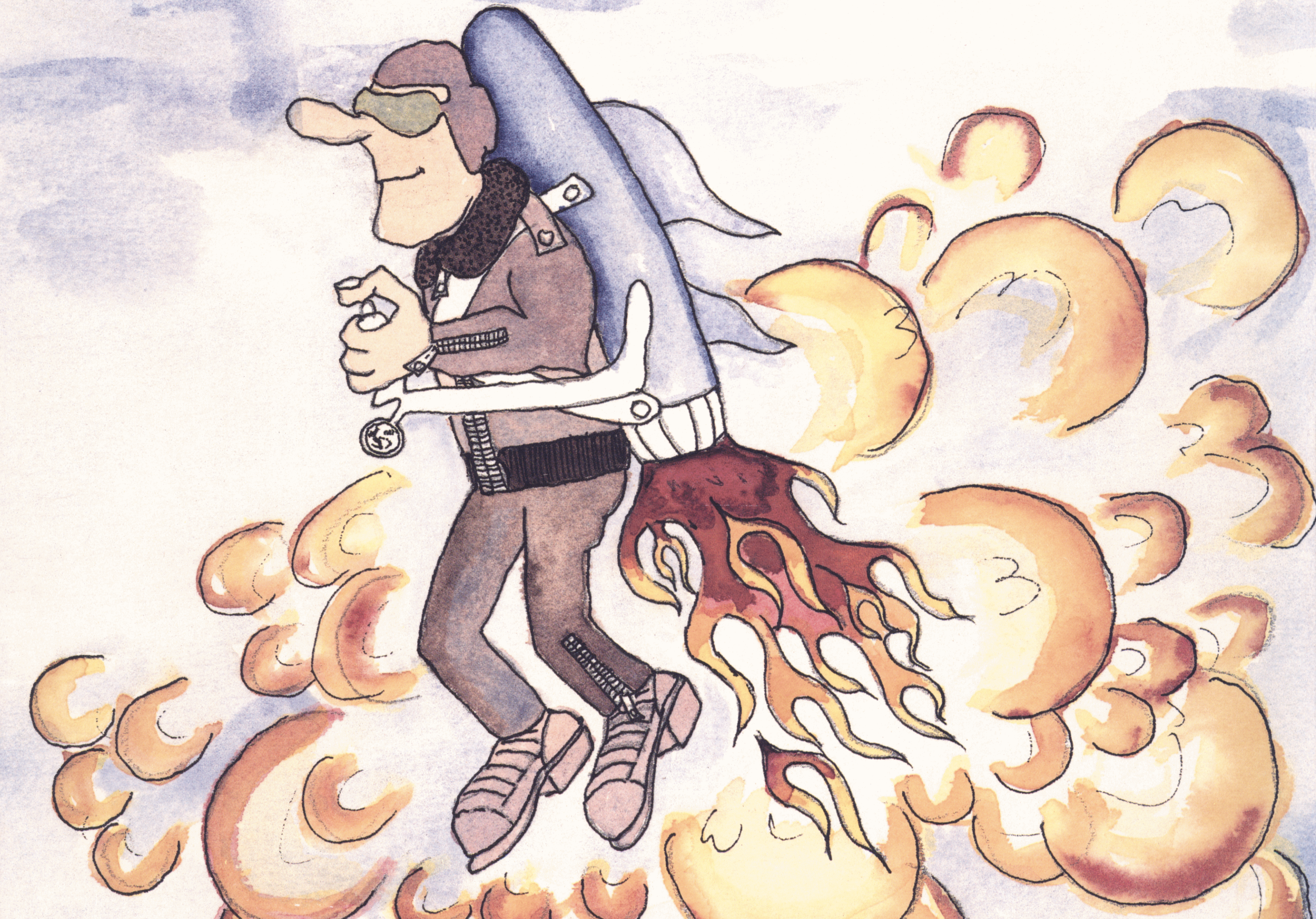
An Easy Course In

Using The HP-22S



A GRAPEVINE PUBLICATION

By Chris Coffin
Illustrated by Robert L. Bloch

An Easy Course In Using The HP-22S

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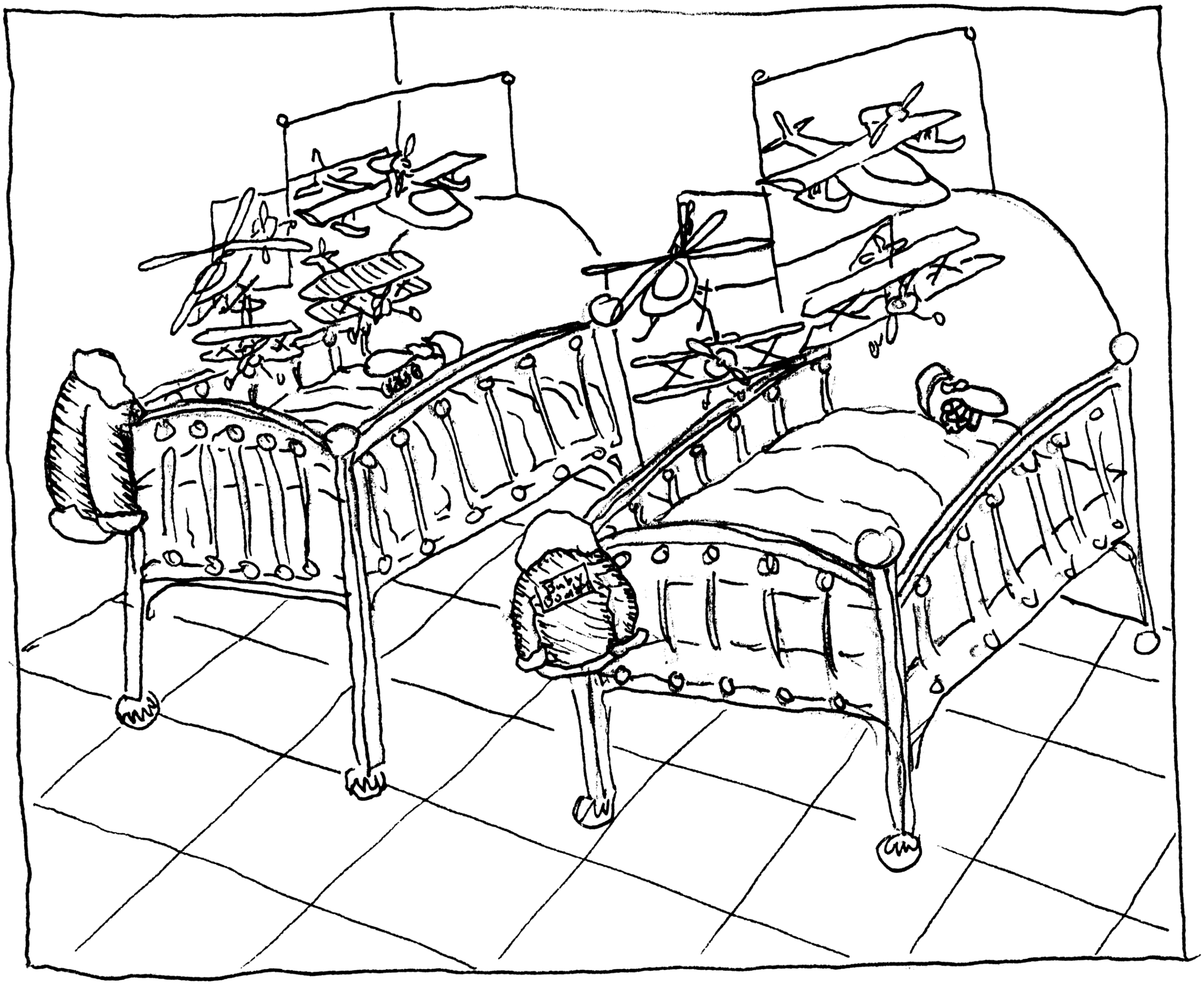


START HERE

…from the ground level…looking up….

Why?

Because the hardest part about getting this machine to fly – in fact, the hardest part of learning anything new – is simply

*Getting Off The Ground**

*Fundamental principle discovered by Wilville and Orbur Rong, at Dogged Dove, South Carolina, 1309.

This means you've just finished the hardest part of this Course!

But it doesn't mean you're done yet. You have embarked on an engrossing calculation adventure simply by buying this book. Now comes the rest of the quest.

Naturally, you're in charge, but your Course will be carefully planned and charted by your Trusted Navigator – this book.

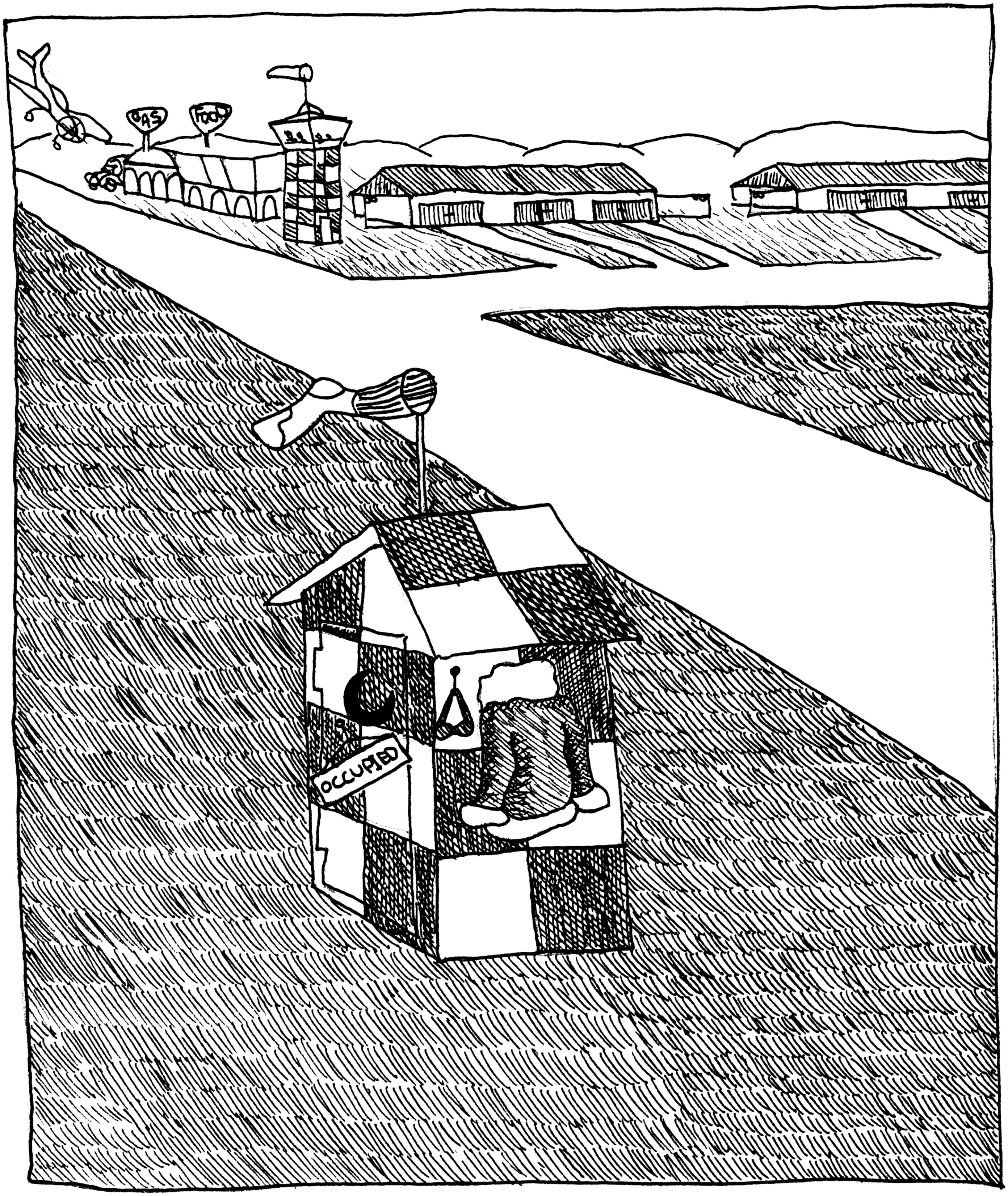
Follow it faithfully!

It's not a long journey – just a few good hours. But *do* it! The rewards will definitely be worth your while; when you finish, you'll know enough to solve problems that haven't even been invented yet!

How's that?

It's like this: The goal here is *not* just to leave you knowing how the buttons work or how to remember a handful of keystrokes and formulas. You'll also get a *fundamental understanding* of problem-solving, so that whenever you encounter a new problem, you'll be able to analyze it and solve it confidently.

Now then....



A Pre-Flight Briefing

What's In This Machine?

It's a loyal and friendly calculator engine – congratulations on a great choice!

Your HP-22S is one of a new generation of calculators, with more power and flexibility than ever before. It can do all sorts of things for you, from computing coordinates to analyzing investments to customizing formulas.

You just need to learn how to control it – which brings up the next question....

What's *Not* In This Machine?

There's no English-speaking person inside your calculator (one of these days, though...). For now, you still need to "speak" your machine's "language" in order to *translate your everyday calculation problems into forms it can understand.*

That's the whole mission right there.

Your part of the work is in *defining* the problem correctly for *yourself* and then *restating* it for your calculator. After all, only a human being is truly able to understand our human world (and even then it's tough sometimes)!

This translation skill isn't really very hard to learn. It's a lot like learning to fly an airplane. Everything seems strange at first, but after some training and practice, you'll be doing things quite proficiently – without even thinking much about them!

And that's where this book comes in....

What's In This Book?

This book is a sort of Pilots' School.

After learning about your craft's controls and buttons, you're going to take it out over the Course for extensive "flight training."

Along the way, you'll encounter lots of explanations, diagrams, quizzes and answers, and once you successfully negotiate these, you'll be ready to fly anywhere your HP-22S can take you (and that's a lot of flying)!

So, what makes the Course so Easy?

It's because *you* choose your airspeed and altitude. Don't worry about how fast or high you're flying; this isn't a competition with anybody – and you're allowed to re-fly the same route repeatedly until you're comfortable with it.

OK?

What's *Not* In This Book?

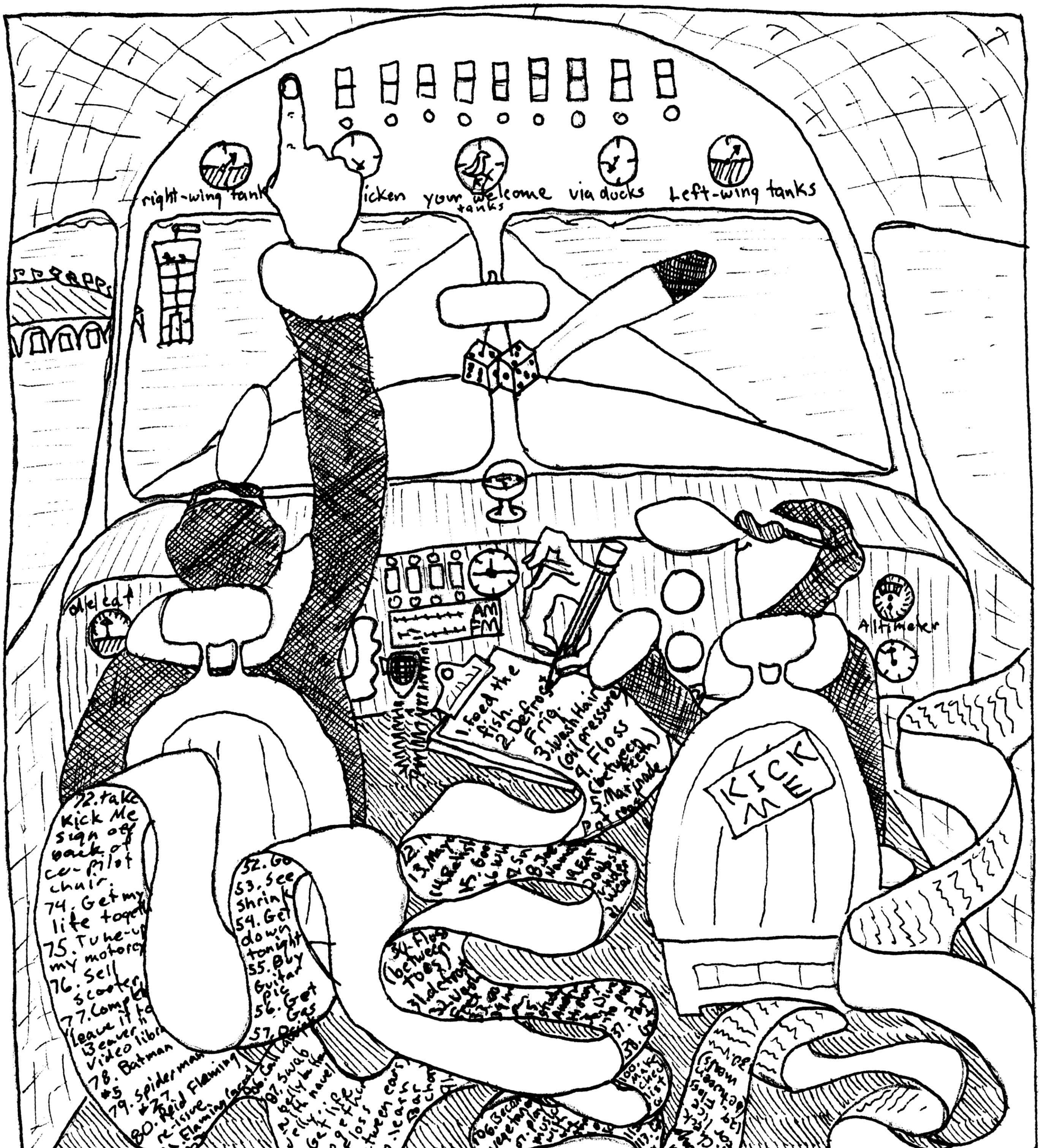
You won't find discussions of *everything,* because some parts of your HP-22S just don't need much explanation at all. Therefore, included here are just those things that *most* "pilots" tend to need some practice on – *most* of the time.

For example, you'll get an extensive look at three subjects in particular (STATistics, the built-in LIBRARY of equations, and your own EQUATIONS list) – but you'll get a shorter look at other things. This isn't because those three topics are any more important in the Great Scheme Of Things; rather, it's because they're the most complicated to learn how to use correctly, so there are entire sections of the Course to cover them.

By contrast, the other menus tend to explain themselves after a few uses of them, so that's what you'll do here – just use them a few times – as a reminder that they're available if you want them.

Anyway, if you want a *complete* description of *everything,* you already have it right there in your Owner's Manual. That's what it's there for – as a reference manual to let you conveniently look up keys, functions, and examples.

This book you're now reading is *not* a reference manual; it's an entirely different approach, with a careful selection of topics that are meant to be taken in order. So start here at the beginning and stay on Course! Then, if this book has been worth its tuition, you'll seldom, if ever, need it all over again. Thereafter, a reference source (such as your Owner's Manual) should be enough to keep your "flying" skills fresh and sharp.



Preparing For Take-Off

The [ON] Button

If you're looking at your HP-22S for the first time, it might seem as complicated as the cockpit of a plane (well...almost). But that's just a first impression, one you'll quickly overcome as you take this Course. After all, nothing is all *that* mysterious – once you know all its parts.

So go ahead and get started – turn it on....

See the [ON] key there at the extreme lower left? It's not just called the [ON] key, is it? It's also the Clear key, which is why it looks like this: [C]

The [OFF] Button

Is your calculator on? All right, now turn it off again, by pressing the [OFF] key....

"It looks like the [OFF] button is that very same [C] key again – but the machine doesn't turn off if I just press [C]!

Why not?"

Since the word OFF is written in *blue* above the [C] key, you need to press the blank, blue "shift" key before pressing [C].

(If you're already quite familiar with the keyboard, the display, and the way menus work, then you can skip ahead now to page 27.)

A Good First Setting

Now, with your calculator turned back on (go ahead and do that), what's in the display?

That's hard to say, actually.

Since your HP-22S has continuous memory, it will look now just as it looked when the dealers or factory technicians finished with it – or when you last used it. So, as a way to be sure that you're starting with a clean slate, here's a procedure for resetting the machine.

Warning: You're only doing this now to be sure that you can follow along on this Course. This procedure completely wipes out all meaningful information that you or someone else may have stored in the machine. This is *not* something you want to do very often. There plenty of other, safer ways to selectively erase parts of its memory. *This* reset is an Equal Opportunity Erasure!

Drum Roll, Please: Press *and hold down* the [C] key. While holding it down, press *and hold down* the [√x] key (upper left-hand corner). While holding *both* those keys down, *press and release* the [Σ+] key (upper right-hand corner). Now release the [√x] key, then the [C] key.

You should see this:

```
MEMORY CLEAR
```

(If this doesn't work the first time, just try again.)

The Display

There you see it, a machine with utter amnesia. But notice that even now, in its most empty state, it uses the display to tell you what's happening. So when in doubt, always look at that display for clues.

(But if you're in no doubt now about the display, what it's telling you, *and* how to adjust it, then skip ahead to page 20.)

The Viewing Angle

Can you see and read the display comfortably? This LCD (Liquid Crystal Display) is hard to read if you're not looking at it from a certain angle – but you can adjust that angle to whatever's the best for you!

Try It: With the machine still on, press and hold down the [C] key. Then press and hold the [+] or the [−] key and watch how the display's best viewing angle varies.... Go ahead and play with it until you find a comfortable angle....

Messages And How To Clear Them

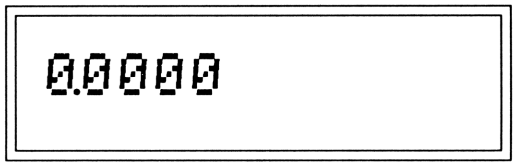
That `MEMORY CLEAR` is a good example of a *message* – something spelled out in English – usually telling you of an error or asking you for something.

The thing to realize is that a message is just a note that *temporarily covers* part of the display. To get rid of it (if it's not already gone by now), all you need to do is press that ever-useful [C] key. Do it now....

...OK?

The Calculator Line

Now your display should look like this:



As you can see, it has one full line for numbers, plus some space above and below it.

Try This: Press [1][2][3][•][4][5]

See how the number keys work? Whenever you're doing calculations, they'll appear in the display on this *Calculator Line.*

Nothing very mysterious yet – right?

Now, what about those spaces above and below the Calculator Line – what are *they* for, anyway?

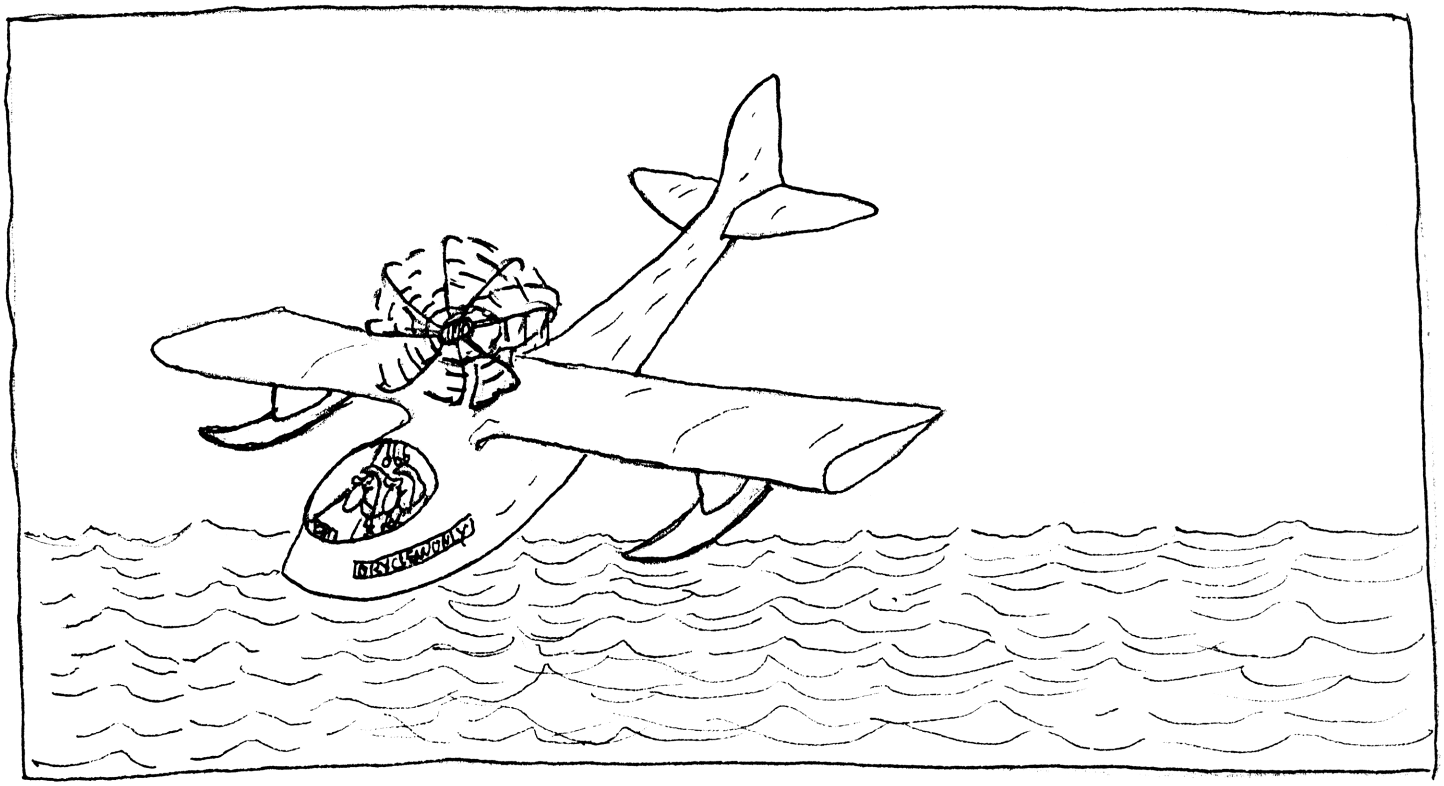
The Annunciators

The space below and above the Calculator Line is for the *annunciators*. Remember that little ⚠ that appeared under the `MEMORY CLEAR` message? That's a good example of an annunciator.

As you might guess from its name, an annunciator "announces" something. In this case, the ⚠ is announcing that you've asked the calculator to do something out of the ordinary, such as to perform an impossible task or (as you did here when you cleared the machine's memory) to do something else that's quite unusual.

Of course, there are lots of other annunciators in the display, but it wouldn't pay to try to learn them all now. Since you'll run across most of them during this Course, you might as well wait until you see each of them in action.

For now, just realize that you may occasionally see something in the display besides the stuff on the Calculator Line. It'll be an annunciator, and you're sure to see and hear more about it then – OK?



The Keyboard

Of course, there's a lot more to "flying" this calculator than just knowing what the display (your "console") is telling you. It's now time to look at all the keys and other controls.

(If you already know the basics of the keyboard, go ahead and jump to page 27).

The Arithmetic Keys – The "Steering Controls"

Concentrate first on the white writing on the actual key-faces themselves. Notice how the digit keys ([0] – [9]) and the four arithmetic functions ([+], [−], [×], and [÷]) are all conveniently placed together on the lower five rows of keys. After all, you certainly shouldn't need to search around just to add 2 and 2!

But (as you'll soon see), also a part of those arithmetic keys are the [INPUT], [+/−], [(], and [)] keys – and the [←] key for correcting mistakes (just in case you should ever make one).

And of course, down there at the bottom, there's the [C] key, which you've seen several times already.

But there's a lot more to the HP-22S than just simple arithmetic. And you can't do very much of it just by sticking to the white-printed keys. In fact, just about every key on the machine has *two* meanings – white and *blue,* so it's time to give that blue "shift" key a closer look....

The ■ Key

Since you've already used it – that blank blue key on the lower left – you've probably figured out by now that it is indeed the "shift" key for the calculator, much like a typewriter's shift key (but, alas, since this book isn't in color, it appears here as ■).

Just like a typewriter, to get the "shifted" (blue) function of any key, you must first press the blue shift key.

Try This: Press the ■ key. What do you notice? The ⬏ *annunciator* appears (another of those little signals above the Calculator Line), announcing that the next key you press will produce that key's *blue* operation.

Now press ■ again. The ⬏ disappears, right? You've just discovered a *toggle key* – a key that alternates its meaning between two opposites, in an on-again-off-again manner.

Keep these things in mind also: *Unlike* a typewriter, you don't need to keep holding the ■ key down while pressing the key you're after. Just press and release the ■ key, and *then* press the key you want. The ⬏ will always tell you when the "shift" is in effect. Notice that the shift is cancelled after every use. You need to re-press the ■ for every blue function you want; ■ is not a "Caps Lock").

You'll find also that the ■ key is more than just a convenient way to cut down on the number of separate keys needed on the calculator. Often, it makes it easier to remember the locations of related operations, since the blue ("shifted") meaning of a key is usually related to its white ("unshifted") meaning.

For example, the [SIN] and [ASIN] operations are a matched set of opposites that you'll find on the same key – one in white, one in blue (second row on the keyboard).

The Higher-Math Keys – For Fancier Maneuvers

Now that you know how to use any of the blue-printed functions on the keys of your HP-22S keyboard, you'll certainly want to take advantage of many of them.

If you think about the number keys ([0] – [9]) and arithmetic keys ([+], [−], [×], and [÷]) on the lower half of your keyboard as your basic controls for "level flight," then you can think about the upper two rows of keys as your "Higher-Math Maneuvers."

Most of the common functions for trigonometry and exponentiation, percentages and statistics "live" here – just waiting for you to use them.

(Don't worry if some or all of these higher-math operations are a little fuzzy in your mind; you'll get plenty of practice and "refresher" problems with them later. Remember that this whole chapter is just a "pre-flight" check so that you'll know your way around the "controls.")

So, what have you seen so far?

- You know how to clear your HP-22S's memory if you ever need to (not often).
- You know about the Calculator Line and the Annunciator Areas – and you know about two annunciators already (▲ and ⬏).
- You know how to gain access to blue-printed operations – with the blue ■ key.
- You know where to find the basic "steering" keys (the *arithmetic* keys). They're the *white*-printed keys on the *lower* half of the keyboard.
- You know where to find the higher-math functions; they're on the *upper* two rows (both *blue* and *white*).

Menus And Their Keys – Your Navigation Charts

Now look at the *blue*-printed functions on the *lower* half of the keyboard. These keys are mostly for *moving and selecting* the numbers and operations in your HP-22S. That is, they're mostly *menu* keys.

There are about a dozen different menus in all, and during this Course you'll see each one. Here's a good one to start with – to get the General Idea:

G.I.: Suppose you want to convert 98.6° F. to its equivalent Celsius temperature (also known as Centigrade). It just so happens that the HP-22S has a UNITS conversion menu (the blue function on the [4] key) to let you do just that.

So, key in [9][8][•][6], then press ■[UNITS]. Here's what you'll see:

```
M   TMP L  VOL
▼     ▼   ▼     ▼
```

The idea is to choose a key directly under one of those ▼ annunciators. Whenever you see a menu with a set of ▼'s in the display above, ignore the functions printed on the actual faces of the top row of keys; those keys then correspond only to menu items – or nothing at all.

Now, since you want to convert TeMPerature, you select the top-row key under the TMP ▼ (this would be the [LN] key in civilian life).

Aha! Yet another menu – this one to let you choose the *direction* of your conversion. So, to convert *to* °C., you press the far-left menu selection....

Voilá! The normal human body temperature is 37° C.

A Sample Menu Chart: The UNITS Menu

That UNITS menu is a good example to start with, because it shows how menus help you "navigate" with your HP-22S.

It's like this: Suppose you had a large atlas – a set of charts and flight routes that you needed to follow to get somewhere. Of course, no cartographer tries to show everything on one map; the details would obscure the bigger picture. So the first chart might show a broad perspective – the whole continent or country. But it would also list down in its key or legend the different sections of the atlas you could turn to for more and more detailed pictures of any part of that whole, right?

So it is with the menus in your HP-22S.

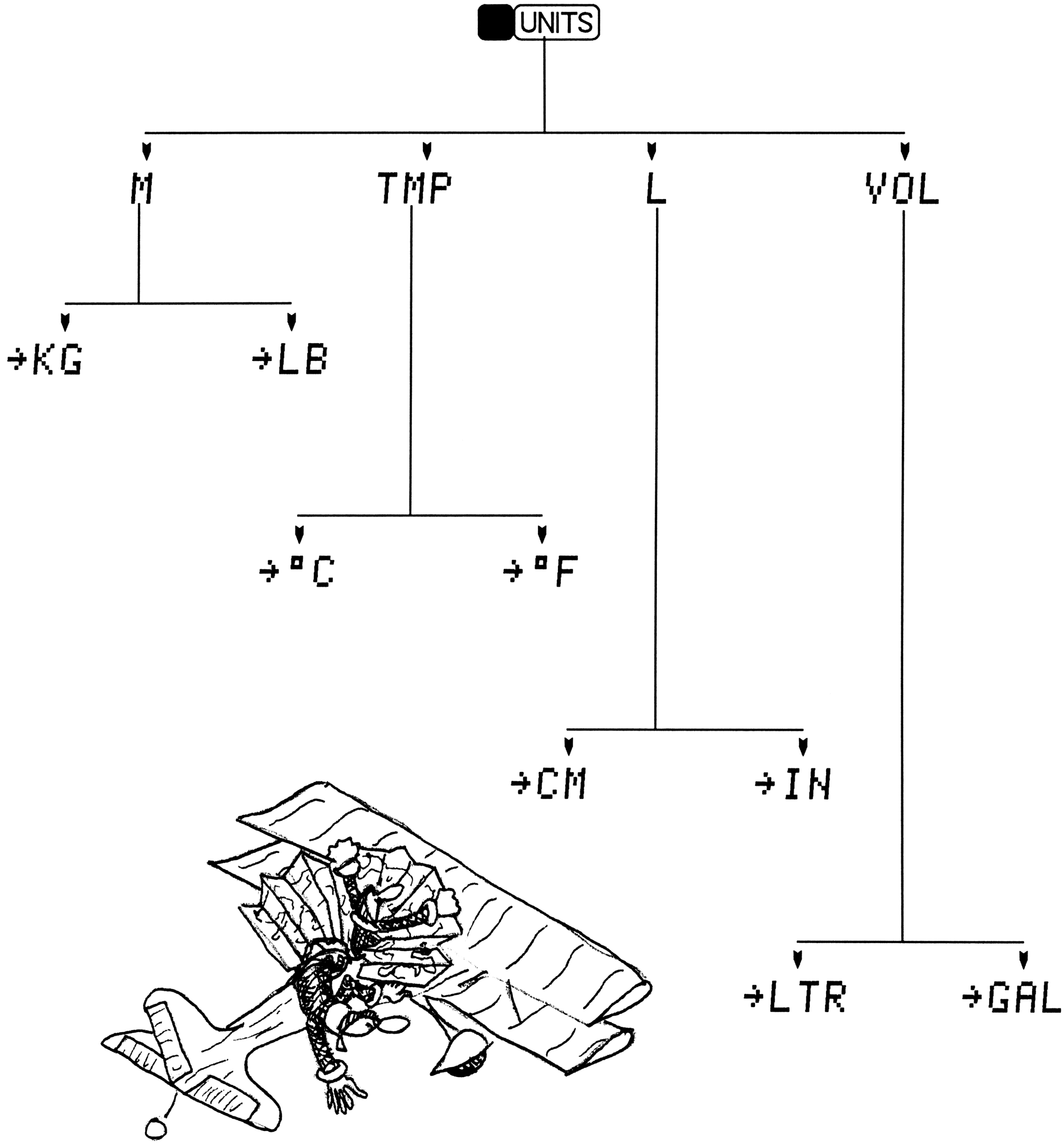
You start at the Calculation Line, which is the biggest possible picture of operations you can do (after all, you have the whole keyboard and all its menus to choose from). From there, you choose a menu, then another menu from *that* menu, and so on, until you reach your destination – the operation you want to perform.

"Does this mean that I'll have to press a whole lot of keys simply to find the operation I need?"

Not really.

Most of the menus on your HP-22s are only one or two "layers" deep, so it's not much trouble to get to any operation you need.

In fact, the UNITS menu is actually one of the more complex menus in the machine – and it's fairly simple. If you were to see the entire UNITS menu – all its possible "routes," all at once – it would look like this:


UNITS
M
TMP
L
VOL
→KG
→LB
→°C
→°F
→CM
→IN
→LTR
→GAL

Review: From looking at the full UNITS menu chart, you can easily see how you did your temperature conversion, right? You keyed in the Fahrenheit temperature, then traversed through this menu until you could select the operation that converts to Celsius (→°C).

Questions: Did you notice how the calculator brought you immediately back to the Calculator Line with the results of your temperature conversion? When you chose the →°C calculation, the machine performed that calculation all right, but then it "jumped" you from the bottom of this menu chart clear up to the "home" position again.

That's the case with many of the operations in your HP-22S: You key in the number(s) to be operated on, then select your operation from some menu. After the machine actually performs that calculation, it immediately returns you "home" to the Calculator Line, ready for your next desired operation.

But do you know how else can you get back "home" again? Suppose you are traversing through this menu and you make a mistake or change your mind and want to "back up." How can you retrace your steps or "jump" home again – without performing any operation?

Answers: To "back up" one step (one *level,* you might say) in a menu Chart, you use the [←] (backspace") key. Try this with the UNITS menu....

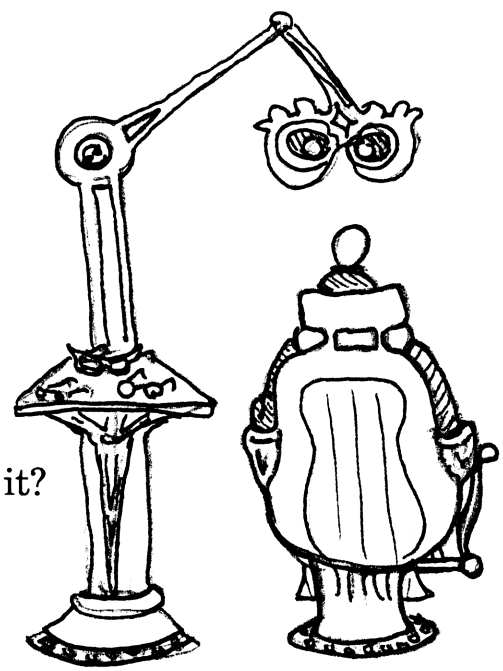
Or, to "jump" all at once back to "home" position, you press the [C] key (in case you haven't noticed by now, that [C] key is definitely the all-purpose, cleaner-upper "beam-me-up" key to get you out of any unusual situation). Try this with the UNITS menu....

Pop Quiz

Yes, things are moving pretty slowly here at first – but never fear – you're going to "take off" just as soon as you're ready. The idea here is to make *sure* you know these things before you start "flying." The answers are on the following pages, along with page numbers to refer you back for re-reading, just in case. So don't hurry; there's plenty of time – and who cares if you re-read? It's *your* Course!

1. What's a toggle key? Name one such key.

2. Name the two major areas of the display.



3. What's an annunciator?

4. What's a message? How do you get rid of it?

5. What does it mean to reset your calculator? When would you do this?

6. What would happen to your calculator if you forgot to turn it off before leaving for Papua New Guinea?

Pop Answers

1. A toggle key is like a light switch, because it alternates between two opposing functions: hit it once to turn something on; hit it again to turn that something off. The key is one such key (see page 21 for review).

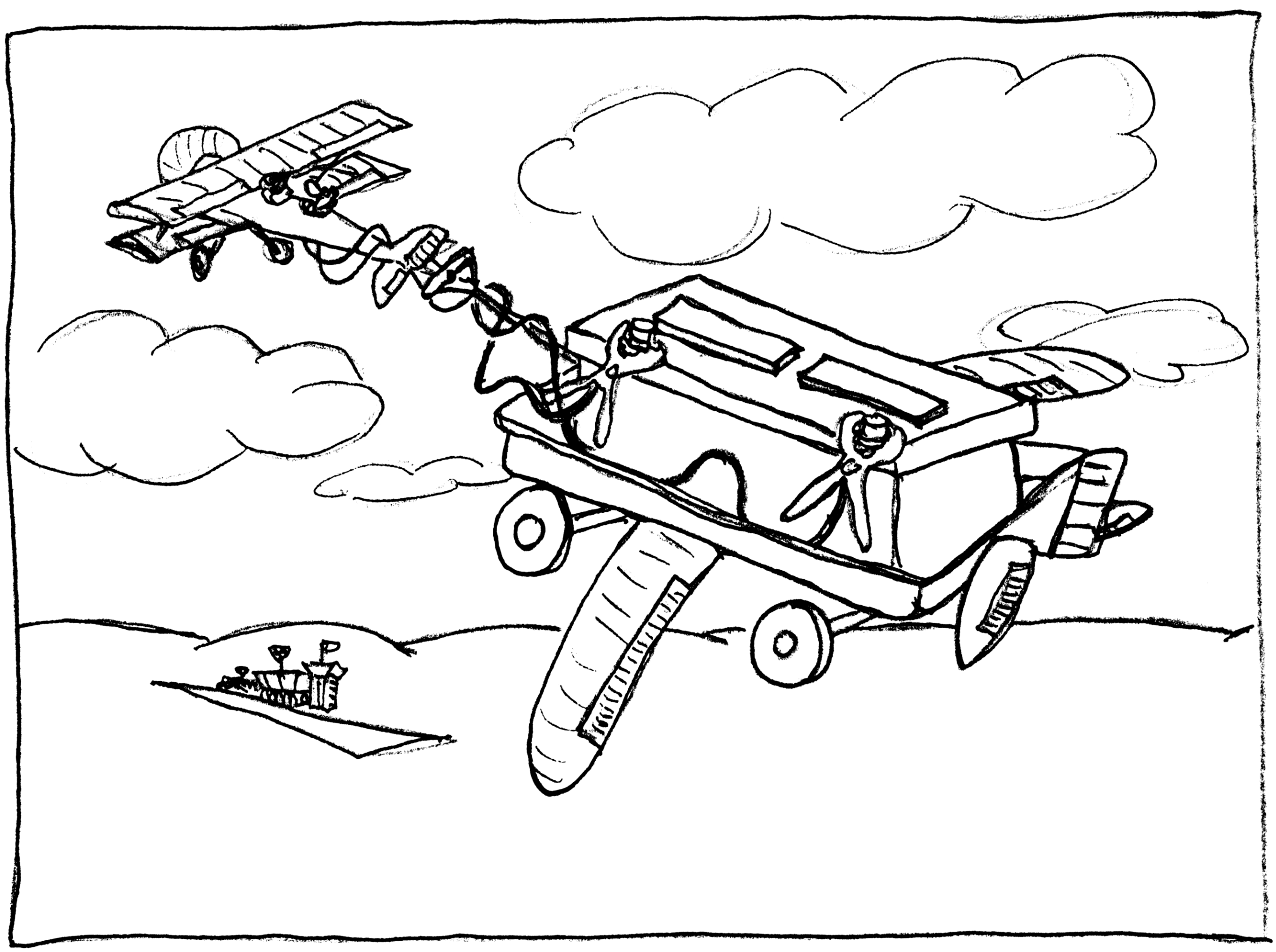
2. The two main areas of the display are the Calculator Line and the Annunciator Area (which is actually split into two parts – above and below the Calculator Line). The Calculator Line (see page 18) is where you'll always see calculations and messages; the Annunciator Area (see page 19) shows the annunciators, naturally.

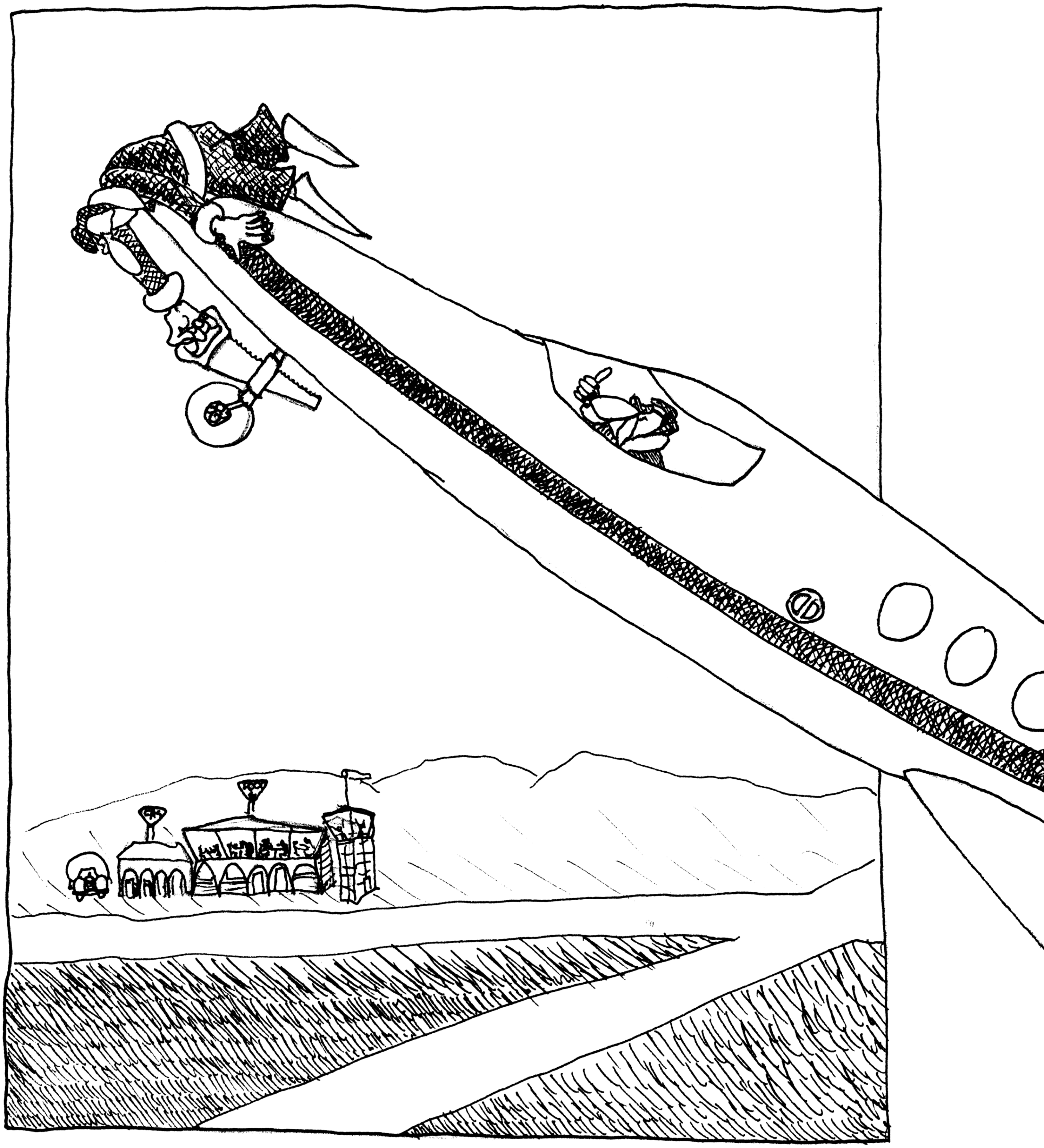
3. An annunciator is a little signal, a status indicator that appears in the Annunciator Area (either above or below the calculator line). The usual function of an annunciator is to keep you informed of the current "doings" of the machine. When you see one, you don't usually need to worry or do anything about it – just understand what it means. For example, the shift annunciator, , means that any key with a blue operation written above it will now produce that blue operation (see page 19 for review of annunciators).

4. A message is a phrase or question that temporarily covers what's on the Calculator Line, to notify you of an error or ask you to do something. When you want it to go away, just press $\boxed{C}$ and see the Calculator Line return undisturbed (recall page 17).

5. To reset the machine means to erase all numbers, letters, formulas, and information of any kind that you may have stored in there. You could lose entire lists, years of statistics, months of data collection and formulas that took weeks to develop. This is not usually a very good idea. Use reset as seldom as possible (page 16).

6. After about 10 minutes, your HP-22S would turn itself off. Then, if you came back and turned it on some time later, its display (and memory) would be the same as it had been when you left. As long as its batteries hold out, your calculator will continuously remember everything you store in it – whether or not the machine is currently turned on.





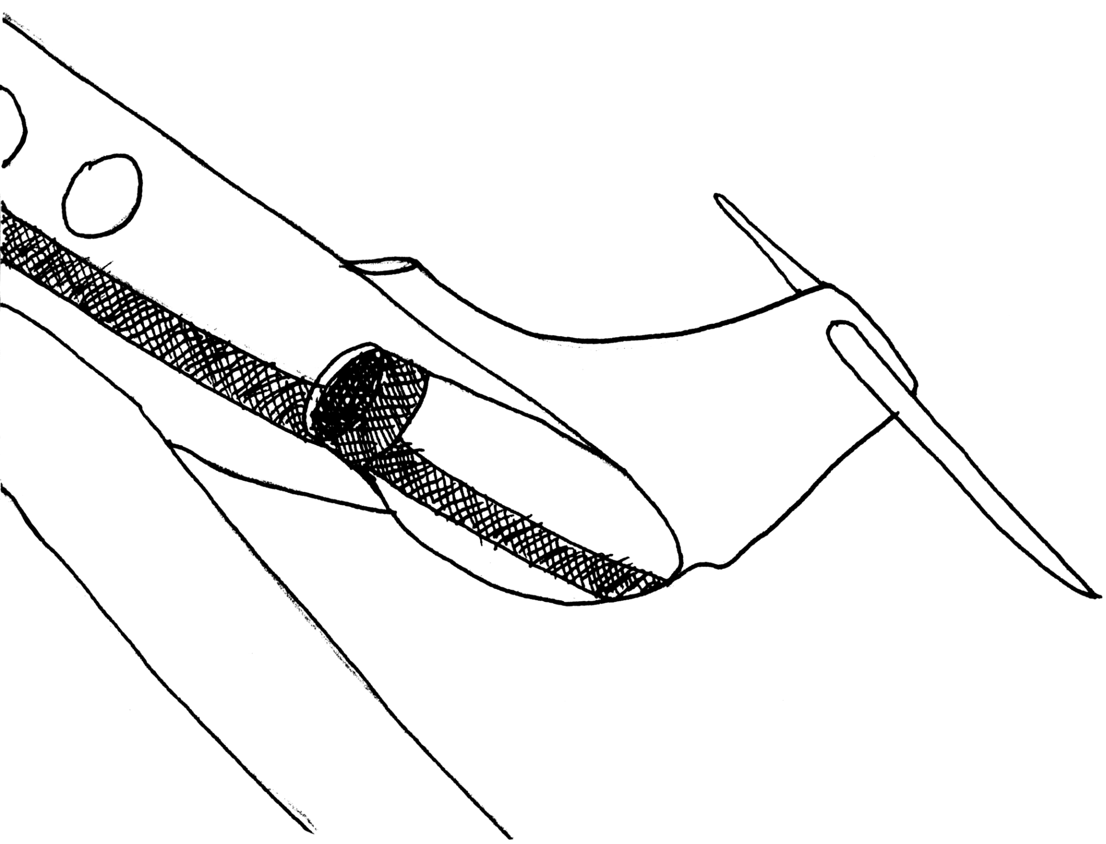
Wheels Up

Basic Flight Maneuvers: "Number Crunching"

It's time now to learn the basics of "flying."

Arithmetic is truly the engine of your calculator, because no matter what other options and accessories come with it, you'll always want a machine that can at least "crunch" numbers.*

As you might guess, most of this "number crunching" happens on the Calculator Line. so turn your machine on (if it's not on already), point it down the runway, and watch that Calculator Line.



*At this point, if you're already quite comfortable with doing arithmetic on your HP-22S, including percentages, negative numbers, scientific notation and various display settings, then feel free to go on to page 49.

Clearing For Departure

First of all, be sure that you're starting from "home" – at the Calculator Line (i.e. there are no menu selections visible in the display). Remember that you can always get there in one "jump" with the [C] key.

Now, you'll notice that you're starting with some non-zero number *already* on the Calculator Line (it's probably the 37.0000 degrees you computed for the normal human body temperature).

Do you need to clear this previous result before beginning a new calculation?

Nope.

For example, press [2][+][3][=] and see the Result: 5.0000

Question: What if you now want to find 64 x 11? Do you need to clear away that 5.0000 somehow?

Answer: Nope. Again, just start your new problem, by pressing [6][4][×][1][1][=], to get 704.0000. Notice how the 5.0000 just moved out of your way.

What's the idea here?

It's this: Anytime you *finish a calculation* (usually by pressing the [=] key), the machine will recognize this and automatically bump that result out of the way when you begin a new calculation. You don't need to clear anything!

OK, fine.

But: What if you decide to scrap a calculation *midway* – when it's not complete? Surely you need to do some clearing then, right? So how do you clear the Calculator Line?

No Problem: Suppose you decide to cancel the calculation 789÷5 just before pressing the [5] key. You would press [7][8][9][÷] – and then there's the point where you want to cancel the whole thing.

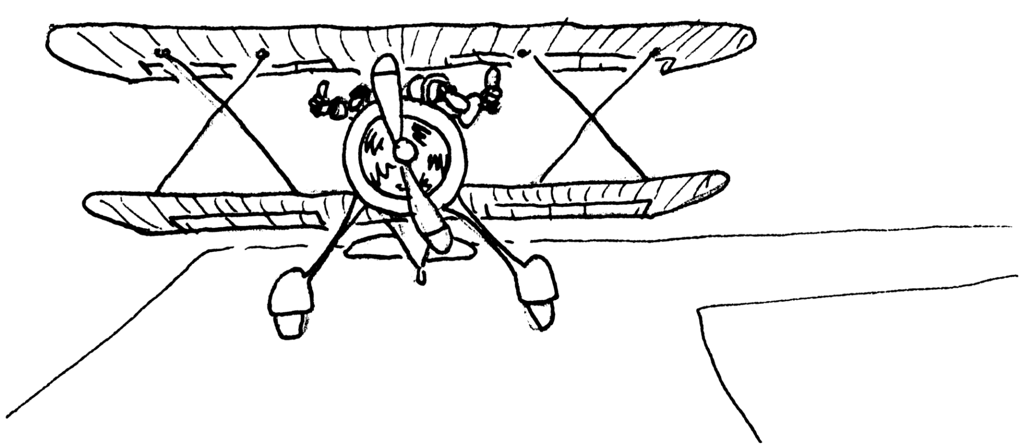
How?

Easy: To clear the entire Calculator Line, you press [C].

Anytime you truly wish to clear that entire Calculator Line, you just press that all-purpose [C] key.

But remember that this is *not necessary* if you've just *completed* a calculation, for then the machine already knows that the next number is part of a new calculation.

OK?



One More Thing: What if you simply make a *little* mistake while keying in an arithmetic problem – so you don't want to start all over again?

Suppose, for example, you want to find 123 + 465.

So you blithely rattle off [1][2][3][+][4][5][6] (oops).

Not To Worry: This is the main reason for that [←] key. When you're keying in digits, it is indeed a simple backspace key, so use it as such....

