HEWLETT-PACKARD

HP-12C

SOLUTIONS HANDBOOK

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HP-12C

Solutions Handbook

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Introduction

This *Solutions Handbook* has been designed to supplement the HP-12C Owner's Handbook by providing a variety of applications in the financial area. Programs and/or step-by-step keystroke procedures with corresponding examples in each specific topic are explained. We hope that this book will serve as a reference guide to many of your problems and will show you how to redesign our examples to fit your specific needs.

Contents

Real Estate

Refinancing

It can be mutually advantageous to both borrower and lender to refinance an existing mortgage which has an interest rate substantially below the current market rate, with a loan at a below-market rate. The borrower has the immediate use of tax-free cash, while the lender has substantially increased debt service on a relatively small cash outlay.

To find the benefits to both borrower and lender:

1. Calculate the monthly payment on the existing mortgage.
2. Calculate the monthly payment on the new mortgage.
3. Calculate the net monthly payment received by the lender (and paid by the borrower) by adding the figure found in Step 1 to the figure found in Step 2.
4. Calculate the Net Present Value (*NPV*) to the lender of the net cash advanced.
5. Calculate the yield to the lender as an *IRR*.
6. Calculate the *NPV* to the borrower of the net cash received.

Example 1: An investment property has an existing mortgage which originated 8 years ago with an original term of 25 years, fully amortized in level monthly payments at 6.5% interest. The current balance is $133,190.

Although the going current market interest rate is 11.5%, the lender has agreed to refinance the property with a $200,000, 17 year, level-monthly-payment loan at 9.5% interest.

What are the *NPV* and effective yield to the lender on the net amount of cash actually advanced?

What is the *NPV* to the borrower on this amount if he can earn a 15.25% equity yield rate on the net proceeds of the loan?

Keystrokes	Display	
g END		
f CLEAR FIN		
17 g 12x		
6.5 g 12÷		
133190 PV		
PMT STO 0	**-1,080.33**	Monthly payment on existing mortgage received by lender.
9.5 g 12÷		
200000 CHS PV		
PMT	**1,979.56**	Monthly payment on new mortgage.
RCL 0 + PMT	**899.23**	Net monthly payment (to lender).
RCL PV		
133190 + STO 0	**-66,810.00**	Net amount of cash advanced (by lender).
11.5 g 12÷ PV	**-80,425.02**	Present value of net monthly payment.
RCL 0 −	**-13,615.02**	*NPV* to lender of net cash advanced.
RCL 0 PV i		
12 ×	**14.83**	% nominal yield (*IRR*).
15.25 g 12÷ PV	**-65,376.72**	Present value of net monthly payment at 15.25%.
RCL 0 −	**1,433.28**	*NPV* to borrower.

Wrap-Around Mortgage

A wrap-around mortgage is essentially the same as a refinancing mortgage, except that the new mortgage is granted by a different lender, who assumes the payments on the existing mortgage, which remains in full force. The new (second) mortgage is thus "wrapped around" the existing mortgage. The "wrap-around" lender advances the net difference between the new (second) mortgage and the existing mortgage in cash to the borrower, and receives as net cash flow, the difference between debt service on the new (second) mortgage and debt service on the existing mortgage.

When the terms of the original mortgage and the wrap-around are the same, the procedures in calculating *NPV* and *IRR* to the lender

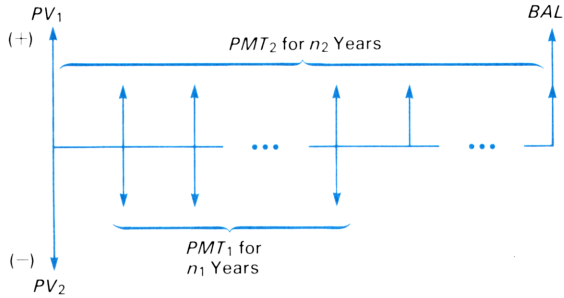
and *NPV* to the borrower are exactly the same as those presented in the preceding section on refinancing.

Example 1: A mortgage loan on an income property has a remaining balance of $200,132.06. When the loan originated 8 years ago, it had a 20-year term with full amortization in level monthly payments at 6.75% interest.

A lender has agreed to "wrap" a $300,000 second mortgage at 10%, with full amortization in level monthly payments over 12 years. What is the effective yield (*IRR*) to the lender on the net cash advanced?

Keystrokes	Display	
[g] [END]		
[f] CLEAR [FIN]		
20 [ENTER]		
8 [-] [g] [12x]	**144.00**	Total number of months remaining in original loan (into n).
6.75 [g] [12÷]	**0.56**	Monthly interest rate (into i).
200132.06 [PV]	**200,132.06**	Loan amount (into PV).
[PMT] [STO] 0	**-2,031.55**	Monthly payment on existing mortgage (calculated).
10 [g] [12÷]	**0.83**	Monthly interest on wrap-around.
300000 [CHS] [PV]	**-300,000.00**	Amount of wrap-around (into PV).
[PMT]	**3,585.23**	Monthly payment on wrap-around (calculated).
[RCL] 0 [+] [PMT]	**1,553.69**	Net monthly payment received (into PMT).
[RCL] [PV]		
200132.06 [+] [PV]	**-99,867.94**	Net cash advanced (into PV).
[i] 12 [×]	**15.85**	Nominal yield (*IRR*) to lender (calculated).

Sometimes the wrap around mortgage will have a longer payback period than the original mortgage, or a balloon payment may exist.



where:

n_1 = number of years remaining in original mortgage

PMT_1 = yearly payment of original mortgage

PV_1 = remaining balance of original mortgage

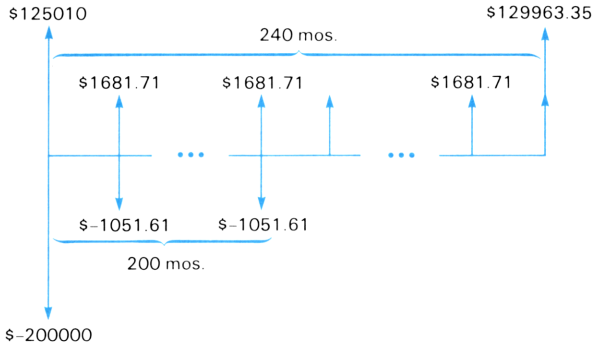
n_2 = number of years in wrap-around mortgage

PMT_2 = yearly payment of wrap-around mortgage

PV_2 = total amount of wrap-around mortgage

BAL = balloon payment

Example 2: A customer has an existing mortgage with a balance of $125,010, a remaining term of 200 months, and a $1051.61 monthly payment. He wishes to obtain a $200,000, 9½% wrap-around with 240 monthly payments of $1681.71 and a balloon payment at the end of the 240th month of $129,963.35. If you, as a lender, accept the proposal, what is your rate of return?



Keystrokes	Display	
g END		
f CLEAR FIN		
200000 CHS ENTER		
125010 + g CFo	**-74,990.00**	Net investment.
1051.61 CHS ENTER		
1681.71 +	**630.10**	Net cash flow received by lender.
g CFj 99 g Nj		
x≷y g CFj		
x≷y g Nj		The above cash flow occurs 200 times.
x≷y g CFj		
2 g Nj		
g LSTX g CFj	**1,681.71**	Next cash flow received by lender.
39 g Nj	**39.00**	Cash flow occurs 39 times.
x≷y 129963.35 +		
g CFj	**131,645.06**	Final cash flow.
f IRR 12 ×	**11.84**	Rate of return to lender.

If you, as a lender, know the yield on the entire transaction, and you wish to obtain the payment amount on the wrap-around mortgage to achieve this yield, use the following procedure. Once the monthly payment is known, the borrower's periodic interest rate may also be determined.

1. Press g END and press f CLEAR FIN.
2. Key in the remaining periods of the original mortgage and press n.
3. Key in the desired annual yield and press g 12÷.
4. Key in the monthly payment to be made by the lender on the original mortgage and press CHS PMT.
5. Press PV.
6. Key in the net amount of cash advanced and press + CHS PV.
7. Key in the total term of the wrap-around mortgage and press n.
8. If a balloon payment exists, key it in and press FV.

9. Press [PMT] to obtain the payment amount necessary to achieve the desired yield.
10. Key in the amount of the wrap-around mortgage and press [CHS] [PV] [i] to obtain the borrower's periodic interest rate.

Example 3: Your firm has determined that the yield on a wrap-around mortgage should be 12% annually. In the previous example, what monthly payment must be received to achieve this yield on a $200,000 wrap-around? What interest rate is the borrower paying?

Keystrokes	Display	
[g] [END]		
[f] CLEAR [FIN]		
200 [n] 12 [g] [12÷]		Number of periods and monthly interest rate.
1051.61 [CHS] [PMT]		Monthly payment.
[PV] 74990 [+]		
[CHS] [PV]	**-165,776.92**	Present value of payments plus cash advanced.
240 [n]		
129963.35 [FV] [PMT]	**1,693.97**	Monthly payment received by lender.
200000 [CHS] [PV] [i]		
12 [×]	**9.58**	Annual interest paid by borrower.

Income Property Cash Flow Analysis

Before-Tax Cash Flows

The before-tax cash flows applicable to real estate analysis and problems are:

- Potential Gross Income
- Effective Gross Income
- Net Operating Income (also called Net Income Before Recapture)
- Cash Throw-off to Equity (also called Gross Spendable Cash)

The derivation of these cash flows follows a set sequence:

1. Calculate Potential Gross Income by multiplying the rent

per unit times the number of units, times the number of rental payment periods per year. This gives the rental income the property would generate if it were fully occupied.

2. Deduct Allowance for Vacancy and Rental Loss. This is usually expressed as a percentage. The result is Rent Collections (which is also Effective Gross Income if there is no "Other Income").
3. Add "Other Income," such as receipts from concessions (laundry equipment, etc.), produced from sources other than the rental office space. This is the Effective Gross Income.
4. Deduct Operating Expenses. These are expenditures the landlord-investor must make, by contract or custom, to preserve the property and keep it capable of producing the gross income. The result is Net Operating Income.
5. Deduct Annual Debt Service on the mortgage. This produces Cash Throw-Off to Equity.

Thus: Effective Gross Income =
Potential Gross Income − Vacancy Loss + Other Income.

Net Operating Income =
Effective Gross Income − Operating Expenses.

Cash Throw-Off =
Net Operating Income − Annual Debt Service.

Example: A 60-unit apartment building has rentals of $250 per unit per month. With a 5% vacancy rate, the annual operation cost is $76,855.

The property has just been financed with a $700,000 mortgage, fully amortized in level monthly payments at 11.5% over 20 years.

a. What is the Effective Gross Income?
b. What is the Net Operating Income?
c. What is the Cash Throw-Off to Equity?

Keystrokes	Display	
9 END		
f CLEAR FIN		
60 ENTER		
250 × 12 ×	**180,000.00**	Potential Gross Income.

Keystrokes	Display	
5 [%]	9,000.00	Vacancy Loss.
[−]	171,000.00	Effective Gross Income.
76855 [−]	94,145.00	Net Operating Income.
20 [g] [12x]		
11.5 [g] [12÷]		
700000 [PV]		
[PMT] 12 [×]	−89,580.09	Annual Debt Service.
[+]	4,564.91	Cash Throw-Off.

Before-Tax Reversions (Resale Proceeds)

The reversion receivable at the end of the income projection period is usually based on forecast or anticipated resale of the property at that time. The before-tax reversion amounts applicable to real estate analysis and problems are:

- Sales Price.
- Cash Proceeds of Resale.
- Outstanding Mortgage Balance.
- Net Cash Proceeds of Resale to Equity.

The derivation of these reversions are as follows:

1. Forecast or estimate Sales Price. Deduct sales and Transaction Costs. The result is the Proceeds of Resale.
2. Calculate the Outstanding Balance of the Mortgage at the end of the Income Projection Period and subtract it from Proceeds of Resale. The result is Net Cash Proceeds of Resale.

Thus: Cash Proceeds of Resale =
Sales Price − Transaction Costs.

Net Cash Proceeds of Resale =
Cash Proceeds of Resale − Outstanding Mortgage Balance.

Example: The apartment property in the preceding example is expected to be resold in 10 years. The anticipated resale price is $800,000. The transaction costs are expected to be 7% of the resale price. The mortgage is the same as that indicated in the preceding example.

- What will the Mortgage Balance be in 10 years?
- What are the Cash Proceeds of Resale and Net Cash Proceeds of Resale?

Keystrokes	Display	
[g] [END]		
[f] CLEAR [FIN]		
20 [g] [12x]	240.00	Mortgage term.
11.5 [g] [12÷]	0.96	Mortgage rate.
700000 [PV]		Property value.
[PMT]	-7,465.01	Monthly payment.
10 [g] [12x]	120.00	Projection period.
[FV]	-530,956.57	Mortgage balance in 10 years.
800000 [ENTER]		Estimated resale.
7 [%]	56,000.00	Transaction costs.
[−]	744,000.00	Cash Proceeds of Resale.
[+]	213,043.43	Net Cash Proceeds of Resale.

After-Tax Cash Flows

The After-Tax Cash Flow (*ATCF*) is found for each year by deducting the Income Tax Liability for that year from the Cash Throw Off.

where Taxable Income =
Net Operating Income − interest − depreciation,

Tax Liability = Taxable Income × Marginal Tax Rate,

and After Tax Cash Flow = Cash Throw Off − Tax Liability

The After-Tax Cash Flow for initial and successive years may be calculated by the following HP-12C program. This program calculates the Net Operating Income using the Potential Gross Income, operational cost and vacancy rate. The Net Operating Income is readjusted each year from the growth rates in Potential Gross Income and operational costs.

The user is able to change the method of finding the depreciation from declining balance to straight line. To make the change, key in [f] [SL] at line 32 of the program in place of [f] [DB].

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		R↓	28– 33
f CLEAR PRGM	00–	g x=0	29– 43 35
0	01– 0	g GTO 36	30–43,33 36
n	02– 11	RCL 1	31– 45 1
STO 1	03– 44 1	f DB	32– 42 25
RCL 7	04– 45 7	STO − 0	33–44 30 0
EEX	05– 26	0	34– 0
2	06– 2	g GTO 17	35–43,33 17
÷	07– 10	n	36– 11
STO 7	08– 44 7	RCL 2	37– 45 2
1	09– 1	RCL 8	38– 45 8
STO + 1	10–44 40 1	%	39– 25
1	11– 1	STO + 2	40–44 40 2
2	12– 2	R↓	41– 33
f AMORT	13– 42 11	RCL . 0	42–45 48 0
STO 0	14– 44 0	%	43– 25
RCL 5	15– 45 5	−	44– 30
n	16– 11	RCL 3	45– 45 3
RCL i	17– 45 12	RCL 9	46– 45 9
RCL 6	18– 45 6	%	47– 25
i	19– 12	STO + 3	48–44 40 3
R↓	20– 33	R↓	49– 33
STO 6	21– 44 6	−	50– 30
R↓	22– 33	1	51– 1
RCL PV	23– 45 13	RCL 7	52– 45 7
RCL 4	24– 45 4	STO × 0	53–44 20 0
PV	25– 13	−	54– 30
R↓	26– 33	×	55– 20
STO 4	27– 44 4	RCL PMT	56– 45 14

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
1	57- 1	RCL 1	63- 45 1
2	58- 2	g PSE	64- 43 31
×	59- 20	$x \geq y$	65- 34
+	60- 40	R/S	66- 31
RCL 0	61- 45 0	g GTO 09	67-43,33 09
−	62- 30	f P/R	

REGISTERS			
n: Used	i: Annual %	PV: Used	PMT: Monthly
FV: 0	R_0: Used	R_1: Counter	R_2: *PGI*
R_3: Oper. cost	R_4: Dep. value	R_5: Dep. life	R_6: Factor (DB)
R_7: Tax Rate	R_8: % gr. (PGI)	R_9% gr. (Op)	$R_{.0}$: Vacancy rt.

1. Press g END and press f CLEAR REG.
2. Key in loan values:
   - Key in annual interest rate and press g 12÷.
   - Key in principal to be paid and press PV.
   - Key in monthly payment and press CHS PMT.
     (If any of the values are not known, they should be solved for.)
3. Key in Potential Gross Income (*PGI*) and press STO 2.
4. Key in operational cost and press STO 3.
5. Key in depreciable value and press STO 4.
6. Key in depreciation life and press STO 5.
7. Key in factor (for declining balance only) and press STO 6.
8. Key in Marginal Tax Rate (as a percentage) and press STO 7.
9. Key in the growth rate in Potential Gross Income (0 for no growth) and press STO 8.
10. Key in growth rate in operational cost (0 if no growth) and press STO 9.
11. Key in vacancy rate (0 for no vacancy rate) and press STO . 0.

12. Key in desired depreciation function at line 32 in the program.
13. Press [R/S] to compute *ATCF*. The display will pause showing the year and then will stop with the *ATCF* for that year. The Y-register contains the year.
14. Continue pressing [R/S] to compute successive After-Tax Cash Flows.

Example 1: A triplex was recently purchased for $100,000 with a 30-year loan at 12.25% and a 20% down payment. Not including a 5% annual vacancy rate, the potential gross income is $9,900 with an annual growth rate of 6%. Operating expenses are $3291.75 with a 2.5% growth rate. The depreciable value is $75,000 with a projected useful life of 20 years. Assuming 125% declining balance depreciation, what are the After-Tax Cash Flows for the first 10 years if the investor's Marginal Tax Rate is 35%?

Keystrokes	Display	
[g] [END]		
[f] CLEAR [REG]		
100000 [ENTER]		
20 [%] [−] [PV]	80,000.00	Mortgage amount.
12.25 [g] [12÷]	1.02	Monthly interest rate.
30 [g] [12×]	360.00	Mortgage term.
[PMT]	−838.32	Monthly payment.
9900 [STO] 2	9,900.00	Potential Gross Income.
3291.75 [STO] 3	3,291.75	1st year operating cost.
75000 [STO] 4	75,000.00	Depreciable value.
20 [STO] 5	20.00	Useful life.
125 [STO] 6	125.00	Declining balance factor.
35 [STO] 7	35.00	Marginal Tax Rate.
6 [STO] 8	6.00	Potential Gross Income growth rate.
2.5 [STO] 9	2.50	Operating cost growth rate.
5 [STO] [.] 0	5.00	Vacancy rate.
[R/S]	1.00	Year 1
	−1,020.88	$ATCF_1$

Keystrokes	Display	
R/S	**2.00**	Year 2
	-822.59	$ATCF_2$
R/S	**3.00**	Year 3
	-598.85	$ATCF_3$
R/S	**4.00**	Year 4
	-348.94	$ATCF_4$
R/S	**5.00**	Year 5
	-72.16	$ATCF_5$
R/S	**6.00**	Year 6
	232.35	$ATCF_6$
R/S	**7.00**	Year 7
	565.48	$ATCF_7$
R/S	**8.00**	Year 8
	928.23	$ATCF_8$
R/S	**9.00**	Year 9
	1,321.62	$ATCF_9$
R/S	**10.00**	Year 10
	1,746.81	$ATCF_{10}$

Example 2: An office building was purchased for $1,400,000. The value of depreciable improvements is $1,200,000 with a 35 year economic life. Straight line depreciation will be used. The property is financed with a $1,050,000 loan. The terms of the loan are 9.5% interest and $9,173.81 monthly payments for 25 years. The office building generates a Potential Gross Income of $175,200 which grows at a 3.5% annual rate. The operating cost is $40,296.00 with a 1.6% annual growth rate. Assuming a Marginal Tax Rate of 50% and a vacancy rate of 7%, what are the After-Tax Cash Flows for the first 5 years?

Keystrokes	Display	
g END		
f CLEAR REG		
1050000 PV		
9173.81 CHS PMT		
9.5 g 12÷		
25 g 12×		
175200 STO 2	**175,200.00**	Potential Gross Income.
40296 STO 3	**40,296.00**	1st year operating cost.

Keystrokes	Display	
1200000 STO 4	1,200,000.00	Depreciable value.
35 STO 5	35.00	Depreciable life.
50 STO 7	50.00	Marginal tax rate.
3.5 STO 8	3.50	Potential Gross Income growth rate.
1.6 STO 9	1.60	Operating cost growth rate.
7 STO · 0	7.00	Vacancy rate.
g GTO 31	7.00	Go to dep. step.
f P/R f SL	32- 42 23	Change to SL.
f P/R R/S	1.00	Year 1
	18,021.07	$ATCF_1$
R/S	2.00	Year 2
	20,014.26	$ATCF_2$
R/S	3.00	Year 3
	22,048.90	$ATCF_3$
R/S	4.00	Year 4
	24,123.14	$ATCF_4$
R/S	5.00	Year 5
	26,234.69	$ATCF_5$

After-Tax Net Cash Proceeds of Resale

The After-Tax Net Cash Proceeds of Resale ($ATNCPR$) is the after-tax reversion to equity; generally, the estimated resale price of the property less commissions, outstanding debt and any tax claim.

The After-Tax Net Cash Proceeds can be found using the HP-12C program which follows. In calculating the owner's income tax liability on resale, this program assumes that the owner elects to have his capital gain taxed at 40% of his Marginal Tax Rate. This assumption is in accordance with a 1978 Federal tax ruling.*

This program uses declining balance depreciation to find the amount of depreciation from purchase to sale. This amount is used to determine the excess depreciation (which is equal to the amount of actual depreciation minus the amount of straight line depreciation).

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* Federal Taxes, code sec. 1202 (¶32,036).

The user may change to a different depreciation method by keying in the desired function at line 35 in place of [f] [DB].

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[i]	25- 12
[f] CLEAR [PRGM]	00-	[RCL] 2	26- 45 2
[g] [END]	01- 43 8	[f] [SL]	27- 42 23
[STO] 2	02- 44 2	[RCL] 2	28- 45 2
[R↓]	03- 33	[×]	29- 20
[%]	04- 25	[.]	30- 48
[−]	05- 30	6	31- 6
[STO] 0	06- 44 0	[×]	32- 20
[−]	07- 30	[STO] [+] 1	33-44 40 1
[.]	08- 48	[RCL] 2	34- 45 2
4	09- 4	[f] [DB]	35- 42 25
[×]	10- 20	[x≷y]	36- 34
[STO] 1	11- 44 1	[RCL] [PV]	37- 45 13
[RCL] [PMT]	12- 45 14	[−]	38- 30
[f] [RND]	13- 42 14	[STO] [+] 1	39-44 40 1
[PMT]	14- 14	[RCL] 6	40- 45 6
[RCL] 2	15- 45 2	[EEX]	41- 26
[g] [12×]	16- 43 11	2	42- 2
[FV]	17- 15	[÷]	43- 10
[STO] [+] 0	18-44 40 0	[RCL] 1	44- 45 1
[f] CLEAR [FIN]	19- 42 34	[×]	45- 20
[RCL] 3	20- 45 3	[RCL] 0	46- 45 0
[PV]	21- 13	[+]	47- 40
[RCL] 4	22- 45 4	[g] [GTO] 00	48-43,33 00
[n]	23- 11	[f] [P/R]	
[RCL] 5	24- 45 5		

	REGISTERS		
n: Used	i: Used	PV: Used	PMT: Used
FV: Used	R_0: Used	R_1: Used	R_2: Desired yr.
R_3: Dep. value	R_4: Dep. life	R_5: Factor	R_6: MTR
R_7—$R_{.3}$: Unused			

1. **Key in the program and press [f] CLEAR [REG].**
2. **Key in loan values:**
   - **Key in annual interest rate and press [g] [12÷].**
   - **Key in mortgage amount and press [PV].**
   - Key in monthly payment and press [CHS] [PMT]. (If any of the values are unknown, they should be solved for.)
3. Key in depreciable value and press [STO] 3.
4. Key in depreciable life in years and press [STO] 4.
5. Key in accelerated depreciation factor for the declining balance method and press [STO] 5.
6. Key in your Marginal Tax Rate as a percentage and press [STO] 6.
7. Key in the purchase price and press [ENTER].
8. Key in the sale price and press [ENTER].
9. Key in the % commission charged on the sale and press [ENTER]*.
10. Key in number of years after purchase and press [R/S].

