HEWLETT-PACKARD

HP-37E &
HP-38E/38C

MARKETING AND FORECASTING

Applications

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HP-37E
&
HP-38E/38C

Marketing & Forecasting Applications

May 1979

00038-90049

Printed in U.S.A.

Introduction

This application book has been designed to supplement the HP-37E and HP-38E/38C Owner's Handbooks by providing a collection of applications specifically associated with marketing and forecasting. Step by step keystroke procedures and/or programs with corresponding examples for 14 problem types are explained. Hopefully, this book will provide a reference guide to the majority of your problems, and show you how to redesign our examples to fit your specific needs.

It is sometimes necessary in these keystroke solutions to include operations which involve prefix keys, namely [f] on the HP-37E and [f] and [9] on the HP-38E/38C. For example, the operation [12×] is performed on the HP-37E as [f] [12×] and on the HP-38E/38C as [9] [12×]. *In such cases, the keystroke solution omits the prefix key and indicates only the operation* (as here, [12×]). As you work through the example problems, take care to press the appropriate prefix keys (if any) for your calculator.

In addition, it should be noted that certain clearing functions on the two calculators have different key mnemonics. Clear finance on the HP-37E is represented as [CL FIN], and is represented as [FIN] on the HP-38E/38C. Clear all is represented as [CL ALL] and [ALL] on the HP-37E and HP-38E/38C respectively. Unless otherwise specified, this book will use the key mnemonics of the HP-37E, although the keystrokes are applicable to both machines.

All results are carried internally to ten significant digits. If intermediate answers are rounded by the user, slightly different final values may be obtained.

Contents

Forecasting

Simple Moving Average

Moving averages are often useful in recording or forecasting sales figures, expenses or manufacturing volume. There are many different types of moving average calculations. An often used, straightforward method of calculation is presented here.

In a moving average a specified number of data points is averaged. When there is a new piece of input data, the oldest piece of data is discarded to make room for the latest input. This replacement scheme makes the moving average a valuable tool in following trends. The fewer the number of data points, the more trend sensitive the average becomes. With a large number of data points, the average behaves more like a regular average, responding slowly to new input data.

A simple moving average may be calculated with your HP-37E or HP-38E/38C as follows:

1) Press [CL ALL].
2) Key in the first m data points (where m is the number of data points in the average); press [Σ+] after each entry.
3) Press [x̄] to obtain the first average.
4) Key in the oldest (first) value entered in step 2); press [Σ−].
5) Key in the newest data point ($m+1$); press [Σ+].
6) Press [x̄] to obtain the next value of the moving average.
7) Repeat steps 4) through 6) for the remaining data.

Example 1: An electronics sales firm wants to calculate a 3-month moving average for the dollar volume of components sold each month. Sales for the first six months of this year were:

January	$211,570	April	$131,760
February	112,550	May	300,500
March	190,060	June	271,120

Keystrokes:	Display:	
[CL ALL]	*0.00*	
211570 [Σ+]	*1.00*	
112550 [Σ+]	*2.00*	
190060 [Σ+]	*3.00*	
[x̄]	*171,393.33*	3-month average for March
211570 [Σ−]	*2.00*	
131760 [Σ+]	*3.00*	
[x̄]	*144,790.00*	3-month average for April
112550 [Σ−]	*2.00*	
300500 [Σ+]	*3.00*	
[x̄]	*207,440.00*	3-month average for May
190060 [Σ−]	*2.00*	
271120 [Σ+]	*3.00*	
[x̄]	*234,460.00*	3-month average for June

For repeated calculations the following HP-38E/38C program can be used for up to a 12 element moving average:

KEY ENTRY	DISPLAY
g P/R g CL P	00–
RCL 1	01– 22 1
RCL 2	02– 22 2
STO 1	03– 21 1
+	04– 51
RCL 3	05– 22 3
STO 2	06– 21 2
+ 3	07– 51
RCL 4	08– 22 4
STO 3	09– 21 3
+ 4	10– 51
RCL 5	11– 22 5
STO 4	12– 21 4
+ 5	13– 51
RCL 6	14– 22 6
STO 5	15– 21 5
+ 6	16– 51
RCL 7	17– 22 7
STO 6	18– 21 6
+ 7	19– 51
RCL 8	20– 22 8

KEY ENTRY	DISPLAY
STO 7	21– 21 7
+ 8	22– 51
RCL 9	23– 22 9
STO 8	24– 21 8
+ 9	25– 51
RCL .0	26– 22 73 0
STO 9	27– 21 9
+ 10	28– 51
RCL .1	29– 22 73 1
STO .0	30– 21 73 0
+ 11	31– 51
RCL .2	32– 22 73 2
STO .1	33– 21 73 1
+ 12	34– 51
RCL 0	35– 22 0
÷	36– 71
R/S	37– 74
STO *m**	38– 21 *m**
g GTO 01	39– 25 7 01
g P/R	

REGISTERS			
R_0 m	R_1 x_1	R_2 x_2	R_3 x_3
R_4 x_4	R_5 x_5	R_6 x_6	R_7 x_7
R_8 x_8	R_9 x_9	$R_{.0}$ x_{10}	$R_{.1}$ x_{11}
$R_{.2}$ x_{12}	$R_{.3}$	$R_{.4}$	$R_{.5}$

* **Note:** At step 38, m = number of elements in the moving average, (i.e. for a 5 element moving average line 38 would be [STO] 5 and for a 12 element moving average line 38 would be [STO] .2).

This program can be used for a moving average of 2 to 12 elements. It may be shortened considerably for moving averages with less than 12 elements. To do this, key in the program, as shown, from line 01 until you reach a [+] superscripted with the number of elements you desire. Key in this line, then skip the rest of the program down to line 35. Then key in lines 35 through 39 being sure to specify the register number at line 38– [STO], corresponding to the number of elements you are using. (For instance, for a 5 element moving average, key in lines 01 through 13 then go to line 35 in the listing, key in the balance of the program. Obviously the program listing line 38, [STO] m, becomes the displayed line 17, [STO] 5).

To run the program:

1) Key in the program.
2) Press [f] [ALL]. Key in the number of elements, m; press [STO] 0.
3) Key in the first data point; press [STO] 1.
4) Key in the second data point; press [STO] 2.
5) Continue as above, keying in and storing each data point in its appropriate register until m data points have been stored.
6) Press [g] [GTO] 00 [R/S] to calculate the first moving average.
7) Key in the next data point; press [R/S] to calculate the next moving average.
8) Repeat step 7) for each new data point.

Example 2: Calculate the 3-element moving average for the data given in Example 1. Your modified program listing will look like this:

KEY ENTRY	DISPLAY
g P/R g CL P	00–
RCL 1	01– 22 1
RCL 2	02– 22 2
STO 1	03– 21 1
+	04– 51
RCL 3	05– 22 3
STO 2	06– 21 2

KEY ENTRY	DISPLAY
+	07– 51
RCL 0	08– 22 0
÷	09– 71
R/S	10– 74
STO 3	11– 21 3
g GTO 01	12– 25 7 01
g P/R	

Keystrokes:	Display:
f ALL	0.00
3 STO 0	3.00
211570 STO 1	211,570.00
112550 STO 2	112,550.00
190060 STO 3	190,060.00
g GTO 00 R/S	171,393.33
131760 R/S	144,790.00
300500 R/S	207,440.00
271120 R/S	234,460.00

Note: This program is based on HP-65 Users' Library program #00756A submitted by Vernon J. Poehls.

Seasonal Variation Factors Based on Centered Moving Averages

Seasonal variation factors are a useful concept in many types of forecasting. There are several methods of developing seasonal moving averages, one of the more common ways being to calculate them as the ratio of the periodic value to a centered moving average for the same period.

