HEWLETT-PACKARD

HP-37E & HP-38E/38C
REAL ESTATE
Applications

For Continuous Memory Models

Although this book refers specifically to the HP-37E or HP-38E, the programs and calculations contained herein apply equally well to the HP-38C. The user should note, however, that the display format and data register contents are retained by the calculator even though it has been turned off. It may be desirable to reset or clear these conditions before running programs or making calculations.

NOTICE

HP-37E & HP-38E/38C

Real Estate Applications

September 1979

00038-90024 Rev. C 9/79

Printed in U.S.A.

Introduction

This **Real Estate Applications** book has been designed to supplement the HP-37E and HP-38E **Owner's Handbooks** providing a collection of key applications specifically associated with real estate transactions. Step by step keystrokes procedures, programs, and examples for over 20 problem types and explained. Hopefully, it will provide a quick and easy reference guide to the majority of your problems, and show you how to redesign our examples to fit your specific needs.

Calculator displays are shown in this book using a point to separate the integer part of a number from the fraction part and a comma to separate thousands, millions, etc. (e.g. 1000 5/10 is shown as 1,000.5). Your calculator may display numbers with the comma and point interchanged (e.g. 1.000,5) depending on the convention of the country where it was purchased.

It is sometimes necessary in these keystroke solutions to include operations which involve prefix keys, namely, [f] on the HP-37E and [f] and [9] on the HP-38E. For example, the operation [12×] is performed on the HP-37E as [f] [12×] and on the HP-38E as [9] [12×]. In such cases, **the keystroke solution omits the prefix key and indicates only the operation** (as here, [12×]). As you work through the example problems, take care to press the appropriate prefix keys (if any) for your calculator.

In addition, it should be noted that certain clearing functions on the two calculators have different key mnemonics. **Clear finance** on the HP-37E is represented as [CL FIN], and is represented as [FIN] on the HP-38E. **Clear all** is represented as [CL ALL] and [ALL] on the HP-37E and HP-38E respectively. Unless otherwise specified, this book will use the key mnemonics of the HP-37E, although the keystrokes are applicable to both machines.

All results are carried internally to ten significant digits. If intermediate answers are rounded by the user, slightly different final values may be obtained.

Contents

Simple Mortgages

Annual Percentage Rate Calculations With Fees

Borrowers are sometimes charged fees in connection with the issuance of a mortgage, which effectively raises the interest rate. The actual amount received by the borrower (PV) is reduced, while the periodic payments remain the same. Given the life or term of the mortgage, the interest rate, the mortgage amount, and the basis of the fee charge (how the fee is calculated), the true Annual Percentage Rate may be calculated. Information is entered as follows:

1) Set the Payment switch to END and press [CL FIN].

2) Calculate and enter the periodic payment amount of the loan.
   a) Key in the total number of payment periods; press [n].
   b) Key in the periodic interest rate; press [i].
   c) Key in the mortgage amount; press [PV].*
   d) To obtain the periodic payment amount press [PMT].*

3) Calculate and key in the actual net amount disbursed.*
   a) If fees are stated as a percentage of the mortgage amount (points), recall the mortgage amount ([RCL] [PV]); key in the fee (percentage) rate; press [%] [−] [PV].
   b) If fees are stated as a flat charge, recall the mortgage amount ([RCL] [PV]); key in the fee amount (flat charge); press [−] [PV].
   c) If fees are stated as a percentage of the mortgage amount plus a flat charge, recall the mortgage amount ([RCL] [PV]); key in the fee (percentage) rate, press [%] [−]; key in the fee amount (flat charge); press [−] [PV].

4) Press [i] to obtain the percentage rate per compounding period.

5) To obtain the annual nominal percentage rate, key in the number of periods per year, and press [×].

* Positive for cash received; negative for cash paid out.

Example 1:

A borrower is charged 2 points for the issuance of his mortgage. If the mortgage amount is $50,000 for 30 years, and the interest rate is 9% per year, with monthly payments, what annual percentage rate is the borrower paying? (1 point is equal to 1% of the mortgage amount.)

Keystrokes	Display	
BEGIN END		
[CL FIN]		
30 [12×]	***360.00***	Months (into n)
9 [12÷]	***0.75***	% monthly interest rate (into i)
50000 [PV]	***50,000.00***	Loan amount (into PV)
[PMT]	***-402.31***	Monthly payment (calculated)
[RCL] [PV] 2 [%] [−] [PV]	***49,000.00***	Actual amount received by borrower (into PV)
[i]	***0.77***	% monthly interest rate (calculated)
12 [×]	***9.23***	Annual percentage rate

Example 2:

Using the same information as given in Example 1, calculate the APR if the mortgage fee is $150 instead of a percentage.

Keystrokes	Display	
BEGIN END		
[CL FIN]		
30 [12×]	***360.00***	Months (into n)
9 [12÷]	***0.75***	% monthly interest rate (into i)
50000 [PV]	***50,000.00***	Loan amount (into PV)
[PMT]	***-402.31***	Monthly payment (calculated)
[RCL] [PV] 150 [−] [PV]	***49,850.00***	Effective mortgage amount (into PV)
[i]	***0.75***	Monthly interest rate (calculated)
12 [×]	***9.03***	Annual percentage rate

Example 3:

Again using the information given in Example 1, what is the APR if the mortgage fee is stated as 2 points plus $150?

Keystrokes	Display	
BEGIN END		
[CL FIN]		
30 [12×]	***360.00***	Months (into n)
9 [12÷]	***0.75***	% monthly interest rate (into i)
50000 [PV]	***50,000.00***	Loan amount (into PV)
[PMT]	***-402.31***	Monthly payment (calculated)
[RCL] [PV] 2 [%] [−]		
150 [−] [PV]	***48,850.00***	Effective mortgage amount (into PV)
[i]	***0.77***	Monthly interest rate (calculated)
12 [×]	***9.26***	Annual percentage rate

Price of a Mortgage Traded at a Discount/Premium

Mortgages may be bought and/or sold at prices lower (discounted) or higher (at a premium) than the remaining balance of the loan at the time of purchase. Given the amount of the mortgage, the periodic payment, the timing and amount of the balloon or prepayment, and the *desired* yield rate, the price of the mortgage may be found. It should be noted that the balloon payment amount (if it exists) occurs coincident with, and does not include, the last periodic payment amount.

Information is entered as follows:

1) Set the Payment switch to END and press [CL FIN].
2) Key in the total number of periods until the balloon payment or prepayment occurs; press [n].
3) Key in the *desired* periodic interest rate (yield) and press [i].
4) Key in the periodic payment amount; press [PMT].*
5) Key in the balloon payment amount and press [FV].*

* Positive for cash received; negative for cash paid out.

6) Press [PV] to obtain the purchase price of the mortgage.

Example 1:

A lender wants to induce the borrower to prepay a low interest rate loan. The interest rate is 5% with 6 years (72 payments) remaining of $137.17 and a balloon payment at the end of the sixth year of $2000. If the lender is willing to discount the future payments at 7½%, how much would the borrower need to prepay the note?

Keystrokes	Display	
BEGIN END		
[CL FIN]		
72 [n] 7.5 [12÷]		
137.17 [PMT]		
2000 [FV] [PV]	***-9,210.48***	Amount necessary to prepay note

Example 2:

A 9½% mortgage with 28 years remaining may be acquired which has a remaining balance of $49,350. Determine the price to pay for this mortgage if the desired annual yield is 12%. (Since the payment amount is not given, it must be calculated.)

Keystrokes	Display	
BEGIN END		
[CL FIN]		
28 [12×]	***336.00***	Months (into n)
9.5 [12÷]	***0.79***	% monthly interest rate (into i)
49350 [CHS] [PV] [PMT]	***420.40***	Monthly payment to be received
12 [12÷]	***1.00***	Desired % monthly interest rate (into i)
[PV]	***-40,555.50***	Purchase price to achieve the desired yield (calculated)

Yield of a Mortgage Traded at a Discount/Premium

The annual yield of a mortgage bought at a discount or premium may be calculated, given the original mortgage amount, interest rate, and periodic payment, as well as the number of payment periods per year, the price paid for the mortgage, and the balloon payment amount (if it exists).

Information is entered as follows:

1) Set the Payment switch to END and press [CL FIN].
2) Key in the total number of periods until the balloon payment occurs and press [n].
3) Key in the periodic payment amount and press [PMT].*
4) Key in the purchase price of the mortgage; press [PV].*
5) Key in the balloon payment amount and press [FV].*
6) Press [i] to obtain the yield per period.
7) Key in the number of periods per year and press [×] to obtain the nominal annual yield.

