## HEWLETT-PACKARD

# HP.33E <br> STUDENT ENGINEERING Applications 



## For Continuous Memory Models


#### Abstract

Although this book refers specifically to the HP-33E or HP-38E, the programs and calculations contained herein apply equally well to the HP-33C or HP-38C. The user should note, however, that the display format and data register contents are retained by the calculator even though it has been turned off. It may be desirable to reset or clear these conditions before running programs or making calculations.




5955-5259

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HP-33E

## Student Engineering Applications

April 1978

## Introduction

This Student Engineering Applications book was written to help you get the most from your HP-33E calculator. The programs were chosen to provide useful calculations for many common problems encountered in engineering.

They will provide you with immediate capabilities in your everyday calculations and you will find them useful as guides to programming techniques for writing your own customized software.

You will find general information on how to key in and run programs under "A Word about Program Usage" in the Applications book you received with your calculator.
We hope that this Student Engineering book will be a valuable tool in your work and would appreciate your comments about it.

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## Electrical Engineering

## Ohm's Law and Reactance Chart

This program provides interchangeable solutions for many of the simple relationships involved in electrical engineering problems. Specifically one may solve for:

Frequency: given inductance and capacitance.
Capacitance: given frequency and inductance
Inductance: given frequency and capacitance
Capacitive reactance: given frequency and capacitance
Inductive reactance: given frequency and inductance
Current or voltage: given resistance and E or I
Resistance: given voltage and current
Power dissipation: given I and R or E and R
Current or voltage: given power and resistance

## Formulas Used:

$$
\begin{array}{ll}
\mathrm{f}=\frac{1}{2 \pi \sqrt{\mathrm{LC}}} & \text { where: } \\
\mathrm{f}=\text { resonant frequency in hertz } \\
\mathrm{X}_{\mathrm{c}}=\frac{1}{2 \pi \mathrm{fC}} & \mathrm{~L}=\text { inductance in henrys } \\
\mathrm{X}_{\mathrm{L}}=2 \pi \mathrm{fL} & \mathrm{C}=\text { capacitance in farads } \\
\mathrm{E}=\mathrm{IR} & \mathrm{X}_{\mathrm{c}}=\text { capacitive reactance in } \Omega \\
\mathrm{P}=\mathrm{I}^{2} \mathrm{R}=\mathrm{E}^{2} / \mathrm{R} & \mathrm{X}_{\mathrm{L}}=\text { inductive reactance in } \Omega \\
\mathrm{I}=\sqrt{\mathrm{P} / \mathrm{R}} & \mathrm{E}=\text { voltage } \\
\mathrm{E}=\sqrt{\mathrm{RP}} & \mathrm{I}=\text { current in amps } \\
& \mathrm{R}=\text { resistance in } \Omega \\
& \mathrm{P}=\text { power in watts }
\end{array}
$$

| KEY ENTRY | DISPLAY |  | KEY ENTRY |  | PLAY |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\square$ CLEAR PRGM | 00 |  | 区 | 19－ | 61 |
| 区 | 01－ | 61 | 2 | 20－ | 2 |
| 9 $\sqrt{x}$ | 02－ | 140 | 区 | 21－ | 61 |
| GSE 18 | 03－ | 1218 | 9 $1 / 1 / x$ | 22－ | 153 |
| GIO 00 | 04－ | 1300 | 9 RTN | 23－ | 1512 |
| $x \geq y$ | 05－ | 21 | 9 1／x | 24－ | 153 |
| GSE 18 | 06－ | 1218 | $\pm$ | 25－ | 61 |
| $9{ }^{(1)}$ | 07－ | 150 | GTO 00 | 26－ | 1300 |
| $x \geq y$ | 08－ | 21 | 9 $1 / 1 / x$ | 27－ | $15 \quad 3$ |
| $\dagger$ | 09－ | 71 | $x \geq y$ | 28－ | 21 |
| GTO 00 | 10－ | 1300 | 9 $x^{2}$ | 29－ | 150 |
| 区 | 11－ | 61 | $x \geq y$ | 30－ | 21 |
| GSE 18 | 12－ | 1218 | 区 | 31－ | 61 |
| GTO 00 | 13－ | 1300 | GTO 00 | 32－ | 1300 |
| 区 | 14－ | 61 | 9 ${ }^{1 / 1 / x}$ | 33－ | 153 |
| GSB 18 | 15－ | 1218 | $\triangle$ | 34－ | 61 |
| 9 $\square^{1 / x}$ | 16－ | 153 | $\square \sqrt{x}$ | 35－ | 140 |
| GTO 00 | 17－ | 1300 | GTO 00 | 36－ | 1300 |
| 9 0 | 18－ | 1573 |  |  |  |


| REGISTERS |  |  |  |
| :--- | :--- | :--- | :--- |
| $R_{0}$ | $R_{1}$ | $R_{2}$ | $R_{3}$ |
| $R_{4}$ | $R_{5}$ | $R_{6}$ | $R_{7}$ |


| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Key in the program |  |  |  |
|  | Reactance Chart |  |  |  |
| 2 | To calculate f: |  |  |  |
|  | Input L and C | L, henrys | ENTER4 |  |
|  |  | C, farads | GSB 01 | f, hertz |
| 3 | To calculate L or C : |  |  |  |
|  | Input f | f, hertz | ENTER4 |  |
|  | Input C or L | C or L, | GSB 05 | L or C, |
|  |  | farads, henrys |  | henrys, farads |
| 4 | To calculate $\mathrm{X}_{\mathrm{c}}$ : |  |  |  |
|  | Input f and C | f , hertz | ENTER4 |  |
|  |  | C, farads | GSB 11 | X |
| 5 | To calculate $\mathrm{X}_{\mathrm{L}}$ : |  |  |  |
|  | Input f and L | f, hertz | ENTER4 |  |
|  |  | L, henrys | GSB 14 | X |
|  | Ohm's Law |  |  |  |
| 6 | To calculate E : |  |  |  |
|  | Input I and R | I, amps | ENTER4 |  |
|  |  | $\mathrm{R}, \Omega$ | GSB 25 | E, volts |
| 7 | To calculate I or R: |  |  |  |
|  | Input E and R or I | E, volts | ENTER |  |
|  |  | R or I, | GSB 24 | I or R, |
|  |  | $\Omega$, amps |  | $\Omega$, amps |
|  | Power |  |  |  |
| 8 | To calculate $P$ : |  |  |  |
|  | a) Input E and R | E, volts | ENTER4 |  |
|  |  | $\mathrm{R}, \Omega$ | GSB 27 | P , watts |
|  | or |  |  |  |


|  | b) Input I and R | I, amps | ENTER4 |  |
| :---: | :--- | :---: | :---: | :---: |
|  |  | $\mathrm{R}, \Omega$ | GSB 28 | P, watts |
| 9 | To calculate I: |  |  |  |
|  | Input P and R | P, watts | ENTER4 |  |
|  |  | $\mathrm{R}, \Omega$ | GSB 33 | I, amps |
| 10 | To calculate E: |  |  |  |
|  | Input P and R | P, watts | ENTER 4 |  |
|  |  | $\mathrm{R}, \Omega$ | GSB 34 | E, volts |

## Example 1:

$\mathrm{C}=0.01 \mu \mathrm{~F}, \mathrm{~L}=160 \mu \mathrm{~h}$, calculate f :

## Keystrokes

## Display

## feng 3

160 EEX CHS 6

## ENIERA

. 01 EEX CHS 6
GSB 01
125.803
or $\left(125.8 \times 10^{3} \mathrm{~Hz}\right)$

Example 2:
$\mathrm{L}=2.5 \mathrm{H}, \mathrm{f}=60 \mathrm{~Hz}$, calculate C and $\mathrm{X}_{\mathrm{L}}$ :

## Keystrokes

60 ENIERA 2.5
60 ENTER 2.5
GSB 14

GSB 05 2.814-06 or $\left(2.814 \times 10^{-6} \mu \mathrm{~F}\right)$

## Display

942.500 or (942.5 $\Omega$ )

## Example 3:

$\mathrm{E}=345 \mathrm{~V}, \mathrm{R}=1.25 \mathrm{M} \Omega$. Calculate I and P :
Keystrokes
345 ENIER4
1.25 EEX 6
GSB 24
276.0-06
or ( $276 \times 10^{-6} \mathrm{amps}$ )

345 ENIER4
1.25 EEX 6

GSB 27
95.22-03
or $\left(95.22 \times 10^{-3}\right.$ watts)

## Resistors in Series or Parallel

This program calculates the total resistance of a group of resistors arranged in parallel or in series.

## Formulas Used:

Resistors in Series: $\mathrm{R}_{\mathrm{T}}=\mathrm{R}_{1}+\mathrm{R}_{2}+\mathrm{R}_{3} \ldots+\mathrm{R}_{\mathrm{n}}$

$$
\text { Resistors in Parallel: } \mathrm{R}_{\mathrm{T}}=\frac{1}{1 / \mathrm{R}_{1}+1 / \mathrm{R}_{2}+1 / \mathrm{R}_{3} \ldots+1 / \mathrm{R}_{\mathrm{n}}}
$$

- Note that this program can be used for summing capacitors in parallel or series instead of resistors. For series capacitors use instructions for parallel resistors. For parallel capacitors use instructions for series resistors.
- Several more advanced programs for circuit analysis may be found in: Anderson, L.H., '"Calculator- Aided Circuit Analysis,’’Ham Radio Magazine, pp. 38-46, October 1977. Although written for the HP-25 they will work equally well on the HP-33E.

