HEWLETT-PACKARD

HP-21 Owner's Handbook

"The success and prosperity of our company will be assured only if we offer our customers superior products that fill real needs and provide lasting value, and that are supported by a wide variety of useful services, both before and after sale."

Statement of Corporate Objectives.
Hewlett-Packard Company

When Messrs. Hewlett and Packard founded our company in 1939, we offered one superior product, an audio oscillator. Today, we offer more than 3,000 quality products, designed and built for some of the world's most discerning customers.

Since we introduced our first pocket calculator in 1972, we've sold over 500,000 worldwide. Their owners include Nobel laureates, astronauts, mountain climbers, businessmen, doctors, students, and housewives.

Each of our pocket calculators is precision crafted and designed to solve the problems its owner can expect to encounter throughout a working lifetime.

HP calculators fill real needs. And they provide lasting value.

HEWLETT hp PACKARD

HP-21
Owner's Handbook

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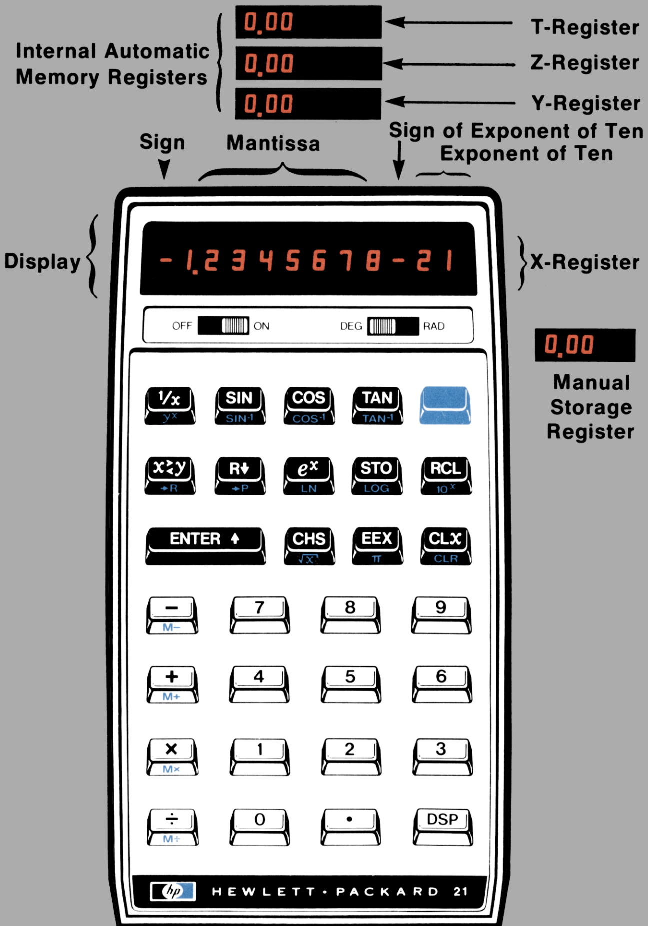

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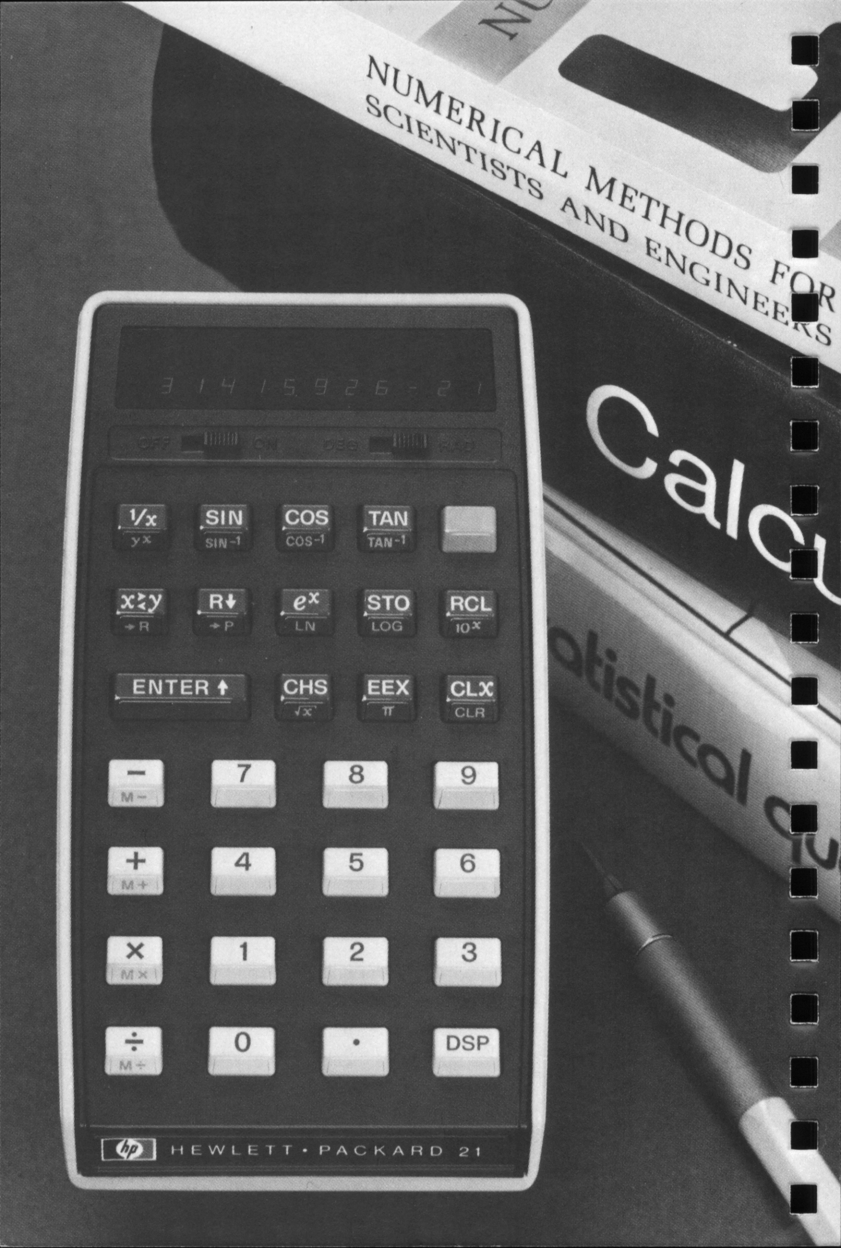
NUMERICAL METHODS FOR
SCIENTISTS AND ENGINEERS
1/x
y^x
SIN
SIN^{-1}
COS
COS^{-1}
TAN
TAN^{-1}
x⇄y
→R
R↓
→P
e^x
LN
STO
LOG
RCL
10^x
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$\sqrt{x}$
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1
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0
•
DSP
HEWLETT · PACKARD 21

Getting Started

Introduction

Congratulations!

Your HP-21 is another professional-quality pocket instrument from the Hewlett-Packard line of calculators—calculators whose durability, small size, and ease of operation have made them the choice for use by climbers on the frigid crags of Mt. Everest and astronauts orbiting the vast depths of space, as well as doctors, engineers, scientists, and other people who require instant, accurate answers to complex and highly technical problems. You're in good company with HP!

This handbook has been designed to help you get the most from your HP-21 and in its pages you'll find a reference guide to every basic operation your calculator can perform.

But HP hopes that you'll use this book as more than a reference. The secret to getting the most from your HP-21 lies in the amount of *confidence* you have in your calculator. And you'll build this confidence and learn the calculator's functions most quickly by sitting down with your HP-21 and working through this handbook page by page.

Even if you've used other pocket calculators, you'll want to take a good, hard look at this handbook. Your HP-21 has unique features which make complex problem solving easy. When you see the simple power of the HP system, you'll become an apostle just as have some half-million HP owners before you.

So get to know your HP-21. It's easy!

Power On

Your HP-21 is shipped fully assembled, including a battery. You can run the calculator on battery power alone or you can connect the battery charger and use the calculator while the battery is charging. If you want to use the calculator on battery power only, charge the battery for 6 hours first. Whether you operate from batteries or from the charger, *the batteries must be in the calculator.*

To begin, slide the On-Off switch to *ON*. Press a few keys at random and watch the display lights change. You can't hurt the HP-21.

CLX Key

Now you're ready to work through a few simple problems. First, clear any numbers now in the display by pressing the CLX (*clear x*) key. This returns the display to 0.00 .

Your HP-21 has a keyboard and a display. The display is used to show you numbers. The keyboard:

- is used to key numbers into the calculator.
- is used to tell the calculator to perform operations upon those numbers.

Keying In Numbers

Key in a number by pressing the number keys in sequence, just as though you were writing on a piece of paper. The decimal point must be keyed in if it is part of the number. For example:

Key in 148.84
by pressing the keys
1 4 8 • 8 4

The resultant number 148.84 is seen in the display.

If you make a mistake when keying in a number, clear the entire number string by pressing CLX. Then key in the correct number.

Keying In Negative Numbers

To enter a negative number, key in the number, then press CHS (*change sign*). The number, preceded by a minus (–) sign, will appear on the display. For example:

Press	**Display**
CHS	–148.84

To change the sign of a negative or positive number on the display, press CHS . For example, to change the sign of the –148.84 now in the display:

Press	**Display**
CHS	148.84

Operations

Most function keys control two functions. One of the functions is written on the top of the key, while the other is written in blue on the slanted face of the key.

