HEWLETT-PACKARD

HP-27 Owner's Handbook

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HP-27
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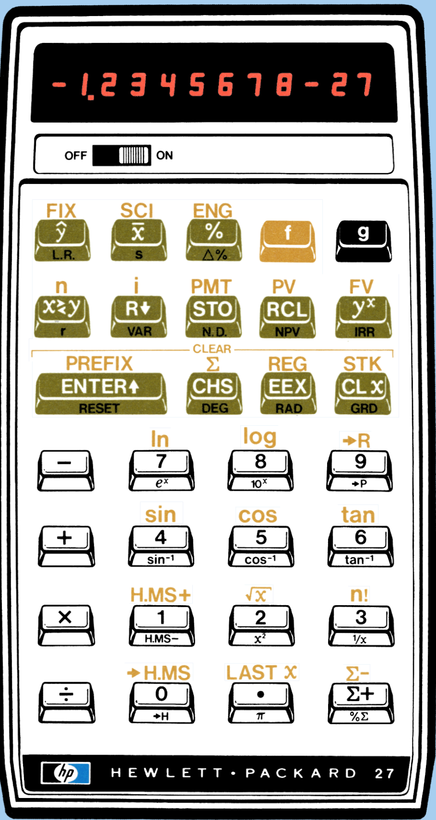


OFF [switch] ON Power switch **(page 11)**.

ŷ Linear estimate **(page 76)**.

FIX Fixed decimal display format **(page 24)**.

L.R. Linear regression. Calculates linear function between two or more points **(page 73)**.

x̄ Computes the mean or arithmetic average of two variables **(page 77)**.

SCI Scientific display format **(page 25)**.

s Calculates the standard deviation for two variables **(page 79)**.

% Calculates x% of y **(page 61)**.

ENG Engineering display format **(page 26)**.

Δ% Computes % difference between two numbers **(page 63)**.

f Prefix key to select alternate function in gold above a key **(page 11)**.

g Prefix key to select alternate function in blue on lower face of a key **(page 11)**.

x≷y Exchanges contents of X- and Y-registers **(page 35)**.

n Number of periods in a financial transaction **(page 66)**.

r Correlation coefficient for L. R. data **(page 75)**.

R↓ Rolls down contents of stack for viewing in display **(page 33)**.

i Interest rate per period **(page 66)**.

VAR Computes the variance (s^2) of two variables **(page 80)**.

STO Stores displayed value in one of 10 storage registers **(page 45)**.

PMT Payment amount per period **(page 66)**.

N.D. Calculates density function and upper tail area under a normal distribution curve **(page 81)**.

RCL Recalls (copies) stored number onto display **(page 45)**.

PV Present value or the amount of money at the start of financial term **(page 66)**.

NPV Calculates the net present value of uneven cash flows **(page 181).**

y^x Raises the number in the Y-register to the power of the number in the display **(page 51)**.

FV Future value or amount received/paid at end of term **(page 67)**.

IRR Calculates the internal rate of return for uneven cash flows **(page 183)**.

ENTER↑ Copies number in displayed X-register into Y-register; also, separates successive numerical entries **(page 36)**.

PREFIX Clears a prefix function pressed erroneously **(page 13)**.

RESET Resets financial status indicators **(page 198)**.

CHS Changes sign of display number **(page 12)**.

Σ Clears statistical data in registers 4 thru 9 **(page 71)**.

DEG Sets decimal degree mode for trigonometric functions **(page 56)**.

EEX Enter exponent. After pressing, next numbers keyed in are exponents of 10 **(page 30)**.

REG Clears storage registers 0 thru 9 **(page 48)**.

RAD Sets radians mode for trigonometric functions **(page 56)**.

CL x Clears display (X-register) **(page 12)**.

[STK] Clears the 4-register stack **(page 35)**.

[GRD] Sets grads mode for trigonometric functions **(page 56)**.

[+] [−] [×] [÷] Arithmetic operations **(page 13)**.

[ln] Computes natural logarithm (base e) of value in display **(page 55)**.

[e^x] Natural antilog. Raises e to the power of value in the display **(page 55)**.

[log] Computes common logarithm (base_{10}) of value in display **(page 55)**.

[10^x] Raises 10 to the power of value in display **(page 55)**.

[→R] Converts polar magnitude and angle in X- and Y-registers to rectangular x and y coordinates **(page 60)**.

[→P] Converts x, y rectangular coordinates to polar magnitude and angle **(page 60)**.

[sin] [cos] [tan] Calculates the sine, cosine or tangent of value in displayed X-register **(page 57)**.

[$\sin^{-1}$] [$\cos^{-1}$] [$\tan^{-1}$] Calculates the arc sine, arc cosine, or arc tangent of value in display **(page 57)**.

[H.MS+] [H.MS−] Adds and subtracts degrees in hours, minutes, seconds **(page 58)**.

[$\sqrt{x}$] Computes positive square root on number in display **(page 53)**.

[x^2] Computes square of number in display **(page 52)**.

[n!] Calculates factorial of any number up to 69 **(page 54)**.

[$1/x$] Calculates reciprocal of the number in the display **(page 52)**.

[→H.MS] Converts decimal degrees to hours, minutes, seconds **(page 57)**.

[→H] Converts hours, minutes, seconds to decimal hours (degrees) **(page 58)**.

[LAST x] Recalls number displayed before the previous operation back into the displayed X-register **(page 43)**.

[π] Places value of pi (3.14159...) into displayed X-register **(page 54)**.

[Σ+] Sums numbers and products of numbers in storage registers 4 thru 9 **(page 72)**.

[Σ−] Subtracts an incorrect value from [Σ+] entries in registers 4 thru 9 **(page 73)**.

[%Σ] Finds what % one number is of another or of the total **(page 64)**.

Introduction

At last, someone has made "a calculator for all seasons." The HP-27 solves the problems that you, the multi-dimensional professional, encounter everyday. Whether you are juggling budgets, answering a technical question, forecasting trends, checking lab results, or analyzing market data, the HP-27 solves problems fast and accurately to make your job easier.

It's three calculators in one. Use it to solve:

- Mathematical and scientific calculations
- Business and financial problems
- Statistical calculations.

At work, at home, or on the road. . .whether you are tackling the company's problems or your own, the HP-27 puts answers at your fingertips. Over a million HP business and scientific pocket calculators are in use throughout the world, so you're in good company with HP!

To get the most from your calculator, take the time to read through this handbook and work the sample problems. The more confidence you have in your HP-27 and your own understanding of it, the more profitably and often you will use it. So, let's get started. . . .


OFF ON
FIX SCI ENG
f
g
%
PMT PV FV
STO
RCL
CLEAR
Σ
REG STK
PREFIX
ENTER ↑
CHS
EEX
CLX
log
ln
7
8
9
tan
cos
sin
4
5
6
n!
H.MS+
1
2
3
LASTx
Σ+
0
HEWLETT • PACKARD 27
hp
1820-1029
C-3540-E

Getting Started

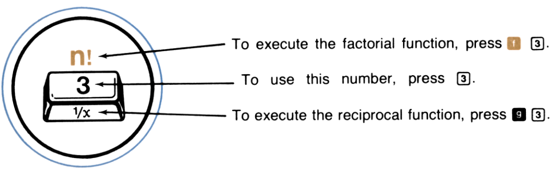
Your HP-27 is shipped fully assembled, including a battery. You can begin using your calculator immediately by connecting the cord from the ac adapter/battery charger to the calculator and plugging the charger into an ac outlet. If you want to use your HP-27 on battery power alone, you should charge the battery for 6 hours first. Whether you operate from battery power or from power supplied by the charger, the battery *must* always be in the calculator.

To begin, slide the OFF-ON switch to ON. The display reads 0.00.

Keyboard

Most keys on the keyboard perform three functions. One function is indicated by the symbol on the flat face of the key, another by the black symbol on the slanted key face, and a third by the gold symbol written above the key.

- To select the function printed in black on the slanted face of the key, first press the black prefix key g, then press the function key.
- To select the function printed on the flat face of the key; press the key.
- To select the function printed in gold above the key, first press the gold prefix key f, then press the function key.



In this handbook, the selected key function will appear in the appropriate color (gold or black) outlined by a box, like this: sin sin^{-1}.

Keying in Numbers

Key in numbers by pressing the number keys in sequence, just as though you were writing on a piece of paper. The decimal point must be keyed in if it is part of the number.

For example, key in 148.84 by pressing the keys:

Press	Display
1 4 8 • 8 4	148.84

The number 148.84 appears in the display.

Negative Numbers

To key in a negative number, press the keys for the number, then press CHS *(change sign)*. The number, preceded by a minus (–) sign, will appear in the display. For example, to change the sign of the number now in the display:

Press	Display
CHS	–148.84

You can change the sign of either a negative or a positive number in the display. For example, to change the sign of the –148.84 now in the display back to positive:

Press	Display
CHS	148.84

Notice that only negative numbers are given a sign in the display.

Clear X

You can clear any numbers that are in the display by pressing CLx *(clear x)*. This key erases the number in the display and replaces it with 0.00.

Press	Display
CLx	0.00

If you make a mistake while keying in a number, clear the entire number string by pressing CLx. Then key in the correct number.

An important point about your HP-27 is that it isn't necessary to clear it between arithmetic calculations. Intermediate answers are saved, but data from previous problems is automatically pushed out of the way.

When you turn the power switch OFF, then ON again, the entire calculator is cleared.

Prefix Clear

If you inadvertently press a prefix key (g or f) when you really want a primary key, press f PREFIX, then press the correct key. If you press the wrong prefix key (f when you actually want g), just pressing g corrects that by overwriting the f.

Performing Simple Arithmetic

Whenever you add, subtract, multiply or divide, you work with two numbers and an arithmetic operation (+, −, × or ÷). Likewise, you cannot add, subtract, multiply or divide unless there are two numbers present in the calculator.

After both numbers are in the calculator, press the operation key. Your answer immediately appears on the display. To place two numbers into the calculator and perform simple arithmetic:

1. Key in the first number.
2. Press ENTER↑ to separate the first number from the second.
3. Key in the second number.
4. Press +, −, × or ÷ to perform the operation.

For example, to add 12 and 3:

Press	Display	
12	12.	The first number.
ENTER↑	12.00	Separates the first number from the second.
3	3.	The second number.
+	15.00	The operation is executed and your answer appears on the display.

The four arithmetic functions are all performed the same way:

To Solve	**Press**	**Display**
24 + 3	24 ENTER↑ 3 +	27.00
24 − 3	24 ENTER↑ 3 −	21.00
24 × 3	24 ENTER↑ 3 ×	72.00
24 ÷ 3	24 ENTER↑ 3 ÷	8.00

In the problems above, you pressed 24 ENTER↑ 3. Try the same number sequence *without* the ENTER↑ step. What appears on the display? It's readily apparent that the ENTER↑ key separates the first number from the second.

Sample Problems: Ready to try some problems on your own? The correct answer is given; try figuring the keystroke sequences by yourself.

Problem	**Answer Displayed**
14 × 6 =	84.00
144 ÷ 6 =	24.00
1/25 =	0.04
A customer buys 12 items at $19.95 each. What is the total sale?	239.40

Chain Calculations

The process of solving a chain calculation is as natural as if you were working it out with pencil and paper. The calculator itself takes care of the hard part. The HP-27 not only displays intermediate answers; but it also retains these results until you need them, then inserts them into the calculation.

For example, solve (12 + 3) × 7.

If you work the problem with a pencil and paper, you would first calculate the intermediate result of (12 + 3). . .

$$\overset{15}{\cancel{(12+3)}} \times 7 =$$

Then you would multiply 15 by 7.

$$\overset{15}{\cancel{(12+3)}} \times 7 = 105$$

With the HP-27 you work through the problem exactly the same way—always concentrating on just two numbers at a time. First, you solve for the intermediate result. . .

To Solve	**Press**	**Display**
$(12 + 3)$	12 ENTER↑ 3 $\boxed{+}$	15.00

. . .then solve for the final answer. You don't need to press ENTER↑ to store the intermediate result—the HP-27 stores it automatically when you key in the next number. To continue. . .

To Solve	**Press**	**Display**
$\overset{15}{\cancel{(12+3)}} \times 7$	7 $\boxed{\times}$	105.00

Now, try this problem. Remember that you have to press ENTER↑ to separate the first number from the second. After that, as the calculator performs each operation, it retains the new number.

To Solve	**Press**	**Display**
$\dfrac{2+3}{10}$	2 ENTER↑ 3 $\boxed{+}$ 10 $\boxed{\div}$	0.50

Following the rules of mathematics, you solve the expressions in innermost parentheses first. Then move through the equation as you did before, one successive number and function at a time.

To Solve	**Press**	**Display**
$3\,(16 - 4)$	16 ENTER↑ 4 $\boxed{-}$ 3 $\boxed{\times}$	36.00
$2\left(\dfrac{50 - 14}{12}\right)$	50 ENTER↑ 14 $\boxed{-}$ 12 $\boxed{\div}$ 2 $\boxed{\times}$	6.00

Problems that are even more complicated can be solved in the same manner, using automatic storage of results. For example, to solve $(2 + 3) \times (4 + 5)$, you would:

First add 2 and 3:

To Solve	Press	Display
$\overset{5}{\cancel{(2+3)}} \times (4+5)$	2 ENTER↑ 3 $\boxed{+}$	$\boxed{5.00}$

Then add 4 and 5:

$\overset{5}{\cancel{(2+3)}} \times \overset{9}{\cancel{(4+5)}}$	4 ENTER↑ 5 $\boxed{+}$	$\boxed{9.00}$

The 5.00 and the 9.00 are already in the calculator; so for the final answer, simply multiply them.

	Press	Display
$\overset{5}{\cancel{(2+3)}} \times \overset{9}{\cancel{(4+5)}}$	$\boxed{\times}$	$\boxed{45.00}$

Not once did you have to write down a "subtotal" or intermediate result. The HP-27 automatically remembered them.

Now, try solving these sample problems. (If you have trouble obtaining the correct answers, review the last few pages.)

Sample Problems:

$$(2 \times 3) + (4 \times 5) = \boxed{26.00}$$

$$\frac{(14 - 12) \times (18 - 12)}{(9 - 7)} = \boxed{6.00}$$

$$(17 - 12) \times 4 \div (10 - 5) = \boxed{4.00}$$

Simple Functions

In spite of the dozens of functions available on the HP-27 keyboard, you will find the calculator simple to operate by using a single, all-encompassing rule: *When you press a function key, the calculator immediately executes the function written on that key.*

> Pressing a function key causes the calculator to immediately perform that function.

For example, to calculate the square root of 148.84 merely:

Press	**Display**
148.84	148.84
f	148.84
√x	12.20

To square the result:

Press	**Display**
g	12.20
x^2	148.84

√x and x^2 are examples of one-number function keys; that is, keys that execute upon a *single* number. Except for the financial and statistical keys, all function keys on the HP-27 operate upon either one or two numbers at a time.

One-Number Functions

To use any one-number function key:

1. Key in the number.
2. Press the function key (or press the applicable prefix key, then the function key).

For example, to use the one-number function [1/x] key, you first key in the number represented by x, then press the function key. To calculate ¼, key in 4 (the x-number) and press [g] [1/x].

Press	Display
4	4.
[g]	4.00
[1/x]	0.25

Now try these other one-number function problems. Remember, *first key in the number, then press the function:*

Problem		Answer	
$\frac{1}{36}$	=	0.03	
$\sqrt{2500}$	=	50.00	
10^5	=	100000.00	(Use the [10^x] key.)
$\sqrt{3204100}$	=	1790.00	
log 12.58925411	=	1.10	
71^2	=	5041.00	

Two-Number Functions

Two-number functions are functions that must have two numbers present in order for the operation to be performed. [+], [−], [×] and [÷] are examples of two-number function keys. So are [y^x], [%], [Δ%], [→R] and [→P].

Two-number functions work the same way as one-number functions—that is, the operation occurs when the function key is pressed. Therefore, *both numbers must be in the calculator before the function key is pressed:*

1. Key in the base number.
2. Press [ENTER↑] to separate the first number from the second.
3. Key in the second number.
4. Press the function key to perform the function.

For example, to calculate 3^4,

Press	Display
3 ENTER↑ 4 y^x	81.00

When working with any function key (including y^x), you should remember that the displayed number is always designated by the x symbol on the function key.

> The number displayed is always x.

Another two-step function is %. You follow the same procedure to find 25% of 167.

Press	Display
167 ENTER↑ 25 %	41.75

(Percentage calculations are explained in detail in section 3.)

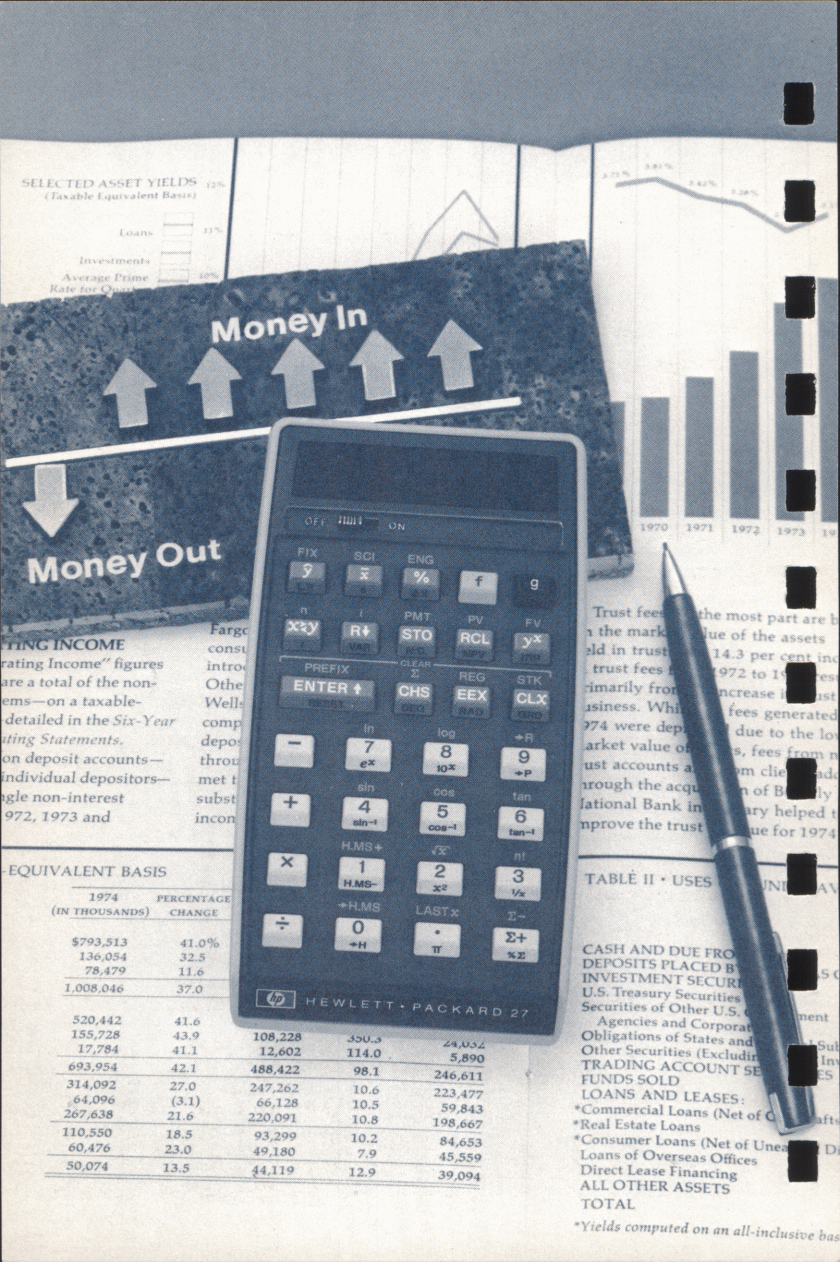
* * * * * * * * * * * * * *

Now that you've learned how to use the calculator, you can begin to fully appreciate the benefits of the Hewlett-Packard logic system. With this system, you enter numbers using a parentheses-free, unambiguous method called RPN (Reverse Polish Notation).

- *You never have to work with more than one function at a time.* The HP-27 cuts problems down to size instead of making them more complex.
- *Pressing a function key immediately executes the function.* You work naturally through complicated problems, with fewer keystrokes and less time spent.
- *Intermediate results appear as they are calculated.* There are no "hidden" calculations, and you can check each step as you go.

- *Intermediate results are automatically handled.* You don't have to write down long intermediate answers when you work a problem.
- *You can calculate in the same order you do with pencil and paper.* You don't have to think the problem through ahead of time.

Before tackling more problems, take a moment to look further at your calculator. The more you understand your HP-27, the more you will use it efficiently and confidently every day.


SELECTED ASSET YIELDS
(Taxable Equivalent Basis)
Money In
Money Out
HEWLETT • PACKARD 27
TABLE II • USES
CASH AND DUE FRO
LOANS AND LEASES:
ALL OTHER ASSETS
TOTAL

Section 2

The Display, Stack and Storage Registers

Controlling the Display

In the HP-27, numbers in the display normally appear rounded to only two decimal places. For example, the fixed constant [π], which is actually in the calculator as 3.141592654, normally appears in the display as [3.14] unless you tell the calculator to show you the number rounded to a greater or lesser number of decimal places.

Although a number is normally shown to only two decimal places, the HP-27 always computes internally using each number as a 10-digit mantissa and a two-digit exponent of 10. For example, when you compute 2 × 3, you see the answer to only two decimal places:

Press	**Display**
2 [ENTER↑] 3 [×]	[6.00]

However, inside the calculator all numbers have 10-digit mantissas and two-digit exponents of 10. So the calculator actually calculates using full 10-digit numbers:

$$2.000000000 \times 10^0 \text{ [ENTER↑] } 3.000000000 \times 10^0 \text{ [×]}$$

yields an answer that is actually carried to 10 digits:

[6.000000000 × 10⁰]

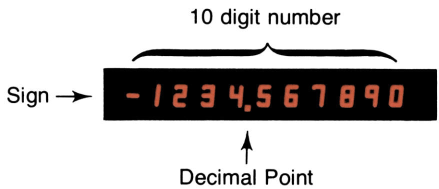
You see only these digits. . .

but these digits are also present.

Display Formatting Keys

There are three keys, [FIX], [SCI], and [ENG] that allow you to control the manner in which numbers are displayed in the HP-27. [FIX] allows numbers to be displayed in "business" (fixed decimal point) format. [SCI] displays numbers in scientific notation. [ENG] displays numbers in engineering notation with exponents of 10 shown in multiples of three (e.g., 10^3, 10^{-6}, 10^9). Display control alters only the *manner* in which numbers are displayed in the HP-27. The actual number itself is not altered by any of the display control keys. No matter what notation you select, these rounding options affect the display only—the HP-27 always calculates internally with a 10-digit number multiplied by 10 raised to a two-digit exponent.

Fixed Point (Business) Display



Using fixed point display, you can specify the number of places to be shown after the decimal point. It is selected by pressing f FIX, followed by a number key to specify the number of decimal places (0 thru 9) to which the display is to be rounded. The displayed number begins at the left side of the display and includes trailing zeros within the setting selected. When the calculator is turned OFF, then ON, it "wakes up" in fixed point notation with the display rounded to two decimal places. For example:

Press	**Display**	
(Turn the calculator OFF, then ON.)	0.00	
123.4567 ENTER↑	123.46	Display is rounded off to two decimal places. Internally, however, the number maintains its original value of 123.4567.
f FIX 4	123.4567	
f FIX 7	123.4567000	
f FIX 0	123.	
f FIX 2	123.46	Normal FIX 2 display.

Scientific Notation Display

(This means $-1.2345678 \times 10^{-23}$)

In scientific notation, each number is displayed with a single digit to the left of the decimal point followed by a specified number of digits (up to seven) to the right of the decimal point and multiplied by a power of 10. It is particularly useful when working with very large or small numbers.

Scientific notation is selected by pressing f SCI followed by a number key to specify the number of decimal places to which the number is rounded. The display is left-justified and includes trailing zeros within the selected setting. For example:

Press	Display	
123.4567 ENTER↑	123.46	Normal FIX 2 display.
f SCI 2	1.23 02	Display 1.23×10^2.
f SCI 4	1.2346 02	Display 1.2346×10^2.
f SCI 7	1.2345670 02	Display 1.2345670×10^2.

In scientific notation, although the calculator displays a maximum of seven digits after the decimal point, it always maintains the full 10-digit number and the two-digit exponent of 10 internally. The portion of the number that is not displayed affects the rounding of the displayed portion.

For example, if you key in 1.000000094 and specify full scientific notation display (f SCI 7), the calculator display rounds off to the seventh digit after the decimal point:

1.000000094

↑

Calculator rounds to this digit in SCI 7.

Press	Display
1.000000094	1.000000094
f SCI 7	1.0000001 00

In SCI 8, the display rounds off to the eighth digit after the decimal point, but you can see only out to seven digits after the decimal:

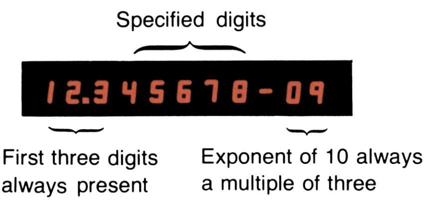
1.000000094

You see to here. . . . . .but the calculator display rounds to here in SCI 8.

Press	Display
f SCI 8	1.0000000 00

You can see that if you had keyed in 1.000000095, SCI 8 would also have caused the display to round the seventh and final digit after the decimal to a one (1).

Engineering Notation Display



Engineering notation allows all numbers to be shown with exponents of 10 that are multiples of three (e.g., 10^3, 10^{-6}, 10^9). This is particularly useful in scientific and engineering calculations, where units of measure are often specified in powers of 10 that are multiples of three. See the following prefix chart.

Factor by which unit is multiplied	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Engineering notation is selected by pressing f ENG followed by a number key. In engineering notation, the first three digits are always present, and the number key specifies the number of *additional* digits displayed after the first three. For example:

Press	Display	
0.000012345	0.000012345	
f ENG 0	12.3 -06	Engineering notation display. First three digits visible and power of 10 is the proper multiple of three.
f ENG 2	12.345 -06	The number key specifies the number of digits displayed *beyond* the first three.
f ENG 4	12.34500 -06	

Notice that because the first three digits are always present, the greatest number of additional digits that can be specified in engineering notation is five.

Press	Display	
f ENG 5	12.345000-06	Maximum number of digits displayed.

f ENG 6	12.345000-06	No change in display.
f ENG 7	12.345000-06	No change in display.

Rounding of displayed numbers in ENG 5 and ENG 6 is similiar to the rounding of numbers in SCI 7 and SCI 8, discussed earlier. As with all display formats, engineering notation display does not affect the *actual* number as it is held internally by the calculator, but only alters the manner in which the number is displayed.

When engineering notation has been selected, the decimal point shifts to show the mantissa as units, tens, or hundreds in order to maintain the exponent of 10 as a multiple of three. For example, multiplying the number now in the calculator by 10 causes the decimal point to shift to the right without altering the exponent of 10:

Press	Display	
f ENG 0	12.3 -06	
10 ×	123. -06	Decimal point shifts. Power of 10 remains at 10^{-6}.

However, multiplying again by 10 causes the exponent to shift to another multiple of three and the decimal point to move to the units position:

Press	Display	
10 ×	1.23 -03	Decimal point shifts. Power of 10 shifts to 10^{-3}.

Automatic Display Switching

The HP-27 switches the display from fixed point notation to full scientific notation (SCI 7) whenever the number is too large or too small to be seen with a fixed decimal point. This feature keeps you from missing unexpectedly large or small answers. For example, if you try to solve $(.05)^3$ in normal FIX 2 display, the answer is automatically shown in scientific notation.

Press	Display	
CLx	0.00 00	ENG 0 from previous example.
f FIX 2	0.00	Normal FIX 2 display.
.05 ENTER↑	0.05	
3 y^x	1.2500000-04	Display automatically switched to SCI 7 to show answer.

Another way of displaying the answer would be 0.000125, but in normal FIX 2 display, you would have seen only 0.00 displayed.

After automatically switching from fixed to scientific, when a new number is keyed in or CLx is pressed, the display automatically reverts back to the fixed point display originally selected.

The HP-27 also switches to scientific notation if the answer is too large ($> 10^{10}$) for fixed point display. The display will not switch from fixed if you solve 1582000 × 1842:

Press	Display	
1582000 ENTER↑	1582000.00	
1842 ×	2914044000.	Fixed decimal point display.

However, if you multiply the result by 10, the answer is too large for fixed point notation, and switches automatically to scientific notation:

Press	Display	
10 ×	2.9140440 10	Scientific notation display.

Notice that automatic switching is between fixed and scientific notation display modes only—engineering notation display must be selected from the keyboard.

Keying in Exponents of Ten

You can key in numbers multiplied by powers of 10 by pressing EEX *(enter exponent of ten)*. For example, to key in 15.6 trillion (15.6×10^{12}), and multiply it by 25:

Press	Display	
15.6	15.6	
EEX	15.6 00	
12	15.6 12	(This means 15.6×10^{12}.)

Now Press	Display
ENTER↑	1.5600000 13
25 [×]	3.9000000 14

You can save time when keying in exact powers of 10 by merely pressing EEX and then pressing the desired power of 10. For example, key in 1 million (10^6) and divide by 52.

Press	Display	
EEX	1. 00	You do not have to key in the number 1 before pressing EEX when the number is an exact power of 10.
6	1. 06	
ENTER↑	1000000.00	Since you have not specified scientific notation, the answer reverts to fixed point notation when you press ENTER↑.
52 [÷]	19230.77	

To see your answer in scientific notation with six decimal places:

Press	Display
f SCI 6	1.923077 04

To key in negative exponents of 10, key in the number, press EEX, press CHS to make the exponent negative, then key in the power of 10. For example, key in Planck's constant (h)—roughly, 6.625×10^{-27} erg sec.—and multiply it by 50.

Press	Display	
CL x	0.000000 00	
f FIX 2	0.00	
6.625 EEX	6.625 00	
CHS	6.625 00	
27	6.625 -27	
ENTER↑	6.6250000 -27	
50 ⊠	3.3125000 -25	Erg sec.

When you use the EEX key, the HP-27 displays each number as an eight-digit mantissa and a two-digit exponent of 10. In a few cases, a number may have to be altered slightly in form before you can key it in using the EEX key:

1. If you key in a number whose mantissa contains more than eight digits to the left of the decimal point, the EEX key is overridden and does not operate. Begin again and key in the number in a form that displays the mantissa with eight digits or less to the left of the decimal point before pressing the EEX key. Thus, $123456789.1 \times 10^{23}$ could be keyed in as $12345678.91 \times 10^{24}$.
2. If you key in a number whose first significant digit occurs *after* the first eight digits of the display (e.g., $0000.000025 \times 10^{55}$) all zeros are displayed but the number is operated on correctly internally. To view the number, you may prefer to key it in as $0000.00025 \times 10^{54}$ or as 0.000025×10^{55}, then proceed using the EEX key.

Overflow and Underflow Displays

When the number in the display is greater than 9.9999999×10^{99}, the HP-27 displays all 9's to indicate that the problem has exceeded the calculator's range:

9.9999999 99

If an excessively large negative number is calculated (greater than $-9.9999999 \times 10^{99}$), the overflow indicator is all 9's preceded by a minus sign.

-9.9999999 99

Very small numbers (less than 10^{-100}) are displayed as 0.00. If you press f SCI 7, the display reads:

00000000 00

Error Display

If you attempt an improper or impossible operation, the word Error appears on the display. For example, enter a number and try to divide by zero. (Go ahead, try it.) The calculator recognizes this as an illegal operation. Other examples of improper operations are square root of a negative number or 0 raised to a negative power.

To clear the error display, press CL x.

Low Power Display

When the batteries get low, several decimal points will appear on the display interspersed among the numbers. Where the true decimal point appeared, there will be a blank. For example, if you have the following number on the display:

Ordinary display: 17.45

the low-power indication will look like this:

Low-power display: 1.7 4.5.........

blank

This means you have approximately one minute of operating time left. Then you must either charge the battery or insert a fully-charged spare battery pack. (Refer to appendix A.)

The Automatic Memory Stack

Automatic storage of intermediate results is the reason that the HP-27 slides so easily through the most complex equations. Automatic storage is made possible by the Hewlett-Packard automatic memory stack.

The X-Register (Display)

When you first switch the calculator ON, the display shows $\boxed{0.00}$. This represents the contents of the display, or "X-register."

Basically, numbers are stored and manipulated in the machine "registers." Each number, no matter how few digits (e.g., 5) or how many (e.g., $2.87148907 \times 10^{27}$), occupies one entire register.

The displayed X-register, which is the only visible register, is one of four registers inside the calculator that are positioned to form the automatic memory stack. We label these registers X, Y, Z, and T. They are "stacked" one on top of the other (like bookshelves) with the displayed X-register on the bottom. When the calculator is switched ON, these four registers are cleared to 0.00.

Name	Register	
T	0.00	
Z	0.00	
Y	0.00	
X	0.00	Always displayed.

Manipulating Stack Contents

The R↓ *(roll down)* and x⇄y *(x exchange y)* keys allow you to review the stack contents or to shift data within the stack for computation at any time.

Reviewing the Stack: To see how the R↓ key works, first load the stack with numbers 1 through 4 by pressing:

4 ENTER↑ 3 ENTER↑ 2 ENTER↑ 1

The numbers that you entered are loaded into the stack, and its contents look like this:

T	4.00	
Z	3.00	
Y	2.00	
X	1.	Display

Each time you press the R↓ key, the stack contents shift downward one register. So the last number that you have keyed in will be rotated around to the T-register when you press R↓.

T	4.00			T	1.00	
Z	3.00		R↓ →	Z	4.00	
Y	2.00			Y	3.00	
X	1.00	Display		X	2.00	Display

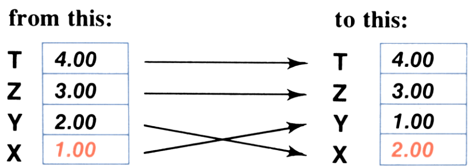
Notice that the *contents* of the registers are shifted. The registers ("shelves") maintain in the same positions. The contents of the X-register are always displayed, so $\boxed{2.00}$ is now visible.

