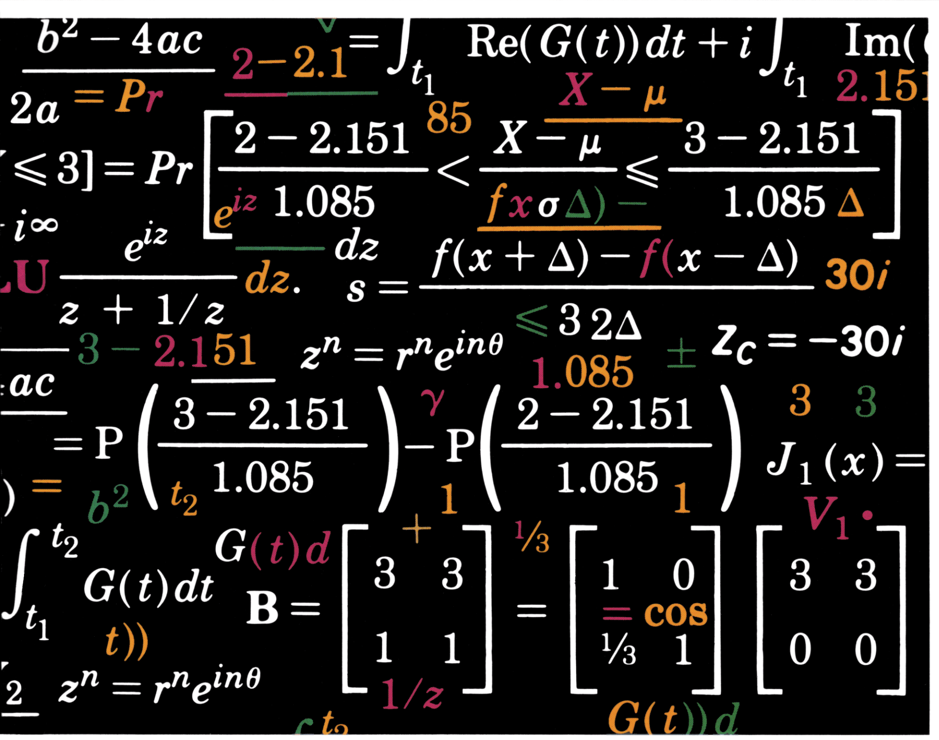
HEWLETT-PACKARD

Step-by-Step Solutions
For Your HP Calculator

Algebra and College Math



HP-28S
HP-28C

HEWLETT PACKARD

Help Us Help You!

Please take a moment to complete this postage-paid card, tear it out and put it in the mail. Your responses and comments will help us better understand your needs and will provide you with the best procedures to solve your problems. Thank you!

HELP US HELP YOU!

Book: **Algebra and College Math** Date acquired: ____________

Name ____________

Street ____________

City, State, Zip ____________

Phone (______) ____________ Business ______ or Home ______

1. What calculator will you use this book with?
   004 ☐ **HP-28S** 005 ☐ **HP-28C** 006 ☐ **Other** ____________
2. How many other HP solution books have you bought for this calculator? ______
3. What is your OCCUPATION?
   101 ☐ **Student** 103 ☐ **Professional** 109 ☐ **Other** ____________
4. Where did you purchase this book?
   403 ☐ **Bookstore** 404 ☐ **Discount or Catalog Store**
   407 ☐ **Mail Order** 410 ☐ **HP Direct** 411 ☐ **Other** ____________
5. How did you first hear about this book?
   501 ☐ **HP Owner** 503 ☐ **Advertising** 506 ☐ **Salesperson** 507 ☐ **Brochure**
   508 ☐ **Other** ____________
6. To what degree did this book influence your calculator purchase decision?
   601 ☐ **Major Influence** 602 ☐ **Minor Influence** 603 ☐ **No Influence**
7. How well does this book cover the material you expected?
   701 ☐ **Good** 702 ☐ **Moderate** 703 ☐ **Low**
8. What level of knowledge is required to make use of the topics in this book?
   801 ☐ **High** 802 ☐ **Medium** 803 ☐ **Low**
9. How clearly was the material in this book presented?
   901 ☐ **Good** 902 ☐ **Moderate** 903 ☐ **Low**
10. How would you rate the value of this book for your money?
    111 ☐ **High** 112 ☐ **Medium** 113 ☐ **Low**

Comments: (Please comment on improvements and additional applications or subjects you would like HP to cover in this or another solution book.) ____________

____

____

____

NO POSTAGE
NECESSARY
IF MAILED
IN THE
UNITED STATES

BUSINESS REPLY CARD

FIRST CLASS PERMIT NO. 40 CORVALLIS, OR

POSTAGE WILL BE PAID BY ADDRESSEE

Hewlett-Packard Company
ATTN: Calculator Market Research
1000 N.E. Circle Blvd.
Corvallis, OR 97330, U.S.A.

Algebra and College Math

Step-by-Step Solutions for Your HP-28S or HP-28C Calculator

HEWLETT PACKARD

Edition 2 June 1988
Reorder Number 00028-90101

Notice

Corvallis Division
1000 N.E. Circle Blvd.
Corvallis, OR 97330, U.S.A.

Printing History

Edition 1	December 1987	Mfg No. 00028-90106
Edition 2	June 1988	Mfg No. 00028-90133

Welcome...

... to the HP-28S and HP-28C Step-by-Step Solution Books. These books are designed to help you get the most from your HP-28S or HP-28C calculator.

This book, *Algebra and College Math*, provides examples and techniques for solving problems on your calculator. A variety of algebraic, trigonometric, and geometric problems are designed to familiarize you with the many functions built into your calculator.

Before you try the examples in this book, you should be familiar with certain concepts from the owner's documentation:

- The basics of your calculator: how to move from menu to menu, how to exit graphics and edit modes, and how to use the menu to assign values to, and solve for, user variables.
- Entering numbers, programs, and algebraic expressions into the calculator.

Please review the section "How To Use This Book." It contains important information on the examples in this book.

For more information about the topics in the *Algebra and College Math* book, refer to a basic textbook on the subject. Many references are available in university libraries and in technical and college bookstores. The examples in the book demonstrate approaches to solving certain problems, but they do not cover the many ways to approach solutions to mathematical problems. *Our thanks to Roseann M. Bate of Oregon State University for developing the problems in this book.*

Contents

How To Use This Book

Please take a moment to familiarize yourself with the formats used in this book.

Keys and Menu Selection: A box represents a key on the calculator keyboard.

[ENTER]
[1/x]
[STO]

[ARRAY]
[PLOT]
[ALGEBRA]

In many cases, a box represents a shifted key on the calculator. In the example problems, the shift key is NOT explicitly shown. (For example, [ARRAY] requires you to press the shift key, followed by the ARRAY key, found above the "A" on the left keyboard.)

The "inverse" highlight represents a menu label:

Key:	**Description:**
DRAW	Found in the [PLOT] menu.
ISOL	Found in the [SOLV] menu.
ABCD	A user-created name. If you created a variable by this name, it could be found in either the [USER] menu or the SOLVR menu. If you created a program by this name, it would be found in the [USER] menu.

Menus typically include more menu labels than can be displayed above the six redefinable menu keys. Press [NEXT] and [PREV] to roll through the menu options. For simplicity, [NEXT] and [PREV] are NOT shown in the examples.

Solving for a user variable within ≡SOLVR≡ is initiated by the shift key, followed by the appropriate user-defined menu key:

[ ] ≡ABCD≡.

The keys above indicate the shift key, followed by the user-defined key labeled "ABCD". Pressing these keys initiates the Solver function to seek a solution for "ABCD" in a specified equation.

The symbol [<>] indicates the cursor-menu key.

Interactive Plots and the Graphics Cursor: Coordinate values you obtain from plots using the [INS] and [DEL] digitizing keys may differ from those shown, due to small differences in the positions of the graphics cursor. The values you obtain should be satisfactory for the Solver root-finding that follows.

Display Formats and Numeric Input: Negative numbers, displayed as

```
-5
-12345.678
[[-1,-2,-3[-4,-5,-6[ ...
```

are created using the [CHS] key.

```
5 [CHS]
12345.678 [CHS]
[[1 [CHS],2 [CHS], ...
```

The examples in this book typically specify a display format for the number of decimal places. If your display is set such that numeric displays do not match exactly, you can modify your display format with the [MODE] menu and the ≡FIX≡ key within that menu. (For example, [MODE] 2 ≡FIX≡ will set the display to the FIX 2 format.)

Programming Reminders: Before you key in the programming examples in this book, familiarize yourself with the locations of programming commands that appear as menu labels. By using the menu labels to enter commands, you can speed keying in programs and avoid errors that might arise from extra spaces appearing in the programs. Remember, the calculator recognizes commands that are set off by spaces. Therefore, the arrow (→) in the command R→C (the real to complex conversion function) is interpreted differently than the arrow in the command → C (create the local variable "C").

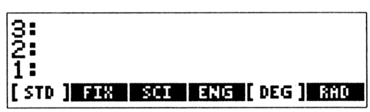
The HP-28S automatically inserts spaces around each operator as you key it in. Therefore, using the [R], [→], and [C] keys to enter the R→C command will result in the expression R → C , and, ultimately, in an error in your program. As you key in programs on the HP-28S, take particular care to avoid spaces inside commands, especially in commands that include an → .

The HP-28C does not automatically insert spaces around operators or commands as they are keyed in.

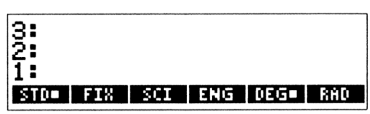
A Note About the Displays Used in This Book: The menus and screens that appear in this book show the HP-28S display. Most of the HP-28C and HP-28S screens are identical, but there are differences in the [MODE] menu and ≣ SOLVR ≣ screen that HP-28C users should be aware of.

For example, the first screen below illustrates the HP-28C [MODE] menu, and the second screen illustrates the same menu as it appears on the HP-28S.

HP-28C [MODE] display.



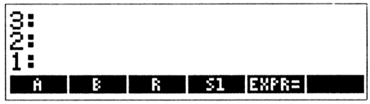
HP-28S [MODE] display.



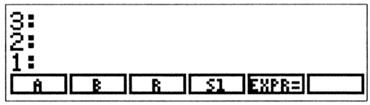
Notice that the HP-28C highlights the entire active menu item, while the HP-28S display includes a small box in the active menu item.

The screens shown below illustrate the HP-28C and HP-28S versions of the SOLVR menu.

HP-28C SOLVR display.



HP-28S SOLVR display.



Both of these screens include the Solver variables A, B, R, S1, and EXPR=. The HP-28C displays Solver variables in gray on a black background. The HP-28S prints Solver variables in black on a gray background.

User Menus: A PURGE command follows many of the examples in this book. If you do not purge all of the programs and variables after working each example, or if your USER menu contains your own user-defined variables or programs, the USER menu on your calculator may differ from the displays shown in this book. Do not be concerned if the variables and programs appear in a slightly different order on your USER menu; this will not affect the calculator's performance.

Functions and Equations

Rational Functions and Polynomial Long Division

The quotient of two polynomials is a rational function. The Taylor series command TAYLR can be used to find the equivalent polynomial if the denominator divides evenly into the numerator. If it does not, then TAYLR gives an expression that approximates the quotient. The following examples show how to evaluate rational functions.

Example: Using the command TAYLR, find the equivalent polynomial for the following rational function.

$$\frac{6x^3-5x^2-8x+3}{2x-3}$$

Press the following keys to put the expression for the numerator in level 1.

′6×X^3−5×X^2−8×X+3 [ENTER]

```
4:
3:
2:
1:   '6*X^3-5*X^2-8*X+3'
```

Duplicate the expression and then store it in a variable named *N* (for "numerator").

[ENTER]
′N [STO]

```
4:
3:
2:
1:   '6*X^3-5*X^2-8*X+3'
```

N has been added to the User menu.

Enter the expression for the denominator and symbolically divide the numerator by the denominator.

[USER]
′2×X−3 [ENTER]
[÷]

```
2:
1:  '(6*X^3-5*X^2-8*X+3)
     /(2*X-3)'
 N
```

Enter the variable to be evaluated.

′X [ENTER]

```
3:
2:  '(6*X^3-5*X^2-8*X+3…
1:                    'X'
 N
```

By inspection, the quotient is of order 2 ($n = 2$). Add the order to the stack to complete the three inputs needed to execute the Taylor series command, and set the display to FIX 2.

2 [ENTER]
[MODE] 2 FIX

```
3: '(6*X^3-5*X^2-8*X+3…
2:                  'X'
1:                 2.00
 STD | FIX■ | SCI | ENG | DEG | RAD■
```

Execute the Taylor function.

[ALGEBRA]
TAYLR

```
3:
2:
1:        '-1+2*X+3*X^2'
TAYLR | ISOL | QUAD | SHOW | OBGET | EXGET
```

The equivalent polynomial for the rational function is $-1+2x+3x^2$.

Example: Find the polynomial quotient and remainder equal to the following rational function.

$$\frac{6x^3-5x^2-8x+3}{3x^2+2x+1}$$

The denominator does not divide evenly into the numerator. The algorithm to solve polynomial long division is included in your calculator's reference manual. The steps of that algorithm will be followed in this example, and referring to them may help you understand the problem better.

Before attempting this example, complete the previous example. The expression $-1+2x+3x^2$ from the previous example must appear in level 1 and $6x^3-5x^2-8x+3$ must be stored in the variable N. Modify the expression in level 1 by substituting "1" for "-1" in the first position of the expression. This is accomplished by pressing the following keys.

1 [ENTER]
{1 [ENTER]

```
3:        '-1+2*X+3*X^2'
2:                 1.00
1:             { 1.00 }
TAYLR | ISOL | QUAD | SHOW | OBGET | EXGET
```

Make the substitution for the first object.

OBSUB

```
3:
2:
1:         '1+2*X+3*X^2'
COLCT | EXPAN | SIZE | FORM | OBSUB | EXSUB
```

Store this expression in a variable named D (for "denominator") and store the initial value of 0 in a variable named Q (for "quotient").

'D [STO]
0'Q [STO]

```
3:
2:
1:
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Recall the numerator N to the stack.

[USER]
[N]

```
3:
2:
1:  '6*X^3-5*X^2-8*X+3'
 Q  D  N
```

Put the denominator D on the stack.

[D]

```
3:
2:  '6*X^3-5*X^2-8*X+3'
1:        '1+2*X+3*X^2'
 Q  D  N
```

By inspection, divide the highest-order term in the numerator ($6x^3$) by the highest-order term in the denominator ($3x^2$). The quotient term is $2x$. Enter $2x$.

'2×X [ENTER]

```
3:  '6*X^3-5*X^2-8*X+3'
2:        '1+2*X+3*X^2'
1:                '2*X'
 Q  D  N
```

Make a copy of the quotient term and return the current quotient variable to the stack.

[ENTER]
[Q]

```
3:                '2*X'
2:                '2*X'
1:                 0.00
 Q  D  N
```

Add this copy to Q.

[+]

```
3:        '1+2*X+3*X^2'
2:                '2*X'
1:                '2*X'
 Q  D  N
```

Store this result in Q.

'Q [STO]

```
3:  '6*X^3-5*X^2-8*X+3'
2:        '1+2*X+3*X^2'
1:                '2*X'
 Q  D  N
```

Multiply the quotient term and the denominator.

[×]

```
2:   '6*X^3-5*X^2-8*X+3'
1:  '(1+2*X+3*X^2)*(2*X)
    '
  Q  |  D  |  N  |     |     |
```

Subtract the result from the numerator.

[−]

```
2:
1:  '6*X^3-5*X^2-8*X+3-(
    1+2*X+3*X^2)*(2*X)'
  Q  |  D  |  N  |     |     |
```

Simplify the result by expanding the expression and then collecting terms.

[ALGEBRA] [EXPAN]

```
1:  '6*(X*X^2)-5*(X*X)-8
    *X+3-((1+2*X)*(2*X)+
    3*X^2*(2*X))'
COLCT|EXPAN|SIZE |FORM |OBSUB|EXSUB
```

By inspection, another expansion is required for the x^2 term.

[EXPAN]

```
1:  '6*(X*(X*X))-5*(X*X)
    -8*X+3-(1*(2*X)+2*X*
    (2*X)+3*(X*X)*(2*X))
COLCT|EXPAN|SIZE |FORM |OBSUB|EXSUB
```

All terms are fully expanded, so now collect terms.

[COLCT]

```
2:
1:  '3+6*X^3-6*X^3-9*X^2
    -10*X'
COLCT|EXPAN|SIZE |FORM |OBSUB|EXSUB
```

Collect terms until complete.

[COLCT]

```
3:
2:
1:          '3-9*X^2-10*X'
COLCT|EXPAN|SIZE |FORM |OBSUB|EXSUB
```

The result is a new and reduced numerator. Since its degree is equal to the denominator's degree, continue this process of finding a quotient term, adding it to Q, and reducing the numerator.

Put D on the stack.

[USER] [D]

```
3:
2:          '3-9*X^2-10*X'
1:           '1+2*X+3*X^2'
  Q  |  D  |  N  |     |     |
```

Divide the highest-order term in the numerator, $-9x^2$, by the highest-order term in the denominator, $3x^2$. By inspection, the result is -3. Enter this quotient term.

3 [CHS] [ENTER]

```
3:          '3-9*X^2-10*X'
2:           '1+2*X+3*X^2'
1:                  -3.00
 Q   D   N
```

Make a copy of the quotient term and return the quotient variable to the stack.

[ENTER]
[Q]

```
3:                  -3.00
2:                  -3.00
1:                  '2*X'
 Q   D   N
```

Add this copy to Q.

[+]

```
3:           '1+2*X+3*X^2'
2:                  -3.00
1:               '-3+2*X'
 Q   D   N
```

Store the result in Q.

'Q [STO]

```
3:          '3-9*X^2-10*X'
2:           '1+2*X+3*X^2'
1:                  -3.00
 Q   D   N
```

Multiply the quotient term and the denominator.

[×]

```
3:
2:          '3-9*X^2-10*X'
1:  '(1+2*X+3*X^2)*(-3)'
 Q   D   N
```

Subtract the resulting expression from the new numerator.

[−]

```
2:
1:  '3-9*X^2-10*X-(1+2*X
    +3*X^2)*(-3)'
 Q   D   N
```

Simplify the expression by expansion and collection of terms.

[ALGEBRA] [EXPAN]

```
1:  '3-9*(X*X)-10*X-((1+
    2*X)*(-3)+3*X^2*(-3)
    )'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Continue until all terms are fully expanded.

EXPAN

```
1: '3-9*(X*X)-10*X-(1*(
   -3)+2*X*(-3)+3*(X*X)
   *(-3))'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Now collect terms.

COLCT

```
3:
2:
1:                 '6-4*X'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

The result is the new numerator. Since its degree is less than the denominator's degree, the iteration process ends. The polynomial quotient is stored in Q, and the remainder equals the final numerator divided by the denominator.

[USER] Q

```
3:
2:                 '6-4*X'
1:                '-3+2*X'
  Q    D    N
```

Thus the answer is

$$-3+2x+\frac{6-4x}{3x^2+2x+1}.$$

The command TAYLR can be used to approximate this result. Executing TAYLR with $n=1$ gives the result $3-14x$.