For instance, to determine the sales for the 3rd quarter of a given year a centered moving average for that quarter would be calculated from sales figures from the 1st, 2nd, 3rd and 4th quarters of that year and the 1st quarter of the following year. The seasonal variation factor for that 3rd quarter would then be the ratio of the actual sales in the 3rd quarter to the centered moving average for that quarter.

While quarterly seasonal variations are commonly used, the HP-38E/38C can also be programmed to calculate monthly seasonal variations using a centered 12-month moving average. Programs for both of these calculations are represented here:

An HP-38E/38C program to calculate quarterly seasonal variations based on a centered 4-point moving average is:

KEY ENTRY	DISPLAY
g P/R g CL P	00–
RCL 1	01– 22 1
2	02– 2
÷	03– 71
RCL 2	04– 22 2
STO 1	05– 21 1
+	06– 51
RCL 3	07– 22 3
STO 2	08– 21 2
+	09– 51
RCL 4	10– 22 4
STO 3	11– 21 3
+	12– 51
RCL 5	13– 22 5

KEY ENTRY	DISPLAY
STO 4	14– 21 4
2	15– 2
÷	16– 71
+	17– 51
4	18– 4
÷	19– 71
R/S	20– 74
RCL 2	21– 22 2
f %T	22– 24 22
R/S	23– 74
STO 5	24– 21 5
g GTO 01	25– 25 7 01
g P/R	

REGISTERS			
R_0	R_1 x_1	R_2 x_2	R_3 x_3
R_4 x_4	R_5 x_5	R_6	R_7

1) Key in the program.

2) Press f ALL.

3) Key in the quarterly sales figures starting with the first quarter:
   a) Key in 1st quarter sales; press STO 1

b) Key in 2nd quarter sales; press [STO] 2
c) Key in 3rd quarter sales; press [STO] 3
d) Key in 4th quarter sales; press [STO] 4
e) Key in the 1st quarter sales for the next year; press [STO] 5.

4) Press [g] [GTO] 00 [R/S] to calculate the centered moving average for the 3rd quarter of the first year.

5) Press [R/S] to calculate the seasonal variation for this quarter.

6) Key in the next quarter's sales; press [R/S] to calculate the moving average for the next quarter.

7) Press [R/S] to calculate the seasonal variation.

8) Repeat steps 6) and 7) for the balance of the data.

Example 1: Econo-Wise Home Appliance Company had quarterly sales for the years 1976 thru 1978 as follows:

SALES (IN $K)

Quarterly	1st	2nd	3rd	4th
1976	397	376	460	501
1977	455	390	530	560
1978	513	434	562	593

Find the centered 4-quarter moving average and seasonal variation factor for each quarter.

Keystrokes:	**Display:**	
[f] [ALL]	*0.00*	
397 [STO] 1	*397.00*	
376 [STO] 2	*376.00*	
460 [STO] 3	*460.00*	
501 [STO] 4	*501.00*	
455 [STO] 5	*455.00*	
[g] [GTO] 00 [R/S]	*440.75*	Centered 4-element average for 3rd quarter, 1976
[R/S]	*104.37*	seasonal variation factor
390 [R/S]	*449.75*	4th quarter, 1976
[R/S]	*111.40*	

Keystrokes:	Display:	
530 R/S	460.25	1st quarter, 1977
R/S	98.86	
560 R/S	476.38	2nd quarter, 1977
R/S	81.87	
513 R/S	491.00	3rd quarter, 1977
R/S	107.94	
434 R/S	503.75	4th quarter, 1977
R/S	111.17	
562 R/S	513.25	1st quarter, 1978
R/S	99.95	
593 R/S	521.38	2nd quarter, 1978
R/S	83.24	

Now average each quarter's seasonal variation for the two years:

f CLEAR Σ	83.24	
98.86 f Σ+	1.00	
99.95 f Σ+	2.00	
g $\bar{x}$	99.41	1st quarter average seasonal variation, %
f CLEAR Σ	99.41	
81.87 f Σ+	1.00	
83.24 f Σ+	2.00	
g $\bar{x}$	82.56	2nd quarter average seasonal variation, %
f CLEAR Σ	82.56	
104.37 f Σ+	1.00	
107.94 f Σ+	2.00	
g $\bar{x}$	106.16	3rd quarter average seasonal variation, %
f CLEAR Σ	106.16	
111.4 f Σ+	1.00	
111.17 f Σ+	2.00	
g $\bar{x}$	111.29	4th quarter average seasonal variation, %

An HP-38E/38C program to calculate a centered 12-month moving average and seasonal variation factors is as follows:

KEY ENTRY	DISPLAY
g P/R g CL P	00–
RCL 1	01– 22 1
2	02– 2
÷	03– 71
RCL 2	04– 22 2
STO 1	05– 21 1
+	06– 51
RCL 3	07– 22 3
STO 2	08– 21 2
+	09– 51
RCL 4	10– 22 4
STO 3	11– 21 3
+	12– 51
RCL 5	13– 22 5
STO 4	14– 21 4
+	15– 51
RCL 6	16– 22 6
STO 5	17– 21 5
+	18– 51
RCL 7	19– 22 7
STO 6	20– 21 6
+	21– 51
RCL 8	22– 22 8
STO 7	23– 21 7
+	24– 51

KEY ENTRY	DISPLAY
RCL 9	25– 22 9
STO 8	26– 21 8
+	27– 51
RCL .0	28– 22 73 0
STO 9	29– 21 9
+	30– 51
RCL .1	31– 22 73 1
STO .0	32– 21 73 0
+	33– 51
RCL .2	34– 22 73 2
STO .1	35– 21 73 1
+	36– 51
RCL .3	37– 22 73 3
STO .2	38– 21 73 2
2	39– 2
÷	40– 71
+	41– 51
RCL 0	42– 22 0
÷	43– 71
R/S	44– 74
RCL 6	45– 22 6
f %T	46– 24 22
R/S	47– 74
STO .3	48– 21 73 3
g GTO 01	49– 25 7 01
g P/R	

REGISTERS			
R_0 n	R_1 x_1	R_2 x_2	R_3 x_3
R_4 x_4	R_5 x_5	R_6 x_6	R_7 x_7
R_8 x_8	R_9 x_9	$R_{.0}$ x_{10}	$R_{.1}$ x_{11}
$R_{.2}$ x_{12}	$R_{.3}$ x_{13}	$R_{.4}$	$R_{.5}$

1) Key in the program.
2) Press [f] [ALL].
3) Key in 12; press [STO] 0.
4) Key in the values for the first 13 months, storing them one at a time in registers 1 through .3; i.e.
   Key in 1st month; press [STO] 1
   Key in 2nd month; press [STO] 2 etc., etc.,
   Key in 10th month; press [STO] .0 etc., etc.,
   Key in 13th month; press [STO] .3
5) Press [g] [GTO] 00 [R/S] to calculate the centered moving average for the 7th month.
6) Press [R/S] to calculate the seasonal variation for that month.
7) Key in the value for the next month (14th); press [R/S] to calculate the moving average for the next month (8th).
8) Repeat steps 6) and 7) for the balance of the data.

These programs may be customized by the user for different types of centered moving average. Inspection of the programs will show how they can readily be modified.

> **Note:** This program is based on HP-67/97 Users' Library program #02643D submitted by Robert M. Olsen.

