [PV] as a positive number when doing payment calculations, thus producing a negative signed payment [PMT]. Some writers, however, suggest that practitioners should enter data into the calculator in accord with whether or not the calculation is performed from the perspective of the borrower or the lender. Thus, this skew on the "convention" says if you are the lender, enter Present Value [PV] as a negative number because it is an **out-of-pocket cash-flow** from that perspective, while the periodic payments (PMTs) received represent an increase in the lender's wealth and thus are positive signed. On the other hand, from the borrower's perspective, enter Present Value [PV] as a positive number because the loan increases your wealth, while a negative signed payment [PMT] represent an outflow of monies from the borrower and thus decreases wealth.

The convention in this book, however, is to show PMT and FV results as positive numbers because this method is more user-friendly toward the client.

COMPUTING A FIXED MONTHLY PAYMENT LOAN

Problem: Purchasers will obtain a $100,000 mortgage on a home selling for $125,000. Assuming the mortgage is a 30 year loan at 7.5% annual interest, how much will they have to pay per month?

Solution methodology: Though nothing is said as to whether the payments occur at the end of the month (as against beginning of the month payments), we readily can assume that the payments indeed occur at the **end** of the month because this is the traditional manner in which real estate loans are structured. Thus, we make sure that the beginning (BEGIN) status indicator does not appear in your display. Further, we should clear all registers (or at least clear the financial registers) before we start solving the problem.

ORGANIZATION OF DATA

Sales price	$125,000
Loan amount	$100,000
Loan term (months)	360
Interest rate	7.5%

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Set 2 decimal places	f 2	
Clear all registers	f CLX	0.00
Make sure end mode set (Could also say "g" "8")	g END	0.00
Enter **amount financed** as a negative number **into [PV]** register	**100,000 CHS PV**	-100,000.00
Enter **monthly interest rate** into [i] register	**7.5 g i**	0.63
Enter **loan term** into **[n]** register	**360 n**	360.00
Compute **monthly payment**	**PMT**	**699.21**

Notes:
(1) As a first step we set 2 decimal places. This is because we typically show **2 decimal** places in loan calculations.
(2) It is a good practice to always make sure you clear all of your calculator's registers (by pressing [f] [CLX]), or at least you should clear the **FIN**ancial registers (by pressing [f] (FIN)).
(3) Since the mortgage loan payments (typically) occur at the end of the month, we made sure the HP 12C was set to **END** mode. Again, we did this by pressing the blue [g] key followed by the [8] key.
(4) The loan amount is always entered into the [PV] register. (Had the loan amount been entered as a positive number, payment would display as **-699.21.**)
(5) Inputs into the [n] and [i] registers must be consistent: If "n" is input as a number of months, then "i" must be entered as a monthly rate.

Problem: Buyers will purchase a home selling for $100,000. The financing is based upon a 30 year, 10%, fixed monthly payment loan. The down payment will be 20% of the purchase price. Points plus loan origination fees total 3% of the mortgage. Compute the monthly payment, last (360th) payment, and the "APR".*

ORGANIZATION OF DATA

Sales price	$100,000
Percent down payment	20%
Loan term (months)	360
Interest rate	10%
Points plus origination fees	3%

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter sales price; compute amount financed; enter result negative-signed in PV register	100,000 ENTER 20 % - CHS PV	100,000.00 80,000.00 -80,000.00
Enter monthly interest rate	10 g i	0.83
Enter loan term	360 n	360.00
Compute monthly payment	PMT	**702.06**
Round payment to 2 decimal places	f RND PMT	702.06
Compute future value	FV	-6.20
Calculate last payment	RCL PMT +	**695.86**
Calculate the "APR":		
Compute cash-flow to borrower	RCL PV 3 % - PV	-77,600.00
Compute monthly "APR"	i	0.86
Convert to annual rate	RCL g i	**10.37**

Question: What if the borrower increases his monthly payment from $702.06 to, say, $800.00 per month. How long will it take for the buyer to pay off the loan? These kinds of questions--and many more like them--can easily be answered using the capabilities of your HP 12C. Let's solve the problem on the next page.

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* In practice, computing a loan's APR requires taking into consideration a number of additional variables not covered in this example, such as loan application and processing fees, and private mortgage insurance premiums (if any).

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear the future value register	0 FV	0.00
Enter monthly interest rate	10 g i	0.83
Input loan amount, negative-signed	80,000 CHS PV	-80,000.00
Enter enhanced monthly payment	800 PMT	800.00
Calculate new loan term	**n**	**216.00**
Calculate 216th payment:		
Find future value residual	FV	-75.10
Add to regular monthly payment	RCL PMT +	**724.90**

Note: Here is how the Financial Registers in your calculator are now loaded:

n	i	PV	PMT	FV	Payment Mode
216	0.8333333333	-80,000.00	800.00	-75.095488	END

Problem extension: At this stage, assume the annual interest rate is lowered from 10% to 7%. Now, ask yourself: (1) If the borrower plans to make payments of $800.00 per month over a term of 20 years (240 months), what is the maximum loan (PV) which can be supported by this level of payment? (2) If the borrower secures a 360 month loan and stays with a monthly payment of $702.06, what is the maximum loan (PV) which will be supported by this combination of interest rate (7%) and term? (Please note that we are not taking into account customary mortgage loan underwriting issues. The sole idea here is to address a few of the "What If" capabilities you have in the financial register function of your HP 12C.)

Solution #1

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear the future value register	0 FV	0.00
Enter new loan term	240 n	240.00
Enter monthly interest rate	7 g i	0.58
Compute (new) loan amount	**PV**	**-103,186.01**

Note: Here is how the Financial Registers in your calculator are now loaded:

n	i	PV	PMT	FV	Payment Mode
240	0.5833333333	-103,186.0052	800.00	0.00	END

Solution #2

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter (new) loan term	360 n	360.00
Enter (new) monthly payment	702.06 PMT	702.06
Compute (new) loan amount	**PV**	**-105,524.93**

Note: Here is how the Financial Registers in your calculator are now set:

n	i	PV	PMT	FV	Payment Mode
360	0.5833333333	-105,524.9312	702.06	0.00	END

Problem extension: Let's assume that the lender offers to loan the borrower $105,000, but requires that the monthly payment be increased from $702.06 to $717.00 (a nice round number!). The term, however, remains the same--360 months. Now, what is the annual interest rate which the lender is charging the borrower? (We fully admit that in practice data such as this will indeed be disclosed on the lender's "HUD-1" statement. This statement is also commonly referred to as the "RESPA" statement.)

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter new loan amount	105,000 CHS PV	-105,000.00
Enter new monthly payment	717 PMT	717.00
Compute (new) interest rate;	**i**	**0.61**
convert to annual rate	**RCL g i**	**7.26**

Note: Here is how the Financial Registers in your calculator are now set:

n	i	PV	PMT	FV	Payment Mode
360	0.6050030189	-105,000.0000	717.00	0.00	END

Problem extension: Let's further assume that the borrower would like to increase his monthly payment by an amount which will enable him to fully pay off the loan at the time the 180th payment is made. What must the payment be raised to? Let's solve for it below.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter new loan term	180 n	180.00
Compute new monthly payment	**PMT**	**959.10**

COMPUTING A FIXED QUARTERLY PAYMENT LOAN

Problem: Buyers will purchase a home selling for $150,000. The Seller will carry a 30% down, **quarterly payment** land contract with a 15 year amortization term, and 11% annual interest rate. In the alternative, if the buyers accept a monthly payment land contract, the down payment will be 25% of the purchase price, with annual interest rate of 10.25%, with a term of 20 years. Compute the installment payments under both alternatives.

ORGANIZATION OF DATA: Option #1

Sales Price	$150,000
Percent down payment	30%
Contract term (years)	15
Total Quarterly payments	60
Annual interest rate	11%

ORGANIZATION OF DATA: Option #2

Sales Price	$150,000
Percent down payment	25%
Contract term (years)	20
Total monthly payments	240
Annual interest rate	10.25%

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter sales price; compute Option #1 contract amount; enter result negative-signed in PV register	150,000 ENTER 30 % - CHS PV	150,000.00 105,000.00 -105,000.00
Enter **quarterly** interest rate	11 ENTER 4 ÷ i	2.75
Enter quarterly term of contract	4 ENTER 15 x n	60.00
Compute quarterly payment	PMT	**3,593.10**
Enter sales price; compute Option #2 contract amount; enter result negative-signed in PV register	150,000 ENTER 25 % - CHS PV	150,000.00 112,500.00 -112,500.00
Enter **monthly** interest rate	10.25 g i	0.85
Enter monthly term of contract	20 g n	240.00
Compute monthly payment	PMT	**1,104.35**

Notes:

(1) In practice you will not need to reset the calculator's payment mode to END each time you perform a financial calculation. Unless your calculator's beginning status indicator (BEGIN) is displayed, it is not necessary to set the payment mode to END each time you perform an end-of-the-period financial calculation. The same applies to setting the number of decimal places.

(2) When entering the term of a contract or mortgage, it is not necessary to use the **BLUE [g]** key as we did in the Option #2 example above. Clearly, the objective is to input the term into the HP 12C's [n] register, no matter how we do it. Since a 20 year monthly payment land contract (or mortgage) requires a total of 240 monthly payments, we could readily type: 240 [n].

COMPUTING THE BI-WEEKLY LOAN PAYMENT

Problem: Compute the bi-weekly payment on a $100,000 loan, with interest of 10% per annum, and an initial computation-term of 30 years. Next, compute (1) the term of the loan and (2) the required last payment.

ORGANIZATION OF DATA

Loan amount	$100,000	Bi-Weekly payments of	?
Annual interest rate	10%	Recomputed term	?
Initial term (years)	30	Last (final) Payment	?

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter initial loan term	30 g n	360.00
Enter monthly interest rate	10 g i	0.83
Enter loan amount, negative-signed	100,000 CHS PV	-100,000.00
Compute monthly payment	PMT	877.57
Compute and round bi-weekly payment and enter into PMT register	2 ÷ f RND PMT	**438.79**
Enter bi-weekly interest rate	10 ENTER 26 ÷ i	0.38
Compute the bi-weekly term	n	545.00
Compute term in years	26 ÷	**20.96**
Compute the last (545th) payment:		
Calculate the "residual" left over after 545 full payments made	FV **FV**	-37.04
Calculate last payment	RCL PMT +	**401.75**

Notes:

(1) The recomputed term in years (20.96) tells us that the loan will be fully amortized in less than 21 years. Precisely, the loan will be amortized after 545 **bi-weekly** payments have been made, subject, however, to an adjustment to the last (545th) bi-weekly payment.

(2) The author's "general rule" is that anytime you compute **[n]** on the HP 12C, you must go back and calculate the Future Value **[FV]**. Calculating the **FV** gives you the "residual" or "balloon payment" that is required or to be "credited" to the borrower who makes **"n"** full payments. In the above example, calculating the **FV** tells us that if the borrower makes 545 full payments, the 545th payment would include an **overpayment** of $37.04. Therefore, we "net" the computed FV (-$37.04) with the regular bi-weekly payment ($438.79), and arrive at a final payment of $401.75.

PERIODIC (End of Period) PAYMENT PRACTICE PROBLEMS

Problems: Using the data in problems **A, B,** and **C,** solve for the periodic payment produced from the data.

Problem A
Loan = $200,000
Term = 30 years
Monthly payments
Interest rate: 9%/yr
Ans: $1,609.25

Problem B (30 Yr Term)
Price = $200,000
LTV Ratio = 80%
Monthly payments
Interest rate: 10%/yr
Ans: $1,404.11

Problem C
Loan = $160,000
Term = 20 years
Quarterly payments
Interest rate: 9.5%/yr
Ans: $4,486.04

Problem: Solve for the unknown variable(s) in the following problems:

Problem D
Loan = $100,000
Monthly PMT = $1,101.09
Term = 240 months
Last PMT = $1,097.27
Annual Interest Rate = ?
Ans: 12%

Problem E
Loan = $200,000
Term = 30 years
Interest rate: 8.25%/yr
Monthly PMT = ?
Ans: $1,502.53

Problem F
Term = 360 months
Monthly PMT = $877.57
Interest rate: 10%/yr
Loan Amount = ?
(To nearest $1.00)
Ans: $100,000

Problem G (i = 9%/yr)
Loan = $65,000
Monthly PMT = $523
Last PMT = $531.61
Term = ?
Ans: 360 months

Problem H
Term = 180 months
Interest rate: 11%/yr
Monthly PMT = $523
Loan Amount = ?
Ans: $46,014.55

Problem I
Loan = $220,000
Term = 240 months
Monthly PMT = $2,315.90
Interest Rate/yr = ?
Ans: 11.30%

Problem J
Interest rate: 14%/yr
Monthly PMT = $998.81
Loan = $75,000
Term = ?
Last Payment = ?
Ans: n = 180
Last Payment = $996.41

Problem K
Interest rate: 11%/yr
Loan = $325,000
Term = 180 months
Quarterly payments = ?
Ans: $11,121.51
Semiannual PMTS = ?
Ans: $22,361.75

Problem L
Interest rate: 11%/yr
Quarterly PMT = $3,422
Loan = $100,000
Term in years = ?
Ans: 15
Last Payment = ?
Ans: $3,422.26

Note: In solving for the Last Payment, we round the periodic payment, enter it into the PMT register, solve for the "residual" Future Value, and add the computed FV to the periodic payment. If, however, we know the amount of the last payment and the periodic payment, and seek another financial variable, such as the interest rate or term, we must first compute the **residual** that will be entered into the FV register. To do this you subtract the **periodic payment** from the **last payment**. The difference, whether positive or negative signed, is then entered into the FV register. To complete the solution, input the remaining known variables and solve for the unknown variable. (This method was used to solve for the interest rate in **Problem D.**)

MATHEMATICS SIDE-BAR FOR COMPUTING INSTALLMENT PAYMENT TO AMORTIZE A LOAN

When solving for the periodic payment required to amortize (payoff) a loan, the calculator is working with the following type of equation:

$$\mathbf{PMT} = \mathbf{PV} \times \left[\frac{i}{1 - (1 + i)^{-n}}\right],$$

where: PMT = Periodic loan payment;
PV = Present Value (Loan Amount);
i = Periodic interest rate; and
n = Number of payments in the loan's term.

For example, let's use the formula to solve for the monthly payment needed to amortize a $200,000 loan, over a 30 year period, at a 9% annual interest rate.

Solution:

$$\mathbf{PMT} = \frac{PV \times i}{1 - \frac{1}{(1 + i)^{n}}}$$

$$= \frac{\$200{,}000 \times 9 \div 1{,}200}{1 - \frac{1}{(1 + 9 \div 1{,}200)^{360}}}$$

$$= \$200{,}000 \times (9 \div 1{,}200) \div (1 - (1 + (9 \div 1{,}200))^{-360})$$

$$= \$200{,}000 \times .0075 \div (1 - (1.0075)^{-360})$$

$= \$1{,}500 \div (1 - .06788600742)$ (Used **[f] PREFIX**)

$= \$1{,}500 \div .9321139926$ (Used **[f] PREFIX**)

$= \$1{,}609.245234$

PMT = $1,609.25 (Rounded to 2 decimal places)

Note: If a balloon payment is integrated into the payment computation, a suitable equation presents as follows:

$$\mathbf{PMT} = (PV - FV \times (1 + i)^{-n}) \times i \div (1 - (1 + i)^{-n})$$

COMPUTING AFTER-TAX PAYMENT COST OF THE LOAN

Problem: A buyer--who is in a 32% tax bracket--gets a $100,000 loan at 10% annual interest. Monthly payments are $877.57. Buyer's annual income does not exceed $100,000. Compute the first year monthly after-tax payment cost.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter monthly interest rate	10 g i	0.83
Enter loan amount, negative-signed	100,000 CHS PV	-100,000.00
Enter monthly loan payment	877.57 PMT	877.57
Amortize 12 payments. Display accumulated interest expense	12 f AMORT	9,974.98
Compute average monthly interest expense for first 12 payments	RCL n ÷	831.25
Calculate tax savings attributed to deductibility of interest expense	32 %	266.00
Subtract average monthly tax savings from monthly payment for P & I	CHS RCL PMT +	611.57

Conclusion: The buyer's average after-tax monthly payment cost of the loan is $611.57. In practice, the amount can vary depending upon the buyer's income tax bracket, the analysis period (12 months; 24 months; and so forth), and as well the after-tax payment cost is impacted by issues concerning the **Revenue Reconciliation Act of 1990.**

Under the Revenue Reconciliation Act of 1990 certain itemized deductions are reduced for "high income" taxpayers. Since the major thrust of this Act is aimed at mortgage interest and real estate taxes, their deductibility will be somewhat limited for so-called "high income" taxpayers.

The Act provides that for the years 1991 and thereafter, an individual whose adjusted gross income (AGI) exceeds the "threshold amount" ($100,000) will be required to reduce the amount of otherwise allowable itemized deductions by three percent (3%) of the "excess" income over $100,000. For example, if a taxpayer has AGI of $125,000, his itemized deductions (which include mortgage interest and real estate taxes) must be reduced by 3% of $25,000 ($125,000 - $100,000). Therefore, his after-tax payment cost will be somewhat higher due to the loss of a portion of the otherwise allowable deduction for mortgage interest and real estate taxes.

Caution: The above method does not adjust for the Revenue Reconciliation Act. If you are computing the after-tax payment cost for a buyer whose income exceeds $100,000, additional adjustments must be made which are not covered in the above routine.

COMPUTING NUMBER OF TIME PERIODS [n]

Introduction

Users of the HP 12C readily discover that the device is programmed in a manner which returns only an **integer number of time periods**, at least when using the Financial Registers to compute the number of time periods required to achieve a given financial result. That is, the calculator is programmed to always return an **integer** solution for the number of time periods. For example: 1 year; 2 years; 300 months; 360 months; but **never**, say, "1.5" years or "350.7" months, and so forth. The calculator always returns a **rounded** number of time periods.

This designed-in feature is not a handicap when the user is concerned with computing the number of time periods to amortize a monthly payment loan. In that case, you simply compute the term [n], and then calculate the "residual", if any, left over at the expiration of the computed term. This is achieved by calculating the future value [FV] and simply adding it to the payment [PMT] to arrive at the last payment in the loan term.

However, suppose we are projecting the **number of years** necessary to achieve a given financial result? In this case the user may find a wide disparity between the HP 12C's internal routine for computing [n] and the realities of a problem's actual solution. For example, assume we are solving for the number of years necessary to achieve a Future Value of $85,950, where the projected annual growth in value is 7% and the purchase price (Present Value) is $50,000. The HP 12C returns 9 years; the mathematically correct answer is "8.006998937" years, which rounds to 8 years.

Clearly, users of the HP 12C will sometimes need to consider a substitute method for computing the number of time periods--particularly the number of years--needed to achieve a required financial result. This problem can be, of course, overcome by simply using a substitute financial calculator--such as the HP 17BII or HP 19BII--when solving for [n], or you can use a suitable program designed for operation in the HP 12C.

The program given in this book will solve for **"actual n"** (exact number of time periods) in every financial situation which can be solved on an algebraic calculator, such as the HP 17BII and HP 19BII. Using the program will not interfere with the operation of your calculator, and you can readily go back and forth between the financial registers and the program.

It is suggested that you set your calculator to display two decimal places (**f, 2**) when using the program. In the alternative, after reviewing the number of time periods [n] computed by the program, always round to two decimal places. This suggestion is made because you will find instances where very slight rounding differences show up in your result. The differences are insignificant even at nine decimal places, and they will round out of the picture at two decimal places.

PROGRAM FOR COMPUTING NUMBER OF TIME PERIODS [n]

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
[f](REG)[f][R/S] [f](PRGM)	00	[RCL] 1	14 45 1
[RCL][PMT]	01 45 14	[+]	15 40
[RCL] 0	02 45 0	[STO][÷] 2	16 44 10 2
[%]	03 25	[RCL] 2	17 45 2
[+]	04 40	[g][LN]	18 43 23
[STO] 1	05 44 1	1	19 1
[STO] 2	06 44 2	[RCL][i]	20 45 12
[RCL][FV]	07 45 15	[%]	21 25
[RCL][i]	08 45 12	[+]	22 40
[%]	09 25	[g][LN]	23 43 23
[STO][-] 2	10 44 30 2	[÷]	24 10
[RCL][PV]	11 45 13	[STO][n]	25 44 11
[RCL][i]	12 45 12	OPTIONAL:	
[%]	13 25	[ENTER]	26 36
		[f] [R/S]	

User Instructions:

1. Store PV in [PV]
   (Follow cash-flow sign convention)
2. Store FV in [FV]
   (Follow cash-flow sign convention)
3. Store PMT in [PMT]
   (Follow cash-flow sign convention)
4. Store periodic "i" in [i]
5. If a Due Annuity (beginning of the time period payment), store [STO] periodic "i" in Memory Register "0".
6. Set payment mode to END.

Beginning of Time Period Example:

[f][CLX][g](BEG) [f] 2
1 [i]
100,000 [CHS][PV]
50,000 [FV]
240 [n]
[PMT]
DISPLAYS: 1,040.14

Utilize Program:

[g](END) 1 [STO] 0
[R/S]
DISPLAYS: 240.00
Check full accuracy:
[f] 9
DISPLAYS: 239.9999996

Note: The example is based upon payments occurring at the **beginning** of the month. This is why we set the payment mode to **BEG** in the keystroke sequence in the left-hand column. However, you must always set the calculator's payment mode to **END** when using the program to calculate the number of time periods necessary to achieve a given financial result. You will note this step in the right-hand column above.

ALWAYS COMPUTE/RECOMPUTE FUTURE VALUE (FV) IF YOU COMPUTE "n" ON THE HP 12C

The HP 12C always rounds the number of **computed time periods** to the highest integer value. This presents a somewhat unique problem for those who use the device in general financial calculations, such as computing the number of time periods to amortize (pay off) a loan, and so forth. The problem is, fortunately, solvable, and must be addressed each time you use this excellent device to compute number of time periods [n].

A general rule followed and taught by the author is that if you compute the number of time periods [n] to do or achieve anything on the HP 12C, you **must** (1) calculate the Future Value (FV) of the residual, if any, remaining due to the calculator's rounding procedure, or (2) you must **recalculate** any Future Value (FV) amount used in arriving at the computed [n]. This procedure assures accuracy in your calculations in that it will prove whether or not the calculation is practical on your HP 12C without using the program, or if you must change over to another device, such as the HP 17BII.

For example, assume you wanted to determine the number of time periods needed to produce $75,000 from an (untaxed) investment of $50,000 drawing compound interest of 10% per year. The FV of the investment is $75,000; the PV is $50,000; and the annual yield is 10%. Inputting the data into your calculator, we compute the number of time periods [n] as follows:

KEYSTROKES:	f CLX	Clears all registers
	10 i	Annual interest rate
	50,000 CHS PV	Inputs investment, negative-signed
	75,000 FV	Required future value
	n	Number of years required
	DISPLAYS: 5.00	
	FV	Recalculate FV
	DISPLAYS: 80,525.50	

On the surface, it appeared that it would take five years to cause $50,000 to increase to $75,000 at an annual interest rate of 10%. However, in checking our result by **recomputing** the future value, we determined that the future value is now $80,525.50, rather than the $75,000 we asked for. Indeed, this tells us that it will not take five years to "grow" $50,000 into $75,000, but rather it will take a little less time. How much less?

You must now decide whether to use the computer program or the HP 17BII (or other suitable device) to solve this problem. No matter what method you use for solving for number of time periods [n], the point is you might not have taken a second look at this issue if you were not on guard for this potential problem area in using the HP 12C. **Always** recompute FV after solving for [n] on the HP 12C.

Solving the problem with the program (or another calculator), we determine that it will take approximately 4.25 years to attain a future value of $75,000 from a $50,000 investment growing at 10% annual compound interest. Indeed, this is 9 months less than the result obtained above.

ADJUSTABLE RATE MORTGAGES

The adjustable rate mortgage (ARM) is a real estate financing device developed by lenders in the interest of transferring to the borrower most, if not all, of the risks associated with interest rate swings in the short-term capital markets. With an ARM the borrower can expect the interest rate to change periodically; accordingly, payments will go up or go down.

Basics of how the ARM works

The Index: Generally speaking, a loan "index rate" is a national indicator of interest rate conditions in the capital markets. The lower the availability of money, the higher the index interest rate. The greater the availability of money, the lower the interest rates.

The most common index rates chosen by lenders are the rates on one-, three-, or five-year U.S. Treasury Securities. The rate most commonly used by lenders in the Detroit metropolitan area is the annualized rate on one-year Treasury bills. This rate, among others, is reported in the Tuesday edition of the Wall Street Journal under the heading of "Key Interest Rates".

The "index" is the base annual interest rate which is used by the lender to calculate the interest rate which the borrower will be required to pay. As discussed below, the lender adds its "margin" to the index rate, but may give a first year "discount" to this rate.

The Margin: To compute the interest rate on an ARM, lenders will add to the index rate an interest rate markup called the "margin" or "spread". The margin is usually two to three percentage points and represents a markup to the lender to cover profit, plus expenses of administering the loan.

The margin rate is established by the lender at the time of making the ARM loan. Contractually it becomes a part of the loan agreement itself. Therefore, the margin rate will not change during the term of the ARM, unless the loan agreement is written in a manner which allows this to occur.

The Adjustment Period: With most ARMS, the interest rate and the monthly payment change every year. Some ARMS, however, are structured to change interest rates and payments every three to five years, though these are less common, as would be the case with an ARM whose interest rate is permitted to change quarterly or biannually.

The period between one interest rate change and the next is called the "adjustment period". Consequently, a loan with an adjustment period of one year is defined as a "one year ARM". Therefore, the interest rate and payment can change once a year. The payment changes, of course, because the interest rate changes (in the most common ARM loan plans) while the remaining term and principal amount of the loan change only in the sense that they are reduced (amortized) as each payment is made.

Interest Rate Caps: A "periodic interest rate cap" places a contractual limit on the amount by which the lender can increase or decrease the ARM's interest rate at the beginning of each adjustment period. For example, if the initial interest rate is 8%, and if the agreed upon annual interest rate cap is 2%, then the maximum interest rate the lender can charge after the first adjustment period in an increasing interest rate market is 10% (8% + 2%). On the other hand, if interest rates were in decline, the maximum the lender could reduce the interest rate would be by the amount of the annual interest rate "cap", being 2% in this example.

An "overall interest rate cap" places a contractual limit on the total interest rate increase over the life of the loan. For example, assuming a yearly ARM has a periodic interest rate "cap" of 2% and an "overall cap" of, say, 6%, the lender could legally increase the ARM's initial interest rate by 2% per year, starting with the second year, until the initial interest rate was increased by a total of 6%.

For example, assuming an initial interest rate of 6%, a periodic annual "cap" of 2%, and an "overall cap" of 6%, the maximum rates the lender could charge the borrower would be: First year, 6%; Second year, 8%; Third year, 10%; and Fourth year, 12%. On the other hand, assume we started with an initial interest rate of 14%, a periodic "cap" of 2% and an "overall cap" of 6%. In a continuously declining interest rate market, the most favorable interest rates the borrower could expect would be: First year, 14%; Second year, 12%; Third year, 10%; and so on.

Negative Amortization: Note that under some ARM plans interest rate changes in excess of the periodic interest rate cap can be carried over by the lender to the next adjustment period. In these cases the composite rate (the sum of the index rate plus the margin) would exceed the interest rate or payment cap, thus causing negative amortization to occur. (This occurs because the payment is not enough to amortize the loan because it is limited by either (1) a payment cap or (2) by an interest rate cap.) When the ARM plan selected allows for negative amortization, the effect is that the loan balance will be increased at the beginning of the next adjustment period. (There are much too many variations on these types of plans and therefore they are clearly outside the scope of this book.)

Initial Interest Rate: The initial interest rate charged on the ARM is usually below that charged on a conventional fixed interest rate mortgage (FRM) loan. The ARM's lower rate, however, is often no more than an introductory or "teaser" rate and can be expected to increase after expiration of the loan's adjustment period. At this time, the interest rate is computed on the basis of the index rate plus the lender's margin (subject to the interest rate cap).

The initial interest rate is generally based upon competitive considerations and thus may be **discounted** by the lender. In these situations, the initial interest rate cannot be determined by simply adding the index interest rate to the lender's required margin. Rather, the total of these two rates will be discounted by a rate which is deemed sufficient to make the lender's ARM package competitive in the loan market being served.

For example, if the index is 6.5% and the lender's required margin is 3%, the borrower's first year interest rate would be 9.5% (6.5% + 3%). If, however, the lender is giving the borrower a "discount" of, say, 2.525% in the first year's rate, the initial annual interest rate would be 6.975% (9.5% - 2.525%). At the first interest adjustment period the new interest rate is determined without regard to a discount rate. Said differently, the lender's original "discount" rate is applied on a one-time only basis and therefore has no bearing on subsequent interest rates which the borrower will obligated for.

Example: In the table given below--as well as the keystroke example that follows on the next page--we do not cover the more complex ARM cases which involve negative amortization or payment caps. Rather, the table (and the keystroke solution example following it) is intended solely to be a starting point for possible study beyond the examples covered in this book.

YEAR *	INDEX	DISCOUNT RATE		MARGIN	CAP ANNUAL/OVERALL			COMPOSITE RATE	FINAL RATE
1992	3.960%	N.A.	+	3%	2%	6%	=	6.960%	6.960%
1993	3.420%	N.A.	+	3%	2%	6%	=	6.420%	6.420%
1994	5.470%	N.A.	+	3%	2%	6%	=	8.470%	8.420%
1995	5.650%	N.A.	+	3%	2%	6%	=	8.650%	8.650%
1996	5.790%	N.A.	+	3%	2%	6%	=	8.790%	8.790%

Comments:

1992: We start with an index rate of 3.96% and a margin rate of 3%. Since the lender is not offering a "discount" (or "teaser") rate in the first year, we do not reduce the composite interest rate. Thus, the final rate is simply 6.96% (3.96% + 3%).

1993: Here the index rate <u>drops</u> from 3.96% to 3.42%. Note further that the final interest rate is 6.42% (index rate of 3.42% + the 3% margin). Since the margin rate drop from 1992 to 1993 is well under the 2% annual cap (it's 0.54%), the final interest rate is the same as the composite rate, 6.42%.

1994: For 1994 the index rate increased by 2.05%, which amount is "0.05%" greater than the periodic interest rate cap of 2%. Thus, the borrower's interest rate increases by 2%, going from 6.42% to 8.42%.

1995: For 1995 the index rate ticks up a bit (0.18%), well within the 2% annual cap. Thus, the final interest rate is 8.65% (5.65% + 3%).

1996: Here we see another uneventful change in the index rate, ticking up by 0.14%, again, well within the 2% annual cap. Thus, the final interest rate is 8.79% (5.79% + 3%).

* **Note:** Data is given for the "annualized rate on one year Treasury Bills" for the first week in July of the years reported. The data is reported in the Tuesday edition of the **Wall Street Journal** (unless Tuesday falls on the 4th of July, in which case the rate will appear in the Wednesday edition) under the heading of "Key Interest Rates".

CALCULATING THE MONTHLY PAYMENT TO AMORTIZE AN ARM

ORGANIZATION OF DATA

1.	MORTGAGE AMOUNT	$100,000
2.	TERM	30 YEARS
3.	INDEX RATE	6.40%
4.	LENDER'S REQUIRED MARGIN	3%
5.	DISCOUNT RATE (FIRST YEAR)	2.5%
6.	ANNUAL INTEREST RATE CAP	2%
7.	OVERALL INTEREST RATE CAP	6%
8.	INTEREST RATE ADJUSTMENT PERIOD	ANNUAL
9.	PROJECTED MONTHLY PAYMENTS TO AMORTIZE (first, second, and third years)	?

Problem: Assuming the loan interest rate increases by the maximum two percent (2%) each year during the second and third year, what will the payments be for the first three years of the loan?

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set END mode and 2 decimal places	f CLX g END f 2	0.00
Compute initial interest rate:		
Index rate + margin less discount	6.4 ENTER 3 + 2.5 -	6.90
Compute 1st year payment:		
Enter mortgage term	360 n	360.00
Enter monthly interest rate	6.9 g i	0.58
Amount financed, negative-signed	100,000 CHS PV	-100,000.00
Compute 1st year monthly payment	**PMT**	**658.60**
Compute 2nd year payment:		
Round and enter rounded payment	f RND PMT	658.60
Compute 1st year loan balance	12 n FV	98,964.46
Enter loan balance in PV	CHS PV	-98,964.46
Clear FV register	0 FV	0.00
Enter number of payments remaining	360 ENTER 12 - n	348.00
Enter 2nd year interest rate	RCL g i 2 + g i	0.74
Compute 2nd year monthly payment	**PMT**	**794.72**
Compute 3rd year payment:		
Round and enter rounded payment	f RND PMT	794.72
Compute 2nd year loan balance	12 n FV	98,205.18
Enter loan balance in PV	CHS PV	-98,205.18

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear FV register	0 FV	0.00
Enter number of payments remaining	348 ENTER 12 - n	336.00
Enter 3rd year interest rate	RCL g i 2 + g i	0.91
Compute 3rd year monthly payment	**PMT**	**936.93**

Conclusion: The monthly payments are as follows: First year, $658.60; Second year, 794.72; and Third year, $936.93.

Calculating the Yield to the lender with early pay off

Problem: Using data from the above problem, calculate the yield to the lender (weighted interest cost to the borrower) if the loan is paid off at the end of the third year. Assume the loan carries an application fee of $350, plus 3% points for incidental loan costs.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Round and enter rounded payment	f RND PMT	936.93
Compute 3nd year loan pay off	12 n FV	97,638.63
Calculate initial cash-flow from lender (loan amount less 3% points and $350 loan application fee)	100,000 ENTER 3 % - 350 -	96,650.00
Enter result as initial cash-flow	CHS g CFo	-96,650.00
Enter first year payment	658.60 g CFj	658.60
Enter 12 cash-flows	12 g Nj	12.00
Enter 2nd year payment	794.72 g CFj	794.72
Enter 12 cash-flows	12 g Nj	12.00
Enter 3rd year payment	936.93 g CFj	936.93
Enter 11 cash-flows	11 g Nj	11.00
Add balloon payment to final PMT	[R↓] 97,638.63 +	98,575.56
Enter final cash-flow	g CFj	98,575.56
Compute monthly IRR	f IRR	0.84
Convert to annual rate	**RCL g i**	**10.07**

Conclusion: The lender's effective yield is 10.07%. That is equivalent to saying that the effective cost of the loan to the borrower is 10.07% per annum.

COMPUTING THE BALLOON PAYMENT

Discussion of the problem: The most common manner in which a real estate loan is structured is to have the borrower/purchaser make equal regular monthly payments over an agreed upon period of time, typically 15, 20, or 30 years. The payments are structured so as to completely pay off ("amortize") the loan over its term. The term of the loan is almost always broken up into monthly periods, with the borrower being obligated to make fixed monthly payments due on a given date, usually the first of the month, with payments starting at least one month after the loan is disbursed to the borrower.

The amortization process, however, rarely goes-to-term, meaning that a real estate loan is typically satisfied at a date earlier than would otherwise be permissible under the loan documents (mortgage and mortgage note or deed of trust). For example, a loan might be structured with a 30 year term, but the borrower may elect to pay it off after 10 years. The final payment which the borrower makes under conditions of an early pay off includes not only the regular payment, but as well includes what is commonly known as a **balloon payment.** Calculating the balloon payment is a straightforward task which is no more complicated than calculating a regular monthly payment on a loan.

Problem: Buyers took out a monthly payment $100,000 mortgage loan, payable over 30 years with an annual interest rate of 7%. If the buyers paid off the loan after making 90 regular monthly payments, what is the required balloon payment and the total final payment required on the loan? In order to calculate the balloon payment we must first determine the regular monthly payment required on this loan. Let's first organize the data.

ORGANIZATION OF DATA

Loan amount	$100,000	Monthly payment	Unknown
Loan term	30 years	Pay off at month	90
Annual interest rate	7%	Balloon & final payment	Unknown

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter loan term	360 n	360.00
Enter monthly interest rate	7 g i	0.58
Enter amount financed into PV as a negative number	100,000 CHS PV	-100,000.00
Compute monthly payment; round and reenter monthly payment	PMT f RND PMT	**665.30** 665.30
Enter number of payments at which time balloon payment becomes due	90 n	90.00
Compute balloon payment	**FV**	**90,334.27**

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Calculate total final payment	RCL PMT +	90,999.57

Note: Leave all data in your calculator.

Note: Here is how the Financial Registers in your calculator are now set:

n	i	PV	PMT	FV	Payment Mode
90	0.5833333333	-100,000.00	665.30	90,334.27069	END

Problem extension: At this stage let's assume that the borrower plans to pay off the loan at the time when the balance owing is $80,000 (not including the regular monthly payment). The question becomes: How long will it take to pay the loan down to $80,000? This is a straightforward problem which simply requires that we tell the calculator what the future balance (FV) is and then ask for a solution for the number of months (n) needed to achieve that balance. From there, a little adjusting of the balloon payment will be needed. Let's solve the problem below.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter required balloon payment	80,000 FV	80,000.00
Calculate number of payments; convert to years (12 years, 9 months)	n 12 ÷	153.00 12.75

Discussion: In order to calculate the final payment on the loan we must decide whether to adjust the final monthly payment (payment number 153) or adjust the balloon payment. From a previous section of this book we know that the HP 12C always displays an "integer" (rounded) result for the number of time periods. Therefore, to test for a residual or change in the future value we should recalculate future value after having calculated "n". In the alternative, we can set the "n" register to "1" less time period--which would make it "152" in the above example--and then solve for the readjusted future value (balloon payment). Let's perform the calculation both ways!

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Compute **adjusted balloon payment**	**FV FV**	**79,837.56**
Determine total final payment	RCL PMT +	80,502.86
Enter 152 month term	152 n	152.00
Compute **adjusted balloon payment**	**FV**	**80,035.99**
Determine total final payment	RCL PMT +	80,701.29

TECHNICAL SIDE-BAR FOR COMPUTING THE BALLOON PAYMENT

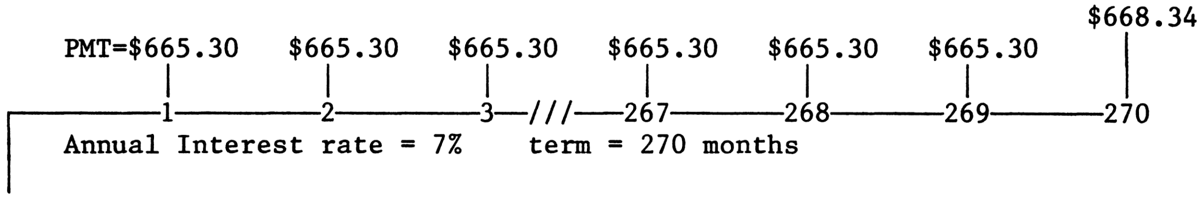
Discussion: On the previous page we determined that the 90th month balloon payment needed to pay off a $100,000 loan written at 7% over a 360 month term with monthly payments of $665.30 was $90,334.27. Technically, where does the balloon payment come from? What is it?

A balloon payment is nothing more than the **discounted present value** of all of the regular payments remaining on a loan at the time the balloon becomes due. For example, if a balloon becomes due with the 90th payment on a 360 month loan, the remaining number of regular payments is 270 (360 - 90). Now, the balloon payment is simply the discounted present value of the 270 remaining payments at the loan's periodic interest rate.

To perform this proof with a high degree of accuracy we need to find the residual, if any, remaining at the time the last regular payment is due on the loan. This number is carried in the FV register when performing the proof. This step is necessary to account for rounding of the monthly payment.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers	f CLX	0.00
Enter original loan term	360 n	360.00
Enter monthly interest rate	7 g i	0.58
Enter amount financed into PV as a negative-signed number	100,000 CHS PV	-100,000.00
Enter monthly payment	665.3 PMT	665.30
Calculate residual due with final payment; leave in FV	FV	3.04
Perform the proof:		
Enter remaining term of loan at time balloon payment due	270 n	270.00
Calculate present value of 270 remaining payments and the $3.04 residual	**PV**	**-90,334.27**

A timeline diagram for the problem follows: Note that the final regular payment due at the 270th month is $3.04 greater than payments 1 through 269.



PV=$90,334.27 = Discounted PV of remaining 270 payments at time balloon due.

COMPUTING BALLOON PAYMENT AND EFFECTIVE INTEREST RATE WITH EARLY PAY OFF

Problem: Buyers took out a monthly payment $80,000 mortgage loan, payable over 30 years with an annual interest rate of 9.5%. Points and loan origination fees total 3% of the loan. Incidental fees total $275. If the buyers paid off the loan after making 72 monthly payments of $672.68, what is the lender's effective annual yield on this loan?

ORGANIZATION OF DATA

Loan amount	$80,000	Monthly payment	$672.68
Annual interest rate	9.5%	Points + Origination fees	3%
Pay off at month	72	Additional closing costs	$275

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter total number of payments made	72 n	72.00
Enter monthly interest rate	9.5 g i	0.79
Enter loan amount, negative-signed	80,000 CHS PV	-80,000.00
Enter monthly payment	672.68 PMT	672.68
Compute balloon payment due with 72nd regular monthly payment	FV	76,201.33
Compute and enter initial cash-flow:		
Reduce loan amount by points,	RCL PV 3 % -	-77,600.00
origination fees and incidentals *	275 + PV	-77,325.00
Compute effective monthly interest	i	0.86
Convert to annual interest rate	RCL g i	10.26

---

* Note that in reducing the amount financed to reflect the payment of points, origination fees and incidentals ($275), we **added** the incidentals to the loan amount as reduced by the payment of points and origination fees. The procedure was necessary since the convention followed throughout this book is to input the amount financed as a negative number, rather than as a positive number. Therefore, in order to further **reduce** the initial cash-flow to the borrower it was necessary to **add** $275 (the incidental fees) to the negative signed amount showing in the display (**-77,600.00**), thus producing a negative signed initial cash-flow of $77,325.00. If, however, we **subtracted** the $275 fee from the interim-displayed "**-77,600.00**", we would produce an out-of-pocket cash-flow from the lender (and to the buyer) of **$77,875.00.** This, of course, would be erroneous and would in turn result in an incorrect effective annual interest rate of 10.10% rather than the actual amount, being 10.26%.

COMPUTING ACCUMULATED INTEREST, PRINCIPAL REDUCTION, AND LOAN BALANCE

Problem: Prepare an amortization statement for the first two months of a $100,000 loan payable at $877.57 per month, with interest at 10% per annum.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter monthly interest rate	10 g i	0.83
Enter loan amount, negative-signed	100,000 CHS PV	-100,000.00
Enter monthly payment	877.57 PMT	877.57
Amortize first month: Display interest expense (1)	1 f AMORT	833.33
Display reduction in loan balance (2)	x≷y	44.24
Display loan balance (3)	RCL PV	-99,955.76
Amortize second month: Display interest expense	1 f AMORT	832.96
Display reduction in loan balance	x≷y	44.61
Display loan balance	RCL PV	-99,911.15

Problem: Use the information from the above problem to prepare an amortization schedule for the first two years of the loan. Assume, however, that seven payments are to be made during the first year.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Reset Present Value register	100,000 CHS PV	-100,000.00
Clear out [n] register	0 n	0.00
Amortize 7 payments: Display accumulated interest expense	7 f AMORT	5,825.47
Display reduction in loan balance	x≷y	317.52
Display loan balance	RCL PV	-99,682.48
Amortize second year: Display accumulated interest expense	12 f AMORT	9,941.74
Display reduction in loan balance	x≷y	589.10
Display loan balance	RCL PV	-99,093.38

Notes:
(1) It is not necessary to set the [n] register when performing an amortization calculation on the HP 12C.
(2) The reduction in loan balance is automatically stored in the "y" register after performing an amortization. We retrieve it with the "x-to-y" key.
(3) Note that the HP 12C automatically reduces the amount stored in PV.

PROGRAM TO PRODUCE SIMPLE LEVEL PAYMENT AMORTIZATION STATEMENT WITH COUNTER

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
[f](REG)[g](END)			
[f](P/R) [f] PRGM	00	[R/S]	13 31
[f] 9	01 42 9	[RCL][PV]	14 45 13
[RCL] 0	02 45 0	[R/S]	15 31
[f][AMORT]	03 42 11	[RCL] 3	16 45 3
[STO] 1	04 44 1	[RCL][n]	17 45 11
[x≷y]	05 34	[-]	18 30
[STO] 2	06 44 2	[g][x=0]	19 43 35
[RCL][n]	07 45 11	[g][GTO] 22	20 43 33 22
[f] 2	08 42 2	[g][GTO] 01	21 43 33 01
[R/S]	09 31	0	22 0
[RCL] 1	10 45 1	[R/S]	23 31
[R/S]	11 31	[g][GTO] 22	24 43 33 22
[RCL] 2	12 45 2	[f][R/S]	

User instructions:

1. Store amount financed in [PV].
2. Store interest rate in [i].
3. Payment, signed opposite of [PV], stores in [PMT].
4. Store number of payments being amortized in Memory Register 0.
5. Store total number of payments to be amortized in Memory Register 3.

Caution: You **must** make sure that the [n] register is set to "0" before you start program operation.

Since you will always use a rounded payment (to 2 decimal places) when preparing a loan amortization schedule, you will find that the **last payment** will almost always be different than the payment amount used to prepare the schedule. To calculate the **last payment**--this is the final payment--once your calculator displays the final balance, recall [RCL] the payment [PMT], press [x≷y], followed by [-]. The sum displayed will be the last payment.

Note that this program is designed to **amortize** at nine (9) decimal places and **displays** the results at two decimal places. This methodology will produce the same loan balance you would otherwise calculate using the [FV] function. However, the results will differ from those produced by your calculator's built-in amortization function, depending upon the number of places you cause it to display when using the **built-in** function. If you want the program consistent with Hewlett-Packard's AMORT function, **delete** steps **01** and **08**. Line 20 will become new line 18, and it will read: **[g][GTO] 20.** Line 24 will become new line 22, and it will read: **[g][GTO] 20.**

OPERATION OF SIMPLE LEVEL PAYMENT LOAN AMORTIZATION PROGRAM

Problem: Prepare an amortization statement for a $100,000 loan at 12% annual interest; payments of $1,955.02 at the end of each month; and a term of 72 months. Amortize twelve payments per year, calculate the last payment (72nd), and have the calculator discontinue the process by storing "72" in Memory Register 3.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Input program; clear all registers	Enter Program f CLX f 2	0.00
Enter amount financed	100,000 CHS PV	-100,000.00
Enter number of payments being amortized each year	12 STO 0	12.00
Enter total number of payments	72 STO 3	72.00
Enter monthly interest rate	12 g i	1.00
Enter monthly payment	1,955.02 PMT	1,955.02
Amortize first year: display number of time periods amortized	R/S	12.00
Display accumulated interest	R/S	11,348.20
Display payment to principal	R/S	12,112.04
Display loan balance	R/S	-87,887.96
Amortize second year: display total number of payments amortized	R/S	24.00
Display accumulated interest	R/S	9,812.09
Display payment to principal	R/S	13,648.15
Display loan balance	R/S	-74,239.80

Note: User should perform calculation up to **n = 60**

Amortize final (6th) year: display total number of payments amortized	R/S	72.00
Display accumulated interest	R/S	1,456.33
Display payment to principal	R/S	22,003.91
Display loan balance	R/S	0.08
Calculate last (72nd) payment	RCL PMT x≷y -	1,954.94
Verify amortization completed	R/S	0.00

Note: To start a **new** amortization calculation after completing the one you were performing, you should turn the calculator off, and then on--by pressing the **[ON]** key twice--or press **[g] [GTO] 00.** Both methods will take the program back to the first line, therefore ready to start a new problem.

COMPUTING A FIXED MONTHLY PAYMENT WITH A BALLOON PAYMENT LOAN

Discussion: A useful and very straightforward technique sometimes employed by the author in structuring a seller-financed sale is to integrate a balloon payment (when acceptable to both parties) into the loan payment stream. This technique can be beneficial to both parties (Seller/Purchaser) insofar as it enables the purchaser to make smaller payments during the balloon's term and as well it may provide a higher yield to a seller who might otherwise not need the enhanced payments.

A balloon payment can be structured at any time during the loan's term. Though it is easiest to have the balloon payment fall due with the last payment in the loan's term, we can, as well, readily structure the balloon to occur **interim** (indeed, at any point in time) to--or concurrent with--the expiration of the loan term.

Problem: Buyer will acquire an income producing business with seller financing. The 20 year monthly payment loan is for $400,000 at 11% annual interest with a balloon in the amount of $100,000 due at the expiration of the term. Compute the monthly payment.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter loan term	240 n	240.00
Enter monthly interest rate	11 g i	0.92
Enter loan amount, negative-signed	400,000 CHS PV	-400,000.00
Enter balloon payment due with loan's last payment	100,000 FV	100,000.00
Compute, round, and reenter payment	PMT f RND PMT	4,013.23
Compute adjusted balloon payment	FV	100,001.60

Comment: Building a balloon payment directly into the loan payment system resulted in a $115.52 per month reduction in the purchaser's payment. Though this amount may not appear significant, the difference in payment amounts grows substantially as the term decreases. For example, if the term is set to 15 years, the monthly payment differential in the above example becomes $219.93. And, with a 10 year term, the payment differential becomes $460.83.

In effect, what happened is that the balloon payment is treated as a **future value** and is then **discounted** back to time period zero (the inception of the loan) at the agreed upon monthly interest rate. That number is effectively deducted from the loan's original amount, and the payment is, as well, effectively based upon the reduced (discounted) loan amount. Understanding this concept is important if you want to structure--in total confidence--loans with interim balloon payments. The next example, along with its proof, will make this concept perfectly clear.

Problem: Using the data from the previous example, compute the loan payment on the assumption that the balloon payment is due along with the 60th payment.

Solution method: When solving these types of problems you should first discount the balloon payment back to "time period zero", being the inception of the loan. This procedure is the same whether you have one balloon payment or many. It is all the same. Just treat the balloon payment as the Future Value (FV) and discount it back to its Present Value. Then, reduce the loan amount (the "real" Present Value) by the discounted present value of the balloon payment (or payments). Let's solve the problem.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter number of months until balloon payment due	60 n	60.00
Enter monthly interest rate	11 g i	0.92
Enter balloon payment as a FV	100,000 FV	100,000.00
Compute discounted present value of the $100,000 balloon payment	PV	-57,839.72
Add loan amount to negative-signed discounted PV of the balloon payment	400,000 +	342,160.28
Enter discounted amount as loan's PV	CHS PV	-342,160.28
Clear future value register	0 FV	0.00
Enter loan term	240 n	240.00
Compute monthly payment	PMT	3,531.74
Round and reenter payment into PMT	f RND PMT	3,531.74
Compute residual at 240th payment	FV	-1.12
Compute last (240th) payment	RCL PMT +	3,530.62

Payment Schedule Summary:

PAYMENT NUMBER	AMOUNT	DESCRIPTION
1 - 59	$3,531.74	Regular payment
60th	$103,531.74	Regular payment + $100,000 balloon
61 - 239	$3,531.74	Regular payment
240th	$3,530.62	Final-adjusted regular payment

Proof

To prove the accuracy of the payment schedule we will use the HP 12C's unequal cash-flow function. What we are looking for is to prove that the internal rate of return (IRR) of the above payment schedule indeed produces the same yield as the loan's annual interest rate (11%).

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers	f CLX	0.00
Input loan amount as initial flow	400,000 CHS g CFo	-400,000.00
Input first group regular payments	3,531.74 g CFj	3,531.74
Input number of payments	59 g Nj	59.00
Input 60th payment (regular payment plus $100,000 interim balloon)	103,531.74 g CFj	103,531.74
Input third group of payments	3,531.74 g CFj	3,531.74
Input maximum number of payments	99 g Nj	99.00
Reenter third group regular payments	3,531.74 g CFj	3,531.74
Input remaining payments in group (179 - 99 = 80)	80 g Nj	80.00
Input final (240th) payment	3,530.62 g CFj	3,530.62
Compute monthly yield (IRR)	f IRR	0.92
Convert to annual yield	RCL g i	11.00
Show all decimal places	f 9	11.00000001

Comment: Interim balloon payments are, admittedly, not common insofar as loans sold in the "Secondary Market" (Fannie Mae, FHLMC, etc.). This, however, should not prevent you from considering these kinds of loans where their use would be advantageous to the purchaser and as well acceptable to the seller. Being able to give a buyer/borrower a start-up cash-flow "boost" or "injection" by maintaining his/her/its periodic payments at a lower level can only benefit your production and the buyer's chances at success in the acquisition.

The author has used these kinds of techniques many times in structuring loan payment systems for (admittedly strong) "anchor tenants". Like all time value of money techniques, the use of interim balloon payments may not be prominent in your financial/real estate work, but access to and familiarity with this area could make a difference in a close situation. Study this technique; master it; and use it. It works!

SALE ON LAND CONTRACT WITH BALLOON PAYMENT AND SUBSEQUENT DISCOUNTING OF CONTRACT

Problem: Assume a property is sold on a monthly payment land contract with the following terms: Contract amount, $135,000; Amortization schedule, 20 years, with a balloon payment due after 10 years; and annual interest rate of 11%. If we assume that one year into the land contract the seller-vendor agrees to assign it over to an investor who offered a discount of 12% off the then-owing balance, what is the anticipated yield to the investor?

Solution methodology: To solve this problem we will first calculate the land contract monthly payment and the balloon payment due along with the 120th monthly installment. Next, we calculate the balance owing after 12 payments have been made. This is the amount which will be **discounted** (by 12%) to the investor. Finally, we determine what interest rate (yield to the investor) mathematically ties together the known terms.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter amortization term	240 n	240.00
Enter monthly interest rate	11 g i	0.92
Enter contract amount into PV register, negative-signed	135,000 CHS PV	-135,000.00
Compute monthly payment	PMT	1,393.45
Round payment to 2 decimal places	f RND PMT	1,393.45
Compute balloon payment at 120th month	120 n FV	101,159.14
Round balloon payment and store in memory register 0	f RND STO 0	101,159.14
Compute balance after 12 payments; store in memory register 1	12 n FV STO 1	133,031.31 133,031.31
Compute discount on land contract	12 %	15,963.76
Determine cash to the seller	-	117,067.55
Enter cash-flow to seller into PV as a negative-signed number	CHS PV	-117,067.55
Set FV to balloon payment due at 120th month	RCL 0 FV	101,159.14
Set [n] register to number of payments remaining on land contract	120 ENTER 12 - n	108.00
Calculate investor's monthly yield	i	1.13
Convert to annual rate	RCL g i	**13.50**

CALCULATION OF DISCOUNT RATE WHERE YIELD SET BY INVESTOR

Problem: Using the data from the prior problem, let's assume the investor set--and the seller agreed to--a yield of 15% per annum, rather than the 13.50% yield which would have been produced by a discount rate of 12% of the contract balance. What discount is required to produce a 15% yield?

Solution methodology: Logically, since the investor requires a greater yield (15% versus 13.5%), the **discount** on the contract must be greater than 12% of the balance owing at the time the 12th payment is made. To solve the problem, all that is required is to set the interest rate register to the monthly equivalent of 15% per annum, and then solve for the present value (out-of-pocket cash-flow) which ties the known data together. We then divide the computed **discounted present value** by the balance known to be owing on the contract ($133,031.31) after 12 payments have been made. The remainder of the solution will then flow quite naturally.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter required monthly yield	15 g i	1.25
Compute cash-flow from investor	PV	-108,778.98
Recall contract balance at 12th payment	RCL 1	133,031.31
Divide (negative-signed) cash-flow from investor by contract balance at 12th month	÷	-0.82
Add "1" and multiply times "100" This is the required **discount rate**	1 + 100 x	**18.23**
Calculate **dollar amount** of discount	RCL 1 x≷y %	**24,252.33**

BEGINNING OF THE PERIOD LOAN PAYMENTS

Problem: Buyers will purchase a parcel of property on which the seller will take back a $100,000 fixed monthly payment mortgage with an annual interest rate of 10% and a term of 180 months. Compute the **beginning** of the month payments on the mortgage.

Discussion: Since the payments are being made at the beginning of the month we set the calculator's payment mode to **BEGIN.** This is done by pressing the blue **[g]** key followed by the **[7]** key. Otherwise, everything else works the same as what we do in computing an <u>end</u> of the time period payment.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and BEGIN mode	f CLX f 2 g BEG	0.00
Enter amortization term	180 n	180.00
Enter monthly interest rate	10 g i	0.83
Enter mortgage amount into PV register, negative-signed	100,000 CHS PV	-100,000.00
Compute monthly payment	PMT	1,065.72

Conclusion: The buyer makes <u>beginning</u> of the month payments of $1,065.72 throughout the term of the mortgage.

Comments: Note that when you pressed the blue [g] key followed by the [7] the calculator's **beginning status indicator** (**BEGIN**) became illuminated. Note further that the **BEGIN** status indicator <u>stayed</u> illuminated <u>throughout</u> the input stage of your solution and <u>after</u> the problem was solved. This feature, in the author's opinion, provides the user with the <u>utmost</u> in safety when performing a payment calculation, whether the payments are beginning of the time period or end of the time period.

<u>Before</u> performing a payment calculation on your HP 12C you should <u>always</u> make sure that the display shows **BEGIN** if you are computing payments that occur at the beginning of the time period. As well, <u>if</u> your payments are to be made at the **end** of the time period--which is the most common loan type you will work with--make sure that the **BEGIN** indicator is <u>not</u> illuminated.

Indeed, the author has seen numerous situations in which real estate practitioners made mistakes in structuring land contracts or mortgages with <u>beginning</u> of the month payment schedules when the contract required the payments to be made at the <u>end</u> of the month. The difference in payments over the term of a contract can be very substantial and <u>potentially</u> costly to the individual who erroneously structured it.

Question: What is the difference between a payment which occurs at the <u>beginning</u> of the month or <u>beginning</u> of the time period and one which occurs at the <u>end</u> of the month or <u>end</u> of the time period? First off, the beginning of the time period payment is <u>always</u> smaller than the end of the time period payment, unless the interest rate is zero. Numerically, the relationship between the two payments looks like this:

$$\textbf{BEGIN PAYMENT} = \frac{\textbf{END PAYMENT}}{\textbf{(1 + i)}} \quad \text{or}$$

$$\textbf{END PAYMENT} = \textbf{BEGIN PAYMENT} \times \textbf{(1 + i)}$$

Where "i" is the periodic interest rate.

Clearly, unless the interest rate is zero--which is highly unlikely--the beginning of the month--or beginning of the time period--payment will **always** be less than the end of the month--or end of the time period--payment.

Problem: Let's assume that the seller takes back a $200,000 land contract (or Deed of Trust) at 11% annual interest over a term of 180 months. In addition, the contract provides for the buyer to make a balloon payment of $50,000 at the expiration of the term. Compute the **beginning** of the month payment. Then, prove that the end of the month payment is exactly equal to the beginning of the month payment multiplied times (1 + i).

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and BEGIN mode	f CLX f 2 g BEG	0.00
Enter amortization term	180 n	180.00
Enter monthly interest rate	11 g i	0.92
Enter mortgage amount into PV register, negative-signed	200,000 CHS PV	-200,000.00
Enter balloon payment	50,000 FV	50,000.00
Compute beginning of month PMT	PMT	2,143.58
Recall monthly interest rate; divide by 100; add "1"; multiply	RCL i 100 ÷ 1 + x	1.01 2,163.23
Reset payment mode to END	g END	2,163.23
Compute end of the month PMT	PMT **PMT**	2,163.23

Conclusion: The buyer is required to make beginning of the month payments of $2,143.58. Secondly, if the payments are made at the end of the month the buyer would pay $2,163.23. The **difference** between the end of the month payment and the beginning of the month payment ($19.65) is exactly the amount which would be produced by multiplying the monthly interest rate (11/1200) times the beginning of the month payment ($2,143.58). Numerically, it looks like this:

DIFFERENCE = PERIODIC INTEREST RATE x BEGINNING OF THE PERIOD PAYMENT
= 11/1200 x $2,143.58
= .0091666667 x $2,143.58
DIFFERENCE = $19.65

PRESENT VALUE

Discussion

Most time value of money problems involve either the input or calculation of the "present value" of money. What is this concept?

Present Value should be thought of as the "existing" or "current" value of something given or exchanged for something else. If I gave you ten dollars ($10.00) for a book, we are, in effect, setting or equating the value of the book to the value of the money exchanged for it. Thus, the "present value" of the book is $10.00.

If a lending institution makes a loan of $100,000 today, at 12% annual interest over a holding period of 360 months, and receives in return the borrower's promise to pay the equivalent of $1,028.612597 per month (on the HP 12C), we can say that the present value of the series of 360 payments is equal or **equivalent to** the $100,000 exchanged therefor.

Present Value must be an **equivalent value.** This always holds true whether the transfers take place immediately, such as a hand-to-hand transfer of $10.00 for a book, or where a time interval separates the exchange of money and receipt of value in the future. The value received in the future--be it property or a series of installment payments--has an **equivalent value today.** The "equivalent value" is its present value.

Computing Present Value of a Future Amount (FV)

When we seek the present value (PV) of a future amount (FV), in effect we seek an amount which is produced by multiplying the <u>reciprocal</u> of the Future Value of One ($1.00) Dollar factor times the Future Amount we seek to **discount** to its present value. Said different, <u>the</u> Present Value of a number can be arrived at by multiplying the Present Value of One ($1.00) Dollar factor times the known Future Value. The Present Value of One ($1.00) Dollar factor is indeed the reciprocal (1/x) of the Future Value of One ($1.00) Dollar factor.

Specifically, if you know the factor for the Future Value of One ($1.00) Dollar, and if you divide that number into the number "1", you arrive at the factor known as the "Present Value of One ($1.00) Dollar". Multiplying this factor times a future value amount gives the equivalent present value. Mathematically, it looks like this:

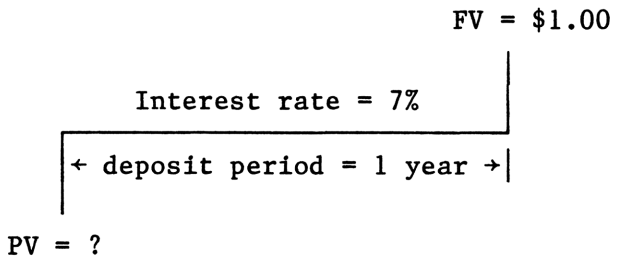
$$\text{PV of \$1.00 Factor} = \frac{1}{\text{FV of \$1.00 Factor}}$$

And, vice versa,

$$\text{FV of \$1.00 Factor} = \frac{1}{\text{PV of \$1.00 Factor}}$$

A PRESENT VALUE OF ONE ($1.00) DOLLAR PROBLEM

We want to know the amount of money which should be deposited into a savings account today in order to produce one dollar ($1.00) 12 months from now. Let's assume the account is not taxed, and further assume that the interest rate is seven percent (7%) per annum and is paid and compounded annually. How much must be deposited? A cash-flow diagram representing this problem follows below.



To solve the problem, let's perform the following keystrokes:

GOLD FIN	Clears financial registers
f 9	Sets all decimal places
1 n	Inputs 1 year as compounding period
7 i	Interest rate per compounding period
1 FV	Sets Future Value of $1.00
PV	Computes Present Value of $1.00
DISPLAYS: -0.934579439	
f PREFIX	(Press [f] and hold down the [ENTER] key)
DISPLAYS: 9345794393	

We read the above result to mean that were you to deposit 93.45794393 cents into a tax-free account earning interest at the rate of 7% per year, with annual compounding and payment of interest, your deposit would be worth one dollar ($1.00) at the end of one year. Of course, you don't work present value problems in the field by going through the mechanics of computing the present value of one ($1.00) dollar, and, as well, you cannot make a "deposit" of approximately 93½ cents into an account. Yet, those extra digits are indeed significant.

For example, if we sought the amount of the "one-time deposit" needed to produce one million ($1,000,000) dollars at the end of one year, with interest paid at the rate of 7% per annum, you simply multiply the factor for the present value of one ($1.00) dollar times the amount needed in the future ($1,000,000), and thus arrive at the amount of the required deposit.

The amount needed is:

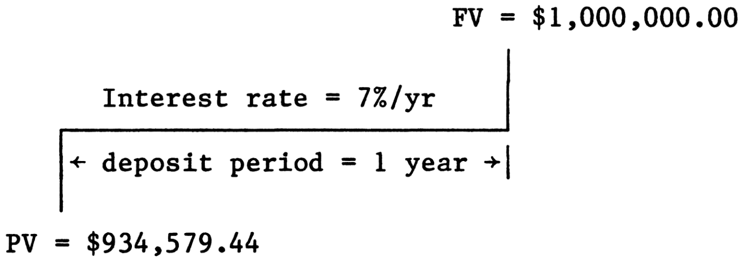
Present Value = PV of \$1.00 x \$1,000,000.00
= .9345794393 x \$1,000,000.00
Present Value = **\$934,579.44**

Let's verify the amount of the one-time deposit (\$934,579.44) needed to produce \$1,000,000 one year into the future. Again, the account draws 7% interest, compounded and paid annually. The keystrokes follow:

GOLD FIN f 2	Clears financial registers; set 2 places
1 n	Inputs 1 year as compounding period
7 i	Interest rate per compounding period
1,000,000 FV	Inputs amount needed in the future
PV	Solves for the present value

DISPLAYS: -934,579.44

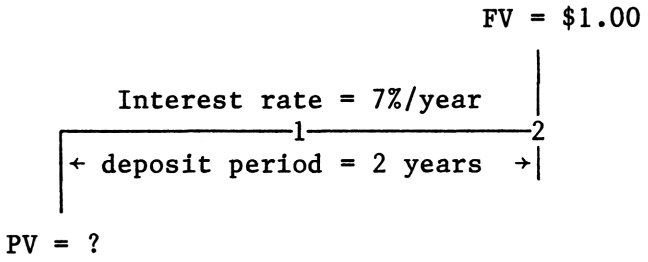
A cash-flow diagram summarizing this problem follows:



Comment: From the above diagram we readily determine that the Future Value of \$1.00 Factor (for a <u>one</u> year analysis period) is indeed "1.07". That is, \$1,000,000 divided by \$934,579.44...equals "1.07". Its reciprocal (1/x) is the Present Value of \$1.00 Factor (1/1.07 = 0.934579439...). Numerically, the problem looks like this:

PV = FV x PV of \$1.00 Factor
= \$1,000,000 x 1/1.07
= \$1,000,000/1.07
PV = \$934,579.44 (to 2 decimal places)

Problem extension: Suppose we want to know the amount of money which must be deposited into a savings account today in order to produce one (\$1.00) dollar two years from now. Again, let's assume the account is not taxed, and further assume that the interest rate is seven percent (7%) per annum and is paid and compounded annually. How much must be deposited? The cash-flow diagram follows:



To solve this problem, let's perform the following keystrokes:

GOLD FIN	Clears financial registers
f 9	Sets all decimal places
2 n	Inputs 2 years as compounding period
7 i	Interest rate per compounding period
1 FV	Sets Future Value of $1.00
PV	Computes Present Value of $1.00
DISPLAYS: -0.873438728	
f PREFIX	(Press [f] and hold down the [ENTER] key)
DISPLAYS: 8734387283	Answer to 10 places is .8734387283

Comment: We read the above result to mean that were you to deposit "87.34387283" cents into a tax-free account earning interest at the rate of 7% per year, with annual compounding and payment of interest, your deposit would be worth one ($1.00) dollar at the end of two years.

Now, if we again sought the amount of the "one-time deposit" needed to accumulate one million ($1,000,000) dollars at a future date, except we extend the period from one to two years (using the same interest rate and compounding period), you would multiply the factor for the present value of one ($1.00) dollar times the amount needed in the future ($1,000,000). The result is the amount of the required deposit.

The amount needed is:

Present Value = PV of $1.00 x $1,000,000.00
= .873438728 x $1,000,000.00 (used 9 places)

Present Value = **$873,438.73**

Comment: You should verify the above result. First, clear your registers by pressing [f] followed by [CLX]. Next, set the [n] register to "2"; set the [i] register to "7"; and set the [FV] register to "1,000,000". Then, press the [PV] key. Your display with show: **-873,438.73** at 2 decimal places.

COMPUTING PRESENT VALUE WHERE PAYMENTS ARE EQUAL

Assume you are making an offer on a land contract (or any other investment, such as a Trust Deed or Mortgage) that pays $1,028.00 per month with a remaining term of 240 months. You require a return of 12% per annum, but open your negotiations at 14% and then go to 13%. What will be your starting offers and ending offer for the 240 payment income stream?

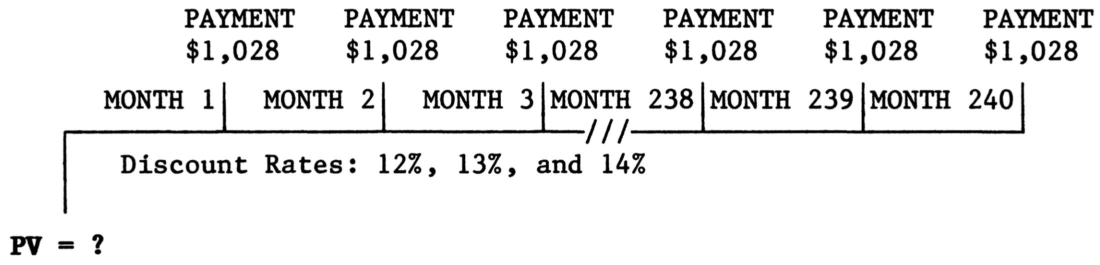
Next, calculate your offers on the basis of the investment requiring a balloon payment of $71,652.14 due in ten years.

Solution Methodology: To solve the problem we will use the Financial Registers of your financial calculator. We are dealing with an income stream which continues for a given length of time. By inputting the known data into the appropriate financial registers and pressing the [PV] key, the calculator discounts the payments back to their **present value.**

Technically, what we seek when solving these kinds of problems is the present value of an "annuity"--of a "stream"--of payments. This is quite different from solving for the present value of a future amount. Remember, in solving for the PV of a Future Value (future amount) we are dealing with just <u>two</u> cash-flows: The PV (negative signed) and the FV (positive signed). On the other hand, solving for the PV of a payment <u>stream</u> involves a series of payments which must be discounted by the calculator back to their one-time equivalent amount (or "deposit").

To work with these types of annuity problems, we make inputs into the payment [PMT] register, [i] register and [n] register, and then press the [PV] key to arrive at the solution. We do not use the [FV] key.

A timeline diagram depicting the first part of the above problem looks like this:



Note the placement of "PV" below the timeline and the PAYMENTs above the line. This is performed to keep the cash-flows consistent with the manner in which they must be entered, or the manner in which they will be displayed on your calculator. Remember: Your HP 12C follows the "cash-flow sign convention". Thus, a positive input for the payment (PMT) produces a negative signed present value (PV).

Let's solve the problem on the next page.

Solution

ORGANIZATION OF DATA

Remaining number of payments	240
Monthly payment	$1,028.00
Negotiated yields	14%, 13%, and 12%
Offers	? ? ?

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Calculate first offer:		
Input remaining number of payments	240 n	240.00
Input required monthly yield	14 g i	1.17
Input monthly payment	1,028 PMT	1,028.00
Compute first offer	PV	-82,668.50
Calculate second offer:		
Input required monthly yield	13 g i	1.08
Compute second offer	PV	-87,745.08
Calculate final offer:		
Input required monthly yield	12 g i	1.00
Compute final offer	PV	-93,362.36
Reset [n] to 120 months	120 n	120.00
Input balloon payment	71,652.14 FV	71,652.14
Input required monthly yield	14 g i	1.17
Calculate alternative offer	PV	-84,021.74
Input second monthly yield	13 g i	1.08
Calculate alternative offer	PV	-88,514.19
Input final monthly yield	12 g i	1.00
Calculate final alternative offer	PV	-93,362.36

Discussion

Using the financial registers to discount a fixed payment stream works well and is within the design of this very powerful function. However, if we were discounting an income or payment stream with variable payments, things get a little more complex. It is the unequal payment stream problems that one customarily comes across in commercial real estate investment analysis. These kinds of problems lend themselves more so to solution using the unequal cash-flow functions (NPV & IRR) of your financial calculator.

COMPUTING PRESENT VALUE WHERE PAYMENTS ARE UNEQUAL

In order to calculate the present value of a series of **unequal payments** we use the calculator's Net Present Value (NPV) function. The study of this function is part of the study of Internal Rate of Return (IRR). The two functions are very useful in the study of income producing property.

Problem: An investor is evaluating an income producing property and estimates the following cash-flows:

1st year NOI (CF1)	-$500.00
2nd year NOI (CF2)	$5,625.00
3rd year NOI (CF3)	$6,875.00
4th year NOI (CF4)	$7,375.00
5th year NOI (CF5)	$7,500.00
5th year sale price (less 6% commission)	$136,170.21 ← Also included in CF5

If the required annual discount (or yield) rate is 14%, what should be offered by the investor seeking to acquire this property? We are not taking into consideration the tax implications of this problem. The core issue addressed here is simply how to use the calculator to solve the problem.

Discussion

Note that in traditional real estate (and appraisal) analysis operating cash-flows are always assumed to occur at the end of the year (EOY). This is not to say that cash-flows cannot be analyzed on a monthly or other basis. Yet, it is convenient to use annual cash-flows, and this practice is followed throughout this book in the interest of keeping consistent with the practices you will experience in most real estate education programs.

Since the cash-flows in the problem are irregular--which is pretty much the case with actual operating flows from real-life properties--they do not neatly fit within the realm of the kinds of things we generally solve with the financial register keys. Indeed, we could solve unequal cash-flow problems with these keys, but that would be tedious, at best.

The flows are **unequal** and therefore fit well within the capabilities of the HP 12C's Unequal Cash-Flow Function. Here we use the CFo input-function to accept the out-of-pocket cash-flow or beginning of the time period cash-flow from an investment. Type the amount; press CHS; then press [g], [CFo].

We use the CFj input-function to enter the amount of the unequal cash-flows which are equal within an uninterrupted time span. For example, if there were 3 cash flows in succession of $10,000 each, we could enter the amount of the flow ($10,000) into the appropriate CFj register and then tell the calculator the number of times this flow occurs. Type the amount of the cash-flow (make sure it is signed correctly), then press [g], [CFj].

Use the Nj register to enter the number of times a cash-flow occurs (up to a maximum of 99) within a cash-flow group. Type the number, then [g], [Nj].

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 places	f CLX f 2	0.00
Enter required annual yield	14 i	14.00
Enter cash-flows 1 through 4:		
First year NOI	500 CHS g CFj	-500.00
Second year NOI	5,625 g CFj	5,625.00
Third year NOI	6,875 g CFj	6,875.00
Fourth year NOI	7,375 g CFj	7,375.00
Calculate year of sale cash-flow:		
Enter final year NOI	7,500 ENTER	7,500.00
Enter sales price	136,170.21 ENTER	136,170.21
Deduct 6% sales commission	6 % -	128,000.00
Add NOI to net sales price	+	135,500.00
Enter year of sale cash-flow	g CFj	135.500.00
Calculate offer (net present value)	f NPV	83,271.13

Problem: Assume the Seller counters with an offer based upon a discount of 12%. Compute the Seller's counter offer.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter counter offer discount rate	12 i	12.00
Compute Seller's counter offer	f NPV	90,504.56

Problem: Let's assume the following changes in the projected cash-flows: First year NOI is "-$1,000.00", and the Second year NOI ($5,625.00) is projected to occur as well in the third year. Compute the net present value (NPV) of the cash-flows when discounted at 12%. (Remember: Since we did not enter an out-of-pocket cash-flow into the CFo register, the computed NPV will indeed be the investment's present value.)

Finally, compute the IRR on an assumption that the investor's cash-outflow for the property is $85,000.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Change 1st year cash-flow	1,000 CHS STO 1	-1,000.00
Change 3rd year cash-flow	RCL 2 STO 3	5,625.00
Compute NPV	**f NPV**	**89,168.41**
Enter out-of-pocket cash-flow	85,000 CHS STO 0	-85,000.00
Compute IRR	**f IRR**	**13.14**

Problem: A property is sold and the seller takes back a monthly payment land contract (mortgage or deed of trust) with the following terms: Contract amount, $100,000; Amortization schedule, 15 years; and annual interest rate of 9%. Assume the seller assigns over to an investor the right to receive the first 60 payments of the land contract (or mortgage or deed of trust) at a price sufficient to provide the investor with a yield of 14% per annum. Calculate the amount paid by the investor.

Solution methodology: We first calculate the monthly payment needed to amortize $100,000 at 9% interest over 180 months. We then discount 60 payments at 14% annual yield. This produces the amount paid by the investor.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter amortization term	15 g n	180.00
Enter monthly interest rate	9 g i	0.75
Enter contract amount into PV register, negative-signed	100,000 CHS PV	-100,000.00
Compute monthly payment	PMT	1,014.27
Round payment to 2 decimal places	f RND PMT	1,014.27
Enter number of payments purchased by the investor into [n] register	60 n	60.00
Investor's required monthly yield	14 g i	1.17
Compute cash-flow from investor	PV	-43,590.30

Comment: The investor is required to pay $43,590.30 if he purchases the right to receive the first 60 installment payments under the land contract. This problem presented a straightforward case of discounting at a yield which is different from that used to produce the required monthly payment.