Press R↓ again and the stack contents are shifted:

T	1.00			T	2.00	
Z	4.00		R↓ →	Z	1.00	
Y	3.00			Y	4.00	
X	2.00	Display		X	3.00	Display

Press R↓ twice more, and the stack shifts back to the start. Once again, the number 1.00 is in the displayed X-register. Now that you know how the stack is rotated, you can use the R↓ key to review the contents of the stack at any time so that you can always tell what is in the calculator. Always remember, though, that it takes four presses of the R↓ key to return the contents to their original registers.

Exchanging X and Y. The [x≷y] *(x exchange y)* key exchanges the contents of the X- and Y-registers without affecting the Z- and T-registers. If you press [x≷y] with data intact from the previous example, the numbers in the X- and Y-registers will be changed



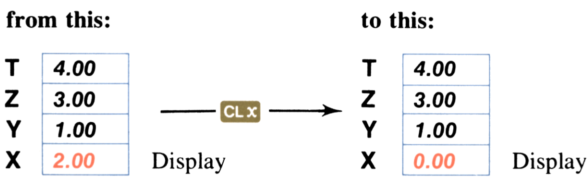
Similarly, pressing [x≷y] again will restore the numbers in the X- and Y-registers to their original places. This key is used to position numbers in the stack or simply to view the Y-register.

Clearing the Stack

To clear the displayed X-register only, press [CLx]. To clear the entire automatic memory stack, including the displayed X-register, press [f] [STK] *(clear stack)*. This replaces all numbers in the stack with zeros. When you turn the calculator OFF, then ON, it "wakes up" with all zeros in the stack registers.

Although it may be comforting, *it is never necessary to clear the stack or the displayed X-register when starting a new calculation*. This will become obvious when you see how old results in the stack are automatically lifted by new entries.

Press [CLx] now, and the stack contents are changed



You can verify that only the X-register contents are affected by [CLx] by using the [R↓] key to review the other stack contents.

If you press f STK, the contents of the entire stack are cleared.

T	4.00		T	0.00	
Z	3.00	— f STK →	Z	0.00	
Y	1.00		Y	0.00	
X	0.00	Display	X	0.00	Display

The ENTER↑ Key

When you key a number into the calculator, its contents are written into the displayed X-register and the other registers remain unchanged. For example, if you keyed in the number 314.32, your stack registers would look like this:

Name	Register	
T	0.00	
Z	0.00	
Y	0.00	
X	314.32	Display

In order to key in a second number at this point, you must separate the digits of the first number from the digits of the second.

One way to separate numbers is to press ENTER↑. Press ENTER↑ to change the contents of the registers:

T	0.00		T	0.00	
Z	0.00		Z	0.00	
Y	0.00	— ENTER↑ →	Y	314.32	
X	314.32	Display	X	314.32	Display

As you can see, the number in the displayed X-register is copied into Y. (The numbers in Y and Z have also been transferred to Z and T, respectively, and the number in T has been lost off the top of the stack. But this will be more apparent when we have different numbers in all four registers.)

Immediately after pressing ENTER↑, the X-register is prepared for a new number, and that new number writes over the number in X. For example, key in the number 543.28 and the contents of the stack registers change. . .

from this:

T	0.00	
Z	0.00	
Y	314.32	
X	314.32	Display

to this:

T	0.00	
Z	0.00	
Y	314.32	
X	543.28	Display

CLx replaces any number in the display with zero. You can then key in a new number and write over the zero in X. For example, if you had meant to key in 689.4 instead of 543.28, you would press CLx:

T	0.00			T	0.00	
Z	0.00			Z	0.00	
Y	314.32		—— CLx ——→	Y	314.32	
X	543.28	Display		X	0.00	Display

and then key in 689.4 to change the stack

from this:

T	0.00	
Z	0.00	
Y	314.32	
X	0.00	Display

to this:

T	0.00	
Z	0.00	
Y	314.32	
X	689.4	Display

Notice that numbers in the stack do not move when a number is keyed in immediately after pressing ENTER↑ or CLx. (However, the numbers in the stack *do* lift when a new number is keyed in immediately after pressing R↓.)

One-Number Functions and the Stack

One-number functions execute upon the number in the X-register only, and the contents of the Y-, Z-, and T-registers are unaffected when a one-number function key is pressed.

For example, with numbers positioned in the stack as in the earlier example, pressing the **f** **√x** keys changes the stack contents, like so:

T	0.00		T	0.00	
Z	0.00		Z	0.00	
Y	314.32	**f** **√x** →	Y	314.32	
X	689.4	Display	X	26.26	Display

The one-number function executes upon only the number in the displayed X-register, and the answer writes over the number that was in the X-register. No other register is affected by a one-number function.

Two-Number Functions and the Stack

Hewlett-Packard calculators do arithmetic by positioning the numbers in the stack the same way you would on paper. For instance, if you wanted to add 34 and 21, you would write 34 on a piece of paper and then write 21 underneath it, like this:

$$\begin{array}{r} 34 \\ \underline{21} \end{array}$$

and then you would add, like this:

$$\begin{array}{r} 34 \\ \underline{+21} \\ 55 \end{array}$$

Numbers are positioned the same way in the HP-27. Here's how it is done.

Press	Display	
f **STK**	0.00	
34	34.	34 is keyed into X.
ENTER↑	34.00	34 is copied into Y.
21	21.	21 writes over the 34 in X.

Now 34 and 21 are sitting vertically in the stack as shown below, so we can add.

T	0.00	
Z	0.00	
Y	34.00	
X	21.	Display

Press	Display	
+	55.00	The answer.

The simple old-fashioned math notation helps explain how to use your calculator. Both numbers are always positioned in the stack in the natural order first; then the operation is executed when the function key is pressed. *There are no exceptions to this rule*. Subtraction, multiplication, and division work the same way. In each case, the data must be in the proper position before the operation can be performed.

To subtract 21 from 34:

$$\begin{array}{r} 34 \\ -21 \\ \hline \end{array}$$

Press	Display	
34	34.	34 is keyed into X.
ENTER↑	34.00	34 is copied into Y.
21	21.	21 writes over the 34 in X.
−	13.00	The answer.

To multiply 34 by 21:

$$\begin{array}{r} 34 \\ \times 21 \\ \hline \end{array}$$

Press	Display	
34	34.	34 is keyed into X.
ENTER↑	34.00	34 is copied into Y.
21	21.	21 writes over the 34 in X.
☒	714.00	The answer.

To divide 34 by 21:

$$\frac{34}{21}$$

Press	Display	
34	34.	34 is keyed into X.
ENTER↑	34.00	34 is copied into Y.
21	21.	21 writes over the 34 in X.
÷	1.62	The answer.

Chain Arithmetic

You've already learned how to key numbers into the calculator and perform calculations with them. In each case you first needed to position the numbers in the stack manually using the ENTER↑ key. However, the stack also performs many movements automatically. These automatic movements add to its computing efficiency and ease of use, and it is these movements that automatically store intermediate results. The stack automatically "lifts" every calculated number in the stack when a new number is keyed in because it knows that after it completes a calculation, any new digits you key in are a part of a new number. Also, the stack automatically "drops" when you perform an arithmetic operation.

Press f STK first, then calculate 16 + 30 + 11 + 17 = ? You've learned to solve such a problem by pressing: 16 ENTER↑ 30 + 11 + 17 +. Now, let's try another method that utilizes the "lift" and "dropping" features of the stack.

Press		Stack Contents	
16	T	0.00	16 is keyed into the displayed X-register.
	Z	0.00	
	Y	0.00	
	X	16.	
ENTER↑	T	0.00	16 is copied into Y.
	Z	0.00	
	Y	16.00	
	X	16.00	
30	T	0.00	30 is written over the 16 in X.
	Z	0.00	
	Y	16.00	
	X	30.	
ENTER↑	T	0.00	30 is entered into Y. 16 is lifted up to Z.
	Z	16.00	
	Y	30.00	
	X	30.00	
11	T	0.00	11 is keyed into the displayed register.
	Z	16.00	
	Y	30.00	
	X	11.	

Press	Stack Contents		
ENTER↑	T	16.00	11 is copied into Y. 16 and 30 are lifted up to Z and T respectively.
	Z	30.00	
	Y	11.00	
	X	11.00	
17	T	16.00	17 is written over the 11 in X.
	Z	30.00	
	Y	11.00	
	X	17.	
+	T	16.00	17 and 11 are added together and the rest of the stack drops. 16 drops to Z and is also duplicated in T. 30 and 28 are ready to be added.
	Z	16.00	
	Y	30.00	
	X	28.00	
+	T	16.00	30 and 28 are added together and the stack drops again. Now 16 and 58 are ready to be added.
	Z	16.00	
	Y	16.00	
	X	58.00	

Press		Stack	Comment
[+]	T	16.00	16 and 58 are added together for the final answer and the stack continues to drop.
	Z	16.00	
	Y	16.00	
	X	74.00	

The same dropping also occurs with [−], [×] and [÷]. The number in T is duplicated in T and Z, the number in Z drops to Y, and the numbers in Y and X combine to give the answer, which is visible in the X-register.

LAST X Register

In addition to the four stack registers, the HP-27 also contains a separate LAST X register that preserves the value displayed in the X-register *before* you performed a function. To place the contents of the LAST X register in the display, press [f] [LAST X]. This register is convenient and helpful when you use a number repeatedly or want to recover from mistakes.

To Avoid Reentering Numbers

The LAST X register is useful in calculations where a number occurs more than once.

For example, to calculate $\dfrac{7.32 + 3.65}{3.65}$

Press	Display	
7.32 [ENTER↑]	7.32	
3.65	3.65	
[+]	10.97	Intermediate answer.
[f] [LAST X]	3.65	Recalls 3.65 to X.
[÷]	3.01	The answer.

Recovering from Mistakes

[LAST X] makes it easy to recover from keystroke mistakes, such as pressing the wrong function or keying in the wrong number.

Example: Divide 12 by 2.157 after you have mistakenly divided by 3.157.

Press	Display	
12	12.	
[ENTER↑]	12.00	
3.157 [÷]	3.80	Oops! You made a mistake.
[f] [LAST X]	3.16	Retrieves that last entry.
[×]	12.00	You're back at the beginning.
2.157 [÷]	5.56	The correct answer.

In the above example, when [f] [LAST X] is pressed, the contents of the stack and LAST X registers are changed:

from this:

T	0.00	
Z	0.00	
Y	0.00	LAST X
X	3.80	◄ 3.16

to this:

T	0.00	
Z	0.00	
Y	3.80	LAST X
X	3.16	◄ 3.16

This makes possible the correction illustrated in the example above.

Ten Storage Registers

Besides the stack, another 10 memory registers (0 thru 9) are provided for manual storage and recalling numbers.

Storing and Recalling Numbers

To store a number, press **STO** then a digit from 0 to 9 to specify the storage location. **STO** 0 places the number on the display in register R_0; **STO** 1, in register R_1; **STO** 2, in register R_2; etc. Data in a storage register is changed by writing over it with a new value or by storage arithmetic.

To recall the number, press **RCL**, then the address digit (0 to 9). When you press **RCL** 3, a copy of the number in register R_3 appears on the display; the original value will remain in the storage location until you write over it or clear it. Recalling data also moves the numbers in the stack upward. (If you press **CLx** or **ENTER↑** just prior to **RCL**, then the stack does *not* lift.)

Example 1: Suppose you want to calculate the cost of buying an item in various quantities. The unit price of the item is $132.57 and the quantities selected are 47, 36 and 29.

One way to solve this is to store the unit price in register R_0. Then recall it to multiply each quantity.

Press	Display	
132.57 **STO** [0]	132.57	
47 [×]	6230.79	First total.
RCL [0]	132.57	
36 [×]	4772.52	Second total.
RCL [0]	132.57	
29 [×]	3844.53	Third total.

The individual totals are still in the stack, so you can easily calculate the combined total cost by adding them.

Press	Display	
[+]	8617.05	
[+]	14847.84	Total cost.

Example 2: Three tanks have capacities in U.S. units of 2.0, 14.4 and 55.0 gallons respectively. If one U.S. gallon is approximately equal to 3.785 liters, what is the capacity of each of the tanks in liters?

Press	Display	
f REG		
f FIX 3	0.000	Display mode set.
3.785 STO 0	3.785	Constant stored in register R_0.
2 ×	7.570	Capacity of first tank in liters.
14.4 RCL 0 ×	54.504	Capacity of second tank in liters.
55 RCL 0 ×	208.175	Capacity of third tank in liters.
f FIX 2	208.18	Display mode reset.

Storage and Recall Register Arithmetic

Arithmetic operations (+, −, ×, ÷) can be performed between a storage register and the X-register in two ways: storage arithmetic and recall arithmetic.

Storage Arithmetic— Arithmetic is performed *upon* the register contents and answers are placed in the storage register—not in the display—so you have to recall them to see or use them.

To perform storage register arithmetic using the number on the display:

1. Press STO .
2. Press the desired arithmetic operation (+, −, × or ÷).
3. Press the storage location number (0 thru 9).

For example, to store 6 in register R_3 and add 5:

Press	Display	
6 STO 3	6.00	Stores 6 in register R_3.
5 STO + 3	5.00	Adds 5 to register R_3.
RCL 3	11.00	Confirms that 11 is stored in register R_3.

If you had pressed: 5 STO [3] [+], that would overwrite the stored 6 with the 5—the value stored in register R_3 would be 5, not 11.

Now, subtract 4 from the number in register R_3:

Press	Display
4 STO [−] [3]	4.00
RCL [3]	7.00

Notice that the general rule is:

Number on display, STO, operation, register number.

Recall Arithmetic—Conversely, to alter the X-register (displayed value) without affecting the contents of the storage register or the stack, press RCL, the arithmetic operation, then the register number. For example, add the current value stored in register R_3 to a new entry (2.00).

Press	Display
2 RCL [+] 3	9.00
RCL 3	7.00

To subtract the contents of register R_3 from the number 11:

Press	Display
11 RCL [−] 3	4.00
RCL 3	7.00

Recall arithmetic does not affect any other stack register except the X-register; i.e., the contents of the stack do not lift.

Remember, like storage arithmetic, the general rule is:

Number on display, RCL operation, register number.

Storage Overflow Indicator

If the value in a storage register exceeds 9.9999999×10^{99}, the following indicator appears on the display:

OF

Clearing the Storage Registers

It is usually unnecessary to clear storage registers R_0 thru R_9 because you can simply write over the old number.

The f REG function clears all 10 storage registers, as well as the financial registers and LAST x.

Availability of Storage Registers

As a general practice, it is wise to use the four storage registers R_0 thru R_3 first to preserve your data.

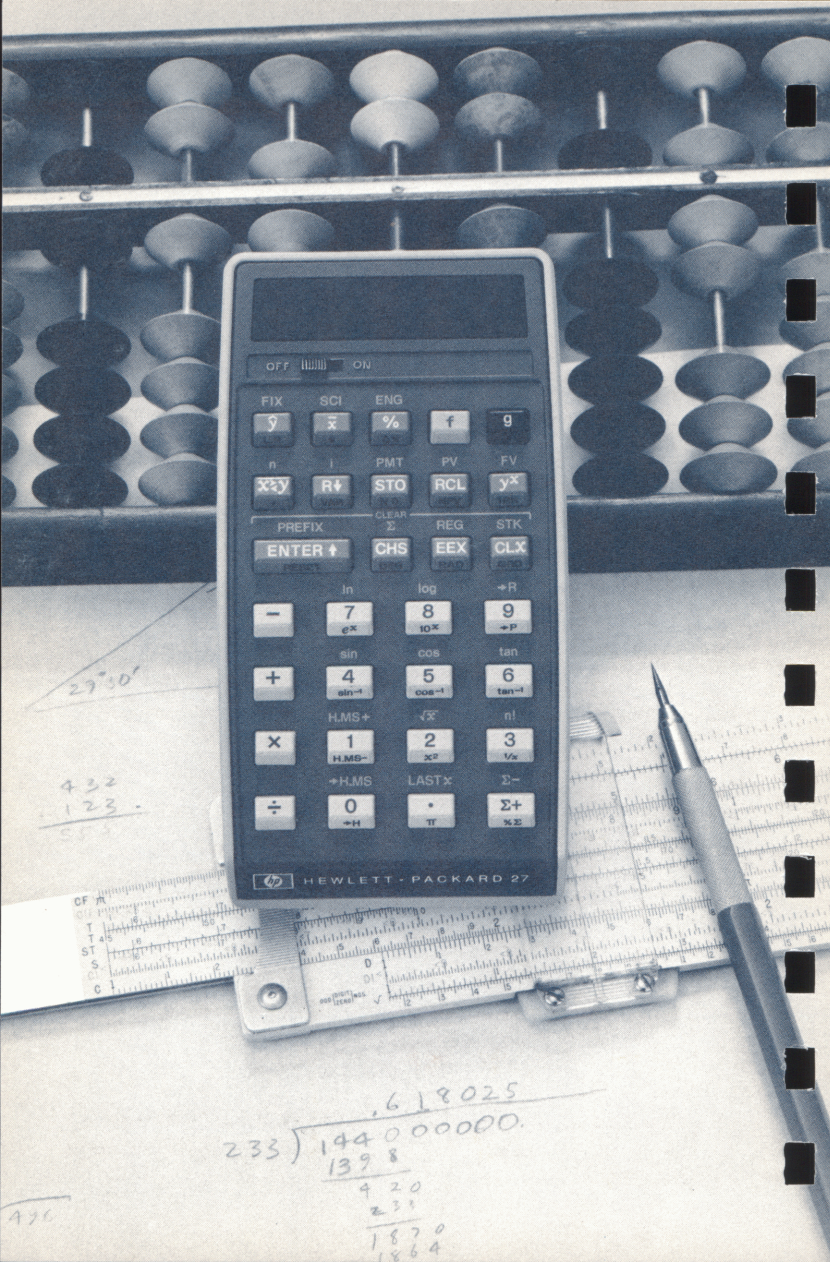
When the Σ+ key is used in statistical problems, the HP-27 uses registers R_4 thru R_9 for its own internal data storage. Therefore, those registers are temporarily not available to you for manual storage.

During iterative i calculations, storage registers R_7, R_8, and R_9 are utilized by the calculator.

Clearing Operations

At this point, it might be useful to review the HP-27 clearing operations:

CL x	Clears the display (X-register).
f PREFIX	Clears the previously pressed prefix key.
f STK	Clears the four-register stack.
f REG	Clears storage registers R_0 thru R_9, the five financial registers, and LAST x.
f Σ	Clears storage registers R_4 thru R_9.


OFF ON
FIX SCI ENG
f g
n i PMT PV FV
x⇄y R↓ STO RCL y^x
CLEAR
PREFIX Σ REG STK
ENTER↑ CHS EEX CLX
ln log →R
− 7 8 9
e^x 10^x →P
sin cos tan
+ 4 5 6
$\sin^{-1}$ $\cos^{-1}$ $\tan^{-1}$
H.MS+ √x n!
× 1 2 3
H.MS− x^2 1/x
→H.MS LASTx Σ−
÷ 0 · Σ+
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.618025
233) 144000000.
1398
420
233
1870
1864

Keyboard Calculations

Exponentiation: Raising a Number to a Power

The y^x key raises a positive number to a positive or negative power or a negative number to an integer power. You use it the same simple way you've performed arithmetic operations; the function is executed immediately when you press the key.

1. Key in the base number. This number is designated y.
2. Press ENTER↑ to separate the first number from the second.
3. Key in the second number (power). This number is designated as x.
4. Press y^x.

To calculate 3^6,

Press	Display
3 ENTER↑ 6 y^x	729.00

To raise a number to a negative power, follow the same procedure and change the sign (CHS) of your exponent before you perform the operation. To solve $4.37^{-2.5}$,

Press	Display
4.37 ENTER↑ 2.5 CHS y^x	0.03

To raise a negative number to a positive or negative integer power, key in the base number, press CHS, press ENTER↑, key in the integer power, and press y^x.

To Solve	Press	Display
-2^3	2 CHS ENTER↑ 3 y^x	-8.00
$-2^{(-3)}$	2 CHS ENTER↑ 3 CHS y^x	-0.13

With a negative base, if the exponent is an odd number, the answer will be negative. If the exponent is an even number, your answer will be positive:

$$(-2)^2 = -2 \times -2 = 4$$

$$(-2)^3 = -2 \times -2 \times -2 = -8$$

You can also use y^x to raise 0 to a positive power; but of course, your answer will always be zero.

Squaring

To square a number in the displayed X-register, press g x^2. For example, to find the square of 45:

Press	Display
45 g x^2	2025.00

To find the square of the result:

Press	Display
g x^2	4100625.00

Reciprocals

To calculate the reciprocal of a number in the displayed X-register, key in the number, then press g $1/x$. For example, to calculate the reciprocal of 25:

Press	Display
25 g $1/x$	0.04

You can also calculate the reciprocal of a value in a previous calculation without re-entering the number. For example, to calculate

$$\frac{1}{1/3 + 1/6}$$

Press	Display	
3 g 1/x	0.33	Reciprocal of 3.
6 g 1/x	0.17	Reciprocal of 6.
+	0.50	Sum of reciprocals.
g 1/x	2.00	Reciprocal of sum.

Extracting Roots

To calculate the square root of a number, key in the number, then press f √x.

To Solve	Press	Display
$\sqrt{25}$	25 f √x	5.00
$\sqrt{81}$	81 f √x	9.00

You must use a positive number. You cannot calculate the square root of a negative number; that's an illegal operation.

You can also extract higher roots, like cube roots and fourth roots; but you use the y^x key, not f √x. The cube root of a number is that number raised to the 1/3 power. Thus, $\sqrt[3]{n}$ is the same as $n^{1/3}$; the fourth root can be written as $n^{1/4}$, etc.

Use the same keystroke sequence that you learned for exponentiation to extract higher roots:

1. Key in the base number and press ENTER↑.
2. Key in the root desired and press g 1/x.
3. Press y^x.

To solve $\sqrt[3]{5}$, $= 5^{1/3}$:

Press	Display
5 ENTER↑ 3 g 1/x y^x	1.71

Now, try $\sqrt[4]{81}$ or $81^{1/4}$:

Press	Display
81 ENTER↑ 4 g 1/x y^x	3.00

Using PI

The value π accurate to 10 places (3.141592654) is provided as a fixed constant in the HP-27. Merely press g π whenever you need it in a calculation. For example, to calculate 3π;

Press	Display
3 g π $\times$	9.42

Example: In the *Guinness Book of Records,* you find that the largest pizza ever baked had a diameter of 21 feet. If your appetite were equal to the task, how many square feet of pizza would you have to devour in order to consume all of the world's largest pizza?

$$\textbf{Area} = \pi \left(\frac{d}{2}\right)^2 = \pi \left(\frac{21}{2}\right)^2$$

Press	Display	
21 ENTER↑	21.00	
2 $\div$	10.50	
g x^2	110.25	
g π	3.14	
$\times$	346.36	Square feet of pizza.

Factorials

The keys f n! enable you to handle permutations and combinations with ease. To calculate the factorial of an integer, key in that number, then press f n!.

Example: Calculate the number of ways 6 people can line up for a photograph. ($6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$)

Press	Display
6	6.
f n!	720.00

The HP-27 can calculate factorials for numbers up to 69.

Logarithms and Antilogs

The HP-27 computes both natural and common logarithms as well as their inverse functions (antilogarithms):

f ln	is $\log_e$ (natural log). It takes the log of the value in the X-register to base e (2.718. . .).
g e^x	is antilog_e (natural antilog). It raises e (2.718. . .) to the power of the value in the X-register.
f log	is $\log_{10}$ (common log). It computes the log of the value in the X-register to base 10.
g 10^x	is antilog_{10} (common antilog). It raises 10 to the power of the value in the X-register.

Example 1: The 1906 San Francisco earthquake, with a magnitude of 8.25 on the Richter Scale is estimated to be 105 times greater than the Nicaragua quake of 1972. What would be the magnitude of the latter on the Richter Scale? The equation is

$$R_1 = R_2 - \log\frac{M_2}{M_1} = 8.25 - \left(\log\frac{105}{1}\right)$$

Solution:

Press	**Display**	
8.25 ENTER↑	8.25	
105 f log	2.02	
−	6.23	Rating on Richter scale.

Example 2: Ace explorer Jason Quarmorte is using an ordinary barometer as an altimeter. After measuring the sea level pressure (30 inches of mercury) he climbs until the barometer indicates 9.4 inches of mercury. Although the exact relationship of pressure and altitude is a function of many factors, Jason knows that an *approximation* is given by the formula:

$$\text{Altitude (feet)} = 25{,}000 \ln\frac{30}{\text{Pressure}} = 25{,}000 \ln\frac{30}{9.4}$$

Where is Jason?

Solution:

Press	Display	
30 ENTER↑	30.00	
9.4 ÷	3.19	
f ln	1.16	
25000	25000	
×	29012.19	Altitude in feet.

He's probably near the summit of Mount Everest (29,028 feet).

Example 3: Logarithms and antilogarithms are also used in continuous compound interest formulas. To compute the continuous effective rate, given the nominal rate, the formula is:

$$\text{Continuous Effective Rate} = \left(e^{\frac{\text{nominal rate}}{100}} - 1 \right) \times 100$$

If a savings institution quotes a nominal rate of 6%, compounded continuously, what is the effective rate?

Press	Display	
6 ENTER↑ 100 ÷	0.06	
g e^x 1 − 100 ×	6.18	% continuous effective rate.

Trigonometric Functions

Your HP-27 provides you with six trigonometric functions. It calculates angles in decimal degrees, radians, or grads. It converts between decimal degrees and degrees, minutes, seconds, and lets you add or subtract degrees, minutes, seconds.

Trigonometric Modes

When the HP-27 is first turned ON, it "wakes up" with angles specified in decimal degrees. To set radians or grads mode, press the g shift key followed by either RAD *(radians)* or GRD *(grads)*. To switch back to the decimal degrees mode again, press the g shift key followed by the DEG *(degrees)* key.

Note: 360 degrees = 2π radians = 400 grads.

Functions

The six trigonometric functions provided by the calculator are:

f [sin] *(sine)*
g [sin⁻¹] *(arc sine)*
f [cos] *(cosine)*
g [cos⁻¹] *(arc cosine)*
f [tan] *(tangent)*
g [tan⁻¹] *(arc tangent)*

Each trigonmetric function assumes angles in decimal degrees, radians, or grads. Trigonometric functions are one-number functions; so to use them, you key in the number, then press the function keys.

Example 1: Find the cosine of 35°.

Press	Display	
35	35.	Calculator "wakes up" in decimal degrees mode.
f [cos]	0.82	The answer.

Example 2: Find the arc sine in grads of .964.

Press	Display	
[CLx]	0.00	
g [GRD]	0.00	Grads mode is set.
.964	0.964	
g [sin⁻¹]	82.87	Grads.

Hours, Minutes, Seconds

The [→H.MS] *(to hours, minutes, seconds)* key converts decimal hours (or degrees) to the format of hours (degrees), minutes and seconds. To see the digits for seconds, you should specify [FIX] 4 display format. For example, to convert 12.56 hours to hours, minutes, seconds:

Press	Display	
f [FIX] 4	0.0000	Sets display format. (Assumes no results remain from previous example.)

12.56	12.56	Decimal hours.
f →H.MS	12.3336	This is read as 12 hours, 33 minutes, 36 seconds.

Conversely, the →H key permits you to change hours (degrees), minutes, seconds to decimal hours (degrees). For example, to change 173°45′ 12″ to decimal degrees:

Press	Display	
137.4512	137.4512	
g →H	137.7533	Decimal degrees.

The conversion is important because trigonometric functions in the HP-27 operate on angles in decimal degrees, but not in *hours, minutes, seconds*.

Two other keys f H.MS+ and g H.MS−, allow you to add and subtract in hours (degrees), minutes, and seconds without converting back to decimal hours (degrees).

Example: Lovesick sailor Oscar Odysseus dwells on the island of Tristan da Cunha (37°03′S, 12°18′W), and his sweetheart, Penelope, lives on the nearest island. Unfortunately for the course of true love, however, Tristan da Cunha is the most isolated inhabited spot in the world. If Penelope lives on the island of St. Helena (15°55′S, 5°43′), calculate the great circle distance that Odysseus must sail in order to court her using the following formula:

$$\text{Distance} = \cos^{-1}\left[\sin(\text{LAT}_s)\sin(\text{LAT}_s)\cos(\text{LAT}_d)\cos(\text{LNG}_d - \text{LNG}_s)\right] \times 60.$$

Where LAT_S and LNG_S = latitude and longitude of the source (Tristan da Cunha).

LAT_D and LNG_d = latitude and longitude of the destination.

Solution: Convert all degrees, minutes, seconds entries into decimal degrees as you key them in. The equation for the great circle distance from Tristan da Cunha to the nearest inhabited land is:

$$\text{Distance} = \cos^{-1}\left[\sin(37°03') \sin(15°55') + \cos(37°03') \cos(15°55') \cos(5°43' - 12°18')\right] \times 60$$

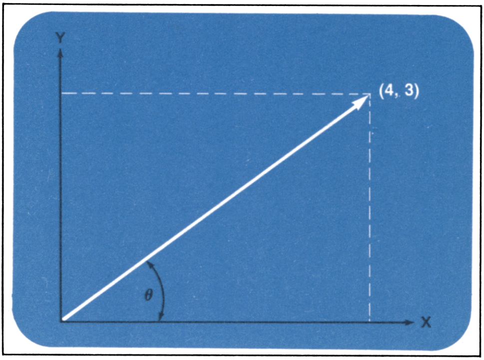
Press	Display	
g DEG		Sets decimal degree mode for trigonometric functions. (Assumes no results remain from previous example.)
f FIX 2		Display mode set.
5.43 ENTER↑	5.43	
12.18 g H.MS− g →H	−6.58	
f cos	0.99	
15.55 g →H STO 1	15.92	
f cos	0.96	
×	0.96	
37.03 g →H STO 0	37.05	
f cos	0.80	
×	0.76	
RCL 0 f sin	0.60	
RCL 1 f sin	0.27	
×	0.17	
+	0.93	
g COS⁻¹	21.92	
60 ×	1315.41	Distance in nautical miles that Odysseus must sail to visit Penelope.

Polar/Rectangular Coordinates Conversion

Two functions are provided for polar/rectangular coordinate conversion. To convert values in the X- and Y-registers, (representing rectangular x, y coordinates, respectively) to polar r, θ coordinates (magntiude and angle, respectively), press g →P. Magnitude r appears in the X-register and angle θ appears in the Y-register.

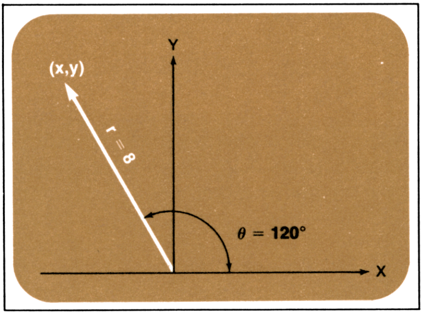
Conversely, to convert values in the X- and Y-registers (representing polar r, θ, respectively) to rectangular coordinates (x, y respectively), press f →R.

Example 1: Convert rectangular coordinates (4, 3) to polar form with the angle expressed in radians.



Press	Display	
g RAD	0.00	Specifies radians mode. (Assumes no results remain from previous example.)
3 ENTER↑ 4	4.	Rectangular coordinates placed in X- and Y-registers.
g →P	5.00	Magnitude r.
x⇄y	0.64	Angle θ in radians.

Example 2: Convert polar coordinates (8, 120°) to rectangular coordinates



Press	**Display**	
g DEG	0.64	Specifies degrees mode. (Results from previous example.)
120 ENTER↑ 8	8.	Polar coordinates θ and r placed in Y- and X-registers, respectively.
f →R	-4.00	X-coordinate.
x⇄y	6.93	Y-coordinate.

Calculating Percentage Problems

There are three keys on your HP-27 for calculating percentage problems: % function is used to find a percentage of a number. The Δ% function is used to find percent differences (% increase/decrease). And the %Σ function is used to find what percent one number is of another or of the total sum (proportion).

With the HP-27, you don't have to convert percents to their decimal equivalents; 4% need not be changed to .04. It can be keyed in the way you see and say it, 4 %.

Percentage %

To find the percentage of a number, key in that base number and press ENTER↑ . Then key in the numerical value of the percent and press % .

For example, find 14% of $300:

Press	Display
300 ENTER↑ 14 %	42.00

Example: Every year you set aside 4% of your company's profits for the employee retirement fund. If your company made a profit last year of $1,576,432, how much money was contributed to the fund?

Press	Display
1576432 ENTER↑ 4 %	63057.28

Net Amount % + or % −

If you buy a new car, you have to figure the sales tax percentage, then add that to the purchase price to find the total cost of the car. It is easy to calculate this net amount with your HP-27 because the calculator holds the base number while you calculate percentages.

For example, if the sales tax on a $6200 car is 5%, what is the amount of the tax and total cost of the car?

Press	Display	
6200 ENTER↑ 5 %	310.00	Percentage amount (sales tax).
+	6510.00	Net amount (base plus percentage amount).

If the dealer gives you a 10% discount on the car, what will your total cost be?

Press	Display	
6200 ENTER↑ 10 %	620.00	Amount of discount.
−	5580.00	Discounted price.
5 % +	5859.00	Net amount (discounted price plus sales tax).

Notice in the last problem that you subtracted a percentage amount and added a percentage amount, without repeating the base number. The calculator stores intermediate answers in the stack until you need them in a calculation.

Percent Difference Between Two Numbers g Δ%

To find the percent difference between two numbers—the ratio of increase or decrease—key in the base number, press ENTER↑, then key in the second number and press g Δ%. Usually, the base amount, or the first number you key in, is the one that occurred first in time.

For example, your rent jumps from $285 a month to $335 a month. What percent is the increase?

Press	Display	
285 ENTER↑ 335 g Δ%	17.54	% increase.

A positive answer denotes an increase, while a negative answer denotes a decrease.

Example: You forgot to place a stop order, and your stock fell from $57.50 to $13.25 a share. What percent is the decrease?

Press	Display	
57.50 ENTER↑ 13.25 g Δ%	-76.96	%

Markup

Markup is a simple percentage calculation using the Δ% function and wholesale or original purchase cost as the base number.