Example 1: An apartment complex, purchased for $900,000 ten years ago, is sold for $1,750,000. The closing costs are 8% of the sale price and the income tax rate is 48%.

A $700,000 loan for 20 years at 9.5% annual interest was used to purchase the complex. When it was purchased the depreciable value was $750,000 with a useful life of 25 years. Using 125% declining balance depreciation, what are the After-Tax Net Cash Proceeds in year 10?

***If a dollar value is desired instead of a commission rate, key in [g] [END] (which doesn't affect register values) at line 04 of the program.**

Keystrokes	Display	
f CLEAR REG	0.00	
700000 PV	700,000.00	Mortgage.
9.5 g 12÷	0.79	Monthly interest.
20 g 12×	240.00	Number of payments.
PMT	-6,524.92	Monthly payment.
750000 STO 3	750,000.00	Depreciable value.
25 STO 4	25.00	Depreciable life.
125 STO 5	125.00	Factor.
48 STO 6	48.00	Marginal Tax Rate.
900000 ENTER	900,000.00	Purchase price.
1750000 ENTER	1,750,000.00	Sale price.
8 ENTER	8.00	Commission rate.
10 R/S	911,372.04	*ATNCPR.*

Notes

Lending

Loan With a Constant Amount Paid Towards Principal

This type of loan is structured such that the principal is repaid in equal installments with the interest paid in addition. Therefore each periodic payment has a constant amount applied toward the principal and a varying amount of interest.

Loan Reduction Schedule

If the constant periodic payment to principal, annual interest rate, and loan amount are known, the total payment, interest portion of each payment, and remaining balance after each successive payment may be calculated as follows:

1. Key in the constant periodic payment to principal and press [STO] 0.
2. Key in periodic interest rate and press [ENTER] [ENTER] [ENTER].
3. Key in the loan amount. If you wish to skip to another time period, press [ENTER]. Then key in the number of payments to be skipped, and press [RCL] 0 [×] [−].
4. Press [$x \geqslant y$] [%] to obtain the interest portion of the payment.
5. Press [RCL] 0 [+] to obtain the total payment.
6. Press [CLX] [RCL] 0 [−] to obtain the remaining balance of the loan.
7. Return to step 4 for each successive payment.

Example 1: A $60,000 land loan at 10% interest calls for equal semi-annual principal payments over a 6-year maturity. What is the loan reduction schedule for the first year? (constant payment to principal is $5000 semi-annually). What is the fourth year's schedule (skip 4 payments)?

Keystrokes	Display	
5000 [STO] 0 10 [ENTER] 2 [÷] [ENTER] [ENTER] [ENTER]	**5.00**	Semi-annual interest rate.
60000 [x≷y] [%]	**3,000.00**	First payment's interest.
[RCL] 0 [+]	**8,000.00**	Total first payment.
[CLX] [RCL] 0 [−]	**55,000.00**	Remaining balance.
[x≷y] [%]	**2,750.00**	Second payment's interest.
[RCL] 0 [+]	**7,750.00**	Total second payment.
[CLX] [RCL] 0 [−]	**50,000.00**	Remaining balance after first year.
4 [RCL] 0 [×] [−] [x≷y] [%]	**1,500.00**	Seventh payment's interest.
[RCL] 0 [+]	**6,500.00**	Total seventh payment.
[CLX] [RCL] 0 [−]	**25,000.00**	Remaining balance.
[x≷y] [%]	**1,250.00**	Eighth payment's interest.
[RCL] 0 [+]	**6,250.00**	Total eighth payment.
[CLX] [RCL] 0 [−]	**20,000.00**	Remaining balance after fourth year.

Add-On Interest Rate Converted to APR

An add-on interest rate determines what portion of the principal will be added on for repayment of a loan. This sum is then divided by the number of months in the loan to determine the monthly payment. For example, a 10% add-on rate for 36 months on $3000 means add one-tenth of $3000 for 3 years (300 × 3)—usually called the "finance charge"—for a total of $3900. The monthly payment is $3900/36.

This keystroke procedure converts an add-on interest rate to an annual percentage rate when the add-on rate and number of months are known.

1. Press [g] [END] and press [f] CLEAR [FIN].
2. Key in the number of months in loan and press [n] [ENTER] [RCL] [g] [12×].

3. Key in the add-on rate and press [×].
4. Key in the amount of the loan and press [PV] * [$x \gtrless y$] [%] [+].
5. Press [$x \gtrless y$] [÷] [CHS] [PMT].
6. Press [i] 12 [×] to obtain the *APR*.

Example 1: Calculate the APR and monthly payment of a 12%, $1000 add-on loan which has a life of 18 months.

Keystrokes	Display	
[g] [END]		
[f] CLEAR [FIN]		
18 [n] [ENTER]		
[RCL] [g] [12×] 12 [×]		
1000 [PV] [$x \gtrless y$] [%] [+]	1,180.00	Amount of loan.
[$x \gtrless y$] [÷] [CHS] [PMT]	-65.56	Month payment.
[i] 12 [×]	21.64	Annual Percentage Rate.

APR Converted to Add-On Interest Rate

Given the number of months and annual percentage rate, this procedure calculates the corresponding add-on interest rate.

1. Press [g] [END] and press [f] CLEAR [FIN].
2. Enter the following information:
   a. Key in number of months of loan and press [n].
   b. Key in *APR* and press [g] [12÷].
   c. Key in 100 and press [PV] [PMT].
3. Press [RCL] [PV] [RCL] [n] [÷] [+] [CHS] 12 [×] to obtain add-on rate.

---

* Positive for cash received; negative for cash paid out.

Example 1: What is the equivalent add-on rate for an 18-month loan with an *APR* of 14%?

Keystrokes	Display	
[g] [END]		
[f] CLEAR [FIN]		
18 [n] 14 [g] [12÷]		
100 [PV] [PMT] [RCL] [PV]		
[RCL] [n] [÷] [+] [CHS]		
12 [×]	**7.63**	Add-On Interest Rate.

Add-On Rate Loan With Credit Life

This HP-12C program calculates the monthly payment amount, credit life amount (an optional insurance which cancels any remaining indebtedness at the death of the borrower), total finance charge, and annual percentage rate (*APR*) for an add-on interest rate (*AIR*) loan. The monthly payment is rounded (in normal manner) to the nearest cent. If other rounding techniques are used, slightly different results may occur.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[g] [LST x]	13– 43 36
[f] CLEAR [PRGM]	00–	[RCL] 1	14– 45 1
[g] [END]	01– 43 8	[×]	15– 20
1	02– 1	[RCL] 4	16– 45 4
[RCL] 0	03– 45 0	[×]	17– 20
1	04– 1	[−]	18– 30
2	05– 2	[RCL] 4	19– 45 4
0	06– 0	[RCL] 1	20– 45 1
0	07– 0	[×]	21– 20
[÷]	08– 10	1	22– 1
[STO] 4	09– 44 4	[+]	23– 40
[RCL] 2	10– 45 2	[x≷y]	24– 34
[×]	11– 20	[÷]	25– 10
[−]	12– 30	[RCL] 3	26– 45 3

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
×	27– 20	RCL 5	54– 45 5
RCL 0	28– 45 0	.	55– 48
÷	29– 10	0	56– 0
f RND	30– 42 14	1	57– 1
CHS	31– 16	+	58– 40
PMT	32– 14	f RND	59– 42 14
R/S	33– 31	STO 5	60– 44 5
RCL PMT	34– 45 14	RCL 5	61– 45 5
RCL 0	35– 45 0	R/S	62– 31
×	36– 20	RCL PV	63– 45 13
CHS	37– 16	$x \gtrless y$	64– 34
PV	38– 13	−	65– 30
RCL PV	39– 45 13	RCL 3	66– 45 3
RCL 2	40– 45 2	−	67– 30
%	41– 25	CHS	68– 16
RCL 0	42– 45 0	R/S	69– 31
×	43– 20	RCL 5	70– 45 5
1	44– 1	RCL 3	71– 45 3
2	45– 2	+	72– 40
÷	46– 10	PV	73– 13
STO 5	47– 44 5	RCL 0	74– 45 0
EEX	48– 26	n	75– 11
2	49– 2	i	76– 12
×	50– 20	RCL g 12÷	77–45,43 12
g FRAC	51– 43 24	g GTO 00	78–43,33 00
g $x=0$	52– 43 35	f P/R	
g GTO 61	53–43,33 61		

	REGISTERS		
n: N	i: i	PV: Used	PMT: PMT
FV: 0	R_0: N	R_1: AIR	R_2: CL (%)
R_3: Loan	R_4: $N/1200$	R_5: Used	R_6—R_9 Unused

1. Key in the program.
2. Press [f] CLEAR [FIN].
3. Key in the number of monthly payments in the loan and press [STO] 0.
4. Key in the annual add-on interest rate as a percentage and press [STO] 1.
5. Key in the credit life as a percentage and press [STO] 2.
6. Key in the loan amount and press [STO] 3.
7. Press [R/S] to find the monthly payment amount.
8. Press [R/S] to obtain the amount of credit life.
9. Press [R/S] to calculate the total finance charge.
10. Press [R/S] to calculate the annual percentage rate.
11. For a new loan return to step 3.

Example 1: You wish to quote a loan on a $3100 balance, payable over 36 months at an add-on rate of 6.75%. Credit life (CL) is 1%. What are the monthly payment amount, credit life amount, total finance charge, and APR?

Keystrokes	Display	
[f] CLEAR [FIN]		
36 [STO] 0	**36.00**	Months.
6.75 [STO] 1	**6.75**	Add-on interest rate.
1 [STO] 2	**1.00**	Credit life (%).
3100 [STO] 3	**3100.00**	Loan.
[R/S]	**-107.42**	Monthly payment.
[R/S]	**116.02**	Credit life.
[R/S]	**-651.10**	Total finance charge.
[R/S]	**12.39**	APR.

Interest Rebate—Rule of 78's

This procedure finds the unearned interest rebate, as well as the remaining principal balance due for a prepaid consumer loan using the Rule of 78's. The known values are the current installment number, the total number of installments for which the loan was written, and the total finance charge (amount of interest). The information is entered as follows:

1. Key in number of months in the loan and press [STO] 1.

2. Key in payment number when prepayment occurs and press [−] [STO] 2 1 [+].

3. Key in total finance charge and press [×] [RCL] 1 [ENTER] [×] [RCL] 1 [+] [÷] [RCL] 2 [×] to obtain the unearned interest (rebate).

4. Key in periodic payment amount and press [RCL] 2 [×] [$x \geqslant y$] [−] to obtain the amount of principal outstanding.

Example 1: A 30 month, $1,000 loan having a finance charge of $180, is being repaid at $39.33 per month. What is the rebate and balance due after the 25th regular payment?

Keystrokes	Display	
30 [STO] 1 25 [−] [STO] 2 1 [+] 180 [×] [RCL] 1 [ENTER] [×] [RCL] 1 [+] [÷] [RCL] 2 [×]	**5.81**	Rebate.
39.33 [RCL] 2 [×] [$x \geqslant y$] [−]	**190.84**	Outstanding principal.

The following HP-12C program can be used to evaluate the previous example.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		ENTER	14- 36
f CLEAR PRGM	00-	×	15- 20
STO 0	01- 44 0	RCL 1	16- 45 1
R↓	02- 33	+	17- 40
STO 2	03- 44 2	÷	18- 10
R↓	04- 33	RCL 2	19- 45 2
STO 1	05- 44 1	×	20- 20
RCL 2	06- 45 2	R/S	21- 31
−	07- 30	RCL 2	22- 45 2
STO 2	08- 44 2	×	23- 20
1	09- 1	x≷y	24- 34
+	10- 40	−	25- 30
RCL 0	11- 45 0	g GTO 00	26-43,33 00
×	12- 20	f P/R	
RCL 1	13- 45 1		

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Fin. charge	R_1: Payment #	R_2: # months
R_3—R_6 Unused			

1. Key in the program.
2. Key in the number of months in the loan and press ENTER.
3. Key in the payment number when prepayment occurs and press ENTER.
4. Key in the total finance charge and press R/S to obtain the unearned interest (rebate).
5. Key in the periodic payment amount and press R/S to find the amount of principal outstanding.
6. For a new case return to step 2.

Keystrokes	Display	
30 [ENTER]		
25 [ENTER]		
180 [R/S]	**5.81**	Rebate.
39.33 [R/S]	**190.84**	Outstanding balance.

Graduated Payment Mortgages

The Graduated Payment Mortgage is designed to meet the needs of young homebuyers who currently cannot afford high mortgage payments, but who have the potential of increased earnings in the years to come.

Under the Graduated Payment Mortgage plan, the payments increase by a fixed percentage at the end of each year for a specified number of years. Thereafter, the payment amount remains constant for the remaining life of the mortgage.

The result is that the borrower pays a reduced payment (a payment which is less than a traditional mortgage payment) in the early years, and in the later years makes larger payments than he would with a traditional loan. Over the entire term of the mortgage, the borrower would pay more than he would with conventional financing.

Given the term of the mortgage (in years), the annual interest rate, the loan amount, the percentage that the payments increase, and the number of years that the payments increase, the following HP-12C program determines the monthly payment and remaining balance for each year until the level payment is reached.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		1	06– 1
[f] CLEAR [PRGM]	00–	[+]	07– 40
[g] [END]	01– 43 8	[STO] 0	08– 44 0
[STO] 2	02– 44 2	[RCL] [n]	09– 45 11
[$x \geq y$]	03– 34	[RCL] 2	10– 45 2
1	04– 1	[–]	11– 30
[%]	05– 25	[g] [12×]	12– 43 11

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
RCL i	13– 45 12	÷	42– 10
g 12÷	14– 43 12	STO 4	43– 44 4
RCL PV	15– 45 13	RCL 3	44– 45 3
STO 3	16– 44 3	PV	45– 13
1	17– 1	1	46– 1
CHS	18– 16	STO 3	47– 44 3
PMT	19– 14	RCL 3	48– 45 3
PV	20– 13	R/S	49– 31
CHS	21– 16	RCL 4	50– 45 4
FV	22– 15	1	51– 1
1	23– 1	RCL 0	52– 45 0
g 12×	24– 43 11	RCL 1	53– 45 1
RCL PMT	25– 45 14	y^x	54– 21
RCL 0	26– 45 0	÷	55– 10
÷	27– 10	×	56– 20
PMT	28– 14	CHS	57– 16
PV	29– 13	f RND	58– 42 14
CHS	30– 16	PMT	59– 14
FV	31– 15	R/S	60– 31
1	32– 1	FV	61– 15
STO + 1	33–44 40 1	FV	62– 15
RCL 1	34– 45 1	f RND	63– 42 14
RCL 2	35– 45 2	R/S	64– 31
−	36– 30	CHS	65– 16
g $x=0$	37– 43 35	PV	66– 13
g GTO 40	38–43,33 40	1	67– 1
g GTO 25	39–43,33 25	STO + 3	68–44 40 3
RCL 3	40– 45 3	STO − 1	69–44 30 1
RCL PV	41– 45 13	RCL 1	70– 45 1

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
g x=0	71- 43 35	CHS	75- 16
g GTO 74	72-43,33 74	R/S	76- 31
g GTO 48	73-43,33 48	g GTO 76	77-43,33 76
RCL 4	74- 45 4	f P/R	

	REGISTERS		
n: Used	i: i/12	PV: Used	PMT: Used
FV: Used	R_0: Used	R_1: Used	R_2: Used
R_3: Used	R_4: Level Pmt.	R_5—R_9 Unused	

1. Key in the program.
2. Press f CLEAR REG.
3. Key in the term of the loan and press n.
4. Key in the annual interest rate and press i.
5. Key in the total loan amount and press PV.
6. Key in the rate of graduation (as a percent) and press ENTER.
7. Key in the number of years for which the loan graduates and press R/S. The following information will be displayed for each year until a level payment is reached.
   a. The current year.
      Then press R/S to continue.
   b. The monthly payment for the current year.
      Then press R/S to continue.
   c. The remaining balance to be paid on the loan at the end of the current year.
      Then press R/S to return to step a. unless the level payment is reached. If the level payment has been reached, the program will stop, displaying the monthly payment over the remaining term of the loan.
8. For a new case press g GTO 00 and return to step 2.

Example: A young couple recently purchased a new house with a Graduated Payment Mortgage. The loan is for $50,000 over a period of 30 years at an annual interest rate of 12.5%. The monthly payments will be graduating at an annual rate of 5% for the first 5 years and then will be level for the remaining 25 years. What are the monthly payment amounts for the first 6 years?

Keystrokes	Display	
f CLEAR REG	0.00	
30 n	30.00	Term
12.5 i	12.50	Annual interest rate
50000 PV	50,000.00	Loan amount
5 ENTER	5.00	Rate of graduation
5 R/S	1.00	Year 1
R/S	-448.88	lst year monthly payment.
R/S	-50,914.67	Remaining balance after 1st year.
R/S	2.00	Year 2
R/S	-471.33	2nd year monthly payment.
R/S	-51,665.07	Remaining balance after 2nd year.
R/S	3.00	Year 3
R/S	-494.89	3rd year monthly payment.
R/S	-52,215.34	Remaining balance after 3rd year.
R/S	4.00	Year 4
R/S	-519.64	4th year monthly payment.
R/S	-52,523.86	Remaining balance after 4th year.
R/S	5.00	Year 5
R/S	-545.62	5th year monthly payment.
R/S	-52,542.97	Remaining balance after 5th year.
R/S	-572.90	Monthly payment for remainder of term.

Variable Rate Mortgages

As its name suggests, a variable rate mortgage is a mortgage loan which provides for adjustment of its interest rate as market interest rates change. As a result, the current interest rate on a variable rate mortgage may differ from its origination rate (i.e., the rate when the loan was made). This is the difference between a variable rate mortgage and the standard fixed payment mortgage, where the interest rate and the monthly payment are constant throughout the term.

Under the agreement of the variable rate mortgage, the mortgage is examined periodically to determine any rate adjustments. The rate adjustment may be implemented in two ways:

1. Adjusting the monthly payment,
2. Modifying the term of the mortgage.

The period and limits to interest rate increases vary from state to state.

Each periodic adjustment may be calculated by using the HP-12C with the following keystroke procedure. The original terms of the mortgage are assumed to be known.

1. Press [g] [END] and press [f] CLEAR [FIN].
2. Key in the remaining balance of the loan and press [PV].
   The remaining balance is the difference between the loan amount and the total principal from the payments which have been made.
   To calculate the remaining balance, do the following:
   a. Key in the previous remaining balance. If this is the first mortgage adjustment, this value is the original amount of the loan. Press [PV].
   b. Key in the annual interest rate before the adjustment (as a percentage) and press [g] [12÷].
   c. Key in the number of years since the last adjustment. If this is the first mortgage adjustment, then key in the number of years since the origination of the mortgage. Press [g] [12×].

d. Key in the monthly payment over this period and press [CHS] [PMT].

e. Press [FV] to find the remaining balance, then press [f] CLEAR [FIN] [CHS] [PV].

3. Key in the adjusted annual interest rate (as a percentage) and press [g] [12÷].

To calculate the new monthly payment:

a. Key in the remaining life of the mortgage (years) and press [g] [12×].

b. Press [PMT] to find the new monthly payment.

To calculate the revised remaining term of the mortgage:

a. Key in the present monthly payment and press [PMT].

b. Press [n] 12 [÷] to find the remaining term of the mortgage in years.

Example: A homeowner purchased his house 3 years ago with a $50,000 variable rate mortgage. With a 30-year term, the current monthly payment is $495.15. When the interest rate is adjusted from 11.5% to 11.75%, what will the monthly payment be? If the monthly payment remained unchanged, find the revised remaining term on the mortgage.

Keystrokes	Display	
[g] [END]		
[f] CLEAR [FIN]		
50000 [PV]	**50,000.00**	Original amount of loan.
11.5 [g] [12÷]	**0.96**	Original monthly interest rate.
3 [g] [12×]	**36.00**	Period.
495.15 [CHS] [PMT]	**-495.15**	Previous monthly payment.
[FV]	**-49,316.74**	
[f] CLEAR [FIN]		
[CHS] [PV]	**49,316.74**	Remaining balance.
11.75 [g] [12÷]	**0.98**	Adjusted monthly interest.
30 [ENTER] 3 [−]	**27.00**	Remaining life of mortgage.

Keystrokes	Display	
g 12×	324.00	
PMT	-504.35	New monthly payment.
495.15 CHS PMT	-495.15	Previous monthly payment.
n 12 ÷	31.67	New remaining term (years).

Skipped Payments

Sometimes a loan (or lease) may be negotiated in which a specific set of monthly payments are going to be skipped each year. Seasonality is usually the reason for such an agreement. For example, because of heavy rainfall, a bulldozer cannot be operated in Oregon during December, January, and February, and the lessee wishes to make payments only when his machinery is being used. He will make nine payments per year, but the interest will continue to accumulate over the months in which a payment is not made.

To find the monthly payment amount necessary to amortize the loan in the specified amount of time, information is entered as follows:

1. Press g END and press f CLEAR FIN.
2. Key in the number of the last payment period before payments close the first time and press n.
3. Key in the annual interest rate as a percentage and press g 12÷ 1 PMT FV.
4. Press CHS PV 12 RCL n - n 0 PMT FV STO 0 RCL n.
5. Key in the number of payments which are skipped and press - n 1 PMT 0 PV FV STO + 0.
6. Press 0 PMT 12 n 100 PV FV RCL PV + CHS f CLEAR FIN i.
7. Key in the total number of years in the loan and press n.
8. Key in the loan amount and press PV PMT RCL 0 ÷ to obtain the monthly payment amount when the payment is made at the end of the month.

9. Press [CHS] [FV] 0 [PMT] 1 [n].
10. Key in the annual interest rate as a percent and press [g] [12÷] [PV] to find the monthly payment amount when the payment is made at the beginning of the month.

Example: A bulldozer worth $100,000 is being purchased in September. The first payment is due one month later, and payments will continue over a period of 5 years. Due to the weather, the machinery will not be used during the winter months, and the purchaser does not wish to make payments during January, February, and March (months 4 thru 6). If the current interest rate is 14%, what is the monthly payment necessary to amortize the loan?

Keystrokes	Display	
[g] [END]		
[f] CLEAR [FIN]		
3 [n]	**3.00**	Number of payments made before a group of payments is skipped.
14 [g] [12÷]		
1 [PMT] [FV] [CHS] [PV]		
12 [RCL] [n] [−] [n]		
0 [PMT] [FV] [STO] 0		
[RCL] [n] 3 [−] [n]		
1 [PMT] 0 [PV] [FV]		
[STO] [+] 0		
0 [PMT] 12 [n]		
100 [PV] [FV]		
[RCL] [PV] [+] [CHS]		
[f] CLEAR [FIN] [i]		
5 [n] 100000 [PV]		
[PMT] [RCL] 0 [÷]	**3,119.89**	Monthly payment in arrears.

Savings

Initial Deposit With Periodic Deposits

Given an initial deposit into a savings account, and a series of periodic deposits coincident with the compounding period, the future value (or accumulated amount) may be calculated as follows:

1. Press [g] [END] and press [f] CLEAR [FIN].

2. Key in the initial investment and press [CHS] [PV].

3. Key in the number of additional periodic deposits and press [n].

4. Key in the periodic interest rate and press [i].

5. Key in the periodic deposit and press [CHS] [PMT].

6. Press [FV] to determine the value of the account at the end of the time period.

Example: You have just opened a savings account with a $200 deposit. If you deposit $50 a month, and the account earns 5¼% compounded monthly, how much will you have in 3 years?

Keystrokes	Display	
[g] [END]		
[f] CLEAR [FIN]		
200 [CHS] [PV]		
3 [g] [12×]		
5.25 [g] [12÷]		
50 [CHS] [PMT] [FV]	**2,178.94**	Value of the account.

Note: If the periodic deposits do not coincide with the compounding periods, the account must be evaluated in another manner. First, find the future value of the initial deposit and store it. Then use the procedure for Compounding Periods Different from Payment Periods to calculate the future value of the periodic deposits. Recall the future value of the initial deposit and add to obtain the value of the account.