- To select the function written on the top face of the key, press the key.
- To select the function written in blue on the slanted face of the key:
    1. First press the blue shift key once.
    2. Then press the function key.

In spite of the dozens of functions available on the HP-21 keyboard, you will find the calculator simple to operate using a single all-encompassing rule: *When you press a function key, the calculator immediately executes the function written on that key.*

> Pressing a function key tells the calculator to immediately perform that function.

For example, to calculate the square root of the 148.84 now in the calculator, merely:

Press	Display	
[blue shift] [√x]	12.20	Square root of 148.84.

[√x] is a one-number function. All function keys operate upon either one number or two numbers at a time. *No function key operates upon more than two numbers at once.*

> Function keys always work with either one number or two numbers.

One-Number Functions

To use one-number functions:

1. Key in the number.
2. Press the function key (or press the blue shift key, then the function key).

For example, to use the one-number function **1/x** key, you key in the x-number and press the function. To calculate 1/4, key in 4 (the x-number) and press the **1/x** key.

Example: Calculate 1/4:

Press	Display
4	4.
1/x	0.25

Now try these other one-number function problems. Remember, first key in the number, then press the function:

$\frac{1}{25}$	=	0.04	
$\sqrt{2500}$	=	50.00	
10^5	=	100000.00	(Use the 10^x key.)
$\sqrt{3204100}$	=	1790.00	
$\log 12.58925411$	=	1.10	

Two-Number Functions

Two-number functions are functions that must have two numbers present in order for the operation to be performed. For example, you cannot add, subtract, multiply, or divide unless there are two numbers present in the calculator, so [+] [−] [×] and [÷] are examples of two-number function keys.

Two-number functions work the same way as one-number functions—that is, the operation occurs when the function key is pressed. Therefore, *both numbers must be in the calculator before the function key is pressed.* To place two numbers into the calculator and perform an operation:

1. Key in the first number.
2. Press **ENTER↑** to separate the first number from the second.
3. Key in the second number.
4. Press the function key to perform the operation.

For example, you add 12 and 3 by pressing:

12	The first number.
ENTER↑	Separates the first number from the second.
3	The second number.
+	The function.

The answer, 15.00, is displayed.

Other arithmetic functions are performed the same way:

To perform	Press	Display
12 − 3	12 ENTER↑ 3 −	9.00
12 × 3	12 ENTER↑ 3 ×	36.00
12 ÷ 3	12 ENTER↑ 3 ÷	4.00

The y^x key is also a two-number operation. It is used to raise numbers to powers, and you can use it in the same simple way that you use every other two-number function key:

1. Key in the first number.
2. Press ENTER↑ to separate the first from the second.
3. Key in the second number (power).
4. Perform the operation (press ■ y^x).

When working with any function key (including y^x), you should remember that the *displayed* number is always designated by *x* on the function keys.

The number displayed is always x.

Thus, to calculate 3^6:

Press	Display	
3	3.	
ENTER↑	3.00	
6	6.	6 is now designated by x.
■ y^x	729.00	The answer.

Now try the following problems using the $\boxed{y^x}$ key, keeping in mind the simple rules for two-number functions:

16^4	(16 to the 4th power) =	65536.00	
81^2	(81 squared) =	6561.00	
$225^{.5}$	(square root of 225) =	15.00	($\boxed{\sqrt{x}}$ also performs this as a one-number function.)
2^{16}	(2 to the 16th power) =	65536.00	

Chain Calculations

The speed and simplicity of operation of the HP-21 become most apparent during chain calculations. Even during the longest of calculations, *you still work with only one or two numbers at a time*—the unique Hewlett-Packard automatic memory stores intermediate results until you need them, then inserts them into the calculation. The process of working through a problem is as natural as it would be if you were working it out with pencil and paper, but the calculator takes care of the hard part.

For example, solve (12 + 3) × 7.

If you were working the problem with a pencil and paper, you would first calculate the intermediate result of (12 +3) . . . .

$$\overset{15}{\cancel{(12 + 3)}} \times 7 =$$

. . . . and then you would multiply the intermediate result by 7.

$$\overset{15}{\cancel{(12 + 3)}} \times 7$$

You work through the problem exactly the same way with the HP-21, never working with more than two numbers at a time. You solve for the intermediate result first. . .

(12 + 3)

Press	Display
12	12.
ENTER↑	12.00
3	3.
$\boxed{+}$	15.00

. . . and then solve for the final answer. You don't need to press **ENTER↑** to store the intermediate result—the HP-21 stores it automatically when you key in the next number. To continue . . .

Press	Display	
7	7.	The intermediate result from the preceding operation is automatically stored when you key in this number.
×	105.00	Pressing the function key gives you the final answer.

Now try these problems. Notice that you only have to press **ENTER↑** to insert the first pair of numbers into the calculator—each subsequent operation is performed using a new number and an automatically stored intermediate result.

To solve:	Press:	
$\frac{(2 + 3)}{10}$	2	
	ENTER↑	
	3	
	+	
	10	
	÷	0.50
$(16 - 4) \times 3$	16	
	ENTER↑	
	4	
	−	
	3	
	×	36.00
$\frac{14 + 7 + 3 - 2}{4}$	14	
	ENTER↑	
	7	
	+	
	3	
	+	
	2	
	−	
	4	
	÷	5.50

Problems that are even more complicated can be solved in the same simple manner, using the automatic storage of intermediate results. For example, to solve $(2 + 3) \times (4 + 5)$ with a pencil and paper, you would:

$$\underbrace{(2 + 3)} \times \underbrace{(4 + 5)}$$

First solve for the contents of these parentheses. . . . . . . and then for these parentheses

. . . . and then you would multiply the two intermediate answers together.

You work through the problem the same way with the HP-21, except that you don't have to write down intermediate answers —the HP-21 remembers them for you.

First add 2 and 3:

Procedure	**Press**	**Display**
$\overset{5}{\cancel{(2 + 3)}} \times (4 + 5)$	2 ENTER↑ 3 +	5.00

Then add 4 and 5:

Procedure	**Press**	**Display**
$\overset{5}{\cancel{(2 + 3)}} \times \overset{9}{\cancel{(4 + 5)}}$	4 ENTER↑ 5 +	9.00

Then multiply the intermediate answers together for the final answer:

Procedure	**Press**	**Display**
$\overset{5}{\cancel{(2 + 3)}} \times \overset{9}{\cancel{(4 + 5)}}$	×	45.00

Notice that you didn't need to write down the intermediate answers from inside the parentheses before you multiplied— the HP-21 automatically stored the intermediate results for you and brought them out when it was time to multiply.

Now try these problems. Remember to work through them as you would with a pencil and paper, but don't worry about intermediate answers—they're remembered automatically by the calculator.

Problems

$(2 \times 3) + (4 \times 5) =$ $\boxed{26.00}$

$$\frac{(14 + 12) \times (18 - 12)}{(9 - 7)} = \boxed{78.00}$$

$$\left(\frac{\sqrt{16.3805 \times 5}}{.05}\right) = \boxed{181.00}$$

$[(17 - 12) \times 4] \div (10 - 5) =$ $\boxed{4.00}$

$\sqrt{(2 + 3) \times (4 + 5)} + \sqrt{(6 + 7) \times (8 + 9)} =$ $\boxed{21.57}$

A Word about the HP-21

Now that you've learned how to use the calculator to solve complicated problems, you can fully appreciate the benefits of the Hewlett-Packard logic system. With this system, you enter numbers using a parenthesis-free unambiguous method called RPN (Reverse Polish Notation).

It is this unique system that gives you all these calculating advantages:

- *You never work with more than two numbers at a time.* The HP-21 cuts problems down to size instead of making them more complex.
- *Pressing a function key immediately executes the function.* You work naturally through complicated problems, with fewer keystrokes and less time spent.
- *Intermediate results appear as they are calculated.* There are no "hidden" calculations, and you can check each step as you go.
- *Intermediate results are automatically stored.* You don't have to write down long intermediate answers when you work a problem.

- *Intermediate answers reappear automatically.* You don't have to remember where they are and then summon them.
- *You can calculate in the same order you do with pencil and paper.* You don't have to think the problem through ahead of time.

The HP system takes a few minutes to learn. But you'll be amply rewarded by the ease with which the HP-21 solves the longest and most complex equations. With HP, the investment of a few moments of learning yields a lifetime of mathematical bliss.

Controlling the Display

In the HP-21, numbers in the display normally appear rounded to only two decimal places. For example, the fixed constant π, which is *actually* used in the calculator as 3.141592654, normally *appears in the display* as `3.14` (unless you tell the calculator to show you the number rounded to a greater or lesser number of decimal places).

Although a number is normally shown to only two decimal places, the HP-21 always computes internally using 10 digits of each number. For example, when you compute 2 × 3, you *see* the answer to only two decimal places:

Press	Display
2 [ENTER↑] 3 [×]	6.00

However, inside the calculator all numbers have 10 digits. So the calculator *actually* calculates using full 10-digit numbers:

2.000000000 [ENTER↑] 3.000000000 [×]

yields an answer that is actually carried to 10 digits:

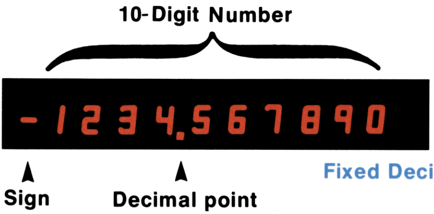
$$\underbrace{6.00}_{\text{You see only these digits. . .}}\underbrace{0000000}_{\text{but these digits are also present.}}$$

Display Key

The [DSP] (*display*) key allows you to control only the *manner* in which a number is displayed. The number itself is not altered by the [DSP] key.