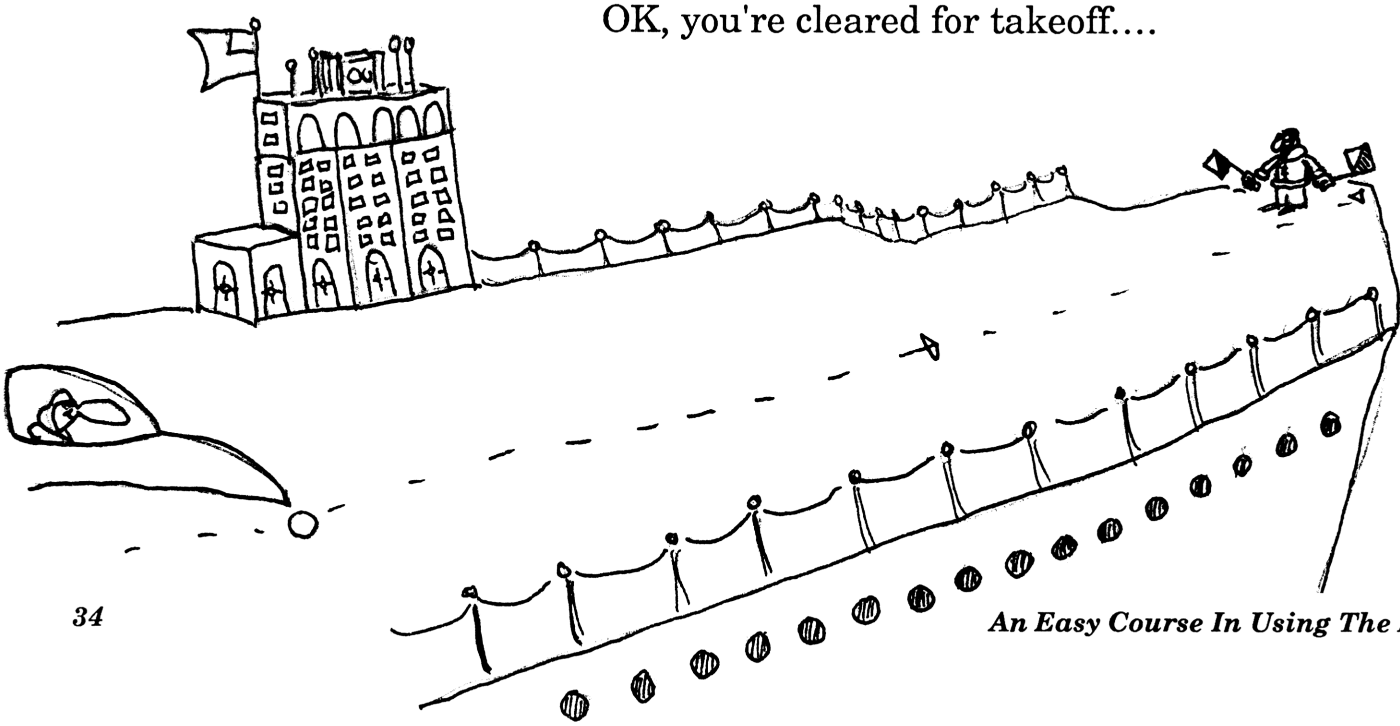
First, back out your erroneous digits: [←][←].

Now key them in correctly: [6][5]

And finish the problem: [=] (Answer: 588.0000)

It's not too tough to clear the display one way or another, is it?

OK, you're cleared for takeoff....



How Many Digits Do You Have?

Now off the runway and climbing well, you begin your flight by noticing that the display is showing exactly four decimal places on every number.

This isn't an accident; four decimal places is usually enough for most technical calculations, and the folks at HP *had* to decide on *some* such number for the HP-22S to "wake up" with after a total memory reset (remember page 16?).

But is four decimal places the best precision you can get?

Not at all.

In fact, every number has a total of 12 digits, no matter how many you can see at the moment.

So the number you're seeing on the Calculator Line right now, 588.0000, is really this:

588.000000000

(Count 'em. There should be a total of 12 digts.)

You just aren't being *shown* anything beyond those four zeroes right now.

And Notice This:

Key in 2 1 0 . 9 8 7 6 5 4 3 2 1 and press − (by selecting any operation – such as this subtraction – you tell the machine that you've finished keying in the first number). What do you see?

```
210.9877-
```

What gives? Where's the rest of the number? Why did the calculator change it?

It didn't change it. It changed only its *presentation* (to you).

The entire 210.987654321 is *still in the machine,* and that is what will be used for the actual subtraction (after you key in another number and press =). But, as you know, the display has been instructed to *show* you only the first four decimal places of any complete number.

It's only this edited, "display" version that is rounded like this. After all, if you're going to see only 4 decimal places, those digits ought to represent the *entire* number as accurately as possible – right?

So the display rounds this number, 210.98765... , *up* to 210.9877 (but if that 5 had instead been a 4 or smaller, the rounding would have been *down* to 210.9876).

Just keep in mind through all this that it's the *display* doing the editing – for your eyes only. Your HP-22S will always do all your arithmetic with all 12 digits in each number.

The DISP Menu: How Many Digits Do You See?

Again, here's how your display probably looks right now:

```
210.9877-
```

Of course, you may want to tell the display to change its editing for you. What if you wanted to see 2 decimal places rather than 4?

Just Say The Word: Press the ■ DISP key (third row, right side – it's another one of the dozen menus in the HP-22S). Here's what you'll see:

```
FX SC EN ALL
```

Now what? Well, you want to FiX the number of decimal places to be 2. So choose the FX selection...then press 2....

Voilá!

Now continue this hypothetical subtraction problem by keying in the other number. The subtraction problem you're doing is this: 210.987654321 - 0.087654321.

So, to key in the other number, press . 0 8 7 6 5 4 3 2 1 (but don't press the = key yet – there's more to talk about first)....

Now your display looks like this:

```
0.087654321_
 ←
```

Notice how the first part of your subtraction problem disappears temporarily so that you can see what you're doing as you key in the second part. But don't worry – your first number is still there; it's just been bumped or "scrolled" off the left end of the display – as the little ← annunciator is telling you.

Now press [=]. Result: 210.90

Just For Laughs: Set your display to show you ALL decimal places that aren't merely extra zeroes.

Solution: Press ■[DISP], then choose ALL. Result: 210.9

See? That extra zero in 210.90 must have nothing but more zeroes beyond it – so none of them alter the true value of the number. When you ask for ALL digits, you're really asking for all *relevant* digits – which excludes such trailing zeroes.

But What If: You want to verify the full precision (all 12 digits) of the number – just to glance at it for a moment – not to FiX every number that way. How do you do this?

Piece O' Cake: See that ■[SHOW] key (the blue function over the [•] key)? Press ■[SHOW] and watch the display temporarily *show* you all the digits of your number – whether or not they're irrelevant trailing zeroes. And you can view this full precision as long as you want – by holding down the [SHOW] key.

When In Rome: Radix Markers And The MODES Menu

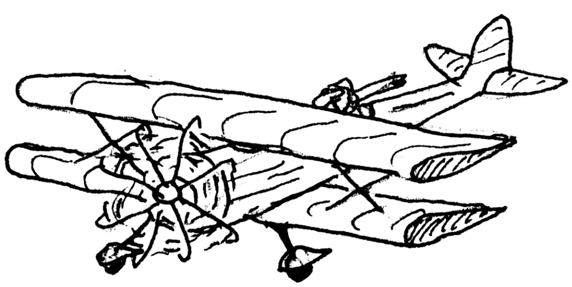
While you're on the subject of numbers and how they appear in the display...

Try This: Key in the number 5,280.35 (press [5][2][8][0][•][3][5]) Now press ■[MODES] to go to the MODES menu.

Now choose the , selection.... Hmmm!...

Bizarre malfunction? Not at all. This is just European numeric notation, where the radix marker (the decimal "point") becomes a comma, and the digit group separators (every three digits) become points. This notation is used conventionally in many countries of the world – quite a few more than those who use the U.S. system.

Of course, you probably also noticed another selection on the MODES menu, the • selection. No prizes for guessing what that one does....



So:

Whenever you want to adjust between U.S. and European numeric notation, you use the MODES menu; whenever you want to adjust the display to show you part or all of each number, you use the DISP menu.

Play around with these some more now, if you wish. When you're ready to go on, though, please be sure to set a FiX 4 display, with U.S. numeric notation....

Level Flight: Simple Arithmetic

Actually, you've already done some arithmetic problems, just to illustrate other things. But now's the time to make sure you're totally comfortable with arithmetic on the HP-22S. Here's a good set of examples. Remember that you can always back out of a mistake or clear the Calculator Line entirely, right?

Example: Find 342 - 173 + 13

Solution: Press [3][4][2][−][1][7][3][+][1][3][=] <u>Answer</u>: 182.0000

See? For simple addition and subtraction, you just press the keys as you would say the problem to yourself – left to right. And notice that as you proceed, you automatically get intermediate results (do this problem again and watch what happens after you press the [+] key). Then the [=] gives you final result.

Yes, But: Find 101.00 - 47.50 x 2

Solution: Press [1][0][1][−][4][7][•][5][×][2][=] <u>Answer</u>: 6.0000

Hmmm...is this right? Which operation were you supposed to do first – the subtraction or the multiplication?

It depends, of course, on the problem as it was stated – whether it *meant* (101.00 - 47.50) x 2 **or** 101.00 - (47.50 x 2). It looks as if your HP-22S has made an assumption, hasn't it? *It doesn't always evaluate an arithmetic problem simply from left to right.*

What About: 342 - (173 + 13) ?

Solution: [3][4][2][−][(][1][7][3][+][1][3][)][=] Answer: 156.0000

Not much mystery here. You can always specify the exact order of evaluation in any kind of arithmetic problem if you use the parenthesis keys (third row down). And notice how the 173.0000+13 changes into 186.0000 when you close the parentheses.

Now Try: 100.00 - 48.0000 x 2 ÷ 64 + 3 - 5 ÷ 11 x 7 + 4

Solution(?): [1][0][1][−][4][8][×][2][÷][6][4][+][3][−][5][÷][1][1][×][7][+][4][=]

Answer (allegedly): 103.3182

Incidentally, notice that you don't need to key in any of those trailing zeroes in the 100.00 or the 48.0000. The machine knows that any unspecified trailing decimal places are zero.

Again, with this last problem, you have The Question: Is the answer correct? Who knows? How can you even know what was *intended* as the correct answer?

Clearly, unless you want to use a whole gob of parentheses, you need some *operator priorities*. That is, when you have a chain calculation like this – with a mixture of different operations (here it's addition, subtraction, multiplication and division), *you need to know which operations come first.*

Well, your HP-22S realizes this, too. And it comes all ready with its own set of assumptions....

Operator Priorities

Here's how your HP-22S interpreted that last problem:

You "said" this: 100 - 48 x 2 ÷ 64 + 3 - 5 ÷ 11 x 7 + 4

But it "heard" this: 100 - (48 x 2 ÷ 64) + 3 - (5 ÷ 11 x 7) + 4

Do you see what's happening? For the HP-22S, multiplication and division have a higher *priority* than addition and subtraction. Those "mental parentheses" the machine inserts into the chain problem are the result of this prioritizing.

Of course, as you can see from the terms inside the parentheses, when you have a part of the chain with operations all of the *same* priority (such as 48 x 2 ÷ 64), then you just go left to right, because mathematically, it doesn't matter; there's no ambiguity – no way to get more than one answer. (Try it: no matter how you slice it, 48 x 2 ÷ 64 always gives you the same result – even if you change the order: 48 ÷ 64 x 2, etc.)

And – as you'll soon discover – there are other operations (things like exponentiation) that have a priority even higher than multiplication and division.

Actually, there's nothing really new or revolutionary about this system of priorities. In fact, you might already be accustomed to using it yourself; it's really quite common. But just in case you're not familiar with it, you'll want to pay close attention as this chapter continues, because these problems will be written with real parentheses only when they're needed to circumvent the automatic mental "parenthesizing." OK?

All right, then, onward and upward – more arithmetic....

Changing The Sign Of A Number

Try This One: Find 34 x -19

Solution: Press [3][4][×][–][1][9][=]   Answer: -646.0000

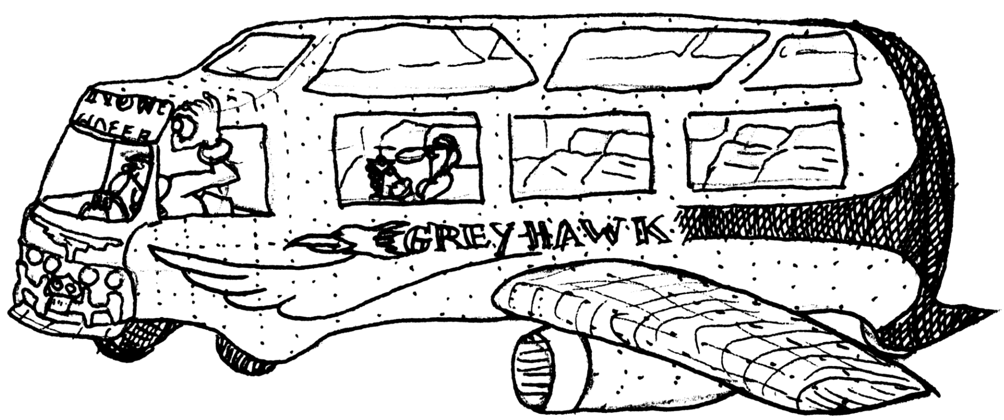
That's the simplest way to key in a negative number: You just key it in as you would say it ("34 times minus 19 equals...").

But there's another way to do it, also.

Like This: Suppose you're not doing any new calculation. You just want to make that -646.0000 into a (positive) 646.0000.

Solution: Press the [+/–] key! That's the "change sign" key, and it's also for changing the sign of whatever you're working on in the Calculator Line. And notice that this [+/–] key is a toggle key – with alternating meanings.

So there are two ways to make a number negative, the [–] key and the [+/–] key.



That E Key

Try This: What's 2,000,000 x 2,000,000 ("two million times two million")?

Solution: [2][0][0][0][0][0][0][×][2][0][0][0][0][0][0][=] <u>Answer</u>: 4.0000E12

This is just a shorthand way of writing very large (or very small numbers), a notation called "scientific notation," since scientists often need to use such numbers.

You'd read this as "four-point-zero-zero-zero-zero times ten to the twelfth power." The **E** is a short way of saying " – times ten to the – ".

Now notice the little blue **E** above the [+/–] key. It means that you can use scientific notation when you key numbers in, also.*

Go For It: Find "2 million times 2 million" once again – but this time, you're not allowed to press the [0] key.

Solution: Press [2][■][E][6][×][2][■][E][6][=]

Saves a few [0]'s, doesn't it? Anyway, whether you like to use this shortcut or not, just be sure to recognize the **E** when your calculator needs to use it.

*And also, you can instruct your display to show you every number in SCientific (or ENgineering) format – with a given number of significant digits. To do this, use the DISP menu and the SC or EN selections (but please don't set your machine that way while following along through this Course unless you're asked to; it makes all results look distractingly different).

Playing The Percentages

Do you realize how easy it is to do percentage calculations on this calculator? "Yes, fans, even these – everybody's least favorite problems – are a real breeze!"

Watch: What's 25% more than 134?

Solution: [1][3][4][+][2][5][■][%][=] Answer: 167.5000

See? Whenever you want to increase or decrease a number by some percentage, you just add or subtract that percentage – just as you would say it.

And how do you simply find a given percentage of any number? Again, it's just as you would say it!

Example: What's 40% of 21.95?

Piece Of Cake: Press [4][0][■][%][×][2][1][•][9][5][=] Answer: 8.7800

To *increase or decrease* by a certain percentage, you *add or subtract;* to simply *find* a percentage *of* some number, you *multiply.* And since you're using prioritized operations (addition/subtraction and multiplication/division) to express these percentages, you know the order in which the machine will evaluate those percentages in a chain calculation, right?

Try a few more problems like these on your own.... What could be easier?

"Hmmm...now how do I use the ■[%CHG] key?"

Well, you use that key to ask for the % of CHanGe between two numbers. That is, it answers this question: "By what percentage has the second number changed from the first one?

OK, but how do I key in two numbers at once? Doesn't the Calculator Line accept only one number at a time?"

Nope – sometimes...

It Takes Two: Press [4][INPUT][5].

See the **:** come on in between these two numbers? This means it recognizes them as an *ordered pair* of numbers (the 4 is first and the 5 is second). You'll find this handy for such calculations as this ■[%CHG] and for statistics (those come in the next chapter).

So that's what the [INPUT] key is generally good for – to create ordered pairs of numbers – usually to use in some calculation that needs two numbers to work properly.

Anyway, where were you? Ah, yes – you're now going to find the percentage change from 4 to 5:

Press ■[%CHG] Answer: 25.0000

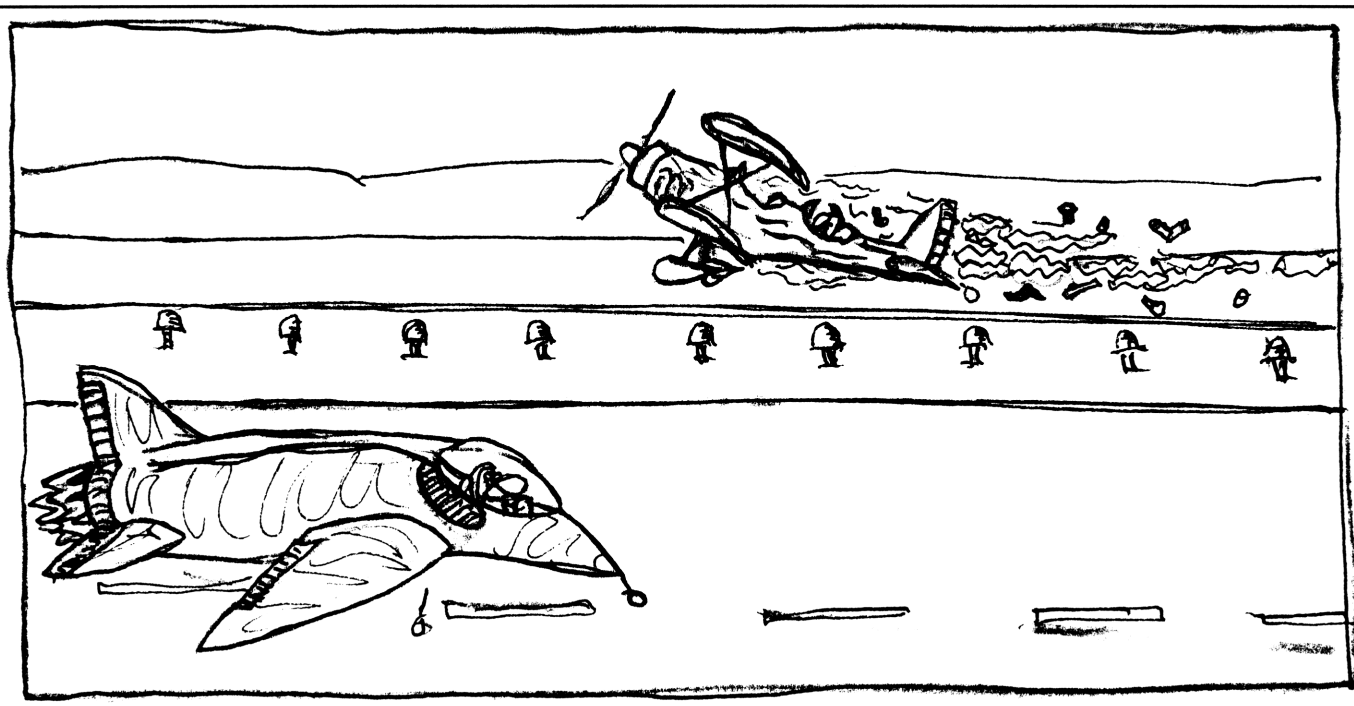
Think you've got the idea of the [INPUT] key and ■[%CHG]? Try...

One More: If the change from 4 to 5 is 25%, then the change from 5 to 4 should be just the *opposite,* a *minus* 25%... ...right?...

Ahem: Press [5][INPUT][4], then ■[%CHG] Answer: -20.0000 (oops.)

Of *course* it's not just the opposite. Remember that in going from 4 to 5, you're changing it by 1/4th (25%) of *what you started with.* But in going from 5 to 4, you're only changing it by *1/5th* (20%) of what you started with.

Always keep in mind with percentage problems that when you say "percentage," you must be aware of what you're saying the percentage "of." In the case of the ■[%CHG] key, it "speaks" (calculates) in terms of the ***percentage of** the first* of the two numbers (the number you "start with").



The ■LAST Key

By now, you've probably wondered what happens to all your previous calculations after you've finished with them. Apparently, whenever you start your next problem, the previous result simply bumps out of the way.

Where does it go? And what's really going on here, anyway? To find out,

---

Do This: Compute 789÷5 + 42 x 789÷5 (remember that if you don't see parentheses, you should assume the operator priorities of your HP-22S).

First, simply calculate 789÷5: Press 7 8 9 ÷ 5 =
Answer: 157.8000

This is now your "previous result." Now you can use that result – as it sits in the *LAST register* – to complete your calculation.

The LAST register is a special memory location in the HP-22S. Its sole purpose in life is to save the calculation or number you finished just prior to the one you're working on now. And to use that saved number, you just press ■LAST!

Thus, to finish this problem, you would now simply press:
4 2 × ■LAST + ■LAST = Answer: 6,785.4000

That ■LAST key and the LAST register are very convenient, no? Just remember that whenever you *finish* a calculation (with the = key or some other function key – such as the LN key) – and then *begin a new one,* the previous result is immediately placed in the LAST register, overwriting (destroying) what was in there before.

A Pause For The Cause

Look at all the stuff you now know about simple arithmetic and number-crunching on that Calculator Line:

- You know when and how to clear the Calculator Line;
- You know that the machine always keeps and uses 12 digits for each number in every arithmetic calculation;
- You know how to change the display setting (with FiX and ALL) to see as many decimal places as you want (up to 11).
- You know that to do arithmetic problems on the HP-22S, you must keep in mind the priorities of the various operations (x and ÷ before + and -);
- You know two different ways to key in negative numbers;
- You know how ridiculously easy it is to do percentage problems now;
- You know what 4.0000E12 means (and how to key it in, too);
- You know about the History Stack and ■ LAST key;

- You know that it's time to "gain some altitude" – climb up and do some higher-math maneuvers (but if you feel you already know how to do exponentiation, circular and hyperbolic trig, number base conversions and probabilities, then try "flying" over to page 66 right now)....

Exponentiation: Roots, Logs And Powers

As you recall, the higher-math functions live mainly on the top two rows of keys. Look first, then, at the very top row. It's time to see how these keys work.

Try One: Find $\frac{1 + \sqrt{5}}{2}$

Solution: Press [(] [1] [+] [5] [√x] [)] [÷] [2] [=] (or you could do it this way instead, without using any parentheses: [1] [÷] [2] [+] [5] [√x] [÷] [2] [=]).

Answer: 1.6180

Notice how this [√x] key operates on the most recent number in the Calculator Line – in this case, the 5. This is how most of the "x-functions" (functions with x in their names) generally work.

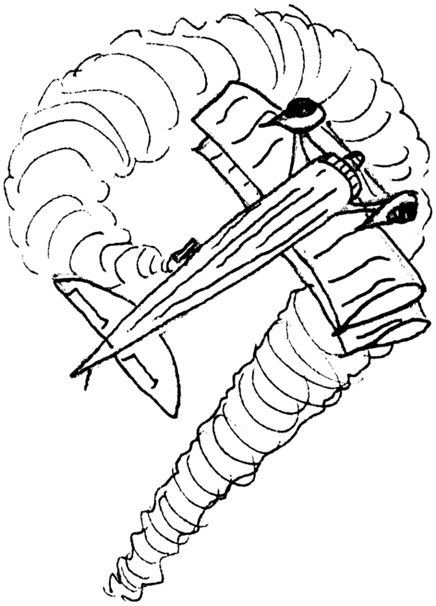
Try Another: Find $(34.19 + 1/11)^2$

Solution: Press [(] [3] [4] [•] [1] [9] [+] [1] [1] [1/x] [)] [■] [x^2]

Answer: 1,175.1807

See? You *reciprocated* the 11 (flipped it over to make it 1/11) with the [1/x] key, then completed the parenthesized portion of the problem. That left only one number on the Calculator Line, which you then *squared* with the [■] [x^2] key.

You're On A Roll:



The LN function is the Natural Logarithm; it uses base **e**. That is, the LN of any number, n, is the power (i.e. the exponent) to which you have to raise **e** to obtain n.

Similarly, the LOG function is the common LOGarithm function, which uses base 10. That means the LOG of n is the power to which you have to raise 10 to get n.

However, there's a handy formula to convert between the logarithm of any base, b, and natural base (**e**):

$$LN(n) = \log_b(n) \times LN(b)$$

Show this to be true for the common LOG base (i.e., b= 10). To do this, find LN(100) by two different methods – once using the LN function directly, and then once indirectly, by using the above formula.

Solution:

First, the easy way: [1][0][0][LN] (<u>Answer</u>: 4.6052)

Then the indirect route: The formula says that

$$LN(100) = \log_{10}(100) \times LN(10)$$

But that's really saying this: LN(100) = LOG(100) x LN(10)

So, go for it: Press [1][0][0][■][LOG][×][1][0][LN][=]

<u>Answer (of course)</u>: 4.6052

Good Questions: What are the easiest ways to obtain the natural LN base (**e**) and the number π on the HP-22S? (That is, obtain the 12-digit *approximations* to **e** and π;– you can't obtain the exact numbers, because they're irrational – their digits go on forever.)

Then how do you compute $\dfrac{e10^{\pi}}{\pi e^{10}}$?

Decent Answers: To get **e**, just compute e^1: [1][e^x] (and then press ■[SHOW] to temporarily peek at all twelve of its digits): 2.71828182846

To get π, just press ■[π] (and ■[SHOW]: 3.14159265359)

As for that bizarre and hairy fraction, notice that both the [LN] and ■[LOG] keys have matching *inverse* functions on the keys just to their left: [e^x] and ■[10^x].

These functions will "undo" the effects of [LN] and ■[LOG] (i.e. they "go the other way"), since they take the "x" that you supply on the Calculator Line and use that as the power to raise the base (either **e** or 10) to arrive at a number, "n."

So, ready to try calculating that fraction? Here goes:

[1][e^x][×]■[π]■[10^x][÷][(]■[π][×][1][0][e^x][=]

<u>Answer</u>: 0.0544, or 5.44242488548E-2 if you display ALL relevant digits (remember how to do this – and what the E notation means? See pages 38 and 44, if you've forgotten).

And Now: Find $(4.4^2)^5$.

Solution(s): You can actually do this a number of different ways. To begin with, you can square the 4.4 (you know how, since you've done this before): $\boxed{4}\boxed{\cdot}\boxed{4}\blacksquare\boxed{x^2}$.

But now, how do you raise a number to any power other than 2?

Simple – you use the $\boxed{y^x}$ key.

Since you now have 4.4^2 (19.3600) on the Calculator Line, just press $\boxed{y^x}$ to see the little exponentiation symbol, ^, appear. This means that you're going to raise the 19.3600 to whatever power you key in next.

So key in the final exponent, and finish the problem off: $\boxed{5}\boxed{=}$

<u>Answer</u>: 2,719,736.0938

Of course, there are other ways to do this problem. Notice that $(4.4^2)^5$ is really the same as $(4.4)^{2\times5}$, or 4.4^{10}. You could even call it $(4.4^5)^2$, if you wanted to – they're all the same. Thus:

$\boxed{4}\boxed{\cdot}\boxed{4}\boxed{y^x}\boxed{1}\boxed{0}\boxed{=}$ is probably the easiest, although

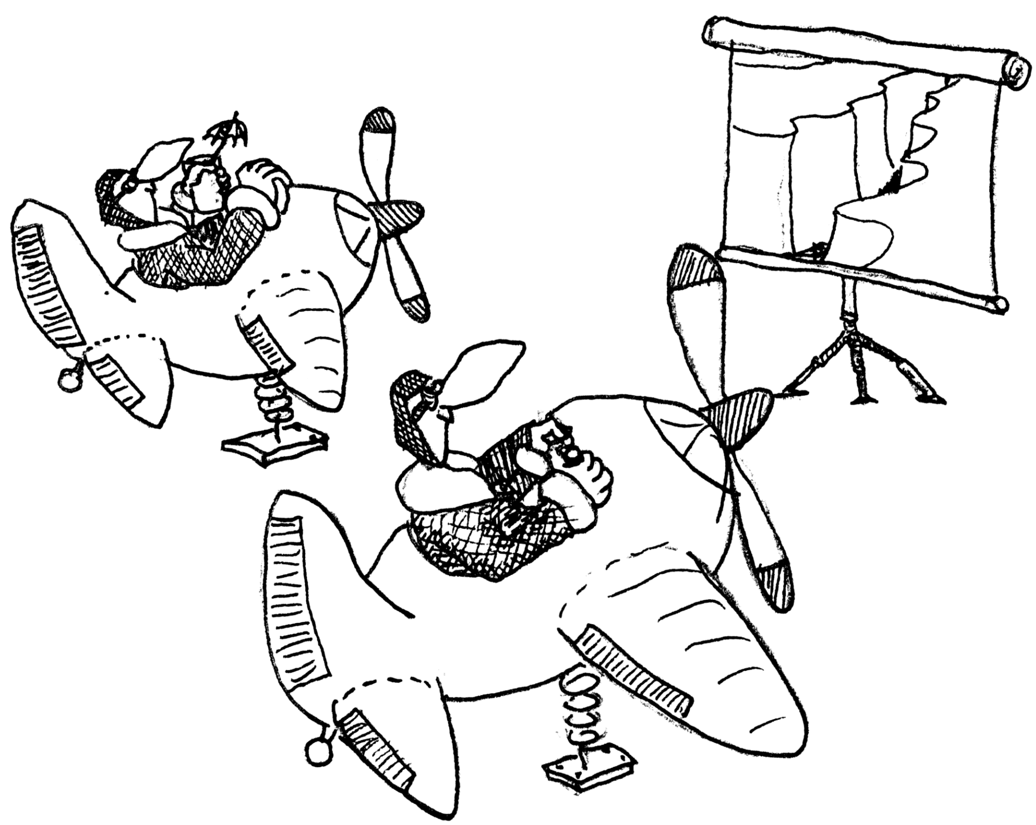
$\boxed{4}\boxed{\cdot}\boxed{4}\boxed{y^x}\boxed{5}\boxed{=}\blacksquare\boxed{x^2}$ works just as well.

Notice that the [y^x] key works a little differently than the other higher-math keys. Most of the others will immediately alter the current number on the Calculator Line. But the [y^x] key behaves more like the [+] or [×] keys; *it acts as an operator, putting its own symbol on the Calculator Line, right after the current number. Its execution is delayed until the machine can get some more information.*

Not only is the ^ an operator, it actually has a higher priority than even the × and ÷ operators (recall, if you will, the discussion of operator priorities back on page 42).

This means that when you have some chain calculation involving exponentiation and any of the other arithmetic operators (+, −, ×, and ÷), the exponentiation will always come first (unless, of course, you specify otherwise with parentheses).

Watch how this works in the following example....



Example: Find 4.25x1.10^12-7.19÷3.34^2.5+40.64

Solution: This problem is purposely stated with the ^ notation for exponentiation – to show you how confused your HP-22S would be without operator priorities. After all, it has to evaluate symbols all on the same line (e.g. 1.10^12); it doesn't know how to read the visual cue of superscripts (e.g. 1.10^{12}).

So, if you interpret this problem with proper HP-22S operator priorities (and all problems in this course will assume this), then you can correctly key it in without parentheses, and here's what that interpretation would effectively be:

(4.25x(1.10^12))-(7.19÷(3.34^2.5))+40.64

or, in more visual terms:

$$(4.25 \times 1.10^{12}) - (7.19 \div 3.34^{2.5}) + 40.64$$

The keystrokes themselves are easy, of course, since you need only key in the original problem as written:

[4][•][2][5][×][1][•][1][y^x][1][2][−][7][•][1][9][÷][3][•][3][4][y^x][2][•][5]
[+][4][0][•][6][4][=]

<u>Answer</u>: 53.6257

Common Trigonometry

That about does it for the top row of higher-math keys. Now move one row down and look at those functions. It's time for some very basic trig....

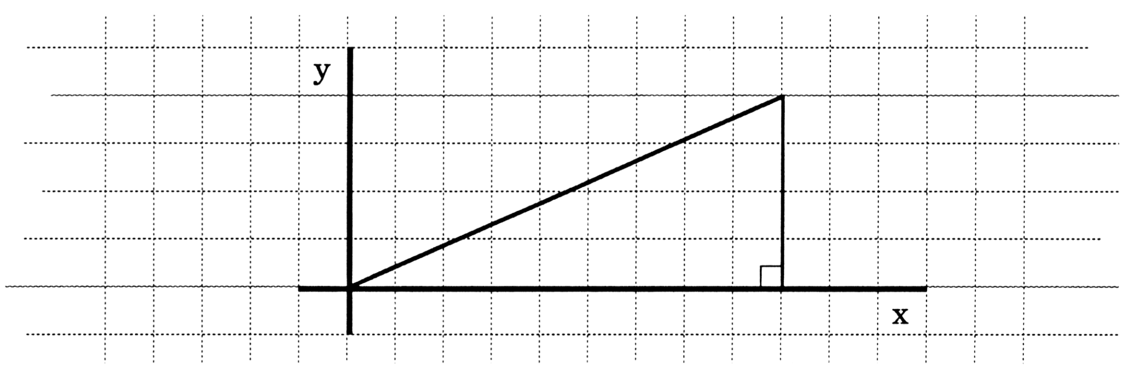
What would you do, for example, if you suddenly encountered a problem like this on some foggy, abandoned airstrip at four in the morning?

$$ATAN(SIN(22°) + COS(\pi/6) \div TAN(-125°))$$

You'd whip out this book to review the basics of trigonometry, wouldn't you?*

The word trigonometry itself means "the measure of the trigon." And if a pentagon is a five-sided figure, then a trigon must be...yep – a three-sided figure. So *trigonometry is "the measure of the triangle"* – the *right* triangle, specifically.

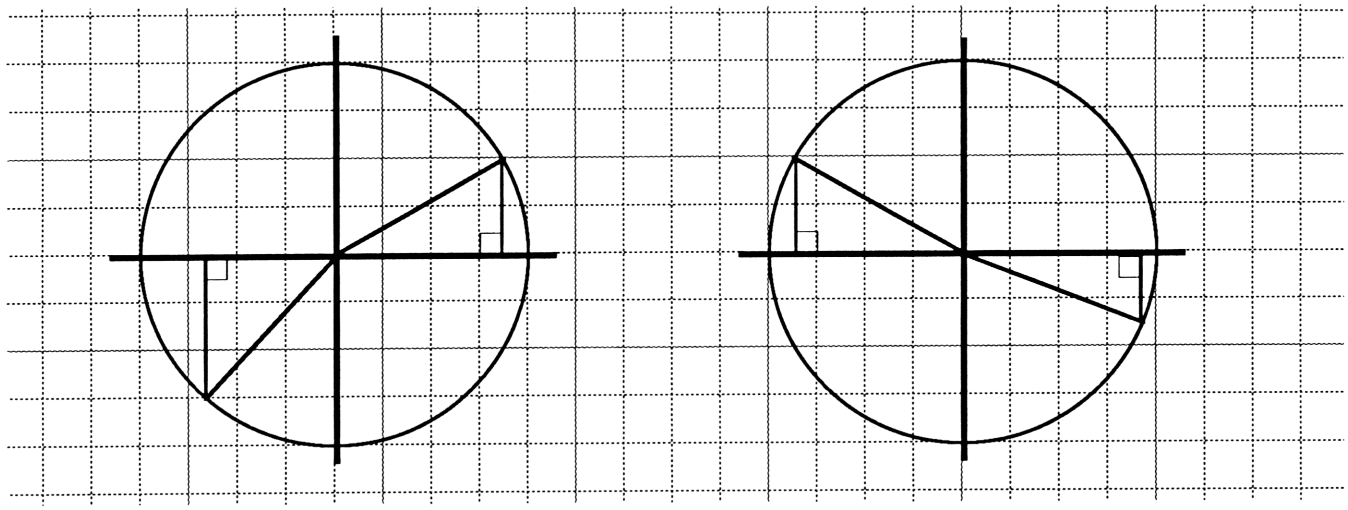
Now, to make trig clearer (and simpler), somebody decided a long time ago that when you're measuring such a triangle on graph paper (with x- and y- coordinates and all that), you should always draw the triangle with the long side (the "hypotenuse") extending out from the origin (P(0,0)) – like this:



Notice how you always place the right (90°) angle along the x-axis, too.

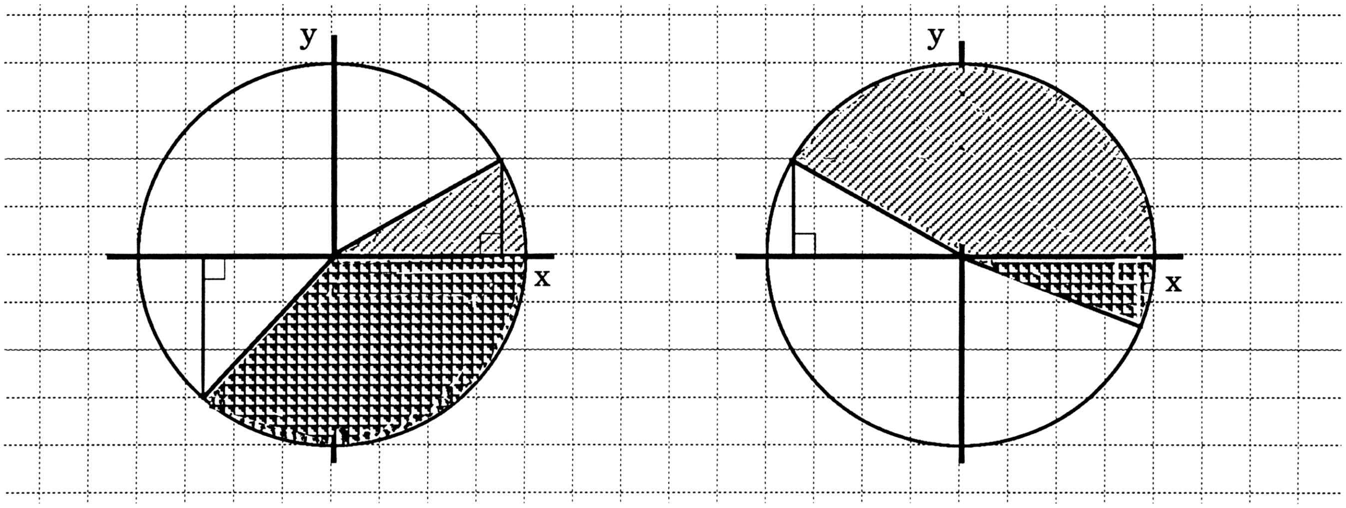
*Of course you would.

Well, the really clever part about all this is that if you draw a circle centered at the origin, then you get fascinating results by drawing any triangle whose hypotenuse forms a radius of this circle:



People discovered different functions that *mathematically described the lengths of (and the relationships between) the three sides of any of these triangles.*

It turned out that all you needed to know was *where* you had touched the circle with the hypotenuse of your triangle; you can tell all sorts of things about your triangle if you know only that. So the question became: How much arc is there on the circle between the point you're touching and some reference point, say, the positive x-axis?



The whole trick of common trigonometry, then, is to express these different functions and relationships within a right triangle *in terms of the arc "swept out" along the circle by the hypotenuse of that triangle.* It all boils down to that.

So how do you measure this arc along the circle ("circular arc"), anyway – in what units – meters? furlongs?

Well, they settled on a couple of different ways:

In any circle, if you were to "wrap" copies of its *radius* along its perimeter, you could fit exactly 2π ($\approx$6.28318530718) of them around the entire circle. So a convenient way to refer to circular arc is in a measurement of the *length of the arc along the circle, expressed in units of its whole radius – called* ***radians***.

There are 2π radians of arc *length* in a full circle.

But another way to measure the arc is to split up the full circle's swept angle into 360 equal pieces.

There are 360 degrees of angle in a full circle.

So you can measure circular arc either by the *angle* it sweeps out or by its *length* (in radians). The point is, you need to tell your HP-22S which measurement you're using:

Press ■MODES. See the two choices, DG and RD? Those stand for DeGrees and RaDians, respectively.

Choose RD. See the RAD annunciator come on in the display? The machine is now reminding you that all of its circular trig calculations will be done using RaDians to measure the arc. That is, the HP-22S is now in RaDians "mode" (which explains why these selections are under the MODES menu).

Now press ■MODES, then DG. When there's *no annunciator* showing, the machine assumes you're "talking" in DeGrees – so it operates in DeGrees mode.

Now Then: No kidding – find ATAN(SIN(22°) + COS(π/6) ÷ TAN(-125°))

Gulp: In this rather ugly problem, you have mixed units. That is, some of the arc is measured in degrees(with the ° symbol), some in radians (without the °).

Another thing: From the problem statement, you can't tell in which of these units the ATAN should be finally calculated.

Finally: Keep in mind that ATAN(x) (the "Arc-TANgent of x") is "the Angle whose TANgent is x." And that could be any one of an infinite family of angles who differ from one another by exact multiples of π radians (and there are similar problems with ACOS and ASIN). But the HP-22S makes the assumption that you're after the simplest of these angles (generally between -90° and 180°).

Now, with all *that* in mind, how do you solve the problem? Like this:

Start in DeGrees mode (press ■[MODES] then DG, to turn off the RAD annunciator, if you haven't already). Then press

[2][2][SIN][+][(]
■[MODES] RD ■[π][÷][6][)][COS][÷]
■[MODES] DG [−][1][2][5][TAN][=]■[ATAN]

Answer: 44.4506 (degrees – 'cuz the RAD annunciator ain't on.)

Notice how all these trig functions always act upon the most recent number in the Calculator Line – just like [√x], ■[x²], [eˣ], etc.

Yes, But: What if you now want to see this result in radians?

Can Do: Notice (and press) the ■D↔RAD key. This menu allows you to actually *change* the number on the Calculator Line from one notation to the other. You'll notice that when you used the ■MODES key to switch in and out of RAD mode, this affected only *subsequent results* – not the number then sitting on the Calculator Line.

By contrast, the ■D↔RAD key has actual functions to convert between the two different angular units.

To convert from degrees to radians now, just choose →RAD and voilá!
Answer: 0.7758

To convert back to degrees again, just press ■D↔RAD again, but this time, choose →DG. What could be simpler?

Get This: You can even convert to and from Degrees, Minutes and Seconds! That is, the 44.4506 you see actually means 44 degrees, plus 45.06/100ths of a degree – "decimal" degrees.

But press ■H↔HMS, then choose →HMS and bingo! You now have 44.2702, which is in HH.MMSS format: 44 degrees 27 minutes, 2 seconds. The reason this menu is called H↔HMS is that H stands for Hours. But you can use it for degrees, because, after all, both hours and degrees have 60 minutes of 60 seconds each, right?

Now press ■H↔HMS and choose →HR to convert this value back to decimal degrees.

In All Likelihood: The PROB Menu

To continue your tour of the higher-math capabilities of the second row of keys, look now at the ■ PROB key and the PROBability menu, with this

---

Example: If you had 6 red marbles, 5 orange marbles, 4 yellow marbles, 3 green marbles, 2 blue marbles and 1 violet marble – all in hand-tooled pouch of tanned water-buffalo leather – how many possible combinations of six marbles (any color and any order) could you draw out of the pouch?

What if you wanted to count permutations – where the order does matter?

Solutions: You're really just asking how many ways you can combine 21 objects, taken 6 at a time. So you'd press 2 1 INPUT 6 ■ PROB, then choose Cn,r, to get the Combinations. Answer: 54,264.0000

To get the permutations, you just press 2 1 INPUT 6 ■ PROB and choose Pn,r. Result: 39,070,080.0000

The PROB menu works in the same manner as most of the others you've seen; first you key in the numbers you want to compute with, and then you select the menu and the item in it that you need. And notice that for each of the Permutations and Combinations calculations, you needed *two* numbers, so you used the INPUT key.

See how easy it is to use the menus on your HP-22S? And can you imagine trying to figure out solutions like these on your own – without all these handy built-in calculations? If you had to do *that,* you might well end up…

…losing your marbles.

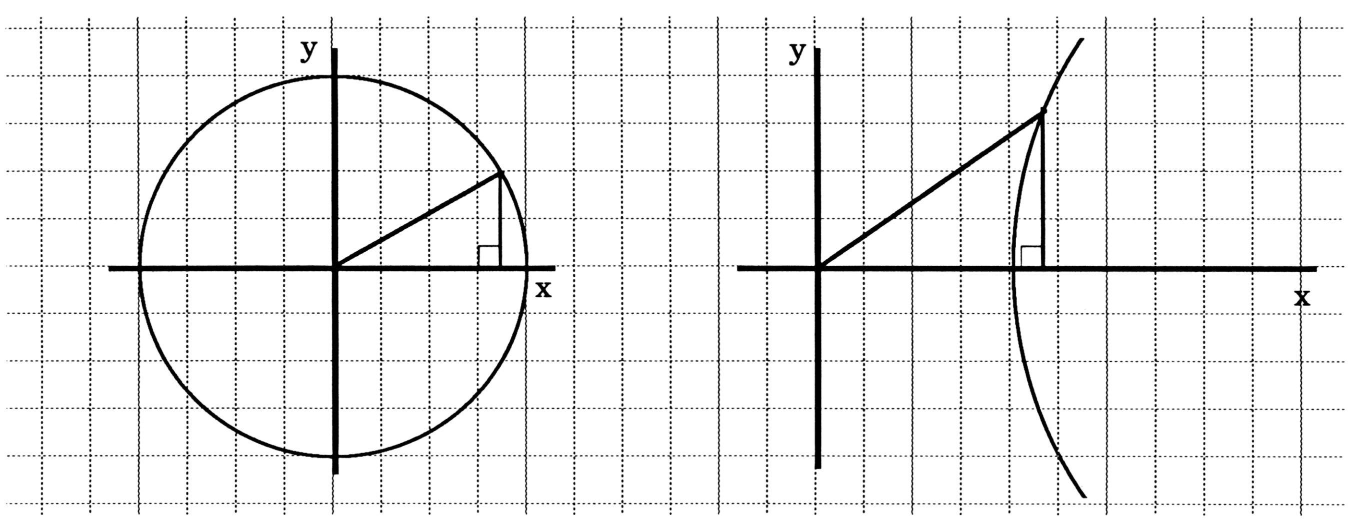
Uncommon Trigonometry: The HYP Key

If you're like most people, you're probably more familiar with the concepts of common *circular* trig (remember back on pages 56-60?) and with the circular trig functions, SIN, COS, and TAN, than with their hyperbolic cousins, SINH, COSH, and TANH.

But each has its own niche and purpose in the Wild Kingdom of numbers, so as another edition in this series of public-service reminders, here's a quick summary of the basic difference between circular and hyperbolic trig...

As you just read a few pages ago, circular trig is based upon drawing a right triangle whose hypotenuse extends from the origin to a point on a circle.

Well, in hyperbolic trig, the hypotenuse ends on a *hyperbola,* instead of a circle. Here's the side-by-side comparison:



(circle) (hyperbola)

So, how do you use the ■ HYP key to compute with SINH, COSH, TANH, etc.?

Like This: Find ATANH(SINH(2.2÷7) x COSH(π/6) + TANH(-1.25))

No Problem: Anytime you see a hyperbolic trig function, you just press ■ [HYP] as you need the function. You'll see the display look like this:

```
HYP _
```

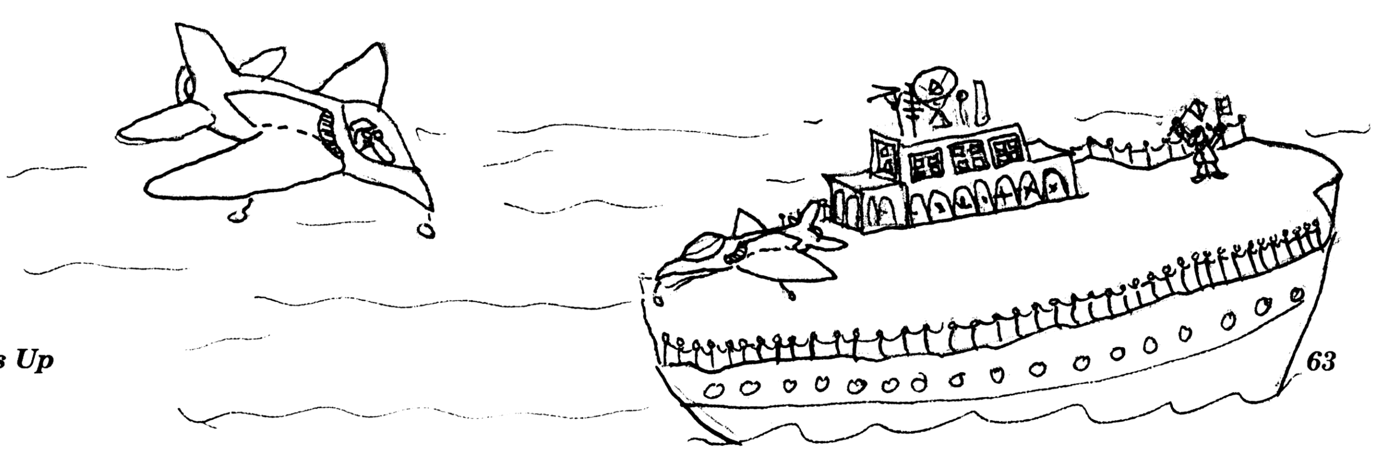
It's telling you now to press one of the "normal" (circular) trig keys to complete the naming of the hyperbolic function.

Thus the solution to this problem would go like this – and watch the display as you complete the naming of each function; its name (e.g. SINH, COSH, etc.) will appear briefly before it does the calculation:

Press [2] [•] [2] [÷] [7] [=] ■ [HYP] [SIN] [×] [(] [π] [÷] [6] [)] ■ [HYP] [COS] [+] [−] [1] [•] [2] [5] ■ [HYP] [TAN] [=] ■ [HYP] ■ [ATAN]

<u>Answer</u>: -0.5282

Not too tough once you get the pattern down, right?



Before you leave the subject of higher math, here are a couple more minor menusthat provide you with handy ways to "crunch" numbers:

Of Odds And Ends: The PARTS Menu

The PARTS menu is a collection of four miscellaneous functions that – like most functions – operate on the current number on the Calculator Line.

Try It: Press ■ PARTS to see a menu that looks like this:

```
IP  FP  RN  ABS
```

What are these functions and what do they do to a number?

- IP takes the Integer Portion of the number. for example, 3.14159265359 would bcome 3.00000000000.

- FP takes the Fractional Portion of a number. Thus, 3.14159265359 would become 0.14159265359

- RN RouNds a number *to the current display setting*. For example, 3.14159265359 would *appear* as 3.1416 under a display setting of FiX 4 (but you know that the full 12 non-zero digits would still be there – just unshown). Upon choosing RN, however, the actual, full-precision number *would be changed* to match the display's precision: 3.14160000000. This is the *only* way in which a display setting can permanently affect the actual value of a number.

- ABS take the ABSolute value of a number. For example, a -2.0000 would become 2.0000, but 7.0000 would remain unchanged.

The BASE Menu: How Many Fingers Do You Have?

You can even do number-base conversions on this machine. The HP-22S has a BASE menu where you can change the numbering "language" in which you do arithmetic. You can use any one of four different number bases: 10 (DECimal), 16 (HeXadecimal), 8 (OCtal), and 2 (BiNary).

Of course, in all but the decimal system, *your display will show you only the integer portion of a value* (since it's very hard to build fractions in other number bases).

Watch: What's e^{π}, as expressed in decimal, hexadecimal, octal and binary?

Solution: Press ■ [π] [e^x] Answer: 23.1407

That's the decimal value – and it shows all the digits it's supposed to. But now press ■ [BASE] and choose HX.

Result: 17 And the little HEX annunciator above the Calculator Line tells you that this is the HEXadecimal representation of (the integer portion of) this value.

Now press ■ [BASE] and choose OC. Result: 27 OCT

Now press ■ [BASE] and choose BN. Result: 10111 BIN

Remember – this is only the display doing this. If you press ■ [BASE] and choose DEC once agin, the entire number will be there – fractional digits and all.

Well, that about covers most of the numerical "maneuvers" you might want to do while "flying" your HP-22S.

Of course, there's plenty more to come, but this is a good spot to "level out" and review in your mind what you've seen and done so far:

- You know how to clear or correct mistakes with the [C] or [←] keys, how to recall previous results with the ■[LAST] key, and how to set the display to your liking with the ■[DISP] and ■[MODES] menu keys;

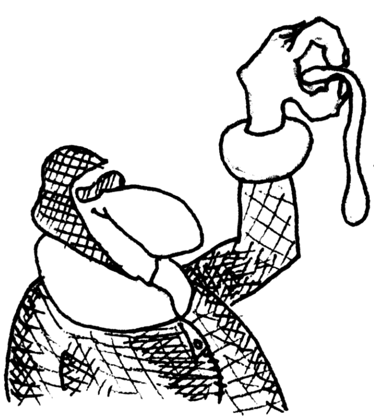
- You know that to do arithmetic problems on this machine, you must keep in mind the priorities of the various operations (^ comes before × and ÷, which, in turn, come before + and −);

- You know how to work with negative numbers, percentage, and scientific notation;

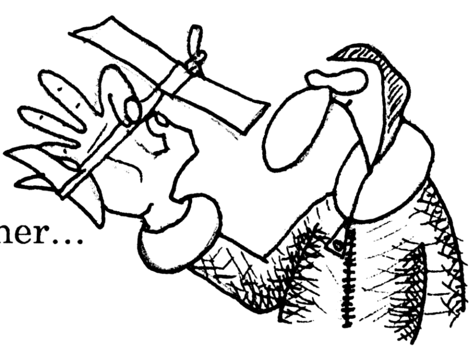
- You know how to compute logarithms, powers, roots, and exponentiations;

- You got a quick refresher course in the fundamentals of circular and hyperbolic trigonometry – and you know how to use the ■[D↔RAD] and ■[H↔HMS] keys to change the angular units the machine uses;

- You even took whirlwind tours through the ■[PROB], ■[PARTS] and ■[BASE] keys and their menus.



Well, then, it's time to put it all together...



...and take your Basic Maneuvers Qualifying Exam....

Basic Maneuvers Qualifying Exam

Solve these your HP-22S – as efficiently and joyfully as you now know how. As usual, the solutions are on the next pages, with page references for review items (and you might see some new variations here, too, so don't "blur" over any of this):

1. Find $\dfrac{100 \div 75}{25 \times 64 + 34 \times -19}$

Then find $\dfrac{25 \times 64 + 34 \times -19}{100 \div 75}$ by two different methods.

