HEWLETT-PACKARD

HP-37E & HP-38E/38C

INVESTMENT ANALYSIS AND STATISTICS

Applications for Business Professionals and Students

For Continuous Memory Models

Although this book refers specifically to the HP-37E or HP-38E, the programs and calculations contained herein apply equally well to the HP-38C. The user should note, however, that the display format and data register contents are retained by the calculator even though it has been turned off. It may be desirable to reset or clear these conditions before running programs or making calculations.

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November 1979

00038-90026 Rev. C 11/79

Printed in U.S.A.

Introduction

This applications book has been designed to supplement the HP-37E and HP-38E *Owner's Handbooks* by providing a collection of applications specifically associated with investment analysis and business statistics. Step by step keystroke procedures and/or programs with corresponding examples for 18 problem types are explained. Hopefully, this book will provide a reference guide to the majority of your problems, and show you how to redesign our examples to fit your specific needs.

It is sometimes necessary in these keystroke solutions to include operations which involve prefix keys, namely, [f] on the HP-37E and [f] and [g] on the HP-38E. For example, the operation [12×] is performed on the HP-37E as [f] [12×] and on the HP-38E as [g] [12×]. In such cases, *the keystroke solution omits the prefix key and indicates only the operation* (as here, [12×]). As you work through the example problems, take care to press the appropriate prefix keys (if any) for your calculator.

In addition, it should be noted that certain clearing functions on the two calculators have different key mnemonics. *Clear finance* on the HP-37E is represented as [CL FIN], and is represented as [FIN] on the HP-38E. *Clear all* is represented as [CL ALL] and [ALL] on the HP-37E and HP-38E respectively. Unless otherwise specified, this book will use the key mnemonics of the HP-37E, although the keystrokes are applicable to both machines.

All results are carried internally to ten significant digits. If intermediate answers are rounded by the user, slightly different final values may be obtained.

Contents

Investment Analysis

Modified IRR—Varying Reinvestment Rate

The traditional IRR technique assumes that all positive cash flows are reinvested at the IRR to earn compound interest over the income projection period. It also assumes that all negative cash flows are to be discounted at the IRR. This means that cash can be invested today to earn compound interest at the IRR until it is needed to cover the forecasted negative cash flows.

Neither of these assumptions is necessarily realistic or valid. It is possible to compensate for either or both by using real-market rates to discount all negative flows (including Capital Outlay) to the present at a ''safe'' rate that will ensure liquidity when funds are needed; and to compound all positive flows at a realistic reinvestment rate to the end of the income projection period.

This procedure results in a single (negative) present value figure, and a single future value figure as well. IRR is then found by solving for [i] in a compounded amount procedure.

Example 1:

Negative Cash Flows, Reinvestment of Positive Flows at IRR.

A development project requires a total capital investment (development costs) of $600,000 staged as follows: $150,000 immediately, plus $150,000 at the end of years 1-3. Net sales proceeds over a total 10-year sellout period are projected as: Year 1—$0; Year 2—$50,000; Years 3-5—$125,000; Year 6—$140,000; Year 7—$150,000; Year 8—$175,000; Year 9—$100,000; Year 10—$50,000.

What is the indicated IRR for the developer, assuming he can earn 5.5% on the money required to cover future cash outlays (negative cash flows)?

The net cash flows projected are:

Year	Cash Flow	Year	Cash Flow
0	–$150,000	6	$140,000
1	–150,000	7	150,000
2	–100,000	8	175,000
3	– 25,000	9	100,000
4	125,000	10	50,000
5	125,000		

The steps in the procedure are:

1) Calculate the present value of the negative cash flows at the ''safe'' rate using the NPV routine.

2) Use the figure obtained in Step 1 as the initial investment in the IRR routine; store it in Register 0.

3) Entering 0 as the cash flow for years with a negative cash flow, find the IRR.

Keystrokes (HP-37E)	Display	
1 [ENTER↑] 5.5 [%] [+]		
[f] [1/x] [ENTER↑]		
[ENTER↑] [ENTER↑]		
25000 [CHS] [×]	–23,696.68	PV at 2nd cash flow
100000 [CHS] [+] [×]	–117,248.04	PV at 1st cash flow
150000 [CHS] [+] [×]	–253,315.68	PV of cash flows
150000 [CHS] [+]		
[STO] 0	–403,315.68	NPV of negative cash flows

Choose 12.5% as first guess:

1 [ENTER↑] 12.5 [%] [+]		
[f] [1/x] [ENTER↑]		
[ENTER↑] [ENTER↑]		
50000 [×]	44,444.44	PV at 9th cash flow
100000 [+] [×]	128,395.06	PV at 8th cash flow

Keystrokes	Display	
175000 [+] [×]	269,684.50	PV at 7th cash flow
150000 [+] [×]	373,052.89	PV at 6th cash flow
140000 [+] [×]	456,047.01	PV at 5th cash flow
125000 [+] [×]	516,486.23	PV at 4th cash flow
125000 [+] [×]	570,209.98	PV at 3rd cash flow
0 [+] [×]	506,853.32	PV at 2nd cash flow
0 [+] [×]	450,536.28	PV at 1st cash flow
0 [+] [×]	400,476.70	PV of cash flows
[RCL] 0 [+]	−2,838.98	NPV at 12.5%

Since the NPV is negative, the IRR is less than 12.5%. Try a lower rate, and repeat the procedure.

Keystrokes (HP-38E)	Display	
[f] [ALL]		
5.5 [i]		
150000 [CHS] [g] [CF_0]		
[g] [CF_j]	−150,000.00	Initial investment and first cash flow
100000 [CHS] [g] [CF_j]	−100,000.00	2nd cash flow
25000 [CHS] [g] [CF_j]	−25,000.00	3rd cash flow
[f] [NPV]	−403,315.68	Present value of negative cash flows at 5.5%
[g] [CF_0]		
0 [g] [CF_j] 3 [g] [N_j]	3.00	Three cash flows of 0
125000 [g] [CF_j]		
[g] [CF_j]	125,000.00	4th & 5th cash flows
140000 [g] [CF_j]	140,000.00	6th cash flow
150000 [g] [CF_j]	150,000.00	7th cash flow
175000 [g] [CF_j]	175,000.00	8th cash flow
100000 [g] [CF_j]	100,000.00	9th cash flow
50000 [g] [CF_j]	50,000.00	10th cash flow
[f] [IRR]	12.38	% annual rate of return

Example 2:

Using the cash flow figures in Example 1, what is the rate of return if the ''safe'' rate for negative cash flows is 5.5% and the reinvestment rate for positive cash flows is 10%.

Here the keystroke procedure is slightly different. The steps are:

1) Calculate the future value of the positive cash flows at the reinvestment rate.
2) Calculate the present value of the negative cash flows at the "safe" rate.
3) Knowing n, PV, and FV, solve for i.

Keystrokes (HP-37E)	Display	
1 [ENTER↑] 10 [%] [+]		
[f] [1/x] [ENTER↑]		
[ENTER↑] [ENTER↑]		
50000 [×]		
100000 [+] [×]		
175000 [+] [×]		
150000 [+] [×]		
140000 [+] [×]		
125000 [+] [×]		
125000 [+] [×]		
0 [+] [×] 0 [+] [×]		
0 [+] [×]	462,317.63	Present value of cash flows
[CL FIN]		
[CHS] [PV]		
10 [n] 10 [i]		
[FV]	1,199,132.88	Future value of positive cash flows at 10%
1 [ENTER↑] 5.5 [%] [+]		
[f] [1/x] [ENTER↑]		
[ENTER↑] [ENTER↑]		
25000 [CHS] [×]		
100000 [CHS] [+] [×]		
150000 [CHS] [+] [×]		
150000 [CHS] [+] [PV]	–403,315.68	Present value of negative cash flows at 5.5%
[i]	11.51	% annual rate of return

Keystrokes (HP-38E)	Display	
f ALL		
3 n	3.00	Skip negative cash flows (1-3)
125000 g CFj		
g CFj		
140000 g CFj		
150000 g CFj		
175000 g CFj		
100000 g CFj		
50000 g CFj	50,000.00	Cash flow in 10th year
10 i	10.00	Reinvestment rate
f NPV	462,317.63	Present value of cash flows
CHS PV FV	1,199,132.88	Future value of positive cash flows at 10%
5.5 i	5.50	Safe rate for negative cash flows
150000 CHS g CF_0		
g CFj		
100000 CHS g CFj		
25000 CHS g CFj		
f NPV PV	–403,315.68	Present value of negative cash flows
10 n		
i	11.51	% annual rate of return

Lease vs Purchase

An investment decision frequently encountered is the decision to lease or purchase capital equipment or buildings. Although a thorough evaluation of a complex acquisition usually requires the services of a qualified accountant, it is possible to simplify a number of assumptions to produce a first approximation.

The following HP-38E program assumes that the purchase is financed with a loan and that the loan is made for the term of the lease. The tax advantages of interest paid, depreciation, and the investment credit which accrues from ownership are compared to the tax advantage of treating the lease payment as an expense. The resulting cash flows are discounted to the present at the firms after tax cost of capital.