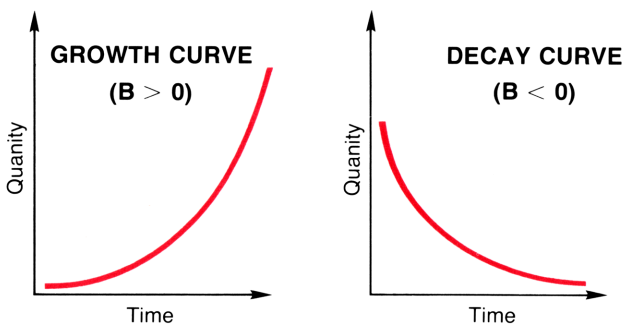
Exponential Curve Fit

Many types of marketing data follow a continuous compounded growth pattern which can be described by the exponential function $Y = Ae^{BX}$. The exponential curve fitting technique is often used to determine the growth rate of a variable over time, when the performance appears to be non-linear, such as the sales figures during the early stages of a new product.

Using the [LN] function on the HP-37E or HP-38E/38C a least squares exponential curve fit may easily be calculated for the equation given above.

The value of the constant B is the continuous growth rate, in decimal form. For instance, if the value of B is 0.10 the continuous growth rate is 10%. This decimal continuous growth rate may easily be converted to the more useful effective growth rate.

If $B > 0$, you will have a growth curve. If $B < 0$, you will have a decay curve. Examples of these are given below:



The procedure is as follows:

1) Press [CL ALL].
2) For each input pair of values, key in the y-value and press [LN]; key in the corresponding x-value and press [Σ+].

3) After all data pairs are input, press [ŷ,r] [x≷y] to obtain the correlation coefficient.

4) Press 1 [ŷ,r] [e^x] 0 [ŷ,r] [e^x] to obtain A in the equation above.

5) Press [x≷y] [R↓] [÷] [LN] to obtain B.

6) Press [e^x] 1 [−] to obtain the effective growth rate (as a decimal).

7) To make a y-estimate, key in the x-value; press [ŷ,r] [e^x].

Example 1: The sales history for a product is given below for the first few months after its introduction. What is the effective growth rate? What are the projected sales for December?

Month		Sales ($K)
June	(1)	31.7
July	(2)	52.2
August	(3)	48.3
September	(4)	56.6
October	(5)	72.7
November	(6)	90.9

Keystrokes:	**Display:**	
[CL ALL]	*0.00*	
31.7 [LN] 1 [Σ+]	*1.00*	
52.2 [LN] 2 [Σ+]	*2.00*	
48.3 [LN] 3 [Σ+]	*3.00*	
56.6 [LN] 4 [Σ+]	*4.00*	
72.7 [LN] 5 [Σ+]	*5.00*	
90.9 [LN] 6 [Σ+]	*6.00*	
[ŷ,r] [x≷y]	*0.95*	Correlation coefficient
1 [ŷ,r] [e^x] 0 [ŷ,r] [e^x]	*29.32*	A
[x≷y] [R↓] [÷] [LN]	*0.18*	B, continuous growth rate (per month)
[e^x] 1 [−]	*0.20*	Effective growth rate (per month)
7 [ŷ,r] [e^x]	*105.86*	Projected sales for December

For repeated use, the following HP-38E/38C program is helpful:

KEY ENTRY	DISPLAY
g P/R g CL P	00–
x≷y	01– 33
g LN	02– 25 23
x≷y	03– 33
f Σ+	04– 24 74
g GTO 00	05– 25 7 00
g ŷ,r	06– 25 2
x≷y	07– 33
R/S	08– 74
1	09– 1
g ŷ,r	10– 25 2
g e^x	11– 25 22
0	12– 0
g ŷ,r	13– 25 2
g e^x	14– 25 22

KEY ENTRY	DISPLAY
R/S	15– 74
x≷y	16– 33
g R↓	17– 25 33
÷	18– 71
g LN	19– 25 23
R/S	20– 74
g e^x	21– 25 22
1	22– 1
−	23– 41
R/S	24– 74
g ŷ,r	25– 25 2
g e^x	26– 25 22
g GTO 24	27– 25 7 24
g P/R	

REGISTERS			
R_0	R_1 n	R_2 Σx	R_3 Σx^2
R_4 Σy	R_5 Σy^2	R_6 Σxy	R_7

1) Key in the program and press f ALL.
2) For each input pair of values, key in the y-value and press ENTER↑; key in the corresponding x-value and press R/S.
3) After all data pairs are input, press g GTO 06 R/S to obtain the correlation coefficient.
4) Press R/S to obtain A.
5) Press R/S to obtain B.
6) Press R/S to obtain the effective growth rate as a decimal.

7) To make a y-estimate, key in the x-value; press [R/S]. For subsequent estimates, key in the x-value and press [R/S].

8) For a different set of data, press [f] [ALL] and proceed at step 2.

Example: With the values given in the previous example use the program to fit the exponential curve and find the projected sales for December and January.

Keystrokes:	**Display:**	
[f] [ALL]	*0.00*	
31.7 [ENTER↑] 1 [R/S]	*1.00*	
52.2 [ENTER↑] 2 [R/S]	*2.00*	
48.3 [ENTER↑] 3 [R/S]	*3.00*	
56.6 [ENTER↑] 4 [R/S]	*4.00*	
72.7 [ENTER↑] 5 [R/S]	*5.00*	
90.9 [ENTER↑] 6 [R/S]	*6.00*	
[g] [GTO] 06 [R/S]	*0.95*	Correlation coefficient
[R/S]	*29.32*	A
[R/S]	*0.18*	B, continuous monthly growth rate
[R/S]	*0.20*	Effective monthly growth rate
7 [R/S]	*105.86*	Projected sales for December
8 [R/S]	*127.17*	Projected sales for January

Gompertz Curve Trend Analysis

A useful curve for evaluating sales trends, etc., is the Gompertz curve. This is a "growth" curve having a general "S" shape and may be used to describe series of data where the early rate of growth is small, then accelerates for a period of time and then slows again as the time grows long. The sales curves for many products follow this trend during the introductory, growth and early mature phases.

The data points to be fit to a Gompertz curve should be equally spaced along the X (or time) axis and all the data points must be positive. The points are divided serially into 3 groups for data entry.

The following HP-38E/38C program processes the data, fits it to a Gompertz curve and calculates estimated values for future data points. The 3 constants which characterize the curve are available to the user if desired. (See appendix for curve equation).

KEY ENTRY	DISPLAY
g P/R g CL P	00–
g LN	01– 25 23
STO + 3	02– 21 51 3
g R↓	03– 25 33
g LN	04– 25 23
STO + 2	05– 21 51 2
g R↓	06– 25 33
g LN	07– 25 23
STO + 1	08– 21 51 1
1	09– 1
STO + 4	10– 21 51 4
RCL 4	11– 22 4
g GTO 00	12– 25 7 00
RCL 3	13– 22 3

KEY ENTRY	DISPLAY
RCL 2	14– 22 2
−	15– 41
RCL 2	16– 22 2
RCL 1	17– 22 1
−	18– 41
÷	19– 71
RCL 4	20– 22 4
f 1/x	21– 24 71
g y^x	22– 25 21
STO 6	23– 21 6
RCL 1	24– 22 1
RCL 3	25– 22 3
×	26– 61
RCL 2	27– 22 2

KEY ENTRY	DISPLAY
ENTER↑	28– 31
×	29– 61
−	30– 41
RCL 1	31– 22 1
RCL 3	32– 22 3
+	33– 51
RCL 2	34– 22 2
2	35– 2
×	36– 61
−	37– 41
÷	38– 71
RCL 4	39– 22 4
÷	40– 71
g e^x	41– 25 22
STO 7	42– 21 7
RCL 6	43– 22 6
1	44– 1
−	45– 41
RCL 6	46– 22 6
RCL 4	47– 22 4
g y^x	48– 25 21
1	49– 1
−	50– 41

