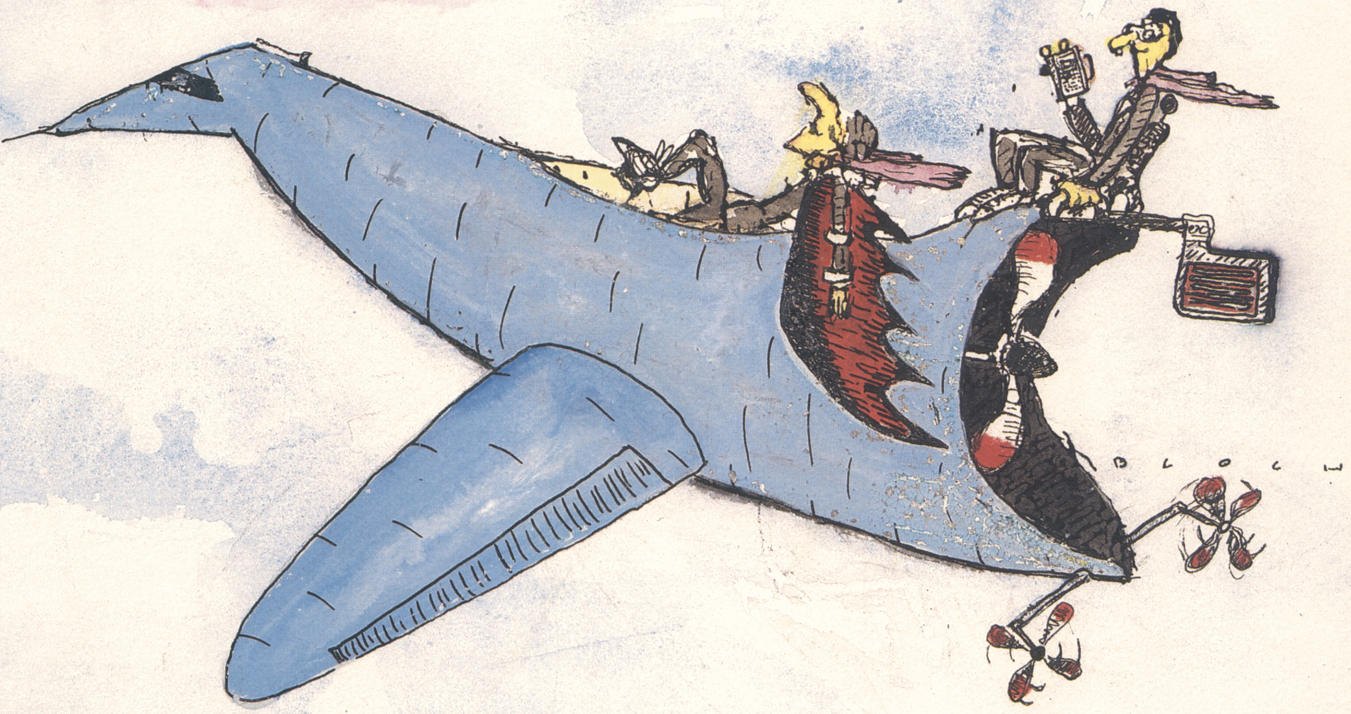
An Easy Course In

Using The HP-27S



A GRAPEVINE PUBLICATION

By Chris Coffin and John W. Loux

An Easy Course In Using The HP-27S

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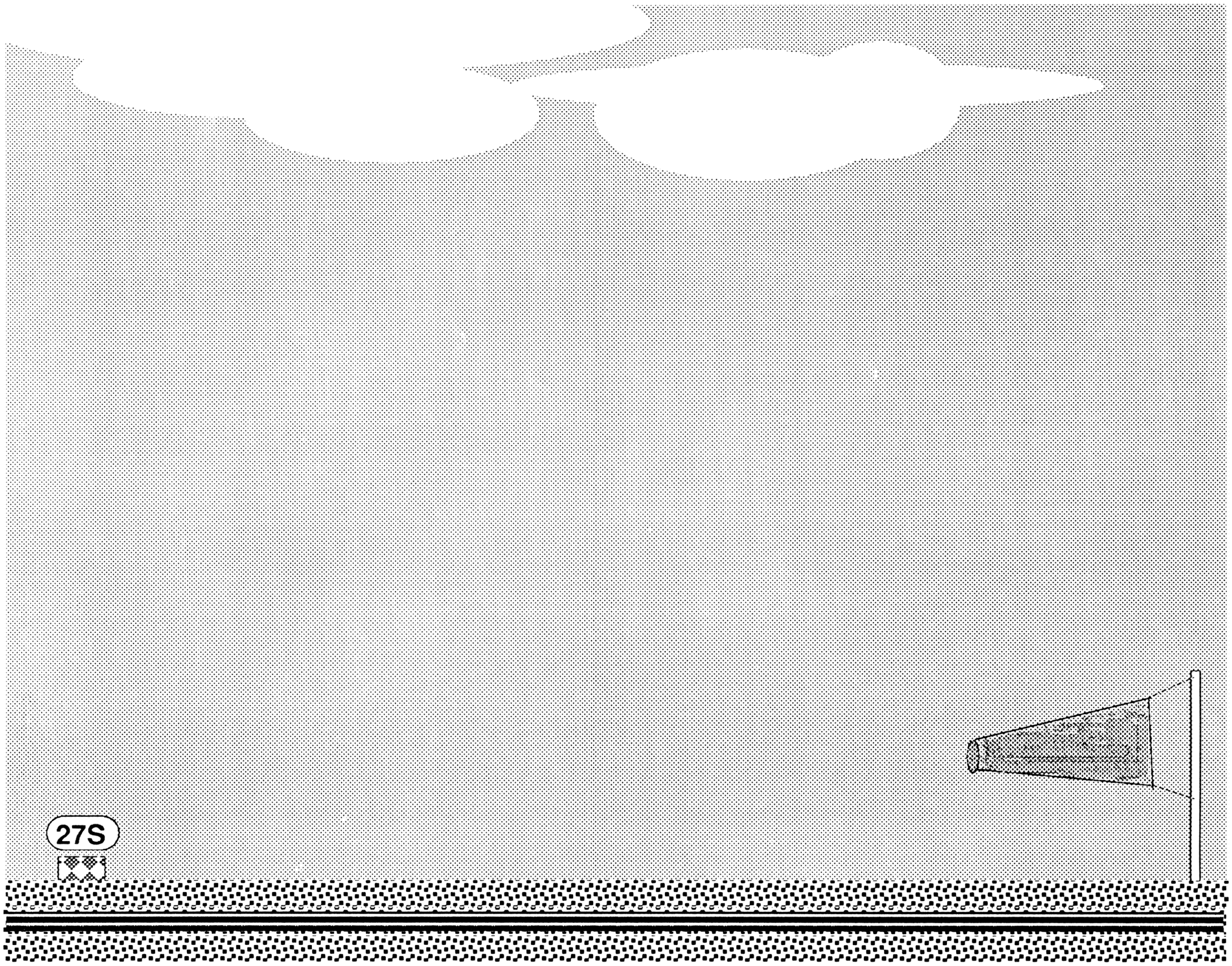


Start Here... from the ground level...looking up....

Why?

Because the hardest part about getting this machine to fly – in fact, the hardest part of learning *anything* new – is simply

*Getting Off The Ground**

*A fundamental principle discovered by the little-known Rong brothers, South Carolina, 1902

This means you've just finished the hardest part of this Course!

But it doesn't mean you're done yet. You've begun an engrossing calculation adventure simply by buying this book. Now comes the rest of the quest.

Naturally, you're in charge, but your Course will be carefully planned and charted by your Trusted Navigator – this book.

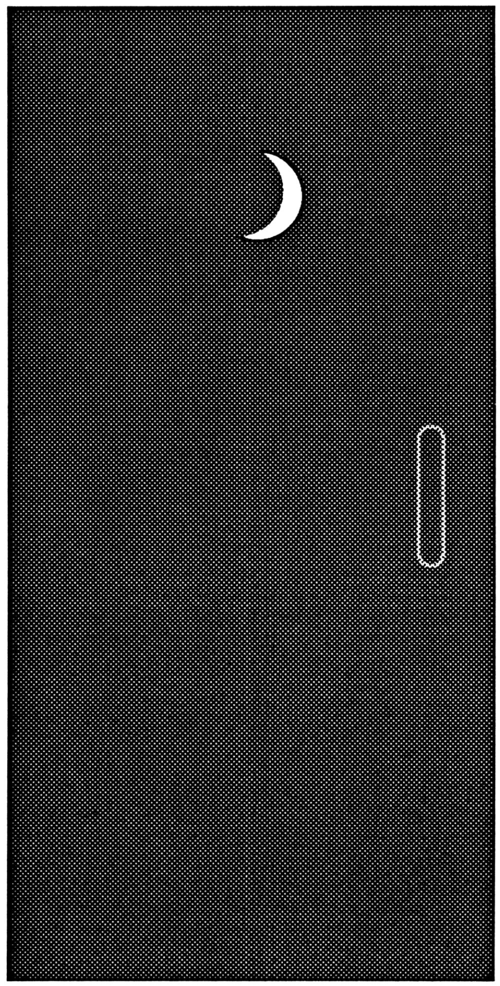
Follow it faithfully!

It's not a long journey – just a few good hours. But *do* it! The rewards will definitely be worth your while; when you finish, you'll know enough to solve problems that haven't even been invented yet!

How's that?

It's like this: The goal here is *not* just to leave you knowing how the buttons work or memorizing a handful of keystrokes and formulas. You'll also get a *fundamental understanding* of problem-solving, so that whenever you encounter a new problem, you'll be able to analyze it and solve it confidently.

Now then....



A Pre-Flight Briefing

What's In This Machine?

It's a loyal and friendly calculator engine – congratulations on a great choice!

Your HP-27S is one of a new generation of calculators, with more power and flexibility than ever before. It can do all sorts of things for you, from computing coordinates to analyzing investments to customizing formulas.

You just need to learn how to control it – which brings up the next question....

What's *Not* In This Machine?

There's no English-speaking person inside your calculator (one of these days, though...). For now, you still need to "speak" your machine's "language" in order to *translate your everyday calculation problems into forms it can understand.*

That's the whole mission right there.

Your part of the work is in *defining* the problem correctly for *yourself* and then *restating* it for your calculator. After all, only a human being is truly able to understand our human world (and even then it's tough sometimes)!

This translation skill isn't really very hard to learn. It's a lot like learning to fly an airplane. Everything seems strange at first, but after some training and practice, you'll be doing things quite proficiently – without even thinking much about them!

And that's where this book comes in....

What's In This Book?

This book is a sort of Pilots' School.

After learning about your craft's controls and buttons, you're going to take it out over the Course for extensive flight training.

Along the way, you'll encounter lots of explanations, diagrams, quizzes and answers, and once you successfully negotiate these, you'll be ready to fly anywhere your HP-27S can take you!

And why is the Course so Easy?

Because you choose your own airspeed and altitude. Don't worry about how fast or high you're flying; this isn't a competition with anybody – and you're allowed to re-fly the same route repeatedly until you're comfortable with it.

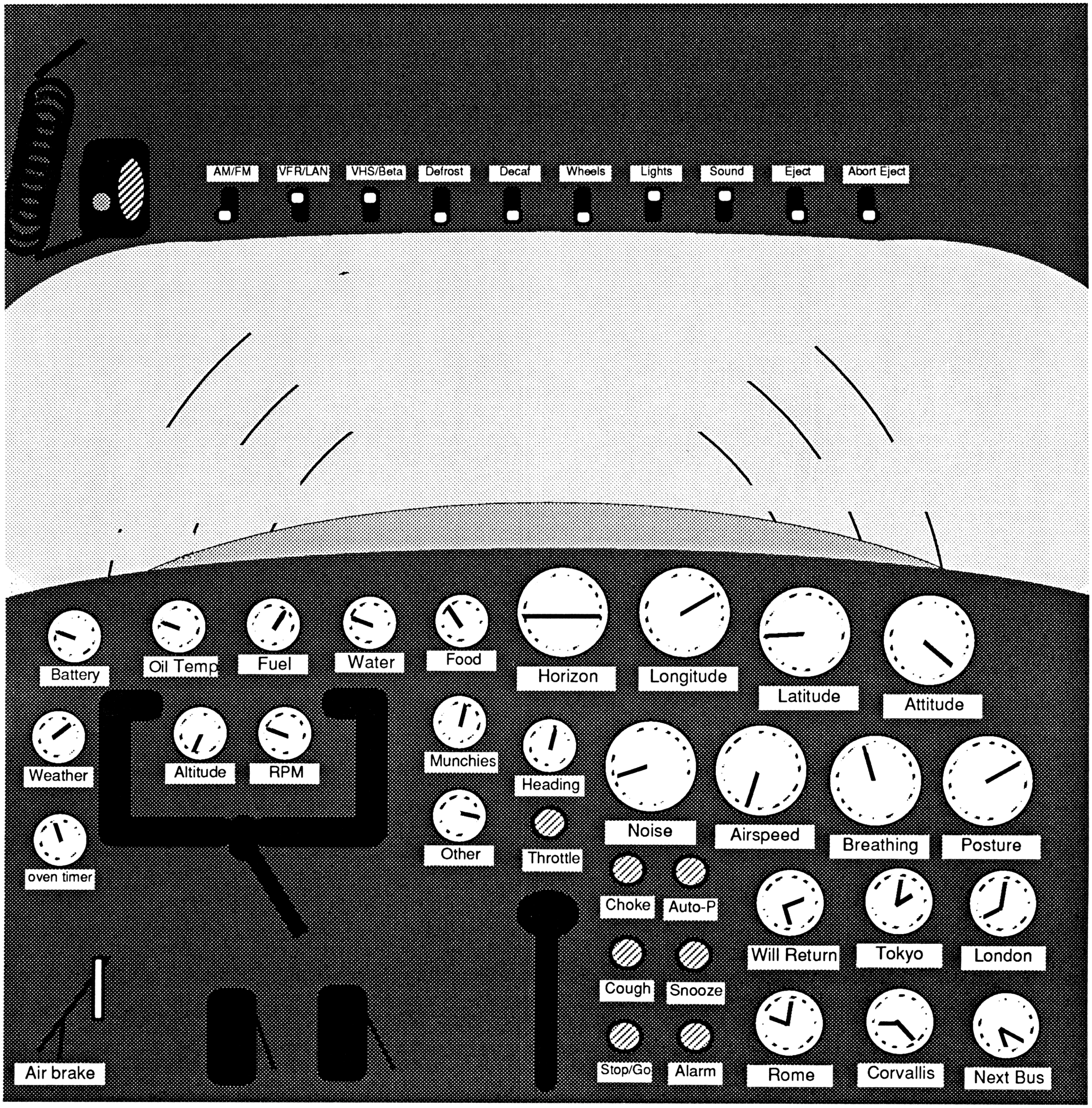
What's *Not* In This Book?

You won't find discussions of *everything,* because some parts of your HP-27S just don't need much explanation at all. Therefore, included here are just those things that *most* "pilots" tend to need some practice on – *most* of the time.

For example, you'll get an extensive look at three of the twelve menus on the HP-27S (SOLVE, STAT and TVM) – but a much shorter look at most of the other nine. This isn't because these three menus are any more important in the Great Scheme of Things; rather, it's because they're the most complicated to learn how to use correctly, so there are entire sections of the course to cover them. By contrast, the other menus tend to explain themselves after a few uses of them, so that's what you'll do here – just use them a few times – as a reminder that they're available if you want them.

Anyway, if you want a complete description of *everything,* you already have it right there in your Owner's Manual. That's what it's there for – as a reference manual to let you conveniently look up keys, functions, and examples.

This book you're now reading is *not* a reference manual; it's an entirely different approach, with a careful selection of topics that are meant to be taken in order. So start here at the beginning and stay on Course! Then, if this book has been worth its tuition, you'll seldom, if ever, need it all over again. Thereafter, a reference source (such as the manual that came with the machine) should be enough to keep your flying skills fresh and sharp.



Preparing For Take-Off

The [ON] Button

If you're looking at your HP-27S for the first time, it might well seem as complicated as the cockpit of a plane (well...almost). But that's just a first impression, one you'll quickly overcome as you take this Course. After all, nothing is all that mysterious – once you know all its parts.

So go ahead and get started – turn it on....

"How? There's no [ON] button!"

Actually, there is, but it's not labelled "ON" as you might expect. It's the [CLR] key at the extreme lower left – named such for a good reason, as you'll see shortly.

The [OFF] Button

Is it on? All right, now turn your calculator off once again, by pressing the [OFF] button....

"It looks like the [OFF] button is that same [CLR] key – true?"

Yes and no.

Yes, you do use the [CLR] key, but since the word OFF is written in gold above it, you need to press the blank, gold "shift" key (one row up) *before* pressing [OFF].

(If you're already quite familiar with the keyboard, the display, and the way menus work, then you can skip ahead now to page 28.)

A Good First Setting

Now, with your calculator turned on, what's in the display? That's hard to say, actually. Since your HP-27S has continuous memory, it will look now just as it looked when the dealers or factory technicians finished with it – or when you last used it.

So as a way to be sure you're starting with a clean slate, here's a procedure for resetting the machine.

Warning: You're only doing this now to be sure that you can follow along on this Course. This procedure completely wipes out all meaningful information that you or someone else may have stored in the machine. This is *not* something you want to do very often. There are plenty of other safer ways to selectively erase parts of its memory. *This* Reset is an Equal Opportunity Erasure!

Drum Roll, Please: Press and *hold down* the [CLR] key. While holding it down, press and *hold down* the far-left key in the top row. While holding *both* those keys down, *press and release* the far-right key in that same top row. Now release the top-left key. Now release the [CLR] key.

You'll hear the machine hiccup and you'll see this:

```
MEMORY LOST
```

(If this doesn't work the first time, just try again.)

The Display

There you see it, a machine with utter amnesia. But notice that even now, in its most empty state, it uses the display to tell you what's happening. So when in doubt, look at that display for clues.

(But if you're in no doubt now about the display, what it's telling you *and* how to adjust it, then skip ahead to page 20).

The Viewing Angle

Can you see and read the display comfortably? The LCD (Liquid Crystal Display) is hard to read if you're not looking at it from a certain angle, but you can adjust that angle!

Try It: With the machine still on, *press and hold down* the [CLR] key. Then press and hold the [+] or the [−] key and watch how the viewing angle varies. Go ahead and play with it until you find a comfortable angle.

Messages And How To Clear Them

That `MEMORY LOST` is a good example of a message – something spelled out in English – usually telling you of an error or asking you for something.

The thing to realize is that a message just *temporarily covers* part of the display. To get rid of it, all you need to do is press the [CLR] key. Do it now....

...OK?

The Calculator Line

Now your display should look like this:

It has two full lines, plus some space above. And, as you can see, when it's just showing numbers (and messages), it usually uses the lower line.

Try This: Press [1][2][3][•][4][5].

See how the number keys work? Whenever you're doing calculations, the line where the arithmetic actually appears in the display is called the *Calculator Line.*

What about the other line in the display? What's it used for – just extra space?

The Menu Line

Do This: Press the gold ("shift") key. Then press the [1] key.

What you've done is select the %CHG Menu (since %CHG was printed in gold above the [1] key).

Notice how the lower line in the display has now become the Menu Line, with the Calculator Line now above it. As you'll soon see, each of the selections noted in boxes on the Menu Line is associated with one of those [△] keys just beneath the display (which, of course, is why the Menu Line has to be the lower line of the display rather than the upper).

That Menu Line is one of the best ways to find out "where" your calculator is. For example, you're seeing this %CHG Menu right now:

Now press [EXIT] to get back to where you started from. This place, where you don't see *any* menu in the display, is your "home base" starting point – the MAIN calculating area. That's why, when you reset (entirely erased) the machine, it "woke up" showing you this MAIN area. Its memory was totally blank, and so it started from its "home base."

Makes sense, right?

The Keyboard

Of course, there's a lot more to this calculator than just the display. It's now time to look at all the keys (but if you already know the basics of the keyboard, go ahead and jump to page 23).

The Arithmetic Keys – The "Steering Controls"

Concentrate first on the white writing on the actual key-faces themselves. Notice how the digit keys and the four arithmetic functions are all conveniently placed together on the lower five rows of keys. You don't need to search around just to add 2 and 2.

But (as you'll soon see), also a part of those arithmetic keys are the [INPUT], [+/−], [(], and [)] keys – and the [←] for correcting mistakes (just in case you should ever make one).

And of course, there's the [CLR] key you've seen already.

But you can't get very far on this machine by just using the white-printed functions of the keys. In fact, just about *every* key on the machine has *two* meanings – white and gold, so it's time to give that gold "shift" key a closer look....

The ■ Key

Of course, you've already used this – that blank gold key on the lower left, which is indeed the "shift" key for the calculator. It's similar to a shift key on a typewriter (but, alas, since this book isn't in color, the shift key appears here as ■).

As you know, this gold ■ key is easy to remember, because any key's "shifted" meaning is generally printed in *gold* right above it. And just like a typewriter, to get the shifted version of any key, you must first press the shift key.

Try This: Press the ■ key. What do you notice? The ↑ *annunciator* appears (a little signal up there in the *Annunciator Area*), telling you that the next key you press will produce its *gold* operation.

Now press ■ again. The ↑ disappears, right? You've just discovered a *toggle key* – a key which alternates its meaning between two opposites in an on-again-off-again manner.

Keep these things in mind, also: *Unlike* a typewriter, you don't need to keep holding the ■ key down while pressing the key you're after. Just press and release the ■ key, *then* press the key you want. The ↑ will always tell you when the "shift" is in effect (and it's cancelled after every use. You need to re-press the ■ for every gold function you want; ■ is *not* a "Caps Lock").

You'll find that the ■ key is more than just a convenient way to cut down on the number of separate keys needed on the calculator. Often, the gold ("shifted") meaning of any key is closely related to its white ("unshifted") meaning.

For example, the [INPUT] key is for putting numbers into the machine, while its gold function, [CLEAR DATA], is for clearing them out again.

The Higher-Math Keys – For Fancier Maneuvers

Now that you can freely find any of the gold-printed functions on the keys, you'll find a *lot* more math "maneuvers" to do with this machine besides $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, and $\boxed{\div}$. Sure – you may not need them as often, but they're available nevertheless....

Look at the functions printed in *gold* on the uppermost three rows of keys.

Most of the common functions for trigonometry and exponentiation "live" here – just waiting for you to use them. And notice on the third row, the gold-printed functions, E (for scientific notation), %, and even π. Not bad, right?

(Don't worry – you'll get plenty of practice with all sorts of math, later on. This is only a "pre-flight" check so that you'll know your way around the controls.)

So what do you know so far? You know where to find the basic "steering" keys of this calculator – the math keys.

You know that for simple *arithmetic,* you need the *white*-printed functions on the *lower* half of the keyboard; and for *higher-math* functions, look for the *gold*-printed functions on the *upper* half of the keyboard.

Menus And Their Keys – Your Navigation Charts

Now reverse your focus: Look at the *gold* printing on the *lower* part of the keyboard and the *white* printing on the *upper* part.

These keys are mostly for *moving and selecting* the numbers and operations in your HP-27S – such as *moving* to the %CHG menu, which you've done already.

In fact, notice that this %CHG menu is one of the 12 different menus you can choose (and the keyboard even reminds you of this – with its little gold bracketing of the menus keys). You're going to see every one of those twelve during the course of this Course (of course).

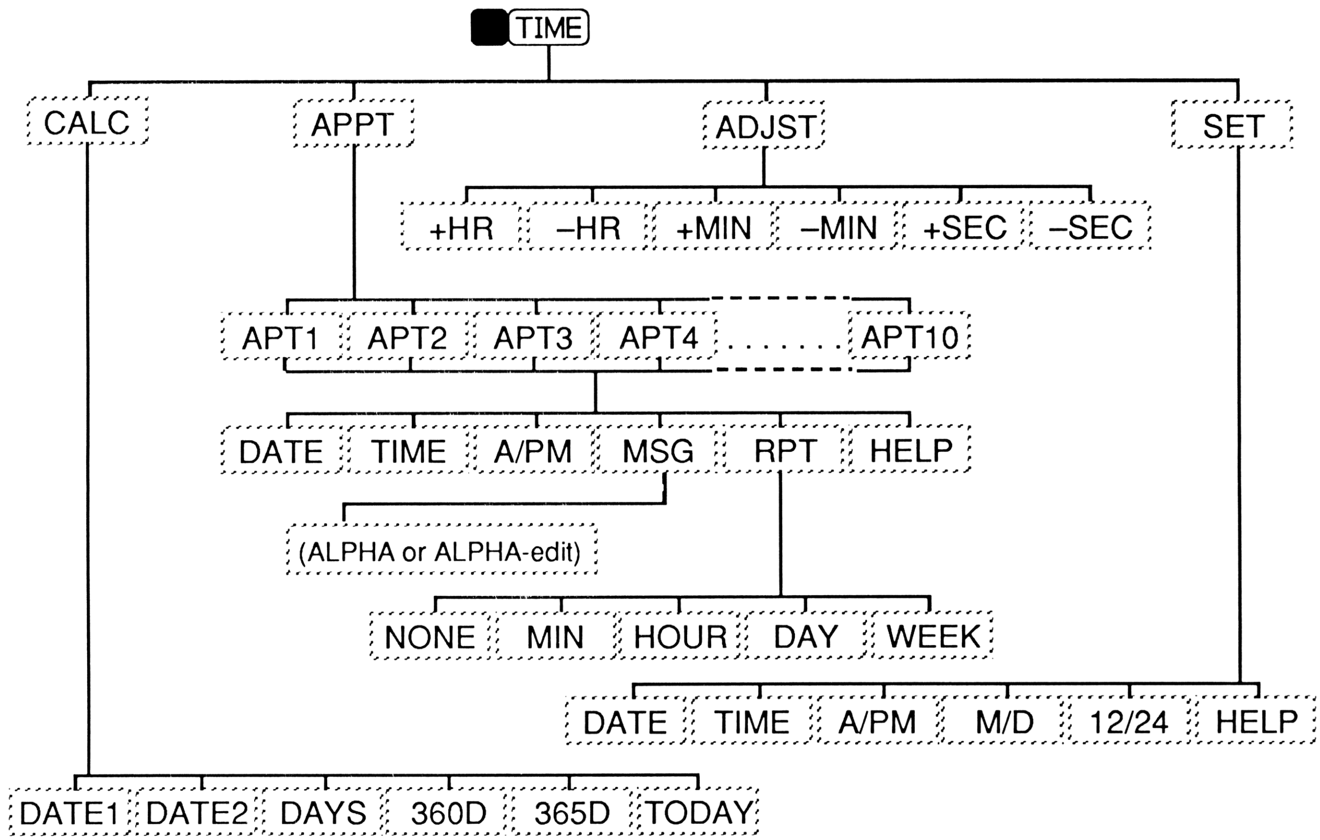
What's the idea behind all these menus?

Well, suppose you had a large atlas – set of charts and flight routes that you follow to get somewhere. Of course, no mapmaker tries to show *everything* on one map; the details would obscure the bigger picture. So the first chart might show a broad perspective – the whole continent or country. Then it would list down in its key or legend the different sections of the atlas you could turn to for a more detailed picture of any given *part* of the whole, right?

So it is with the menus in your HP-27S. You start from the MAIN area, which is like the biggest possible "picture" of the machine. From there, you can choose any menu, which shows you increasingly more detailed routes and destinations in any one part of the machine.

Simple, right?

A Sample Menu Chart: The TIME Menu



Try This: Look at the chart above. Of course, it isn't a complete chart of *all* the places you can take your machine; it's one "major route" (TIME) – and its "sub-routes." Use this chart to plot a quick practice trip to the "place" called **TODAY**, starting from the MAIN area.

Solution: Press TIME **CALC** **TODAY**. *Notice* that there's no key called **CALC** or **TODAY**. When you need to select from a menu in the display, *you press the key immediately under that selection.*

See what happens in this exercise? **TODAY** is a *calculation* – a destination where you get a result – (in this case it's today's date and day of the week – don't worry whether it's right or not; you'll fix that later). But some menu selections (such as **CALC**) are merely relay points to send you on toward your final destination.

Next Question: How do you get "home" again – back to your largest-scale chart – the MAIN calculating area?

Next Answer: You can either *retrace* through all the charts and routes you went through to get here ***or*** you can flip all the way back to the MAIN chart immediately....

To retrace your routes, use the [EXIT] key. Every time you press [EXIT], you retrace your route one "chart-scale" back upwards (recall how you [EXIT]ed the %CHG menu on page 19).

Thus, since you're now at the TIME CALCulations menu, you would press [EXIT] once to go from

here ⇒

to here ⇒

and then [EXIT] once more to get to the MAIN level once again!

To see how you can do the same thing *all at once,* first travel out to TODAY once again....

Now press ■[MAIN]. ■[MAIN] is the "When-All-Else-Fails-And-You-Can't-Figure-Out-Where-You-Are" key.

Pages Of The Menu

One thing you need to notice about these various menus: Sometimes there's just not enough room in one display for all your possible choices. The "cartographers" needed *more than one page* for certain charts, so the other pages are simply continuations of the first page – the same "scale" and everything.

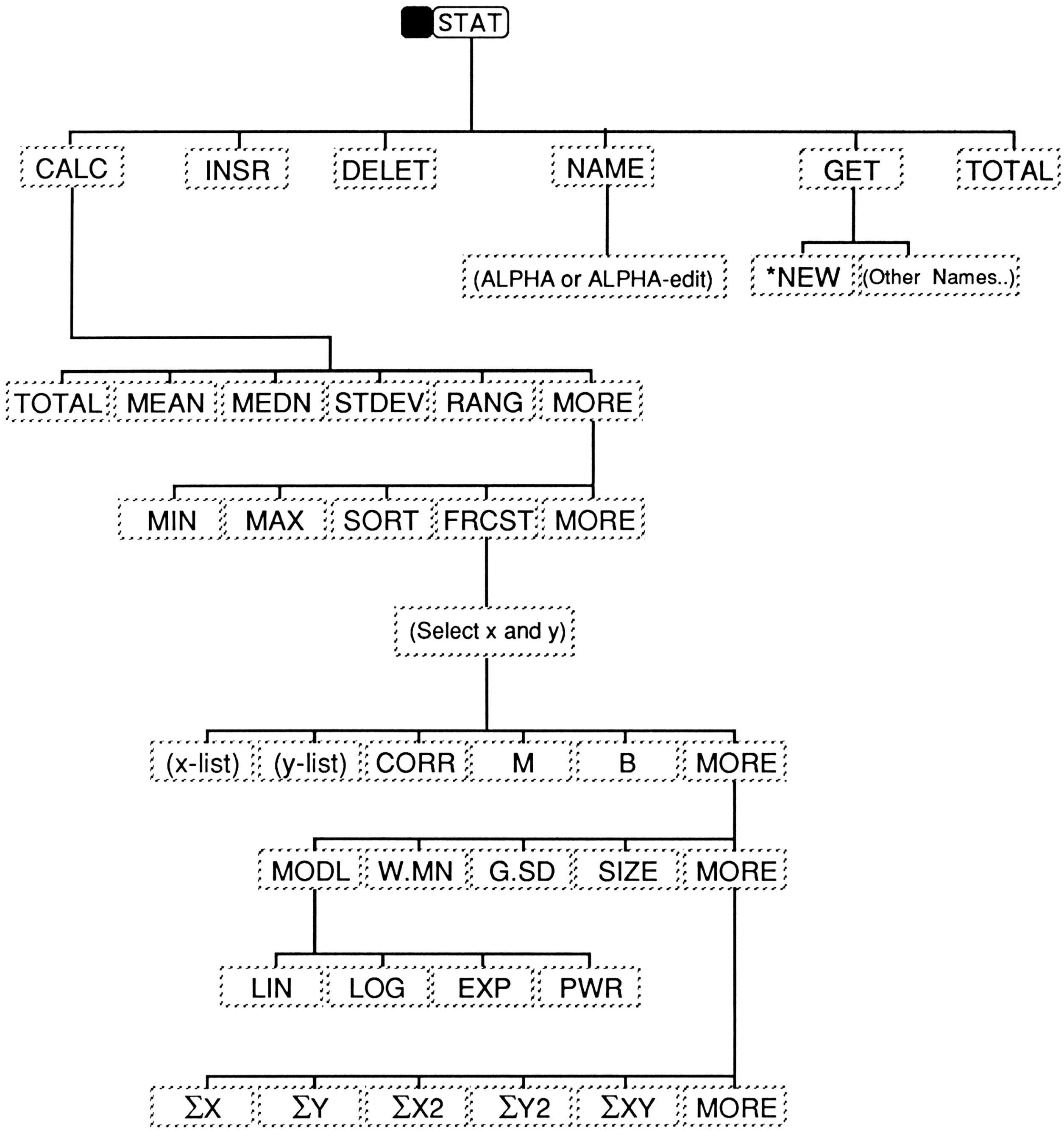
For Example: Look at the menu map on the opposite page. Now, starting from your MAIN (blank) menu, "move" through this map until your Menu Line looks like this:

MIN | MAX | SORT | FRCST | MORE

Solution: Press ■ STAT CALC, then MORE. That MORE is an additional item that appears on any menu with more than one page; it appears on *every* page of a multiple-page menu. OK?

So just remember these things about menus and "moving around" your HP-27S:

- When in doubt, look at your display! It will always orient you, showing you your current menu choices, error messages, instructions, etc.
- You can always get "Home" to the MAIN menu quickly, by pressing ■ MAIN.
- You can always retrace your steps "backward" (toward Home) with EXIT.
- Whenever there are more choices than places for them, you'll find another "page" for that menu, by choosing the MORE choice on the current "page."


STAT
CALC
INSR
DELET
NAME
GET
TOTAL
(ALPHA or ALPHA-edit)
*NEW
(Other Names..)
TOTAL
MEAN
MEDN
STDEV
RANG
MORE
MIN
MAX
SORT
FRCST
MORE
(Select x and y)
(x-list)
(y-list)
CORR
M
B
MORE
MODL
W.MN
G.SD
SIZE
MORE
LIN
LOG
EXP
PWR
ΣX
ΣY
ΣX2
ΣY2
ΣXY
MORE

Pop Quiz

Yes, things are moving pretty slowly here at first – but never fear, you're going to take off just as soon as you're ready. The idea here is to make *sure* you know these things before you start flying.

The answers are on the following pages, but don't look at them until you need to. Along with those answers you'll find a page number to refer you back for re-reading, just in case.

Above all, don't hurry; there's plenty of time, and who cares if you re-read? It's *your* Course!

1. What's a toggle key? Name one such key.

2. Name the two Lines of the display, and give their respective uses.

3. What's an annunciator?

4. What's a message? How do you get rid of it?

5. What does it mean to reset your calculator? When would you do this?

6. What would happen to your calculator if you forgot to turn it off before leaving for Tibet?

7. Using what you now know about menus (and referring to the ■TIME menu map on page 24, find the RePeaT selection menu that you would use to define APpointmenT number 7 (but don't bother to define anything; just locate this particular page of this particular menu), then report back to the MAIN menu As Soon As Possible (ASAP).

8. Figure out how to set the correct time and date in your calculator.

Pop Answers

1. A toggle key is like a light switch: hit it once to turn something on; hit it again to turn that something off. ■ is one such key (see page 21 for review).

2. They are the Calculator Line and the Menu Line, respectively. The Calculator Line is for calculating with numbers (and editing letters, as you'll see). The Menu Line tells "where" you are and offers your current choices of operations (pages 18-19).

3. An annunciator is a little signal, a status indicator that appears in the Annunciator Area, above the Calculator Line. Its usual function is to keep you informed of the current "doings" of the machine. You generally can't and shouldn't do anything about it – just understand what it means. For example, the shift annunciator, ⮥ means that any key with a gold operation written above it will now produce that gold operation (page 21).

4. A message is a phrase or question that temporarily *covers* what's on the Calculator Line, to notify you of an error or ask you to do something. When you want it to go away, just press CLR and see your display undisturbed once again (page 17).

5. By resetting the machine, you effectively erase all numbers, letters, formulas, and appointments that you may have stored in the machine. You could lose entire phone directories, years of statistics, months of data collection, and formulas that took weeks to develop. This is usually not too good an idea. Use reset as seldom as possible (page 16).

6. After about 10 minutes, it would turn itself off. Then, if you came back and turned it on, its display (and memory) would be the same as it had been when you left.

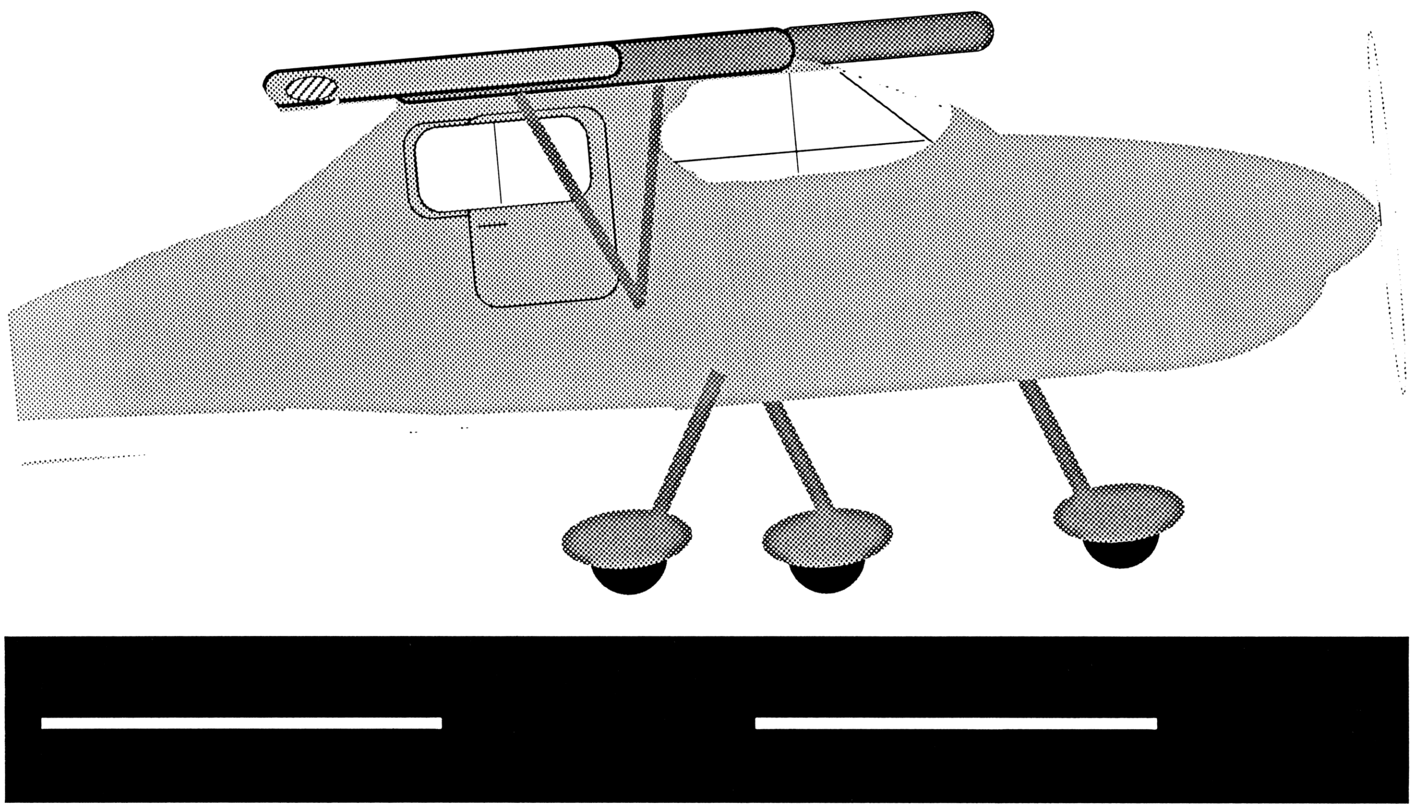
7. Follow along, if you need to, with the map on page 24: From the MAIN menu, press ■[TIME], then APPT MORE APT? RPT, and there you will find the proper menu (don't bother to use it right now).

   Notice that if you press [EXIT] [EXIT] at this point, you'll "back up" to the point where you selected which appointment to define. That menu is a multiple-page menu, where the MORE key will flip-flop you between its two pages (see page 26 if you don't remember this). Now press ■[MAIN] to "fly home" again.

8. From the Main Menu, press ■[TIME] (obviously), then SET (because you want to set the time and date). That brings you to another menu.

   Of course, if you merely press DATE or TIME on that menu, this probably won't set the correct time or date in the display. What do you do? You ask for HELP!

   See? It shows you how to key in the number to represent the correct time and date. For example, if today is March 14, 1988, and it's 2:45 p.m., you would press [3][•][1][4][1][9][8][8] DATE, then 2.45 (and A/PM if necessary) and TIME to get 2:45 in the afternoon. Now set your *correct* date and time, noticing how the information in the display changes as you do so. Play around with the other choices on this menu and see what you find.



Wheels Up

Basic Flight Maneuvers: "Number-Crunching"

It's time to master the basics.

Arithmetic is truly the engine of your calculator, because no matter what other options and accessories come with it, you'll always want a machine that can at least "crunch" numbers.*

As you might guess, most of this "number crunching" happens on the Calculator Line. So turn your machine on (if it's not already), point it down the runway, and watch that Calculator Line.

*At this point, if you're already quite comfortable with doing arithmetic on your HP-27S, including percentages, negative numbers, scientific notation and various display settings, then feel free to go on to page 51.

Clearing For Departure

First of all, start at the MAIN menu (where there's no menu visible in your display – remember how to get there with ■MAIN?).

Now, you'll notice that you're starting with some non-zero number *already* on the Calculator Line (it's probably the number you used to set the correct time – from the last problem of the previous quiz).

Do you need to clear this previous result before beginning a new calculation?

Nope.

For example, press 2 + 3 = and see the result: 5.0000
That previous number didn't mess anything up, did it?

Question: What if you now want to find 64 x 11? Do you need to clear away that 5.0000 somehow?

Answer: Nope. Again, just start your new problem, by pressing 6 4 × 1 1 =, and get 704.0000. And notice how the 5.0000 just moved up out of your way.

What's the idea here?

It's this: Anytime you finish a calculation (usually by pressing the = key), the machine knows it and will automatically bump that result out of the way when you begin a new calculation. You don't need to clear anything!

Fine.

But: What if you decide to scrap a calculation midway – when it's *not* complete? How do you clear the Calculator Line?

No Problem: Suppose you decide to abort the calculation 789÷5 just before pressing the [5] key. You would press [7][8][9] [÷] and then there's the point where you want to cancel the whole thing.

How?

Easy: *To clear the entire Calculator Line,* you press [CLR].

Anytime you truly wish to clear the entire Calculator Line, you just press [CLR].

But remember that this *isn't necessary* if you've just *completed* a calculation.

OK?

One More Thing: What if you simply make a little mistake while keying in an arithmetic problem – but you don't want to start all over again?

Suppose, for example, you want to find 123 + 465.

So you blithely rattle off [1][2][3] [+] [4][5][6] (oops).

Not To Worry: This is the main reason for that [←] key. It's a simple backspace key, so use it as such....

First, back out your erroneous digits: [←][←].

Now key them in correctly: [6][5]

And finish the problem: [=] (Answer: 588.0000)

Not too tough, eh? Now that you're cleared and ready to go,...

How Many Digits Do You Have?

Now off the runway and climbing well, you begin your flight by noticing that the display is showing exactly four digits past the decimal point on every number.