Number of Periods to Deplete a Savings Account or to Reach a Specified Balance

Given the current value of a savings account, the periodic interest rate, the amount of the periodic withdrawal, and a specified balance, this procedure determines the number of periods to reach that balance (the balance is zero if the account is depleted).

1. Press [g] [END] and press [f] CLEAR [FIN].
2. Key in the value of the savings account and press [CHS] [PV].
3. Key in the periodic interest rate and press [i].
4. Key in the amount of the periodic withdrawal and press [PMT].
5. Key in the amount remaining in the account and press [FV]. This step may be omitted if the account is depleted (FV = 0).
6. Press [n] to determine the number of periods to reach the desired balance.

Example: Your savings account presently contains $18,000 and earns 5½% compounded monthly. You wish to withdraw $300 a month until the account is depleted. How long will this take? If you wish to reduce the account to $5,000, how many withdrawals can you make?

Keystrokes	Display	
[g] [END]		
[f] CLEAR [FIN]		
18000 [CHS] [PV]		
5.5 [g] [12÷]		
300 [PMT] [n]	**71.00**	Months to deplete account.

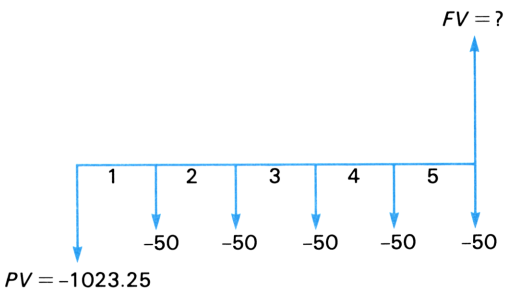
Keystrokes	Display	
5000 [FV] [n]	**53.00**	Months to reduce the account to $5,000.

Periodic Deposits and Withdrawals

This section is presented as a guideline for evaluating a savings plan when deposits and withdrawals occur at irregular intervals. One problem is given, and a step-by-step method for setting-up and solving the problem is presented:

Example: You are presently depositing $50 at the end of each month into a local savings and loan, earning 5½% compounded monthly. Your current balance is $1023.25. How much will you have accumulated in 5 months?

The cash flow diagram looks like this:



Keystrokes	Display	
[g] [END]		
[f] CLEAR [FIN]		
50 [CHS] [PMT]		
5.5 [g] [12÷]		
1023.25 [CHS] [PV]		
5 [n] [FV]	**1,299.22**	Amount in account.

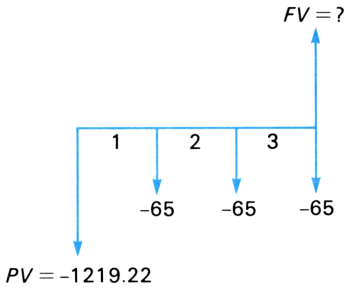
Now suppose that at the beginning of the 6th month you withdraw $80.

What is the new balance?

Keystrokes	Display	
80 [−]	**1,219.22**	New balance.

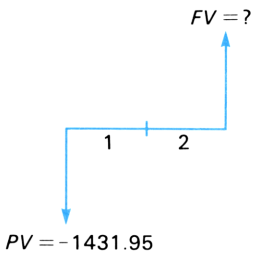
You increase your monthly deposit to $65. How much will you have in 3 months?

The cash flow diagram looks like this:



Keystrokes	Display	
[CHS] [PV]		
65 [CHS] [PMT]		
3 [n] [FV]	**1,431.95**	Account balance.

Suppose that for 2 months you decide not to make a periodic deposit. What is the balance in the account?



Keystrokes	Display	
[CHS] [PV] 2 [n]		
0 [PMT] [FV]	**1,445.11**	Account balance.

This type of procedure may be continued for any length of time, and may be modified to meet the user's particular needs.

Savings Account Compounded Daily

This HP-12C program determines the value of a savings account when interest is compounded daily, based on a 365 day year. The user is able to calculate the total amount remaining in the account after a series of transactions on specified dates.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		g △DYS	19- 43 26
f CLEAR PRGM	00-	n	20- 11
CHS	01- 16	FV	21- 15
PV	02- 13	f RND	22- 42 14
R↓	03- 33	FV	23- 15
3	04- 3	ENTER	24- 36
6	05- 6	RCL PV	25- 45 13
5	06- 5	+	26- 40
÷	07- 10	STO + 3	27-44 40 3
i	08- 12	RCL FV	28- 45 15
R↓	09- 33	RCL 2	29- 45 2
STO 0	10- 44 0	+	30- 40
RCL PV	11- 45 13	CHS	31- 16
CHS	12- 16	PV	32- 13
R/S	13- 31	RCL 1	33- 45 1
STO 2	14- 44 2	STO 0	34- 44 0
R↓	15- 33	RCL PV	35- 45 13
STO 1	16- 44 1	CHS	36- 16
RCL 0	17- 45 0	g GTO 13	37-43,33 13
RCL 1	18- 45 1	f P/R	

	REGISTERS		
n: △days	i: i/365	PV: Used	PMT: 0
FV: Used	R_0: Initial date	R_1: Next date	R_2: $ amount
R_3: Interest	R_4—R_4 Unused		

1. Key in the program.
2. Press [f] CLEAR [REG] and press [g] [M.DY].
3. Key in the date (MM.DDYYYY) of the first transaction and press [ENTER].
4. Key in the annual nominal interest rate as a percentage and press [ENTER].
5. Key in the amount of the initial deposit and press [R/S].
6. Key in the date of the next transaction and press [ENTER].
7. Key in the amount of the transaction (positive for money deposited, negative for cash withdrawn) and press [R/S] to determine the amount in the account.
8. Repeat steps 6 and 7 for subsequent transactions.
9. To see the total interest to date, press [RCL] 3.
10. For a new case press [f] [PRGM] and go to step 2.

Example: Compute the amount remaining in this 5.25% account after the following transactions:

1. January 19, 1981 deposit $125.00
2. February 24, 1981 deposit $60.00
3. March 16, 1981 deposit $70.00
4. April 6, 1981 withdraw $50.00
5. June 1, 1981 deposit $175.00
6. July 6, 1981 withdraw $100.00

Keystrokes	Display	
[f] CLEAR [REG]		
[g] [M.DY]		
1.191981 [ENTER]		
5.25 [ENTER]		
125 [R/S]	**125.00**	Initial deposit.
2.241981 [ENTER]		
60 [R/S]	**185.65**	Balance in account February 24, 1981.
3.161981 [ENTER]		
70 [R/S]	**256.18**	Balance in account March 16, 1981.

Keystrokes	Display	
4.061981 [ENTER] 50 [CHS] [R/S]	**206.95**	Balance in account April 6, 1981.
6.011981 [ENTER] 175 [R/S]	**383.62**	Balance in account June 1, 1981.
7.061981 [ENTER] 100 [CHS] [R/S]	**285.56**	Balance in account July 6, 1981.
[RCL] 3	**5.56**	Total interest.

Compounding Periods Different From Payment Periods

In financial calculations involving a series of payments equally spaced in time with periodic compounding, both periods of time are normally equal and coincident. This assumption is preprogrammed into the HP-12C.

In savings plans however, money may become available for deposit or investment at a frequency different from the compounding frequencies offered. The HP-12C can easily be used in these calculations. However, because of the assumptions mentioned, the periodic interest rate must be adjusted to correspond to an equivalent rate for the payment period.

Payments deposited for a partial compounding period will accrue simple interest for the remainder of the compounding period. This is often the case, but may not be true for all institutions.

These procedures present solutions for future value, payment amount, and number of payments. In addition, it should be noted that only annuity due (payments at the beginning of payment period) calculations are shown since this is most common in savings plan calculations.

To calculate the equivalent payment period interest rate, information is entered as follows:

1. Press [g] [BEG] and press [f] CLEAR [FIN].
2. Key in the annual interest rate (as a percent) and press [ENTER].
3. Key in the number of compounding periods per year and press [n] [÷] [i].
4. Key in 100 and press [PV] [FV].
5. Key in the number of payments (deposits) per year and press [n] [i] [f] CLEAR [FIN] [i].

The interest rate which corresponds to the payment period is now in register "i" and you are ready to proceed.

Example 1: Solving for future value.

Starting today you make monthly deposits of $25 into an account paying 5% compounded daily (365-day basis). At the end of 7 years, how much will you receive from the account?

Keystrokes	Display	
[g] [BEG]		
[f] CLEAR [FIN]		
5 [ENTER]		
365 [n] [÷] [i]		
100 [PV] [FV]		
12 [n] [i]		
[f] CLEAR [FIN] [i]	**0.42**	Equivalent periodic interest rate.
7 [g] [12×]		
25 [CHS] [PMT]		
[FV]	**2,519.61**	Future value.

Example 2: Solving for payment amount.

For 8 years you wish to make weekly deposits in a savings account paying 5.5% compounded quarterly. What amount must you deposit each week to accumulate $6000?

Keystrokes	Display	
[g] [BEG]		
[f] CLEAR [FIN]		
5.5 [ENTER]		
4 [n] [÷] [i]		
100 [PV] [FV]		
52 [n] [i]		
[f] CLEAR [FIN] [i]	**0.11**	Equivalent periodic interest rate.
8 [ENTER] 52 [×] [n]		
6000 [FV] [PMT]	**−11.49**	Periodic payment.

Example 3: Solving for number of payment periods.

You can make weekly deposits of $10 into an account paying 5.25% compounded daily (365-day basis). How long will it take you to accumulate $1000?

Keystrokes	Display	
[g] [BEG]		
[f] CLEAR [FIN]		
5.25 [ENTER]		
365 [n] [÷] [i]		
100 [PV] [FV]		
52 [n] [i]		
[f] CLEAR [FIN] [i]	**0.10**	Equivalent periodic interest rate.
10 [CHS] [PMT]		
1000 [FV] [n]	**96.00**	Weeks.

Investment Analysis

Lease vs. Purchase

An investment decision frequently encountered is the decision to lease or purchase capital equipment or buildings. Although a thorough evaluation of a complex acquisition usually requires the services of a qualified accountant, it is possible to simplify a number of assumptions to produce a first approximation.

The following HP-12C program assumes that the purchase is financed with a loan and that the loan is made for the term of the lease. The tax advantages of interest paid, depreciation, and the investment credit which accrues from ownership are compared to the tax advantage of treating the lease payment as an expense. The resulting cash flows are discounted to the present at the firms after-tax cost of capital.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		RCL PV	11- 45 13
f CLEAR PRGM	00-	STO 9	12- 44 9
−	01- 30	RCL PMT	13- 45 14
1	02- 1	STO . 0	14-44 48 0
STO + 0	03-44 40 0	RCL n	15- 45 11
RCL 3	04- 45 3	STO . 1	16-44 48 1
−	05- 30	RCL i	17- 45 12
×	06- 20	STO . 2	18-44 48 2
STO 8	07- 44 8	RCL 5	19- 45 5
1	08- 1	PV	20- 13
f AMORT	09- 42 11	RCL 6	21- 45 6
STO 1	10- 44 1	n	22- 11

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
RCL 7	23- 45 7	RCL 3	37- 45 3
i	24- 12	×	38- 20
RCL 0	25- 45 0	RCL PMT	39- 45 14
f SOYD	26- 42 24	−	40- 30
STO + 1	27-44 40 1	RCL 8	41- 45 8
RCL 9	28- 45 9	−	42- 30
PV	29- 13	RCL 4	43- 45 4
RCL · 0	30-45 48 0	RCL 0	44- 45 0
PMT	31- 14	y^x	45- 21
RCL · 1	32-45 48 1	÷	46- 10
n	33- 11	STO + 2	47-44 40 2
RCL · 2	34-45 48 2	g GTO 00	48-43,33 00
i	35- 12	f P/R	
RCL 1	36- 45 1		

	REGISTERS		
n: Used	i: Used	PV: Used	PMT: Used
FV: 0	R_0: Used	R_1: Used	R_2: Purch. adv.
R_3: Tax	R_4: Discount	R_5: Dep. value	R_6: Dep. life
R_7: Factor (DB)	R_8: Used	R_9: Used	R_0: Used
R_1: Used	R_2: Used	R_3: Unused	

Instructions:

1. Key in the program.
   -Select depreciation function and key in at line 26.
2. Press g END and press f CLEAR REG.
3. Input the following information for the purchase of the loan:
   -Key in the number of years for amortization and press n.
   -Key in the annual interest rate and press i.

-Key in the loan amount (purchase price) and press [CHS] [PV].

-Press [PMT] to find the annual payments.

4. Key in the marginal effective tax rate* and press [STO] 3.
5. Key in the discount rate or cost of capital* and press [ENTER] 1 [+] [STO] 4.
6. Key in the depreciable value and press [STO] 5.
7. Key in the depreciable life and press [STO] 6.
8. For declining balance depreciation, key in the depreciation factor (as a percentage) and press [STO] 7.
9. Key in the total first lease payment (including any advance payments) and press [ENTER] 1 [RCL] 3 [−] [×] [STO] 2.
10. Key in the first year's maintenance expense that would be anticipated if the asset was owned and press [ENTER]. If the lease contract does not include maintenance, then it is not a factor in the lease vs. purchase decision and 0 expense should be used.
11. Key in the next lease payment and press [R/S]. During any year in which a lease payment does not occur (e.g., the last several payments of an advance payment contract), use 0 for the payment.
12. Repeat steps 10 and 11 for all maintenance expenses and lease payments over the term of the analysis.

    Optional—If the investment tax credit is taken, key in the amount of the credit after finishing steps 10 and 11 for the year in which the credit is taken and press [g] [GTO] 43 [R/S]. Continue steps 10 and 11 for the remainder of the term.
13. After all lease payments and expenses have been entered (steps 10 and 11), key in the lease buy back option and press [ENTER] 1 [RCL] 3 [−] [×] [g] [GTO] 43 [R/S]. If no buy back option exists, use the estimated salvage value of the purchased equipment at the end of the term.
14. To find the net advantage of owning, press [RCL] 2. A negative value represents a net lease advantage.

* Key in as a decimal (e.g., 5% as .05).

Example: Home Style Bagel Company is evaluating the acquisition of a mixer which can be leased for $1700 a year with the first and last payments in advance and a $750 buy back option at the end of 10 years (maintenance is included).

The same equipment could be purchased for $10,000 with a 12% loan amortized over 10 years. Ownership maintenance is estimated to be 2% of the purchase price per year for the first four years. A major overhaul is predicted for the 5th year at a cost of $1500. Subsequent yearly maintenance of 3% is estimated for the remainder of the 10 year term. The company would use sum-of-the-years-digits depreciation on a 10 year life with $1500 salvage value. An accountant informs management to take the 10% capital investment tax credit at the end of the second year and to figure the cash flows at a 48% tax rate. The after tax cost of capital (discounting rate) is 5 per cent.

Because lease payments are made in advance and standard loan payments are made in arrears the following cash flow schedule is appropriate for a lease with the last payment in advance.

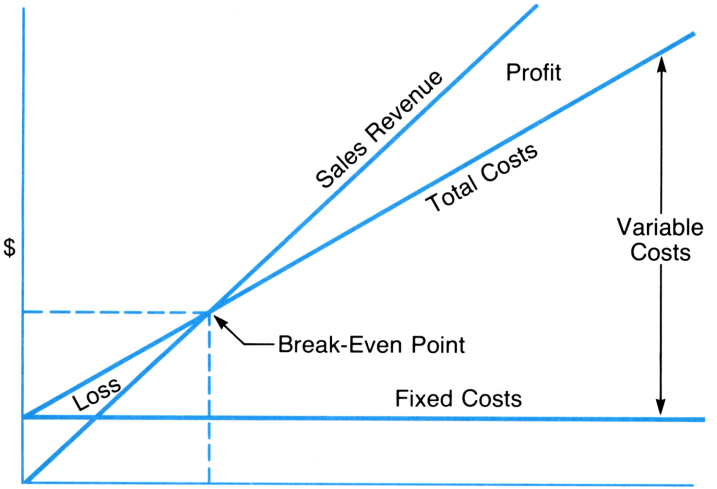
Year	Maintenance	Lease Payment	Tax Credit	Buy Back
0		1700 + 1700		
1	200	1700		
2	200	1700	1000	
3	200	1700		
4	200	1700		
5	1500	1700		
6	300	1700		
7	300	1700		
8	300	1700		
9	300	0		
10	300	0		750

Keystrokes	Display	
g END		
f CLEAR REG	0.00	
10 n 12 i		Always use negative loan amount.
10000 CHS PV	-10,000.00	
PMT	1,769.84	Purchase payment.
.48 STO 3	0.48	Marginal tax rate.

Keystrokes	Display	
.05 [ENTER]		
1 [+] [STO] 4	**1.05**	Discounting factor.
10000 [ENTER]		
1500 [−] [STO] 5	**8,500.00**	Depreciable value.
10 [STO] 6	**10.00**	Depreciable life.
1700 [ENTER] [+]	**3,400.00**	1st lease payment.
1 [RCL] 3 [−] [×] [STO] 2	**1,768.00**	After-tax expense.
200 [ENTER]		
1700 [R/S]	**312.36**	Present value of 1st year's net purchase.
200 [ENTER]		
1700 [R/S]	**200.43**	2nd year's advantage.
1000 [g] [GTO] 43	**1,000.00**	Tax credit.
[R/S]	**907.03**	Present value of tax credit.
200 [ENTER]		
1700 [R/S]	**95.05**	3rd year.
200 [ENTER]		
1700 [R/S]	**−4.38**	4th year.
1500 [ENTER]		
1700 [R/S]	**−628.09**	5th year.
300 [ENTER]		
1700 [R/S]	**−226.44**	6th year.
300 [ENTER]		
1700 [R/S]	**−309.48**	7th year.
300 [ENTER]		
1700 [R/S]	**−388.81**	8th year.
300 [ENTER] 0 [R/S]	**−1,034.72**	9th year.
300 [ENTER] 0 [R/S]	**−1,080.88**	10th year.
750 [ENTER]	**750.00**	Buy back.
1 [RCL] 3 [−] [×]	**390.00**	After-tax buy back expense.
[g] [GTO] 43 [R/S]	**239.43**	Present value.
[RCL] 2	**−150.49**	Net lease advantage.

Break-Even Analysis

Break-even analysis is basically a technique for analyzing the relationships among fixed costs, variable costs, and income. Until the break-even point is reached at the intersection of the total income and total cost lines, the producer operates at a loss. After the break-even point each unit produced and sold makes a profit. Break-even analysis may be represented as follows:



The variables are: fixed costs (F), sales price per unit (P), variable costs per unit (V), number of units sold (U), and gross profit (GP). One can readily evaluate GP, U or P given the other four variables. To calculate the break-even volume, simply let the gross profit equal zero and calculate the number of units sold (U).

To calculate the break-even volume:

1. Key in the fixed costs and press [ENTER].
2. Key in the unit price and press [ENTER].
3. Key in the variable costs per unit and press [−].
4. Press [÷] to calculate the break-even volume.

To calculate the gross profit at a given volume:

1. Key in the unit price and press [ENTER].
2. Key in the variable costs per unit and press [−].
3. Key in the number of units sold and press [×].
4. Key in the fixed costs and press [−] to calculate the gross profit.

To calculate the sales volume needed to achieve a specified gross profit:

1. Key in the desired gross profit and press [ENTER].
2. Key in the fixed costs and press [+].
3. Key in the sales price per unit and press [ENTER].
4. Key in the variable costs per unit and press [−].
5. Press [÷] to calculate the sales volume.

To calculate the required sales price to achieve a given gross profit at a specified sales volume:

1. Key in the fixed costs and press [ENTER].
2. Key in the gross desired and press [+].
3. Key in the specified sales volume in units and press [÷].
4. Key in the variable costs per unit and press [+] to calculate the required sales price per unit.

Example 1: The E.Z. Sells company markets textbooks on salesmanship. The fixed costs involved in setting up to print the books are $12,000. The variable costs per copy, including printing and marketing the books, are $6.75 per copy. The sales price per copy is $13.00. How many copies must be sold to break even?

Keystrokes	Display	
12000 [ENTER]	**12,000.00**	Fixed cost.
13 [ENTER]	**13.00**	Sales price.
6.75 [−] [÷]	**1,920.00**	Break-even volume.

Find the gross profit if 2500 units are sold.

13 [ENTER]	**13.00**	Sales price.
6.75 [−]	**6.25**	Profit per unit.
2500 [×]	**15,625.00**	
12000 [−]	**3,625.00**	Gross profit.

If a gross profit of $4,500 is desired at a sales volume of 2500 units, what should the sales price be?

12000 [ENTER]	**12,000.00**	Fixed cost.
4500 [+]	**16,500.00**	
2500 [÷]	**6.60**	
6.75 [+]	**13.35**	Sales price per unit to achieve desired gross profit.

For repeated calculations the following HP-12C program can be used:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[+]	12- 40
[f] CLEAR [PRGM]	00-	[x≷y]	13- 34
[RCL] 3	01- 45 3	[÷]	14- 10
[RCL] 2	02- 45 2	[g] [GTO] 00	15-43,33 00
[−]	03- 30	[RCL] 1	16- 45 1
[g] [GTO] 00	04-43,33 00	[RCL] 5	17- 45 5
[RCL] 4	05- 45 4	[+]	18- 40
[×]	06- 20	[RCL] 4	19- 45 4
[RCL] 1	07- 45 1	[÷]	20- 10
[−]	08- 30	[RCL] 2	21- 45 2
[g] [GTO] 00	09-43,33 00	[+]	22- 40
[RCL] 5	10- 45 5	[g] [GTO] 00	23-43,33 00
[RCL] 1	11- 45 1	[f] [P/R]	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: *F*	R_2: *V*
R_3: *P*	R_4: *U*	R_5: *GP*	R_6—$R_{.6}$ Unused

1. Key in the program and store the known variables as follows:
   a. Key in the fixed costs, *F* and press [STO] 1.
   b. Key in variable costs per unit, *V* and press [STO] 2.
   c. Key in the unit price, *P* (if known) and press [STO] 3.
   d. Key in the sales volume, *U*, in units (if known) and press [STO] 4.
   e. Key in the gross profit, *GP* (if known) and press [STO] 5.
2. To calculate the sales volume to achieve a desired gross profit:
   a. Store values as shown in 1a., 1b., and 1c.
   b. Key in the desired gross profit (zero for break-even) and press [STO] 5.
   c. **Press [R/S] [g] [GTO] 10 [R/S] to calculate the required volume.**
3. To calculate the gross profit at a given sales volume.
   a. Store values as shown in 1a., 1b., 1c., and 1d.
   b. **Press [R/S] [g] [GTO] 05 [R/S] to calculate the gross profit.**
4. To calculate the sales price per unit to achieve a desired gross profit at a specified sales volume:
   a. Store values as shown in 1a., 1b., 1d. and 1e.
   b. Press [g] [GTO] 16 [R/S] to calculate the required sales price.

Example 2: A manufacturer of automotive accessories produces rearview mirrors. A new line of mirrors will require fixed costs of $35,000 to produce. Each mirror has a variable cost of $8.25. The price of the mirrors is tentatively set at $12.50 each. What volume is needed to break-even?

Keystrokes	Display	
35000 [STO] 1	**35,000.00**	Fixed cost.
8.25 [STO] 2	**8.25**	Variable cost.
12.5 [STO] 3	**12.50**	Sales price.
0 [STO] 5	**0.00**	
[R/S] [g] [GTO] 10 [R/S]	**8,235.29**	**Break-even volume is between 8,235 and 8,236 units.**

What would be the gross profit if the price is raised to $14.00 and the sales volume is 10,000 units?

Keystrokes	Display	
14 [STO] 3	**14.00**	Sales price.
		F and *V* are already stored.
10000 [STO] 4	**10,000.00**	Volume.
[R/S] [g] [GTO] 05 [R/S]	**22,500.00**	Gross profit.

