HEWLETT-PACKARD

HP-22 Owner's Handbook

HEWLETT PACKARD

HP-22
Owner's Handbook

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How To Use This Handbook

This handbook is divided into four sections as follows:

Section 1—Getting started, basic arithmetic, the calculator display, and the operational stack.

Section 2—Concepts of time and money, a tutorial explanation of the financial topics and terminology covered in section 3.

Section 3—Financial applications: keystroke procedures for solving more than 50 different financial problems.

Section 4—Mathematical and statistical functions.

If this is your first Hewlett-Packard calculator, we encourage you to read the entire handbook. If you are already familiar with HP calculators and their logic system, then you may choose to skip some material in the first section.

Before using the calculator, you may need to charge its battery pack as described in appendix A. The calculator can then be operated while the battery is charging or, later, on battery power alone.

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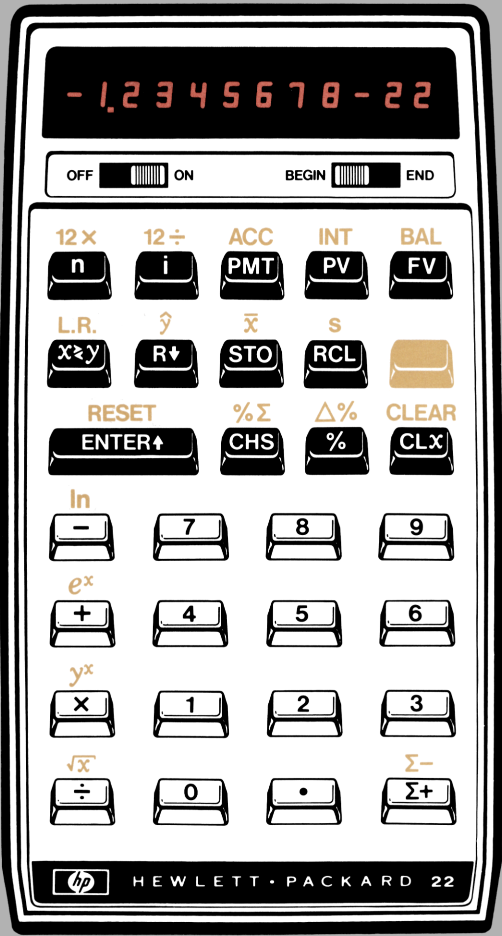
HP-22 Functions and Keyboard Index



OFF ON
Power switch
(page 9).

BEGIN END
Annuity switch
(page 54).

n Number of periods in a financial transaction **(page 52).**

12× Converts yearly periods to monthly payment periods **(page 52).**

i Interest rate per period **(page 52).**

12÷ Converts annual interest to interest rate/month **(page 52).**

PMT Payment amount per period **(page 52).**

ACC Computes accumulated interest between any two time periods of a loan **(page 53).**

PV Present value or the amount of money at the start of financial term **(page 52).**

INT Calculates simple interest **(page 53).**

FV Future value or amount received/paid at end of term **(page 52).**

BAL Gives remaining loan balance at any point in time **(page 53).**

x⇄y Exchanges contents of X- and Y-registers **(page 19).**

L.R. Linear regression. Calculates linear function between two or more points **(page 111).**

R↓ Rolls down contents of stack for viewing in display **(page 19).**

$\hat{y}$ Linear estimate **(page 113).**

STO Stores displayed value in one of 10 storage registers **(page 22).**

$\bar{x}$ Computes the mean or arithmetic average **(page 109).**

RCL Recalls (copies) stored number from a storage register onto display **(page 22).**

s Calculates standard deviation of a sample **(page 110).**

Shift key. Selects alternate function in gold above function keys. **(page 9).**

ENTER↑ Copies number in displayed X-register into Y-register; also, separates successive numerical entries **(page 10).**

RESET Resets financial status indicators and clears statistical data **(page 54).**

CHS Changes sign of displayed number **(page 9).**

%Σ Finds what % one number is of another or of the total **(page 50).**

% Converts a number to a percentage **(page 47).**

Δ% Computes % difference between two numbers (ratio of increase/decrease) **(page 48).**

CLX Clears display (X-register) **(page 10).**

CLEAR Clears the display, stack and storage registers; resets financial status indicators **(page 54).**

− + × ÷ Arithmetic operations **(page 10).**

ln Computes natural logarithm (base e) of value in display **(page 103).**

e^x Natural antilog. Raises e to the power of value in display **(Page 104).**

y^x Raises number in Y-register to power of number in display **(page 105).**

$\sqrt{x}$ Computes positive square root of number in display **(page 106)**

Σ+ Total sums, products and squares of numbers **(page 108).**

Σ− Subtracts a sum, product or square from the summation in Σ+ **(page 109).**

Introduction

Congratulations!

You've just made a wise investment, one that will save you money today, next week, and next year. Your HP-22 is another professional-quality business calculator from Hewlett-Packard—calculators whose small size, internal financial power, and ease of operation have made them the choice of realtors, bankers, stockbrokers, investment consultants, and other people who require fast, accurate answers. Over a million HP business and scientific pocket calculators are in use throughout the world, so you're in good company with HP!

Like any investor, you want to obtain the maximum return from your investment. This handbook will help you get the most from your new HP-22 calculator.

It's really four books in one. The first section covers basic arithmetic and the operational stack; the second section explains important financial concepts; the third section describes how to solve simple and complex application problems; and the fourth section explains the mathematical and statistical functions on your calculator.

The secret to getting the most from your HP-22 lies in taking the time to read through this handbook page by page and working the problems as you go along. The more confidence you have in your calculator and your own understanding of it, the more profitably you will use it. Your HP-22 has several unique features that simplify problem-solving. So, let's get started.

Section 1

Getting Started

The Keyboard

An illustration of the HP-22 keyboard follows the Contents. The unlabelled gold key [ ] extends selected keys to the functions noted in gold above the keys. To use the alternate function (the one in gold letters), merely press [ ] before pressing the associated function key.

Power On

Your HP-22 is fully assembled with the battery pack inside. You can run the calculator on battery power alone, or you can connect the battery charger and use the calculator while the battery is charging. To use the calculator on battery power only, first charge the batteries for 6 hours. (Refer to appendix A.)

Whether you operate from batteries or from the charger, the batteries *must* be in the calculator.

To begin, turn the power switch to ON. The display reads 0.00. Now you are ready to work a few simple problems.

Keying In Numbers

Key in a number by pressing the digit keys in order, left to right. If a decimal point is part of the number, you must key in the decimal point in the appropriate position. For example, key in 148.84 by pressing the keys:

[1] [4] [8] [•] [8] [4]

The number 148.84 appears in the display.

If you make a mistake, clear the incorrect number by pressing [CLx]. Then start again and key in the correct number.

Keying In Negative Numbers

To key in a negative number, key in the digits, then press the [CHS] (*change sign*) key. The number on the display will be preceded by a minus (−) sign.

You already have 148.84 on the display. Press [CHS] and see what happens. If you press [CHS] again, the −148.84 on the display returns to a positive number.

The CLx Key

Suppose you keyed in 149.84 instead of keying 148.84. That's easy to correct using the CLx (*clear X*) key.

CLx clears the display. If you make a mistake keying in a number, press CLx to eliminate that number then continue by keying in the correct value. There are two other clear keys on your HP-22: ▬ CLEAR and ▬ RESET. These are associated with the financial keys and are explained in section 3 and appendix D.

An important point about your HP-22 is that it isn't necessary to clear it between arithmetic calculations. Previous data is automatically pushed out of the way.

When you turn the power switch OFF, then ON again, the entire calculator is cleared.

Performing Simple Arithmetic

Whenever you add, subtract, multiply or divide, you work with two numbers and an arithmetic operation (+, −, × or ÷). Likewise, you cannot add, subtract, multiply or divide unless there are two numbers present in the calculator.

After both numbers are in the calculator, press the operations key. Your answer immediately appears on the display. To place two numbers into the calculator and perform simple arithmetic:

1. Key in the first number.
2. Press ENTER↑ to separate the first number from the second.
3. Key in the second number.
4. Press +, −, × or ÷ to perform the operation.

For example, you add 12 and 3 by pressing:

Press	Display	
12	12.	The first number.
ENTER↑	12.00	Separates the first number from the second.
3	3.	The second number.
+	15.00	The operation is executed and your answer appears on the display.

The four arithmetic functions are all performed the same way:

To Solve	Press	Display
24 + 3	24 ENTER↑ 3 [+]	27.00
24 − 3	24 ENTER↑ 3 [−]	21.00
24 × 3	24 ENTER↑ 3 [×]	72.00
24 ÷ 3	24 ENTER↑ 3 [÷]	8.00

In the problems above, you pressed 24 ENTER↑ 3. Try the same number sequence *without* the ENTER↑ step. What appears on the display? It's readily apparent that the ENTER↑ key separates the first number from the second.

If you have several numbers in a problem, you don't have to press ENTER↑ after every single one. A number may also be followed by an arithmetic operation ([+] [−] [×] or [÷]) or by one of the keyboard functions, such as [√x].

Sample Problems: Ready to try some problems on your own? The correct answer is given; try figuring the keystroke sequences by yourself.

Problem	Answer
14 × 6 =	84.00
144 ÷ 6 =	24.00
1/25 =	0.04
43 87 + 455 =	585.00
A customer buys 12 items at $19.95 each. What is the total sale?	239.40

Chain Calculations

Chain calculations are operations like adding the results of multiplication operations (the sum of products) or multiplying the results of addition operations (the product of sums). Sound complicated? Not so—it's easy to do with your HP-22.

The process of solving a problem is as natural as if you were working it out with pencil and paper. The calculator itself takes care of the hard part. The HP-22 not only displays intermediate answers; but it also retains these results until you need them, then inserts them into the calculation.

For example, solve (12 + 3) × 7.

If you work the problem with a pencil and paper, you would first calculate the intermediate result of (12 + 3). . .

$$\overbrace{(12 + 3)}^{15} \times 7 =$$

Then you would multiply 15 by 7.

$$\overbrace{(12 + 3)}^{15} \times 7 = 105$$

With the HP-22 you work through the problem exactly the same way—always concentrating on just two numbers at a time. First, you solve for the intermediate result. . .

To Solve	Press	Display
(12 + 3)	12 ENTER↑ 3 +	15.00

. . . then solve for the final answer. You don't need to press ENTER↑ to store the immediate result—the HP-22 stores it automatically when you key in the next number. To continue. . .

To Solve	Press	Display
15 × 7	7 ×	105.00

Now, try this problem. Remember that you have to press ENTER↑ to separate the first number from the second. After that, as the calculator performs each operation, it stores the new number.

To Solve	Press	Display
$\frac{2 + 3}{10}$	2 ENTER↑ 3 + 10 ÷	0.50

Following the rules of mathematics, you solve the expressions in parentheses first. Then move through the equation as you did before, one successive number and function at a time.

To Solve	Press	Display
$3\,(16 - 4)$	16 **ENTER↑** 4 $\boxed{-}$ 3 $\boxed{\times}$	36.00
$2\left(\dfrac{50 - 14}{12}\right)$	50 **ENTER↑** 14 $\boxed{-}$ 12 $\boxed{\div}$ 2 $\boxed{\times}$	6.00

Problems that are even more complicated can be solved in the same manner, using automatic storage of results. For example, to solve $(2 + 3) \times (4 + 5)$, you would:

First add 2 and 3:

	Press	Display
$(2 \overset{5}{\cancel{+}} 3) \times (4 + 5)$	2 **ENTER↑** 3 $\boxed{+}$	5.00

Then add 4 and 5:

	Press	Display
$(2 \overset{5}{\cancel{+}} 3) \times (4 \overset{9}{\cancel{+}} 5)$	4 **ENTER↑** 5 $\boxed{+}$	9.00

The 5.00 and the 9.00 are already in the calculator; so for the final answer, simply multiply them:

	Press	Display
$(2 \overset{5}{\cancel{+}} 3) \times (4 \overset{9}{\cancel{+}} 5)$	$\boxed{\times}$	45.00

Not once did you have to write down a "subtotal" or intermediate result. The HP-22 automatically remembered them.

Now, try solving these sample problems. (If you have trouble obtaining the correct answers, review the last few pages.)

Sample Problems

$(2 \times 3) + (4 \times 5) =$ 26.00

$$\frac{(14 - 12) \times (18 - 12)}{(9 - 7)} = \boxed{6.00}$$

$$(17 - 12) \times 4 \div (10 - 5) = \boxed{4.00}$$

You want to purchase two calculators at \$395 each. Your state has a 4% sales tax. How much will they cost?

$$2\ (\$395 \times 1.04) = \boxed{821.60}$$

Before proceeding to the financial problems, take a moment to look further at your calculator. The more you understand your HP-22, the more you will use it efficiently and confidently every day.

The Display

The display is your viewing window into the calculator. The display is also the means by which the calculator communicates with you—to tell you that the power is low or that you attempted an illegal operation.

Low Power Display

When the batteries get low, several decimal points will appear on the display interspersed among the numbers. Where your true decimal point appeared, there will be a blank. For example, if you have the following number on the display:

Ordinary display: 17.45

the low-power indication will look like this:

Low-power display: 1.7 4.5........

blank▲

This means you have approximately one minute of operating time left. Then you must either charge the battery or insert a fully-charged spare battery pack. (Refer to appendix A.)

Error Display

If you attempt an improper or impossible operation, the word Error appears on the display. For example, enter a number and try to divide by zero. (Go ahead, try it.) The calculator recognizes this as an illegal operation. Other improper operations include the square root of a negative number or 0 raised to a negative power.

To clear the error display, press **CLx**.

Rounding Numbers

If you key in a long numerical sequence, such as:

19.785234

when you press ENTER↑, the display reads: 19.79 . Because most financial calculations deal with dollars and cents, numbers in the display usually are rounded to two decimal places.

The HP-22 always computes internally using 10 digits of each number. Rounding affects the display only; it does not affect the accuracy of the number internally. While you see 19.79 on the display, the number inside the calculator is: 19.78523400.

Display Formatting

Sometimes you want to see more than two decimal places. You have 19.79 on the display. Now, press 4.

The gold key lets you view the number inside the calculator to more than (or fewer than) two decimal places. Using the key, you can display the number in business fixed decimal notation or scientific notation. No matter which form you choose, remember that the HP-22 always calculates internally with the entire 10-digit number.

Fixed Decimal Point Display

To view other than two decimal places, press followed by the appropriate number key (0 thru 9) to specify the number of decimal places. After keying in 19.785234, try the following:

Press	Display
4	19.7852
6	19.785234
2	19.79
0	20.

A display setting remains until a different one is specified by the user. If the calculated number is greater than the 10 digits of fixed notation can display, the HP-22 will automatically switch to scientific notation.

If you turn the calculator OFF, then ON, the display always returns to two decimal places: 0.00 for normal business operations.

Scientific Display

Scientific notation displays the number as N × 10^n where N is between ± 1 and ± 10. For example, 1.9785234–05 means 1.9785234 × 10^{-5} or .000019785234. 1.9785234 05 means 1.9785234 × 10^5 or 197852.34.

Scientific notation is useful when you are working with very large or very small numbers. The first eight positions in the display are reserved for the number (N), the last two positions are reserved for the exponent of 10. As you have seen, the exponent can be positive (05) or negative (−05).

To view answers in scientific notation, press [gold key] [•].

Press	Display	
CLx	0.00	
197.85234 ENTER↑	197.85	
[gold key] [•]	1.9785234 02	Represents 1.9785234 × 10^2.
[gold key] 2	197.85	

If you are using scientific notation and turn the calculator OFF, then ON, the display returns to two decimal places: 0.00.

Automatic Display Switching

The HP-22 automatically switches the display to scientific notation whenever the number is too large or too small for fixed decimal display. For example, if you are figuring a 5% discount on a nickel item (.05 × .05):

Press	Display	
CLx	0.00	Normal fix 2 display.
.05 ENTER↑	0.05	
.05 [×]	2.5000000–03	Scientific notation.

Another way of displaying the answer would be to press [gold key] 4 to obtain 0.0025. But in the normal fix 2 (business) notation, you would have only seen 0.00 so the display automatically switches to scientific notation to let you view the answer.

Overflow and Underflow Displays

When the number in the display is greater than 9.9999999×10^{99} (and that's a very large number), the HP-22 displays all 9's to indicate that the problem has exceeded the calculator's display abilities.

`9.9999999  99`

If an excessively large negative number is calculated (greater than -9.999999×10^{99}), the overflow indicator is all 9's preceded by a minus sign.

`-9.9999999  99`

There is also an underflow indicator. Very small numbers (less than 10^{-100}) are displayed as 0.00. If you press [•], the display reads:

`00000000  00`

The Operational Stack

The HP operational stack and its associated logic system is the most efficient method available for solving complex problems. You saw how easy it was to perform chain calculations because the calculator retained intermediate answers. This section describes how the HP-22 does that.

The X-Register (The Display)

Inside your calculator is a memory of four registers, "stacked" one on top of each other, like shelves. These are labelled X, Y, Z and T, with T at the top and X at the bottom, like so:

Stack

T	0.00	Top
Z	0.00	
Y	0.00	
X	0.00	Display

When the calculator is switched ON, the entire machine is cleared so all four stack registers are set to 0.00. You can also press [CLEAR] to clear the stack. When you key in a number and it appears on the display, it can be a simple digit, like 1 or 2, or it can be a long numerical sequence, like 3.141592654. Each number, no matter how simple or complex, occupies one register.

The X-register is the only one that's visible. What you see on the display is also written into the X-register. For example, if you keyed in 21, the stack would look like this:

T	0.00	
Z	0.00	
Y	0.00	
X	21.	Display reads: 21.

Because the X-register and the display are the same, pressing **CLx** clears the display.

Now, add 21 and 34. In order to key in a second number, you must separate the first number from the second by pressing **ENTER↑**. When you press **ENTER↑** the first number is entered up into the Y-register to make room for your next entry.

T	0.00	
Z	0.00	
Y	21.00	
X	21.00	Display

The number 21 also still appears on the display (X-register) until you write over it by keying in a new number. If you are interrupted in the middle of a calculation, the number still on the display tells you where you left off. (For example, if the number on the display is 21 without trailing zeros, you probably haven't entered it yet. If you had hit the **ENTER↑** key, the display would read 21.00.)

Since you are adding 21 and 34, key in the second number. The memory registers look like this:

T	0.00	
Z	0.00	
Y	21.00	
X	34.	Display

The numbers in the stack are positioned vertically the same way you might perform the arithmetic on paper:

$$\begin{array}{r} 21 \\ +\ 34 \\ \hline ? \end{array}$$

The simple old-fashioned math notation explains what is happening inside your calculator. Both numbers are positioned in the stack first; then the operation is executed when you press the proper key (+, −, × or ÷).

To add the two numbers, press [+]. The display reads [55.00]. What number is now in the X-register?

Is anything in the Y-register? Let's take a look.

x⇄y Key

To verify what is in the Y-register at any time, simply press the x⇄y (*x exchange y*) key. The x⇄y key exchanges the contents of the X- and Y-registers while the top two registers remain unchanged.

		What was			Becomes
	T	0.00		T	0.00
	Z	0.00		Z	0.00
What was	Y	0.00	**Becomes**	Y	55.00
	X	55.00	Display	X	0.00

Notice that the numbers move, but the registers themselves (the storage "shelves") remain in the same position.

Pressing x⇄y again restores the numbers in the X- and Y-registers to their original positions.

Another benefit of x⇄y is that you can use it correct your own transposition errors. Suppose you want to divide 25 by 5, but you inadvertently key in 5 ENTER↑ 25. You want to solve $\frac{25}{5}$, not $\frac{5}{25}$.

Since both numbers are in the stack, press x⇄y to transpose them to the right positions, then press [÷].

R↓ Key

The R↓ (*roll down*) key lets you review the entire stack contents at any time. Each time you press R↓, the stack contents shift down one register.

The X-register reads `55.00`. When you press **R↓**, the number in the X-register rotates to the top of the stack so that the other numbers can drop down.

T	0.00			T	55.00	
Z	0.00	→ **R↓** →		Z	0.00	
Y	0.00			Y	0.00	
X	55.00	Display		X	0.00	Display

Like **x⇄y**, the *contents* of the registers shift, but the registers themselves remain in the same position. Only the numbers move.

Press **R↓** again and the contents shift down once more.

T	0.00	
Z	55.00	
Y	0.00	
X	0.00	Display

Press **R↓** twice again (four times all together). The 55.00 should appear on the display (X-register). It takes four presses of the **R↓** key to examine the entire stack once and return the contents to their original registers.

Numbers in the stack do not move when a new number is keyed in immediately after pressing **ENTER↑** or **CLX**. However, if you press **R↓** or **x⇄y** immediately after pressing **ENTER↑** or **CLX**, the numbers in the stack will move upwards when a new number is keyed in.

Other Operations and Functions: Subtraction, multiplication, division, and exponentiation work the same way as the addition problem you just solved. In each case, the numbers must be in the proper position before the operation or function can be performed.

For example, to solve 34 − 21:

Press			
	T	0.00	
	Z	0.00	
34 **ENTER↑** 21	Y	34.00	
	X	21.	Display

Press

[−]

T	0.00	
Z	0.00	
Y	0.00	
X	13.00	Display

Suppose you want to add: 27 + 14 + 15 + 38. You have already learned how to solve it such that after entering the first number, you proceed one number and operation at a time (27 [ENTER↑] 14 [+] 15 [+] 38 [+]).

Now that you have learned the stack, another method would be to enter all four numbers into the stack, then add:

Press

27 [ENTER↑] 14 [ENTER↑]
15 [ENTER↑] 38

T	27.00	
Z	14.00	
Y	15.00	
X	38.	Display

[+]

T	27.00	
Z	27.00	
Y	14.00	
X	53.00	Display

The number in T is duplicated in T and Z, the number in Z drops to Y, and the numbers in Y and X combine to give the sum visible on the display. The same duplicating and dropping action also occurs with [−], [×] and [÷]. To continue with the addition. . .