Example 1:

Find the annual yield of a 7%, 21 year mortgage prepaid in full at the end of the 12th year, if the mortgage amount is $100,000, the purchase price is $86,000, and equal monthly payments of $758.47 are received. The remaining balance at the end of the 12th year is $60,647.67.

Keystrokes	Display	
BEGIN END [CL FIN]		
12 [12×]	***144.00***	Balloon payment occurs at end of 144th month (into n)
758.47 [PMT]	***758.47***	Monthly payment received (into PMT)
86000 [CHS] [PV]	***-86,000.00***	Amount loaned (into PV)
60647.67 [FV] [i]	***0.77***	Percent monthly yield (calculated)
12 [×]	***9.23***	Percent annual yield

* Positive for cash received; negative for cash paid out.

Present Value of a Mortgage With Balloon Payment One Period After Last Payment

Sometimes the balloon payment is due and payable one period after the last periodic payment in the contract maturity. If this is encountered, the present value (or purchase price) of the mortgage loan may be calculated as follows:

1) Set the Payment switch to END and press [CL FIN].
2) Key in the total number of payment periods until the balloon payment occurs; press [n].
3) Key in the periodic interest rate; press [i].
4) Key in the periodic payment amount; press [PMT].*
5) Key in the balloon amount, less the periodic payment amount, and press [FV].*
6) Press [PV] to obtain the present value of the mortgage.

Example 1:

A 20-year $60,000 mortgage loan with a balloon payment of $23,507.58 has monthly payments of $483.20. The balloon payment is to be made one month after the last monthly payment. What is the present worth or market value to the purchaser of the mortgage loan if he desires a 9.5% annual yield?

Keystrokes	Display	
BEGIN END		
[CL FIN]		
20 [ENTER↑] 12 [×]		
1 [+] [n]	***241.00***	The balloon occurs at the end of the 241st month (into n)
9.5 [12÷]	***0.79***	Monthly interest rate (into i)
483.20 [PMT]	***483.20***	Monthly payment (into PMT)

* Positive for cash received; negative for cash paid out.

Keystrokes	Display	
23507.58 RCL PMT		
− FV	*23,024.38*	Adjusted balloon amount (into FV)
PV	*–55,352.77*	Present value (the amount to pay for the opportunity to receive the cash flows)

Yield of a Mortgage With Balloon Payment One Period After Last Payment

Given the periodic payment amount, total number of periods in mortgage life, mortgage price, and the balloon payment amount which occurs one period after the last payment, the yield is calculated as follows:

1) Set the Payment switch to END and press CL FIN.
2) Key in the total number of payment periods until the balloon payment occurs; press n.
3) Key in the periodic payment amount; press PMT.*
4) Key in the price of the mortgage; press PV.*
5) Key in the balloon amount, less the periodic payment amount, and press FV.*
6) Press i to obtain the periodic yield.
7) Key in the number of periods per year and press × to obtain the annual yield.

Example 1:

What is the annual yield to the lender of a $7900 mortgage which has monthly payments of $80 for 5 years and a balloon payment of $7000 occurring one period after the last periodic payment?

Keystrokes

BEGIN END

CL FIN

5 ENTER↑ 12 ×

* Positive for cash received; negative for cash paid out.

Keystrokes	Display	
1 [+] [n]	***61.00***	The balloon occurs at the end of the 61st payment (into n)
80 [PMT]	***80.00***	Monthly payment received (into PMT)
7900 [CHS] [PV]	***-7,900.00***	Loan amount (into PV)
7000 [RCL] [PMT] [−] [FV]	***6,920.00***	Adjusted balloon payment amount received (into FV)
[i]	***0.86***	Percent monthly yield (calculated)
12 [×]	***10.28***	Percent annual yield

Deferred Annuities

Sometimes transactions are established where payments do not begin for a specified number of periods (the payments are deferred). To determine the present value of such an annuity, the following keystrokes may be used:

1) Set the Payment switch to the appropriate position and press [CL FIN].
2) Key in the total number of payments to be made and press [n].
3) Key in the periodic interest rate and press [i].
4) Key in the periodic payment amount, press [PMT].*
5) To calculate the "deferred" present value of the annuity, press [PV].
6) Press [STO] [FV].
7) Key in the total number of periods the payments were deferred and press [n].
8) Key in 0 and press [PMT].
9) Press [PV] to determine today's present value of the deferred annuity (or the amount needed today to meet the requirements of the future).

* Positive for cash received; negative for cash paid out.

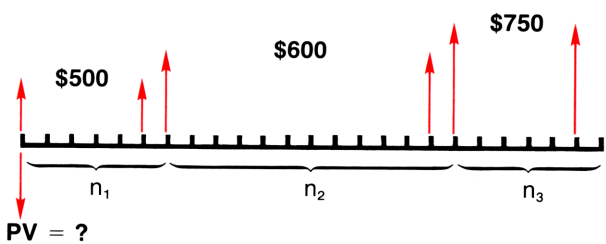
Example 1:

Your firm is responsible for the upkeep of a newly built bridge. No repairs are required until the beginning of the fourth year, when $2000 will be needed for repainting. From then on, it is estimated that $2000 will be needed at the beginning of each year for the next 26 years. You wish to establish a fund which earns 6% annually, specifically for bridge upkeep. How much do you need to deposit in the fund today to meet the upkeep requirements?

Keystrokes	Display	
BEGIN END		
CL FIN		
27 n 6 i		
2000 CHS PMT PV	***28,006.33***	Value of the annuity at the end of the 3rd year
STO FV 3 n		
0 PMT PV	***-23,514.66***	Today's value of the annuity

Leases often call for periodic contractual adjustments of rental payments. For example, a 2-year lease calls for monthly payments (at the beginning of the month) of $500 per month for the first 6 months, $600 per month for the next 12 months, and $750 per month for the last 6 months. This situation illustrates what is called a "step-up" lease. A "step-down" lease is similar, except that rental payments are decreased periodically according to the lease contract. Lease payments are made at the beginning of the period.

In the example cited, the rental payment stream for months 7–24 are "deferred annuities", as they start at some time in the future. The cash flow diagram from the investor's viewpoint looks like this:



To find today's present value (the amount of the investment) the following keystrokes may be used:

1) Set the Payment switch to BEGIN and press [CL FIN].

2) Calculate the present value of the payments in the last time span (here, n_3).
   a) Key in the total number of periods in the last time span and press [n].
   b) Key in the periodic discount (interest) rate and press [i].
   c) Key in the periodic payment amount; press [PMT].*
   d) To calculate the present value press [PV].

3) The present value of the last group of cash flows can now be considered as a balloon payment for the next to last group of cash flows (n_2); press [FV].*

4) Calculate the present value of the payments in time span n_2 (as in step 2) and press [FV].*

5) Calculate the present value of the payments in time span n_1. The display shows the present value of the entire group of cash flows.

Example 2:

A 2-year lease calls for monthly payments (at the beginning of the month) of $500 per month for the first 6 months, $600 per month for the next 12 months, and $750 per month for the last 6 months. If you wish to earn 13.5% annually on these cash flows, how much should you invest?

Keystrokes	Display	
BEGIN END		
[CL FIN]		
6 [n] 13.5 [12÷]		
750 [PMT] [PV]	**–4,376.69**	Present value of last group of payments
[CHS] [FV] 12 [n]		
600 [PMT] [PV]	**–10,602.25**	Present value of last 2 groups of cash flows
[CHS] [FV] 6 [n]		
500 [PMT] [PV]	**–12,831.75**	Amount to invest to achieve a 13.5% yield

* Positive for cash received; negative for cash paid out.

In some instances, the net present value technique may be used to find the present value of a deferred annuity.

Present Value of Increasing/Decreasing Annuity

All of the previous problems have dealt either with equal periodic payments, or payments that are held constant for a period of time, as in the step-up or step-down lease. It is possible, however, to consider a payment stream where the payment amounts change, such as income, repair costs or price changes reflecting inflation or deflation. The present value of such a payment stream can be calculated by summing each individual payment moved in time. A quicker and easier way to sum the payments is with the following keystrokes:

1) Set the Payment switch to END and press [CL FIN].
2) Key in the total number of payment periods; press [n].
3) Key in the payment percentage increase per period expressed as one plus the decimal interest rate; press [ENTER↑]. If there is a percentage decrease, express it as one minus the decimal interest rate.
4) Key in the discount (interest) rate per period expressed as one plus the decimal interest rate; press [△%] [i].
5) Press [CLx]; key in the starting payment; press [x≷y] [÷] [PMT].*
6) Press [PV] to obtain the present value of the payment stream.*

> **Note:** This procedure assumes that payments occur at the end of the period.