Using the [DSP] key, you can choose either of two ways to display a number; fixed decimal point notation or scientific notation. No matter what notation you choose, these rounding options affect the display only—the HP-21 always calculates internally with the entire number.

Fixed Decimal Point Display

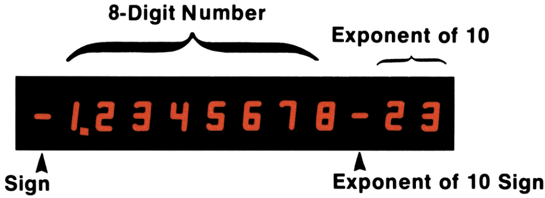


Fixed Decimal Point Display

Fixed decimal point display is selected by pressing DSP · followed by the appropriate number key to specify the number of decimal places (0–9) to which the display is to be rounded. Fixed point display allows answers to be shown with the same number of places after the decimal point. The number begins at the left side of the display and includes trailing zeros within the setting selected. When the calculator is turned OFF, then ON, it always returns back to fixed point notation with the display rounded to two decimal places.

Press	Display	
(Turn the calculator OFF, then ON.)	0.00	
123.4567 ENTER↑	123.46	Display is rounded off to 2 decimal places. Internally, however, the number maintains its original value to 10 digits.
DSP · 4	123.4567	
DSP · 6	123.456700	
DSP · 2	123.46	Normal fix 2 display.
DSP · 0	123.	

Scientific Display



(This means $-1.2345678 \times 10^{-23}$)

Scientific Notation Display

Scientific notation display is useful when you are working with large or very small numbers and allows all answers to be displayed with a specific number of digits after the decimal point. It is selected by pressing DSP followed by the appropriate number key to specify the number of decimal places to which the number is seen rounded. Again, the display is left-justified and includes trailing zeros within the selected setting. For example:

Press	Display	
(Turn the calculator OFF, then ON.)	0.00	
123.4567 ENTER↑	123.46	
DSP 2	1.23 02	Equals 1.23×10^2.
DSP 4	1.2346 02	Equals 1.2346×10^2.
DSP 7	1.2345670 02	Equals 1.2345670×10^2.

In scientific notation display, the HP-21 shows only eight digits plus the two-digit exponent of ten. So even though you may try to carry the operation out farther, the calculator will display a maximum of eight digits. For example, continuing the above operation results in no apparent change in the display:

Press	Display	
DSP 8	1.2345670 02	No change in display.
DSP 9	1.2345670 02	No change in display.

In scientific notation display, the HP-21 contains a ten-digit number and a two-digit exponent of ten inside the calculator, even though it displays only out to seven digits after the decimal point. So although DSP 7, DSP 8, and DSP 9 may affect the rounding of the display, the calculator maintains full accuracy internally.

For example, if you key in 1.000000094 and specify full scientific notation display (DSP 7), the calculator rounds off to the seventh digit after the decimal point: 1.000000094

Calculator rounds to this digit in DSP 7.

Press	Display
1.000000094	1.000000094
DSP 7	1.0000001 00

In DSP 8 , the calculator rounds off to the eighth digit after the decimal point, but you see only out to seven digits after the decimal: 1.000000094

You see to here . . . . . . but the calculator rounds to here in DSP 8.

Press	Display
DSP 8	1.0000000 00

You can see that if you had keyed in 1.000000095, DSP 8 would also have caused the seventh and final displayed digit to be rounded to a one (1).

Automatic Display Switching

The HP-21 features automatic overflow and underflow that switch the display to full scientific whenever the number is too large or too small for fixed decimal point display. For example, solve $(.05)^2$:

Press	Display	
CLx	0.0000000 00	DSP 7 from previous example.
DSP • 2	0.00	Normal fix 2 display.
.05 ENTER↑	0.05	
.05 ×	2.5000000 -03	

Another way of displaying the answer would be `0.0025`, but in normal fix 2 display, you would have seen only `0.00` as the displayed part of the answer. So the display automatically switches to scientific notation display to let you see the answer.

The HP-21 also switches to scientific notation if the answer is too large for fixed decimal point display ($> 10^{10}$). For example, solve 1582000 × 1842.

Press	Display	
1582000 ENTER↑		
1842 ×	2914044000.	Fixed decimal point display.

The answer does not overflow, so it remains in fixed notation. However, solve 1582000 × 18420.

Press	Display	
1582000 ENTER↑		
18420 ×	2.9140440 10	Scientific display.

The number is too large for fixed decimal point display, so it is automatically displayed in scientific notation.

Keying In Exponents of Ten

You can key in numbers multiplied by powers of ten by pressing EEX (*enter exponent*). For example, key in 15.6 trillion (15.6×10^{12}), and multiply it by 25:

Press	Display	
15.6	15.6	
EEX	15.6 00	
12	15.6 12	(This means 15.6×10^{12}.)

Now Press

ENTER↑	1.5600000 13	
25 ×	3.9000000 14	

You can save time when keying in exact powers of ten by pressing EEX and then pressing the desired power of ten. For example, key in 1 million (10^6) and divide by 52.

Press	Display	
EEX	1. 00	You do not have to key in the number 1 before pressing EEX when the number is an exact power of 10.
6	1. 06	
ENTER↑	1000000.00	Since you have not specified scientific notation, the answer reverts to fixed point display when you press ENTER↑ .
52 ÷	19230.77	

To see your answer in scientific notation with six decimal places:

Press	Display
DSP 6	1.923077 04

To key in negative exponents of ten, key in the number, press EEX , press CHS to make the exponent negative, then key in the power of 10. For example, key in Planck's constant (h)—roughly, 6.625×10^{-27} erg sec.— and multiply it by 50.

Press	Display	
CLX	0.0000000 00	
DSP · 2	0.00	
6.625 EEX	6.625 00	
CHS	6.625 –00	
27	6.625 –27	
ENTER↑	6.625000 –27	
50 ×	3.3125000 –25	Erg sec.

Using the EEX key, you can key in numbers made up of 10-digit mantissas and two-digit exponents of 10. However, when you use the EEX key, the HP-21 displays each number as an eight-digit mantissa and a two-digit exponent of 10. In a few cases, a number may have to be altered slightly in form before you can key it in using the EEX key:

- If you key in a number whose mantissa contains more than eight digits to the left of the decimal point, the EEX key is overridden and does not operate. Begin again and key in the number in a form that displays the mantissa with eight digits or less to the left of the decimal point before pressing the EEX key. (Thus, $123456789.1 \times 10^{23}$ could be keyed in as $12345678.91 \times 10^{24}$.)
- If you key in a number whose first significant digit occurs *after* the first eight digits of the display, the EEX key should not be used. To key in the number correctly, begin again and place the number in a form such that its first significant digit is one of the first eight digits of the display, then proceed using the EEX key.(Thus, $0000.000025 \times 10^{55}$ must not be keyed in in that form. It could be keyed in as $0000.00025 \times 10^{54}$, or as 0.000025×10^{55}, for example.)

EEX and y^x

Do not confuse the use of the EEX (*enter exponent*) key with the y^x key. EEX is used to specify the power of ten by which a number is multiplied. y^x is used to raise a number to a power.

For example, compute the cube of Avogadro's number: $(6.02 \times 10^{23})^3$.

Press	Display	
6.02	6.02	
EEX 23	6.02 23	
ENTER↑	6.0200000 23	
3	3.	
y^x	2.1816721 71	2.1816721×10^{71} is the cube of Avogadro's number.

Overextended Calculation

When the display would be greater than 9.9999999×10^{99}, the HP-21 displays all 9's to indicate that the problem has exceeded the calculator's abilities for display. For example, if you solve $(1 \times 10^{49}) \times (1 \times 10^{50})$, the HP-21 will display the answer.

Press	Display
EEX 49 ENTER↑	1.0000000 49
EEX 50 ×	1.0000000 99

But if you attempt to multiply the above result by 100, the HP-21 display indicates that it has overextended by showing you all 9's.

Press	Display
100 ×	9.9999999 99

How the HP-21 Works

The Stack

Automatic storage of intermediate results is the reason that the HP-21 slides so easily through the most complex equations. And the key to automatic storage is the Hewlett-Packard automatic memory stack.

Initial Display

When you first switch the calculator ON, the display shows `0.00`. This represents the contents of the display, or "X-register."

Basically, numbers are stored and manipulated in the machine "registers." Each number, no matter how simple (e.g., 0, 1, or 5) or how complex (e.g., 3.141592654, –23.28362, or $2.87148907 \times 10^{27}$), occupies one entire register.

The displayed X-register, which is the only visible register, is one of four registers inside the calculator that are positioned to form the automatic memory stack. We label these registers X, Y, Z, and T. They are "stacked" one on top of the other with the displayed X-register on the bottom. When the calculator is switched ON, these four registers are cleared to 0.00.

Name	Register	
T - (top)	0.00	
Z	0.00	
Y	0.00	
X	0.00	Always displayed.

You can also place all 0's in the stack registers by pressing [ ] [CLR]. Do this now.