Basic Program

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[g] [P/R] [g] [CL P]	00-	[RCL] 1	12- 22 1
[−]	01- 41	[×]	13- 61
1	02- 1	[RCL] [PMT]	14- 22 14
[STO] [+] 3	03- 21 51 3	[−]	15- 41
[RCL] 1	04- 22 1	[RCL] 7	16- 22 7
[−]	05- 41	[−]	17- 41
[×]	06- 61	[RCL] 2	18- 22 2
[STO] 7	07- 21 7	[RCL] 3	19- 22 3
1	08- 1	[g] [y^x]	20- 25 21
[f] [AMORT]	09- 24 11	[÷]	21- 71
[g] [GTO] 24	10- 25 7 24	[STO] [+] 0	22- 21 51 0
[+]	11- 51	[g] [GTO] 00	23- 25 7 00

Depreciation Options

Declining Balance

KEY ENTRY	DISPLAY
[RCL] 5	24- 22 5
[RCL] 6	25- 22 6
[%]	26- 23
[−]	27- 41
[STO] 5	28- 21 5
[g] [R↓]	29- 25 33
[g] [LAST X]	30- 25 31
[g] [GTO] 11	31- 25 7 11
[g] [P/R]	

Sum-of-the-Years'-Digits

KEY ENTRY	DISPLAY
[RCL] 5	24- 22 5
[RCL] 4	25- 22 4
[RCL] 6	26- 22 6
[÷]	27- 71
[×]	28- 61
1	29- 1
[STO] [−] 4	30- 21 41 4
[g] [R↓]	31- 25 33
[g] [GTO] 11	32- 25 7 11
[g] [P/R]	

Straight Line

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[RCL] 5	24- 22 5		
[g] [GTO] 11	25- 25 7 11		
[g] [P/R]			

REGISTERS			
R_0 Purch. Adv.	R_1 Tax	R_2 Discount	R_3 Used
R_4 SOYD	R_5 Depr.	R_6 Factor	R_7 Used

1) Key in the basic program (23 steps).

2) Select the depreciation option and continue keying in the program.

3) Set payment switch to END and press [f] [ALL].

4) Input the following information for the purchase loan:
   - Key in number of years for amortization; press [n].
   - Key in the yearly interest rate; press [i].
   - Key in the loan amount (purchase price); press [CHS] [PV]. Press [PMT].

5) Key in the marginal effective tax rate[1]; press [STO] 1.

6) Key in the discount rate or cost of capital[1]; press [ENTER↑] 1 [+] [STO] 2.

7) Select and use the keystrokes which correspond to the depreciation option chosen.

   Straight-Line

   - Key in the depreciable value; press [ENTER↑].
   - Key in the salvage value; press [−].
   - Key in the life of the asset in years; press [÷] [STO] 5.

   Sum-of-the-Years'-Digits

   - Key in the depreciable value; press [ENTER↑].
   - Key in the salvage value; press [−] [STO] 5.
   - Key in the life of the asset (in years); press [ENTER↑] [ENTER↑] [STO] 4.
   - Key in 1 [+] [×] 2 [÷] [STO] 6.

[1] Key in as a decimal (e.g., 5% as .05)

Declining Balance

- Key in the depreciable value; press [STO] 5.
- Key in the depreciation factor (as a percent); press [ENTER↑].
- Key in the life of the asset (in years); press [÷] [STO] 6.

8) Key in the total first lease payment (including any advance payments); press [ENTER↑] 1 [RCL] 1 [−] [×] [STO] 0.

9) Key in the first year's maintenance expense that would be anticipated if the asset was owned; press [ENTER↑]. If the lease contract does not include maintenance, then it is not a factor in the lease vs purchase decision and 0 expense is used.

10) Key in the next lease payment; press [R/S]. During any year in which a lease payment does not occur (e.g., the last several payments of an advance payment contract), use 0 for the payment.

11) Repeat steps 9 and 10 for all maintenance expenses and lease payments over the term of the analysis.

    Optional—If the investment Tax credit is taken, key in the amount of the credit after finishing steps 9 and 10 for the year in which the credit is taken; press [g] [GTO] 18 [R/S]. Continue steps 9 and 10 for the remainder of the term.

12) After all lease payments and expenses have been entered (steps 9 and 10), key in the lease buy back option; press [ENTER↑] 1 [RCL] 1 [−] [×] [g] [GTO] 18 [R/S]. If no buy back option exists, use the estimated salvage value of the purchased equipment at the end of the term.

13) To find the net advantage of owning press [RCL] 0. If the recalled value is *negative* it represents a net lease advantage.

Example:

Home Style Bagel Company is evaluating the acquisition of a mixer which can be leased for $1700 a year with the first and last payments in advance and a $750 buy back option at the end of 10 years (maintenance is included).

The same equipment could be purchased for $10,000 with a 12% loan amortized over 10 years. Ownership maintenance is estimated to be 2% of the purchased price per year for the first four years. A major overhaul is predicted for the 5th year at a cost of $1500. Subsequent yearly maintenance of 3% is estimated for the remainder of the 10 year term. The company would use sum-of-the-years'-digits depreciation on a 10 year life with $1500 salvage value. An accountant informs management to take the 10% capital investment tax credit at the end of the second year and to figure the cash flows at a 48% tax rate. The after tax cost of capital (discounting rate) is 5 per cent.

Because lease payments are made in advance and standard loan payments are made in arrears the following cash flow schedule is appropriate for a lease with the last payment in advance.

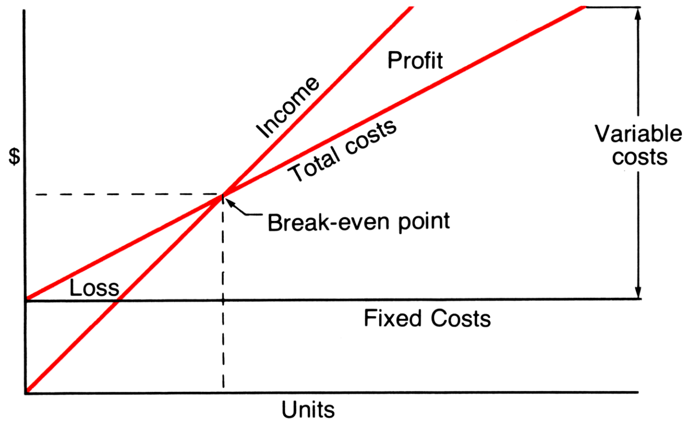
Year	Maintenance	Lease Payment	Tax Credit	Buy Back
0		1700 + 1700		
1	200	1700		
2	200	1700	1000	
3	200	1700		
4	200	1700		
5	1500	1700		
6	300	1700		
7	300	1700		
8	300	1700		
9	300	0		
10	300	0		750

Keystrokes	Display	
BEGIN [switch] END		
[f] [ALL]	0.00	
10 [n] 12 [i]		
10000 [CHS] [PV]	-10,000.00	Always use negative loan amount
[PMT]	1,769.84	Purchase payment
.48 [STO] 1	0.48	Tax rate
.05 [ENTER↑]		
1 [+] [STO] 2	1.05	Discounting factor
10000 [ENTER↑]		
1500 [−] [STO] 5	8,500.00	Depreciable value

Keystrokes	Display	
10 [ENTER↑] [ENTER↑]		
[STO] 4	10.00	Depreciable life
1 [+] [×] 2 [÷]		
[STO] 6	55.00	SOYD base
1700 [ENTER↑] [+]	3,400.00	1st lease payment
1 [RCL] 1 [−] [×]		
[STO] 0	1,768.00	After tax expense
200 [ENTER↑]		
1700 [R/S]	312.36	Present value of 1st year's net purchase advantage
200 [ENTER↑]		
1700 [R/S]	200.43	2nd year's advantage
1000 [g] [GTO] 18		
[R/S]	907.03	Present value of tax credit
200 [ENTER↑]		
1700 [R/S]	95.05	3rd year
200 [ENTER↑]		
1700 [R/S]	−4.38	4th year
1500 [ENTER↑]		
1700 [R/S]	−628.09	5th year
300 [ENTER↑]		
1700 [R/S]	−226.44	6th year
300 [ENTER↑]		
1700 [R/S]	−309.48	7th year
300 [ENTER↑]		
1700 [R/S]	−388.80	8th year
300 [ENTER↑] 0 [R/S]	−1,034.72	9th year
300 [ENTER↑] 0 [R/S]	−1,080.88	10th year
750 [ENTER↑]		
1 [RCL] 1 [−] [×]	390.00	After tax buy back expense
[g] [GTO] 18 [R/S]	239.43	Present value
[RCL] 0	−150.49	Net *lease* advantage

Break-Even Analysis

Break-even analysis is basically a technique for analyzing the relationships among fixed costs, variable costs, and income. Until the break-even point is reached, at the intersection of the total income and total cost lines, the producer operates at a loss. After the break-even point, each unit produced and sold makes a profit. Break-even analysis may be represented as follows:



Given four of these variables: fixed costs, sales price per unit, variable costs per unit, number of units sold, and gross profit, the following procedures evaluate the remaining variable. To calculate the break-even values, simply let the gross profit equal zero.

Gross Profit

1) Key in the sales price per unit, press [ENTER↑].
2) Key in the variable costs per unit, press [−].
3) Key in the number of units sold, press [×].
4) To find the gross profit, key in the fixed costs and press [−].

