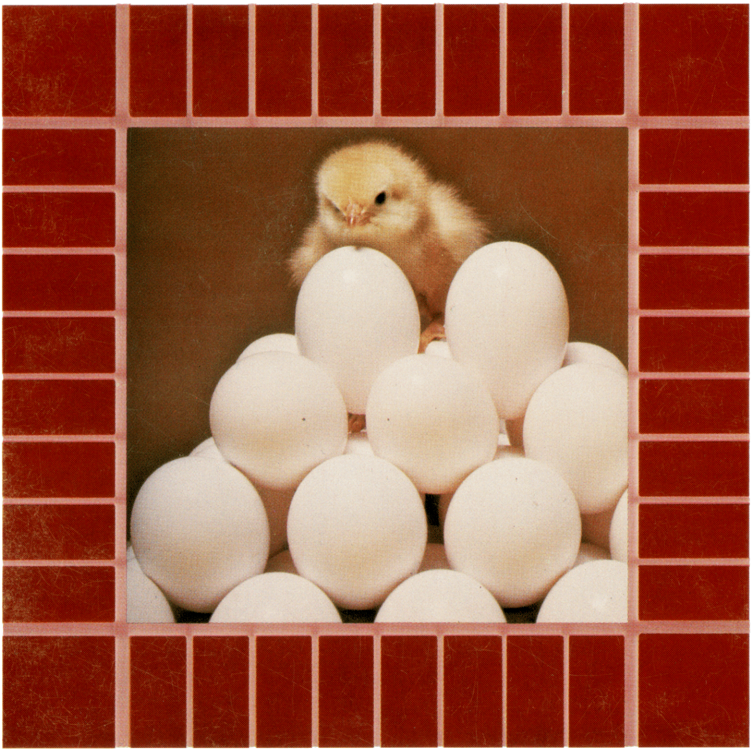
HEWLETT-PACKARD

HP-34C

STATISTICS Applications



NOTICE

HEWLETT PACKARD

HP-34C

Statistics Applications

March 1982

00034-90033 Rev. D

Printed in U.S.A.

Introduction

This Statistics Applications book was written to help you get the most from your HP-34C calculator. The programs were chosen to provide useful calculations for many of the common problems encountered in statistics.

They provide you with immediate problem solving capabilities in your everyday work. You may also find them useful as guides to programming techniques for writing your own customized software.

You will find general information on how to key in and run programs under ''A Word About Program Usage'' in the Applications book you received with your calculator.

We hope this Statistics Applications book will be a valuable tool in your work and appreciate your comments about it.

Contents

General Statistics

Basic Statistics for Two Variables

This program calculates means, standard deviations, covariance, correlation coefficient, coefficients of variation, sums of data points, sum of multiplication of data points, and sums of squares of data points derived from a set of ungrouped data points $\{(x_i, y_i), i = 1, 2, \ldots, n\}$, or grouped data points $\{(x_i, y_i, f_i)\ i = 1, 2, \ldots, n\}$. f_i denotes the frequency of repetition of (x_i, y_i).

$$\text{means } \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \qquad \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

$$\text{standard deviations } s_x = \sqrt{\frac{\Sigma x_i^2 - n\bar{x}^2}{n-1}}$$

$$\left(\text{or } s_x' = \sqrt{\frac{\Sigma x_i^2 - n\bar{x}^2}{n}}\right)$$

$$s_y = \sqrt{\frac{\Sigma y_i^2 - n\bar{y}^2}{n-1}}$$

$$\left(\text{or } s_y' = \sqrt{\frac{\Sigma y_i^2 - n\bar{y}^2}{n}}\right)$$

$$\text{covariance } s_{xy} = \frac{1}{n-1}\left(\Sigma x_i y_i - \frac{1}{n}\Sigma x_i \Sigma y_i\right)$$

$$\left(\text{or } s_{xy}' = \frac{1}{n}\left[\Sigma x_i y_i - \frac{1}{n}\Sigma x_i \Sigma y_i\right]\right)$$

$$\text{correlation coefficient } \gamma_{xy} = \frac{s_{xy}}{s_x s_y}$$

Coefficients of variation $V_x = \frac{s_x}{\bar{x}} \cdot 100, \ V_y = \frac{s_y}{\bar{y}} \cdot 100$

Notes:

1. n is a positive integer and $n > 1$.

2. For grouped data $\bar{x} = \frac{\sum_{i=1}^{m} f_i x_i}{\sum_{i=1}^{m} f_i}$, etc.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	000–	RCL 8	018– 24 8
h LBL B	001–25, 13, 12	g x^2	019– 15 3
STO 9	002– 23 9	RCL 9	020– 24 9
h F? 0	003–25, 71, 0	×	021– 61
CHS	004– 32	h F? 0	022–25, 71, 0
STO + 0	005–23, 51, 0	CHS	023– 32
g R↓	006– 15 22	STO + 4	024–23, 51, 4
STO 8	007– 23 8	RCL 8	025– 24 8
g R↓	008– 15 22	RCL 9	026– 24 9
STO 7	009– 23 7	×	027– 61
f R↑	010– 14 22	h F? 0	028–25, 71, 0
f R↑	011– 14 22	CHS	029– 32
h ABS	012– 25 34	STO + 3	030–23, 51, 3
×	013– 61	RCL 7	031– 24 7
×	014– 61	g x^2	032– 15 3
h F? 0	015–25, 71, 0	RCL 9	033– 24 9
CHS	016– 32	×	034– 61
STO + 5	017–23, 51, 5	h F? 0	035–25, 71, 0

KEY ENTRY	DISPLAY
CHS	036- 32
STO + 2	037-23, 51, 2
RCL 7	038- 24 7
RCL 9	039- 24 9
×	040- 61
h F? 0	041-25, 71, 0
CHS	042- 32
STO + 1	043-23, 51, 1
RCL 0	044- 24 0
h CF 0	045-25, 61, 0
R/S	046- 74
h LBL A	047-25, 13, 11
x⇄y	048- 21
f Σ+	049- 14 74
R/S	050- 74
h LBL 1	051-25, 13, 1
h x̄	052- 25 8
R/S	053- 74
x⇄y	054- 21
R/S	055- 74
h LBL 2	056-25, 13, 2
h s	057- 25 9
R/S	058- 74
GTO 9	059- 22 9
h LBL 3	060-25, 13, 3
h s	061- 25 9
x⇄y	062- 21
R/S	063- 74

KEY ENTRY	DISPLAY
h LBL 9	064-25, 13, 9
RCL 0	065- 24 0
ENTER↑	066- 31
x⇄y	067- 21
1	068- 1
−	069- 41
÷	070- 71
f √x	071- 14 3
÷	072- 71
R/S	073- 74
h LBL 4	074-25, 13, 4
h s	075- 25 9
h x̄	076- 25 8
x⇄y	077- 21
g R↓	078- 15 22
÷	079- 71
EEX	080- 33
2	081- 2
×	082- 61
R/S	083- 74
h s	084- 25 9
h x̄	085- 25 8
x⇄y	086- 21
f R↑	087- 14 22
x⇄y	088- 21
÷	089- 71
EEX	090- 33
2	091- 2

KEY ENTRY	DISPLAY
[×]	092– 61
[R/S]	093– 74
[h] [LBL] 5	094– 25, 13, 5
[h] [x̄]	095– 25 8
[x⇄y]	096– 21
[STO] 6	097– 23 6
[RCL] 5	098– 24 5
[RCL] 1	099– 24 1
[RCL] 6	100– 24 6
[×]	101– 61
[−]	102– 41
[RCL] 0	103– 24 0
1	104– 1
[−]	105– 41
[÷]	106– 71
[STO] [•] 0	107– 23 .0
[R/S]	108– 74
[RCL] 0	109– 24 0
[ENTER↑]	110– 31
[x⇄y]	111– 21
1	112– 1
[−]	113– 41
[÷]	114– 71
[÷]	115– 71
[R/S]	116– 74
[h] [LBL] 6	117– 25, 13, 6
[h] [s]	118– 25 9
[RCL] [•] 0	119– 24 .0
[÷]	120– 71
[×]	121– 61
[h] [1/x]	122– 25 2
[R/S]	123– 74
[GTO] 1	124– 22 1