Press

[+] [+]

T	27.00	
Z	27.00	
Y	27.00	
X	94.00	Display

Storing and Recalling Numbers

Besides the stack, another 10 memory registers are provided for manual storage and recalling numbers.

To store a number, press **STO** then a digit from 0 to 9 to specify the storage location. **STO** 0 places the number on the display in register 0; **STO** 1, in register 1; **STO** 2, in register 2; etc. Data in a storage register is changed by writing over it with a new value or by storage arithmetic.

To recall the number, press **RCL**, then the address digit (0 to 9). When you press **RCL** 3, a copy of the number in register 3 appears on the display; the original value will remain in the storage location until you write over it or clear it. Recalling data also moves the numbers in the stack upward.

Sample Problem: Suppose you want to calculate the cost of buying an item in various quantities. The unit price of the item is $132.57 and the quantities selected are 47, 36 and 29.

One way to solve this is to store the unit price in register 0. Then recall it to multiply each quantity.

Press	Display	
132.57 STO 0	132.57	
47 ×	6230.79	First total.
RCL 0	132.57	
36 ×	4772.52	Second total.
RCL 0	132.57	
29 ×	3844.53	Third total.

The individual totals are still in the stack, so you can easily calculate the combined total cost by adding them.

Press	Display	
+	8617.05	
+	14847.84	Total cost.

Storage Register Arithmetic

Arithmetic can also be performed using the number on the display and a number in a storage register. The arithmetic is performed *upon* the register contents, and answers are placed in the storage register—not on the display—so you have to recall them to see or use them.

To perform storage register arithmetic using the number on the display:

1. Press STO .
2. Press the desired arithmetic operation (+, −, × or ÷).
3. Press the storage location number (0 thru 9).

For example, to store 6 in register 3 and add 5:

Press	Display	
6 STO 3	6.00	Stores 6 in register 3.
5 STO + 3	5.00	Adds 5 to register 3.
RCL 3	11.00	Confirms that 11 is stored in register 3.

If you had pressed: 5 STO 3 +, that would overwrite the stored 6 with the 5—the value stored in register 3 would be 5, not 11.

Now, subtract 4 from the number in register 3:

Press	Display
4 STO − 3	4.00
RCL 3	7.00

Notice that the general rule is:

Number on display, STO , operation, location.

Clearing the Storage Registers

It is usually unnecessary to clear storage registers 0 thru 9 because you can simply write over the old number. (Previous data is pushed out of the way.)

The ▮ CLEAR function clears all 10 storage registers and the operational stack.

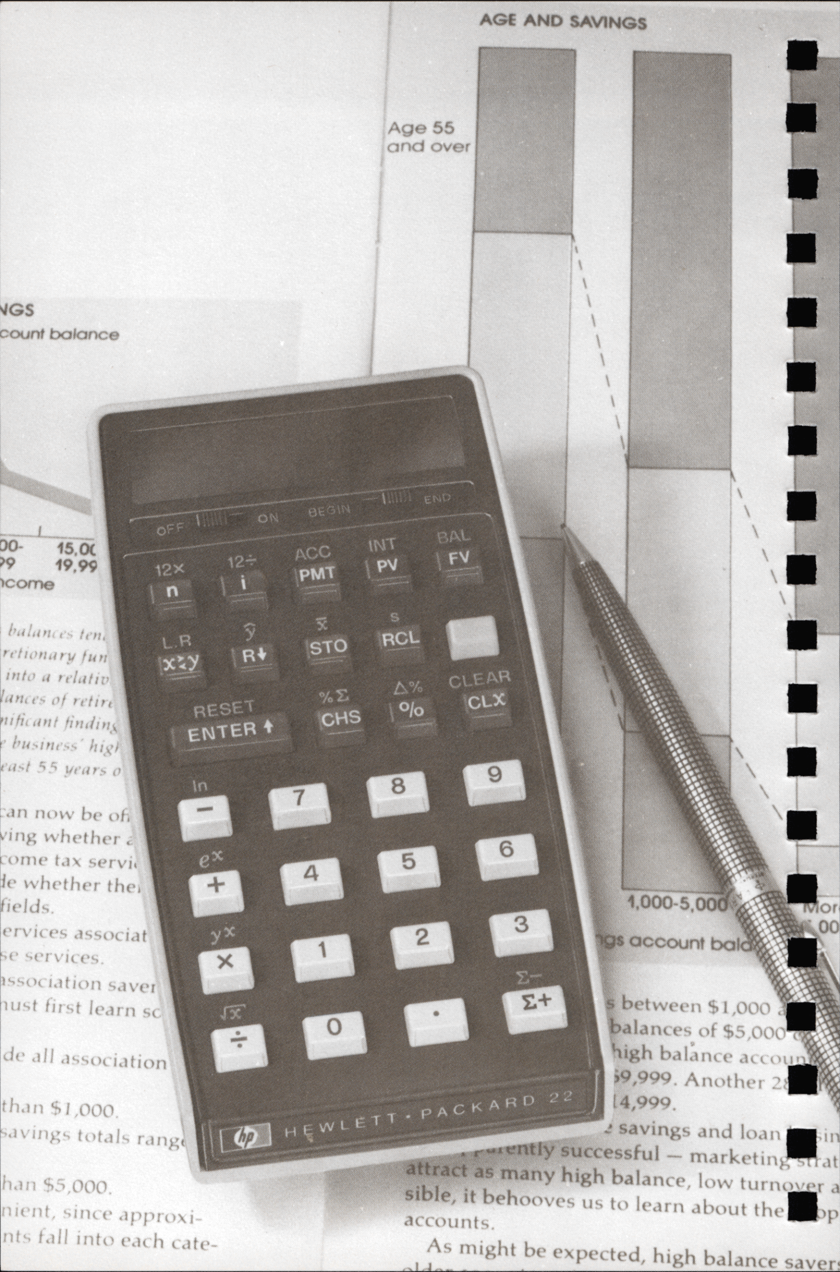
Availability of Storage Registers

As a general practice, it is wise to use storage registers 0 thru 4 first to preserve your data.

When the [Σ+] key is used in statistical problems, the HP-22 uses registers 5 thru 9 for its own internal data storage. Therefore, those registers are temporarily not available to you for manual storage.

During financial calculations that solve for interest, storage registers 8 and 9 are utilized by the calculator. When solving for accumulated interest, the HP-22 clears register 7.

For a detailed pictorial explanation of the HP-22 storage registers, read appendix D, *How the HP-22 Registers Work*.


AGE AND SAVINGS
Age 55
and over
1,000-5,000
OFF ON BEGIN END
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Section 2

Concepts of Time and Money

Essentially, there are two things you can do with money: spend it or invest it. A savings account in a bank is considered an investment, but putting your money under the mattress would be a bad investment. Whether you spend or invest it, you want to receive something worthwhile in return.

This section looks at the nature of cash flows, or how time and money relate to one another. And time *does* influence the value of money. After all, $1000 today has a different value than $1000 in 1945. Likewise, $1000 in 1945 had a different value than the same amount in 1930.

The formulas for solving common time-and-money problems are given throughout this section, but don't dwell on them or attempt to memorize them. Your HP-22 calculator has all these formulas built-in and ready at your fingertips, so you are free to concentrate on the concepts themselves.

Percent: the Universal Yardstick

Percentage is the universal yardstick—the common standard of measurement—in the financial world. If your money increases or decreases, the gain or loss is measured in percent as well as in dollars. Taxes, interest rates, discounts, inflation, appreciation, depreciation, even the last raise you got, or the typewriter you bought last week for 40% off—all are expressed in terms of percent.

Percent, denoted by the symbol %, simply means hundredth. When you see 25%, it's the same as 25/100 or 0.25 or ¼. The actual mechanics of how to solve percentage problems are explained in section 3 of this handbook; here, we are concerned with the concept itself.

Percentage is a dynamic relationship, a comparison or ratio of two numbers that often signifies that a change has taken place. "Third-quarter earnings are down 27% from last year" may be cause for concern, while "a 12% raise effective today" may be cause for celebration.

Likewise, when you start with a given amount of money and receive money in return, the difference—whether it's a gain or loss—is viewed in relation to the original amount and expressed as a percentage. If you start out with one share of stock worth \$100 and sell it for \$125, you have earned 25/100 or a 25% return.

When you superimpose that gain or loss against time, it's called the **rate of return.** The time period most commonly used in business is one year. So if you earned that \$25 in one year, that's a 25% annual rate of return.

Interest

Percent is also used to calculate interest. **Interest** is a charge for the use of money. In a sense, you ''rent'' the money or someone ''rents'' it from you.

Interest is based on three things:

1. The amount of money borrowed or saved.
2. The length of time.
3. The interest rate (a percentage).

This makes sense because the longer you rent something, the more you pay for it. If you rent a car for a week, it will cost more than if you rent it one day.

You can charge for money by the day, the week, the month, etc., but usually money is loaned or borrowed at a yearly rate. This annual interest rate is called the **annual percentage rate** or **APR** and is expressed as a percent. If a certain investment pays 9% yearly, that means \$9 per year for every \$100 invested.

But there are other considerations, too, when you pay or receive interest—namely, what *type* of interest and how often it is paid.

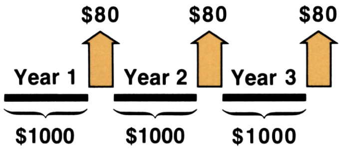
Simple Interest

With **simple interest,** only the principal—i.e., the original amount of money—earns interest for the entire life of the transaction. For example, suppose you put \$1000 in the bank at 8% simple interest for 3 years. The formula for calculating simple interest is:

$$\text{Interest}_{(\text{simple})} = \text{Principal x interest rate x time}$$

So, you would earn: $I = \$1000 \times 8\% \times 3 = \$240$ during that time period.

Simple Interest



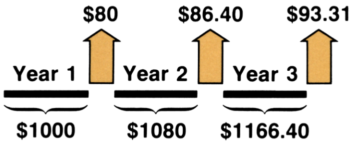
In essence, you receive $80 in interest at the end of the first year. Since only the principal ($1000) earns interest, you would receive another $80 at the end of the second year, and another $80 at the end of the third year.

Could you earn more than $240 in those three years with the same $1000? Yes, by adding the interest to the principal each year.

Compound Interest

Suppose at the end of the first year, you withdraw the $1080, go to another bank, and deposit a balance of $1080. The second year, you will earn $I = \$1080 \times 8\% \times 1$ or $86.40. You do the same thing again and, at the end of the third year, earn $I = \$1166.40 \times 8\% \times 1$, or $93.31.

Compound Interest



Instead of $240, you receive $259.71. By adding the simple interest to the principal each year, you increased the earning power of your money by $19.71.

Fortunately, you don't have to run all over town withdrawing and depositing money. Banks have an efficient way of handling that for you. With **compound interest,** each time the interest is paid, it is added to the balance or principal. Compound interest is usually stated as an annual rate, although it may be paid (compounded) daily, monthly, quarterly, or semi-annually.

So if you have \$1000 at present, before you invest you can calculate how much money you will receive in the future. This is called **future value.** The formula for computing the future value of money with compound interest is:

$$FV = PV \times (1 + i)^n$$

or

$$\text{Future Value} = \text{Present Value x} \left(1 + \begin{array}{l}\text{interest rate}\\ \text{per compounding}\\ \text{period}\end{array}\right)^{\begin{array}{l}\text{no. of}\\ \text{compounding}\\ \text{periods.}\end{array}}$$

Or perhaps you want \$1000 in the future—to take a trip to Acapulco next year—and want to know how much you have to invest to reach that goal. Since you have established the desired future value, you are solving for the money required right now or the **present value.** The formula for computing present value with compound interest is:

$$PV = \frac{FV}{(1 + i)^n}$$

or:

$$\text{Present Value} = \frac{\text{Future Value}}{\left(1 + \begin{array}{l}\text{interest rate}\\ \text{per compounding}\\ \text{period}\end{array}\right)^{\text{no. of compounding periods.}}}$$

Notice that both formulas use compounding "period" as the time element. If interest is compounded annually, then one year is the period of time. But when interest is compounded more often, the time element or compounding period affects the value of your money.

Compounding Periods

Let's go back to your original \$1000, invested at 8% compounded annually for 3 years. Using the future value formula, you can calculate what you will receive at the end of those 3 years:

$$FV = \$1000 \times (1 + .08)^3 = \$1259.71$$

Is there a way to earn more money with that \$1000 at the same 8% interest rate? Yes, by compounding or adding the interest to the principal more than once a year. Suppose you put that money into an account where the 8% interest is compounded quarterly. How much do you have at the end of a year? (Note: in the formula, you *must* use "interest rate per period" for i. Since 8% is the annual rate, you have to divide by the number of compounding periods, in this case, 4.)

$$FV = \$1000 \times (1 + .02)^4 = \$1082.43$$

How much will you have after 3 years?

$$FV = \$1000 \times (1 + .02)^{4 \times 3} = \$1268.24$$

Now, instead of \$240 or \$259.71, you earned \$268.24 in interest. It becomes apparent that the more often interest is compounded, the more money you receive in return.

Try figuring \$1000 at 8%, compounded monthly for 3 years:

$$FV = \$1000 \times \left(1 + \frac{.08}{12}\right)^{12 \times 3} = \$1270.24$$

Try the same problem again, compounded daily this time:

$$FV = \$1000 \times \left(1 + \frac{.08}{365}\right)^{365 \times 3} = \$1271.22$$

Just by more frequent compoundings, you have increased your earnings. Notice, in the above problems, that *interest rate per period MUST correspond with the compounding period interval*. Don't mix monthly interest with quarterly periods or daily interest with semi-annual compounding periods.

Is there a limit to the amount of money you can earn by increasing the frequency of compounding? Of course, and it's called **continuous compounding.** We've seen that compounding more and more frequently increases your earnings. If you compound continuously (more often than daily or hourly or every second), you reach the maximum mathematical limit. In other words, you reach the point where you just can't compound any more often.

Section 3 describes how to calculate compound interest and continuous compounding with the HP-22, so you'll encounter this topic again in the financial applications problems.

Effective Annual Rate

Something interesting is emerging here. Even though 8% is stated as the annual rate, you actually receive more than 8% interest with compounding more than once a year.

$1000 at 8% for 1 Year

COMPOUNDED	RETURN	%INTEREST
Quarterly	$1082.43	8.243%
Monthly	$1083.00	8.300%
Daily	$1083.28	8.328%
Continuous	$1083.29	8.329%

When interest is compounded more often than once a year, the stated annual rate (8% in the example above) is called the **nominal rate.** The rate of interest actually earned in one year (8.328%) is called the **effective rate.**

Many savings institutions quote both the nominal rate and the effective rate. And your calculator can quickly convert one to the other. As the chart shows, the effective rate may differ considerably from the nominal rate, so it pays (literally!) to know what it is.

* * * * * * * * * * *

So far, we have been dealing with four basic elements: the amount of money you have, right now, to invest (present value); the interest rate (i); the number of compounding periods (n); and the amount of money you will get back (future value). Notice that these four elements correspond to four out of the top five keys on your calculator.

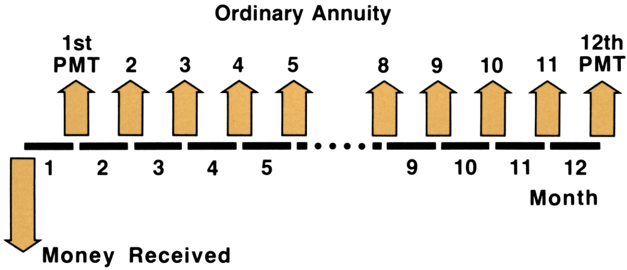
The fifth key (PMT) stands for payment. We just used an example where money sat in the bank. Now, let's look at a few situations where you are required to make payments.

Annuities

An **annuity** is a series of equal payments made at regular intervals. Your paycheck is an example of an annuity—also your monthly rent or mortgage payment, the premiums on an insurance policy, the installments on a car loan, or regular deposits into a savings account.

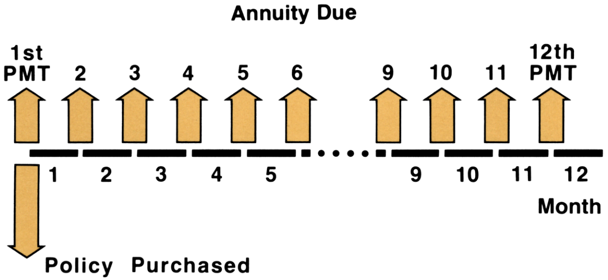
The time between annuity payments is called the **payment interval** or **payment period.** If your payment is due at the *end* of each payment period, it's called an **ordinary annuity** or **payments in arrears.** Examples of ordinary annuities are a car loan (where you drive away now and pay later) or a mortgage (where the payments start one month after you get your loan).

The time/money relationship for an ordinary annuity with monthly payments for a year—a loan, for example—would look like this:



But with some annuities—like insurance premiums or a lease—the payment is due at the beginning of the month. This is called an **annuity due** because the payment falls at the beginning of the payment period. Other terms are **payments in advance** or **anticipated payments.**

An annuity due with monthly payments for a year—say, a car insurance policy—looks like this:



Notice that with an annuity due, you have a payment right away at the beginning of the first interval. With an ordinary annuity, your payment isn't due until the end of the first period, but you also have a payment at the end of the entire term.

What difference does it make whether you pay at the beginning of the month or at the end of the month? It's still 12 payments so it's all the same, right? WRONG! It does make a difference.

Suppose you are undecided whether to buy or lease a car. Either way it will cost you \$165 a month for 24 payments at 12% interest. So you can check to see whether the loan or lease would be less expensive.

The formula for the present value of an ordinary annuity (loan) is:

$$PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

Substituting the known values, you calculate the present value of that series of payments, i.e., if you paid a cash lump sum instead of making payments, you'd pay \$3505.16.

The formula for the present value of an annuity due (lease) is:

$$PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right] (1 + i)$$

Substituting the values for PMT, i and n, you discover that the present value of the lease payment would be \$3540.21. An annuity due will cost you \$35.05 more.

That was a borrowing example. Now, look at a savings situation or money in the bank. The sooner you make a deposit, the sooner you start collecting interest. If you wait a month before making a deposit, you will lose a month's interest.

Consider a saving plan in which you deposit \$100 a month and earn 8% interest compounded monthly. The formula for future value of an ordinary annuity is:

$$FV = PMT \times \left[\frac{(1 + i)^n - 1}{i} \right]$$

So, if you deposit the money at the end of each month, after a year you will have:

$$FV = \$100 \times \frac{\left(1 + \frac{.08}{12}\right)^{12} - 1}{\frac{.08}{12}} = \$1244.99$$

The formula for future value of an annuity due is:

$$FV = PMT \times \left[\frac{(1+i)^n - 1}{i}\right](1+i)$$

So, if you deposit at the beginning of each month, after a year you will have more:

$$FV = \$100 \times \frac{\left(1 + \frac{.08}{12}\right)^{12} - 1}{\frac{.08}{12}} \left(1 + \frac{.08}{12}\right) = \$1253.29$$

Although it's the same number of payments, the same amount, and the same interest rate, as you can see, investing at the beginning of the payment period does make a difference: you earn more money!

It's important to remember that if you are *borrowing* or paying money, an *ordinary annuity* is to your advantage because it will cost you less. If you are *investing* or receiving money, an *annuity due* is preferrable because it is more profitable.

Loans and Amortization

Many of the annuities that you will encounter are loans, so here's a word or two concerning interest on loans.

Annual Percentage Rate

The federal "truth in lending" laws require a lender to state interest in terms of the annual percentage rate or APR.

If previously an interest rate was quoted as "1-1/2% a month," now it is quoted as "1-1/2% a month or 18% annual percentage rate." Whenever you encounter a daily, monthly or add-on rate—anything other than the annual interest rate—for your own protection, you should also know the APR.

Add-On Interest

Occasionally you will encounter a loan that has **add-on interest.** This means that simple interest is computed for the life of the loan and added to the loan.

Add-on interest can be deceptive, so be sure you also know the annual percentage rate. Add-on interest sounds attractive because the rate quoted is usually low. However, a 5% add-on loan of $1500 for 18 months actually comes to 9.27% APR.

With add-on interest, you pay interest on the entire loan amount for the entire term. As you shall learn in the next discussion, with a direct reduction loan you pay interest only on the unpaid loan balance which gradually decreases.

The Financial Applications section describes how to convert add-on interest rates to APR.

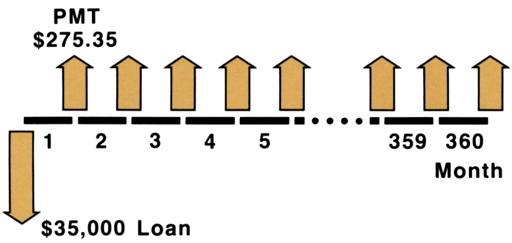
Amortization

If a loan or interest-bearing debt is discharged by (usually) equal payments, then it is said to be **amortized.** The word amortization comes from the French ''a mort'' meaning ''at the point of death'' Likewise, you are ''killing'' a loan by paying it off.

Most simple mortgages and installment loans are called **direct reduction loans.** The debt is discharged by equal periodic payments although varying portions of each payment are applied toward principal and interest.

The interest is paid first, then the remainder of the payment is used to reduce the debt. The time frame over which you make payments is called the **schedule of payments.** The breakdown of payments into interest portions and principal portions is called an **amortization schedule.**

Suppose you find your dream house. If you take out a $35,000 mortgage for 30 years at 8-3/4%, your monthly payments are $275.35. Your payment schedule would look like this:



At the end of the first month, interest is calculated on the entire \$35,000:

$$\frac{8.75\%}{12} \times 35{,}000 = \$255.21$$

and is added to the balance:

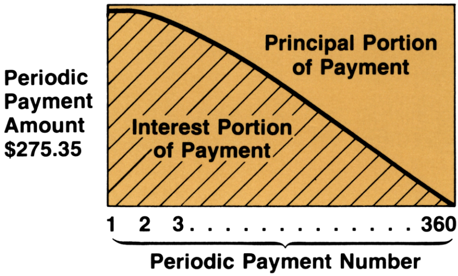
$$\$35{,}000 + \$255.21 = \$35{,}255.21$$

Then your first payment is deducted to obtain your new balance:

$$\$35{,}255.21 - \$275.35 = \$34{,}979.86$$

The next month and every month thereafter, the same procedure is followed, i.e., interest is calculated first and added to the balance before your payment is subtracted.