2. Which is greater: $4 \div 7$ or $6.285 \div 11$? What's the difference?

3. 4.0000E12 is "4 trillion" (a very large number). So, what is 4.0000E-12? And how would you key this in?

4. This year's salmon run is 35% more than last year's, but next year's will be only 85% of this year's. If last year's peak count through the fish ladder was about 2500 per hour, what might next year's peak count be?

5. Find -19 x -19. Compute it three different ways, two of which *don't* use the $\boxed{\times}$ key.

6. Find $\sqrt{4096}$ by two different methods. Then find $\sqrt[12]{4096}$.

7. The most severe drop in the history of the U.S. stock market occurred between September 3, 1929 and July 8, 1932. During that time, one stocks average fell from 452 to 58. If the same percentage drop happened with a beginning average level of, say, 2735, where would the bottom level be?

8. A standard football field is 100 yeards long (never mind the end zones). If it's 45 yards wide, and you add 9% to both its width and its length, by what percentage do you increase its area?

9. By what percentage must you change $\frac{\sqrt{5+1}}{2}$ to get $\frac{\sqrt{5-1}}{2}$?

10. If 18 U.S. states and 3 Canadian provinces meet to decide how to stop acid rain, and a simple majority can veto any proposal, how many ways are there to cast just the minimum 11 "no" votes that would prevent any action from being taken?

11. Find $((1 - 1.1^{-10}) \div .1 + 100) \times 1.1^{10}$

12. Show that $\cos^2\theta + \sin^2\theta = 1$ for $\theta = (\pi/4 + 1)$ and $-22.5°$

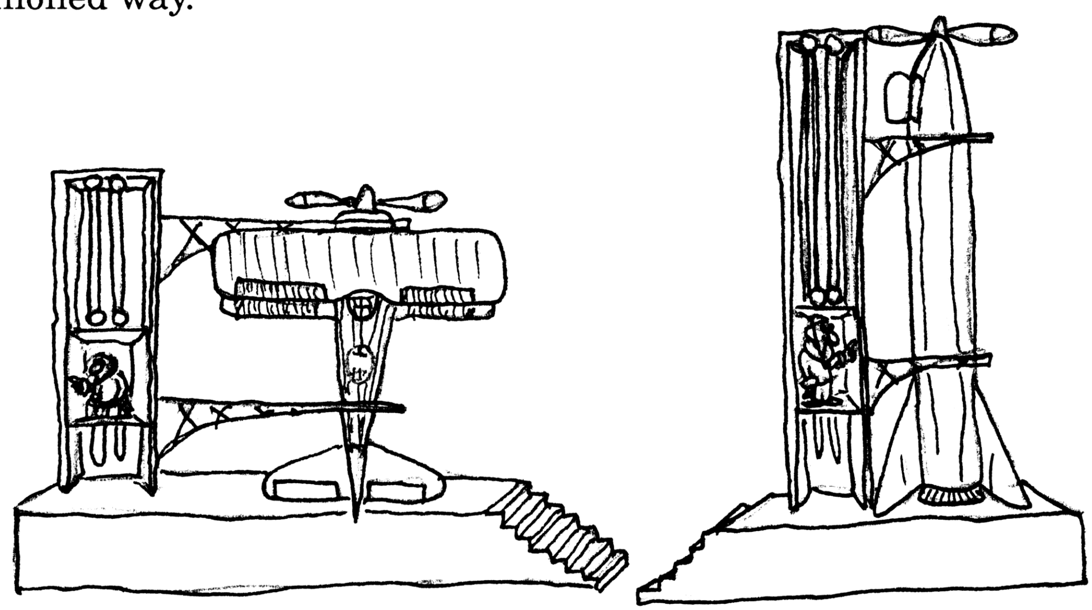
Show that $\cosh^2\theta - \sinh^2\theta = 1$ for $\theta = 4.4$

Question: Which of the circular arc unit denotations (radians or degrees) are applicable to hyperbolic trig? What does comon sense – with help from the diagram on page 62 – tell you? Test this common sense by twice computing the ASINH or ACOSH of some randomly chosen number – once while in degrees mode, once while in radians mode. What's the difference in results?

13. Compute $\sin(\pi/2)$, $\sin(\pi)$, $\cos(\pi/2)$, and $\cos(\pi)$.

14. Show that 23.4567° is the same as 23°27'24.12".

Do this twice – once with the help of the ■ H↔HMS key menu and once "the old fashioned way."



15. You're piloting your rebuilt F-104 on a non-stop, 25,000-mile, around-the-world flight (that new carburetor sure does improve the gas mileage, eh?).

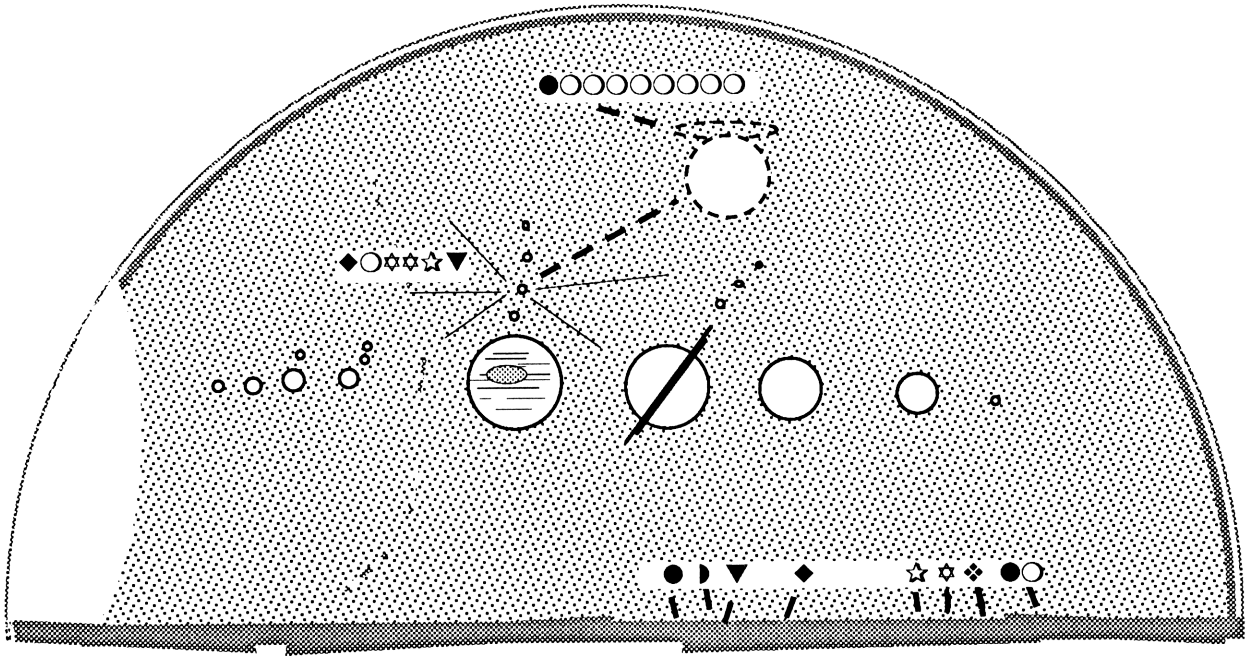
You're flying level at 130,000 feet (she ain't half bad at altitude performance, either) due westward into a constantly-setting sun, when you see, in the dark blue above you, a meteorite appear, glowing and growing, as it slowly approaches...on a *near-parallel* collision course with your aircraft – like a merging vehicle on a supersonic freeway.

Fascinated by the improbability of it all, you hold to your course until the near-molten object is almost touching your canopy. You're just preparing to turn away to avoid the collision when suddenly your slipstream disturbs and slows the meteor. That's the last you see of it, as it slips and tumbles to the rear. But there, unbeknownst to you, it collides with the onrushing tail of your jet, and though part of it falls away, another portion fuses onto the tail, cooling and remaining there for the rest of the trip.

Unaware of all this, you complete your epic journey and thoroughly enjoy your ticker-tape parade, etc., etc. Then one day, after the excitement has died down, you're doing a routine inspection of your aircraft and you notice a fused chunk of material at the base of the plane's tail.

You can't imagine how it got there, and you use a welding torch to cut free the chunk of now-cold metal and examine it. Only then does it dawn on you where this came from – it's part of that freak meteorite!

It resembles a small discus, cut neatly in two – with one half missing. It's very heavy – about 64 pounds – being composed mainly of iron, but with a mysterious outer coating, upon which you see some pictures and symbols etched....



Your scalp tightens and your face pales as an enormous rush of adrenalin hits you with the realization of what this must be....

A *message probe!* – from...*Somewhere Else!* But where? And what does the message say? You need the other half of the probe....

Where would it have hit the Earth? Losing no time, you look up some thumbnail formulae for the free fall of a comparable (same weight and minimum profile) smooth sphere through atmosphere (those'll have to do; there's nothing listed for a red-hot half-discus):

$y_{feet} \approx y_0 - 256t - 1100$ for $t>8$ seconds

$x_{feet} \approx x_0 + v_0t - (6400/w)t^{4/3}$ (v_0 is the initial velocity in feet per second; w is the weight of the object in pounds)

Reckoning back to the moment when you lost sight of the meteorite, you estimate your position at the time to be about 65 miles east of Isla Isabela in the Galapagos Islands (0° latitude, 90°44' W. longitude). You need to estimate the approximate longitude and latitude to begin the search for the other half of the discus. Then, *if* you find it, you must try to decipher the message.

B. M. Q. E. Solutions

1. Press 1 0 0 ÷ 7 5 ÷ (2 5 × 6 4 + 3 4 × − 1 9 =
<u>Answer (■ SHOW to briefly view all the decimal places)</u>: 1.39762404E-3

The easy way to do the second part is to notice that it's the "flip" of the first, which you can find by using the 1/x key. <u>Answer (FiX 4 places)</u>: 715.0000

Of course, the other way is to start all over from scratch: Press (2 5 × 6 4 + 3 4 × − 1 9) ÷ (1 0 0 ÷ 7 5 = (to review your basic arithmetic, flip back and re-read pages 40-43).

2. To compare the two, you need to find both answers:
Press 4 ÷ 7 = <u>Result</u>: 0.5714

Now press 6 • 2 8 5 ÷ 1 1 =. Hmmm...just from looking at the first four decimal places, these two results seem identical. But they may not be the same out in the other decimal places.

How do you compare? Well, remember that your first result is saved in the LAST register, so if you subtract that from this second result, you should be able to see the difference: Press − ■ LAST = ...

<u>Answer</u>: -0.0001 *Aha* (you suspected as much)! There *is* a difference – and it's *negative,* so that means your first number (4/7) was greater.

3. This is "four times ten to the minus twelfth power," or "four trillionths." Written out fully, it would be 0.000000000004.

To key this in, you would press [4] ■ [E] [−] [1] [2] (see page 44 for a re-read of E, if you wish).

4. Say it to yourself: "2,500 plus 35%, times 85%."

Now press keys as you say it: [(] [2] [5] [0] [0] [+] [3] [5] ■ [%] [)] [×] [8] [5] ■ [%] [=]

<u>Answer</u>: 2,868.7500 That's nearly three thousand fish per hour (see pages 45-47 to review % calculations).

5. First way: [1] [9] [+/−] [×] [−] [1] [9] [=]

Second way: [1] [9] [+/−] ■ [x^2]

Third way: [1] [9] [+/−] [y^x] [2] [=] (in all cases, you get the same result: 361.0000).

6. First way: [4] [0] [9] [6] [$\sqrt{x}$]

Second way: [4] [0] [9] [6] [y^x] [•] [5] [=], because when you take the square root, you're really taking the one-half (.5) power (and the result either way is 64.0000).

Similarly, $\sqrt[12]{4096}$ is really $4096^{1/12}$. Thus: [4] [0] [9] [6] [y^x] [1] [2] [1/x] [=]
<u>Answer</u>: 2.0000

7. This is a %CHG problem, isn't it? But it's in two parts: First, you need to find the percentage of that historical (hysterical) market drop. Then you need to *use* that result on the hypothetical level of 2735 – in a simple percentage calculation (i.e. "what's 'x' percent of 2735?").

So, first press [4][5][2][INPUT][5][8][■][%CHG] <u>Result</u>: -87.1681.
That is, the market *changed* (went down) *by* over 87%.

Now how can you use this number to find the theoretical low for an equivalent crash that starts at a high of 2735?

Consider this: If a number changes by, say, a positive 25%, then it is 125% of what it use to be, right? And if the change is a -87%, then it is *13%* of what it used to be. The resulting percentage is always the *sum* of what you started with (100%) *plus* the percentage it changed (regardless whether that change was an increase or a decrease – the sign of the change will take care of this for you).

So if you *add* your result here (-87.1681) to 100, you'll get the percentage of the original number that is *remaining* as a result– and it's *that* percentage you actually need to finish the calculation.

Thus, you press [+][1][0][0][=] <u>Result</u>: 12.8319

This says that the market fell to under 13% *of* its peak value. And now to finish, all you need to do is to ask, "What's 12.8319% of 2735?"

Press [■][%][×][2][7][3][5][=] <u>Answer</u>: 350.9513 So a market drop comparable to the 1929-32 crash would take a high today of 2735 and send it all the way down to under 351.*

*"Toto, I don't think we're in Kansas anymore...."

8. This is another %CHG problem – but this time, there's no second part; the only real challenge is to figure out the "before" and "after" areas of your football field.

The "before" area is easy: [4][5][×][1][0][0][=] Result: 4,500.0000

Now, since this is going to be one of the two necessary numbers you need to supply to do a %CHG calculation, press [INPUT] so that the machine will hold this number on the Calculator Line while you compute the second number.

Now for that second number: Press [(][1][0][0][+][9][■][%][)][×][(][4][5][+][9][■][%][=]

Result: 5,346.4500

Notice that while you're doing this calculation, there's a **:** in front of it to tell you that this is still just the second of *two* separate numbers on the Calculator Line. And if you want even more evidence, you can use the [■][SWAP] key to exchange their places (just be sure to press [■][SWAP] again to switch them back)!

Now you're all set for the %CHG calculation: you have the "before" number and the "after" number on the Calculator Line – just as if you had entered them together (i.e. "before" [INPUT] "after").

So press [■][%CHG] Answer: 18.8100

By increasing both the length and width of the field by 9%, you increase the area by 18.81%.

9. You're probably getting tired of these %CHG problems – but they're very good practice, so here's one more (and it's the last one, so take heart):

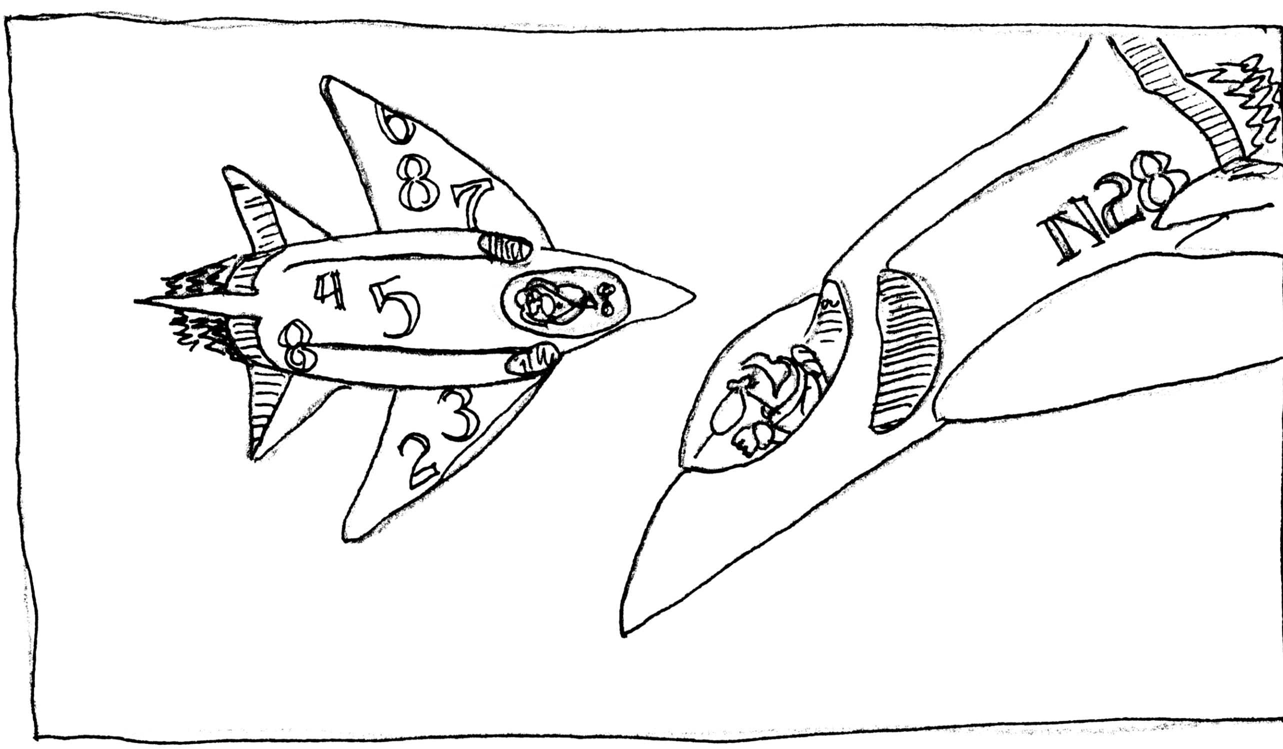
The only real trick here is in calculating and preserving both the "before" and "after" numbers, so that they'll be ready on the Calculator Line when you're ready to go for the final answer.

To get the "before" number, press: [(] [5] [√x] [+] [1] [)] [÷] [2] [=], then [INPUT] to hold it on the Calculator Line (result: 1.6180:)

As for the "after" number, press: [(] [5] [√x] [−] [1] [)] [÷] [2] [=] (result: :0.6180)

Now just finish off the problem by pressing ■ [%CHG] Answer: -61.8034

Do you notice anything unusual about these numbers? (Hint: You might want to do the problem again after using the ■ [DISP] menu to set the display to show you ALL relevant digits of each number.



10. What you're really asking here is "How many different combinations of 11 "no" votes are possible among 21 voters?"

Well, that's not too tough, is it? On the PROB menu on your HP-22S, there's the Cn,r selection that's tailor-made for this.

So key in [2][1][INPUT][1][1] and press [■][PROB]. Then choose Cn,r ...Poof!

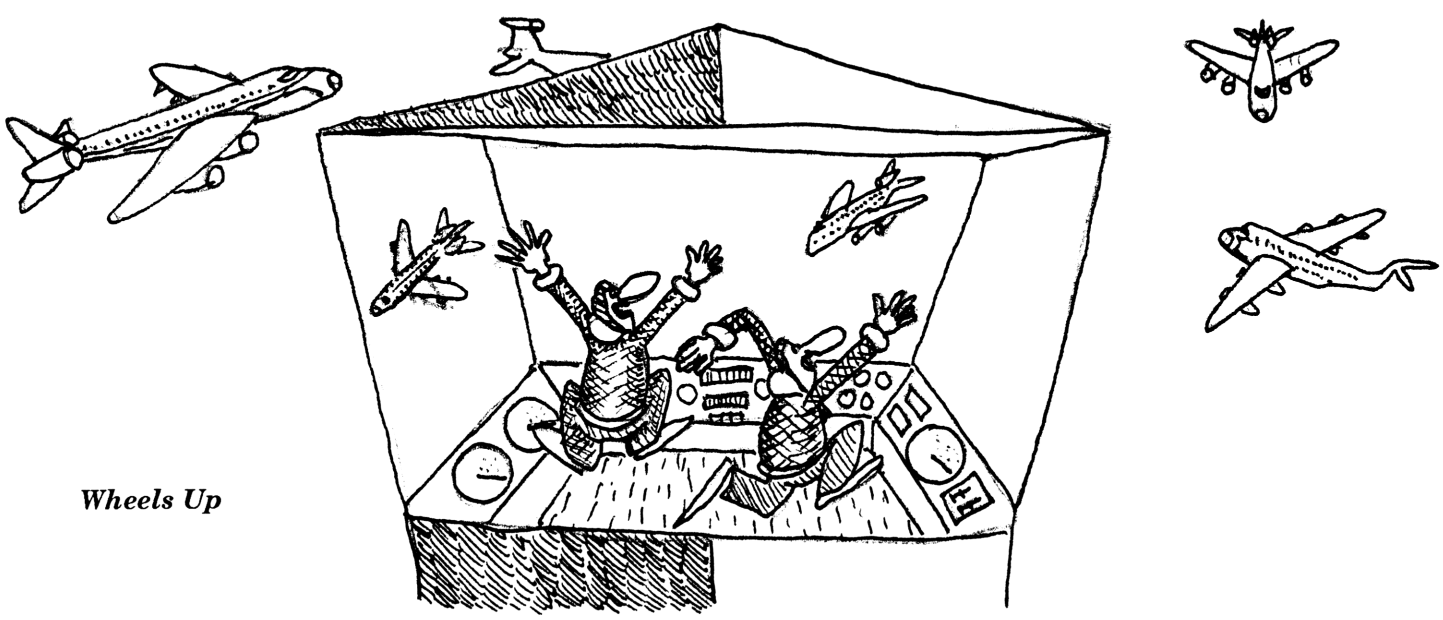
<u>Answer</u>: 352,716 (this assumes that your machine's display is still set to show ALL relevant digits; otherwise, you'll have some trailing zeroes. At this point in the quiz, it might be better to leave your machine at the display ALL setting for the next few problems).

So there are well over a quarter-million ways to stymie any acid-rain policy.

11. This is just a friendly reminder on operator priorities – how to read them and then how to use them on your HP-22S. Here's how to do it:

Press [(][(][1][−][1][•][1][y^x][−][1][0][)][÷][•][1][+][1][0][0][)][×][1][•][1][y^x][1][0][=]

<u>Answer</u>: 275.311670612 (see page 42 to review operator priorities)



12. You'll need to switch to radians mode for the first calculation, so if the little RAD annunciator is not already showing, press ■ MODES and choose RD.

Then: · 2 5 × ■ π + 1 = (result: 1.7853981634)

Since you need to use this number twice, it's very convenient to have it saved in the LAST register. You've just finished a calculation with an = keystroke, so if you now begin an entirely new calculation now (as opposed to continuing to modify this one), the machine will immediately put this result into the LAST register.

So start an entirely new calculation: 0 +. This is an easy and harmless way to start one, isn't it? Now you know that you have your result from above safely copied in the LAST register, so you can continue now and finish off this calculation: ■ LAST COS ■ X^2 + ■ LAST SIN ■ X^2 =

Answer: 1

Yep – that's as the formula says it should be – no need to reinvent trigonometry yet (but you can review it on pages 56-60, if you want).

Is the same true for -22.5° (isn't the suspense awful)?

Press ■ MODES and DG. Then 2 2 · 5 +/– COS ■ X^2 + – 2 2 · 5 SIN ■ X^2 =

Answer: 1 (What a relief.)

As long as you keep your units (degrees and radians) straight, the square of the sine plus the square of the cosine – of any circular angle – gives you 1.

Now what about that *hyperbolic* trig identity? Can you demonstrate that the square of the hyperbolic cosine *minus* the square of the hyperbolic sine should also give you 1?

Press [4][•][4][■][HYP][COS][■][X²][−][4][•][4][■][HYP][SIN][■][X²][=]

Answer: **1** , as predicted.

But what are the *units* of hyperbolic trig functions, degrees? radians? You just performed this calculation in degrees mode, and it seemed to work all right, but how does its behavior compare to radians mode?

To experiment, take that number, 4.4, again, and while still in "degrees" mode, find ACOSH(4.4) by pressing [4][•][4][■][HYP][COS] (answer: **40.7315730024**).

Now switch to radians mode (press [■][MODES] and choose **RD**), then repeat the calculation: [4][•][4][■][HYP][COS]

Same answer!

Conclusion: For hyperbolic trigonometry, you must specify arc lengths in "radians" (they're not radii, actually, but multiples of the shortest distance between the hyperbola and the origin – at the x-axis).

This makes some sort of sense, for if you think about it, it's not nearly as convenient to specify a point on a hyperbola by giving the angle this forms with the (positive) x-axis. After all, a hyperbola doesn't sweep all the way around like a circle does; in fact, it's always within ±45° of the x-axis. So it's far better to specify a point on it directly – by giving the actual distance along its curve, from the x-axis to the point itself.

13. First, be sure to set your HP-22S to radians mode (if you're not already in that mode, press ■ [MODES], then choose RD).

Then: ■ [π] [÷] [2] [=] [SIN]   Result: 1

OK – it's what it's supposed to be, right? The SIN of π/2 radians *is* 1.

So try ■ [π] [SIN]   Result: -2.06761537357E-13

What's this? A wrong answer?! The SIN of π radians is *supposed* to be *zero!*

Try ■ [π] [÷] [2] [=] [COS]   Result: -5.10338076868E-12   *Aaaack!*
Try ■ [π] [COS]   Result: -1

Good Grief! *What* is going on??!!?

It's very simple, actually: In all these calculations, *you're not actually using the number* π. Nobody does – because nobody can. Remember that π has an infinite number of digits. You're using an *approximation* of π that is rounded to 12 digits (that's all the calculator can hold). And it so happens that at the twelfth digit of π, a proper rounding is an *upward* rounding:

3.14159265358979323846... becomes 3.14159265359.

So you're actually working with a number that's slightly more than π. and thus you'll get corresponding results when you use this in any calculation. Sometimes (as with sin(π/2)), this difference is too tiny to affect the first 12 places of the answer, and therefore, the answer is the same as if you were indeed working with π itself. At other times, however, the difference is enough to show up in your result. But no matter what, you may rest assured that your calculator is being *extremely accurate to the numbers it is given to use.* You don't *want* your machine to tell you that cos(1.5707963268) is exactly 0 – because it's *not.*

14. Just to appreciate the technology of it all, do this "the old-fashioned way" first:

Key in the number in question: [2][3][•][4][5][6][7]. Now, you know that the degrees portion of this (the 23) is correct already, so lop it off by taking the Fractional Portion: ■[PARTS] FP (of course, your result is 0.4567).

This is a (decimal) *fraction* of a degree, so if you now multiply this by 60, you'll see how many minutes this is: [×][6][0][=] (<u>result</u>: 27.402)

Aha! So far you've established that 23.4567° is the same as 23°27.402'.

Now how do you convert that fraction of a minute into seconds? The same way you converted a fraction of a degree into minutes: ■[PARTS] FP [×][6][0][=]

<u>Bingo</u>: 24.12   Now you know that 23.4567° = 23°27'24.12"

Compare all that labor with the "new-fangled" "high-tech" approach:

Press [2][3][•][4][5][6][7] ■[H↔HMS], then choose →HMS

<u>Answer</u>: 23.272412   Not bad, eh?

Notice that although this is just a single decimal number (like the 23.4567 you keyed in) it has actually been *formatted* for you to read it mentally as degrees, minutes, seconds *and decimal fractions of seconds*. That is, the conversion extends out only as far as seconds; everything beyond that is just in decimal fractions.

And of course, this format is understood by the machine when it's going the *other* way too: You could now press ■[H↔HMS] and →HR to re-convert this formatted number back to decimal degrees.

15. What a puzzle, eh? All right, first things first: You'd better figure out where the other half of this object landed....

You were flying due westward along the equator (notice the 0° latitude of your estimated position). Not only that, you know your speed, since you were keeping the setting sun constantly on your horizon. That is, your speed was actually matching the speed of the Earth's rotation – about 1050 miles an hour. Now assuming that you cut the meteor pretty cleanly in half (and it looks that way from the half you now have), the other half probably didn't get bounced sideways too far; it must have dropped off toward the ground in a path more or less aligned with your plane's – along the equator (very convenient, no?).

The question is, how far west did that half travel while it dropped – assuming that it began its final fall at roughly the speed and altitude of your plane? Start by figuring out how *long* it took to fall the 130,000 feet:

y_{feet} (which is zero at the end) = y_0 (which is 130,000) - 1100 - 256t

$$\text{or} \qquad t = \frac{130{,}000 - 1100}{256}$$

Aha! (some actual calculating to do)! (1 3 0 0 0 0 − 1 1 0 0) ÷ 2 5 6 =
<u>Answer</u>: t = 503.515625 seconds

Of course, you shouldn't believe more than the first three digits of any number you get here, since that's the precision to which you're sure of your speed and altitude (and the formulae probably aren't even *that* good).

So while you're thinking about it, set your display to SC 2 (press ■ DISP, then SC). See? This actually shows you the three total digits of precision, one before the decimal point – as is conventional in SCientific notation – and then the specified two places after it.

OK, so this poor, abused space-probe fell for about 500 seconds, give or take a few. That's a long time. And how far west did it drift during those 500 seconds (of course, you can't know what the winds were doing to it, but you have to start *somewhere,* so what would it have done in perfectly still air)?

Your horizontal (x-distance) formula will tell you this:

$$x_{feet} = x_0 + v_0 t - (6400/w)t^{4/3}$$

x_0 is the position at the time of collision; call it zero for now, and then later you can add it to your assumed position at that point.

Of course, w is 64 pounds – that much you do know. And v_0 is 1050 miles per hour, but you need to convert this to feet per second for use in your formula: 1 0 5 0 (miles per hour) × 5 2 8 0 (feet per mile) ÷ 3 6 0 0 seconds per hour = 1.54E3 (1,540 feet per second).

All right, now that you have your v_0 there on the Calculator Line – and your time, t, sitting in the LAST register, you can just grind away at that x-distance formula (remember that x_0 is zero):

× ■ LAST − 6 4 0 0 ÷ 6 4 × ■ LAST y^x (4 ÷ 3) =

<u>Answer</u>: 3.75E5

That's about 370,000 feet – assuming absolutely still air, of course. Naturally, this isn't really the case, but it's at least a "ball-park" figure to work from. At this point, you could call the weather bureau in Ecuador to try to establish what winds were prevailing, but that gets very complicated, since the winds vary a lot according to altitude. Better to just go with a thumbnail calculation first – just to see first if there's any hope that this thing might have hit land rather than ocean.

So you'll live with this 370,000-foot drift estimate, but you'd rather express it in miles, so you can start figuring the rough latitude and longitude of impact:

[÷][5][2][8][0][=]

Answer: 7.10E1 About 71 miles to the west.

Of course, since you were on the equator when this whole thing happened, if you're assuming a due-westward drift, the *latitude* of impact is still on the equator (0°), but how many degrees of longitude are in your 71 (statute) miles of westward drift – at the equator)?

Well, there are 360° in about 25,000 equatorial miles, so now multiply ([×]) your 71-odd miles by [3][6][0][÷][2][5][0][0][0] (degrees per mile) [=]

Answer: 1.02E0 (degrees)

Fine. You think the probe hit your plane at about 90°44' W. longitude. And by your rough calculations here, it might have struck the Earth's surface a little over a degree west of there. What longitude would that be – exactly?

You need to convert 90°44' into simple decimal degrees – so you can then add your drift, right?

OK: [9][0][•][4][4][■][H↔HMS] →HR (go to FiX 4 display notation now: 90.7333)

Now add the computed drift: [+][■][LAST][=] (ain't that LAST register handy?)...

...and convert back to degrees, minutes and seconds: [■][H↔HMS] →HMS

Answer: 91.4520

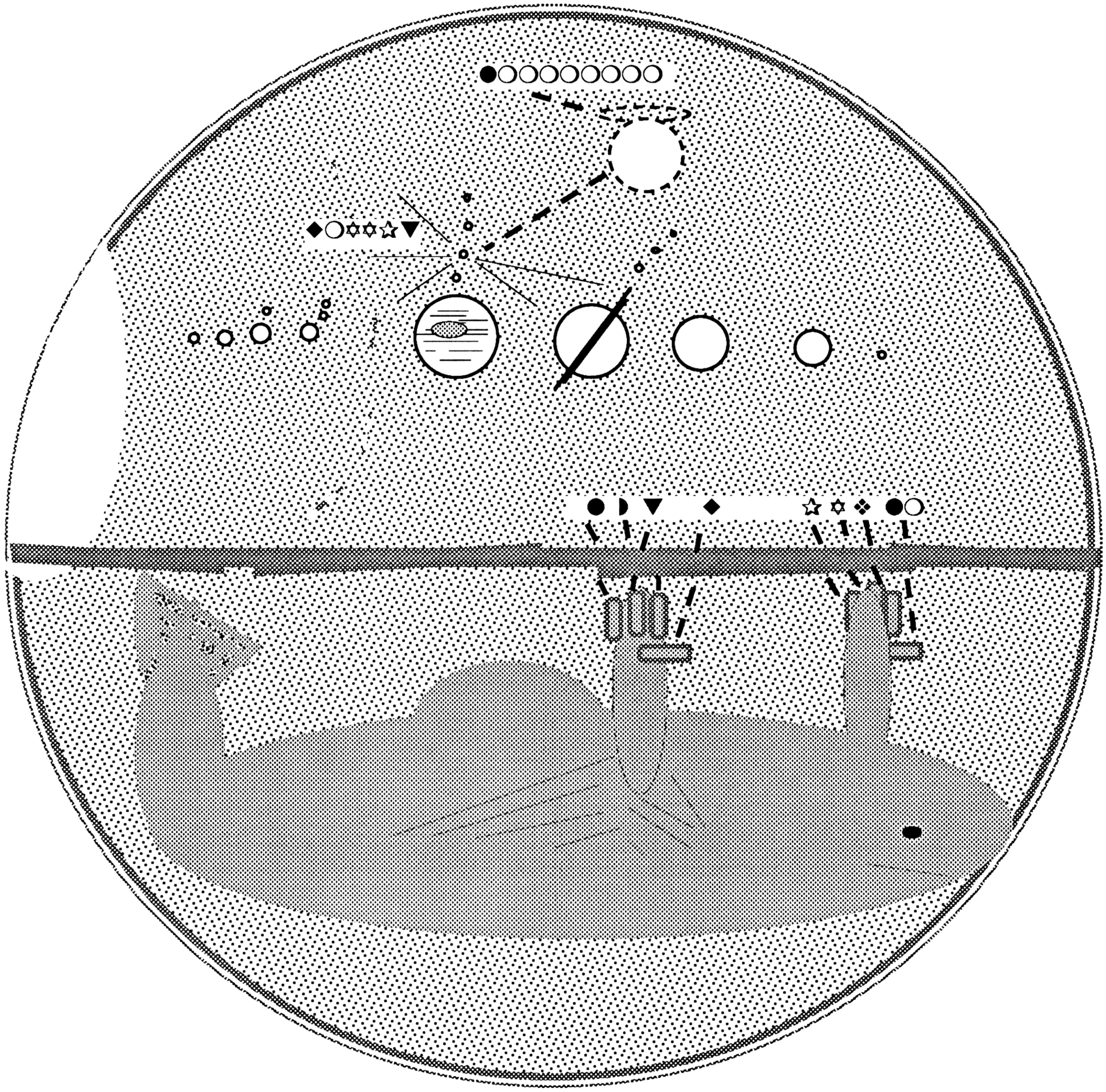
So with any luck, this meteorite must have made impact somewhere around 91°45' W. longitude.

And (would you believe it?) that's not in the ocean; it's on Isla Isabela! So you direct your search teams (you've got a lot of friends who will do just about anything for science – and pizza) to begin combing that island, searching in a circle centered on your computed impact point (and with a radius of several miles to allow for a large cumulative error in all these shaky assumptions).

As luck would have it (and luck seems to be having a lot of it in this little adventure), residents on one part of the island report seeing a twinkling flash overhead just after sundown that evening – and with their help, one of your party at last finds the other half of this space-discus embedded in soft ground among a grove of trees on a hillside.

Hardly daring to hope that it's intact, let alone readable, you hustle over there to see it, bringing with you the other half of the probe, your trusty HP-22S, and a copious amount of pizza.

While your ravenous crew attacks the food, you can hardly eat from the excitement and curiosity provoked by the now-reunited halves of the probe....



Does it begin to make any sense to you?

It *is* apparently some message – maybe a greeting like the Voyager probes.

But Voyager was meant to probe beyond our solar system, and from the looks of this, its origin is much closer than that. Those upper drawings are too familiar: First is the huge sun, then Mercury, Venus, Earth (with its Moon), Mars (with its two moons), the asteroid belt, then Jupiter with its big red spot, Saturn with its rings, Uranus, Neptune and Pluto!

And there's a lot of markings surrounding the second large moon of Jupiter – that's Europa. From the emanating lines, it looks as though this may be the source of the message! Notice the enlarged picture of Europa, rotating on its axis at the upper right.

But what do those symbols mean? And is this whale-like image a picture of the species that made the probe? Could creatures who swim also know how to send space probes? If so, the they must know how to reason and calculate at least as well as we do (even without HP-22S's)!

How do you suppose they do it?

Well, how did we humans first learn to reason and calculate? Probably with our fingers, eh? Now you're getting somewhere: Look at this "whale's" eight "fingers." Notice the inscribed symbols associated with each one....

This is exactly how we associate symbols with our fingers! We use the symbol 1 for the first finger, 2 for the second finger, and so on, until we get to the tenth finger, at which point we run out of symbols and start over, denoting a completion of the set with an extra digit (1). Thus we have no single symbol for "ten." Rather, we use a composite symbol, made up of a 1 and 0, meaning "one complete *set* of symbols, plus 0 extra counts beyond that."

Then on we go with the next set: 11, 12, etc....And we add a third column ("hundreds") when we've completed a set of sets (ten tens), and so on.

This is our base-ten system – *because we use ten unique symbols (0-9).* But from the looks of it, it made more sense for these whale-folks to count in base *eight*, eh? And look – there are seven single symbols for the first seven fingers, then a composite – which uses the first symbol again, plus an eighth symbol!

Hmmm...what if you were to replace each of those symbols next to the "whale's" fingers with more familiar ones – the first eight digits of our own number system? Like this:

●	◗	▼	◆	✭	✡	❖	●❍
1	2	3	4	5	6	7	10_8

(Notice how the 10 is marked with a subscript to denote that this is "one-zero, octal" – not "ten.")

So, what do you think about the coding on the discus? It does indeed look like base eight, doesn't it? And if you use that assumption and make the same substitution with the other two series of symbols, you get these two octal numbers:

●❍❍❍❍❍❍❍❍ (marked near the large schematic of Europa's rotation)
100000000_8

◆❍✡✡✭▼ (marked near the lines emanating from Europa)
$40\ 66\ 53_8$

To make things even easier, why not convert these (apparently) octal values into their decimal equivalents? Press ■ BASE and choose OC. Then key in 1 0 0 0 0 0 0 0 0 and switch back to decimal: ■ BASE and DEC.

Result: 16,777,216.0000

OK, fine. But what does this number mean? It's associated in the drawing with Europa's axial rotation. What number would that be?

Well, what special numbers or significance do we attach to the rotation of the Earth? After all, it happens every day....

Every *day!* It's how we measure *time!* Maybe these whale-folks do it the same way! Then that number might signify their subdividing units of time. That is, what if they have 100000000_8 (that's 16,777,216) units of time for each of their days? After all, 100000000_8 is exactly 8^8 – very convenient if you have 4 fingers on each hand, eh?

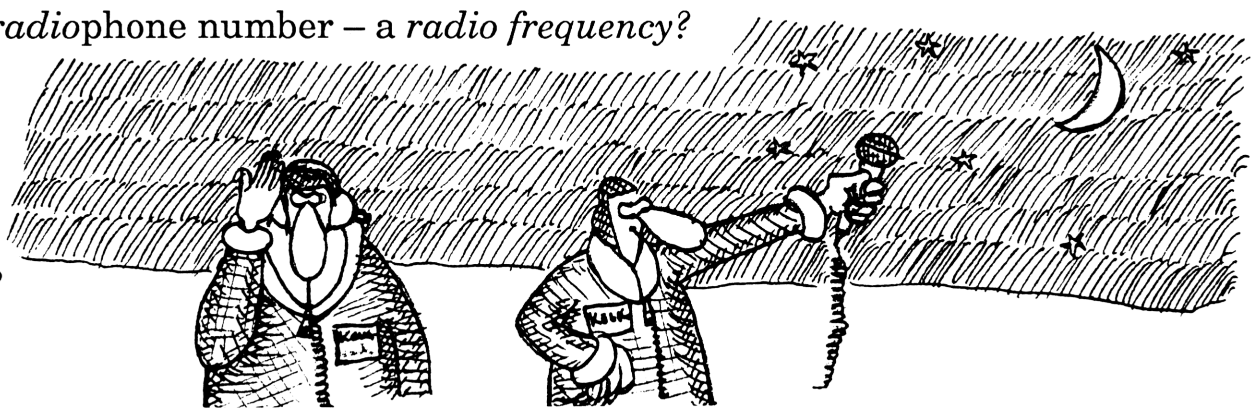
What you have might have here is a way to convert from their time units to Earth units! Looking up in another one of your handy references (you sure do have some convenient books), you find that the length of Europa's day is about 3.5 Earth days (307,076 of our seconds). Could it be that there are 16,777,216 "Europa-seconds" per 307,076 "Earth-seconds?"

OK, *suppose* that's what this means. What good does that do? It still doesn't explain what the other number means – and why would these whale-people want us to know their time system in the first place? That's not exactly a great descriptive introduction of a civilization – especially since to translate this we need to be well enough aware of Europa already to know how long its day is.

What is that other number? Clearly, it's shown as coming from Europa itself – with all those emanating lines. Is this supposed to be a message of Univeral Peace or something? If so, what message could be so universal and profound as to lie in a single number? Can you think of any just off the cuff?...

Maybe it's not so profound. Maybe it's just the key – like the number on your charge card, or your phone number or....– a phone number!?!?! ("for a good time, call Europa...406653_8 ?)

Hmm, that seems like an intelligent message to try to send – where to "call" for more messages – but of course, it can't really be a *telephone* number. How about a *radio*phone number – a *radio frequency?*



AHA! That would explain why the time units were needed – to convert radio frequencies between their system and ours!   Eureka!*

With heightening anticipation, you pound away furiously on your HP-22S, the clicking of its keys the only sound on this clear, hushed night in the Galapagos (except for your crew of searchers, who have now polished off the pizza and are lying around, groaning comfortably and looking at the stars):

Your theory is that this alleged radio frequency is expressed in "Cycles-per Europa-second," – and expressed in octal, too – and you need to see it in "Cycles-per Earth-second" (Hz) – expressed in decimal, of course.

First, you convert 406653_8 to decimal: ■ [BASE] OC [4][0][6][6][5][3] ■ [BASE] DEC

<u>Result</u>: 134,571.0000 This is still "Cycles-per-Europa-second," but at least it's in decimal.

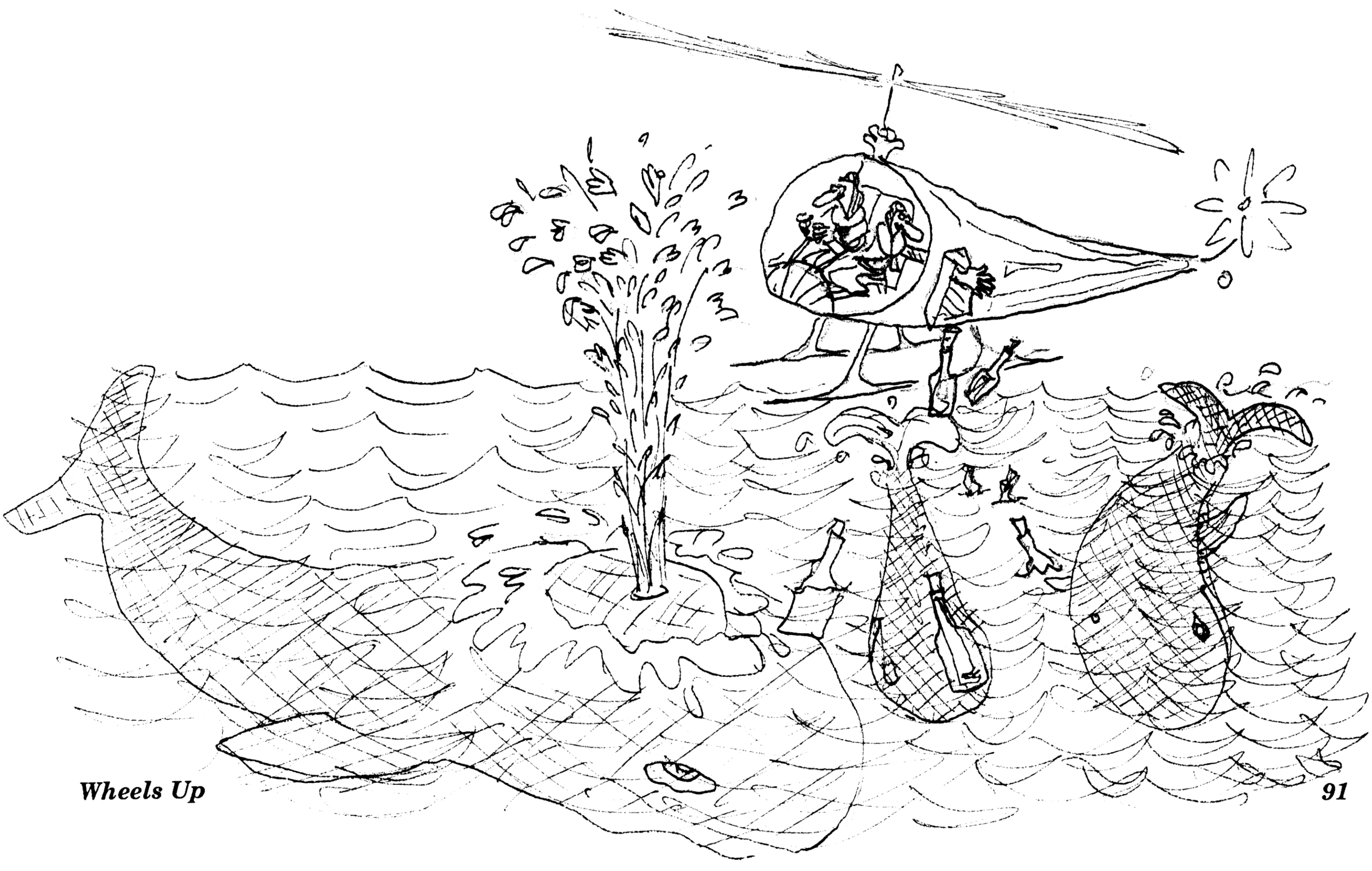
Now there's only one calculating step left – to convert this frequency to Earth-seconds: [×][1][6][7][7][7][2][1][6][÷][3][0][7][0][7][6][=]

<u>Answer</u>: 7,352,338.6209

Egads! That's just a little over 7.35 Megahertz – it *is* a radio wavelength! ("Hey, somebody, aim a ham radio at Europa and start listening for whales!...")

*(no – Europa.)

Notes And Messages



A Menu Summary: Whaddya Got?

Well, nobody can say that you haven't "earned your wings" so far, eh? That quiz took you through just about every calculation trick and every menu you've seen up to now, didn't it?

OK, ok – back down to Earth for awhile....

By now, you're no doubt *quite* practiced with many of the keys on your HP-22S – certainly with all the arithmetic and higher math keys, and the eight function menus you've learned so far.

Before you move on to the really "juicy stuff" about your calculator, there's something you ought to notice now – if you haven't already: All the menu keys that you've explored so far are blue, "shifted" keys, and their names are printed on a slightly different texture and color than the other shifted keys, to denote that they are indeed *menus:*

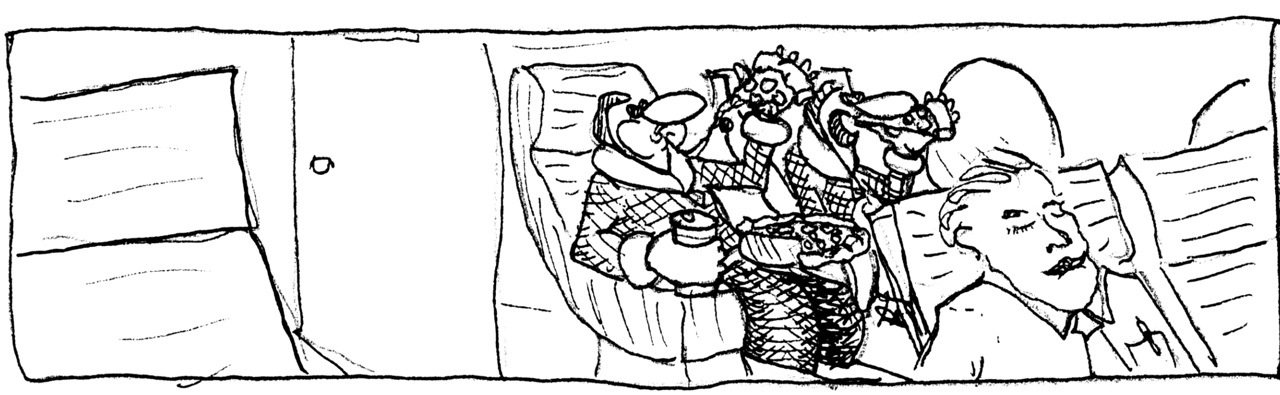
PROB

DISP

MODES PARTS

UNITS H↔HMS D↔RAD BASE

This is to remind you that these are *not* function keys that *do* something immediately (like SIN, for example). Rather, they are menus that *lead* to such functions.



What's Next?

The remainder of this course is all about the use of a few certain keys that are the most powerful of all:

- The [STAT] key is a menu key, but it's powerful enough to be granted status as a key unto itself – no shifting needed. And notice that the two keys you use along with it, [Σ+] and ■[Σ–], are conveniently located right above it. The next chapter is all about the STAT menu.

- The [STO] and [RCL] keys are your tickets to the use of the 26 storage registers that are named with the letters A-Z. You'll learn about them and the ■[CLEAR] menu and ■[MEM] key in an in-depth look at the memory in your HP-22S (you'll notice that the ■[CLEAR] menu key is the only function menu key that you *haven't* encountered yet – because it makes much more sense to see it as you learn about your machine's memory).

- Then comes the world of "What-If?", as you begin to explore how your HP-22S lets you evaluate an equation and solve for any variable in it – using the [EVAL] and ■[SOLVE] keys. You'll start with equations that have already been programmed (permanently) into calculator – equations you'll find on the keyboard and in the built-in equation LIBRARY (notice the ■[LIBRARY] key).

- Then you'll *really* open things up when you learn to invent your own custom equations, as you create and store a list of them with the help of the ■[EQUATIONS] key. You'll see that you can use any of the keyboard or menu functions to help you build these equations!

So that's what's still to come: statistics, machine memory, built-in equations and custom equations. If you're ready, then, just open that throttle up and...

go!

ANALYZING YOUR FLIGHT DATA:
The STAT Menu

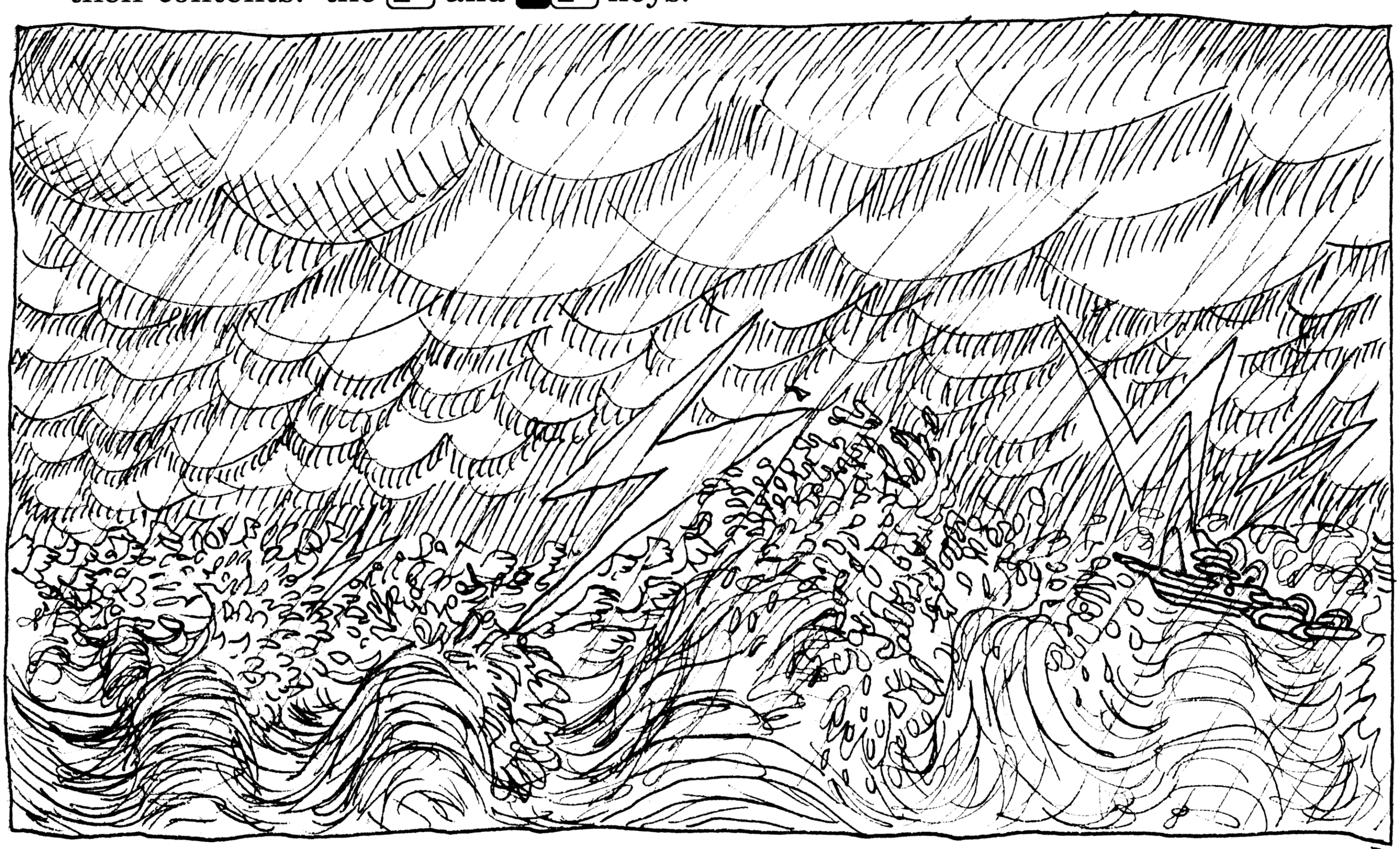
The Statistical Registers In Your HP-22S

Remember the LAST register – that specialized storage register that automatically saved your last completed calculation? Well, it's now time to learn how to use some more specialized registers in your HP-22S – the STATistical Registers.

These registers are specialized, because – like the LAST register – *their contents change after you do certain operations.*

In the case of the LAST register, those "certain operations" were any keystrokes that began a new calculation after another one had been completed.

In the case of the STATistical registers, there are two specific keys you use to change their contents: the [Σ+] and ■[Σ−] keys.*



*And if you're already sure you know how to use these keys – and the [STAT] key menu – to calculate means, standard deviations, and linear regressions, then you can skip right now over to page 120.