REGISTERS			I
R_0 n	R_1 Σx	R_2 Σx^2	R_3 Σy
R_4 Σy^2	R_5 Σxy	R_6 $\bar{y}$	R_7 x_i
R_8 y_i	R_9 f_i	$R_{.0}$ s_{xy}	$R_{.1}$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program. (For un-			
	grouped data only: line 47–			
	line 124)			
2	Initialize.		[f] CLEAR [REG]	
	Ungrouped Data			
3	Repeat step 3–4 for i = 1, 2,			
	…, n			
	Input: x_i	x_i	[ENTER↑]	
	y_i	y_i	[A]	i
4	If you made a mistake in			
	inputting x_k and y_k, then			
	correct by →	x_k	[ENTER↑]	
		y_k	[x⇄y] [g] [Σ−]	k–l
5	Go to step 8 for basic stat-			
	istic calculations.			
	Grouped Data			
6	Repeat step 6–7 for i = 1, 2,			
	…, n			
	Input: x_i	x_i	[ENTER↑]	
	y_i	y_i	[ENTER↑]	
	f_i	f_i	[B]	Σf_i
7	If you made a mistake in			
	inputting x_k and y_k, then			
	correct by →	x_k	[ENTER↑]	
		y_k	[ENTER↑]	
		f_k	[h] [SF] 0	
			[B]	$\Sigma f_i - f_k$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
8	To calculate basic statistics:			
	$\bar{x}$		[GSB] 1	$\bar{x}$
	$\bar{y}$		[R/S]	$\bar{y}$
	s_x		[GSB] 2	s_x
	s_x'		[R/S]	s_x'
	s_y		[GSB] 3	s_y
	s_y'		[R/S]	s_y'
	V_x*		[GSB] 4	V_x*
	V_y*		[R/S]	V_y*
	s_{xy}		[GSB] 5	s_{xy}
	s'_{xy}		[R/S]	s'_{xy}
	γ_{xy}		[GSB] 6	γ_{xy}
9	To recall sums:			
	Σx_i		[RCL] 1	Σx_i
	Σx_i^2		[RCL] 2	Σx_i^2
	Σy_i		[RCL] 3	Σy_i
	Σy_i^2		[RCL] 4	Σy_i^2
	$\Sigma x_i y_i$		[RCL] 5	$\Sigma x_i y_i$
10	Repeat step 8 if you want			
	the results again.			
11	For a new case, go to step 2.			
	*"Error 0" will be displayed			
	if $\bar{x}$ or $\bar{y}$ is zero. Press any key			
	and proceed.			

Example 1:

Example 1:

For the following set of data, find the means, standard deviations, covariance, correlation coefficient, coefficients of variation, and the sums.

x_i	26	30	44	50	62	68	74
y_i	92	85	78	81	54	51	40

Keystrokes	Display	
f CLEAR REG		
26 ENTER↑ 92 A	1.0000	
100 ENTER↑ 99 A	2.0000	(error)
100 ENTER↑ 99		
x⇄y g Σ−	1.0000	(correction)
30 ENTER↑ 85 A		
44 ENTER↑ 78 A		
50 ENTER↑ 81 A		
62 ENTER↑ 54 A		
68 ENTER↑ 51 A		
74 ENTER↑ 40 A	7.0000	($n = 7$)
GSB 1	50.5714	($\bar{x}$)
R/S	68.7143	($\bar{y}$)
GSB 2	18.5010	(s_x)
R/S	17.1286	(s'_x)
GSB 3	19.9976	(s_y)
R/S	18.5142	(s'_y)
GSB 4	36.5838	(V_x)
R/S	29.1026	(V_y)
GSB 5	−354.1429	(s_{xy})
R/S	−303.5510	(s'_{xy})
GSB 6	−0.9572	(γ_{xy})

Example 2:

Apply the program to the following set of grouped data.

x_i	4.8	5.2	3.8	4.4	4.1
y_i	15.1	11.5	14.3	13.6	12.8
f_i	1	3	1	6	2

Keystrokes	Display	
[f] CLEAR [REG]		
4.8 [ENTER↑]		
15.1 [ENTER↑] 1 [B]	1.0000	
10 [ENTER↑] 10 [ENTER↑]		
4 [B]	5.0000	(error)
10 [ENTER↑] 10 [ENTER↑]		
4 [h] [SF] 0 [B]	1.0000	(correction)
5.2 [ENTER↑]		
11.5 [ENTER↑] 3 [B]		
3.8 [ENTER↑]		
14.3 [ENTER↑] 1 [B]		
4.4 [ENTER↑]		
13.6 [ENTER↑] 6 [B]		
4.1 [ENTER↑]		
12.8 [ENTER↑] 2 [B]	13.0000	(Σf_i)
[GSB] 1	4.5231	($\bar{x}$)
[R/S]	13.1615	($\bar{y}$)
[GSB] 2	0.4494	(s_x)
[R/S]	0.4317	(s'_x)
[GSB] 3	1.1087	(s_y)
[R/S]	1.0652	(s'_y)
[GSB] 4	9.9348	(V_x)
[R/S]	8.4238	(V_y)
[GSB] 5	−0.3065	(s_{xy})
[R/S]	−0.2830	(s'_{xy})
[GSB] 6	−0.6153	(γ_{xy})

Factorial, Permutations and Combinations

This program finds the extended range factorial (n can be greater than 69), permutations and combinations. Permutations and combinations are functions of the factorial, but this program does not use the factorial key, so that better accuracy and larger range can be obtained.

The equations are:

Factorial $$n! = n\ (n-1)\ (n-2) \cdot \ldots \cdot 2 \cdot 1\ ;\ n \geqslant 1$$

Permutations $${}_mP_n = \frac{m!}{(m-n)!} = m(m-1) \ldots (m-n+1)$$

Combinations $${}_mC_n = \frac{m!}{(m-n)!\ n!} = \frac{m(m-1) \ldots (m-n+1)}{1 \cdot 2 \cdot \ldots \cdot n}$$

where m, n are integers and $0 \leqslant n \leqslant m$.

Notes:

1. ${}_mP_0 = 1$, ${}_mP_1 = m$, ${}_mP_m = m!$
2. ${}_mC_0 = {}_mC_m = 1$
3. ${}_mC_1 = {}_mC_{m-1} = m$
4. ${}_mC_n = {}_mC_{m-n}$
5. In calculating $n!$, the accuracy will be reduced for $n > 69$, since it is calculated using logarithms.

KEY ENTRY	DISPLAY
f CLEAR PRGM	000–
h LBL A	001– 25, 13, 11
f FIX 9	002– 14, 11, 9
STO 1	003– 23 1
STO 3	004– 23 3
6	005– 6
9	006– 9
x⇄y	007– 21
f x≤y	008– 14 41
GTO 9	009– 22 9
x⇄y	010– 21
−	011– 41
STO f I	012– 23, 14, 23
h LST x	013– 25 0
h x!	014– 25 1
f LOG	015– 14 2
STO 2	016– 23 2
RCL 1	017– 24 1
ENTER↑	018– 31
f LOG	019– 14 2
STO + 2	020– 23, 51, 2
h LBL 7	021– 25, 13, 7
g DSE	022– 15 23
GTO 8	023– 22 8
RCL 2	024– 24 2
h FRAC	025– 25 33
g 10^x	026– 15 2
f FIX 9	027– 14,11, 9
R/S	028– 74

KEY ENTRY	DISPLAY
RCL 2	029– 24 2
f FIX 0	030– 14, 11, 0
h INT	031– 25 32
h RTN	032– 25 12
h LBL 8	033– 25, 13, 8
RCL 3	034– 24 3
1	035– 1
−	036– 41
STO 3	037– 23 3
f LOG	038– 14 2
STO + 2	039– 23, 51, 2
GTO 7	040– 22 7
h LBL 9	041– 25, 13, 9
h x!	042– 25 1
h RTN	043– 25 12
h LBL B	044– 25, 13, 12
f FIX 0	045– 14, 11, 0
x⇄y	046– 21
STO 0	047– 23 0
x⇄y	048– 21
f x>y	049– 14 51
GTO 2	050– 22 2
f x=y	051– 14 71
GTO 9	052– 22 9
ENTER↑	053– 31
0	054– 0
f x=y	055– 14 71
GTO 3	056– 22 3
CLX	057– 34

KEY ENTRY	DISPLAY
1	058– 1
[f] [x=y]	059– 14 71
[GTO] 5	060– 22 5
[−]	061– 41
[STO] [f] [I]	062– 23, 14, 23
[g] [R↓]	063– 15 22
[STO] 4	064– 23 4
[h] [LBL] 4	065– 25, 13, 4
[RCL] 4	066– 24 4
1	067– 1
[−]	068– 41
[STO] 4	069– 23 4
[×]	070– 61
[g] [DSE]	071– 15 23
[GTO] 4	072– 22 4
[h] [RTN]	073– 25 12
[h] [LBL] 2	074– 25, 13, 2
0	075– 0
[+]	076– 71
[h] [LBL] 3	077– 25, 13, 3
1	078– 1
[h] [RTN]	079– 25 12
[h] [LBL] 1	080– 25, 13, 1
[f] [FIX] 0	081– 14, 11, 0
[f] [x>y]	082– 14 51
[GTO] 2	083– 22 2
[−]	084– 41
[h] [LST x]	085– 25 0
[f] [x≤y]	086– 14 41

KEY ENTRY	DISPLAY
[GSB] 6	087– 13 6
[STO] 4	088– 23 4
1	089– 1
[STO] [f] [I]	090– 23, 14, 23
[+]	091– 51
[STO] 0	092– 23 0
[CLx]	093– 34
[f] [x=y]	094– 14 71
[GTO] 3	095– 22 3
[h] [LBL] 0	096– 25, 13, 0
[g] [R↓]	097– 15 22
1	098– 1
[RCL] [f] [I]	099– 24, 14, 23
[+]	100– 51
[STO] [f] [I]	101– 23, 14, 23
[f] [x>y]	102– 14 51
[GTO] 5	103– 22 5
[RCL] 4	104– 24 4
[x≷y]	105– 21
[+]	106– 51
[h] [LST x]	107– 25 0
[÷]	108– 71
[RCL] 0	109– 24 0
[×]	110– 61
[STO] 0	111– 23 0
[GTO] 0	112– 22 0
[h] [LBL] 6	113– 25, 13, 6
[STO] 0	114– 23 0
[x≷y]	115– 21

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
h RTN	116– 25 12	RCL 0	118– 24 0
h LBL 5	117– 25, 13, 5		

REGISTERS			I $n-69$
R_0 Used	R_1 m, n	R_2 Log(69!)+ ...	R_3 $(n-i)$
R_4 m	R_5 $n-1$	R_6–$R_{.2}$ Unused	

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program (For			
	factorial only: key in line 1–			
	line 43. For permutations			
	and combinations only: key in			
	line 41-line 118).			
	Factorial (n!)			
2	For $n \leq 69$.	n	A	$n!$
3	For $n > 69$.	n	A	decimal no.
			R/S	exp. of 10
	Permutations			
4	Calculate ${}_mP_n$.	m	ENTER↑	
		n	B	${}_mP_n$
	Combinations			
5	Calculate ${}_mC_n$.	m	ENTER↑	
		n	GSB 1	${}_mC_n$
6	Repeat any of the above steps			
	if desired.			