Fixed Costs

1) Key in the sales price per unit, press [ENTER↑].
2) Key in the variable costs per unit, press [−].
3) Key in the number of units sold, press [×].
4) To find the amount of fixed costs, key in the gross profit and press [−].

Sales Price Per Unit

1) Key in the gross profit, press [ENTER↑].
2) Key in the fixed costs, press [+].
3) Key in the number of units sold, press [÷].
4) To find the sales price per unit, key in the variable costs per unit and press [+].

Variable Costs Per Unit

1) Key in the sales price per unit, press [ENTER↑].
2) Key in the gross profit, press [ENTER↑].
3) Key in the fixed costs, press [+].
4) Key in the number of units sold, press [÷].
5) Press [−] to find the variable costs per unit.

Number of Units Sold

1) Key in the fixed costs, press [ENTER↑].
2) Key in the gross profit, press [+].
3) Key in the sales price per unit, press [ENTER↑].
4) Key in the variable costs per unit, press [−].
5) Press [÷] to obtain the number of units sold.

Example 1:

The Cooper Company sells finance textbooks at $13 each. Given the costs and revenues below, how many textbooks must be sold to break even?

Fixed Costs	
Typesetting	$ 4,000
Graphics production	5,000
Printing and binding	3,000
Total fixed costs	$12,000

Variable Costs Per Copy

Distribution	$ 1.00
Commissions	3.75
Royalties	2.00
Total variable cost per copy	$ 6.75
Sales price per copy	$ 13.00

Keystrokes	Display	
12000 [ENTER↑] 0 [+]		
13 [ENTER↑] 6.75 [−] [÷]	1,920.00	Number of units

The degree of operating leverage at a point is defined as the ratio of the percentage change in net operating income to the percentage change in units sold. The greatest degree of operating leverage is found near the break-even point, where a small change in sales may produce a very large increase in profits. This happens because the profits are close to zero near the break-even point. Likewise, firms with a small degree of operating leverage are operating farther from the break-even point, and they are relatively insensitive to changes in sales volume.

The necessary inputs to calculate the degree of operating leverage are fixed costs, sales price per unit, variable costs per unit, and number of units sold.

The keystrokes are:

1) Key in the sales price per unit, press [ENTER↑].
2) Key in the variable costs per unit, press [−].
3) Key in the number of units, press [×] [ENTER↑] [ENTER↑].
4) Key in the fixed costs, press [−].
5) Press [÷] to determine the degree of operating leverage.

Example 2:

In the above example, what is the Cooper Company's degree of operating leverage at 2000 units? At 5000 units?

Keystrokes	Display	
13 [ENTER↑] 6.75 [−] 2000 [×] [ENTER↑] [ENTER↑] 12000 [−] [÷]	25.00	This is close to the break-even point.
13 [ENTER↑] 6.75 [−] 5000 [×] [ENTER↑] [ENTER↑] 12000 [−] [÷]	1.62	The company is farther from the break-even point and less sensitive to changes in sales volume

Bonds

A bond is a contract to pay interest, usually semi-annually, at a given rate (coupon), and to pay the principal of the bond at some specified future date. The value or price of a bond is the present value of the coupon payments plus the present value of the principal or redemption value, at a given interest rate (yield).

Bond Purchased on Coupon Date

When a bond is purchased on a coupon date, the price may be found as follows:

1) Set the payment switch to END and press [CL FIN].
2) Key in the total number of *periodic* coupons; press [n].
3) Key in the desired *periodic* yield (yield to maturity) as a percent; press [i].
4) Key in the amount of the *periodic* coupon; press [PMT].*
5) Key in the redemption value; press [FV].*
6) Press [PV]* to obtain the purchase price of the bond.

* Positive for cash received; negative for cash paid out.

Example:

Interest on a 4½%, $1000 bond due in 25 years is payable semi-annually with the first payment in 6 months. What will be the price of this bond to yield 5½% compounded semi-annually?

Keystrokes	Display	
BEGIN ▮▮ END		
[CL FIN]		
25 [ENTER↑] 2 [×] [n]	50.00	Total number of coupons
5.5 [ENTER↑] 2 [÷] [i]	2.75	Periodic yield
1000 [ENTER↑] 4.5 [%]		
2 [÷] [PMT]	22.50	Periodic coupon (received)
1000 [FV]	1,000.00	Redemption value (received)
[PV]	-865.01	Purchase price

Likewise, the yield of a bond purchased on a coupon date may be found as follows:

1) Set the Payment switch to END and press [CL FIN].
2) Key in the total number of *periodic* coupons; press [n].
3) Key in the amount of the *periodic* coupon; press [PMT].*
4) Key in the purchase price; press [PV].*
5) Key in the redemption value; press [FV].*
6) Press [i] to obtain the *periodic* yield as a percent.

Example:

A buyer pays $850 for a 4¾%, $1000 bond with semi-annual coupons for the next 15 years. If he holds the bond to maturity, what is his annual yield?

* Positive for cash received; negative for cash paid out.

Keystrokes	Display	
BEGIN ▮ END		
[CL FIN]		
15 [ENTER↑] 2 [×] [n]	30.00	Total number of coupons
1000 [ENTER↑] 4.75 [%]		
2 [÷] [PMT]	23.75	Periodic coupon received
850 [CHS] [PV]	-850.00	Purchase price
1000 [FV]	1,000.00	Redemption value
[i] 2 [×]	6.31	Annual yield as a percent

Bond Purchased Between Coupons

Bonds are frequently purchased at some time between coupon periods. In such cases, the Securities Industry Association has established certain formulae[1] to determine their price and yield. For semi-annual bonds held for more than 6 months, the following HP-38E program evaluates bond price and accrued interest on an Actual/Actual day basis:

[1] Standard Securities Calculation Methods; Securities Industry Association; 1973.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
g P/R g CL P	00–	PMT	28– 14
f FIN	01– 24 32	RCL 4	29– 22 4
RCL 1	02– 22 1	i	30– 12
f INTGR	03– 24 61	RCL 5	31– 22 5
6	04– 6	FV	32– 15
.	05– 73	PV	33– 13
0	06– 0	RCL 6	34– 22 6
1	07– 1	RCL 1	35– 22 1
g x≤y	08– 25 5	f ΔDAYS	36– 24 41
g GTO 21	09– 25 7 20	STO 7	37– 21 7
RCL 1	10– 22 1	RCL 6	38– 22 6
5	11– 5	RCL 0	39– 22 0
.	12– 73	f ΔDAYS	40– 24 41
9	13– 9	RCL 7	41– 22 7
9	14– 9	÷	42– 71
9	15– 9	n	43– 11
9	16– 9	0	44– 0
9	17– 9	PMT	45– 14
9	18– 9	FV	46– 15
+	19– 51	CHS	47– 32
g GTO 24	20– 25 7 23	RCL n	48– 22 11
RCL 1	21– 22 1	RCL 3	49– 22 3
6	22– 6	CHS	50– 32
−	23– 41	×	51– 61
STO 6	24– 21 6	R/S	52– 74
RCL 2	25– 22 2	−	53– 41
n	26– 11	g GTO 00	54– 25 7 00
RCL 3	27– 22 3	g P/R	

REGISTERS			
R_0 Settlement	R_1 Next coupon	R_2 # periods	R_3 Coupon
R_4 Yield	R_5 Redemption	R_6 Last coupon	R_7 Used

For bonds calculated on a 30/360 day basis, two additional program steps are needed. Insert [g] [R↓] after [f] [ΔDAYS] at steps 36 and 40.

1) Key in the program.
2) Set the Payment switch to END.
3) Key in the settlement (purchase) date (MM.DDYYYY); press [STO] 0.
4) Key in the date (MM.DDYYYY) of the next coupon; press [STO] 1.
5) Key in the *total* number of coupons which are received; press [STO] 2.
6) Key in the amount of the *semi-annual* coupon; press [STO] 3.*
7) Key in the *semi-annual* yield as a percent; press [STO] 4.
8) Key in the redemption value; press [STO] 5.*
9) Press [R/S] to obtain the amount of accrued interest (a share of the next coupon to which the seller is entitled).
10) Press [R/S] to determine the price of the bond.

Note: If a coupon is received on the last day of the month (i.e., October 31), an error condition may result. This happens because the count-back routine determines the last coupon date to be exactly 6 months earlier (i.e., April 31), and this may be an illegal date.

Example:

Given the following U.S. Treasury Bond, find its price.

Settlement date January 3, 1977; maturity date December 14, 1990 (28 coupon periods); next coupon date June 14, 1977; coupon rate 4.75%; yield 5%.

* Positive for cash received; negative for cash paid out.

Keystrokes	Display	
BEGIN [switch] END		
1.031977 [STO] 0	1.03	Settlement date
6.141977 [STO] 1	6.14	Next coupon date
28 [STO] 2	28.00	Total number of coupons
4.75 [ENTER↑] 2 [÷]		
[STO] 3	2.38	Semi-annual coupon
5 [ENTER↑] 2 [÷]		
[STO] 4	2.50	Semi-annual yield
100 [STO] 5	100.00	Redemption value is not specified and is assumed to be 100
[R/S]	-0.26	Accrued interest
[R/S]	-97.51	Purchase price

To find bond yield, an iteration (trial and error) approach may be used with the bond price program. The user inputs successive ''guesses'' at the periodic yield into register 4 and solves for price. If the calculated price is the actual price paid, the yield is in register 4. If not, adjust the yield and repeat the procedure until the desired accuracy is obtained.