This isn't an accident; four decimal places is usually enough for most technical calculations, and the folks at HP *had* to decide on *some* such number for the HP-27S for it to "wake up" with after a total memory reset (remember page 16?).

But is this the best precision you can get?

Not at all.

In fact, *every* number has a total of 12 digits, *no matter how many you can see at the moment.*

So that number you're seeing on the Calculator Line right now, `588.0000`, is really

`588.0000`00000.

(Count 'em. There should be a total of 12 digits.)

You just aren't being shown anything beyond those first four zeros right now.

And Notice This: Key in [2][1][0][•][9][8][7][6][5][4][3][2][1] and press [INPUT] (this is another way to tell the machine you've "finished" with the number on the Calculator Line). What do you see?

210.9877

What gives? Why did the calculator change the number?

It didn't change it. It changed only its *presentation* (to you).

The entire **210.987654321** is *still in the machine,* but, as you know, the display has been instructed to *show* you only the first 4 places past the decimal point.

And it's only this edited version that is rounded like this. After all, if you're going to see only 4 decimal places, those digits ought to represent the *entire* number as accurately as possible, right?

So the display rounds a number like **210.98765**... *up* to **210.9877** (but if that **5** had been a **4** or smaller, the number would have been rounded *down* to **210.9876**).

Just keep in mind through all this that it's the *display* doing the editing for your eyes only. Your machine will always do all your arithmetic with 12 digits in each number.

The MODES Menu: How Many Digits Do You See?

Here's how your display probably looks right now:

```
588.0000
210.9877
```

Of course, you may want to tell the display to change its editing for you. What if you wanted to see 2 decimal places rather than 4?

Just Say The Word: Press the ■ MODES key (second row up from the bottom – this is another one of those twelve menus). Here's what you'll see:

```
SELECT DISPLAY FORMAT
 FIX | SCI | ENG | ALL | . /, | MORE
```

When in doubt, follow the display's directions: You want to FIX the number of decimal places to be 2. So press the ⌂ key under the box marked **FIX**.

Then there's another set of directions to follow. OK, choose 2 and press INPUT.

Voila!

Your display now shows this:

```
588.00
210.99
```

Just For Laughs: Set your display to show you ALL decimal places that aren't merely extra zeros.

Solution: Press ■(MODES), then ALL. Here's the result:

```
588
210.987654321
```

See? For each number, you get ALL the decimal places that aren't just extra zeros. You don't need to see any decimal places of the 588 to know all of its digits, but the 210.987654321 unveils its full splendor.

But What If: You want to merely *check* on the full precision of a number – just for a brief glimpse – not officially set the display there. How do you do this?

Piece O' Cake: See that (SHOW) key (the gold function over the (.) key)? Press ■(SHOW) and watch the display *temporarily* show you the full precision (equivalent to the ALL selection on the MODES menu, it omits trailing zeroes). And you can view this full precision as long as you want by holding down the (SHOW) key.

While you're in the neighborhood, take a look at what else that MODES Menu can do for you:

Try This: From the MAIN menu, key in 5,280 (press [5][2][8][0]). Now go to the MODES menu and press the **. /,** key.... Well, now!...

Bizarre malfunction? Not at all. This is just European numeric notation, where the *radix* (the decimal "point") becomes a comma, and the digit separators (every three digits) become periods. This notation is used conventionally in many countries of the world – quite a few more than those who use the U.S. system.

And notice this: That key you pressed to alter this radix notation, **. /,**, is another good example of a *toggle* key. That is, if you now select it again (please do so; your display won't follow along with the rest of this Course if you don't)...zap – you're back in the USA once again.

So whenever you want to adjust something about the display – or some other adjustable aspect of the machine – chances are, you'll be able to do so with that MODES menu.

Play around with it some more now, if you wish. And when you're through, please be sure to set a FIX 4 display, with U.S. radix notation....

Level Flight: Simple Arithmetic

Actually, you've already done some arithmetic problems, just to illustrate other things. But now's the time to make sure you're totally comfortable with arithmetic on the HP-27S. Here's a good set of examples. Remember that you can back out of a mistake or clear the Calculator Line entirely, right?

Example: Find 342 - 173 + 13

Solution: Press [3][4][2][−][1][7][3][+][1][3][=] Answer: **182.0000**

See? For just addition and subtraction, you just press the keys as you would say the problem to yourself – left to right. And notice that as you proceed, you automatically get intermediate results (do this problem again and watch what happens after you press the [+] key). Then the [=] gives your final result.

Yes, But: Find 101.00 - 47.50 x 2

Solution: Press [1][0][1][−][4][7][•][5][×][2][=] Answer: **6.0000**

Hmmm...is this right? Which operation were you supposed to do first – the subtraction or the multiplication?

It depends, of course, on the problem as it was stated – whether it *meant* (101.00 - 47.50) x 2 **or** 101.00 - (47.50 x 2). Looks like your HP-27S has made an assumption, hasn't it? *It doesn't always evaluate an arithmetic problem simply from left to right.*

What About: 342 - (173 + 13) ?

Solution: [3][4][2] [−] [(][1][7][3] [+] [1][3][)] [=] Answer: **156.0000**

Not much mystery here. You can always specify the exact order of evaluation in *any* kind of arithmetic problem by using parentheses. And notice how the **173+13** changes into **186.00** when you close the parentheses.

Now Try: 100.00 - 48.0000 x 2 ÷ 64 + 3 - 5 ÷ 11 x 7 + 4

Solution(?): [1][0][1] [−] [4][8] [×] [2] [÷] [6][4] [+] [3] [−] [5] [÷] [1][1] [×] [7] [+] [4] [=]

Answer (allegedly): **103.3182**

Incidentally, notice that you don't need to key in any of those trailing zeroes in the 100.00 or the 48.0000. The machine knows that any unspecified trailing decimal places are zero.

Again, with this last problem, you have The Question: Is the answer correct? Who knows? How can you even know what was *intended* as the correct answer?

Clearly, if you're not going to use a whole gob of parentheses, you need some *operator priority*. That is, when you have a chain calculation like this – with a mixture of different operations (here it's addition, subtraction, multiplication, and division), *you need to know which operations come first.*

Well, your HP-27S realizes this, too. And it comes all ready with its own set of assumptions....

Operator Priorities

Here's how your HP-27S interpreted that last problem:

You "said" this: 100 - 48 x 2 ÷ 64 + 3 - 5 ÷ 11 x 7 + 4

But it "heard" this: 100 - (48 x 2 ÷ 64) + 3 - (5 ÷ 11 x 7) + 4

Do you see what's happening? For the HP-27S, multiplication and division have a higher priority than addition and subtraction. Those "mental parentheses" the machine inserts into the chain problem are the result of this prioritizing.

Of course, as you can see from the terms inside the parentheses, when you have a part of the chain with operations all of the *same* priority (such as 48 x 2 ÷ 64), then you just go left to right, because mathematically, it doesn't matter; there's no ambiguity – no way to get more than one answer.

(Try it: no matter how you slice it, 48 x 2 ÷ 64 always gives you the same result – even if you change the order: 48 ÷ 64 x 2, etc.).

And – as you'll soon discover, there are other operations (things like exponentiation) that have a priority *higher* than multiplication and division.

Actually, there's nothing really new or revolutionary about this system of priorities. In fact, you might already be accustomed to using it yourself; it's really quite common. But just in case you're not, you'll want to pay close attention as this chapter continues, because these problems use real parentheses only when they're needed to circumvent the automatic "mental parenthesizing." OK?

All right, then, onward and upward – more arithmetic....

Changing The Sign Of A Number

Try This One: Find 34 x -19

Solution: Press [3][4] [×] [−][1][9] [=] Answer: **-646.0000**

That's the simplest way to key in a negative number: You just key it in as you would say it ("34 times minus 19 equals...").

But there's another way to do it, also.

Like This: Suppose you're not doing any new calculation. You just want to make that **-646.0000** into a (positive) **646.0000**.

Solution: Press the [+/−] key! That's the "change sign" key, and it's also for changing the sign of whatever you're working on in the Calculator Line. Notice that this [+/−] key is a toggle key – with alternating meanings.

So there are two ways to make a number negative, the [−] key and the [+/−] key.

Playing The Percentages

Do you realize how easy it is to do percentage calculations on this calculator? "Yes, fans, even these – everybody's *least* favorite problems – are a real breeze!"

Watch: What's 25% more than 134?

Solution: [1][3][4] [+] [2][5] ■ [%] [=]   Answer: 167.5000

See? Whenever you want to *increase or decrease* a number by some percentage, you just *add or subtract that percentage* – just as you would say it.

And how do you simply find a percentage of a number?

Example: What's 40% of 21.95?

Piece of Cake: Press [2][1][•][9][5] [×] [4][0] ■ [%] [=]   Answer: 8.7800

To *increase or decrease* by a certain percentage, you *add or subtract;* to simply *find* a percentage *of* some number, you *multiply.* And since you're using prioritized operations (addition/subtraction and multiplication/division) to express these percentages, you know the order in which the machine will evaluate those percentages in a chain calculation, right?

Try a few more problems on your own.... What could be easier?

That E Key

Try This: What's 2,000,000 x 2,000,000 ("two million times two million")?

Solution: [2][0][0][0][0][0][0] [×] [2][0][0][0][0][0][0] [=] Answer: **4.0000E12**

This is just a shorthand way of writing very large (or very small numbers, a notation called "scientific notation," since scientists often need to use such numbers.

You'd read this as "four-point-zero-zero times ten to the twelfth power." That E is a short way of saying " – times ten to the – "

Now notice that little gold E above the [+/–] key. It means that you can use scientific notation when you key numbers in, also.*

Go For It: Find "2 million times 2 million" once again – but this time, you're not allowed to press the [0] key.

Solution: Press [2] ■ [E] [6] [×] [2] ■ [E] [6] [=]

Saves a few [0]'s, doesn't it? Anyway, whether you like to use this shortcut or not, just be sure to recognize the E when your calculator needs to use it.

*And also, you can instruct your display to show you *every* number in SCIentific (or ENGineering) format – and with a given number of significant digits. To do this, use the MODES menu and SCI or ENG (but please don't set your machine that way while following along through this Course unless you're asked to; it makes all results look distractingly different).

The History Stack And The ■LAST Key

By now, you've probably wondered what happens to all your previous calculations after you've finished with them: Apparently, whenever you start your next problem, those previous results simply bump up out of the way.

What's *really* going on here, anyway?

Do This: Press 1 INPUT 2 INPUT 3 INPUT 4 INPUT.

You are "pretending" that you've just finished four problems, one after another (besides the = key, that INPUT key is the other common way to tell the machine that you're finished with the number on the Calculator Line).

Now notice the arrow keys, ▲ and ▼.

If you press them, you'll find that the four most recent results you've completed will "roll around," so that you can make any one of them the "most recent" again!

Try it! (With much research, your four most recent results were carefully planned so that you can watch the rolling more easily.)

Roll them back to the way they started (with 4.0000 on the Calculator line), then press 5 INPUT. Can you now roll *five* results around? Give it a shot.

Nope. This *History Stack* is good only for the last four. So now it has 5.0000, 4.0000, 3.0000, and 2.0000, from "bottom to top."

It is indeed a "Stack" of your recent calculations History, isn't it? And notice that it doesn't matter whether or not you can see all levels of that Stack. They're still there – all four of them. But what good are they?

Glad You Asked: On the last three flights you took in your own personal DC-3 (not really, just pretend), you logged the following air hours and fuel consumption totals:

Air Hours	Total Fuel Consumed (kg)
81.53	318,852.50
103.70	388,888.75
96.85	399,970.85

On which of these flights did you get the best "mileage" ("hourage?") – the least fuel per hour of flight?

Solution: You'll need 3 separate calculations, one for each trip....

For the first trip, press [3][1][8][8][5][2][•][5] [÷] [8][1][•][5][3] [=] (3,910.8610);

Second trip: Press [3][8][8][8][8][8][•][7][5] [÷] [1][0][3][•][7] [=] (3,750.1326).

Third trip: Press [3][9][9][9][7][0][•][8][5] [÷] [9][6][•][8][5] [=] (4,129.7971);

Just looking through the History Stack (pressing [▲] or [▼]) tells you that your second trip had the best "hourage!"

Not Only That: Suppose you now want to find out the *difference* between the fuel-per-hour usages in your best flight and your worst. How can you use your History Stack to do this?

How About This: If you start with the History Stack back in its original position from the previous problem, your display will look something like this:

```
3,750.1326
4,129.7971
```

You have your worst flight's data right where you want it – with your "best year" right above it (use ▼ and ▲ if you want to verify this). So press − to prepare to subtract...

Now press ■LAST (bottom row, second from right).

Presto! That ■LAST key brings a *copy* of the second level of the History Stack back to the Calculator Line. See how that works?

Now finish the subtraction (press =) to find the difference in hourly fuel usage: 379.6645.

So there you have it – a four-level History Stack that you can roll around, look at, and even recall (from the second level to the first) with the ■LAST key!

A Pause For The Cause

Look at all the stuff you now know about simple arithmetic and number-crunching on that Calculator Line:

- You know when and how to clear the Calculator Line;
- You know that the machine always keeps and uses 12 digits for each number in every arithmetic calculation;
- You know how to change the display setting (with FIX and ALL) to see however many decimal places you want (up to 11).
- You know that to do arithmetic problems on this machine, you must keep in mind the priorities of the various operations (x and ÷ before + and -);
- You know two different ways to work with negative numbers;
- You know how ridiculously easy it is to do percentage problems now;
- You know what 4.0000E12 means (and how to key it in, too);
- You know about the History Stack and the ■LAST key;
- You know that it's time to gain some more altitude – climb up and do some higher-math maneuvers (but if you feel you already know how to do exponentiation, circular and hyperbolic trig, and number base conversions, then try page 64)....

Exponentiation: Roots, Logs, And Powers

As you recall, the higher-math functions live mainly as gold-printed functions on the upper half of the keyboard. Look first, then, at the very top row of keys. See all those functions printed in gold?

Try One: Find $\dfrac{1 + \sqrt{5}}{2}$

Solution: Press [(] [1] [+] [5] ■[√x] [)] [÷] [2] [=] (or – alternatively – do it this way: [1] [÷] [2] [+] [5] ■[√x] [÷] [2] [=]).

<u>Answer:</u> **1.6180**

Notice how this ■[√x] function operates on the most recent number in the Calculator Line – in this case, the **5**. This is how most of the "x-functions" (functions with x in their names) generally work.

Try Another: Find $(34.19 + 1/11)^2$

Solution: Press [(] [3] [4] [·] [1] [9] [+] [1] [1] ■[1/x] [)] ■[x^2] [=]

<u>Answer:</u> **1,175.1807**

See? You reciprocated the 11 (flipped it over, to be 1/11) with the ■[1/x] key, then completed the parenthesized portion of the problem. That left only one number on the Calculator Line, which you then squared with the ■[x^2] key.

You're On A Roll: The LN function is the Natural Logarithm (base e); the LOG function is the common LOGarithm (base 10). *But* there's a formula to convert between the logarithm of *any* base, b, and the natural base:

$$LN(x) = \log_b(x) \times LN(b)$$

Show this is true for the common LOG base (i.e. b = 10): Find LN(100) by two different methods – once using the LN function directly, once indirectly.

Solution: First, the easy way: [1][0][0] ■[LN] (Answer: **4.6052**)
Then the indirect route: [1][0][0] ■[LOG] [×] [1][0] ■[LN] [=]

Good Questions: What's the easiest way to obtain the natural LN base, e (to twelve digits, of course) on the HP-27S? How about the same approximation for π? And how much is $\frac{e10^{\pi}}{\pi e^{10}}$?

Decent Answers: To get e, just compute e^1: [1] ■[e^x] (then press ■[SHOW] and hold it down, as needed, to peek at all 12 digits): **2.71828182846**

To get π, just press ■[π]. (**3.14159265359**)

As for that bizarre and hairy fraction, here goes:
Press [1] ■[e^x] [×] ■[π] ■[10^x] [÷] [(] ■[π] [×] [1][0] ■[e^x] [=]
Answer: **0.0544**, or **5.44242488548E-2**
(remember what this notation means?)

And Now: Find $(4.4^2)^5$.

Solution: The first part is easy; you've seen it before: Press [4][•][4] ■[x²]. But now, how do you raise a number to any power other than 2? Simple – you use the ■[yˣ] key:

You have 4.4^2 (which is **19.3600**) on the Calculator Line, so press ■[yˣ] to see the little exponentiation symbol, ^, appear. This means that you're going to raise the **19.3600** to whatever power you key in next.

So key in the final exponent, and finish the problem off: [5][=]
Answer: **2,719,736.0938**

Notice that the [yˣ] key works a little differently than the other gold-printed, higher-math keys. Most of them will *immediately* alter the current number on the Calculator Line. But the [yˣ] key behaves more like the [+] or [×] keys; *it acts as an operator, putting its own symbol on the line following the current number. Its execution is delayed until the machine can get some more information.*

Not only that, the ^ operator has a *higher priority* than even × and ÷. Remember the discussion of operator priorities (back on page 44)?

This means that when you have some chain calculation involving exponentiation and any other arithmetic operations (+, -, x, and ÷), the exponentiation will always come first (unless, of course, you specify otherwise with parentheses).

Watch: Find 4.25x1.10^12-7.19÷3.34^2.5+40.64

Solution: This problem is purposely stated with the ^ notation for exponentiation – to show you how confused your HP-27S would be without operator priorities. After all, it has to evaluate symbols all on the same line (e.g. 1.10^12); it doesn't know how to read the visual cue of superscripts (e.g. 1.10^{12}).

So, if you interpret this problem with proper HP-27S operator priorities (and all problems in this Course will assume this), then you can correctly key it in *without* any parentheses, and here's what that interpretation would effectively be:

(4.25x(1.10^12))-(7.19÷(3.34^2.5))+40.64

or, in more visual terms:

$$(4.25 \times 1.10^{12}) - (7.19 \div 3.34^{2.5}) + 40.64$$

The keystrokes themselves are easy, of course, since you need only key in the original problem as written:

[4][•][2][5] [×] [1][•][1][■][yˣ][1][2] [−] [7][•][1][9] [÷] [3][•][3][4][■][yˣ][2][•][5] [+]
[4][0][•][6][4] [=]

<u>Answer</u>: **53.6257**

Common Trigonometry

That about does it for the top row of math functions. Now move one row down and look at those functions (actually, you've already made a dent in exploring that second row – with [10^x] and [LOG]). It's time for some very basic trig...

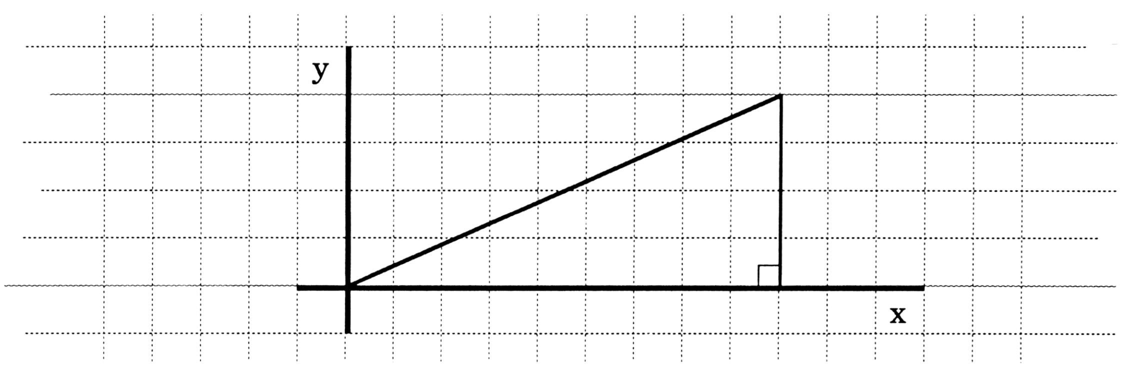
What would you do, for example, if you encountered a problem like this on some foggy, abandoned airstrip at four in the morning?

$$\text{ATAN}(\text{SIN}(22°) + \text{COS}(\pi/6) \div \text{TAN}(-125°))$$

You'd whip out this book to review the basics of trigonometry, wouldn't you?*

The word trigonometry itself means "the measure of the trigon." And if a pentagon is a five-sided figure, then a trigon must be...yep – a three-sided figure. So *trigonometry is "the measure of the triangle"* – the *right* triangle, actually.

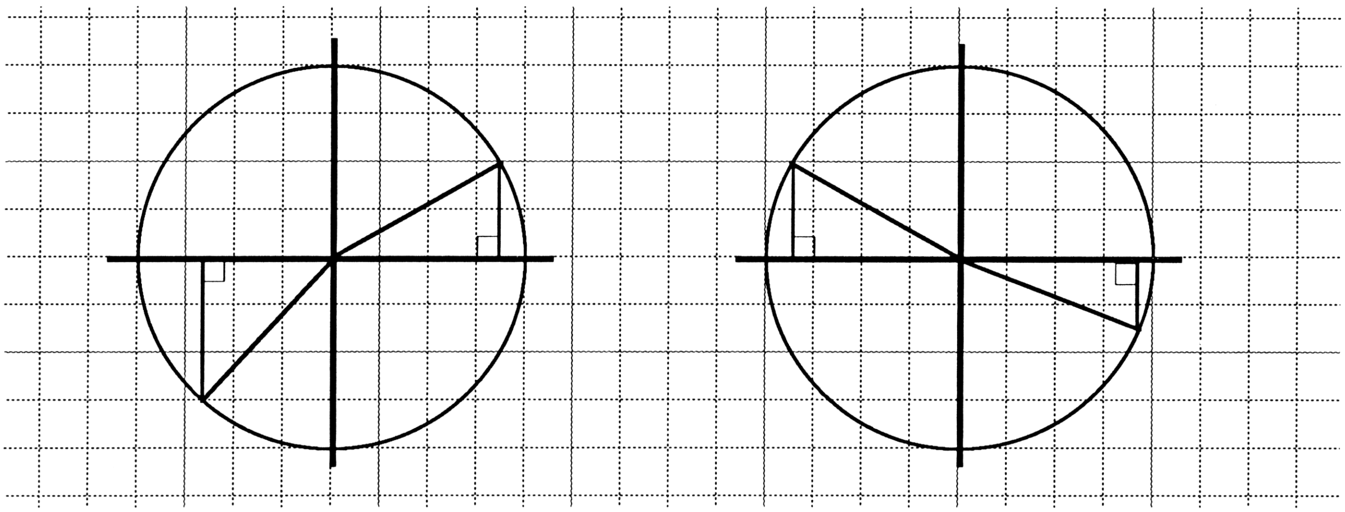
And to make trig clearer (and simpler), somebody decided a long time ago that when you're measuring such a triangle on graph paper (with x- and y- coordinates and all that), you should always draw the triangle with the long side (the "hypotenuse") extending out from the origin (P(0,0)) – like this:



Notice how you always place the right angle along the x-axis, too.

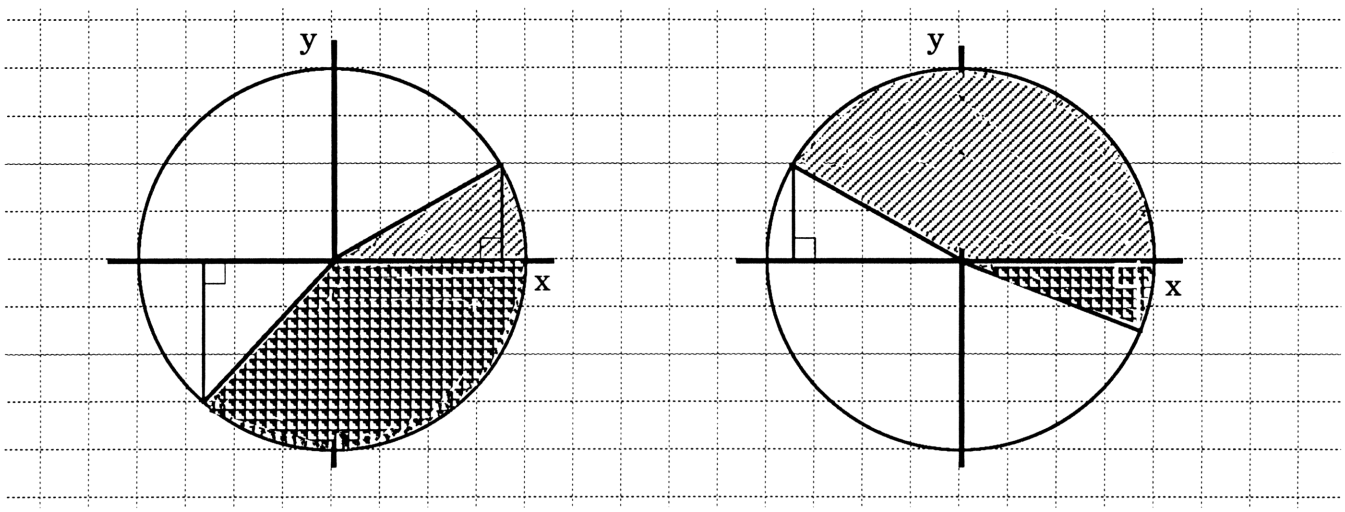
*Of course you would.

Well, the really clever part about all this is that if you draw a circle centered at the origin, then you get fascinating results by drawing any triangle whose hypotenuse forms a radius of this circle:



People discovered different functions that *mathematically described the lengths of (and the relationships between) the three sides of any of these triangles.*

It turned out that all you needed to know was *where* you had touched the circle with the hypotenuse of your triangle; you can tell all sorts of things about your triangle if you know only that. So the question becomes: how much arc is there on the circle between the the point you're touching and some reference starting point, say, the positive x-axis?



The whole trick of common trigonometry is to express these different functions and relationships within a right triangle in terms of the arc "swept out" along the circle by the hypotenuse of that triangle. It all boils down to that.

So how do you measure circular arc, anyway – in what units?

Well, they settled on a couple different ways:

In any circle, if you were to "wrap" copies of its *radius* along its perimeter, you could fit exactly 2π (≈6.28318530718) of them around the entire circle. So a convenient way to refer to circular arc is in a measurement of the length of the arc along the circle, expressed in units of its whole radius – called *radians.*

There are 2π *radians* of arc *length* in a circle.

But another way to measure arc is to split up the circle's swept angle into 360 equal pieces.

There are 360 *degrees* of *angle* in a circle.

You can measure circular arc either by the angle it sweeps out or by its length – as measured against that circle's radius.

And you need to tell your HP-27S which measurement you're using:

Press ■ (MODES) MORE, then D/R. See the RADians annunciator appear? When it's showing, the trig functions will assume radians measure; when the RAD is gone, the machine assumes degrees – and D/R is the toggle key for choosing between these 2 options.

Now Then: No kidding – find ATAN(SIN(22°) + COS(π/6) ÷ TAN(-125°))

Gulp: In this rather ugly problem, you have mixed units. When you see the little ° mark, you know that the arc is expressed in degrees; otherwise, it's meant as radians.

Another thing: From the problem statement, you can't tell in which of these units the ATAN should be finally calculated.

Finally: Keep in mind that ATAN(x) ("Arc-TANgent of x") is "the Angle whose TANgent is x." *And that could be any one of an infinite family of angles* who differ from one another by exact multiples of 2π radians (360°). The same is true for ACOS and ASIN. But the HP-27S makes the assumption that you're after the simplest of these angles (generally between -90° and 180°).

Now, knowing all *that,* how do you solve the problem? Like this:

Start in degrees mode (press ■ [MODES] **MORE** **D/R** [EXIT], to turn off the RAD annunciator, if necessary). Then press

[2] [2] [SIN] [+] [(]
■ [MODES] **MORE** **D/R** [EXIT] ■ [π] [÷] [6] [)] [COS] [÷]
■ [MODES] **MORE** **D/R** [EXIT] [−] [1] [2] [5] [TAN] [=] ■ [ATAN]

Answer: **44.4506** (degrees – 'cuz the annunciator ain't on)

Notice how these functions always act upon the most recent number in the Calculator line – just like [√x], [x²], [eˣ], etc.

Yes, But: What if you now want to see this result in radians?

Can Do: Notice (and press) the ■(CONVERT) key. The CONVERT menu allows you to actually change the number on the Calculator Line from one notation to the other. You'll notice that when you switched in and out of RAD mode, this affected only *subsequent* results – not the number then sitting on the Calculator Line.

By contrast, this CONVERT menu has actual functions to convert between different number formats and units.

To convert from degrees to RADians, just press >RAD and voila!
Answer: 0.7758

To convert back again, press >DEG. What could be simpler?

Get This: You can even convert to and from Degrees, Minutes and Seconds! That is, the 44.4506 degrees you see now means 44 degrees, plus 45.06/100ths of a degree – "decimal" degrees.

But press >HMS and bingo! You have 44.2702, which you read in HH.MMSS format (44 degrees, 27 minutes, 2 seconds). The reason it says HMS rather than DMS is that H stands for Hours (remember how to set the time on this machine? It's exactly the same format here). After all, hours and degrees behave identically: each has 60 minutes, and each minute has 60 seconds.

Now press >HR to convert this value back to decimal degrees.

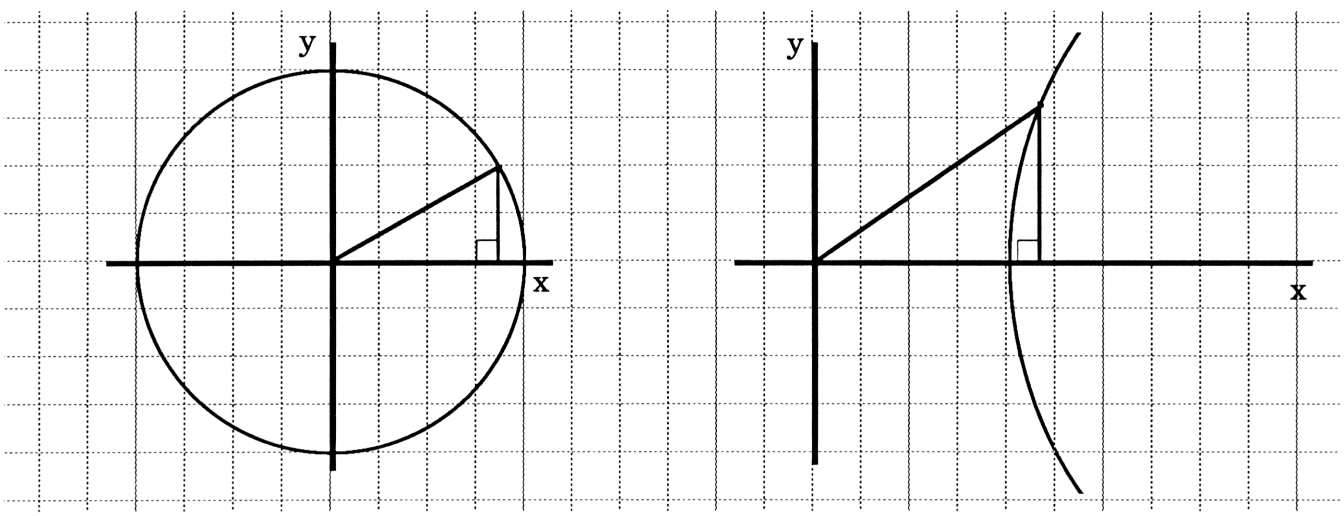
Uncommon Trigonometry: The HYP Menu

If you're like most people, you're probably more familiar with the common *circular* trig functions, SIN, COS, and TAN than you are with their hyperbolic counterparts, SINH, COSH, and TANH.

But each has its own niche and purpose in the Wild Kingdom of numbers, so as the second in this series of public-service reminders, here's a quick summary of the basic difference between circular and hyperbolic trig....

As you just read a couple of pages ago, circular trig is based upon drawing a right triangle whose hypotenuse extends from the origin to a point on a circle. Well, no prizes for guessing where the hypotenuse ends in hyperbolic trig.

Here's the side-by-side comparison:



(circle) (hyperbola)

Anyway, the bottom line in all of this is that when you press ■ HYP from the MAIN menu on your HP-27S, you'll suddenly have access to a complete set of hyperbolic trig functions – which you use in exactly the same manner as you use the circular set that appears on your keyboard. OK?

Of Odds And Ends: The PARTS menu

The PARTS menu is a collection of four miscellaneous functions that – like most functions – operate on the most recent number in the Calculator Line.

Try It: Press ■ PARTS and you'll see a menu that looks like this:

What are these functions and what do they do to a number?

- **IP** takes the Integer Portion of the number.
  For example, 3.14159265359 would become 3.00000000000.

- **FP** takes the Fractional Portion of the number. For example, 3.14159265359 would become 0.14159265359.

- **RND** RouNDs a number *to the current display setting.* For example, 3.14159265359 would *appear* as 3.1416 under a display setting of FIX 4. Upon pressing **RND** then, the actual, full-precision number would be *modified* to match the display's precision: 3.14160000000. This is the *only* way in which a display setting can permanently affect the actual value of a number.

- **ABS** takes the ABSolute value of a number.
  For example, a -2 would become a 2, but a 7 would remain unchanged.

The BASE menu

"But wait! – There's more...– still!" You can even do number base conversions on this machine. The HP-27S has a BASE menu, where you can see the (integer portions of) a number in any one of four number bases: 10 (DECimal), 16 (HEXadecimal), 8 (OCTal), and 2 (BINary).

Watch: What's e^{π} expressed in decimal, hexadecimal, octal, and binary?

Solution: Press [1] ■[e^x] ■[y^x] ■[π] [=] Answer: 23.1407

■[BASE] DECimal Answer: Same as above.

Remember: There's no hiding of the fractional portion when you're in DECimal format – no need.

Press **HEX**. HEXadecimal Answer: 17

Notice how the menu changes in HEX mode: You're given a way to key in the other possible hex digits (A-F) – in case you want to do some arithmetic rather than just view a result. So to get back to the BASE menu, press [EXIT].

Press **OCT**. OCTal Answer: 27

Press **BIN**. BINary Answer: 10111

Remember – this is only the display doing this. If you [EXIT] or return to DEC format, the entire number will be there – fractional digits and all.

That covers most of the numerical "maneuvers" you might want to do while flying your HP-27S.

Of course, there's plenty more to come, but this is a good spot to level out and review in your mind what you've seen so far:

- You know how to: clear or correct mistakes, recall previous results with the History Stack or the LAST key, and set the display to your liking;

- You know that to do arithmetic problems on this machine, you must keep in mind the priorities of the various operations (^ comes before x and ÷ , which, in turn, come before + and -);

- You know how to work with negative numbers, percentages, and scientific notation;

- You know how to compute logarithms, powers, roots, exponentiation;

- You got a quick refresher in the fundamentals of circular and hyperbolic trigonometry – and you know how the CONVERT menu lets you change the units of angles (and hours) used by the machine;

- You even took a whirlwind tour of the PARTS and BASE menus.

Well, then, it's time to put it all together…

…and take your Basic Maneuvers Qualifying Exam.…

B. M. Q. E.

Solve these on your HP-27S – as efficiently and joyfully as you now know how. As usual, the solutions are on the next pages, with page references for review items (and you might see some new variations here, too):

1. Find $\dfrac{100 \div 75}{25 \times 64 + 34 \times -19}$

   Then find $\dfrac{25 \times 64 + 34 \times -19}{100 \div 75}$ by two different methods.

2. Which is greater: $4 \div 7$ or $6.285 \div 11$? What's the difference?

3. **4.0000E12** is "4 trillion" (a very large number).

   But how much is **4.0000E-12**? How would you key this in?

4. This year's salmon run is 35% more than last year's, but next year's will be only 85% of this year's. If last year's peak count through the fish ladder was about 2500 per hour, what will next year's count be?

5. Find -19 x -19. Find it 3 different ways, two of which *don't* use the $\boxed{\times}$ key.

6. Find $\sqrt{4096}$ two different ways. Then find $\sqrt[12]{4096}$.

7. Find $((1 - 1.1^{-10}) \div .1 + 100) \times 1.1^{10}$

8. Show that $\cos^2\theta + \sin^2\theta = 1$ for $\theta = .25\pi + 1$ radians and -22.5°
   Show that $\cosh^2\theta - \sinh^2\theta = 1$ for $\theta = 4.4$

   Which of the circular arc unit denotations (radians or degrees) are applicable to hyperbolic trig? What does common sense – with help from the diagram on page 61 – tell you? Test this common sense by *twice* computing the ASiNH or ACOSH of some randomly chosen number – once while in degrees mode, once while in RADians mode. What's the difference in results?

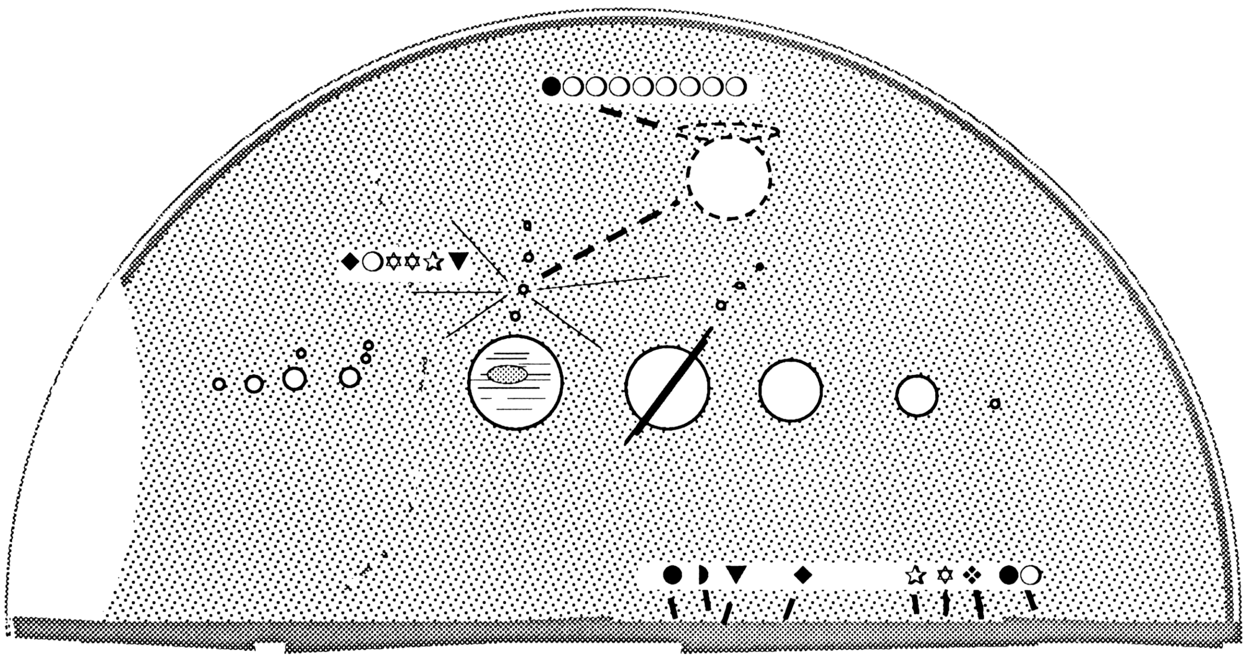
9. Compute $\sin(\pi/2)$, $\sin(\pi)$, $\cos(\pi/2)$, and $\cos(\pi)$.

10. Prove that 23.4567° is the same as 23°27'24.12". Do this twice – once with the CONVERT menu and once "the old fashioned way."

11. You're piloting your rebuilt F-104 on a non-stop, 25,000-mile, around-the-world flight (that new carburetor sure does improve the gas mileage, eh?). Flying at 130,000 feet due westward into a constantly-setting sun, you see, in the dark blue above you, a meteorite appear, glowing and growing, as it slowly approaches on a *near-parallel* collision course with your aircraft – like a merging vehicle on a supersonic freeway.

    Fascinated by the improbability of it all, you hold your course until the near-molten object virtually touches your canopy. Suddenly, your slipstream disturbs it, sending it slipping and tumbling to the rear, where it's cut apart by the onrushing tail of your jet. Part of the meteorite falls away, but part of it fuses onto the tail, cooling and remaining there for the rest of the trip.

Unaware of this, you complete of your journey and notice the fused fragment only upon routine inspection afterward. You cut it free and examine it. It resembles a small discus, cut neatly in two – with one half missing. It's very heavy – about 64 pounds – being composed mainly of iron, but with a mysterious outer coating, upon which you see some pictures and symbols etched....



You need the other half of the discus! Where would it have hit the Earth? Losing no time, you look up some thumbnail formulae for the free fall of a comparable (same weight and minimum profile) smooth sphere through atmosphere (those'll have to do; there's nothing listed for a red-hot half-discus):

$y_{feet} = y_0 - 256t - 1100$ for $t > 8$ seconds

$x_{feet} = x_0 + v_0 t - (6400/w)t^{4/3}$ (v_0 is the initial velocity in feet per second; w is the weight of the object in pounds)

You estimate your position at the time of the collision to be about 65 miles east of Isla Isabela in the Galapagos Islands (0° latitude, 90°44' W. longitude). Can you estimate the approximate longitude and latitude coordinates to begin the search to find the other half of the discus? Can you then decipher the message?

B. M. Q. E. Solutions

1. Press [1][0][0] [÷] [7][5] [÷] [(][2][5] [×] [6][4] [+] [3][4] [×] [−][1][9] [=]
   Answer (use ■[SHOW] to see this): **1.39762403913E-3**

   The easy way to do the second part is to notice that it's the "flip" of the first, which you can find by using the ■[1/x] key. Answer: **715.5000**

   Of course, the other way is to start all over from scratch: Press [(][2][5] [×] [6][4] [+] [3][4] [×] [−][1][9][)] [÷] [(][1][0][0] [÷] [7][5] [=] (to review your basic arithmetic, flip back and re-read pages 42-45).

2. To compare the two, you need to find both answers: Press [4] [÷] [7] [=] and [6][•][2][8][5] [÷] [1][1] [=]. Now compare the two answers as they sit at the first and second levels of your History Stack. If you're looking only at the first four decimal places, they'll look identical (**0.5714**). But now set the display to show you ALL of the relevant digits, and you'll see the difference (see pages 39-41 to brush up on display settings).

   Then, to compute this difference, just subtract: [−] ■[LAST] [=] (remember how that ■[LAST] key works? See pages 48-50). Answer: **-6.4935065E-5**

3. This is "four times ten to the minus twelfth power," or "four trillionths." Written out fully, it would be 0.000000000004. To key this in, you would press [4]■[E][−][1][2] (see page 47 for a re-read of **E**, if you want).

4. Say to yourself: "2,500 plus 35%, times 85%." Now press keys as you say it: [(][2][5][0][0] [+] [3][5]■[%][)] [×] [8][5]■[%] [=] Answer: **2,868.75** – nearly three thousand fish per hour (see page 46 to review % calculations).

5. First way: [1][9][+/−] [×] [−][1][9] [=]. Second way: [1][9][+/−] ■[x²]; Third way: [1][9][+/−] ■[yˣ][2] [=]. (In all cases, you get the same result: **361**)

6. First way: [4][0][9][6] ■[√x]. Second way: [4][0][9][6] ■[yˣ][•][5] [=]. The square root is the one-half (.5) power (result either way: **64**). Similarly, $\sqrt[12]{4096}$ is $4096^{1/12}$. So: [4][0][9][6] ■[yˣ][1][2] ■[1/x] [=] Answer: **2**

7. Press [(][(][1] [−] [1][•][1] ■[yˣ][−][1][0][)] [÷] [•][1] [+] [1][0][0][)] [×] [1][•][1] ■[yˣ][1][0] [=]
Answer: **275.311670612** (see page 44 to review operator priorities).