Example: You purchase typewriters at $159.95 wholesale and retail them for $195.00. What percent is your markup?

Press	Display	
159.95 ENTER↑ 195.00 g Δ%	21.91	% markup.

Perhaps you have a fixed profit rate in mind and wish to establish your retail price. You would solve for the retail price the same as you solved for net amount.

Example: You purchase several valves @ $2.26 from the manufacturer and wish to sell them at 25% profit. How much should you mark them up?

Press	Display	
2.26 ENTER↑ 25 %	0.57	Amount of markup.
+	2.83	Retail price.

Margin

Margin also is a simple percentage problem with the Δ% function, only this time you use the selling price as the base number. Returning to the typewriters that you bought for $159.95 and sold for $195, the margin is calculated as follows:

Press	Display	
195 ENTER↑ 159.95 g Δ% CHS	17.97	% margin.

So, the markup is 21.91% and the margin is 17.97%.

Finding Percent of Total g %Σ

Σ is the Greek symbol "sigma" which we use to mean sum or total. To find what percentage one number is of the total, add the numbers first by keying in each number followed by Σ+. Then key in the particular number you wish to convert to a percentage and press g %Σ.

Example 1: You own 150 shares of stock in Sleepy-Head Waterbeds, 52 shares of Flickering Films Inc., and 200 shares of Raucous Records Company. What percent of your portfolio does each represent?

Press	Display	
150 Σ+	1.00	First entry.
52 Σ+	2.00	Second entry.
200 Σ+	3.00	Third entry.
150 g %Σ	37.31	% Sleepy-Head Waterbeds.
52 g %Σ	12.94	% Flickering Films.
200 g %Σ	49.75	% Raucous Records.

The [Σ+] key displays the number of entries rather than the numerical value keyed in. When you use the [Σ+] key, press [f] [Σ] *(clear [Σ])* between problems.

Example 2: Assume that the Sleepy-Head Waterbeds stock is worth $450; Flickering Films stock, $1404; and Raucous Records, $1500. What percent of the total value of your portfolio does each represent?

Press	Display	
[f] [Σ]		
450 [Σ+]	1.00	
1404 [Σ+]	2.00	
1500 [Σ+]	3.00	Number of entries.
450 [g] [%Σ]	13.42	% Sleepy-Head Waterbeds.
1404 [g] [%Σ]	41.86	% Flickering Films Inc.
1500 [g] [%Σ]	44.72	% Raucous Records Co.

Proportions [g] [%Σ]

To find what percent one number is of another (proportion), state the problem as "A is what percent of B?" Key in B (the base number), press [Σ+], then key in A (the number to be converted), and press [g] [%Σ]. (An alternative solution would be to key in B, press [STO] 5, key in A, and press [g] [%Σ].)

For example, 64 is what percent of 340?

Press	Display	
[f] [Σ]		
340 [Σ+]	1.00	
64 [g] [%Σ]	18.82	%

Or the alternative procedure:

Press	Display	
[f] [Σ]		
340 [STO] 5	340.00	
64 [g] [%Σ]	18.82	%

Example: To purchase that $47,000 lakefront cabin that you've had your eye on, a $9400 down payment is required. What percentage of the price does your down payment represent?

Press	Display	
f Σ		
47000 Σ+	1.00	
9400 g %Σ	20.00	%

The Financial Function Keys

Your HP-27 has the most frequently used business calculations (including the associated formulas) preprogrammed into the second row of keys:

(f) n i PMT PV FV

The nature of these calculations is to use known data and solve for an unknown value. Three of these values must be entered to solve for one or both of the other values.

The n value represents the total number of compounding or payment periods. To enter 30 years, press 30 f n. If you wish to input that in monthly periods, press 30 ENTER↑ 12 × f n.

The i value is the interest rate per period. If interest is expressed as an annual rate but your problem involves monthly payments or daily compounding, then you must convert the annual rate to rate per period. To key in 9% annual interest, press 9 f i. To input the monthly rate, press 9 ENTER↑ 12 ÷ f i. To input the daily rate, press 9 ENTER↑ 365 ÷ f i.

Remember that n and i *must* correspond to the same time frame. If n is months, then i must be the monthly interest rate.

The PMT function stands for payment or deposit amount. It assumes equal payments and must correspond to the same time frame as n or i. Don't mix monthly payments with annual periods or daily interest.

The PV function stands for present value or the amount of money at the start of the transaction.

The [FV] function represents the future value of money or the amount you will obtain/pay at the end of the term.

Both [n] and [i] are involved in all financial calculations. The nature of financial calculations on the HP-27 requires that given three of the above values (including n or i), you can solve for a fourth value. You can enter the values in any order.

Displaying Financial Values

To see what number is in a particular financial register, press [RCL] [f] then the desired location key ([n], [i], [PMT], [PV], or [FV]).

Changing Financial Values

You may solve a problem and want to explore a different alternative by changing only one of the three known values—perhaps using a different interest rate or a greater/smaller payment amount.

With the HP-27 you don't have to re-enter the numbers all over again. Simply key in the new value and press the appropriate financial function key. The new value will be substituted for the old value.

Example: You borrow $9000 at 9½% interest, to put a new swimming pool in the backyard. What will your monthly payments be if you pay the loan in 2 years?

Press	Display	
[g] [RESET]		
2 [ENTER↑] 12 [×] [f] [n]	24.00	Number of months.
9.5 [ENTER↑] 12 [÷] [f] [i]	0.79	Interest rate/month.
9000 [f] [PV]	9000.00	Amount of your loan.
[f] [PMT]	413.23	Your monthly payment.

What if you take out a 3-year loan?

Press	Display	
3 [ENTER↑] 12 [×] [f] [n]	36.00	Number of months.
[f] [PMT]	288.30	Alternative monthly payment.

This is considered an alternative of the same problem because given n, i and PV, you solved for payment more than once.

If you decide "but I can only afford $200 a month, what would my loan be?"—that's a new problem. It requires a different set of three input values (now n, i and PMT to solve for PV), so you solve it as a new problem.

Press g [RESET] between problems to reset the five financial keys for new values. Your previous answer is left on the display so that you can use it again or store it, if you wish. (For a detailed explanation, refer to appendix B.)

The following examples quickly illustrate use of the financial keys. Section 4 describes how to solve loan, savings, lease, and investment problems.

Solving for n

Given the periodic interest rate (i) and two of the following: initial principal (PV), periodic payment (PMT), or future value (FV), you can calculate the number of periods in a compound amount or a loan.

Example 1: Fur trapper Bill Buckskin wishes to buy a $22,000 log cabin. A local bank offered to loan Bill the $22,000 at 9¼% interest. Making $200 monthly payments, how long will it take Bill to repay his mortgage?

Press	Display	
g [RESET]		
9.25 [ENTER↑] 12 [÷] f [i]	0.77	Monthly interest rate.
200 f [PMT]	200.00	Monthly payment.
22000 f [PV]	22000.00	Amount of loan.
f [n]	245.26	Months.
12 [÷]	20.44	Years.

Example 2: With an eye toward the future, you put $5,000 into an education fund for your child. If the money earns 6% interest compounded daily, how many years before the fund reaches $10,000?

Press	Display	
g RESET		
5000 f PV	5000.00	Initial principal.
6 ENTER↑ 365 ÷ f i	0.02	Daily interest rate.
10000 f FV	10000.00	Desired amount.
f n	4216.98	Days.
365 ÷	11.55	Years.

Solving for i

Given the number of periods and two of the following: initial principal (PV), periodic payment (PMT) or future value (FV), you can calculate the periodic annual interest rate, annual yield, or rate of return.

Example: What is the annual interest rate on a 25-year $32,500 mortgage with $230 monthly payments?

Press	Display	
g RESET		
25 ENTER↑ 12 × f n	300.00	Number of months.
230 f PMT	230.00	Monthly payment.
32500 f PV	32500.00	Amount of loan.
f i	0.58	% monthly interest rate.
12 ×	7.01	% annual interest rate.

Solving for PMT

Given the number of periods, the interest rate and either present value (PV) or future value (FV), you can calculate the payment amount.

Example: Find the monthly payments on a 30-year, $47,000 mortgage at 8.5% annual interest rate.

Press	Display	
g RESET		
30 ENTER↑ 12 × f n	360.00	Number of months.

8.5 ENTER↑ 12 ÷ f i	0.71	% monthly interest rate.
47000 f PV	47000.00	Principal of loan.
f PMT	361.39	Monthly payment.

Solving for PV

Given the number of periods, the interest rate, and either payment (PMT) or future value (FV), you can calculate the initial principal.

Example: Junior Engineer wants to buy an oscilloscope. He can afford $60 a month for 36 months and is willing to pay 9% annual interest. How much can he afford to pay for the scope?

Press	Display	
g RESET		
36 f n	36.00	Number of months.
9 ENTER↑ 12 ÷ f i	0.75	% monthly interest rate.
60 f PMT	60.00	Monthly payment.
f PV	1886.81	Maximum he can afford.

Solving for FV

Given the number of periods, the interest rate, and either the payment (PMT) or the initial amount (PV), you can calculate the amount of money you will pay/receive in the future.

Example: A house purchased 3 years ago for $42,500 is located in an area where property values are appreciating at 12% a year. What is the current approximate value of the house?

Press	Display	
g RESET		
3 f n	3.00	Number of years.
12 f i	12.00	% annual interest.
42500 f PV	42500.00	Original price.
f FV	59709.44	Current value.

There are two other financial keys on your HP-27: g NPV solves for net present value, and g IRR solves for internal rate of return. Both of these are sophisticated methods of evaluating investments and are explained in *Discounted Cash Flow Analysis,* section 4.

Statistical Functions

The following keys are used in statistical calculations:

Σ+	Sigma plus. Totals sums, products and squares of numbers.
f Σ-	Sigma minus. Subtracts a sum, product or square from the summation in Σ+. Useful for subtracting out an incorrect entry.
g L.R.	Linear regression. Plots a straight line that best fits a set of data points by finding the slope and y-intercept.
$\hat{y}$	Linear estimate. Given a value for x, this computes the predicted value for y or where the point will fit on a plotted linear function.
g r	Correlation coefficient measures goodness of fit for linear regression data. r = +1 (perfect fit, positive slope) r = −1 (perfect fit, negative slope) r = 0 (no fit)
g N.D.	Normal distribution. Calculates the density function and the upper tail area under a normal distribution curve.
$\bar{x}$	Mean. Computes the mean or arithmetic average for both x and y.
g s	Standard deviation. Calculates the dispersion around the mean for both x and y.
g VAR	Variance. Computes the variances of x and y, such that: $VAR_x = (s_x)^2$ and $VAR_y = (s_y)^2$

When completely finished with a statistical problem and before starting a new problem, press f Σ. This clears old data from the statistical registers.

Summations [Σ+]

Pressing the [Σ+] key automatically gives you several different sums and products of values in the X- and Y-registers. The calculator stores these values in registers R_4 thru R_9.

When you key in one or two numbers and press [Σ+], the following happens:

1. The number of entries is stored in register R_4.
2. The number (x) on the display is added to the contents of storage register R_5.
3. The square (x^2) of the display number is added to the contents of register R_6.
4. The number in the Y-register of the stack is added to the contents of storage register R_7.
5. The square of y (y^2) is added to the contents of register R_8.
6. The product of x and y (xy) is added to the contents of register R_9.

When you input paired data (x and y), you must key in the y value first and separate the two numbers by [ENTER↑]. The general rule is:

y value [ENTER↑] x value [Σ+]

To recap, this is where values are stored inside your calculator:

Register	Contents
R_4	Number of entries (n).
R_5	Summation of x values (Σx).
R_6	Summation of x^2 values (Σx^2).
R_7	Summation of y values (Σy).
R_8	Summation of y^2 values (Σy^2).
R_9	Summation of products of x and y values (Σxy).
Display (X-register)	Number of entries (n).

Immediately, you have a powerful data bank for statistical calculations. To see any of these summations at any time, simply recall the contents of the desired register. Remember, when you press RCL 4 thru 9, you will not lose that data. Only a copy of the number (sum) appears in the display. (Note that the n in register R_4 has nothing to do with the financial n.)

Deleting and Correcting Data

If you key an incorrect entry into Σ+, you don't have to start over again. If you keyed it before pressing Σ+, simply press CLx to clear the display, then continue on with the correct value.

If you had already added in the wrong value, simply key in that wrong number, subtract it out by pressing f Σ-, then continue with the correct number.

This applies to two variables, as well as one. Suppose you key in 10 ENTER↑ 20 Σ+ and discover that the y value is wrong. Delete the data pair by pressing 10 ENTER↑ 20 f Σ-, then continue with the correct numbers.

Linear Regression L.R.

Linear regression is a statistical method for finding a straight line that best fits a set of data points, thus providing a relationship between two variables. If there is equal time or space between data points, then this is called a trend line.

Naturally, at least two data points must be in the machine before a line can be drawn or fitted to them. After you have totaled the data points using the Σ+ key, you can calculate the coefficients of the linear equation

$$y = A + Bx$$

by pressing g L.R.. A is the y-intercept, and appears in the display. B represents the slope of the line and is stored in the Y-register.

Example: A commercial land appraiser has examined six vacant lots in a local community, all of which have the same depths but different frontages and values. Based on the following input data, what is the relationship between frontage and lot value?

Input Data:

(x) Lot frontage (feet)	(y) Lot value ($)
70.8	10100
60.0	9000
85.0	12700
75.2	11120
69.5	11000
84.0	12500

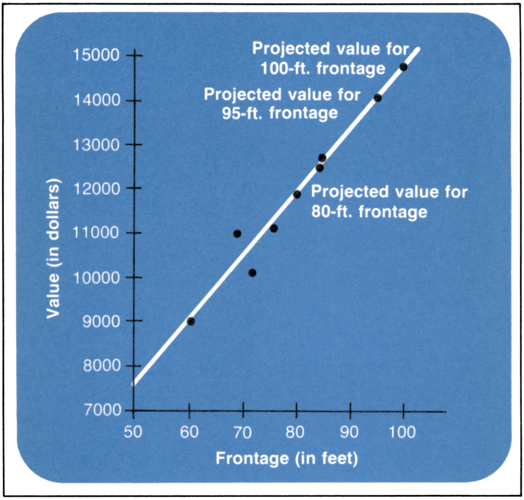
Accumulate the data using [Σ+]. (Remember that when you enter two values, x and y, you must enter the y value first.)

Press	Display	
[f] [Σ]		
10100 [ENTER↑] 70.8 [Σ+]	1.00	First entry.
9000 [ENTER↑] 60 [Σ+]	2.00	Second entry.
12700 [ENTER↑] 85 [Σ+]	3.00	Third entry.
11120 [ENTER↑] 75.2 [Σ+]	4.00	Fourth entry.
11000 [ENTER↑] 69.5 [Σ+]	5.00	Fifth entry.
12500 [ENTER↑] 84 [Σ+]	6.00	Sixth entry.
[g] [L.R.]	393.90	A = y-intercept.
[x⇄y]	144.11	B = slope of line.

Thus, the equation of the regression line is:

$$y = 393.90 + 144.11\ x$$

The y-intercept value represents the projected value for x = 0. The slope indicates the change in the projected value caused by an incremental change in the x value. Plotting this example, you see that a 1-foot increase in the frontage results in a projected increase in value of $144.11.



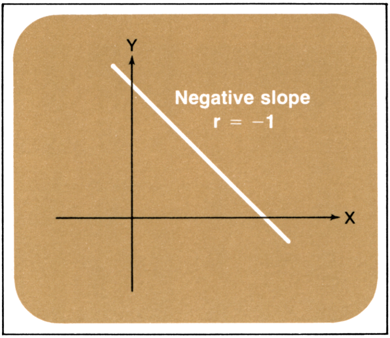
Correlation Coefficient [r]

In the diagram, the solid line is the best fit for the given data points.

It's a good idea to check the "goodness of fit" of the linear function by calculating the correlation coefficient. This tells you how close to a straight line the data points lie. Since all the data is in your HP-27, to calculate the correlation coefficient,

Press	Display
[g] [r]	0.97

If r = +1, then the line has a positive slope and the data fits perfectly. If r = −1, the data still is a perfect fit but the line has a negative slope, like so:



(An example of a negative trend line is declining property values or declining sales.) If r = 0, the data values are spread out all over and do not come close to a straight line.

In the example of lot frontage related to value, the correlation coefficient is close to 1, so we can feel comfortable using linear regression. Suppose, though, that the correlation coefficient was not close to 1 but instead was 0.5 or 0.6. This would indicate that a straight line is not a very good fit to the data. Then you might try to fit the data to a curve.

In the statistical applications in section 4, three different types of curve fits are described: exponential, logarithmic, and power. A correlation coefficient may be calculated for each of these curves and should be interpreted similarly: if r is close to ±1, the curve is a reasonable approximation to the data; if not, try a different fit.

Linear Estimate

Having plotted a line, you can quickly estimate other values. With the data totaled in registers R_4 thru R_9, a predicted y (designated $\hat{y}$) can be calculated by keying in an x value and pressing $\hat{y}$.

Example 1: For the previous example, find projected values for 80-, 95-, and 100-foot frontages.

Press	Display	
80 ŷ	11922.65	80-foot frontage projected value.
95 ŷ	14084.29	95-foot frontage projected value.
100 ŷ	14804.83	100-foot frontage projected value.

To find an estimated value, it is not necessary to calculate L.R. first. In the next example, you key in the known data and solve for a projected unknown.

Example 2: You bought a house 3 years ago for $47,500. The first year it appreciated $5,000. The second year its value rose to $60,000. Today you figure the market price to be $64,000 if you were to sell. What will your house be worth next year?

Press	Display
f Σ	
47500 ENTER↑ 1 Σ+	1.00
52500 ENTER↑ 2 Σ+	2.00
60000 ENTER↑ 3 Σ+	3.00
64000 ENTER↑ 4 Σ+	4.00

To make a projection for next year (year 5), simply solve for ŷ:

Press	Display
5 ŷ	70250.00

Mean

Your HP-27 can quickly calculate the means or arithmetic averages of two variables. Whether it's the average of tests scores or last month's sales figures, given one or two sets of numbers, your HP-27 will calculate the mean of those samples.

1. Press f Σ *(Clear Σ).*

2. If you are summing *one* set of numbers, key in the first number and press [Σ+]; then the second number and [Σ+] again; the third number, etc. Continue until you have entered all the values.

3. If you are summing *two* sets of numbers, key in the y value and press [ENTER↑]; key in the x value, then press [Σ+]. Key in the second y value, press [ENTER↑], key in the second x value, and press [Σ+]. Continue until you have entered all the values.

4. Press [$\bar{x}$] for the mean of the x values.

5. Press [x⇄y] for the mean of the other set of values ($\bar{y}$).

Example: A survey of seven salesmen in your company reveals that they work the following hours a week and sell the following dollar volume each month. How many hours does the average salesman work each week? How much does the average salesman sell each month?

Salesman	Hours/Week	Sales/Month
1	32	$17,000
2	40	$25,000
3	45	$26,000
4	40	$20,000
5	38	$21,000
6	50	$28,000
7	35	$15,000

To find the average workweek and sales of this sample:

Press	Display	
[f] [Σ]		Storage registers cleared.
32 [ENTER↑] 17000 [Σ+]	1.00	First entry.
40 [ENTER↑] 25000 [Σ+]	2.00	Second entry.
45 [ENTER↑] 26000 [Σ+]	3.00	
40 [ENTER↑] 20000 [Σ+]	4.00	
38 [ENTER↑] 21000 [Σ+]	5.00	
50 [ENTER↑] 28000 [Σ+]	6.00	

35 ENTER↑ 15000 Σ+	7.00	Total number of entries in each sample.
$\bar{x}$	21714.29	Mean dollar sales/month ($\bar{x}$).
x⇄y	40.00	Mean workweek in hours ($\bar{y}$).

Note that x and y values overwrite the X- and Y-registers.

Standard Deviation

You already have data from the previous example in Σ+ so to calculate the standard deviation (a measure of dispersion around the mean), simply:

Press	Display	
g s	4820.59	Dollars (s_x)
x⇄y	6.03	Hours (s_y)

The HP-27 calculates standard deviation according to the formulas:

$$s_x = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}} \qquad s_y = \sqrt{\frac{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}{n-1}}$$

Notice that the seven salesmen that we used is a sample. If we used *all the salesmen,* then the data would be considered a population rather than a sample.

The relationship between *sample* standard deviation (s) and *population* standard deviation (s′) is given by:

$$s' = s\sqrt{\frac{n-1}{n}}$$

Since n (number of entries) is stored in register R_4, you can convert sample standard deviation to population standard deviation. You already have (s) 6.03 on the display, so simply:

Press	Display	
x̄	21714.29	Mean (dollars).
Σ+	8.00	Number of entries + 1.
g s	4463.00	s'_x
x⇄y	5.58	s'_y

To continue summing data pairs, press x̄ f Σ- before entering more data.

Variance

The variance of a set of data is the square of the standard deviation. Just as you can calculate the mean and standard deviation of two variables, you can compute the variances of both x and y:

$$VAR_x = (s_x)^2$$

$$VAR_y = (s_y)^2$$

Example: Eight students have the following I.Q.s and received the following grades on their chemistry final. What are the mean I.Q. and exam scores? What is the variance of each?

I.Q.	Exam Grade
112	75
142	78
120	85
110	78
120	88
132	95
116	60
114	80

Press	Display	
f Σ		
112 ENTER↑ 75 Σ+	1.00	First data pair.

142 ENTER↑ 78 Σ+		
120 ENTER↑ 85 Σ+		
110 ENTER↑ 78 Σ+		
120 ENTER↑ 88 Σ+		
132 ENTER↑ 95 Σ+		
116 ENTER↑ 60 Σ+		
114 ENTER↑ 80 Σ+	8.00	Last data pair.
$\bar{x}$	79.88	Mean (exam scores).
x⇄y	120.75	Mean (I.Q.)
g VAR	106.70	Variance of x (exam scores).
x⇄y	119.93	Variance of y (I.Q.).

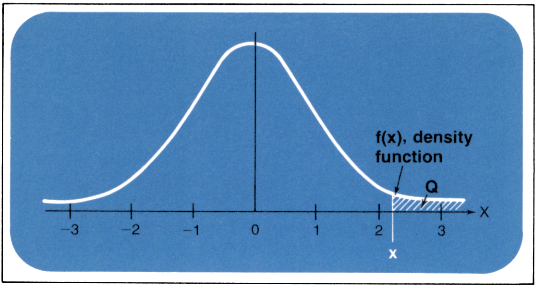
Just as you were able to calculate the standard deviation of a population as well as a sample, you can also calculate population variance. Press $\bar{x}$ Σ+ g VAR.

Press	Display	
$\bar{x}$	79.88	Mean (exam scores).
Σ+	9.00	Number of entries +1.
g VAR	93.36	Population variance (exam scores).
x⇄y	104.94	Population variance (I.Q.).

To continue summing data pairs, press $\bar{x}$ f Σ- before entering more data.

Normal Distribution

The N.D. key calculates the density function f(x) and the upper tail area Q under a standardized normal distribution curve, given x. The density function is the relative probability that a particular x value will occur. The upper tail area signifies the probability of occurrence of all values ⩾ x. The value of Q is returned to the X-register and the value f(x), to the Y-register.



The value input to N.D. is the standard variable. Given the mean ($\bar{x}$) and standard deviation (s) of a normal population, any value x' is converted to the standard variable by the following formula:

$$x = \frac{x' - \bar{x}}{s}$$

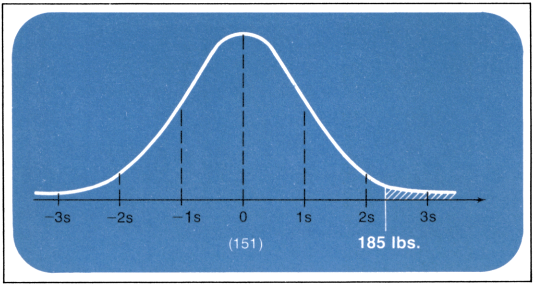
Example: The mean weight of 500 male students is 151 lbs. and the standard deviation is 15. Assuming that the weights are normally distributed, how many students weigh 185 or more?

First, find x according to the following formula:

$$x = \frac{x' - \bar{x}}{s} = \frac{185 - 151}{15}$$

Press	**Display**
185 ENTER↑ 151 −	
15 ÷	2.27

This means the standard variable (185 lbs) is 2.27 standard deviations to the right of the mean.



You already have 2.27 in the display, so to continue. . .

Press	Display	
g N.D.	0.01	Upper tail area (Q).
f FIX 4	0.0117	Displayed to four places.
x⇄y	0.0306	Density function.

To find approximately how many students weigh 185 or more, multiply the total number in the sample by the factor 0.0117.

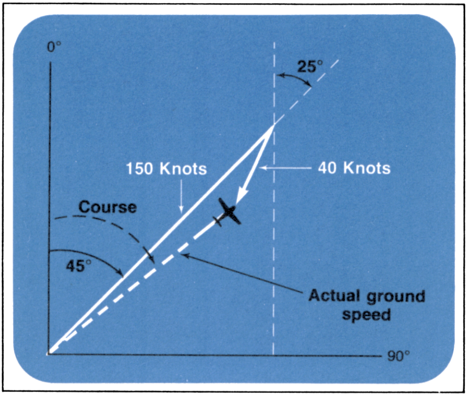
$$500 \times 0.0117 = 5.85 \text{ students}$$

Vector Summations

You have learned rectangular to polar coordinate conversions and you have learned how to sum quantities using Σ+. Combining these functions, you can perform vector addition and subtraction.

Example: In his converted Swordfish aircraft, bush pilot Apeneck Sweeney reads an air speed of 150 knots and a heading of 045° from his instruments. The Swordfish is also being buffeted by a headwind of 40 knots from a bearing of 025°. What should Sweeney use to have a true heading of 45°? What is the ground speed of Swordfish?

Method: The true heading and ground speed are equal to the sum of the instrument vector and the wind vector. The vectors are converted to rectangular coordinates and summed using the [Σ+] and [Σ-] keys. Their sum is recalled by recalling the values in storage registers R_5 (Σx) and R_7 (Σy), and the new rectangular coordinates are then converted back to polar coordinates to give the vector of the actual ground speed and true heading.



Press	Display	
[f] [REG]	0.00	Clears storage registers. (Display assumes no results remain from previous examples.)
[g] [DEG]	0.00	Sets degrees mode.
45 [ENTER↑]	45.00	θ for the Swordfish instrument vector.
150	150.	r for the Swordfish instrument vector.
[f] [→R]	106.07	Converted to rectangular coordinates.

Σ+	1.00	Instrument coordinates accumulated in storage registers R_5 and R_7.
25 ENTER↑	25.00	θ for wind vector.
40	40.	r for wind vector.
f →R	36.25	Converted to rectangular coordinates.
f Σ-	0.00	Coordinates for wind vector subtracted from coordinates for Swordfish's instrument vector.
RCL 7	89.16	Recalls sum of y-coordinates from register R_7.
RCL 5	69.81	Recalls sum of x-coordinates from register R_5. (Sum of y-coordinates lifted to Y-registers.)
g →P	113.24	Actual ground speed of the Swordfish in knots.
x⇄y	51.94	True heading of the Swordfish in degrees.

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FIX
SCI
ENG
PMT
PV
FV
PREFIX
CLEAR
REG
STK
ENTER ↑
CHS
EEX
CLX
STO
RCL
ln
log
sin
cos
tan
H.MS+
LAST x
Σ+
№ 74495
TASMAN SEA
NEW ZEALAND
PHILIPPINE SEA
CORAL SEA

Section 4

Applications

This section covers both common and complex problems encountered in the fields of statistics, mathematics, navigation, surveying, and finance. Problems, such as these, demonstrate the amazing versatility of your HP-27.

Statistical Applications

Permutations

A permutation of m different objects, taken n at a time, is an *arrangement* of n out of m objects with several possible ways of ordering the arrangement.

$$_m\text{Permutation}_n = \frac{m!}{(m - n)!}$$

where m and n are integers and $0 \leqslant n \leqslant m \leqslant 69$.

For example, how many ways can 10 people be seated on a bench if only 4 seats are available?

$$_{10}P_4 = \frac{10!}{6!} = 5040$$

Note:

$$_mP_o = \frac{m!}{m!} = 1$$

$$_mP_1 = \frac{m!}{(m - 1)!} = m$$

$$_mP_m = m!$$

The keystrokes are as follows:

1. Key in the value m, and press f n! f LAST X.
2. Key in the value n.
3. Press − f n! ÷.

Example: A child has a set of toy blocks with the 26 letters of the alphabet each appearing once. How many different three-letter permutations ($_{26}P_3$) can be made with these blocks?

Press	Display	
26 f n! f LAST X		
3 − f n! ÷	15600.00	Number of permutations.

Combinations

A combination of m different objects taken n at a time is a *selection* of n out of m objects with no attention given to the order of arrangement. The number of possible combinations is given by:

$$_mC_n = \frac{m!}{(m-n)!\,n!}$$

where m and n are integers and $0 \leqslant n \leqslant m \leqslant 69$.

In the permutation example, you arranged 10 people in different order, four at a time. If you were to *select* four out of the same 10 people for a committee, how many combinations could you pick?

$$_{10}C_4 = \frac{10!}{6!\,4!} = 210$$

Note:

$$_mC_0 = \frac{m!}{m!} = 1$$

$$_mC_1 = \frac{m!}{(m-1)!1!} = m$$

The following keystrokes give the number of possible combinations:

1. Key in the value m. Press f n! f LAST X.
2. Key in the value n.
3. Press − f LAST X f n!.
4. Press x⇄y f n! × ÷.

Example: A manager wants to choose a committee of three people from the seven engineers working for him. In how many different ways can the committee be selected?

Press	Display
7 f n! f LAST X	
3 − f LAST X f n!	
x⇄y f n! × ÷	35.00

The committee may be chosen in ${}_7C_3 = 35$ different ways.

* * * * * *

In section 3, linear regression was used as an analysis tool for plotting and projecting values. But perhaps your data doesn't fit a straight line. The three applications that follow are the methods of fitting your data to three different types of curves: exponential, logarithmic and power curves.

Exponential Curve Fit

The exponential curve is representative of situations where an increase or decrease in a quantity is "compounded" over time; e.g., financial growth curve, compounded amount, or radioactive decay.

Using natural logarithms, a least squares exponential curve fit may be calculated according to the equation $y = ae^{bx}$. The keystrokes given here compute the estimates of the constants a and b by rewriting the equation $y = ae^{bx}$ as

$$\ln y = bx + \ln a$$

and solving this equation as a linear regression problem. (The y values must be positive.) Here, a is the y-intercept.

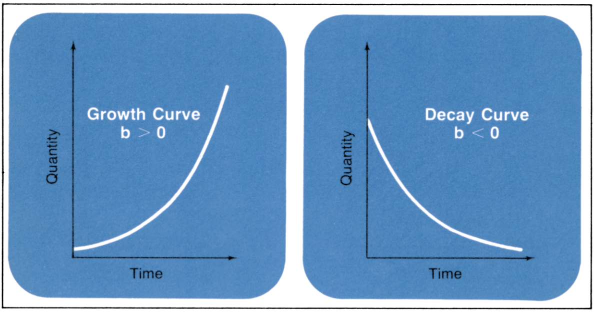
$$b = \frac{\Sigma x_i \ln y_i - \frac{1}{n}(\Sigma x_i)(\Sigma \ln y_i)}{\Sigma x_i^2 - \frac{1}{n}(\Sigma x_i)^2}$$

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - b\frac{\Sigma x_i}{n}\right]$$

$$\hat{y} = ae^{bx}$$

$$x = \frac{1}{b}(\ln y - \ln a)$$

If $b > 0$, you will have a growth curve. If $b < 0$, you will have a decay curve. Examples of these are given below:



The keystrokes are as follows:

1. Press f Σ.
2. For each pair of values, key in y, press f ln, key in x, press Σ+. Repeat this step for all data pairs.

3. Press g L.R. g e^x. The value of a is displayed.
4. Press x⇄y to obtain the slope of the curve.
5. Press g e^x 1 − 100 × to obtain the percentage growth rate.
6. Press g r to obtain the correlation coefficient.
7. To solve for a projected value of y, key in the x value and press ŷ g e^x. The value ŷ is displayed.
8. To find a projected x value, press g L.R.. Key in the y value, and press f ln x⇄y − x⇄y ÷. The value x̂ is displayed.

Example 1: A stock's price history is listed below. If the stock continues this growth rate, what is the price projected to be at the end of 1976 (year 5)? *(Hint: Let y be the price of the stock and x, the year. Solve for ŷ.)*

End of Year	Price
1972 (1)	52-½
1973 (2)	55-¼
1974 (3)	(missing data)
1975 (4)	61
1976 (5)	?

Press	Display	
f Σ		
52.5 f ln 1 Σ+	1.00	First data pair input.
55.25 f ln 2 Σ+	2.00	Second data pair input.
61 f ln 4 Σ+	3.00	Third data pair input.
g L.R. g e^x	49.96	Value of a.
x⇄y	0.05	Slope of curve.
g e^x 1 − 100 ×	5.12	% growth rate.
5 ŷ g e^x	64.14	Projected price at the end of year 1976.

Example 2: A company's growth over a period of several years is measured by its net earnings, in thousands of dollars, at the end of each fiscal year. The data below shows this growth:

Fiscal year	1	2	3	4	5	6	?
Earnings (K$)	179.6	215.1	260.7	301.2	368.5	424.1	1000

If the earnings (y) are assumed to be growing exponentially with time (x), at what time in the future will earnings hit $1 million? (Here you are solving for $\hat{x}$.)

Press	**Display**	
f [Σ]		
179.6 f [ln] 1 [Σ+]	1.00	First data pair.
215.1 f [ln] 2 [Σ+]	2.00	
260.7 f [ln] 3 [Σ+]	3.00	
301.2 f [ln] 4 [Σ+]	4.00	
368.5 f [ln] 5 [Σ+]	5.00	
424.1 f [ln] 6 [Σ+]	6.00	
g [L.R.] g [e^x]	152.40	Value of a.
[x⇄y]	0.17	Slope of curve.
g [e^x] 1 [−] 100 [×]	18.89	% growth rate.
g [r]	1.00	Correlation coefficient.
g [L.R.]		
1000 f [ln] [x⇄y] [−] [x⇄y] [÷]	10.87	Years.