When a number is keyed in, its contents are written into the displayed X-register, and the other registers remain unchanged. For example, if you keyed in the number 314.32, your stack registers would look like this:

Name	Register	
T	0.00	
Z	0.00	
Y	0.00	
X	314.32	Display.

Reviewing the Stack

The R↓ (*roll down*) key lets you review the entire stack contents at any time. Each time you press the key, the stack contents shift downward one register.

So the number as you've keyed it in will be rotated around to the T-register by pressing R↓.

When you press the R↓ key the stack contents are rotated from this . . . to this:

	from this			to this	
T	0.00		T	314.32	
Z	0.00		Z	0.00	
Y	0.00		Y	0.00	
X	314.32	Display.	X	0.00	Display.

Notice that the *contents* of the registers are shifted. The registers themselves maintain their positions. The X-register is always displayed, so you can see 0.00 now.

Pressing the R↓ key again rotates the contents once more.

Press R↓ again and the stack contents are shifted from this . . . . . . to this .

	from this			to this	
T	314.32		T	0.00	
Z	0.00		Z	314.32	
Y	0.00		Y	0.00	
X	0.00	Display.	X	0.00	Display.

Press R↓ twice more . . . and the stack shifts . . .

. . . through this . . .

T	0.00	
Z	0.00	
Y	314.32	
X	0.00	Display

. . . back to the start again .

T	0.00	
Z	0.00	
Y	0.00	
X	314.32	Display.

Once again the number 314.32 is in the displayed X-register.

Now that you know how the stack is rotated, you can use the **R↓** key to review the contents of the stack at any time. Always remember, though, that it takes four presses of the **R↓** key to return the contents to their original registers.

Now ensure that the original number, 314.32, is again in the displayed X-register. In order to key in a second number, you must first tell the calculator that you are finished writing the first number.

Pressing the **R↓** key told the calculator that you were finished writing the number 314.32, and you could key in a new number right now. But in actual practice, as you already know, you use the **ENTER↑** key to separate numbers.

So to see how the **ENTER↑** key normally works, press **CLX** and then key the number 314.32 back into the X-register. Then press **ENTER↑** to change the contents of the registers . . .

from this:

T	0.00	
Z	0.00	
Y	0.00	
X	314.32	Display.

to this:

T	0.00	
Z	0.00	
Y	314.32	
X	314.32	Display.

As you can see, the number in the displayed X-register is copied into Y. (The numbers in Y and Z have also been transferred to Z and T, respectively, and the number in T has been lost off the top of the stack. But this will be more apparent when we have different numbers in all four registers.)

Immediately after pressing ENTER↑, the X-register is prepared for a new number. And that new number writes over the number in X. For example, key in the number 543.28 and the contents of the stack registers change . . .

	from this:			to this:	
T	0.00		T	0.00	
Z	0.00		Z	0.00	
Y	314.32		Y	314.32	
X	**314.32**	Display.	X	**543.28**	Display.

CLX replaces any number in the display with zero. Any new number then writes over the zero in X.

For example, if you had meant to key in 689.4 instead of 543.28, you would press CLX now to change the stack . . .

	from this:			to this:	
T	0.00		T	0.00	
Z	0.00		Z	0.00	
Y	314.32		Y	314.32	
X	**543.28**	Display.	X	**0.00**	Display.

And then key in 689.4 to change the stack . . .

	from this:			to this:	
T	0.00		T	0.00	
Z	0.00		Z	0.00	
Y	314.32		Y	314.32	
X	**0.00**	Display.	X	**689.4**	Display.

Notice that numbers in the stack do not move when a new number is keyed in immediately after pressing ENTER↑ or CLX. (However, numbers in the stack *do* move upward when a new number is keyed in immediately after pressing R↓.)

Clearing

To clear the display only, press **CLX**. To clear the entire automatic memory stack, including the displayed X-register (but not the manual storage register—more about that later), press [blue prefix key] **CLR**. (Notice that it isn't necessary—although it may be comforting—to clear the calculator when starting a new calculation.) To clear everything, *including* the manual storage register, turn the HP-21 OFF, then ON.

Arithmetic—How the Stack Does It

In Hewlett-Packard calculators, arithmetic is performed by first positioning the numbers in the stack the same way you would on paper. For instance, if you wanted to add 34 and 21 you would write 34 on a piece of paper and then write 21 underneath it like this:

$$\begin{array}{r} 34 \\ \underline{21} \end{array}$$

and *then* you'd add like this:

$$\begin{array}{r} 34 \\ \underline{+21} \\ 55 \end{array}$$

Numbers are positioned the same way in the HP-21. Here's how it is done. (Clear the previous number entry first by pressing **CLX**.)

Press	Display	
34	34.	34 is keyed into X.
ENTER↑	34.00	34 is copied into Y.
21	21.	21 writes over the 34 in X.

Now 34 and 21 are sitting vertically in the stack as shown below, so we can add.

T	0.00	
Z	0.00	
Y	34.00	
X	21.	Display.

Press	Display	
+	55.00	The answer.

The simple old-fashioned math notation explains how to use your calculator. Both numbers are always positioned in the stack first; then the operation is executed when the key is pressed. *There are no exceptions to this rule.*

Subtraction, multiplication, and division work the same way. In each case, the data must be in the proper position before the operation can be performed.

To subtract 21 from 34:

$$\begin{array}{r} 34 \\ -21 \\ \hline \end{array}$$

Press	Display	
34	34.	34 is keyed into X.
ENTER↑	34.00	34 is copied into Y.
21	21.	21 writes over the 34 in X.
−	13.00	The answer.

To multiply 34 by 21:

$$\begin{array}{r} 34 \\ \times 21 \\ \hline \end{array}$$

Press	Display	
34	34.	34 is keyed into X.
ENTER↑	34.00	34 is copied into Y.
21	21.	21 writes over the 34 in X.
×	714.00	The answer.

To divide 34 by 21:

$$\frac{34}{21}$$

Press	Display	
34	34.	34 is keyed into X.
ENTER↑	34.00	34 is copied into Y.
21	21.	21 writes over the 34 in X.
÷	1.62	The answer.

Chain Arithmetic

You've already learned how to enter numbers into the calculator and perform calculations with them. In each case you first needed to position the numbers in the stack manually using the ENTER↑ key. However, the stack also performs many movements automatically. It's these automatic movements that give it tremendous computing efficiency and ease of use, and it's these movements that automatically store intermediate results. The stack automatically "lifts" every calculated number in the stack when a new number is keyed in because it *knows* when it completes a calculation that any new digits you key in are a part of a new number. Also, the stack automatically "drops" when you perform an operation. For example, calculate 16 + 30 + 11 + 17 = ?

Note: Press ▮ CLR to insure that the stack initially contains all zeros.

Press	Stack	Contents	
	T	0.00	
	Z	0.00	16 is keyed into the displayed X-register.
16	Y	0.00	
	X	16.	
	T	0.00	
ENTER↑	Z	0.00	16 is copied into Y.
	Y	16.00	
	X	16.00	

Press	Register	Stack	Comment
30	T	0.00	30 writes over the 16 in X.
	Z	0.00	
	Y	16.00	
	X	30.	
$\boxed{+}$	T	0.00	16 and 30 are added together. The answer, 46, is displayed.
	Z	0.00	
	Y	0.00	
	X	46.00	
11	T	0.00	11 is keyed into the displayed X-register. The 46 in the stack is automatically raised.
	Z	0.00	
	Y	46.00	
	X	11.	
$\boxed{+}$	T	0.00	46 and 11 are added together. The answer, 57, is displayed.
	Z	0.00	
	Y	0.00	
	X	57.00	
17	T	0.00	17 is keyed into the X-register 57 is automatically entered into Y.
	Z	0.00	
	Y	57.00	
	X	17.	
$\boxed{+}$	T	0.00	57 and 17 are added together for the final answer.
	Z	0.00	
	Y	0.00	
	X	74.00	

After any calculation or number manipulation, the stack automatically lifts when a new number is keyed in. Because operations are performed when the operations are pressed, the length of such chain problems is unlimited until the answer exceeds the range of the calculator (up to $9.999999999 \times 10^{99}$).

In addition to the automatic stack lift after a calculation, the stack automatically drops *during* calculations involving both X- and Y-registers. It happened in the above example, but let's do the problem differently to see this feature more clearly. First press **CLX** to clear the X-register.

Press	Stack	Contents	
	T	0.00	
16	Z	0.00	16 is keyed into the displayed X-register.
	Y	0.00	
	X	16.	
	T	0.00	
ENTER↑	Z	0.00	16 is copied into Y.
	Y	16.00	
	X	16.00	
	T	0.00	
30	Z	0.00	30 is written over the 16 in X.
	Y	16.00	
	X	30.	
	T	0.00	
ENTER↑	Z	16.00	30 is entered into Y. 16 is lifted up to Z.
	Y	30.00	
	X	30.00	
	T	0.00	
11	Z	16.00	11 is keyed into the displayed register.
	Y	30.00	
	X	11.	
	T	16.00	
ENTER↑	Z	30.00	11 is copied into Y. 30 and 16 are lifted up to Z and T respectively.
	Y	11.00	
	X	11.00	

Press	Register	Contents	
17	T	16.00	17 is written over the 11 in X.
	Z	30.00	
	Y	11.00	
	X	17.	
[+]	T	16.00	17 and 11 are added together and the rest of the stack drops. 16 drops to Z and is also duplicated in T. 30 and 28 are ready to be added.
	Z	16.00	
	Y	30.00	
	X	28.00	
[+]	T	16.00	30 and 28 are added together and the stack drops again. Now 16 and 58 are ready to be added.
	Z	16.00	
	Y	16.00	
	X	58.00	
[+]	T	16.00	16 and 58 are added together for the final answer and the stack continues to drop.
	Z	16.00	
	Y	16.00	
	X	74.00	

This same dropping action also occurs with [−], [×], and [÷]. The number in T is duplicated in T and Z, the number in Z *drops* to Y, and the numbers in Y and X *combine* to give the answer, which is visible in the X-register.

