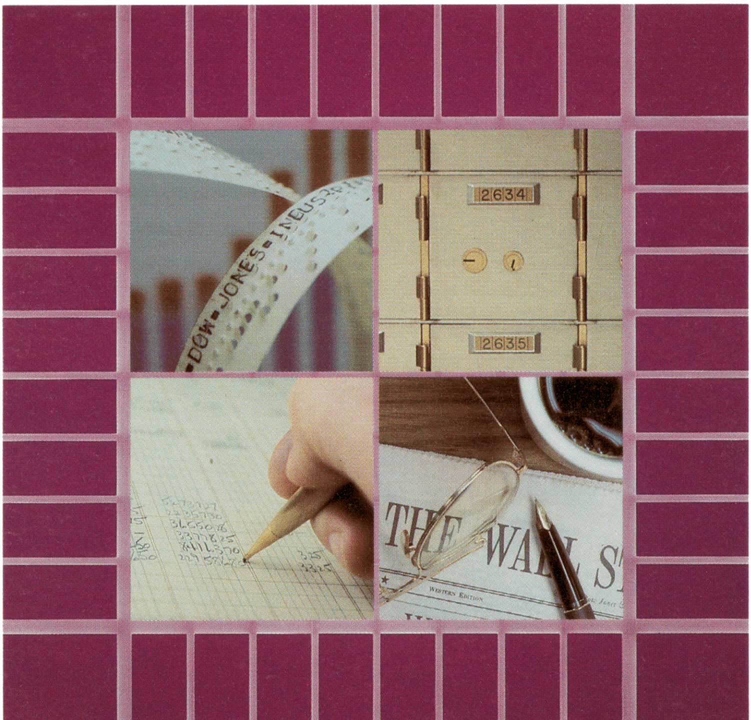
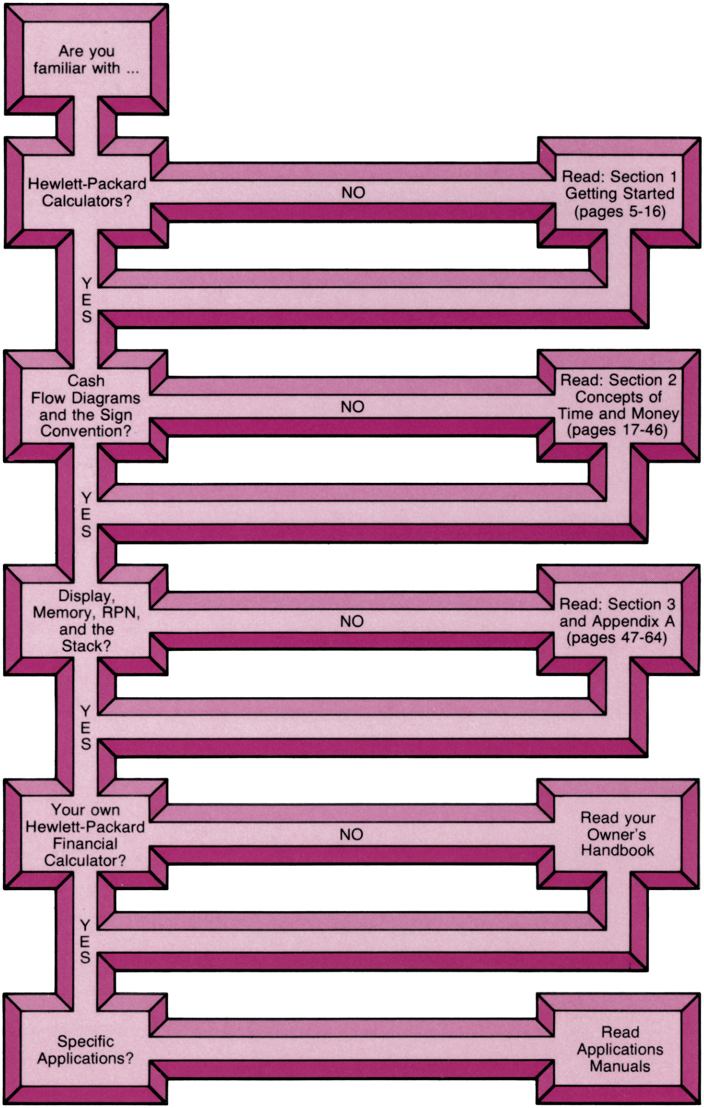
HEWLETT-PACKARD

YOUR HP FINANCIAL CALCULATOR: AN INTRODUCTION TO FINANCIAL CONCEPTS AND PROBLEM SOLVING




Are you familiar with ...
Hewlett-Packard Calculators?
NO
Read: Section 1 Getting Started (pages 5-16)
YES
Cash Flow Diagrams and the Sign Convention?
NO
Read: Section 2 Concepts of Time and Money (pages 17-46)
YES
Display, Memory, RPN, and the Stack?
NO
Read: Section 3 and Appendix A (pages 47-64)
YES
Your own Hewlett-Packard Financial Calculator?
NO
Read your Owner's Handbook
YES
Specific Applications?
Read Applications Manuals

HEWLETT
PACKARD

Your HP Financial Calculator:

An Introduction to Financial Concepts and Problem Solving

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Contents

Introduction

At last, you have an aid to your financial problem solving; a professional-quality business calculator from Hewlett-Packard. Realtors, investment counselors, mortgage and commercial bankers, stockbrokers, students of business, and managers, have asked for:

- **A business calculator capable of solving complex financial problems with up to five financial values included in the computation.** You simply key in the known data in a business problem, using the sign convention, and the calculator solves for the unknown value. No more hunting through tables, schedules, and supplementary books, no more long hours of problem solving by hand or days of waiting for computer outputs—just one calculator to help you find the answers.
- **A financial calculator capable of solving complex mathematical and statistical problems, with the most efficient method available today.** You work every problem just as you would with paper and pencil—but the calculator takes care of the longhand and automatically holds answers until you are ready to use them. If you are new to HP calculators, take the time to read this book thoroughly. Once through, you may never have to refer to it again, leaving you free to concentrate on solving problems.
- **A business machine so easy to use, so simple and powerful, you'll never be content with another system.** Due to RPN, the logic system used in HP calculators, there are just a few simple rules of operation that apply every time you solve a problem—rules that are consistent and unambiguous. RPN enables you to see all intermediate answers, to recover easily from errors, to reuse numbers without reentering them, to solve problems one step at a time, and to evaluate virtually any expression without copying and remembering parentheses.

All of these desires have been incorporated in the design of your HP financial calculator. This book will supplement your owner's handbook and applications manuals to lead you down the path of financial problem solving. So you see that you've made a good investment—one that will help you make other good investments.

Section 1

Getting Started

Power On

You can begin using your Hewlett-Packard financial calculator immediately by connecting the ac adapter/recharger to the calculator and plugging the recharger into an outlet. If you want to use your calculator on battery power alone, you may have to charge the battery. The battery will be fully charged after 6 hours on the recharger. Whether you operate from battery power or from the ac adapter/recharger, *the battery must always be in the calculator*. The battery is never in danger of being overcharged, even if it gets warm.

Feel at ease with your calculator. Remember, *you can't hurt it,* even by pressing improper key sequences. If the word **Error** appears on your display, press any key to clear the error. The key pressed restores the calculator to the condition that existed prior to the error. If you are unsure of what caused the error, refer to your owner's handbook for error conditions.

To begin, turn the power switch to ON. The display reads **0.00**.

Keying In Numbers

Key in numbers by pressing the number keys in sequence just as though you were writing on a piece of paper. The decimal point must be keyed in if it is part of the number (unless it is to the right of the last digit).

For example, key in 1,234,567.89

Keystrokes	Display
1234567.89	1,234,567.89

The number 1,234,567.89 appears in the display. Notice that commas are automatically inserted for you. Numbers can be read quickly and easily.

Negative Numbers

To key in a negative number press the keys for the number, then press [CHS] *(change sign)*. The number preceded by a minus (−) sign will appear in the display. For example, change the sign of the number now in the display.

Keystroke	**Display**
[CHS]	**-1,234,567.89**

Now change it back to a positive number:

Keystroke	**Display**
[CHS]	**1,234,567.89**

You can change the sign of either a negative or a positive number in the display. Notice that only negative numbers are given a sign (−).

Clearing

There are several clearing options available with your financial calculator. In this manual, we shall use the following mnemonics for the clearing keys: [CLX] *(clear x)*, [CL ALL] *(clear all)*, and [CL FIN] *(clear finance)*. Although the mnemonics of the clearing keys may be different on your calculator (depending on the model), the clearing functions are equivalent. Refer to your owner's handbook for the full range of clearing functions.

Press [CLX] *(clear x)* to clear only the *display*. This key erases the number in the display and replaces it with **0.00** (zero). If you make a mistake while keying in a number, press [CLX] to clear the entire string of digits. Then key in the correct number.

It isn't necessary to clear the display between arithmetic calculations. Previous data is automatically pushed out of the way when you begin a new step.

Press [CL ALL] (or CLEAR [ALL]) to clear the entire calculator.

Press [CL FIN] (or CLEAR [FIN]) to erase previous financial values and replace them with 0.00 (zero). We will discuss [CL FIN] further in section 2.

Performing Simple Arithmetic

Whenever you add, subtract, multiply, or divide, you work with two numbers and an arithmetic operator (+, −, ×, or ÷). Before you can perform any of these operations, two numbers must be present in the calculator, just as two numbers must be written down on paper before you can perform an arithmetic operation by hand. The operation is performed as soon as you press the operation key. To perform simple arithmetic:

1. Key in the first number.
2. Press ENTER↑ to separate the first number from the second.
3. Key in the second number.
4. Press +, −, ×, or ÷ to perform the operation.

All arithmetic operations are performed in the same way:

Solve	Keystrokes	Display
13 + 2	13 ENTER↑ 2 +	15.00
13 − 2	13 ENTER↑ 2 −	11.00
13 × 2	13 ENTER↑ 2 ×	26.00
13 ÷ 2	13 ENTER↑ 2 ÷	6.50

In the problems above, you pressed 13 ENTER↑ 2. Try the same number sequence *without* the ENTER↑ step. What appears in the display? It is readily apparent that the ENTER↑ key separates the first number from the second. Now clear the display for the next example by pressing CLX.

Exchange Key

Suppose you want to find the total number of units sold by Telemat Company during the first two months of this year. There were 1,563,830 units sold the first month and 1,872,434 sold the second. After keying in the two numbers (using the ENTER↑ key as a separator), you suddenly realize that you may have keyed in the first number incorrectly. As usual you can see the last number on the display. Is there any way you can check the first number *before* pressing the + key?

Yes, by pressing the [x⇄y] *(exchange)* key. The exchange key exchanges the two numbers so that the display shows the first number entered. With the exchange key, you can review both numbers before you perform an arithmetic operation. Try it:

Keystrokes	Display
1563830 [ENTER↑]	1,563,830.00
1872434	1,872,434.