After 10.87 years, earnings will be $1000K or a million dollars.

Logarithmic Curve Fit

If your data doesn't fit linear regression or an exponential curve, try logarithmic curve fit. This is calculated according to the equation $y = a + b (\ln x)$, and all x values must be positive.

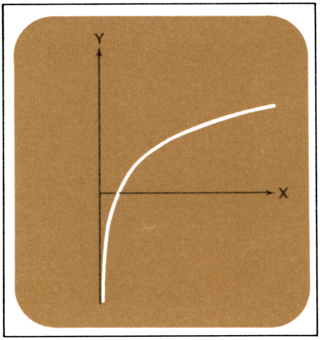
$$b = \frac{\Sigma y_i \ln x_i - \frac{1}{n} \Sigma \ln x_i \Sigma y_i}{\Sigma (\ln x_i)^2 - \frac{1}{n} (\Sigma \ln x_i)^2}$$

$$a = \frac{1}{n} (\Sigma y_i - b \Sigma \ln x_i)$$

$$\hat{y} = a + b (\ln x)$$

$$x = e^{[(y - a)/b]}$$

A typical logarithmic curve is shown below.



The following keystrokes calculate logarithmic curve fit according to the equation $y = a + b(\ln x)$:

1. Press f Σ.
2. Key in the first y value, press ENTER↑. Key in the first x value, press f ln Σ+. Repeat this step for each data pair.
3, After all data pairs are input, press g L.R. to obtain a in the equation above.
4. Press x⇄y to obtain b.

5. Press g r to obtain the correlation coefficient.
6. To solve for a projected y value, key in the x value, and press f ln ŷ.
7. To obtain a projected x value, press g L.R.. Key in the y value, and press x⇄y − x⇄y ÷ g e^x. The value $\hat{x}$ is displayed.

Example: A manufacturer observes declining sales of a soon-to-be obsoleted product, of which there were originally 10,000 units in inventory. The cumulative sales figures over a number of months, given below, may be fit by a logarithmic curve of the form $y = a + b \ln x$, where y represents cumulative sales in units and x the number of months since the beginning. Find what month the last unit (number 10,000) is projected to be sold.

Month	1	2	3	4	5	6
Cumulative Sales (units)	1431	3506	5177	6658	7810	8592

Press	Display	
f Σ		
1431 ENTER↑ 1 f ln Σ+	1.00	First data pair.
3506 ENTER↑ 2 f ln Σ+	2.00	
5177 ENTER↑ 3 f ln Σ+	3.00	
6658 ENTER↑ 4 f ln Σ+	4.00	
7810 ENTER↑ 5 f ln Σ+	5.00	
8592 ENTER↑ 6 f ln Σ+	6.00	
g L.R.	1066.15	Value of a.
x⇄y	4069.93	Value of b.
g r	0.99	Correlation coefficient.
g L.R. 10000 x⇄y −		
x⇄y ÷ g e^x	8.98	Months.

Considering that you start at 0 and 0.9 is near the end of the first month (it's like centuries), then $\hat{x}$ = 8.98 means that the 10,000th unit will be sold near the end of the ninth month.

Power Curve Fit

Another method of analysis is the power curve or geometric curve. The equation $y = ax^b$ is linearized to the form

$$\ln y = \ln a + b (\ln x)$$

and values a and b are computed by calculations similar to linear regression.

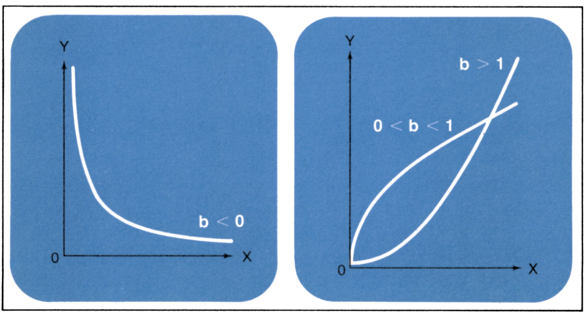
$$b = \frac{\Sigma (\ln x_i)(\ln y_i) - \dfrac{(\Sigma \ln x_i)(\Sigma \ln y_i)}{n}}{\Sigma (\ln x_i)^2 - \dfrac{(\Sigma \ln x_i)^2}{n}}$$

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - b\,\frac{\Sigma \ln x_i}{n}\right]$$

$$\hat{y} = ax^b$$

$$\hat{x} = e^{[(y - \ln a)/b]}$$

Some examples of power curves are shown below.



The following keystrokes fit a power curve according to the equation $\ln y = a + b (\ln x)$:

1. Press f Σ.
2. Key in the first y value, press f ln. Key in the first x value, press f ln Σ+. Repeat this step for all data pairs.
3. Press g L.R. g e^x to obtain a in the above equation.
4. Press x⇄y to obtain b.
5. Press g r to obtain the correlation coefficient.
6. To find a projected y, key in the x value and press ŷ g e^x to obtain ŷ.
7. To find a projected x, press g L.R. f ln. Key in the y value, and press x⇄y − x⇄y ÷ g e^x. The x̂ value is displayed.

Example: If Galileo had wished to investigate quantitatively the relationship between the time (t) for a falling object to hit the ground and the height (h) it has fallen, he might have released a rock from various levels of the Tower of Pisa (which was leaning even then) and timed its descent by counting his pulse. The following data are measurements Galileo might have made.

t (pulses)	2	2.5	3.5	4	4.5
h (feet)	30	50	90	130	150

Find the power curve formula that best expresses h as a function of t ($h = at^b$).

Press	**Display**	
f Σ		
30 f ln 2 f ln Σ+	1.00	First data pair.
50 f ln 2.5 f ln Σ+	2.00	
90 f ln 3.5 f ln Σ+	3.00	
130 f ln 4 f ln Σ+	4.00	
150 f ln 4.5 f ln Σ+	5.00	

g L.R. g e^x	7.72	Value of a.
x⇄y	1.99	Value of b.
g r	1.00	Correlation coefficient.

So the formula that best expresses h as a function of t is

$$h = 7.72\ t^{1.99}$$

We know, as Galileo did not, that in fact $h \propto t^2$.

Chi-Square Statistic

The chi-square statistic measures the goodness of fit between two sets of frequencies. It's used to test whether a set of observed frequencies differ from a set of expected frequencies sufficiently to reject the hypothesis under which the expected frequencies were obtained.

In other words, you are testing whether discrepancies between the observed frequencies (O_i) and the expected frequencies (E_i) are significant, or whether they may reasonably be attributed to chance. The formula generally used is:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

If there is a close agreement between the observed and expected frequencies, χ^2 will be small. If the agreement is poor, χ^2 will be large.

The following keystrokes calculate the χ^2 statistic:

1. Press CLX ENTER↑.
2. Key in the first O_i value, press ENTER↑.
3. Key in the first E_i value, press STO 0 − ENTER↑ × RCL 0 ÷ +.
4. Repeat steps 2 and 3 for all data pairs. The χ^2 value is displayed.

Example: A suspect die from a Las Vegas casino is brought to an independent testing firm to determine its bias, if any. The die is tossed 120 times and the following results obtained.

Number	1	2	3	4	5	6
Observed Frequency	25	17	15	23	24	16

The expected frequency = 120 throws/6 sides, or E = 20 for each number, 1 thru 6. (Since E is a constant in this example, there's no need to store it in R_0 each time.)

Press	Display	
CLx ENTER↑ 25 ENTER↑ 20 STO 0		
− ENTER↑ × RCL 0 ÷ +	1.25	
17 ENTER↑ 20 −		
ENTER↑ × RCL 0 ÷ +	1.70	
15 ENTER↑ 20 −		
ENTER↑ × RCL 0 ÷ +	2.95	
23 ENTER↑ 20 −		
ENTER↑ × RCL 0 ÷ +	3.40	
24 ENTER↑ 20 −		
ENTER↑ × RCL 0 ÷ +	4.20	
16 ENTER↑ 20 −		
ENTER↑ × RCL 0 ÷ +	5.00	χ^2.

The number of degrees of freedom is (n − 1). Since n = 6, the degrees of freedom = 5.

Consulting statistical tables, you look up χ^2 to a 0.05 significance level with 5 degrees of freedom, and see that $\chi^2_{0.05,\ 5} = 11.07$. Since $\chi^2 = 5$ is within 11.07, we may conclude that to a 0.05 significance level (Probability = .95), the die is fair.

Paired t Statistic

There are other tests of significance besides chi-square. Suppose you have two samples drawn from two normal populations. The t test is the most commonly used method of testing the differences between the two means.

An example might be the "before" and "after" characteristics of a group that has been subjected to some treatment. The t statistic tests the equality of the means, in this case, to determine whether the treatment has resulted in an "after" group whose characteristics differ significantly from the "before" group.

Let

$$D_i = x_i - y_i \text{ (the difference of all the x's and y's)}$$

$$\overline{D} = \frac{1}{n} \sum_{i=1}^{n} D_i$$

$$s_D = \sqrt{\frac{\Sigma D_i^2 - \frac{1}{n}(\Sigma D_i)^2}{n-1}}$$

$$s_{\overline{D}} = \frac{s_D}{\sqrt{n}}$$

The test statistic

$$t = \frac{\overline{D}}{s_{\overline{D}}}$$

which has n − 1 degrees of freedom (df) can be used to test the null hypothesis

$$H_0: \overline{x} = \overline{y}$$

In other words, the hypothesis to be tested is that the two means are *not* significantly different. The following keystrokes calculate test statistic t and the degrees of freedom (df):

1. Press f Σ.
2. Key in the first x value, press ENTER↑. Key in the first y value, press − Σ+. Repeat this step for all data pairs. *(Note that the order of entry differs from previous problems. Here you key in the x value first, then the y value.)*

3. Press g s STO 3. The s_D value is displayed.
4. Press $\bar{x}$. The $\overline{D}$ value is displayed.
5. Press RCL 3 ÷ RCL 4 f √x ×. The test statistic t is displayed.
6. Press RCL 4, press 1 − to obtain degrees of freedom (df).

Example: An electronics firm makes an efficiency study of its production line. One of the factors to be evaluated is the ambient temperature in the building. The firm drops the temperature from the standard 23°C (73.4°F) to 21.5°C (70.7°F). The total outputs at each of 7 production stations are recorded for one day with ambient temperature 23°C, and then for a day at 21.5°C, and are given below.

23°C	147	38	103	96	62	117	98
21.5°C	153	35	94	101	60	106	90

Press	**Display**	
f Σ		
147 ENTER↑ 153 − Σ+	1.00	First data pair.
38 ENTER↑ 35 − Σ+	2.00	
103 ENTER↑ 94 − Σ+	3.00	
96 ENTER↑ 101 − Σ+	4.00	
62 ENTER↑ 60 − Σ+	5.00	
117 ENTER↑ 106 − Σ+	6.00	
98 ENTER↑ 90 − Σ+	7.00	Total number of entries.
g s STO 3	6.72	Standard deviation is calculated and stored.
$\bar{x}$	3.14	$\overline{D}$
RCL 3 ÷ RCL 4 f √x ×	1.24	t (test statistic).
RCL 4	7.00	
1 −	6.00	df (degrees of freedom).

Consulting statistical tables, we find that $t_{0.1,\ 6} = 1.44$; i.e., t to a 0.1 significance level, with 6 degrees of freedom, equals 1.44. Since $t = 1.24$ is within that limit, we conclude that the change in temperature did not appreciably affect the production line output.

t Statistic for Two Means

Suppose there are two normal populations with unknown means and the same variance. One may wish to examine the relationship between the two means, and hypothesize that there *is* a difference:

$$H_0: \mu_1 - \mu_2 = D$$

where D is some constant, and μ_1 and μ_2 are the means of the two populations.

$$\bar{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

$$\bar{y} = \frac{1}{n_2} \sum_{i=1}^{n_2} y_i$$

$$t = \frac{\bar{x} - \bar{y} - D}{\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}\ \sqrt{\dfrac{\Sigma x_i^2 - n_1\bar{x}^2 + \Sigma y_i - n_2\bar{y}^2}{n_1 + n_2 - 2}}}$$

We can use this t statistic which has the t distribution with $n_1 + n_2 - 2$ degrees of freedom (df) to test the null hypothesis H_0.

The procedure below calculates the t statistic for two means to measure the accuracy of this hypothesis.

1. Press f Σ.
2. Key in the first x value, press Σ+. Repeat this step for all x values.
3. Press $\bar{x}$ STO 0 g VAR RCL 4 STO 2.

4. Press 1 [−] [×] [STO] 1.
5. Press [f] [Σ].
6. Key in the first y value, press [Σ+]. Repeat this step for all y values.
7. Press [x̄] [CHS] [RCL] 0 [+].
8. Key in the constant D, and press [−] [STO] 0 [g] [VAR] [RCL] 4. Press 1 [−] [×] [RCL] 1 [+].
9. Press [RCL] 4 [RCL] 2 [+] 2 [−]. The degrees of freedom (df) are displayed.
10. Press [÷] [RCL] 4 [g] [1/x] [RCL] 2 [g] [1/x] [+] [×] [f] [√x] [RCL] 0 [x⇄y] [÷]. The t statistic is displayed.

Example: Figures indicate that, at age 12, girls have a mean height that is 2.3 centimeters greater than the mean height of boys that age. A sixth-grade class measures the heights of its 12-year-olds and finds these data for 11 girls and 7 boys (all heights in cm):

Girls:	158,	144,	139,	154,	149,	150,	154,	160,	147,	155,	143
Boys:	152,	138,	154,	147,	158,	148,	147				

Find the t statistic for this data to test the hypothesis that $\mu_1 - \mu_2 = 2.3$ cm.

Press	**Display**	
[f] [Σ]		
158 [Σ+] 144 [Σ+] 139 [Σ+]	3.00	
154 [Σ+] 149 [Σ+] 150 [Σ+]	6.00	
154 [Σ+] 160 [Σ+] 147 [Σ+]	9.00	
155 [Σ+] 143 [Σ+]	11.00	Total number of girls.
[x̄] [STO] 0 [g] [VAR]	43.62	Variance $_x$.
[RCL] 4 [STO] 2		
1 [−] [×] [STO] 1	436.18	

f [Σ]		
152 [Σ+] 138 [Σ+] 154 [Σ+]	3.00	
147 [Σ+] 158 [Σ+] 148 [Σ+]	6.00	
147 [Σ+]	7.00	Total number of boys.
[x̄] CHS RCL 0 [+]	1.13	
2.3 [−] STO 0 g [VAR]	40.81	Variance $_y$.
RCL 4		
1 [−] [×] RCL 1 [+] RCL 4		
RCL 2 [+] 2 [−]	16.00	df.
[÷] RCL 4 g [1/x] RCL 2 g [1/x]		
[+] [×] f [√x] RCL 0 [x⇄y] [÷]	−0.37	t statistic.

Consulting statistical tables, we find that $t_{0.1,\ 16} = 1.337$. Since $t = -0.37$ falls within that limit, the hypothesis of height difference is not rejected at the 0.1 level of significance.

Analysis of Variance (One-Way)

The one-way analysis of variance tests the differences among the means of several samples to see whether the variability is due to chance or due to real differences in the populations from which the samples were drawn.

Variability by chance is measured by the Error SS (sum of the squares). Variability caused by real differences is measured by Treat SS. Error SS and Treat SS are normalized to mean square value to obtain an F ratio.

Let

x_{ij} = j^{th} observation in i^{th} treatment group

n_i = number of observations in group i

$\bar{x}_i$ = mean of group i $= \dfrac{1}{n_i} \sum_{j=1}^{n_i} x_{ij}$

$$\bar{x} = \text{grand mean of observations} = \frac{\sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij}}{\sum_{i=1}^{k} n_i}$$

$$\text{Then Total SS} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (x_{ij} - \bar{x})^2 = \sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij}{}^2 - \frac{\left(\sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij}\right)^2}{\sum_{i=1}^{k} n_i}$$

$$\text{Treat SS} = \sum_{i=1}^{k} n_i (\bar{x}_i - \bar{x})^2 = \sum_{i=1}^{k} \frac{\left(\sum_{j=1}^{n_i} x_{ij}\right)^2}{n_i} = \frac{\left(\sum_{i=1}^{k} \sum_{j=1}^{n_i}\right)^2}{\sum_{i=1}^{k} n_i}$$

$$\text{Error SS} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2 = \text{Total SS} - \text{Treat SS}$$

$$df_1 = \text{Treat df} = k - 1$$

$$df_2 = \text{Error df} = \sum_{i=1}^{k} n_i - k$$

$$\text{Treat MS} = \frac{\text{Treat SS}}{\text{Treat df}}$$

$$\text{Error MS} = \frac{\text{Error SS}}{\text{Error df}}$$

$$F = \frac{\text{Treat MS}}{\text{Error MS}} \quad \left(\text{with } k - 1 \text{ and } \sum_{i=1}^{k} n_i - k \text{ degrees of freedom}\right)$$

The following keystrokes yield the analysis of a variance table: sum of squares, mean squares, degrees of freedom, and the F ratio.

1. Press f REG.
2. Press 1 STO + 3.
3. Key in the first value in the first row across and press STO + 0 Σ+ 1 STO + 1. Repeat this step for the second value in the first row across, the third value, etc. to the end of the first row.
4. Press RCL 0 ENTER↑ × RCL 1 ÷ STO + 2.
5. Press 0 STO 0 STO 1.
6. Repeat steps 2 thru 5 for each row of the table.
7. Press RCL 6 RCL 5 ENTER↑ ×.
8. Press RCL 4 ÷ −. The Total SS value is displayed.
9. Press RCL 2 RCL 5 ENTER↑ × RCL 4 ÷ −. The Treat SS value is displayed.
10. Press − to obtain Error SS.
11. Press f LAST X RCL 3, press 1 −. The Treat df value is displayed.
12. Press ÷ to obtain Treat MS.
13. Press x⇄y RCL 4 RCL 3 −. The Error df value is displayed.
14. Press ÷ to obtain Error MS.
15. Press ÷ to obtain F.

Example: Lucy Shopper is choosing a new car from among three different makes of comparable price. One of the factors she wishes to evaluate is the expected resale value of the car after 2 years. After identifying a number of two-year-old cars of each make, she compiles the table of prices below (all prices in thousands of dollars):

		Cars				
		1	2	3	4	5
	1	3.4	4.0	3.9	3.2	3.3
Make	2	4.1	4.0	3.7	4.1	
	3	3.6	2.9	3.0	3.5	3.4

It is not clear whether the variation in prices among the makes is a true difference, or due to randomness within the sample. To get a more quantitative answer to this question, Lucy runs an analysis of variance on the data.

Press	Display	
f REG		
1 STO + 3	1.00	
3.4 STO + 0 Σ+ 1 STO + 1	1.00	
4.0 STO + 0 Σ+ 1 STO + 1		
3.9 STO + 0 Σ+ 1 STO + 1	1.00	
3.2 STO + 0 Σ+ 1 STO + 1		
3.3 STO + 0 Σ+ 1 STO + 1	1.00	
RCL 0 ENTER↑ × RCL 1 ÷ STO		
+ 2	63.37	
0 STO 0 STO 1	0.00	
1 STO + 3		
4.1 STO + 0 Σ+ 1 STO + 1	1.00	
4.0 STO + 0 Σ+ 1 STO + 1		
3.7 STO + 0 Σ+ 1 STO + 1	1.00	
4.1 STO + 0 Σ+ 1 STO + 1		
RCL 0 ENTER↑ × RCL 1 ÷ STO		
+ 2	63.20	
0 STO 0 STO 1	0.00	
1 STO + 3		
3.6 STO + 0 Σ+ 1 STO + 1	1.00	
2.9 STO + 0 Σ+ 1 STO + 1		
3.0 STO + 0 Σ+ 1 STO + 1	1.00	
3.5 STO + 0 Σ+ 1 STO + 1		
3.4 STO + 0 Σ+ 1 STO + 1	1.00	
RCL 0 ENTER↑ × RCL 1 ÷ STO		
+ 2	53.79	
0 STO 0 STO 1	0.00	
RCL 6 RCL 5 ENTER↑ ×		
RCL 4 ÷ −	2.10	Total SS.

Keystrokes	Display	
RCL 2 RCL 5 ENTER↑ ×		
RCL 4 ÷ −	1.08	Treat SS.
−	1.03	Error SS.
f LAST X RCL 3		
1 −	2.00	Treat df.
÷	0.54	Treat MS.
x⇄y RCL 4 RCL 3 −	11.00	Error df.
÷	0.09	Error MS.
÷	5.76	F (with 2 and 11 degrees of freedom).

Consulting statistical tables, we find that F to a .05 significance level, with 2 and 11 degrees of freedom, equals 3.98 ($F_{.05,\ 2,\ 11} = 3.98$).

If the F ratio obtained above fell within this limit, the differences in resale price would not be significant. Since F = 5.76 exceeds the value F = 3.98, we conclude that the resale prices of different makes are significantly different.

Covariance

Covariance is a measure of the interdependence between paired variables (x and y). Like standard deviation, covariance may be defined for either a sample (S_{xy}) or a population (S'_{xy}) as follows:

$$S_{xy} = r \times s_x \times s_y$$

$$S'_{xy} = r \times s'_x \times s'_y$$

The following keystrokes solve for covariance of a sample (S_{xy}) and of a population (S'_{xy}):

1. Press f Σ.
2. Key in y, press ENTER↑, key in x, and press Σ+. Repeat this step for all data pairs.
3. Press g s × ENTER↑ g r ×. The value of S_{xy} appears in the display.
4. With S_{xy} on the display, press RCL 4, press 1 − RCL 4 ÷ × to obtain S'_{xy}.

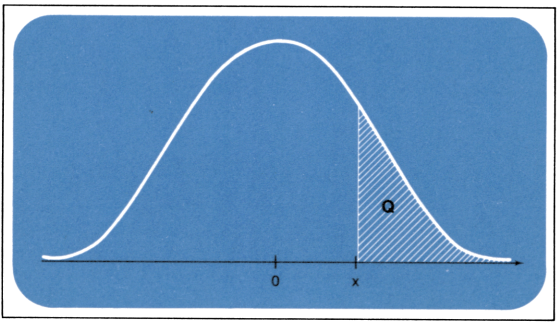
Example: Find the sample covariance (S_{xy}) and population covariance (S'_{xy}) for the following paired variables:

x_i	26	30	44	50	62	68	74
y_i	92	85	78	81	54	51	40

Press	Display	
f Σ		
92 ENTER↑ 26 Σ+	1.00	First entry.
85 ENTER↑ 30 Σ+	2.00	
78 ENTER↑ 44 Σ+	3.00	
81 ENTER↑ 50 Σ+	4.00	
54 ENTER↑ 62 Σ+	5.00	
51 ENTER↑ 68 Σ+	6.00	
40 ENTER↑ 74 Σ+	7.00	Total number of entries.
g s × ENTER↑ g r ×	-354.14	S_{xy}
RCL 4	7.00	Number of entries.
1 − RCL 4 ÷ ×	-303.55	S'_{xy}

Inverse Normal Integral

With the preprogrammed normal distribution function N.D. in section 3, you found the upper tail area Q, given a point x.



This procedure performs the inverse or opposite operation. That is, given an upper tail area Q, you can determine the value of x. (Note: $0 < Q \leqslant 0.5$)

$$x = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(Q)$$

where $\left|\epsilon(Q)\right| < 4.5 \times 10^{-4}$

$$t = \sqrt{\ln \frac{1}{Q^2}}$$

$c_0 = 2.515517$ $\quad d_1 = 1.432788$
$c_1 = 0.802853$ $\quad d_2 = 0.189269$
$c_2 = 0.010328$ $\quad d_3 = 0.001308$

This keystroke procedure solves for point x, given the upper tail area Q:

1. Input and store the following constants:
   - Key in 2.515517 (c_0) and press STO 0.
   - Key in 0.802853 (c_1) and press STO 1.
   - Key in 0.010328 (c_2) and press STO 2.
   - Key in 1.432788 (d_1) and press STO 3.
   - Key in 0.189269 (d_2) and press STO 4.
   - Key in 0.001308 (d_3) and press STO 5.
2. Key in the given Q value, press ENTER↑ ⊠ g 1/x f ln f √x STO 6.
3. Press ENTER↑ ENTER↑ ENTER↑ RCL 5 ⊠ RCL 4 ⊞ ⊠ RCL 3 ⊞ ⊠ 1 ⊞ STO 7.
4. Press CLX RCL 2 ⊠ RCL 1 ⊞ ⊠ RCL 0 ⊞ RCL 7 ⊟÷ ⊟.
5. For a new Q, go to step 2.

If less accuracy is acceptable, it may be faster to arrive at x by successive approximations, using the N.D. function to calculate Q for each estimate of x. Continue to revise your estimate of x until the calculated Q is within an acceptable tolerance.

Example: Find point x if Q = 0.12.

Press	Display	
2.515517 STO 0	2.52	
0.802853 STO 1	0.80	
0.010328 STO 2	0.01	
1.432788 STO 3	1.43	
0.189269 STO 4	0.19	
0.001308 STO 5	1.308000–03	
0.12 ENTER↑ ⊠ g 1/x		
f ln f √x STO 6	2.06	
ENTER↑ ENTER↑ ENTER↑		
RCL 5 ⊠ RCL 4 ⊞ ⊠		
RCL 3 ⊞ ⊠ 1 ⊞ STO 7	4.76	
CLx	0.00	
RCL 2 ⊠ RCL 1 ⊞ ⊠		
RCL 0 ⊞ RCL 7 ⊟ ⊟	1.18	Value of x.

Mathematical Applications

The following calculations solve problems commonly encountered in mathematical and scientific applications.

Quadratic Equation

This calculation solves for the roots x_1, x_2 of the quadratic equation

$$ax^2 + bx + c = 0$$

1. Key in a, press STO 0 ENTER↑ ⊞.
2. Key in b, press x⇄y ÷ CHS ENTER↑ ENTER↑ ⊠.
3. Key in c, press RCL 0 ÷ STO 1 ⊟.
4. If the displayed number is positive or 0, the roots are real. Skip to step 5 for the solutions.

   If the displayed number is negative, this means the roots are complex (u ± iv). Press CHS f √x x⇄y for the value of u, and press x⇄y to obtain v.

5. Press **f** **√x** **x⇄y** . If the displayed number is ⩾ 0, press **+** to obtain the first root x_1. If the displayed number is < 0, press **−** **CHS** to obtain x_1.
6. Press **RCL** 1 **x⇄y** **÷** to obtain the second root x_2.

Example: A man stands at the top of a cliff 110 meters (361 feet) high. He throws a rock off the cliff with a velocity in the vertical direction of 20 m/s. How long will it take the rock to hit the ground? (Take the acceleration due to gravity as 9.8 m/s².)

The equation to be solved is $s = 1/2\ gt^2 + v_o t + s_o$ where $s = 0$, $g = -9.8$, $v_o = 20$, and $s_o = 110$. Rewrite the equation as $4.9\ t^2 - 20t - 110 = 0$.

Press	Display	
4.9 **STO** 0 **ENTER↑** **+**	9.80	
20 **CHS** **x⇄y** **÷** **CHS** **ENTER↑**		
ENTER↑ **×**	4.16	
110 **CHS** **RCL** 0 **÷** **STO** 1 **−**	26.61	Roots are real.
f **√x** **x⇄y** **+**	7.20	Value of t_1.
RCL 1 **x⇄y** **÷**	−3.12	Value of t_2.

The primary value for t is 7.20 seconds. (The secondary root, −3.12, is not meaningful here.)

Polynomial Evaluation

This calculation solves a polynomial of the form:

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$$

where n can be any positive integer.

The following routine calculates f(x):

1. Key in the value of x. Press **ENTER↑** three times.
2. Key in the value of a_n and press **×**.

3. Key in the value of a_{n-1}, press $\boxed{+}$ $\boxed{\times}$. Repeat this step for all the a values thru a_1.
4. Key in the value of a_0 and press $\boxed{+}$. The value of f(x) is displayed.

Example: Let $f(x) = 4x^3 - x^2 + 17x - 5$. Find f (7).

Press	Display	
7 ENTER↑ ENTER↑ ENTER↑	7.00	
4 $\boxed{\times}$	28.00	
1 CHS $\boxed{+}$ $\boxed{\times}$	189.00	
17 $\boxed{+}$ $\boxed{\times}$	1442.00	
5 CHS $\boxed{+}$	1437.00	f(7).

Complex Arithmetic

This keystroke procedure is designed to perform chain calculations on complex numbers of the form a + ib. The operations involved are the basic four functions: +, −, ×, ÷. For a calculation such as $(a_1 + ib_1) \times (a_2 + ib_2)$, a_1, b_1, a_2, and b_2 are stored in registers R_1, R_2, R_3, and R_4, respectively. At the end of a calculation, the result u + iv is stored in registers R_1 (u) and R_2 (v). The next entry in the chain, $a_3 + ib_3$ is then keyed in and stored in registers R_3 (a_3) and R_4 (b_3).

Addition

$$(a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + (b_1 + b_2)i$$

Subtraction

$$(a_1 + ib_1) - (a_2 + ib_2) = (a_1 - a_2) + (b_1 - b_2)i$$

Multiplication

$$(a_1 + ib_2) \times (a_2 + ib_2) = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

Division

$$\frac{(a_1 + ib_1)}{(a_2 + ib_2)} = \frac{r_1}{r_2} e^{i(\theta_1 + \theta_2)}, a_2 + ib_2 \neq 0$$

where $r_1 e^{i\theta_1}$ is the polar representation of $a_1 + ib_1$, and $r_2 e^{i\theta_2}$ is the representation of $a_2 + ib_2$.

These keystrokes cover addition, subtraction, multiplication, and division of complex numbers. After the first step, skip to the desired function.

1. Input and store the following:
   - Key in a_1 and press STO 1.
   - Key in b_1 and press STO 2.
   - Key in a_2 and press STO 3.
   - Key in b_2 and press STO 4.

Addition

2. Press RCL 1 RCL 3 + to obtain the value of u.
3. Press RCL 2 RCL 4 + to obtain v.

Subtraction

2. Press RCL 1 RCL 3 − to obtain the value of u.
3. Press RCL 2 RCL 4 − to obtain v.

Multiplication

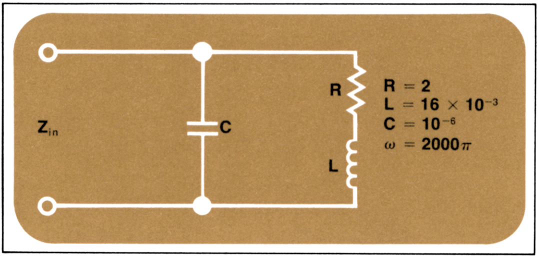
2. Press RCL 2 RCL 1 g →P.
3. Press RCL 4 RCL 3 g →P.
4. Press x≷y R↓ × R↓ + R↓ R↓ R↓ f →R to obtain the value of u.
5. Press x≷y to obtain v.

Division

2. Press RCL 2 RCL 1 g →P.
3. Press RCL 4 RCL 3 g →P.
4. Press x≷y R↓ ÷ R↓ − CHS R↓ R↓ R↓ f →R to obtain the value of u.
5. Press x≷y to obtain v.

To perform another operation, press STO 2 x≷y STO 1. Key in a_3, press STO 3, key in b_3, press STO 4, then go back to step 2 of the appropriate function.

Example:



In the above circuit, find Z_{in} if the equation is:

$$Z_{in} = \frac{-\frac{500}{\pi} i\,(2 + 32\,\pi i)}{2 + \left(32\pi - \frac{500}{\pi}\right) i}$$

(First, multiply the complex numbers in the numerator, then divide.)

Press	Display	
0 STO 1	0.00	a_1
500 CHS g π ÷ STO 2	–159.15	b_1
2 STO 3	2.00	a_2
32 g π × STO 4	100.53	b_2
RCL 2 RCL 1 g →P	159.15	
RCL 4 RCL 3 g →P	100.55	
x⇄y R↓ × R↓ +		
R↓ R↓ R↓ f →R	16000.00	Value of u.
x⇄y	–318.31	Value of v.

Now the problem becomes:

$$Z_{in} = \frac{16000 - 318.31\, i}{2 + \left(32\pi - \dfrac{500}{\pi}\right) i}$$

Press	Display	
STO 2 x⇄y STO 1		u_1 and v_1 stored.
2 STO 3	2.00	a_2.
32 g π × 500 g π ÷		
− STO 4	-58.62	b_2.
RCL 2 RCL 1 g →P	16003.17	
RCL 4 RCL 3 g →P	58.66	
x⇄y R↓ ÷ R↓ − CHS		
R↓ R↓ R↓ f →R	14.72	Value of u.
x⇄y	272.42	Value of v.

$$Z_{in} = 14.72 + 272.42\, i.$$

Complex Functions

This routine calculates various functions of a complex number. Let the complex number $z = a + ib$ have polar representation $re^{i\theta}$. Then the functions and their formulas are as follows:

1. $|z| = r.$
2. $z^2 = r^2 e^{i2\theta}.$
3. $1/z = \dfrac{1}{r} e^{-i\theta}, z \neq 0.$
4. $\sqrt{z} = \pm(\sqrt{r}\, e^{i\theta/2}) = \pm(u + iv).$

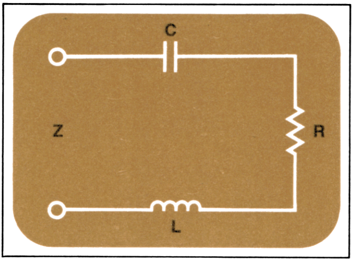
Let the answer be represented by $u + iv$.

The following procedures solve for $|z|$, z^2, $1/z$ and $\sqrt{z}$.

1. Key in b, press ENTER↑. Key in a, press g →P. The absolute value of z is displayed.
2. To calculate z^2, go to step 3. To calculate $1/z$, go to step 4. To calculate $\sqrt{z}$, go to step 5.