Direction of Execution

The automatic stack lift and automatic stack drop let you retain and position intermediate results without reentering the numbers. This is an advantage the stack has over other data handling methods. Problems can be solved by keying in the numbers in any logical order you choose. You can even key in numbers in left-to-right order.

Try left-to-right order with the following expression:

$$(35 + 45) \times (55 + 65)$$

Press	Display	
35	35.	The left-most number is keyed into the X-register.
ENTER↑	35.00	No operations can be performed so you press ENTER↑ .

Press	Display	
45	45.	The next number is keyed into X.
[+]	80.00	The intermediate result of the addition operation is displayed.
55	55.	The next number is keyed into X.
ENTER↑	55.00	The multiplication operation cannot be performed yet, so you press ENTER↑.
65	65.	The next number is keyed into X.
[+]	120.00	The addition operation is performed next.
[×]	9600.00	The answer is calculated without repositioning the numbers.

Of course, you don't have to work problems from left to right. Most people start at the innermost parentheses and work outward, keying in numbers as they need them. Either way, the more complex the problem, the more you'll appreciate the capabilities of the automatic memory stack. Try this additional example.

Example: Calculate $5 \times [(3 \div 4) + (5 \div 2) + (4 \div 3)] \div (3 \times .213)$.

3	3.	
ENTER↑	3.00	
4.	4.	
[÷]	0.75	$(3 \div 4)$
5	5.	
ENTER↑	5.00	
2	2.	
[÷]	2.50	$(5 \div 2)$
[+]	3.25	$(3 \div 4) + (5 \div 2)$
4	4.	
ENTER↑	4.00	
3	3.	
[÷]	1.33	$(4 \div 3)$
[+]	4.58	$(3 \div 4) + (5 \div 2) + (4 \div 3)$

Press	Display	
3	3.	
ENTER↑	3.00	
.213	0.213	
×	0.64	(3 × .213)
÷	7.17	[(3÷4)+(5÷2)+(4÷3)]÷(3×.213).
5	5.	The first number is keyed in.
×	35.86	The answer.

You could also work this problem from left to right.

Constant Arithmetic

You have probably noticed that whenever the stack drops because of a two-number operation (not because of R↓), the number in the T-register is copied into the Z-register and also rewritten into the T-register. This stack operation can be used to insert a constant into a problem.

Example: Bacteriologist Martin Arrowsmith tests a certain strain whose population increases by 15% each day under ideal conditions. If he starts a sample culture of 1000, what will be the bacteria population at the end of each day for six consecutive days?

Method: Put the growth factor (1.15) in the Y-, Z-, and T-registers and put the original population (1000) in the X-register. Thereafter, you get the new population whenever you press ×.

1.15	1.15	Growth factor.
ENTER↑	1.15	
ENTER↑	1.15	
ENTER↑	1.15	Growth factor now in T.
1000	1000.	Starting population.
×	1150.00	Population after 1st day.
×	1322.50	Population after 2nd day.
×	1520.88	Population after 3rd day.
×	1749.01	Population after 4th day.
×	2011.36	Population after 5th day.
×	2313.06	Population after 6th day.

When you press [×] the first time, you calculate 1.15 × 1000. The result (1150.00) is displayed in the X-register and a new copy of the growth factor drops into the Y-register. Since a new copy of the growth factor is duplicated from the T-register each time the stack drops, you never have to reenter it.

Notice that performing a two-number operation such as [×] causes the number in the T-register to be duplicated there each time the stack is dropped. However, the [R↓] key, since it *rotates* the contents of the stack registers, cannot *rewrite* any number, but merely shifts the numbers that are already in the stack.

Exchanging x and y

Another key that manipulates the stack contents is the [x⇄y] (*x exchange y*) key. The [x⇄y] key exchanges the contents of the X- and Y-registers without affecting the Z- and T-registers. For example:

Press

1 [ENTER↑] 2 [ENTER↑] 3 [ENTER↑] 4

The stack contents now look like this.

T	1.00	
Z	2.00	
Y	3.00	
X	4.	Display

Press

[x⇄y]

And the stack contents are changed to this.

T	1.00	
Z	2.00	
Y	4.00	
X	3.00	Display

Pressing [x⇄y] again will restore the numbers in the X- and Y-registers to their original places. You can always verify the position of the data in the stack by reviewing with the [R↓] key.

Error Display

If you happen to key in an improper operation, the word *Error* will appear in the display.

For example, try to divide 1 by 0 (the HP-21 will recognize this as an illegal operation):

$$\frac{1}{0} =$$

Press	Display
1 ENTER↑	1.00
0 ÷	Error

You can clear the error by pressing CLX or by keying another number into the displayed X-register.

Press	Display
CLX	0.00

All those operations that cause the word Error to appear in the display are listed in appendix B.

Shift Key Cancel

If you press the blue shift key and wish to cancel before another key is operated, press ENTER↑ and continue. For example:

Press	Display
2	2.00

Whoops! You wanted to CHS, not $\sqrt{x}$, so:

Press	Display
ENTER↑ CHS	–2.00

In this case, the normal ENTER↑ function is *not* performed. Pressing ENTER↑ after the blue shift key does nothing but cancel the shift key.

Function Keys

Finding Reciprocals

To calculate reciprocals of a displayed number, key in the number, then press **1/x**.

Example: Find the reciprocal of 25.

Press	Display
25 **1/x**	0.04

You can also calculate the reciprocal of a value in a previous calculation without reentering the number.

Example: Calculate $\dfrac{1}{1/3 + 1/6}$.

Press	Display	
3 **1/x**	0.33	Reciprocal of 3.
6 **1/x**	0.17	Reciprocal of 6.
+	0.50	Sum of reciprocals.
1/x	2.00	Reciprocal of sum.

Finding Square Roots

To calculate the square root of any displayed value, press ■ **√x**.

Example: Find the square root of 16.

Press	Display
16 ■ **√x**	4.00

Now find the square root of the result.

Press	Display
■ **√x**	2.00

Using π

π, 3.141592654, is provided as a fixed constant in the HP-21. Merely press [blue] [π] whenever you need it in a calculation before executing the applicable operation.

Example 1: Calculate 3π.

Press	Display
3 [blue] [π] [×]	9.42

Example 2: Find the area A of a circle with 3-foot radius r, where

$$A = \pi r^2$$
$$r = 3$$

Solution:

Press	Display	
3 [ENTER↑] [×]	9.00	
[blue] [π]	3.14	
[×]	28.27	Square feet.

Example 3: Finding himself wafted dangerously close to a series of jagged mountain peaks, intrepid balloonist Ike Daedalus frantically cranks open the helium valve on his spherical balloon. Gas from the helium tank increases the balloon's radius from 25 feet to 27 feet, and Daedalus safely clears the mountaintops. By how much did Daedalus increase the volume of the balloon?

Solution: Volume of a sphere is equal to $4/3\pi r^3$. The increase in volume is therefore $(4/3)\ \pi\ (27)^3$ minus $(4/3)\ \pi\ (25)^3$. This can also be written

$$\frac{4}{3}\pi[\ (27)^3 - (25)^3\ ]$$

Press	Display
27 [ENTER↑] 3 [blue] [y^x]	19683.00
25 [ENTER↑] 3 [blue] [y^x]	15625.00
[−]	4058.00
4 [×]	16232.00

Press	Display	
3 ÷	5410.67	
π	3.14	
×	16998.11	Cubic feet that Daedalus has increased the balloon's volume.

Manual Storage Register

In addition to the four automatic memories of the stack registers, the HP-21 also has a storage register that is *manually* operated, and unaffected by operations within the stack. Keys used to manipulate the manual storage register are STO, RCL, M−, M+, M×, M÷.

Storing and Recalling Data

To store a number appearing in the display (whether the result of a calculation or a keystroke entry), merely press STO. If the storage register already has a number in it, the old number will be overwritten by the new one. The value in the displayed X-register will remain unchanged.

To recall a number from the manual storage register, press the RCL key. Recalling a number does not *remove* it from the storage register, but only *copies* the stored number into the displayed X-register. Recalling a number will cause the stack to lift (unless RCL follows a CLX or an ENTER↑).

The original number will remain in the storage register until either:

1. A new number is placed in the storage register over the old one.
2. The calculator is turned off.

To clear only the manual storage register, press 0 STO.

Example: A customer at Hepzibah Pyncheon's Emporium has bought three items priced at \$1000, \$2000, and \$3000, respectively. Hepzibah's policy is to grant a 5% discount on all purchases over \$500. How much will the customer pay for each of the three items? What is the total cost?