8. If necessary, press ■[MODES] **MORE** **D/R** to go to RADians mode (annunciator now showing). Then [EXIT] and [•][2][5] [×] ■[π] [+] [1] [=]. Now make another copy of this result (leaving the first one to loiter in the History Stack): [0] [+] ■[LAST] [=]. Then: [COS] ■[x²] [+] ■[LAST] [SIN] ■[x²] [=] Answer: **1** No need to rewrite math history yet (but review trig on pages 56-61, if you want).

Now try the other one: Press ■[MODES] **MORE** **D/R** for degrees mode (no annunciator showing). Then [EXIT] and [2][2][•][5][+/−] [COS] ■[x²] [+] [−][2][2][•][5] [SIN] ■[x²] [=] Answer: **1** What a relief. Now what about the hyperbolic identity?

Press ■[HYP], then [4][•][4] **COSH** ■[x²] [−] [4][•][4] **SINH** ■[x²] [=] Answer: **1**
Now pick any number – your birth date, say: If you were born on March 18, 1988 (awfully precocious, aren't you?), key in [3][•][1][8][1][9][8][8] and press **ACOSH** Result: **1.82499393149** The ACOSH of 3.181988, in "degrees." Now switch the RAD mode on (■[MODES] **MORE** **D/R** [EXIT]), and try your birth date again: [3][•][1][8][1][9][8][8] **ACOSH** *Same Result:* **1.82499393149**

Bottom Line: For hyperbolic trig, you must specify arc length in "radians" (they're not radii, actually, but multiples of the shortest distance between the hyperbola and the origin – at the x-axis). Since a hyperbola's curve is always within 45° of the x-axis, specifying a point on the curve by giving the *angle* it sweeps out just doesn't make much sense.

9. First, be sure to set RADians mode (if necessary, press ■(MODES) **MORE** **D/R** (EXIT)). Then: ■(π) (÷) (2) (=) (SIN) Result: **1** OK – it's what it's supposed to be, right? So try ■(π) (SIN) Result: **-2.06761537357E-13**

What's this? A wrong answer?! The SIN of π radians is supposed to be *zero.* What gives? Try ■(π) (÷) (2) (=) (COS) Result: **-5.10338076868E-12** *Aaaack!* Try ■(π) (COS) Result: **-1** Good Grief! *What* is going on??!!?

It's very simple actually: You're *not using π itself;* nobody does, because π has an infinite number of digits. You're using an approximation of π that is *rounded* to 12 digits. And it so happens that at the twelfth digit of π, a proper rounding dictates an *upward* rounding: 3.14159265358979323846... becomes 3.14159265359. You're actually working with a number that's *slightly more* than π, and thus you'll get corresponding results. Sometimes (as with sin(π/2)), the difference is too tiny to affect the answer; at other times this difference does indeed enter into it. But no matter what, *you can count on your calculator to be extremely accurate and faithful to the numbers it uses.*

10. Just to appreciate the technology of it all, do it the "old fashioned" way first: Key in the number in question: (2)(3)(•)(4)(5)(6)(7). Now, you know the **23** part is right just by looking at it, so lop it off by taking only the Fractional Portion: ■(PARTS) **FP** (**0.4567**). This is a *fractional portion* of a degree, so multiply by 60 to find the number of minutes: (×) (6)(0) (=) (**27.402**)

Aha! There's your 27 whole minutes staring at you. But you also have some *fraction* of a minute there, too (.402 minutes, to be precise). You need to convert this to seconds and fractional seconds – and it's the same process as with a fractional degree: **FP** (×) (6)(0) (=) (**24.12**) Bingo!

Now compare all that with the modern, easy approach: Press (2)(3)(•)(4)(5)(6)(7) ■(CONVERT) **▸HMS** ...*That's* why they call it technological progress.

11. What a puzzle, eh? All right, first things first. You'd better figure out where the other half of the discus may have landed:

You were flying due westward *along the equator* (notice the 0° latitude of your position at the time of collision). Not only that, you know your speed, since you were keeping the setting sun constantly on your horizon. That is, your speed was actually matching the speed of the Earth's rotation – about 1050 miles per hour. Now, assuming that you cut this discus pretty cleanly in half, the missing half probably didn't get bounced sideways too far; it must have dropped off toward the ground in a path more or less aligned with your plane's – along the equator (very convenient, no?).

The question is: How far west did it travel while it dropped – assuming that it began its final fall at roughly the speed and altitude of your aircraft. Start by figuring out *how long* it took to fall the 130,000 feet:

y_{feet} (which is zero at the end) = y_0 (which is 130,000) - 1100 - 256t

$$\text{or} \qquad t = \frac{130{,}000 - 1100}{256}$$

(Aha! Some actual calculating to do! (1 3 0 0 0 0 − 1 1 0 0) ÷ 2 5 6 =)
Answer: t = 503.515625 seconds

Of course, you shouldn't believe more than the first three digits of any number you get here, since that's the precision to which you're sure of your speed and altitude (and the formulae probably aren't even *that* good).

So while you're thinking about it, set your display to SCI 2 (press ■ MODES SCI 2 INPUT). See? This actually shows you three total digits of precision, one before the decimal point – as is conventional in scientific notation – and then the specified two places after it.

OK, so this poor abused discus fell for about 500 seconds, give or take a few. That's a long time. So how far west did it drift in this time (you can't know what the winds were doing to it, but what would it have done in still air)?

Now prepare to plug this t into your x-distance estimation formula:

$$x_{feet} = x_0+v_0t - (6400/w)t^{4/3}$$

x_0 is zero for now, so you can ignore that term in the formula (you'll add the final result here to the assumed position of the collision);

w is 64 pounds;

v_0 is 1050 miles per hour, but you need to convert this to feet per second for use in your formula: [1][0][5][0] (miles per hour) [×] [5][2][8][0] (feet per mile) [÷] [3][6][0][0] seconds per hour [=] **1.54E3** (feet per second) – about 1,540 feet per second.

Now that you have your v_0, just grind away at that x-distance formula:

[×] [■][LAST] [−] [6][4][0][0] [÷] [6][4] [×] [■][LAST] [■][y^x] [(][4][÷][3][=]

Answer: **3.75E5**

That's about 370,000 feet, but now you'd like to know it in miles, so you can start figuring the rough latitude and longitude of impact:

[÷][5][2][8][0][=]   Answer: **7.10E1**   About 71 miles to the west.

Of course, since you were on the equator when this whole thing happened, if you're assuming a due-westward drift, the latitude of impact is still on the equator (0°), but how many degrees of longitude are in 71 (statue) miles?

Well, there are 360° in 25,000 equatorial miles, so now multiply ([×]) your 71-odd miles by [3][6][0] [÷] [2][5][0][0][0] (degrees per mile) [=]

Answer: **1.02E0** (degrees)

Fine. So the discus hit your plane at about 90°44" W. longitude. Then it hit the ground (or the ocean) a little over a degree west of there. What longitude would that be – exactly?

You need to convert 90°44" into just decimal degrees first, right?

OK: [9][0][·][4][4] [■][CONVERT] ▶HR (go to FIX 4 now: **90.7333**)

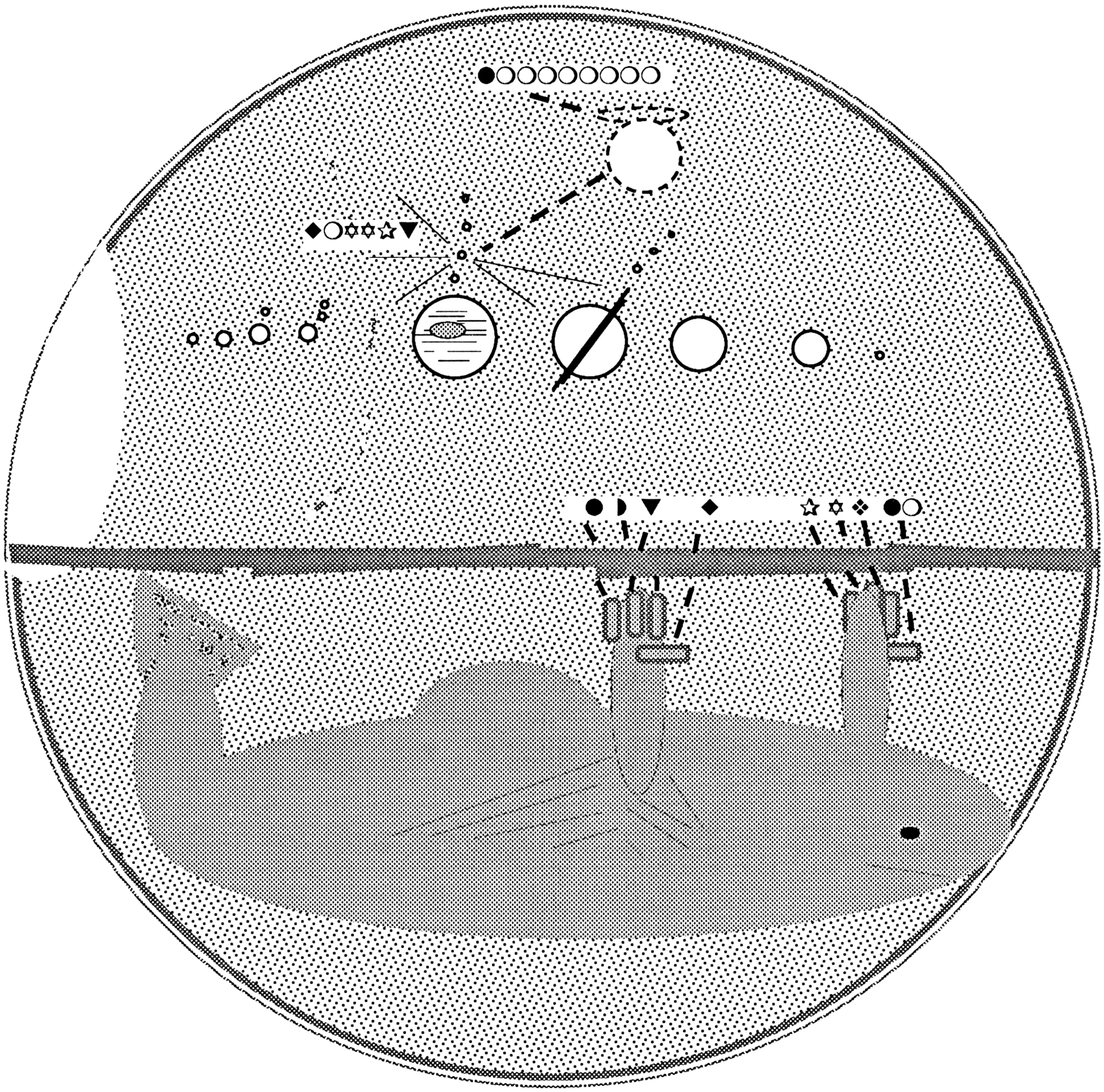
Now add the computed westward drift: [+] [■][LAST] [=]

Now convert back to degrees, minutes and seconds: ▶HMS (**91.4520**)

So, 91° 45" W. longitude must be somewhere near the actual impact point – and fortunately, it's not ocean – it's on Isla Isabela. So you direct the search teams to begin combing that island, searching a circle centered on your computed impact point (and with a radius of several miles to allow for a large cumulative error in all these shaky assumptions).

As luck would have it (and luck seems to be having it a lot in this little adventure), residents on the island saw a twinkling flash over the island just after sundown that evening – and with their help, you at last find the other half of the discus embedded among a grove of trees....

Here's how the two halves look when reunited:



Does it begin to make any sense to you?

It's apparently a greeting probe – similar to the messages inscribed upon the Voyager probes being sent to places beyond our solar system. But that's not where this message apparently originated; those upper drawings are too familiar: First is the huge sun, then Mercury, Venus, Earth (with its Moon), Mars (with its two moons), the asteroid belt, then Jupiter with its big red spot, Saturn with its rings, Uranus, Neptune and Pluto.

And there's a lot of markings surrounding the second large moon of Jupiter – Europa. From the emanating lines, it looks as though this may be the source of the message! Notice the enlarged picture of Europa, rotating on its axis at the upper right. But what do the symbols mean?

And is this whale-like image a picture of what the species looks like – with arms and fingers, no less? Could creatures who still swim (in water – or in some other fluid?) also be technically advanced enough to construct and send space probes? If so, then they must know how to reason and calculate at least as well as we humans do (even without HP-27S's)!

How do you suppose they might calculate?

Well, how did we humans first learn? Probably with our fingers, right?

Now you're getting somewhere: Look at this "whale's" eight fingers. Notice the inscribed symbols associated with each one. This is exactly how *we* associate symbols with *our* digits!

We use the symbol 1 for the first finger, 2 for the second finger, and so on, until we get to the tenth finger, at which point we run out of symbols and start over, denoting a completion of the set with an extra digit (1). Thus, we have no symbol for "ten." Rather it's a *composite* symbol made up of a 1 and a 0, meaning "one complete set of symbols plus 0 extra counts beyond that."

Then on we go with the next set: 11, 12, etc....And we add a third column ("hundreds") when we've completed ten sets of ten.

This is our base-ten system – *because we use ten unique symbols (0-9).* But from the looks of it, it made more sense for these whale-folks to count in base *eight,* eh? There are 7 single symbols for the first 7 fingers; then a composite, which uses the first symbol again – plus the missing eighth symbol!

If you were to replace those symbols with the familiar numerals you know and love, the fingers would indeed be numbered as follows:

●	◗	▼	◆	☆	✡	❖	●○
1	2	3	4	5	6	7	10_8

(Notice how the 10 is marked with a subscript to denote that this is "one-zero, octal" – not "ten.") So what do think about the coding on the discus? It looks like base eight! And if you use that assumption and make the same substitution with the other two series of symbols, you get these two numbers:

●○○○○○○○○ (marked near the large schematic of Europa's rotation)
100000000_8

◆○✡✡☆▼ (marked near the lines emanating from Europa)
406653_8

Well, to make things even easier, why not convert these to decimal values, so that you can get a feel for the numbers you're working with? Press ■ BASE, then **OCT**, and key in 1 0 0 0 0 0 0 0 0. Now press **DEC**. The number in base ten is **16,777,216.0000**).

Now what does this number mean? It's associated (apparently) with Europa's rotation on its axis. Hmmm....What special numbers or significance do *we* attach to the rotation of the Earth? After all, it happens every day...

Every *day!* It's one way to measure *time!* Maybe these whale-folks measure time similarly! Then that number might signify their subdividing units of time. That is, what if they have 16,777,216 units of time for each of their days (after all, 100000000 is exactly 8^8 – very convenient if you have 4 fingers on each hand, eh)? *What you might have here is a way to convert from their time units to Earth units (seconds, hours, fortnights, whatever)!*

Looking up in some handy astronomical reference, you find that Europa's day is about 307,076 seconds (about 3.5 Earth days). To be more precise, then, according to the whale-folks' alleged definition, *there are 16,777,216 "Europa-seconds" per 307,076 "Earth-seconds."*

OK, *suppose* that's true. What good does that do you? Maybe it's time to look at that other inscribed number (406653_8)....The whole idea of this discus is a message. And it seems pretty clear that those lines emanating from Europa are meant to show that it comes from Europa – or so you've assumed so far. But then what about that number? What message could be so universally meaningful and profound as to lie in a single number? Can you think of any just off the top of your head?...   ...no?

Then maybe it's not so profound. Maybe it's just the key to unlock the real message – like the numbers in your automatic teller machine...or a phone number – a phone number!?!?! ("for a good time, call Europa....406653_8")

Hmmm – can't exactly be a telephone - there's no cable laid out that far yet. How about a *radio*phone? Could this be a radio frequency? Hey – that would explain why the time units were needed – to convert frequencies! Eureka!*

Convert to decimal: OCT 4 0 6 6 5 3 DEC Result: 134,571.0000. Presumably, that's a radio frequency in "cycles-per-Europa-second." Now EXIT the BASE menu and multiply this "cycles-per-Europa-second" by "Europa-seconds-per-Earth-seconds" to get "cycles-per-Earth-seconds:"

× 1 6 7 7 7 2 1 6 ÷ 3 0 7 0 7 6 = Answer: 7,352,338.6209

That's just a little over 7 megahertz – it *is* a radio wavelength! ("Somebody aim a ham radio at Europa and start listening for whales....")

*(No – Europa.)

A Menu Summary

Well, nobody can say that you haven't "earned your wings" so far, eh? That last quiz took you through just about every calculation trick and every menu you've seen up to now, didn't it? OK, ok – back down to Earth for a while....

Maybe while you were doing the quiz, you might have noticed something very convenient about the twelve menus on your HP-27S:

MENUS

[SOLVE] [STAT] [TVM] [TIME]

[BASE] [PROB] [HYP] [CONVERT]

[%CHG] [PARTS] [MODES] [PRINTER]

The thing to notice is that these menus have a certain "pecking order." That is, while you're working in certain menus, you can take little side trips to other menus, then return *directly* to the original menus by pressing the [EXIT] key – without being bumped back out to the MAIN menu.

For Example: Press ■[TIME] to go to the TIME menu. Now suppose you wanted to do a quick hyperbolic trig side calculation. Just press ■[HYP]. Now suppose you wanted to return to the TIME menu to resume what your were doing. [EXIT] is all it takes!

By Contrast: This doesn't work for all menus. Starting from the MAIN menu, press ■[HYP] as your first choice. Then suppose you wanted the BASE menu for a minute – just to do a side calculation: ■[BASE]. Now try to return to the HYP menu: [EXIT]. It won't do it, will it? It kicks you back out to the MAIN menu; you have to re-select ■[HYP] yourself.

So which menus let you go off on side trips and return directly? Actually, there are three different levels:

Level 1: [SOLVE] [STAT] [TVM] [TIME]

Level 2: [BASE] [PROB] [HYP] [CONVERT]
[%CHG] [PARTS]

Level 3: [MODES] [PRINTER]

The rule of thumb is this: *You can take convenient (i.e. direct-return) side trips to any menu on a lower level than where you are now.*

Thus, for example, you can go from [TIME] to [PROB] and back directly, or from [BASE] to [MODES] and back directly, or even from [TIME] to [HYP] to [MODES], back to [HYP], and back to [TIME] – directly – returning with just the [EXIT] key! But you *can't* go from [TIME] to [STAT], nor from [BASE] to [HYP], *and then back,* without begin bumped out to the MAIN menu.

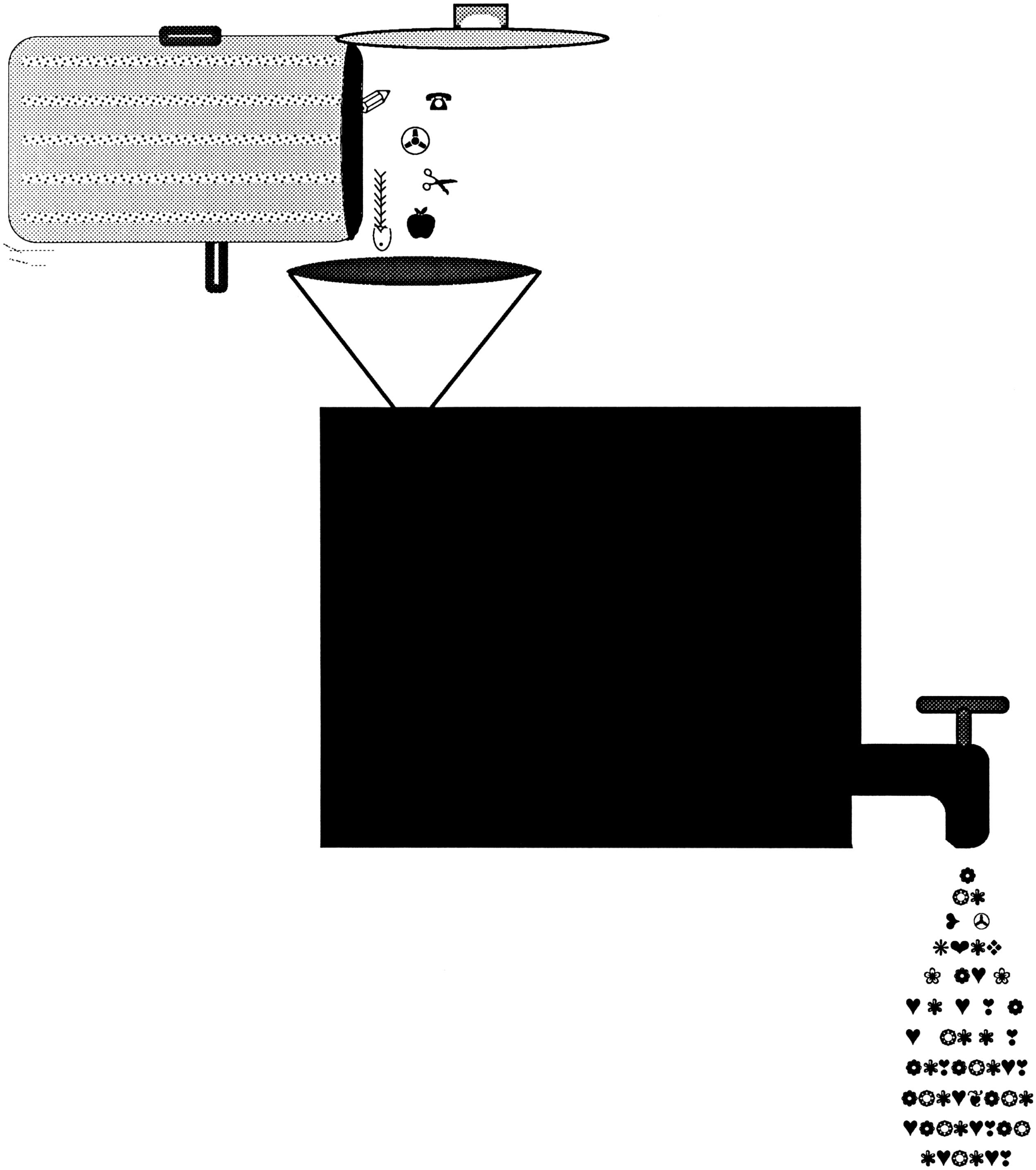
Get the idea?

Of course, you haven't yet been introduced to some of the lower-level menus (that'll happen in due Course). But does this menu hierarchy suggest to you just which ones are the real "biggies"?

That's right – those top-level menus.

There's not that much mystery about the TIME menu; you've seen it as an example here so far, and you probably won't hear too much more about it. So the remaining 3 chapters of this Course are really all about the other 3 "biggies:"

STAT, SOLVE, and TVM



ANALYZING YOUR FLIGHT DATA:
The STAT Menu

Of course, the whole idea of all these menus is to arrive at places where you can do calculations.

But what happens if you want to use results from one calculation in some other calculation – maybe in some menu that's "way across the country?"

You'll remember an instance in that last quiz, for example, when you took a moment to *make yourself another copy of some result* – so it would float in the History Stack until you were ready to use it. But what if you had had 2 or 3 – or 50 such results? There's got to be a better way, eh?

The question is: How can you *save* your answers for later use?

After all, the HP-27S has continuous memory; it won't forget any number that you've properly saved in it. You just need to know where the cubby-holes are – and that's what this chapter is all about.

Well, there are two ways to save numbers: in *registers* and in *lists.**

*and if you already know how to use both these methods, then go right ahead to page 91.

The Numbered Registers

The simplest way to save the result of any calculation is in a *numbered storage register.*

It's called a numbered storage register because that's how you refer to it when you want to store into it or recall from it.

Try This: Store the result of 789÷5 into register 4.

Solution: Press [7][8][9] [÷] [5] [=] to get **157.8000**, then [STO] [4].

What could be simpler? Just as with arithmetic, you do what you say.

Now Try This: Clear your Calculator Line (by pressing [CLR]). Then recall that result you just stored in register 4.

Nothing To It: Press [RCL] [4].

See how these numbered registers can act as your holding bins? And there are ten such registers, numbered 0 – 9, so you have room for quite a few results.

So that's one option you have for saving results – and it's the quickest, simplest way to do that. But there's another way....

Lists Of Numbers

What if you had a set of numbers that represent, say, your income from flight instruction for the past 11 years? You can't store these in the numbered registers because you'd run out of registers (there are only 10 registers).

Besides, it would surely be handy to keep all of these figures in one list, so you can compare them, sum them, etc.

Good news – you can do this! Suppose that the numbers below are your annual incomes for the past 11 years:

Year	Gross Income
1977	17,000.00
1978	22,000.00
1979	21,500.00
1980	24,000.00
1981	14,500.00
1982	19,000.00
1983	23,000.00
1984	24,000.00
1985	24,500.00
1986	18,000.00
1987	27,000.00

Your Goal: Figure out where in your HP-27S you need to "go" to build a list in which you can save these numbers.

Solution: Starting from the MAIN menu, press the ■STAT key. You'll see something similar to this:

```
ITEM(1)=?
CALC INSR DELET NAME GET TOTAL
```

This menu is for lists – and you're about to build one.

Next Question: How do you start a new list?

Next Answer: By GETting a *NEW (blank) list to fill. From the above menu, you press GET . The machine then asks you which list you want to get, and it gives you a choice on the Menu Line. If you had already saved some lists of your own, their names would now appear alongside *NEW.

But right now, *NEW is the only game in town, so you choose it (actually, therefore, you didn't need to GET it at all, since a *NEW list was already in your display, above. But now you know how to GET any list, new or named).

That takes you back to the STAT menu and the above display. You're all ready to begin keying in numbers.

Time To Fill In The Numbers: Put all 11 years' worth of income data in this list.

Solution: (Be sure to watch your display as you do these keystrokes. That will tell you a lot about how to work with a list.)

Press

[1][7][0][0][0] [INPUT]
[2][2][0][0][0] [INPUT]
[2][1][5][0][0] [INPUT]
[2][4][0][0][0] [INPUT]
[1][4][5][0][0] [INPUT]
[1][9][0][0][0] [INPUT]
[2][3][0][0][0] [INPUT]
[2][4][0][0][0] [INPUT]
[2][4][5][0][0] [INPUT]
[1][8][0][0][0] [INPUT]
[2][7][0][0][0] [INPUT]

Not too tough, right?

And notice that below the Calculator Line, you'll begin to see the running total of all the numbers in your list.

But here are some important things to notice:

1. Items in a list are numbered, so you made each calendar year correspond (chronologically) to a number from 1 to 11.

2. You always signal to your calculator to "accept" your number by pressing [INPUT].

3. Each time you press [INPUT], the machine accepts the number *and* re-computes its running total of all items. Notice, by the way, that since you're working with dollars and cents here, it might be a good idea to change the display to FIX 2 notation (press ■[MODES] **FIX** [2] [INPUT]).

4. After you've keyed in all your eleven numbers, the machine still sits there and waits for the twelfth item. What do you do now?

(Read on)

Editing And Using Your List

First of all, don't worry about your machine's insistence on a twelfth item. It's always going to allow you to add another item to the bottom of your list, but just because there's this unfilled item down there doesn't mean you can't work with your 11-item list anyway.

Secondly, are you sure all your items are keyed in correctly? How about a quick review?

Go For It: Jump to the beginning of your list and step down through it to check each entry.

How? First, you move the pointer back up to the top. Remember the two keys, ▲ and ▼ that roll your History Stack around? Well, they're really good for all sorts of lists and stacks – including this kind of storage list.

So you *could* press ▲ ten times to get the pointer back to item 1.

Or you could press ■▲. That's a quick way to *jump* to the beginning of the list, all in one motion (no prizes for guessing how to jump to the end of the list again).

So you might as well go first class – use the ■▲ method. Then use ▼ to walk down the list and check your numbers.

A good clue is that your TOTAL should be 234,500.00

Next Problem: Suppose you want this 11-item list to keep a *moving* record (i.e. always for the most *recent* eleven years). What would you do to change the list at the end of 1988, when you had another year's income to record?

Next Solution: You would delete the first item (1977's income) and then tack on the 1988 income (say, 34,000) at the end of the list.

First, you would press ■▲ to jump to the top of the list, if you're not there now. Next, to get back to that STAT menu, press EXIT.

Then press the DELET selection from that STAT menu. *This deletes whatever item you're now looking at on the Calculator Line* (and that's always what determines which item is affected when you edit your list on an item-by-item basis).

So the first item in your list will now be your 1978 income (22,000.00), and there should be only ten items altogether, right?

Now jump to the end (press ■▼), to check that theory...

...sure enough, Item(11) is now blank.

So key in 34,000 and press INPUT to store it there.

Voila! You've just updated your 11-year income statement! And notice that your running TOTAL (251,500.00) is always up to date, too.

Naming And Saving Your List

So, now that you're through with your income statement for another year, can you [EXIT] the STAT menu and do other calculations? Not quite yet. You haven't NAMEd your list. After all, how else could you go GET this list again later?

Name It: Name your list INCOME.

Solution: First, press [EXIT] (to get back to the STAT menu again), then NAME. Aha! – the secret to typing alphabetical characters on the HP-27S! Since there aren't enough keys for all of them, they're in *menus!* For example, to type INCOME, you'd press menu keys as follows:

FGHI I NOPQ N ABCDE C NOPQ O JKLM M ABCDE E.

Get the idea? It's not the most convenient way ever invented for typing, but it gets the job done on those few occasions when you need it, so grin and bear it. Then, when you finish, press [INPUT], to finalize the name. Your list is now named INCOME, and you can now [EXIT] to the MAIN menu (go ahead).

Verify: First, GET a *NEW list (press ■ [STAT] GET *NEW), just for the sake of comparison. It's all blank, just as you would expect, right? But notice that when you press GET, your named list is right there on the menu, too! So go GET it now: GET INCO.... Voilá!

Notice that the display can't fit all the letters of INCOME into its menu box. Usually it can fit four or five, so it's wise to choose short names that are unique somewhere in their first 4 or 5 letters.

Next Job: Create a new list, showing your own personal flight hours (as shown here) for the last eleven years. Name this list HOURS:

Year	Hours	Year	Hours
1978	150	1984	200
1979	330	1985	175
1980	170	1986	300
1981	500	1987	160
1982	250	1988	125
1983	220		

Solution: First, GET a *NEW list (press GET *NEW). Then – before you forget – NAME it HOURS: Press NAME, then FGHI H NOPQ O RSTUV U RSTUV R RSTUV S [INPUT]. Now fill this list with your flying-hours data:

[1][5][0] [INPUT]
[3][3][0] [INPUT]
[1][7][0] [INPUT]
[5][0][0] [INPUT]
[2][5][0] [INPUT]
[2][2][0] [INPUT]
[2][0][0] [INPUT]
[1][7][5] [INPUT]
[3][0][0] [INPUT]
[1][6][0] [INPUT]
[1][2][5] [INPUT]

Flying By The Seat Of Your Pants: A Sort-Of-Pop Quiz On The STAT Menu

Now that you know about the STAT menu, why not go explore it on your own? Here you'll find a few things to review...and a few new twists to discover....

1. Fill up your History Stack with the numbers **1.00**, **2.00**, **3.00**, and **4.00** (reading top to bottom). Now, using the numbered storage registers as temporary holding bins, see if you can create a new History Stack that is in reverse order to the current one – without keying in any more numbers.

2. Find the average, minimum, maximum, and median values in your INCOME list.

3. What was your average yearly earnings *increase* (dollar amount – not percentage) over this 11-year period? (Reminder: This is the yearly average of the net increase from the *first year to the last – not* from the minimum year to the maximum year!)

4. Rename your INCOME list. Call it $/YR instead.

5. Does your annual flight instruction income list ($/YR) have any *correlation* with your air-hours (the list called HOURS)? Find out by fitting a smooth curve between those lists. Then use that fit to predict how many air-hours you might log in a year where you earn $40,000. How much might you have earned in a year in which you logged 350 hours? Are these good guesses?

Sort-Of-Pop Answers

1. First, to fill up the History Stack, press 1 INPUT, 2 INPUT, 3 INPUT, 4 INPUT (just as you did back on page 48). Next, break it down into four pieces and store the pieces in storage registers: STO 1 ▼ STO 2 ▼ STO 3 ▼ STO 4

   Now recall them back in opposite order: RCL 1 RCL 2 RCL 3 RCL 4. Bingo! Notice that you didn't need to worry about the old Stack. Those numbers were "popped off the top" when you began to recall other numbers.

2. From the STAT menu, GET your INCOME list (if you're not there already), by pressing GET INCO. Now, notice that first item in the STAT menu: Press CALC and see some of the different ways to analyze your data.

   Press MEAN to get the average: MEAN=22,863.64
   Press MORE MIN to get the minimum: MIN=14,500.00
   Press MAX to get the maximum: MAX=34,000.00
   Press MORE MEDN to get the median: MEDIAN=23,000.00

3. This average increase is going to be the *difference* between the first year's income and the 11th year's income – divided by 11 – right? OK, from the STAT menu, GET your INCOME list (if you're not there already), by pressing GET INCO. Now move to the 11th year, ■ ▼ ▲, and put a copy of this onto the Calculator Line: RCL INPUT (this is a handy thing to know!).

   What you're doing here is building yourself a simple subtraction problem by extracting from your list data the two numbers to be subtracted. Since you now have the first number on the Calculator Line, put in the minus now: −
   Then ■ ▲ RCL INPUT to get the second number. Finally, press = ÷ 1 1 =, The result: 1,090.91 – your average yearly income rise over 11 years!

4. From the STAT menu, GET your INCOME list (if you're not there already), by pressing **GET** **INCO**. Now, even though the list has a name already, press **NAME** so you can change it. See how the cursor appears over the current name? And play with those menu selections that show arrows. See how they move the cursor around for you?

   Now type in your new name right over the existing one: Press **ALPHA** to get to the character sets, and...hmm... – how do you get a $??

   No problem: Just choose any character set, **WXYZ**, then **OTHER**. Here's where all the non-letter characters live – and you'll notice there are several pages to this menu, so press **MORE** and there, on the second page, is the **$** character. Type it and notice that you're instantly back to the ALPHA menu once again.

   Did you notice where the / character was? Same place as the $, right? So go find it and type it in: **WXYZ** **OTHER** **MORE** **/**. Now complete the new name: **WXYZ** **Y** **RSTUV** **R** .

   Next, to get rid of the rest of the old name: press [EXIT] to get to the editing menu. Then, since the cursor is already just where you want it, press **DEL** and watch the I disappear! Keep pressing **DEL** (six times in all) until INCOME disappears altogether.

   Now, to accept this new name and "make it official," just press [INPUT].

   Poof! You're back at the STAT menu. Did the new name "take?" Press **GET** to find out. Sure 'nuf – it's right there on the menu!

5. You've seen some of the calculating tools available to you on the STAT menu under the **CALC** selection – things like MEAN and MIN and MAX. And there are others you can try out later, on your own – SORT, TOTAL, STDEV, RANG, etc. – all quite straightforward.

But all those tools are merely ways to analyze the data in one list at a time. What you're looking for here is a way to compare *two lists to each other* – a correlation analysis – to see if there's some steady, mathematical relationship between corresponding numbers in each list.

Well, there *are* such tools (also in the CALC section of the STAT menu) for "crunching" 2-variable (i.e. 2-list) statistics. And among these is the **FRCST** selection, for doing correlation analyses like this problem.

So select **FRCST** (it's on the second page of the CALC menu under the STAT menu). What happens?

First of all, you're asked to SELECT X VARIABLE. What do you do?

Well, what is this problem all about? You've been asked to find a correlation between your income from flight instruction and your own logged air-hours. Does such a correlation make any sense? As a start, look back for a moment at your list of air-hours (page 90) and compare it briefly with your instruction income list (page 83 – remember that you updated it, by omitting your 1977 income and adding your 1988 income).

What do you observe? It seems that the more money you earned as an instructor, the fewer personal air-hours you logged, right? Does that make some sense? Sure it does: since more of your time was taken up instructing someone else, you had less time for your own flying.

All right, so there's a good chance that you'll be able to find some kind of numerical relationship between your instruction income and your own flight hours. And that relationship will be *negative.* That is, the *more* income you had, the *fewer* flight hours you logged, and vice versa.

OK, that's all well and good. But which one determines the other? Which activity has a higher priority for you? Do you work in your instruction around your own flight hours – or is it the other way around?

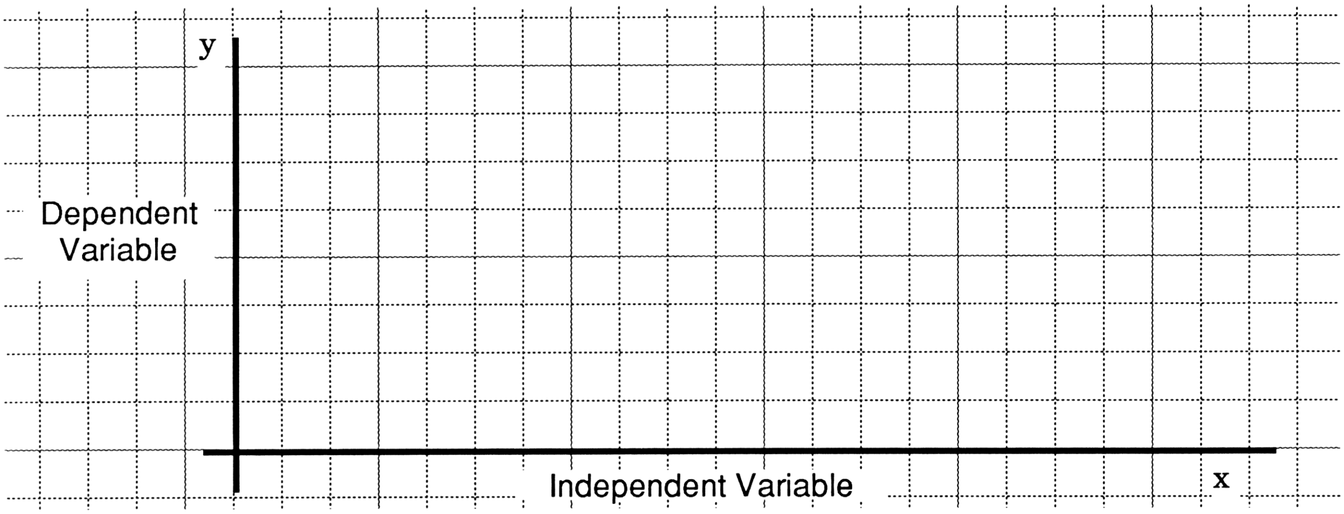
It's probably the other way around, isn't it? Alas, you just gotta earn your keep before you can go larking around in an airplane for the sheer fun of it.

So your own flight hours are *dependent* upon how much free time you have – how much time you're *not* giving instruction. But you *can't* dictate or predict that instruction time; it's simply the amount of business you happen to get from flight students. That's up to them. It's *independent* of how you spend your free time (after all, you might spend it watching TV, or writing symphonies, or something else).

To sum up what you've said here: Your flight hours *depend* upon your instruction income, but your instruction income does *not* depend upon your flight hours.

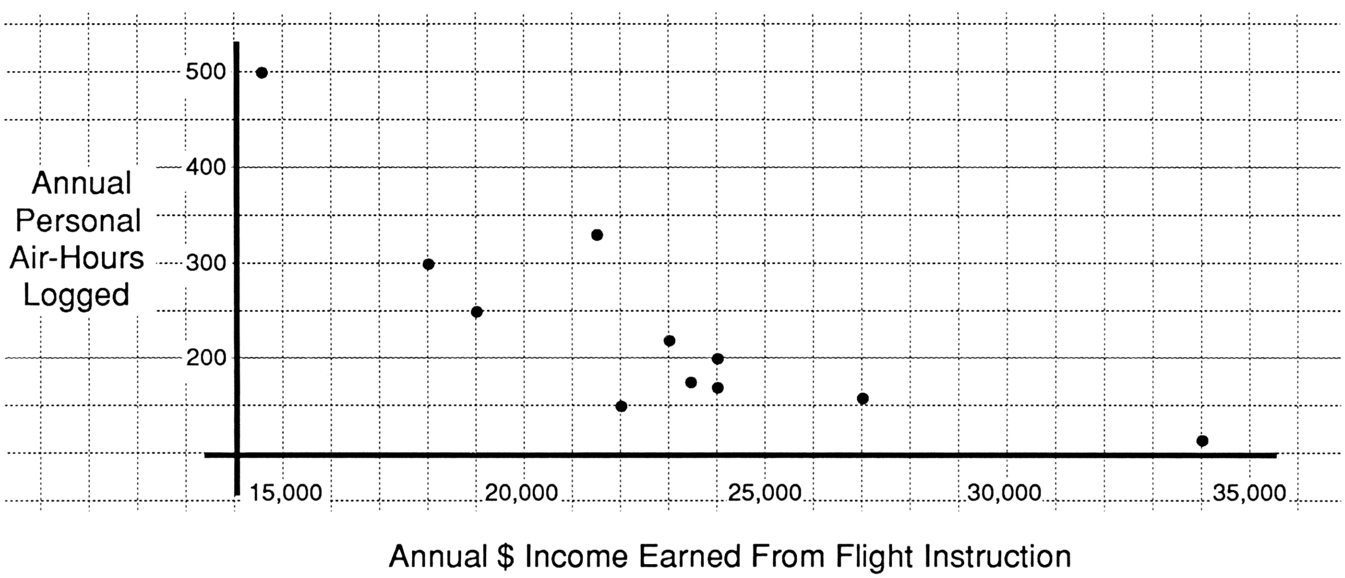
So in this correlation analysis, your income ($/YR) will be your independent variable, and your flight hours (HOURS) will be your dependent variable.

Now, as you may know, when you're graphing one variable against another, it's conventional to do it like this:



The independent variable is assigned to the x-axis, and the dependent variable is assigned to the y-axis – so you can see how the dependent variable wobbles up or down as the independent variable proceeds from left to right.

So here's the plot you're going to ask the HP-27S to make and analyze with your income and hours lists:



The position of each of the eleven data points is determined by the eleven pairs of corresponding values in the two lists.

Now you know what your calculator is asking in the display:

```
SELECT X VARIABLE
HOURS $/YR
```

You need to tell it that your $/YR list is your x-variable list. So press **$/YR**.

Then, naturally, it will want to know what your y-variable list is. So press **HOURS**.

Now you're on to yet *another* menu: This is where you'll try to fit various-curve shapes to your data.

If you go to the next page of this menu (press **MORE**), you'll find a selection called **MODL**. Press this to see the four different curve MODeLs (the shapes and their corresponding equations) that you can choose from:

LINear	equation:	$y = B + Mx$
LOGarithmic	equation:	$y = B + M\ln(x)$
EXPonential	equation:	$y = Be^{Mx}$
PoWeR	equation:	$y = Bx^{M}$

Guess you'd better try each model, to see which fits the best, eh? For starters, choose **LIN**.

This takes you back to the first page of that menu, and the display shows you which curve shape you've chosen.

So how does the machine actually fit the curve to your data? And how can you use the fitting to predict other values on the curve? And how do you know how good the fit is?

The answers to all of these questions are right before your eyes – on this menu:

When you press **M**, for example, the HP-27S then calculates the value of M (remember the LINear equation on the previous page?) that would give you the best fit for your data for this shape of a curve (<u>result</u> here: M=-0.02).

And pressing **B** gives the best value of B (<u>result</u> here: B=624.77).

To see how good a fit this is, calculate the CORRelation coefficient (press **CORR**), which is an indicator of exactly that – the goodness of the fit (result here: CORR=-0.80). Now, what does this mean?

Remember what "negative correlation" means? You already observed that as your independent variable – income – *increases* (proceeds left to right), your dependent variable – air-hours – *decreases* (goes basically "downhill").

A CORRelation coefficient near 1 indicates a good "uphill" fit; near -1, it means it's a good "downhill" fit. So you should *expect* any good fit of your data to have a *negative* CORRelation coefficient, right?