Operating Leverage

The degree of operating leverage (*OL*) at a point is defined as the ratio of the percentage change in net operating income to the percentage change in units sold. The greatest degree of operating leverage is found near the break-even point where a small change in sales may produce a very large increase in profits. Likewise, firms with a small degree of operating leverage are operating farther from the break-even point, and they are relatively insensitive to changes in sales volume.

The necessary inputs to calculate the degree of operating leverage are fixed costs (*F*), sales price per unit (*P*), variable costs per unit (*V*), and number of units (*U*).

The operating leverage may be readily calculated as follows:

1. **Key in the sales price per unit and press [ENTER].**
2. **Key in the variable costs per unit and press [−].**
3. **Key in the number of units and press [×] [ENTER] [ENTER].**
4. **Key in the fixed cost and press [−] [÷] to obtain the operating leverage.**

Example 1: For the data given in example 1 of the Break-Even Analysis section, calculate the operating leverage at 2000 units and at 5000 units when the sales price is $13 a copy.

Keystrokes	Display	
13 [ENTER]	**13.00**	Price per copy.
6.75 [−]	**6.25**	Profit per copy.
2000 [×] [ENTER] [ENTER] 12000 [−] [÷]	**25.00**	Close to break-even point.
13 [ENTER]	**13.00**	Price per copy.
6.75 [−]	**6.25**	Profit per copy.
5000 [×] [ENTER] [ENTER] 12000 [−] [÷]	**1.62**	Operating further from the break-even point and less sensitive to changes in sales volume.

For repeated calculations the following HP-12C program can be used:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[ENTER]	06– 36
[f] CLEAR [PRGM]	00–	[RCL] 1	07– 45 1
[RCL] 3	01– 45 3	[−]	08– 30
[RCL] 2	02– 45 2	[÷]	09– 10
[−]	03– 30	[g] [GTO] 00	10–43,33 00
[×]	04– 20	[f] [P/R]	
[ENTER]	05– 36		

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: *F*	R_2: *V*
R_3: *P*	R_4—R_8 Unused		

1. Key in the program.
2. Key in and store input variables F, V, and P as described in the Break-Even Analysis program.
3. Key in the sales volume and press [R/S] to calculate the operating leverage.
4. To calculate a new operating leverage at a different sales volume, key in the new sales volume and press [R/S].

Example 2: For the figures given in example 2 of the Break-Even Analysis section, calculate the operating leverage at a sales volume of 9,000 and 20,000 units if the sales price is $12.50 per unit.

Keystrokes	Display	
35000 [STO] 1	**35,000.00**	Fixed costs.
8.25 [STO] 2	**8.25**	Variable cost.
12.5 [STO] 3	**12.50**	Sales price.
9000 [R/S]	**11.77**	Operating leverage near break-even.
20000 [R/S]	**1.70**	Operating leverage further from break-even.

Profit and Loss Analysis

The HP-12C may be programmed to perform simplified profit and loss analysis using the standard profit income formula and can be used as a dynamic simulator to quickly explore ranges of variables affecting the profitability of a marketing operation.

The program operates with net income return and operating expense as percentages. Both percentage figures are based on *net* sales price.

It may also be used to simulate a company wide income statement by replacing list price with gross sales and manufacturing cost with cost of goods sold.

Any of the five variables; a) list price, b) discount (as a percentage of list price), c) manufacturing cost, d) operating expense (as a percentage), e) net profit after tax (as a percentage), may be calculated if the other four are known.

Since the tax rate varies from company to company, provision is made for inputting your applicable tax rate. (The example problem uses a tax rate of 48%).

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		1	24- 1
f CLEAR PRGM	00-	+	25- 40
RCL 5	01- 45 5	RCL 0	26- 45 0
RCL 6	02- 45 6	×	27- 20
÷	03- 10	g GTO 00	28-43,33 00
RCL 4	04- 45 4	÷	29- 10
+	05- 40	CHS	30- 16
CHS	06- 16	RCL 1	31- 45 1
RCL 0	07- 45 0	+	32- 40
+	08- 40	RCL 1	33- 45 1
RCL 0	09- 45 0	÷	34- 10
÷	10- 10	RCL 0	35- 45 0
g GTO 00	11-43,33 00	×	36- 20
RCL 3	12- 45 3	g GTO 00	37-43,33 00
RCL 1	13- 45 1	RCL 5	38- 45 5
RCL 2	14- 45 2	RCL 6	39- 45 6
RCL 0	15- 45 0	÷	40- 10
÷	16- 10	−	41- 30
CHS	17- 16	g GTO 00	42-43,33 00
1	18- 1	RCL 4	43- 45 4
+	19- 40	−	44- 30
×	20- 20	RCL 6	45- 45 6
R/S	21- 31	×	46- 20
÷	22- 10	g GTO 00	47-43,33 00
CHS	23- 16	f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: 100	R_1: list price	R_2: % discount
R_3: mfg. cost	R_4: % op. exp.	R_5: % net profit	R_6: 1—% tax
R_7—$R_{.3}$ Unused			

1. Key in the program and press [f] CLEAR [REG], then key in 100 and press [STO] 0.
2. Key in 1 and press [ENTER], then key in your appropriate tax rate as a decimal and press [−] [STO] 6.
3. a. Key in the list price in dollars (if known) and press [STO] 1.
   b. Key in the discount in percent (if known) and press [STO] 2.
   c. Key in the manufacturing cost in dollars (if known) and press [STO] 3.
   d. Key in the operating expense in percent (if known) and press [STO] 4.
   e. Key in the net profit after tax in percent (if known) and press [STO] 5.
4. To calculate list price:
   a. Do steps 2 and 3b, c, d, e, above.
   b. Press [RCL] 3 [R/S] [÷] 1 [g] [GTO] 14 [R/S] [÷] [g] [GTO] 00.
5. To calculate discount:
   a. Do steps 2 and 3a, c, d, e, above.
   b. Press [RCL] 3 [R/S] [g] [GTO] 29 [R/S].
6. To calculate manufacturing cost:
   a. Do steps 2 and 3a, b, d, e, above.
   b. Press [g] [GTO] 13 [R/S] [g] [GTO] 01 [R/S] [×].
7. To calculate operating expense:
   a. Do steps 2 and 3a, b, c, e, above.
   b. Press [g] [GTO] 12 [R/S] [R/S] [g] [GTO] 38 [R/S].

8. To calculate net profit after tax:
   a. Do steps 2 and 3a, b, c, d, above.
   b. Press [g] [GTO] 12 [R/S] [R/S] [g] [GTO] 43 [R/S].

Example: What is the net return on an item that is sold for $11.98, discounted through distribution an average of 35% and has a manufacturing cost of $2.50? The standard company operating expense is 32% of net shipping (sales) price and the tax rate is 48%.

Keystrokes	Display	
[f] CLEAR [REG]		
100 [STO] 0	**100.00**	
1 [ENTER] .48 [−] [STO] 6	**0.52**	48% tax rate.
11.98 [STO] 1	**11.98**	List price ($).
35 [STO] 2	**35.00**	Discount (%).
2.50 [STO] 3	**2.50**	Manufacturing cost ($).
32 [STO] 4	**32.00**	Operating expenses (%).
[g] [GTO] 12 [R/S] [R/S]	**67.90**	
[g] [GTO] 43 [R/S]	**18.67**	Net profit (%).

If manufacturing expenses increase to $3.25, what is the effect on net profit?

3.25 [STO] 3	**3.25**	Manufacturing cost.
[g] [GTO] 12 [R/S] [R/S]	**58.26**	
[g] [GTO] 43 [R/S]	**13.66**	Net profit reduced to 13.66%.

If the manufacturing cost is maintained at $3.25, how high could the overhead (operating expense) be before the product begins to lose money?

0 [STO] 5	**0.00**	
[g] [GTO] 12 [R/S] [R/S]	**58.26**	
[g] [GTO] 38 [R/S]	**58.26**	Maximum operating expense (%).

At 32% operating expense and $3.25 manufacturing cost, what should the list price be to generate 20% net profit?

20 STO 5	**20.00**	
RCL 3 R/S ÷	**11.00**	
1 g GTO 14 R/S ÷	**16.93**	List price ($).

What reduction in manufacturing cost would achieve the same result without necessitating an increase in list price above $11.98?

g GTO 13 R/S	**7.79**	
g GTO 01 R/S ×	**2.30**	Manufacturing cost ($).

Securities

After-Tax Yield

The following HP-12C program calculates the after-tax yield to maturity of a bond held for more that one year. The calculation assumes an actual/actual day basis. For after-tax computations, the interest or coupon payments are considered income, while the difference between the bond or note face value and its purchase price is considered capital gains.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		STO 0	16- 44 0
f CLEAR PRGM	00-	RCL 3	17- 45 3
f CLEAR FIN	01- 42 34	RCL 5	18- 45 5
STO 7	02- 44 7	%	19- 25
R↓	03- 33	−	20- 30
STO 6	04- 44 6	RCL 0	21- 45 0
RCL 2	05- 45 2	÷	22- 10
RCL 1	06- 45 1	PMT	23- 14
−	07- 30	RCL 1	24- 45 1
RCL 4	08- 45 4	RCL 0	25- 45 0
%	09- 25	÷	26- 10
RCL 2	10- 45 2	PV	27- 13
$x \geq y$	11- 34	RCL 6	28- 45 6
−	12- 30	RCL 7	29- 45 7
EEX	13- 26	f YTM	30- 42 22
2	14- 2	g GTO 00	31-43,33 00
÷	15- 10	f P/R	

	REGISTERS		
n: Unused	i: Yield	PV: Used	PMT: Used
FV: Used	R_0: Used	R_1: Purchase price	R_2: Sales price
R_3: Coupon rate	R_4: Capital rate	R_5: Income rate	R_6: Used
R_7: Used	R_8—$R_{.5}$ Unused		

1. Key in the program.
2. Key in the purchase price and press [STO] 1.
3. Key in the sales price and press [STO] 2.
4. Key in the annual coupon rate (as a percentage) and press [STO] 3.
5. Key in capital gains tax rate (as a percentage) and press [STO] 4.
6. Key in the income tax rate (as a percentage) and press [STO] 5.
7. Press [g] [M.DY].
8. Key in the purchase date (MM.DDYYYY) and press [ENTER].
9. Key in the assumed sell date (MM.DDYYYY) and press [R/S] to find the after-tax yield (as a percentage).
10. For the same bond but different dates return to step 8.
11. For a new case return to step 2.

Example: You can buy a 7% bond on October 1, 1981 for $70 and expect to sell it in 5 years for $90. What is your net (after-tax) yield over the 5-year period if interim coupon payments are considered as income, and your tax bracket is 50%?

(One-half of the long term capital gain is taxable at 50%, so the tax on capital gains alone is 25%).

Keystrokes	Display	
70 [STO] 1	70.00	Purchase price.
90 [STO] 2	90.00	Selling price.
7 [STO] 3	7.00	Annual coupon rate.
25 [STO] 4	25.00	Capital gains tax rate.
50 [STO] 5	50.00	Income tax rate.
[g] [M.DY]		
10.011981 [ENTER]	10.01	Purchase date.
10.011986 [R/S]	8.53	% after-tax yield.

Discounted Notes

A note is a written agreement to pay a sum of money plus interest at a certain rate. Notes do not have periodic coupons, since all interest is paid at maturity.

A discounted note is a note that is purchased below its face value. The following HP-12C program finds the price and/or yield* of a discounted note.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[x≷y]	09– 34
[f] CLEAR [PRGM]	00–	[−]	10– 30
[RCL] 1	01– 45 1	[RCL] 4	11– 45 4
[RCL] 2	02– 45 2	[×]	12– 20
[g] [△DYS]	03– 43 26	[STO] 5	13– 44 5
[RCL] 3	04– 45 3	[R/S]	14– 31
[÷]	05– 10	[RCL] 1	15– 45 1
[RCL] 5	06– 45 5	[RCL] 2	16– 45 2
[%]	07– 25	[g] [△DYS]	17– 43 26
1	08– 1	[RCL] 3	18– 45 3

* The yield is a reflection of the return on an investment.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[x≷y]	19– 34	[×]	26– 20
[÷]	20– 10	[EEX]	27– 26
[RCL] 4	21– 45 4	2	28– 2
[RCL] 5	22– 45 5	[×]	29– 20
[÷]	23– 10	[g] [GTO] 00	30–43,33 00
1	24– 1	[f] [P/R]	
[−]	25– 30		

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: Settl. date	R_2: Mat. Date
R_3: 360 or 365	R_4: redemp. value	R_5: dis./price	R_6—R_5 Unused

1. Key in the program.
2. Press [g] [M.DY].
3. Key in the settlement date (MM.DDYYYY) and press [STO] 1.
4. Key in the maturity date (MM.DDYYYY) and press [STO] 2.
5. Key in the number of days in a year (360 or 365) and press [STO] 3.
6. Key in the redemption value per $100 and press [STO] 4.
7. To calculate the purchase price:
   a. Key in the discount rate and press [STO] 5.
   b. Press [R/S] to calculate the purchase price.
   c. Press [R/S] to calculate the yield.
   d. For a new case, go to step 3.
8. To calculate the yield when the price is known:
   a. Key in the price and press [STO] 5.
   b. Press [g] [GTO] 15 [R/S] to calculate the yield.
   c. For a new case, go to step 3.

Example 1: Calculate the price and yield on this U.S. Treasury Bill: settlement date October 8, 1980; maturity date March 21, 1981; discount rate 7.80%. Compute on 360 day basis.

Keystrokes	Display	
[g] [M.DY]		
10.081980 [STO] 1	**10.08**	Settlement date.
3.211981 [STO] 2	**3.21**	Maturity date.
360 [STO] 3	**360.00**	Actual day basis.
100 [STO] 4	**100.00**	Redemption value per $100.
7.8 [STO] 5	**7.80**	Discount rate.
[R/S]	**96.45**	Price.
[R/S]	**8.09**	Yield.

Example 2: Determine the yield of this security: settlement date June 25, 1980; maturity date September 10, 1980; price $99.45; redemption value $101.33. Assume 360 day basis.

Keystrokes	Display	
6.251980 [STO] 1	**6.25**	Settlement date.
9.101980 [STO] 2	**9.10**	Maturity date.
360 [STO] 3	**360.00**	360 day basis.
101.33 [STO] 4	**101.33**	Redemption value per $100.
99.45 [STO] 5	**99.45**	Price.
[g] [GTO] 15 [R/S]	**8.84**	Yield.

Forecasting

Simple Moving Average

Moving averages are often useful in recording or forecasting sales figures, expenses or manufacturing volume. There are many different types of moving average calculations. An often used, straightforward method of calculation is presented here.

In a moving average a specified number of data points are averaged. When there is a new piece of input data, the oldest piece of data is discarded to make room for the latest input. This replacement scheme makes the moving average a valuable tool in following trends. The fewer the number of data points, the more trend sensitive the average becomes. With a large number of data points, the average behaves more like a regular average, responding slowly to new input data.

A simple moving average may be calculated with your HP-12C as follows:

1. Press [f] CLEAR [REG].
2. Key in the first m data points (where m is the number of data points in the average) and press [Σ+] after each entry.
3. Press [g] [$\bar{x}$] to obtain the first average.
4. Key in the oldest (first) value entered in step 2 and press [g] [Σ−].
5. Key in the newest data point ($m + 1$) and press [Σ+].
6. Press [g] [$\bar{x}$] to obtain the next value of the moving average.
7. Repeat steps 4 through 6 for the remaining data.

Example: An electronics sales firm wishes to calculate a 3-month moving average for the dollar volume of components sold each month. Sales for the first six months of this year were:

January	$211,570	April	$131,760
February	112,550	May	300,500
March	190,060	June	271,120

Keystrokes	Display	
f CLEAR REG	0.00	
211570 Σ+	1.00	
112550 Σ+	2.00	
190060 Σ+	3.00	
g $\bar{x}$	171,393.33	3-month average for March.
211570 g Σ-	2.00	
131760 Σ+	3.00	
g $\bar{x}$	144,790.00	3-month average for April.
112550 g Σ-	2.00	
300500 Σ+	3.00	
g $\bar{x}$	207,440.00	3-month average for May.
190060 g Σ-	2.00	
271120 Σ+	3.00	
g $\bar{x}$	234,460.00	3-month average for June.

For repeated calculations the following HP-12C program can be used for up to a 12 element moving average:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		RCL 5	11- 45 5
f CLEAR PRGM	00-	STO 4	12- 44 4
RCL 1	01- 45 1	+ 5	13- 40
RCL 2	02- 45 2	RCL 6	14- 45 6
STO 1	03- 44 1	STO 5	15- 44 5
+	04- 40	+ 6	16- 40
RCL 3	05- 45 3	RCL 7	17- 45 7
STO 2	06- 44 2	STO 6	18- 44 6
+ 3	07- 40	+ 7	19- 40
RCL 4	08- 45 4	RCL 8	20- 45 8
STO 3	09- 44 3	STO 7	21- 44 7
+ 4	10- 40	+ 8	22- 40

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
RCL 9	23- 45 9	RCL . 2	32-45 48 2
STO 8	24- 44 8	STO . 1	33-44 48 1
+9	25- 40	+12	34- 40
RCL . 0	26-45 48 0	RCL 0	35- 45 0
STO 9	27- 44 9	÷	36- 10
+10	28- 40	R/S	37- 31
RCL . 1	29-45 48 1	STO m*	38- 44 --
STO . 0	30-44 48 0	g GTO 01	39-43,33 01
+11	31- 40	f P/R	

REGISTERS			
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: m	R_1: X_1	R_2: X_2
R_3: X_3	R_4: X_4	R_5: X_5	R_6: X_6
R_7: X_7	R_8: X_8	R_9: X_9	R_0: X_{10}
$R_{.1}$: X_{11}	$R_{.2}$: X_{12}	R_3—R_4 Unused	

This program can be used for a moving average of 2 to 12 elements. It may be shortened considerably for moving averages with less than 12 elements. To do this, key in the program, as shown, from line 01 until you reach a + superscripted with the number of elements you desire. Key in this line, then skip the rest of the program down to line 35. Then key in lines 35 through 39, being sure to specify the register number at line 38, STO m, corresponding to the number of elements you are using. (For instance, for a 5 element moving average, key in lines 01 through 13 then go to line 35 in the listing and key in the balance of the program. Obviously the program listing line 38, STO m, becomes the displayed line 17, STO 5.)

* At step 38, m = number of elements in the moving average, i.e. for a 5 element moving average line 38 would be STO 5 and for a 12-element moving average line 38 would be STO . 2).

To run the program:

1. Key in the program.
2. Press [f] CLEAR [REG]. Key in the number of elements, m, and press [STO] 0.
3. Key in the first data point and press [STO] 1.
4. Key in the second data point and press [STO] 2.
5. Continue as above, keying in and storing each data point in its appropriate register until m data points have been stored.
6. Press [g] [GTO] 00 [R/S] to calculate the first moving average.
7. Key in the next data point and press [R/S] to calculate the next moving average.
8. Repeat step 7 for each new data point.

Example 2: Calculate the 3-element moving average for the data given in example 1. Your modified program listing will look like this:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[+]	07– 40
[f] CLEAR [PRGM]	00–	[RCL] 0	08– 45 0
[RCL] 1	01– 45 1	[÷]	09– 10
[RCL] 2	02– 45 2	[R/S]	10– 31
[STO] 1	03– 44 1	[STO] 3	11– 44 3
[+]	04– 40	[g] [GTO] 01	12–43,33 01
[RCL] 3	05– 45 3	[f] [P/R]	
[STO] 2	06– 44 2		

Keystrokes	Display
[f] CLEAR [REG]	0.00
3 [STO] 0	3.00
211570 [STO] 1	211,570.00
112550 [STO] 2	112,550.00
190060 [STO] 3	190,060.00

Keystrokes	Display	
[g] [GTO] 00 [R/S]	**171,393.33**	3-month average for March.
131760 [R/S]	**144,790.00**	3-month average for April.
300500 [R/S]	**207,440.00**	3-month average for May.
271120 [R/S]	**234,460.00**	3-month average for June.

Seasonal Variation Factors Based on Centered Moving Averages

Seasonal variation factors are useful concepts in many types of forecasting. There are several methods of developing seasonal moving averages, one of the more common ways being to calculate them as the ratio of the periodic value to a centered moving average for the same period.

For instance, to determine the sales for the 3rd quarter of a given year a centered moving average for that quarter would be calculated from sales figures from the 1st, 2nd, 3rd and 4th quarters of the year and the 1st quarter of the following year. The seasonal variation factor for that 3rd quarter would then be the ratio of the actual sales in the 3rd quarter to the centered moving average for that quarter.

While quarterly seasonal variations are commonly used, the HP-12C can also be programmed to calculate monthly seasonal variations using a centered 12-month moving average. Programs for both of these calculations are represented here:

An HP-12C program to calculate quarterly seasonal variations based on a centered 4-point moving average is:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[RCL] 2	**04– 45 2**
[f] CLEAR [PRGM]	**00–**	[STO] 1	**05– 44 1**
[RCL] 1	**01– 45 1**	[+]	**06– 40**
2	**02– 2**	[RCL] 3	**07– 45 3**
[÷]	**03– 10**	[STO] 2	**08– 44 2**

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[+]	09– 40	4	18– 4
[RCL] 4	10– 45 4	[÷]	19– 10
[STO] 3	11– 44 3	[R/S]	20– 31
[+]	12– 40	[RCL] 2	21– 45 2
[RCL] 5	13– 45 5	[%T]	22– 23
[STO] 4	14– 44 4	[R/S]	23– 31
2	15– 2	[STO] 5	24– 44 5
[÷]	16– 10	[g] [GTO] 01	25–43,33 01
[+]	17– 40	[f] [P/R]	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: X_1	R_2: X_2
R_3: X_3	R_4: X_4	R_5: X_5	R_6—$R_{.6}$ Unused

1. Key in the program.
2. Press [f] CLEAR [REG].
3. Key in the quarterly sales figures starting with the first quarter:
   a. Key in 1st quarter sales and press [STO] 1.
   b. Key in 2nd quarter sales and press [STO] 2.
   c. Key in 3rd quarter sales and press [STO] 3.
   d. Key in 4th quarter sales and press [STO] 4.
   e. Key in the 1st quarter sales for the next year and press [STO] 5.
4. Press [g] [GTO] 00 [R/S] to calculate the centered moving average for the 3rd quarter of the first year.
5. Press [R/S] to calculate the seasonal variation for this quarter.
6. Key in the next quarter's sales and press [R/S] to calculate the moving average for the next quarter.

7. Press [R/S] to calculate the seasonal variation.
8. Repeat steps 6 and 7 for the balance of the data.

Example: Econo-Wise Home Appliance Company had quarterly sales for the years 1978 thru 1980 as follows:

SALES (IN $K)

Quarterly	1st	2nd	3rd	4th
1978	397	376	460	501
1979	455	390	530	560
1980	513	434	562	593

Find the centered 4-quarter moving average and seasonal variation factor for each quarter.

Keystrokes	Display	
[f] CLEAR [REG]		
397 [STO] 1	**397.00**	
376 [STO] 2	**376.00**	
460 [STO] 3	**460.00**	
501 [STO] 4	**501.00**	
455 [STO] 5	**455.00**	
[g] [GTO] 00 [R/S]	**440.75**	Centered 4-element average for 3rd quarter, 1978 seasonal variation factor.
[R/S]	**104.37**	
390 [R/S]	**449.75**	4th quarter, 1978.
[R/S]	**111.40**	
530 [R/S]	**460.25**	1st quarter, 1979.
[R/S]	**98.86**	
560 [R/S]	**476.38**	2nd quarter, 1979.
[R/S]	**81.87**	
513 [R/S]	**491.00**	3rd quarter, 1979.
[R/S]	**107.94**	
434 [R/S]	**503.75**	4th quarter, 1979.
[R/S]	**111.17**	
562 [R/S]	**513.25**	1st quarter, 1980.
[R/S]	**99.95**	
593 [R/S]	**521.38**	2nd quarter, 1980.
[R/S]	**83.24**	

Now average each quarter's seasonal variation for the two years?

f CLEAR Σ	0.00	
98.86 Σ+	1.00	
99.95 Σ+	2.00	
g x̄	99.41	1st quarter average seasonal variation, %.
f CLEAR Σ	0.00	
81.87 Σ+	1.00	
83.24 Σ+	2.00	
g x̄	82.56	2nd quarter average seasonal variation, %.
f CLEAR Σ	0.00	
104.37 Σ+	1.00	
107.94 Σ+	2.00	
g x̄	106.16	3rd quarter average seasonal variation, %.
f CLEAR Σ	0.00	
111.4 Σ+	1.00	
111.17 Σ+	2.00	
g x̄	111.29	4th quarter average seasonal variation, %.