Solution:

Press	Display	
1 ENTER↑ .05 − STO	0.95	Stores constant 0.95 (95%) in storage register.
1000 ×	950.00	Amount customer will pay for first item.
2000 RCL ×	1900.00	Amount customer will pay for second item.
3000 RCL ×	2850.00	Amount customer will pay for third item.
+ +	5700.00	Total cost.

Storage Register Arithmetic

Arithmetic is performed *using* the storage register by pressing the RCL key as illustrated above. Arithmetic is performed *upon* the contents of the storage register by pressing the blue shift key and either M+, M−, M×, or M÷.

Press	Result
■ M+	Number in X-register added to contents of the storage register.
■ M−	Number in X-register subtracted from the storage register contents.
■ M×	Number in X-register multiplied by the number in the storage register, and the product placed into the storage register.
■ M÷	Storage contents divided by number in X-register and the quotient placed into the storage register.

Example: During harvest, farmer Flem Snopes hauls tomatoes to the cannery for three days. On Monday and Tuesday he hauls loads of 45 tons, 47 tons, 49 tons, and 43 tons, for which the cannery pays him $55 per ton. On Wednesday the price rises to $57.50 per ton, and Snopes ships loads of 46 tons and 48 tons. If the cannery deducts 2% of the price on Monday and Tuesday because of blight on the tomatoes, and 3% of the price on Wednesday, what is Snopes' total net income?

Method: Keep total amount in the storage register while using the stack to add tonnages and calculate amounts of loss.

Press	Display	
45 ENTER↑ 47 [+] 49 [+] 43 [+]	184.00	Total of Monday and Tuesday's tonnage.
55 [×]	10120.00	Monday and Tuesday's gross amount.
STO	10120.00	Gross placed in storage register.
.02 [×]	202.40	Monday and Tuesday's deductions.
[ ] [M−]	202.40	Monday and Tuesday's deductions subtracted from total in storage register.
46 ENTER↑ 48 [+]	94.00	Total of Wednesday's tonnage.
57.50 [×]	5405.00	Wednesday's gross amount.
[ ] [M+]	5405.00	Wednesday's gross amount added to total in storage register.
.03 [×]	162.15	Amount of Wednesday's deduction.
[ ] [M−]	162.15	Wednesday's deductions subtracted from total in storage register.
RCL	15160.45	Snopes' total net income.

Trigonometric Functions

The following trigonometric functions are provided:

SIN (sine)
[ ] SIN^{-1} (arc sine)
COS (cosine)
[ ] COS^{-1} (arc cosine)
TAN (tangent)
[ ] TAN^{-1} (arc tangent)

To use the **SIN**, **COS** and **TAN** functions, key in the number and press the appropriate function key. To use the arc functions, press then press the associated function key. For example, find **SIN⁻¹** (.866):

Press	Display		
.866 **SIN⁻¹**	60.00	degrees	depending upon which angular mode you are in.
	or		
	1.05	radians	

Trigonometric functions can be performed in either of two angular modes: decimal degrees or decimal radians. Note that trigonometric functions assume decimal angles regardless of angular mode. To select a mode, use the DEG RAD slide switch.

Example 1: Find the cosine of 35°. If the HP-21 is not already in Degrees mode, switch to DEG RAD before performing the calculation.

Solution:

Press	Display
35 **COS**	0.82

Example 2: Find the tangent of 6 radians.

Solution:

Switch DEG RAD

Press	Display
6 **TAN**	–0.29

Example 3: In the midst of his transpacific crossing, bronzed surfer Ishmael Queequeg begins to tire, and he decides to paddle his board for either Honolulu, Hawaii (21° 18′ N, 157° 52′ W), or Anchorage, Alaska (61° 13′ N, 149° 54′ W). Queequeg knows his own position (45° 37′ N, 150° 12′ W), and he knows that the great circle distance between two points is given by the formula

$$\text{Distance} = \cos^{-1}\left[\sin(\text{LAT}_s)\sin(\text{LAT}_d) + \cos(\text{LAT}_s)\cos(\text{LAT}_d)\cos(\text{LNG}_d - \text{LNG}_s)\right] \times 60.$$

Where LAT_s and LNG_s = latitude and longitude of the source (Queequeg).

LAT_d and LNG_d = latitude and longitude of the destination.

Which port will be closer for Queequeg and his surfboard?

Method: Enter degrees, minutes, seconds as a decimal; for example, 21° 18′ is computed as:

$$21° \ 18' = 21 + \frac{18}{60} = 21.30°$$

For ease of computation, all minutes have been converted to decimal degrees in the following equations.

The equation for the distance to Honolulu is:

$$\cos^{-1}\left[(\sin 45.62°)(\sin 21.30°) + (\cos 45.62°)(\cos 21.30°)\cos(157.87° - 150.20°)\right] \times 60$$

Slide DEG RAD switch to Degrees:

Press	Display	
157.87 ENTER↑	157.87	
150.20 −	7.67	
COS	0.99	
21.30 COS ×	0.92	
45.62 COS ×	0.65	
45.62 SIN	0.71	
21.30 SIN ×	0.26	
+	0.91	
$\cos^{-1}$	25.12	
60 ×	1507.13	Distance from Queequeg's position to Honolulu, Hawaii.

The equation for the distance to Anchorage is:

$$\cos^{-1}\left[(\sin 45.62°)(\sin 61.22°) + (\cos 45.62°)(\cos 61.22°)\cos(149.90° - 150.20°)\right] \times 60$$

Press	Display
149.90 ENTER↑	149.90
150.20 −	−0.30
COS	1.00
61.22 COS ×	0.48
45.62 COS ×	0.34
45.62 SIN	0.71

Press	Display	
61.22 SIN ×	0.63	
+	0.96	
■ COS⁻¹	15.60	
60 ×	936.06	Distance from Queequeg's position to Anchorage, Alaska.

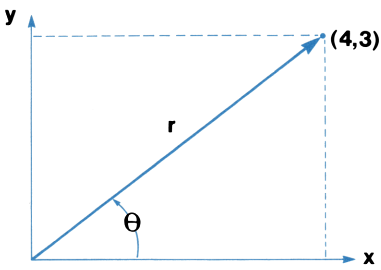
Queequeg's shortest, although not his warmest, journey will be to Anchorage.

Polar/Rectangular Coordinate Conversion

Two functions are provided for polar/rectangular coordinate conversion. To convert values in the X- and Y-registers, (representing rectangular x, y coordinates, respectively) to polar r, θ coordinates (magnitude and angle, respectively), press ■ →P. Magnitude r then appears in the X-register and angle θ appears in the Y-register.

Conversely, to convert values in the X- and Y-registers (representing polar coordinates, r, θ, respectively) to rectangular coordinates (x, y, respectively), press ■ →R.

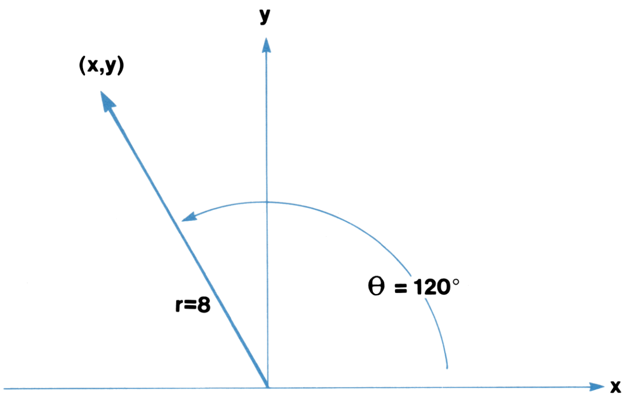
Example 1: Convert rectangular coordinates (4,3) to polar form with the angle expressed in radians.



Solution: Make certain the Degree/Radian switch is in the Radians position: DEG ▮ RAD .

Press	Display	
3 ENTER↑ 4 ■ →P	5.00	Magnitude.
x⇄y	0.64	Angle in radians.

Example 2: Convert polar coordinates (8,120°) to rectangular coordinates.

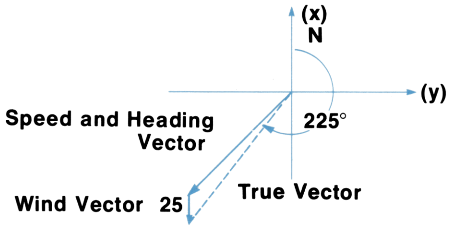


Solution:
Slide DEG [switch] RAD switch to Degrees.

Press	Display	
120 ENTER↑ 8 [blue key] →R	–4.00	x-coordinate
x⇄y	6.93	y-coordinate

Example 3: The instruments in fearless bush pilot Apeneck Sweeney's converted P-41 indicate an airspeed of 125 knots and a heading of 225°. However the aircraft is also being buffeted by a steady 25-knot wind that is blowing from north to south. What is the actual course and speed of the aircraft?

Method: Combine the vector indicated on the P-41's instruments with the wind vector to yield the actual course and speed. Convert the vectors to rectangular, then combine the x-coordinates in the storage register and the y-coordinates in the stack. Finally, bring the summed x- and y-coordinates back out again and convert them to polar coordinates giving the actual vector of the aircraft. (North becomes the x-coordinate in order that the problem corresponds with navigational convention.)