This LINear fit seems pretty good, then. But could you do better with some other shape of curve?

Try them all – right in order....

MORE MODL LOG CORR Result: CORR=-0.86
MORE MODL EXP CORR Result: CORR=-0.85
MORE MODL PWR CORR Result: CORR=-0.88

All of these are pretty decent fits, but it looks like the best of these four models is a PoWeR curve, no? So suppose you assume that your air hours total, H *is* related to your instruction income, $, by the power curve equation: $H = B\$^{M}$ (instead of x and y, you use $ and H). OK, then what are the B and M values?

Press M and get M=-1.61 (press [STO] [0] to save this for future use).
Press B and get B=2,266,291,206.79 (press [STO] [1] to save it).

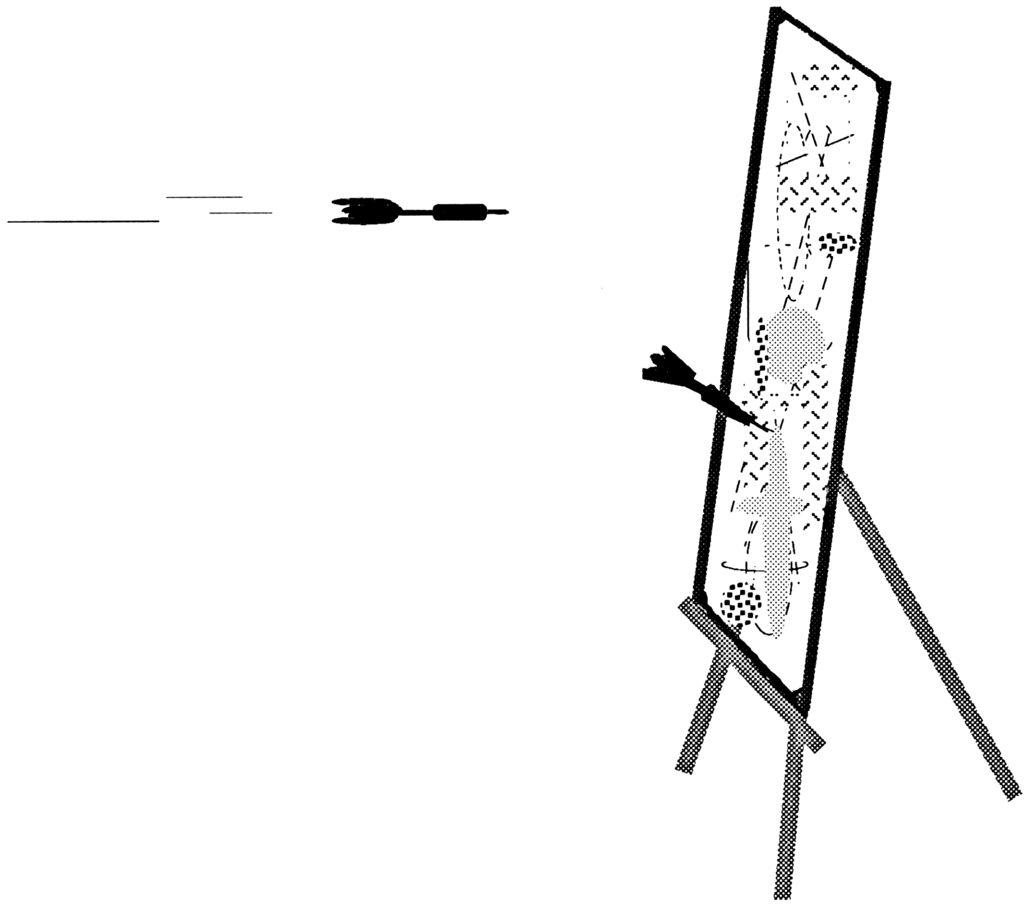
So, according to the best fit your HP-27S can find, $H = 2{,}266{,}291{,}206.79\$^{-1.61}$

You can now use this to predict any point on this curve, couldn't you? As the problem stated, you want to know your likely air-hours would be for a year where you earn $40,000; and you want to guess how much you might have earned in a year when you logged 350 air-hours.

Of course, you can always figure this out *manually,* using your stored values, M and B (e.g. you would press [RCL] [1] [×] [4] [0] [0] [0] [0] [■] [y^x] [RCL] [0] [=], to get your predicted air-hours for a $40,000 year: 84.62 hours).

But why do that when there's a shortcut provided for you right there on the menu? Just press [4] [0] [0] [0] [0], $/YR, then HOURS and poof! HOURS=84.62
Likewise, press [3] [5] [0] HOURS, then $/YR, to get $/YR=16,597.79

This prediction business is now *simple,* eh? You can play "What-If?" like this all day long – predicting hypothetical incomes or air-hours. And you'll get results that make some real sense: If you're going to make a pile of money, you probably won't have as much time to fly on your own; if you're going to goof off a lot, you probably won't make as much money.



MAKING YOUR OWN FLIGHT PLANS:
The SOLVE Menu And "What-If?"

Playing "What-If?" With Menu Keys

Take a deep breath – you've earned it.

After all, you've mastered quite a bit of the HP-27S. You know about the keyboard, the display, arithmetic, higher math, storage registers, and most of the menus, including one of the "biggies" (STAT).

So by now, you've probably had about enough of this "practice flying." Getting to know the basics of controlling your calculator is all pretty tame, no?

The truth is, probably the most exciting calculation you've done so far was analyzing the correlation between your income and flight hours – that last quiz problem.

But do you know *why* that was more exciting?

It's because of the way that FRCST menu worked. Stop and think for a moment about all the different menu keys you've seen so far:

- Some were merely *"navigational" keys* – ways to get somewhere else – such as the **CALC** and **MORE** selections on many of the menus;
- Some were *toggle keys,* such as the **D/R** selection;
- Some were *functions* that operated on the number(s) in the Calculator Line – like the HYPerbolic trig menu selections.
- Some were places into which you could only *store values* for various reasons – usually to control the machine. The SET (time and date) menu is a good example of this.

But nowhere – until that FRCST menu – had you run across any menu with selections that could both *store* <u>*and*</u> *calculate* a given quantity for you.

You used that menu to predict new values – by *varying* one parameter ($/YR) and *solving* immediately for another (HOURS), right there on the menu – no [+]'s or [−]'s needed!

To put it another way, you had two *variables* to play with – a "predicted x" and a "predicted y." And each of these variables had its own key on the menu. You could vary either one to see how it affected the other! The machine was actually "plugging" the values you gave it into the variables of the equation (in this case, it was the equation for a power curve).

That's what the HP-27S is all about – playing with variables in an equation.

But here's the **real** bombshell:

You can do the same thing with equations that *you write yourself – not just those already built into the machine.*

Thus, you can invent your own solutions, and begin to ask questions like "*what if* the longitude correction goes up by 2 minutes?" And *"what if* that line's slope were 5 percent lower?" And your calculator will show you the answers!

You can play *and customize* this "What-If" game for your own purposes!

So it's now time to practice this very fruitful little game....

Before you start building your own equations, try a few more examples of the "What-If" game with some equations that are already built-into the HP-27S. Now you'll see examples from some of the other "minor" menus, because they all use this "What-If" technique.

Watch This: From the Main Menu, press ■ [%CHG], and see this:

```
16,597.79
 OLD | NEW | %CH
```

And suppose you want to solve this problem:

Today's high temperature was 75°F. But yesterday it peaked at 64°. By what percentage did the high temperature increase in one day?

If the *two*-day high-temperature increase was 20%, what was the third day's high? (There's your "What-Iffing!")

Solution: [6][4] OLD [7][5] NEW %CH

Answer: %CHANGE=17.19

Now specify the percentage you want: [2][0] %CH

And ask for what the NEW high should be: NEW

Answer: NEW=76.80

See the basic pattern?

And did you notice that you're pressing the same key both to *specify* a variable's value and to *calculate* it?

With the first calculation, you specified the values of the variables OLD and NEW. Then you calculated the variable %CH.

Next, you turned around and specified your desired %CH, you left OLD just as it was before, and calculated NEW.

How does the machine know when you want to specify a value and when you want it to calculate it for you?

The General Idea In 59 Words Or Less

Whenever you *key in a number* and then press a variable's key, you will be *specifying* that value for that variable.

Whenever you press a variable's key *without having keyed in a number just previously,* you will be asking the HP-27S to *calculate that variable's value – based upon the values of all the other variables in the menu.*

Just remember: When you use only the menu keys to store values into variables, *you're just using a shortcut to store values into registers.* That is,

[6][4] **OLD**
[7][5] **NEW**

is exactly the same as

[6][4] [STO] **OLD**
[7][5] [STO] **NEW**

You can [STO] and [RCL] to and from variable registers just as you can from the ten numbered registers.

(But you're going to become so proficient at menus and variables that you'll want to use the shortcut method all the time; therefore, this book will show the upper keystroke pattern only.)

Try This: Review the values in each of the variables.

Solution: Press RCL OLD and see: OLD=64.00

Press RCL NEW and see: NEW=76.80

Press RCL %CH and see: %CHANGE=20.00

Remember that RCL key? It's the one you use to recall values from numbered storage registers. Is that what it's doing here, too?

Yep – except that these registers aren't named with numbers but with the variables they represent. That is, there are actually three storage registers *built into the machine,* named OLD, NEW, and %CH. To get a better idea, here's a "picture" of the machine's memory, as you've discovered it so far:

Numbered Registers

0
1
2
3
4
5
6
7
8
9

STAT Lists

$/YR
HOURS

Built-In Variables

predicted x
predicted y
OLD
NEW
%CH

Your Formulas

Variables For Your Formulas

<- - - - - - - - - - (nothing here so far.) - - - - - - - - - ->

One More: If the polar ozone layer decreased by 36% this year and 25% last year, what was its total percentage decrease over two years?

Solution: Again, this is a job for the %CHG menu, so stay right there. And remember, you don't have to clear these registers out from the previous calculation; your new values will simply *write over* them.

Now, although you don't have any real ozone numbers to work with, you can start with a convenient number, 100, to represent the ozone level two years ago: Press [1][0][0] [OLD]

Now, the level one year later was 36% less than that. In other words, the %CH was -36. So press [3][6] [+/–] [%CH].

Now solve for the NEW level by pressing [NEW]: NEW=64.00
So if the ozone level was 100 two years ago, then it was just 64 one year later (you could have done this one in your head, right?).

Now for the second year: The NEW level from the first year is the OLD level for the second year, right? So: [STO] [OLD]

Now give the decrease for the second year: [2][5] [+/–] [%CH]

And solve for the NEW level by pressing [NEW]: NEW=48.00

And what was the percent change for the two years as a whole? Your NEW level is now correct, but your OLD level needs to be 100 for the entire two-year period, right?

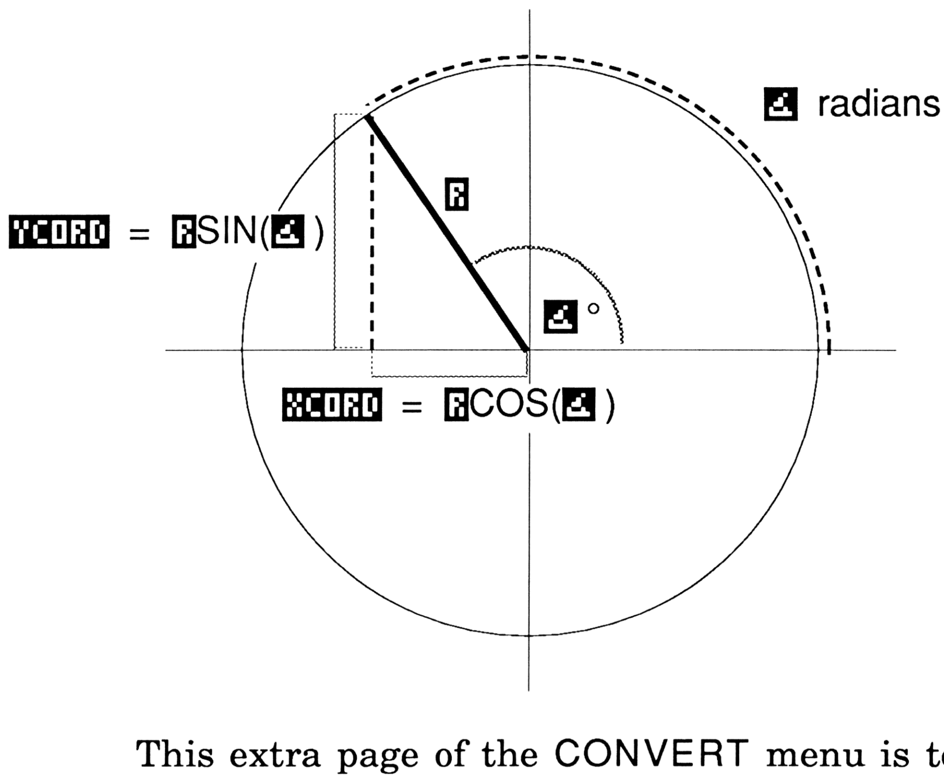
Press [1][0][0] [OLD], then [%CH] <u>Answer</u>: %CHANGE=-52.00

But there are other menus where you can play a little "What-If?" also....

For Example: Press ■CONVERT MORE. Look at all the functions on this second page of the CONVERT menu....

Actually, that D/R selection there on the right is just a duplicate of the one in the MODES menu (remember?). HP knew you might find it handy here, also – so you don't have to keep traipsing off to the MODES menu every five minutes.

But as for the other selections, study this picture of the relationships between a circle's coordinates, sides, and angles:



This extra page of the CONVERT menu is to allow you to more easily study *and play "What-If?"* with the different parameters in trig – and to convert back and forth between rectangular (X- and Y-) and polar (R and ∡) coordinates.

Remember the difference between rectangular and polar plane coordinates?

To specify the position of any point in a plane, you need two numbers (one for each dimension).

- You can give the horizontal and vertical distances from the origin (the point (0,0)) – rectangular coordinates.

- *OR,* you can give the direct distance from the origin (the *radius*) and its *direction* (measured as an angle counterclockwise from the x-axis), which are polar coordinates.

And with a little bit of trigonometry, you can always convert between these alternative notations:

If you know the XCORD and YCORD, you can solve for either R or ∠.

If you know R and ∠, you can solve for either XCORD or YCORD.

Example(s): What is the actual distance from the origin of the point (5,-12)? What is the vertical coordinate of the point (99,45°)?

Solution(s): Press [5] XCORD [1][2][+/−] YCORD, then R
Answer: RADIUS=13.00

Now, what if you know the radius and angle (set degrees mode with D/R, if necessary)? Just press: [9][9] R [4][5] ∠...

...and solve for the corresponding YCORD: YCORD
Answer: YCORD=70.00

One more minor ("smallie"?) menu to tour. It too has "What-If?" capabilities:

Go For It: Press ■ PROB to see the PROBability menu. Here's where you can do all those PROBlems with permutations and combinations and random numbers.

Example: If you had 6 red marbles, 5 orange marbles, 4 yellow marbles, 3 green marbles, 2 blue marbles, and 1 violet marble – all in a leather pouch – how many possible *combinations* of six marbles (any color and never mind the order) could you draw out of the pouch?

What if you wanted to count *permutations* – where the order does matter – but you took them out only 3 at a time?

Solution: You're really just asking how many ways you can combine 21 objects, taking them 6 at a time. So you'd press 2 1 X, 6 Y, then C X,Y.

Result: `C X,Y=54,264.00`

Now if it's permutations in sets of three that you're after, then just change what you need to: 3 Y and press P X,Y.

Result: `P X,Y=7,980.00`

Now, could you imagine trying to figure all that out by yourself? And what if you changed your mind midway? In this "What-If?" menu format, it's simple and painless to re-calculate, but if you were doing this by hand, you might well end up… … losing your marbles.

Memory Space: The Final Frontier

"These are the voyages...

...of your own choosing."

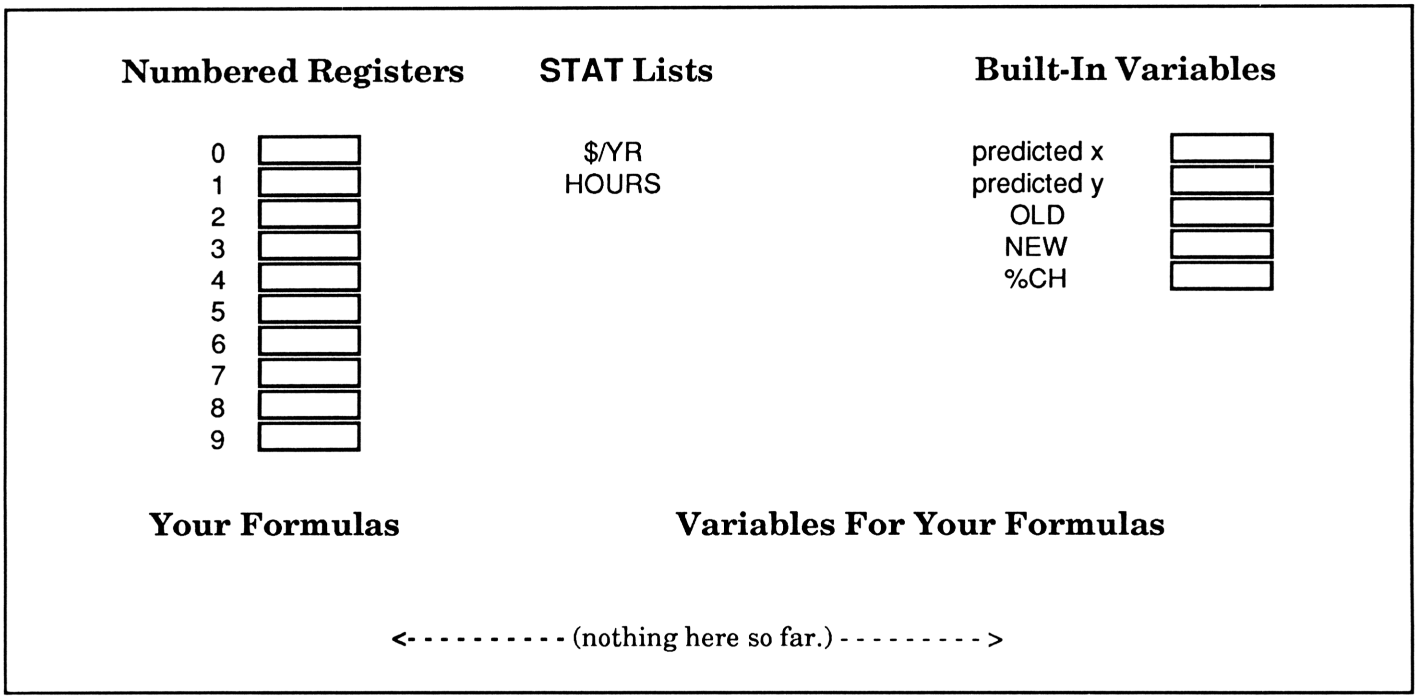
Now you need to learn how to solve a problem for which there is no calculation already built into your HP-27S – a place that just isn't "on the map" – yet. You're going to literally add to your calculator's "atlas."

And once you've invented these new equations, you'll *store* them in the memory of the machine, in your personal *list* of solutions, where they'll reside thereafter until you need them again (of course, you *can* erase them, also – either intentionally or... otherwise).

But most importantly, since these customized solutions take up space in the memory of your HP-27S, you'll need to know a little more about that memory before you start building your personalized list....

A Picture Of Memory

Here again is the diagram of the different parts of the HP-27S's memory that you have discovered up to now:



Besides those ten numbered registers, you had also discovered some other registers: The names OLD, NEW and %CH that you saw in your display in the %CHG menu were actually names of registers associated with that menu. And because you often *vary* the contents of these registers in order to play "What-If", they're called "variable registers," or just "variables" for short.

But those registers are *created* from formulas that are *built into the machine.* You can clear these variables, *but you can't erase the formulas themselves.*

So they're a bit different and are separate from the memory space shown at the bottom of this diagram, where you can create (and delete) your own formulas – and their registers. It's this memory space you're now going to start using....

Full Speed Ahead: From the MAIN menu, press ■ SOLVE, and enter the world of your HP-27S's equation SOLVEr:

```
{NEW} FOR NEW EQUATION
CALC  EDIT  DELET  NEW
```

This is the menu where you need to go whenever you want to create or use a menu of your own.

Now press **NEW**.

Here's a menu you'll certainly recognize! Remember how to type characters with this ALPHA menu?

So there you are, ready to type in an equation – a useful formula that creates its own menu for you....

Creating Your Own Formulas

For openers, since you've already messed about with fuel calculations (remember your DC-3?), try a simple little formula to figure your *car's* gas mileage (after all, you gotta get to the airstrip somehow).

Have At It: Key in an equation for gas mileage. Then figure your mileage for a trip of 350 miles that used 12 gallons of gas. With a 15-gallon tank, how far could you have gone before the last "fumes" ran out?

Solution: From the SOLVE menu, press JKLM M NOPQ P FGHI G = JKLM M FGHI I JKLM L ABCDE E ÷ FGHI G ABCDE A JKLM L INPUT.

Then press CALC....The machine "verifies" your formula, checking to make sure that it understands the equation you gave it. Then....voila! Your own mileage menu:

```
14.15
MPG MILE GAL
```

And you know what to do now, right?

3 5 0 MILE
1 2 GAL
MPG <u>Answer</u>: MPG=29.17

Now for that old familiar "What-Iffing:" Change the number of gallons to the maximum: 1 5 GAL. And figure how many miles you could have gone: MILE <u>Answer</u>: MILE=437.50

Fine. You're happy with the equation – but suspect you'd like to rename the variables in it....

Edit Your Formula: Instead of **MILE**, you want **MILS**; and instead of **GAL**, you want **GALS**.

Solution: EXIT to the SOLVE menu. Now press **EDIT** and notice that the blinking cursor is back.

Time to edit: Press **-->>**. Of course, this jumps you to the far right-hand end of the equation. So, since you're "in the neighborhood" anyway, you might as well type **ALPHA** **RSTUV** **S**, to make GAL into GALS.

Next, press EXIT to get back to the editing menu, then **<--** **<--** **<--** **<--** **<--** **<--** to move the cursor over the E in MILE. You want to insert an S, so again: **ALPHA** **RSTUV** **S** EXIT.

And now, of course, you need to DELete that pesky E, but since the cursor is still sitting over it, you just press **DEL**. Now INPUT to make it official, and there you are!

Finally, press **CALC**, to allow the calculator to verify the formula again, and you'll see your modified variable names come up in the menu.

Next Problem: Many service stations sell gasoline by the liter instead of the gallon. Write yourself an equation to help you convert between liters and gallons so that you can still use your mileage formula (there are about 3.785 liters in one U.S. gallon).

Solution: From the SOLVE menu, press NEW and type:

JKLM L RSTUV T RSTUV R RSTUV S [=]
FGHI G ABCDE A JKLM L RSTUV S
[×] [3] [•] [7] [8] [5] [INPUT].
Now CALC ... taa-daa! A conversion formula!

Try It: Another trip uses 40.7 liters in 306 miles, so the mileage is...?

Solution: From your LTRS-GALS menu, press [4] [0] [•] [7] LTRS, then GALS.
Answer: GALS=10.75

Now [EXIT] to SOLVE, *and press* [▲]. Why? Because you now have a collection – a list– of equations, and you move through that list in the same way you move through a STAT list – with the [▲] and [▼] keys ([■] [▲] and [■] [▼], too)!

So CALC with this equation...*Notice...* if you now [RCL] GALS, the gallons are *already set* at 10.75. By spelling the GALS variable the same in each equation, you have actually *shared* that variable between the two; *only one GALS register is created.*

So just key in what you need in order to finish the calculation:
[3] [0] [6] MILS and MPG   Answer: MPG=28.46

Take a look at a memory diagram of your machine now:

Numbered Registers

0
1
2
3
4
5
6
7
8
9

STAT Lists

$/YR
HOURS

Built-In Variables

predicted x
predicted y
OLD
NEW
%CH

Your Formulas

MPG=MILS÷GALS
LTRS=GALSx3.785

Variables For Your Formulas

MPG
MILS
GALS
LTRS

If you're curious about how much more room you have, just press ■ MEM to read off the percentage of the total available memory that's still available....

Did you realize what an incredible arsenal of calculating power you had here? You've got 94% of your memory still available – you've barely made a dent!

Now try another problem, just to get more familiar with the rules of the SOLVE menu....

??: A construction contractor often needs to quote the square-footage area of the rectangular concrete slabs he lays, and he then needs to find the cubic yardage of mixed concrete needed for the job. The only things he knows are the length and width (in feet) of the slab, and its depth in inches – and he can order concrete only in whole cubic yards (no extra fractions of yards). Write two SOLVE formulas to help him. Could you do it in one formula?

»: Hmm...First of all, you get an area by multiplying a length by a width, so the area formula isn't too much of a problem:

AREA=LONG×WIDE

But (happily enough) you also need to know the area to find the volume, right? After all, volume is what you get when you multiply length by width by depth, or – to put it another way – multiply *area* by depth:

VOL=AREA×DEEP

Nothing to this formula business, right? Not really – but here's a caution: These formulas *won't work* if you don't keep your units consistent. That is, you can't expect to come up with a volume in cubic yards by giving the length and width in feet (for example) and the depth in inches. To use the above formulas *as is,* you need to key in all your dimensions *in the same unit* – feet, inches, furlongs, light years, whatever.

Don't mix units! After all what kind of unit is a "foot x foot x inch?" It's not a cubic foot or a cubic inch, or a cubic anything, really. It's some kind of hybrid mix – and they just don't sell batches of concrete measured that way.

So you need to decide the units you're going to use when keying in your length, width and depth.

Suggestion: Make it easy on yourself and let the calculator do some converting for you. Use the units you would most likely use in reality – depth in inches and length and width in feet. And you'll want to see the area in (square) feet and the volume in (cubic) yards.

Now how do your formulas need to look?

Well, your area formula doesn't need to change at all, does it? If you multiply feet by feet, you're going to get an area in (square) feet:

```
FTAR=FTLG×FTWD
```

You've just changed the names of the dimensions to remind yourself what units to use. This is a very good habit to develop as you write more of your own formulas – try to make the name of a variable tell you as much as possible about what it means.

As for the depth formula, it will need a little more "massaging" to get it right. You can convert inches to feet by dividing the number of inches by 12, right?

Like this:

```
VLYD=FTLG×FTWD×INDP÷12
```

But now you have the volume in cubic *feet,* don't you? To convert to cubic yards, you'll need to divide the whole mess by 27 (because there are 27 cubic feet in one cubic yard). Thus:

```
VLYD=FTLG×FTWD×INDP÷12÷27
```

A few words about those equations: You don't need any parentheses here. Do you remember why?

It's because × and ÷ have the same *priority of evaluation* in a formula (recall those operator priorities you saw back on pages 44, 54-55, and 64).

Thus, this SOLVE formula: A=2+3×5

produces the same result as this one: A=2+(3×5)

And it's because the × has a *higher priority of evaluation* than the +.

Here's the list of priorities the HP-27S uses when evaluating a SOLVE formula:

- First come the *functions,* e.g. SQRT(A+B) – "the square root of A+B."
- Next comes *exponentiation,* such as A^5 – "A to the fifth power."
- Next in line are multiplication (×) and division (÷).
- Last of all come addition (+) and subtraction (−).

(And of course, whenever there's more than one operation with the same priority, then the calculator works left-to-right.)

Notice that there's an extra category at the top of this list (compare this list to the summary on page 64). Functions are generally anything that would immediately modify the current number on the Calculator Line – only there's no Calculator Line in a SOLVE equation. Instead, the argument (whatever you want the function to operate on) appears in parentheses right there in the formula. And those parentheses signal that this operation comes *first.* OK?

Now then – back to the problem of those concrete slabs. Here are the two formulas you've developed by now:

FTAR=FTLG×FTWD

VLYD=FTLG×FTWD×INDP÷12÷27

But isn't dividing something by 12 and then by 27 the same as dividing it by 324 (which is 12 x 27)? (righto) Well, then:

FTAR=FTLG×FTWD

VLYD=FTLG×FTWD×INDP÷324

Another thing: Notice that you could *share* the variable called FTAR between these two formulas:

FTAR=FTLG×FTWD

VLYD=FTAR×INDP÷324

See how you substituted FTAR for FTLG×FTWD?

Now you can conveniently use the first equation to quote the area of a given slab, then jump right over to the second equation and compute the yards of concrete necessary – simply by keying in the inches of depth – because the AREA will already be calculated and sitting there!

So that's it then....

...oops...didn't the problem state that the contractor could order concrete loads only in whole cubic yards? (Yep)

You need to round your calculation *up* to the next whole cubic yard (you don't round it down because then you wouldn't have enough for the job), right?

How are you going to adjust the equation for that?

Like this: Take your raw cubic yards requirement, add 1 whole yard, and then keep just the whole-yards portion of the result.

Thus, if your raw needs were 3.4 yards, you'd calculate that to be IP(3.4+1), where IP stands for the *Integer Portion.*

The IP of (3.4+1) is the IP of 4.5, which is just 4. So your calculator would give you an order recommendation of 4 cubic yards, when the slab called for 3.4.

That's correct.

How about 1.9 yards? IP(1.9+1) = 2. That checks, too.

How about 2.0 yards? IP(2.0 +1) = 3.

Hmmm... if the slab is going to need *exactly* 2.0 yards, you don't want to order an entire extra yard....the formula doesn't seem to be perfect yet.

To fix it, just use some common sense: If the raw requirements turned out to be 2.01 yards, would you then order 3 yards? Probably not.

How about for 2.1 yards? ...mmm... – guess you'd better, right?

So change your formula to reflect this judgement – adding just a hair *less* than a whole yard, like this: IP(2.0+.9) That fixes your problem, doesn't it? Therefore:

FTAR=FTLG×FTWD

VLYD=IP(FTAR×INDP÷324+.9)

Key these in and test them out: From the SOLVE menu, press ■ ▼ NEW to begin a new entry at the bottom of your growing list of formulas. Then:

FGHI F RSTUV T ABCDE A RSTUV R = FGHI F RSTUV T
JKLM L FGHI G × FGHI F RSTUV T WXYZ W ABCDE D

Finally, INPUT CALC ....Looks OK on the menu, doesn't it?

Try it: 4 0 FTLG 3 0 FTWD FTAR <u>Answer</u>: FTAR=1,200.00

Looks good. Now EXIT and ▼ and NEW to do the bigger equation...(by now you've surely got a very clear idea about how to key in characters, so instead of trying to read a lot of keystrokes, see if you can type just by looking at the written version of the equation. And keep in mind that the digits, math operations, and parentheses are right on the keyboard anyway):

VLYD=IP(FTAR×INDP÷324+.9)

Then INPUT, CALC, and press RCL FTAR, just to see if your shared FTAR variable is working correctly: FTAR=1,200.00 Right on!

So you've got this slab that's 30 feet by 40 feet. Suppose it's 6 inches deep. How many cubic yards of concrete should you order?

Press 6 INDP and VLYD <u>Answer</u>: VLYD=23.00 Bingo!

So there you have it. Now, to answer the last part of the problem, *could you* put both of these calculations in the same formula?

Stop and think for a minute: Isn't every formula an *equation* (i.e. "something *equals* something else")? Right now, for example, you've been developing these two equations:

```
FTAR=FTLG×FTWD

VLYD=IP(FTAR×INDP÷324+.9)
```

And when you CALCulate with either of these, it comes up on its own menu, of course. But notice that you always solve for one of the variables by using the given values of *all* the others – every one of them is used in the calculation to find the "unknown."

It *seems,* therefore, as if you must have only one "unknown;" the rest must all be "knowns." And that's why it seems that you *cannot* have a single formula for *both* the FTAR or the VLYD – because that would be two unknowns in the same equation.

Ah, but that's where the HP-27S "cheats" a bit – to make such things possible....

There *is* a way to write *two separate, unrelated formulas into one* – so that all the variables of both formulas appear on one menu!

To use this method, first you must hearken back (for one brief shining moment) to those happy, golden days of algebra class, and *rewrite* both of your concrete formulas so that everything is on one side of the = and a zero is on the other:

FTAR-FTLG×FTWD=0

VLYD-IP(FTLG×FTWD×INDP÷324+.9)=0

Remember how to do this ("adding the negative to both sides of the equation")?

And notice that the FTAR is a variable of the first equation only. If you were to keep it as a shared variable, it would be the *unknown* in the first equation and a *known* in the second one. There's nothing wrong with that, but it would mean that you would always have to solve for the FTAR *before* solving for the VLYD, and that would be defeating the purpose here – to allow you to use just one menu to solve for either unknown, *in any order you want.*

Therefore, you'll need to be able to find VLYD from your FTLG and FTWD, which are always *knowns* that you must key in anyway.

This is true in general: If you have a set of equations with one or more shared variables that are normally *unknowns,* then it's not as useful to combine them as it would be if they're all knowns – because it limits the order in which you may calculate.

OK, so you've got these two formulas that you want to somehow combine into one. All you need now is to know about IF(S)....*

*In fact, for the rest of this chapter, you may find it useful to have your owner's manual handy and opened to Table 5-3 – just to get an idea of all the different handy "phrases" in your HP-27S's SOLVE vocabulary.

Here's the way to combine them:

IF(S(FTAR):FTAR-FTLG×FTWD:VLYD-
IP(FTLG×FTWD×INDP÷324+.9))=0

This formula is saying:

"IF you're Solving for FTAR, *then* (and here's the first :)

FTAR-FTLG×FTWD=0

otherwise (IF you're Solving for something *other* than FTAR – this is the second :),

VLYD-IP(FTLG×FTWD×INDP÷324+.9)=0

See? You had to make the two equations equal the same thing (zero) so that you could combine them into one formula with this IF(S... thing.

Ready to try keying this in?

All right, first DELETe your separate FTAR equation: From the SOLVE menu, move the pointer with the ▲ key until it's pointing to your FTAR formula. Then press DELET...

Notice that you then have the option to delete the formula and its variables or just the variables. If you were to choose just the variables, the formula would remain in your list, but none of its variables would exist in the machine's memory until the next time you CALCulated with it, at which time they would be *re-created.* This is just to save memory. Anyway, you definitely want to delete both the formula and its variables here.

You should now be looking at the VLYD formula. Hmm...Since *this* long thing is a pain to key in, why not EDIT it to create your combined formula?

Press EDIT then ALPHA, and type the portion of the combined formula that you need to insert: IF(S(FTAR):FTAR-FTLG×FTWD: (recall that the : is in the OTHER portion of the ALPHA menu. And notice how the display shows you an ellipsis (…) when a formula runs off either end.)

Now press EXIT to return to the editing menu, and move the cursor over 4 places: --> --> --> -->. Replace the = with - (by pressing − and DEL). Next, replace the other FTAR with FTLG×FTWD : --> --> --> --> --> DEL DEL and then ALPHA and type LG×FTWD.

Now EXIT and go to the end of the formula to finish it: -->>) = 0 INPUT
And CALC to check it with your 30- by 40- foot slab:

3 0 FTWD 4 0 FTLG 6 INDP and VLYD   Answer: VLYD=23.00

Now find the FTAR   Answer: FTAR=1,200.00

See? You can now calculate either unknown – in either order you want!

Think you got the hang of this IF(S)... business? Try one more...

For The Road: Remember your gas mileage formula and your liters-gallons conversion formula? Can you combine them into a single formula that allows you to compute your mileage (miles per gallon) no matter whether you buy gasoline in liters or gallons?

A Good Route: Here are two formulas now in your HP-27S's memory.

MPG=MILS÷GALS

LTRS=GALS×3.785

Of course, you could have written the second one as

GALS=LTRS÷3.785

And so what would the formula for mileage be if you bought gasoline by the liter? It would be this:

MPGL=MILS÷(LTRS÷3.785)

How's that? Because gallons are simply "liters divided by 3.785."

Notice that you need to use a second unknown (MPGL) to solve for the mileage in the second formula. That's the pattern of the IF(S) – you have two formulas with two separate unknowns. You rearrange each of them to get zero on the right side, and then – zingo! – you scoop them both into a single formula....

Rearranging:

MPG-MILS÷GALS=0

MPGL-MILS÷LTRS×3.785=0

Now comes the "zingo" part:

```
IF(S(MPG)
 :MPG-MILS÷GALS
 :MPGL-MILS÷LTRS×3.785)=0
```

Key this in: At the SOLVE menu, delete the two separate formulas shown on page 128, by pressing [▲] and [▼] until you're "pointing" to each of them. Then use **DELET**.

Now you're ready to begin a new equation, so press [■][▼] to find the bottom of your list, and **NEW** to begin. Then start hammering away, typing in the above equation....

When you finish, press [INPUT] and **CALC** to test it – try that first mileage problem you did back on page 114:

[3][5][0] **MILS** [1][2] **GALS** **MPG** Answer: MPG=29.17

And what if you went the same distance on 50 liters?

[5][0] **LTRS** **MPGL** Answer: MPGL=26.50

Well, you've probably grasped the idea by now, so it's time to turn you loose at the controls and go places.

Now all you need is lots of practice in building your own equations. And that means thinking in terms of "What-If?".

So try these problems. The first few will simply be more lessons in using the built-in equations you saw in this chapter.

But as for the rest...

well, that's a different story. They're all new problems to solve, like uncharted territory into which you must now fly on your own....

"What-If"
You Had A SOLVE Quiz Now?

1. If you have 112.3 ml of pure H_2O and you boil off 40% of it, how much do you have if you then increase the remainder by 40%? Can you answer this by mere inspection?

2. The most severe drop in the history of the U.S. stock market occurred between September 3, 1929 and July 8, 1932. During that time, one stocks average fell from 452 to 58. If the same percentage drop happened with a beginning average level of, say, 2735, where would the bottom level be?

3. A standard football field is 100 yards long (never mind the end zones). If it's 45 yards wide, and you add 9% to both its width and its length, by what percentage do you increase its area?

4. By what percentage must you *decrease* $\frac{\sqrt{5}+1}{2}$ to get $\frac{\sqrt{5}-1}{2}$?

5. If 18 U.S. states and 3 Canadian provinces meet to decide how to stop acid rain, and a simple majority can veto any proposal, how many ways are there to prevent any action from being taken?

6. What angles (in degrees) form a right triangle with sides of 3, 4, and 5 ? And if one angle is 60° and the hypotenuse is 2, what are the other two sides?

7. Write a unit conversion formula allowing up to four conversion factors.

8. Your friend is an obstetrician with a brand-new HP-27S, and she asked you to help her write two formulas. One formula takes the estimated starting date for a woman's pregnancy and computes her delivery date (280 days later). The other formula uses today's date and the expected delivery date to compute the number of weeks and days the pregnancy has progressed so far.

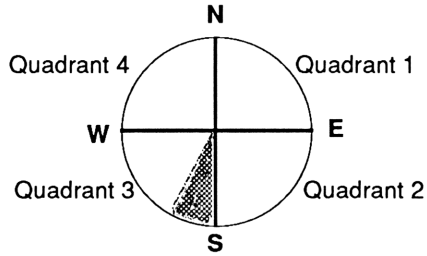
9. You are a lineman for the county, and you drive the main road an awful lot. Fortunately, the county reimburses you for the use of your own car (and fuel) – generally $.21/mile, reimbursed monthly. Write and test a SOLVE equation to compute this reimbursement.

10. You do direct-mail surveys on solid-waste disposal/recycling (yes, you use recycled paper in your flyers). You explain 4 proposals, to which a person then responds on an approval scale of 0 to 10 (0 = "Not on your life!" and 10 = "When can we start?"). So a household response varies from 0 to 40.

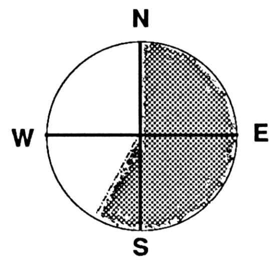
    If the overall *typical* household response is over 25, then the proposals are considered politically feasible, but most households just don't respond, and the volume of responses you *do* receive varies a lot. You have thus observed that this volume itself indicates relative interest (either positive or negative): the higher the fraction of households responding, the more political validity those responses seem to have. So you *weight* each mailing's response average with the volume of its response. Develop and test an equation to calculate a typical household's attitude over four such mailings.

11. Write a formula for numerical integration. Then show that $\int_1^x \frac{1}{x}\,dx = \ln(x)$.

12. In land surveying there are two common notations for horizontal direction. One method is the quadrant-bearing method, where you specify the quadrant and the angle "bearing" into it from the north-south meridian, like this:



The other method simply measures the compass angle or "azimuth" as measured (in degrees, minutes, and seconds) clockwise from north, like this:



Write and test a formula to convert between these two notations.

13. Write and test a formula that will sum the moments and forces acting upon a rigid free body. The formula should analyze two lists of equal length, one being a list of three-dimensional *force* vectors, the other a list of the *distance* vectors (measured to an origin) from which those forces act upon the body.

14. Suppose that in a certain species of codfish, each male-female pair spawns about 20 billion eggs per year. If an average fish is about 0.1 cubic feet in volume, write a formula to compute the current volume of fish and the volumetric growth rate of this species' population if it all started "y" years ago with one male and one female and *all* of the spawned offspring lived for just one year – and reproduced once.

Bonus question: What might this do to the market price of this fish?

"What-If" These Were The Answers?

1. Use the %CHG menu: Press ■ [%CHG], then [1][1][2][•][3] OLD [4][0][+/-] %CH then NEW to get the reminder after the boil-off: NEW=67.38

   Now this becomes the OLD amount for the second part of the calculation: [STO] OLD. Next, change your %CH to reflect an *increase* this time: [4][0] %CH. Then press NEW. Result: NEW=94.33.

   The moral of this story: Subtracting x% from a number and then adding back x% of the result does *not* get you back to where you started from, because the two *percentages* are the same, but they're based on different *numbers*. (See pages 103-107 to review %CHG calculations.)

2. Again, you need to recognize that this is a %CHG problem. So, from the %CHG menu, press [4][5][2] OLD [5][8] NEW %CH.

   There's the percent change of the 1929-1932 crash (notice that it's negative because the change was downward): %CHANGE=-87.17

   Next, key in the 2735 to reflect the hypothetical high level: [2][7][3][5] OLD Now see what the corresponding low level would be. Press NEW

   NEW=350.95*

*"Toto, I don't think we're in Kansas anymore...."

3. First, you'd better figure out what the starting area is:

[4][5] [×] [1][0][0] [=] Answer: 4,500.00

Next, you need to find the dimensions of the enlarged field:

[(][1][0][0] [+] [9] ■ [%] [)] [×] [(][4][5] [+] [9] ■ [%] [=] Answer: 5,346.45

Now you need a %CHG calculation, so go to the %CHG menu and press: [STO] NEW (since your NEW area is now sitting there on the Calculator Line);

Then press [RCL] ■ [LAST] OLD (that's the starting area that was floating up above in the History Stack);

Finally, %CH gives you the increase in area: %CHANGE=18.81

(If you had *decreased* it, the %CH would have been negative).

4. Again, the strategy is to calculate the two values you're going to compare, then use the %CHG menu to get the final answer:

Press [(][5] ■ [√x] [+] [1] [)] [÷] [2] [=] (1.61803398875 – set ■ [MODES] ALL),
and [(][5] ■ [√x] [−] [1] [)] [÷] [2] [=] (0.61803398875)

Now, at the %CHG menu and press [STO] NEW ■ [LAST] OLD %CH

Answer: %CHANGE=-61.803398875

(Notice anything unusual about these numbers?)

5. If you stop and think about this, what you're really asking is "How many different *combinations* of votes are there that would defeat a proposal?" That is, you need to add together the sum of all the different possible combinations of 11 "no" votes, 12 "no" votes, 13 "no" votes, etc.