The amortization of your mortgage would look like this:

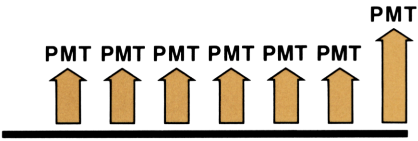


As you reduce the size of the loan, the interest decreases. . .and a gradually higher percentage of each payment goes toward the debt itself or outstanding principal. By the time you reach your last payment, very little is deducted for interest.

With your HP-22 calculator, at any point in time you can easily figure the accumulated interest and remaining balance of your loan. The keystroke procedure is described in the Financial Applications section.

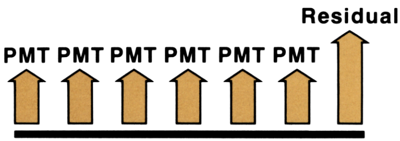
Balloon Payments

In real life, cash flows (money in and money out) are not always so simple that they can be divided neatly into equal amounts of money or equal periods of time. A common financial occurrence is an annuity that has a large payment at the end, like so:



The last payment—usually considerably larger although it could also be smaller than the others—is called a **balloon payment** or balloon. For example, if you inherit a windfall and pay off your mortgage before the 30 years of the loan have elapsed, that's a balloon. In some instances, when your money is tied up elsewhere, it is to your advantage to make small installment payments, then later pay the major portion of the debt when money is available.

In the leasing and real estate industries, this large amount at the end is often called the **residual value.** Even after several years of use or depreciation, a building (and especially the land that it is on) still has some cash value. For example, an owner can lease an office building for 25 years, and then still sell it. The cash flow for residual value fits a similar pattern to the balloon payment:



There are other financial situations that fit the same pattern. For example, you lease a piece of capital equipment—say, a computer—for 5 years. At the end of the leasing period, the equipment still has a certain residual or fair market value.

Regardless of the specific name, just remember the concept of a balloon payment: a different amount, usually larger, at the end of the term.

Sinking Funds

Another method of discharging a debt is called the sinking fund method and is commonly used with bond issues.

Put yourself in the position of a bond issuer. Since millions of dollars are often borrowed by the sale of bonds, that amount of money just can't be plucked out of thin air when the bonds mature. In order to be able to meet the redemption price of the bond, the bond issuer creates a separate fund into which he makes equal periodic deposits over the term so that just after the last deposit, the fund amounts to the debt (the face values or redemption prices of the bonds). This is called a **sinking fund** because its purpose is to sink—or shrink—the debt. Usually the sinking fund itself earns interest but not necessarily the same rate of interest that the bond issuer is paying.

How does this differ from putting money aside in a regular savings account? Mainly, because a sinking fund is an ordinary annuity—not annuity due—and payments into the fund are made at the *end* of each payment period.

Let's look at an example: a 10-year bond issue of $100,000 is to be discharged by the sinking fund method. If 20 semiannual deposits, the first due in 6 months, are made into a fund that pays 5% compounded semiannually, find out how much you have to deposit into the fund each time.

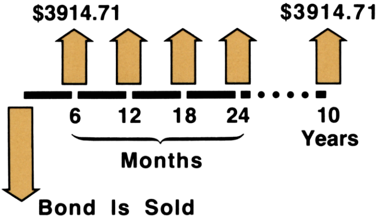
To solve this, consider a sinking fund the same as future value of an ordinary annuity.

$$FV = PMT \times \left[\frac{(1 + i)^n - 1}{i}\right]$$

$$\$100{,}000 = PMT \times \left[\frac{(1 + .025)^{20} - 1}{.025}\right]$$

PMT = $3914.71 (Your semiannual deposit into the sinking fund.)

The bond issuer's deposits into the sinking fund look like this:



In summary, sinking fund means that you discharge a debt by accumulating funds.

Discounted Cash Flow Analysis

By now, you have a good grasp of several financial terms and concepts. But we haven't answered the questions you will ask over and over again: "What should I invest in?" "Is investment A better than investment B?" "Where will I make the greatest profit?" "Should I buy securities or a shopping center?"

For all the other wonderful things your HP-22 does, it is not a crystal ball that can give you a ready answer. However, your calculator can help you evaluate future cash demands and returns to see which scheme or investment best meets your profit objectives.

One method of evaluation is called **discounted cash flow analysis.** There are two ways to use discounted cash flow analysis: the net present value approach or finding the discounted rate of return.

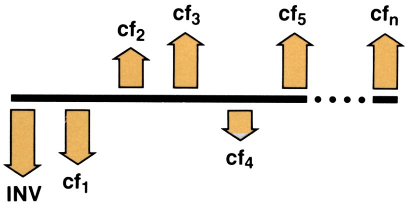
Solving for Net Present Value (NPV)

Suppose you invest a large amount of money (INV) into a scheme that generates a cash flow cf_1 the first year, cf_2 the second year, and so on, up to cf_n in the n^{th} year when the cash flow ends. You could write it down like this:

$$-\text{INV} + \text{cash flow first year} + \text{cash flow second year} + \ldots$$
$$\ldots + \text{cash flow } n^{th} \text{ year}$$

or: $$-\text{INV} + cf_1 + cf_2 + \ldots + cf_n$$

INV, the original investment, is negative because it represents a cash outlay. A diagram of the cash flows might look like this:



Notice that the cash flows may not necessarily be positive. Maybe, in a new business, you have a loss the first year. Or perhaps after you've been in business a while, a recession causes you to have a bad year.

You also have to consider the *time* value of money, not just the dollar value. (Would you rather have someone give you $10,000 today or 5 years from today?) The cash flows (cf_1, cf_2, etc.) are mini-future values that you are going to receive. But realistically they have to be translated back (discounted) to present value for you to accurately assess the investment.

In the compound interest discussion, you learned the formula for present value:

$$PV = \frac{FV}{(1 + i)^n}$$

You can translate the future cash flows to present values:

$$NPV = -INV + \frac{cf_1}{(1 + i)^1} + \frac{cf_2}{(1 + i)^2} + \ldots + \frac{cf_n}{(1 + i)^n}$$

This method of analysis is called solving for **net present value** because you are comparing the sum of the present values of all the future cash flows (all the cf's) to the initial investment (−INV). For i, substitute the rate of return that you want from the investment.

At the start, the net present value (NPV) is negative because you've put out a large amount of money (INV). As the cash returns flow in, NPV will increase. Eventually—hopefully—NPV will turn positive. When NPV = 0, you have reached the break-even point on the investment.

If you have a specified period of time in mind and NPV is still negative after that time has elapsed, then forget that investment scheme and move on to another.

It's a simple, clear-cut analysis: if NPV is negative, the net present value is less than the investment and the investment is NOT profitable. Assuming a desired minimum yield, if NPV is positive, the investment does meet your profit objectives.

For example, you are thinking of buying an apartment building for $100,000. Based on the anticipated cash flows below, will this investment return 10% a year?

Year		**Cash Flow**
1		$7000
2		$8500
3		$9000
4	(You intend to sell it.)	$120,000

Simply substitute these values into the equation:

$$\text{NPV} = -\$100{,}000 + \frac{\$7000}{(1 + .10)^1} + \frac{\$8500}{(1 + .10)^2} + \frac{\$9000}{(1 + .10)^3} + \frac{\$120{,}000}{(1 + .10)^4} = \$2111.88$$

The net present value after the fourth year is positive, so the investment does return 10% or greater per year.

Suppose you sell the building in the second year for $110,000. Would that be more profitable?

$$\text{NPV} = -\$100{,}000 + \frac{\$7000}{(1 + .10)^1} + \frac{\$110{,}000}{(1 + .10)^2} = \$-2727.27$$

Answer: No. The net present value is negative so you won't even meet your desired 10% rate of return.

Solving for Internal Rate of Return (IRR)

Sometimes if you know your initial investment and can predict the periodic cash flows, you want to find the rate of return that you will earn. This is also called **yield, discounted rate of return, or internal rate of return (IRR).**

The same formula applies, only NPV = 0.

$$NPV = -INV + \frac{cf_1}{(1 + i)^1} + \frac{cf_2}{(1 + i)^2} + \ldots + \frac{cf_n}{(1 + i)^h}$$

Solving for i gives you the rate of return on your investment.

The easiest way is to estimate the rate of return you are looking for. Substitute that percentage for i, then solve the equation. *When NPV is 0, your "guesstimate" is the actual IRR*. If NPV is negative, your estimated percentage is higher than the actual IRR—try a lower interest rate. If NPV is positive, the actual IRR is even better or higher than your desired rate.

For example, what is the estimated rate of return for a restaurant costing $200,000 that produces the following cash flows?

Year	**Cash Flow**
1	−$4000
2	$20,000
3	$27,000
4	$42,000
5	$56,500
6 (you sell it)	$230,000

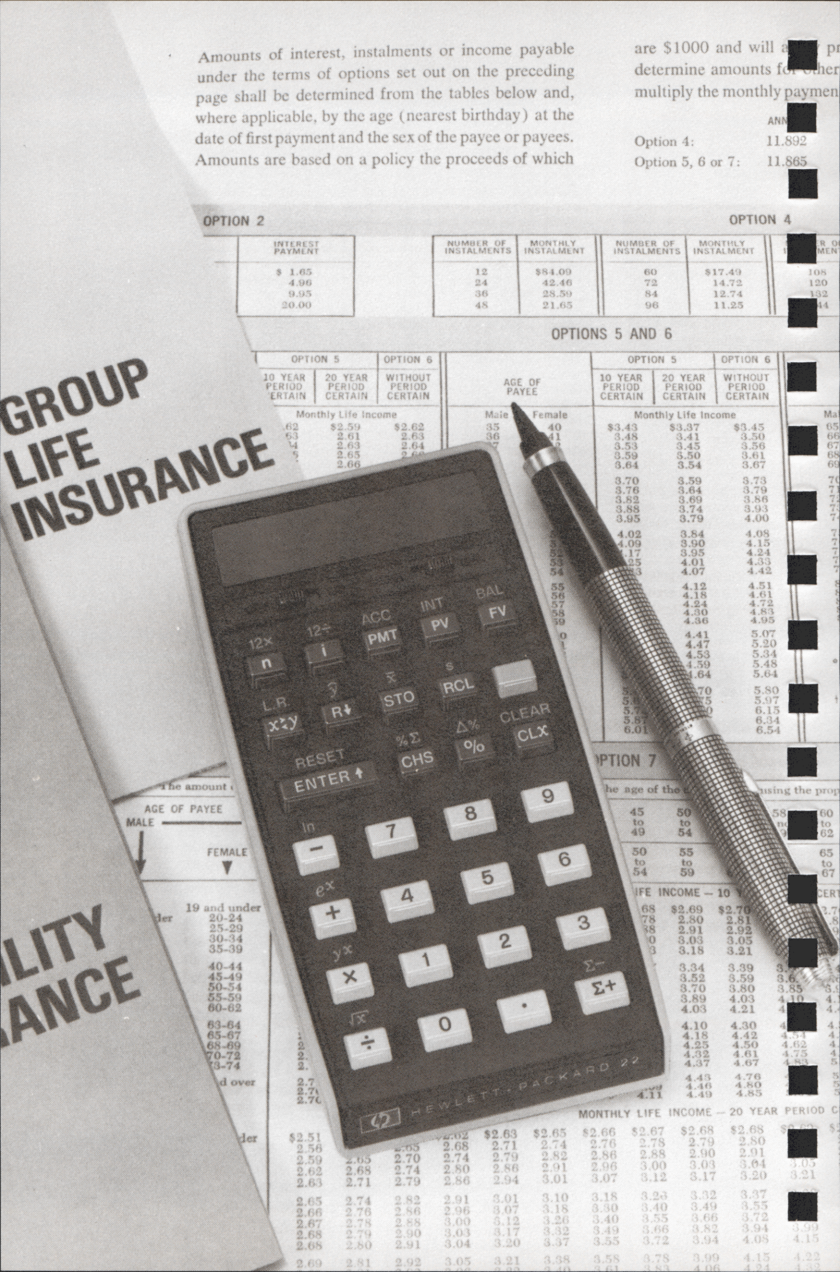
If you try 12%, the NPV after the sixth year is $6867.05 so the actual IRR is higher than 12%.

Next, try 13%. This time the NPV is negative (−$2265.95) so the IRR must be less than 13%.

As a result of these two iterations, the IRR must be between 12% and 13%. Since the NPV for 13% is closer to zero than the NPV for 12%, the IRR must be closer to 13%. The actual yield or IRR on this restaurant investment is approximately 12.75%.

* * * * * * * * * * *

Purchasing an HP-22 calculator is just the first of many profitable decisions. Now, you can use your HP-22 to evaluate time and money relationships *before* you invest, and to explore several financial alternatives. The next section describes the keystrokes used to solve the financial problems discussed here and specialized application problems. If you have understood these concepts, the mechanics of solving a problem—be it in real estate, banking, leasing, insurance or investments—will not be difficult.


Amounts of interest, instalments or income payable under the terms of options set out on the preceding page shall be determined from the tables below and, where applicable, by the age (nearest birthday) at the date of first payment and the sex of the payee or payees. Amounts are based on a policy the proceeds of which
Option 4:
11.892
Option 5, 6 or 7:
11.865
OPTION 2
OPTION 4
INTEREST PAYMENT
NUMBER OF INSTALMENTS
MONTHLY INSTALMENT
OPTIONS 5 AND 6
OPTION 5
OPTION 6
10 YEAR PERIOD CERTAIN
20 YEAR PERIOD CERTAIN
WITHOUT PERIOD CERTAIN
AGE OF PAYEE
Monthly Life Income
GROUP
LIFE
INSURANCE
OPTION 7
AGE OF PAYEE
MALE
FEMALE
MONTHLY LIFE INCOME — 20 YEAR PERIOD
HEWLETT · PACKARD 22
12x
n
12÷
i
ACC
PMT
INT
PV
BAL
FV
STO
RCL
L.R.
x⇄y
R↓
CHS
%
CLEAR
CLX
RESET
ENTER ↑
ln
e^x
y^x
√x
Σ−
Σ+

Section 3

Financial Applications

Calculating Percentage Problems

There are three function keys on your HP-22 for calculating percentage problems: %, Δ% and %Σ. Each serves a different purpose. The % function is used to find a percentage of a number. The Δ% function is used to find percent differences (% increase/decrease). And the %Σ function is used to find what percent one number is of another or of the total sum (proportion).

With the HP-22, you don't have to convert percents to their decimal equivalents; 4% need not be changed to .04. It can be keyed in the way you see and say it, 4 %.

Finding Percentage %

To find the percentage of a number, key in that base number and press ENTER↑. Then key in the numerical value of the percent and press %.

For example, find 14% of $300:

Press	Display
300 ENTER↑ 14 %	42.00

Notice that the keystroke sequence is similiar to the keystrokes you learned for arithmetic operations. The function happens immediately when you press the % key.

Example: Every year you set aside 4% of your company's profits for the employee retirement fund. If your company made a profit last year of $1,576,432, how much money was contributed to the fund?

Press	Display
1576432 ENTER↑ 4 %	63057.28

Finding Net Amount % + or % −

If you buy a new car, you have to figure the sales tax amount, then add that to the purchase price to find the total cost of the car. It is easy to calculate this net amount with your HP-22 because the calculator holds the base number while you calculate percentages.

For example, if the sales tax on a $6200 car is 5%, what is the amount of the tax and total cost of the car?

Press	Display	
6200 ENTER↑ 5 %	310.00	Percentage amount (sales tax).
+	6510.00	Net amount (base plus percentage amount).

If the dealer gives you a 10% discount on the car, what will your total cost be?

Press	Display	
6200 ENTER↑ 10 %	620.00	Amount of discount.
−	5580.00	Discounted price.
5 % +	5859.00	Net amount (discounted price plus sales tax).

Notice in the last problem that you subtracted a percentage amount and added a percentage amount, without repeating the base number. Once it is figured, the calculator stores it in the stack until you need it in a calculation.

Finding Percent Difference Between Two Numbers ▭ Δ%

To find the percent difference between two numbers—the ratio of increase or decrease—key in the base number, press ENTER↑, then key in the second number and press ▭ Δ%. Usually, the base amount, or the first number you key in, is the one that occurred first in time.

For example, your rent jumps from $285 a month to $335 a month. What percent is the increase?

Press	Display	
285 ENTER↑ 335 [gold] [△%]	17.54	% increase.

A positive answer denotes an increase, while a negative answer denotes a decrease.

Examples: Within a given period of time, the Dow-Jones average dropped from 985 to 695. What was the percentage decrease?

Press	Display	
985 ENTER↑ 695 [gold] [△%]	−29.44	% decrease.

You forgot to place a stop order and your stock fell from $57.50 to $13.25 a share. What percent is the decrease?

Press	Display	
57.50 ENTER↑ 13.25 [gold] [△%]	−76.96	% decrease.

Markup

Markup is a simple percentage calculation using the [△%] function and the wholesale or original purchase cost as the base number.

Example: You purchase typewriters at $159.95 wholesale and retail them for $195.00. What percent is your markup?

Press	Display	
159.95 ENTER↑ 195.00 [gold] [△%]	21.91	% markup.

Perhaps you have a fixed profit rate in mind and wish to establish your retail price. You would solve for the retail price the same as you solved for net amount.

Example: You purchase several valves at $2.26 from the manufacturer and wish to sell them at 25% profit. How much should you mark them up?

Press	Display	
2.26 ENTER↑ 25 [%]	$ 0.57	Amount of markup.
[+]	$ 2.83	Retail price.

Margin

Margin also is a simple percentage problem with the [△%] function, only this time you use the selling price as the base number. Returning to the previous typewriter example, the margin is calculated as follows:

Press	Display	
195 ENTER↑ 159.95		
[ ] [△%] CHS	17.97	% margin.

So, the markup is 21.91% and the margin is 17.97%.

Finding Percent of Total [ ] [%Σ]

∑ is the Greek symbol "sigma" which we use to mean sum or total. To find what percentage one number is of the total, add the numbers first by keying in each number followed by [Σ+]. ([Σ+] adds them in register 9.) Then key in the particular number you wish to convert to a percentage and press [ ] [%Σ].

Example 1: You own 150 shares of stock in Sleepy-Head Waterbeds, 52 shares of Flickering Films Inc., and 200 shares of Raucous Records Company. What percent of your portfolio does each represent?

Press	Display	
150 [Σ+]	1.00	First entry.
52 [Σ+]	2.00	Second entry.
200 [Σ+]	3.00	Third entry.
150 [ ] [%Σ]	37.31	% Sleepy-Head Waterbeds.
52 [ ] [%Σ]	12.94	% Flickering Films.
200 [ ] [%Σ]	49.75	% Raucous Records.

Why did 1.00 appear on the display when you actually keyed in 150? The [Σ+] key displays the number of entries rather than the numerical value itself.

When you use the [Σ+] key, press [ ] [RESET] between problems. Your previous answer still remains on the display, but register 9 is cleared and ready for new data. (Refer to appendix D for a detailed explanation.)

Example 2: Assume that your Sleepy-Head Waterbeds stock is worth $450; Flickering Films stock, $1404; and Raucous Records, $1500. What percent of the total value of your portfolio does each represent?

Press	Display	
▬ [RESET]		
450 [Σ+]	1.00	
1404 [Σ+]	2.00	
1500 [Σ+]	3.00	Number of entries.

This time, figure the percentage for each stock-holding yourself. The answers are given below. If you encounter problems or get an incorrect answer, review the last page.

13.42	% Sleepy-Head Waterbeds
41.86	% Flickering Films Inc.
44.72	% Raucous Records Co.

Finding Proportions ▬ [%Σ]

To find what percent one number is of another (proportion), simply state the problem as "A is what percent of B?" Key in B (the base number), press [Σ+], then key in A (the number to be converted), and press ▬ [%Σ].

For example, 64 is what percent of 340?

Press	Display	
▬ [RESET]		
340 [Σ+]	1.00	
64 ▬ [%Σ]	18.82	%.

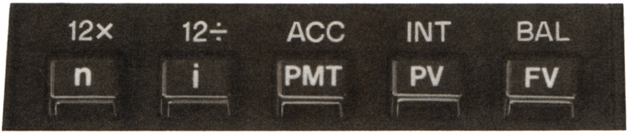
To purchase that $47,000 lakefront cabin that you've had your eye on, a $9400 down payment is required. What percentage of the price does your down payment represent?

Press	Display	
▬ [RESET]		
47000 [Σ+]	1.00	
9400 ▬ [%Σ]	20.00	%.

The Financial Function Keys

Your HP-22 has the most frequently used business calculations (including the associated formulas) preprogrammed into the top row of keys. The nature of these calculations is to use known data and solve for an unknown value.

The financial function keys are shown below:



The financial symbols (n, i, PMT, PV and FV) were explained in section 2. Three of these values must be entered to solve for one or both of the other values.

The n value represents the total number of compounding or payment periods. The alternate function above, [12×], converts yearly periods to monthly periods (12x) then automatically stores that number in **n**. To enter 30 years, press 30 **n**. If you wish to input that in monthly periods, press 30 [12×]. The HP-22 converts 30 (years) to 360 (months) and stores it automatically in **n**. There is no need to press **n** again.

The i value is the interest rate per period. If interest is expressed as an annual rate, pressing [12÷] calculates the interest rate per month and automatically stores it in **i**. To key in 9% annual interest, press 9 **i**. To input the monthly rate, press 9 [12÷]. There is no need to press **i** again, the monthly rate is automatically stored.

Remember that **n** and **i** *must* correspond to the same time frame. If n is months, then i must be the monthly interest rate.

The **PMT** key stands for payment or deposit amount. It assumes equal payments and must correspond to the same time frame as **n** or **i**. Don't mix monthly payments with annual periods or daily interest.