KEY ENTRY	DISPLAY
ENTER↑	51– 31
×	52– 61
÷	53– 71
RCL 6	54– 22 6
÷	55– 71
RCL 2	56– 22 2
RCL 1	57– 22 1
−	58– 41
×	59– 61
g e^x	60– 25 22
STO 5	61– 21 5
R/S	62– 74
RCL 6	63– 22 6
$x \rightleftarrows y$	64– 33
g y^x	65– 25 21
RCL 5	66– 22 5
$x \rightleftarrows y$	67– 33
g y^x	68– 25 21
RCL 7	69– 22 7
×	70– 61
g GTO 62	71– 25 7 62
g P/R	

REGISTERS			
R_0	R_1 s_1	R_2 s_2	R_3 s_3
R_4 n	R_5 a	R_6 b	R_7 c

1) Key in the program and press [f] [ALL] .
2) Divide the data points to be input into 3 equal consecutive groups. Label them group I, II and III for convenience.
3) Key in the first point of group I; press [ENTER↑] .
4) Key in the first point of group II; press [ENTER↑] .
5) Key in the first point of group III; press [R/S] .
6) Repeat steps 3), 4) and 5) for the balance of the data in each group. After executing step 5) the display shows how many sets of data have been entered.
7) To fit the data to a Gompertz curve, press [g] [GTO] 13 [R/S] . The resultant display is the curve constant "a". Constants "b" and "c" may be obtained by pressing [RCL] 6 and [RCL] 7 respectively.
8) To calculate a projected value, key in the number of the period and press [R/S] .
9) Repeat step 8) for each period desired.

Example 1: The X-Presso Company marketed a revolutionary new coffee brewing machine in 1968. Sales grew at a steady pace for several years then began to slow. The sales records for the first 9 years of the product's life were as follows:

Year	Sales ($K)	Year	Sales ($K)
1	18	6	260
2	41	7	282
3	49	8	322
4	151	9	340
5	188		

What are the projected sales volumes for this product in its 10th and 12th year? What is the maximum yearly sales volume for this product if the present trend continues? What annual sales rate would the curve have predicted for the 5th year of the product's life? (Arrange the data as follows:)

Group I	Group II	Group III
18	151	282
41	188	322
49	260	340

Keystrokes:	Display:	
f ALL	*0.00*	
18 ENTER↑	*18.00*	
151 ENTER↑	*151.00*	
282 R/S	*1.00*	
41 ENTER↑	*41.00*	
188 ENTER↑	*188.00*	
322 R/S	*2.00*	
49 ENTER↑	*49.00*	
260 ENTER↑	*260.00*	
340 R/S	*3.00*	
g GTO 13 R/S	*0.004*	a
RCL 6	*0.65*	b
RCL 7	*373.92*	c
10 R/S	*349.09*	Sales in 10th year, ($K)
12 R/S	*363.36*	Sales in 12th year, ($K)
100 R/S	*373.92*	Maximum annual sales (after very long product life)
5 R/S	*202.60*	Sales in 5th year ($K) (actual sales were $188K)

Forecasting With Exponential Smoothing

A common method for analyzing trends in sales, inventory and securities data is the moving average. Exponential smoothing is a version of the weighted moving average which is readily adaptable to programmable calculator forecasting.

Exponential smoothing is often used for short term sales and inventory forecasts. Typical forecast periods are monthly or quarterly.

Unlike a moving average, exponential smoothing does not require a great deal of historical data. However, it should not be used with data which has more than a moderate amount of up or down trend.

In the use of exponential smoothing, a smoothing factor is chosen which affects the sensitivity of the average much the same way as the length of the standard moving average period. The correspondence between the two techniques can be represented by the formula

$$\alpha = \frac{2}{n + 1}$$

where α is the exponential smoothing factor (with values from 0 to 1) and n is the length of the standard moving average. As the equation shows, the longer the moving average period, the smaller the equivalent α and the less sensitive the average becomes to fluctuations in current values.

Forecasting with exponential smoothing involves selecting the best smoothing factor based on historical data and then using the factor for updating subsequent data and forecasting. This procedure uses the following HP-38E/38C program:

KEY ENTRY	DISPLAY
g P/R g CL P	00–
ENTER↑	01– 31
ENTER↑	02– 31
RCL 6	03– 22 6
−	04– 41
ENTER↑	05– 31
×	06– 61
STO + 5	07– 21 51 5
g LAST x	08– 25 31
R/S	09– 74
g R↓	10– 25 33
g R↓	11– 25 33
RCL 0	12– 22 0
×	13– 61
RCL 2	14– 22 2
RCL 1	15– 22 1
×	16– 61
+	17– 51
RCL 2	18– 22 2
CHS	19– 32
x≤y	20– 33
STO 2	21– 21 2

KEY ENTRY	DISPLAY
+	22– 51
RCL 0	23– 22 0
×	24– 61
RCL 1	25– 22 1
RCL 3	26– 22 3
×	27– 61
+	28– 51
STO 3	29– 21 3
RCL 1	30– 22 1
×	31– 61
RCL 0	32– 22 0
÷	33– 71
RCL 2	34– 22 2
+	35– 51
STO 4	36– 21 4
RCL 3	37– 22 3
RCL 0	38– 22 0
÷	39– 71
RCL 2	40– 22 2
+	41– 51
STO 6	42– 21 6
g P/R	

REGISTERS			
R_0 α	R_1 $1-\alpha$	R_2 S_{t-1}	R_3 T_{t-1}
R_4 D_t	R_5 Σe^2	R_6 $\hat{D}_{t+1}$	R_7

Selecting the "best" smoothing constant (α):

1) Key in the program and press [f] [ALL].
2) Key in the number 1; press [ENTER↑].
3) Key in the "trial α"; press [STO] 0 [−] [STO] 1.
4) Key in the first historical value (X_1); press [STO] 2.
5) Key in the second historical value (X_2); press [STO] 6 [R/S]. The result is the error between the forecast value ($\hat{D}_{t+1}$) and the true value (X_{t+1}).
6) Press [R/S]; the display shows the next forecast ($\hat{D}_{t+2}$).
7) Optional: Press [RCL] 4 to display the smoothed estimate of current demand.
8) Continue steps 5) and 6) for X_3, X_4, ..., X_n until all historical values have been entered, except when doing step 5) merely key in the value and press [R/S]. (Do not press [STO] 6).
9) Press [RCL] 5. This value represents the cumulative forecasting error (Σe^2). Record the value and the following additional values; press [RCL] 0 (α), [RCL] 2 (smoothed average S_{t-1}), [RCL] 3 (trend T_{t-1}) and [RCL] 6 (forecast $\hat{D}_{t+1}$).
10) Press [f] [ALL].
11) Repeat steps 1 through 10 until a "best" α is selected based on the lowest cumulative forecasting error (Register 5).

Forecasting:

1) Key in the number 1; press [ENTER↑].
2) Key in the selected α; press [STO] 0 [−] [STO] 1.
3) From the α selection routine or from a previous forecast:
   - Key in the smoothed average S_{t-1}; press [STO] 2.
   - Key in the trend T_{t-1}; press [STO] 3.
   - Key in the forecast $\hat{D}_{t+1}$; press [STO] 6.
4) Key in the current data value; press [R/S]. The output is the error in forecasting the value just entered.
5) Press [R/S]. The displayed value represents the forecast for the next period.