An HP-12C program to calculate a centered 12-month moving average and seasonal variation factor is as follows:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		STO 2	08- 44 2
f CLEAR PRGM	00-	+	09- 40
RCL 1	01- 45 1	RCL 4	10- 45 4
2	02- 2	STO 3	11- 44 3
÷	03- 10	+	12- 40
RCL 2	04- 45 2	RCL 5	13- 45 5
STO 1	05- 44 1	STO 4	14- 44 4
+	06- 40	+	15- 40
RCL 3	07- 45 3	RCL 6	16- 45 6

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
STO 5	17- 44 5	RCL . 2	34-45 48 2
+	18- 40	STO . 1	35-44 48 1
RCL 7	19- 45 7	+	36- 40
STO 6	20- 44 6	RCL . 3	37-45 48 3
+	21- 40	STO . 2	38-44 48 2
RCL 8	22- 45 8	2	39- 2
STO 7	23- 44 7	÷	40- 10
+	24- 40	+	41- 40
RCL 9	25- 45 9	RCL 0	42- 45 0
STO 8	26- 44 8	÷	43- 10
+	27- 40	R/S	44- 31
RCL . 0	28-45 48 0	RCL 6	45- 45 6
STO 9	29- 44 9	%T	46- 23
+	30- 40	R/S	47- 31
RCL . 1	31-45 48 1	STO . 3	48-44 48 3
STO . 0	32-44 48 0	g GTO 01	49-43,33 01
+	33- 40	f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: n	R_1: X_1	R_2: X_2
R_3: X_3	R_4: X_4	R_5: X_5	R_6: X_6
R_7: X_7	R_8: X_8	R_9: X_9	$R_{.0}$: X_{10}
$R_{.1}$: X_{11}	$R_{.2}$: X_{12}	$R_{.3}$: X_{13}	

1. Key in the program.
2. Press f CLEAR REG.
3. Key in 12 and press STO 0.

4. Key in the values for the first 13 months, storing them one at a time in registers 1 through .3; i.e.
   Key in the 1st month and press [STO] 1.
   Key in the 2nd month and press [STO] 2, etc.,
   Key in the 10th month and press [STO] [.] 0, etc.,
   Key in the 13th month and press [STO] [.] 3.
5. Press [g] [GTO] 00 [R/S] to calculate the centered moving average for the 7th month.
6. Press [R/S] to calculate the seasonal variation for that month.
7. Key in the value for the next month (14th) and press [R/S] to calculate the moving average for the next month (8th).
8. Repeat steps 6 and 7 for the balance of the data.

These programs may be customized by the user for different types of centered moving averages. Inspection of the programs will show how they can be modified.

Gompertz Curve Trend Analysis

A useful curve for evaluating sales trends, etc., is the Gompertz curve. This is a "growth" curve having a general "S" shape and may be used to describe series of data where the early rate of growth is small, then accelerates for a period of time and then slows again as the time grows long. The sales curves for many products follow this trend during the introductory, growth, and maturity phases.

The data points to be fit to a Gompertz curve should be equally spaced along the x (or time) axis and all the data points must be positive. The points are divided serially into 3 groups for data entry.

The following HP-12C program processes the data, fits it to a Gompertz curve and calculates estimated values for future data points. The 3 constants which characterize the curve are available to the user if desired.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		ENTER	28– 36
f CLEAR PRGM	00–	×	29– 20
g LN	01– 43 23	−	30– 30
STO + 3	02–44 40 3	RCL 1	31– 45 1
R↓	03– 33	RCL 3	32– 45 3
g LN	04– 43 23	+	33– 40
STO + 2	05–44 40 2	RCL 2	34– 45 2
R↓	06– 33	2	35– 2
g LN	07– 43 23	×	36– 20
STO + 1	08–44 40 1	−	37– 30
1	09– 1	÷	38– 10
STO + 4	10–44 40 4	RCL 4	39– 45 4
RCL 4	11– 45 4	÷	40– 10
g GTO 00	12–43,33 00	g e^x	41– 43 22
RCL 3	13– 45 3	STO 7	42– 44 7
RCL 2	14– 45 2	RCL 6	43– 45 6
−	15– 30	1	44– 1
RCL 2	16– 45 2	−	45– 30
RCL 1	17– 45 1	RCL 6	46– 45 6
−	18– 30	RCL 4	47– 45 4
÷	19– 10	y^x	48– 21
RCL 4	20– 45 4	1	49– 1
$1/x$	21– 22	−	50– 30
y^x	22– 21	ENTER	51– 36
STO 6	23– 44 6	×	52– 20
RCL 1	24– 45 1	÷	53– 10
RCL 3	25– 45 3	RCL 6	54– 45 6
×	26– 20	÷	55– 10
RCL 2	27– 45 2	RCL 2	56– 45 2

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
RCL 1	57- 45 1	y^x	65- 21
−	58- 30	RCL 5	66- 45 5
×	59- 20	$x \gtrless y$	67- 34
g e^x	60- 43 22	y^x	68- 21
STO 5	61- 44 5	RCL 7	69- 45 7
R/S	62- 31	×	70- 20
RCL 6	63- 45 6	g GTO 62	71-43,33 62
$x \gtrless y$	64- 34	f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: S_1	R_2: S_2
R_3: S_3	R_4: n	R_5: a	R_6: b
R_7: c	R_8—R_0 Unused		

1. Key in the program and press f CLEAR REG.
2. Divide the data points to be input into 3 equal consecutive groups. Label them group I, II and III for convenience.
3. Key in the first point of group I and press ENTER.
4. Key in the first point of group II and press ENTER.
5. Key in the first point of group III and press R/S.
6. Repeat steps 3, 4 and 5 for the balance of the data in each group. After executing step 5 the display shows how many sets of data have been entered.
7. To fit the data to a Gompertz curve, press g GTO 13 R/S. The resultant display is the curve constant "a". Constants "b" and "c" may be obtained by pressing RCL 6 and RCL 7 respectively.
8. To calculate a projected value, key in the number of the period and press R/S.
9. Repeat step 8 for each period desired.

Example: The X-Presso Company marketed a revolutionary new coffee brewing machine in 1968. Sales grew at a steady pace for several years, then began to slow. The sales records for the first 9 years of the product's life were as follows:

Year	Sales ($K)	Year	Sales ($K)
1	18	6	260
2	41	7	282
3	49	8	322
4	151	9	340
5	188		

What are the projected sales volumes for this product in its 10th and 12th year? What is the maximum yearly sales volume for this product if the present trend continues? What annual sales rate would the curve have predicted for the 5th year of the product's life? (Arrange the data as follows:)

Group I	Group II	Group III
18	151	282
41	188	322
49	260	340

Keystrokes	Display	
f CLEAR REG	0.00	
18 ENTER	18.00	
151 ENTER	151.00	
282 R/S	1.00	
41 ENTER	41.00	
188 ENTER	188.00	
322 R/S	2.00	
49 ENTER	49.00	
260 ENTER	260.00	
340 R/S	3.00	Total number of entries.
g GTO 13 R/S	0.004	a
RCL 6	0.65	b
RCL 7	373.92	c
10 R/S	349.09	Sales in 10th year, ($K).
12 R/S	363.36	Sales in 12th year, ($K).

Keystrokes	Display	
100 R/S	**373.92**	Maximum annual sales (after very long product life).
5 R/S	**202.60**	Sales in 5th year (actual sales were $188K).

Forecasting With Exponential Smoothing

A common method for analyzing trends in sales, inventory and securities data is the moving average. Exponential smoothing is a version of the weighted moving average which is readily adaptable to programmable calculator forecasting.

Exponential smoothing is often used for short term sales and inventory forecasts. Typical forecast periods are monthly or quarterly.

Unlike a moving average, exponential smoothing does not require a great deal of historical data. However, it should not be used with data which has more than a moderate amount of up or down trend.

When using exponential smoothing, a smoothing factor is chosen which affects the sensitivity of the average much the same way as the length of the standard moving average period. The correspondence between the two techniques can be represented by the formula:

$$\alpha = \frac{2}{n+1}$$

where α is the exponential smoothing factor (with values from 0 to 1) and n is the length of the standard moving average. As the equation shows, the longer the moving average period, the smaller the equivalent α and the less sensitive the average becomes to fluctuations in current values.

Forecasting with exponential smoothing involves selecting the best smoothing factor based on historical data and then using the factor for updating subsequent data and forecasting. This procedure uses the following HP-12C program:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		+	22– 40
f CLEAR PRGM	00–	RCL 0	23– 45 0
ENTER	01– 36	×	24– 20
ENTER	02– 36	RCL 1	25– 45 1
RCL 6	03– 45 6	RCL 3	26– 45 3
−	04– 30	×	27– 20
ENTER	05– 36	+	28– 40
×	06– 20	STO 3	29– 44 3
STO + 4	07–44 40 4	RCL 1	30– 45 1
g LST x	08– 43 36	×	31– 20
R/S	09– 31	RCL 0	32– 45 0
R↓	10– 33	÷	33– 10
R↓	11– 33	RCL 2	34– 45 2
RCL 0	12– 45 0	+	35– 40
×	13– 20	STO 5	36– 44 5
RCL 2	14– 45 2	RCL 3	37– 45 3
RCL 1	15– 45 1	RCL 0	38– 45 0
×	16– 20	÷	39– 10
+	17– 40	RCL 2	40– 45 2
RCL 2	18– 45 2	+	41– 40
CHS	19– 16	STO 6	42– 44 6
x≷y	20– 34	g GTO 00	43–43,33 00
STO 2	21– 44 2	f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: α	R_1: $1-\alpha$	R_2: S_{t-1}
R_3: T_{t-1}	R_4: Σe^2	R_5: D_t	R_6: $\hat{D}_{t+1}$
R_7—$R_{.4}$ Unused			

Selecting the "best" smoothing constant (α):

1. Key in the program and press [f] CLEAR [REG].
2. Key in the number 1 and press [ENTER].
3. Key in the "trial α" and press [STO] 0 [−] [STO] 1.
4. Key in the first historical value (X_1) and press [STO] 2.
5. Key in the second historical value (X_2) and press [STO] 6 [R/S]. The result is the error between the forecast value ($\hat{D}_{t+1}$) and the true value (X_{t+1})
6. Press [R/S]; the display shows the next forecast ($\hat{D}_{t+2}$).
7. Optional: Press [RCL] 5 to display the smoothed estimate of current demand.
8. Continue steps 5 and 6 for X_3, X_4, ... , X_n until all historical values have been entered. When doing step 5 merely key in the value and press [R/S]. (Do not press [STO] 6).
9. Press [RCL] 4. This value represents the cumulative forecasting error (Σe^2). Record the value and the following additional values: press [RCL] 0 (α), [RCL] 2 (smoothed average S_{t-1}), [RCL] 3 (trend T_{t-1}) and [RCL] 6 (forecast $\hat{D}_{t+1}$).
10. Press [f] CLEAR [REG].
11. Repeat steps 2 through 10 until a "best" α is selected based on the lowest cumulative forecasting error (Register 4).

Forecasting:

1. Key in the number 1 and press [ENTER].
2. Key in the selected α and press [STO] 0 [−] [STO] 1.
3. From the α selection routine or from a previous forecast:
   Key in the smoothed average S_{t-1} and press [STO] 2.
   Key in the trend T_{t-1} and press [STO] 3.
   Key in the forecast $\hat{D}_{t+1}$ and press [STO] 6.
4. Key in the current data value and press [R/S]. The output is the error in forecasting the value just entered.
5. Press [R/S]. The displayed value represents the forecast for the next period.
6. Record the following values: [RCL] 0 (α), [RCL] 2 (S_{t-1}), [RCL] 3 (T_{t-1}) and [RCL] 6 ($\hat{D}_{t+1}$) for use as initial values in the next forecast. Your may also wish to record [RCL] 5 (D_t).
7. Repeat steps 4, 5 and 6 for the next forecast if available.

Example: Select the best smoothing constant based on sales (in thousands of dollars) of 22, 23, 23, 25, 23, 27, 25. Given the current sales in month 8 of 26, forecast the following month. Select the smoothing constant (α):

Keystrokes	Display	
f CLEAR REG	0.00	
1 ENTER	1.00	
.5 STO 0 −	0.50	
STO 1	0.50	
22 STO 2	22.00	
23 STO 6 R/S	0.00	
R/S	23.00	
23 R/S R/S	23.25	
25 R/S R/S	25.25	
23 R/S R/S	23.69	
27 R/S R/S	27.13	
25 R/S R/S	25.95	
RCL 4	23.61	Cumulative error (Σe^2).
RCL 0	0.50	Smoothing constant (α).
RCL 2	25.11	Smoothing average (S_{t-1}).
RCL 3	0.42	Trend (T_{t-1}).
RCL 6	25.95	Last forecast ($\hat{D}_{t+1}$).

The procedure is repeated for several α's.

Smoothing Constant (α)	.5	.1	.25	.2
Cumulative Error (Σe^2)	23.61	25.14	17.01	18.03

For the selected $\alpha = .25$

$S_{t-1} =$ 24.28

$T_{t-1} =$ 0.34

$\hat{D}_{t+1} =$ 25.64

Forecasting:

Keystrokes	Display
f CLEAR REG	0.00
1 ENTER	1.00
.25 STO 0 −	0.75
STO 1	0.75

Keystrokes	Display	
24.28 [STO] 2	**24.28**	
.34 [STO] 3	**0.34**	
25.64 [STO] 6	**25.64**	
26 [R/S]	**0.36**	
[R/S]	**26.16**	Forecast for month 9, ($\hat{D}_{t+1}$).
[RCL] 5	**25.80**	Expected usage for current (month 8) period, (Smoothed D_t).
[RCL] 0	**0.25**	α
[RCL] 2	**24.71**	Record for initial values when month 9 actual figures become available.
[RCL] 3	**0.36**	
[RCL] 6	**26.16**	

Note: At least 4 periods of current data should be entered before forecasting is attempted.

Pricing Calculations

Markup and Margin Calculations

Sales work often involves calculating the various relations between markup, margin, selling price, and costs. Markup is defined as the difference between selling price and cost, divided by the cost. Margin is defined as the difference between selling price and cost, divided by selling price. In other words, markup is based on cost and margin is based on selling price.

The following keystroke sequences are given to readily make these calculations on the HP-12C:

CALCULATE	GIVEN	KEYSTROKES
Selling Price	Cost & Markup	Key in cost, [ENTER], key in markup (in %), [%] [+].
Selling Price	Cost & Margin	Key in cost, [ENTER] 1 [ENTER], key in margin (in %), [%] [−] [÷].
Cost	Selling Price & Markup	Key in selling price, [ENTER] 1 [ENTER], key in markup (in %), [%] [+] [÷].
Cost	Selling Price & Margin	Key in selling price, [ENTER] 1 [ENTER], key in margin (in %), [%] [−] [×].
Markup	Cost & Selling Price	Key in cost, [ENTER], key in selling price, [Δ%].
Markup	Margin	Key in margin, [ENTER] [ENTER] 1 [$x \gtrless y$] [%] [−] [÷].

CALCULATE	GIVEN	KEYSTROKES
Margin	Selling Price & Cost	Key in selling price, [ENTER], key in cost, [△%] [CHS].
Margin	Markup	Key in markup, [ENTER] [ENTER] 1 [$x \geq y$] [%] [+] [÷].

Example 1: If the cost of an item is $160 and the margin is 20%, what is the selling price? What is the markup?

Keystrokes	Display	
160 [ENTER]	**160.00**	Cost.
1 [ENTER] 20	**20.**	Margin (%).
[%] [−] [÷]	**200.00**	Selling price.
20 [ENTER] [ENTER]	**20.00**	
1 [$x \geq y$] [%] [−] [÷]	**25.00**	Markup (%).

Example 2: If an item sells for $21.00 and has a markup of 50%, what is its cost? What is the margin?

Keystrokes	Display	
21 [ENTER]	**21.00**	Selling price.
1 [ENTER] 50	**50.**	Markup (%).
[%] [+] [÷]	**14.00**	Cost.
50 [ENTER] [ENTER]	**50.00**	
1 [$x \geq y$] [%] [+] [÷]	**33.33**	Margin (%).

The following HP-12C program may be helpful for repetitive calculations of selling price and costs as well as conversions between markup and margin:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		÷	08– 10
f CLEAR PRGM	00–	R/S	09– 31
ENTER	01– 36	g LST x	10– 43 36
g GTO 04	02–43,33 04	×	11– 20
CHS	03– 16	g LST x	12– 43 36
1	04– 1	×	13– 20
x ≷ y	05– 34	g GTO 00	14–43,33 00
%	06– 25	f P/R	
+	07– 40		

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0—R_8 Unused		

1. Key in program.
2. To calculate selling price, given the markup, key in the cost, press ENTER, key in the markup and press g GTO 00 R/S R/S.
3. To calculate cost, given the markup, key in the selling price, press ENTER, key in the markup and press g GTO 00 R/S.
4. To calculate selling price, given the margin, key in the cost, press ENTER, key in the margin and press g GTO 03 R/S.
5. To calculate cost, given the margin, key in the selling price, press ENTER, key in the margin and press g GTO 03 R/S R/S.
6. To calculate markup from the margin, key in the margin and press ENTER g GTO 03 R/S.
7. To calculate margin from the markup, key in the markup and press ENTER g GTO 00 R/S.

Example: Find the cost of an item selling for $38.00 with a margin of 30%. What is the markup on the item? If the markup is raised to 50%, what will the selling price be?

Keystrokes	Display	
38 [ENTER]	**38.00**	Selling price.
30 [g] [GTO] 03	**30.00**	Margin (%).
[R/S] [R/S]	**26.60**	Cost.
30 [ENTER] [g] [GTO] 03 [R/S]	**42.86**	Markup (%).
26.6 [ENTER]	**26.60**	Cost.
50 [g] [GTO] 00 [R/S] [R/S]	**39.90**	New selling price.

Calculations of List and Net Prices With Discounts

It is often useful to be able to quickly calculate list or net price when the other price and a series of discount rates are known. Alternatively, if the list and net and several discounts are known it may be desirable to calculate a missing discount. The following series of keystrokes may be used:

1. Key in 1, press [ENTER] [ENTER] [ENTER] [STO] 1.
2. Key in the first discount (as a percentage) and press [%] [−] [STO] [×] 1 [R↓].
3. Repeat step 2 for each of the remaining known discount rates.
4. To calculate the list price, key in the net price and press [RCL] 1 [÷].
5. To calculate the net price, key in the list price and press [RCL] 1 [×].
6. To calculate an unknown discount rate, immediately after doing step 3 (display should show 1.00), key in the net price, press [ENTER] and key in the list price.
7. Press [RCL] 1 [×] [÷] [−] 100 [×].

Example: The list price of an item is \$3.28 and the net price is \$1.45. Two of the discount rates are 48% and 5%. What is the third discount rate?

Keystrokes	Display	
1 [ENTER] [ENTER]		
[STO] 1	1.00	
48 [%] [−] [STO] [×] 1		
[R↓]	1.00	
5 [%] [−] [STO] [×] 1		
[R↓]	1.00	
1.45 [ENTER]		
3.28 [RCL] 1	0.49	
[×] [÷] [−] 100 [×]	10.51	3rd discount rate (%).

The following program for the HP-12C will be helpful in performing the calculations:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[÷]	09– 10
[f] CLEAR [PRGM]	00–	1	10– 1
1	01– 1	[$x \geq y$]	11– 34
[$x \geq y$]	02– 34	[−]	12– 30
[%]	03– 25	[EEX]	13– 26
[−]	04– 30	2	14– 2
[STO] [×] 1	05–44 20 1	[×]	15– 20
[g] [GTO] 00	06–43,33 00	[g] [GTO] 00	16–43,33,00
[RCL] 1	07– 45 1	[f] [P/R]	
[×]	08– 20		

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: $R_1 D_1 \times D_2 \ldots D$	R_2—R_7 Unused

1. Key in the program.
2. Key in 1 and press [STO] 1.
3. Key in the first discount rate (as a percentage) and press [R/S].
4. Repeat step 2 for each of the remaining discount rates.
5. To calculate the list price, key in the net price and press [RCL] 1 [÷].
6. To calculate the net price, key in the list price and press [RCL] 1 [×].
7. To calculate the unknown discount rate, key in the net price, press [ENTER], key in the list price and press [g] [GTO] 07 [R/S].

Example: Calculate the unknown discount rate for the previous example. If the list price is now raised to $3.75 what is the new net price?

Keystrokes	Display	
1 [STO] 1	**1.00**	
48 [R/S]	**0.52**	
5 [R/S]	**0.95**	
1.45 [ENTER]		
3.28 [g] [GTO] 07 [R/S]	**10.51**	3rd discount rate (%).
[R/S]	**0.89**	Include 3rd discount rate in calculation.
3.75 [RCL] 1 [×]	**1.66**	New net price.

Statistics

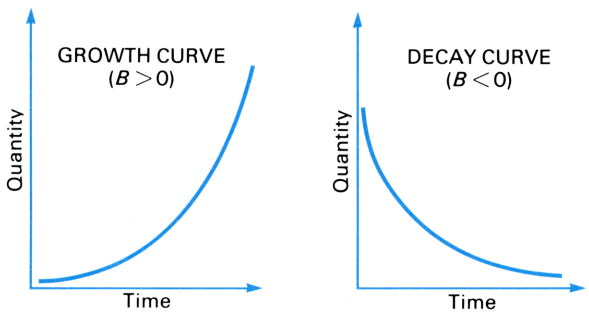
Curve Fitting

Exponential Curve Fit

Using the [LN] function of the HP-12C, a least squares exponential curve fit may be easily calculated according to the equation $y = Ae^{Bx}$. The exponential curve fitting technique is often used to determine the growth rate of a variable such as a stock's value over time, when it is suspected that the performance is non-linear. The value for B is the decimal value of the continuous growth rate. For instance, assume after keying in several end-of-month price quotes for a particular stock it is determined that the value for B is 0.10. This means that over the measured period the stock has experienced a 10% continuous growth rate.

If $B > 0$, you will have a growth curve. If $B < 0$, you will have a decay curve.

Examples of these are given below.



The procedure is as follows:

1. Press [f] CLEAR [REG].
2. For each input pair of values, key in the y-value and press [g] [LN], key in the corresponding x-value, and press [Σ+].

3. After all data pairs are input, press [g] [ŷ,r] [x≷y] to obtain the correlation coefficient (between ln y and x).
4. Press 1 [g] [ŷ,r] [g] [e^x] 0 [g] [ŷ,r] [g] [e^x] to obtain A in the equation above.
5. Press [x≷y] [R↓] [÷] [g] [LN] to obtain B.
6. Press [g] [e^x] 1 [−] to obtain the effective growth rate (as a decimal).
7. To make a y-estimate, key in the x-value and press [g] [ŷ,r] [g] [e^x].

Example 1: A stock's price history is listed below. What effective growth rate does this represent? If the stock continues this growth rate, what is the price projected to be at the end of 1982 (year 7)?

End of Year	Price
1976 (1)	45
1977 (2)	51.5
1978 (3)	53.75
1979 (4)	80
1980 (5)	122.5
1981 (6)	210
1982 (7)	?