Purge the variables created in this example and clear the stack.

{ 'Q' 'D' 'N' [PURGE]

Complex Numbers

Complex numbers, $x+iy$, can be represented in two ways: as an object or as an algebraic. A complex number object has the form (x,y). As an algebraic, the complex number is represented by $'x+iy'$, where x and y are real numbers and i is a constant equal to the complex number (0,1). Calculations with complex numbers are easily solved on the HP-28S.

Example: Evaluate the following expression.

$$\frac{\sin(.5+.3i)+(3-4i)*(2+i)^{1/3}}{\ln(5-8i)-\operatorname{arccosh}(2+9i)}.$$

First, set the display for FIX 4.

[CLEAR]
[MODE] 4 FIX

```
3:
2:
1:
 STD  FIX■  SCI  ENG  DEG  RAD■
```

Calculate $\sin(.5+.3i)$.

`(.5,.3` [TRIG] SIN

```
3:
2:
1:          (0.5012,0.2672)
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Key in the complex number $3-4i$.

`(3,-4` [ENTER]

```
3:
2:          (0.5012,0.2672)
1:        (3.0000,-4.0000)
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Key in the complex number $2+i$.

`(2,1` [ENTER]

```
3:          (0.5012,0.2672)
2:        (3.0000,-4.0000)
1:          (2.0000,1.0000)
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Take the inverse of the number 3.

3 [1/x]

```
3:        (3.0000,-4.0000)
2:          (2.0000,1.0000)
1:                  0.3333
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Calculate the third root of $2+i$.

[^]

```
3:        (0.5012,0.2672)
2:       (3.0000,-4.0000)
1:        (1.2921,0.2013)
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Multiply the resulting complex number by $3-4i$.

[×]

```
3:
2:        (0.5012,0.2672)
1:       (4.6814,-4.5644)
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Add the two numbers in levels 1 and 2. The sum is equal to the numerator.

[+]

```
3:
2:
1:       (5.1826,-4.2972)
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Calculate the denominator by entering it in as an algebraic expression and then converting the expression into a number.

'LN(5−8×i)−ACOSH(2+9×i)
[→NUM]

```
3:
2:       (5.1826,-4.2972)
1:      (-0.6728,-2.3656)
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Divide the numerator by the denominator to obtain the final result.

[÷]

```
3:
2:
1:        (1.1041,2.5049)
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Example: Verify the following definition by showing that both sides of the equation are equal for the case $x=3$ and $y=4$.

$$\tan(x+iy)=\frac{\sin(x)\cos(x)+i*\sinh(y)\cosh(y)}{\sinh(y)^2+\cos(x)^2}$$

Set the calculator to radians mode and key in the algebraic expression.

[MODE] [RAD] [CLEAR]
'TAN(x+y×i)=(SIN(x)×
COS(x)+SINH(y)×COSH(y)×
i)÷(SINH(y)^2+COS(x)^2)'
[ENTER] [< >]

```
1:  'TAN(x+y*i)=(SIN(x)*
    COS(x)+SINH(y)*COSH(
    y)*i)/(SINH(y)^2+COS
    (x)^2)'
```

Store the equation in the variable EQ and display the Solver menu.

SOLV STEQ
SOLVR

```
3:
2:
1:
 X  Y  LEFT= RT=
```

Store the number 3 in the variable *x*.

3 X

```
 x: 3.0000
2:
1:
 X  Y  LEFT= RT=
```

Store the number 4 in the variable *y*.

4 Y

```
 y: 4.0000
2:
1:
 X  Y  LEFT= RT=
```

Evaluate the left-hand side of the expression.

LEFT=

```
LEFT='TAN(3+4*i)'
2:
1:           'TAN(3+4*i)'
 X  Y  LEFT= RT=
```

Convert this expression into a number.

→NUM

```
3:
2:
1:      (-0.0002,0.9994)
 X  Y  LEFT= RT=
```

Evaluate the right-hand side of the expression.

RT=

```
RIGHT='(-0.1397+745.2...
1: '(-0.1397+745.2394*i
   )/745.7197'
 X  Y  LEFT= RT=
```

Convert this expression into a number to show that the right and left sides of the equation are equal.

→NUM

```
3:
2:      (-0.0002,0.9994)
1:      (-0.0002,0.9994)
 X  Y  LEFT= RT=
```

Exit from the Solver, clear the stack, and purge the following variables.

SOLV CLEAR
{'y''x''EQ' PURGE

```
3:
2:
1:
STEQ RCEQ SOLVR ISOL QUAD SHOW
```

Example: Express the following complex numbers in polar notation.

a. $3-2\sqrt{3}i$

b. $-1/2+\frac{\sqrt{3}}{2}i$

c. $3+4i$

First, set the angle mode to degrees.

[MODE] ▤DEG▤

```
3:
2:
1:
 STD | FIX■ | SCI | ENG | DEG■ | RAD
```

a. Enter the number 3.

3 [ENTER]

```
3:
2:
1:                           3.0000
 STD | FIX■ | SCI | ENG | DEG■ | RAD
```

Enter the number −2.

−2 [ENTER]

```
3:
2:                           3.0000
1:                          -2.0000
 STD | FIX■ | SCI | ENG | DEG■ | RAD
```

Take the square root of the number 3.

3 [√]

```
3:                           3.0000
2:                          -2.0000
1:                           1.7321
 STD | FIX■ | SCI | ENG | DEG■ | RAD
```

Multiply −2 by the square root of 3.

[×]

```
3:
2:                           3.0000
1:                          -3.4641
 STD | FIX■ | SCI | ENG | DEG■ | RAD
```

Combine the two numbers in levels 1 and 2 into a complex number.

[TRIG] ▤R→C▤

```
3:
2:
1:              (3.0000,-3.4641)
 P→R | R→P | R→C | C→R | ARG |
```

Convert the complex number in rectangular notation to polar notation.

[R→P]

```
3:
2:
1:     (4.5826,-49.1066)
 P→R | R→P | R→C | C→R | ARG |
```

b. Enter the complex number $-1/2+\frac{\sqrt{3}}{2}i$ as an algebraic expression.

Convert the expression into a number.

[CLEAR]
'-1÷2+√3÷2×i' [→NUM]

```
3:
2:
1:       (-0.5000,0.8660)
 P→R | R→P | R→C | C→R | ARG |
```

Convert the complex number from rectangular form to polar form.

[R→P]

```
3:
2:
1:     (1.0000,120.0000)
 P→R | R→P | R→C | C→R | ARG |
```

c. Enter the complex number $3+4i$ in rectangular form and take the absolute value of it. The magnitude is returned.

[CLEAR]
(3,4 [REAL] [ABS]

```
3:
2:
1:                 5.0000
 ABS |SIGN|MANT|XPON|    |
```

Return (3,4) to the stack. (If LAST is disabled, you must re-enter (3,4)).

[LAST]

```
3:
2:                 5.0000
1:        (3.0000,4.0000)
 ABS |SIGN|MANT|XPON|    |
```

Press [ARG]. The polar angle is returned.

[TRIG] [ARG]

```
3:
2:                 5.0000
1:                53.1301
 P→R | R→P | R→C | C→R | ARG |
```

Combine the magnitude and the polar angle into a complex number.

[R→C]

```
3:
2:
1:       (5.0000,53.1301)
 P→R | R→P | R→C | C→R | ARG |
```

Hyperbolic and Inverse Hyperbolic Functions

The LOGS menu contains hyperbolic and inverse hyperbolic functions. The arguments to these functions can be either numeric or symbolic.

Example: Given $Z = 4/\sqrt{(7)}$, find sinh Z, csch Z, cosh Z, sech Z, tanh Z, and coth Z.

Clear the display and set the number of display digits to 3.

[CLEAR] [MODE] 3 [FIX]

```
3:
2:
1:
 STD | FIX■ | SCI | ENG | DEG | RAD■
```

Calculate $4/\sqrt{(7)}$ and store it in the variable Z.

4 [ENTER]
7 [√]

```
3:
2:                          4.000
1:                          2.646
 STD | FIX■ | SCI | ENG | DEG | RAD■
```

[÷]

```
3:
2:
1:                          1.512
 STD | FIX■ | SCI | ENG | DEG | RAD■
```

'Z [STO]

```
3:
2:
1:
 STD | FIX■ | SCI | ENG | DEG | RAD■
```

Calculate sinh Z.

Z [LOGS] [SINH]

```
3:
2:
1:                          2.157
 SINH | ASINH | COSH | ACOSH | TANH | ATANH
```

Calculate csch Z. The csch Z is equal to the inverse of sinh Z.

[1/x]

```
3:
2:
1:                          0.464
 SINH | ASINH | COSH | ACOSH | TANH | ATANH
```

Calculate cosh Z.

Z [COSH]

```
3:
2:                  0.464
1:                  2.378
SINH ASINH COSH ACOSH TANH ATANH
```

Calculate sech Z. The sech Z is equal to the inverse of cosh Z.

[1/x]

```
3:
2:                  0.464
1:                  0.421
SINH ASINH COSH ACOSH TANH ATANH
```

Calculate tanh Z.

Z [TANH]

```
3:                  0.464
2:                  0.421
1:                  0.907
SINH ASINH COSH ACOSH TANH ATANH
```

Calculate coth Z. The coth Z is equal to the inverse of tanh Z.

[1/x]

```
3:                  0.464
2:                  0.421
1:                  1.102
SINH ASINH COSH ACOSH TANH ATANH
```

Example: Verify that acosh(2.378) = 1.512 using the definition

$$\text{acosh}(x) = \ln(x + \sqrt{x^2 - 1}), \text{ for } x \geq 1.$$

Key in the equation for the definition and store it in the variable EQ.

[CLEAR]
`'ACOSH(X)=LN(X+√(X^2 -1))'` [SOLV] [STEQ]

```
3:
2:
1:
STEQ RCEQ SOLVR ISOL QUAD SHOW
```

Display the Solver menu, key in the number 2.378, and assign it to the variable X.

[SOLVR] `2.378` [X]

```
X: 2.378
2:
1:
X LEFT= RT=
```

Now check if the left side of the equation acosh(x) equals 1.512.

[LEFT=]

```
LEFT=1.512
2:
1:                  1.512
X LEFT= RT=
```

Now check if the right side of the equation is 1.512.

RT=

Exit from the Solver menu and purge the variables used in these examples.

Note to HP-28S users: If you do not exit from the Solver before attempting to purge EQ, the calculator will display the message `EQUATION NOT FOUND`. (EQ *will* be cleared even though the message is displayed.) To avoid displaying this message, always exit from the Solver before purging equations and variables.

[SOLV] `{'X''EQ''Z'` [PURGE]

Function Evaluation

The Solver can find the values of a function (be it of one variable or of several variables) given the values of the independent variables. The values can be real or complex numbers or symbolic expressions.

Given the function $f(x,y)=2\pi x^2\,|\,\sqrt{y^2-x^2}\,|$ find $f(1,\sqrt{2})$, $f(\sin T,1)$, and $f(3,5)$.

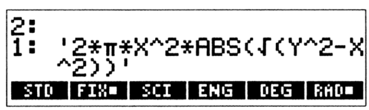
Clear the stack, set the display format, and set the symbolic evaluation flag.

[CLEAR]
[MODE] 4 [FIX]
36 [ENTER] SF [ENTER]

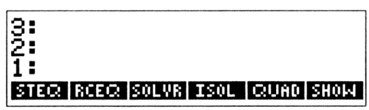
Note in the keystrokes above, you could also use [SF] within the [TEST] menu as an alternative to typing the letters 'SF' and the [ENTER] key.

Put the expression for the function in level 1 and store it in the variable EQ.

' 2×π×X^2×
ABS(√(Y^2-X^2)) [ENTER]

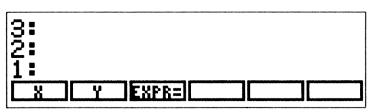


[SOLV]
[STEQ]



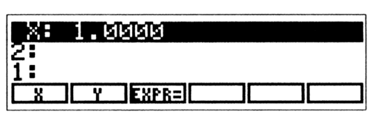
From the SOLV menu, press the [SOLVR] key to display a menu of the independent variables.

[SOLVR]



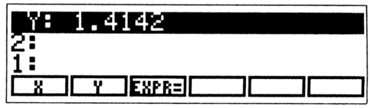
Store the number 1 in the variable X.

1 [X]



Store the square root of two in the variable Y.

2 [√]
[Y]

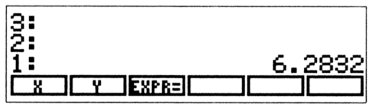


Evaluate the expression.

[EXPR=]

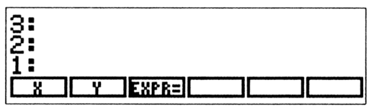
Convert this expression into a number.

[→NUM]



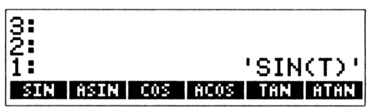
Clear the previous result and evaluate f(sin$T$,1).

[DROP]



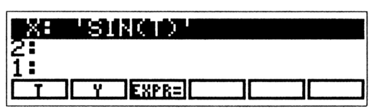
Put sin$T$ on the stack. Notice that in this instance we use the [SIN] key in the [TRIG] menu to enter the function.

[TRIG]
' [SIN] T [ENTER]



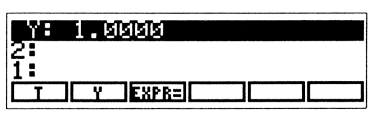
Store the expression in the variable X.

[SOLV] [SOLVR] [X]



Note the Solver variable X has been replaced by the variable T. Store the number one in the variable Y.

1 [Y]



Now compute the function value.

[EXPR=]

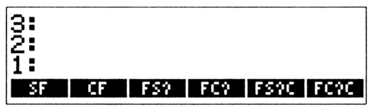
To redisplay the variable X, its current symbolic value must be purged.

'X [PURGE]

Note that the variable X is again displayed in the Solver menu.

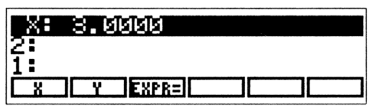
For the last part of the example, clear flag 36 to set the calculator in the numerical evaluation mode and force numeric evaluation of π in the expression.

[DROP]
36 [TEST] [CF]



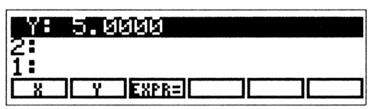
Put a 3 on the stack and store it in X.

3 [ENTER]
[SOLV] [SOLVR] [X]



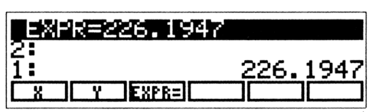
Store 5 in Y.

5 [Y]



Evaluate the expression.

[EXPR=]



With flag 36 set, the result would have been '2*π*9*4'.

To insure that the variables X and Y are not inadvertently incorporated in other calculations, exit from the Solver and purge the variables from memory. You may also wish to set flag 36 to its default setting.

[SOLV] {'Y''X''EQ' [PURGE] 36 SF [ENTER]

Graphs of Algebraic Functions

This section illustrates a number of algebraic function plots including manipulation of plot parameters for enhanced representation of the function characteristics.

Example: Plot the power function $y = x^{-3}$.

Purge any plot parameters that may be stored in the variable PPAR.

[CLEAR] [ATTN]
'PPAR [PURGE]

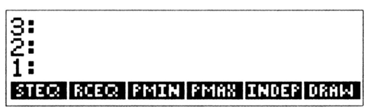
Store x^{-3} in the variable EQ.

[CLEAR] [MODE] 4 [FIX]
'X^(-3) [PLOT] [STEQ]

Note to HP-28C users: Version 1BB of the HP-28C will give an "INFINITE RESULT" error unless flag 59 is clear, or you take steps to avoid evaluation of the function at $x = 0$. *HP-28C users only* perform one of the following two steps to avoid the INFINITE RESULT error.

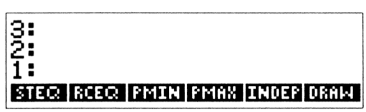
To clear flag 59, enter:

59 CF [ENTER]



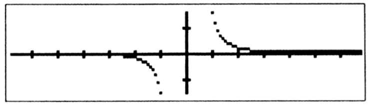
To avoid evaluation of the function at $x = 0$, change the plot minima and maxima (PMIN and PMAX) such that [DRAW] does not evaluate the function at the point of the error. Let PMIN be (-6,-1.5) and PMAX be (6, 1.5).

(-6,-1.5 [PMIN]
(6,1.6 [PMAX]



Plot the expression.

DRAW



Example: Plot the power function $y = \pm\sqrt{x}$. The solution for this example depends upon whether you use an HP-28C or an HP-28S.

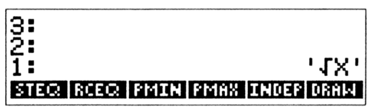
Store $\sqrt{x}$ in the variable EQ, then proceed to the appropriate solution method below.

'√X STEQ

HP-28C Method. If you plot the expression now, your HP-28C will trap an error and display the message "Non-real Result" because y is imaginary for $x < 0$. To avoid this error, take only the real part of the function y.

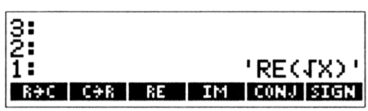
Recall the equation that you just stored.

RCEQ



Take the real part of the function.

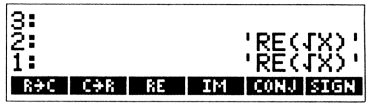
CMPLX RE



If you plot the function now, only positive values of y will appear. A trick to plot both positive and negative values of y at the same time is to make a copy of the function, negate the coy and set both functions equal to each other. (They are not really equal to each other – this is just a way to plot two functions at the same time on the HP-28C.)

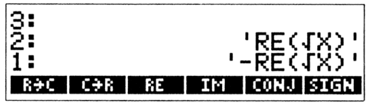
Duplicate the function.

[ENTER]



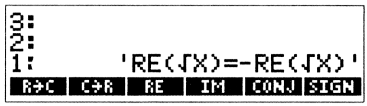
Negate the function.

[CHS]



Set the two functions equal to each other.

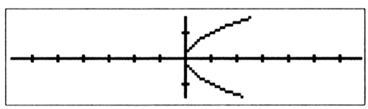
[=] [ENTER]



Store this equation in EQ and plot it.

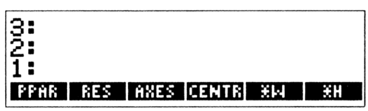
[PLOT] [STEQ]

[DRAW]



Exit from the plot screen and proceed to the next example.

[ATTN]



HP-28S Method. If you plot the function now, only positive values of *y* will appear in the graph. A trick to plot both positive and negative values of *y* at the same time is to make a copy of the function, negate the copy, and set both functions equal to each other. (They really are not equal to each other – this is just a way to plot two functions at the same time on the HP-28S.)

Recall the expression.

RCEQ

Duplicate the expression.

ENTER

Negate the expression.

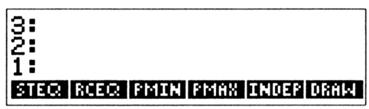
CHS

Now set the two expressions equal to each other.

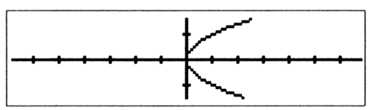
= ENTER

Store this equation in EQ and plot it.

STEQ



DRAW



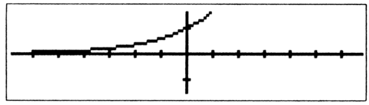
Exit from the plot screen to prepare for the next example.

ATTN

Example: Plot the exponential function $y = e^{x/2}$.

Enter the function exp(x /2) and store it in the variable EQ. Then plot the function.

'EXP(X÷2) [STEQ]
[DRAW]

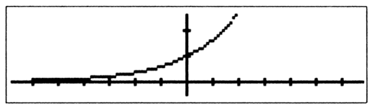


Press [ATTN] to return back to the stack display. This time let the point (0,1) be the center of the display.

[ATTN] (0,1 [CENTR]

Plot the function again.

[DRAW]



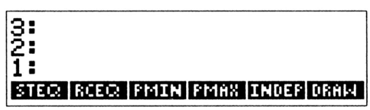
Purge the plot parameters.

[ATTN] 'PPAR [PURGE]

Example: Plot the logarithmic function $y = x\log(x^2+2)$.

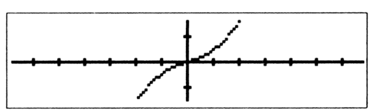
Enter the expression and store it in EQ.

'X×LOG(X^2+2) [STEQ]



Plot the function.

[DRAW]



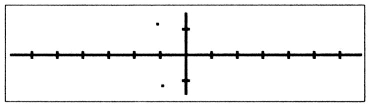
Example: Plot the polynomial function $y = x^3 + 2x^2 - 11x - 12$.

Enter the expression and store it in the variable EQ.

`'X^3+2×X^2-11×X-12`
STEQ

Plot the function.

DRAW

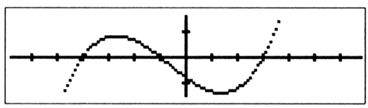


Much of the graph is not shown on the display. To see more of the graph adjust the plot parameters by multiplying the height by 15.

[ATTN] `15` *H

Draw the function again.

DRAW



Purge the variables created in this example.

[ATTN] `{'PPAR''EQ'` [PURGE]

Quadratic Equations

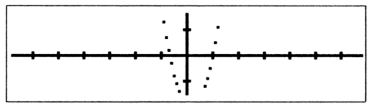
The zeros of a quadratic equation can be found using the QUAD command. Plotting the equation is not necessary, but you may be interested in seeing what the graph looks like and checking whether there are two real roots, two complex roots, or a double root.

For example, solve $3x^2 - x - 2 = 0$. First plot the equation.

CLEAR MODE 4 FIX
'3×X^2−X−2 ENTER

