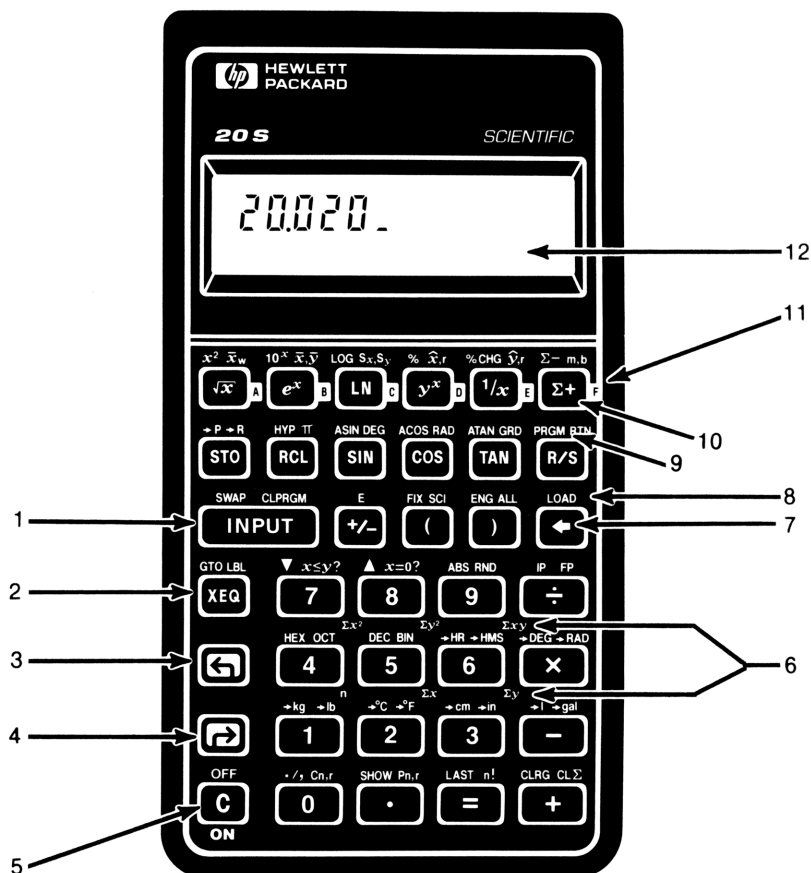# HEWLETT-PACKARD

Scientific Calculator

Owner's Manual

HP-20S

HEWLETT PACKARD

1. Separates two numbers.

2. Executes a program.

3. Activates blue-labeled keys.

4. Activates yellow-labeled keys.

5. On; clears display, cancels operation.

6. n through Σxy are statistical summation memory aids.

7. Backspace.

8. Loads built-in programs.

9. Enters Program mode.

10. Accumulates statistical data.

11. A through F keys for labels, built-in programs, hexadecimal digits.

12. Annunciator Line.

HP-20S Scientific Calculator

Owner's Manual

HEWLETT PACKARD

The contents of this manual are printed on recycled paper.

Notice

For warranty and regulatory information for this calculator, see pages 117 and 120.

This manual and any examples contained herein are provided "as is" and are subject to change without notice. **Hewlett-Packard Company makes no warranty of any kind with regard to this manual, including, but not limited to, the implied warranties of merchantability and fitness for a particular purpose.** Hewlett-Packard Co. shall not be liable for any errors or for incidental or consequential damages in connection with the furnishing, performance, or use of this manual or the keystroke programs contained herein.

Printing History

Welcome to the HP-20S

Your HP-20S is another example of the superior quality and attention to detail in engineering and manufacturing that have marked Hewlett-Packard products for more than 40 years. Hewlett-Packard stands behind this calculator. We offer worldwide service and expertise to support its use.

Hewlett-Packard Quality

Our calculators are made to excel, to last, and to be easy to use.

- This calculator is designed to withstand the drops, vibrations, pollutants (smog, ozone), temperature extremes, and humidity variations that it can be exposed to in everyday worklife.

- The calculator and its manual have been designed and tested for ease of use. We added examples to the manual to highlight the varied uses of this calculator.

- Advanced materials and permanent, molded-in key lettering provide a long keyboard life and a positive feel to the keyboard.

- CMOS (low-power) electronics and the liquid-crystal display allow data to be retained even when the calculator is off and let the batteries last a long time.

- The microprocessor has been optimized for fast and reliable computations using 15 digits internally for precise results.

- Extensive research has created a design that has minimized the adverse effects of static electricity (a potential cause of malfunctions and data loss in calculators).

Features

- Large 12-character display.
- Ten data registers and 99 program lines.
- One- and two-variable statistics with linear regression.
- Probability functions.
- Unit and base conversions.
- Polar/rectangular conversions.
- Hyperbolic functions.
- Accurate math, 12-digits with a $10^{\pm 499}$ exponent range.
- Keystroke programming.
- Six built-in programs:
  - Root finder.
  - Numerical integration.
  - Complex number operations.
  - 3×3 matrix solutions.
  - Quadratic equation.
  - Curve fitting.

Contents

Appendix

1

Getting Started

Power On and Off
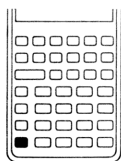
To turn on your HP-20S, press $\boxed{C}$ (the key above the "ON" label). To turn the calculator off, press either shift key ($\boxed{\leftarrow}$ or $\boxed{\rightarrow}$), then $\boxed{C}$ (also written $\boxed{\leftarrow}$ $\boxed{OFF}$ or $\boxed{\rightarrow}$ $\boxed{OFF}$).

Since the calculator has *Continuous Memory*, turning it off does not affect the information you've stored. To conserve energy, the calculator turns itself off approximately 10 minutes after you stop using it. The calculator's three alkaline batteries last approximately a year. If you see the low-battery symbol ($\boxed{}$) in the display, replace the batteries as soon as possible. Refer to the appendix for more information.

Adjusting the Display Contrast

To change the display contrast, hold down $\boxed{C}$ and press $\boxed{+}$ or $\boxed{-}$.

Simple Arithmetic Calculations

If you make a typing mistake while entering a number, press $\boxed{\blacktriangleleft}$ to erase the incorrect digits.

Arithmetic Operators. The following examples demonstrate using the arithmetic operators ⊞, ⊟, ⊠, ⊡, and $\boxed{y^x}$ (exponentiation)*.

Keys:	Display:	Description:
24.715 ⊞ 62.471 ⊟	**87.1860**	Adds 24.715 and 62.471.

When a calculation has been completed (by pressing ⊟), pressing a number key starts a new calculation:

19 ⊠ 12.68 ⊟	**240.9200**	Calculates 19×12.68.

$\boxed{y^x}$ is the exponentiation operator:

4.7 $\boxed{y^x}$ 3 ⊟	**103.8230**	Calculates 4.7^3.

If you press an operator key after completing a calculation, the calculation is continued:

⊞ 115.5	**115.5_**	Continues the calculation.
⊟	**219.3230**	Completes the calculation of $4.7^3 + 115.5$.

You can do chain calculations without using ⊟ after each step. Calculate $6.9 \times 5.35 \div 0.918$:

6.9 ⊠ 5.35 ⊡	**36.9150**	Pressing ⊡ displays the intermediate answer, showing result of 6.9×5.35.
.918	**0.918_**	Continues the calculation.
⊟	**40.2124**	Completes the calculation.

---

* If you press more than one operator, for example ⊞ ⊟ ⊞ ⊠ ⊞, all are ignored except the last one.

Chain calculations are interpreted according to the priority of the operators in the expression. Calculate $4 + (9 \times 3)$:

4 [+] 9 [×]	**9.0000**	The addition is delayed; [×] has higher priority than [+].
3 [=]	**31.0000**	Calculates $4 + (9 \times 3)$.

Negative Numbers. Enter the number and press [+/–].

Calculate $-75 \div 3$:

Keys:	**Display:**	**Description:**
75 [+/–]	**–75–**	Changes the sign of 75.
[÷] 3 [=]	**–25.0000**	Calculates the result.

Calculate $0.4 - e^{-1.1}$:

.4 [–] 1.1 [+/–]	**–1.1–**	
[eˣ]	**0.3329**	Calculates $e^{-1.1}$.
[=]	**0.0671**	Completes the calculation.

Understanding the Display and Keyboard

The Cursor

The cursor (_) is visible when you are in the process of entering a number.

Clearing the Calculator

When the cursor is on, ◄ erases the last digit you entered. While you are entering a number, pressing C clears it to 0. Otherwise, C clears the display of its current contents and cancels the current calculation.

While you are entering a number, pressing C clears it to 0. Otherwise, C clears the display of its current contents and cancels the current calculation.

Clearing Messages. ◄ and C also clear messages. When the HP-20S is displaying an error message, ◄ or C clears the message and restores the original contents of the display.

Clearing Memory
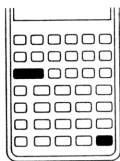
To clear portions of memory:

Keys	Description
⬅ CLRG	Clears registers R_0 through R_9.
➡ CLΣ	Clears statistical registers R_4 through R_9.
➡ CLPRGM	Clears programs when in Program mode.

To clear all memory and reset the calculator, press and hold down C, then press and hold down both √x̄ and Σ+. When you release them, all memory is cleared. The **ALL CLr** message is displayed.

Annunciators

Annunciators are symbols in the display that indicate the status of the calculator.

Annunciator	Status
⬑	Left shift is active. When you press a key, the function labeled in blue above the key is executed (page 14).
⬏	Right shift is active. When you press a key, the function labeled in yellow above the key is executed (page 14).
:	INPUT has been pressed, or two values have been entered or returned (page 14).
PEND	An arithmetic operation is pending in addition to what shows in the display.
⬛	Battery power is low (page 9).
GRAD	The calculator is in Grads mode for trigonometric calculations (page 35).
RAD	The calculator is in Radians mode for trigonometric calculations (page 35).
HEX	The calculator is in Hexadecimal mode (page 44).
OCT	The calculator is in Octal mode (page 44).
BIN	The calculator is in Binary mode (page 44).
PRGM	The calculator is in Program mode. (Refer to chapters 6 and 7.)

The Shift Keys

Most keys have blue or yellow functions printed above the key. The shift keys access these labeled operations: the blue shift key executes a blue labeled operation; the yellow shift key executes a yellow labeled operation. To perform a shifted operation, press ◄ or ► to turn on the shift annunciator (◄ or ►). Then, press the key that has the desired label above it.

For example, pressing ◄ followed by [HEX] (also written ◄[HEX]) puts the calculator in Hexadecimal mode. Pressing ◄ [DEC] puts the calculator in Decimal mode.

To perform consecutive shifted operations, hold down the shift key.

If you accidentally press ◄ or ►, just press it again to turn off the shift annunciator. If you press the wrong shift key, press the other one to cancel it and display the correct one.

The INPUT Key

The [INPUT] key is used to separate two numbers when using two-number functions or two-variable statistics.

The : annunciator is displayed if [INPUT] has been pressed. If a number is in the display, press [C] to erase the : annunciator and the display. If the cursor or an error message is visible in the display, press [C] twice to erase the : annunciator.

The SWAP Key
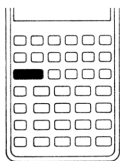
Pressing ◄ [SWAP] exchanges:

- The last two numbers that you entered; for instance, the order of division or subtraction.

- The results of functions that return two values. The : annunciator indicates that two results have been returned; press ◄] [SWAP] to see the hidden result.

- The *x*- and *y*-values when using statistics.

The Alpha Keys

The A, B, C, D, E, and F labels have several functions. They are used as program labels and as digits in Hexadecimal mode.

Introducing the Math Functions
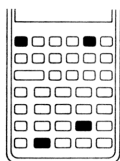
One-Number Functions. Math functions involving one number use the number in the display:

Keys:	Display:	Description:
89.25 [√x]	9.4472	Calculates $\sqrt{89.25}$.
3.57 [+] 2.36 [¹/x]	0.4237	1/2.36 is calculated first.
[=]	3.9937	Adds 3.57 and 1/2.36.
180 [►] [►in]	70.8661	Converts 180 centimeters to inches.

Two-Number Functions. When a function requires two numbers, the numbers are entered like this: *number1* [INPUT] *number2*. Pressing [INPUT] evaluates the current expression and displays **:**. For example, the following keystrokes calculate the percent change between 17 and 29:

Keys:	Display:	Description:
17 [INPUT]	**17.0000**	Enters *number1*, displays **:** annunciator.
29	**29**	Enters *number2*.
[◄] [%CHG]	**70.5882**	Calculates the percent change.

Calculate the number of combinations of four items taken two at a time:

4 [INPUT] 2 [►] [Cn,r]	**6.0000**	Calculates number of combinations.

If you enter *number1*, then press a two-number function key without pressing [INPUT], the calculator supplies a zero as *number2*. If you enter a number, press [INPUT], and then press a two-number function key, the calculator uses the same number for both *number1* and *number2*.

Display Format of Numbers

When you turn on the HP-20S for the first time, numbers are displayed with four decimal places and a period as the decimal point. The *display format* controls how many digits appear in the display.

Regardless of the current display format, each number is stored as a signed, 12-digit mantissa with a signed, three-digit exponent. For example, pressing ☞ π in FIX 4 (four decimal places) displays 3.1416. Internally, the number is stored as 3.14159265359 × 10^{000}.

If the result of a calculation is a number containing more significant digits than can be displayed in the current display format, the displayed number is rounded to fit.

Specifying the Number of Displayed Decimal Places (FIX)
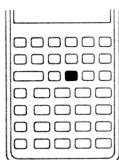
To specify the number of displayed decimal places:

1. Press ◄ FIX.
2. Enter the number of digits (0 through 9) that you wish to appear after the decimal point.

Keys:	Display:	Description:
C ◄ FIX 3	0.000	Displays three decimal places.
45.6 × .1256 =	5.727	
◄ FIX 9	5.727360000	Displays nine decimal places.
◄ FIX 4	5.7274	Restores four decimal places.

When a number is too large or too small to be displayed in FIX format, it is automatically displayed in scientific notation.

Displaying the Full Precision of Numbers (ALL)
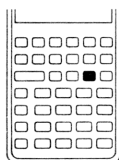
To set your calculator to display numbers as precisely as possible, press ☞ ALL. Trailing zeros are not displayed.

Scientific and Engineering Notation
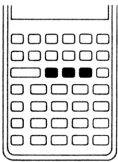
Scientific and engineering notation express the number as a mantissa multiplied by a power of 10. The letter **E** separates the exponent from the mantissa.

Scientific Notation (SCI). Scientific notation uses a mantissa with one digit to the left of the decimal point. For example, this is SCI 6:

Digits after decimal point | Sign of exponent | Power of 10

$$-1.234567\text{E}-5$$

Sign of mantissa   Mantissa
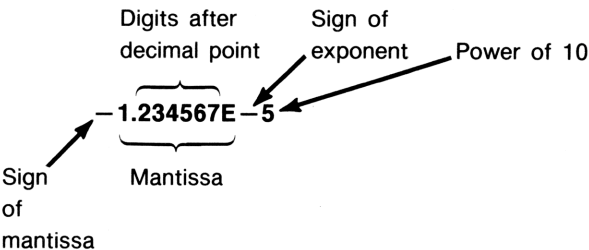
To specify scientific notation:

1. Press [➔] [SCI].

2. Enter the number of digits that you wish to appear after the decimal point.

Engineering Notation (ENG). Engineering notation expresses a number as a mantissa with one, two, or three digits to the left of the decimal point, multiplied by 10 raised to a power that is a multiple of 3. For example, this is ENG 4:

Significant digits after first digit | Sign of exponent | Power of 10 (multiples of 3)

$$-12.345\text{E}-12$$

Sign of mantissa   Mantissa
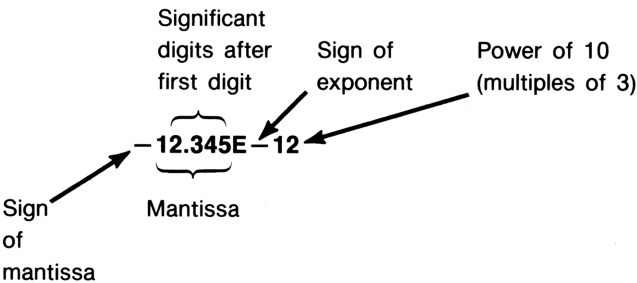
To specify engineering notation:

1. Press ⏎ [ENG].

2. Enter the number of significant digits that you wish to appear after the first digit.

Entering Numbers With Exponents (E). Regardless of the current display format, you can always enter a number as a mantissa followed by an exponent:

1. Enter the mantissa. If the mantissa is negative, use +/₋ to change the sign.

2. Press ⏎ [E] (or ➤ [E]) to start the exponent.

3. If the exponent is negative, press +/₋ or [-].

4. Enter the exponent.

Calculate $4.78 \times 10^{13} \div 8 \times 10^{25}$:

Keys:	Display:	Description:
4.78 ⏎ [E] 13 [÷]	**4.7800E13**	
8 ⏎ [E] 25 [=]	**5.9750E−13**	5.975×10^{-13}.

Calculate $-2.36 \times 10^{-15} \times 12$:

2.36 +/₋ ⏎ [E] +/₋ 15		-2.832×10^{-14}.
[×] 12 [=]	**−2.8320E−14**	

Interchanging the Period and Comma
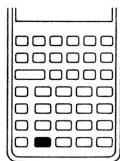
You can interchange the period and comma used as the decimal point and digit separator. For example, one million can be displayed:

1,000,000.0000 or 1.000.000,0000

To toggle between the period and comma, press ⏎ [·/,].

Full Precision of a Number (SHOW)
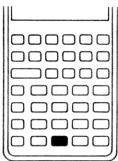
To temporarily view all 12 stored mantissa digits of the number in the display, press ◄ and then hold down [SHOW]. The 12 digits are shown without the decimal point.

Starting with four decimal places (◄ [FIX] 4):

Keys:	Display:	Description:
10 [÷] 7 [=]	1.4286	
◄ [SHOW]	142857142857	Displays 12 digits.
1 [÷] 80 [+/-] [=]	−0.0125	
◄ [SHOW]	−125000000000	Displays 12 digits.

Range of Numbers

The range of numbers the HP-20S can store is shown below. Underflow displays zero. Overflow displays the **OFLO** message for a moment, then the largest positive or negative number possible.

Numbers the HP 20S Can Store

Overflow — Underflow — Overflow

$-9.99999999999 \times 10^{499}$

-1×10^{-499} 0 1×10^{-499}

$9.99999999999 \times 10^{499}$
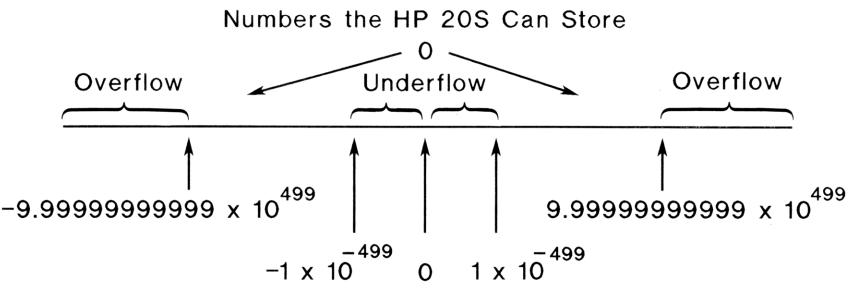
Messages

The HP-20S displays messages about the status of the calculator or informs you that you have attempted an incorrect operation. To clear a message from the display, press $\boxed{C}$ or $\boxed{\blacklozenge}$. Refer to page 122 for a list of messages and their meanings.