Keystrokes	Display	
[f] CLEAR [REG]		
45 [g] [LN] 1 [Σ+]	**1.00**	First data pair input.
51.5 [g] [LN] 2 [Σ+]	**2.00**	Second data pair input.
53.75 [g] [LN] 3 [Σ+]	**3.00**	Third data pair input.
80 [g] [LN] 4 [Σ+]	**4.00**	Fourth data pair input.
122.5 [g] [LN] 5 [Σ+]	**5.00**	Fifth data pair input.
210 [g] [LN] 6 [Σ+]	**6.00**	Sixth data pair input.
[g] [ŷ,r] [x≷y]	**0.95**	Correlation coefficient (between ln y and x).
1 [g] [ŷ,r] [g] [e^x]		
0 [g] [ŷ,r] [g] [e^x]	**27.34**	A
[x≷y] [R↓] [÷] [g] [LN]	**0.31**	B

Keystrokes	Display	
[g] [e^x] 1 [−]	0.36	Effective growth rate.
7 [g] [$\hat{y}$,r] [g] [e^x]	232.35	Projected price at end of year 7 (1982).

For repeated use of this routine, the following HP-12C program will be useful.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[g] [e^x]	14– 43 22
[f] CLEAR [PRGM]	00–	[R/S]	15– 31
[$x \geqslant y$]	01– 34	[$x \geqslant y$]	16– 34
[g] [LN]	02– 43 23	[R↓]	17– 33
[$x \geqslant y$]	03– 34	[÷]	18– 10
[Σ+]	04– 49	[g] [LN]	19– 43 23
[g] [GTO] 00	05–43,33 00	[R/S]	20– 31
[g] [$\hat{y}$,r]	06– 43 2	[g] [e^x]	21– 43 22
[$x \geqslant y$]	07– 34	1	22– 1
[R/S]	08– 31	[−]	23– 30
1	09– 1	[R/S]	24– 31
[g] [$\hat{y}$,r]	10– 43 2	[g] [$\hat{y}$,r]	25– 43 2
[g] [e^x]	11– 43 22	[g] [e^x]	26– 43 22
0	12– 0	[g] [GTO] 00	27–43,33 00
[g] [$\hat{y}$,r]	13– 43 2	[f] [P/R]	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: n	R_2: Σx
R_3: Σx^2	R_4: Σy	R_5: Σy^2	R_6: Σxy
R_7—R_9 Unused			

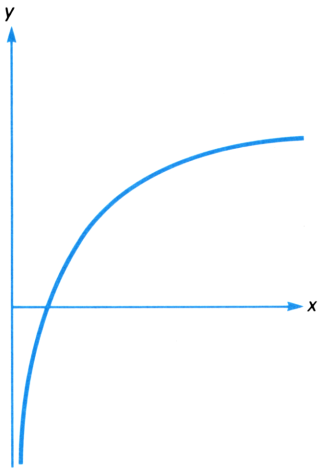
1. Key in the program and press [f] CLEAR [REG].
2. For each input pair of values, key in the y-value and press [ENTER], key in the corresponding x-value and press [R/S].
3. After all data pairs are input, press [g] [GTO] 06 [R/S] to obtain the correlation coefficient (between ln y and x).
4. Press [R/S] to obtain A.
5. Press [R/S] to obtain B.
6. Press [R/S] to obtain the effective growth rate as a decimal.
7. To make a y-estimate, key in the x-value and press [R/S]. For subsequent estimates, key in the x-value and press [g] [GTO] 25 [R/S].
8. For a different set of data, press [f] CLEAR [REG] and go to step 2.

Keystrokes	Display	
[f] CLEAR [REG]		
45 [ENTER] 1 [R/S]	**1.00**	First data pair input.
51.5 [ENTER] 2 [R/S]	**2.00**	Second data pair input.
53.75 [ENTER] 3 [R/S]	**3.00**	Third data pair input.
80 [ENTER] 4 [R/S]	**4.00**	Fourth data pair input.
122.5 [ENTER] 5 [R/S]	**5.00**	Fifth data pair input.
210 [ENTER] 6 [R/S]	**6.00**	Sixth data pair input.
[g] [GTO] 06 [R/S]	**0.95**	Correlation coefficient (between ln y and x).
[R/S]	**27.34**	A
[R/S]	**0.31**	B
[R/S]	**0.36**	Effective growth rate.
7 [R/S]	**232.35**	Projected price at the end of year 7 (1982).

Logarithmic Curve Fit

If your data does not fit a line or an exponential curve, try the following logarithmic curve fit. This is calculated according to the equation $y = A + B(\ln x)$, and all x values must be positive.

A typical logarithmic curve is shown below.



The procedure is as follows:

1. Press [f] CLEAR [REG].
2. Key in the first y-value and press [ENTER]. Key in the first x-value and press [g] [LN] [$\Sigma+$]. Repeat this step for each data pair.
3. After all data pairs are input, press [g] [$\hat{y},r$] [$x\gtrless y$] to obtain the correlation coefficient (between y and $\ln x$).
4. Press 1 [g] [$\hat{y},r$] 0 [g] [$\hat{y},r$] to obtain A in the equation above.
5. Press [$x\gtrless y$] [R↓] [−] to obtain B.
6. To make a y-estimate, key in the x-value and press [g] [LN] [g] [$\hat{y},r$].

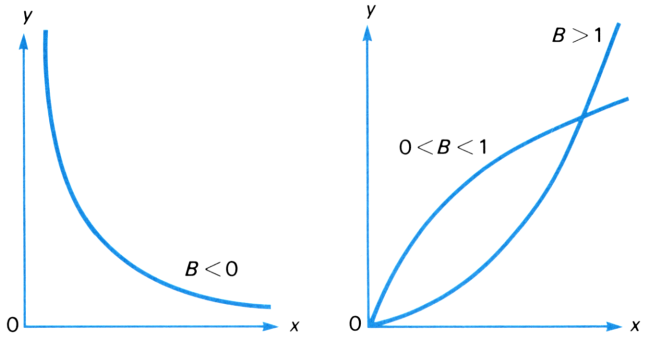
Example 1: A manufacturer observes declining sales of a soon-to-be obsolete product, of which there were originally 10,000 units in inventory. The cumulative sales figures over a number of months, given below, may be fit by a logarithmic curve of the form $y = A + B\ (\ln x)$, where y represents cumulative sales in units and x the number of months since the beginning. How many units will be sold by the end of the eighth months?

Month	1	2	3	4	5	6
Cumulative Sales (units)	1431	3506	5177	6658	7810	8592

Keystrokes	Display	
f CLEAR REG 1431 ENTER 1 g LN Σ+	1.00	First pair data input.
3506 ENTER 2 g LN Σ+	2.00	Second pair data input.
5177 ENTER 3 g LN Σ+	3.00	Third pair data input.
6658 ENTER 4 g LN Σ+	4.00	Fourth pair data input.
7810 ENTER 5 g LN Σ+	5.00	Fifth pair data input.
8592 ENTER 6 g LN Σ+	6.00	Sixth pair data input.
g ŷ,r x≷y	0.99	Correlation coefficient (between y and $\ln x$).
1 g ŷ,r 0 g ŷ,r	1,066.15	Value of A.
x≷y R↓ −	4,069.93	Value of B.
8 g LN g ŷ,r	9,529.34	Total units sold by end of eighth month.

Power Curve Fit

Another method of analysis is the power curve or geometric curve. The equation of the power curve is $y = Ax^B$, and the values for A and B are computed by calculations similar to linear regression. Some examples of power curves are shown below.



The following keystrokes fit a power curve according to the equation $\ln y = \ln A + B(\ln x)$:

1. Press [f] CLEAR [REG].
2. Key in the first y-value and press [g] [LN]. Key in the first x-value and press [g] [LN] [Σ+]. Repeat this step for all data pairs.
3. Press [g] [x̂,r] [x≷y] to obtain the correlation coefficient (between $\ln y$ and $\ln x$).
4. Press 0 [g] [x̂,r] [g] [e^x] to obtain A in the above equation.
5. Press 1 [g] [x̂,r] 0 [g] [x̂,r] [x≷y] [R↓] [−] to obtain B.
6. To make a y-estimate, key in the x-value and press [g] [LN] [g] [x̂,r] [g] [e^x].

Example: If Galileo had wished to investigate quantitatively the relationship between the time (t) for a falling object to hit the ground and the height (h) it has fallen, he might have released a rock from various levels of the Tower of Pisa (which was leaning even than) and timed its descent by counting his pulse. The following data are measurements Galileo might have made.

t(pulses)	2	2.5	3.5	4	4.5
h(feet)	30	50	90	130	150

Find the power curve formulas that best expresses h as a function of t ($h = At^B$).

Keystrokes	Display	
[f] CLEAR [REG]		
30 [g] [LN]		
2 [g] [LN] [Σ+]	**1.00**	First pair data input.
50 [g] [LN]		
2.5 [g] [LN] [Σ+]	**2.00**	Second pair data input.
90 [g] [LN]		
3.5 [g] [LN] [Σ+]	**3.00**	Third pair data input.
130 [g] [LN]		
4 [g] [LN] [Σ+]	**4.00**	Fourth pair data input.
150 [g] [LN]		
4.5 [g] [LN] [Σ+]	**5.00**	Fifth pair data input.
[g] [r̂,r] [x≷y]	**1.00**	Correlation coefficient (between ln y and ln x).
0 [g] [r̂,r] [g] [e^x]	**7.72**	Value of A.
1 [g] [r̂,r] 0 [g] [r̂,r] [x≷y] [R↓] [−]	**1.99**	Value of B.

The formula that best expresses h as a function of t is

$$h = 7.72\, t^{1.99}$$

We know, as Galileo did not, that in fact h is proportional to t^2.

Standard Error of the Mean

The standard error of the mean is a measure of how reliable the mean of a sample ($\bar{x}$) is as an estimator of the mean of the population from which the sample was drawn.

To calculate the standard error of the mean:

1. Press [f] CLEAR [REG].
2. If you are summing one set of numbers, key in the first number and press [Σ+]. Continue until you have entered all of the values.

3. If you are summing two sets of numbers, key in the y-value and press [ENTER], key in the x-value and press [Σ+]. Continue until you have entered all of the values.
4. Press [g] [$\bar{x}$] to obtain the mean of the x-values.
5. Press [g] [s] [RCL] 1 [g] [$\sqrt{x}$] [÷] to obtain the standard error of the mean of the x-values.
6. Alternatively, press [g] [s] [$x \gtrless y$] [RCL] 1 [g] [$\sqrt{x}$] [÷] to obtain the standard error of the mean of the y-values.

Example: A sample of 6 one-bedroom apartment rentals reveals that one rents for $190 per month unfurnished, one rents for $200 per month, two rent for $205 per month, one rents for $216 per month, and one rents for $220 per month. What are the mean monthly rental and the standard deviation? What is the standard error of the mean?

Keystrokes	Display	
[f] CLEAR [REG]		
190 [Σ+] 200 [Σ+]		
205 [Σ+] 205 [Σ+]		
216 [Σ+] 220 [Σ+]	**6.00**	Total number of inputs.
[g] [$\bar{x}$]	**206.00**	Average monthly rent.
[g] [s]	**10.86**	Standard deviation.
[RCL] 1 [g] [$\sqrt{x}$] [÷]	**4.43**	Standard error of the mean.

Mean, Standard Deviation, Standard Error for Grouped Data

Grouped data are presented in frequency distributions to save time and effort in writing down (or entering) each observation individually. Given a set of data points

$$x_1, x_2, \ldots, x_n$$

with respective frequencies

$$f_1, f_2, \ldots, f_n$$

this procedure computes the mean, standard deviation, and standard error of the mean.

1. Press [f] CLEAR [REG].
2. Key in the first value and press [ENTER] [ENTER].
3. Key in the respective frequency and press [STO] [+] 0 [×] [Σ+]. The display shows the number of data points entered.
4. Repeat steps 2 and 3 for each data point.
5. To calculate the mean (average) press [RCL] 0 [STO] 1 [RCL] 6 [STO] 3 [g] [$\bar{x}$].
6. Press [g] [s] to find the standard deviation.
7. Press [RCL] 0 [g] [$\sqrt{x}$] [÷] to find the standard error of the mean.

Example 1: A survey of 266 one-bedroom apartment rentals reveals that 54 rent for $190 per month unfurnished, 32 rent for $195 per month, 88 rent for $200 per month, and 92 rent for $206 per month. What are the average monthly rental, the standard deviation, and the standard error of the mean?

Keystrokes	Display	
[f] CLEAR [REG]		
190 [ENTER] [ENTER]		
54 [STO] [+] 0 [×] [Σ+]	**1.00**	First data pair entered.
195 [ENTER] [ENTER]		
32 [STO] [+] 0 [×] [Σ+]	**2.00**	Second data pair entered.
200 [ENTER] [ENTER]		
88 [STO] [+] 0 [×] [Σ+]	**3.00**	Third data pair entered.
206 [ENTER] [ENTER]		
92 [STO] [+] 0 [×] [Σ+]	**4.00**	Fourth data pair entered.
[RCL] 0 [STO] 1 [RCL] 6		
[STO] 3 [g] [$\bar{x}$]	**199.44**	Average monthly rent.
[g] [s]	**5.97**	Standard deviation.
[RCL] 0 [g] [$\sqrt{x}$] [÷]	**0.37**	Standard error of the mean.

Use the following HP-12C program for the previous example:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		g $\bar{x}$	09- 43 0
f CLEAR PRGM	00-	R/S	10- 31
STO + 0	01-44 40 0	g s	11- 43 48
×	02- 20	R/S	12- 31
$\Sigma+$	03- 49	RCL 0	13- 45 0
g GTO 00	04-43,33 00	g $\sqrt{x}$	14- 43 21
RCL 0	05- 45 0	÷	15- 10
STO 1	06- 44 1	g GTO 00	16-43,33 00
RCL 6	07- 45 6	f P/R	
STO 3	08- 44 3		

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Σf_i	R_1: Σf_i	R_2: $\Sigma f_i x_i$
R_3: $\Sigma f_i x_i^2$	R_4: Σx_i	R_5: Σx_i^2	R_6: $\Sigma f_i x_i^2$
R_7—$R_{.7}$ Unused			

1. Key in the program.
2. Press f CLEAR REG.
3. Key in the first value and press ENTER ENTER.
4. Key in the respective frequency and press R/S. The display shows the number of data points entered.
5. Repeat steps 3 and 4 for each data point.
6. To calculate the mean, press g GTO 05 R/S.
7. Press R/S to find the standard deviation.
8. Press R/S to find the standard error of the mean.
9. For a new case, go to step 2.

Keystrokes	Display	
f CLEAR REG		
190 ENTER ENTER		
54 R/S	1.00	First data pair.
195 ENTER ENTER		
32 R/S	2.00	Second data pair.
200 ENTER ENTER		
88 R/S	3.00	Third data pair.
206 ENTER ENTER		
92 R/S	4.00	Total number of data sets.
g GTO 05 R/S	199.44	Average monthly rent (mean).
R/S	5.97	Standard deviation.
R/S	0.37	Standard error of the mean.

Chi-Square Statistics

The chi-square statistic is a measure of the goodness of fit between two sets of frequencies. It is used to test whether a set of observed frequencies differ from a set of expected frequencies sufficiently to reject the hypothesis under which the expected frequencies were obtained.

In other words, you are testing whether discrepancies between the observed frequencies (O_i) and the expected frequencies (E_i) are significant, or whether they may reasonably be attributed to chance. The formula generally used is:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

If there is a close agreement between the observed and expected frequencies, χ^2 will be small. If the agreement is poor, χ^2 will be large.

The following keystrokes calculate the χ^2 statistic:

1. Press f CLEAR REG.
2. Key in the first O_i value and press ENTER.

3. Key in the first E_i value and press [STO] 0 [−] [ENTER] [×] [RCL] 0 [÷] [+].
4. Repeat steps 2 and 3 for all data pairs. The χ^2 value is displayed.

Example 1: A suspect die from a Las Vegas casino is brought to an independent testing firm to determine its bias, if any. The die is tossed 120 times and the following results obtained.

Number	1	2	3	4	5	6
Observed Frequency	25	17	15	23	24	16

The expected frequency = 120 throws/6 sides, or $E = 20$ for each number, 1 thru 6. (Since E is a constant in this example, there's no need to store it in R_0 each time.)

Keystrokes	Display	
[f] CLEAR [REG]		
25 [ENTER]		
20 [STO] 0 [−] [ENTER]		
[×] [RCL] 0 [÷] [+]	1.25	
17 [ENTER] 20 [−]		
[ENTER] [×] [RCL] 0 [÷] [+]	1.70	
15 [ENTER] 20 [−]		
[ENTER] [×] [RCL] 0 [÷] [+]	2.95	
23 [ENTER] 20 [−]		
[ENTER] [×] [RCL] 0 [÷] [+]	3.40	
24 [ENTER] 20 [−]		
[ENTER] [×] [RCL] 0 [÷] [+]	4.20	
16 [ENTER] 20 [−]		
[ENTER] [×] [RCL] 0 [÷] [+]	5.00	χ^2

The number of degrees of freedom is $(n - 1)$. Since $n = 6$, the degrees of freedom = 5.

Consulting statistical tables, you look up χ^2 to a 0.05 significance level with 5 degrees of freedom, and see that $\chi^2_{0.05,5} = 11.07$. Since $\chi^2 = 5$ is within 11.07, we may conclude that to a 0.05 significance level (probability = .95), the die is fair.

Try the following HP-12C program with the same example.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		RCL 0	05- 45 0
f CLEAR PRGM	00-	÷	06- 10
STO 0	01- 44 0	+	07- 40
−	02- 30	g GTO 00	08-43,33 00
ENTER	03- 36	f P/R	
×	04- 20		

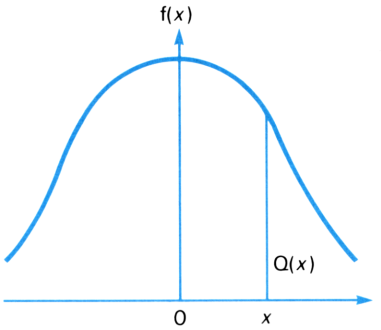
	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: E_i	R_1—R_9 Unused	

1. Key in the program.
2. Press f CLEAR REG.
3. Key in the first O_i value and press ENTER.
4. Key in the first E_i value and press R/S.
5. Repeat steps 3 and 4 for all data pairs. The χ^2 value is displayed.
6. For a new case, go to step 2.

Keystrokes	Display	
f CLEAR REG		
25 ENTER		
20 R/S	1.25	
17 ENTER		
20 R/S	1.70	
15 ENTER		
20 R/S	2.95	
23 ENTER		
20 R/S	3.40	
24 ENTER		
20 R/S	4.20	
16 ENTER		
20 R/S	5.00	χ^2

Normal Distribution

The normal (or Gaussian) distribution is an important tool in statistics and business analysis. The following HP-12C program gives an *approximation* to the upper tail area Q under a standardized normal distribution curve, given x. The upper tail area signifies the probability of occurrence of all values $\geqslant x$.



$$Q(x) \cong \frac{1}{2}\,\mathrm{EXP}\left[-\frac{(83x+351)x+562}{703/x+165}\right]$$

Relative error less than 0.042% over the range $0 < x < 5.5$

Reference:

Stephen E. Derenzo, "Approximations for Hand Calculators Using Small Integer Coefficients," *Mathematics of Computation*, Vol. 31, No. 137, page 214-225; January 1977.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		8	02- 8
f CLEAR PRGM	00-	3.	03- 3
STO 0	01- 44 0	×	04- 20

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
3	05– 3	RCL 0	18– 45 0
5	06– 5	÷	19– 10
1	07– 1	1	20– 1
+	08– 40	6	21– 6
RCL 0	09– 45 0	5	22– 5
×	10– 20	+	23– 40
5	11– 5	÷	24– 10
6	12– 6	CHS	25– 16
2	13– 2	g e^x	26– 43 22
+	14– 40	2	27– 2
7	15– 7	÷	28– 10
0	16– 0	g GTO 00	29–43,33 00
3	17– 3	f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: x	R_1—R_6 Unused	

1. Key in program.
2. Key in x and press R/S to compute $Q(x)$.
3. Repeat step 2 for each new case.

Example: Find $Q(x)$ for $x = 1.18$ and $x = 2.1$.

Keystrokes	Display	
1.18 R/S	**0.12**	$Q(1.18)$
2.1 R/S	**0.02**	$Q(2.1)$

Covariance

Covariance is a measure of the interdependence between paired variables (x and y). Like standard deviation, covariance may be defined for either a sample (S_{xy}) or a population (S'_{xy}) as follows:

$$S_{xy} = r \cdot s_x \cdot s_y$$

$$S'_{xy} = r \cdot s'_x \cdot s'_y$$

The following procedure finds the covariance of a sample (S_{xy}) and of a population (S'_{xy}):

1. Press [f] CLEAR [REG].
2. Key in the y-value and press [ENTER].
3. Key in the x-value and press [Σ+]. Repeat steps 2 and 3 for all data pairs.
4. Press [g] [s] [×] [ENTER] [g] [ŷ,r] [R↓] [×] to obtain the value of S_{xy}.
5. Press [RCL] 1 1 [−] [RCL] 1 [÷] [×] to obtain S'_{xy}.

Example 1: Find the sample covariance (S_{xy}) and population covariance (S'_{xy}) for the following paired variables:

x_i	26	30	44	50	62	68	74
y_i	92	85	78	81	54	51	40

Keystrokes	Display	
[f] CLEAR [REG]		
92 [ENTER] 26 [Σ+]		
85 [ENTER] 30 [Σ+]		
78 [ENTER] 44 [Σ+]		
81 [ENTER] 50 [Σ+]		
54 [ENTER] 62 [Σ+]		
51 [ENTER] 68 [Σ+]		
40 [ENTER] 74 [Σ+]	**7.00**	Total number of entries.
[g] [s] [×] [ENTER]		
[g] [ŷ,r]		
[R↓] [×]	**−354.14**	S_{xy}

Keystrokes	Display	
[RCL] 1 1 [−] [RCL] 1 [÷] [×]	−303.55	S'_{xy}

Try the previous example using the following HP-12C program:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[R/S]	09– 31
[f] CLEAR [PRGM]	00–	[RCL] 1	10– 45 1
[Σ+]	01– 49	1	11– 1
[g] [GTO] 00	02–43,33 00	[−]	12– 30
[g] [s]	03– 43 48	[RCL] 1	13– 45 1
[×]	04– 20	[÷]	14– 10
[ENTER]	05– 36	[×]	15– 20
[g] [ŷ,r]	06– 43 2	[g] [GTO] 00	16–43,33 00
[R↓]	07– 33	[f] [P/R]	
[×]	08– 20		

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: n	R_2: Σx
R_3: Σx^2	R_4: Σy	R_5: Σy^2	R_6: Σxy
R_7—$R_{.7}$ Unused			

1. Key in the program.
2. Press [f] CLEAR [REG].
3. Key in the y-value and press [ENTER].
4. Key in the x-value and press [R/S]. Repeat steps 3 and 4 for all data pairs.
5. Press [g] [GTO] 03 [R/S] to obtain the value of S_{xy}.
6. Press [R/S] to obtain S'_{xy}.
7. For a new case, go to step 2.

Keystrokes	Display	
f CLEAR REG		
92 ENTER 26 R/S		
85 ENTER 30 R/S		
78 ENTER 44 R/S		
81 ENTER 50 R/S		
54 ENTER 62 R/S		
51 ENTER 68 R/S		
40 ENTER 74 R/S	7.00	Total number of entries.
g GTO 03 R/S	-354.14	S_{xy}
R/S	-303.55	S'_{xy}

Permutation

A permutation is an *ordered* subset of a set of distinct objects. The number of possible permutations, each containing n objects, that can be formed from a collection of m distinct objects is given by:

$$ {}_mP_n = \frac{m!}{(m-n)!} $$

where m, n are integers and $0 \leqslant n \leqslant m \leqslant 69$.

Use the following HP-12C program to calculate the number of possible permutations.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		RCL 0	05- 45 0
f CLEAR PRGM	00-	−	06- 30
STO 0	01- 44 0	g n!	07- 43 3
$x \gtrless y$	02- 34	÷	08- 10
g n!	03- 43 3	g GTO 00	09-43,33 00
g LSTx	04- 43 36	f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: n	R_1—R_8 Unused	

1. Key in the program.
2. Key in m and press [ENTER].
3. Key in n and press [R/S] to calculate ${}_mP_n$.
4. For a new case go to step 2.