3. Press ENTER↑ × x⇄y ENTER↑ + x⇄y f →R. The value of u is displayed. Press x⇄y to obtain v.
4. Press g 1/x x⇄y CHS x⇄y f →R. The value of u is displayed. Press x⇄y to obtain v.
5. Press f √x x⇄y 2 ÷ x⇄y f →R. The value of u is displayed. Press x⇄y to obtain v.

Example:



In the circuit above, the impedance Z is given by

$$z = 2 + i\left(\omega L - \frac{1}{\omega C}\right)$$

Given R = 2, L = 16×10^{-3}, C = 1.5×10^{-6}, $\omega = 2000\pi$, the formula becomes

$$z = 2 + i\left(32\pi - \frac{1000}{3\pi}\right)$$

The admittance Y of the circuit is defined as

$$Y = 1/z$$

Find Y and $|Y|$.

Press	Display	
32 [g] [π] [×] 1000 [ENTER↑] 3 [÷]		
[g] [π] [÷] [−] 2 [g] [→P]	5.92	$\lvert z \rvert$.
[g] [1/x] [x⇄y] [CHS] [x⇄y] [f] [→R]	0.06	Value of u.
[x⇄y]	0.16	Value of v.
[x⇄y] [g] [→P]	0.17	$\lvert Y \rvert$.

$$Y = 1/z = 0.06 + 0.16\,i.$$

$$\lvert Y \rvert = 0.17.$$

Vector Cross Product

If $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ are two three-dimensional vectors then the cross product of A and B is calculated as follows:

$$A \times B = \left(\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}, - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}, \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \right) =$$
$$(a_2 b_3 - a_3 b_2,\ a_3 b_1 - a_1 b_3,\ a_1 b_2 - a_2 b_1)$$

and the solution is represented by (c_1, c_2, c_3).

This procedure calculates the cross product (c_1, c_2, c_3) of two vectors.

1. Input and store the following:
   - Key in a_1, press [STO] 1.
   - Key in a_2, press [STO] 2.
   - Key in a_3, press [STO] 3.
   - Key in b_1, press [STO] 4.
   - Key in b_2, press [STO] 5.
   - Key in b_3, press [STO] 6.

2. Press [RCL] 2 [RCL] 6 [×] [RCL] 3 [RCL] 5 [×] [−].
   The value of c_1 is displayed.

3. Press RCL 3 RCL 4 ⊠ RCL 1 RCL 6 ⊠ ⊟.
   The value of c_2 is displayed.
4. Press RCL 1 RCL 5 ⊠ RCL 2 RCL 4 ⊠ ⊟.
   The value of c_3 is displayed.

Example: The force F on a particle with charge q which is moving with a velocity $\vec{v}$ through a magnetic field $\vec{B}$ is given by $F = q\,\vec{v} \times \vec{B}$. Suppose a proton ($q = 1.6 \times 10^{-19}$ coulomb) is moving with $\vec{v} = (0.4, 2.8, -1.2) \times 10^7$ m/s. A uniform magnetic field surrounding the proton is of a strength $\vec{B} = (1.3, -0.3, 0.7)$ tesla. Calculate the force on the proton.

This can be written as $F = q\,\vec{v} \times \vec{B} = (1.6 \times 10^{-19})\,(10^7)\,(0.4, 2.8, -1.2) \times (1.3, -0.3, 0.7)$ N.

Press	Display	
0.4 STO 1	0.40	a_1.
2.8 STO 2	2.80	a_2.
1.2 CHS STO 3	-1.20	a_3.
1.3 STO 4	1.30	b_1.
0.3 CHS STO 5	-0.30	b_2.
0.7 STO 6	0.70	b_3.
RCL 2 RCL 6 ⊠ RCL 3		
RCL 5 ⊠ ⊟	1.60	c_1.
RCL 3 RCL 4 ⊠ RCL 1		
RCL 6 ⊠ ⊟	-1.84	c_2.
RCL 1 RCL 5 ⊠ RCL 2		
RCL 4 ⊠ ⊟	-3.76	c_3.

The solution then becomes:

$$\begin{aligned} F &= (1.6 \times 10^{-12})\,(1.60, -1.84, -3.76)\text{ N} \\ &= (2.56, -2.94, -6.02) \times 10^{-12}\text{ N}. \end{aligned}$$

This is about 4×10^{14} times the weight of the proton.

Vector Dot Product

Let $\vec{a} = (a_1, a_2, \ldots, a_n)$ and $\vec{b} = (b_1, b_2, \ldots b_n)$ be two vectors. The norm of a is denoted by $|\vec{a}|$ and the norm of $\vec{b}$ is denoted by $|\vec{b}|$ where

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + \ldots + a_n^2}$$

$$|\vec{b}| = \sqrt{b_1^2 + b_2^2 + \ldots + b_n^2}$$

The dot product of $\vec{a}$ and $\vec{b}$ ($\vec{a} \cdot \vec{b}$) is calculated by the following formula:

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + \ldots + a_n b_n.$$

The angle between a and b is denoted by θ and is calculated by the following formula:

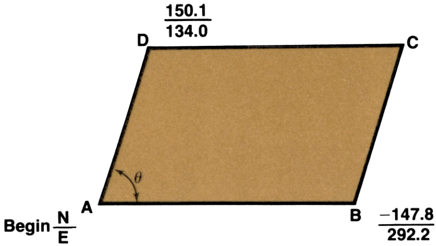
$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}\right)$$

The angle is calculated in any angular mode. When calculated in degrees, decimal degrees are assumed.

This procedure calculates the vector dot product and angle θ.

1. Press **f** **Σ**.
2. Key in a_1, press **ENTER↑**. Key in b_1, press **Σ+**. Repeat this step for all values of a and b.
3. Press **RCL** 8 **f** **√x** to obtain $|\vec{a}|$.
4. Press **RCL** 6 **f** **√x** to obtain $|\vec{b}|$.
5. Press **RCL** 9 for the dot product $\vec{a} \cdot \vec{b}$.
6. Press **RCL** 8 **RCL** 6 **×** **f** **√x** **÷** **g** **COS⁻¹** for angle θ.

Example: A surveyor has determined the coordinates of three corners of a plot of land in the shape of a parallelogram. He now wants to find the area of the plot, the lengths of the sides, and the angle θ.



Solution:

Area $= \vec{AB} \cdot \vec{AD} = (-147.8,\ 292.2) \cdot (150.1,\ 134.0)$

Press	Display	
f Σ		
147.8 CHS ENTER↑ 150.1 Σ+	1.00	First entry.
292.2 ENTER↑ 134 Σ+	2.00	Second entry.
RCL 8 f √x	327.45	Side AB $\lvert\vec{AB}\rvert$.
RCL 6 f √x	201.21	Side AD $\lvert\vec{AD}\rvert$.
RCL 9	16970.02	Area of plot.
RCL 8 RCL 6 ×		
f √x ÷ g COS⁻¹	75.07	°angle θ.

Determinant and Inverse of a 2 × 2 Matrix

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a 2 × 2 matrix.

The determinant of A denoted by Det A or $|A|$ is evaluated by the following formula:

$$\text{Det } A = a_{22}\, a_{11} - a_{12}\, a_{21}.$$

The HP-27 also calculates the multiplicative inverse A^{-1} of A according to the following formula:

$$A^{-1} = \begin{bmatrix} a_{22}/\text{Det A} & -a_{12}/\text{Det A} \\ -a_{21}/\text{Det A} & a_{11}/\text{Det A} \end{bmatrix}$$

This procedure calculates the determinant of a 2 × 2 matrix and an inverse matrix such that you can calculate A^{-1}.

1. Input and store the following:
   - Key in a_{11} and press STO 1.
   - Key in a_{12} and press STO 2.
   - Key in a_{21} and press STO 3.
   - Key in a_{22} and press STO 4.
2. Press RCL 1 RCL 4 ⊠ RCL 2 RCL 3 ⊠ ⊟ STO 5. The determinant of A is displayed.
3. Press RCL 4 RCL 5 ⊞÷. The inverse of a_{11} is displayed.
4. Press RCL 2 RCL 5 ÷ CHS . The inverse of a_{12} is displayed.
5. Press RCL 3 RCL 5 ÷ CHS . The inverse of a_{21} is displayed.
6. Press RCL 1 RCL 5 ÷. The inverse of a_{22} is displayed.

Example: Given a pair of simultaneous linear equations in two unknowns,

$$2x + 3y = 21$$

$$4x - y = 7$$

Another way to write the equations is in terms of matrices:

$$\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 21 \\ 7 \end{bmatrix}$$

Rewriting the matrix equation gives

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} 21 \\ 7 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$$

and A^{-1} is the multiplicative inverse of A.

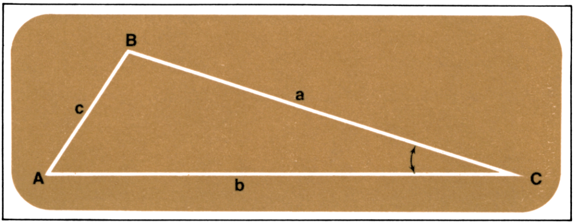
Find Det A and A^{-1}.

Press	Display	
2 STO 1	2.00	
3 STO 2	3.00	
4 STO 3	4.00	
1 CHS STO 4	-1.00	
RCL 1 RCL 4 ⊠	-2.00	
RCL 2 RCL 3 ⊠ ⊟ STO 5	-14.00	Determinant of A.
f FIX 4	-14.0000	Display to 4 places.
RCL 4 RCL 5 ÷	0.0714	Inverse of a_{11}.
RCL 2 RCL 5 ÷ CHS	0.2143	Inverse of a_{12}.
RCL 3 RCL 5 ÷ CHS	0.2857	Inverse of a_{21}.
RCL 1 RCL 5 ÷	-0.1429	Inverse of a_{22}.

$$A^{-1} = \begin{bmatrix} 0.0714 & 0.2143 \\ 0.2857 & -0.1429 \end{bmatrix}$$

Triangle Solution a,b,C

Given two sides (a and b) and their included angle C, this calculation solves for the third side and other two angles.



$$c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

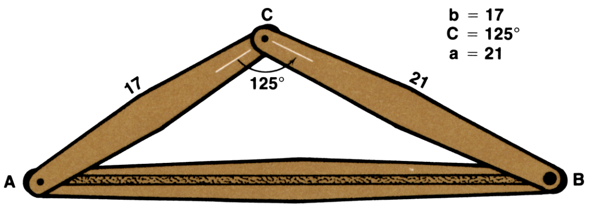
$$A = \tan^{-1}\left(\frac{a \sin C}{b - a \cos C}\right)$$

$$B = \cos^{-1}\left[-\cos(A + C)\right]$$

These keystrokes work in any angular mode. However, if in degrees, decimal degrees are assumed.

1. Key in b, press STO 0.
2. Key in angle C, press STO 1.
3. Key in a, press f →R RCL 0 x⇄y − g →P. The length of the third side (c) is displayed.
4. Press x⇄y to obtain angle A.
5. Press RCL 1 ÷ f COS CHS g COS⁻¹ to obtain angle B.

Example: A mechanical linkage is to be designed so that arms AC and CB are pinned at point C. Point B is able to slide along the base, hence changing the angle C. At its maximum extension, angle C is to have a value of 125°. If the arms AC and BC have lengths 17 cm and 21 cm, respectively, what is the minimum length of the base (side AB)?

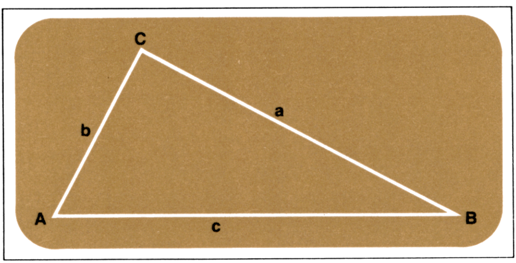


Press	Display	
17 STO 0	17.00	
125 STO 1	125.00	
21 f →R RCL 0 x⇄y		
− g →P	33.76	Value of c.
x⇄y	30.64	°angle A.
RCL 1 + f COS CHS		
g COS⁻¹	24.36	°angle B.

The side AB must be at least 33.76 cm in length.

Triangle Solution a,b,c

Given three sides (a, b and c), of a triangle, this calculation solves for the three angles.



$$A = 2\cos^{-1}\left(\sqrt{\frac{S(S-a)}{bc}}\right)$$

$$\text{where } S = (a+b+c)/2$$

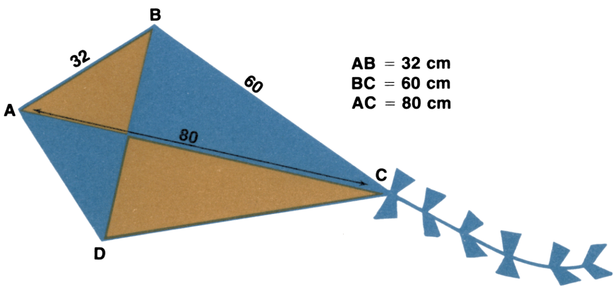
$$B = \tan^{-1}\left(\frac{b \sin A}{c - b\cos A}\right)$$

$$C = \cos^{-1}\left[-\cos(A+B)\right]$$

These keystrokes work in any angular mode. However, if in degrees, all angles are assumed to be in decimal degrees.

1. Input and store the following:
   - Key in side a, press STO 0.
   - Key in side b, press STO 1.
   - Key in side c, press STO 2.
2. Press + + 2 ÷ ENTER↑ ENTER↑ RCL 0 − ×.
3. Press RCL 1 ÷ RCL 2 ÷ f √x g COS⁻¹ 2 × STO 3. The value of angle A is displayed.
4. Press RCL 1 f →R RCL 2 x⇄y − g →P x⇄y. The value of angle B is displayed.
5. Press RCL 3 + f COS CHS g COS⁻¹ to obtain angle C.

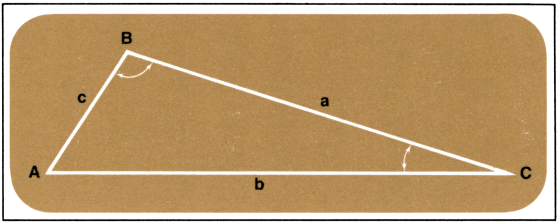
Example: You decide to make a kite with the dimensions shown below. Calculate the angles <ACB, <BAC, and <ABC.



Press	Display	
60 STO 0	60.00	Side BC (a).
80 STO 1	80.00	Side AC (b).
32 STO 2	32.00	Side AB (c).
+ + 2 ÷ ENTER↑ ENTER↑	86.00	
RCL 0 − ×	2236.00	
RCL 1 ÷ RCL 2 ÷	0.87	
f √x g COS⁻¹ 2 × STO 3	41.68	°<BAC (A).
RCL 1 f →R RCL 2 x⇄y		
− g →P x⇄y	117.55	°<ABC (B).
RCL 3 + f COS CHS		
g COS⁻¹	20.77	°<ACB (C).

Triangle Solution a,A,C

Given two angles (A and C) and an opposite side (a), this calculation solves for the third angle and the other two sides.



$$B = 180° - (A + C) = 200 \text{ grads} - (A + C)$$

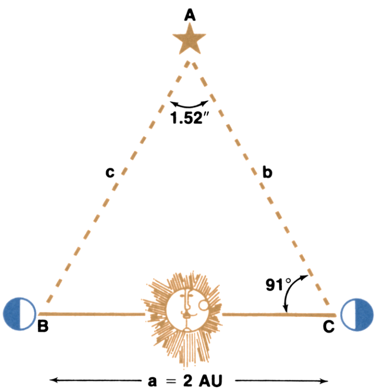
$$b = \frac{a \sin B}{\sin A}$$

$$c = \frac{a \sin C}{\sin A}$$

These keystrokes work in any angular mode. However, if in degree mode, all angles are assumed to be in decimal degrees.

1. Input and store the following:
   - Key in side a, press STO 0.
   - Key in angle C, press STO 1.
   - Key in angle A, press STO 2.
2. Press + f sin RCL 2 f sin ÷ RCL 0 × to obtain side B.
3. Press RCL 1 x⇄y f →R RCL 0 x⇄y − g →P to obtain side c.
4. Press x⇄y to obtain angle B.

Example: An astronomer trying to determine the distance from the earth to a nearby star, photographs the star at approximately six-month intervals and records the data as shown below.



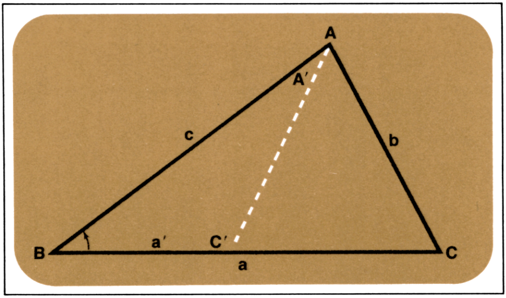
Find the distance b from the earth to the star, given that the mean distance of the earth from the sun is 1 astronomical unit (AU).

(Note that A = 1.52 seconds must be converted to decimal degrees.)

Press	Display	
2 STO 0	2.00	Side A.
91 STO 1	91.00	°angle C.
.000152 g →H STO 2	4.2222222–04	°angle A.
+ f sin RCL 2 f sin		
÷ RCL 0 ×	271359.69	AU side b, distance to the star.
RCL 1 x⇄y f →R		
RCL 0 x⇄y − g →P	271359.73	AU side c.
x⇄y	89.00	°angle B.

Triangle Solution B,b,c,

Given two sides (b and c) and a non-included angle (B), this calculation solves for the remaining two angles and the third side.



$$C = \sin^{-1}\left(\frac{c \sin B}{b}\right)$$

$$A = 2 \sin^{-1} - (B + C) = \pi \text{ radians} - (B + C) = 180° - (B + C)$$

$$= 200 \text{ grads} - (B + C)$$

$$a = \frac{b \sin A}{\sin B}$$

If angle B is acute ($<90°$) and side b $<$ side c, a second set of solutions exists. Therefore, you would also solve for A′, C′ and a′.

$$C' = 2 \sin^{-1} 1 - C$$

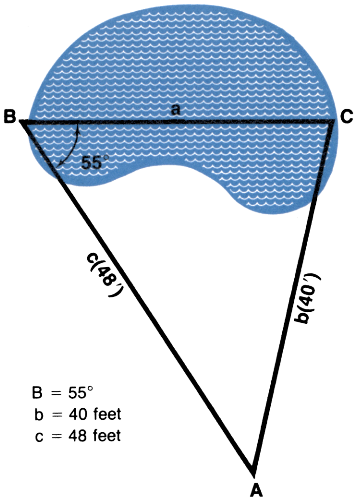
$$A' = 2 \sin^{-1} 1 - (B + C')$$

$$a' = \frac{b \sin A'}{\sin B}$$

These keystrokes work in any angular mode. However, if in degrees, decimal degrees are assumed.

1. Press 1 g sin⁻¹ ENTER↑ + STO 0.
2. Input and store the following:
   - Key in angle B, press STO 1.
   - Key in side b, press STO 2.
   - Key in side c, press STO 3.
3. Press x⇄y ÷ RCL 1 f sin × g sin⁻¹ STO 4. Angle C is displayed.
4. Press RCL 1 + RCL 0 x⇄y − to obtain angle A.
5. Press f sin RCL 2 × RCL 1 f sin ÷ to obtain side a.
6. If side b $<$ side c, a second set of solutions exist. Press RCL 0 RCL 4 − to find C′. Repeat steps 4 and 5 to find A′ and a′.

Example: A landscape designer discovers Zen and decides to transform the backyard of his palacial Beverly Hills mansion into a Japanese garden. The heart of the transformation will be the conversion of his kidney-shaped swimming pool into a goldfish pond spanned by a stone bridge. To find the length of the pool and still stay dry, the designer makes measurements as shown in the diagram.



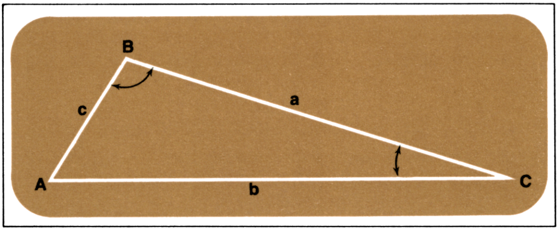
Calculate angles A and C and find the length of the pool.

Press	Display	
1 [g] [sin⁻¹] [ENTER↑] [+]		
[STO] 0		
55 [STO] 1	55.00	°angle B.
40 [STO] 2	40.00	Side b.
48 [STO] 3	48.00	Side c.
[x⇄y] [÷] [RCL] 1 [f] [sin]		
[×] [g] [sin⁻¹] [STO] 4	79.41	°angle C.
[RCL] 1 [+] [RCL] 0 [x⇄y] [−]	45.59	°angle A.
[f] [sin] [RCL] 2 [×]		
[RCL] 1 [f] [sin] [÷]	34.88	Feet (length of pool).

The alternate solution is C′ = 100.59°, A′ = 24.41°, and a′ = 20.18 ft. This solution, however, does not fit the diagram.

Triangle Solution a, B, C

Given two angles (B and C) and their included side (a), this calculation solves for the third angle and the other two sides.



$$A = 180° - (B + C) = 200 \text{ grads} - (B + C)$$

$$b = \frac{a \sin B}{\sin A}$$

$$c = \frac{a \sin C}{\sin A}$$

These keystrokes work in any angular mode. However, if in degrees, decimal degrees are assumed.

1. Press 1 [g] [sin⁻¹] [ENTER↑] [+].
2. Key in angle B, press [STO] 0 [−].
3. Key in angle C, press [STO] 1 [−]. Angle A is displayed.
4. Key in side a. Press [x⇄y] [f] [sin] [÷] [STO] 2 [RCL] 0 [f] [sin] [×]. Side b is displayed.
5. Press [RCL] 2 [RCL] 1 [f] [sin] [×] to obtain side c.

Example: Alice and her mother were lying in the new spring grass on a hill overlooking the town. It was early morning and Venus was still visible near the Eastern horizon.

"Mommy," Alice mused, "how far away is that star?"

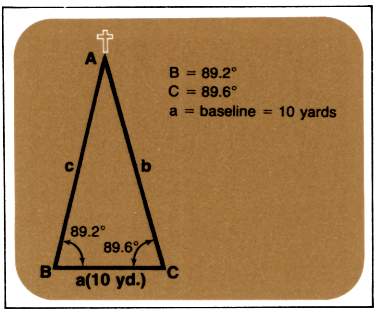
"I don't know, dear. But it's not really a star; it's a planet."

"Oh. Well, how far away is the sky?"

"The sky is everywhere. It starts right here and keeps going up and up and up."

"Oh, Mommy!" Alice sounded exasperated. "Then how far away is that church?" She pointed to a steeple that rose above the trees not far from the base of their hill.

"Ah, that we can find," her mother said, and jumped up. "Come on." They ran back to the house and returned shortly with a yard-stick, string, a protractor, and some stakes. Within minutes they'd staked out a baseline 10 yards long, and measured the angle from each end point of the baseline to the steeple. The measurements they made are shown below:



With these measurements, can you tell Alice how far away the steeple is?

Press	**Display**	
1 g sin⁻¹ ENTER↑ +		
89.2 STO 0 −		
89.6 STO 1 −	1.20	°angle A.
10 x⇄y f sin ÷ STO 2		
RCL 0 f sin ×	477.45	Yards (side b).
RCL 2 RCL 1 f sin ×	477.49	Yards (side c).

The steeple is about 475 yards away.

Angle Conversions: degrees ⇄ radians

Let D be an angle in degrees, R the same angle in radians.

$$D = \frac{180}{\pi} R.$$

$$R = \frac{\pi}{180} D.$$

To convert degrees to radians:

1. Press g π 180 ÷. Press ENTER↑ three times.
2. Press CLx, key in the degree value and press ×. The value of the angle in radians is displayed.
3. For a new degree value, go to step 2.

To convert radians to degrees:

1. Key in 180 and press g π ÷. Press ENTER↑ three times.
2. Press CLx, key in the radian value, and press ×. The value of the angle in degrees is displayed.
3. For a new radian value, go to step 2.

Examples: Convert 1° to radians.

Press	**Display**
g π 180 ÷	
ENTER↑ ENTER↑ ENTER↑ CLx	0.00

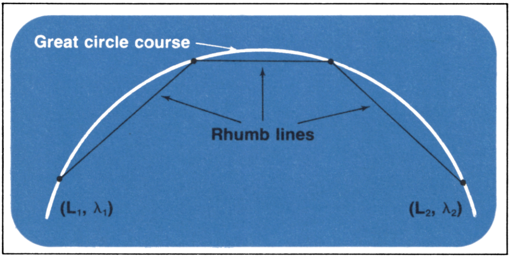
1 [×]	0.02	Radians.
Convert 240° to radians.		
[CLx] 240 [×]	4.19	Radians.
Convert 1 radian to degrees.		
180 [g] [π] [÷]		
[ENTER↑] [ENTER↑] [ENTER↑] [CLx]	0.00	
1 [×]	57.30	Degrees.
Convert ¾ π radians to degrees.		
[CLx] 3 [ENTER↑] 4 [÷]		
[g] [π] [×] [×]	135.00	Degrees.

Navigation Applications

The following procedures describe how to calculate a rhumb line course, great circle navigation, and a sight reduction table.

Rhumb Line Navigation

A rhumb line is a curve on the surface of a sphere that cuts all meridians at the same angle.



Your HP-27 can calculate the rhumb line distance and course between two points on the earth. You can link successive legs without keying in the initial latitude and longitude again. (Accuracy deteriorates for very short legs.)

Northern latitudes and western longitudes are keyed in and displayed as positive values. Southern latitudes and eastern longitudes are keyed in and displayed as negative values.

To avoid getting incorrect results computing distances due east or due west across the international dateline, compute up to the dateline, then proceed on the other side.

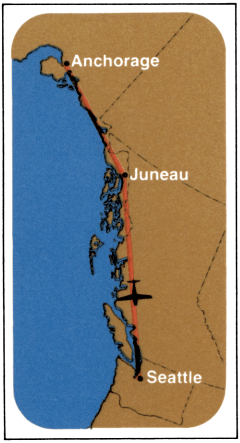
No course should pass through the North or South Pole. Also, as the rhumb line course approaches 90° or 270°, you may encounter errors in the distance calculations.

The following keystrokes calculate a rhumb line course (C) and rhumb line distance (DIST):

1. Key in latitude of initial point (L_1). Press g →H STO 1.
2. Key in longitude of initial point (λ_1). Press g →H STO 3.
3. Key in the next latitude, and press g →H STO 2.
4. Key in the next longitude, and press g →H STO 4.
5. Press RCL 3 RCL 4 − STO 7.
6. Press 2 ÷ f sin g sin⁻¹ 90 ÷ g π × RCL 2.
7. Press 2 ÷ 45 + f tan RCL 1.
8. Press 2 ÷ 45 + f tan ÷ f ln.
9. Press g →P x⇄y RCL 7 f sin g sin⁻¹.
10. If the number on the display is positive, press x⇄y to obtain C (rhumb line course) in decimal degrees.

    If the displayed number is negative, press x⇄y 360 + to obtain C in decimal degrees.
11. Press f cos. If cos (c) is 0, skip to step 13.
12. Press RCL 2 RCL 1 − x⇄y ÷ 6 × to obtain the distance (DIST) in nautical miles. Go to step 14.
13. Press RCL 7 RCL 2 f cos × 60 × to obtain the distance (DIST) in nautical miles.
14. To solve for the course and distance of another leg, press RCL 2 STO 1 RCL 4 STO 3 and return to step 3.

Example: Find the distances and headings for a flight from Anchorage, Alaska, to Juneau, Alaska, to Seattle, Washington.



Anchorage	**L 61°13′N**	**λ 149°54′W**
Juneau	**L 58°18′N**	**λ 134°25′W**
Seattle	**L 47°36′N**	**λ 122°20′W**

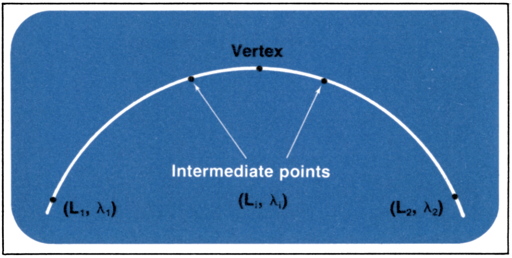
Press	Display	
61.13 g →H STO 1	61.22	Anchorage L.
149.54 g →H STO 3	149.90	Anchorage λ.
58.18 g →H STO 2	58.30	Juneau L.
134.25 g →H STO 4	134.42	Juneau λ.
RCL 3 RCL 4 − STO 7	15.48	
2 ÷ f sin		
g sin^{-1} 90 ÷		
g π × RCL 2	58.30	
2 ÷ 45 + f tan RCL 1	61.22	
2 ÷ 45 + f tan		
÷ f ln	−0.10	
g →P x⇄y RCL 7 f sin		

g sin⁻¹ x⇄y	110.52	°C_1
f cos RCL 2 RCL 1 −		
x⇄y ÷ 60 ×	499.22	$DIST_1$
RCL 2 STO 1 RCL 4 STO 3		Prepare to solve for next leg.
47.36 g →H STO 2	47.60	Seattle L.
122.20 g →H STO 4	122.33	Seattle λ.
RCL 3 RCL 4 − STO 7	12.08	
2 ÷ f sin g sin⁻¹ 90 ÷		
g π × RCL 2	47.60	
2 ÷ 45 + f tan RCL 1	58.30	
2 ÷ 45 + f tan ÷ f ln	-0.31	
g →P x⇄y RCL 7 f sin		
g sin⁻¹ x⇄y	145.94	°C_2.
f cos RCL 2 RCL 1 −		
x⇄y ÷ 60 ×	774.90	$DIST_2$.

Anchorage to Juneau	C = 110.52°	DIST = 499.22 naut. miles.
Juneau to Seattle	C = 145.94°	DIST = 774.90 naut. miles.

Great Circle Navigation

This method of navigation involves travelling between two points according to the arc of a great circle.



This procedure calculates the great circle distance and initial course for the great circle track between two points on the earth. You can also calculate the coordinates of the vertex, the distance from the initial point to the vertex, and the latitude where a longitude line and the great circle track intersect.

Northern latitudes and western longitudes are keyed in and displayed as positive values; southern latitudes and eastern longitudes, as negative values.

No point on a leg should be at either the North or South Pole. Nor should a leg pass more than halfway around the earth. Points located at diametrically opposite sides of the earth should not be used because there are an infinite number of great circle courses through such points.

These keystrokes solve for the great circle distance (DIST) in nautical miles; the initial course (C_i); the longitude (λ_v) and latitude (L_v) of the vertex; the distance ($DIST_v$) from the initial point to the vertex; and the latitude (L_i) when an intermediate longitude is known.

1. Key in the latitude of your destination. Press [g] [→H] [STO] 2, then press 1 [f] [→R].
2. Key in the latitude of your starting point. Press [g] [→H] [STO] 1, then press 1 [f] [→R] [STO] 5.
3. Press [x⇄y] [STO] 6 [R↓] [×] [R↓] [STO] 7 [×] [R↓] [R↓] [R↓].
4. Key in the longitude of the starting point, and press [f] [→H] [STO] 3.
5. Key in the longitude of the destination, and press [g] [→H] [STO] 4.
6. Press [−] [STO] 0 [f] [cos] [×] [+] [g] [cos⁻¹] [STO] 8. Press 60 [×]. The great circle distance (DIST) is displayed.
7. Press [RCL] 7 [RCL] 6 [RCL] 8 [f] [cos] [×] [−].
8. Press [RCL] 8 [f] [sin] [÷] [RCL] 5 [÷] [g] [cos⁻¹] [RCL] 0 [f] [sin] [STO] 0.
9. If the number on the display is positive, press [x⇄y] [STO] 9 to obtain the great circle course (C_i). If the displayed number is negative, press [x⇄y] 360 [x⇄y] [−] [STO] 9 to obtain C_i.
10. Press [RCL] 1 [f] [tan] [STO] 1.

11. Press RCL 2 f tan STO 2. If you don't want to find the coordinates of the vertex, skip to step 17.
12. Press RCL 4 RCL 1 f →R x⇄y RCL 3 RCL 2 f →R R↓ − R↓ − R↓ R↓ R↓ ÷ g tan⁻¹ ENTER↑ f →H.MS. The longitude of the vertex (λ_V) is displayed.
13. If λ_V is negative, press x⇄y 180 + f →H.MS to obtain the alternate vertex longitude (λ_V'). If λ_V is positive, press x⇄y 180 − f →H.MS to obtain the alternate vertex longitude (λ_V').
14. To calculate the latitude of the vertex (L_V), let $\lambda_i = \lambda_V$ or λ_V'. Key in λ_i and press g →H ENTER↑ ENTER↑ .
15. Press RCL 4 − f sin RCL 1 × x⇄y .
16. Press RCL 3 − f sin RCL 2 × − RCL 0 ÷ g tan⁻¹ f →H.MS. The latitude of the vertex (L_V) or of any intermediate point is displayed (depending whether you keyed in λ_V or λ_i).
17. To calculate the distance to the vertex, press g →H f sin RCL 9 f cos RCL 5 × ÷ g 1/x g sin⁻¹ 60 ×. The distance from the initial point to the vertex ($DIST_V$) in nautical miles is displayed.

Example: A ship is proceeding from Manila to Los Angeles. The captain wishes to sail a great circle course from the entrance of San Bernardino Strait, L12°45′12″N, λ124°20′06″E (input as negative), to L33°48′48″N, λ120°07′06″W, five miles south of Santa Rosa Island.

Find the initial great circle course and great circle distance, the latitude and longitude of the vertex, and the distance from the initial point to the vertex.