Example 1:

Repair costs on the new car that you just purchased are expected to start at $200 one year from now and increase at the rate of 12% per year. What sum must be put in the bank today to cover the next three years of maintenance, if the bank pays 5.5% interest?

* Positive for cash received; negative for cash paid out.

Keystrokes	Display	
BEGIN END		
CL FIN		
3 n	3.00	Years
1.12 ENTER↑	1.12	Periodic increase factor (1 + .12)
1.055 △% i	-5.80	Adjust i for interest rate
CLx 200 x≷y ÷		
PMT	178.57	Adjusted starting payment
PV	-604.48	Present value of payments

When the payment amounts change by a constant amount instead of by a constant ratio, as in the previous problem, the following keystrokes will give the present value:

1) Set the Payment switch to END and press CL FIN.
2) Key in the periodic discount (interest) rate as a percent; press i.
3) Key in the starting payment; press ENTER↑.
4) Key in the amount that the payment increases each period; press ENTER↑.
5) Key in the periodic discount (interest) rate as a decimal; press ÷ STO 0 + PMT.
6) Key in the total number of payment periods; press ENTER↑ n RCL 0 × CHS FV.
7) Press PV to obtain the present value of the payments.

Example 2:

If the repair costs in the previous example increased each year by $35, what would be the sum required?

Keystrokes	Display	
BEGIN END		
CL FIN		
5.5 i	5.50	Periodic discount (interest rate)
200 ENTER↑	200.00	Starting payment
35 ENTER↑	35.00	Periodic payment increase

Keystrokes	Display	
.055 ÷ STO 0 + PMT	836.36	Adjusted payment
3 ENTER↑ n RCL 0 × CHS FV	−1,909.09	Adjusted FV
PV	−630.65	Present value of the payments

Both of these procedures work for decreasing as well as increasing payments. In addition, the above have wide application in after tax cash flow analysis. The projections of income, expense, debt service, amortization, and depreciation all fit the above procedures and the tax consequences can be calculated without resorting to year by year summations.

Equity Investment Analysis

Equity Investment Analysis is a method of evaluating income producing real estate investment alternatives on a pretax basis. Two key factors in this type of analysis are the anticipated income stream that the property will provide and the property's projected resale value at the end of the investment horizon. Based on this and the current price of the property, an equity yield rate can be found giving an indication of the profitability of the investment.

A brief explanation of terms frequently used in real estate analysis is given here in order to aid in understanding the problems and results more fully.*

Annual Net Cash Flow is the annual net operating income without depreciation minus the annual debt service (i.e., annual mortgage payments).

Reversion is the future sales price minus the mortgage balance at the end of the projection period.

Equity yield rate is that annual rate at which the present value of the net annual cash flows plus the present value of the equity reversion equals the equity investment value.

Equity investment value is the equity in the property at the beginning of the projection period.

Overall Capitalization Rate is the net operating income divided by the selling price.

Equity Yield Rate

Given the projection period in years, reversion amount, annual net cash flow, and equity investment value, the equity yield rate may be calculated as follows:

1) Set the Payment switch to END and press [CL FIN].

2) Key in the reversion; press [FV].

* For further information, refer to ELLWOOD TABLES, American Institute for Real Estate Appraisers, 1970.

3) Key in the number of years projection; press [n].

4) Key in the net annual cash flow; press [PMT].

5) Key in the equity investment value; press [CHS] [PV].

6) Press [i] to obtain the equity yield rate.

Example 1:

An apartment complex is listed for $1,960,500 and has an annual net operating income of $166,315.37. The prospective buyer is considering a down payment of $572,500 and will finance the remaining $1,388,000 for 29 years at 8%. If the property appreciates a total of 20% over the next 10 years, what would the equity yield rate be?

Using calculations described in the Owner's Handbook, it is found that the monthly mortgage payments are $10,270.45 and therefore the annual net cash flow is $43,069.97 (the monthly payment is rounded).

(NOI − debt service = net cash flow)

The remaining mortgage balance at the end of 10 years will be $1,201,922.54.

To calculate the reversion at the end of the tenth year, find the future sales price and subtract the remaining balance.

Keystrokes	Display	
BEGIN END		
[CL FIN]		
1960500 [ENTER↑]		
20 [%] [+]	***2,352,600.00***	Future sales price
1201922.54 [−] [FV]	***1,150,677.46***	Reversion
10 [n] 43069.97 [PMT]		
572500 [CHS] [PV] [i]	***13.00***	Equity yield rate

Equity Investment Value and Present Value

Given the desired equity yield rate, projection period, annual net cash flow, and the reversion, this procedure solves for the equity investment value and present value of the investment (current sales price). Information is entered as follows:

1) Set the Payment switch to END and press [CL FIN].

2) Key in the reversion; press [FV].
3) Key in the projection period in years; press [n].
4) Key in the equity yield rate; press [i].
5) Key in the annual net cash flow; press [PMT].
6) Press [PV] for the equity investment value.
7) Key in the mortgage amount; press [−] to obtain the current sales price or present value.

Example 1:

An investor has some money he wants to invest in real estate. One of his alternatives is a warehouse, currently leased for 10 years, which generates $26,460 annually before debt service (NOI). Because the warehouse is located in a growth area, he estimates the property should sell for $420,000 at the end of 10 years. He can obtain an 8½%, 20 year mortgage for $240,000 which would have monthly payments of $2,082.78. If his desired yield is 11% over 10 years, what would his equity investment value be and how much could he pay for the property (what is the current sales price)?

Keystrokes	Display	
BEGIN END		
[CL FIN]		
10 [12×] 8.5 [12÷]		
2082.78 [CHS] [PMT]		
240000 [PV] [FV]	***-167,984.38***	Remaining loan balance after 10 years
420000 [+] [FV]	***252,015.62***	Reversion value
10 [n] 11 [i]		
[RCL] [PMT] 12 [×]		
26460 [+] [PMT] [PV]	***-97,393.37***	Equity investment value
240000 [−]	***-337,393.37***	Current sales price

Future Sales Price and Overall Depreciation/Appreciation Rate

This calculation solves for the sales price at the end of the projection period given the desired equity yield rate, annual net cash flow, equity

investment value, projection period, and the mortgage balance at the end of the projection period. Information is entered as follows:

1) Set the Payment switch to END and press [CL FIN].
2) Key in the projection period in years; press [n].
3) Key in the equity yield rate; press [i].
4) Key in the annual net cash flow; press [PMT].
5) Key in the equity investment value; press [CHS] [PV].
6) Press [FV] to compute the reversion amount.
7) Key in the mortgage balance at the end of the projection period and press [+] to obtain the required future sales price.
8) Key in the purchase price; press [x≷y] [Δ%] to obtain the overall appreciation (if the answer is positive) or depreciation (if the answer is negative).

Example 1:

A shopping center has an annual net cash flow of $14211.24. The desired equity yield rate is 14% over a 9 year period. If the current asking price is $616,000 what must the sales price at the end of year 9 be in order to achieve the desired 14% return? What overall appreciation does this represent?

(Assume 25% equity ($154,000), 25 year mortgage at 8%, monthly payment of $3,565.79, with a remaining balance of $385,522.31 at the end of year 9).

Keystrokes	Display	
BEGIN END		
[CL FIN]		
9 [n] 14 [i]		
14211.24 [PMT]		
154000 [CHS] [PV] [FV]	***272,207.35***	Reversion
385522.31 [+]	***657,729.66***	Future sales price
616000 [x≷y] [Δ%]	***6.77***	Overall appreciation

Mortgage Constant

The mortgage constant is the total annual debt service (principal and interest) on a mortgage with level payments, divided by the loan amount. The formula is:

$$f = \frac{ADS}{PV}$$

Example:

What is the mortgage constant for a $50,000 loan at 9.25% interest, with monthly payments over 30 years?

Keystrokes	Display	
BEGIN END		
CL FIN		
50000 PV		
9.25 12÷ 30 12×		
PMT	**-411.34**	Monthly payment
12 ×	**-4,936.05**	Annual debt service
RCL PV ÷		
CHS f 6	**0.098721**	Mortgage constant

FINANCIAL ANALYSIS

Refinancing

It can be mutually advantageous to both borrower and lender to refinance an existing mortgage which has an interest rate substantially below the current market rate, with a loan at a below-market rate. The borrower has the immediate use of tax-free cash, while the lender has substantially increased debt service on a relatively small cash outlay.