| KEY ENTRY | DISPLAY |  |
| :---: | :---: | :---: |
| ( CLEAR PRGM | 00 |  |
| 0 | 01- | 0 |
| GTO 04 | 02- | 1304 |
| 1 | 03- | 1 |
| STOO 0 | 04- | 230 |
| R/S | 05- | 74 |
| RCL 0 | 06- | 240 |
| 9 $x=0$ | 07- | 1571 |
| GSB 20 | 08- | 1220 |
| R $\sim_{0}$ | 09- | 22 |
| STO日 +1 | 10-23 | 51 |
| GTo 05 | 11- | 1305 |


| KEY ENTRY | DISPLAY |  |  |
| :---: | :---: | :---: | :---: |
| ECL 1 | 12- | 24 | 1 |
| (RCL 0 | 13- | 24 | 0 |
| 9 $x=0$ | 14- | 15 |  |
| GSE 20 | 15- | 12 | 20 |
| CLX | 16- |  | 34 |
| STO 1 | 17- | 23 | 1 |
| not | 18- |  | 22 |
| GTO 00 | 19- | 13 | 00 |
| Rot | 20- |  | 22 |
| 9 11/x | 21- | 15 | 3 |
| ENIER | 22- |  | 31 |
| 9 RTN | 23- | 15 | 12 |


| REGISTERS |  |  |  |
| :--- | :--- | :--- | :--- |
| $\mathrm{R}_{0}$ Code: 0 or 1 | $\mathrm{R}_{1} \quad \mathrm{R}, \Sigma 1 / \mathrm{R}$ | $\mathrm{R}_{2}$ | $\mathrm{R}_{3}$ |
| $\mathrm{R}_{4}$ | $\mathrm{R}_{5}$ | $\mathrm{R}_{6}$ | $\mathrm{R}_{7}$ |


| STEP | INSTRUCTIONS | $\begin{gathered} \text { INPUT } \\ \text { DATA/UNITS } \end{gathered}$ | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Key in the program |  |  |  |
| 2 | Initialize |  | (feg PRGM |  |
| 3 | a) Set for parallel resistors |  | R/S | 0.0000 |
|  | or |  |  |  |
|  | b) Set for series resistors |  | GSE 03 | 1.0000 |
| 4 | Optional: Retrieve last $\mathrm{R}_{T}$ |  |  |  |
|  | for use in next calculation |  | R* | Previous $\mathrm{R}_{\mathrm{T}}$ |
| 5 | Input individual resistor values | $\mathrm{R}_{\mathrm{i}}$ | R/S |  |
|  | (Repeat until all resistors in |  |  |  |
|  | group have been input.) |  |  |  |
| 6 | Calculate total resistance of |  |  |  |
|  | the group |  | GSB 12 | $\mathrm{R}_{\mathrm{T}}, \Omega$ |
| 7 | Optional: Store $\mathrm{R}_{\mathrm{T}}$ for use in |  |  |  |
|  | next calculation | $\mathrm{R}_{\mathrm{T}}$ | STO 2 |  |
| 8 | Go to step 3a or 3b for next |  |  |  |
|  | group. |  |  |  |

## Example:



Determine the total circuit resistance from A to B.

## Keystrokes

f REG $f$ PRGM
Group 1: R/S
680 R/S 120 R/S

| GSB 12 |
| :--- |
| Group 2: GSB |
| Rt |
| R/S 330 R/S |

680 R/S
GSB 12
Group 3: R/S
R
R/S 220 R/S
GSB 12
STO 2
Group 4: R/S
680 R/S 470 R/S

## GSB 12

Total R

| GSB 03 |  | 1.0000 |
| :--- | :--- | :--- |
| $R 力$ | 277.9130 | Series mode <br> Retrieve $R_{4}$ |
| R/S RCL 2 R/S |  |  |
| GSB 12 |  | 461.5767 |

## Exponential Growth or Decay

Many growth or decay phenomena encountered in science and engineering obey an exponential law of the general form:

$$
\mathrm{X}_{\mathrm{t}}=\mathrm{X}_{\mathrm{ss}}-\left(\mathrm{X}_{\mathrm{ss}}-\mathrm{X}_{\mathrm{o}}\right) \mathrm{e}^{-\mathrm{t}} \tau
$$

where: $\quad X_{t}=$ Value at any time, $t$, (i.e., the instantaneous value)
$X_{\text {ss }}=$ Steady state value (i.e., at $t=\infty$ )
$X_{o}=$ Initial value (i.e., at $t=0$ )
$\mathrm{t}=$ Elapsed time (time after $\mathrm{t}=0$ )
$\tau=$ Exponential time-constant for specific phenomena
This program provides interchangeable solutions for any one of the four variables $\mathrm{X}_{\mathrm{t}}, \mathrm{X}_{\mathrm{ss}}, \mathrm{X}_{\mathrm{o}}$ or t provided three variables and $\tau$ are known.

| KEY ENTRY | DISPLAY |  |  | KEY ENTRY | DISPLAY |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| fCLEAR PRGM | 00 |  |  | (RCL 2 | 23- |  | 2 |
| RCL 0 | 01- | 24 |  | $\square$ | 24- |  | 41 |
| RCL 2 | 02- | 24 |  | 区 | 25- |  | 61 |
| $\square$ | 03- |  | 41 | RCL 2 | 26- |  | 2 |
| GSE 40 | 04- | 124 |  | $\pm$ | 27- |  | 51 |
| $\dagger$ | 05- |  | 71 | GTO 00 | 28- | 13 | 00 |
| RCL 2 | 06- | 24 |  | RCL 0 | 29- |  | 0 |
| $\pm$ | 07- |  | 51 | (RCL 2 | 30- |  | 2 |
| GTO 00 | 08- | 130 |  | $\square$ | 31- |  | 41 |
| GSE 40 | 09- |  |  | RCL 1 | 32- |  | 1 |
| ENTERA | 10- |  | 31 | RCL 2 | 33- | 24 | 2 |
| ENITER | 11- |  | 31 | $\square$ | 34- |  | 41 |
| RCL 1 | 12- | 24 | 1 | $\bigcirc$ | 35- |  | 71 |
| 区 | 13- |  | 61 | [1 LT | 36- |  | 1 |
| RCL 0 | 14- | 24 |  | RCL 4 | 37- |  | 4 |
| $\square$ | 15- |  | 41 | $\pm$ | 38- |  | 61 |
| $x_{i} ; y^{\text {a }}$ | 16- |  | 21 | GTO 00 | 39- | 13 | 00 |
| 1 | 17- |  | 1 | RCL 3 | 40- |  | 3 |
| $\square$ | 18- |  | 41 | RCL 4 | 41- |  | 4 |
| $\dagger$ | 19- |  | 71 | $\dagger$ | 42- |  | 71 |
| GTO 00 | 20- | 1300 |  | $9 e^{\text {a }}$ | 43- |  | 1 |
| GSE 40 | 21- | 124 |  | 9 RTN | 44- | 15 | 12 |
| RCL 1 |  | 24 |  |  |  |  |  |


| REGISTERS |  |  |  |
| :--- | :--- | :--- | :--- |
| $\mathrm{R}_{0} \mathrm{X}_{0}$ | $\mathrm{R}_{1} \mathrm{X}_{\mathrm{t}}$ | $\mathrm{R}_{2} \mathrm{X}_{\text {ss }}$ | $\mathrm{R}_{3} \mathrm{t}$ |
| $\mathrm{R}_{4} \tau$ | $\mathrm{R}_{5}$ | $\mathrm{R}_{6}$ | $\mathrm{R}_{7}$ |


| STEP | INSTRUCTIONS | INPUT <br> DATA/UNITS | KEYS | OUTPUT <br> DATA/UNITS |
| :---: | :--- | :---: | :---: | :---: |
| 1 | Key in the program |  |  |  |
| 2 | Store time constant | $\tau$ | STO 4 |  |
| 3 | Store variables: |  |  |  |
|  | Initial value | $\mathrm{X}_{\mathrm{o}}$ | STO 0 |  |
|  | Instantaneous value | $\mathrm{X}_{\mathrm{t}}$ | STO 1 |  |
|  | Steady state value | $\mathrm{X}_{\mathrm{ss}}$ | STO 2 |  |
|  | Elapsed time | t | STO 3 |  |
|  | (Store any 3 of the 4 |  |  |  |
| 4 | variables) | To calculate: |  |  |
|  | $\mathrm{X}_{0}$, initial value |  | GSB 21 | $\mathrm{X}_{0}$ |
|  | $\mathrm{X}_{\mathrm{t}}$, instantaneous value |  | GSB 01 | $\mathrm{X}_{\mathrm{t}}$ |
|  | $\mathrm{X}_{\text {ss }}$, steady state value |  | GSB 09 | $\mathrm{X}_{\mathrm{ss}}$ |
|  | t, elapsed time | GSB 29 | t |  |
| 5 | For a new case go to step 2. |  |  |  |

## Example 1:

Given a $5 \mu \mathrm{~F}$ capacitor in series with a 1 megohm resistor. 1.5 seconds after the circuit is completed 125 volts are measured across R . To what voltage was the capacitor originally charged?

## Note:

$\pi=$ the RC time-constant, and the voltage at $\mathrm{t}=\infty$ is zero.

## Keystrokes

## Display

5 EEX CHS 6
ENIERA EEX 6
$x$ STO 4
125 STO 1
0 sTO 2
1.5 STO 3

## Example 2:

A cobalt 60 source (half life $=5.26$ years) had an activity of 3.54 curies when purchased 8.5 years ago. What is its present activity?

## Note:

Activity at $\mathrm{t}=\infty$ will be zero, $\pi=$ half life $/ \ln 2$.

## Keystrokes

## Display

### 5.26 ENTER4

2 난
STO 4
$3 . 5 4 \longdiv { \text { STO } } 0$
0 STO 2
8.5 STO 3

GSB 01
1.1549
curies

## Heat and Thermal Engineering

## Black Body Thermal Radiation

Bodies with finite temperatures emit thermal radiation. The higher the absolute temperature, the more thermal radiation emitted. Bodies which emit the maximum possible amount of energy at every wavelength for a specified temperature are said to be black bodies. While black bodies do not actually exist in nature, many surfaces may be assumed to be black for engineering considerations.

Black body monochromatic emissive power


Figure 1 is a representation of black body thermal emission as a function of wavelength. Note that as temperature increases the area under the curves (total emissive power $\mathrm{E}_{\mathrm{b}(0-x)}$ ) increases. Also note that the wavelength of maximum emissive power $\lambda_{\max }$ shifts to the left as temperature increases.
This program can be used to calculate the wavelength of maximum emissive power for a given temperature, the temperature corresponding to a particular wavelength of maximum emissive power, the total emissive power for all wavelengths and the emissive power at a particular wavelength.