Did you key in the first number correctly? Check by pressing the exchange key:

Keystroke	Display
[x⇄y]	1,563,830.00

If the first number was keyed in correctly, you can add by pressing [+]. If not, press [CLX] to clear the display, key in the correct number, then add.

Keystroke	Display
[+]	3,436,264.00

Example: Suppose you wish to subtract $25.83 from $144.25. By mistake you enter $25.83 first, pressing 25.83 [ENTER↑] 144.25. There's no need to clear the display and start again. Simply press the exchange key [x⇄y] to reverse the order of the numbers in the calculator, and subtract:

Keystrokes	Display
25.83 [ENTER↑] 144.25	144.25
[x⇄y]	25.83
[−]	118.42

The order of numbers is important in subtraction and division. But you never have to worry about entering the numbers in the wrong order. By pressing the exchange key you can change the order. You will see later that this becomes an extremely helpful function when doing long financial calculations.

Chain Calculations

You don't have to press [ENTER↑] after every number that you key in. A number may be followed by a function key, such as [÷], [%], or [√x]. All keys that execute operations on numbers are function keys. So function keys are also separators between numbers. When you press a function key, the calculator "knows" that you have finished an operation and is ready for a new number to be keyed in. The only time you need to use the [ENTER↑] key as a separator is when you want to *key in* two numbers, one immediately following the other, enabling you to perform an operation on both numbers.

Example: Let's say you've written three checks without updating your checkbook and you've just deposited your paycheck for $1,053.00 into your checking account. Assuming that your last balance was $58.33 and your checks were written for $22.95, $13.70, and $10.14, how much do you have in your account?

Press [ENTER↑] to separate the first number from the second and subtract. The intermediate result is displayed. You do not have to press [ENTER↑] before you key in the next number because the calculator "knows" that a new number will be keyed in after an arithmetic operation has been performed. Since intermediate results are automatically held by the calculator, you simply key in the next number and press the proper function key.

Keystrokes	Display
58.33 [ENTER↑]	58.33
22.95 [−]	35.38
13.70 [−]	21.68
10.14 [−]	11.54
1053 [+]	1,064.54

Let's look at an equation for what you've done:

$$58.33 - 22.95 - 13.70 - 10.14 + 1{,}053 = 1{,}064.54$$

or:

```
    58.33
   −22.95
  ---------
    35.38
   −13.70
  ---------
    21.68
   −10.14
  ---------
    11.54
+1,053.00
  ---------
 1,064.54
```

- You did the arithmetic just as you would when you balance your checkbook, but the calculator did the calculating and held the intermediate results for you.
- The intermediate results in the running checkbook balance are the same intermediate results that are displayed (and automatically held) in your calculator.

Let's look at another example in chain arithmetic.

Example: Over the course of a month you drive the following distances between fill-ups: 260 miles, 315 miles, 275 miles, 342 miles. In driving those distances you use 12.7 gallons, 13.7 gallons, 13.2 gallons and 14.8 gallons of gasoline. What is the average number of miles per gallon obtained by your car? This problem involves dividing the sum of the distances by the sum of the gallons of gasoline used.

If one were to write the problem down it would look something like:

$$\frac{260 + 315 + 275 + 342}{12.7 + 13.7 + 13.2 + 14.8} = \frac{1{,}192}{54.40} = 21.91$$

Solving this problem with pencil and paper involves summing the dividend (the top half of the problem), summing the divisor (the bottom half of the problem), and then dividing.

The problem is solved the same way using your calculator.

First add the top half:

Keystrokes	**Display**	
260 [ENTER↑] 315 [+]		Use [ENTER↑] to separate the 1st number from the 2nd.
275 [+] 342 [+]	*1,192.00*	Then add.

Now add the bottom half of the problem. Don't worry about the first result (1,192). The calculator will automatically hold it until you are ready to divide.

Since another pair of numbers must be keyed in before you can add, you must use the [ENTER↑] key again to separate the first two numbers.

Keystrokes	Display
12.7 [ENTER↑] 13.7 [+]	
13.2 [+] 14.8 [+]	54.40

Now you are ready to divide.

If you doubt that the calculator is still holding the first result, press [x⇄y] *(exchange)*. Be sure to change it back by pressing [x⇄y] again so that division will take place in the proper order. The number in the display is always the divisor.

Keystrokes	Display	
[x⇄y]	1,192.00	First intermediate result (total miles travelled).
[x⇄y]	54.40	Second intermediate result (total gallons used).
[÷]	21.91	Miles per gallon.

Your car gets 21.91 miles per gallon on the average.

Example: A well-esteemed, Nobel Prize-winning friend of yours is coming to town next month. You want to host a banquet/reception in your friend's honor, budget permitting. Your favorite restaurant can provide a suitable banquet room on the desired evening for $54.99 and cater the dinner for $12.75/adult and $6.50/child. Looking through your invitation list, there are at most 63 adults and 21 children who might attend. Champagne would be nice; you'd order 15 bottles at $9.80/bottle. Approximately how much would this social gala cost?

The problem looks like this:

$$(63 \times \$12.75) + (21 \times \$6.50) + (15 \times \$9.80) + \$54.99 = ?$$

Solving this problem with pencil and paper, you need to obtain the intermediate results before you can add:

$$\underbrace{\cancel{(63 \times \$12.75)}}_{}^{803.25} + \cancel{(21 \times \$6.50)}^{136.50} + \cancel{(15 \times \$9.80)}^{147.00} + \$54.99 = ?$$

Your HP calculator makes solving this problem easy because it follows the same logic you would use in solving the problem with pencil and paper and automatically holds results for you until you are ready to use them.

Before you can add, you must first compute the total cost of the adult dinners (63 × \$12.75).

Solve	Keystrokes	Display
63 × \$12.75	63 [ENTER↑] 12.75 [×]	803.25

Then compute the total cost of the child dinners (21 × \$6.50). Don't worry about the last number you calculated; your calculator will hold it until you're ready to add.

Solve	Keystrokes	Display
21 × \$6.50	21 [ENTER↑] 6.50 [×]	136.50
Now add.	[+]	939.75

Compute the cost of the champagne (15 × \$9.80) and add:

Solve	Keystrokes	Display
15 × \$9.80	15 [ENTER↑] 9.80 [×]	147.00
Now add.	[+]	1,086.75

Finally, key in the cost of the banquet room and add.

Keystrokes	Display	
54.99 [+]	1,141.74	Final answer.

If everyone attended, you could expect to pay close to \$1,150 for the reception.

In the last few problems, the calculator held the first and subsequent results while you calculated another result. HP financial calculators hold up to three such results automatically.

For a more complete understanding of the way your calculator handles numbers, refer to appendix A, RPN and the Automatic Memory Stack. The more you understand your calculator, the more efficiently and confidently you will use it.

Note: Whether you are adding or subtracting, multiplying or dividing, taking the square root of a number or raising a number to a power, there are only two rules to remember:

1. Use the ENTER↑ key when you wish to separate one number from another.
2. When you press an arithmetic operator or function key, the calculator performs that operation immediately.

Sample problems: Total the following list of expenditures for office supplies purchased in one month:

284 binders @ $1.79 each	284 × 1.79
33 reams typing paper @ $2.55/ream	33 × 2.55
39 dozen pencils @ $1.20/dozen	39 × 1.20
82 boxes paper clips @ $.35/box	82 × .35
28 staplers @ $1.75 each	28 × 1.75
1 typewriter @ $568.00	1 × 568.00
	Subtotal $?
4% sales tax	?
	Total $?

Let's run through the keystrokes:

Keystrokes	Display	
284 ENTER↑ 1.79 ×	508.36	
33 ENTER↑ 2.55 ×	84.15	
+	592.51	
39 ENTER↑ 1.20 ×	46.80	
+	639.31	
82 ENTER↑ .35 ×	28.70	
+	668.01	
28 ENTER↑ 1.75 ×	49.00	
+	717.01	
568 +	1,285.01	Subtotal.
4 %	51.40	Tax (more about the % key in section 2).
+	1,336.41	Total.

Solve the following problems yourself. If you have trouble obtaining the correct answers given, review the last few pages.

$$(4 \times 1.50) + (3 \times .75) = 8.25$$

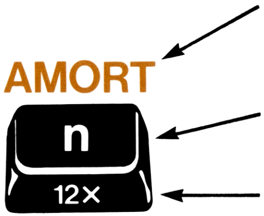
$$\frac{(14.50 - 2.75) + (18 - 12.25)}{(17 + 18)} = .50$$

$$\frac{13.89 + (3 \times 6.20)}{(42 - 15) - (8 + 16)} = 10.83$$

Prefix Keys

Most keys on your financial calculator have two or three functions, depending on your calculator model. One function is indicated by the symbol on the key face; other functions are written in color immediately above the key or, in some models, on the slanted face of the key. To select a function above or below the face of the key, first press the prefix key matching that function in color, then press the function key.

Some keystroke sequences may require the use of prefix keys to make them applicable to your calculator. Check your calculator keyboard for proper execution sequences.



- To select the function printed in gold above the key, first press the gold prefix key [f], then press the function key.
- To select the function printed on the flat face of the key, press the key.
- To select the function printed in blue* on the slanted face of the key, first press the blue prefix key [g], then press the function key.

For example, you will see [Δ%] under a **Keystrokes** column in the following section. In order to execute the [Δ%] function you must press [f] then the [%] key, above which is written [Δ%] in the same color as the prefix key [f].

Calculator Self-Check

Should you feel that your calculator is malfunctioning, press [STO] [ENTER↑]. This tells the calculator to run through an electronic check of all internal components.

If something is not working properly, the display will show **Error 9**; your calculator needs repair.

If nothing is wrong, ***-8,8,8,8,8,8,8,8,8,8,*** will show in the display. Press any key to clear the display.

> **Note:** In *E*-version calculators, pressing [STO] [ENTER↑] for self-check clears the entire calculator. In *C*-version (Continuous Memory) calculators, pressing [STO] [ENTER↑] for self-check clears all *except* the storage registers, financial registers, and program memory.

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* Some models only.

Section 2

Concepts of Time and Money

Essentially, there are three things you can do with money: spend it, invest it, or sit on it. A savings account in a bank is considered an investment and we all know what spending entails, but putting your money under a cushion...? Whether you spend or invest money, you want to receive something worthwhile in return.

This section looks at the nature of cash flows, or how time and money relate to one another when you borrow or invest. Your HP financial calculator has the common time-and-money formulas built-in and ready at your fingertips, so you are free to concentrate on the concepts themselves.

Percent: The Universal Yardstick

Percentage is the universal yardstick—the common standard of measurement—in the financial world. If your money increases or decreases, the gain or loss is measured in percent as well as in dollars. Taxes, interest rates, discounts, inflation, appreciation, depreciation, even the last raise you got, or the typewriter you bought last week for 40% off—all are expressed in terms of percent.

Percent, denoted by the symbol %, simply means "for each hundred." When you see 25%, it's the same as $^{25}/_{100}$ or 0.25 or ¼.

Percentage is a dynamic relationship, a comparison or ratio of two numbers that often signifies that a change has taken place. "Third-quarter earnings are down 27% from last year" may be cause for concern, while "a 12% raise effective today" may be cause for celebration.