Examples:

1. $5! = 120$
2. $69! \simeq 1.711224524 \times 10^{98}$
3. $70! \simeq 1.197857069 \times 10^{100}$
4. $100! \simeq 9.332622518 \times 10^{157}$
5. ${}_{27}P_5 = 9687600.00$
6. ${}_{73}C_4 = 1088430.00$

Keystrokes	Display	
5 [A]	120.0000000	(5!)
69 [A]	1.711224 98	(69!)
70 [A]	1.197857069	(decimal no.)
[R/S]	100.	(10^{100})
100 [A]	9.332622518	(decimal no.)
[R/S]	157	(10^{157})
27 [ENTER↑] 5 [B]	9,687,600.	(${}_{27}P_5$)
73 [ENTER↑] 4 [GSB] 1	1,088,430.	(${}_{73}C_4$)

Moments, Skewness and Kurtosis

For grouped or ungrouped data, moments are used to describe sets of data, skewness is used to measure the lack of symmetry in a distribution, and kurtosis is the relative peakness or flatness of a distribution. For a given set of data

$$\{x_1, x_2, \ldots, x_n\}:$$

$$1^{st} \text{ moment} \quad \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$2^{nd} \text{ moment} \quad m_2 = \frac{1}{n} \Sigma x_i^2 - \bar{x}^2$$

$$3^{rd} \text{ moment} \quad m_3 = \frac{1}{n} \Sigma x_i^3 - \frac{3}{n} \bar{x} \Sigma x_i^2 + 2\bar{x}^3$$

$$4^{th} \text{ moment} \quad m_4 = \frac{1}{n} \Sigma x_i^4 - \frac{4}{n} \bar{x} \Sigma x_i^3 + \frac{6}{n} \bar{x}^2 \Sigma x_i^2 - 3\bar{x}^4$$

Moment coefficient of skewness

$$\gamma_1 = \frac{m_3}{m_2^{3/2}}$$

Moment coefficient of kurtosis

$$\gamma_2 = \frac{m_4}{m_2^2}$$

This program also provides the option for calculating those statistics for grouped data (using similar formulas as for ungrouped data):

data	x_1	x_2	...	x_m
frequency	f_1	f_2	...	f_m

Note that for this case, 1^{st} moment

$$\bar{x} = \frac{\sum_{i=1}^{m} f_i x_i}{\sum_{i=1}^{m} f_i}$$

Reference:

Theory and Problems of Statistics, M. R. Spiegel, Schaum's Outline, McGraw-Hill, 1961.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM		h LST x	016– 25 0
h LBL A	001–25, 13, 11	×	017– 61
ENTER↑	002– 31	STO + 3	018–23, 51, 3
1	003– 1	h LST x	019– 25 0
GSB 0	004– 13 0	×	020– 61
R/S	005– 74	STO + 4	021–23, 51, 4
h LBL B	006–25, 13, 12	h LST x	022– 25 0
GSB 0	007– 13 0	×	023– 61
R/S	008– 74	STO + 5	024–23, 51, 5
h LBL 0	009–25, 13, 0	1	025– 1
h F? 0	010–25, 71, 0	h F? 0	026–25, 71, 0
CHS	011– 32	CHS	027– 32
STO + 1	012–23, 51, 1	STO + 0	028–23, 51, 0
x⇄y	013– 21	RCL 0	029– 24 0
×	014– 61	h CF 0	030–25, 61, 0
STO + 2	015–23, 51, 2	h RTN	031– 25 12

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
h LBL 1	032– 25, 13, 1	+	055– 71
RCL 2	033– 24 2	RCL 6	056– 24 6
RCL 1	034– 24 1	RCL 8	057– 24 8
+	035– 71	×	058– 61
STO 6	036– 23 6	2	059– 2
R/S	037– 74	×	060– 61
RCL 3	038– 24 3	+	061– 51
RCL 1	039– 24 1	STO 9	062– 23 9
+	040– 71	R/S	063– 74
RCL 6	041– 24 6	RCL 5	064– 24 5
g x^2	042– 15 3	RCL 6	065– 24 6
STO 8	043– 23 8	RCL 4	066– 24 4
−	044– 41	×	067– 61
STO 7	045– 23 7	4	068– 4
R/S	046– 74	×	069– 61
RCL 4	047– 24 4	−	070– 41
RCL 3	048– 24 3	RCL 8	071– 24 8
RCL 6	049– 24 6	RCL 3	072– 24 3
×	050– 61	×	073– 61
3	051– 3	6	074– 6
×	052– 61	×	075– 61
−	053– 41	+	076– 51
RCL 1	054– 24 1	RCL 1	077– 24 1

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[+]	078– 71	1	089– 1
[RCL] 8	079– 24 8	[•]	090– 73
[g] [x^2]	080– 15 3	5	091– 5
3	081– 3	[h] [y^x]	092– 25 3
[×]	082– 61	[+]	093– 71
[−]	083– 41	[R/S]	094– 74
[STO] 6	084– 23 6	[RCL] 6	095– 24 6
[R/S]	085– 74	[RCL] 7	096– 24 7
[h] [LBL] 2	086–25, 13, 2	[g] [x^2]	097– 15 3
[RCL] 9	087– 24 9	[+]	098– 71
[RCL] 7	088– 24 7	[R/S]	099– 74

REGISTERS			I
R_0 n	R_1 Σf_i	R_2 $\Sigma f_i x_i$	R_3 $\Sigma f_i x_i^2$
R_4 $\Sigma f_i x_i^3$	R_5 $\Sigma f_i x_i^4$	R_6 x, m_4	R_7 m_2
R_8 $\bar{x}^2$	R_9 m_3	$R_{.0}$–$R_{.4}$ Unused	

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	Initialize the program.		[f] CLEAR [REG]	
	Ungrouped Data			
3	Repeat step 3–4 for i = 1, 2,			
	..., n.			
	Input x_i.	x_i	[A]	i
4	If you made a mistake in			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	inputting x_k, then correct			
	by →	x_k	h SF 0	
			A	$k-1$
5	Go to step 8 for moments			
	calculations.			
	Grouped Data			
6	Repeat step 6–7 for i = 1, 2,			
	..., m.			
	Input x_i	x_i	ENTER↑	
	f_i	f_i	B	i
7	If you made a mistake in			
	inputting x_h and f_h, then			
	correct by →	x_h	ENTER↑	
		f_h	h SF 0	
			B	$h-1$
8	Calculate moments etc: $\bar{x}$		GSB 1	$\bar{x}$
	m_2		R/S	m_2
	m_3		R/S	m_3
	m_4		R/S	m_4
	γ_1		GSB 2	γ_1
	γ_2		R/S	γ_2
9	Repeat step 8 if you want the			
	results again.			
10	For a new case, go to step 2.			

Examples:

1. Ungrouped data

i	1	2	3	4	5	6	7	8	9
x_i	2.1	3.5	4.2	6.5	4.1	3.6	5.3	3.7	4.9

Keystrokes	Display	
f FIX 4		
f CLEAR REG		
2.1 A	1.0000	(i)
4 A	2.0000	(error)
4 h SF 0 A	1.0000	(correction)
3.5 A 4.2 A		
6.5 A 4.1 A		
3.6 A 5.3 A		
3.7 A 4.9 A	9.0000	(n)
GSB 1	4.2111	($\bar{x}$)
R/S	1.3899	(m_2)
R/S	0.3864	(m_3)
R/S	5.4894	(m_4)
GSB 2	0.2358	(γ_1)
R/S	2.8417	(γ_2)

2. Grouped data

i	1	2	3	4	5
x_i	3	2	4	6	1
f_i	4	5	3	2	1

Keystrokes	Display	
f CLEAR REG		
3 ENTER↑ 4 B	1.0000	(i)
5 ENTER↑ 5 B	2.0000	(error)
5 ENTER↑ 5		
h SF 0 B	1.0000	(correction)
2 ENTER↑ 5 B		
4 ENTER↑ 3 B		
6 ENTER↑ 2 B		

1 [ENTER↑] 1 [B]	5.0000	(m)
[GSB] 1	3.1333	($\overline{x}$)
[R/S]	1.9822	(m_2)
[R/S]	2.1381	(m_3)
[R/S]	11.0479	(m_4)
[GSB] 2	0.7661	(γ_1)
[R/S]	2.8117	(γ_2)

Distribution Functions

Normal and Inverse Normal Distribution

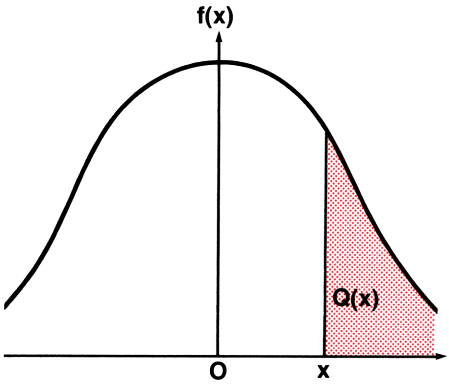
This program evaluates the standard normal density function $f(x)$ and the normal integral $Q(x)$ for given x. If Q is given, x can also be found.