2

Arithmetic and Storage Registers

Chain Calculations

Chain calculations do a sequence of operations without pressing $\boxed{=}$ after each operation. The HP-20S interprets expressions using the system of *operator priority* described in the next section.

Keys:	Display:	Description:
750 $\boxed{\times}$ 12 $\boxed{\div}$	**9,000.0000**	Calculates intermediate value. **PEND** annunciator is on.
360 $\boxed{=}$	**25.0000**	Completes the calculation. **PEND** annunciator is off.

Operator Priority and Pending Operations

Some chain calculations might be interpreted several different ways. For example, $9 + 12 \div 3$ has two interpretations:

$$9 + \frac{12}{3} = 13 \quad \text{or} \quad \frac{9 + 12}{3} = 7$$

The HP-20S uses a system of operator priority to evaluate expressions:

$\boxed{y^x}$ (exponentiation)   Highest priority

$\boxed{\times}$ $\boxed{\div}$

$\boxed{+}$ $\boxed{-}$ Lowest priority

The HP-20S calculates an intermediate result when the next operator you enter has lower or equal priority. When the next operator has higher priority, the HP-20S retains the previous number(s). For example, in the calculation:

9 $\boxed{+}$ 12 $\boxed{\div}$ 3 $\boxed{=}$

division has a higher priority than additon. Thus, the 9 and $\boxed{+}$ are retained as a pending operation until the division is completed:

Keys:	Display:	Description:
9 $\boxed{+}$ 12 $\boxed{\div}$	12.0000	Pressing $\boxed{\div}$ *does not* add 9 + 12.
3 $\boxed{=}$	13.0000	

Calculate 4×7^3 plus 5×7^2 plus 6.

4 $\boxed{\times}$ 7 $\boxed{y^x}$	7.0000	$\boxed{y^x}$ has higher priority than $\boxed{\times}$.
3 $\boxed{+}$	1,372.0000	Calculates 4×7^3.
5 $\boxed{\times}$	5.0000	$\boxed{\times}$ has higher priority than $\boxed{+}$.
7 $\boxed{y^x}$	7.0000	$\boxed{y^x}$ has higher priority than $\boxed{\times}$.
2	2_	
$\boxed{+}$	1,617.0000	Adds 5×7^2 to 1,372.
6 $\boxed{=}$	1,623.0000	Completes the calculation.

If a calculation requires that operations be done in an order inconsistent with operator priority (for example, addition *before* multiplication), use parentheses. You can use a maximum of five pending operations.*

Using Parentheses

Use parentheses to group operations and to specify the order in which they are performed.† For example, you can calculate:

$$\frac{9 + 12}{3}$$

by placing parentheses around the addition so that it is done before the division:

Keys:	Display:	Description:
⟨(⟩ 9 ⟨+⟩ 12 ⟨)⟩	**21.0000**	⟨)⟩ evaluates expression inside parentheses.
⟨÷⟩ 3 ⟨=⟩	**7.0000**	

Calculate $\dfrac{30}{85 - 12} \times \sqrt{16.9 - 8}$:

30 ⟨÷⟩ ⟨(⟩	**30.0000**	
85	**85_**	
⟨−⟩	**85.0000**	⟨(⟩ prevents division of 30 by 85.
12 ⟨)⟩	**73.0000**	⟨)⟩ evaluates expression inside parentheses.
⟨×⟩	**0.4110**	Calculates 30 ÷ 73.

---

* There are less than five pending operations available if you have more than four pending left parentheses. For example, you can calculate 1 + (2 + (3 + (4 + (5 + 6.

† Closing parentheses at the end of the expression can be omitted. For example, 25 ÷ (3 × (9 + 12 ⟨=⟩ is equivalent to 25 ÷ (3 × (9 + 12)) ⟨=⟩.

⑴ 16.9	**16.9_**	
⊟ 8 ⑴	**8.9000**	⑴ evaluates expression inside parentheses.
⟦√x̄⟧	**2.9833**	Calculates $\sqrt{8.9}$.
⟦=⟧	**1.2260**	Completes the calculation.

Reusing the Previous Result (LAST)
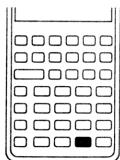
When you start a new calculation, a copy of the last result is stored in the LAST register. To recall that value to the display, press ⟦◥⟧ ⟦LAST⟧. For example, LAST shortens the following two calculations:

$$0.0821 \times (18 + 273.1)$$

$$2 + \frac{13}{0.0821 \times (18 + 273.1)}$$

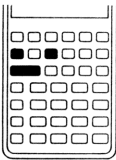| **Keys:**	**Display:**	**Description:**
.0821 ⟦×⟧ ⟦⑴⟧ 18 ⟦+⟧ 273.1 ⟦⑴⟧ ⟦=⟧	**23.8993**	Displays first result, which is stored in LAST, when next calculation is started. Closing parenthesis is optional.
2 ⟦+⟧ 13 ⟦÷⟧ ⟦◥⟧ ⟦LAST⟧	**23.8993**	⟦◥⟧ ⟦LAST⟧ recalls the previous result.
⟦=⟧	**2.5439**	Second result.

Exchanging Two Numbers (SWAP)

Pressing ⬅ SWAP exchanges the last two numbers that you entered during a calculation. For example, if you have entered 44 ÷ 75, ⬅ SWAP reverses the order of the numbers to 75 ÷ 44.

Keys:	Display:	Description:
44 ÷ 75	75_	Oops; you meant to enter 75 ÷ 44.
⬅ SWAP	44.0000	Swaps the 75 and 44.
=	1.7045	Completes the calculation.
8 + 4 ÷ 5	5_	Stop! You really wanted to add 8 + 5 ÷ 4.
⬅ SWAP	4.0000	Swaps the 5 and 4.
=	9.2500	Completes the calculation.

When a function returns two results, the **:** annunciator comes on. Pressing ⬅ SWAP exchanges the two results. For example, to convert the rectangular coordinates $(10, -15)$ to polar coordinates:

Keys:	Display:	Description:
➡ DEG		Sets Degrees mode.
10 INPUT	10.0000	Stores x.
15 +/− ⬅ →P	−56.3099	Displays the angle. **:** indicates another result was calculated.
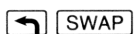
<img> ⬅ SWAP   18.0278   Displays the radius.

C   0.0000   Clears the display.

Another use of ⬅ SWAP is with functions that require two numbers separated by INPUT. For example, to accumulate (x,y) data pairs in the statistical registers, enter x-value INPUT y-value Σ+. Pressing ⬅ SWAP (before pressing Σ+) exchanges the x-value and y-value. Refer to page 56 for an example.

Using Storage Registers
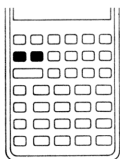
Registers R_0 through R_9 are for storing numbers. They are accessed using STO and RCL. When you are using the statistics functions, R_4 through R_9 are used to store summation data.

- STO n, where n is an integer 0 through 9, copies the number in the display to the designated register. The number is copied with full precision.

- RCL n copies the contents of R_n to the display. The number is displayed in the current display format.

The following keystrokes use R_1 and R_2 to calculate:

$$\frac{(27.1 + 35.6) \times 1.0823}{(27.1 + 35.6)^{1.0823}}$$

Keys:	Display:	Description:
27.1 + 35.6 =	62.7000	
STO	62.7000	Calculator awaits register number.
1	62.7000	Stores 62.7 in R_1.

$\boxed{\times}$ 1.0823 $\boxed{\text{STO}}$ 2	**1.0823**	Stores 1.0823 in R_2.
$\boxed{\div}$	**67.8602**	
$\boxed{\text{RCL}}$	**67.8602**	Calculator awaits register number.
1	**62.7000**	Recalls contents of R_1.
$\boxed{y^x}$ $\boxed{\text{RCL}}$ 2	**1.0823**	Recalls contents of R_2.
$\boxed{=}$	**0.7699**	Exponentiation is done before division.

To cancel store or recall after pressing $\boxed{\text{STO}}$ or $\boxed{\text{RCL}}$, press $\boxed{\text{C}}$ or $\boxed{\blacklozenge}$.

Clearing Registers. Press $\boxed{\text{◄}}$ $\boxed{\text{CLRG}}$ to clear all registers. To clear an individual register, store 0 in it. It is unnecessary to clear a register before storing a value since $\boxed{\text{STO}}$ n replaces the previous value with the new value.

Storage Register Arithmetic. This table describes the arithmetic operations that can be performed on numbers stored in registers. The result is stored in the register.

Keys	New Number in Register *n*
$\boxed{\text{STO}}\boxed{+}$ *n*	old number + displayed number
$\boxed{\text{STO}}\boxed{-}$ *n*	old number − displayed number
$\boxed{\text{STO}}\boxed{\times}$ *n*	old number × displayed number
$\boxed{\text{STO}}\boxed{\div}$ *n*	old number ÷ displayed number

The following keystrokes use two registers to calculate:

$$1.097 \times 25.6671 = ?$$

$$1.097 \times 35.6671 = ?$$

Keys:	Display:	Description:
1.097 [STO] 0	**1.0970**	Stores 1.097 in R_0.
[×] 25.6671 [STO] 1	**25.6671**	Stores 25.6671 in R_1.
[=]	**28.1568**	First answer.
[RCL] 0	**1.0970**	Recalls contents of R_0 and starts a new calculation.
[×] 10 [STO] [+] 1	**10.0000**	Adds 10 to contents of R_1.
[RCL] 1	**35.6671**	Contents of R_1 replace right-most number of pending expression.
[=]	**39.1268**	Second answer.

3

Numeric Functions

HP-20S functions require either one or two arguments (an argument is a number acted upon by a function):

- Functions with one argument act on the number in the display. For example, 6 $\boxed{\sqrt{x}}$ calculates the square root of 6.

- Functions with two arguments use $\boxed{\text{INPUT}}$ to separate the arguments. For example, 4 $\boxed{\text{INPUT}}$ 5 $\boxed{\blacktriangleleft}$ $\boxed{\text{\%CHG}}$ calculates the percent change between 4 and 5. The arguments can be expressions. For example, 1 $\boxed{+}$ 3 $\boxed{\text{INPUT}}$ 2 $\boxed{+}$ 3 $\boxed{\blacktriangleleft}$ $\boxed{\text{\%CHG}}$ also calculates the percent change between 4 and 5.

- Polar/rectangular coordinate conversions use two arguments and return two results.

General and Logarithmic Functions
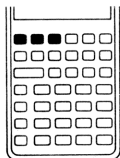
Key(s)	Description
$\boxed{\sqrt{x}}$	Square root.
$\boxed{\blacktriangleleft}$ $\boxed{x^2}$	Square.
$\boxed{e^x}$	Natural antilogarithm.
$\boxed{\blacktriangleleft}$ $\boxed{10^x}$	Base 10 antilogarithm.
$\boxed{LN}$	Natural logarithm.
$\boxed{\blacktriangleleft}$ $\boxed{LOG}$	Base 10 logarithm.

Keys:	Display:	Description:
45 $\boxed{\sqrt{x}}$	**6.7082**	$\sqrt{45}$.

Calculate $10^{-4.5} \times 10^{-3.7}$:

4.5 $\boxed{+/-}$ $\boxed{\blacktriangleleft}$ $\boxed{10^x}$	**3.1623E−5**	Calculates base 10 antilogarithm of −4.5.
$\boxed{\times}$ 3.7 $\boxed{+/-}$ $\boxed{\blacktriangleleft}$ $\boxed{10^x}$	**0.0002**	Calculates base 10 antilogarithm of −3.7.
$\boxed{=}$	**6.3096E−9**	Multiplies the two antilogarithms.

Reciprocal
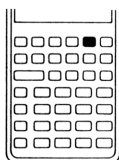
Press $\boxed{1/x}$ to calculate the reciprocal of the number in the display. Calculate ⅓ + ¼:

Keys:	Display:	Description:
3 ⌈¹/x⌋ ⌈+⌋ 4 ⌈¹/x⌋	**0.2500**	Calculates $1 \div 3$ and $1 \div 4$. Addition is deferred.
⌈=⌋	**0.5833**	Adds the two reciprocals.

The exponentiation operator, ⌈yˣ⌋, can also be used to find roots of positive numbers. For example, find $\sqrt[4]{3}$ (which is equivalent to $3^{1/4}$):
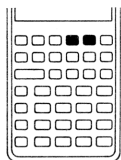
Keys:	Display:	Description:
3 ⌈yˣ⌋	**3.0000**	Exponentiation.
4 ⌈¹/x⌋ ⌈=⌋	**1.3161**	Reciprocal of power will calculate the root.

Percent Functions

Percent

The ⌈◄⌋ ⌈%⌋ function performs two different operations:

- When there is no pending operator, or the last operator you entered was ⌈×⌋, ⌈÷⌋, or ⌈yˣ⌋, pressing ⌈◄⌋ ⌈%⌋ divides the displayed number by 100.

- When ⌈+⌋ or ⌈−⌋ is the pending operator, ⌈◄⌋ ⌈%⌋ interprets the displayed number as a percent and returns that percent of the number preceding the ⌈+⌋ or ⌈−⌋.

Example: Percent Calculations. Find 27% of 85.3.

Keys:	Display:	Description:
85.3 $\boxed{\times}$ 27 $\boxed{\leftarrow}$ $\boxed{\%}$	**0.2700**	Divides 27 by 100.
$\boxed{=}$	**23.0310**	Calculates 27% of 85.3.

Find the number that is 25% less than 200.

200 $\boxed{-}$ 25 $\boxed{\leftarrow}$ $\boxed{\%}$	**50.0000**	Calculates 25% of 200.
$\boxed{=}$	**150.0000**	Completes the calculation.

Percent Change

To calculate the percent change between two numbers, n_1 and n_2, expressed as a percentage of n_1, enter:
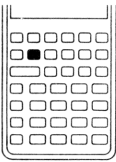
$$n_1 \boxed{\text{INPUT}} \ n_2 \boxed{\leftarrow} \boxed{\text{\%CHG}}$$

Example: Percent Change Calculations. Calculate the percent change between 291.7 and 316.8.

Keys:	Display:	Description:
291.7 $\boxed{\text{INPUT}}$	**291.7000**	Enters n_1.
316.8 $\boxed{\leftarrow}$ $\boxed{\text{\%CHG}}$	**8.6047**	Calculates percent change.

Calculate the percent change between (12 × 5) and (65 + 18).

12 $\boxed{\times}$ 5 $\boxed{\text{INPUT}}$	**60.0000**	Calculates and enters n_1.
65 $\boxed{+}$ 18 $\boxed{\leftarrow}$ $\boxed{\text{\%CHG}}$	**38.3333**	Percent change between 60 and (65 + 18).

Pi (π)

Pressing $\boxed{\rightarrow}$ $\boxed{\pi}$ displays the value of π. Although the displayed value is rounded to the current display format, the 12-digit value is actually used.

Example: Surface Area of a Sphere. Find the surface area of a sphere with radius=4.5 inches (surface area = $4\pi r^2$).

| Keys:	Display:	Description:
4 $\boxed{\times}$ $\boxed{\rightarrow}$ $\boxed{\pi}$	3.1416	Displays π.
$\boxed{\times}$ 4.5 $\boxed{\leftarrow}$ $\boxed{x^2}$	20.2500	Displays 4.5^2.
$\boxed{=}$	254.4690	Surface area in square inches.

Trigonometric Modes and Functions

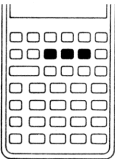## Changing the Trigonometric Mode
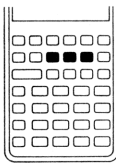
The trigonometric mode determines how numbers are interpreted when using the trigonometric and coordinate conversion functions.

| Keys	Description	Annunciator
⟨🢂⟩ ⟨DEG⟩	Sets *Degrees* mode. There are 360 degrees in a circle. Angles are measured in decimal degrees (rather than degrees-minutes-seconds).	None
⟨🢂⟩ ⟨RAD⟩	Sets *Radians* mode. There are 2π radians in a circle.	**RAD**
⟨🢂⟩ ⟨GRD⟩	Sets *Grads* mode. There are 400 grads in a circle.	**GRAD**

To exit RAD or GRAD mode press ⟨🢂⟩ ⟨DEG⟩.

Trigonometric Functions

Angles are interpreted in decimal degrees, radians, or grads depending on the current trigonometric mode.

Keys	Function	Keys	Function
⟨SIN⟩	sine	⟨🢀⟩ ⟨ASIN⟩	arc sine
⟨COS⟩	cosine	⟨🢀⟩ ⟨ACOS⟩	arc cosine
⟨TAN⟩	tangent	⟨🢀⟩ ⟨ATAN⟩	arc tangent

Keys:	Display:	Description:
⟨🢂⟩ ⟨DEG⟩		Sets Degrees mode.
15 ⟨SIN⟩	**0.2588**	Sine of 15°.
1 ⟨+⟩ 60 ⟨TAN⟩	**1.7321**	Tangent of 60°.
⟨=⟩	**2.7321**	Calculates 1 + tan 60°.

.35 🡔 ACOS	69.5127	Arc cosine of 0.35.
─ .62 🡔 ACOS	51.6839	Arc cosine of 0.62.
=	17.8288	Arc cosine of 0.35 — arc cosine of 0.62.