Likewise, when you start with a given amount of money and receive money in return, the difference—whether it's a gain or loss—is viewed in relation to the original amount and expressed as a percentage. If you start out with one share of stock worth $100 and sell it for $125, you have earned $^{25}/_{100}$ or a 25% return.

When you superimpose that gain or loss against time, it's called the **rate of return.** The time period most commonly used in business is one year. So if you earned that $25 in one year, that's a 25% annual rate of return.

Percent and Your Financial Calculator

Hewlett-Packard financial calculators provide you with three separate functions for calculating percentage problems: [%], [Δ%], and [%T]. With the [%] key, you can key in a number and a percent and find that percent of the number. With the [Δ%] key, you can find the percent change (increase or decrease) between an old value and a new value. And with the [%T] key, you can find what percent one number is of another number or of the sum of several numbers. You are finding proportions when you use [%T].

When calculating percentage problems, you don't have to convert percents to their decimal equivalents: 10% need not be changed to .10. It can be keyed in the way you see it and say it, 10 [%].

Look at the following example, illustrating the use of percentage keys and concepts.

Example: Suppose you own 150 shares of stock in Coakley Laboratories, 52 shares of Idylwild Aircraft, and 200 shares of Burrell Industries.

If you sell 30% of your stock in Coakley Laboratories, how many shares would you have left?

Keystrokes	Display	
150 [ENTER↑] 30 [%]	45.00	30% of 150 is 45 (# shares to be sold).
[−]	105.00	Shares left.

Notice you didn't have to reenter 150 before subtracting; your calculator automatically retains the base number in percentage calculations.

Assume your Burrell Industries stock is now selling at $7.50 per share, down from $10.75 per share last year at this time. What is the percent decrease in cost per share over the last year? Notice that the keystroke sequence is "old value-enter-new value."

Keystrokes	Display	
10.75 [ENTER↑]	10.75	Cost per share last year.
7.50 [Δ%]	-30.23	% decrease.

Your calculator automatically displays a minus sign to show percentage decreases.

What percent of your portfolio does each corporation represent at this time?

Keystrokes	Display	
150 [ENTER↑] 52 [+]		
200 [+]	402.00	Sum of shares.
150 [%T]	37.31	% Coakley Laboratories.
[CLx]	0.00	Clear the display.
52 [%T]	12.94	% Idylwild Aircraft.
[CLx]	0.00	Clear the display.
200 [%T]	49.75	% Burrell Industries.

After you sell 45 shares of Coakley Laboratories?

Keystrokes	Display	
105 [ENTER↑] 52 [+]		
200 [+]	357.00	Sum of shares.
105 [%T]	29.41	% Coakley Laboratories.
[CLx]	0.00	
52 [%T]	14.57	% Idylwild Aircraft.
[CLx]	0.00	
200 [%T]	56.02	% Burrell Industries.

Interest

Percent is also used to calculate interest. Interest is a charge for the use of money. In a sense, you "rent" the money or someone "rents" it from you.

Interest is based on three things:

1. The amount of money borrowed or saved.
2. The length of time.
3. The interest rate (a percentage).

This makes sense because the longer you rent something, the more you pay for it. If you rent a car for a week, it will cost more than if you rent it one day.

You can charge for money by the day, the week, the month, etc., but usually money is loaned or borrowed at a yearly rate. This annual interest rate is expressed as a percent. If a certain investment pays 9% yearly, that means $9 per year for every $100 invested.

But there are other considerations, too, when you pay or receive interest—namely, what *type* of interest and how often it is paid.

Simple Interest

With **simple interest,** the principal—i.e., the original amount of money—earns interest for the entire life of the transaction. For example, suppose you borrow $1,000 at an 8% interest rate ($8 for each $100) for one year. The formula for calculating simple interest is:

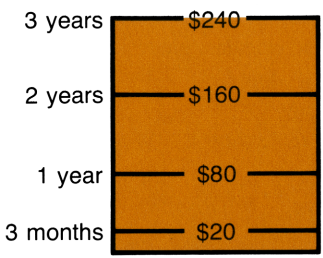
$$\text{Interest}_{(\text{simple})} = \text{Principal} \times \text{Interest rate} \times \text{Time}$$

The interest amount would be $80 ($1,000 × 8% ×1 year).

Borrowing the same amount of money for only 3 months would cost one-fourth as much, or $20 ($1,000 × 8% × ¼th of a year). And borrowing $1,000 for 3 years would cost 3 times as much, or $240 ($1,000 × 8% × 3 years).

Simple Interest

"Rent" on $1,000 at 8% simple interest:



On your calculator:

Keystrokes	Display	
1000 [ENTER↑]	1,000.00	Principal.
8 [%]	80.00	Interest amount (one year).
3 [ENTER↑] 12 [÷]	0.25	Portion of year (¼).
[×]	20.00	Interest for ¼th of a year.

Borrowing $1,000 for 3 years:

Keystrokes	Display	
1000 [ENTER↑]	1,000.00	Principal.
8 [%]	80.00	Interest amount (one year).
3 [×]	240.00	Interest in 3 years.

Remember, percentages are like hundredths. Taking 8% of $1,000 is the same as multiplying $^8/_{100}$ (.08) times $1,000. Your calculator simplifies calculations by taking 8% of $1,000 automatically, as shown in the previous section.

Compound Interest

Although the concept of simple interest underlies most financial transactions, its use in the business world usually differs somewhat from the problem presented in the previous paragraph. For example, suppose you invested $1,000 for 2 years at 8%. How much interest would your investment earn? Using simple interest, the answer would be $160; you receive $80 at the end of the first year, and $80 at the end of the second year.

What would happen if at the end of the first year, the $80 of interest earned was invested for the second year along with the initial $1,000? At the end of the second year the $1,000 and $80 together would earn $86.40, $6.40 more than the initial $1,000 earned the first year. Each time the interest is paid, it is added to the balance. In effect, the interest is earning interest. Continuing this procedure, year after year, both the amount invested and the interest earned continue to grow.

This method of reinvesting earned interest, referred to as **compounding the interest,** is much more common in business transactions than earning interest on the principal alone (simple interest). **Compound interest** is usually stated as an annual rate, although it may be compounded (calculated) continuously, daily, monthly, quarterly, or semi-annually.

So if you have $1,000 at present, you can calculate how much money you will receive in the future, before you invest. This is called its **future value**[1]. Or perhaps you want $1,000 in the future—to take a trip to Acapulco next year—and want to know how much you have to invest to reach that goal. Since you have established the desired future value, you are solving for the money right now or its **present value.**[2]

What if you invested $1,000 at 8% *compounded annually* for 3 years? Let's use the calculator to compute the future value, or the amount of money you will have at the end of 3 years.

Keystrokes	**Display**	
[CL FIN]		Clear finanical entries.
1000 [CHS] [PV]	-1,000.00	Key in the initial investment and change the sign since you pay out money when you invest it. Press [PV] for present value.
8 [i]	8.00	Key in 8 and press [i] for 8% annual interest rate.
3 [n]	3.00	You want to find the value of your investment in 3 years. Press [n] for the number of compounding periods.
[FV]	1,259.71	Simply press [FV] to find the future value of your investment.

1. The formula for computing the future value of money with compound interest is:

$$FV = PV \times (1 + i)^n$$

2. The formula for computing the present value of money with compound interest is:

$$PV = \frac{FV}{(1 + i)^n}$$

The display shows that when you invest $1,000 at 8% compounded annually for 3 years, you will receive $1,259.71 at the end of those 3 years.

Is there a way to earn more money with that $1,000 at the same 8% interest rate? Yes, by compounding or adding the interest to the principal more than once a year. Suppose you put that money into an account where the 8% interest is *compounded quarterly*. How much do you have at the end of a year?

The $1,000 remains the same in our calculations, so you don't need to key the present value in again. But the number of compounding periods have changed from one per year to four per year.

Keystrokes	Display	
4 [n]	4.00	The number of compounding periods [n], has changed from 3 in 3 years to 4 in 1 year.
8 [ENTER↑] 4 [÷] [i]	2.00	Since the interest rate must always correspond with the compounding interval, you must divide the 8% annual interest rate by the number of compounding periods in a year, 4.
[FV]	1,082.43	When you press [FV], the calculator shows that you will have $1,082.43 at the end of one year.

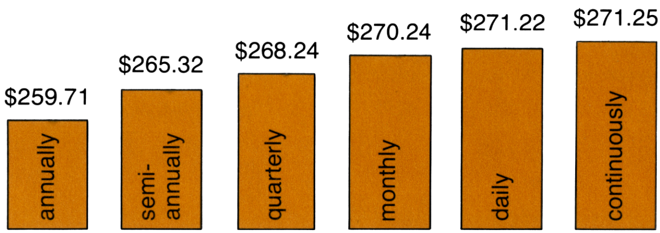
And after 3 years?

Keystrokes	Display	
4 [ENTER↑] 3 [×] [n]	12.00	Simply change [n] to 12; 4 compounding periods per year × 3 years.
[FV]	1,268.24	The future value of your investment of $1,000 at 8% compounded quarterly at the end of 3 years.

Now instead of $240.00 (no compounding) or $259.71 (compounding annually), you earned $268.24 in interest. It becomes apparent that the more often interest is compounded, the more money you receive in return.

Compound Interest

3 years "rent" on $1,000 at 8% compounded...



There is a limit, called *continuous compounding*, to the amount of money you can earn by increasing the frequency of compounding. If you compound continuously (more often than daily or hourly or every second), you reach the maximum mathematical limit. In other words, you reach the point where you just can't compound any more often.

Something interesting is emerging here. Even though 8% is stated as the annual rate, you actually receive more than 8% interest with compounding more than once a year.

$1,000 at 8% for 1 Year

COMPOUNDED	RETURN	% INTEREST
Quarterly	$1,082.43	8.243%
Monthly	$1,083.00	8.300%
Daily	$1,083.28	8.328%
Continuously	$1,083.29	8.329%

When interest is compounded more often than once a year, the stated annual rate (8% in the example above) is called the **nominal rate.** The rate of interest actually earned in one year is called the **effective rate** (i.e., 8.328% compounding daily).

Many savings institutions quote both the nominal rate and the effective rate. And your calculator can quickly convert one to the other. (Refer to the applications manuals for interest rate conversions.) As the chart shows, the effective rate may differ considerably from the nominal rate, so it pays (literally!) to know what it is.

Clearing financial entries: Each time you begin a new problem press [CL FIN] (or CLEAR [FIN], depending on the model of your calculator) to erase previous financial values. When you press [CL FIN], all values for [n], [i], [PV], [PMT], and [FV] are replaced with 0.00 (zero). If you want to change some, but not all, of the values in a financial problem it is not necessary to press [CL FIN] and reenter all of the values again. Simply key in the new data and press the appropriate financial keys to change particular financial values, as we did in the last subsection.

You can view particular financial values held in the calculator at any time, by pressing the [RCL] *(recall)* key and then the desired financial key ([n], [i], [PV], [PMT], or [FV]). The designated financial value will then be displayed.