The standard normal distribution has mean 0 and standard deviation 1.

Equations:

1. Standard normal density

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



2. Normal integral

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt$$

The integral is calculated by using the integrate key $\boxed{\int_y^x}$.

3. Inverse normal

For a given $Q > 0$, x can be found such that

$$Q = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{u^2}{2}} du$$

The following rational approximation is used:

Define $y = t - \dfrac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(Q)$

where: $|\epsilon(Q)| < 4.5 \times 10^{-4}$

$$t = \begin{cases} \sqrt{\ln \dfrac{1}{Q^2}} & \text{if } 0 < Q \leqslant 0.5 \\ \sqrt{\ln \dfrac{1}{(1-Q)^2}} & \text{if } 0.5 < Q < 1 \end{cases}$$

$c_0 = 2.515517$ $d_1 = 1.432788$
$c_1 = 0.802853$ $d_2 = 0.189269$
$c_2 = 0.010328$ $d_3 = 0.001308$

Then $x = \begin{cases} y & \text{if } 0 < Q \leqslant 0.5 \\ -y & \text{if } 0.5 < Q < 1 \end{cases}$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1970.

Normal Distribution

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	000–	+	014– 71
h LBL A	001–25, 13, 11	h RTN	015– 25 12
STO 0	002– 23 0	h LBL B	016–25, 13, 12
h LBL 0	003–25, 13, 0	GSB A	017– 13 11
ENTER↑	004– 31	RCL 0	018– 24 0
×	005– 61	CHS	019– 32
2	006– 2	RCL 0	020– 24 0
+	007– 71	f y^x 0	021–14, 72, 0
CHS	008– 32	1	022– 1
g e^x	009– 15 1	x⇄y	023– 21
h π	010– 25 73	−	024– 41
2	011– 2	2	025– 2
×	012– 61	+	026– 71
f $\sqrt{x}$	013– 14 3	R/S	027– 74

REGISTERS		I
R_0 x	R_1—$R_{.9}$ Unused	

Inverse Normal Distribution

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	000–	GTO 0	006– 22 0
h LBL 1	001–25, 13, 1	g R↓	007– 15 22
g x<0	002– 15 41	•	008– 73
GTO 0	003– 22 0	5	009– 5
1	004– 1	x⇄y	010– 21
f x≤y	005– 14 41	f x>y	011– 14 51

KEY ENTRY	DISPLAY
GSB 8	012– 13 8
ENTER↑	013– 31
×	014– 61
h 1/x	015– 25 2
f LN	016– 14 1
f √x	017– 14 3
STO f I	018–23, 14, 23
RCL 2	019– 24 2
×	020– 61
RCL 1	021– 24 1
+	022– 51
RCL f I	023–24, 14, 23
×	024– 61
RCL 0	025– 24 0
+	026– 51
RCL f I	027–24, 14, 23
RCL 5	028– 24 5
×	029– 61
RCL 4	030– 24 4
+	031– 51
RCL f I	032–24, 14, 23

KEY ENTRY	DISPLAY
×	033– 61
RCL 3	034– 24 3
+	035– 51
RCL f I	036–24, 14, 23
×	037– 61
1	038– 1
+	039– 51
÷	040– 71
RCL f I	041–24, 14, 23
x⇄y	042– 21
−	043– 41
h F? 1	044–25, 71, 1
CHS	045– 32
h CF 1	046–25, 61, 1
R/S	047– 74
h LBL 8	048–25, 13, 8
h SF 1	049–25, 51, 1
1	050– 1
−	051– 41
CHS	052– 32

REGISTERS			I t
R_0 c_0	R_1 c_1	R_2 c_2	R_3 d_1
R_4 d_2	R_5 d_3	R_6–$R_{.9}$ Unused	

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	Normal Distribution			
1	Key in the program.			
2	(Optional) Input x to calculate			
	f(x)	x	[A]	f(x)
3	Input x to calculate $Q(x)$	x	[B]	$Q(x)$
4	For a new x, go to step 2.			
	Inverse Normal			
	Distribution			
1	Key in the program.			
2	Store constants for inverse			
	normal distribution.	c_0	[STO] 0	
		c_1	[STO] 1	
		c_2	[STO] 2	
		d_1	[STO] 3	
		d_2	[STO] 4	
		d_3	[STO] 5	
3	Input Q to calculate x.	Q	[GSB] 1	x
4	For a new case, go to step 2.			

Example 1:

Find $f(x)$ and $Q(x)$ for $x = 1.18$ and $x = -2.28$.

Keystrokes	Display		
(Key in normal distribution program)			
[f] [SCI] 2			
1.18 [A]	1.99	-01	($f(x)$)
1.18 [B]	1.19	-01	($Q(x)$)
2.28 [CHS] [B]	9.89	-01	($Q(x)$)
2.28 [CHS] [A]	2.97	-02	($f(x)$)

Example 2:

Given $Q = 0.12$ and $Q = 0.95$, find x.

Keystrokes	Display	
(Key in inverse normal distribution program)		
[f] [FIX] 4		
2.515517 [STO] 0		
0.802853 [STO] 1		
0.010328 [STO] 2		
1.432788 [STO] 3		
0.189269 [STO] 4		
0.001308 [STO] 5		
0.12 [GSB] 1	1.1751	(x)
0.95 [GSB] 1	-1.6452	(x)

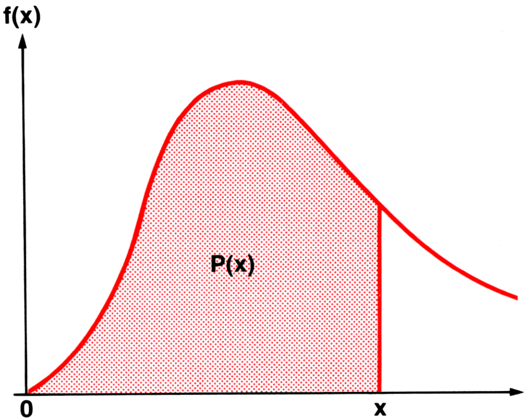
Chi-Square Distribution

This program evaluates the chi-square density

$$f(x) = \frac{1}{2^{\frac{v}{2}}\,\Gamma\left(\frac{v}{2}\right)}\, x^{\frac{v}{2}-1} e^{-\frac{x}{2}}$$

where: $x \geqslant 0$

v is the degrees of freedom.



The integrate key $\boxed{\int_y^x}$ is used to evaluate the cumulative distribution

$$P(x) = \int_0^x f(t)\, dt$$

Notes:

1. Program requires $v < 141$. If $v \geqslant 141$, erroneous overflow will result.

2. If both x and v are large, $x^{\frac{v}{2}-1}$ may overflow the machine.

3. If v is even,

$$\Gamma\left(\frac{v}{2}\right) = \left(\frac{v}{2} - 1\right)!$$

If v is odd,

$$\Gamma\left(\frac{v}{2}\right) = \left(\frac{v}{2} - 1\right)\left(\frac{v}{2} - 2\right) \cdots \left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)$$

4. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1970.