Angle and Hour Conversions

Keys	Function
🡔 [➔HR]	*To hours*; converts the number from hours(degrees)-minutes-seconds-decimal seconds format (H.MMSSss or D.MMSSss) to decimal hours (or degrees) format.
🡒 [➔HMS]	*To hours-minutes-seconds*; converts the number from decimal hours (or degrees) to hours(degrees)-minutes-seconds-decimal seconds format (H.MMSSss or D.MMSSss).
🡔 [➔DEG]	*To degrees*; converts the number from a radian value to its decimal degree equivalent.
🡒 [➔RAD]	*To radians*; converts the number from a decimal degree value to its radian equivalent.
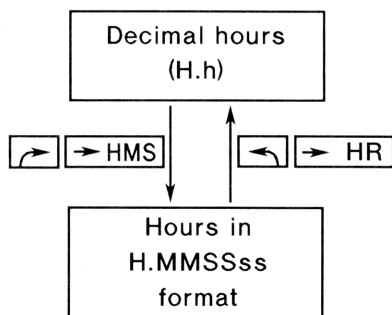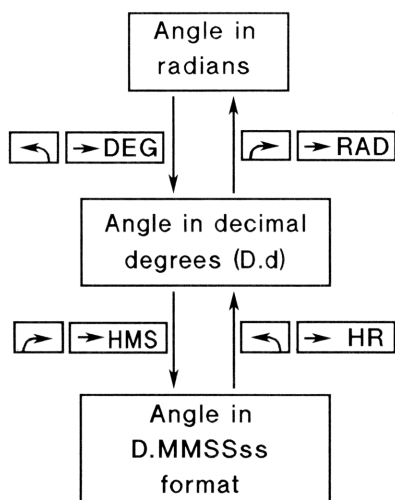
```
         ┌──────────────┐
         │   Angle in   │
         │   radians    │
         └──────────────┘
   ◄┐ ►DEG        ┌► ►RAD
         ┌──────────────┐            ┌──────────────┐
         │Angle in decimal│          │ Decimal hours │
         │ degrees (D.d) │            │    (H.h)     │
         └──────────────┘            └──────────────┘
   ┌► ►HMS   ◄┐ ► HR        ┌► ►HMS    ◄┐ ► HR
         ┌──────────────┐            ┌──────────────┐
         │  Angle in    │            │  Hours in    │
         │  D.MMSSss    │            │  H.MMSSss    │
         │   format     │            │   format     │
         └──────────────┘            └──────────────┘
```

Keys:	Display:	Description:
1.79 ⊠ ┌► π =	**5.6235**	Calculates 1.79π.
◄┐ ►DEG	**322.2000**	Converts 1.79π radians to degrees.
90.2015 ◄┐ ►HR	**90.3375**	Converts 90 degrees, 20 minutes, 15 seconds to decimal degrees.
25.2589 ┌► ►HMS	**25.1532**	25.2589 degrees = 25 degrees, 15 minutes, 32 seconds.
◄┐ SHOW	**251532040000**	Shows decimal seconds (32.04 seconds).

Coordinate Conversions
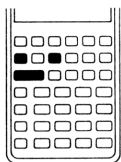
Coordinate conversions require pairs of data separated by INPUT; θ is interpreted according to the current trigonometric mode.


(x-coordinate, y-coordinate)
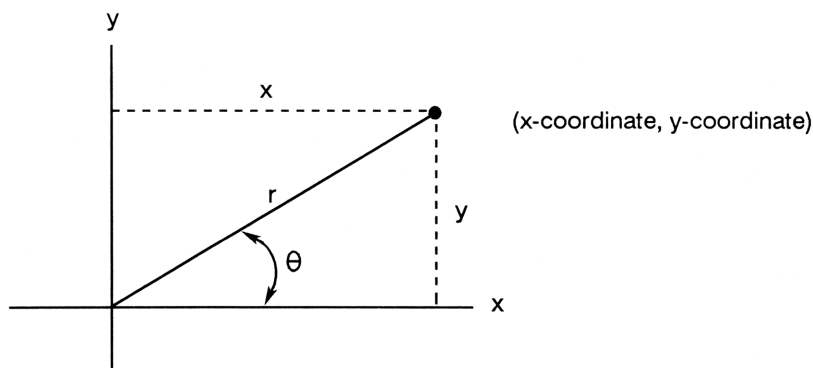
Converting From Rectangular to Polar Coordinates:

1. Enter x INPUT y ◂ ▸P to display θ.

2. Press ◂ SWAP to display r.

Converting From Polar to Rectangular Coordinates:

1. Enter r INPUT θ ▸ ▸R to display y.

2. Press ◂ SWAP to display x.

Example: Coordinate Conversions. Convert the rectangular coordinates $(10, -15)$ to polar coordinates:

Keys:	Display:	Description:
⌐► DEG		Sets Degrees mode.
10 INPUT	10.0000	Enters x.
15 +/- ◄⌐ ►P	−56.3099	Enters y, calculates r and θ, and displays θ.
◄⌐ SWAP	18.0278	Displays r.

Convert the polar coordinates (7, 30°) to rectangular coordinates:

|		
7 INPUT	7.0000	Enters r.
30 ⌐► ►R	3.5000	Enters θ, calculates x and y, and displays y.
◄⌐ SWAP	6.0622	Displays x.

Probability Functions
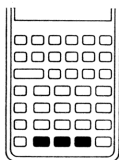
Your HP-20S can calculate factorials, combinations, and permutations.

Factorial. Pressing ⌐► n! calculates the factorial of the number in the display. The number must be an integer in the range 0 through 253.

**Combinations and Permutations.** The keystrokes for calculating combinations and permutations are:

n-value INPUT r-value ⌐► Cn,r

or

n-value INPUT r-value ⌐► Pn,r

The number of *combinations* of n objects taken r at a time is the number of different sets containing r items that can be taken from a larger group of n items. No item occurs more than once in the set of r items, and different orders of the same r items *are not* counted separately.

The number of *permutations* of n objects taken r at a time is the number of different arrangements of r items that can be taken from a larger group of n items. No item can occur more than once in an arrangement, and different orders of the same r items *are* counted separately.

Keys:	Display:	Description:
5 [INPUT]	5.0000	Enters the n-value.
3 [►] [Cn,r]	10.0000	Enters the r-value; calculates combinations of 5 objects, 3 at a time.
5 [INPUT]	5.0000	Enters the n-value.
3 [►] [Pn,r]	60.0000	Enters the r-value; calculates permutations of 5 objects, 3 at a time.

Probability Formulas

$$Cn,r = \frac{n!}{(n - r)! \, r!}$$

$$Pn,r = \frac{n!}{(n - r)!}$$

Hyperbolic Functions
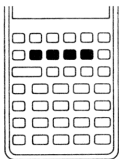
Keys	Function
⬅ HYP SIN	Hyperbolic sine.
⬅ HYP ⬅ ASIN	Inverse hyperbolic sine.
⬅ HYP COS	Hyperbolic cosine.
⬅ HYP ⬅ ACOS	Inverse hyperbolic cosine.
⬅ HYP TAN	Hyperbolic tangent.
⬅ HYP ⬅ ATAN	Inverse hyperbolic tangent.

Keys:	Display:	Description:
5 ⬅ HYP SIN	**74.2032**	Hyperbolic sine.
540.25 ⬅ HYP ⬅ ACOS	**6.9852**	Inverse hyperbolic cosine.

Parts of Numbers
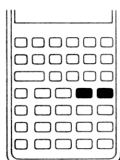
| Keys	Function
⬅ IP	Integer part of the number.
➡ FP	Fractional part of the number (the number without its integer part).
⬅ ABS	Absolute value of the number.
➡ RND	Rounds the number internally to the number of digits specified in the current FIX, SCI, or ENG display format. (No rounding occurs in ALL mode.)

Keys:	Display:	Description:
12.3456789 $\boxed{=}$	**12.3457**	Enters a nine-digit number.
$\boxed{\blacktriangleleft}$ $\boxed{\text{SHOW}}$	**123456789000**	Displays full precision of number.
$\boxed{\blacktriangleright}$ $\boxed{\text{RND}}$ $\boxed{\blacktriangleleft}$ $\boxed{\text{SHOW}}$	**123457000000**	Number is rounded internally.

Unit Conversions
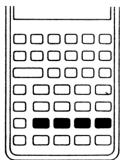
Keys:	Converts:
$\boxed{\blacktriangleleft}$ $\boxed{\text{→kg}}$	lb (pounds) to kg (kilograms)
$\boxed{\blacktriangleright}$ $\boxed{\text{→lb}}$	kg (kilograms) to lb (pounds)
$\boxed{\blacktriangleleft}$ $\boxed{\text{→°C}}$	°F (Fahrenheit) to °C (Celsius)
$\boxed{\blacktriangleright}$ $\boxed{\text{→°F}}$	°C (Celsius) to °F (Fahrenheit)
$\boxed{\blacktriangleleft}$ $\boxed{\text{→cm}}$	in (inches) to cm (centimeters)
$\boxed{\blacktriangleright}$ $\boxed{\text{→in}}$	cm (centimeters) to in (inches)
$\boxed{\blacktriangleleft}$ $\boxed{\text{→l}}$	gal (gallons) to l (liters)
$\boxed{\blacktriangleright}$ $\boxed{\text{→gal}}$	l (liters) to gal (gallons)

Example: Unit Conversions. Convert 100 pounds to kilograms.

Keys:	Display:	Description:
100 ⤶ ⟨→kg⟩	**45.3592**	Converts 100 pounds to kilograms.

Convert 6 feet to centimeters.

6 ⟨×⟩ 12 ⟨=⟩	**72.0000**	Converts 6 feet to inches.
⤶ ⟨→cm⟩	**182.8800**	Converts 72 inches to centimeters.

4

Base Conversions and
Base Arithmetic

The HP-20S enables you to switch between four number-base modes—decimal, hexadecimal, octal, and binary. You can convert numbers from one base to another and perform arithmetic operations in any of the four bases. The **HEX**, **OCT**, and **BIN** annunciators indicate the current (nondecimal) mode.

Switching Bases
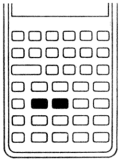
To switch to a different base mode, press:

Mode	Keys	Annunciator
Hexadecimal	↰ HEX	**HEX**
Octal	↱ OCT	**OCT**
Decimal	↰ DEC	None
Binary	↱ BIN	**BIN**

When you switch to a new base:

- The number in the display is converted to the new mode.

- When you switch *from* decimal to another base, the *integer part* of the number is displayed in the new base. Internally, the 12-digit representation of the decimal number is preserved. When you switch back to decimal base, the full decimal number is displayed in the current display format. Numbers are *truncated* to integers internally only when they are used in an arithmetic operation in hexadecimal, octal, or binary base.

- Hexadecimal, octal, and binary numbers are right-justified in the display—that is, they are displayed as far to the right as possible.

- In Octal and Binary modes: certain keys are inactive. For example, 8 and 9 do not function in Octal mode; 2 through 9 do not function in Binary mode. If you press an inactive key, the base annunciator will blink.

- In Hexadecimal mode: The top-row keys become the hexadecimal digits A through F.

- In Binary mode: If the binary number is longer than eight bits, the right-most (least significant) eight bits are shown, and a window number appears at the left of the display.

Window number appears if number has more than 8 digits. → 0   11111011 ← Least significant 8 digits of binary number.

Press ⊡ to view the other eight-bit segments.

The binary number 1011011011111101101111000010111010111 looks like this in the windows:

4 1011	3 01101111	2 11011011	1 11000101	0 11010111
↑	↑	↑	↑	↑

Window Numbers

Example: Converting Between Bases. The following keystrokes do a series of base conversions. Convert 125_{10} to binary, octal, and hexadecimal numbers:

Keys:	Display:	Description:
125 $\boxed{\blacktriangleright}$ $\boxed{\text{BIN}}$	**1111101**	Switches to binary base; $125_{10} = 1111101_2$.
$\boxed{\blacktriangleright}$ $\boxed{\text{OCT}}$	**175**	Switches to octal base; $1111101_2 = 175_8$.
$\boxed{\blacktriangleleft}$ $\boxed{\text{HEX}}$	**7d**	Switches to hexadecimal base; $175_8 = 7D_{16}$.
$\boxed{\blacktriangleleft}$ $\boxed{\text{DEC}}$	**125.0000**	Restores decimal base.

Convert $24FF_{16}$ to binary base.

$\boxed{\blacktriangleleft}$ $\boxed{\text{HEX}}$	**7d**	Sets hexadecimal base.
24FF $\boxed{\blacktriangleright}$ $\boxed{\text{BIN}}$	**0 11111111**	Converts $24FF_{16}$ to binary base and displays least-significant eight digits.
$\boxed{\cdot}$	**1 100100**	Displays six remaining digits in window 1.
$\boxed{\cdot}$	**0 11111111**	Back to window 0.
$\boxed{\cdot}$	**1 100100**	Back to window 1, again.

The binary number is 10010011111111. Now, convert to decimal base:

$\boxed{\blacktriangleleft}$ $\boxed{\text{DEC}}$	**9,471.0000**	Restores decimal base.

Representation of Numbers

The internal representation of a number does not change when a number is converted to another base. When a number is converted from its decimal value to a different base, the integer part of the number is represented as a 36-bit binary number.

In Hexadecimal, Octal, and Binary modes, numbers are displayed in 2's complement format. The left-most bit of the binary representation of a number is the sign bit. It is set to 1 for negative numbers.

Keys:	Display:		Description:
8738 $\boxed{\Leftarrow}$ $\boxed{\text{HEX}}$		**2222**	Converts 8738_{10} to hexadecimal base.
$\boxed{+/-}$		**FFFFFdddE**	2's complement.
$\boxed{\Leftarrow}$ $\boxed{\text{DEC}}$	**−8,738.0000**		Negative decimal number.

Range of Hexadecimal, Octal, and Binary Numbers

The 36-bit word size determines the range of numbers that can be represented in hexadecimal, octal, or binary base, and the range of decimal numbers that can be converted to other bases.

Range of Numbers for Base Conversions

Base	Largest Positive Integer		Largest Negative Integer	
DEC	34,359,738,367		−34,359,738,368	
HEX	7FFFFFFFF		800000000	
OCT	377777777777		400000000000	
BIN (dis-	4	111	4	1000
played in	3	11111111	3	00000000
windows 0	2	11111111	2·	00000000
through 4)	1	11111111	1	00000000
	0	11111111	0	00000000

When you enter numbers in hexadecimal, octal, or binary base, digit entry halts if you attempt to enter too many digits. For example, if you attempt to enter a 10-digit hexadecimal number, digit entry halts after the ninth digit.

If the display contains a decimal number outside the range, switching to another base displays **too big**.

Keys:	Display:	Description:
◤ $\boxed{\text{E}}$ 20 ◥ $\boxed{\text{OCT}}$	**too big**	1×10^{20} cannot be converted to octal base.
◤ $\boxed{\text{DEC}}$	**1.0000E20**	Restores decimal base.

Numbers that are outside the conversion range are represented by the message **too big**.

3 ◤ $\boxed{\text{E}}$ 11 $\boxed{-}$ 3 ◤ $\boxed{\text{E}}$ 8 ◤ $\boxed{\text{HEX}}$	**11E1A300**	3×10^8 is $11E1A300_{16}$ in Hexadecimal mode.
◤ $\boxed{\text{SWAP}}$	**too big**	3×10^{11} is outside the base-conversion range.
◤ $\boxed{\text{DEC}}$	**300,000,000,000.**	Restores decimal base.
$\boxed{\text{C}}$	**0.0000**	Clears the display.

Arithmetic Operations

All functions are active in all bases (except nonshifted functions on the top row keys).

All arithmetic operations in hexadecimal, octal, and binary base use 2's complement arithmetic. When a division produces a remainder, only the integer portion of the number is retained.

Example: Arithmetic in Hexadecimal, Octal, and Binary Bases. Calculate $12F_{16} + E9A_{16}$:

Keys:	Display:	Description:
⬛ HEX	**0**	Sets hexadecimal base.
12F + E9A =	**FC9**	Adds hexadecimal numbers.

Calculate $7760_8 - 4326_8$:

⬛ OCT	**7711**	Switches to octal base ($FC9_{16} = 7711_8$).
7760 − 4326 =	**3432**	Subtracts octal numbers.

Calculate $100_8 \div 5_8$:

100 ÷ 5 =	**14**	Integer part of result.

Compare the previous result to the decimal division shown below:

100 ÷ 5 ⬛ DEC	**5.0000**	Converts all values in the expression to Decimal mode.
=	**12.8000**	Division of $64_{10} \div 5_{10}$. ($100_8 = 64_{10}$).
⬛ OCT	**14**	Integer portion of 12.8_{10} in octal base.

Add $5A0_{16}$ plus 1001100_2.

◄┐ HEX 5A0	**5A0_**	Enters hexadecimal number.
►┐ BIN	**0 10100000**	Switches to binary base.
+ 1001100 =	**0 11101100**	Calculates result in binary base. Displays window 0.
·	**1 101**	Displays window 1.

Arithmetic results that cannot be represented in 36 bits display an overflow warning and the largest positive or negative number:

◄┐ HEX	**5EC**	Switches to hexadecimal base.
5AAAAAAA × 4 =	**OFLO** **7FFFFFFFF**	Temporary message. Largest positive number.
EBBBBBBBB − 6CCCCCCCC =	**OFLO** **800000000**	Temporary message. Largest negative number.

Press ◄┐ DEC to return to Decimal mode.

5

Statistical Calculations
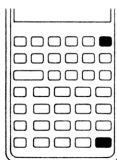
The $\boxed{\Sigma+}$ and $\boxed{\blacktriangleleft}$ $\boxed{\Sigma-}$ keys are used to enter and delete statistical data for one- and two-variable statistics. Summation data is accumulated in registers R_4 through R_9. Once you enter the data, you can use the statistical functions to calculate:

- Mean and standard deviation.
- Linear regression and linear estimation.
- Weighted mean.
- Summation statistics: n, Σx, Σx^2, Σy, Σy^2, and Σxy.

Entering Statistical Data

There is no limit to the number of values you can accumulate in the statistical registers. However, if statistical data causes the value of a statistical register to exceed $\pm 9.99999999999 \times 10^{499}$, the HP-20S displays a temporary overflow warning (**OFLO**).

The statistical registers, R_4 through R_9, can be used to store data for other than statistical use. To clear any data that may have been previously stored, press $\boxed{\blacktriangleright}$ $\boxed{CL\Sigma}$.

Entering Data for One-Variable Statistics

To enter x data for one-variable statistics:

1. Clear any previous contents of R_4 through R_9 by pressing $\boxed{\blacktriangleright}$ $\boxed{CL\Sigma}$.

2. Enter the first value and press $\boxed{\Sigma+}$. The HP-20S displays the number of items (n) accumulated; in this case, **1.0000**.

3. Continue accumulating values by entering the numbers and pressing $\boxed{\Sigma+}$. The n-value is updated with each entry.

Entering Data for Two-Variable Statistics or Weighted Mean

To enter x,y-pairs of statistical data:

1. Clear any previous contents of R_4 through R_9 by pressing $\boxed{\mathbf{r}}$ $\boxed{\text{CL}\Sigma}$.

2. Enter the first x-value and press $\boxed{\text{INPUT}}$. The HP-20S displays the x-value.

3. Enter the corresponding y-value and press $\boxed{\Sigma+}$. The HP-20S displays the number of pairs of items (n) accumulated; in this case, **1.0000**.

4. Continue entering x,y-pairs. The n-value is incremented with each entry.

To enter data for calculating the weighted mean, enter each data value as x, and its corresponding weight as y.

Correcting Statistical Data

Incorrect entries can be deleted using $\boxed{\mathbf{\leftarrow}}$ $\boxed{\Sigma-}$. If either value of an x,y-pair is incorrect, you must delete and reenter both values.

To delete and reenter statistical data:

1. Enter the x-value to be deleted. If the data consists of x,y-pairs, press $\boxed{\text{INPUT}}$ and then enter the y-value.