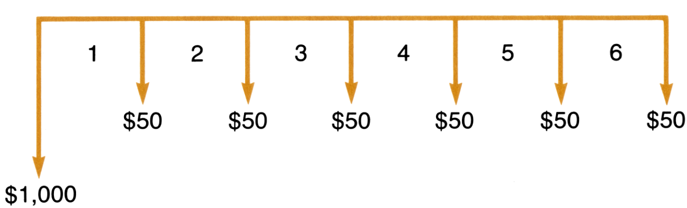
Compound Interest and the Cash Flow Diagram

The concept of compound interest is not difficult. The computations involved, however, can become exceedingly complex. Problems encountered often involve numerous payments and receipts before the transaction is concluded. Your financial calculator is designed to solve many of the most complicated calculations, but it requires a precise format for describing the problem. Such a format can be represented pictorially in the form of a cash flow diagram. The diagram is nothing more than a description of the timing and direction in which cash changes hands using terms that correspond to your calculator's financial keyboard. As long as you can picture your problem with a cash flow diagram and label it, your calculator can find the answers.

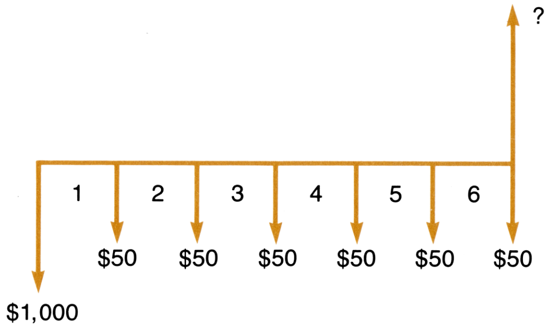
The diagram starts with a horizontal line called the time line. It represents the duration of a financial problem and is divided into compounding periods. For example a financial problem that transpires over 6 months with monthly compounding would look like this:

The exchange of money in a problem is pictured with vertical arrows; money received is represented with an arrow pointing up from the time line where the transaction occurred and money paid out is represented by an arrow pointing down.

For example, if you deposited (paid out) $1,000 at the beginning of the time period and then deposited an additional $50 at the end of each month for the remaining 6 months, you would label the diagram like this:



At the end of the period your account would have a balance that included the initial deposit, the subsequent payments, and any interest paid. This balance could be withdrawn (received), if necessary, and would represent a final cash exchange, completing the problem and the cash flow diagram.

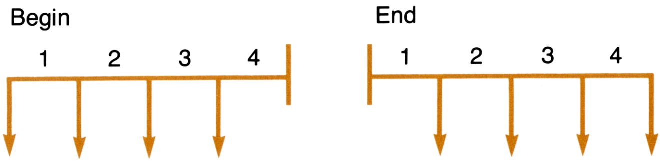


The following keys in the first row of your financial keyboard correspond exactly with our cash flow diagram: n, i, PV, PMT, and FV. The number of compounding periods* in a financial problem is represented by n: n would be 6 in our example. Interest rate *per compounding period* is denoted by i on the keyboard. (The interest rate must correspond with the compounding interval. Don't mix monthly interest with quarterly periods or daily interest with semiannual compounding periods.) The different cash flows are represented by PV, PMT, and FV. PV *(present value)* represents the cash flow at the start of the time line. In our example PV would be the $1,000 initial deposit. FV stands for future value and represents the cash flow at the end of the time line; the amount that could be withdrawn at the end of six months, indicated by the question mark in the above cash flow diagram. PMT *(payment)*

* Some financial problems involve a portion of a payment period as well as a series of whole payment periods. This occurs whenever a transaction begins on a date that does not correspond to the beginning of the usual payment period. Although there is no standard convention that applies to every problem of this kind, certain problems—such as purchasing a house in the middle of the month when regular payments are made at the beginning of each month—must be separated into two parts: one with the fractional portion of a payment period and one with the remaining whole number of payment periods. The payments made during the whole number of payment periods are calculated using *compound* interest, while the interest accumulated during the fractional portion of a payment period is often calculated using *simple* interest. Be sure that you partition problems when necessary, and calculate accordingly.

represents a series of cash exchanges of the same direction and amount, i.e., annuities. In our example $50 payments are deposited at the end of each month.

Payments can either start at the beginning of each period (BEGIN), or start at the end of each period (END). There are always the same number of payments as periods.



Whenever payments (PMT) are involved, it is necessary to specify which of the alternatives is applicable by setting the payment switch BEGIN END, found above the financial keys, to the proper position. BEGIN is for payments in advance and END is for payments in arrears. Or BEGIN is for annuities due and END is for ordinary annuities. The payment switch setting *does* make a difference in your calculated results. That's because interest accumulates on different amounts depending on whether payments are made at the beginning or the end of a compounding period. In our example, the payments occur at the end of each period, so the payment switch must be placed in the END position before starting calculations.

> **The Sign Convention:** Cash received (arrow pointing up) is represented by a positive value (+), and cash paid out (arrow pointing down) is represented by a negative value (−).

In our example, the $1,000 initial transaction (PV), and the periodic $50 payments would both be negative values. The amount received at the end of the time span would be positive.

The sign convention allows you to solve financial problems with 4 *or* 5 variables. (For instance, we shall soon solve for FV, given values for n, i, PV, and PMT.*) In fact, you can easily solve for any of the financial

* Some earlier handheld calculators could handle only two of the three kinds of cash flows at a time. The sign did not need to be specified because the cash flows were necessarily of opposite sign (e.g., PV positive, PMT negative; or PV negative, FV positive). Since your calculator can handle three kinds of cash exchanges (PV, PMT, and FV) at a time, their direction is no longer obvious. Thus, the sign needs to be specified using the described sign convention.

values above as long as you specify the values of at least three other financial keys (and include n or i as one of them).

Remember:

n = number of compounding periods
i = interest rate per compounding period
PV = present value
PMT = periodic payment
FV = future value

BEGIN = payments made at the beginning of the period
END = payments made at the end of the period

Now let's do the problem represented by the cash flow diagram and calculate the FV. Before beginning the calculation, one additional piece of information is necessary; the interest rate paid each compounding period. For this example let the interest rate be .75% per period (or 9% nominal interest). Remember, all cash that is paid out has a negative value.

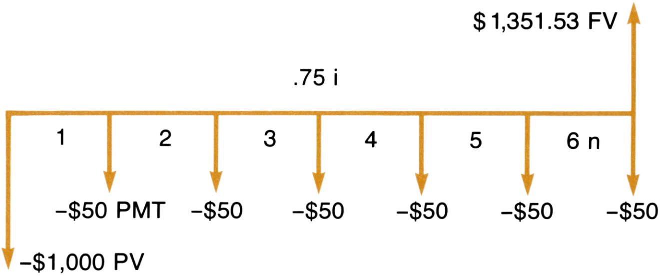
Since the $50 payments are made at the end of each period, set the payment switch BEGIN ▮▮▮ END to END.

Keystrokes	Display	
[CL FIN]		Clear previous financial values.
6 [n]	6.00	
.75 [i]	0.75	
1000 [CHS] [PV]	-1,000.00	Negative for cash paid out.
50 [CHS] [PMT]	-50.00	

The calculator now has all of the necessary information to solve for FV, which is the last key pressed.

Keystroke	Display	
[FV]	1,351.53	The calculated value is positive indicating we receive this amount.

As you can see, the keys on HP financial calculators and the signs of the values entered correspond precisely to the problem as represented by the cash flow diagram.



Suppose you wanted to increase your initial investment (PV) sufficiently to create an ending balance (FV) of $2,000 with the same interest rate, number of periods, and payments. What present value would be necessary?

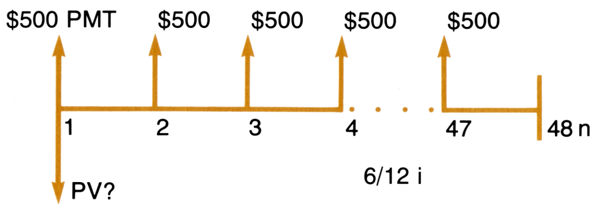
There is no need to start the entire problem over again. The n, i, and PMT are unchanged and therefore do not have to be reentered. The only value that needs to be entered is the new desired FV. Enter the FV and solve for PV.

Keystrokes	Display	
2000 [FV]	2,000.00	Desired cash receipt.
[PV]	-1,620.04	Necessary cash paid out.

Looking over our example we find that with only a few easy keystrokes we have solved problems that would have required a great deal of time had we attempted to answer them by evaluating the complex mathematical formulas involved. The financial calculator's power allows you to consider numerous investment alternatives while concerning yourself only with the underlying concepts and the practicality of the values used.

Let's try another problem. Suppose you are concerned about providing for your daughter's college education 14 years from today. You expect that the cost will be about $6,000 a year or about $500 a month. If you withdrew the monthly payments for 4 years from a bank account paying 6% a year, compounded monthly, how much must you deposit in the bank at the start of the college years (PV) to make the monthly payments?

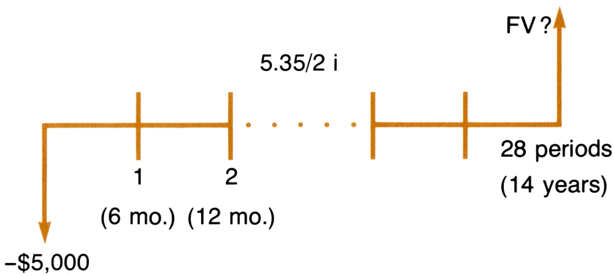
A cash flow diagram of the problem would look like the following:



The periodic interest rate must correspond to the time span between payments (compounding periods), so you must divide the yearly rate (6%) by 12 in order to produce a monthly rate [i]. As you can see from the diagram, the payments of $500 a month (PMT) start with the beginning of the time span; so you should set the payment switch BEGIN END to BEGIN. Since we are beginning a new problem, it is best to clear out any values remaining from the previous problem by pressing [CL FIN].

Keystrokes	**Display**	
[CL FIN]		Clears previous financial values.
6 [ENTER↑] 12 [÷] [i]	0.50	Calculate and enter interest rate per period.
4 [ENTER↑] 12 [×] [n]	48.00	Calculate and enter the number of compounding periods.
500 [PMT]	500.00	Amount received each period.
[PV]	–21,396.61	Total deposit required.

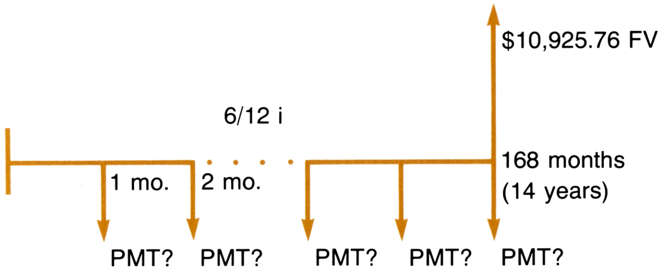
The next question we might ask is how do we accumulate such a sum by the time she enters college. We have several possibilities. Your daughter has a \$5,000 paid up insurance policy that pays 5.35% (nominal) a year compounded semiannually. How much would it be worth by the time she enters college?