Problem extension: What is the seller's overall yield on the contract over its complete term?

Solution methodology: To solve this problem (in an efficient manner) we must use the unequal cash-flow function because the payments expected on the contract are unequal. Indeed, though the investor receives the first 60 payments on the contract we still view these payments as a form of "payment" to the seller, except the "payments" are zero dollars. That is, the first 60 payments are **skipped** as to the seller.

Therefore, what we have is a payment stream in which the first 60 payments to the seller are **skipped**, followed by 120 payments of $1,014.27. The yield to the seller must be less than the 9% he started with because he will be receiving 60 less payments than would otherwise be dictated by the land contract's term. Note as well that since the 60 payments sold to the investor were sold at a price sufficient to produce a 14% yield, the proceeds of this sale (assignment), being $43,590.30, are not sufficient to offset the loss of the first 60 regular payments.

Finally, since the seller receives $43,590.30 as an up-front payment from the investor at the inception of the land contract, his out-of-pocket cash-flow is not $100,000, but rather is $56,409.70 ($100,000 - $43,590.30). Thus, the seller's actual out-of-pocket cash-flow will be input into your calculator's CFo register. We then enter 60 skipped "payments" of $0.00 into the calculator, followed by a total of 120 payments of $1,014.27.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Input investor's payment into STACK	43,590.30 ENTER	43,590.30
Subtract value of land contract	100,000 -	-56,409.70
Input seller's out-of-pocket cash-flow at inception of contract	g CFo	-56,409.70
Input zero for skipped regular payment on the land contract	0 g CFj	0.00
Input 60 skipped payments	60 g Nj	60.00
Input regular payment	1,014.27 g CFj	1,014.27
Input 99 regular payments	99 g Nj	99.00
Reenter regular payment	[R↓] g CFj	1,014.27
Input remaining term of contract (180 - (60 + 99) = 21)	21 g Nj	21.00
Calculate seller's monthly yield	f IRR	0.66
Convert to annual yield	RCL g i	7.94

Comment: The seller receives an overall yield of 7.94% on the land contract. The overall yield must be lower than the contract rate (9%) because we discounted the first 60 payments at 14% annual interest.

For practice, you should discount the first 60 payments to yield the contract rate of interest, 9% annually. You will determine this amount to be $48,860.81. Then, assume that the seller receives this amount as an up-front payment at the closing of the sale. This leaves him with an out-of-pocket cash-flow of $51,139.19 ($100,000 - $48,860.81). Next, input "-51,139.19" into Memory Register 0. Then, set your [n] register to "3" (because we are working with three cash-flow groups). Finally, calculate the IRR. You will determine the IRR to be 9.000% to three decimal places.

For further practice, discount the first 60 payments on the contract to yield 6% annual interest. Then calculate the seller's overall yield on the contract. (Ans: 9.79%)

Problem: Contract amount, $200,000; Amortization schedule, 20 years; annual interest rate, 8.5%; and a balloon payment due along with the 120th payment. Assume that 12 months into the contract the seller assigns payments 13 through 48 over to an investor who requires a yield of 13% per annum on the acquisition. Calculate (1) the monthly payment on the contract; (2) the balloon payment; (3) the amount paid by the investor for the 36 payments; and (4) calculate the overall yield to the seller.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter amortization term	20 g n	240.00
Enter monthly interest rate	8.5 g i	0.71
Enter contract amount into PV register, negative-signed	200,000 CHS PV	-200,000.00
Compute monthly payment	**PMT**	**1,735.65**
Round payment to 2 decimal places	f RND PMT	1,735.65
Balloon payment due at 120th month	120 n	120.00
Calculate balloon payment	**FV**	**139,986.98**
Clear FV register	0 FV	0.00
Enter number of payments purchased by the investor into [n] register	36 n	36.00
Investor's required monthly yield	13 g i	1.08
Compute cash-flow from investor	**PV**	**-51,512.21**
Seller's out-of-pocket cash-flow	200,000 CHS g CFo	-200,000.00
Input regular monthly payment	1,735.65 g CFj	1,735.65
Input 11 payments	11 g Nj	11.00
Bring back regular monthly payment	[R↓]	1,735.65
Add investor's payment; input as 2nd cash-flow group	51,512.21 + g CFj	53,247.86
Input zero for skipped regular payment on the land contract	0 g CFj	0.00
Input 36 skipped payments	36 g Nj	36.00
Input regular payment	1,735.65 g CFj	1,735.65
Input remaining number of regular payments, less final cash-flow	71 g Nj	71.00
Bring back regular payment	[R↓]	1,735.65
Add balloon payment and input	139,986.98 + g CFj	141,722.63
Calculate seller's monthly yield	f IRR	0.68
Convert to seller's annual yield	**RCL g i**	**8.22**

HOME BUYER INCOME QUALIFICATION METHODOLOGY

The basis for buyer qualification has its roots in the concept of mortgage loan underwriting. To **underwrite** a real estate loan a lender must determine (1) that the borrower has the ability to pay the debt, (2) is willing to pay the debt, and (3) the property to be pledged as collateral is sufficient security to assure repayment of the mortgage in the event of purchaser default. The process is, for sure, a speciality unto itself--to much for any book, other than one dedicated solely to mortgage loan underwriting.

In the paragraphs which follow we touch upon a number of the financial and computational issues which impact the process of home buyer income qualification in a <u>conventional</u> mortgage loan setting. By a "conventional" mortgage loan we mean a loan which is made by an institutional lender (bank, mortgage loan company, etc.) and is <u>not</u> insured or guaranteed by a governmental agency, such as The Federal Housing Administration (FHA).

DOWN PAYMENT REQUIREMENTS

Generally, residential real estate buyers must make a minimum down payment of 5% from their <u>own</u> cash. In accord with customary underwriting guidelines, the lender must verify that the down payment in fact came from the borrower's personal savings or other liquid assets. The balance of a larger down payment may come from sources <u>other than</u> the borrower's personal assets, such as a gift, trade equity, rent credit, etc. In addition, when the down payment is 20% or <u>more</u>--hence, the LTV ratio is 80% or <u>less</u>--the whole down payment may come from a gift from a relative (spouse, parent, stepparent, legal guardian, grandparents, brother, sister, or child).

Note that the property seller may take the borrower's exiting real estate, or an asset other than real estate, in trade as part of the down payment, as long as (1) the borrower has made at least a 5% cash down payment and (2) his or her equity contribution for the traded property is a true value consideration that is supported by a current, full appraisal.

CUSTOMARY LENDING RATIOS

In making home buyer income qualification decisions, the mortgage lender, investor, and real estate practitioner must take into consideration three lending ratios which are customarily used to assist decision makers in determining whether a buyer income-qualifies. The ratios are setforth below:

1. Loan-to-Value Ratio (LTV);
2. Basic Housing Expense-to-Gross Income Ratio; and the
3. Total Debt-to-Gross Income Ratio.

THE LOAN-TO-VALUE RATIO

The loan-to-value ratio (LTV) addresses the relationship between the mortgage applied for (or taken) and the property's value (primarily defined as the lower of the sales price or appraised value). The LTV ratio is arrived at by dividing the amount of the mortgage loan by the value of the property.

The lower the ratio--such as 80%, 75%, etc.--the greater the equity cushion to the lender; the higher the ratio, the greater the risk assumed by the lender. Generally, you will find the lender's maximum for this ratio falls within a range of 80% to 97% of a property's sales price or appraised value.

For example, assume a borrower secures a $95,000 mortgage on a home which sold for $118,750, but appraised at $125,000. The LTV ratio is 80% ($95,000/$118,750), not 76% ($95,000/$125,000). Note that it is customary to round the result of this calculation up to the nearest whole number. Therefore, a LTV ratio of 89.2% would be rounded to 90%; a LTV ratio of 85.3% would be rounded to 86%, and so forth.

THE BASIC HOUSING EXPENSE RATIO

Lenders qualifying the residential buyer for a mortgage loan limit the buyer's Basic Housing Expense Ratio to 25-33 percent of the buyer's stable monthly gross income. Monthly Basic Housing Expense includes the principal and interest payment on the mortgage loan, plus escrow deposits (or direct payments) for prorata real estate taxes and homeowner's insurance (liability, fire, etc.). This is commonly known as "PITI". In addition, the Basic Housing Expense includes any owner's association dues (including utility charges that are attributed to the common areas), ground rent, and payments for special assessments. Note that in some areas estimated utility and maintenance costs may be considered as part of the Basic Housing Expense.

In addition, private mortgage insurance (PMI) is part of this expense when the LTV ratio exceeds 80% of the price or appraised value. Basically, PMI carriers--such as Mortgage Guaranty Insurance Company ("MGIC") and General Electric Mortgage Insurance Companies ("GE")--write insurance which enables lenders to bring the risks of high LTV ratio loans into mathematically predetermined ranges. You should become familiar with the most common PMI loan underwriting standards used by lenders in your area.

THE TOTAL DEBT RATIO

For an upper limit, most lenders require the borrower's Total Debt Ratio---that is, total monthly installment payments, including the Basic Housing Expense components--to be a maximum of 38% of stable monthly gross income. As with just about everything, there are exceptions. Lenders can accept Total Debt Ratios as high as 41% of the borrower's gross monthly income, though a number of "compensating factors" must exist.

The borrower's monthly total obligations generally consist of the sum of:

1. The monthly basic housing expense (as detailed above).
2. Monthly payments on installment and revolving debt that extend beyond 10 months (though often lenders follow different requirements and may include debts which are expected to be outstanding for as little as one month.)
3. Monthly mortgage payments on any non-income producing real estate.
4. Monthly child support, alimony, or spousal maintenance payments.

COMPENSATING FACTORS FOR HIGHER QUALIFYING RATIOS

There are numerous underwriting considerations which justify a lender's use of a higher Basic Housing Expense Ratio or a higher Total Debt Ratio, or both. Generally, to go beyond 28% for the Basic Housing Expense Ratio or 36% for the Total Debt Ratio, one or more of the following conditions must exist:

1. The buyer makes a larger down payment toward the price of the property. Generally, this means the down payment exceeds 20% of the purchase price. The higher the down payment, the greater the likelihood of exceeding the customary 28%/36% (Basic/Total Debt) ratios.
2. The property qualifies as an "energy efficient" home. What is required here is that the home be certified in accord with the HUD or Fannie Mae or FHLMC guidelines. This certification is "good" for up to an additional 2% in the housing/debt expense ratios.
3. The buyer has demonstrated the ability to save money and has a good credit history or is free of debt and appears capable of remaining debt-free. There are no real or quantifiable rules or guidelines here, other than to suggest that if one has been prudent in the management of his or her financial affairs and is debt-free, higher lending ratios are a strong possibility.
4. The buyer has strong potential for greater earnings and employment advancement because of his or her education or job training.
5. The buyer has considerable savings or net worth such that his or her ability to repay the mortgage loan can be considered more secure. Again, there are no quantifiable rules or guidelines here. Basically, if the buyer has considerable cash or other liquid assets in relationship to the price or value of the home, and if he or she appears capable of maintaining their current level of solvency, they likely will qualify for higher ratios--possibly as high as 41% for the Total Debt Ratio.
6. The buyer has demonstrated the ability to apply a greater portion of income to basic needs like housing expenses.

ALL LENDING RATIOS MUST BE SATISFIED TO QUALIFY THE BUYER

To qualify a buyer for a mortgage loan, both the LTV ratio and the two expense-to-income ratios (Basic & Total Debt) must be satisfied. That is:

1. The ratio of the mortgage loan to the value of the property must fall within the lender's specified guidelines for the risk incurred.
2. The ratio of "PITI" (or "PITI" plus "other" expenses, such as PMI, special assessments, association dues, etc.)-to-gross income must not exceed 28-33% (subject to the compensating factors exceptions).
3. The Total Debt Ratio must not exceed 36-38% of the buyer's gross income (subject to certain compensating factors exceptions).

APPLYING THE LENDING RATIOS

To illustrate, assume a buyer has gross monthly income of $6,462.46 and makes application for a $150,000 mortgage on a home selling for $190,000. In order to bypass a requirement for private mortgage insurance (PMI), the lender permits a maximum LTV ratio of 80%. It also limits the PITI to 29% of gross income, and further limits the Total Debt Ratio (PITI + consumer installment debts) to 37% of the borrower's gross income.

From the above information, we calculate the LTV ratio and the dollar limits for the borrower's obligations as follows:

(1)
$$\text{LTV RATIO} = \frac{\text{MORTGAGE LOAN}}{\text{PURCHASE PRICE}} \times 100$$

$$= \frac{\$150,000}{\$190,000} \times 100$$

$$\text{LTV RATIO} = 78.95\% = 79\% \qquad \text{(always round the LTV up)}$$

Since the computed LTV ratio is less than 80%--thus requiring the buyer to make a down payment of $40,000 ($190,000 - $150,000)--the lender's guideline for this ratio is satisfied.

Next, compute the dollar limits for the Basic Housing Expense Ratio and the Total Debt Ratio.

(2) MAXIMUM MONTHLY PAYMENT FOR PITI = 29% x MONTHLY GROSS INCOME

= .29 x $6,462.46

MAXIMUM PITI PMT = $1,874.11

(3) MAXIMUM MONTHLY PAYMENT FOR PITI + CONSUMER DEBT = 37% x MONTHLY GROSS INCOME

= .37 x $6,462.46

MAXIMUM TOTAL DEBT PMT = $2,391.11

From the above calculations we conclude the borrower's maximum monthly payment for PITI (including any owners association dues, prorata payments on special assessments, etc.) would be limited to $1,874.11, and the maximum monthly payment for Total Debt (includes the "PITI" plus monthly installment debts, child support and alimony, etc.) would be limited to $2,391.11.

Note: The above example is just that: It is an "example", and therefore may not be fully representative of the lending ratios customarily used in your locality. As well, in practice, the ratios are always open to negotiation, and thus you should never take any one ratio as controlling. In a close call, always make a further investigation of the buyer's financial status in order to ascertain whether or not he or she could come within an exception which may allow for application of higher lending ratios.

REAL ESTATE TAXES

Real Estate taxes are generally set by state, county, or municipal government, and are determined by use of a rate expressed in dollars and cents or as a percent. Tax rates in dollars and cents are usually specified as a rate per $1,000 or $100 of assessed valuation, while rates expressed as a percent are applied as a percentage of a property's sales price or fair market value for assessment purposes.

COMPUTING THE ASSESSED VALUE

The assessed value of real property is customarily determined by multiplying a specific tax assessment ratio times the sales price or market value of the property as appraised for tax purposes. For example, if a property sold for or appraised at a market value of $150,000, and if assessed value is determined on the basis of a tax assessment ratio equal to 50% of market value, then the assessed value for tax purposes would be $75,000 (50% of $150,000).

COMPUTING THE TAX ASSESSMENT

The amount of tax imposed against taxable real estate is determined by multiplying a given tax rate times the property's assessed value. For example, if the assessed value of a property is $75,000, and if the applicable tax rate is $50 per $1,000 of assessed value (which is equivalent to $5.00 per $100 of assessed value), the tax assessment would be computed as follows:

$$\text{TAX ASSESSMENT} = \text{TAX RATE} \times \frac{\text{ASSESSED VALUE}}{\$1{,}000}$$

$$= \$50 \times \frac{\$75{,}000}{\$1{,}000}$$

$$= \$50 \times 75$$

$$\text{TAX ASSESSMENT} = \$3{,}750 \quad \text{(per year)}$$

On the other hand, we can look at the problem in this fashion:

$$\text{TAX ASSESSMENT} = \frac{\text{TAX RATE}}{\$1{,}000} \times \text{ASSESSMENT RATIO} \times \text{MARKET VALUE}$$

$$= \frac{\$50.00}{\$1{,}000} \times 50\% \times \$150{,}000$$

$$= .05 \times .5 \times \$150{,}000$$

$$\text{TAX ASSESSMENT} = \$3{,}750 \quad \text{(per year)}$$

Instead of working with an assessment ratio and rate per $1,000 of assessed value, let's assume that the applicable tax rate was 2.5% of a property's sales price or market value. If we again assume a market value of $150,000, the tax assessment would be computed as follows:

TAX ASSESSMENT = TAX RATE x SALES PRICE OR MARKET VALUE

= 2.5% x $150,000

= .025 x $150,000

TAX ASSESSMENT = $3,750 (per year)

SUGGESTED METHOD FOR HANDLING UNDER ASSESSMENTS

An issue which sometimes arises in home buyer income qualification problems concerns the possibility of an under (or over) assessment of a property by a taxing authority. Clearly, if you are qualifying a residential buyer in a locality where you expect the assessing authority to under assess the property, then, the buyer's anticipated monthly payment for PITI (or PITI plus "other" expenses) will be less than that which the sales price, mortgage inputs, insurance and "other" expenses would otherwise dictate. In these cases, since the buyer's anticipated monthly payment will be lower, the annual income needed to qualify the buyer should also be lower.

To compensate for an anticipated under assessment of a property, the user can adjust the tax input into the income qualification programs as follows:

1. For jurisdictions which use ASSESSMENT RATIOS and TAX RATES PER $1,000 of ASSESSED VALUE, adjust either the assessment ratio or the tax rate per $1,000. **Do not adjust both.**

   For example, assume the assessment ratio is 50% of the sales price or fair market value and the tax rate is $50 per $1,000 of assessed value. Let's further assume the taxing authorities typically under assess by 10%. In this case, we adjust the assessment ratio as follows:

   Adjusted Ratio = Assessment Ratio x (100% - % Under Assessment)

   = 50% x (100% - 10%)

   Adjusted Ratio = 45%. (The adjusted assessment ratio)

2. On the other hand, if the taxing authority uses a tax ratio, simply multiply the ratio times 100% less the percentage of under assessment.

   For example, let's assume a tax ratio of 2% of sales price or market value, and further assume a traditional under assessment by 10%. In this case, we adjust the tax ratio as follows:

   Adjusted Tax Ratio = Tax Ratio x (100% - % Under Assessment)

   = 2% x (100% - 10%)

   Adjusted Tax Ratio = 1.8%

PRIVATE MORTGAGE INSURANCE (PMI)

As a matter of policy, the secondary market (FNMA, FHLMC, etc.) considers loans with LTVs in excess of 80 percent to be essentially "high risk". Clearly, the higher the LTV, the lower the buyer/borrower's equity in the subject property, and therefore the greater the risk to the lender in cases of default. These kinds of higher risk loans gave rise to the need for lender protection beyond the traditional loan qualification factors, like the buyer's income, asset base, and perceived desirability of the property.

Private Mortgage Insurance (or "PMI") is written by private companies and typically covers, on the low end, from 12 to 17 percent, and on the high end, from 25 to 30 percent of a lender's insurable risks associated with a high LTV loan. The insurable risk includes the mortgage balance at the time of default, plus accumulated interest and associated loan foreclosure costs, such as attorneys fees, court costs, and property preservation expenses.

There are numerous variations in PMI coverages and policies, much too many for any book, short of one dedicated strictly to PMI underwriting. However, in general, many PMI companies have "Standard Coverages" plans. In the table below we give examples of the premium charges associated with typical standard plans for 30 percent, 25 percent, 17 percent, and 12 percent coverage on 30 year loans at the given LTVs. (The table is by no means exhaustive!)

Standard Coverages
Owner Occupied Properties - Annual Premiums

FIXED PAYMENT MORTGAGES

Coverage	LTV	Reduces Exposure To	1st year Premium Refund	1st year Premium No Refund	Constant/ Declining Renewals
30%	95%-90.01	67%	1.50%	1.45%	0.49%
22%	95%-90.01	75%	1.00%	0.95%	0.49%
25%	90%-85.01	68%	0.65%	0.60%	0.34%
17%	90%-85.01	75%	0.40%	0.35%	0.34%
12%	85% & Under	75%	0.29%	0.24%	0.29%

* Data taken from "MGIC" ALL STATES rate card (except SC, dated 3/97.)
Notes: Constant Renewals: The renewal rate is applied to the original insured loan balance for years 2 through 10. For years 11-through-term, the renewal rate is reduced to .25%. Declining Renewals: The renewal rate is applied to the outstanding insured loan balance for years 2-through-term.

Determining the PMI premium costs

Problem: Assume a borrower obtains a $50,000, conventional thirty year, fixed rate, fixed payment mortgage loan with an 85.5% LTV ratio. First, since the LTV is in excess of 80%, the loan requires private mortgage insurance. Next, since the LTV ratio contains a fractional component--0.5%--we need to round it up to the next highest integer; that is, we must round the LTV to the next highest whole number. In this case, it is 86%. Thus, we must determine a rate for a loan with an LTV between 86% and 90%.

The next question is, who determines the amount of coverage (30%, 25%, etc.) needed on the loan and how long must it continue? The answer is that the lender (in concurrence with secondary market guidelines) is the one who decides how much insurance it requires, not the PMI company, nor the borrower. The decision is solely the lender's, and it is largely based upon its actual experience in the market and the perceived risks associated with the particular loan.

Insofar as the length of time the borrower must continue with PMI coverage, current secondary market guidelines provide that coverage must be discontinued at the borrower's request at the time the LTV drops below 80%. At the time this example was prepared, the onus of approaching the lender to request that PMI be discontinued is the borrower's responsibility. Generally, even if the borrower's LTV ratio drops below 80% on the basis of the original sales price, the lender must (and indeed should!) satisfy itself that the property is worth at least 125% of the loan balance at the time application is made to discontinue the PMI. Thus, this places the responsibility on the borrower to secure a current appraisal of the property.

The Coverage: Let's assume the lender requires 17 percent coverage on a 30 year fixed payment loan with an LTV of 86%. Looking at the table on the previous page we see that a loan with 17% coverage on an 86% LTV ratio reduces the lender's exposure to "75%". To back into what this means, assume the loan's LTV was 80%. Under this scenario, the lender's risk is just that, 80%; that is, it has 80% of the value of the property into the transaction. On the other hand, PMI mathematically reduces the amount of the lender's exposure on a loan. In the within example, purchasing 17% coverage on a loan with an upper limit LTV ratio of 90% will indeed reduce the lender's exposure to something similar to what it would have were the loan written with an LTV ratio of 75%.

If we reduced a 90% LTV ratio loan by 17%, the resulting LTV ratio would be 74.70%, rounding to 75%. That is, take 83% (100%-17%) of 90%; the answer is 74.70%. This is basically where the 75% exposure comes from in the table.

Looking at the issue from a slightly different perspective, suppose a secondary market purchaser requires PMI on 90% LTV loans for the amount of the mortgage in excess of 75% of property value. Let's further assume that a $100,000 property is to be financed with a $90,000 loan. Therefore, PMI coverage of 17% satisfies this situation under the stated assumptions. To verify that 17% mortgage insurance coverage reduces the lender's exposure on a 90% LTV ratio loan to 75%, let's work through the following:

> Take the unpaid mortgage balance ($90,000), subtract 75% of the property value ($90,000 - 75% x $100,000 = $15,000), and divide the result by the original loan balance ($15,000/$90,000). The result (16.67%), rounded to the next highest whole number, 17%, is the amount of private mortgage insurance required to be carried by the purchaser.

The 1st Year Premium: Note the first year (refund) premium is established at "0.40%". This represents the first year premium payment on the loan. Continuing with the above example, a $50,000 loan requires a first year premium charge of $200 (0.40% x $50,000), and so on. The first year premium is usually paid in advance at the closing of the loan, or, on loans with an original LTV of 90% or less, the first year premium can be included in the insured loan balance. For example, if we financed-in the loan's first year premium, the loan amount goes from $50,000 to $50,200.

Note that under customary PMI underwriting policies, financing-in a loan's first year premium does not increase its LTV ratio. However, though the LTV ratio remains the same **for PMI underwriting purposes**, the mortgage payment (including points and origination fees) indeed increases because the loan amount has increased by the amount of the financed-in first year premium.

The Renewal Premium: In addition to the first year premium, the borrower pays an annual charge, commonly called a renewal or recurring premium. This expense is determined on an annual basis and typically paid in monthly installments along with the loan's regular monthly payment. Note further that the premium can be (1) declining (amortized) or (2) constant. If we amortize a premium, we calculate a fixed percent of the annual outstanding balance on the mortgage; that is the annual obligation for the premium. On the other hand, if we used a constant premium plan, a fixed percent is applied to the original loan balance each year to determine the annual obligation for the premium. Again, it is prorated and paid on a monthly basis.

In our example, let's elect a "constant" premium payment plan. Thus, the annual rate is "0.34%" of the loan's principal amount. Note that the principal amount of the loan includes not only the base amount ($50,000), but as well it includes the financed-in first year PMI premium, $200 in this example. Therefore, the annual renewal premium on the loan would be $170.68 ($50,200 x 0.34%), which results in a monthly payment of $14.22. This amount is paid monthly, and indeed in most, if not all cases, would start immediately with the borrower's first monthly installment payment on the loan.

Qualifying the high LTV ratio buyer

In using two of the HP 12C programs in this book to perform buyer income qualification, you can take into account PMI, both renewal premiums and financed-in rates. The process used in the programs is admittedly two-step, attributed to the complexity of the problem in relationship to the memory limits of the calculator. The programs which handle PMI accept a financed-in rate for the first year premium, such as "0.40%". The constant renewal premium rate is handled by the programs as a monthly "other" expense and therefore must be added in with similar expenses.

For example, with the **buyer qualification program** for PITI and annual income needed to qualify (page 88), you can take into account both (1) financed-in and (2) renewal premiums for PMI. The financed-in first year PMI premium rate (for loans with LTVs which do not exceed 90%) will be stored in Memory Register 9. The rate used effectively increases the loan amount, thus increasing the amount of the loan payment (for principal and interest) utilized within the program.

Further, if you input an amount for the "renewal premium" into Memory Register 7, you are indeed telling the program what the monthly expense is for renewing the PMI insurance. To arrive at this amount (as is pointed out in greater detail in the program instructions) you will first add the amount of the financed-in premium percent to the loan amount, just as we did above. For example, if the loan is for $100,000 and the first year premium is "0.40%", you add 0.40% of $100,000 and arrive at an effective loan amount of $100,400.

Knowing the effective amount of the loan we can determine the monthly obligation for the PMI renewal premium. You will do this calculation exactly as we did above: Multiply the renewal premium percentage ("0.34%", etc.) times the total amount of the loan (which may also include a charge for financed-in first year PMI if the LTV ratio does not exceed 90%). Finally, divide this amount by "12" to arrive at the monthly obligation. The result would then be stored in Memory Register 7 as an "other" expense.

As stated earlier, in cases where the buyer's LTV exceeds 90 percent, you generally cannot finance-in the first year PMI premium. Therefore, it must be paid at the closing, thus increasing the buyer's out-of-pocket expense for the purchase. Insofar as inputs into the HP 12C programs in this book, if the buyer pays the first year premium in advance, you will not make an input into the Memory Register which is dedicated to holding these rates (Memory Register 9). You will, however, make an input into the memory register which holds the monthly renewal premium expense (Memory Register 7), which amount, in this case, would solely be based upon the applicable rate (0.34%, etc.) times the amount of the base loan, divided by 12.

A word about "Monthly Premium Plans": From the standpoint of working high-level buyer income qualification problems with a financial calculator, a less onerous PMI plan is the standard "Monthly Premiums" plan, such as those offered by "MGIC". Unlike the Standard Annual examples we covered in the opening PMI section of this book, in general, Monthly Premium plans do not involve the issue of (1) paying up-front or (2) financing-in a first year premium. Rather, in the Monthly Premium plan, premiums are paid monthly, with the lender--on behalf of the borrower--electing one of two straightforward renewal payment plans. Under a **constant renewal** plan the annualized renewal rate for years 2 through 10 is the same as the first year rate (such as "0.78%" on a 30 year, 95% LTV ratio loan, where the lender's exposure is reduced to 67%) and is applied to the original loan balance, with the annualized rate later reduced to, for example, "0.20%" for years 11-through-term. On the other hand, if a **declining renewal** plan is elected by the lender, the annualized renewal rate for years 2-through-term is the same as the annualized first year rate, and it is applied to the outstanding insured loan balance, which is adjusted each year.

It is well-admitted that using a financial calculator to work with a "Standard Annual" PMI plan is pedagogically much more difficult versus the much more straightforward "Monthly Premium Plans" which do not require a separate first year premium (financed-in or paid in advance). However, since the relevant HP 12C programs in this book indeed have the capability of handling the more complex financing issue associated with the "Standard Annual" PMI plan, they were given heavy emphasis.

The following is an example of a PMI claim submission:

Original price of property	$100,000	
Original loan (10% down) (1)	90,000	
Balance due at time of default (2)		$ 88,849.58
Accumulated interest (3)		7,400.06
Subtotal		$ 96,249.64
Attorney fees (4)		2,887.49
Property taxes paid (5)		1,000.00
Hazard insurance (6)		250.00
Property preservation/maintenance (7)		165.00
Disbursements/foreclosure expense (8)		615.00
Subtotal		$101,167.13
Less:		
Escrow balance (if any) (9)		0.00
Rental & other income from property (10)		0.00
Grand total claim		**$101,167.13**

The lender's total claim is therefore $101,167.13. The PMI company has two options when a claim is submitted:

First, it can pay the insured claim up to the amount of the lender's **insured** actual losses. For example, if the PMI policy covers the top 20 percent of

Notes:

(1) Conventional loan, 360 month term; annual fixed interest rate, 10%; monthly payment $789.81.

(2) Loan default occurred after 26 payments were made.

(3) Borrower was able to remain in the property (hold over) 304 days after the 26th payment was made. Daily interest on the loan balance equals $24.3423.

(4) Attorneys fees are generally limited by the PMI carrier to 2 to 5 percent of the first subtotal on the claim. Example used 3% of $96,249.64.

(5) Taxes were $3,000 per year, and the escrow account held six months of prepaid taxes. Thus, the lender was obligated to pay four months of real property taxes. (304 days holdover/365 days per year x 12 months/year - 6 months of taxes held in escrow by the lender.)

(6) Generally paid from escrowed funds. The example, however, assumed the fund is depleted.

(7) Property maintenance (lawn care, winterizing, snow removal, etc.).

(8) Includes court filing fees, publication of the default/foreclosure in legal news, plus incidental out-of-pocket expenses.

(9) If the lender pays monies for property taxes or hazard insurance, the escrow account most likely has been depleted to zero.

(10) Any income derived from the property during the period of default goes to mitigate the PMI carrier's exposure on the loss. Example is based upon a single family owner-occupied structure; hence no rental income.

the loan, the company's maximum exposure is for twenty percent (20%) of the lender's claim. Using the above example, if we assume the lender was able to sell the property for $90,000 on the open market, the PMI carrier would be obligated to pay the difference between the claim amount--$101,167.13--and the sales price, $90,000, for a total of $11,167.13. Note that this amount is well within the lender's maximum exposure of 20 percent of the claim, $20,233.43 (20% of $101,167.13).

On the other hand, let's assume that the lender sells the property for $110,000. What are its losses? None! Therefore, the PMI carrier is not required to pay anything on this loan. Of course, this is just an example and in practice the absolute lion's share of defaulted high LTV loans with PMI coverage result in the carrier making a payment to the lender.

Secondly, the PMI carrier could elect to take over the property at the time a claim is submitted. If it elects to do this, the lender must provide it with marketable title. Under this option, the carrier would pay the lender the amount of the full claim, $101,167.13. The carrier, however, will take over the property--and thus become obligated to pay the full amount of the lender's claim--only when it sees this avenue as a means of minimizing its losses.

Computing the break-even sales price after borrower default

A question which arises in the decision process concerns how we determine the **break-even point** at which the lender can sell the property at a loss and still break-even because it receives the maximum claim allowed under the insurance coverage on the property. Indeed, selling the property involves exposure for commissions (and incidental costs), though it as well involves a positive cash-flow from the PMI carrier. How do we join all facets together to arrive at a break-even sales price? A suggested equation follows below.

Example: Using the above facts, determine what the minimum selling price for the property should be for the lender to break-even. Assume a sales commission of seven percent (7%) on the transaction. We start with the equation:

$$\text{BREAK-EVEN PRICE} = \frac{\text{CLAIM x (1 - \%PMI COVERAGE)}}{\text{1 - \% SALES COMMISSION}}$$

Where: CLAIM = lender's claim against the PMI company
%PMI COVERAGE = Amount of PMI insurance elected by the lender
% SALES COMMISSION = Real estate commission on sale of property

$$= \frac{\$101{,}167.13 \text{ x } (1 - 20\%)}{1 - 7\%}$$

$$= \frac{\$101{,}167.13 \text{ x } .8}{.93}$$

BREAK-EVEN PRICE = $87,025.49

HOME BUYER INCOME QUALIFICATION PROGRAM
Monthly PITI & Annual Income To Qualify

KEYSTROKE	DISPLAY		KEYSTROKE	DISPLAY	
[f](REG)[f][R/S] [f](PRGM)	00		g GTO 00	28	43 33 00
0	01	0	%	29	25
STO FV	02	44 15	x	30	20
RCL 1	03	45 1	÷	31	10
RCL 0	04	45 0	1	32	1
-	05	30	2	33	2
RCL 9	06	45 9	x	34	20
%	07	25	R/S	35	31
+	08	40	RCL 6	36	45 6
STO PV	09	44 13	g [x=0]	37	43 35
PMT	10	14	g GTO 00	38	43 33 00
RCL 7	11	45 7	x≷y	39	34
+	12	40	RCL 4	40	45 4
RCL 2	13	45 2	RCL 5	41	45 5
RCL 3	14	45 3	÷	42	10
+	15	40	x	43	20
1	16	1	RCL 6	44	45 6
2	17	2	1	45	1
0	18	0	RCL 5	46	45 5
0	19	0	%	47	25
÷	20	10	x	48	20
RCL 0	21	45 0	÷	49	10
x	22	20	1	50	1
+	23	40	2	51	2
R/S	24	31	x	52	20
1	25	1	+	53	40
RCL 4	26	45 4	f R/S		
g [x=0]	27	43 35			

Note: Memory used: P - 57, r - 13. Therefore, you could have an additional four lines in the **buyer income qualification program** and still have 13 storage registers available. The 13 storage registers are consistent with keys "0" through and including ".2".

Also, note that the program uses storage registers 0, 1, 2, 3, 4, 5, 6, 7, and 9. In addition, financial registers [n] and [i] are used. The storage registers used in this program are kept consistent with those used by the **Maximum Price Program.** In order to achieve this, Memory Register 8 had to be left out of this **buyer income qualification program** since it is more convenient to use storage register 8 for another purpose (storing the buyer's income) in the **maximum price** program.

Storage Registers Used:

1. Store term in [n].
2. Monthly interest rate in [i].
3. Price of property in Memory Register 0.
4. Down payment store in Memory Register 1.
5. Real estate tax, as a percentage of price, store in Memory Register 2.
6. Hazard (Homeowner's) insurance, as a percentage of price, store in Memory Register 3.
7. Basic Housing Expense Ratio store in Memory Register 4.
8. Total Debt Ratio store in Memory Register 5.
9. Monthly Consumer debts store in Memory Register 6.
10. Monthly "other" expenses (renewal premiums for PMI, utilities, etc.), store in Memory Register 7.
11. Financed-in PMI as a percentage of the base mortgage, store in Memory Register 9.

Description of the Program

Make sure the **payment mode** on your calculator is set to **END** before operating the program. If the **BEGIN** status indicator appears in the display, you must press: **[g] END** before operating the program. If preferred, make the first program line: **g END**; every line then moves ahead by one program line.

The program's operational sequence requires input of the financial variables needed to income qualify the residential real estate buyer, after which you will press the Run/Stop Key **[R/S]**. The first number displayed will be the buyer's monthly payment for PITI, including financed-in PMI, if any, plus monthly "other" expenses, such as utilities, maintenance and renewal premiums for PMI.

Pressing **[R/S]** a second time computes the annual income needed to qualify the buyer using the lender's Basic Housing Expense Ratio (typically 28% to 30% of the buyer's gross annual income). If your inputs include the buyer's monthly consumer installment debt and the lender's required Total Debt Ratio (typically 36% to 38%), pressing **[R/S]** a third time will compute the annual income needed to qualify using the higher ratio; otherwise pressing **[R/S]** causes the calculator to go back to step "00" and recalculate the monthly

PITI, and so on.

In order to calculate the annual income needed to qualify using the Total Debt Ratio, you **must** calculate (1) the buyer's monthly payment for PITI and (2) the income needed to qualify using the Basic Housing Expense Ratio. Otherwise, by leaving out monthly consumer installment debts (Memory Register 6), the program calculates PITI (etc.) and the annual income needed to qualify using the Basic Housing Expense Ratio.

Finally, a "Conditional Test" (lines 27 and 28) enables the user to limit the program's operation to computing PITI (etc.) by leaving out the Basic Housing Expense Ratio (Memory Register 4). That is, if you do not input a ratio into Memory Register 4, the program will compute monthly PITI--including "other" expenses and any financed-in PMI--and immediately afterwards returns to step "00".

Required Inputs

When computing the buyer's monthly payment for PITI (or PITI plus "other" expenses, including financed-in PMI, if any), the program uses the following inputs into the storage and financial registers:

* Number of monthly payments	[n]
* Monthly mortgage interest rate	[i]
* Price of property	STO [0]
* Down payment	STO [1]
* Tax Ratio (annual) as a percentage of sales price	STO [2]
* Estimated annual insurance expense as a percentage of sales price	STO [3]
* Estimated monthly "other" expenses (utilities, maintenance, renewal premiums for PMI, etc.)	STO [7]
* PMI financed into the base mortgage, expressed on an annual percentage basis	STO [9]

To compute the annual income needed to qualify, you must have the following additional inputs at the time you calculate PITI (etc.):

* Lender's guideline ratio for the Basic Housing Expense Ratio (typically 28% to 30% of buyer's annual gross income)	STO [4]
* Lender's guideline ratio for the Total Debt Ratio (typically 36% to 38% of buyer's annual gross income)	STO [5]
* Buyer's estimated monthly consumer installment debt payments	STO [6]

Problem 1

The following information is given:

1.	SALES PRICE OF HOME	$118,500
2.	DOWN PAYMENT	$25,000
3.	ANNUAL (FIXED) INTEREST RATE	9.75%
4.	LOAN TERM IN MONTHS (FIXED PAYMENT)	360
5.	TAX RATIO AS A PERCENTAGE OF PRICE	1.50%
6.	ESTIMATED COST OF HOMEOWNER'S INSURANCE AS A PERCENTAGE OF PRICE	.25%
7.	LENDER'S BASIC HOUSING EXPENSE RATIO	28%
8.	MONTHLY CONSUMER INSTALLMENT DEBT	$375
9.	LENDER'S TOTAL DEBT RATIO	36%

(a) Calculate the estimated monthly payment for PITI and the annual gross income needed to qualify the buyer using both housing expense ratios.

Solution (a)

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter program into your HP 12C		
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter loan term	360 n	360.00
Enter monthly interest rate	9.75 g i	0.81
Store price of home	118,500 STO 0	118,500.00
Store down payment	25,000 STO 1	25,000.00
Store annual tax ratio	1.5 STO 2	1.50
Store homeowner's insurance ratio	.25 STO 3	0.25
Store Basic Housing Expense Ratio	28 STO 4	28.00
Store Total Debt Ratio	36 STO 5	36.00
Store monthly consumer debt	375 STO 6	375.00
Compute monthly PITI	R/S	**976.12**
Compute annual income to qualify:		
Basic Housing Expense Ratio	R/S	**41,833.79**
Total Debt Ratio	R/S	**45,037.40**

Conclusion: The buyer requires a minimum annual gross income of $45,037.40 to qualify for the purchase. Both housing expense ratios must be satisfied.

Note: If the tax data is given as an assessment ratio and rate per $1,000 of Assessed Value, to convert to a tax ratio (expressed as a percent), do the following: **Assessment Ratio x Rate Per $1,000 of A.V. ÷ $1,000.**

(b) Qualify the buyer using the following changes: Price = $100,000. Down Payment = $10,000. Financed-into the mortgage PMI rate = 0.40%; annual renewal premium = 0.34% of the Total Loan Amount. **Note:** The Total Loan amount equals the Base Loan ($90,000) plus the financed-in premium of 0.40%.

Solution (b)

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Store price of home	100,000 STO 0	100,000.00
Store down payment	10,000 STO 1	10,000.00
Store financed-in PMI rate	.4 STO 9	0.40
Financed-in PMI	RCL 0 RCL 1 - RCL 9 %	360.00
Compute and store monthly PMI renewal premium	+ .34 % 12 ÷ STO 7	25.60
Compute monthly PITI	R/S	**947.77**
Compute annual income to qualify:		
Basic Housing Expense Ratio	R/S	**40,618.60**
Total Debt Ratio	R/S	**44,092.24**

Conclusion: The buyer requires a minimum gross annual income of $44,092.24 to qualify for the purchase. Both housing expense ratios must be satisfied.

Mathematics Side-Bar - Converting to Tax Ratio Method Used in Program

The real estate tax computation method used in the program must be limited to one and only one method due to the memory limitations of the HP 12C. Therefore, the method used in the buyer qualification programs in this book is the **tax ratio method** (tax as a percentage of price). Under this method we express annual taxes as a percentage of price or assessed value.

To convert to the **tax ratio** method from the common **assessment ratio** and **tax rate per $1,000 of assessed value method**, we perform a straightforward calculation. For example, if we assume an **assessment ratio** of 50% of price (or fair market value), and a **tax rate per $1,000 of assessed value** of $30.00, we readily convert to the equivalent **tax ratio** of 1.5% of price (or fair market value), as follows:

Tax Ratio = Assessment Ratio x Tax Rate Per $1,000 of A.V. ÷ $1,000

= 50% x $30 ÷ $1,000

= .50 x .03

Tax Ratio = 0.015 = 1.5% (of sale price or fair market value)

(The calculated **tax ratio** stores in Memory Register 2 of your HP 12C.)

Problem 2

The following information is given:

1.	SALES PRICE OF HOME	$175,000
2.	DOWN PAYMENT	$35,000
3.	ANNUAL (FIXED) INTEREST RATE	9.65%
4.	LOAN TERM IN MONTHS (FIXED PAYMENT)	360
5.	TAX RATIO AS A PERCENTAGE OF PRICE (ASSESSMENT RATIO OF 50%, TAX RATE PER $1,000 OF ASSESSED VALUE EQUAL TO $40.)	2.00%
6.	ESTIMATED COST OF HOMEOWNER'S INSURANCE AS A PERCENTAGE OF PRICE	.25%
7.	LENDER'S BASIC HOUSING EXPENSE RATIO	28%
8.	MONTHLY CONSUMER INSTALLMENT DEBT	$375
9.	LENDER'S TOTAL DEBT RATIO	36%

(a) Calculate the estimated monthly payment for PITI and the annual gross income needed to qualify the buyer using both housing expense ratios.

Solution (a)

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers	f CLX	0.00
Enter loan term	360 n	360.00
Enter monthly interest rate	9.65 g i	0.80
Store price of home	175,000 STO 0	175,000.00
Store down payment	35,000 STO 1	35,000.00
Store annual tax ratio	2 STO 2	2.00
Store homeowner's insurance ratio	.25 STO 3	0.25
Store Basic Housing Expense Ratio	28 STO 4	28.00
Store Total Debt Ratio	36 STO 5	36.00
Store monthly consumer debt	375 STO 6	375.00
Compute monthly PITI	R/S	1,520.67
Compute annual income to qualify:		
Basic Housing Expense Ratio	R/S	65,171.70
Total Debt Ratio	R/S	63,189.10

Conclusion: The buyer requires a minimum annual gross income of $65,171.70 to qualify for the purchase. Both housing expense ratios must be satisfied.

Note: To convert the assessment ratio (50% of PRICE or FMV) and TAX RATE PER $1,000 of assessed value ($40.00) to a **tax ratio**, you would do the following: **50 ENTER 1,000 ÷ 40 x.** Your display will show: **2.00.**

Problem 2 Continued

Assume the following changes in the problem:

1. DOWN PAYMENT	$45,000.00
2. LENDER'S BASIC HOUSING EXPENSE RATIO	30%
3. LENDER'S TOTAL DEBT RATIO	38%
4. MONTHLY "OTHER" EXPENSES (REQUIRED BY LENDER TO SECURE HIGHER RATIOS)	$75.00

(b) Calculate the estimated monthly payment for PITI plus "other" expenses and the annual gross income needed to qualify the buyer using both housing expense ratios.

Solution (b)

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Revised down payment	45,000 STO 1	45,000.00
Revised Basic Housing Expense Ratio	30 STO 4	30.00
Revised Total Debt Ratio	38 STO 5	38.00
Input monthly "other" expenses	75 STO 7	75.00
Compute monthly PITI plus "other"	R/S	1,510.49
Compute annual income to qualify:		
Basic Housing Expense Ratio	R/S	60,419.64
Total Debt Ratio	R/S	59,541.82

(c) Assume the lender's underwriting guidelines allow for the enhanced qualification ratios (30%/38%) without the need for taking into account estimated "other" expenses, such as heat, and so on. Delete "other" expenses and re-qualify the buyer.

Solution (c)

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Delete "other" expenses	0 STO 7	0.00
Compute monthly PITI	R/S	1,435.49
Compute annual income to qualify:		
Basic Housing Expense Ratio	R/S	57,419.64
Total Debt Ratio	R/S	57,173.40

Conclusion: The buyer requires a minimum annual gross income of $57,419.64 to qualify for the purchase. Both housing expense ratios must be satisfied.

Comment: When qualifying a buyer who is marginal at a twenty percent (20%) down payment, you should consider, if possible, a larger down payment sufficient to warrant the higher qualification ratios (30%/38%).

PRACTICE

		Problem #1	**Problem #2**	**Problem #3**
PRICE OF HOME	STO 0	$100,000	$150,000	$250,000
DOWN PAYMENT	STO 1	$20,000	$35,000	$75,000
TAX RATIO	STO 2	1%	50%AV/$45/$1M	1.25%
INSURANCE RATIO	STO 3	.25%	.25%	.20%
HOUSING EXPENSE RATIO	STO 4	28%	29%	30%
TOTAL DEBT RATIO	STO 5	36%	37%	38%
MONTHLY CONSUMER DEBT	STO 6	$200	$300	$400
LOAN TERM IN MONTHS	[n]	360	360	240
ANNUAL INTEREST RATE	[g][i]	8%	8.25%	7.75%

Answers:

Monthly PITI	[R/S]	$691.18	$1,176.46	$1,738.74
Housing ratio income	[R/S]	$29,621.93	$48,680.96	$69,549.73
Total debt ratio income	[R/S]	$29,705.94	$47,885.08	$67,539.26
Income needed to qualify		$29,705.94	$48,680.96	$69,549.73

		Problem #4	**Problem #5**	**Problem #6**
PRICE OF HOME	STO 0	$100,000	$150,000	$250,000
DOWN PAYMENT	STO 1	$10,000	$30,000	$50,000
TAX RATIO	STO 2	2%	40%AV/$85/$M	2.25%
INSURANCE RATIO	STO 3	.25%	.25%	.20%
HOUSING EXPENSE RATIO	STO 4	28%	28%	28%
TOTAL DEBT RATIO	STO 5	36%	36%	36%
MONTHLY CONSUMER DEBT	STO 6	$500	$600	$697
MONTHLY OTHER EXPENSE	STO 7	$25.60	$17.31	$26.10
(PMI RENEWAL PREMIUM?)		YES	NO	NO
FINANCED-IN FIRST YEAR PMI PREMIUM	STO 9	0.40%	---	---
LOAN TERM IN MONTHS	[n]	360	360	360
ANNUAL INTEREST RATE	[g][i]	7.75%	8.25%	8%

Answers:

Monthly PITI + PMI	[R/S]	$860.45	$1,375.08	$2,004.05
Housing ratio income	[R/S]	$36,876.43	$58,932.00	$85,887.68
Total debt ratio income	[R/S]	$45,348.34	$65,836.00	$90,034.86
Income needed to qualify		$45,348.34	$65,836.00	$90,034.86

Note: In problem #4 we assumed the lender requires a PMI policy which insures the top 17% of the mortgage loan. Using the MGIC "ALL STATES" "Standard Annuals" rate program for a 90% LTV ratio loan (85.01-90%), we establish the first year (refund) premium at 0.40%, and the renewal at 0.34%.

In arriving at the total for PITI, both the financed-in PMI rate (0.40%) and the renewal premium of 0.34% of the total amount financed were taken into account. The base loan is $90,000 ($100,000 - $10,000) and its base monthly payment is $644.77. Adding $360 for the financed-in first year PMI premium adds $2.58 to the P & I. Adding one-twelfth of 0.34% of the total amount financed ($90,360) increases the P & I an additional $25.60.

HOME BUYER INCOME QUALIFICATION PROGRAM
Maximum Sales Price Routine

KEYSTROKE	DISPLAY		KEYSTROKE	DISPLAY	
[f](REG)[f][R/S] [f](PRGM)	00		+	30	40
0	01	0	RCL 2	31	45 2
STO FV	02	44 15	RCL 3	32	45 3
1	03	1	+	33	40
CHS	04	16	1	34	1
ENTER	05	36	2	35	2
1	06	1	0	36	0
RCL 9	07	45 9	0	37	0
%	08	25	÷	38	10
x	09	20	RCL PV	39	45 13
1	10	1	[1/x]	40	22
+	11	40	+	41	40
÷	12	10	STO . 0	42	44 48 0
STO PMT	13	44 14	÷	43	10
PV	14	13	STO FV	44	44 15
RCL 8	15	45 8	RCL 8	45	45 8
RCL 5	16	45 5	RCL 4	46	45 4
x	17	20	x	47	20
1	18	1	1	48	1
2	19	2	2	49	2
0	20	0	0	50	0
0	21	0	0	51	0
÷	22	10	÷	52	10
RCL 6	23	45 6	RCL 7	53	45 7
-	24	30	-	54	30
RCL 7	25	45 7	RCL 1	55	45 1
-	26	30	RCL PV	56	45 13
RCL 1	27	45 1	÷	57	10
RCL PV	28	45 13	+	58	40
÷	29	10	RCL . 0	59	45 48 0

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
÷	60 10	g GTO 00	64 43 33 00
STO 0	61 44 0	x≷y	65 34
RCL FV	62 45 15	f [R/S]	
g x≷y	63 43 34		

Note: Memory used: P - 71, r - 11. Therefore, you could have an additional six lines in the **maximum price program** and still have 11 memory storage registers available. The 11 storage registers are consistent with keys "0" through and including "point zero" (**.0**).

Note that the program uses storage registers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and .0. Therefore, all available storage registers are used by the program. In addition, financial registers [n], [i], and [FV] are used.

Storage Registers Used:

1. Store term in [n].
2. Monthly interest rate in [i].
3. Down payment store in Memory Register 1.
4. Real Estate tax, as a percentage of price, store in Memory Register 2.
5. Hazard (Homeowner's) insurance, as a percentage of price, store in Memory Register 3.
6. Basic Housing Expense Ratio, store in Memory Register 4.
7. Total Debt Ratio store in Memory Register 5.
8. Monthly Consumer debts, store in Memory Register 6.
9. Monthly "other" expenses (renewal premiums for PMI, utilities, etc.), store in Memory Register 7.
10. Buyer's gross annual income, store in Memory Register 8.
11. Financed-in PMI, as a percentage of the base mortgage, store in Memory Register 9.
12. Maximum Price produced with the Basic Housing Expense Ratio may be recalled from Memory Register 0.
13. Maximum Price produced with the Total Debt Ratio may be recalled from the FV register.

Description of the Program

Make sure the **payment mode** on your calculator is set to **END** before operating the program. If the **BEGIN** status indicator appears in the display, you must press: **[g] END** before operating the program. If preferred, make the first program line: **g END**; every line then moves ahead by one program line.

To effectively operate the program you must input both housing expense ratios (Basic Housing Expense, Mem. Reg. 4, and Total Debt, Mem. Reg. 5) plus the buyer's estimated monthly consumer installment debt (Mem. Reg. 6). That is, in addition to the financial inputs needed to compute a **maximum sales price** using the Basic Housing Expense Ratio, you must input the lender's Total Debt Ratio and the buyer's monthly consumer installment debt to effectively qualify the buyer. However, if you prefer to qualify using one ratio, you must input that ratio into Memory Registers 4 and 5.

The program's operational sequence requires input of the financial variables needed to income-qualify a residential real estate buyer, except we do not input "price", because we are solving for this component, but we do input the buyer's annual gross income. After inputting the variables, you press the Run/Stop key **[R/S]**; the number displayed is the maximum sales price for which the buyer income-qualifies.

The **maximum price** displayed will be the lesser of the prices computed using the two housing expense ratios. That is, the program computes the maximum price supported by the financial inputs using both housing expense ratios, but returns the lower of the two amounts. This is consistent with accepted mortgage loan underwriting practices (FNMA, FHLMC, etc.).

After calculating the **maximum price** supported by the buyer's annual gross income, you must test to determine whether the loan-to-value ratio (LTV) is greater than eighty percent (80%). If the LTV ratio is over 80% of the computed **maximum price,** and if the buyer seeks a purchase at or near this price, a decision must be made between (1) making a larger down payment or (2) securing private mortgage insurance (PMI). (You should become familiar with the PMI underwriting rates for your locale.)

To calculate the LTV, you must first determine the **maximum price** supported by the data-inputs. You do this by making the required inputs into the program, pressing the Run/Stop key [R/S], after which the maximum affordable sales price will display in your calculator's window. Now, with the maximum price **displayed,** perform the following keystrokes:

ENTER ENTER	Pushes maximum price into the "y" and "z" STACK registers
RCL 1	Recalls the down payment
-	Calculates amount of mortgage
x≷y	Brings price into "x" register; mortgage goes to "y" register
÷ 100 x	Calculates the LTV

The result of the above calculation will be the loan's LTV ratio.

The program handles PMI premiums which are **financed-into** the base mortgage loan (Mem. Reg. 9), and also handles estimated monthly PMI renewal premiums (Mem. Reg. 7). It should be noted that where the LTV ratio does not exceed 90% of the purchase price or appraised value, whichever is less, PMI can generally be financed into the base mortgage under most "Standard Coverages" plans. It should also be noted that PMI "renewal premiums" are based upon the **total** loan amount, which includes the base mortgage **plus** the financed-in PMI, if any.

Required Inputs

When computing the **maximum sales price**, the program uses the following inputs into the storage and financial registers:

* Number of monthly payments	[n]
* Monthly mortgage interest rate	[i]
* Down payment	STO [1]
* Tax Ratio (annual) as a percentage of sales price	STO [2]
* Estimated annual insurance expense as a percentage of sales price	STO [3]
* Lender's guideline ratio for the Basic Housing Expense Ratio (typically 28% to 30% of buyer's annual gross income)	STO [4]
* Lender's guideline ratio for the Total Debt Ratio (typically 36% to 38% of buyer's annual gross income)	STO [5]
* Buyer's estimated monthly consumer installment debt payments	STO [6]
* Estimated monthly "other" expenses (utilities, maintenance, renewal premiums for PMI, etc.)	STO [7]
* Annual gross income	STO [8]
* PMI financed into the base mortgage, expressed on an annual percentage basis	STO [9]

Problem 1

The following information is given:

1.	ANNUAL GROSS INCOME	$45,000
2.	DOWN PAYMENT	$25,000
3.	ANNUAL (FIXED) INTEREST RATE	9.75%
4.	LOAN TERM IN MONTHS (FIXED PAYMENT)	360
5.	TAX RATIO AS A PERCENTAGE OF PRICE (50% assessment ratio; $30 per $1,000 of assessed value)	1.50%
6.	ESTIMATED COST OF HOMEOWNER'S INSURANCE AS A PERCENTAGE OF PRICE	.25%
7.	LENDER'S BASIC HOUSING EXPENSE RATIO	28%
8.	MONTHLY CONSUMER INSTALLMENT DEBT	$375
9.	LENDER'S TOTAL DEBT RATIO	36%

(a) Calculate the estimated **maximum** home **price** for which the buyer will income-qualify.

Solution (a)

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter program into your HP 12C		
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter loan term	360 n	360.00
Enter monthly interest rate	9.75 g i	0.81
Store down payment	25,000 STO 1	25,000.00
Store annual tax ratio	1.5 STO 2	1.50
Store homeowner's insurance ratio	.25 STO 3	0.25
Store Basic Housing Expense Ratio	28 STO 4	28.00
Store Total Debt Ratio	36 STO 5	36.00
Store monthly consumer debt	375 STO 6	375.00
Store annual gross income	45,000 STO 8	45,000.00
Compute maximum purchase price	R/S	**118,388.37**
Determine Housing Expense Ratio which produced maximum price:		
Basic Housing Expense price	RCL 0	125,851.15
Total Debt Ratio price	RCL FV	**118,388.37**
Calculate the LTV Ratio:		
Enter price into "y" and "z"	ENTER ENTER	118,388.37
Determine mortgage amount	RCL 1 -	93,388.37
Reverse "x" and "y" registers	x≷y	118,388.37
Complete LTV Ratio calculation	÷ 100 x	**78.88**

(b) Qualify the buyer using the following changes: Down payment = $20,000. Consumer debt = $325 per month.

Solution (b)

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Store down payment	20,000 STO 1	20,000.00
Store monthly debt	325 STO 6	325.00
Compute maximum purchase price	R/S	**119,089.10**
Determine Housing Expense Ratio which produced maximum price:		
Basic Housing Expense price	RCL 0	121,576.69
Total Debt Ratio price	RCL FV	**119,089.10**
Calculate the LTV Ratio:		
Enter price into "y" and "z"	ENTER ENTER	119,089.10
Determine mortgage amount	RCL 1 -	99,089.10

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Reverse "x" and "y" registers	x≷y	119,089.10
Complete LTV Ratio calculation	÷ 100 x	83.21

Conclusion: The buyer qualifies for a maximum purchase price of approximately $119,000. Since the loan-to-value ratio (LTV) is estimated at <u>over</u> eighty percent (80%) of the maximum price, private mortgage insurance (PMI) is required, or a larger down payment would be needed.

Note that in order to purchase a $119,089.10 home--this odd number is carried for extreme accuracy--<u>without</u> PMI, the buyer would have to pay an additional $3,817.82 towards the purchase price. This would be far superior to attempting to make the sale through the use of private mortgage insurance. To make this point, we continue with the example below.

(c) Assume the buyer considers private mortgage insurance (PMI). We estimate the first year's financed-in premium at 0.35% of the base loan, plus an annual "renewal premium" equal to 0.34% of the <u>total</u> loan amount. Calculate the maximum price for which the buyer <u>now</u> qualifies.

Solution (c)

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Input financed-in annual PMI rate	.35 STO 9	0.35
Calculate maximum price without monthly renewal premium	R/S	118,793.50
Estimate amount of mortgage loan	RCL 1 - RCL 9 % +	99,139.28
Estimate annual renewal premium	.34 %	337.07
Estimate monthly renewal premium	12 ÷	28.09
Store in "other/month" location	STO 7	28.09
Recalculate maximum price	R/S	116,006.83

Conclusion: Using the private mortgage insurance (PMI) plan in this example added nothing to the buyer's ability to purchase a slightly more expensive home ($119,000). In fact, integrating the cost of PMI into the buyer's overall expenses produced a maximum price ($116,000) that decreased by approximately $3,000, though the monthly cost of ownership remained **the same.** This seems a substantial price to pay for the right to reduce one's down payment by approximately $3,800 (3.21% of $119,089.10 = $3,817.82).

Clearly, using PMI must be weighed very carefully. As well, each buyer's situation must be weighed with an equally high degree of care when the anticipated LTV ratio is over 80% of the price or fair market value.

Problem: Prove that the computed maximum sales price of $116,006.83 is supported by an income of $45,000. Then, determine the accuracy of the computed annual income to qualify by dividing the result by $45,000 and multiplying by 100. You will find the computed maximum price to be supported by an income level ($44,973.59) which is 99.94% of the amount we used in the computation, $45,000.00. (This variance is due to the procedure used for handling PMI in the program's methodology.)

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers	f CLX	0.00
Calculate P & I plus financed-in PMI:		
Enter mortgage term	360 n	360.00
Enter monthly interest rate	9.75 g i	0.81
Mortgage amount plus financed-in PMI	116,006.83 ENTER 20,000 - .35 % + CHS PV	116,006.83 96,006.83 -96,342.85
Compute monthly payment; store in register 0	PMT STO 0	827.73
Compute monthly real estate taxes:		
Enter sales price	116,006.83 ENTER	116,006.83
Calculate monthly tax and add to register 0	1.5 % 12 ÷ STO + 0	145.01
Compute monthly insurance expense:		
Roll down price into display	[R↓]	116,006.83
Calculate monthly insurance and add to register 0	.25 % 12 ÷ STO + 0	24.17
Calculate monthly PMI renewal premium and add to register 0	RCL PV CHS .34 % 12 ÷ STO + 0	327.57 27.30
Recall monthly PITI plus PMI	RCL 0	1,024.21
Add monthly consumer debt	325 +	1,349.21
Determine annual income to qualify:		
Enter Total Debt Ratio	36 ENTER	36.00
Convert to decimal and invert	.01 x [1/x]	2.78
Monthly income to qualify	x	3,747.80
Annual income to qualify	**12 x**	**44,973.59**
Determine accuracy of result:		
Divide by income used to generate the $116,006.83 sales price	45,000 ÷	1.00
Accuracy as a percentage of income used to generate the maximum price	**100 x**	**99.94**

Problem 2

The following information is given:

1.	ANNUAL GROSS INCOME	$65,171.70
2.	DOWN PAYMENT	$35,000
3.	ANNUAL (FIXED) INTEREST RATE	9.65%
4.	LOAN TERM IN MONTHS (FIXED PAYMENT)	360
5.	TAX RATIO AS A PERCENTAGE OF PRICE (ASSESSMENT RATIO OF 50%, TAX RATE PER $1,000 OF ASSESSED VALUE EQUAL TO $40)	2.00%
6.	ESTIMATED COST OF HOMEOWNER'S INSURANCE AS A PERCENTAGE OF PRICE	.25%
7.	LENDER'S BASIC HOUSING EXPENSE RATIO	28%
8.	MONTHLY CONSUMER INSTALLMENT DEBT	$375
9.	LENDER'S TOTAL DEBT RATIO	36%

(a) Calculate the estimated **maximum** home **price** for which the buyer will income-qualify.

Solution (a)

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers	f CLX	0.00
Enter loan term	360 n	360.00
Enter monthly interest rate	9.65 g i	0.80
Store down payment	35,000 STO 1	35,000.00
Store annual tax ratio	2 STO 2	2.00
Store homeowner's insurance ratio	.25 STO 3	0.25
Store Basic Housing Expense Ratio	28 STO 4	28.00
Store Total Debt Ratio	36 STO 5	36.00
Store monthly consumer debt	375 STO 6	375.00
Store annual gross income	65,171.7 STO 8	65,171.70
Compute maximum purchase price	R/S	**174,999.99**
Determine Housing Expense Ratio which produced maximum price:		
Basic Housing Expense price	RCL 0	**174,999.99**
Total Debt Ratio price	RCL FV	180,722.77
Calculate the LTV Ratio:		
Bring back maximum price	RCL 0	174,999.99
Enter price into "y" and "z"	ENTER ENTER	174,999.99
Determine mortgage amount	RCL 1 -	139,999.99
Reverse "x" and "y" registers	x≷y	174,999.99

Solution (a) continued

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Complete LTV Ratio calculation	÷ 100 x	**80.00**

Conclusion: The buyer qualifies for a maximum purchase price of approximately $175,000. Since the loan-to-value ratio (LTV) is estimated at eighty percent (80%) of the maximum price, private mortgage insurance (PMI) is not required, nor would a larger down payment be needed.

(b) Assume the lender will increase the income-qualifying ratios from 28%/36% to 30%/38% if the buyer makes a $45,000 down payment. Recompute the maximum price which the buyer may qualify for. **Note**, however, that you must always recompute the LTV ratio after performing this--or any other--maximum price calculation.

Solution (b)

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Store down payment	45,000 STO 1	45,000.00
Store Basic Housing Expense Ratio	30 STO 4	30.00
Store Total Debt Ratio	38 STO 5	38.00
Compute maximum purchase price	R/S	**193,646.95**
Determine Housing Expense Ratio which produced maximum price:		
Basic Housing Expense price	RCL 0	**193,646.95**
Total Debt Ratio price	RCL FV	199,369.73
Calculate the LTV Ratio:		
Bring back maximum price	RCL 0	193,646.95
Enter price into "y" and "z"	ENTER ENTER	193,646.95
Determine mortgage amount	RCL 1 -	148,646.95
Reverse "x" and "y" registers	x≷y	193,646.95
Complete LTV Ratio calculation	÷ 100 x	**76.76**

Conclusion: The buyer qualifies for a maximum purchase price of approximately $193,000. Since the loan-to-value ratio (LTV) is estimated at considerably less than eighty percent (80%) of the maximum price, private mortgage insurance (PMI) is not required.

PRACTICE

		Problem #1	**Problem #2**	**Problem #3**
DOWN PAYMENT	STO 1	$50,000	$45,000	$100,000
TAX RATIO	STO 2	1.25%	45%AV/$70/1M	2%
INSURANCE RATIO	STO 3	.25%	.3%	.25%
HOUSING EXPENSE RATIO	STO 4	28%	29%	30%
TOTAL DEBT RATIO	STO 5	36%	36%	41%
MONTHLY CONSUMER DEBT	STO 6	$1,200	$900	$1,743.75
ANNUAL GROSS INCOME	STO 8	$85,000	$50,000	$165,540.05
ANNUAL INTEREST RATE	[g][i]	8%	7.5%	7.775%
LOAN TERM IN MONTHS	[n]	360	360	360

Answers:

Maximum sales price	[R/S]	$199,924.68	$92,696.17	$511,278.13
Housing ratio price	RCL 0	$273,674.03	$154,348.58	$536,265.91
Total debt ratio price	RCL FV	$199,924.68	$92,696.17	$511,278.13
Loan-to-value ratio		75%	52%	81%

Note: In practice, the example in problem #3 would need to be further refined since the data triggered an LTV ratio slightly in excess of 80% (80.44%). As well, many lenders require a LTV ratio below 80% for loans on residential homes priced in excess of one-half million dollars.

		Problem #4	**Problem #5**	**Problem #6**
DOWN PAYMENT	STO 1	$10,000	$12,500	$10,000
TAX RATIO	STO 2	1.25%	25%AV/$99/M$	2%
INSURANCE RATIO	STO 3	.22%	.35%	.25%
HOUSING EXPENSE RATIO	STO 4	28%	28%	28%
TOTAL DEBT RATIO	STO 5	36%	36%	36%
MONTHLY CONSUMER DEBT	STO 6	$300	$900	$450.00
MONTHLY OTHER EXPENSE (PMI RENEWAL PREMIUM)	STO 7	$39.91	$17.25	$21.14
ANNUAL GROSS INCOME	STO 8	$35,000	$50,000	$39,000
FINANCED-IN FIRST YEAR PMI PREMIUM	STO 9	0.50% (1)	0.34% (2)	0.40% (3)
ANNUAL INTEREST RATE	[g][i]	8%	7.5%	7.775%
LOAN TERM IN MONTHS	[n]	360	360	360

Answers:

Maximum sales price	[R/S]	$91,150.47	$71,552.07	$84,859.67
Housing ratio price	RCL 0	$98,903.01	$132,028.22	$105,772.97
Total debt ratio price	RCL FV	$91,150.47	$71,552.07	$84,859.67
Loan-to-value ratio (approx.)		89.5%	82.5%	88.6%

(1) 20% coverage: LTV range: 85.01-90%. 1st year premium .50%, renewal .34%.
(2) 17% coverage: LTV range: 85%&Under. 1st year premium .34%, renewal .34%.
(3) 17% coverage: LTV range: 85.01-90%. 1st year premium .40%, renewal .34%.

HOME BUYER INCOME QUALIFICATION PROGRAM
Maximum Affordable Mortgage, Maximum Price & Required Down Payment

KEYSTROKE	DISPLAY	
[f][R/S][f](PRGM)		
g 8	01	43 8
1	02	1
CHS	03	16
STO PMT	04	44 14
PV	05	13
[1/x]	06	22
RCL 4	07	45 4
RCL 5	08	45 5
+	09	40
RCL 6	10	45 6
÷	11	10
1	12	1
2	13	2
÷	14	10
+	15	40
[1/x]	16	22
STO 7	17	44 7
RCL 0	18	45 0
1	19	1
2	20	2
0	21	0
0	22	0
÷	23	10
STO 8	24	44 8
RCL 2	25	45 2
x	26	20
x	27	20
STO 9	28	44 9
RCL 8	29	45 8
RCL 3	30	45 3
x	31	20
RCL 1	32	45 1
-	33	30
RCL 7	34	45 7
x	35	20
g x≷y	36	43 34
g GTO 39	37	43 33 39
x≷y	38	34
ENTER	39	36
ENTER	40	36
STO PV	41	44 13
R/S	42	31
RCL 6	43	45 6
EEX	44	26
2	45	2
÷	46	10
÷	47	10
R/S	48	31
-	49	30
CHS	50	16
[f][R/S]		

Note: Memory used: P - 50, r - 14. Therefore, if you added one (1) line to the program, your calculator would allocate an additional storage register. Your memory usage would then be: P - 57, r - 13.

The program uses storage registers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. In addition, financial registers [n], [i], and [PV] are used.

Storage Registers Used:

1. Store term in [n].
2. Monthly interest rate in [i].
3. Buyer's gross annual income, store in Memory Register 0.
4. Monthly consumer debts, store in Memory Register 1.
5. Basic Housing Expense ratio store in Memory Register 2.
6. Total Debt Ratio store in Memory Register 3.
7. Real estate tax as a percentage of price, store in Mem. Reg. 4.
8. Hazard (Homeowner's) insurance as a percentage of price, store in Memory Register 5.
9. Loan-to-Value Ratio (LTV) store in Memory Register 6.
10. Maximum affordable mortgage displays and is stored in [PV] register.
11. Maximum mortgage produced under the Basic Housing Expense Ratio may be recalled from Memory Register 9.

Description of the Program

Note that the program sets the **payment mode** to **END** mode (see step "01"). Therefore, it doesn't matter if the **BEGIN** status indicator appears in your display before you start a problem with this program. **However**, since it is a good practice to make sure that the payment mode is set to END when performing buyer income qualification problems, we set the mode to END in any event in the solution keystrokes in this section!

To operate the program you must input both housing expense ratios (Basic Housing Expense, Mem. Reg. 2, and Total Debt, Mem. Reg. 3) plus the buyer's monthly consumer installment debt (Mem. Reg. 1).

The operational sequence requires input of the financial variables commonly used in the qualification process, except we are not solving for the required income, but rather solve for the maximum affordable mortgage. This is the primary variable which the program was designed to solve. However, once you know the maximum affordable mortgage, and if you know the LTV (as this program requires), you can readily solve for the maximum affordable price. Finally, if you know (1) the maximum mortgage and (2) maximum price, you readily determine the amount of the down payment. The program performs each of these calculations.

After inputting the data, you press the Run/Stop [R/S] key. The first number displayed is the maximum affordable mortgage loan. This number will store automatically in the [PV] register.

Pressing [R/S] a second time calculates the maximum affordable sales price. This calculation assumes that the buyer will have available sufficient funds needed to make the down payment. If the buyer lacks sufficient funds to make the down payment, he does not qualify for the maximum sales price, though he "technically" qualifies for the mortgage loan. At times, this will require that you make a few additional attempts at the qualification process, whether performed with the current program or another.

Please note that the program does not take into account PMI or "other" expenses.

Problems

The following information is given:

	PROBLEM #1	PROBLEM #2
1. BUYER'S ANNUAL INCOME	$45,000	$75,000
2. MONTHLY CONSUMER INSTALLMENT DEBT	$375	$500
3. LENDER'S BASIC HOUSING EXPENSE RATIO	28%	30%
4. LENDER'S TOTAL DEBT RATIO	36%	39%
5. TAX RATIO AS A PERCENTAGE OF PRICE	1.5%	1.5%
6. HAZARD INSURANCE AS A PERCENT OF PRICE	.25%	.225%
7. LOAN-TO-VALUE RATIO	80%	80%
8. ANNUAL (FIXED) INTEREST RATE	8%	7.85%
9. LOAN TERM IN MONTHS (FIXED PAYMENT)	360	360

Calculate the maximum affordable mortgage, maximum sales price, and required down payment needed to justify the maximum price.

Problem #1

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter program into your HP 12C		
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter loan term	360 n	360.00
Enter monthly interest rate	8 g i	0.67
Store buyer's annual income	45,000 STO 0	45,000.00
Store monthly consumer debt	375 STO 1	375.00
Store Basic Housing Expense Ratio	28 STO 2	28.00
Store Total Debt Ratio	36 STO 3	36.00
Store annual tax ratio	1.5 STO 4	1.50
Store homeowner's insurance ratio	.25 STO 5	0.25
Store Loan-to-Value Ratio	80 STO 6	80.00
Compute maximum mortgage	R/S	**106,434.51**
Compute maximum sales price	R/S	**133,043.14**
Compute required down payment	R/S	**26,608.63**
Determine maximum mortgage supported by Basic Housing Ratio	RCL 9	114,621.78

Conclusion: The buyer qualifies for a maximum mortgage of $106,400 (rounded). If he has approximately $26,600 available for the down payment, he qualifies for a home selling for $133,000. Finally, the maximum mortgage is determined by the Total Debt Ratio since the Basic Housing Expense Ratio (Mem. Reg. 9) produced a higher loan amount.

Problem #2

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers	f CLX	0.00
Enter loan term	360 n	360.00
Enter monthly interest rate	7.85 g i	0.65
Store buyer's annual income	75,000 STO 0	75,000.00
Store monthly consumer debt	500 STO 1	500.00
Store Basic Housing Expense Ratio	30 STO 2	30.00
Store Total Debt Ratio	39 STO 3	39.00
Store annual tax ratio	1.5 STO 4	1.50
Store homeowner's insurance ratio	.225 STO 5	0.23
Store Loan-to-Value Ratio	80 STO 6	80.00
Compute maximum mortgage	R/S	**207,636.05**
Compute maximum sales price	R/S	**259,545.06**
Compute required down payment	R/S	**51,909.01**
Determine maximum mortgage supported by Basic Housing Ratio	RCL 9	207,636.05

Conclusion: The buyer qualifies for a maximum mortgage of $207,600 (rounded). If he has approximately $52,000 available for the down payment, he qualifies for a home selling for $259,500. Finally, the maximum loan amount was produced by the Basic Housing Expense Ratio. This tells us the buyer's debts in relationship to his income are low, at least insofar as mortgage underwriting consideration go.

Problem: By keystroke procedures, prove the accuracy of the results obtained in Problem #2.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers	f CLX	0.00
Calculate "P & I" Payment:		
Enter mortgage term	360 n	360.00
Enter monthly interest rate	7.85 g i	0.65
Enter "mortgage", negative-signed	207,636.05 CHS PV	-207,636.05
Compute monthly P & I payment and store in register 0	PMT STO 0	1,501.90 1,501.90
Compute monthly real estate taxes:		
Enter maximum sales price	259,545.06 ENTER	259,545.06

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Convert tax ratio to decimal	1.5 ENTER .01 x	0.02
Determine annual real estate taxes	x	3,893.18
Convert to monthly obligation and add to register 0	12 [1/x] x STO + 0	324.43 324.43
Compute monthly insurance expense:		
Enter maximum sales price	259,545.06 ENTER	259,545.06
Convert insurance ratio to decimal	.225 ENTER .01 x	.002
Determine annual insurance expense	x	583.98
Convert to monthly obligation and add to register 0	12 [1/x] x STO + 0	48.66 48.66
Determine monthly PITI	RCL 0	1,875.00
Determine annual income to qualify:		
Enter Basic Housing Expense Ratio	30 ENTER	30.00
Convert to decimal equivalent	.01 x	0.30
Determine monthly income to qualify	÷	6,250.00
Convert to annual income to qualify	12 x	**75,000.00**

Conclusion: The buyer's annual income used in the maximum mortgage qualification was $75,000. We further determined that the maximum loan amount was produced by the Basic Housing Expense Ratio and thus used 30% in proving the accuracy of the calculation. Indeed, the annual income needed to qualify a buyer using the above data was determined to be exactly $75,000, the exact amount of the buyer's income used in Problem #2 above.

PRACTICE

		Problem #1	**Problem #2**	**Problem #3**
BUYER'S ANNUAL INCOME	STO 0	$77,549.51	$100,000	$55,000
MONTHLY CONSUMER DEBT	STO 1	$500	$1,500	$250
HOUSING EXPENSE RATIO	STO 2	28%	30%	29%
TOTAL DEBT RATIO	STO 3	36%	40%	37%
TAX RATIO	STO 4	3.1795%	1.25%	2%
INSURANCE RATIO	STO 5	.267%	.200%	.30%
LOAN-TO-VALUE RATIO	STO 6	80%	75%	80%
LOAN TERM IN MONTHS	[n]	360	360	360
ANNUAL INTEREST RATE	[g][i]	10.75%	7.5%	8.5%
Answers:				
Maximum loan	[R/S]	$140,000	$213,097.61	$131,796.81
Maximum sales price	[R/S]	$175,000	$284,130.15	$164,746.02
Required down payment	[R/S]	$35,000	$71,032.54	$32,949.20

ESTIMATING THE AMOUNT OF BUYER'S FUNDS AVAILABLE TO MAKE THE DOWN PAYMENT

This section is devoted to a program which enables you to take the **amount of funds available to a buyer** and allocate them over:

(1) The number of months of reserves of PITI required by the lender.
(2) Number of months of prepaid real estate taxes **over and above** the number of months of reserves for taxes included as part of the PITI reserves.
(3) Points and/or loan origination fees expressed as a percentage of the mortgage loan and paid in advance at the closing.
(4) Estimated number of days of prepaid mortgage interest paid in advance and attributed to closing on a date other than exactly one month before the first mortgage payment becomes due.
(5) Estimated "other" expenses of the sale, such as mortgage-title insurance, surveys, closing fees, and so forth.
(6) Private mortgage insurance (PMI) **financed-into** the loan.
(7) The **amount of available funds** allocated to, or available for, the down payment.