2. Press $\boxed{\mathbf{\leftarrow}}$ $\boxed{\Sigma-}$ to delete the value(s). The n-value is decreased by 1.

3. Enter the correct value or x,y-pair using $\boxed{\Sigma+}$.

Clearing Statistical Data
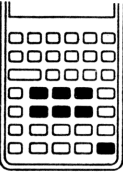
Clear the statistical registers before entering new data so that R_4 through R_9 are zero when you begin. If you don't clear the registers, data currently stored in R_4 through R_9 is automatically included in the summation calculations. To clear the statistical registers, press [➤] [CLΣ]. The display and any pending operations are also cleared.

Summary of Statistical Calculations

Some functions return two values. The **:** annunciator indicates that two values have been returned. Press [↰] [SWAP] to see the hidden value.

Keys	Description	[↰] [SWAP] to Display
[RCL] 4 (n)	Number of data points entered.	
[RCL] 5 (Σx)	Sum of the x-values.	
[RCL] 6 (Σy)	Sum of the y-values.	
[RCL] 7 (Σx^2)	Sum of the squares of the x-values.	
[RCL] 8 (Σy^2)	Sum of the squares of the y-values.	
[RCL] 9 (Σxy)	Sum of the products of the x- and y-values.	
[➤] [$\bar{x},\bar{y}$]	Arithmetic mean (average) of the x-values.	Mean (average) of the y-values if you entered y-data.
[➤] [$\bar{x}$w]	Mean of the x-values weighted by the y-values.	

Keys	Description	◄┐ SWAP to Display
► Sx,Sy	Standard deviation of the x-values.*	Standard deviation of the y-values if you entered y-data.*
y-value ► x̂,r	Estimate of x̂ for a given value of y.	Correlation coefficient.†
x-value ► ŷ,r	Estimate of ŷ for a given value of x.	Correlation coefficient.†
► m,b	Slope (m) of the calculated line.	y-intercept (b) of the calculated line.

* The standard deviation is a measure of how dispersed the numbers are about the mean. The HP-20S calculates the *sample standard deviation*, which assumes the data is a sampling of a larger, complete set of data. If the data constitutes the entire population of data, the *true population*, refer to page 55, "Calculating the Population Standard Deviation."

† The correlation coefficient is a number in the range −1 through +1 that measures how closely the data fits the calculated line. A value of +1 indicates a perfect positive correlation, and −1 indicates a perfect negative correlation. A value close to zero indicates the curve is a poor fit.

Mean, Standard Deviation, and Summation Statistics
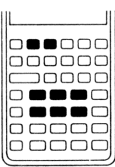
You can calculate the mean, standard deviation, n, Σx, and Σx^2 of x-data. For x,y-data, you can also calculate the mean and standard deviation of the y-data and Σy, Σy^2, and Σxy.

Example: Calculating the Mean, Standard Deviation, and Root Mean Square. A yacht captain wants to determine how long it takes to change a sail. She randomly chooses six members of her crew, observes them as they carry out the sail change, and records the number of minutes required: 4.5, 4, 2, 3.25, 3.5, 3.75.

Calculate the mean and standard deviation of the times. Also, calculate the root mean square, using the formula $\sqrt{\Sigma x^2/n}$.

Keys:	Display:	Description:
[↱] [CLΣ]	0.0000	Clears the statistical registers.
4.5 [Σ+]	1.0000	Enters the first time.
4 [Σ+] 2 [Σ+] 3.25 [Σ+] 3.5 [Σ+] 3.75 [Σ+]	6.0000	Enters the remaining data.
[↱] [x̄,ȳ]	3.5000	Calculates the mean.
[↱] [Sx,Sy]	0.8515	Calculates the standard deviation.
[RCL] 7	77.1250	Displays Σx^2.
[÷] [RCL] 4	6.0000	Displays n.
[=] [√x̄]	3.5853	Calculates the root mean square.

Calculating the Population Standard Deviation
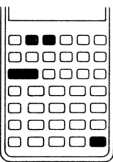
The standard deviations calculated by [↱] [Sx,Sy] and [↱] [Sx,Sy] [↰] [SWAP] are the sample standard deviations. They assume that the data is a sampling of a larger, complete set of data. If the data constitutes the entire population of data, the *true population standard deviation* can be calculated by calculating the mean of the original data, adding the mean to the statistical data using [Σ+], and then calculating the standard deviation. For two-variable statistics, after calculating the mean of the original data, press [↰] [SWAP] to put the data in the proper order (ȳ in the display) then press [Σ+].

Example: Population Standard Deviation. The coach has four new players on the team with heights of 193, 182, 177, and 185 centimeters and weights of 90, 81, 83, and 77 kilograms. Find the mean and population standard deviations of their heights and weights.

Keys:	Display:	Description:
⬅ CLΣ	0.0000	Clears the statistical registers.
193 INPUT 90 Σ+	1.0000	Enters height and weight of player 1.
182 INPUT 81 Σ+	2.0000	Enters height and weight of player 2.
177 INPUT 83 Σ+	3.0000	Enters height and weight of player 3.
185 INPUT 77 Σ+	4.0000	Enters height and weight of player 4.
⬅ $\bar{x},\bar{y}$	184.2500	Calculates mean of heights (x).
⬅ SWAP	82.7500	Displays mean of weights (y).
Σ+	5.0000	Adds means to data. (Data must be in x,y order with y in the display.)
⬅ Sx,Sy	5.8041	Calculates population standard deviation for heights (x).
⬅ SWAP	4.7104	Displays population standard deviation for weights (y).

Linear Regression and Estimation
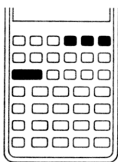
Linear regression is a statistical method for finding a straight line that best fits a set of x,y-data. There must be at least two different x,y-pairs. The straight line provides a relationship between the x- and y-variables: $y = mx + b$, where m is the slope and b is the y-intercept.

Linear Regression. To do a linear regression calculation:

1. Enter the x,y-data using the instructions on page 52.
2. Press:

   ■ ⟦►⟧ ⟦x̂,r⟧ ⟦◄⟧ ⟦SWAP⟧ (or ⟦►⟧ ⟦ŷ,r⟧ ⟦◄⟧ ⟦SWAP⟧) to display r, the correlation coefficient.

   ■ ⟦►⟧ ⟦m,b⟧ to display m, the slope of the line, then ⟦◄⟧ ⟦SWAP⟧ to display b, the y-intercept.

Linear Estimation. The straight line calculated by linear regression can be used to estimate a y-value for a given x-value, or vice versa. To do linear estimation calculations:

1. Enter the x,y-data using the instructions on page 52.
2. Enter the known x-value or y-value.

   ■ To estimate x for the given y, enter the y-value, then press ⟦►⟧ ⟦x̂,r⟧.

   ■ To estimate y for the given x, enter the x-value, then press ⟦►⟧ ⟦ŷ,r⟧.

Example: Linear Regression and Estimation. The rate of a certain chemical reaction depends on the initial concentration of one chemical. When the reaction is run repeatedly, varying only the initial concentration of the chemical, the following rates are observed:

Concentration X (moles per liter)	0.050	0.075	0.10	0.125	0.20
Rate Y (moles per liter-seconds)	0.0062	0.00941	0.0140	0.0146	0.023

Calculate the slope and y-intercept of the best straight line fitted to the data. Also calculate the correlation coefficient.

Keys:	Display:	Description:
⌐► CLΣ	0.0000	Clears the statistical registers.
.05 INPUT .0062 Σ+ .075 INPUT .00941 Σ+ .1 INPUT .014 Σ+ .125 INPUT .0146 Σ+ .2 INPUT .023 Σ+	5.0000	Enters the x,y-data.
⌐► m,b	0.1093	Displays the slope.
◄┑ SWAP	0.0014	Displays the y-intercept. : indicates another result.
⌐► x̂,r ◄┑ SWAP	0.9890	Displays the correlation coefficient.

Estimate the rate of the reaction when the concentration equals 0.09 moles per liter.

.09 ⌐► ŷ,r	0.0113	Calculates estimate of y for $x=0.09$.

What concentration is necessary for the rate to equal 0.0200?

.02 ⌐► x̂,r	0.1700	Calculates estimate of x for $y=0.02$.
C	0.0000	Clears display and : annunciator.

Weighted Mean
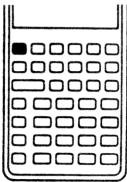
$x_1, x_2, \ldots, x_n$ occurring with weights $y_1, y_2, \ldots, y_n$.

1. Use [Σ+] to enter the data as x,y-pairs. The y-values are the weights of the x-values.

2. Press [➡] [x̄w].

Example: Weighted Mean. Your manufacturing company purchases a certain part four times a year. Last year's purchases were:

Price/Part	$4.25	$4.60	$4.70	$4.10
Number of Parts	250	800	900	1000

Calculate the average price per part.

Keys:	Display:	Description:
[➡] [CLΣ]	0.0000	Clears the statistical registers.
4.25 [INPUT] 250 [Σ+] 4.6 [INPUT] 800 [Σ+] 4.7 [INPUT] 900 [Σ+] 4.1 [INPUT] 1000 [Σ+]	4.0000	Enters the data and their weights.
[➡] [x̄w]	4.4314	Calculates weighted mean (average part price).

Statistical Formulas

$$\bar{x} = \frac{\Sigma x}{n}, \quad \bar{y} = \frac{\Sigma y}{n}, \quad \bar{x}_w = \frac{\Sigma xy}{\Sigma y}$$

$$S_x = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}}$$

$$S_y = \sqrt{\frac{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}{n-1}}$$

$$m = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$$

$$b = \bar{y} - m\bar{x} \qquad \hat{x} = \frac{y - b}{m} \qquad \hat{y} = mx + b$$

$$r = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)\left(\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right)}}$$

6

Programming

A program lets you repeat calculations without repeating keystrokes. To enter a program, you use the same keystrokes that you use manually, but press the keys while you are in Program mode. Your calculator will then repeat them on command.

The HP-20S enables you to use its programming features in two ways. You can write original programs by having the calculator record and repeat your keystrokes, or you can run any one of six built-in programs.

Any program, regardless of whether you entered it yourself or loaded it from the built-in program library, can be run and edited. This chapter explains how to do original programming and editing. Chapter 7 gives instructions on using the built-in programs.

Before the programming concepts and commands are explained in detail, try this quick example. Start by writing out the formula, then solve the problem from the keyboard.

A Simple Programming Example. To find the cross-sectional area of a pipe with a diameter of 5 inches, use the formula

$$A = \frac{\pi d^2}{4}.$$

Before doing the calculation, rearrange the equation in this order $d^2 \times \pi \div 4 = A$.

Enter 5 in the display and press:

$\boxed{\leftharpoondown}$ $\boxed{x^2}$ $\boxed{\times}$ $\boxed{\rightharpoondown}$ $\boxed{\pi}$ $\boxed{\div}$ 4 $\boxed{=}$

giving a result of 19.6350 square inches.
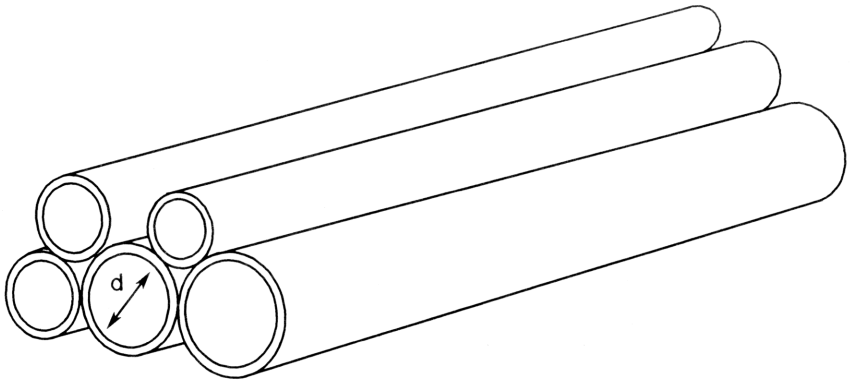
But what if you wanted to find the area of many different pipes? Rather than repeat the keystrokes each time (varying only the "5" for the different diameters), you can put the repeatable keystrokes into a program that would look like this:

01 x^2
02 $\times$
03 π
04 $\div$
05 4
06 $=$

This program assumes that the value for the diameter is in the display when the program starts to run. It calculates and displays the area. To enter this program into program memory, press the following keys. (Don't worry about the numbers that appear in your display—they're called keycodes and are explained later.) If you notice a mistake while typing a line, press ◀ to erase the line, then type it over.
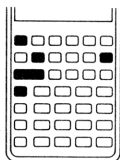
Keys:	Display:	Description:
⬅ PRGM		Enters Program mode.
➡ CLPRGM	00-	Clears any previously stored programs.
⬅ x²	01- 51 11	Enters the keystrokes that create the program.
×	02- 55	
➡ π	03- 61 22	
÷	04- 45	
4	05- 4	
=	06- 74	
⬅ PRGM		Exits Program mode.

Now try running this program to find the area of a pipe with a diameter of 5 inches.

Keys:	Display:	Description:
C	0.0000	Clears the display.
⬅ GTO · ·	0.0000	Goes to the first line of the program.
5 R/S	19.6350	The answer!

Creating Programs

The steps you follow to create programs are:

1. Enter Program mode.
2. Enter the repeatable keystrokes.
3. Exit Program mode.
4. Run the program.

We will continue using the pipe area program to illustrate programming concepts. As you were programming, you may have noticed the numbers in the display. They are line numbers and keycodes.

Line Numbers. *Line numbers* appear left-justified in the display as you are entering your program. The numbers, 00 through 99, are followed by a dash. The dash separates the line numbers from keycodes.

Keycodes. The numbers to the right of the line number are called *keycodes.* A keycode indicates which key you pressed. The first digit indicates which row on the keyboard the key is in. The second digit indicates which column the key is in. A line contains one or more keycodes which together represent a single operation. Labels and number keys don't appear as keycodes, but instead as **A** through **F** or **0** through **9**.

Columns

	1	2	3	4	5	6

x^2 $\bar{x}_w$ 10^x $\bar{x},\bar{y}$ LOG S_x,S_y % $\hat{x},r$ %CHG $\hat{y},r$ $\Sigma-$ m,b

1 [√x] A [e^x] B [C] C [14] D [1/x] E [Σ+] F

→P →R   HYP π   ASIN DEG   ACOS RAD   ATAN GRD   PRGM RTN

2 [21] [RCL] [SIN] [COS] [TAN] [R/S]

SWAP   CLPRGM   E   FIX SCI   ENG ALL   LOAD

3 [31] [+/−] [( ] [ )] [←]

GTO LBL   ▼ $x \le y$?   ▲ $x = 0$?   ABS RND   IP FP

4 [41] [7] [8] [9] [÷]

HEX OCT   DEC BIN   →HR →HMS   →DEG →RAD

Σx^2 Σy^2 Σxy

5 [51] [52] [5] [6] [×]

→kg →lb   →°C →°F   →cm →in   →l →gal

n Σx Σy

6 [61] [1] [2] [3] [−]

OFF   •/, Cn,r   SHOW Pn,r   LAST n!   CLRG CLΣ

7 [C] [0] [•] [=] [75]

ON

Rows

1	2	3	4	5

[INPUT] = 31

[◀] [GTO] C = 51 41 C

[◀] [HEX] = 51 52

[▶] [$\hat{x}$,r] = 61 14

[STO] [+] 3 = 21 75 3

2 = 2

Checksum. After you have entered a program you can check to see if the keystrokes are entered correctly by comparing the *checksum* listed in this manual to the checksum created by your program. The checksum is a unique hexadecimal value assigned to the specific keystrokes that you entered. To view the checksum, press and hold $\boxed{\blacktriangleleft}$ $\boxed{\text{SHOW}}$ for a moment while you are in Program mode. The checksums for the examples in this manual are valid if there is only one program in memory. The checksum for the pipe area program on page 63 is **9Ad7**.

Program Boundaries (LBL and RTN)
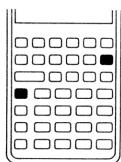
If you want to store more than one program in your HP-20S, then the program needs boundaries—a *label* to mark its beginning and a *return* to mark its end.

Program Labels. Programs and segments of programs (called *routines*) start with a label that acts as a name. Use a label to separate programs any time you have more than one program in memory. The keystrokes to create a label are $\boxed{\blacktriangleright}$ $\boxed{\text{LBL}}$ followed by A through F or 0 through 9. A label is used to execute a specific program or routine.

When you press $\boxed{\text{XEQ}}$ *label*, the program pointer moves to the specified label and begins execution. (The *program pointer* is an internal pointer that marks the line that is displayed while in Program mode.) All of memory is searched for the specified label, starting at the program pointer. If no label is found, the message **Error - LbL** is displayed.

Return. Programs end with a return ($\boxed{\blacktriangleright}$ $\boxed{\text{RTN}}$) instruction. When a program finishes running, the RTN instruction returns the program pointer to line 00. If the last line of the program is not a RTN instruction, the program pointer automatically returns to line 00. The keystrokes are $\boxed{\blacktriangleright}$ $\boxed{\text{RTN}}$. Using $\boxed{\blacktriangleright}$ $\boxed{\text{RTN}}$ in subroutines is discussed on page 76.

Entering Programs

Pressing ⬅ PRGM toggles the calculator into and out of Program mode (**PRGM** annunciator on). While the HP-20S is in Program mode, keystrokes that you enter are stored as program lines. The calculator has enough memory for 99 program lines. Each function and each digit of a number occupy one program line.

To enter a program into memory:

1. Press ⬅ PRGM to enter Program mode. The **PRGM** annunciator appears in the display.

2. Press ⬅ GTO ·· to display line 00. This sets the program pointer to line 00 without affecting other programs.

   If you don't need any other programs that might be in memory, clear program memory by pressing ➡ CLPRGM. This sets the program pointer to line 00 since there are no other lines to display.

3. To start entering the program, press ➡ LBL followed by the label you wish to assign; A through F or 0 through 9.

4. To enter program instructions, press the same keys you would use to do an operation manually.

5. End the program with a return instruction by pressing ➡ RTN.

6. Press ⬅ PRGM to exit Program mode.

Data Input. There are many ways to supply a program with data. Here are two ways to supply data to a program that expects one data item:

■ Enter the number in the display before you run the program.

■ Store the number into a register before you run the program, then recall it from within the program.

Here are two ways to supply data to a program that expects two data items:

■ Enter data in the display before you run the program by using $number_1$ INPUT $number_2$. The program can store $number_2$ then do a ⬅ SWAP to access $number_1$.

■ Store both items in registers before you run the program, then recall them from within the program.

Example: This example clears the pipe area program and enters a new version of the program that includes a label and a return instruction. (Refer to page 71 if you don't want to clear all of program memory.) If you make a mistake during entry, press ⬅ to delete the current program line, then reenter it correctly.