First of all, you'd better get a picture and a general "feel" for these six STATistical registers. Here they are, by name:

n	Σx	Σy	Σx²	Σy²	Σxy

And what goes into each of these registers when you press [Σ+] or ■[Σ−]? Surely those names indicate *something* about the use of the registers.

What *are* statistics, anyway?

To put it as simply as possible, statistics are calculation methods in which you use *collections* of numeric observations to detect patterns or correspondences among real-world events or phenomena.

That may sound like a mouthful, but it's not, really.

For example, everybody's familiar with the idea of an *average*. If you want to predict the amount of rain that's going to fall this year, one good way is to add up all the rainfall amounts for the past few years and then divide by that number of years.

In other words, you'd be doing this calculation: $\dfrac{\Sigma x}{n}$,

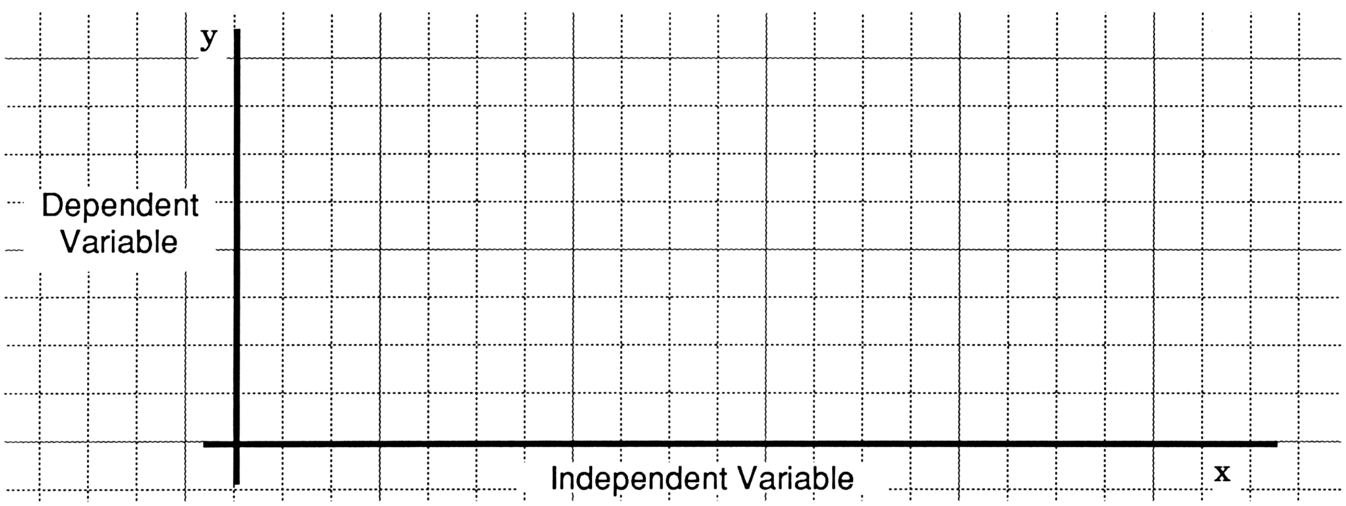
where each individual x (remember that Σ means to *sum* all the individual x's together) is the rainfall amount for one of those past years – and n is the number of years of such data that you're using.

All right, that might explain the contents of the first two registers, n and Σx: those are where you store the *number* of data (n) and the *sum* of the data (Σx).

But what about the other four registers? Why would you need Σx^2 ? And what's all this about y data?

Well, suppose someone suggests that the amount of rainfall you have every year *depends* on the average daily temperature for that year. How could you test this? Clearly, you now have *two* sets of observations (rainfall and temperature), the one supposedly depending on the other.

It's just like the graph of a mathematical function, where each y-value (the *height* of any point on the curve) is a function of (i.e. it *depends upon*) the corresponding x-value (the horizontal position of that point on the curve):

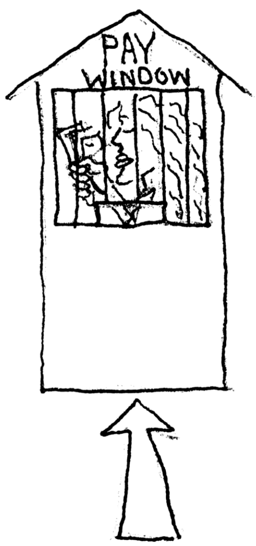


This is what *two*-variable statistics is all about – finding various relationships between an independent variable (the x-data) and an allegedly dependent variable (the y-data).

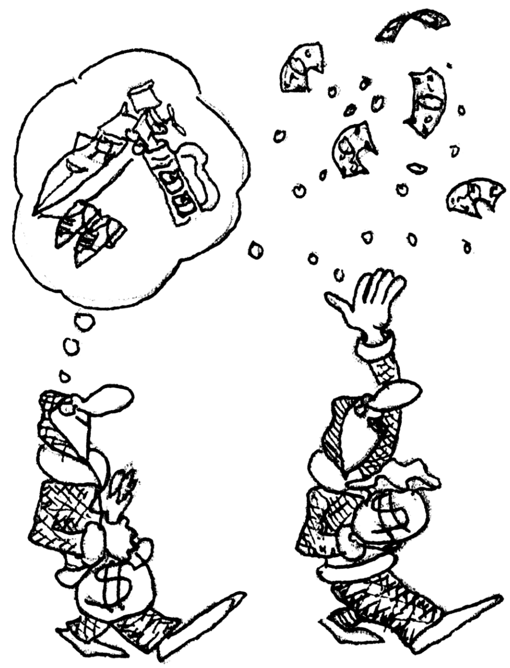
And that's why you have those other statistical registers on your HP-22S. It can do either one *or* two-variable statistics – and the necessary formulas require not just Σx and Σy but also Σx^2, Σy^2, and Σxy. The machine needs to keep running tallies of *all* these sums in order to be able to do such curve-fitting and other one- and two-variable statistics. OK? Then it's time to try...

Entering Some Statistical Data

Example: Suppose you've been a flight instructor for the past 11 years, with the following tuition income history:



Year	Income
1977	$17,000
1978	22,000
1979	21,500
1980	24,000
1981	14,500
1982	19,000
1983	23,000
1984	24,000
1985	24,500
1986	18,000
1987	27,000



You want to use the statistical capabilities of your HP-22S to tell you some things about this income history. How do you do this?

Solution: First, press ■CLEAR, then choose the Σ option.

You haven't seen this ■CLEAR menu key before, and it really doesn't make sense to explore it any further until later, when you know about all the different parts of the memory of your calculator. For right now, just realize that this menu lets you *selectively* clear various portions of the calculator's memory – instead of using that ominous reset procedure back on page 16. Here, for example, you're selectively clearing (setting to zero) just the six STATistical registers.

Now you're ready to begin.

The idea is to key in all of these values. Keep in mind that your HP-22S will *not* remember the individual values themselves. What it *will* remember (in the STAT registers, of course) are those various *sums* from which it can calculate all sorts of useful statistics for you.

So here you go: Press [1][7][0][0][0][Σ+]

(You might want to change the display format to FiX two decimal places here, since you're working with dollars and cents in this example. Press ■[DISP] FX [2].)

Notice that the machine helps you keep your place in your list by echoing back to you the current value in its n-register – which is, after all, the number of data you have entered thus far, right?

So keep going: [2][2][0][0][0][Σ+]
[2][1][5][0][0][Σ+]
[2][4][0][0][0][Σ+]
[1][4][5][0][0][Σ+]
[1][9][0][0][0][Σ+]
[2][3][0][0][0][Σ+]
[2][4][0][0][0][Σ+]
[2][4][5][0][0][Σ+]
[1][8][0][0][0][Σ+]
[2][7][0][0][0][Σ+]

And as you do this, keep in mind what's actually going on inside your calculator....

Every time you press [Σ+], the machine looks for *two* values on the Calculator Line. If it finds only one value, it takes this as your x - data and assumes that the y -value is zero.

If you *have* put two values on the Calculator Line – by using the [INPUT] key (remember how to do that? see page 47 for a reminder), then the values will be taken as x - and y - data respectively (x first).

Then, no matter whether some values are zero or not, your HP-22S adds each x -value to the Σx register, and the *square* of that value to the Σx² register. It does the same for the y -data (adding to the Σy and Σy² registers), and then adds the *product* of x and y (that's "x times y ") to the Σxy register.

Finally, it increments the n -register by one, to count the total number of data that you have now *accumulated** in this way – and it echoes the new n -value in the display to help you keep your place.

All this – in just one little [Σ+] keystroke!

Thus, if you've done everything right, at this point you should get the message in the display that n=11.00, since you now have 11 years' worth of income accumulated in the STAT registers.

---

*It's better to use this term, "accumulate," rather than "store," because it's not quite accurate to say that you've "stored" your data in the STAT registers. After all, once you enter them there with the [Σ+] key, *you cannot retrieve them again as individual values;* it's only their *cumulative sums* that are stored.

Checking And Editing Your Data

...Hmm... but what if you're not sure that you've done everything right (or, what if you're *sure* that you've *not* done everything right)?

The first clue that you may have messed up would be that the display didn't end up telling you that $n=11.00$ (if not, then just start over on page 98). But assuming you got *that* much right, how *else* might you check the accuracy of your entered data?

Try This: Press the [STAT] menu key. Here's what you'll see:

Σ	$\bar{x},\bar{y}$	s	L.R.

Now choose the Σ selection and presto! There's another menu that let's you check the current totals in any of the six STAT registers. Choose the n selection and reconfirm: $n=11.00$ But you knew that.

Ah, but what about Σx? Press [STAT], then Σ, then x. If you've correctly keyed in your data, you'll see your 11-year total income as:

$$\Sigma x = 234{,}500.00$$

If you see this, then you can assume that everything's all right. If not, then go back and start again at the top of page 98.

This is one of the drawbacks to *cumulative* statistics (as opposed to storing the data values themselves): If you've mis-entered something, there's not much to do except to start over – unless you know exactly *how* you went wrong.

All right, suppose that you have a correct set of data entered in the statistical registers of your HP-22S. Now what if you want to *edit* it?

"Uh......hmmm......can that even be done? How can you change individual values that aren't even stored as such?"

It's not obvious, but you *can* – by "backing out" ("un-entering") the value(s) you want to delete, then entering any new values you want to add.

Like This: Suppose you want to keep a running record of only the most recent *ten* years' worth of flight instruction income. And suppose you now know your 1988 income to be $34,000.00. How would you edit your statistical accumulation so that it contains only data from 1979 -1988?

Nuthin' To It: First, you use the ■[Σ−] key to unenter your incomes for the years 1977 and 1978 (literally, you're just subtracting from each of the statistical registers the same amounts that the [Σ+] key originally added. This effectively erases those data values from your cumulative totals):

[1][7][0][0][0] ■[Σ−]
[2][2][0][0][0] ■[Σ−]

Notice how the echoed n-value confirms that you are indeed subtracting data values here (so n=9.00). Now add your 1988 income: [3][4][0][0][0][Σ+]

(If you did this correctly, then n=10.00, and Σx=229,500.00 Remember how to use the [STAT] key menu and the Σ selection to check this? See page 101.)

Just Plain Mean

All right – now that you have the corrected set of income figures all accumulated properly into the STATistical registers of your HP-22S, what do you want to know about your income?

Might as well start with something simple....

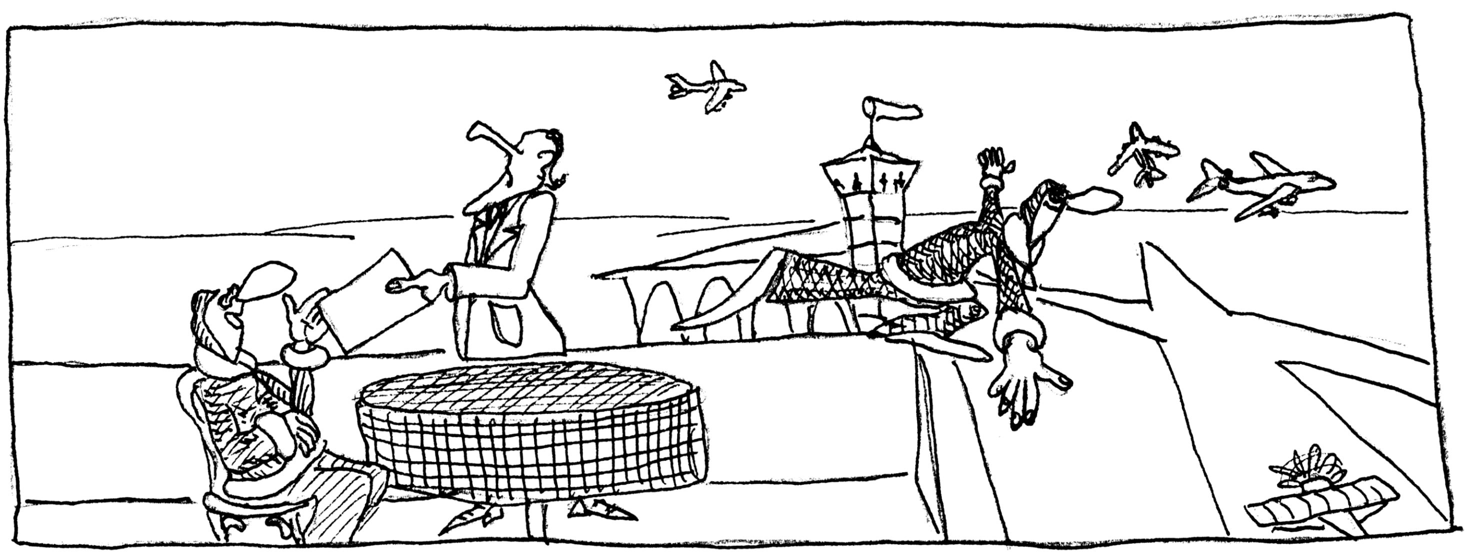
F'rinstance: What was your average (also known as your mean) annual income over these ten years?

Solution: Press [STAT], then choose $\bar{x},\bar{y}$, to arrive at the menu where you calculate $\bar{x}$ (this is the symbol for the average of x).

Now simply press $\bar{x}$ and you're done: $\bar{x}$=22,950.00

(You can see this is right, since you knew that the 10-year total was $229,500.00 – and it's not too tough to mentally divide by ten, is it?)

Notice that on that same menu, you could also choose to calculate $\bar{y}$, also.



A Moving Average

Time Flies: Now suppose it's the end of 1989 – and you're curious how this year's earnings ($29,000.00) affected your ten-year *moving* average. That is, since you're always concerned with the most recent ten years, you need to do your editing trick on the STAT registers once again – to subtract your 1979 income and add your 1989 income – and *then* find the average of *those* ten values.

So what are the keystrokes? By now these ought to be

Old Hat: Press [2][1][5][0][0] ■ [Σ−], then [2][9][0][0][0][Σ+].

You've just updated the data (i.e. you've moved the average forward) – and you can tell from the n -value in the display that you still have ten values, so that checks.

Now find that average: [STAT] x̄,ȳ x̄ Result: x̄=23,700.000

And does this answer make sense?

Sure it does: You deleted a value of 21,500.00 and replaced it with a value of 29,000.00. You'd therefore *expect* the average to go up – and so it did, from 22,950.00 to 23,700.00.

A Weighted Average

You've seen a plain old, garden-variety, vanilla-flavored average – and one that "moves" in time (a moving average).

But what's a weighted average?

Try This: Suppose that your *monthly* flight-instruction income for 1989 went something like this (and this ought to add up to $29,000, which is what you used for your 1989 yearly income value):

January	0.00
February	1,000.00
March	1,000.00
April	2,000.00
May	2,000.00
June	4,000.00
July	7,000.00
August	7,000.00
September	3,000.00
October	2,000.00
November	0.00
December	0.00

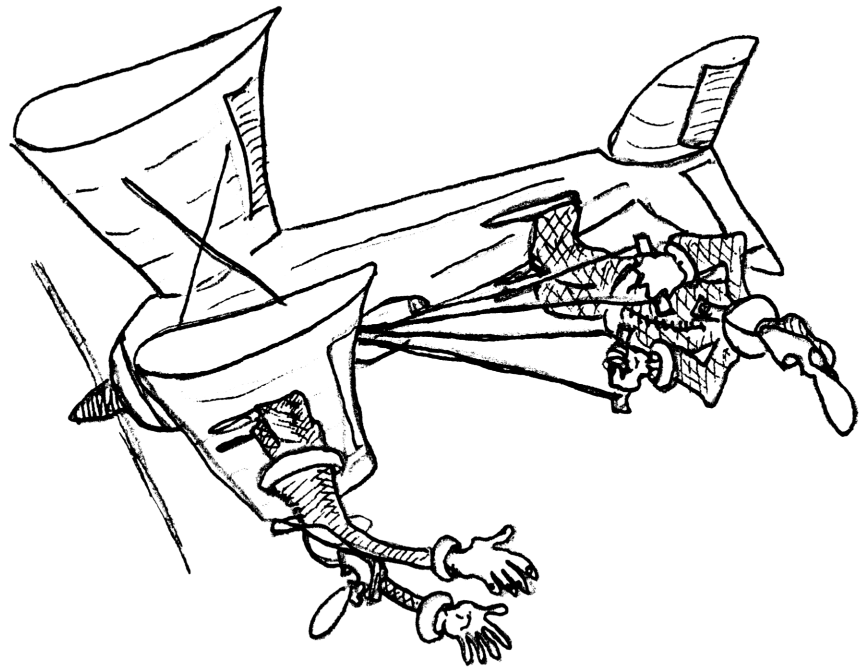
<u>Burning Question:</u> What was your monthly average income?

Strategy: You could just key all this into the STATistics registers and find $\bar{x}$, exactly as you've done with your ten years of annual income.

Or you could observe that since there are *repeated* values here, it might be easier to use a *weighted* average calculation ($\bar{x}w$).

Compare: First, do it the "old-fashioned" way: Press ■ CLEAR Σ, then

0 Σ+
1 0 0 0 Σ+
1 0 0 0 Σ+
2 0 0 0 Σ+
2 0 0 0 Σ+
4 0 0 0 Σ+
7 0 0 0 Σ+
7 0 0 0 Σ+
3 0 0 0 Σ+
2 0 0 0 Σ+
0 Σ+
0 Σ+



Notice that you must *not* skip the zero entries; even if they don't contribute anything to most of the STAT registers, they still must be counted in the n-value (the total number of data values) in order to make the average accurate, right?

Now find the average: STAT $\bar{x},\bar{y}$ $\bar{x}$ Result: $\bar{x}$=2,416.67

Yes, But: There's the quicker way – with a weighted average. The idea is to key in each different value just once – but also note how many times that value occurs in your collection. Thus, here's a summary of your monthly income, according to the frequency of the given monthly level:

Monthly Income	# Of Times Occurring
0.00	3
1,000.00	2
2,000.00	3
3,000.00	1
4,000.00	1
7,000.00	2

To give this picture to your HP-22S, you key in two numbers for each item. The first (the x-value) is the income level; the second (the y-value) is the frequency – the number of times that x-value occurs. Thus, your keystrokes would go like this:

■ [CLEAR] Σ
[0] [INPUT] [3] [Σ+] (remember how to use that [INPUT] key to separate two numbers on the Calculator Line? If not, see page 47.)
[1] [0] [0] [0] [INPUT] [2] [Σ+]
[2] [0] [0] [0] [INPUT] [3] [Σ+]
[3] [0] [0] [0] [INPUT] [1] [Σ+]
[4] [0] [0] [0] [INPUT] [1] [Σ+]
[7] [0] [0] [0] [INPUT] [2] [Σ+]
[STAT] $\bar{x},\bar{y}$ $\bar{x}w$ Result: $\bar{x}w$=2,416.67 Look familiar?

So that's what a weighted average is: For every x-value you key in, you must also specify (with a corresponding y-value) how "heavily" that x-value should be "weighted," that is, how many times in this data set it can be considered to have "occurred."

Deviating From The Average

Of course, an average can only tell you a certain amount about your data. To say that you "earned an average of $2,416.67 per month in 1989" makes it sound as if you just strolled down to the mailbox once a month and pulled out a paycheck for that gross amount.

Sure, it *might* have happened that way. Or you *might* have earned the entire year's pay ($29,000.00) in one weekend – and earned nothing the rest of the year. You *can't tell* merely by looking at the average, because no matter how uneven and irregular your income actually was, as long as it adds up to $29,000.00 for the year, your monthly average will always calculate out to be the same number.

An average just doesn't tell you anything about the *distribution* of your values.

Well, after a quick glance at your actual monthly income history for the year, you *know* what your income distribution was; it was neither smooth-and-regular nor "all-in-one-weekend." In the winter months, you didn't have any work or income at all, while in the summer, you were swamped with student pilots wanting your instruction.

So exactly *how* even or uneven was it? How can you measure this from a set of values?

That's one of the uses of the *standard deviation* – to measure how widely dispersed or varied from the average your actual data values are.

To get the idea,

Try This: Key in a set of *identical* values to the STATistical registers, say four 4's: ■ CLEAR Σ 4 Σ+ 4 Σ+ 4 Σ+ 4 Σ+

Now, what's the average ($\bar{x}$) of this set of values? It's 4.00, of course (press STAT $\bar{x}$,$\bar{y}$ $\bar{x}$ if you want to see for yourself).

And what's the standard deviation – the measure of how far a typical value tends to vary from this average?

To find out press STAT s sx. <u>Result:</u> sx=0.00

Since every value was an average value, they didn't vary from that average at all. Thus, the standard deviation is zero.

Now Try This: Key in these irregular values to the STAT registers: 11, 0, 1, 4 (press ■ CLEAR Σ 1 1 Σ+ 0 Σ+ 1 Σ+ 4 Σ+).

Find the average: STAT $\bar{x}$,$\bar{y}$ $\bar{x}$ <u>Result:</u> $\bar{x}$=4.00 – same as before.

But now find the *standard deviation* of this set of data:
STAT s sx <u>Result:</u> sx=4.97

That's a large deviation, compared to the average itself – telling you that the actual values vary quite widely from that average.

So the more uneven and irregular your data values are, the larger the standard deviation.

This is how you could measure how "even" or uneven your flight instruction income was over the year 1989: Take the standard deviation of your monthly income levels for that year....

Like So: (Unfortunately, because you have since entered other statistical data, you'll now need to re-key in that list of 12 values again; see page 105 if you want to review the list.)

■ CLEAR Σ
0 Σ+ 1 0 0 0 Σ+ 1 0 0 0 Σ+
2 0 0 0 Σ+ 2 0 0 0 Σ+ 4 0 0 0 Σ+
7 0 0 0 Σ+ 7 0 0 0 Σ+ 3 0 0 0 Σ+
2 0 0 0 Σ+ 0 Σ+ 0 Σ+

Just to refresh your memory, find the average income again:
STAT $\bar{x},\bar{y}$ $\bar{x}$ (Result: $\bar{x}$=2,416.67)

Now for the "unevenness test:" STAT s sx Result: sx=2,466.44

That's quite a large deviation in comparison to the average itself – so it was quite an up-and-down year for you, income-wise. And if you were to compare this to the same calculation for other income years, you could tell immediately in which years you had smoother, more uniform earnings – the years with smaller standard deviations.

Now, to be more exact, there are at least *two kinds* of standard deviations – and they mean different things, because they use different formulas....

The formula programmed into your HP-22S assumes that the data you key in is really a *sample* of a much larger set of data – as if you were catching and weighing a few fish to get an idea of the typical weight of any such fish in the ocean. This is useful, obviously, because you can't actually catch and weigh every fish in the ocean.

But for other situations, you *can* actually analyze all the possible data. This is what you were doing when you were analyzing your monthly flight instruction income for 1989. You didn't take just a random sampling of a few months; you analyzed the entire year – all possible data.

The standard deviation (s) formula for this *whole-population* kind of analysis is just a little bit different than for a sampling (and the whole-population is the kind *not* programmed into your HP-22S' statistical calculations). But you *can* get your HP-22S to calculate the whole-population s by using its built-in (sample population) s formula – with a little adjustment to your data.

Like This: To get the true standard deviation for your 1989 monthly income, you must add one extra entry to your STAT accumulation. That entry is the *average* for the actual data. Thus, you would first calculate this (press [STAT] x̄,ȳ x̄), then enter it as a "13th value:" [Σ+]

Now you use the built-in formula: [STAT] s sx Result:* sx=2,361.44

So any time you want to find the s value for a set of data representing *all* the possible values (the "entire population of fish," if you will) – rather than just a sampling of them – you should first *add the average* to the data itself, and *then* compute s.

*As you can see, this is somewhat less than what you got on the previous page (and for your purposes, it probably wouldn't matter which formula you use, since all you're interested in is how this would *change* from year to year – to indicate the relative smoothness of each year's income).

Getting Trendy: Regression And Estimation

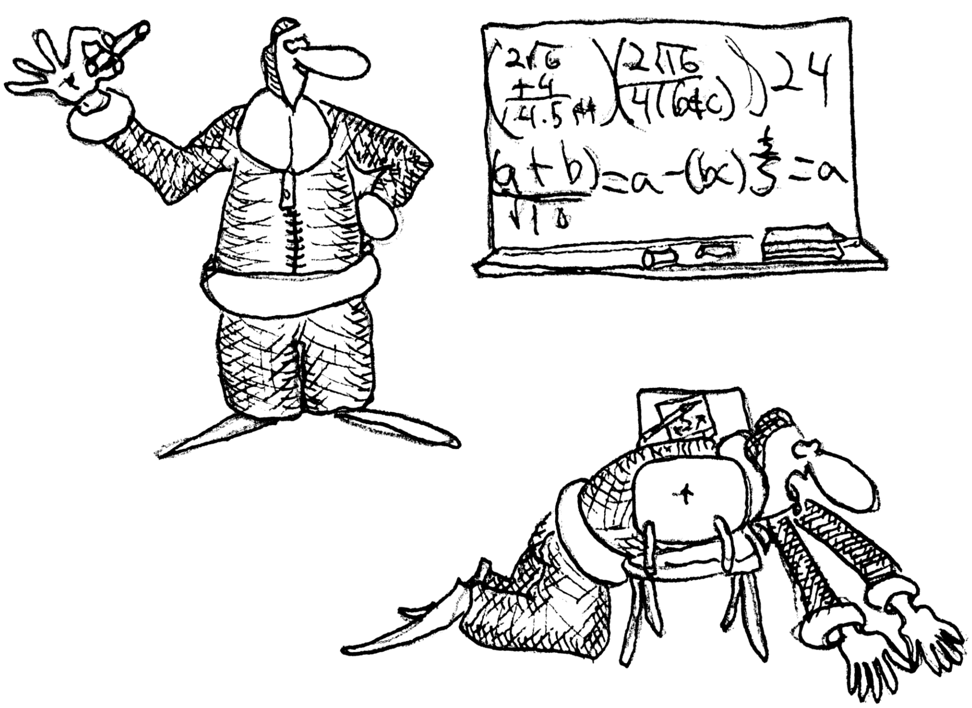
So now you know a little bit more about your 1989 flight-instruction income: You know the average monthly level, and you know from the standard deviation that it varied from that average quite a bit.

But do you have any idea *why* it varied like that?

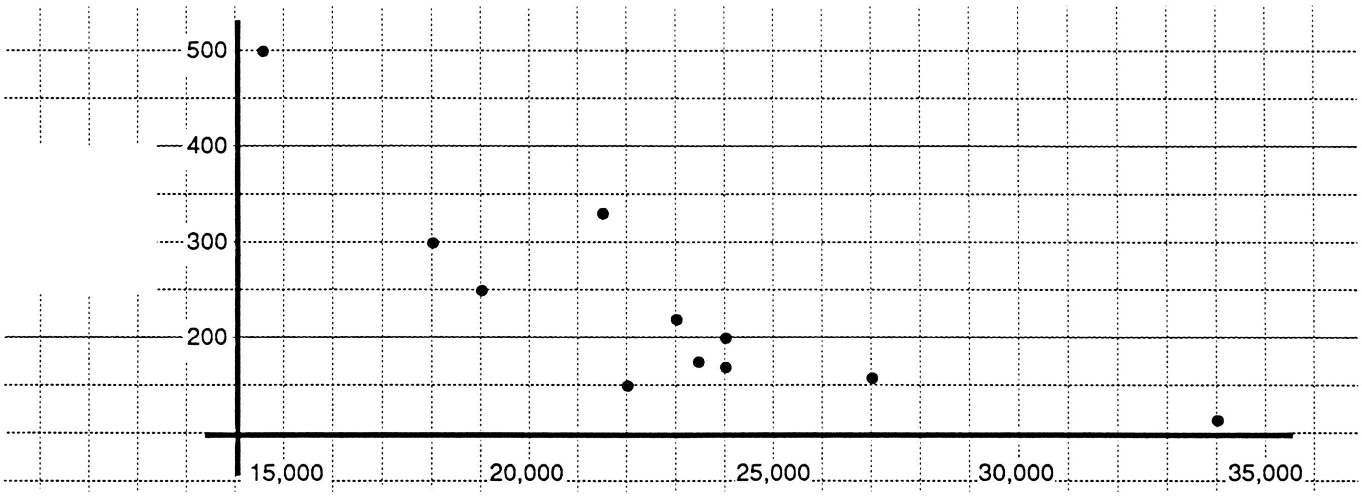
Sure – because more you worked far more hours in the summer months than in the winter. After all, you charge by the hour, right?

OK, but why do you work so many more hours in the summer? Does everybody's interest in flying suddenly increase in June or July? Is that when everyone can afford the time or the money? Would it have anything to do with the weather or the availability of airplanes – or the availability of you as an instructor?

Well, you might have all sorts of ideas as to just why your income varies in this pattern, but no matter what your favorite theory is, how would you demonstrate whether or not it's a good explanation?



Here's where you really start to use statistics to help you answer questions and test possibilities – and one very useful tool is already built into your HP-22S: linear regression and estimation. Your calculator can actually use the x- and y- values in your data set to plot points on a graph like this:



Well that's all good and well, you might say, but *what* y-values? There are only x-values in this list of monthly incomes.

Ah, but didn't you say you might have some ideas as to why you give so much flight instruction in the summer months – and so little in the winter? There's some other factor – some *variable* condition upon which your income seems to *depend.*

What's your best guess? TV reruns? Probably not. You'll notice that over the last ten *years,* your income varied quite a bit – and yet there were those boring reruns every summer. What kind of condition might vary *overall,* from year to year, and also *within* a given year (such as 1989), to produce the kind of seasonal pattern you observe here in your flight instruction income.

Mmm...seasonal, eh? How about the *weather* – say, the amount of rainfall? Most students cannot learn in foul weather; even the most experienced pilots of small planes avoid it, if possible, with all its poor visibility, winds, rain/ice, etc. ...Mmm...could be, could be. So how might you test this theory?

First, you'd better state your theory in so many words, just so the folks at home know what it is that you're trying to prove:

"The amount of annual income I earned from flight instruction over the past ten years (1980-1989) depended upon the amount of rainfall in the vicinity of the airfield. The rainier it was during any particular year, the less income I earned for that year."

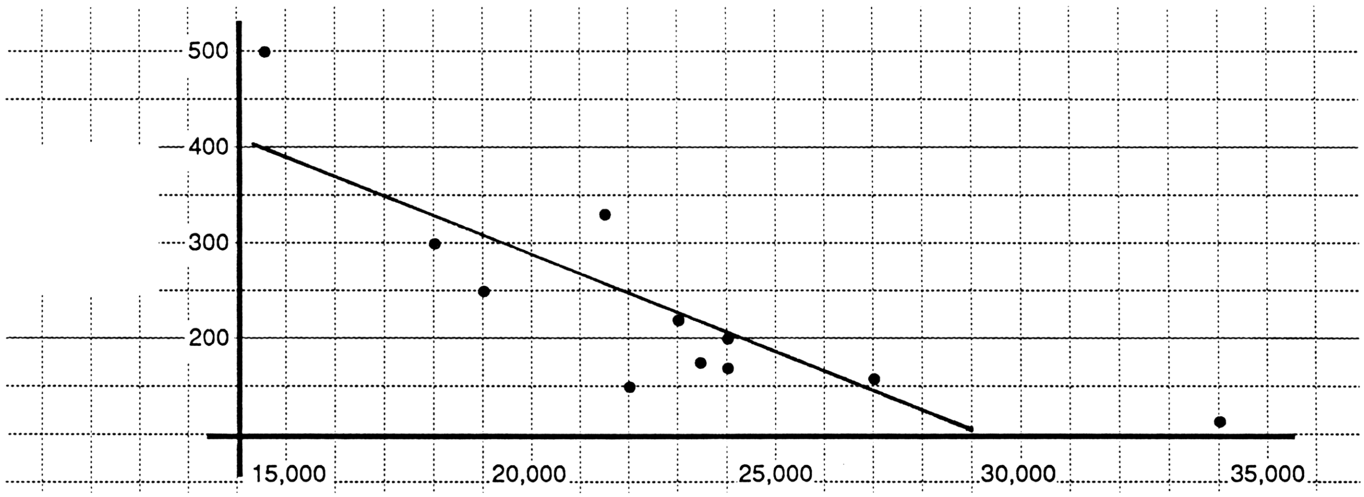
OK, now when you say "depended upon," you are saying that the rain and nothing else was by-and-large responsible for your varying flight instruction hours. You're saying that the times when you were sick, or on vacation – or when your pet rhino ate the keys to the Cessna – didn't really cause these variations. Sure, these unforeseeable little episodes made you lose some income, but you're claiming here that they were insignificant in comparison to the governing condition – the *independent variable* – rainy weather.

Fine. And now what do you mean by the phrase, "the rainier it was, the less I earned"? Do you mean that if the rainfall doubled, your income was cut by half? Two-thirds? If the rainfall is an inch more in any given year, do you make $1 000 less for that year? What?

Well, that's a tough one. You've only just now suggested that there is *any* kind of numeric connection – a correlation – between rain and income, and now you're already trying to decide what the correlating numbers or relationships might be.

Why not let you HP-22S decide that for you? Why not try, say, a very *simple* correlation and see how well it fits your data?

As you might suspect, the HP-22S can indeed help you test such a correlation. On the [STAT] key menu, you've probably noticed a selection called L.R. – Linear Regression – a fancy name for fitting a *straight line* along and among your data points:



Remember how to mathematically describe straight lines like these? They all obey equations of this form:

$$y = mx + b$$

You pick an x-value, multiply it by a certain number, m, add another certain number, b, and zap! that's your y-value.

You might also recall that the number, m, is called the *slope* of the line, because it determines its "steepness" as it appears on the graph. For every 1 unit you change the x-coordinate, you change the y-coordinate by "m" units. So the higher the m-value, the steeper the slope of the line. And if m is negative, the line slopes "downhill" to the right; if m is positive, the line slopes "uphill" to the right.

As for the number, b, notice what happens if you ask what y-value you get when x is zero: By calculating with the above formula, you find that y = b, right? But x is zero *only along the y-axis,* so the y-value you plot for that point on the line would also be on the y-axis. Thus, the line would cross the y-axis at that point. For this reason, b is called the *y-intercept* value.

So *if* your incomes and the rainfall levels are *linearly* related, then for any horizontal point, x (any given rainfall amount), there should be a unique corresponding y -value (your income), which you can compute using this formula and your HP-22S.

That would be the ideal case – where every pair of corresponding income-rainfall values produced a point that fit *exactly* on such a line. Of course, as the diagrams on the previous page show, real data almost never fits any such linear correlation theory *that* accurately (after all, remember, this is only a theory dreamed up in your head – that the rainfall and your income are neatly related in this simple, numerical way).

So even the line that best fits your data probably won't touch all your data points. Nevertheless, that line *is* the best fit, because it's the "happy medium" line that's closest to all of them. This is the actual calculation called Linear Regression.

So after you've keyed in all your rainfall-income data, what your HP-22S is going to tell you are the values of m and b for this "best-fitting" line, *plus* it will give you a measure of how well the line fits your data. This measure is called the *correlation coefficient,* r, which is always a number between -1 and 1. If r is close to 1 (for a positive m) or -1 (for a negative m), then the fit is good. If r is nearer to 0, then the fit is not good.

Ready to test your theory? All right, then, to put it a little more mathematically, what you're really proposing on page 114 is this:

"For the last ten years, my annual income from flight instruction has been directly proportional to the amount of rainfall. That is:

$$y_{income} = mx_{rainfall} + b$$

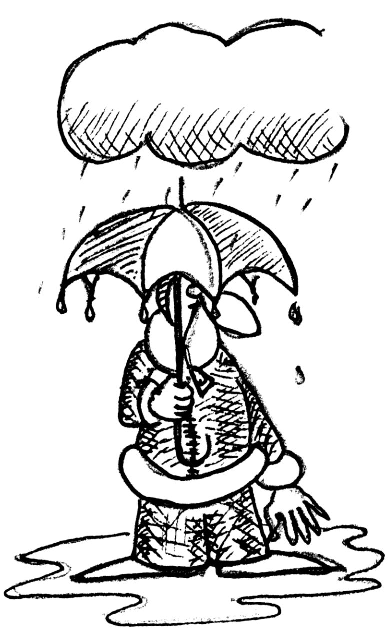
where m *and* b *are constants."*

Test It: Here are your rainfall and income histories for the last ten years:

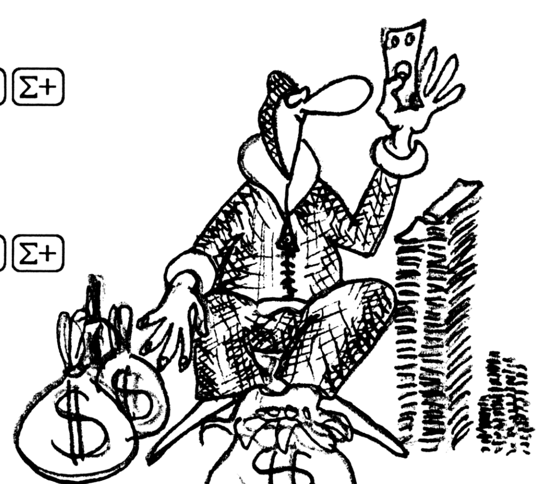
Year	Rainfall (inches)	Income ($)
1980	17.0	24,000
1981	50.0	14,500
1982	25.0	19,000
1983	22.0	23,000
1984	20.0	24,000
1985	17.5	24,500
1986	30.0	18,000
1987	16.0	27,000
1988	12.5	34,000
1989	19.0	29,000

Is there a linear correlation between annual rainfall and your flight instruction income?

Press ■ CLEAR Σ, then



1 7 INPUT 2 4 0 0 0 Σ+
5 0 INPUT 1 4 5 0 0 Σ+
2 5 INPUT 1 9 0 0 0 Σ+
2 2 INPUT 2 3 0 0 0 Σ+
2 0 INPUT 2 4 0 0 0 Σ+
1 7 • 5 INPUT 2 4 5 0 0 Σ+
3 0 INPUT 1 8 0 0 0 Σ+
1 6 INPUT 2 7 0 0 0 Σ+
1 2 • 5 INPUT 3 4 0 0 0 Σ+
1 9 INPUT 2 9 0 0 0 Σ+



(Your display should now show n=10.00)

Now press [STAT] and choose L.R. It's time to see what kind of straight line you can draw through these points.

First, choose m Result: m=-441.79

This is the slope of your line. Notice that it's negative, meaning that the line goes "down-hill" from left to right. Does this make sense? Sure it does: you've already observed that the more rain you had, the less income you earned – and that's exactly what the downhill slope is telling you here: "As x (rainfall) increases, y (income) decreases."

Next, calculate b (press [STAT], then choose L.R. and b).
Result: b=33,817.08

So the best-fitting line through your ten data points is

$$y_{\text{income}} = -441.79(x_{\text{rainfall}}) + 33{,}817.08$$

And just how good a fit is this (the Big Question)? Time to find out:

Press [STAT], then choose L.R. and r. Result: r=-0.84

This r, remember, is the correlation coefficient, and while it's nowhere near a perfect -1 here (recall that the minus sign indicates the sign/direction of the slope, m), it's a lot closer to -1 than it is to 0. So it's fair to say that there is a strong *linear* correlation between the annual rainfall and your annual flight-instruction income.

Question: Now that you're satisfied that there probably is some kind of linear correlation, if someone came up to you in the hangar after-hours and whispered mysteriously that next year there would be 32.0 inches of rain, what would your best guess be for your income next year?

Or if this soothsayer instead predicted that you'd gross $22,000, how much rain would you expect?

Answer: You'd whip out your trusty HP-22S (having already performed the Linear Regression from the previous page) and press

[3][2][STAT] L.R. ŷ   Result: ŷ=19,679.67

This ŷ is the *predicted* y-value (income level) *corresponding* to an x-value (rainfall level) of 32.0 inches *on that best-fitting line* whose constants you just determined (if you want to verify this by hand, you can: use 32 as the x-value in the formula on the preceding page).

So, judging from your past ten years' experience, you might expect your income next year to be somewhere in the neighborhood of $19,600.

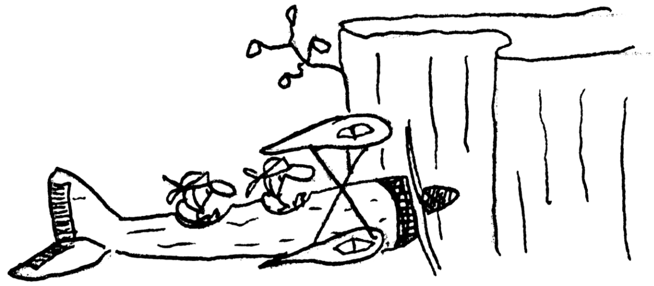
What if you want to go the other way – predict the rainfall from your alleged $22,000 income next year?

Press [2][2][0][0][0][STAT] L.R. x̂   Result: x̂=26.75 – about 27" of rain.

Notice that, to use the L.R. calculations to make guesses like this, you key in the known amount *first,* and *then* press [STAT] L.R., etc., to calculate the predicted value of the other coordinate.

STAT Quiz

Now test yourself to make sure that you know your way around the STATistics registers and calculations on your HP-22S. As usual, the answers are on the following pages (and as usual, you shouldn't peek prematurely).



1. Using the rainfall/income data you've already accumulated in the STAT registers (from page 117), calculate the total rainfall over the last ten years (1980-1989). Also, compute your total income over this same period.

2. How do the standard deviations for these rainfall and income records compare (as fractions of their averages)? Is this any clue as to whether or not there is a correlation between them?

3. Go back and re-enter the data on page 117 – with one big change: This time, pretend that you're trying to prove a dependence going the *other* way – that the area rainfall *depends* on your income (i.e. income is your x-data and the rainfall is the y-data).

   Does this give the same results for m and b, and the same predictions for 32 inches of rain or $22,000 in income? Is the fit, r, the same or different?

   If r is just as good, then what should you conclude about this hypothesis – as opposed to your original (the reverse) hypothesis?

4. Of course, the linear correlation with which you have tested your rainfall/income theory isn't the only possible kind of correlation. It's merely one of the simplest – which is why you chose it to start with – and it's the *only* kind of correlation your HP-22S can actually calculate.

But this doesn't mean you can't test for other kinds of correlations with your HP-22S; you merely need to *adjust* the data – to *transform* the problem into a linear problem so that your calculator can actually handle it.

For example, there are three other commonly used correlations ("shapes of curves"), each of which uses the parameters m and b – but not just as a linear slope and y-intercept. Here are those correlation equations, alongside their *adjusted* versions that are indeed equations of *lines* – provided that you "do certain things" to the x- and y-values:

Name Of Curve	Actual Form Of Equation	Adjusted Form Of Equation
Logarithmic	$y = m\ln(x) + b$	$y = m\ln(x) + b$ (same)
Exponential	$y = e^b e^{mx}$	$\ln(y) = mx + b$
Power	$y = e^b x^m$	$\ln(y) = m\ln(x) + b$

Looking at the adjusted forms, can you see how to adjust your raw data (page 117) so that the HP-22S can calculate a slope (m), y-intercept (b), and correlation coefficient (r), for each kind of "adjusted" line?

And if you do this, which of these forms (including the simple, linear form you've already calculated) produces the best fit to your rainfall/income data?

[STAT] Quiz Answers

1. Press [STAT] Σx Result: Σx=229.00 (inches of rain over ten years)
Press [STAT] Σy Result: Σy=237,000.00 ($ of income over ten years)

2. Because these data represent the entire body ("whole population") of possible values, you can't simply compute sx and sy. Remember (from page 111) that those formulas assume your data is only a *sample* of a larger population. To transform your data so that the HP-22S formulas *are* applicable to your whole-population case, you must *add* to your accumulation an extra data point consisting of the averages of the x- and y- data. Thus you would do this:

[STAT] x̄,ȳ x̄ [INPUT] Result: 22.90: (the [INPUT] preserves the x̄ on the Calculator Line, while you now go compute the ȳ to accompany it....)

[STAT] x̄,ȳ ȳ Result: ȳ=23,700.00

Now, even though it may not be obvious, you *do* have both averages there on the Calculator Line, so now press [Σ+] to accumulate them into your data.

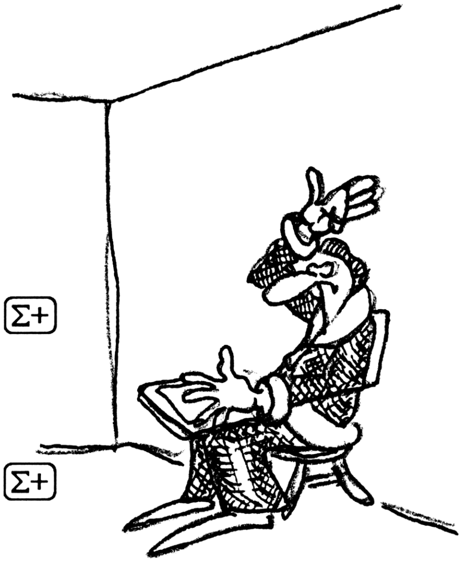
Now find the normalized s's (i.e., taken as fractions of their averages):

[STAT] s sx [÷][2][2][•][9][=] Result: 0.44
[STAT] s sy [÷][2][3][7][0][0][=] Result: 0.23

So, what do standard deviations tell you about the likelihood of correlations? *Nothing,* obviously. Here the correlation coefficient, r, was fairly good (if you'll recall, it was -0.84), and yet these deviations differ widely from each other.

3. Press ■ [CLEAR] Σ, then

[2][4][0][0][0] [INPUT] [1][7] [Σ+]
[1][4][5][0][0] [INPUT] [5][0] [Σ+]
[1][9][0][0][0] [INPUT] [2][5] [Σ+]
[2][3][0][0][0] [INPUT] [2][2] [Σ+]
[2][4][0][0][0] [INPUT] [2][0] [Σ+]
[2][4][5][0][0] [INPUT] [1][7][•][5] [Σ+]
[1][8][0][0][0] [INPUT] [3][0] [Σ+]
[2][7][0][0][0] [INPUT] [1][6] [Σ+]
[3][4][0][0][0] [INPUT] [1][2][•][5] [Σ+]
[2][9][0][0][0] [INPUT] [1][9] [Σ+]



OK, *this* time, you're analyzing the theory that

$$y_{rainfall} = mx_{income} + b$$

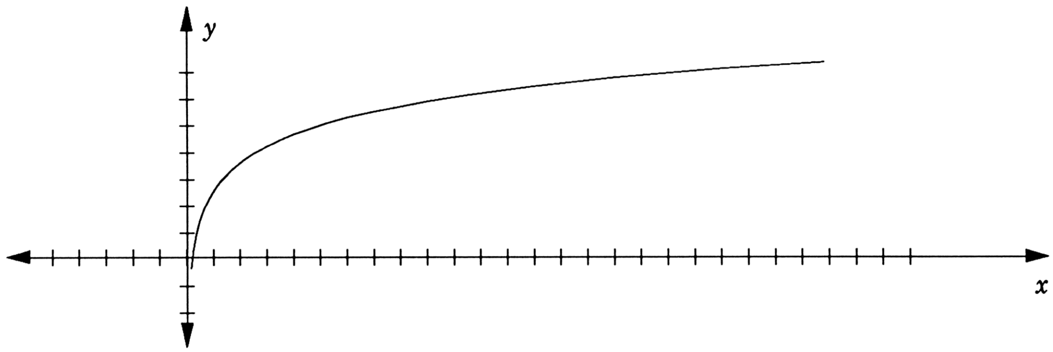
Press [STAT] L.R. m   Result: m=-1.60E-3
Press [STAT] L.R. b   Result: b=60.79
Press [STAT] L.R. r   Result: r=-0.84

Hmm...the constants, m and b are very different than on page 118. But the correlation coefficient is *the same* (a good fit)! And what would this best-fitting line "predict" for rainfall when income was $22,000 – or income when the rainfall was 32 inches?

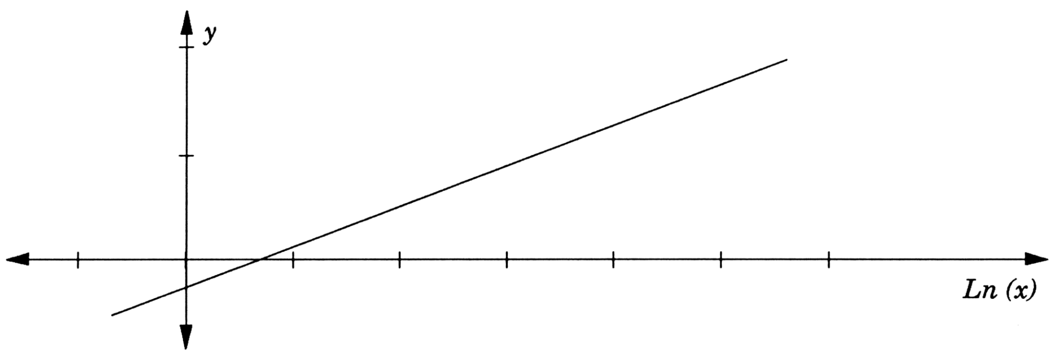
Press [2][2][0][0][0] [STAT] L.R. ŷ   Result: ŷ=25.62
Press [3][2] [STAT] L.R. x̂   Result: x̂=18,007.39

These are only slightly different results than on page 119, aren't they? And both curves fit the data equally well. So which predictions are more probable? Well, just using common sense, which *theory* are you ready to believe – that the rain depends on your income, or vice versa? Remember that statistics can *indicate correlations,* but they can't *identify real-world causes.* <u>You</u> must do that.

4. Take the logarithmic curve as an example. Here's what the curve looks like when you simply plot y-values and x-values:



It's not a straight line, is it? So there's no way to use the HP-22S' built-in *Linear* Regression to analyze such a model. Ah, but here's the picture when you plot y-values against *ln(x-values)*..



See? This *is* a straight line – as you could probably guess from looking at the formula on page 121 (or on the y-axes in the above diagrams). So this *is* a correlation your HP-22S can compute – and it's *equivalent* to the above logarithmic curve – because you've *adjusted* your x-data.

This means that as you key in your data, if you take the natural log (LN) of each x-value (but don't alter the corresponding y-value), and *then* accumulate the resulting data pairs (with the Σ+ key), you will be effectively asking your HP-22S to analyze a linear relationship between the y-value (your income) and the *natural log* of the x-value (the rainfall). And the "goodness" of that fit tells you how good your actual data fits the logarithmic curve, because a test for *linear* fit of this *altered* data is mathematically the same as a test for *logarithmic* fit of the *actual* x- and y- data!

So what are your keystrokes? First, of course, you press ■ CLEAR Σ. Then:

1 7 LN INPUT 2 4 0 0 0 Σ+
5 0 LN INPUT 1 4 5 0 0 Σ+
2 5 LN INPUT 1 9 0 0 0 Σ+
2 2 LN INPUT 2 3 0 0 0 Σ+
2 0 LN INPUT 2 4 0 0 0 Σ+
1 7 • 5 LN INPUT 2 4 5 0 0 Σ+
3 0 LN INPUT 1 8 0 0 0 Σ+
1 6 LN INPUT 2 7 0 0 0 Σ+
1 2 • 5 LN INPUT 3 4 0 0 0 Σ+
1 9 LN INPUT 2 9 0 0 0 Σ+

(Of course, at this point, the n -value is 10.00. Also, in case you want to check the accuracy of your entries – by using the Σ selection from the STAT key menu – the correct values for Σx and Σy are 30.56 and 237,000.00, respectively.)

Now you can solve for r , the correlation coefficient indicating the goodness of this logarithmic fit: STAT L.R. r Result: r=-0.91

Aha! The logarithmic model is actually *better than your original linear model* (remember that its r was only -0.84)!

So, what would this more-believable model predict, given a rainfall of 32 inches or income of $22,000.00?

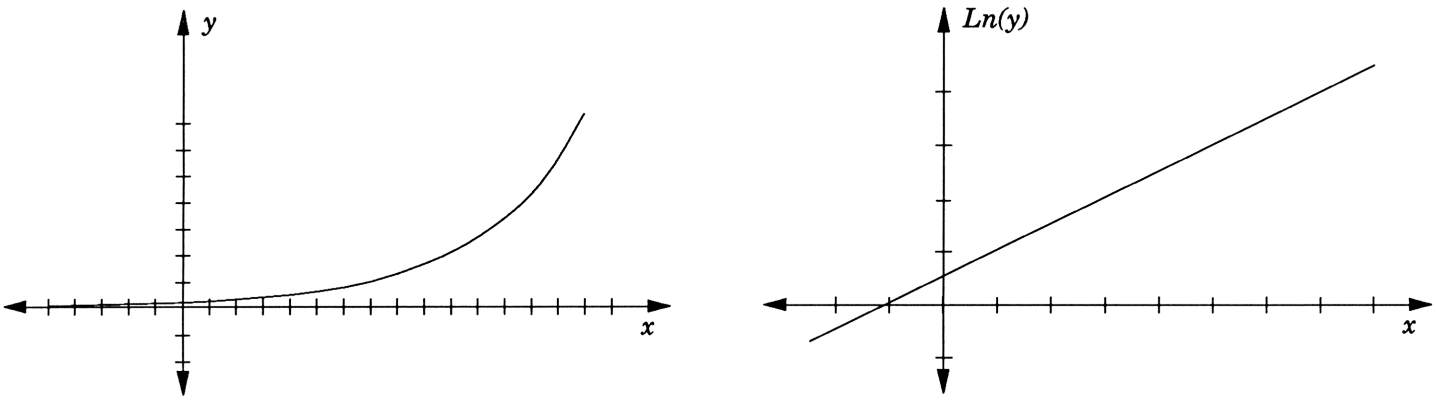
Remember that you must alter each x-value you enter: 3 2 LN STAT L.R. ŷ Result: ŷ=18,252.71 – a little *less* income than the linear model.

As for a rainfall prediction from $22,000.00 income, remember that any x-value you compute must then be "unaltered" (to "undo" a natural log, you *exponentiate* with the e^x key): 2 2 0 0 0 STAT L.R. x̂ e^x Result: 24.14 inches.