Press	Display	
225 ENTER↑ 125		
→R	-88.39	Instrument x-coordinate.
STO CLX	0.00	
180 ENTER↑ 25		
→R	-25.00	Wind x-coordinate.
M+	-25.00	
R↓	0.00	Wind y-coordinate (the wind is blowing from due north).
+	-88.39	Sum of y-coordinates.
RCL	-113.39	Sum of x-coordinates.
→P	143.77	Speed of aircraft.
x⇄y	-142.06	Angle of aircraft.
360 +	217.94	Angle changed to positive value to agree with navigational convention.

Sweeney is actually flying at 143.77 knots on a course of 217.94°.

Logarithmic and Exponential Functions

Logarithms

The HP-21 computes both natural and common logarithms as well as their inverse functions (antilogarithms):

LN is $\log_e$ (natural log); takes log of value in X-register to base e (2.718 . . .).

e^x is antilog_e (natural antilog); raises e (2.718 . . .) to the power of value in X-register. (To display the value of e, press 1 e^x).

[blue shift key] [LOG] is $\log_{10}$ (common log); computes log of value in X-register to base 10.

[blue shift key] [10^x] is antilog_{10} (common antilog); raises 10 to the power of value in X-register.

Example 1: The 1906 San Francisco earthquake, with a magnitude of 8.25 on the Richter Scale is estimated to be 105 times greater than the Nicaragua quake of 1972. What would be the magnitude of the latter on the Richter Scale? The equation is

$$R_1 = R_2 - \log \frac{M_2}{M_1} = 8.25 - (\log \frac{105}{1})$$

Solution:

Press	Display	
8.25 ENTER↑	8.25	
105 [blue shift key] [LOG]	2.02	
[−]	6.23	Rating on Richter Scale.

Example 2: Having lost most of his equipment in a blinding snowstorm, ace explorer Buford Eugobanks is using an ordinary barometer as an altimeter. After measuring the sea level pressure (30 inches of mercury) he climbs until the barometer indicates 9.4 inches of mercury. Although the exact relationship of pressure and altitude is a function of many factors, Eugobanks knows that an *approximation* is given by the formula:

$$\text{Altitude (feet)} = 25{,}000 \ln \frac{30}{\text{Pressure}} = 25{,}000 \ln \frac{30}{9.4}$$

Where is Buford Eugobanks?

Solution:

Press	Display	
25000 ENTER↑	25000.00	
30 ENTER↑	30.00	
9.4 [÷]	3.19	
[blue shift key] [LN]	1.16	
[×]	29012.19	Feet altitude.

Eugobanks is probably near the summit of Mount Everest (29,028 ft.)

Raising Numbers to Powers

[ ] [y^x] permits you to raise a positive number (either an integer or a decimal) to any power. For example, calculate 2^9 $(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)$.

Press	Display
2 [ENTER↑] 9 [ ] [y^x]	512.00

Now find $8^{1.2567}$.

Press	Display
8 [ENTER↑]	8.00
1.2567 [ ] [y^x]	13.64

In conjunction with [1/x], [ ] [y^x] provides a simple way to extract roots. For example, find the cube root of 5. (This may also be written as $5^{1/3}$):

Press	Display	
5 [ENTER↑]	5.00	
3 [1/x]	0.33	Reciprocal of 3.
[ ] [y^x]	1.71	Cube root of 5.

Example 1: A particularly hopped-up Vincent Black Shadow motorcycle rockets along a straight line according to the equation

$$\text{Distance} = \tfrac{1}{2}(t^6 + 8t).$$

Determine the Shadow's velocity (V) and acceleration (A) after two seconds (t = 2), according to the formulas:

$V\ (\text{ft./sec.}) = 3t^5 + 4 = (3 \times 2^5) + 4$
$A\ (\text{ft./sec.}^2) = 15t^4 = 15 \times 2^4$

Solution:

Press	Display	
2 [ENTER↑] 5 [ ] [y^x]	32.00	
3 [×]	96.00	
4 [+]	100.00	Ft/sec. The Shadow's velocity.
2 [ENTER↑] 4 [ ] [y^x]	16.00	
15 [×]	240.00	Ft/sec.² The Shadow's acceleration.

Example 2: An aircraft pilot reads a pressure altitude (PALT) of 25,500 feet with a calibrated airspeed (CAS) of 350 knots. What is the flight mach number

$$(M) = \frac{\text{speed of aircraft}}{\text{speed of sound}}$$

if the following formula is applicable?

M =

$$\sqrt{5\left[\left(\left\{\left[\left(1+0.2\left[\frac{CAS}{661.5}\right]^2\right)^{3.5}-1\right]\left[1-6.875\times10^{-6}\,PALT\right]^{-5.2656}\right\}+1\right)^{0.286}-1\right]}$$

Method: The most efficient place to begin work on this problem is at the innermost set of brackets. So begin by solving for the quantity $\left[\frac{CAS}{661.5}\right]^2$ and proceed outward from there.

Press	Display	
350 [ENTER↑] 661.5 [÷]	0.53	
[ENTER↑] [×]	0.28	Square of bracketed quantity.
.2 [×] 1 [+]	1.06	
3.5 [yˣ] 1 [−]	0.21	Contents of left-hand set of brackets are in the stack.
1 [ENTER↑] 6.875 [EEX]	6.875 00	
[CHS] 6 [ENTER↑]	6.8750000 -06	
25500 [×] [−]	0.82	
5.2656 [CHS] [yˣ]	2.76	Contents of right-hand set of brackets are in the stack.
[×] 1 [+]	1.58	
.286 [yˣ] 1 [−]	0.14	
5 [×] [√x]	0.84	Mach number of the flight.

Afterword

If you have worked completely through this handbook, you should have a very good knowledge of all of the basic functions of the HP-21. But in fact you've only begun to see the power of the calculator. You'll come to understand it better and appreciate it more as you use the HP-21 daily to solve even the most complex mathematical expressions. At your fingertips you have a tool that was unavailable to Archimedes, Galileo, or Einstein. The only limits to the flexibility of the HP-21 are the limits of your own mind.

Appendix A

Accessories, Service, and Maintenance

Standard Accessories

Your HP-21 comes complete with one each of the following standard accessories:

Battery Pack
Soft Carrying Case
HP-21 Owner's Handbook
Battery Charger

Optional Accessories

Other accessories are specified on the Accessory Order Form.

To order additional standard or optional accessories for your HP-21 see your nearest dealer or fill out an Accessory Order Form and return it with check or money order to:

HEWLETT-PACKARD
Advanced Products Division
19310 Pruneridge Avenue
Cupertino, CA 95014

If outside the U.S., please contact the Hewlett-Packard Sales Office nearest you.

Calculator Operation

Note: Charge battery pack before portable use.

CAUTION

Use of any batteries other than the Hewlett-Packard battery pack may result in damage to your calculator.

Your calculator contains a rechargeable battery pack. *The batteries must be in the calculator for it to operate.* With the batteries in the calculator and the charger connected between the calculator and the line, the battery will charge with the calculator OFF or ON. Normal charging times from dead battery to full charge are:

	Calculator ON	17 hours
	Calculator OFF	6 hours

Shorter charge periods will reduce battery operating time. Whether the calculator is OFF or ON, the HP-21 battery pack is never in danger of becoming overcharged with the charger connected to the ac line. *It is normal for both the calculator and the charger to be warm to the touch during charging.*

CAUTION

Attempting to operate the HP-21 from the ac line with the battery pack removed may result in damage to your calculator.

The procedure for using the battery charger is as follows:

1. If your charger has a line voltage select switch, make sure it is set to the proper voltage. The two line voltage ranges are 100 to 127 volts and 200 to 254 volts.

CAUTION

Your HP-21 may be damaged if it is connected to the charger when the charger is not set for the correct line voltage.

2. Set the HP-21 power switch to OFF.
3. Insert the battery charger plug into the rear connector of the HP-21 and insert the power plug into a live ac power outlet.
4. At the end of the charging period, you may continue to use your HP-21 with ac power or proceed to the next step for battery-only operation.
5. With the HP-21 power switch turned OFF, disconnect the battery charger from both the power receptacle and the HP-21.

CAUTION

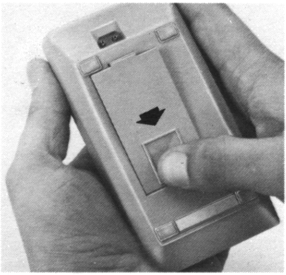
The use of a charger other than the HP battery charger supplied with the calculator may result in damage to your calculator.

Battery Operation

Use only the HP battery pack. A fully charged battery pack provides approximately 3 to 5 hours of continuous operation. By turning the power OFF when the calculator is not in use, the HP-21's battery pack should easily last throughout a normal working day.

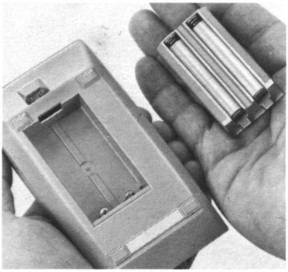
Battery Pack Replacement

To replace your battery pack use the following procedure:

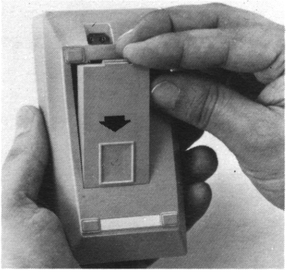


1. Turn the Power Switch to OFF and disconnect the battery charger from the calculator.

2. Press the thumbset on the rear of the calculator in the direction of the arrow.



3. Let the battery pack fall into the palm of your hand.



4. Insert the new battery pack in the direction of the arrow. Slant the leading edge of the pack into the edge of the doorway and snap the battery pack into place.