```
3:
2:
1:                '3*X^2-X-2'
 STO | FIX■ | SCI | ENG | DEG | RAD■
```

PLOT STEQ
'PPAR PURGE DRAW



You can easily see that the equation has two real roots. Now use QUAD to find those roots. First, recall the equation and put *X* on the stack to indicate that this is the variable for which you are solving (the coefficients could be variables, in which case the solution is symbolic).

ATTN RCEQ
'X ENTER

```
3:
2:                '3*X^2-X-2'
1:                        'X'
STEQ | RCEQ | PMIN | PMAX | INDEP | DRAW
```

Find the roots:

ALGEBRA QUAD

```
3:
2:
1:             '(1+s1*5)/6'
TAYLR | ISOL | QUAD | SHOW | OBGET | EXGET
```

The QUAD function can also be found in the SOLV menu.

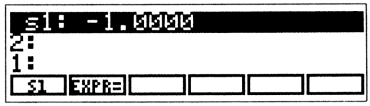
The resulting expression represents both roots. "s1" is a variable whose value is either +1 or −1. Store this expression in the variable EQ and use the Solver to find the numerical solutions.

SOLV STEQ
SOLVR

```
3:
2:
1:
 S1 | EXPR= |  |  |  |
```

Let *s1* be negative by entering a −1 and pressing the S1 menu key.

`-1` S1

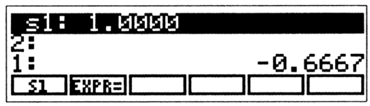


Press EXPR= to get the first root.

EXPR=

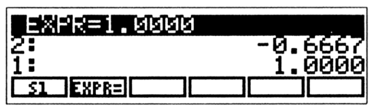
Let *s1* be equal to +1.

`1` S1



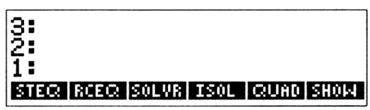
Solve for the second root.

EXPR=



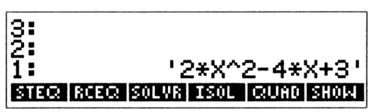
Exit from the Solver and clear the stack and all the variables used in this example.

[SOLV] [CLEAR]
`{'s1''PPAR''EQ'` [PURGE]

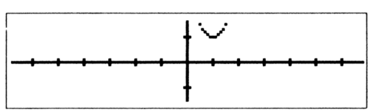


Example: Find the roots for $2x^2-4x+3$. First store the equation in the variable EQ, then draw it.

`'2×X^2-4×X+3` [ENTER]



[PLOT] STEQ DRAW

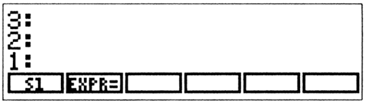


Since the graph of this equation does not intersect the x-axis, there are no real roots; the roots are complex. Solve for these roots using the QUAD command.

ATTN RCEQ
'X SOLV QUAD

Now use the Solver to get the numeric solutions.

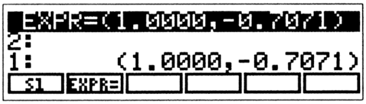
STEQ
SOLVR



Let *s1* equal −1 and solve for one of the roots.

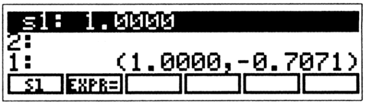
-1 S1

EXPR=

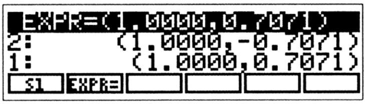


Let *s1* equal +1 and solve for the second root.

1 S1



EXPR=



The roots for this equation are $1 \pm 0.7071i$.

Exit from the Solver and purge the variables created in this example.

SOLV {'s1''PPAR''EQ' PURGE

Polynomial Equations

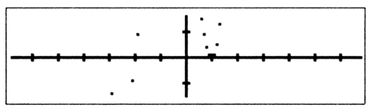
The roots of polynomial equations can be found by several methods. Graphing the polynomial enables you to estimate the roots. The estimations can then be used as guesses for the Solver or for the ROOT command. An alternative to graphing the polynomial to obtain the "guesses" is using $\pm p\,/q$ where the values of p are the positive divisors of the constant term and the values of q are the positive divisors of the coefficient of the highest-powered term. In most cases it is easier and quicker to graph the polynomial to find the approximate roots.

Example: Plot the graph and find the roots of

$$x^4+3x^3-3x^2-7x+6=0$$

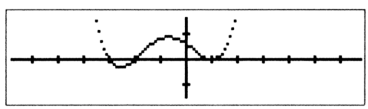
First, clear the display and any current plot parameters. Then, enter the expression, store it in the variable EQ, and plot it.

[CLEAR] 'PPAR [PURGE]
'X^4+3×X^3-3×X^2-7×X+6
[PLOT] STEQ
DRAW



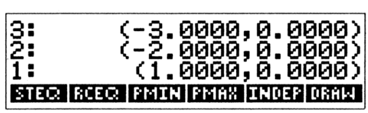
Multiply the height by 10 and plot the graph again.

[ATTN] 10 *H
DRAW



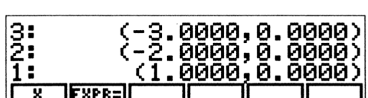
Digitize the three points where the function equals zero (i.e., where the graph intersects or touches the x-axis) by moving the cross hairs to each of the three points and pressing [INS]. When you press the [ATTN] key, the coordinates of the three points are displayed. The x coordinate of each point will be used as initial estimates for the Solver.

[<] ... [<] [INS]
[>] ... [>] [INS]
[>] ... [>] [INS]
[ATTN]



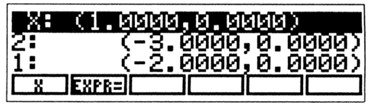
Now use these values in the Solver.

[SOLV] SOLVR



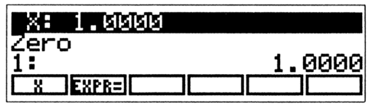
Store the point in level 1 in the variable X.

X



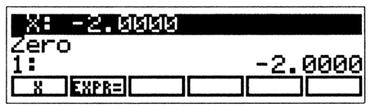
Now solve for X by pressing the shift key followed by the X key in the Solver menu. The result is shown in level 1.

[ ] X



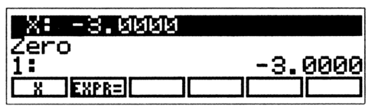
Clear this result and find the next root.

DROP
X [ ] X



Clear this result and find the last root.

DROP
X [ ] X



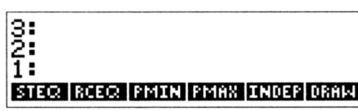
The three roots are -3, -2, and 1.

Example: Plot the graph and find one of the roots of

$$x^3 - 3x^2 - 1.5x + 6 = 0$$

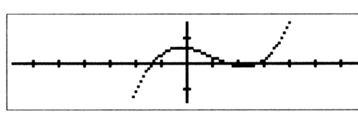
For this example you will again plot the function to get the initial guesses and then use the ROOT command to find the roots. First, enter the expression and store it in the variable EQ.

CLEAR
'X^3-3×X^2-1.5×X+6
PLOT STEQ



Plot the graph.

DRAW

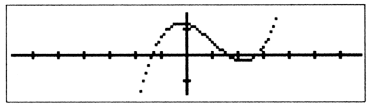


Since the plotting parameters from example 1 were not purged, the height is still multiplied by 10. Decrease the vertical scale by multiplying the height by .5.

ATTN
.5 *H

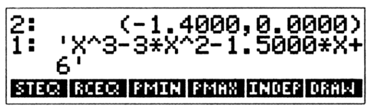
Draw the graph again. Use the cross hairs and the INS key to digitize the left-most point that crosses the x-axis.

DRAW
< . . . < INS



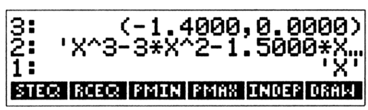
The ROOT command requires three inputs in this case, the polynomial expression, the name of the variable you are solving for, and the initial guess. The polynomial is in level 3, the name is in level 2, and the guess is in level 1. The digitized guess is in level 1 after the INS key above. Now recall the expression.

ATTN RCEQ



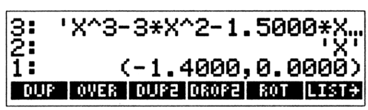
Put the variable name *X* on the stack.

'X ENTER



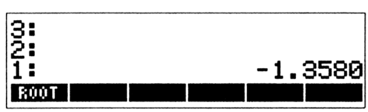
To move the coordinates for the initial guess to level 1, rotate the stack.

STACK ROT



Solve for *X* and find one of the roots of the equation.

SOLV ROOT



Purge the variables used in these two examples.

{'X''PPAR''EQ' PURGE

Simultaneous Linear Equations

A system of two linear equations in two unknowns can be solved by first plotting the graphs of the two lines, finding the point of intersection (if one exists), and then solving for the unknown variables by using the Solver with the intersection point as the initial guess. The system can also be solved using matrices, but this method won't work if the lines are parallel or coincident. A third method is to isolate one of the variables for one of the equations, plug this expression into the other equation (giving you one equation in one unknown), and then solving for that one unknown by using the Solver.

For example, solve the following system

$$\begin{Bmatrix} 2x + 1y = 6 \\ 5x - 4y = 3 \end{Bmatrix}.$$

Clear the display and set the mode to FIX 4.

[CLEAR] 4 [MODE] ≡FIX≡

```
3:
2:
1:
 STO | FIX■ | SCI | ENG | DEG | RAD■
```

Method 1: Using PLOT. To graph the system, first isolate the variable y in both of the equations and then set both of these expressions equal to each other.

'2×X+Y=6' 'Y [ENTER]

```
3:
2:                          '2*X+Y=6'
1:                                'Y'
 STO | FIX■ | SCI | ENG | DEG | RAD■
```

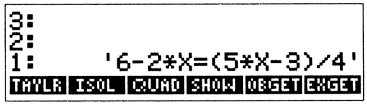
[ALGEBRA] ≡ISOL≡

```
3:
2:
1:                           '6-2*X'
TAYLR| ISOL | QUAD | SHOW |OBGET|EXGET
```

'5×X−4×Y=3' 'Y ≡ISOL≡

```
3:
2:                           '6-2*X'
1:                       '(5*X-3)/4'
TAYLR| ISOL | QUAD | SHOW |OBGET|EXGET
```

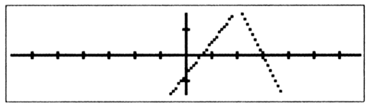
[=] [ENTER]



Prepare to plot the lines by purging any prior plot parameters. Store the equation in EQ and draw it.

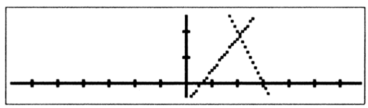
[PLOT] 'PPAR [PURGE]
[STEQ]

[DRAW]



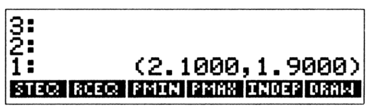
Exit from the plot display. Move the center of the plot to (0,1) and draw the graph again.

[ATTN]
(0,1 [CENTR]
[DRAW]



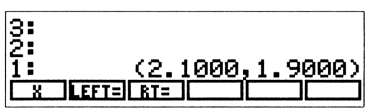
Move the cursor to the approximate point of intersection and digitize the point by pressing [INS]. Press [ATTN] to return to the stack display. The coordinates of the point are returned to the stack.

[>] [>] . . . [^] [^] . . . [INS]
[ATTN]



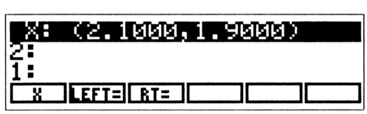
Display the Solver menu. The menu consists of the variable X, $LEFT =$, and $RT =$.

[SOLV] [SOLVR]



Store the digitized point in the variable X as the initial estimate. (The Solver only uses the first coordinate.)

[X]



Solve for X.

[ ] [X]

The variable X equals 2.0769. Since both sides of the equation are a symbolic solution for Y, pressing [LEFT=] or [RT=] will give you the numerical solution for Y.

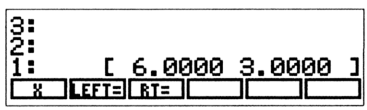
[LEFT=]

[RT=]

The variable Y equals 1.8462.

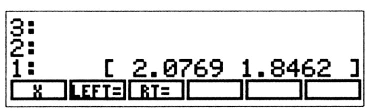
Method 2: Using Matrices. Key in the constant vector (the right side of both equations).

[CLEAR]
`[6 3` [ENTER]



Key in the coefficient matrix. The coefficients of the first equation make up the first row of the matrix. The coefficients of the second equation make up the second row. Divide the constant vector by the coefficient matrix.

`[[2 1[5 -4` [÷]



The same results as the graphing method are obtained: $X = 2.0769$ and $Y = 1.8462$.

Exit from the Solver, clear the stack, and purge all the variables that were used in this example.

[SOLV] [CLEAR] `{'X''PPAR''EQ'` [PURGE]

Method 3: Using Solver. First, enter the first equation and isolate the variable Y. The result is an expression for Y.

'2×X+Y=6' 'Y
[SOLV] [ISOL]

Enter the second equation and store it in the variable EQ.

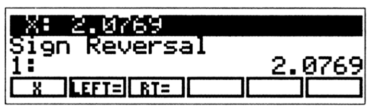
'5×X−4×Y=3' [STEQ]

Display the Solver menu and store the expression for Y in the variable Y. This gives you one equation in one unknown.

[SOLVR] [Y]

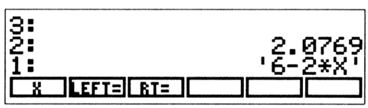
Now solve for X. The same result as the two previous methods is returned to level 1.

[ ] [X]



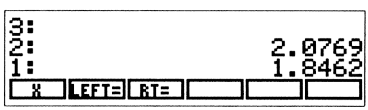
Put the expression for Y on the stack.

Y [ENTER]



Convert this expression into a number.

[→NUM]



The value for Y is returned to level 1.

Exit from the Solver and purge the variables created in this example.

[SOLV] {'X''Y''EQ' [PURGE]

Systems of Linear Equations

Using matrices, solve the following system.

$$\begin{Bmatrix} 6x + 1y - 3z + 0w = 37 \\ -2x + 3y + 5z - 7w = 6 \\ 8x + 0y + 4z - 5w = 75 \\ 0x - 7y - 4z + 1w = 7 \end{Bmatrix}$$

Clear the display, set the display mode, and key in the constant vector.

[CLEAR] [MODE] 1 [FIX]
[37 6 75 7 [ENTER]

```
3:
2:
1: [ 37.0 6.0 75.0 7.0…
STD | FIX■ | SCI | ENG | DEG | RAD■
```

Key in the coefficient matrix and divide the constant vector by the coefficient matrix.

[[6 1 -3 0[-2 3 5 -7
[8 0 4 -5[0 -7 -4 1 [÷]

```
3:
2:
1: [ 7.0 -2.0 1.0 -3.0…
STD | FIX■ | SCI | ENG | DEG | RAD■
```

The solution to the system is $x = 7, y = -2, z = 1,$ and $w = -3.$

Infinite Sequences and Series

Infinite Sequences and Series

Calculations involving infinite sequences and series are best solved by writing programs. By using FOR loops in programs, calculations can be repeated as many times as desired.

Example: Find the first 10 terms of the sequence whose general term is the following.

$$\frac{x!}{e^x}$$

A general program that calculates any number of terms for this sequence is listed below. Enter the program and store it in the variable *FDE* (for "factorial divided by exponent"). To run the program, press [USER] and then press the user variable key [FDE]. When you run the program, the calculator displays a prompt that asks for the number of terms you want calculated. Enter a number, such as 10, then press [ ] [CONT] (the shift key followed by the continue key) to continue running the program. The program returns a list of the first 10 numbers in the sequence.

After entering the program, store it in the variable *FDE*.

Program:	**Comments:**
`« 2 FIX`	Set the display format to two digits.
`"# OF TERMS?"`	
`CLLCD 1 DISP`	Prompt message.
`HALT`	Program halts. (Key in a number and press [CONT].)
`→ n «`	The number is stored in the variable *n*.
`1 n FOR x`	Loop: do for *x* from 1 to *n*.
`x FACT`	Calculate the factorial of *x*.
`x EXP`	Take the exponent of *x*
`÷`	and divide the two numbers.
`NEXT`	Increment *x* and repeat until $x > n$.
`n →LIST »»`	Put the *n* terms into a list.

[ENTER] `'FDE` [STO]

Clear the display, then run the program.

[CLEAR] [USER] FDE