6) Record the following values: RCL 0 (α), RCL 2 (S_{t-1}), RCL 3 (T_{t-1}) and RCL 6 ($\hat{D}_{t+1}$) for use as initial values in the next forecast. You may also wish to record RCL 4 (D_t).

7) Repeat steps 4, 5 and 6 for the next forecast if available.

Example: Select the best smoothing constant based on sales (in thousands of dollars) of 22, 23, 23, 25, 23, 27, 25. Given the current sales in month 8 of 26, forecast the following month. Select the smoothing constant (α):

Keystrokes:	**Display:**	
f ALL	*0.00*	
1 ENTER↑	*1.00*	
.5 STO 0 −	*0.50*	
STO 1	*0.50*	
22 STO 2	*22.00*	
23 STO 6 R/S	*0.00*	
R/S	*23.00*	
23 R/S R/S	*23.25*	
25 R/S R/S	*25.25*	
23 R/S R/S	*23.69*	
27 R/S R/S	*27.13*	
25 R/S R/S	*25.95*	
RCL 5	*23.61*	Cumulative error (Σe^2)
RCL 0	*0.50*	Smoothing constant (α)
RCL 2	*25.11*	Smoothed average (S_{t-1})
RCL 3	*0.42*	Trend (T_{t-1})
RCL 6	*25.95*	Last forecast ($\hat{D}_{t+1}$)

The procedure is repeated for several α's.

Smoothing Constant (α)	.5	.1	.25	.2
Cumulative Error (Σe^2)	23.61	25.14	17.01	18.03

For the selected $\alpha = .25$ $S_{t-1} = 24.28$
$T_{t-1} = 0.34$
$\hat{D}_{t+1} = 25.64$

Forecasting:

Keystrokes:	Display:	
f ALL	0.00	
1 ENTER↑	1.00	
.25 STO 0 −	0.75	
STO 1	0.75	
24.28 STO 2	24.28	
.34 STO 3	0.34	
25.64 STO 6	25.64	
26 R/S	0.36	
R/S	26.16	Forecast for month 9, ($\hat{D}_{t+1}$)
RCL 4	25.80	Expected usage for current (month 8) period, (Smoothed D_t)
RCL 0	0.25	
RCL 2	24.71	Record for initial values
RCL 3	0.36	when month 9 actual figures
RCL 6	26.16	become available

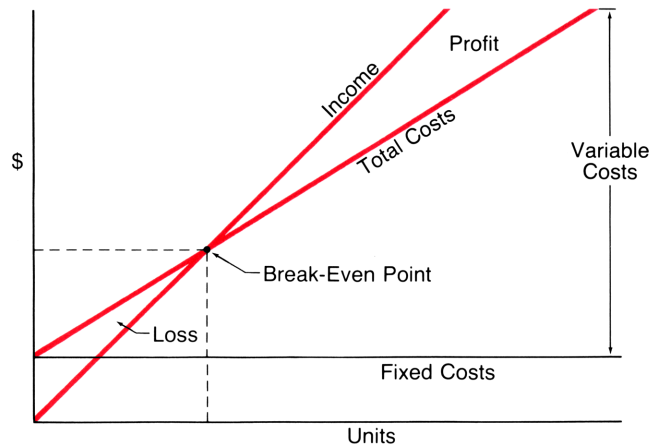
Note: At least 4 periods of current data should have been entered before forecasting is attempted.

This program is based in part on a HP Users' Library program by Robert M. Olsen.

Financial Analysis

Break-Even Analysis

Break-even analysis is basically a technique for analyzing the relationships among fixed costs, variable costs, and income. Until the break-even point is reached at the intersection of the total income and total cost lines, the producer operates at a loss. After the break-even point each unit produced and sold makes a profit. Break-even analysis may be represented as follows:



The variables are: fixed costs (F), sales price per unit (P), variable costs per unit (V), number of units sold (U), and gross profit (GP). One can readily evaluate GP, U or P given the other four variables. To calculate the break-even volume simply let the gross profit equal zero and calculate the number of units sold (U).

The following keystroke procedure may be used:

To calculate the break-even volume:

1) Key in the fixed costs; press [ENTER↑] .
2) Key in the unit price; press [ENTER↑] .
3) Key in the variable costs per unit; press [−] .
4) Press [÷] to calculate the break-even volume.

To calculate the gross profit at a given volume:

1) Key in the unit price; press [ENTER↑].
2) Key in the variable costs per unit; press [−].
3) Key in the number of units sold; press [×].
4) Key in the fixed costs and press [−] to calculate the gross profit.

To calculate the sales volume needed to achieve a specified gross profit:

1) Key in the desired gross profit; press [ENTER↑].
2) Key in the fixed costs; press [+].
3) Key in the sales price per unit; press [ENTER↑].
4) Key in the variable costs per unit; press [−].
5) Press [÷] to calculate the gross profit.

To calculate the required sales price to achieve a given gross profit at a specified sales volume:

1) Key in the fixed costs; press [ENTER↑].
2) Key in the gross profit desired; press [+].
3) Key in the specified sales volume in units; press [÷].
4) Key in the variable costs per unit and press [+] to calculate the required sales price per unit.

Example 1: The E.Z. Sells company markets textbooks on salesmanship. The fixed costs involved in setting up to print the books were $12,000. The variable costs per copy, including printing and marketing the books, are $6.75 per copy. The sales price per copy is $13.00. How many copies must be sold to break even?

Keystrokes:	**Display:**	
12000 [ENTER↑]	*12,000.00*	
13 [ENTER↑]	*13.00*	
6.75 [−] [÷]	*1,920.00*	Break-even volume

Find the gross profit if 2500 units are sold.

13 ENTER↑	*13.00*	
6.75 −	*6.25*	
2500 ×	*15,625.00*	
12000 −	*3,625.00*	Gross profit

If a gross profit of $4,500 is desired at a sales volume of 2500 units, what should the sales price be?

12000 ENTER↑	*12,000.00*	
4500 +	*16,500.00*	
2500 ÷	*6.60*	
6.75 +	*13.35*	Sales price per unit to achieve desired gross profit

For repeated calculations the following HP-38E/38C program can be used:

KEY ENTRY	DISPLAY
g P/R g CL P	*00–*
RCL 3	*01– 22 3*
RCL 2	*02– 22 2*
−	*03– 41*
g GTO 00	*04– 25 7 00*
RCL 4	*05– 22 4*
×	*06– 61*
RCL 1	*07– 22 1*
−	*08– 41*
g GTO 00	*09– 25 7 00*
RCL 5	*10– 22 5*
RCL 1	*11– 22 1*
+	*12– 51*

KEY ENTRY	DISPLAY
x≷y	*13– 33*
÷	*14– 71*
g GTO 00	*15– 25 7 00*
RCL 1	*16– 22 1*
RCL 5	*17– 22 5*
+	*18– 51*
RCL 4	*19– 22 4*
÷	*20– 71*
RCL 2	*21– 22 2*
+	*22– 51*
g GTO 00	*23– 25 7 00*
g P/R	

REGISTERS			
R_0	R_1 F	R_2 V	R_3 P
R_4 U	R_5 GP	R_6	R_7

1) Key in the program and store the known variables as follows:
   a) Key in fixed costs, F; press [STO] 1.
   b) Key in variable costs per unit, V; press [STO] 2.
   c) Key in the unit price, P (if known); press [STO] 3.
   d) Key in the sales volume, U, in units (if known); press [STO] 4.
   e) Key in the gross profit, GP, (if known); press [STO] 5.

2) To calculate the sales volume to achieve a desired gross profit:
   a) Store values as shown in 1a), 1b) and 1c).
   b) Key in the desired gross profit (zero for break-even); press [STO] 5.
   c) Press [R/S] [g] [GTO] 10 [R/S] to calculate the break-even volume.

