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CALCULATOR SUPPORT
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HP 18C SOLUTIONS

The following solutions have been developed as a continuing effort by Hewlett-Packard to meet the needs of our customers.

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18-7 <u>Duration of a Bond</u>

Formula to calculate the duration of a bond (the average time that elapses until the various cash flows are received).

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Bond Price and Yield

The following formulas calculate bond price, yield to maturity, and accrued interest. The computations assume that the price is quoted as a percentage and redemption can be at maturity or at call. Semi-annual or annual coupon payments can be specified. Either calendar basis (30/360 or Actual/Actual) can be accommodated.

Entering and Using the Formulas.

1. From the MAIN menu, press SOLVE to display the SOLVE menu.
2. Type in the following formulas. After typing in each formula, press [INPUT]. (To key in the < character, press ■ [Y].)

```
ACTUAL: N=-DDAYS(PREV:SETT:1)÷DDAYS(PREV:NEXT:1)
          +RND(DDAYS(PREV:MAT:1)×M÷365:0)

360: N=DDAYS(SETT:MAT:3)×M÷360

(PRICE+CPN÷M×(1-FP(N)))×SPFV(YLD÷M×MIN(N:1):IF(N<1:1:FP(N)))
=CPN÷M×(1+USPV(YLD÷M:IP(N)))+RV×SPPV(YLD÷M:IP(N))

AI=CPN÷M×(1-FP(N))×SGN(FP(N))
```

3. For Actual/Actual calendar bonds, select the ACTUAL formula and press CALC to display the custom menu. Store the following variables:
   - Date of the previous coupon in PREV.
   - Settlement date in SETT.
   - Date of the next coupon in NEXT.
   - Maturity date in MAT.
   - Number of coupon payments per year in M.
4. For 30/360 calendar bonds, select the 360 formula and press CALC to display the custom menu. Store the following variables:
   - Settlement date in SETT.
   - Maturity date in MAT.
   - Number of coupon payments per year in M.
5. Press N to calculate the "life" of the bond. Press [EXIT] to continue the calculation.

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6. Select the PRICE formula and press **CALC** to display the custom menu.
7. Store the following variables:
   - Annual coupon rate as a percent in **CPN**.
   - Redemption value or call price in **RV**.
8. To calculate the purchase price, store the annual yield in **YLD** and press **PRICE**.
9. To calculate the annual yield, store the purchase price (as a percentage) in **PRICE** and press **YLD**.
10. Press [EXIT] [↓] **CALC** to designate AI (accrued interest) as the current formula and display the custom menu.
11. Press **AI** to calculate the amount of accrued interest.

The following examples assume that the four formulas have been entered in the order given. In addition, two decimal places are displayed.

Example 1: Bond Price. What price should you pay on April 28, 1986 for a 6¾% U.S. Treasury Bond that matures at 100 on December 4, 1996 if you want a yield of 8¼%? There are two coupons each year and the Actual/Actual calendar is used.

Start from the ACTUAL menu.

Keys:	Display:	Description:
12.041985 **PREV**	PREV=12.04	Stores previous coupon date.
4.281986 **SETT**	SETT=4.28	Stores settlement date.
6.041986 **NEXT**	NEXT=6.04	Stores next coupon date.
12.041996 **MAT**	MAT=12.04	Stores maturity date.
2 **M**	M=2.00	Stores number of coupons per year.
N	N=21.20	Calculates bond life.
[EXIT] [↓] [↓] **CALC**		Displays PRICE menu.
6.75 **CPN**	CPN=6.75	Stores annual coupon.
100 **RV**	RV=100.00	Stores redemption value.
8.25 **YLD**	YLD=8.25	Stores annual yield.
PRICE	PRICE=89.52	Calculates bond price.

What is the amount of accrued interest?

[EXIT] [↓] **CALC**		Displays AI custom menu.
AI	AI=2.69	Calculates accrued interest.

Example 2: Bond Yield. Calculate the yield of a municipal bond purchased on May 1, 1986 that matures at 100 on July 1, 1995. The coupon rate is 6.5% and the purchase price is 99.525. There are two coupons each year and the 30/360 calendar is used.

Start from the 360 menu.