```
# OF TERMS?
```

Enter the number 10 and press [CONT] to continue running the program. The list of the first 10 terms of the sequence is displayed.

10 [CONT]

```
1: { 0.37 0.27 0.30
   0.44 0.81 1.78 4.60
   13.53 44.78 164.75 }
FDE
```

Run the program again.

FDE

```
# OF TERMS?
```

Enter the number 5 (or any other integer) and continue running the program.

5 [CONT]

```
2: { 0.37 0.27 0.30 0....
1: { 0.37 0.27 0.30
   0.44 0.81 }
FDE
```

Example: Find the sum of the first 100 terms of the series

$$\sum_{x=1}^{x=n} \frac{1}{x(x+1)} \text{ where } n \text{ is the total number of terms.}$$

The program that finds the sum of the first n terms is listed below. When this program is run, a prompt asking for the number of terms is displayed. After entering the number and continuing the program, the prompt message and the number n is displayed in level 3 and the sum of the first n terms is in level 1.

Enter the program below and store it in the variable *ONE*. (The series converges to one for large n.)

Program:	Comments:
« STD	Standard display format.
CLLCD "# OF TERMS? "	Prompt message.
DUP 1 DISP	Make a copy and display line 1.
HALT	Program halts (you key in a number).
→ n «	Store one copy of the number in n.
n →STR +	Convert the number into a string and concatenate with the prompt.
0 1 n FOR x	Loop: do for x from 1 to n with initial zero sum.
'INV((x × (x+1))' EVAL	$1/((x)(x+1))$.
+	Add to the accumulating total.
NEXT	Increment x and repeat until $x > n$.
CLLCD DUP 3 DISP SWAP 1 DISP »»	Generate final display.

[ENTER] 'ONE [STO]

Run the program.

[USER] [ONE]

```
# OF TERMS?
```

Enter the number 100 and continue running the program. The sum of the first 100 terms is returned to level 1.

100 [CONT]

```
# OF TERMS?100
.990099009897
```

If desired, purge the two programs created in these examples.

{'ONE''FDE' [PURGE]

Determinants of Matrices

Determinants of Matrices

The HP-28S and HP-28C do calculations using matrices whose elements are real and/or complex numbers. The determinant of a matrix is easily found by using the command DET. But since DET is a command, it cannot be used in algebraics.

Example: Find the determinant of the following matrix.

$$\begin{bmatrix} 2 & 6 & 1 & -2 \\ -3 & 4 & 5 & 7 \\ 4 & -2 & 1 & 3 \\ 5 & 3 & -4 & 6 \end{bmatrix}$$

Key in the matrix and find the determinant.

[CLEAR] [MODE] 2 FIX
`[[2 6 1 -2[-3 4 5 7[4`
`-2 1 3[5 3 -4 6` [ENTER]

```
1: [[ 2.00 6.00 1.00 -...
    [ -3.00 4.00 5.00 ...
    [ 4.00 -2.00 1.00 ...
 STD | FIX■ | SCI | ENG | DEG | RAD■
```

[ARRAY] DET

```
3:
2:
1:                2439.00
CROSS | DOT | DET | ABS | RNRM | CNRM
```

Example: Solve for x and y.

$$\begin{vmatrix} 7 & 6 & 5 \\ 1 & 2 & 1 \\ y & -2 & x \end{vmatrix} = 0 \text{ and } \begin{vmatrix} x & 2 & y \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{vmatrix} = 2$$

Using the definition of the determinant of a 3×3 matrix, these two equations can also be written as the following:

$$14x + 6y - 10 - (10y - 14 + 6x) = 0 \text{ and } 21x + 8 + 10y - (3y + 20x + 28) = 2$$

The problem reduces to a system of two equations in two unknowns. To find y, isolate x in one of the equations, then substitute this expression for x in the other equation. To find x, substitute the value for y in the expression for x.

First, key in one of the equations and simplify it by collecting terms.

CLEAR
'14×X+6×Y−10−(10×Y−14 +6×X)=0 ALGEBRA COLCT

```
3:
2:
1:              '4+8*X-4*Y=0'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Store this equation in the variable EQ.

SOLV STEQ

```
3:
2:
1:
STEQ RCEQ SOLVR ISOL QUAD SHOW
```

Key in the other equation and simplify it also.

'21×X+8+10×Y−(3×Y+20×X +28)=2 ALGEBRA COLCT

```
3:
2:
1:             '-20+X+7*Y=2'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Obtain a symbolic expression for x by isolating the variable.

'X ISOL

```
3:
2:
1:               '2-7*Y+20'
TAYLR ISOL QUAD SHOW OBGET EXGET
```

Use the Solver to substitute the expression for x in the equation that is already stored in the variable EQ and solve for y. First, display the Solver menu.

SOLV SOLVR

```
3:
2:
1:               '2-7*Y-20'
X Y LEFT= RT=
```

Press X. The expression from level 1 is stored in the variable X. Notice that the variable X disappears from the Solver menu.

X

```
X: '2-7*Y-20'
2:
1:
Y LEFT= RT=
```

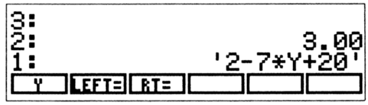
Now solve for y. Press the shift key followed by Y from the Solver menu.

□ Y

```
Y: 3.00
Zero
1:                      3.00
Y LEFT= RT=
```

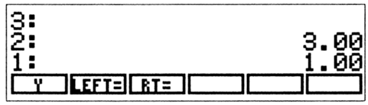
Recall the expression for x .

X [ENTER]



Find the numerical value for x by evaluating the expression.

[EVAL]



Thus, $x = 1$ and $y = 3$.

Exit from the Solver and purge the variables created in this example.

[SOLV] { 'Y' 'X' 'EQ' [PURGE]

Logarithms

Logarithms

This series of examples illustrates manipulation of numeric and algebraic expressions using logarithms.

Example: Use logarithms to evaluate the following.

$$N = \frac{3.271 * \sqrt{48.17}}{2.94^3}$$

First, enter the equation and then take the logarithm of both sides by pressing LOG.

CLEAR MODE 4 FIX
'N=3.271×√48.17÷2.94^3'
LOGS LOG

```
2:
1: 'LOG(N)=LOG(3.2710*√
   48.1700/2.9400^3)'
 LOG  ALOG  LN  EXP  LNP1 EXPM
```

Expand the equation so that the right side of the equation is expressed as the sum or difference of several logarithms. (This involves using the fundamental laws of logarithms, but is easily accomplished using the EXPAN command.)

ALGEBRA EXPAN

```
1: 'LOG(N)=LOG(3.2710*√
   48.1700)-LOG(2.9400^
   3)'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

EXPAN

```
1: 'LOG(N)=LOG(3.2710)+
   LOG(√48.1700)-LOG(
   2.9400)*3'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Now evaluate this equation.

EVAL

```
3:
2:
1:        'LOG(N)=-0.0490'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Solve for N by taking the antilogarithm of both sides of the equation.

LOGS ALOG

```
3:
2:
1:       'N=ALOG(-0.0490)'
 LOG  ALOG  LN  EXP  LNP1 EXPM
```

Press [EVAL] to get the numerical solution.

[EVAL]

```
3:
2:
1:                'N=0.8934'
 LOG  ALOG  LN  EXP  LNP1 EXPM
```

Example: Solve for x by using logarithms.

$$a^{2x-3}=b^x$$

Enter the equation and take the logarithm of both sides.

[CLEAR]
'A^(2×X-3)=B^X' [LOG]

```
2:
1:  'LOG(A^(2*X-3))=LOG(
    B^X)'
 LOG  ALOG  LN  EXP  LNP1 EXPM
```

Expand the equation.

[ALGEBRA] [EXPAN]

```
2:
1:  'LOG(A)*(2*X-3)=LOG(
    B)*X'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

[EXPAN]

```
2:
1:  'LOG(A)*(2*X)-LOG(A)
    *3=LOG(B)*X'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

The object is to isolate x on the left side (or right side, if you wish) of the equation by first moving all the terms with x to the left side and all the terms with no x to the right side.

Add 3log(A) to both sides of the equation. Rather than entering this term, retrieve the term by using EXGET. First duplicate the equation.

[ENTER]

```
2:  'LOG(A)*(2*X)-LOG(A…
1:  'LOG(A)*(2*X)-LOG(A)
    *3=LOG(B)*X'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Enter the position of the third multiplication sign, which, in this case, is 10. (To determine the position, count each operator or number, excluding parentheses and quotes. The first position is LOG, the second position is the variable *A* , '*' is in the third position, and so on.)

Execute the EXGET command. The expression 3log(*A*) is returned to the stack.

10 EXGET

```
3:
2:  'LOG(A)*(2*X)-LOG(A…
1:              'LOG(A)*3'
TAYLR ISOL QUAD SHOW OBGET EXGET
```

Add 3log(*A*) to both sides of the equation and collect the terms.

[+]

```
1:  'LOG(A)*(2*X)-LOG(A)
    *3+LOG(A)*3=LOG(B)*X
    +LOG(A)*3'
TAYLR ISOL QUAD SHOW OBGET EXGET
```

COLCT

```
2:
1:  '2*LOG(A)*X=LOG(B)*X
    +3*LOG(A)'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Now move *x* log(*B*) to the left side of the equation by subtracting it from both sides of the equation. This can be accomplished using the EXGET command.

ENTER

```
2:  '2*LOG(A)*X=LOG(B)*…
1:  '2*LOG(A)*X=LOG(B)*X
    +3*LOG(A)'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

10 EXGET

```
3:
2:  '2*LOG(A)*X=LOG(B)*…
1:              'LOG(B)*X'
TAYLR ISOL QUAD SHOW OBGET EXGET
```

[−]

```
1:  '2*LOG(A)*X-LOG(B)*X
    =LOG(B)*X+3*LOG(A)-
    LOG(B)*X'
TAYLR ISOL QUAD SHOW OBGET EXGET
```

COLCT

```
2:
1:  '2*LOG(A)*X-LOG(B)*X
    =3*LOG(A)'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Use the FORM editor to merge $2x\log(A)$ and $x\log(B)$ into $(2\log(A) - \log(B))x$. Press [FORM], move the cursor to the minus sign, press [M→] (merge right), then press [ATTN] to exit FORM and return the modified equation to the stack.

[FORM] [[→]] . . . [[→]]

```
((((2*LOG(A))*X)[-](LOG(B
)*X))=(3*LOG(A)))
```

[M→] [ATTN]

```
2:
1:  '(2*LOG(A)-LOG(B))*X
    =3*LOG(A)'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Divide $2\log(A) - \log(B)$ into both sides of the equation, first using EXGET to retrieve the subexpression.

[ENTER]

```
2:  '(2*LOG(A)-LOG(B))*…
1:  '(2*LOG(A)-LOG(B))*X
    =3*LOG(A)'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

5 [EXGET]

```
3:
2:  '(2*LOG(A)-LOG(B))*…
1:       '2*LOG(A)-LOG(B)'
TAYLR ISOL QUAD SHOW OBGET EXGET
```

[÷]

```
1:  '(2*LOG(A)-LOG(B))*X
    /(2*LOG(A)-LOG(B))=3
    *LOG(A)/(2*LOG(A)-
TAYLR ISOL QUAD SHOW OBGET EXGET
```

Collect the terms.

[COLCT]

```
2:
1:  'X=3/(2*LOG(A)-LOG(B
    ))*LOG(A)'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

The resulting equation is the solution to this example.

$$x = \frac{3\log(A)}{2\log(A) - \log(B)}$$

Example: Solve for x in the following expression.

$$\log(x+3)=0.7$$

The goal is to isolate x, which is easily done using the isolate command ISOL. First put the equation on the stack.

CLEAR LOGS
'LOG(X+3)=.7' ENTER

```
3:
2:
1:      'LOG(X+3)=0.7000'
 LOG | ALOG | LN | EXP | LNP1 | EXPM
```

Enter the variable to be isolated (X) and execute ISOL.

'X ALGEBRA ISOL

```
3:
2:
1:                2.0119
TAYLR | ISOL | QUAD | SHOW | OBGET | EXGET
```

The result is $x=2.0119$.

Example: Find $\log_7 36$.

The HP-28S and HP-28C calculate logarithms to base 10 and base e (the LN function). You can write a program to calculate the logarithms to any given base using the following formula.

$$\log_a t = \frac{\log_{10} t}{\log_{10} a}$$

Key in the following program that returns the logarithm of a given number to a given base (provided the base is in level 2 and the number in level 1 of the stack).

CLEAR LOGS
« LOG SWAP LOG ÷ ENTER

```
3:
2:
1:    « LOG SWAP LOG / »
 LOG | ALOG | LN | EXP | LNP1 | EXPM
```

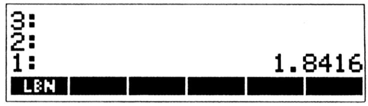
Store this program in the variable LBN.

'LBN STO USER

```
3:
2:
1:
 LBN |  |  |  |  |
```

Now compute $\log_7 36$.

7 ENTER
36 LBN



The program LBN will calculate the logarithm to a given base of a given number and may be stored in the calculator's memory for your convenience.

Trigonometry

Trigonometric Relations and Identities

This section illustrates calculations involving simple trigonometric relations and identities.

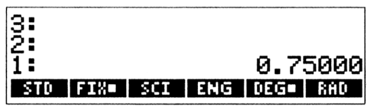
Example: Given cot $(x) = 0.75$, find tan (x), sec(x), cos(x), sin(x), and csc(x) without solving for x.

Set degrees mode and the number of display digits to FIX 5.

[CLEAR] [MODE] DEG
5 FIX

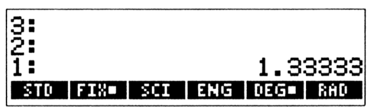
Enter the number .75, which is equal to cot(x).

.75 [ENTER]



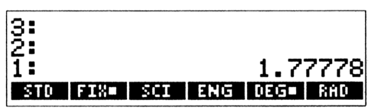
Take the inverse to calculate tan(x), since tan$(x) = 1/\cot(x)$.

[1/x]



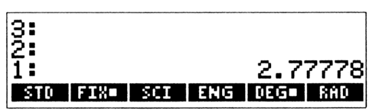
Calculate sec(x) using the relation $\sec(x) = \sqrt{\tan^2(x) + 1}$. First, calculate the square of tan(x).

[x^2]



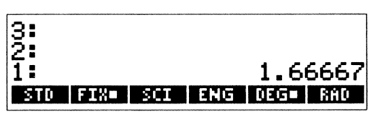
Add 1 to the square of tan(x).

1 [+]



Take the square root of the number to calculate sec(x).

[√]



Calculate cos(x) by taking the inverse of sec(x).

[1/x]

```
3:
2:
1:                0.60000
 STD | FIX■ | SCI | ENG | DEG■ | RAD
```

Calculate sin(x) by using the relation sin(x) = $\sqrt{1-\cos^2(x)}$. First, calculate the square of cos(x).

[x^2]

```
3:
2:
1:                0.36000
 STD | FIX■ | SCI | ENG | DEG■ | RAD
```

Enter the number 1 and switch the order of the 1 and the square of cos(x).

1 [SWAP]

```
3:
2:                1.00000
1:                0.36000
 STD | FIX■ | SCI | ENG | DEG■ | RAD
```

Subtract the square of cos(x) from 1.

[−]

```
3:
2:
1:                0.64000
 STD | FIX■ | SCI | ENG | DEG■ | RAD
```

Take the square root of this number to calculate sin(x).

[√]

```
3:
2:
1:                0.80000
 STD | FIX■ | SCI | ENG | DEG■ | RAD
```

Take the inverse of sin(x) to calculate csc(x).

[1/x]

```
3:
2:
1:                1.25000
 STD | FIX■ | SCI | ENG | DEG■ | RAD
```

Clear the stack.

[DROP]