3) To calculate the gross profit at a given sales volume:
   a) Store values as shown in 1a), 1b, 1c) and 1d).
   b) Press [R/S] [g] [GTO] 05 [R/S] to calculate the gross profit.

4) To calculate the sales price per unit to achieve a desired gross profit at a specified sales volume:
   a) Store values as shown in 1a), 1b), 1d) and 1e).
   b) Press [g] [GTO] 16 [R/S] to calculate the required sales price.

Example 2: A manufacturer of automotive accessories produces rear-view mirrors. A new line of mirrors will require fixed costs of $35,000 to produce. Each mirror has a variable cost of $8.25. The price of the mirrors is tentatively set at $12.50 each. What volume is needed to break even?

Keystrokes:	Display:	
35000 STO 1	*35,000.00*	
8.25 STO 2	*8.25*	
12.5 STO 3	*12.50*	
0 STO 5	*0.00*	
R/S g GTO 10 R/S	*8,235.29*	Break-even volume is between 8,235 and 8,236 units

What would be the gross profit if the price is raised to $14.00 and the sales volume is 10,000 units?

Keystrokes:	Display:	
14 STO 3	*14.00*	NOTE: F and V are already stored
10000 STO 4	*10,000.00*	
R/S g GTO 05 R/S	*22,500.00*	Gross profit

Operating Leverage

The degree of operating leverage (OL) at a point is defined as the ratio of the percentage change in net operating income to the percentage change in units sold. The greatest degree of operating leverage is found near the break-even point where a small change in sales may produce a very large increase in profits. Likewise, firms with a small degree of operating leverage are operating farther from the break-even point, and they are relatively insensitive to changes in sales volume.

The necessary inputs to calculate the degree of operating leverage are fixed costs (F), sales price per unit (P), variable costs per unit (V), and number of units (U).

The operating leverage may be readily calculated as follows:

1) Key in the sales price per unit; press [ENTER↑].
2) Key in the variable costs per unit; press [−].
3) Key in the number of units, press [×] [ENTER↑] [ENTER↑].
4) Key in the fixed cost and press [−] [÷] to obtain the operating leverage.

Example 1: For the data given in Example 1 of the Break-even Analysis calculate the operating leverage at 2000 units and at 5000 units when the sales price is $13 a copy.

Keystrokes:	**Display:**	
13 [ENTER↑]	*13.00*	
6.75 [−]	*6.25*	
2000 [×]	*12,500.00*	
[ENTER↑] [ENTER↑]		
12000 [−] [÷]	*25.00*	Close to the break-even point
13 [ENTER↑]	*13.00*	
6.75 [−]	*6.75*	
5000 [×]	*31,250.00*	
[ENTER↑] [ENTER↑]		
12000 [−] [÷]	*1.62*	Operating further from the break-even point and less sensitive to changes in sales volume

A convenient HP-38E/38C program for repetitive calculations is:

KEY ENTRY	DISPLAY
g P/R g CL P	00–
RCL 3	01– 22 3
RCL 2	02– 22 2
−	03– 41
×	04– 61
ENTER↑	05– 31

KEY ENTRY	DISPLAY
ENTER↑	06– 31
RCL 1	07– 22 1
−	08– 41
÷	09– 71
g GTO 00	10– 25 7 00
g P/R	

REGISTERS			
R_0	R_1 F	R_2 V	R_3 P

1) Key in the program.
2) Key in and store input variables F, V, and P as described in step 1a) thru c) of the Break-even Analysis program.
3) Key in the sales volume and press R/S to calculate the operating leverage.
4) To calculate a new operating leverage at a different sales volume, key in the new sales volume and press R/S .

Example 2: For the figures given in Example 2 of the Break-even Analysis section, calculate the operating leverage at a sales volume of 9,000 and 20,000 units if the sales price is $12.50 per unit.

Keystrokes:	**Display:**	
35000 STO 1	***35,000.00***	
8.25 STO 2	***8.25***	
12.5 STO 3	***12.50***	
9000 R/S	***11.77***	Operating leverage near break-even.
20000 R/S	***1.70***	Operating leverage further from break-even.

Profit and Loss Analysis

The HP-38E/38C may be programmed to perform simplified profit and loss analysis using the standard profit income formula and can be used as a dynamic simulator to quickly explore ranges of variables affecting the profitability of a marketing operation.

The program operates with net income return and operating expense as percentages. Both percentage figures are based on **net** sales price.

It may also be used to simulate a companywide income statement by replacing list price with gross sales and manufacturing cost with cost of goods sold.

Any of the five variables;a) list price, b) discount (as a percentage of list price), c) manufacturing cost, d) operating expense (as a percent) e) net profit after tax (as a percent), may be calculated if the other four are known.

Since the tax rate varies from company to company, provision is made for inputting your applicable tax rate. (The example problem uses a tax rate of 48%).

KEY ENTRY	DISPLAY
g P/R g CL P	00–
RCL 5	01– 22 5
RCL 6	02– 22 6
÷	03– 71
RCL 4	04– 22 4
+	05– 51
CHS	06– 32
RCL 0	07– 22 0
+	08– 51
RCL 0	09– 22 0
÷	10– 71

KEY ENTRY	DISPLAY
g GTO 00	11– 25 7 00
RCL 3	12– 22 3
RCL 1	13– 22 1
RCL 2	14– 22 2
RCL 0	15– 22 0
÷	16– 71
CHS	17– 32
1	18– 1
+	19– 51
×	20– 61
R/S	21– 74

KEY ENTRY	DISPLAY
÷	22- 71
CHS	23- 32
1	24- 1
+	25- 51
RCL 0	26- 22 0
×	27- 61
g GTO 00	28- 25 7 00
÷	29- 71
CHS	30- 32
RCL 1	31- 22 1
+	32- 51
RCL 1	33- 22 1
÷	34- 71
RCL 0	35- 22 0

KEY ENTRY	DISPLAY
×	36- 61
g GTO 00	37- 25 7 00
RCL 5	38- 22 5
RCL 6	39- 22 6
÷	40- 71
−	41- 41
g GTO 00	42- 25 7 00
RCL 4	43- 22 4
−	44- 41
RCL 6	45- 22 6
×	46- 61
g GTO 00	47- 25 7 00
g P/R	

REGISTERS			
R_0 100	R_1 list price	R_2 % discount	R_3 mfg. cost
R_4 % op. exp.	R_5 % net profit	R_6 1−% tax	R_7

1) Key in the program; press f ALL, then key in 100; press STO 0.

2) Key in 1; press ENTER↑, then key in your appropriate tax rate, as a decimal; press − STO 6.

3) a) Key in the list price in dollars (if known); press STO 1.
   b) Key in the discount in percent (if known); press STO 2.
   c) Key in the manufacturing cost in dollars (if known); press STO 3.
   d) Key in the operating expense in percent (if known); press STO 4.
   e) Key in the net profit after tax in percent (if known); press STO 5.

4) To calculate **list price**:

   a) Do steps 2 and 3b), c), d), e), above.
   b) Press [RCL] 3 [R/S] [÷] 1 [g] [GTO] 14 [R/S] [÷] [g] [GTO] 00.

5) To calculate **discount**:

   a) Do steps 2 and 3a), c), d), e), above.
   b) Press [RCL] 3 [R/S] [g] [GTO] 29 [R/S].

6) To calculate **manufacturing cost**:

   a) Do steps 2 and 3a), b), d), e), above.
   b) Press [g] [GTO] 13 [R/S] [g] [GTO] 01 [R/S] [×].