To find the benefits to both borrower and lender:

1) Calculate the monthly payment on the existing mortgage.
2) Calculate the monthly payment on the new mortgage.
3) Calculate the net monthly payment received by the lender (and paid by the borrower) by adding the figure found in Step 1 to the figure found in Step 2.
4) Calculate the net present value (NPV) to the lender of the net cash advanced.
5) Calculate the yield to the lender as an IRR.
6) Calculate the NPV to the borrower of the net cash advanced.

Example 1:

An investment property has an existing mortgage which originated 8 years ago with an original term of 25 years, fully amortized in level monthly payments at 6.5% interest. The current balance is $133,190.

Although the going current market interest rate is 10.5%, the lender has agreed to refinance the property with a $200,000, 17-year, level-monthly-payment loan at 9% interest.

What are the NPV and effective yield to the lender on the net amount of cash actually advanced?

What is the NPV to the borrower on this amount if he can earn a 14% equity yield rate on the net proceeds of the loan?

Keystrokes	Display	
BEGIN END		
CL FIN		
17 12× 6.5 12÷		
133190 PV		
PMT STO 0	**-1,080.33**	Monthly payment on existing mortgage
9 12÷		
200000 CHS PV		
PMT	**1,917.61**	Monthly payment on new mortgage
RCL 0 + PMT	**837.28**	Net monthly payment
RCL PV		
133190 + STO 0	**-66,810.00**	Net amount of cash advanced
10.5 12÷ PV	**-79,507.22**	Present value of net monthly payment
RCL 0 −	**-12,697.22**	NPV to lender of net cash advanced
RCL 0 PV i		
12 ×	**13.51**	% nominal yield (IRR) to lender on new money
14 12÷ PV	**-65,032.32**	Present value of net monthly payment at 14%
RCL 0 −	**1,777.68**	NPV to borrower

Wrap-Around Mortgages

A wrap-around mortgage is essentially the same as a refinancing mortgage, except that the new mortgage is granted by a different lender, who assumes the payments on the existing mortgage, which remains in full force. The new (second) mortgage is thus "wrapped around" the existing mortgage. The "wrap-around" lender advances the net difference between the new (second) mortgage and the existing mortgage in cash to the borrower, and receives as net cash flow the difference between debt service on the new (second) mortgage and debt service on the existing mortgage.

When the terms of the original mortgage and the wrap-around are the same, the procedures in calculating NPV and IRR to the lender and NPV

to the borrower are exactly the same as those presented in the preceding section on Refinancing.

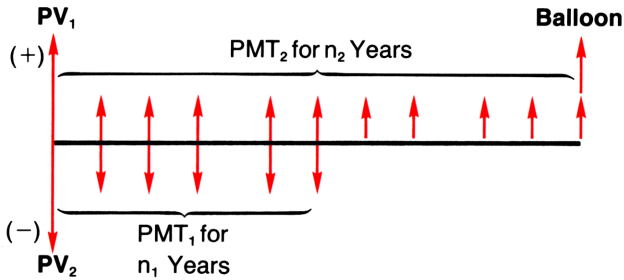
Example 1:

A mortgage loan on an income property has a remaining balance of $200,132.06. When the loan originated 8 years ago, it had a 20-year term with full amortization in level monthly payments at 6.75% interest.

A lender has agreed to "wrap" a $300,000 second mortgage at 9.5%, with full amortization in level monthly payments over 12 years. What is the effective yield (IRR) to the lender on net cash advanced?

Keystrokes	Display	
BEGIN END		
CL FIN		
20 ENTER↑ 8 − 12×	*144.00*	Total number of months remaining in original loan (into n)
6.75 12÷	*0.56*	Monthly interest rate (into i)
200132.06 PV	*200,132.06*	Loan amount (into PV)
PMT STO 0	*-2,031.55*	Monthly payment on existing mortgage (calculated)
9.5 12÷	*0.79*	Monthly interest on wrap-around (into i)
300000 CHS PV	*-300,000.00*	Amount of wrap-around (into PV)
PMT	*3,499.12*	Monthly payment on wrap-around (calculated)
RCL 0 + PMT	*1,467.57*	Net monthly payment received (into PMT)
RCL PV		
200132.06 + PV	*-99,867.94*	Net cash advanced (into PV)
i RCL 12÷	*14.51*	Nominal yield (IRR) to lender (calculated)

Sometimes, the wrap-around mortgage will have a longer pay-back period than the original mortgage, or a balloon payment may exist.



where:

n_1 = number of years remaining in original mortgage
PMT_1 = yearly payment of original mortgage
PV_1 = remaining balance of original mortgage
n_2 = number of years in wrap-around mortgage
PMT_2 = yearly payment of wrap-around mortgage
PV_2 = total amount of wrap-around mortgage
BAL = balloon payment

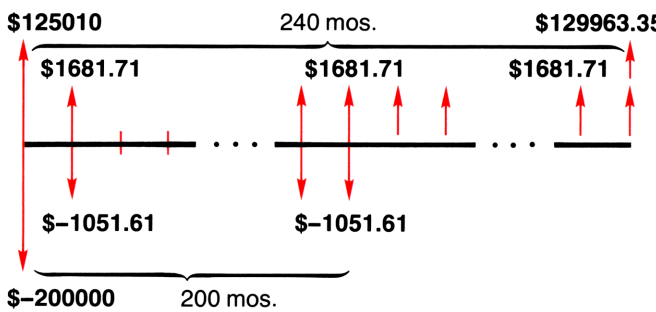
To find the annual yield to the lender, a trial and error (iterative) approach must be used on the HP-37E. An initial guess is entered for the periodic IRR, and the present value of the cash flows is found. By subtracting the initial cash output, the net present value is found. If the NPV is equal to 0, the initial guess *is* the IRR. Otherwise, adjust the "guess" and repeat the procedure until the desired accuracy is achieved. The keystrokes are:

1) Set the Payment switch to END and press [CL FIN].
2) Key in a "best guess" periodic internal rate of return; press [i].
3) To find the present value of the original mortgage:
   a) Key in the total number of periods remaining; press [n].
   b) Key in the periodic payment amount; press [CHS] [PMT]. (The lender makes the payments on the original mortgage.)
   c) Press [PV] to find the present value, then press [STO] 0.

4) Find the present value of the wrap-around mortgage:
   a) Key in the total number of periods in the wrap-around; press [n].
   b) Key in the periodic payment *received* by the lender; press [PMT].
   c) Key in the balloon amount (if it exists) at the end of the payback period; press [FV].
   d) Press [PV] to find the present value.
5) Press [RCL] 0 [+] [CHS] to obtain the present value of the cash flows.
6) Key in the remaining balance on the original mortgage; press [+].
7) Key in the total amount of the wrap-around mortgage; press [−] to obtain the NPV.
8) If the net present value is not equal to zero, adjust the guess and repeat steps 1-7 until the desired accuracy is obtained.

Example 2:

A customer has an existing mortgage with a balance of $125,010, a remaining term of 200 months, and a $1051.61 monthly payment. He wishes to obtain a $200,000, 9½% wrap-around with 240 monthly payments of $1681.71 and a balloon payment at the end of the 240th month of $129,963.35. If you, as a lender, accept the proposal, what is your rate of return?



Keystrokes	Display	
BEGIN END		
CL FIN		

Choose a 12% annual yield as the first IRR guess.

12 12÷	*1.00*	% monthly IRR (into i)
200 n	*200.00*	Months (into n)
1051.61 CHS PMT	*-1,051.61*	Payment (into PMT)
PV STO 0	*90,786.92*	Present value of original mortgage at 12% annual IRR (calculated)
240 n	*240.00*	Months (into n)
1681.71 PMT	*1,681.71*	Payment (into PMT)
129963.35 FV PV	*-164,663.31*	Present value of wrap-around at 12% annual IRR (calculated)
RCL 0 + CHS	*73,876.39*	Present value of the cash flows
125010 +		
200000 −	*-1,113.61*	Net present value

Since the NPV is negative, the IRR is too high. Choose a lower rate of return, say 11.75%, and repeat the procedure. The actual IRR is 11.8391%.