For example, if we assume a buyer has $25,000 **available** for the down payment and associated prepaids on his or her mortgage loan, what is the relative allocation of the available funds? That is, how much of the $25,000 will go for points and/or loan origination fees? How much for prepaid taxes or PITI reserves required by the lender? The program answers the bulk of these kinds of questions faced by the residential real estate practitioner whose client is acquiring property subject to a **conventional mortgage loan.**

Note: The program in this section is strictly designed to handle conventional mortgage loan financing. You cannot use it for any governmental insured loans. This limitation applies to all forms of financing which do not meet strict conventional mortgage loan origination requirements.

Please note further that the program does not directly take into consideration private mortgage insurance premiums which are paid on an **up-front** basis over and above PMI premiums which are financed-into the mortgage loan. Put differently, the program cannot accept separate rate inputs for private mortgage insurance premiums (PMI) which are not a part of the monthly mortgage payment. If you have a need to refine the calculation beyond this level, you can estimate the amount of the first-year up-front PMI premium and treat it as an "other" expense, thus lumping-in the estimate with mortgage-title insurance and similar costs.

These limitations aside, you will find that this program will help you zero-in on the relative allocation of a buyer's available funds in a manner which will save time and coordination effort with the lender and client.

PROGRAM FOR ALLOCATING BUYER'S AVAILABLE FUNDS OVER THE DOWN PAYMENT, PREPAIDS, PITI RESERVES, AND FINANCED-IN FIRST YEAR PMI PREMIUM

```
KEYSTROKE         DISPLAY               KEYSTROKE         DISPLAY

f R/S f PRGM                            +                 32        40
1                 01         1          STO 8             33      44  8
CHS               02        16          RCL 3             34      45  3
STO PMT           03     44 14          RCL 4             35      45  4
PV                04        13          +                 36        40
[1/x]             05        22          RCL 5             37      45  5
RCL . 0           06  45 48  0          x                 38        20
%                 07        25          RCL 3             39      45  3
+                 08        40          RCL 9             40      45  9
RCL 5             09     45  5          x                 41        20
x                 10        20          +                 42        40
RCL g i           11  45 43 12          1                 43         1
RCL . 0           12  45 48  0          2                 44         2
%                 13        25          0                 45         0
+                 14        40          0                 46         0
3                 15         3          ÷                 47        10
6                 16         6          RCL 8             48      45  8
5                 17         5          +                 49        40
0                 18         0          RCL 1             50      45  1
0                 19         0          x                 51        20
÷                 20        10          RCL 7             52      45  7
RCL 6             21     45  6          +                 53        40
x                 22        20          CHS               54        16
+                 23        40          RCL 0             55      45  0
RCL 2             24     45  2          +                 56        40
RCL . 0           25  45 48  0          1                 57         1
%                 26        25          RCL 8             58      45  8
+                 27        40          -                 59        30
1                 28         1          ÷                 60        10
0                 29         0          STO 8             61      44  8
0                 30         0          f R/S
÷                 31        10
```

Required Information and Memory Register Storage Locations Used

1. Input mortgage loan term into [n].
2. Input monthly mortgage interest rate on the loan into [i].
3. Store the amount of funds available for the purchase into Memory Register 0.
4. Purchase price of the home is stored in Memory Register 1.
5. Points and/or loan origination fees expressed as a percentage of the mortgage loan, store in Memory Register 2.
6. Real estate taxes, expressed as a percentage of the price of the property, store in Memory Register 3.
7. Hazard (Homeowner's) insurance expense, expressed as a percentage of the price, store in Memory Register 4.

8. Number of months reserves of PITI required by the lender, store in Memory Register 5. You will generally use two months for the reserves, unless another amount is required by lenders in your area. The reserves for monthly PITI could be more, though most likely less than six months reserves. (Per the MGIC "MGIC/Fannie Mae/Freddie Mac Underwriting Guidelines Reference", dated February, 1997, Fannie Mae and the Freddie Mac are noted as requiring two months of PITI reserves.)
9. Estimated number of days of prepaid mortgage interest needed at closing, store in Memory Register 6. This charge typically arises because mortgage interest is a back bill, that is, it is paid in arrears. Therefore, unless the mortgage closing takes place exactly one month prior to the date the first payment is due, a charge will be added in the closing statement (HUD-1) for prepaid interest.

   A general practice followed by this author is to estimate every closing as requiring 15 days of prepaid interest. On average, this figure would be accurate if the practitioner's sample was large enough. There are, of course, no hard and fast rules on this issue.
10. Estimated "other" expenses, such as mortgage-title insurance, surveys, pest inspections, and the like, store in Memory Register 7. Practical necessity requires that we lump all of these costs together rather than have each track the price at different rates. You simply cannot estimate the buyer's down payment with 100% accuracy. It is impractical to attempt it, and if one were inclined to try, the HP 12C lacks sufficient memory to go much beyond the length of the program covered here.
11. Estimated number of months of prepaid taxes paid or required at the closing. Depending upon the part of the year a property is closed, the buyer could be obligated to prepay up to one full year of real estate property taxes. Check the convention in your area. The author's experience in Michigan is that four to five months of taxes are prepaid.
12. Financed-into-the-mortgage private mortgage insurance (PMI) rate. This category of expense is stored in Memory Register "point zero"; that is, ". 0". This category of expense has very little impact on the amount of funds which can be placed as a down payment on the residential purchase. However, since memory was available to include this variable, it was indeed included.

    Check the PMI rates in your area. Generally, you will find this rate to be approximately one-half of one percent of the amount of the mortgage loan. When the first-year PMI premium is financed into the mortgage loan, its impact on the overall financial picture of the buyer is admittedly minimal.
13. The amount of the estimated funds available for the down payment will display and can be recalled from Memory Register 8.
14. Overall memory usage: **P - 64 r - 12.**

Summary of registers used:

Memory Registers: Memory storage registers 0 through and including .0 are used in the program. Therefore, you have one memory storage register available over and above those allocated for the program.

Financial Registers: Three financial registers are used by the program. You make inputs into [n] and [i]; the program computes the present value [PV] of a uniform payment stream of $1.00 per month over the term of the loan [n] at the given monthly mortgage interest rate [i].

Caution: The program is designed to work exclusively in cases where the buyer is financing the property with a mortgage loan. The program cannot work if you do not make inputs for a mortgage term [n] and monthly interest rate [i]. If you look at step "05" you will note that I am inverting [1/x] the present value computed in step "04". If the computed PV was zero, we would be attempting to divide by zero at step "05", an impossibility.

You can override this requirement by giving the program an exceptionally long mortgage term, such as 100,000 months, and by inputting an exceedingly small interest rate, such as ".0000001", followed by pressing [g] [i]. What this does is set the mortgage payment component of the equation to a minimal amount such that its impact on the total equation will be insignificant. Under this scenario you can (technically) compute the amount of funds available for the down payment without taking a mortgage loan into consideration. The reality is, however, that if you do not need a mortgage, you probably don't have to concern yourself about allocating funds!

Discussion of the program

The program takes a given amount of funds available for the purchase of real property and breaks it up--distributes it--over points, prepaids for real estate taxes, number of months of PITI (principal, interest, taxes and insurance) required by the mortgage lender, prepaid interest, and general estimated prepaids which do not track price on a percentage basis. After this process is completed, the program gives you back the amount of the available funds which can be directly applied as the down payment.

Knowing the amount of the down payment, we readily calculate the amount of the mortgage loan. Of course, you must test for the loan-to-value ratio (LTV) after you calculate the amount of the mortgage. For example, if your mortgage loan is large in relationship to the price of the property, and in particular if it exceeds 80% of the purchase price, private mortgage insurance (PMI) will be needed, at the minimum.

However, in cases where the LTV ratio exceeds anything acceptable to the mortgage lender, say 95%-plus, the sale may fail. Just take your time when doing the calculation and work back and forth, alternating between changes in your amount available for the down payment (the Memory Register 0 input) and the LTV you will compute manually.

In proofing the program we will take the amount calculated for the down payment and determine the amount of the mortgage itself. From there we calculate the loan payment (P & I), points, prepaids for PITI and advance payments for taxes, prepaid interest and other expenses, and add these figures to the amount of the down payment which was produced by the program; the result will be exactly the amount which was available for purchase of the property.

We will do two examples. The first example does not take into account a first year financed-in PMI premium, while the second example does.

Problem 1

The following information is given:

AMOUNT AVAILABLE FOR THE PURCHASE	$50,000
PRICE OF THE PROPERTY	$189,500
MORTGAGE TERM	360 months
ANNUAL INTEREST RATE	10%
POINTS AND/OR ORIGINATION FEES	3%
TAXES AS A PERCENTAGE OF PRICE	2%
HAZARD INSURANCE AS A PERCENTAGE OF PRICE	0.25%
REQUIRED PITI RESERVES	3 months
ESTIMATED PREPAID MORTGAGE INTEREST	15 days
"OTHER" ESTIMATED PREPAID EXPENSES	$650
ESTIMATED PREPAID REAL ESTATE TAXES	4 months

Calculate the amount of funds directly available to make the down payment. Then compute the loan-to-value ratio (LTV).

Later, you will prove the accuracy of the computed down payment by determining and totalling the expenses associated with each category in the problem. Subject to a minor rounding difference, you will find that the down payment we compute below will be exactly the amount which is left after deducting the above expenses from the buyer's available funds.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter program into your HP 12C		
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter loan term	360 n	360.00
Enter monthly interest rate	10 g i	0.83
Store amount of funds available	50,000 STO 0	50,000.00
Store price of home	189,500 STO 1	189,500.00
Total of points/origination fees	3 STO 2	3.00
Store annual tax ratio	2 STO 3	2.00
Store homeowner's insurance ratio	.25 STO 4	0.25
Store required months PITI reserves	3 STO 5	3.00
Number of days of prepaid interest	15 STO 6	15.00
Store other estimated prepaids	650 STO 7	650.00
Number of months of prepaid taxes	4 STO 9	4.00
Compute the amount available to make the down payment	R/S	37,855.85

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Determine the LTV Ratio:		
Bring up purchase price	RCL 1	189,500.00
Hypothetical mortgage amount	- CHS	151,644.15
Calculate LTV Ratio	RCL 1 ÷ 100 x	80.02

Comment: The buyer has available $37,855.85 for the down payment; the balance of his available funds go to the expenses. The LTV ratio is estimated at approximately 80% of the purchase price. In practice you do not make down payments in odd amounts such as that computed in this example.

Problem extension: Lower the PITI reserves requirement from three months to two months. Then redetermine the potential down payment, calculate the LTV ratio, and calculate the amount for each expense category used in the problem.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Reset PITI reserve requirement	2 STO 5	2.00
Compute amount available for the down payment	R/S	39,633.80
Determine the LTV Ratio:		
Bring up purchase price	RCL 1	189,500.00
Hypothetical mortgage amount	- CHS	149,866.20
Calculate LTV Ratio	RCL 1 ÷ 100 x	79.09

Discussion: To prove the accuracy of the computed down payment ($39,633.80) we must determine the amount allocated by the program for each expense category used in the problem. We know that many of the expenses track the amount of the mortgage loan (the "P & I" portion of the PITI reserves; points; prepaid interest). And, the price of the property ($189,500) is the base number from which we determine both the reserve for taxes and insurance (the "T & I" portion of the PITI reserves) and the prepaid taxes. Finally, we add in the lump-all category of "other" expenses in order to complete the totalling process used to prove the accuracy here.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Determine mortgage loan payment:		
Bring up down payment	RCL 8	39,633.80
Subtract purchase price	RCL 1 -	-149,866.20
Set to PV	PV	-149,866.20
Calculate monthly payment (P & I)	PMT	1,315.18
Calculate reserves for P & I and store in Mem. Reg. 0	2 x STO 0	2,630.37

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Calculate monthly tax obligation:		
Bring up purchase price	RCL 1	189,500.00
Multiply times tax ratio	RCL 3 %	3,790.00
Determine monthly tax obligation	12 ÷	315.83
(Enter twice)	ENTER ENTER	315.83
Determine tax component of reserves	2 x	631.67
Add to Mem. Reg. 0	STO + 0	631.67
Determine and add prepaid taxes:		
Bring back monthly tax obligation	R↓	315.83
Times number months prepaid	4 x	1,263.33
Add to Mem. Reg. 0	STO + 0	1,263.33
Determine monthly insurance expense:		
Bring up purchase price	RCL 1	189,500.00
Multiply times insurance ratio	RCL 4 %	473.75
Determine monthly insurance cost	12 ÷	39.48
Insurance component for reserves	2 x	78.96
Add to Mem. Reg. 0	STO + 0	78.96
Determine and add points:		
Bring up mortgage and change sign	RCL PV CHS	149,866.20
Multiply times points percentage	RCL 2 %	4,495.99
Add to Mem. Reg. 0	STO + 0	4,495.99
Determine and add prepaid interest:		
Bring up mortgage and change sign	RCL PV CHS	149,866.20
Bring up annual interest rate	RCL g i	10.00
Multiply times annual percentage	%	14,986.62
Determine daily interest expense	365 ÷	41.06
Interest component for reserves	15 x	615.89
Add to Mem. Reg. 0	STO + 0	615.89
Add estimated "other" expenses	650 STO + 0	650.00
Determine total expenses	RCL 0	10,366.20
Deduct from amount available	50,000 x≷y -	39,633.80

Summary of Data:

Amount available for purchase		$50,000.00
2 months P & I reserves	$ 2,630.37	
2 months tax reserves	631.67	
2 months insurance reserves	78.96	
4 months prepaid taxes	1,263.33	
Points & Origination fees	4,495.99	
Prepaid interest (15 days)	615.89	
"Other" expenses	650.00	
Down payment	39,633.80	
Total	$50,000.01	(Slight rounding error)

Problem 2

The following information is given:

AMOUNT AVAILABLE FOR THE PURCHASE	$15,000
PRICE OF THE PROPERTY	$80,000
MORTGAGE TERM	360 months
ANNUAL INTEREST RATE	10%
POINTS AND/OR ORIGINATION FEES	2%
TAXES AS A PERCENTAGE OF PRICE	2%
INSURANCE AS A PERCENTAGE OF PRICE	0.25%
REQUIRED PITI RESERVES	2 months
ESTIMATED PREPAID MORTGAGE INTEREST	15 days
"OTHER" ESTIMATED PREPAID EXPENSES	$400
ESTIMATED PREPAID REAL ESTATE TAXES	4 months
FINANCED-IN PMI RATE	0.40%

Calculate the amount of funds directly available to make the down payment. Then compute the loan-to-value ratio (LTV). Keep in mind as you go through this example that when we finance PMI into the mortgage loan we add the first year PMI premium charge to the base amount of the mortgage which we will arrive at below. (Remember: The program does not automatically add the financed-in PMI first year premium. You will do this manually.)

Again, you will prove the accuracy of the computed down payment by determining and totalling the expenses associated with each category in the problem. You will find the results obtained to be extremely accurate.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter loan term	360 n	360.00
Enter monthly interest rate	10 g i	0.83
Store amount of funds available	15,000 STO 0	15,000.00
Store price of home	80,000 STO 1	80,000.00
Total of points/origination fees	2 STO 2	2.00
Store annual tax ratio	2 STO 3	2.00
Store homeowner's insurance ratio	.25 STO 4	0.25
Store required months PITI reserves	2 STO 5	2.00
Number of days of prepaid interest	15 STO 6	15.00
Store other estimated prepaids	400 STO 7	400.00
Number of months of prepaid taxes	4 STO 9	4.00
Store financed-in PMI rate	.4 STO . 0	0.40
Compute the down payment	R/S	10,875.34

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Determine the LTV Ratio:		
Bring up purchase price	RCL 1	80,000.00
Mortgage without PMI percentage	- CHS	69,124.66
Add PMI percentage	RCL . 0 % +	69,401.15
Complete LTV calculation	RCL 1 ÷ 100 x	86.75

Comment: The buyer has available $10,875.34 for the down payment. The LTV ratio is estimated at approximately 87% of the purchase price. This confirms the need for private mortgage insurance. In practice, you will first calculate the down payment; determine if the LTV ratio exceeds 80%; and then utilize an appropriate rate for the first year private mortgage insurance premium.

Discussion: To prove the accuracy of the computed down payment ($10,875.34) we must determine the amount allocated by the program for each expense category used in the problem. What is different about this proof is that we have to add the financed-in private mortgage insurance (0.40%) to the base amount of the mortgage loan. We then use this amount to determine the points, prepaid interest, and the monthly mortgage payment.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Determine mortgage loan payment:		
Bring up down payment	RCL 8	10,875.34
Subtract purchase price	RCL 1 -	-69,124.66
Bring up financed-in PMI rate	RCL . 0	0.40
Add financed-in PMI to base loan	% +	-69,401.15
Set to PV	PV	-69,401.15
Calculate monthly payment (P & I)	PMT	609.04
Calculate reserves for P & I and store in Mem. Reg. 0	2 x STO 0	1,218.09
Calculate monthly tax obligation:		
Bring up purchase price	RCL 1	80,000.00
Multiply times tax ratio	RCL 3 %	1,600.00
Determine monthly tax obligation	12 ÷	133.33
(Enter twice)	ENTER ENTER	133.33
Determine tax component of reserves	2 x	266.67
Add to Mem. Reg. 0	STO + 0	266.67
Determine and add prepaid taxes:		
Bring back monthly tax obligation	R↓	133.33
Times number months prepaid	4 x	533.33
Add to Mem. Reg. 0	STO + 0	533.33

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Determine monthly insurance expense:		
Bring up purchase price	RCL 1	80,000.00
Multiply times insurance ratio	RCL 4 %	200.00
Determine monthly insurance cost	12 ÷	16.67
Insurance component for reserves	2 x	33.33
Add to Mem. Reg. 0	STO + 0	33.33
Determine and add points:		
Bring up mortgage and change sign (includes financed-in up-front PMI)	RCL PV CHS	69,401.15
Multiply times points percentage	RCL 2 %	1,388.02
Add to Mem. Reg. 0	STO + 0	1,388.02
Determine and add prepaid interest:		
Bring up mortgage and change sign	RCL PV CHS	69,401.15
Bring up annual interest rate	RCL g i	10.00
Multiply times annual percentage	%	6,940.12
Determine daily interest expense	365 ÷	19.01
Interest component for reserves	15 x	285.21
Add to Mem. Reg. 0	STO + 0	285.21
Add estimated "other" expenses	400 STO + 0	400.00
Determine total expenses	RCL 0	4,124.66
Bring back down payment	RCL 8	10,875.34
Add down payment to total expenses	+	15,000.00

Summary of Data:

Amount available for purchase		$15,000.00
2 months P & I reserves	$ 1,218.09	
2 months tax reserves	266.67	
2 months insurance reserves	33.33	
4 months prepaid taxes	533.33	
Points & Origination fees	1,388.02	
Prepaid interest (15 days)	285.21	
"Other" expenses	400.00	
Down payment	10,875.34	
Total	$14,999.99	(Slight rounding error)

Comment: Treating the up-front first year mortgage insurance premium as a financed-in rate could diminish, somewhat, the accuracy of the computed down payment. This is the case where the first-year PMI premium **cannot** be financed-into the mortgage loan. For example, if we assumed a mortgage loan of $69,000, with a first year PMI premium of 0.40% of the loan, the first year up-front premium would be $276.00. If this amount is not financed-into the mortgage, the amount of the computed down payment will be off by approximately the amount of the PMI premium. The difference is not significant.

SOLUTION TEMPLATE FOR ESTIMATING BUYER'S FUNDS AVAILABLE TO MAKE THE DOWN PAYMENT

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END MODE	f CLX f 2 g END	0.00
Enter mortgage loan term	n	
Type annual interest rate	g i	
Store amount of available funds	STO 0	
Store price of property	STO 1	
Total percentage of points & loan origination fees	STO 2	
Store annual tax ratio (taxes as a percentage of price)	STO 3	
Hazard insurance ratio as a percentage of price	STO 4	
Required number of months of PITI reserves	STO 5	
Estimated number of days of prepaid mortgage interest	STO 6	
Other estimated prepaids as a gross amount	STO 7	
Number of months of prepaid taxes	STO 9	
Financed-into the mortgage first year PMI rate	STO . 0	
Compute the down payment	R/S	

ESTIMATING SELLER'S NET PROCEEDS

The program which follows was primarily designed to estimate the net proceeds to a seller of real estate. The **net** is arrived at by deducting from the sale price:

(1) commissions, expressed as a percentage of the sale price;
(2) mortgage loan balance, either estimated or computed internal to the program; and
(3) incidental costs, such as title insurance, professional fees, and so forth.

The program also solves for:

(1) estimated sales price needed to produce a required **net proceeds,** taking into account a given commission rate, loan balance, and incidental expenses; and
(2) estimated commission (expressed as a percentage of the sale price), taking into account the seller's desired net proceeds, mortgage loan balance, and incidental expenses.

The need for this type of solution method became apparent during one of the author's HP 17B in-house seminars for a firm in Grosse Pointe Woods, Michigan. Indeed, it is not surprising that some sellers approach price from the perspective of estimating the **net proceeds** available from their current residence so they can estimate how much money they have available for their next home purchase. You may find this technique beneficial at an Open House.

The program is long, at least in relationship to the HP 12C's available memory. Therefore, I could not take into account additional variables which track sales price on a percentage basis. For example, title insurance costs are clearly correlated with sale price; the higher the price, within certain ranges, the greater the insurance expense. The program must use these kinds of costs on an estimated basis, and they must all be lumped together with the other incidental costs.

The limitations of this program aside, you will find that it gives you more information in much less time than you likely experienced in the past if you are one of many who "pencil-in" these kinds of calculations. Many real estate agents and brokers who attended the author's seminars--both HP 12C and HP 17BII/19BII--requested a solution program or HP SOLVE equation to perform the exact kinds of problems covered in this section of the book.

Please note that the program is designed around the following general equation:

NET = PRICE x (1 - %COMMISSION/100) - (INCIDENTALS + LOAN BALANCE)

The program follows on the next page. After the program we work through several examples, and conclude the section with a keystroke template.

PROGRAM FOR COMPUTING NET SALES PROCEEDS, OR REQUIRED SALES PRICE, OR MAXIMUM COMMISSION RATE

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
f R/S f PRGM		g [x=0]	44 43 35
RCL 5	01 45 5	g GTO 66	45 43 33 66
g [x=0]	02 43 35	R↓	46 33
g GTO 05	03 43 33 05	RCL 3	47 45 3
g GTO 12	04 43 33 12	+	48 40
RCL n	05 45 11	RCL 7	49 45 7
STO 4	06 44 4	+	50 40
PMT	07 14	1	51 1
PMT	08 14	RCL 1	52 45 1
RCL 2	09 45 2	%	53 25
STO n	10 44 11	x	54 20
FV	11 15	CHS	55 16
CHS	12 16	1	56 1
RCL 7	13 45 7	+	57 40
g [x=0]	14 43 35	÷	58 10
g GTO 17	15 43 33 17	STO 6	59 44 6
g GTO 41	16 43 33 41	RCL 4	60 45 4
x≷y	17 34	STO n	61 44 11
RCL 1	18 45 1	0	62 0
g [x=0]	19 43 35	STO FV	63 44 15
g GTO 62	20 43 33 62	RCL 6	64 45 6
R↓	21 33	g GTO 00	65 43 33 00
1	22 1	R↓	66 33
RCL 1	23 45 1	g GTO 68	67 43 33 68
%	24 25	RCL 3	68 45 3
x	25 20	+	69 40
CHS	26 16	RCL 7	70 45 7
1	27 1	+	71 40
+	28 40	CHS	72 16
RCL 0	29 45 0	RCL 0	73 45 0
x	30 20	+	74 40
RCL 3	31 45 3	RCL 0	75 45 0
-	32 30	÷	76 10
+	33 40	1	77 1
STO 6	34 44 6	0	78 0
0	35 0	0	79 0
STO FV	36 44 15	x	80 20
RCL 4	37 45 4	STO 6	81 44 6
STO n	38 44 11	0	82 0
RCL 6	39 45 6	STO FV	83 44 15
g GTO 00	40 43 33 00	RCL 4	84 45 4
x≷y	41 34	STO n	85 44 11
CHS	42 16	RCL 6	86 45 6
RCL 1	43 45 1	f R/S	

Required Information and Memory Storage Locations Used

1. Input the original term on the seller's outstanding mortgage loan into [n]. Note that the program is limited to handling only one underlying mortgage loan.
2. Input monthly mortgage interest rate on the loan into [i].
3. Input the original amount of the underlying loan, negative-signed, into [PV]. Also, make sure you are in END mode (g END).
4. Sales price for the property, store in Memory Register 0.
5. Commission expressed as a percentage of the sale price, store in Memory Register 1.
6. Estimated number of monthly payments to be made on the seller's underlying mortgage loan by the time of closing, store in Memory Register 2.
7. Estimated incidental costs, such as professional fees, survey, title insurance, fixed commissions paid as an incentive to agents and/or others, store in Memory Register 3.
8. Estimated underlying mortgage loan balance, if you know it, can be stored in Memory Register 5.

   Note that the program **tests** to see if the user input an amount (or had a number previously stored) for the mortgage balance. This is performed as a safety measure just in case you did not clear the Financial Registers before starting a problem in which you know the loan balance and enter it into Memory Register 5. If you know the estimated mortgage loan balance, you do not need to input data on the underlying loan. Indeed, the program gives priority to a loan balance which is stored in Memory Register 5, and thus bypasses the loan balance computation performed in program steps 05 through 11.
9. Seller's desired net proceeds from the sale, store in Memory Register 7.
10. Overall memory usage: **P - 92 r - 08.**

Summary of registers used:

Memory Registers: 0, 1, 2, 3, 4, 5, 6, and 7 are used by the program. The remaining 12 storage registers (8 through and including .9) are used to generate the program internally.

Financial Registers: All five financial registers (n, i, PV, PMT, and FV) are used if you cause the program to solve for the estimated loan balance.

On the other hand, if the property is not subject to an existing mortgage, or if you input an estimated amount for the mortgage loan into Memory Register 5, the financial registers will not be used, even if you unknowingly had data stored in them from a previous problem. However, you should still clear all registers (f, CLX) before you start a problem.

Caution: If you stop and then restart any HP 12C program before it completes it cycle, you can get incorrect results, depending upon design of the program. The above program is one of those which **may give you incorrect** results if you stop it and then restart it before it completes its cycle. **Always reconfirm your results** by pressing the Run/Stop (R/S) key a second time after your results are first displayed. Though we do not give proofs of the results obtained in the examples that follow, in practice you should always proof your results.

Problems

The following information is given:

	PROBLEM #1	PROBLEM #2
PRICE OF PROPERTY [STO] 0	$200,000	$125,000
COMMISSION AS PERCENT OF PRICE [STO] 1	6%	7%
AGE OF MORTGAGE AT TIME OF CLOSING [STO] 2	60 months	not needed
INCIDENTAL EXPENSES [STO] 3	$850	$500
ORIGINAL MORTGAGE LOAN [CHS] [PV]	$150,000	not needed
MORTGAGE TERM [n]	360 months	not needed
ANNUAL MORTGAGE INTEREST RATE [g] [i]	11%	not needed
LOAN BALANCE AT TIME OF CLOSING [STO] 5	not needed	$94,202

Calculate the seller's estimated net proceeds from the sale.

Problem #1

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter program into your HP 12C		
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter loan term	360 n	360.00
Enter monthly interest rate	11 g i	0.92
Enter loan amount, negative-signed	150,000 CHS PV	-150,000.00
Store estimated price of property	200,000 STO 0	200,000.00
Store commission as % of price	6 STO 1	6.00
Store number of payments made on loan by time of closing	60 STO 2	60.00
Store incidental expenses	850 STO 3	850.00
Compute net sales proceeds	R/S	41,403.03
Reconfirm net sales proceeds	**R/S**	**41,403.03**

Comment: The program computed the loan balance due after sixty payments are made on the mortgage, deducted six percent commission from the estimated $200,000 sales price, and also deducted $850 in estimated incidental expenses. The seller can expect a net of $41,403 from the sale.

Extension of Problem #1: Keeping the same data in your calculator, tell the calculator that the seller requires a net of $41,403.03 from the sale. Do this by storing the computed net into Memory Register 7. Then solve for the required sales price. (Note that since the program now knows the commission rate (it's in Memory Register 1) and required net (stored in Memory Register 7), the only variable left which can be solved for is the selling price. Let's solve it on the next page.

Problem #1 Extension

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Store computed net	STO 7	41,403.03
Compute required sales price	R/S	200,000.00
Reconfirm required sales price	**R/S**	**200,000.00**

Comment: The required sales price of $200,000 is confirmed. Make sure that you always reconfirm the accuracy of your computed result by pressing the Run/Stop (R/S) key after your answer first displays. Remember: If you stop the program before you get an answer, the result that will display could be in error.

Second Extension of Problem #1: Retain the data in your calculator. Then tell the calculator that the commission percentage is unknown. We do this by setting Memory Register 1 to zero. Everything else remains the same. Since the program is designed to solve for (1) net sales price, and it knows this amount, and is also designed to solve for (2) the required sales price, and it knows that amount, the only variable left is (3) the percentage of commission. Let's solve for it below.

Problem #1 Second Extension

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear Memory Register 1	0 STO 1	0.00
Solve for commission percentage	R/S	6.00
Reconfirm commission percentage	**R/S**	**6.00**

Comment: The required commission of six percent of sale price is confirmed. Again, always reconfirm the accuracy of your computed result by pressing the Run/Stop (R/S) key after your answer first displays. Remember: If you stop the program before you get an answer, the result that will display could be in error. Once you press [R/S], let the calculator complete the process.

Question: What happens if you have Memory Register 7 (holds required net from the sale), Memory Register 1 (holds required commission), and Memory Register 0 (holds sales price) loaded with data before you operate the program? The program will generate the sales price. As you please, you might read through the program and see why this is so. There is no way around this issue since there is not enough memory remaining to enable us to cause the program to discontinue operation if registers 0, 1, and 7 have data stored in them. Overall, this is a minor inconvenience, and indeed these minor programming limitations make all of us think more carefully when making inputs into a program.

Let's solve Problem #2 on the next page.

Problem #2

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers	f CLX	0.00
Store estimated price of property	125,000 STO 0	125,000.00
Store commission as % of price	7 STO 1	7.00
Store incidental expenses	500 STO 3	500.00
Store estimated loan	94,202 STO 5	94,202.00
Compute net sales proceeds	R/S	21,548.00
Reconfirm net sales proceeds	**R/S**	**21,548.00**

Comment: The program determined that the loan balance was $94,202. It then deducted this amount from the sales price ($125,000) net of seven percent commission ($8,750). Next, it deducted the estimated incidental expenses of $500.

Extension of Problem #2: Keeping the same data in your calculator, tell the calculator that the seller requires a net of $21,548.00 from the sale. You do this by storing the computed net into Memory Register 7. Then solve for the required sales price. Note that since the program now knows the commission (it is stored in Memory Register 1) and required net (stored in Memory Register 7), the only variable left which it can solve for is the selling price. Let's solve it below.

Problem #2 Extension

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Store computed net	STO 7	21,548.00
Compute required sales price	R/S	125,000.00
Reconfirm required sales price	**R/S**	**125,000.00**

Comment: The required sales price of $125,000 is confirmed. Again, make sure that you always reconfirm the accuracy of your computed result by pressing the Run/Stop (R/S) key after your answer first displays. Remember: If you stop the program before you get an answer, the result that will display could be in error.

Second Extension of Problem #2: Keeping the same data in your calculator, tell the calculator that the commission percentage is unknown. We do this by setting Memory Register 1 to zero. Everything else remains the same. Since the program is designed to solve for (1) net sales price, and it knows this amount, and is designed to solve for (2) the required sales price, and it knows that amount, the only variable left is (3) the commission expressed as a percentage of sales price. Let's solve the problem on the next page.

Problem #2 Second Extension

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Store zero in Mem. Reg. 1	0 STO 1	0.00
Solve for commission percentage	R/S	7.00
Display all digits	f 9	7.000000000
Reconfirm commission percentage	**R/S**	**7.000000000**

Comment: The required commission of seven percent of sale price is confirmed. Again, always reconfirm the accuracy of your computed result by pressing the Run/Stop (R/S) key after your answer first displays. Remember: If you stop the program before you get an answer, the result that displays could be in error. Once you press [R/S], let the calculator complete the process.

Question: What happens if you have Memory Registers 7 (holds required net from the sale), "1" (holds required commission), and "0" (holds sales price) loaded with data before you operate the program? The program will generate the sales price. If you haven't read through the program to figure out why this happens, try it, or follow the next few paragraphs.

Answer: Look at the first "test" the program undergoes; it's at step "01". Here we test to see if a loan balance has been stored in Memory Register 5. The reason for this is to pretty much assure that you won't get into trouble by having the program take into account two loan balances in the rare instance where you might have data stored in the Financial Registers and Memory Register 5 at the same time. At step "02", we test to see if the register is empty. If so, we start to calculate the loan's balance at step "05".

One way or the other, we get to step "12". The next thing we do is to check to see if you told the program the amount of the required net from the sale. If you did, we surely don't want to solve for this, so the program ushers us along to step "41". We don't stay there very long because at step "43" we determine whether or not you gave the program a percentage amount for the commission. If you did, we sort of "fail" the test at step "44", because register 1 was not set to zero, so we go directly to step "46".

At line "46" we have to get things a bit realigned (because the loan balance keeps trying to get away from us), that is why we use the Roll Down Key to **roll** the data in the STACK down one line. I could as well have used the "x-to-y" key; both keys aid in moving the STACK down. At this stage the reality of a 99 program line calculator comes to light! We have five more program lines available before the program uses Memory Register 7. Therefore there are not enough lines available to enable us to perform another test to see if we indeed entered the sales price and still be able to cause several zeroes ("0") to appear in the display after making this determination.

Thus, at program line "46" we have to assume that we are solving for the required sales price needed to make financial sense out of the commission rate, loan balance (if any), and incidental expenses.

PRACTICE

	Problem #1	**Problem #2**
NET PROCEEDS TO THE SELLER	unknown	unknown
PRICE OF PROPERTY [STO] 0	$85,000	$195,000
COMMISSION AS PERCENT OF PRICE [STO] 1	8%	5%
AGE OF MORTGAGE AT TIME OF CLOSING [STO] 2	67 months	not needed
INCIDENTAL EXPENSES [STO] 3	$500	$750
ORIGINAL MORTGAGE LOAN [CHS] [PV]	$59,500	not needed
MORTGAGE TERM [n]	360 months	not needed
ANNUAL MORTGAGE INTEREST RATE [g][i]	11%	not needed
LOAN BALANCE AT TIME OF CLOSING [STO] 5	not needed	$95,550.00
Net to the Seller [R/S]	**$20,150.97**	**$88,950.00**

	Problem #3	**Problem #4**
DESIRED NET PROCEEDS TO THE SELLER [STO] 7	$50,000.00	$50,000.00
PRICE OF PROPERTY [STO] 0	unknown	unknown
COMMISSION AS PERCENT OF PRICE [STO] 1	8%	6%
AGE OF MORTGAGE AT TIME OF CLOSING [STO] 2	67 months	not needed
INCIDENTAL EXPENSES [STO] 3	$900	$750
ORIGINAL MORTGAGE LOAN [CHS] [PV]	$159,500	not needed
MORTGAGE TERM [n]	360 months	not needed
ANNUAL MORTGAGE INTEREST RATE [g][i]	12%	not needed
LOAN BALANCE AT TIME OF CLOSING [STO] 5	not needed	$55,550.00
Required Price of Property [R/S]	**$223,994.31**	**$113,085.11**

	Problem #5	**Problem #6**
DESIRED NET PROCEEDS TO THE SELLER [STO] 7	$50,000.00	$100,000.00
PRICE OF PROPERTY [STO] 0	$214,000	$404,500
COMMISSION AS PERCENT OF PRICE [STO] 1	unknown	unknown
AGE OF MORTGAGE AT TIME OF CLOSING [STO] 2	100 months	98 months
INCIDENTAL EXPENSES [STO] 3	$900	$950
ORIGINAL MORTGAGE LOAN [CHS] [PV]	$159,500	$300,000
MORTGAGE TERM [n]	360 months	360
ANNUAL MORTGAGE INTEREST RATE [g][i]	11%	11.125%
LOAN BALANCE AT TIME OF CLOSING [STO] 5	not needed	not needed
Commission as percent of price [R/S]	**6%**	**4.96%**
Commission to 9 decimal places	**6.003541215%**	**4.958919110%**

SOLUTION TEMPLATE FOR COMPUTING SELLER'S ESTIMATED NET PROCEEDS

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END MODE	f CLX f 2 g END	0.00
Enter existing mortgage term (Not needed if balance known)	n	
Type annual interest rate (Not needed if balance known)	g i	
Enter loan amount, negative-signed (Not needed if balance known)	CHS PV	
Store estimated price of property	STO 0	
Store commission as % of price	STO 1	
Store estimated # of payments made on loan by time of closing (Not needed if balance known)	STO 2	
Store estimated incidental expenses (title costs, etc.)	STO 3	
Store estimated loan balance at time of closing, if known	STO 5	
Store required net, if known (Do not use if solving for net)	STO 7	
Compute unknown variable: (1) Net proceeds of sale; or (2) Required sale price; or (3) Maximum % commission.	R/S	
Reconfirm unknown variable	**R/S**	

BUYDOWN MORTGAGES

A buydown or "bought down" mortgage is traditionally a fixed interest rate, level payment mortgage loan which has the potential for making it easier for the residential property buyer to income-qualify for a loan. With the buydown loan a third party--usually the seller, builder, parents, or relatives of the buyer--makes an up-front payment to the lender at the closing of the mortgage loan, the result of which is to lower the borrower's mortgage interest rate for a specified number of years, usually from three to five years, though possibly as long as ten years. The result of lowering the borrower's loan interest rate is to lower the monthly mortgage payment over the length of the buydown term.

The up-front payment to the lender financially parallels what we do in making a payment of points to a lender in order to raise its yield beyond the nominal interest rate stated in the mortgage loan documents. With the buydown, however, the "points" or "buydown funds" which are paid to the lender indeed are exactly the amount of money needed to raise the lender's yield up to the nominal interest rate stated in the mortgage loan documents. This is needed because the borrower will have a reduced monthly mortgage payment over the course of the agreed upon buydown term.

Note that it is also possible for the buyer to pay the buydown fee rather than a third party, although it is typically a third party who makes the required payment to the lender. No matter who makes the payment, the lender is generally required to deposit the buydown funds or subsidy payments into a separate custodial account and cannot include them in an account with other corporate funds.

The mortgage loan documents must reflect the permanent payment terms rather than the terms of the buydown agreement itself. That is, the terms of the buydown agreement cannot change the terms of the primary mortgage loan documents--the note and the mortgage. Therefore, the borrower could theoretically have an excess of buydown funds in the lender's custodial account in the event the mortgage loan is paid off or the property is sold before the buydown funds are depleted. If this occurs, the terms of the buydown agreement control the disposition of the buydown funds remaining in the account. In practice, always concern yourself with the issue of who gets the buydown funds if the mortgage is paid off before all of the funds have been applied.

Permanent buydowns

A buydown can be (1) permanent or (2) temporary. With the permanent buydown the annual interest rate is effectively lowered for the full length of the mortgage term. The effect of this is to permanently lower the borrower's monthly loan payment during the complete term of the loan (e.g., 30 years). Note that since the interest rate is **permanently** "bought down", it is questionable to call this type of financing arrangement a "buydown" in the true sense of its meaning.

Since customary loan underwriting standards provide for a maximum buydown term of ten years, you simply cannot have a true buydown loan with a term which exceeds ten years. Therefore, it is incorrect to designate a permanently "bought down" loan a "buydown" loan within the context of what the secondary mortgage market considers acceptable.

Financially, what the buyer gets in the "permanent buydown" is a lower interest rate. This is achieved by paying a certain amount of "points" up-front. For example, if we assumed the lender's nominal annual interest rate is 10% on a $100,000, 30 year loan, the fixed monthly payment would be $877.57. However, if the interest rate was "bought down" to, say, 9% for the complete term of the loan, the borrower's monthly payment would be $804.62, which represents a difference or "shortfall" of $72.95 per month. This difference would be made up by making an up-front, one-time payment to the lender in an amount which is **equivalent to** the discounted present value of the stream of 360 "shortfalls" of $72.95 per month. Thus, the lender would be paid **$8,312.71** at the closing of the loan.

In keystroke-form, the problem looks like this:

KEYSTROKES:	f CLX f 2	Clear all registers; set 2 places
	360 n	Loan term
	10 g i	Monthly interest rate
	100,000 CHS PV	Loan amount
	PMT	Monthly payment at 10% interest
	DISPLAYS: 877.57	
	9 g i	Bought down interest rate (monthly)
	PMT	Monthly payment at 9% interest
	DISPLAYS: 804.62	
	-	Nets the two mortgage payments
	DISPLAYS: 72.95	
	f RND PMT	Rounds and enters difference
	10 g i	Monthly loan interest rate
	PV	Discounted present value of "shortfall"
	DISPLAYS:	
	-8,312.71	Amount needed to fund the buydown

Conclusion: To permanently "buy down" the 360 month, $100,0000 mortgage loan from 10% annual interest to 9%, the lender must receive $8,312.71 at--or before--the closing of the loan. There are, however, possible exceptions to the requirement that the lender be in receipt of the buydown funds at or before the closing. One such exception concerns funds agreed to be paid by the borrower's employer when it moves its headquarters and transfers employees from one location to another. (Always check with your local lender in order to determine if and how this exception might impact a purchase or sale you are working on.)

Temporary buydowns

With the temporary buydown the annual interest rate is effectively lowered for an agreed upon period **less than** the full term of the mortgage loan. For example, the interest rate--and therefore the monthly mortgage payment--

might be lowered for three to five or even ten years. Indeed, there are numerous variations in these plans, though a common plan involves a "3-2-1 percent" reduction. With this plan the borrower's annual interest rate is effectively lowered by three percent in the first year, by two percent in the second year, and by one percent in the third year. Therefore, the regular monthly payment reflected by the loan's full annual interest rate would be made starting with the fourth year and continues throughout the balance of the mortgage term.

For example, if we again assumed the lender's nominal annual interest rate is 10% on a $100,000, 30 year loan, the fixed monthly payment would be $877.57. However, under a "3-2-1 percent" plan, the interest rate would be bought down to 7% for the first year, 8% in the second year, and 9% in the third year. The resulting monthly mortgage payments would be $665.30 in the first year, $733.76 in the second year, $804.62 in the third year, and $877.57 for the balance of the term (years 4 through 30).

The monthly "shortfalls" to the lender under the above "3-2-1 percent" buydown plan would be $212.27 in the first year, $143.81 in the second year, and $72.95 in the third year. In tabular form, it looks like this:

YEAR	**Note Rate**	**Buydown Percent**	**Effective Rate**	**Monthly PMT @ 10%**	**Borrower's Payment**	**Monthly Shortfall**
1	10%	3%	7%	$877.57	$665.30	$212.27
2	10%	2%	8%	877.57	733.76	143.81
3	10%	1%	9%	877.57	804.62	72.95
4-30	10%	-0-	10%	$877.57	$877.57	0.00

Computing the amount needed to fund the buydown agreement

To calculate the amount of money which must be paid to fund the buydown agreement we discount the stream of "shortfalls" at the mortgage interest rate back to their present value. Thus, this method assumes that the lender is crediting or "paying" the same yield on the buydown fund as that received on its mortgage loan.

In effect, you should think of the buydown fund itself as a sort of bank account or trust which the lender calls upon every month to supply the "shortfall" attributed to the borrower making a "bought down" payment which is indeed lower than that required at the mortgage note interest rate. Using the above example, during the first year of the loan the lender would call upon the buydown fund to provide twelve payments of $212.27, followed by twelve payments of $143.81 in the second year, and finally twelve payments of $72.95 in the third year.

In the keystroke procedure which follows we will determine that the amount needed to fund the buydown agreement (at the mortgage interest rate) is $4,575.11. This is accomplished by simply discounting all the "shortfalls" at the mortgage interest rate back to the inception of the loan ("time period zero").

KEYSTROKES:	f CLX f 2	Clear all registers; set 2 places
	10 g i	Inputs monthly note interest rate
	212.27 g CFj	Inputs first year shortfall
	12 g Nj	Inputs 12 payments
	143.81 g CFj	Inputs second year shortfall
	12 g Nj	Inputs 12 payments
	72.95 g CFj	Inputs third year shortfall
	12 g Nj	Inputs 12 payments
	f NPV	Computes discounted present value
	DISPLAYS:	
	4,575.11	Amount needed to fund the buydown

Comment: To fund the buydown agreement, the lender will (generally) require a payment of $4,575.11 to be made at the closing of the loan. This will create a fund which will be sufficient to provide the lender with exactly the amount of the shortfalls we calculated above. Looking at this issue a little differently, we can say that if the lender receives $4,575.11 as an up-front payment at the closing and in turn lends $100,000, and thereafter receives 12 monthly payments of $665.30 in the first year, followed by 12 payments of $733.76 in the second year, followed by 12 payments of $804.62 in the third year, and finally 324 consecutive monthly payments of $877.57, its yield (IRR) will be 10%. Let's prove it below:

KEYSTROKES:	f CLX	Clears all registers
	4,575.11 ENTER	Loads buydown payment into the STACK
	100,000 -	Subtracts loan from buydown payment
	g CFo	Enters lender's out of pocket cash-flow (-$95,424.89) into calculator
	665.30 g CFj	First year bought-down payment
	12 g Nj	Enters 12 payments
	733.76 g CFj	Second year bought-down payment
	12 g Nj	Enters 12 payments
	804.62 g CFj	Third year bought-down payment
	12 g Nj	Enters 12 payments
	877.57 g CFj	Enters 4th through 30th year payment
	99 g Nj	Enters maximum number of cash-flows
	[R↓] g CFj	Enters $877.57, 5th cash-flow group
	99 g Nj	Enters maximum number of cash-flows
	[R↓] g CFj	Enters $877.57, 6th cash-flow group
	99 g Nj	Enters maximum number of cash-flows
	3 x 36 + 360 -	We need 27 more $877.57 payments
	[R↓] g CFj	Enters $877.57, 7th cash-flow group
	27 g Nj	Enters final number of payments
	f IRR	Computes lender's yield
	DISPLAYS: 0.83	
	RCL g i	Converts to annual yield
	DISPLAYS:	
	10.00	

Comment: The above example assumed that the lender granted the borrower the same interest rate on the buydown fund as the interest rate stated in the mortgage or promissory note. If this is not the case, then you would not use the mortgage interest rate as the discount rate when computing the discounted present value (NPV) of the "shortfalls" to the lender. Instead, you would use the monthly equivalent of the annual interest rate expected to be earned by the buydown fund.

For example, if the buydown funds are to be supplied from an account (trust, etc.) expected to earn, say, an annual rate of 5%, with monthly compounding, you would input the monthly equivalent of 5% into your [i] register when discounting the shortfalls back to their present value. For practice, you should use 5% annual interest to discount the "shortfalls" we worked with on the previous page. You will determine that the buydown amount increases from $4,575.11 (at 10% discount rate) to $4,848.90. The keystrokes follow:

KEYSTROKES:	f CLX f 2	Clear all registers; set 2 places
	5 g i	Inputs monthly yield to the borrower
	212.27 g CFj	Inputs first year shortfall
	12 g Nj	Inputs 12 payments
	143.81 g CFj	Inputs second year shortfall
	12 g Nj	Inputs 12 payments
	72.95 g CFj	Inputs third year shortfall
	12 g Nj	Inputs 12 payments
	f NPV	Computes discounted present value
	DISPLAYS:	
	4,848.90	Amount needed to fund the buydown

Comment: Since we used a lower yield or discount rate, the amount needed to fund the buydown must be greater.

Of course, if the borrower is not allowed a discount on the buydown "shortfalls", the amount needed to fund the buydown agreement increases substantially. Using the above example, and assuming the "shortfalls" are not discounted by the lender, the borrower would be required to pay $5,148.36 at the closing of the loan. We arrive at this figure as follows:

KEYSTROKES:	212.27 ENTER 143.81 + 72.95 + 12 x	Add 1st year, 2nd year, and 3rd year "shortfalls"; multiply times 12.
	DISPLAYS:	
	5,148.36	Amount needed to fund the buydown

Common underwriting considerations

In structuring or reviewing a buydown agreement you must take into account numerous underwriting requirements necessary to make the loan salable in the secondary mortgage market. However, it would be impossible to try to cover these requirements in this type of book. This is particularly so since the underwriting requirements for "buydowns" in the secondary mortgage market are not only voluminous but as well they are always subject to change, and in fact they vary (somewhat) from one secondary market purchaser to another. Thus, in practice, always check with your local lender or call your regional FNMA or FHLMC office so you can familiarize yourself with the most recent underwriting requirements for the type of buydown loan you are working with.

BLENDED RATE MORTGAGE

A parcel of real estate sold for $250,000. The buyer will make a down payment of $50,000 (20% of $250,000) and secures a **blended rate loan** from the seller's bank. Seller's loan--the underlying loan--is a 30 year, 12% annual rate, $100,000, first mortgage on which 120 payments of $1,028.61 have been made. The bank's current market rate for loans of this type is 14% annual interest rate. Compute the blended rate and the buyer's monthly payment.

ORGANIZATION OF DATA

Sale price	$250,000.00
Down payment	$50,000
Underlying loan balance	?
Underlying loan interest rate	12%
Underlying loan monthly payment	$1,028.61
Interest rate on "new money"	14%
Amount of new money	?
Blended interest rate	?
Term of blended rate mortgage (360 - 120 = 240)	240 months

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Compute balance on underlying loan:	120 n	120.00
	12 g i	1.00
	100,000 CHS PV	-100,000.00
	1,028.61 PMT	1,028.61
	FV	93,418.59
Compute new money:		
Enter sales price	250,000 ENTER	250,000.00
Less: Underlying loan balance	RCL FV -	156,581.41
Less: Down payment	50,000 -	106,581.41
Compute payment on new money:		
Enter new money in PV	CHS PV	-106,581.41
Clear FV register	0 FV	0.00
Enter blended rate loan term	240 n	240.00
Enter monthly interest rate	14 g i	1.17
Compute and round payment	PMT f RND	1,325.36
Add payment on underlying loan to blend payment; store in PMT	1,028.61 + PMT	2,353.97
Enter amount financed in PV	250,000 ENTER 50,000 - CHS PV	-200,000.00
Compute blended rate	i	1.09
Convert to annual rate	RCL g i	13.08

INTERNAL RATE OF RETURN

The Internal Rate of Return (IRR) is defined as the interest rate that equates the present value of the expected future cash-flows to the initial cost outlay. Put slightly different, Internal Rate of Return is the discount or yield rate that equates the sum of the present values of all cash-flows to the amount of the initial investment outlay.

The concept of IRR can also be analogized with a passbook savings account. From this perspective you should think of the investor's out-of-pocket cash-flow (the initial cash outlay) for the project as being equivalent to the initial deposit into an interest bearing savings account. The interest rate earned on the account is equivalent to the IRR of the investment. In fact, the combination of (1) investment in the account (includes initial deposit, plus later deposits, if any) and (2) interest paid at the IRR rate is sufficient to generate the property's projected (or actual) annual cash-flows.

After we complete the keystrokes necessary to solve the problem which follows we will show that (1) the computed IRR fits nicely into the concept of interest paid on an interest bearing passbook account, and we will further show that (2) the computed IRR is exactly the interest rate needed to discount the cash-flows back to the amount of the initial investment outlay.

In solving for IRR your calculator **must** use iterative solution methods since there are no known methods for directly solving for this variable. Therefore, you will find it takes a little longer to solve for IRR on your HP 12C than, for example, the time needed to solve for payment (PMT) or future value (FV). Both of these functions can be solved with little mathematical sophistication. This is clearly not the case when solving for IRR.

Problem: Assume an investor purchased an income producing property at a discount rate of 14% per annum. The out-of-pocket cash-flow was $83,271.13. Five years later the property was sold. The investor now looks over the cash-flows to see whether they produced a yield superior to that used in making the investment decision. The investment's cash-flows follow:

Initial cash-flow (CFo)	$ 83,271.13
1st year NOI (CF1)	-1,000.00
2nd year NOI (CF2)	6,975.00
3rd year NOI (CF3)	6,975.00
4th year NOI (CF4)	7,986.00
5th year NOI (CF5)	8,619.00
Sales price (end of 5th year)	$149,750.00
Less 7% commission	

Solution

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 places	f CLX f 2	0.00
Enter investment outlay	83,271.13 CHS g CFo	-83,271.13
Enter first year NOI	1,000 CHS g CFj	-1,000.00

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter 2nd and 3rd year NOI;	6,975 g CFj	6,975.00
enter number of cash-flows	2 g Nj	2.00
Enter fourth year NOI	7,986 g CFj	7,986.00
Compute sale price less commission:		
Enter sales price into STACK;	149,750 ENTER	149,750.00
compute and deduct commission	7 % -	139,267.50
Add fifth year NOI and enter total for fifth year cash-flow	8,619 + g CFj	147,886.50
Compute internal rate of return;	**f IRR**	**16.10**
store in memory register 6;	STO 6	16.10
display all digits	f 9	16.10077996

Note: You will use the computed IRR in the passbook analogy example.

Discussion: It is apparent that our hypothetical investor did better than expected since the computed IRR (16.10%) is greater than the yield decision criteria (14%) used in acquiring the property.

Of course, the results here do not take into account the tax implications of the acquisition. A detailed tax-oriented analysis would be outside the scope of this book. See, however, the SPREAD SHEET on page 154 and the detailed analysis which follows it.

Insofar as decision criteria associated with the concept of internal rate of return (IRR), if the projected IRR for a project is equal to or greater than the investor's required yield, the project is accepted, or at least it merits further investigation. If, however, the projected IRR is less than the investor's required yield, the project is rejected, or the price should be reconsidered and possibly negotiated to a level sufficient to increase the projected IRR up to the level required by the investor. In this regard the principal decision criteria associated with IRR is really no different than that applied when analyzing an investment on the basis of determining its Net Present Value (NPV). (Our coverage of NPV starts on page 147.)

The Passbook Analogy

Using a passbook analogy it is easy to see that the computed IRR (16.10078%) in the above problem parallels the interest rate which must be paid on a savings account where the depositor ("investor") makes an initial deposit of $83,271.13 at the time of opening the account, deposits an additional $1,000 at the end of the first year (this is the $1,000 loss), receives cash-flows (payments) of $6,975 at the end of the second and third years, followed by a payment of $7,986 at the end of the fourth year, and finishes with a withdrawal of $147,886.50 (the fifth year NOI of $8,619 plus the net sales price of $139,267.50) at the time the account is closed out.

Clearly, implied in the concept of internal rate of return is (1) the recovery of the investor's capital (usually in the form of the down payment, plus later cash investments into the property), plus (2) interest received from period-to-period. Subsequent investments into a property financially parallel the incurrence of a loss (such as the $1,000 loss in the above example).

To clarify the problem, you should verify the accuracy of the cash-flow illustration which follows. You can recall (RCL) the computed IRR from your memory storage register 6. We will run through the keystrokes for all five years in the cash-flow illustration.

KEYSTROKES:	83,271.13 ENTER RCL 6 % DISPLAYS: 13,407.30	Determine interest earned during first year of ownership
	+ 1,000 + DISPLAYS: **97,678.43**	Investment value at end of first year of ownership
	RCL 6 % DISPLAYS: 15,726.99	Determine interest earned during second year of ownership
	+ 6,975 - DISPLAYS: **106,430.42**	Investment value at end of second year of ownership
	RCL 6 % DISPLAYS: 17,136.13	Determine interest earned during third year of ownership
	+ 6,975 - DISPLAYS: **116,591.55**	Investment value at end of third year
	RCL 6 % DISPLAYS: 18,772.15	Determine interest earned during fourth year of ownership
	+ 7,986 - DISPLAYS: **127,377.70**	Investment value at end of fourth year
	RCL 6 % DISPLAYS: 20,508.80	Determine interest earned during fifth year
	+ 8,619 - 139,267.50 - DISPLAYS: **0.0001**	Investment value at end of fifth year. ("Account" fully depleted!)

Illustration of the IRR Problem and its Cash-Flow Components

YEAR	**1**	**2**	**3**	**4**	**5**
VALUE AT BEGINNING OF YEAR	$83,271.13	97,678.43	106,430.42	116,591.55	$127,377.70
IRR @ 16.10078%	13,407.30	15,726.99	17,136.13	18,772.15	20,508.80
CASH RECEIVED/ INVESTED	+1,000.00	-6,975.00	-6,975.00	-7,986.00	-8,619.00 -$139,267.50
VALUE AT END OF YEAR	$97,678.43	106,430.42	116,591.55	127,377.70	$ 0.00

Mathematics Side-Bar for Internal Rate of Return

The calculator most likely used any number of straightforward numerical analysis techniques (Bisection and/or Secant) to **wring** IRR out of the cash-flows we input. In effect, the calculator <u>very logically</u> **searched** for a discount rate which would equate the cash-flow equation you see below. Indeed, we have little problem calculating the IRR with the HP 12C because modern financial calculators have solution algorithms programmed into them which take the drudgery out of using "tables" to solve IRR problems.

The IRR equation discounts all projected cash-flows at a rate sufficient to cause the <u>sum</u> of the discounted present value of the cash flows to exactly equal the amount of the initial cash outlay. One form of the standard IRR equation follows below.

$$\mathbf{PV = CFo = \underset{t\,=\,1}{\overset{t\,=\,n}{SUMMATION}} \frac{CFt}{(1 + IRR\%)^{t}}} \quad \text{which is a shortened version of:}$$

$$\mathbf{CFo = \frac{CF1}{(1 + IRR\%)^{1}} + \frac{CF2}{(1 + IRR\%)^{2}} + \ldots + \frac{CFn}{(1 + IRR\%)^{n}}}$$

Let's use the data from the problem on page 137 to show that the IRR is indeed the discount rate which equates the discounted present value of a series of cash-flows to the value of the initial cash outlay for a project. Plug-in the cash-flows and the computed IRR (use 7 decimal places for extreme accuracy). From there we readily verify that the present value of the cash-flows is $83,271.13. (If you still have the computed IRR stored in memory register 6, you might recall it, convert it to its decimal equivalent, and carry the computation through using 10 place accuracy. The results, however, will be the same as you will attain in this example using 7 decimal place accuracy.)

$$\mathbf{CFo = \frac{-\$1{,}000}{(1 + .1610078)^{1}} + \frac{\$6{,}975}{(1 + .1610078)^{2}} + \frac{\$6{,}975}{(1 + .1610078)^{3}}}$$

$$\mathbf{+ \frac{\$7{,}986}{(1 + .1610078)^{4}} + \frac{\$8{,}619}{(1 + .1610078)^{5}} + \frac{\$139{,}267.50}{(1 + .1610078)^{5}}}$$

$$\mathbf{= -\$861.32 + \$5{,}174.57 + \$4{,}456.96 + \$4{,}395.30 + \$4{,}085.84 + \$66{,}019.78}$$

CFo = $83,271.13, or

PV = $83,271.13

Conclusion: It can readily be seen that the internal rate of return (IRR) of the investment's cash-flows is in fact the discount rate which equates the sum of the present values of all cash-flows back to the amount of the initial investment outlay.

MODIFIED INTERNAL RATE OF RETURN

Certain deficiencies in the Internal Rate of Return (IRR) method of analyzing projects with multiple negative cash-flows, or projects whose cash-flows are expected to produce questionably high rates of return, gave rise to the necessity of developing additional project discounting cash-flow techniques. One such technique, the Modified Internal Rate of Return (MIRR) method, and some reasons for its development are discussed below.

Discussion of the problem

When using the Internal Rate of Return (IRR) method to analyze a project's expected or actual cash-flows, it is assumed that all positive cash-flows are, in effect, reinvested at the computed discount rate and that all negative cash-flows are theoretically funded at the same rate. The IRR method further assumes that the recipient of a project's positive cash-flows can, in effect, reinvest these funds into an account--or other investment--and earn the same yield as the project's computed internal rate of return. Negative cash-flows are, in effect, assumed to be funded from a hypothetical investment (bank account, etc.) that earns interest or generates monies at the same internal rate of return (IRR) as the project itself. However, these assumptions are erroneous unless the internal rate of return falls within a realistic borrowing or investment opportunity yield range.

A more technical, though equally relevant, problem associated with the internal rate of return method concerns the possibility of computing multiple solutions when analyzing an investment which produces, or is expected to produce, multiple negative cash-flows. The more cash-flow sign changes (positive to negative and negative to positive) found or anticipated in a project, the greater the chance of finding more than one discount rate that mathematically solves the IRR problem. While multiple IRR solutions make mathematical sense, they make little, if any, financial sense and therefore must be used guardedly, or a method of analysis other than the IRR method should be considered.

MIRR methodology

The modified internal rate of return (MIRR) method of project cash-flow analysis assumes that all positive cash-flows are immediately reinvested at rates equal to the investor's (owner's) "cost of capital" or assumed reinvestment rate. In theory, this means depositing a project's positive cash-flows into investments yielding the highest rates of return for risks comparable to those of the original investment. Procedurally, this is carried out by computing the future value (FV) of all positive cash-flows at the assumed cost of capital rate over the term of the project being analyzed.

Negative cash-flows (anticipated or actual losses) are assumed to be funded (paid for) through investments made at a safe or sure rate of return, such as the United States Treasury Bill rate, at the beginning of the project. Procedurally, this is achieved by discounting all projected (or actual) negative cash-flows, at a "safe rate", back to the present value (PV) necessary to fund the anticipated future negative cash-flows. This would be equivalent to the investor establishing a contingency fund at the beginning of the project large enough to fund all anticipated negative cash-flows.

Having computed the future value (FV) of all positive cash-flows and the present value (PV) of all anticipated negative cash-flows, we add the project's initial cost outlay (negative signed) to the present value of the negative cash-flows (also negative signed) to arrive at the project's combined present value.

We can now compute the interest (discount) rate necessary to produce the future value of all positive cash-flows from the project's (negative signed) combined present value. Put another way, we seek the interest rate necessary to generate the future value (FV) of all positive cash-flows from the negative signed combined present value (PV). The computed interest rate is the project's modified internal rate of return (MIRR).

The MIRR method can readily be applied using the HP 12C's unequal cash-flow function, followed by application of the financial function keys. An example follows.

Example of a problem with multiple IRRs

	Initial Cash Outlay	End of First Year	End of Second Year
Cash-Flows:	-$50,000	$250,000	-$150,000
The Flows:	Initial	Positive	Negative
IRRs:	**-30.28 and 330.28**		

Keystroke solution

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 places	f CLX f 2	0.00
Enter investment outlay	50,000 CHS g CFo	-50,000.00
Enter first year cash-flow	250,000 g CFj	250,000.00
Enter final cash-flow	150,000 CHS g CFj	-150,000.00
Compute internal rate of return	**f IRR**	**-30.28**
(Store in Register 3)	STO 3	-30.28

Caution: At this stage it would appear that the internal rate of return (IRR) of this investment is approximately negative thirty percent (-30.28%). This, however, is clearly not the case. Indeed, the above problem--in respects we will not detail--falls into a range within which the HP 12C (as well as the HP 17BII and HP 19BII) cannot provide a complete answer. In effect, we must give the calculator's solution algorithms a little assistance in solving the above problem (as well as similar problems!). To do this we must make a **guess** at what the IRR is and then "run that number" through the calculator's internal programming. Let's try the following guesses and see what results the calculator provides: 1%, 10%, 50%, and 100%.

Note: The procedure used to assist the calculator's programming with these kinds of problems is to (1) type the guess, followed by (2) pressing the [RCL] key, (3) followed by pressing the BLUE [g] key, (4) followed by pressing the Run/Stop key [R/S].

Keystroke solution

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Use first guess	1 RCL g [R/S]	-30.28
Try second guess	10 RCL g [R/S]	-30.28
Try third guess	50 RCL g [R/S]	-30.28
Try fourth guess	**100 RCL g [R/S]**	**330.28**
(Store in Register 4)	STO 4	330.28

Discussion: The cash-flow sequence has two solutions to the IRR problem: 330.28% and -30.28%. In a financial sense, clearly, something is wrong here, though the mathematical answers provided are indeed correct. In practice one generally does not run into bizarre cash-flow scenarios such as the sequence used in this problem. Indeed, in this author's practise as a real estate attorney, I have never seen a cash-flow scenario as exaggerated as the one setforth above, short of similar cash-flow scenarios in finance texts and calculator books. In truth, I had to make up the cash-flows in the interest of emphasizing the multiple IRR problem which MIRR was intended to address.

There will be instances when unequal cash-flow sequences will cause your calculator to display **"Error 3"**. If this occurs, you should first check your input data to make sure it was typed correctly. If it was, you are placed on notice that the problem may have more than one IRR solution.

To solve an IRR problem which causes your HP 12C to display **"Error 3"**, you should follow the same keystroke sequence outlined above. That is, make a **first guess**; press [RCL], then press [g], and then press [R/S]. Repeat the process with a fairly wide range of **new guesses** until you begin to repeat the solutions. You must then decide if the answers are realistic or if you should switch to MIRR or some other project discounting method.

Note that the **first guess** used can also be a negative number if the cash-flows are expected to produce a yield (IRR) of less than zero. Said differently, if you expect the cash-flows to produce a loss, and therefore produce a negative IRR, you can start your guess procedure with a negative signed number, such as **-5, -20, -90**, and so forth. Even if your IRR turns out to be positive, in most cases this will not cause you any difficulties.

Proof of multiple IRRs generated above:

For IRR equals "-30.28%"

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear financial registers	f FIN	330.28
First year cash-flow into PV	250,000 CHS PV	-250,000.00
First IRR into [i] register	RCL 3 i	-30.28
Set [n] register to "1"	1 n	1.00
Value at <u>end</u> of 2nd year	FV	174,306.09
Add 2nd year cash-flow	RCL 2 +	24,306.09
Enter net of cash-flows as FV	FV	24,306.09
Enter cash outlay as PV	RCL 0 PV	-50,000.00
Enter investment/analysis term	2 n	2.00
Solve for yield (IRR)	i	-30.28
Verified result is correct	RCL 3 -	0.00000004

For IRR equals "330.28%"

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear financial registers	f FIN	0.00000004
First year cash-flow into PV	250,000 CHS PV	-250,000.00
Second IRR into [i] register	RCL 4 i	330.28
Set [n] register to "1"	1 n	1.00
Value at <u>end</u> of 2nd year	FV	1,075,693.91
Add 2nd year cash-flow	RCL 2 +	925,693.91
Enter net of cash-flows as FV	FV	925,693.91
Enter cash outlay as PV	RCL 0 PV	-50,000.00
Enter investment/analysis term	2 n	2.00
Solve for yield (IRR)	i	330.28
Verified result is correct	RCL 4 -	-0.0000001

Comment: It is unlikely that a potential investor could feel comfortable with an analysis which produces two separate yields for the internal rate of return, particularly so when the range in computed yields is as wide as that produced above. Modified Internal Rate of Return (MIRR) might be the financial tool of choice in these kinds of situations. Let's run a more civilized MIRR problem in the next section and leave it to your imagination as to how far to go with the MIRR technique in your business.

Problem: An investor evaluates an income producing property and projects the cash-flows setforth below. The internal rate of return (IRR) for these cash-flows is 16% annually. Using a **safe rate** of 5.5% and a **reinvestment rate** of 11.25%, calculate the project's expected internal rate of return (IRR) and modified internal rate of return (MIRR).

Initial cash-flow (CFo)	$ 83,317.21
1st year NOI (CF1)	-17,000.00
2nd year NOI (CF2)	15,000.00
3rd year NOI (CF3)	-9,000.00
4th year NOI (CF4)	17,500.00
5th year NOI (CF5)	19,000.00
6th year income + Sale (CF6)	$180,000.00

IRR Verification

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 places	f CLX f 2	0.00
Enter investment outlay	83,317.21 CHS g CFo	-83,317.21
Enter first year NOI	17,000 CHS g CFj	-17,000.00
Enter 2nd year NOI	15,000 g CFj	15,000.00
Enter 3rd year NOI	9,000 CHS g CFj	-9,000.00
Enter 4th year NOI	17,500 g CFj	17,500.00
Enter 5th year NOI	19,000 g CFj	19,000.00
Enter 6th year cash-flow	180,000 g CFj	180,000.00
Compute internal rate of return.	**f IRR**	**16.00**

MIRR Solution

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 places	f CLX f 2	0.00
Compute NPV of positive cash-flows:		
Enter initial investment as "0"	0 g CFo	0.00
Enter cash-flows:		
1st year -	0 g CFj	0.00
2nd year -	15,000 g CFj	15,000.00
3rd year -	0 g CFj	0.00
4th year -	17,500 g CFj	17,500.00
5th year -	19,000 g CFj	19,000.00

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
6th year -	180,000.00 g CFj	180,000.00
Enter reinvestment rate	11.25 i	11.25
Compute NPV; store in Memory Register 7	f NPV STO 7	129,638.72 129,638.72
Compute NPV of negative cash-flows:		
Enter initial investment as "0"	0 g CFo	0.00
Enter cash-flows:		
1st year -	17,000 CHS g CFj	-17,000.00
2nd year -	0 g CFj	0.00
3rd year -	9,000 CHS g CFj	-9,000.00
Enter safe investment rate	5.5 i	5.50
Compute NPV; store in available register	f NPV STO 8	-23,778.27 -23,778.27
Clear financial registers	f FIN	-23,778.27
Compute FV of positive cash-flows:		
Recall NPV of positive cash-flows, change sign and store in PV	RCL 7 CHS STO PV	129,638.72 -129,638.72
Enter number of cash-flows	6 n	6.00
Enter reinvestment rate	11.25 i	11.25
Compute FV	FV	245,773.38
Add initial cost outlay to NPV of negative cash-flows; store in PV	83,317.21 CHS RCL 8 + STO PV	-107,095.48
Compute MIRR	**i**	**14.85**

Comment: The modified internal rate of return (MIRR) is therefore 14.85% annually. By implication, the IRR of 16.00% overstates the yield on the investment to this investor by approximately 1.15%.

Caution should be exercised in making any representations as to the benefits of using MIRR versus IRR when the computed rates are as close as those arrived at in the above problem. Specifically, whether or not an investor can secure an additional 1.15% yield on his cash-flows when they fall within a range of possible investment opportunities is trying to walk a financial tightrope. Close calls like the one in this problem drive home the importance of being able to draw upon your sheer feel for an investment's desirability over and above what the "numbers" show. Clearly, the strongest of financial tools and techniques can never take the place of judgment, experience, and investment-prowess.

NET PRESENT VALUE

Discussion

Net Present Value (NPV) can be defined as the **difference in value**--either positive or negative--between the **discounted present value** of an investment's projected cash-flows (at the investor's required yield) and the initial cost outlay paid for the investment. For example, if you paid $100,000 for an investment which is expected to provide you with an income stream which, when discounted at your required yield, is worth--or equivalent to--$125,000 **today,** we can say that the **net present value** of the investment **in your hands** is $25,000. That is, you expect to receive a $25,000 "bonus" or "windfall" **at the inception (beginning)** of this particular investment over and above what you would otherwise expect from a similar investment.

This result, of course, obtains as follows:

Discounted Present Value of Cash-Flows at your required yield	$125,000
less:	
Cash Outflow for Investment	- 100,000
Equals a **NPV** of	**$ 25,000**

On the other hand, suppose your required investment yield is high enough to cause the discounted present value of the investment's projected income stream to equal $90,000. Under this scenario, the difference **(net)** between the present value of the cash-flows ($90,000) **and** the cost outlay ($100,000) is a negative number (-$10,000). Therefore, as far as you are concerned this would not be a good investment since its net present value (NPV) to you **is a negative number.**

Net Present Value (NPV) is nothing more than the **difference** between (1) the discounted present value of an investment's projected cash-flows (at your required yield rate) and (2) the initial cost outlay paid for or down on the investment. Insofar as general decision criteria built around this concept, if the NPV is equal to or greater than zero, you accept the investment; otherwise you reject it.

Problem: Let's compute the NPV and the IRR of an investment with the following projected cash-flows and required yield to the investor set at a discount (yield) rate of 14% per annum.

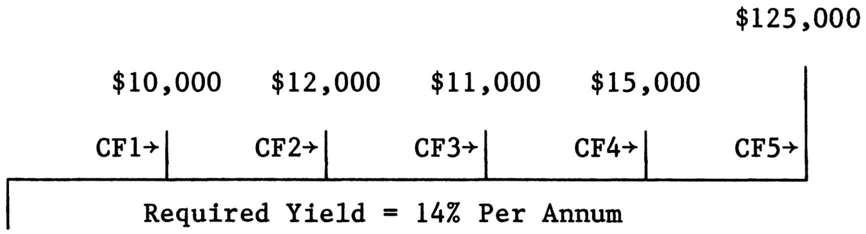
REQUIRED CASH-OUTLAY	$100,000	FOURTH YEAR NOI	$ 15,000
FIRST YEAR NOI	10,000	FIFTH YEAR CASH-FLOW (includes sale price less commissions, plus NOI)	$125,000
SECOND YEAR NOI	12,000		
THIRD YEAR NOI	11,000		

Remember: The internal rate of return (IRR) of an investment is **independent** of the net present value (NPV) though the two concepts are closely interrelated. Note that were we to use a **discount rate** equal to an investment's internal rate of return (IRR), the investment's net present value (NPV) will be **zero** since this would cause the cash-flows to discount to **exactly** the amount of the initial cash outlay for the investment. Thus, the **net** of the two amounts will be zero.

Cash-Flow Diagram

Let's first look at the above problem by utilizing a cash-flow diagram. This method is beneficial in that it gives the practitioner a much better feel for the flow of the numbers (cash-flows) through a problem.

Cash-Flow Diagram



$ 99,232.51 Discounted Present Value of Cash-Flows at 14% Yield
- $100,000.00 Out-of-Pocket Cash-Flow
NPV = -$767.49

Symbolically, the problem looks like this:

$$\mathbf{NPV} = \mathbf{SUMMATION}_{t=1}^{t=n} \frac{\mathbf{CFt}}{(\mathbf{1 + Yield})^{t}} - \textbf{Out-of-Pocket Cash-Flow}$$

$$= \$10{,}000/(1.14) + \$12{,}000/(1.14)^2 + \$11{,}000/(1.14)^3$$

$$+ \$15{,}000/(1.14)^4 + \$125{,}000/(1.14)^5 - \$100{,}000$$

$$= \$8{,}771.93 + \$9{,}233.61 + \$7{,}424.69 + \$8{,}881.20 + \$64{,}921.08 - \$100{,}000$$

$$= \$99{,}232.51 - \$100{,}000$$

NPV = -$767.49

Comment: It is apparent that the **net present value (NPV)** is indeed the <u>difference</u> between (1) the discounted present value of the projected cash-flows and (2) the investor's out-of-pocket cash outlay. NPV is a dynamic concept in that it can vary from investor to investor depending upon the required annual yield. However, this is not the case with IRR since it always remains the same, no matter who acquires the asset.

Keystroke Solution

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 places	f CLX f 2	0.00
Enter investment outlay	100,000 CHS g CFo	-100,000.00
Enter first year NOI	10,000 g CFj	10,000.00
Enter second year NOI	12,000 g CFj	12,000.00
Enter third year cash-flow	11,000 g CFj	11,000.00
Enter fourth year cash-flow	15,000 g CFj	15,000.00
Enter fifth year cash-flow	125,000 g CFj	125,000.00
Compute investment's yield	f IRR	**13.79**
Enter investor's required yield	14 i	14.00
Compute net present value	**f NPV**	**-767.49**

Comment: Since the investment's projected cash-flows are estimated to produce a yield (IRR) of 13.79%, which amount is <u>less than</u> the investor's required yield (14%), the investment fails. That is, it does not meet the yield criteria established by the investor. On the other hand, if we assumed the investor was agreeable to a yield of, say, 13.5%, which amount is lower than the investment's projected yield, the cash-flows input above will indeed produce a <u>positive</u> NPV, and therefore the investment will pass the investor's yield criteria.

Problem: Let's prove that the NPV will indeed be positive when the yield [i] is set to 13.5%. Then, reconfirm that the IRR remains 13.79%. Finally, set the required yield [i] to the computed IRR. Then confirm that the NPV will equal zero when the discount (yield) rate is set to the IRR%.