There are no payments so the payment switch **BEGIN** ▮▮ **END** has no effect. In this problem our compounding periods occur semiannually so the yearly rate must be divided in half to obtain [i]. The value of [n] is 14 years times 2 periods per year. This is another new problem, so be sure to clear previous financial entries.

Keystrokes	**Display**	
[CL FIN]		
14 [ENTER↑] 2 [×] [n]	28.00	Total number of periods.
5.35 [ENTER↑] 2 [÷] [i]	2.68	Periodic interest rate.
5000 [CHS] [PV]	-5,000.00	Deposited (a negative value).
[FV]	10,470.85	Value of policy.

The insurance policy will supply about half of the needed amount. An additional amount must be set aside to make up the \$10,925.76 deficit (21,396.61 − 10,470.85). Beginning next month, if we made monthly payments into a special college account, how large would the payments have to be to accumulate the necessary future value of $10,925.76 in the 14 years remaining? Assume the account would pay 6% a year, compounded monthly.



Rather than multiplying 14 times 12 to get the proper number of compounding periods for [n] and dividing 6 by 12 for [i], we can use a shortcut provided on the financial keys for making quick conversions from years and yearly rates to months and monthly rates.

> **Remember:** [n] must always be the total number of compounding periods in the time span.
> [i] must always be the interest rate per compounding period.

Set the payment switch BEGIN ▮▮▮ END to END. (Check your keyboard for correct prefix keys.)

Keystrokes	Display	
[CL FIN]		
14 [12×]	*168.00*	Automatically carries out the multiplication by 12 and stores the answer in [n].
6 [12÷]	*0.50*	Divides by 12 and stores in [i].
10925.76 [FV]	*10,925.76*	Future value desired.
[PMT]	*–41.65*	Necessary deposit each period (each month).

Note that we used the [12×] key to automatically compute and store the value of [n], and the [12÷] key to automatically compute and store the value of [i].

If we made the payment only $35 a month, how long (n) would it be before we reached the desired amount?

Keystrokes	Display	
35 [CHS] [PMT]	-35.00	
[n]	188.54	Number of periods.

In order to find the number of years, divide by 12.

Keystrokes	Display	
12 [÷]	15.71	Years.

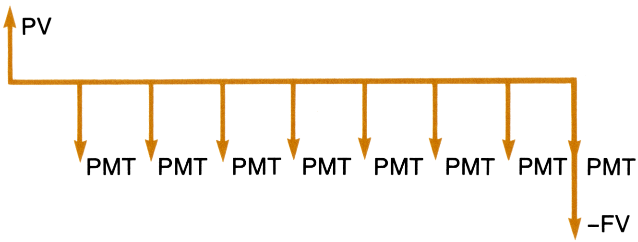
If, on the other hand, the monthly payment were increased to $45, with the 14-year term, the excess could be used as a contingency fund. For instance, with a $45 a month payment, what interest rate could the bank pay, while still enabling us to meet our goal?

Keystrokes	Display	
14 [12×]	168.00	Original term.
45 [CHS] [PMT]	-45.00	New monthly deposit.
[i]	0.42	Monthly interest rate.
12 [×]	5.01	Nominal yearly interest rate.

Note that it was necessary to reenter the length of the original term. Our previous computation of [n] (188.54) was stored in the calculator and would have otherwise been used for the term of this calculation.

In the preceding sample problems, we have seen how cash flow diagrams can be useful in representing a wide range of compound interest problems, and how the diagrams can be translated directly into solutions on an HP financial calculator. The diagrams are helpful tools that describe complex business and financial problems in a manner suitable for calculation. In addition, the cash flow diagram can be applied in other ways to become a valuable aid.

As we are all too often aware, each segment of the business community has its own special vocabulary. When considering compound interest problems of the kind we have been discussing, there are often numerous terms used throughout the business world describing the same problem, but which are not familiar outside a particular segment. For instance, this diagram:



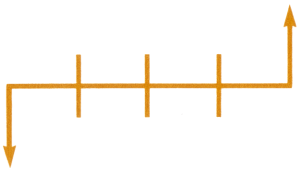
might represent a mortgage with a balloon payment in the terminology of the banking and real estate industries or a lease with a buy back (residual) in the leasing industry. There are probably many other terms in other industries as well as countries for describing this cash transaction. But regardless of the language, the essential problem is the same. In providing a means of describing business financial problems without using terminology specific to a particular segment, the cash flow diagram becomes, in a sense, a universal language.

The cash flow diagrams for four basic compound interest problems are presented in the following table along with some of the more common terminology.

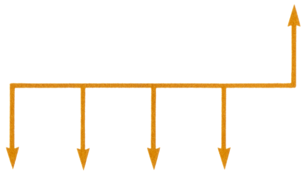
Some of the terms you see listed in the table may be common to your industry and some may not. There also may be diagrams represented that correspond to familiar transactions, but which do not bear familiar names. The important point to remember is that for financial calculations, it is the **magnitude** and **timing** of the cash exchanges represented by the cash flow diagram that are important, not the industry-dependent terminology.

Generalized Net Cash Flow Diagrams and Terminology

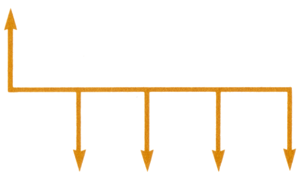
(Note that diagrams involving payments may be represented with payments at the beginning or end of the period.)



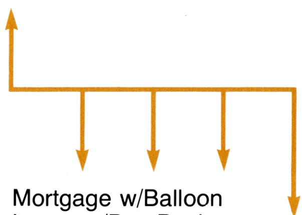
Compound Growth
Savings Account
Appreciation



Savings Plan
Sinking Fund
Pension Fund
Annuity (series of payments)



Mortgage
Lease
Direct Reduction Loan
Direct Reduction Installment
Amortization
Annuity



Mortgage w/Balloon
Lease w/Buy Back
Lease w/Residual
Annuity

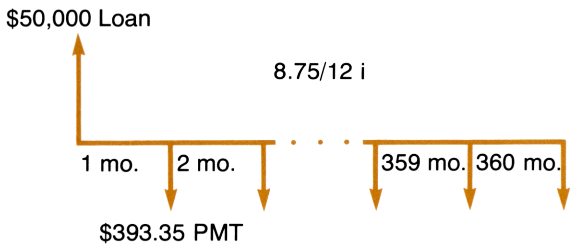
Amortization

If a loan or interest-bearing debt is paid off by (usually) equal payments, then it is said to be **amortized**. The word amortization comes from the French ''a mort'' meaning ''at the point of death.'' Likewise, you are ''killing'' a loan by paying it off.

Most simple mortgages and installment loans are called **direct reduction loans**. The debt is discharged by equal periodic payments although varying portions of each payment are applied toward principal and interest.

The interest is paid first, then the remainder of the payment is used to reduce the debt. The time frame over which you make payments is called the **schedule of payments**. The breakdown of payments into interest portions and principal portions is called an **amortization schedule**.

Suppose you find your dream house. If you take out a $50,000 mortgage for 30 years at 8¾% with monthly payments of $393.35, your payment schedule would look like this:



At the end of the first month, interest is calculated on the entire $50,000:

$$\frac{8.75\%}{12} \times 50{,}000 = \$364.58$$

and is added to the balance:

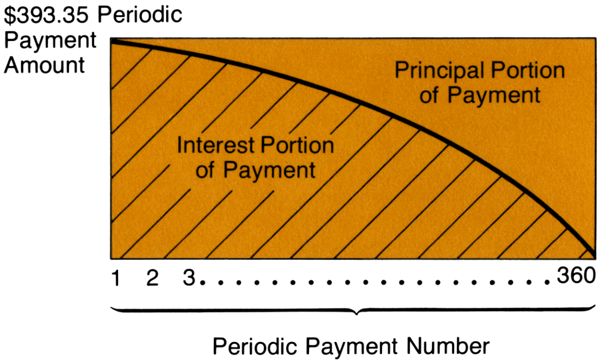
$$\$50{,}000 + \$364.58 = \$50{,}364.58$$

Then your first payment is deducted to obtain your new balance:

$$\$50{,}364.58 - \$393.35 = \$49{,}971.23$$

The next month and every month thereafter, the same procedure is followed, i.e., interest is calculated first and added to the balance before your payment is subtracted.

The amortization of your mortgage would look like this:



As you reduce the size of the loan, the interest decreases. Gradually a higher percentage of each payment goes toward the debt itself or outstanding principal. By the time you reach your last payment, very little is deducted for interest.

With your HP financial calculator, you can easily compute the accumulated interest and remaining balance of your loan at any point in time. All you need to do is key in the principal [PV], periodic interest rate [i], and periodic payment [PMT]. Then key in the number of periods to be amortized and press [AMORT].

Your calculator will display the total interest portion of the payments. Simultaneously, your calculator generates the total principal portion of the payments (press [x⇄y]) and the remaining balance of the loan (press [RCL] [PV]).

When working amortization problems, remember to abide by the *sign convention* (cash received is positive, cash paid out is negative) and be sure to set the payment switch to the proper position.

When using the [AMORT] function, all calculated values are automatically rounded to match the display setting. The normal display shows numbers as dollars and cents. If your problem requires other rounding, refer to section 3 to set the display to the number of digits you wish carried.

Example: Generate an amortization schedule for the first 2 months of a 30-year mortgage for $50,000 at 8¾% annual interest with monthly payments of $393.35. Then find the balance on the loan after 1 year.

Set the payment switch BEGIN ▮▮ END to END.

Keystrokes	Display	
[CL FIN]		Clear old financial values.
50000 [PV]	50,000.00	Loan amount.
393.35 [CHS] [PMT]	-393.35	Periodic payment.
8.75 [12÷]	0.73	Periodic interest rate.

The [AMORT] function returns two values; the interest portion and the principal portion of the periodic payment. Since the display can only show one answer at a time, the second value is held in the automatic memory. It can be viewed by pressing the exchange key [x⇄y].

Keystrokes	Display	1st month of schedule.
1 [AMORT]	-364.58	Interest portion of payment.
[x⇄y]	-28.77	Principal portion of payment.
[RCL] [PV]	49,971.23	Remaining balance.
[RCL] [n]	1.00	Number of periods calculated (one month).

Notice that the [AMORT] function changes both the [n] and [PV] values: [PV] brings back the new balance, [n] provides the total number of periods amortized.

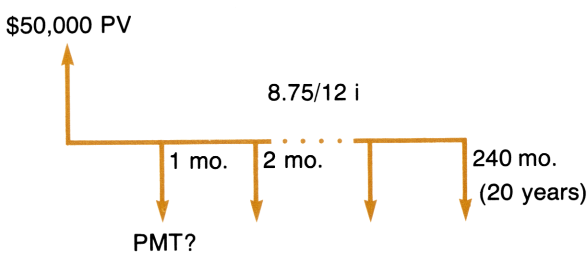
To generate the next period of the schedule simply press 1 AMORT again:

Keystrokes	Display	2nd month.
1 AMORT	–364.37	Interest portion of payment.
x≷y	–28.98	Principal portion of payment.
RCL PV	49,942.25	Remaining balance.
RCL n	2.00	2nd payment period.