On the HP-38E, the following procedure could be used to give a direct solution to the previous problem:

Keystrokes	Display	
200000 CHS ENTER↑		
125010 + g CF_0	*-74,990.00*	Net investment
1051.61 CHS		
ENTER↑ 1681.71 +	*630.10*	Net cash flow received by lender
g CF_j 99 g N_j		
x≷y g CF_j		The above cash flow
99 g N_j		occurs 200 times
x≷y g CF_j 2 g N_j		
g LAST x g CF_j	*1,681.71*	Next cash flow received by lender
39 g N_j	*39.00*	Cash flow occurs 39 times

Keystrokes	Display	
x≷y 129963.35 +		
g CF$_j$	***131,645.06***	Final cash flow
f IRR 12 ×	***11.84***	Rate of return to lender

If you, as a lender, know the yield on the entire transaction, and you wish to obtain the payment amount on the wrap-around mortgage to achieve this yield, use the following procedure. Once the monthly payment is known, the borrower's periodic interest rate may also be determined.

1) Set the Payment switch to END and press CL FIN.
2) Key in the remaining term of the original mortgage and press n.
3) Key in the desired annual yield and press 12÷.
4) Key in the monthly payment to be made by the lender on the original mortgage and press CHS PMT.
5) Press PV.
6) Key in the *net* amount of cash advanced and press + CHS PV.
7) Key in the total term of the wrap-around mortgage and press n.
8) If a *balloon payment exists,* key it in and press FV.
9) Press PMT to obtain the payment amount necessary to achieve the desired yield.
10) Key in the amount of the wrap-around mortgage and press CHS PV i to obtain the borrower's periodic interest rate.

Example 3:

Your firm has determined that the yield on a wrap-around mortgage should be 12% annually. In the previous example, what monthly payment must be received to achieve this yield on a $200,000 wrap-around? What interest rate is the borrower paying?

Keystrokes

BEGIN END

CL FIN

200 n 12 12÷

1051.61 CHS PMT

PV 74990 +

Keystrokes	Display	
CHS PV	**-165,776.92**	PV of payments plus cash advanced
240 n		
129963.35 FV PMT	**1,693.97**	Monthly payment received by lender
200000 CHS PV i		
12 ×	**9.58**	Annual interest paid by borrower

Modified IRR—Varying Reinvestment Rate

The traditional IRR technique assumes that all positive cash flows are reinvested at the IRR to earn compound interest over the income projection period. It also assumes that all negative cash flows are to be discounted at the IRR. This means that cash can be invested today to earn compound interest at the IRR until it is needed to cover the forecasted negative cash flows.

Neither of these assumptions is necessarily realistic or valid. It is possible to compensate for either or both by using real-market rates to discount all negative flows (including Capital Outlay) to the present at a "safe" rate that will ensure liquidity when funds are needed; and to compound all positive flows at a realistic reinvestment rate to the end of the income projection period.

This procedure results in a single (negative) present value figure, and a single future value figure as well. IRR is then found by solving for i in a compounded amount procedure.

Example 1:

Negative Cash Flows, Reinvestment of Positive Flows at IRR.

A development project requires a total capital investment (development costs) of $600,000 staged as follows: $150,000 immediately, plus $150,000 at the end of years 1-3. Net sales proceeds over a total 10-year sellout period are projected as: Year 1—$0; Year 2—$50,000; Years 3-5—$125,000; Year 6—$140,000; Year 7—$150,000, Year 8—$175,000; Year 9—$100,000; Year 10—$50,000.

What is the indicated IRR for the developer, assuming he can earn 5.5% on the money required to cover future cash outlays (negative cash flows)?

The net cash flows projected are:

Year	Cash Flow	Year	Cash Flow
0	$–150,000	6	$140,000
1	–150,000	7	150,000
2	–100,000	8	175,000
3	– 25,000	9	100,000
4	125,000	10	50,000
5	125,000		

The steps in the procedure are:

1) Calculate the present value of the negative cash flows at the "safe" rate using the NPV routine.

2) Use the figure obtained in Step 1 as the initial investment in the IRR routine; store it in Register 0.

3) Entering 0 as the cash flow for years with a negative cash flow, find the IRR.

Keystrokes (HP-37E)	Display	
1 [ENTER↑] 5.5 [%] [+] [f] [1/x] [ENTER↑] [ENTER↑] [ENTER↑]		
25000 [CHS] [×]	***–23,696.68***	PV at 2nd cash flow
100000 [CHS] [+] [×]	***–117,248.04***	PV at 1st cash flow
150000 [CHS] [+] [×]	***–253,315.68***	PV of cash flows
150000 [CHS] [+]		
[STO] 0	***–403,315.68***	NPV of negative cash flows

Choose 12.5% as first guess:

1 [ENTER↑] 12.5 [%] [+] [f] [1/x] [ENTER↑] [ENTER↑] [ENTER↑]		
50000 [×]	***44,444.44***	PV at 9th cash flow
100000 [+] [×]	***128,395.06***	PV at 8th cash flow
175000 [+] [×]	***269,684.50***	PV at 7th cash flow
150000 [+] [×]	***373,052.89***	PV at 6th cash flow
140000 [+] [×]	***456,047.01***	PV at 5th cash flow
125000 [+] [×]	***516,486.23***	PV at 4th cash flow
125000 [+] [×]	***570,209.98***	PV at 3rd cash flow
0 [+] [×]	***506,853.32***	PV at 2nd cash flow

0 [+] [×]	***450,536.28***	PV at 1st cash flow
0 [+] [×]	***400,476.70***	PV of cash flows
[RCL] 0 [+]	***–2,838.98***	NPV at 12.5%

Since the NPV is negative, the IRR is less than 12.5%. Try a lower rate, and repeat the procedure.

Keystrokes (HP-38E)	**Display**	
[f] [ALL]		
5.5 [i]		
150000 [CHS]		
[g] [CF_0] [g] [CF_j]	***–150,000.00***	Initial investment and first cash flow
100000 [CHS]		
[g] [CF_j]	***–100,000.00***	2nd cash flow
25000 [CHS]		
[g] [CF_j]	***–25,000.00***	3rd cash flow
[f] [NPV]	***–403,315.68***	Present value of negative cash flows at 5.5%
[g] [CF_0]		
0 [g] [CF_j] 3 [g] [N_j]	***3.00***	Three cash flows of 0
125000 [g] [CF_j]		
[g] [CF_j]	***125,000.00***	4th & 5th cash flows
140000 [g] [CF_j]	***140,000.00***	6th cash flow
150000 [g] [CF_j]	***150,000.00***	7th cash flow
175000 [g] [CF_j]	***175,000.00***	8th cash flow
100000 [g] [CF_j]	***100,000.00***	9th cash flow
50000 [g] [CF_j]	***50,000.00***	10th cash flow
[f] [IRR]	***12.38***	% annual rate of return

Example 2:

Using the cash flow figures in Example 1, what is the rate of return if the "safe" rate for negative cash flows is 5.5% and the reinvestment rate for positive cash flows is 10%?

Here the keystroke procedure is slightly different. The steps are:

1) Calculate the future value of the positive cash flows at the reinvestment rate.

2) Calculate the present value of the negative cash flows at the "safe" rate.

3) Knowing n, PV, and FV, solve for i.

Keystrokes (HP-37E)	**Display**	
1 [ENTER↑] 10 [%]		
[+] [f] [1/x] [ENTER↑]		
[ENTER↑] [ENTER↑]		
50000 [×]		
100000 [+] [×]		
175000 [+] [×]		
150000 [+] [×]		
140000 [+] [×]		
125000 [+] [×]		
125000 [+] [×]		
0 [+] [×] 0 [+] [×]		
0 [+] [×]	***462,317.63***	Present value of cash flows
[CL FIN]		
[CHS] [PV]		
10 [n] 10 [i]		
[FV]	***1,199,132.88***	Future value of positive cash flows at 10%
1 [ENTER↑] 5.5 [%]		
[+] [f] [1/x] [ENTER↑]		
[ENTER↑] [ENTER↑]		
25000 [CHS] [×]		
100000 [CHS] [+] [×]		
150000 [CHS] [+] [×]		
150000 [CHS] [+] [PV]	***–403,315.68***	Present value of negative cash flows at 5.5%
[i]	***11.51***	% annual rate of return

Keystrokes (HP-38E)	Display	
f ALL		
3 n	*3.00*	Skip negative cash flows (1-3)
125000 g CFj		
g CFj		
140000 g CFj		
150000 g CFj		
175000 g CFj		
100000 g CFj		
50000 g CFj	*50,000.00*	Cash flow in 10^{th} year
10 i	*10.00*	Reinvestment rate
f NPV	*462,317.63*	Present value of cash flows
CHS PV FV	*1,199,132.88*	Future value of positive cash flows at 10%
5.5 i	*5.50*	Safe rate for negative cash flows
150000 CHS g CF0		
g CFj		
100000 CHS g CFj		
25000 CHS g CFj		
f NPV PV	*–403,315.68*	Present value of negative cash flows
10 n i	*11.51*	% annual rate of return

Canadian Mortgages

In Canada, interest is compounded semi-annually with payments made monthly. This results in a different monthly mortgage factor than is used in the United States, and is programmed into the HP-37E and HP-38E. This difference can be handled easily on these calculators by the addition of a few keystrokes. For any problem requiring an input for [i], the Canadian mortgage factor is calculated first and then this value is entered for [i] in the calculation to give the answer for Canada. The keystrokes to calculate the Canadian mortgage factor are:

1) Press [CL FIN].
2) Key in 6; press [n].
3) Key in 200; press [ENTER↑] [PV].
4) Key in the annual interest rate as a percent; press [+] [CHS] [FV].
5) Press [i].