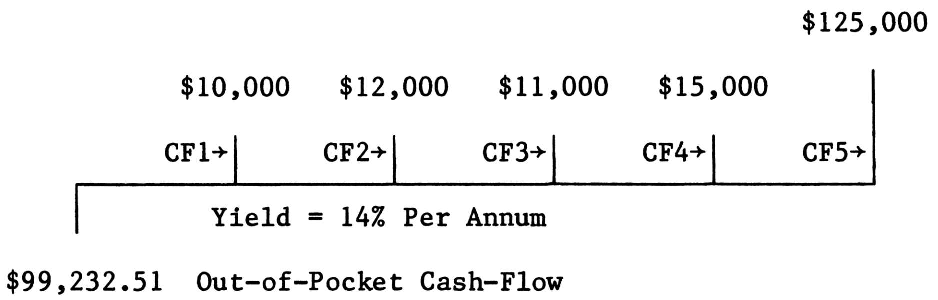
Solution

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter investor's required yield	13.5 i	13.50
Compute net present value	f NPV	**1,051.41**
Reconfirm the IRR equals 13.79%	f IRR	13.79
Set discount rate to the IRR *	i	13.79
Compute NPV	**f NPV**	**-0.000002**
Clear all registers	f CLX	0.00

* **Note:** The [i] register is automatically set to the computed "IRR". Thus, once you have a feel for this procedure, eliminate the step altogether.

Problem: Assume the investor secures the property for an out-of-pocket cash-flow ($99,232.51) that makes possible an internal rate of return of 14% annually. At this stage one might ask: What **fixed** income stream would be **equivalent** to the unequal cash-flows expected from the investment? Put slightly different, what fixed annual payment would give the investor benefits **equivalent** to the anticipated irregular cash-flow stream? This question can best be answered by employing the financial analysis technique known as computing the **uniform series payment.**

When we compute a **uniform series** of unequal cash-flows we seek a level payment stream which is financially equivalent to the discounted present value (NPV) of the stream of unequal cash-flows. This problem is solvable with the financial registers. In effect, we seek a **uniform** payment stream which takes the "bumps" out of the following cash-flow scenario:



Let's first verify that the discounted present value (NPV) of the income stream is $99,232.51 at a 14% yield to the investor.

KEYSTROKES:	f CLX	Clear all registers
	10,000 g CFj	First year cash-flow
	12,000 g CFj	Second year cash-flow
	11,000 g CFj	Third year cash-flow
	15,000 g CFj	Fourth year cash-flow
	125,000 g CFj	Final year cash-flow
	14 i	Required annual yield
	f NPV	Net present value of cash-flows
	DISPLAYS: 99,232.51	

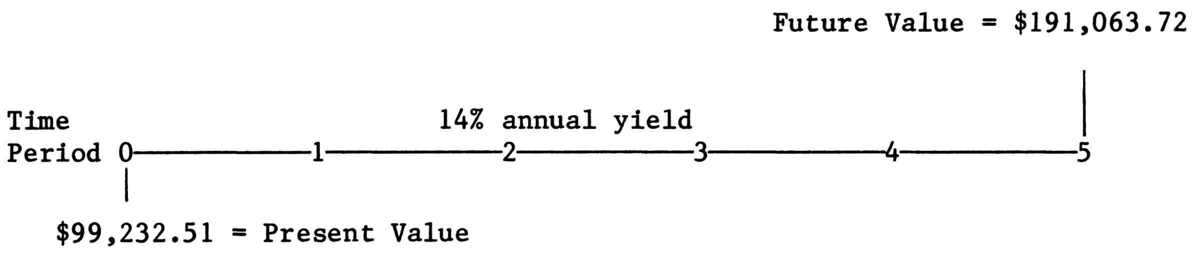
To calculate the **uniform series** of the discounted present value, we treat this amount ($99,232.51) as an out-of-pocket cash-flow and then calculate the fixed payment (PMT) which is **equivalent** to it. The keystrokes follow:

KEYSTROKES:	CHS PV	Input negative-signed present value
	PMT	Computes the uniform series payment
	DISPLAYS: 28,904.80	

Discussion: Theoretically, we should be indifferent in choosing between the unequal cash-flow stream--presented above--and its **uniform series payment** of $28,904.80. Both income streams are financial equivalents of each other because they produce the same discounted present value ($99,232.51). Therefore, both income streams should produce identical **future values** were we to project their cash-flows into the future.

Specifically, if we project a **present value (PV)** of $99,232.51 five years into the future at an annual compound yield of 14%, we should get the same result as projecting end of the year **payments (PMT)** of $28,904.80 five years into the future. As well, projecting the individual unequal cash-flows five years into the future should result in an identical future value. Let's perform the projections:

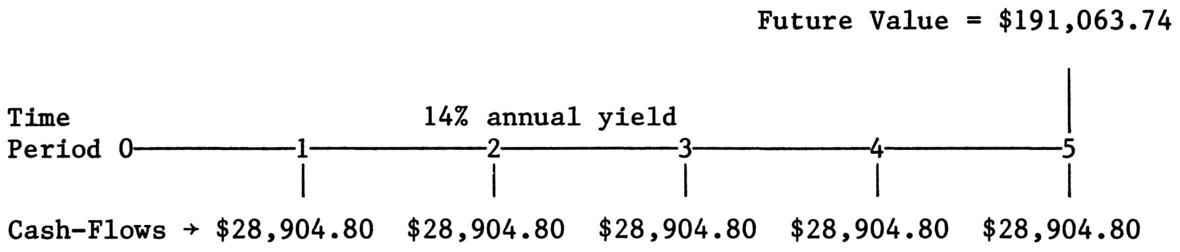
Question One: What is the Future Value of $99,232.51 "deposited" today at 14% annual yield if allowed to remain "on deposit" for 5 years in a tax-free account? The timeline diagram looks like this:



Solving by keystroke, we have:

KEYSTROKES:	f CLX	Clear all registers
	99,232.51 CHS PV	Inputs discounted PV of cash-flows
	5 n	Inputs investment ("deposit") term
	14 i	Input annual yield (growth rate)
	FV	Computes Future Value
	DISPLAYS: 191,063.72	

Question Two: What is the Future Value of a stream of $28,904.80 cash-flows (this is the same as a series of "deposits") over a five year period at a 14% annual compounded yield? The cash-flow diagram looks like this:

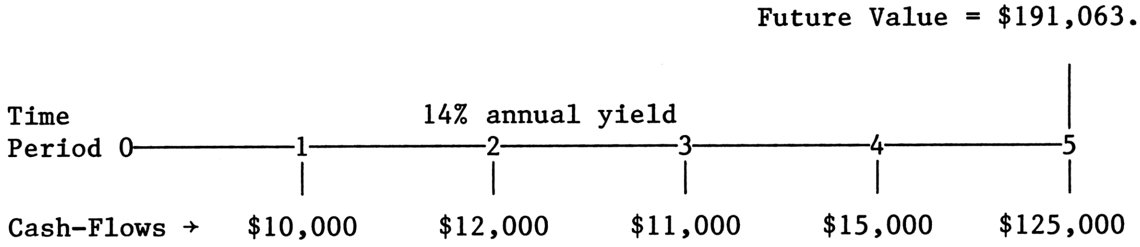


Comment: The cash-flows in the above diagram are below the timeline diagram in order to keep consistent with the traditional cash-flow sign convention.

Solving by keystroke, we have:

KEYSTROKES:	f CLX g END	Clear all registers; set END mode
	5 n	Inputs investment ("deposit") term
	14 i	Input annual yield (growth rate)
	28,904.80 CHS PMT	Inputs end of year cash-flow
	FV	Computes Future Value
	DISPLAYS:	
	191,063.74	(Rounding difference of "$0.02")

Question Three: What is the Future Value of the stream of unequal cash-flows ($10,000, $12,000, $11,000, $15,000, and $125,000)--this is the same as a series of "deposits"--over their five year term at a 14% annual compounded yield? The cash-flow diagram looks like this:



Solving by keystrokes, we have:

KEYSTROKES:	f CLX	Clear all registers
	4 n	Inputs term of $10,000 cash-flow
	14 i	Input annual yield (growth rate)
	10,000 CHS PV	First year cash-flow as present value
	FV	Compute future value
	STO 0	Store in Memory Register 0
	DISPLAYS:	
	16,889.60	
	3 n	Inputs term of $12,000 cash-flow
	12,000 CHS PV	Second year cash-flow as PV
	FV	Computes future value
	STO + 0	Adds to Memory Register 0
	DISPLAYS:	
	17,778.53	
	2 n	Inputs term of $11,000 cash-flow
	11,000 CHS PV	Third year cash-flow as PV
	FV	Computes future value
	STO + 0	Adds to Memory Register 0
	DISPLAYS:	
	14,295.60	

KEYSTROKES:	1 n	Inputs term of $15,000 cash-flow
	15,000 CHS PV	Fourth year cash-flow as PV
	FV	Computes future value
	STO + 0	Adds to Memory Register 0
	DISPLAYS: 17,100.00	
	RCL 0	Bring back FV of the four cash-flows
	DISPLAYS: 66,063.73	
	125,000 +	Adds final year cash-flow
	DISPLAYS: 191,063.73	(Again, slight rounding difference)

Comment: Subject to slight rounding differences, all three cash-flow streams produce the same Future Value. We note, of course, that in order for an investment which consists of **deposits** to generate the future values computed above, the yield on the investment must consistently be 14% per annum.

Using the technique of computing the **uniform series payment** has numerous applications in leasing, income property appraisal and real estate financial analysis. It is no longer necessary that you discount individual unequal cash-flows back to a present value and sum them to arrive at the value of an income producing investment, such as a leasehold interest. Indeed, using the unequal cash-flow function of any high level financial calculator makes short-work out of this problem, the HP 12C well-included.

In the author's opinion, **uniform series** analysis is the financial analysis technique of choice when trying to average-out or settle upon a damage amount for breach of a lease which includes payments structured as unequal cash-flows. Many commercial leases contain payments that are structured with skipped or missed payments (as pointed out in the lease section of this book), and indeed some will have increasing payment obligations throughout the term of the lease. Knowing how to **smooth out** such payment/income streams is invaluable in reaching a settlement in breach of lease disputes. This is not to say that the **uniform series** technique is the only issue to be considered in such matters, though it is and should be one.

SPREAD SHEET: Manual Cash-Flow Projection

	Rate of Increase	1995	1996	1997	1998
Potential Gross Income (PGI)	3%	$156,300	$160,989	$165,819	$170,793
Less: Vacancy & Bad Debts (3% of PGI)		4,689	4,830	4,975	5,124
Plus: Miscellaneous	4%	3,900	4,056	4,218	4,387
Gross Operating Income		$155,511	$160,215	$165,062	$170,056
Less: Operating Expenses					
Real & Personal taxes	2%	16,340	16,667	17,000	17,340
Management	4%	12,241	12,731	13,240	13,769
Property insurance	5%	6,961	7,309	7,675	8,058
Contract labor	6%	10,962	11,620	12,317	13,056
Repairs & Maintenance	7%	9,135	9,774	10,459	11,191
Supplies	5%	6,833	7,175	7,533	7,910
Utilities	3%	14,616	15,054	15,506	15,971
Professional fees	2%	2,284	2,330	2,376	2,424
Miscellaneous	2%	731	746	761	776
Total Operating Expense		80,103	83,406	86,867	90,495
NET OPERATING INCOME (NOI) =		**$75,408**	**$76,809**	**$78,195**	**$79,561**
Less: Debt Service:					
$500,500 @ 10%, 20 year term		57,959	57,959	57,959	57,959
NET or BEFORE-TAX CASH-FLOW (BTCF) =		**$17,449**	**$18,850**	**$20,236**	**$21,602**
Income Subject to Tax:					
Add principal payment		8,282	9,149	10,107	11,166
Minus depreciation		14,667	14,667	14,667	14,667
TAXABLE INCOME =		$11,064	$13,332	$15,676	$18,101
Less Taxes @ 36%		3,983	4,800	5,643	6,516
AFTER-TAX CASH-FLOW (ATCF) =		**$13,466**	**$14,050**	**$14,593**	**$15,086**

REVERSION

Original Price	$715,000	(Assumes 5% annual growth)
Mortgage	500,500	(Assumed 70% LTV Ratio)
Equity	214,500	(Land value, 20%. Depreciation, S.L. 39 years)

Expected Cash-Flow From Sale:

Expected Selling Price	$869,000
- Selling expenses (7%)	60,830
= Net sale proceeds	808,170
- Mortgage balance	461,796
= Before tax equity Reversion (ER)	$346,374
- Taxes due on sale	54,662
= After-Tax Equity Reversion	**$291,712**

Expected taxes Due on Sale:

Expected Selling Price	$869,000
- Selling expenses	60,830
= Realized on sale	808,170
- Original price	715,000
+ Depreciation	58,668
= Total gain from sale	$151,838
x Investor's tax bracket (36%)	0.36
= Taxes Due on Sale	**$ 54,662**

IRR AFTER TAX	IRR BEFORE TAX	KEYSTROKES (Combined)
f CLX f 2	f CLX f 2	Clear all registers; set 2 decimal places
214,500 CHS g CFo	214,500 CHS g CFo	Inputs investor's equity in property
13,466 g CFj	17,449 g CFj	Inputs first year ATCF/BTCF
14,050 g CFj	18,850 g CFj	Inputs second year ATCF/BTCF
14,593 g CFj	20,236 g CFj	Inputs third year ATCF/BTCF
15,086 ENTER	21,602 ENTER	Year of sale ATCF/BTCF
291,712 + g CFj	346,374 + g CFj	Add and input year of sale cash-flows
f IRR	f IRR	Computes AFTER/BEFORE-tax yield to investor
DISPLAYS: 13.95	**DISPLAYS: 20.35**	

Caution: This example is not totally representative of the tax aspects of a sale for each and every investor. Taxpayer-investors face an almost constantly changing tax system. Check with your tax advisor before making any tax representations as to a real estate transaction.

ANALYZING PROJECTS WITH VARIABLE CASH-FLOWS

Introduction

In determining the overall feasibility of an income producing property, the investor must make an analysis of the investment's profitability in relationship to his/her/its financial objectives. Thus, we must take into account the investment's historical as well as projected cash-flows, both positive (income from operations and sale proceeds) and negative (operating expense, taxes, etc.).* Insofar as our treatment of the federal income tax impact on a real estate investment, it must be kept to the absolute minimum. The process covered, however, should be instructive of the relative effect this component has on an investment's after-tax yield (IRR).

Insofar as an investment's cash-flows, we generally project and analyze two cash-flow sources: (1) the pre-tax and after-tax income produced through rental operations, and (2) the pre-tax and after-tax proceeds from the sale of the property at the expiration of the holding period. From our objective in **real estate problem solving**, we are concerned with a specific application of the Unequal Cash-Flow function of your HP 12C.

Clearly, all cash-flows arising from and through an income producing property are not "regular"! Indeed, in real estate investment analysis, the "Rule" is that cash-flows are irregular; that is, they are invariably **unequal** from year to year and period to period. To analyze irregular cash-flows in an efficient manner, we must use the IRR or NPV function; there is no getting around it. For the typical cash-flow patterns found in real estate investment analysis, the HP 12C will do an excellent job with a minimum of keystroke hassles. Since the calculator accepts up to 20 unequal cash-flow groups, it has plenty of reserve power to perform these calculations.

The Cash-Flows

Please refer to the **SPREAD SHEET: Manual Cash-Flow Projection** on the opposite page. It summarizes where we are headed and should be carefully followed along with the discussion which follows. In the following paragraphs we will calculate, or discuss, the cash-flows needed to construct the spread sheet. We will not, however, do each and every math operation since that would take considerable space and would add little to the coverage.

Projecting the Potential Gross Income (PGI): We start our analysis with a

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* The tax aspects of the process can be complicated by the specifics of the investor's financial situation as well as the seemingly endless flow of changes in tax laws and IRS regulations interpreting these laws. Therefore, a detailed treatment of the tax issues which an investor could face (alternative minimum tax (AMT), "passive losses", etc.) are well outside the scope of this book and cannot be covered.

description of this concept: Potential Gross Income (also called "Scheduled Gross Income", "Gross Potential Income", "Gross Rent Roll") is the maximum income which the property is capable of producing at 100% occupancy. It does not take into account operating expenses or bad debt losses; it is purely a gross amount, and it is "potential" because it is rarely, if ever, achieved.

In the spread sheet, our example property has the capability of producing $156,300 in potential gross income (PGI), starting with the calendar year 1995. We carry this amount through our calculation. We further assumed that the PGI would grow by three percent per year. Thus, note the projection, starting with the year 1995 ($156,300), through to $170,793 for the year 1998. In each instance we added 3% to a prior year's cash-flow in order to arrive at the next year's projected cash-flow. For example,

```
156,300 [ENTER] 3 [%] +    DISPLAYS: 160,989.00,
```

which is the second year cash-flow for the PGI. We perform this calculation for each year, in the same manner we did above, and round as we go along. (You should project--and round--the PGI for the remaining years.)

The Vacancy & Bad Debt Losses: Make sure that every statement you review or prepare has an estimate for vacancies and bad debts (uncollectible rent obligations). There are always possible exceptions, such as a single family rental property. Otherwise you will consistently find credit and vacancy estimates, or losses associated with taking a rental unit out of service due to renovations, taken into account.

Note that the spread sheet used an estimate of three percent for vacancy and bad debt losses. To arrive at the estimated amount of the vacancy and credit losses for a given year, you multiply the estimated or given percentage (3% in this example) times the PGI. For example, vacancy and bad debt expenses for 1995 are estimated at $4,689. We determined this as follows:

```
156,300 [ENTER] 3 [%]    DISPLAYS: 4,689.00    @ [f] 2
```

The next step is to project this amount into future years. This is the same procedure you already used in projecting the PGI.

In practice your estimates for this component will come from historical data for the kinds of properties you are analyzing. Much data is available from national real estate associations.

Miscellaneous Income: Just about every real estate investment produces "miscellaneous" income. This can come from vending and washing machines, telephone service, garage space, cable TV connections, and so forth. In the spread sheet, we used $3,900 as the project's estimated full year income for the miscellaneous category for 1995, and it was assumed to increase by four percent per year. Note that I did not use a deduction for estimated losses associated with this income category. In practice, you may use such estimates; every property, every deal is different; there are absolutely no hard and fast rules, and nobody has all the answers.

The Gross Operating Income: This figure can be the actual income produced by the project for the base year used in the analysis, or it can be an estimate. In the spread sheet example, it is indeed based upon a projection of eight months of data over the calendar year 1995. (The example was prepared in September of 1995.)

The Operating Expenses: Operating expenses traditionally consist of all the periodic cash outlays needed to maintain the property in a condition where it can continue to produce the projected gross operating income.

Although our spread sheet example does not specifically categorize expenses as fixed or variable, you should note that this is a customary method of designating the expenses associated with the operating expense category. On closer scrutiny, however, even so-called "fixed expenses", such as real property taxes and insurance do indeed vary. This is so because as the value of the property increases, real estate taxes generally follow. And, if the value increases, then the dollar amount of the risks covered by the insurance company also increase, thus resulting in increased premiums.

Note that we project each expense category into the future at separate growth rates. This is admittedly more time consuming but will give you the most accurate picture of the impact of operating expenses on a property's profitability. Again, you should run through the numbers in each expense category and project them out as I did with the PGI and the Vacancy and Bad Debts loss categories.

The Net Operating Income (NOI): Net operating income is the amount of income remaining after deducting the operating expenses from the gross operating income.

Please note that in arriving at a property's NOI, we do not take into account the annual debt service (ADS) associated with the mortgage (or other) financing against the property. NOI is strictly a measure of investment performance based upon income and expenses arising from **operations**. Thus, "NOI" is just what it "says" it is: It's a **net** amount for a property's operating income. It is arrived at by deducting the **operating expenses** from the **gross operating income.**

The concept of net operating income is probably the most critical of all financial criteria associated with income producing real estate. We use it in estimating value (by applying various techniques, such as Income Multipliers, Mortgage-Equity Analysis, "Cap Rates", and so forth), and more particularly it is the base number with which a lender is most likely concerned since--as stated in other sections of this book, and surely other books--it gives the lender a picture of the relative financial risks associated with a given income producing property. Clearly, the higher the ratio of net operating income (NOI) to annual debt service (ADS), the greater the protection for the lender, and the better able to maintain the property in the investor's hands as against losing it through a foreclosure or similar process.

The Debt Service: You have seen this before. Since we are analyzing an income producing property on the basis of annual cash-flows, we use the annual debt service (ADS) required by the loan(s) on the property.

In the spread sheet example, we used a $500,500 loan. This was based upon two additional factors: First, the property is assumed to be suitable for a loan-to-value ratio of 70%, with a 20 year term. Second, I determined that historical income capitalization rates ("cap rates") for similar properties was "10.547%". Thus, a sales price or value was estimated as follows:

VALUE = NOI/Cap Rate = $75,408/(10.547/100) = $715,000. (rounded)

The mortgage loan of $500,500 was determined by multiplying the LTV ratio times the estimated value ($715,000 x 70%). Finally, the annual debt service (ADS) is computed by multiplying the monthly loan payment times 12.

The NET CASH-FLOW: After estimating the cash inflows and cash outflows through a property, we deduct the annual debt service to arrive at a figure which tells us how much money is "flowing" to the investor (be it an individual, partnership, etc.). In the example, the first year NET CASH-FLOW was estimated at $17,449. Note that this figure does not take into account items which are more properly associated with the income tax aspects of the transaction in the investor's hands. Rather, NET CASH-FLOW strictly represents the amount of cash-flow, positive or negative, produced by the property.

Income Subject to Tax: To determine the income subject to tax, as well as the amount of the income tax obligation, you have to make a few adjustments in your financial data. One adjustment squarely centers on the fact that the income tax laws do not recognize the amount you pay to **principal** on a loan. That is, what you pay toward reducing a mortgage loan balance is not considered a deductible item on your federal income tax return. Thus, we **add back** this amount to the NET CASH-FLOW category.

Please note the adjustments in the spread sheet for the "principal payment". To calculate this amount, we used the calculator's amortization (AMORT) function and rounded the results. You should verify the data for each and every year. Here is how we did the principal payment projection:

```
f CLX f 2 240 n 10 g i        Input loan data; compute the
500,500 CHS PV PMT            monthly mortgage payment
12 f AMORT                    Amortized 12 payments for 1995
[x≷y]                         Brings up total principal payment (8,282)

12 f AMORT                    Amortized 12 payments for 1996
[x≷y]                         Brings up total principal payment (9,149)

12 f AMORT                    Amortized 12 payments for 1997
[x≷y]                         Brings up total principal payment (10,107)

12 f AMORT                    Amortized 12 payments for 1998
[x≷y]                         Brings up total principal payment (11,166)
```

Depreciation, however, is an allowable expense in computing taxable income and therefore must be **deducted** from the NET CASH-FLOW amount in order to arrive at the taxable income generated from the property. In the example I used a nonresidential asset life of 39 years, with straight-line (S.L.)

depreciation. Since land is not considered a depreciable asset--it is considered incapable of wearing out--we must remove its cost, or estimated value, if we do not have an actual cost, from the price of the property. The example assumes the land to be twenty percent of the value of the property; hence I used $572,000 for the depreciable "basis" of the property.

Please note that we did not take into account so-called "mid-month" conventions for determining fractional depreciation for the first month the asset is put into service. Rather, I used a full 12 months, an easier number to with. Annual depreciation was calculated as follows:

$715,000 x 80%/39 = $14,666.67 (rounded to $14,667.00)

Taxable income is then determined by adding the principal payment to the estimated before-tax cash-flow (BTCF) for the year, and from that amount we deduct the allowable depreciation deduction. You should verify each figure in the spread sheet.

The AFTER-TAX CASH-FLOW: Knowing the projected taxable income for each year, we deduct 36% of the amount from the before-tax cash-flow (BTCF) in order to arrive at the after-tax cash-flow (ATCF). We used a weighted income tax rate of 36%. (It is conceded that using a tax rate of 36% is solely grounded on convenience and is not meant to imply that every investor is in a 36% federal or weighted average state and federal tax bracket!) If you live in a state with a state income tax, you should take into account an estimate of the weighted average rate for these taxes. This takes into consideration the deductibility of the state income taxes on the investor's federal income tax return. For our purposes, however, we are mainly concerned with running an IRR on investment data with a pretty good, though not necessarily 100% accurate representation of the investor's financial situation.

Some writers prefer the **after-tax cash-flow (ATCF)** as the principal measure of an asset's performance. Without challenging this position, let's suggest that in your activities you should be highly concerned with an asset's potential for positive cash-flow generation and neighborhood stability as against investing an excessive amount of time with the tax issues. Indeed, I have seen high income individuals miss a quality property because they pondered the tax aspects of the transaction until the opportunity was gone.

Please note the relatively wide spread between the investment's after-tax yield (13.95%) versus its pre-tax yield (20.35%). Both methods of determining yield are sound; it is for the practitioner to judge which method best fits his or her situation.

For our example, however, we are indeed taking the after-tax cash-flows (ATCF) as our **unequal periodic cash-flows** which will ultimately produce the projected annual after-tax IRR. In addition, the spread sheet analyzes the investment on the basis of its yield being computed on the basis of the investor's projected net cash-flow (also called the "before-tax cash-flow (BTCF)"). This method puts the data on the same footing with investment yield criteria which most of us are more familiar with, passbook savings and money market funds accounts, mutual investment funds, and the like.

We project below the After-Tax Cash Flow (ATCF) for the first year of the investment:

Gross Operating Income (1995)	$155,511	
Less: Total Operating Expenses	80,103	
Equals: Net Operating Income	$ 75,408	
Less: Annual Debt Service	57,959	
Equals: Before-Tax Cash-Flow	**$ 17,449**	**$17,449**
Income Subject to Tax:		
Add: Principal payment	8,282	
Less: Depreciation	14,667	
Equals: Taxable Income	$ 11,064	
Taxes @36% =	**3,983**	**-3,983**
Equals: After-Tax Cash-Flow		**$13,466**

The Property Reversion: For the most part, it is impractical to try to analyze an income producing property without taking into account its projected reversion value. Calculating (actually, "estimating"!) the probable sales price of the investment at the expiration of the holding period speaks to both an art and an investment science.

The method used in arriving at the reversion price in the spread sheet example was to project an annual growth rate of five percent throughout the holding period of the investment. That is, we estimated that the property would increase in value from its original price of $715,000 at the rate of five percent (5%) per year over the projected four year holding period.

To calculate the reversion value of $869,000, perform the following keystrokes:

f CLX f 0	Clear registers; set "0" decimal places
4 n	Enters investment holding period
5 i	Projected annual compound growth rate
715,000 CHS PV	Inputs original value of property
FV	Projects future value reversion of $869,087

Another method of projecting the sales price is to capitalize the net operating income stream into the future. This can be achieved with different capitalization methods. The first involves capitalization "in perpetuity". With this method we use an estimate of the "cap rate" (R) derived from existing sales of comparable properties. This method differs from the mortgage-equity income capitalization techniques covered in this book since it does not take into account the impact of loan financing or the contribution to value made by reducing the mortgage during the holding period. Thus, the standard capitalization model may fail to meet the accuracy of methods which use an Overall Capitalization Rate (RO) to estimate a property's value.

A second method involves traditional mortgage-equity analysis, such as the models covered in this book. These models indeed take a "snap shot" of a property's value over a predetermined length of time, for example, four years. This is clearly not the case with a standard capitalization model since these kinds of valuation models assume that the property is to be held for an unlimited period of time, 100 years, 200 years, an infinite holding period. In this respect, a standard capitalization model can be deficient when compared with models which use a limited investment holding period.

The Equity Reversion: The Equity Reversion (ER)--also called the Before-Tax Equity Reversion--is simply the amount of money which is left after selling the property at the expiration of the projected holding or analysis period. We arrive at this figure by deducting from the sales proceeds (1) the cost of the sale, such as brokerage and advertising expenses, (2) mortgage loan balance, and (3) incidental costs, such as accounting and legal fees.

It is called an **"Equity Reversion"** because the proceeds of a sale of real property typically go to the holder of the "equity" in the property. That is, the person or persons who make the "downpayment" on the property indeed are the equity holders. Hence, in most situations, the proceeds of the sale of real property "revert to" the true owners, being the equity owners.

In our example, we arrived at the equity reversion ($346,374) by deducting the estimated selling expenses of $60,830 (7% of $869,000) and mortgage balance of $461,796 from the estimated reversion price of $869,000.

We determined the mortgage balance as follows:

f CLX f 2	Clears registers; set 2 decimal places
20 g n 10 g i 500,500 CHS PV PMT	Input loan data and compute monthly mortgage payment
48 f AMORT	Amortize holding period of 48 months
RCL PV	Brings up loan balance (-461,796.17)

The equity reversion (ER) was determined as follows:

Property Reversion	$869,000
less: Selling Expenses	60,830
less: Loan Balance	461,796
Equals Equity Reversion	$346,374

The Taxes Due on Sale: The final step in determining the net cash-flow from the sale of the property is to deduct the income taxes which are projected to be due as a result of the investor recognizing a gain on the sale. This expense item consists of the income taxes charged against the profit made from the sale of the property. Of course, if the property is sold at a loss, there are no taxes due on sale, though there may be tax benefits which the investor receives due to the loss. This area, as previously stated, is well outside the scope of this book.

In estimating the taxes due on sale you first determine the **amount realized** from the sale of the subject property. For our purposes, it was derived by deducting $60,830 in sales commission and incidental expenses from the reversion price of $869,000. The remainder, $808,170, represents the amount realized on the sale.

The amount realized stacks-up as follows:

Property Reversion	$869,000
Less: Selling Expenses	60,830
Equals: Amount Realized	$808,170

The next step requires that we calculate the property's **adjusted basis.** Under current tax law the original price (cost basis) of the investment property is reduced in value for income tax purposes by the amount of depreciation taken during the holding period. In the example, we assumed a straight-line depreciation procedure and took a total of $58,668 in depreciation. Therefore, for federal income tax purposes, the original cost basis of the property ($715,000) must be reduced by the depreciation. This results in an adjusted basis of $656,332 ($715,000 - $58,668). This amount is then deducted from the amount realized on the sale, $808,170, to arrive at the amount of total gain to be recognized, $151,838.

The Recognized Gain summarizes below:

Amount Realized	$808,170
Less: Original Price	715,000
Plus: Depreciation	58,668
Equals: Recognized Gain	$151,838

Estimating a gain on the sale of $151,838, we determine the **taxes due on sale** by multiplying the investor's marginal tax bracket times the amount of gain recognized. Thus, the taxes due on sale are $54,662 ($151,838 x .36).

The Taxes Due at Sale summarizes below:

Recognized Gain on Sale	$151,838	
Times: Marginal Tax Rate	.36	
Equals: Taxes Due at Sale	$ 54,662	(rounded)

This amount is applied in determining the After-Tax Equity Reversion to the investor.

> **Caution:** In the example we assumed that the investor was in a 36% income tax bracket (federal <u>or</u> weighted average). Care must be exercised when performing such estimates since you may not have all the facts on the investor's financial and tax affairs and, even if you had this information, the complexity of the tax laws impacting real estate transactions likely requires an analysis by a tax practitioner. Be very guarded in making <u>any</u> representations of a tax nature.

The After-Tax Equity Reversion: This is the last step before we calculate the project's yield. We previously established the before-tax equity reversion (ER) at $346,374. From this amount we deduct the estimated income taxes

($54,662) attributed to the sale of the property; the result is the after-tax equity reversion (ATER). Said differently, this is the after-tax net amount received by the investor as a result of selling the property.

The calculation summarizes as follows:

Before-Tax Equity Reversion (BTER)	$346,374
Less: Taxes Due on Sale	54,662
Equals: After-Tax Equity Reversion (ATER)	$291,712

The after-tax equity reversion (ATER) indeed parallels a "balloon payment" in that it is assumed to be received along with the final year cash-flow. In determining the final after-tax cash-flow (ATCF) in the year of sale, we add the after-tax equity reversion (ATER) to the after-tax operating cash-flow. This figure is then input into your calculator as the final year ATCF.

Computing the Investment Yield: Note that we calculated the yield to the investor on the basis of (1) the after-tax cash-flows and (2) on the basis of the expected before tax cash-flow (BTCF). There are no hard and fast rules on which method to use, though you will most likely use both methods since this appears to be the practice in the prominent real estate investment analysis texts. However, in practice you may alternate between the two methods, depending, of course, upon which method best suits your financial picture and interests or those of your client.

Note also that in computing the investment yield (IRR) we ran two separate sets of data. There are common entries between the methods. First, the out-of-pocket cash-flow (CFo) is the same for both methods; it is the amount of the investor's equity (the downpayment) in the property ($214,500). In the example, the $214,500 is entered as a negative signed number (use the CHS key) into the BLUE [g] [CFo] register. This keeps the input consistent with the solution methodology programmed into your HP 12C.

Next, you make separate inputs for the after-tax cash-flows (ATCF) into the BLUE [g], [CFj] registers, one entry at a time, but do not enter the final after-tax operating cash-flow until you add to it the after-tax equity reversion (ATER) of $291,712. (Please note the example below as well as the spread sheet example.) At this stage you can compute the investment's projected yield using the IRR function, GOLD [f] key followed by the [IRR] key. The yield is 13.95%.

The After-Tax Investment Yield calculation follows:

f CLX f 2	Clear all registers; sets 2 decimal places
214,500 CHS g CFo	Investor's equity in property
13,466 g CFj	First year operating after-tax cash-flow
14,050 g CFj	Second year operating after-tax cash-flow
14,593 g CFj	Third year operating after-tax cash-flow
15,086 ENTER	Fourth year operating ATCF into the stack
291,712 +	Adds After-Tax Equity Reversion to fourth year after-tax cash-flow, totalling $306,798

g CFj	Enters final year after-tax cash-flow
f IRR	Computes After-Tax investment yield
DISPLAYS: 13.95	

To compute the investment's before-tax yield, you use the same out-of-pocket cash-flow ($214,500), but enter the projected **before-tax cash-flows** and projected before-tax equity reversion ($346,374) from the property, in proper sequence, of course. The yield is 20.35%, thus placing the investment in a position where the investor can double his or her money in about four years, assuming the investment was untaxed.

The Before-Tax Investment Yield calculation follows:

f CLX f 2	Clear all registers; set 2 decimal places
214,500 CHS g CFo	Investor's equity in the property
17,449 g CFj	First year before-tax cash-flow
18,850 g CFj	Second year before-tax cash-flow
20,236 g CFj	Third year before-tax cash-flow
21,602 ENTER	Fourth year before-tax cash-flow into the stack
346,374 +	Adds Equity Reversion to fourth year before-tax cash-flow, totalling $367,976
g CFj	Enters final year before-tax cash-flow
f IRR	Computes Before-Tax investment yield
DISPLAYS: 20.35	

Comment: The yield calculations we performed are surely critical steps in determining whether an investment meets the financial requirements of the investor. You should note, however, that there is a line of thinking in this field which argues that the **net present value (NPV)** is a more appropriate method to be used in determining if an investment meets the financial needs or requirements of the investor.

To be sure, both methods of income property analysis (IRR & NPV) should be considered. The problem we get into, however, in trying to run an **accurate** net present value (NPV) on an investment like the one we just completed is that you cannot simply input a suggested (required) yield needed by the investor and get back from the calculator--any calculator, for that matter--the exact amount by which the price of the investment must be lowered (adjusted) in order to meet the yield required by the investor.

For example, using the inputs from the Before-Tax investment yield calculation, if we input a required yield of, say, 21.5% into the [i] register and then compute the net present value (NPV) of the cash-flows, we determine that the investor does not meet his yield requirements because the NPV is less than zero (-7,232.20). Thus, it appears the investor is out-of-pocket $7,232.20 if he accepts the investment. This is, however, a very loose analysis since if the property sold for $7,232.20 less than the original price ($715,000), **everything else changes**. Indeed the mortgage and the operating expenses change; NOI changes; and, of course, the equity reversion changes. Thus, to efficiently perform this level of calculation you really need a computer and an appropriate spread sheet program.

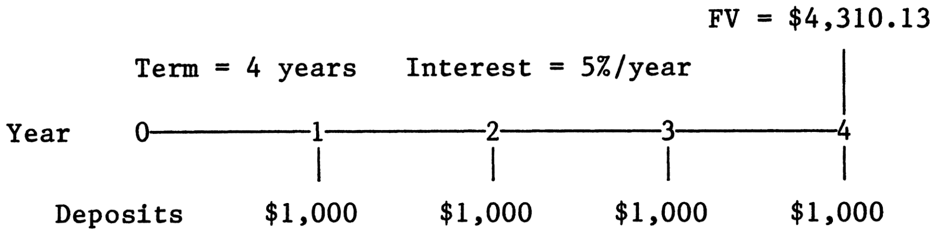
ANNUITIES-THE BASICS

An **annuity** represents a specified payment, or income stream, payable, or received, over a specified period of time. Put differently, an annuity can be described as a stream of payments or cash-flows either paid or received over a fixed period of time. The cash-flows can be the same from period to period, such as month to month or year to year, or they can be different. We first cover basic annuities with fixed payment streams, and then conclude the topic with a straightforward "step-up" annuity.

The first thing to fix in our minds about an annuity is that it is a **stream** of payments. Indeed, it can represent a stream of withdrawals flowing out of a **one-time deposit** into a savings account, or it can be a series of payments going into an account or other investment. The payments (or cash-flows) can move in both directions: Into or out of an investment.

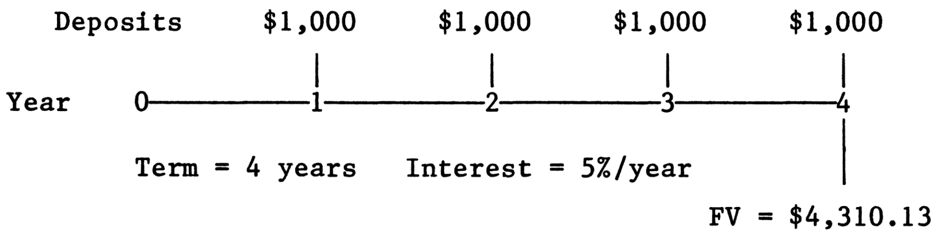
The easiest way to analyze an annuity with fixed payments is to use the financial registers of your calculator, [n], [i], [PV], [PMT], and [FV]. The key, of course, is to decide which financial registers are required to analyze your particular annuity problem. As well, we must make decisions as to the manner in which the inputs into the financial registers are signed, positive or negative.

We start with a straightforward problem with a series of $1,000 deposits made into an account at the **end of each year** for a term of four years. Let's further assume that the deposits earn compound interest at the rate of 5 percent per year. For the purposes of the cash-flow diagram given below, we take the position that the payments are **negative** cash-flows.



Note the placement of the deposits **below** the timeline and the future value of the account--indeed, the future value of the annuity--placed **above** the timeline. This placement is consistent with the cash-flow sign convention system around which your HP 12C is programmed. As previously discussed, this convention tells us that if the PMT is entered as a positive number, the computed future value [FV] will be a negative number, while entering a negative number for the PMT produces a positive result for the future value [FV]. Since we entered the payment [PMT] as a **negative** number, the computed future value [FV] displays as a positive signed number.

A suitable timeline diagram depicting the cash-flows input as **positive** numbers follows on the next page.



To solve the problem, let's perform the following keystrokes:

KEYSTROKES:	f CLX f 2	Clear all registers; set 2 places
	g END	Sets END mode
	4 n	Inputs term of annuity payments
	5 i	Annual interest rate
	1,000 CHS PMT	Inputs deposit, negative signed
	FV	Computes future value of the annuity
	DISPLAYS:	
	4,310.13	

For practice, let's change the sign of the $1,000 deposits from a negative number to a positive number. Then, recalculate the future value of the deposits.

KEYSTROKES:	1,000 PMT	Inputs deposits, positive signed
	FV	Computes future value of the annuity
	DISPLAYS:	
	-4,310.13	

Discussion: The cash-flows of $1,000 should be thought of as payments; thus we use the [PMT] key to enter the amount of the deposits into your calculator. If the payments are entered as **negative numbers**--and thus conceptually placed below a timeline diagram--, the computed future value [FV] of the annuity of deposits will be **positive signed**; otherwise entry of a positive signed payment produces a negative signed future value.

The term (4 years in this example) must be entered into the [n] register. This tells the calculator the number of payments or number of deposits which make up the annuity. Specifically, in our problem we entered "4" into the [n] register to tell the calculator that we are making four annual payments into the annuity. On the other hand, if payments were made twice per year over a 4 year period, we would enter "8" into the [n] register to confirm that we are making eight semiannual payments into the annuity.

Since interest in the example is paid and compounded annually, we tell the calculator what the annual interest rate is by inputting the number "5" into the [i] register. If, on the other hand, payments were made twice a year, and if interest was paid and compounded semiannually, we would have entered "2.5" (5/2) into the [i] register because the HP 12C is designed to accept only **periodic interest**, meaning interest per compounding period. (This factor distinguishes the HP 12C from the HP 17BII and HP 19BII which accept inputs for interest on an annual basis.)

Next, since we did not make a deposit into the account at the beginning of the year, we made sure that the present value [PV] register was set to zero. This can be done any number of ways: First, by pressing **f CLX**; second, by pressing **f FIN**; and third, by simply storing a "0" in the [PV] register before you make your data inputs and calculate the future value [FV].

The final consideration in analyzing the annuity is to determine where to set the payment mode [BEG, END]?

Note that **"BEG"**, printed in blue under the "7", stands for **beginning** of the time period. That is, if the **BEG** mode is set by pressing [g] [7], the calculator's financial registers are configured--that is, they are set--to analyze financial problems on the basis of fixed payments or fixed deposits occurring at the beginning of the time period. Accordingly, since the payments into our annuity did not occur at the beginning of the year, it was not necessary to set the beginning mode [BEG].

When payments into or out of an annuity occur at the end of the time period, we set the calculator to **END** mode. This is carried out by pressing the blue [g] key followed by the [8]. Since our annuity required payments to be made at the end of the year, we set the END mode.

Determining the accuracy of the computed future value

Our series of end of the year deposits of $1,000 over a 4 year period at 5 percent compound interest gives us a total of $4,310.13. How do we know this to be accurate? Think of each payment as constituting a separate present value [PV] which in turn produces a total amount, a future value [FV] at the expiration of its specific holding period. For example, the deposit at the end of year 1 draws interest for 3 years; the deposit at the end of year 2 draws interest for 2 years; the deposit at the end of year 3 draws interest for 1 year; and the deposit at the end of year 4 does not draw interest. If we total the individual future values, we have:

Year	Deposit	#Years on-Deposit	Future Value	Cumulative Total
1	$1,000	3 years	$1,157.63	$1,157.63
2	$1,000	2 years	$1,102.50	$2,260.13
3	$1,000	1 year	$1,050.00	$3,310.13
4	$1,000	---	$1,000.00	$4,310.13

Comment: The future value of the series of deposits is proven. In the next section we will determine what happens to the future value of the annuity of deposits when the payments are made at the beginning of the year.

What happens if the payments are made at the beginning of the year?

Before we start the keystroke procedure, we should state that the relationship between the future value of a **due annuity** and an **ordinary annuity** is determined by the fact that the **due annuity** draws interest on each deposit for exactly one additional interest compounding period. In our example, this tells us that the deposit in year 1 will draw interest for 4 years instead of 3; the deposit in year 2 will draw interest for 3 years instead of 2; the deposit in year 3 will draw interest for 2 years instead of 1; and the deposit in year 4 will draw interest for 1 year instead of none.

In effect, there is a **shifting** of the application of the periodic interest rate backwards by one interest rate compounding period. The net effect of this is that the future value of the **due annuity** will be exactly equal to the future value of the **ordinary annuity** plus the interest which would be earned by having the ordinary annuity's future value on deposit for one interest rate compounding period. Therefore, the future value of the due annuity we calculate below should be exactly 5 percent greater than the $4,310.13 we arrived at for the ordinary annuity, or $4,525.63 ($4,310.13 + 5%). Let's do the keystrokes:

KEYSTROKES:	f CLX f 2	Clear all registers; set 2 places
	g BEG	Sets beginning mode. Note the **BEGIN** status indicator light up in the display
	4 n	Inputs term of annuity
	5 i	Annual interest rate
	1,000 CHS PMT	Inputs deposit, negative signed
	FV	Computes future value of the annuity
	DISPLAYS:	
	4,525.63	

Comment: The relationship between the future value of a **due annuity** and the future value of the **ordinary annuity** is controlled by the fact that each deposit into the due annuity earns interest for one more interest compounding period. Mathematically, it looks like this:

FV DUE ANNUITY = FV ORDINARY ANNUITY x (1 + periodic interest rate/100)

= $4,310.13 x (1 + .05)

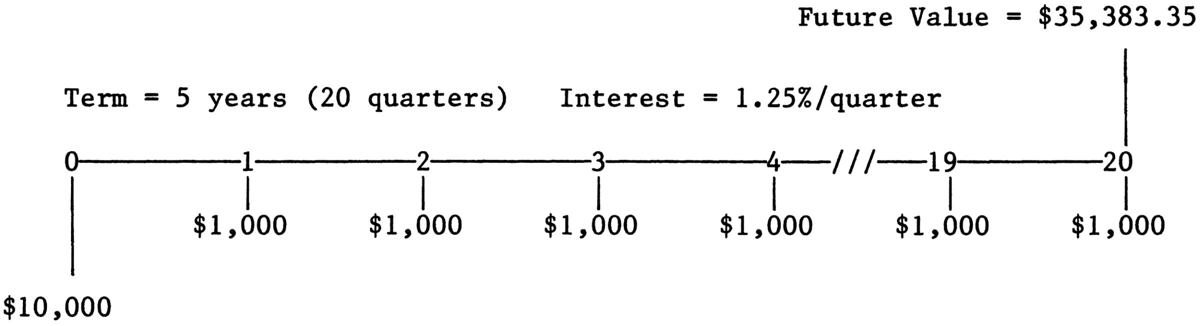
= $4,310.13 x 1.05

FV DUE ANNUITY = $4,525.64 (Slight rounding difference compared with above result)

What happens if we make an initial deposit along with the periodic deposits?

Let's assume that we make a $10,000 initial deposit, plus $1,000 deposits at the **end** of each quarter into an account drawing 5 percent interest per year, compounded quarterly, and continue the $1,000 deposits for five years. What is the future value of the annuity at the end of five years?

Discussion: From an analytical point of view, this problem breaks down into two separate future value problems. The calculator, of course, eliminates the complexity and gives us an answer in seconds! Let's draw a timeline diagram to see what is happening, and then we will perform the keystrokes.



To solve the problem, let's perform the following keystrokes:

KEYSTROKES:	f CLX f 2	Clear all registers; set 2 places
	g END	Sets END mode
	20 n	Inputs number of quarters in the annuity
	5 ENTER 4 ÷ i	Quarterly interest rate
	10,000 CHS PV	Inputs one-time initial deposit
	1,000 CHS PMT	Inputs periodic deposit, negative signed
	FV	Computes future value of the annuity
	DISPLAYS:	
	35,383.35	

Discussion: As in the case of the annual payment example we previously calculated, the cash-flows of $1,000 in the above problem should be thought of as negative cash-flows and thus entered into the [PMT] register as negative signed numbers. Further, since the initial deposit made at the beginning of the year is also considered an out-of-pocket cash-flow, we must enter it as a negative signed number into the present value [PV] register. It goes into [PV] because it is properly designated as a one-time deposit. That is, it is made only once, and appropriately goes into [PV] as a negative signed entry.

The term of the annuity is 5 years. However, the payments are made quarterly, with interest also paid and compounded quarterly. This tells us that we are dealing with a total of 20 quarterly payments (5 years x 4 quarters/ year). Following the cash-flow sign convention, if payments [PMT] are entered as a negative number, future value [FV] will be positive. And, if the present value [PV] is entered as a negative number, the future value [FV] it generates will also be positive. Putting the two deposit-components together, we have a positive future value [FV].

Determining the accuracy of the computed future value

Our annuity, consisting of a one-time deposit of $10,000 made at the beginning of the year, plus the 20 end of the quarter deposits of $1,000 at five percent, compounded quarterly, gives us a total of $35,383.35. Again, how do we know this to be accurate?

Think of each payment as constituting a separate present value [PV] which in turn produces a separate future value [FV] at the end of its deposit period. For example, the deposit at the end of the first quarter draws interest for 19 quarters, while the deposit made at the end of the second quarter draws interest for 18 quarters, and so forth. In addition, the one-time deposit of $10,000 made at the beginning of the first year draws interest for the full 20 quarters (5 years x 4 quarters/year).

If we were to project the future value of each deposit (a rather tedious process we will pass on), and add their sum to the future value of the one-time deposit of $10,000, we would arrive at a total future value of $35,383.35. For those of you would like the math-side of this problem, read the following; otherwise skip over to the next page.

Solution by equation

We know that the computed future value ($35,383.35) is the sum of two future values, each produced by a different system: one being a series of deposits; the other a one-time deposit at the beginning of the year. Mathematically, it looks like this:

FV of the Annuity = FV of One-Time Deposit + FV of Periodic Deposits

$$= PV \times (1 + i/100)^n + \frac{(1 + i/100)^n - 1}{i/100} \times PMT$$

$$= \$10{,}000 \times (1 + 1.25/100)^{20} + \frac{(1 + 1.25/100)^{20} - 1}{1.25/100} \times \$1{,}000$$

$$= \$10{,}000 \times 1.282037232 + \frac{1.282037232 - 1}{.0125} \times \$1{,}000$$

$$= \$12{,}820.37 + 22.56297856 \times \$1{,}000$$

$$= \$12{,}820.37 + \$22{,}562.98$$

$$FV = \$35{,}383.35$$

The result is proven.

What happens if the payments into the annuity are unequal in amount?

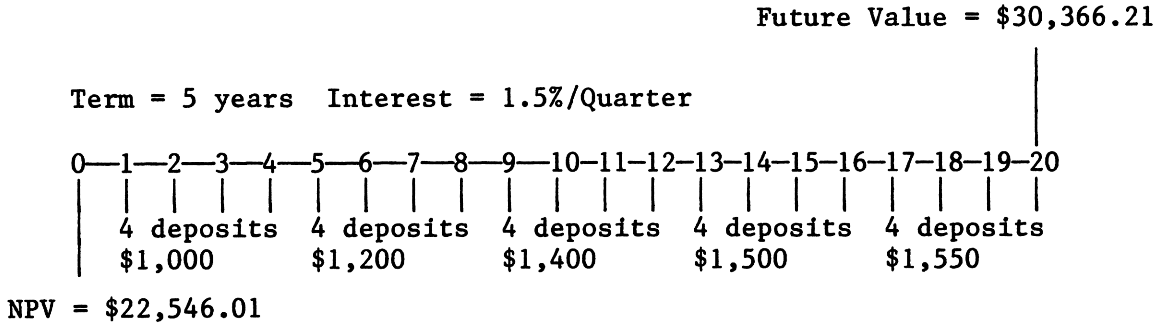
Let's consider a straightforward annuity into which we make irregular payments over a 5 year period. The payments are to be made at the end of each quarter and draw interest at the rate of 6 percent per year, paid and compounded quarterly. We detail the payment schedule below:

First Year	$1,000 per quarter
Second Year	$1,200 per quarter
Third Year	$1,400 per quarter
Fourth Year	$1,500 per quarter
Fifth Year	$1,550 per quarter

We ask: What is the future value [FV] of this annuity? Indeed, we could ask: What is the future value of this **increasing annuity**, because that is exactly what it is. Payment streams which **increase** in amount over the term or holding period of an annuity (or asset) are indeed properly designated as **increasing annuities**. On the other hand, annuities with payment streams which **decrease** at some predetermined rate or amount are designated as **decreasing annuities**.*

To solve this problem we must first calculate the discounted present value (NPV) of the stream of payments. To do this we use the Unequal Cash-Flow Function of your HP 12C (CFj, Nj, and NPV). We do this because the payments (the deposits) are **unequal**; that is, they are irregular and thus do not readily lend themselves to easy analysis with the financial registers. Next, after calculating the net present value (NPV) of the deposits, we will project them forward to the end of the annuity's deposit period, 20 quarters.

Let's start with a timeline diagram:



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* An in-depth study of this area is beyond the scope of this book. Suffice it to say that "increasing/decreasing annuity" applications from finance mathematics have applications in leasing, income property appraisal, financial planning, and other areas. It is a substantial tool whose applications are finding their way into financial and real estate problem solving methods taught in advanced seminars.

To solve the problem, let's perform the following keystrokes:

KEYSTROKES:	f CLX f 2	Clear all registers; set 2 places
	6 ENTER 4 ÷ i	Annual quarterly interest rate (1.5%)
	1,000 g CFj	Enters first cash-flow amount
	4 g Nj	Enters four cash-flows
	1,200 g CFj	Enters second cash-flow amount
	4 g Nj	Enters four cash-flows
	1,400 g CFj	Enters third cash-flow amount
	4 g Nj	Enters four cash-flows
	1,500 g CFj	Enters fourth cash-flow amount
	4 g Nj	Enters four cash-flows
	1,550 g CFj	Enters fifth cash-flow amount
	4 g Nj	Enters four cash-flows
	f NPV	Computed discounted present value
	DISPLAYS: **22,546.01**	

Knowing the discounted present value (NPV) of the cash-flows, we readily calculate the future value (FV) of the annuity of deposits by using the financial register keys. The keystrokes follow:

KEYSTROKES:	g END	Sets END mode
	CHS PV	Enters negative signed PV **(-22,546.01)**
	20 n	Inputs term of deposits in quarters
	FV	Computes future value of deposits
	DISPLAYS: **30,366.21**	

Note: In practice, you must make sure that the payment [PMT] register is cleared out--that is, set to zero--before you calculate the future value [FV] of the annuity. We did not perform the extra steps above because we previously cleared all registers by pressing **f CLX**. Therefore, it was not necessary for us to clear the PMT register by pressing "0" followed by the [PMT] key. However, always make sure the [PMT] register is set to "zero" before running the above kinds of calculations.

Caution: The computed future value of the annuity of deposits did not take into account income taxes which might impact the interest earned during the term of the annuity, nor did it take into consideration an adjustment for inflation. In practice, we would take into account taxes as well as an adjustment for inflation.

Clearly, the actual growth in value of the annuity is slowed by the payment of taxes, assuming the account is not free of income taxation during its term. Therefore, the effective growth in value of the deposits might not be "1.5%" (or what have you) per quarter. Rather, the effective growth rate could indeed be less. It is determined by the following relationship:

Effective Yield = Nominal Yield x (1 - % Income Taxes/100)

Always be guarded in making representations which involve tax decisions.

DEFERRED PAYMENT MORTGAGES

A **deferred payment mortgage** is a real estate financing device which indeed **defers** an agreed upon number of payments immediately after the lender makes the loan. Said a little differently, "deferral" in the context of this type of loan means that the payments are **postponed** for a given number of time periods, typically an agreed upon number of **months.** The period of deferral does not reduce the number of payments required on the loan. Rather, the deferral period is **tacked** onto the original term of the loan. For example, if a loan requires 240 monthly payments, but grants the borrower a 5 month deferral, the total term of the loan would be 245 months.

Interest is charged and added to the principal balance during the deferral period. In fact, the periodic interest rate is compounded during the deferral period, thus causing the loan's principal balance to increase at an increasing rate during the period of deferral.

Keeping straight the number of deferred payment periods

A concept which must be clear when working with a deferred payment mortgage loan is that the number of deferred payments in fact equals the number of deferred time periods. Put slightly different, if your loan defers the first six payments, you have six payment periods deferred, **not seven.** This is an easy concept to confuse. Stressing the point, in determining the number of **deferred periods** on the loan you do not reduce the number of time periods by "one" before the first payment comes due. Use the number of **deferred payments.**

By the very nature of a regular end of the month--end of the period--mortgage loan the first payment always falls due one month--or one time period--after the loan is issued. This is so because an end of the month--end of the period--loan by its very design, by its very structure, requires the borrower to pay interest in **arrears.** That is, interest is a **back bill** in the customary end of the period mortgage loan. Thus, by **design,** this type of loan **automatically** (and, indeed, mathematically) defers the first payment for one interest period, typically one month. Thus, your first payment is due one month after you receive the lender's money, and your second payment is due one month after your first payment, and so forth.

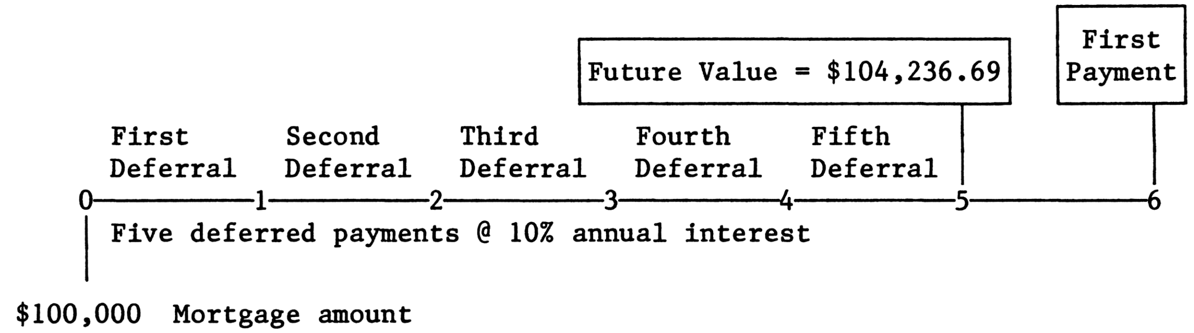
Remember: The number of deferred payments equals the number of deferred time periods.

Problem

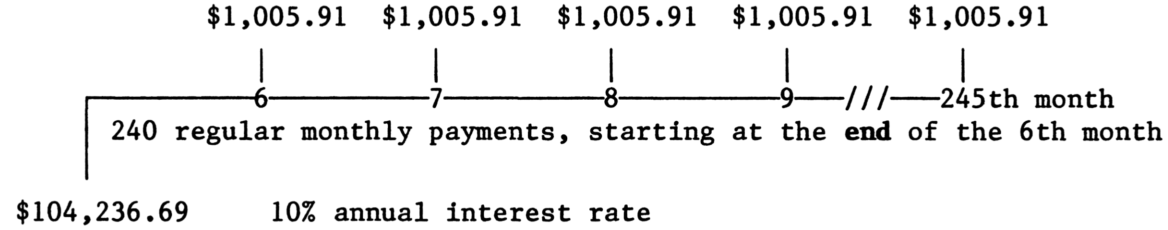
Let's assume a $100,000 mortgage is written at 10% annual interest over a 240 month term. Further, the loan provides that the first five payments will be deferred. What is the monthly mortgage payment?

To solve these kinds of problems we must first determine the future value of the original loan amount projected **for the complete** period of deferral. In effect, think of making a deposit of the principal amount of the loan ($100,000 in this example) and allowing it to remain "on deposit" for the period of the deferral, being five months in this example. The "deposit" earns or draws interest at the rate of the annual mortgage interest (10% in our example), compounded in accord with the manner in which payments are made on the loan, being monthly. By the end of the deferral period the borrower is clearly behind in what would normally be his or her regular payment obligation, so the amount owing must be greater than the original amount of the loan.

Think of the deferral period in terms of the timeline diagram below:



The timeline diagram tells us that the principal amount of the loan increases from $100,000 to $104,236.69 **at the expiration of the deferral period**; that is, after **five months** have elapsed. Flipping the timeline diagram over, and inserting the regular monthly payment which we will shortly calculate, we have the following:



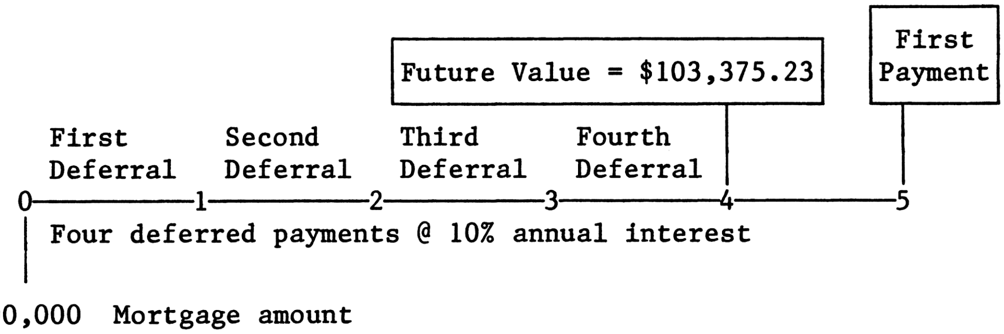
KEYSTROKES:	f CLX f 2	Clear all registers; set 2 places
	5 n	Input number of deferred payments
	10 g i	Inputs monthly interest rate
	100,000 CHS PV	Loan amount, negative-signed
	FV	Computed future value of the principal amount of the loan
	DISPLAYS: 104,236.69	
	CHS PV	Inputs loan amount, increased by interest accrued during 5 month deferral period
	240 n	Inputs loan term
	0 FV	Clear future value register
	PMT	Computes monthly loan payment
	DISPLAYS: 1,005.91	

Comment: The regular monthly payment is $1,005.91. It starts at the **end** of the sixth month because there were five deferred payments on the loan.

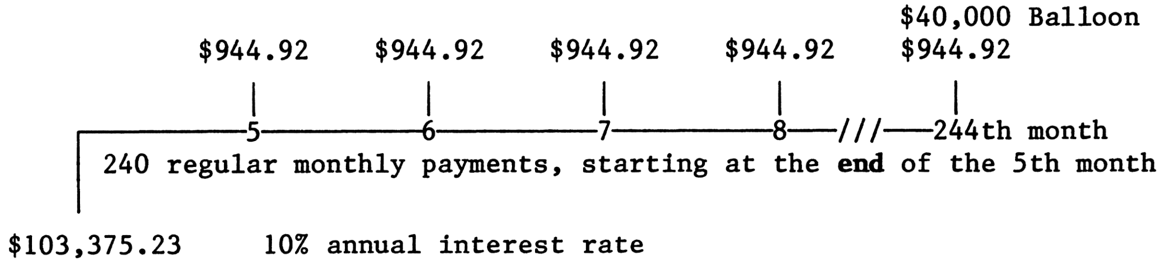
Problem: Let's do a similar problem, except we will build a balloon payment into the loan. We again work with a $100,000 loan at 10% annual interest, but change the number of deferred periods from five to four and add a balloon payment of $40,000 due along with the last regular payment.

The problem adds a little different perspective because now we have a payment due at the end of the term along with the regular payment. The added payment--it's just like any other balloon payment--easily integrates into the loan payment calculation once we know what our original loan amount has grown to at the end of the four month deferral period.

Again, the timeline diagram looks like this:



The timeline diagram tells us that the principal amount of the loan increases from $100,000 to $103,375.23 **at the expiration of the deferral period;** that is, after **four months** have elapsed. Flipping the timeline diagram over, and inserting the regular monthly payment which we will shortly calculate, we have the following:



Conclusion: The regular monthly payment is $944.92. It starts at the **end** of the fifth month because there were four deferred payments on the loan. The balloon payment of $40,000 is due with the 240th regular payment of $944.92. The 240th payment occurs at the **end** of the 244th month.

We calculate the monthly payment on the next page.

```
KEYSTROKES:     f CLX f 2           Clear registers; set 2 places
                4 n                 Inputs number of deferred payments
                10 g i              Inputs monthly interest rate
                100,000 CHS PV      Loan amount, negative-signed
                FV                  Computed future value of the principal
                                    amount of the loan
                DISPLAYS: 103,375.23
                CHS PV              Inputs loan amount, increased by interest
                                    accrued during 4 month deferral period
                240 n               Inputs loan term
                40,000 FV           Inputs balloon payment
                PMT                 Computes monthly loan payment
                DISPLAYS:
                944.92
```

Proof

Our method follows what we will do throughout this book. We compute the internal rate of return (IRR) of the cash-flows. If the computed annual yield (IRR) is 10 percent, the calculation is correct.

```
KEYSTROKES:     f CLX f 2           Clears all registers; set 2 places
                100,000 CHS g CFo   Inputs amount of loan, negative-signed
                0 g CFj             Inputs deferred payment
                4 g Nj              Inputs 4 deferred payment periods
                944.92 g CFj        Inputs regular payment
                99 g Nj             Inputs 99 payments
                R↓ g CFj            Brings back and inputs regular payment
                99 g Nj             Inputs 99 regular payments
                R↓ g CFj            Brings back and inputs regular payment
                99 ENTER 2 x        Total number of PMTS in Mem. Reg. 2 & 3
                1 +                 Adds final payment (the 240th)
                DISPLAYS: 199.00    (Tells us to enter 41 payments)
                41 g Nj             Inputs 41 regular payments
                R↓ R↓               Brings back regular payment
                40,000 +            Adds balloon payment
                g CFj               Inputs $40,944.92 for the final payment
                f IRR               Compute the yield
                DISPLAYS:  0.83
                RCL g i             Convert to annual yield
                DISPLAYS:
                10.00
```

Comment: The loan's annual interest rate of 10 percent is proven. Therefore the payment schedule is correct.

Note that we could as well prove the accuracy of the payment schedule by computing the net present value (NPV) of the payment stream. That is, with the same data we used for the IRR calculation, we can check to see if the payments over or under generate the loan amount ($100,000), or if they come close. Just press 10 [g] [i] [f] [NPV]. Your display will show: "0.20".

GRADUATED PAYMENT MORTGAGES

A **Graduated Payment Mortgage** ("GPM") is nothing more than a special case of an annuity utilized to satisfy a borrower's obligations under a mortgage loan. It is an "annuity" because a GPM consists of a "series" of payments during its term, typically monthly. As the description implies, the payments on a GPM are **graduated.** That is, they are not all the same and in fact increase during the term of the loan, typically once per year for an agreed upon number of years. For example, a borrower may agree to a payment schedule whereby the monthly payment made during the loan's second year will be five (5%) percent greater than the payment made during the first year; the third year's payments being five (5%) percent greater than those of the second year, with the process continuing for the agreed upon number of years.

Typically, a GPM loan might be structured to provide for a five percent (5%) (or 7.5%) increase in monthly payments starting with the second year, with the process continuing through the sixth year. Therefore, all payments from and after the sixty-first through and including the last payment would be the same, excepting the final payment, which is usually different due to rounding of the payment amounts during the loan's term.

The GPM offers the borrower the advantage of a significantly lower payment during the first few years of the loan term as compared with the payment on the more traditional fixed payment loan. It therefore can be of considerable advantage to the borrower who **reasonably** anticipates either an increase in personal earnings or increased cash-flows from the property. On the other hand, total interest paid on the GPM can <u>exceed</u> that of a fixed payment loan because the payments during the first few years are always less than those of a comparable fixed payment loan.

Insofar as the lender is concerned--be it a traditional lending institution or an owner/mortgagee--the GPM can be advantageous in that it adopts the fundamental financial realities of the market place by enabling the purchaser/mortgagor to share the start-up cash-flow interference of a purchase without any out-of-pocket cost whatsoever to the seller. The lender (or seller/mortgagee) still receives the agreed upon yield on the GPM as would be the case with a traditional fixed payment loan.

It must be recognized, however, that the buyer's true equity in the transaction can actually decrease during the first few years of the GPM because the loan's balance <u>may</u> increase during this period rather than go down as is the case with a fixed payment mortgage. This occurs because the payments made during the first few years of a <u>long-term</u> GPM (typically 30 years) are not enough to cover the interest obligation on the loan. Therefore the loan balance increases because the unpaid interest obligation during the first few years is added to the principal each month.

Exactly how much the GPM's loan balance will increase, and for how long it will increase, are questions which can only be answered by saying that it is a function of (1) the term of the loan, (2) the percentage growth in each year's payment, and (3) the number of years of agreed upon increase in payments. Basically, the greater the agreed upon growth in monthly payments, the greater the length of the loan term, and the greater the number of years of increase in payment, the longer it will take before the balance begins to decrease.

From the Seller's perspective a GPM has at least two significant, though opposed implications. First, where the sale is properly structured, it should be easier to effect a higher sales price since all parties to the transaction are, in a very loose sense, sharing in the realities of the market forces affecting the transaction. Secondly, and most critical in terms of potential risk to the seller/mortgagee or mortgage lender, this financing technique can be, but is not necessarily, more risky.

The risk associated with the GPM clearly stems from the potential for "inverse equity" buildup during the first few years of the loan. In effect, if the down payment was not sufficient, or if the property's natural appreciation in value is not enough to overtake the increases in the loan's balance during the initial few years after closing, the mortgage holder will be in a less secure position than would exist were a fixed payment loan the principal source of financing.

For example, a $100,000 GPM, written at 10% annual interest, for a term of 360 months, with monthly payments increasing five (5%) percent per year in the second-through sixth years, carries the following monthly payment schedule:

First year	$730.59	Fourth year	$845.75	
Second year	$767.12	Fifth year	$888.04	
Third year	$805.48	Sixth year	$932.44	(360th PMT = $925.30)

Below we give the computed loan balance for each year (using the AMORT function with the display set at f, 2), until the balance is approximately equal to the original amount of the loan, which is $100,000 in this example:

First year	$101,291.02	Fifth year	$102,611.81	
Second year	$102,258.21	Sixth year	$101,639.97	
Third year	$102,844.66	Seventh year	$100,566.37	
Fourth year	$102,986.51	Eight year	$ 99,988.11	(90th payment)

It is clear that the loan balance will not drop below $100,000 until the borrower has made the ninetieth payment. The implication of this is that the lender--or seller/mortgagee--will be at risk for a longer period of time when financing with the GPM, unless the down payment is sufficient to cushion the difference between the anticipated year-by-year market value and the highest anticipated loan balance for any given year.

When structuring or reviewing a GPM loan, the real estate practitioner must consider the equity build-up issue as well as the amount of down payment needed to supply a cushion for the party supplying the financing. Of course, this is somewhat academic in cases where a traditional lending institution provides the financing since they will consider the issue. Yet, prudence would seem to dictate that we should still consider the issue no matter who supplies the funds on a GPM-financed real estate purchase.

MANUAL PROCEDURE FOR COMPUTING GRADUATED PAYMENT MORTGAGE SCHEDULE

Example: Assume a $100,000, 30 year GPM at 10% annual interest, with monthly payments increasing 5% per year for five years immediately following the first 12 payments. We will compute six different payments, plus the final (360th) payment.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Set 2 decimal places; clear registers	f 2 f CLX	0.00
Enter "1" as 1st payment amount in series of grouped cash-flows	1 g CFj	1.00
Enter number of payments in first group of cash-flows	12 g Nj	12.00
Recall 1st year payment and increase by growth percentage; enter in CFj	R↓ 5 % + g CFj	1.05
Enter 12 PMTS in 2nd cash-flow group	12 g Nj	12.00
Recall 2nd year payment and increase by growth percentage; enter in CFj	R↓ 5 % + g CFj	1.10
Enter 12 PMTS in 3rd cash-flow group	12 g Nj	12.00
Recall 3rd year payment and increase by growth percentage; enter in CFj	R↓ 5 % + g CFj	1.16
Enter 12 PMTS in 4th cash-flow group	12 g Nj	12.00
Recall 4th year payment and increase by growth percentage; enter in CFj	R↓ 5 % + g CFj	1.22
Enter 12 PMTS in 5th cash-flow group	12 g Nj	12.00
Recall 5th year payment and increase by growth percentage; enter in CFj	R↓ 5 % + g CFj	1.28
Enter # of PMTS in final cash-flow group in maximum of 99 cash-flows	99 g Nj	99.00
Recall 6th group PMT and reenter	R↓ g CFj	1.28
Enter # of PMTS in cash-flow group	99 g Nj	99.00
Recall 7th group PMT and reenter	R↓ g CFj	1.28
Enter # of PMTS in cash-flow group	99 g Nj	99.00
Recall 8th group PMT and reenter	R↓ g CFj	1.28
Enter final # of cash-flows [360 - (60 + 99 + 99 + 99)] = 3	3 g Nj	3.00
Enter monthly interest rate	10 g i	0.83
Compute net present value (NPV)	f NPV	136.88
Invert NPV	[1/x]	0.01

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Input loan amount; compute and round first year payment	100,000 x f RND	730.59
Compute and round second year PMT	5 % + f RND	767.12
Compute and round third year PMT	5 % + f RND	805.48
Compute and round fourth year PMT	5 % + f RND	845.75
Compute and round fifth year PMT	5 % + f RND	888.04
Compute and round 61st through 360th PMT (subject to last PMT calculation for 360th month)	5 % + f RND	932.44

Computing the final (360th) payment

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers	f CLX	0.00
Enter first year PMT as 1st cash-flow group. Input 12 PMTS	730.59 g CFj 12 g Nj	730.59 12.00
Enter second year PMT Input 12 PMTS	767.12 g CFj 12 g Nj	767.12 12.00
Enter third year PMT Input 12 PMTS	805.48 g CFj 12 g Nj	805.48 12.00
Enter fourth year PMT Input 12 PMTS	845.75 g CFj 12 g Nj	845.75 12.00
Enter fifth year PMT Input 12 PMTS	888.04 g CFj 12 g Nj	888.04 12.00
Enter final computed PMT Input 99 PMTS	932.44 g CFj 99 g Nj	932.44 99.00
Reenter final computed PMT Input 99 PMTS	R↓ g CFj 99 g Nj	932.44 99.00
Reenter final computed PMT Input 99 PMTS	R↓ g CFj 99 g Nj	932.44 99.00
Reenter final computed PMT Input final number of PMTS [360 - (60 + 99 + 99 + 99)] = 3	R↓ g CFj 3 g Nj	932.44 3.00
Enter monthly interest rate	10 g i	0.83
Compute net present value (Result tells us we have over-generated PV by approximately $0.36)	f NPV	100,000.36

Compute future value of the excess at the end of the loan term using the mortgage yield rate of 10%:

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Input "excess" PV, negative-signed	100,000 - CHS PV	-0.36
Input loan term	360 n	360.00
Make sure PMT register clear	0 PMT	0.00
Compute Future Value of excess PV	FV	7.17
Change sign of computed FV	CHS	-7.17
Add negative-signed excess to final computed PMT. Amount displayed is the final (360th) payment	RCL 8 +	925.27

Alternative procedure for Computing the final (360th) payment

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter monthly interest rate	10 g i	0.83
Enter loan amount, negative-signed	100,000 CHS PV	-100,000.00
Enter first year payment;	730.59 PMT	730.59
amortize 12 payments	12 f AMORT	10,058.10
Enter second year payment;	767.12 PMT	767.12
amortize 12 payments	12 f AMORT	10,172.63
Enter third year payment;	805.48 PMT	805.48
amortize 12 payments	12 f AMORT	10,252.21
Enter fourth year payment;	845.75 PMT	845.75
amortize 12 payments	12 f AMORT	10,290.85
Enter fifth year payment;	888.04 PMT	888.04
amortize 12 payments	12 f AMORT	10,281.78
Enter final computed payment;	932.44 PMT	932.44
amortize 300 payments	300 f AMORT	177,112.32
Recall loan balance from PV register	RCL PV	7.87 *
Subtract excess PV from payment	CHS RCL PMT +	924.57

---

* You will note that the computed loan balance using the amortization procedure results in an "overpayment" of $7.87, whereas we computed $7.17 using the net present value (NPV) procedure. The two procedures produce a difference of $.70 in the computed last (360th) payment. Why?

The variance in amounts produced by the two procedures is squarely attributed to the fact that the AMORT function uses a rounding procedure which produces slightly different results depending upon the number of decimal places you caused your calculator to display.

GPM EXAMPLE II
COMPUTING THE PAYMENT SCHEDULE, PRICE AND YIELD OF A GPM

Example: Lender will issue a 20 year, $100,000, 10.5% annual interest graduated payment mortgage with payments increasing 6% per year for four years after the first year. Prepare a payment schedule, including the last payment calculation.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Set 2 decimal places; clear registers	f 2 f CLX	0.00
Enter "1" as 1st payment amount in series of grouped cash-flows	1 g CFj	1.00
Enter number of payments in first group of cash-flows	12 g Nj	12.00
Recall 1st year payment and increase by growth percentage; enter in CFj	R↓ 6 % + g CFj	1.06
Enter 12 PMTS in 2nd cash-flow group	12 g Nj	12.00
Recall 2nd year payment and increase by growth percentage; enter in CFj	R↓ 6 % + g CFj	1.12
Enter 12 PMTS in 3rd cash-flow group	12 g Nj	12.00
Recall 3rd year payment and increase by growth percentage; enter in CFj	R↓ 6 % + g CFj	1.19
Enter 12 PMTS in 4th cash-flow group	12 g Nj	12.00
Recall 4th year payment and increase by growth percentage; enter in CFj	R↓ 6 % + g CFj	1.26
Enter final # of PMTS in maximum groups of 99 (the HP 12C's limit)	99 g Nj	99.00
Recall 5th year PMT and reenter	R↓ g CFj	1.26
Enter final number of cash-flows [240 - (48 + 99)] = 93	93 g Nj	93.00
Enter monthly interest rate	10.5 g i	0.88
Compute net present value	f NPV	119.54
Invert NPV	[1/x]	0.01
Input loan amount; compute and round first year payment	100,000 x f RND	836.57
Compute and round second year PMT	6 % + f RND	886.76
Compute and round third year PMT	6 % + f RND	939.97
Compute and round fourth year PMT	6 % + f RND	996.37
Compute and round 49th through 240th PMT (subject to last payment calculation for 240th month)	6 % + f RND	1,056.15

Computing the final (240th) payment

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers	f CLX	0.00
Enter first year PMT as 1st cash-flow group. Input 12 PMTS	836.57 g CFj 12 g Nj	836.57 12.00
Enter second year PMT Input 12 PMTS	886.76 g CFj 12 g Nj	886.76 12.00
Enter third year PMT Input 12 PMTS	939.97 g CFj 12 g Nj	939.97 12.00
Enter fourth year PMT Input 12 PMTS	996.37 g CFj 12 g Nj	996.37 12.00
Input final computed PMT Input 99 PMTS	1,056.15 g CFj 99 g Nj	1,056.15 99.00
Reenter final computed PMT Input final number of PMTS [240 - (48 + 99)] = 93	R↓ g CFj 93 g Nj	1,056.15 93.00
Enter monthly interest rate	10.5 g i	0.88
Compute net present value (Result tells us that we have under-generated PV by $0.46.)	f NPV	99,999.54
Compute future value of $.46 at end of loan term using the mortgage yield rate of 10.5%:		
Input deficit PV, negative-signed	100,000 - PV	-0.46
Input loan term	240 n	240.00
Make sure PMT register clear	0 PMT	0.00
Compute FV of PV deficit	FV	3.68
Add FV to PMT	RCL 6 +	1,059.83

Note: To recap the above procedure, we first computed the discounted present value of the computed cash-flows. Our objective here was to determine the amount which must be projected forward in order to adjust the final (240th) payment. We noted that the computed present value of the payment schedule produced $99,999.54. This tells us that rounding cost us approximately $0.46 **at the present value level.**

Since the payment schedule is off by approximately $0.46 at the time the loan is originated, we must project this amount **forward** in order to compute the amount needed to be **added** to the final (240th) payment. We compute the needed amount to be $3.68, which amount must be paid with the last payment. This is really nothing more than a small "balloon payment" to be made along with the final regular payment.