5. Secure the battery pack by pressing it gently.

Note: If you use your HP-21 extensively in field work or during travel, you may want to order the optional Reserve Power Pack, consisting of a battery charging attachment and spare battery pack. This enables you to charge one pack while using the other.

If the battery pack won't hold a charge, it may be defective. If the warranty is in effect, return the pack to Hewlett-Packard according to the shipping instructions. If the battery pack is out of warranty, see your nearest dealer or use the Accessory Order Form provided with your HP-21 to order a replacement.

Service

Low Power

All decimal points except the true one light to warn you that you have a minimum of 1 minute of operating time left.

`1.0.0.0.0.0 0.0.` **Low Power Display**

True decimal point

You must then either:

1. Turn the calculator OFF and connect it to a charger that is plugged into the ac line.
2. Insert a fully charged battery pack.

Blank Display

If the display blanks out, turn the HP-21 OFF, then ON. If `0.00` does not appear in the display, check the following:

1. If battery charger is attached to the HP-21, make sure it is plugged into an ac outlet. If not, turn the calculator OFF before plugging the charger into the ac outlet.
2. Examine battery pack to see if the contacts are dirty.
3. Substitute a fully charged battery pack, if available, for the one that was in the calculator.
4. If display is still blank, try operating the HP-21 using the charger (with the batteries in the calculator).
5. If, after step 4, display is still blank, service is required. (Refer to Warranty paragraphs.)

Temperature Range

Temperature ranges for the calculator are:

Operating	0° to 45° C	32° to 113°F
Charging	15° to 40° C	59° to 104°F
Storage	−40° to +55° C	−40° to +131°F

Warranty

In-Warranty

The HP-21 is warranted against defect in materials and workmanship for one year from date of delivery. During the warranty period, Hewlett-Packard will repair or, at its option, replace components that prove to be defective when the calculator is returned, shipping prepaid, to a Hewlett-Packard Customer Service Facility (refer to Shipping Instructions).

This warranty does not apply if the calculator has been damaged by accident, or misuse, or as a result of service or modification by other than an authorized Hewlett-Packard Customer Service Facility. No other warranty is expressed or implied. Hewlett-Packard is not liable for consequential damage.

Out-of-Warranty

Beyond the one-year warranty period, calculators will be repaired for a moderate charge. *All* repair work performed beyond the warranty period is warranted for a 90-day period.

Obligation to Make Changes

Products are sold on the basis of specifications applicable at the time of sales. Hewlett-Packard shall have no obligation to modify or update products once sold.

Shipping Instructions

Whether the unit is in warranty or out of warranty, it is the customer's responsibility to pay charges for shipping to the applicable service facility listed on the *Service Card*. During warranty, the service facility will, in turn, ship the unit back to the customer prepaid, via the fastest economical means.

On out-of-warranty repairs, the customer will pay shipping charges both ways.

Malfunctions traced to the calculator, batteries, or battery charger require that you return the following to us:

Calculator with all standard accessories.
Completed *Service Card*.

Send returned items safely packaged to the address shown on the *Service Card*.

Under normal conditions, calculators will be repaired and reshipped within five (5) working days of receipt at any Hewlett-Packard Service Facility listed on the *Service Card*.

Should other problems or questions arise regarding service, please call your nearest Hewlett-Packard sales or service facility.

Appendix B

Improper Operations

If you attempt a calculation containing an improper operation —say, division by zero—the display will show **Error**. To clear, press **CLX**.

The following are improper operations:

- **÷**, where $x = 0$
- **y^x**, where $y \leqslant 0$
- **$\sqrt{x}$**, where $x < 0$
- **$1/x$**, where $x = 0$
- **LOG**, where $x \leqslant 0$
- **LN**, where $x \leqslant 0$
- **SIN^{-1}**, where $|x|$ is > 1
- **COS^{-1}**, where $|x|$ is > 1
- **M÷**, where $x = 0$

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Service Card

Refer to the appendix of your Owner's Handbook to diagnose a calculator malfunction. The warranty period for your calculator is one year from date of purchase. Unless **Proof Of Purchase** is enclosed (sales slip or validation) Hewlett-Packard will assume any unit over 12 months old is out of warranty. **Proof Of Purchase** will be returned with your calculator. Should service be required, please return your calculator, charger, batteries and this card protectively packaged to avoid in-transit damage. Such damage is not covered under warranty.

Inside the U.S.A.

Return items safely packaged directly to:

Hewlett-Packard
APD Service Department
1000 N.E. Circle Boulevard
Corvallis, Oregon 97330

We advise that you insure your calculator and use priority (AIR) mail for distances greater than 300 miles to minimize transit times. All units will be returned via priority mail.

Outside the U.S.A.

Where required please fill in the validation below and return your unit to the nearest designated Hewlett-Packard Sales and Service Office. Your warranty will be considered invalid if this completed card is not returned with the calculator.

Model No. ____________ Serial No. ____________

Date Received ____________

Invoice No./Delivery Note No. ____________

Sold by:

Service Information

Must be **completed** and **returned** with your calculator, charger and batteries.

Name

Company

Street Address

City

State Zip Date

Home Phone Work Phone

Describe Problem:

Model No. Serial No.

HEWLETT hp PACKARD

HP-21 Warranty Card

Please fill in and return this card
within 10 days of receipt of calculator

Your Name _______________ Title _______________

Street Address _______________

City _______________ State _______________ Zip _______________

Date Received _______________ Serial Number _______________

(1) Where Purchased

- [1] Retail Store
- [2] Direct Mail
- [3] HP Salesperson
- [4] Gift
- [5] Other _______________

(2) Source of information leading to your purchase

- [1] Retail Store
- [2] HP Owner
- [3] HP Ad
- [4] Mail Brochure
- [5] HP Salesperson
- [6] Other _______________

(3) Job Category

- [1] Design/Development
- [2] Research
- [3] Educator
- [4] Student
- [5] Administration
- [6] Other _______________

Type of business you are in _______________

(4) Type of problems you plan to solve with your HP-21

____

____

Note: Save your sales receipt as proof of date of purchase

If you are outside the United States:

- **Return this card in the enclosed warranty envelope.**
- **If no envelope, please mail this card to the nearest Hewlett Packard Sales and Service Office**

FIRST CLASS
Permit No. 33
Corvallis, Oregon

BUSINESS REPLY MAIL

No postage stamp is necessary if mailed in United States

Postage will be paid by:

Hewlett-Packard
1000 N.E. Circle Boulevard
Corvallis, Oregon 97330

Calculator Catalog and Buying Guide Request Card

Primary Interest:

- ☐ *Scientific Calculators*
- ☐ *Business Calculators*
- ☐ *Both*

Valid in U.S. only

HEWLETT hp PACKARD

430C

Thank you for your order. A friend or associate might also like to know about Hewlett-Packard calculators. If you would like us to send him the **New** Hewlett-Packard Calculator **Catalog and Buying Guide,** please write his or her name and address on this postage paid Request Card.

Name ______________________

Title ______________________

Company ______________________

Street ______________________

City ______________ State ________ Zip ______

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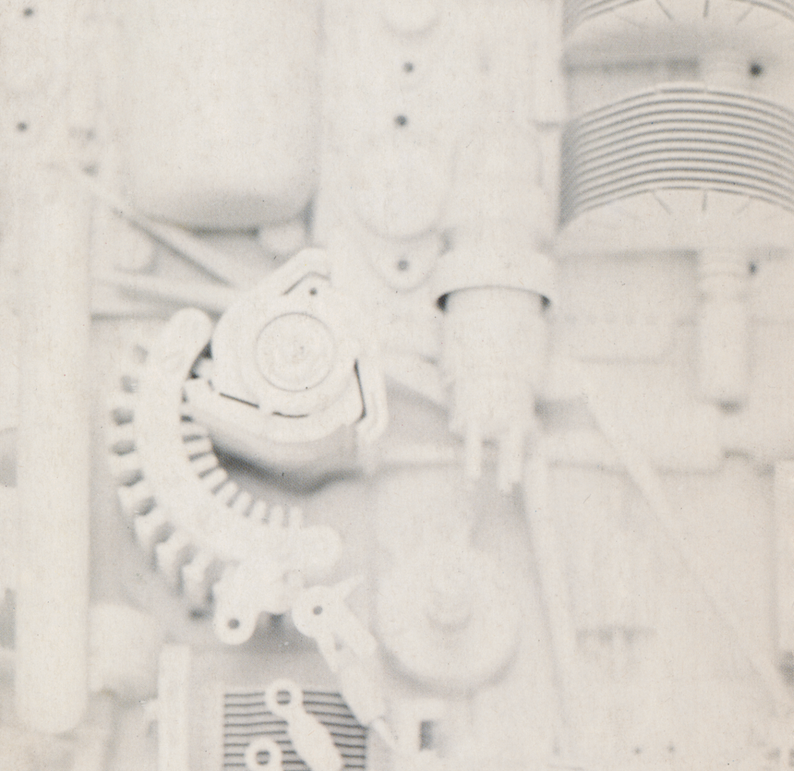
1000 N.E. Circle Blvd., Corvallis, OR 97330



For additional Sales and Service Information Contact Your Local Hewlett-Packard Sales Office or Call 408/996-0100 (Ask for Calculator Customer Service).

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