Keys:	Display:	Description:
🔽 PRGM		Enters Program mode (**PRGM** annunciator on).
🔼 CLPRGM	00-	Clears program memory.
🔼 LBL A	01- 61 41 A	Labels this program routine "A".
🔽 x^2	02- 51 11	Enters the program lines.
×	03- 55	
🔼 π	04- 61 22	
÷	05- 45	
4	06- 4	
=	07- 74	
🔼 RTN	08- 61 26	Ends the program.
🔽 SHOW	CF08	Checksum (page 66).
🔽 PRGM		Exits Program mode (**PRGM** annunciator off).

Positioning the Program Pointer

Program memory starts at line 00. The list of program lines is circular, so you can wrap the program pointer from the bottom to the top. There are several ways to move the program pointer to view different lines:

Whether you are in Program mode or not:

- ⬅ GTO ·· to move to line 00.
- ⬅ GTO · *line-number* to move to a specified line.
- ⬅ ▲ or ⬅ ▼ to move one line at a time.
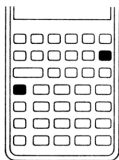- Hold ⬅ and press ▲ or ▼ to move up or down.

When in Program mode:

- Hold ⬅ ▲ or ⬅ ▼ to move up or down rapidly.

When not in Program mode:

- ⬅ GTO *label* to move to a specified label.

Running Programs

There are two ways to run a program:

- Use XEQ.
- Use GTO and R/S (Run/Stop).

The **PRGM** annunciator blinks on and off, and the message **running** appears in the display while the program is running.

Starting Programs With XEQ

To execute a program using XEQ:

- Enter data required by the program, if necessary.
- Press XEQ *label*.
- If you hold down *label* after pressing XEQ, the line where execution will begin is displayed. The program starts to run when *label* is released.

Example: Run the program labeled A to find the areas of three different pipes with diameters of 5, 2.5, and 2π. Remember to enter the diameter before executing label A.

Keys:	Display:	Description:
5 [XEQ] A	19.6350	Enters the diameter, then starts program A. The resulting area is displayed.
2.5 [XEQ] A	4.9087	Area of second pipe.
2 [×] [⬆] [π]	3.1416	Diameter of third pipe.
[=]	6.2832	Area of third pipe.
[XEQ] A	31.0063	

Starting Programs With GTO and R/S

To execute a program using [GTO] and [R/S]:

- Use [GTO] to position the program pointer where you want to begin (page 68).
- Enter data required by the program, if necessary.
- Press [R/S]. If you hold down [R/S], the line where execution will begin is displayed. The program starts to run when [R/S] is released.

Stopping Programs

When a program is running, you can stop it by pressing [R/S] or [C].

Programming a Stop. Pressing [R/S] while in Program mode inserts a STOP instruction. This halts a running program until you press [R/S] again. You can stop a program to enter data. You can use [R/S] rather than RTN to end a program. When the program halts, the program pointer will not return to the top of the program.

Error Stops. If an error occurs while a program is running, program execution halts at the point the error occurred, and an error message is displayed. (There is a list of error messages and conditions on page 122.) Press [C] or [◀] to clear the display. To see the program line containing the error-causing instruction, press [◥] [PRGM].

Clearing Programs

You must be in Program mode (the **PRGM** annunciator must be on) to clear programs. Press [▸] [CLPRGM] to clear all programs from memory.

To clear a specific program you must delete each line individually. Position the pointer at the last line of the program that you want to delete and press [◀] repeatedly. Refer to page 68 for more information about how to position the pointer.

Editing Programs

You can modify a program by inserting and deleting program lines. Even if a program line requires only a minor change, you must delete the old line and insert a new one.

Deleting program lines:

1. Enter Program mode.

2. Position the pointer where you want to begin. (If you are deleting more than one consecutive program line, start with the *last* line in the group.)

3. Delete the line you want to change by pressing [◀]. Succeeding lines are automatically renumbered.

4. To exit Program mode, press [◥] [PRGM].

For example, if you want to delete lines 05 through 08, you first display line 08, then press [◀] four times. Subsequent program lines are moved up and automatically renumbered.

Inserting program lines:

1. Enter Program mode.

2. Position the pointer to the line before where you want to add lines.

3. Enter the new lines. They are inserted after the displayed line. Succeeding lines are automatically renumbered.

4. To exit Program mode, press ◄ [PRGM].

For example, if you want to insert several new lines between lines 04 and 05 of a program, you first display line 04, then enter the instructions. Subsequent program lines, starting with the original line 05, are moved down and renumbered accordingly.

Stepping Through Programs

You can test programs by stepping through them. The program executes one line at a time as you step through it. The result is displayed after each program line is executed, so you can verify the progress of calculations. To execute a program one line at a time:

1. Exit Program mode.

2. Position the pointer where you want to begin.

3. Enter data in the display, if necessary.

4. Press ◄, then press and hold ▼. This displays the current program line. When you release ▼, the line is executed. The result of that execution is then displayed, and the program pointer moves to the next line.

5. Repeat step 4 until you find an error or reach the end of the program.

To move to the *preceding* line, you can press ◄ ▲. No execution occurs.

Example: Step through the execution of the program labeled A. Use a diameter of 5 for the test data. Check that the **PRGM** annunciator is off before you start.

Keys:	Display:	Description:
⬅ GTO A		Moves program pointer to label A.
5	5_	Enters 5 in the display.
⬅ ▼ *(hold)* *(release)*	01- 61 41 A 5.0000	Label A.
⬅ ▼ *(hold)* *(release)*	02- 51 11 25.0000	Squares input.
⬅ ▼ *(hold)* *(release)*	03- 55 25.0000	Multiplies 25 by ...
⬅ ▼ *(hold)* *(release)*	04- 61 22 3.1416	... π.
⬅ ▼ *(hold)* *(release)*	05- 45 78.5398	Calculates intermediate result.
⬅ ▼ *(hold)* *(release)*	06- 4 4_	... ÷ 4.
⬅ ▼ *(hold)* *(release)*	07- 74 19.6350	... =.
⬅ ▼ *(hold)* *(release)*	08- 61 26 19.6350	End of program. Result is correct.

Sample Program: Pythagorean Theorem

You can use most of the HP-20S features in Program mode just like you use them manually. To illustrate how STO and RCL are used to recall data from registers in a program, enter the following Pythagorean theorem program. It calculates the length of the hypotenuse (side c) of a right triangle, given the lengths of sides a and b. The formula used is $c = \sqrt{a^2 + b^2}$. Assume that the calculation begins with side a in R_1 and side b in R_2.

Keys:	Display:	Description:
◀ PRGM		Enters Program mode.
▶ CLPRGM	00-	Clears program memory. (Skip this step to leave programs intact.)
▶ LBL E	01- 61 41 E	Labels program "E".
RCL 1	02- 22 1	Recalls a from R_1.
◀ x^2	03- 51 11	a^2.
+	04- 75	
RCL 2	05- 22 2	Recalls b from R_2.
◀ x^2	06- 51 11	b^2.
=	07- 74	$a^2 + b^2$.
$\sqrt{x}$	08- 11	$\sqrt{a^2 + b^2}$
▶ RTN	09- 61 26	
◀ SHOW	3902	Checksum (page 66).
◀ PRGM		Exits Program mode.

Now store the a and b values of 22 and 9 into R_1 and R_2 then run the program:

Keys:	Display:	Description:
22 STO 1	22.0000	Stores a in R_1.
9 STO 2	9.0000	Stores b in R_2.
XEQ E	23.7697	Length of the hypotenuse.

Sample Program: Random Number Generator

The following program generates random numbers in the range $0 < r_i < 1$. The program uses a starting value between 0 and 1. For a different sequence of random numbers, use a different starting value.*

Keys:	Display:	Description:
⬅ PRGM		Enters Program mode.
➡ CLPRGM	00-	Clears program memory. (Skip this step to leave programs intact.)
➡ LBL A	01- 61 41 A	Names program "A".
RCL 0	02- 22 0	Get r_i.
×	03- 55	Multiplies...
9	04- 9	
9	05- 9	
7	06- 7	... by 997.
=	07- 74	Equals $997r_i$.
➡ FP	08- 61 45	$r_{i+1} = $ FP $(997r_i)$.
STO 0	09- 21 0	Saves r_{i+1}.

* The program uses the algorithm: $r_{i+1} = $ FP $(997r_i)$, where r_0 is a starting value between 0 and 1 (for example, 0.5284163). The random number generator passes the chi-square frequency tests for uniformity, and the serial and run tests for randomness. The most significant digits are more random than the least significant digits.

If the starting value is between 0 and 1, and if the starting value $\times 10^7$ is not divisible by 2 or 5, then the generator produces 500,000 different random numbers before repeating.

[→] [RTN]	**10- 61 26**	Ends program.
[◄] [SHOW]	**7Ab8**	Checksum (page 66).
[◄] [PRGM]		Exits Program mode.

To store the starting value in R_0 and run the program:

Keys:	Display:	Description:
.5284163	**0.5284163_**	Enters starting value in display.
[STO] 0	**0.5284**	Stores starting value in R_0.
[XEQ] A	**0.8311**	Generates first random number.
[XEQ] A	**0.5579**	Generates second random number.

Continue pressing [XEQ] A to continue generating random numbers.

If you want to scale the random numbers to within the range *lower limit* < R_i < *upper limit*, add program lines to multiply the random number by the difference between the limits, and add the product to the lower limit. That is:

$$scaled\ R_i = (upper\ limit - lower\ limit)r_i + lower\ limit.$$

Subroutines

A program is composed of one or more *routines*. A routine is a functional unit that accomplishes a specific task. As programs get more complicated, it helps to break them into smaller pieces. This makes a program easier to write, read, understand, and alter.

A routine typically starts with a label (LBL) and ends with an instruction that alters or stops program execution, such as RTN or GTO.

A subroutine is a routine that is called from (executed by) another routine and returns control to that same routine when it finishes. The subroutine must start with a LBL and end with a RTN. A subroutine can call other subroutines.

If a subroutine is at the end of program memory and does not end with ⟨►⟩ ⟨RTN⟩, control is still transferred to the step after the originating ⟨XEQ⟩ when the routine completes. It is as if the subroutine had ended with ⟨►⟩ ⟨RTN⟩.

Calling Subroutines (XEQ)

Use ⟨XEQ⟩ *label* to call a specific subroutine. The subroutine must start with the label A through F or 0 through 9. Searching begins at the ⟨XEQ⟩ and proceeds down the program, wrapping around through line 00 until the label is found. Within a program, ⟨XEQ⟩ *label* transfers execution of a running program to the program line containing that label, wherever it may be. The program continues running from the new location. Then, at the next ⟨►⟩ ⟨RTN⟩ statement, execution returns to the line after the originating ⟨XEQ⟩ and continues.

For example, to write a program that calculates the average slope between x_1 and x_2 on the graph, where $y = x^2 - \sin x$, you would use the formula:

$$slope = \frac{(x_2^2 - \sin x_2) - (x_1^2 - \sin x_1)}{x_2 - x_1}$$
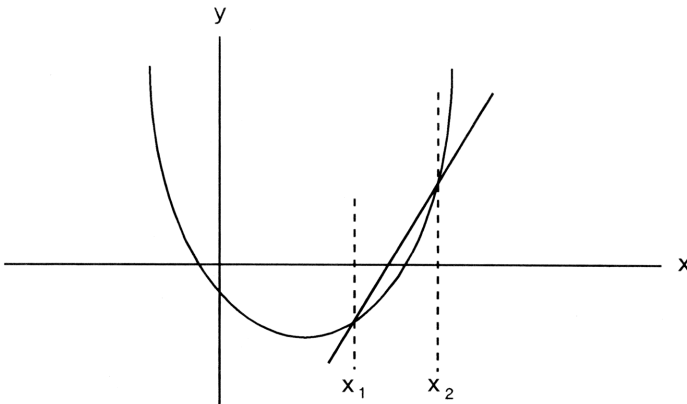
The solution requires two calculations of the expression $x^2 - \sin x$ (for $x = x_1$ and for $x = x_2$). Since the solution includes an expression that must be repeated for both values of x, you can create a subroutine to execute the repeated keystrokes and save space in program memory. The program assumes that x_1 [INPUT] x_2 has been entered before executing the program and that the calculator is in Radians mode ([◄►] [RAD]).

Keys:	Display:	Description:
[◄] [PRGM]		Enters Program mode.
[►] [CLPRGM]	00-	Clears program memory.
[►] [LBL] C	01- 61 41 C	Names program "C".
[STO] 2	02- 21 2	Stores the displayed value (x_2) in R_2.
[◄] [SWAP]	03- 51 31	Swaps (x_2 for x_1).
[STO] 1	04- 21 1	Stores displayed value (x_1) in R_1.
[C]	05- 71	Clears display so there is no hidden value or : annunciator when program is complete.
[RCL] 2	06- 22 2	Recalls x_2.
[XEQ] 5	07- 41 5	Executes subroutine to calculate $x_2^2 - \sin x_2$.
[−]	08- 65	$(x_2^2 - \sin x_2) - \ldots$
[RCL] 1	09- 22 1	Recalls x_1.
[XEQ] 5	10- 41 5	Executes subroutine again to calculate $x_1^2 - \sin x_1$.
[=]	11- 74	$(x_2^2 - \sin x_2) - (x_1^2 - \sin x_1)$.

$\boxed{\div}$	**12- 45**	Divides result by...
$\boxed{(\,)}$	**13- 33**	Reorders precedence.
$\boxed{\text{RCL}}$ 2	**14- 22 2**	Recalls x_2.
$\boxed{-}$	**15- 65**	$x_2 - \ldots$
$\boxed{\text{RCL}}$ 1	**16- 22 1**	Recalls x_1. Closing parenthesis not required because = follows.
$\boxed{=}$	**17- 74**	$((x_2{}^2 - \sin x_2) - (x_1{}^2 - \sin x_1))/ (x_2 - x_1)$.
$\boxed{\text{R/S}}$	**18- 26**	Stops.
$\boxed{\blacktriangleright}$ $\boxed{\text{LBL}}$ 5	**19- 61 41 5**	Label 5 starts the subroutine.
$\boxed{\text{STO}}$ 0	**20- 21 0**	Stores the displayed value in R_0.
$\boxed{(\,)}$	**21- 33**	Reorders priority.
$\boxed{\blacktriangleleft}$ $\boxed{x^2}$	**22- 51 11**	Squares the displayed value.
$\boxed{-}$	**23- 65**	Subtracts.
$\boxed{\text{RCL}}$ 0	**24- 22 0**	Recalls contents of R_0.
$\boxed{\text{SIN}}$	**25- 23**	Calculates the sine.
$\boxed{)}$	**26- 34**	Closing parenthesis required to evaluate $x^2 - \sin x$.
$\boxed{\blacktriangleright}$ $\boxed{\text{RTN}}$	**27- 61 26**	Ends subroutine and returns to line following originating $\boxed{\text{XEQ}}$.
$\boxed{\blacktriangleleft}$ $\boxed{\text{SHOW}}$	**7EE9**	Checksum (page 66).
$\boxed{\blacktriangleleft}$ $\boxed{\text{PRGM}}$		Exits Program mode.

To execute the program using 3 and 4 as x_1 and x_2, press
3 $\boxed{\text{INPUT}}$ 4 $\boxed{\text{XEQ}}$ C. The result is 7.8979. To exit Radians mode, press
$\boxed{\blacktriangleright}$ $\boxed{\text{DEG}}$.

Nested Subroutines. A subroutine can call another subroutine, and that subroutine can call yet another subroutine. This "nesting" of subroutines—the calling of a subroutine from within another subroutine—is limited to four levels of subroutines. The operation of nested subroutines is shown below:

MAIN PROGRAM

(Top level)   (1st level)   (2nd level)   (3rd level)   (4th level)

END OF PROGRAM
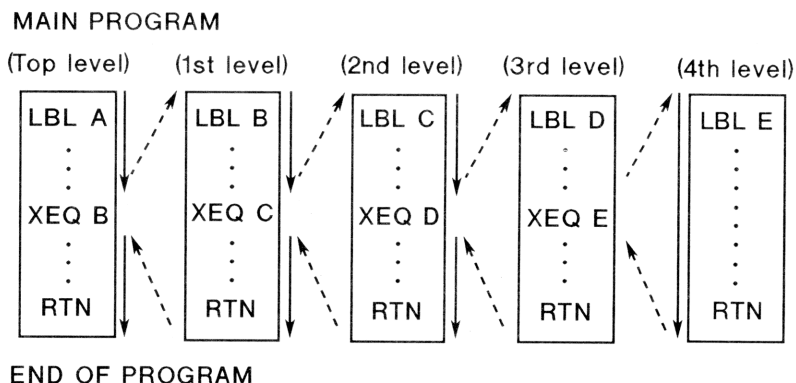
If you attempt to execute a subroutine nested more than four levels deep, the message **Error - Sub** appears in the display.

Branching and Conditionals

Branching (GTO)
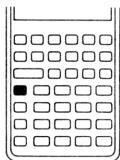
As we have seen with subroutines, it is often desirable to transfer execution to a part of the program other than the next line. This is called *branching*.

Unconditional branching uses the GTO *(go to)* instruction to branch to a program *label*. Use the keys: $\boxed{\blacktriangleleft}$ $\boxed{\text{GTO}}$ *label*.

The ◄ GTO label instruction transfers the execution of a running program to the program line containing that label, wherever it may be. Searching starts at ◄ GTO and continues throughout all of program memory. The program continues running from the new location. It does not automatically return to its point of origin when a ► RTN is encountered. Consequently, ◄ GTO is not used for subroutines.

Conditional Instructions—Decisions and Control
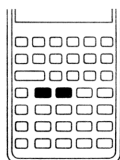
In addition to subroutines, another way to control program execution is with a *conditional test*—a true/false test that compares two numbers and skips the next program instruction if the comparison is false.

The HP-20S has two conditional statements: they are ► x≤y? and ► x=0?. $x \leq y$? asks the question, "Is x less than or equal to y?" $x = 0$? asks the question, "Is x equal to 0?" If the answer is true, the program continues execution with the line immediately following the question. If the answer is false, the program skips one line and continues from there.

For instance, if a conditional instruction is $x = 0$?, then the program compares the contents of the display to zero. If there is a zero in the display, then the program goes on to the next line. If there is *not* a zero in the display, then the program *skips* one line and continues from there. This rule is commonly known as "Do if true."

For $x \leq y$?, the program compares y (the displayed value) with x (the hidden value). Use INPUT or any other operator (for example, + or ÷) to separate x and y. If x is less than or equal to y, then the program goes on to the next line. If x is *not* less than or equal to y (that is, x is greater than y), then the program *skips* one line and continues from there.

The following example illustrates conditional branching and a GTO statement.

Example: Your accountant asks you to write a program that calculates the amount of tax a person is required to pay. You know that if the income exceeds $30,000.00, then the tax rate is 38%. If the income is less than or equal to $30,000.00, the tax rate is 28%. The question is: is *income* ≤ 30,000.00? Or to state it a different way: is $x \leqslant y$?