Annual Coupon Bonds

For bonds which have annual coupons, use the following HP-38 program to evaluate price and accrued interest on an Actual/Actual day basis:

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[g] [P/R] [g] [CL P]	00-	[STO] 7	20- 21 7
[f] [FIN]	01- 24 32	[RCL] 6	21- 22 6
[RCL] 0	02- 22 0	[RCL] 4	22- 22 4
[n]	03- 11	[f] [△DAYS]	23- 24 41
[RCL] 1	04- 22 1	[RCL] 7	24- 22 7
[PMT]	05- 14	[÷]	25- 71
[RCL] 2	06- 22 2	[n]	26- 11
[i]	07- 12	0	27- 0
[RCL] 3	08- 22 3	[PMT]	28- 14
[FV]	09- 15	[FV]	29- 15
[PV]	10- 13	[CHS]	30- 32
[R/S]	11- 74	[RCL] [n]	31- 22 11
[RCL] 5	12- 22 5	[RCL] 1	32- 22 1
[g] [EEX]	13- 25 32	[CHS]	33- 32
6	14- 6	[×]	34- 61
[CHS]	15- 32	[R/S]	35- 74
[−]	16- 41	[−]	36- 41
[STO] 6	17- 21 6	[g] [GTO] 00	37- 25 7 00
[RCL] 5	18- 22 5	[g] [P/R]	
[f] [△DAYS]	19- 24 41		

REGISTERS			
R_0 # periods	R_1 Coupon	R_2 Yield	R_3 Redemption
R_4 Settlement	R_5 Next coupon	R_6 Last coupon	R_7 Used

For annual coupon bonds calculated on a 30/360 day basis, insert [g] [R↓] after [f] [ΔDAYS] at steps 19 and 23.

1) Key in the program.
2) Set the Payment switch to END.
3) Key in the *total* number of coupons which are received; press [STO] 0.
4) Key in the amount of the *annual* coupon; press [STO] 1.*
5) Key in the *annual* yield as a percent; press [STO] 2.
6) Key in the redemption value; press [STO] 3.*
7) Press [R/S]. The present value of the total coupons and redemption value is displayed.
8) Set the Payment switch to BEGIN. This changes the calendar input format to DD.MMYYYY.
9) Key in the settlement (purchase) date (DD.MMYYYY); press [STO] 4.
10) Key in the date (DD.MMYYYY) of the next coupon; press [STO] 5.
11) Press [R/S] to obtain the amount of accrued interest.
12) Press [R/S] to determine the price of the bond.
13) For a new case, return to step 2.

Example:

What is the price and accrued interest of a 20 year Eurobond with annual coupons of 5.25% purchased on August 15, 1977 to yield 5%? The next coupon is received on December 1, 1977.

Keystrokes	Display	
BEGIN [switch] END		
20 [STO] 0	20.00	Total number of coupons
5.25 [STO] 1	5.25	Annual coupon
5 [STO] 2	5.00	Annual yield
100 [STO] 3	100.00	Redemption value
[R/S]	–103.12	PV of 20 coupons and redemption value

* Positive for cash received; negative for cash paid out.

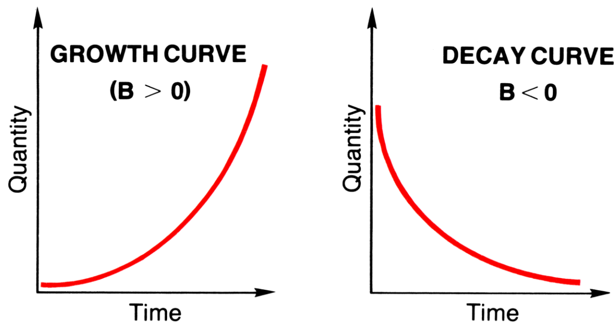
Keystrokes	Display	
BEGIN [switch] END		
15.081977 [STO] 4	15.08	Settlement date
1.121977 [STO] 5	1.12	Next coupon date
[R/S]	-3.70	Accrued interest
[R/S]	-103.02	Purchase price

Statistics

Exponential Curve Fit

Using the LN function of the HP-37E and HP-38E, a least squares exponential curve fit may easily be calculated according to the equation $y = Ae^{Bx}$. The exponential curve fitting technique is often used to determine the growth rate of a variable such as a stock's value over time, when it is suspected that the performance is non-linear. The value for B is the decimal value of the continuous growth rate. For instance, assume after keying in several end-of-month price quotes for a particular stock, it is determined that the value for B is 0.10. This means that over the measured period the stock has experienced a 10% continuous growth rate. This decimal continuous growth rate may then be converted to an effective growth rate.

If $B > 0$, you will have a growth curve. If $B < 0$, you will have a decay curve. Examples of these are given below.



The procedure is as follows:

1) Press CL ALL.
2) For each input pair of values, key in the y-value and press LN; key in the corresponding x-value and press Σ+.

3) After all data pairs are input, press [ŷ,r] [x≷y] to obtain the correlation coefficient.
4) Press 1 [ŷ,r] [e^x] 0 [ŷ,r] [e^x] to obtain A in the equation above.
5) Press [x≷y] [R↓] [÷] [LN] to obtain B.
6) Press [e^x] 1 [−] to obtain the effective growth rate (as a decimal).
7) To make a y-estimate, key in the x-value; press [ŷ,r] [e^x].

Example 1:

A stock's price history is listed below. What effective growth rate does this represent? If the stock continues this growth rate, what is the price projected to be at the end of 1978 (year 7)?

End of Year	Price
1972 (1)	45
1973 (2)	51.5
1974 (3)	53.75
1975 (4)	80
1976 (5)	122.5
1977 (6)	210
1978 (7)	?

Keystrokes	Display	
[CL ALL]		
45 [LN] 1 [Σ+]	1.00	First data pair input
51.5 [LN] 2 [Σ+]	2.00	Second data pair input
53.75 [LN] 3 [Σ+]	3.00	Third data pair input
80 [LN] 4 [Σ+]	4.00	Fourth data pair input
122.5 [LN] 5 [Σ+]	5.00	Fifth data pair input
210 [LN] 6 [Σ+]	6.00	Sixth data pair input
[ŷ,r] [x≷y]	0.95	Correlation coefficient
1 [ŷ,r] [e^x] 0 [ŷ,r] [e^x]	27.34	A
[x≷y] [R↓] [÷] [LN]	0.31	B
[e^x] 1 [−]	0.36	Effective growth rate
7 [ŷ,r] [e^x]	232.35	Projected price at the end of year 7 (1978)

For repeated use of this routine, the following HP-38E program could be beneficial.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[g] [P/R] [g] [CL P]	00-	[R/S]	15- 74
[x⇄y]	01- 33	[x⇄y]	16- 33
[g] [LN]	02- 25 23	[g] [R↓]	17- 25 33
[x⇄y]	03- 33	[÷]	18- 71
[f] [Σ+]	04- 24 74	[g] [LN]	19- 25 23
[g] [GTO] 00	05- 25 7 00	[R/S]	20- 74
[g] [ŷ,r]	06- 25 2	[g] [e^x]	21- 25 22
[x⇄y]	07- 33	1	22- 1
[R/S]	08- 74	[−]	23- 41
1	09- 1	[R/S]	24- 74
[g] [ŷ,r]	10- 25 2	[g] [ŷ,r]	25- 25 2
[g] [e^x]	11- 25 22	[g] [e^x]	26- 25 22
0	12- 0	[g] [GTO] 00	27- 25 7 00
[g] [ŷ,r]	13- 25 2	[g] [P/R]	
[g] [e^x]	14- 25 22		

REGISTERS			
R_0	R_1 n	R_2 Σx	R_3 Σx^2
R_4 Σy	R_5 Σy^2	R_6 Σxy	R_7

1) Key in the program and press [f] [ALL].

2) For each input pair of values, key in the y-value and press [ENTER↑]; key in the corresponding x-value and press [R/S].

3) After all data pairs are input, press [g] [GTO] 06 [R/S] to obtain the correlation coefficient.

4) Press [R/S] to obtain A.

5) Press [R/S] to obtain B.

6) Press [R/S] to obtain the effective growth rate as a decimal.

7) To make a y-estimate, key in the x-value; press [R/S]. For subsequent estimates, key in the x-value and press [g] [GTO] 25 [R/S].

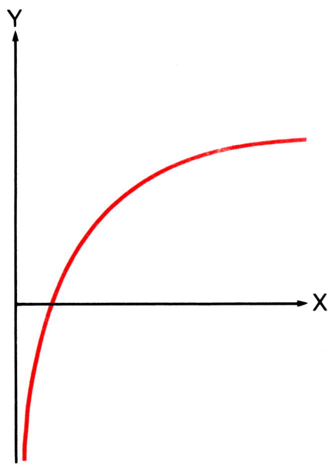
8) For a different set of data, press [f] [ALL] and proceed at step 2.