No doubt about it – you need the PROB menu (so get there: press ■ PROB). Now then: To find out how many ways 21 votes could produce 11 "no" votes, press 2 1 X 1 1 Y and C X,Y. <u>Result</u>: C X,Y=352,716 (See page 110 if you want to review the PROB menu.)

Store this as your running total of different possible combinations: STO 0

Now do the same for 12 "no" votes: 1 2 Y C X,Y (C X,Y=293,930); and add these two results: STO + 0 (did you realize that you could add to a storage register *directly* like this – a handy thing to know, eh?).

Keep going for the remainder of the possible "no" outcomes, each time adding the result to your total in Register 0:

1 3 Y C X,Y (C X,Y=203,490) STO + 0
1 4 Y C X,Y (C X,Y=116,280) STO + 0
1 5 Y C X,Y (C X,Y=54,264) STO + 0
1 6 Y C X,Y (C X,Y=20,349) STO + 0
1 7 Y C X,Y (C X,Y=5,985) STO + 0
1 8 Y C X,Y (C X,Y=1,330) STO + 0
1 9 Y C X,Y (C X,Y=210) STO + 0
2 0 Y C X,Y (C X,Y=21) STO + 0
2 1 Y C X,Y (C X,Y=1) STO + 0

Now see the total: RCL 0: 1,048,576

There are literally more than a million ways to bollix things up.

6. One more "quickie" involving some built-in menus:

Now, this *is* a trig problem, obviously, but before you go pounding on those second-row keys, recall that this kind of conversion – back and forth between polar and rectangular notations – is available and at your fingertips in the CONVERT menu (see pages 108-109 to review this).

So press ■ (CONVERT) **MORE** and have at it:

First of all, be sure you're set to degrees mode (i.e. be sure that the RADians annunciator is *not* showing in the display; press **D/R** if necessary).

Then (3) **XCORD** (4) **YCORD** and **∡** to see: ∡=53.1301023542

For the other (non-right) angle, just reverse the coordinates, like this: (4) **XCORD** (3) **YCORD** and **∡**. You'll get: ∡=36.8698976458

Now go the other way, specifying an angle and radius to find the corresponding X and YCOoRDinates:

(2) **R** (6)(0) **∡**,

XCORD <u>Result</u>: XCORD=1

YCORD <u>Result</u>: YCORD=1.73205080757

Not too surprising, right? After all , you've just given the HP-27S a 30-60-90 triangle, so it gave you that well-known relationship among the three sides:

"If the hypotenuse is 2, then the shorter leg is 1 and the longer leg is $\sqrt{3}$."

7. "Units, units, blah, blah.... What's the big deal about units? Everybody knows how to convert between miles and feet, etc. Who needs a formula?"

Yes, but do you know how to convert from meters per second to, say, furlongs per fortnight?

In general, it's not just a one-time, easy-to remember, one-number conversion factor – because the units you're using aren't always simple, single dimensions such as length or time; sometimes they're combined in all sorts of bizarre but useful(?) ways.

And do you remember the sure-fire technique for converting units correctly? You multiply them in a chain of fractions, like this:

$$\frac{1 \text{ meters}}{1 \text{ seconds}} \times \frac{1 \text{ furlongs}}{201.168 \text{ meters}} \times \frac{1{,}209{,}600 \text{ seconds}}{1 \text{ fortnights}} = \frac{6{,}012.88\ldots\text{furlongs}}{1 \text{ fortnights}}$$

And you can always tell when you've done it correctly, because *all the units except the ones you want will "cancel themselves out:"*

$$\frac{1 \cancel{\text{meters}}}{1 \cancel{\text{seconds}}} \times \frac{1 \text{ furlongs}}{201.168 \cancel{\text{meters}}} \times \frac{1{,}209{,}600 \cancel{\text{seconds}}}{1 \text{ fortnights}} = \frac{6{,}012.88\ldots\text{furlongs}}{1 \text{ fortnights}}$$

Treat the unit names themselves as if they were numbers, and you should find that each unwanted name appears twice – once in the numerator and once in the denominator. Since the whole mess is a string of fractions being multiplied, you can "cancel like names" in the numerator and denominator. This leaves you with only the numerical factors to crunch – voila!

So the idea of this problem is to illustrate this and provide a quicker way to convert between such compound units.

(Of course, it isn't going to save you any of the trouble of looking up the conversion factors for the individual simple units involved, but such is life.)

According to the problem as stated, you should allow for up to four intermediate conversion factors, as in this conversion of 60 mph:

$$\frac{60\text{ Miles}}{1\text{ Hours}} \times \frac{24\text{ Hours}}{1\text{ Days}} \times \frac{7\text{ Days}}{1\text{ Weeks}} \times \frac{2\text{ Weeks}}{1\text{ Fortnight}} \times \frac{8\text{ Furlongs}}{1\text{ Miles}} = \frac{161{,}280\ldots\text{Furlongs}}{1\text{ Fortnight}}$$

Here are your beginning units; Here are your ending units;

and the four fractions in between are your four intermediate factors.

If you called these four factors A, B, C, and D, then you would read this entire conversion as "60 Miles per Hour times A Hours per Day, times B days per Week, times C Weeks per Fortnight, times D Furlongs per Mile equals how many Furlongs per Fortnight?"

So what's the big deal, right? Your equation is simple arithmetic:

$$U2=U1\times A\times B\times C\times D$$

And you could add more factors or change their names if you wanted, right?

(Are you getting this sneaking suspicion that the point of all this wasn't necessarily just the equation-building – that this was a quick reminder of how to do unit conversions in general?)

8. This is another one of those combination-formula problems – where you have two distinct formulas, each with its own single unknown.

As usual, your strategy is to develop each formula separately – in the proper format, with everything on the left and a zero on the right– and then combine them with the IF(S) language.

OK, but how do you get the HP-27S to use a calendar, anyway?

Simple: Use the calendar functions it has in its SOLVE vocabulary: For example, DATE(12.191959 : 44) is a phrase that means this:

"The date which is 44 days after December 19th, 1959"

(Remember the format with which you set the time – 'way back at the beginning of the course? You key in dates in MM.DDYYYY format, where the MM are the digits representing the month, the DD the day of the month, and the YYYY the year).

So the first formula isn't very tough at all:

DUE=DATE(START:280)

or, in proper ready-to-be-combined format:

DUE-DATE(START:280)=0

As for the second formula, that's a bit tougher.

Fortunately, there's another phrase in the HP-27S's calendar vocabulary. It works like this:

DDAYS(12.191959:3.311960:1)

This is the Difference in DAYS (the number of days) between the two given dates, according to one of three possible calendars: Calendar 1 (as in this example) is the actual calendar – leap years and all; Calendar 2 doesn't recognize leap years; and Calendar 3 assumes that a year is made up of 12 identical 30-day months.

Of course, you and your obstetrician friend are interested in reality, so you'll be using Calendar 1 in your formula.

Now, if a normal term of pregnancy is 280 days, then if you found out how many more days the pregnancy was expected to go (from TODAY) before the DUE date and subtracted that number from 280, you'd have the number of days it has progressed so far, correct?

OK, and how would you write this? How about this?

PROG = 280 - (DDAYS(TODAY:DUE:1))

That will work just fine – except that it's going to give you the number of *days* the pregnancy has progressed up to now. That isn't quite what your friend needs; she'd like this to be expressed in terms of *weeks and days.*

That's different....

How are you going to produce two pieces of information from the single number (the number of days, PROG)?

You're going to take a lesson from your HP-27S – cheat a little.

Notice how the machine can actually get *three* pieces of information out of one number when it's expressing a date: 12.191959. In this one number, which is just an untidy little fraction between 12 and 13, you can represent the month, day and year.

Well then, why couldn't you convert your PROG days into a number of the format WW.D, where the WW would represent the number of whole weeks progressed so far, and the D would give the odd days (no more than 6, right)?

Your doctor friend could then easily read off each part of the answer – especially if you name it W.D as a reminder.

So "W.D = something" is your second formula.

Next problem: How do you actually do the conversion?

No problem: Dipping once again into your machine's seemingly bottomless bag of SOLVE tricks, you come up with IDIV and MOD.

IDIV is Integer DIVision, where the first number is divided by the second, but any remainder is thrown out. For example,

IDIV(10:3) = 3 and IDIV(13:5) = 2

So you can easily extract the number of whole weeks contained in PROG days:

W = IDIV(PROG:7)

And what about the odd days? That's what MOD is all about. MOD gives the *remainder* of any division:

MOD(10:3) = 1 and MOD(13:5) = 3

Your odd days, D, will therefore be D = MOD(PROG:7)

OK, are you all set *now?*

Not quite. Your final number must be of the form W.D, so you need to divide D by 10 (to get it beyond the decimal point), then add this to the W to get the correct format. Your final second formula will therefore look something like this:

W.D = IDIV(PROG:7) + (MOD(PROG:7) ÷ 10)

Now replace PROG with what it really is: 280 - (DDAYS(TODAY:DUE:1)) ; and here's your second formula:

```
W.D=IDIV(280-(DDAYS(TODAY:DUE:1)):7)+
 (MOD(280-(DDAYS(TODAY:DUE:1)):7)÷10)
```

Now format it so that you can properly put it into a combination formula:

```
W.D-IDIV(280-(DDAYS(TODAY:DUE:1)):7)-
(MOD(280-(DDAYS(TODAY:DUE:1)):7)÷10)=0
```

Ready to combine this and the simple DUE-date-prediction formula (shown again below)?

```
DUE-DATE(START:280)=0
```

Here goes nothing:

```
IF(S(DUE)
:DUE-DATE(START:280)
:W.D-IDIV(280-(DDAYS(TODAY:DUE:1)):7)-(MOD
(280-(DDAYS(TODAY:DUE:1)):7)÷10))=0
```

It's broken up here so that you can see the two alternatives to the IF(S) more easily. Remember how that works: IF you press **DUE** to Solve for the DUE date, the first equation (after the first :) will be used; if you solve for anything else, the second equation (after the second :) will govern.

But in any case, the variables for both will appear on one menu.

Go for it. ☞

Here it is again so that it's easier to type from:

```
IF(S(DUE)
:DUE-DATE(START:280)
:W.D-IDIV(280-(DDAYS(TODAY:DUE:1)):7)-(MOD
(280-(DDAYS(TODAY:DUE:1)):7)÷10))=0
```

Start in the usual way – at the SOLVE menu with a fresh formula at the bottom of your list (press ■ ▼ **NEW**). Then type away!...When you're through, press INPUT, of course, then **CALC** and try a couple of problems (you might want to set the display to FIX 6 for these):

If a woman's due date is thought to be April 22, 1988, and today is November 25, 1987, how many weeks and days is she into her pregnancy?

Press 4 • 2 2 1 9 8 8 **DUE** 1 1 • 2 5 1 9 8 7 **TODAY**, then **W.D**

<u>Answer</u>: `W.D=18.500000`

She's 18 weeks and 5 days into her pregnancy.

If a revised estimate puts the start of the pregnancy at July 11, 1987, what is a better estimate of the due date?

Press 7 • 1 1 1 9 8 7 **START** **DUE** <u>Answer</u>: `DUE=4.161988`

A good guess for a birth day is April 16, 1988.

9. The actual arithmetic for figuring your monthly reimbursement is pretty trivial, isn't it?

$RMB=MILS x .21

The real problem is to figure out how to keep track of that mileage all through the month, right?

To get a handle on this, talk yourself through an example. Formulas like these can come to you easily if you take a little time, sit down with a pencil and paper, and talk to yourself. After all, before you can tell the calculator how to do it, you have to decide how you would do it manually – on paper, right?

"Hmm...I'd use the odometer on the car...and every time I made a business-related trip, I'd have to note the starting mileage, the ending mileage – and then subtract the two to get the net reimbursable mileage.

"But how many trips will I take in any month? I don't know – it varies. So I can't exactly use an equation like this:

MILS=(END1-START1)+(END2-START2)+(END3-START3)...

because I don't know how long it has to be!"

It *is* a problem, isn't it?

But wait...isn't this kind of sequential record-keeping problem just tailor-made for using *lists of numbers* (i.e. from the STAT menu)?

Great news: *You can use lists in SOLVE formulas!!*

Picture this: You have two lists, named START and END. ITEM(1) in your start list is your odometer reading at the start of your first business trip of the month; ITEM(1) in your END list is the reading at the end of that trip, etc.

You simply sum up each list, subtract the total of the START list from the total of the END list, and there's your mileage for the month!

After figuring your reimbursement amount, you can clear each of the lists (or start new ones with different names if you need to keep your records for awhile).

So how do you refer to and use STAT lists in a SOLVE formula? Like this:

ITEM(START:3) would denote the third entry in your START list; ITEM (END:1) would mean the first entry in your END list, etc. Get the idea? This is the SOLVE menu's language for referring to items in a STAT list.

OK, but how do you sum all these items? You don't even know how long each list is – the same problem that made just a plain old formula out of the question.

Aha! Also in the SOLVE menu's vocabulary is the phrase SIZES(). It understands that to mean the *current length* of the list given in the parentheses. Thus, SIZES(START) would stand for the number of items in your START list, for example.

"Fine, but how do I sum all of these items – even if I know how many there are?"

How sweet it is: There's also a phrase for summing! (They sure have thought of everything, haven't they?)

To use this SOLVE language to sum all the items in your END list, for example, you would do it like this:

TOTAL=Σ(N:1:SIZES(END):1:ITEM(END:N))

Here's how your machine "reads" this:

TOTAL is going to be the result of a sum of numbers. That sum uses the variable N as its counter. In other words, since it's supposed to sum only a certain number of items, the N is how it remembers which item it's *currently adding* to its sum.

The summing should start with N at 1 (this is the first 1 inside the parentheses), and continue until N is equal to SIZES(END), which is the length of the list called END; and N should proceed 1 at a time (as opposed to skipping every other item or something like that). That's the meaning of the second 1.

Notice how each item in this summation command is separated by a :
– a fairly standard thing with SOLVE, as you've discovered by now, right?

Finally, the items to be summed are ITEM(END:N), *which will be every item in the END list, as N proceeds from 1 to the length of that list,* right?

There you have it – the total of the entire list!

So, knowing all that, how would you sum the *difference* between corresponding items in two lists? Like this:

```
Σ(N:1:SIZES(END):1:ITEM(END:N)-ITEM(START:N))
```

(You can use the SIZES of the END list as the length of both the END and the START lists, because they'll be the same length, right?)

Therefore, here's how the final reimbursement calculation should look:

```
$RMB=.21×
Σ(N:1:SIZES(END):1:ITEM(END:N)-ITEM(START:N))
```

Be sure that all your parentheses match! This looks like a mess, but you now know how it all works, right? You've simply said – in SOLVE language – a statement that goes like this:

"Reimbursement is equal to twenty-one cents times the *sum of the differences* of each pair of corresponding END and START odometer readings as they appear in the two lists named such."

So key it in and try a problem:

From the SOLVE menu, press ■▼ to find the bottom of your list, and **NEW** to begin. Then type in the above equation (and remember that any strange, non-alphabetical characters will probably be hiding in that **OTHER** menu)....

Now press INPUT,then **CALC**. Incidentally (as you may have already discovered), if the machine beeps at you and rejects your equation, it automatically goes into the edit menu and points with the cursor at what it doesn't understand – so you can correct the typo with a minimum of searching or rekeying!

All set? Then it's time to test it!

Suppose you had 4 trips in a given month, and that the odometer readings were as follows:

Trip	START Reading	END Reading
1	45,678.9	46,111.0
2	47,142.8	47,376.5
3	48,123.4	48,571.4
4	49,012.3	50,987.6

How much should you be reimbursed?

First, you'd better establish your two lists.

Press ■ STAT GET *NEW, then

4 5 6 7 8 . 9 INPUT
4 7 1 4 2 . 8 INPUT
4 8 1 2 3 . 4 INPUT
4 9 0 1 2 . 3 INPUT EXIT NAME START (type this) INPUT

Next, GET *NEW and

4 6 1 1 1 INPUT
4 7 3 7 6 . 5 INPUT
4 8 5 7 1 . 4 INPUT
5 0 9 8 7 . 6 INPUT EXIT NAME END (type this) INPUT

Now head for the SOLVE menu (■ SOLVE) and "point" to the $RMB formula, CALC and $RMB (you may need to press $RMB twice to get it to calculate).

Answer: $RMB=648.711

(Ain't it wunnerful?)

10. What kind of computation are you going to do here? It's a weighted mean, right? Therefore, you actually have several options with your HP-27S....

First of all, *were you aware* that you can compute a weighted mean directly, by building two parallel STAT lists? The first list would give the values themselves; the second list would give the corresponding frequencies ("weights") for each of those values. And the FRCST menu has a selection called W.MN ("Weighted MeaN") that will give you just that, using the first (x) list as the values and the second (y) list as the weights (make a note to yourself to explore this on your own, if you wish).

But that's probably not the best choice for this problem.

Why not?

Because part of the usefulness of the "What-If?" idea is that you can play with *any* parameter in the problem – including the weighted mean itself. That is, you might want to specify a certain *result* and then backtrack to see what level of response was necessary in any given one of the four mailings.

By contrast, if you use STAT lists and the built-in W.MN calculation to try to do this, you can only guess at the response(s) necessary to achieve a specified result – a trial-and-error process that is unnecessary with "What-If?"

Sure, you used STAT lists with your mileage reimbursement problem, but with that particular problem, nobody was very interested in playing "What-If?" with your mileage. That equation was built merely as a way to sum and summarize the records kept in those STAT lists. You want to be able to do a little more hypothesizing here.

So that's the case for using an equation of your own design.

Now it's time to envision all the different variables you're going to need.

First of all, remember that there are exactly four mailings (which is another good reason why you can decline the use of STAT lists here: if there were 20 mailings or an unknown number of mailings, it would be totally unworkable to try to put everything into one equation; you'd *have* to use lists and the list notations – SIZE, ITEM, etc.).

For each mailing, you're going to need to know these things:

- How many flyers you sent out ("OUT");
- How many responses you got back ("IN");
- The summed response point total (1-40) of the responses ("PTS").

- And of course, you'll need your final result, the weighted mean of all four mailings ("SCORE"), which will indicate feasibility if it's above 25.

So it looks as if you'll have about 13 variables in your equation:

OUT1	IN1	PTS1
OUT2	IN2	PTS2
OUT3	IN3	PTS3
OUT4	IN4	PTS4

SCORE

All right, now what math is involved here? What is a weighted mean?

Well, what's a normal mean? You find it by summing together a set of values and then dividing by the number of values you summed like this:

$$\begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ +\ 5 \\ \hline 15 \end{array} \div 5 \text{ values} = 3.0 \text{ (the simple mean)}$$

Each value is summed *once.* That is, each number carries a *weight of 1.*

In a weighted mean, it's as if the values appear *more than once;* when you're summing, you repeatedly add each value a number of times, and that number is its weight:

Value	Weight	is the same as
1	2	1 + 1
2	4	2 + 2 + 2 + 2
3	1	3
4	2	4 + 4
5	1	+ 5
	Total:	26

÷ 10 values = 2.6 (the weighted mean)

So to get a weighted mean, you multiply each value by its weight, then add these products and divide by the sum of the weights. You can see that if all weights are just 1, this gives you just the simple mean, right?

So how does all this translate into an equation for your direct-mail survey?

Your raw values are going to be your averages for each "nth" mailing: (PTSn÷INn) – right?

And your weights will be the fraction of households responding to each mailing: (INn÷OUTn) – right?

So (as you saw on the previous page) to get the weighted mean, you sum the product of each value and its weight, then divide by the sum of the weights:

```
SCORE=
((PTS1÷IN1)x(IN1÷OUT1)+(PTS2÷IN2)x(IN2÷OUT2)
+(PTS3÷IN3)x(IN3÷OUT3)+(PTS4÷IN4)x(IN4÷OUT4))
÷((IN1÷OUT1)+(IN1÷OUT1)+(IN1÷OUT1)+(IN1÷OUT1))
```

Here all the parentheses have been left in so that you can identify each value and weight. But (happily), knowing what you do about the priority of operations of the HP-27S, you can omit a lot of them. Not only that, you'll notice that some of the terms simply cancel each other out. For example, (PTS1÷IN1)x(IN1÷OUT1) simplifies to just PTS1÷OUT1, doesn't it?

So after you do this simplifying and omitting of parentheses, things look a lot better – good enough to key in (go for it – press ■ SOLVE ■ ▼ **NEW**, then):

```
SCORE=
(PTS1÷OUT1+PTS2÷OUT2+PTS3÷OUT3+PTS4÷OUT4)
÷(IN1÷OUT1+IN2÷OUT2+IN3÷OUT3+IN4÷OUT4)
```

Notice, by the way, that this is just a single equation; you don't need to mess with any of that IF(S.... business this time!

Time to test it (press [INPUT] after you've keyed it in, then **CALC**). Notice that when the menu appears, the variables are listed in the order they appear in the equation itself – food for future thought and planning. Now then:

	Flyers Sent (Out)	Responses Received (In)	Total Attitude (Points)
1	20,000	651	14,895
2	75,000	4,073	112,653
3	48,000	1,177	39,448
4	35,000	1,435	27,542

Suppose that these were your results of four mailings.

Questions:

- What was the overall attitude per household?
- To complete a barely-feasible overall score of 25.0, how many incoming responses were necessary in mailing number 4 (no change in total points)?
- If you had sent out 10,000 more flyers (but with no more returning) in mailing number 2, how would this have affected the overall score?

First things first – find the overall resulting score for these four mailings:

[1][4][8][9][5] **PTS1** [2][0][0][0][0] **OUT1** [1][1][2][6][5][3] **PTS2** [7][5][0][0][0] **OUT2** **MORE** [3][9][4][4][8] **PTS3** [4][8][0][0][0] **OUT3** [2][7][5][4][2] **PTS4** [3][5][0][0][0] **OUT4** [6][5][1] **IN1** **MORE** [4][0][7][3] **IN2** [1][1][7][7] **IN3** [1][4][3][5] **IN4**.

Then **MORE** to wrap back around to the first page of the menu and **SCORE** to calculate....Result: `SCORE=25.3025388856`

Looks like you can probably start some recycling.

Now play "What-If?" with that fourth mailing:

Press [2][5] SCORE, then MORE MORE and IN4.

Result: `IN4=1,499.54016667`

It would have taken about 1,500 responses on this 4th mailing to lower the overall score to exactly 25.

More responses to *lower* the score? Yes, because this mailing brought back a per-piece point total that was lower than average. Thus, lending any *more weight* to this mailing (through added responses) – without changing that point total – *lowers* the overall score.

And what if the second mailing had sent out 10,000 more no-response flyers?

First, change the other data back to the true numbers: [1][4][3][5] IN4. Then MORE and [1][0][0][0][0] [STO][+] OUT1 (after all, these variables are just registers that you can use for arithmetic, right?).

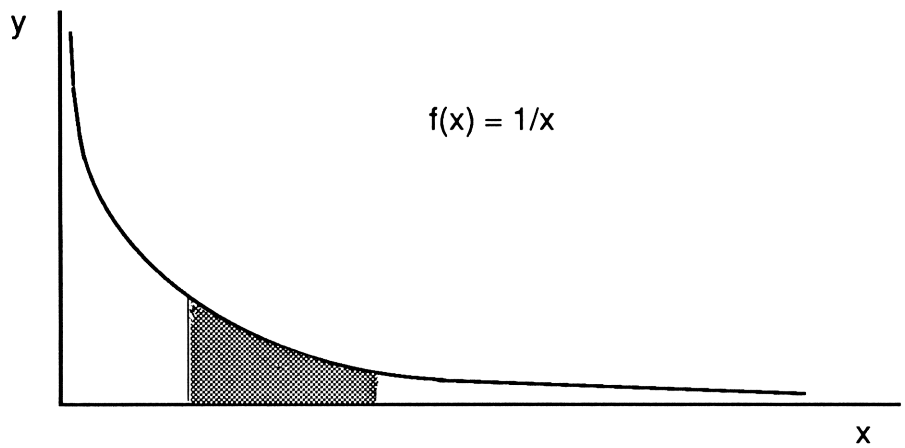
Finally, SCORE to get: `SCORE=25.4882451752`

Does this make sense? A slight *rise* in the score – caused by a *drop* in the fraction responding to one of the mailings??

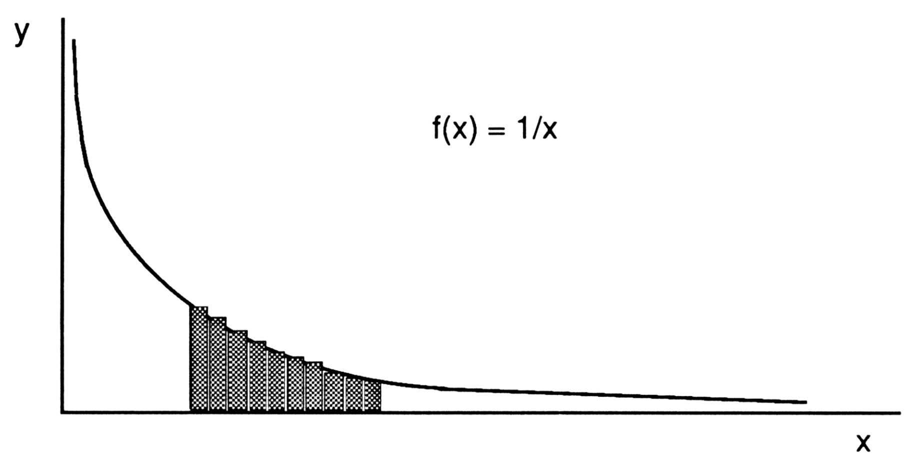
Yep, it can happen. Notice the two places in the formula where OUT1 appears: As a part of the numerator and a part of the denominator. It just worked out that its increase had a bigger effect on decreasing the denominator than decreasing the numerator. Think about it....

11. Numerical integration, eh? Hmmm....

Remember the principal behind integration? In the simple, single-variable form, it's the way to find an area beneath the curve of a function:



And you can approach this numerically by dividing the horizontal distance to be covered into small slices, compute the approximate area of each slice, and add all these approximate areas together. The smaller the slices, the more accurate the approximation:



In the limit, as you chop up the area into thinner and thinner slices, the width of each slice approaches zero and the number of them approaches infinity. You then have the actual integral.

Of course, you can't actually divide an area into an infinite number of slices, so any numerical integration (i.e. done without symbolic calculus) must necessarily be approximate.

All right, fine. So how do you go about computing the area of any given slice? It shouldn't be all that tough, should it? After all, each slice is just a rectangle, so all you need are its width and height.

The width is simple: Take the *horizontal distance* that you're slicing up and *divide it by the number of slices.* Thus, if you're "integrating" from "a" to any number, "b," by dividing that area up into "n" slices, then the width of *any* such slice is just (b-a)/n

What about the height of each rectangle? That's not necessarily going to be the same for any two slices, is it? In order to best approximate the area under the curve, the top of each rectangle should touch the function's curve, of course, but *which part* of the rectangle should touch? The upper left corner? The upper right corner? The middle of the top?

Looking at the diagram on the previous page, it's the upper left corner of each rectangle, and you can see that in this case, this will result in a slight overestimate of the area under the curve (because the upper right corners extend above the curve). In other functions (e.g. those that are increasing – going "uphill" in this interval), it might have been an underestimate.

Well, some ways are better than others, because for a given number of slices (n), they tend to give more accurate estimates of area. But all techniques agree on one thing: The more slices you make, the better the estimate becomes. If you're not happy with any estimate, you can keep your technique and just re-compute it with an increased number of slices. So just pick a simple one here – the one pictured – where the height of the rectangle is established by its touching the function with its *upper left corner.*

So that means that the height of any given "jth" slice is the value (the "height" or "y-coordinate") of the function, f(x), at the horizontal position of the left side of that jth slice.

Well, what is that x-position? If you think about it for a minute, you'll agree that it's this:

$$x_j = a + (b\text{-}a)/n \times (j\text{-}1)$$

a is your starting x-position, and from there you want to move over a certain whole number of slice-widths, where each such width is (b-a)/n. And for the jth slice, you move over j-1 slice widths (because you don't move at all for slice number 1), right?

So the *entire* area is going to be this sum of rectangle areas:

$$A = \sum_{j=1}^{n} (b\text{-}a)/n \times f[a+(b\text{-}a)/n \times (j\text{-}1)]$$

You can see the width and height components clearly now, can't you? And keep in mind that f [ ] means "compute the function value at the x-coordinate given by what's inside the [ ]." In the problem you're trying to solve here, f(x) is 1/x.

So now the only task remaining is to translate this nice, tidy mathematical expression into something your HP-27S can understand and crunch for you in a SOLVE equation.

But you already know how to do summations with Σ notation (from the mileage reimbursement problem). So this one ought to be a snap...

...and it is....

AREA=Σ(J:1:N:1:(B-A)÷N÷(A+(B-A)÷N×(J-1)))

Remember how this notation works? You're saying, essentially, "Compute the sum of the terms described in general by the following mess, using J as a counter and counting from 1 to N, in increments of 1." And notice how the formula has been slightly simplified after "plugging in" the function, 1/x.

Now to key this in and test it (from the SOLVE menu, press ■▼ and NEW, then type away)....

When you're finished, press INPUT and CALC, and there you have it – a quick and dirty way to compute the area under the curve 1/x between any two x-coordinates, A and B, by slicing up this area into N rectangles of equal width.

You now need to verify that this area gives the natural logarithm (LN) of any number, x, if you use 1 as the lower limit (A) and x as the upper limit (B). What's a good choice for x? How about the natural log base, e? You know that the natural log of e is, by definition, 1, right? So if you compute

$$\int_1^e \frac{1}{x}\,dx$$

with a sufficient number of those rectangular slices, you should be able to come pretty close to 1.00000000000. Try it with, say, 10 slices:

1 A ■ e^x B 1 0 N AREA. Result: AREA=1.0564284611

That's a bit high (but from the way you chose to draw your rectangular slices, you knew it would be). Not close enough to make you a believer? OK...

Try 100 slices: 1 0 0 N AREA. Result: AREA=1.00545207993

It takes longer, but it's a lot more accurate. And if you've got the time, you can get a lot closer.

12. An simple, innocent-looking little problem, right? Uh...well...take a look:

First of all, if you know the AZimuth (AZ), then that one number is enough to let you calculate *either* the BeaRiNG (BRNG) or the QUADrant (QUAD). But the reverse is tougher: In order to calculate AZ, you need to use *both* the BRNG *and* the QUAD.

So it looks like you're going to have one of those IF(S... combination equations – only this one is a *three*-way combination, since you'll want to be able to solve for any one of those three numbers: AZ, QUAD, or BRNG.

And how are you going to do that? With *nested* IF's – that's how. For example, you could approach it like this:

```
IF(S(AZ)
   :solve for AZ
   :IF(S(QUAD)
       :solve for QUAD
       :solve for BRNG))
```

Your HP-27S would interpret this just as you would expect:

"IF I'm Solving for AZ
    then I solve for AZ
    else IF I'm Solving for QUAD
            then I solve for QUAD
            else I solve for BRNG."

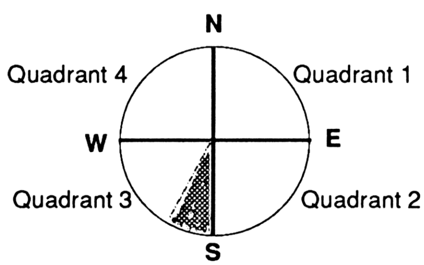
Remember how the colons (:) are used to signal the **then** and **else** portions of a simple IF statement? It's no different here; it's just that the **else** portion of the first statement happens to contain another entire IF-**then**-**else** statement. This is called a "nested" IF statement, since there's one inside another.

So you have the general form of the solution decided. Now how exactly do you solve for each of these three numbers?

Take the lonely-only AZ first: You need to transform the numbers BRNG and QUAD into a meaningful AZ – and don't forget that AZ and BRNG are angles that must be in degrees, minutes and seconds.

Stumped for a way to start?

Take another look at what you know:



The angle shown here is in QUADrant 3. Now, if you suppose just for laughs that its BRNG is 30°, then you could immediately figure the AZ by inspection: 180 + 30 or 210° – no sweat, right?

But how did you know that? You looked at the two full quadrants that this azimuth angle had already swept through (quadrants 1 and 2), and in your mind you allowed for 90° per complete quadrant – and that's exactly right. That made 180°, then adding the additional BRNG angle (30°) gave you the true AZ, 210°.

To sum it up: If the QUAD was 3, that meant the AZ was 3-1 = 2 full quadrants of 90° each, plus the BRNG angle.

So it looks like an easy formula for AZ is this:

AZ = (QUAD-1)x90+BRNG

Ah, but does this work for all four quadrants?

Try QUAD = 2 and BRNG = 30. The correct AZ is 150°, but the above formula gives you 120°.

Not so good.

It turns out that your formula works only for quadrants 1 and 3. You need to do something different for 2 and 4.

But that actually makes sense, doesn't it? After all, a BRNG is measured *clockwise* in quadrants 1 and 3, but *counterclockwise* in quadrants 2 and 4. No wonder, eh?

Well, what would work in the other two quadrants? How about this?

AZ = (QUADx90-BRNG)

"Mmm...could be, could be....let's see...2 times 90 minus 30 is...150....– yep, it checks – and it's even simpler than the other formula, too!"

Sometimes, things just work themselves out – despite everything you do.

Now you have just two things left to do with this AZ calculation:

- Put these two pieces together with an IF test to determine which quadrant you're dealing with;

- Convert the given BRNG to decimal degrees with the HRS function and the resulting AZ back to degrees, minutes and seconds with the HMS function.

And after doing those things, you *might* come up with something that looks like this:

```
AZ=IF(QUAD=1 OR QUAD=3:HMS((QUAD-1)x90+HRS(BRNG))
                        :HMS(QUADx90-HRS(BRNG)))
```

You probably haven't seen or used this kind of test before, with an = in it, have you?

Well, the IF(S... isn't the only kind of IF test you can have; there's a whole slew of different tests you can perform (e.g. with =, < , > , etc.).

But in every case, the idea is the same: you state the test (or tests – you can bunch them up with AND's and OR's – like this one), then give two options, the first of which is done only if the answer to the test is "yes;" the second option is done only if it's "no."

So that about wraps up the first of three cases – solving for the AZ – except for one minor adjustment:

Since you're going to combine all three cases eventually, you'd better put this in its =0 form, so that it's easier to splice together with the others when the time comes:

```
AZ-IF(QUAD=1 OR QUAD=3
     :HMS((QUAD-1)×90+HRS(BRNG))
     :HMS(QUAD×90-HRS(BRNG)))=0
```

(You're not exactly looking forward to keying this in, are you?)

(Yep, it's a dirty job, all right, but once it's in there, it's in there.)

Now then...what about solving for QUAD – using a known AZ?

That ought to be simple enough: Again, the idea is to look at how many whole quadrants AZ sweeps through – but this time you want to add one to that number, to indicate the partial quadrant where the AZ actually lies. So you divide AZ by 90 degrees and then *round up,* don't you?

Thus, for example, an AZ of 107° would be 107÷90 = 1.1889, which rounds up to 2. 107° is indeed in quadrant 2; it looks like this is a good method.

But how do you go about rounding a fraction up to the next whole number? There just doesn't seem to be any ready-made SOLVE function to do that.

One good way is to round *down* – by taking the Integer Portion (remember IP?) – and then adding 1. So you could use this formula:

QUAD=IP(HRS(AZ)÷90)+1

(Don't neglect the fact that you need to convert AZ from degrees, minutes and seconds to decimal degrees with that HRS function.)

That's fine. But since you know a better way already, why not use it? Remember IDIV ("Integer DIVision") – that function that will do your division and take the integer portion of the result – all at once? Here's how you would use it:

QUAD=IDIV(HRS(AZ):90)+1

Well, why not? And if you you're gong to use this in the big, hairy, final formula, you'd better put this in properly combinable form:

QUAD-IDIV(HRS(AZ):90)-1=0

Two down, one to go; now all you need to do is figure out how to solve for BRNG from a known AZ.   Hmmm....

Observation #1: Since you're talking about measuring and converting angles here, it seems that trig functions are a natural place to look for solutions. *However,* the HP-27S doesn't measure its angles the same way that surveyors measure their azimuths. Surveyors use a geological convention, starting at North and measuring clockwise. The HP-27S uses the mathematical convention of starting at East and measuring counterclockwise. So whatever else is involved in converting an AZ into a BRNG, if you're going to use the HP-27S's trig functions to do it, you'll need to first adjust the AZ so that your machine knows what angle you're really talking about:

Since the two conventions measure in opposite directions, as you increase AZ, you'll decrease the corresponding angle, α, in the HP-27S. So the basic idea is that α = -AZ. But the two conventions are also offset; they differ by one quadrant, or 90°. So the relation must be something like α = -AZ ± 90°.

Which is it – plus or minus 90°? A quick mental example will show that it's *plus,* so α = -AZ + 90, or, more conveniently, α = 90 - AZ. This is how you must adjust AZ – subtract it from 90° – to even begin to compute the BRNG on your HP-27S.

Observation #2: The BRNG is always the angle as measured from the nearest *vertical axis,* isn't it? What *trigonometric* quantity in a circle is measured from the *vertical* axis? It's the x-coordinate – the *cosine* of an angle, isn't it?

"Yes, but BRNG is an *angle,* and a cosine is an x-coordinate – a straight line measured along the x-axis. You still can't directly compare them."

True, true. But there's at least *some* connection, so just keep that on the back burner for a bit.

Observation #3: Whenever you give the HP-27S a cosine and ask what angle it belongs to (by finding the ACOS), it will give you the simplest answer it can – an angle between 0° and 180°.

But you want to convert to a BRNG, which is always between 0° and 90°.

Well, what about an ASIN? If you give your calculator a number that represents the sine of some angle, and you then ask for the angle itself (with the ASIN function), you're also going to get the simplest possible answer – but *this* function's range is somewhere between -90° and 90°. Hmmm...and if you then took the absolute value, you could get the possible range down to what you want – 0° to 90°. OK....

Observation #4: If you take the cosine of any angle less than 90°, then take the *ASIN* of the result, you don't get the angle back, of course (ACOS does this), but you *do* transform the x-coordinate (the cosine) into the y-coordinate (the sine) and vice versa. That means that you can transform a BRNG measured from the vertical axis into a BRNG measured from the horizontal axis.

Putting all the observations together (drum roll, please), try this on for size:

BRNG=HMS(ASIN(ABS(COS(90-HRS(AZ)))))

See how this set of nested functions works? Working, as always from the innermost parentheses outward, you first convert AZ to a usable decimal α, then take its cosine to find its horizontal coordinate. Next, you get rid of any minus sign, then find the angle that has this as its *vertical* coordinate. Finally, you reconvert back to degrees, minutes and seconds.

Now polish it for the big day:

```
BRNG-HMS(ASIN(ABS(COS(90-HRS(AZ)))))=0
```

The Big Day dawns....Here are the three separate formulas:

```
AZ-IF(QUAD=1 OR QUAD=3
      :HMS((QUAD-1)×90+HRS(BRNG))
      :HMS(QUAD×90-HRS(BRNG)))=0

QUAD-IDIV(HRS(AZ):90)-1=0

BRNG-HMS(ASIN(ABS(COS(90-HRS(AZ)))))=0
```

And here is the pattern in which you must combine them all:

```
IF(S(AZ)
  :solve for AZ
  :IF(S(QUAD)
     :solve for QUAD
     :solve for BRNG))
```

The Inevitable Result:

```
IF(S(AZ)
  :AZ-IF(QUAD=1 OR QUAD=3
        :HMS((QUAD-1)×90+HRS(BRNG))
        :HMS(QUAD×90-HRS(BRNG)))
  :IF(S(QUAD)
     :QUAD-IDIV(HRS(AZ):90)-1
     :BRNG-HMS(ASIN(ABS(COS(90-HRS(AZ)))))))=0
```

(You may now pause for a long moment of silence to key this in: From the SOLVE menu, press ■ ▼ NEW, then contemplate, meditate, and digitate....)

(Comment: Notice that the space key, [space], is next to the Σ key.)

Now test this massive undertaking (press INPUT, then CALC) with the following table of equivalencies.

Be sure to test solutions going in either direction (from AZ to QUAD and BRNG *or* vice versa) – and remember that all angles you see will be in degrees, minutes and seconds, so set your HP-27S that way, too (press ■ MODES MORE D/R MORE FIX 4 INPUT):

AZ	QUAD	BRNG
30.0000	1	30.0000
60.3419	1	60.3419
92.4500	2	87.1500
150.3000	2	29.3000
210.0000	3	30.0000
246.4437	3	66.4437
345.0000	4	15.0000
358.5959	4	1.0001

Not a bad little formula, eh?

13. Recalling (as you undoubtedly can) from your basic static-mechanics course in physics or engineering, you know that in any rigid free body, the resulting total linear *force* tending to push the body in its direction is the vector sum of all "n" individual linear forces:

$$\mathbf{F}_T = \mathbf{F}_1 + \mathbf{F}_2 \ldots + \mathbf{F}_n$$

Of course, that's why it would be so handy to have this formula to give you a total *resultant* force.

Similarly, the total *moment* (i.e. the total *angular* force) tending to rotate the body about some origin (a point of reference) is given by the vector sum of each linear force, **F**, *"times" its distance, **R**, from the origin:*

$$\mathbf{M}_T = \mathbf{M}_1 + \mathbf{M}_2 \ldots + \mathbf{M}_n = \mathbf{R}_1 x \mathbf{F}_1 + \mathbf{R}_2 x \mathbf{F}_2 \ldots + \mathbf{R}_n x \mathbf{F}_n$$

Keep in mind that all these boldface quantities are 3-dimensional **vectors,** each of which has components in the x-, y-, and z- directions. Or, as you may prefer to think about it, each has components *in the directions of* the **unit vectors:** **i** = (1,0,0), **j** = (0,1,0), and **k** = (0,0,1).

And another thing: That "multiplication" you see in the above definition is put in quotes because it's not just like 2x3 or something. Since these forces, distances and moments all have directions associated with them, you need to keep these directions straight, and so this vector "multiplication" (the cross product) is *not commutative;* you can't arbitrarily reverse the order of multiplication without really messing things up.

That is, you can't change $\mathbf{R}_1 x \mathbf{F}_1$ into $\mathbf{F}_1 x \mathbf{R}_1$ *because this also reverses the direction of the resulting moment,* $\mathbf{M}_1$.

Well, it's easy to add vectors; you just add their respective components:

If you have any two vectors, $\mathbf{V}_A = (Vx_A, Vy_A, Vz_A)$ and $\mathbf{V}_B = (Vx_B, Vy_B, Vz_B)$

then $\mathbf{V}_A + \mathbf{V}_B = (Vx_A+Vx_B,\ Vy_A+Vy_B,\ Vz_A+Vz_B)$.

No sweat, right? But what about the cross product of two vectors? If you have those same two vectors,

then $\mathbf{V}_A \times \mathbf{V}_B \neq (Vx_AVx_B,\ Vy_AVy_B,\ Vz_AVz_B)$.