```
3:
2:
1:
 STD | FIX■ | SCI | ENG | DEG■ | RAD
```

Example: Plot the unit circle $\sin^2(x)+\cos^2(x)=1$.

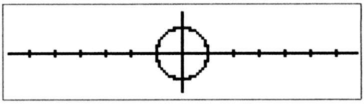
The program to plot the unit circle is listed below. Key in the program and store it in the variable "UCIR".

Program:	**Comments:**
« DEG	Set the angle mode to degrees.
CLLCD DRAX	Clear the display and draw the axes.
0 360 FOR x	Loop: do for x from 0 to 360 degrees.
x SIN	Calculate $\sin(x)$.
x COS	Calculate $\cos(x)$.
R→C	Form a coordinate pair $(\sin(x),\cos(x))$.
PIXEL	Plot the point.
5 STEP »	Increment x by 5 and repeat until $x>360$.

[ENTER] 'UCIR [STO]

Run the program.

[USER] [UCIR]



If desired, purge the program created in this section.

[ATTN] 'UCIR [PURGE]

Trigonometric Functions for One and Two Angles

Trigonometric relations, such as the law of cosines or the identity for the cosine of the sum of two angles, are not built into the HP-28S or HP-28C. However, the algebraic formula for the relations can be stored in a variable. Then by using the Solver, you can solve for any unknown in the formula.

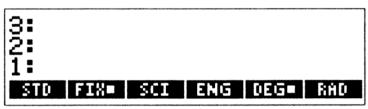
Example: Given an oblique triangle XYZ with the following parameters

$$x=3n$$
$$y=n^2-1$$
$$z=20$$
$$Z=94.9 \text{ degrees,}$$

where n is a positive integer, solve for n and then find sides x and y and angles X and Y.

First, set the number of display digits to 2 and select the degree mode.

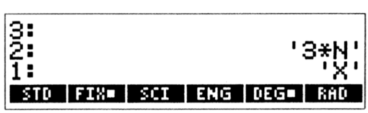
[CLEAR]
[MODE] 2 [FIX]
[DEG]



Normally, capital letters denote the angles of the triangle and lower case letters denote the corresponding opposite sides. Since capital and lower case letters are indistinguishable in the Solver and User menus, let X, Y, and Z be called $ANGX$, $ANGY$, and $ANGZ$, respectively. Also, let n, x, y, and z be represented by capital letters.

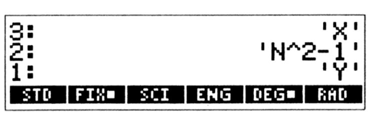
Enter '3*N' and the variable X.

'3×N' 'X' [ENTER]



Enter 'N^2 – 1' and the variable Y.

'N^2-1' 'Y' [ENTER]

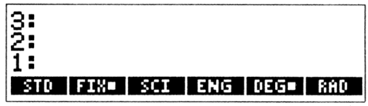


Enter the number 20 and the variable Z.

20'Z [ENTER]

Store the numbers in the variables X,Y, and Z.

[STO]
[STO]
[STO]



Store the number 94.9 in the variable $ANGZ$.

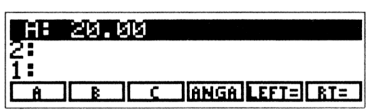
94.9'ANGZ [STO]

You can solve for N by using the law of cosines and the Solver. Enter the formula for the law of cosines and store it in EQ. (Since capital and lower case letters are indistinguishable in the Solver menu, let the angle variable be $ANGA$ rather than A.) Display the Solver menu.

'A^2=B^2+C^2-2×B×C×
COS(ANGA)' [SOLV] STEQ
SOLVR

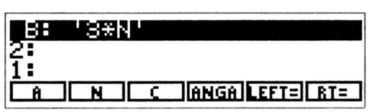
Store the value of the variable Z in the variable A. (Note: Only press [Z]. If you include the single quote, then the letter Z will be stored in the variable A.)

Z A



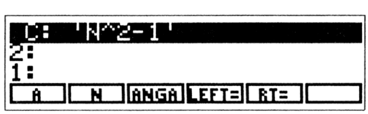
Store the value of the variable X in the variable B. (Notice that the Solver menu changes – the variable B is replaced by the variable N.)

X B



Store the value of the variable Y in the variable C.

Y C



Store the value of the variable *ANGZ* in the variable *ANGA*.

ANGZ ANGA

Since *N* is a positive integer, let the number 1 be an initial guess for *N*.

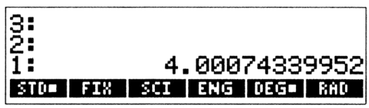
1 N

Solve for *N*.

[ ] N

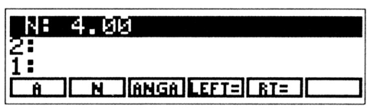
Display all digits of the computed result.

[MODE] STD



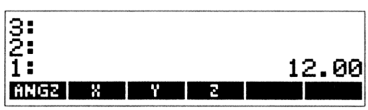
Since *N* is defined to be a positive integer, store the integer 4 in the variable *N*.

2 FIX [DROP]
[SOLV] SOLVR
4 N



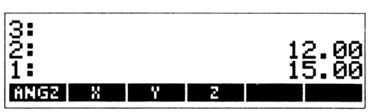
Solve for side *X* by pressing X and then [EVAL]. The same result can be obtained by pressing the letter [X] followed by [EVAL].

[USER] X
[EVAL]



Solve for side *Y* by pressing Y followed by [EVAL].

Y
[EVAL]

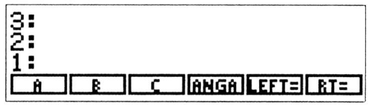


Purge the variables that were used in the law of cosines formula. Clear the stack.

{'ANGA''C''B''A' [PURGE]
[CLEAR]

Use the law of cosines again to find *ANGX* and *ANGY*. First, solve for *ANGX*.

[SOLV] [SOLVR]



Store *X* in the variable *A*. Notice that '3*N' is still stored in *X*.

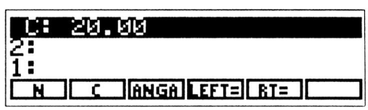
X [A]

Store *Y* in the variable *B*.

Y [B]

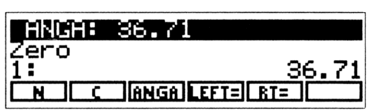
Store *Z* in the variable *C*.

Z [C]



You have just substituted *X*, *Y*, and *Z* into the law of cosines equation giving $X^2 = Y^2 + Z^2 - 2XY\cos(ANGA)$. Find angle *X* by solving for *ANGA*.

[ ] [ANGA]

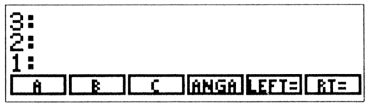


Purge the following variables. Rather than typing the variable names, display the User menu and press [{] followed by ['] ANGA ['], ['] C ['], and so forth.

[USER]
{ 'ANGA' 'C' 'B' 'A' [PURGE]
[CLEAR]

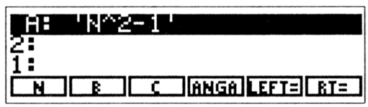
Display the Solver menu again.

[SOLV] SOLVR



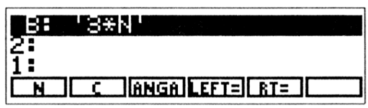
Find angle Y in a similar manner. Store Y in the variable A .

Y A



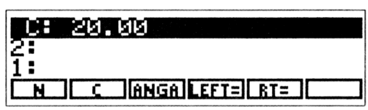
Store X in the variable B .

X B



Store Z in the variable C .

Z C



The resulting equation is now $Y^2=X^2+Z^2-2XZ\cos(ANGA)$. Find *ANGY* by solving for *ANGA* .

[ ] ANGA

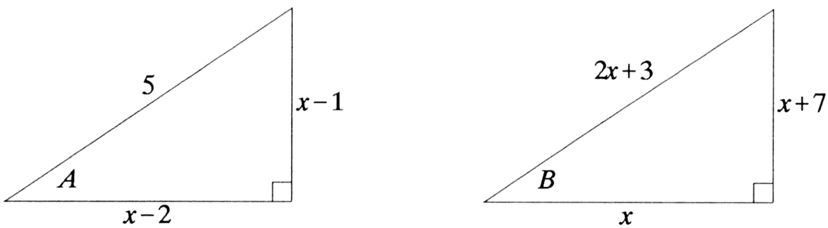
Exit from the Solver and purge the variables used in this example.

[SOLV]
{ 'ANGA' 'ANGZ' 'C' 'B' 'A' 'EQ' 'Z' 'Y' 'X' 'N'
[PURGE]

Example: Given the two right triangles shown below, and the relationships $\cos(A+B) = -0.5077$ and $0 < x < 10$, find x.

Use the following trigonometric identity.

$$\cos(A+B) = \cos(A)\times\cos(B) - \sin(A)\times\sin(B)$$



From the diagram, $\cos(A) = (x-2)/5$, $\cos(B) = x/(2x+3)$, $\sin(A) = (x-1)/5$, and $\sin(B) = (x+7)/(2x+3)$.

Substituting into the trigonometric identity equation that appears above results in the following:

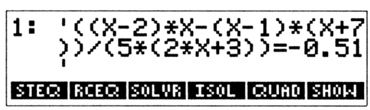
$$\cos(A+B) = \frac{x-2}{5}\times\frac{x}{2x+3} - \frac{x-1}{5}\times\frac{x+7}{2x+3} = -0.5077.$$

After simplifying this equation we obtain,

$$\frac{(x-2)\times x - (x-1)\times(x+7)}{5\times(2x+3)} = -0.5077.$$

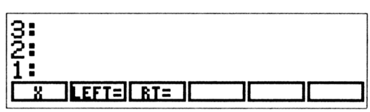
Enter this equation.

CLEAR
'((X−2)×X−(X−1)×(X+7))
÷(5×(2×X+3))=−.5077
ENTER



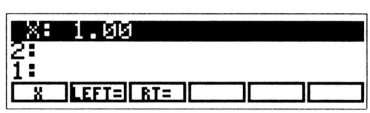
Store the equation and display the Solver menu.

STEQ
SOLVR



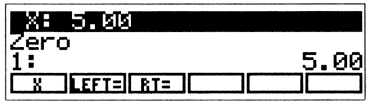
Store the initial guess of 1 in the variable X.

1 X



Solve for X.

[ ] [X]



Exit from the Solver and purge the variables created in this example.

[SOLV] { 'EQ' 'X [PURGE]

Graphs of Trigonometric Functions

This section illustrates how to plot various trigonometric functions.

Example: Plot the function $y = \sin(x)/x$. The technique for this example depends upon whether you are using an HP-28C or HP-28S.

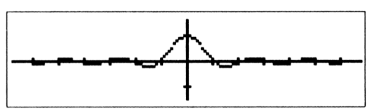
HP-28C Method. Version "1BB" of the HP-28C will generate an error when the DRAW function evaluates the example function at $x = 0$. The following program checks for evaluation at zero and avoids the error that would occur.

Program:	**Comments:**
`« CLLCD RAD`	Clear the display and set the angular mode to radians.
`'IFTE(X==0, 1, SIN(X)÷X)'`	Evaluate the function for X not equal to zero.
`STEQ DRAW`	Store the fuction and draw it.

[MODE] STD [<>]

Restore the default plot parameters, expand the width by a factor of three, and press [EVAL] to run the program.

[PURGE]
3 *W
[EVAL]



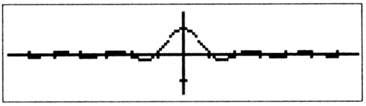
HP-28S Method. On the HP-28S it is not necessary to avoid evaluation at zero.

Set the calculator to radians mode, then key in the function and store it in EQ.

[MODE] RAD
'SIN(X)÷X
[PLOT] STEQ

Restore the default plot parameters, expand the width by a factor of three, and press [DRAW] to plot the function.

'PPAR [PURGE]
3 [*W]
[DRAW]



Example: Plot the first 10 terms of the Fourier series.

$$\sin(x)+\sin\frac{(3x)}{3}+\sin\frac{(5x)}{5}+\sin\frac{(7x)}{7}+\sin\frac{(9x)}{9}+\cdots$$

A general program can be written that plots a specified number of terms. The program below assumes you key in the desired number of terms, and then execute the program.

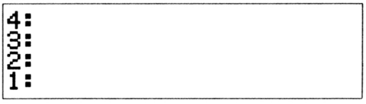
Key in the program and store it in the variable name "SQWV". (The graph is an approximation of a square wave.)

Program:	**Comments:**
`« CLLCD RAD`	Clear the display and set the mode to radians.
`0 1 ROT 2 × FOR n`	Loop: do for n from 1 to 2N.
`n X × SIN n ÷`	Calculate $\sin(n*x)/n$.
`+`	Add the sine term.
`2 STEP`	Increment n by 2 and repeat until $n > 2N$.
`STEQ DRAW »`	Store the equation and draw the function.

[ENTER] 'SQWV [STO]

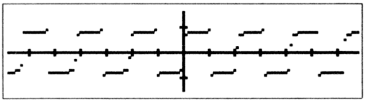
Set the display to standard mode and purge any existing variable named X.

[CLEAR] [MODE] [STD]
[<>] 'X [PURGE]



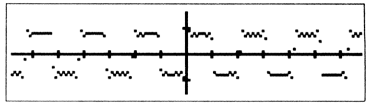
Display the User menu and execute the program for 10 terms.

[USER] 10 [SQWV]



Run the program again, this time for 5 terms.

ATTN
5 SQWV



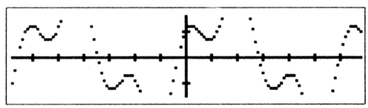
Example: Plot the function $y = 2\sin(x) + \cos(3x)$. If you have the HP 82240A printer, also print the graph.

Key in the function and store it in EQ.

'2×SIN(X)+COS(3×X)
PLOT STEQ

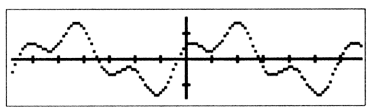
Purge the plot parameters and plot the function.

'PPAR PURGE
DRAW



Double the height parameter and plot the function again.

ATTN
2 *H
DRAW



Printing the Graph with the HP-28S. To print the graph using the HP-28S, press the ON key, and, while still holding the ON key, press the L key. Release both keys. The printer annunciator will appear on your display while the printer prints the graph.

Purge the variables used in this section.

{'SQWV''PPAR''EQ' PURGE

Printing the Graph with the HP-28C. If you are using an HP-28C, key in the following program to print the graph on your printer.

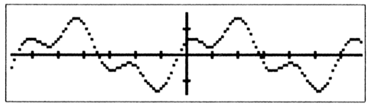
ATTN
«CLLCD DRAW PRLCD »
ENTER

Store the program in the variable *PRPLT*.

'PRPLT [STO]

Execute the program *PRPLT* which draws the graph of the expression stored in EQ and then prints it.

[USER] PRPLT



Purge the variables used in this section.

{'SQWV''PPAR''EQ''PRPLT [PURGE]

Inverse Trigonometric Functions

The inverse trigonometric functions arc sine, arc cosine, and arc tangent are built into the HP-28S and HP-28C. To calculate arc cosecant, arc secant, and arc cotangent of a number, simply take the inverse of the number and calculate the arc sine, arc cosine, or arc tangent, respectively.

Example: Find the principal values of

a. arcsin(.5),
b. arccos(−.95),
c. arctan(−8.98),
d. arccsc(−7.66),
e. arcsec(2), and
f. arccot(2.75) in HMS format.

First set the angle mode to degrees and the display setting to FIX 5.

CLEAR MODE DEG
5 FIX

```
3:
2:
1:
 STD | FIX■ | SCI | ENG | DEG■ | RAD
```

a. Compute arcsin(.5) in HMS format.

.5 TRIG ASIN

```
3:
2:
1:                      30.00000
 SIN | ASIN | COS | ACOS | TAN | ATAN
```

Since the angle is an integer, pressing HMS does not change the display.

→HMS

```
3:
2:
1:                      30.00000
 →HMS | HMS→ | HMS+ | HMS- | D→R | R→D
```

b. Compute arccos(−.95) in HMS format.

.95 CHS ACOS

```
3:
2:                      30.00000
1:                     161.80513
 SIN | ASIN | COS | ACOS | TAN | ATAN
```

→HMS

```
3:
2:                      30.00000
1:                     161.48185
 →HMS | HMS→ | HMS+ | HMS- | D→R | R→D
```

c. Compute arctan(−8.98) in HMS format.

8.98 [CHS] [ATAN]

```
3:                 30.00000
2:                161.48185
1:                -83.64580
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

[→HMS]

```
3:                 30.00000
2:                161.48185
1:                -83.38449
 →HMS  HMS→  HMS+  HMS-  D→R  R→D
```

d. Compute arccsc(−7.66). Note that arccsc(−7.66) = arcsin(−1/7.66). Calculate the inverse of −7.66.

7.66 [CHS] [1/x]

```
3:                161.48185
2:                -83.38449
1:                 -0.13055
 →HMS  HMS→  HMS+  HMS-  D→R  R→D
```

Press [ASIN] to find arccsc(−7.66) = arcsin(−1/7.66).

[ASIN]

```
3:                161.48185
2:                -83.38449
1:                 -7.50128
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Convert the resulting angle to HMS format.

[→HMS]

```
3:                161.48185
2:                -83.38449
1:                 -7.30046
 →HMS  HMS→  HMS+  HMS-  D→R  R→D
```

e. Compute arcsec(2). First, find the inverse of 2.

2 [1/x]

```
3:                -83.38449
2:                 -7.30046
1:                  0.50000
 →HMS  HMS→  HMS+  HMS-  D→R  R→D
```

Calculate the arccosine of the number since arcsec(2) = arccos(1/2).

[ACOS]

```
3:                -83.38449
2:                 -7.30046
1:                 60.00000
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Since the resulting angle is an integer, there is no need to convert it to HMS format.

f. Compute arccot(2.75) in HMS format.

`2.75` [1/x]

```
3:          -7.30046
2:          60.00000
1:           0.36364
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Calculate the arctangent of the resulting number to find arccot(2.75).

ATAN

```
3:          -7.30046
2:          60.00000
1:          19.98311
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

→HMS

```
3:          -7.30046
2:          60.00000
1:          19.58592
 →HMS  HMS→  HMS+  HMS-  D→R  R→D
```

Example: Evaluate sin(arccos(− .9) − arcsin(.6))

First, calculate arccos(− .9).

[CLEAR]
`.9` [CHS] ACOS

```
3:
2:
1:         154.15807
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Next, calculate arcsin(.6).

`.6` ASIN

```
3:
2:         154.15807
1:          36.86990
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Subtract arcsin(.6) from arccos(− .9).

[−]

```
3:
2:
1:         117.28817
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Calculate the sine of the resulting number.

SIN

```
3:
2:
1:           0.88871
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Trigonometric Equations

Solutions to trigonometric equations can be found by graphing the equation, by using the Solver, or both. This section demonstrates one way to solve a trigonometric equation.

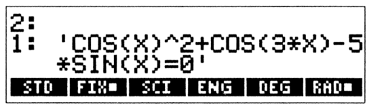
Solve $\cos^2(x)+\cos(3x)-5\sin(x)=0,\ 0 \le x \le 2\pi$.

First, set the angle mode to radians and set the display to FIX 2.

CLEAR
MODE ≣RAD≣
2 ≣FIX≣

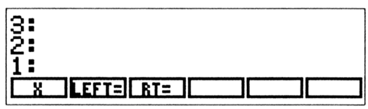
Key in the expression.

'COS(X)^2+COS(3×X)
-5×SIN(X)=0' ENTER



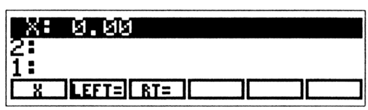
Store the equation and display the Solver menu. The menu shows X as the only variable.

SOLV ≣STEQ≣
≣SOLVR≣



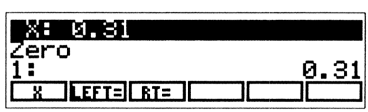
Let 0 be an initial estimate for X.

0 ≣X≣



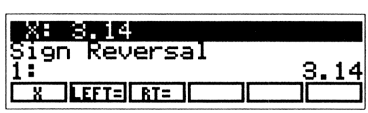
Solve for X.

☐ ≣X≣



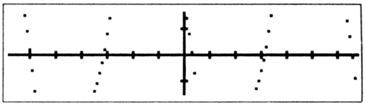
Try solving for X again with the number 3.14 as the initial estimate.

3.14 ≣X≣
☐ ≣X≣



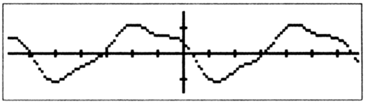
Check your results by plotting the function.

[PLOT] 'PPAR [PURGE]
DRAW



Increase the height by 5 and draw the function again.

[ATTN] 5 *H
DRAW



Between $x = 0$ and $x = 6.28$, the graph intersects the x-axis at approximately $x = .3$ and $x = 3.1$.

Exit from the graph and purge the variables used in this example.

[ATTN] {'X' 'EQ' 'PPAR' [PURGE]

Geometry

Rectangular Coordinates

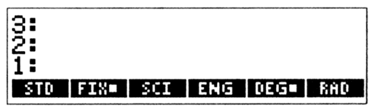
This section illustrates how to solve various problems dealing with rectangular coordinates. The object (x, y) represents either a complex number or the coordinates of a point; thus it is an acceptable argument to all of the arithmetic functions.

Example: Given triangle ABC with vertices $A\,(x1, y1) = (-4,3)$, $B\,(x2, y2) = (2,5)$, and $C\,(x3, y3) = (-3,-1)$, find

a. the length of side AC,
b. the coordinates of the midpoint of side AB,
c. the slope of side BC and the inclination,
d. the area of triangle ABC, and
e. the equivalent polar coordinates of the three points.

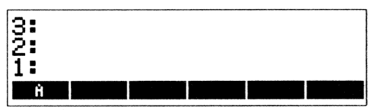
First, set the angle mode to degrees and the display to FIX 2.

CLEAR
MODE DEG
2 FIX



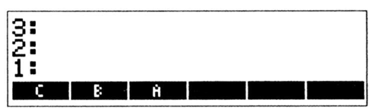
Next, enter the coordinates of point A and store it in the variable A.

`(-4,3)'A` STO
USER



Do the same for points B and C.