Keys:	Display:	Description:
◄ PRGM		Enters Program mode.
► CLPRGM	00-	Clears previous programs.
► LBL A	01- 61 41 A	Names program.
INPUT	02- 31	Enters display value into x-position for conditional test.
3	03- 3	Enters first digit of 30,000.
0	04- 0	
0	05- 0	
0	06- 0	
0	07- 0	Enters last digit of 30,000.
► x≤y?	08- 61 42	Conditional test: is $x \leqslant$ 30,000? Does next line if true, otherwise skips one line.
◄ GTO 0	09- 51 41 0	Goes to label 0 if *income* ≤ 30,000.
◄ SWAP	10- 51 31	Exchanges 30,000 and *income*.
×	11- 55	Multiplies x-value.
3	12- 3	Each digit uses one program line.

8	**13- 8**	Enters tax rate.
◀ %	**14- 51 14**	
=	**15- 74**	38% of x-value.
R/S	**16- 26**	Halts program.
▶ LBL 0	**17- 61 41 0**	Starts routine for $income \leqslant 30{,}000$.
◀ SWAP	**18- 51 31**	Exchanges 30,000 and x-value.
×	**19- 55**	Multiplies x-value.
2	**20- 2**	Each digit uses one program line.
8	**21- 8**	Enters tax rate.
◀ %	**22- 51 14**	
=	**23- 74**	28% of x-value.
R/S	**24- 26**	Halts the program.
◀ SHOW	**d6b6**	Checksum (page 66).
◀ PRGM		Exits Program mode.

Test the program by comparing samples done manually. For example, 15000 × 28 ◀ % = 4,200.0000. Test a few more incomes manually, then run the program and compare them. To run the program, enter the income value in the display and press XEQ A.

The : that appears in the display after the program is complete is caused by the INPUT that separates *income* from 30,000 for the conditional test in line 08. The program can be rewritten so that × is used to separate *income* from 30,000 for the conditional test. Also, to save program lines, the common keystrokes can be grouped together. The following program uses conditional branching and unconditional branching for the common keystrokes.

Keys:	Display:	Description:
◄┐ PRGM		Enters Program mode.
┌► CLPRGM	00-	Clears any existing programs.
┌► LBL A	01- 61 41 A	Names program A.
×	02- 55	Puts display value into x-position for conditional. It will be used later to multiply by the tax rate.
3	03- 3	One digit per line.
0	04- 0	
0	05- 0	
0	06- 0	
0	07- 0	y-value is 30,000.
┌► x≤y?	08- 61 42	If yes, goes to next line; if no, skips one line.
◄┐ GTO 1	09- 51 41 1	Goes to label 1 if *income* ≤ 30,000.
3	10- 3	Replaces 30,000 by the tax rate.
8	11- 8	
◄┐ GTO 2	12- 51 41 2	Goes to label 2 for common steps.
┌► LBL 1	13- 61 41 1	Starts routine 1 for $x \leq 30,000$.
2	14- 2	One digit per line.

8	15- 8	
⮐ LBL 2	16- 61 41 2	Starts routine 2 with common lines.
⬅ %	17- 51 14	Calculates 38% or 28% ...
=	18- 74	... of income.
R/S	19- 26	End of program.
⬅ SHOW	CbCA	Checksum (page 66).
⬅ PRGM		Exits Program mode.

Test this program the same way you tested the previous program on page 83. Press C to remove the : from the previous example.

Keystrokes for Other Conditionals

The HP-20S provides two of many possible conditionals using x, y, and zero. The following table shows examples of the keystrokes that you can use to create some other conditionals in a program:

Conditional	Program Steps	Explanation
$n=0?$, $n\neq0?$	n ⮐ x=0? ⬅ GTO 1 (Lines for $n\neq0$) ⋮ ⮐ LBL 1 (Lines for $n=0$)	n is x. Is $n=0?$ Yes. Go to LBL 1. No. Continue here.
$n\geq0?$, $n<0$	C INPUT n ⮐ x≤y? ⬅ GTO 1 (Lines for $n<0$) ⋮ ⮐ LBL 1 (Lines for $n\geq0$)	0 is x. n is y. Is $0\leq n?$ (is $n\geq0?$). Yes. Go to LBL 1. No. Continue here.

Conditional	Program Steps	Explanation
$n \leqslant 0?$, $n > 0?$	n [INPUT] 0 [▶] [x≤y?] [◀] [GTO] 1 *(Lines for $n > 0$)* ⋮ [▶] [LBL] 1 *(Lines for $n \leqslant 0$)*	n is x. 0 is y. Is $n \leqslant 0$? Yes. Go to LBL 1. No. Continue here.
$n_1 = n_2?$, $n_1 \neq n_2?$	n_1 [−] n_2 [=] [▶] [x=0?] [◀] [GTO] 1 *(Lines for $n_1 \neq n_2$)* ⋮ [▶] [LBL] 1 *(Lines for $n_1 = n_2$)*	 Is $n_1 - n_2 = 0$? (is $n_1 = n_2$?) Yes. Go to LBL 1. No. Continue here.
$n_1 \geqslant n_2?$, $n_1 < n_2?$	n_2 [INPUT] n_1 [▶] [x≤y?] [◀] [GTO] 1 *(Lines for $n_1 < n_2$)* ⋮ [▶] [LBL] 1 *(Lines for $n_1 \geqslant n_2$)*	n_2 is x. n_1 is y Is $n_2 \leqslant n_1$? (is $n_1 \geqslant n_2$?) Yes. Go to LBL 1. No. Continue here.
$n_1 \leqslant n_2?$, $n_1 > n_2$	n_1 [INPUT] n_2 [▶] [x≤y?] [◀] [GTO] 1 *(Lines for $n_1 > n_2$)* ⋮ [▶] [LBL] 1 *(Lines for $n_1 \leqslant n_2$)*	n_1 is x. n_2 is y. Is $n_1 \leqslant n_2$? Yes. Go to LBL 1. No. Continue here.

Available Program Memory

Program memory can have up to 99 lines. If you attempt to add program lines (anywhere in program memory) after 99 lines have been entered, the message **Error - FuLL** is displayed.

Nonprogrammable Functions

The following HP-20S functions are not programmable:

◀	▶ CLPRGM
◀ ▼	◀ SHOW
◀ ▲	◀ LOAD
◀ GTO · *line-number*	◀ PRGM
◀ GTO ··	▶ OFF
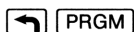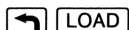
7

Built-in Program Library
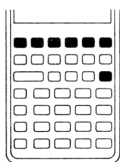
Your HP-20S has six built-in programs that can be copied into program memory using ⤶ [LOAD]. To load a program, press ⤶ [PRGM], then ⤶ [LOAD] followed by A through F. An abbreviation of the program name is displayed for a moment, then the program pointer is set to line 00. The built-in programs are:

Program Name	Title	Message
A	Root Finder	root
B	Numerical Integration	int
C	Complex Operations	CPL
D	3 × 3 Matrix Operations	3 bY 3
E	Quadratic Equation	qUAd
F	Curve Fitting	Fit

The built-in programs are designed to save keystrokes when entering a program. These programs can be edited and run just like programs that you entered yourself. When a new program is loaded, it clears any other programs that may be in memory. This chapter gives instructions and an example for each program in the library.

Root Finder (root)

This program finds a solution for $f(x) = 0$ using the secant method, which is derived from Newton's method with a numerical approximation for the derivative $f'(x)$. You must define the function $f(x)$ by entering the program lines to calculate $f(x)$, assuming x is in the display. You must also supply an initial guess, x_0, for the solution. The closer the initial guess is to the actual solution, the faster the program converges to an answer.

The main program is 62 lines long, and uses registers R_5 through R_9 and labels A, F, 8, and 9. The remaining program lines, registers, and labels can be used for defining $f(x)$. You can replace the default values of Δx limit (relative error), ϵ ($f(x)$ tolerance), and count (number of iterations) with different values depending on the desired accuracy and solution speed. Refer to the equations on page 91 to see how these values are used.

Program Instructions:

1. Press 🠔 [PRGM], then 🠔 [LOAD] A to load the program. Then press 🠔 [▲] once to move to the last line of the program.
2. After the LBL F on line 62 (**62- 61 41 F**), enter the keystrokes to calculate the value of $f(x)$ using a value of x in the display. See the example below.
3. Press 🠔 [PRGM].
4. To calculate a root, enter your initial guess (x_0) and press [XEQ] A.
5. To enter a new function, repeat the instructions starting at step 1.
6. Optional: To change ϵ, change the default value of 10^{-2} at lines 51 through 53.
7. Optional: To change Δx limit, change the default value of 10^{-10} at lines 39 through 42.
8. Optional: To change count, change the default value of 100 at lines 09 through 10.

Example: Find the root of $f(x) = x^6 - x - 1 = 0$ using an initial guess of $x_0 = 2$.

Keys:	Display:	Description:
◄⅂ PRGM ◄⅂	**root**	Loads program.
LOAD A	**00-**	
◄⅂ ▲	**62- 61 41 F**	Displays label for beginning of $f(x)$ routine.
STO 0	**63- 21 0**	Starts $f(x)$ routine; saves x.
y^x	**64- 14**	
6	**65- 6**	
−	**66- 65**	x^6.
RCL 0	**67- 22 0**	x.
−	**68- 65**	$x^6 - x$.
1	**69- 1**	
=	**70- 74**	$x^6 - x - 1$.
◄⅂ SHOW	**46b5**	Checksum (page 66).
◄⅂ PRGM		Exits Program mode.
2 XEQ A	**1.1347**	Enters x_0 and calculates root.

Remarks:

After the program has finished, the value of x such that $f(x) \approx 0$ is displayed and stored in R_6. To calculate the corresponding value of $f(x)$, press XEQ F with x in the display.

Error - Func is displayed if the equation for finding x_{i+1} divides by zero or causes some other improper math operation. Try a new guess that is closer to the root. If ϵ or Δx *limit* needs to be increased, refer to program instruction steps 6 and 7.

Error - LbL is displayed if the iteration count is exceeded. This means that for the initial guess provided, the program cannot converge on a root within *count* iterations. Try a new guess that is closer to the root, examine the function to see if it has no real roots, or increase the iteration count, ϵ, or Δx *limit*. (Refer to program instruction steps 6 through 8.)

If an error occurs, check the root approximation (in R_6) to see if it is close enough.

If the function being solved has multiple roots, you can use this program to find each root by selecting different initial guesses that are close to each of the different roots.

The value of $f(x)$ for any x can be calculated by entering the value for x and pressing XEQ F.

If there is a pending expression when the initial guess (x_0) is entered, it is ignored.

The program uses the equations:

Newton's Method: $$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Derivative Approximation: $$f'(x_i) \approx \frac{f(x_i + \delta_i) - f(x_i)}{\delta_i}$$

where $\delta_i = x_{i-1} - x_i$, $\delta_0 = 10^{-5}x_0$ if $x_0 \neq 0$ and $\delta_0 = 10^{-5}$ if $x_0 = 0$.

Convergence Criteria: $\left| \dfrac{x_{i+1} - x_i}{x_i} \right| < \Delta x$ *limit* or $|x_{i+1} - x_i| = 0$, and $|x_i| \neq 0$ and $|f(x_i)| < \epsilon$, within *count* iterations

---

Numerical Integration (int)

This program calculates an approximate integral for $f(x)$ using Simpson's rule. You must define the function $f(x)$ by entering the program lines that are required to calculate $f(x)$, assuming x is in the display. You must also supply the number of intervals, n, for the integral. The larger the number of intervals, the more accurate the answer is, but the more slowly the program will calculate an answer.

The main program is 58 lines long and uses registers R_5 through R_9 and labels A, F, 7, 8, and 9. The remaining program lines, registers, and labels can be used for defining $f(x)$.

Program Instructions:

1. Press ⬅ PRGM , then ⬅ LOAD B to load the program. Then press ⬅ ▲ once to move to the last line of the program.

2. After the LBL F on line 58 (**58- 61 41 F**), enter the keystrokes to calculate the value of $f(x)$ using a value of x in the display. See the example below.

3. Press ⬅ PRGM .

4. Enter the lower limit of integration (x_0) and press STO 5 to save it in R_5.

5. Enter the upper limit of integration (x_n) and press STO 6 to save it in R_6.

6. To calculate the integral, enter the number of integration intervals and press XEQ A . This number must be an even, positive integer.

7. To enter a new function, repeat the instructions starting at step 1.

Example: Calculate the integral of $f(x) = x^6 - x - 1$ from $x_0 = 0$ to $x_n = 3$ using 8 integration intervals.

Keys:	Display:	Description:
⬅ PRGM ⬅ LOAD B	int 00-	Loads program.
⬅ ▲	58- 61 41 F	Displays label for beginning of $f(x)$ routine.
STO 0	59- 21 0	Starts $f(x)$ routine; saves x.
y^x	60- 14	
6	61- 6	
−	62- 65	x^6.

[RCL] 0	63- 22 0	x.
[−]	64- 65	$x^6 - x$.
1	65- 1	
[=]	66- 74	$x^6 - x - 1$.
[◀] [SHOW]	b62E	Checksum (page 66).
[◀] [PRGM]		Exits Program mode.
0 [STO] 5	0.0000	Saves x_0 (lower limit).
3 [STO] 6	3.0000	Saves x_n (upper limit).
8 [XEQ] A	305.2806	Enters number of integration intervals and calculates integral.

Remarks:

The Simpson's rule integral can only be calculated if the number of integration intervals is an even, positive integer. The message **Error - Func** appears immediately after starting the program if an odd, negative, or noninteger number of integration intervals is used.

After calculating the integral, the lower and upper limits of integration (x_0 and x_n) are still in R_5 and R_6. The integral can be calculated with a different number of integration intervals by entering the new number of intervals and pressing [XEQ] A, without reentering the integration limits.

The function $f(x)$ for any x can be calculated by entering the value for x and pressing [XEQ] F.

This program uses the following equations:

Simpson's Rule: $\int_{x_0}^{x_n} f(x)\, dx \approx \dfrac{h}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + \ldots$
$$+ 4f(x_{n-3}) + 2f(x_{n-2}) + 4f(x_{n-1})$$
$$+ f(x_n)]$$

where $h = \dfrac{x_n - x_0}{n}$, and n is an even, positive integer.

Complex Operations (CPL)

This program permits chained calculations involving complex numbers in rectangular form. Five complex operators are provided (add, subtract, multiply, divide, and power), as well as two commonly used functions (reciprocal and magnitude). Functions and operators may be mixed in the course of a calculation to allow evaluation of certain expressions such as $z_1/(z_2 + z_3)$ and $(z_1 + z_2)/z_3$, where z_1, z_2, and z_3 are complex numbers in rectangular form $z = a + bi$.

This program uses R_0 through R_4.

Program Instructions:

1. Press ⬅ PRGM to enter Program mode, ⬅ LOAD C to load the program, and ⬅ PRGM to exit Program mode.

2. Enter the keystrokes for the desired complex operation. The imaginary part of the result is displayed. For each operation except magnitude, press ⬅ SWAP to see the real part.

Operation	Keystrokes
Addition $(a_1 + b_1 i) + (a_2 + b_2 i)$	a_1 INPUT b_1 XEQ A a_2 INPUT b_2 R/S
Subtraction $(a_1 + b_1 i) - (a_2 + b_2 i)$	a_1 INPUT b_1 XEQ B a_2 INPUT b_2 R/S
Multiplication $(a_1 + b_1 i) \times (a_2 + b_2 i)$	a_1 INPUT b_1 XEQ C a_2 INPUT b_2 R/S
Division $(a_1 + b_1 i) \div (a_2 + b_2 i)$	a_1 INPUT b_1 XEQ D a_2 INPUT b_2 R/S
Reciprocal $1 \div (a + bi)$	a_1 INPUT b_1 XEQ E
Integer power $(a + bi)^n$	a_1 INPUT b_1 XEQ F n R/S
Magnitude $\sqrt{a^2 + b^2}$	a INPUT b XEQ 9

Example 1: Calculate $(2 + 3i) - (6 + 4i)$:

Keys:	Display:	Description:
🔽 PRGM 🔽 LOAD C	CPL 00-	Loads program.
🔽 PRGM		Exits Program mode.
2 INPUT 3	3_	Enters first complex number.
XEQ B	3.0000	Complex subtract.
6 INPUT 4	4_	Enters second complex number.
R/S	−1.0000	Calculates difference. Displays imaginary part.
🔽 SWAP	−4.0000	Displays real part.

Using the result of the previous example, calculate $((2 + 3i) - (6 + 4i))/(1 - i)$:

🔽 SWAP	−1.0000	Restores original order of result.
XEQ D	−1.0000	Complex divide. It is not necessary to re-enter the result of the previous calculation.
1 INPUT 1 +⁄−	−1_	Enters divisor.
R/S	−2.5000	Calculates quotient. Displays imaginary part.
🔽 SWAP	−1.5000	Displays real part.

Example 2: Calculate the magnitude of $(3 + 6i)^2$.

Keys:	Display:	Description:
3 [INPUT] 6 [XEQ] F	6.0000	Enters complex number.
2 [R/S]	36.0000	Enters power and calculates. Displays imaginary part.
[XEQ] 9	45.0000	Calculates magnitude.

Example 3: Evaluate the expression: $\dfrac{z_1}{z_2 + z_3}$

where $z_1 = 23 + 13i$, $z_2 = -2 + i$, and $z_3 = 4 - 3i$. Since the program does not allow for parentheses, perform the calculation as $z_1 \times [1 / (z_2 + z_3)]$.

Keys:	Display:	Description:
2 [+/-] [INPUT] 1 [XEQ] A	1.0000	Enters z_2, complex add.
4 [INPUT] 3 [+/-] [R/S]	-2.0000	Enters z_3; calculates $z_2 + z_3$.
[XEQ] E	0.2500	Calculates $1/(z_2 + z_3)$.
[XEQ] C	0.2500	Complex multiply.
23 [INPUT] 13 [R/S]	9.0000	Displays imaginary part of $z_1/(z_2 + z_3)$.
[↰] [SWAP]	2.5000	Displays real part of $z_1/(z_2 + z_3)$.

Remarks:

The complex power can only be calculated for an integer exponent. The message **Error - Func** appears if a noninteger exponent is used. The same error message will appear if the magnitude of the complex number in the denominator is zero when dividing or taking the reciprocal.

If there is a pending expression when complex numbers are entered, it is evaluated before the numbers are used for complex operations.

This program uses the following equations:

Addition: $\quad\quad\quad\quad\quad z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)i$

Subtraction: $\quad\quad\quad\quad z_1 - z_2 = (a_1 - a_2) + (b_1 - b_2)i$

Multiplication: $\quad\quad\quad\quad\quad z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$

Division: $\quad\quad\quad\quad\quad z_1 / z_2 = \dfrac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$

Power: $\quad\quad\quad z^n = r^n e^{in\theta}$

Reciprocal: $\quad\quad\quad 1/z = \dfrac{a}{r^2} - \dfrac{b}{r^2}i$

Magnitude: $\quad\quad\quad |z| = \sqrt{a^2 + b^2}$

---

3 × 3 Matrix Operations (3 bY 3)

This program uses Cramer's rule (the method of determinants) to solve systems of linear equations with three unknowns:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

The program also calculates the determinant of the system and can be used to calculate each element of the inverse.