Well, then, what about those two other models (exponential and power curves)? Might they prove to be even better fits than the logarithmic curve?

There's only one way to find out....

Here's what an exponential curve would look like before and after the "alteration" to its equivalent linear form:



Notice that this time, it's y-values that you're going to "mess with:"

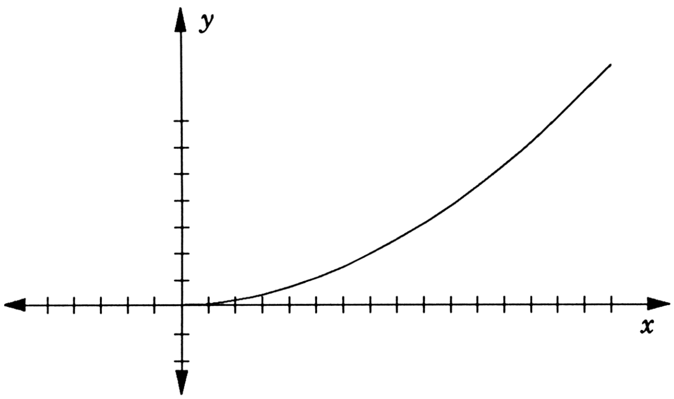
■ CLEAR Σ, then

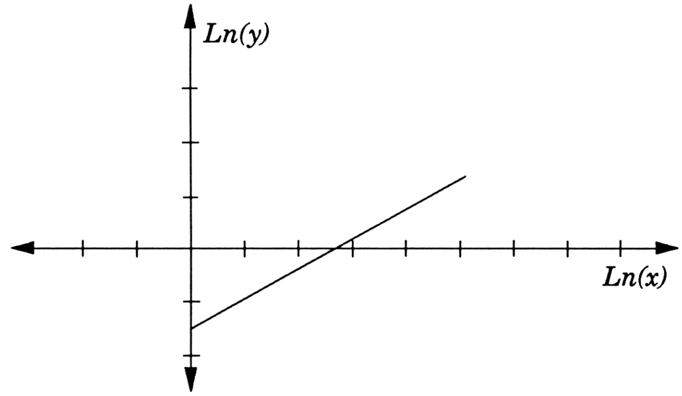
17 INPUT 24000 LN Σ+
50 INPUT 14500 LN Σ+
25 INPUT 19000 LN Σ+
22 INPUT 23000 LN Σ+
20 INPUT 24000 LN Σ+
17.5 INPUT 24500 LN Σ+
30 INPUT 18000 LN Σ+
16 INPUT 27000 LN Σ+
12.5 INPUT 34000 LN Σ+
19 INPUT 29000 LN Σ+

(Data check: Σx=229.00 Σy=100.47)

Now check the fit: STAT L.R. r Result: r=-0.90 Well, that's better than the linear model, but the logarithmic fit still "leads the league."

Finally, here's what a power curve would look like before and after the "alteration" to its equivalent linear form:





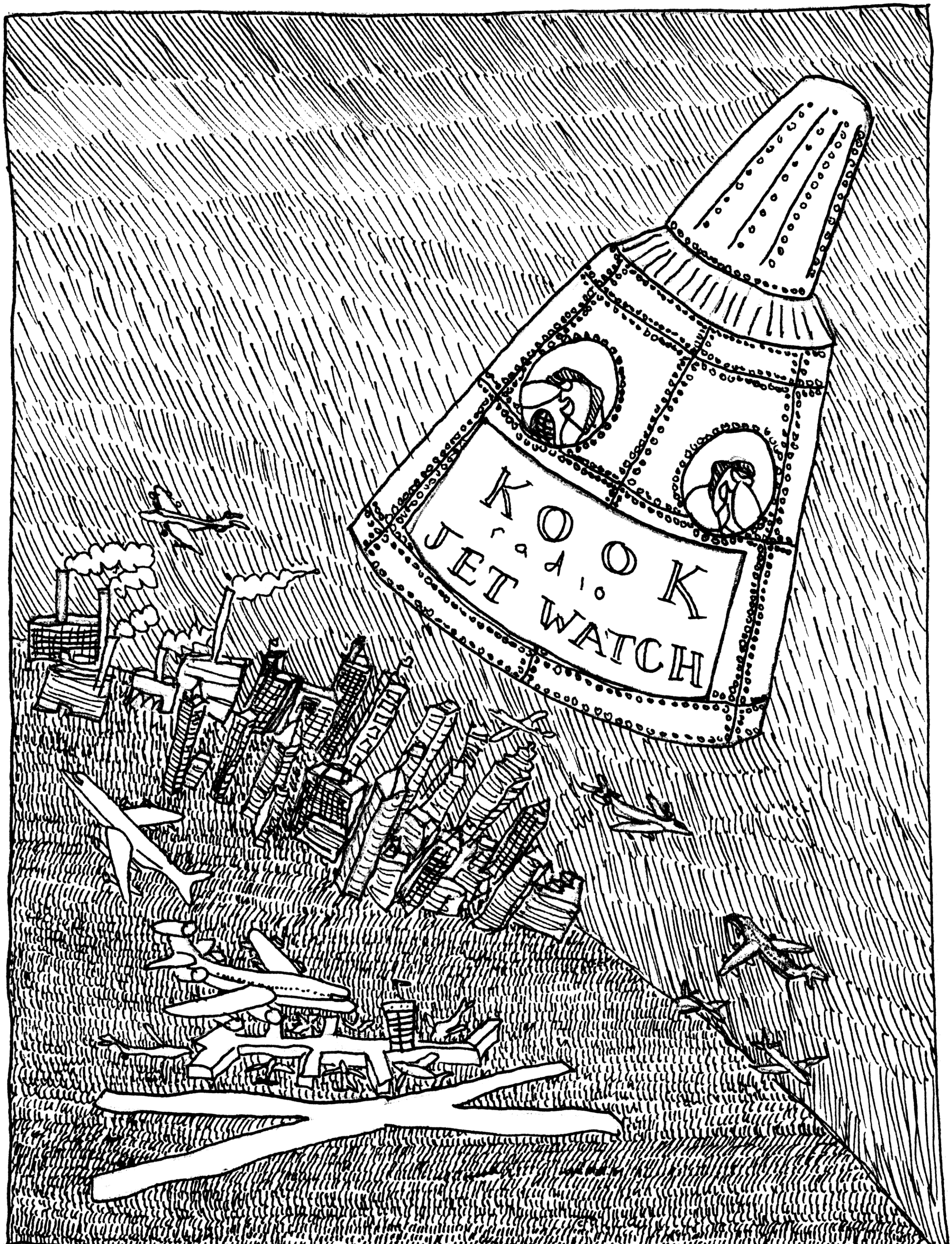
Notice that for this kind of curve, you have to alter *both* the x- and y-values.

■ CLEAR Σ, then
1 7 LN INPUT 2 4 0 0 0 LN Σ+
5 0 LN INPUT 1 4 5 0 0 LN Σ+
2 5 LN INPUT 1 9 0 0 0 LN Σ+
2 2 LN INPUT 2 3 0 0 0 LN Σ+
2 0 LN INPUT 2 4 0 0 0 LN Σ+
1 7 • 5 LN INPUT 2 4 5 0 0 LN Σ+
3 0 LN INPUT 1 8 0 0 0 LN Σ+
1 6 LN INPUT 2 7 0 0 0 LN Σ+
1 2 • 5 LN INPUT 3 4 0 0 0 LN Σ+
1 9 LN INPUT 2 9 0 0 0 LN Σ+

(Data check: Σx=30.56 Σy=100.47)
Now check the fit: STAT L.R. r Result: r=-0.95

Bingo! An even better fit than the logarithmic model! So use this model to get the *best* set of predictions corresponding to 32 inches of rain and $22,000.00:

3 2 LN STAT L.R. ŷ eˣ Result: 18,007.23 (dollars)
2 2 0 0 0 LN STAT L.R. x̂ eˣ Result: 22.98 (inches)



SOME WELL-TRAVELLED ROUTES:
Equations And The Built-In LIBRARY

Congratulations!

You've just reached the halfway point in this Easy Course in "flying" your HP-22S.

You now know all the basics of level flight (arithmetic), plus a number of standard maneuvers (higher math – including trig and exponentiation – plus the various menu keys: ■ PROB, ■ MODES, ■ PARTS, ■ UNITS, ■ H↔HMS, ■ D↔RAD, and ■ BASE).

You've also learned about the uses of a number of *special storage registers,* including the LAST register and, more recently, the STAT registers.

So in the first three chapters, you've been learning all about how to calculate with various *keys and menus* on your HP-22S, but you haven't seen very much about *equations and variables.* Now it's time to look at how they make your calculating life *much* more pleasant.

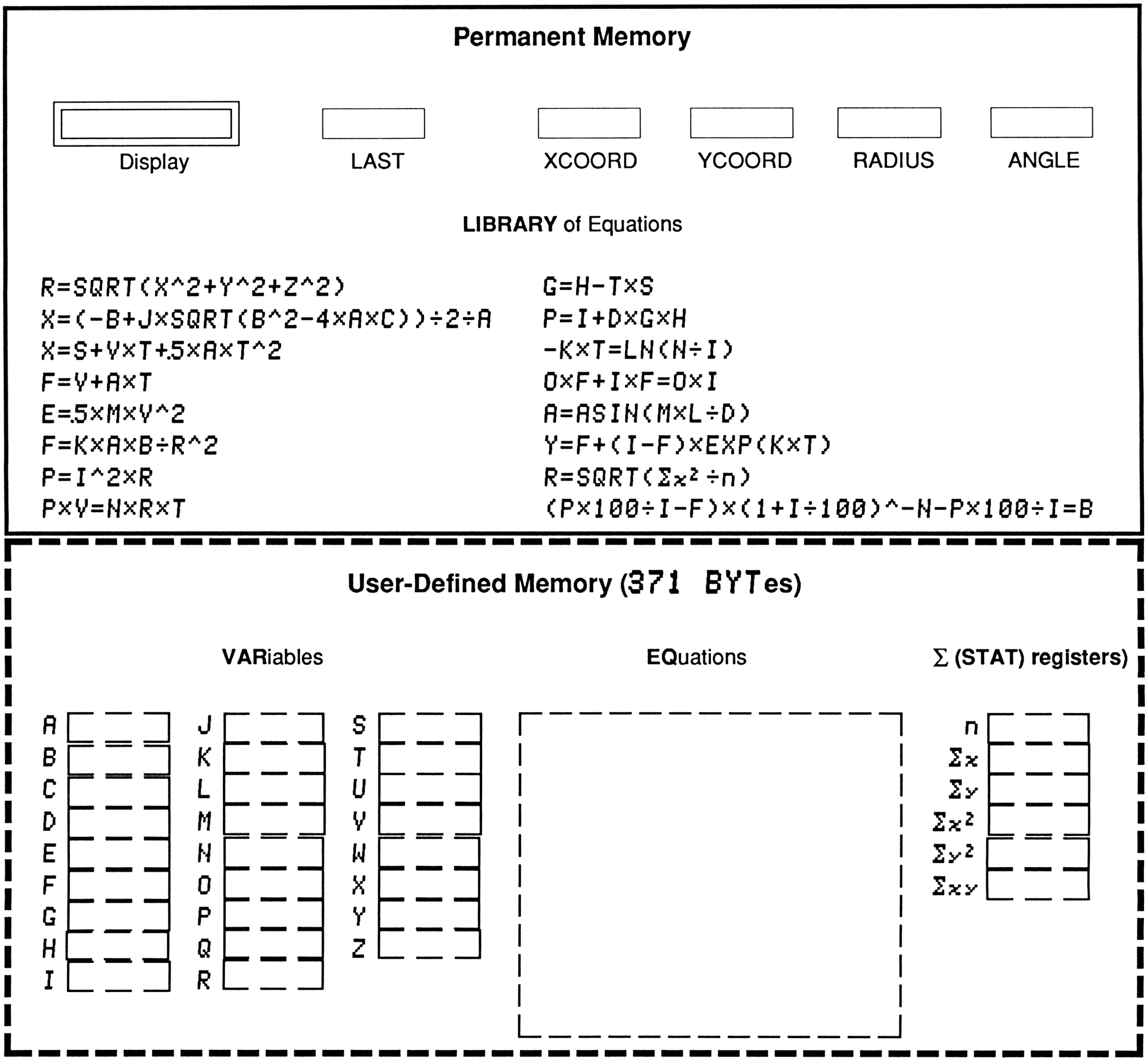
But first things first.

In order to be able to envision what's happening as you use variables and equations, you need to have a more complete picture of the memory in your HP-22S....*

*But if you already know all about variables, the alphabetic keyboard, and the STO and RCL keys, then now's the time to turn to page 139.

The Memory In Your HP-22S

Take a look at this:



You'll certainly recognize some things on this diagram – the display, the LAST register, etc.

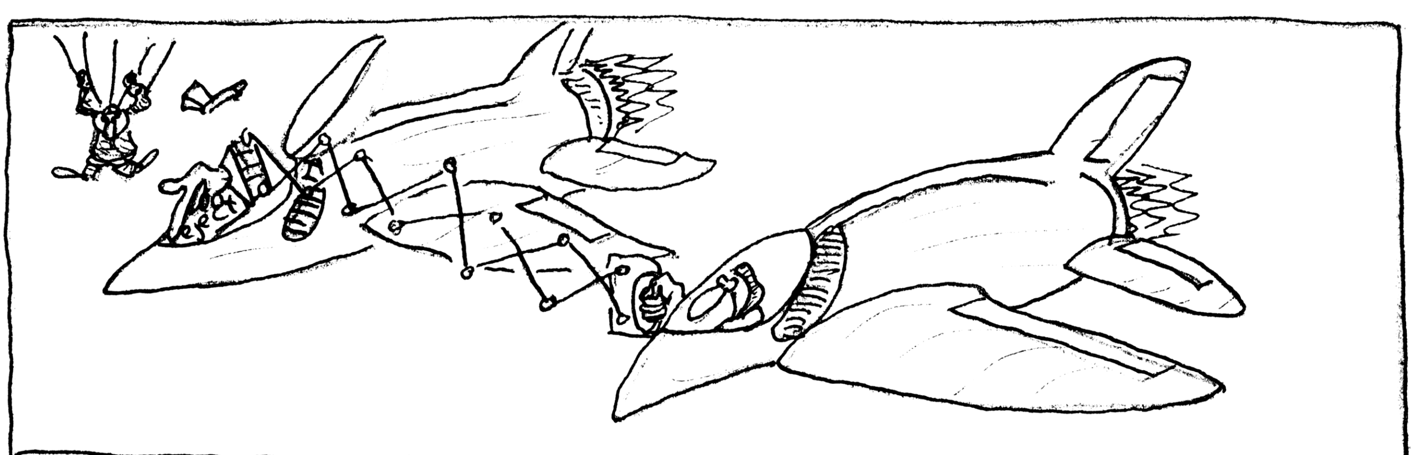
And you also know what those STAT registers are for – but notice how they're drawn here on the diagram: They're really a specially-defined and *temporary* part of a *larger* area of memory that you can fill in any way you choose – which is why it's called User-Defined Memory here – and this is why it's shown in dotted lines on the diagram.

"Huh? The STAT registers are temporary? How so? They always seemed to be there when I needed them."

Ah – that's true. But here's the part you may not have realized: The STAT registers literally *disappear* whenever you don't want them; they "melt" into that big undefined area of available memory *whenever you clear them*.

Remember that ■ CLEAR Σ you were doing regularly in the previous chapter? Normally, you were doing that just to "re-zero" the registers before keying in a new batch of data. But whenever you did that clearing – even if it was for only a moment before you started accumulating again – the STAT registers were undefined for that time; that is, they took up *none* of those 371 bytes you see listed on the diagram.

This is why you see a dotted line around the STAT registers: they're there only when you ask them to be there (by pressing Σ+ or ■ Σ−). And by pressing ■ CLEAR Σ, you tell them to "go away" so that you can use the extra memory for something else.



A couple of questions are undoubtedly thrashing around in your mind right now, so now's a good time to let'em out:

Question: Out of the 371 total possible bytes available in your HP-22S User-Defined Memory, how many do the STAT registers use when they're there?

Answer: You can actually demonstrate this to yourself in a little experiment: Press ■CLEAR ALL. Now confirm your wishes by choosing Y (the machine gives you this second chance to change your mind, just in case it's not what you really wanted to do). You're slowly but surely learning all about that ■CLEAR key menu. There's really not much to it. As you might suspect, the ALL selection on that menu clears ALL the User-Defined Memory – giving you a total of 371 bytes.

To see this, just press ■MEM and there it is: 371 BYT

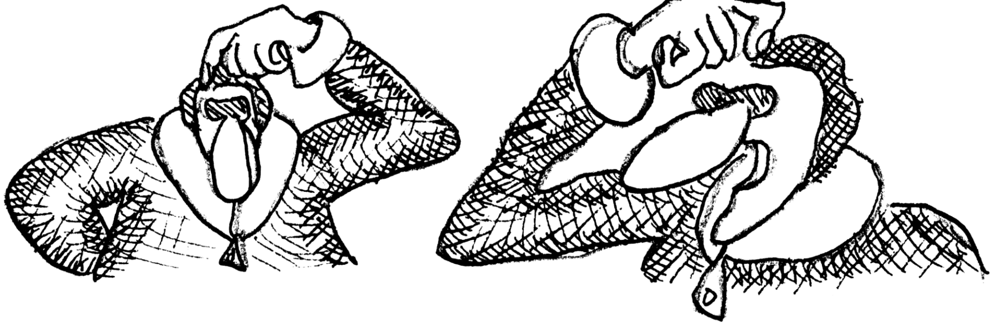
(Ignore that VARS selection for a little while.)

Now, the very second you start to accumulate any data into the STAT registers, the machine will instantly grab enough of the User-Defined Memory to build all six of those registers. If you then press ■MEM once again to check how much MEMory you have left, you ought to be able to tell exactly how much the STAT registers require:

Press C to get back to the Calculator Line (because you're looking at a *message* right now – remember those?), then Σ+ (to cause the STAT registers to be built), then ■MEM. Result: 323 BYT

Aha! 371-323=48! So the six STAT registers need 48 bytes of the User-Defined Memory. Sounds like about 8 bytes per register, eh?

OK..., but...



Question: What's a byte, anyway?

Answer: A byte is an individual unit measure of memory in a computer or calculator.

OK, ok, you probably knew *that* much. What you're really asking is: How *much* is a byte? How big is it? And just what does it mean to you in terms of storing numbers in your HP-22S?

That's easy: If it takes eight bytes per STAT register (as you've just demonstrated), and each register holds just one number (which is true for all such registers – STAT, LAST, etc.), then it must take *eight bytes to store one number.*

That's right. Then how many numbers could you theoretically hold in your HP-22S' User-Defined Memory at one time?

[C][3][7][1][÷][8][=] Result: 46.38

So, assuming that six of these are the numbers in the STAT registers, you've got room for about 40 more such numbers in your HP-22S.

Good grief! That surely doesn't sound like very much! Is it enough?

You betcha – and now's when you start learning why....

The Alphabetic Keyboard And Variables

Of course, you noticed a lot of other "registers" with dotted lines around them sitting in the User-Defined Memory area in the diagram on page 130.

These are the *alphabetic storage registers,* also called *alphabetic variable registers.* And, true to their name, there are 26 of them – one for each letter of the alphabet.

Try This: Clear everything out of the User Defined Memory. Then store the number 26 in the Z-register. How much memory would you guess you've used?

Solution: Press ■ CLEAR ALL Y. (Now press ■ MEM to verify, if you wish: Sure enough – you're starting with all 371 BYTes.)

Now press C to erase the message and you're ready to store. So key in 2 6 and press the STO key (second row, left side).

Notice how the little _ prompt comes on, along with the A..Z annunciator, telling you that the HP-22S is now ready to know *which* register you want to store into. This means that the keyboard has *redefined* itself to be mostly the *alphabetic letters* that you see off to the lower right of each key.

Notice the Z key down at the bottom (it's otherwise known as the 0 key or the ■ MEM key). Press it now....

Voilá – the display confirms what you just did: The Z- register now contains the number 26!

Now, to prove it, clear the Calculator Line: [←]...

...and recall the value from the Z-register: [RCL][Z]

There it is – 26.00 – back for more fun and games. You can now calculate with this, store it somewhere else, erase it, whatever.

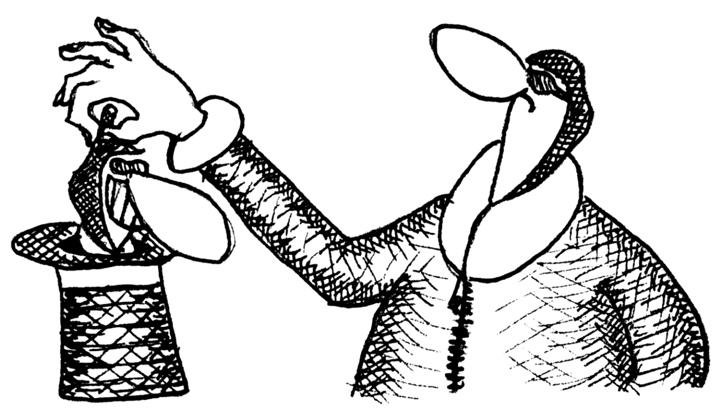
Keep in mind also, that both [STO] and [RCL] are *copying* processes; that is, [STO] sends a *copy* of the current number on the Calculator Line to the register you specify. And [RCL] brings a *copy* of a register's contents back; there is still a 26 sitting in the Z-register.

Now, how many bytes do you think you've just committed to the storage of this one number in this one register?

Eight, right? A register is a register. Just as the STAT registers needed eight bytes each, so do each of the alphabetic registers.

Prove it: Press ■[MEM] and see it for yourself: 363 BYT.

Before you stored something non-zero into the Z-register, it didn't exist (i.e. the eight bytes it needed were not reserved or *allocated* for it) – and you had 371 bytes of free memory. But now it does exist – because it has a *non*-zero value in it, and so you have 8 fewer bytes of free memory.



Challenge: Store a 1 in the A-register and a 13 in the M-register. Can you then review the contents of any existing alphabetic register *without using* the [RCL] key?

Yep: First, press ([C], if necessary, then) [1][STO][A] and [1][3][STO][M].

(Any problems finding the various alphabetic keys? Notice that the [M] key is otherwise known as the [INPUT] key.)

How many bytes do you think you now have remaining in your User-Defined Memory? You now have three alphabetic registers defined, each of which take eight bytes.

Thus: [3][7][1][−][3][×][8][=] 347.00 There should be 347 bytes left.

Confirm this, by pressing ■[MEM] . Shurr 'nuff.

Now then, what was that Challenge again? To review the contents of the *existing* alphabetic registers (there's three of them here) – without using the [RCL] key? Hmmm...!

Aha! – Try that VARS selection on the ■[MEM] menu. *This* is what VARS does: It gives you an alphabetized listing of each existing alphabetic register (remember that the alphabetic registers are also called alphabetic VARiableS – hence the name, VARS).

So now you should be seeing a display that looks like this:

```
▼▲
 A=1.00
```

That ▼▲ annunciator is telling you that you're looking at just one entry of a *list,* and that if you now care to use the [▼] and [■][▲] keys (just above the [■] key), you can move down or up this list, reviewing all the entries at your leisure.

So, just work your way down the list: Press [▼] to see `M=13.00`. Press [▼] again to see `Z=26.00`.

If you now press [▼] once more, you'll "wrap around," back up to the top of the list once more. What could be easier? And if you had wanted to, you could have "gone around" the whole list in the other direction, by using the [■][▲] key.

You'll find that this pattern holds true for all sorts of lists in your HP-22S – and you're going to encounter some other kinds soon, so play around with this one some more now, if you wish.

When you're through, press [C] to return to your normal view of the Calculator Line.

Why [C]? Because this whole `VARS` list review was a series of *messages* appearing in the display (remember page 17?); none of the values you saw were actually sitting on the Calculator Line ready for computation. They were merely messages that temporarily covered the display's true contents on the Calculator Line (`347.00`).

One More Thing: You now have those three registers (A-, M-, and Z-) with numbers in them. Try storing a zero in the M-register now:

Easy, Right? Press [0][STO][M]. No sweat....

OK, but now check your remaining MEMory and the values of your VARS: Press ■[MEM]

Egads! 355 BYTes?!? But you had just *347* bytes a minute ago! What gives?

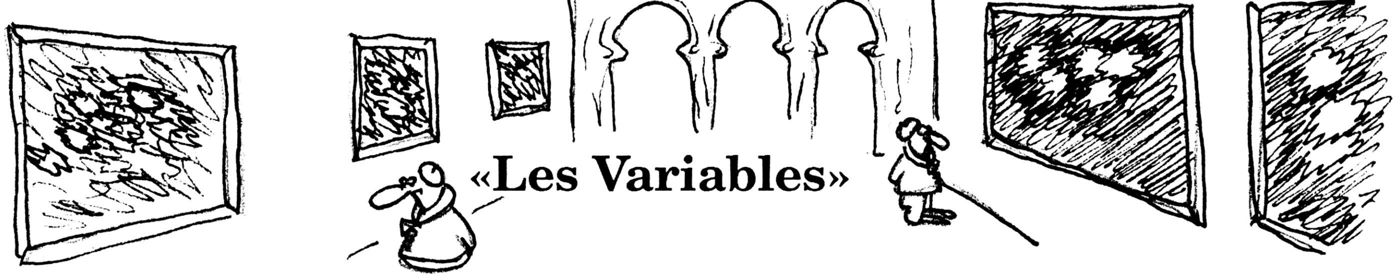
With mounting suspicions, you choose the VARS selection to check the contents of your three registers....

A=1.00 OK, that's fine....

[▼] What's this? Where's the M-register?

It's been "disappeared" – deallocated – *because its contents were set to exactly zero.* It doesn't matter how; in this case, you literally [STO]red a zero in there, but in most cases it happens when you use the ■[CLEAR] menu. No matter how it happens, any time an alphabetic register's contents are set to zero, its eight bytes of memory are "re-melted" back into the User-Defined "pot."

So you now have only two alphabetic registers allocated – the A- and Z- registers.



«Les Variables»

OK, ok, you're just dying to know why the 26 alphabetic registers are also called variable registers – right?

Well, speculate for a minute....

Could it be because their existences are variable? Whenever they contain zeroes, their memory allocations are automatically revoked; they disappear.

Mmmm...could be... – but nope.

Might it be because the alphabetic keys by which they're named *vary* their meanings between normal math functions/menus and the alphabetic letters?

Nice try.

Give up?

It's because these 26 registers will be storing the values of ***variables*** *in the* ***equations*** *you'll be using and writing.**

*And if you already know all about variables – and how to use the [EVAL], ■[SOLVE], and ■[LIBRARY] keys with the built-in LIBRARY of formulas, then you've ruined all the fun for yourself, and there's just nothing more for you to do except flip over now to page 161.

Equations: How You'd Solve Them On Paper

First of all, what *is* an equation, anyway? The very word tells you something about itself, doesn't it?

It's a mathematical staement – a sentence – that *equates* the value of two different things. If you say that

$$a = b$$

you're saying that *no matter what you call them,* both a and b have the *same numerical value.* It's like putting two differently-labelled containers on opposite pans of a balance scale: you really don't know – or care – what's in each container, but you *do* know they are numerically equal in weight.

Well, this idea extends, of course, to *combinations* of unknown objects. So if you say that

$$y + z = u + v$$

then you may not know how much y or u "weighs" all by itself, but you do know that the weights of y and z together exactly balance the combined weights of u and v.

Of course, all of this is really the fundamental idea behind algebra, right? You give unknown numbers these letter-names, so that you can add and subtract and do all manner of arithmetic and operations with them – without ever knowing what values they have.

As long as you *start* with a true equation and then make sure to *perform the same operation(s) to each side of the equation,* it's just like adding the same thing to each pan of the scale: *You'll always preserve the balance; you'll always be able to equate the two sides with an = sign.*

Question: Suppose you really did want to know how much z "weighs" in that equation on the previous page. What would you do?

Answer: You'd subtract the y "weight" from both sides of the balance – so that you could see what z would "balance" all by itself. And here's what you'd get:

$$y + z - y = u + v - y$$

or $$z = u + v - y$$

This would be the most useful way of putting things – if you already *knew* the "weights" of u, v and y – so that you could then use those *knowns* to *calculate* the weight of the *unknown* z, right?

Yes, But: What if it so happened that the "weights" you actually knew were of z, v and y, instead? How would you then find out what u was?

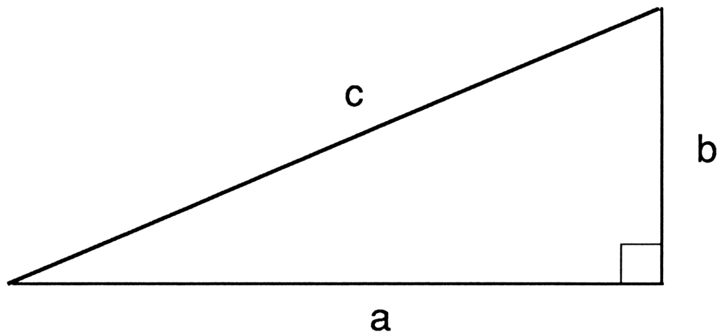
Ah, Yes: You'd have to rearrange the above equation, so that you had just u on one side and everything else on the other – then you could just read off the answer. So, start rearranging:

	z	=	$u + v - y$
or	$z - v$	=	$u + v - y - v$
or	$z - v$	=	$u - y$
or	$z - v + y$	=	$u - y + y$
or	$z - v + y$	=	u

Now simply read off the final equation: "If you subtract v from z and then add y to the result, you'll get u." Simple, right?

Sure it's simple – as long as you can get all your *known* values on one side and just the one *unknown* on the other.

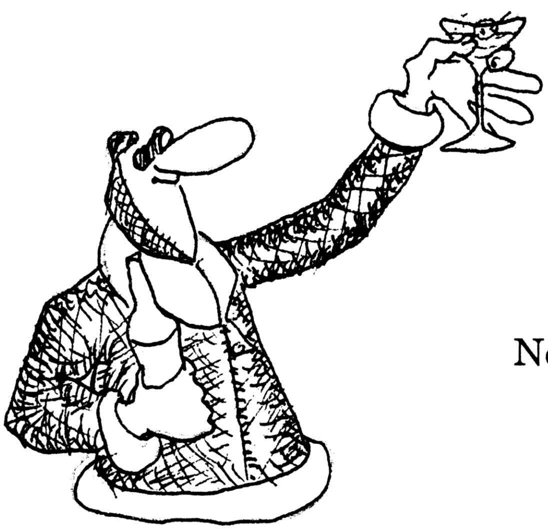
But what if you have an equation where the unknown isn't always the same variable? Remember all those problems, for example where you needed to solve for one side of a right triangle – given the other two sides – but it wasn't always the same side you were after?



According to the world-famous Pythagoras,* the equation relating these sides is:

$$a^2 + b^2 = c^2$$

The problem is, sometimes you knew a and b, so c was your unknown; but other times you'd know b and c, so a was your unknown, etc. But each time there was a *different unknown,* in order to be able to "read it off" to yourself for convenient calculation, you first had to *rearrange* the above equation to *isolate* it:



$$c = \sqrt{a^2 + b^2}$$
$$a = \sqrt{c^2 - b^2}$$
$$b = \sqrt{c^2 - a^2}$$

Not any more!...

*Not only was he the first known writer of this equation, he even did so in Greek!

Playing "What-If?" With Variables In Equations

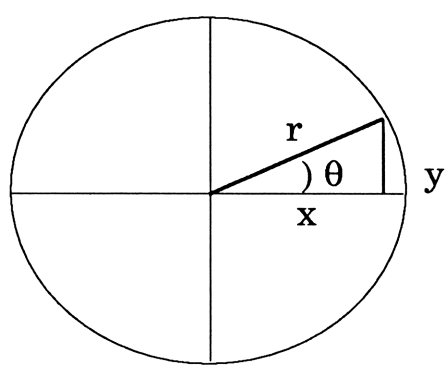
In case you haven't noticed by now, there are few more keys on your HP-22S that you haven't explored yet – but now you're ready....

Have you noticed, for example, the four blue ("shifted") functions on the second row of keys from the bottom of the keyboard? Those blue names are all linked with lines, indicating some kind of relationship....

To Wit: These four keys form a good first example of how your HP-22S makes it *unnecessary to rearrange equations* simply because you want to calculate a different unknown. The equations used by the four keys are:

$$\text{XCOORD} = \text{RADIUS} \times \text{COS(ANGLE)}$$
$$\text{YCOORD} = \text{RADIUS} \times \text{SIN(ANGLE)}$$
$$\text{RADIUS} = \sqrt{\text{XCOORD}^2 + \text{YCOORD}^2}$$
$$\text{ANGLE} = \text{ATAN(YCOORD} \div \text{XCOORD)}$$

These equations refer to a right triangle inside a circle – oriented exactly according to the conventions of trigonometry (as you saw on pages 56-60). So the RADIUS (r) that you're talking about here is just the *hypotenuse* of a right triangle whose legs are the XCOORD (x) and the YCOORD (y). And the ANGLE (θ) is the "central angle" – between the x-leg and r-leg:



Watch: If the shortest side of a right triangle is 3 meters long, and the next shortest is 4 meters long, then how long is the hypotenuse?

"What If" the 4-meter side were lengthened to 7.5 meters – but the shortest side remained at 3 meters)? How long would the hypotenuse then be?

No Sweat: Key in [3] and press ■[XCOORD]. See how the display confirms that you have now stored this number into the XCOORD variable?

Next, press [4], then ■[yCOORD]. That's your other known value.

Now, what do you think you need to do to calculate the RADIUS? Just press that key: ■[RADIUS]! Result: r=5.00

(So how does the HP-22S know you want to *calculate* this unknown value rather than *store* a known value there? It's because you had not keyed in a value *just prior* to pressing ■[RADIUS].)

Now comes the good part: The values in each of the three variables will stay there until you change them (either by storing new values into them, clearing them, or calculating new values in their places).

So right now, there's a 3 in the XCOORD register – leave it there so you can use it to answer the second question (the "What-If?" question). Change only what you need to:

[7][•][5] ■[yCOORD]

Now calculate your unknown: ■[RADIUS] Answer: r=8.08

Notice these things about what you just did:

1. You just used a built-in *set* of equations in your HP-22S.

2. In this particular case, the equations are so useful they were given their own set of keys – and the four variables (XCOORD, YCOORD, RADIUS and ANGLE) were given their own *specially defined registers* (similar to the LAST register and the STAT registers.

3. These four equations (the ones you see on page 143) are really rearrangements and recombinations of one another – but *you* don't have to do that; the machine knows which value you're solving for and it picks out the equation that isolates that value. All you need to do here is to key in two (known) values to solve for a third (unknown). And you can easily play "What-If?" by keeping one known constant, varying the other, and then re-solving for your unknown.

4. Which values *do* you need in order to calculate each variable? It's easiest to remember if you look at the keyboard: The HP-22S will use the current values for *both* of the *lefthand* two variables (XCOORD and YCOORD) to *calculate either* of the righthand two variables (RADIUS or ANGLE)

   And vice versa: You need to specify *both* RADIUS and ANGLE (on the right) to calculate *either* XCOORD or YCOORD (on the left).

Try It Again: How long is the x-leg of a triangle whose hypotenuse (radius) is 8 cm and whose central angle is 37°? How long is the y-leg?

Easy: Press [8] ■[RADIUS] ■[MODES] DG [3][7] ■[ANGLE], then ■[XCOORD].
Answer: x=6.39

Notice that the ANGLE variable always assumes the angular units that are currently set: The RAD annunciator was probably still on in your display, so you needed to change to DeGrees mode.

And Again: What's the central angle of a triangle whose x-leg is 19 and whose y-leg is 11? Give the answer in radians and degrees. Then calculate the length of the hypotenuse, just for the heck of it.

Can Do: Press ■[MODES], then choose RD for radians mode. Then:

[1][9] ■[XCOORD] [1][1] ■[YCOORD] ■[ANGLE] Answer: θ=0.52 (radians)

Now do it again in degrees mode: ■[MODES] DG ■[ANGLE]

Answer: θ=30.07 (degrees)

Notice there's no need to re-key in anything. Every known value is still in its register; you're only changing the units of the answer.

Finally, to get the length of the hypotenuse (again, no need to change or repeat anything; you've already keyed in all the information you need to calculate the answer immediately): ■[RADIUS]

Answer: r=21.95

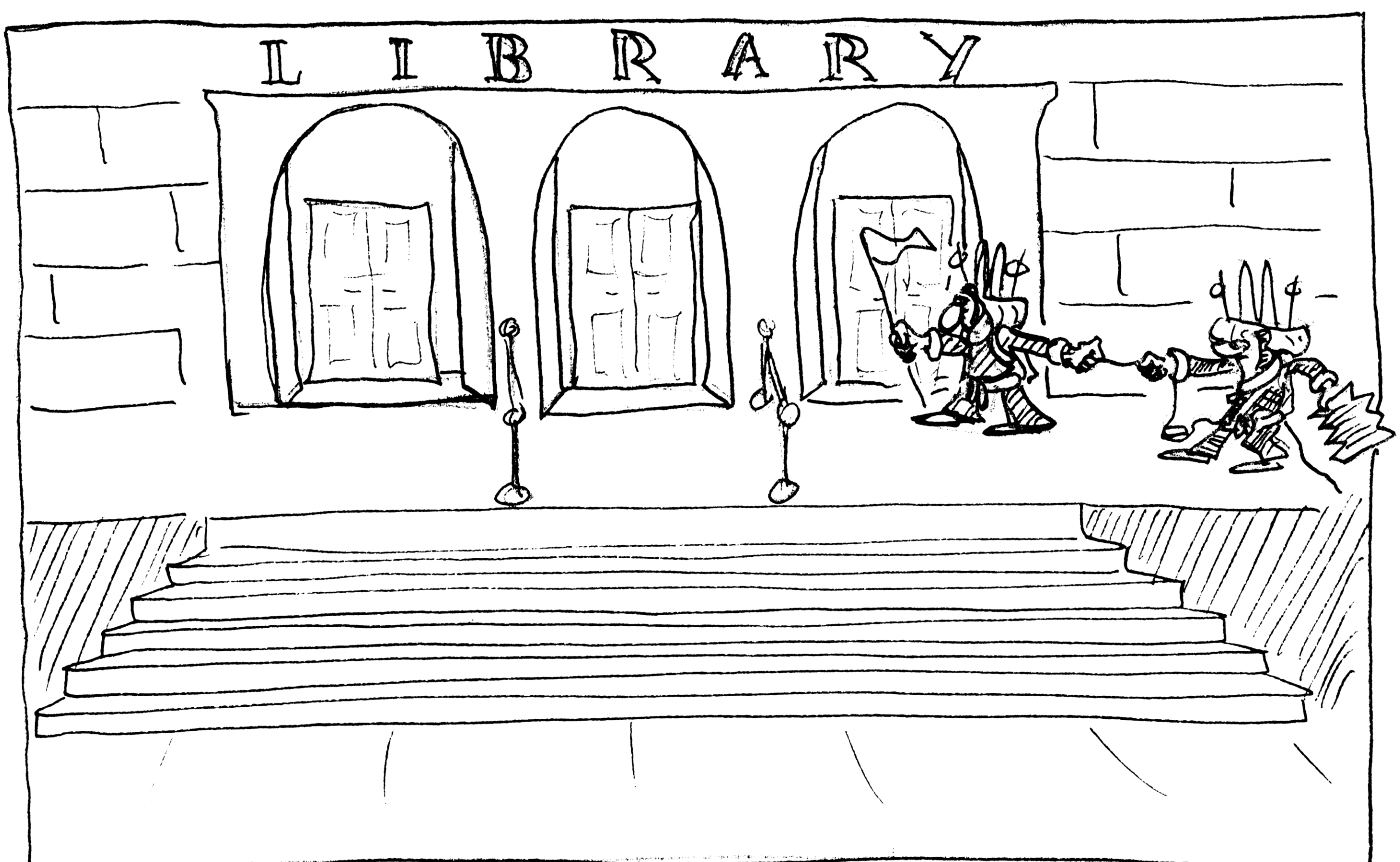
As you can see, this "What-Iffing" business with equations is *very* useful. As long as you give enough knowns, you can calculate any variable as the unknown.

Not bad, eh?

But are these triangle equations the only ones HP has built into this calculator?

Not by a long shot.

They *are* the only ones found in a *related set* like this (all others are just individual equations) – and as you read before, these are the only ones honored with keys and registers of their own. But there are plenty of others, and now it's time now to explore them....



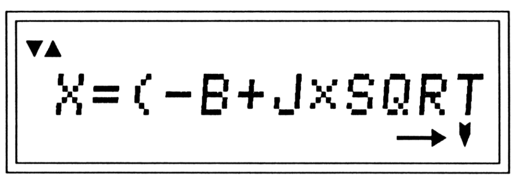
Checking Out A Few Things From The LIBRARY

The rest of the built-in equations in your HP-22S are all in the *Equation LIBRARY.*

Quiet Please: Enter the Equation LIBRARY by pressing ■ LIBRARY. Here you'll find equations in a list – much like the list of variable values you saw when you pressed VARS in the ■ MEM menu key (see pages 136-137). And – as in any list – you can "scroll" or "walk" through it by using the ▼ and ■ ▲ keys, as indicated by the ▼▲ annunciator.

So do that – press ▼ or ■ ▲ repeatedly, to browse through the equations and get a feel for what you have (16 equations in all).

Notice This: Move through the list until you come to an equation that looks like this:



See the one menu choice (→) there on the right? When you press that selection, you'll "scroll" the display to the right so that you can look at the rest of such any equation like this that's too long for the display. Try it now....

See how it goes one character at a time? And notice that whenever the display is somewhere in the middle of the equation (i.e. either end is "off the screen," you'll have *two* such scrolling keys – one to the left and the other to the right.

The EVAL Key

OK, fine – it's a Library of Equations, all right. Now, how do you *use* them?

Like This: Take as your first example the equation you're probably still "pointing to" with your display – that long one from the previous page. If you've looked at the whole thing (with the help of those scrolling keys), then you've discovered that the equation in its entirety is:

```
X=(-B+J×SQRT(B^2-4×A×C))÷2÷A
```

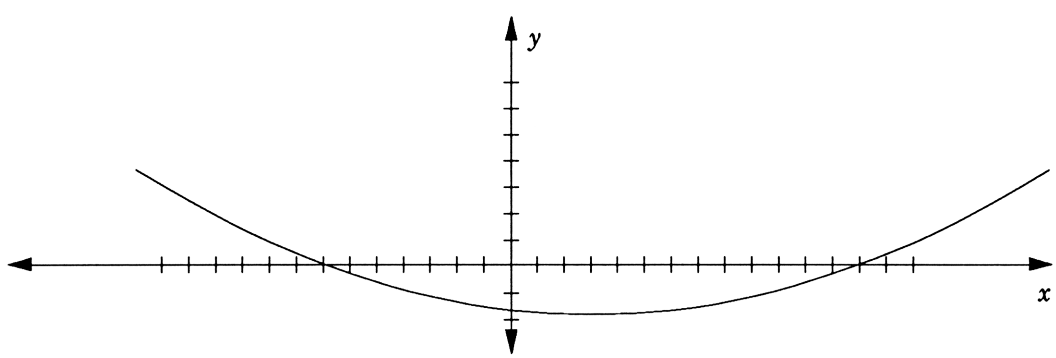
First of all, do you recognize this? You might. Maybe looking at it in this form will help to jog your memory:

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

It's a useful formula when you're working with a *parabola,* which is a certain variety of X-Y curve described by this equation:

$$Y = Ax^2 + Bx + C$$

...and this general shape:

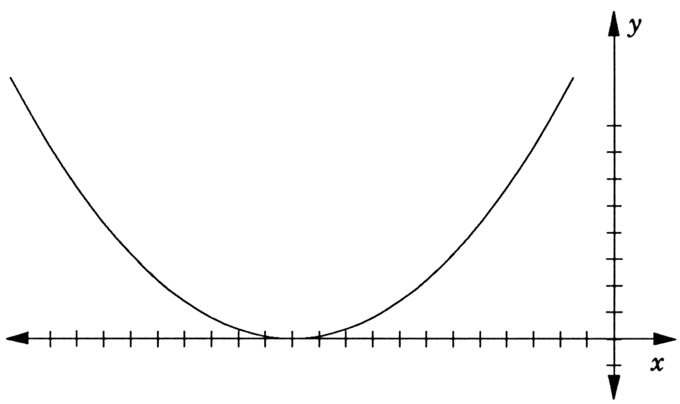


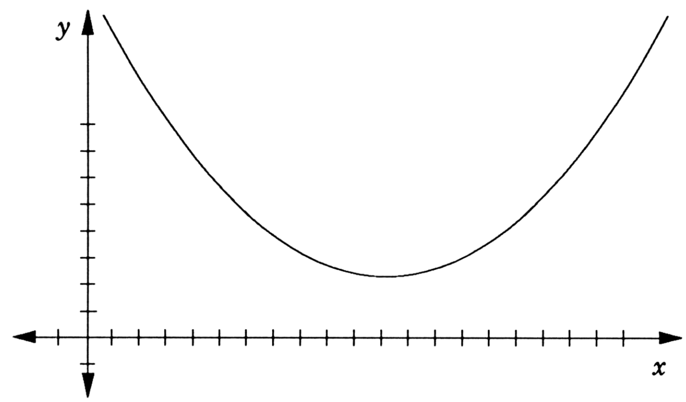
You may have also heard this parabolic kind of equation referred to as a *quadratic equation.*

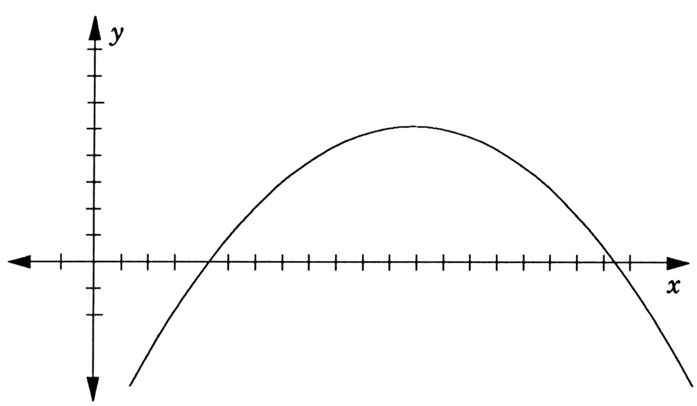
But no matter what you call it, by far the most interesting and useful information you can get from it is when you can get the Y-value to become zero:

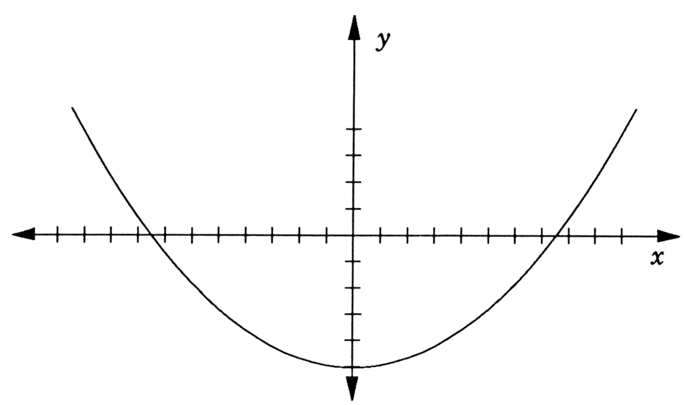
$$Y = AX^2 + BX + C = 0$$

And when would this happen? Whenever this equations X-Y graph crosses the X-axis, like this:









The X-values at these points are called "roots," and as you can tell from these pictures, there can be two roots, just one, or even none at all.

So the equation you see there in your HP-22S's LIBRARY is the formula for calculating *where* any given parabola ($Y = AX^2 + BX + C$) would cross the X-axis. That is, it *solves* this quadratic equation for the value(s) of the unknown X('s) when Y is zero: $AX^2 + BX + C = 0$

And so the solution is indeed the quadratic equation you may remember from math class – which explains why it might look familiar:

$$X = \frac{-B + J x \sqrt{B^2 - 4AC}}{2A}$$

Well...*sort of* familiar...but there's something odd about it. It doesn't quite look like the more familiar version on page 149, does it?

Instead of the ± to signify the two different possible solutions, you have this J sitting there. Does it do the same thing?

Yes, it does. Notice that J can indeed serve as a ± sign *if you restrict its value to either 1 or -1.*

But why can't you have the ± operation instead? Because it's actuallya short form of writing *two* entirely different equations:

$$X = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \quad \text{and} \quad X = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

Remember that in your HP-22S's *LIBRARY* of equations, there are only single equations – no "matched sets" of related equations (unlike the XCOORD/YCOORD/RADIUS/ANGLE set-up). So, to get around this limitation you just introduce an extra variable, J. Then as long as you restrict its value to 1 or -1, it will let you "switch" back and forth between the two different forms of the quadratic solution!

Now knowing all *that,* you're ready to try using this equation from the LIBRARY (at this point, you should still be looking at it in your display)....

Suppose you have this quadratic equation:

$$10X^2 - 39X + 7 = 11$$

What is X?

Hmmm....The first thing to do is to somehow get a *zero* over on the right side of this equation, so that you can use the famous formula (remember: that formula is good *only* when you have the equation in the form $AX^2 + BX + C = \mathbf{0}$ It *must* equal zero).

Fortunately, it's always easy to do this. In this case, all you do is subtract 11 from each side of the equation, and you get

$$10X^2 - 39X - 4 = 0$$

Now you know the values of A, B and C to use in the formula: A = 10, B = -39, and C = -4. You can just read these off from the equation, right?

Now, notice the [EVAL] key on the left side of the keyboard. Notice also its shifted counterpart, the ■[SOLVE] key.

These two keys tell the HP-22S to help you key in the known variable values (in this case they are A, B and C) – and then solve for the unknown variable (in this case, X).

Try one: With the quadratic solution equation in the display, press [EVAL]....

See? Immediately the machine begins to ask you for all the known values. And notice that it asks for them *in the order in which they appear in the LIBRARY formula* – not alphabetically.

Notice also that little INPUT annunciator comes on, telling you that after you key what the display is prompting for, you should press [INPUT] to continue.

So, do it: Key in the B-value by pressing [3][9][+/–], then [INPUT].

Next up: The J-value (that substitute for a ± operation – so it has to be either 1 or -1, as you know). Remember that to get both possible values for X, you're going to need to solve this formula twice – once with J = 1 and once with J = -1.

For this first solution, let J be positive, so press [1][INPUT]

Next, A is 10, so press [1][0][INPUT]. Then [4][+/–][INPUT] for the C-value....

Poof!

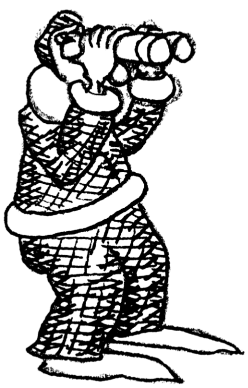
Instantly, you have your answer (so quickly, in fact, that you hardly realize what you're seeing): X=4.00

So this is *one* value of X that satisfies the equation

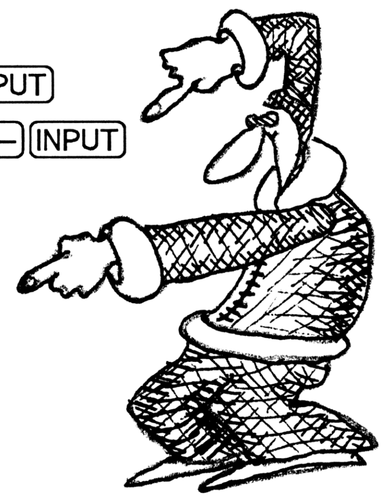
$$10X^2 - 39X - 4 = 0$$

What's the other? Well, all you need to do is to change the sign of J to get the other solution, so press [EVAL] once again....

The good news is, you don't need to re-key in all the other values that are the same as before; just press [INPUT] to confirm the current value that the machine shows you, changing only the J-value:



You See	You Press
B?-39.00	[INPUT]
J?1.00	[+/-][INPUT]
A?10.00	[INPUT]
C?-4.00	[INPUT]
X=-0.10	



So that's your other solution: -0.10

And that's how to use the [EVAL] key: It's good whenever the *unknown* you're trying to solve for appears *by itself* (isolated) on the *left* side of the equation.

But what about when the unknown is all buried and mixed up with other "stuff" on the right side of the equation? Or what about times when you may change what's known and what's unknown.

Mmm...that calls for stronger medicine, eh? That's when you reach for....

The ■SOLVE Key

Here's where the "What-Iffing" *really* starts to cook: With the ■SOLVE key, you get to choose which variable is your unknown. All the possibilites show up on a menu, and you simply choose which one you want.

But once you do that, the calculator goes through the same process as with the EVAL key: It prompts you for values for all the other (known) variables, then instantly calculates the only one left – the unknown.

Watch: Search through the LIBRARY list (press ■LIBRARY, then use the ▼ or ■▲ keys) until you find this equation:

P×V=N×R×T

Most of the equations in your HP-22S's LIBRARY are commonly found and used in some branch of science, and this one is no exception. It's the so-called Ideal Gas Law, derived and used in thermal mechanics, physical chemistry, and all sorts of chemical/mechanical engineering applications.

The equation says that if you have some pure gas in some closed container, then you ought to find this particular mathematical relationship between its Pressure (P), its Volume (V), its Temperature (T), and how much of it you have – the Number of moles (N).* The R is a constant number – like the slope of correlation that ties them all together.

Now, in the real world, no gas behaves *exactly* according to this simple equation (although some come quite close), so this is the Ideal case – hence its name – the Ideal Gas Law.

*A mole is a certain *number* of molecules – a number defined long ago to be about 6.02×10^{23}.

Sometimes you know everything but the temperature; other times, it's the pressure you need to calculate, etc. The point is, because the equation is in your HP-22S, you need only use the ■ SOLVE key and tell the machine which is your unknown.

Suppose, for example, that you have a 10-Liter vessel containing 2 moles of gaseous Nitrogen (N_2) at 300 K (that's the Kelvin temperature scale – beginning at Absolute Zero). But you don't know the pressure in that vessel – until you do this:

With the Ideal Gas Equation showing in the display, press ■ SOLVE and see how the display lets you choose any one of the variables as your unknown:

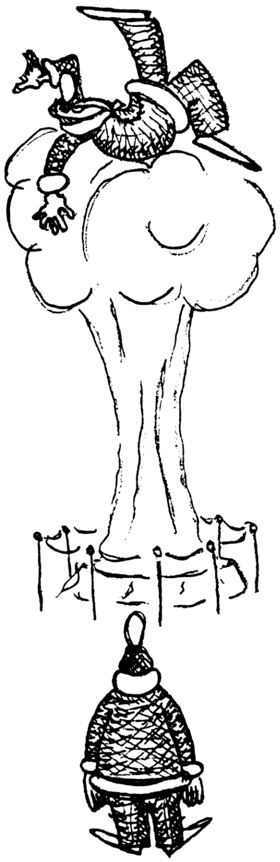
```
N  P  R   T  V
▼  ▼  ▼   ▼  ▼
```

So choose the P selection, because you want to *calculate* the Pressure. And instantly, your HP-22S will begin to ask you for the values of all the *other* variables, because you have declared them all to be "knowns."