## Equations:

$$
\begin{gathered}
\lambda_{\max } \mathrm{T}=\mathrm{c}_{3} \\
\mathrm{E}_{\mathrm{b}(0-x)}=\sigma \mathrm{T}^{4} \\
\mathrm{~Eb} \lambda=\frac{2 \pi \mathrm{c}_{1}}{\lambda_{5}\left(\mathrm{e}_{2} / \lambda \mathrm{T}-1\right)}
\end{gathered}
$$

where: $\quad \lambda_{\max }$ is the wavelength of maximum emissivity in microns; T is the absolute temperature in ${ }^{\circ} \mathrm{R}$ or K ;
$\mathrm{E}_{\mathrm{b}(0-x)}$ is the total emissive power in $\mathrm{Btu} / \mathrm{hr}-\mathrm{ft}^{2}$ or watts $/ \mathrm{cm}^{2}$; Ebd is the emissive power at $\lambda$ in $\mathrm{Btu} / \mathrm{hr}^{-} \mathrm{ft}{ }^{2}-\mu \mathrm{m}$ or watts/ $\mathrm{cm}^{2}-\mu \mathrm{m}$;

$$
c_{1}=1.8887982 \times 10^{7} \mathrm{Btu}-\mu \mathrm{m}^{4} / \mathrm{hr}-\mathrm{ft}^{2}
$$

$$
=5.9544 \times 10^{3} \mathrm{~W} \mu \mathrm{~m}^{4} / \mathrm{cm}^{2}
$$

$$
\mathrm{c}_{2}=2.58984 \times 10^{4} \mu \mathrm{~m}-{ }^{\circ} \mathrm{R}=1.4388 \times 10^{4} \mu \mathrm{~m}-\mathrm{K}
$$

$$
\mathrm{c}_{3}=5.216 \times 10^{3} \mu \mathrm{~m}-{ }^{\circ} \mathrm{R}=2.8978 \times 10^{3} \mu \mathrm{~m}-\mathrm{K}
$$

$$
\sigma=1.713 \times 10^{-9} \mathrm{Btu} /{\mathrm{hr}-\mathrm{ft}^{2}{ }^{-}{ }^{\circ} \mathrm{R}^{4}=5.6693 \times 10^{-12} \mathrm{~W} / \mathrm{cm}^{2}-\mathrm{K}^{4} .4 .}^{4}
$$

$$
\sigma_{\text {exp }}=1.731 \times 10^{-9} \mathrm{Btu} /{\mathrm{hr}-\mathrm{ft}^{2}-{ }^{\circ} \mathrm{R}^{4}=5.729 \times 10^{-12} \mathrm{~W} / \mathrm{cm}^{2} .}_{2}
$$ $-\mathrm{K}^{4}$

- Sources differ on values for constants. This could yield small discrepancies between published tables and HP-33E outputs.


## Reference:

Robert Siegel and John R. Howell, Thermal Radiation Heat Transfer, Volume 1, National Aeronautics and Space Administration, 1968.

| KEY ENTRY | DISPLAY |  | KEY ENTRY | DISPLAY |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| fCLEAR PRGM | 00 |  | $\pm$ | 19- | 61 |  |
| STO 5 | 01- | 235 | (RCL 6 | 20- | 24 | 6 |
| GSB 34 | 02- | 1234 | 5 | 21- |  | 5 |
| STO 6 | 03- | 236 | fy | 22- |  | 3 |
| R/S | 04- | 74 | $\bigcirc$ | 23- |  | 71 |
| STO 6 | 05- | 236 | RCL 2 | 24- | 24 | 2 |
| GSE 34 | 06- | 1234 | RCL 6 | 25- |  | 6 |
| STO 5 | 07- | $23 \quad 5$ | $\dagger$ | 26- |  | 71 |
| R/S | 08- | 74 | RCL 5 | 27- |  | 5 |
| (RCL 5 | 09- | 245 | $\dagger$ | 28- |  | 71 |
| 4 | 10- | 4 | $9 e^{\text {ex }}$ | 29- |  | 1 |
|  | 11- | 143 | 1 | 30- |  | 1 |
| RCL 4 | 12- | 244 | $\square$ | 31- |  | 41 |
| 区 | 13- | 61 | $\square$ | 32- |  | 71 |
| P PaUSE | 14- | 1474 | GTO 00 | 33- |  | 00 |
| RCL 1 | 15- | 241 | RCL 3 | 34- |  | 3 |
| 2 | 16- | 2 | x $\quad 2 y$ | 35- |  | 21 |
| 区 | 17- | 61 | $\square$ | 36- |  | 71 |
| 9\% | 18- | 1573 | 9 RTN | 37- | 15 | 12 |


| REGISTERS |  |  |  |
| :--- | :--- | :--- | :--- |
| $\mathrm{R}_{0}$ | $\mathrm{R}_{1} \mathrm{c}_{1}$ | $\mathrm{R}_{2} \mathrm{c}_{2}$ | $\mathrm{R}_{3} \mathrm{c}_{3}$ |
| $\mathrm{R}_{4} \sigma$ | $\mathrm{R}_{5} \mathrm{~T}$ | $\mathrm{R}_{6} \lambda$ | $\mathrm{R}_{7}$ |


| STEP | INSTRUCTIONS | INPUT <br> DATA/UNITS | KEYS | OUTPUT <br> DATA/UNITS |
| :---: | :--- | :---: | :---: | :---: |
| 1 | Key in the program |  |  |  |
| 2 | Set display |  |  |  |
| 3 | Store constants |  |  |  |
|  | a) For SI $(\mathrm{W}, \mu \mathrm{m}, \mathrm{cm}$, | K) |  |  |


| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT <br> DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
|  | $c_{1}=5.9544 \times 10^{3}$ | $\mathrm{C}_{1}$ | STO 1 |  |
|  | $c_{2}=1.4388 \times 10^{4}$ | $\mathrm{C}_{2}$ | STO 2 |  |
|  | $\mathrm{c}_{3}=2.8978 \times 10^{3}$ | $\mathrm{C}_{3}$ | STO 3 |  |
|  | $\sigma=5.6693 \times 10^{-12}$ | $\sigma$ | STO 4 |  |
|  | or |  |  |  |
|  | $\sigma_{\text {exp }}=5.729 \times 10^{-12}$ | $\sigma_{\text {exp }}$ | STO 4 |  |
|  | b) For English (Btu, $\mu \mathrm{m}$, hr, |  |  |  |
|  | $\mathrm{ft},{ }^{\circ} \mathrm{R}$ ) |  |  |  |
|  | $\mathrm{c}_{1}=1.8887982 \times 10^{7}$ |  | STO 1 |  |
|  | $\mathrm{c}_{2}=2.58984 \times 10^{4}$ |  | STO 2 |  |
|  | $\mathrm{c}_{3}=5.216 \times 10^{3}$ |  | STO 3 |  |
|  | $\sigma=1.713 \times 10^{-9}$ |  | STO 4 |  |
|  | or |  |  |  |
|  | $\sigma_{\text {exp }}=1.731 \times 10^{-9}$ |  | STO 4 |  |
| 4 | To calculate $\lambda_{\text {max }}$ | T | GSB 01 | $\lambda_{\max }, \mu \mathrm{m}$ |
| 5 | To calculate temp. at which |  |  |  |
|  | $\lambda$ is maximum | $\lambda$ | GSB 05 | T, ${ }^{\circ}$ |
| 6 | To calculate black |  |  |  |
|  | body total emissive |  |  |  |
|  | power and total |  |  |  |
|  | emissive power at any $\lambda$. |  |  |  |
|  | a) For $\lambda_{\text {max }}$, do step 3 or |  |  |  |
|  | step 4 then |  | GSB 09 | $\left(\mathrm{E}_{\mathrm{b}(0-\infty)}\right)^{*}$ |
|  |  |  |  | $\mathrm{E}_{\mathrm{b}} \lambda_{\text {max }}$ |
|  | b) For other $\lambda$ | $\lambda$ | STO 6 GSB 09 | $\left(\mathrm{E}_{\mathrm{b}(0-\infty)}\right)^{*}$ |
|  |  |  |  | Eb $\lambda$ |
|  | * ( $E_{\text {b }(0-\infty)}$ ) displayed |  |  |  |
|  | by pause only. |  |  |  |

## Example:

If the human eye was designed to work most efficiently in sunlight and the visible spectrum peaks at about $.550 \mu \mathrm{~m}$, what is the sun's temperature in K ? Assume that the sun is a black body. What is the total emissive power and the emissive power at $\lambda_{\max }$ ? What is the emissive power at $\lambda=0.400 \mu \mathrm{~m}$ (ultraviolet limit) and $0.700 \mu \mathrm{~m}$ (infrared limit).

Keystrokes
Display

| - 5 SCI |  |  |
| :---: | :---: | :---: |
| 5.9544 EEX 3 |  |  |
| STO) 1 |  |  |
| 1.4388 EEX 4 |  |  |
| STO 2 |  |  |
| 2.8978 EEX 3 |  |  |
| STO 3 |  |  |
| 5.6693 EEX CHS 12 |  |  |
| STO 4 |  |  |
| . 55 GSB 05 | 5.268703 | $\left(5.2698 \times 10^{3} \mathrm{~K}\right)$ |
| GSB 09 | 4.368703 | $\left(4.3687 \times 10^{3}\right.$ watts/ $\mathrm{cm}^{2}$, |
|  |  | $\left.\mathrm{E}_{\mathrm{b}(0-\infty)}\right)$ <br> (Pause only) |
|  | 5.222903 | $\begin{aligned} & \left(5.2229 \times 10^{3} . \text { watts } / \mathrm{cm}^{2}\right. \\ & -\mu \mathrm{m}, \mathrm{E} \backslash \mu \mu \alpha \chi) \end{aligned}$ |
| . $4 \widehat{\text { STO } 6 \text { GSB } 09}$ | 4.368703 | (Ignore) |
|  | 3.964903 | (3.9649 $\times 10^{3}$ watts $/ \mathrm{cm}^{2}$ |
|  |  | $-\mu \mathrm{m}, \mathrm{Eb} \mathrm{\lambda)}$ |
| . 7 STO 6 GSB 09 | 4.368703 | (Ignore) |
|  | 4.593403 | (4.5934 $\times 10^{3}$ watts $/ \mathrm{cm}^{2}$ |
|  |  | $-\mu \mathrm{m}, \mathrm{E}$ ¢) |

## Ideal Gas Equation of State

Many gases obey the ideal gas laws quite closely at reasonable temperatures and pressures. This program calculates any one of the four variables when data for the other three and the universal gas constant are entered. Likewise, the value of the universal gas constant can be determined by entering data for the four variables.