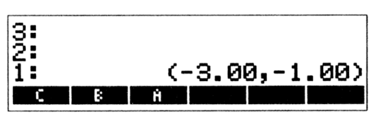
`(2,5)'B` STO
`(-3,-1)'C` STO



a. The length of side AC is $\sqrt{(x3-x1)^2+(y3-y1)^2}$. The easiest way to find the length is to subtract A from C and calculate the absolute value of the difference. (The absolute value of the complex argument (x, y) is $\sqrt{x^2+y^2}$.)

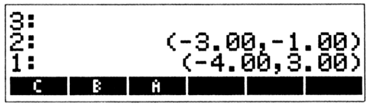
Put C on the stack.

C



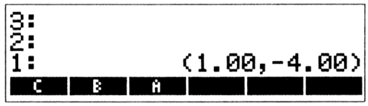
Put point A on the stack.

A



Subtract point A from point C.

[−]



Calculate the absolute value by pressing ABS. The resulting number is the length of side AC.

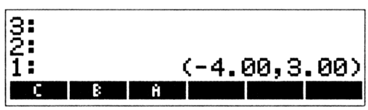
[REAL] ABS

b. The coordinates of the midpoint $M(x,y)$ of side AB is $x = (x1+x2)/2$ and $y = (y1+y2)/2$. Thus

$$M(x,y) = ((x1+x2)/2, (y1+y2)/2) = (x1+x2, y1+y2)/2 = (A+B)/2.$$

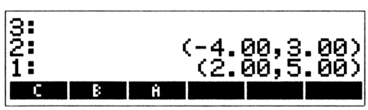
Put the coordinates for point A on the stack.

[CLEAR]
[USER] A



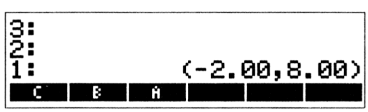
Put the coordinates for point B on the stack.

B



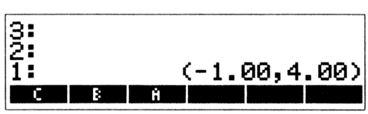
Add the two coordinates.

[+]



Divide the sum by 2 to obtain the coordinates for the midpoint.

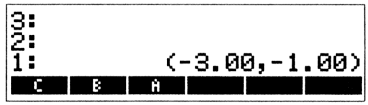
2 [÷]



c. The slope m of line BC is $m = (y3 - y2)/(x3 - x2)$. The slope is also equal to tan(θ) where θ is the inclination. To calculate the slope, subtract B from C, separate the result, swap the order, and divide the two numbers.

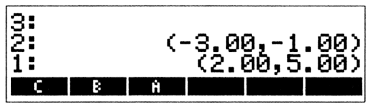
First, put the coordinates for C on the stack.

CLEAR
C



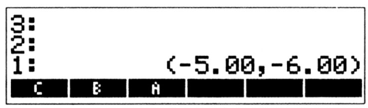
Put the coordinates for B on the stack.

B



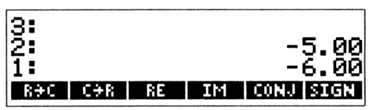
Calculate $C - B$.

−



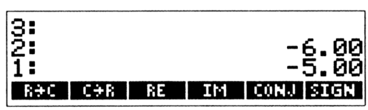
Separate the coordinates.

CMPLX C→R



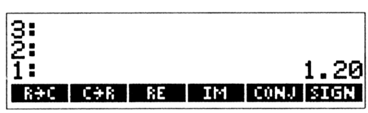
Swap the order of the x and y coordinates.

SWAP



Calculate the slope by dividing the y coordinate in level 2 by the x coordinate in level 1.

÷



The slope is equal to 1.20.

Compute the inclination by taking the arctangent of the slope.

[TRIG] ATAN

```
3:
2:
1:                        50.19
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

d. The area of the triangle formed by the three points is the absolute value of the following:

$$1/2 \begin{vmatrix} x1 & y1 & 1 \\ x2 & y2 & 1 \\ x3 & y3 & 1 \end{vmatrix}$$

To put the three points in a matrix, separate the coordinates then put the number 1 on the stack for each of the three points.

Separate the coordinates of point A .

[CLEAR]
A C→R

```
3:
2:                        -4.00
1:                         3.00
 P→R  R→P  R→C  C→R  ARG
```

Complete row 1 of the matrix.

1 [ENTER]

```
3:                        -4.00
2:                         3.00
1:                         1.00
 P→R  R→P  R→C  C→R  ARG
```

Separate the coordinates of point B and complete row 2 of the matrix.

B C→R
1 [ENTER]

```
3:                         2.00
2:                         5.00
1:                         1.00
 P→R  R→P  R→C  C→R  ARG
```

Separate the coordinates of C and complete row 3 of the matrix.

C C→R
1 [ENTER]

```
3:                        -3.00
2:                        -1.00
1:                         1.00
 P→R  R→P  R→C  C→R  ARG
```

Put the nine numbers into a three-by-three matrix.

{3,3 [ARRAY] →ARRY

```
1: [[ -4.00 3.00 1.00 ]
    [ 2.00 5.00 1.00 ]
    [ -3.00 -1.00 1.00…
→ARRY ARRY→ PUT  GET  PUTI  GETI
```

Compute the determinant of the matrix.

DET

```
3:
2:
1:                  -26.00
CROSS DOT DET ABS RNRM CNRM
```

Divide the determinant by 2 and take the absolute value of the result. The area of the triangle is returned to level 1.

2 ÷
ABS

```
3:
2:
1:                   13.00
CROSS DOT DET ABS RNRM CNRM
```

e. To convert the points from rectangular to polar form, simply key in the variable name and press R→P.

Key in the variable name *A* and convert point *A* to polar form.

CLEAR
A TRIG R→P

```
3:
2:
1:          (5.00,143.13)
P→R R→P R→C C→R ARG
```

Key in the variable *B* and convert point *B* to polar form.

B R→P

```
3:
2:          (5.00,143.13)
1:           (5.39,68.20)
P→R R→P R→C C→R ARG
```

Do the same for point *C*.

C R→P

```
3:          (5.00,143.13)
2:           (5.39,68.20)
1:        (3.16,-161.57)
P→R R→P R→C C→R ARG
```

Purge the three variables used in this example.

{ 'C' 'B' 'A' PURGE

Polar Coordinates

A point in a plane can be represented in rectangular notation or polar notation. To draw a point that is described in polar notation on the HP-28S or HP-28C, first convert it to rectangular form and then plot it. You can either write a program to draw the graph of a polar equation or convert the equation to rectangular form before attempting to draw it.

Example: Convert the following polar coordinates (whose angles are expressed in degrees) to rectangular coordinates, then plot the points.

$$A(4,-15)\quad B(-4,380)\quad C(-2,570)\quad D(2,-195)$$

Converting polar coordinates is easily accomplished by executing the Polar-to-Rectangular function P→R. One way to plot the four points is to put the four points on the stack and use the PIXEL command four times, being sure to clear the display first by pressing CLLCD. You may also wish to draw the axes by executing the DRAX command. Another way to plot the points is to separate the coordinates, put them in a four-by-two matrix, and then use the statistical scatter plot commands STOΣ and DRWΣ.

To illustrate the first approach, set the angle mode to degrees, and set the display to FIX 2.

CLEAR
MODE DEG
2 FIX

```
3:
2:
1:
 STD | FIX■ | SCI | ENG | DEG■ | RAD
```

Key in point A and convert it to rectangular coordinates.

(4,-15 TRIG P→R

```
3:
2:
1:               (3.86,-1.04)
 P→R | R→P | R→C | C→R | ARG |
```

Enter the coordinates for point B and convert it to rectangular form.

(-4,380 P→R

```
3:
2:               (3.86,-1.04)
1:              (-3.76,-1.37)
 P→R | R→P | R→C | C→R | ARG |
```

Do the same for points C and D.

(-2,570 P→R

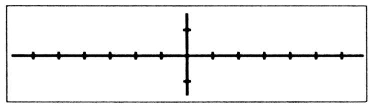
```
3:               (3.86,-1.04)
2:              (-3.76,-1.37)
1:                (1.73,1.00)
 P→R | R→P | R→C | C→R | ARG |
```

`(2,-195` [P→R]

The rectangular form of the four points are A (3.86, − 1.04), B (− 3.76, − 1.37), C (1.73,1.00), and D (− 1.93,0.52).

Clear the plot parameters, clear the display and draw the axes. Note: The soft key labeled [↓] will execute the [DRAX] function after [CLLCD] eliminates the menu display.

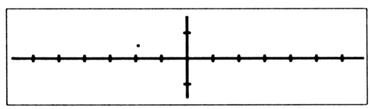
[PLOT] 'PPAR [PURGE]
[CLLCD]
[DRAX]



Although you can't see them, the coordinates for the four points are still on the stack. Therefore, they are still available for use.

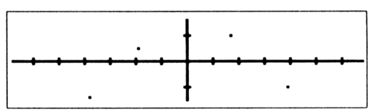
Draw point D (which is in level 1 of the stack) by executing the PIXEL command. (Press the soft key labeled [^].)

[PIXEL]



Draw points C, B, and A by executing the PIXEL command three more times.

[PIXEL]
[PIXEL]
[PIXEL]



Press [ATTN] to exit from the plot display.

Example: Sketch the rose $r = 2sin\ (2\theta)$ for $0<\theta<360$.

The following program draws the graph of a polar equation. The program assumes that the equation is in the form $r = f\ (\theta)$, where $f\ (\theta)$ is an expression with θ as the unknown variable. The input to the program is the expression $f\ (\theta)$.

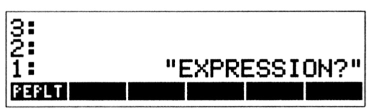
Key in the program listed below and store it in the variable *PEPLT* (for "polar equation plot.")

Program:	**Comments:**
`« "EXPRESSION?"`	Prompt message.
`HALT`	Program stops (Enter the expression).
`→ r`	Store the expression in the local variable *r*.
`« DROP`	Drop the prompt message.
`DEG`	Set the angle mode to degrees.
`CLLCD`	Clear the display.
`0 360 FOR j`	Loop: do for *j* from 0 to 360.
`j 'theta' STO`	Store the current *j* in the variable *theta*.
`r EVAL`	Evaluate the expression for *r*.
`theta`	Put *theta* on the stack.
`R→C`	Combine *r* and *theta*.
`P→R`	Convert (*r*,*theta*) to rectangular form.
`PIXEL`	Draw the point.
`3 STEP`	Increment *j* by 3 and repeat until *j* >360.
`{ PPAR theta }`	Purge the plot parameters
`PURGE » »`	and *theta*.

[ENTER] 'PEPLT [STO]

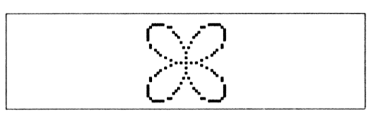
Display the User menu and execute the program.

[USER] [PEPLT]



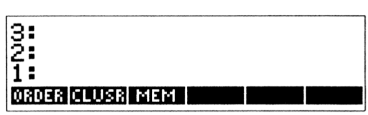
Key in the expression 2×SIN(2×theta) and press [CONT].

'2×SIN(2×theta) [CONT]



If you do not want to save the program, purge *PEPLT*.

[ATTN] 'PEPLT' [PURGE]



Example: Transform $r(1-\sin(\theta))=2$ into its rectangular form, substituting x^2+y^2 for r^2 and y for $r\sin(\theta)$.

Key in the equation. Let the angle be called "th".

[TRIG]
'r×(1-SIN(th))=2 [ENTER]

```
3:
2:
1:      'r*(1-SIN(th))=2'
 SIN  ASIN  COS  ACOS  TAN  ATAN
```

Display the Algebra menu. Expand the equation to get $r - r\sin(\theta) = 2$.

[ALGEBRA] [EXPAN]

```
3:
2:
1:      'r*1-r*SIN(th)=2'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Add $r\sin(\theta)$ to both sides of the equation. To do this, press the [ENTER] key to duplicate the expanded equation.

[ENTER]

```
3:
2:      'r*1-r*SIN(th)=2'
1:      'r*1-r*SIN(th)=2'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Next, enter the number 6 and press [EXGET]. The subexpression $r\sin(\theta)$ is returned.

6 [EXGET]

```
3:
2:      'r*1-r*SIN(th)=2'
1:            'r*SIN(th)'
TAYLR ISOL QUAD SHOW OBGET EXGET
```

Then, add this subexpression to the expresssion in level 2.

[+]

```
2:
1:  'r*1-r*SIN(th)+r*SIN
    (th)=2+r*SIN(th)'
TAYLR ISOL QUAD SHOW OBGET EXGET
```

Simplify the expression.

[COLCT]

```
3:
2:
1:         'r=2+SIN(th)*r'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Square both sides of the equation. The equation $r^2=(2+r\sin(\theta))^2$ is returned to level 1.

[x^2]

```
2:
1:  'SQ(r)=SQ(2+SIN(th)*
    r)'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Now you can substitute x^2+y^2 for r^2 and y for $r\sin(\theta)$. The Expression Substitute command EXSUB can accomplish this task.

Since "SQ(r)" is in the first position of the equation, put the number 1 on the stack.

1 [ENTER]

```
3:
2:  'SQ(r)=SQ(2+SIN(th)…
1:                  1.00
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Enter X^2+Y^2 and press [EXSUB].

'X^2+Y^2 [EXSUB]

```
2:
1:  'X^2+Y^2=SQ(2+SIN(th
    )*r)'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

The subexpression "SIN(th)*r" is in the fourteenth position; therefore, key in the number 14.

14 [ENTER]

```
3:
2:  'X^2+Y^2=SQ(2+SIN(t…
1:                 14.00
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Substitute "Y" for "SIN(th)*r".

'Y [EXSUB]

```
3:
2:
1:      'X^2+Y^2=SQ(2+Y)'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

To simplify this equation, subtract "SQ(2+Y)" from both sides of the equation, expand the equation, then collect terms.

First, duplicate the equation by pressing the [ENTER] key.

[ENTER]

```
3:
2:      'X^2+Y^2=SQ(2+Y)'
1:      'X^2+Y^2=SQ(2+Y)'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Enter the number 9 and press [EXGET]. The subexpression 'SQ(2+Y)' is returned to level 1.

9 [EXGET]

```
3:
2:      'X^2+Y^2=SQ(2+Y)'
1:              'SQ(2+Y)'
TAYLR ISOL QUAD SHOW OBGET EXGET
```

Subtract 'SQ(2+Y)' from both sides of the equation.

[−]

```
3:
2:
1:   'X^2+Y^2-SQ(2+Y)=0'
TAYLR ISOL QUAD SHOW OBGET EXGET
```

Expand the equation.

EXPAN

```
2:
1:  'X*X+Y*Y-(2^2+2*2*Y+
    Y^2)=0'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Simplify the equation by collecting terms.

COLCT

```
2:
1:  '-4+X^2+Y^2-Y^2-4*Y=
    0'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Collect terms.

COLCT

```
3:
2:
1:          '-4+X^2-4*Y=0'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

The final result is the equation of a parabola.

The Straight Line

This section includes some basic analytic geometry problems for the straight line and methods to solve them on the HP-28S or HP-28C.

Example: Given the line passing through points $A\ (8,-10)$ and $B\ (-10,26)$, find

a. the y-intercept and slope of the line, and,
b. the corresponding value for y, given $x = -4$.

First, set the display to FIX 2.

CLEAR
MODE 2 FIX

```
3:
2:
1:
 STO | FIX■ | SCI | ENG | DEG■ | RAD
```

The solutions to this example can all be found by using the commands in the Statistics menu. Since statistical data points are entered as arrays, use brackets around the coordinates instead of parentheses.

Key in point A and press Σ+. The matrix ΣDAT is created with point A as the first entry in the matrix.

STAT
[8,-10 Σ+

```
3:
2:
1:
 Σ+ | Σ- | NΣ | CLΣ | STOΣ | RCLΣ
```

Add point B to the matrix.

[-10,26 Σ+

```
3:
2:
1:
 Σ+ | Σ- | NΣ | CLΣ | STOΣ | RCLΣ
```

a. Find the y-intercept and the slope by executing the Linear Regression function LR. The y-intercept is returned to level 2 and the slope to level 1.

LR

```
3:
2:                                6.00
1:                               -2.00
 COLΣ | CORR | COV | LR | PREDV |
```

b. To find the corresponding value for y given $x = -4$, enter the number -4 and compute the predicted value. The value for y is returned to level 1.

−4 PREDV

```
3:                6.00
2:               -2.00
1:               14.00
COLΣ CORR COV LR PREDV
```

Clear the display and purge the variables that were created in this example.

CLEAR { 'ΣPAR' 'ΣDAT' PURGE

Example: Given the vertices $D(-4,3)$, $E(2,5)$, and $F(-3,-1)$ of triangle DEF, find

a. the equation of lines DE and DF in the normal form, and,
b. the equation of the bisector of angle D.

a. Given two points $(x1,y1)$ and $(x2,y2)$, the normal form of the equation of the line connecting the two points is $s\times(Ax+By+C)/(\sqrt{A^2+B^2})=0$, where $s=\{-1 \text{ or } 1\}$, $A=y1-y2$, $B=x2-x1$, and $C=x1\times y2-x2\times y1$.

If $C>0$, then $s=-1$.
If $C<0$, then $s=1$.
If $C=0$ and B is non-zero, then the sign of s agrees with the sign of B.
If $C=B=0$, then the sign of s agrees with the sign of A.

First, store 'Y1−Y2' in the variable A.

'Y1-Y2' 'A STO USER

```
3:
2:
1:
A
```

Store 'X2−X1' in the variable B.

'X2-X1' 'B STO

```
3:
2:
1:
B A
```

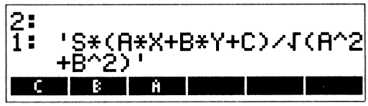
Store 'X1×Y2−X2×Y1' in the variable C.

'X1×Y2-X2×Y1' 'C STO