7) To calculate **operating expense**:

   a) Do steps 2 and 3a), b), c), e), above.
   b) Press [g] [GTO] 12 [R/S] [R/S] [g] [GTO] 38 [R/S].

8) To calculate **net profit after tax**:

   a) Do steps 2 and 3a), b), c), d), above.
   b) Press [g] [GTO] 12 [R/S] [R/S] [g] [GTO] 43 [R/S].

Example: What is the net return on an item that is sold for $11.98, discounted through distribution at an average of 35% and has a manufacturing cost of $2.50? The standard company operating expense is 32% of net shipping (sales) price and the tax rate is 48%.

Keystrokes:	**Display:**	
[f] [ALL] 100 [STO] 0	*100.00*	
1 [ENTER↑] .48 [−] [STO] 6	*0.52*	48% tax rate
11.98 [STO] 1	*11.98*	List price ($)
35 [STO] 2	*35.00*	Discount (%)
2.50 [STO] 3	*2.50*	Manufacturing cost ($)
32 [STO] 4	*32.00*	Operating expense (%)
[g] [GTO] 12 [R/S] [R/S]	*67.90*	
[g] [GTO] 43 [R/S]	*18.67*	Net profit (%)

If manufacturing expense increases to $3.25 what is the effect on net profit?

3.25 [STO] 3	*3.25*	
[g] [GTO] 12 [R/S] [R/S]	*58.26*	
[g] [GTO] 43 [R/S]	*13.66*	Net profit reduced to 13.66%

If the manufacturing cost is maintained at $3.25 how high could the overhead (operating expense) go before the product begins to lose money?

0 STO 5	*0.00*	
g GTO 12 R/S R/S	*58.26*	
g GTO 38 R/S	*58.26*	Maximum operating expense (%)

At 32% operating expense and $3.25 manufacturing cost what should the list price be to generate 20% net profit?

20 STO 5	*20.00*	
RCL 3 R/S ÷	*11.00*	
1 g GTO 14 R/S ÷	*16.93*	List price ($)
g GTO 00	*16.93*	

What reduction in manufacturing cost would achieve the same result without necessitating an increase in list price above $11.98?

g GTO 13 R/S	*7.79*	
g GTO 01 R/S ×	*2.30*	Manufacturing cost ($)

Pricing Calculations

Markup and Margin Calculations

Sales work often involves calculating the various relations between markup, margin, selling price and costs. Markup is defined as the difference between selling price and cost, divided by the cost. Margin is defined as the difference between selling price and cost, divided by selling price. In other words, markup is based on cost and margin is based on selling price.

The HP-37E has a built in feature, the [PRICE] key, which allows ready calculations of the selling price or the cost (see page 10 of the HP-37E Owner's Handbook). Simple keystroke sequences are available to make this calculation with the HP-38E/38C. In addition, the other variables may be readily calculated by means of simple keystroke sequence on either the HP-37E or the HP-38E/38C:

CALCULATE	GIVEN	KEYSTROKES
Selling Price	Cost & Markup	Key in cost, [ENTER↑], key in markup (in %), [%] [+].
Selling Price	Cost & Margin *	Key in cost, [ENTER↑] 1 [ENTER↑], key in margin (in %), [%] [−] [÷].
Cost	Selling Price & Markup *	Key in selling price, [ENTER↑] 1 [ENTER↑], key in markup (in %), [%] [+] [÷].
Cost	Selling Price & Margin	Key in selling price, [ENTER↑] 1 [ENTER↑], key in margin (in %), [%] [−] [×].
Markup	Cost & Selling Price	Key in cost, [ENTER↑], key in selling price, [f] [Δ%].

CALCULATE	GIVEN	KEYSTROKES
Markup	Margin	Key in margin, [ENTER↑] [ENTER↑] 1 [x≷y] [%] [−] [÷] .
Margin	Selling Price & Cost	Key in selling price, [ENTER↑] , key in cost, [f] [Δ%] [CHS] .
Margin	Markup	Key in markup, [ENTER↑] [ENTER↑] 1 [x≷y] [%] [+] [÷] .

* For the HP-37E see use of the [PRICE] key for this calculation.

Example 1: If the cost of an item if $160 and the margin is 20%, what is the selling price? What is the markup?

Keystrokes:	Display:	
160 [ENTER↑]	*160.00*	
1 [ENTER↑] 20	*20.00*	
[%] [−] [÷]	*200.00*	Selling price
20 [ENTER↑] [ENTER↑]	*20.00*	
1 [x≷y] [%] [−] [÷]	*25.00*	Markup (%)

Example 2: If an item sells for $21.00 and has a markup of 50%, what is its cost? What is the margin?

Keystrokes:	Display:	
21 [ENTER↑]	*21.00*	
1 [ENTER↑] 50	*50.*	
[%] [+] [÷]	*14.00*	Cost
50 [ENTER↑] [ENTER↑]	*50.00*	
1 [x≷y] [%] [+] [÷]	*33.33*	Margin (%)

A short HP-38E/38C program may be helpful for repetitive calculations of selling price and costs as well as conversions between markup and margin:

KEY ENTRY	DISPLAY
[g] [P/R] [g] [CL P]	00–
[ENTER↑]	01– 31
[g] [GTO] 04	02– 25 7 04
[CHS]	03– 32
1	04– 1
[X≷Y]	05– 33
[%]	06– 23
[+]	07– 51

KEY ENTRY	DISPLAY
[÷]	08– 71
[R/S]	09– 74
[g] [LAST x]	10– 25 31
[×]	11– 61
[g] [LAST x]	12– 25 31
[×]	13– 61
[g] [P/R]	

1) Key in program.

2) To calculate selling price, given the markup; key in the cost, press [ENTER↑], key in the markup (in %), press [g] [GTO] 00 [R/S] [R/S].

3) To calculate cost, given the markup; key in the selling price, press [ENTER↑], key in the markup, press [g] [GTO] 00 [R/S].

4) To calculate selling price, given the margin; key in the cost, press [ENTER↑], key in the margin, press [g] [GTO] 03 [R/S].

5) To calculate cost, given the margin; key in the selling price, press [ENTER↑], key in the margin, press [g] [GTO] 03 [R/S] [R/S].

6) To calculate markup from the margin; key in the margin, press [ENTER↑] [g] [GTO] 03 [R/S].

7) To calculate margin from the markup; key in the markup, press [ENTER↑] [g] [GTO] 00 [R/S].

Example: Find the cost of an item selling for $38.00 with a margin of 30%. What is the markup on the item? If the markup is raised to 50%, what will the selling price be?

Keystrokes:	**Display:**	
38 [ENTER↑]	***38.00***	
30 [g] [GTO] 03		
[R/S] [R/S]	***26.60***	Cost
30 [ENTER↑] [g] [GTO] 03		
[R/S]	***42.86***	Markup (%)
26.6 [ENTER↑]	***26.60***	
50 [g] [GTO] 00		
[R/S] [R/S]	***39.90***	New selling price

Calculations of List and Net Prices With Discounts

It is often useful to be able to quickly calculate list or net price when the other price and a series of discount rates are known. Alternatively, if the list and net and several discounts are known it may be desirable to calculate a missing discount. The following series of keystrokes may be used:

1) Key in 1, press [ENTER↑] [ENTER↑] [STO] 1.
2) Key in the first discount (in %), press [%] [−] [STO] [×] 1 [R↓].
3) Repeat step 2) for each of the remaining known discount rates.
4) To calculate the list price; key in the net price and press [RCL] 1 [÷].
5) To calculate the net price; key in the list price and press [RCL] 1 [×].
6) To calculate an unknown discount rate; immediately after doing step 3 (display should show 1.00), key in the net price, press [ENTER↑] and key in the list price.
7) Press [RCL] 1 [×] [÷] [−] 100 [×].