Keystrokes	Display	
[f] [ALL]		
45 [ENTER↑] 1 [R/S]		
51.5 [ENTER↑] 2 [R/S]		
53.75 [ENTER↑] 3 [R/S]		
80 [ENTER↑] 4 [R/S]		
122.5 [ENTER↑] 5 [R/S]		
210 [ENTER↑] 6 [R/S]		
[g] [GTO] 06 [R/S]	0.95	Correlation coefficient
[R/S]	27.34	A
[R/S]	0.31	B
[R/S]	0.36	Effective growth rate
7 [R/S]	232.35	Projected price at the end of year 7 (1978)

Logarithmic Curve Fit

If your data does not fit a line or an exponential curve, try the following logarithmic curve fit. This is calculated according to the equation $y = A + B (\ln x)$, and all x values must be positive.

A typical logarithmic curve is shown below.



The procedure is as follows:

1) Press [CL ALL].
2) Key in the first y-value; press [ENTER↑]. Key in the first x-value; press [LN][Σ+]. Repeat this step for each data pair.
3) After all data pairs are input, press [ŷ,r] [x⇄y] to obtain the correlation coefficient.
4) Press 1 [ŷ,r] 0 [ŷ,r] to obtain A in the equation above.
5) Press [x⇄y][R↓][−] to obtain B.
6) To make a y-estimate, key in the x-value; press [LN][ŷ,r].

Example 1:

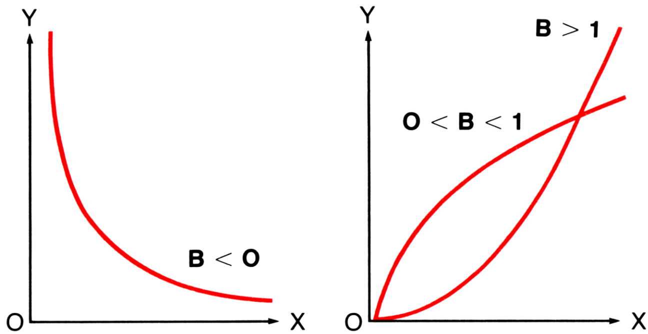
A manufacturer observes declining sales of a soon-to-be obsoleted product, of which there were originally 10,000 units in inventory. The cumulative sales figures over a number of months, given below, may be fit by a logarithmic curve of the form y = A + B ln x, where y represents cumulative sales in units and x the number of months since the beginning. How many units will be sold by the end of the eighth month?

Month	1	2	3	4	5	6
Cumulative Sales (units)	1431	3506	5177	6658	7810	8592

Keystrokes	**Display**	
[CL ALL]		
1431 [ENTER↑] 1 [LN][Σ+]		
3506 [ENTER↑] 2 [LN][Σ+]		
5177 [ENTER↑] 3 [LN][Σ+]		
6658 [ENTER↑] 4 [LN][Σ+]		
7810 [ENTER↑] 5 [LN][Σ+]		
8592 [ENTER↑] 6 [LN][Σ+]	6.00	Six data pairs
[ŷ,r] [x⇄y]	0.99	Correlation coefficient
1 [ŷ,r] 0 [ŷ,r]	1,066.15	Value of A
[x⇄y][R↓][−]	4,069.93	Value of B
8 [LN][ŷ,r]	9,529.34	Total units sold by end of eighth month

Power Curve Fit

Another method of analysis is the power curve or geometric curve. The equation of the power curve is $y = Ax^B$, and the values for A and B are computed by calculations similar to linear regression. Some examples of power curves are shown below.



The following keystrokes fit a power curve according to the equation $\ln y = \ln A + B (\ln x)$:

1) Press [CL ALL].
2) Key in the first y-value; press [LN]. Key in the first x-value; press [LN][Σ+]. Repeat this step for all data pairs.
3) Press [ŷ,r] [x⇄y] to obtain the correlation coefficient.
4) Press 0 [ŷ,r] [e^x] to obtain A in the above equation.
5) Press 1 [ŷ,r] 0 [ŷ,r] [x⇄y][R↓][−] to obtain B.
6) To find a projected y, key in the x-value and press [LN][ŷ,r] [e^x] to obtain y.

Example 1:

If Galileo had wished to investigate quantitatively the relationship between the time (t) for a falling object to hit the ground and the height (h) it has fallen, he might have released a rock from various levels of the Tower of Pisa (which was leaning even than) and timed its descent by counting his pulse. The following data are measurements Galileo might have made.

t (pulses)	2	2.5	3.5	4	4.5
h (feet)	30	50	90	130	150

Find the power curve formulas that best expresses h as a function of t ($h = At^B$).

Keystrokes	**Display**	
[CL ALL]		
30 [LN] 2 [LN] [Σ+]		
50 [LN] 2.5 [LN] [Σ+]		
90 [LN] 3.5 [LN] [Σ+]		
130 [LN] 4 [LN] [Σ+]		
150 [LN] 4.5 [LN] [Σ+]	*5.00*	Five data pairs
[ŷ,r] [x⇄y]	*1.00*	Correlation coefficient
0 [ŷ,r] [e^x]	*7.72*	Value of A
1 [ŷ,r] 0 [ŷ,r]		
[x⇄y] [R↓] [−]	*1.99*	Value of B

So, the formula that best expresses h as a function of t is

$$h = 7.72\ t^{1.99}$$

We know, as Galileo did not, that in fact $h \propto t^2$ (h is proportional to t^2).

Exponential Smoothing

A common method for analyzing trends in sales, inventory and securities time series data is the moving average. Exponential smoothing is a version of the weighted moving average which is readily adaptable to programmable calculator forecasting.

In the use of exponential smoothing, a smoothing factor is chosen which effects the sensitivity of the average much the same way as the length of the standard moving average period. The correspondence between the two techniques can be represented by the formula

$$\alpha = \frac{2}{n + 1}$$

where α is the exponential smoothing factor (with values from 0 to 1) and n is the length of the standard moving average. As the equation shows, the longer the moving average period, the smaller the equivalent α, and the less sensitive the average becomes to fluctuations in current values.

Forecasting with exponential smoothing involves selecting the best smoothing factor based on historical data and then using the factor for updating subsequent data and forecasting. This procedure uses the following HP-38E program.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[g] [P/R] [g] [CL P]	00-	[CHS]	19- 32
[ENTER↑]	01- 31	[x⇄y]	20- 33
[ENTER↑]	02- 31	[STO] 2	21- 21 2
[RCL] 4	03- 22 4	[+]	22- 51
[−]	04- 41	[RCL] 0	23- 22 0
[ENTER↑]	05- 31	[×]	24- 61
[×]	06- 61	[RCL] 1	25- 22 1
[STO] [+] 5	07- 21 51 5	[RCL] 3	26- 22 3
[g] [LAST X]	08- 25 31	[×]	27- 61
[R/S]	09- 74	[+]	28- 51
[g] [R↓]	10- 25 33	[STO] 3	29- 21 3
[g] [R↓]	11- 25 33	[RCL] 1	30- 22 1
[RCL] 0	12- 22 0	[×]	31- 61
[×]	13- 61	[RCL] 0	32- 22 0
[RCL] 2	14- 22 2	[÷]	33- 71
[RCL] 1	15- 22 1	[RCL] 2	34- 22 2
[×]	16- 61	[+]	35- 51
[+]	17- 51	[STO] 4	36- 21 4
[RCL] 2	18- 22 2	[g] [P/R]	

REGISTERS			
R_0 α	R_1 $1-\alpha$	R_2 S_{t-1}	R_3 T_{t-1}
R_4 D_t	R_5 Σe^2	R_6	R_7

Selecting the "best" smoothing constant (α).

1) Key in the number 1; press [ENTER↑].
2) Key in the "trial α"; press [STO] 0 [−] [STO] 1.
3) Key in the first historical value (X_1); press [ENTER↑] [STO] 2.

4) Key in the second historical value (X_2); press [STO] 4 [x⇄y] [−] [RCL] 0 [×] [RCL] 1 [÷] [STO] 3.

5) Key in X_2 again; press [R/S]. The result is the error (e_n) between the forecast value (D_n) and the true value (X_n).

6) Press [R/S]. The display shows the next forecast.

7) Continue steps 5 and 6 for X_3, X_4... X_n until all historical values have been entered.

8) Press [RCL] 5. This values represents the cummulative forecasting error (Σe^2). Record the value and the following additional values; press [RCL] 0 (α), [RCL] 2 (smoothed average S_{t-1}), [RCL] 3 (trend T_{t-1}) and [RCL] 4 (forecast D_t).

9) Press [f] [ALL].

10) Repeat steps 1 through 9 until a "best" α is selected based on the lowest cummulative forecasting error (Register 5).

Forecasting

1) Key in the number 1; press [ENTER↑].

2) Key in the selected α; press [STO] 0 [−] [STO] 1.

3) From the α selection routine or from a previous forecast
   - Key in smoothed average S_{t-1}; press [STO] 2.
   - Key in trend T_{t-1}; press [STO] 3.
   - Key in forecast D_t; press [STO] 4.

4) Key in the current data value; press [R/S]. The output is the error in forecasting the value just entered.

5) Press [R/S]. This represents the forecast for the next period.

6) Record the following values: [RCL] 0 (α), [RCL] 2 (S_{t-1}), [RCL] 3 (T_{t-1}) and [RCL] 4 (D_n) for use as initial values in the next forecast.