The Canadian mortgage factor is now stored in [i] for future use. The examples below show how this factor is used for [i] in Canadian mortgage problems.

Example 1—Periodic Payment Amount

What is the monthly payment required to fully amortize a 30-year, $30,000 Canadian mortgage if the interest rate is 9%?

Keystrokes	Display	
BEGIN END		
[CL FIN]		
6 [n] 200 [ENTER↑] [PV]		
9 [+] [CHS] [FV] [i]	***0.74***	Canadian mortgage factor
30 [12×]	***360.00***	Total monthly periods in mortgage life (into n)
30000 [PV] 0 [FV]		
[PMT]	***-237.85***	Monthly payment (calculated)

Example 2—Number of Periodic Payments to Fully Amortize a Mortgage

An investor can afford to pay $440 per month on a $56,000 Canadian mortgage. If the annual interest rate is 9¼%, how long will it take to completely amortize this mortgage?

Keystrokes	Display	
BEGIN END		
CL FIN		
6 n 200 ENTER↑ PV		
9.25 + CHS FV i	***0.76***	Canadian monthly mortgage factor
440 CHS PMT	***-440.00***	Monthly payment (into PMT)
56000 PV 0 FV n	***436.34***	Total number of monthly payments
12 ÷	***36.36***	Total years

Example 3—Effective Interest Rate (Yield)

A Canadian mortgage has monthly payments of $612.77 with a maturity of 25 years. The principal amount is $75,500. What is the annual interest rate?

Keystrokes	Display	
BEGIN END		
CL FIN		
25 12×		
612.77 CHS PMT		
75500 PV i	***0.72***	Canadian monthly mortgage factor
6 n 0 PMT		
200 CHS PV		
FV RCL PV +	***8.75***	Annual interest rate

Example 4—Balance Remaining at End of Specified Period

A Canadian mortgage has monthly payments of $612.77 at 8.75% interest. The principal amount is $75,500. What will be the outstanding balance remaining at the end of 10 years?

Keystrokes	Display	
BEGIN END		
CL FIN		
6 n 200 ENTER↑ PV		
8.75 + CHS FV i	***0.72***	Canadian monthly mortgage factor
612.77 CHS PMT		
10 12×		
75500 PV FV	***-61,877.18***	Remaining balance outstanding at the end of 10 years

Depreciation Calculations

The three most common accounting methods are straight line, declining balance, and sum-of-the-years'-digits (SOYD) depreciation. Declining balance and SOYD are accelerated methods, providing higher depreciation amounts initially than the straight line method.

To illustrate the procedures, and to show the differences resulting from the application of different depreciation methods, a continuing example will be used.

Straight Line Method

The annual depreciation allowance using this method is determined by dividing the cost or other basis of the property valuation (excluding land costs) less its estimated salvage value by its useful life expectancy. Information is entered as follows:

1) Key in the depreciable amount (improvements cost less salvage value); press [ENTER↑] [ENTER↑].
2) Key in estimate of useful life; press [÷] [STO] 0.
3) Key in the number of the year for which data is desired; press [×] to obtain total straight line depreciation to date.
4) Press [−] to obtain remaining depreciable value to date.
5) Press [RCL] 0 [−] to obtain remaining depreciable value for each subsequent year.

Example:

A property has just been acquired for $150,000. The purchase price is allocated between $25,000 for land and $125,000 for improvements (building). The remaining useful life of the building is agreed to be 25 years. There is no salvage value forecast at the end of the useful life of the building. Using the straight line method of depreciation, what are the building's annual depreciation allowance and remaining depreciable value for the first three years of its useful life?

Keystrokes	Display	
125000		
[ENTER↑] [ENTER↑]	*125,000.00*	Depreciable amount
25 [÷] [STO] 0	*5,000.00*	Annual depreciation allowance
1 [×] [−]	*120,000.00*	Remaining depreciable value, year 1
[RCL] 0 [−]	*115,000.00*	Remaining depreciable value, year 2
[RCL] 0 [−]	*110,000.00*	Remaining depreciable value, year 3

If you own an HP-38E, you might wish to try the previous example using this program for straight line depreciation.

KEY ENTRY	DISPLAY
[g] [P/R] [g] [CL P]	*00–*
[STO] 1	*01– 21 1*
[g] [R↓]	*02– 25 33*
[÷]	*03– 71*
[STO] 0	*04– 21 0*
[R/S]	*05– 74*
[RCL] 1	*06– 22 1*
[×]	*07– 61*

KEY ENTRY	DISPLAY
[−]	*08– 41*
[R/S]	*09– 74*
[RCL] 0	*10– 22 0*
[−]	*11– 41*
[R/S]	*12– 74*
[g] [GTO] 10	*13– 25 7 10*
[g] [P/R]	

REGISTERS			
R_0 Annual Depr.	R_1 Year	R_2	R_3

1. Key in the program.
2. Key in depreciable amount (cost less salvage value); press [ENTER↑] [ENTER↑].
3. Key in asset's useful life (number of years); press [ENTER↑].
4. Key in year for which depreciation is to be calculated; press [R/S] to calculate the annual depreciation.

5. To calculate the total depreciation to date; press [R/S].
6. To calculate the remaining depreciable value for subsequent years; press [R/S].
7. For a new case, go to step 2.

Keystrokes	Display	
125000		
[ENTER↑] [ENTER↑]		
25 [ENTER↑]		
1 [R/S]	***5,000.00***	Annual depreciation
[R/S]	***120,000.00***	Remaining depreciable value, year 1
[R/S]	***115,000.00***	Remaining depreciable value, year 2
[R/S]	***110,000.00***	Remaining depreciable value, year 3

Declining Balance Method

The declining balance method is one form of accelerated depreciation; as such it provides for more depreciation in the earlier years of ownership and less depreciation in the later years than the straight line method. The following calculations find the depreciation and remaining book value for each year of an asset's depreciable life when the cost or other valuation basis, salvage value, and life expectancy are known. Calculations under the section entitled Full Year are valid when an asset is held for a full twelve months in the first year of depreciation, while the calculation under the section entitled Partial Year is used in cases where the asset is held less than twelve months in its first year of depreciable life.

Full Year

To find the depreciation and remaining balance for each year, information is entered as follows:

1) Press [CL FIN].
2) Key in the year for which the depreciation and book value are desired; press [ENTER↑] 1 [−] [n].
3) Key in the declining factor as a percent; press [ENTER↑].

4) Key in useful life expectancy (number of years); press [÷] [STO] 0 [CHS] [i].

5) Key in cost or other basis; press [CHS] [PV] [FV] to obtain the remaining book value at the beginning of the specified year.

6) Press [RCL] 0 [%] to obtain depreciation in the specified year.

7) Press [−] to obtain remaining book value at the end of the specified year.

8) Repeat steps 6 and 7 to obtain each succeeding year's depreciation and remaining book value.

Example:

What is the 150% declining balance depreciation schedule for the example property over the first 3 years?