```
3:
2:
1:
C B A
```

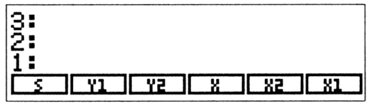
Key in the normal form of the equation.

'S×(A×X+B×Y+C)÷
√(A^2+B^2)' [ENTER]



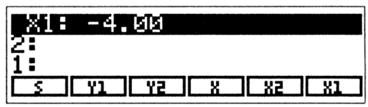
Store the equation in the variable EQ and display the Solver menu. A menu of the variables is shown in the display.

[SOLV] STEQ
SOLVR



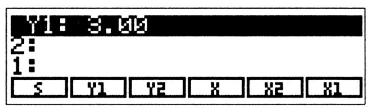
Find the equation for line *DE*. Let point *D* be the first point and *E* be the second. First, enter the coordinate −4 and press the X1 soft key.

−4 X1



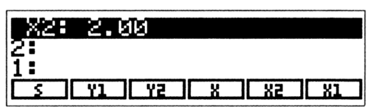
Enter the number 3 and store it in *Y*1.

3 Y1



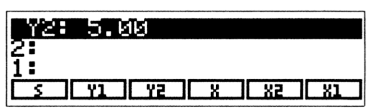
Enter the number 2 and store it in *X2*.

2 X2



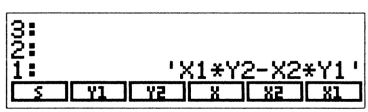
Enter the number 5 and store it in *Y2*.

5 Y2



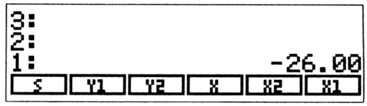
Determine the sign of the variable *S*.

C [ENTER]



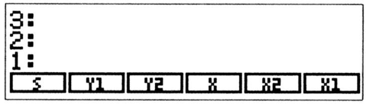
Evaluate *C*.

[EVAL]



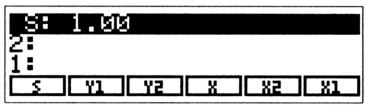
The value of *C* is returned to level 1, and it is negative. Drop the value of *C* from the stack.

[DROP]



Since *C* is negative, *S* is equal to 1. Enter the number 1 into the variable *S*.

1 [S]

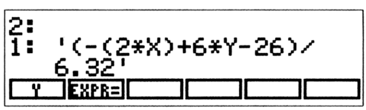


Display the resulting expression.

[EXPR=]

Evaluate the expression by pressing [EVAL]. The left side of the normal form of the equation of line *DE* is returned to level 1. (The right side is equal to zero.)

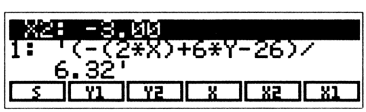
[EVAL]



Now find the equation for line *DF*.

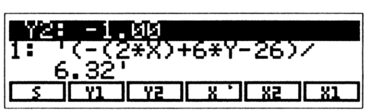
Store the coordinate −3 in the variable *X2*.

-3 [X2]



Store the coordinate −1 in the variable *Y2*.

-1 [Y2]



Press [C] followed by the [ENTER] key.

C [ENTER]

```
3:
2: '(-(2*X)+6*Y-26)/6.…
1:            'X1*Y2-X2*Y1'
[ S ][ Y1 ][ Y2 ][ X ][ X2 ][ X1 ]
```

Evaluate C.

[EVAL]

```
3:
2: '(-(2*X)+6*Y-26)/6.…
1:                  13.00
[ S ][ Y1 ][ Y2 ][ X ][ X2 ][ X1 ]
```

C is positive. Drop the value of C from the stack.

[DROP]

```
2:
1: '(-(2*X)+6*Y-26)/
   6.32'
[ S ][ Y1 ][ Y2 ][ X ][ X2 ][ X1 ]
```

Since $C > 0$, then $S = -1$. Enter a -1 and press [S].

-1 [S]

```
S: -1.00
1: '(-(2*X)+6*Y-26)/
   6.32'
[ S ][ Y1 ][ Y2 ][ X ][ X2 ][ X1 ]
```

Display the resulting expression.

[EXPR=]

```
EXPR='-(((Y1-Y2)*X+(X…
   )*Y+(X1*Y2-X2*Y1))/√
   ((Y1-Y2)^2+(X2-X1)^2
[ Y ][EXPR=][ ][ ][ ][ ]
```

Evaluate the expression to obtain the normal form of the equation of line DF. This is also only the left side of the equation; the right side is equal to zero.

[EVAL]

```
3:
2: '(-(2*X)+6*Y-26)/6.…
1: '-((4*X+Y+13)/4.12)'
[ S ][ Y1 ][ Y2 ][ X ][ X2 ][ X1 ]
```

b. To find the equation of the bisector of angle D, simply equate the two expressions in levels 1 and 2 and simplify. To simplify this process even more, subtract the two expressions and equate the difference to zero.

[-]

```
1: '(-(2*X)+6*Y-26)/
   6.32+(4*X+Y+13)/4.12
   '
[ Y ][EXPR=][ ][ ][ ][ ]
```

Key in the number 0 and set the expression in level 2 equal to the number in level 1.

0 [ENTER]
[=] [ENTER]

```
1:  '(-(2*X)+6*Y-26)/
    6.32+(4*X+Y+13)/4.12
    =0'
 S    Y1   Y2   X    X2   X1
```

Expand the equation.

[ALGEBRA] EXPAN

```
1:  '(-(2*X)+6*Y)/6.32-
    26/6.32+((4*X+Y)/
    4.12+13/4.12)=0'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Expand it again.

EXPAN

```
1:  '-(2*X)/6.32+6*Y/
    6.32-26/6.32+(4*X/
    4.12+Y/4.12+13/4.12)
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Simplify the equation by collecting terms. The final result is the equation of the bisector of angle *D* .

COLCT

```
2:
1:  '-0.96+0.65*X+1.19*Y
    =0'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Purge the variables used in this example.

`{'S''Y2''X2''Y1''X1''EQ''C''B''A'` [PURGE]

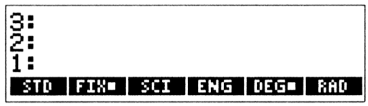
The Circle

Finding the points of intersection of two equations is a common problem in analytic geometry. In this section you'll work through the steps to find the points of intersection of two circles.

Example: Given two circles $x^2+y^2-5=0$ and $(x+2)^2+(y-1)^2-20=0$, find the point(s) of intersection, if any exist.

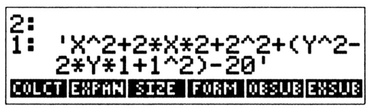
First, set the display to FIX 2.

CLEAR
MODE 2 FIX



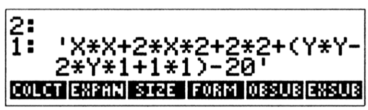
Key in the expression for the second circle as shown below, and simplify it by expansion and collection of terms.

`'(X+2)^2+(Y-1)^2-20'`
ALGEBRA EXPAN



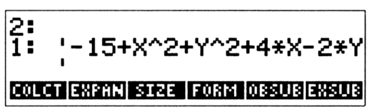
Expand again.

EXPAN



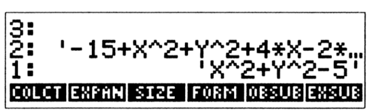
Simplify the expression by collecting terms.

COLCT



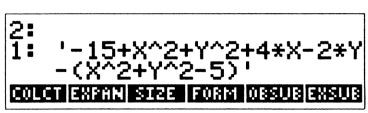
Key in the expression for the first circle as shown below and press ENTER.

`'X^2+Y^2-5` ENTER



Find the equation for the radical axis by subtracting the expression in level 1 from the expression in level 2.

−



Expand the expression.

EXPAN

```
2:
1: '-15+X*X+Y*Y+4*X-2*Y
   -(X*X+Y*Y-5)'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

Simplify the expression by collecting terms. The result is the left side of the equation for the radical axis. (The right side is equal to zero.)

COLCT

```
3:
2:
1:          '-10+4*X-2*Y'
COLCT EXPAN SIZE FORM OBSUB EXSUB
```

To find the point(s) where the two circles intersect, simultaneously solve the equation for the radical axis and either one of the equations for the circles. In this example, take the equation for the radical axis and solve for the variable Y. Then substitute the resulting expression for Y in the equation for the first circle. This gives an equation with one unknown, namely, X. Solve for X, then find the corresponding value(s) for Y.

Solve for the variable Y.

`'Y` ISOL

```
3:
2:
1:          '(-10+4*X)/2'
TAYLR ISOL QUAD SHOW OBGET EXGET
```

Store this expression in the variable Y.

`'Y` STO

```
3:
2:
1:
TAYLR ISOL QUAD SHOW OBGET EXGET
```

Key in the equation for the first circle. Then use the command SHOW to substitute the expression stored in Y into the equation of the circle. The resulting equation is a function of one variable, X.

`'X^2+Y^2-5=0' 'X` SHOW

```
2:
1: 'X^2+((-10+4*X)/2)^2
   -5=0'
TAYLR ISOL QUAD SHOW OBGET EXGET
```

Since the equation in level 1 is a quadratic, use the QUAD command to find the value(s) of X.

`'X` QUAD

```
3:
2:
1:                  2.00
TAYLR ISOL QUAD SHOW OBGET EXGET
```

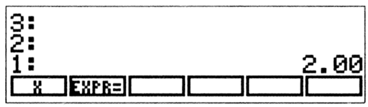
The single number $X = 2$ is returned to level 1; thus the circles intersect in one point. If there were two values of X, then the circles intersect in two points. A complex value of X means there are no intersection points.

Now use the Solver to find the corresponding value of Y. First, put the expression stored in the variable Y on the stack.

'Y [RCL]

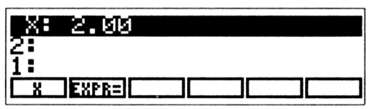
Store this expression in the variable EQ and display the Solver menu.

[SOLV] [STEQ]
[SOLVR]



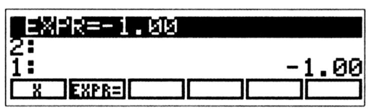
Store the value that you just found in the variable X.

[X]



Press [EXPR=] to get the corresponding value of Y.

[EXPR=]



Thus the circles intersect at the point $(2, -1)$.

Exit from the Solver and purge the variables that were created in this example.

[SOLV] { 'X' 'EQ' 'Y' [PURGE]

The Parabola

This section describes how to plot the graph of a parabola. Vertical parabolas are plotted as you would expect – solve for y, store the expression, and draw with the DRAW key. If you attempt to draw a horizontal parabola in the same manner, an error will result. This section demonstrates a program to draw a horizontal parabola.

Example: Plot the graph of $x^2=4(y+1)$.

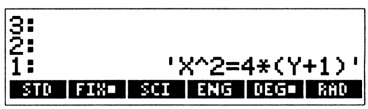
First, set the display to FIX 2.

[CLEAR]
[MODE] 2 FIX

The semireduced form of the equation of a vertical parabola is $(x-h)^2=4p(y-k)$, where (h,k) is the vertex, $x=h$ is the axis, $(h,k+p)$ is the focus, and $y=k-p$ is the directrix. In this example, $h=0$, $k=-1$, and $p=1$. Therefore, the vertex is V(0, −1); the axis is $x=0$; the focus is F(0,0); and the directrix is $y=-2$.

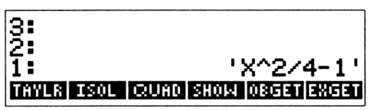
Key in the equation for the parabola.

'X^2=4×(Y+1) [ENTER]



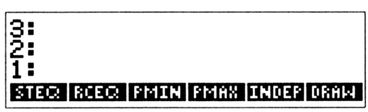
Isolate the variable Y.

'Y [ALGEBRA] ISOL



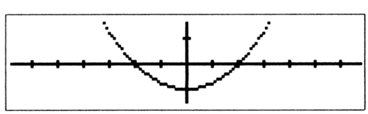
Store the expression for Y in the variable EQ.

[PLOT] STEQ



Draw the graph of the parabola.

DRAW



Exit from the graph and purge the variables created in this example.

ATTN { 'PPAR' 'EQ' PURGE

Example: Plot the graph of the horizontal parabola $y^2 = -4(x - 1)$.

The general equation of a horizontal parabola is $(y - k)^2 = 4p(x - h)$. The vertex is (h, k); the axis is $y = k$; the focus is $(h + p, k)$; and the directrix is $x = h - p$. Therefore, in this case, h, k, and p are equal to 1, 0, and -1, respectively. The vertex is V(1,0); the axis is $y = 0$; the focus is at (0,0); and the directrix is $x = 2$.

The following program plots a horizontal parabola. The program expects three numbers to be entered onto the stack as inputs into the program: the values of h, k, and p. (A prompt message is displayed requesting you to enter the numbers.) Given these three numbers, the program draws the graph of the parabola with the vertex at the center of the display, and each tic mark on the axes represents 10 units.

Key in the program below and store it in the variable HPAR (for "horizontal parabola").

Program:	**Comments:**
`« "ENTER h,k,p"`	Prompt message.
`HALT`	Program halts (you key in 3 numbers).
`→ h k p «`	Store the 3 numbers in h ,k and p .
`DROP`	Drop the prompt message.
`CLLCD`	Clear the display.
`10 *H 10 *W`	Multiply the height and width by 10.
`h k R→C CENTR`	The center of the display is (h ,k).
`DRAX`	Draw the axes.
`'(Y-k)^2=4×p×(X-h)'`	Equation for a horizontal parabola.
`'X' ISOL`	Isolate X in the above equation.
`'X' STO`	Store the expression in the variable X .
`k 20 - k 20 + FOR j`	Loop: do for j from $k-20$ to $k+20$.
`j 'Y' STO`	Store the current j in variable Y .
`X EVAL Y R→C`	Evaluate X and form point (X ,Y).
`PIXEL`	Draw point (X ,Y).
`NEXT`	Increment j by 1 and repeat until $j>(k+20)$.
`{ X Y PPAR } PURGE »»`	Purge variables X, Y, and PPAR.

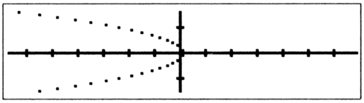
[ENTER] `'HPAR` [STO]

Display the User menu and execute the program.

[USER] HPAR

Enter the values for h, k, and p. Continue running the program by pressing [CONT]. The graph of the parabola is drawn.

`1,0,-1` [CONT]



Press [ATTN] to exit from the plot display.

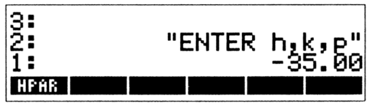
Example: Plot the graph of $(y+10)^2=12(x+35)$.

This is the equation of a horizontal parabola with the vertex at $V(h,k)=(-35,-10)$ and $p=3$. Run the program HPAR.

HPAR

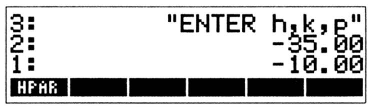
Key in the value of h.

`-35` ENTER



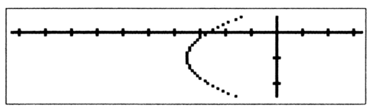
Key in the value of k.

`-10` ENTER



Key in the value for p and continue running the program. The graph of the parabola is drawn.

`3` CONT



Exit from the graphics display and purge the program HPAR, if you wish.

ATTN `'HPAR` PURGE

Example: Horizontal Parabolas Using DRAW. The program below is an alternate approach from the point-by-point function plot in program HPAR. This program takes h, k, and p from the stack, creates an equation representing the upper and lower halves of the parabola, and uses the DRAW command to create the plot. Note for $y^2(x) < 0$, the DRAW routine produces a line intersecting the curve at the vertex.

Key in the following program.

```
« 'X' PURGE 10 *H 10 *W
→ h k p «
'2×√((X-h)×p)'
EVAL DUP NEG = k + RE
STEQ CLLCD DRAW [ENTER] [<>]
```

```
1: « 'X' PURGE 10 *H 10
   *W → h k p « '2*√((X
   -h)*p)' EVAL DUP NEG
   = k + RE STEQ CLLCD
```

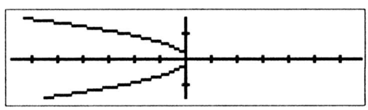
Store the program by the name HPAR2 and purge the current plot parameters.

```
'HPAR2 [STO]
'PPAR [PURGE]
```

```
4:
3:
2:
1:
```

Execute the program for the previous horizontal parabola.

```
1,0,-1 [USER] [HPAR2]
```



Exit from the plot display and purge program HPAR2 if you wish.

```
[ATTN] 'HPAR2 [PURGE]
```

The Ellipse and Hyperbola

This section describes the procedure for drawing the graphs of ellipses and hyperbolas.

Example: Plot the graph of the following ellipse.

$$\frac{(x+2)^2}{9}+\frac{(y-1)^2}{4}=1$$

The general equation of an ellipse is

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

The center is at the point (h,k). If $a>b$, then the major axis is parallel to the x-axis. The vertices are at points $(h\pm a,k)$; the foci are at points $(h\pm c,k)$, where $c=\sqrt{a^2-b^2}$; and the ends of the minor axis are at points $(h,k\pm b)$. If $b>a$, then the major axis is parallel to the y-axis; the vertices are at points $(h,k\pm b)$; the foci are at points $(h,k\pm c)$; and the ends of the minor axis are at points $(h\pm a,k)$.

For this example, $h=-2$, $k=1$, $a=3$, $b=2$, $c=2.24$, and the major axis is parallel to the x-axis. The center is at (3,2); the vertices are at points (1,1) and (−5,1); the foci are at (0.24,1) and (−4.24,1); and the ends of the minor axis are at points (−2,3) and (−2,−1).

The following program draws the graph of an ellipse. After a prompt message is displayed, the program expects the values of h, k, a, and b to be entered onto the stack. The graph of the ellipse is drawn with its center in the center of the display. Each tic mark on the axes represents two units.

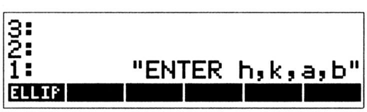
Key in the program and store it in the variable *ELLIPSE*.

Program:	**Comments:**
`« "ENTER h,k,a,b"`	Prompt message.
`HALT`	Program halts (Enter the 4 values).
`→ h k a b`	Values are stored in *h,k,a,* and *b*.
`« DROP`	Drop the prompt message.
`CLLCD`	Clear the display.
`2 *H 2 *W`	Multiply the height and width by 2.
`h k R→C CENTR`	The center of the display is (*h*,*k*).
`DRAX`	Draw the axes.
`'(X-h)^2÷a^2+ (Y-k)^2÷b^2=1'`	The general equation of an ellipse.
`'Y' ISOL`	Isolate *Y* from the equation.
`'Y' STO`	Store the expression in the variable *Y*.
`-1 1 FOR j`	Loop1: do for *j* from −1 to 1.
`j 's1' STO`	Store the current *j* in variable *s* 1.
`h a - h a + FOR n`	Loop2: do for *n* from *h* −*a* to *h* +*a*.
`n 'X' STO`	Store the current *n* in variable *X*.
`X Y EVAL R→C`	Form the point (*X*,*Y*).
`PIXEL`	Plot the point (*X*,*Y*).
`.2 STEP`	Increment *n* by .2 and repeat until *n* >*h* +*a*.
`2 STEP`	Increment *j* by 2 and repeat loop1.
`{ PPAR X Y s1 }`	Purge the variables
`PURGE »»`	created by this program.

[ENTER] 'ELLIPSE [STO]

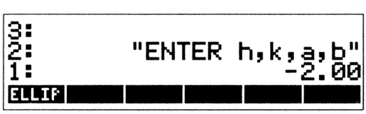
Display the User menu and run the program. The prompt message is returned to level 1.

[USER] [ELLIP]



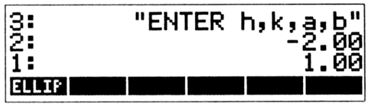
Enter the value for *h* .

−2 [ENTER]



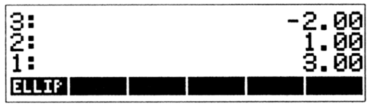
Key in the value for k.

1 [ENTER]



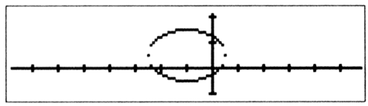
Enter the value for a.

3 [ENTER]



Enter the value for b and press [CONT] to continue running the program. The graph of the ellipse is drawn.

2 [CONT]



Press [ATTN] to exit from the plot display and, if desired, purge the program.

[ATTN] 'ELLIPSE [PURGE]

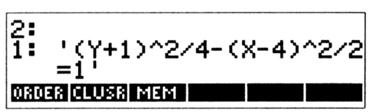
Example: Plot the graph of the vertical hyperbola

$$\frac{(y+1)^2}{4} - \frac{(x-4)^2}{2} = 1.$$

The graph of the vertical hyperbola can be drawn by first isolating the variable y. Since y is a squared term, the result of isolating y is an expression representing the two solutions. One solution represents the top half of the hyperbola, and the other solution represents the lower half. Use the Solver to find the two solutions. After the two expressions for y are found, set them equal to each other and draw their graphs. (This technique is used to draw two functions simultaneously.)

Enter the equation as shown below.

'(Y+1)^2÷4-(X-4)^2÷2=1'
[ENTER]



Isolate the variable *Y*. The result is an expression representing two solutions. The variable *s1* can be either +1 or −1.

`'Y` [SOLV] [ISOL]