That's *not* the definition of a cross product; the true cross product is a bit more involved than that. You might have seen it expressed in matrix form as a determinant, with the **i**, **j**, and **k** unit vectors, like this:

$$\mathbf{V}_A \times \mathbf{V}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ Vx_A & Vy_A & Vz_A \\ Vx_B & Vy_B & Vz_B \end{vmatrix}$$

But according to the rules for figuring determinants, that's just another way to say

$$\mathbf{V}_A \times \mathbf{V}_B = (\ Vy_AVz_B - Vy_BVz_A\ ,\ Vx_BVz_A - Vx_AVz_B\ ,\ Vx_AVy_B - Vx_BVy_A\ )$$

So if you apply all this to the problem of summing forces and moments on a free body, the math will look like this:

$$\mathbf{F}_A + \mathbf{F}_B = (\ Fx_A+Fx_B,\ Fy_A+Fy_B,\ Fz_A+Fz_B\ )$$

$$\mathbf{M}_A = \mathbf{R}_A \times \mathbf{F}_A = (\ Ry_AFz_A - Fy_ARz_A\ ,\ Fx_ARz_A - Rx_AFz_A\ ,\ Rx_AFy_A - Fx_ARy_A\ )$$

$$\mathbf{M}_A + \mathbf{M}_B = (\ Mx_A+Mx_B,\ My_A+My_B,\ Mz_A+Mz_B\ )$$

All right, fine. But the problem says to use just two lists to contain all the forces and their distances acting on the free body. From the looks of all those x's and A's on the previous page, don't you need about six lists – one for each component of each force and each distance?

Well, yes, you *could* do it that way, but then it might be a rather large pain to key in or edit your forces and distances (after all, the idea here is to be able to play "What-If?" by varying your parameters). To edit a single force or a single distance, you'd have to edit up to three separate lists.

There's gotta be a better way.

There is. You *can* do it all in just two lists, as the problem requests. It just means that every force in your force vector list will *take up three list items in a row* – one for each force component. And the same will be true of your list of distance vectors.

So here's how the two lists will look:

ITEM	FORCE	DIST
1	Fx_1	Rx_1
2	Fy_1	Ry_1
3	Fz_1	Rz_1
4	Fx_2	Rx_2
5	Fy_2	Ry_2
6	Fz_2	Rz_2
7	Fx_3	Rx_3
8	Fy_3	Ry_3
9	Fz_3	Rz_3
⋮	⋮	⋮

Next thing to observe: You are going to need a total of six variables in your equation – to give you the final totals for each component of the resultant force and moment:

FX FY FZ MX MY MZ

All right, now take a look at how you're actually going to sum all the like components of a given resultant. For example, how are you going to calculate FX?

It's a summation, all right, so you need to use the Σ( ) notation – remember back on page 148?

But it's slightly different here: To sum all the x-Force components, *instead* of using your counter, N, to proceed from Item 1 to the *last* Item in the list *in steps of 1,* like this:

Σ(N:1:SIZES(FORCE):1:ITEM(FORCE:N))

you need to proceed from Item 1 to the *third-to-the-last* Item (because that will be the x-Force component of the final force vector), and you'll move through the list *in steps of 3,* like this:

FX=Σ(N:1:SIZES(FORCE)-2:3:ITEM(FORCE:N))

Similarly, FY=Σ(N:2:SIZES(FORCE)-1:3:ITEM(FORCE:N))
FZ=Σ(N:3:SIZES(FORCE):3:ITEM(FORCE:N))

See how to adjust the starting and stopping points for the other components? To get FY, you start with Item 2 and end with the second-to-last Item; for FZ, you start with Item 3 and end with the last Item. And in each case you skip along, summing every third Item. Piece of cake!

Of course, by now it's probably occurred to you that this problem is yet another one where you have multiple independent equations, and if you want to be able to solve for any one of them from a single SOLVE menu, you need to patch them together with that IF(S... business.

So before you go on and figure out the solution to the Moment components, you'd better adjust the three equations you've got so far:

```
FX-Σ(N:1:SIZES(FORCE)-2:3:ITEM(FORCE:N))=0

FY-Σ(N:2:SIZES(FORCE)-1:3:ITEM(FORCE:N))=0

FZ-Σ(N:3:SIZES(FORCE):3:ITEM(FORCE:N))=0
```

And it would also be a good idea to note the final form of your equation. It's going to be a rather extensive *nesting* of IF(S... statements, isn't it? In fact, it'll go something like this:

```
IF(S(FX)
  :solve for FX
  :IF(S(FY)
    :solve for FY
    :IF(S(FZ)
      :solve for FZ
      :IF(S(MX)
        :solve for MX
        :IF(S(MY)
          :solve for MY
          :solve for MZ)))))
```

Now for the hairier calculations – those cross products that you need for the Moment components. Here is the general formula you decided upon:

$$\mathbf{M} \quad = \quad \mathbf{R} \times \mathbf{F} \; = \; (R_yF_z - F_yR_z \,,\; F_xR_z - R_xF_z \,,\; R_xF_y - F_xR_y)$$

And here's how you would formulate the solution to the x-component of the total Moment. Notice how you start and end the N-count in a way similar to your Force equations – and that also you jump along three items at a time:

```
MX=∑(N:1:SIZES(FORCE)-2:3
:ITEM(DIST:N+1)xITEM(FORCE:N+2)-ITEM(DIST:N+2)xITEM(FORCE:N+1))
```

Do you see how to adjust the index for each list item to refer to any of the x-, y-, or z- components of either list – as you need them in the formula? You just add 1 or 2 to the value of N, and that effectively moves you down the list to the y- or z- component, respectively.

Now it's easy to write the other two equations as well:

```
MY=∑(N:1:SIZES(FORCE)-2:3
:ITEM(DIST:N+2)xITEM(FORCE:N)-ITEM(DIST:N)xITEM(FORCE:N+2))
```

```
MZ=∑(N:1:SIZES(FORCE)-2:3
:ITEM(DIST:N)xITEM(FORCE:N+1)-ITEM(DIST:N+1)xITEM(FORCE:N))
```

Now put all your properly-formatted equations on one page, along with the format in which you must combine them:

```
FX-Σ(N:1:SIZES(FORCE)-2:3:ITEM(FORCE:N))=0

FY-Σ(N:2:SIZES(FORCE)-1:3:ITEM(FORCE:N))=0

FZ-Σ(N:3:SIZES(FORCE):3:ITEM(FORCE:N))=0

MX-Σ(N:1:SIZES(FORCE)-2:3
:ITEM(DIST:N+1)×ITEM(FORCE:N+2)
-ITEM(DIST:N+2)×ITEM(FORCE:N+1))=0

MY-Σ(N:1:SIZES(FORCE)-2:3
:ITEM(DIST:N+2)×ITEM(FORCE:N)
-ITEM(DIST:N)×ITEM(FORCE:N+2))=0

MZ-Σ(N:1:SIZES(FORCE)-2:3
:ITEM(DIST:N)×ITEM(FORCE:N+1)
-ITEM(DIST:N+1)×ITEM(FORCE:N))=0
```

```
IF(S(FX)
  :solve for FX
  :IF(S(FY)
    :solve for FY
    :IF(S(FZ)
      :solve for FZ
      :IF(S(MX)
        :solve for MX
        :IF(S(MY)
          :solve for MY
          :solve for MZ)))))
```

Now combine them:

```
IF(S(FX)
  :FX-Σ(N:1:SIZES(FORCE)-2:3:ITEM(FORCE:N))
  :IF(S(FY)
    :FY-Σ(N:2:SIZES(FORCE)-1:3:ITEM(FORCE:N))
    :IF(S(FZ)
      :FZ-Σ(N:3:SIZES(FORCE):3:ITEM(FORCE:N))
      :IF(S(MX)
        :MX-Σ(N:1:SIZES(FORCE)-2:3:
            ITEM(DIST:N+1)×ITEM(FORCE:N+2)-
            ITEM(DIST:N+2)×ITEM(FORCE:N+1))
        :IF(S(MY)
          :MY-Σ(N:1:SIZES(FORCE)-2:3:
              ITEM(DIST:N+2)×ITEM(FORCE:N)-
              ITEM(DIST:N)×ITEM(FORCE:N+2))
          :MZ-Σ(N:1:SIZES(FORCE)-2:3:
              ITEM(DIST:N)×ITEM(FORCE:N+1)-
              ITEM(DIST:N+1)×ITEM(FORCE:N))
                                   )))))=0
```

Go ahead and key this in (from the SOLVE menu press ■▼ and **NEW**. Then when you finish, press INPUT and **CALC**).

Remember to be patient with yourself and your mistakes. With an equation of this size, you're bound to make a couple of errors on the first pass through. But that's why the HP-27S has that built-in error checker and pointer: if you do mess up, it'll lead you right to the source of the problem. So keep on plugging until it's right. The time you invest now will be paid back to you by ten- or a hundred- fold later!

Now test your formula with this problem:

A certain rigid free body has the following forces acting upon it, at the following distance from its center of mass (which is called point (0,0)):

FORCE (Newtons)	DISTance (Meters)
(100, 200, 500)	(-3, 4, 5)
(-75, 440, -93)	(6, -10, 15)
(34, -11, 19)	(25, 64, -100)

Find the total Force and Moment acting upon the body.

Now that your equation is all ready, you just need to create your FORCE and DISTance lists with the STAT menu. So press STAT, then:

GET *NEW NAME FORCE INPUT and
1 0 0 INPUT 2 0 0 INPUT 5 0 0 INPUT
7 5 +/– INPUT 4 4 0 INPUT 9 3 +/– INPUT
3 4 INPUT 1 1 +/– INPUT 1 9 INPUT

Then EXIT and

GET *NEW NAME DIST INPUT and
3 +/– INPUT 4 INPUT 5 INPUT
6 INPUT 1 0 +/– INPUT 1 5 INPUT
2 5 INPUT 6 4 INPUT 1 0 0 +/– INPUT

And for the grand finale: ■ SOLVE CALC, then

FX Result: FX=59
FY Result: FY=629
FZ Result: FZ=426

MX Result: MX=-4,554
MY Result: MY=-2,442
MZ Result: MZ=-1,561

So the total or resultant **F**orce on the body is (59, 629, 426) Newtons;

and the resultant **M**oment is (-4554, -2442, -1561) Newton-Meters.

And you can go back to your FORCE and DISTance lists to edit them or lengthen them anytime you want (just be sure, of course, that each list has the same number of items in it, and that that number is a multiple of three)!

14. Just for laughs (and that's what this final problem is really all about), suppose you express this codfish population volume not simply in cubic meters but in terms of the resulting rise in the world sea level.

The oceans' surface areas total roughly 140 million (that's 1.4×10^8) square miles. For a 1-mile rise in this ocean, then, you'd need an overall volume increase of about 140 million cubic miles, right? So, at .1 cubic foot per fish, which is $5280^3 \div .1$, or 1.5 trillion fish per cubic *mile,* it would take 140 million x 1.5 trillion, or *210 quintillion* (2.1×10^{20}) fish to raise the world ocean level by one mile.

Surely there'd be enough room....

Well, assuming that about half of each new generation of codfish is male and half is female, that's about 10 billion new spawnings every year *for every spawning of the previous year.* So every year, the population increases about 10-billion-fold. A very simple function for the codfish population, then, is

$$P(y)=(10\text{ billion})^y,$$

where y is the time expressed in years. And since 10 billion is really 10^{10}, and since $(10^{10})^y$ is the same as 10^{10y}, you get

$$P(y)=10^{10y}$$

And the volume, V, taken up by these fish – expressed as a vertical rise in the world ocean – would then be their population divided by 210 quintillion:

$$V=10^{10y}\div(2.1\times10^{20}) \quad \text{miles.}$$

Or, in combinable HP-27S SOLVE lingo: `V-1E10^Y÷2.1E20=0`

Fine. Now what about a formula for the *rate* of this rise at any year, y?

A little differential calculus will tell you that the *rate of change* of the formula, V(y), with respect to the time variable, y, as given by its derivative with respect to y (if you've never done any calculus, don't worry about it; just go with this), is:

$R(y) = V'(y) = LN(10^{10}) \div (2.1x10^{20}) x10^{10y}$ *miles of ocean rise per year,* or

$23 \div (2.1x10^{20}) x10^{10y}$ miles per year, or

$1.1x10^{-19} x10^{10y}$ miles per year, or

$1.1x10^{-19} \div (365x24x3600) x10^{10y}$ *miles per second,* or

$3.5x10^{-27} x10^{10y}$ miles of ocean rise per second.

So your rate formula in its final form would be:

```
R-3.5E-27×1E10^Y=0
```

And the combination formula would be (old hat by now, eh?):

```
IF(S(V):V-1E10^Y÷2.1E20:R-3.5E-27×1E10^Y)=0
```

So key this in (from the SOLVE menu, press ■▼ and NEW, then go for it)....

...Then, when you finish, of course, press INPUT and CALC.

Now ask some pertinent questions: By how much and how fast would the ocean be rising after each of the first 3 years? (With all the imprecision here, use a display format of SCI 1; press ■ [MODES] [SCI] [1] [INPUT].)

Press [1] [Y], then

[V] Result: V=4.8E-11 (miles).
That's about 3 millionths of an inch – not so you'd notice.
[R] Result: R=3.5E-17 (miles per second).
That's about 70 millionths of an inch per year – not a spectator sport.

Press [2] [Y], then

[V] Result: V=4.8E-1 (miles).
That's about 2500 feet (Florida real estate values drop sharply).

[R] Result: R=3.5E-7 (miles per second).
That's about 1.3 inches per minute (Aspen condos soar in price).

Press [3] [Y], then

[V] Result: V=4.8E9 (nearly 5 billion miles).
[R] Result: R=3.5E3 (3,500 miles per second).

"And after about 3.14 years (less than 38 months) of unlimited codfish reproduction, the earth would be 120 billion miles deep in cod, and the radius of this galactic fish-ball would be expanding outward at half the speed of light."

"This would tend to reduce the market price of codfish."

So... those are the rudiments of the SOLVE menu in your HP-27S.

But there's a lot more you could do with it, if you wanted to learn some more complicated ways of building the equations.

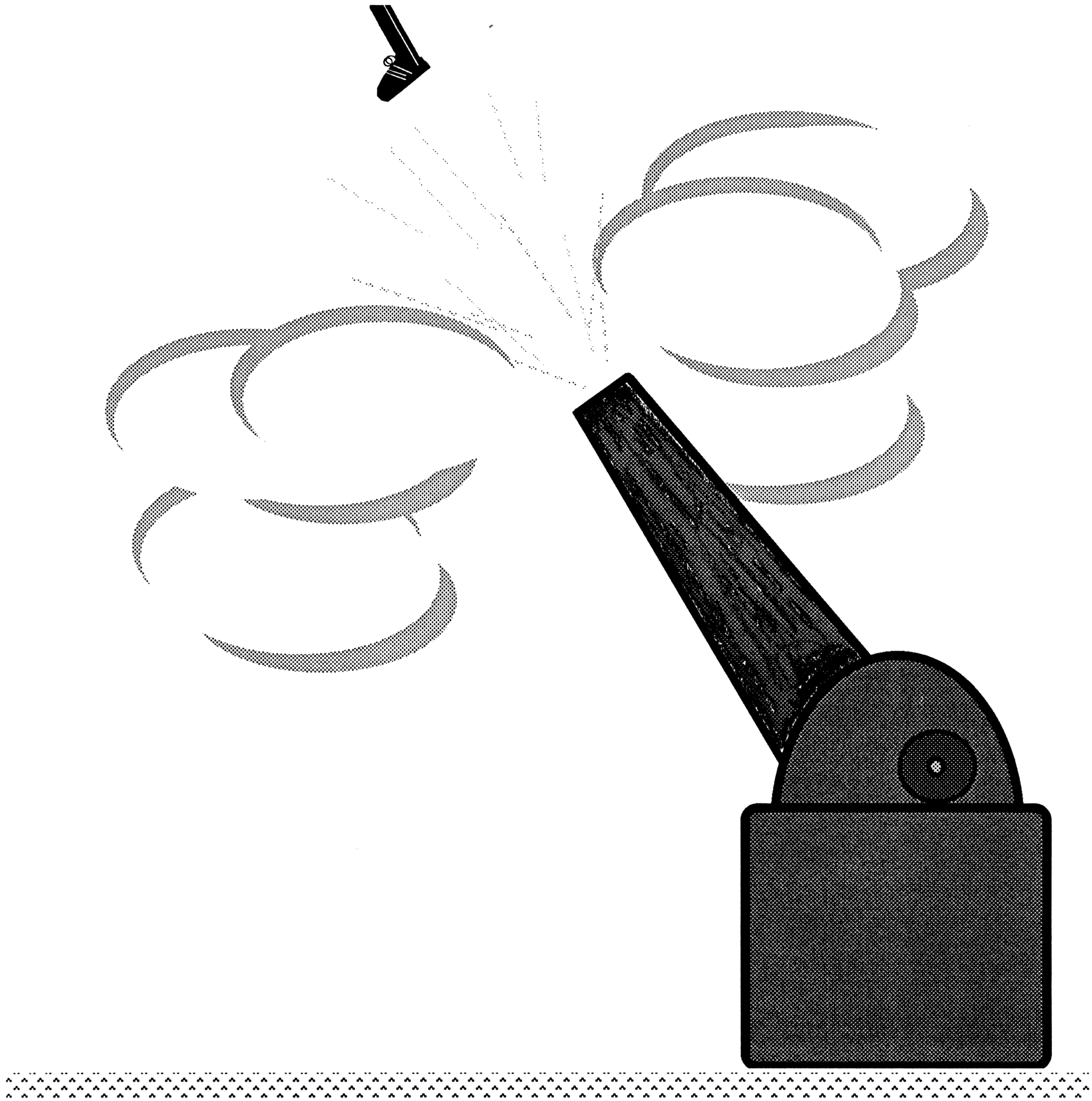
A good place to start looking for more ideas is your owner's manual. There you'll find in Chapter 5 a full description of all the different functions and formulas you can use to build equations– along with some very impressive examples of what you can do with such calculating power!

And if that's not enough, see Chapter 9 for Additional Examples of all kinds – many of them using the SOLVE menu.

And if *that's* not enough, there's another HP book you might be interested in, called "Technical Applications." Its a Step-By-Step Solutions Book for your HP-27S (it applies also to the HP-19B). It carries HP's manufacturing part number 00027-90045 (but the reorder number is 00027-90044, for some reason).

That book is full of some very powerful math and engineering equations, plus some good tips and lessons on how to use the SOLVEr more efficiently and powerfully. See your dealer or call HP if you're interested.

But even if you don't need to get any more complicated than the problems you've just seen here, even *that* much power in your pocket is sure handy, eh? After all, you don't need to be stunt-certified before your "flying" skills pay off, right?



FLYING ECONOMICALLY:
The TVM Menu

Problem Understanding = Problem Solving

It's time to take a good hard look at that TVM menu, which is probably the least understood menu on the HP-27S.

Similar to other "What-If?" menus, it lets you experiment and vary the parameters in your calculations – so you can ask (and answer) questions such as *"what if* the rent goes up by $200?" And *"what if* that interest rate were a couple points lower or higher?"

But before getting started on all this, consider a little proverb about teaching (maybe you've heard it before):

"You never really understand something
until you can explain it to someone else."

This is certainly true for your HP-27S. In order for you to get it to solve a financial calculation problem, you need to explain the problem to it – in the "picture language" it understands. So of course, you need to figure out that picture *for yourself* first. That's the

Bottom Line:

You must be able to understand and define the problem for yourself
before you can possibly "explain" it to your calculator.

This final chapter of the Course will train you to *understand and define* the financial calculation problems you encounter. If you can do that, the actual button-pushing is easy. So don't be too impatient to start hitting keys here. For a little while, just park your calculator, sit back, and consider carefully the details that surround even simple finance calculations. Even if you've seen some of this before, it will pay you to review it now....

What is TVM?

TVM, of course, stands for the Time Value of Money.

The *assumption* of TVM is that money – any money – earns *interest* simply through the passage of time, because when you're not using it, you're renting it out (loaning it) for someone else to use.

Therefore, whenever you ask "How much money do I have?", you must also ask "What time is it?" Under the TVM assumption, the first question is meaningless without the second one.

And because time does affect the value of money in all such evaluations, it's actually useful to say that time *is* money (as long as you don't actually start believing that). That is, when comparing two different *amounts* of money, you can still *equate* them by considering the *time* difference between them.

That is, if \$1.00 today will be worth \$1.10 this time next year, then you can say it with an "equals" in it:

"$\$1.00_{\text{now}}$ *equals* $\$1.10_{\text{next year}}$."

The Time literally adds the extra Value to the Money.

So when you reach into your pocket and find a dollar bill, you can view that as

$\$1.00_{\text{now}}$, $\$1.10_{\text{next year}}$, or $\$2.59_{\text{ten years from now}}$.

Assuming a 10% annual interest rate, each of these is really the *same amount of money.*

How Does Interest Behave?

OK, so borrowed money earns interest over time. Everybody knows that.

But exactly how does that work? If you had nothing better to do but sit and watch the pennies of interest in your bank account accrue, what would the pattern look like?

It all depends on what kind of interest you were earning.

Basically, there are two forms of interest – **Simple** and **Compound**. Here's how they differ:

Simple Interest is the less common method nowadays: The amount of money earned as interest *per period* is defined as a set percentage of the *amount originally loaned.*

Example: Suppose you're earning **Simple Interest** of 1% per month in your bank account (nice if you can get it), and you have $100.00 sitting in there for 6 months. How much interest will you earn over the entire 6 months?

Solution: There will be exactly $6.00 in interest, because you earn 1% of the original $100.00 every month. That's one dollar per month for six months.

Compound Interest, on the other hand, is much more widely used: The amount of money earned as interest per period is defined as a set percentage of the *amount owed as of the beginning of that period – including any past interest earned.*

Example: Take that same \$100.00 for six months, earning 1% per month, but this time it's **Compound Interest**. How much is earned over 6 months?

Solution: The first month is easy: 1% of \$100.00 is 1.00. So after one month, the total amount owed is \$101.00.

Now comes the second month: 1% of \$101.00 is \$1.01. And so on.

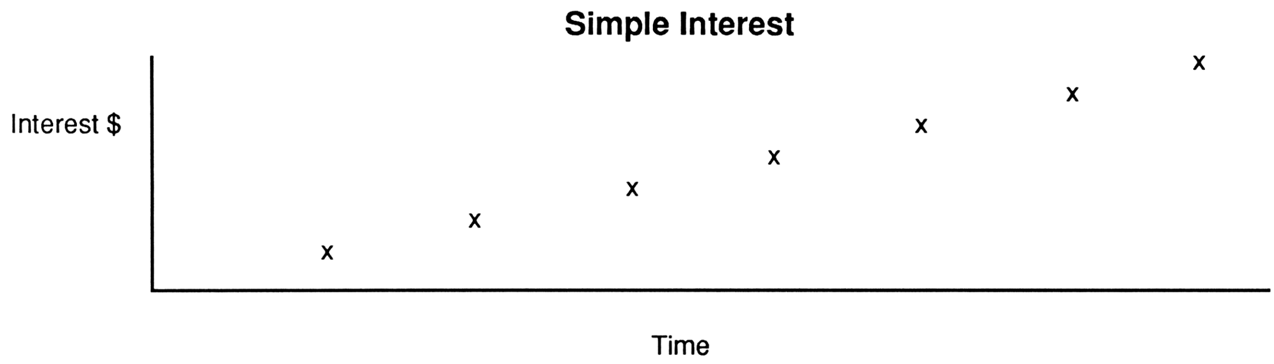
After 6 months, you'll have (roughly) \$106.15 – more than the \$106.00 you would have earned with Simple Interest.

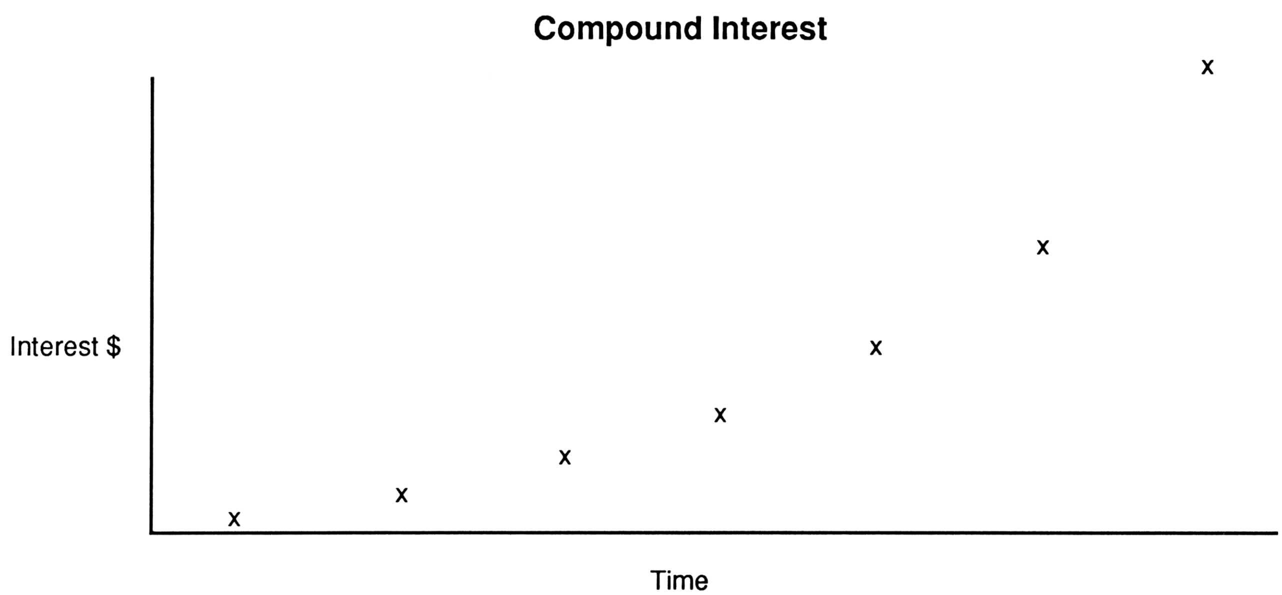
See the difference between this and the previous case?

With **Simple Interest**, no matter what month you're talking about, the interest earned is always based upon the amount owed at the *beginning* of the loan – that one *stationary point in time*. Interest is earned only on the original amount.

With **Compound Interest**, the interest earned is based upon the amount owed at some *other* point in time, and this point *moves*. Interest is earned not only on the original amount but also on all other interest already earned. Thus, you get the name, ***Compound*** **Interest**.

Here's a pictorial summary of the behaviors of each of these types of interest:





As you can see, compound interest grows faster than simple interest – which may help to explain why Compound Interest is used so much more widely.

In fact, since the TVM calculations on your HP-27S use Compound Interest, you're not going to hear a whole lot more about Simple Interest in this book. Anytime you see the word "interest" from here on out, you can take it to mean Compound Interest.

OK?

Now, all this may seem fairly obvious so far. But here are some subtleties that you should not overlook:

Subtlety #1: In either form of interest, there is *exactly one* time period which is the Defined Interest Period (D.I.P.), and *exactly one* Defined Interest Rate (D.I.R.) corresponding to that period. No other approximations should be used in their places when you're calculating interest.

The examples with the $100.00 for six months stated specifically that interest accrues at 1% per month. So the D.I.P. is 1 month, and the corresponding D.I.R. is 1%.

However, it's conventional to quote interest rates on an annual basis. A bank would take that 1% per month, multiply it by the 12 months in a year, and say "12% Annual Percentage Rate."

But this A.P.R. is only a convenient approximation!

If you were to ask that bank to calculate your (compound) interest on $100.00 over 6 months, it would *not* use that 12% annual rate as is. It would always divide it by 12 to arrive at the D.I.P. and the D.I.R. Then it would go through exactly the same calculation as you did on page 190 to get your answer.

Always make sure that you know the D.I.R. and the the D.I.P. before you start a financial calculation.

Subtlety #2: In your 6-month account problem, you never stopped to ponder how much interest you had earned after, say, 2.5 months, or 3.79 months, or 4.61 months.

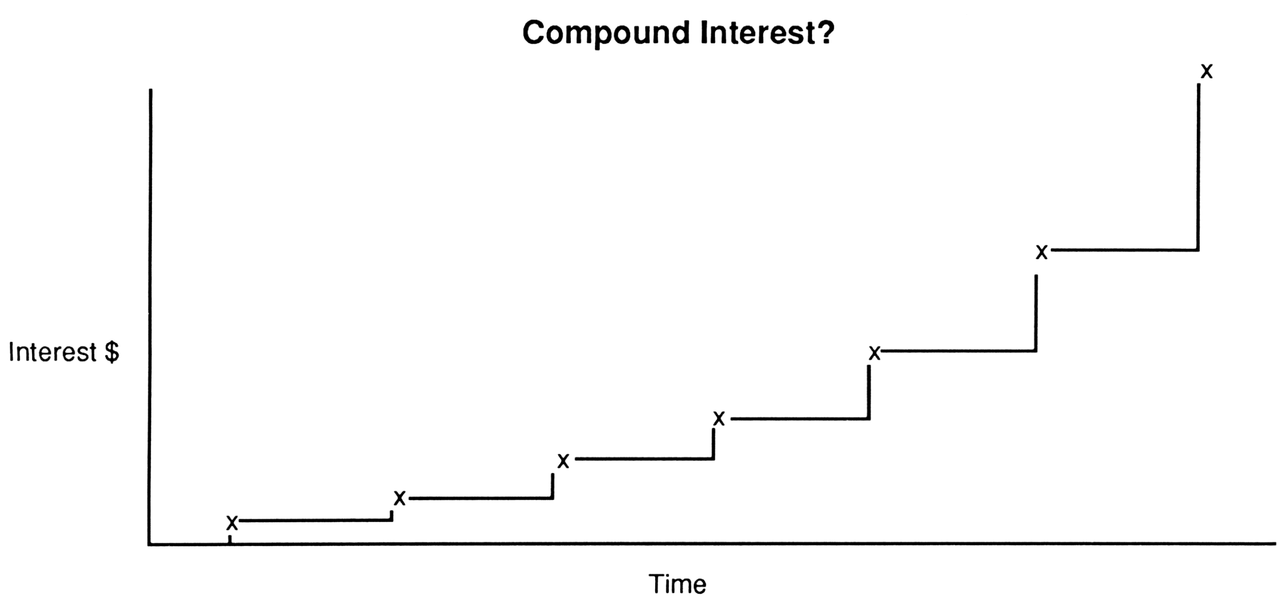
Why not?

Because you haven't been given the rules.

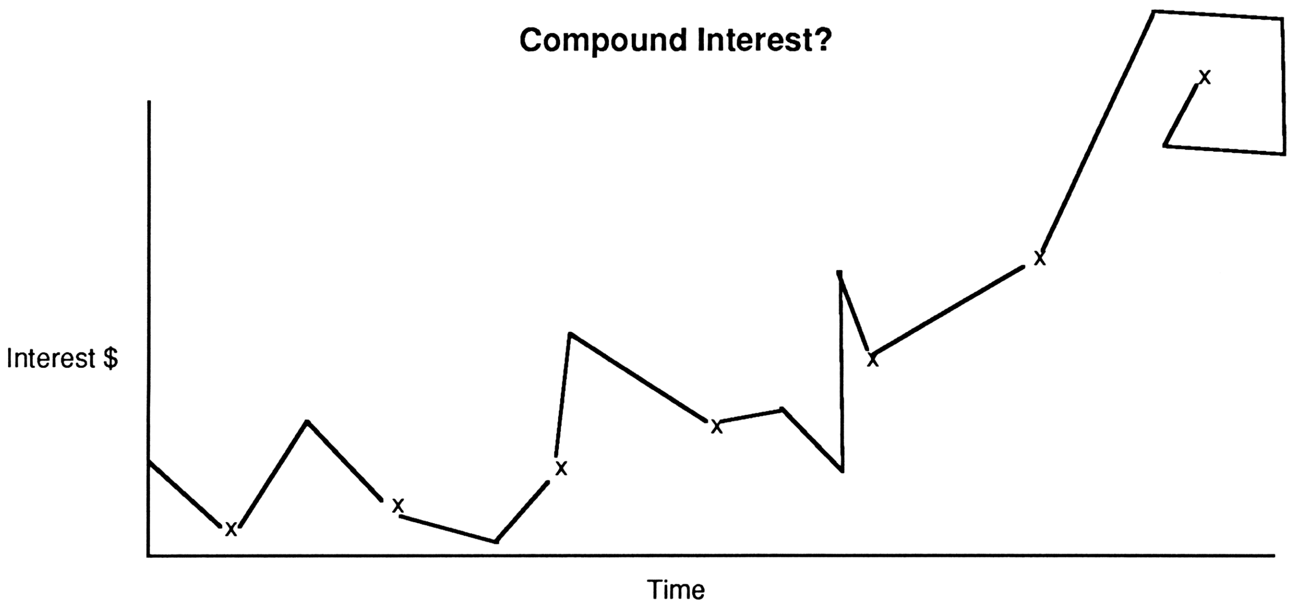
The D.I.R. and D.I.P. define only what the loan balance will be at *one point in each period* (usually the beginning or the end). There must be other definitions to determine how that balance proceeds from one point to the next.

That's why the points on the graph on page 191 aren't connected with lines. Knowing only the D.I.R. and the D.I.P., nobody can tell how those lines should be drawn.

For all you know, those lines might be drawn like this:



or this:



or who-knows-what-else.

All you know is the balance at *one point* in each Defined Interest Period.

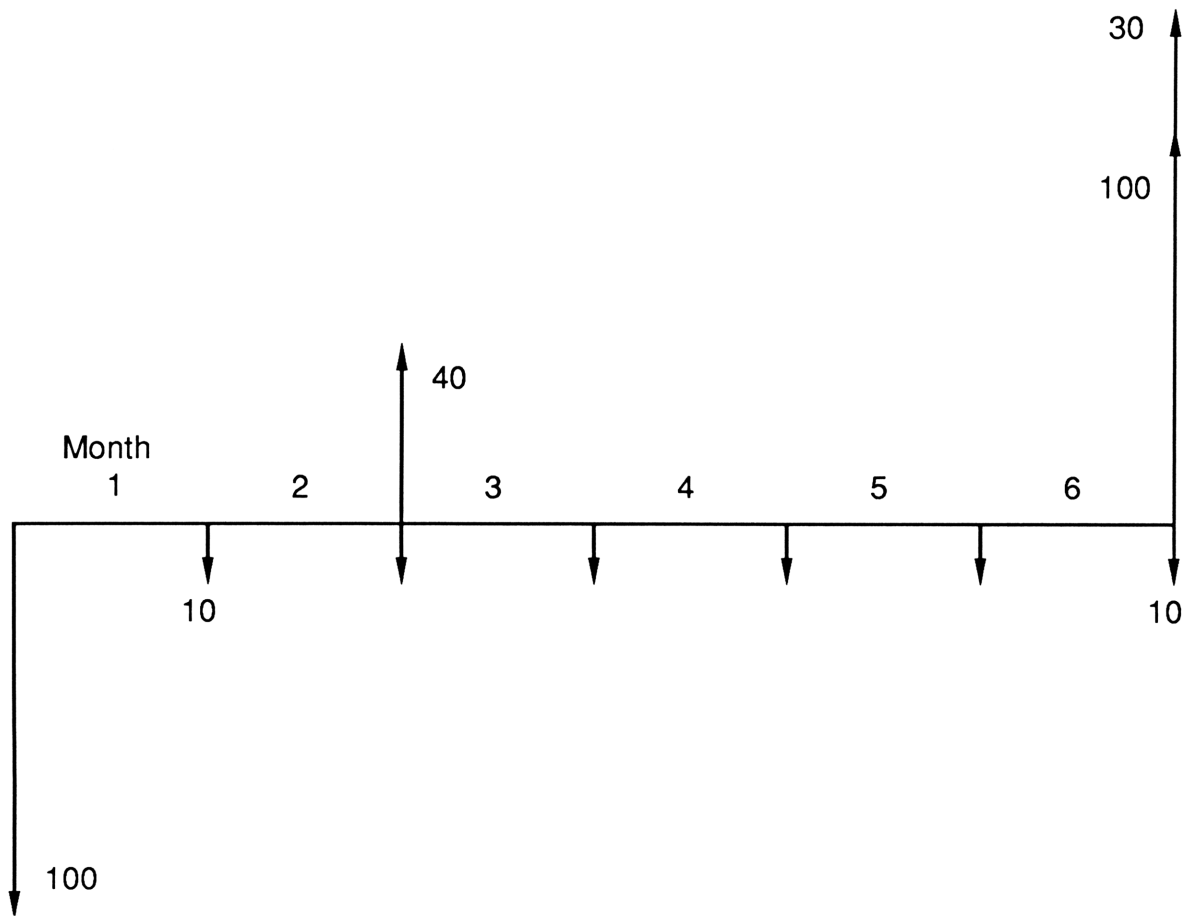
"Subtleties, subtleties...hmmm....Then what's the best way to picture my bank account with its $100.00 and this compound interest – if I can't draw a nice smooth curve of some kind?

And what if I deposit more into – or withdraw from – that account during those 6 months? How do I represent all *that* in one picture?"

Funny you should ask.... ☞

Cash-Flow Diagrams

This is a cash-flow diagram. It's the easiest way to define and understand any financial calculation:



The first thing to do in performing any financial calculation is...

Draw A Cash-Flow Diagram!

Write this in stone somewhere. It should become a reflex.

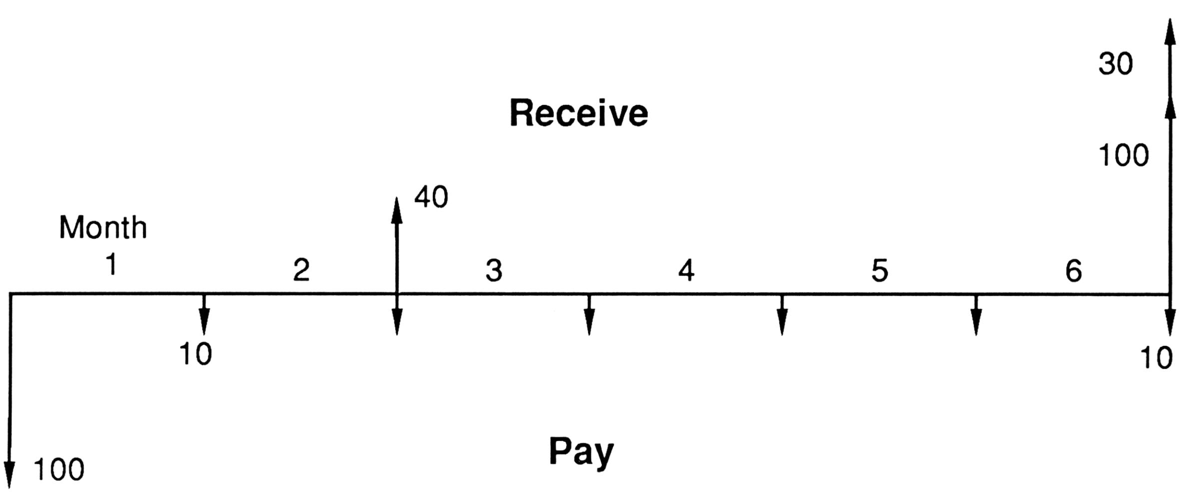
You don't need to be an artist to draw these diagrams. After all, they're just rough pictures to help you visualize the problem. But to make them really useful, it's best to stick to these rules:

1. Always pick the perspective of either a borrower or a lender (you might say "either a borrower or a lender be" – or you might not – depending upon your tolerance for ballistic vegetables).

   If you're buying money market shares, or putting money in a savings account, you should consider yourself the lender. If you're taking out a loan to buy a house or something, then obviously you're the borrower.

   The same loan will look very different on a cash-flow diagram, depending upon whether you're the lender or the borrower. So in drawing the picture, pick one perspective and stick with it!

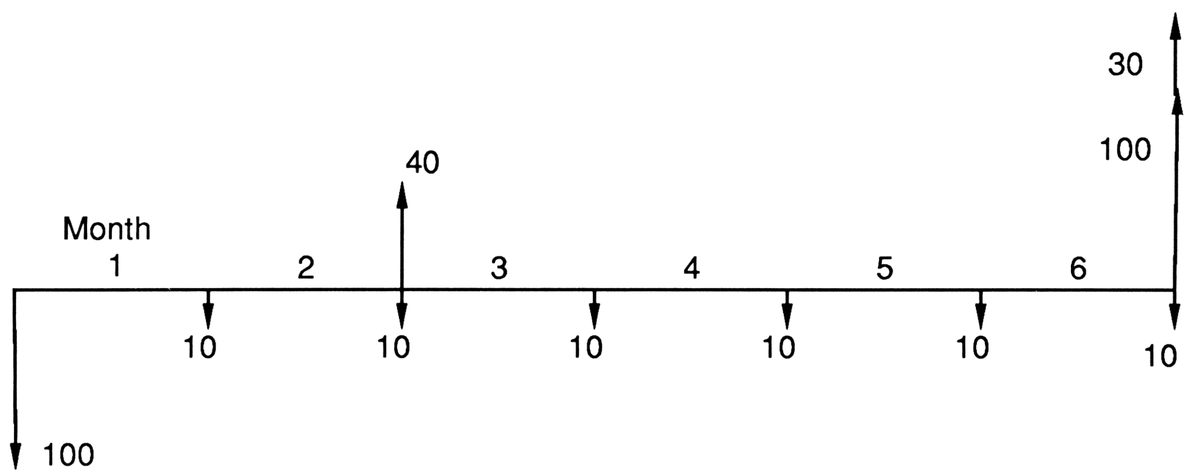
2. The vertical arrows on the diagram represent cash-flows – moments when you literally pull out your wallet to pay or receive cash. And once you've picked your perspective, the *directions* of the vertical arrows then denote the directions of the transactions. An *upward* arrow means that you *receive* money; a *downward* arrow means that you *pay* money:



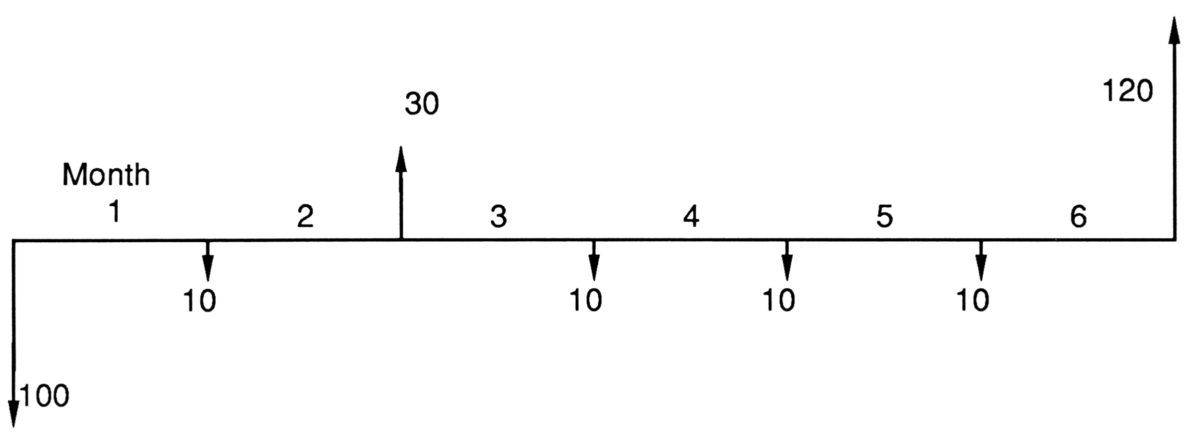
3. The lengths of the vertical arrows in your diagram should reflect the amounts of the transactions: A longer arrow means more money (but don't quibble over millimeters if the general idea is clear).