The program uses R_0 through R_9.

Program Instructions:

1. Press ⬦ [PRGM] to enter Program mode, ⬦ [LOAD] D to load the program, and ⬦ [PRGM] to exit Program mode.

2. Use the diagram as a typing aid to store the coefficients of the equations in R_1 through R_9.

R_7 a_{11}	R_8 a_{12}	R_9 a_{13}
R_4 a_{21}	R_5 a_{22}	R_6 a_{23}
R_1 a_{31}	R_2 a_{32}	R_3 a_{33}

3. To solve the system of equations, enter b_1 and press [STO] 0. Enter b_2 and press [INPUT], then enter b_3 and press [XEQ] A. x_1 is displayed. Press [R/S] to see x_2, then press [R/S] to see x_3. The **:** annunciator appears in the display when x_1, x_2, or x_3 is displayed. It should be ignored—it does not imply that there is a second result available.

4. To calculate the determinant, press [XEQ] D. You can do this anytime after step 2.

5. To calculate the first column of the inverse, calculate the system solution using the first column of the identity matrix (1 [STO] 0, 0 [INPUT] 0, [XEQ] A). a_{11}' is displayed. Press [R/S] to see a_{21}', then [R/S] to see a_{31}'.

   To calculate the second column of the inverse, calculate the system solution using the second column of the identity matrix (0 [STO] 0, 1 [INPUT] 0, [XEQ] A). a_{12}' is displayed. Press [R/S] to see a_{22}', then [R/S] to see a_{32}'.

To calculate the third column of the inverse, calculate the system solution using the third column of the identity matrix (0 [STO] 0, 0 [INPUT] 1, [XEQ] A). a_{13}' is displayed. Press [R/S] to see a_{23}', then [R/S] to see a_{33}'.

Example 1: Find the solution to the following set of equations:

$$19x_1 - 4x_2 + 4x_3 = 5$$

$$5x_1 - 12x_2 - 10x_3 = -3$$

$$-15x_1 + 8x_2 + 3x_3 = 4$$

Keys:	Display:	Description:
[↰][PRGM][↰][LOAD]D	**3 bY 3** **00-**	Loads program.
[↰] [PRGM]		Exits Program mode.
19 [STO] 7	**19.0000**	Stores a_{11}.
4 [+/−] [STO] 8	**−4.0000**	Stores a_{12}.
4 [STO] 9	**4.0000**	Stores a_{13}.
5 [STO] 4	**5.0000**	Stores a_{21}.
12 [+/−] [STO] 5	**−12.0000**	Stores a_{22}.
10 [+/−] [STO] 6	**−10.0000**	Stores a_{23}.
15 [+/−] [STO] 1	**−15.0000**	Stores a_{31}.
8 [STO] 2	**8.0000**	Stores a_{32}.
3 [STO] 3	**3.0000**	Stores a_{33}.
5 [STO] 0	**5.0000**	Stores b_1.
3 [+/−] [INPUT]	**−3.0000**	Enters b_2.
4 [XEQ] A	**−1.6667**	Enters b_3 and calculates x_1.

Keys:	Display:	Description:
R/S	−4.4091	Calculates x_2.
R/S	4.7576	Calculates x_3.

Example 2: Find the determinant and inverse of the matrix stored example 1.

Keys:	Display:	Description:
XEQ D	−264.0000	Calculates $det\ A$.
1 STO 0	1.0000	Stores i_{11}.
0 INPUT	0.0000	Enters i_{21}.
0 XEQ A	−0.1667	Enters i_{31} and calculates a_{11}'.
R/S	−0.5114	Calculates a_{21}'.
R/S	0.5303	Calculates a_{31}'.
0 STO 0	0.0000	Stores i_{12}.
1 INPUT	1.0000	Enters i_{22}.
0 XEQ A	−0.1667	Enters i_{32} and calculates a_{12}'.
R/S	−0.4432	Calculates a_{22}'.
R/S	0.3485	Calculates a_{32}'.
0 STO 0	0.0000	Stores i_{13}.
0 INPUT	0.0000	Enters i_{23}.
1 XEQ A	−0.3333	Enters i_{33} and calculates a_{13}'.
R/S	−0.7955	Calculates a_{23}'.
R/S	0.7879	Calculates a_{33}'.

Remarks:

If the determinant equals zero, the system of equations is linearly dependent, and this program cannot be used to find a solution. The message **Error - Func** will appear if you attempt to solve for x_1, x_2, or x_3.

To solve two equations in two unknowns, the last column and row of A should be set to 0 0 1, and the last element of B should be set to 0. The resulting system of three equations and three unknowns can be solved as indicated in the program instructions.

When calculating the system solution, no operations are allowed while the x-values are displayed. If you do any operation other than $\boxed{\text{R/S}}$, you must reenter b_2 and b_3 and restart the system solution according to step 3 of the program instructions (b_2 $\boxed{\text{INPUT}}$ b_3 $\boxed{\text{XEQ}}$ A).

If there is a pending expression for b_3 when starting the system solution ($\boxed{\text{XEQ}}$ A), it is evaluated before the solution is calculated. If there is a pending expression when calculating the determinant, the determinant *will be calculated incorrectly.*

This program uses the following equations:

System of Equations: $\qquad\qquad AX = B$

$$\text{where } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Determinant: $\quad det\ A = a_{21}mn_2 - a_{31}mn_3 + a_{11}mn_1$

where mn_i are the minors $mn_1 = a_{22}a_{33} - a_{32}a_{23}$, $mn_2 = a_{32}a_{13} - a_{12}a_{33}$, $mn_3 = a_{22}a_{13} - a_{12}a_{23}$

System Solution: $x_1 = \dfrac{det_1}{det\ A}$, $x_2 = \dfrac{det_2}{det\ A}$, $x_3 = \dfrac{det_3}{det\ A}$

where det_i is the determinant of A with its ith column replaced by B, and $det\ A \neq 0$.

Inverse and Identity: $A^{-1} = \begin{bmatrix} a_{11}' & a_{12}' & a_{13}' \\ a_{21}' & a_{22}' & a_{23}' \\ a_{31}' & a_{32}' & a_{33}' \end{bmatrix}, I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

where the ith column of the inverse is calculated by solving the system of equations with B replaced by the ith column of I.

Quadratic Equation (qUAd)

This program uses the quadratic formula to solve for the real and complex roots of a second-degree polynomial in the form $ax^2 + bx + c = 0$. If two real roots exist, the program first calculates the root with the larger absolute value, then the root with the smaller absolute value. If only complex roots exist (when $b^2 - 4ac < 0$), the program calculates both the real and imaginary parts of the roots.

The program uses R_0 through R_5.

Program Instructions:

1. Press ◄ PRGM to enter Program mode, ◄ LOAD E to load the program, and ◄ PRGM to exit Program mode.
2. Enter a and press XEQ A.
3. Enter b and press XEQ B.
4. Enter c and press XEQ C.
5. To calculate the roots, press XEQ D.

   ■ If the : annunciator does not come on, the displayed number is the first real root. Press R/S to see the second real root.

   ■ If the : annunciator comes on, the displayed number is the value of the imaginary part of the complex root. Press ◄ SWAP to see the real part of the complex root. The second complex root is the same as the first except for the sign of the imaginary part.

Example 1: A ball is thrown straight up at an initial velocity of 20 meters per second from a height of 2 meters. Ignoring air resistance, when will it reach the ground? The acceleration due to gravity is approximately 9.81 meters per second^2.

According to Newtonian mechanics, this problem may be expressed as the second degree polynomial $f(t) = -\frac{1}{2}(9.81)t^2 + 20t + 2$, where t is time in seconds. When the ball hits the ground, $f(t) = 0$.

Keys:	Display:	Description:
⬅ PRGM ⬅ LOAD E	qUAd 00-	Loads program.
⬅ PRGM		Exits Program mode.
9.81 ÷ 2 +/-	-2_	
XEQ A	-4.9050	Enters a.
20 XEQ B	20.0000	Enters b.
2 XEQ C	2.0000	Enters c.
XEQ D	4.1751	Calculates t_1.
R/S	-0.0977	Calculates t_2.

Since a negative time has no meaning in the context of this problem, the first result, 4.1751 seconds, is the meaningful answer.

Example 2: Find the roots of $3x^2 + 5x + 3 = 0$.

Keys:	Display:	Description:
3 XEQ A	3.0000	Enters a.
5 XEQ B	5.0000	Enters b.
3 XEQ C	3.0000	Enters c.

| $\boxed{\text{XEQ}}$ D | 0.5528 | Calculates x_1. The : an-nunciator indicates that this is the positive-valued imaginary part of the complex root. |
| $\boxed{\text{◀}}$ $\boxed{\text{SWAP}}$ | −0.8333 | Displays real part of complex root. |

Remarks:

This program can be used in conjunction with the root finder program to solve cubic equations. Since a cubic equation always has at least one real root, the root finder program could be used to find the root. Then synthetic division could reduce the cubic equation to a quadratic equation, which could then be solved by this program.

The message **Error - Func** appears if the coefficient of the quadratic term (a) is zero.

If there is a pending expression when the coefficients a, b, and c are entered, it is evaluated before the coefficients are saved.

This program uses the following equations:

Quadratic Formula:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Real Roots: If $-b \geq 0$,
$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

If $-b \leq 0$,
$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$x_2 = \frac{c}{ax_1}$$

Real Part of Complex Root:
$$r = \frac{-b}{2a}$$

Imaginary Part of Complex Root:
$$i = \frac{\pm \sqrt{|b^2 - 4ac|}}{2a}$$

Curve Fitting (Fit)

This program fits x,y-data to one of three curve-fitting models: power, exponential, or logarithmic. The program calculates the correlation coefficient r and the two regression coefficients m and b. It includes routines to calculate x given a known y and y given a known x.

The program uses R_2 through R_9.

Power Curve: $y = bx^m$

Exponential Curve: $y = be^{mx}$

Logarithmic Curve: $y = m \ln x + b$

General Linear Equation: $y = mx + b$
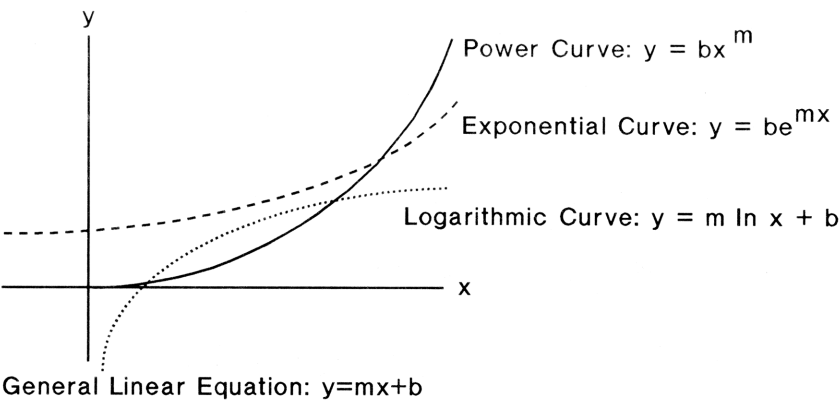
The program uses a transformation of the curve fit equations into a general linear form. The (x,y) data pairs are transformed to this linear form as they are entered into the statistical registers. This allows the program to use the built-in statistical functions described in chapter 5 to calculate the statistical variables.

Curve Fitting Models and Transformations

Model	Equation	Transformed Equation	Transformed Data
Logarithmic	$y = m \ln x + b$	$y = m \ln x + b$	$\ln x, y$ $(x > 0)$
Exponential	$y = be^{mx}$	$\ln y = mx + \ln b$	$x, \ln y$ $(y > 0)$
Power	$y = bx^m$	$\ln y = m \ln x + \ln b$	$\ln x, \ln y$ $(x > 0, y > 0)$

Program Instructions:

1. Press ◄⊣ PRGM to enter Program mode, ◄⊣ LOAD F to load the program, and ◄⊣ PRGM to exit Program mode.

2. Clear the statistical registers by pressing ▸ CLΣ.

3. Select the desired curve fit by pressing XEQ A (power), XEQ B (exponential), or XEQ C (logarithmic).

4. Enter each x,y data pair (x INPUT y) and press R/S. The total number of data pairs is displayed. (If you get an error, reselect the curve fit.)

5. Optional: To delete an x,y data pair, enter x INPUT y, then press XEQ 9. The total number of data pairs is displayed. (If you get an error, reselect the curve fit.)

6. To calculate $\hat{x}$ and r, enter the y-value and press XEQ D. $\hat{x}$ is displayed. Press ◄⊣ SWAP to see r.

7. To calculate $\hat{y}$ and r, enter the x-value and press XEQ E. $\hat{y}$ is displayed. Press ◄⊣ SWAP to see r.

8. To calculate m and b, press XEQ F. m is displayed. Press ◄⊣ SWAP to see b.

Example: Use the data below to calculate m, b, and r for a power curve. Estimate y for an x-value of 37 and x for a y-value of 101.

X	40.5	38.6	37.9	36.2	35.1	34.6
Y	104.5	102	100	97.5	95.5	94

Keys:	Display:	Description:
◄⊣ PRGM ◄⊣ LOAD F	Fit 00-	Loads program.
◄⊣ PRGM		Exits Program mode.
▸ CLΣ	0.0000	Clears statistical registers.
XEQ A	0.0000	Selects power curve fit.
40.5 INPUT	40.5000	Enters x_1.

104.5 $\boxed{\text{R/S}}$	1.0000	Enters y_1.
38.6 $\boxed{\text{INPUT}}$	38.6000	Enters x_2.
102 $\boxed{\text{R/S}}$	2.0000	Enters y_2.
37.9 $\boxed{\text{INPUT}}$	37.9000	Enters x_3.
100 $\boxed{\text{R/S}}$	3.0000	Enters y_3.
36.2 $\boxed{\text{INPUT}}$	36.2000	Enters x_4.
97.5 $\boxed{\text{R/S}}$	4.0000	Enters y_4.
35.1 $\boxed{\text{INPUT}}$	35.1000	Enters x_5.
95.5 $\boxed{\text{R/S}}$	5.0000	Enters y_5.
34.6 $\boxed{\text{INPUT}}$	34.6000	Enters x_6.
94 $\boxed{\text{R/S}}$	6.0000	Enters y_6.
$\boxed{\text{XEQ}}$ F	0.6640	Calculates m.
$\boxed{\blacktriangleleft}$ $\boxed{\text{SWAP}}$	8.9730	Displays b.
37 $\boxed{\text{XEQ}}$ E	98.6845	Calculates $\hat{y}$.
$\boxed{\blacktriangleleft}$ $\boxed{\text{SWAP}}$	0.9959	Displays r.
101 $\boxed{\text{XEQ}}$ D	38.3151	Calculates $\hat{x}$.
$\boxed{\blacktriangleleft}$ $\boxed{\text{SWAP}}$	0.9959	Displays r.

If you want to repeat this example for exponential and logarithmic curves, the table below lists the starting key sequence and results for m, b, r, $\hat{y}$, and $\hat{x}$. After performing the starting key sequence, you must reenter the data before calculating the results.

Item	Exponential	Logarithmic
To start:	⎡►⎤ ⎡CLΣ⎤ ⎡XEQ⎤ B	⎡►⎤ ⎡CLΣ⎤ ⎡XEQ⎤ C
m	0.0177	65.8446
b	51.1312	−139.0088
r	0.9945	0.9965
$\hat{y}$ (x = 37)	98.5870	98.7508
$\hat{x}$ (y = 101)	38.3628	38.2857

Remarks:

The message **Error - Func** is displayed if $x_i \leqslant 0$ for logarithmic curves, if $y_i \leqslant 0$ for exponential curves, or if either x_i or $y_i \leqslant 0$ for power curves. If you get an error (**Error - Func**), reselect the curve fit type by pressing ⎡XEQ⎤ A, B, or C. Using valid data, repeat the operation that caused the error.

Data values of large magnitude but relatively small differences can cause problems in the precision of the calculated results, as can data values of greatly different magnitudes.

If there is a pending expression when the data pairs are input, it is evaluated before the data pairs are used for regression calculations.

Assistance, Batteries, Memory, and Service

Obtaining Help in Operating the Calculator

We at Hewlett-Packard are committed to providing you with ongoing support. You can obtain answers to questions about using the calculator from our Calculator Support department.

Please read "Answers to Common Questions" before contacting us. Our experience has shown that many of our customers have similar questions about our products. If you don't find an answer to your question, you can contact us using the address or phone number listed on the inside back cover.

Answers to Common Questions

Q. I'm not sure if the calculator is malfunctioning or if I'm doing something incorrectly. How can I determine if the calculator is operating properly?

A. Refer to page 116, which describes the diagnostic self-test.

Q. My numbers contain commas instead of periods as decimal points. How do I restore the periods?

A. Press ⬅ ⎡·/,⎤ (page 19).

Q. How do I change the number of decimal places the HP-20S displays?

A. The procedure is described in "Display Format of Numbers" on page 16.

Q. How do I clear all or portions of memory?

A. See page 12 to clear portions of memory. To clear all memory, press and hold down [C], then press and hold down both [√x̄] and [Σ+]. When you release them, all memory is cleared.

Q. What does an "E" in a number (for example, 2.51E−13) mean?

A. Exponent of ten (for example, 2.51×10^{-13}). Refer to "Scientific and Engineering Notation" on page 18.

Q. Why does calculating the sine of π radians display a very small number instead of zero?

A. The calculator is *not* malfunctioning. π cannot be expressed *exactly* with the 12-digit precision of the calculator.

Q. Why do I get incorrect answers when I use the trigonometric functions?

A. You must make sure you are in the correct trigonometric mode (page 34).

Q. What does PEND in the display mean?

A. An arithmetic operation is pending (in progress).

Q. What does : in the display mean?

A. The [INPUT] key has been pressed, or two values have been returned (page 14).

Power and Batteries

The HP-20S is powered by 3 button-cell batteries. Expected battery life depends on how the calculator is used and the chemical content of the batteries.

Use only fresh button-cell batteries. Do not use rechargeable batteries.

Low Power Annunciator (▇▭)

When the low battery annunciator (▇▭) comes on, you should replace the batteries as soon as possible.

If you continue to use the calculator after the battery annunciator comes on, power can eventually drop to a level at which the display becomes dim, and stored data may be affected. If this happens, the calculator requires fresh batteries before it will operate properly. If stored data has not been preserved due to extremely low power, the HP-20S displays **ALL CLr**.

Installing Batteries

Once the batteries are out, you must replace the batteries within one minute to prevent loss of Continuous Memory.

To install batteries:

1. Have three fresh button-cell batteries at hand. Hold batteries by the edges. Do not touch the contacts. Wipe each battery with a clean, lint-free cloth to remove dirt and oil.