Press	Display
33.4848 g →H STO 2	33.81
1 f →R	0.83
12.4512 g →H STO 1	12.75

1 g →H STO 5	0.98	
x⇄y STO 6		
R↓ × R↓ STO 7		
× R↓ R↓ R↓	0.81	
124.2006 CHS g →H STO 3	–124.34	
120.0706 g →H STO 4	120.12	
− STO 0	–244.45	
f cos × + g cos⁻¹ STO 8	103.10	
60 ×	6185.88	Great circle distance.
RCL 7 RCL 6 RCL 8		
f cos × −	0.61	
RCL 8 f sin ÷ RCL 5 ÷		
g cos⁻¹ RCL 0 f sin STO 0	0.90	
x⇄y STO 9	50.32	Great circle course (decimal degrees).
RCL 1 f tan STO 1	0.23	
RCL 2 f tan STO 2	0.67	
RCL 4 RCL 1 f →R		
x⇄y RCL 3 RCL 2 f →R		
R↓ − R↓ − R↓ R↓ R↓	0.75	
÷ g TAN⁻¹ ENTER↑		
f →H.MS	–19.26	λ_v.
x⇄y 180 + f →H.MS	160.34	λ'_v. (Leave in display for L_v calculation.)
g →H ENTER↑ ENTER↑	160.57	
RCL 4 − f sin RCL 1	0.23	
× x⇄y RCL 3 − f sin		
RCL 2	0.67	
× − RCL 0 ÷ g tan⁻¹		
f →H.MS	41.21	L_v (L_i).
g →H f sin RCL 9	50.32	
f cos RCL 5 × ÷ g 1/x	0.94	
g sin⁻¹ 60 ×	4228.83	Distance_v.

Sight Reduction Table

Given the observer's latitude and both the declination and local hour angle (LHA) of a celestial body, you can calculate the azimuth and altitude of the body.

Northern latitude, northern declinations and western hour angles are input as positive values. Southern latitudes, southern declinations, and eastern hour angles are input as negative values.

The keystrokes are as follows:

1. Key in the declination, and press g →H 1 f →R.
2. Key in the observer's latitude, and press g →H 1 f →R.
3. Press STO 1 x⇄y STO 2 R↓ × R↓ STO 3 × R↓ R↓ R↓.
4. Key in the local hour angle (LHA), and press g →H 1 f →R.
5. Press x⇄y STO 4 R↓ × + g sin⁻¹ STO 5 f →H.MS. The computed altitude of the celestial body is displayed.
6. Press RCL 3 RCL 2 RCL 5. Press 1 f →R R↓ × − x⇄y.
7. Press RCL 1 × ÷ g cos⁻¹ STO 6 RCL 4.
8. If the displayed value is positive, press 360 RCL 6 − to obtain the azimuth of the body in decimal degrees.

   If the displayed value is negative, press RCL 6 to obtain the azimuth.

This procedure may also be used for star identification. Input the azimuth in place of LHA and the observed altitude in place of declination, and your outputs are declination and LHA respectively. You can then identify the star by comparing this computed declination to the list of stars in *The Nautical Almanac*.

Example: Compute the altitude and azimuth of the sun if its LHA is 333°01′54″W and its declination is 12°28′06″S (input as negative). The assumed latitude is 34°11′06″S (input as negative).

Press	**Display**
12.2806 CHS g →H 1 f →R	0.98

Keystrokes	Display	
34.1106 CHS g →H 1 f →R	0.83	
STO 1 x⇄y STO 2 R↓ ×		
R↓ STO 3 × R↓ R↓ R↓	0.81	
333.0154 g →H 1 f →R	0.89	
x⇄y STO 4 R↓ × +		
g sin⁻¹ STO 5 f →H.MS	57.16	Altitude.
f FIX 4	57.1558	Displayed to four places.
f FIX 2	57.16	
RCL 3 RCL 2 RCL 5	57.27	
1 f →R R↓ × − x⇄y		
RCL 1 ×		
÷ g cos⁻¹ STO 6 RCL 4	−0.45	
RCL 6	54.97	Azimuth.

Altitude = 57°15′58″ Azimuth = 54.97°

Surveying Applications

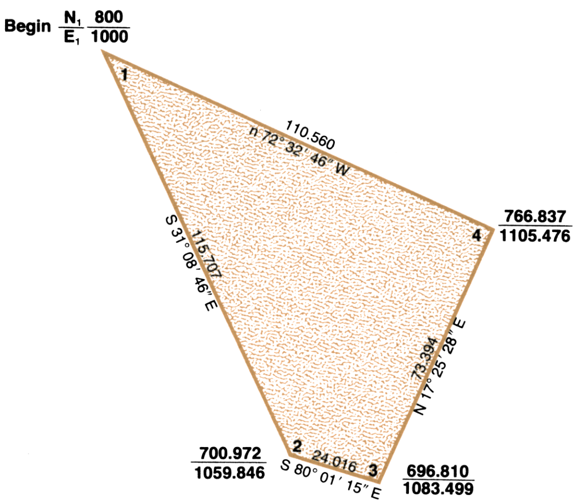
Bearing Traverse

This procedure uses bearing and distances to calculate the coordinates of successive points in a traverse. Area, closing distance, and closing azimuth can be calculated for a closed traverse.

1. Press f REG.
2. Key in the northing of your starting point (N_1) and press STO 1.
3. Key in the easting of the starting point (E_1) and press STO 2 x⇄y Σ+.
4. Input one of the following:
   - Key in the NE bearing, and press g →H.
   - Key in the SE bearing, and press g →H CHS 180 +.
   - Key in the NW bearing, and press g →H CHS.
   - Key in the SW bearing, and press g →H 180 +.
5. Input either of the following:
   - Key in the horizontal distance.
   - Key in the slope distance, and press ENTER↑. Next, key in the zenith angle, and press g →H f sin ×.

6. Press f →R Σ+ RCL 3 f LAST X + STO 3.
7. Press f LAST X 2 ÷ − × STO + 0.
8. Press RCL 5 to obtain northing of the second point. Press RCL 7 to obtain the easting of the second point.
9. Repeat steps 4 thru 8 for each successive leg on the traverse.
10. Press RCL 0 to obtain the area. (Ignore the sign if your answer is negative.)
11. Press RCL 2 RCL 1 f Σ- RCL 7 RCL 5 g →P to obtain the closing horizontal distance.
12. Press x⇄y f →H.MS to obtain the closing azimuth. (If your answer is negative, add 360°.)

Example: Given the following closed traverse, find the closing northing, easting, azimuth, and distance, as well as the area.



Press	Display	
f REG		
800 STO 1	800.00	N_1
1000 STO 2	1000.00	E_1
x⇄y Σ+	1.00	
31.0846 g →H CHS 180 +	148.85	
115.707 f →R Σ+		
RCL 3 f LAST X + STO 3	–99.03	
f LAST X 2 ÷ – ×		
STO + 0 RCL 5	700.97	N_2
RCL 7	1059.85	E_2
80.0115 g →H CHS 180 +	99.98	
24.016 f →R Σ+ RCL 3		
f LAST X + STO 3	–103.19	
f LAST X 2 ÷ – ×		
STO + 0 RCL 5	696.81	N_3
RCL 7	1083.50	E_3
17.2528 g →H	17.42	
73.394 f →R Σ+		
RCL 3 f LAST X + STO 3	–33.16	
f LAST X 2 ÷ – ×		
STO + 0 RCL 5	766.84	N_4
RCL 7	1105.48	E_4
72.3246 g →H CHS	–72.55	
110.560 f →R Σ+	5.00	
RCL 3 f LAST X + STO 3	–2.3909900–03	
f LAST X 2 ÷ – ×		
STO + 0 RCL 5	800.00	Closing N.
RCL 7	1000.01	Closing E.
RCL 0	–5104.08	Area.
RCL 2 RCL 1 f Σ–	4.00	
RCL 7 RCL 5 g →P	0.01	Closing HD.
x⇄y f →H.MS	109.05	Closing azimuth.
f FIX 4	109.0549	

Closing N = 800
Closing E = 1000.01
Area = 5104.08 sq. feet
Closing Horizontal Distance = 0.01 feet
Closing Azimuth = 109°05′49″

Field Angle Traverse

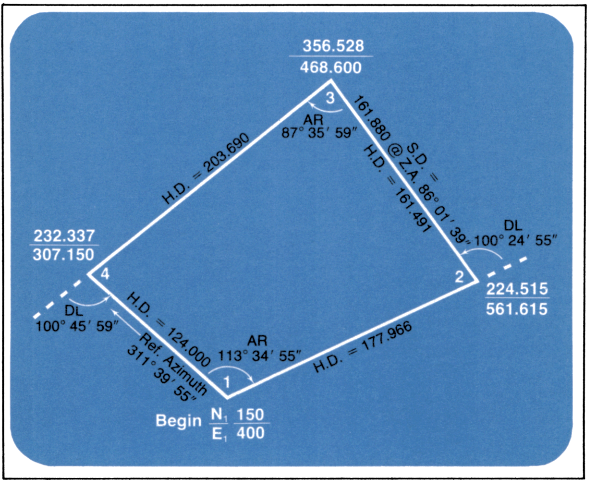
In the previous application, you used a bearing with respect to a reference line to calculate the coordinates of a point. With field angle traverse, you use an angle with respect to the previous side.

This procedure uses angles or deflections and distances to calculate coordinates of successive points in a traverse. Area, closing distance, and closing azimuth can be calculated for a closed traverse.

1. Press [f] [REG].
2. Key in the initial easting (E_1), and press [ENTER↑].
   Key in the initial northing (N_1), and press [Σ+].
3. Key in the reference azimuth, and press [g] [→H] 180 [+] [STO] 1.
4. Input *one* of the following:
   - Key in the right angle, press [g] [→H] 180 [+].
   - Key in the left angle, press [g] [→H] [CHS] 180 [+].
   - Key in the right deflection, press [g] [→H].
   - Key in the left deflection, press [g] [→H] [CHS].
5. Press [RCL] 1 [+] [STO] 1.
6. Input *one* of the following:
   - Key in the horizontal distance.
   - Key in the slope distance, and press [ENTER↑]. Key in the zenith angle, and press [g] [→H] [f] [sin] [×].
7. Press [f] [→R] [Σ+].
8. Omit this step if area is not calculated. Press [RCL] 3 [f] [LAST X] [+] [STO] 3 [f] [LAST X] 2 [÷] [−] [×] [STO] [+] 2.
9. Press [RCL] 5 for N.

10. Press RCL 7 for E.
11. Repeat steps 4 thru 10 for all points.
12. Press RCL 2 for area.
13. For closing horizontal distance and azimuth, key in E_1 again and press ENTER↑. Key in N_1 again and press f Σ-. Press RCL 7 RCL 5 g →P to obtain closing HD.
14. Press x⇄y f →H.MS to obtain the closing azimuth. (If negative, add 360° by pressing 360 f H.MS+.)

Example: Given the following traverse, calculate the northings, eastings, closing horizontal distance, and closing azimuth.



Press	Display	
f REG		
400 ENTER↑ 150 Σ+	1.00	
311.3955 g →H 180 +		
STO 1	491.67	
113.3455 g →H 180 +	293.58	
RCL 1 + STO 1	785.25	
177.966 f →R Σ+	2.00	
RCL 5	224.52	N_2
RCL 7	561.61	E_2
100.2455 g →H CHS	-100.42	
RCL 1 + STO 1	684.83	
161.880 ENTER↑ 86.0139		
g →H	86.03	
f sin ×	161.49	
f →R Σ+	3.00	
RCL 5	356.53	N_3
RCL 7	468.60	E_3
87.3559 g →H 180 +	267.60	
RCL 1 + STO 1	952.43	
203.690 f →R Σ+	4.00	
RCL 5	232.34	N_4
RCL 7	307.15	E_4
100.4559 g →H CHS	-100.77	
RCL 1 + STO 1	851.67	
124.00 f →R Σ+	5.00	
RCL 5	149.90	N_1
RCL 7	399.78	E_1
400 ENTER↑ 150 f Σ-	4.00	
RCL 7 RCL 5 g →P	0.24	Closing HD.
x⇄y f →H.MS	-113.40	
360 f H.MS+	246.19	Closing azimuth.
f FIX 4	246.1943	Displayed as HH.MMSS.

Inverse from Coordinates

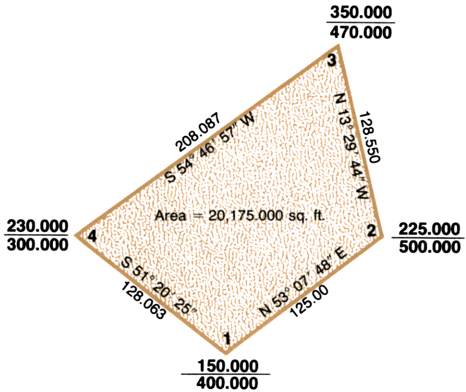
The previous two applications solved for coordinates of a traverse. This procedure uses the coordinates to calculate the distance and azimuth between points. Area can be calculated for a closed traverse.

1. Press f REG.
2. Key in N_1, press ENTER↑, key in E_1, and press Σ+.
3. Press RCL 7.
4. Key in the next N, and press ENTER↑. Key in the next E, and press RCL 5 − STO 1 R↓ x⇄y − STO 2 x⇄y R↓ x⇄y Σ+.
5. If area is to be calculated, skip to step 6. If area is not calculated, press f LAST X g →P to obtain the horizontal distance. Press x⇄y f →H.MS to obtain the azimuth. (If negative, add 360°.) Return to step 3 for next N and next E.
6. Press RCL 3 f LAST X + STO 3.
7. Press f LAST X 2 ÷ − × STO + 0 RCL 1 RCL 2 g →P to obtain the horizontal distance.
8. Press x⇄y f →H.MS to obtain the azimuth. (If negative, add 360°.) Return to step 3 for next N and E, and repeat until N_1 and E_1 have been keyed in again.
9. Press RCL 0 to obtain the area.

Once the azimuth is calculated, you can convert azimuth (in H.MS) to bearing (in H.MS) as follows:

Azimuth	Bearing
0° to 90°	NE.
90° to 180°	SE, press CHS 180 f H.MS+.
180° to 270°	SW, press 180 −.
270° to 360°	NW, press CHS 360 f H.MS+.

Example: Calculate the distance and azimuth between points for the following traverse and solve for area.

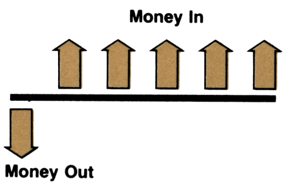


Press	Display	
f REG		
f FIX 4	0.0000	Display to 4 places.
150 ENTER↑ 400 Σ+	1.0000	
RCL 7	150.0000	
225 ENTER↑ 500 RCL 5 − STO 1	100.0000	
R↓ x⇄y − STO 2	75.0000	
x⇄y R↓ x⇄y Σ+	2.0000	
RCL 3 f LAST X + STO 3	100.0000	
f LAST X 2 ÷ − × STO + 0	3750.0000	
RCL 1 RCL 2 g →P	125.0000	$HD_{1,2}$.
x⇄y f →H.MS	53.0748	$Azimuth_{1,2}$.
RCL 7	225.0000	
350 ENTER↑ 470 RCL 5 − STO 1	–30.0000	
R↓ x⇄y − STO 2	125.0000	
x⇄y R↓ x⇄y Σ+	3.0000	

RCL 3 f LAST x + STO 3	70.0000	
f LAST x 2 ÷ − × STO + 0	10625.0000	
RCL 1 RCL 2 g →P	128.5496	$HD_{2,3}$.
x⇄y f →H.MS	−13.2945	
360 f H.MS+	346.3015	$Azimuth_{2,3}$.
RCL 7	350.0000	
230 ENTER↑ 300 RCL 5 − STO 1	−170.0000	
R↓ x⇄y − STO 2	−120.0000	
x⇄y R↓ x⇄y Σ+	4.0000	
RCL 3 f LAST x + STO 3	−100.0000	
f LAST x 2 ÷ − × STO + 0	1800.0000	
RCL 1 RCL 2 g →P	208.0865	$HD_{3,4}$.
x⇄y f →H.MS	−125.1303	
360 f H.MS+	234.4657	$Azimuth_{3,4}$.
RCL 7	230.0000	
150 ENTER↑ 400 RCL 5 − STO 1	100.0000	
R↓ x⇄y − STO 2	−80.0000	
x⇄y R↓ x⇄y Σ+	5.0000	
RCL 3 f LAST x + STO 3	0.0000	
f LAST x 2 ÷ − × STO + 0	4000.0000	
RCL 1 RCL 2 g →P	128.0625	$HD_{4,1}$.
x⇄y f →H.MS	128.3935	$Azimuth_{4,1}$.
RCL 0	20175.0000	Area.

Financial Applications

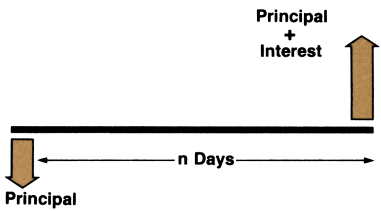
Throughout the following text are cash flow diagrams associated with the financial problems. A cash flow diagram is simply a picture of money in and money out.



If the solution to your problem isn't evident, ask yourself, "What are the cash flows?" Construct a cash flow diagram; then look for a similar diagram in this applications section.

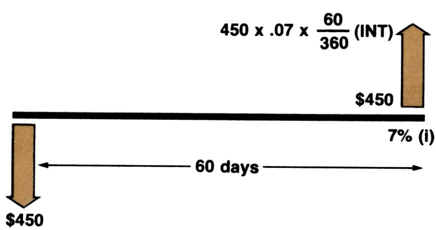
Simple Interest—360 and 365 Days

This calculation finds the amount of accrued simple interest on either a 360-day or 365-day basis when the number of days, interest rate, and principal (present value) are known.



1. Key in the number of days, press ENTER↑.
2. Key in 360 or 365 (whichever base you prefer to use), press ÷.
3. Key in principal, press ×.
4. Key in interest rate (as a decimal), press ×.

Example 1: Your good friend needs a loan to start his latest enterprise. He has requested that you lend him $450 for 60 days. You lend him the money at 7% simple interest, based on a 360-day year. What is the amount of accrued interest he will owe you in 60 days?



Press	Display	
60 ENTER↑ 360 ÷	0.17	
450 ×	75.00	
.07 ×	5.25	Accrued interest.

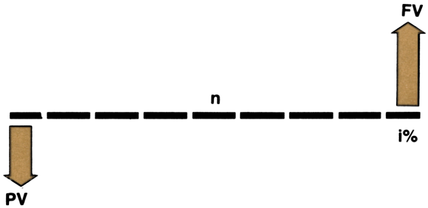
Example 2: What is the accrued interest on $450 for 60 days at 7%, figured on a 365-day year?

Press	Display	
60 ENTER↑ 365 ÷	0.16	
450 ×	73.97	
.07 ×	5.18	Accrued interest.

Compound Interest

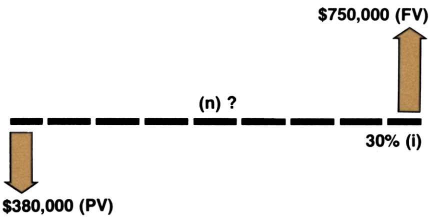
The following calculations deal with a lump sum deposit or investment, subject to multiple compounding of interest.

Number of Periods in a Compounded Amount. This calculation finds the number of compounding periods (n) when the interest rate (i), initial principal (present value) and compounded amount (future value) are given.



1. Press g RESET.
2. Input the following in any order:
   - Key in the periodic interest rate, press f i.
   - Key in the present value, press f PV.
   - Key in the future value, press f FV.
3. Press f n to obtain the number of periods.

Example: A potential development site currently appraised at $380,000 appreciates at 30% per year. If this rate continues, how many years will it be before this land is worth $750,000?

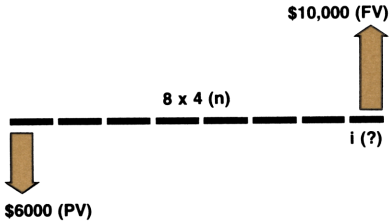


Press	Display	
g RESET		
30 f i	30.00	% annual interest rate.
380000 f PV	380000.00	Current value.
750000 f FV	750000.00	Future value.
f n	2.59	Years.

Interest Rate for Compounded Amounts. Given the initial amount, future value, and number of time periods, this calculates the interest rate:

1. Press g RESET.
2. Input the following in any order:
   - Key in the number of periods, press f n.
   - Key in the present value, press f PV.
   - Key in the future value, press f FV.
3. Press f i to obtain the periodic interest rate.
4. Key in the number of periods per year, press × to obtain an annual interest rate.

Example: What annual interest rate must be obtained to amass $10,000 in 8 years on an investment of $6000, with quarterly compounding?

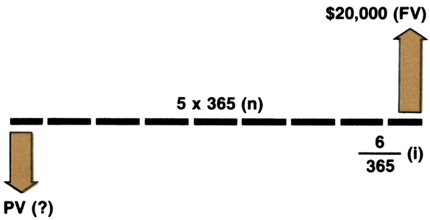


Press	Display	
g RESET		
8 ENTER↑ 4 × f n	32.00	Quarters.
6000 f PV	6000.00	Present value.
10000 f FV	10000.00	Desired amount.
f i	1.61	% quarterly interest rate.
4 ×	6.44	% annual interest rate.

Present Value of a Compounded Amount. This calculation finds the present value of a future amount "discounted back" at a given rate for a specified number of periods.

1. Press g RESET.
2. Input the following in any order:
   - Key in the number of periods, press f n.
   - Key in the periodic interest rate, press f i.
   - Key in the future value, press f FV.
3. Press f PV to obtain the present value.

Example: In 5 years when your son starts college, you will need $20,000. You deposit a lump sum in a certificate account that earns 6% compounded daily. How much do you need to deposit today to reach that goal?



Press	Display	
g RESET		
5 ENTER↑ 365 × f n	1825.00	Days.
6 ENTER↑ 365 ÷ f i	0.02	% daily interest rate.
20000 f FV	20000.00	Desired amount.
f PV	14816.73	To be deposited.

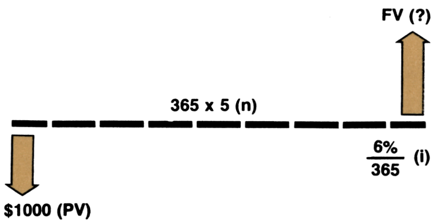
Future Value of a Compounded Amount. This calculation finds the future value of an initial amount compounded at a given rate for a specified number of periods.

1. Press g RESET.
2. Input the following in any order:
   - Key in the number of periods, press f n.
   - Key in the periodic interest rate, press f i.
   - Key in the present value, press f PV.
3. Press f FV to obtain the future value.

Example 1: Property values in an unattractive area are declining at the rate of 2% per year. If your property is presently valued at $32,000, what will it be worth in 6 years if this trend continues?

Press	Display	
g RESET		
6 f n	6.00	Years.
2 CHS f i	-2.00	% depreciation rate.
32000 f PV	32000.00	Present value.
f FV	28346.96	Value in 6 years.

Example 2: The local trading post manager opened up a savings operation 5 years ago, offering 6% compounded daily. Gold miner Yellowstone Sam deposited $1000 at that time, and now wants to know his present balance and the total accrued interest after all this time.



Press	Display	
g RESET		
5 ENTER↑ 365 × f n	1825.00	Days.
6 ENTER↑ 365 ÷ f i	0.02	% daily interest rate.
1000 f PV	1000.00	Original deposit.
f FV	1349.83	Present balance.
RCL f PV −	349.83	Accrued interest.

Interest Rate Conversions

Nominal Rate Converted to Effective Rate. Given a nominal interest rate and the number of compounding periods per year, this keystroke procedure computes the effective annual interest rate.

1. Press [g] [RESET].
2. Key in the number of compounding periods per year, press [f] [n].
3. Key in the nominal rate, press [x≷y] [÷] [f] [i].
4. Key in 100, press [f] [PV].
5. Press [f] [FV].
6. Press [RCL] [f] [PV] [−] to obtain effective annual rate.

Example: What is the effective annual rate of interest if the annual nominal rate of 12% is compounded quarterly?

Press	Display	
[g] [RESET]		
4 [f] [n]	4.00	Compounding periods.
12 [x≷y] [÷] [f] [i]	3.00	% quarterly interest rate.
100 [f] [PV] [f] [FV]		
[RCL] [f] [PV] [−]	12.55	% effective interest rate.

Add-on Interest Rate Converted to APR. An add-on interest rate determines what portion of the principal will be added on for repayment of a loan. This sum is then divided by the number of months in the loan to determine the monthly payment. For example, a 10% add-on rate for 36 months on $3000 means add one-tenth of $3000 for 3 years (300 × 3). This is usually called the "finance charge." The total ($3900) is divided by the number of payments (36) to obtain the monthly payment ($108.33).

This keystroke procedure converts an add-on interest rate to an annual percentage rate and calculates the monthly payment.

1. Press g RESET.
2. Key in number of *months* in loan. Press f n ENTER↑ ENTER↑ 12 ÷.
3. Key in add-on rate, press ENTER↑ 100 ÷ × 1 + ÷ f PV 1 f PMT.
4. Press f i to obtain the monthly interest rate.
5. Press g RESET f i.
6. Press 12 × to obtain the APR.
7. Press RCL f n f n.
8. Key in the principal and press f PV.
9. Press f PMT to obtain the monthly payment.

Example: Calculate the APR and monthly payment of a 5%, $1000 add-on loan which has a life of 18 months.

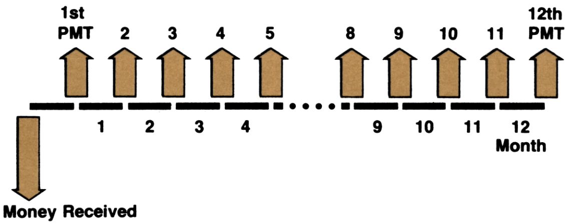
Press	**Display**	
g RESET		
18 f n ENTER↑ ENTER↑	18.00	Months.
12 ÷		
5 ENTER↑ 100 ÷ ×		
1 + ÷ f PV 1 f PMT		
f i	0.77	% monthly interest rate.
g RESET f i	0.77	Reuse i.
12 ×	9.27	% APR.
RCL f n f n	18.00	Recall and reuse n.
1000 f PV	1000.00	Loan principal.
f PMT	59.72	Monthly payments.

What's Really Happening? *A word of explanation about those last keystrokes: pressing* g RESET *prepares the calculator to solve for a different variable. Since n is unchanged in both parts of the calculation, you can reuse that value by pressing* RCL f n f n. *For a detailed explanation, refer to appendix B.*

Ordinary Annuities (Payments in Arrears)

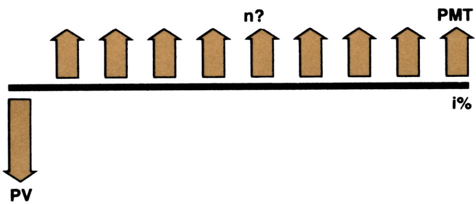
An annuity is a series of equal payments made at regular intervals. The time between annuity payments is called the payment interval or payment period. If your payment is due at the *end* of each payment period, it's called an ordinary annuity or payment in arrears. Examples of ordinary annuities are a car loan (where you drive away now and pay later) or a mortgage (where the payments start one month after you get your loan).

The time/money relationship for an ordinary annuity with monthly payments for a year would look like this:



The following problems all pertain to ordinary annuities, e.g., loans, mortgages, and sinking funds.

Number of Periods (Ordinary Annuity). This procedure calculates the number of periods for an ordinary annuity, when the present value and interest rate are known.



1. Press **g** [RESET].
2. Input the following in any order:
   - Key in the periodic interest rate, press **f** [i].
   - Key in the payment, press **f** [PMT].
   - Key in the present value, press **f** [PV].
3. Press **f** [n] to obtain the number of periods.

Example 1: A. Hunter borrows $2000 at 10% interest to go on an African safari. His payments are $100 a month. How long will it take him to pay off the loan?

Press	Display	
g [RESET]		
10 [ENTER↑] 12 [÷] **f** [i]	0.83	% monthly interest rate.
100 **f** [PMT]	100.00	Monthly payment.
2000 **f** [PV]	2000.00	Principal of loan.
f [n]	21.97	Months.

Example 2: Through an insurance fund, you have accumulated $50,000 for your retirement. How long can you withdraw $3000 every 6 months (starting 6 months from now) if the fund earns 5% per annum compounded semiannually?

Press	Display	
g [RESET]		
5 [ENTER↑] 2 [÷] **f** [i]	2.50	% semiannual interest rate.
3000 **f** [PMT]	3000.00	
50000 **f** [PV] **f** [n]	21.83	Semiannual withdrawals.

Interest Rate or APR (Ordinary Annuity). This routine computes the periodic interest rate for an ordinary annuity, given the number of periods, payment amount, and initial principal.

1. Press g RESET.
2. Input the following in any order:
   - Key in number of periods, press f n.
   - Key in payment amount, press f PMT.
   - Key in present value, press f PV.
3. Press f i to obtain the periodic interest rate.

The computed periodic rate is multiplied by the number of periods per year to obtain an annual rate.

Example 1: What is the annual interest rate on a 2-year, $1775 loan with $83.65 monthly payments?

Press	Display	
g RESET		
2 ENTER↑ 12 × f n	24.00	Number of payments.
83.65 f PMT		
1775 f PV	1775.00	Amount of loan.
f i	1.01	% monthly interest rate.
12 ×	12.11	% annual interest rate.

Borrowers are sometimes charged fees related to the issuance of a mortgage, which effectively raises the interest rate. Given the basis of the fee charge, the true annual percentage rate may be calculated.

Example 2: A borrower is charged 2 points for the issuance of his mortgage. If the mortgage amount is $50,000 for 30 years, and the interest rate is 9% per year, with monthly payments, what annual percentage rate is the borrower paying? (1 point is equal to 1% of the mortgage amount.)

First, compute the payment amount which is based on $50,000.

Press	Display	
g RESET		
30 ENTER↑ 12 × f n	360.00	Months.
9 ENTER↑ 12 ÷ f i	0.75	% monthly interest rate.
50000 f PV f PMT	402.31	Monthly payment.
g RESET		Clear status for APR calculation.
f PMT	402.31	Reuse payment.
RCL f n f n	360.00	Recall and reuse time periods.
RCL f PV 2 % −		
f PV	49000.00	Effective amount received.
f i	0.77	% monthly percentage rate.
12 ×	9.23	% APR.

What's Really Happening? *For a mortgage with fees, the borrower is making payments on the original loan amount, which corresponds with the initial calculation of the payment amount. If you borrow $10,000, but are immediately charged $500 in fees, you really only receive $9500. But, your payments are based on $10,000. With fees, then, you're really paying the same for less money, which generates the need to compute the true APR.*

Payment Amount (Ordinary Annuity). This routine calculates the payment amount for an ordinary annuity given the number of periods, the initial principal, and the interest rate.

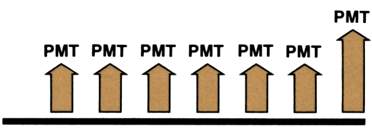
1. Press g RESET.

2. Input the following in any order:
   - Key in the number of periods, press f [n].
   - Key in the periodic interest rate, press f [i].
   - Key in the present value, press f [PV].
3. Press f [PMT] to obtain the payment amount.

Example: Find the monthly payment amount on a 30-year, $52,000 mortgage at 9.75% annual interest rate.

Press	Display	
g [RESET]		
30 ENTER↑ 12 [×] f [n]	360.00	Number of payments.
9.75 ENTER↑ 12 [÷] f [i]	0.81	% monthly interest rate.
52000 f [PV] f [PMT]	446.76	Monthly payment.

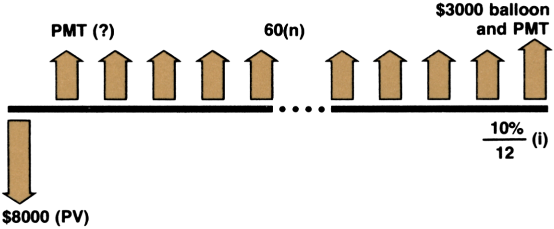
Payment Amount (Ordinary Annuity with Balloon). A common financial occurrence is an annuity that has a large payment at the end, like so:



The last payment—usually considerably larger although it could also be smaller than the others—is called a balloon payment or balloon.

By subtracting the present value of the balloon payment from the loan amount, the problem effectively becomes "What is the monthly payment on a direct reduction loan?"

Example: Yellowstone Sam is heading north, and will invest in an \$8000 dog sled and team. His loan specifies 60 monthly payments at 10% with a balloon payment in the 60th month of \$3000. What will his monthly payments be?

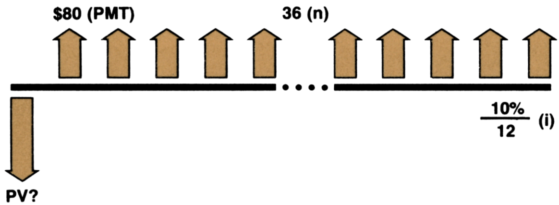


Press	Display	
g RESET		
60 f n	60.00	Months.
10 ENTER↑ 12 ÷ f i	0.83	% monthly interest rate.
3000 f FV f PV	1823.37	Present value of balloon.
g RESET		
f PV	1823.37	Input balloon PV.
RCL f n f n	60.00	Recall and reuse n.
RCL f i f i	0.83	Recall and reuse i.
8000 RCL f PV −		
f PV	6176.63	Net present value of loan amount less balloon.
f PMT	131.24	Monthly payment.

Present Value (Ordinary Annuity). This calculation finds the principal amount of a direct reduction loan when the interest rate, payment amount and number of payments are known.

1. Press g RESET.
2. Input the following in any order:
   - Key in number of periods, press f n.
   - Key in the periodic interest rate, press f i.
   - Key in the payment amount, press f PMT.
3. Press f PV to obtain the present value.

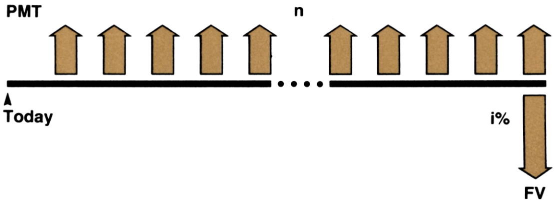
Example: Yellowstone Sam decides to purchase a snowmobile. He plans to pay $80 per month for 3 years, and he's willing to pay 10% annual interest. How much can he afford to pay for the snowmobile?



Press	Display	
g RESET		
36 f n	36.00	Number of payments.
80 f PMT	80.00	Monthly payment.
10 ENTER↑ 12 ÷ f i	0.83	% monthly interest rate.
f PV	2479.30	Price he can pay for snowmobile.

Future Value (Ordinary Annuity). With loan calculations, you generally solve for n, i, PMT, or PV. There is another type of ordinary annuity called a "sinking fund," where you make payments at regular intervals into a fund to discharge a debt (for example, to pay off a bond issue at maturity). With sinking fund calculations, you solve for n, i, PMT, or FV (how much you will have in the fund at a future date).