KEY ENTRY	DISPLAY
f CLEAR PRGM	000–
h LBL B	001–25, 13, 12
GSB A	002– 13 11
0	003– 0
RCL 2	004– 24 2
f $\sqrt[x]{y}$ 3	005–14, 72, 3
R/S	006– 74
h LBL A	007–25, 13, 11
STO 2	008– 23 2
x⇄y	009– 21

KEY ENTRY	DISPLAY
1	010– 1
STO 3	011– 23 3
x⇄y	012– 21
2	013– 2
÷	014– 71
STO 1	015– 23 1
h INT	016– 25 32
h LST x	017– 25 0
f x≠y	018– 14 61
GTO 1	019– 22 1

KEY ENTRY	DISPLAY
1	020– 1
−	021– 41
h $x!$	022– 25 1
STO 3	023– 23 3
RCL 2	024– 24 2
GTO 3	025– 22 3
h LBL 1	026– 25, 13, 1
•	027– 73
5	028– 5
f $x=y$	029– 14 71
GTO 2	030– 22 2
$x \gtrless y$	031– 21
1	032– 1
−	033– 41
STO × 3	034– 23, 61, 3
GTO 1	035– 22 1
h LBL 2	036– 25, 13, 2
h π	037– 25 73
f $\sqrt{x}$	038– 14 3
STO × 3	039– 23, 61, 3

KEY ENTRY	DISPLAY
RCL 2	040– 24 2
h LBL 3	041– 25, 13, 3
ENTER↑	042– 31
ENTER↑	043– 31
RCL 1	044– 24 1
1	045– 1
−	046– 41
h y^x	047– 25 3
$x \gtrless y$	048– 21
2	049– 2
÷	050– 71
CHS	051– 32
g e^x	052– 15 1
×	053– 61
2	054– 2
RCL 1	055– 24 1
h y^x	056– 25 3
÷	057– 71
RCL 3	058– 24 3
÷	059– 71

REGISTERS			I
R_0	R_1 $\nu/2$	R_2 x	R_3 1, $\Gamma(\nu/2)$
R_4—$R_{.9}$ Unused			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	(Optional) Input degrees of			
	freedom ν and x to calculate			
	$f(x)$.	ν	ENTER↑	
		x	A	$f(x)$
3	Input degrees of freedom ν			
	and x to calculate $P(x)$.	ν	ENTER↑	
		x	B	$P(x)$
4	For a different case, go to			
	step 2.			

Example 1:

If degrees of freedom $\nu = 20$, find $f(x)$, $P(x)$ for $x = 9.6$ and $x = 15$.

Keystrokes	Display		
f SCI 2			
20 ENTER↑			
9.6 A	1.53	-02	($f(9.6)$)
20 ENTER↑ 9.6 B	2.51	-02	($P(9.6)$)
20 ENTER↑ 15 A	5.72	-02	($f(15)$)
20 ENTER↑ 15 B	2.24	-01	($P(15)$)

Example 2:

If $\nu = 3$, find $P(x)$ for $x = 7.82$.

Keystrokes	Display		
f SCI 2			
3 ENTER↑			
7.82 B	9.50	-01	($P(7.82)$)

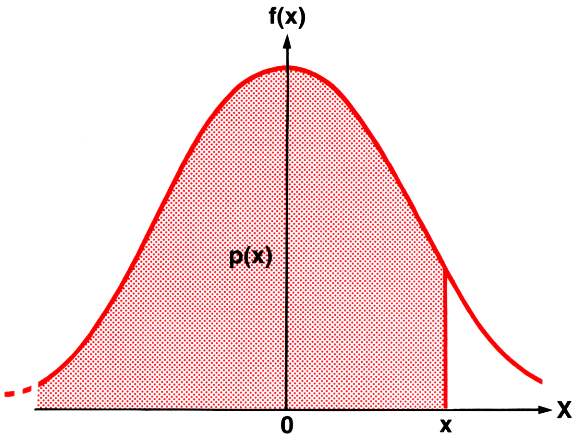
t Distribution

This program evaluates the cumulative distribution $P(x, \nu)$ for a given x and degrees of freedom ν.

Equations:

1. Density function

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$



2. Cumulative distribution function

$$P(x) = \int_{-\infty}^{x} f(y)dy$$

The integral is calculated by using the integrate key $\boxed{\int_y^x}$.

Remark:

The program requires $v < 141$ for $f(x)$, otherwise erroneous overflow will result. (The normal distribution is a good approximation for large v).

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1970.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	000–	STO 3	016– 23 3
h LBL A	001–25, 13, 11	RCL 2	017– 24 2
STO 1	002– 23 1	+	018– 71
x⇄y	003– 21	h π	019– 25 73
STO 0	004– 23 0	RCL 0	020– 24 0
x⇄y	005– 21	×	021– 61
h LBL 0	006–25, 13, 0	f √x	022– 14 3
GSB 1	007– 13 1	+	023– 71
STO 4	008– 23 4	RCL 4	024– 24 4
RCL 0	009– 24 0	×	025– 61
GSB 3	010– 13 3	h RTN	026– 25 12
STO 2	011– 23 2	h LBL 1	027–25, 13, 1
RCL 0	012– 24 0	g x^2	028– 15 3
1	013– 1	RCL 0	029– 24 0
+	014– 51	+	030– 71
GSB 3	015– 13 3	1	031– 1

KEY ENTRY	DISPLAY
[+]	032– 51
[RCL] 0	033– 24 0
1	034– 1
[+]	035– 51
2	036– 2
[÷]	037– 71
[CHS]	038– 32
[h] [y^x]	039– 25 3
[h] [RTN]	040– 25 12
[h] [LBL] 3	041– 25, 13, 3
1	042– 1
[STO] 5	043– 23 5
[x⇄y]	044– 21
2	045– 2
[÷]	046– 71
[h] [INT]	047– 25 32
[h] [LST X]	048– 25 0
[f] [x≠y]	049– 14 61
[GTO] 4	050– 22 4
1	051– 1
[−]	052– 41
[h] [x!]	053– 25 1
[STO] 5	054– 23 5
[h] [RTN]	055– 25 12
[h] [LBL] 4	056– 25, 13, 4
[•]	057– 73
5	058– 5

KEY ENTRY	DISPLAY
[f] [x=y]	059– 14 71
[GTO] 2	060– 22 2
[x⇄y]	061– 21
1	062– 1
[−]	063– 41
[STO] [×] 5	064– 23, 61, 5
[GTO] 4	065– 22 4
[h] [LBL] 2	066– 25, 13, 2
[h] [π]	067– 25 73
[f] [$\sqrt{x}$]	068– 14 3
[STO] [×] 5	069– 23, 61, 5
[RCL] 5	070– 24 5
[h] [RTN]	071– 25 12
[h] [LBL] [B]	072– 25, 13, 12
[GSB] [A]	073– 13 11
[h] [CF] 0	074– 25, 61, 0
[RCL] 1	075– 24 1
[g] [x=0]	076– 15 71
[GTO] 5	077– 22 5
[g] [x<0]	078– 15 41
[h] [SF] 0	079– 25, 51, 0
[CHS]	080– 32
[RCL] 1	081– 24 1
[f] [$\int_y^x$] 0	082– 14, 72, 0
[h] [F?] 0	083– 25, 71, 0
[CHS]	084– 32
[ENTER↑]	085– 31

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
ENTER↑	086– 31	R/S	094– 74
ENTER↑	087– 31	+	095– 51
1	088– 1	R/S	096– 74
x⇄y	089– 21	h LBL 5	097–25, 13, 5
−	090– 41	•	098– 73
2	091– 2	5	099– 5
+	092– 71	R/S	100– 74
h F? 0	093–25, 71, 0		

REGISTERS			I
R_0 ν	R_1 x	R_2 $\Gamma(\nu/2)$	R_3 $\Gamma[(\nu+1)/2]$
R_4 Used	R_5 Used	R_6—$R_{.4}$ Unused	

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	(Optional) Input degrees of			
	freedom ν and x to calculate			
	$f(x, \nu)$.	ν	ENTER↑	
		x	A	$f(x, \nu)$
3	Input degrees of freedom ν and			
	x to calculate $P(x, \nu)$.	ν	ENTER↑	
		x	B	$P(x, \nu)$
4	For a different case, go to			
	step 2.			

Example 1:

Find $f(x, v)$ and $P(x, v)$ for $x = 2.2$, $v = 11$.

Keystrokes	Display		
[f] [SCI] 2			
11 [ENTER↑] 2.2 [A]	4.37	-02	($f(x, v)$)
11 [ENTER↑] 2.2 [B]	9.75	-01	($P(x, v)$)

Example 2:

Find $P(x, v)$ for $x = -1.75$, $v = 30$.

Keystrokes	Display		
[f] [SCI] 2			
30 [ENTER↑]	30.00		(v)
1.75 [CHS] [B]	4.51	-02	($P(x, v)$)

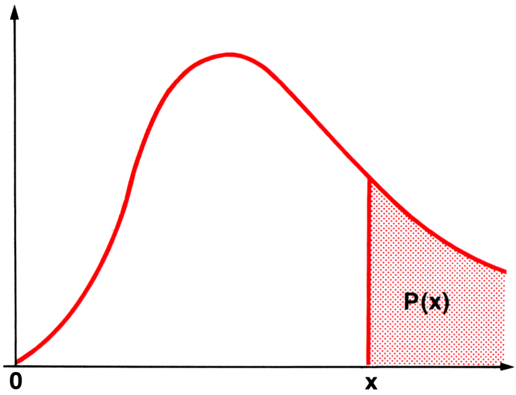
F Distribution

This program evaluates the integral of the F distribution

Equations:

1. Density function

$$f(y) = \frac{\Gamma\left(\frac{v_1 + v_2}{2}\right) y^{\frac{v_1}{2} - 1} \left(\frac{v_1}{v_2}\right)^{\frac{v_1}{2}}}{\Gamma\left(\frac{v_1}{2}\right) \Gamma\left(\frac{v_2}{2}\right) \left(1 + \frac{v_1}{v_2} y\right)^{\frac{v_1 + v_2}{2}}}$$



2. Cumulative distribution function

$$P(x) = \int_x^\infty f(y)dy$$

The integral $P(x)$ is calculated by using the integrate key [$\int_y^x$] .