Example: Assume an investor will purchase the right to receive the 240 month graduated payment loan computed on the prior two pages. If the investor will receive a yield of 11% annually, what price would be offered? After you compute the price ($96,576.61), verify that the investor's yield equals 11%.

We will use the unequal cash-flow function of your HP 12C to solve this problem.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers	f CLX	0.00
Enter required monthly yield	11 g i	0.92
Enter first year PMT as 1st cash-flow group. Input 12 PMTS	836.57 g CFj 12 g Nj	836.57 12.00
Enter second year PMT Input 12 PMTS	886.76 g CFj 12 g Nj	886.76 12.00
Enter third year PMT Input 12 PMTS	939.97 g CFj 12 g Nj	939.97 12.00
Enter fourth year PMT Input 12 PMTS	996.37 g CFj 12 g Nj	996.37 12.00
Enter 49th - 239th PMT Input 99 PMTS	1,056.15 g CFj 99 g Nj	1,056.15 99.00
Reenter 49th - 239th PMT Input 92 PMTS	R↓ g CFj 92 g Nj	1,056.15 92.00
Enter final (240th) PMT	1,059.83 g CFj	1,059.83
Compute net present value (We are discounting the payment schedule at an 11% yield)	f **NPV**	**96,576.61**
Change sign of computed NPV and store in memory register 0 *	CHS STO 0	-96,576.61
Compute the internal rate of return of the cash-flows (takes over 1 minute)	f IRR	0.92
Convert monthly yield to annual	RCL g i	11.00
Show 9 place accuracy	f 9	10.99999999

---

* Note that when we entered the computed net present value (96,576.61) into the calculator's Memory Register 0, we could just as well have used another procedure which would have required typing: [g] [CFo]. We did not use this procedure, however, since using the CFo function to input an out-of-pocket cash-flow sets the HP 12C's [n] register back to zero. Following this procedure would require that we input a "7" into the [n] register before starting the internal rate of return (IRR) calculation.

PROGRAM FOR COMPUTING GRADUATED PAYMENT MORTGAGE SCHEDULE

This program is designed to compute the payment schedule necessary to amortize a graduated payment loan. The program is restricted to loans with twelve payments per year, with annual payment increases starting after the first twelve payments.

KEYSTROKE	DISPLAY		KEYSTROKE	DISPLAY	
f P/R f PRGM			RCL 1	28	45 1
RCL i	01	45 12	STO i	29	44 12
STO 1	02	44 1	1	30	1
1	03	1	2	31	2
RCL 7	04	45 7	STO n	32	44 11
%	05	25	0	33	0
+	06	40	STO FV	34	44 15
1	07	1	PV	35	13
RCL 1	08	45 1	STO x 3	36	44 20 3
%	09	25	RCL 4	37	45 4
+	10	40	RCL 2	38	45 2
1	11	1	1	39	1
2	12	2	2	40	2
y^x	13	21	x	41	20
÷	14	10	-	42	30
1	15	1	STO n	43	44 11
-	16	30	PV	44	13
EEX	17	26	1	45	1
2	18	2	RCL 7	46	45 7
x	19	20	%	47	25
STO i	20	44 12	+	48	40
1	21	1	RCL 2	49	45 2
CHS	22	16	y^x	50	21
STO PMT	23	44 14	x	51	20
RCL 2	24	45 2	1	52	1
STO n	25	44 11	RCL 1	53	45 1
FV	26	15	%	54	25
STO 3	27	44 3	+	55	40

KEYSTROKE	DISPLAY
RCL 2	56 45 2
1	57 1
2	58 2
x	59 20
y^x	60 21
÷	61 10
STO + 3	62 44 40 3
RCL 5	63 45 5
RCL 3	64 45 3
÷	65 10
R/S	66 31
f 2	67 42 2
f RND	68 42 14
STO 3	69 44 3
1	70 1
STO 0	71 44 0
1	72 1
STO + 0	73 44 40 0
RCL 3	74 45 3
RCL 7	75 45 7
%	76 25
+	77 40
STO 3	78 44 3
R/S	79 31
RCL 2	80 45 2
RCL 0	81 45 0
g x≷y	82 43 34
g GTO 72	83 43 33 72
f FIN	84 42 34
CLX	85 35
f R/S	

Required Information and Memory Storage Locations Used

1. Monthly interest rate, store in [i].
2. Number of years payments increase, store in memory register 2.
3. Loan term (months), store in memory register 4.
4. Loan amount, store in memory register 5.
5. Percentage annual payment growth, store in memory register 7.
6. Set END mode (g END).
7. Overall memory usage: **P - 85 r - 09.**

Note: Look at steps 67 **(f 2)** and 68 **(f RND)**. Here we are causing the calculator's display to be set to two decimal places (f, 2), and immediately thereafter round to the number of places set (f, RND). The purpose of doing this is to produce a more accurate amortization schedule, though the difference without this procedure can be no more than one or two pennies. In this regard, the above program **may** produce results which differ by one or two pennies when compared with other published GPM programs.

If the user prefers to eliminate the rounding steps, delete steps 67 and 68. You will then move every step from lines 69 through 85 back by two lines. For example, "old" step 69 will be "new" step 67, and so forth. Also, you will change the GOTO step by having the program GTO "new" line 70, instead of "old" line 72.

COMPUTING A GPM SCHEDULE BY HP 12C PROGRAM

Example: Assume a $100,000, 30 year GPM at 10% annual interest, with monthly payments increasing five percent (5%) per year for five years immediately following the first twelve payments. We will compute six different payments. (You will recognize this example from a prior GPM manual keystroke procedure.)

ORGANIZATION OF DATA/INPUTS

1. Monthly interest rate stores in [i].
2. Number of years of payment increase stores in memory register 2.
3. Loan term, in months, stores in memory register 4.
4. Loan amount stores in memory register 5.
5. Percentage annual growth in payment stores in memory register 7.

PROCEDURE	KEYSTROKE/INPUTS	DISPLAY
Enter GPM program into your HP 12C		
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter monthly interest rate	10 g i	0.83
Enter number of years of payment increase after first year	5 STO 2	5.00
Enter total number of payments	360 STO 4	360.00
Enter loan amount	100,000 STO 5	100,000.00
Enter annual growth in payments	5 STO 7	5.00
Assure program in starting position	g GTO 00	5.00
Compute first year payment	R/S	730.59
Compute second year payment	R/S	767.12
Compute third year payment	R/S	805.48
Compute fourth year payment	R/S	845.75
Compute fifth year payment	R/S	888.04
Compute 61st - 360th payment, subject to last PMT calculation	R/S	932.44
Confirm computation completed	R/S	0.00

Comment: The results obtained are consistent with the payment schedule computed using the manual keystroke procedure covered on pages 179 and 180.

SOLVING FOR PRICE (NPV) AND YIELD (IRR) OF A GPM

Example: Assume a lender issued a 30 year, $100,000, 12%, monthly payment GPM with the end of month payment schedule given below. Let's also assume an investor desires to purchase the loan immediately after the tenth payment is made. The lender offers a price which will produce an 11% annual yield. The investor desires a yield of 13%. Compute the investor's offer and the lender's counter offer.

ORIGINAL # OF PAYMENTS	MONTHLY PAYMENT	# PAYMENTS DUE AT OFFER
12	$ 863.48	2
12	906.66	12
12	951.99	12
12	999.59	12
12	1,049.57	12
299	1,102.05	299
1	$1,094.55	1

Solution methodology: The problem presents a straightforward application of discounting an unequal income stream by manual keystroke procedure on your HP 12C. We use the unequal cash-flow function.

Note: You cannot perform this calculation if you have the GPM program in your calculator since it requires ten free storage registers.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Set 2 places; clear all registers	f 2 f CLX	0.00
Input investor's required monthly yield	13 g i	1.08
Enter Grouped Payments:		
1st year - PMT AMOUNT	863.48 g CFj	863.48
# PMTS REMAINING	2 g Nj	2.00
2nd year - PMT AMOUNT	906.66 g CFj	906.66
# PMTS REMAINING	12 g Nj	12.00
3rd year - PMT AMOUNT	951.99 g CFj	951.99
# PMTS REMAINING	12 g Nj	12.00
4th year - PMT AMOUNT	999.59 g CFj	999.59
# PMTS REMAINING	12 g Nj	12.00
5th year - PMT AMOUNT	1,049.57 g CFj	1,049.57
# PMTS REMAINING	12 g Nj	12.00

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
61st - 299th PMT	1,102.05 g CFj	1,102.05
(Use maximum groups of 99)	99 g Nj	99.00
	R↓ g CFj	1,102.05
	99 g Nj	99.00
	R↓ g CFj	1,102.05
	99 g Nj	99.00
	R↓ g CFj	1,102.05
	2 g Nj	2.00
Enter last payment	1,094.55 g CFj	1,094.55
Compute net present value (Investor's offer)	f NPV	94,073.18
Input lender's offered monthly yield	11 g i	0.92
Compute net present value	f NPV	109,821.94

Problem: Let's use the data already in your calculator to prove the yield of the GPM was originally 12% per year. To do this we will reset the number of payments in the first year to 12. We then compute the loan's internal rate of return (IRR).

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Verify number of cash-flow groups input into the [n] register	RCL n	10.00
Set [n] register to "1"	1 n	1.00
Verify number of cash-flows input into 1st cash-flow group	RCL g Nj	2.00
Input 12 cash-flows into first cash-flow group	12 g Nj	12.00
Input present value of loan as negative-signed initial cash-flow	100,000 CHS g CFo	-100,000.00
(Note that [n] register was set back to 0 by use of CFo register)	RCL n	0.00
Reset [n] register to 10 cash-flow groups	10 n	10.00
Compute original monthly yield (Requires 1 minute, 50 seconds)	f IRR	1.00
Convert to annual yield	RCL g i	12.00
Verify result at 9 decimal places	f 9	12.00000000

PREPARING ANNUAL AMORTIZATION STATEMENT FOR THE GPM

Preparing the GPM's amortization statement with the HP 12C presents a special need which can best be met by using the program setforth below.

Graduated Payment Loan Amortization Program

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
f R/S f PRGM		RCL 0	28 45 0
RCL 0	01 45 0	f AMORT	29 42 11
f AMORT	02 42 11	STO + 3	30 44 40 3
STO 1	03 44 1	x≷y	31 34
x≷y	04 34	STO + 4	32 44 40 4
STO 6	05 44 6	RCL n	33 45 11
RCL n	06 45 11	R/S	34 31
R/S	07 31	RCL 3	35 45 3
RCL 1	08 45 1	R/S	36 31
R/S	09 31	RCL 4	37 45 4
RCL 6	10 45 6	R/S	38 31
R/S	11 31	RCL PV	39 45 13
RCL PV	12 45 13	R/S	40 31
R/S	13 31	RCL n	41 45 11
RCL n	14 45 11	RCL 2	42 45 2
RCL 7	15 45 7	+	43 40
-	16 30	RCL 7	44 45 7
g [x=0]	17 43 35	-	45 30
g GTO 63	18 43 33 63	g [x=0]	46 43 35
RCL 2	19 45 2	g GTO 49	47 43 33 49
g [x=0]	20 43 35	g GTO 19	48 43 33 19
g GTO 01	21 43 33 01	RCL 5	49 45 5
f AMORT	22 42 11	STO PMT	50 44 14
STO 3	23 44 3	RCL 2	51 45 2
x≷y	24 34	f AMORT	52 42 11
STO 4	25 44 4	STO 3	53 44 3
RCL 5	26 45 5	x≷y	54 34
STO PMT	27 44 14	STO 4	55 44 4

KEYSTROKE	DISPLAY		KEYSTROKE	DISPLAY	
RCL n	56	45 11	R/S	63	31
R/S	57	31	0	64	0
RCL 3	58	45 3	R/S	65	31
R/S	59	31	0	66	0
RCL 4	60	45 4	R/S	67	31
R/S	61	31	g GTO 00	68	43 33 00
RCL PV	62	45 13	f R/S		

Required Information and Memory Storage Locations Used

1. Input loan amount, negative-signed, into [PV].
2. Input monthly interest rate in [i].
3a. First year payment, store in [PMT] register.
3b. Payments number 2 through and including final year payment store in Mem. Reg. 5. If program is used to amortize a fixed payment loan with less than 12 payments in the first year, you will input the payment amount into both the [PMT] register and Mem. Reg. 5.
4a. Number of payments in first year, store in Mem. Reg. 0. For example, if the loan requires 4 installments of the first year's payment, enter "4" into Mem. Reg. 0; there will be 8 remaining payments of the same amount to be made in the second year of the loan, so enter "8" into Mem. Reg. 2. If 12 payments made in first year, enter "12" into Mem Reg 0, and enter "0" in Mem. Reg. 2.
4b. Remaining number of payments from first year, store in Mem. Reg. 2.
5. Total number of payments in amortization schedule, store in Mem. Reg. 7. This amount is not necessarily the loan's original term. Rather, it is the total number of payments which you plan to amortize.
6. Set END (g END) mode.
7. Program automatically amortizes final year of the GPM, and displays "0".
8. **Caution:** You must make sure the [n] register is set to "0".
9. Overall memory usage: **P - 71, r - 11.**

Summary of registers used:

Memory Registers: 0, 1, 2, 3, 4, 5, 6, and 7.

Financial Registers: [i], [PV], and [PMT].

Description: The program enables you to prepare an amortization schedule for a GPM. The payment schedule can consist of less than 12 payments in the first year, and therefore less than 12 payments in the final year amortized. For example, assume the loan requires four payments of $1,000 per month to be made in the first year, and as well requires a second year payment of $1,200 per month. The program will amortize four payments of $1,000 in the first year, plus the remaining eight in the second year, and then amortizes four payments of $1,200 per month during the second year, and so forth.

Problem: Prepare an amortization schedule for the first 72 payments of a 30 year, $100,000 GPM, written at 10% annual interest. Assume the borrower makes five payments during the first calendar year of the loan. Therefore, he makes five payments of $730.59 in the first year, seven of the same amount in the second year, and makes five payments of $767.12 in the second year, and so forth. The loan's partial payment schedule is setforth below.

YEAR	PAYMENT 1	NUMBER	PAYMENT 2	NUMBER
1	$730.59	5	---	---
2	730.59	7	$767.12	5
3	767.12	7	805.48	5
4	805.48	7	845.75	5
5	845.75	7	888.04	5
6	888.04	7	$932.44	5
7	$ 932.44	7		

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter AMORT Program into your HP 12C		
Clear all registers; set 2 places and END mode	f CLX f 2 g END	0.00
Enter monthly interest rate	10 g i	0.83
Enter loan amount, negative-signed	100,000 CHS PV	-100,000.00
Enter 1st year loan payment	730.59 PMT	730.59
Enter 2nd year loan payment	767.12 STO 5	767.12
Number of payments in first year	5 STO 0	5.00
Remaining payments from first year	7 STO 2	7.00
Total number payments to amortize	72 STO 7	72.00
1st calendar year amortization:		
Confirm number of payments made	R/S	5.00
Interest paid in first year	R/S	4,175.29
Negative amortization in first year	R/S	-522.34
Remaining loan balance	R/S	-100,522.34
Enter 3rd year loan payment	805.48 STO 5	805.48
2nd calendar year amortization:		
Confirm total payments to date	R/S	17.00
Interest paid in 2nd year	R/S	10,106.52
Negative amortization in 2nd year	R/S	-964.99
Remaining loan balance	R/S	-101,487.33
Enter 4th year loan payment	845.75 STO 5	845.75

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
3rd calendar year amortization:		
Confirm total payments to date	R/S	29.00
Interest paid in 3rd year	R/S	10,168.11
Negative amortization in 3rd year	R/S	-301.00
Remaining loan balance	R/S	-101,788.33
Enter 5th year loan payment	888.04 STO 5	888.04
4th calendar year amortization:		
Confirm total payments to date	R/S	41.00
Interest paid in 4th year	R/S	10,176.69
Principal paid in 4th year	R/S	183.76
Remaining loan balance	R/S	-101,604.57
Enter 6th year loan payment	932.44 STO 5	932.44
5th calendar year amortization:		
Confirm total payments to date	R/S	53.00
Interest paid in 5th year	R/S	10,133.34
Principal paid in 5th year	R/S	745.14
Remaining loan balance	R/S	-100,859.43
6th calendar year amortization:		
Confirm total payments to date	R/S	65.00
Interest paid in 6th year	R/S	10,033.96
Principal paid in 6th year	R/S	1,155.32
Remaining loan balance	R/S	-99,704.11
7th calendar year amortization:		
Confirm total payments to date	R/S	72.00
Interest paid in 7th year	R/S	5,798.05
Principal paid in 7th year	R/S	729.03
Remaining loan balance	R/S	-98,975.08
Confirm final payment amortized	R/S	0.00
Reconfirm problem completed	R/S	0.00

Note: Please note that the program discontinues operation after you have amortized the required number of payments. However, to take advantage of this feature you must store [STO] the number of payments to be amortized into Memory Storage Register 7. Otherwise, if this register is empty, the program will operate until you stop it. Just press any key.

Problem: Let's amortize the first 72 payments of a fixed payment $100,000 loan written at 8% annual interest. The monthly payment is $733.76. Assume two payments are made in the first calendar year. The loan's partial payment schedule is setforth below.

YEAR	PAYMENT	NUMBER
1	$733.76	2
2	733.76	12
3	733.76	12
4	733.76	12
5	733.76	12
6	733.76	12
7	$733.76	10

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 places and END mode	f CLX f 2 g END	0.00
Enter monthly interest rate	8 g i	0.67
Enter loan amount, negative-signed	100,000 CHS PV	-100,000.00
Enter fixed loan payment	733.76 PMT	733.76
Enter 2nd year loan payment	STO 5	733.76
Number of payments in first year	2 STO 0	2.00
Remaining payments from first year	10 STO 2	10.00
Total number payments to amortize	72 STO 7	72.00
1st calendar year amortization:		
Confirm number of payments made	R/S	2.00
Interest paid in first year	R/S	1,332.89
Principal paid in first year	R/S	134.63
Remaining loan balance	R/S	-99,865.37
2nd calendar year amortization:		
Confirm total payments to date	R/S	14.00
Interest paid in 2nd year	R/S	7,958.64
Principal paid in 2nd year	R/S	846.48
Remaining loan balance	R/S	-99,018.89
3rd calendar year amortization:		
Confirm total payments to date	R/S	26.00
Interest paid in 3rd year	R/S	7,888.39
Principal paid in 3rd year	R/S	916.73
Remaining loan balance	R/S	-98,102.16

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
4th calendar year amortization:		
Confirm total payments to date	R/S	38.00
Interest paid in 4th year	R/S	7,812.30
Principal paid in 4th year	R/S	992.82
Remaining loan balance	R/S	-97,109.34
5th calendar year amortization:		
Confirm total payments to date	R/S	50.00
Interest paid in 5th year	R/S	7,729.89
Principal paid in 5th year	R/S	1,075.23
Remaining loan balance	R/S	-96,034.11
6th calendar year amortization:		
Confirm total payments to date	R/S	62.00
Interest paid in 6th year	R/S	7,640.64
Principal paid in 6th year	R/S	1,164.48
Remaining loan balance	R/S	-94,869.63
7th calendar year amortization:		
Confirm total payments to date	R/S	72.00
Interest paid in 7th year	R/S	6,293.69
Principal paid in 7th year	R/S	1,043.91
Remaining loan balance	R/S	-93,825.72
Confirm final payment amortized	R/S	0.00
Reconfirm problem completed	R/S	0.00

Comment: Readers who are interested in increasing the versatility of their HP 12C programs should study the GPM amortization program to determine how the author caused it to flip-flop back and forth and amortize the required number of payments of each loan payment in the correct year. Equally important, the **test** used to enable the program to discontinue operation after the last payment has been made is believed to be novel. This technique should expand the flexibility of your HP 12C programs.

THE GROWING EQUITY MORTGAGE (GEM)

The GEM is a real estate financing device which enables the lender to recoup the principal amount of its loan much quicker when compared with a conventional 30 year fixed payment loan. This feature has clear benefit to the borrower in that the total interest paid on a GEM loan will be far less than would otherwise be paid with a conventional fixed payment 25-to-30 year loan. This occurs because the monthly payment on the GEM grows annually, thereby contributing to a much faster reduction in the loan balance. This feature has a flip-side, which is that the risk to the borrower is substantially increased if his income does not keep pace with the increasing annual financial demands of the GEM.

The first year's monthly payment on the GEM is calculated exactly like that of any traditional fixed payment loan, though the GEM's initial payment is typically based upon a 25-to-30 year amortization period. After the first year the payment increases by an agreed upon percentage, usually from 2 to 5 percent--though it could be as high as 7 percent in exceptional situations--and continues to increase at the same rate for a specified length of time during the loan's term.

The GEM closely parallels the graduated payment mortgage (GPM) in that its payments increase annually. However, while the typical GPM undergoes negative amortization and thus experiences an increase in its principal balance during its early years, this feature is not characteristic of a GEM. Indeed, since the GEM's first year payment is set to **exactly** the amount needed to pay off the loan over a 25-to-30 year term, negative amortization is impossible if the payments are made timely.

The monthly payments on the GEM might be structured to increase for ten years and thereafter remain constant for the balance of the term. This arrangement is much less burdensome to the borrower than being subject to a payment schedule which undergoes an increase each and every year after the first year of the GEM's term. Insofar as the relationship between the GEM's term and the number of years of agreed upon payment increase, the greater the number of years of payment increase, the shorter the term of the loan, and the shorter the number of years of payment increase, the longer the term of the loan.

The GEM's term usually ranges from 15 to 20 years. From the perspective of most practitioners, the difficulty in dealing with this powerful financing tool is clearly found in the complexity of trying to calculate the GEM's term by traditional calculator methods. The program in this book, however, is designed to give the reader the GEM's term in a matter of seconds. There are, for sure, a number of potential pitfalls to watch for when computing the GEM's term, but this is a small price to pay for performing it on an inexpensive--though exceptionally useful--hand-held calculating device.

The GEM loan is more attractive to the home buyer who anticipates an increasing level of income over the term of the loan. You should be guarded in recommending a GEM unless the buyer expects his income to increase substantially over the term of the GEM.

PROGRAM FOR COMPUTING NUMBER OF MONTHS TO AMORTIZE THE GEM

KEYSTROKE	DISPLAY		KEYSTROKE	DISPLAY	
f R/S f PRGM			y^x	33	21
RCL 0	01	45 0	÷	34	10
1	02	1	1	35	1
0	03	0	-	36	30
0	04	0	÷	37	10
STO 1	05	44 1	RCL 3	38	45 3
÷	06	10	1	39	1
1	07	1	+	40	40
+	08	40	1	41	1
RCL i	09	45 12	2	42	2
RCL 1	10	45 1	CHS	43	16
÷	11	10	y^x	44	21
STO 3	12	44 3	CHS	45	16
1	13	1	1	46	1
+	14	40	+	47	40
1	15	1	RCL 3	48	45 3
2	16	2	÷	49	10
y^x	17	21	x	50	20
÷	18	10	RCL PMT	51	45 14
RCL 2	19	45 2	x	52	20
y^x	20	21	CHS	53	16
1	21	1	RCL PV	54	45 13
-	22	30	+	55	40
RCL 0	23	45 0	RCL 3	56	45 3
RCL 1	24	45 1	1	57	1
÷	25	10	+	58	40
1	26	1	RCL 2	59	45 2
+	27	40	1	60	1
RCL 3	28	45 3	2	61	2
1	29	1	x	62	20
+	30	40	y^x	63	21
1	31	1	RCL 3	64	45 3
2	32	2	x	65	20

KEYSTROKE	DISPLAY		KEYSTROKE	DISPLAY	
x	66	20	RCL 3	81	45 3
RCL 0	67	45 0	1	82	1
RCL 1	68	45 1	+	83	40
÷	69	10	g LN	84	43 23
1	70	1	÷	85	10
+	71	40	CHS	86	16
RCL 2	72	45 2	RCL 2	87	45 2
y^x	73	21	1	88	1
RCL PMT	74	45 14	2	89	2
x	75	20	x	90	20
÷	76	10	+	91	40
CHS	77	16	STO n	92	44 11
1	78	1	ENTER	93	36
+	79	40	g GTO 00	94	43 33 00
g LN	80	43 23	f R/S		

Required Information and Memory Storage Locations Used

1. Input loan amount, positive-signed, in [PV].
2. Input monthly interest rate in [i].
3. Input first year monthly payment, positive-signed, in [PMT].
4. Store annual growth in monthly payment in Mem. Reg. 0.
5. Store number of years of increase in payment amount in Mem. Reg. 2. In many cases you will use a number of years of increase in payment amount which are considerably less than the term of the GEM. In cases where the payments will increase throughout the GEM's term, you will have to estimate the number of years of increase (likely starting with 12 and working up to 15, 16, 17, possibly 18, depending upon the annual interest rate used) and start with that amount as your estimated number of years of growth. This issue will be further clarified below.
6. Set END (g END) mode for all calculations. You are limited to computing the term of a GEM with end of the month payments.
7. The GEM's computed term may be recalled from the [n] register, and displays at termination of the program's operation.
8. Overall memory usage: **P - 99 r - 07.**

Summary of registers used:

Memory Registers: 0, 1, 2, and 3. Registers 4, 5, and 6 are available.

Financial Registers: [n], [i], [PV].

Note: To determine the number of months to amortize a GEM, it is not necessary to use an actual payment amount; the payment is not relevant. Instead of the monthly payment, you can use the "installment to amortize $1.00" (ITAO) as the payment and "1" as the "PV". The program is designed to accept the amount financed and its equivalent payment as **positive numbers,** and thus accepts a "PV of $1.00" and its equivalent "ITAO" as **positive numbers.** Make sure you are consistent in your inputs into the program.

Problem: Let's compute the number of years (and the equivalent number of months) necessary to amortize a 360 month, 8% annual interest GEM loan which is projected to grow by 2%, 3%, 4%, and 5% per year, with growth periods of 10, 12, and 14 years. the results are setforth in the grid. The keystrokes follow the grid.

8% GEM, 30 YEAR INITIAL TERM

Time to Amortize

	10 YEARS OF PAYMENT GROWTH	12 YEARS OF PAYMENT GROWTH	14 YEARS OF PAYMENT GROWTH	
ANNUAL GROWTH	MONTHS/YEARS	MONTHS/YEARS	MONTHS/YEARS	
2%	248.03/20.67	241.86/20.16	237.82/19.82	
3%	218.45/18.20	212.46/17.71	209.06/17.42	
4%	196.43/16.37	191.14/15.93	188.73/15.73	
5%	179.35/14.95	174.99/14.58	**173.74/14.48**	**Caution**

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear registers; set 2 places and END mode	f CLX f 2 g END	0.00
Input initial loan term	360 n	360.00
Input monthly interest rate	8 g i	0.67
Input "1" for loan amount	1 PV	1.00
Compute ITAO. Change sign and reenter as payment amount	PMT CHS PMT	-0.01 0.01
Enter first annual PMT growth rate	2 STO 0	2.00
Number of years of payment growth	10 STO 2	10.00
Compute months needed to amortize	R/S	248.03
Convert to number of years	RCL g n	20.67
Assume 12 years payment growth	12 STO 2	12.00
Compute months needed to amortize	R/S	241.86

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Convert to number of years	RCL g n	20.16
Assume 14 years payment growth	14 STO 2	14.00
Compute months needed to amortize	R/S	237.82
Convert to number of years	RCL g n	19.82
Enter 3% annual PMT growth rate	3 STO 0	3.00
Number of years of payment growth	10 STO 2	10.00
Compute months needed to amortize	R/S	218.45
Convert to number of years	RCL g n	18.20
Assume 12 years payment growth	12 STO 2	12.00
Compute months needed to amortize	R/S	212.46
Convert to number of years	RCL g n	17.71
Assume 14 years payment growth	14 STO 2	14.00
Compute months needed to amortize	R/S	209.06
Convert to number of years	RCL g n	17.42
Enter 4% annual PMT growth rate	4 STO 0	4.00
Number of years of payment growth	10 STO 2	10.00
Compute months needed to amortize	R/S	196.43
Convert to number of years	RCL g n	16.37
Assume 12 years payment growth	12 STO 2	12.00
Compute months needed to amortize	R/S	191.14
Convert to number of years	RCL g n	15.93
Assume 14 years payment growth	14 STO 2	14.00
Compute months needed to amortize	R/S	188.73
Convert to number of years	RCL g n	15.73

Comment: Looking at the data immediately above, it is apparent that were we to try 15 years of payment growth we could be asking for a nonexistent solution, though, indeed, the program will return an "answer" to this question. The problem here is that we cannot expect a sound answer to the GEM's term if we cause--inadvertently or otherwise--the payment increase period to extend beyond the actual time required to amortize the loan.

For example, using the data for 4% annual payment growth, if we set a payment growth period of 15 years, the program returns "15.70" as the number of years to amortize the loan. This tells us that "15" years of payment growth could be too much since that would take us into the 16th year, though the loan would be paid off in "15.70" years, also included in the 16th year.

You must be on guard for the above issue. Do not automatically use a computed term for a GEM if the number of years of increase are equal to the **integer** value of the computed term. For example, if you used 15 years as the increase period, and if the computed term came to **"15.SOMETHING"**, the **integer** value of the computed term would be 15, therefore equalling the number of years of increase. At this point you would have to use an IRR check to see if the payments balance out, thereby generating the exact IRR as the loan's annual interest rate.

Let's compute the 5% annual growth problem.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter 5% annual PMT growth rate	5 STO 0	5.00
Number of years of payment growth	10 STO 2	10.00
Compute months needed to amortize	R/S	179.35
Convert to number of years	RCL g n	14.95
Assume 12 years payment growth	12 STO 2	12.00
Compute months needed to amortize	R/S	174.99
Convert to number of years	RCL g n	14.58
Assume **14 years payment growth**	14 STO 2	14.00
Compute months "needed" to amortize	R/S	173.74
Convert to **number of years**	RCL g n	**14.48**

Comment: The result obtained immediately above is within the range of answers which require **caution** and an IRR test to make certain that the payment stream is correct. The result obtained tells us that it would take approximately **14.48** years to pay off the loan, though the period of payment increase indeed takes the "payment" into the 15th year. This brings home the critical issue you must always be on guard for when computing a GEM with this program: If the integer value of the computed term is equal to the number of years of payment increases used in the problem, you must test the accuracy of the payment stream with the IRR function.

If your payment increase period does not fall (1) under or (2) to the left of the integer (means rounded down to the whole number) value of the computed term, your answer could be incorrect; therefore, you must use the IRR test if you plan to use the payment schedule.

INTEGER NUMBER OF YEARS TO AMORTIZE	14	15	16	17	18	19	20
YEARS OF GROWTH	13	14	15	16	17	18	19

Conventional Fixed Payment Loan Compared with GEM Financing

Monthly payment for $100,000 loan at 8%
Initial Amortization Term of 30 Years

YEAR	FIXED PAYMENT	3% GEM, 5 year growth 252.16 mo.	3% GEM, 7 year growth 234.4 mo.	3% GEM, 10 year growth 218.45 mo.	3% GEM, full term growth 207.53 mo.
1	$733.76	$733.76	$733.76	$733.76	$ 733.76
2	733.76	755.78	755.78	755.78	755.78
3	733.76	778.45	778.45	778.45	778.45
4	733.76	801.80	801.80	801.80	801.80
5	733.76	825.86	825.86	825.86	825.86
6	733.76	850.63	850.63	850.63	850.63
7	733.76	850.63	876.15	876.15	876.15
8	733.76	850.63	902.44	902.44	902.44
9	733.76	850.63	902.44	929.51	929.51
10	733.76	850.63	902.44	957.40	957.40
11	733.76	850.63	902.44	986.12	986.12
12	733.76	850.63	902.44	986.12	1,015.70
13	733.76	850.63	902.44	986.12	1,046.17
14	733.76	850.63	902.44	986.12	1,077.56
15	733.76	850.63	902.44	986.12	1,109.88
16	733.76	850.63	902.44	986.12	1,143.18
17	733.76	850.63	902.44	986.12	1,177.48
18	733.76	850.63	902.44	986.12	1,212.80
19	733.76	850.63	902.44	986.12	--------
20	733.76	850.63	902.44	------	**208th PMT**
21	733.76	850.63	------	**219th PMT**	**$639.15**
22	733.76	------	**235th PMT**	**$446.38**	
23	733.76	**252nd PMT**	**$361.61**		Requires
24	733.76	**$986.32**		Requires	17 years,
25	733.76		Requires	18 years,	4 months
26	733.76	Requires	19 years,	3 months	to amortize
27	733.76	21 years,	7 months	to amortize	
28	733.76	0 months	to amortize		
29	733.76	to amortize			
30	733.76				

Problem: Verify the payment schedule for the full-term growth GEM.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear registers; set 2 places and END mode	f CLX f 2 g END	0.00
Input initial loan term	360 n	360.00
Input monthly interest rate	8 g i	0.67
Input loan amount	100,000 PV	100,000.00
Compute first year monthly payment; change sign; reenter as PMT	PMT CHS PMT	-733.76 733.76
Enter annual payment growth rate	3 STO 0	3.00
Enter estimate of required number of years of growth in payment	12 STO 2	12.00
Compute months needed to amortize Convert to number of years	R/S RCL g n	212.46 17.71
Enter "16" years growth as guess	16 STO 2	16.00
Compute months needed to amortize Convert to number of years	R/S RCL g n	207.63 17.30
Note: The next answer puts you into the range where an IRR test is mandatory		
Enter "17" years growth as guess	17 STO 2	17.00
Compute months needed to amortize Convert to number of years	R/S RCL g n	207.53 17.29

Discussion: At this stage we know that causing the first year's payment to grow at 3% per year for 17 years will produce a complete amortization of the loan in "17.29" years. The "17.29" years tells us that the payment schedule goes **beyond** the 17th year and covers about "29%" of the 18th year. Looking at it a little differently, there will be 208 payments on the loan, but the last payment (the 208th) will be smaller than the payment which immediately precedes it because we rounded up the loan's computed term from "207.53" months to "208" months.

To verify that the last payment equals $639.15 (see chart on previous page), we will need 19 storage registers open on your HP 12C. Therefore, you will need to purge the GEM program from your calculator. (Sorry!)

Solution methodology: We use the Unequal Cash-Flow Function and the Financial Registers to solve for the last payment in the GEM's payment stream. As well, we could verify the accuracy of the loan's payment stream with the unequal cash-flow function, though we will not do the actual keystrokes, but we will run through the problem in sentence form.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear registers; set 2 places and END mode	f CLX f 2 g END	0.00
Enter loan amount, negative-signed	100,000 CHS g CFo	-100,000.00
Enter first year payment	733.76 g CFj	733.76
Enter 12 payments	12 g Nj	12.00
Enter second year payment	755.78 g CFj	755.78
Enter 12 payments	12 g Nj	12.00
Enter third year payment	778.45 g CFj	778.45
Enter 12 payments	12 g Nj	12.00
Enter fourth year payment	801.80 g CFj	801.80
Enter 12 payments	12 g Nj	12.00
Enter fifth year payment	825.86 g CFj	825.86
Enter 12 payments	12 g Nj	12.00
Note: You should input payment-groups 6 through 17		
Enter 18th year payment	1,212.80 g CFj	1,212.80
Enter 4 payments (208 - 12 x 17 = 4)	4 g Nj	4.00
Input monthly interest rate	8 g i	0.67
Compute net present value of the payment stream	f NPV	144.02

Discussion: The positive NPV tells us that we have produced a payment schedule which, when discounted at the loan's monthly interest rate, produces a present value (NPV) in excess of the actual loan amount. That is, we started with a loan of $100,000, but the present value of the loan's payment schedule is actually $144.02 **greater.**

This is squarely attributed to the fact that the computed term of the GEM loan was "207.53" months, but we had to use "208 payments" because customary loan repayment practices do not involve fractional months. Therefore, were we to make 208 full payments on the loan, we would, in effect, generate a discounted present value which exceeds the loan's original amount by $144.02.

Since the excess loan amount ($144.02) is produced at the inception of the loan, being time period zero, we must project this amount forward in order to determine its full impact on the payment schedule. That is, by projecting the NPV of $144.02 forward by 208 months (the term used to amortize the loan) we will arrive at the future value of this amount. The computed future value (FV) represents the excess which exists in the last payment. By netting the amounts (payment - "excess") we arrive at the final (208th) payment, and indeed arrive at a perfect payment schedule. (Let's solve for the final payment on the next page.)

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Set NPV to negative-signed PV	CHS PV	-144.02
Input loan's recomputed term	208 n	208.00
Make sure PMT register clear	0 PMT	0.00
Compute future value of the NPV	FV	573.65
Subtract FV from last regular payment on the loan	1,212.80 RCL FV -	639.15

Discussion: The 208th payment on the loan is verified at $639.15. You could prove the accuracy of the payment schedule by calculating the IRR or the NPV. The NPV method would be much quicker, and indeed you have almost all the data already in your calculator, assuming you did not clear out the unequal cash-flow registers.

If the loan's payment schedule is still in your calculator, you might set the "$1,212.80" per month payment to occur "3" times instead of "4". To do this, enter the number "18" into your "n" register. We do this because the loan, as currently configured, has "18" unequal cash-flow groups, so if we are to do anything to or with the number of unequal cash-flows stored in the [Nj] register, we have to tell this calculator that the [n] register is indeed set consistent with the cash-flow group we intend to change.

Once you set [n] to "18", just tell the calculator that the number of "$1,212.80" payments are now "3" (instead of "4"). All you have to do is type: 3 g Nj. Next, you must input the final payment, which is the amount we calculated above. To do this, simply type "639.15" g CFj. Your registers are now fully loaded with the payment schedule from this problem and you can run the net present value of the loan payments.

Since we already input the loan amount ($100,000) into the calculator as the initial cash-flow (-100,000.00), by pressing **f NPV** you can calculate the loan's net present value (NPV). If the NPV comes close to zero, the data is correct. Indeed, when you run this problem you will determine that the NPV is ".0000135000" (using [f] [PREFIX] to bring up all the digits).

And, if you have about 2 minutes and 45 seconds, you can verify the loan's yield by simply calculating the IRR of the data in your calculator. Simply press: **f IRR.** Your calculator will produce an annual yield of "7.999999993%".

Comment: Whether or not you regularly compute a GEM payment schedule, the technology learned from mastering this technique has applications in many areas of time value of money. As you please, and with practice, you can design payment or loan payment systems or lease/annuity-analysis techniques that will not only be technically correct, but as well they will add a measure of artistry to your work. It just takes practice and getting a feel for the **flow** of the numbers **through** the problem you are working with. Practice! There are no "easy" avenues to working level competence in this field!

WRAPAROUND MORTGAGES

A wraparound mortgage is one in which a lender makes a loan secured by a mortgage on real estate that is already subject to a first or possibly second mortgage--the "underlying loan". The face amount of the wraparound loan is written for the remaining balance on the underlying loan, plus the amount advanced by the wraparound lender, who may also be the seller. The term of the wraparound loan is either equal to or greater than that of the term remaining on the underlying loan.

Amortization takes place as if the wraparound loan is the first mortgage. The wraparound loan agreement typically requires the lender to make installment payments required on the senior mortgage out of payments received on the wraparound mortgage. Balloon payments, if any, required on the underlying loan are paid by the wraparound lender whether or not an equal payment is required on the wraparound mortgage at the time the balloon payment becomes due on the underlying loan.

The interest rate on the wraparound loan is always higher than the rate on the underlying mortgage. Therefore, the wraparound lender realizes a higher yield than that specified in the wraparound mortgage agreement since it collects the higher rate on the full outstanding balance, but only pays the senior mortgage holder the lower interest rate required on the underlying loan.

The most common wraparound loan calculation involves computing the yield realized by the wraparound lender on its investment. The lender's net investment is its out-of-pocket cash-flow at the time of making the wraparound loan. That is, it is the difference between the face amount of the wraparound mortgage and the balance on the underlying loan, plus the excess of any balloon payments received on the wraparound loan. This method is covered in the examples that follow.

Rather than work backwards from a given payment to the wraparound lender's yield, it is submitted that the more appropriate procedure is to calculate the **payment necessary** to produce the yield required by the wraparound lender on its investment. That is, we should first determine the yield required by the wraparound mortgage lender and then calculate the payment necessary to produce the required yield. Making full disclosure of the lender's true yield--and the APR of the loan--makes for not only good business practice in a commercial loan transaction, but also satisfies one of the lender's disclosure responsibilities in a consumer-borrower loan transaction under the Federal Truth in Lending Act. This method is covered following the wraparound loan yield examples.

After we go through the underlying fundamentals and keystrokes of how to calculate the yield on a wraparound loan we will work with a HP 12C program and effortlessly run through the WRAP's required monthly payment.

SOLVING FOR YIELD ON A SIMPLE WRAPAROUND MORTGAGE WITH BALLOON PAYMENT

Solution Objective: To calculate the lender's yield on a simple wraparound mortgage.

Summary of Information Required:

1. Annual interest rate charged on both loans.
2. Monthly payment on underlying loan.
3. Number of payments remaining on underlying loan.
4. Balance on underlying loan at time wraparound mortgage made.
5. Amount borrowed on wraparound loan.
6. Monthly payment on wraparound loan.
7. Balloon payment, if any, on wraparound loan.
8. Total number of payments on wraparound loan.

Assumptions--Limitations--Based Upon Solution Methodology

We assume that all payments, both underlying and wraparound, are made at the end of the month and that interest on both loans is computed and compounded on a monthly basis. Further, the balloon payment due on the wraparound loan must be made with the final payment on the wraparound loan. Finally, no variable payments (cash-flows) are allowed; that is, all payments must be the same within each loan type.

Example:

Borrower has a 10.68% mortgage (the "underlying loan") with 120 remaining payments of $808.40 per month. The loan has an outstanding balance of $59,465.28 at the time the wraparound mortgage is made.

The borrower applied for, and the lender issued, a $157,000, 11.6%, wraparound mortgage which requires 120 payments of $2,062.81 per month, plus a balloon payment of $34,500 due with the final payment.

Compute the lender's yield.

Solution:

Our first step requires that we calculate the lender's out-of-pocket cash-flow as well as the net monthly payments to the lender. We will use two different solution methods: The first uses the Financial Register Keys; the second uses the unequal cash-flow function (IRR).

Calculation of Input Data for Simple Wraparound Mortgage with Balloon

A. Lender's out-of-pocket cash-flow:

1. Wraparound mortgage	- $157,000.00
2. Plus: Underlying mortgage balance	+ 59,465.28
3. Equals Lender's net cash-flow	- $ 97,534.72

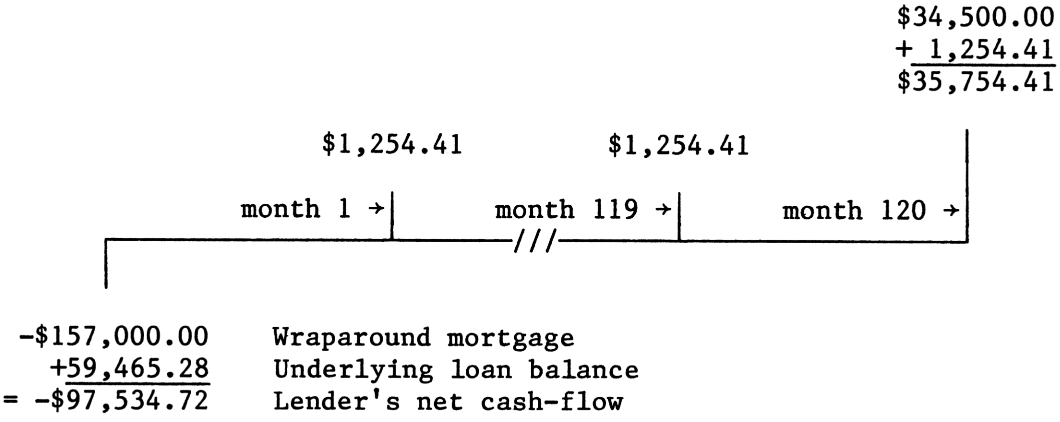
B. Net monthly cash-flows:

1. Monthly payment on wraparound loan	$ 2,062.81
2. Less: Underlying loan payment	- 808.40
3. Equals monthly cash-flow to lender	$ 1,254.41

C. Summary of cash-flows:

1. Lender's initial cash-flow	- $97,534.72
2. Net monthly cash-flow to lender Number of payments = 119	1,254.41
3. Last monthly payment plus balloon payment ($1,254.41 + $34,500.00) Number of payments = 1	$ 35,754.41

D. A summary Timeline Diagram follows:



KEYSTROKE ROUTINE TO SOLVE FOR YIELD ON A SIMPLE WRAPAROUND PAYMENT MORTGAGE WITH BALLOON

PROCEDURE: FINANCIAL REGISTERS	KEYSTROKE/INPUT	DISPLAY
Set 2 decimal places; clear all registers; set END mode	f 2 f CLX g END	0.00
Input number of payments remaining on underlying and wraparound loans	120 n	120.00
Set PV to lender's out-of-pocket cash-flow	97,534.72 CHS PV	-97,534.72
Set PMT to lender's net cash-inflow	1,254.41 PMT	1,254.41
Set FV to balloon payment on wraparound loan	34,500 FV	34,500.00
Calculate lender's monthly yield	i	1.01
Convert to annual yield	RCL g i	12.08

PROCEDURE: UNEQUAL CASH-FLOW METHOD	KEYSTROKE/INPUT	DISPLAY
Clear all registers	f CLX	0.00
Input lender's out-of-pocket cash-flow as initial cash-flow	97,534.72 CHS g CFo	-97,534.72
Enter grouped cash-flows:		
1st through 99th payment	1,254.41 g CFj	1,254.41
# of PMTS	99 g Nj	99.00
100th through 119th payment	R↓ g CFj	1,254.41
# of PMTS	20 g Nj	20.00
Add balloon payment to lender's net cash-flow; enter final flow	R↓ 34,500 + g CFj	35,754.41
Compute monthly yield	f IRR	1.01
Convert to annual yield (You could also press: **RCL g i**)	12 x	12.08

SOLVING FOR YIELD ON A COMPLEX WRAPAROUND MORTGAGE WITH BALLOON PAYMENT

Solution Objective: To calculate the lender's yield on a complex wraparound mortgage.

Summary of Information Required:

1. Annual interest rate on both loans.
2. Monthly payment on underlying loan.
3. Number of payments remaining on underlying loan.
4. Balance on underlying loan at time wraparound mortgage made.
5. Balloon payment, if any, on underlying loan.
6. Amount borrowed on wraparound loan.
7. Monthly payment on wraparound loan.
8. Balloon payment on wraparound loan.
9. Total number of payments on wraparound loan.

Assumptions--Limitations--Based Upon Solution Methodology

We assume that all payments, both underlying and wraparound, are made at the end of the month and that interest on both loans is computed and compounded on a monthly basis. Further, the balloon payment due on the wraparound loan must be made with the final payment on the wraparound loan, and as well the balloon payment, if any, due on the underlying loan must be made with the final payment on the underlying loan. Finally, no variable payments (cash-flows) are allowed; that is, all payments must be the same within each loan type.

Example:

Borrower has a 10.68% mortgage (the "underlying loan") with 120 remaining payments of $808.40 per month. The loan has an outstanding balance of $59,465.28 at the time the wraparound mortgage is made.

The borrower applied for, and the lender issued, a $200,000, 11.6%, wraparound mortgage which requires 180 payments of $2,292.98 per month, plus a balloon payment of $27,000 due with the final payment.

Compute the lender's yield.

Solution:

We first calculate the lender's out-of-pocket cash-flow as well as the net monthly payments to the lender. Since the term of the wraparound loan is not equal to the remaining number of payments on the underlying loan, we must use the unequal cash-flow function of your calculator to solve this problem.

Calculation of Input Data for Complex Wraparound Mortgage with Balloon

A. Lender's out-of-pocket cash-flow:

1. Wraparound mortgage	- $200,000.00
2. Plus: Underlying mortgage balance	+ 59,465.28
3. Equals Lender's net cash-flow	- $140,534.72

B. Net monthly cash-flows:

1. Monthly payment on wraparound loan	$ 2,292.98
2. Less: Underlying loan payment	- 808.40
3. Equals monthly cash-flow to lender	$ 1,484.58

Number of payments at above amount = 120

C. Monthly cash-flows during remaining term of wraparound mortgage:

1. Monthly payment on wraparound loan	$ 2,292.98

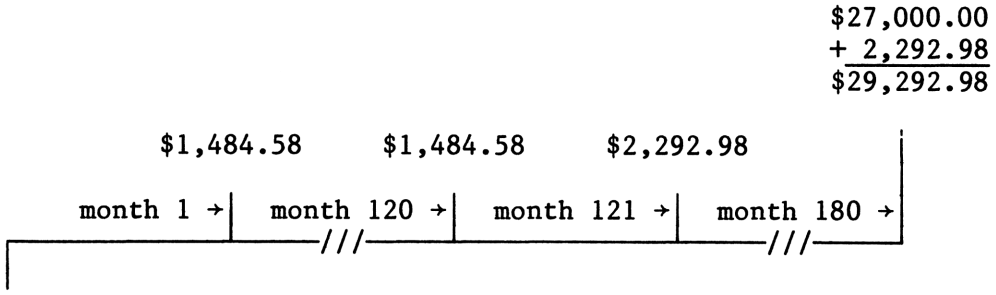
Number of payments at above amount = 59

D. Final cash-flow on wraparound mortgage:

1. Monthly payment plus balloon payment ($2,292.98 + $27,000.00)	$ 29,292.98

Number of cash-flows at above amount = 1

E. A summary Timeline Diagram follows:



-$200,000.00	Wraparound mortgage
+59,465.28	Underlying loan balance
-$140,534.72	Lender's net cash-flow

KEYSTROKE ROUTINE TO SOLVE FOR YIELD ON A COMPLEX WRAPAROUND PAYMENT MORTGAGE WITH BALLOON PAYMENT

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Set 2 decimal places; clear all registers *	f 2 f CLX	0.00
Input lender's out-of-pocket cash-flow as initial cash-flow	140,534.72 CHS g CFo	-140,534.72
Input net payments during remaining term of underlying mortgage:		
1st through 99th payment	1,484.58 g CFj	1,484.58
# of PMTS	99 g Nj	99.00
100th through 120th payment	R↓ g CFj	1,484.58
# of PMTS	21 g Nj	21.00
Input payments made after underlying mortgage satisfied:		
121st through 179th payment	2,292.98 g CFj	2,292.98
# of PMTS	59 g Nj	59.00
Balloon payment + final PMT	R↓ 27,000 + g CFj	29,292.98
Compute monthly yield	f IRR	0.99
Convert to annual yield	RCL g i	11.85

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* You will note that when performing an internal rate of return (IRR) problem it is not necessary to set the HP 12C's payment mode to any particular setting. That is, you may perform the computation with the payment mode set to either beginning (BEG) or ending (END) mode. There is no interface between the payment mode and the operation of the unequal cash-flow function on your HP 12C.

PROGRAM FOR COMPUTING PAYMENT NECESSARY TO PRODUCE A REQUIRED YIELD ON A WRAPAROUND PAYMENT MORTGAGE

This program is designed to compute the wraparound mortgage loan payment necessary to produce the yield required by a lender. The program will handle a two mortgage complex wraparound case.

KEYSTROKE	DISPLAY		KEYSTROKE	DISPLAY	
f P/R f PRGM			+	27	40
1	01	1	STO 7	28	44 7
CHS	02	16	RCL 3	29	45 3
STO PMT	03	44 14	STO n	30	44 11
RCL 3	04	45 3	FV	31	15
STO n	05	44 11	[1/x]	32	22
PV	06	13	RCL 6	33	45 6
RCL 2	07	45 2	x	34	20
x	08	20	RCL 7	35	45 7
RCL 0	09	45 0	+	36	40
+	10	40	STO 7	37	44 7
RCL 1	11	45 1	0	38	0
-	12	30	STO FV	39	44 15
STO 7	13	44 7	1	40	1
1	14	1	CHS	41	16
CHS	15	16	STO PMT	42	44 14
STO PV	16	44 13	RCL 4	43	45 4
RCL 4	17	45 4	STO n	44	44 11
STO n	18	44 11	PV	45	13
0	19	0	RCL 7	46	45 7
STO PMT	20	44 14	x≷y	47	34
FV	21	15	÷	48	10
[1/x]	22	22	STO PMT	49	44 14
RCL 5	23	45 5	0	50	0
x	24	20	STO 7	51	44 7
CHS	25	16	RCL PMT	52	45 14
RCL 7	26	45 7	f R/S		

Required Information and Memory Storage Locations Used

1. Wraparound mortgage amount store in memory register 0.
2. Underlying mortgage balance store in memory register 1.
3. Underlying mortgage payment store in memory register 2.
4. Lender's required monthly yield store in financial register [i].
5. Number of payments remaining on underlying loan store in memory register 3.
6. Wraparound mortgage term (months) store in memory register 4.
7. Balloon payment, if any, on wraparound mortgage store in memory register 5.
8. Balloon payment, if any, on underlying loan store in memory register 6.
9. Set END mode (g END).
10. Overall memory usage: **P - 57 r - 13.**

Note: The program is designed to compute the required wraparound mortgage payment on the assumption that all payments occur at the end of the month. No other payment solution is possible with the program as written.

The program will handle a payment stream in which the last payment is not equal to those which precede it. You recall that you will almost always find that a loan's payment stream will have its final payment slightly higher or slightly lower than the other payments. This, of course, is linked to the effects of rounding the monthly payment when first calculated.

To adjust for payment rounding, the program treats the difference between the last payment and the one immediately preceding it as a **balloon payment.** For example, if the loan payment stream consists of 359 monthly payments of $1,000, followed by a final (360th) payment of $1,010, the difference between the two payments ($1,010 - $1,000 = $10) would be treated as a **positive balloon payment** on the loan.

In the alternative, if a loan's last payment is less than the monthly payment which immediately precedes it, we treat the difference as a **negative signed balloon payment.** For example, if the loan's payment schedule consisted of 359 payments of $862.83, followed by a final payment of $859.86, we would treat the difference of **-$2.97** as a **negative signed balloon payment.** Again, the difference would be entered and treated as a "balloon payment", appropriately signed.

When working with a complex wraparound loan where there are balloon payments for both the wraparound loan and the underlying loan as well as last payment issues for both loans, we simply **net** each loan's last payment difference with the balloon payment, if any, due on the loan. This procedure will become clearer when working through the two examples that follow.

COMPUTING THE WRAPAROUND MORTGAGE PAYMENT NECESSARY TO PRODUCE A REQUIRED YIELD

Example: A $127,628.00 mortgage will be "wrapped around" a $100,000, 12%, 360 payment underlying loan. The underlying loan requires 359 payments of $1,028.62, plus a last payment (360th) of $1,002.75.* The wraparound mortgage lender requires a yield of 14% per annum on the out-of-pocket cash-flow. The term of the wraparound loan is equal to the remaining term on the underlying loan. If 60 monthly payments were made on the underlying loan at the time the wraparound loan is issued, compute the wrap's monthly payment and its last payment.

Since the 360th payment on the underlying loan is $25.87 **less** than the payments that precede it, the program requires this amount to be treated as a "balloon payment" of **-$25.87.** As far as the program is designed, what we have are 300 remaining payments of $1,028.62 on the underlying loan, **plus a separate balloon payment of -$25.87.****

Solution:

The first step requires that we compute the balance owing on the underlying mortgage after 60 payments have been made.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Enter monthly interest rate on underlying loan	12 g i	1.00
Enter amount of underlying loan	100,000 CHS PV	-100,000.00
Enter monthly payment	1,028.62 PMT	1,028.62
Enter number of payments made	60 n	60.00
Compute underlying loan balance	FV	97,662.61

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* To verify that the last payment on the underlying loan is $1,002.75, set [n] to "360", [i] to "1", [PV] to "-100,000", and [PMT] to "1,028.62". Now compute the future value [FV]. Your display will show: **-25.87.** Add the FV to the payment (RCL PMT +). Your display will show: **1,002.75.**

** If the final payment on the underlying loan is **greater** than the payments that precede it, you will treat the difference between the last payment and its prior payment as a **positive** balloon payment on the underlying loan. The difference will then be stored in the memory register allocated for holding the balloon payment on the underlying loan.

Computing the Wraparound Mortgage Payment

PROCEDURE	KEYSTROKE/DISPLAY	DISPLAY
Enter wrap-payment program		
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Input wraparound loan amount	127,628 STO 0	127,628.00
Input balance on underlying loan	97,662.61 STO 1	97,662.61
Input underlying loan payment	1,028.62 STO 2	1,028.62
Input remaining term on underlying loan	300 STO 3	300.00
Input total number of payments on wraparound loan	300 STO 4	300.00
Input "balloon payment" on underlying loan	25.87 CHS STO 6	-25.87
Input lender's required monthly yield	14 g i	1.17
Compute monthly wraparound mortgage payment	R/S	**1,389.32**
Display all digits in answer	f 9	1,389.322095

Discussion and Solving for the Last Payment

We could stop here and conclude that the required yield to the lender would be achieved by the borrower making 300 payments of $1,389.32 per month. However, this would not be taking into account the final payment on the loan, which in this example equals $1,394.97. Therefore, the payment schedule would consist of 299 monthly payments of $1,389.32, followed by the final payment of $1,394.97.

Solving for the final payment presents a straightforward case of computing the Future Value of an assumed "deposit" of $.002095 per month over a period of 300 months, with interest of 14% (the lender's required yield on the wraparound loan) per annum. The hypothetical payment derives from the difference between the mathematically correct payment needed ($1,389.322095) and the rounded payment ($1,389.32). The "deposits" occur at the end of the month, just like the payments on the loans.

To solve for the last payment, perform the following keystrokes:

f CLX 300 n 14 g i .002095 CHS PMT FV

Your display will show: **5.65**. This would be the "shortfall" to the lender if the borrower made 300 payments of $1,389.32. Now, add **$5.65** to $1,389.32. The result is the required final payment, **$1,394.97**.

COMPUTING THE WRAPAROUND MORTGAGE PAYMENT NECESSARY TO PRODUCE A REQUIRED YIELD: COMPREHENSIVE ROUTINE

Solution Objective: To calculate the monthly payment necessary to produce a required annual yield on a complex wraparound payment mortgage.

Summary of Information Required:

1. Annual yield required by the wraparound lender.
2. Annual interest rate charged on underlying loan.
3. Original amount of underlying loan.
4. Monthly payment on underlying loan.
5. Balance on underlying loan at time wraparound mortgage issued.
6. Balloon payment, if any, on underlying loan.
7. Amount borrowed on wraparound mortgage.
8. Balloon payment, if any, on wraparound mortgage.
9. Total number of payments on wraparound mortgage.

Assumptions--Limitations--Based Upon Design of Program

The program design requires that all payments--both underlying and wraparound--must be made at the end of the month and that interest on both loans also be applied on a monthly basis. Further, any balloon payment--whether on the wraparound or underlying loan--must be made with the final payment on its respective loan. No variable (unequal) cash-flows are allowed. That is, all payments must be the same within each loan type, except the program will handle a different final payment by treating the difference between the last payment--within each loan--and the payment which precedes it as a "balloon payment". The last payment-generated "balloon payment" must be properly signed when entered into the appropriate storage register, positive for a last payment excess, and negative for a last payment which is less than the payment which precedes it.

Example: Borrower applies for a 20 year, $200,000, wraparound mortgage which requires a balloon payment of $50,000 to be made with the final (240th) payment. The lender requires a yield of 14% per annum on its out-of-pocket cash-flow. The wraparound loan will be issued at the time the 60th payment has been made on the borrower's underlying loan.

The underlying loan is a 12%, $100,000 mortgage with payments of $1,028.62 per month. It requires a balloon payment due with the 180th payment. Thus, the loan will be fully amortized in fifteen years (180 payments).

Solution:

The first step requires that we compute the balance owing on the underlying mortgage after 60 payments have been made. This gives us the outstanding obligation at the time the wraparound loan is issued. We next compute the balloon payment due on the underlying loan along with its 180th regular monthly installment.

Computation of Loan Balance and Balloon Payment on Underlying loan

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear financial registers; set 2 decimal places and END mode	f FIN f 2 g END	0.00
Enter number of payments made on underlying loan at time wraparound loan is issued	60 n	60.00
Enter monthly interest rate	12 g i	1.00
Enter amount of underlying loan, negative-signed	100,000 CHS PV	-100,000.00
Enter monthly payment	1,028.62 PMT	1,028.62
Compute underlying loan balance	FV	97,662.61
Enter term of underlying loan	180 n	180.00
Compute balloon payment due at time 180th regular payment due	FV	85,702.01

Computation of Wraparound Mortgage Payment

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter WRAP program into your HP 12C		
Clear all registers	f CLX	0.00
Enter lender's required monthly yield on out-of-pocket cash-flow	14 g i	1.17
Input amount of wraparound loan	200,000 STO 0	200,000.00
Input balance on underlying loan	97,662.61 STO 1	97,662.61
Input underlying loan payment	1,028.62 STO 2	1,028.62
Input number of payments remaining on underlying loan *	120 STO 3	120.00
Input term (months) of wrap-loan	240 STO 4	240.00
Input balloon payment on wrap-loan	50,000 STO 5	50,000.00

---

* The number of payments remaining on the underlying loan equals the difference between the term of the loan (180 months) and the number of payments made at the time the wraparound loan is issued.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Input balloon payment on underlying loan	85,702.01 STO 6	85,702.01
Compute monthly payment on wraparound loan	R/S	**2,322.92**

Solving for the Last Payment

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter computed monthly payment into calculator's "Y" register	ENTER	2,322.92
Set display to "2" places, if not already set, round and store	f 2 f RND STO 7	2,322.92
Subtract rounded payment from computed payment in "Y" register	-	-0.002
Change sign of computed "excess" rounded payment; store in PMT	CHS STO PMT	0.002
Reset PV register to "0"	0 PV	0.00
Compute Future Value component of last payment *	FV	-2.20
Add negative-signed potential overpayment to rounded monthly payment	RCL 7 +	**2,320.72**

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* Here we are computing the Future Value of the imaginary deposits of the "excess" payments being made on the wraparound loan. Since we rounded the wraparound loan's payment **up**, we are, in a loose sense, making a larger monthly payment than would otherwise be required. The "excess" monthly payment builds up to a potential total overpayment of $2.20 at the time the final payment is made. Therefore, we reduce the rounded monthly payment ($2,322.92) by $2.20 to arrive at the final payment of $2,320.72.

Note also that we did not set the [n] register to the number of payments made on the wraparound loan, nor did we need to reset the [i] register to the lender's required monthly yield. This data remained in the appropriate storage registers since the program's operation did not disturb what we previously input into [i], nor did it disturb the data stored in the [n] register.

SOLUTION TEMPLATE FOR COMPUTATION OF WRAPAROUND LOAN PAYMENT

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
1. Set payment mode to END and display 2 decimal places	g END f 2	0.00
2. Clear all registers	f CLX	0.00
3. Input Wraparound Mortgage PRGM		
4. Input amount of wraparound loan	STO 0	
5. Input balance on underlying loan	STO 1	
6. Input underlying loan payment	STO 2	
7. Remaining term on underlying loan	STO 3	
8. Total number of payments on wraparound loan	STO 4	
9. Input balloon payment, if any, on wraparound loan	STO 5	
10. Input balloon payment, if any, on underlying loan	STO 6	
11. Type annual yield required by wraparound lender	g i	
12. **Compute monthly payment**	R/S	
13. Enter and round PMT; store in memory register 7	ENTER f 2 f RND STO 7	
14. Subtract; CHS; store in PMT	- CHS STO PMT	
15. Set PV to "0"; Compute last payment component	0 STO PV FV	
16. **Compute last payment**	RCL 7 +	

SKIPPED PAYMENT CASH-FLOW ANALYSIS

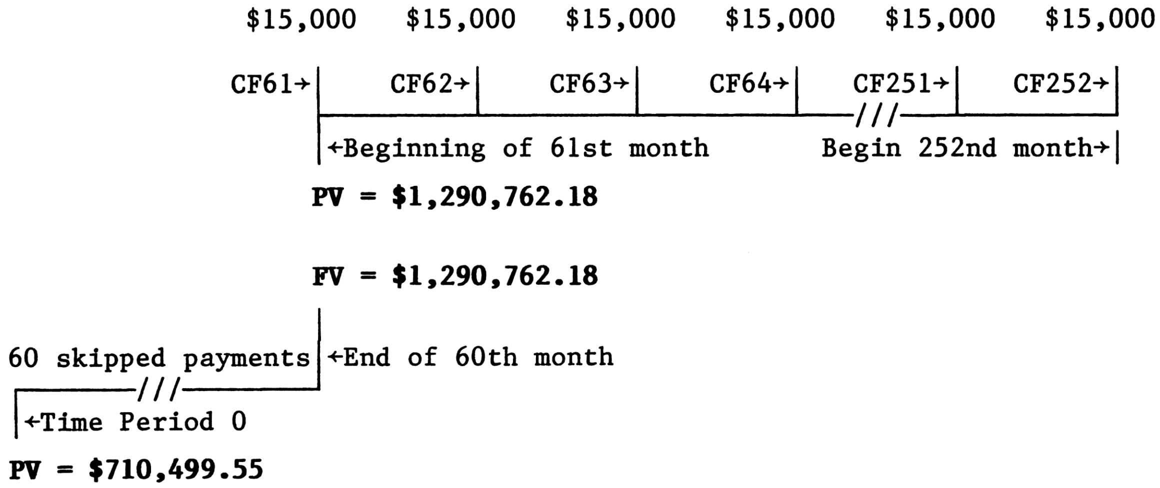
Problem: You are analyzing a 21 year lease to determine its present value. The first sixty "payments" are skipped. That is, during the first sixty months there are no payments made. (This is obviously a strong "anchor tenant".) The lease payment schedule then provides for 192 beginning of the month payments of $15,000 each. If we discount the payment stream at 12% per annum, what is the present value of the lease?

Solution Method #1: Using the Financial Registers

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Set beginning mode; clear registers, and set 2 decimal places	g BEG f CLX f 2	0.00
Enter monthly discount rate	12 g i	1.00
Set monthly payment to $15,000	15,000 PMT	15,000.00
Set number of **regular** payments	192 n	192.00
Compute discounted present value of stream of 192 **beginning** of the month payments	PV	-1,290,762.18

Interpretation

What we have done so far is to discount a series of 192 beginning of the month payments of $15,000 at the rate of one percent (1%) per month. The computed present value ($1,290,762.18) is, in effect, the **future value** of a present value which sits **back** an additional 60 months, discounted at the rate of one percent (1%) per month. (See timeline diagram below.)



Solution continued

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Change sign of computed PV and load into FV register	CHS FV	1,290,762.18
Clear PMT register	0 PMT	0.00
Set number of months	60 n	60.00
Compute discounted PV of cash-flows	PV	-710,499.55

Interpretation

We first discounted 192 beginning of the month payments--of $15,000--back to **a** present value. It makes little difference that **the** present value sits someplace other than at time period zero. In effect, we have what is in effect an "interim present value".

The "interim present value"--now the **future value**--must be discounted from the **beginning** of the 61st month (end of 60th month) back to time period zero. The process is no different than solving for the amount of a "one-time deposit"--into an untaxed account--needed to produce $1,290,762.18 sixty months into the future.

Mathematical Proof of Results

Mathematically, we know that:

$$PV = PMT \times \frac{1 - (1 + i)^{-n}}{i}$$

$$PV_{\mathbf{due}} = (1 + i) \times PMT \times \frac{1 - (1 + i)^{-n}}{i}$$

Therefore,

$$PV_{\mathbf{due}} = (1 + .01) \times \$15,000 \times \frac{1 - (1 + .01)^{-192}}{.01}$$

$$PV_{\mathbf{due}} = 1.01 \times \$15,000 \times 85.19882363$$

$$PV = \$1,290,762.18 \quad \text{(to 2 decimal places)}$$

Discounting $1,290,762.18 at one percent (1%) per month over a sixty month period, we have:

$$PV = FV \times (1 + i)^{-n}$$

$$PV = \$1,290,762.18 \times 1.01^{-60}$$

$$\mathbf{PV = \$710,499.55.} \quad \text{(The result is proven.)}$$

Solution Method #2: Using the Unequal Cash-Flow Function

The Unequal Cash-Flow Function of your calculator is more suited for quick solution of problems involving skipped--or missed--payments.

In the keystroke sequence that follows you will note that we input a "skipped payment" directly into the initial cash-flow (CFo) register of the HP 12C. This procedure is followed since the lease payments are made at the **beginning** of the month. Since one skipped payment is entered into the CFo register, there remains fifty-nine missed payments to be entered into the CFj register.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places. (Note that it does not matter if you are in BEG or END mode when doing NPV or IRR calculations)	f CLX f 2	0.00
Enter first skipped payment	0 g CFo	0.00
Enter skipped payment into first regular cash-flow group	g CFj	0.00
Enter balance of missed PMTS	59 g Nj	59.00
Enter regular payment (which occurs at the beginning of the 61st month)	15,000 g CFj	15,000.00
Enter cash-flows	99 g Nj	99.00
Reenter regular payment	R↓ g CFj	15,000.00
Enter remaining number of payments. (192 - 99 = 93)	93 g Nj	93.00
Enter monthly discount rate	12 g i	1.00
Compute net present value *	f NPV	710,499.55

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* Since we did not enter an initial out-of-pocket cash-flow into the CFo register, the computed net present value (NPV) is in fact the discounted present value of the cash-flows.

COMPUTATION OF EQUIVALENT LEVEL MONTHLY PAYMENT (Uniform Series Payment)

Problem: The financial issue presented here concerns the computation of the **uniform series payment** which would be equivalent to the skipped payment lease schedule just analyzed. What level monthly payment would be financially equivalent to a beginning of the month lease payment schedule consisting of sixty skipped payments followed by 192 payments of $15,000, with all payments discounted at 1% per month?

In effect, what we are looking for is the level beginning of the month payment necessary to produce a present value of $710,499.55, where the payments are discounted at 1% per month.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and BEG mode	f CLX f 2 g BEG	0.00
Set [n] register to total number of payments	252 n	252.00
Enter monthly discount rate	1 i	1.00
Enter present value, negative-signed	710,499.55 CHS PV	-710,499.55
Compute uniform monthly lease payment	PMT	7,658.62
Round payment to 2 decimal places and load into PMT register	f RND PMT	7,658.62
Compute residual ("balloon payment") due with 252nd payment	FV	-0.13
Compute last payment	RCL PMT +	7,658.49

One Application for Computing the Uniform Series Lease Payment

Assume you were computing the unpaid lease obligation of a client which is attempting to satisfy its obligation under an unexpired skipped payment lease. If the "uniform series" monthly obligation is $7,658.62, and if we assume the lessor will be without a tenant for 18 months, we could readily compute the **discounted present value** of the unexpired lease term. The only financial issue remaining would be the yield or discount rate to be used in the computation.

Let's discount the monthly payments at 1%. The answer is: $126,843.99.

KEYSTROKE	DISPLAY
f CLX g BEG 18 n	18.00
1 i	1.00
7,658.62 PMT	7,658.62
PV	-126,843.99

INTERIM BALLOON-MULTIPLE PAYMENT-LEASE PAYMENT SCHEDULE PREPARATION

Problem: You are structuring a payment schedule for a 60 month lease that requires ten (10) payments to be made at the beginning of the 30th month. (Note that this will be the 30th payment made on the lease.) The lessor's interest is valued at $100,000. Compute the beginning of the month payments necessary to produce an annual yield of 15%.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places	f CLX f 2	0.00
Input "1" for the advanced payment	1 g CFo	1.00
Input "1" for the regular payment	g CFj	1.00
Input number of payments to be made before interim balloon payment due at the 30th month	28 g Nj	28.00
Input number of regular payments to be made at beginning of 30th month	10 g CFj	10.00
Input "1" for regular payment	1 g CFj	1.00
Input balance of payments to be made on the lease	30 g Nj	30.00
Input monthly discount rate	15 g i	1.25
Compute net present value	f NPV	48.84
Compute monthly payment required to produce 15% annual yield	100,000 RCL PV ÷	2,047.61

Interpretation: What we did was to first compute the **present value** (NPV) of a hypothetical series of one dollar ($1.00) cash-flows discounted at 15% per annum. We did this by assuming that the lease payment schedule consisted of a one dollar ($1.00) payment made at the beginning of the first month (CFo), followed by equivalent beginning of the month payments for the next 28 months. At the beginning of the 30th month--which is the time when the 30th "payment" is due--the lease requires 10 regular monthly payments to be made. We treat this as ten "regular" payments of one dollar ($1.00) and therefore input the number "10" for the 30th cash-flow. Finally, we input the final one dollar ($1.00) regular payment into CFj (type: "1", [g], [CFj]), and cause it to occur 30 times by typing 30 followed by the [g] and [Nj] keys.

We next input the required monthly yield, and then calculated the net present value (NPV) of the hypothetical one dollar ($1.00) cash-flows. This produced a NPV of $48.84 (or to 8 place accuracy: 48.83752222).

Intuitively, if a "stream" of one dollar cash-flows, discounted at 15% per annum, produced a **present value (NPV)** of $48.83752222, then, to produce an

equivalent present value (NPV) of $100,000, we must compute a regular payment which has the same ratio of payment-to-PV as that produced by the stream of $1.00 payments. That is: $1.00 is-to-$48.83752222 as the unknown payment is-to-$100,000. Therefore: **Lease PMT = $100,000/48.83752222.**

Mathematically, the ratio addressed above looks like this:

$$\frac{\$1.00}{\$48.83752222} = \frac{\text{Lease Payment}}{\$100{,}000}$$

so,

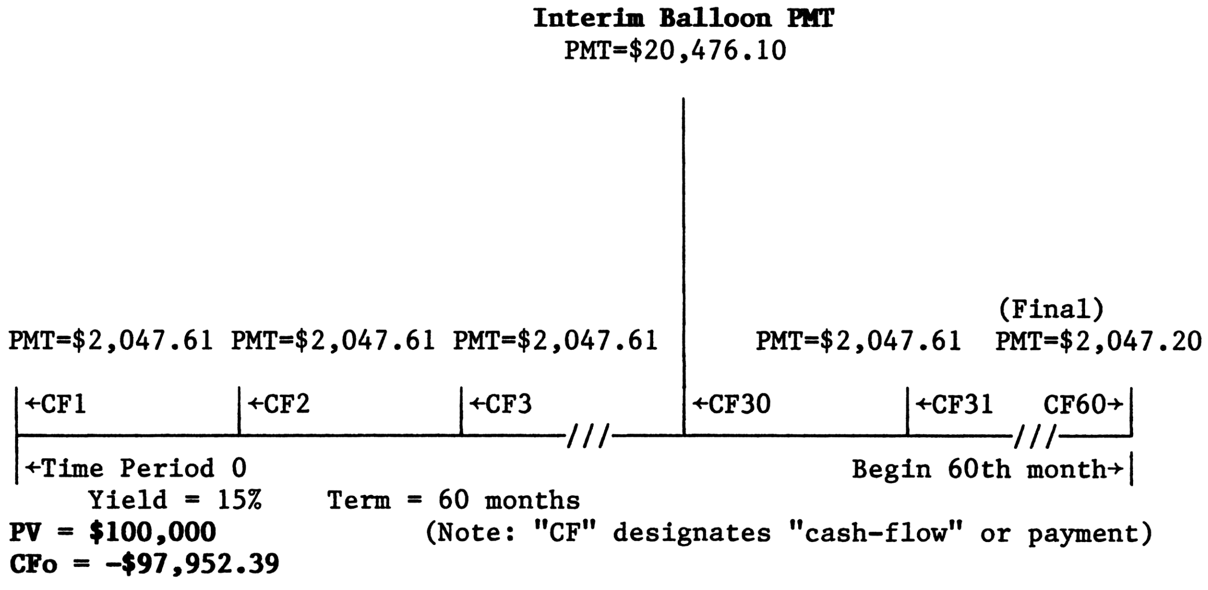
$$\text{Lease Payment} \times 48.83752222 = \$100{,}000$$

$$\text{Lease Payment} = \frac{\$100{,}000}{48.83752222}$$

$$\text{Lease Payment} = \$2{,}047.61 \quad \text{(to 2 decimal places)}$$

Timeline Diagram

A timeline diagram depicting the cash-flows for the above problem is set-forth below. Note that the last payment is given as $2,047.20. The computation of the last payment, however, is not shown. You should verify the result on your own.