Example:

Select the best smoothing constant based on sales (in thousands of dollars) of 22, 23, 23, 25, 23, 27, 25.

Given the current sales in month 8 of 26, forecast the following month.

Selecting The Smoothing Constant (α).

Keystrokes	Display	
Trial α = .5		
1 [ENTER↑]	1.00	
.5 [STO] 0 [−]		
[STO] 1	0.50	
22 [ENTER↑] [STO] 2	22.00	
23 [STO] 4 [x⇄y]		
[−] [RCL] 0		
[×] [RCL] 1		
[÷] [STO] 3	1.00	
23 [R/S]	0.00	
[R/S]	23.25	
23 [R/S] [R/S]	23.25	
25 [R/S] [R/S]	24.69	
23 [R/S] [R/S]	23.69	
27 [R/S] [R/S]	26.20	
25 [R/S] [R/S]	25.55	
[RCL] 5	18.81	Cummulative error (Σe^2)
[RCL] 0	0.50	Smoothing constant (α)
[RCL] 2	25.11	Smoothed average (S_{t-1})
[RCL] 3	0.44	Trend (T_{t-1})
[RCL] 4	25.55	Last forecast (D_t)
[f] [ALL]	0.00	

The procedure is repeated for several α's.

Smoothing Constant (α)	.5	.1	.2	.25
Cummulative Error (Σe^2)	18.81	15.81	14.13	14.31

For the selected α (.2)
$S_{t-1} = 23.98$
$T_{t-1} = 0.33$
$D_n = 25.30$

Forecasting

Keystrokes	Display
1 [ENTER↑]	1.00
.2 [STO] 0 [−]	0.80
[STO] 1	0.80
23.98 [STO] 2	23.98

Keystrokes	Display	
.33 [STO] 3	0.33	
25.3 [STO] 4	25.30	
26 [R/S]	0.70	Error in forecasting current month
[R/S]	25.76	Forecast for month 9
[RCL] 0	0.20	Used for initial values when month 9 actual figures become available
[RCL] 2	24.38	
[RCL] 3	0.34	
[RCL] 4	25.76	

Standard Error of the Mean

The standard error of the mean is a measure of how reliable the mean of a sample (x) is as an estimator of the mean of the population from which the sample was drawn.

To calculate the standard error of the mean:

1) Press [CL ALL].

2) If you are summing one set of numbers, key in the first number and press [Σ+]. Continue until you have entered all of the values.

3) If you are summing two sets of numbers, key in the y-value and press [ENTER↑]; key in the x-value and press [Σ+]. Continue until you have entered all of the values.

4) Press [$\bar{x}$] to obtain the mean of the x-values.

5) Press [s] [RCL] 1 [$\sqrt{x}$] [÷] to obtain the standard error of the mean of the x-values.

6) Alternatively, press [s] [x⇄y] [RCL] 1 [$\sqrt{x}$] [÷] to obtain the standard error of the mean of the y-values.

Example:

A sample of 6 one-bedroom apartment rentals reveals that 1 rents for $190 per month unfurnished, 1 rents for $200 per month, 2 rent for $205 per month, 1 rents for $216 per month, and 1 rents for $220 per month. What are the mean monthly rental and the standard deviation? What is the standard error of the mean?

Keystrokes	Display	
[CL ALL]		
190 [Σ+] 200 [Σ+]		
205 [Σ+] 205 [Σ+]		
216 [Σ+] 220 [Σ+]	6.00	Total number of data inputs
[x̄]	206.00	Average monthly rent
[s]	10.86	Standard deviation
[RCL] 1 [√x] [÷]	4.43	Standard error of the mean

Mean, Standard Deviation, Standard Error for Grouped Data

Grouped data are presented in frequency distributions to save time and effort in writing down (or entering) each observation individually. Given a set of data points

$$x_1, x_2, \ldots, x_n$$

with respective frequencies

$$f_1, f_2, \ldots, f_n$$

this procedure computes the mean, standard deviation, and standard error of the mean.

1) Press [CL ALL].
2) Key in the first value and press [ENTER↑] [ENTER↑].
3) Key in the respective frequency and press [STO] [+] 0 [×] [Σ+]. The display shows the number of data points entered.
4) Repeat steps 2 and 3 for each data point.
5) To calculate the mean (average) press [RCL] 0 [STO] 1 [RCL] 6 [STO] 3 [x̄].
6) Press [s] to find the standard deviation.
7) Press [RCL] 0 [√x] [÷] to find the standard error of the mean.

Example 1:

A survey of 266 one-bedroom apartment rentals reveals that 54 rent for

$190 per month unfurnished, 32 rent for $195 per month, 88 rent for $200 per month, and 92 rent for $206 per month. What are the average monthly rental, the standard deviation, and the standard error of the mean?

Keystrokes	Display	
[CL ALL]		
190 [ENTER↑] [ENTER↑] 54		
[STO] [+] 0 [×] [Σ+]	1.00	First data pair entered
195 [ENTER↑] [ENTER↑] 32		
[STO] [+] 0 [×] [Σ+]	2.00	Second data pair entered
200 [ENTER↑] [ENTER↑] 88		
[STO] [+] 0 [×] [Σ+]	3.00	Third data pair entered
206 [ENTER↑] [ENTER↑] 92		
[STO] [+] 0 [×] [Σ+]	4.00	Fourth data pair entered
[RCL] 0 [STO] 1 [RCL] 6		
[STO] 3 [x̄]	199.44	Average monthly rent
[s]	5.97	Standard deviation
[RCL] 0 [√x] [÷]	0.37	Standard error of the mean

Use the following HP-38E program to evaluate the previous example:

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[g] [P/R] [g] [CL P]	00-	[g] [x̄]	09- 25 0
[STO] [+] 0	01- 21 51 0	[R/S]	10- 74
[×]	02- 61	[g] [s]	11- 25 73
[f] [Σ+]	03- 24 74	[R/S]	12- 74
[g] [GTO] 00	04- 25 7 00	[RCL] 0	13- 22 0
[RCL] 0	05- 22 0	[f] [√x]	14- 24 21
[STO] 1	06- 21 1	[÷]	15- 71
[RCL] 6	07- 22 6	[g] [GTO] 00	16- 25 7 00
[STO] 3	08- 21 3	[g] [P/R]	

REGISTERS			
R_0 Σf_i	R_1 Σf_i	R_2 $\Sigma f_i x_i$	R_3 $\Sigma f_i x_i^2$
R_4 Σx_i	R_5 Σx_i^2	R_6 $\Sigma f_i x_i^2$	R_7

1) Key in the program.
2) Press [f] [ALL].
3) Key in the first value and press [ENTER↑] [ENTER↑].
4) Key in the respective frequency and press [R/S]. The display shows the number of data points entered.
5) Repeat steps 3 and 4 for each data point.
6) To calculate the mean, press [g] [GTO] 05 [R/S].
7) Press [R/S] to find the standard deviation.
8) Press [R/S] to find the standard error of the mean.
9) For a new case, go to step 2.

Keystrokes	Display	
[f] [ALL]		
190 [ENTER↑] [ENTER↑]		
54 [R/S]	1.00	
195 [ENTER↑] [ENTER↑]		
32 [R/S]	2.00	
200 [ENTER↑] [ENTER↑]		
88 [R/S]	3.00	
206 [ENTER↑] [ENTER↑]		
92 [R/S]	4.00	
[g] [GTO] 05 [R/S]	199.44	Average monthly rent
[R/S]	5.97	Standard deviation
[R/S]	0.37	Standard error of the mean

Chi-Square Statistic

The chi-square statistic means the goodness of fit between two sets of frequencies. It is used to test whether a set of observed frequencies differ from a set of expected frequencies sufficiently to reject the hypothesis under which the expected frequencies were obtained.

In other words, you are testing whether discrepancies between the observed frequencies (O_i) and the expected frequencies (E_i) are significant, or whether they may reasonably be attributed to chance. The formula generally used is:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

If there is a close agreement between the observed and expected frequencies, χ^2 will be small. If the agreement is poor, χ^2 will be large.

The following keystrokes calculate the χ^2 statistic:

1) Press [CL ALL].
2) Key in the first O_i value, press [ENTER↑].
3) Key in the first E_i value, press [STO] 0 [−] [ENTER↑] [×] [RCL] 0 [÷] [+].
4) Repeat steps 2 and 3 for all data pairs. The χ^2 value is displayed.

Example 1:

A suspect die from a Las Vegas casino is brought to an independent testing firm to determine its bias, if any. The die is tossed 120 times and the following results obtained.

Number	1	2	3	4	5	6
Observed Frequency	25	17	15	23	24	16

The expected frequency = 120 throws/6 sides, or E = 20 for each number, 1 thru 6. (Since E is a constant is this example, there's no need to store it in R_0 each time.)