4. The horizontal direction represents time (flowing from left to right). Usually, this line is marked at regular intervals to denote the D.I.P.'s. That makes a lot of sense, because *your HP-27S is equipped to calculate compound interest problems where cash-flows occur once every D.I.P.*

5. Whenever you have multiple transactions that occur simultaneously, you can add them all together to obtain one net transaction. In other words, this:



is the same as this:



Those are the basic rules, all right? Now, it still may not seem very obvious why these diagrams are so useful. Maybe this will help:

The real beauty of a cash-flow diagram is that *you can make it simpler,* actually *adjusting* it to get a clearer picture of the situation.

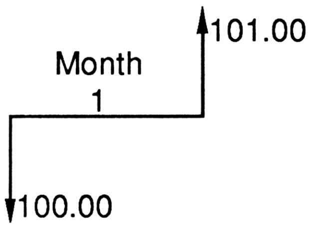
How does this work?

Go back to your $100.00 sitting in the bank account for 6 months. Under the rules of compound interest, here's a month-by-month summary of your account balance:

At The End Of	Your Balance Is
Month 1	$101.00
Month 2	102.01
Month 3	103.03
Month 4	104.06
Month 5	105.10
Month 6	106.15

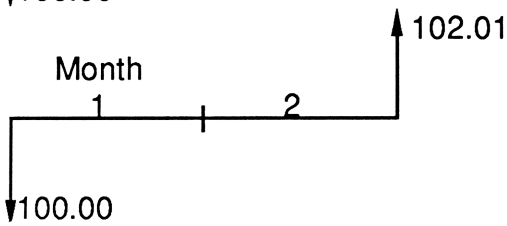
Now think about that for a minute. Since you have the option of withdrawing your money at the end of any of these months, each of these withdrawal amounts are *equivalent to any other,* after taking into account the time involved.

And look at how these options look on cash-flow diagrams:

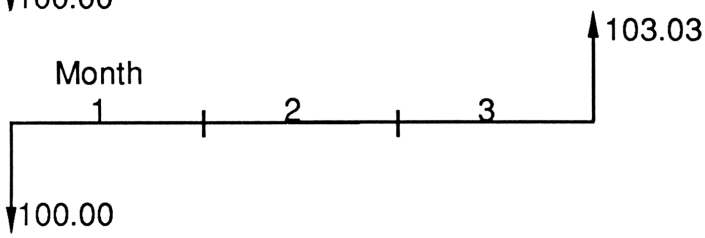


Think of this as playing "What-If?" with your withdrawal:

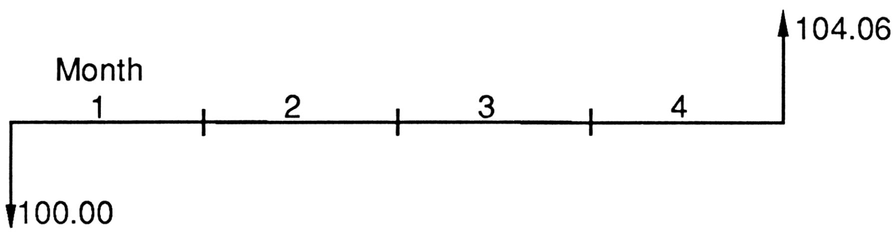
"What if I withdrew my money now?"



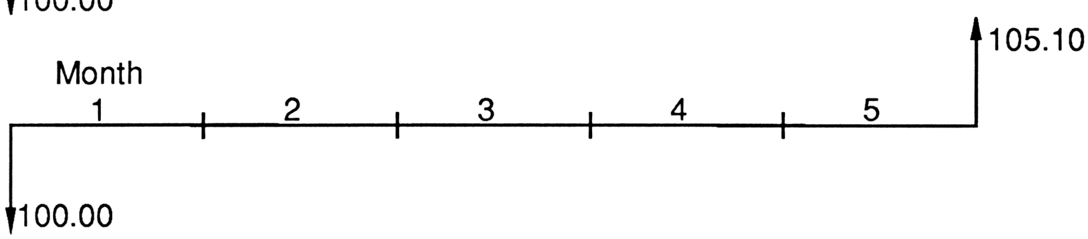
"What if I waited...



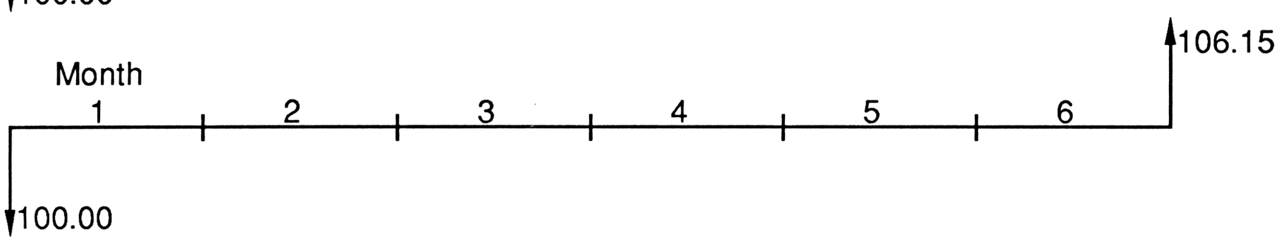
three



more



months?"



Since each of these pictures is entirely equivalent to the others, what you're really doing is *sliding* a single cash-flow up and down the timeline, seeing how it would look at various times. And that's the real beauty of a cash-flow diagram:

*You can **slide** any cash-flow along the timeline of a cash-flow diagram and retain complete accuracy as long as you **increase** it (when sliding to the right) or **decrease** it (when sliding to the left), according to the **prevailing interest rate!***

(Don't worry – you're going to put all these ideas to practice in just a few more pages. But keep grinning and bearing this, because it's very important to cover these things first, OK?)

Now then, look at what you know about the Time Value of Money so far:

- You know the difference between the two common types of interest;

- You know that of these two types, Compound Interest is much more widely used (and your HP-27S's TVM functions use it);

- You know how to draw cash-flow diagrams, using the five rules for making it the most clear and useful;

- You know that you can slide cash-flows along the timeline and keep the picture entirely accurate, as long as you adjust those cash-flows according to the prevailing interest rate.

What you *don't* know (yet) is how to draw a cash-flow diagram for your HP-27S. After all, it's supposed to do the calculating around here.

How can a machine possibly "see" or "understand" a picture that you've drawn on a piece of paper?

Drawing The Picture For Your Calculator

Time to rev up the machine again. From the MAIN menu, press ■ TVM.

You'll see something like this:

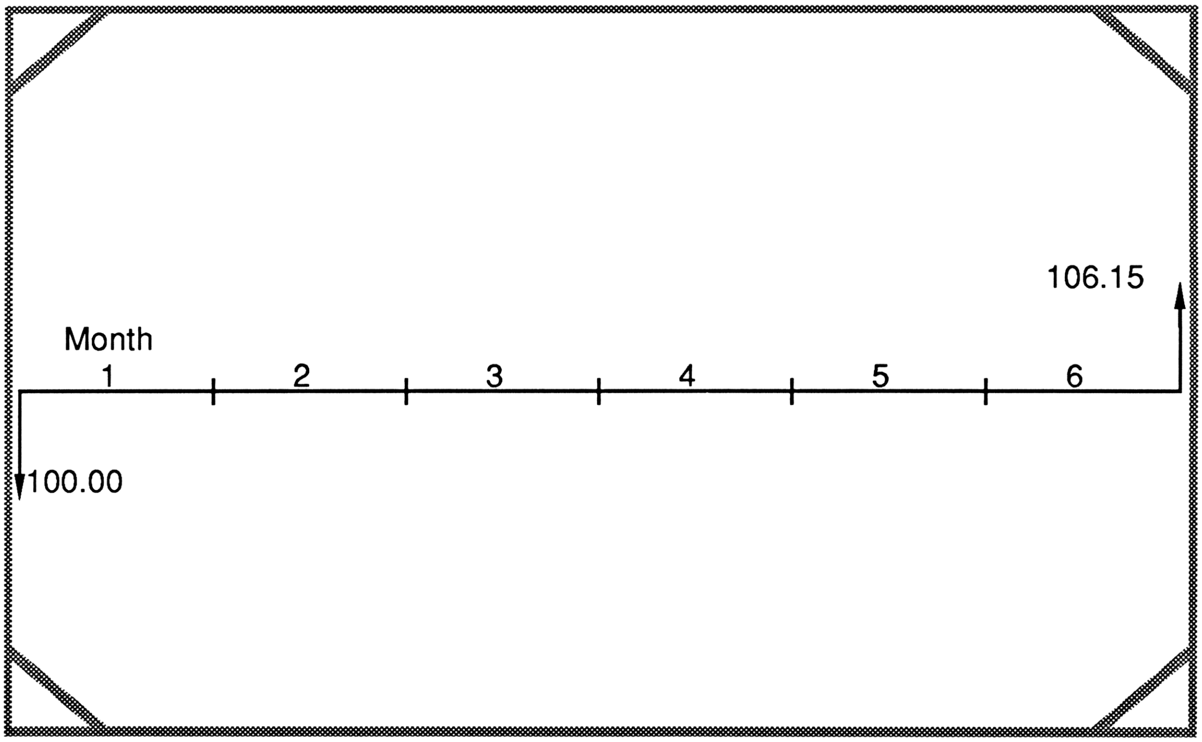
```
12 P/YR          END MODE
 N   I%YR  PV  PMT  FV  OTHER
```

(At this point, it's probably best to change the display setting to FIX 2 – for dollars and cents. Do that now; remember how – with the MODES menu?)

This is the TVM Menu. The first five selections here are the keys for drawing a cash-flow diagram for your calculator:

N	Number of Defined Interest Periods (D.I.P.'s)
I%YR	annualized Interest % rate
PV	Present Value
PMT	PayMenT
FV	Future Value

Now, the best way to think about these keys is that they form a picture frame that you can place over the paper drawing of a cash-flow diagram:



First of all, you need to realize that the TVM keys are good *only* when your diagram shows steady, level cash-flows, *one for every period throughout the timeline* – as you see above.

If your cash-flow diagram has uneven or irregular cash-flows along its timeline, you can't use TVM.

The reason for this is the **PMT** (PayMenT) key.

When describing the diagram to your calculator, the amount of that steady, level cash-flow goes into the PMT register.

TVM calculations always have a PMT amount (it can even be $0.00, but it must be level and regular throughout the picture).

As for the other four TVM parameters:

N represents the Number of periods (D.I.P.'s) from one side of the picture frame to the other.

PV is the Present Value, which is the net cash-flow (if any) that occurs at the left side of the picture frame *over and above* any PMT cash-flow that may also occur there.

FV is the Future Value, which is the net cash-flow (if any) that occurs at the right side of the picture frame *over and above* any PMT cash-flow that may also occur there.

So, for example, to describe the diagram over there on page 202 (it's the picture of your $100 bank account, assuming you wait until after month 6 to withdraw), here's what you would tell your HP-27S:

N = 6
PMT = 0.00
PV = -100.00
FV = 106.15

Notice the pattern here: Anytime you have a cash-flow paid *out* (i.e. from you to the other party), you show it on your diagram with a *downward* arrow, and you *tell it to your calculator with a minus sign* (remember the [+/–] key?).

By opening your bank account, you actually *loan* the bank $100.00, so it appears on your diagram as an outflow of cash; in your calculator as a -100.00

But you haven't completed that picture yet. What's I%YR?

Hmm...All you really know is that the D.I.R. is 1% per month, right? But notice that **OTHER** item on your menu. Press it. Here's what you'll see:

This is yet another level of menu beyond the TVM menu. Here you can vary certain parameters that affect the outcome of your TVM calculations.

For example, notice that message that says 12 P/YR (you've been seeing this message from the TVM menu, too, but here's where you can see what it means).

Your HP-27S is telling you that its current assumption about the "picture" you're "drawing" for it: 12 payments per year.

Is this correct for your bank account problem? You don't really make any PMT's (withdrawals or deposits), do you? No, but there *are* 12 PMT's per year, nevertheless.

And how can you know that? *Because the number of payments per year is always the number of periods (D.I.P.'s) per year.* There's *always* one PMT per period; that's what you mean by a PMT, right?

And you *do* know the D.I.P. is one month, so there *are* 12 payments per year – no need to change that message (which you could do by using the **P/YR** key).

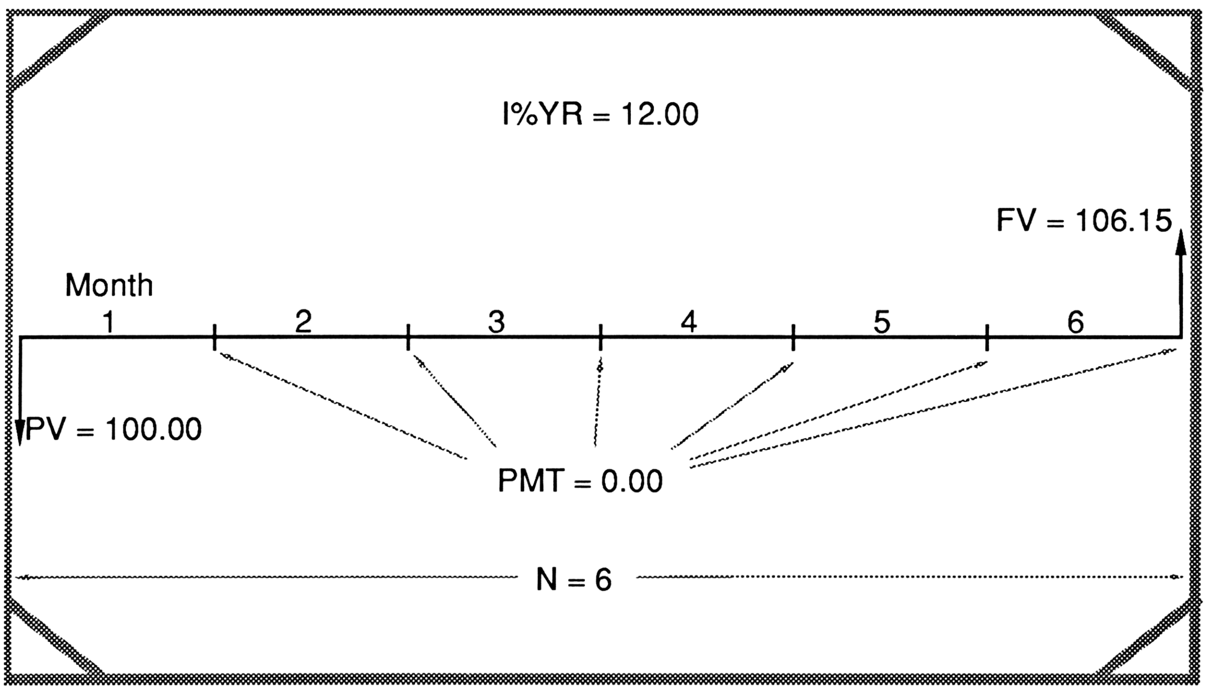
Now press EXIT to return back to the TVM menu.

Now you know that there are 12 PMT's per year.

But how does this help you to verify what I%YR should be? Because if you take the D.I.R. and multiply it by the number of D.I.P.'s per year, you'll get the I%YR. In other words,

1%	x	12	=	12%
D.I.R.	x	D.I.P.'s/year	=	I%YR

So here's the complete TVM picture of your 6-month bank account, which you've figured out in your head this first time:



Now test your calculator to see if it understands the "picture" you're "trying" to draw for it....

Verify: From the TVM menu, press

[6] N
[1][2] I%YR
[1][0][0] [+/-] PV
[0] PMT FV Result: FV=106.15 Right on the money!

Things to notice and remember:

1. Remember how pressing a menu key can mean either to store or to calculate depending upon what you did just previously?

2. You could have keyed in the N, the I%YR, the PV, and the PMT in any order – just so you have them all in there when you ask for the FV. *You must always specify four out of the five.* Never ignore one of the variables or assume that it's zero and therefore irrelevant!

3. The machine obeyed its own sign convention: Since your PV was negative, meaning cash out of your pocket, it figured that FV must be the pay-off, when the cash comes back – therefore, positive.

   In fact, the machine insists that PV and FV always be of opposite sign – to keep this idea of investment and return. If you accidentally give it a PV and FV of the same sign (when solving for PMT or something else), it'll beep at you and tell you NO SOLUTION, because it must have both an investment and a return to be a meaningful TVM problem.

4. Now that you have all the parameters in the TVM registers, you can play "What-If" simply by changing the number of months you leave your money in there.

For example, by pressing [1] N and then FV, you are changing the analysis to the case where you withdraw your money after just one month. By pressing [2] N FV, you do the two-month scenario, and so on.

See? You're playing the TVM version of the "What-If" game with the menu keys. Here you're varying one parameter (N, the number of periods), preserving three others, and checking to see how this affects the fifth parameter (Future Value) – and you never need to re-key in any number once it's stored in a TVM register. It's there to be used – or varied – over and over again.

So, what if you let your account grow for a year (12 months)? How about 50 years?

As you can see, the answers to each of these questions are merely variations on the same procedure you've already done!

And notice, also, that by playing this TVM "What-If," you're actually using the TVM keys to "slide" your last (withdrawal) cash-flow up and down the timeline – just as you did on paper (page 199)!

A Typical Loan Problem

Yes, fans, at long last, it's time to put TVM into practice with something more exciting than a bank account. After all, what's life without a good mortgage?

Like This One: You're buying a $110,000.00 home, putting down $10,000.00 and financing the rest in a 30-year mortgage with monthly payments. The quoted A.P.R. is 12.00%. What will your monthly payments be?

Solution: (If you already know how to solve this, go to page 215.)

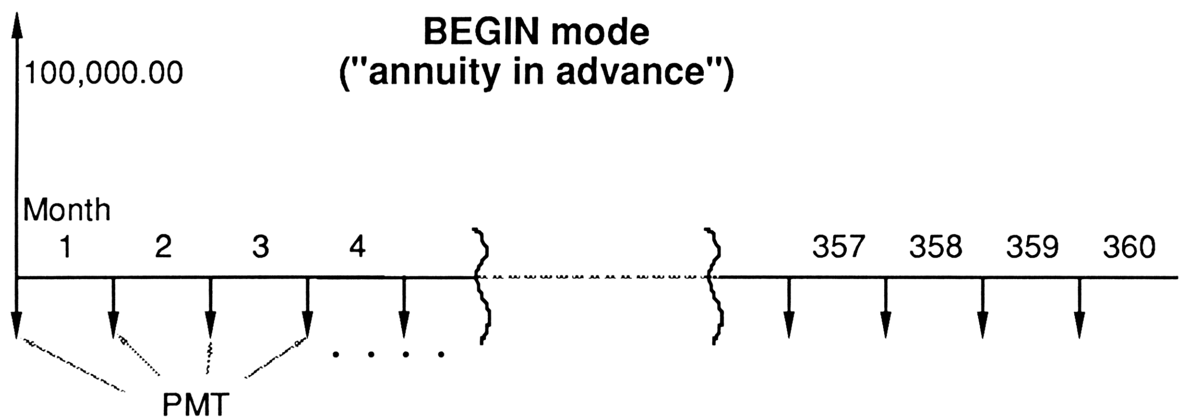
You can't solve this problem until you fill in some details that aren't specifically spelled out:

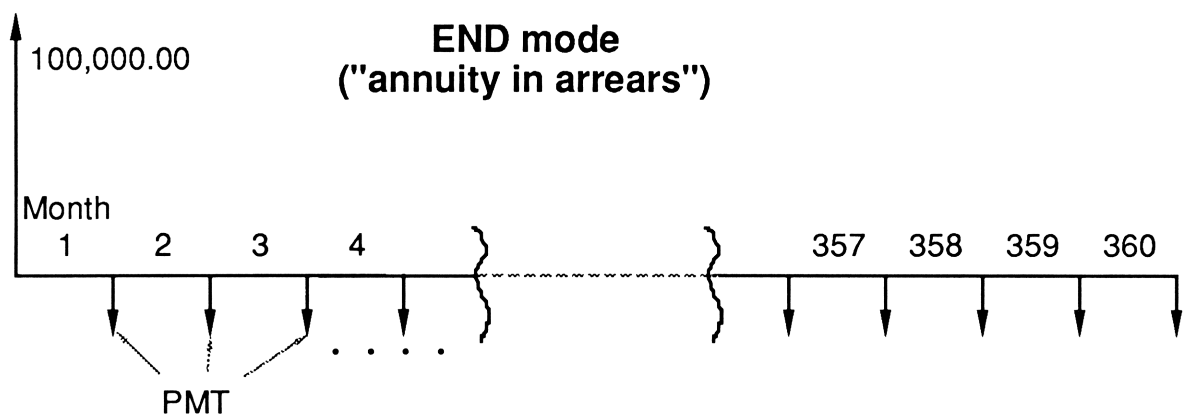
- Is this a `BEGIN MODE` or an `END MODE` problem?
- What is the D.I.P.?
- What is the D.I.R.?
- Why do you care about these things?

BEGIN Or END?

Does it really make any difference whether the payment is at the beginning or at the end of a month? After all, isn't the beginning of one month the same as the end of the previous one?

Well, compare the two pictures on a cash-flow diagram:





As you can see, in the first case, a payment is due at the beginning of the loan – right after you sign the papers. This reduces the balance sooner, so *there is less interest to be paid on the borrowed money.*

With less interest to be paid, the PMT amount (which covers both principal and interest) will be less. So it *does* matter whether the payment is at the beginning or at the end of the month.

So what about your mortgage? Which is it going to be – BEGIN or END?

In a real contract, it would have to be stated, of course, but for this problem, just assume the more common case – the END of the month.

Fine. Now, how do you tell your HP-27S to assume this too?

Like This: From the TVM menu, press the OTHER key and see this again:

```
12 P/YR              END MODE
P/YR  BEG  END           AMRT
```

Notice the two keys, BEG and END. Press them alternately and notice the change in the message in your display (but then be sure to leave it in END MODE for this problem).

Enough said, right?

Just don't forget to check this "little detail" for each new TVM problem! Usually, when you're at the TVM Menu itself, you won't be able to tell which mode your calculator is using (the only time you can is when you've just moved to that menu from somewhere else; you'll see a message on the Calculator Line. Of course, if you press CLR or start keying in numbers, this message goes away).

Next detail to clear up:

What's the D.I.P.?

The problem says 12% A.P.R., and it's safe to assume compound interest, but it doesn't say how *often* this interest compounds. Is it yearly? After all, A.P.R. does stand for Annual Percentage Rate.

Yes'n no: Yes, that's what A.P.R. means, but no, the compounding isn't yearly.

It's monthly.

How do you know? Because the payments are specified as monthly, and unless explicitly stated otherwise, the payment period *is* the compounding period (and vice versa).

Thus, the Defined Interest Period (D.I.P.) – the period over which interest *actually* compounds – is a month (wasn't the suspense killing you, though?)

And now you need to tell your HP-27S this detail also....

Clue It In: From the second "page" of the TVM menu, press 1 2 P/YR

(Your display has probably already been showing 12 P/YR, but do this anyway, just for practice.)

Next mystery: What's the D.I.R.? That is, what's the monthly rate of interest that corresponds to an A.P.R. of 12.00%?

It's "12 percent divided by 12 months per year," or 1% per month (remember the little discussion of A.P.R. back in Subtlety #1, on page 192?).

And now that you've decided this, you'll probably sleep better tonight, but you don't need to tell your calculator about your conclusion; it will *assume* it.

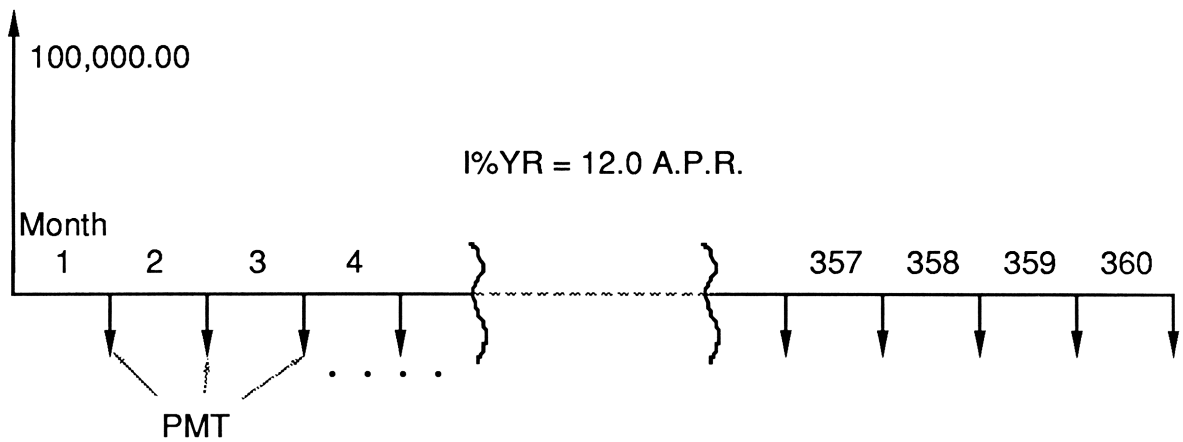
The nominal A.P.R. used to compute a loan payment is exactly what I%YR means to the HP-27S.

That is, to arrive at the actual interest rate to use in its calculations, a bank would take this *nominal* A.P.R. and divide it by the number of compounding periods in a year.

So when you see a nominal A.P.R. specified for the purposes of computing a payment or a balance, just make sure that your P/YR is set properly and then use that A.P.R. as your I%YR.

A Typical Loan Solution

Are you all set to solve this mortgage problem (finally)? Fine, but don't forget to draw it in its final form on a cash-flow diagram:



Notice how it's drawn from the perspective of you – the borrower. Your loan is shown as positive (you *receive* it, right?). Then, of course, each payment is negative – because you *pay* it.

Now, simply by looking at the picture, you can just read off the values you need for the TVM registers:

Read 'em and Whip: From the TVM menu (of course), press 3 0 × 1 2 N 1 2 I%YR 1 0 0 0 0 0 PV 0 FV.

Then press PMT to calculate the payment.
Answer: PMT=-1,028.61

Notice how your calculator knows that this PMT must be negative (a cash outflow). It's obeying its own sign convention, of course, and thus it agrees with your diagram!

Question: How did you know that FV (Future Value) was zero?

Answer: The problem didn't specifically *say* that there's anything left to pay on the loan at the end of the 30 years, so you can safely assume there isn't. In other words, the payments completely *amortize* (literally, "kill off") the loan in 360 months.

If it were otherwise, the problem would have specified an amount due in a lump-sum payment – called a *balloon* payment – at the end of the 360-month term.

All right, so you know that FV is zero. *But even so, you couldn't ignore it.*

Zero is a number just like any other, and your calculator always uses four TVM numbers to solve for the fifth. If you had forgotten to specify your FV for this problem, your machine would *not* have assumed zero. It simply would have used whatever number was last stored into the FV register – no matter when that was.

Remember! A number in a continuous-memory machine like the HP-27S is like a budget deficit: It won't melt back to zero just because you ignore it.

A Typical Loan Quiz*

1. Review in your mind the checklist of all the little invisible steps you took to actually calculate the payment on that $100,000.00 mortgage.

*Pretty merciful quiz, don't you think? Actually, this mental checklist is a very important habit for you to develop, so take a bit of time here and see if you can tick off all the particulars you need to remember. A suggested version is on the next page in case you want to compare it against your checklist.

A Typical Loan Checklist

1. You got a verbal description of the loan (page 208);

2. You decided on the *annuity mode.* In other words, do the payments come at the END of the month (also called "annuity in arrears") or at the BEGINning of the month (also called "annuity in advance")? (See pages 209-210.)

3. You interpreted the terms of the loan in order to arrive at the proper D.I.P. and thus the D.I.R. You then set the P/YR accordingly (pages 211-212). And you observed that the nominal A.P.R. given in the problem can be used directly as your I%YR.

4. You drew the correct picture of the problem from your perspective as a borrower, thereby establishing the directions of the cash-flows (up or down on the drawing, + or - in your calculator) – page 213.

5. You observed that since no mention was made of any balloon payment, the mortgage must be completely amortized after the 360th payment. Therefore, the Future Value (FV) must be zero (page 214).

6. You plugged in all your known information (N = 360 months, I%YR = 12.00, PV = 100,000.00, FV = 0.00) and solved for PMT.

7. You realized that most of the real problem is in defining it; the keystrokes are easy.

Variations On A Theme

So there you have it – a $1,028.61 monthly mortgage payment. And that's assuming that your payment comes at the END of the month.

Question: What if it were at the BEGINning of the month? What would your payment amount be? By looking at the comparison diagrams on page 209, can you tell whether it's more or less than the END mode payment?

Solution: Don't touch any of the values you now have in the five TVM registers. Just go to the OTHER TVM menu and change the annuity mode from END to BEGIN.

Then [EXIT] back to the TVM menu, and solve for PMT once again.

Answer: PMT=-1,018.43

About ten bucks a month *less* – not a whole lot, you might think. But over 360 months, that works out to be over $3,600 in interest saved.

How are you saving this interest? Is the rate lower?

No. With BEGIN mode, you're still paying the same interest *rate,* but you are saving money, because you're *borrowing for a slightly shorter time.* From the diagram on page 209, you can see that you're actually making each payment one month earlier than in the END mode scenario.

Well, in either BEGin *or* END mode, that payment's a little steep for your budget, but you do want the house, so now you're going to go shopping for better mortgage terms....

O Happy Day: Up jumps a lender who offers you all the same terms – 360 months, a payment at the end of each month – *except* it's only a 10.5% A.P.R.(!) What will your payment be?

A Mere Pittance: Again, leave all your TVM parameters alone, except for the ones you're now going to vary:

Press OTHER,then END, to set the annuity mode properly. Then EXIT. Now, the only thing you need to change is the I%YR, right?

Right. But just this once, prove it to yourself by recalling each of the five TVM registers (this is just another friendly public service reminder that all your menu variables are just registers that you STO into and RCL from):

RCL N	(N=360.00 Don't change it.)
RCL I%YR	(I%YR=12.00 That's what to change.)
RCL PV	(PV=100,000.00 Don't change it.)
RCL PMT	(PMT=-1018.43 You're going to recalculate this after changing the I%YR.)
RCL FV	(FV=0.00 Don't change it.)

OK, so change the interest rate: Press 1 0 · 5 I%YR

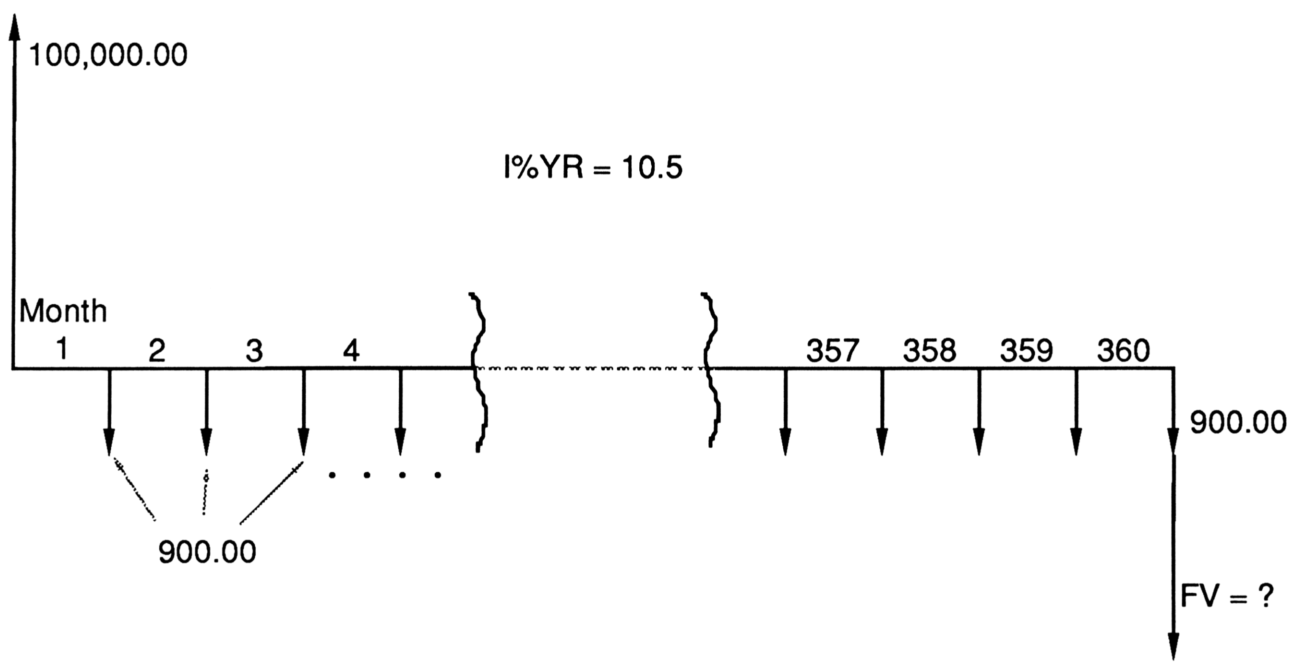
Then press PMT to see the good news: PMT=-914.74

Well...that's better – about $100.00 better.

But what if it's still too high? What if your absolute limit were $900.00 even?

That's OK. But it will mean that at the end of the 360th month, you'll have a lump-sum balloon payment to make – to pay off the remaining balance. You *know* there'll be this remaining balance, because the calculator just told you that it will require $914.74 per month to completely amortize this loan. If you pay even slightly less than that, there'll be some left over to pay at the end.

So here's the way this looks on a cash-flow diagram:



How much is that final balloon payment?

Find Out: Again, three of the five TVM parameters are already correct (the term, N, the interest rate, I%YR, and the Present Value, PV). What you're going to do now is specify the payment (rather than calculate it) and then find the Future Value.

Press 9 0 0 +/- PMT (remember why this should be negative?), and solve for the remaining balance: FV

Answer: FV=-37,089.98

Remember the definition of Future Value! This represents the amount you would have to pay *over and above* your final (360th) payment – and it's negative, as it should be, to indicate who's paying it. And since this is an END mode problem, your last PMT occurs at the same time as this balloon payment.

(Looking back at that diagram on the previous page, can you see how clearly and succinctly all this information is shown?)

As you can see, then, you *could* lump those last two cash-flows together and write just one check if you wanted. Just be sure that you don't forget what each part means. Sometimes, the terms of a contract won't itemize this final combined payment, and if you try to confirm such terms by using that *combined* total as your Future Value ... well, that's not what your HP-27S means by Future Value. You need to subtract out the amount of the final PMT.

You'll never get confused if you draw the cash-flow diagram.

So, what have you decided up to now?

You're going to borrow $100,000.00 at 10.5 A.P.R., repaying it in 360 end-of-the-month installments of $900.00 each, plus a final balloon payment of $37,089.98.

OK?

No? *Now* what's wrong?

Oh – the balloon's a bit much, eh? It's amazing what a little less each month adds up to over thirty years, isn't it?

All right, how big a balloon payment could you stand? You realize, of course, that a lower balloon will demand a slightly higher monthly payment; you'll be breaking your $900.00 monthly payment ceiling.

True, but if a few bucks a month now can save thousands in thirty years....

How about, say, a $20,000.00 balloon?

Fine. Now check to see what PMT this will demand.

Not Much More: As usual, vary only what you want to (this should be getting quite routine by now). Press 2 0 0 0 0 +/- FV to specify the balloon. Then just press PMT, to find the corresponding payment.

Answer: PMT=-906.79

So those are the numbers you can live with, eh? Review them now with the RCL key (RCL N, RCL I%YR, etc.).

They should give you this summary of the TVM registers:

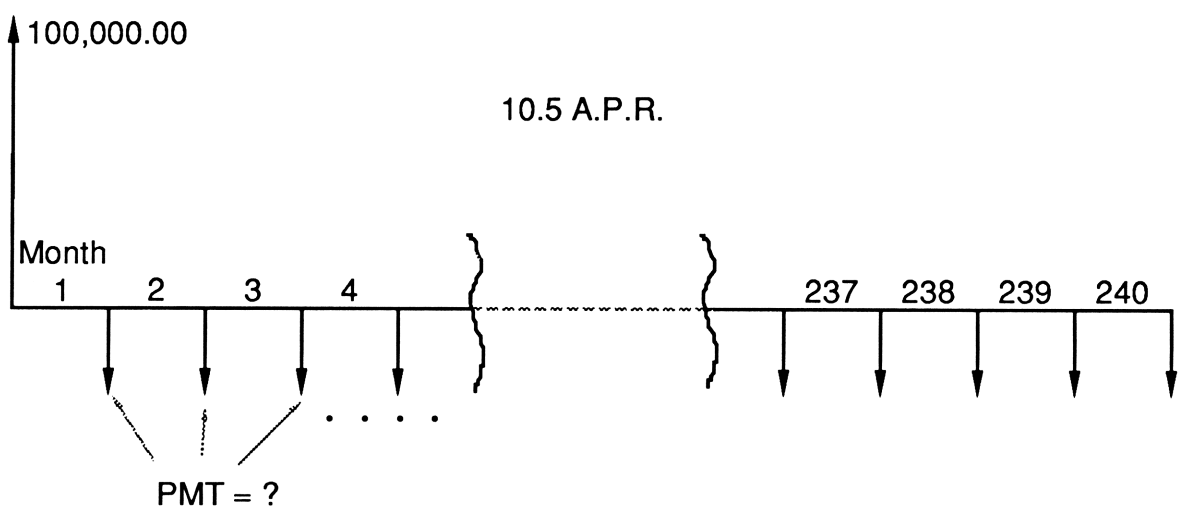
360	10.50	100,000.00	–906.79	–20,000.00
N	I%YR	PV	PMT	FV

The only problem is, your lender with the 10.5 A.P.R. won't agree to anything less than full amortization – no balloons allowed.

In fact, he's actually encouraging you to consider even higher payments – to shorten the term of the loan down to 25, 20, even 15 years.

Of course, you know you can't afford this much every month, but now that you've got a machine like the HP-27S, it doesn't hurt to play "What-If," right?

What if you had the same financing rate and amount, but, only a 20-year term (for example)? Your cash-flow diagram would look like this:



Play With The Term: Find the payment amount necessary to fully amortize this loan in 25, 20, and 15 years.

Solution: Change what you need to; leave the rest of the five parameters alone. Press [2][5] [×] [1][2] [N] (shorter term) [0] [FV] (no balloon allowed) and then solve for [PMT]

Answer: PMT=-944.18 Not too much more than your $914.74 – for five years' fewer payments!

Now try 20 years: Press [2][0] [×] [1][2] [N], and then [PMT].

Answer: PMT=-998.38

Finally, try 15 years: Press [1][5] [×] [1][2] [N], then [PMT].

Answer: PMT=-1,105.40

Did you realize that for a payment of about $200 more per month, you could cut your mortgage term *in half?*

Not bad – if you could scrape together that extra $200/month! But you can't, so you shake the fellow's hand, thank him for his eye-opening offer, and head on down the road....

Feeling slightly discouraged at this point, you start to wonder if you can really afford this house after all.

Sigh: You had set your upper monthly payment limit at $900.00, but even that was pushing it. What you *really* wanted to get away with was about $800.00 or less – and no balloon.

How much *could* you afford to finance at 10.5 A.P.R., with 30 years of $800 payments (end of the month, as usual)?

Check and See: Here's how your TVM registers look at this point (and you know how to verify it, right?):

N	I%YR	PV	PMT	FV
180	10.50	100,000.00	–1,105.40	0.00

This time, you're varying the PMT and calculating the Present Value – the amount you finance:

[3][0] [×] [1][2] **N** (to return to the original concept of a 30-year term)
[8][0][0] [+/–] **PMT** (what you'd like to get away with)
PV (find how much you could thus finance)

Answer: `PV=87,456.61`

So if you could come up with about $12,500 more in a down-payment, you could swing the $800.00 monthly installment. But you can't do that, and you *do* want the house, so keep your $900 limit, with a $20,000 balloon – and keep looking....

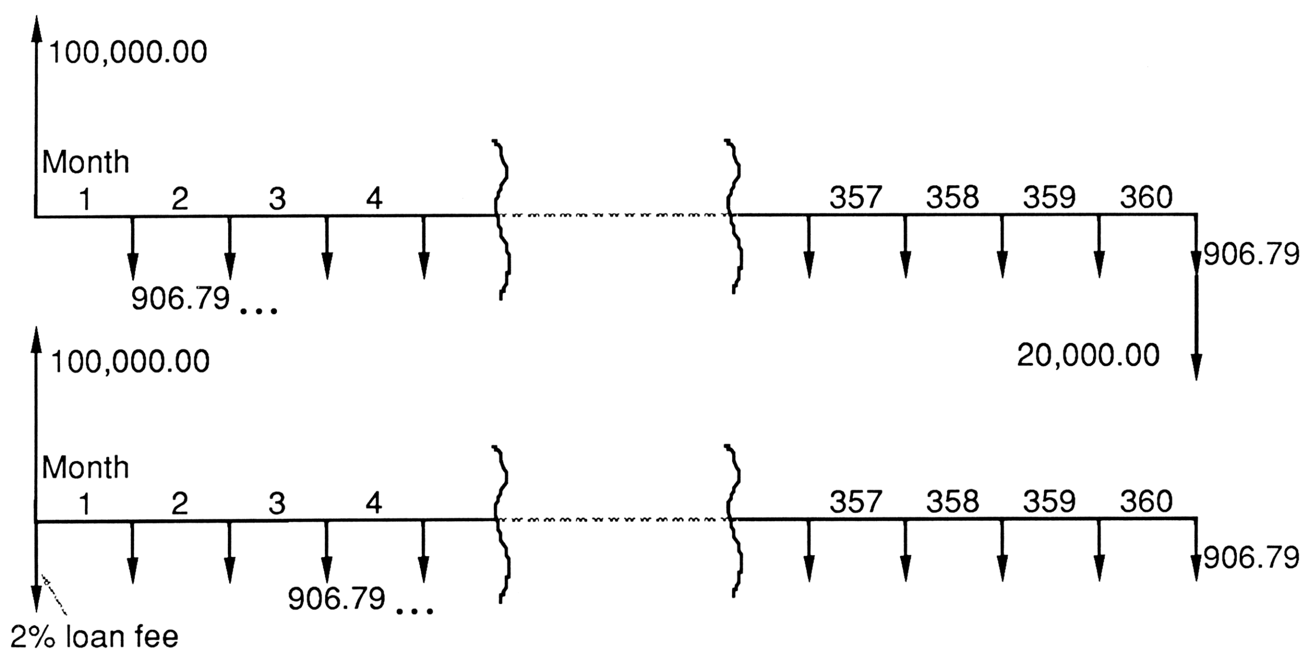
On The Road Again: Here's a lender who will loan you $100,000 in exchange for $906.79 payments (end of the month, as usual) for 360 months – period.

Hmm...that sounds at lot better than $906.79 a month, plus a $20,000.00 balloon, as you had back on page 221. What's the catch?

It's this: There's a *loan fee* (also known as "points up front") due and payable at the moment you sign the papers. This fee is 2% of the amount financed, and it's *not* an early installment on the loan; you still have to pay 360 full payments of $906.79 each.

Is this *really* a better deal than the $20,000.00 balloon?

Solution: Whenever you're trying to compare deals, of course, you should first diagram them:



But it's still hard to tell which is the better deal for you. How can you use your TVM calculations to help?

Think about it: On page 221, you *know* you're paying 10.5 A.P.R., because that's what you *specified* when calculating your PMT. But this lender here just pulled that payment amount out of the air (what a coincidence, eh?).

If you give your HP-27S this scenario to crunch, what will it tell you is the prevailing interest rate?

Here are the current contents of the TVM registers:

N	I%YR	PV	PMT	FV
360	10.50	87,456.61	–800.00	0.00

So change only the PV and PMT.

First, press 9 8 0 0 0 PV. Do you see why? Because you can net together simultaneous cash-flows on a cash-flow diagram.