STEP-UP CONSTANT RATIO LEASE PAYMENT SCHEDULE PREPARATION

Problem: You are structuring a 60 month step-up lease which requires beginning of the month payments to increase by 5% per year. The lessor values the leasehold interest at $100,000. Compute a payment schedule for the following discount rates: 12%; 14%; and 15%.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places	f CLX f 2	0.00
Input "1" for the advanced payment	1 g CFo	1.00
Input "1" for regular payment	g CFj	1.00
Input number of regular payments	11 g Nj	11.00
Increase 1st year payment by 5%	R↓ 5 % + g CFj	1.05
Input number of regular payments	12 g Nj	12.00
Increase 2nd year payment by 5%	R↓ 5 % + g CFj	1.10
Input number of regular payments	12 g Nj	12.00
Increase 3rd year payment by 5%	R↓ 5 % + g CFj	1.16
Input number of regular payments	12 g Nj	12.00
Increase 4th year payment by 5%	R↓ 5 % + g CFj	1.22
Input number of regular payments	12 g Nj	12.00
Input (1st) monthly discount rate	12 g i	1.00
Compute net present value	f NPV	49.60
Divide lease value by the NPV attained at 12% annual yield. This is the monthly lease payment.	100,000 RCL PV ÷	**2,016.19**
Input (2nd) monthly discount rate	14 g i	1.17
Compute net present value	f NPV	47.40
Compute monthly payment required to produce 14% annual yield	100,000 RCL PV ÷	**2,109.52**
Input (3rd) monthly discount rate	15 g i	1.25
Compute net present value	f NPV	46.36
Compute monthly payment required to produce 15% annual yield	100,000 RCL PV ÷	**2,157.08**

Note: In this problem we are not concerning ourselves with computing the last (final) payment in the lease payment schedule. The procedure for doing this has been adequately covered in the Graduated Payment Mortgage (GPM) problems as well as the Wraparound Payment Mortgage (WRAPS) problems.

COMPUTING LEASE PAYMENT SCHEDULES WITH REGULAR OR ADVANCE PAYMENTS, WITH SECURITY DEPOSIT CAPABILITY

The program that follows is designed to compute the periodic payment (monthly; quarterly; semi-annual; annual; etc.) on a fixed payment lease. The program handles leases with any number of advance payments, or you can use it to compute payments occurring at the end of the payment period. In addition, it is structured to enable you to take into account a refundable security deposit.

The program also enables you to take into consideration a "reversion value" ("residual value") at the expiration of the lease. For example, if you use the program for computing an equipment lease, you can take into consideration an assumed value or purchase price for the equipment at the end of the lease term. This technique can be applied in real estate lease analysis, though an in-depth coverage of that area is outside the scope of this book. You **must** be guarded and do additional research if you need a program for this purpose. This program will not directly handle these kinds of financial issues, nor is the documentation broad enough to address the issue.

The program also computes the **adjusted residual value** of a lease payment stream. This capability was designed into the program to give the user total accuracy in determining either the adjusted residual value or a high degree of accuracy in determining what financial "residual" should be added to or subtracted from the last payment in your payment stream in order to come up with an extremely accurate schedule. This capability is exactly the same as the technique for calculating the future value (FV) remaining in a traditional mortgage's payment stream in order to determine an exact amount for the loan's last payment.

The program is designed in accord with traditional methods of handling refundable security deposits. This method effectively gives the lessee (tenant, etc.) the same yield on his/her/its security deposit as the yield received by the lessor (owner) on the lease. The program can be adjusted to use a different discount rate, and though the author has written such a program, it was considered outside the scope of this book and therefore was not included.

You should note that the program is designed to accept the lease amount (also considered "value" of the equipment or value of the lease for which a payment amount is sought) as a positive number and as well returns a positive signed payment amount. Thus, it does not follow the traditional "cash-flow sign convention". Consequently, there is no need to input a negative signed lease amount in order to produce a positive signed payment; just punch-in a positive signed lease amount and residual value, if any, and receive in return a positive signed payment amount.

The program will not handle payment (income) streams which increase or decrease periodically. That is, you cannot calculate an increasing or decreasing payment stream lease or loan with this program. Your results are limited to straightforward fixed payment streams, and therefore must always be used as fixed payment amounts. **Do not** go beyond the scope of the program.

PROGRAM FOR COMPUTING LEASE PAYMENT SCHEDULES WITH REGULAR OR ADVANCE PAYMENTS, WITH SECURITY DEPOSIT CAPABILITY

KEYSTROKE	DISPLAY		KEYSTROKE	DISPLAY	
f (P/R) f (PRGM)			RCL 2	31	45 2
RCL PV	01	45 13	÷	32	10
RCL 1	02	45 1	RCL 0	33	45 0
-	03	30	+	34	40
RCL FV	04	45 15	RCL 3	35	45 3
RCL 1	05	45 1	$x \gtrless y$	36	34
-	06	30	÷	37	10
1	07	1	f 2	38	42 2
RCL i	08	45 12	f RND	39	42 14
%	09	25	STO PMT	40	44 14
x	10	20	R/S	41	31
STO 2	11	44 2	RCL PMT	42	45 14
1	12	1	RCL 2	43	45 2
+	13	40	1	44	1
RCL n	14	45 11	+	45	40
CHS	15	16	RCL n	46	45 11
y^x	16	21	RCL 0	47	45 0
x	17	20	-	48	30
-	18	30	CHS	49	16
STO 3	19	44 3	y^x	50	21
RCL 2	20	45 2	CHS	51	16
1	21	1	1	52	1
+	22	40	+	53	40
RCL n	23	45 11	RCL 2	54	45 2
RCL 0	24	45 0	÷	55	10
-	25	30	RCL 0	56	45 0
CHS	26	16	+	57	40
y^x	27	21	x	58	20
CHS	28	16	RCL PV	59	45 13
1	29	1	-	60	30
+	30	40	RCL 1	61	45 1

KEYSTROKE	DISPLAY		KEYSTROKE	DISPLAY	
+	62	40	÷	69	10
RCL 2	63	45 2	RCL 1	70	45 1
1	64	1	-	71	30
+	65	40	CHS	72	16
RCL n	66	45 11	STO 4	73	44 4
CHS	67	16	R/S	74	31
y^x	68	21	g GTO 00	75	43 33 00
			f R/S		

Required Information and Memory Storage Locations Used

1. Total number of payments (lease term) store in [n].
2. Periodic yield (also called "implicit lease rate") store in [i]. For example if the lease requires monthly payments, the "periodic" yield will be the monthly interest rate. Similarly, if the payments are made quarterly, the "periodic yield" will be based upon the annual rate divided by four.
3. Asset value store positive signed in [PV].
4. Residual value (if any) store positive signed in [FV].
5. Number of advanced payments, store in Mem. Reg. 0. If lease requires only one advance payment, and therefore is a traditional "due annuity", store a "1" in memory register 0. If the lease requires more than one advance payment, store that number in memory register 0. If payments occur at the end of the period, make sure memory register 0 is empty.
6. Security deposit (if any) store positive signed in Mem. Reg. 1.
7. Recomputed residual value [FV] displays and can be recalled from memory register 4 after completion of the calculation.
8. Set END (g END) mode for all computations. (You adjust for a beginning of the period lease by inputting the number of advance payments into memory register 0.)
9. Overall memory usage: **P - 78, r - 10.**

Summary of registers used:

Memory Registers: 0, 1, 2, 3, and 4.

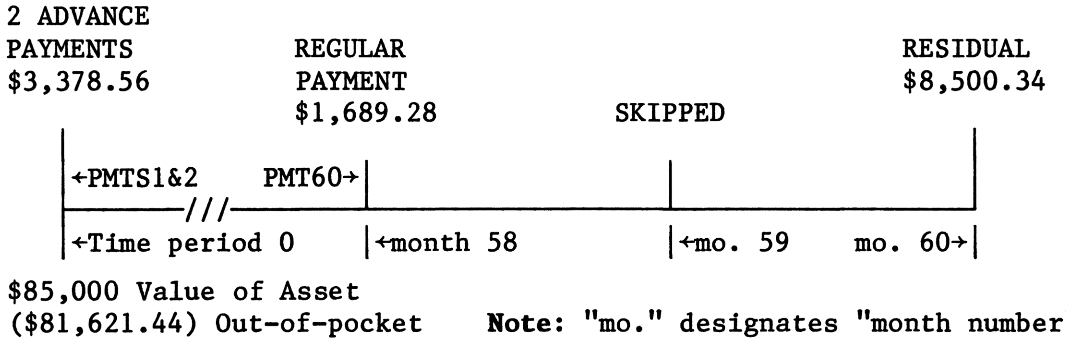
Financial Registers: [n], [i], [PV], and [FV].

Lease payment may be recalled from the [PMT] register.

Problem: A piece of equipment valued at $85,000 will be leased for 60 months. The lessor requires two payments in advance, sets the residual value of the equipment at $8,500, and requires a yield of 10.5% annually. Compute the fixed monthly payment and the adjusted residual value of the equipment.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter program into your HP 12C		
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Input lease term	60 n	60.00
Input monthly yield to lessor	10.5 g i	0.88
Input value of equipment	85,000 PV	85,000.00
Input equipment residual value	8,500 FV	8,500.00
Input number of advance payments	2 STO 0	2.00
Compute monthly payment	R/S	1,689.28
Compute residual's adjusted value	R/S	8,500.34

Conclusion: The lessee makes two advance payments totalling $3,378.56, and afterwards makes regular beginning of the month payments of $1,689.28 for the next fifty eight months. The "residual" of $8,500.34 would be due one month after the last regular payment. A cash-flow diagram follows:



Note: "mo." designates "month number".

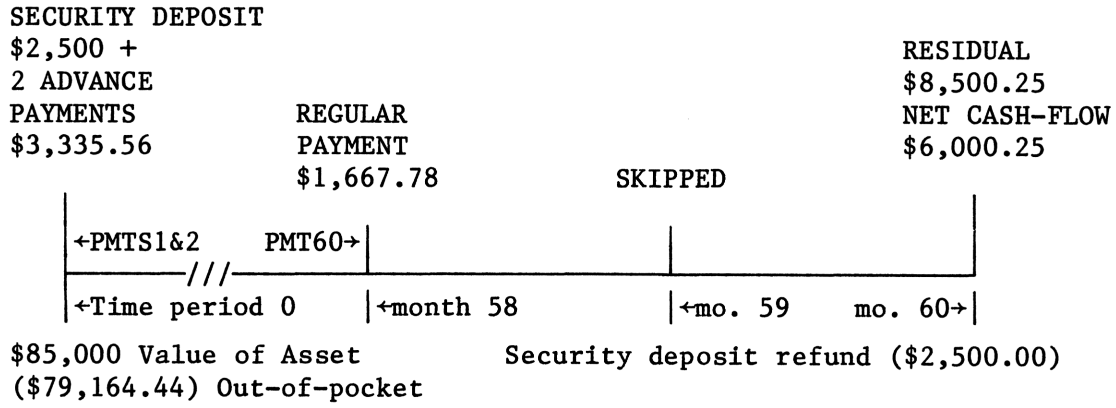
Verification Using Unequal Cash-Flow Function:

KEYSTROKES:	f CLX	Clear all registers
	81,621.44 CHS g CFo	Enters out-of-pocket cash-flow
	1,689.28 g CFj	Enters regular monthly payment
	58 g Nj	Enters 58 regular payments
	0 g CFj	Enters one missed payment
	8,500.34 g CFj	Enters residual value
	f IRR	Computes monthly yield
	RCL g i	Converts to annual yield
	DISPLAYS: 10.50	(The annual yield is verified)

Problem: Using the data from the previous problem, integrate a refundable security deposit of $2,500 into the transaction. Thus, the relevant facts are: (1) Value of equipment, $85,000. (2) Residual value of $8,500. (3) Two Advance payments required. (4) Required annual yield equals 10.5%. (5) Term of 60 months. (6) And, again, a refundable security deposit of $2,500.00. Compute the monthly lease payment, the adjusted residual value, and then prove the accuracy of the results with the unequal cash-flow (IRR) function.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Input lease term	60 n	60.00
Input monthly yield to lessor	10.5 g i	0.88
Input value of equipment	85,000 PV	85,000.00
Input equipment residual value	8,500 FV	8,500.00
Input number of advance payments	2 STO 0	2.00
Input refundable security deposit	2,500 STO 1	2,500.00
Compute monthly payment	R/S	1,667.78
Compute residual's adjusted value	R/S	8,500.25

Conclusion: In addition to the refundable $2,500 security deposit, the lessee makes two advance payments totalling $3,335.56, and thereafter makes monthly payments of $1,667.78 for the next fifty eight months. This gives us a total of 60 regular payments. For the final cash-flow--due after one skipped month--, the $2,500 security deposit would be net against the "residual" payment ($8,500.25) on the lease. The net residual value--payment on the lease--would therefore be $6,000.25. A cash-flow diagram follows:



Note: "mo." designates "month number".

Verification Using Unequal Cash-Flow Function:

```
KEYSTROKES:  f CLX                        Clear all registers
             1,667.78 ENTER 2 x           Calculate advance payment
             2,500 +                      Adds refundable security deposit
             85,000 -                     Subtract equipment value, get out
                                          of pocket cash-flow -$79,164.44
             g CFo                        Enters -$79,164.44 cash outflow
             1,667.78 g CFj               Enters regular monthly payment
             58 g Nj                      Enters 58 regular payments
             0 g CFj                      Enters one missed payment
             8,500.25 ENTER 2,500 -       Residual less security deposit
             g CFj                        Enters $6,000.25 final cash-flow
             f IRR                        Computes monthly yield
             RCL g i                      Converts to annual yield
             DISPLAYS:  10.50             (The annual yield is verified)
             f 9
             DISPLAYS:  10.49999961
```

Problem: Change the data in the above problem as follows: (1) Set the annual yield to 14%, and (2) increase the number of advance payments from two to three. Recompute the monthly lease payment, the adjusted residual value, and then prove the accuracy of the results with the unequal cash-flow (IRR) function.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places (make sure you are in END)	f CLX f 2 g END	0.00
Input lease term	60 n	60.00
Input monthly yield to lessor	14 g i	1.17
Input value of equipment	85,000 PV	85,000.00
Input equipment residual value	8,500 FV	8,500.00
Input number of advance payments	3 STO 0	3.00
Input refundable security deposit	2,500 STO 1	2,500.00
Compute monthly payment	R/S	1,788.16
Compute residual's adjusted value	R/S	8,500.39

Verification Using Unequal Cash-Flow Function:

```
f CLX f 2                                    Clear registers and set 2 places
1,788.16 ENTER 3 x 2,500 + 85,000 - g CFo    Enters -77,135.52 out-of-pocket
1,788.16 g CFj  57 g Nj                      Enter 57 regular payments
0 g CFj  2 g Nj                              Enter 2 skipped payments
8,500.39 ENTER 2,500 - g CFj                 Enter $6,000.39 final cash-flow
f IRR   (r u n n i n g)   RCL g i            DISPLAYS 14.00
```

COMPUTING LEASE PAYMENT SCHEDULES WITH REGULAR OR ONE ADVANCE PAYMENT, RESIDUAL VALUE CAPABILITY, AND PAYMENT STREAM INCREASING OR DECREASING BY A CONSTANT AMOUNT

The program which follows on the next page is designed to compute the monthly payment on a (1) lease or (2) mortgage loan where it is desired to have the payments increase (or decrease) annually by **a constant amount.** Payments can occur at the **beginning** or **end** of the month. You are, however, limited to computing leases (or loans) with **a maximum of** one advance payment.

The **minimum term** which can be used in the program is **36 months,** being 36 payments. From that point, you can increase the term by adding any (whole number) multiple of 12 months. For example, your term can range from 36 months to 48 to 60 to 120...to 360...to 1,200, and so forth, but always in multiples of 12. You should note, however, that the greater the term, the longer it takes for the calculator to solve for the payment.

If you try to use a term of 24 or 12 months, the program will not discontinue operation; it will continue to run until you stop it. To stop the program, press the Run/Stop (R/S) key.

The program also accepts inputs for a residual value (or balloon payment). Due to memory limits, refundable security deposits cannot be integrated into the program. This limitation aside, the program will give you enormous computing capability and will enhance your payment schedules beyond anything likely published to date for use in this excellent calculator.

The payment schedules you produce with this program will take into account **arithmetic** (constant amount) increases. This is different from what you experience when computing a payment schedule for the GEM or GPM. In those mortgages, the payment increases **geometrically,** that is, by an increasing amount each and every year during the loan's increase period. However, in the case of a loan payment system which increases by a **constant amount** ("arithmetically"), your monthly payments will increase (or decrease) by the same amount each and every year. For example, by $100 per month per year, by $1.00 per month, and so forth.

The **decay** feature of the program enables you to cause the payment stream to **decrease**--that is, go down--annually. Indeed, the scope of applications for this technology are very broad. For example, the author designed lease payment schedules for motor vehicles (mainly trucks) in which the payments are highest during the first year and decrease annually. As well, this type of payment schedule is readily adaptable to mortgage loan financing, at least from the quantitative side of the business.

The program accepts inputs for the lease or loan amount as a positive signed number and as well returns a positive signed monthly payment.

Finally, **do not** go beyond the scope of the program and its documentation; make sure your lease or loan term is integer divisible by "12"; and, be sure you proof each problem before accepting a payment result. (You may consider the HP 17BII for proofing a payment schedule which exceeds 18 years.)

KEYSTROKE	DISPLAY	
f R/S f PRGM		
3	01	3
STO 3	02	44 3
1	03	1
2	04	2
STO n	05	44 11
1	06	1
CHS	07	16
STO PMT	08	44 14
PV	09	13
1	10	1
RCL i	11	45 12
%	12	25
x	13	20
1	14	1
+	15	40
STO 7	16	44 7
RCL 6	17	45 6
CHS	18	16
y^x	19	21
RCL 4	20	45 4
x	21	20
CHS	22	16
RCL 8	23	45 8
+	24	40
RCL PV	25	45 13
÷	26	10
STO PV	27	44 13
RCL 7	28	45 7
RCL n	29	45 11
CHS	30	16
y^x	31	21
STO 0	32	44 0
STO 1	33	44 1

KEYSTROKE	DISPLAY	
1	34	1
STO + 1	35	44 40 1
RCL 7	36	45 7
2	37	2
4	38	4
STO 2	39	44 2
CHS	40	16
y^x	41	21
STO + 1	42	44 40 1
2	43	2
x	44	20
STO + 0	45	44 40 0
1	46	1
2	47	2
STO + 2	48	44 40 2
RCL 6	49	45 6
RCL 2	50	45 2
-	51	30
g [x=0]	52	43 35
g GTO 73	53	43 33 73
RCL 7	54	45 7
RCL 2	55	45 2
y^x	56	21
[1/x]	57	22
STO + 1	58	44 40 1
RCL 3	59	45 3
1	60	1
STO + 3	61	44 40 3
x	62	20
x	63	20
STO + 0	64	44 40 0
RCL n	65	45 11
STO + 2	66	44 40 2
RCL 2	67	45 2

KEYSTROKE	DISPLAY
RCL 6	68 45 6
-	69 30
g [x=0]	70 43 35
g GTO 73	71 43 33 73
g GTO 49	72 43 33 49
RCL PV	73 45 13
RCL 0	74 45 0
RCL 5	75 45 5
x	76 20
-	77 30
RCL 1	78 45 1
÷	79 10
ENTER	80 36
STO PMT	81 44 14
f R/S	

Required Information and Memory Storage Locations Used

1. Monthly yield or monthly interest rate, store in financial register [i]. For example, if the annual yield on a lease is 14%, you enter the monthly yield by pressing: 14 [g] [i].
2. Value of the lease or amount of the loan, store positive signed in memory register 8.
3. Lease or loan term in <u>months</u>, store in memory register 6. Remember: The <u>minimum</u> acceptable term is 36 months. Thereafter, the term <u>must</u> increase by a whole number multiple of "12", such as "48", "60", etc.
4. Annual (constant amount) growth in the monthly payment, store in memory register 5.
5. Residual value of a lease, or balloon payment on a mortgage loan, store positive signed in memory register 4.
6. Set END (g END) mode if your payments occur at the end of the month. If they occur at the beginning of the month, set BEG (g BEG) mode. The program is capable of handling one and only one advance payment.
7. Overall memory usage: **P - 85, r - 09.**

Summary of registers used:

Memory Registers: 0, 1, 2, 3, 4, 5, 6, 7, and 8.

Financial Registers: Automatically set and utilized by program, [n], and [PV].

You <u>must</u> make sure the [FV] register is clear before you start a problem.

You <u>must</u> input the monthly interest rate into the [i] financial register.

The lease or loan payment will display and can be recalled from the [PMT] register after you complete a computation.

Caution: <u>Always</u> <u>verify</u> the accuracy of a computation by running an internal rate of return (IRR) or net present value (NPV) test on the completed data. If the loan term exceeds 18 years, use the HP 17BII to check your data.

Problem: A piece of equipment valued at $85,000, with a residual value of $8,500, will be leased for 60 months. The lessor requires a yield of 10.5% annually. The lease payments are to increase by $100.00 per month after the first year. First, compute the **end of the month** lease payments. Next, compute the **beginning of the month** lease payments, and then prove the IRR.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter program into your HP 12C		
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Input monthly yield to lessor	10.5 g i	0.88
Input value of equipment	85,000 STO 8	85,000.00
Input lease term in months	60 STO 6	60.00
Input annual monthly payment growth	100 STO 5	100.00
Input equipment residual value	8,500 STO 4	8,500.00
Compute 1st year monthly payment	R/S	1,539.47
Calculate 2nd year monthly payment	100 +	1,639.47
Calculate 3rd year monthly payment	100 +	1,739.47
Calculate 4th year monthly payment	100 +	1,839.47
Calculate 5th year monthly payment	100 +	1,939.47

Problem: Now let's do the beginning of month payment and prove the IRR.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Set for beginning mode	g BEG	1,939.47
Compute 1st year monthly payment	R/S	1,524.56
Calculate 2nd year monthly payment	100 +	1,624.56
Calculate 3rd year monthly payment	100 +	1,724.56
Calculate 4th year monthly payment	100 +	1,824.56
Calculate 5th year monthly payment	100 +	1,924.56

Comment: In proving the accuracy of the payment schedule in this problem we will not compute the rounded last (final) payment in the lease term. In practice you should do this, if for no other reason than to assure complete accuracy in your schedule of payments. As well, it is suggested that a payment schedule which takes into account rounding of the last payment not only builds your confidence as a practitioner, but equally important, your clients will generally feel more confident when they are exposed to an extremely accurate and carefully designed payment schedule.

Verification Using Unequal Cash-Flow Function

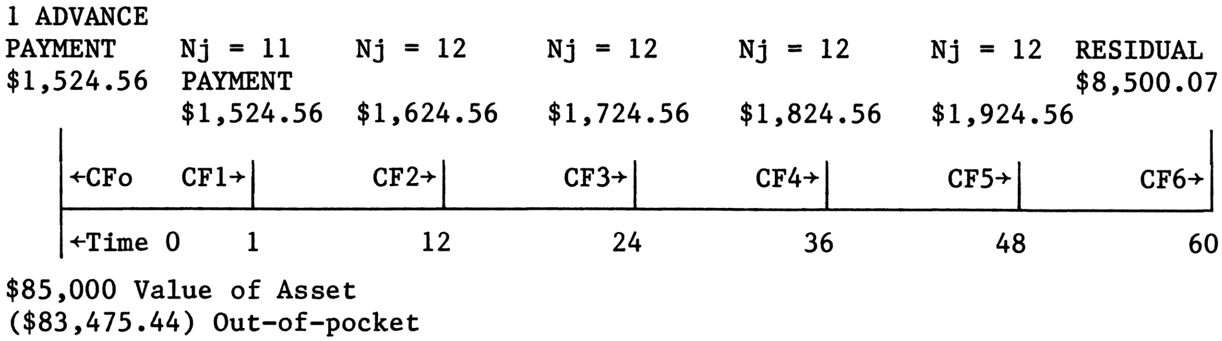
KEYSTROKES:	f CLX	Clear all registers
	1,524.56 ENTER 85,000 -	Out-of-pocket (-$83,475.44) flow
	g CFo	Enters out-of-pocket cash-flow
	1,524.56 g CFj	First year payment of $1,524.56
	11 g Nj	Enters 11 remaining payments
	R↓ 100 + g CFj	Enters $1,624.56 2nd year payment
	12 g Nj	Enters 12 payments
	R↓ 100 + g CFj	Enters $1,724.56 3rd year payment
	12 g Nj	Enters 12 payments
	R↓ 100 + g CFj	Enters $1,824.56 4th year payment
	12 g Nj	Enters 12 payments
	R↓ 100 + g CFj	Enters $1,924.56 5th year payment
	12 g Nj	Enters 12 payments
	8,500 g CFj	Enters residual value
	f IRR	Computes monthly yield
	RCL g i	Converts to annual yield
	DISPLAYS: 10.50	The annual yield is verified to 2 decimal places

Our monthly payment schedule is summarized below:

PAYMENTS	AMOUNT	
1 - 12	$1,524.56	
13 - 24	1,624.56	
25 - 36	1,724.56	
37 - 48	1,824.56	
49 - 60	1,924.56	
Residual	$8,500.00	(Adjusted residual is $8,500.07)

Note: The **residual value** ($8,500.07) was adjusted with techniques covered in other sections of this book. The adjustment is minor, but it is still a good practice to calculate it. It takes only a few minutes.

Timeline Diagram



Problem: Let's work with the data from the previous problem, <u>except</u> we will cause the monthly payment to decrease by $100.00 per month. To summarize the data: Lease value of $85,000; residual value, $8,500; term, 60 months; annual yield, 10.5%; payments <u>decrease</u> by $100.00 per month starting in the second year; and there will be one advance payment made at the beginning of the lease. Compute monthly payment schedule and prove the IRR.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and BEG mode	f CLX f 2 g BEG	0.00
Input monthly yield to lessor	10.5 g i	0.88
Input value of equipment	85,000 STO 8	85,000.00
Input lease term in months	60 STO 6	60.00
Input annual monthly payment decay	100 CHS STO 5	-100.00
Input equipment residual value	8,500 STO 4	8,500.00
Compute 1st year monthly payment	R/S	1,882.94
Calculate 2nd year monthly payment	100 -	1,782.94
Calculate 3rd year monthly payment	100 -	1,682.94
Calculate 4th year monthly payment	100 -	1,582.94
Calculate 5th year monthly payment	100 -	1,482.94

Comment: Again, in proving the accuracy of the payment schedule in <u>this</u> problem we will <u>not</u> compute the rounded last (final) payment in the lease term. However, In practice you should compute the final payment.

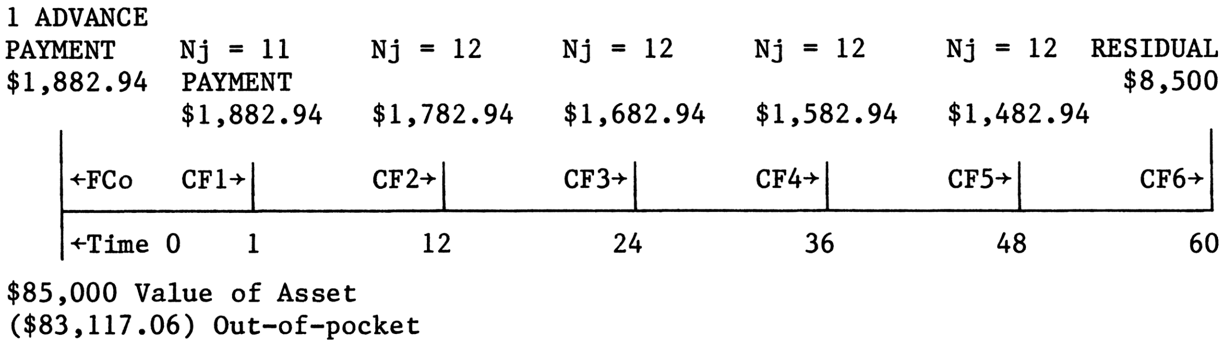
Verification Using Unequal Cash-Flow Function

KEYSTROKES:	f CLX	Clear all registers
	1,882.94 ENTER 85,000 -	Out-of-pocket (-$83,117.06) flow
	g CFo	Enters out-of-pocket cash-flow
	1,882.94 g CFj	First year payment of $1,882.94
	11 g Nj	Eleven remaining payments
	R↓ 100 - g CFj	Enters $1,782.94 2nd year payment
	12 g Nj	Enters 12 payments
	R↓ 100 - g CFj	Enters $1,682.94 3rd year payment
	12 g Nj	Enters 12 payments
	R↓ 100 - g CFj	Enters $1,582.94 4th year payment
	12 g Nj	Enters 12 payments
	R↓ 100 - g CFj	Enters $1,482.94 5th year payment
	12 g Nj	Enters 12 payments
	8,500 g CFj	Enters residual value
	f IRR	Computes monthly yield
	RCL g i	Converts to annual yield
	DISPLAYS: 10.50	The yield is verified to 2 places

Our monthly payment schedule is summarized below:

PAYMENTS	AMOUNT	
1 - 12	$1,882.94	
13 - 24	1,782.94	
25 - 36	1,682.94	
37 - 48	1,582.94	
49 - 60	1,482.94	
Residual	$8,500.00	(Adjusted residual is $8,500.02)

Timeline Diagram



Comment: Lease or loan payment schedules with arithmetically changing payments may not be common in your current applications. This is, however, not to say that these kinds of schedules are not adaptable to the specific needs of your clients. Indeed, the need has always been there for techniques which enable the finance practitioner to design payment schedules which change arithmetically. The problem has been, where does the real estate practitioner turn for this technology? It is the author's belief that the program in this section (along with its counterpart in the author's HP 17BII book) will fill the need for this type of technology at the level of working through the problem with a financial calculator.

The reader who is most likely a practicing real estate professional will undoubtedly find applications for many of the techniques covered in this book. As well, you will likely be able to tailor additional applications for the specialized routine covered in this section; just take your time and think the issues out carefully.

In the author's experience, he has been called upon many times times to structure increasing or decreasing arithmetic gradient leases. The examples in this book are squarely out of the author's practice.

Note: Always make sure that the payment schedules you generate with this program are perfectly sound and prepared in total conformity with the instructions in this book. And, never go beyond the scope of the program or its documentation.

Problem: You are structuring a payment schedule for a mortgage loan with the following characteristics: The amount is $100,000; term, 180 end of the month payments; annual interest rate, 8%; annual growth in monthly payment, $10.00. Compute the monthly payment schedule and prove its accuracy with the internal rate of return function (IRR).

Note: We will show the first five payments on the mortgage in the calculation immediately below. To run the IRR of this problem you will have to purge the program from your calculator, or do it on the HP 17BII.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode	f CLX f 2 g END	0.00
Input monthly yield to lender	8 g i	0.67
Input loan amount	100,000 STO 8	100,000.00
Input loan term in months	180 STO 6	180.00
Input annual monthly payment growth	10 STO 5	10.00
Compute 1st year monthly payment	R/S	900.19
Calculate 2nd year monthly payment	10 +	910.19
Calculate 3rd year monthly payment	10 +	920.19
Calculate 4th year monthly payment	10 +	930.19
Calculate 5th year monthly payment	10 +	940.19

Comment: In proving the accuracy of the payment schedule in this problem we could compute and use the rounded last (final) payment in the loan term. If you prefer, use the adjusted last payment in the proof given below. We adjust the **final payment** with the techniques covered in other parts of this book. The adjusted 180th payment is $1,040.59, accounting for a forty cent ($0.40) addition to the 180th payment.

Verification Using Unequal Cash-Flow Function

KEYSTROKES:	f R/S f PRGM ON ON	Clears program from calculator and returns to normal display
	f CLX	Clear all registers
	100,000 CHS g CFo	Inputs loan amount ($100,000.00)
	900.19 g CFj	First year payment of $900.19
	12 g Nj	Enters 12 payments
	R↓ 10 + g CFj	Enters $910.19 2nd year payment
	12 g Nj	Enters 12 payments
	R↓ 10 + g CFj	Enters $920.19 3rd year payment
	12 g Nj	Enters 12 payments
	R↓ 10 + g CFj	Enters $930.19 4th year payment
	12 g Nj	Enters 12 payments

KEYSTROKES:	R↓ 10 + g CFj	Enters $940.19 5th year payment
	12 g Nj	Enters 12 payments
	R↓ 10 + g CFj	Enters $950.19 6th year payment
	12 g Nj	Enters 12 payments
	R↓ 10 + g CFj	Enters $960.19 7th year payment
	12 g Nj	Enters 12 payments
	R↓ 10 + g CFj	Enters $970.19 8th year payment
	12 g Nj	Enters 12 payments
	R↓ 10 + g CFj	Enters $980.19 9th year payment
	12 g Nj	Enters 12 payments
	R↓ 10 + g CFj	Enters $990.19 10th year payment
	12 g Nj	Enters 12 payments
	R↓ 10 + g CFj	Enters $1,000.19 11th year payment
	12 g Nj	Enters 12 payments
	R↓ 10 + g CFj	Enters $1,010.19 12th year payment
	12 g Nj	Enters 12 payments
	R↓ 10 + g CFj	Enters $1,020.19 13th year payment
	12 g Nj	Enters 12 payments
	R↓ 10 + g CFj	Enters $1,030.19 14th year payment
	12 g Nj	Enters 12 payments
	R↓ 10 + g CFj	Enters $1,040.19 15th year payment
	12 g Nj	Enters 12 payments
	f IRR	Computes monthly interest rate (Requires 2 minutes, 15 seconds)
	RCL g i	Converts to annual yield
	DISPLAYS: 8.00	

Comment: The yield is proven at 8% per annum (to two decimal places).

Problem: Let's quickly determine the amount needed to adjust the loan's final (180th) payment in order to have a perfect payment schedule. To do this we simply calculate the net present value (NPV) of the cash-flows already in your calculator. From there, we set the number of time periods register [n] to 180 (to reflect the fact that this is a 180 month loan), make sure the PMT register is clear, and compute the future value (FV) of the net present value. Let's do it.

KEYSTROKES:	8 g i	Enter the monthly yield
	f NPV	Computes net present value of PMTS
	DISPLAYS: -0.12	This tells us that the payments under generate the loan amount by $.12. This can be made up later.
	15 g n	Entering loan term
	0 PMT	Make sure PMT register clear
	FV	Computes the residual needed with the 180th payment
	DISPLAYS: 0.40	This tells us the last payment must be $.40 greater if we want 100% accuracy in the PMT schedule.
	1,040.19 +	Adds $.40 residual to last PMT
	DISPLAYS: 1,040.59	This is the 180th payment

STRUCTURING A MONTHLY PAYMENT LEASE HAVING MULTIPLE SKIPPED PAYMENTS, REFUNDABLE SECURITY DEPOSIT, AND A RESIDUAL VALUE

It is sometimes necessary for a lessor to structure a lease with multiple skipped payments. This situation exists where the lessor is willing to forego lease payments during the months in which the lessee's operating cash-flows are expected to be low. For example, because of sometimes heavy snowfall, a construction company might have reduced cash-flows during the months of January, February, and March. This situation presents a potentially ideal opportunity to accommodate a credit worthy lessee with a skipped payment lease.

Suggested Solution Methodology

You can use the grouped-uneven cash-flow functions of your financial calculator to solve these kinds of problems.

The following procedure is suggested for structuring a skipped payment lease where payments (1) are made at the end of the month or (2) where one advance payment will be made. **Note** that the procedure covered below will not work in cases where the lease payment schedule contains more than one advance payment, nor will it work in cases where the sequence of regular and skipped payments is not identical for each year of the lease. (For these special cases the HP 17BII and HP 19BII are recommended due to their enhanced memory capacity.)

1. Calculate the required **lease rate factor.** To do this, perform the following steps:

   a) For the first year of the lease, use CFo to enter $1.00 for the advance payment, if any, and use CFj to enter $1.00 for the regular payment, followed by Nj to enter the number of consecutive payments. Next, use CFj to enter $0.00 for the skipped payment, followed by Nj to enter the number of skipped (missed) payments. Repeat the process until the sequence of payments represented by the first year's cash-flow pattern is fully input.

   b) Enter the required monthly yield into the interest rate register [i]. Next, calculate the net present value (NPV) of the cash-flows. (This is the present value of the **first year's** skipped payment cash-flow pattern.)

   c) Enter "12" into the [n] register, set the payment mode to begin (g BEG) if an advance payment is to be made (otherwise set END mode), and calculate the equivalent **uniform installment payment** [PMT] necessary to amortize the present value calculated in step "b" above.

   d) Enter the number of months in the lease term into the [n] register, and calculate the present value [PV] of the hypothetical uniform cash-flows over the term of the lease.

e) Invert [1/x] the present value calculated in "d" and store in an available memory register. This is the **lease rate factor.**

2. Calculate the present value of the **net residual.** To do this, perform the following steps:

   a) Subtract the amount of the refundable security deposit, if any, from the residual value of the equipment. Enter this sum into the future value [FV] register.

   b) Make sure the payment [PMT] register is set to "0". (Do "0" PMT.)

   c) Calculate the discounted present value [PV] of the net residual.

3. Calculate the lessor's net cash-flow at the inception of the lease. To do this, perform the following:

   a) Change the sign of the discounted present value of the net residual value--calculated in step "2c"--and add the refundable security deposit.

   b) In situations where the user intends to integrate into the lease payment calculation the lessor's initial **direct costs** associated with initiating the lease, you would subtract those costs from the result obtained in "a" above.

   c) Subtract the results obtained in step "a" above-or from step "b" if direct costs are taken into consideration-from the value (or cost) of the equipment.

4. Calculate the monthly lease payment. To do this, perform:

   a) Recall the stored **lease rate factor** and multiply it times the lessor's net cash-flow at the inception of the lease.

   b) Round the computed payment to two decimal places. Subject to any last payment calculation, this is the monthly lease payment. (Note that the last payment in the schedule of payments will generally be slightly higher or slightly lower than the payment which precedes it. This is due to the effects of rounding the monthly payment to two decimal places.)

Problem: A piece of equipment valued at $85,000 is to be leased on the 1st day of October. The first payment is due in advance, and payments will continue over the next 59 months. The lessor requires a refundable security deposit of $2,000, and sets the equipment's residual value at $8,500. In addition, due to weather conditions, the equipment cannot be used during the months of January, February and March. Therefore, the lessor agrees that payments will be skipped during these months. If the lessor's required annual yield is 10.5%, compute the regular monthly payment necessary to amortize the lease.

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and beginning (BEG) mode since an advance payment is made	f CLX f 2 g BEG	0.00
Calculate the lease rate factor:		
Input "1" for the advance payment	1 g CFo	1.00
Input "1" for regular payment	g CFj	1.00
Input number of regular payments	2 g Nj	2.00
Input "0" for skipped payment	0 g CFj	0.00
Input number of skipped payments (January, February and March)	3 g Nj	3.00
Input "1" for regular payment	1 g CFj	1.00
Input number of remaining payments	6 g Nj	6.00
Input required monthly yield	10.5 g i	0.88
Calculate net present value	f NPV	8.55
Input "12" into [n] register	12 n	12.00
Calculate uniform installment PMT	PMT	-0.75
Number of payments in lease term	60 n	60.00
Calculate present value	PV	35.05
Invert present value and store (This is the **lease rate factor**, shown to 2 decimal places.)	[1/x] STO 0	0.03
Calculate present value of the net residual:		
Subtract security deposit from residual value of equipment; store in FV register	8,500 ENTER 2,000 - FV	6,500.00
Set the payment register to "0" (Can also do: "0" PMT)	0 STO PMT	0.00
Calculate present value	PV	-3,853.90

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Calculate lessor's net cash-outflow:		
Change sign of present value and add refundable security deposit	CHS 2,000 +	5,853.90
Subtract results from equipment value	CHS 85,000 +	79,146.10
Calculate monthly lease payment:		
Recall lease rate factor	RCL 0	0.03
Multiply times net cash-outflow	x	2,258.11
Round and store in PMT register	f RND PMT	2,258.11

Optional: Calculate Last (60th) Payment. (13 storage registers needed)

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Check number of available storage registers. If less than 13 you must clear all programs.	g MEM f R/S f PRGM f R/S	
Clear all registers	f CLX	0.00
Calculate and input net cash-outflow at inception of lease	2,000 ENTER 2,258.11 + 85,000 - g CFo	4,258.11 -80,741.89
Input regular payment	2,258.11 g CFj	2,258.11
Input number of payments	2 g Nj	2.00
Input skipped payment	0 g CFj	0.00
Input number of skipped payments	3 g Nj	3.00
Recall and input regular payment	RCL 1 g CFj	2,258.11
Input number of payments	9 g Nj	9.00
Input skipped payment	0 g CFj	0.00
Input number of skipped payments	3 g Nj	3.00
Recall and input regular payment	RCL 1 g CFj	2,258.11
Input number of payments	9 g Nj	9.00
Input skipped payment	0 g CFj	0.00
Input number of skipped payments	3 g Nj	3.00
Recall and input regular payment	RCL 1 g CFj	2,258.11
Input number of payments	9 g Nj	9.00
Input skipped payment	0 g CFj	0.00
Input number of skipped payments	3 g Nj	3.00
Recall and input regular payment	RCL 1 g CFj	2,258.11
Input number of payments	9 g Nj	9.00

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Input skipped payment	0 g CFj	0.00
Input number of skipped payments	3 g Nj	3.00
Recall and input regular payment	RCL 1 g CFj	2,258.11
Input **remaining number of payments**	6 g Nj	6.00
Input net residual to lessor: equipment residual value less refundable security deposit	8,500 ENTER 2,000 - g CFj	6,500.00
Enter monthly yield to lessor	10.5 g i	0.88
Calculate net present value	f NPV	-0.06
Set [n] register to number of months in lease, less advance PMT	59 n	59.00
Make sure PMT set to "0"	0 PMT	0.00
Compute future value of residual attributed to rounding of PMT	FV	0.11
Calculate last payment by adding future value to regular payment	RCL 1 +	2,258.22

Payment Schedule Summary

Security deposit	$2,000.00
Advance payment	2,258.11
2 regular payments	2,258.11
3 skipped months	0.00
9 regular payments	2,258.11
3 skipped payments	0.00
9 regular payments	2,258.11
3 skipped payments	0.00
9 regular payments	2,258.11
3 skipped payments	0.00
9 regular payments	2,258.11
3 skipped payments	0.00
5 regular payments	2,258.11
last regular payment	2,258.22
Residual less security deposit, made at beginning of 60th month.	$6,500.00

IRR Proof

f CLX
2,000 ENTER 2,258.11 +
85,000 - g CFo
2,258.11 g CFj 2 g Nj
0 g CFj 3 g Nj
RCL 1 g CFj 9 g Nj
0 g CFj 3 g Nj
RCL 3 g CFj 9 g Nj
0 g CFj 3 g Nj
RCL 5 g CFj 9 g Nj
0 g CFj 3 g Nj
RCL 7 g CFj 9 g Nj
0 g CFj 3 g Nj
RCL 9 g CFj 5 g Nj
2,258.22 g CFj
8,500 ENTER 2,000 - g CFj
f IRR
(Takes 1 minute, 40 seconds)
RCL g i
DISPLAYS: **10.50**
f 9
DISPLAYS: **10.50000098**

INTEREST RATE CONVERSIONS

Determining the annual effective interest rate

There are times when we seek to determine the **effective annual interest rate** produced by compounding a known **annual nominal interest rate.** The most common application can be found in determining the **effective yield** on a savings account in which the annual nominal interest rate is paid and compounded a given number of times per year.

For example, let's assume a savings account pays a five percent (5%) annual nominal interest rate per year, with the interest paid and compounded monthly. Therefore, each month's interest in turn earns interest at the monthly nominal rate (5%/12). Thus, the account **effectively** pays more than five percent interest per year. Let's calculate the effective yield below:

KEYSTROKES:	f CLX f 2	Clear all registers; set 2 places
	12 n	Number of interest compounding periods per year
	5 g i	Enters monthly interest rate
	100 CHS PV	Inputs hypothetical deposit as negative-signed PV
	FV	Computes Future Value of hypothetical deposit of $100.00
	DISPLAYS: 105.12	
	RCL PV + DISPLAYS:	Subtracts $100 from $105.12...
	5.12	Effective annual interest rate

Discussion: Since the annual nominal interest rate (5%) is paid and compounded more than once per year--twelve times in this example--the overall **effective yield** on the account must be greater than the nominal interest rate. Thus, if you start with a nominal rate of 5% per year and pay and compound interest monthly, your effective yield is (approximately) 5.12%.

Theoretically, someone who makes a deposit into an account--and leaves it on deposit for a full year--should be indifferent as to whether the account pays 5.12% annual nominal interest with annual compounding, or whether the account pays 5% annual nominal interest with monthly compounding. Both options produce the same **effective yield** at the end of a full year, 5.12%.

Determining the annual nominal interest rate

There are times when we must convert a given effective interest rate or total overall yield into a different rate which, when compounded, pays or produces the same total yield as the given interest rate. Financially, the compounded rate we seek is **equivalent to** the given interest rate or total overall yield we started with. Secondly, these types of interest rate conversion problems arise when we seek a **nominal interest rate** which, when compounded, produces a required overall yield. The overall yield is technically (or more commonly) known as an effective yield or **effective interest rate.**

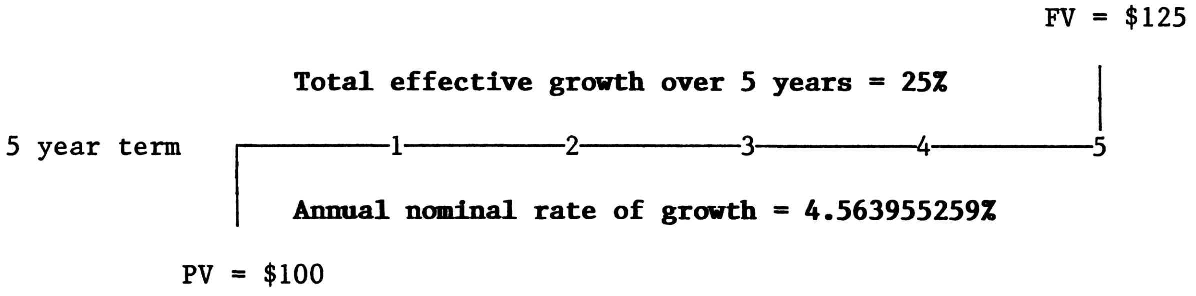
For example, let's assume that you seek to achieve a 25 percent growth in value of a property over a 5 year holding period. The total (or overall) growth of 25 percent clearly does not occur at any given moment in time; that is, it does not take place on any particular day or during any particular part of the year. Indeed, in real estate problem solving it is assumed that a property's growth in value occurs on a **periodic basis,** traditionally assumed to occur on an annual basis.

To better understand a 25 percent total or effective growth in property value over a 5 year holding period we might seek to know the **equivalent** annual compound growth of the property. Said differently, we would seek to determine the **nominal rate of growth** which, when compounded annually, produces an effective or total appreciation in value of exactly 25 percent at the end of five years. Knowing the equivalent annual growth of a property, or indeed any investment, gives us an opportunity to better compare competing investments.

Problem

Let's determine the **annual nominal rate of growth** required to produce a total or **effective growth** in property value of 25 percent over a 5 year holding period. To do this in a more user-friendly manner on the HP 12C, it is best that we take the position that the initial value of the investment--its Present Value [PV]--is $100.00. Therefore, its final--or Future Value [FV]--must be $125 if it undergoes a total growth of 25 percent over the holding period. Since the growth takes place over a five year period, the term or holding period [n] is 5.

A timeline diagram depicting the problem looks like this:



Keystroke solution: To solve this by keystroke, we set the [FV] to 125; [PV] is set to 100 CHS; and [n] is set to 5. We then solve for [i].

KEYSTROKES:	f CLX f 9	Clear all registers; set 9 places
	5 n	Inputs holding period
	100 CHS PV	Inputs hypothetical value as PV, negative-signed
	125 FV	Inputs future value of investment
	i	Computed equivalent annual growth

KEYSTROKES: DISPLAYS:
4.563955259

Interpretation: The above result tells us that if we were paid interest--or if an investment property grew in value--at an annual compound rate of "4.563955259%", at the end of five years our original investment or original property value would increase from $100 to $125. Indeed, the investment **effectively** grew by a total of 25%, which growth is **equivalent** to annual compounding at a **nominal interest rate** of 4.563955259%.

Looking at this problem a little differently, it tells us that if we **added** 4.563955259% compound growth per year to any investment, at the end of five years we would have an investment which grew in total value by 25%. Compounding is nothing more than a series of additions. Let's try it below: Add the computed annual growth of 4.563955259% to the investment five times in succession, each time adding growth to the prior year's enhanced value attributed to growth having been applied in the prior year(s).

KEYSTROKES:	RCL PV CHS	Brings up $100, positive-signed
	RCL i % +	Value at end of first year ($104.56)
	RCL i % +	Value at end of second year ($109.34)
	RCL i % +	Value at end of third year ($114.33)
	RCL i % +	Value at end of fourth year ($119.54)
	RCL i % +	Value at end of fifth year ($125.00)
	DISPLAYS: 125.0000000	

Conclusion: An overall growth in value of 25 percent over a five year holding period is equivalent to a compound annual growth rate of 4.563955259%. To convert the annual total or effective growth to an annual nominal growth rate, set the [FV] register to 100 plus the total or effective growth; set the [PV] register to 100, negative signed; set the term to the number of years in the holding or projection period; and solve for the annual nominal interest rate by pressing the [i] key.

Problem

Let's assume an individual is considering an investment which pays interest at the nominal rate of 10% per year, with monthly compounding. Further assume that a competing investment also pays interest, but that it compounds the interest on a quarterly basis rather than on a monthly basis. Forgetting about "risk", what is the minimum **nominal interest rate** which must be paid on the investment with quarterly compounding for the investor to be indifferent when comparing one investment to the other?

Here we will be converting an annual nominal interest rate to its **effective interest rate**, and then determine what the **nominal interest rate** must be on a separate investment which compounds or pays interest on a different basis. Specifically, we have one investment which pays or compounds interest on a **monthly** basis, while the other investment pays or compounds interest on a **quarterly** basis. How do we, in effect, equalize the overall effective yield on these investments by adjusting one investment's annual nominal interest rate to reflect the fact that it pays or compounds interest on a less frequent basis than that of the other investment?

The solution method requires that we first calculate the **annual effective interest rate** for the investment with monthly compounding of interest. Knowing the annual effective yield, we can work backwards to determine the **equivalent annual nominal interest rate** of the second investment whose compounding periods are different from those of the first.

KEYSTROKES:	f CLX f 9	Clear registers; set 9 places
	12 n	12 compounding periods per year
	10 g i	Convert annual nominal interest rate to monthly nominal rate
	100 CHS PV	Inputs 100, negative-signed for PV
	FV	Computes future value
	DISPLAYS: 110.4713067	(Effective yield is 10.4713067%)
	4 n	Sets quarterly compounding
	i	Computes equivalent nominal quarterly interest rate
	DISPLAYS: 2.520891193	

Conclusion: An investment which pays a quarterly compounded interest rate of 2.520891193% produces the same **effective yield** as an investment paying a nominal interest rate of 10% per year with monthly compounding. Saying it somewhat different, if you invested $100 into an investment paying 10% per year, with monthly compounding of interest, the investment's total value at the end of the first year would be $110.4713067. To produce the same total value from a competing investment which pays and compounds interest quarterly, the investment must pay an **annual nominal interest rate** of 10.08356477% (4 x 2.520891193%).

Note that the annual nominal interest rate of the investment with quarterly compounding must be greater than that of the investment with monthly compounding of interest. This is so because when interest is paid more frequently it produces a higher effective yield at the end of the year. As well, the less frequent interest is paid, the lower the effective yield at the end of the year. Since an investment paying interest quarterly surely is paying less often than one with monthly compounding, its annual nominal interest rate must be higher to produce the same end results.

(The topic's coverage is by no means meant to be exhaustive.)

A justification for the technique

Short of being a finance or math "prof" or calculator book writer, how do we really apply this topic in the real world of real estate problem solving? One key to its use falls squarely with problems where we know the total growth in value of a property over a given holding period and seek to find an equivalent annual nominal rate which, when compounded, produces the exact increase in value attributed to the property. Indeed, we did this a few pages ago. Now let's tie it into a specific application from this book.

In appraisal analysis you are likely to find courses or techniques which present the overall growth in value of a property on a gross or total basis. For example, the property might be projected to grow by 25% or 30% over the holding period. The growth rate may not, however, be given to you on an equivalent annual nominal basis. This can be a deficit since there appears to be a trend toward treating growth in value of a property as occurring on an annual or even monthly basis instead of treating the growth as a lump sum number produced over the holding period.

Indeed, given an overall projected growth in value of a property of, say, 40% over the expected holding period, it can be instructive to think of the growth as occurring on an annual or monthly basis, thus keeping it consistent with the methods commonly used for computing the debt service on a mortgage loan.

Take, for example, the program covered in the section dealing with "deriving an overall capitalization rate (RO) by mortgage-equity build-up: one LTV model", covered on page 270. In that program we treat the projected growth in value of the property as a gross amount. Thus, you enter an overall growth rate for the change in value of the property over the holding period. On the other hand, to give the reader a different view of how we handle projected growth in value of a property over its holding period, the "equity residual analysis using a discounted cash-flow model routine" (covered on page 282) and the "debt service coverage ratio (DSCR) constraint routine" (covered on page 291) treat growth as occurring on a compounded annual basis.

Thinking of growth in value of a property as occurring on an annual basis as against being given on a lump sum basis helps the practitioner in at least two ways. First, we keep the concept of yield or growth in value consistent with the way we think of traditional investment yield concepts, such as the equity yield (YE), which is an annual rate. Secondly, and more importantly from an analytical perspective, it gives us the flexibility to change the projected holding period--and other data--of the investment and retain a growth rate which still fits into whatever holding period or data is chosen.

Specifically, if you assumed a property holding period of 5 years, with a projected total growth of 25%, what do you do if the client requests an analysis based upon, say, a six year holding period? Do you use a growth rate of 30% (25%/5 years x 6 years), or is it more instructive to the client for you to project the growth on an annual basis? Using techniques (programs, etc.) which enable you to **readily** change investment criteria (such as loan term, interest rates, and holding period) without having to juggle the total projected growth keeps your problem solving techniques consistent with what is expected of the real estate practitioner by the more sophisticated investor.

APPRAISAL--AN INTRODUCTION

To **appraise** real estate means to **estimate** its value. By the very nature of this concept, an appraisal involves giving an **opinion** of value. As such, appraisals are consistently relied upon by lenders, individuals and investors, businesses, governmental agencies, and others.

There are three traditional approaches to estimating the value of real property: The **cost approach**; the **sales comparison approach**; and the **income capitalization**, or "income approach", for short. Of the three methods, only the income approach is touched upon in this book.

Why an income approach to value? This technique is grounded on the proposition that an income producing property's value is reflected in its ability to generate income. This is particularly so when you figure that (for all practical purposes) the primary objective in the ownership of income producing real property is to maximize one's wealth. Thus, the greater the contribution made by a property to your wealth, the greater its value to you and hence the greater its value to others.

No matter how sophisticated the income capitalization approach to estimating value, it really boils down to just one thing:

VALUE = PRESENT VALUE OF A PROPERTY'S ANTICIPATED CASH-FLOWS

THE GENERAL INCOME CAPITALIZATION MODELS

There are two general categories of income capitalization valuation models. The models are broken down into **Direct Capitalization** (or "ratio") models and **Discounted Cash-Flow** (DCF) models. DCF models are also commonly known as "Ellwood" (after L.W. Ellwood who designed the technique) or mortgage-equity models. To work with these models we need estimates of the first year net operating income and a suitable estimate for a capitalization rate for the Direct Capitalization model and a suitable estimate for the Yield or Equity Yield (YE) rate (plus additional inputs, such as the mortgage interest rate, holding period, etc.) for the Discounted Cash-Flow models.

Direct Capitalization models: With the Direct Capitalization model we derive a property's value by taking the ratio of the estimated (or actual) first year net operating income (stabilized or otherwise) and a suitable capitalization (or "cap") rate. That is, with this model we capitalize the first year **net operating income** by a suitable capitalization rate. Note that "capitalization" in this context simply means to divide by. Thus, we divide the first year NOI by the decimal equivalent of the capitalization rate.

Basically, a "cap" rate can be defined as any rate which is used to convert or change an estimate of income into value. More particularly, an "overall cap rate" is a rate for a property which reflects the relationship between its sales price or value and its year of sale net operating income. For example, if a property sold for $1,000,000 in a year in which it produced net operating income of $120,000, its **overall cap rate** would be 12% ($120,000/$1,000,000).

Symbolically, this method for estimating value can be expressed as follows:

$$\textbf{VALUE} = \frac{\textbf{NOI}}{\textbf{CAP RATE}}$$

where:
VALUE = estimated value of the property
NOI = first year net operating income
CAP RATE = income capitalization rate (as a decimal)

For example, if an income producing property is projected to generate $50,000 in net operating income (NOI) for any given year, and if we "capitalize" the income for that year **in perpetuity** at 15%, the property is thus valued at $333,333.33, rounded to $300,000. We arrive at this result by dividing the decimal equivalent of the "cap" rate into the projected net operating income. Operationally, it looks like this:

VALUE = NOI/CAP RATE
= $50,000/15%
= $50,000/.15
VALUE = $333,333.33 (rounded to $300,000)

To work with this model we need an estimate of the NOI and we also need a suitable cap rate. The net operating income estimate is commonly projected based upon historical or empirical data about the property or the kinds of properties which can be considered "comparable" to the subject property. Deriving the cap rate, however, can be a little more involved, particularly if a suitable rate cannot be extracted from comparable sales data.

To determine a suitable cap rate for use in the Direct Capitalization model, one would generally: (1) derive the rate from comparable sales transactions data, or (2) construct the rate as a weighted average cost of capital through what is called the band-of-investment analysis (or built-up) method.

The built-up method is generally used if sufficient competent sales data are not available from truly comparable properties to allow for derivation of the cap rate, "R". (Please note that "R" is sometimes designated as "RO" to describe it as an "overall rate" or "overall capitalization rate".)

To derive a suitable overall cap rate from comparable sales transactions the "comparable" properties must indeed be comparable to the subject property. Generally, the following attributes must exist before we can consider deriving a capitalization rate for a subject property from comparables:

1. The financing terms must be similar. More particularly, this means the debt-to-equity ratios, annual interest rate, loan terms, and terms of sale of the "comps" must be similar to the financing and financing terms available for the subject property.
2. The operating expense ratios of the "comps" must be similar to that of the subject property. That is, the operating expenses of the "comps" as a percentage of the effective gross income must be similar to that of the subject property. How close is "similar"? Nobody knows for sure, but a variance of, say, five to ten

percentage points would seem to eliminate a property from being considered as "comparable" to a property which one is appraising or has under review.

3. The physical condition of the "comps" should be similar to that of the subject property. This means that the expected useful lives of the "comps" (taking into account neighborhood stability as well as physical condition and estimated remaining useful lives of the improvements) should be truly comparable to that of the subject property.

Example: You are analyzing a property which is expected to produce first year NOI in the amount of $110,000. Relevant data on four comparable sales are setforth below.

Sale #	**NOI**	**PRICE**	**CAP RATE** (NOI/PRICE x 100)
1	$100,000	$ 970,000	10.309%
2	90,000	830,000	10.843%
3	115,000	1,045,000	11.005%
4	135,000	1,380,000	9.783%

Value the subject property using direct capitalization with an overall rate selected from the comparable sales.

Solution: First, we are assuming that the four sale properties are indeed the best comparables to the subject property. Next, the data suggests the following range of capitalization rates: 9.783% to 11.005%. Indeed, this is a significant variance in cap rates and one which would produce a substantial range of values for the subject property, as shown below.

Highest Value = $110,000/9.783% = **$1,124,399** (rounded to $1,124,000)

Lowest Value = $110,000/11.005% = **$999,546** (rounded to $999,500)

At this stage, the best that can be said is that the estimate of final value must lie with and within the experience of the investor and his advisors on the transaction (appraiser, commercial broker, attorney, etc.). Indeed, wide variances in value are not uncommon in practice. The seller might argue for a valuation closer to (if not in fact) the highest estimate; the investor might argue for the lowest estimate. For real estate taxation purposes, the owner might be arguing closer to the lowest value! Clearly, negotiation and compromise are sometimes our best and final defense in these situations.

The conclusion offered on this issue is that the financial techniques of appraisal are but one facet of the total valuation process: negotiation, common sense and, at times, even litigation enters this process, though, fortunately, this is rare. Notwithstanding the level of HP 12C appraisal programs covered in this book, never lose sight of the compromise aspects of this business, or at least never lose sight of the possibility of compromising on valuation issues. Choose your valuation techniques and, keeping the best interests of your investor or client in mind, make or attempt to make a competent, but common sense estimate of value.

One potential limitation with the direct capitalization method

One potential limitation associated with the direct capitalization method is that it assumes that the income stream will continue **indefinitely.** That is, the technique assumes that the property can maintain a level net operating income cash-flow for an **infinite** (unlimited) length of time. Of course, this is impossible, and indeed it represents the major flaw in the traditional assumptions underlying this handy technique. Yet, you should not rule out this method as a technique for estimating the value of an income producing property since its use is widespread throughout the appraisal industry.

For example, if we assume a net operating income stream of $50,000 for the year of analysis, and if we estimate the cap (or discount) rate at 10%, direct capitalization values the property at $500,000 ($50,000/.10). Indeed, this is capitalization in perpetuity. Taking, however, a slightly different look at direct capitalization, it might be instructive to think of this concept as one which argues for the complete recapture of the price or value of the property at the end of a **finite holding period.**

To get a better feel for the fact that this concept is indeed based upon the idea that the income stream will continue for an unlimited length of time, try these inputs into your calculator:

KEYSTROKES:	f CLX g END	Clear registers & set END mode
	f 2	Set 2 decimal places
	1 n	Assume a one year holding period
	10 i	The cap rate is 10%
	50,000 PMT	NOI is $50,000/year
	500,000 **FV**	There is full recapture of the value at end of holding period
	PV	Solve for the Present Value
	DISPLAYS: **-500,000.00**	(Same value as capitalizing the income stream in perpetuity)
	10 n	Assume a 10 year holding period
	PV	Again, solve for Present Value
	DISPLAYS: **-500,000.00**	Again, same value

What happened? We assumed an income stream of $50,000 per year, and further assumed the sales price was $500,000 at the end of the two target holding periods (one year and ten years). We then discounted the two cash-flow components ($50,000 NOI stream and the $500,000 reversion) at a cap rate of 10%. Each time the calculator told us the value of the property was $500,000 (-500,000.00). Indeed, capitalizing an income stream "in perpetuity" is procedurally equivalent to assuming that the value of the property (PV) at time period zero will indeed be the value of the property (FV) at the expiration of the holding period. It is submitted that this might be an easier way to view the concept of direct capitalization. Just think of it as a valuation procedure which is applied to a limited property holding period with full recapture of the value of the property at the end of the holding period.

However, let's not lose sight of the mathematical reality here, which is that we are still capitalizing the income stream for an unlimited/infinite period of time. To show this, let's attempt to solve for the holding period [n]; it cannot be done, and your calculator bears this out nicely!

KEYSTROKES:	n		We are attempting to verify the holding period equals "10 years".
	DISPLAYS:	Error 5	Means, no solution exists for "n"!
	100 n		Making a "point" about "n"!
	PV		Solve for the Present Value.
	DISPLAYS:	**-500,000.00**	Again, same Present Value.
	n		We again try to solve for "n".
	DISPLAYS:	Error 5	Again, no solution exists for "n"!
	f CLX		Cleared all registers

Clearly, [n] cannot be solved for, though, indeed, you can solve for every other variable in the problem (i, PV, PMT, and FV). In fact, this is true even though **"no solution exists"** for [n]. Indeed, there are an unlimited number of solutions for [n], and it is for this reason that the calculator's programming cannot provide us with an answer. For practice, you might use the same data, but set [n] to 1,000 years. Again, verify that the value of the property is indeed $500,000. Then try to solve for [n]; same result! Capitalization in perpetuity clearly assumes an infinite holding period or an infinite number of solutions for the holding period. Therefore, your calculator cannot return an answer for the holding period [n].

The difficulty in finding truly comparable sales data often diminishes the prospects for successful extraction of an overall cap rate from market data. In these cases we can turn to the band-of-investment (built-up) technique. (For more thorough coverage of how we derive an overall cap rate from comparable sales transactions, please see The Appraisal of Real Estate, 10th edition, published in 1992 by The Appraisal Institute, Chicago.)

Deriving the overall cap rate (RO) with the band-of-investment method

This section deals with a technique for developing an estimate of the overall capitalization rate (RO) by the band-of-investment or mortgage-equity technique. Specifically, the technique covered below addresses only One Loan-to-Value Ratio problems. However, please note that the same methodology can be used to derive an overall capitalization rate where more than one LTV ratio--and therefore more than one mortgage loan--is being considered as the source of financing for the property being analyzed.

The equation given below is a variant of the traditional "Ellwood" mortgage-equity equation. What follows on the next page is the author's version of the technique originally designed by Charles B. Akerson.

$$\boxed{RO = M \times RM + (1 - M) \times YE - M \times p \times SFF - \Delta O \times SFF}$$

The Ellwood equation, under the Akerson format, is equivalent to:

RO = %LTV Ratio x ALC	RO = M x RM
+ (1 - %LTV Ratio) x Equity Yield	+ (1 - M) x YE
- %LTV Ratio x %Reduction in Mortgage x SFF	- M x p x SFF
- ΔValue of Property x SFF	- ΔO x SFF

Where: RO = Overall Capitalization Rate

%LTV Ratio = Loan-to-Value Ratio (as a decimal)

ALC = Annual Loan Constant

Equity Yield = Return on Equity (YE)
(Also called the Equity Yield "YE")

%Reduction in Mortgage = Percent by which the mortgage loan will be reduced over the holding period, expressed as a decimal

ΔO = Total percentage change in value of the property (either positive or negative) over the projected holding period, expressed as a decimal

SFF = Sinking Fund Factor computed at the Equity Yield rate

Additional data required:

1. Interest rate on Mortgage Loan.
2. Term of Mortgage Loan (in months).
3. Holding Period at time of expected sale of property.

Problem

Assume:
LTV Ratio = 75% (.75)
Mortgage Interest Rate = 14% (Annual)
Mortgage Term = 25 years (300 months)
Equity Yield Rate = 17% (Annual)
Projected Investment Holding Period = 9 years
Change in Value of Property at time of sale = 55.13%
(Equivalent to approximately 5% compound annual growth)
Net Operating Income (NOI) = $100,000

Calculate the Overall Cap Rate (RO) and the Value of the Property (V). **Note:** The procedure used in this book is to calculate the monthly mortgage payment, with the balloon payment calculated on the same basis. This method gives slightly different results compared with methods which use annual mortgage payments. For example, in the above problem you would figure the value of the property at $848,571 (vs. $851,590) if we used annual mortgage payments instead of monthly, a difference of "-0.35%". However, in practice, each method would most likely give the same results, $850,000.

(a) Compute the Annual Loan Constant (ALC):

Solution by Formula

$$ALC = 12 \times \frac{i}{1 - (1 + i)^{-n}}$$

$$= 12 \times \frac{14/1200}{1 - (1 + 14/1200)^{-(25 \times 12)}}$$

$$= 12 \times .01203761040 \quad \text{(used f PREFIX)}$$

ALC = .144451325 (@ f 9)

Keystroke Solution

f CLX f 9 g END

25 g n 14 g i

1 CHS PV

PMT (running) 12 x

DISPLAYS: 0.144451325

(b) Compute percent reduction in mortgage loan over 9 years:

KEYSTROKES:

f CLX f 9	Clear registers; set 9 decimal places
25 g n	Mortgage term of 300 months
14 g i	Monthly interest rate
1 CHS PV	One dollar ($1.00) as the Present Value
PMT	Computes the Installment to Amortize $1.00
9 g n	This is the holding period (108 months)
FV	Balloon payment, expressed as a decimal
100 x	Percent of loan outstanding (92.05%)
100 x≷y -	Percent reduction in mortgage loan

DISPLAYS: 7.948168600

Comment: The mortgage will be reduced by 7.9481686% at the end of the ninth year. You will use 0.079481686 as the %Reduction in Mortgage in the formula.

(c) Compute the Sinking Fund Factor (SFF):

Solution by Formula

$$SFF = i/((1 + i)^{n} - 1)$$

$$SFF = .17/((1 + .17)^{9} - 1)$$

$$SFF = .17/(4.108400333 - 1)$$

$$SFF = .17/3.108400333$$

SFF = 0.054690510 (@ 9 places)

Keystroke Solution

f CLX f 9 g END

9 n 17 i

1 FV

PMT

DISPLAYS: -0.054690510

(d) Working with the solution equation, we have:

RO = %LTV x ALC
= .75 x .144451325 = 0.108338494
+ (1 - %LTV) x Equity Yield +

= (1 - .75) x .17
= .25 x .17 = 0.0425

\- %LTV x %Reduction in Mortgage x SFF -
= .75 x .079481686 x .05469051
= .059611265 x .05469051 = 0.003260170

\- ΔValue x SFF -
= .5513 x .05469051 = 0.030150878
Overall Capitalization Rate (RO) = 0.117427446

(e) Compute the Income Multiplier and the Value of the Property (V):

Multiplier = 1/RO
= 1/.117427446
Multiplier = 8.515896701

Value of Property (V) = NOI x Multiplier
= $100,000 x 8.515896701
Value of Property (V) = **$851,590** (rounded)

or:

Value of Property (V) = NOI/RO = $100,000/.117427446
Value of Property (V) = **$851,590** (rounded)

Discounted Cash-Flow Models (DCF): DCF routines can be used to estimate the present value of an income producing property and, as well, they may be used to extract the equity yield rate (YE) and the overall capitalization rate from a sale. When estimating the value of an income producing property, each cash-flow is discounted and summed along with the discounted present value of the property reversion. Thus, the DCF methodology is to convert NOI into its before-tax cash-flow (BTCF)--also called the "cash throw-off to equity (CTO)"--, with the cash-flows then discounted at an appropriate equity yield rate (YE). In addition, the property reversion is also discounted at the same discount rate and added to the discounted present value of the before-tax cash-flows. The result is a composite estimate of the value of the property (V).

To estimate value with a DCF model we need an estimate of the NOI (or operating income and operating expenses) projected for the first year of ownership, projected holding period and change in value of the property over the holding period, terms of financing (if any), LTV ratio or amount of the mortgage loan, and a suitable equity yield rate. The equity yield rate is synonymous with a discount rate in that it is used to discount all cash-flows back to their present value. The equity yield is indeed a compound annual yield expressed as a percentage of the investor's down payment. It represents the overall yield to the equity investors and in this respect it takes into account both the return of and the return on the investor's equity in the property.

From the perspective of the investor, it is likely that the selection of the equity yield rate (YE) will be dictated by his minimum yield requirements--sometimes called a "hurdle rate". This would include a suitable interest markup premium over and above a "riskless rate" (such as the U.S. Treasury Bill Rate) to cover the risks associated with being in a much less secure position than a mortgage lender (whose funds are generally secured by a first mortgage against the property), plus a suitable markup to cover social and/or political risks associated with the ownership of real property.

From the perspective of an appraiser attempting to render a fair opinion as to the market value of a property, selection--indeed, the derivation--of the equity yield rate (YE) can be much more challenging. In general, if we are trying to determine the market value of an income producing property, finding a suitable equity yield rate can often be achieved (or certainly zeroed-in) by:

1. Asking market participants (brokers, lenders, appraisers and others who counsel commercial real estate investors) what rates are being sought by investors in similar properties.
2. Deriving the rates from data available from recent sales of comparable properties. (The equity yield rate is synonymous with the internal rate of return (IRR) of an investment. Therefore, you could project the property's before-tax cash-flows and equity reversion against the investor's down payment in the property using the IRR function. The yield readily follows by calculating the IRR.)
3. Seeking information from surveys conducted by large commercial and industrial real estate firms, such as Cushman & Wakefield, or working through real estate counseling firms, such as the Real Estate Research Corporation.

The principal income capitalization techniques covered in this book are grounded on the assumption that the property will be held for a **limited** number of years. As stated earlier, these kinds of techniques are commonly referred to as "Ellwood" or mortgage-equity or discounted cash-flow (DCF) methods. (The "CONSTANT RATIO CHANGE PER PERIOD MODEL"--covered on page 262--is not an "Ellwood" DCF technique because it does not take into account contribution to an investment property's value made by reducing a mortgage loan over the projected holding period.) Overall, the field is quite extensive and contains an almost endless number of techniques, some of which are admittedly more complex than those published in this book--such as "multi-period" discounted cash-flow models.

In the following pages we cover four income capitalization techniques: CONSTANT RATIO CHANGE PER PERIOD MODEL; ONE LTV MODEL; EQUITY RESIDUAL ANALYSIS USING A DISCOUNTED CASH-FLOW MODEL; and the DEBT SERVICE COVERAGE RATIO CONSTRAINT MODEL. To build confidence in their use, several proofs are given. The proofs, in and of themselves, are instructive of the flow of dollars through an income producing property.

CONSTANT RATIO (GEOMETRIC) CHANGE PER PERIOD MODEL

When income--and at times Value--is expected to grow at a constant rate of change, the income stream being projected represents an "increasing annuity". The simplest form of this annuity is found in end of the year growth or decay models, such as the one covered in this part of the book.

The following equation can be used to compute the present value of a step-up (increasing) or step-down (decreasing) constant ratio change per year annuity where the cash-flows are assumed to occur at the end of the year.

$$\text{Present Value} = \frac{1 - (1 + g)^n/(1 + i)^n}{i - g} \times \text{NOI}$$

Where: $g \neq i$ (growth in income does <u>not</u> equal the discount rate)
g = annual rate of growth (or decay) in income
i = discount or yield rate
n = number of end-of-year (EOY) cash-flows
NOI = net operating income, or, in general, income per year

Problem:

Assume: NOI = $10,000 per year (EOY)
n = 10 years
i = 10%
g = 3% growth in income per year

What is the present value of the cash-flows?

Solution by Formula:

$$\text{Present Value} = \frac{1 - (1 + g)^n/(1 + i)^n}{i - g} \times \text{NOI}$$

$$\text{PV} = \frac{1 - (1 + .03)^{10}/(1 + .10)^{10}}{.10 - .03} \times \$10{,}000$$

$$= 6.883743693 \times \$10{,}000$$

$$\text{PV} = \$68{,}837.44 \quad \text{@ 2 decimal places}$$

KEYSTROKES:	1.03 ENTER 10 [y^x]	(Raises 1.03 to the 10th power)
	1.10 ENTER 10 [y^x]	(Raises 1.10 to the 10th power)
	÷ CHS 1 +	(Completes the numerator)
	.1 ENTER .03 -	(Completes denominator)
	÷	(Numerator divided by denominator)
	10,000 x	
	DISPLAYS: 68,837.44	@ f 2

Comment: We can also solve these kinds of problems using the calculator's unequal cash-flow function. Since the example on the previous page assumes the cash-flows occur at the end of the year--at the end of the time period--we must input the initial cash-flow into the CFj register. We do not make an input into CFo because all cash-flows occur at the end of the time period.

Solution Using the Unequal Cash-Flow Function

KEYSTROKES:	f CLX f 2	Clears all registers; sets 2 places
	10 i	Inputs required yield (discount) rate
	10,000 g CFj	Inputs first year cash-flow ($10,000.00)
	3 % + g CFj	Inputs 2nd year cash-flow ($10,300.00)
	3 % + g CFj	Inputs 3rd year cash-flow ($10,609.00)
	3 % + g CFj	Inputs 4th year cash-flow ($10,927.27)
	3 % + g CFj	Inputs 5th year cash-flow ($11,255.09)
	3 % + g CFj	Inputs 6th year cash-flow ($11,592.74)
	3 % + g CFj	Inputs 7th year cash-flow ($11,940.52)
	3 % + g CFj	Inputs 8th year cash-flow ($12,298.74)
	3 % + g CFj	Inputs 9th year cash-flow ($12,667.70)
	3 % + g CFj	Inputs final year cash-flow ($13,047.73)
	f NPV	Computes net present value of cash-flows
	DISPLAYS: 68,837.44	

Discussion: The unequal cash-flow function of your HP 12C readily lends itself to solving annuity problems in which the income increases or decreases by a given percentage every period. For example, though the above problem assumed a growth rate of 3% each year, we could just as easily have used different rates of growth for each year or series of years. As well, we could cause the income to grow by a certain percentage for a given number of years and then have the income decay (decrease) by a given percentage for any number of years.