The **PV** key stands for present value or the amount of money at the start of the transaction.

The **FV** key represents the future value of money or the amount you will obtain/pay at the end of the term.

Both [n] and [i] are involved in all financial calculations. The nature of financial calculations on the HP-22 requires that given three of the above values (including n or i), you can solve for a fourth value. You can enter the values in any order.

The other three alternate functions are [ ] [ACC], [ ] [INT], and [ ] [BAL]. Often you may want to find out the accumulated interest and remaining balance on a loan between two points in time. The [ACC] and [BAL] functions provide this information quickly. The [INT] function enables you to calculate simple interest.

Displaying Financial Values

To see what number is in a particular financial register, press [RCL] then the desired location key ([n], [PV], [PMT], [PV], or [FV]).

Changing Financial Values

You may solve a problem and want to explore a different alternative by changing only one of the three known values—perhaps using a different interest rate or a greater/smaller payment amount.

With the HP-22 you don't have to re-enter all the numbers all over again. Simply key in the new value and press the appropriate financial function key. The new value will be substituted for the old value.

Let's take a sample problem. You borrow \$9000 at 9½% interest, to put a new swimming pool in the backyard. What will your monthly payments be if you pay the loan in 2 years? (For this example, set the switch in the upper right to BEGIN ▬▯ END. The switch will be explained shortly.)

Press	Display	
[ ] [RESET]		
2 [ ] [12×]	24.00	Number of months.
9.5 [ ] [12÷]	0.79	Interest rate/month.
9000 [PV]	9000.00	Amount of your loan.
[PMT]	413.23	Your monthly payment.

What if you take out a 3-year loan?

Press	Display	
3 [ ] [12×]	36.00	Number of months.
[PMT]	288.30	Alternative monthly payment.

This is considered an alternative of the same problem because given n, i and PV, you are solving for payment more than once.

If you decided "but I can only afford $200 a month, what would my loan be?"—that's a new problem. It requires a different set of three input values (now n, i and PMT to solve for PV), so you must press [ ] [RESET] and treat it as a new problem.

[RESET] and [CLEAR] Functions

Press [ ] [RESET] before starting a new financial calculation. The [RESET] function "resets" the financial registers (associated with the top-row keys), clears storage registers 5 thru 9 (associated with the [Σ+] key), and tells the calculator that these registers are ready to accept a new problem. Your previous answer is still left on the display so that you can use it elsewhere if you wish—either in the stack or store it in one of the manual storage registers. (For a more detailed explanation, refer to appendix D.)

Pressing [ ] [CLEAR] resets the financial registers and also clears all 10 storage registers and the stack.

The Annuity Switch: BEGIN ▯▮ END

A valuable aid is the annuity switch in the upper right corner of the keyboard. Section 2, Concepts of Time and Money, explains the difference between an ordinary annuity and an annuity due; the formulas for each are already built into your calculator. Simply set the annuity switch, and the HP-22 automatically selects the correct formula to solve your problem.

For annuity due problems, set the switch to BEGIN.

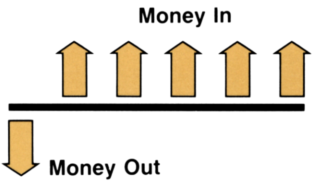
For ordinary annuity problems, set the switch to END. Since ordinary annuity problems are more common, you may want to leave the switch set to END until you specifically encounter an annuity due problem.

Switching from one mode to another does not erase data.

Reading This Applications Section

The applications and examples here are representative of a wide range of possible calculations. If your specific problem is not included in the pages that follow, *don't* assume that your HP-22 won't solve it. Many problems in finance involve terminology that differs from industry to industry. The basic concepts are the same, but we all speak our own language, the vernacular of our profession.

If the solution to your problem isn't evident at first, construct a cash flow diagram—a picture of money in and money out.



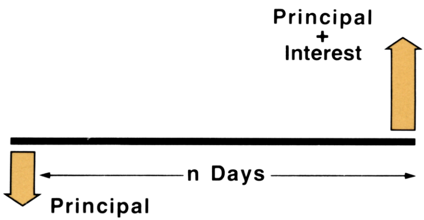
Once you've done this, look for a diagram like yours in the following pages. Instead of "Where is my problem?" ask yourself, "What are the cash flows?" You'll see that your problems, your cash flows, look like those that follow, and you'll soon grasp the power of your HP-22.

The Application Reference Guide, appendix C, can also help you find the keystrokes you need for a particular problem.

Simple Interest

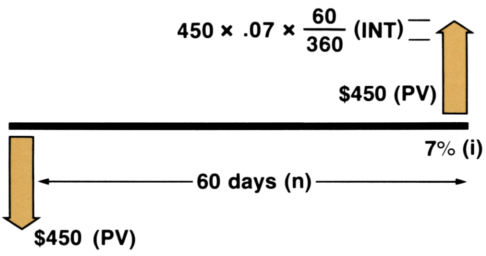
Accrued Interest—360-Day Basis

This calculation finds the amount of accrued simple interest on a 360-day basis when the number of days, interest rate, and principal (present value) are known. This calculation returns the principal to the Y-register.



1. Press [gold] RESET.
2. Input the following in any order:
   - Key in number of days, press n.
   - Key in annual interest rate, press i.
   - Key in principal, press PV.
3. Press [gold] INT to obtain the amount of accrued interest on a 360-day basis.
4. Press + to obtain the net amount (principal + interest) on a 360-day basis.

Example: Your good friend needs a loan to start his latest enterprise. He has requested that you lend him $450 for 60 days. You lend him the money at 7% simple interest, based on a 360-day year. What is the amount of accrued interest he will owe you in 60 days? What is the net amount?



Press	**Display**	
[gold] RESET		
60 n 7 i 450		
PV [gold] INT	5.25	Accrued interest.
+	455.25	Net amount.

Accrued Interest—365-day Basis

This calculation finds the amount of accrued simple interest on a 365-day basis when the number of days, interest rate, and principal (present value) are known.

1. Press RESET.
2. Input the following in any order:
   - Key in number of days, press n.
   - Key in annual interest rate, press i.
   - Key in principal, press PV.
3. Press INT R↓ x⇄y to obtain the accrued interest on a 365-day basis.
4. Press + to obtain the net amount (principal + interest) on a 365-day basis.

Example: What is the accrued interest and net amount on $450 for 60 days at 7%, figured on a 365-day year?

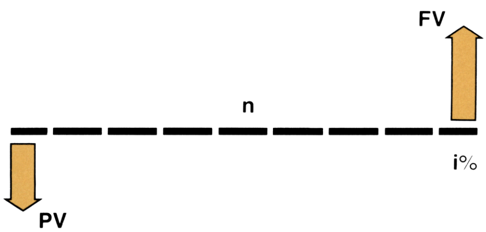
Press	Display	
RESET		
60 n 7 i 450 PV		
INT R↓ x⇄y	5.18	Accrued interest.
+	455.18	Net Amount.

Compound Interest

The position of the annuity switch has no effect on the following compound interest problems because you are not dealing with a series of payments. (Remember the definition of an annuity.)

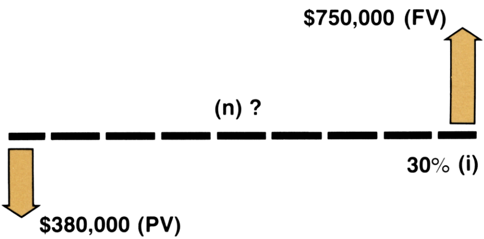
Number of Periods in a Compounded Amount

This calculation finds the number of compounding periods (n) when the interest rate, initial principal (present value) and compounded amount (future value) are given.



1. Press RESET.
2. Input the following in any order:
   - Key in the periodic interest rate, press **i**.
   - Key in the present value, press **PV**.
   - Key in the future value, press **FV**.
3. Press **n** to obtain the number of periods.

Example: A potential oil field site currently appraised at $380,000 appreciates at 30% per year. If this rate continues, how many years will it be before this land is worth $750,000?



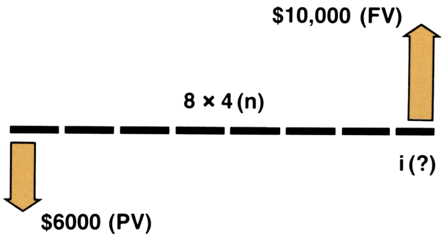
Press	Display	
RESET		
30 **i** 380000 **PV**		
750000 **FV** **n**	2.59	Years.

Interest Rate for Compounded Amounts

Given the initial amount, future value, and number of time periods, this calculates the interest rate:

1. Press RESET.
2. Input the following in any order:
   - Key in the number of periods, press **n**.
   - Key in the present value, press **PV**.
   - Key in the future value, press **FV**.
3. Press **i** to obtain the periodic interest rate.
4. Key in the number of periods per year, press **×** to obtain an annual interest rate.

Example: What annual interest rate must be obtained to amass $10,000 in 8 years on an investment of $6000, with quarterly compounding?



Press	Display	
RESET		
8 ENTER↑ 4 × n	32.00	Quarters.
6000 PV 10000		
FV i	1.61	% quarterly interest rate.
4 ×	6.44	% annual interest rate.

What if the compounding were monthly?

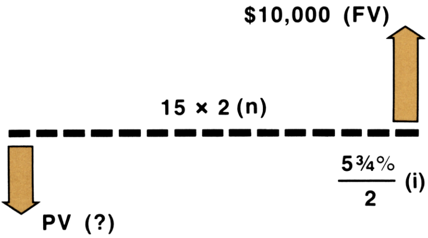
Press	Display	
8 n	96.00	Months.
i	0.53	% monthly interest rate.
12 ×	6.40	% annual interest rate.

Present Value of a Compounded Amount

Sometimes present value/future value calculations are thought of as moving money forward in time or, conversely, bringing money back to the present from some future date. This calculation finds the present value of a future amount "discounted back" at a given rate for a specified number of periods.

1. Press [RESET].
2. Input the following in any order:
   - Key in the number of periods, press [n].
   - Key in the periodic interest rate, press [i].
   - Key in the future value, press [FV].
3. Press [PV] to obtain the present value.

Example: You look forward to retirement in 15 years and wish to deposit one lump sum which will grow to $10,000 at that time, earning 5¾% interest compounded semiannually. How much do you need to deposit today to reach that goal?



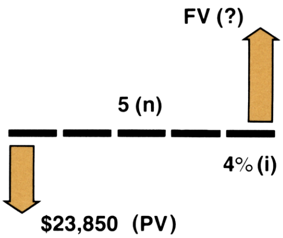
Press	Display	
[RESET]		
15 [ENTER↑] 2 [×] [n]	30.00	Semiannual periods.
5.75 [ENTER↑] 2 [÷] [i]	2.88	% semiannual interest rate.
10000 [FV] [PV]	4272.72	Principal to be invested.

Future Value of a Compounded Amount

This calculation finds the future value of an initial amount compounded at a given rate for a specified number of periods.

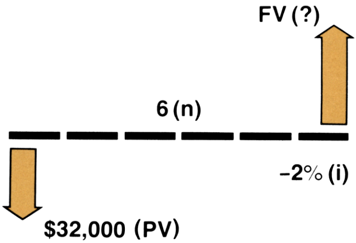
1. Press [shift] RESET.
2. Input the following in any order:
   - Key in the number of periods, press **n**.
   - Key in the periodic interest rate, press **i**.
   - Key in the present value, press **PV**.
3. Press **FV** to obtain the future value.

Example 1: A cabin purchased 5 years ago for $23,850 is located in an area where similar models have been appreciating at 4% per year. What is the current approximate value of the cabin?



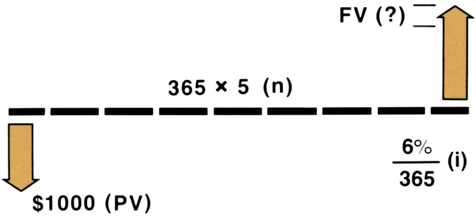
Press	Display	
[shift] RESET		
5 **n** 4 **i** 23850 **PV** **FV**	29017.17	Current value.

Example 2: Property values in an unattractive area are declining at the rate of 2% per year. What will property presently valued at $32,000 be worth in 6 years if this trend continues?



Press	Display	
RESET		
6 n 2 CHS i	-2.00	% interest rate.
32000 PV FV	28346.96	Property value.

Example 3: The local trading post manager opened up a savings operation 5 years ago, offering 6% compounded daily. Gold miner Yellowstone Sam deposited $1000 at that time, and now wants to know his present balance and the total accrued interest after all this time.



Press	Display	
RESET		
5 ENTER↑ 365 × n	1825.00	Days.
6 ENTER↑ 365 ÷ i	0.02	% daily interest rate.
1000 PV FV	1349.83	Present balance.
RCL PV −	349.83	Accrued interest.

Interest Rate Conversions

Nominal Rate Converted to Effective Rate

Given a nominal interest rate and the number of compounding periods per year, this keystroke procedure computes the effective annual interest rate.

1. Press RESET.
2. Key in the number of compounding periods per year, press n.

3. Key in the nominal rate, press RCL n ÷ i.
4. Key in 100, press PV.
5. Press FV.
6. Press RCL PV − to obtain effective annual rate.

Example: What is the effective annual rate of interest if the annual nominal rate of 12% is compounded quarterly?

Press	Display	
■ RESET		
4 n 12 RCL n		
÷ i	3.00	% quarterly interest rate.
100 PV FV		
RCL PV −	12.55	% effective interest rate.

Effective Rate Converted to Nominal Rate

Given an effective interest rate and the number of compounding periods per year, this routine calculates the nominal interest rate.

1. Press ■ RESET.
2. Key in number of periods per year, press n.
3. Key in 100, press ENTER↑ PV.
4. Key in effective annual rate, press + FV i.
5. Press RCL n × to get nominal rate.

Example: Find the nominal rate if the effective annual rate is 12.55% compounded quarterly.

Press	Display	
■ RESET		
4 n 100 ENTER↑ PV		
12.55 + FV i RCL n ×	12.00	% nominal interest rate.

Nominal Rate Converted to Continuous Effective Rate

This procedure converts a nominal annual interest rate to the continuous effective rate.

1. Key in the nominal rate, press ENTER↑.

2. Key in 100, press ⊟÷.
3. Press [■] [e^x] 1 [−] 100 [×] to obtain the continuous effective rate.

Example: Compute the continuous effective rate corresponding to a nominal rate of 8.75%.

Press	Display	
8.75 [ENTER↑] 100 [÷]		
[■] [e^x] 1 [−] 100 [×]	9.14	% continuous effective rate.

What's Really Happening? *The effective interest rate might be regarded as an interest rate that includes the "effect" of compounding for one year. Some interesting things to note about nominal and effective rates are:*

1. *If the number of compounding periods in a year is one, the nominal and effective rates are identical.*
2. *For a lump sum deposit, the compounded amount after 1 year is identical to the net amount using the effective annual rate. For example $1000 deposited at 5% compounded quarterly (5.0945% effective) for 1 year:*

Press	Display
[■] [RESET]	
4 [n] 5 [ENTER↑] 4 [÷] [i] 1000 [PV] [FV]	1050.95
or	
1000 [ENTER↑] 5.0945 [%] [+]	1050.95

3. *The more often a nominal rate is compounded, the larger its effective rate.*

Add-on Interest Rate Converted to APR

An add-on interest rate determines what portion of the principal will be added on for repayment of a loan. This sum is then divided by the number of months in the loan to determine the monthly payment. For example, a 10% add-on rate for 36 months on $3000 means add one-tenth of $3000 for 3 years (300 × 3)—usually called the "finance charge"—for a total of $3900. The monthly payment is $3900/36.

This keystroke procedure converts an add-on interest rate to an annual percentage rate when the add-on rate and number of months are known.

1. Press [gold] [RESET]. Set switch to END.
2. Key in number of months in loan, press [n] [ENTER↑] [ENTER↑] 12 [÷].
3. Key in add-on rate, press [ENTER↑] 100 [÷] [×] 1 [+] [÷] [PV] 1 [PMT].
4. Press [i] 12 [×] to obtain the APR.

Example: Calculate the APR and monthly payment of a 5%, $1000 add-on loan which has a life of 18 months.

Press	Display	
BEGIN [switch] END		
[gold] [RESET]		
18 [n] [ENTER↑] [ENTER↑]		
12 [÷] 5 [ENTER↑] 100 [÷]		
[×] 1 [+] [÷] [PV] 1 [PMT] [i]		
12 [×]	9.27	% annual percentage rate.
[gold] [RESET]		
[RCL] [n] [n] [RCL] [i] [i]		
1000 [PV] [PMT]	59.72	Monthly payment.

What's Really Happening? *A word of explanation about those last keystrokes: pressing [gold] [RESET] prepares the calculator to solve for a different variable. Since two of the values, n and i, are unchanged in both parts of the calculation, you can reuse them in the second part by pressing [RCL] [n] [n] and [RCL] [i] [i]. For a more detailed discussion, refer to appendix D.*

APR Converted to Add-On Interest Rate

Given the number of months and annual percentage rate, this procedure calculates the corresponding add-on interest rate.

1. Press [gold] [RESET]. Set switch to END.
2. Enter the following information in any order:
   - Key in number of months of loan, press [n].
   - Key in APR, press [gold] [i].
   - Key in 100, press [PV] [PMT].

3. Press 100 [RCL] [n] [÷] [−] 12 [×] to obtain add-on rate.

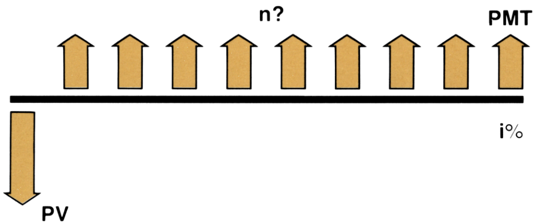
Example: What is the equivalent add-on rate for an 18-month loan with an APR of 14%?

Press	Display	
BEGIN ▬ END		
[RESET]		
18 [n] 14 [i]		
100 [PV] [PMT] 100		
[RCL] [n] [÷] [−] 12 [×]	7.63	% add-on rate.

Ordinary Annuities (Payments in Arrears)

Number of Periods—Ordinary Annuity

This procedure calculates the number of periods for an ordinary annuity when the present value and interest rate are known.



1. Set annuity switch to END and press [RESET].
2. Input the following in any order:
   - Key in the periodic interest rate, press [i].
   - Key in the payment, press [PMT].
   - Key in the present value, press [PV].
3. Press [n] to obtain the number of periods.

Example 1: Fur trapper Bill Buckskin wishes to invest in a $22,000 log cabin to keep the rain off. A local merchant has offered to loan Bill the $22,000 at 10.5% interest. Making $200 monthly payments, how long will it take Bill to repay his mortgage?

Press	Display	
BEGIN END		
RESET		
10.5 i	0.88	% monthly interest rate.
200 PMT 22000 PV n	376.89	Months.

Example 2: Through an insurance fund, you have accumulated $50,000 for your retirement. How long can you withdraw $3000 every 6 months (starting 6 months from now) if the fund earns 5% per annum compounded semiannually?

Press	Display	
BEGIN END		
RESET		
5 ENTER↑ 2 ÷ i 3000 PMT		
50000 PV n	21.83	Semiannual withdrawals.

Number of Periods to Reach a Specified Balance

This calculation gives the time to reduce an initial principal to a specified remaining balance with equal payments, as in amortization of a loan to a specified balloon payment.

1. Set annuity switch to END and press RESET.
2. Input the following in any order:
   - Key in the periodic interest rate, press i.
   - Key in the payment amount, press PMT.
   - Key in the specified remaining balance, press PV.
3. Press n to obtain the number of periods required to amortize the remaining balance.
4. Key in the total number of payments.
5. Press x≷y − to obtain the number of periods required to reach the specified balance.

Example 1: How long does it take to reduce a 30-year $40,000 mortgage with annual interest of 9% and payments of $321.85 to a remaining balance of $20,000?

Press	Display	
BEGIN END		
RESET		
9 i 321.85		
PMT 20000		
PV n	83.98	Months.
360 x≷y −	276.02	Months to reduce $40,000 to $20,000.
12 ÷	23.00	Years.

Example 2: If you begin annual withdrawals of $2500 next year from a $40,000 fund earning 6% annual interest, how long will it be before the fund is reduced to $25,000? (It would take 55.24 years to entirely use up the fund with $2500 withdrawals.)

Press	Display	
BEGIN END		
RESET		
6 i 2500 PMT		
25000 PV n	15.73	Years to use up $25,000.
55.24 x≷y −	39.51	Years to reduce $40,000 fund to $25,000.

Interest Rate (APR)-Ordinary Annuity

This routine computes the periodic interest rate for an ordinary annuity, given the number of periods, payment amount, and intial principal.

1. Set annuity switch to END and press RESET.
2. Input the following in any order:
   - Key in number of periods, press n.
   - Key in payment amount, press PMT.
   - Key in present value, press PV.
3. Press i to obtain the periodic interest rate.

The computed periodic rate is multiplied by the number of periods per year to give an annual rate.

Example 1: What is the annual interest rate on a 25-year, $32,500 mortgage with $230 monthly payments?

Press	Display	
BEGIN ■■ END		
▬ [RESET]		
25 ▬ [n]	300.00	Months.
230 [PMT] 32500 [PV] [i]	0.58	% monthly interest rate.
12 [×]	7.01	% annual interest rate.

Example 2: What is the yield on a mortgage purchased for $32,000 if the monthly payments are $318.89 and there are 300 payments remaining?

Press	Display	
BEGIN ■■ END		
▬ [RESET]		
300 [n] 318.89 [PMT]		
32000 [PV] [i]	0.94	% monthly interest rate.
12 [×]	11.23	% annual yield.

Borrowers are sometimes charged fees related to the issuance of a mortgage, which effectively raises the interest rate. Given the basis of the fee charge, the true annual percentage rate may be calculated.

Example 3: A borrower is charged 2 points for the issuance of his mortgage. If the mortgage amount is $50,000 for 30 years, and the interest rate is 9% per year, with monthly payments, what annual percentage rate is the borrower paying? (1 point is equal to 1% of the mortgage amount.)