You See	You Press
V?0.00	1 0 INPUT
N?0.00	2 INPUT
R?0.00	Hmm....

...the value of R is *physically* a constant, but the *number* you use to represent it here will depend on the units of your measurement (it's like describing the length of a football field. It has only one physical, *actual* length, but you can represent that one unique length with many different numbers: 100 *yards,* 300 *feet,* 3,600 *inches,* 91.44 *meters,* etc.).

So you need to choose a value for R that agrees with your other units of measure – Liters, Kelvins (those are temperature degrees, remember), and then whatever units you want to use for the pressure you're going to calculate – say, atmospheres (atm. – the average pressure of the earth's atmosphere at sea level).



Well, if you were to look up the various values for R in the many different possible combinations of units, you'd find among them this value:

$$R = 0.082053 \quad L \bullet atm/mol \bullet K$$

(Admittedly, those *are* strange-looking units, but they are the correct ones for this problem.)

So, to finish the problem:

You See	You Press
R?0.00	[•] [0] [8] [2] [0] [5] [3] [INPUT]
T?0.00	[3] [0] [0] [INPUT]
P=4.92 (atm)	

That's roughly equivalent to the pressure you'd feel by diving down to 150 feet underwater.

OK – that's *one* way to use this equation – solving for pressure. But now...

What If: You raised the temperature of that nitrogen you had in the previous problem – from 300 K to 500 K – but kept the pressure constant? How much *volume* would you then need for this gas?

Simple: Don't change anything you don't need to.

Press ■ SOLVE again (and as you can see, as long as you don't go moving around and browsing in the LIBRARY with the ▼ and ■ ▲ keys, you'll continue to be pointing to your last LIBRARY selection – whether or not it shows in the display).

Now, since you're going to need to calculate a Volume, choose the V selection...

...and confirm all the variables that aren't changing from the previous problem, and change the Temperature:

You See	You Press
P?4.92	INPUT
N?2.00	INPUT
R?0.08	INPUT
T?300.00	5 0 0 INPUT

V=16.67 (Liters)

Do you see how easy it is to play "What-If?" with equations on your HP-22S? All it needs to know is the equation and the name of the unknown variable; then it takes over, asking you all the right questions, and calculating the resulting answer.

Hmm... ...by now, something should be bothering you – some question or idea that needs to be cleared up....

"Oh, yeah – wasn't there something about the alphabetic registers and variables – and how they're connected?"

Yep (pages 134-139).

It's a fact: Whenever you use the [INPUT] key to put a value into any letter-variable of an equation from the LIBRARY, *that value is stored in the corresponding alphabetic register.*

Look: To see for yourself, use the ■[MEM] key and the VARS selection to review the current contents of any *existing* (i.e. *allocated* – with non-zero values) alphabetic registers:

You Press	You See	You Press	You See
■[MEM] VARS	A=10.00	▼	P=4.92
		▼	R=0.08
▼	B=-39.00	▼	T=500.00
▼	C=-4.00	▼	V=16.67
▼	J=-1.00	▼	X=-0.10
▼	N=2.00	▼	Z=26.00

The Z-value is left over from your first explorations of the alphabetic registers, but *all the others were stored in the process of using equations to solve problems* (the A, B, C, J and X came from the quadratic equation exercises; the rest appeared during the Ideal Gas problems).

Now you can see why the VARS selection on the ■[MEM] menu is named such – it's a quick way to review *all* the currently existing variable values!

So, what does this tell you about alternative ways to change or store alphabetic variable values?

Since those values are simply kept in the 26 alphabetic registers, you don't necessarily have to use the [EVAL] or ■[SOLVE] keys to store variable values; you can use [STO] and [RCL] directly!

Like So: Referring back to the previous problem, change the Temperature back to 300 K – without using the [EVAL] or ■[SOLVE] keys.

Solution: Press [3][0][0][STO][T]. That's all there is to it!

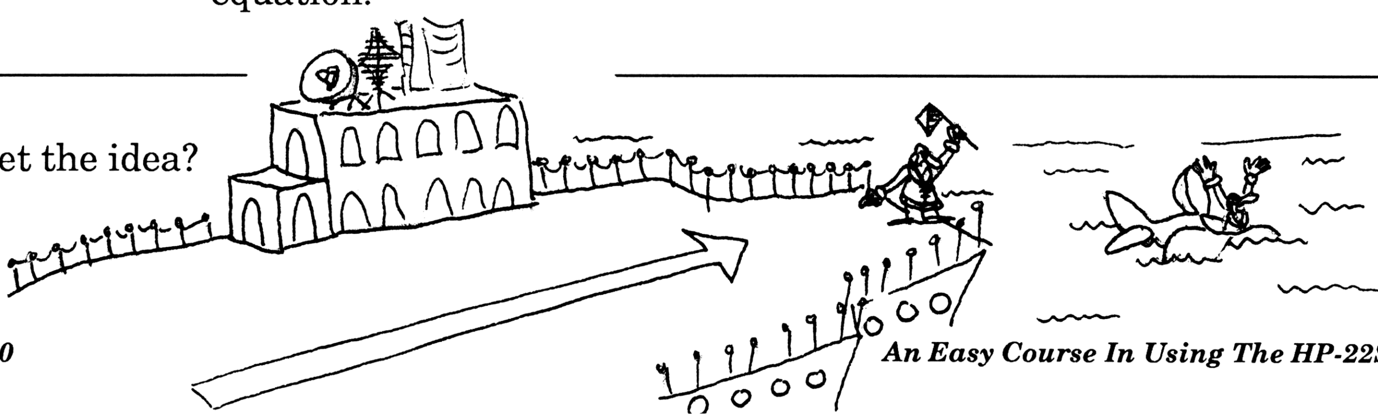
You have truly changed the current value of that variable – just as if you had done so by using [EVAL] or ■[SOLVE]. Thus, if you were to go back and use the Idea Gas equation with either of those keys now, when prompting you for the Temperature, T?, your HP-22S would show its current "suggested" value to be *this* value, 300.00 K!

Also: What if you want to review the current value of, say, the Volume, V?

No Sweat: Just press [RCL][V] (Result: V=16.67).

You have just recalled (a copy of) the value in the T-register – which will always be taken as the current value of any variable called T in any equation.

Get the idea?



A Quiet But Helpful LIBRARY Quiz: Memory, Variables, Registers And Built-In Equations

1. As you know, if you have just cleared the STAT registers, the memory they required will be returned ("re-melted" back into) the 371-byte User-Defined Memory "pot."

   Then, the next time you press [Σ+], the machine will instantly set aside 6 registers' worth (6 x 8 = 48 bytes) of memory for the six STAT registers.

   Also, through the Σ selection on the STAT menu, you can actually view the current values of those six STAT registers – look at them without changing them.

   Questions: Does some similar deallocation/allocation process happen when first you clear and then subsequently use the other special registers: XCOORD, YCOORD, RADIUS and ANGLE? How would you test this? And can you view the current values of these registers?

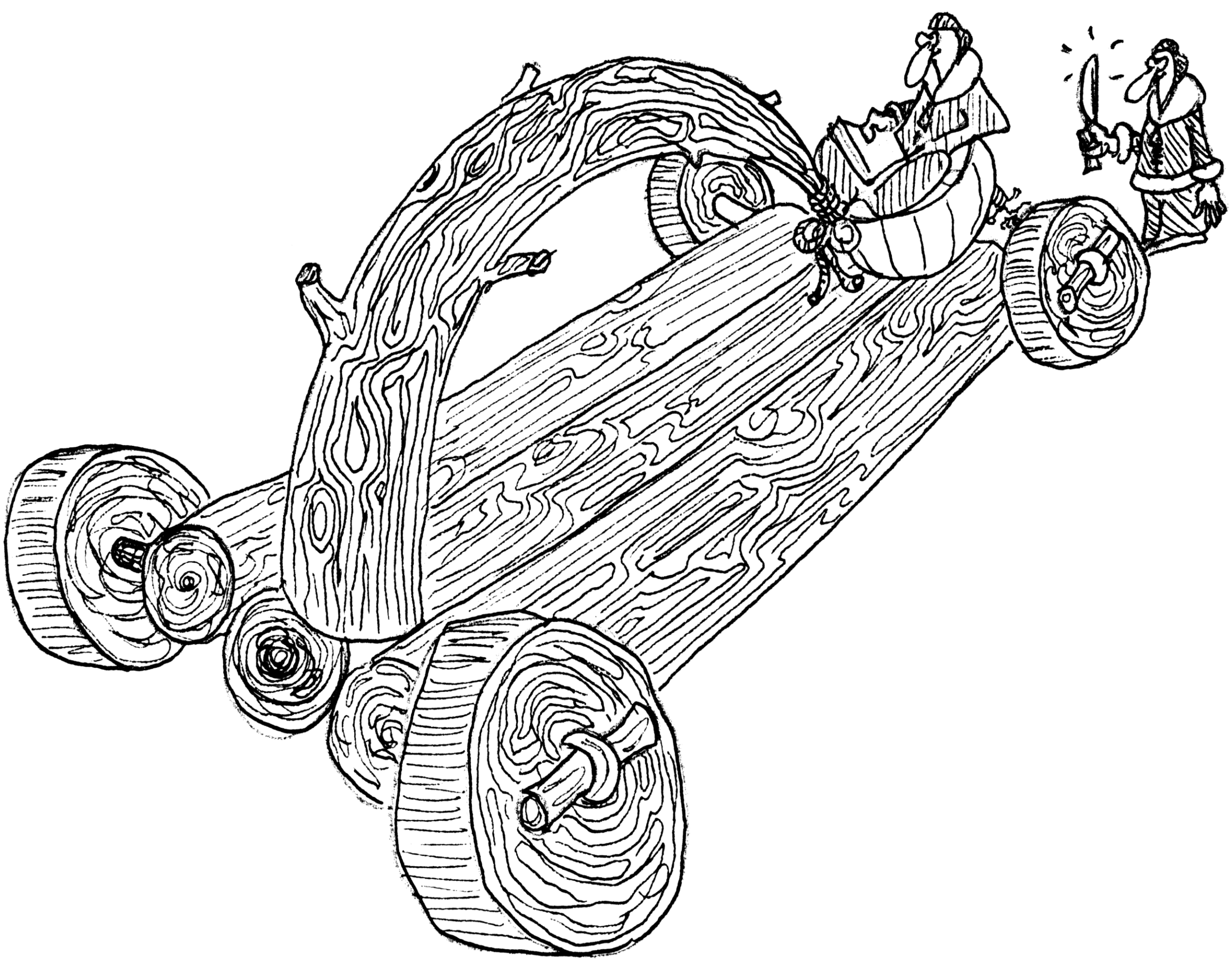
2. How would you answer the above questions for the one other special register – the LAST register?

3. To actually calculate an unknown by using any equation in the LIBRARY, when would you use the [EVAL] key and when would you use the ■[SOLVE] key?

4. Which equation is the "first" (i.e. "at the top of the list") in the LIBRARY?

5. If you were to start with a fully-cleared User-Defined Memory and then do one calculation with each of the 16 LIBRARY equations, can you tell how many free bytes of User-Defined Memory you'd have left? Can you tell how many of the 26 alphabetic variable registers would remain *un*-allocated?

Are these loaded questions?



Hushed But Courteous LIBRARY Answers

1. No. And yes, you can.

That is, the XCOORD, YCOORD, RADIUS and ANGLE registers are indeed special registers because they have their own names – and because they're not considered variables by the HP-22S (i.e. it *won't* clear them if you tell it to ■CLEAR VAR).

In fact, you *could* argue that they're even more special than the STAT registers because they are *always allocated* and they take up *none* of the 371 bytes of User-Defined Memory.

So, no – there's no allocation/deallocation going on. You could demonstrate this by clearing them (■CLEAR ALL Y), checking the User-Defined Memory (■MEM should show 371 BYT), then C to return to the Calculator Line. Now if you key some value into one of those registers (say, 3 ■XCOORD) and then check the memory again (■MEM), you'll *still* find 371 BYTes available.

And yes – you can check the contents of any of these four registers without changing them; you use the RCL key. For example, press RCL ■YCOORD to confirm that y=0.00 (you haven't given it a value since you cleared them all).

2. No – The LAST register is not part of User-Defined Memory, either (i.e. no allocation/deallocation) – and you could perform a ■CLEAR ALL test similar to the one described above if you were really in the mood.

And *of course* you can examine the contents of the LAST register. That's what the ■LAST key does – brings a copy of those contents to the Calculator Line.

3. Most equations you will encounter or store in your HP-22S will probably be of this form:

"One Variable" = "Everything Else"

That is, you'll have a single alphabetic variable (A or B or something) on the left side of the = sign and everything else on the right.

If this is the case and *if* your unknown happens to be that lonely variable on the left, then [EVAL] is the key you want, because it conveys all this information in a flash to your HP-22S, which therefore begins correctly to prompt you for the known values of all the variables that appear on the right of the = sign.

Otherwise (i.e. if your unknown appears on the *right* or if the left side is *not* an isolated variable), you should use the ■[SOLVE] key and then choose among the menu options you'll then be offered – to designate which variable is your unknown.

Of course, if you forget this sometimes, it's not the end of the world. You can always abort what you're doing and escape to the Calculator Line with the help of the [C] key, right?

4. There really is no "top" of the list, since you can wrap around endlessly through the list in either direction. However, if you want to get *technical,* you *might* be able to argue that the vector-length equation (R=SQRT(X^2+Y^2+Z^2)) is "first," since that's what you'll see if you press ■[LIBRARY] immediately after a total machine reset (don't do a reset; just take this little trivia fact on faith).

The point is: "It don't make no diff'rence."

5. Yes sirree, ma'am – these *are* loaded questions. Take a look:

As you know, each variable register requires eight bytes whenever it's allocated – that is, whenever it contains some value *other than zero.*

But the problem stipulates only that you use each formula once; it doesn't say what values you should use for each of the variables. It might well be that some values – either the knowns or the unknown – turn out to be zero, in which case those variables wouldn't cost you any bytes.

So the only entirely correct answer to the problem is: "There's not enough information given to solve it."

However, if you're like most mortals, when you first read the problem, you probably went ahead and tried to figure out how many of the 26 possible variables actually appear in the 16 built-in LIBRARY problems.

So, all right – you now have Official Permission to *assume* that none of your variable values is zero. How many variables would then be allocated in User-Defined Memory by your using each equation once?

Well, at last count, it looks as if only Q, U and W weren't invited to the variables party, so there would be 23 alphabetic variable registers allocated.

Plus, the STAT registers *would* be allocated, because there's one equation (R=SQRT(Σx²÷n)) that refers to those registers. So that would be six more registers to count.

So you'd have a grand total of 29 eight-byte registers allocated, leaving you with 371 - (29 x 8) = 139 unused bytes.

A Tip-Toe Through The LIBRARY

That's really all there is to the button-pushing mechanics of using equations on your HP-22S. The machine really does make it easy for you, doesn't it?

That being the case, there's really no point in testing you on each and every LIBRARY equation. Your HP manual gives you good examples for each equation, and besides, you may not ever learn about or use them all.

So what follows here on the next 30 pages is *optional.*

These *won't* be examples of each equation. Rather, you'll see some reminders about physical units and how to work with them, plus a review of the standard units and general ideas for each equation. And by the way – don't be put off by the huge-and-important-sounding title given to each of the equations. It's just another way to remind you of the limitations and definitions of each formula, and that's what this section is intended to do.

So feel free to browse in any order you wish among the topics between here and page 195. Just make sure that everybody "synchronizes watches" to meet together again on page 196 – where the real "free-flying" creativity starts.

OK?

A Units Reminder

"Units, units, blah, blah.... What's the big deal about units? Everybody knows how to convert between feet and miles, etc. And anyway, there's even a menu key (■ UNITS) on the HP-22S to do conversions, right?"

Sure, you can do a *few* conversions – things like kilograms (kg) to pounds (lb) and centimeters (cm) to inches (in).

But how do you convert

$$\frac{\text{l-atm}}{\text{mol-K}} \quad \text{to} \quad \frac{\text{Btu}}{\text{Mol-R}} \quad ?$$

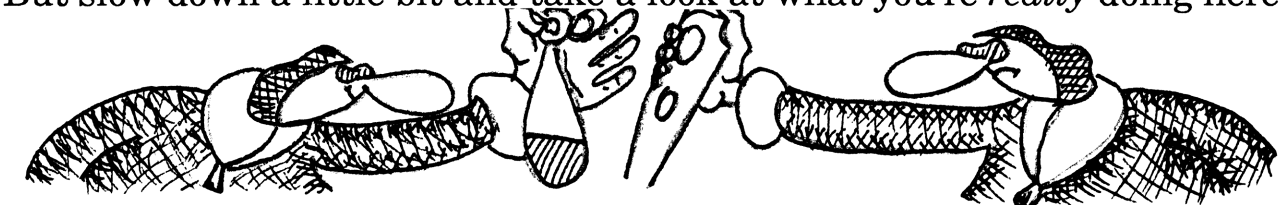
That's another story, eh?

Not really – it's just a longer version of the more familiar conversions you make. For example, think about how you convert units of time:

Suppose you're writing one of those torrid romance novels where the two star-crossed lovers are separated for some reason and so the "hours seem like years." Being a very *accurate* and expressive romance writer, naturally, you really want to *express,* say, a horrible three-hour separation *in terms of years.* How do you do this?

"Well, let's see....there are 24 hours a day and 365 days a year...so three hours is 3/24ths of a day, and a day is 1/365th of a year....Hmm...so I have 3/24ths of 1/365th, so I multiply 3/24 by 1/365... ...right?"

Right. But slow down a little bit and take a look at what you're *really* doing here:



The conversion *factor* between hours and days is "24 *hours per day,"* which you should write like this:

$$\frac{24 \text{ hours}}{1 \text{ day}}$$

You could also write this as "1 day per 24 hours:" $\frac{1 \text{ day}}{24 \text{ hours}}$

The point is, each of these fractions is really equal to 1, *because the numerator and the denominator are the same – even if they look different* (and when you divide any non-zero number, x, by itself, x/x, you get 1, right?).

So if you start with some given amount of time (those 3 hours, say) then to preserve the actual *amount of time* you're talking about, you should always multiply by a conversion factor where the numerator is *equal in time* to the denominator – *because then you're actually just multiplying by 1* (i.e. "x over x"):

$$3 \text{ hours} \quad \times \quad \frac{1 \text{ day}}{24 \text{ hours}} \quad = \quad \frac{3}{24} \quad \text{days}$$

The only tricky part to remember is that while you really *are* multiplying your starting amount by this conversion factor that's equal to 1, the *reason* it's 1 is because of the *units* you attach to both the numerator and denominator. After all, if you look only at the *numbers* above, it *looks* as if you're *changing* the amount of time you're talking about – multiplying 3 by 1/24 – which is certainly *not* the same as multiplying by one, is it?

You can't separate the numbers from their units; those units are part of the calculation!

You must <u>multiply</u> both the numbers and their units!

"Multiply units? How can you multiply units? They're just words that you write down next to the numbers, aren't they?"

No – they're actually *algebraic quantities, <u>represented</u> by words.*

This becomes a lot clearer if you let each unit be represented by a single letter – as you're probably more used to seeing them in algebra. If you do that, then your 3-hour, "sweet-sorrow" separation becomes this not-quite-so-romantic algebra problem:

> "I have an amount called 3h, and I want to know how much this is *in terms of y.*
> I know that 24h = 1d and I know that 365d = 1y."

Well, when you put it like *that,* it's probably clearer what you need to do, isn't it?

If $24h = 1d$, then $\frac{1d}{24h} = 1$.   Likewise, if $365d = 1y$, then $\frac{1y}{365d} = 1$.

So, *instead of* writing 3h, you can write 3h x 1 x 1 (which is just 3h again, obviously – so you've preserved its value). Then, because you know these fractions are each equal to 1, instead of writing those 1's, you can write this:

$$3h \times \frac{1d}{24h} \times \frac{1y}{365d}$$

Now can you see how the units multiply also? In fact, if you've set up the conversion correctly, all the "in-between" units (i.e. all the units except what you want to end up with) will cancel themselves out; you'll have *one occurence of each* in the numerator *and one* in the denominator. Watch the whole thing:

$$3h = 3h \times 1 \times 1 = 3h \times \frac{1d}{24h} \times \frac{1y}{365d} = \frac{3\cancel{h}\cancel{d}y}{(24)(365)\cancel{h}\cancel{d}} = \frac{3\ y}{(24)(365)}$$

Bingo! You end up with *only* the units you want - the "y's" (years)!

Try Another: Convert the *speed* of 60 mph to units of speed that are a little less common – say, "furlongs per fortnight."

No Problem: All you need to know are the conversions for each unit involved. That is, how many miles are in one furlong? And how many hours are in one fortnight?

Hmm...it's not just a one-step conversion, is it? Still, you can use the same units-multiplying rules repeatedly, as you *work toward the units you want:*

"There are 24 hours per day, and 7 seven days per week, and 2 weeks per fortnight. And there are 8 furlongs per mile." Fine. Now express these same equivalencies in fractions that are each equal to 1:

$$\frac{24 \text{ hours}}{1 \text{ day}} \qquad \frac{7 \text{ days}}{1 \text{ week}} \qquad \frac{2 \text{ weeks}}{1 \text{ fortnight}} \qquad \frac{8 \text{ furlongs}}{1 \text{ mile}}$$

OK, now start with your known quantity, 60 mph, and multiply it by all these 1's:

$$\frac{60 \cancel{\text{miles}}}{1 \cancel{\text{hour}}} \times \frac{24 \cancel{\text{hours}}}{1 \cancel{\text{day}}} \times \frac{7 \cancel{\text{days}}}{1 \cancel{\text{week}}} \times \frac{2 \cancel{\text{weeks}}}{1 \text{ fortnight}} \times \frac{8 \text{ furlongs}}{1 \cancel{\text{mile}}} = \frac{161{,}280 \text{ furlongs}}{\text{fortnight}}$$

Again, notice how you arrange the fractions so that their units all cancel out except the ones you're aiming for. If this *isn't* the case then you haven't arranged the fractions correctly (i.e. you need to invert – "flip over" –one or more of the fractions so that the units *do* cancel out correctly. This is perfectly "legal," of course, since both the numerator and denominator are the same amounts, right?).

Of course, even with this great, fool-proof method, you still need to get from one unit to another; you still had to *know* (or look up) that there are 8 furlongs per mile, right?

Well, both miles and furlongs are units of length. But there are several dozen other such units. And that's just for *length*. What about area, volume, pressure, energy, temperature, mass, time, charge, current, etc., etc.?

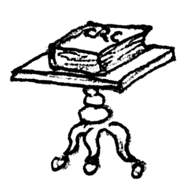
The point is, unless you can do a *lot* of memorizing, it's very inconvenient to try to work out problems when any given measurement could be expressed in so many different units.*

That's exactly why there are a few *standard systems* of units. In fact, the ■ UNITS key menu on your HP-22S offers a few conversions between two of the more common systems, the "English system" (with pounds, inches, gallons and °F) and the "metric system" (with kilograms, centimeters, liters and °C).

"And what does 'system' mean here? When you say 'a standard *system* of units,' what exactly are you saying?"

A *system* of units is a collection of *fundamental and coherent* units.

"Terrific. Now what do 'fundamental' and 'coherent' mean?"



(Thought you'd never ask....) →

*To get a better idea, look up the subjects of "units" and "conversions" in the nearest copy of the CRC Handbook of Chemistry and Physics, published by CRC Press, Inc., Cleveland, Ohio. The discussions you read here use data from the 56th edition (1975-76) of that incredible book.

Take the idea of "fundamental" first: In this universe, it seems as if there are very few quantities that must be *defined* in terms of measureable *observations.*

To define a unit of **length**, for example, you must find some "thing" in the physical universe to measure against your unit – some metal rod in a vault in France, the wavelength of a certain frequency of light, the king's forearm – *something.*

So **length** is indeed a fundamental *physical quantity;* to define it, you must agree upon some unique, measureable "thing" somewhere as your standard. But area is not a fundamental physical quantity – because *you can define it in terms of a fundamental quantity – length* (area is "length times length," or length^2). And volume is also non-fundamental, because it's simply length^3. Get the idea?

"Sure. So, how many truly fundamental quantities are there?"

Good question. The best answer is: "**It Depends.**"

For example, most of the time, you may be used to regarding length, time, and mass as completely separate ideas – separate and fundamental quantities. But Einstein (and lots of others) showed this is not the case. *It all depends on how you measure them.* A yardstick is not a yardstick if it's moving past you in a pickup truck; it's actually a little shorter than a yard and a little harder to move – more massive – to you (and yet it's fully one yard long to the driver of that truck)!

The point is, space (as represented by length) , time and mass are ultimately and intimately connected(!), it seems, *but the connection doesn't become apparent in our observations on an everyday scale.* So we *think* of them as separate and fundamental quantities simply because it's not convenient (nor usually even possible) to measure them according to their theoretical "connectedness."*

*Incidentally, the main thrust of most of modern physics is to try to establish such links between all the apparently separate and unrelated phenomona we see: Time, space, gravity, electrical and nuclear forces, etc. It ain't easy – but there's been some partial success. The idea is that someday, you could define just *one* "quantity" which is so Fundamental that from it you could define *all* the other quantities.

So you don't really mean Fundamental fundamental. You're asking how many "basically separate" quantities you need in a system of physical units in order to get along in today's everyday science. Well, there are six such *basic* quantities:

Length	**Electrical Current**
Mass	**Temperature**
Time	**Luminous Intensity**

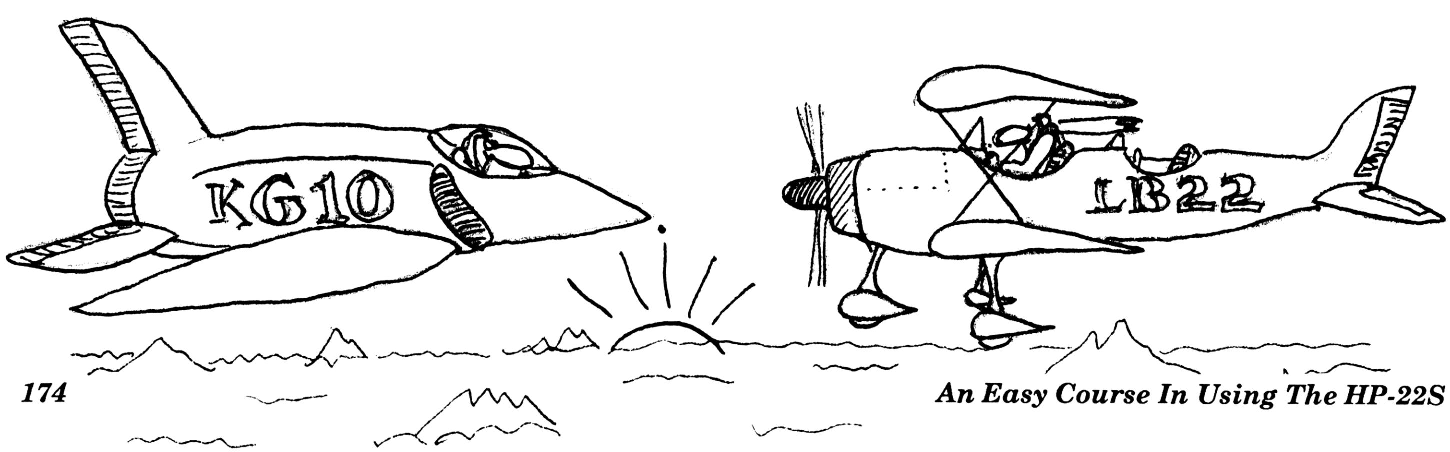
As you know, practically speaking, you must define **length** in terms of some physical reference observation– and the same is true for these others. For **mass,** somewhere there's an Official Bar of metal with a certain 1-kg Reference Mass; for **time**, a "second" is some precise multiple of the period of a certain radiation in a cesium clock; and **electrical current** (the Ampere) is measured as that flow of electrical charge which produces a certain force* between two parallel conductors that are each carrying this current.

Temperature and Luminous Intensity are good examples of other practical concessions. The theory is that Temperature is related to the energy of individual particles in a substance – but who wants to volunteer to try to put a stopwatch on all those atoms? So you have the idea of temperature to somehow indicate the average energy of a particle. And it's a similar case for Luminous Intensity, which is a way to measure the radiant energy of photons (light "waves" or particles") emitted from a surface: Nobody wants to try to count the individual photons, so you refer instead to the intensity of the luminous energy emitted.

*The quantity called force may seem to be a fundamental quantity, but it's not; you can define force as that which you must exert in order to *accelerate* a given mass – and since acceleration is defined in terms of length and time, clearly force is a quantity that's totally defined *in terms of* length mass and time – so it's not fundamental here. It may seem backwards, then, to use this non-fundamental quantity to help measure the truly fundamental quantity – flowing electrical charge – but that's simply because it's *easier* to measure a macroscopic force than it is to count each electrical charge as it passes by in a current. Incidentally, the units you just read about (meter, kilogram, second, and Ampere) belong to the most widely used unit system in the world today, called the International System (its common designation is SI, which is its French acronym).

Here are afew of the more common quantities, both basic and "compound" and their coherent, SI units:

Quantity	**Formula In Terms Of Basic Quantities**	**SI Units**
Length	Length (L)	meter (m)
Mass	Mass (M)	kilogram (kg)
Time	Time (t)	second (s)
Current	Current (I)	Ampere (Amp)
Temperature	Temperature (T)	Kelvin (K)
Luminous Intensity	Luminous Intensity (i)	candela (cd)
Area	L^2	m^2
Volume	L^3	m^3
Speed	L/t	m/s
Acceleration	L/t^2	m/s^2
Force	$M \cdot L/t^2$	$kg \cdot m/s^2$ = Newton (N)
Energy	$M \cdot L^2/t^2$	$kg \cdot m^2/s^2$ = Joule (J)
Power	$M \cdot L^2/t^3$	$kg \cdot m^2/s^3$ = Watt (W)
Density	M/L^3	kg/m^3
Pressure	$M/t^2 \cdot L$	$kg/s^2 \cdot m$ = Pascal (Pa)
Electrical Charge	$I \cdot t$	$Amp \cdot s$ = Coulomb
Electrical Potential	$M \cdot L^2/I \cdot t^3$	$kg \cdot m^2/Amp \cdot s^3$ = Volt



But that's not the whole story. Notice that what makes this system so convenient is that it is *coherent*. That means that when you combine simple, whole amounts of the *basic* units, you usually get simple, whole amounts of the *compound* units.

To measure electrical charge, for example, you have an SI unit called a Coulomb. And what is *1* Coulomb? Simple: It's the amount of charge flowing in *1* second past any point in a conductor carrying *1* Ampere of current.

This is *much* more convenient than having to remember some *irregular* number, like 1 Coulomb = 2.34067 Amp-seconds, or something equally arbitrary.

So when your scientific calculations involve a lot of different units, try to express everything in terms of one system's units. Then, when you're finished, you'll be more easily able to simplify those units as far as possible (using the cancellation rules you saw back on pages 167-170) into units you can understand.

Like This: Confirm that you can actually calculate pressure change due to water depth by using the formula $p = dgh$, where p is the pressure change, d is the density of water, g is the acceleration of gravity, and h is the height (or depth) of the water.

Solution: From that SI units table on the previous page, you can see that "density x acceleration x length"

$$= \frac{kg}{m^3} \times \frac{m}{s^2} \times m = \frac{kg}{m{\cdot}s^2}$$

Are those *really* units of pressure? What *are* units of pressure?

$$\text{pressure} = \frac{N}{m^2} = \frac{kg{\cdot}m/s^2}{m^2} = \frac{kg}{m{\cdot}s^2} \qquad \text{Sure enough!}$$

Some Equation Reminders And Cautions

So that's a quick review of units and unit conversion – something you should always bear in mind when you're working with the built-in LIBRARY equations (or any equations, for that matter) in your HP-22S.

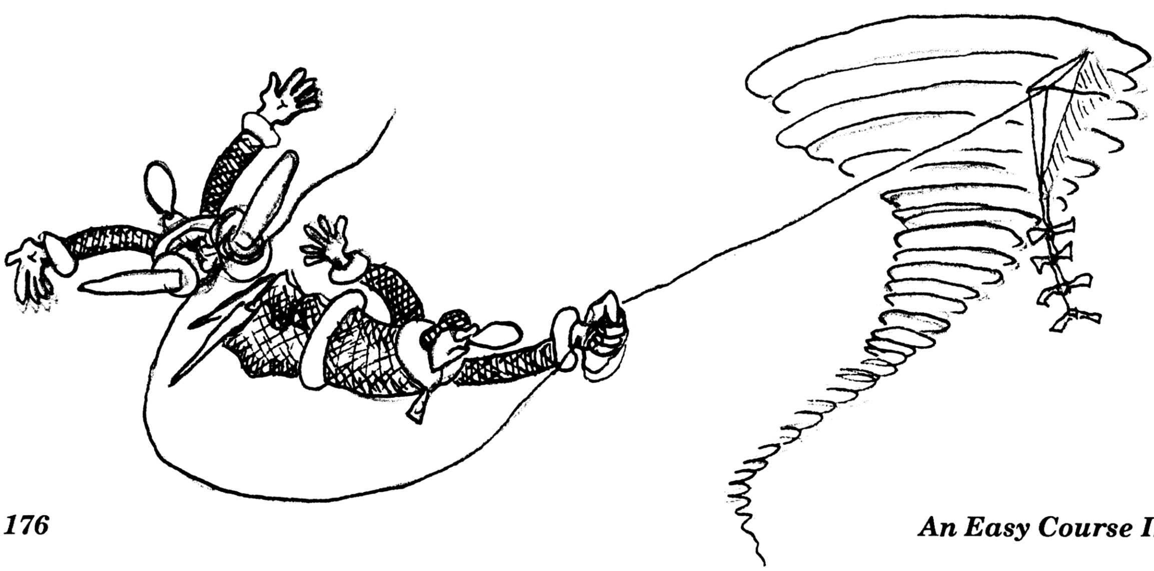
Remember: *These equations will indeed be reliable and convenient, but only if you use consistent units – and only if you use the equations when they truly apply.*

You still need to do the real *thinking* when solving problems with these equations. The only thing your calculator knows is how to crunch the numbers you give it; it *cannot* tell whether those numbers or units are correct and applicable. The very best equation in all the world is useless if you supply incorrect values – and it's *worse* than useless if you mis-apply it.

So the following pages give you some reminders about the units and the general ideas behind each of the LIBRARY equations. As a matter of fact, you should probably develop the habit of doing this kind of review/reminder mentally *whenever* you use any kind of formula:

"When does it apply?"

"What forms and units do the equation's assumptions require?"



The Vector Magnitude Equation:

```
R=SQRT(X^2+Y^2+Z^2)
```

This formula is fairly straightforward, isn't it? This is the general formula for the length of a hypotenuse (the "diagonal") of a *rectangular vector.* A *vector* is any quantity or state of an object that can be described by a *collection of coordinates.*

Caution #1: Not all vectors are rectangular. For example, a ship's position on the ocean is usually expressed in latitude and longitude, a good example of a *two-dimensional* vector (and it could be three-dimensional if you also gave the ship's altitude – but this isn't necessary, since the ocean is usually at sea level, except in rare emergencies). So the longitude and latitude coordinates tell you how to go in circular ("angular") paths – not straight ("rectangular") lines – but the above equation is for *rectangular vectors only* (and up to a maximum of three coordinates). *So make sure your vector is in rectangular coordinates before using this equation.*

Caution #2: To use the above equation, you must also be sure that the coordinates within your (rectangular) vector all have the same *units.* For example, if you live at 1234 N.E. 19th street, 11th floor, then this tells the postperson to start at the "center" of the city (i.e. the origin, where the street numbers begin at zero), then go 19 blocks out to 19th street, then about 12.34 blocks along 19th street to your building, then (if you're lucky) climb up to the 11th floor.

So you *could* describe your postal address as (19, 12.34, 11) – and that would indeed be a three-dimensional rectangular vector. But if the post office wanted to use this address vector and an HP-22S to compute the delivery distance for their new ATCF (As The Crow Flies) intra-city carrier-crow service, they'd be in big trouble, because two of the coordinates are expressed in units of city blocks (whatever *those* are), but the third coordinate is in terms of floors or stories on a building. They'd have to convert stories to blocks; *then* they could figure how far a crow would need to fly to deliver your mail from their office to your window.

The Quadratic Equation Solutions:

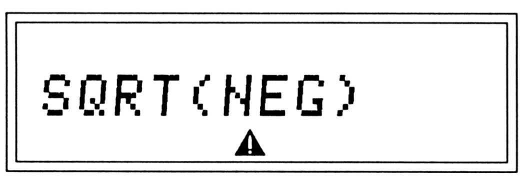
X=(-B+JxSQRT(B^2-4xAxC))÷2÷A

You've already seen a pretty good discussion of this one (back on pages 149-154), so without going into all the excruciating details again, here are a few basic reminders:

Caution #1: You *must* have the equation in the form $AX^2 + BX + C = 0$ in order to correctly read off and use the coefficients, A, B and C, to solve for X. You may *not* have any value except zero on the other side of the equals sign. The values of A, B and C may be any real numbers, of course, except that A cannot be zero (for then you would be asking the calculator to divide by zero).

Caution #2: There are usually *two* real solutions for X (one for when J is +1, one for when it's -1, as you read back on page 151). Don't make the mistake of assuming that you've seen the only answer; try the formula again with the other value of J (if it gives you the same result, this is a special case where both solutions are the same – i.e. where the parabola touches the x-axis at just one point – see page 150).

Caution #3: Sometimes there are *no* real solutions (also pictured on page 150). This happens when the portion of the formula under the square-root radical turns out to be a negative number (that is, when B^2 is less than 4AC). If so, your HP-22S will tell you with an error message:



This tells you that it can't find a *real* answer because you've asked it to find the square root of a negative number. So you'd need to work out the answer yourself, using whatever you know about complex numbers – which is more than the HP-22S knows.

The Classical, One-Dimensional Equation Of Position Of A Body Under Constant Acceleration:

X=S+V×T+.5×A×T^2

You may recognize this better when it's written like this:

$$x = x_0 + v_0 t + at^2/2$$

This is the equation that will give you the final position, X, in any given direction of any object that has been under acceleration in that direction for some time, T, after having started at position, S (x_0), and at some initial velocity, V (v_0), in that direction..

It tells what happens, for instance, when you drop a rock off a cliff (but the cliff has to be on the moon or somewhere, because you must neglect air resistance and any other friction or effect that tends to retard motion and/or cause the acceleration – that's the local gravitational acceleration – to be non-constant). It's called *classical* because it hearkens back to Sir Isaac Newton, who could not yet know all the weird things that happen as the velocity, V, of an object approaches **c**, the speed of light.

So, **Caution #1:** Know the formula's practical limits. Don't use it when friction is too high or when acceleration is not constant enough for your purposes – and don't use it when V is more than 10-25% of **c**, the speed of light.

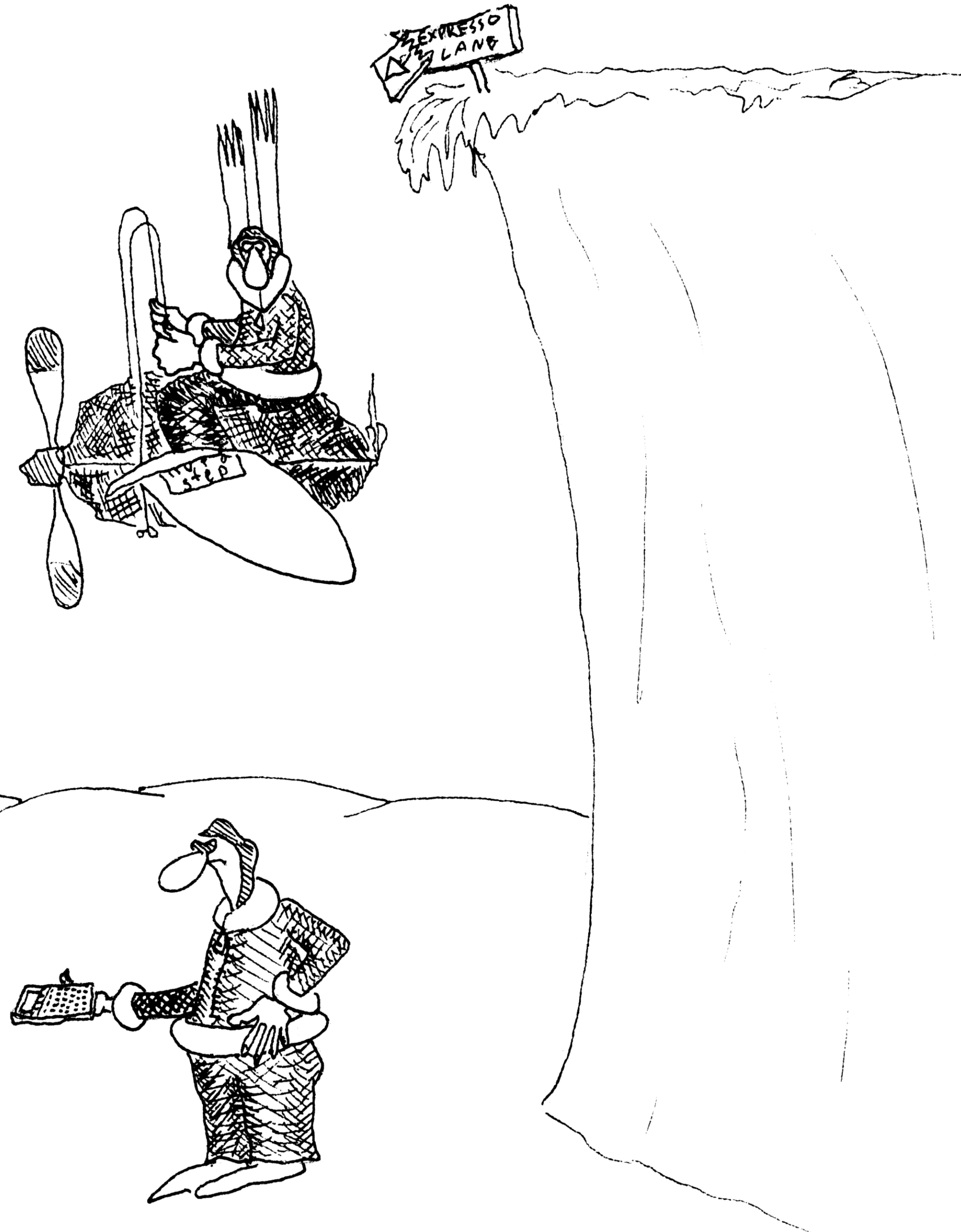
And, **Caution #2:** Keep your units straight and consistent! If your positions (S and X) are expressed in furlongs and the time (T) in fortnights, you'd better not give an acceleration (A) in units of "inches per square year."

Remember: In SI units (which are *always* recommended), lengths are in meters (m); velocities are in meters per second (m/s), time is in seconds (s), and accelerations are in meters per square second (m/s^2).

The Classical One-Dimensional Equation Of Velocity Of A Body Under Constant Acceleration:

$$F=V+A\times T$$

This is another part of the equation on the previous page. This one, however, gives you F, the final *velocity* (instead of X, the final *position*) of that falling rock at any time, T, under the same set of assumptions, cautions and units you just saw.



The Classical Equation Of Kinetic Energy Of A Body:

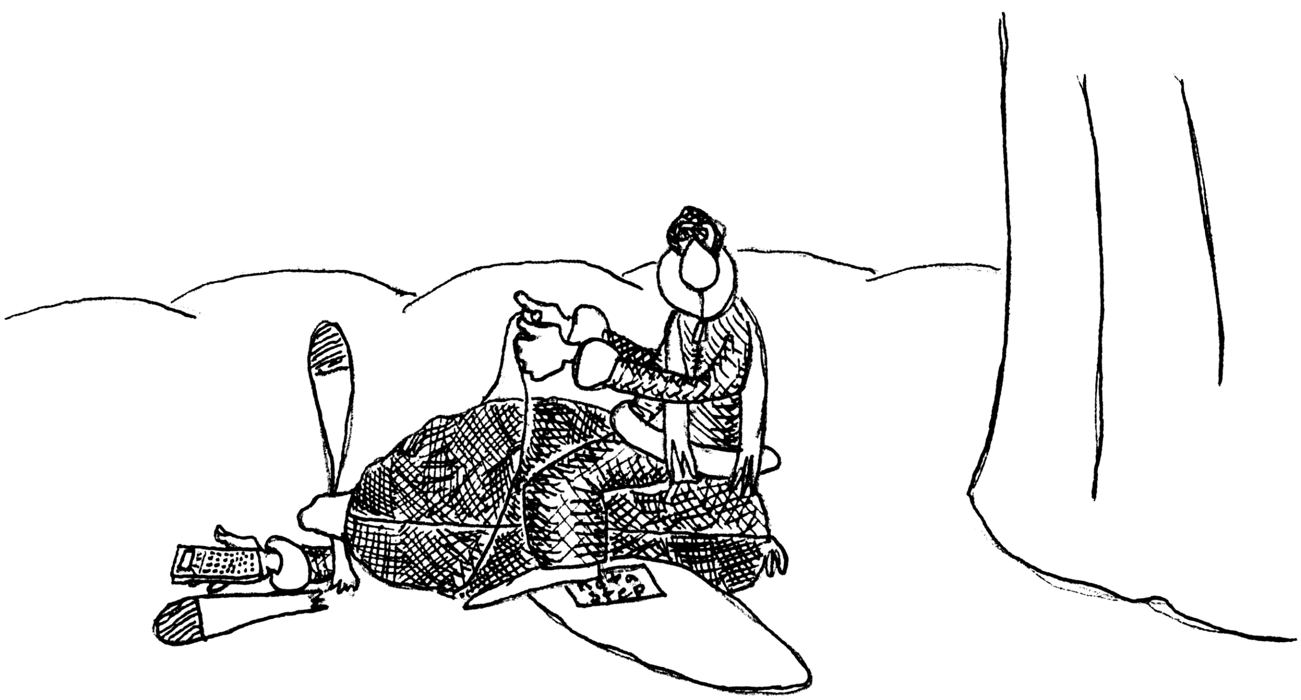
E=.5×M×V^2

You may recognize this better when it's written as: $E = mv^2/2$

Caution #1: Remember that the velocity, V, is the *total* velocity (i.e. you need to combine component velocities in different coordinate directions, if that is how they are expressed.

Caution #2: The energy here, E, is only the *kinetic* energy due to the *translational* velocity, V. There could be other kinds of kinetic energy, due to the object's rotation or vibration. And of course, there are plenty of other forms of energy (electrical or gravitational potential, thermal, chemical, etc.), about which this formula says nothing at all.

Caution #3: Use SI units or some other coherent system! If the mass, M, is in kg and the velocity, V is in m/s, then the energy, E, will be in J (Joules).



The Force-Distance Equation
For Gravitational And Electrical Forces:

F=KxAxB÷R^2

Actually, this equation is good for any situation where you have *two objects that repel or attract each other in inverse proportion to the square of the distance, R, between them.* In nature, the two primary phenomena that behave like this are gravity and electrical attraction/repulsion; hence the name.

Now then, what are those quantities, K, A and B? Well, that depends on what the equation is describing.

If you're describing gravitational attraction, then A and B are the *gravitational masses* of the the two objects, and K is the proportionality constant that describes how much they attract each other.

If you're describing electrical attraction, then A and B are the *electrical charges* of the the two objects, and K is then the proportionality constant that describes how much they attract each other (in the case of opposite charges) or repel each other (in the case of like charges).

In general, then, if you have a situation where the intrinsic properties of two bodies cause them to attract or repel each other in *exactly* this fashion (proportional to the degree of the intrinsic property present in each body and *inversely* proportional to the *square* of the distance between them), then you may use this equation.

Caution: Be very careful of your units! The SI units for graviational or electrical interactions, for example, are Newtons for the force (F), meters for the distance (R), kilograms for the masses (A and B) *or* coulombs for the charges (A and B), and then the proportionality constant, K, would be *either* $N{\cdot}m^2/kg^2$ *or* $N{\cdot}m^2/coulomb^2$.

The Instantaneous Power Equation For Simple Electrical Resistances:

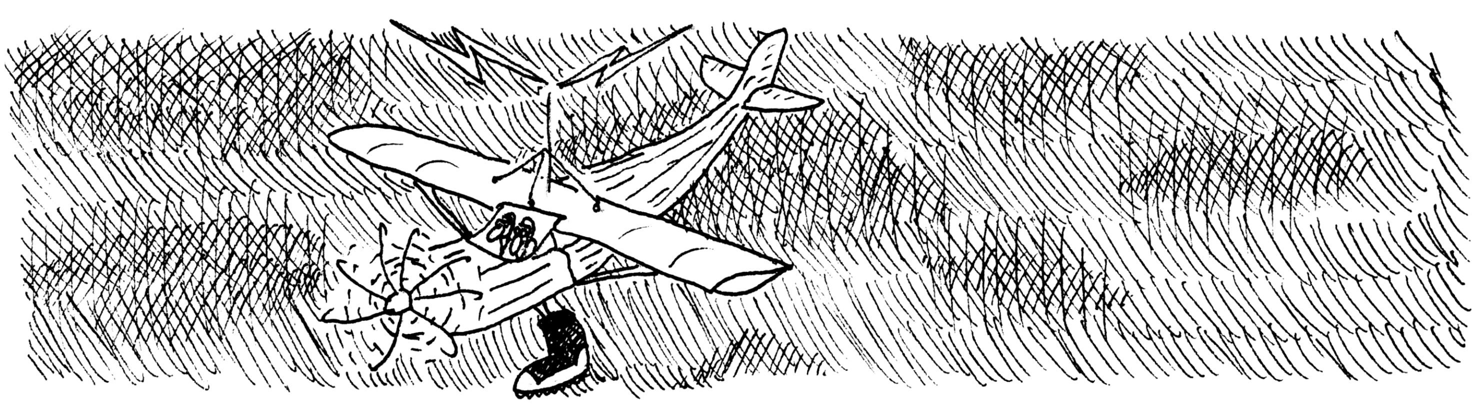
$$P=I^2\times R$$

This equation gives the instantaneous (at some time, t) rate of use (or dissipation) of electrical energy in a simple electrical load or resistance.

The SI units are easy to remember (and they are used almost universally): P is the power in Watts; I is the current in Amperes; R is the resistance in *Ohms* (1 Ohm = 1 Volt/Ampere).

Caution #1: The above equation says nothing about DC (Direct Current) versus AC (Alternating Current) – or whether the current is at a steady state or not. It may very well be that the current is still increasing during that short time after you flip the switch to close the circuit. That's why this power is called the *instantaneous* power (at some instant) – not the average or effective power, which is often used when referring to steady-state sinusoidal AC circuits, for example.

Caution #2: Remember also that in the typical AC circuit there are usually elements that are not *resistors* (such as capacitors and inductors) but which also serve to impede a time-varying current. In such cases, there are several different names and definitions of power that are usually of interest (beasts such as the total power, average power, peak-to-peak power, real power, effective power, and reactive power).



The "Ideal Gas" Law:

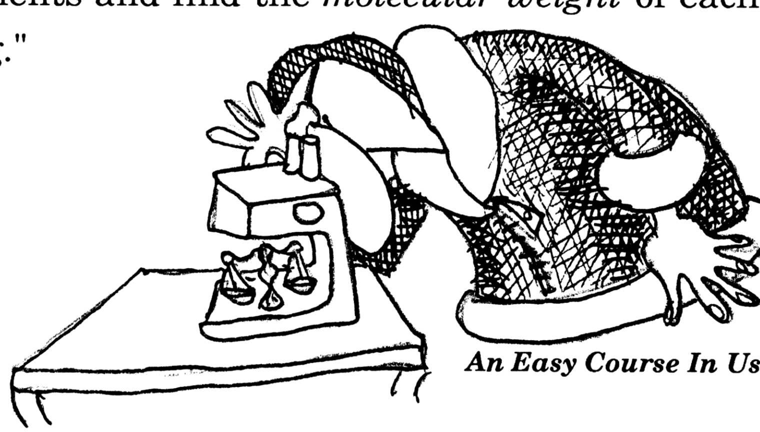
$$P \times V = N \times R \times T$$

The ideal gas law basically says that if it weren't for the messy, untidy aspects of reality, the particles of a gas – any gas– would behave like infinitesimally small billiard balls (the balls themselves would in fact have zero volume), just jiggling randomly around, not attracting each other but bouncing elastically off of each other whenever a chance collision occurred.

Now, mind you, no gas really behaves this way, but this is a *starting point* for being able to predict the quantitative relationships between the amount (N – the number of moles), the pressure (P), the volume (V), and the temperature (T) of a gas. Physical chemists then introduce all sorts of terms to *correct* this equation so that it more nearly describes real gas(es). (Of course, some gases are already a lot more "ideal" than others, and it's interesting to note that all gases do indeed approach "ideality" as the products of their pressures and their volumes approach zero: $P \times V \longrightarrow 0$).

Caution #1: The ideal gas law says nothing about what *kind* of billiard balls you have – only *how many* you have. Thus, the number N is a measure of the gas in *moles,* which is a measure of the *number of particles* (molecules) of the substance. One mole of anything is about 6.02×10^{23} particles (quite a lot, really).

This means that one mole of gas A may have a weight *totally different* than a mole of gas B – but you're not comparing them by weight but by their numbers of particles. To convert from a substance's weight to its mole measure, you need to look up in the Periodic Table of the elements and find the *molecular weight* of each atom in the molecule you're "weighing."



For example, the molecular weight ("grams per mole") of water vapor is about 18.0152, because water is made up of two hydrogen atoms (1.0079 grams per mole) and one Oxygen atom (15.9994 grams per mole).

So if you know you have 100 grams of water vapor, you can convert this to moles by dividing 100 *grams* by 18.0152 *grams per mole* to get 5.55 *moles* of water vapor (check the units of this conversion by using the multiply-and-cancel process that you saw back on pages 167-170, if you want to prove this to yourself).

Caution #2: Units! The constant, R, is (for some strange reason) called the ideal gas constant. Its units are totally dependent on the units you choose for the other values. It's common to find R expressed in terms of

- Liters, atmospheres, moles and Kelvins
- Liters, Pascals, moles and Kelvins
- Joules, moles and Kelvins

...and many others.

The point is, you need to use the value for R that *contains the same units for pressure, temperature and volume as you are using.* For example, you *cannot* use

$$R = 0.08206 \text{ Liter}\cdot\text{atm/mol}\cdot\text{K}$$

if you have expressed your pressure in Pascals and your volume in cubic meters.