Sinking fund payments start at the end of the first period, like so:



This is different from opening a savings account with a starting deposit *today*. Savings are annuity due calculations and will be described later in this section.

The following procedure finds the future value of an ordinary annuity:

1. Press [g] [RESET].
2. Input the following in any order:
   - Key in the number of periods, press [f] [n].
   - Key in the periodic interest rate, press [f] [i].
   - Key in the payment amount, press [f] [PMT].
3. Press [f] [FV] to obtain the future value.

Example: A $100,000 bond is to be discharged by the sinking fund method. If, starting 6 months from now, you deposit $3914.75 twice a year into a sinking fund that pays 5% compounded semiannually, will you be able to pay off the bond in 10 years?

Press	Display	
[g] [RESET]		
10 [ENTER↑] 2 [×] [f] [n]	20.00	Semiannual periods.
5 [ENTER↑] 2 [÷] [f] [i]	2.50	% semiannual interest.
3914.75 [f] [PMT]	3914.75	Semiannual deposit.
[f] [FV]	100000.95	Balance of fund after 10 years.

Just barely!

Amortization Schedule

This calculation generates an amortization schedule for a direct reduction loan (mortgage) from the first payment to a given payment, when the loan amount, periodic interest rate, and payment amount are known.

1. Key in payment amount, press STO 1.
2. Key in periodic interest rate, press ENTER↑ ENTER↑.
3. Key in loan amount.
4. Press x⇄y % to obtain the interest portion of the payment.
5. Press RCL 1 x⇄y − to obtain the principal portion of the payment.
6. Press − to obtain the remaining loan balance.

Repeat steps 4 through 6 for all desired periods.

Example: Generate an amortization schedule for the first two months of a $30,000 loan, at 7%, with monthly payments of $200.

Press	Display	
200 STO 1	200.00	Payment.
7 ENTER↑ 12 ÷ ENTER↑		
ENTER↑	0.58	% monthly interest rate.
30000 x⇄y %	175.00	Interest portion of first payment.
RCL 1 x⇄y −	25.00	Principal portion of first payment.
−	29975.00	Remaining balance.
x⇄y %	174.85	Interest portion of second payment.
RCL 1 x⇄y −	25.15	Principal portion of second payment.
−	29949.85	Remaining balance.

Accumulated Interest and Remaining Balance. This calculation finds the accumulated interest and remaining balance at any point in the life of a fully amortized mortgage, when the mortgage amount, payment amount, and periodic interest rate are known.

1. Press g RESET f REG.
2. Input the following in any order:
   - Key in periodic interest rate, press f i.
   - Key in payment amount, press f PMT.
   - Key in amount of loan, press f PV.
3. Press f n to obtain total number of periods.
4. Press STO 1 g RESET.
5. Key in the beginning payment number *less one*. Press − f n.
6. Press RCL f i f i.
7. Press RCL f PMT f PMT.
8. Press f PV to obtain the remaining balance at the beginning of the time frame.
9. Press RCL 1 to recall the total number of periods.
10. Key in the last payment number of the time frame, press − f n.
11. Press f PV for the remaining balance at the end of the time frame.
12. Press − RCL f PMT. Key in the number of payments made during the time frame, press ×.
13. Press x⇄y − to obtain accumulated interest paid during the time frame.

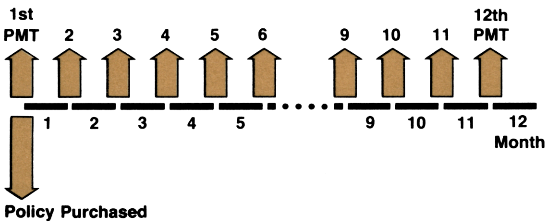
Example: For tax purposes, a home owner wants to know the accumulated interest paid on his 8.75%, $45,000 mortgage between payments 22 and 33 inclusive. His payments are $354; calculate the interest paid.

Press	Display	
g RESET f REG		
8.75 ENTER↑ 12 ÷ f i	0.73	% monthly interest.
354 f PMT		
45000 f PV f n	360.07	Total number of periods.
STO 1 g RESET	360.07	
21 − f n	339.07	
RCL f i f i	0.73	Recall and reuse i.
RCL f PMT f PMT	354.00	Recall and reuse PMT.
f PV	44415.11	Balance at beginning of 22nd payment.
RCL 1	360.07	
33 − f n f PV	44038.57	Remaining balance after 33rd payment.
− RCL f PMT 12 ×		
x≷y −	3871.46	Accumulated interest.

Annuity Due (Payments in Advance)

With some annuities—like insurance premiums or a lease—the payment is due at the beginning of the month. This is called an annuity due because the payment falls at the beginning of the payment period. Other terms are payments in advance or anticipated payments.

An annuity due with monthly payments for a year—say, a car insurance policy—looks like this:



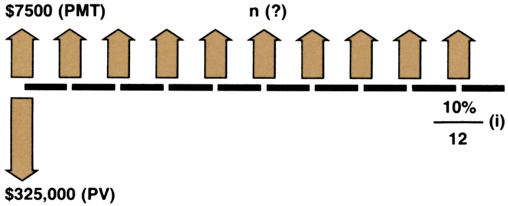
Notice that with an annuity due, you have a payment right away at the beginning of the first interval. (With an ordinary annuity, your payment isn't due until the end of the first period, but you also have a payment at the end of the entire term.)

The following calculations all deal with annuity due problems, e.g., savings, insurance, leases, and rents.

Number of Periods (Annuity Due). This calculation finds the total number of periods required for an initial amount when the payment amount (paid at the beginning of each period) and interest rate are known.

1. Press g RESET.
2. Key in periodic interest rate, press f i.
3. Key in payment amount, press x⇄y % + f PMT.
4. Key in initial amount, press f PV, or key in desired amount and press f FV.
5. Press f n.

Example 1: Given an investment possibility of $325,000 that will immediately produce rental income of $7500 per month, how long must the investment be held to yield 10% per annum?



Press	Display	
g RESET		
10 ENTER↑ 12 ÷ f i	0.83	% monthly interest rate.

7500 x⇄y % +		
f PMT	7562.50	
325000 f PV		
f n	53.43	Months.

Example 2: If you deposit $50 a month in a savings account that pays 6% interest, how long will it take to reach $1000?

Press	Display	
g RESET		
6 ENTER↑ 12 ÷ f i	0.50	% monthly interest rate.
50 x⇄y % + f PMT		
1000 f FV	1000.00	Desired amount.
f n	19.02	Months.

Interest Rate (Annuity Due). This routine calculates the interest rate on an initial amount, given the number of periods and payment amount (paid at the beginning of each period).

1. Press g RESET.
2. Key in *one less* than the total number of periods, press f n.
3. Key in payment amount, press f PMT.
4. Key in initial amount, press x⇄y − f PV.
5. Press f i to obtain the periodic rate. Multiply by the number of payment periods per year to obtain the annual rate or yield.

Example: Equipment worth $12,000 is leased for 8 years with monthly payments in advance of $200. The equipment is assumed to have no salvage value at the end of the lease. What yield rate does this represent?

Press	Display	
g RESET		
8 ENTER↑ 12 × 1 − f n	95.00	Number of periods less one.

200 f PMT	200.00	Payment amount.
12000 x⇄y − f PV		
f i	1.09	% monthly yield.
12 ×	13.07	% annual yield.

Payment Amount (Annuity Due). This calculation finds the payment amount (paid at the beginning of each period) given the initial amount, number of periods and interest rate.

1. Press g RESET.
2. Key in the number of payment periods, press f n.
3. Key in the periodic interest rate, press f i.
4. Press 1 x⇄y % +.
5. Key in initial value, press x⇄y ÷ f PV.
6. Press f PMT.

Example: The owner of a building presently worth $70,000 intends to lease it for 20 years at the end of which time he assumes the building will be worthless (i.e., has no residual value). How much must the quarterly payments (in advance) be to achieve a 10% annual yield?

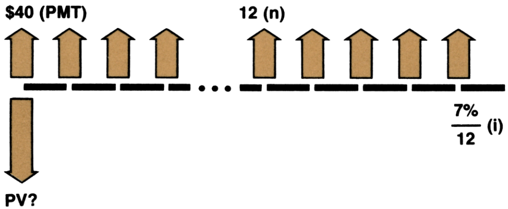
Press	Display	
g RESET		
20 ENTER↑ 4 × f n	80.00	Number of quarterly payments.
10 ENTER↑ 4 ÷ f i	2.50	% quarterly interest.
1 x⇄y % +		
70000 x⇄y ÷ f PV		
f PMT	1982.27	Quarterly payment.

Present Value (Annuity Due). This calculation finds the present value of a series of payments (made at the beginning of each period) given the number of payments, interest rate, and payment amount.

1. Press g RESET.
2. Key in the number of payment periods, press f n.

3. Key in the periodic interest rate, press f i.
4. Key in the payment amount, press x⇄y % + f PMT.
5. Press f PV.

Example: The owner of a downtown parking lot has achieved full occupancy and a 7% annual yield by renting parking spaces for $40 per month payable in advance. Several regular customers want to rent their spaces on an annual basis. What annual rent, also payable in advance, will maintain a 7% annual yield rate?



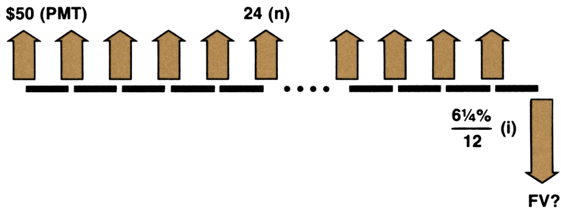
Press	Display	
g RESET		
12 f n	12.00	Months.
7 ENTER↑ 12 ÷ f i	0.58	% monthly interest rate.
40 x⇄y % + f PMT		
f PV	464.98	Equivalent annual payment.

Future Value (Annuity Due). This calculation finds the future value of a series of payments in advance, given the number of periods and interest rate.

1. Press g RESET.
2. Key in the number of periods, press f n.

3. Key in the periodic interest rate, press [f] [i].
4. Key in the payment amount, press [x⇄y] [%] [+] [f] [PMT].
5. Press [f] [FV].

Example: If you can afford to deposit $50 per month in an account with 6¼% interest compounded monthly, how much will you have 2 years from now?



Press	Display	
[g] [RESET]		
24 [f] [n]	24.00	Number of periods.
6.25 [ENTER↑] 12 [÷] [f] [i]	0.52	% montly interest rate.
50 [x⇄y] [%] [+] [f] [PMT]		
[f] [FV]	1281.34	Savings.

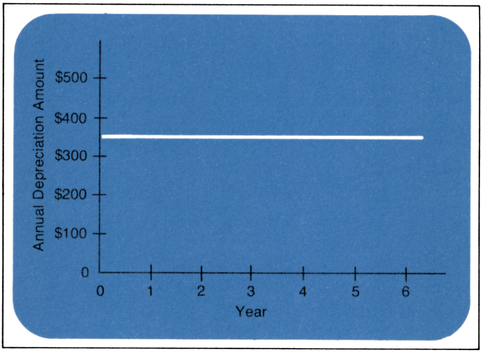
Depreciation

Machines, buildings, delivery trucks, showcases, tools and other tangible assets all decline in value with the passing of time. To provide for the replacement of obsolete or worn-out equipment, you usually set aside a fixed amount of money each year that is equal to the loss in value of that article during the year.

There are three methods of depreciation commonly used: straight-line, sum-of-the-digits, and declining-balance. Let's take a popcorn machine that costs $2500 brand new and has a salvage value of $400. We will depreciate the $2100 over a life expectancy of 6 years, using each of these three methods.

Straight-Line Method—The straight-line method (SL) is simply a matter of dividing the total depreciable amount by the number of useful years, then subtracting that amount each year from the item's value. The depreciation on the popcorn machine is $2100, divided by 6 years = $350 a year.

If you plot this depreciation amount on a graph, it looks like a straight line, hence, the name.



The advantage of the straight-line method is its simplicity—it's easy to figure and it's consistent. Your deduction is always the same.

To calculate the annual depreciation allowance on your HP-27:

1. Calculate and key in depreciable amount (cost less salvage value), press ENTER↑ ENTER↑.
2. Key in asset's useful life (number of years), press ÷ to obtain each year's depreciation.
3. Press STO 1 − to obtain depreciable value after the first year.
4. Continue pressing RCL 1 − to obtain the remaining depreciable value for each subsequent year. If book value is needed, add salvage value to depreciable value.

Example: A duplex costing $41,500 (exclusive of land) is depreciated over 25 years, using the straight-line method. What is the annual depreciation amount and remaining depreciable value for years 1 and 2 if it has no salvage value?

Press	Display	
41500 ENTER↑ ENTER↑	41500.00	Depreciable value.
25 ÷	1660.00	Annual depreciation amount.
STO 1 −	39840.00	Remaining depreciable value year 1.
RCL 1 −	38180.00	Remaining depreciable value year 2.

In reality, some items depreciate the most during their initial periods of use. For example, the value of your car declines the most in the first two years. Or perhaps your equipment doesn't wear out most in the early years, but you want to increase the initial depreciation for the financial advantages. This is called accelerated depreciation, and you would use one of the two following methods.

Sum-of-the-Years-Digits Method—The sum-of-the-years-digits method (SOYD) is based on the sum of the digits from one to the number of years of the asset's life. For the popcorn machine, the life is 6 years, so:

$$6 + 5 + 4 + 3 + 2 + 1 = 21 \text{ (sum of the years digits)}$$

Theoretically, the first year you use up 6/21 of the asset's life; the second year, 5/21 of the asset's life, etc. So, the first year, you multiply the depreciation amount ($2100) by that year's use (6/21).

$$\frac{6}{21} \times \$2100 = \$600$$

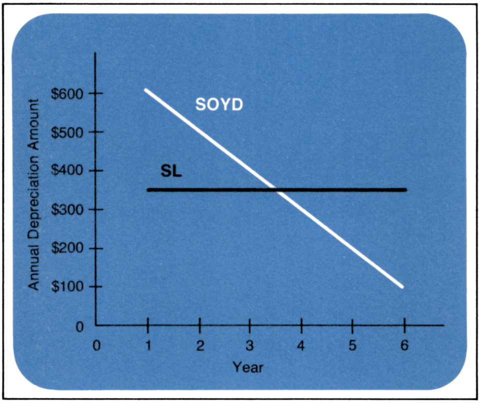
The second year, by that year's use:

$$\frac{5}{21} \times \$2100 = \$500$$

The third year:

$$\frac{4}{21} \times \$2100 = \$400$$

If you plot the depreciation on a graph, you can see the difference between the straight-line method and the sum-of-the-years-digits method.



The keystroke procedure below finds the depreciation and remaining depreciable value using the SOYD method for each year of an asset's depreciable life when its useful life expectancy and cost (less salvage value) are known.

1. Key in the beginning depreciable value, press ENTER↑.
2. Key in asset's depreciable life, press ENTER↑.
3. Press ENTER↑ 1 + CHS STO 1 × 2 ÷ ÷ STO 2 CL x.
4. Press 1 RCL 1 + STO 1 RCL 2 × to obtain the depreciation amount.

5. Press ⊟ to obtain the remaining depreciable value at the end of the year.
6. Repeat steps 4 and 5 for subsequent year's depreciation and remaining depreciable value.

Example: Apartments valued at $88,000 are depreciated over 25 years using SOYD depreciation. What is the depreciation amount and remaining depreciable value for the first 2 years?

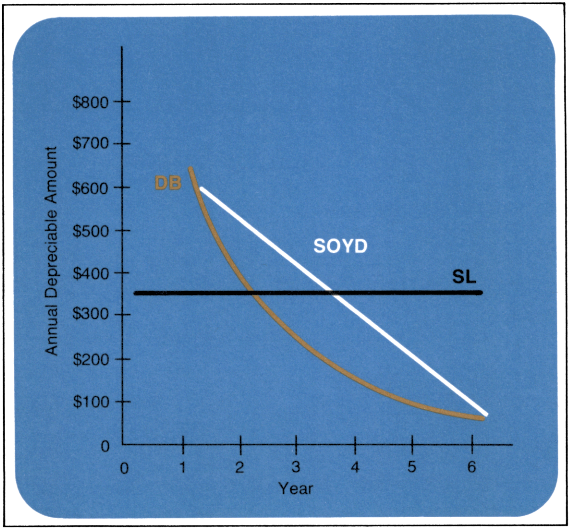
Press	Display	
88000 ENTER↑	88000.00	Depreciable value.
25 ENTER↑	25.00	Depreciable life.
ENTER↑ 1 ⊞ CHS		
STO 1 ⊠		
2 ⊡ ⊡		
STO 2 CLX	0.00	
1 RCL 1 ⊞ STO 1		
RCL 2 ⊠	6769.23	Depreciation amount year 1.
⊟	81230.77	Remaining depreciable value, year 1.
1 RCL 1 ⊞ STO 1		
RCL 2 ⊠	6498.46	Depreciation amount year 2.
⊟	74732.31	Remaining depreciable value, year 2.

Declining Balance Method—With the declining balance method (sometimes called the fixed-rate method), a constant percentage is applied each year to the remaining balance (book value) to find the depreciation amount. The salvage value is not subtracted intially, but the asset may not be depreciated below this salvage value.

Certain declining balance "factors" are authorized for income tax purposes. A factor of 1.25 simply means 125% declining balance; 2.00 means double-declining balance. To compute the annual depreciation rate, divide the factor by the asset's estimated life in years. If you use a factor of 1.50 (150%) for the popcorn machine, then you will depreciate it $\frac{150}{6}$ or 25% each year.

	Depreciation	Balance (Book Value)
1st year 25% of \$2500 =	\$625.00	\$1875.00
2nd year 25% of \$1875 =	\$468.75	\$1406.25
3rd year 25% of \$1406.25 =	\$351.56	\$1054.69
4th year 25% of \$154.69 =	\$263.67	\$ 791.02
5th year 25% of \$791.02 =	\$197.75	\$ 593.27
6th year \$593.27 − \$400 (salvage value) =	\$193.27	\$ 400.00

You can plot the declining balance depreciation on the graph to compare it with the other two methods.



The following keystroke procedure finds the depreciation and remaining book value for each year of an asset's depreciable life when the declining factor, cost, salvage value, and life expectancy are known:

1. Key in declining factor (1.25 for 125% declining balance, 2.00 for double declining etc.), press ENTER↑ 100 ×.
2. Key in number of years of useful life, press ÷ STO 1.
3. Key in cost (do not deduct salvage value).
4. Press RCL 1 % to obtain first year's depreciation.
5. Press − to obtain remaining book value after first year.
6. Repeat steps 4 and 5 to obtain each succeeding year's depreciation and remaining book value until book value is equal to or less than the salvage value. In the period when the remaining book value is less than the salvage value, the previous book value should be reduced by the salvage value to obtain the final year's depreciation.

Example: The Drifter Apartments have a depreciable value of $86,000. The owner wishes to use 125% declining balance depreciation over 20 years. What is the annual depreciation amount and remaining book value in years 1 and 2?

Press	**Display**	
1.25 ENTER↑		
100 ×	125.00	
20 ÷ STO 1	6.25	
86000	86000	Depreciable value.
RCL 1 %	5375.00	First year depreciation.
−	80625.00	Remaining depreciable value, year 1.
RCL 1 %	5039.06	Second year depreciation.
−	75585.94	Remaining depreciable value, year 2.

Discounted Cash Flow Analysis

Discounted cash flow analysis is a way of evaluating investment alternatives in which investments are compared on the same basis—their present dollar worth. Two forms of discounted cash flow analysis are the net present value approach and the discounted or internal rate of return approach. These two functions, NPV and IRR, are preprogrammed on your HP-27.

Net Present Value. The g NPV function solves for the net present value according to the formula:

$$NPV = -INV + \frac{cf_1}{(1+i)^1} + \frac{cf_2}{(1+i)^2} + \ldots . \frac{cf_n}{(1+i)^n}$$

NPV = net present value.

INV = your initial investment or cash outlay.

$\frac{cf_1}{(1+i)^1}$ = cash flow generated the first year.

$\frac{cf_2}{(1+i)^2}$ = cash flow generated the second year.

i = the rate of return or desired yield.

INV, the original investment, is negative because it represents a cash outlay. With the net present value approach, you are comparing the sum of the present values of all the future cash flows (all the cf's) to that initial investment.

At the start, NPV is negative because you've put out a large amount (INV). As the cash returns flow in, NPV will increase. Eventually—hopefully—NPV will turn positive. When NPV = 0, you have reached the break-even point on the investment. If NPV is positive, your investment is profitable.

The following procedure is used to find the net present value when the assumed yield rate (interest rate) and periodic cash flows are known.

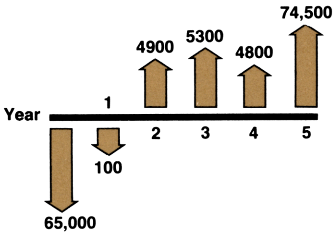
1. Press g RESET. Press f Σ.
2. Key in the initial investment and press f PV.
3. Key in the desired or known yield rate and press f i.
4. Key in the first cash flow and press g NPV. The number displayed is the net present value. Repeat this step for all successive cash flows.

As soon as the display shows a positive number, the investment is recovered on a discounted cash flow basis.

The number of cash flows and the sum of their present values are accumulated in registers 8 and 9. Hence, you must clear the registers by pressing f Σ before solving for NPV.

Example 1: An investor pays $65,000 for a duplex which he intends to keep 5 years and then sell. The first year he knows he will have to spend a considerable amount for repairs. If he desires a 9% after-tax yield rate and the after-tax cash flows are as follows, will he achieve this yield?

Year	Cash Flows ($)
1	−100
2	4900
3	5300
4	4800
5	74500



Press	Display	
g RESET f Σ		
65000 f PV	65000.00	Initial investment
9 f i	9.00	% annual yield.
100 CHS g NPV	-65091.74	First year NPV.
4900 g NPV	-60967.51	Second year NPV.
5300 g NPV	-56874.94	Third year NPV.
4800 g NPV	-53474.50	Fourth year NPV.
74500 g NPV	-5054.61	Fifth year NPV.

Since the final NPV is negative, the investment does *not* achieve the desired yield.

Internal Rate of Return. The g IRR function solves for the internal or discounted rate of return for a maximum of 10 uneven cash flows. The same formula applies, only NPV = 0.

$$0 = -\text{INV} + \frac{cf_1}{(1+i)^1} + \frac{cf_2}{(1+i)^2} + \ldots \frac{cf_{10}}{(1+i)^{10}}$$

Solving for i gives you the rate of return (IRR) on your investment. This is actually quite a complex calculation, but your HP-27 handles it with ease. In fact, the HP-27 is the *only* pocket calculator that's preprogrammed to solve IRR.

Key in the cash flows as follows:

1. Press g RESET.
2. Key in the amount of the initial investment, and press f PV.
3. Key in the total number of cash flows and press f n.
4. Key in the first cash flow (followed by CHS if it is negative, i.e., if you had a loss that year). Store that number in register 0 by pressing STO 0.

5. Key in the second cash flow and press STO 1. Continue keying in cash flows (up to 10) and store them in successive storage registers, up to register 9.
6. Press g IRR.

If each of the cash flows = 0 or if there are no cash flows, an Error message is displayed. If you haven't recovered even part of the initial investment, then logically there is no "return" and you can't solve for IRR.

If all the cash flows are negative, your answer is meaningless.

Example 1: What is the estimated rate of return on a restaurant costing $200,000 that produces the following cash flows?

Year	Cash Flow
1	$ −4,000
2	$ 20,000
3	$ 27,000
4	$ 42,000
5	$ 56,000
6 (You sell it.)	$230,000

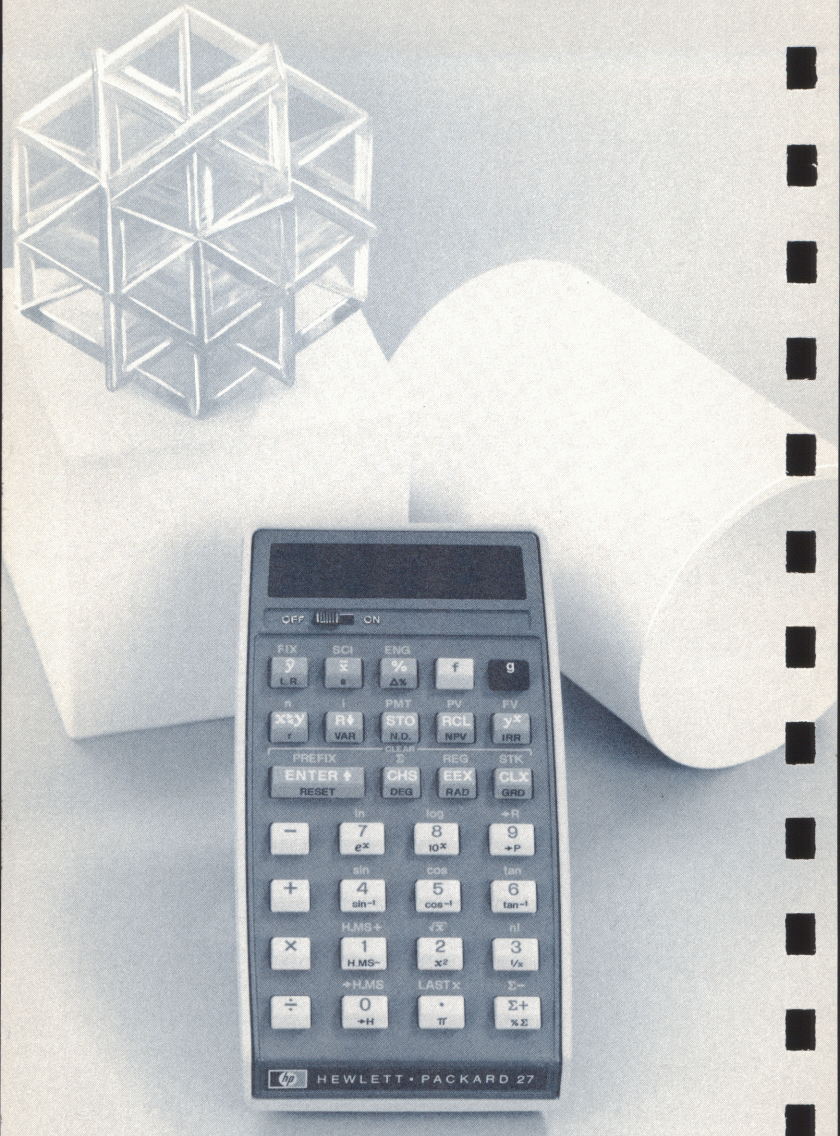
Press	Display	
g RESET		
200000 f PV	200000.00	Initial investment.
6 f n	6.00	Number of cash flows.
4000 CHS STO 0	-4000.00	First cash flow.
20000 STO 1	20000.00	Second cash flow.
27000 STO 2	27000.00	Third cash flow.
42000 STO 3	42000.00	Fourth cash flow.
56000 STO 4	56000.00	Fifth cash flow.
230000 STO 5	230000.00	Sixth cash flow.
g IRR	12.72	% annual yield or rate of return on investment.

Example 2: In a moment of altruism, Joe Karnegie loaned his nephew, Winston, $8000 to finish his last two years of college. Winston agreed to pay back the original amount plus $300 in interest, within five years after graduation.

If Winston repaid the loan as follows, what was benevolent Uncle Joe's rate of return?

Year	Payment
Junior year	0.
Senior year	0.
Year 1	$ 500.00
Year 2	$1300.00
Year 3	$1000.00
Year 4	$1000.00
Year 5	$4500.00

Press	Display	
g RESET		
8000 f PV	8000.00	Initial investment.
7 f n	7.00	Number of cash flows.
0 STO 0	0.00	First cash flow.
0 STO 1	0.00	Second cash flow.
500 STO 2	500.00	Third cash flow.
1300 STO 3	1300.00	Fourth cash flow.
1000 STO 4	1000.00	Fifth cash flow.
1000 STO 5	1000.00	Sixth cash flow.
4500 STO 6	4500.00	Seventh cash flow.
g IRR	0.62	% rate of return.


OFF ON
FIX SCI ENG
% f g
L.R. s Δ%
n i PMT PV FV
STO RCL
VAR N.D. NPV IRR
CLEAR
PREFIX Σ REG STK
ENTER
CHS EEX CLX
RESET DEG RAD GRD
ln log →R
7 8 9
e^x 10^x →P
sin cos tan
4 5 6
$\sin^{-1}$ $\cos^{-1}$ $\tan^{-1}$
H.MS+ $\sqrt{x}$ n!
1 2 3
H.MS− x^2 1/x
→H.MS LAST x Σ−
0 Σ+
→H π %Σ
HEWLETT • PACKARD 27

Appendix A

Accessories, Maintenance, and Service

Standard Accessories

Your HP-27 comes complete with the following standard accessories: battery pack, ac adapter/recharger, soft carrying case, and the *HP-27 Owner's Handbook*.

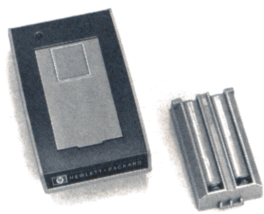
> **CAUTION:**
> Use of any batteries other than the Hewlett-Packard battery pack may damage your calculator.

Optional Accessories

Other accessories are shown here. To order additional standard or optional accessories, see your nearest dealer or send the Accessory Order Form, along with a check or money order, to:

Hewlett-Packard Company
19310 Pruneridge Avenue
Cupertino, CA 95014

Security Cradle



Reserve Power Pack

Outside the United States, mail the Accessory Order Form to the Hewlett-Packard Sales Office nearest you.

Calculator Operation

Charging Times

Your calculator contains a rechargeable battery pack. *The batteries must be in the calculator for it to operate.* With the recharger connected to the calculator and to an ac power outlet, the batteries will charge, whether the calculator is ON or OFF. Normal charging times from dead battery to full charge are:

Calculator ON	17 hours
Calculator OFF	6 hours

Charge the battery before portable use. Shorter charge periods than the above will reduce battery operating time.

Whether the calculator is OFF or ON, the battery pack will never overcharge with the ac adapter/recharger connected to the ac line.

Traveling with the HP-27

Taking a business trip abroad? If so, take your calculator with you. The recharger has a line-voltage select switch for two ranges: 100 to 127 volts and 200 to 254 volts. Check the line voltage of the country you intend to visit, and set the switch to the appropriate range.

The only other item you will need is an ac plug adapter for the wall outlet (the same adapter you would use for an electric shaver or other personal appliance). These are available at some hotels, or you may purchase the adapter in the country of your destination.

Charging the Battery

CAUTION:
Your HP-27 may be damaged if the ac adapter/recharger is not set for the correct line voltage, or if you use any charger other than the HP ac adapter/recharger supplied with your calculator.

The procedure for using the battery charger is as follows:

1. Make sure the voltage select switch on the recharger is set to the proper voltage range. (In the U.S., set it to 100 to 127 volts.)
2. Turn the HP-27 power switch OFF.

3. Insert the recharger plug into the rear connector of the HP-27, and insert the power plug into a live power outlet. The HP-27 may feel warm to the touch while recharging. Don't be concerned; this is normal because part of the charging circuit is inside the calculator.
4. At the end of the charging period, you may continue to use your HP-27 with ac power; or turn the power switch OFF and disconnect the recharger for battery-only operation.

If the battery pack won't hold a charge, it may be defective. If the one-year warranty is in effect, return the pack to Hewlett-Packard according to the shipping instructions. If your warranty has expired, see your nearest dealer or use the Accessory Order Form to purchase a replacement.

Battery Operation

Use only the HP battery pack. A fully-charged pack provides approximately 3 to 5 hours of continuous operation. By turning the power OFF when the calculator is not in use, you can conserve energy and make the HP-27 battery pack last easily through a normal working day.

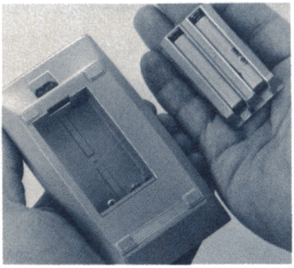
Low Power Display

When the batteries get low, several decimal points will appear on the display. This means you have approximately one minute of operating time left. Then you must either charge the battery or insert a fully-charged spare battery pack.

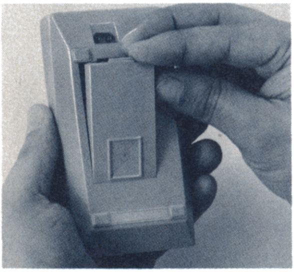
Replacing the Battery Pack

To replace your battery pack, use the following procedure:

1. Turn the HP-27 power switch to OFF, and disconnect the recharger from the calculator.
2. With your thumb on the square inset on the rear of the calculator, press down and slide the door in the direction of the arrow. The latch will spring open.



3. Remove the battery pack.



4. To insert the new pack, slant the leading edge of the pack into the upper edge of the doorway, then snap the latch down into place.

If you use your HP-27 extensively on batteries, you may want to order the Reserve Power Pack. This optional accessory lets you charge one pack while using the other.

Blank Display

If the display blanks out, turn the calculator OFF, then ON. If 0.00 does not appear on the display, check the following:

1. Make sure the ac adapter/recharger is plugged into an ac outlet. If not, turn the calculator OFF before plugging in the recharger.
2. Examine the battery pack to see if the contacts are dirty.
3. Substitute a fully-charged battery pack, if available, for the one that was in the calculator.

If the display is still blank, your calculator should be serviced. (Refer to *Warranty* in this appendix.)

Temperature Range

Temperature ranges for the calculator are:

Operating:	0° to 45°C	32° to 113°F
Charging:	15° to 40°C	59° to 104°F
Storage:	−40° to +55°C	−40° to +131°F

Temperatures above or below these specified limits may not cause permanent damage to your calculator, but you probably will get incorrect answers. Also, temperature extremes will damage the batteries.

Serial Number

The serial number of your calculator is located on the back of the case, right above the battery door and below the recessed plug. It's advisable to keep a separate record of your serial number.