## Equation:

$$
\mathrm{PV}=\mathrm{n} \mathrm{RT}
$$

where: $\quad \mathrm{P}$ is the absolute pressure
V is the volume
n is the number of moles present
R is the universal gas constant
T is the absolute temperature

## TABLE 1

Values of the Universal Gas Constant

| Value of R | Units of R | Units of P | Units of V | Units of T |
| :---: | :---: | :---: | :---: | :---: |
| 8.314 | $\mathrm{N}-\mathrm{m} / \mathrm{g}$ mole- K | $\mathrm{N} / \mathrm{m}^{2}$ | $\mathrm{m}^{3} / \mathrm{g}$ mole | K |
| 83.14 | $\mathrm{cm}^{3}$-bar/g mole- K | bar | $\mathrm{cm}^{3} / \mathrm{g}$ mole | K |
| 82.05 | $\mathrm{cm}^{3}$-atm/g mole- K | atm | $\mathrm{cm}^{3} / \mathrm{g}$ mole | K |
| 0.08205 | $\ell$-atm/g mole- K | atm | $\ell / \mathrm{g}$ mole | K |
| 0.7302 | atm-ft ${ }^{3} / \mathrm{lb}$ mole- ${ }^{\circ} \mathrm{R}$ | atm | $\mathrm{ft}^{3} / \mathrm{lb}$ mole | ${ }^{\circ} \mathrm{R}$ |
| 10.73 | psi-ft ${ }^{3} / \mathrm{lb}$ mole- ${ }^{\circ} \mathrm{R}$ | psi | $\mathrm{ft}^{3} / \mathrm{lb}$ mole | ${ }^{\circ} \mathrm{R}$ |
| 1545 | psf-ft ${ }^{3} / \mathrm{lb}$ mole- ${ }^{\circ} \mathrm{R}$ | psf | $\mathrm{ft}^{3} / \mathrm{lb}$ mole | ${ }^{\circ} \mathrm{R}$ |

## Remarks:

- At low temperatures or high pressures the ideal gas law does not represent the behavior of real gases.
- The value of R used must be compatible with the units of $\mathrm{P}, \mathrm{V}, \mathrm{T}$.
- In running the program be sure to enter zero for the variable to be calculated.

| KEY ENTRY | DISPLAY |  | KEY ENTRY | DISPLAY |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| fCLEAR PRGM | 00 |  | RCL 4 | 17- | 244 |
| GSB 28 | 01- | 1228 | $x$ | 18- | 61 |
| STO 2 | 02- | 232 | RCL 5 | 19- | 245 |
| R+ | 03- | 22 | $x$ | 20- | 61 |
| GSB 28 | 04- | 1228 | RCL 1 | 21- | 241 |
| STO 1 | 05- | 231 | RCL 2 | 22- | 242 |
| R | 06- | 22 | $x$ | 23- | 61 |
| R/S | 07- | 74 | $\div$ | 24- | 71 |
| GSB 28 | 08- | 1228 | R/S | 25- | 74 |
| STO 5 | 09- | $23 \quad 5$ | 9 $1 / x$ | 26- | $15 \quad 3$ |
| R* | 10- | 22 | GTO 00 | 27- | 1300 |
| GSB 28 | 11- | 1228 | 9 $\quad x \neq 0$ | 28- | 1561 |
| STO 4 | 12- | 234 | 9 RTN | 29- | 1512 |
| R* | 13- | 22 | R* | 30- | 22 |
| GSB 28 | 14- | 1228 | 1 | 31- | 1 |
| STO 3 | 15- | 233 | 9 RTN | 32- | 1512 |
| RCL 3 | 16- | 243 |  |  |  |


| REGISTERS |  |  |  |
| :--- | :--- | :--- | :--- |
| $R_{0}$ | $R_{1} P$ | $R_{2} V$ | $R_{3} n$ |
| $R_{4} R$ | $R_{5} T$ | $R_{6}$ | $R_{7}$ |


| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Key in the program |  |  |  |
| 2 | Set display and initialize |  | (f) FIX 2 PRGM |  |
| 3 | Input variables* |  |  |  |
|  | Pressure | P | ENTER ${ }^{\text {a }}$ |  |
|  | Volume | V | R/S |  |
|  | Number of moles | n | ENTER4 |  |
|  | Universal gas constant** | R | ENTER4 |  |
|  | Absolute temp. | T |  |  |
| 4 | a) To calculate P or V |  | R/S | P or V |
|  | or |  |  |  |
|  | b) To calculate $n, R$ or $T$ |  | R/S R/S | $\mathrm{n}, \mathrm{R}$ or T |
| 5 | To change conditions: |  |  |  |
|  | a) Go to step 3, or, |  |  |  |
|  | b) Store new variable in |  |  |  |
|  | in proper register (see |  |  |  |
|  | register contents) and |  |  |  |
|  | store 1 in register of |  |  |  |
|  | variable to be calculated, |  |  |  |
|  | then, |  |  |  |
|  | for $P$ or V |  | GSB 16 | P or V |
|  | or, |  |  |  |
|  | for $\mathrm{n}, \mathrm{R}$ or T |  | GSB $16 \mathrm{R} / \mathrm{S}$ | $\mathrm{n}, \mathrm{R}$ or T |
|  | * Note: variables must be in- |  |  |  |
|  | put in order shown. Input |  |  |  |
|  | zero for variable to be calcul | ated |  |  |
|  | ** Be sure R is in units com- |  |  |  |
|  | patible with units of |  |  |  |
|  | variables |  |  |  |

## Example:

0.63 moles of air are enclosed in $25000 \mathrm{~cm}^{3}$ at 1200 K . What is the pressure in bars? In atmospheres? Assume an ideal gas.
( $\mathrm{R}=83.14 \mathrm{~cm}^{3}$-bar $/ \mathrm{g}$ mole-K or $82.05 \mathrm{~cm}^{3}$ - $\mathrm{atm} / \mathrm{g}$ mole-K)

## Keystrokes $\pm$ FIX $2 \oplus$ PRGM

0 ENIERA
25000 R/S
.63 ENTER
83.14 ENTERA

| 1200 R/S | 2.51 | bars |
| :--- | :--- | :--- |
| 82.05 STO 4 |  |  |
| GSB 16 | 2.48 | atm. |

## Mechanical Engineering

## Equations of Motion

This program provides solutions for many problems involving motion of an object given a constant acceleration and initial velocity. Velocity, distance traveled and acceleration may be found if time is known. Given the distance traveled, velocity or time may be calculated or, given velocity, time or acceleration may be calculated.

## Equations:

$$
\begin{gathered}
V=V_{0}+a t \\
V=\left(V_{0}^{2}+2 a S\right)^{12} \\
S=V_{0}+1 / 2 a t^{2} \\
t=\frac{V-V_{0}}{a} \text { or } a=\frac{V-V_{0}}{t}
\end{gathered}
$$

where: $\quad \mathrm{V}=$ velocity

$$
\begin{aligned}
& V_{0}=\text { initial velocity } \\
& a=\text { acceleration } \\
& S=\text { distance } \\
& t=\text { time }
\end{aligned}
$$

These same equations also hold for circular motion where:
$\omega[$ angular velocity (radians $/ \mathrm{sec}$ ) $]$ replaces V $\omega_{0}$ replaces $\mathrm{V}_{0}$ $\alpha$ replaces a
$\theta$ [angular displacement (radians)] replaces S

- Generally accepted values for the frequently used gravitational constant are $980.665 \mathrm{~cm} / \mathrm{sec}^{2}$ or $32.17398 \mathrm{ft} / \mathrm{sec}^{2}$.

| KEY ENTRY | DISPLAY |  |  | KEY E | DISPLAY |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| fCLEAR PRGM | 00 |  |  | RCL 3 | 23- | 24 | 3 |
| STO 2 | 01- | 23 |  | $x$ | 24- |  | 61 |
| R ${ }^{\text {d }}$ | 02- |  | 22 | R/S | 25- |  | 74 |
| STO 0 | 03- | 23 | 0 | $x \geq y$ | 26- |  | 21 |
| R/S | 04- |  | 74 | RCL 0 | 27- | 24 | 0 |
| RCL 2 | 05- | 24 | 2 | $\square$ | 28- |  | 41 |
| $x$ | 06- |  | 61 | $x \geq y$ | 29- |  | 21 |
| RCL 0 | 07- | 24 | 0 | $\div$ | 30- |  | 71 |
| $\pm$ | 08- |  | 51 | R/S | 31- |  | 74 |
| (9TN | 09- | 15 | 12 | GSB 10 | 32- | 12 |  |
| 2 | 10- |  | 2 | STO 7 | 33- | 23 | 7 |
| $x$ | 11- |  | 61 | RCL 0 | 34- | 24 | 0 |
| RCL 2 | 12- | 24 | 2 | CHS | 35- |  | 32 |
| $x$ | 13- |  | 61 | $\pm$ | 36- |  | 51 |
| RCL 0 | 14- | 24 | 0 | RCL 2 | 37- | 24 | 2 |
| $9 x^{2}$ | 15- | 15 | 0 | $\div$ | 38- |  | 71 |
| $\pm$ | 16- |  | 51 | 9 $x<0$ | 39- | 15 | 41 |
| $\square \sqrt{x}$ | 17- | 14 | 0 | GTO 42 | 40- | 13 | 42 |
| 9 RTN | 18- | 15 | 12 | R/S | 41- |  | 74 |
| STO 3 | 19- | 23 | 3 | RCL 7 | 42- | 24 | 7 |
| 2 | 20- |  | 2 | CHS | 43- |  | 32 |
| $\div$ | 21- |  | 71 | GTO 34 | 44- | 13 | 34 |
| GSB 05 | 22- | 12 | 05 |  |  |  |  |


| REGISTERS |  |  |  |
| :--- | :--- | :--- | :--- |
| $R_{0} V_{0}$ | $R_{1}$ | $R_{2} a$ | $R_{3} t$ |
| $R_{4}$ | $R_{5}$ | $R_{6}$ | $R_{7}$ Used |


| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Key in the program |  |  |  |
| 2 | Input initial velocity | V | ENTER4 |  |
|  | and acceleration | a | GSB 01 |  |
|  | To solve for velocity: |  |  |  |
| 3 | Input time | t | GSB 05 | V |
|  | or |  |  |  |
|  | Input distance | S | GSB 10 | V |
|  | To solve for distance: |  |  |  |
| 4 | Input time | t | GSB 19 | S |
|  | To solve for time: |  |  |  |
| 5a | Input velocity | V | ENTER4 |  |
|  | and acceleration | a | GSB 26 | $t$ |
|  | or |  | RCL 2 GSB 26 | t |
|  | or |  |  |  |
| 5 b | Input distance | S | GSB 32 | t |
|  | To solve for acceleration: |  |  |  |
| 6 | Input velocity | V | ENTER ${ }^{\text {a }}$ |  |
|  | and time | t | GSB 26 | a |

## Example:

A stone is thrown from a 100 meter high bridge with an initial velocity of 15 meters per second. What will be the velocity when it strikes the river below? How long will it take to hit the water? (The acceleration of gravity is $9.80665 \mathrm{~m} / \mathrm{sec}^{2}$ )

## Keystrokes

Display
15 ENTERA
9.80665 GSB 01
100 GSB 10
46.7582
V, meters/sec
100 GSB 32
3.2384
t , seconds

## Natural Frequency of Oscillators

This program solves for the natural frequency, the spring constant or the mass (alternatively, weight) of a simple oscillator. Examples of this are a spring or torsional pendulum obeying Hooke's law or a pendulum undergoing small oscillations.