First, compute the payment amount which is based on $50,000.

Press	Display	
BEGIN ▮ END		
▮ RESET		
30 ▮ n	360.00	Months.
9 ▮ i	0.75	% monthly interest rate.
50000 PV PMT	402.31	Monthly payment.
▮ RESET	402.31	Clear status for APR calculation.
RCL n n	360.00	Recall and reuse time periods.
RCL PMT PMT	402.31	Recall and reuse payment amount.
RCL PV 2 % − PV	49000.00	Effective amount received.
i	0.77	% monthly percentage rate.
12 ×	9.23	% annual percentage rate.

What's Really Happening? *For a mortgage with fees, the borrower is making payments on the original loan amount, which corresponds with the initial calculation of the payment amount. However, when the borrower pays fees, he effectively walks out of the lending institution door with less money. If you borrow $10,000, but are immediately charged $500 in fees, you really only receive $9500. But, your payments are based on $10,000. With fees, then, you're really paying the same for less money, which generates the need to compute the true APR.*

Payment Amount—Ordinary Annuity

This routine calculates the payment amount for an ordinary annuity given the number of periods, the initial principal, and the interest rate.

1. Set annuity switch to END and press ▮ RESET.
2. Input the following in any order:
   - Key in the number of periods, press n.
   - Key in the periodic interest rate, press i.
   - Key in the present value, press PV.
3. Press PMT to obtain the payment amount.

Example 1: Find the monthly payment amount on a 20-year, $27,000 mortgage with an 8.5% annual interest rate.

Press	Display	
BEGIN ▮▮ END		
■ RESET		
20 ■ n 8.5 ■ i		
27000 PV PMT	234.31	Monthly payment.

Example 2: An engineering firm is considering a potential investment of $700,000. If the company's minimum rate of return must be 16% per annum for a projected 20-year life, what must the annual cash flow be to justify the investment, assuming equal cash flows each year?

Press	Display	
BEGIN ▮▮ END		
■ RESET		
20 n 16 i		
700000 PV PMT	118066.92	Annual cash flow.

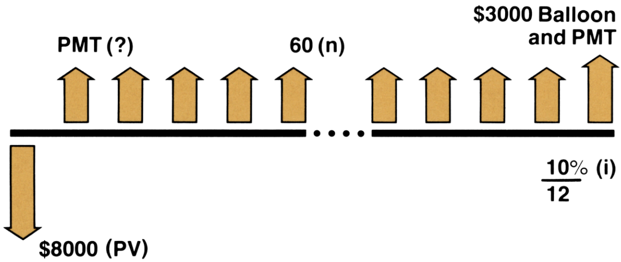
Example 3: XYZ Insurance Company manages funds that earn 6¼% annual interest for the depositor. How much can one withdraw annually from a $20,000 fund if the fund must last 15 years?

Press	Display	
BEGIN ▮▮ END		
■ RESET		
15 n 6.25 i		
20000 PV PMT	2093.02	Annual withdrawal.

Payment Amount—Ordinary Annuity with Balloon

By subtracting the present value of the balloon payment from the loan amount, this problem effectively becomes "What is the monthly payment on a direct reduction loan?"

Example: Yellowstone Sam is heading north, and will invest in an $8000 dog sled and team. His loan specifies 60 monthly payments at 10% with a balloon payment in the 60th month of $3000. What will his monthly payments be?



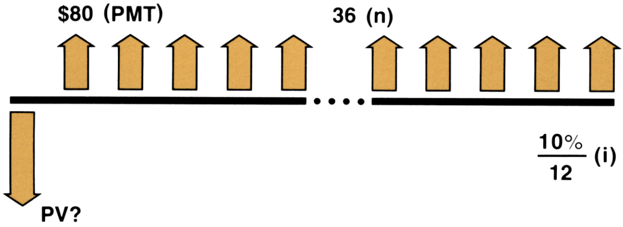
Press	Display	
BEGIN ▮ END		
RESET		
60 n 10 i		
3000 FV PV	1823.37	Present value of balloon.
RESET		
RCL n n	60.00	Months.
RCL i i	0.83	% monthly interest rate.
8000 RCL PV − PV	6176.63	Net present value of loan amount less balloon.
PMT	131.24	Monthly payment.

Present Value—Ordinary Annuity

This calculation finds the principal amount of a direct reduction loan when the interest rate, payment amount and number of payments are known.

1. Set annuity switch to END and press RESET.
2. Input the following in any order:
   - Key in number of periods, press n.
   - Key in the periodic interest rate, press i.
   - Key in the payment amount, press PMT.
3. Press PV to obtain the present value.

Example 1: Rather than buying a dog sled, if Yellowstone Sam decides to purchase a snowmobile and plans to pay $80 per month for 3 years, and he's willing to pay 10% annual interest, how much can he afford to pay for the snowmobile?



Press	Display	
BEGIN ▮▮▮ END		
▭ RESET		
36 n 10 ▭ i	0.83	% monthly interest rate.
80 PMT PV	2479.30	

Example 2: An investor wishes to purchase a mortgage with a balloon payment to yield him 13.5% per annum. What maximum price can he pay if there are 48 quarterly payments of $275 and a $10,000 balloon at the end of year 12?

Press	Display	
BEGIN ▮▮▮ END		
▭ RESET		
48 n 13.5 ENTER↑		
4 ÷ i	3.38	% quarterly interest rate.
10000 FV PV STO 1	2032.61	Present value of balloon.
▭ RESET		
RCL n n RCL i i	3.38	Recall and reuse n and i.

275 PMT PV	6491.94	Present value of payments.
RCL 1 +	8524.56	Mortgage purchase amount.

This example represents the sum of the present values of the balloon amount and the payments at the specified interest rate.

Accumulated Interest

This routine finds the accumulated interest on a loan between two points in time when the mortgage amount, payment amount, and interest rate are known. The exact value for n, the number of periods, is computed and stored at the same time the accumulated interest calculation is made. (The HP-22 always assumes this is ordinary annuity so there is no need to set the annuity switch.)

1. Press ▇ RESET.
2. Input the following in any order:
   - Key in the periodic interest rate, press i.
   - Key in the payment amount, press PMT.
   - Key in the present value, press PV.
   - Key in the number of the beginning payment in the time frame, press STO 8.
   - Key in the number of the last payment in the time frame, press STO 9.
3. Press ▇ ACC to obtain the accumulated interest between two points in time.
4. RCL n gives the exact term of the loan.

Example: For tax purposes, a home owner wishes to know the accumulated interest paid on his 8.75%, $45,000 mortgage between payments 22 and 33 inclusive (includes payments 22 and 33). He makes payments of $354; calculate the interest paid.

Press	Display
▇ RESET	
8.75 ▇ i	
354 PMT	
45000 PV	
22 STO 8	22.00
33 STO 9	33.00

Press	Display	
[ACC]	3871.46	Accumulated interest between payments 22 and 33.
RCL n	360.07	Total term of loan (exact number of months to amortize with $345 payments).

What's Really Happening? *For most calculations, the number computed for* n *is not an integer. This means that the last payment amount is somewhat smaller or larger than the normal payment amount. For example, on a typical 30-year mortgage, the computed n might be 359.58. This means that the mortgage is fully amortized after 359.58 months.*

Remaining Balance

This calculation finds the remaining balance of a fully amortized mortgage when the mortgage amount, payment amount, and interest rate are known. The actual value for n, the number of periods, is computed and stored at the time the calculation for remaining balance is made.

1. Press [RESET].
2. Input the following in any order:
   - Key in the periodic interest rate, press i.
   - Key in the payment amount, press PMT.
   - Key in the present value, press PV.
   - Key in the payment number associated with desired remaining balance, press STO 9.
3. Press [BAL] to obtain the remaining balance.

Example: For the previous example, what is the remaining balance after the 33rd payment is made? If the HP-22 still contains the necessary information, [BAL] may immediately follow [ACC]. Otherwise, solve as follows:

Press	Display	
[RESET]		
8.75 i		
354 PMT 45000 PV		
33 STO 9	33.00	
[BAL]	44038.57	Remaining balance after the 33rd payment.

The previous routine may be used to find the last payment amount, which often differs from the normal payment due to rounding of the accumulated interest over the life of the loan.

Example: Having executed the previous example, RCL n gives 360.07 payments. This means that it would take 360.07 payments of $354 to amortize the loan. In practice the 360th payment would be increased to take care of the ".07" portion. Find the amount of the 360th payment.

Press	Display	
360 STO 9	360.00	Last payment number.
BAL	26.38	Remaining balance corresponding to .07.
RCL PMT +	380.38	Last payment amount.

Amortization Schedule

This calculation generates an amortization schedule for a fully amortized loan from the first payment to a given payment, when the mortgage amount, interest rate and payment amount are known. (This routine does not depend on the position of the annuity switch.)

1. Press RESET
2. Input the following in any order.
   - Key in the periodic interest rate, press i.
   - Key in the payment amount, press PMT.
   - Key in the initial principal, press PV.
3. Key in the number of the period desired, press STO 8 STO 9.
4. Press RCL PMT ENTER↑ ACC to obtain the interest portion of the payment.
5. Press − to obtain the principal portion of the payment.
6. Press BAL to obtain the remaining balance.
7. Repeat steps 3 thru 6 for all desired periods.

Example: If Yellowstone Sam buys a snowmobile for $2479.30 at 10% interest per annum and pays $80 per month, generate an amortization schedule for him for the first 3 months of the loan.

Press	Display	
[ ] RESET		
10 [ ] i 80 PMT		
2479.30 PV		
1 STO 8 STO 9		
RCL PMT ENTER↑		
[ ] ACC	20.66	Interest portion 1^{st} payment.
−	59.34	Principal portion of 1^{st} payment.
[ ] BAL	2419.96	Remaining balance after 1^{st} payment.
2 STO 8 STO 9		
RCL PMT ENTER↑		
[ ] ACC	20.17	Interest portion of 2^{nd} payment.
−	59.83	Principal portion of 2^{nd} payment.
[ ] BAL	2360.13	Remaining balance after 2^{nd} payment.
3 STO 8 STO 9		
RCL PMT ENTER↑		
[ ] ACC	19.67	Interest portion of 3^{rd} payment.
−	60.33	Principal portion of 3^{rd} payment.
[ ] BAL	2299.79	Remaining balance after 3^{rd} payment.

Values on a bank amortization schedule may occasionally differ by a few cents because of rounding. For example, the accumulated interest on a \$12345 remaining balance for one month with an annual interest rate of 8.5% is $\$12345 \times \frac{8.5\%}{12} = \$87.44375$. The bank rounds this value to \$87.44, while the HP-22 computes the results exactly to 10 digits.

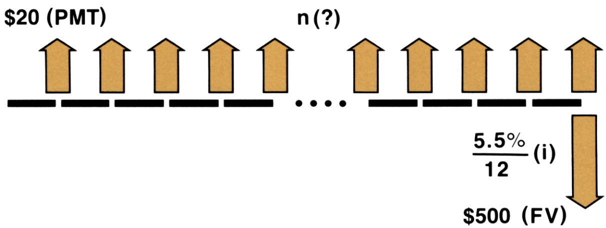
Sinking Funds—Ordinary Annuity

Number of Periods for Sinking Fund

This calculation finds the number of periods required to amass a specific sum when the payment amount and interest rate are known.

1. Set annuity switch to END and press [ ] [RESET].
2. Input the following in any order:
   - Key in the periodic interest rate, press [i].
   - Key in the payment amount, press [PMT].
   - Key in the future value, press [FV].
3. Press [n] to obtain the number of periods.

Example: Bill Buckskin is setting up a travel fund for a trip to Alaska. If he starts in a month, depositing $20 per month in a 5½% account, compounded monthly, how long will it take him from today to accumulate $500 for the trip?



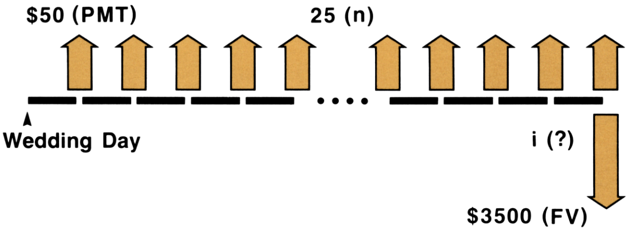
Press	Display	
BEGIN ▬ END		
[ ] [RESET]		
5.5 [ ] [i]	0.46	% monthly interest rate.
20 [PMT] 500 [FV] [n]	23.72	Months.

Interest Rate for Sinking Fund

This procedure finds the periodic interest rate for a sinking fund given the future value, number of time periods, and payment amount.

1. Set annuity switch to END and press [ ] RESET.
2. Input the following in any order:
   - Key in the number of periods, press n.
   - Key in the payment amount, press PMT.
   - Key in the future value, press FV.
3. Press i to obtain the periodic interest rate.

Example: You told your wife on your wedding day you would give her $50 every anniversary, but you forgot. She didn't, and on your 25th anniversary demands a $3500 fur coat, or she will file for breech of contract. What annual interest rate relates the 25 missed payments to $3500? Should you give her the coat?



Press	Display	
BEGIN ▮ END		
[ ] RESET		
25 n 50 PMT 3500 FV i	7.70	% annual interest rate.

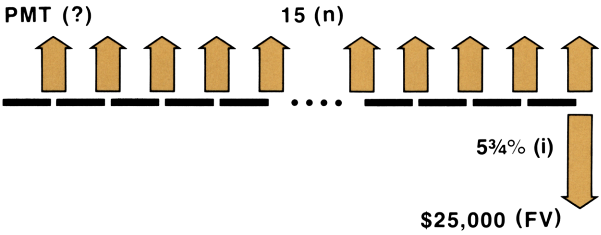
We don't know, but we never said the HP-22 would solve *all* your problems.

Payment Amount for Sinking Fund

This calculation finds the payment for a sinking fund, when the future value, number of periods, and interest rate are known.

1. Set annuity switch to END and press [ ] RESET.
2. Input the following in any order:
   - Key in the number of periods, press n.
   - Key in the periodic interest rate, press i.
   - Key in the future value, press FV.
3. Press PMT to obtain the payment amount.

Example: Calculate the sinking fund annual payment amount necessary to amass $25,000 in 15 years at 5¾%.



Press	**Display**
BEGIN ▮▮ END	
RESET	
15 n 5.75 i 25000 FV PMT	1094.69

Future Value for Sinking Fund

This routine finds the amount amassed at the end of a specified number of periods when the payment amount and interest rate are known.

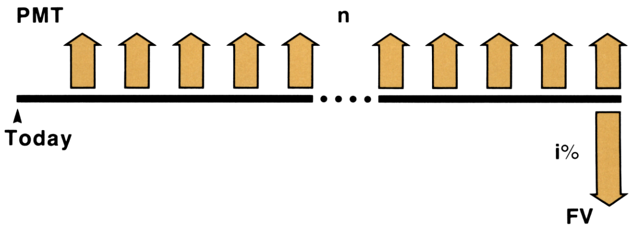
1. Set annuity switch to END and press RESET.
2. Input the following in any order:
   - Key in the number of periods, press n.
   - Key in the periodic interest rate, press i.
   - Key in the payment amount, press PMT.
3. Press FV to obtain the future value.

Example: If you had secretly deposited $50 each anniversary from your wedding date for 25 years in an 8% certificate account, how much would you have before and after buying a $3500 fur coat for your wife?

Press	**Display**
BEGIN ▮▮ END	
RESET	

25 **n** 8 **i** 50 **PMT** **FV**	3655.30	Before buying the coat.
3500 **−**	155.30	If you get to keep what's left over.
CLx	0.00	If your wife gets to keep what's left over.

What's Really Happening? *There is an important point to note about this group of calculations–it is that the payments start at the end of the first period. This is an ordinary annuity and is not like opening a savings account with a starting deposit today. The cash flow diagram is:*



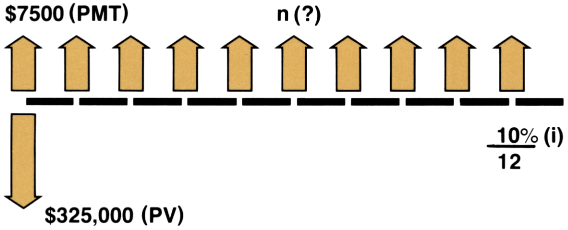
Annuities Due (Payments in Advance)

Number of Periods—Annuity Due

This calculation finds the total number of periods required for an initial amount when the payment amount (paid at the beginning of each period) and interest rate are known.

1. Set annuity switch to BEGIN and press ▭ RESET.
2. Input the following in any order:
   - Key in the periodic interest rate, press **i**.
   - Key in the payment amount, press **PMT**.
   - Key in the present value, press **PV**.

Example: Given an investment possibility of $325,000 which will immediately produce rental income of $7500 per month, how long must the investment be held to yield 10% per annum?



Press	Display	
BEGIN ▯▯▯ END		
▭ RESET		
10 ▭ i	0.83	% monthly interest rate.
7500 PMT 325000 PV n	53.43	Months.

Interest Rate—Annuity Due

This routine calculates the interest rate on an initial amount, given the number of periods and payment amount (paid at the beginning of each period).

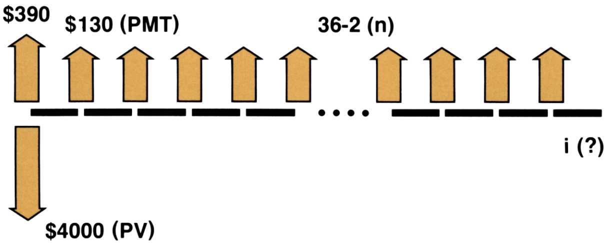
1. Set annuity switch to BEGIN and press ▭ RESET.
2. Input the following in any order:
   - Key in the number of periods, press n.
   - Key in the payment amount, press PMT.
   - Key in the present value, press PV.
3. Press i to obtain the periodic interest rate.

The periodic interest rate is multiplied by the number of periods per year to obtain an annual interest rate.

Example 1: Equipment worth $12,000 is leased for 8 years with monthly payments in advance of $200. The equipment is assumed to have no salvage value at the end of the lease. What yield rate does this represent?

Press	Display	
BEGIN ▮ END		
▭ RESET		
8 ▭ n	96.00	Months.
200 PMT 12000 PV i	1.09	% monthly yield.
12 ×	13.07	% annual yield.

Example 2: An auto leasing firm will lease a $4000 station wagon for 36 months assuming no salvage value, for $130 per month. The company specifies that the 35th and 36th payments are to be prepaid at the same time as the first payment. What is the annual yield for the leasing firm?



Press	Display	
BEGIN ▮ END		
▭ RESET		
34 n	34.00	Payments.
130 PMT	130.00	
4000 RCL PMT 2 ×	260.00	Payments 35 and 36.
− PV	3740.00	Subtracted from the initial amount.
i	1.05	% monthly yield.
12 ×	12.60	% annual yield.

Prepayment of the 35th and 36th payments effectively reduces the initial value by $260 and reduces the number of payments to 34.

Payment Amount—Annuity Due

This calculation finds the payment amount (paid at the beginning of each period) given the initial amount, number of periods and interest rate.

1. Set the annuity switch to BEGIN and press [gold key] RESET.
2. Input the following in any order:
   - Key in the number of periods, press n.
   - Key in the periodic interest rate, press i.
   - Key in the present value, press PV.
3. Press PMT to obtain the amount.

Example 1: The owner of a building presently worth $70,000 intends to lease it for 20 years at the end of which time he assumes the building will be worthless (i.e., has no residual value). How much must the quarterly payments (in advance) be to achieve a 10% annual yield?

Press	Display	
BEGIN [switch] END		
[gold key] RESET		
20 ENTER↑ 4 × n	80.00	Number of quarterly payments.
10 ENTER↑ 4 ÷ i	2.50	% quarterly interest rate.
70000 PV PMT	1982.27	Quarterly payment.

Example 2: If, in the previous example, the building is assumed to have a value of $15,000 at the end of the 80^{th} quarter, find the quarterly payment amount to yield 10%.

Press	Display	
BEGIN [switch] END		
[gold key] RESET		
80 n 10 ENTER↑ 4 ÷ i	2.50	% quarterly interest rate.
15000 FV PV	2080.57	Present value of residual value.
[gold key] RESET		
RCL n n	80.00	Number of quarters.

RCL i i	2.50	% quarterly interest rate.
70000 RCL PV − PV	67919.43	Lease amount less present value of residual value.
PMT	1923.35	Quarterly payment.

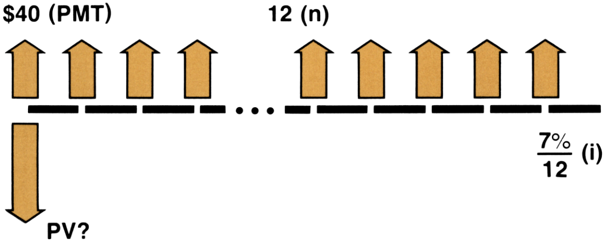
By subtracting the present value of the residual value from the present value of the building, this problem effectively becomes "What is the quarterly lease amount on a $67919.43 building for 20 years at a 10% annual interest rate?"

Present Value—Annuity Due

This calculation finds the present value of a series of payments made at the beginning of each period given the number of payments, interest rate, and payment amount.

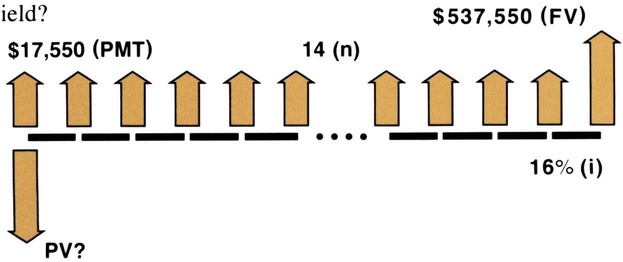
1. Set the annuity switch to BEGIN and press ▭ RESET.
2. Input the following in any order:
   - Key in the number of periods, press n.
   - Key in the periodic interest rate, press i.
   - Key in the payment amount, press PMT.
3. Press PV to obtain the present value.