Note:

Usually v_1 is identified as the degree of freedom for numerator, and v_2 is identified as the degree of freedom for denominator.

KEY ENTRY	DISPLAY
[f] CLEAR [PRGM]	000–
[h] [LBL] [A]	001– 25, 13, 11
[STO] 2	002– 23 2
[g] [R↓]	003– 15 22
[STO] 1	004– 23 1
[x⇄y]	005– 21
[STO] 0	006– 23 0
[GSB] 3	007– 13 3
[STO] 3	008– 23 3
[RCL] 1	009– 24 1
[GSB] 3	010– 13 3
[STO] 4	011– 23 4
[RCL] 0	012– 24 0
[RCL] 1	013– 24 1
[+]	014– 51
[GSB] 3	015– 13 3
[STO] 5	016– 23 5
[RCL] 2	017– 24 2
[h] [LBL] 0	018– 25, 13, 0

KEY ENTRY	DISPLAY
[ENTER↑]	019– 31
[ENTER↑]	020– 31
[ENTER↑]	021– 31
[RCL] 0	022– 24 0
[RCL] 1	023– 24 1
[÷]	024– 71
[STO] 7	025– 23 7
[×]	026– 61
1	027– 1
[+]	028– 51
[RCL] 0	029– 24 0
[RCL] 1	030– 24 1
[+]	031– 51
2	032– 2
[÷]	033– 71
[h] [y^x]	034– 25 3
[x⇄y]	035– 21
[RCL] 0	036– 24 0
2	037– 2

KEY ENTRY	DISPLAY
+	038– 71
1	039– 1
−	040– 41
h yˣ	041– 25 3
x⇄y	042– 21
+	043– 71
RCL 7	044– 24 7
RCL 0	045– 24 0
2	046– 2
+	047– 71
h yˣ	048– 25 3
×	049– 61
RCL 5	050– 24 5
×	051– 61
RCL 3	052– 24 3
+	053– 71
RCL 4	054– 24 4
+	055– 71
h RTN	056– 25 12
h LBL 3	057– 25, 13, 3
1	058– 1
STO 6	059– 23 6
x⇄y	060– 21
2	061– 2
+	062– 71
h INT	063– 25 32
h LST x	064– 25 0

KEY ENTRY	DISPLAY
f x≠y	065– 14 61
GTO 1	066– 22 1
1	067– 1
−	068– 41
h x!	069– 25 1
STO 6	070– 23 6
h RTN	071– 25 12
h LBL 1	072– 25, 13, 1
•	073– 73
5	074– 5
f x=y	075– 14 71
GTO 2	076– 22 2
x⇄y	077– 21
1	078– 1
−	079– 41
STO × 6	080– 23, 61, 6
GTO 1	081– 22 1
h LBL 2	082– 25, 13, 2
h π	083– 25 73
f √x	084– 14 3
STO × 6	085– 23, 61, 6
RCL 6	086– 24 6
h RTN	087– 25 12
h LBL B	088– 25, 13, 12
GSB A	089– 13, 11
0	090– 0
RCL 2	091– 24 2

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[f] [$\sqrt[x]{y}$] 0	092– 14, 72, 0	[CHS]	095– 32
1	093– 1	[R/S]	096– 74
[–]	094– 41		

REGISTERS			I
R_0 ν_1	R_1 ν_2	R_2 x	R_3 $\Gamma(\nu_1/2)$
R_4 $\Gamma(\nu_2/2)$	R_5 $\Gamma[(\nu_1+\nu_2)/2]$	R_6 Used	R_7 ν_1/ν_2
R_8—$R_{.5}$ Unused			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
2	(Optional) Input ν_1, ν_2, and			
	x to calculate $f(x)$.	ν_1	[ENTER↑]	
		ν_2	[ENTER↑]	
		x	[A]	$f(x)$
3	Input ν_1, ν_2 and x to calculate			
	$P(x)$.	ν_1	[ENTER↑]	
		ν_2	[ENTER↑]	
		x	[B]	$P(x)$
4	For a different case, go to			
	step 2.			

Examples:

1. $v_1 = 6,\ v_2 = 7$
   $P(3.87) = 0.0499$

2. $v_1 = 4,\ v_2 = 20$
   $P(2.25) = 0.0999$

Keystrokes	Display		
f SCI 2			
6 ENTER↑ 7 ENTER↑			
3.87 B	4.99	−02	($P(3.87)$)
4 ENTER↑ 20 ENTER↑			
2.25 B	9.99	−02	($P(2.25)$)

Test Statistics

t Statistics

I. Paired t Statistic

Given a set of paired observations from two normal populations with means μ_1, μ_2 (unknown)

x_i	x_1	x_2	$\cdots$	x_n
y_i	y_1	y_2	$\cdots$	y_n

let

$$D_i = x_i - y_i$$

$$\overline{D} = \frac{1}{n} \sum_{i=1}^{n} D_i$$

$$s_D = \sqrt{\frac{\Sigma D_i^2 - \frac{1}{n} (\Sigma D_i)^2}{n - 1}}$$

The test statistic

$$t = \frac{\overline{D}}{s_D} \sqrt{n}$$

which has $n - 1$ degrees of freedom (*df*) can be used to test the null hypothesis

$$H_0\colon\ \mu_1 = \mu_2$$

II. t Statistic for Two Means

Suppose $\{x_1, x_2, \ldots, x_{n1}\}$ and $\{y_1, y_2, \ldots, y_{n2}\}$ are independent random samples from two normal populations having means μ_1, μ_2 (unknown) and the same unknown variance σ^2.

We want to test the null hypothesis

$$H_0\colon\ \mu_1 - \mu_2 = d$$

Define

$$\bar{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

$$\bar{y} = \frac{1}{n_2} \sum_{i=1}^{n_2} y_i$$

$$t = \frac{\bar{x} - \bar{y} - d}{\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}\ \sqrt{\dfrac{\Sigma x_i^2 - n_1\bar{x}^2 + \Sigma y_i^2 - n_2\bar{y}^2}{n_1 + n_2 - 2}}}$$

We can use this t statistic which has the t distribution with $n_1 + n_2 - 2$ degrees of freedom (df) to test the null hypothesis H_0.

References:

1. Statistics in Research, B. Ostle, Iowa State University Press, 1963.
2. Statistical Theory and Methodology in Science and Engineering, K. A. Brownlee, John Wiley & Sons, 1965.

KEY ENTRY	DISPLAY
f CLEAR PRGM	000–
h LBL A	001– 25, 13, 11
÷	002– 41
f Σ+	003– 14 74
R/S	004– 74

KEY ENTRY	DISPLAY
h LBL 1	005– 25, 13, 1
h x̄	006– 25 8
STO 6	007– 23 6
R/S	008– 74
h s	009– 25 9

KEY ENTRY	DISPLAY
R/S	010– 74
RCL 6	011– 24 6
RCL 0	012– 24 0
f √x	013– 14 3
×	014– 61
x⇄y	015– 21
÷	016– 71
R/S	017– 74
RCL 0	018– 24 0
1	019– 1
−	020– 41
R/S	021– 74
GTO 1	022– 22 1
h LBL B	023– 25, 13, 12
f Σ+	024– 14 74
R/S	025– 74
RCL 1	026– 24 1
STO • 1	027– 23 .1
RCL 2	028– 24 2
STO • 2	029– 23 .2
RCL 0	030– 24 0
STO • 0	031– 23 .0
0	032– 0
f CLEAR Σ	033– 14 34
R/S	034– 74
h LBL 2	035– 25, 13, 2
STO 6	036– 23 6

KEY ENTRY	DISPLAY
RCL • 1	037– 24 .1
RCL • 2	038– 24 .2
STO 4	039– 23 4
x⇄y	040– 21
STO 3	041– 23 3
h x̄	042– 25 8
STO 8	043– 23 8
x⇄y	044– 21
RCL 0	045– 24 0
×	046– 61
RCL • 0	047– 24 .0
÷	048– 71
STO 7	049– 23 7
x⇄y	050– 21
−	051– 41
RCL 6	052– 24 6
−	053– 41
RCL 4	054– 24 4
RCL 3	055– 24 3
RCL 7	056– 24 7
×	057– 61
−	058– 41
RCL 2	059– 24 2
+	060– 51
RCL 1	061– 24 1
RCL 8	062– 24 8
×	063– 61

KEY ENTRY	DISPLAY
[−]	064– 41
[RCL] 0	065– 24 0
[RCL] [•] 0	066– 24 .0
[+]	067– 51
2	068– 2
[−]	069– 41
[STO] 9	070– 23 9
[÷]	071– 71
[f] [√x]	072– 14 3
[RCL] 0	073– 24 0
[h] [1/x]	074– 25 2

KEY ENTRY	DISPLAY
[RCL] [•] 0	075– 24 .0
[h] [1/x]	076– 25 2
[+]	077– 51
[f] [√x]	078– 14 3
[×]	079– 61
[÷]	080– 71
[R/S]	081– 74
[RCL] 9	082– 24 9
[R/S]	083– 74
[GTO] 2	084– 22 2