Warranty

Full One-Year Warranty

The HP-27 is warranted against defects in materials and workmanship for one (1) year from date of delivery. During the warranty period, Hewlett-Packard will repair or, at its option, replace at no charge components that prove to be defective, provided the calculator is returned, shipping prepaid, to Hewlett-Packard's Customer Service Facility. (Refer to Shipping Instructions.)

This warranty does not apply if the calculator has been damaged by accident or misuse, or as a result of service or modification by other than an authorized Hewlett-Packard Customer Service Facility. No other express warranty is given by Hewlett-Packard. **HEWLETT-PACKARD SHALL NOT BE LIABLE FOR CONSEQUENTIAL DAMAGES.**

Warranty Transfer

The warranty is recorded by calculator serial number, as well as date of purchase. If you sell your calculator or give it as a gift, the warranty is transferrable and remains in effect for the new owner until the original one-year expiration date. It is not necessary for the owner to notify Hewlett-Packard of the transfer.

Shipping Instructions

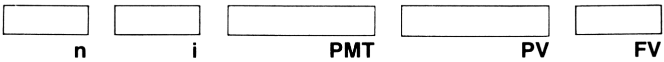
Whether the unit is under warranty or not, it is the customer's responsibility to pay shipping charges to the Hewlett-Packard Customer Service Facility. Please return the calculator with all standard accessories and a completed Service Card, describing the problem.

Under normal conditions, your calculator will be repaired and reshipped within five working days of receipt at the Hewlett-Packard Customer Service Facility.

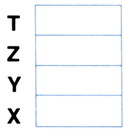
After warranty repairs are completed, the Service Center returns the unit with postage prepaid. On out-of-warranty repairs, the unit is returned C.O.D. (covering shipping costs and the service charge).

How the HP-27 Registers Work

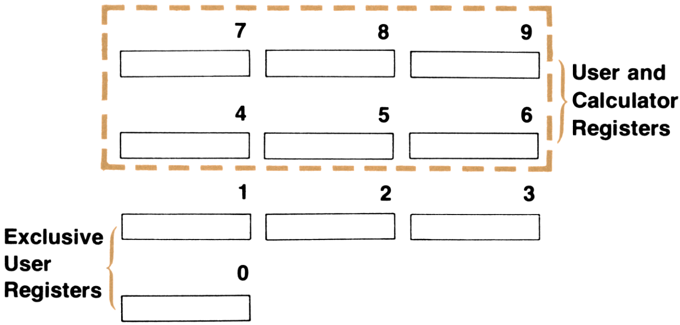
Inside your calculator are five financial registers (associated with the top row keys):



There are also four registers in the operational stack:

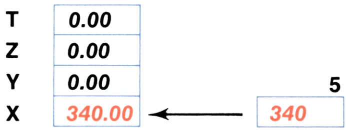


And 10 general-purpose storage registers. Four of these (registers 0 thru 3) are dedicated exclusively for your own use. The other six registers (4 thru 9) are shared by the user and the calculator.



If you press 340 STO 5, that number is transferred from the display to register 5:

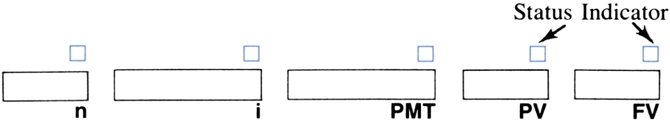
When you press RCL 5, a copy of that number appears on the display but the value is still stored in register 5:



When you press f [REG], all 10 storage registers are cleared. If you press f [Σ], just the statistical registers (4 thru 9) are cleared.

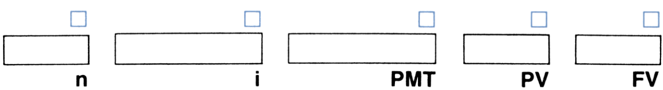
The Financial Registers

Associated with each of the financial registers is a status indicator. (You can't see them but they're there.)



When a status indicator is on (■), the value in the particular register is to be used as data for the next financial calculation. When a status indicator is reset or off (□), the register value cannot be used.

Let's take an example calculating the monthly payment on a 20-year $27,000 mortgage with 8.5% annual interest. Press g [RESET] to reset the status indicators:



Key in 20 (the number of yearly payments):

T	0.00	
Z	0.00	
Y	0.00	
X	20.	(Display)

Since the n value must be monthly payment periods, convert the yearly figure by pressing ENTER↑ 12 ×, then input it by pressing f n. The converted value on the display is copied into the n register and the first status indicator is turned on.

T	0.00	
Z	0.00	
Y	0.00	
X	240.00	(Display)

n	i	PMT	PV	FV
240				

The stack lift indicator is set after a financial value has been keyed into a financial register.

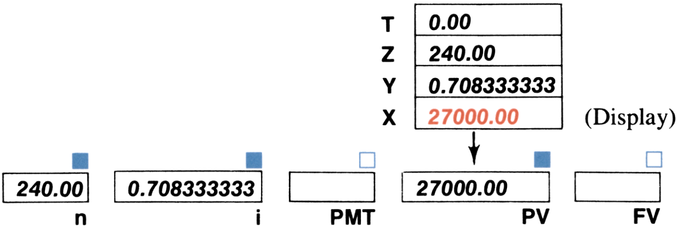
Next, key in the monthly interest rate: 8.5 ENTER↑ 12 ÷ f i. Again, the value on the display is copied into the i register and that status indicator is turned on.

T	0.00	
Z	0.00	
Y	240.00	
X	0.71	(Display)

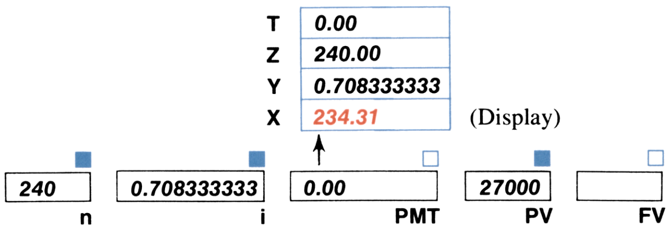
n	i	PMT	PV	FV
240	0.708333333			

Remember that although the value on the display is rounded off to two decimal places, your calculator always figures internally to 10 places, so the number placed in [i] is really 0.708333333. This is extremely important when you stop to think that those extra decimal places are used over 240 months!

To solve a top-row financial function, three status indicators must be on. Key in 27000 and press [f] [PV]:

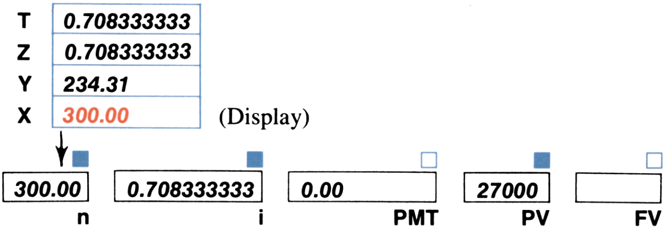


The calculator sees that n, i and PV status indicators are turned on. Once three status indicators are on, pressing one of the other two financial keys will cause the calculator to execute and display the new value. Press [f] [PMT]:

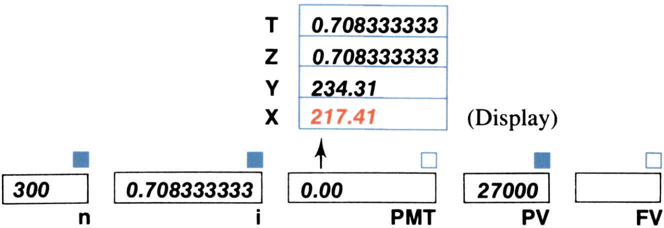


There are two important points to remember: (1) the HP-27 will never allow you to turn on more than three status indicators; and (2) of the three status indicators that are on, at least one of them must be n or i.

Suppose you want to solve for payment again but with a different time value—say, a 25-year loan instead of 20 years. Since only one value changes and the necessary status indicators are already turned on, you can simply overwrite the new number in the n register. Press 25 ENTER↑ 12 × f n.

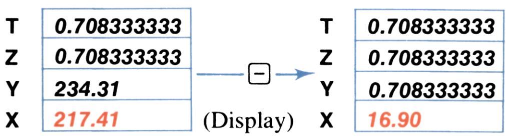


Solve for the new payment by pressing f PMT:



Notice that the two calculated answers are stored in the X and Y registers of the stack for your convenience. For example, you can review both payment amounts by pressing x⇄y to look back at the first answer.

Or, with both answers positioned like that in the stack, you can quickly subtract to calculate the difference between the two payment amounts:



$ per month difference in payments.

The RESET Function

The power of RESET becomes apparent when you do a problem that requires a different status indicator. Before, the prerequisite status indicators were n, i and PV in order to obtain PMT.

Suppose you can afford $225 a month and wish to find the length of the loan. This involves a different set of known values. Now, the prerequisite status indicators are i, PMT and PV, and you want to find n.

Because this is a new financial calculation involving different input registers, reset the status indicators by pressing g RESET.

T	0.708333333	
Z	0.708333333	
Y	0.708333333	
X	16.90	(Display)

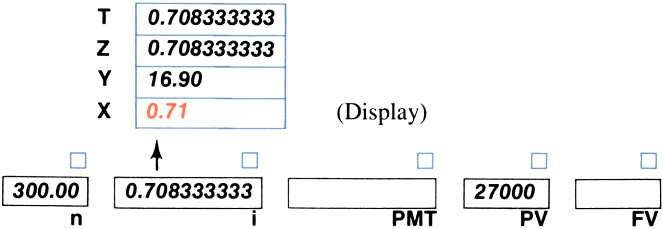
n	i	PMT	PV	FV
300.00	0.708333333	0.00	27000	

Notice that your previous answer is still on the display, and hence, in the stack. If you want to use this number in a future problem, you can ENTER↑ it up in the stack, store it in one of the 10 manual storage registers, or use it in another financial problem.

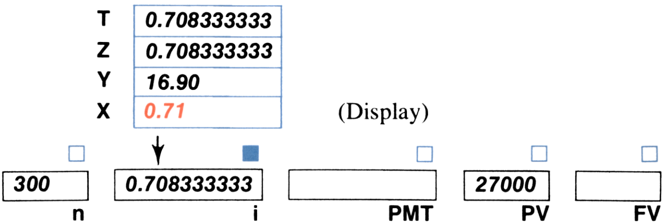
The five status indicators are reset by g RESET so the HP-27 is ready for a new problem. The values are still in the financial register—you can use them again or write over them with new values.

The i and PV values are the same as the original problem so we'll use that data over again. But how do you turn on the status indicators?

Press RCL f i to place that number in the display:

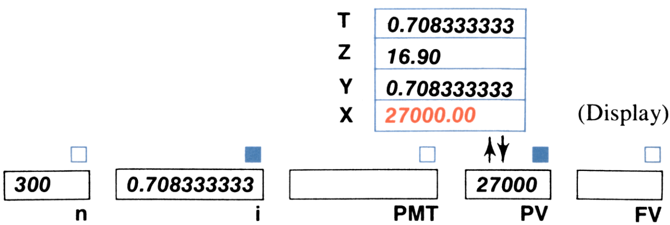


Then press [f] [i] again to transfer the number into the i register and turn on the status indicator:

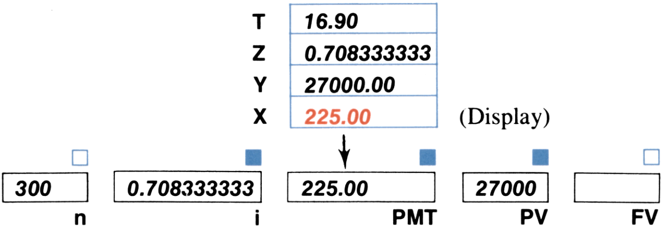


Herein lies one advantage of the [RESET] function: You don't have to calculate i again! (You may be tempted just to key in 0.71 [f] [i], but multiply that rounded-off figure over 25 years and your answer would be wrong.)

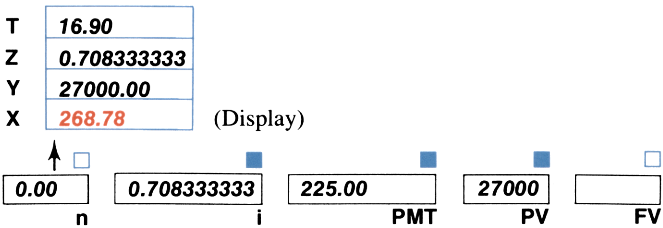
Press [RCL] [f] [PV] [f] [PV] to do the same for the PV value:



Turn on the third status indicator by inputting the new payment amount, 225 [f] [PMT].



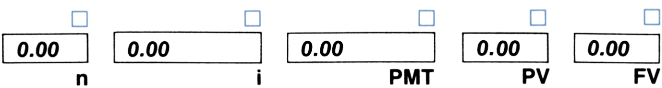
And solve for the length of the loan by pressing [f] [n]:



In summary, [g] [RESET] resets all five top-row status indicators so that the HP-27 is ready for a new financial problem. It does not erase the values in those registers.

The [f] [REG] Function

Pressing [f] [REG] clears the five financial registers, clears all 10 storage registers, and clears LAST X. It does not clear the stack.



T	16.90
Z	0.708333333
Y	27000.00
X	268.78 (Display)

7	8	9
0.00	0.00	0.00
4	**5**	**6**
0.00	0.00	0.00
1	**2**	**3**
0.00	0.00	0.00
0		
0.00		

Any data in the storage registers is eliminated.

The f Σ Function

Pressing f Σ clears the shared user-calculator registers 4 thru 9. These are the statistical registers, so you should press f Σ before starting any new statistical problem.

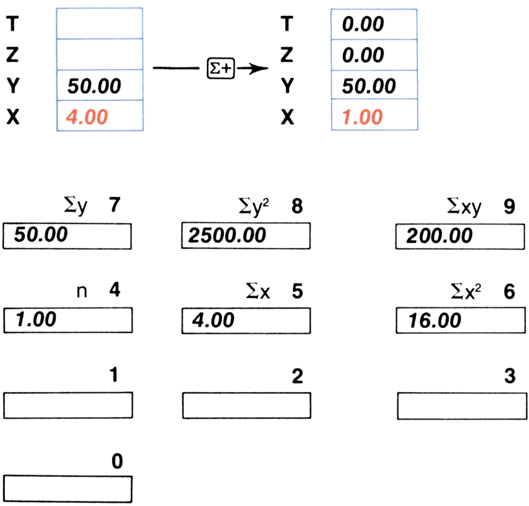
7	8	9
0.00	0.00	0.00
4	**5**	**6**
0.00	0.00	0.00
1	**2**	**3**
0		

The Statistical Registers

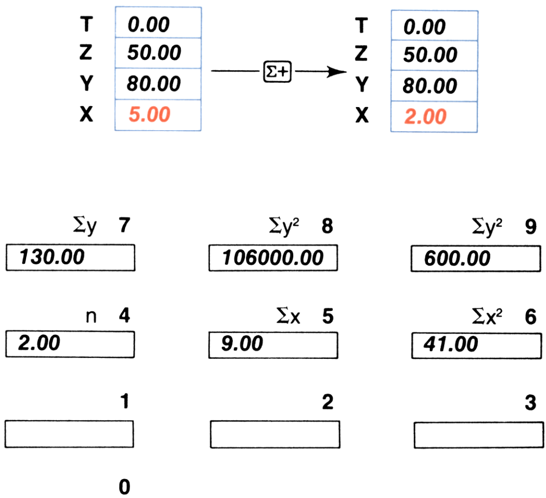
Pressing the [Σ+] key produces several different sums and products of values in the X- and Y-registers and stores this data in registers 4 thru 9:

Register	Contents
R_4	Number of entries (n).
R_5	Summation of x values (Σx).
R_6	Summation of x^2 values (Σx^2).
R_7	Summation of y values. (Σy).
R_8	Summation of y^2 values (Σy^2).
R_9	Summation of products of x and y values (Σxy).
Display (X-register)	Number of entries (n).

If you input 50 [ENTER↑] 4 [Σ+], the values are stored as follows:



If the next data pair were 80 ENTER↑ 5 Σ+, the contents of registers 4 thru 9 would change as follows:



Availability of Registers

Some functions utilize or overwrite other registers when solving a calculation. Any data stored in those registers will be lost, so you should be aware of this effect.

All statistical calculations overwrite the X-register or both the X- and Y-registers. The previous X value can be salvaged from LAST x.

When solving for ŷ, L.R. and r, the calculator uses the PV and FV registers as "scratchpads." After execution, PV and FV are cleared. (If an error occurs, PV and FV are not cleared.)

The calculation for iterative i uses registers R_7, R_8 and R_9. After execution, these three registers are cleared (unless an error occurred).

The IRR function uses the i, PMT and FV registers as "scratchpads." After execution or if an error occurs, these three financial registers are cleared.

Value Limits

The algorithms in the HP-27 can accommodate a wide range of values; however, the accuracy of your answer also depends on the values you key in (e.g., the "garbage in, garbage out" syndrome.)

A negative number for n will produce an answer, but one that's meaningless because you used a negative time period.

You cannot have a negative number of entries for statistical problems. That is, you cannot key three inputs into [Σ+], then subtract out four with [Σ-].

Iterative solutions for i are accurate to ±.0001%. Extreme input values cannot be used. For example, do not use interest rates smaller than i = −0.000001% or +0.000001%.

Negative interest rates close to −100% produce a meaningless answer or a continually blinking display. Do not use a value <−99.9% for iterative i or IRR. Examples of interest rates close to −100% are situations where you have a large PV but very small PMT, or where you have large payments that result in a small FV.

When solving for internal rate of return (IRR), the formula used is a polynomial equation. Therefore, a condition can be created such that more than one interest rate is considered correct in a mathematical sense. Your HP-27 calculates and displays one of these answers, but the calculator has no way of indicating when more than one answer exists or what the other values might be. This phenomenon may occur when there are multiple negative cash flows interspersed with positive cash flows, so caution should be exercised. Remember the answer displayed on your HP-27 is *a* correct answer; it may not be *the only* answer.

Appendix C

Formulas

The following formulas are preprogrammed in your HP-27 calculator.

Statistical Formulas

Mean

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{y} = \frac{\Sigma y}{n}$$

Standard Deviation

$$s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n - 1}}$$

Variance

$$VAR_x = (s_x)^2$$

$$VAR_y = (s_y)^2$$

Normal Distribution

$f(x)$ = density function
Q = upper tail area

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-x^2}{2}\right)}$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{\left(\frac{-t^2}{2}\right)} dt$$

Correlation Coefficient

$$r = \frac{\Sigma xy - \frac{1}{n}\Sigma x \Sigma y}{(n - 1)\, s_x\, s_y}$$

Linear Regression

$$y = A + Bx$$

where A = y intercept of the line
B = slope of the line

$$A = \frac{\Sigma y \Sigma x^2 - \Sigma x \Sigma xy}{n\Sigma x^2 - (\Sigma x)^2}$$

$$B = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

Linear Estimate

$$\hat{y} = A + Bx$$

Financial Formulas

Percentage

$$\% = \frac{\text{Base} \times \text{Rate}}{100}$$

$$\text{Percent Difference} = \Delta\% = \left(\frac{\text{New Amount} - \text{Base}}{\text{Base}}\right) \times 100$$

Unless otherwise stated:

n = number of time periods
i = periodic interest rate, expressed as a decimal value
PMT = periodic payment
PV = present value or principal
FV = future value
I = interest amount

Compound Amount

$$FV = PV\,(1 + i)^n$$

Savings (Annuity Due)

$$FV = PMT \left[\frac{(1 + i)^n - 1}{i} \right] (1 + i)$$

Rents, Leases (Annuity Due)

$$PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right] (1 + i)$$

Direct Reduction Loans (Ordinary Annuity)

$$PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

Direct Reduction Loan with Balloon Payment (Ordinary Annuity)

$$PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right] + BAL\,(1 + i)^{-n}$$

Sinking Fund (Ordinary Annuity)

$$FV = PMT \left[\frac{(1 + i)^n - 1}{i} \right]$$

Discounted Cash Flow Analysis

NPV = Net present value
INV = Initial investment
cf = cash flow

$$NPV = -INV + \frac{cf_1}{(1 + i)^1} + \frac{cf_2}{(1 + i)^2} + \ldots + \frac{cf_n}{(1 + i)^n}$$

Depreciation

L = asset's useful life expectancy
SBV = starting book value
SAL = salvage value
DEP_k = depreciation for year k
RBV_k = remaining book value at the end of year k
RDV_k = remaining depreciable value at end of year k

- Straight-Line Depreciation

$$DEP_k = \frac{SBV - SAL}{L}$$

$$RDV_k = RDV_{k-1} - DEP_k$$

- Sum-of-the-Digits Depreciation

$$DEP_k = \frac{2(L - k + 1)}{L(L + 1)} \times (SBV - SAL)$$

$$RDV_k = RDV_{k-1} - DEP_k$$

- Declining Balance Depreciation

F = declining balance factor

$$DEP_k = RBV_{k-1} \times \frac{F}{L}$$

$$RDV_k = RDV_{k-1} - DEP_k$$

Mathematical Formulas

Polar to Rectangular

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Rectangular to Polar

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \left(\frac{y}{x}\right)$$

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Ile de la Reunion
Tel: 21-13-75
Cable: ZOOM

RHODESIA

*Field Technical Sales
45 Kelvin Road North
P.O. Box 3458
Salisbury
Tel: 705231 (5 lines)
Telex: RH 4122

RWANDA

*Buromeca
R.C. Kigali 1228
B.P. 264 Kigoli
Rwanda

SAUDI ARABIA

*Modern Electronic Establishment (M.E.E.)
P.O. Box 1228
Jeddah
Tel: 27798/31173
Telex: 40035

M.E.E.
Riyadh
Tel: 62596/29269

M.E.E.
Al Khobar
Tel: 44678/44813

Riyadh House Establishment
Riyadh
Tel: 21741/27360

*Service

SINGAPORE

*Hewlett-Packard Singapore
(Pte.) Ltd.
Blk. 2, 6th Floor, Jalan
Bukit Merah
Redhill Industrial Estate
Alexandra P.O. Box 87,
Singapore 3
Tel: 633022
Telex: HPSG RS 21486
Cable: HEWPACK, Singapore

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*Hewlett-Packard South Africa
(Pty.), Ltd.
P.O. Box 31716,
Braamfontein Transvaal
Milnerton
30 DeBeer Street
Johannesburg
Street Delivery Zip Code: 2001
P.O. Box Delivery Zip Code: 2017
Tel: 725-2030, 725-2080,
725-2081
Telex: 0226 JH
Cable: HEWPACK Johannesburg

*Hewlett-Packard South Africa
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Breecastle House
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Cape Town
Street Delivery Zip Code: 8001
P.O. Box Delivery Zip Code: 8018
Tel: 2-6941/2/3
Telex: 0006 CT
Cable: HEWPACK Cape Town

*Hewlett-Packard South Africa
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641 Ridge Road, Durban
P.O. Box 37099
Overport, Natal
Street Delivery Zip Code: 4001
P.O. Box Delivery Zip Code: 4067
Tel: 88-6102
Telex: 67954
Cable: HEWPACK

Hewlett-Packard South Africa
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Sandton, Transvaal 2001
Tel: 802-1040/6

SYRIA

Sawah and Company
Damascus
Tel: 16367/19697

Suleiman Hilal el Mlawi
Damascus
Tel: 114663

TAHITI

*Metagraph
B.P. 1741
Papeete
Tahiti
Tel: 20/320, 29/979
Cable: METAGRAPH PAPEETE
Telex: SOMAC 033 F.P.

TAIWAN

*Hewlett-Packard Taiwan
39 Chung Hsiao West Road
Section 1
Overpass Insurance Corp. Bldg.
7th Floor
Taipei
Telex: TP824 HEWPACK
Cable: HEWPACK Taipei
Tel: 3819160, 3819161, 3819162

Hewlett-Packard Taiwan
Kaohsiung
Tel: 297319

THAILAND

*UNIMESA Co., Ltd.
Elsom Research Building
Bangjak Sukumvit Avenue
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Tel: 932387, 930338
Cable: UNIMESA Bangkok

*Service

TUNISIA

Societe Samos
Tunis
Tel: 284 355

TURKEY

Melodi Records
Istanbul
Tel: 442636

Talekom
Istanbul
Tel: 494040

UNITED ARAB EMIRATES

*Emitac Limited
P.O. Box 1641
Sharjah
Tel: Sharjah 22779
Dubai 25795
Telex: Sharjah 8033

YEMEN

A. Besse and Co. Yemen Ltd.
Sanaa
Tel: 2182/2342

ZAMBIA

*R.J. Tilbury (Zambia) Ltd.
P.O. Box 2792
Lusaka
Tel: 73793
Cable: ARJAYTEE, Lusaka

FOR AREAS NOT LISTED, CONTACT:

Hewlett-Packard Export
Trade Company
3200 Hillview Avenue
Palo Alto, California 94034
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TWX: 910-373-1260
Telex: 034-8300, 034-8493
Cable: HEWPACK Palo Alto

EUROPE

AUSTRIA

*Hewlett-Packard Ges. m.b.H.
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P.O. Box 7
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DENMARK

*Hewlett-Packard A/S
Datevej 52
DK-3460 Birkerod
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Cable: HEWPACK AS
Telex: 16640 hp as

Hewlett-Packard A/S
DK-8600 Silkeborg
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FINLAND

*Hewlett-Packard Oy
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P.O. Box 185
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*Service

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*Hewlett-Packard France
Quartier de Courtaboeuf
Boite Postale No. 6
F-91401 Orsay
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Cable: HEWPACK Orsay
Telex: 60048

Hewlett-Packard France
F-69130 Ecully
Tel: (78) 33 81 25/83 65 25

Hewlett-Packard France
F-31770 Colomiers
Tel: (61) 78 11 55

Hewlett-Packard France
F-13271 Aéroport de Marignane
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Hewlett-Packard France
F-35000 Rennes
Tel: (99) 36 33 21

Hewlett-Packard France
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Tel: (88) 35 23 20/21

GERMAN FEDERAL REPUBLIC

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D-6000 Frankfurt 56
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Hewlett-Packard GmbH
D-1000 Berlin 30
Tel: (030) 24 90 86

Hewlett-Packard GmbH
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Tel: (07031) 66 72 87

Hewlett-Packard GmbH
D-4000 Dusseldorf
Tel: (0211) 63 80 31/5

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Hewlett-Packard GmbH
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Hewlett-Packard GmbH
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ITALY

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1-10121 Turin
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1-95126 Catania
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SPAIN

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Telex: 23515 hpe

Hewlett-Packard Española S.A.
E. Seville
Tel: 64 44 54/58

Hewlett-Packard Española S.A.
E-Barcelona, 17
Tel: (3) 2036200-08 & 2044098/9

Hewlett-Packard Española S.A.
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*Hewlett-Packard Sverige AB
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*Service

*SWITZERLAND

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CH-8952 Schlieren-Zürich
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Telex: 53933 hpag ch

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Tel: 847178&9

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Hewlett-Packard Ltd.
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*Hewlett-Packard
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Tel: 221-79-71

*Service

EUROPEAN AREAS NOT LISTED, CONTACT:

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7, Rue du Bois-du-Lan
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Telex: 2 24 86

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Platia Kefallariou
GR-Kifissia-Athens
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8080429, 8018693
Telex: 21-6588
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P.O. Box 7
A-1205 Vienna
Tel: (0222) 33 66 06 to 09
Cable HEWPACK Vienna
Telex: 75923 hewpak a

Service Card

Refer to the appendix of your Owner's Handbook to diagnose a calculator malfunction. The warranty period for your calculator is one year from date of purchase. Unless **Proof Of Purchase** is enclosed (sales slip or validation) Hewlett-Packard will assume any unit over 12 months old is out of warranty. **Proof Of Purchase** will be returned with your calculator. Should service be required, please return your calculator, charger, batteries and this card protectively packaged to avoid in-transit damage. Such damage is not covered under warranty.

Inside the U.S.A.

Return items safely packaged directly to:

Hewlett-Packard
APD Service Department
P.O. Box 5000
Cupertino, Calif. 95014

We advise that you insure your calculator and use priority (AIR) mail for distances greater than 300 miles to minimize transit times. All units will be returned by fastest practical means.

Outside the U.S.A.

Where required please fill in the validation below and return your unit to the nearest designated Hewlett-Packard Sales and Service Office. Your warranty will be considered invalid if this completed card is not returned with the calculator.

Model No. ______________ Serial No. ______________

Date Received ______________

Invoice No./Delivery Note No. ______________

Sold by:

Service Information

Must be **completed** and **returned** with your calculator, charger and batteries.

Owner's Name — Date

Home Phone — Work Phone

Ship-to address for returning repaired calculator

Street Address

City — State — Zip

Describe Problem:

Model No. — **Serial No.**

Preferred method of payment for out of warranty repairs.

If not specified, unit will be returned C.O.D.

☐ BankAmericard ☐ Master Charge

Card No. — **Expiration Date**

Name appearing on credit card

☐ Purchase Order, Companies with established Hewlett-Packard credit only. (Include copy of Purchase Order with shipment)

P.O. Number

Authorized Signature

HEWLETT hp PACKARD

Calculator Catalog and Buying Guide Request Card

Primary Interest:

☐ *Scientific Calculators*

☐ *Business Calculators*

☐ *Both*

Valid in U.S. only

HEWLETT hp PACKARD

430A

A friend or associate might also want to know about Hewlett-Packard calculators. If you would like us to send him the Hewlett-Packard Calculator **Catalog and Buying Guide,** please mail his name and address on this postage paid Request Card.

Name ______________________

Title ______________________

Company ______________________

Street ______________________

City ____________ State ________ Zip ______

FIRST CLASS

Permit No.
232

Cupertino
California

BUSINESS REPLY MAIL

No postage stamp is necessary if mailed in United States

Postage will be paid by:

Hewlett-Packard
19310 Pruneridge Avenue
Cupertino, CA 95014

HP-27 Warranty Card

Please fill in and return this card within 10 days of receipt of calculator.

Your Name ____________ Title ____________ Street Address ____________

Date Received ____________ Serial Number ____________ City ____________ State ____________ Zip ____________

① Where Purchased:

11 Department Store	20 Direct Mail	50 From Employer
12 College Bookstore	30 HP Salesperson	99 Other ________
13 Other Retail Store	40 Received as gift	

② Source of Information Leading to Your Purchase:

10 Retail Store	40 Direct Mail Brochure	80 Professional Association
20 HP Owner	50 HP Salesperson	99 Other ________
31 Retail Store Advertisement	60 Previous Use	
32 HP Advertisement	70 Employer	

③ Areas of Application for Your HP-27:

01 Architecture	08 Engineering/Technical Management	15 Plant Operations
02 Aviation	09 Financial Management	16 Pollution Control
03 Capital Budgeting	10 Forecasting/Planning	17 Quality Control
04 Consulting	11 Investment Analysis	18 Research

04	Consulting	11	Investment Analysis	18	Research
05	Data Processing	12	Navigation/Guidance	19	Statistical Analysis
06	Design/Development	13	Nuclear Science	20	Surveying
07	Engineering	14	Physics	99	Other ________

(4) Category Best Describing Your Industry:

01	Aerospace	06	Computers	11	State or Local Gov't
02	Armed Services/DOD	07	Education	12	Student
03	Chemical Products	08	Federal Gov't	13	Transportation
04	Construction	09	Food Production	14	Utilities
05	Communications	10	Leasing/Renting	15	Other ________

(5) Number of Employees At Your Location:

01	1-10	02	11-50	03	51-100	04	101-1000	05	1001-10,000	06	over 10,000

If you are outside the United States:

- **Return this card in the enclosed warranty envelope.**
- **If no envelope, please mail this card to the nearest Hewlett-Packard Sales and Service Office.**

(Fold, moisten and seal to form mailing envelope.)

FIRST CLASS
Permit No.
232
Cupertino
California

BUSINESS REPLY MAIL

No postage stamp is necessary if mailed in United States

Postage will be paid by:

Hewlett-Packard
19310 Pruneridge Avenue
Cupertino, CA 95014

Useful Conversion Factors

The following factors are provided to 10 digits of accuracy where possible. Exact values are marked with an asterisk. For more complete information on conversion factors, refer to *Metric Practice Guide E380-74* by the American Society for Testing and Materials (ASTM).

Length

1 inch	=	25.4 millimeters*
1 foot	=	0.304 8 meter*
1 mile (statute)†	=	1.609 344 kilometers*
1 mile (nautical)†	=	1.852 kilometers*
1 mile (statute)†	=	1.150 779 448 miles (nautical)†

Area

1 square inch	=	6.451 6 square centimeters*
1 square foot	=	0.092 903 04 square meter*
1 acre	=	43 560 square feet
1 square mile†	=	640 acres

Volume

1 cubic inch	=	16.387 064 cubic centimeters*
1 cubic foot	=	0.028 316 847 cubic meter
1 ounce (fluid)†	=	29.573 529 56 cubic centimeters
1 ounce (fluid)†	=	0.029 573 530 liter
1 gallon (fluid)†	=	3.785 411 784 liters*

Mass

1 ounce (mass)	=	28.349 523 12 grams
1 pound (mass)	=	0.453 592 37 kilogram*
1 ton (short)	=	0.907 184 74 metric ton*

Energy

1 British thermal unit	=	1 055.055 853 joules
1 kilocalorie (mean)	=	4 190.02 joules
1 watt-hour	=	3 600 joules*

Force

1 ounce (force)	=	0.278 013 85 newton
1 pound (force)	=	4.448 221 615 newtons

Power

1 horsepower (electric)	=	746 watts*

Pressure

1 atmosphere	=	760 mm Hg at sea level
1 atmosphere	=	14.7 pounds per square inch
1 atmosphere	=	101 325 pascals

Temperature

Fahrenheit	=	1.8 Celsius + 32
Celsius	=	5/9(Fahrenheit − 32)
kelvin	=	Celsius + 273.15
kelvin	=	5/9 (Fahrenheit + 459.67)
kelvin	=	5/9 Rankine

† U.S. values chosen. * Exact values.

HEWLETT hp PACKARD

Sales and Service from 172 offices in 65 countries.
19310 Pruneridge Avenue, Cupertino, CA 95014

For additional Sales and Service Information Contact Your Local Hewlett-Packard Sales Office or Call 408/996-0100 (Ask for Calculator Customer Service).

00027-90001

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