Now we want to find the balance on the loan after one year. We have already calculated the first 2 periods, so press 10 AMORT to compute the amount of interest and the amount of principal paid in the next 10 periods, leaving a balance on the loan after 1 year in PV.

Keystrokes	Display	
10 AMORT	–3,631.86	Interest portion of payments made from month 3 through month 12.
x≷y	–301.64	Principal portion of payments made from month 3 through month 12.
RCL PV	49,640.61	Balance after 12 months or 1 year.
RCL n	12.00	

What would the monthly payments be if we decide to pay off the mortgage in 20 years? The cash flow diagram would look like this:



Keystrokes	Display	
50000 [PV]	50,000.00	Reenter initial principal.
20 [12×]	240.00	# monthly payments in 20 years.
[PMT]	–441.86	Periodic payment.

It was not necessary to reenter the periodic interest rate as it remained the same in our last calculation.

How much interest and how much of the principal will be paid after 1 year with this 20-year mortgage? (Since we are starting the amortization schedule from the beginning we must set [n], the number of periods, to zero. We did this in our first amortization example by pressing [CL FIN], but this time we want to preserve the value of [i].)

Keystrokes	Display	
0 [n]	0.00	Set n to zero. (n was 240 from the last calculation.)
12 [AMORT]	–4,336.87	Interest portion of payments for 1 year.
[x⇄y]	–965.45	Principal portion of payments for 1 year.
[RCL] [PV]	49,034.55	Remaining balance.

How many payment periods have we calculated?

Keystrokes	Display
[RCL] [n]	12.00

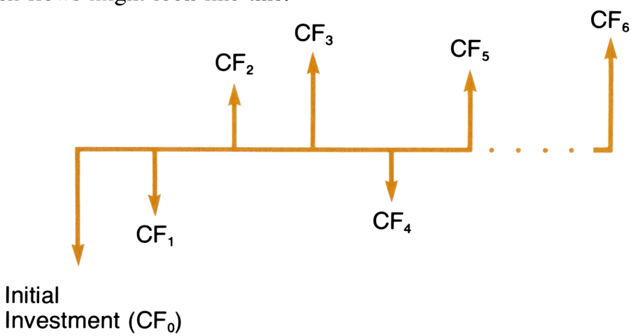
Remember: Set [n] to zero (by pressing 0 [n] or [CL FIN]) whenever you begin to calculate a new amortization schedule, to keep track of your payment number. When the payment switch is set to BEGIN, the whole first payment goes to pay off the principal (no interest has yet accumulated). In this case, it is essential to change [n] to zero so that interest is not calculated on the principal before the first payment.

Discounted Cash Flow Analysis

Except for PV and FV, the cash flow diagrams on page 36 contain *even* (equal) cash flows. Discounted cash flow analysis is a way of evaluating investments with *uneven* cash flows. Your calculator can help you evaluate future cash demands and returns to see which scheme or investment best meets your profit objectives. Two forms of discounted cash flow analysis are the net present value (NPV) approach and the internal rate of return (IRR) approach.

Net Present Value (NPV)

Suppose you invest a large amount of money into a scheme that generates a cash flow CF_1 (cash $flow_1$) the first year, CF_2 the second year, and so on, up to CF_n in the n^{th} year when the cash flow ends. A diagram of the cash flows might look like this:



Notice that the original investment (CF_0) is negative because it represents a cash outlay. Also note that the cash flows may not necessarily be positive. Maybe, in a new business, you have a loss the first year. Or perhaps after you've been in business a while, a recession causes you to have a bad year.

You also have to consider the *time* value of money, not just the dollar value. (Would you rather have someone give you $10,000 today or 5 years from today?) The cash flows (CF_1, CF_2, etc.) are mini-future values that you are going to receive. But realistically these future cash flows have to be translated back (discounted) to present value on the basis of a given rate of interest, for you to accurately assess the investment.

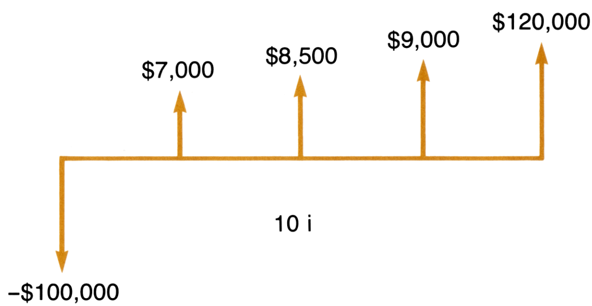
This method of analysis is called solving for **net present value*** because you are comparing the sum of the present values of all the future cash flows (CF_j) to the initial investment (CF_0). For the interest rate (i), use the rate of return that you want from the investment.

At the start, the net present value (NPV) is negative because you've put out a large amount of money in the original investment. As the cash returns flow in, NPV will increase. Eventually—hopefully—NPV will turn positive. When NPV=0, you have reached the break-even point in the investment.

It's a simple, clear-cut analysis: assuming a desired minimum yield, if NPV is *negative*, the investment DOES NOT meet your profit objectives; if NPV is *positive*, the investment DOES meet your profit objectives.

For example, you are thinking of buying an apartment building for $100,000. Based on the anticipated cash flows below, will this investment return 10% a year?

Year		Cash Flow
1		$7,000
2		$8,500
3		$9,000
4	(You intend to sell it.)	$120,000



* The formula for net present value is:

$$NPV = CF_0 + \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \ldots + \frac{CF_n}{(1+i)^n}$$

or net present value = initial investment + sum of present values of future cash flows. Notice that we are translating each cash flow from a predicted future value to a present value using the formula for finding present value:

$$PV = \frac{FV}{(1+i)^n}$$

Using the formula for NPV, we find that after the 4th year the net present value is positive ($2,111.88) so the investment does return 10% or greater per year.

Suppose you sell the building in the second year for $110,000. Would that be more profitable? No, the net present value is negative (–$2,727.27) so you won't even meet your desired 10% rate of return.

Internal Rate of Return (IRR)

Sometimes if you know your initial investment and can predict the periodic cash flows, you want to find the rate of return that you will earn. Internal rate of return (IRR) is the interest rate that equates the present value of all cash flows with an initial cash flow. IRR is also called the **yield** or **discounted rate of return**.

The formula (page 43) for finding NPV also applies for finding IRR, only we let NPV=0 and solve for i. The easiest way to find IRR is to choose a best-guess interest rate (i) and find the net present value. *When NPV is 0, your "guesstimate" is the actual IRR*. If NPV is negative, your estimated percentage is higher than the actual IRR—try a lower interest rate. If NPV is positive, the actual IRR is even better or higher than the rate you've chosen.

For example, what is the estimated rate of return for a restaurant costing $200,000 that produces the following cash flows?

Year		Cash Flow
1		–$4,000
2		$20,000
3		$27,000
4		$42,000
5		$56,500
6	(you sell it)	$230,000

If you try 12%, the NPV after the sixth year is $6,867.05 so the actual IRR is higher than 12%.

Next, try 13%. This time the NPV is negative (–$2,265.95) so the IRR must be less than 13%.

As a result of these two iterations, the IRR must be between 12% and 13%. Since the NPV for 13% is closer to zero than the NPV for 12%, the IRR must be closer to 13%. The actual yield or IRR on this restaurant investment is approximately 12.75%.

Specific keystrokes for NPV and IRR are described in your owner's handbook. If you have understood these concepts, the mechanics of solving a problem—be it in real estate, banking, leasing, insurance or investments—will not be difficult. You can use your calculator to evaluate time and money relationships before you invest and to explore several financial alternatives.

Section 3

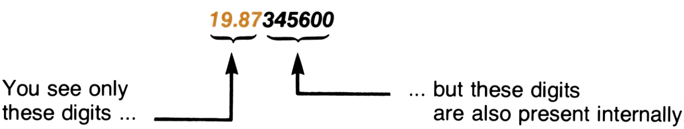
The Display and Memory

The Display

When the HP financial calculator is switched on, its displayed numbers are rounded to two decimal places. That's because most of your calculations deal with dollars and cents.

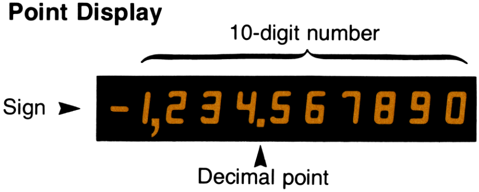
Keystrokes	**Display**
19.873456 [ENTER↑]	19.87

Although you only see two decimal places, the HP financial calculator actually calculates using a full 10-digit number internally so that you are assured of greater accuracy.



If you want to see more than two decimal places, you may select either of two formats: fixed point display or scientific notation display. *No matter which format or how many displayed digits you choose, the actual number itself inside your calculator is not altered unless you use the* [RND] *or the* [AMORT] *function*. HP calculators always calculate internally with full 10-digit accuracy.

Fixed Point Display



Using fixed point display, you can specify the number of digits to be shown after the decimal point. Press the gold prefix key [f] followed by a number (0 through 9) to specify the number of decimal places.

You already have 19.87 in the display, now try the following:

Keystrokes	Display
$\boxed{f}$4	19.8735
$\boxed{f}$6	19.873456
$\boxed{f}$0	20.

The display setting remains in effect until you specify a different one. Notice that digits in the display are *rounded up* at a value of 5 and over (*not truncated*).

If a calculated answer is greater or smaller than can be fully displayed, the calculator will automatically switch to scientific notation.

However, when you work with decimal numbers that are smaller than your specified fixed point display, yet less than 10 digits in length, your calculator will automatically round and display the *first significant digit* of that number (the first digit to the right of the decimal place other than zero).

For example: .025 × .025 = .000625. If you calculate this in fixed 2 notation your calculator will display .001 rather than .00, so that you know a nonzero number has been calculated. To see the full number, change to a larger fixed point display by pressing $\boxed{f}$9 for instance (to display the fractional portion to 9 places), *or* switch to scientific notation (see below).

Scientific Notation Display

In scientific notation, each number is displayed with a single digit to the left of the decimal point followed by six digits to the right of the decimal point. The two digits and sign at the right of the display indicate that the number is multiplied by that power of 10. Scientific notation is nothing more than a mathematical "shorthand."

	Scientific Notation Display	
\$62,500,000 = 6.25×10^7	6.250000 07	(Move the decimal point 7 places to the right to see the number in its original format.)
.0000000625 = 6.25×10^{-8}	6.250000-08	(Move the decimal point 8 places to the left to see the number in its original format.)

The two-digit exponent of 10, displayed on the right, will be positive if you are using large numbers and will be negative if you are using very small numbers.

To select scientific notation, press [f] [•].

Keystrokes	Display	
[f] 2 [CLx]	0.00	Fixed 2 display.
4750000 [ENTER↑]	4,750,000.00	
[f] [•]	4.750000 06	Scientific notation display.
[f] 2	4,750,000.00	Return to fixed 2 display.