```
2:
1: 's1*√((1+(X-4)^2/2)*
   4)-1'
STEQ RCEQ SOLVR ISOL QUAD SHOW
```

Store the expression for *Y* in the variable EQ and display the Solver menu.

[STEQ]
[SOLVR]

```
3:
2:
1:
S1  X  EXPR=
```

Store the number 1 in the variable *s1*.

`1` [S1]

```
s1: 1.00
2:
1:
S1  X  EXPR=
```

[EXPR=]

```
EXPR= √((1+(X-4)^2/2)…
1: '√((1+(X-4)^2/2)*4)-
   1'
S1  X  EXPR=
```

Store the number −1 in the variable *s1*.

`-1` [S1]

```
s1: -1.00
1: '√((1+(X-4)^2/2)*4)-
   1'
S1  X  EXPR=
```

[EXPR=]

```
EXPR= '-√((1+(X-4)^2/2…
1: '-√((1+(X-4)^2/2)*4)
   -1'
S1  X  EXPR=
```

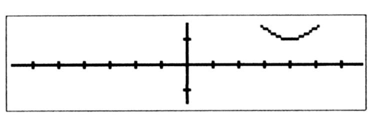
Set the expression in level 2 equal to the one in level 1.

[=] [ENTER]

```
1: '√((1+(X-4)^2/2)*4)-
   1=-√((1+(X-4)^2/2)*4
   )-1'
S1  X  EXPR=
```

Store this equation in the variable EQ, and plot the graph of the hyperbola.

[PLOT] [STEQ]
[DRAW]



Press [ATTN] to exit from the plot display, and multiply the height by 10.

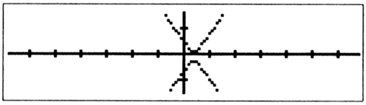
[ATTN] 10 *H

Multiply the width by 10.

10 *W

Draw the graph again. Each tic mark represents 10 units.

DRAW



Exit from the plot display, and purge the variables used in this example.

[ATTN]
{'PPAR''s1''EQ' [PURGE]

Example: Plot the graph of the horizontal hyperbola

$$\frac{(x-4)^2}{4} - \frac{(y+1)^2}{2} = 1.$$

The general equation of a hyperbola is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1.$$

For this example, $h = 4$, $k = -1$, $a = 2$, and $b = \sqrt{2}$.

A combination of the program to draw a horizontal parabola and the program to draw an ellipse can be used to draw the horizontal hyperbola. (A listing and explanation is not given here. Refer to the section entitled "The Parabola" for an explanation of specific program steps.)

Key in the program as shown below.

```
«"ENTER h,k,a,b" HALT
→ h k a b « DROP
CLLCD 2 *H 2 *W h k
R→C CENTR DRAX
'(X-h)^2÷a^2-(Y-k)^2÷
b^2=1' 'X' ISOL 'X' STO
-1 1 FOR j j 's1' STO
k 4 - k 4 + FOR n n 'Y'
STO X EVAL Y R→C PIXEL
.2 STEP 2 STEP { X Y s1
PPAR } PURGE »»
```
[ENTER] [<>]

```
1: « "ENTER h,k,a,b"
   HALT → h k a b «
   DROP CLLCD 2.00 *H
   2.00 *W h k R→C
```

Store the program in the variable HHYPE (for "horizontal hyperbola").

'HHYPE' [STO]

```
4:
3:
2:
1:
```

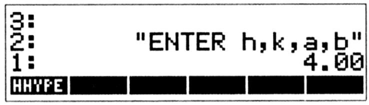
Display the User menu and execute the program. A prompt message is displayed requesting you to enter the values for h, k, a, and b.

[USER] [HHYPE]

```
3:
2:
1:        "ENTER h,k,a,b"
HHYPE
```

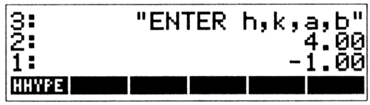
Enter the value for *h* .

4 [ENTER]



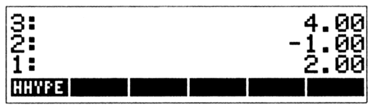
Enter the value for *k* .

-1 [ENTER]



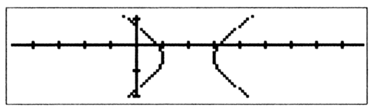
Key in the value for *a* .

2 [ENTER]



Calculate the value of *b* by entering the number 2 and taking the square root of it. Press [CONT] to continue running the program. The graph of the horizontal hyperbola is drawn.

2 [√] [CONT]



If desired, purge the program.

[USER] 'HHYPE [PURGE]

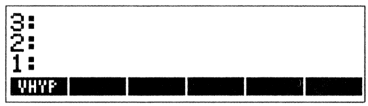
Example: Plotting the General Form of the Equation. As an alternative to point-by-point plotting of the functions, the DRAW command can be used by separating the ellipse and hyperbola equations into upper and lower halves. The following programs take h, k, a, and b from the stack and produce an equation representing the ellipse and hyperbola

equations. The two halves are then drawn in parallel. The program HHYP and MELL will draw horizontal lines at points where $y^2(x) < 0$.

Key in the programs below.

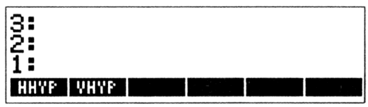
The first program's parameters specify a vertical hyperbola.

```
« -1 1 MCON [ENTER]
'VHYP [STO]
```



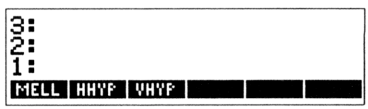
The second program's parameters specify a horizontal hyperbola.

```
« 1 -1 MCON [ENTER]
'HHYP [STO]
```



An ellipse has both squared terms positive, and, thus, parameters 1,1.

```
« 1 1 MCON [ENTER]
'MELL [STO]
```

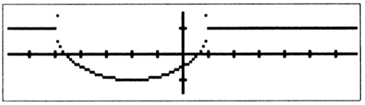


The last program implements the general form of the equation for an ellipse and hyperbola, with parameters input from programs VHYP, HHYP, and MELL.

```
« {X Y s1} PURGE
→ h k a b sx sy «
'sx×SQ((X-h)÷a)+
sy×SQ((Y-k)÷b)=1'
EVAL 'Y' ISOL DUP 1 's1'
STO EVAL SWAP 's1' SNEG
EVAL = RE STEQ CLLCD
DRAW 's1' PURGE »»
[ENTER]
'MCON [STO]
```

Now try the previous examples from this section. Purge any plot parameters that have been specified.

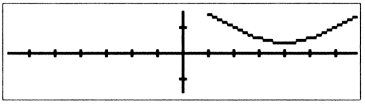
'PPAR [PURGE]
-2,1,3,2 [ENTER]
[USER] [MELL]



Note the difference in the centering of the ellipse from the previous program in the section.

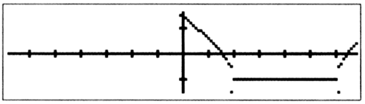
Now draw the vertical hyperbola.

[ATTN]
4,-1,2,'√2 [ENTER]
[VHYP]



The horizontal hyperbola has the same parameters as the preceding graph.

[ATTN]
4,-1,2,'√2 [ENTER]
[HHYP]



Exit from the plot display and purge the programs above if desired.

[ATTN] {'VHYP''HHYP''MELL''MCON' [PURGE]

Parametric Equations

Typical parametric equation problems include plotting the graph described by the equations and describing the path of a projectile. Examples of these two problems are included in this section.

Example: Make a table of values and plot the points for

$$x = 2 - 3\cos(t) \text{ and } y = 4 + 2\sin(t), 0 \le t \le 360.$$

First, set the angle mode to degrees.

[CLEAR]
[MODE] DEG

The following program creates a table of values and plots the points. The program assumes the expression for the x coordinate is stored in variable X and the expression for the y coordinate is stored in the variable Y. The program also assumes that the variable for time is capital T. The inputs to the program are the range (the low and high values) and the increment of T.

Key in the program and store it in the variable *PAREQ* (for "parametric equations").

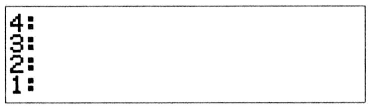
Program:	**Comments:**
`"LO,HI,INC?"`	Prompt message.
`HALT`	Program halts (Enter the 3 inputs).
`→ lo hi inc`	Inputs stored in respective variables.
`« DROP`	Drop the prompt message.
`lo hi FOR n`	Loop: do for n from *lo* to *hi*.
`n 'T' STO`	Store the current n in the variable T.
`T X EVAL Y EVAL`	Take T, X, and Y and put them in a vector.
`{3} →ARRY`	
`Σ+`	Add the vector to the ΣDAT matrix.
`inc STEP`	Increment n by the value *inc* and repeat loop.
`CLLCD`	Clear the display.

`2 3 COLΣ`	Denote which columns to plot.
`SCLΣ DRWΣ`	Scale the coordinates and draw the points.
`{T ΣPAR PPAR} PURGE`	Purge the variables created by the program.

[ENTER] 'PAREQ [STO]

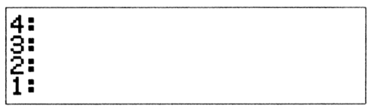
Key in the expression for the x coordinate and store it in the variable X.

'2-3×COS(T)' 'X [STO]



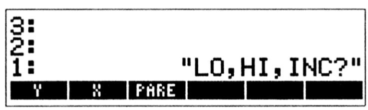
Key in the expression for the y coordinate and store it in the variable Y.

'4+2×SIN(T)' 'Y [STO]



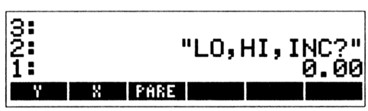
Display the User menu and execute the program. The prompt message is returned to level 1.

[USER] [PARE]



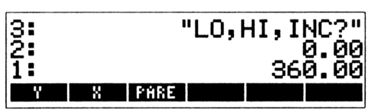
Enter the low value of T.

0 [ENTER]



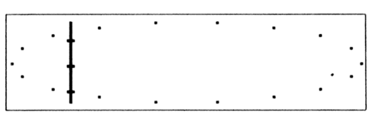
Enter the high value of T.

360 [ENTER]



Let the value for the increment be 20. Continue running the program.

20 [CONT]



The graph of the parametric equations is plotted. Press [ATTN] to exit from the plot display. The table of values is stored in ΣDAT. T is in column 1; X is in column 2; and Y is in column 3. You can see the first few entries to the matrix by pressing the soft key labeled ΣDAT. To see the individual entries, use the GETI command.

Purge the variables used in this example.

{'ΣDAT''Y''X''PAREQ' [PURGE]

Example: An archer stands 200 meters from a target. (The target is at the same height as the archer.) The archer shoots the arrow at an initial velocity of 170 miles per hour. At what angle should the archer aim the arrow in order to hit the target?

First, set the angle mode to degrees and the display to FIX 2.

[MODE] DEG
2 FIX

```
3:
2:
1:
 STO  FIX■  SCI  ENG  DEG■  RAD
```

The parametric equations for the path of a projectile moving in a plane at time t with the origin as the starting point are

$$x = v_i t \cos(\alpha) \text{ and } y = v_i t \sin(\alpha) - .5gt^2$$

where v_i is the initial velocity, α is the angle from the horizontal at which the projectile starts, and g is the force due to gravity. (All other forces are assumed negligible.)

When the arrow hits the target, the height y is zero and the range x is 200 meters. The initial velocity is $v_i = 170$ mph. Thus there are two equations in two unknowns (the angle and time). To find the angle, first isolate t in the first parametric equation. The result is an expression for t. Substitute the expression in the second parametric equation. Now you have one equation in one unknown. Use the Solver to find the angle.

Key in the first parametric equation and isolate T.

'X=V×T×COS(A)''T
[SOLV] ISOL

```
3:
2:
1:            'X/COS(A)/V'
 STEQ RCEQ SOLVR ISOL QUAD SHOW
```

Store the resulting expression for T in the variable T.

'T [STO]

```
3:
2:
1:
STEQ RCEQ SOLVR ISOL QUAD SHOW
```

Key in the second parametric equation with $g = 9.8m/s^2$. Substitute the expression for T in the equation by using the SHOW command so that all implicit references to X are made explicit. The result is the equation for the path in rectangular coordinates.

'Y=V×T×SIN(A)-.5×
9.8×T^2''X [SHOW]

```
1:  'Y=V*(X/COS(A)/V)*
    SIN(A)-0.50*9.80*(X/
    COS(A)/V)^2'
STEQ RCEQ SOLVR ISOL QUAD SHOW
```

Store the equation in the variable EQ and display the Solver menu.

[STEQ]
[SOLVR]

```
3:
2:
1:
Y V X A LEFT= RT=
```

Store the number 0 in the variable Y.

0 [Y]

```
Y: 0.00
2:
1:
Y V X A LEFT= RT=
```

Store the number 200 in the variable X.

200 [X]

```
X: 200.00
2:
1:
Y V X A LEFT= RT=
```

Since this problem uses SI units, convert mph to m/s. Enter the number 170.

170 [ENTER]

```
3:
2:
1:                170.00
Y V X A LEFT= RT=
```

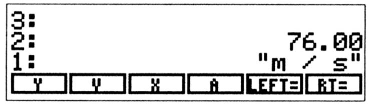
Key in the units "mph."

[LC] 'mph [ENTER]

```
3:
2:                170.00
1:                 'mph'
Y V X A LEFT= RT=
```

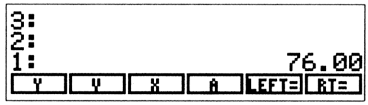
Convert 170 mph to m/s. Key in the units "m/s". Since m/s is not in the Units catalog, use double quotes around the units. CONVERT recognizes multiplicative combinations of the units listed in the catalog.

[LC] "m ÷ s" [ENTER]
[CONVERT]



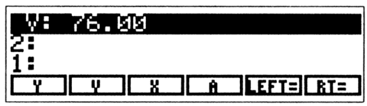
Drop "m/s".

[DROP]



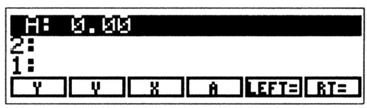
Store the velocity 76 m/s in the variable *V*.

[V]



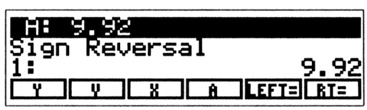
Let the number 0 be an initial estimate for the angle *A* .

0 [A]



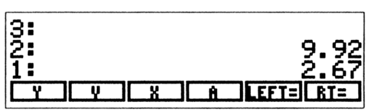
Find the angle.

[ ] [A]



Thus the archer must aim the arrow at an angle of 9.92 degrees to hit the target. How long will it take for the arrow to hit the target? To find the time, simply press [T] followed by [→NUM]. (Equivalently, T [ENTER] [EVAL] will recall the expression and then evaluate it with the current variable assignments).

T [→NUM]



Exit from the Solver and purge the following variables.

[SOLV] { 'A' 'V' 'X' 'Y' 'EQ' 'T' [PURGE]

More Step-by-Step Solutions for Your HP-28S or HP-28C Calculator

These additional books offer a variety of examples and keystroke procedures to help set up your calculations the way *you* need them.

Practical routines show you how to use the built-in menus to solve problems more effectively, while easy-to-follow instructions help you create personalized menus.

Calculus (00028-90102)

- Perform function operations: definition, composition, analysis, angles between lines, and angles between a line and a function.
- Solve problems of differential calculus: function minimization, computing tangent lines and implicit differentiation.
- Obtain symbolic and numerical solutions for integral calculus problems: polynomial integration, area between curves, arc length of a function, surface area, and volume of a solid of revolution.

Vectors and Matrices (00028-90105)

- Perform general matrix operations: summation, multiplication, determinant, inverse, transpose, conjugate, and minor rank.
- Solve a system of linear equations.
- Calculate several important vector operations.
- Learn methods for calculating eigenvalues and eigenvectors.
- Perform the method of least squares and Markov Chain calculations.

Probability and Statistics (00028-90104)

- Set up a statistical matrix.
- Calculate basic statistics: mean, standard deviation, variance, covariance, correlation coefficient, sums of products, normalization, delta percent on paired data, moments, skewness, and kurtosis.

- Perform regression techniques: curve fitting, multiple linear, and polynomial regression.
- Compute several test statistics.

And Specifically for Your HP-28S...

Mathematical Applications (00028-90111)

- Find the area and all sides and angles of any plane triangle.
- Perform synthetic division on polynomials of arbitrary order.
- Calculate all the roots of a first, second, third, and fourth degree polynomial, with real or complex coefficients.
- Solve first- and second-order differential equations.
- Convert the coordinates of two- or three-dimensional vectors between two coordinate systems, where one system is translated and/or rotated with respect to the other.
- Collect statistical data points, and fit curves to the data.

How to Order...

For the location and number of the U.S. dealer nearest you, call toll-free 1-800-752-0900. To order a book your dealer does not carry, call toll-free 1-800-538-8787 and refer to call code P270. Master Card, Visa, and American Express cards are welcome. For countries outside the U.S., contact your local Hewlett-Packard sales office.

Step-by-Step Solutions for Your HP-28S or HP-28C Calculator

Algebra and College Math contains a variety of examples and solutions to show how you can solve your technical problems more easily.

- **Functions and Equations**
  Rational Functions and Polynomial Long Division • Complex Numbers • Hyperbolic and Inverse Hyperbolic Functions • Function Evaluation • Graphs of Algebraic Functions • Quadratic Equations • Polynomial Equations • Simultaneous Linear Equations • Systems of Linear Equations
- **Infinite Sequences and Series**
- **Logarithms**
- **Trigonometry**
  Trigonometric Relations and Identities • Trigonometric Functions for One and Two Angles • Graphs of Trigonometric Functions • Inverse Trigonometric Functions • Trigonometric Equations
- **Geometry**
  Rectangular Coordinates • Polar Coordinates • The Straight Line • The Circle • The Parabola • The Ellipse and Hyperbola • Parametric Equations

HEWLETT PACKARD

Reorder Number
00028-90101

00028-90133 English
Printed in Canada 6/88

0 88698 00021 2