Keystrokes	Display	
[CL FIN]		
1 [ENTER↑] 1 [−] [n]		
150 [ENTER↑]		
25 [÷] [STO] 0 [CHS] [i]		
125000 [CHS] [PV]		
[FV]	***125,000.00***	Remaining book value at beginning of 1st year
[RCL] 0 [%]	***7,500.00***	Depreciation in 1st year
[−]	***117,500.00***	Remaining book value at end of 1st year
[RCL] 0 [%]	***7,050.00***	Depreciation in 2nd year
[−]	***110,450.00***	Remaining book value at end of 2nd year
[RCL] 0 [%]	***6,627.00***	Depreciation in 3rd year
[−]	***103,823.00***	Remaining book value at end of 3rd year

The following HP-38E program for declining balance depreciation can also be used to evaluate the previous example:

KEY ENTRY	DISPLAY
g P/R g CL P	00–
f FIN	01– 24 32
CHS	02– 32
PV	03– 13
g R↓	04– 25 33
÷	05– 71
STO 0	06– 21 0
CHS	07– 32
i	08– 12
x≷y	09– 33
1	10– 1
−	11– 41
n	12– 11
FV	13– 15

KEY ENTRY	DISPLAY
R/S	14– 74
RCL 0	15– 22 0
%	16– 23
R/S	17– 74
−	18– 41
R/S	19– 74
1	20– 1
STO + 1	21– 21 51 1
CLX	22– 34
RCL 1	23– 22 1
g PSE	24– 25 4
g R↓	25– 25 33
g GTO 15	26– 25 7 15
g P/R	

REGISTERS			
R_0 Fact/Life	R_1 Year	R_2	R_3

1. Key in the program.
2. Key in year for which depreciation is to be calculated; press STO 1.
3. Key in the declining factor as a percent; press ENTER↓.
4. Key in asset's useful life; press ENTER↓.
5. Key in cost and press R/S to calculate remaining book value at start of the schedule.
6. To calculate each year's depreciation; press R/S.
7. To calculate the remaining book value; press R/S.

8. Repeat steps 6 and 7 for subsequent years. The year number will be "paused" in the display.
9. For a new case, press [9] [CL P] and go to step 2.

Keystrokes	Display	
1 [STO] 1		
150 [ENTER↑]		
25 [ENTER↑] 125000		
[R/S]	***125,000.00***	
[R/S]	***7,500.00***	
[R/S]	***117,500.00***	
[R/S]	***2.00***	"Paused"
	7,050.00	
[R/S]	***110,450.00***	
[R/S]	***3.00***	"Paused"
	6,627.00	
[R/S]	***103,823.00***	

Partial Year

If the asset is held for less than twelve months in the first year, the depreciation using the declining balance method can be found as follows:

1) Key in the declining factor as a percent; press [ENTER↑].
2) Key in the depreciable life; press [÷] [STO] 0.
3) Key in the initial book value; press [RCL] 0 [%].
4) Key in the number of months held in the first year; press [ENTER↑] 12 [÷] [×] to obtain first year's depreciation.
5) Press [−] to see the remaining book value.
6) Press [RCL] 0 [%] to obtain the second year's depreciation.
7) Press [−] for remaining book value.
8) Repeat steps 6 and 7 for successive years depreciation and remaining book value.

Example:

An asset is valued at $50,000 with an expected life of 16 years. It is held for 6 months the first year and double declining balance depre-

ciation (200%) is used. What are the depreciation and remaining book value for the first two years?

Keystrokes	Display	
200 [ENTER↑]		
16 [÷] [STO] 0		
50000 [RCL] 0 [%]		
6 [ENTER↑] 12 [÷]		
[×]	***3,125.00***	First year's depreciation
[−]	***46,875.00***	Remaining depreciation
[RCL] 0 [%]	***5,859.38***	Second year's depreciation
[−]	***41,015.63***	Remaining book value

Sum-of-the-Years'-Digits Method

Like the declining balance method, the sum-of-the-years'-digits method (SOYD) is an accelerated form of depreciation, allowing more depreciation in the early years of an asset's life than allowed under the straight line method. The calculations below find the depreciation and remaining depreciable value using the SOYD method for each year of an asset's depreciable life when its useful life expectancy and cost or other basis (less salvage value) are known. The section entitled Full Year is used if the asset is held the full twelve months of the first year and the section entitled Partial Year is used if the asset is held for less than twelve months the first year of depreciation.

Full Year

To find the depreciation and remaining depreciable value, information is entered as follows:

1) Key in the cost or other depreciable basis of asset; press [STO] 0.

2) Key in asset's useful life expectancy (number of years); press [STO] 1.

3) Key in the beginning year number (i.e., year 1); press [STO] 2.

4) Press [RCL] 1 [RCL] 2 [−] 1 [+] 2 [×] [RCL] 1 1 [+] [÷] [RCL] 1 [÷] [RCL] 0 [×] to obtain the beginning year's depreciation.

5) Press [RCL] 1 [RCL] 2 [−] 2 [÷] [×] to obtain the remaining depreciable value.

6) For each subsequent year's depreciation and remaining depreciable value, press 1 [STO] [+] 2 and repeat steps 4 and 5.

Example:

An apartment house has a cost basis (excluding land cost and salvage value) of $210,000.00 and a useful life expectancy of 25 years. Using the SOYD method, what are the depreciation allowances and remaining depreciable values for each of the first two years?

Keystrokes	Display	
210000 STO 0		
25 STO 1		
1 STO 2		
RCL 1 RCL 2 −		
1 + 2 × RCL 1		
1 + ÷ RCL 1		
÷ RCL 0 ×	***16,153.85***	Year 1 depreciation
RCL 1 RCL 2 −		
2 ÷ ×	***193,846.15***	Remaining depreciable value, year 1
1 STO + 2		
RCL 1 RCL 2 −		
1 + 2 × RCL 1		
1 + ÷ RCL 1		
÷ RCL 0 ×	***15,507.69***	Year 2 depreciation
RCL 1 RCL 2 −		
2 ÷ ×	***178,338.46***	Remaining depreciable value, year 2

If you have an HP-38E, try this program to evaluate the previous example:

KEY ENTRY	DISPLAY
g P/R g CL P	00–
RCL 1	01– 22 1
RCL 2	02– 22 2
g PSE	03– 25 4
−	04– 41
1	05– 1
+	06– 51
2	07– 2
×	08– 61
RCL 1	09– 22 1
1	10– 1
+	11– 51
÷	12– 71
RCL 1	13– 22 1
÷	14– 71

KEY ENTRY	DISPLAY
RCL 0	15– 22 0
×	16– 61
R/S	17– 74
RCL 1	18– 22 1
RCL 2	19– 22 2
−	20– 41
2	21– 2
÷	22– 71
×	23– 61
R/S	24– 74
1	25– 1
STO + 2	26– 21 51 2
g GTO 01	27– 25 7 01
g P/R	

REGISTERS			
R_0 Dep. Value	R_1 Life	R_2 Year	R_3

1. Key in the program.
2. Key in the depreciable amount (cost less salvage value); press STO 0.
3. Key in the asset's useful life (number of years); press STO 1.
4. Key in the year for which depreciation is to be calculated; press STO 2.
5. To calculate the year's depreciation, press R/S.
6. To calculate the remaining depreciable value, press R/S.
7. Repeat steps 5 and 6 for subsequent years.
8. For a new case, press g CL P and go to step 2.

Keystrokes	Display	
210000 STO 0		
25 STO 1		
1 STO 2		
R/S	*1.00*	"Paused"
	16,153.85	Depreciation
R/S	*193,846.15*	Remaining depreciable value
R/S	*2.00*	"Paused"
	15,507.69	Depreciation
R/S	*178,338.46*	Remaining depreciable value

Partial Year

Sometimes an asset's acquisition date is different from the beginning of the accounting year. To find the amount of depreciation in that partial year, first calculate the normal SOYD depreciation schedule. Then calculate the fraction of a year from the acquisition date to the beginning of the next accounting year (factor 1), and calculate the fraction of a year remaining from the beginning of the accounting year to the first full year of holding (factor 2 = 1 − factor 1). After these factors have been obtained, apply them to the normal SOYD schedule as follows:

1) Key in number of months from acquisition date to the beginning of the next accounting year; press ENTER↑ 12 ÷ STO 3 to obtain and store factor 1.

2) Key in 1; press x≷y − STO 4 to obtain and store factor 2.

3) Key in normal first year scheduled depreciation; press STO 5 RCL 3 × to obtain year 1 depreciation.

4) Press RCL 5 RCL 4 ×.

5) Key in normal second year scheduled depreciation; press STO 5 RCL 3 × + to obtain year 2 depreciation.

6) Repeat steps 2 through 5 to obtain each succeeding year's depreciation.

In the final year, which is one year beyond the useful life due to the partial years at the beginning and end of the schedule, the depreciation is simply:

Normal year N schedule depreciation [×] factor 2 = Year N + 1 value.