## Equations:

$$
\begin{gathered}
\mathrm{f}=\frac{1}{2 \pi} \sqrt{\frac{\mathrm{k}}{\mathrm{~m}}} \text { for spring } \\
\mathrm{f}=\frac{1}{2 \pi} \sqrt{\frac{\mathrm{k}_{\mathrm{T}}}{\mathrm{~J}}} \text { for torsional pendulum }
\end{gathered}
$$

$\mathrm{f}=\frac{1}{2 \pi} \sqrt{\frac{1}{\ell / \mathrm{g}}}$ for pendulum in small oscillation (i.e., $\theta \simeq \sin \theta$ )
$\mathrm{m}=\frac{\mathrm{W}}{\mathrm{g}}$
where: $\mathrm{m}=$ mass, $\mathrm{W}=$ weight

$$
\begin{aligned}
& \mathrm{g}=\text { acceleration due to gravity } \\
& \mathrm{f}=\text { natural frequency of the oscillator (hertz) } \\
& \mathrm{k}=\text { spring constant }(\mathrm{wt} / \ell) \\
& \mathrm{k}_{\mathrm{T}}=\text { torsional constant }(\mathrm{wt} \ell / \text { radian }) \\
& \mathrm{J}=\text { mass moment of inertia }\left(\mathrm{wt} \ell \sec ^{2}\right) \\
& \ell=\text { length }
\end{aligned}
$$

- Note that for a simple pendulum, length is equivalent to weight.

[^0]This program is based on an HP-65 Users' Library program by Lane R. Pendleton.

| KEY ENTRY | DISPLAY |  |  | KEY ENTRY | DISPLAY |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\pm$ CLEAR PRGM | 00 |  |  | $\dagger$ | 20－ | 71 |  |
| STOO 3 | 01－ | 23 | 3 | 9 $\sqrt{x}$ | 21－ | 14 | 0 |
| ［80 |  |  | 22 | 9 0 | 22－ | 15 | 73 |
| STO 2 | 03－ | 23 | 2 | 2 | 23－ |  | 2 |
| （6．） | 04－ |  | 22 | 区 | 24－ |  | 61 |
| STO 1 |  | 23 | 1 | $\square$ | 25－ |  | 71 |
| $9 \mathrm{x}=0$ |  |  | 71 | GTO 00 | 26－ | 13 | 00 |
| GTO 18 | 07－ | 13 | 18 | RCL 1 | 27－ | 24 | 1 |
| RCL 2 |  | 24 | 2 | 9 $\pi$ | 28－ | 15 | 73 |
| 9 $x=0$ | 09－ | 15 | 71 | $\pm$ | 29－ |  | 61 |
| GTO 14 | 10－ | 13 | 14 | 2 | 30－ |  | 2 |
| GSB 27 | 11－ | 12 | 27 | 区 | 31－ |  | 61 |
| $\dagger$ | 12－ |  | 71 | 9 $x^{2}$ | 32－ | 15 | 0 |
| GTO 00 | 13－ | 130 | 00 | 9 RTN | 33－ | 15 | 12 |
| GSE 27 | 14－ |  | 27 | RCL 0 | 34－ | 24 | 0 |
| RCL 3 | 15－ | 24 | 3 | $\dagger$ | 35－ |  | 71 |
| 区 | 16－ |  | 61 | GTO 00 | 36－ | 13 | 00 |
| GTO 00 | 17－ | 130 | 00 | RCL 0 | 37－ | 24 | 0 |
| RCL 2 |  | 24 | 2 | $\pm$ | 38－ |  | 61 |
| RCL 3 |  | 24 | 3 | GTO 00 | 39－ | 13 | 00 |


| REGISTERS |  |  |  |
| :--- | :--- | :--- | :--- |
| $R_{0} g$ | $R_{1} f$ | $R_{2} k$ | $R_{3} m$ |
| $R_{4}$ | $R_{5}$ | $R_{6}$ | $R_{7}$ |


| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Key in the program |  |  |  |
| 2 | Store the gravitational |  |  |  |
|  | acceleration constant: |  |  |  |
|  | a) If SI | 980.665, | STO 0 | g, cm/sec ${ }^{2}$ |
|  |  | (cm/ $\mathrm{sec}^{2}$ ) |  |  |
|  | b) If English | 386.088, | STO 0 | $\mathrm{g}, \mathrm{in} / \mathrm{sec}^{2}$ |
|  |  | (in/sec ${ }^{2}$ ) |  |  |
| 3 | Input data in order shown |  |  |  |
|  | (use zero for unknown |  |  |  |
|  | variable): |  |  |  |
|  | Frequency | $\mathrm{f},\left(\sec ^{-1}\right)$ | ENTER4 |  |
|  | Spring constant | $k(w t / l)$ | ENTER4 |  |
|  | Mass | m | 9 PRGM |  |
|  | or Weight | W | GSB 34 |  |
| 4 | Calculate unknown variable: |  | R/S |  |
|  | Frequency |  |  | f, Hz. |
|  | or spring const. |  |  | k, (wt/l) |
|  | or mass |  |  | m |
| 5 | To convert mass to weight | m | GSB 37 | W |
| 6 | To convert weight to mass | W | GSB 34 | m |

## Example 1:

A weight of 10 lbs . is attached to a spring whose constant is $100 \mathrm{lbs} / \mathrm{in}$. Find the frequency of the system.

## Keystrokes

## Display

386.088 STO 0

0 ENIER4
100 ENTERA 10
GSB 34
R/S $9.8892 \quad \mathrm{~Hz}$

## Example 2:

A torsional pendulum has a natural frequency of 200 Hz . Its mass moment of inertia is 400 kg in. $\mathrm{sec}^{2}$. Find the torsional constant.

## Keystrokes

## Display

980.665 STO 0

200 ENIERA
0 ENTERA
400 PRGM
R/S $9 \mathbf{S C l} 4$
6.316508
$\left(6.3165 \times 10^{8}\right)$
kg in/radian

## Example 3:

Find the length of a pendulum which has a 1 Hz natural frequency.

## Note:

The length of the pendulum is equivalent to its weight.)

## Keystrokes

Display
386.088 STO 0

1 ENTER4
1 ENTER4 0 R/S
GSB 37
9.7797
inches

## Kinetic Energy

This program calculates an interchangeable solution among the variables weight (or mass), velocity, and kinetic energy, for an object moving at constant velocity. The program operates in either English or metric units. For metric units, any consistent set of units may be used; the quantity mass must be used. For English units, the energy must be in foot-pounds, the velocity in feet per second, and the quantity weight in pounds.
where: K.E. = kinetic energy

$$
\begin{aligned}
& \mathrm{W}=\text { weight }(\mathrm{lb}) \\
& \mathrm{m}=\text { mass }(\mathrm{kg}, \mathrm{~g}) \\
& \mathrm{v}=\text { velocity } \\
& \mathrm{g}=\text { acceleration due to gravity }=980.665 \mathrm{~cm} / \mathrm{sec}^{2} \\
& \\
& \\
& \text { or } 32.17398 \mathrm{ft} / \mathrm{sec}^{2}
\end{aligned}
$$

## Equations:

$$
\begin{gathered}
\text { English } \\
\text { K.E. }=\frac{1}{2} \frac{\mathrm{~W}}{\mathrm{~g}} \mathrm{v}^{2} \quad \text { K.E. }=\frac{1}{2} \mathrm{mv}^{2} \\
1 \mathrm{ft}-\mathrm{lb}=1.98 \times 10^{6} \mathrm{hp}
\end{gathered}
$$

| KEY ENTRY | DISPLAY |  | KEY ENTRY | DISPLAY |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| OCLEAR PRGM | 00 |  | 1 | 19- |  | 1 |
| 2 | 01- | 2 | 9 | 20- |  | 9 |
| STO 0 | 02- | 230 | 8 | 21- |  | 8 |
| GTO 13 | 03- | 1313 | EEX | 22- |  | 33 |
| 6 | 04- | 6 | 4 | 23- |  | 4 |
| 4 | 05- | 4 | $\dagger$ | 24- |  | 71 |
| . | 06- | 73 | GTO 13 | 25- | 13 | 13 |
| 3 | 07- | 3 | (9) $x^{2}$ | 26- | 15 | 0 |
| 4 | 08- | 4 | $x<y$ | 27- |  | 21 |
| 7 | 09- | 7 | RCL 0 | 28- | 24 | 0 |
| 9 | 10- | 9 | 区 | 29- |  | 61 |
| 6 | 11- | 6 | $x \geq y$ | 30- |  | 21 |
| STO 0 | 12- | 230 | $\square$ | 31- |  | 71 |
| R/S | 13- | 74 | GTO 13 | 32- | 13 | 13 |
| (9) $x^{2}$ | 14- | 150 | $\dagger$ | 33- |  | 71 |
| $\triangle$ | 15- | 61 | RCL 0 | 34- | 24 | 0 |
| RCL 0 | 16- | 240 | $\pm$ | 35- |  | 61 |
| $\dagger$ | 17- | 71 | 9 $\sqrt{x}$ | 36- | 14 | 0 |
| GTO 13 | 18- | 1313 | GTO 13 | 37- | 13 | 13 |


| REGISTERS |  |  |  |
| :--- | :--- | :--- | :--- |
| $\mathrm{R}_{0}$ Used | $\mathrm{R}_{1}$ | $\mathrm{R}_{2}$ | $\mathrm{R}_{3}$ |
| $\mathrm{R}_{4}$ | $\mathrm{R}_{5}$ | $\mathrm{R}_{6}$ | $\mathrm{R}_{7}$ |


| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Key in the program |  |  |  |
| 2 | Initialize: |  |  |  |
|  | for metric (SI) |  | GSB 01 | 2.0000 |
|  | for English |  | GSB 04 | 64.3480 |
|  | To calculate Kinetic Energy: |  |  |  |
| 3 | Input weight (mass) | W (m) | ENTER 4 |  |
|  | and velocity | V | R/S | K.E. |
|  | To calculate Weight (mass): |  |  |  |
| 4 | Input kinetic energy | K.E. | ENTER 4 |  |
|  | and velocity | V | GSB 26 | W (m) |
|  | To calculate Velocity: |  |  |  |
| 5 | Input kinetic energy | K.E. | ENTER |  |
|  | and weight (mass) | W (m) | GSB 33 | V |
|  | Optional: |  |  |  |
|  | Convert K.E. (ft-lb) to K.E. |  |  |  |
|  | (hp) | K.E. (ft-lb) | GSB 19 | K.E. (hp) |
| 7 | For a new case go to step 2. |  |  |  |

## Example 1:

The slider of a slider-crank mechanism is used to punch holes in a slab of metal. It is found that the work required to punch a hole is 775 ft -lb. If the slider weighs 5.25 lbs ., how fast must it be moving when it strikes the metal? What is the required work in horsepower?