The Equation Of Change In The Gibb's Free Energy Of A System During An Isothermal Process:

$$G=H-T\times S$$

That's quite a mouthful for this simple little formula, no? Well, it's all necessary to remind you what this equation is doing for you. In a nutshell, the change in the Gibb's Free Energy of a system is a measure of the maximum useful work you can get out of the system during a process – "useful" meaning "besides the work associated with expanding the system ."

Notice that you're always talking about the *change* in G and about a *process*. That's what this equation is all about: To be more accurate, the formulas should really use the Δ symbol to indicate "the change in" G, H and S:

$$\Delta G=\Delta H-T\Delta S$$

Of course, H and S represent the *enthalpy* and *entropy* of the system (but you're not going to see a review of what they mean physically, since there are entire courses and textbooks devoted to such arguments. Suffice it to say that H basically represents the heat of formation of a substance, while S is some measure of its "orderliness").

Caution #1: This equation is valid only if the process is *isothermal* (at constant temperature, T). If the temperature varies during the process, all bets are off.

Caution #2: G, H and S are usually expressed in *molar* form (i.e. so many "somethings" *per mole*). In SI molar units, then, G and H are J/mol, while T is in Kelvins and S is in J/mol·K.

The Equation Of Pressure
In A Non-Compressible Fluid:

$$P=I+D\times G\times H$$

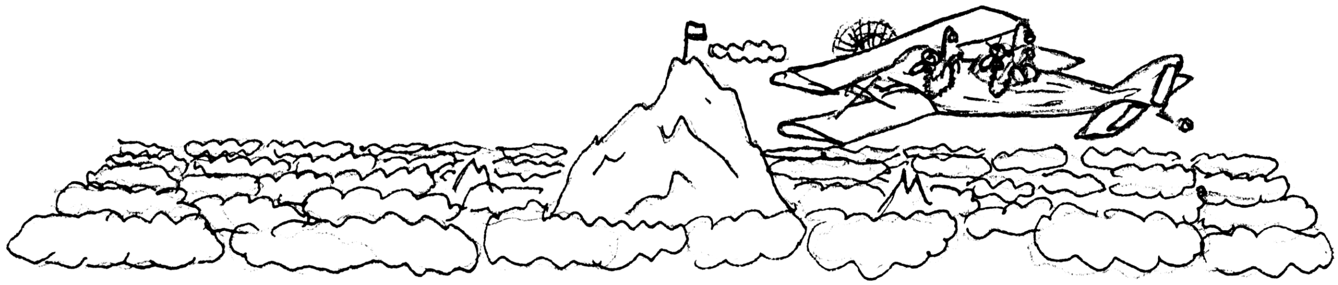
You've already seen this formula and verified its units (back on page 175), but here it's been more generalized so that you can also include the "starting pressure," I, i.e. the pressure at the surface or "top" of the fluid in question.

The one caution is this:

Caution: As the above heading suggests, this formula is accurate only when the fluid is more or less *non-compressible.* That is, it must be such that it does not "squash" under its own weight – where molecules at greater depths become much closer to one another. If a fluid does significantly compress, then of course, this means that the density would vary with depth, and so you'd have varying "weight" of fluid according to depth. And since a fluid's pressure is caused by its own weight, this would bollix up the pressure calculation, as well.

Water is probably the most common example of a relatively non-compressible fluid; the above formula works quite well (just remember that seawater has a greater density than freshwater).

By contrast, most gases are quite compressible. Air, for example, is so compressible that its own weight causes its density at sea level to be nearly three times that at an altitude of 10 km (≈33,000 feet). That's why it's so hard to breathe up there: there's just a lot less air per cubic meter!



The Equation For Exponential Growth Or Decay:

-K×T=LN(N÷I)

This may look a little strange in this form (which happens to be the most efficient one for the purposes of your HP-22S), so look at it like this (with a little algebra, you can change this into the above form):

N=I×EXP(-K×T)   (in other words, $N = Ie^{-KT}$)

N is the amount of something you have now; I is what you started with when the time, T, was zero (T is, of course, time, which starts at zero, at the beginning of your analysis and should always increase thereafter – never go negative). K is the all-important rate constant: If K is more than zero, this means that N is less than I (i.e. your amount is *shrinking* exponentially). But if K is less than zero, then N is greater than I (your amount is growing).

Of course, the most common application of this form is in the radioactive decay of a substance. In that case, as time goes by, the amount (# of particles) you have of the stuff will *decrease*. But just bear in mind that by making K a negative number, you in turn make the exponent positive, causing an increase in the amount (a bank balance under continuous compounding is a good example of such positive exponential growth).

Caution: Notice the units implied here. Since N and I have the same units (they're both amounts of the same "stuff" – radioactive carbon, money, whatever), the exponent (power) of the exponentiation is just a simple number ("unitless"). Therefore the units of K must cancel out the units of T, making that side of the equation also a unitless quantity. So K must be the *inverse* units of T ("inverse years," "inverse seconds" – whatever time units you're using).

The Equation For Image Location In Thin Lenses:

$$O \times F + I \times F = O \times I$$

This equation is a good rule of thumb to help you locate the image through a thin lens.

And what's a *thin* lens? It's a lens where the thickness of the actual glass is "much thinner" (1% or less is probably a good rule of thumb) than the distance, O, from the lens to the object.

Caution #1: To correctly use this equation, be sure that the lens is "thin" enough, and that it's not actually a multiple or compound lens (made out of two different materials having different indices of refraction – in which case all bets are off).

Caution #2: The units of O, F and I are all in length (meters, inches, leagues, whatever). Just be sure they match.

Caution #3: Be sure that you observe the sign conventions built into the formula. O is defined as the distance between the object and the lens, and this is a *positive* quantity for a real object.

By contrast, F, the focal length of the lens, can often be *either* positive or negative. It's *positive* if the lens is *convex* (where parallel light *converges to* a focal point on the *side opposite* the object); it's *negative* if the lens is convex (where parallel light *diverges from* the focal point on the same side as the object).

And I, the distance from the image to the lens, has a sign convention similar to the focal length: I is *positive* if it appears on the side *opposite the object,* negative if it appears on the same side.

The Equation For Predicting Angles Of Constructive Interference From A Uniform Diffraction Grating:

A=ASIN(M×L÷D)

Just in case you ever want to be able to predict the angles at which all those light/dark fringe patterns show up on the neighbors' garage when you shine your homemade laser through your bedroom window screen at midnight, this equation might be just what you need (it also comes in handy in optics class, but it's not nearly as much fun there).

Caution #1: This works well only when your light source is more or less monochromatic (one wavelength).

Caution #2: Be sure to set the mode in which you want your resulting angle, A, to be displayed. Most mortals prefer degrees to radians, but it's totally up to you. Also, note that this angle is measured as the *deflection* from the path of the same beam unobstructed.

Caution #3: Both L and D are lengths and must be expressed in the same units. Problem is, that garage is several meters away, but your light's wavelength is just several ten-thousandths of a millimeter. Something's gotta give – namely, you. You gotta give your HP-22S *both* lengths in meters (or in ten thousandths of millimeters, or rods or blocks or whatever).

Caution #4: M is simply the *numbering* of the bright (maximum positive) interference fringes you see – numbered from the center outward, and getting dimmer as you go; it's a lot easier to see the first fringe (M=1) than the 10th (M=10).

The Exponential Equation For Growth Or Decay With An Expected Final Value:

Y=F+(I-F)×EXP(K×T)

This is the more generalized form of the equation you saw on page 188. In that case, you simply started your clock at some point and designated the time, T, at that point to be zero. And you started with an initial amount, I, of some ("stuff") and proceeded until you stopped the clock and checked to see how much "stuff" you had then.

Well, it's really the same here – only different:

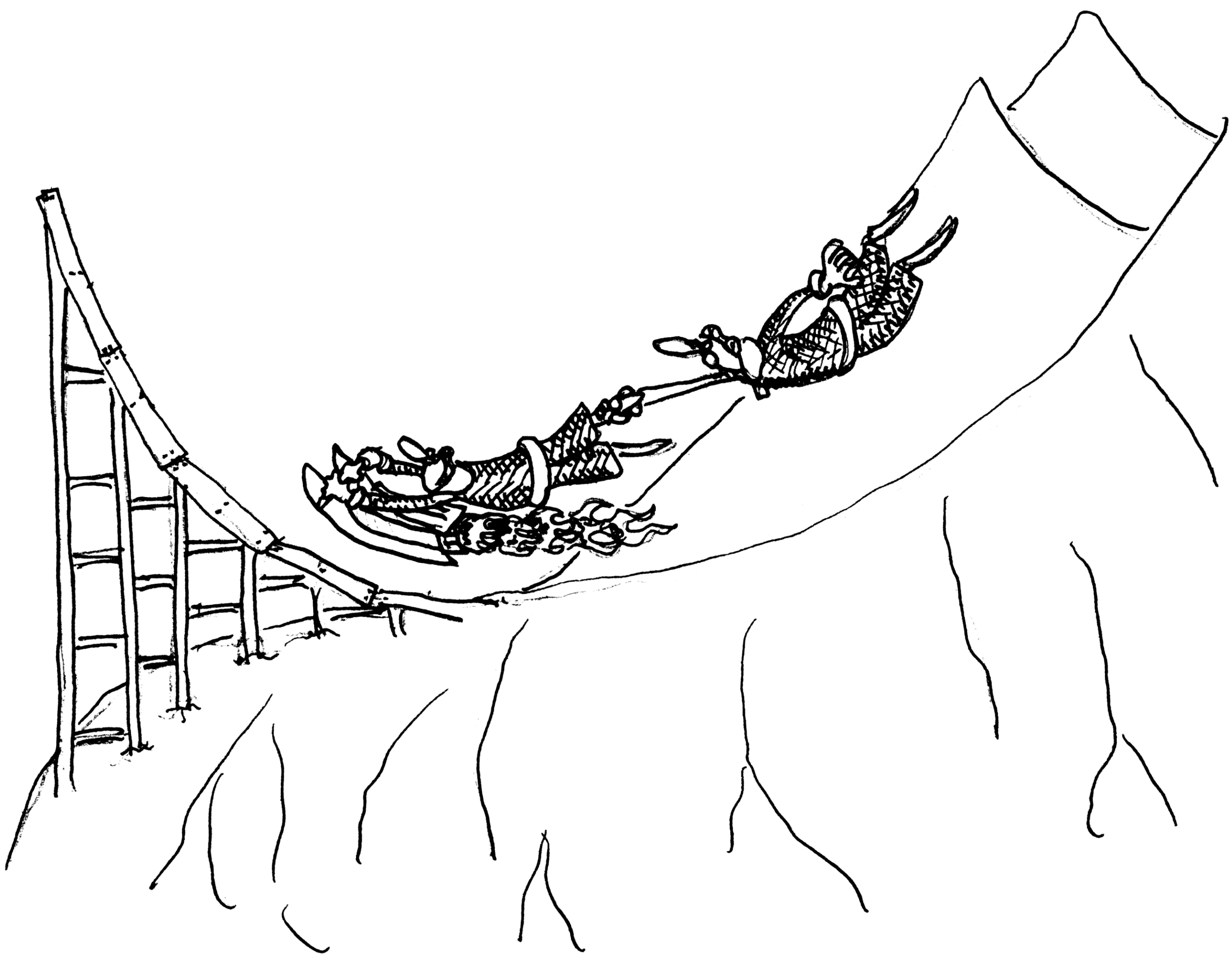
Caution #1: You can let T be *any* time – either in the future, when T is a positive number – or the past, when T is negative. (Actually you could do this in the other equation, too, and it would also signify the amount of stuff you had in the past – but you'd generally use this equation if you're interested in the past).

Caution #2: You're assuming something about this "stuff" of yours. You're assuming that either in the distant future (when T is a very large *positive* number) or the distant past (when T is a very large *negative* number), *the amount eventually reaches some limitingvalue,* F. You may not know *what* F is (i.e. it could be the unknown you choose to solve for by selecting F after pressing the ■ SOLVE key), but you're assuming that there *is* such a limiting value.*

*By way of comparison, incidentally, notice that in the other exponential equation, you were simply assuming that F was always equal to zero – so you ignored it completely; if you let F be zero in this equation here, you have basically the same equation as the other one.

Caution #3: There's no minus sign on the growth constant, K. The minus sign was on the other K because that equation is more often used for *decay* – so they built the sign into the equation, so that when you key in a positive K (more convenient then pressing [+/–] every time), it's still taken to mean *negative growth* (i.e. decay). But here, you have to say what you mean – and be sure to pay attention to what sign you give T, also.

Caution #4: As always, watch your units! K is in units of "inverse time," just as in the other equation. And F, Y and I are all in the same units of "stuff," right?



The Equation For The Root Mean Square (RMS) Of A Single-Variable Set Of Data:

R=SQRT(Σx²÷n)

Many fields of science and statistics use the Root Mean Square (RMS) calculation, which is "the square-root of the average square," if you want to think about it like that. It's useful for analyzing the *magnitudes* (not the *signs*) of data; by squaring each datum, you erase all evidence of whether it was positive or negative.

"Well," you might say, "couldn't I do that simply by taking the absolute value of each datum – then take the mean of that sum?" Yes – but finding the mean of the magnitudes of the data is *not* the Root Mean Square's only job. If it were, then this would be its equation:

R=(ΣSQRT(x²))÷n

As you can see, here you'd be taking the average of the data's magnitudes, whereas the RMS takes the average of their *squares* and *then* takes a final square root. What this does is give *less weight to extreme values,* since $\sum(x^2) < (\sum|x|)^2$. So RMS's other function is to smooth the data a bit, by reducing the effect of a few "way-out" values.

Caution #1: In using this formula, notice that the only value you can solve for is R, because the other values in the equation are taken from the HP-22S's STATistical registers, and the machine will not solve for those quantities; it assumes they are "knowns." Therefore, both the [EVAL] and ■[SOLVE] keys will do the same thing – solve immediately for R – so you'd better have your statistics all keyed into the STAT registers first.

Caution #2: This formula solves only for the RMS of x-data; it won't tell you anything about any y-data you may also have keyed in. Notice that the above heading reminds you: this is a formula for *single-variable* data.

The "Time Value Of Money" Equation, Relating Beginning And Ending Balances, Periodically Compounding Interest, And Uniform Periodic Payments:

(P×100÷I-F)×(1+I÷100)^-N-P×100÷I=B

From the looks of its name and its length, this must be the most complicated formula of them all, eh?

Well...now that you mention it, that could very well be true. This formula contains some of the most subtle math and is probably the least understood of any of the formulas in the LIBRARY.

This formula is tied to the idea of interest earned on money loaned. It doesn't matter who the lender is (if you borrow money to buy a house, then the bank is the lender; if you open a savings account, then *you* are the lender).

The assumption this formula makes is that a certain beginning amount (B) is loaned for a certain number (N) of periods of time. And over each period of time, interest (I) accrues *on the balance owed at the start of that period.*

The formula also assumes that there is one payment (P) per period – at the *end* of the period. In other words, the interest accrues all period long, and then the payment goes toward paying that interest plus some of the principal, thus establishing a new starting balance for the next period. And so on. This is the way most mortgages and car loans work.

And then at the end of the last period – right after the last payment is made – there could be a final remaining balance (F) owed.

Caution #1: To use this formula accurately, you need to *pick the perspective of either the lender or the borrower* – and stick with it. The reason for this is that the signs of your dollar amounts (B, P and F) indicate which ways the loan and repayments are going – *to* you or *from* you. A positive sign on any dollar amount says that the money is coming *to* you; a negative sign says that it's being paid out *from* you.

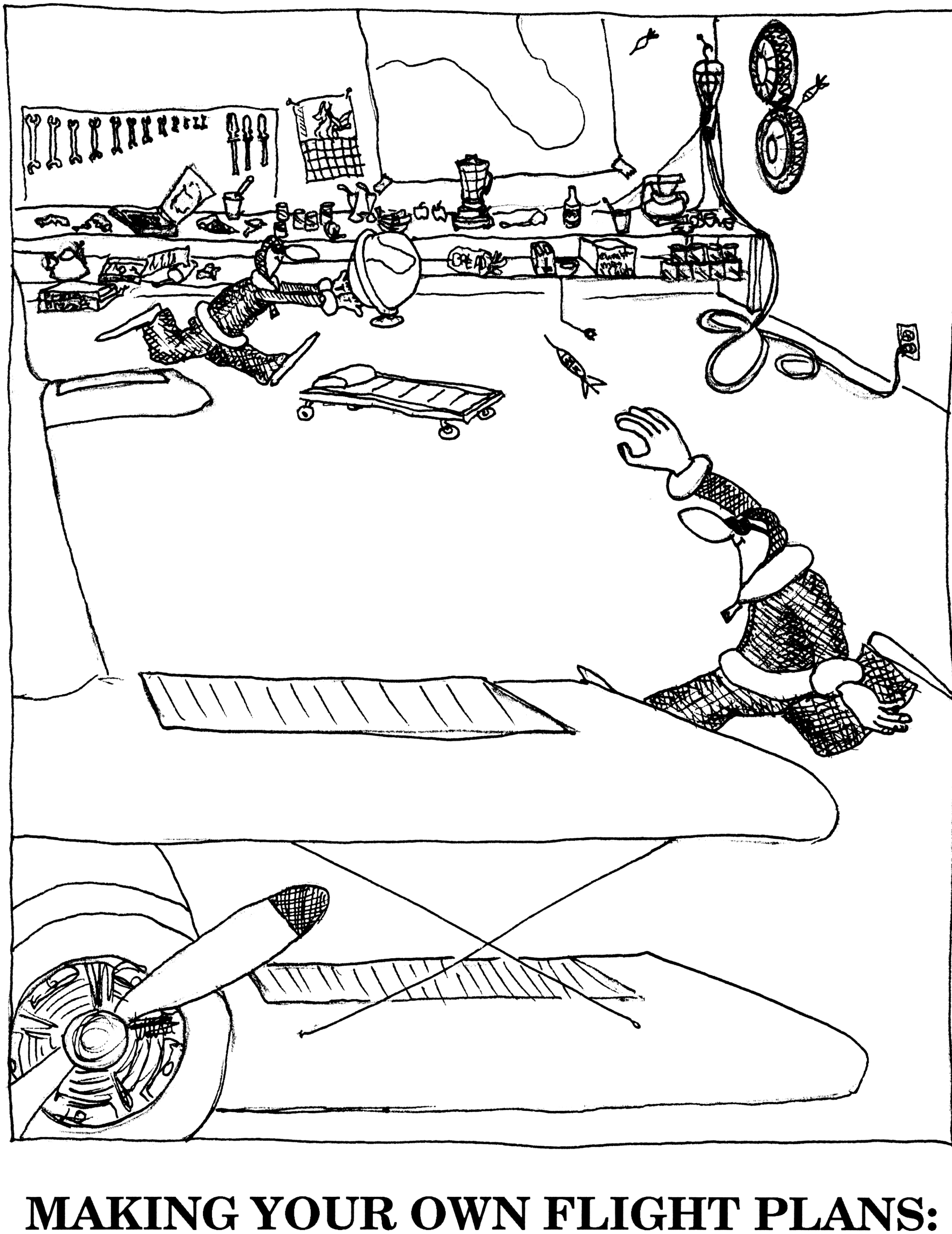
This means, for example, that if you borrow money for a car, the B amount will be *positive,* since that's when the bank hands *you* the money to buy the car. But then your payment amount (P) will be *negative* as you repay the loan – and so will any final remaining balance payment (F) that you must make at the end.

The formula will always use the signs of its answers to tell you which direction money is flowing. For example, if you're solving for P, the payment, and it comes out positive, this says you'll be *receiving* those payments (and if that doesn't square with the perspective you've chosen to set up the problem, you'd better check the signs you put on B and F! An *excellent* way to keep this all straight is to draw yourself a cash-flow diagram, such as the ones in the examples in your HP manual – pp 110-111).

Caution #2: I is the *periodic* interest rate – *not* the annual interest rate. The "period" in "periodic" is the *period of your payment.* Thus, if your car loan requires *monthly* payments, then you need to give this formula a *monthly* interest rate.

How do you know what that rate is? Well, most home or car loans will specify an annual interest rate and then a monthly payment schedule. So you usually just divide the annual rate by 12 to get the monthly rate to use in this formula.

Caution #3: N is the number of *periods* (i.e. the number of *payments*) in the life of the loan – *not* the number of years. This idea exactly matches the way I works. And if you solve for either N or I as your unknown, you'll get your answer in periods too – not in years.



MAKING YOUR OWN FLIGHT PLANS:

Writing Your Own Equations

A Memory Reminder: The CLEAR Menu

Remember what happens when you go to the CLEAR menu (by pressing ■ CLEAR)? You see this:

```
VAR EQ ALL Σ
```

By selecting one of these menu items, you'll clear that part of User-Defined Memory (and clearing ALL actually "re-zeroes" the Permanent Memory registers, too):

Permanent Memory

Display LAST XCOORD YCOORD RADIUS ANGLE

LIBRARY of Equations

```
R=SQRT(X^2+Y^2+Z^2)
X=(-B+J×SQRT(B^2-4×A×C))÷2÷A
X=S+V×T+.5×A×T^2
F=V+A×T
E=.5×M×V^2
F=K×A×B÷R^2
P=I^2×R
P×V=N×R×T
G=H-T×S
P=I+D×G×H
-K×T=LN(N÷I)
O×F+I×F=O×I
A=ASIN(M×L÷D)
Y=F+(I-F)×EXP(K×T)
R=SQRT(Σx²÷n)
(P×100÷I-F)×(1+I÷100)^-N-P×100÷I=B
```

User-Defined Memory (371 BYTes)

VARiables

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

EQuations

Σ **(STAT) registers)**

n Σx Σy Σx² Σy² Σxy

"Thinking Through" An Equation Or Two

Up to now, you've seen just about all parts of this memory *except* that big void called

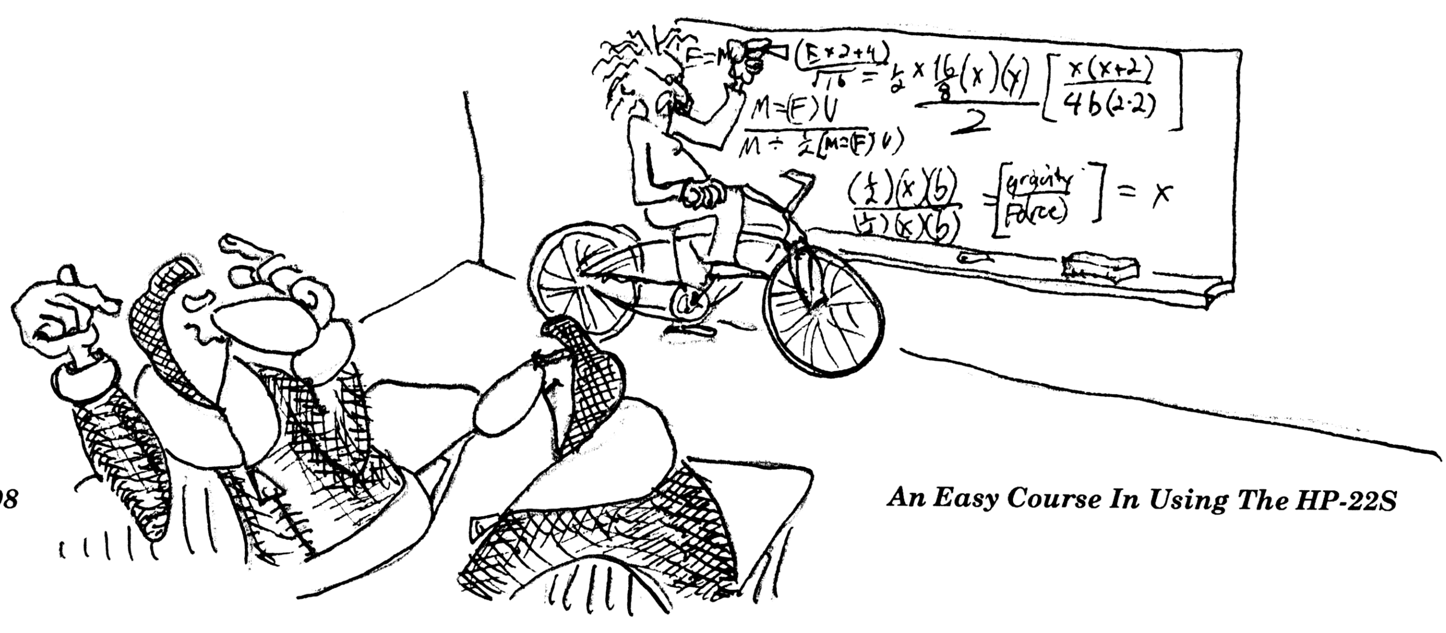
EQUATIONS.

Oh, sure, you've seen the *built-in* equations in the LIBRARY, but they don't even appear in the picture of User-Defined Memory; they're *not* User-Defined, because you can't clear them or modify them. Ah, but in your *own* EQUATIONS – in their own list – you can do anything you want with them: clear them, edit them, whatever. And that's what this last chapter is all about (incidentally, you might want to change your display to FIX 4 decimal places to follow along here).

OK, so when would you want to build your own equations?

Whenever you need to do *repeated calculations*
or play "What-If?" with the different variables in a formula.

For example, take something relatively simple – such as percentage calculations: Of course, there are keys right on the keyboard to let you do this (■ % and ■ %CHG). But if have a lot of questions to answer – with the same repetitive calculation – or if the unknown changes from one variable to another, it's probably easier to write yourself a little formula....



Questions: Today's high temperature was 25° above today's low, but yesterday's high-low variation was only 14°. By what percentage did this temperature range change in one day?

If the range changed by 40% in two days, what was the third day's range?

If the polar ozone layer decreased by 25% this year and 36% last year, what was its total percentage decrease over these two years?

Answers: Well, you *could* work these out with the ■ [%CHG] and ■ [%] keys. For example:

[1][4][INPUT][2][5] ■ [%CHG] gives 78.5714

And then [1][4][+][4][0] ■ [%][=] 19.6000

And so on. But that's not very convenient – especially when you get to questions like that ozone problem. The ■ [%CHG] key is great, for example, for answering just that particular question; knowing A and B, you can always find C.

But what if you want to play ***"What-If?"***

Sometimes you may already know what C is and instead you want to work backwards to solve for the original amount, A. Or maybe it's B you don't know. In other words, you'd like to be able to solve for *any* of the three variables, knowing the other two.

(In other words, you want to build your own SOLVE EQUATION.)

Try It: Build your own SOLVE EQUATION to let you play "What-If?" with the percentage change between two numbers, A and B.

Solution: Press ■ EQUATIONS and you'll immediately see what you need to do next:

TYPE NEW EQN

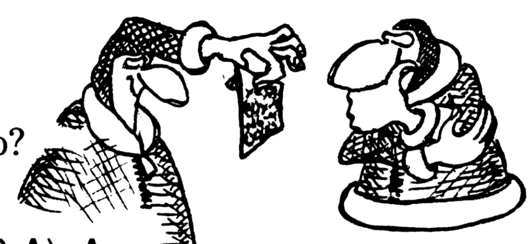
OK, so what *is* this new equation you're going to type? Before you begin, you'd better know where you're going.

How do you figure the percentage change between two numbers? You have some starting value, A, and some other ending value, B. And the question is, by what percentage, C, do you have to change A to get B?

Hmmm...It must have something to do with the *difference* between them – that's subtraction, right? C=B-A

And then you relate that difference to A – which is what you *started with* (recall page 47) – by dividing by A: C=(B-A)÷A

That'll get you the change as a decimal fraction of the original A, but you want the *percentage* change. How do you convert from a decimal fraction to a percentage?



You multiply by 100, no?

So there you have it: C=100x(B-A)÷A

And Now: Key this equation into your HP-22S:

`C=100×(B-A)÷A`

Here's How: It doesn't seem as if the machine will let you type anything except numbers, does it?

Well, there is a little trick to it: Anytime you want to type the name of one of the alphabetic variable registers, you need to press either the [STO] or the [RCL] key.

So your actual keystrokes here would be:

[RCL][C][=][1][0][0][×][(][RCL][B][−][RCL][A][)][÷][RCL][A][INPUT]

Of course, you could have just as well used the [STO] key instead of the [RCL] key – or any combination of the two. Just bear in mind that you're not doing any actual storing or recalling here; it's just the way you have to tell the machine that you want to type a letter rather than a number. And notice that the machine will respond with the A..Z annunciator whenever it's ready for you to type a letter key.

Notice also that as soon as you started typing the new equation, the INPUT annunciator came on – to remind you to press the [INPUT] key when you're all ready for the machine to examine your equation (to make sure that you haven't goofed and that it understands what you mean).

Remember, you can always use the [←] key if you bollix it up, OK?

When all goes well, this equation will now appear in your display, looking like one of the built-in equations you saw in the LIBRARY, with part of it extending to the right, beyond the end of the display.

But this isn't the LIBRARY. The ▼▲ annunciator says that you *are* in a list (which you can move through with the [▼] and [■][▲] keys – same as usual), but this is your *own* EQUATION list.

Test It: Try using your new equation on those questions from page 199.

Keystrokes: Remember that to use an equation, you must either press [EVAL] or [■][SOLVE]. The [EVAL] key will let you solve only for the variable that appears by itself in the equation (that would be C in your equation). If you want any other variable to be your unknown, you must press [■][SOLVE] and then select your unknown before the machine will begin prompting you.

So, to answer the first question, with your equation showing in the display, press [EVAL]. Then [2][5][INPUT] for the B? prompt and [1][4] [INPUT] for A?. Result: C=78.5714

Looks good so far. Now for the real payoff – knowing C and A, solve for B, to find the third-day's high temperature that would make for a two-day increase of 40%:

[■][SOLVE] B [4][0][INPUT][INPUT]   Result: B=19.6000

Notice that you can just press [INPUT] when prompted for A?, since the suggested value is already correct from the previous problem. After all, these variable values are just sitting there in their respective storage registers, and they won't change or clear until you tell them to, right?

Keep Going: Now do the ozone problem....But how? After all, you don't have any real ozone data to work with, do you?

No, but you can still do the problem: Suppose the ozone level two years ago was 100. Last year, it decreased by 36%. So use your newly-built equation to find the ozone level at the end of last year.

While pointing to that equation in your list, press ■ SOLVE and select B as your unknown. Then respond to the prompts with 3 6 +/– INPUT for C? (remember that a decrease is a *negative* percentage change) and 1 0 0 INPUT for A?. Result: B=64.0000

Now, this ending level for last year is the starting level for this year, right? You're going to use the same equation again to find the ending level (B) for this year – and when prompted for A?, you're going to give it the B-value from last year....

■ SOLVE B, then 2 5 +/– INPUT for C? and RCL B INPUT for A?. Notice how you can simply recall the value out of one variable's register and use it for the value in another. Result: B=48.0000

OK, that's the ending level after this year – assuming that you started two years ago with a level of 100. So, what's the total percentage change over those two years? Use that equation one more time – this time solving for C....

As always just change what you need to. Your B-value is already correct, so let it be, and just put 100 into A: EVAL INPUT 1 0 0 INPUT Result: C=-52.0000

That ozone layer depleted 52% in two years (and may the real one never be allowed anywhere near that level of depletion)!

Did you notice something about the way your equation prompts you for the known values? Here's the equation as you now have it in the machine:

$$C=100\times(B-A)\div A$$

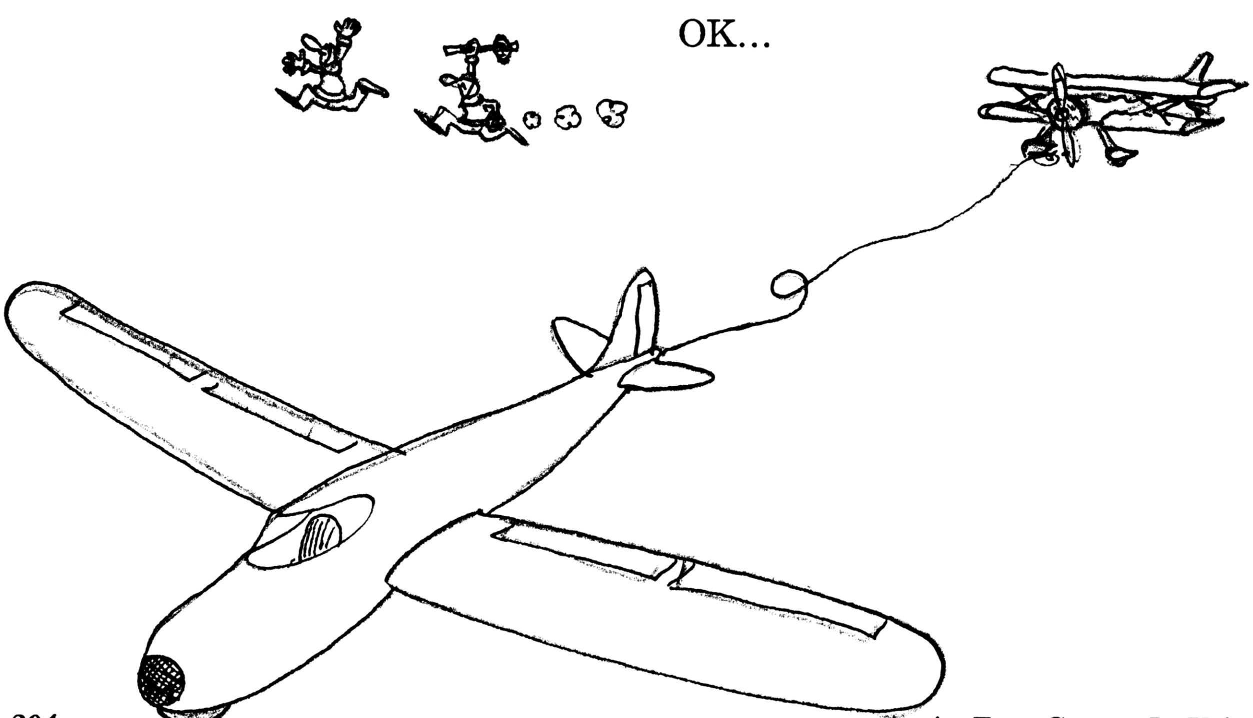
If C was your unknown, the machine asked for B, then A.

If B was your unknown, the machine asked for C, then A.

If A was your unknown, the machine asked for C, then B.

As you saw in the built-in equation LIBRARY, *the HP-22S will always prompt for values in the order in which those variables first appear in the equation.*

So, although your equation is working just fine, maybe it would make a little more intuitive sense to you to switch the order of appearance of the variables A and B. After all, when you're asking the mathematical question, "How much do I need to change A to get B?" you naturally begin with A, right?



Challenge: Edit your equation so that it prompts you for A? before B?.

Solution: First, you'd better decide what the edited version is going to look like. You need A to appear before B in the equation – but you still want to be subtracting A from B.

How about -A+B? That's the same as B-A, isn't it? OK, then here's what you want the edited version to be:

C=100×(-A+B)÷A

OK, how do you edit your equation – just erase it and start over?

Nope. Just press ■ EDIT while the display is "pointing to" (showing) this particular equation in your list, and see this:

INPUT
100×(B-A)÷A_
←

Notice that you're now looking at the far (right-hand) end of the formula, where there's a cursor (_) to indicate you may now add on (or use the ← key to delete) characters in the formula.

Well, you want to change what's inside those parentheses, so you need to press the ← key six times, to back up to that point....

Now key in the new version: − RCL A + RCL B) ÷ RCL A; then INPUT signals that you're finished and lets the machine inspect and verify your formula.

Test It: Re-work the percentage problems posed on page 199 – just to see if you get the same answers as on pages 202-203.

Answers: Press [EVAL], then [1][4][INPUT] for A? and [2][5][INPUT] for B?
Result: C=78.5714 OK so far....

Next, find the third day's temperature range that would give a two-day increase of 40%:

■[SOLVE] B [4][0] [INPUT] (for C?) and [INPUT] (for A?, since it's already correct from the previous problem). Result: B=19.6000

Fine. Now do that ozone problem again: ■[SOLVE] B, then [3][6][+/−] [INPUT] for C?, and [1][0][0][INPUT] for A? (Result: B=64.0000). This is the ozone level at the end of last year (i.e. the beginning of this year).

Next, figure the level at the *end* of this year: ■[SOLVE] B [2][5][+/−][INPUT] for C? and [RCL][B][INPUT] for A?. Result: B=48.0000

One more calculation (where you put the original starting ozone level of 100 back into A, but leave the B-value as it is, to figure the percentage decrease over the entire two years): [EVAL][1][0][0][INPUT][INPUT].

Result: C=-52.0000

So your modified version of the formula is working just fine. And – especially in this last problem – it does seem to make a little more intuitive sense to prompt for A? before B?, don't you think?

Question: If it made so much sense to put A before B in the formula, why not complete the logical progression by putting C last (making it appear *after* the first appearances of A and B in the formula)?

Answer: Remember the difference between the [EVAL] key and the ■[SOLVE] key? The [EVAL] key will automatically *assume* you want to solve for the variable that appears *by itself to the left of the = sign* in the formula. If there is no such isolation of one variable on the left, then [EVAL] works exactly the same as the ■[SOLVE] key.

So if you changed the formula to read like this, for example:

100×(-A+B)÷A=C

that would indeed guarantee that the prompts would always appear in alphabetical order, but at the same time, this would eliminate the usefulness of the [EVAL] key as such, since there would be no isolated variable to the left of the = sign.

Well, it doesn't make sense to throw away the use of a perfectly good [EVAL] key; you might as well have *some* variable isolated on the left. And since the percentage change (C) is probably the unknown more often than the starting or ending amounts (A or B), it's still probably the best candidate for isolation and quick solution via [EVAL].

But if you *really* want to make for alphabetical prompting *and* still have some convenience with the [EVAL] key, you *could* use a little algebra and rewrite the formula like this, having isolated A instead of C:

A=100×B÷(C+100)

Now that you have the hang of it, try developing some more of your own formulas....

F'rInstance: Write an equation for your car's gas mileage. Then figure your mileage for a trip of 350 miles that used 12 gallons of gas.

With a 15-gallon fuel tank, how far could you have gone before the last "fumes" ran out?

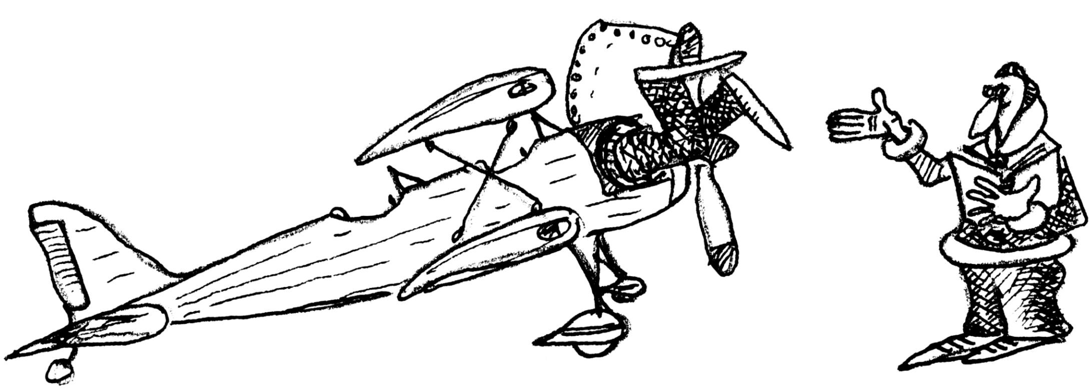
Solution: Mileage is expressed as "miles per gallon," right? That is, your car's fuel Economy is Miles divided by Gallons:

E=M÷G

(Although the word mileage begins with an M, so does the word Miles – which you want M to represent. Obviously, you can't use the same letter for different variables, since each variable must have its own register – so choose E, or some other semi-suitable substitute.)

Key this in: Press ■ EQUATIONS, then ▼ to find the bottom of your growing list of equations.

Now RCL E = RCL M ÷ RCL G INPUT should do the trick.



Now use this formula to figure the answers to the questions:

Press [EVAL], then press [3][5][0][INPUT] for M? and [1][2][INPUT] for G?.
Answer: E=29.1667

Now for the "What-Iffing" that makes the SOLVE idea so great:

Using the result of this mileage computation, find how far you could have gone on 15 gallons. As usual, change only those values you need to; leave everything else as a continuation from the previous calculation:

■[SOLVE] M (solving for the Miles this time). Then [INPUT] for E?, since the suggested (current) value is what you want to use; and [1][5] [INPUT] for G?.

Answer: M=437.5000

So you could have gone 437.5 miles on 15 miles of gas – assuming that your fuel consumption were the same as for your 350-mile trip that used 12 gallons.

See how easy it is to develop a handy little formula like this? And in this one, the prompts already seem to come in a fairly logical order, no? And since the one you're most often interested in is the mileage, it makes sense to isolate it on the left, where the [EVAL] key lets you get to it quickly and conveniently.

This is the way you'll probably develop most of your own formulas: Even though the idea is to play "What-If" and be able to solve for *any* variable as your unknown, it's best to isolate (on the left) the unknown you'll want most often.

Next Task: Some gas stations in the U.S. sell gasoline by the liter, rather than by the gallon (especially when the price per gallon exceeds $1.00 by quite a lot). Yet distance is still measured in miles, and so a vehicle's fuel economy is expressed in "miles per gallon," as in the formula you just developed.

By contrast, in Canada – and most of the rest of the world, of course – gasoline is generally sold by the liter, and distance is measured in kilometers. A vehicle's fuel economy would therefore more naturally be expressed in "kilometers per liter."

Develop a pair of formulas to allow a Canadian traveler to compute comparable fuel economy in the U.S. – and to allow a U.S. traveler to do the same in Canada (for the Canadian traveler's version, modify the mileage formula you already have).

Then use the formulas to answer these questions:

During their trip along the Washington and Oregon coasts, one family from Campbell River, B.C., travelled 490 miles on 21 gallons of gasoline. Is their car operating within their expected range of fuel economy (7-12 kilometers per liter)?

Another family, from Philomath, Oregon, drove through southern British Columbia, and on one part of the trip, they covered 656 kilometers on 58 liters of fuel. What was their mileage for this part of the trip?

Next Solution: Start by modifying the current mileage formula.

The idea here is that if you're traveling outside of the U.S., you'll see distances in kilometers and gasoline sold in liters – but you still want to know your fuel economy in miles per gallon.

So this is merely a two-step units-conversion problem (remember these? See pages 167-175 for a reminder):

$$\frac{1\ \cancel{\text{mile}}}{\cancel{\text{gallon}}} \times \frac{??\ \text{kilometers}}{1\ \cancel{\text{mile}}} \times \frac{1\ \cancel{\text{gallon}}}{??\ \text{liters}} = \frac{??\ \text{kilometers}}{\text{liter}}$$

OK, so now all you need to know is how many liters are in one gallon – and how many kilometers are in one mile?

Hmm...doesn't this HP-22S of yours have a UNITS menu to help you with this kind of thing?

Yup. (Review pages 23-25 if you need to).

So key in [1] (to convert 1 gallon to liters), then press ■ [UNITS] and select VOL and →LTR. <u>Result</u>: 3.7854

(If you use ■ [SHOW], you'll see that to be more exact, you could use many more digits than this – but for the purposes of your fuel economy calculations, four decimal places is plenty).

So there are about 3.7854 liters per U.S. gallon.

And how many kilometers are in one U.S. (statute) mile?

Again, you can use the UNITS menu...except that the only length conversion on that menu is between *inches* and *centimeters*. Hmmm.... Well, you can always do a "conversion within a conversion," can't you? Like this:

$$\frac{??\ \cancel{\text{centimeters}}}{\cancel{\text{inch}}} \times \frac{1\ \cancel{\text{meter}}}{100\ \cancel{\text{centimeters}}} \times \frac{1\ \text{kilometer}}{1000\ \cancel{\text{meters}}}$$

$$\times \frac{12\ \cancel{\text{inches}}}{1\ \cancel{\text{foot}}} \times \frac{5{,}280\ \cancel{\text{feet}}}{1\ \text{mile}} = \frac{??\ \text{kilometers}}{\text{mile}}$$

The only thing you may not know off the top of your head is how many centimeters are in one inch. But the UNITS menu will do that one for you.

So key in [1] (preparing to convert 1 inch to centimeters) and press ■[UNITS], then L, then →CM. <u>Result</u>: 2.5400

(And by using the ■[SHOW] key, you'll see that this is an *exact* relationship.)

Now take this result and proceed through the above conversion:
[÷][1][0][0][÷][1][0][0][0][×][1][2][×][5][2][8][0][=] <u>Result</u>: 1.6093(44)

So there are about 3.7854 liters per gallon and about 1.6093 kilometers per mile.

Fine. Now plug these conversion factors into the final conversion calculation you need to write your formulas:

$$\frac{1\ \cancel{\text{mile}}}{\cancel{\text{gallon}}} \times \frac{1.6093\ \text{kilometers}}{1\ \cancel{\text{mile}}} \times \frac{1\ \cancel{\text{gallon}}}{3.7854\ \text{liters}} = \frac{??\ \text{kilometers}}{\text{liter}}$$

Doing the arithmetic: $\boxed{1}\boxed{\cdot}\boxed{6}\boxed{0}\boxed{9}\boxed{3}\boxed{\div}\boxed{3}\boxed{\cdot}\boxed{7}\boxed{8}\boxed{5}\boxed{4}\boxed{=}$

Result: 0.4251

This means that 1 mile-per-gallon (mpg) is the same as 0.4251 kilometers-per-liter (kpl). So you'll need to multiply mpg by 0.4251 to get kpl.

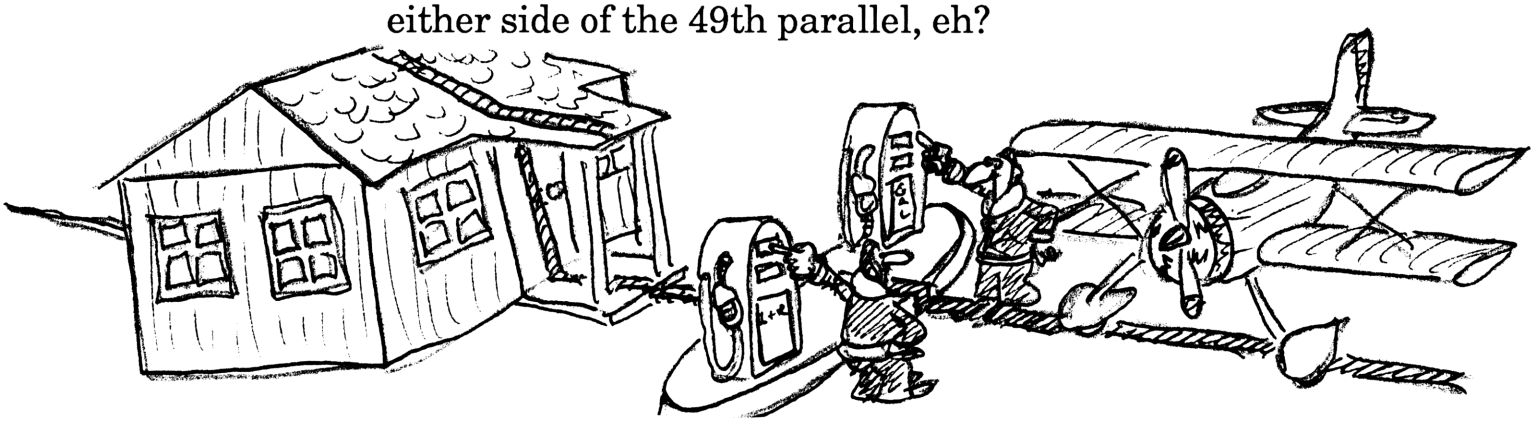
That is, if E_{US} is fuel economy expressed in mpg, and E_{CAN} is the same fuel economy expressed in kpl, then

$$E_{CAN} = E_{US} \times .4251$$

or, to put it the other way,

$$E_{US} = E_{CAN} \div .4251$$

Now you're ready to write the formulas for those travelers from either side of the 49th parallel, eh?



If you buy fuel and read distances in liters and kilometers, then you can still get mileage in mpg:

First, find your "mileage" as "kilometerage:"

$$E_{CAN} = K \div L$$

Now substitute the above expression for E_{CAN} into your conversion factor ($E_{US}=E_{CAN} \div 0.4251$), to get the final formula:

$$E_{US} = K \div L \div .4251$$

Not bad, eh?

How about going the other way? What if you're living in a world of miles and gallons, but you think in kilometers per liter?

Well, $E_{US} = M \div G$, right?

And $E_{CAN} = E_{US} \times .4251$, right?

So $E_{CAN} = M \div G \times .4251$ ...bingo! – there you have your second formula!

So, to put this in the exact notation your HP-22S needs to see:

E=K÷L÷.4251

and

E=M÷G×.4251

Hmmm...Is this a smart thing to do – using the same variable, E, in both equations?

As you know, there's only one E-register in your HP-22S; if you first solve for E in the mpg-to-kpl conversion, then switch over to the other formula, the machine will, of course, retain and use that previous value of E, unless you change it.

Ah, but any one person isn't likely to need both equations in his/her HP-22S at the same time, right? And with only 26 variable registers to choose from, you'd better get used to sharing some variables between different formulas.

OK, then, type these formulas in and answer those questions from page 210:

Press ■ EQUATIONS and ▼ until you're pointing to your original mileage formula (E=M÷G). Now edit it (■ EDIT) and add onto it for the Canadian traveler's formula: × · 4 2 5 1 INPUT.

It should now look like this: E=M÷G×.4251

Next, INPUT and ▼ and type the U.S. traveler's formula: RCL E = RCL K ÷ RCL L ÷ · 4 2 5 1 INPUT.

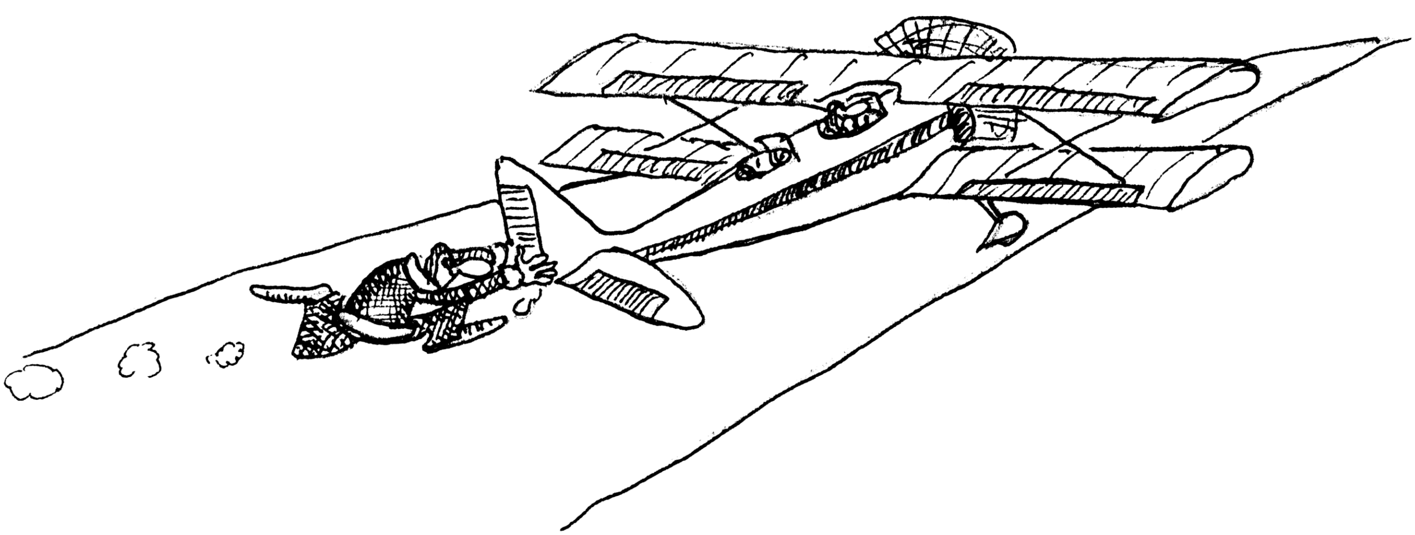
It should now look like this: E=K÷L÷.4251

Finally, you're ready to calculate with them (refer to page 210 for the problems posed): For the Canadian family of travelers, use the ■[▲] (or [▼]) key to "point to" that formula (it's the one with M and G), then press [EVAL]. Now key in [4][9][0][INPUT] for M? and [2][1] [INPUT] for G?.

Answer: E=9.9190 (this is now converted to *kilometers per liter,* remember – so it looks like the family car is doing just fine.)

For the U.S. family of travelers, press ■[EQUATIONS] and use the [▼] (or ■[▲]) key to "point to" that formula (it's the one with K and L), then press [EVAL]. Now key in [6][5][6][INPUT] for K? and [5][8][INPUT] for L?.

Answer: E=26.6063 (and this is already converted to *miles per hour,* remember.)



Now, how do you suppose the memory of your HP-22S looks? Schematically, of course, it still looks something like this (except at this point, the STAT registers and most of the VARiable registers contain "something" – non-zero values – and are therefore "allocated" – reserving 8 bytes each of the User-Defined Memory)....

Permanent Memory

Display | LAST | XCOORD | YCOORD | RADIUS | ANGLE

LIBRARY of Equations

```
R=SQRT(X^2+Y^2+Z^2)
X=(-B+J×SQRT(B^2-4×A×C))÷2÷A
X=S+V×T+.5×A×T^2
F=V+A×T
E=.5×M×V^2
F=K×A×B÷R^2
P=I^2×R
P×V=N×R×T
G=H-T×S
P=I+D×G×H
-K×T=LN(N÷I)
0×F+I×F=0×I
A=ASIN(M×L÷D)
Y=F+(I-F)×EXP(K×T)
R=SQRT(Σx²÷n)
(P×100÷I-F)×(1+I÷100)^-N-P×100÷I=B
```

User-Defined Memory (371 BYTes)

VARiables

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

EQuations

```
C=100×(-A+B)÷A
E=M÷G×4251
E=K÷L÷.4251
```

Σ (STAT) registers)

n Σx Σy Σx² Σy² Σxy

But the question is: *How much of your User-Defined Memory do EQUATIONS use?* What does it cost to "do-it-yourself" – like the three you've built so far (shown above)?

To find out, try an experiment: First, clear all your variables (press ■CLEAR VAR Y and ■CLEAR Σ). Now press ■MEM. If you've followed along exactly with all the keystrokes of this Easy Course up to this point, you should have 332 BYTes left in your User-Defined Memory. The idea now is to key in one more equation and see how much this reduces your available User-Defined Memory....

Hmm...: A construction contractor often needs to quote the square-footage area of the rectangular concrete slabs he lays, and he then needs to find the cubic yardage of mixed concrete needed for the job. The only things he knows are the length and width (in feet) of the slab, and its depth in inches – and he can order concrete only in whole cubic yards (no extra fractions of yards). Write an HP-22S SOLVE formula to help him.