Example:

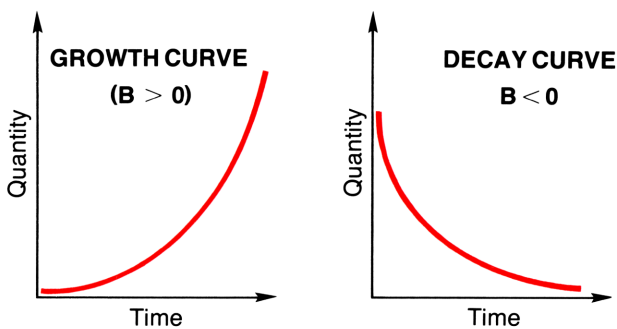
Generate the first 2 years SOYD depreciation schedule for the previous example if the property was purchased on September 1 and depreciation is charged on a calendar year basis.

Keystrokes	Display	
4 [ENTER↑] 12 [÷]		
[STO] 3	***0.33***	Factor 1
1 [x≷y] [−] [STO] 4	***0.67***	Factor 2
16153.85 [STO] 5		
[RCL] 3 [×]	***5,384.62***	Year 1 depreciation
[RCL] 5 [RCL] 4 [×]		
15507.69 [STO] 5		
[RCL] 3 [×] [+]	***15,938.46***	Year 2 depreciation

Exponential Curve Fit

Using the [LN] function of the HP-37E and HP-38E, a least squares exponential curve fit may easily be calculated according to the equation $y = Ae^{Bx}$. The exponential curve fitting technique is often used to determine the growth rate of a variable such as a stock's value over time, when it is suspected that the performance is non-linear. The value for B is the decimal value of the continuous growth rate. For instance, assume after keying in several end-of-month price quotes for a particular stock, it is determined that the value for B is 0.10. This means that over the measured period the stock has experienced a 10% continuous growth rate. This decimal continuous growth rate may then be converted to an effective growth rate.

If $B > 0$, you will have a growth curve. If $B < 0$, you will have a decay curve. Examples of these are given below.



The procedure is as follows:

1) Press [CL ALL].

2) For each input pair of values, key in the y-value and press [LN]; key in the corresponding x-value and press [Σ+].

3) After all data pairs are input, press [ŷ,r] [x⇄y] to obtain the correlation coefficient.

4) Press 1 [ŷ,r] [e^x] 0 [ŷ,r] [e^x] to obtain A in the equation above.

5) Press [x≷y] [R↓] [÷] [LN] to obtain B.

6) Press [e^x] 1 [−] to obtain the effective growth rate (as a decimal).

7) To make a y-estimate, key in the x-value; press [$\hat{y}$,r] [e^x].

Example 1:

A stock's price history is listed below. What effective growth rate does this represent? If the stock continues this growth rate, what is the price projected to be at the end of 1978 (year 5)?

End of Year	Price
1974 (1)	52½
1975 (2)	55¼
1976 (3)	(missing data)
1977 (4)	61
1978 (5)	?

Keystrokes	Display	
[CL ALL]		
52.5 [LN] 1 [Σ+]	***1.00***	First data pair input
55.25 [LN] 2 [Σ+]	***2.00***	Second data pair input
61 [LN] 4 [Σ+]	***3.00***	Third data pair input
[$\hat{y}$,r] [x≷y]	***1.00***	Correlation coefficient
1 [$\hat{y}$,r] [e^x]		
0 [$\hat{y}$,r] [e^x]	***49.96***	A
[x≷y] [R↓] [÷] [LN]	***0.05***	B
[e^x] 1 [−]	***0.05***	Effective growth rate
5 [$\hat{y}$,r] [e^x]	***64.14***	Projected price at the end of year 5 (1978)

If you have an HP-38E, try this example using the following program:

KEY ENTRY	DISPLAY
g P/R g CL P	00–
x⇄y	01– 33
g LN	02– 25 23
x⇄y	03– 33
f Σ+	04– 24 74
g GTO 00	05– 25 7 00
g ŷ,r	06– 25 2
x⇄y	07– 33
R/S	08– 74
1	09– 1
g ŷ,r	10– 25 2
g e^x	11– 25 22
0	12– 0
g ŷ,r	13– 25 2
g e^x	14– 25 22

KEY ENTRY	DISPLAY
R/S	15– 74
x⇄y	16– 33
g R↓	17– 25 33
÷	18– 71
g LN	19– 25 23
R/S	20– 74
g e^x	21– 25 22
1	22– 1
−	23– 41
R/S	24– 74
g ŷ,r	25– 25 2
g e^x	26– 25 22
g GTO 00	27– 25 7 00
g P/R	

REGISTERS			
R_0	R_1 n	R_2 Σx	R_3 Σx^2
R_4 Σy	R_5 Σy^2	R_6 Σxy	R_7

1) Key in the program and press f ALL.
2) For each input pair of values, key in the y-value and press ENTER↑; key in the corresponding x-value and press R/S.
3) After all data pairs are input, press g GTO 06 R/S to obtain the correlation coefficient.
4) Press R/S to obtain A.
5) Press R/S to obtain B.
6) Press R/S to obtain the effective growth rate as a decimal.

7) To make a y-estimate, key in the x-value; press [R/S]. For subsequent estimates, key in the x-value and press [g] [GTO] 25 [R/S].

8) For a different set of data, press [f] [ALL] and proceed at step 2.

Keystrokes	Display	
[f] [ALL]		
52.5 [ENTER↑] 1 [R/S]		
55.25 [ENTER↑] 2 [R/S]		
61 [ENTER↑] 4 [R/S]		
[g] [GTO] 06 [R/S]	***1.00***	Correlation coefficient
[R/S]	***49.96***	A
[R/S]	***0.05***	B
[R/S]	***0.05***	Effective growth rate
5 [R/S]	***64.14***	Projected price at the end of year 5 (1978)

Appendix

Financial Formulas

Compound Interest

n = number of compounding periods

i = periodic interest rate, expressed as %

PV = present value

FV = future value or balance

PMT = periodic payment

δ = BEGIN/END switch position factor (0 or 1) indicating treatment of PMT; 0 corresponds to END, 1 to BEGIN

r = i/100, periodic interest rate expressed as decimal.

$$0 = PV + (1 + r\delta)\,PMT\left[\frac{1 - (1 + r)^{-n}}{r}\right] + FV\,(1 + r)^{-n}$$

Increasing/Decreasing Annuity

I_D = discount rate (as decimal)

I_G = growth rate (as decimal)

G = constant increment

Constant ratio payments

$$PV = \frac{PMT}{I_D - I_G}\left[1 - \left(1 + \frac{I_D - I_G}{1 + I_G}\right)^{-n}\right]$$

Constant increment payments

$$PV = \left(\frac{G}{I_D} + PMT\right)\left[\frac{(1 + I_D)^n - 1}{I_D\,(1 + I_D)^n}\right] - \frac{G \cdot n}{I_D\,(1 + I_D)^n}$$

Wrap-Around Mortgage

$$PV_2 - PV_1 = \frac{PMT_2\left[1 - (1 + r)^{-n_2}\right]}{r}$$

$$- \frac{PMT_1\left[1 - (1 + r)^{-n_1}\right]}{r} + FV(1 + r)^{-n_2}$$

Canadian Mortgages

$$\text{monthly factor} = \left[\left(1 + \frac{r}{2}\right)^{1/6} - 1\right] \times 100$$

Depreciation

L = asset's useful life expectancy

SBV = starting book value

SAL = salvage value

DEP_k = depreciation for year k

RBV_k = remaining book value at end of year k

RDV_k = remaining depreciable value at end of year k.

Straight-Line Depreciation

$$DEP_k = \frac{SBV - SAL}{L}$$

$$RDV_k = RDV_{k-1} - DEP_k$$

Sum-of-the-Years'-Digits Depreciation

$$DEP_k = \frac{2(L - k + 1)}{L(L + 1)} \quad (SBV - SAL)$$

$$RDV_k = RDV_{k-1} - DEP_k$$

Declining Balance Depreciation

F = declining balance factor (as decimal)

$$DEP_k = RBV_{k-1} \frac{F}{L}$$

$$RDV_k = RDV_{k-1} - DEP_k$$

Exponential Curve Fit

$$y = Ae^{Bx}$$

$$B = \frac{\Sigma x_i \ln y_i - \frac{1}{n}(\Sigma x_i)(\Sigma \ln y_i)}{\Sigma x_i^2 - \frac{1}{n}(\Sigma x_i)^2}$$

$$A = \exp\left[\frac{\Sigma \ln y_i}{n} - B\frac{\Sigma x_i}{n}\right]$$

$$\hat{y} = Ae^{Bx}$$

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00038-90024 Rev. C 9/79

Printed in U.S.A.