## Keystrokes

Display

## GSB 04

775 ENIER4
5.25 GSB 33
97.4627
$\mathrm{ft} / \mathrm{sec}$

775 ENTERA
GSB 19
3.9141-04
$3.9141 \times 10^{-4} \mathrm{hp}$

## Example 2:

An object weighing 4.8 kg is moving with constant velocity of 3.5 $\mathrm{m} / \mathrm{sec}$. Find its kinetic energy.

## Keystrokes Display

ff FIX 4 GSB 01
4.8 ENTERA 3.5 R/S 29.4000 joules

## RPM/Torque/Power

This program provides an interchangeable solution for RPM, torque, and power in both Système International (metric) and English units.

|  | SI | English |
| :---: | :---: | :---: |
| RPM | RPM | RPM |
| Torque | $\mathrm{nt}-\mathrm{m}$ | $\mathrm{ft}-\mathrm{lb}$ |
| Power | watts | hp |

## Equations:

$$
\begin{aligned}
& \text { RPM } \times \text { Torque }=\text { Power } \\
& 1 \mathrm{hp}=745.7 \text { watts } \\
& 1 \mathrm{ft}-\mathrm{lb}=1.356 \text { joules } \\
& 1 \mathrm{hp}=550 \frac{\mathrm{ft}-\mathrm{lb}}{\mathrm{sec}}
\end{aligned}
$$

$$
1 \mathrm{RPM}=\pi / 30 \text { radians } / \mathrm{sec}
$$

| KEY ENTRY | DISPLAY |  | KEY ENTRY | DISPLAY |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| fCLEAR PRGM | 00 |  | $x \geq y$ | 21- |  | 21 |
| 3 | 01- | 3 | 9. $1 / x$ | 22- | 15 | 3 |
| 0 | 02- | 0 | STO 5 | 23- | 23 | 5 |
| $9 \pi$ | 03- | 1573 | $\div$ | 24- |  | 71 |
| $\div$ | 04- | 71 | $x$ | 25- |  | 61 |
| STO 7 | 05- | 237 | STO 7 | 26- | 23 | 7 |
| 7 | 06- | 7 | R/S | 27- |  | 74 |
| 4 | 07- | 4 | $\div$ | 28- |  | 71 |
| 5 | 08- | 5 | RCL 7 | 29- | 24 | 7 |
| - | 09- | 73 | $x$ | 30- |  | 61 |
| 7 | 10- | 7 | R/S | 31- |  | 74 |
| STO 5 | 11- | 235 | RCL 6 | 32- | 24 | 6 |
| 1 | 12- | 1 | $\div$ | 33- |  | 71 |
| - | 13- | 73 | GTO 27 | 34- | 132 | 27 |
| 3 | 14- | 3 | $\triangle$ | 35- |  | 61 |
| 5 | 15- | 5 | RCL 7 | 36- | 24 | 7 |
| 6 | 16- | 6 | $\div$ | 37- |  | 71 |
| STO 6 | 17- | 236 | R/S | 38- |  | 74 |
| R/S | 18- | 74 | RCL 5 | 39- | 24 | 5 |
| 9 $1 / x$ | 19- | 153 | $\div$ | 40- |  | 71 |
| STO 6 | 20- | 236 | GTO 27 | 41- | 13 | 27 |


| REGISTERS |  |  |  |
| :--- | :--- | :--- | :--- |
| $R_{0}$ | $R_{1}$ | $R_{2}$ | $R_{3}$ |
| $R_{4}$ | $R_{5}$ Used | $R_{6}$ Used | $R_{7}$ Used |


| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Key in the program |  |  |  |
| 2 | Initialize: |  |  |  |
|  | For metric (SI) |  | GSB 01 | 1.3560 |
|  | For English |  | GSB 01 R/S | 5,251.4089 |
|  | To calculate RPM |  |  |  |
| 3 | Input power | power | ENTER 4 |  |
|  | Input torque | torque | GSB 28 | RPM |
|  | To calculate Torque: |  |  |  |
| 4 | Input power | power | ENTER + |  |
|  | Input RPM | RPM | GSB 28 | torque |
|  | Optional: |  |  |  |
| 4b | Convert torque to other |  |  |  |
|  | system |  | R/S | torque |
|  | To calculate Power: |  |  |  |
| 5a | Input torque | torque | ENTER4 |  |
|  | Input RPM | RPM | GSB 35 | power |
|  | Optional: |  |  |  |
| 5b | Convert power to other |  |  |  |
|  | system |  | R/S | power |

## Example 1:

Compute the torque from an engine developing 11 hp at 6500 RPM. Find the SI equivalent.

Keystrokes
Display
GSB 01 R/S
11 Entert
6500 GSB 28
8.8870

R/S
12.0508
ft-lb, English
nt-m, SI

## Example 2:

A generator is turning at 1600 RPM with a torque of $20 \mathrm{nt}-\mathrm{m}$. If it is $90 \%$ efficient what is the power input in watts? In horsepower?

## Keystrokes <br> Display

GSB 01

| 20 ENTERA $.9 母$ |  |  |
| :--- | :--- | :--- |
| 1600 GSB 35 | $3,723.3691$ | watts |
| R/S | 4.9931 | hp |

## Stress Analysis

## Static Equilibrium at a Point

This program calculates the two reaction forces necessary to balance a given two-dimensional force vector. The direction of the reaction forces may be specified as a vector of arbitrary length or by Cartesian coordinates using the point of force application as the origin.


## Equations:

$$
\begin{aligned}
\mathrm{R}_{1} \cos \theta_{1}+\mathrm{R}_{2} \cos \theta_{2} & =\mathrm{F} \cos \phi \\
\mathrm{R}_{1} \sin \theta_{1}+\mathrm{R}_{2} \sin \theta_{2} & =\mathrm{F} \sin \phi
\end{aligned}
$$

where: F is the known force; $\phi$ is the direction of the known force;
$\mathrm{R}_{1}$ is one reaction force;
$\theta_{1}$ is the direction of $\mathrm{R}_{1}$;
$R_{2}$ is the second reaction force;
$\theta_{2}$ is the direction of $\mathrm{R}_{2}$;
The coordinates $x_{1}$ and $y_{1}$ are referenced from the point where $F$ is applied to the end of the member along which $\mathrm{R}_{1}$ acts; $\mathrm{x}_{2}$ and $\mathrm{y}_{2}$ are the coordinates referenced from the point where $F$ is applied to the end of the member along which $\mathrm{R}_{2}$ acts.

## Remarks:

This program assumes the calculator is set in DEG mode.

| KEY ENTRY | DISPLAY |  | KEY ENTRY | DISPLAY |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| fCLEAR PRGM | 00 |  | ECL 2 | 25－ |  | 2 |
| （9）P | 01－ | 154 | 区 | 26－ |  | 61 |
| $x=y$ | 02－ | 21 | $\square$ | 27－ |  | 41 |
| 1 | 03－ | 1 | RCL 1 | 28－ | 24 | 1 |
| ＋+ B | 04－ | 144 | RCL 2 | 29－ | 24 | 2 |
| STO 0 | 05－ | 230 | x | 30－ |  | 61 |
| $x=y$ | 06－ | 21 | RCL 0 | 31－ | 24 | 0 |
| STO 1 | 07－ | 231 | RCL 3 | 32－ | 24 | 3 |
| 9 RTN | 08－ | 1512 | 区 | 33－ |  | 61 |
| 9 0 P | 09－ | 154 | $\square$ | 34－ |  | 41 |
| $x \geq y$ | 10－ | 21 | $\square$ | 35－ |  | 71 |
| 1 | 11－ | 1 | R／S | 36－ |  | 74 |
| －$\square^{8}$ | 12－ | 144 | thstx | 37－ |  | 73 |
| ［sTO 2 | 13－ | $23 \quad 2$ | STO 6 | 38－ | 23 | 6 |
| $x \geq y$ | 14－ | 21 | （RCL 5 | 39－ | 24 | 5 |
| STOO 3 | 15－ | $23 \quad 3$ | RCL 0 | 40－ |  | 0 |
| 9 RTN | 16－ | 1512 | 区 | 41－ |  | 61 |
| － 0 － | 17－ | 144 | ECL 4 | 42－ | 24 | 4 |
| STOO 4 | 18－ | 234 | RCL 1 | 43－ | 24 | 1 |
| $x \geq y$ | 19－ | 21 | 区 | 44－ |  | 61 |
| STO 5 | 20－ | 235 | $\square$ | 45－ |  | 41 |
| RCL 4 | 21－ | 244 | ECL 6 | 46－ |  | 6 |
| RCL 3 | 22－ | 243 | $\square$ | 47－ |  | 71 |
| $\pm$ | 23－ | 61 | 9 RTN | 48－ | 15 | 12 |
| RCL 5 | 24－ | 245 |  |  |  |  |


| REGISTERS |  |  |  |
| :--- | :--- | :--- | :--- |
| $\mathrm{R}_{0} \cos \theta_{1}$ | $\mathrm{R}_{1} \sin \theta_{1}$ | $\mathrm{R}_{2} \cos \theta_{2}$ | $\mathrm{R}_{3} \sin \theta_{2}$ |
| $\mathrm{R}_{4} \mathrm{~F} \cos \phi$ | $\mathrm{R}_{5} \mathrm{~F} \sin \phi$ | $\mathrm{R}_{6}$ Used | $\mathrm{R}_{7}$ |


| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Key in the program |  |  |  |
| 2 | Define reaction directions as |  |  |  |
|  | Cartesian coordinates or as |  |  |  |
|  | vectors of arbitrary magnitude. |  |  |  |
|  | (Use the point of force |  |  |  |
|  | application as the origin): |  |  |  |
|  | Define direction one in |  |  |  |
|  | rectangular form | $y_{1}$ | ENTER4 | $y_{1}$ |
|  |  | $\mathrm{x}_{1}$ | GSB 01 | $\sin \theta_{1}$ |
|  | or |  |  |  |
|  | in polar form | $\theta_{1}$ | GSB 03 | $\sin \theta_{1}$ |
|  | and |  |  |  |
|  | Define direction two in |  |  |  |
|  | rectangular form | $\mathrm{y}_{2}$ | ENTER4 |  |
|  |  | $\mathrm{x}_{2}$ | GSB 09 | $\sin \theta_{2}$ |
|  | or |  |  |  |
|  | polar form | $\theta_{2}$ | GSB 11 | $\sin \theta_{2}$ |
| 3 | Key in known force: direction, | $\phi$ | ENTER4 |  |
|  | then magnitude and |  |  |  |
|  | compute reactions. | F | GSB 17 | $\mathrm{R}_{1}$ |
|  |  |  | R/S | $\mathrm{R}_{2}$ |
| 4 | To change force, go to step 3 . |  |  |  |
|  | To change either or both |  |  |  |
|  | reaction directions, go to |  |  |  |
|  | step 2. |  |  |  |