Keystrokes	Display	
CL ALL 25 ENTER↑		
20 STO 0 − ENTER↑		
× RCL 0 ÷ +	1.25	
17 ENTER↑ 20 −		
ENTER↑ × RCL 0 ÷ +	1.70	
15 ENTER↑ 20 −		
ENTER↑ × RCL 0 ÷ +	2.95	
23 ENTER↑ 20 −		
ENTER↑ × RCL 0 ÷ +	3.40	
24 ENTER↑ 20 −		
ENTER↑ × RCL 0 ÷ +	4.20	
16 ENTER↑ 20 −		
ENTER↑ × RCL 0 ÷ +	5.00	χ^2

The number of degrees of freedom is (n − 1). Since n = 6, the degrees of freedom = 5.

Consulting statistical tables, you look up χ^2 to a 0.05 significance level with 5 degrees of freedom, and see that $\chi^2_{0.05,5} = 11.07$. Since $\chi^2 = 5$ is within 11.07, we may conclude that to a 0.05 significance level (Probability = .95), the die is fair.

Try the following HP-38E program with the same example:

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
g P/R g CL P	00-	RCL 0	05- 22 0
STO 0	01- 21 0	÷	06- 71
−	02- 41	+	07- 51
ENTER↑	03- 31	g GTO 00	08- 25 7 00
×	04- 61	g P/R	

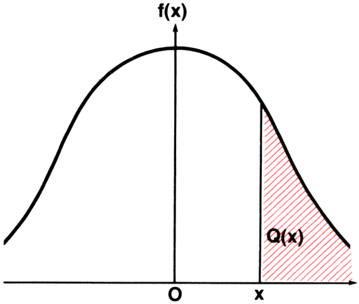
REGISTERS			
R_0 Used	R_1	R_2	R_3
R_4	R_5	R_6	R_7

1) Key in the program.
2) Press [f] [ALL].
3) Key in the first O_i value; press [ENTER↑].
4) Key in the first E_i value; press [R/S].
5) Repeat steps 3 and 4 for all data pairs. The χ^2 value is displayed.
6) For a new case, go to step 2.

Keystrokes	Display	
[f] [ALL]		
25 [ENTER↑]		
20 [R/S]	1.25	
17 [ENTER↑]		
20 [R/S]	1.70	
15 [ENTER↑]		
20 [R/S]	2.95	
23 [ENTER↑]		
20 [R/S]	3.40	
24 [ENTER↑]		
20 [R/S]	4.20	
16 [ENTER↑]		
20 [R/S]	5.00	χ^2

Normal Distribution

The normal (or Gaussian) distribution is an important tool in statistics and business analysis. The following HP-38E program gives an **approximation** to the upper tail area Q under a standardized normal distribution curve, given x. The upper tail area signifies the probability of occurrence of all values $\geq$ x.



$$Q(x) \cong \frac{1}{2} \text{EXP} \left[- \frac{(83x + 351)x + 562}{703/x + 165} \right]^{*}$$

relative error less than 0.042% over the range $0 < x < 5.5$

***Reference**

Stephen E. Derenzo, "Approximations for Hand Calculators Using Small Integer Coefficients," *Mathematics of Computation*, Vol. 31, No. 137, page 214-225; January 1977.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
g P/R g CL P	00-	0	16- 0
STO 0	01- 21 0	3	17- 3
8	02- 8	RCL 0	18- 22 0
3	03- 3	÷	19- 71
×	04- 61	1	20- 1
3	05- 3	6	21- 6
5	06- 5	5	22- 5
1	07- 1	+	23- 51
+	08- 51	÷	24- 71
RCL 0	09- 22 0	CHS	25- 32
×	10- 61	g e^x	26- 25 22
5	11- 5	2	27- 2
6	12- 6	÷	28- 71
2	13- 2	g GTO 00	29- 25 7 00
+	14- 51	g P/R	
7	15- 7		

REGISTERS			
R_0 x	R_1	R_2	R_3
R_4	R_5	R_6	R_7

1) Key in program.

2) Key in x and press R/S to compute Q(x).

3) Repeat step 2 for each new case.

Find Q(x) for x = 1.18 and x = 2.1.

Keystrokes	**Display**	
1.18 R/S	0.12	Q(1.18)
2.1 R/S	0.02	Q(2.1)

Covariance

Covariance is a measure of the interdependence between paired variables (x and y). Like standard deviation, covariance may be defined for either a sample (S_{xy}) or a population (S'_{xy}) as follows:

$$S_{xy} = r \cdot s_x \cdot s_y$$

$$S'_{xy} = r \cdot s'_x \cdot s'_y$$

The following procedure finds the covariance of a sample (S_{xy}) and of a population (S'_{xy}):

1) Press [CL ALL].
2) Key in the y-value; press [ENTER↑].
3) Key in the x-value; press [Σ+]. Repeat steps 2 and 3 for all data pairs.
4) Press [s] [x] [ENTER↑] [ŷ,r] [R↓] [×] to obtain the value of S_{xy}.
5) Press [RCL] 1 1 [−] [RCL] 1 [÷] [×] to obtain S'_{xy}.

Example 1:

Find the sample covariance (S_{xy}) and population covariance (S'_{xy}) for the following paired variables:

x_i	26	30	44	50	62	68	74
y_i	92	85	78	81	54	51	40

Keystrokes	Display	
[CL ALL]		
92 [ENTER↑] 26 [Σ+]		
85 [ENTER↑] 30 [Σ+]		
78 [ENTER↑] 44 [Σ+]		
81 [ENTER↑] 50 [Σ+]		
54 [ENTER↑] 62 [Σ+]		
51 [ENTER↑] 68 [Σ+]		
40 [ENTER↑] 74 [Σ+]	7.00	Total number of entries

Keystrokes	**Display**	
[s] [×] [ENTER↑] [ŷ,r]		
[R↓] [×]	-354.14	S_{xy}
[RCL] 1 1 [−] [RCL] 1		
[÷] [×]	-303.55	S'_{xy}

Try the previous example using the following HP-38E program:

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[g] [P/R] [g] [CL P]	00-	[R/S]	09- 74
[f] [Σ+]	01- 24 74	[RCL] 1	10- 22 1
[g] [GTO] 00	02- 25 7 00	1	11- 1
[g] [s]	03- 25 73	[−]	12- 41
[×]	04- 61	[RCL] 1	13- 22 1
[ENTER↑]	05- 31	[÷]	14- 71
[g] [ŷ,r]	06- 25 2	[×]	15- 61
[g] [R↓]	07- 25 33	[g] [GTO] 00	16- 25 7 00
[×]	08- 61	[g] [P/R]	

REGISTERS			
R_0	R_1 n	R_2 Σx	R_3 Σx^2
R_4 Σy	R_5 Σy^2	R_6 Σxy	R_7

1) Key in the program.
2) Press [f] [ALL].
3) Key in the y-value; press [ENTER↑].
4) Key in the x-value; press [R/S]. Repeat steps 3 and 4 for all data pairs.
5) Press [g] [GTO] 03 [R/S] to obtain the value of S_{xy}.
6) Press [R/S] to obtain S'_{xy}.
7) For a new case, go to step 2.

Keystrokes	Display	
[f] [ALL]		
92 [ENTER↑] 26 [R/S]		
85 [ENTER↑] 30 [R/S]		
78 [ENTER↑] 44 [R/S]		
81 [ENTER↑] 50 [R/S]		
54 [ENTER↑] 62 [R/S]		
51 [ENTER↑] 68 [R/S]		
40 [ENTER↑] 74 [R/S]	7.00	Total number of entries
[g] [GTO] 03 [R/S]	-354.14	S_{xy}
[R/S]	-303.55	S'_{xy}

Permutation

A permutation is an **ordered** subset of a set of distinct objects. The number of possible permutations, each containing n objects, that can be formed from a collection of m distinct objects is given by

$$_mP_n = \frac{m!}{(m-n)!}$$

where m, n are integers and $0 \leq n \leq m \leq 69$.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[g] [P/R] [g] [CL P]	00-	[−]	06- 41
[STO] 0	01- 21 0	[g] [n!]	07- 25 3
[x⇄y]	02- 33	[÷]	08- 71
[g] [n!]	03- 25 3	[g] [GTO] 00	09- 25 7 00
[g] [LAST X]	04- 25 31	[g] [P/R]	
[RCL] 0	05- 22 0		

REGISTERS			
R_0 n	R_1	R_2	R_3
R_4	R_5	R_6	R_7

1) Key in the program.
2) Key in m; press [ENTER↑].
3) Key in n; press [R/S] to calculate ${}_mP_n$.
4) For a new case, go to step 2.

Example:

How many ways can 10 people be seated on a bench if only 4 seats are available?

Keystrokes	Display	
10 [ENTER↑]		
4 [R/S]	5,040.00	${}_{10}P_4$

Combination

A combination is a selection of one or more of a set of distinct objects without regard to order. The number of possible combinations, each containing n objects, that can be formed from a collection of m distinct objects is given by

$$ {}_mC_n = \frac{m!}{(m-n)!\ n!} $$

where m, n are integers and $0 \leqslant n \leqslant m \leqslant 69$.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[g] [P/R] [g] [CL P]	00-	[g] [n!]	07- 25 3
[STO] 0	01- 21 0	[RCL] 0	08- 22 0
[x⇄y]	02- 33	[g] [n!]	09- 25 3
[g] [n!]	03- 25 3	[×]	10- 61
[g] [LAST X]	04- 25 31	[÷]	11- 71
[RCL] 0	05- 22 0	[g] [GTO] 00	12- 25 7 00
[−]	06- 41	[g] [P/R]	

REGISTERS			
R_0 n	R_1	R_2	R_3
R_4	R_5	R_6	R_7

1) Key in the program.
2) Key in m; press ENTER↑.
3) Key in n; press R/S to calculate ${}_mC_n$.
4) For a new case, go to step 2.