If you are using scientific notation and turn the calculator OFF, then ON again, the display automatically returns to two decimal places: 0.00.

The display is also the means by which the calculator communicates with you—to tell you that you attempted to calculate a number smaller or larger than the machine's capacity, that the battery power is low, or that you attempted an illegal operation.

Overflow and Underflow Displays

Any attempt to enter or calculate numbers larger in magnitude than $9.999999999 \times 10^{99}$ will stop the calculation and display 9.999999 99 for positive numbers or -9.999999 99 for negative numbers.

Any attempt to enter or calculate numbers closer to zero than 10^{-99} will produce zero.

Low Power Display

When you are operating the calculator from battery power, a red dot inside the display will glow to warn you that the battery is almost exhausted.

• 19.87

You must then connect the ac adapter/recharger to the calculator and operate from ac power, or you must substitute a fully charged battery for the one that is in the calculator. Refer to your owner's handbook for a description of these operations.

Error Display

If you attempt an improper or impossible operation, the word ***Error*** and a digit (0 through 8) appear in the display. For example, enter a number and try to divide by zero. (Go ahead, try it.) The calculator recognizes this as an improper operation. Other such operations include the square root of a negative number or 0 raised to a negative power. To clear the error display, press any key. The key you press is not executed and does nothing but restore the calculator to the condition that existed prior to the error. Refer to your owner's handbook to interpret error messages.

Storing and Recalling Numbers

You have seen that your HP calculator automatically "remembers" up to three intermediate results until you are ready to use them. (Refer to section 1: Getting Started and appendix A.) You can also store and recall numbers manually in the memory (storage) registers. Storage capacities differ from model to model; your owner's handbook describes the exact quantity and location of memory registers specific to your own calculator. Here's how they work:

To store a number, press [STO] then a digit to specify the storage location. [STO] 0 places the number on the display in register 0; [STO] 1, in register 1; [STO] 2, in register 2; etc. Data in a storage register is changed by storing a new value in the same register ("writing over" old data) or by storage register arithmetic.

To recall the number, press RCL, then the register number. When you press RCL 3, a copy of the number in register R_3 appears in the display The previous display value will be retained in automatic memory; to recall it press x⇄y or R↓. The original value will remain in the storage location until you write over it or clear it.

Example: Suppose you want to calculate the cost of buying an item in various quantities. The unit price of the item is $132.57 and the quantities selected are 47, 36, and 29.

One way to solve this is to store the unit price in register 0. Then recall it to multiply each quantity.

Keystrokes	Display	
132.57 STO 0	132.57	
47 ×	6,230.79	First total.
RCL 0	132.57	
36 ×	4,772.52	Second total.
RCL 0	132.57	
29 ×	3,844.53	Third total.

The individual totals are still in the automatic memory, so you can easily calculate the combined total cost by adding them.

Keystrokes	Display	
+	8,617.05	
+	14,847.84	Total cost.

Storage Register Arithmetic

Arithmetic can also be performed using the number in the display and a number in a storage register. The arithmetic is performed *upon* the register contents, and answers are placed in the storage register—not in the display—so you have to recall them to see or use them.

To perform storage register arithmetic using the number in the display:

1. Press STO.
2. Press the desired arithmetic operation (+, −, ×, or ÷).
3. Press the storage location number.

For example, to store 6 in register R_3 and add 5:

Keystrokes	Display	
6 [STO] 3	6.00	Stores 6 in register R_3.
5 [STO] [+] 3	5.00	Adds 5 to register R_3.
[RCL] 3	11.00	Confirms that 11 is stored in register 3.

If you had pressed: 5 [STO] 3 [+], that would overwrite the stored 6 with the 5—the value stored in register R_3 would be 5, not 11.

Now, subtract 4 from the number in register R_3:

Keystrokes	Display
4 [STO] [−] 3	4.00
[RCL] 3	7.00

Notice that the general rule is:

Number in display, [STO], operation, location.

Clearing the Storage Registers

It is usually unnecessary to clear storage registers because you can simply write over the old number. (Previous data is pushed out of the way.)

The [CL ALL] function clears all storage registers and the automatic memory. The key serves to change all register values to zero.

Appendix A

RPN and the Automatic Memory Stack

The HP automatic memory stack and its associated logic, RPN*, is the most efficient method available for solving complex problems. The memory stack, composed of four distinct registers (memories) ''stacked'' on top of each other, allows the automatic storage of three intermediate results. Thus your calculator simplifies even the most complicated problems. A full understanding of the automatic memory is not necessary to perform simple arithmetic and financial calculations. But we strongly encourage you to *optimize your use of the calculator* by reading this appendix.

The Display

You can think of the stack as four shelves, one above the other. These shelves or memory registers are labelled X, Y, Z, and T, with T at the top and X at the bottom. Each register may contain one and only one number. The number itself, of course, may be comprised of up to 10 digits or it may be specified in scientific notation.

When the calculator is switched on, all four stack registers are set to 0.00, like this:

The Stack Registers

Name	Contents	
T	*0.00*	Top
Z	*0.00*	
Y	*0.00*	
X	*0.00*	Display (bottom)

The contents of the X-register are always visible: the X-register is the display on your calculator.

* RPN, HP's logic system, was named in honor of a logic notation devised by the Polish mathematician Jan Lukasiewicz. In 1951, he showed that arbitrary mathematical expressions could be specified unambiguously without parentheses. Originally developed to make the evaluation of algebraic expressions as simple as possible, RPN has been implemented in HP calculators by means of the stack and postfix operators. (The term ''postfix operators'' simply means that you specify the operation to be performed *after*, rather than before, you specify the number or numbers involved.) RPN enables you to see all intermediate answers, to recover easily from errors, to reuse numbers without reentering them, to solve problems one step at a time, and to evaluate virtually any expression without copying and remembering parentheses.

The ENTER↑ Key

When you key a number into the calculator, its contents are written into the displayed X-register. In order to key in another number, you must indicate to the calculator that you have completed keying in the first number and that any new digits you key in are part of a new number.

You use the ENTER↑ key to separate the first number from the second.

When you press the ENTER↑ key, the number in the displayed X-register is copied into Y.

Key in 314.32 and then press ENTER↑. The contents of the stack registers change...

...from this...

T	*0.00*	
Z	*0.00*	
Y	*0.00*	
X	*314.32*	Display

...to this.

T	*0.00*	
Z	*0.00*	
Y	*314.32*	
X	*314.32*	Display

Immediately after pressing ENTER↑, the X-register is prepared for a new number, and that new number *writes over* the number in X. Key in the number 543.28 and the contents of the stack registers change...

...from this...

T	*0.00*	
Z	*0.00*	
Y	*314.32*	
X	*314.32*	Display

...to this.

T	*0.00*	
Z	*0.00*	
Y	*314.32*	
X	*543.28*	Display

Now the calculator is ready to perform an arithmetic operation like $+$, $-$, $\times$, or $\div$, with the two numbers in the stack. Every arithmetic operation involving two numbers is done with the contents of the X- and Y-registers.

Clearing

The CLx key replaces any number in the display (X-register) with zero. If a new number is keyed in, it writes over the zero in X.

For example, if you had meant to key in 689.4 instead of 543.28, you would press CLx to change the stack...

...from this...			...to this.		
T	0.00		T	0.00	
Z	0.00		Z	0.00	
Y	314.32		Y	314.32	
X	543.28	Display	X	*0.00*	Display

and then key in 689.4 to change the stack...

...from this...			...to this.		
T	*0.00*		T	*0.00*	
Z	*0.00*		Z	*0.00*	
Y	*314.32*		Y	*314.32*	
X	*0.00*	Display	X	*689.4*	Display

The CL ALL key clears the entire calculator by replacing every number in the stack, storage registers, and financial registers with zero.

Arithmetic and the Stack

Hewlett-Packard calculators do arithmetic by positioning the numbers in the stack the same way you would on paper. For instance, if you wanted to add 34 and 21 you would write 34 on a piece of paper and then write 21 underneath it, like this:

$$\begin{array}{r} 34 \\ \underline{21} \end{array}$$

and then you would add, like this:

$$\begin{array}{r} 34 \\ \underline{+\ 21} \\ 55 \end{array}$$

As you can see, numbers are positioned the same way in the automatic memory stack. (If you clear the previous number entries first, the numbers in the stack will correspond to those shown in the example below.

Pressing CL ALL will set the stack register values to zero.)

Keystrokes	Display	
34	34.	34 is keyed into X.
ENTER↑	34.00	34 is copied into Y.
21	21.	21 writes over the 34 in X.

Now 34 and 21 are positioned vertically in the stack, so we can add.

Keystroke	Display	
+	55.00	The answer.

When you added, the contents of the stack registers changed...

...from this...		...to this.	
T	0.00	T	0.00
Z	0.00	Z	0.00
Y	34.00	Y	0.00
X	21.	X	55.00

Simple math notation helps explain how to use your calculator. Both numbers are always positioned in the stack in the natural order first then the operation is executed when the function key is pressed. *There are no exceptions to this rule*. Subtraction, multiplication, and division work the same way. In each case, the data must be in the proper position before the operation can be performed.

Manipulating Stack Contents

The x⇆y (*exchange*) and R↓ (*roll down*) keys allow you to review the stack contents or to shift data within the stack for computation at any time.

Exchanging X and Y

To see how the x⇆y key works, first load the stack with numbers 1 through 4 by pressing:

4 ENTER↑ 3 ENTER↑ 2 ENTER↑ 1

The numbers that you keyed in are now loaded into the stack, and its contents look like this:

T	4.00	
Z	3.00	
Y	2.00	
X	1.	Display

The [x⇄y] key exchanges the contents of the X- and Y-registers without affecting the Z- and T-registers. If you press [x⇄y], the numbers in the X- and Y-registers will be changed...

...from this...		...to this.	
T	4.00	T	4.00
Z	3.00	Z	3.00
Y	2.00	Y	1.00
X	1.	X	2.00

Similarly, pressing [x⇄y] again will restore the numbers in the X- and Y-registers to their original places. This key is used to position numbers in the stack or simply to view the Y-register.

Reviewing the Stack

Each time you press the [R↓] key, the stack contents shift downward one register. The last number that you have keyed in will be rotated around to the T-register when you press [R↓].

When you press the [R↓] key with the data intact from our example above, the stack contents are rotated...

...from this...			...to this.		
T	4.00		T	1.00	
Z	3.00		Z	4.00	
Y	2.00		Y	3.00	
X	1.00	Display	X	2.00	Display

Notice that the *contents* of the registers are shifted. The registers themselves maintain their positions. The contents of the X-register are always displayed, so 2.00 is now visible.

Press R↓ again and the stack contents are rotated...

...from this...			**...to this.**		
T	1.00		T	2.00	
Z	4.00		Z	1.00	
Y	3.00		Y	4.00	
X	2.00	Display	X	3.00	Display

Press R↓ twice more...and the stack rolls down...