## Example 1:

Find the reaction forces in the pin-jointed structure shown below.


| Keystrokes | Display |  |
| :--- | :--- | :--- |
| 8 CHS ENTER4 |  |  |
| 7 CHS |  |  |
| GSB 01 | -0.7526 | $\sin \theta_{1}$ |
| 0 ENTER4 10 CHS |  |  |
| GSB 09 | 0.0000 | $\sin \theta_{2}$ |
| 90 CHS ENTER4 |  |  |
| 500 GSB 17 | -664.3841 | $\mathrm{R}_{1}$ |
| R/S | 437.5000 | $\mathrm{R}_{2}$ |

## Example 2:

Find the reaction forces for the diagram below:


Keystrokes
30 GSB 03 125 GSB 11 240 ENTER4

| 100 GSB 17 | 90.9770 | $\mathrm{R}_{1}$ |
| :--- | :--- | :--- |
| R/S | 50.1910 | $\mathrm{R}_{2}$ |

Display
0.5000
0.8192

## Vector Cross Product

If $A=\left(a_{1}, a_{2}, a_{3}\right)$ and $B=\left(b_{1}, b_{2}, b_{3}\right)$ are two dimensional vectors then the cross product of A and B is denoted by $\mathrm{A} \times \mathrm{B}$ and is calculated as follows：

$$
\begin{aligned}
A \times B= & \left(\left|\begin{array}{cc}
a_{2} & a_{3} \\
b_{2} & b_{3}
\end{array}\right|,-\left|\begin{array}{cc}
a_{1} & a_{3} \\
b_{1} & b_{3}
\end{array}\right|,\left|\begin{array}{cc}
a_{1} & a_{2} \\
b_{1} & b_{2}
\end{array}\right|\right)= \\
& \left(a_{2} b_{3}-a_{3} b_{2}, a_{3} b_{1}-a_{1} b_{3}, a_{1} b_{2}-a_{2} b_{1}\right)
\end{aligned}
$$

Let the solution be represented by $\left(c_{1}, c_{2}, c_{3}\right)$ ．

| KEY ENTRY | DISPLAY |  |
| :---: | :---: | :---: |
| （f）CLEAR PRGM | 00 |  |
| ［ RCL 2 | 01－ | 242 |
| （RCL 6 | 02－ | 246 |
| 区 | 03－ | 61 |
| ECL 3 | 04－ | 243 |
| （RCL 5 | 05－ | 245 |
| 区 | 06－ | 61 |
| $\square$ | 07－ | 41 |
| R／S | 08－ | 74 |
| RCL 3 | 09－ | 243 |
| ［RCL 4 | 10－ | 244 |
| 区 | 11－ | 61 |
| RCL 1 | 12－ | 241 |


| KEY ENTRY | DISPLAY |  |  |
| :---: | :---: | :---: | :---: |
| ［RCD 6 | 13－ | 24 | 6 |
| 区 | 14－ |  | 61 |
| $\square$ | 15－ |  | 41 |
| R／S | 16－ |  | 74 |
| RCL 1 | 17－ |  | 1 |
| RCL 5 | 18－ | 24 | 5 |
| 区 | 19－ |  | 61 |
| ［CL 2 | 20－ | 24 | 2 |
| ［RCL 4 | 21－ | 24 | 4 |
| 区 | 22－ |  | 61 |
| $\square$ | 23－ |  | 41 |
| GTO 00 | 24－ | 130 | 00 |
|  |  |  |  |


| REGISTERS |  |  |  |
| :--- | :--- | :--- | :--- |
| $R_{0}$ | $R_{1} a_{1}$ | $R_{2} a_{2}$ | $R_{3} a_{3}$ |
| $R_{4} b_{1}$ | $R_{5} b_{2}$ | $R_{6} b_{3}$ | $R_{7}$ |

## 44 Stress Analysis

| STEP | INSTRUCTIONS | INPUT <br> DATA/UNITS | KEYS | OUTPUT <br> DATA/UNITS |
| :---: | :--- | :---: | :---: | :---: |
| 1 | Key in the program |  |  |  |
| 2 | Store A | $\mathrm{a}_{1}$ | STO 1 |  |
|  |  | $\mathrm{a}_{2}$ | STO 2 |  |
|  |  | $\mathrm{a}_{3}$ | STO 3 |  |
| 3 | Store B | $\mathrm{b}_{1}$ | STO 4 |  |
|  |  | $\mathrm{b}_{2}$ | STO 5 |  |
|  |  | $\mathrm{b}_{3}$ | STO 6 |  |
| 4 | Calculate cross product |  | GSB 01 | $\mathrm{c}_{1}$ |
|  |  |  | R/S | $\mathrm{C}_{2}$ |
|  |  |  | R/S | $\mathrm{c}_{3}$ |
| 5 | For a new case, go to step 2. |  |  |  |

## Example:

Let $\mathrm{A}=(2,5,2)$ and $\mathrm{B}=(3,3,-4)$.

## Solution:

$\mathrm{A} \times \mathrm{B}=(-26,14,-9)$
Keystrokes
Display
2 STO 1
5 STO 2
2 STO 3
3 STO 4
3 STO 5
4 CHS STO 6

| GSB | 01 | -26.0000 |
| :--- | ---: | :--- |
| R/S | 14.0000 | $c_{1}$ |
| R/S | -9.0000 | $c_{2}$ |

## Angle Between, Norm and Dot Product of Vectors

Let $\vec{a}=\left(a_{1}, a_{2} \ldots, a_{n}\right)$ and $\vec{b}=\left(b_{1}, b_{2}, \ldots, b_{n}\right)$ be two vectors.
The norm of a is denoted by $|\vec{a}|$ and is calculated by the following
formula：

$$
|\vec{a}|=\sqrt{a_{1}^{2}+a_{2}^{2}+\ldots+a_{n}^{2}}
$$

similarly，

$$
|\vec{b}|=\sqrt{b_{1}{ }^{2}+b_{2}{ }^{2}+\ldots+b_{n}{ }^{2}}
$$

The dot product of $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} \cdot \vec{b}$ and is calculated by the following formula：

$$
\vec{a} \cdot \vec{b}=a_{1} b_{1}+a_{2} b_{2}+\ldots+a_{n} b_{n}
$$

The angle between $\vec{a}$ and $\vec{b}$ is denoted by $\theta$ and is calculated by the following formula：

$$
\theta=\cos ^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\overrightarrow{\mathrm{a}}| \cdot|\overrightarrow{\mathrm{b}}|}\right)
$$

The angle is calculated in any angular mode．When calculated in degrees， decimal degrees are assumed．

| KEY ENTRY | DISPLAY | KEY ENTRY | DISPLAY |
| :---: | :---: | :---: | :---: |
| fCLEAR PRGM | 00 | STOQ 2 | 11－23 512 |
| Entre | 01－ 31 | GTo 00 | 12－ 1300 |
| 9 $x^{2}$ | 02－ 150 | RCL 2 | 13－ 242 |
| STO +1 | 03－23 51 | RCL 0 | 14－ 240 |
| R0） | 04－ 22 | RCL 1 | 15－ 241 |
| $x \leq y$ | 05－ 21 | 区 | 16－ 61 |
| ENTER | 06－ 31 | T $\sqrt{x}$ | 17－ 140 |
| 9 $x^{2}$ | 07－ 150 | $\dagger$ | 18－ 71 |
| STO +0 | 08－23 510 | $9 \mathrm{cos}^{-1}$ | 19－ 158 |
| ［日 | 09－ 22 | GTO 00 | 20－ 1300 |
| 区 | 10－ 61 |  |  |


| REGISTERS |  |  |  |
| :--- | :--- | :--- | :--- |
| $R_{0} \Sigma a_{i}{ }^{2}$ | $R_{1} \Sigma b_{i}{ }^{2}$ | $R_{2} \Sigma a_{i} b_{i}$ | $R_{3}$ |
| $R_{4}$ | $R_{5}$ | $R_{6}$ | $R_{7}$ |


| STEP | INSTRUCTIONS | INPUT DATA／UNITS | KEYS | OUTPUT DATA／UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Key in the program |  |  |  |
| 2 | Initialize |  | （ 7 REG PRGM |  |
| 3 | Perform for $\mathrm{i}=1, \ldots, \mathrm{n}$ ： |  |  |  |
|  | Key in $\mathrm{a}_{\mathrm{i}}$ and $\mathrm{b}_{\mathrm{i}}$ | $\mathrm{a}_{\mathrm{i}}$ | ENTER4 |  |
|  |  | $\mathrm{b}_{1}$ | R／S |  |
| 4 | Find norm of $\vec{a}$ |  | RCL 0 㹉 $\sqrt{x}$ | $\|\vec{a}\|$ |
| 5 | Find norm of $\vec{b}$ |  | RCL 1 ¢ $\sqrt{x}$ | $\|\vec{b}\|$ |
| 6 | Find $\|\vec{a} \cdot \vec{b}\|$ |  | RCL 2 | $\|\vec{a} \cdot \vec{b}\|$ |
| 7 | Calculate angle between |  |  |  |
|  | $\vec{a}$ and $\vec{b}$ |  | GSB 13 | $\theta$ |

## Example：

Let $\vec{a}=(2,5,2)$ and $\vec{b}=(3,3,-4)$

## Solution：

$|\vec{a}|=5.7446$
$|\vec{b}|=5.8310$
$\vec{a} \cdot \vec{b}=13.0000$
$\theta=67.1635$

## Keystrokes

## Display

（ REG PRGM
2 ENTERA 3 R／S
5 ENTER4 3 R／S
2 ENTER4 4 CHS
R／S

RCL 0 龱
5.7446

RCL 1 片
5.8310

RCL 2
GSB 13
13.0000
67.1635

## Engineering Economics

## Discounted Cash Flow: Net Present Value, Internal Rate of Return

The primary purpose of this program is to compute the net present value of a series of cash flows. In general, an initial investment $V_{0}$ is made in some enterprise which is expected to bring in periodic cash flows $\mathrm{C}_{1}, \mathrm{C}_{2}, \ldots, \mathrm{C}_{\mathrm{n}}$. Given a discount rate i , which must be entered as a decimal, then for each cash flow $\mathrm{C}_{\mathrm{k}}$, the program will compute the net present value at period $k, N_{k}$. A negative value for $\mathrm{NPV}_{\mathrm{k}}$ indicates that the enterprise has not yet been profitable. A positive $\mathrm{NPV}_{\mathrm{k}}$ means that the enterprise has been profitable, to the extent that a rate of return i on the original investment has been exceeded.