Keys:	**Display:**	**Description:**
5.011986 SETT	SETT=5.01	Stores known values.
7.011995 MAT	MAT=7.01	
2 M	M=2.00	
N	N=18.33	Calculates bond life.
EXIT ↓ CALC		Displays PRICE menu.
99.525 PRICE	PRICE=99.53	Stores known values.
6.5 CPN	CPN=6.50	
100 RV	RV=100.00	
YLD	YLD=6.57 *	Calculates bond yield.

Example 3: Yield to Call. A 10% coupon bond was purchased October 14, 1980 for 97.25. If the bond is called on September 16, 1986 for 103, what is the yield to call? There are two coupons each year and the calendar basis is 30/360.

Start from the 360 menu.

Keys:	**Display:**	**Description:**
10.141980 SETT	SETT=10.14	Stores known values.
9.161986 MAT	MAT=9.16	
2 M	M=2.00	
N	N=11.84	Calculates bond life.
EXIT ↓ CALC		Displays PRICE menu.
97.25 PRICE	PRICE=97.25	Stores known values.
10 CPN	CPN=10.00	
103 RV	RV=103.00	
YLD *	YLD=11.01 *	Calculates yield to call.

* The solver searches for a numerical solution and displays intermediate estimates.

Additional Information

- Since the formulas are long, you may need to clear most of memory before keying them in.
- If you always use the same calendar, choose the appropriate calendar formula (either ACTUAL or 360) and omit the other formula.
- Use the maturity date to determine the previous coupon date (PREV) and the next coupon date (NEXT). PREV and NEXT are one coupon period apart and the following applies:

$$PREV \le SETT < NEXT$$

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Weighted Mean

The following routine calculates the weighted mean of a set of numbers, assuming that you know the corresponding weights of each item.

1. From the MAIN menu, press SUM to select the SUM menu.
2. Press ■ [CLEAR ALL] YES to clear the list. (If you do not want to delete the list, name the old list and get a new list.)
3. Press 0 [STO] 0 to clear register 0.
4. Key in the value of the item and press [×].
5. Key in the weight of the item and press [STO] [+] 0 [INPUT].
6. Repeat steps 4 and 5 until you have entered all the values of the items and their corresponding weights.
7. Press [÷] [RCL] 0 [=] to calculate the weighted mean.

Example. A survey of 266 one-bedroom apartment rentals reveals that 54 rent for $190 per month, 32 rent for $195 per month, 88 rent for $200 per month, and 92 rent for $206 per month. What is the average monthly rental?

Start from the MAIN menu.

Keys:	**Display:**	**Description:**
SUM *		Displays the SUM menu.
■ [CLEAR ALL] YES	▶ITEM(1)= TOTAL=0.00	Clears the list.
0 [STO] 0		Clears register 0.
190 [×] 54 [STO] [+] 0 [INPUT]	▶ITEM(2)= TOTAL=10,260.00	Stores first data pair.
195 [×] 32 [STO] [+] 0 [INPUT]	▶ITEM(3)= TOTAL=16,500.00	Stores second data pair.
200 [×] 88 [STO] [+] 0 [INPUT]	▶ITEM(4)= TOTAL=34,100.00	Stores third data pair.
206 [×] 92 [STO] [+] 0 [INPUT]	▶ITEM(5)= TOTAL=53,052.00	Stores fourth data pair.
[÷] [RCL] 0 [=]	199.44	Calculates average monthly rent.

* If you want to preserve the current list, skip the next step (pressing 0 ■ [CLEAR ALL]), name the list, and press [illegible] [illegible].

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Simple Interest

With simple interest, only the principal (the original loan amount) earns interest for the entire life of the transaction. The interest earned, plus the principal, is repaid in one lump sum. The following routines calculate the amount of simple interest on a 360-day basis and a 365-day basis. Once the interest is calculated, the total payment (loan amount plus interest) can also be calculated.

1. From the MAIN menu, press SOLVE to display the SOLVE menu.
2. Type in the simple interest formula as follows:

   SINT=N÷YEAR×PV×I÷100
3. Press CALC to verify the formula and to display the custom menu.
4. Store or calculate the following variables:
   - Amount of simple interest in SINT.
   - Total number of days in N.
   - Number of days in the calendar year (either 360 or 365) in YEAR.
   - Loan amount in PV.
   - Annual interest rate in I.
5. Press + RCL PV = to calculate the total payment (interest plus principal).