Example: How many ways can 10 people be seated on a bench if only 4 seats are available?

Keystrokes	Display	
10 [ENTER]		
4 [R/S]	**5,040.00**	${}_{10}P_4$.

Combination

A combination is a selection of one or more of a set of distinct objects without regard to order. The number of possible combinations, each containing n objects, that can be formed from a collection of m distinct objects is given by:

$$ {}_mC_n = \frac{m!}{(m-n)!n!} $$

Where m, n are integers and $0 \leqslant n \leqslant m \leqslant 69$.

Use the following HP-12C to calculate the number of possible combinations.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[g] [n!]	03- 43 3
[f] CLEAR [PRGM]	00-	[g] [LST x]	04- 43 36
[STO] 0	01- 44 0	[RCL] 0	05- 45 0
[x ≷ y]	02- 34	[-]	06- 30

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
g n!	07– 43 3	÷	11– 10
RCL 0	08– 45 0	g GTO 00	12–43,33 00
g n!	09– 43 3	f P/R	
×	10– 20		

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: n	R_1—R_8 Unused	

1. Key in the program.
2. Key in m and press ENTER.
3. Key in n and press R/S to calculate ${}_mC_n$.
4. For a new case, go to step 2.

Example: A manager wants to choose a committee of three people from the seven engineers working for him. In how many different ways can the committee be selected?

Keystrokes	Display	
7 ENTER		
3 R/S	35.00	${}_7C_3$

Random Number Generator

This HP-12C program calculates uniformly distributed pseudo-random numbers u_i in the range

$$0 < u_i < 1$$

The following method is used:

u_{i+1} = fractional part of $(997u_i)$
where $i = 0, 1, 2, \ldots$
$u_0 = 0.5284163$* (seed)

* Other seeds may be selected but the quotient of (seed $\times 10^7$) divided by two or five must not be an integer. Also, it would be wise to statistically test other seeds before using them.

The period of this generator has a length of 500,000 numbers and the generator passes the frequency test (chi square) for uniformity, the serial test and the run test. The most significant digits (the left hand digits) are the most random digits. The right most digits are significantly less random.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		STO 0	09– 44 0
f CLEAR PRGM	00–	9	10– 9
.	01– 48	9	11– 9
5	02– 5	7	12– 7
2	03– 2	×	13– 20
8	04– 8	g FRAC	14– 43 24
4	05– 4	STO 0	15– 44 0
1	06– 1	R/S	16– 31
6	07– 6	g GTO 10	17–43,33 10
3	08– 3	f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: U_i	R_1—R_7 Unused	

1. Key in the program.
2. To generate a random number, press R/S.
3. Repeat step 2 as many times as desired.

Example: Generate a sequence of 5 random numbers.

Keystrokes	Display
R/S	0.83
R/S	0.56
R/S	0.27
R/S	0.04
R/S	0.20

Personal Finance

Homeowner's Monthly Payment Estimator

It is often useful, when comparison shopping for a mortgage or determining the appropriate price range of houses to consider, to be able to quickly estimate the monthly payment. This procedure calculates the approximate mortgage payment given the purchase price, tax rate per $1000, percent down, interest rate and term of the loan.

The calculation assumes that the assessed value is 100% of the sales price and does not take into account financing of the closing costs.

A simple keystroke procedure may be used to calculate the monthly payment:

1. Press [g] [END] and press [f] CLEAR [FIN].
2. Key in the annual interest rate and press [g] [12÷].
3. Key in the term of the loan (in years) and press [g] [12×].
4. Key in the purchase price and press [STO] 1.
5. Key in the percent down and press [%] [−] [PV].
6. Key in the tax rate in dollars per thousand and press [RCL] 1 [×] 12000 [÷] [CHS] [ENTER] [PMT] [PMT] [+]. (A negative sign is the convention for cash paid out).

Example: What would your monthly payments be on a $65,000 house in a neighborhood with a $25 per thousand tax rate and a 10¾% interest rate on a 35 year loan with 10% down?

Keystrokes	**Display**	
[g] [END]		
[f] CLEAR [FIN]		
10.75 [g] [12÷]	**0.90**	Monthly interest rate.
35 [g] [12×]	**420.00**	Months of loan.
65000 [STO] 1	**65,000.00**	Purchase price.

10 [%] [−] [PV] 25 [RCL] 1 [×]	**58,500.00**	Mortgage balance.
12000 [÷] [CHS]	**−135.42**	Approximate monthly taxes.
[ENTER] [PMT] [PMT] [+]	**−672.16**	Approximate monthly payment.

The following HP-12C program may be used instead of the above.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		1	12– 1
[f] CLEAR [PRGM]	00–	2	13– 2
[g] [END]	01– 43 8	[EEX]	14– 26
[RCL] 1	02– 45 1	3	15– 3
[RCL] 2	03– 45 2	[÷]	16– 10
[%]	04– 25	[CHS]	17– 16
[−]	05– 30	[ENTER]	18– 36
[PV]	06– 13	[PMT]	19– 14
[ENTER]	07– 36	[PMT]	20– 14
[g] [LST x]	08– 43 36	[+]	21– 40
[+]	09– 40	[g] [GTO] 00	22–43,33 00
[RCL] 3	10– 45 3	[f] [P/R]	
[×]	11– 20		

	REGISTERS		
n: Term	i: Interest	PV: Loan	PMT: Loan PMT
FV: 0	R_0: Unused	R_1: Purch. Price	R_2: % Down
R_3: Tax rate	R_4—R_7 Unused		

1. Key in the program.
2. Press [f] CLEAR [FIN].
3. Key in the annual interest rate and press [g] [12÷].
4. Key in the term of the loan in years and press [g] [12×].
5. Key in the purchase price and press [STO] 1.
6. Key in the percent down and press [STO] 2.
7. Key in the tax rate in dollars per thousand and press [STO] 3.
8. To calculate the approximate monthly payment, press [R/S].
9. For a new case, store only the new variables by performing steps 3 thru 7 as needed. Press [R/S] for the new approximate monthly payment.

Example: Solve the previous example using the HP-12C program.

Keystrokes	Display	
[f] CLEAR [FIN]		
10.75 [g] [12÷]	**0.90**	Monthly interest.
35 [g] [12×]	**420.00**	Months of loan.
65000 [STO] 1	**65,000.00**	Purchase price.
10 [STO] 2	**10.00**	Percent down.
25 [STO] 3	**25.00**	Tax rate per thousand.
[R/S]	**-672.16**	Approximate monthly payment.

What would the approximate payment be if the loan was at 10% interest?

10 [g] [12÷] [R/S]	**-638.33**	Approximate monthly payment.

What if the down payment is increased to 20%?

20 [STO] 2 [R/S]	**-582.45**	Approximate monthly payment.

Tax-Free Individual Retirement (IRA) or Keogh Plan

The advent of tax-free retirement accounts (IRA or Keogh) has resulted in considerable benefits for many persons who are not able to participate in group profit sharing or retirement plans. The savings due to tax-free status are often considerable, but complex to calculate. Required data are: the years to retirement, the total annual investment, the compound annual interest rate of the investment, and an assumed tax rate which would be paid on a similar non tax-free investment. This program calculates:

1. The future cash value of the tax-free investment.
2. The total cash paid in.
3. The total dividends paid.
4. The future value of the investment at retirement, assuming that after retirement you withdrew the money at a rate which causes the money to be taxed at ½ the rate at which it would otherwise have been taxed during the pay-in period.
5. The diminished purchasing power assuming a given annual inflation rate.
6. The future value of a comparable taxable investment.
7. The diminished purchasing power of 6 above.

Notes:

- The calculations run from the beginning of the first year to the end of the last year.
- The interest (annual yield), i, should be entered to as many significant figures as possible for maximum accuracy.
- The assumed 10% annual inflation rate may be changed by modifying the program at lines 19 and 20.
- The assumed tax rate used to calculate the after-tax value of the tax-free investment may be changed by modifying the program at line 9.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		1	19- 1
f CLEAR PRGM	00-	0	20- 0
RCL n	01- 45 11	RCL n	21- 45 11
RCL PMT	02- 45 14	y^x	22- 21
×	03- 20	÷	23- 10
R/S	04- 31	R/S	24- 31
+	05- 40	RCL i	25- 45 12
R/S	06- 31	1	26- 1
RCL 1	07- 45 1	RCL 1	27- 45 1
.	08- 48	%	28- 25
5	09- 5	−	29- 30
%	10- 25	×	30- 20
CHS	11- 16	i	31- 12
1	12- 1	FV	32- 15
+	13- 40	R/S	33- 31
RCL FV	14- 45 15	g GTO 17	34-43,33 17
×	15- 20	f P/R	
R/S	16- 31		
1	17- 1		
.	18- 48		

	REGISTERS		
n: Years	i: Used	PV: 0	PMT: Yearly Pmt
FV: Used	R_0: Unused	R_1: tax %	R_2–R_5 Unused

1. Key in the program.
2. Press f CLEAR REG and press g BEG.
3. Key in the tax rate as a percentage and press STO 1.
4. Key in years to retirement and press n.

5. Key in the interest rates as a percentage and press [i].
6. Key in the annual payment and press [CHS] [PMT].
7. Press [FV] to calculate the future value of the tax-free investment.
8. Press [R/S] to compute the total cash paid in.
9. Press [R/S] to compute the total dividends paid.
10. Press [R/S] to compute the future value when, after retirement, money is withdrawn at a rate causing the tax rate to equal ½ the rate paid during the pay-in period.
11. Press [R/S] to compute the diminished purchasing power, in terms of today's dollars, of the future value assuming a 10% annual inflation rate.
12. Press [R/S] to compute the future value of an ordinary tax investment.
13. Press [R/S] to compute the diminished purchasing power of the amount in step 12.

Example: Assuming a 35-year investment period with a dividend rate of 8.175% and a tax rate of 40%:

1. If you invest $1500 each year in a tax-free account, what will its value be at retirement?
2. How much cash will be paid in?
3. What will be the value of the earned dividends?
4. After retirement, if you withdraw cash from the account at a rate such that it will be taxed at a rate equal to one-half the rate paid during the pay-in period, what will be the after-tax value?
5. What is the diminished purchasing power of that amount, in today's dollars, assuming 10% annual inflation?
6. If you invest the same amount ($1500)* each year with dividends taxed as ordinary income, what will be the total tax-paid cash at retirement?
7. What is the purchasing power of that figure in terms of today's dollars?

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* $1500 *after taxes* for a non-Keogh or IRA account.

Keystrokes	Display	
f CLEAR REG g BEG		
40 STO 1	40.00	Tax rate.
35 n	35.00	Years to retirement.
8.175 i	8.18	Dividend rate.
1500 CHS PMT	-1,500.00	Annual payment.
FV	290,730.34	Future value at retirement.
R/S	-52,500.00	Cash paid in.
R/S	238,230.34	Earned dividends.
R/S	232,584.27	After-tax value.
R/S	8,276.30	Diminished purchasing power.
R/S	139,360.09	Tax-paid cash at retirement.
R/S	4,959.00	Purchasing power of tax-paid cash at retirement.

Stock Portfolio Evaluation and Analysis

This program evaluates a portfolio of stocks given the current market price per share and the annual dividend. The user inputs the initial purchase price of a stock, the number of shares, the beta coefficient*, the annual dividend, and the current market price for a portfolio of any size.

The program returns the percent change in value of each stock and the valuation and beta coefficient* of the entire portfolio. Output includes the original portfolio value, the new portfolio value, the percent change in value and the annual dividend and yield as a percent of the current market value. The overall beta coefficient of the portfolio is also calculated.

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* The beta coefficient is a measure of a stock variability (risk) compared to the market in general. Beta values for individual stocks can be acquired from brokers, investment publications or the local business library.

Notes:

- Prices are input in the form *XXX.ND* where *N* is the numerator and *D* the denominator of the fractional portion of the price, e.g., 25⅝ is input as 25.58.
- The beta coefficient analysis is optional. Key in 1.00 if beta is not to be analyzed.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		STO − 4	21-44 30 4
f CLEAR PRGM	00-	×	22- 20
STO 6	01- 44 6	RCL 7	23- 45 7
g FRAC	02- 43 24	×	24- 20
g x=0	03- 43 35	STO + 0	25-44 40 0
g GTO 15	04-43,33 15	x≷y	26- 34
1	05- 1	RCL 7	27- 45 7
0	06- 0	×	28- 20
×	07- 20	STO + 1	29-44 40 1
g INTG	08- 43 25	R↓	30- 33
g LSTx	09- 43 36	×	31- 20
g FRAC	10- 43 24	STO + 3	32-44 40 3
÷	11- 10	RCL 5	33- 45 5
1	12- 1	g LSTx	34- 43 36
0	13- 0	△%	35- 24
÷	14- 10	R/S	36- 31
RCL 6	15- 45 6	g GTO 01	37-43,33 01
g INTG	16- 43 25	+	38- 40
+	17- 40	x≷y	39- 34
RCL 4	18- 45 4	STO 7	40- 44 7
g x=0	19- 43 35	×	41- 20
g GTO 38	20-43,33 38	STO 5	42- 44 5

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
STO + 2	43-44 40 2	RCL 0	54- 45 0
1	44- 1	RCL 1	55- 45 1
STO 4	45- 44 4	R/S	56- 31
R/S	46- 31	%T	57- 23
g GTO 01	47-43,33 01	R/S	58- 31
RCL 2	48- 45 2	RCL 3	59- 45 3
R/S	49- 31	RCL 0	60- 45 0
RCL 0	50- 45 0	÷	61- 10
R/S	51- 31	g GTO 00	62-43,33 00
Δ%	52- 24	f P/R	
R/S	53- 31		

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: ΣPV	R_1: ΣDIV	R_2: Σ Orig. Val.
R_3: $\Sigma P_i S_i \beta_i$	R_4: Flag	R_5: P_i	R_6: *XXX.ND*
R_7: S_i	R_8—$R_{.1}$ Unused		

Instructions:

1. Key in the program.
2. Initialize the program by pressing f CLEAR REG.
3. Key in the number of shares of a stock and press ENTER.
4. Key in the initial purchase of the stock and press R/S.
5. Key in the beta coefficient of the stock and press ENTER.
6. Key in the annual dividend of the stock and press ENTER.
7. Key in the present price of the stock and press R/S. The display will show the percent change in the stock value.
8. Repeat steps 3 through 7 until all the stocks are entered.
9. Next, to evaluate the entire portfolio, press g GTO 48.

10. Press [R/S] to see the initial portfolio value.
11. Press [R/S] to see the present portfolio value.
12. Press [R/S] to see the percent change in value.
13. Press [R/S] to see the total yearly dividend.
14. Press [R/S] to see the annual dividend yield as a percent of the current market value.
15. Press [R/S] to see the beta coefficient of the portfolio.
16. For a new case return to step 2.

Example: Evaluate the following portfolio:

Number of Shares Held	Initial Purchase Price	Beta Coefficient	Annual Dividend	Present Market Price	Stock
100	25⅝	.8	$1.70	27¼	Int'l Heartburn
200	30¼	1.2	$2.10	33½	P.D.Q.
50	89⅞	1.3	$4.55	96⅛	Datacrunch
500	65¼	.6	$3.50	64⅜	N.W. Sundial

Keystrokes	Display	
[f] CLEAR [REG]	0.00	
100 [ENTER]	100.00	
25.58 [R/S]	1.00	
.8 [ENTER]	0.80	Int'l Heartburn
1.70 [ENTER]	1.70	
27.14 [R/S]	6.34	Percent change in stock's value.
200 [ENTER]	200.00	
30.14 [R/S]	1.00	
1.2 [ENTER]	1.20	P.D.Q.
2.10 [ENTER]	2.10	
33.12 [R/S]	10.74	Percent change in stock's value.

Keystrokes	Display	
50 [ENTER]	**50.00**	
89.78 [R/S]	**1.00**	
1.3 [ENTER]	**1.30**	Datacrunch
4.55 [ENTER]	**4.55**	
96.18 [R/S]	**6.95**	Percent change in stock's value.
500 [ENTER]	**500.00**	
65.14 [R/S]	**1.00**	
.6 [ENTER]	**0.60**	N.W. Sundial
3.50 [ENTER]	**3.50**	
64.38 [R/S]	**-1.34**	Percent change in stock's value.
[g] [GTO] 48		
[R/S]	**45,731.25**	Original value.
[R/S]	**46,418.75**	Present value.
[R/S]	**1.50**	Percent change in value.
[R/S]	**2,567.50**	Total yearly dividend.
[R/S]	**5.53**	Annual dividend yield.
[R/S]	**0.77**	Portfolio beta coefficient.

Canadian Mortgages

In Canada, interest is compounded semi-annually with payments made monthly. This results in a different monthly mortgage factor than is used in the United States and preprogrammed into the HP-12C. This difference can be easily handled by the addition of a few keystrokes. For any problem requiring an input for [i], the Canadian mortgage factor is calculated first and then this value is entered for [i] in the calculation to give the answer for Canada. The keystrokes to calculate the Canadian mortgage factor are:

1. Press [f] CLEAR [FIN] [g] [END].
2. Key in 6 and press [n].
3. Key in 200 and press [ENTER] [PV].
4. Key in the annual interest rate as a percentage and press [+] [CHS] [FV].
5. Press [i].

The Canadian mortgage factor is now stored in [i] for future use. The examples below show how this factor is used for [i] in Canadian mortgage problems.

Periodic Payment Amount

Example 1: What is the monthly payment required to fully amortize a 30-year, $30,000 Canadian mortgage if the interest rate is 9%?

Keystrokes	Display	
[f] CLEAR [FIN]		
[g] [END]		
6 [n] 200 [ENTER] [PV]		
9 [+] [CHS] [FV] [i]	**0.74**	Canadian mortgage factor
30 [g] [12×]	**360.00**	Total monthly periods in mortgage life
30000 [PV] 0 [FV] [PMT]	**−237.85**	Monthly payment

Number of Periodic Payments to Fully Amortize a Mortgage

Example 2: An investor can afford to pay $440 per month on a $56,000 Canadian mortgage. If the annual interest rate is 9¼%, how long will it take to completely amortize this mortgage?

Keystrokes	Display	
f CLEAR FIN g END 6 n 200 ENTER PV 9.25 + CHS FV i	**0.76**	Canadian mortgage factor.
440 CHS PMT	**-440.00**	Monthly payment.
56000 PV 0 FV n	**437.00**	Total number of monthly payments.

Effective Interest Rate (Yield)

Example 3: A Canadian mortgage has monthly payments of $612.77 with a maturity of 25 years. The principal amount is $75,500. What is the annual interest rate?

Keystrokes	Display	
f CLEAR FIN g END 25 g 12×		
612.77 CHS PMT 75500 PV i	**0.72**	Canadian mortgage factor.
6 n 0 PMT 200 CHS PV FV RCL PV +	**8.75**	Annual interest rate.

Balance Remaining at End of Specified Period

Example 4: A Canadian mortgage has monthly payments of $612.77 at 8.75% interest. The principal amount is $75,500. What will be the outstanding balance remaining at the end of 10 years?

Keystrokes	Display	
f CLEAR FIN		
g END		
6 n 200 ENTER PV		
8.75 + CHS FV i	**0.72**	Canadian mortgage factor.
612.77 CHS PMT		
10 g 12×		
75500 PV FV	**-61,877.18**	Outstanding balance remaining at the end of 10 years.

Miscellaneous

Learning Curve for Manufacturing Costs

Many production process costs vary with output according to the "learning curve" equation. The production team becomes more proficient in manufacturing a given item as more and more of them are fabricated and costs may be expected to decrease by a predictable amount. The learning factor, r, characterizes the learning curve. For instance, if $r = .80$ the curve is called an 80% learning curve.

It is readily apparent that the learning, or experience curve, has many uses in setting production standards, forecasting costs, setting prices, etc. Note, however, that the learning factor may change, especially after large numbers have been produced.

If the cost of the first unit of a run, C_1, and the learning curve factor, r, are known, the following procedure can be used to calculate the cost of the nth item:

1. Key in the cost of the first item, C_1, and press [ENTER].
2. Key in the number of units produced, n, and press [ENTER].
3. Key in the learning factor, r, and press [g] [LN] 2 [g] [LN] [÷].
4. Then press [y^x] [×] to calculate the cost of the nth unit, C_n.

Example 1: An electronic manufacturer begins a pilot run on a new instrument. From past experience he expects the process to have a learning factor, r, of 0.90. If the first unit costs $875 to produce, what is the expected cost of the 100th unit?

Keystrokes	Display	
875 [ENTER]	875.00	
100 [ENTER]	100.00	
.9 [g] [LN] 2 [g] [LN] [÷]	-0.15	
[y^x] [×]	434.51	Cost of the 100th unit.

If the costs of the first unit, C_1, and the nth unit, C_n, are known the learning factor may be calculated. In addition, it is possible to calculate C_{ij}, the average cost of the ith thru jth unit. These calculations may be rapidly done with the following HP-12C program:

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
f P/R		×	23- 20
f CLEAR PRGM	00-	g GTO 00	24-43,33 00
g LN	01- 43 23	STO 3	25- 44 3
2	02- 2	$x \geq y$	26- 34
g LN	03- 43 23	STO 4	27- 44 4
÷	04- 10	RCL 2	28- 45 2
STO 2	05- 44 2	g LN	29- 43 23
R↓	06- 33	2	30- 2
$x \geq y$	07- 34	g LN	31- 43 23
STO 1	08- 44 1	÷	32- 10
÷	09- 10	1	33- 1
g LN	10- 43 23	+	34- 40
RCL 2	11- 45 2	STO 0	35- 44 0
÷	12- 10	y^x	36- 21
g e^x	13- 43 22	RCL 3	37- 45 3
STO 2	14- 44 2	RCL 0	38- 45 0
g GTO 00	15-43,33 00	y^x	39- 21
RCL 2	16- 45 2	−	40- 30
g LN	17- 43 23	RCL 0	41- 45 0
2	18- 2	÷	42- 10
g LN	19- 43 23	RCL 4	43- 45 4
÷	20- 10	RCL 3	44- 45 3
y^x	21- 21	−	45- 30
RCL 1	22- 45 1	÷	46- 10

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
RCL 1	47- 45 1	g GTO 00	49-43,33 00
×	48- 20	f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: $K+1$	R_1: C_1	R_2: r
R_3: i	R_4: j	R_5—R_3 Unused	

1. Key in the program, (Note: If average costs are not going to be calculated, lines 25 through 48 need not be keyed in).
2. To calculate r, the learning factor, if C_1 and C_n are known:
   a. Key in C_1, the cost of the first unit and press ENTER.
   b. Key in C_n, the cost of the nth unit and press ENTER.
   c. Key in n, the number of units and press R/S to calculate r, the learning factor.
3. To calculate the cost of the nth unit when C_1 and r are known:
   a. Key in C_1 and press STO 1. Key in r and press STO 2. (Note: This step may be skipped if step 2 has just been done).
   b. Key in the number of units, n and calculate C_n, the cost of the nth unit by pressing g GTO 16 R/S.
4. To calculate the average cost per unit of the ith through jth unit, C_{ij}, if C_1 and r are known:
   a. Key in C_1 and press STO 1. Key in r and press STO 2. (Note: This step may be skipped if step 2 has just been done).
   b. Key in the number of the last unit of the batch, j and press ENTER.
   c. Key in the number of the first unit of the batch, i, and calculate the average cost per unit by pressing g GTO 25 R/S.

Example 2: The electronic manufacturer cited in example 1 found that the 100th instrument actually cost $395 to manufacture. Find the actual learning factor, r, the cost of the 500th unit and the average cost of units 500 thru 1000. (Recall that C_1 was $875).

Keystrokes	Display	
875 [ENTER]	**875.00**	
395 [ENTER]	**395.00**	
100 [R/S]	**0.89**	Actual r.
500 [g] [GTO] 16 [R/S]	**299.14**	Cost of the 500th unit.
1000 [ENTER]	**1000.00**	
500 [g] [GTO] 25 [R/S]	**280.00**	Average cost of the 500th thru 1000 unit.