2. Make sure the calculator is *off*. **Do not press** C **again until the entire procedure for changing batteries is completed. Changing batteries with the calculator on can erase the contents of Continuous Memory.**

3. Hold the calculator as shown. To remove the battery-compartment door, press down and outward on it until it slides off (away from the center).
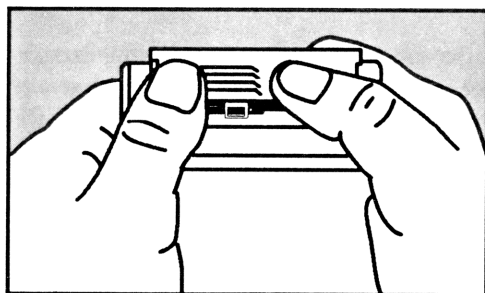
4. Turn the calculator over and shake the batteries out.

Warning Do not mutilate, puncture, or dispose of batteries in fire. The batteries can burst or explode, releasing hazardous chemicals.

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5. Hold the calculator as shown and stack the batteries, one at a time, in the battery compartment. Orient the batteries according to the diagram inside the battery compartment. Be sure the raised and flat ends match the diagram.
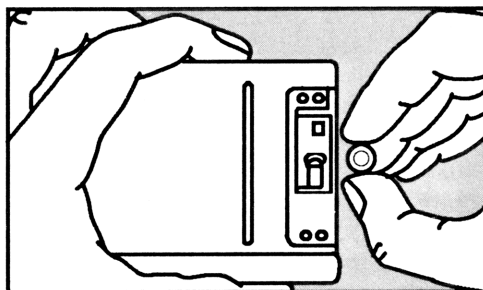
6. Slide the tab of the battery-compartment door into the slot in the calculator case.

Now turn the calculator back on. If it does not function, check that the orientation of the batteries is correct. If the calculator still does not function, you might have taken too long to change the batteries or inadvertently turned the calculator on while the batteries were out. *Remove the batteries* again and lightly press a coin against both battery contacts in the calculator *for a few seconds.* Put the batteries back in and turn the calculator on. It should display **ALL CLr**.

Resetting the Calculator

If the calculator doesn't respond to keystrokes or if it is otherwise behaving unusually, you should attempt to reset it. Resetting the calculator halts the current calculation and clears the display. Stored data remains intact.

To reset the calculator, hold down the C key and press LN at the same time. It may be necessary to repeat the reset keystrokes several times. If you are unable to reset the calculator, try installing fresh batteries. If the calculator still fails to operate properly, you should attempt to clear all of memory using the procedure described in the next section.

Erasing Continuous Memory

If the calculator fails to respond to keystrokes and you are unable to restore operation by following the reset instructions, clearing memory may restore calculator operation. Press and hold down $\boxed{C}$, then press and hold down both $\boxed{\sqrt{x}}$ and $\boxed{\Sigma +}$. When you release them, all memory is cleared. The **ALL CLr** message is displayed.

Memory can inadvertently be cleared if the calculator is dropped or if power is otherwise interrupted.

Environmental Limits

To maintain product reliability, you should avoid getting the calculator wet and observe the following temperature and humidity limits:

- Operating temperature: 0° to 45°C (32° to 113°F).
- Storage temperature: −20° to 65°C (−4° to 149°F).
- Operating and storage humidity: 90% relative humidity at 40°C (104°F) maximum.

Determining if the Calculator Requires Service

Use these guidelines to determine if the calculator requires service. If these procedures confirm that the calculator is not functioning properly, read the section "If the Calculator Requires Service" on page 118.

■ **If the calculator won't turn on (nothing is visible in the display):**

1. Attempt to reset the calculator (page 113).

2. Attempt to erase Continuous Memory (page 114).

3. If the calculator fails to respond after step 1 or 2, replace the batteries (page 111).

4. If the calculator fails to respond after step 3, remove the batteries (page 111) and lightly press a coin against both calculator battery contacts. Put the batteries back in and turn on the calculator. It should display **ALL CLr**.

   If steps 1 through 4 fail to restore calculator operation, it requires service.

■ **If the calculator doesn't respond to keystrokes (nothing happens when you press any of the keys):**

1. Attempt to reset the calculator (page 113).

2. If the calculator fails to respond after step 1, attempt to erase Continuous Memory (page 114). This will erase all the information you've stored.

3. If the calculator fails to respond after steps 1 and 2, remove the batteries (page 111) and lightly press a coin against both calculator battery contacts. Put the batteries back in and turn on the calculator. It should display **ALL CLr**.

   If steps 1 through 3 fail to restore calculator function, the calculator requires service.

■ **If the calculator responds to keystrokes but you suspect that it is malfunctioning:**

1. Do the self-test (described below). If the calculator fails the self test, it requires service.

2. If the calculator passes the self-test, it is quite likely that you've made a mistake in operating the calculator. Try rereading portions of the manual, and check "Answers to Common Questions" on page 109.

3. Contact the Calculator Support department. The address and phone number are listed on the inside back cover.

Confirming Calculator Operation—
the Self-Test

If the display can be turned on, but it appears that the calculator is not operating properly, you can do a diagnostic self-test. To run the self-test:

1. First, hold down the $\boxed{\text{C}}$ key, then press and hold $\boxed{y^x}$. (A *continuous* self-test can be performed by holding down $\boxed{\text{C}}$ as you press $\boxed{1/x}$. This test displays various patterns and the copyright message, then automatically repeats. The test continues until you halt it by pressing $\boxed{\text{C}}$.)

2. Press any key four times, and watch the display as various patterns are displayed. After pressing the key four times, the calculator displays the copyright message **COPr. HP 1987** momentarily, and then the message **01**. This indicates the calculator is ready for the key test.

3. Starting at the upper left corner ($\boxed{\sqrt{x}}$) and moving from left to right, press each key in the top row. Then, moving left to right, press each key in the second row, third row, etc., until you've pressed each key.

   ■ If you press the keys in the proper order, and they are functioning properly, the calculator displays two-digit numbers. (The calculator is counting the keys using hexadecimal base.)

   ■ If you press a key out of order, or if a key isn't functioning properly, the next keystroke displays **20 - FAIL**, followed by a one-digit number. If you received the message because you pressed a key out of order, you should reset the calculator (hold down $\boxed{\text{C}}$ and press $\boxed{\text{LN}}$) and start the self-test over. If you pressed the keys in order, but got this message, the calculator requires service.

4. When the keyboard test has been completed, the calculator displays a message:

   ■ The calculator displays **20 - Good** if it passed the self-test.

   ■ The calculator displays **20 - FAIL**, followed by a one-digit hexadecimal number 1 through F, if it failed the self-test. If the calculator failed the self-test, it requires service (page 118). Include a copy of the fail message with the calculator when you ship it for service.

5. To exit the self-test, reset the calculator (hold down $\boxed{\text{C}}$ and press $\boxed{\text{LN}}$).

6. If the calculator failed the self-test, rerun the test to verify the results.

Limited One-Year Warranty

What Is Covered

The calculator (except for the batteries, or damage caused by the batteries) *is warranted by Hewlett-Packard against defects in materials and workmanship for one year from the date of original purchase.* If you sell your unit or give it as a gift, the warranty is automatically transferred to the new owner and remains in effect for the original one-year period. During the warranty period, we will repair or, at our option, replace at no charge a product that proves to be defective, provided you return the product, shipping prepaid, to a Hewlett-Packard service center. (Replacement may be with a newer model of equivalent or better functionality.)

This warranty gives you specific legal rights, and you may also have other rights that vary from state to state, province to province, or country to country.

What Is Not Covered

Batteries, and damage caused by the batteries, are not covered by the Hewlett-Packard warranty. Check with the battery manufacturer about battery and battery leakage warranties.

This warranty does not apply if the product has been damaged by accident or misuse or as the result of service or modification by other than an authorized Hewlett-Packard service center.

No other express warranty is given. The repair or replacement of a product is your exclusive remedy. **ANY OTHER IMPLIED WARRANTY OF MERCHANTABILITY OR FITNESS IS LIMITED TO THE ONE-YEAR DURATION OF THIS WRITTEN WARRANTY.** Some states, provinces, or countries do not allow limitations on how long an implied warranty lasts, so the above limitation may not apply to you. **IN NO EVENT SHALL HEWLETT-PACKARD COMPANY BE LIABLE FOR CONSEQUENTIAL DAMAGES.** Some states, provinces, or countries do not allow the exclusion or limitation of incidental or consequential damages, so the above limitation or exclusion may not apply to you.

Products are sold on the basis of specifications applicable at the time of manufacture. Hewlett-Packard shall have no obligation to modify or update products once sold.

Consumer Transactions in the United Kingdom

This warranty shall not apply to consumer transactions and shall not affect the statutory rights of a consumer. In relation to such transactions, the rights and obligations of Seller and Buyer shall be determined by statute.

If the Calculator Requires Service

Hewlett-Packard maintains service centers in many countries. These centers will repair a calculator, or replace it with the same model or one of equal or greater value, whether it is under warranty or not. There is a service charge for service after the warranty period. Calculators normally are serviced and reshipped within five working days.

Obtaining Service

- **In the United States:** Send the calculator to the Calculator Service Center listed on the inside of the back cover.

- **In Europe:** Contact your Hewlett-Packard sales office or dealer, or Hewlett-Packard's European headquarters for the location of the nearest service center. *Do not ship the calculator for service without first contacting a Hewlett-Packard office.*

Hewlett-Packard S.A.
150, Route du Nant-d'Avril
P.O. Box CH 1217 Meyrin 2
Geneva, Switzerland
Telephone: (022) 780 81 11

- **In other countries:** Contact your Hewlett-Packard sales office or dealer or write to the Corvallis Service Center (listed on the inside of the back cover) for the location of other service centers. If local service is unavailable, you can ship the calculator to the Corvallis Service Center for repair.

All shipping, reimportation arrangements, and customs costs are your responsibility.

Service Charge

There is a standard repair charge for out-of-warranty service. The Corvallis Service Center (listed on the inside of the back cover) can tell you how much this charge is. The full charge is subject to the customer's local sales or value-added tax wherever applicable.

Calculator products damaged by accident or misuse are not covered by the fixed service charges. In these cases, charges are individually determined based on time and material.

Shipping Instructions

If your calculator requires service, ship it to the nearest authorized service center or collection point.

- Include your return address and description of the problem.
- Include proof of purchase date if the warranty has not expired.
- Include a purchase order, check, or credit card number plus expiration date (VISA or MasterCard) to cover the standard repair charge.

- Ship the calculator in adequate protective packaging to prevent damage. Such damage is not covered by the warranty, so we recommend that you insure the shipment.

- Pay the shipping charges for delivery to the Calculator Service Center, whether or not the calculator is under warranty.

Warranty on Service

Service is warranted against defects in materials and workmanship for 90 days from the date of service.

Service Agreements

In the U.S., a support agreement is available for repair and service. Refer to the form in the front of the manual. For additional information, contact the Calculator Service Center (see the inside of the back cover).

Regulatory Information

U.S.A. The HP-20S generates and uses radio frequency energy and may interfere with radio and television reception. The calculator complies with the limits for a Class B computing device as specified in Subpart J of Part 15 of FCC Rules, which provide reasonable protection against such interference in a residential installation. In the unlikely event that there is interference to radio or television reception (which can be determined by turning the HP-20S off and on or by removing the batteries), try:

- Reorienting the receiving antenna.

- Relocating the calculator with respect to the receiver.

For more information, consult your dealer, an experienced radio/television technician, or the following booklet, prepared by the Federal Communications Commission: *How to Identify and Resolve Radio-TV Interference Problems*. This booklet is available from the U.S. Government Printing Office, Washington, D.C. 20402, Stock Number 004-000-00345-4. At the first printing of this manual, the telephone number was (202) 783-3238.

West Germany. The HP-20S complies with VFG 1046/84, VDE 0871B, and similar noninterference standards. If you use equipment that is not authorized by Hewlett-Packard, that system configuration has to comply with the requirements of Paragraph 2 of the German Federal Gazette, Order (VFG) 1046/84, dated December 14, 1984.

Messages

Press $\boxed{C}$ or $\boxed{\blacktriangleleft}$ to clear a message from the display.

ALL CLr (All Clear). Continuous memory has been erased (page 114).

COPr. HP 1987 (Copyright HP 1987). Copyright is displayed during self-test.

CPL (Complex Operations). Built-in program (page 94).

Error - Func (Error - Function).

- Attempt to divide by zero.
- Attempt to calculate combinations or permutations with $n < r$, n or r not positive integer or $\geq 10^{12}$.
- Attempt to use a trigonometric or hyperbolic function with an illegal argument.
- Attempt to calculate the logarithm of zero or a negative number.
- Attempt to calculate 0^0 or 0 raised to a negative power.
- Attempt to raise a negative number to a noninteger power.
- Attempt to calculate the square root of a negative number.

Error - FuLL (Error - Full). Attempt to calculate an expression with more than five pending operations (page 24), or attempt to enter more than 99 program lines.

Error - LbL (Error - Label). Attempt to $\boxed{\text{XEQ}}$ or $\boxed{\text{GTO}}$ a label that is not in the program.

Error - StAt (Error - Statistics).

- Attempt to calculate $\bar{x}_w$, $\hat{x}$, $\hat{y}$, or r with x-data only (all y-values equal to zero).
- Attempt to calculate $\hat{x}$, $\hat{y}$, r, m, or b with all x-values equal.
- Attempt to calculate with n equal to zero.
- Attempt to calculate S_x, S_y, $\hat{x}$, $\hat{y}$, r, m, or b, with n $\leqslant$ 1, or when a division by zero or square root of a negative number occurred. Also, attempt to calculate $\bar{x}$, $\bar{y}$ with n = 0, or $\hat{x}_w$ with $\Sigma y = 0$.

Error - Sub (Error - Subroutine). Subroutines nested more than four levels deep (page 80).

Fit (Curve Fitting). Built-in program (page 105).

int (Numerical Integration). Built-in program (page 91).

OFLO (Overflow). The magnitude of a result is too large for the calculator to handle. **OFLO** is displayed for a moment (or remains in the display when **OFLO** occurs in a running program), then the HP-20S returns $\pm 9.99999999999E499$ in the current display format. If **OFLO** is caused by storage register arithmetic, the display value remains unchanged. If **OFLO** is caused by $\Sigma +$, n appears in the display. (Refer to "Range of Numbers" on page 20).

root (Root Finder). Built-in program (page 89).

running (Running). A program or a long calculation is running.

too big (Too Big). The magnitude of the number is too large to be converted to hexadecimal, octal, or binary base. The number must be in the range $-34,359,738,368 \leqslant n \leqslant 34,359,738,367$ (page 48).

qUAd (Quadratic Equation). Built-in program (page 102).

3 bY 3 (3 × 3 Matrix Operations). Built-in program (page 97).

20 - FAIL n (HP-20S Fail). The self-test failed; n is the fail code (page 116).

20 - Good (HP-20S Good). The self-test is complete (page 116).

Index

Bold type indicates the main page reference if a topic is discussed in more than one place.

Special Characters

⤴, 14
⤵, 14
▲, 69
▼, 69
·/,, **19**, 109
√x̄, 31
π, **34**, 110
%, 32
%CHG, 33
(, 24
), 24
+/−, 11
¹/x, 15
10ˣ, 31
Σ+, 51
Σ−, 51
◀, **9**, 68, 71
=, 10
:, **13**, 110
n, **51**, 53, 54
Σx, **51**, 53, 54
Σx², **51**, 53, 54
Σy, **51**, 53, 54
Σy², **51**, 53, 54
Σxy, **51**, 53, 54
θ, 38
12-digit representation, 45
ABS, 41
2's complement, 47, **49**
3 × 3 matrix operations, 97

A

Absolute value, 41
ACOS, 35, **41**
ALL, 17
ALL CLr, 114
Alpha characters, 15
Angle conversions, 36
Angles, 35
Annunciators, **13**, 67
Answers to questions, 109-110
Arc sine, 35
  cosine, 35
  tangent, 35
Arithmetic operators, 10
ASIN, 35
ATAN, 35
Auto-off, 9

B

Back step, **69**, 72
Base arithmetic, 44
  conversions, 44
  illegal keys, 45
  2's complement, 47
▭, 9
Batteries, 9, **111**
BIN, 13, **44**
BIN, 44
Binary, 44
  windows, 45

N

[n!], 39
Negative numbers, 11
Nested subroutines, 80
Newton's method, 89
Nonprogrammable functions, 87
Number base modes, 44
Number of lines, 64, **87**
Number order, 26
Numerical integration, 91
  equations, 93

O

OCT, 13, 44
[OCT], 44
Octal, 44
Off, 9
[OFF], 9
On, 9
[ON], 9
One-number functions, 15
One-variable statistics, 51
Operator keys, 10
Operator priority, 22
Operators, arithmetic, 10
Overflow, 20, **123**

P

[→P], 38
Parentheses, 24
PEND, 13
PEND, 13, 110
Pending operations, 24, 110
Percent change, 30, **33**
Period, **16**, 19, 109
Permutations, **39**, 40
Pi, 34
[Pn,r], 39-40
Polar to rectangular, 38
Pounds, 42
Power on and off, 9
Precedence, 22
Priority, 22
PRGM, 13
[PRGM], 67

Probability formulas, 40
Programming, 61
  accessing, **67**, 72
  built-in, 88
  error stops, 71
  labels, 66
  lines, number of, 67
  memory, 87
  mode, 61, 67
  number of lines, 87
  original, 61
  pointer, 66
  pointer, positioning, 68
  quick example, 61
  step-through, 72
  stopping, 70
  testing, 72
  viewing, 68
Pythagorean theorem, 73

Q

Quadratic equation, 102
  equations, 104

R

[→R], 38
[R/S], 70,
[→RAD], 36
RAD, 13
[RAD], 35
Radians, 35, 36
Random number generator, 75
Range of numbers, 20
  in different bases, 48
[RCL], **27**, 73
Real parts of roots, 102
Reciprocal, 31
Rectangular to polar, 38
Registers, **27**, 51-52, 73
Reset, **13**, 113
Return, **66**, 68
  end of program, 66
  end of subroutine, 76
[RND], 41
Root finder, 89
  equations, 91

Contacting Hewlett-Packard

For Information About Using the Calculator. If you have questions about how to use the calculator, first check the table of contents, the index, and "Answers to Common Questions" in the Appendix. If you can't find an answer in the manual, you can contact the Calculator Support department:

Hewlett-Packard
Calculator Support
1000 N.E. Circle Blvd.
Corvallis, OR 97330, U.S.A.

(503) 757-2004
8:00 a.m. to 3:00 p.m. Pacific time
Monday through Friday

For Service. If your calculator doesn't seem to work properly, refer to appendix A for diagnostic instructions and information on obtaining service. If you are in the United States and your calculator requires service, mail it to the Corvallis Service Center:

Hewlett-Packard
Corvallis Service Center
1030 N.E. Circle Blvd.
Corvallis, OR 97330, U.S.A.
(503) 757-2002

If you are outside the United States, refer to appendix A for information on locating the nearest service center.

Contents