REGISTERS			I
R_0 d	R_1 x	R_2 y	R_3 df
R_4 Σx, Σy	R_5 Σx^2, Σy^2	R_6 Σy Σx	R_7 Σy^2, Σx^2
R_8 Σxy	R_9 n_1, n_2	$R_{.0}$ n_1	$R_{.1}$ Σx
$R_{.2}$ Σx^2	$R_{.3}$–$R_{.7}$ Unused		

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	Paired t Statistics			
1	Key in the program from			
	line 1–line 22.			
2	Initialize the program.		[f] CLEAR [REG]	
3	Repeat steps 3–4 for i = 1, 2,			
	…, *n*.			
	Input: x_i	x_i	[ENTER↑]	
	y_i	y_i	[A]	i
4	If you made a mistake in			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	inputting x_k and y_k, then			
	correct by →	x_k	ENTER↑	
		y_k	− g Σ−	$k-1$
5	To calculate test statistics:			
	$\overline{D}$		GSB 1	$\overline{D}$
	S_D		R/S	S_D
	t		R/S	t
	df		R/S	df
6	Repeat step 5 if you want the			
	results again.			
	t Statistics for			
	Two Means			
1	Key in the program from			
	line 23–line 84.			
2	Initialize the program.		f CLEAR REG	
3	Repeat step 3–4 for i = 1, 2,			
	…, n_1.			
	Input x_i.	x_i	B	i
4	If you made a mistake in			
	inputting x_k, then correct			
	by →	x_k	g Σ−	$k-1$
5	Initialize for the 2nd array			
	of data.		R/S	0.0000
6	Repeat step 6–7 for i = 1, 2,			
	…, n_2.			
	Input y_i.	y_i	B	i
7	If you made a mistake in			
	inputting y_k, then correct			
	by →	y_k	g Σ−	$k-1$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
8	Input *d* to calculate test			
	statistic: *t*	*d*	[GSB] 2	*t*
	df		[R/S]	*df*
9	Repeat step 8 if you want			
	the results again.			

Example 1:

x_i	14	17.5	17	17.5	15.4
y_i	17	20.7	21.6	20.9	17.2

$$\bar{D} = -3.2000$$
$$s_D = 1.0000$$
$$t = -7.1554$$
$$df = 4.0000$$

Keystrokes	Display	
[f] [FIX] 4		
[f] CLEAR [REG]		
14 [ENTER↑] 17 [A]	1.0000	
17 [ENTER↑] 15 [A]	2.0000	(error)
17 [ENTER↑] 15 [−]		
[g] [Σ−]	1.0000	(correction)
17.5 [ENTER↑] 20.7 [A]		
17 [ENTER↑] 21.6 [A]		
17.5 [ENTER↑] 20.9 [A]		
15.4 [ENTER↑] 17.2 [A]	5.0000	
[GSB] 1	−3.2000	($\bar{D}$)
[R/S]	1.0000	(s_D)
[R/S]	−7.1554	(t)
[R/S]	4.0000	(df)

Example 2:

x	79	84	108	114	120	103	122	120		
y	91	103	90	113	108	87	100	80	99	54

$n_1 = 8$
$n_2 = 10$
If $d = 0$ (i.e., H_0: $\mu_1 = \mu_2$)
then $t = 1.73$, $df = 16.00$

Keystrokes	**Display**	
[f] CLEAR [REG]		
79 [B] 84 [B] 99		
[B] 99 [g] [Σ-]	2.0000	
108 [B] 114 [B] 120		
[B] 103 [B] 122		
[B] 120 [B]	8.0000	
[R/S]	0.0000	
91 [B] 103 [B] 90		
[B] 113 [B] 108		
[B] 87 [B] 100 [B]		
80 [B] 99 [B] 54 [B]	10.0000	
0 [GSB] 2	1.7316	(t)
[R/S]	16.0000	(df)

Chi-Square Evaluation

This program calculates the value of the χ^2 statistic for the goodness of fit test by the equation

$$\chi_1^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i = observed frequency

E_i = expected frequency

If the expected values are equal

$$\left(E = E_i = \frac{\Sigma O_i}{n} \text{ for all } i\right)$$

then

$$\chi_2^2 = \frac{n \Sigma O_i^2}{\Sigma O_i} - \Sigma O_i$$

Note:

In order to apply the goodness of fit test to a set of given data, combining some classes may be necessary to make sure that each expected frequency is not too small (say, not less than 5).

Reference:

Mathematical Statistics, J. E. Freund, Prentice Hall, 1962.

KEY ENTRY	DISPLAY
f CLEAR PRGM	000–
h LBL A	001–25, 13, 11
ENTER↑	002– 31
g R↓	003– 15 22
−	004– 41
g x^2	005– 15 3
f R↑	006– 14 22
+	007– 71
h F? 0	008–25, 71, 0
CHS	009– 32
STO + 1	010–23, 51, 1
1	011– 1
h F? 0	012–25, 71, 0
CHS	013– 32
STO + 0	014–23, 51, 0
RCL 0	015– 24 0
h CF 0	016–25, 61, 0
R/S	017– 74
h LBL 1	018–25, 13, 1
RCL 1	019– 24 1

KEY ENTRY	DISPLAY
R/S	020– 74
GTO 1	021– 22 1
h LBL B	022–25, 13, 12
f Σ+	023– 14 74
R/S	024– 74
h LBL 2	025–25, 13, 2
RCL 0	026– 24 0
RCL 0	027– 24 0
RCL 2	028– 24 2
×	029– 61
RCL 1	030– 24 1
÷	031– 71
RCL 1	032– 24 1
−	033– 41
R/S	034– 74
RCL 1	035– 24 1
RCL 0	036– 24 0
+	037– 71
R/S	038– 74
GTO 2	039– 22 2

REGISTERS			I
R_0 Index	R_1 x_2^2, ΣO_i	R_2 ΣO_i^2	R_3 Used
R_4 Used	R_5 Used	R_6 Unused	R_7—$R_{.9}$ Unused

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	Unequal Expected			
	Frequency			
1	Key in the program from			
	line 1–line 21.			
2	Initialize the program.		[f] CLEAR [REG]	
3	Repeat steps 3–4 for i = 1, 2,			
	..., n.			
	Input: O_i	O_i	[ENTER↑]	
	E_i.	E_i	[A]	i
4	If you made a mistake in			
	inputting O_k and E_k, then			
	correct by →	O_k	[ENTER↑]	
		E_k	[h] [SF] 0 [A]	$k-1$
5	Calculate χ_1^2.		[GSB] 1	χ_1^2
6	Repeat step 5 if you want			
	the result again.			
	Equal Expected			
	Frequency			
1	Key in the program from			
	line 22–line 39.			
2	Initialize the program.		[f] CLEAR [REG]	
3	Repeat steps 3–4 for i = 1, 2,			
	..., n.			
	Input: O_i	O_i	[B]	i
4	If you made a mistake in			
	inputting O_h, then correct			
	by →	O_h	[g] [Σ-]	$h-1$
5	Calculate: χ_2^2		[GSB] 2	χ_2^2
	E		[R/S]	E

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
6	Repeat step 5 if you want			
	the results again.			

Example 1:

Find the value of χ^2 statistic for the goodness of fit for the following data set:

O_i	8	50	47	56	5	14
E_i	9.6	46.75	51.85	54.4	8.25	9.15

$\chi_1^2 = 4.8444$ df = 5

Keystrokes	Display	
[f] CLEAR [REG]		
8 [ENTER↑] 9.6 [A]		
50 [ENTER↑] 46.75 [A]		
47 [ENTER↑] 51.85 [A]		
56 [ENTER↑] 54.4 [A]		
100 [ENTER↑] 100 [A]	5.0000	(error)
100 [ENTER↑] 100		
[h] [SF] 0 [A]	4.0000	(correction)
5 [ENTER↑] 8.25 [A]		
14 [ENTER↑] 9.15 [A]	6.0000	
[GSB] 1	4.8444	(χ_1^2)

Example 2:

The following table shows the observed frequencies in tossing a die 120 times. χ^2 can be used to test if the die is fair.

Note:

Assume that the expected frequencies are equal.

number	1	2	3	4	5	6
frequency O_i	25	17	15	23	24	16

$\chi_2^2 = \ 5.00 \qquad \text{df} = 5$

$E \ \ = 20.00$

Keystrokes	**Display**	
f CLEAR REG		
25 B 17 B 19 B	3.0000	(error)
19 g Σ-	2.0000	(correction)
15 B 23 B 24 B		
16 B	6.0000	
GSB 2	5.0000	(χ_2^2)
R/S	20.0000	(E)

Since 5.00 is less than 11.07 (5% significance level), no actual differences exist between the observed and expected frequencies.

Contingency Table

Contingency tables can be used to test the null hypothesis that two variables are independent.

This program calculates the χ^2 statistic for testing the independence of the two variables. Also Pearson's coefficient of contingency C_c, which measures the degree of association between the two variables, is calculated.