Queuing and Waiting Theory

Waiting lines, or queues, cause problems in many marketing situations. Customer goodwill, business efficiency, labor and space considerations are only some of the problems which may be minimized by proper application of queuing theory.

Although queuing theory can be a complex and complicated subject, handheld calculators can be used to arrive at helpful decisions.

One common situation that we can analyze involves the case of several identical stations serving customers, where the customers arrive randomly in unlimited numbers. Suppose there are n (1 or more) identical stations serving the customers. λ is the arrival rate (Poisson input) and μ is the service rate (exponential service). We will assume that all customers are served on a first-come, first-served basis and wait in a single line (queue) then are directed to whichever station is available. We also will assume that no customers are lost from the queue. This situation, for instance, would be closely approximated by customers at some banking operations.

The formulas for calculating some of the necessary probabilities are too complex for simple keystroke solution. However, tables listing these probabilities are available and can be used to aid in quick solutions. Using the assumptions outlined above and a

suitable table giving mean waiting time as a multiple of mean service (see page 512 of the Reference) the following keystroke solutions may be obtained:

1. Key in the arrival rate of customers, λ, and press [ENTER].
2. Key in the service rate, μ, and press [÷] to calculate ρ, the **intensity factor**. (Note ρ must be less than n for valid results, otherwise the queue will lengthen without limit).
3. Key in n, the number of servers and press [÷] to calculate ρ/n.
4. For a given n and ρ/n find the mean waiting time as a multiple of mean service time from the table. Key it in and press [ENTER].
5. Calculate the **average waiting time** in the queue by keying in the service rate, μ, and pressing [STO] 1 [÷] [STO] 2.
6. Calculate the **average waiting time** in the system by pressing [RCL] 1 [1/x] [+].
7. Key in λ and press [RCL] 2 [×] to calculate the **average queue length**.
8. Key in ρ, the intensity factor (from step 2 above) and press [+] to calculate the **average number of customers in the system**.

Reference:

Richard E. Trueman, "An Introduction to Quantitative Methods for Decision Making," Holt, Rinehart and Winston, New York, 1977.

Example 1: Bank customers arrive at a bank on an average of 1.2 customers per minute. They join a common queue for three tellers. Each teller completes a transaction at the rate of one customer every 2 minutes (0.5 customers per minute). What is the average waiting time in the queue? In the system? What is the average number of customers in the queue? In the system?

Keystrokes	Display	
1.2 [ENTER]	**1.20**	
.5 [÷]	**2.40**	ρ, intensity factor.
3 [÷]	**0.80**	ρ/n

From Table 12.2, page 512 of the reference, the mean waiting time as a multiple of mean service time for $n = 3$, $\rho/n = 0.8$ is 1.079. (Note S is used instead of n in the reference's notation).

1.079 [ENTER]	**1.08**	
.5 [STO] 1 [÷] [STO] 2	**2.16**	Average wait in queue (min).
[RCL] 1 [1/x] [+]	**4.16**	Average wait in system (min).
1.2 [RCL] 2 [×]	**2.59**	Average queue length.
2.4 [+]	**4.99**	Average # customers in system.

If the number of servers is limited to one, with other conditions remaining the same (unlimited queue, Poisson arrival, exponential service), the average queue length can be readily calculated without reference to tables:

1. Key in the arrival rate, λ, and press [STO] 1.
2. Key in the service rate, μ, and press [STO] 2 [÷] [ENTER] [ENTER] 2 [y^x] [$x \geqslant y$] 1 [$x \geqslant y$] [−] [÷] to calculate the average number of customers waiting in queue at any one time.
3. Press [RCL] 1 [÷] to calculate the average waiting time.
4. Press [RCL] 2 [1/x] [+] to calculate the average total time the customer spends in the system.
5. Press [RCL] 1 [×] to calculate the average number of customers in the system.

Example 2: A small grocery store has but a single check-out counter. Customers arrive at a rate of 1 every 2 minutes ($\lambda = .5$) and, on the average, customers can be checked out at a rate of .9 per minute (μ). What is the average number of customers in the waiting line at any time? The average waiting time? What is the average total time for a customer to wait and be checked out? The average number of customers in the system?

Keystrokes	**Display**
.5 [STO] 1	**0.50**
.9 [STO] 2 [÷] [ENTER] [ENTER] 2 [y^x] [$x \geqslant y$]	**0.56**

Keystrokes	Display	
1 [x≷y] [−] [÷]	0.69	Average # customers waiting in queue.
[RCL] 1 [÷]	1.39	Average waiting time.
[RCL] 2 [1/x] [+]	2.50	Average total time in the system.
[RCL] 1 [×]	1.25	Average # customers in the system.

With an HP-12C program one can readily calculate the necessary probabilities for this type of problem (dispensing with the use of tables) and perform additional calculations as well.

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
[f] [P/R]		[RCL] 7	17- 45 7
[f] CLEAR [PRGM]	00-	[y^x]	18- 21
1	01- 1	1	19- 1
[STO] [−] 0	02-44 30 0	[RCL] [.] 0	20-45 48 0
[RCL] [.] 0	03-45 48 0	[RCL] 7	21- 45 7
[RCL] 0	04- 45 0	[÷]	22- 10
0	05- 0	[−]	23- 30
[g] [x≤y]	06- 43 34	[÷]	24- 10
[g] [GTO] 09	07-43,33 09	[RCL] 7	25- 45 7
[g] [GTO] 16	08-43,33 16	[g] [n!]	26- 43 3
[+]	09- 40	[÷]	27- 10
[y^x]	10- 21	[STO] 6	28- 44 6
[g] [LSTx]	11- 43 36	[RCL] 2	29- 45 2
[g] [n!]	12- 43 3	[+]	30- 40
[÷]	13- 10	[1/x]	31- 22
[Σ+]	14- 49	[STO] 1	32- 44 1
[g] [GTO] 01	15-43,33 01	[RCL] 6	33- 45 6
[RCL] [.] 0	16-45 48 0	[×]	34- 20

KEYSTROKES	DISPLAY	KEYSTROKES	DISPLAY
STO 2	35- 44 2	RCL 8	50- 45 8
RCL . 0	36-45 48 0	÷	51- 10
×	37- 20	STO 6	52- 44 6
RCL 7	38- 45 7	R/S	53- 31
RCL . 0	39-45 48 0	RCL 8	54- 45 8
−	40- 30	RCL 7	55- 45 7
÷	41- 10	RCL 9	56- 45 9
STO 3	42- 44 3	×	57- 20
RCL . 0	43-45 48 0	−	58- 30
+	44- 40	×	59- 20
STO 4	45- 44 4	g e^x	60- 43 22
RCL 8	46- 45 8	RCL 2	61- 45 2
÷	47- 10	×	62- 20
STO 5	48- 44 5	g GTO 53	63-43,33 53
RCL 3	49- 45 3	f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: K	R_1: P_0	R_2: P_b
R_3: L_q	R_4: L	R_5: T	R_6: Used, T_q
R_7: n	R_8: λ	R_9: μ	$R_{.0}$: ρ
$R_{.1}$ Unused			

1. Key in the program and press f CLEAR REG.
2. Key in the number of servers, n, and press STO 0 STO 7.
3. Key in the arrival rate of customers, λ, and press STO 8.
4. Key in the service rate of each server, μ, and press STO 9.
5. Press ÷ STO . 0 to calculate and store ρ, the intensity factor.

6. Press [R/S] to see T_q, the average waiting time in the queue. Display P_0, probability that all servers are idle, by pressing [RCL] 1. Display P_b, probability that all servers are busy, by pressing [RCL] 2. Display L_q, average number waiting in the queue, by pressing [RCL] 3. Display L, average number in the system (waiting and being served), by pressing [RCL] 4.

   Display T, average total time through the system, by pressing [RCL] 5. T_q, the average waiting time in the queue, may again be displayed by pressing [RCL] 6.

7. If desired, calculate $P(t)$, the probability of waiting longer than a given time, by keying in the time and pressing [R/S].

8. Repeat step 7 for other times of interest.

Example 3: Using the data from example 1 of the keystroke solutions verify the data obtained. In addition, obtain P_0, the probability that none of the tellers are busy, and P_b, the probability that all the tellers are busy. What is the probability that a customer will have to wait 2 minutes or more?

Keystrokes	Display	
[f] CLEAR [REG]	**0.00**	
3 [STO] 0 [STO] 7	**3.00**	n
1.2 [STO] 8	**1.20**	λ
.5 [STO] 9	**0.50**	μ
[÷] [STO] [.] 0	**2.40**	ρ
[R/S]	**2.16**	T_q, average waiting time in queue.
[RCL] 1	**0.06**	P_0, probability all servers are idle.
[RCL] 2	**0.65**	P_b, probability all servers are busy.
[RCL] 3	**2.59**	L_q, average # waiting in queue.
[RCL] 4	**4.99**	L, average # waiting in system.
[RCL] 5	**4.16**	T, average total time in the system.
2 [R/S]	**0.36**	Probability of having to wait 2 minutes or more.

Appendix

Real Estate

Wrap-Around Mortgage

n_1 = number of years remaining in original mortgage.

PMT_1 = yearly payment of original mortgage.

PV_1 = remaining balance of original mortgage.

n_2 = number of years in wrap-around mortgage.

PMT_2 = yearly payment of wrap-around mortgage.

PV_2 = total amount of wrap-around mortgage.

r = interest rate of wrap-around mortgage as a decimal.

FV = balloon payment.

$$PV_2 - PV_1 = \frac{PMT_2[1-(1+r)^{-n2}]}{r}$$

$$-\frac{PMT_1[1-(1+r)^{-n1}]}{r} + FV(1+r)^{-n2}$$

After-Tax Cash Flows

$ATCF_k$ = After-Tax Cash Flow for kth year.

Int_k = interest for kth year.

Dep_k = depreciation for kth year.

r = appropriate tax rate.

NOI = Net Operating Income.

$ATCF_k = NOI(1-r) - 12 \times PMT + r \times (Int_k + Dep_k)$.

After-Tax Net Cash Proceeds of Resale

CO = capital purchase.

CPR = sales price − closing costs.

r = marginal tax rate.

$NCPR = CPR$ − remaining balance of mortgage.

$$ATNCPR = NCPR + r \times [(.6 \text{ SL Dep.} - \text{Total Dep}) + .4 \times (CO - CPR)]$$

Lending

Loans With a Constant Amount Paid Towards Principal

BAL_k = remaining balance after time period k.

$CPMT$ = constant payment to principal.

$BAL_k = PV - (k \times CPMT)$

Kth payment to interest $= i(BAL_k) = (PMT_i)_k$

Kth total payment $= CPMT + (PMT_i)_k$

Add-On Interest Rate to APR

r = add-on rate as a decimal.

n = number of monthly payments.

$APR = 1200i$, where i is the solution in the following equation:

$$\frac{n}{1 + \frac{n}{12}r} = \frac{1 - (1+i)^{-n}}{i}$$

Add-On to APR With Credit Life

CL = credit life as decimal.

AMT = loan amount.

FC = finance charge.

$$\left[\frac{1+\left(\frac{n}{12}\right)r}{1-\left(\frac{n}{12}\right)CL-\left(\frac{n}{12}\right)^{2}CL\cdot r}\right] \quad AMT = G$$

$$\frac{G}{n} = PMT$$

$$\frac{G \times CL \times n}{12} = \text{amount of credit life}$$

$$FC = (G - AMT - CL)$$

Rule of 78's Rebate

PV = finance charge.

I_k = interest charged at month k.

n = number of months in loan.

$$I_k = \frac{2(n-k+1)}{n(n+1)} PV$$

$$Rebate = \frac{(n-k)\,I_k}{2}$$

$$BAL_k = (n-k) \times PMT - Rebate_k$$

Skipped Payments

A = number of payment periods per year.

B = number of years.

C = annual percentage rate (as decimal).

D = periodic payment amount.

E = loan amount.

K = number of last payment before payments close the first time.

L = number of skipped payments.

$$D_{END} = \frac{E}{\left[1 - \left(1 + \frac{C}{A}\right)^{-AB}\right]}$$

$$\times \frac{\left[\left(1 + \frac{C}{A}\right)^{A} - 1\right]\frac{C}{A}}{\left[\left(1 + \frac{C}{A}\right)^{A} - \left(1 + \frac{C}{A}\right)^{A-K} + \left(1 + \frac{C}{A}\right)^{A-L-K} - 1\right]}$$

$$D_{BEGIN} = \frac{D_{END}}{1 + \frac{C}{A}}$$

Savings

Compounding Periods Different From Payment Periods

C = number of compounding periods per year.

P = number of payment periods per year.

i = periodic interest rate, expressed as a percentage.

$r = i/100$, periodic interest rate expressed as a decimal.

$i_{PMT} = ((1 + r/C)^{C/P} - 1)\,100$

Investment Analysis

Lease vs. Purchase

PMT_p = loan payment for purchase.

PMT_L = lease payment.

I_n = interest portion of PMT_p for period n.

D_n = depreciation for period n.

M_n = maintenance for period n.

T = marginal tax rate.

Net purchase advantage =

$$\sum_{n=1}^{k} \frac{\text{cost of leasing } (n) - \text{cost of owning } (n)}{(1+i)^n}$$

Cost of leasing $(n) = (1 - T)\, PMT_L$

Cost of owning $(n) = PMT_p - T(I_n + D_n) + (1 - T)M_n$

Break-Even Analysis and Operating Leverage

GP = Gross Profit.

P = Price per unit.

V = Variable costs per unit.

F = Fixed costs.

U = number of Units.

OL = Operating Leverage.

$GP = U(P - V) - F$

$$OL = \frac{U(P - V)}{U(P - V) - F}$$

Profit and Loss Analysis

Net income = (1 − tax) (net sales price − manufacturing expense − operating expense)

Net sales price = list price (1 − discount rate)

where operating expense represents a percentage of net sales price.

Securities

Discounted Notes

Price (given discount rate)

B = number of days in year (annual basis).

DR = discount rate (as decimal).

DSM = number of days from settlement date to maturity date.

P = dollar price per \$100 per value.

RV = redemption value per \$100 par value.

$$P = \left[RV\right] - \left[DR \times RV \times \frac{DSM}{B}\right]$$

Yield (given price)

B = number of days in year (annual basis).

DSM = number of days from settlement date to maturity date.

P = dollar price per \$100 par value.

RV = redemption value per \$100 par value.

Y = annual yield of investment with security held to maturity (as a decimal).

$$Y = \left[\frac{RV - P}{P}\right] \times \left[\frac{B}{DSM}\right]$$

Forecasting

Simple Moving Average

$\bar{x}$ = moving average.

m = number of elements in the moving average.

$$\bar{x}_1 = \frac{x_1 + x_2 + x_3 + \ldots x_m}{m}$$

$$\bar{x}_2 = \frac{x_2 + x_3 + x_4 + \ldots x_{m+1}}{m}$$

etc.

Seasonal Variation Factors Based on a Centered Moving Average

$\bar{x}_c$ = centered moving average

m = number of elements in the centered moving average.

$$\bar{x}_c = \frac{\frac{x_1}{2} + (x_2 + x_3 + \ldots x_m) + \frac{x_{m+1}}{2}}{m}$$

SV = seasonal variation factor.

x_i = value of the ith data point.

$\bar{x}_i$ = centered moving average of the ith data point.

$$SV = \frac{x_i}{\bar{x}_i}$$

Gompertz Curve Trend Analysis

$y = ca^{(b^x)}$

where x, y, a, b and c are positive

$$b = \left(\frac{S_3 - S_2}{S_2 - S_1} \right)^{1/n}$$

$$c = \exp\left[\frac{1}{n} \left(\frac{S_1 S_3 - S_2^2}{S_1 + S_3 - 2S_2} \right) \right]$$

$$a = \exp\left[\frac{(b-1)(S_2 - S_1)}{b(b^n - 1)^2} \right]$$

where S_1, S_2 and S_3 are

$$S_1 = \sum_{i=1}^{n} \ln y_i = n \ln c + b (\ln a) \frac{b^n - 1}{b - 1}$$

$$S_2 = \sum_{i=n+1}^{2n} \ln y_i = n \ln c + b^{n+1} (\ln a) \frac{b^n - 1}{b - 1}$$

$$S_3 = \sum_{i=2n+1}^{3n} \ln y_i = n \ln c + b^{2n+1} (\ln a) \frac{b^n - 1}{b - 1}$$

a, b and c are determined by solving the three equations above simultaneously.

Forecasting With Exponential Smoothing

α = smoothing constant $(0 < \alpha < 1)$

X_t = actual current period usage

Smoothed average $S_t = \alpha X_t + (1 - \alpha) S_{t-1}$

Change, $C_t = S_t - S_{t-1}$

Trend, $T_t = \alpha C_t + (1 - \alpha) T_{t-1}$

Current period expected usage, $D_t = S_t + \dfrac{(1 - \alpha)}{\alpha} T_t$

Forecast of next period expected usage, $\hat{D}_{t+1} = S_t + \left(\dfrac{1}{\alpha}\right) T_t$

Error, $e_t = \hat{D}_t - X_t$

Cumulative error $= \sum_{t=1}^{m} e_t^2$

Initial conditions: $S_{t-1} = X_{t-1}$

$T_{t-1} = 0$

Pricing Calculations

Markup and Margin Calculations

Ma = margin (%).

Mu = markup (%).

S = selling price.

C = cost.

$$Ma = 100\,\frac{S - C}{S}$$

$$Mu = 100\,\frac{S - C}{C}$$

$$S = \frac{C}{1 - \dfrac{Ma}{100}}$$

$$S = C\left(1 + \frac{Mu}{100}\right)$$

$$C = S\left(1 - \frac{Ma}{100}\right)$$

$$C = \frac{S}{1 + \dfrac{Mu}{100}}$$

$$Ma = \frac{Mu}{1 + \dfrac{Mu}{100}}$$

$$Mu = \frac{Ma}{1 - \dfrac{Ma}{100}}$$

Calculation of List and Net Prices With Discounts

L = List price.

N = Net price.

D = Discount (%).

$$D' = 1 - \frac{D}{100}$$

$$L = \frac{N}{D'_1 \times D'_2 \times \ldots D'_x}$$

$$D_x = 100 \left(1 - \frac{N}{L\,(D_1 \times D_2 \times \ldots \times D_{x-1}}\right)$$

Statistics

Exponential Curve Fit

$$y = Ae^{Bx}$$

$$B = \frac{\Sigma x_i \ln y_i - \frac{1}{n}(\Sigma x_i)(\Sigma \ln y_i)}{\Sigma {x_i}^2 - \frac{1}{n}(\Sigma x_i)^2}$$

$$A = \exp\left[\frac{\Sigma \ln y_i}{n} - B\frac{\Sigma x_i}{n}\right]$$

$$\hat{y} = Ae^{Bx}$$

Logarithmic Curve Fit

$$y = A + B(\ln x)$$

$$B = \frac{\Sigma y_i \ln x_i - \frac{1}{n}\Sigma \ln x_i \Sigma y_i}{\Sigma (\ln x_i)^2 - \frac{1}{n}(\Sigma \ln x_i)^2}$$

$$A = \frac{1}{n}(\Sigma y_i - B\Sigma \ln x_i)$$

$$\hat{y} = A + B(\ln x)$$

Power Curve Fit

$$y = Ax^B \qquad (A > 0)$$

$$\ln y = \ln A + B \ln x$$

$$\mathrm{B} = \frac{\Sigma (\ln x_i)(\ln y_i) - \frac{(\Sigma \ln x_i)(\Sigma \ln y_i)}{n}}{\Sigma (\ln x_i)^2 - \frac{(\Sigma \ln x_i)^2}{n}}$$

$$A = \exp\left[\frac{\Sigma \ln y_i}{n} - B\frac{\Sigma \ln x_i}{n}\right]$$

$$\hat{y} = Ax^B$$

Standard Error of the Mean

$$S_x = \frac{S_x}{\sqrt{n}} \qquad S_y = \frac{S_y}{\sqrt{n}}$$

Mean, Standard Deviation, Standard Error For Grouped Data

$$\text{mean } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\text{standard deviation } S_x = \sqrt{\frac{\Sigma f_i x_i^2 - (\Sigma f_i)\,\bar{x}^2}{\Sigma f_i - 1}}$$

$$\text{standard error } S_{\bar{x}} = \frac{S_x}{\sqrt{\Sigma f_i}}$$

Personal Finance

Tax-Free Retirement Account (IRA) or Keogh Plan

n = the number of years to retirement.

i = the compounded annual interest.

PMT = the earnings used for investment (and taxes).

FV = future value.

tax = the percent tax (expressed as a decimal).

For ordinary taxable investment:

$$FV = \frac{PMT}{i(1-\text{tax})}[1 + i\,(1 - \text{tax})]\,\{[1 + i\,(1 - \text{tax})]^n - 1\}$$

For tax-free investment:

$$FV = \frac{PMT}{i}(1 + i)\,[(1 + i)^n - 1]$$

Stock Portfolio Evaluation and Analysis

n = the number of issues held.

P_i = the current market price/share of a stock.

S_i = the number of shares of a stock held.

β_i = the beta coefficient of an individual stock.

T = the total present value of a portfolio.

Portfolio beta coefficient:

$$\beta = \sum_{i=1}^{n} \frac{P_i\,S_i\,\beta_i}{T}$$

Canadian Mortgages

r = annual interest rate expressed as a decimal.

$$\text{monthly factor} = \left(\left(1 + \frac{r}{2} \right)^{1/6} - 1 \right) \times 100$$

Miscellaneous

Learning Curve for Manufacturing Cost

C_n = Cost of nth unit.

C_1 = Cost of first unit.

n = number of units.

r = learning factor.

$k = \ln r / \ln 2$

$C_n = C_1 n^k$

$\bar{C}_{ij}$ = the average cost of the ith through jth unit:

$$\bar{C}_{ij} = \frac{C_1}{j - i} \left[\frac{j^{k+1} - i^{k+1}}{k + 1} \right]$$

This formula is only approximate and may give appreciable error at small i.

Queuing and Waiting Theory

n = number of servers.

λ = arrival rate of customers (Poisson input).

μ = service rate of each server (exponential service).

ρ = Intensity factor = λ / μ ($\rho < n$ for valid results).

P_0 = Probability that all servers are idle.

P_b = Probability that all servers are busy.

L_q = average number of customers in queue.

L = average number of customers in system (waiting and being served).

T_q = average waiting time in queue.

T = average total time through the system.

$P(t)$ = Probability of waiting longer than time t.

$$P_0 = \left[\sum_{k=0}^{n-1} \frac{\rho^k}{k!} + \frac{\rho^n}{n!\left(1 - \frac{\rho}{n}\right)} \right]^{-1}$$

$$P_b = \frac{\rho^n P_0}{n!\left(1 - \frac{\rho}{n}\right)}$$

$$L_q = \frac{\rho P_b}{n - \rho} \quad L = L_q + \rho, \qquad T = L/\lambda, \; T_q = \frac{L_q}{\lambda}$$

$$P(t) = P_b e^{-(n\mu - \lambda)t}$$

Graduated Payment Mortgage

$$\mathrm{PV} = \mathrm{PMT}_1 \left\{ \left[\frac{1 - (1 + \mathrm{I})^{-\mathrm{A}}}{\mathrm{I}} \right] \left[\frac{(1 + \mathrm{Q})^{\mathrm{B}} - 1}{\mathrm{Q}} \right] + \frac{(1 + \mathrm{C})^{\mathrm{B}} \left[\frac{1 - (1 + \mathrm{I})^{-(\mathrm{n} - \mathrm{AB})}}{\mathrm{I}} \right]}{(1 + \mathrm{I})^{\mathrm{AB}}} \right\}$$

where:

$$\mathrm{Q} = \frac{1 + \mathrm{C}}{(1 + \mathrm{I})^{\mathrm{A}}} - 1$$

A = number of payments per year

B = number of years that payments increase

C = percent increase in periodic payment (as decimal)

PMT_1 = amount of first payment

Real Estate

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Pricing Calculations

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Miscellaneous

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