Example 1: The owner of a downtown parking lot has achieved full occupancy and a 7% annual yield by renting parking spaces for $40 per month payable in advance. Several regular customers want to rent their spaces on an annual basis. What annual rent, also payable in advance, will maintain a 7% annual yield rate?



Press	Display	
BEGIN ▭ END		
[RESET]		
12 [n]	12.00	Monthly payments.
7 [i]	0.58	% monthly interest rate.
40 [PMT] [PV]	464.98	Equivalent annual payment.

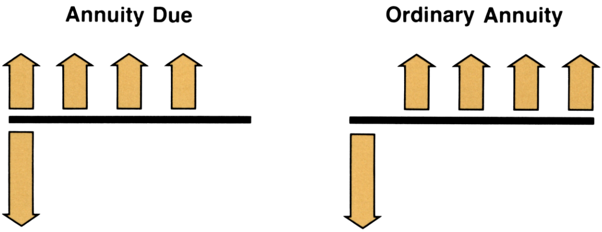
Example 2: A development company is selling a group of condominiums that rent for a total annual fee of $17,550 (in advance). The expected lifetime is 14 years with an equity value of $537,550 at that time. What price must the company receive to realize a 16% yield?



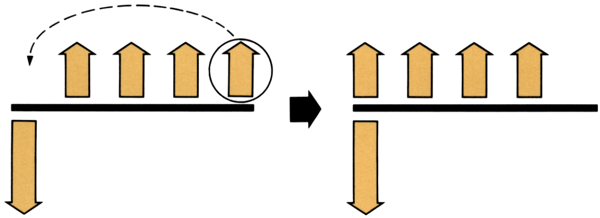
Press	Display	
BEGIN ▭ END		
[RESET]		
14 [n] 16 [i]		
17550 [PMT] [PV]	111307.96	Present value of annual rent.
[STO] 1		
[RESET]		
14 [n] 16 [i]		
537550 [FV] [PV]	67298.75	Present value of equity value.
[RCL] 1 [+]	178606.71	Selling price.

The calculation above finds the present value of both the rental payments and the equity value. The sum represents the price necessary to achieve the desired yield.

What's Really Happening? *By now you should feel comfortable with cash flow diagrams. For lease and rental calculations, the payment occurs at the beginning of the period. Take a look at the diagrams for annuity due and ordinary annuity:*



Sometimes a lease is phrased to effectively say ''ordinary annuity with the last payment at the front''. What does that mean? Look at the diagram:



That's annuity due! The switch on your HP-22 takes case of positioning the payments at the beginning or end of the period.

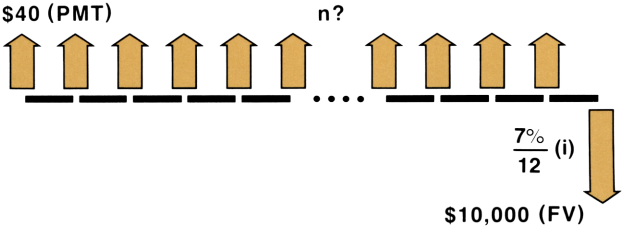
Savings and Insurance—Annuity Due

Number of Periods—Annuity Due

This calculation finds the number of periods required to amass a sum given the interest rate and payment amount (in advance).

1. Set the annuity switch to BEGIN and press RESET.
2. Input the following in any order:
   - Key in the periodic interest rate, press **i**.
   - Key in the payment amount, press **PMT**.
   - Key in the future value, press **FV**.
3. Press **n** to obtain the number of periods.

Example: If you deposit $40 per month in a savings account that pays 7% interest compounded monthly, how long will it take to amass $10,000?



Press	Display	
BEGIN ▯ END		
RESET		
7 **i**	0.58	% monthly interest rate.
40 **PMT** 10000 **FV** **n**	154.05	Months.
12 ÷	12.84	Years.

Interest Rate—Annuity Due

This routine finds the interest rate needed to amass a sum of money given the number of periods and payment amount (in advance).

1. Set the annuity switch to BEGIN and press RESET.

2. Input the following in any order:
   - Key in the number of periods, press **n**.
   - Key in the payment amount, press **PMT**.
   - Key in the future value, press **FV**.
3. Press **i** to obtain the periodic interest rate.

Example: If you require $10,000 at the end of 15 years, and can deposit $30 per month starting today, what monthly compound interest rate must you have?

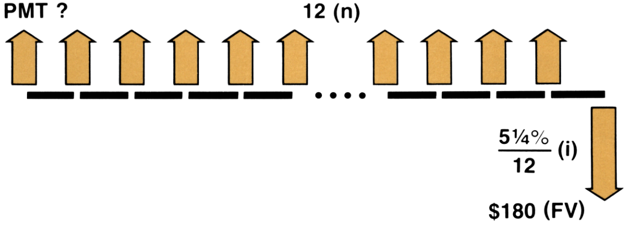
Press	Display	
BEGIN END		
RESET		
15 **n**	180.00	Months.
30 **PMT** 10000 **FV** **i**	0.63	% monthly interest rate.
12 ×	7.51	% annual interest rate.

Payment Amount—Annuity Due

This calculation finds the payment amount required to amass a sum in a specified number of periods given the interest rate.

1. Set the annuity switch to BEGIN and press RESET.
2. Input the following in any order:
   - Key in the number of periods, press **n**.
   - Key in the periodic interest rate, press **i**.
   - Key in the future value, press **FV**.
3. Press **PMT** to obtain the payment amount.

Example: If you want to save money to make a $180 purchase and can receive 5¼% interest compounded monthly, how much do you need to deposit monthly to make the purchase in one year?



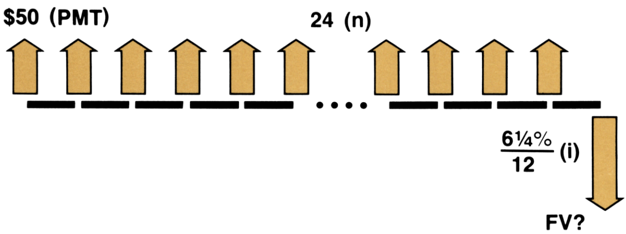
Press	Display	
BEGIN ▮▮ END		
RESET		
12 n 5.25 i	0.44	% monthly interest rate.
180 FV PMT	14.58	Monthly deposit.

Future Value—Annuity Due

This calculation finds the future value of a series of payments in advance, given the number of periods and interest rate.

1. Set the annuity switch to BEGIN and press RESET.
2. Input the following in any order:
   - Key in the number of periods, press n.
   - Key in the periodic interest rate, press i.
   - Key in the payment amount, press PMT.
3. Press FV to find the future value.

Example: If you can afford to deposit $50 per month in an account with 6¼% interest compounded monthly, how much will you have 2 years from now?



Press	Display	
BEGIN ▮▮ END		
RESET		
24 n 6.25 i	0.52	% monthly interest rate.
50 PMT FV	1281.34	Savings.

Depreciation

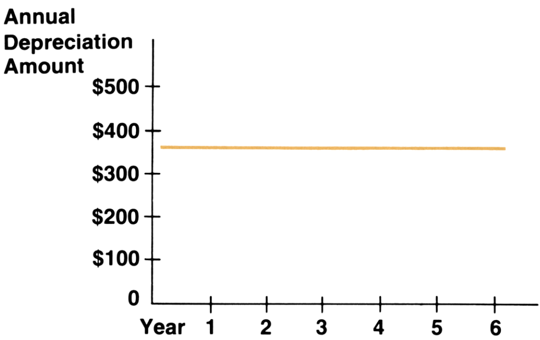
Machines, buildings, delivery trucks, showcases, tools and other tangible assets all decline in value with the passing of time. To provide for the replacement of obsolete or worn-out equipment, you usually set aside a fixed amount of money each year that is equal to the loss in value of that article during the year.

There are three methods of depreciation commonly used: straight-line, sum-of-the-years-digits, and declining-balance. Let's take a popcorn machine that costs $2500 brand new and has a salvage value of $400. We will depreciate the $2100 over a life expectancy of 6 years, using each of these three methods.

Straight-Line Method

The straight-line method (SL) is simply a matter of dividing the total depreciable amount by the number of useful years, then subtracting that amount each year from the item's value. The depreciation on the popcorn machine is $2100, divided by 6 years = $350 a year.

If you plot this depreciation amount on a graph, it looks like a straight line, hence, the name.



Straight-Line Depreciation

The advantage of the straight-line method is its simplicity—it's easy to figure and it's consistent. Your deduction is always the same.

To calculate the annual depreciation allowance on your HP-22:

1. Calculate and key in depreciable amount (cost less salvage value), press ENTER↑ ENTER↑.
2. Key in asset's useful life (number of years), press ÷ to obtain each year's depreciation.
3. Press STO 1 − to obtain depreciable value after the first year.
4. Continue pressing RCL 1 − to obtain the remaining depreciable value for each subsequent year. If book value is needed, add salvage value to depreciable value.

Example: A duplex costing $41,500 (exclusive of land) is depreciated over 25 years, using the straight-line method. What is the annual depreciation amount and remaining depreciable value for years 1 and 2 if it has no salvage value?

Press	Display	
41500 ENTER↑ ENTER↑	41500.00	Depreciable value.
25 ÷	1660.00	Annual depreciation amount.
STO 1 −	39840.00	Remaining depreciable value year 1.
RCL 1 −	38180.00	Remaining depreciable value year 2.

In reality, some items depreciate the most during their initial periods of use. For example, the value of your car declines the most in the first two years. Or perhaps your equipment doesn't wear out most in the early years, but you want to increase the initial depreciation for the financial advantages. This is called accelerated depreciation, and you would use one of the other two following methods.

Sum-of-the-Years-Digits Method

The sum-of-the-years-digits method (SOYD) is based on the sum of the digits from one to the number of years of the asset's life. For the popcorn machine, the life is 6 years, so:

$$6 + 5 + 4 + 3 + 2 + 1 = 21 \text{ (sum of the years digits)}$$

Theoretically, the first year you use up 6/21 of the asset's life; the second year, 5/21 of the asset's life, etc. So, the first year, you multiply the depreciation amount ($2100) by that year's use (6/21).

$$\frac{6}{21} \times \$2100 = \$600$$

The second year, by that year's use:

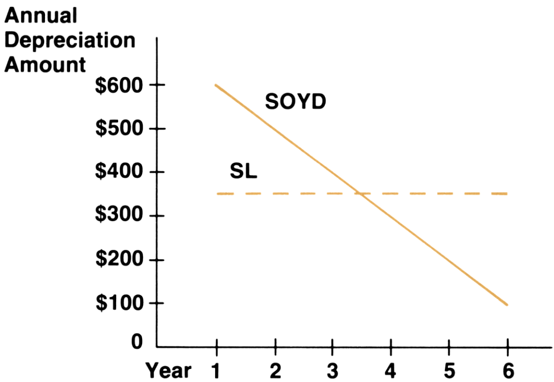
$$\frac{5}{21} \times \$2100 = \$500$$

The third year:

$$\frac{4}{21} \times \$2100 = \$400$$

Etc.

If you plot the depreciation on a graph, you can see the difference between the straight-line method and the sum-of-the-years-digits method.



Sum-of-the-Years-Digits Depreciation

The keystroke procedure below finds the depreciation and remaining depreciable value using the SOYD method for each year of an asset's depreciable life when its useful life expectancy and cost (less salvage value) are known.

1. Key in the beginning depreciable value, press ENTER↑.

2. Key in asset's depreciable life, press ENTER↑.
3. Press ENTER↑ 1 + CHS STO 1 × 2 ÷ ÷ STO 2 CLx.
4. Press 1 RCL 1 + STO 1 RCL 2 × to obtain the depreciation amount.
5. Press − to obtain the remaining depreciable value at the end of the year.
6. Repeat steps 4 and 5 for subsequent year's depreciation and remaining depreciable value.

Example: Apartments valued at $88,000 are depreciated over 25 years using SOYD depreciation. What is the depreciation amount and remaining depreciable value for the first 2 years?

Press	Display	
88000 ENTER↑	88000.00	Depreciable value.
25 ENTER↑	25.00	Depreciable life.
ENTER↑ 1 + CHS		
STO 1 ×		
2 ÷ ÷		
STO 2 CLx	0.00	
1 RCL 1 + STO 1		
RCL 2 ×	6769.23	Depreciation amount year 1.
−	81230.77	Remaining depreciable value, year 1.
1 RCL 1 + STO 1		
RCL 2 ×	6498.46	Depreciation amount year 2.
−	74732.31	Remaining depreciable value, year 2.

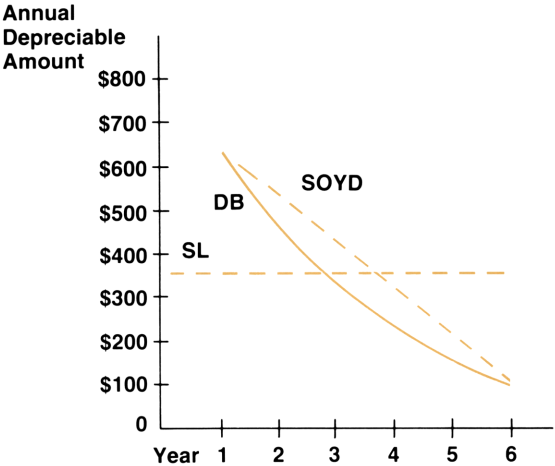
Declining Balance Method

With the declining balance method (sometimes called the fixed-rate method), a constant percentage is applied each year to the remaining balance (book value) to find the depreciable amount. The salvage value is not subtracted initially, but the asset may not be depreciated below this salvage value.

Certain declining balance "factors" are authorized for income tax purposes. A factor of 1.25 simply means 125% declining balance, 2.00 means double-declining balance. To compute the annual depreciation rate, divide the factor by the asset's estimated life in years. If you use a factor of 1.50 (150%) for the popcorn machine, then you will depreciate it $\frac{150}{6}$ or 25% each year.

	Depreciation	Balance (Book Value)
1st year 25% of \$2500 =	\$625.00	\$1875.00
2nd year 25% of \$1875 =	\$468.75	\$1406.25
3rd year 25% of \$1406.25 =	\$351.56	\$1054.69
4th year 25% of \$1054.69 =	\$263.67	\$ 791.02
5th year 25% of \$791.02 =	\$197.75	\$ 593.26
6th year \$593.26 − \$400 (salvage value) =	\$ 93.26	\$ 400.00

You can plot the declining balance depreciation on the graph to compare it with the other two methods.



Declining-Balance Depreciation

The following keystroke procedure finds the depreciation and remaining book value for each year of an asset's depreciable life when the declining factor, cost, salvage value, and life expectancy are known:

1. Key in declining factor (1.25 for 125% declining balance, 2.00 for double declining etc.), press ENTER↑ 100 ×.

2. Key in number of years of useful life, press ÷ STO 1.
3. Key in cost (do not deduct salvage value).
4. Press RCL 1 % to obtain first year's depreciation.
5. Press − to obtain remaining book value after first year.
6. Repeat steps 4 and 5 to obtain each suceeding year's depreciation and remaining book value until book value is equal to or less than the salvage value. In the period when the remaining book value is less than the salvage value, the previous book value should be reduced by the salvage value to obtain the final year's depreciation.

Example: The Drifter Apartments have a depreciable value of $86,000. The owner wishes to use 125% declining balance depreciation over 20 years. What is the annual depreciation amount and remaining book value in years 1 and 2?

Press	Display	
1.25 ENTER↑		
100 ×	125.00	
20 ÷ STO 1	6.25	
86000	86000	Depreciable value.
RCL 1 %	5375.00	First year depreciation.
−	80625.00	Remaining depreciable value, year 1.
RCL 1 %	5039.06	Second year depreciation.
−	75585.94	Remaining depreciable value, year 2.

Discounted Cash Flow Analysis

Discounted cash flow analysis is a way of evaluating investment alternatives in which investments are compared on the same basis—their present dollar worth. Two forms of discounted cash flow analysis are the net present value approach and the discounted or internal rate of return approach.

Net Present Value (NPV)

Assuming a minimum desired yield, the net present value method finds the present value of the future cash flows generated by the investment and compares this value to the initial investment. If this present value is greater than or equal to the investment, the investment meets your profit objectives. If the net present value is less than the investment, it is not profitable to the extent of the desired yield.

The following procedure is used to find the net present value of an investment, when the assumed yield rate (interest rate or cost of capital), periodic cash flows and time of occurrence are known. The same procedure is also used if you need to find the present value of a series of cash flows with no initial investment; simply key in zero for the initial investment.

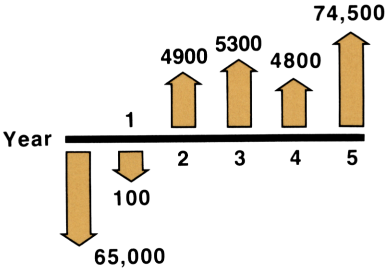
1. Press [ ] [RESET].
2. Key in the desired yield rate and press [i].
3. Key in the initial investment, press [CHS] [Σ+] and display 1.00.
4. Calculate the present value of the next cash flow:
   - Key in the period of the cash flow and press [n]. ([n] may be pressed immediately after [Σ+] if the period number correponds to the value returned to the display after [Σ+] is pressed.)
   - Key in the cash flow, press [FV].
   - Press [PV] to obtain the present value of the cash flow.
5. Press [Σ+] to accumulate the net present value of the cash flows and obtain n for the next period's cash flow.
6. Repeat steps 4 and 5 for all cash flows.
7. Press [RCL] 9 to obtain the present value of the cash flows.

As soon as the display shows a positive number after pressing [RCL] 9, the investment is recovered on a discounted cash flow basis. If the final NPV (after [RCL] 9 is pressed following last cash flow) is negative, the investment is not recovered on a discounted cash flow basis assuming the yield used for [i].

The position of the annuity switch for this calculation is irrelevant because these are not annuity calculations.

Example 1: An investor pays $65,000 for a duplex which he intends to keep 5 years and then sell. The first year he knows he will have to spend a considerable amount for repairs. If he desires a 9% after-tax yield rate and the after-tax cash flows are as follows, will he achieve this yield?

Year	Cash Flows ($)
1	-100
2	4900
3	5300
4	4800
5	74500



Press	Display	
RESET		
9 i		
65000 CHS	-65000.00	Initial investment
Σ+	1.00	First cash flow.
n 100 CHS		
FV PV	-91.74	Present value of first cash flow.
Σ+	2.00	Second cash flow.
n 4900 FV PV	4124.23	Present value of second cash flow.
Σ+	3.00	Third cash flow.
n 5300 FV PV	4092.57	Present value of third cash flow.
Σ+	4.00	Fourth cash flow.
n 4800 FV PV	3400.44	Present value of fourth cash flow.
Σ+	5.00	Fifth cash flow.
n 74500 FV PV	48419.89	Present value of fifth cash flow.
Σ+ RCL 9	-5054.61	Net present value.

Since the final NPV is negative, the investment does not achieve the desired yield.

Discounted or Internal Rate of Return (IRR)

The interest rate that equates the present value of all future cash flows with the original investment is known as the internal rate of return (also called discounted rate of return). The method here is to use the net present value approach, trying various rates until a rate is found which causes the NPV to be zero or close to it.

1. Choosing a best-guess interest rate, compute the net present value by the previous procedure for net present value.
2. If the net present value is negative, the internal rate of return is lower than the value chosen in step 1. If the net present value is positive, the internal rate of return is higher than the value chosen in step 1.
3. Repeat steps 1 and 2 until the net present value is sufficiently close to zero.

Example: What is the internal rate of return (yield on investment) for an office building costing $115,000 if the cash flows over the next 4 years are as follows:

Year	Cash Flow
1	$10,000
2	$9,500
3	$9,000
4	0
5	$140,000 (property sold in 5th year)

Try 10%:

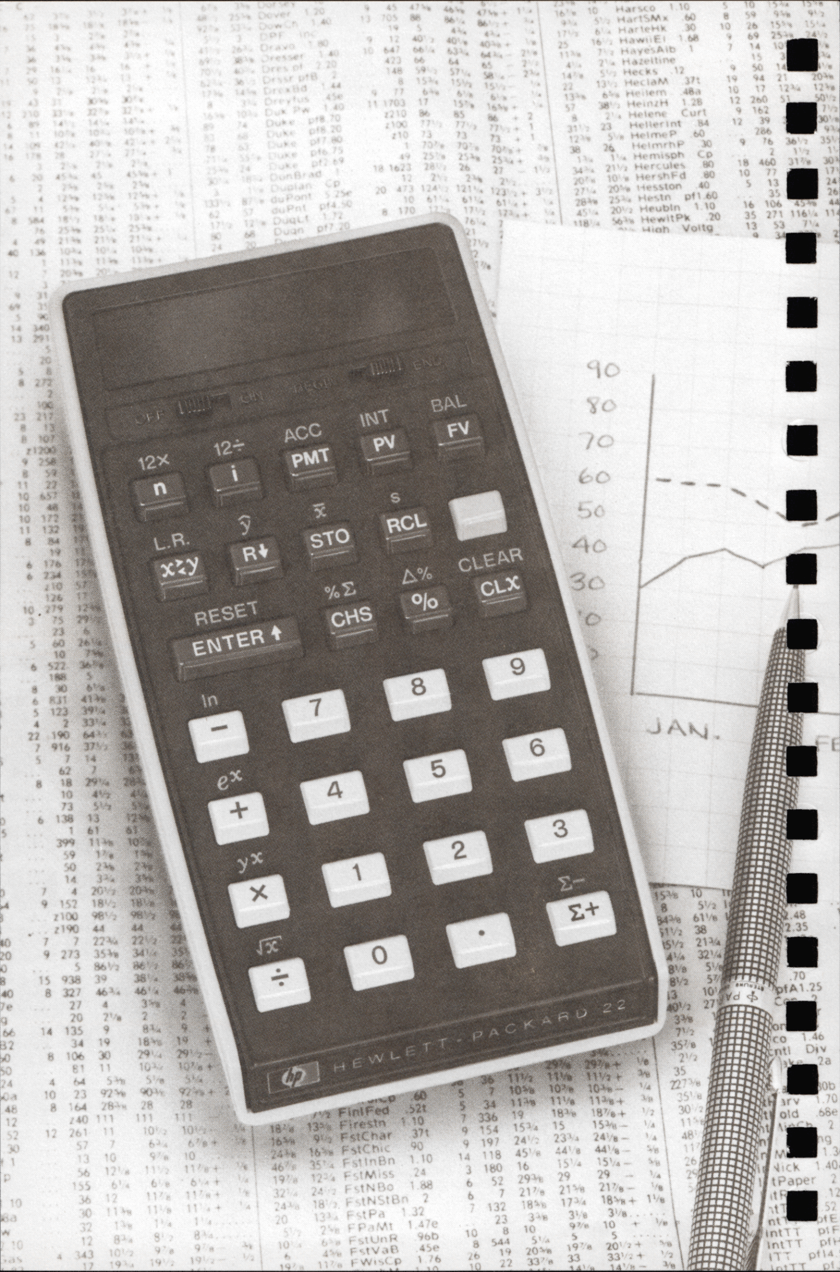
Press	Display	
[gold] RESET		
10 i	10.00	% interest rate.
115000 CHS	–115000.00	Initial investment.
Σ+	1.00	First cash flow.
n 10000 FV PV	9090.91	Present value of first cash flow at 10%.
Σ+	2.00	Second cash flow.
n 9500 FV PV	7851.24	Present value of second cash flow at 10%.