The IRR function is very powerful and should be mastered. It enables the most math-shy individual to crunch numbers in any number of advanced functions whose equations occupy one-half a page the size of this one. With a few simple keystrokes and inputs into the unequal cash-flow registers of your HP 12C we can achieve financial results which otherwise might not have been within your reach if you had to work strictly with complex equations.

Computing the Present Value With Property Reversion Capabilities

Suppose you are satisfied that the discounted present value of the projected annual cash-flows gives you a suitable estimate of the value of an income producing property. Let's further assume that the property will increase in value on an annual basis and that this increase is independent of the annual increases in operating income. To value such an income stream, consisting of both increasing (or decreasing) operating income followed by a possible reversion (sale) of the property at the end of the holding period, we can use the equation which follows on the next page.

Note that after we work with the equation we will use a program to obtain the same results. (If you prefer to bypass the mathematics of this routine, skip over to the HP 12C program. It starts on page 267.)

Solution by Formula:

$$\text{Present Value} = \frac{\dfrac{1 - (1 + g)^n/(1 + i)^n}{i - g}}{1 - (1 + gv)^n/(1 + i)^n} \times \text{NOI}$$

Where:
NOI = net operating income, or, in general, income per year
g = annual rate of growth (or decay) in income
i = discount or yield rate
$g \neq i$ (growth in income does not equal the discount rate)
n = number of end-of-year (EOY) cash-flows
gv = rate of growth (or decay) in property value per year

Note: Where income growth per year equals the growth or "natural appreciation" per year of the property, the above equation reduces to something much simpler.

$$\text{Present Value} = \frac{\text{NOI}}{i - g}$$

Problem:

Assume:
NOI = $45,000 per year (EOY)
i = 15% and 13% required yields
g = 3% projected annual growth in NOI
gv = 3% projected annual "natural appreciation" in property value each year over the holding period
n = holding period is not relevant here because we are using the same growth rate for income and the property

What is the present value of the cash-flows at yields of 15% and 13%?

Solution:

$$\text{Present Value} = \frac{\text{NOI}}{i - g}$$

$$= \frac{\$45{,}000}{.15 - .03} = \frac{\$45{,}000}{.12}$$

Present Value = $375,000 @ 15% Annual Yield

$$\text{Present Value} = \frac{\text{NOI}}{i - g}$$

$$= \frac{\$45{,}000}{.13 - .03} = \frac{\$45{,}000}{.10}$$

Present Value = $450,000 @ 13% Annual Yield

Problem:

Assume: NOI = \$45,000 per year (EOY)
i = 13% required yield
g = 3% projected annual growth in NOI
gv = 1% projected annual appreciation in value of property
n = 10 year holding period

What is the present value of the property?

Discussion: What we have here is an income producing property which is projected to produce \$45,000 in net operating income (NOI) the first year. The income is projected to grow by 3% per year during the ten year holding period. Clearly, the increasing income stream impacts the value of the property. The problem, however, has an added twist, which is that the property is expected to grow in value at the rate of 1% per year, which growth is independent of the value generated by the income stream itself.

What the equation does--and as well what the program that follows does--is to compute the present value of the cash-flows. This is the value of the property, assuming we do not cause the property value to increase independent of the value produced by discounting the projected cash-flows.

The equation is flexible enough to (1) recapture the value of the property at the expiration of the holding period, or (2) cause the present value produced by the discounted cash-flows to **grow** at a given rate (gv) over the holding period (n), and then be discounted and become part of an enhanced value, or (3) we can simply discount a stream of cash-flows--such as lease payments--back to a present value, thus assuming there is no recapture of value at the expiration of the holding (or lease) period. These capabilities will become clearer as we work with the equation and its counterpart program.

Solution by Formula:

Note: The keystrokes are performed on the next page.

$$\text{Present Value} = \frac{\dfrac{1 - (1 + g)^n/(1 + i)^n}{i - g}}{1 - (1 + gv)^n/(1 + i)^n} \text{ x NOI}$$

$$= \frac{\dfrac{1 - (1 + .03)^{10}/(1 + .13)^{10}}{.13 - .03}}{1 - (1 + .01){}^\wedge 10/(1 + .13)^{10}} \text{ x } \$45,000$$

$$= \frac{6.040978939}{.674591193} \text{ x } \$45,000$$

$$= \$402,975.99$$

Present Value = \$403,000 (Rounded)

KEYSTROKES:	1.03 ENTER 10 $[y^{x}]$	(Raises 1.03 to the 10th power)
	1.13 ENTER 10 $[y^{x}]$	(Raises 1.13 to the 10th power)
	÷ CHS 1 +	(Numerator of first fraction)
	.13 ENTER .03 -	(Denominator of first fraction)
	÷	(Numerator divided by denominator)
	1.01 ENTER 10 $[y^{x}]$	(Raises 1.01 to the 10th power)
	1.13 ENTER 10 $[y^{x}]$	(Raises 1.13 to the 10th power)
	÷ CHS 1 + ÷	(Completes the division process)
	45,000 x	(Multiplies by first year NOI)
	DISPLAYS: 402,975.99	@ f 2

Comment: The value of the property is approximately $403,000. In the steps which follow below we will add the discounted present value of the increasing net operating income stream to the discounted present value of the property after causing it to grow by one percent (1%) per year for ten years. The proof will show that the total of these discounted income streams is exactly $402,975.99

KEYSTROKES:	f CLX	Clears all registers
	13 i	Inputs required yield (discount) rate
	45,000 g CFj	Inputs first year cash-flow ($45,000.00)
	3 % + g CFj	Inputs 2nd year cash-flow ($46,350.00)
	3 % + g CFj	Inputs 3rd year cash-flow ($47,740.50)
	3 % + g CFj	Inputs 4th year cash-flow ($49,172.72)
	3 % + g CFj	Inputs 5th year cash-flow ($50,647.90)
	3 % + g CFj	Inputs 6th year cash-flow ($52,167.33)
	3 % + g CFj	Inputs 7th year cash-flow ($53,732.35)
	3 % + g CFj	Inputs 8th year cash-flow ($55,344.32)
	3 % + g CFj	Inputs 9th year cash-flow ($57,004.65)
	3 % +	Final year operating cash-flow ($58,714.79)
	1.01 ENTER	Enters 1 + 1% growth rate into the STACK
	10 $[y^{x}]$	Raises 1.01 to the 10th power
	402,975.99 x	Projected sales price of property after 10 years ($445,136.19)
	+	Adds sales price to final year cash-flow
	g CFj	Enters $503,850.99 as final year cash-flow
	f NPV	Computes net present value of cash-flows
	DISPLAYS: 402,975.99	

Comment: In practice it is well admitted that one rarely sees an acquisition the size of the above example handled as a cash deal. However, the technology in the equation (and its counterpart program) is well suited for discounting lease payments which grow by a constant ratio over the lease term.

In using this, or any other equation or program in this book, always make sure that your application is suited to the kinds of problems solved in the examples covered for each program or equation. And, be sure you proof your results by running the internal rate of return (IRR) of the results you obtain from the equation or program.

PROGRAM FOR COMPUTING PRESENT VALUE OF STEP-UP OR STEP-DOWN CONSTANT RATIO CHANGE PER YEAR ANNUITY WITH RESIDUAL VALUE CAPABILITY

```
KEYSTROKE        DISPLAY              KEYSTROKE        DISPLAY

f R/S f PRGM                          -                23       30
RCL 0            01     45  0         ÷                24       10
EEX              02         26        RCL 1            25    45  1
2                03          2        x                26       20
÷                04         10        RCL 2            27    45  2
STO 3            05     44  3         EEX              28       26
1                06          1        2                29        2
+                07         40        ÷                30       10
RCL i            08     45 12         1                31        1
EEX              09         26        +                32       40
2                10          2        RCL 4            33    45  4
÷                11         10        1                34        1
STO 4            12     44  4         +                35       40
1                13          1        ÷                36       10
+                14         40        RCL n            37    45 11
÷                15         10        y^x              38       21
RCL n            16     45 11         CHS              39       16
y^x              17         21        1                40        1
CHS              18         16        +                41       40
1                19          1        ÷                42       10
+                20         40        ENTER            43       36
RCL 4            21     45  4         STO PV           44    44 13
RCL 3            22     45  3         f R/S
```

Required Information and Memory Storage Register Locations Used

1. Set END (g END) mode.
2. Growth or decay in NOI, expressed as a percentage, store in Mem. Reg. 0.
3. NOI or annual income stores in Mem. Reg. 1.
4. Annual growth or decay in value of the property, expressed as a percentage, store in Mem. Reg. 2. If there is no residual (reversion) value at the expiration of the holding period, store "-100" (100 CHS) in Mem. Reg. 2. If there is full recapture of the value of the property or leasehold interest, store a "0" in Mem. Reg. 2.
5. Number of years in holding period, store in [n] register.
6. Required annual yield--or discount rate--store in [i] register.

Overall memory usage: **P - 50, r - 14.**

Utilizing the Program

Problem:

Assume: NOI = \$45,000 per year (EOY)
i = 15% or 13% annual yield
g = 3% per year projected annual growth in NOI
gv = 3% per year projected natural appreciation in value of the property over the holding period
n = 10 years holding (analysis period)

Compute the present value of the cash-flows at the given yield rates.

KEYSTROKES:	f CLX f 0	Clears all registers; sets 0 places
	10 n	Inputs holding period
	15 i	Inputs required yield (discount) rate
	3 STO 0	Stores annual growth in NOI as a percent
	45,000 STO 1	Stores annual NOI
	3 STO 2	Stores percentage annual growth in value of the property over the holding period
	R/S	Starts program operation
	DISPLAYS: 375,000.	@ f 0

Comment: Note that since the annual growth in value of the property is equal to the annual growth in the NOI, the design of the routine effectively forces an infinite holding period. That is, when both growth rates are the same, as long as the holding period [n] is not set to zero (0), it doesn't matter what the holding period is set to. The routine will always return the same value for the discounted presented value of the cash flows. To show this, if we set the holding period [n] to any positive number, other than zero, the routine returns the same value.

KEYSTROKES:	1 n	Sets holding period to one year
	R/S	Starts program operation
	DISPLAYS: 375,000.	
	100 n	Sets "holding period" to 100 years
	R/S	Starts program operation
	DISPLAYS: 375,000.	@ f 0

Comment: Discount the cash-flows at 13%. Use a 10 year and a 100 year holding period. The value produced by the cash-flows will be identical for both holding periods.

KEYSTROKES:	10 n	Set holding period to original period
	13 i	Inputs required yield (discount) rate
	R/S	Starts program operation
	DISPLAYS: 450,000.	
	100 n	Sets "holding period"
	R/S	Starts program operation
	DISPLAYS: 450,000.	@ f 0

Problem: Let's prove that the $450,000 discounted present value we calculated in the previous example is mathematically correct. To do this we use the unequal cash-flow function of your calculator. Keep in mind as we go through the keystrokes that we are discounting an increasing income stream back to its present value. In addition, we are discounting the terminal value of the property at the end of the holding period back to time period zero. The sum of these two components will indeed be $450,000, the initial value of the property.

KEYSTROKES:	f CLX f 2	Clears all registers; sets 2 places
	13 i	Inputs required yield (discount) rate
	45,000 g CFj	Inputs 1st year cash-flow ($45,000.00)
	3 % + g CFj	Inputs 2nd year cash-flow ($46,350.00)
	3 % + g CFj	Inputs 3rd year cash-flow ($47,740.50)
	3 % + g CFj	Inputs 4th year cash-flow ($49,172.72)
	3 % + g CFj	Inputs 5th year cash-flow ($50,647.90)
	3 % + g CFj	Inputs 6th year cash-flow ($52,167.33)
	3 % + g CFj	Inputs 7th year cash-flow ($53,732.35)
	3 % + g CFj	Inputs 8th year cash-flow ($55,344.32)
	3 % + g CFj	Inputs 9th year cash-flow ($57,004.65)
	3 % +	Final year operating cash-flow ($58,714.79)
	1.03 ENTER	Enters 1 + 3% growth rate into the STACK
	10 [y^x]	Raises 1.03 to the 10th power
	450,000 x	Projected sales price of property after 10 years ($604,762.37 @ 2 places)
	+	Adds sales price to final year cash-flow
	g CFj	Enters $663,477.16 as final year cash-flow
	f NPV	Computes net present value of cash-flows
	DISPLAYS: 450,000.00	

Problem: Let's assume that a lease is expected to produce $60,000 in revenue in the first year, with income expected to increase by 3% per year over a five year holding period. Value the lease to produce the following annual yields: 10%; and 20%. Remember, since there is no recapture of value at the expiration of the five year holding period we must input "-100" into memory register 2.

KEYSTROKES:	f CLX f 0	Clears all registers; sets 0 places
	5 n	Inputs term of the lease
	10 i	Inputs required yield
	3 STO 0	Stores annual growth in income
	60,000 STO 1	Stores annual income
	100 CHS STO 2	There is no recapture of value at the expiration of the holding period
	R/S	Starts program operation
	DISPLAYS: 240,156.	
	20 i	Inputs required yield
	R/S	Starts program operation
	DISPLAYS: 188,511.	@ f 0

PROGRAM FOR COMPUTING OVERALL CAPITALIZATION RATE (RO) BY MORTGAGE-EQUITY BUILD-UP TECHNIQUES: ONE LTV MODEL

KEYSTROKE	DISPLAY	
f R/S f PRGM		
RCL i	01	45 12
STO 8	02	44 8
RCL n	03	45 11
STO 9	04	44 9
1	05	1
CHS	06	16
STO PV	07	44 13
PMT	08	14
1	09	1
2	10	2
x	11	20
STO 4	12	44 4
RCL 3	13	45 3
g n	14	43 11
FV	15	15
RCL PV	16	45 13
÷	17	10
1	18	1
+	19	40
STO 5	20	44 5
1	21	1
CHS	22	16
STO FV	23	44 15
0	24	0
STO PV	25	44 13
RCL 1	26	45 1
STO i	27	44 12
RCL 3	28	45 3
STO n	29	44 11
PMT	30	14
CHS	31	16
1	32	1
RCL 0	33	45 0
%	34	25
x	35	20
STO 6	36	44 6
RCL 5	37	45 5
x	38	20
1	39	1
RCL 2	40	45 2
%	41	25
x	42	20
+	43	40
x	44	20
RCL 6	45	45 6
RCL 4	46	45 4
x	47	20
+	48	40
RCL 6	49	45 6
CHS	50	16
1	51	1
+	52	40
1	53	1
RCL 1	54	45 1
%	55	25
x	56	20
x	57	20
+	58	40
STO 7	59	44 7
0	60	0
STO FV	61	44 15
RCL 8	62	45 8
STO i	63	44 12
RCL 9	64	45 9
STO n	65	44 11
RCL 7	66	45 7
f R/S		

Required Information and Memory Storage Locations Used

1. Set END (g END) mode.
2. Input monthly mortgage interest rate in [i].
3. Input mortgage term (months) in [n].
4. Loan-to-Value Ratio (LTV), as a percent, store in Mem. Reg. 0.
5. Required Equity Yield (YE) store in Mem. Reg. 1.
6. Change in value of property (ΔVO) over holding period, STO Mem. Reg. 2.
7. Holding period (in years), store in Mem. Reg. 3.
8. % Reduction in Mortgage can be recalled from Mem. Reg. 5.
9. Overall Capitalization Rate (RO) displays and stores in Mem. Reg. 7.

Overall memory usage: **P - 71, r - 11.**

Problem:

Assume:	LTV Ratio = 75%
	NOI = $50,000
	Annual Mortgage Interest Rate = 14%
	Mortgage term = 25 years
	Required Annual Equity Yield = 17%
	Projected Investment Holding Period = 9 years
	Projected Change in Value of Property = 55.13%

Step 1: Use the program to compute the Overall Capitalization Rate (RO). Then calculate the Income Multiplier and estimate the Value of the Property.

KEYSTROKES:	f CLX	Clear all registers
	f 6	Display 6 decimal places
	25 g n	Inputs mortgage term
	14 g i	Inputs monthly interest rate
	75 STO 0	Stores Loan-to-Value Ratio
	17 STO 1	Stores required Equity Yield Rate (YE)
	55.13 STO 2	Stores property appreciation over 9 years
	9 STO 3	Stores Holding Period
	R/S	Starts program operation
	DISPLAYS:	
	0.117427	Overall Capitalization rate (RO)

Step 2: Compute the Income Multiplier.

KEYSTROKES:	[1/x]	Inverts the Overall Capitalization Rate
	DISPLAYS:	
	8.515897	Income Multiplier

Step 3: Compute the estimated Value of the Property (VO).

KEYSTROKES:	50,000 x	Multiplies NOI times Income Multiplier
	DISPLAYS:	
	425,794.8376	Estimated Value of the Property

Problem: Assume the required Equity Yield (YE) is 19% per annum. Recompute the Overall Capitalization Rate (RO) and the Value of the Property (VO).

KEYSTROKES:	19 STO 1	Stores amended Equity Yield
	R/S	Recomputes Overall Capitalization rate
	DISPLAYS:	
	0.125176	Overall Capitalization Rate (RO)
	[1/x]	Inverts RO - Gives Income Multiplier
	50,000 x	NOI times the Income Multiplier
	DISPLAYS:	
	399,439.1485	Estimated Value of the Property = $399,439

Conclusion: The Value of the Property (VO) is approximately $400,000.

Problem

Using the data from the above example, show the comparative cash-flows, significant appraisal ratios, and <u>prove</u> the Equity Yield (YE) to the investor equals 19% per annum. To make the proof, we use the IRR function.

Solution

(a) Compute the Value of the Mortgage (VM) loan. (In traditional mortgage-equity analysis the mortgage itself is assumed to add a measure of value to the property. Knowing the computed Value of the Property, we multiply the LTV ratio times it to arrive at the mortgage amount.)

VM = %LTV Ratio x VO

= .75 x $399,439.15 (2 places for extreme accuracy)

VM = $299,579.36 (Value of the mortgage loan)

(b) Compute the investor's Equity Position (VE). (Knowing the Value of the Property and the amount of the mortgage loan, we find the equity by subtracting the loan from the Value of the Property.)

VE = VO - VM

= $399,439.15 - $299,579.36

VE = $99,859.79

(c) Compute the Cash Throw-Off to Equity (CTO). (The Cash Throw-Off is a measure of the before-tax, before-depreciation profitability of an investment property. It is simply the net operating income less the annual debt service on the mortgage loan.)

1. Calculate the annual debt service (ADS).

ADS = VM x ALC (ALC may be recalled from Mem. Reg. 4)

= $299,579.36 x .144451325

ADS = $43,274.64

2. Subtract ADS from NOI to get Cash Throw-Off to Equity (CTO).

CTO = NOI - ADS

= $50,000 - $43,274.64

CTO = $6,725.36

(d) Compute the Property Reversion (PR). (This is the projected sales price at the expiration of the holding period. It is considered a "reversion" since the property, in effect, reverts to--goes to--another at the expiration of the projected holding period.)

$$PR = VO \times (1 + \%\Delta Value/100)$$

$$= \$399,439.15 \times (1 + 55.13/100)$$

$$PR = \$619,649.95$$

(e) Compute the Mortgage Balance (BAL) after 9 years. (I use "BAL" here to designate the mortgage balance, and use it in the equations which follow this subsection.)

KEYSTROKES:	f FIN	Clears financial registers
	f 2	Sets 2 decimal places
	25 g n	Inputs mortgage term
	14 g i	Inputs monthly interest rate
	299,579.36 CHS PV	Inputs loan amount, negative-signed
	PMT	Monthly payment to amortize loan
	9 g n	Inputs number of payments made
	FV	Loan balance after 108 payments
	DISPLAYS:	
	275,768.29	Balance "BAL" equals $275,768.29

(f) Compute the Equity Reversion (ER). (This is the amount of money which in effect "reverts" to the equity investor/owner of the property. It is determined by subtracting the loan balance from the Property Reversion (PR).)

Equity Reversion = Property Reversion - Mortgage Balance

$$ER = PR - BAL$$

$$= \$619,649.95 - \$275,768.29$$

$$ER = \$343,881.66$$

(g) Compute the Equity Dividend Rate (RE). (The Equity Dividend Rate is another name for the "Cash-on-Cash" Return or Rate. We arrive at this by dividing the first-year cash throw-off to equity (CTO) by the amount of the investor's equity (VE) in the property. It is a measure of profitability, though it does not take into account the overall yield to the investor, the Equity Yield (YE). The figure is likely more important to a lender than it is to the investor.)

$$RE = CTO/VE$$

$$= \$6,725.36/\$99,859.79$$

$$= .067348...$$

$$RE = 6.73\%$$

(h) Compute the Debt Service Coverage Ratio (DSCR). (This is a measure which lenders are most concerned with. It tells us the number of times over the annual debt service (ADS) can be paid from the property's first year net operating income (NOI). Indeed, the DSCR is one of the most significant criteria considered by a lender on commercial real estate.)

DSCR = NOI/ADS

= $50,000/$43,274.64

DSCR = 1.16 (to 2 decimal places)

Summary

VO	$399,439.15	Value of the Property
VM	$299,579.36	Value of the Mortgage Loan
VE	$99,859.79	Value of the Investor's Equity in the Property
CTO	$6,725.36	Annual Cash Throw-Off to Equity
PR	$619,649.95	Property Reversion: Projected Sales Price
BAL	$275,768.29	Loan balance
ER	$343,881.66	Equity Reversion: Sales Price less Mortgage Balance
RE	6.73%	Return on Equity: Also called Cash-on-Cash Rate
DSCR	1.16	Debt Service Coverage Ratio
RO	0.125176	Overall Cap Rate
YE	19%	Equity Yield: Compound annual return on Equity

Problem: Prove the yield to the investor (YE) is 19%. This is equivalent to proving the internal rate of return (IRR) of any investment. Input the Investor's equity as a negative cash-flow, followed by the balance of flows.

```
KEYSTROKES:   f CLX                  Clear all registers
              f 2                    Set 2 decimal places
              99,859.79 CHS g CFo    Inputs equity investment as a
                                     negative signed cash-flow
              6,725.36 g CFj         Inputs Cash Throw-Off to Equity
              8 g Nj                 Inputs 8 cash-flows
              R↓                     Brings back $6,725.36 cash-flow
              343,881.66 +           Adds Equity Reversion to cash-flow
              g CFj                  Inputs 9th year total cash-flow
              f IRR                  Computes Equity Yield (IRR)
              DISPLAYS:
              19.00
```

Comment: The Equity Yield of 19% per annum is proven. Note that this is a compound annual yield. We achieved extreme accuracy because we carried all inputs out to two decimal places. In practice, you will not carry out your data to two decimal places. Thus, your IRR proofs will fail to perfectly duplicate the required equity yield. If, however, your IRR is off by, say, more than one-eighth percent (0.125%), recheck your data very carefully.

PRACTICE

		PROBLEM 1	PROBLEM 2	PROBLEM 3
LTV RATIO	STO 0	60%	70%	75%
NOI (1st year)		$45,000	$100,000	$40,000
Required Equity Yield	STO 1	12%	13%	17%
Mortgage Interest rate	[g][i]	10.25%	11%	11.25%
Mortgage Term	[n]	240 mo.	240 mo.	180 mo.
Growth in Value of Property	STO 2	3%/year * compounded	2%/year ** compounded	25% in total
Holding Period	STO 3	7 years	5 years	5 years

Answers:

Overall Capitalization Rate (RO)	.0866360	.0997198	.0921813
Income Multiplier (1/RO)	11.54254	10.02809	10.84819
Value of the Property (VO)	$519,414	$1,002,809	$433,928

Data/Significant Ratios

Value of the property (VO)	$519,414	$1,002,809	$433,928
Value of the Mortgage (VM)	$311,648	$701,966	$325,446
Value of Equity Position (VE)	$207,766	$300,843	$108,482
Annual Debt Service (ADS)	$36,711	$86,947	$45,003
Cash Throw-Off to Equity (CTO)	$8,289	$13,053	-$5,003
Mortgage Balance (BAL)	$263,134	$637,483	$269,475
Property Reversion (PR)	$633,814	$1,107,182	$542,410
Equity Reversion (ER)	$375,680	$469,699	$272,935
Equity Yield (VE)	12%	13%	17%
Equity Dividend Rate (RE)	4%	4.3%	-4.6%
Debt Service Coverage ratio (DSCR)	1.23	1.15	0.89

IRR Proof:

	DISPLAY	**DISPLAY**	**DISPLAY**
f CLX f 0	0.	0.	0.
Value of Equity (VE) CHS g CFo	-207,766	-300,843	-108,482
Cash Throw-Off (CTO) g CFj	8,289	13,053	-5,003
Holding Period Less "1" g Nj	6	4	4
Roll Down [R↓]	8,289	13,053	-5,003
Equity Reversion (ER)	375,680	469,699	272,935
+ g CFj (final year cash-flow)	383,969	482,752	267,932
f IRR (running)	**12.**	**13.**	**17.**
f 2 (Investor's Equity Yield)	**12.00**	**13.00**	**17.00**

---

* Growth = $[(1.03)^{7} - 1] \times 100 = 22.9873865\%$

** Growth = $[(1.02)^{5} - 1] \times 100 = 10.40808030\%$

Problem: Working with irregular cash-flow streams

Assume:
LTV Ratio = 60%
First year NOI = $100,000
Annual compound growth in NOI = 2%
Mortgage interest rate = 10.25%
Mortgage term = 20 years
Compound annual growth in Value of Property = 3%
Holding period = 7 years
Required Equity Yield (YE) = 16%

Calculate:
Uniform Series of the Increasing NOI stream (NOI(US))
Overall Capitalization Rate (RO)
Value of the Property (VO)
Value of the Mortgage (VM)
Value of the Equity Investment (VE)
The Equity Yield (YE) after computing the above

Note: The following data may be recalled after operating the program:

(1) Annual Loan Constant (ALC) recalls from Memory Register 4.
(2) Percent reduction in mortgage loan (expressed as a decimal) recalls from Memory Register 5.
(3) Overall Capitalization Rate (RO) may be recalled from Memory Register 7.

Solution methodology: Note that we are causing the net operating income (NOI) to **increase** annually. The first year NOI of $100,000 will be increased by two percent in the second year, for a total NOI of $102,000, and this amount will be increased by two percent in the third year, for a total of $104,040, and so forth. These kinds of increasing income streams cannot be entered directly into the calculator because memory limitations prevent input of the complex program necessary to solve this problem in one pass through a program. Fortunately, the Unequal Cash-Flow Function of your calculator reduces this problem to nothing more than a few extra keystrokes in our solution. (The HP 12C clearly **shines** when used to solve these kinds of problems.)

To stabilize an **increasing** (or for that matter, **decreasing**) stream of payments or cash-flows, we compute the **uniform series** of the cash-flows. That is, we calculate a **uniform** payment or income stream which is **equivalent** to the irregular cash-flows--the "irregular" NOIs--which are produced by the income producing property. This is accomplished by using the net present value function (NPV) to solve for the **discounted present value** of the unequal income stream. From there, we simply compute the **equivalent** annual (periodic) payment. It is this amount which will be divided by the Overall Capitalization Rate in order to produce the estimated Value of the Property.

Step 1: Input ONE LTV OVERALL CAPITALIZATION RATE program into calculator.

Step 2: Compute the Uniform Series (US) of the irregular cash-flows.

KEYSTROKES:	f CLX	Clears all registers
	f 2	Sets 2 decimal places
	16 i	Inputs required annual Equity Yield (YE)
	100,000 g CFj	Inputs 1st year NOI
	2 % + g CFj	Inputs $102,000 for 2nd year NOI
	2 % + g CFj	Inputs $104,040 for 3rd year NOI
	2 % + g CFj	Inputs $106,120.80 for 4th year NOI
	2 % + g CFj	Inputs $108,243.22 for 5th year NOI
	2 % + g CFj	Inputs $110,408.08 for 6th year NOI
	2 % + g CFj	Inputs $112,616.24 for 7th year NOI
	f NPV	Computes discounted Present Value of End-of-Year Net Operating Incomes
	DISPLAYS: 423,972.21	
	CHS PV	Sets NPV to PV, negative-signed
	PMT	Computes the Uniform Series Equivalent PMT
	DISPLAYS: 104,980.89	

Discussion: What we did was to input an **increasing annuity**--an increasing payment stream--into the Unequal Cash-Flow Function of your calculator. By discounting the stream of payments (NOIs, to be exact) we determine the present value of the cash-flows. It does not matter whether the flows come from an irregular stream of payments--like those in this example--or if they are produced by a fixed stream of payments; the present value of the cash-flows can be readily calculated in either instance.

Knowing the NPV ("Present Value", since we did not input an out-of-pocket cash-flow) of the payment stream, we can calculate the fixed "payment" needed to duplicate the irregular payment stream. That payment is exactly equivalent to the irregular stream of cash-flows used to generate the Present Value. This technique saves you time in working these kinds of problems.

Step 3: Compute the Overall Capitalization Rate (RO).

KEYSTROKES:	f CLX f 9	Clear registers; set 9 places
	20 g n	Inputs mortgage term
	10.25 g i	Inputs monthly interest rate
	60 STO 0	Stores Loan-to-Value Ratio
	16 STO 1	Stores required Equity Yield Yield Rate (YE)
	1.03 ENTER 7 y^x	Computes property appreciation over 7 year
	1 - 100 x	holding period (22.99%)
	STO 2	Stores property appreciation over 7 years
	7 STO 3	Stores Holding Period
	R/S	Starts program operation
	DISPLAYS:	
	0.106355329	Overall Capitalization Rate (RO)

Comment: Knowing the Overall Capitalization Rate (RO), we can determine the Value of the income property. The fact that the income stream we use has the "bumps" taken out does not change anything; the procedure works the same.

Step 4: Compute the Value of the Property (VO).

Value of the Property = Uniform Series NOI/Overall Capitalization Rate

VO = NOI(US)/RO

= $104,980.89/.106355329 (9 places used for extreme accuracy)

VO = $987,076.91

Step 5: Compute the Value of the Mortgage (VM).

VM = %LTV RATIO x VO

= 60% x $987,076.91

VM = $592,246.15

Step 6: Compute the Value of the Equity Investment (VE).

VE = VO - VM

= $987,076.91 - $592,246.15

VE = $394,830.76

Step 7: Determine the Percent Reduction in the loan balance after 7 years.

KEYSTROKES:	f 9	Sets 9 decimal places
	RCL 5	Recalls decimal equivalent reduction in loan
	DISPLAYS:	
	0.155668678	(This is "p" in the general equation)

Step 8: Compute the loan balance after 7 years.

BALANCE = (1 - %Reduction in Loan Balance) x VM

= (1 - .155668678) x $592,246.15

BALANCE = $500,051.97

Step 9: Determine the Annual Loan Constant (ALC).

KEYSTROKES:	RCL 4	Recalls ALC from Memory Register 4
	DISPLAYS:	
	0.117797206	

Step 10: Compute the Uniform Series Cash Throw-Off to Equity.

CTO(US) = NOI(US) - ADS

CTO(US) = NOI(US) - ALC x VM

= \$104,980.89 - .117797206 x \$592,246.15

= \$104,980.89 - \$69,764.94

CTO(US) = \$35,215.95

Comment: It is not necessary to individually discount each year's cash throw-off to equity (CTO) back to a present value in order to determine the income contribution aspect of an income producing property's value. Just net the uniform series net operating income [NOI(US)] with the annual debt service (ADS). It works fine.

Step 11: Compute the Property Reversion (PR).

PR = VO$(1 + gv/100)^n$ (Where "gv" equals annual compound growth.)

= \$987,076.91 x $(1 + 3/100)^7$

= \$987,076.91 x 1.03^7

= \$987,076.91 x 1.229873865

PR = \$1,213,980.09

Comment: The computed property reversion (\$1,213,980.09) is the projected price of the property if sold at the expiration of the seven year holding period. Annual compound growth in value was assumed at three percent per year. Assuming we used a gross amount representing the total projected growth in value over the holding period, you would add the decimal equivalent of this number to one ("1") in the above equation and proceed in the same manner as we did above.

Step 12: Compute the Equity Reversion (ER).

ER = Property Reversion - Mortgage Loan Balance

ER = PR - BAL

= \$1,213,980.09 - \$500,051.97

ER = \$713,928.12

Comment: The Equity Reversion (ER) represents the projected final cash-flow from the property. The procedure used in the HP 12C program assumes that the user took into account all factors impacting the value of the property throughout the holding period. This is, admittedly, a bit of a juggling act, though one can surely argue that the purpose of using income capitalization techniques--and programs--is to enable the investor or appraiser to zero-in on a good **estimate** of value. However, it is inconceivable that any method or program can produce a totally 100% accurate estimate of a property's value well into the future.

Step 14: Recap data needed to prove internal rate of return of the investment property.

Note: We use 2 decimal places to assure extreme accuracy in proving the internal rate of return (IRR) of our data.

VO	$987,076.91	(Value of the Property)
VM	$592,246.15	(Value of the Mortgage Loan)
VE	$394,830.76	(Value of the Equity)
CTO(US)	$35,215.95	(Uniform Series Cash Throw-Off to Equity)
ER	$713,928.12	(Equity Residual)
YE	16%	(Equity Yield)
n	7 year Holding Period	

Step 15: Prove the yield to the investor in this property is indeed a compound annual growth of 16%.

KEYSTROKES:	f CLX	Clear all registers
	f 2	Set 2 decimal places
	394,830.76 CHS g CFo	Inputs equity investment as a negative-signed number
	35,215.95 g CFj	Inputs Uniform Series CTO
	6 g Nj	Inputs 6 cash-flows
	R↓ 713,928.12 +	Total cash-flow in 7th year of $749,144.07
	g CFj	Inputs Uniform CTO(US) + ER
	f IRR	Computes Equity Yield (IRR)
	DISPLAYS:	
	16.00	

Comment: The Equity Yield (YE) of 16% per annum is proven. The yield tells us that over a seven year holding period an investor who places $394,830.76 into this particular property will earn a compound annual return of 16% from and on the investment.

PRACTICE

		PROBLEM 1	PROBLEM 2	PROBLEM 3
LTV RATIO	STO 0	70%	75%	75%
NOI (1st year)		$45,000	$100,000	$40,000
Annual growth in NOI		2%	---	2.25%
Annual <u>decay</u> in NOI		---	2%	---
Required Equity Yield	STO 1	13%	16%	17%
Mortgage Interest rate	[g][i]	9.25%	10%	11.25%
Mortgage Term	[n]	300 mo.	300 mo.	240 mo.
Growth in Value of Property	STO 2	25% in total	2%/year compounded	25% in total
Holding Period	STO 3	5 years	5 years	5 years

Answers:

Overall Capitalization Rate	.065341594	.100283921	.091726767
Uniform Series NOI	$46,611.33	$96,650.45	$41,552.11
Value of the Property	$713,348.53	$963,768	$452,999

Data/Significant Ratios

Value of the property (VO)	$713,348.53	$963,768	$452,999
Value of the Mortgage (VM)	$499,343.97	$722,826	$339,749
Value of Equity Position (VE)	$214,004.56	$240,942	$113,250
Annual Debt Service (ADS)	$51,315.49	$78,819.90	$42,778.04
Uniform Series Cash Throw-Off	-$4,704.16	$17,830.55	-$1,225.93
Mortgage Balance (BAL)	$466,911.88	$680,640.20	$309,355
Property Reversion (PR)	$891,685.66	$1,064,078	$566,249
Equity Reversion (ER)	$424,773.78	$383,438	$256,894
Equity Yield (YE)	13%	16%	17%
Equity Dividend Rate (RE)	-2.95%	8.79%	-2.45%
First year DSCR (DSCR)	0.88	1.27	0.94
Uniform Series DSCR (DSCR(US))	0.91	1.23	0.97

IRR Proof:

	DISPLAY	**DISPLAY**	**DISPLAY**
f CLX f 2	0.00	0.00	0.00
Value of Equity CHS g CFo	-214,004.56	-240,942	-113,250
Uniform Series Cash Throw-Off g CFj	-4,704.16	17,830.55	-1,225.93
Holding Period less "1" g Nj	4	4	4
Roll Down [R↓]	-4,704.16	17,830.55	-1,225.93
Equity Reversion	424,773.78	383,438	256,894
+ g CFj	420,069.62	401,268.55	255,668.07
f IRR (running)	**13.00**	**16.00**	**17.00**

EQUITY RESIDUAL ANALYSIS USING A DISCOUNTED CASH-FLOW MODEL

Discussion

In this section we will value/determine the **required equity position** of an investor in income producing property when the mortgage amount is known. In this regard, we are working in reverse when compared to what we did when solving valuation problems with the One LTV Model technique.

In income capitalization theory, property value is a composite of a number of financial attributes of an income producing property. Thus, a property can be valued by summing the total of the following components of value:

1) the present value of the cash throw-off (CTO) to equity;
2) the present value of the equity reversion (ER); and
3) the present value of the mortgage loan (VM).

The present value of the mortgage loan, is, of course, the principal amount of the loan. Symbolically, the model looks like this:

Value of Property = PV of Cash Throw-Off + PV of Equity Reversion + Mortgage

Condensing the nomenclature, we have:

VALUE = PV of CTO + PV of ER + VM (Discounted at Equity Yield Rate)

Indeed, if we know the amount of the mortgage loan, and if the cash-flows and anticipated net sales price of the property are sufficient to support the loan's debt service, the equation will be in balance. Think about this: If the mortgage amount **increases,** the annual debt service (ADS) increases, thus causing the present value of the cash throw-off (CTO) to **decrease.** This keeps the equation in balance.

And, the same analogy applies to the present value of the property's equity reversion (ER). If the mortgage amount **increases,** there will be a greater annual debt service (ADS) and the loan's balance will as well be greater at the expiration of the holding period. If the mortgage loan balance is higher, because the loan was greater, then the amount of the equity reversion (ER) **decreases,** again balancing off the equation.

Specifically, if we discounted the projected cash-flows (and this includes the equity reversion) at the required yield rate (YE), add the results to the amount of the mortgage, we arrive at the Value of the Property (V).

Look at the problem on page 271. In that example, we used a required equity yield (YE) of 19% and a holding period of nine years. We determined the following data which are relevant to the approach we are now taking:

Value of the Property (V) = $399,439.15
Value of the Mortgage (VM) = $299,579.36
Cash Throw-Off to Equity (CTO) = $6,725.36
Equity Reversion (ER) = $343,881.66

The model we now work with tells us this: If we add

PV of CTO + PV of ER + VM

we should get the property's value, $399,439.15. Let's perform the calculation.

KEYSTROKES:	f CLX f 2	Clear all registers; set 2 places
	9 n	Inputs holding period
	19 i	Equity Yield Rate (YE)
	6,725.36 PMT	Sets CTO to annual payment
	PV	Calculates discounted PV of CTO
	DISPLAYS: -27,999.91	
	CHS STO 0	Stores positive amount for PV CTO
	0 STO PMT	Clears payment register
	343,881.66 FV	Sets Equity Reversion to FV
	PV	Calculated discounted PV of ER
	DISPLAYS: -71,859.86	
	CHS RCL 0 +	Adds PV of CTO to PV of ER
	DISPLAYS: 99,859.77	Looks familiar? Its the (VE).
	299,579.36	Amount of mortgage loan (VM)
	+	Sums up: PV of CTO + PV of ER + VM
	DISPLAYS:	
	399,439.13	(Rounding difference of two cents)

Comment: Our calculation shows several things. First, the value of an income producing property can indeed be determined by adding (1) the value of its mortgage loan to the sum of the present values of its (2) cash throw-off and (3) equity reversion. Next, an interesting concept comes to light which is that if you add the discounted present value of the Equity Reversion (PV of ER) to the discounted present value of the Cash Throw-Off (PV of CTO) you arrive at the Value of the Equity Investment (VE) in the property.

Summing the various components, we have:

	PROPERTY VALUE		CASH THROW-OFF TO EQUITY		EQUITY REVERSION		VALUE OF MORTGAGE
GROSS VALUE	$399,439.13		$6,725.36		$343,881.66		$299,579.36
DISCOUNTED PV	$399,439.13		$27,999.91		$71,859.86		$299,579.36
Composite	$399,439.13	=	$27,999.91	+	$71,859.86	+	$299,579.36

Note that the present value of both the mortgage loan and the property itself is the same as the actual or gross amount of the quantities. Indeed, both the mortgage loan and the value of the property "occur" at time period zero were you to plot them on a timeline diagram. Thus, their present values must equal their gross values. Let's work with a HP 12C program which should make short-work of the kind of valuation discussed above.

PROGRAM FOR COMPUTING VALUE OF THE REQUIRED EQUITY (VE) USING A DISCOUNTED CASH-FLOW MODEL: ONE MORTGAGE MODEL

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
f R/S f PRGM		x	34 20
0	01 0	RCL PV	35 45 13
STO FV	02 44 15	CHS	36 16
RCL n	03 45 11	+	37 40
STO 9	04 44 9	STO 8	38 44 8
RCL i	05 45 12	RCL 2	39 45 2
STO . 0	06 44 48 0	g n	40 43 11
1	07 1	FV	41 15
RCL 1	08 45 1	CHS	42 16
%	09 25	RCL 7	43 45 7
x	10 20	x	44 20
STO 6	11 44 6	RCL 8	45 45 8
1	12 1	+	46 40
+	13 40	1	47 1
RCL 2	14 45 2	RCL 5	48 45 5
CHS	15 16	%	49 25
y^x	16 21	x	50 20
STO 7	17 44 7	1	51 1
CHS	18 16	+	52 40
1	19 1	RCL 2	53 45 2
+	20 40	y^x	54 21
RCL 6	21 45 6	RCL 7	55 45 7
÷	22 10	x	56 20
RCL 0	23 45 0	CHS	57 16
ENTER	24 36	1	58 1
PMT	25 14	+	59 40
PMT	26 14	÷	60 10
f RND	27 42 14	STO 4	61 44 4
PMT	28 14	RCL 9	62 45 9
RCL PMT	29 45 14	STO n	63 44 11
1	30 1	RCL . 0	64 45 48 0
2	31 2	STO i	65 44 12
x	32 20	RCL 4	66 45 4
-	33 30	f P/R	

Required Information and Memory Storage Locations Used

1. Set END (g END) mode.
2. Input monthly mortgage interest rate in [i].
3. Input mortgage term (months) in [n].
4. Value of Mortgage (VM), negative-signed, stores in [PV].
5. Net Operating Income (NOI) store in Mem. Reg. 0.
6. Required Equity Yield (YE) store in Mem. Reg. 1.
7. Holding Period (in years), store in Mem. Reg. 2.
8. Annual compound growth in Value of the Property, store in Mem. Reg 5.
9. Value of the Property (V) displays and stores in Mem. Reg. 4.

Overall Memory Usage: **P - 71, r - 11**

Problem:

Assume:
NOI = $150,000 (Uniform Series)
Annual Growth in NOI = 0.00
Value of the Mortgage loan = $700,000
Annual Mortgage Interest Rate = 10%
Mortgage term = 28 years
Growth in Value of the Property over Holding Period = 1%/year
Holding Period = 5 years
Required Equity Yield = 17%

Use the program to solve for the Value of the Property (V) and then determine the amount of the required equity investment (VE).

Part 1: Compute the Value of the Property (V).

KEYSTROKES:

f CLX f 2	Clear registers; set 2 decimal places
28 g n	Inputs mortgage term
10 g i	Inputs monthly interest rate
700,000 CHS PV	Inputs amount of mortgage loan
150,000 STO 0	Stores net operating income (NOI)
17 STO 1	Stores required Equity Yield Rate (YE)
5 STO 2	Stores Holding Period
1 STO 5	Inputs compound annual growth rate
R/S	Starts program operation
DISPLAYS:	
1,220,644.49	Estimated Value of the Property

Part 2: Compute the required equity investment (VE).

VE = V - VM

= $1,220,644.49 - $700,000 (2 decimal places for extreme accuracy)

VE = $520,644.49

Problem: Using the data still in your calculator, change the Equity Yield (YE) from 17% to 19%. Then compute the Value of the Property (V) and the required Equity Investment (VE).

KEYSTROKES:

19 STO 1	Inputs amended Equity Yield (YE)
R/S	Starts program operation
DISPLAYS:	
1,160,978.23	Estimated Value of the Property

Compute the Value of the Equity Investment (VE):

VE = V - VM

= $1,160,978.23 - $700,000

VE = $460,978.23

Problem: Change the Equity Yield (YE) to 20% and set the Holding Period to ten years. Again, compute the Value of the Property (V) and the required Equity Investment (VE).

KEYSTROKES:	20 STO 1	Inputs new Equity Yield (YE)
	10 STO 2	Inputs new Holding Period
	R/S	Starts program operation
	DISPLAYS:	
	1,114,605.39	Estimated Value of the Property

VE = V - VM

= $1,114,605.39 - $700,000

VE = $414,605.39

Data Summary

V = $1,114,605.39
VM = $700,000.00
Loan Interest Rate = 10%
VE = $414,605.39
Equity Yield = 20%
Holding Period = 10 yrs
NOI = $150,000
Mortgage term = 28 yrs
Growth in V = 1%/yr

Problem: Show that (subject to $0.03 rounding error):

VALUE = PV of CTO + PV of ER + VM (Discounted at Equity Yield Rate)

Financial Component	Discounted Present Value	How Determined
VM = $700,000.00	$700,000.00	Given
ADS = $74,588.67	---	28 g n 10 g i 700,000 CHS PV PMT 12 x
BAL = $621,669.44	---	Using ADS inputs: 10 g n FV
CTO = $75,411.33	$316,159.90	$150,000 NOI - $74,588.67 ADS = $75,411.33: 10 n 20 i 75,411.33 PMT 0 FV PV Displays: -316,159.90
PR = $1,231,217.78	---	10 n 1 i 1,114,605.39 CHS PV 0 PMT FV
ER = $609,548.34	$98,445.46	$1,231,217.78 PR - $621,669.44 BAL = $609,548.34: STO FV 10 n 20 i 0 PMT PV: Displays: -98,445.46

Property Value of $1,114,605.36 = $700,000 + $316,159.90 + $98,445.46

Problem:

Assume: NOI = $150,000
Annual Growth in NOI:
 2nd & 3rd years = 2%
 4th - 6th years = 3%
 7th - 10th years = 4%
Value of the Mortgage loan = $700,000
Annual Mortgage Interest Rate = 11%
Mortgage term = 25 years
Growth in Value of the Property over Holding Period = 2%/year
Holding Period = 10 years
Required Equity Yield = 16%

Use the program to solve for the Value of the Property (V). Then determine the amount of the required equity investment (VE).

Note: This problem covers a slightly different aspect of increasing income (increasing annuity) net operating income (NOI) streams in that here we cause the NOI to increase at different rates over different years. Indeed, were we to manually discount each cash-flow back to time period zero it would make the calculation much more tedious. Let's use the calculator's Unequal Cash-Flow Function; it cuts right through this aspect of the problem.

Part 1: Compute the Uniform Series of the net operating income stream. To do this we first calculate the NPV of the increasing income stream. We then calculate the annual income which is equivalent to the NPV.

```
KEYSTROKES:  f CLX f 2          Clear registers; set 2 decimal places
             16 i               Inputs required Equity Yield Rate (YE)
             150,000 g CFj      Inputs 1st year NOI
             2 % + g CFj        Inputs 2nd & 3rd year NOI  ($153,000.00)
             2 g Nj             Inputs 2 cash-flows (2nd & 3rd years)
             R↓ 3 % + g CFj     Inputs 4th - 6th year NOI  ($157,590.00)
             3 g Nj             Inputs 3 cash-flows (4th, 5th, and 6th yrs)
             R↓ 4 % + g CFj     Inputs 7th - 10th year NOI ($163,893.60)
             4 g Nj             Inputs 4 cash-flows (7, 8, 9, and 10th yrs)
             f NPV              Discounted present value of cash-flows

             DISPLAYS: 756,013.11

             CHS PV             Sets NPV to PV, negative-signed
             10 n               Inputs number of years in Holding Period
             PMT                Computes the Uniform Series Equivalent PMT

             DISPLAYS: 156,419.93
```

Comment: The uniform series net operating income (NOI(US)) is $156,419.93. This level payment is exactly equivalent to the "step-up" annuity stream of NOIs we input above. We use the NOI(US) for our input into the program. Remember: You store this variable in Memory Register 0.

Part 2: Utilize the program to compute the Value of the Property.

KEYSTROKES:	f CLX f 2	Clear registers; set 2 decimal places
	25 g n	Inputs mortgage term
	11 g i	Inputs monthly interest rate
	700,000 CHS PV	Inputs amount of mortgage loan
	156,419.93 STO 0	Stores uniform series NOI
	16 STO 1	Stores required Equity Yield Rate (YE)
	10 STO 2	Stores Holding Period
	2 STO 5	Inputs <u>compound</u> <u>annual</u> <u>growth</u> <u>rate</u>
	R/S	Starts program operation
	DISPLAYS:	
	1,273,037.21	Estimated Value of the Property

Part 3: Compute the required equity investment (VE).

VE = V - VM

= $1,273,037.21 - $700,000   (2 decimal places for extreme accuracy)

VE = $573,037.21   (Required Equity Position of the Investor)

Problem: Using the above data, verify that the Equity Yield (YE) equals 16%.

Part 4: Compute annual debt service (ADS) and loan balance after ten years.

KEYSTROKES:	f CLX f 2	Clears registers; set 2 decimal places
	25 g n	Inputs mortgage term
	11 g i	Inputs monthly interest rate
	700,000 CHS PV	Inputs loan amount, negative-signed
	PMT	Monthly payment to amortize loan
	12 x	Annual Debt Service

DISPLAYS: 82,329.50

10 g n	Inputs number of payments made
FV	Loan balance after 120 payments

DISPLAYS: 603,625.73

Part 5: Compute the Uniform Series Cash Throw-Off to Equity.

CTO(US) = NOI(US) - ADS

= $156,419.93 - $82,329.50

CTO(US) = $74,090.43

Part 6: Compute the Equity Reversion (ER).

$$ER = V \times (1 + gv/100)^{\text{Holding Period}} - BAL$$

$$\begin{aligned} ER &= \$1,273,037.21 \times (1 + 2/100)^{10} - \$603,625.73 \\ &= \$1,551,825.26 - \$603,625.73 \\ ER &= \$948,199.53 \end{aligned}$$

Part 7: Recap relevant data.

V	$1,273,037.21	
VM	$700,000.00	
VE	$573,037.21	(Needed to prove the Equity Yield)
NOI	$150,000.00	(First year net operating income)
NOI(US)	156,419.93	(Needed to prove the Equity Yield)
ADS	$82,329.50	
CTO(US)	$74,090.43	(Needed to prove the Equity Yield)
PR	$1,551,825.26	
BAL	$603,625.73	
ER	$948,199.53	(Needed to prove the Equity Yield)
YE	16%	
HOLD	10 years	(Needed to prove the Equity Yield)

Part 8: Prove the Equity Yield (YE).

KEYSTROKES:	f CLX f 2	Clear all registers; set 2 decimal
	573,037.21 CHS g CFo	Input equity investment as a negative-signed cash-flow
	74,090.43 g CFj	Inputs Uniform Series CTO
	9 g Nj	Inputs 9 cash-flows
	R↓	Brings back $74,090.43 cash-flow
	948,199.53 +	Adds Equity Reversion to cash-flow
	g CFj	Inputs 10th year cash-flow of $1,022,289.96
	f IRR	Computes Equity Yield (IRR)
	DISPLAYS: 16.00	

Comment: The Equity Yield of 16% per annum is proven. In practice, each time you perform a calculation similar to what we did above, you should make certain that the required Equity Yield (YE) in fact is supported by the cash-flows used in your analysis.

One final point: In analyzing an income producing property we often use different growth (or decay, for that matter) rates for the annual Operating Income and the annual Operating Expenses. This method is significantly more accurate where the growth rates of income and expenses differ by as little as one percent. You can solve these kinds of problems with the program and techniques covered in this section. Just compute the Uniform Series of the income; then do the same for the operating expenses; subtract the two, and you have the Uniform Series NOI. Everything else remains the same. (For a more comprehensive treatment of this area, please see Professional Real Estate Problem Solving Using the HP 17BII, published by the author.

PRACTICE

		PROBLEM 1	PROBLEM 2	PROBLEM 3
Uniform Series NOI	STO 0	$175,000	140,000	$70,000
Required Equity Yield	STO 1	12%	13%	17%
Holding period	STO 2	5 years	6 years	7 years
Mortgage loan	CHS PV	$1,000,000	$800,000	$500,000
Mortgage term	[n]	180 months	240 months	300 mo.
Mortgage interest rate	[g][i]	11%	10.5%	12%
Growth in Value of Property	STO 5	3%/year compounded	2%/year compounded	25% in * total

Answers:

Value of the Property (@ f 0)	$1,960,807	$1,393,062	$637,016

Data/Significant Ratios

Value of the property (V)	$1,960,807	$1,393,062	$637,016
Value of the Mortgage (VM)	$1,000,000	$800,000	$500,000
Value of Equity Position (VE)	$960,807	$593,062	$137,016
Annual Debt Service (ADS)	$136,392	$95,844	$63,193
Cash Throw-Off to Equity (CTO)	$38,608	$44,156	$6,807
Mortgage Balance (BAL)	$825,116	$701,581	$465,225
Property Reversion (PR)	$2,273,113	$1,568,814	$796,270
Equity Reversion (ER)	$1,447,997	$867,233	$331,045
Equity Yield (YE)	12%	13%	17%
Equity Dividend Rate (RE)	4.02%	7.45%	4.97%
Debt Service Coverage ratio (DSCR)	1.28	1.46	1.11

IRR Proof:

	DISPLAY	**DISPLAY**	**DISPLAY**
f CLX f 0	0.	0.	0.
Value of Equity CHS g CFo	-960,807	-593,062	-137,016
Cash Throw-Off g CFj	38,608	44,156	6,807
Holding Period Less "1" g Nj	4	5	6
Roll Down [R↓]	38,608	44,156	6,807
Equity Reversion	1,447,997	867,233	331,045
+ g CFj	1,486,605	911,389	337,852
f IRR (running)	**12.**	**13.**	**17.**
f 2	**12.00**	**13.00**	**17.00**

---

* To compute the equivalent annual compound growth, perform the following: 100 [ENTER] 25 [%] + [FV] 100 [CHS] [PV] 7 [n] 0 [PMT] [i]: DISPLAYS 3.24 @ f, 2. The compound annual growth is therefore 3.24% per year at two decimal places. Store "3.24" in Memory Register 5.

LIMITING THE MAXIMUM MORTGAGE WITH A DEBT SERVICE COVERAGE RATIO CONSTRAINT

Limiting the LTV ratio is the lender's first line of defense insofar as protecting its security in an investment property. However, what happens if the first year net operating income (NOI) is still insufficient to carry the debt load against the property? That is, if the annual debt service (ADS) cannot be supported by the projected--or actual--first year net operating income, the lender's security in the property could be jeopardized. Therefore, the investor's capital investment in the property could be diminished (or lost) should it become necessary for the lender to protect its interest in the property through judicial means.

Since the primary business of lenders is to loan money--rather than obtain ownership of a customer's property through default--it is necessary to evaluate the riskiness of a loan on the basis of not only the LTV ratio and the stability of the NOI stream, but also on the basis of the projected debt service coverage ratio (DSCR). By establishing a DSCR constraint the lender provides itself (and derivatively the debtor) with an operating income cushion. This helps to minimize the risks associated with making the **maximum allowable mortgage loan**. Clearly, the higher the DSCR, the lower the allowable maximum mortgage loan and the greater the protection to the lender. Conversely, the lower the DSCR, the higher the maximum allowable mortgage loan, with resulting higher risk assumed by the both the investor and the lender.

Debt service coverage ratios generally range from as low as 1.1 for very low risk projects--or projects where the investor has sufficient capital reserves to weather a potential drop in projected operating income, etc.--, to as high as 1.5 or more for projects where the risks are perceived to be high. For example, a required ratio of 1.5 tells us that the property must produce \$1.50 of <u>credible</u> (and stable) net operating income for every \$1.00 of anticipated annual debt service. And, a DSCR of 1.00 tells us that the net operating income is break-even and thus just enough to cover the annual debt service. Conversely, if the DSCR drops <u>below</u> 1.00, the investor would have to fund the annual debt service from his own funds, a condition which lenders prefer to avoid.

As was previously shown in this book, the DSCR is defined as the ratio of the first year (generally <u>unstabilized</u>) net operating income (NOI) to the annual debt service (ADS). Symbolically, it looks like this:

$$DSCR = NOI/ADS$$

where:

NOI = First year Net Operating Income
ADS = Annual Debt Service
DSCR = Debt Service Coverage Ratio

In appraisal or investment analysis situations where the NOI, loan terms ("n" and "i") and the lender's required DSCR are known, we can easily compute the maximum allowable mortgage loan supported by the data. (We derive the equation for the maximum allowable loan on the next page.)

Step 1: We know that:

$$DSCR = NOI/ADS$$

Step 2: We further know that:

$$ADS = ALC \times VM$$

Substituting terms of the ADS equation into the DSCR equation, we have:

$$DSCR = \frac{NOI}{ALC \times VM}$$

Therefore,

$$VM = \frac{NOI}{ALC \times DSCR}$$

Example: Let's assume a lender's underwriting guidelines require a DSCR of 1.5 times the projected first year NOI on an income producing property you are evaluating. First year NOI is projected at $250,000. The loan carries an annual interest rate of 10% and requires monthly payments over a 25 year term. (In practice it is more likely that the loan would also require a balloon payment after 5 or possibly 10 years.)

Step 1: Compute the Annual Loan Constant (ALC).

KEYSTROKES	COMMENTS
f CLX f 9	Clear registers; set 9 decimal places
25 g n	Inputs mortgage term (300 months)
10 g i	Inputs monthly interest rate
1 CHS PV	Inputs -$1.00 for the present value
PMT	Computes Installment to Amortize $1.00
12 x	Computes the Annual Loan Constant (ALC)
DISPLAYS: 0.109044090	
f PREFIX	Brings up <u>all</u> 10 digits
DISPLAYS: 1090440895	(Equivalent to 1.090440895/10)
f 2	Set 2 decimal places
1.5 x	DSCR x ALC
[1/x]	Inverts "DSCR x ALC"
250,000 x	Computes maximum mortgage loan
DISPLAYS:	
1,528,433.75	Maximum mortgage loan

Comment: The maximum mortgage loan supported by the data (first year NOI, lender's required DSCR and the mortgage loan terms) is approximately $1,500,000.

On the next page we solve the problem with the equation.

By Equation

$$VM = \frac{NOI}{ALC \times DSCR}$$

$$= \frac{\$250,000}{.1090440895 \times 1.5}$$

$$= \frac{\$250,000}{.1635661343}$$

$$VM = \$1,528,433.75$$

Keystrokes

250,000 ENTER

1.090440895 ENTER

10 ÷

1.5 x ÷

DISPLAYS:
1,528,433.75 (@ 2 decimal places)

Comment: The maximum loan supported by the projected first year net operating income is approximately $1,500,000.

Example: Assume the asking price for a property is $2,500,000. The lender allows a 65% LTV ratio and a DSCR of 1.3. If the annual mortgage interest rate is 10.5% and the amortization period is 25 years, estimate the <u>minimum</u> first year net operating income needed to carry the financing.

Solution methodology: First, determine the ALC and the Value of the Mortgage (VM). The VM is simply the price times the LTV ratio. Then multiply VM times the ALC and the DSCR. The result is the required NOI.

KEYSTROKES	COMMENTS
f CLX	Clear registers
25 g n	Inputs mortgage term
10.5 g i	Inputs monthly interest rate
1 CHS PV	Inputs -$1.00 for the present value
PMT	Computes Installment to Amortize $1.00
12 x STO 0	Compute annual loan constant (ALC)
DISPLAYS: 0.11	(@ f, 9: 0.113301805)
2,500,000 ENTER 65 %	Value times LTV ratio equals loan amount of $1,625,000.
RCL 0 x	Multiplied mortgage loan times ALC
DISPLAYS: 184,115.43	
1.3 x	Completes ALC x mortgage loan x DSCR
DISPLAYS: **239.350.06**	

Comment: The required <u>minimum</u> net operating income (NOI) is approximately $239,000. The equation used to produce the above result follows.

Required 1st year NOI = PRICE x LTV ratio x Annual Loan Constant x DSCR

Condensing the nomenclature, we have:

Required NOI = PRICE x LTV x ALC x DSCR

PROGRAM FOR DETERMINING VALUE BY MORTGAGE-EQUITY BUILD-UP: DEBT SERVICE COVERAGE RATIO (DSCR) CONSTRAINT

KEYSTROKE	DISPLAY
f R/S f PRGM	
0	01 0
STO FV	02 44 15
RCL n	03 45 11
STO 8	04 44 8
1	05 1
CHS	06 16
STO PV	07 44 13
PMT	08 14
1	09 1
2	10 2
x	11 20
RCL 3	12 45 3
x	13 20
[1/x]	14 22
RCL 1	15 45 1
x	16 20
STO 0	17 44 0
CHS	18 16
STO PV	19 44 13
PMT	20 14
RCL 4	21 45 4
g n	22 43 11
FV	23 15
RCL 2	24 45 2
EEX	25 26
2	26 2
÷	27 10
STO 7	28 44 7
1	29 1
+	30 40
RCL 4	31 45 4
CHS	32 16
y^x	33 21
STO 6	34 44 6
RCL FV	35 45 15
x	36 20
STO - 0	37 44 30 0
RCL 6	38 45 6
CHS	39 16
1	40 1
+	41 40
RCL 7	42 45 7
÷	43 10
RCL PMT	44 45 14
1	45 1
2	46 2
x	47 20
CHS	48 16
RCL 1	49 45 1
+	50 40
x	51 20
STO + 0	52 44 40 0
RCL 5	53 45 5
EEX	54 26
2	55 2
÷	56 10
1	57 1
+	58 40
RCL 4	59 45 4
y^x	60 21
RCL 6	61 45 6
x	62 20
CHS	63 16
1	64 1
+	65 40
STO ÷ 0	66 44 10 0
RCL 8	67 45 8
STO n	68 44 11
RCL 0	69 45 0
f R/S	

Required Information and Memory Storage Locations Used

1. Set END mode (g END) before operating the program.
2. Input mortgage term (months) in [n].
3. Input monthly mortgage interest rate in [i].
4. Net Operating Income (NOI) store in Mem. Reg. 1
5. Required Equity Yield (YE) store in Mem. Reg. 2.
6. Debt Service Coverage Ratio (DSCR) store in Mem. Reg. 3
7. Holding period (in years), store in Mem. Reg. 4.
8. Annual compound growth in value of the property store in Mem. Reg. 5.

9. Value of the property displays/can be recalled from Mem. Reg. 0.
10. Maximum mortgage loan can be recalled from PV register.
11. Monthly mortgage loan payment can be recalled from PMT register.

Overall memory usage: **P - 71, r - 11.**

Comment

It is traditional for the lender to limit the debt service coverage ratio (DSCR) based upon an investment property's projected first year net operating income (NOI). This method (also considered an "underwriter's method") generally does not take into account projected future growth in NOI when determining the amount of the maximum mortgage loan with a DSCR constraint. The program in this section follows this procedure.

The program was designed primarily to solve for the Value of the Property (V) using a DSCR constraint. Therefore, the higher the input for the DSCR, the lower will be the Value (V) generated by the program, and vice versa.

Problem:

Assume:
First year NOI = \$55,000
Minimum debt service coverage ratio (DSCR) = 1.25
Annual Mortgage Interest Rate = 10%
Mortgage Term = 20 years
Required Annual Equity Yield = 17%
Projected Investment Holding Period = 7 years
Estimated annual growth in Value of Property = 5%

Use the DSCR program to (1) calculate the estimated value of the Property (VO) and (2) then determine the amount of the maximum allowable mortgage loan.

Step 1: Compute the Value of the Property (VO).

KEYSTROKES:	f CLX f 2	Clear all registers; set 2 decimal places
	g END	Set END mode
	20 g n	Inputs mortgage term
	10 g i	Inputs monthly interest rate
	55,000 STO 1	Store 1st year net operating income
	17 STO 2	Store required Equity Yield Rate (YE)
	1.25 STO 3	Store maximum debt service coverage ratio
	7 STO 4	Store Holding Period
	5 STO 5	Store compound annual property growth rate
	R/S	Starts program operation
	DISPLAYS:	
	596,180.74	Estimated Value of the Property

Step 2: Determine the maximum allowable mortgage loan by recalling the result from the PV register.

KEYSTROKES:	RCL PV	Maximum mortgage loan in PV register
	DISPLAYS:	
	-379,956.94	Maximum mortgage loan

Conclusion: The estimated value of the property is approximately $596,000. The maximum mortgage loan supported by a DSCR of 1.25 (and the other data inputs) is approximately $380,000.

Problem extension: Increase the DSCR from 1.25 to 1.50. Again, estimate the Value of the Property and the maximum mortgage loan supported by the inputs.

KEYSTROKES:	1.5 STO 3	Store maximum debt service coverage ratio
	R/S	Starts program operation
	DISPLAYS:	
	564,508.95	Estimated Value of the Property
	RCL PV	Maximum mortgage loan in PV register
	DISPLAYS:	
	-316,630.78	Maximum mortgage loan

Conclusion: The estimated value of the property is approximately $564,000. The maximum mortgage loan supported by a DSCR of 1.5 (and the other data inputs) is approximately $316,000.

Problem extension: Assume the required Equity Yield (YE) is 19% per annum. Again, estimate the Value of the Property and the maximum mortgage loan supported by the inputs. (Verify that the program stored the projected loan balance in the FV register, and further verify that the Value of the Property is stored in Memory Register 0.)

KEYSTROKES:	19 STO 2	Stores amended Equity Yield Rate (YE)
	R/S	Starts program operation
	DISPLAYS:	
	523,969.11	Estimated Value of the Property
	RCL PV	Determine maximum mortgage loan
	DISPLAYS:	
	-316,630.78	Maximum mortgage loan
	RCL FV	Recall loan balance after 84 payments
	DISPLAYS:	
	266,198.60	Loan balance after 84 payments
	RCL 0	Recall estimated Value of the Property
	DISPLAYS:	
	523,969.11	Estimated Value of the Property

Conclusion: The estimated value of the property is approximately $524,000. The maximum mortgage loan supported by a DSCR of 1.5 (and the other data inputs) is (again!) approximately $316,000.

Comment: In moving from the second last problem extension to the problem immediately above all we changed was the required Equity Yield (YE). Intuitively, since the only input we changed was the required equity yield to the investor, the maximum mortgage loan should not change, and indeed it remained the same ($316,630.78). However, since the investor required a higher yield (19% instead of 17%), the value of the property to him must be less, and this is supported by the results produced by the program.

Problem

Using the data from the above example, (1) verify that the maximum mortgage loan is $316,630.78; (2) verify the DSCR at 1.50; and (3) prove the Equity Yield (YE) to the investor equals 19% per annum.

Solution

Step 1: We show that the maximum mortgage loan supported by the data inputs is $316,630.78. We start with a recital of the equation used to determine the maximum mortgage loan supported by a given first year NOI, DSCR, and mortgage loan data.

$$VM = \frac{NOI}{ALC \times DSCR}$$

We now calculate the Annual Loan Constant (ALC):

KEYSTROKES:	f (FIN) g END	Clear FINancial registers; set END mode
	20 g n 10 g i	Input loan term and monthly interest rate
	1 CHS PV	Inputs $1.00 for the PV
	PMT	Computes Installment to Amortize $1.00
	12 x	Computes ALC (DISPLAYS @ f 9: 0.115802597)
	1.5 x	Computes DSCR times the ALC
	[1/x]	Inverts "DSCR x ALC"
	55,000 x	NOI times 1/(DSCR x ALC)
	DISPLAYS:	
	316,630.78	**Maximum loan** is proven

Step 2: Compute the investor's Equity Position (VE). (Knowing the Value of the Property and the amount of the mortgage loan, we find the required equity (down payment) by subtracting the loan from the Value of the Property.) Work to 2 decimal places for extreme accuracy in proving the IRR.

VE = VO - VM

= $523,969.11 - $316,630.78

VE = $207,338.33

Step 3: Compute the Cash Throw-Off to Equity (CTO). Remember: The CTO is determined by subtracting the annual debt service (ADS) from the first year net operating income (NOI).

a) Calculate the annual debt service (ADS).

ADS = VM x ALC

= $316,630.78 x .115802597

ADS = 36,666.67

b) Subtract the ADS from the NOI to get Cash Throw-Off to Equity (CTO).

CTO = NOI - ADS

= $55,000 - $36,666.67

CTO = $18,333.33

c) Show that the DSCR equals 1.50.

DSCR = NOI/ADS

= $55,000/$36,666.67

DSCR = 1.50

Step 4: Compute the Property Reversion (PR). Remember: This is the projected price at the expiration of the holding period.

$PR = VO \text{ x } (1 + gv/100)^n$ (Where "gv" equals annual compound growth)

$= \$523,969.11 \text{ x } (1 + 5/100)^7$

= $523,969.11 x 1.407100423

PR = $737,277.16

Step 5: Compute the Equity Reversion (ER). Remember: The Equity Reversion represents the projected cash-flow from the property determined by subtracting the anticipated mortgage loan balance from the projected sales price of the property at the expiration of the holding period. Note that we are not building in a factor for expenses associated with the sale of the property. However, since the main thrust of the program is to estimate value within the context of the maximum mortgage loan supported by a lender's required DSCR, this deficit is not believed to be material.

We now calculate the projected mortgage loan balance:

KEYSTROKES:	f (FIN) g END	Clear FINancial registers; set END mode
	f 2	Set 2 decimal places
	20 g n	Input loan term
	10 g i	Input monthly interest rate
	316,630.78 CHS PV	Inputs maximum mortgage loan into PV
	PMT	Computes monthly payment
	7 g n	Inputs number of payments made

KEYSTROKES:	FV	Loan balance after 7 years
	DISPLAYS:	
	266,198.60	**Balance** at expiration of holding period

Let's complete the calculation of the Equity Reversion:

Equity Reversion = Property Reversion - Mortgage Balance

ER = PR - BAL

ER = $737,277.16 - $266,198.60

ER = $471,078.56

Step 6: Let's summarize the data.

Summary

VO	$523,969.11	Value of the Property
VM	$316,630.78	Value of the Mortgage Loan
VE	$207,338.33	Value of the Investor's Equity in the Property
CTO	$18,333.33	Annual Cash Throw-Off to Equity
PR	$737,277.16	Property Reversion: Projected Sales Price
BAL	$266,198.60	Loan balance
ER	$471,078.56	Equity Reversion: Sales Price less Mortgage Balance
DSCR	1.50	Debt Service Coverage Ratio
YE	19%	Equity Yield: Compound annual return on Equity
HOLD	7 years	Holding period

Step 7: Prove the yield to the investor (YE) is 19%. This is equivalent to proving the internal rate of return (IRR) of any investment. Input the investor's equity as a negative cash-flow, followed by the balance of flows.

KEYSTROKES:	f CLX	Clear all registers
	f 2	Set 2 decimal places
	207,338.33 CHS g CFo	Inputs equity investment as a negative signed cash-flow
	18,333.33 g CFj	Inputs Cash Throw-Off to Equity
	6 g Nj	Inputs 6 cash-flows
	[R↓]	Brings back $18,333.33 cash-flow
	471,078.56 +	Adds Equity Reversion to cash-flow
	g CFj	Inputs 7th year total cash-flow
	f IRR	Computes Equity Yield (IRR)
	DISPLAYS:	
	19.00	

Comment: The Equity Yield of 19% per annum is proven. Note that this is a compound annual yield. We achieved extreme accuracy because we carried all inputs out to two decimal places.

PRACTICE

		PROBLEM 1	PROBLEM 2	PROBLEM 3
NOI (1st year)	STO 1	$100,000	$100,000	$75,000
Required Equity Yield	STO 2	15%	15%	19%
Debt Service Coverage Ratio	STO 3	1.25	1.50	1.20
Holding Period	STO 4	7 years	7 years	7 years
Growth in Value of Property	STO 5	3%/year	3%/year	3%/year
Mortgage Interest rate	[g][i]	10.25%	10.25%	11%
Mortgage Term	[n]	180 mo.	180 mo.	180 mo.

Answers:

Value of the Property (VO)	$987,868	$952,194	$639,006
Maximum Mortgage Loan (VM)	$611,648	$509,707	$458,239
Value of Equity position (VE)	$376,220	$442,487	$180,767

Data/Significant Ratios

Loan-to-Value Ratio (LTV)	62%	54%	72%
Annual Debt Service (ADS)	$80,000	$66,667	$62,500
Cash Throw-Off to Equity (CTO)	$20,000	$33,333	$12,500
Mortgage Balance (BAL)	$435,537	$362,948	$331,563
Property Reversion (PR)	$1,214,953	$1,171,079	$785,897
Equity Reversion (ER)	$779,416	$808,131	$454,334
Equity Yield (VE)	15%	15%	19%
Equity Dividend Rate (RE)	5.32%	7.53%	6.91%
Debt Service Coverage ratio (DSCR)	1.25	1.50	1.20

IRR Proof:

	DISPLAY	**DISPLAY**	**DISPLAY**
f CLX f 0	0.	0.	0.
Value of Equity CHS g CFo	-376,220	-442,487	-180,767
Cash Throw-Off g CFj	20,000	33,333	12,500
Holding Period Less "1" g Nj	6	6	6
Roll Down [R↓]	20,000	33,333	12,500
Equity Reversion	779,416	808,131	454,334
+ g CFj	799,416	841,464	466,834
f IRR (running)	**15**	**15**	**19**
f 2	**15.00**	**15.00**	**19.00**

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Professional Real Estate Problem Solving Using the HP 12C
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HP 12C Platinum Partial Programming Supplement

By John A. Tirone

Introduction

The HP-12C Platinum is capable of holding 400 lines of program versus 99 lines in the original HP-12C. Because the new "Platinum" holds more than 99 program lines each step in a program will appear in your display as a three digit number (a three digit location), such as **001**, **005**, **090**, **100**, and so forth. This can readily be seen when you type a program into your HP-12C Platinum--the program line counter will always appear with three digits followed by a comma (","). Let's compare the two HP-12C calculator displays (HP-12C Platinum versus the original HP-12C) below:

KEYSTROKES	**"PLATINUM" DISPLAY SHOWS**	**HP-12C DISPLAY SHOWS**
ON [f] [R/S] [f] [PRGM]	**000,**	**00**
RCL 0	**001, 45 0**	**01 45 0**
RCL 1	**002, 45 1**	**02 45 1**
1	**003, 1**	**03 1**
2	**004, 2**	**04 2**
x	**005, 20**	**05 20**
STO 2	**006, 44 2**	**06 44 2**
ENTER	**007, 36**	**07 36**

Note: For the 8th "program" step we will cause the calculator to return to its first programming line. To do this we use a GTO (go-to) step. Please note that the first line of the HP-12C Platinum is "000"; however, the first program line on the original HP-12C is "00". Thus, if you tried to key-in GTO 000 on the original HP-12C you will get a different result when compared to the HP-12C Platinum. Let's perform the keystrokes below.

KEYSTROKES	**"PLATINUM" DISPLAY SHOWS**	**HP-12C DISPLAY SHOWS**
g GTO 000	**008,43,33,000**	**08 43,33 00** **09 0**
g GTO 003	**009,43,33,003**	**10 43,33 00** **11 3**
g GTO 299	**010,43,33,299**	**12 43,33 29** **13 9**
g GTO 005	**011,43,33,005**	**14 43,33 00** **15 5**
g GTO 400	**Error 4**	**16 43,33 40** **17 0**

Note: Line 000 is the first line in a HP-12C Platinum program. Therefore you can "only" have an additional 399 steps after step "000". Since we committed an error in trying to send the "Platinum" program to line 401 (line "000" plus an additional 400 lines would be 401 lines of programming!) we have to remove the error by typing the "CLX" key (Clear the display). This returns the HP-12C Platinum's display to the last program line typed. However, typing CLX while in the programming mode on the original HP-12C causes it to "clear the displayed x register" as you will see below.

KEYSTROKES	**"PLATINUM" DISPLAY SHOWS**	**HP-12C DISPLAY SHOWS**
CLX	**011,43,33,005**	**18 35**
[f] [PRGM]	**000,**	**00**
[f] [R/S]	**0.00**	**0.00**

Note: We cleared out the "program" (quotes used because obviously the keystrokes make no sense as far as a workable program goes!) and left the programming mode on both HP-12C financial calculators.

Storage Registers Used And Available

Please note that after each HP-12C program in my book I note the storage memory used by the particular program. For example, if you turn to page 89 of my book you will see that the memory used on the "HOME BUYER INCOME QUALIFICATION PROGRAM - Monthly PITI & Annual Income To Quality" program is **"P - 57, r - 13"**. What this means is that this particular program could have been 57 lines long (instead of 53!) and and you would still have 13 memory storage registers available (registers R0 through and including R.2).

Typing the program from page 88 of my book into an HP-12C Platinum would result in the calculator showing **"P - 057, r - 20"** as the memory used (assuming this were the only program in the calculator).

Recall that the original HP-12C gives us 8 "free" lines of program before it starts to use the memory storage registers available for your programs (Registers R.9 through and including R7). The HP-12C Platinum gives us 8 "free" program lines, 43 undesignated registers (each register capable of holding 7 programming lines) plus 13 memory storage registers (R.9 through and including R7). Therefore the HP-12C Platinum will hold a 309 line program before it starts to allocate memory storage registers R.9 through R7.