The program may also be used iteratively to calculate an internal rate of return. The objective here is to find the discount rate i which will make the final net present value, $\mathrm{NPV}_{\mathrm{n}}$, equal to zero. The procedure, then, is to store $\mathrm{V}_{0}$ and a first guess at the rate of return i , input the cash flows $\mathrm{C}_{1}$ through $\mathrm{C}_{\mathrm{n}}$, and thus find $\mathrm{NPV}_{\mathrm{n}}$. If $\mathrm{NPV}_{\mathrm{n}}$ is negative, the estimated rate of return was too high; if $\mathrm{NPV}_{\mathrm{n}}$ is positive, the estimate for i was too low. Adjust the estimate for i accordingly, store the new i, and input the cash flows again. Inspect the new value of $N P V_{n}$ to obtain a new estimate for $i$ and repeat the process. The entire procedure is repeated until $\mathrm{NPV}_{\mathrm{n}}$ is zero, or very close to it. The last value of i input is then regarded as the internal rate of return.
Each figure for net present value is found by

$$
\mathrm{NPV}_{\mathrm{k}}=\mathrm{V}_{0}+\sum_{\mathrm{j}=1}^{\mathrm{k}} \frac{\mathrm{C}_{\mathrm{j}}}{(1+\mathrm{i})^{\mathrm{j}}}
$$

This program employs the convenient sign convention used in the most recent HP calculators and programs. Cash received is represented by a positive value (+). Cash paid out is represented by a negative value (-).

| KEY ENTRY | DISPLAY |  | KEY ENTRY | DISPLAY |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (f)CLEAR PRGM | 00 |  | STO 3 | 12- | 233 |
| RCL 1 | 01- | 241 | R/S | 13- | 74 |
| 1 | 02- | 1 | RCL 2 | 14- | 242 |
| STO 4 | 03- | 234 | RCL 4 | 15- | 244 |
| $\pm$ | 04- | 51 | 1 | 16- | 1 |
| STO 2 | 05- | 232 | $\pm$ | 17- | 51 |
| $\div$ | 06- | 71 | STO 4 | 18- | 234 |
| RCL 0 | 07- | 240 | ¢ $y^{x}$ | 19- | 143 |
| $\pm$ | 08- | 51 | $\div$ | 20- | 71 |
| RCL 4 | 09- | 244 | RCL 3 | 21- | 243 |
| f PaUSE | 10- | 1474 | $\pm$ | 22- | 51 |
| $x \geq y$ | 11- | 21 | GTO 09 | 23- | 1309 |


| REGISTERS |  |  |  |
| :--- | :--- | :--- | :--- |
| $R_{0} V_{0}$ | $R_{1} i$ | $R_{2}(1+i)$ | $R_{3} N P V_{k}$ |
| $\mathrm{R}_{4} \mathrm{k}$ | $\mathrm{R}_{5}$ | $\mathrm{R}_{6}$ | $\mathrm{R}_{7}$ |


| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Key in the program |  |  |  |
| 2 | Store initial investment and |  |  |  |
|  | discount rate |  |  |  |
|  | Initial investment | $\mathrm{V}_{0}$ * | STO 0 |  |
|  | Interest rate | i (decimal) | STO 1 |  |
| 3 | Perform for $\mathrm{k}=1, \ldots, \mathrm{n}$ : |  | (1) PRGM |  |
|  | Input $\mathrm{C}_{\mathrm{k}}$ cash flow and |  |  |  |
|  | compute $\mathrm{NPV}_{\mathrm{k}}$ | $\mathrm{C}_{\mathrm{k}}$ * | R/S | (k) |
|  |  |  |  | $\mathrm{NPV}_{\mathrm{k}}$ * |
| 4 | For a new case, go to step 2. |  |  |  |
|  | * Note: Cash received is re- |  |  |  |
|  | presented by a positive |  |  |  |
|  | value ( + ). Cash paid out is |  |  |  |
|  | represented by a negative |  |  |  |
|  | value ( - ). |  |  |  |

## Example:

You are contemplating installing a processing machine for $\$ 150,000$ at a capital cost of $10 \%$ after taxes. Based on the following cash flows, will this investment be profitable?

| Year | Cash Flow |
| :---: | :---: |
| 1 | $\$ 30,000$ |
| 2 | 26,300 |
| 3 | 50,000 |
| 4 | 55,600 |
| 5 | 45,200 |

## Keystrokes Display



Since $C_{5}$ is positive the investment is profitable to the extent that the cost of capital is $10 \%$.

## Compound Amount



This program applies to an amount of principal that has been placed into an account and compounded periodically, with no further deposits. The important variables in this case are the number of compounding periods $n$, the periodic interest rate $i$, the principal or present value PV , the future value of the account FV, and the amount of interest accrued I. Any of these may be calculated from the others by these formulas:

$$
\begin{gathered}
\mathrm{n}=\frac{\ln |\mathrm{FV} / \mathrm{PV}|}{\ln (1+\mathrm{i})} \\
\mathrm{i}=\left|\frac{\mathrm{FV}}{\mathrm{PV}}\right|^{1 \mathrm{n}}-1 \\
\mathrm{PV}=-\mathrm{FV}(1+\mathrm{i})^{-\mathrm{n}} \\
\mathrm{FV}=-\mathrm{PV}(1+\mathrm{i})^{\mathrm{n}} \\
\mathrm{I}=-\mathrm{PV}\left[(1+\mathrm{i})^{\mathrm{n}}-1\right]
\end{gathered}
$$

where: $n=$ total number of periods
$\mathrm{i}=$ periodic interest rate (expressed as decimal)
i.e., an annual interest rate of $6 \%$ is expressed as 0.06 , which is a monthly rate of $\frac{0.06}{12}=0.005$
$\mathrm{PV}=$ present value (value at beginning of first period)
$F V=$ future value (value at end of last period)

This program employs the convenient sign convention used in the most recent HP calculators and programs. Cash received is represented by a positive value $(+)$. Cash paid out is represented by a negative value $(-)$.

| KEY ENTRY | DISPLAY |  | KEY ENTRY | DISPLAY |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| f CLEAR PRGM | 00 |  | RCL 5 | 24- | 24 | 5 |
| RCL 5 | 01- | 245 | CHS | 25- |  | 32 |
| (RCL 4 | 02- | 244 | $x$ | 26- |  | 61 |
| $\div$ | 03- | 71 | GTO 00 | 27- | 13 | 00 |
| 9 ABS | 04- | 1534 | GSB 44 | 28- | 12 | 44 |
| $\square$ LN | 05- | 141 | RCL 1 | 29- | 24 | 1 |
| GSB 44 | 06- | 1244 | 9 $y^{x}$ | 30- | 14 | 3 |
| T LN | 07- | 141 | RCL 4 | 31- | 24 | 4 |
| $\div$ | 08- | 71 | CHS | 32- |  | 32 |
| GTO 00 | 09- | 1300 | $x$ | 33- |  | 61 |
| RCL 5 | 10- | 245 | GTO 00 | 34- | 13 | 00 |
| RCL 4 | 11- | 244 | GSB 44 | 35- | 12 | 44 |
| $\div$ | 12- | 71 | RCL 1 | 36- | 24 | 1 |
| (9) $A B S$ | 13- | 1534 | 9 $y^{x}$ | 37- | 14 | 3 |
| RCL 1 | 14- | 241 | 1 | 38- |  | 1 |
| (9) $1 / x$ | 15- | 153 | $\square$ | 39- |  | 41 |
| 4 $y^{x}$ | 16- | 143 | RCL 4 | 40- | 24 | 4 |
| 1 | 17- | 1 | CHS | 41- |  | 32 |
| $\square$ | 18- | 41 | $x$ | 42- |  | 61 |
| GTO 00 | 19- | 1300 | GTO 00 | 43- | 13 | 00 |
| GSB 44 | 20- | 1244 | RCL 2 | 44- | 24 | 2 |
| RCL 1 | 21- | 241 | 1 | 45- |  | 1 |
| CHS | 22- | 32 | $\pm$ | 46- |  | 51 |
| $4 y^{x}$ | 23- | 143 | 9 RTN | 47- | 15 | 12 |


| REGISTERS |  |  |  |
| :--- | :--- | :--- | :--- |
| $R_{0}$ | $R_{1} n$ | $R_{2} i$ | $R_{3}$ |
| $R_{4} \mathrm{PV}$ | $R_{5} \mathrm{FV}$ | $R_{6}$ | $R_{7}$ |


| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Key in the program |  |  |  |
| 2 | To compute number of |  |  |  |
|  | periods | i (decimal) | STO 2 |  |
|  |  | PV* | STO 4 |  |
|  |  | FV* | STO 5 |  |
|  |  |  | GSB 01 | n |
| 3 | To compute periodic interest |  |  |  |
|  | rate | $n$ | STO 1 |  |
|  |  | PV* | STO 4 |  |
|  |  | FV* | STO 5 |  |
|  |  |  | GSB 10 | i (decimal) |
| 4 | To compute principal | n | STO 1 |  |
|  |  | i (decimal) | STO 2 |  |
|  |  | FV* | STO 5 |  |
|  |  |  | GSB 20 | PV |
| 5 | To compute future value | n | STO 1 |  |
|  |  | i (decimal) | STO 2 |  |
|  |  | PV* | STO 4 |  |
|  |  |  | GSB 28 | FV |
| 6 | To compute accrued interest | n | STO 1 |  |
|  |  | (i decimal) | STO 2 |  |
|  |  | PV* | STO 4 |  |
|  |  |  | GSB 35 | I |
| 7 | For a new case, go to step 2, |  |  |  |
|  | $3,4,5$, or 6. |  |  |  |
|  | * Note: Cash received is re- |  |  |  |
|  | presented by a positive value |  |  |  |
|  | (+). Cash paid out is repre- |  |  |  |
|  | sented by a negative value ( - ). |  |  |  |

54 Engineering Economics

## Example 1:

Find the rate of return on $\$ 1000$ compounded quarterly if it amounts to $\$ 1500$ in 5 years.

## Note:

$\mathrm{n}=20$

## Keystrokes Display

| 20 STO 1 |  |  |  |
| :--- | :--- | :--- | :--- |
| 1000 CHS STO 4 |  |  |  |
| 1500 STO 5 GSB 10 | 0.0205 | (quarterly) |  |
| $4 \times$ |  | 0.0819 | (8.19\% annually) |

## Example 2:

How much will you need to invest today at $53 / 4 \%$ compounded quarterly to have $\$ 3000$ in 5 years?

Note:
$\mathrm{n}=20$
Keystrokes: Display
f) FIX 2

20 STO 1
. 0575 ENTER4
$4 \div$ STO 2
3000 STO 5 GSB $20-2,255.02$

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[^0]:    - In running the program be sure to enter zero for the variable to be calculated.