Σ+	3.00	Third cash flow.
n 9000 FV PV	6761.83	Present value of third cash flow at 10%.
Σ+	4.00	This period has 0 cash flow, so skip to 5.
5 n 140000 FV PV	86928.99	Present value of fifth cash flow at 10%.
Σ+ RCL 9	-4367.03	Net present value at 10%.

Since the NPV is negative, the actual internal rate of return is lower than 10%; therefore, try 9% and repeat the procedure.

Press	Display	
RESET		
9 i	9.00	% interest rate.
115000 CHS	-115000.00	Initial investment.
Σ+	1.00	
n 10000 FV PV	9174.31	Present value of first cash flow at 9%.
Σ+	2.00	
n 9500 FV PV	7995.96	Present value of second cash flow at 9%.
Σ+	3.00	
n 9000 FV PV	6949.65	Present value of third cash flow at 9%.
Σ+	4.00	
5 n	5.00	
140000 FV PV	90990.39	Present value of fifth cash flow at 9%.
Σ+ RCL 9	110.32	Net present value at 9%.

This time the NPV is positive, so the IRR is greater than 9%. As a result of these two trials, we know that the IRR is between 10% and 9%. The actual internal rate of return is about 9.02%.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Section 4

Mathematical and Statistical Functions

Just as your HP-22 has built-in financial formulas, it also contains several preprogrammed mathematical and statistical functions. These are useful in several areas—for example, real estate appraisal, forecasting, predicting trends, and investment analysis.

Performing Mathematical Functions

On the [−], [+], [×] and [÷] keys, you will find these alternative functions, used with the ■ key:

■ [ln]	Log_e (natural logarithm); this takes the log of the value on the display to base e (2.718281828).
■ [e^x]	Antilog_e (the natural antilog); this raises e (2.718281828) to the power of the value on the display.
■ [y^x]	Raises a positive number to a positive or negative power; raises 0 to a positive power; or raises a negative number to an integer power.
■ [$\sqrt{x}$]	Finds the square root of the number on the display.

Logarithms

To calculate the natural logarithm of a number, simply key in the number and press ■ [ln]. To find the $\log_e$ of 30.00:

Press	Display
30 ■ [ln]	3.40

Now, press ■ [e^x]. Because e^x is the antilog, you return to the original number, 30.

To calculate the common logarithm (base 10) of a number, key in the number, then press ■ [ln] 10 ■ [ln] [÷]. To find $\log_{10}$ 5:

Press	Display
5 ■ [ln] 10 ■ [ln] [÷]	0.70

Example 1: The solution for n in the future value/present value formulas may be found by the following:

$$n = \frac{\ln(FV/PV)}{\ln\left(1 + \frac{i}{100}\right)}$$

Find n where PV = 100, FV = 150 and i = 8%.

Press	Display
150 ENTER↑ 100 ÷ ln	0.41
8 ENTER↑ 100 ÷ 1 +	1.08
ln ÷	5.27

Example 2: Logarithms are also used in continuous compound interest formulas. If a savings institution offers a 7.79% effective rate on savings compounded continuously, what nominal rate does this represent?

$$\text{Nominal rate} = 100 \times \ln\left(\frac{7.79}{100} + 1\right)$$

Press	Display	
7.79 ENTER↑ 100 ÷	0.08	
1 + ln 100 ×	7.50	% nominal rate.

Antilogs

To calculate the antilog of a number, key in the number and press e^x. This raises e (2.718. . .) to the power of the value on the display:

Press	Display
30 e^x	1.0686475 13

Since you used the natural logarithm to convert continuous effective interest to the nominal rate, it follows that the antilog or e^x is used for the opposite conversion.

Example: To compute the continuous effective rate, given the nominal rate, the formula is:

$$\text{Continuous Effective Rate} = \left(e^{\frac{\text{nominal rate}}{100}} - 1 \right) \times 100$$

So, if a savings institution quotes a nominal rate of 6%, compounded continuously, what is the effective rate?

Press	Display	
6 ENTER↑ 100 ÷	0.06	
■ e^x 1 −		
100 ×	6.18	% continuous effective rate.

Exponentiation: Raising a Number to a Power

The y^x key raises a positive number to a positive or negative power or a negative number to an integer power. You use it the same simple way you've performed arithmetic and percentage operations; the function is executed immediately when you press the key.

1. Key in the base number. This number is designated y.
2. Press ENTER↑ to separate the first number from the second.
3. Key in the second number (power). This number is designated as x.
4. Press ■ y^x.

To calculate 3^6,

Press	Display
3 ENTER↑ 6 ■ y^x	729.00

To raise a number to a negative power, follow the same procedure and change the sign (CHS) of your exponent before you perform the operation. To solve $4.37^{-2.5}$,

Press	Display
4.37 ENTER↑ 2.5 CHS ■ y^x	0.03

You can also raise a negative number to an integer power. Key in the base number, press CHS, press ENTER↑, key in the integer power, and press ■ y^x. To solve $(-2)^3$,

Press	Display
2 CHS ENTER↑ 3 [gold] y^x	-8.00

With a negative base, if the exponent is an odd number, the answer will be negative. If the exponent is an even number, your answer will be positive:

$$(-2)^2 = -2 \times -2 = 4$$

$$(-2)^3 = -2 \times -2 \times -2 = -8$$

You can also use y^x to raise 0 to a positive power; but of course, your answer will always be zero.

Sample Problems: Solve the following problems with the y^x key:

$$(-2)^{-2} = [.25]$$

$$16^4 = [65536.00]$$

$$(-3)^3 = [-27.00]$$

Extracting Roots

To calculate the square root of a number, key in the number, then press [gold] $\sqrt{x}$.

To Solve	Press	Display
$\sqrt{25}$	25 [gold] $\sqrt{x}$	5.00
$\sqrt{81}$	81 [gold] $\sqrt{x}$	9.00

You must use a positive number. You cannot calculate the square root of a negative number; that's an illegal operation.

You can also extract higher roots, like cube roots and fourth roots; but you use the y^x key, not $\sqrt{x}$. The cube root of a number is that number raised to the 1/3 power. Thus, $\sqrt[3]{n}$ is the same as $n^{1/3}$; the fourth root can be written as $n^{1/4}$, etc.

Use the same keystroke sequence that you learned for exponentiation to extract higher roots:

1. Key in the base number and press ENTER↑.
2. Key in the number 1 and press ENTER↑. Key in the root desired, then press ÷.
3. Press [gold] y^x.

To solve $\sqrt[3]{5} = 5^{1/3}$:

Press	**Display**
5 ENTER↑ 1 ENTER↑ 3 ÷ [gold key] yˣ	1.71

Now, try $\sqrt[4]{81}$ or $81^{1/4}$:

Press	**Display**
81 ENTER↑ 1 ENTER↑ 4 ÷ [gold key] yˣ	3.00

Statistical Functions

The following keys are used in statistical calculations:

Σ+	Sigma plus. Totals sums, products and squares of numbers.
[gold key] Σ−	Sigma minus. Subtracts a sum, product or square from the summation in Σ+. Useful for subtracting out an incorrect entry.
[gold key] L.R.	Linear regression. Plots a straight line that best fits a set of data points (trend line) by finding the slope and y-intercept.
[gold key] $\hat{y}$	Linear estimate. Given a value for x, this computes the predicted value for y or where the point will fit on a plotted linear function.
[gold key] $\bar{x}$	Mean. Computes the mean or arithmetic average of a sample set of numbers.
[gold key] s	Standard deviation. Calculates the dispersion around the mean, or how the sample data relates to the arithmetic average.

If you have completely finished a statistical problem and wish to start a new problem, press [gold key] RESET to clear the old values from the storage registers. The display still contains your previous answer (*not* 0.00) so that it is in the stack, and you can store the answer or use it in other calculations. At the same time, the statistical registers (5 thru 9) are ready to accept new data.

Summations: Σ+

Pressing the Σ+ key automatically gives you several different sums and products of values in the X- and Y-registers. The calculator stores these values in registers 5 thru 9.

When you key a number into the display and press the Σ+ key, the following happens:

1. The number (x) on the display is added to the contents of storage register R_9.
2. The square (x^2) of the displayed number is added to the contents of register R_8.
3. The number 1 is added to storage register R_7.
4. The number in the Y-register of the stack is added to the contents of storage register R_6.
5. The displayed number that you keyed in is multiplied by the contents of the Y-register and that product is added to storage register R_5.
6. The total number in R_7 appears on the display. This indicates entry number 1, entry number 2, entry number 3, etc.

All that when you press Σ+! You can also input paired data using the Σ+ key. When you enter two values at a time (x and y), you must key in the y value first and separate the two numbers by **ENTER↑**. The general rule is:

y value **ENTER↑** x value Σ+

To recap, this is where values are stored inside your calculator:

Register	**Contents**
R_9	Summation of x values ($\sum x$).
R_8	Summation of x^2 values ($\sum x^2$).
R_7	Number of entries (n).
R_6	Summation of y values ($\sum y$).
R_5	Summation of products of x and y values ($\sum xy$).
Display (X-register)	Number of entries (n).

Immediately, you have a powerful data bank for statistical calculations. To see any of these summations at any time, simply recall the contents of the desired register. Remember, when you press **RCL** 5 thru 9, you will not lose that data. Only a copy of the number (sum) appears in the display.

The powerful capability of the [Σ+] key becomes readily apparent as you solve some application problems.

Mean

Given a set of numbers, you can find the mean or arithmetic average of that sample. Whether it's the average of test scores, grades, or last month's sales figures, your HP-22 handles it easily. The keystroke procedure is:

1. Press [ ] [RESET].
2. Key in the first number and press [Σ+]; then the second number and [Σ+] again; the third, etc. Continue until you have entered all the values.
3. Press [ ] [$\bar{x}$].

Example: A survey showed 10 of the wealthiest persons in the world were the following ages:

62 84 47 58 68 60 62 59 71 73

To find the average (mean) age of this sample of wealthy persons:

Press	Display	
[ ] [RESET]		Storage registers cleared.
62 [Σ+]	1.00	First entry.
84 [Σ+] 47 [Σ+] 58 [Σ+]		
68 [Σ+] 60 [Σ+] 62 [Σ+]		
59 [Σ+] 71 [Σ+] 73 [Σ+]	10.00	Total number of entries.
[ ] [$\bar{x}$]	64.40	Average (mean) age in years.

For the moment, leave that number on the display and don't reset or clear the calculator.

Deleting and Correcting Data

If you inadvertently keyed in an incorrect number in the age series, you don't have to start over again. If you keyed it before pressing [Σ+], simply press [CLx] to clear the display, then continue on with the correct value.

If you had already added in the wrong value, simply key in that wrong number, subtract it out by pressing [ ] [Σ−], then continue on with the correct number and the rest of the series.

For example, suppose the 62-year old member of the sample were to lose his position as one of the wealthiest persons because of a series of ill-advised investments in cocoa futures. He is replaced by a 21-year old rock musician, so the data changes as follows:

Press	Display	
62	62.00	Data to be replaced.
[gold] Σ-	9.00	Number of entries (n) is now nine.
21	21.00	The new data.
Σ+	10.00	Number of entries (n) is 10 again.

The new data has been calculated into each of the summations present in the storage registers. To see the new mean:

Press	Display	
[gold] $\bar{x}$	60.30	The new average (mean) age.

Standard Deviation

You already have the mean figure from the previous example on the display. To calculate the standard deviation (a measure of dispersion around the mean), simply:

Press	Display	
[gold] s	17.09	Years (standard deviation)

The HP-22 calculates standard deviation according to the formula:

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1}}$$

Notice that the 10 wealthy people that we used is a sample. If the 10 people used were actually the *10 wealthiest persons in the world* or if we used *all the wealthy people,* then the data would be considered a population rather than a sample.

The relationship between *sample* standard deviation (s) and population standard deviation (s′) is given.

$$s' = s\sqrt{\frac{n - 1}{n}}$$

Since n (number of entries) is stored in register R_7, you can convert sample standard deviation to population standard deviation. You already have (s) 17.09 on the display, so simply:

Press	Display	
RCL 7	10.00	Number of entries.
1 [−]	9.00	Calculates n − 1.
RCL 7 [÷]	0.90	
[ ] [√x] [×]	16.21	Population standard deviation (s′).

Linear Regression

Linear regression is a statistical method for finding a straight line that best fits a set of data points, thus providing a relationship between two variables. Sometimes this is also called a "trend line" or "least squares line." (Trend line assumes that there is equal time between data points.)

What is an application of linear regression in business? Forecasting and market projections can be made based on a line fit to a group of historical data. One example would be an appraiser who wants to know how much the value of an office building may change if its square footage is increased. If the appraiser has data on several similar buildings with different square footages and values, he can compute the regression line which gives an approximate relationship between the two variables, square footage and value, (i.e., he may find that the building value increases by $200 for each additional square foot of space).

After a group of data points have been totaled in registers R_5 thru R_9 using the [Σ+] key, you can calculate the coefficients of the linear equation $y = A + Bx$ using the least squares method by pressing [ ] [L.R.]. Naturally, at least two data points must be in the machine before a least squares line can be fitted to them. The y intercept of the least squares line (A) is calculated using the equation:

$$A = \frac{\sum y \sum x^2 - \sum x \sum xy}{n \sum x^2 - (\sum x)^2}$$

The slope of the least squares line (B) is calculated using the equation:

$$B = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$$

These formulas are preprogrammed in your HP-22 calculator, so you don't need to memorize them—just key in the data.

Example: A commercial land appraiser has examined six vacant lots in the downtown section of a local community, all of which have the same depths but different frontages and values. Based on the following input data, what is the relationship between frontage and lot value?

Input Data:

(x) Lot frontage (feet)	(y) Lot value ($)
70.8	10,100
60.0	9,000
85.0	12,700
75.2	11,120
69.5	11,000
84.0	12,500

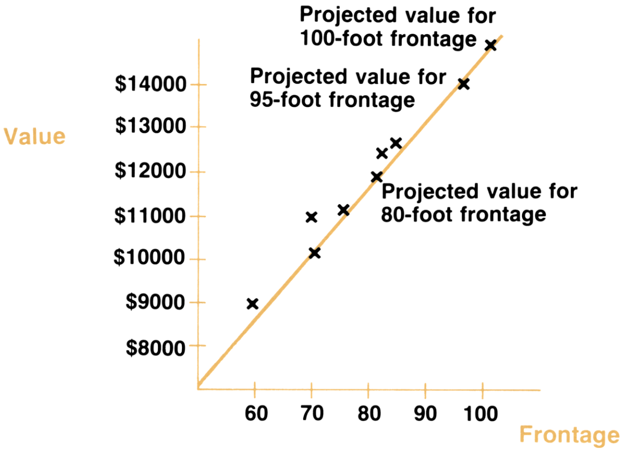
Accumulate the data by the summation procedure previously outlined. (Notice that when you enter two values, x and y, you must enter the y value first.)

Press	Display	
■ RESET		
10100 ENTER↑ 70.8 Σ+	1.00	First entry.
9000 ENTER↑ 60 Σ+	2.00	Second entry.
12700 ENTER↑ 85 Σ+	3.00	Third entry.
11120 ENTER↑ 75.2 Σ+	4.00	Fourth entry.
11000 ENTER↑ 69.5 Σ+	5.00	Fifth entry.
12500 ENTER↑ 84 Σ+	6.00	Sixth entry.
■ L.R.	393.90	A = y-intercept.
x⇄y	144.11	B = slope of line.

Thus the equation of the regression line is:

$$y = 393.90 + 144.11\,x$$

The y-intercept value represents the projected value for x = 0. The slope indicates the change in the projected value caused by an incremental change in the x value. Plotting this example, you see that a 1-foot increase in the frontage results in a projected increase in value of $144.11.



Linear Estimate

Having plotted a line, you can quickly estimate other values. With the data totaled in registers R_5 thru R_9, a predicted y ($\hat{y}$) can be calculated by keying in an x value and pressing [$\hat{y}$]. This will help you plot the least squares line.

Example 1: For the previous example, find projected values for 80-, 95-, and 100-foot frontages.

Press	Display	
80 [$\hat{y}$]	11922.65	80-foot frontage projected value.
95 [$\hat{y}$]	14084.29	95-foot frontage projected value.
100 [$\hat{y}$]	14804.83	100-foot frontage projected value.

Example 2: You bought a house three years ago for $47,500. The first year it appreciated $5,000. The second year its value rose to $60,000. Today you figure the market price to be $64,000 if you were to sell. What will your house be worth next year?

Press	Display
[RESET]	
47500 [ENTER↑] 1 [Σ+]	1.00

52500 ENTER↑ 2 Σ+	2.00
60000 ENTER↑ 3 Σ+	3.00
64000 ENTER↑ 4 Σ+	4.00

To make a projection for next year (year 5), you don't need to calculate the linear regression function first. You simply solve for $\hat{y}$:

Press	Display
5 ■ $\hat{y}$	70250.00

Exponential Curve Fit (Growth Curve)

Using the ln function of the HP-22, a least squares exponential curve fit may easily be calculated according to the equation $y = be^{mx}$. The procedure is as follows:

1. For each input pair of values, calculate the ln of the y-value, as the pairs are input with the Σ+ key.
2. After all data pairs are input, press ■ L.R. ■ e^x to obtain b in the equation above.
3. Press x⇄y ■ e^x 1 − to obtain the growth rate.
4. To make an estimate, key in the x-value, press ■ $\hat{y}$ ■ e^x.

This technique is often used to determine the growth rate of a variable such as a stock's value over time, when it is suspected that the performance is nonlinear. The value for m ($y = be^{mx}$) is the decimal value of the continuous growth rate. For instance, assume after keying in several end-of-month price quotes for a particular stock, it is determined that the value for m is .10. This means that over the measured period the stock has experienced a 10% continuous growth rate.

Example: A stock's price history is listed below. What continuous growth rate does this represent? If the stock continues this growth rate, what is the price projected to be at the end of 1974 (year 5)?

End of Year	Price
1970 (1)	52½
1971 (2)	55¼
1972 (3)	(missing data)
1973 (4)	61
1974 (5)	?

Press	Display	
RESET		
52.5 ln 1 Σ+	1.00	First data pair input.
55.25 ln 2 Σ+	2.00	Second data pair input.
61 ln 4 Σ+	3.00	Third data pair input.
L.R. e^x	49.96	y-intercept.
x≷y e^x 1 −	0.05	5% growth rate.
5 $\hat{y}$ e^x	64.14	Projected price at the end of year 1974.

Appendix A

Accessories, Maintenance and Service

Standard Accessories

Your HP-22 comes complete with the following standard accessories: battery pack, ac adapter/recharger, soft carrying case, and the *HP-22 Owner's Handbook.*

CAUTION:
Use of any batteries other than the Hewlett-Packard battery pack may damage your calculator.

Optional Accessories

Other accessories are described on the Accessory Order Form. To order additional standard or optional accessories, see your nearest dealer or send the Accessory Order Form, along with a check or money order, to:

Hewlett-Packard Company
Corvallis Division
1000 N.E. Circle Blvd.
Corvallis, OR 97330

Outside the United States, mail the Accessory Order Form to the Hewlett-Packard Sales Office nearest you.

Calculator Operation

Charging Times

Your calculator contains a rechargeable battery pack. *The batteries must be in the calculator for it to operate.* With the ac adapter/recharger connected to the calculator and to an ac power outlet, the batteries will charge, whether the calculator is ON or OFF. Normal charging times from dead battery to full charge are:

Calculator ON	17 hours
Calculator OFF	6 hours

Charge the battery before portable use. Shorter charge periods than the above will reduce battery operating time.

Whether the calculator is OFF or ON, the battery pack will never overcharge with the charger connected to the ac line.

Traveling with the HP-22

Taking a business trip aboard? If so, take your calculator with you. An optional recharger is available with a voltage select switch for two ranges: 100 to 127 volts and 200 to 254 volts. Check the line voltage of the country you intend to visit, and set the switch to the appropriate range.

The only other item you will need is an ac plug adapter for the wall outlet (the same adapter you would use for an electric shaver or other personal appliance). These are available at some hotels, or you may purchase the adapter in the country of your destination.

Charging the Battery

CAUTION:
Your HP-22 may be damaged if the ac adapter/recharger is not set for the correct line voltage, or if you use any recharger other than the HP recharger supplied with your calculator.