Example. A friend has requested that you lend him $450 for 60 days. You lend him the money at 7% simple interest, to be calculated on a 360-day basis. How much interest will he owe you in 60 days? What is the total amount owed?

Start from the SINT menu.

Keys:	**Display:**	**Description:**
450 PV 60 N 7 I 360 YEAR	PV=450.00 N=60.00 I=7.00 YEAR=360.00	Stores known values.
SINT	SINT=5.25	Calculates amount of simple interest.
+ RCL PV =	455.25	Calculates total payment amount.

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Random Number Generator

This routine calculates uniformly distributed pseudo-random numbers u_i in the range:

$$0 < u_i < 1$$

The following method is used:

$$u_{i+1} = \text{fractional part of } (997u_i)$$

where $i = 0, 1, 2, \ldots$
$u_0 = 0.5284163$ (seed)

Other seeds may be selected, but the quotient of (seed $\times$ 10^7) divided by two or five must not be an integer. Also, it would be wise to statistically test other seeds before using them.

The period of this generator has a length of 500,000 numbers and the generator passes the frequency test (chi square) for uniformity, the serial test, and the run test. The most significant digits (the left hand digits) are the most random digits. The right most digits are significantly less random.

1. From the MAIN menu, press SOLVE to display the SOLVE menu.
2. Type in the random number formula as follows:
   RDM=FP(997×SEED)
3. Press CALC to verify the formula and to display the custom menu.
4. Store the value of the seed in SEED.
5. Press RDM to determine the first random number.
6. Press [STO] SEED to store the random number as the seed for the next random number.
7. Press RDM to determine the next random number.
8. Repeat steps 6 and 7 as many times as desired.

Example. Generate a sequence of 5 random numbers.

Start from the RDM menu.

Keys:	**Display:**	**Description:**
[DISP] ALL		Sets display mode to display all digits.
.5284163 SEED	SEED=0.5284163	Stores initial seed.
RDM	RDM=0.8310511	Calculates first random number.
[STO] SEED	SEED=0.8310511	Stores new seed.
RDM	RDM=0.5579467	Calculates second random number.
[STO] SEED RDM	RDM=0.2728599	Calculates third random number.
[STO] SEED RDM	RDM=0.0413203	Calculates fourth random number.
[STO] SEED RDM	RDM=0.1963391	Calculates fifth random number.

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Amount of Periodic Payment When Balloon Payment Includes Final Payment

The TVM menu assumes that the balloon payment amount is in addition to any periodic payment. However, given the loan amount, total number of payment periods, annual interest rate, and the amount of the balloon payment, the periodic payment amount can be calculated. Remember to use the cash flow sign convention (money paid out is negative, money received is positive).

1. From the MAIN menu, press FIN, then TVM.
2. Clear the TVM variables, store the number of payments per year in P/YR, and set the payment mode (Begin or End, as appropriate).
3. Store 1 in N.
4. Store the balloon payment amount in FV.
5. Store the annual interest rate in I%YR.
6. Press PV to discount the balloon for one period.
7. Press +/- FV to store the adjusted balloon payment in FV.
8. Store the total number of periodic payments in N.
9. Store the loan amount in PV.
10. Press PMT to calculate the periodic payment amount.

Example. A $10,000, 12% loan has 35 monthly payments plus a $5,000 balloon payment at the end of the 36th month. What is the monthly payment amount?

Start from the MAIN menu.

Keys:	**Display:**	**Description:**
FIN TVM		Displays the TVM menu.
■ CLEAR ALL	0.00	Clears the TVM variables.
OTHER ■ CLEAR ALL EXIT		Sets 12 payments per year; End mode.
1 N	N=1.00	Stores 1 in N.
5000 +/- FV	FV=-5,000.00	Stores balloon payment.
12 I%YR	I%YR=12.00	Stores annual interest rate.
PV	PV=4,950.50	Calculates adjusted balloon payment.
+/- FV	FV=-4,950.50	Stores adjusted balloon payment in FV.
35 N	N=35.00	Stores total number of payments in N.
10000 PV	PV=10,000.00	Stores loan amount in PV.
PMT	PMT=-221.21	Calculates monthly payment.

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Discounted Notes

A note is a written agreement to pay a sum of money plus interest at a certain rate. Notes do not have periodic coupons, since all interest is paid at maturity.