So actually, with the $2,000 loan fee (2% of $100,000), you're *not* getting a $100,000 loan. It's a $98,000 loan – but you're still paying $906.79 a month for it.

So press 9 0 6 • 7 9 +/– PMT and then I%YR, to figure the *real* A.P.R.... Answer: I%YR=10.64

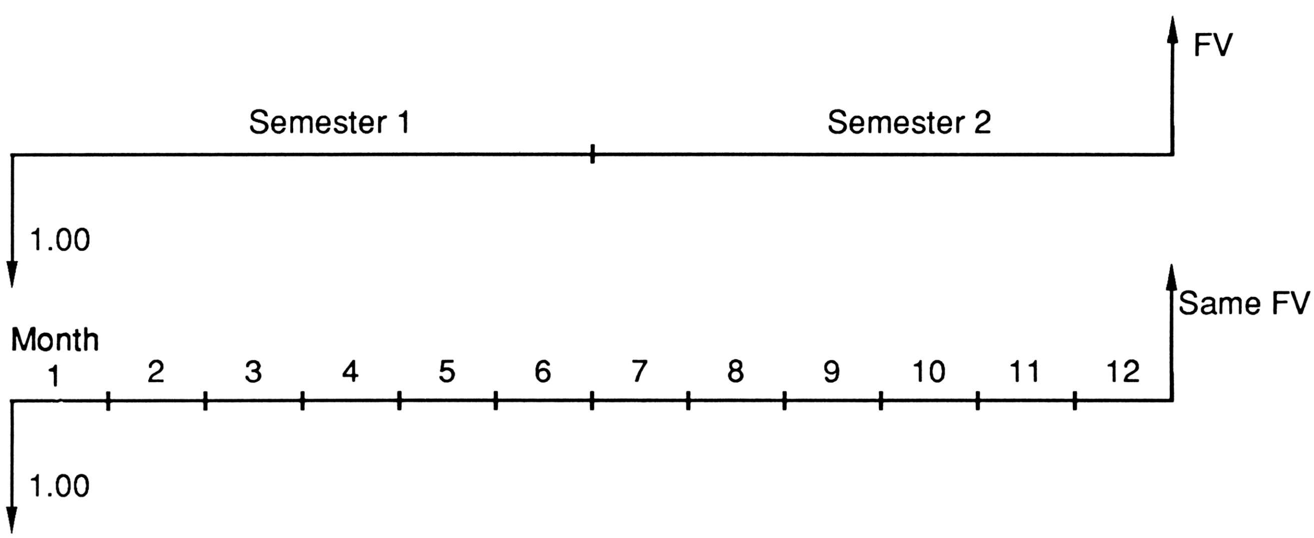
Aha! That loan fee effectively jacked up the rate *above* 10.5 A.P.R. Not so good!

Down But Not Out: Off you go again, still in search of that elusive Perfect Mortgage. Around the corner is yet another bank, which goes for all your terms – monthly payments, balloon, A.P.R., everything – *except* ...they are a Canadian lender, so their interest compounds *semiannually.*

Hmm... How do you compute a monthly payment with interest that compounds semiannually? Doesn't the payment period *always* need to match the interest period (the D.I.P.) on the HP-27S? ...Yep.

Time to punt? Nope. You merely need to *convert the semiannual compounding rate to its equivalent monthly rate. Then you can use TVM as usual.*

Think about it: If you can find a *monthly* interest rate that accrues in one year *exactly* the same amount of interest on a dollar as this *semiannual* rate, then they're really the same rate with different names, aren't they?



"If you can't tell the difference, there *is* no difference."

Solution:

The key here is to recognize that you need to do a side calculation first – to convert to an equivalent monthly rate. Then once you do that, you'll plug that rate into the I%YR register, and away you go.

So start by solving the first of those two diagrams on the previous page: Ask how much that $1 grows in one year, assuming *semi-annual* compounding of 10.5 A.P.R.:

Press [2] **N** [1][0][•][5] **I%YR** [1][+/−] **PV** [0] **PMT**, then **OTHER** and [2] **P/YR** to specify the compounding frequency.

Now [EXIT] to TVM, and solve for Future Value: **FV**

Answer: `FV=1.11`

Again, this is what that $1 becomes after a year of semi-annual compounding – OK?

Now for the "coope de grayce:" You want to find the I%YR that will produce these exact same circumstances ($1.00 up front and $1.11...exactly one year later), *except that you want this to happen in twelve periods of monthly compounding.*

So just change P/YR to 12: **OTHER** [1][2] **P/YR**, then [EXIT], change the N to 12 (press [1][2] **N**), then solve for I%YR (press **I%YR**). *This gives you the equivalent monthly compounding rate.*

Answer: `I%YR=10.28`

Leave that result right there in the I%YR register.

Now you're ready to figure your payment:

N = 360.00 (key this in)
I%YR = 10.28 (you just put this there – don't touch it)
PV = 100,000.00 (you'll need to key this in)
FV = -20,000.00 (this needs changing, too)

Solve for PMT: PMT=-889.80

That's less than your 906.79 for the monthly compounding case – which makes sense, right? After all, your equivalent monthly A.P.R. turned out to be 10.28, rather than 10.5.

"At last – a mortgage with a $20,000.00 balloon *and* payments of less than $900.00!"

All your shopping around and "What-Iffing" with your HP-27S has paid off!

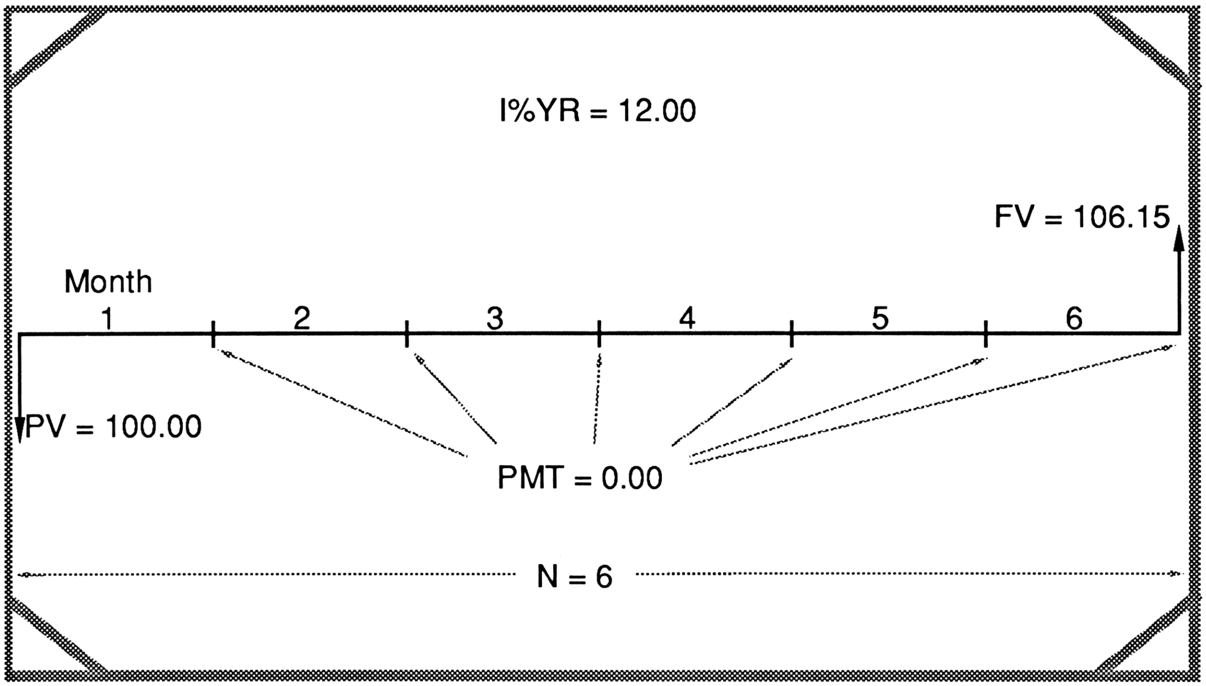
Review again exactly what you had to do here with this Canadian mortgage:

To convert the *quoted* A.P.R. into a *usable* A.P.R. (one matching the payment period), you had to do a side calculation.

In that side calculation, you found out what the quoted A.P.R. effectively does to a dollar in one year. Then you simply changed the number of compounding periods – to reflect the period you want to convert to – and asked what A.P.R. does the very same thing to a dollar in *that* number of periods.

Once you got your result, you used it as your I%YR in a straightforward TVM calculation.

And *all* this comes from your basic knowledge of the "picture" analysis of the Time Value of Money – where your five TVM variables form a "picture frame" of understanding that you place over your cash-flow diagram:



So by now, you should know how to solve these types of problems:

- A simple, fully-amortized loan with annuity either in arrears or in advance;
- A loan with a balloon payment;
- Conversion of an interest rate that compounds over a different period of time than the payment period;
- A loan with prepaid loan fees (also called "prepaid finance charges" or "points up front").

Ready to solo?...

A TVM Quiz

(You just *knew* this was coming, didn't you? Be sure to diagram each situation.)

1. A mortgage is written at 15.5% A.P.R. It amortizes totally in 30 years of $400 monthly payments (in arrears). What is the loan amount?

2. If the above loan had $350 payments – and annuity in advance – what would the remaining balance be after the 360th month? After the 120th month?

3. A finance company agrees to loan $100,000 at 17% A.P.R., compounded daily (on a 360-day year), in exchange for quarterly payments (in arrears) that will amortize the loan in 15 years. A 2% finance charge is due and payable at the beginning of the loan. (Good grief!)

   a. What is the payment amount?

   b. What is the remaining balance after 10 years?

   c. If the balloon is actually paid at that ten-year point, what's the quarterly-compounded A.P.R. really earned by the finance company?

   d. Would that A.P.R. have been different if the loan had continued on to full term (15 years)?

4. You want to buy an $86,000 home, with 10% down and the balance financed at 11% A.P.R., with monthly payments in arrears. Your maximum payment is $750. Can you totally amortize this loan in 20 years? If you were to make *whatever* payment is necessary to do so, what is the face value of the interest you would save – compared to a 30-year total amortization?

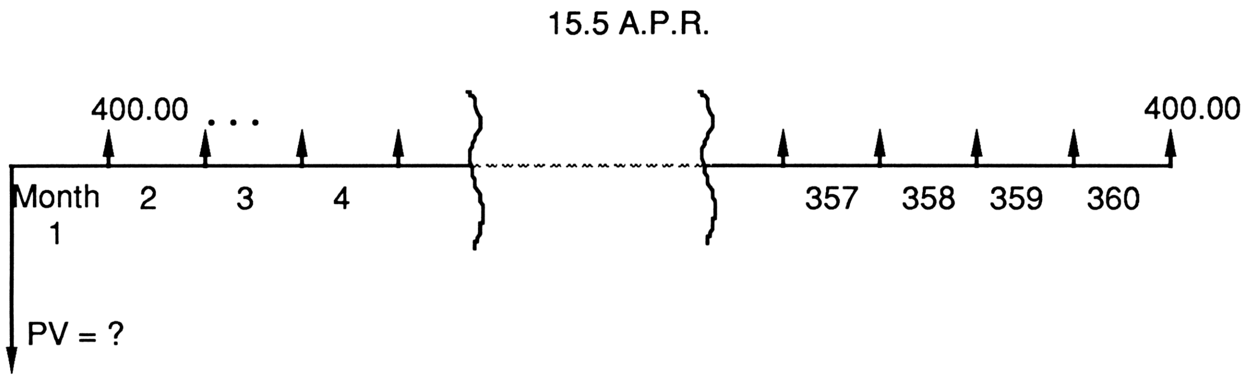
5. Which carries a higher interest rate: A $20,000 car loan with monthly payments of $440 (in arrears) and a balloon of $5000 in five years; or a credit card that charges 16.5% A.P.R., compounded daily?

6. If inflation is an *effective* 4% per year (compounded daily), and your tax bracket is 27%, what will be the true value (i.e. the buying power in terms of today's dollars) of a taxable account earning an *effective* 10% per year (compounded daily) into which you are paying $5.82 per day – every day – for 25 years? For 40 years? For 50 years?

7. If, at the beginning of your 26th year (happy birthday), you pay $2000 per year for 7 years into a (tax-deferred) IRA yielding a 10% A.P.R., and then you *stop* paying into that account for 28 subsequent years, how does the resulting balance compare with a similar account that you open at the beginning of your 33rd year and pay $2000 for *every year* up through your 60th year?

TVM Quiz Solutions

1. Here's the picture of the problem:



This is drawn from the perspective of the lender, right?

It's a pretty straightforward problem – just asking you to backtrack and figure out what amount was financed. That unknown would be the Present Value, wouldn't it?

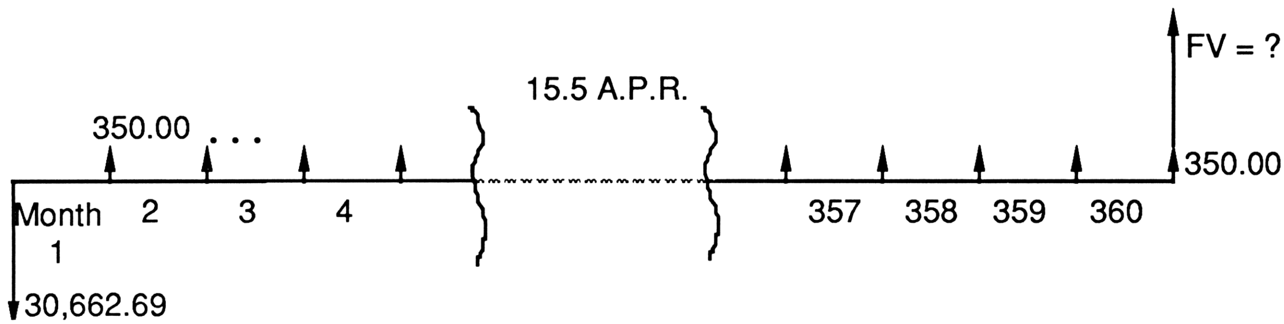
From the TVM Menu, press OTHER and check your annuity mode (should be END MODE) and P/YR (should be 12).

Then EXIT to TVM, and go for it:

3 0 × 1 2 N
1 5 • 5 I%YR
4 0 0 PMT
0 FV Then solve: PV Answer: PV=-30,662.69

(Sort of a weird amount for a mortgage)

2. Here is the variation on the previous problem:



Since you've just completed the original version of the problem, all those TVM parameters are still conveniently sitting in their registers. So change only what you have to: First, from the TVM Menu, press OTHER BEG EXIT

Then 3 5 0 PMT and FV to get the balloon left to pay after 30 years:

FV=353,916.61

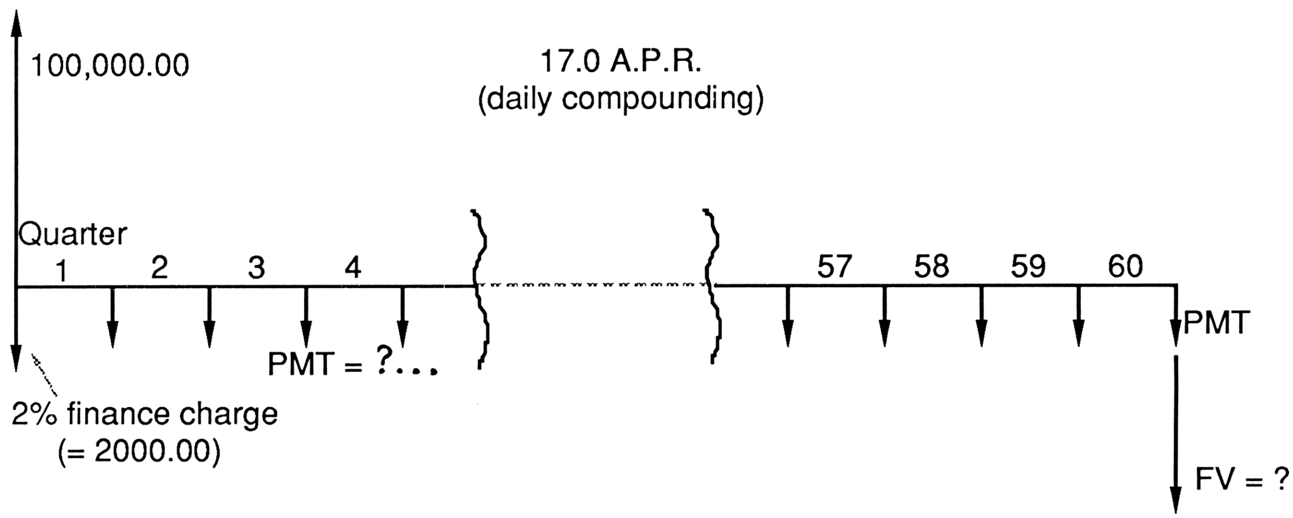
Yikes! What's *that*? How could an innocent little $30,662.69 mortgage produce this howling monster balloon? ("But officer, there *must* be some mistake!...")

Nope: It turns out that $350 a month *doesn't even cover the interest* on the loan. Therefore, instead of covering all interest and eating away a little of the principal, which is what a normally amortizing payment is supposed to do, this anemic little payment *never touches* the principal and even fails to cover all the interest. So all that growth is the "uncovered" interest compounding for all those years. This is called *negative amortization.**

Look how far it gets in just 10 years: 1 2 0 N FV
FV=42,448.84

*Asked to name the most awesome force in the universe, Albert Einstein is said to have once replied, "Compound interest."

3. Here's the situation:



Your first problem is to match the interest rate (daily) to the payment schedule (quarterly). That's a side calculation (remember?):

From the TVM menu, press [3][6][0] **N** [1][7] **I%YR** [1] [+/–] **PV** [0] **PMT**, then **OTHER** **END** and [3][6][0] **P/YR** to specify the compounding frequency. Now [EXIT] to TVM, and solve for Future Value: **FV** <u>Answer</u>: `FV=1.19`

Now just change the P/YR to 4: **OTHER** [4] **P/YR**, then [EXIT] , change N to 4 (press [4] **N**), and solve for I%YR (press **I%YR**), to get the equivalent quarterly-compounding rate. <u>Answer</u>: `I%YR=17.36`

There you have it – the quarterly-compounding A.P.R. that does the same thing to a dollar in one year as does a daily-compounded 17% A.P.R. And your correct quarterly A.P.R. is now in I%YR, safe and sound.

a. All right, the first question is, what's the payment amount? That's now a straightforward calculation:

1 5 × 4 N	(15 years of quarterly payments to amortize totally)
1 0 0 0 0 0 PV	(this is the amount being amortized; it's not reduced by the finance charge)
0 FV	(payment amount must totally amortize in 15 years)
PMT	PMT=-4,708.42

b. Now it's easy to find the remaining balance after 10 years (40 payments):

4 0 N FV   FV=-62,101.87

c. To figure the finance company's *true* A.P.R., you now need to consider that they didn't really loan $100,000, but only $98,000. But they're still receiving the *payments* as calculated on $100,000, so the only thing you need to change before calculating I%YR is the PV:

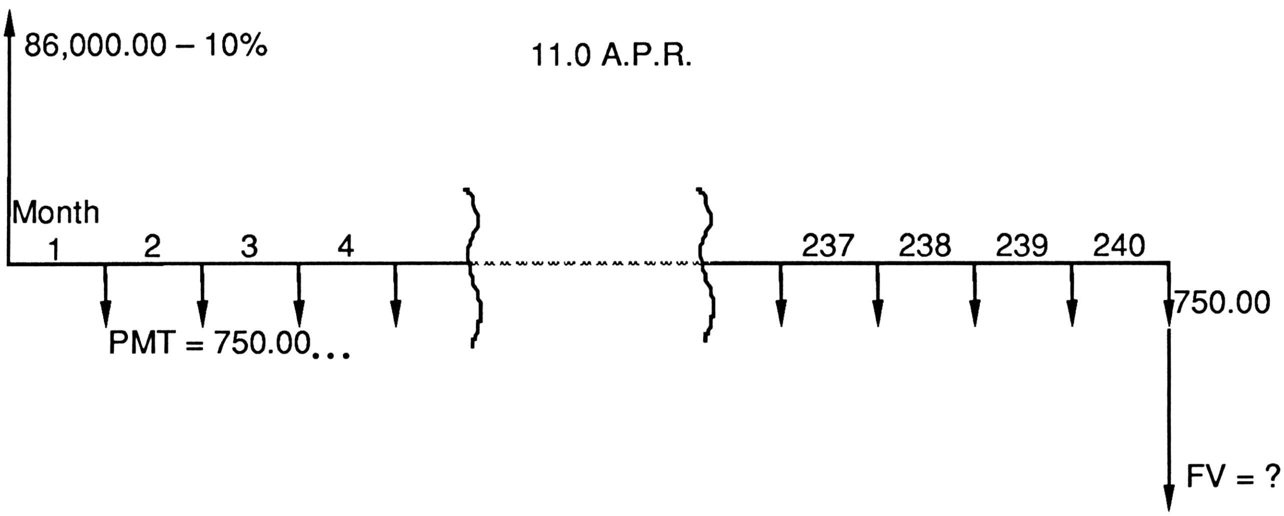
9 8 0 0 0 PV I%YR   I%YR=17.84

d. If you take the loan to term instead, N is now 60, and the FV is now zero, but all the other parameters apply from the calculation you just did:

6 0 N 0 FV I%YR   I%YR=17.81

It *does* make a difference! Why? Because the finance charge is the same in either case, but its impact on the yield is "distributed" over more time in this latter case, thus "diluting" the "boosting" effect on the A.P.R.

4. Here's the scenario:



You want to vary N and see how it affects FV. If FV is a negative quantity, then that represents a balloon you'll have to pay at the end of the term. If FV is zero or positive, it means you either totally amortized the loan or actually *overpaid* it (thus showing a positive balance); the $750 payment is sufficient in that case. If not, then the payment must be higher – or the term must be longer.

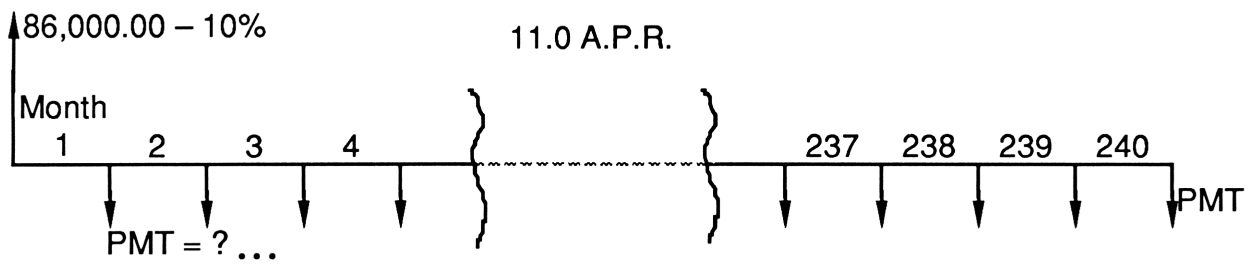
First, be sure you're set to **12 P/YR** and **END** mode.

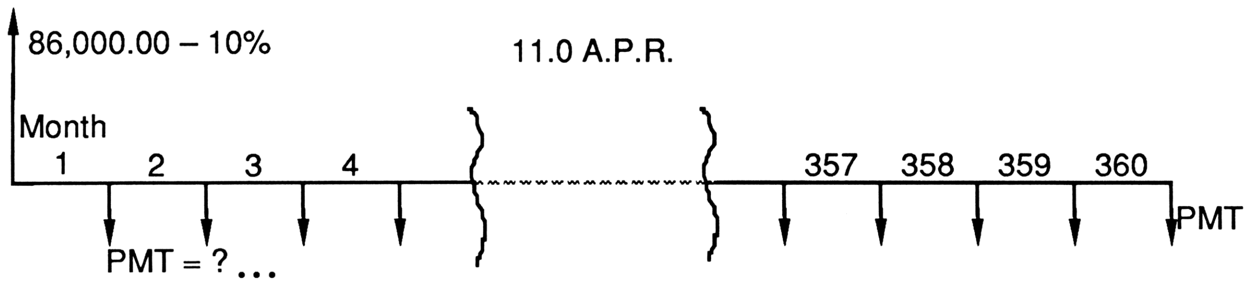
Then, from the TVM menu, press

[2][0][×][1][2] **N**
[1][1] **I%YR**
[8][6][0][0][0][−][1][0][■][%] **PV**
[7][5][0][+/−] **PMT**
FV Answer: **FV=-42,341.66**

That's a balloon payment that you would need to *pay* (it's negative), so $750 is *not* enough to amortize the mortgage in 20 years.

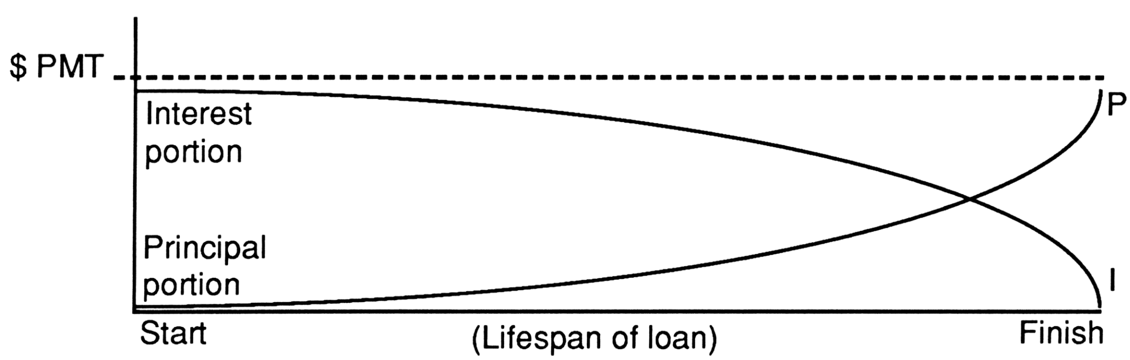
As for the second part of the problem, you're comparing these two scenarios:





The question is, what is the face value of the *difference in interest* paid over these two terms?

Of course, every amortizing PMT does two things. It completely pays all interest accrued on the loan in that month, *and* it pays a little bit of the principal amount. Thus, the next month, there's a little less interest to pay and therefore a little more room to pay on principal, etc. Here's a rough picture of how the "P" and "I" portions of a payment change over the term of a mortgage:



Now, since both of the above loans are totally amortized, the principal paid in each case is the same – the total amount financed, $77,400. Thus, the difference in interest paid is simply the difference in total payments.

So to get the answer here is quite easy. Just solve for the payments in each case, multiply your answers by the *number* of payments in each case (giving you the total P & I paid in each case) and subtract:

Since everything is set up from the first part of the problem, press [0] **FV** to specify total amortization, then **PMT** . (Answer: PMT=-798.91)

Then [×] [RCL] **N** [=] [STO] [1] to save the total paid for that case (see how that arithmetic works – recalling from and storing to registers?)

Next, press [3][0] [×] [1][2] **N** **PMT** (Answer: PMT=-737.10)

And [×] [RCL] **N** [−] [RCL] [1] [=] (see how this arithmetic works?)

Answer: -73,616.08

That's the *face value of the difference* in payments for the two loans (it's negative because you subtracted the lesser, the 20-year amortization, from the greater, the 30-year case).

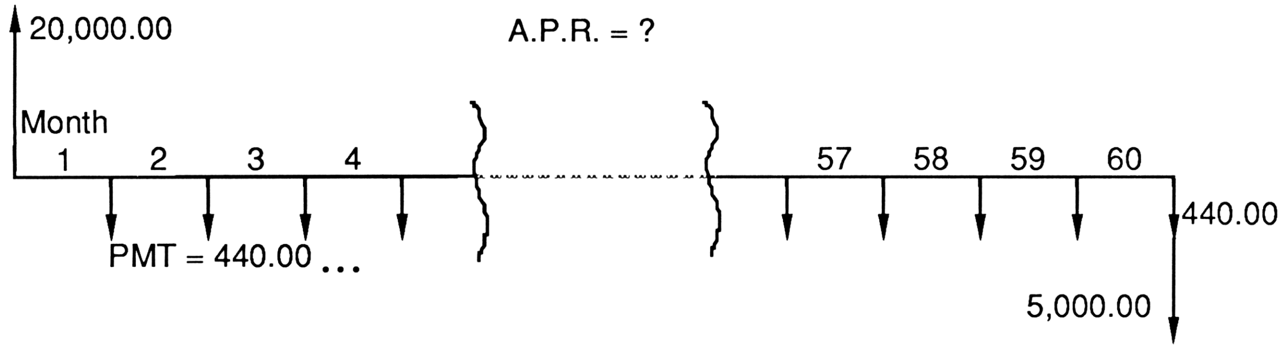
But why this phrase, "face value?"

It's because with the TVM assumption, you really can't equate amounts of money paid *unless they are transacted at the same time.* Clearly that's not the case here, since one loan goes on ten years longer than the other.

So it's not really correct to say that the 20-year case saves $73,616.08 over the 30-year case. After all, if you had a spare $737.10 per month for ten years, you'd be putting that money away in an interest-bearing account, wouldn't you? And after ten years, it would be worth *much more* than its face value.

It would have the *added value of time.*

5. This is just a comparison of two I%YR's, but you'll need some figgerin' with each before you can compare them directly. Here's the car loan diagram:



Just plug in all the knowns and solve for the unknown – the I%YR: From the TVM menu (after setting a monthly END mode, as usual), press

6 0 **N**	(five years' worth of months)
2 0 0 0 0 **PV**	(pretend you're the borrower; you receive the loan)
4 4 0 +/– **PMT**	(negative because you pay this every month)
5 0 0 0 +/– **FV**	(there's your balloon at the end)
I%YR	<u>Answer</u>: I%YR=16.70

As for the credit card rate, all you need is one of those $1 side calculations to convert the card's rate to the same compounding cycle as the car loan:

Press 3 6 5 **N** 1 6 • 5 **I%YR** 1 +/– **PV** 0 **PMT**, **OTHER** 3 6 5 **P/YR**.
Now EXIT and solve for FV: **FV** <u>Answer</u>: FV=1.18

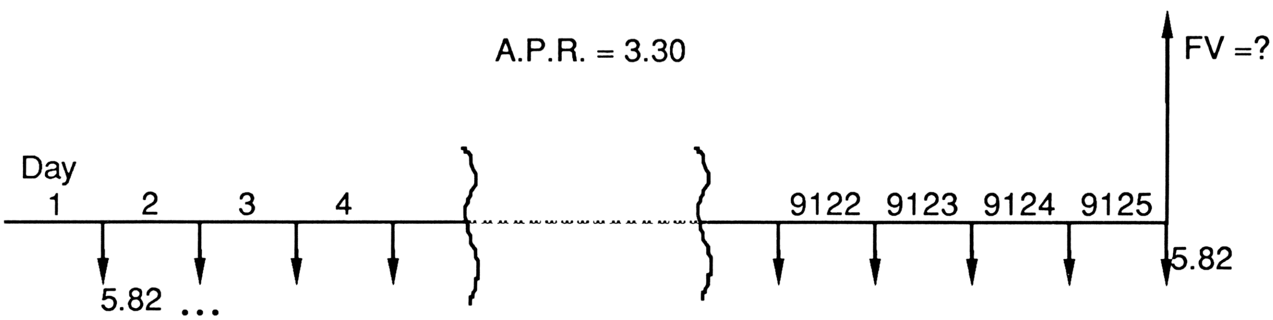
Now change the P/YR: **OTHER** 1 2 **P/YR**, then EXIT 1 2 **N**, and solve for I%YR: **I%YR**), to get the equivalent monthly rate. <u>Answer</u>: I%YR=16.61

That's slightly better than the 16.70% of your car loan. So actually, you would have done better to have charged your car on your credit card!

6. The first thing to realize is that inflation is a tax on the time value of your money. In other words, if you're money's face value is earning 10% per year, but inflation is raising the price of everything by 4% per year, then your interest is really accumulating you *actual* income at the rate of 6%, right?

Secondly, something similar happens with an income tax. If all your interest is taxable, that means that 27 cents of every additional interest dollar goes you-know-where. So forgetting about inflation for a minute, the face value of your interest is decreased through taxation by 27% of 10%, right?

You need to combine these two to find the effective "today's-dollar" rate at which you can accrue your savings: [1][0] [−] [2][7][%] [−] [4] [=] **3.30** Isn't real great, is it? It's reality, though – so it's what you use in the TVM calculation:

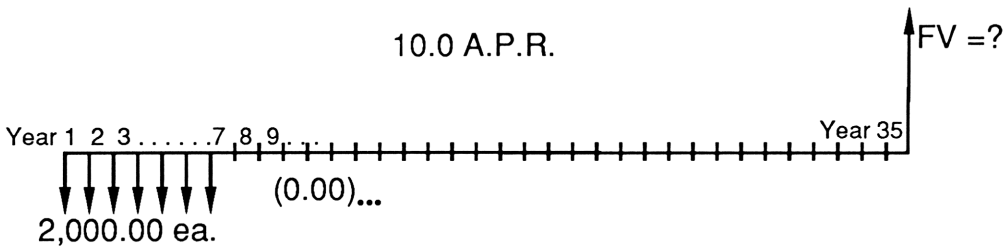


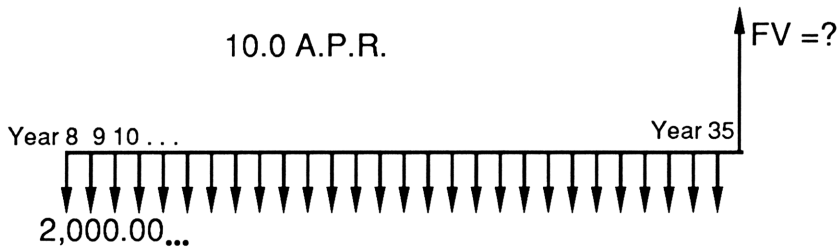
I%YR	(stores this real interest rate)
OTHER [3][6][5] **P/YR** [EXIT]	(END mode is OK)
[3][6][5] [×] [2][5] **N**	(ignore leap years)
[0] **PV**	(start with nothing saved)
[5][•][8][2] [+/−] **PMT**	(your daily savings)
FV	<u>Answer</u>: **FV=82,512.68**

Now vary the number of years: [3][6][5] [×] [4][0] **N** **FV**
<u>Answer</u>: **FV=176,587.14**

[3][6][5] [×] [5][0] **N** **FV** <u>Answer</u>: **FV=270,789.77**

7. This is a comparison of between the Future Values of two different scenarios:





Notice that the yearly payments are at the beginning of each year. The first thing to do, then, is to set your OTHER parameters appropriately: From TVM, press OTHER 1 P/YR BEG EXIT.

After that, it's easy. First, you need to figure out what the account balance is at the end of your 7 years of saving: 7 N 1 0 I%YR 0 PV 2 0 0 0 +/- PMT FV <u>Answer</u>: FV=20,871.78

Now let that amount "ride" for another 28 years, with no further payments into the account. To do that, you need to recognize that the answer you just got for FV will become your PV now (you're moving the entire TVM "picture frame" forward in time): So press +/- PV 0 PMT 2 8 N FV
<u>Answer</u>: FV=300,991.75

The second scenario: 2 8 N (unnecessary if you've just finished the first part of this problem) 0 PV 2 0 0 0 +/- PMT FV
<u>Answer</u>: FV=295,261.86

Just 7 early years of saving are better than 28 later years!
That's the Time Value of Money.

By now you should be pretty comfortable with those TVM keys. It's amazing, all the different things they can tell you – if you set the problems up correctly, of course! Now, here's some space for your very own…

Notes And Doodles On TVM

...and here are a couple of good things to complete your N & D collection ☞

Amortization Schedules

Try This: From the TVM menu, go to the OTHER menu. There's one item there that you haven't used yet: **AMRT**.

Press it now; you should see this:

```
KEY #PMTS; PRESS {#P}
 #P | INT | PRIN | BAL | NEXT | TABLE
```

An AMoRTization schedule is an itemized listing of the principal and interest ("P and I") paid over any given number of periods within the term of a mortgage. Typically, for example, you would need to know the amount of interest you've paid in any tax year, because that interest is tax-deductible.

The thing to realize is that *the AMRT menu is a set of side calculations that use the values currently sitting in your TVM registers.*

Like This: Just to get a feel for this, suppose you had a straightforward, 30-year, fully amortized mortgage for $100,000 at 10.5% A.P.R., with monthly payments in arrears. Use the AMRT menu to find the principal and interest you paid in each of the first three years, and what remaining balance was still due after the three years.

Solution: First, you fill in all the usual TVM values and parameters.

From the TVM menu, you press [3][6][0] [N] [1][0][•][5] [I%YR], then [1][0][0][0][0][0] [PV] [0] [FV] [OTHER] [1][2] [P/YR] [END] [EXIT] [PMT]

Result: PMT=-914.74 (This should look vaguely familiar)

Now go to AMRT, by pressing [OTHER] [AMRT], and follow the display's directions. To amortize 12 payments at once, press [1][2] [#P]:

```
#P=12 PMTS: 1-12
 #P | INT | PRIN | BAL | NEXT |TABLE
```

Now solve for whatever you need:

[BAL]: Bingo! BALANCE=99,499.48
[INT]: INTEREST=-10,476.36 (this is negative because AMRT follows the sign conventions you set up with TVM).
[PRIN]: PRINCIPAL=-500.52

To amortize the NEXT set of 12 payments, just press [NEXT]....etc.

Not bad, eh? Play around with the AMRT menu items (e.g. [TABLE] will let you print out any portion of the amortization schedule if you have a printer)!

Pennies And Particulars

There's one other important thing to note about the TVM keys: As you know, your HP-27S will use 12 digits in all of its arithmetic – indeed, in most of its calculations, including the TVM calculations.

For Example: Recall the current value that's sitting in the PMT register (from TVM, press RCL PMT). You'll see PMT=-914.74

But now set the display to show you all of its digits (press ■ MODES ALL RCL PMT). This is how big the payment amount *really* is (to 9 places): PMT=-914.739294493

Problem is, nobody really writes a check for that much, right? So even though this *is* the amount you would need to pay every month to *exactly* amortize that 30-year mortgage, you won't be paying that. To be accurate, you need to use the true amount to the nearest penny (here it would be rounded up: 914.740000000).

So key it in (■ MODES FIX 2 INPUT) 9 1 4 • 7 4 +/– PMT

Now *re-*calculate the FV. What do you get and what does it mean?

You get FV=1.78, and it means that since you had to round your payment *up* slightly (by a fraction of a penny), you've technically *overpaid* your loan by $1.78, (assuming, of course that you haven't also rounded off any other numbers, such as the interest rate or remaining balance).

You can tell that this is an over-payment – due back to you – since it's positive. If you had rounded the payment *down,* you would probably have an underpaid balance – and the FV would have signified this with a minus sign.

So when using the TVM keys, remember to use real-life payment amounts (dollars and cents only) to finalize terms and balances.

Now, what about AMRT? After all, it uses the TVM values, too. Does it use any unrealistic penny fragments the TVM registers may hold?

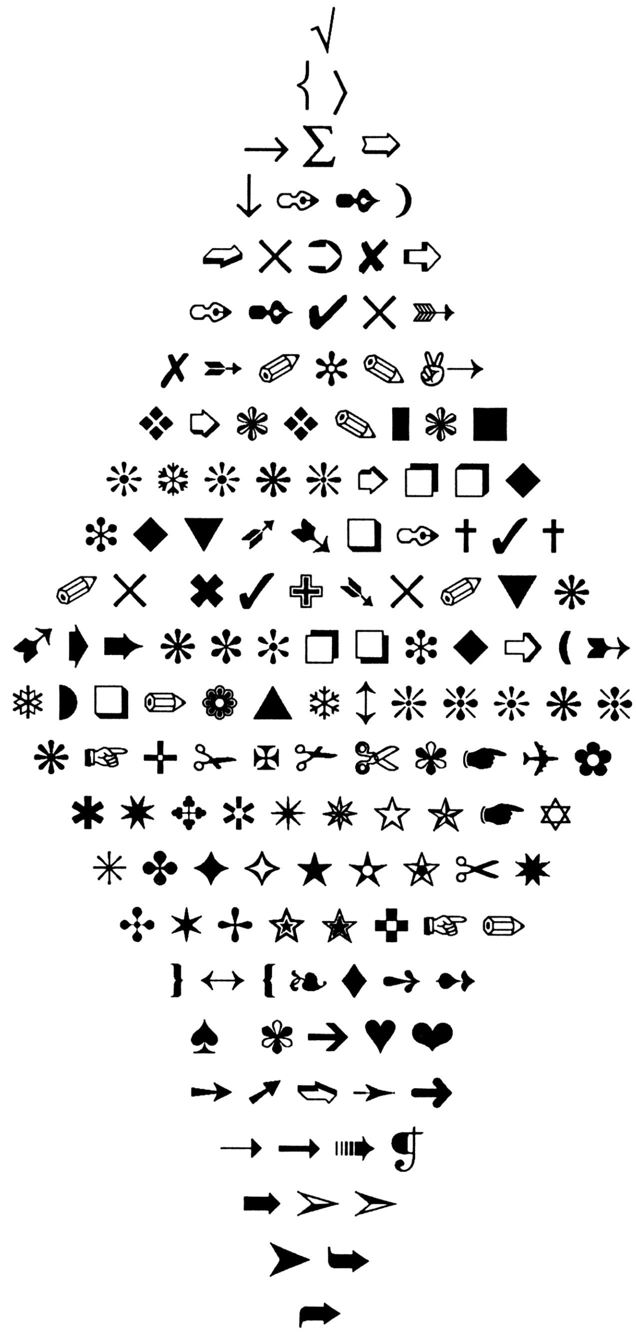
Nope.

With AMRT, HP basically took care of this concern for you: *Except for the I%YR variable (which is always used to its complete 12 digits), the AMRT menu uses TVM values only up to the number of digits the display is set to show.*

In other words, as long as you have your display set to FIX 2 digits (dollars and cents), the AMRT calculations will automatically round its own copies of PV and PMT in the same way – so you always end up with a realistic and verifiable amortization schedule!

Prove It: Go back on your own and re-run the AMRT example on page 247, except this time, run it with a "display-ALL" setting.... Then try it with a FIX 4 setting....

See how the results differ slightly? Of course, you'll usually want the FIX 2 setting, but it's good to know and see the difference.



Points Of Departure

"The Book Stops Here"

That's about it – the end of the Easy Course, the hangar door – for now. But *you* have only just begun. You've just finished your basic training and become a fully-qualified HP-27S pilot!

But of course, it *doesn't* mean you've seen it *all*.

As we said at the start, this book doesn't even pretend to cover all of the many uses of your machine. In fact, we've totally ignored many of its useful functions, because (as we also promised at the beginning), we thought those areas were very well-explained in the manual that came with your calculator. Because you now know how to work with menus and play "What-If?", they should all be fairly straightforward.

So if you now want to explore those other topics, here's the list for you, along with the chapter numbers where they appear in your owner's manual:

Chapter 7: **Time, Appointments, and Date Arithmetic**
Chapter 8: **Printing**
Chapter 9: **Additional Examples**

Explore them – enjoy them – and get the most out of your HP-27S!

Notes (Yours)

So how did you like this book? Do you find yourself wishing we had covered other things? More of the same things?

Or did you find any mistakes, typos, or other little mysteries we ought to know about (yes, we usually have a few innocent-looking little boo-boos. Did any of them leap out and grab you by the lapels)?

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Anyway, thanks for going along for the ride. When all is said and done, we hope that this book will have said and done a lot of it for you, helping you and your HP-27S explore all sorts of calculating routes and adventures – and stay good friends all along the way.

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