The programming memory of the original HP-12C versus the HP-12C Platinum stacks up as follows:

HP-12 Platinum	**Original HP-12C**
8 "free" lines of program.	8 "free" lines of program.
43 undesignated storage registers.	Registers R0 through R6.
Registers R0 through R6.	
AVAILABLE PROGRAMMING LINES:	AVAILABLE PROGRAMMING LINES:
8 + (7 x 43) + (7 x 13) =	8 + (7 x 13) =
8 + 301 + (7 x 13) =	8 + 91 = 99
309 + (7 x 13) =	
309 + 91 = 400	

PROGRAM FOR COMPUTING NUMBER OF TIME PERIODS [n]

KEYSTROKE	DISPLAY
[f](REG)[f](RPN)	
[f][R/S][f](PRGM)	000,
[RCL][PMT]	001, 45 14
[RCL] 0	002, 45 0
[%]	003, 25
[+]	004, 40
[STO] 1	005, 44 1
[STO] 2	006, 44 2
[RCL][FV]	007, 45 15
[RCL][i]	008, 45 12
[%]	009, 25
[STO][-] 2	010, 44 30 2
[RCL][PV]	011, 45 13
[RCL][i]	012, 45 12
[%]	013, 25

KEYSTROKE	DISPLAY
[RCL] 1	014, 45 1
[+]	015, 40
[STO][÷] 2	016, 44 10 2
[RCL] 2	017, 45 2
[g][LN]	018, 43 23
1	019, 1
[RCL][i]	020, 45 12
[%]	021, 25
[+]	022, 40
[g][LN]	023, 43 23
[÷]	024, 10
[STO][n]	025, 44 11
OPTIONAL:	
[ENTER]	026, 36
[f] [R/S]	

Memory used: P - 029, r - 20.

User Instructions:

1. Store PV in [PV]
   (Follow cash-flow sign convention)
2. Store FV in [FV]
   (Follow cash-flow sign convention)
3. Store PMT in [PMT]
   (Follow cash-flow sign convention)
4. Store periodic "i" in [i]
5. If a Due Annuity (beginning of the time period payment), store [STO] periodic "i" in Memory Register "0".
6. Set payment mode to END.

Beginning of Time Period Example:

[f][CLX][g](BEG) [f] 2
1 [i]
100,000 [CHS][PV]
50,000 [FV]
240 [n]
[PMT]
DISPLAYS: 1,040.14

Utilize Program:

[g](END) 1 [STO] 0
[R/S]
DISPLAYS: 240.00
Check full accuracy:
[f] 9
DISPLAYS: 239.9999996

Note: The example is based upon payments occurring at the **beginning** of the month. This is why we set the payment mode to **BEG** in the keystroke sequence in the left-hand column. However, you must always set the calculator's payment mode to **END** when using the program to calculate the number of time periods necessary to achieve a given financial result. You will note this step in the right-hand column above.

PROGRAM TO PRODUCE SIMPLE LEVEL PAYMENT AMORTIZATION STATEMENT WITH COUNTER

KEYSTROKE	DISPLAY		
[f] (REG) [g] (END) [f] (RPN) [f] [R/S] [f] (PRGM)	000,		
[f] 9	001,	42	9
[RCL] 0	002,	45	0
[f][AMORT]	003,	42	11
[STO] 1	004,	44	1
[x y]	005,		34
[STO] 2	006,	44	2
[RCL][n]	007,	45	11
[f] 2	008,	42	2
[R/S]	009,		31
[RCL] 1	010,	45	1
[R/S]	011,		31
[RCL] 2	012,	45	2
[R/S]	013,		31
[RCL][PV]	014,	45	13
[R/S]	015,		31
[RCL] 3	016,	45	3
[RCL][n]	017,	45	11
[-]	018,		30
[g][x=0]	019,	43	35
[g][GTO] 022	020,	43,33,022	
[g][GTO] 001	021,	43,33,001	
0	022,		0
[R/S]	023,		31
[g][GTO] 022	024,	43,33,022	
[f][R/S]			

Memory used: P - 029, r - 20.

User instructions:

1. Store amount financed in [PV].
2. Store interest rate in [i].
3. Payment, signed opposite of [PV], stores in [PMT].
4. Store number of payments being amortized in Memory Register 0.
5. Store total number of payments to be amortized in Memory Register 3.

Caution: You **must** make sure that the [n] register is set to "0" before you start program operation.

Since you will always use a rounded payment (to 2 decimal places) when preparing a loan amortization schedule, you will find that the **last payment** will almost always be different than the payment amount used to prepare the schedule. To calculate the **last payment**--this is the final payment--once your calculator displays the final balance, recall [RCL] the payment [PMT], press [x y], followed by [-]. The sum displayed will be the last payment.

Note that this program is designed to **amortize** at nine (9) decimal places and **displays** the results at two decimal places. This methodology will produce the same loan balance you would otherwise calculate using the [FV] function. However, the results will differ from those produced by your calculator's built-in amortization function, depending upon the number of places you cause it to display when using the **built-in** function. If you want the program consistent with Hewlett-Packard's AMORT function, **delete** steps **001** and **008**. Line 020 will become new line 018, and it will read: **[g][GTO] 020.** Line 024 will become new line 022, and it will read: **[g][GTO] 020.**

HOME BUYER INCOME QUALIFICATION PROGRAM
Monthly PITI & Annual Income To Qualify

```
KEYSTROKE              DISPLAY               KEYSTROKE         DISPLAY

[f](REG)[f](RPN)
[f][R/S][f](PRGM)      000,                  g GTO 000         028, 43,33,000
0                      001,          0       %                 029,         25
STO FV                 002,    44   15       x                 030,         20
RCL 1                  003,    45    1       ÷                 031,         10
RCL 0                  004,    45    0       1                 032,          1
-                      005,         30       2                 033,          2
RCL 9                  006,    45    9       x                 034,         20
%                      007,         25       R/S               035,         31
+                      008,         40       RCL 6             036,    45    6
STO PV                 009,    44   13       g [x=0]           037,    43   35
PMT                    010,         14       g GTO 000         038, 43,33,000
RCL 7                  011,    45    7       x≷y               039,         34
+                      012,         40       RCL 4             040,    45    4
RCL 2                  013,    45    2       RCL 5             041,    45    5
RCL 3                  014,    45    3       ÷                 042,         10
+                      015,         40       x                 043,         20
1                      016,          1       RCL 6             044,    45    6
2                      017,          2       1                 045,          1
0                      018,          0       RCL 5             046,    45    5
0                      019,          0       %                 047,         25
÷                      020,         10       x                 048,         20
RCL 0                  021,    45    0       ÷                 049,         10
x                      022,         20       1                 050,          1
+                      023,         40       2                 051,          2
R/S                    024,         31       x                 052,         20
1                      025,          1       +                 053,         40
RCL 4                  026,    45    4       f R/S
g [x=0]                027,    43   35
```

Memory used: P - 057, r - 20.

HOME BUYER INCOME QUALIFICATION PROGRAM
Maximum Sales Price Routine

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
[g](END) [f](REG) [f](RPN) [f][R/S]			
[f](PRGM)	000,	+	030, 40
0	001, 0	RCL 2	031, 45 2
STO FV	002, 44 15	RCL 3	032, 45 3
1	003, 1	+	033, 40
CHS	004, 16	1	034, 1
ENTER	005, 36	2	035, 2
1	006, 1	0	036, 0
RCL 9	007, 45 9	0	037, 0
%	008, 25	÷	038, 10
x	009, 20	RCL PV	039, 45 13
1	010, 1	[1/x]	040, 22
+	011, 40	+	041, 40
÷	012, 10	STO . 0	042, 44 48 0
STO PMT	013, 44 14	÷	043, 10
PV	014, 13	STO FV	044, 44 15
RCL 8	015, 45 8	RCL 8	045, 45 8
RCL 5	016, 45 5	RCL 4	046, 45 4
x	017, 20	x	047, 20
1	018, 1	1	048, 1
2	019, 2	2	049, 2
0	020, 0	0	050, 0
0	021, 0	0	051, 0
÷	022, 10	÷	052, 10
RCL 6	023, 45 6	RCL 7	053, 45 7
-	024, 30	-	054, 30
RCL 7	025, 45 7	RCL 1	055, 45 1
-	026, 30	RCL PV	056, 45 13
RCL 1	027, 45 1	÷	057, 10
RCL PV	028, 45 13	+	058, 40
÷	029, 10	RCL . 0	059, 45 48 0

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
÷	060, 10	g GTO 000	064, 43,33,000
STO 0	061, 44 0	x≷y	065, 34
RCL FV	062, 45 15	f [R/S]	
g x≷y	063, 43 34		

Memory used: P - 071, r - 20.

Note that the program uses storage registers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and .0. In addition, financial registers [n], [i], and [FV] are used.

Storage Registers Used:

1. Store term in [n].
2. Monthly interest rate in [i].
3. Down payment store in Memory Register 1.
4. Real Estate tax, as a percentage of price, store in Memory Register 2.
5. Hazard (Homeowner's) insurance, as a percentage of price, store in Memory Register 3.
6. Basic Housing Expense Ratio, store in Memory Register 4.
7. Total Debt Ratio store in Memory Register 5.
8. Monthly Consumer debts, store in Memory Register 6.
9. Monthly "other" expenses (renewal premiums for PMI, utilities, etc.), store in Memory Register 7.
10. Buyer's gross annual income, store in Memory Register 8.
11. Financed-in PMI, as a percentage of the base mortgage, store in Memory Register 9.
12. Maximum Price produced with the Basic Housing Expense Ratio may be recalled from Memory Register 0.
13. Maximum Price produced with the Total Debt Ratio may be recalled from the FV register.

Make sure the **payment mode** on your calculator is set to **END** before operating the program. If the **BEGIN** status indicator appears in the display, you must press: **[g] END** before operating the program. If preferred, make the first program line: **g END**; every line then moves ahead by one program line.

HOME BUYER INCOME QUALIFICATION PROGRAM
Maximum Affordable Mortgage, Maximum Price & Required Down Payment

```
KEYSTROKE             DISPLAY

[f] (REG) [f] (RPN)
[f][R/S][f](PRGM)     000,
g 8                   001,    43    8
1                     002,          1
CHS                   003,         16
STO PMT               004,    44   14
PV                    005,         13
[1/x]                 006,         22
RCL 4                 007,    45    4
RCL 5                 008,    45    5
+                     009,         40
RCL 6                 010,    45    6
÷                     011,         10
1                     012,          1
2                     013,          2
÷                     014,         10
+                     015,         40
[1/x]                 016,         22
STO 7                 017,    44    7
RCL 0                 018,    45    0
1                     019,          1
2                     020,          2
0                     021,          0
0                     022,          0
÷                     023,         10
STO 8                 024,    44    8
RCL 2                 025,    45    2

KEYSTROKE             DISPLAY

x                     026,           20
x                     027,           20
STO 9                 028,    44      9
RCL 8                 029,    45      8
RCL 3                 030,    45      3
x                     031,           20
RCL 1                 032,    45      1
-                     033,           30
RCL 7                 034,    45      7
x                     035,           20
g x≷y                 036,    43     34
g GTO 039             037, 43,33,039
x≷y                   038,           34
ENTER                 039,           36
ENTER                 040,           36
STO PV                041,    44     13
R/S                   042,           31
RCL 6                 043,    45      6
EEX                   044,           26
2                     045,            2
÷                     046,           10
÷                     047,           10
R/S                   048,           31
-                     049,           30
CHS                   050,           16
[f][R/S]
```

Memory used: P - 050, r - 20.

The program uses storage registers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. In addition, financial registers [n], [i], and [PV] are used.

PROGRAM FOR ALLOCATING BUYER'S AVAILABLE FUNDS OVER THE DOWN PAYMENT, PREPAIDS, PITI RESERVES, AND FINANCED-IN FIRST YEAR PMI PREMIUM

KEYSTROKE	DISPLAY
f (RPN)	
f R/S f PRGM	000,
1	001, 1
CHS	002, 16
STO PMT	003, 44 14
PV	004, 13
[1/x]	005, 22
RCL . 0	006, 45 48 0
%	007, 25
+	008, 40
RCL 5	009, 45 5
x	010, 20
RCL g i	011, 45,43 12
RCL . 0	012, 45 48 0
%	013, 25
+	014, 40
3	015, 3
6	016, 6
5	017, 5
0	018, 0
0	019, 0
÷	020, 10
RCL 6	021, 45 6
x	022, 20
+	023, 40
RCL 2	024, 45 2
RCL . 0	025, 45 48 0
%	026, 25
+	027, 40
1	028, 1
0	029, 0
0	030, 0
÷	031, 10
+	032, 40
STO 8	033, 44 8
RCL 3	034, 45 3
RCL 4	035, 45 4
+	036, 40
RCL 5	037, 45 5
x	038, 20
RCL 3	039, 45 3
RCL 9	040, 45 9
x	041, 20
+	042, 40
1	043, 1
2	044, 2
0	045, 0
0	046, 0
÷	047, 10
RCL 8	048, 45 8
+	049, 40
RCL 1	050, 45 1
x	051, 20
RCL 7	052, 45 7
+	053, 40
CHS	054, 16
RCL 0	055, 45 0
+	056, 40
1	057, 1
RCL 8	058, 45 8
-	059, 30
÷	060, 10
STO 8	061, 44 8
f R/S	

Memory used: P - 064, r - 20.

Appendix 9

PROGRAM FOR COMPUTING NET SALES PROCEEDS, OR
REQUIRED SALES PRICE, OR MAXIMUM COMMISSION RATE

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
f RPN f R/S f PRGM	000,	g [x=0]	044, 43 35
RCL 5	001, 45 5	g GTO 066	045, 43,33,066
g [x=0]	002, 43 35	R↓	046, 33
g GTO 005	003, 43,33,005	RCL 3	047, 45 3
g GTO 012	004, 43,33,012	+	048, 40
RCL n	005, 45 11	RCL 7	049, 45 7
STO 4	006, 44 4	+	050, 40
PMT	007, 14	1	051, 1
PMT	008, 14	RCL 1	052, 45 1
RCL 2	009, 45 2	%	053, 25
STO n	010, 44 11	x	054, 20
FV	011, 15	CHS	055, 16
CHS	012, 16	1	056, 1
RCL 7	013, 45 7	+	057, 40
g [x=0]	014, 43 35	÷	058, 10
g GTO 017	015, 43,33,017	STO 6	059, 44 6
g GTO 041	016, 43,33,041	RCL 4	060, 45 4
x≷y	017, 34	STO n	061, 44 11
RCL 1	018, 45 1	0	062, 0
g [x=0]	019, 43 35	STO FV	063, 44 15
g GTO 062	020, 43,33,062	RCL 6	064, 45 6
R↓	021, 33	g GTO 000	065, 43,33,000
1	022, 1	R↓	066, 33
RCL 1	023, 45 1	g GTO 068	067, 43,33,068
%	024, 25	RCL 3	068, 45 3
x	025, 20	+	069, 40
CHS	026, 16	RCL 7	070, 45 7
1	027, 1	+	071, 40
+	028, 40	CHS	072, 16
RCL 0	029, 45 0	RCL 0	073, 45 0
x	030, 20	+	074, 40
RCL 3	031, 45 3	RCL 0	075, 45 0
-	032, 30	÷	076, 10
+	033, 40	1	077, 1
STO 6	034, 44 6	0	078, 0
0	035, 0	0	079, 0
STO FV	036, 44 15	x	080, 20
RCL 4	037, 45 4	STO 6	081, 44 6
STO n	038, 44 11	0	082, 0
RCL 6	039, 45 6	STO FV	083, 44 15
g GTO 000	040, 43,33,000	RCL 4	084, 45 4
x≷y	041, 34	STO n	085, 44 11
CHS	042, 16	RCL 6	086, 45 6
RCL 1	043, 45 1	f R/S	

Memory used: P - 092, r - 20

PROGRAM FOR COMPUTING GRADUATED PAYMENT MORTGAGE SCHEDULE

This program is designed to compute the payment schedule necessary to amortize a graduated payment loan. The program is restricted to loans with twelve payments per year, with annual payment increases starting after the first twelve payments.

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
f RPN f P/R f PRGM	000,	RCL 1	028, 45 1
RCL i	001, 45 12	STO i	029, 44 12
STO 1	002, 44 1	1	030, 1
1	003, 1	2	031, 2
RCL 7	004, 45 7	STO n	032, 44 11
%	005, 25	0	033, 0
+	006, 40	STO FV	034, 44 15
1	007, 1	PV	035, 13
RCL 1	008, 45 1	STO x 3	036, 44 20 3
%	009, 25	RCL 4	037, 45 4
+	010, 40	RCL 2	038, 45 2
1	011, 1	1	039, 1
2	012, 2	2	040, 2
y^x	013, 21	x	041, 20
÷	014, 10	-	042, 30
1	015, 1	STO n	043, 44 11
-	016, 30	PV	044, 13
EEX	017, 26	1	045, 1
2	018, 2	RCL 7	046, 45 7
x	019, 20	%	047, 25
STO i	020, 44 12	+	048, 40
1	021, 1	RCL 2	049, 45 2
CHS	022, 16	y^x	050, 21
STO PMT	023, 44 14	x	051, 20
RCL 2	024, 45 2	1	052, 1
STO n	025, 44 11	RCL 1	053, 45 1
FV	026, 15	%	054, 25
STO 3	027, 44 3	+	055, 40

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
RCL 2	056, 45 2	1	72 1
1	057, 1	STO + 0	73 44 40 0
2	058, 2	RCL 3	74 45 3
x	059, 20	RCL 7	75 45 7
y^x	060, 21	%	76 25
÷	061, 10	+	77 40
STO + 3	062, 44 40 3	STO 3	78 44 3
RCL 5	063, 45 5	R/S	79 31
RCL 3	064, 45 3	RCL 2	80 45 2
÷	065, 10	RCL 0	81 45 0
R/S	066, 31	g x≷y	82 43 34
f 2	067, 42 2	g GTO 072	83 43,33,072
f RND	068, 42 14	f FIN	84 42 34
STO 3	069, 44 3	CLX	85 35
1	070, 1	f R/S	
STO 0	071, 44 0		

Required Information and Memory Storage Locations Used

1. Monthly interest rate, store in [i].
2. Number of years payments increase, store in memory register 2.
3. Loan term (months), store in memory register 4.
4. Loan amount, store in memory register 5.
5. Percentage annual payment growth, store in memory register 7.
6. Set END mode (g END).
7. Memory used: P - 085 r - 20.

Note: Look at steps 067 **(f 2)** and 068 **(f RND)**. Here we are causing the calculator's display to be set to two decimal places (f, 2), and immediately thereafter round to the number of places set (f, RND). The purpose of doing this is to produce a more accurate amortization schedule, though the difference without this procedure can be no more than one or two pennies. In this regard, the above program **may** produce results which differ by one or two pennies when compared with other published GPM programs.

If the user prefers to eliminate the rounding steps, delete steps 067 and 068. You will then move every step from lines 069 through 085 back by two lines. For example, "old" step 069 will be "new" step 067, and so forth. Also, you will change the GTO step by having the program GO TO "new" line 070, instead of "old" line 072.

PREPARING ANNUAL AMORTIZATION STATEMENT FOR THE GPM

Preparing the GPM's amortization statement with the HP 12C presents a special need which can best be met by using the program setforth below.

Graduated Payment Loan Amortization Program

KEYSTROKE	DISPLAY
f RPN f R/S f PRGM	000,
RCL 0	001, 45 0
f AMORT	002, 42 11
STO 1	003, 44 1
x≷y	004, 34
STO 6	005, 44 6
RCL n	006, 45 11
R/S	007, 31
RCL 1	008, 45 1
R/S	009, 31
RCL 6	010, 45 6
R/S	011, 31
RCL PV	012, 45 13
R/S	013, 31
RCL n	014, 45 11
RCL 7	015, 45 7
-	016, 30
g [x=0]	017, 43 35
g GTO 063	018, 43,33,063
RCL 2	019, 45 2
g [x=0]	020, 43 35
g GTO 001	021, 43,33,001
f AMORT	022, 42 11
STO 3	023, 44 3
x≷y	024, 34
STO 4	025, 44 4
RCL 5	026, 45 5
STO PMT	027, 44 14
RCL 0	28 45 0
f AMORT	29 42 11
STO + 3	30 44 40 3
x≷y	31 34
STO + 4	32 44 40 4
RCL n	33 45 11
R/S	34 31
RCL 3	35 45 3
R/S	36 31
RCL 4	37 45 4
R/S	38 31
RCL PV	39 45 13
R/S	40 31
RCL n	41 45 11
RCL 2	42 45 2
+	43 40
RCL 7	44 45 7
-	45 30
g [x=0]	46 43 35
g GTO 049	47 43,33,049
g GTO 019	48 43,33,019
RCL 5	49 45 5
STO PMT	50 44 14
RCL 2	51 45 2
f AMORT	52 42 11
STO 3	53 44 3
x≷y	54 34
STO 4	55 44 4

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
RCL n	056, 45 11	R/S	063, 31
R/S	057, 31	0	064, 0
RCL 3	058, 45 3	R/S	065, 31
R/S	059, 31	0	066, 0
RCL 4	060, 45 4	R/S	067, 31
R/S	061, 31	g GTO 000	068, 43,33,000
RCL PV	062, 45 13	f R/S	

Required Information and Memory Storage Locations Used

1. Input loan amount, negative-signed, into [PV].
2. Input monthly interest rate in [i].

3a. First year payment, store in [PMT] register.

3b. Payments number 2 through and including final year payment store in Mem. Reg. 5. If program is used to amortize a fixed payment loan with less than 12 payments in the first year, you will input the payment amount into both the [PMT] register and Mem. Reg. 5.

4a. Number of payments in first year, store in Mem. Reg. 0. For example, if the loan requires 4 installments of the first year's payment, enter "4" into Mem. Reg. 0; there will be 8 remaining payments of the same amount to be made in the second year of the loan, so enter "8" into Mem. Reg. 2. If 12 payments made in first year, enter "12" into Mem Reg 0, and enter "0" in Mem. Reg. 2.

4b. Remaining number of payments from first year, store in Mem. Reg. 2.

5. Total number of payments in amortization schedule, store in Mem. Reg. 7. This amount is not necessarily the loan's original term. Rather, it is the total number of payments which you plan to amortize.
6. Set END (g END) mode.
7. Program automatically amortizes final year of the GPM and displays "0".
8. **Caution:** You must make sure the [n] register is set to "0".
9. Memory used: P - 071, r - 20.

Summary of registers used:

Memory Registers: 0, 1, 2, 3, 4, 5, 6, and 7.

Financial Registers: [i], [PV], and [PMT].

Description: The program enables you to prepare an amortization schedule for a GPM. The payment schedule can consist of less than 12 payments in the first year, and therefore less than 12 payments in the final year amortized. For example, assume the loan requires four payments of $1,000 per month to be made in the first year, and as well requires a second year payment of $1,200 per month. The program will amortize four payments of $1,000 in the first year, plus the remaining eight in the second year, and then amortizes four payments of $1,200 per month during the second year, and so forth.

PROGRAM FOR COMPUTING NUMBER OF MONTHS TO AMORTIZE THE GEM

KEYSTROKE	DISPLAY			KEYSTROKE	DISPLAY		
f RPN f R/S f PRGM	000,			y^x	033,		21
RCL 0	001,	45	0	÷	034,		10
1	002,		1	1	035,		1
0	003,		0	-	036,		30
0	004,		0	÷	037,		10
STO 1	005,	44	1	RCL 3	038,	45	3
÷	006,		10	1	039,		1
1	007,		1	+	040,		40
+	008,		40	1	041,		1
RCL i	009,	45	12	2	042,		2
RCL 1	010,	45	1	CHS	043,		16
÷	011,		10	y^x	044,		21
STO 3	012,	44	3	CHS	045,		16
1	013,		1	1	046,		1
+	014,		40	+	047,		40
1	015,		1	RCL 3	048,	45	3
2	016,		2	÷	049,		10
y^x	017,		21	x	050,		20
÷	018,		10	RCL PMT	051,	45	14
RCL 2	019,	45	2	x	052,		20
y^x	020,		21	CHS	053,		16
1	021,		1	RCL PV	054,	45	13
-	022,		30	+	055,		40
RCL 0	023,	45	0	RCL 3	056,	45	3
RCL 1	024,	45	1	1	057,		1
÷	025,		10	+	058,		40
1	026,		1	RCL 2	059,	45	2
+	027,		40	1	060,		1
RCL 3	028,	45	3	2	061,		2
1	029,		1	x	062,		20
+	030,		40	y^x	063,		21
1	031,		1	RCL 3	064,	45	3
2	032,		2	x	065,		20

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
x	066, 20	RCL 3	081, 45 3
RCL 0	067, 45 0	1	082, 1
RCL 1	068, 45 1	+	083, 40
÷	069, 10	g LN	084, 43 23
1	070, 1	÷	085, 10
+	071, 40	CHS	086, 16
RCL 2	072, 45 2	RCL 2	087, 45 2
y^x	073, 21	1	088, 1
RCL PMT	074, 45 14	2	089, 2
x	075, 20	x	090, 20
÷	076, 10	+	091, 40
CHS	077, 16	STO n	092, 44 11
1	078, 1	ENTER	093, 36
+	079, 40	g GTO 000	094, 43,33,000
g LN	080. 43 23	f R/S	

Required Information and Memory Storage Locations Used

1. Input loan amount, positive-signed, in [PV].
2. Input monthly interest rate in [i].
3. Input first year monthly payment, positive-signed, in [PMT].
4. Store annual growth in monthly payment in Mem. Reg. 0.
5. Store number of years of increase in payment amount in Mem. Reg. 2. In many cases you will use a number of years of increase in payment amount which are considerably less than the term of the GEM. In cases where the payments will increase throughout the GEM's term, you will have to estimate the number of years of increase (likely starting with 12 and working up to 15, 16, 17, possibly 18, depending upon the annual interest rate used) and start with that amount as your estimated number of years of growth. This issue will be further clarified below.
6. Set END (g END) mode for all calculations. You are limited to computing the term of a GEM with end of the month payments.
7. The GEM's computed term may be recalled from the [n] register, and displays at termination of the program's operation.
8. Memory used: P - 099 r - 20.

Summary of registers used:

Memory Registers: 0, 1, 2, and 3. Registers 4, 5, and 6 are available.

Financial Registers: [n], [i], [PV].

PROGRAM FOR COMPUTING PAYMENT NECESSARY TO PRODUCE A REQUIRED YIELD ON A WRAPAROUND PAYMENT MORTGAGE

This program is designed to compute the wraparound mortgage loan payment necessary to produce the yield required by a lender. The program will handle a two mortgage complex wraparound case.

KEYSTROKE	DISPLAY			KEYSTROKE	DISPLAY		
f RPN f P/R f PRGM	000,			+	027,		40
1	001,		1	STO 7	028,	44	7
CHS	002,		16	RCL 3	029,	45	3
STO PMT	003,	44	14	STO n	030,	44	11
RCL 3	004,	45	3	FV	031,		15
STO n	005,	44	11	[1/x]	032,		22
PV	006,		13	RCL 6	033,	45	6
RCL 2	007,	45	2	x	034,		20
x	008,		20	RCL 7	035,	45	7
RCL 0	009,	45	0	+	036,		40
+	010,		40	STO 7	037,	44	7
RCL 1	011,	45	1	0	038,		0
-	012,		30	STO FV	039,	44	15
STO 7	013,	44	7	1	040,		1
1	014,		1	CHS	041,		16
CHS	015,		16	STO PMT	042,	44	14
STO PV	016,	44	13	RCL 4	043,	45	4
RCL 4	017,	45	4	STO n	044,	44	11
STO n	018,	44	11	PV	045,		13
0	019,		0	RCL 7	046,	45	7
STO PMT	020,	44	14	x≷y	047,		34
FV	021,		15	÷	048,		10
[1/x]	022,		22	STO PMT	049,	44	14
RCL 5	023,	45	5	0	050,		0
x	024,		20	STO 7	051,	44	7
CHS	025,		16	RCL PMT	052,	45	14
RCL 7	026,	45	7	f R/S			

Required Information and Memory Storage Locations Used

1. Wraparound mortgage amount store in memory register 0.
2. Underlying mortgage balance store in memory register 1.
3. Underlying mortgage payment store in memory register 2.
4. Lender's required monthly yield store in financial register [i].
5. Number of payments remaining on underlying loan store in memory register 3.
6. Wraparound mortgage term (months) store in memory register 4.
7. Balloon payment, if any, on wraparound mortgage store in memory register 5.
8. Balloon payment, if any, on underlying loan store in memory register 6.
9. Set END mode (g END).
10. Memory used: P - 057 r - 20.

Note: The program is designed to compute the required wraparound mortgage payment on the assumption that all payments occur at the end of the month. No other payment solution is possible with the program as written.

The program will handle a payment stream in which the last payment is not equal to those which precede it. You recall that you will almost always find that a loan's payment stream will have its final payment slightly higher or slightly lower than the other payments. This, of course, is linked to the effects of rounding the monthly payment when first calculated.

To adjust for payment rounding, the program treats the difference between the last payment and the one immediately preceding it as a **balloon payment.** For example, if the loan payment stream consists of 359 monthly payments of $1,000, followed by a final (360th) payment of $1,010, the difference between the two payments ($1,010 - $1,000 = $10) would be treated as a **positive balloon payment** on the loan.

In the alternative, if a loan's last payment is less than the monthly payment which immediately precedes it, we treat the difference as a **negative signed balloon payment.** For example, if the loan's payment schedule consisted of 359 payments of $862.83, followed by a final payment of $859.86, we would treat the difference of **-$2.97** as a **negative signed balloon payment.** Again, the difference would be entered and treated as a "balloon payment", appropriately signed.

When working with a complex wraparound loan where there are balloon payments for both the wraparound loan and the underlying loan as well as last payment issues for both loans, we simply **net** each loan's last payment difference with the balloon payment, if any, due on the loan. This procedure will become clearer when working through the two examples that follow.

PROGRAM FOR COMPUTING LEASE PAYMENT SCHEDULES WITH REGULAR OR ADVANCE PAYMENTS, WITH SECURITY DEPOSIT CAPABILITY

KEYSTROKE	DISPLAY			KEYSTROKE	DISPLAY		
f RPN f P/R f PRGM	000,			RCL 2	031,	45	2
RCL PV	001,	45	13	÷	032,		10
RCL 1	002,	45	1	RCL 0	033,	45	0
-	003,		30	+	034,		40
RCL FV	004,	45	15	RCL 3	035,	45	3
RCL 1	005,	45	1	x≷y	036,		34
-	006,		30	÷	037,		10
1	007,		1	f 2	038,	42	2
RCL i	008,	45	12	f RND	039,	42	14
%	009,		25	STO PMT	040,	44	14
x	010,		20	R/S	041,		31
STO 2	011,	44	2	RCL PMT	042,	45	14
1	012,		1	RCL 2	043,	45	2
+	013,		40	1	044,		1
RCL n	014,	45	11	+	045,		40
CHS	015,		16	RCL n	046,	45	11
y^x	016,		21	RCL 0	047,	45	0
x	017,		20	-	048,		30
-	018,		30	CHS	049,		16
STO 3	019,	44	3	y^x	050,		21
RCL 2	020,	45	2	CHS	051,		16
1	021,		1	1	052,		1
+	022,		40	+	053,		40
RCL n	023,	45	11	RCL 2	054,	45	2
RCL 0	024,	45	0	÷	055,		10
-	025,		30	RCL 0	056,	45	0
CHS	026,		16	+	057,		40
y^x	027,		21	x	058,		20
CHS	028,		16	RCL PV	059,	45	13
1	029,		1	-	060,		30
+	030,		40	RCL 1	061,	45	1

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
+	062, 40	÷	069, 10
RCL 2	063, 45 2	RCL 1	070, 45 1
1	064, 1	-	071, 30
+	065, 40	CHS	072, 16
RCL n	066, 45 11	STO 4	073, 44 4
CHS	067, 16	R/S	074, 31
y^x	068, 21	g GTO 000	075, 43,33,000
		f R/S	

Required Information and Memory Storage Locations Used

1. Total number of payments (lease term) store in [n].
2. Periodic yield (also called "implicit lease rate") store in [i]. For example if the lease requires monthly payments, the "periodic" yield will be the monthly interest rate. Similarly, if the payments are made quarterly, the "periodic yield" will be based upon the annual rate divided by four.
3. Asset value store positive signed in [PV].
4. Residual value (if any) store positive signed in [FV].
5. Number of advanced payments, store in Mem. Reg. 0. If lease requires only one advance payment, and therefore is a traditional "due annuity", store a "1" in memory register 0. If the lease requires more than one advance payment, store that number in memory register 0. If payments occur at the end of the period, make sure memory register 0 is empty.
6. Security deposit (if any) store positive signed in Mem. Reg. 1.
7. Recomputed residual value [FV] displays and can be recalled from memory register 4 after completion of the calculation.
8. Set END (g END) mode for all computations. (You adjust for a beginning of the period lease by inputting the number of advance payments into memory register 0.)
9. Memory used: P - 078, r - 20.

Summary of registers used:

Memory Registers: 0, 1, 2, 3, and 4.

Financial Registers: [n], [i], [PV], and [FV].

Lease payment may be recalled from the [PMT] register.

PROGRAM FOR COMPUTING LEASE PAYMENT SCHEDULES WITH REGULAR OR ONE ADVANCE PAYMENT, RESIDUAL VALUE CAPABILITY, AND PAYMENT STREAM INCREASING OR DECREASING BY A CONSTANT AMOUNT

KEYSTROKE	DISPLAY
f RPN f R/S f PRGM	000,
3	001, 3
STO 3	002, 44 3
1	003, 1
2	004, 2
STO n	005, 44 11
1	006, 1
CHS	007, 16
STO PMT	008, 44 14
PV	009, 13
1	010, 1
RCL i	011, 45 12
%	012, 25
x	013, 20
1	014, 1
+	015, 40
STO 7	016, 44 7
RCL 6	017, 45 6
CHS	018, 16
y^x	019, 21
RCL 4	020, 45 4
x	021, 20
CHS	022, 16
RCL 8	023, 45 8
+	024, 40
RCL PV	025, 45 13
÷	026, 10
STO PV	027, 44 13
RCL 7	028, 45 7
RCL n	029, 45 11
CHS	030, 16
y^x	031, 21
STO 0	032, 44 0
STO 1	033, 44 1
1	034, 1
STO + 1	035, 44 40 1
RCL 7	036, 45 7
2	037, 2
4	038, 4
STO 2	039, 44 2
CHS	040, 16
y^x	041, 21
STO + 1	042, 44 40 1
2	043, 2
x	044, 20
STO + 0	045, 44 40 0
1	046, 1
2	047, 2
STO + 2	048, 44 40 2
RCL 6	049, 45 6
RCL 2	050, 45 2
-	051, 30
g [x=0]	052, 43 35
g GTO 073	053, 43,33,073
RCL 7	054, 45 7
RCL 2	055, 45 2
y^x	056, 21
[1/x]	057, 22
STO + 1	058, 44 40 1
RCL 3	059, 45 3
1	060, 1
STO + 3	061, 44 40 3
x	062, 20
x	063, 20
STO + 0	064, 44 40 0
RCL n	065, 45 11
STO + 2	066, 44 40 2
RCL 2	067, 45 2
RCL 6	068, 45 6
-	069, 30
g [x=0]	070, 43 35
g GTO 073	071, 43,33,073
g GTO 049	072, 43,33,049
RCL PV	073, 45 13
RCL 0	074, 45 0
RCL 5	075, 45 5
x	076, 20
-	077, 30
RCL 1	078, 45 1
÷	079, 10
ENTER	080, 36
STO PMT	081, 44 14
f R/S	

Required Information and Memory Storage Locations Used

1. Monthly yield or monthly interest rate, store in financial register [i]. For example, if the annual yield on a lease is 14%, you enter the monthly yield by pressing: 14 [g] [i].
2. Value of the lease or amount of the loan, store positive signed in memory register 8.
3. Lease or loan term in months, store in memory register 6. Remember: The minimum acceptable term is 36 months. Thereafter, the term must increase by a whole number multiple of "12", such as "48", "60", etc.
4. Annual (constant amount) growth in the monthly payment, store in memory register 5.
5. Residual value of a lease, or balloon payment on a mortgage loan, store positive signed in memory register 4.
6. Set END (g END) mode if your payments occur at the end of the month. If they occur at the beginning of the month, set BEG (g BEG) mode. The program is capable of handling one and only one advance payment.
7. Memory used: P - 085, r - 20.

Summary of registers used:

Memory Registers: 0, 1, 2, 3, 4, 5, 6, 7, and 8.

Financial Registers set and utilized by the program, [n] and [PV].

Note:

You must make sure the [FV] register is clear before you start a problem.

You must input the monthly interest rate into the [i] financial register.

The lease or loan payment will display and can be recalled from the [PMT] register after you complete a computation.

Caution: Always verify the accuracy of a computation by running an internal rate of return (IRR) or net present value (NPV) test on the completed data. This calculation can be performed on the HP-12C Platinum using up to a maximum of thirty (30) unequal cash-flow groups.

PROGRAM FOR COMPUTING PRESENT VALUE OF STEP-UP OR STEP-DOWN CONSTANT RATIO CHANGE PER YEAR ANNUITY WITH RESIDUAL VALUE CAPABILITY

KEYSTROKE	DISPLAY			KEYSTROKE	DISPLAY		
f RPN f R/S f PRGM	000,			-	023,		30
RCL 0	001,	45	0	÷	024,		10
EEX	002,		26	RCL 1	025,	45	1
2	003,		2	x	026,		20
÷	004,		10	RCL 2	027,	45	2
STO 3	005,	44	3	EEX	028,		26
1	006,		1	2	029,		2
+	007,		40	÷	030,		10
RCL i	008,	45	12	1	031,		1
EEX	009,		26	+	032,		40
2	010,		2	RCL 4	033,	45	4
÷	011,		10	1	034,		1
STO 4	012,	44	4	+	035,		40
1	013,		1	÷	036,		10
+	014,		40	RCL n	037,	45	11
÷	015,		10	y^x	038,		21
RCL n	016,	45	11	CHS	039,		16
y^x	017,		21	1	040,		1
CHS	018,		16	+	041,		40
1	019,		1	÷	042,		10
+	020,		40	ENTER	043,		36
RCL 4	021,	45	4	STO PV	044,	44	13
RCL 3	022,	45	3	f R/S			

Required Information and Memory Storage Register Locations Used

1. Set END (g END) mode.
2. Growth or decay in NOI, expressed as a percentage, store in Mem. Reg. 0.
3. NOI or annual income stores in Mem. Reg. 1.
4. Annual growth or decay in value of the property, expressed as a percentage, store in Mem. Reg. 2. If there is no residual (reversion) value at the expiration of the holding period, store "-100" (100 CHS) in Mem. Reg. 2. If there is full recapture of the value of the property or leasehold interest, store a "0" in Mem. Reg. 2.
5. Number of years in holding period, store in [n] register.
6. Required annual yield--or discount rate--store in [i] register.

Memory used: P - 050, r - 20.

PROGRAM FOR COMPUTING OVERALL CAPITALIZATION RATE (RO)
BY MORTGAGE-EQUITY BUILD-UP TECHNIQUES: ONE LTV MODEL

```
KEYSTROKE           DISPLAY                 KEYSTROKE           DISPLAY

f RPN f R/S f PRGM  000,                    %                   034,          25
RCL i               001,     45   12        x                   035,          20
STO 8               002,     44    8        STO 6               036,     44    6
RCL n               003,     45   11        RCL 5               037,     45    5
STO 9               004,     44    9        x                   038,          20
1                   005,           1        1                   039,           1
CHS                 006,          16        RCL 2               040,     45    2
STO PV              007,     44   13        %                   041,          25
PMT                 008,          14        x                   042,          20
1                   009,           1        +                   043,          40
2                   010,           2        x                   044,          20
x                   011,          20        RCL 6               045,     45    6
STO 4               012,     44    4        RCL 4               046,     45    4
RCL 3               013,     45    3        x                   047,          20
g n                 014,     43   11        +                   048,          40
FV                  015,          15        RCL 6               049,     45    6
RCL PV              016,     45   13        CHS                 050,          16
÷                   017,          10        1                   051,           1
1                   018,           1        +                   052,          40
+                   019,          40        1                   053,           1
STO 5               020,     44    5        RCL 1               054,     45    1
1                   021,           1        %                   055,          25
CHS                 022,          16        x                   056,          20
STO FV              023,     44   15        x                   057,          20
0                   024,           0        +                   058,          40
STO PV              025,     44   13        STO 7               059,     44    7
RCL 1               026,     45    1        0                   060,           0
STO i               027,     44   12        STO FV              061,     44   15
RCL 3               028,     45    3        RCL 8               062,     45    8
STO n               029,     44   11        STO i               063,     44   12
PMT                 030,          14        RCL 9               064,     45    9
CHS                 031,          16        STO n               065,     44   11
1                   032,           1        RCL 7               066,     45    7
RCL 0               033,     45    0        f R/S
```

Required Information and Memory Storage Locations Used

1. Set END (g END) mode.
2. Input monthly mortgage interest rate in [i].
3. Input mortgage term (months) in [n].
4. Loan-to-Value Ratio (LTV), as a percent, store in Mem. Reg. 0.
5. Required Equity Yield (YE) store in Mem. Reg. 1.
6. Change in value of property (ΔVO) over holding period, STO Mem. Reg. 2.
7. Holding period (in years), store in Mem. Reg. 3.
8. % Reduction in Mortgage can be recalled from Mem. Reg. 5.
9. Overall Capitalization Rate (RO) displays and stores in Mem. Reg. 7.

Memory used: P - 071, r - 20.

PROGRAM FOR COMPUTING VALUE OF THE REQUIRED EQUITY (VE) USING A DISCOUNTED CASH-FLOW MODEL: ONE MORTGAGE MODEL

```
KEYSTROKE           DISPLAY                 KEYSTROKE           DISPLAY

f RPN f R/S f PRGM  000,                    x                   034,         20
0                   001,          0         RCL PV              035,     45  13
STO FV              002,     44  15         CHS                 036,         16
RCL n               003,     45  11         +                   037,         40
STO 9               004,     44   9         STO 8               038,     44   8
RCL i               005,     45  12         RCL 2               039,     45   2
STO . 0             006,  44 48   0         g n                 040,     43  11
1                   007,          1         FV                  041,         15
RCL 1               008,     45   1         CHS                 042,         16
%                   009,         25         RCL 7               043,     45   7
x                   010,         20         x                   044,         20
STO 6               011,     44   6         RCL 8               045,     45   8
1                   012,          1         +                   046,         40
+                   013,         40         1                   047,          1
RCL 2               014,     45   2         RCL 5               048,     45   5
CHS                 015,         16         %                   049,         25
y^x                 016,         21         x                   050,         20
STO 7               017,     44   7         1                   051,          1
CHS                 018,         16         +                   052,         40
1                   019,          1         RCL 2               053,     45   2
+                   020,         40         y^x                 054,         21
RCL 6               021,     45   6         RCL 7               055,     45   7
÷                   022,         10         x                   056,         20
RCL 0               023,     45   0         CHS                 057,         16
ENTER               024,         36         1                   058,          1
PMT                 025,         14         +                   059,         40
PMT                 026,         14         ÷                   060,         10
f RND               027,     42  14         STO 4               061,     44   4
PMT                 028,         14         RCL 9               062,     45   9
RCL PMT             029,     45  14         STO n               063,     44  11
1                   030,          1         RCL . 0             064,  45 48   0
2                   031,          2         STO i               065,     44  12
x                   032,         20         RCL 4               066,     45   4
-                   033,         30         f P/R
```

Required Information and Memory Storage Locations Used

1. Set END (g END) mode.
2. Input monthly mortgage interest rate in [i].
3. Input mortgage term (months) in [n].
4. Value of Mortgage (VM), negative-signed, stores in [PV].
5. Net Operating Income (NOI) store in Mem. Reg. 0.
6. Required Equity Yield (YE) store in Mem. Reg. 1.
7. Holding period (in years), store in Mem. Reg. 2.
8. Annual <u>compound</u> growth in Value of the Property, store in Mem. Reg 5.
9. Value of the Property (V) displays and stores in Mem. Reg. 4.

Memory used: P - 071, r - 20

Use with HP-12C Platinum 25th Anniversary Edition

PROGRAM FOR COMPUTING VALUE OF THE REQUIRED EQUITY (VE)
USING A DISCOUNTED CASH-FLOW MODEL: ONE MORTGAGE MODEL

KEYSTROKE	DISPLAY	KEYSTROKE	DISPLAY
f RPN f R/S f PRGM	000,	CHS	035, 16
0	001, 0	RCL 3	036, 45 3
STO FV	002, 44 15	x	037, 20
RCL n	003, 45 11	RCL PV	038, 45 13
STO 9	004, 44 9	CHS	039, 16
RCL i	005, 45 12	+	040, 40
STO . 0	006, 44 48 0	STO 8	041, 44 8
1	007, 1	RCL 2	042, 45 2
RCL 1	008, 45 1	g n	043, 43 11
%	009, 25	FV	044, 15
x	010, 20	CHS	045, 16
STO 6	011, 44 6	RCL 7	046, 45 7
1	012, 1	x	047, 20
+	013, 40	RCL 8	048, 45 8
RCL 2	014, 45 2	+	049, 40
CHS	015, 16	1	050, 1
y^x	016, 21	RCL 5	051, 45 5
STO 7	017, 44 7	%	052, 25
CHS	018, 16	x	053, 20
1	019, 1	1	054, 1
+	020, 40	+	055, 40
RCL 6	021, 45 6	RCL 2	056, 45 2
÷	022, 10	y^x	057, 21
STO 3	023, 44 3	RCL 7	058, 45 7
0	024, 0	x	059, 20
STO PMT	025, 44 14	CHS	060, 16
PMT	026, 14	1	061, 1
f RND	027, 42 14	+	062, 40
PMT	028, 14	÷	063, 10
RCL PMT	029, 45 14	STO 4	064, 44 4
1	030, 1	RCL 9	065, 45 9
2	031, 2	STO n	066, 44 11
x	032, 20	RCL . 0	067, 45 48 0
RCL 0	033, 45 0	STO i	068, 44 12
-	034, 30	RCL 4	069, 45 4
		f P/R	

Required Information and Memory Storage Locations Used

1. Set END (g END) mode.
2. Input monthly mortgage interest rate in [i].
3. Input mortgage term (months) in [n].
4. Value of Mortgage (VM), negative-signed, stores in [PV].
5. Net Operating Income (NOI) store in Mem. Reg. 0.
6. Required Equity Yield (YE) store in Mem. Reg. 1.
7. Holding period (in years), store in Mem. Reg. 2.
8. Annual compound growth in Value of the Property, store in Mem. Reg 5.
9. Value of the Property (V) displays and stores in Mem. Reg. 4.

Memory used: P - 071, r - 20

PROGRAM FOR DETERMINING VALUE BY MORTGAGE-EQUITY BUILD-UP:
DEBT SERVICE COVERAGE RATIO (DSCR) CONSTRAINT

KEYSTROKE	DISPLAY			KEYSTROKE	DISPLAY		
f RPN f R/S f PRGM	000,			x	036,		20
0	001,		0	STO - 0	037,	44 30	0
STO FV	002,	44	15	RCL 6	038,	45	6
RCL n	003,	45	11	CHS	039,		16
STO 8	004,	44	8	1	040,		1
1	005,		1	+	041,		40
CHS	006,		16	RCL 7	042,	45	7
STO PV	007,	44	13	÷	043,		10
PMT	008,		14	RCL PMT	044,	45	14
1	009,		1	1	045,		1
2	010,		2	2	046,		2
x	011,		20	x	047,		20
RCL 3	012,	45	3	CHS	048,		16
x	013,		20	RCL 1	049,	45	1
[1/x]	014,		22	+	050,		40
RCL 1	015,	45	1	x	051,		20
x	016,		20	STO + 0	052,	44 40	0
STO 0	017,	44	0	RCL 5	053,	45	5
CHS	018,		16	EEX	054,		26
STO PV	019,	44	13	2	055,		2
PMT	020,		14	÷	056,		10
RCL 4	021,	45	4	1	057,		1
g n	022,	43	11	+	058,		40
FV	023,		15	RCL 4	059,	45	4
RCL 2	024,	45	2	y^x	060,		21
EEX	025,		26	RCL 6	061,	45	6
2	026,		2	x	062,		20
÷	027,		10	CHS	063,		16
STO 7	028,	44	7	1	064,		1
1	029,		1	+	065,		40
+	030,		40	STO ÷ 0	066,	44 10	0
RCL 4	031,	45	4	RCL 8	067,	45	8
CHS	032,		16	STO n	068,	44	11
y^x	033,		21	RCL 0	069,	45	0
STO 6	034,	44	6	f R/S			
RCL FV	035,	45	15				

Required Information and Memory Storage Locations Used

1. Set END mode (g END) <u>before</u> operating the program.
2. Input mortgage term (months) in [n].
3. Input monthly mortgage interest rate in [i].
4. Net Operating Income (NOI) store in Mem. Reg. 1
5. Required Equity Yield (YE) store in Mem. Reg. 2.
6. Debt Service Coverage Ratio (DSCR) store in Mem. Reg. 3
7. Holding period (in years), store in Mem. Reg. 4.
8. Annual compound growth in value of the property store in Mem. Reg. 5.

9. Value of the property displays/can be recalled from Mem. Reg. 0.
10. Maximum mortgage loan can be recalled from PV register.
11. Monthly mortgage loan payment can be recalled from PMT register.

Memory used: P - 071, r - 20.

Comment

It is traditional for the lender to limit the debt service coverage ratio (DSCR) based upon an investment property's projected first year net operating income (NOI). This method (also considered an "underwriter's method") generally does not take into account projected future growth in NOI when determining the amount of the maximum mortgage loan with a DSCR constraint. The program in this section follows this procedure.

The program was designed primarily to solve for the Value of the Property (V) using a DSCR constraint. Therefore, the higher the input for the DSCR, the lower will be the Value (V) generated by the program, and vice versa.

Appraisal Insert

A clarification of an issue with respect to handling projected growth in value of a property over its holding period when using my HP-12C (or HP 17BII/HP 19BII, for that matter!) algorithms.

Please note that my HP-12C appraisal routines use ANNUAL GROWTH or projected ANNUAL GROWTH IN VALUE of a property over the/a holding period. For example, I may use 4% a year, 5%, and you can use use an annual decay, such as **-3%**, etc. The "problem", I suppose, for those of you who will be taking appraisal classes is that **growth in value of a property** may be given as TOTAL GROWTH over the projected holding period. For example, 40% over a 5 year holding period; 35% over a 6 year holding period, and so forth.

Thus, if you are taking a course where growth in value of a property is given as a **total or gross amount** over the complete holding period you will need to convert that rate to a rate per year in order to be able to use my algorithms. Here is how it is done (it is truly no problem whatsoever!):

Let's assume that the projected holding period (or projected analysis period) is 5 years. Let's also assume that the projected TOTAL CHANGE IN VALUE or GROWTH IN VALUE OF THE PROPERTY OVER THE 5 YEAR HOLDING PERIOD is 40%. To convert to an annual (compound!) rate of growth, do the following:

KEYSTROKES	DISPLAY	WHAT WE ARE DOING
f 5		I set 5 decimal places. Could just as well set 2 or 3 or whatever number of places you like!
f CLX	0.00000	I cleared all registers.
g 8	0.00000	I set device to END MODE.
5 n	5.00000	5 year holding period in "n" register.
1 INPUT	1.00000	I need to work with "1" for starters!
40 [%] [+]	1.40000	I added 40% to "1"; result is 1.40!
FV	1.40000	Input 1.40 into the Future Value Register.
1 CHS PV	-1.00000	Initial value of the property into PV, negative signed because if we did not do this the problem cannot be solved as entered.
[i]	**6.96104**	This is equivalent annual growth rate.

What the result tells us is that if a problem states a total projected growth in value of 40% over a 5 year holding period, the equivalent annual compound rate of growth would be 6.96104 percent (actually, "6.961037573%").

This also tells us that if, for example, we were to project the initial value--or purchase price--of a property forward at an annual compound growth of "6.96104%" over a 5 year holding period, we should wind up with a total growth in value of 40%. Let's prove this by projecting forward $1.00 at a compound annual growth rate of "6.961037573%" for 5 years. But first, let's do it by the math!

KEYSTROKES	DISPLAY	WHAT WE ARE DOING
1 ENTER	1.00000	Call this the initial value of the property!
RCL [i]	6.96104	Recalled annual compound growth rate. **REMEMBER: YOUR CALCULATOR HOLDS "i" TO 10 PLACE ACCURACY.**
100 ÷	0.06961	Converted growth rate to a decimal.
1 +	1.06961	Added "1" to annual growth rate.
5 y^x	1.40000	Raised 1.06961... to 5th power.
1 -	0.40000	Subtract "1": This leaves the total growth in value of the property over the holding period!
100 x	**40.00000**	Converted the decimal to a percent.

The result shows that compounding the initial value of a property at 6.96104...% over a 5 year holding period produces a total growth of 40%.

Of course, if we know the growth rate per year and holding period we can use the TVM registers to project the value of the property at the expiration of the holding period.

KEEP THE ANNUAL GROWTH RATE IN YOUR [i] register; don't change anything!

KEEP THE HOLDING PERIOD IN YOUR [n] REGISTER; don't change anything!

HOWEVER, ASSUME THAT THE INITIAL VALUE OF THE PROPERTY IS $100,000!

Clearly, if a $100,000 property is to increase in value by 40% at the end of the holding period, its "terminal" or final value should be $140,000. Let's check this by doing the keystrokes!

KEYSTROKES	DISPLAY	WHAT WE ARE DOING
		REMEMBER: THE "i" REGISTER ALREADY HOLDS 6.961037573 AND THE "n" REGISTER HOLDS 5!
f 2	40.00	Getting all lined up here!
100,000 CHS PV	-100,000.00	Initial value into PV register.
FV	**140,000.00**	It works!

Use only annual compound growth rate with the equations and HP-12C programs I published in my **Professional Real Estate Problem Solving** books as currently published.

Any questions? (jatirone@yahoo.com)

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ALSO ADMITTED IN ARIZONA

Dear Purchaser of my Professional Real Estate Problem Solving Using the HP 12C:

Thanks so much for acquiring my HP-12C real estate book. I appreciate this very much.

Please note the following:

MORTGAGE INTEREST DEDUCTIBILITY CAUTION

PLEASE NOTE: The IRS currently allows for deductibility of interest on mortgage loans up to a maximum of one million ($1,000,000.00) dollars. Thus, in the event that you should try to estimate the after-tax payment cost of a mortgage loan where the loan amount exceeds $1,000,000.00, the results will be incorrect if you use the program and technique in my book. Thus, do not use the technique or program in this book to estimate the monthly after-tax payment cost to the borrower/whoever where the amount of the mortgage loan exceeds one million ($1,000,000.00) dollars.

PLEASE NOTE FURTHER: If you or a client or customer (borrower or whoever) is subject to the Alternative Minimum Tax (AMT), the after-tax payment cost of a loan or mortgage to that type of borrower cannot be estimated with the HP-12C Programs or techniques covered in this book.

Thanks again for acquiring my book and have a fine day.

Sincerely,

John A. Tirone

JOHN A. TIRONE

PLATINUM#1

LOAN PAY OFF EXAMPLE: INTEREST COMPOUNDED DAILY; PAYMENTS MADE AT IRREGULAR INTERVALS.

Problem: Calculate the loan pay off amount due on 11/15/2006.

LOAN AMOUNT	$75,000.00
LOAN ORIGINATION DATE	5/11/2006
INTEREST RATE, COMPOUNDED DAILY	12%
1st payment made 6/27/2006	$750.00
2nd payment made 7/18/2006	$750.00
3rd payment made 8/16/2006	$750.00
4th payment made 9/02/2006	$750.00
5th payment made 10/04/2006	$750.00
LOAN PAY OFF DUE ON 11/15/2006	?

OF DAYS BETWEEN DATES

5/11/06 to 6/27/06 = 47
6/27/06 to 7/18/06 = 21
7/18/06 to 8/16/06 = 29
8/16/06 to 9/02/06 = 17
9/02/06 to 10/04/06 = 32
10/04/06 to 11/15/06 = 42

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Clear all registers; set 2 decimal places and END mode.	f CLX f 2 g END	0.00
Enter daily interest rate.	12 ENTER 365 ÷ i	0.03
Enter loan amount in PV.	75,000 CHS PV	-75,000.00
Calculate 1st payment days-between-dates (5/11/06 to 6/27/06).	5.112006 ENTER 6.272006 g EEX	5.11 47.00
Enter # of days into "n" register.	n	47.00
Calculate amount due @ 6/27/06;	FV	76,167.71
round; reduce by 1st payment.	f RND 750 -	75,417.71
Change sign and enter into PV.	CHS PV	-75,417.71
Calculate 2nd payment days-between-dates (6/27/06 to 7/18/06).	6.272006 ENTER 7.182006 g EEX	6.27 21.00
Enter # of days into "n" register.	n	21.00
Calculate amount due @ 7/18/06;	FV	75,940.12
round; reduce by 2nd payment.	f RND 750 -	75,190.12
Change sign and enter into PV.	CHS PV	-75,190.12
Calculate 3rd payment days-between-dates (7/18/06 to 8/16/06).	7.182006 ENTER 8.162006 g EEX	7.18 29.00
Enter # of days into "n" register.	n	29.00
Calculate amount due @ 8/16/06;	FV	75,910.31
round; reduce by 3rd payment.	f RND 750 -	75,160.31
Change sign and enter into PV.	CHS PV	-75,160.31
Calculate 4th payment days-between-dates (8/16/06 to 9/02/06).	8.162006 ENTER 9.022006 g EEX	8.16 17.00

PROCEDURE	KEYSTROKE/INPUT	DISPLAY
Enter # of days into "n" register.	n	17.00
Calculate amount due @ 9/02/06;	FV	75,581.49
round; reduce by 4th payment.	f RND 750 -	74,831.49
Change sign and enter into PV.	CHS PV	-74,831.49
Calculate 5th payment days-between-	9.022006 ENTER	9.02
dates (9/02/06 to 10/04/06).	10.042006 g EEX	32.00
Enter # of days into "n" register.	n	32.00
Calculate amount due @ 10/04/06;	FV	75,622.78
round; reduce by 5th payment.	f RND 750 -	74,872.78
Change sign and enter into PV.	CHS PV	-74,872.78
Calculate final payment days-	10.042006 ENTER	10.04
between-dates (10/04/06 to 11/15/06).	11.152006 g EEX	42.00
Enter # of days into "n" register.	n	42.00
Calculate amount due on 11/15/06.	**FV**	**75,913.64**

Conclusion: The balance owing on the loan on 11/15/2006 is $75,913.64.

Problem: Let's prove the IRR of the cash-flows equals 12%.

KEYSTROKES	Comments
f CLX	Clears all registers.
75,000 CHS g CFo	Amount of loan, negative-signed, into CFo.
0 g CFj 46 g Nj	46 days precede the 1st payment.
750 g CFj	1st payment, $750 paid on the 47th day.
0 g CFj 20 g Nj	20 days precede the 2nd payment.
750 g CFj	2nd payment, $750 paid on the 68th day.
0 g CFj 28 g Nj	28 days precede the 3rd payment.
750 g CFj	3rd payment, $750 paid on the 97th day.
0 g CFj 16 g Nj	16 days precede the 4th payment.
750 g CFj	4th payment, $750 paid on the 114th day.
0 g CFj 31 g Nj	31 days precede the 5th payment.
750 g CFj	5th payment, $750 paid on the 146th day.
0 g CFj 41 g Nj	41 days precede the final payment.
75,913.64 g CFj	Final payment made on the 188th day.
f IRR (running)	Calculating the internal rate of return.
365 x	Convert daily IRR result to annual result.
f 9	Show all decimal places.
Displays: 11.99999692	
f 2	Show 2 decimal places.
Displays: 12.00	

Comment: The cash-flow (payment) sequence produces an internal rate of return of 12%, which is the loan interest rate we started with. Thus, the final payment--the loan pay off--we arrived at ($75,913.64) is correct.

Professional Real Estate Problem Solving Usng the HP 12C
FUTURE VALUE PRACTICE PROBLEMS
Page 31 Solution Set

Problem A:

An investor has an opportunity to invest $15,000 for five years. Assuming the investment is not taxed during the holding period, what is the value of the investment under the following conditions:

Interest rate of 7%
Compounding:
(1) Annually;
Ans: FV = $21,038.28

Solution:

f CLX f 2
5 n 7 [i]
15,000 CHS PV
FV
Displays: 21,038.28

(2) Quarterly;
Ans: $21,221.67

Solution:

f CLX
5 ENTER 4 x [n]
7 ENTER 4 [÷] [i]
15,000 CHS PV
FV
Displays: 21,221.67

(3) Daily;
Ans: $21,285.30

Solution:

f CLX 15,000 CHS PV
5 ENTER 365 x [n]
7 ENTER 365 [÷] [i]
FV
Displays: 21,285.30

Growth rate of 9%
Compounding:
(1) Semi-annual;
Ans: FV = $23,294.54

Solution:

f CLX f 2 15,000 CHS PV
5 ENTER 2 x [n]
9 ENTER 2 [÷] [i]
FV
Displays: 23,294.54

(2) Daily;
Ans: $23,523.38

Solution:

f CLX
5 ENTER 365 x [n]
9 ENTER 365 [÷] [i]
15,000 CHS PV
FV
Displays: 23,523.38

(3) Annually;
Ans: $23,079.36

Solution:

f CLX
5 [n] 9 [i]
15,000 CHS PV
FV
Displays: 23,079.36

Decay rate of 3%
Compounding:
(1) Annually;
Ans: FV = $12,881.01

Solution:

f CLX f 2
5 [n] 3 CHS [i]
15,000 CHS PV
FV
Displays: 12,881.01

Problem B:

Solve for the unknown financial variable in the following problems:

PV of $200,000
FV of $322,102
Annual compounding
5 year deposit term
Annual Interest Rate?
Ans: 10%

FV of $162,046.95
Monthly compounding
1 year deposit term
7.75% interest rate
Amount of deposit?
Ans: PV of $150,000

PV of $150,000.00
FV of $162,086.00
1 year deposit term
7.75% interest rate
Compounding periods?
Ans: See below.

Solution:	**Solution:**	**Solution:**
f CLX	f CLX	See below.
5 [n]	1 [g] [n]	
200,000 CHS PV	7.75 [g] [i]	
322,102 FV	162,046.95 FV	
[i] (running)	PV	
Displays: <u>**10.00**</u>	**Displays:** <u>**-150,000.00**</u>	

The problem above in which we seek a solution for the <u>number</u> <u>of</u> <u>compounding</u> <u>periods</u> per year <u>does</u> <u>not</u> have a direct solution in finance mathematics, nor has a solution method for this type of problem been preprogrammed into your HP 12C calculator. Though we could write a program to solve this on the HP 12C, it is best to utilize the substantial computing power programmed into the equation SOLVER of the HP 17BII and HP 19BII (including the HP 17B, HP 19B, and the HP 18C) Financial Calculators--when properly programmed--to solve this type of problem.

The answer to the problem is that the 7.75% annual interest rate is being compounded daily. Therefore, we have 365 compounding periods. To solve this on the above noted Hewlett-Packard products, you would enter the following equation into the device's equation SOLVER:

$$\mathbf{FV = PV \times (1 + I\%YR \div P/YR \div 100)\hat{}(\#YRS \times P/YR)}$$

After inputting the equation, the keystroke procedure would be:

162,086 |FV| 150,000 |PV| 7.75 |I%YR| 1 |#YRS| |P/YR|

Set to zero (0) decimal places, the display will show: **P/YR = 365.**

Verification:

Let's verify the number of compounding periods equals 365. Implicit in this is that we are paying interest--or the growth rate is applied--daily. Thus, we prorate the growth rate (7.75%) over 365 days (time periods). The number of time periods, the number of <u>compounding</u> periods must also be 365. Let's verify that below.

HP-12C Verification Keystrokes:

f CLX f 2	Clear the calculator; set 2 decimal places.
7.75 ENTER	Input the annual growth rate.
365 [÷] [i]	Prorate growth rate over 365 compounding periods.
150,000 CHS PV	Enter the present value.
162,086.00	Enter the future value.
[n]	Verify "n" must be 365.
Displays: <u>**365.00**</u>	

Comment: The "truth be told", if we followed the "Cash-Flow Sign Convention" the above equation would be written as you see below. This "convention" is really a non-math explanation for <u>forcing</u> financial equations "to zero".

$$\mathbf{FV - PV \times (1 + I\%YR \div P/YR \div 100)\hat{}(\#YRS \times P/YR) = 0}$$

PERIODIC (End of Period) PAYMENT PRACTICE PROBLEMS

Page 40 Solution Set

Preliminary: Before solving the following problems let's first set your HP-12C--"Platinum" or GOLD case!--to END mode, two decimal places.

KEYSTROKES: [g] (END) [f] 2

Problems: Using the data in problems **A, B,** and **C,** solve for the periodic payment produced from the data.

Problem A
Loan = $200,000
Term = 30 years
Monthly payments = ?
Interest rate: 9%/yr
Ans: $1,609.25

Problem B (30 Yr Term)
Price = $200,000
LTV Ratio = 80%
Monthly payments = ?
Interest rate: 10%/yr
Ans: $1,404.11

Problem C
Loan = $160,000
Term = 20 years
Quarterly payments = ?
Interest rate: 9.5%/yr
Ans: $4,486.04

Solution A:

f CLX
200,000 CHS PV
30 [g] [n]
9 [g] [i]
PMT
Displays: **<u>1,609.25</u>**

Solution B:

f CLX
30 [g] [n]
200,000 ENTER 80 [%]
CHS PV
10 [g] [i] PMT
Displays: **<u>1,404.11</u>**

Solution C:

f CLX
9.5 ENTER 4 [÷] [i]
160,000 CHS PV
20 ENTER 4 [x] [n]
PMT
Displays: **<u>4,486.04</u>**

Problem: Solve for the unknown variable(s) in the following problems:

Problem D
Loan = $100,000
Monthly PMT = $1,101.09
Term = 240 months
Last PMT = $1,097.27
Annual Interest Rate = ?
Ans: 12%

Problem E
Loan = $200,000
Term = 30 years
Interest rate: 8.25%/yr
Monthly PMT = ?
Ans: $1,502.53

Problem F
Term = 360 months
Monthly PMT = $877.57
Interest rate: 10%/yr
Loan Amount = ?
(To nearest $1.00)
Ans: $100,000

Solution D:

f CLX
100,000 CHS PV
1,101.09 PMT
1,097.27 RCL PMT [-]
FV 240 [n]
[i] (running)
RCL [g] [i]
Displays: **<u>12.00</u>**

Solution E:

f CLX
200,000 CHS PV
30 [g] [n]
8.25 [g] [i]
PMT
Displays: **<u>1,502.53</u>**

Solution F:

f CLX
360 [n]
877.57 PMT
10 [g] [i]
PV
Displays: **<u>-99,999.82</u>**

Note: Doing "RCL" "g" "i" causes the HP-12C to multiply whatever <u>periodic interest</u> rate is stored in the "i" register to be multiplied times "12". Thus, this procedure <u>only</u> works when we are dealing with a <u>monthly payment loan</u>. On the "flip-side", doing "RCL" "g" "n" causes the calculator to

divide whatever is in the "n" register by 12.

Problem G (i = 9%/yr)
Loan = $65,000
Monthly PMT = $523
Last PMT = $531.61
Term = ?
Ans: 360 months

Solution G:

f CLX
9 [g] [i]
65,000 CHS PV
523 PMT
531.61 RCL PMT [-]
FV [n]
Displays: 360.00

Problem H
Term = 180 months
Interest rate: 11%/yr
Monthly PMT = $523
Loan Amount = ?
Ans: $46,014.55

Solution H:

f CLX
180 [n]
11 [g] [i]
523 PMT
PV
Displays: -46,014.55

Problem I
Loan = $220,000
Term = 240 months
Monthly PMT = $2,315.90
Interest Rate/yr = ?
Ans: 11.30%

Solution I:

f CLX
220,000 CHS PV
240 [n]
2,315.9 PMT
[i] (running)
RCL [g] [i]
Displays: 11.30

Problem J

Interest rate: 14%/yr
Monthly PMT = $998.81
Loan = $75,000
Term = ?
Last Payment = ?
Ans: n = 180
Last Payment = $996.41

Solution J:

f CLX
14 [g] [i]
998.81 PMT
75,000 CHS PV
[n]
Displays: 180.00
FV
RCL PMT [+]
Displays: 996.41

Problem K

Interest rate: 11%/yr
Loan = $325,000
Term = 180 months
Quarterly payments = ?
Ans: $11,121.51
Semiannual PMTS = ?
Ans: $22,361.75

Solution K:

f CLX
11 ENTER 4 [÷] [i]
180 ENTER 12 [÷]
4 [x] n (Note: n=60.00)
325,000 CHS PV PMT
Displays: 11,121.51
11 ENTER 2 [÷] [i]
180 ENTER 12 [÷] 2 [x]
[n] (Note: n=30.00)
PMT
Displays: 22,361.75

Problem L

Interest rate: 11%/yr
Quarterly PMT = $3,422
Loan = $100,000
Term in years = ?
Ans: 15
Last Payment = ?
Ans: $3,422.26

Solution L:

f CLX
11 ENTER 4 [÷] [i]
3,422 PMT
100,000 CHS PV
[n] (Note: n=60.00)
FV FV (Note: FV=0.26)
RCL PMT [+]
Displays: 3,422.26
[f] CLX

Note: In solving for the Last Payment, we round the periodic payment, enter it into the PMT register, solve for the "residual" Future Value, and add the computed FV to the periodic payment. If, however, we know the amount of the last payment and the periodic payment, and seek another financial variable, such as the interest rate or term, we must first compute the **residual** that will be entered into the FV register. To do this you subtract the **periodic payment** from the **last payment.** The difference, whether positive or negative signed, is then entered into the FV register. To complete the solution, input the remaining known variables and solve for the unknown variable. (This method was used to solve for the interest rate in **Problem D.**)

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More Practice!

Problem M
Loan = $200,000
Term = 10 years
Balloon PMT = $100,000
Monthly payments = ?
Interest rate: 6.25%/yr
Ans: $1,643.63

Solution M:

f CLX
200,000 CHS PV
10 [g] [n]
6.25 [g] [i]
100,000 FV PMT
Displays: 1,643.63

Problem N
Price = $450,000
LTV Ratio = 80%
Term = 300 months
Monthly payments = ?
Interest rate: 7%/yr
Ans: $2,544.41

Solution N:

f CLX
300 [n]
450,000 ENTER 80 [%]
CHS PV
7 [g] [i] PMT
Displays: 2,544.41

Problem O
Loan = $160,000
Term = ? months
Interest rate: 6.5%/yr
Monthly PMT = $1,192.92
Ans: 240 months

Solution O:

f CLX
6.5 [g] [i]
160,000 CHS PV
1,192.92 PMT
[n]
Displays: 240.00

Problem P
Loan Amount = ?
Monthly PMT = $2,047.39
Term = 240 months
Interest rate = 10 7/8%
Ans: $200,000

Solution P:

f CLX
7 ENTER 8 [÷] 10 [+]
[g] [i]
2,047.39 PMT
240 [n]
PV
Displays: -200,000.00

Problem Q
Loan = $200,000
Term = 7 years
Interest rate: 7%/yr
Monthly PMT = $2,092.60
Balloon Payment = ?
Ans: $100,000.14

Solution Q:

f CLX
200,000 CHS PV
7 [g] [n]
7 [g] [i]
2,092.60 PMT
FV
Displays: 100,000.14

Problem R
Term = 240 months
Monthly PMT = $3,504.35
Interest rate: 7%/yr
Loan Amount = ?
(To nearest $1.00)
Ans: $452,000

Solution R:

f CLX **f 0**
240 [n]
3,504.35 PMT
7 [g] [i]
PV
Displays: -452,000.

Problem S
Loan = $216,000
Monthly payments = ?
Term = 5 years
Interest rate = 6.125%
Balloon PMT = $125,000
Ans: $2,402.60

Solution S:

f CLX **f 2**
5 [g] [n]
6.125 [g] [i]
216,000 CHS PV
125,000 FV
PMT
Displays: 2,402.60

Problem T
Loan = $300,000
Term = 7 years
Interest rate: 7%/yr
Monthly PMT = $2,592.60
Balloon Payment = ?
Ans: $209,000.06

Solution T:

f CLX f 2
300,000 CHS PV
7 [g] [n]
7 [g] [i]
2,592.60 PMT
FV
Displays: 209,000.06

Problem U
Term = 300 months
Monthly PMT = $1,194.00
Interest rate: 6 1/4%
Loan Amount = ?
(To nearest $1.00)
Ans: $181,000

Solution U:

f CLX f 2
300 [n]
1,194 PMT
6.25 [g] [i]
PV
Displays: -180,999.76

ADDITIONAL IRR PROOFS OF EXAMPLE/PROBLEM RESULTS

Page 226: Verification Using the Unequal Cash-Flow Function:

KEYSTROKES:	f CLX; f 2	Clear all registers; set 2 places.
	100,000 ENTER 2,047.61 [-]	Out-of-pocket cash-flow
	CHS g CFo	Input -97,952.39 into CFo register
	2,047.61 g CFj	Input regular monthly payment
	28 g Nj	Enters **28** regular payments!
	2,047.61 10 [x] g CFj	Enters interim balloon: 20,476.10
	2,047.61 g CFj	Back to the regular payments!
	29 g Nj	Input regular monthly payment
	2,047.20 g CFj	Input final payment
	f IRR	Computes monthly yield
	12 [x]	Converts to annual yield
	DISPLAYS: 15.00	(The annual yield is verified)

Page 227: Verification of **12% Discount Rate** Using the Unequal Cash-Flow Function:

KEYSTROKES:	f CLX f 2	Clear all registers; set 2 places.
	100,000 ENTER 2,016.19 [-]	Out-of-pocket cash-flow
	CHS g CFo	Input -97,983.81 into CFo register
	2,016.19 g CFj	Input 1st yr regular monthly payment
	11 g Nj	Enters 11 regular payments!
		REMEMBER: This is an advance payment system so the "first payment" was essentially made at the start of the lease.
	RCL **1** 5 [%] [+] f RND	Recall **1st yr** payment; increase by 5%. Round to 2 places.
	g CFj	Inputs 2,117.00 as **2nd yr** monthly lease payment.
	12 g Nj	Enters 12 regular payments!
	RCL **2** 5 [%] [+] f RND	Recall **2nd yr** payment; increase by 5%. Round to 2 places.
	g CFj	Inputs 2,222.85 as **3rd yr** monthly lease payment.
	12 g Nj	Enters 12 regular payments.
	RCL **3** 5 [%] [+] f RND	Recall **3rd yr** payment; increase by 5%. Round to 2 places.
	g CFj	Inputs 2,333.99 as **4th yr** monthly lease payment.
	12 g Nj	Enters 12 regular payments.
	RCL **4** 5 [%] [+] f RND	Recall **4th yr** payment; increase by 5%. Round to 2 places.
	g CFj	Inputs 2,450.69 as **5th yr** monthly lease payment.
	12 g Nj	Enters 12 regular payments.
	f IRR	Computes monthly discount rate
	12 [x]	Converts to annual discount rate
	DISPLAYS: 12.00	(Annual discount rate is verified)
	f 9	Show all places.
	DISPLAYS: 12.00000707	Not bad!

THE ROUNDING FUNCTION, "RND"

Discussion: Please note that the display format of your HP-12C simply controls what you see in the displayed x-register but it does not control the number inside the calculator. For example, let's set the calculator to display 9 decimal places (press f 9); then type the number **1.123456789** and then hit the ENTER key. Your display will show as follows:

1.123456789

Now, set 2 decimal places (press **f 2**). Your display will show as follows:

1.12

Now, multiply the displayed number (1.12) by 100 (press 100 [x]). What your display now shows is **112.35.** However, the number that is actually inside the calculator is **112.3456789.** Verify this by setting the calculator to 9 decimal places by pressing **f 9.** Your display will show:

112.3456789

At this stage let's assume that you wanted to round the above result to three decimal places. To do this we first set the display to 3 places and then use the Round (RND) function. The keystrokes follow.

f 3	We set the display to 3 decimal places. Your display shows: **112.346.**
f RND	This operates the rounding (RND) function. Your display still shows: **112.346**!

Let's verify that your HP-12C rounded the internally stored number (**112.3456789**) to **112.346.** The keystroke follows:

f 9	We set all places your calculator can display. Your display shows: **112.3460000.**

Ok; a little practice!

f CLX f 5	Cleared the calculator and set 5 decimal places.
1.123456789	Typed 1.123456789 into the displayed "x" register.
ENTER	Your display shows: **1.12346.**
f RND	Rounded number stored internally in the "x" register to what was displayed. Your display now shows: **1.12346.**
f 9	Your display shows: **1.123460000.**
f 4	Set 4 decimal places. Your display shows: **1.1235.**
f RND	Rounded number stored internally in the "x" register to what was displayed. Your display now shows: **1.1235.**
f 9	Your display shows: **1.123500000.**

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>
> Dr. Edward J. Farragher, Associate Professor of Finance
> Oakland University, Rochester, Michigan

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