...through this...			**...back to the start again.**		
T	3.00		T	4.00	
Z	2.00		Z	3.00	
Y	1.00		Y	2.00	
X	4.00	Display	X	1.00	Display

Once again the number 1.00 is in the displayed X-register. Now that you know how the stack is rotated, you can use the R↓ key to review the contents of the stack at any time so that you can always tell what is in the calculator. Always remember, though, that it takes four presses of the R↓ key to return the contents to their original registers.

Chain Arithmetic

You've already learned how to key numbers into the calculator and perform calculations with them. In each case you first needed to position the numbers in the stack manually using the ENTER↑ key. However, the stack also performs many movements automatically. These automatic movements add to its computing efficiency and ease of use, and it is these movements that automatically store intermediate results. The stack automatically "lifts" every calculated number up into the stack when a new number is keyed in because it "knows" that after it completes a calculation, any new digits you key in are a part of a new number. Also, the stack automatically "drops" when you perform an arithmetic operation.

To see how it works, let's solve

$$16 + 30 + 11 + 17 = ?$$

Keystrokes	Stack Contents		
16	T	0.00	
	Z	0.00	16 is keyed into the displayed X-register.
	Y	0.00	
	X	16.	
ENTER↑	T	0.00	
	Z	0.00	16 is copied into Y.
	Y	16.00	
	X	16.00	
30	T	0.00	
	Z	0.00	30 writes over the 16 in X.
	Y	16.00	
	X	30.	
+	T	0.00	
	Z	0.00	16 and 30 are added together. The answer, 46, is displayed.
	Y	0.00	
	X	46.00	
11	T	0.00	11 is keyed into the displayed X-register. The 46 in the stack is automatically raised.
	Z	0.00	
	Y	46.00	
	X	11.	
+	T	0.00	
	Z	0.00	46 and 11 are added together. The answer, 57, is displayed.
	Y	0.00	
	X	57.00	
17	T	0.00	
	Z	0.00	17 is keyed into the X-register. 57 is automatically entered into Y.
	Y	57.00	
	X	17.	
+	T	0.00	
	Z	0.00	57 and 17 are added together for the final answer.
	Y	0.00	
	X	74.00	

Notice that the stack automatically lifts when a new number is keyed in and automatically drops during calculations involving the X- and Y-registers. This automatic lift and drop of the stack give you tremendous computing power since you can retain and position intermediate results in long calculations without the necessity of reentering numbers.

To illustrate how numbers are retained in the stack, let's do the same problem a different way. Solve

$$16 + 30 + 11 + 17 = ?$$

Load the stack just as we did in the [x⇄y] example, by pressing:

16 [ENTER↑] 30 [ENTER↑] 11 [ENTER↑] 17

(Note that if you press [ENTER↑] after 17 you will lose 16 off the top of the stack.) Now the stack looks like this:

Stack Contents

T	***16.00***
Z	***30.00***
Y	***11.00***
X	***17.***

You can add by pressing [+] three times.

Each time you press [+], the contents of the X- and Y-registers are added together and the result is displayed (and stored) in the X-register. The contents of the Z-register drop into the Y-register so that the calculator is ready to perform arithmetic again with the contents of X and Y. But when the contents of the T-register (in our example, 16.00) drop into the Z-register, what should be in T? Actually, the designer had two choices: either put 0.00 (zero) in T or put the same number (16.00) in T. As we shall see, there is good reason to duplicate the 16.00 in the T-register (constant arithmetic).

Keystrokes	Stack Contents		
[+]	T	16.00	17 and 11 are added together and the rest of the stack drops. 16 drops to Z and is duplicated in T. 30 and 28 can be added.
	Z	16.00	
	Y	30.00	
	X	28.00	
[+]	T	16.00	30 and 28 are added together. The stack drops again and 16 drops into Y. Now 16 and 58 can be added.
	Z	16.00	
	Y	16.00	
	X	58.00	
[+]	T	16.00	
	Z	16.00	16 and 58 are added together for the final answer.
	Y	16.00	
	X	74.00	

This same dropping action also occurs with [−], [×], and [÷].

Constant Arithmetic

You have noticed that whenever the stack drops because of an arithmetic operation (not because of [R↓]), the number in the T-register is reproduced in T. Since the T-register will continue to copy this number (unless it is pushed off the top of the stack or written over by a new number), the number will drop down through the stack each time an arithmetic operation is performed. Thus, you can use this number "constantly" without reentering it each time you want to use it. Let's take the following example:

The production manager at Permex Pipes decides to package pipe products in quantities of 50, 75, 100, 250, and 500. She knows that the production cost per unit is $4.38. Using the procedure for constant arithmetic with her HP calculator, she can quickly calculate the total cost of the contents in each size package, without repeatedly keying in $4.38.

Keystrokes	Display	
4.38 [ENTER↑]		
[ENTER↑] [ENTER↑]	4.38	Enter the unit price into Y-, Z-, and T-registers.
50 [×]	219.00	Product cost; package of 50.
[CLx]	0.00	Clear previous answer.
75 [×]	328.50	Product cost; package of 75.
[CLx]	0.00	Clear previous answer.
100 [×]	438.00	Product cost; package of 100.
[CLx]	0.00	Clear previous answer.
250 [×]	1,095.00	Product cost; package of 250.
[CLx]	0.00	Clear previous answer.
500 [×]	2,190.00	Product cost; package of 500.

When you press [×] the first time, you calculate 4.38 times 50. The result (219.00) is displayed in the X-register and a new copy of the unit price (4.38) drops into the Y-register. Since a new copy of the unit price is duplicated from the T-register each time the stack drops, you never have to reenter it.

Notice that performing an arithmetic operation such as [×] causes the number in the T-register to be duplicated each time the stack drops. However, the [R↓] key, since it *rotates* the contents of the stack registers, does not *rewrite* any number, but merely shifts numbers that are already in the stack.

Another way to do constant arithmetic is to store the constant in a storage register and recall it each time you want to use it. (Refer to Storing and Recalling Numbers for an example.)

Order of Execution

If you run into a problem like this one:

$$5 \times \frac{[(3 \div 4) - (5 \div 2) + (4 \times 3)]}{(3 \times .213)}$$

you must decide where to begin before you press a key.

By starting *every* problem at its innermost number or parentheses and working outward, you maximize the efficiency and power of your HP calculator and reduce the number of keystrokes you need to use to solve a problem.

You *could* work the problem above by beginning at the left side of the equation and simply working through it in left-to-right order. All problems cannot be solved using left-to-right order, however, and the *best* order for solving any problem is to begin with the innermost parentheses and work outward just as you would on paper. So, to solve the problem above:

Keystrokes	Display	
3 [ENTER↑] 4 [÷]	0.75	Result of (3 ÷ 4).
5 [ENTER↑] 2 [÷]	2.50	Result of (5 ÷ 2).
[−]	-1.75	Result of (3 ÷ 4) − (5 ÷ 2).
4 [ENTER↑] 3 [×]	12.00	Result of (4 × 3).
[+]	10.25	Result of (3 ÷ 4) − (5 ÷ 2) + (4 × 3).
3 [ENTER↑] .213 [×]	0.64	Result of (3 × .213).
[÷]	16.04	Last division performed.
5 [×]	80.20	Finally, multiply by 5 for the answer.

Now try these problems. Remember to work through them as you would with a pencil and paper, but don't worry about intermediate answers—they're handled automatically by the calculator.

$$(2 \times 3) + (4 \times 5) = 26.00$$

$$\frac{(14 + 12) \times (18 - 12)}{(9 - 7)} = 78.00$$

$$4 \times (17 - 12) \div (10 - 5) = 4.00$$

Appendix B

Financial Formulas

For the following formulas, let:

n = number of compounding periods
i = periodic interest rate, expressed as a decimal
PV = present value
FV = future value or balance
PMT = periodic payment
S = payment switch position factor (0 or 1) indicating treatment of PMT; 0 corresponds to END, 1 to BEGIN.
I = interest amount

Percentage

$$\% = \frac{\text{Base (y)} \times \text{Rate (x)}}{100}$$

$$\Delta\% = \left(\frac{\text{New Amount (x)} - \text{Base (y)}}{\text{Base (y)}}\right) \times 100$$

$$\%T = \frac{\text{Amount (x)}}{\text{Total (y)}} \times 100$$

Simple Interest

$$I_{360} = \frac{n}{360} \times PV \times i$$

$$I_{365} = \frac{n}{365} \times PV \times i$$

Compound Interest

General equation:

$$0 = PV + (1 + iS)\,PMT\left[\frac{1 - (1 + i)^{-n}}{i}\right] + FV\,(1 + i)^{-n}$$

When we set PMT = 0, the general equation reduces to the familiar formulas:

$$PV = \frac{-\,FV}{(1 + i)^{n}} \quad \text{and} \quad FV = -\,PV \times (1 + i)^{n}$$

(The minus sign before PV, FV, and PMT, implies the sign convention.)

When the payment switch **BEGIN** **END** is set to END (ordinary annuity, payment in arrears), S = 0.

Solving for PV, when FV = 0 in our general equation, yields:

$$PV = -PMT\left[\frac{1 - (1 + i)^{-n}}{i}\right]$$

Solving for FV, when PV = 0 in our general equation, yields:

$$FV = -PMT \times \left[\frac{(1 + i)^{n} - 1}{i}\right]$$

When the payment switch **BEGIN** **END** is set to BEGIN (annuity due, payments in advance), S = 1.

Solving for PV, when FV = 0 in our general equation, yields:

$$PV = -PMT\left[\frac{1 - (1 + i)^{-n}}{i}\right](1 + i)$$

Solving for FV, when PV = 0 in our general equation, yields:

$$FV = -PMT \times \left[\frac{(1 + i)^{n} - 1}{i}\right](1 + i)$$

Amortization

n = number of payment periods to be amortized
INT_j = amount of PMT applied to interest in period j
PRN_j = amount of PMT applied to principal in period j
PV_j = present value (balance) of loan after payment in period j
j = period number

$$INT_1 = \begin{cases} 0 \text{ if } n = 0 \text{ and payment switch is set to BEGIN} \\ \left| PV_0 \times i \right|_{RND} \times (\text{sign of PMT}) \end{cases}$$

$$PRN_1 = PMT - INT_1$$
$$PV_1 = PV_0 + PRN_1$$

•
•
•

$$INT_j = \left| PV_{j-1} \times i \right|_{RND} \times (\text{sign of PMT})$$
$$PRN_j = PMT - INT_j$$
$$PV_j = PV_{j-1} + PRN_j$$

$$\Sigma INT = \sum_{j=1}^{n} INT_j = INT_1 + INT_2 + \ldots + INT_n$$

$$\Sigma PRN = \sum_{j=1}^{n} PRN = PRN_1 + PRN_2 + \ldots + PRN_n$$

$$PV_n = PV_0 + \Sigma PRN$$

Net Present Value

NPV = net present value of a discounted cash flow

CF_j = cash flow at period j

$$NPV = CF_0 + \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \ldots + \frac{CF_n}{(1+i)^n}$$

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