I. 2 x *k* Contingency Table

$i \backslash j$	1	2	...	k	Totals
1	x_{11}	x_{12}	...	x_{1k}	R_1
2	x_{21}	x_{22}	...	x_{2k}	R_2
Totals	C_1	C_2	...	C_k	T

II. 3 x *k* Contingency Table

$i \backslash j$	1	2	...	k	Totals
1	x_{11}	x_{12}	...	x_{1k}	R_1
2	x_{21}	x_{22}	...	x_{2k}	R_2
3	x_{31}	x_{32}	...	x_{3k}	R_3
Totals	C_1	C_2	...	C_k	T

Equations:

$$\text{Row sum } R_i = \sum_{j=1}^{k} x_{ij} \qquad \begin{array}{l} i = 1, 2 \text{ (for } 2 \times k) \\ i = 1, 2, 3 \text{ (for } 3 \times k) \end{array}$$

$$\text{Column sum } C_j = \sum_{i=1}^{n} x_{ij} \qquad \begin{array}{l} j = 1, 2, \ldots, k \\ n = 2 \text{ (for } 2 \times k) \\ n = 3 \text{ (for } 3 \times k) \end{array}$$

$$\text{Total } T = \sum_{i=1}^{n} \sum_{j=1}^{k} x_{ij} \qquad \begin{array}{l} n = 2 \text{ (for } 2 \times k) \\ n = 3 \text{ (for } 3 \times k) \end{array}$$

Chi-square statistic

$$\chi^2 = \sum_{i=1}^{n} \sum_{j=1}^{k} \frac{(x_{ij} - E_{ij})^2}{E_{ij}} \qquad df = (n - 1)(k - 1)$$

$$= T \left(\sum_{i=1}^{n} \sum_{j=1}^{k} \frac{x_{ij}^2}{R_i C_j} \right) - T \qquad \begin{array}{l} n = 2 \text{ (for } 2 \times k) \\ n = 3 \text{ (for } 3 \times k) \end{array}$$

Contingency coefficient

$$C_c = \sqrt{\frac{\chi^2}{T + \chi^2}}$$

Reference:

B. Ostle, *Statistics in Research*, Iowa State University Press, 1972.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	000-	GSB 9	012- 13 9
h LBL 0	001-25, 13, 0	R/S	013- 74
f CLEAR REG	002- 14 33	RCL 7	014- 24 7
h CF 0	003-25, 61, 0	R/S	015- 74
h CF 1	004-25, 61, 1	h LBL 1	016-25, 13, 1
0	005- 0	h F? 1	017-25, 71, 1
R/S	006- 74	1	018- 1
h LBL B	007-25, 13, 12	h F? 1	019-25, 71, 1
h SF 0	008-25, 51, 0	STO 3	020- 23 3
h LBL A	009-25, 13, 11	RCL 4	021- 24 4
h F? 1	010-25, 71, 1	RCL 1	022- 24 1
0	011- 0	+	023- 71

KEY ENTRY	DISPLAY
STO 9	024– 23 9
RCL 5	025– 24 5
RCL 2	026– 24 2
+	027– 71
STO + 9	028–23, 51, 9
RCL 6	029– 24 6
RCL 3	030– 24 3
+	031– 71
STO + 9	032–23, 51, 9
RCL 9	033– 24 9
1	034– 1
−	035– 41
RCL 0	036– 24 0
×	037– 61
R/S	038– 74
ENTER↑	039– 31
ENTER↑	040– 31
RCL 0	041– 24 0
÷	042– 51
+	043– 71
f √x	044– 14 3
R/S	045– 74
h LBL 2	046–25, 13, 2
RCL 1	047– 24 1
R/S	048– 74
RCL 2	049– 24 2
R/S	050– 74
h F? 1	051–25, 71, 1
GTO 3	052– 22 3

KEY ENTRY	DISPLAY
RCL 3	053– 24 3
R/S	054– 74
h LBL 3	055–25, 13, 3
RCL 0	056– 24 0
R/S	057– 74
GTO 1	058– 22 1
h LBL 9	059–25, 13, 9
h F? 0	060–25, 71, 0
CHS	061– 32
STO + 3	062–23, 51, 3
STO + 0	063–23, 51, 0
STO 7	064– 23 7
g x^2	065– 15 3
STO 8	066– 23 8
g R↓	067– 15 22
h F? 0	068–25, 71, 0
CHS	069– 32
STO + 2	070–23, 51, 2
STO + 0	071–23, 51, 0
STO + 7	072–23, 51, 7
g x^2	073– 15 3
STO 9	074– 23 9
g R↓	075– 15 22
h F? 0	076–25, 71, 0
CHS	077– 32
STO + 1	078–23, 51, 1
STO + 0	079–23, 51, 0
STO + 7	080–23, 51, 7
g x^2	081– 15 3

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
RCL 7	082– 24 7	+	091– 71
+	083– 71	STO + 6	092–23, 51, 6
STO + 4	084–23, 51, 4	RCL f I	093–24, 14, 23
RCL 9	085– 24 9	1	094– 1
RCL 7	086– 24 7	h F? 0	095–25, 71, 0
+	087– 71	CHS	096– 32
STO + 5	088–23, 51, 5	+	097– 51
RCL 8	089– 24 8	STO f I	098–23, 14, 23
RCL 7	090– 24 7	h CF 0	099–25, 61, 0

REGISTERS			I Index
R_0 T	R_1 R_1	R_2 R_2	R_3 R_3
R_4 $\Sigma x_{1j}^2/c_j$	R_5 $\Sigma x_{2j}^2/c_j$	R_6 $\Sigma x_{3j}^2/c_j$	R_7 c_j
R_8 x_{3j}^2	R_9 x_{2j}^2	$R_{.0}$–$R_{.4}$ Unused	

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program.			
	2xk (go to step 8 for *3xk*).			
2	Initialize the program.		GSB 0	
			h SF 1	
3	Repeat steps 3–6 for j = 1, 2,			
	…, k.			
	Input: x_{1j}	x_{1j}	ENTER↑	
	x_{2j}	x_{2j}	A	j
4	(OPTIONAL) Calculate			
	column sum C_j.		R/S	C_j

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
5	If you made a mistake in			
	inputting x_{1h} and x_{2h}, then			
	correct by →	x_{1h}	[ENTER↑]	
		x_{2h}	[B]	$h-1$
6	(OPTIONAL) Calculate			
	column sum C_h.			
	(correction)		[R/S]	$-C_h$
7	Go to step 13 for contingency			
	table calculations.			
	3xk			
8	Initialize the program.		[GSB] 0	
9	Repeat steps 9-12 for j = 1, 2,			
	..., k.			
	Input: x_{1j}	x_{1j}	[ENTER↑]	
	x_{2j}	x_{2j}	[ENTER↑]	
	x_{3j}	x_{3j}	[A]	j
10	(OPTIONAL) Calculate			
	column sum C_j.		[R/S]	C_j
11	If you made a mistake in			
	inputting x_{1h}, x_{2h}, and x_{3h},			
	then correct by →	x_{1h}	[ENTER↑]	
		x_{2h}	[ENTER↑]	
		x_{3h}	[B]	$(h-1)$
12	(OPTIONAL) Calculate			
	column sum C_h (correction).		[R/S]	C_h
13	Calculate: Test Statistics χ^2		[GSB] 1	χ^2
	Coefficients C_c		[R/S]	C_c
	Row Sum 1 R_1		[GSB] 2	R_1
	Row Sum 2 R_2		[R/S]	R_2

STEP	INSTRUCTION	KEYS	INPUTS DATA/UNITS	OUTPUT DATA/UNITS
	Row Sum 3 R_3 (3xk only)		[R/S]	R_3
	Total T		[GSB] 3	T
14	Repeat step 13 if you want			
	the results again.			
15	For another case, go to step 2			
	or step 8.			

Example 1:

i \ j	1	2	3
A	2	5	4
B	3	8	7

Keystrokes:	Display:	
[GSB] 0 [h] [SF] 1	0.0000	
2 [ENTER↑] 3 [A]		
5 [ENTER↑] 8 [A]		
6 [ENTER↑] 9 [A]	3.0000	(error)
[R/S]	15.0000	(C_3)
6 [ENTER↑] 9 [B]	2.0000	(correction)
[R/S]	–15.0000	($-C_3$)
4 [ENTER↑] 7 [A]	3.0000	
[GSB] 1	0.0221	(x^2)
[R/S]	0.0276	(C_c)
[GSB] 2	11.0000	(R_1)
[R/S]	18.0000	(R_2)
[GSB] 3	29.0000	(T)

Example 2:

Find test statistic x^2 and coefficient of contingency C_c for the following set of data

i \ j	1	2	3	4
1	36	67	49	58
2	31	60	49	54
3	58	87	80	68

Keystrokes:	Display:	
GSB 0	0.0000	
36 ENTER↑ 31 ENTER↑		
58 A	1.0000	
67 ENTER↑ 60 ENTER↑		
87 A	2.0000	
4 ENTER↑ 49 ENTER↑		
80 A	3.0000	(error)
R/S	133.0000	(C_3)
4 ENTER↑ 49 ENTER↑		
80 B	2.0000	
R/S	-133.0000	($-C_3$)
49 ENTER↑ 49 ENTER↑		
80 A	3.0000	
58 ENTER↑ 54 ENTER↑		
68 A	4.0000	
GSB 1	3.3574	(χ^2)
R/S	0.0692	(C_c)
GSB 2	210.0000	(R_1)
R/S	194.0000	(R_2)
R/S	293.0000	(R_3)
GSB 3	697.0000	(T)

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00034-90033 Rev. D

Printed in U.S.A. 3/82