Example:

A manager wants to choose a committee of three people from the seven engineers working for him. In how many different ways can the committee be selected?

Keystrokes	Display	
7 ENTER↑		
3 R/S	35.00	${}_7C_3$

Random Number Generator

This HP-38E program calculates uniformly distributed pseudo-random numbers u_i in the range

$$0 < u_i < 1$$

The following method is used:

u_{i+1} = fractional part of $(997u_i)$

where $i = 0, 1, 2, \ldots$

$u_0 = 0.5284163$* (seed)

The period of this generator has a length of 500,000 numbers and the generator passes the frequency test (chi square) for uniformity, the serial test and the run test. The most significant digits (the left hand digits) are the most random digits. The right most digits are significantly less random.

* Other seeds may be selected but the quotient of (seed $\times 10^7$) divided by two or five must not be an integer. Also, it would be wise to statistically test other seeds before using them.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
g P/R g CL P	00-	9	10- 9
.	01- 73	9	11- 9
5	02- 5	7	12- 7
2	03- 2	×	13- 61
8	04- 8	g FRAC	14- 25 61
4	05- 4	STO 0	15- 21 0
1	06- 1	R/S	16- 74
6	07- 6	g GTO 10	17- 25 7 10
3	08- 3	g P/R	
STO 0	09- 21 0		

REGISTERS			
R_0 u_i	R_1	R_2	R_3
R_4	R_5	R_6	R_7

1) Key in the program.
2) To generate a random number; press R/S.
3) Repeat step 2 as many times as desired.

Example 1:

Generate a sequence of 5 random numbers.

Keystrokes	Display
R/S	0.83
R/S	0.56
R/S	0.27
R/S	0.04
R/S	0.20

Appendix

Formulas

Lease vs. Purchase

PMT_p = loan payment for purchase

PMT_ℓ = lease payment

I_n = interest portion of PMT_p for period n

D_n = depreciation for period n

M_n = maintenance for period n

T = marginal tax rate

Net purchase advantage =

$$\sum_{n=1}^{k} \frac{\text{cost of leasing (n)} - \text{cost of owning (n)}}{(1 + i)^n}$$

Cost of leasing (n) = $(1 - T)\, PMT_\ell$

Cost of owning (n) = $PMT_p - T(I_n + D_n) + (1 - T)M_n$

Bonds

Reference:

Spence, Bruce M. and others, STANDARD SECURITIES CALCULATION METHODS, Securities Industry Association, 1973.

DIM/b = days between issue date and maturity date/day basis

DSM/b = days between settlement date and maturity date/day basis

DIS/b = DIM/b − DSM/b

E = number of days in coupon period where settlement occurs

DSC = E − DIS = days from settlement date to next six month coupon date

N = number of semi-annual coupons payable between settlement date and maturity date or call date

$$\text{CPN} = \frac{\text{CPN} \cdot \text{CALL}}{100}$$

CALL = redemption value per \$100 par value (call price)

Price (given yield) with more than 6 months to maturity.

$$\text{PRICE} = \frac{\text{CALL}}{\left(1 + \frac{\text{YIELD}}{200}\right)^{N-1+\frac{\text{DSC}}{E}}} + \left[\sum_{K=1}^{N} \frac{\frac{\overline{\text{CPN}}}{2}}{\left(1 + \frac{\text{YIELD}}{2}\right)^{K-1+\frac{\text{DSC}}{E}}}\right] - \left[\frac{\overline{\text{CPN}}}{2} \cdot \frac{\text{DIS}}{E}\right]$$

Exponential Curve Fit

$$y = Ae^{Bx}$$

$$B = \frac{\Sigma x_i \ln y_i - \frac{1}{n}(\Sigma x_i)(\Sigma \ln y_i)}{\Sigma x_i^2 - \frac{1}{n}(\Sigma x_i)^2}$$

$$A = \exp\left[\frac{\Sigma \ln y_i}{n} - B\frac{\Sigma x_i}{n}\right]$$

$$\hat{y} = Ae^{Bx}$$

Logarithmic Curve Fit

$$y = A + B(\ln x)$$

$$B = \frac{\Sigma y_i \ln x_i - \frac{1}{n} \Sigma \ln x_i \Sigma y_i}{\Sigma(\ln x_i)^2 - \frac{1}{n} (\Sigma \ln x_i)^2}$$

$$A = \frac{1}{n}(\Sigma y_i - B\Sigma \ln x_i)$$

$$\hat{y} = A + B(\ln x)$$

Power Curve Fit

$$y = Ax^B \qquad (A > 0)$$

$$\ln y = \ln A + B\ln x$$

$$B = \frac{\Sigma(\ln x_i)(\ln y_i) - \frac{(\Sigma \ln x_i)(\Sigma \ln y_i)}{n}}{\Sigma(\ln x_i)^2 - \frac{(\Sigma \ln x_i)^2}{n}}$$

$$A = \exp\left[\frac{\Sigma \ln y_i}{n} - B\frac{\Sigma \ln x_i}{n}\right]$$

$$\hat{y} = Ax^B$$

Exponential Smoothing

α = smoothing constant

smoothed average $S_t = \alpha X_t + (1 - \alpha) S_{t-1}$

trend $T_t = \alpha(S_t - S_{t-1}) + (1 - \alpha) T_{t-1}$

forecast $D_{t+1} = S_t + \frac{(1 - \alpha)}{\alpha} T_t$

error $e = D_{t+1} - X_{t+1}$

cummulative error $= \sum_{t=1}^{n} e_t^2$

Standard Error of The Mean

$s_{\bar{x}} = \frac{s_x}{\sqrt{n}}$ $\quad s_{\bar{y}} = \frac{s_y}{\sqrt{n}}$

Mean, Standard Deviation, Standard Error For Grouped Data

mean $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

standard deviation $s_x = \sqrt{\frac{\Sigma f_i x_i^2 - (\Sigma f_i)\bar{x}^2}{\Sigma f_i - 1}}$

standard error $s_{\bar{x}} = \frac{s_x}{\sqrt{\Sigma f_i}}$

NOTES

OTHER APPLICATIONS BOOKS WHICH ARE AVAILABLE

LENDING, SAVINGS, AND LEASING APPLICATIONS (00038-90025)

APR with Fees; Discounted Mortgages; Constant Principal Loans; Add-On Rate Converted to APR; Add-On Loan With Credit Life; Rule of 78's; Nominal Rate to Effective Rate; Number of Periods to Deplete a Savings Account; Periodic Deposits and Withdrawals; Savings Account Compounded Daily; Compounding Periods Different from Payment Periods; Advance Payments With Residual; Skipped Payments

REAL ESTATE APPLICATIONS (00038-90024)

APR With Fees; Discounted Mortgages; Present Value and Yield of a Mortgage With Balloon Payment One Period After Last Payment; Deferred Annuities; Present Value of Increasing/Decreasing Annuity; Equity Yield Rate; Equity Investment Value and Present Value; Future Sales Price and Overall Depreciation/Appreciation Rate; Mortgage Constant; Refinancing; Wrap-Around Mortgages; Modified IRR (FMRR); Canadian Mortgages; Depreciation; Exponential Curve Fit

REAL ESTATE II: INCOME PROPERTY ANALYSIS APPLICATIONS (00038-90051)

Annual Property Cash Flow Analysis: Before-Tax Cash Flows and Reversions; After-Tax Cash Flows (including Multiple Mortgages); After-Tax Net Cash Proceeds of Resale. **Mortgage-Equity (Ellwood) Analysis:** Basic Rate and Overall Rate; Value (Present Worth) with R; Equity Dividend Rate; Cash Throw-Off to Equity; Value (Present Worth) with Dollar Amounts Given; Capital Appreciation or Depreciation on Resale; Equity Yield Rate from Dollar Figures. **Investment and Feasibility Analysis:** Feasibility Tests; Present Worth; Net Present Value; Profitability Index; Internal Rate of Return; Payback Period.

MARKETING AND FORECASTING APPLICATIONS (00038-90049)

Moving Average; Seasonal Variation Factors; Exponential Curve Fit; Gompertz Curve Fit; Forecasting with Exponential Smoothing; Breakeven Analysis; Operating Leverage; Profit and Loss Analysis; Markup and Margin Calculations; List and Net Prices; Learning Curve; Queuing and Waiting Line Theory; Cash Flow Loader; Percentage Tabulator.

PERSONAL FINANCE APPLICATIONS (00038-90052) (HP-38E/38C ONLY)

IRA or Keogh Plan; Stock Portfolio Evaluation; U.S. Treasury Bill Valuation; True Annual Growth Rate of an Investment Portfolio; Bond Purchased Between Coupons; The True Cost of an Insurance Policy; Real Estate Equity Investment Analysis; Homeowner's Monthly Payment Estimator; True Annual Percentage Interest Rate on a Mortgage with Fees; Rent versus Buy.

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00038-90026 Rev. C 11/79

Printed in U.S.A.