The procedure for using the ac adapter/recharger is as follows:

1. If the recharger has a voltage select switch (optional), make sure it is set to the proper voltage range. (In the U.S., set it to 100 to 127 volts.)
2. Set the HP-22 power switch to OFF.
3. Insert the recharger plug into the rear connector of the HP-22, and insert the power plug into a live power outlet. The HP-22 may feel warm to the touch while recharging. Don't be concerned; this is normal because part of the charging circuit is inside the calculator.
4. At the end of the charging period, you may continue to use your HP-22 with ac power; or turn the power switch OFF and disconnect the recharger for battery-only operation.

If the battery pack won't hold a charge, it may be defective. If the one-year warranty is in effect, return the pack to Hewlett-Packard according to the shipping instructions. If your warranty has expired, see your nearest dealer or use the Accessory Order Form to purchase a replacement.

Battery Operation

Use only the HP battery pack. A fully-charged pack provides approximately 3 to 5 hours of continuous operation. By turning the power OFF when the calculator is not in use, you can conserve energy and make the HP-22 battery pack last easily through a normal working day.

Low Power Display

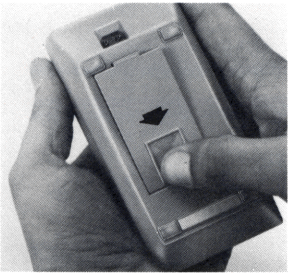
When the batteries get low, several decimal points will appear on the display. This means you have approximately one minute of operating time left. Then you must either charge the battery or insert a fully-charged spare battery pack.

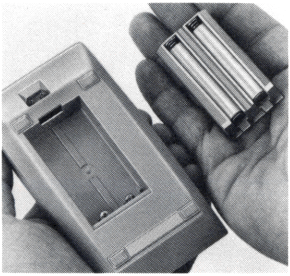
6 0.2. . . . . . 23 **Low Power Display**

True Decimal Point

Replacing the Battery Pack

To replace your battery pack, use the following procedure:

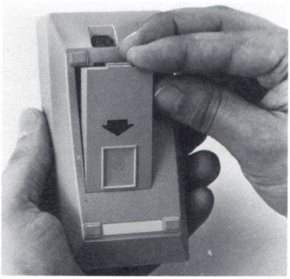




1. Turn the HP-22 power switch to OFF, and disconnect the ac adapter/recharger from the calculator.

2. With your thumb on the square inset on the rear of the calculator, press down and slide the door in the direction of the arrow. The latch will spring open.

3. Remove the battery pack.



4. To insert the new pack, slant the leading edge of the pack into the edge of the doorway, then snap the latch down into place.

If you use your HP-22 extensively on batteries, you may want to order the Reserve Power Pack. This optional accessory lets you charge one pack while using the other.

Blank Display

If the display blanks out, turn the calculator OFF, then ON. If 0.00 does not appear on the display, check the following:

1. Make sure the ac adapter/recharger is plugged into an ac outlet. If not, turn the calculator OFF before plugging in the recharger.
2. Examine the battery pack to see if the contacts are dirty.
3. Substitute a fully-charged battery pack, if available, for the one that was in the calculator.

If the display is still blank, your calculator should be serviced. (Refer to *Warranty* in this appendix.)

Temperature Range

Temperature ranges for the calculator are:

Operating:	0° to 45°C	32° to 113°F
Charging:	15° to 40°C	59° to 104°F
Storage:	−40° to +55°C	−40° to +131°F

Temperatures above or below these specified limits may not cause permanent damage to your calculator, but you probably will get incorrect answers. Also, temperature extremes will damage the batteries.

Warranty

Full One-Year Warranty

All Hewlett-Packard calculators and their accessories are warranted against defects in materials and workmanship for one (1) year from the date of delivery. During the warranty period, Hewlett-Packard will repair or, at its option, replace at no charge, components that prove to be defective, provided the calculator or accessory is returned, shipping prepaid, to Hewlett-Packard's Customer Service Facility. (Refer to Shipping Instructions).

This warranty does not apply if the calculator or accessory has been damaged by accident or misuse, or as a result of service or modification by other than an authorized Hewlett-Packard Customer Service Facility. No other express warranty is given by Hewlett-Packard. **HEWLETT-PACKARD SHALL NOT BE LIABLE FOR CONSEQUENTIAL DAMAGES.**

Some states do not allow the exclusion or limitation of incidental or consequential damages, so the above limitation or exclusion may not apply to you.

This warranty gives you specific legal rights, and you may also have other rights which vary from state to state.

Warranty Information Toll Free Number

800-648-4711 (in Nevada, call collect 702-323-2704.)

Warranty Transfer

The warranty is recorded by calculator serial number, as well as date of purchase. If you sell your calculator or give it as a gift, the warranty is transferrable and remains in effect for the new owner until the original one-year expiration date. It is not necessary for the owner to notify Hewlett-Packard of the transfer.

Shipping Instructions

Whether the unit is under warranty or not, it is the customer's responsibility to pay shipping charges to the Hewlett-Packard Customer Service Facility. Please return the calculator with all standard accessories and a completed Service Card, describing the problem.

Under normal conditions, your calculator will be repaired and reshipped within five working days of receipt at the Hewlett-Packard Customer Service Facility.

After warranty repairs are completed, the Customer Service Facility returns the unit with postage prepaid. On out-of-warranty repairs, the unit is returned C.O.D. (covering shipping costs and the service charge).

Appendix B

Financial Formulas

Percentage

$$\% = \frac{\text{Base} \times \text{Rate}}{100}$$

$$\text{Percent Difference} = \Delta\% = \left(\frac{\text{New Amount} - \text{Base}}{\text{Base}} \right) \times 100$$

Unless otherwise stated:

n = number of time periods
i = periodic interest rate, expressed as a decimal value
PMT = periodic payment
PV = present value or principal
FV = future value
I = interest amount

Simple Interest

$$I_{360} = \frac{n}{360} \times PV \times i$$

$$I_{365} = I_{360} \times \frac{360}{365}$$

Compound Amount

$$FV = PV\,(1 + i)^n$$

Savings (Annuity Due)

$$FV = PMT \left[\frac{(1 + i)^n - 1}{i} \right] (1 + i)$$

Rents, Leases (Annuity Due)

$$PV = PMT \left[\frac{1-(1+i)^{-n}}{i} \right] (1+i)$$

Direct Reduction Loans (Ordinary Annuity)

$$PV = PMT \left[\frac{1-(1+i)^{-n}}{i} \right]$$

Direct Reduction Loan with Balloon Payment (Ordinary Annuity)

$$PV = PMT \left[\frac{1-(1+i)^{-n}}{i} \right] + BAL\,(1+i)^{-n}$$

Sinking Fund (Ordinary Annuity)

$$FV = PMT \left[\frac{(1+i)^{n}-1}{i} \right]$$

Accumulated Interest/Remaining Balance

- Remaining balance after payment H:

$$BAL_H = PMT \left[\frac{1-(1+i)^{H-n}}{i} \right]$$

- Amount of a loan amortized from payments J through K inclusive:

$$AMORT_{J \text{ thru } K} = BAL_{J-1} - BAL_K$$

- Amount paid to interest by payments J through K inclusive:

$$I_{J \text{ thru } K} = (K - J + 1)\,PMT - (BAL_{J-1} - BAL_K)$$

Add-On Interest

r = add-on rate as a decimal value
N = number of monthly payments

APR = 1200i, where i is the solution in the following equation

$$\frac{n}{1 + \frac{n}{12} r} = \frac{1 - (1 + i)^{-n}}{i}$$

Discounted Cash Flow Analysis

NPV = Net present value
INV = Initial investment
cf = cash flow

$$NPV = -INV + \frac{cf_1}{(1 + i)^1} + \frac{cf_2}{(1 + i)^2} + \ldots + \frac{cf_n}{(1 + i)^n}$$

Depreciation

L = asset's useful life expectancy
SBV = starting book value
SAL = salvage value
DEP_k = depreciation for year k
RBV_k = remaining book value at the end of year k
RDV_k = remaining depreciable value at end of year k

- Straight-Line Depreciation

$$DEP_k = \frac{SBV - SAL}{L}$$

$$RDV_k = RDV_{k-1} - DEP_k$$

- Sum-of-the-Years-Digits Depreciation

$$DEP_k = \frac{2(L - k + 1)}{L(L + 1)} \times (SBV - SAL)$$

$$RDV_k = RDV_{k-1} - DEP_k$$

- Declining Balance Depreciation

$$\text{F} = \text{declining balance factor}$$

$$\text{DEP}_k = \text{RBV}_{k-1} \times \frac{\text{F}}{\text{L}}$$

$$\text{RDV}_k = \text{RDV}_{k-1} - \text{DEP}_k$$

Appendix C

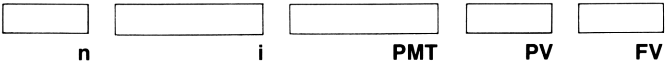
Applications Reference Guide

Need help finding the keystrokes to solve your particular problem? Perhaps we can help you. If your problem deals with:

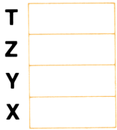
Appendix D

How the HP-22 Registers Work

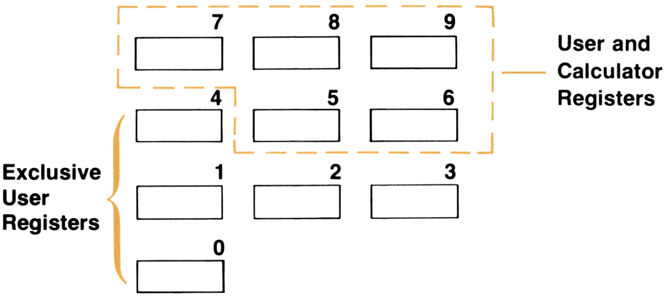
Inside your calculator are 5 financial registers (associated with the top row keys):



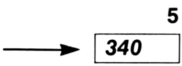
There are also four registers in the operational stack:



And 10 general-purpose storage registers. Five of these (registers 0 thru 4) are dedicated exclusively for your own use. The other five registers (5 thru 9) are shared by the user and the calculator.



If you press 340 **STO** 5, that number is transferred from the display to register 5:



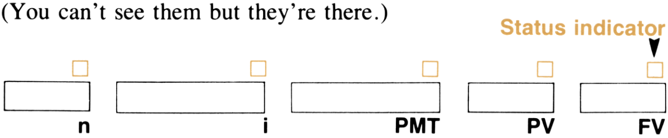
When you press RCL 5, a copy of that number appears on the display but the value is still stored in register 5:

T	0.00		
Z	0.00		
Y	0.00		5
X	340.00		340

When you press RESET, the shared user-calculator registers (5 thru 9) are cleared. If you press CLEAR, all 10 storage register are cleared.

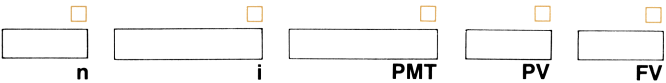
The Financial Registers

Associated with each of the financial registers is a status indicator. (You can't see them but they're there.)



When a status indicator is on (■), the value in the particular register is to be used as data for the next financial calculation. When a status indicator is *reset* or off (□), the register value cannot be used.

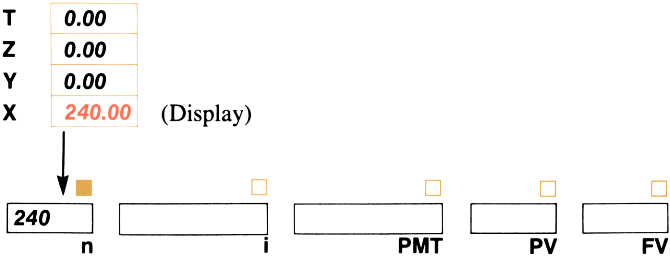
Let's take an example calculating the monthly payment on a 20-year $27,000 mortgage with 8.5% annual interest. Press RESET to reset the status indicators:



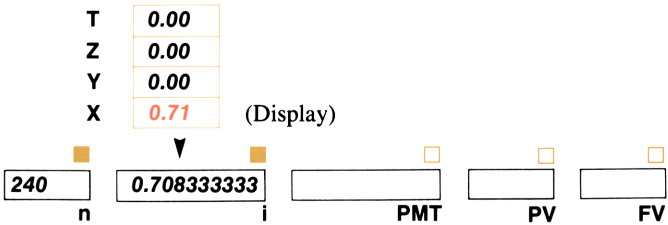
Key in 20 (the number of yearly payments):

T	0.00	
Z	0.00	
Y	0.00	
X	20.	(Display)

Since the n value must be *monthly* payment periods, convert the yearly figure by pressing n. The converted value on the display is copied into the n register and the first status indicator is turned on.

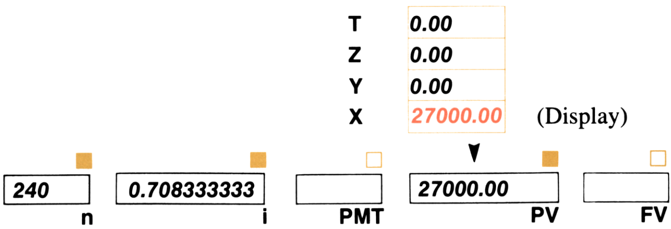


Next, key in 8.5 **i**. Again, the value on the display is copied into the i register and that status indicator is turned on.



Remember that although the value on the display is rounded off to two decimal places, your calculator always figures internally to 10 places. So the number placed in **i** is really 0.708333333. This is extremely important when you stop to think that those extra decimal places are used over 240 months! By having internal accuracy to 10 places, you are assured of correct answers.

To solve a top-row financial function, three status indicators must be on. Key in 27000 and press **PV**:



The calculator sees that n, i and PV status indicators are turned on. Once three status indicators are on, pressing one of the other two financial keys will cause the calculator to execute and store the new value in the appropriate register. You also see this new value on the display. Press **PMT**:

T	0.00	
Z	0.00	
Y	0.00	
X	234.31	(Display)

n	i	PMT	PV	FV
■	■	□	■	□
240	0.708333333	234.312273	2700	

There are two important points to remember: (1) the HP-22 will never allow you to turn on more than three status indicators; and (2) of the three status indicators that are on, at least one of them must be n or i.

Suppose you want to solve for payment again but with a different time value—say, a 25-year loan instead of 20 years. Since only one value changes and the necessary status indicators are already turned on, you can simply overwrite the new number in the n register. Press 25 ▬ **n**:

T	0.00	
Z	0.00	
Y	234.31	
X	300.00	(Display)

n	i	PMT	PV	FV
■	■	□	■	□
300.00	0.708333333	234.312273	2700	

Solve for the new payment by pressing **PMT**:

T	0.00	
Z	0.00	
Y	234.31	
X	217.41	(Display)

n	i	PMT	PV	FV
■	■	□	■	□
300	0.708333333	217.411313	27000	

Notice that the two calculated answers are stored in the X and Y registers of the stack for your convenience. For example, you can review both payment amounts by pressing **x⇄y** to look back at the first answer.

Or, with both answers positioned like that in the stack, you can quickly subtract to calculate the difference between the two payment amounts:

T	0.00		T	0.00	
Z	0.00	⟶ [−] ⟶	Z	0.00	
Y	234.31		Y	0.00	
X	217.41	(Display)	X	16.90	$ per month difference in payments.

The Power of the [RESET] Function

The power of RESET becomes apparent when you do a problem that requires a different status indicator. Before, the prerequisite status indicators were n, i and PV in order to obtain PMT.

Suppose you can afford $225 a month and wish to find the length of the loan. This involves a different set of known values. Now, the prerequisite status indicators are i, PMT and PV, and you want to find n.

Because this is a new financial calculation involving different input registers, reset the status indicators by pressing [ ] [RESET].

T	0.00	
Z	0.00	
Y	0.00	
X	16.90	(Display)

n	i	PMT	PV	FV
300.00	0.708333333	217.41	2700	

Notice that your previous answer is still on the display, and hence, in the stack. If you want to use this number in a future problem, you can **ENTER↑** it up in the stack, store it in one of the 10 manual storage registers, or use it in another financial problem.

The five status indicators are reset, so the HP-22 is ready for a new problem. The values are still in the financial registers—you can use them again or write over them with new values.

The **i** and **PV** values are the same as the original problem so we'll use that data over again. But how do you turn on the status indicators?

Press **RCL** **i** to place that number in the display:

T	0.00	
Z	0.00	
Y	16.90	
X	0.71	(Display)

n	i	PMT	PV	FV
300.00	0.708333333	217.41	27000	

Then press **i** again to transfer the number into the i register and *turn on the status indicator*:

T	0.00	
Z	0.00	
Y	16.90	
X	0.71	(Display)

n	i	PMT	PV	FV
300	0.708333333	217.41	2700	

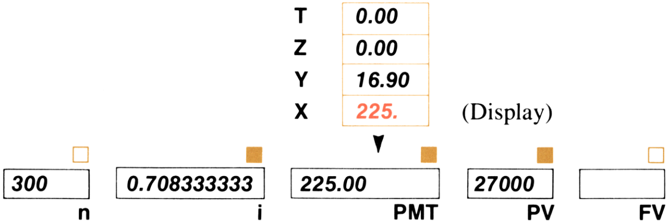
Herein lies one advantage of the RESET function: *You don't have to calculate i again!* (You may be tempted just to key in 0.71 **i**, but multiply that rounded-off figure over 25 years and your answer would be wrong.)

Press **RCL** **PV** **PV** to do the same for the PV value:

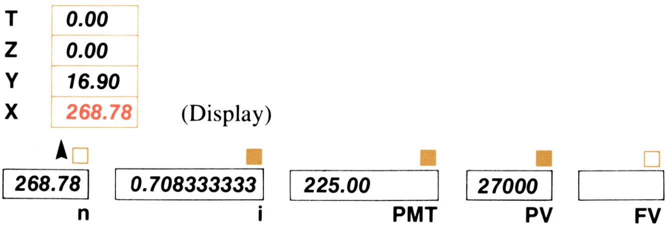
	T	0.00
	Z	0.00
	Y	16.90
(Display)	X	27000.00

n	i	PMT	PV	FV
300	0.708333333	217.41	27000	

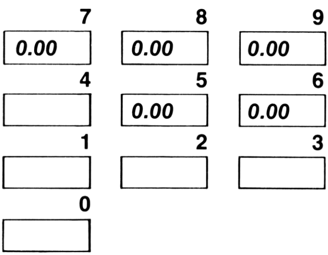
Turn on the third status indicator by inputting the new payment amount:



And solve for the length of the loan by pressing **n**:



In summary, RESET resets all five top-row status indicators so that the HP-22 is ready for a new financial problem. It also places zeros in the user-machine storage registers (5 thru 9) so that they are ready for a new statistical problem:



The CLEAR Function

Pressing CLEAR resets the five top-row status indicators just like RESET, but it also clears all four stack registers and the 10 storage registers:

n	i	PMT	PV	FV
268.78	0.708333333	225	27000	

T	0.00	
Z	0.00	
Y	0.00	
X	0.00	(Display)

7	8	9
0.00	0.00	0.00
4	**5**	**6**
0.00	0.00	0.00
1	**2**	**3**
0.00	0.00	0.00
0		
0.00		

Any data in the stack or storage registers is eliminated.

The Storage Registers

Pressing the Σ+ key produces several different sums and products of values in the X- and Y-registers and stores this data in registers 5 thru 9:

Register	Contents
R_9	Summation of x values ($\sum x$)
R_8	Summation of x^2 values ($\sum x^2$)
R_7	Number of entries (n)
R_6	Summation of y values ($\sum y$)
R_5	Summation of products of x and y values ($\sum xy$)
Display (X-register)	Number of entries (n)

Let's take a look inside at these registers, using some sample data. For example, if you input 50 ENTER↑ 2 Σ+, the values are stored as follows:

T	0.00			T	0.00	
Z	0.00	→ Σ+ →		Z	0.00	
Y	50.00			Y	50.00	
X	2.00	(Display)		X	1.00	(Display)

n 7	$\sum x^2$ 8	$\sum x$ 9
1.00	4.00	2.00
4	$\sum xy$ 5	$\sum y$ 6
	100.00	50.00
1	2	3
0		

If the next data pair were 80 ENTER↑ 3 Σ+, the contents of registers 5 thru 9 would change, like so:

T	0.00			T	0.00	
Z	50.00	→ Σ+ →		Z	50.00	
Y	80.00			Y	80.00	
X	3.00	(Display)		X	2.00	(Display)

n 7	$\sum x^2$ 8	$\sum x$ 9
2.00	13.00	5.00
4	$\sum xy$ 5	$\sum y$ 6
	340.00	130.00
1	2	3
0		

After all the numbers are stored, you have a powerful, organized data bank for statistical calculations. The statistical keys draw on this data bank as follows:

Σ+
Σ− } Update the values in registers 5 thru 9.

[$\bar{x}$]	Uses the data in registers 7, 8 and 9.
[s]	Uses the data in registers 7, 8 and 9.
[L.R.]	Uses the data in registers 5 thru 9.
[$\hat{y}$]	Uses the data in registers 5 thru 9.
[%Σ]	Uses the data in the X-register (of the stack) and register 9.

Execution Time

Execution time varies according to the numbers entered. This is especially true for i in annuity problems. Some solutions for i may require several seconds, during which time you will see a blinking display.

Input Values

The algorithms in the HP-22 can accommodate a wide range of values; however, the accuracy of your answer also depends on the input values. A negative number for n will produce an answer, but one that's meaningless because you used a negative time period. Extreme input values sometimes cannot be used, for example, rates between i = −0.000001% and i = +0.000001%.

Sometimes if you key in identical or very close numbers and try to solve for s (standard deviation), you will receive an error message when the answer should be zero or very close to it. This is caused by the rounding off of intermediate answers.

Miscellaneous

Executing [L.R.] or [$\hat{y}$] resets all five top-row status indicators.

Some of the gold-key functions also affect the storage registers:

[ACC]	Uses data found in registers 8 and 9. Status indicators for i, PMT and PV must be turned on. Register 7 is cleared.
[BAL]	Uses the data found in register 9. Status indicators for i, PMT and PV must be turned on.
[INT]	Status indicators for n, i, and PV must be turned on.

Solving for i clears storage registers 8 and 9 except in compound interest problems.

General Index

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