A discounted note is a note that is purchased below its face value. The following formulas find the price and/or yield of a discounted note. The formulas assume the calendar basis is Actual/360.

Entering and Using the PRICE and YIELD Formulas:

1. From the MAIN menu, press SOLVE to display the SOLVE menu.
2. Type in the PRICE formula as follows.
   PRICE=RV-(DISC×RV×DDAYS(SETL:MAT:1)÷36000)
3. Press INPUT to verify the formula.
4. Type in the YIELD formula as follows.
   YIELD=(RV-PRICE)÷PRICE×36000÷DDAYS(SETL:MAT:1)
5. Press INPUT to verify the formula.
6. Press ↑ CALC to select PRICE as the current formula and display the custom menu.
7. Store the following variables:
   - Redemption value per $100 in RV.
   - Discount rate as a percent in DISC.
   - Settlement date (in MM.DDYYYY format) in SETL.
   - Maturity date (in MM.DDYYYY format) in MAT.
8. To calculate the purchase price, press PRICE.
9. To calculate the yield when the price is known, press EXIT ↓ CALC to select YIELD as the current formula and display the custom menu. If you have not just calculated the price, store the price in PRICE.
10. Press YIELD to calculate the yield.

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Example 1. Calculate the price and yield of the following U.S. Treasury Bill: settlement date October 14, 1986; maturity date March 17, 1987; discount rate 8.7%.

Start from the PRICE menu.

Keys:	**Display:**	**Description:**
10.141986 SETL	SETL=10.14	Stores known values.
3.171987 MAT	MAT=3.17	
8.7 DISC	DISC=8.70	
100 RV	RV=100.00	
PRICE	PRICE=96.28	Calculates price.
EXIT ⬇ CALC		Displays YIELD custom menu.
YIELD	YIELD=9.04	Calculates yield.

Example 2. Determine the yield of the following discounted note: settlement date June 25, 1986; maturity date September 10, 1986; price 99.45; redemption value 101.33.

Start from the YIELD menu.

Keys:	**Display:**	**Description:**
6.251986 SETL	SETL=6.25	Stores known values.
9.101986 MAT	MAT=9.10	
99.45 PRICE	PRICE=99.45	
101.33 RV	RV=101.33	
YIELD	YIELD=8.84	Calculates yield.

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Duration of a Bond

The duration of a bond is the average time that elapses until the various cash flows (coupons and redemption value) are received. In other words, the duration is a weighted average of the number of periods until the payments occur. For coupon-bearing bonds, the duration is always shorter than the term to maturity. (For zero-coupon bonds, duration is equal to maturity.)

Entering and Using the DURATION **Formula:**

1. From the MAIN menu, press SOLVE to display the SOLVE menu.
2. Type in the DURATION formula as follows. Since the formula is long, you may need to clear most of memory before keying it in.

```
(CPN×((1+YLD÷100)^(N+1)-(1+YLD÷100)-YLD÷100×N)
÷(YLD÷100)^2÷(1+YLD÷100)^N+RDV×N÷(1+YLD÷100)^N)=
(USPV(YLD:N)×CPN+SPPV(YLD:N)×RDV)×DUR
```

3. Press CALC to verify the formula and to display the custom menu.
4. Store the following variables:
   - Dollar amount of each coupon in CPN.
   - Periodic yield to maturity (as a percentage) in YLD.
   - Total number of coupons in N.
   - Redemption value in RDV.
5. Press DUR to calculate the number of periods in the duration.

Example. Calculate the duration of the following bond: $60 coupon paid semi-annually for 5 years, 13.5% annual yield, $1000 redemption value.

Start from the DURATION custom menu.

Keys:	Display:	Description:
60 CPN	CPN=60.00	Stores amount of periodic coupon.
13.5 ÷ 2 YLD	YLD=6.75	Stores periodic yield.
5 × 2 N	N=10.00	Stores total number of coupons.
1000 RDV	RDV=1,000.00	Stores redemption value.
DUR	DUR=7.73	Calculates duration in semi-annual periods.
÷ 2 =	3.87	Duration in years.

Reference: Jess H. Chua, *A Closed-Form Formula for Calculating Bond Duration*, Financial Analysts Journal, May-June 1984.

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