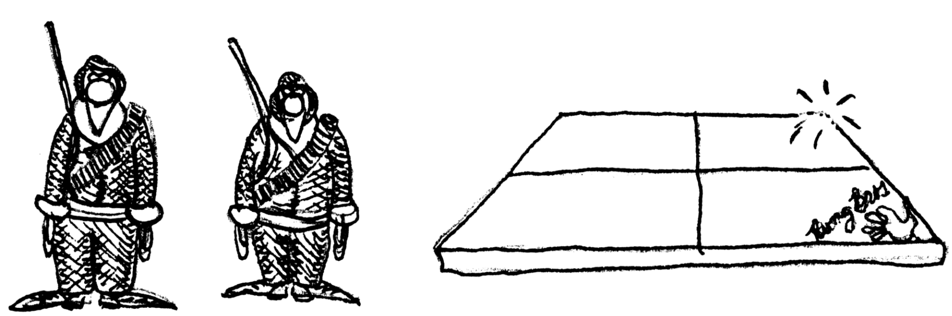
Ahh...: First of all, you get the area of a rectangle by multiplying its length by its width (A=LxW), right?

But that's not the real problem; most people can do that much figgerin' in their heads. What you really want is the cubic yards of concrete you need – and that means you need to figure the *volume* of the slab, *in cubic yards.*

Well, most people *also* know that a rectangular volume is equal to its length multiplied by its width, multiplied by its depth (V=LxWxD), no?

Seems straightforward enough. So if you're putting down a slab of concrete that is 30 feet by 40 feet by 6 inches deep, then it ought to have a volume of 30x40x6, or 7,200....

...7,200 *what?* Cubic *yards?* You'd better hope not – that would be the world's most expensive 30x40-foot slab.



Obviously, there's a problem with units again. To get cubic yards, you have to multiply *yards by yards by yards* – not feet by feet by inches. Your 7,200 answer was 7,200 feet^2•inches (whatever *those* are) and they just don't sell batches of concrete measured that way.

So, are you going to insist that the contractor (or whomever is going to use this formula) mentally convert from feet and inches into yards before keying in the dimensions of the slab?

No – you can do better than that. Make it easy on the user by letting the calculator do the converting:

$$1\ \cancel{\text{feet}^2}\cdot\cancel{\text{inch}} \times \frac{1\text{ yard}}{3\ \cancel{\text{feet}}} \times \frac{1\text{ yard}}{3\ \cancel{\text{feet}}} \times \frac{1\text{ yard}}{36\ \cancel{\text{inches}}} = \frac{1\text{ yard}^3}{324}$$

So, assuming that you're still going to key in the length and width in feet and the depth in inches, here's your formula:

V=L×W×D÷324

That ought to do the trick, right? But before keying this into your EQUATIONS list, check it by hand on the calculator (with your 30' x 40' example):

3 0 × 4 0 × 6 ÷ 3 2 4 =

<u>Answer</u>: 22.2222 (cubic yards of concrete)

That's more like it. Does this mean it's ready to key in as a formula?

Er...well...haven't you forgotten one small detail? Don't you have to order in *whole-yard* loads of concrete? (Sigh)

Sure – the contractor (or whoever's in charge of this slab) *could* easily "eyeball" that answer you got (22.2222) and know that it calls for a 23-cubic-yard load. But why not go the whole distance and do this rounding up as part of your formula?

How are you going to adjust the formula for that?

Suggestion: Take your actual exact volume requirement, add one whole cubic yard to it, then drop any fractional portion, like this:

$$22.2222 + 1 = 23.2222 \longrightarrow 23.0000$$

(drop the digits after the decimal point)

Why this approach? Well, it just so happens that one of the operations your HP-22S will let you do in your formulas is to take the Integer Portion (IP) of a number.* So if you take the integer portion of "one more than the actual volume," you ought to get the whole number of cubic yards you're looking for:

```
V=IP(L×W×D÷324+1)
```

*There are a lot of nifty operations you can use in your equations; turn to chapter 6 (around page 80) of your HP Owner's Manual to see the complete list.

OK, that would work for this particular problem, all right, but what if it happened that your exact volume requirement was *already* a whole number of cubic yards – say, 22.0000 cubic yards? Wouldn't this formula still tell you to order another whole cubic yard?

IP(22.0000+1) = 23.0000 — Yep – and that would be an entire cubic yard too much, right?

Hmmm...looks like you're not quite through here, yet. To fix this, just use some common sense: If the actual requirements turned out to be 22.01 cubic yards, would you then order 23 yards?

Probably not.

How about for 22.1 yards? ...mmm...– guess you'd better, right? So anything less than about a tenth of a cubic yard should be ignored by the formula. Aha! Then *this* is what you want:

V=IP(L×W×D÷324+.9)

See how this works? If you need anything less than a tenth of an extra yard, then by adding 0.9 to this and rounding down (taking the Integer Portion), the formula *won't* recommend an extra yard:

IP(22.09+.9) = 22.0000

but

IP(22.1+.9) = 23.0000

Now you're ready to key in this final version and try out the numbers with [EVAL]:

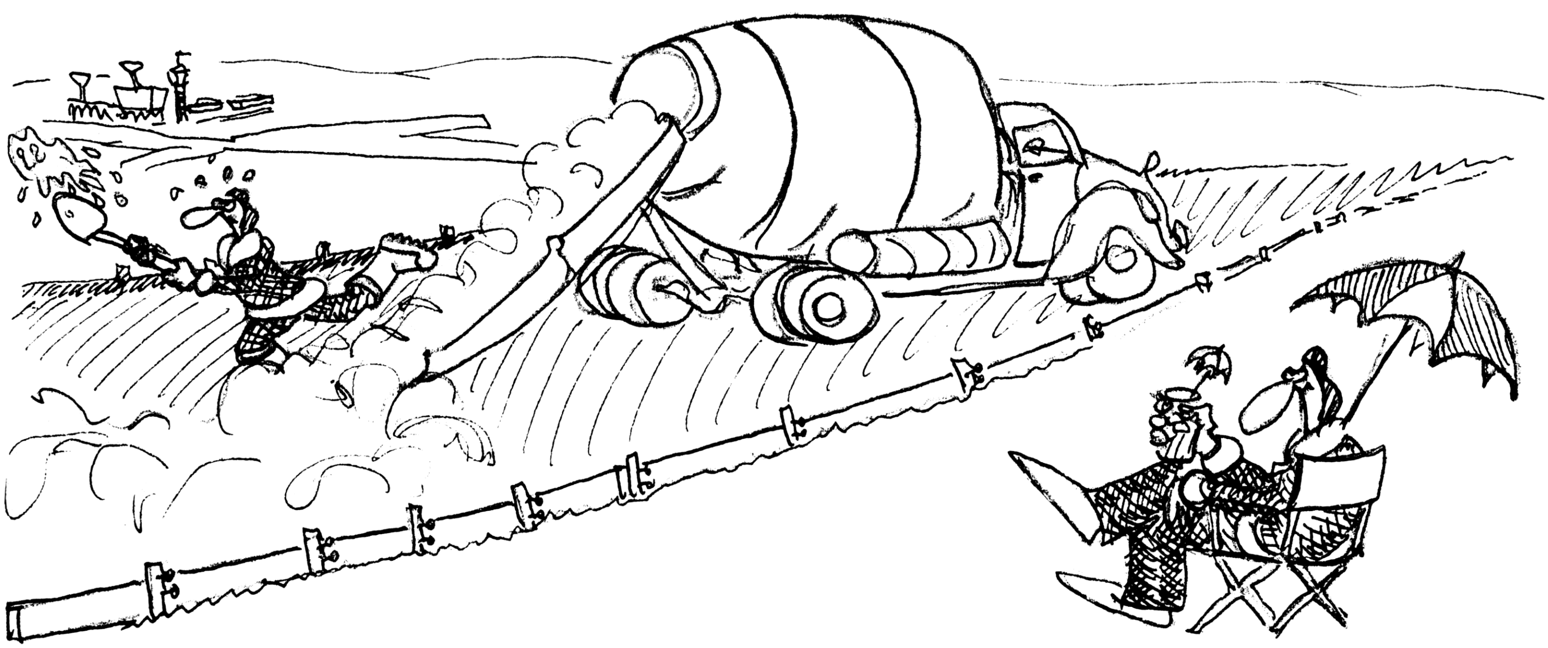
Press ■[EQUATIONS] and [▼][▼]... to get to the bottom of your list. Now type the finished formula:

[RCL][V][=]■[PARTS] IP [RCL][L][×][RCL][W][×][RCL][D][÷][3][2][4][+][•][9][)][INPUT]

Notice how you can use a menu key and its menu selections (■[PARTS] and IP in this case) from the keyboard right there in the middle of keying in a formula. This is how you can include your HP-22S keyboard operations in your formulas – you select those operations as you key in the formula.

Now test your formula: Press [EVAL], then [4][0][INPUT] for L?, [3][0][INPUT] for W?, [6][INPUT] for D?, and...

Bingo! <u>Result</u>: V=23.0000



Now to complete your little experiment: As you recall, you were using this final EQUATION example to test how much memory a given formula uses out of your unused pool of bytes in User-Defined Memory.

Before you began that last example, you had 332 bytes, right? After keying in this concrete formula *and using it,* you have not only used some bytes for the formula itself, you've also grabbed 8 bytes each for V, L, W and D. So before you check your MEMory, you'd better clear your variables again – just so the change you see in memory is attributable only to the new formula itself: Press ■CLEAR VAR Y.

Now check your memory (press ■MEM) and see that you have 315 bytes. That means your formula must have used 332-315 or 17 bytes.

Look at the formula: V=IP(L×W×D÷324+.9)

You'll find that there are 18 characters in this formula (counting the decimal point), so it's not *strictly* a matter of counting characters – but that's usually a good way to "guesstimate" a formula's memory usage, if you're worried about it.

To be more precise, Appendix A in your HP Owner's Manual has a little table (somewhere around page 145) that tells you exactly what's costing what here:

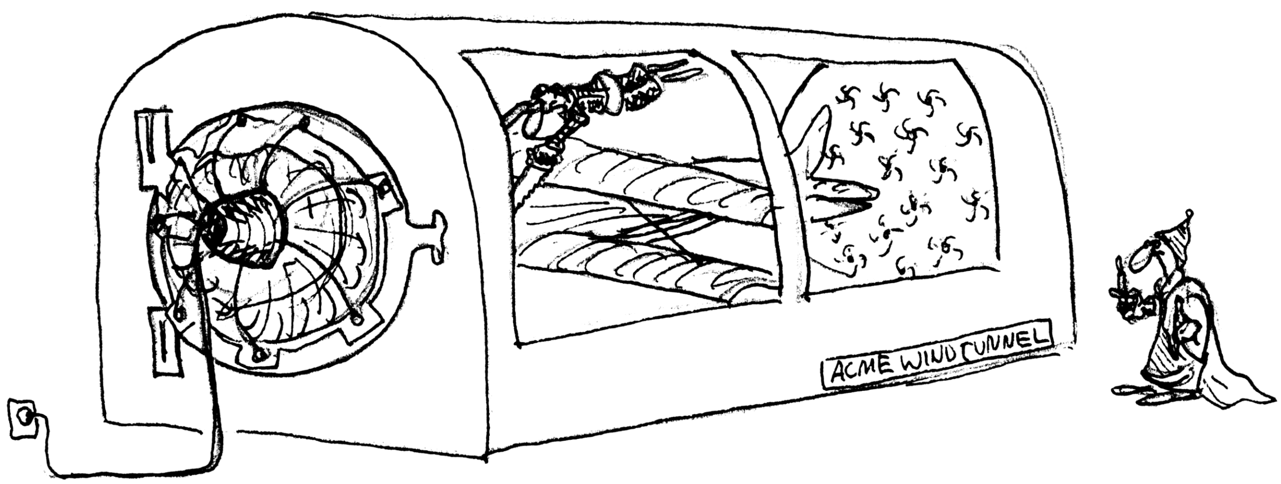
It costs you 1 byte just to *have* a formula. Then there's 1 byte for each digit (3, 2, 4, the decimal point, and 9 – that's 5), 1 byte for each variable (V, L, W, and D – that's 4 more), 1 byte for each function (IP(– that's 1 more), and 1 byte for each operator (=, ×, ×, ÷, +, and) – that's 6 more).

Sure enough – that adds up to 17. But again, if you can't remember all this, just count the characters in your formula, and that's a good safe guess (safe because it's usually an *over*estimate). This means, of course, that you you shouldn't use any more digits or parentheses than you really need in your formulas, because they cost you bytes, right?

Again, remember that this is the memory that each *equation* requires. When you want to use that equation, you also need enough free space to create the registers for *variables it uses* (eight bytes per variable). Of course, when you're not using a given equation, you can clear its variables to allow for *another equations'* variables.

So remember: *If you ever get an error message* telling you that your HP-22S's memory is full, you should first clear the *variables* (with ■ CLEAR VAR Y) – not any equations. Only then – if clearing variables doesn't buy you the memory you need – should you contemplate clearing one or more *equations* (to do this you would press ■ EQUATIONS, then ▼ or ■ ▲ to point to the unwanted equation, then ■ CLEAR Y to clear it).

Bottom-Line Rule Of Thumb: *Write your equations to be as short as possible and use as few variables as possible.* Now, don't lie awake nights and rack your brains trying to figure out ways to save memory. Generally, you'll have enough memory to do what you want – but it's a good idea to try to "streamline" things as much as you can.



Well, you've probably grasped the idea of SOLVE by now – how to build your own equations (and conserve memory). It's now time to turn you loose at the controls and see if you can "go places" all on your own. That's all you need now – just lots of practice in building your own equations. And that means thinking in terms of "What-If?"

So try these problems. Some will feel familiar – merely variations on what you've already seen. As for the rest...you're going to have to fly into "uncharted territory."...

Your Final Flight Test

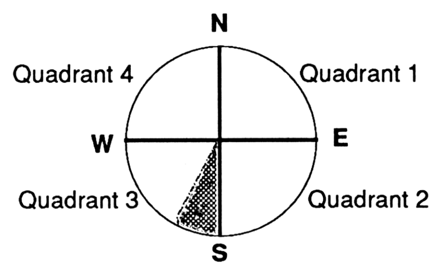
1. You are a lineman for the county, and you drive the main road an awful lot. Fortunately, the county reimburses you for the use of your own car and fuel – generally $.21/mile, reimbursed monthly. Find a way to use the STAT registers and a SOLVE equation to compute the monthly reimbursement amount.

2. You do direct-mail surveys on solid-waste disposal/recycling (yes, you use recycled paper in your flyer). The flyer explains 4 proposals, to which a person then responds on an approval scale of 0 to 10 (where 0 = "Not on your life!" and 10 = "When can we start?"). So a household response varies from 0 to 40.

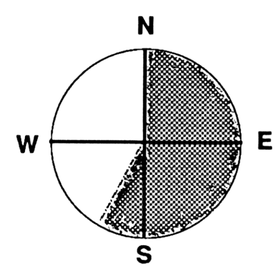
   If the overall typical household response is over 25, then the proposals are considered politically feasible. The problem is, most households just don't respond at all, and when you do get responses, their volume varies a lot (depending on the time of year, the local economy, etc.). In fact, you have observed that this volume itself indicates relative interest (either positive or negative). That is, the higher the fraction of households responding, the more political validity those responses seem to have.

   So as a way to make the overall survey smoother and more accurate, you decide to *weight* each mailing's response average with the volume of its response. Can you build a SOLVE equation to calculate the typical household's attitiude over four such mailings?

3. In land surveying, there are two common notations for horizontal direction. One method is the quadrant-bearing method, where you specify the quadrant and the angle "bearing" into it from the north-south meridian, like this:



The other method simply measures the compass angle or "azimuth" as measured (in degrees, minutes, and seconds) clockwise from north, like this:



Build a set of SOLVE equations to help convert between these two notations.

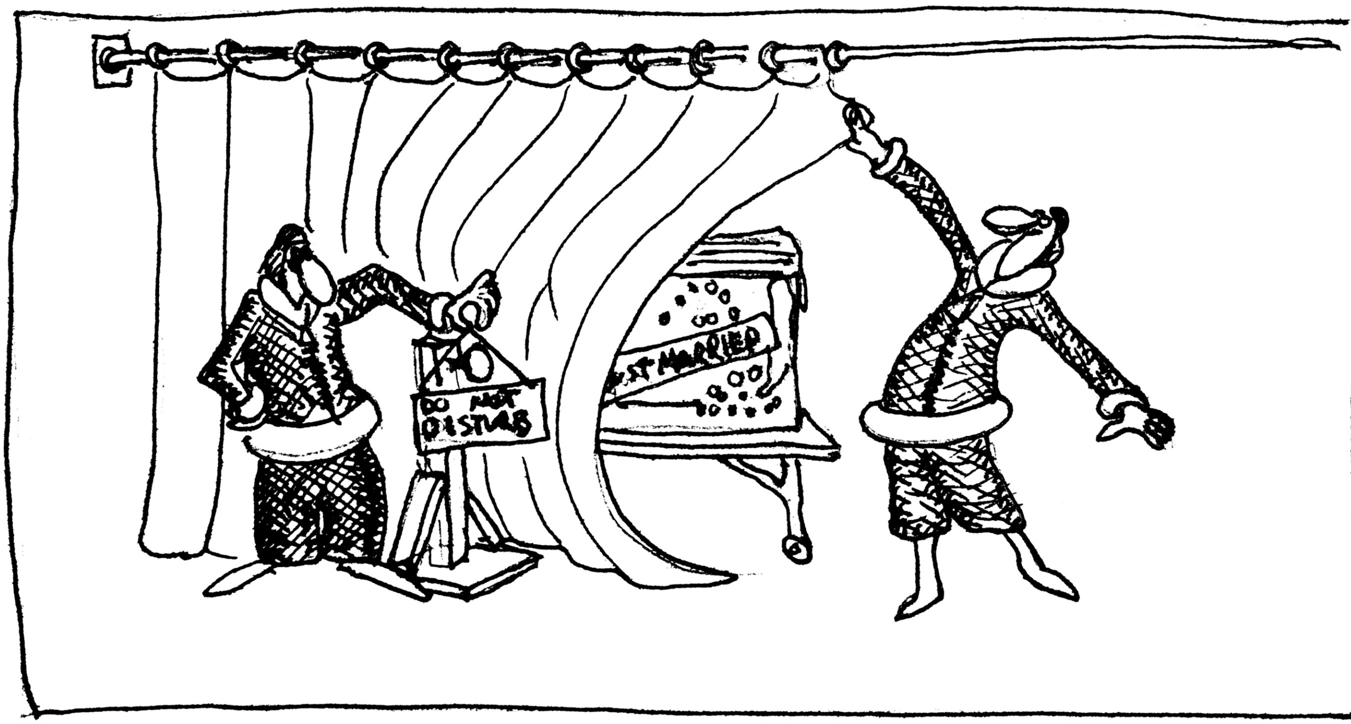
4. Suppose that in a certain species of codfish, each male-female pair spawns about 20 billion eggs per year (and though this problem is fictitious, certain species of fish do actually spawn eggs by the billion!).

Start with one happily-married pair of codfish. Then assume that they live only one year (alas!) and spawn their 20 billion fertilized eggs only once.

But then assume that each egg hatches and that all their offspring do the same – live one year and participate in spawning their *own* 20 billion eggs, and so on for each successive year.

Finally, assume that an average fish of this species is about 0.1 cubic feet in volume.

Using all these (highly questionable) assumptions, write a formula to compute the current *volume* of these fish in the ocean and also a formula for the growth rate of that volume, assuming that it all started "Y" years ago with just the one set of newlyweds.



Bonus question: How might this growth pattern affect the market price of this fish?

Your Final Flight Test Results

1. The actual arithmetic for figuring your monthly reimbursement is pretty trivial, isn't it?

$$R=Mx.21$$

The real problem is to figure out how to keep track of the mileage all through the month. How are you going to get the value for M?

To get a handle on this, talk yourself through an example. Formulas like these can come to you easily if you take a little time, sit down with a pencil and paper, and talk to yourself. After all, before you can tell the calculator how to do it, you have to decide how you would do it manually – on paper, right?

"Hmm...I'd use the odometer on my car...and every time I made a business-related trip, I'd have to note the starting mileage, the ending mileage – and then subtract the two to get the net reimbursable mileage.

"But how many trips will I take in any month? I don't know – it varies a lot from month to month! So I *can't use* an equation like this:

$$M = (B-A) + (D-C) + (F-E) + (H-G) + \ldots$$

where A and B are the starting and ending odometer readings (respectively) for the first trip, C and D are for the second trip, etc. – *because I don't know how long this equation has to be to cover all the trips in the month!"*

It *is* a bit of a problem, isn't it?

Fear not – all is not lost....

First of all, *notice* that

$$(B-A) + (D-C) + (F-E) + H-G) + \ldots$$

is the same as

$$(B+D+F+H+\ldots) - (A+C+E+G+\ldots)$$

In other words, you can get the total mileage of all your trips for the month by subtracting the *sum* of all starting readings from the *sum* of all ending readings.

So, picture this: You have two lists of numbers. The first list is a collection of all the *starting* odometer readings for all your trips in the month (in the previous equation those were represented by the variables A, C, etc.); the second list is a collection of all the ending readings (represented in the equation by B, D, etc.):

First List	Second List
A	B
C	D
E	F
G	H
.	.
.	.
.	.

If you can find a way to keep such a handy pair of *open-ended* lists (i.e. they can be as long as you need them to be) to record and sum these starting and ending odometer readings, then you're all set.

Now stop and think for a minute: In this entire Course, have you seen anything about your HP-22S that could hold a *list* of numbers?

Nope. And that's a fact: The HP-22S has no way to store a list of numbers. The best you could do is to designate successive alphabetic registers to be successive items in the list (A, B, C, etc.). But then you're back to that formula,

(B+D+F+H+...) - (A+C+E+G+...)

and the same problem of not knowing how long this equation must be. Sure – you could use all 26 variables, but then this means you'd need to clear all those registers anytime you wanted to use this formula even for a very small list.

There's gotta be a better way.

Well, consider this: In this mileage problem, *do you really need the list itself?* After all, the only reason you want it is to sum its total, right? Well, why not keep track only of that total – *keep a running total rather than the list itself?*

Aha! That's where the STAT registers can help....

As you know, the STAT registers will accumulate sums: The sum of the x-data (Σx), the sum of the y-data (Σy), the sum of the products of the x- and y- data (Σxy), and so on.

And these sums are accumulated in six specially designated registers, right? You don't have to use up a lot of variable registers – and yet you can use these special registers' names as variables in a SOLVE equation – just like you do with the alphabetic registers (did you realize this?)!

Watch:

Suppose you let the Σx register sum all the starting odometer readings; and the Σy register would sum all your ending odometer readings. Then your SOLVE equation to compute your mileage is just this simple:

R=.21×(Σy−Σx)

Key this into your list of equations: ■ EQUATIONS and ▼ until you get to the bottom of the list. Then RCL R = • 2 1 × (STAT Σy − STAT Σx) INPUT.

Do you see how to use the STAT variables just like any others in a formula? You just invoke their names (from the STAT menu, of course). And now, to use this equation, you simply accumulate your odometer readings into those STAT registers. Try it with these four "county trips:"

Trip	Starting Odometer Reading	Ending Odometer Reading
1	45,678.9	46,111.0
2	47,142.8	47,376.5
3	48,123.4	48,571.4
4	49,012.0	50,987.6

Remember that your starting and ending readings will be the x-data and y-data respectively (review pages 95-102 if you've forgotten how to use the STAT registers and the Σ+ and ■ Σ− keys). So your keystrokes would be C ■ CLEAR Σ to clear the STAT registers. Then ■ DISP FX 2 (for dollars and cents), and

4 5 6 7 8 • 9 INPUT 4 6 1 1 1 Σ+ 4 7 1 4 2 • 8 INPUT 4 7 3 7 6 • 5 Σ+
4 8 1 2 3 • 4 INPUT 4 8 5 7 1 • 4 Σ+ 4 9 0 1 2 INPUT 5 0 9 8 7 • 6 Σ+

Now simply ■ EQUATIONS and EVAL. Bingo! R=648.77

This is what the county owes you as reimbursement for this month's travels.

2. What kind of computation are you going to do here? It's a weighted mean, isn't it? Now, you've done this before – back on pages 105-107, when you were analyzing rainfall versus flight instruction income – where it came in handy for summing up multiple occurrences of the same value.

So you *could* use the STAT registers' capabilities to do something similar here. After all, this *is* true weighting, because you don't actually have multiple occurrences of the same value; you're *pretending* you do, because you've decided that a response's larger magnitude carries more weight simply by being larger.

Should you use the STAT registers in this problem?

Hmm...recall from the previous problem that you didn't need the actual lists of beginning and ending mileages – just their totals – *because you didn't want to know anything else or play "What-If" with any of the list items.* Your mileage was your mileage, and that was that. And besides, since you didn't know how long such lists would need to be anyway, this made the STAT registers your only real choice.

That's not the case here: First of all, the problem says to develop a formula for *four* mailings – so you know exactly how long the lists will need to be. And secondly, with something like this political surveying, it would be *very* useful to be able to play "What-If" with different responses and scores, no? So you do indeed want to preserve the raw data itself – not just sum the totals.

<u>Conclusion</u>:

This problem calls for an equation that uses the alphabetic registers, not the STAT registers.

All right, then, it's time to envision all the different variables you're going to need for this massive formula, eh? For each mailing, "n," of the four mailings, you're going to need to know these things:

- How many flyers you sent out for the "nth" mailing (OUT_n);
- How many responses you got back in for the "nth" mailing (IN_n);
- The summed response point total for the "nth" mailing ($POINTS_n$).

And then, of course, you'll need your final result (SCORE) – the weighted mean of all four mailings – which will indicate feasibility if it's above 25. So it looks as if you'll have about 13 variables in your equation:

Name of Variable	Variable Represents
A	OUT_1
B	IN_1
C	$POINTS_1$
D	OUT_2
E	IN_2
F	$POINTS_2$
G	OUT_3
H	IN_3
I	$POINTS_3$
J	OUT_4
K	IN_4
L	$POINTS_4$
S	SCORE

Note: This is a convenient, intuitive order of variables (i.e. this is the way you might naturally wish to key them in), but as you'll recall, the calculator will actually prompt you for them *in the order they appear in the equation.* Since you haven't written the equation yet, it's hard to say whether this order can be preserved. Maybe later it will be possible to adjust the equation to do so.

The next step, of course, is the equation: What math is involved? What's a weighted mean?

As you might remember from pages 105-107, a weighted mean is a mean (an average) where each value carries a weight that tells how many times that value is considered to have "occurred" (even when it didn't actually occur in the raw data more than once).

Thus, to get a weighted mean, you *multiply each value by its weight,* then *add these products* and *divide by the sum of the weights* (not merely the number of raw values), since these weights tell you how many values you're "pretending" you have.

So how does this translate into an equation for your direct-mail survey?

Your raw data will be the "points per responding household" for each mailing, right? For example, your raw data for mailing #1 will be $\mathrm{POINTS}_1 \div \mathrm{IN}_1$. That would give you the simple average household attitude for that mailing, true?

But you also feel that the magnitude of the response lends credence to that response, so you're weighting each mailing's average response by the fraction of households that contributed to it:

$$\mathrm{WEIGHT}_1 = \mathrm{IN1} \div \mathrm{OUT}_1 \quad \text{, for example.}$$

So the first mailing's *total* contribution to the weighted mean would be the product of the raw (simple) average multiplied by its weight:

$$(\mathrm{POINTS}_1 \div \mathrm{IN}_1) \mathrm{x} (\mathrm{IN1} \div \mathrm{OUT}_1)$$

If you sum up the corresponding total contributions from all four mailings...

$$(\quad (\text{POINTS}_1 \div \text{IN}_1) x (\text{IN}_1 \div \text{OUT}_1) \quad + \quad (\text{POINTS}_2 \div \text{IN}_2) x (\text{IN}_2 \div \text{OUT}_2)$$
$$+ \quad (\text{POINTS}_3 \div \text{IN}_3) x (\text{IN}_3 \div \text{OUT}_3) \quad + \quad (\text{POINTS}_4 \div \text{IN}_4) x (\text{IN}_4 \div \text{OUT}_4) \quad)$$

or, in other words,

$$(\quad (C \div B) x (B \div A) \quad + \quad (F \div E) x (E \div D)$$
$$+ \quad (I \div H) x (H \div G) \quad + \quad (L \div K) x (K \div J) \quad)$$

...and divide by the sum of the weights:

$$\div \quad (\quad (\text{IN}_1 \div \text{OUT}_1) + (\text{IN}_2 \div \text{OUT}_2) + (\text{IN}_3 \div \text{OUT}_3) + (\text{IN}_4 \div \text{OUT}_4) \quad)$$

or, in other words,

$$\div \quad (\quad (B \div A) + (E \div D) + (H \div G) + (K \div J) \quad)$$

...you'll get the total weighted average—the SCORE (S). Here's the whole thing put together more compactly:

$$S = ((C \div B) x (B \div A) + (F \div E) x (E \div D) + (I \div H) x (H \div G) + (L \div K) x (K \div J))$$
$$\div ((B \div A) + (E \div D) + (H \div G) + (K \div J))$$

Then, of course, you can remove most of the parentheses, since the calculator's own Operator Priorities make them unnecessary (they were here just to make it clearer to you how the equation came about). You can also do some simplifying to reduce the equation as much as possible. After all of that, here's the result:

```
S=(C÷A+F÷D+I÷G+L÷J)÷(B÷A+E÷D+H÷G+K÷J)
```

Think it'll work? There's only one way to find out: To key this in, press ■ EQUATIONS then ▼ to get to the bottom of the list. Then start plugging away:

RCL S = (RCL C ÷ RCL A + RCL F ÷ RCL D + RCL I ÷ RCL G + RCL L ÷ RCL J) ÷ (RCL B ÷ RCL A + RCL E ÷ RCL D + RCL H ÷ RCL G + RCL K ÷ RCL J) INPUT

Now to test it:

Mailing #	Flyers Sent Out	Responses Received	Total Attitude
1	A = 20,000	B = 651	C = 14,895
2	D = 75,000	E = 4,073	F = 112,653
3	G = 48,000	H = 1,177	I = 39,448
4	J = 35,000	K = 1,435	L = 27,542

OK, suppose those were the results of your four mailings. Questions:

- What was the overall (weighted) attitude per household?
- To make a barely-feasible overall score of 25.0, how many incoming responses were necessary in mailing number 4 (no change in total points)?
- If you had sent out 10,000 more flyers (but with no more returning in mailing number 1, how would this have affected the overall score?

First things first – find the overall score (S) for these four mailings: Press EVAL and key in the variable values as you're prompted for them (sorry, – it just didn't work out to have them appear in the most convenient order):

1 4 8 9 5 INPUT 2 0 0 0 0 INPUT 1 1 2 6 5 3 INPUT 7 5 0 0 0 INPUT 3 9 4 4 8 INPUT 4 8 0 0 0 INPUT 2 7 5 4 2 INPUT 3 5 0 0 0 INPUT 6 5 1 INPUT 4 0 7 3 INPUT 1 1 7 7 INPUT 1 4 3 5 INPUT

Result: S=25.30 Looks like you can probably start some recycling.

Now play "What-If?" to answer the other two questions: ■SOLVE → → (– to move to the correct "page" of the menu). Then select K (since this is what you're solving for here – the incoming responses in mailing #4).

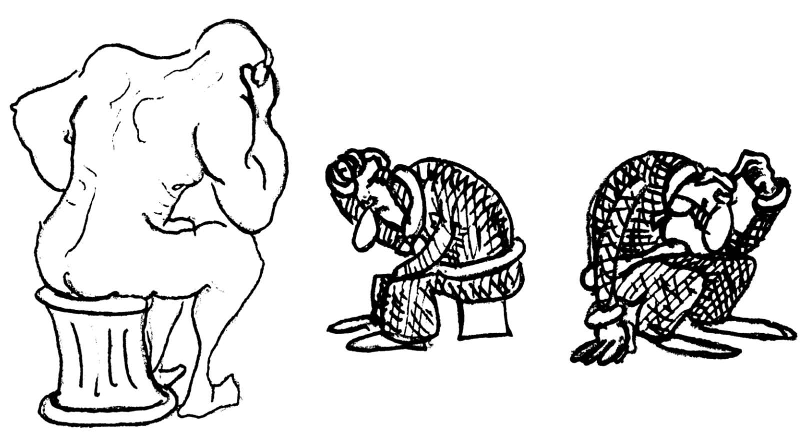
Now give 2 5 INPUT for the S? prompt and simply INPUT for all the others, since they don't change.... Result: K=1,499.54

It would have taken about 1,500 responses (instead of 1,435) on this 4th mailing to lower the overall score to exactly 25.... Say *what? More* responses to *lower* the score? Yes, because this mailing brought back a per-piece total that was lower than average. Thus, lending any *more weight* to this mailing through added responses) – without changing its point total – lowers the overall score.

And what if the first mailing had sent out 10,000 more no-response flyers? That is, what if A had been 10,000 more? No problem – you're going to solve for S again, but before you do, be sure to change K back to its actual data value – and change A to 30,000:

EVAL and INPUT though all value prompts, except A? (3 0 0 0 0 INPUT) and K? (1 4 3 5 INPUT)... Result: S=25.49

Does this make sense – a slight *rise* in the score, caused by a *drop* in the fraction responding to one of the mailing?? Yep, it can happen – if the newly-reduced weight *drops* its below-average *contribution* to the total. Think about it....



3. A simple, innocent-looking little conversion problem, right? Uh...well... take a look. This is going to call for a good dose of your reasoning power (and a good portion of your HP-22S's SOLVE power, too):

First of all, this particular conversion isn't quite the same as converting between units or something like that. In a simple units case, you would have one SOLVE equation that would relate both units and therefore do either conversion – Gallons to Liters or Liters to Gallons, for example:

G=L÷3.7854

This is simple because all you need to know is the one unit (and a conversion constant) to calculate the other.

But that's not quite the case with this surveyors' notation: Sure – if you know the Azimuth (A) then you can calculate either the Bearing (B) or the Quadrant (Q) – and the other number (Q or B, respectively) is useless and merely in the way.

But the reverse is not true: In order to calculate A, you definitely need to use *both* B and Q.

Well, that's quite a problem when it comes to writing a single, tidy SOLVE equation for these three variables. Suppose you *did* put all three variables together in one equation that correctly related them all (this one doesn't really, but pretend that it does):

A=(25xB)-(64÷Q)

This would be fine if A is the only unknown you ever want to SOLVE for – because you do indeed need both other variables to do it. But after all, this is supposed to be a conversion equation – you gotta be able to go both ways and SOLVE for any variable, right?

And look what happens if you want to SOLVE for, say, B, instead:

The only know value you need to use is A; *you don't need – and you don't want – to use Q at all.* And yet, according to the formula, the HP-22S is still going to use *both* A and Q to give you a value for B. That's wrong; it'll be the wrong answer!

So how do you tell your HP-22S to "sometimes use only certain variables" in a formula?

You don't.

It will always use the current values of *all variables except the unknown* – the one you're solving for.

Hmm!...It looks as if you won't be able to get away with just a single equation for these surveyors' conversions.

How many will you need?

Well, you definitely need one with all the variables in it – for when you *do* want to solve for A – right? That one would "Convert B and Q to A."

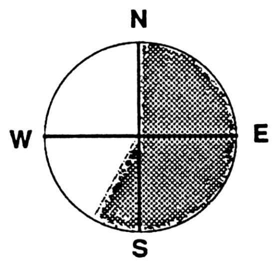
And then you also need two other separate equations that don't include the unnecessary third variable: "Convert A to B" (omitting Q) and "Convert A to Q" (omitting B).

That's three equations, instead of one.

Now it's not quite so trivial, is it? OK, so you'd better get started....

Take a simpler one first – one involving only two of the three variables: Solve for Q (the Quadrant), using a known A (Azimuth)....

...Stumped for a way to start? Then begin by noticing that the Azimuth sweeps through one quadrant for every 90°, and it's always between 0° and 360°, right? OK, maybe the answer's really simple: If you simply divide the Azimuth by 90° what will you get? That is, Q=A÷90   Try it.... An Azimuth of, say, 210° would put you somewhere in Quadrant 3, like this:



But 210÷90=2.3333.... – not 3. It looks like you'll need to *round up* the result of your division, no? And how do you round up (recall your concrete-slab-estimation days, back on pages 218-222)? You *add one to the result and then take the Integer Portion,* don't you? OK:

Q=IP(A÷90+1)

Hmm...looks pretty good – but is there any way this could fail? What about (again, as in your concrete problem) when the result of the division is already an exact integer? Do you still want to add one to it? ...Hmm...that would happen only if the Azimuth were *exactly* 0°, 90°, 180°, or 270°, in which case it's not clear exactly which Quadrant you're in anyway (i.e. you're on the line between two of them). The formula would tell you that you're in Quadrant 1 at 0°, 2 at 90°, 3 at 180°, and 4 at 270° – which makes at least as much sense as saying that 0° is in Quadrant 4, 90° in Quadrant 1, etc.

So there you go – a working formula to key in: Press ■ EQUATIONS then ▼ to the bottom of the list. Then RCL Q = ■ PARTS I P RCL A ÷ 9 0 + 1) INPUT.

Next up: Convert a known Azimuth (A) into a Bearing (B). Hmm...

Observation #1: Since you're talking about measuring and converting angles here, trig functions are natural places to look for solutions. *However,* the HP-22S doesn't measure angles the same way that surveyors measure their azimuths. As you can tell from the diagram on page 226, surveyors use a geological convention, starting at North and measuring clockwise. But the HP-22S uses the mathematical convention (see page 57) of starting at "East" and measuring counterclockwise. So whatever else is involved in converting an Azimuth into a Bearing, if you're going to use the HP-22S's trig functions to do it, you'll first need to adjust the Azimuth so that your machine knows what angle you're really talking about:

Since the two conventions measure in *opposite* directions, as you increase A, you'll be *decreasing* the corresponding mathematical angle, α, in the HP-22S. so the basic idea is that α = -A.

OK, but the two conventions are also *offset;* they differ by one quadrant, or 90°. so the relation must be something like α = -A ± 90°. But which is it – plus or minus 90°? A quick mental example will show you that it's *plus,* so α = -A + 90, or more conveniently, α = 90 - A. This is how you must adjust the Azimuth – subtract it from 90° – to even *begin* to compute the Bearing on your HP-22S.

Observation #2: Surveyors like to express angles in degrees, minutes, and seconds, but the trig functions on your calculator need to see the angles expressed in decimal form (degrees and decimal fractions of degrees). So you'd better convert A to decimal form right away; then when you finally arrive at B, convert it back to degrees, minutes and seconds:

B=HMS(...(HR(90-A))

(In case you're wondering, HMS and HR are the SOLVE-language notation for the operations you've seen in the ■ H↔HMS menu – see page 81.)

Observation #3: The Bearing is always measured from the nearest vertical axis, isn't it? And what trigonometric quantity is measured from the vertical axis? It's the horizontal distance – also known as the the x-coordinate or the *cosine* of an angle isn't it?

OK, but when you give the HP-22S a cosine and ask what angle it belongs to, (i.e. you find the ACOS), you'll get some angle between 0 and 180° – and yet the Bearing is always between 0 and 90°. Not so good....Ah – but what about an ASIN? It will give you an anwer between -90° and 90° – and if you then took an absolute value, you'd have it converted to *something equivalent* in the range of 0 to 90°, no? You're getting there....

Observation #4: If you take the cosine of any angle less than 90°, then take the ASIN of the result, you don't get the original angle back again, of course (after all, that's what ACOS does). But you do get another very useful result: You transform the x-coordinate (the cosine) into the y-coordinate (the sine) and vice versa. That means that you can transform a Bearing measured from the vertical axis into a Bearing measured from the horizontal axis!

Putting all the observations together (drum roll, please), try this on for size:

B=HMS(ASIN(ABS(COS(90-HR(A)))))

See how this set of nested functions works? Working, as always, from the innermost parentheses outward, you first convert A to a usable decimal angle, α, then take its cosine (and drop the minus sign if there is one), to find the horizontal distance this point is from the vertical axis. Next, you use ASIN to find the angle that has this distance as its *vertical* distance from the *horizontal* axis – and then you reconvert the result back to degrees, minutes, and seconds!

So key it in: Press ■ EQUATIONS then ▼, then RCL B = ■ H↔HMS →HMS ■ ASIN ■ PARTS ABS COS 9 0 − ■ H↔HMS →HR RCL A))))) INPUT.

Two down, one to go. Now that you're all warmed up, it's time for the main event – find the Azimuth by using a known Bearing and Quadrant....

Start by using what you've learned from the other two equations: You know that you can get the azimuth angle of the *beginning* of the correct Quadrant simply by subtracting 1 from the Quadrant number and then multiplying by 90°. For example, if the Quadrant is 3, you immediately know that the azimuth angle of the *beginning* of that quadrant is (3-1)x90=180°, right?

Then, notice that in *certain quadrants* (1 and 3), you can find the Azimuth itself merely by *adding the Bearing angle* to the azimuth angle of the beginning of that quadrant. That is, if the Quadrant is 1 or 3, then the beginning of the Quadrant is 0° or 180° (as you figured above) – and so A=(Q-1)x90+B.

But in the *other* two quadrants (2 and 4) – which begin at 90° and 270°, respectively – you must add the *complement* of the Bearing angle (the complement is the *difference* between the angle itself and 90°): A=(Q-1)x90+(90-B).

Hmm...how do you tell your HP-22S how to determine which formula to use? Try this: A=HMS((Q-1)x90+JxHR(B)+Kx(90-HR(B))).
And how does this help? First, notice that you need to convert B to decimal degrees (that's what the HR function does); then, at the end, you need to convert A back to HMS (degrees, minutes and seconds), the way a surveyor likes it.

But then what about those variables, J and K? What do *they* do? Well, notice that this formula would be just perfect if you could think of a way to make J=1 and K=0 whenever the Quadrant was 1 or 3 – *and* make K=1 and J=0 whenever the Quadrant was 2 or 4....

See? In each case, *only the part you want* will be added to (Q-1)x90 – because it's multiplied by 1 – while the other (unwanted) part won't be added because it's multiplied by zero! But what kind of operation could you do to get, say, J, which must 1 whenever Q is odd (1 or 3) but zero whenever Q is even (2 or 4)?

Since the distinction is between odd and even values of Q, how about the *remainder* of division by 2? Like this: J=FP(Q÷2)
FP is the Fractional Portion function, which extracts only the digits to the *right* of the decimal point (as opposed to the IP function, which – as you know – keeps only the digits to the *left* of the decimal point).

Now FP(Q÷2) gives 0 for even values of Q, all right, but it gives 0.5 for odd values of Q. So, just multiply it by 2: J=2xFP(Q÷2) Bingo!

And what about K? You want it to behave in a manner just the *opposite* to J – giving 0 for odd Q and 1 for even Q – right? That's easy: Just add 1 to Q, *changing what was odd to even and vice versa* – then perform the same test as for J: K=2xFP((Q+1)÷2) Pretty slick, eh?

All right, now put it all together:

```
A=HMS((Q-1)×90
          +2×FP(Q÷2)×HR(B)
                  +2×FP((Q+1)÷2)×(90-HR(B)))
```

And key it in: Press ■ EQUATIONS then ▼, then

RCL A = ■ H↔HMS →HMS (RCL Q − 1) × 9 0 + 2 × ■ PARTS FP RCL Q ÷ 2) × ■ H↔HMS →HR RCL B) + 2 × ■ PARTS FP (RCL Q + 1) ÷ 2) × (9 0 − ■ H↔HMS →HR RCL B)))) INPUT.

Well now...do these three formulas (to SOLVE for each of A, B and Q) actually work? How do you use them?

Like this:

Here's a table of correct equivalencies. Use these to test each of your three formulas (since you'll be working with degrees, minutes and seconds, it would be a good idea to set your display to FIX 4 decimal places – [C] ■ [DISP] FX [4]):

Azimuth	Quadrant	Bearing
60.3419	1	60.3419
92.4500	2	87.1500
150.3000	2	29.3000
246.4437	3	66.4437
345.0000	4	15.0000
358.5959	4	1.0001

Remember that in each formula, the unknown you're solving for appears by itself on the left – so you can use the [EVAL] key. Remember also that when solving for Q (the Quadrant), or B (the Bearing), the only input required will be A (the Azimuth), whereas you'll be prompted for *both* B and Q to solve for A.

Here, for example, would be the keystrokes you would use to test, say, the third entry in the table:

■ [EQUATIONS] then ■ [▲] to the Q=... formula. Then [EVAL] and [1] [5] [0] [•] [3] [INPUT] for the A? prompt. <u>Result</u>: Q=2.0000 Yep – that checks.

Next, press ■ [EQUATIONS] then [▼] to the B=... formula. Then [EVAL] and just [INPUT] for the A? prompt, since the A-register keeps its value from the preceding problem. <u>Result</u>: B=29.3000 That looks OK, too.

Finally, press ■ [EQUATIONS] then [▼] to the A=... formula. Then [EVAL] and [INPUT] for both the Q? and B? prompts, since they both now have correct values from the previous problems. <u>Result</u>: A=150.3000 Everything checks – at least for this set of values. Now try the others on your own....

4. Just for laughs (and that's what this final problem is really all about), suppose you express this codfish population volume not simply in cubic meters but in terms of the resulting rise in the world sea level....

All right: The oceans' surface areas total roughly 140 million (that's 1.4×10^8) square miles. For a 1-mile rise in the world's oceans, then, you'd need an overall volume increase of about 140 million cubic miles, right?

So, at .1 cubic feet per fish, which comes to $5280^3 \div .1$, or 1.5 trillion fish per cubic mile, it would therefore take 140 million x 1.5 trillion, or *210 quintillion* (that's 2.1×10^{20}) codfishes to raise the world's ocean level by just one mile. That is, the level, L, could be expressed as a function of the codfish population, P:

L=P÷2.1E20 miles (remember how this E notation works – page 44?)

That's an awful lot of codfish to raise the sea level by even one mile. Surely there'd be enough room....

OK, so what's the population of the codfish going to be at the end of any given year, Y? Well, assuming that about half of each new generation of codfish is male and half is female – and that this new generation pairs up in happily wedded bliss (never mind the incest and the hassles at family reunions) – that's about 10 billion new spawnings every year *for every spawning of the previous year*. So every year, the population increases about 10-billion-fold. You could therefore write an expression for the codfish population, P, as a function of the number of years, Y, since the original pair, like this:

$$P = (10 \text{ billion})^Y, \text{ or } P=(10^{10})^Y, \text{ or } P=1E10\text{^}Y$$

Now put this expression for P into the one above, for L, and here's what you get:

L=1E10^Y÷2.1E20

All right – that's one of the formulas you need – the volume of codfish (expressed in miles of sea level rise). But you also want to find a formula which tells you how *fast* this volume is expanding.

This isn't quite as easy, because it takes a little knowledge of differential calculus. But don't panic – there's a fairly simple result to it all:

It turns out that if the volume formula is V=1E10^Y÷2.1E20 miles, then the rate of increase of this volume is

R=23÷(2.1E20)x1E10^Y miles per year, which (if you do the division) comes to

1.1E-19x1E10^Y miles per year, or (converting years to seconds)

1.1E-19÷(365x24x3600)x1E10^Y *miles per second,* which is

3.5E-27x1E10^Y miles of ocean rise per second.

So your rate formula in its simplest form would be

R=3.5E-27x1E10^Y

So key in this formula and the one on the bottom of the previous page (you may need to clear variables or even other equations to make room). To do so, press:

■ [EQUATIONS] [▼] [RCL] [V] [=] [1] ■ [E] [1] [0] [y^x] [RCL] [Y] [÷] [2] [•] [1] ■ [E] [2] [0] [INPUT], and

[▼] [RCL] [R] [=] [3] [•] [5] ■ [E] [+/−] [2] [7] [×] [1] ■ [E] [1] [0] [y^x] [RCL] [Y] [INPUT]

Now you're ready to test these formulas....

Begin the tests by asking some pertinent questions: By how much and how fast would the ocean be rising after each of, say, the first three years? (With all the "thumb-nail" numbers used in the formulas, change your display format to something more appropriate to the relative imprecision here – like SCientific format, with 1 decimal place – press [C] to escape the EQUATIONS menu and then ■[DISP], SC [1].)

Now find the ocean rise due to codfish after one year. Press: ■[EQUATIONS] ■[▲] (to point to the Volume formula). Then [EVAL] and [1][INPUT] for Y?

Results: V=4.8E-11 (miles).
That's about 3 millionths of an inch in sea level rise – not so you'd notice.

How about the rate of this rise? Press ■[EQUATIONS] [▼] (to point to the Rate formula). Then [EVAL] and [INPUT] for Y?

Results: R=3.5E-17 (miles per second).
That's about 70 millionths of an inch in sea level rise per year – not exactly a spectator sport.

What about after the second year? Press: ■[EQUATIONS] ■[▲] (to point to the Volume formula). Then [EVAL] and [2][INPUT] for Y?

Results: V=4.8E-1 (miles).
That's a 2500-foot rise in sea level (Florida real estate market slumps badly).

And how about the rate of this second-year rise? Press ■[EQUATIONS][▼] (to point to the Rate formula). Then [EVAL] and [INPUT] for Y?

Results: R=3.5E-7 (miles per second).
That's rising at about 1.3 inches per minute (Tibetan tourism booms).

And then after the *third* year?

Press: ■EQUATIONS ■▲ (to point to the V olume formula). Then EVAL and 3 INPUT for Y?

Results: V=4.8E9 (miles – nearly 5 billion miles).

And how fast is this going? Press ■EQUATIONS ▼ (to point to the R ate formula). Then EVAL and INPUT for Y?

Results: R=3.5E3 (3500 miles per second).

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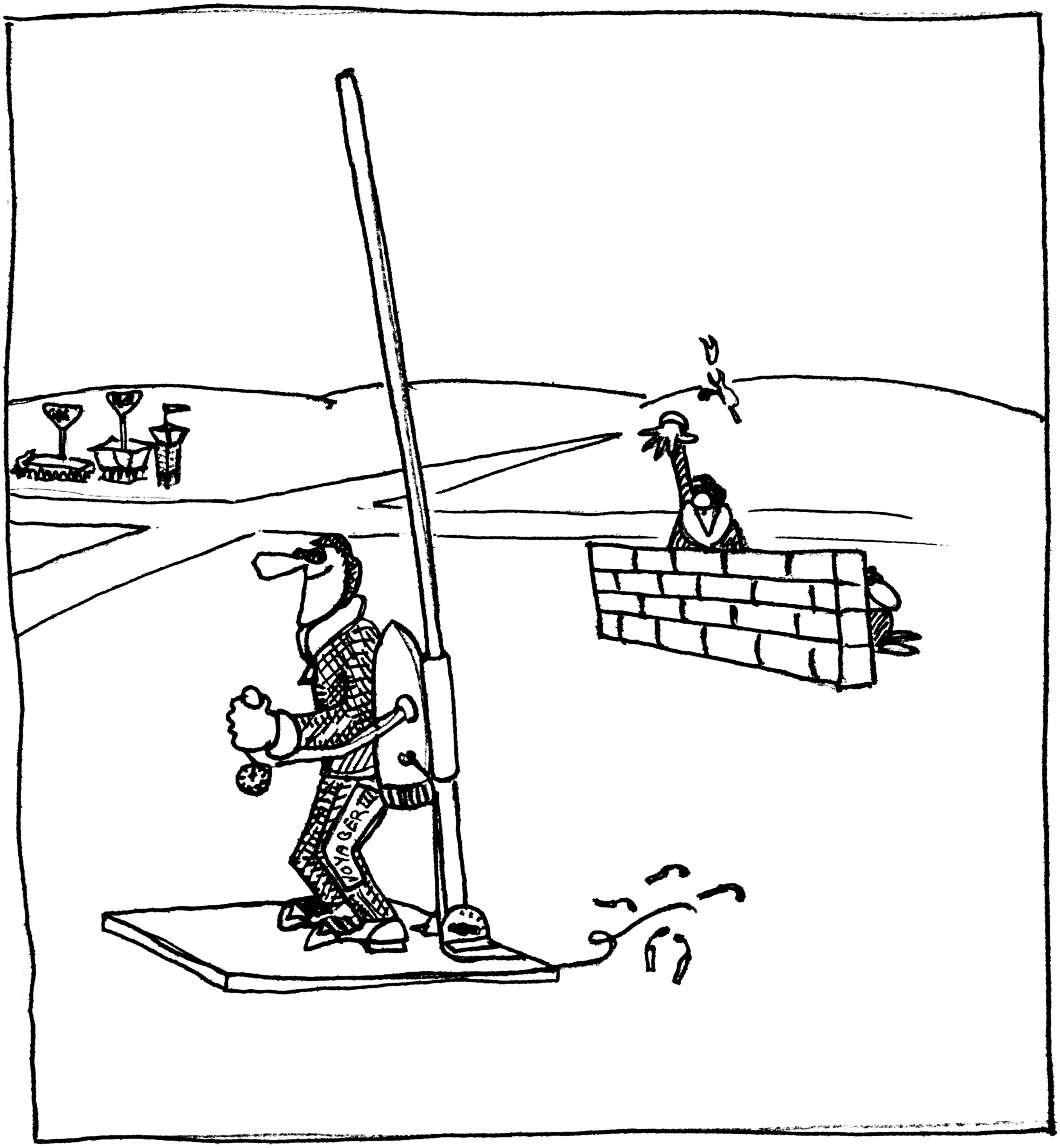
And so on.... In fact, after just 3.14 years (less than 38 months) of such unlimited codfish reproduction, the earth would be buried 120 billion miles deep in cod, and the radius of this galactic fish-ball would be expanding outward at half the speed of light.

This would tend to reduce the market price of codfish.



Points Of Departure

"The Book Stops Here"

That's about it – the end of the Easy Course, the hangar door – for now. But you have only just begun. You've just finished your basic training and become a fully-qualified HP-22S pilot.

But of course, it doesn't mean you've seen it all.

As we said at the start, this book doesn't even pretend to cover all of the many uses of your machine. In fact, we've totally ignored many of its useful functions (as we also promised at the beginning), because we thought those areas were already very well-explained in the manual that came with your calculator. Because you know how to work with menus and play "What-If?", they should now all be fairly straightforward.

So if you want to explore those other topics, you're now ready and able, so do it! Explore them – enjoy them – and get the most out of your HP-22S!

Notes (Yours)

So how did you like this book? Do you find yourself wishing we had covered other things? More of the same things?

Or did you find any mistakes, typos, or other little mysteries we ought to know about (yes, we usually have a few innocent-looking little boo-boos. Did any of them leap out and grab you by the lapels)?

Please let us hear from you. Your comments are our only way of knowing whether these books help or not. And we always read and reply to our mail – so drop us a note (there's room for comments on the back of the order forms ☞):

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Anyway, thanks for going along for the ride. When all is said and done, we hope that this book will have said and done a lot of it for you, helping you and your HP-22S explore all sorts of calculating routes and adventures – and remain good friends all along the way.

By the way, if you liked this book, here's a full list of books that you or someone you know might enjoy

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