Example: The list price of an item is $3.28 and the net price is $1.45. Two of the discount rates are 48% and 5%. What is the third discount rate?

Keystrokes:	**Display:**	
1 [ENTER↑] [ENTER↑] [STO] 1	*1.00*	
48 [%] [−] [STO] [×] 1		
[R↓]	*1.00*	
5 [%] [−] [STO] [×] 1		
[R↓]	*1.00*	
1.45 [ENTER↑]		
3.28 [RCL] 1	*0.49*	
[×] [÷] [−] 100 [×]	*10.51*	3rd discount rate (%)

The following program for the HP-38E/38C will be helpful in performing the calculations:

KEY ENTRY	DISPLAY
[g] [P/R] [g] [CL P]	00–
1	01– 1
[x≷y]	02– 33
[%]	03– 23
[−]	04– 41
[STO] [×] 1	05– 21 61 1
[g] [GTO] 00	06– 25 7 00
[RCL] 1	07– 22 1
[×]	08– 61

KEY ENTRY	DISPLAY
[÷]	09– 71
1	10– 1
[x≷y]	11– 33
[−]	12– 41
[g] [EEX]	13– 25 32
2	14– 2
[×]	15– 61
[g] [GTO] 00	16– 25 7 00
[g] [P/R]	

REGISTERS			
R_0	R_1 $D_1'xD_2'x \ldots D_n'$	R_2	R_3

1) Key in the program.
2) Key in 1, press [STO] 1.
3) Key in the first discount rate (in %), press [R/S].
4) Repeat step 2) for each of the remaining discount rates.
5) To calculate the list price; key in the net price and press [RCL] 1 [÷].
6) To calculate the net price; key in the list price and press [RCL] 1 [×].
7) To calculate the unknown discount rate; key in the net price, press [ENTER↑] then key in the list price and press [g] [GTO] 07 [R/S].

Example: Calculate the unknown discount rate for the previous example. If the list price is now raised to $3.75 what is the new net price?

Keystrokes:	**Display:**	
1 [STO] 1	*1.00*	
48 [R/S]	*0.52*	
5 [R/S]	*0.95*	
1.45 [ENTER↑]	*1.45*	
3.28 [g] [GTO] 07 [R/S]	*10.51*	3rd discount rate (%)
[R/S]	*0.89*	Include 3rd discount rate in calculation
3.75 [RCL] 1 [×]	*1.66*	New net price

Miscellaneous

Learning Curve for Manufacturing Cost

Many production process costs vary with output according to the "learning curve" equation. The production team becomes more proficient in manufacturing a given item as more and more of them are fabricated and costs may be expected to decrease by a predictable amount. The learning factor, r, characterizes the learning curve. For instance, if r = .80 the curve is called an 80% learning curve.

It is readily apparent that the learning, or experience curve, has many uses in setting production standards, forecasting costs, setting prices, etc. Note, however, that the learning factor may change, especially after large numbers have been produced.

If the cost of the first unit of a run, C_1, and the learning curve factor, r, are known, the following procedure can be used to calculate the cost of the nth item:

1) Key in the cost of the first item, C_1; press [ENTER↑].
2) Key in the number of units produced, n; press [ENTER↑].
3) Key in the learning factor, r; press [LN] 2 [LN] [÷].
4) Then press [y^x] [×] to calculate the cost of the nth unit, C_n.

Example 1: An electronic manufacturer begins a pilot run on a new instrument. From past experience he expects the process to have a learning factor, r, of 0.90. If the first unit costs $875 to produce, what is the expected cost of the 100th unit?

Keystrokes:	**Display:**	
875 [ENTER↑]	*875.00*	
100 [ENTER↑]	*100.00*	
.9 [LN] 2 [LN] [÷]	*-0.15*	
[y^x] [×]	*434.51*	Cost of the 100th unit

If the costs of the first unit, C_1, and the nth unit, Cn, are known the learning factor may be calculated. In addition, it is possible to calcu-

late C_i, the average cost of the ith thru jth unit. These calculations may be rapidly done with the following HP-38E/38C program:

KEY ENTRY	DISPLAY
g P/R g CL P	00-
g LN	01- 25 23
2	02- 2
g LN	03- 25 23
÷	04- 71
STO 2	05- 21 2
g R↓	06- 25 33
x≷y	07- 33
STO 1	08- 21 1
÷	09- 71
g LN	10- 25 23
RCL 2	11- 22 2
÷	12- 71
g e^x	13- 25 22
STO 2	14- 21 2
g GTO 00	15- 25 7 00
RCL 2	16- 22 2
g LN	17- 25 23
2	18- 2
g LN	19- 25 23
÷	20- 71
g y^x	21- 25 21
RCL 1	22- 22 1
×	23- 61
g GTO 00	24- 25 7 00

KEY ENTRY	DISPLAY
STO 3	25- 21 3
x≷y	26- 33
STO 4	27- 21 4
RCL 2	28- 22 2
g LN	29- 25 23
2	30- 2
g LN	31- 25 23
÷	32- 71
1	33- 1
+	34- 51
STO 0	35- 21 0
g y^x	36- 25 21
RCL 3	37- 22 3
RCL 0	38- 22 0
g y^x	39- 25 21
−	40- 41
RCL 0	41- 22 0
÷	42- 71
RCL 4	43- 22 4
RCL 3	44- 22 3
−	45- 41
÷	46- 71
RCL 1	47- 22 1
×	48- 61
g GTO 00	49- 25 7 00
g P/R	

REGISTERS			
R_0 K+ 1	R_1 C_1	R_2 r	R_3 i
R_4 j	R_5	R_6	

1) Key in the program. (NOTE: if average costs are not going to be calculated, lines 25 thru 49 need not be keyed in.)

2) **To calculate r, the learning factor, if C_1 and Cn are known:**
   a) Key in C_1, the cost of the first unit; press [ENTER↑].
   b) Key in Cn, the cost of the n^{th} unit; press [ENTER↑].
   c) Key in n, the number of units and press [R/S] to calculate r, the learning factor.

3) **To calculate the cost of the n^{th} unit when C_1 and r are known:**
   a) Key in C_1; press [STO] 1. Key in r; press [STO] 2. (NOTE: This step may be skipped if step 2) has just been completed.)
   b) Key in the number of units, n; calculate Cn, the cost of the n^{th} unit by pressing [g] [GTO] 16 [R/S].

4) **To calculate the average cost per unit of the i^{th} through j^{th} unit, C_i, if C_1 and r are known:**
   a) Key in C_1; press [STO] 1. Key in r; press [STO] 2. (NOTE: This step may be skipped if step 2) has just been done.)
   b) Key in the number of the last unit of the batch, j; press [ENTER↑].
   c) Key in the number of the first unit of the batch, i, and calculate the average cost per unit by pressing [g] [GTO] 25 [R/S].

Example 2: The electronic manufacturer cited in example 1 found that the 100th instrument actually cost $395 to manufacture. Find the actual learning factor, r, the cost of the 500th unit and the average cost of units 500 thru 1000. (Recall that C_1 was $875.)

Keystrokes:	**Display:**	
875 [ENTER↑]	*875.00*	
395 [ENTER↑]	*395.00*	
100 [R/S]	*0.89*	Actual r
500 [g] [GTO] 16 [R/S]	*299.14*	Cost of the 500th unit
1000 [ENTER↑]	*1,000.00*	
500 [g] [GTO] 25 [R/S]	*280.00*	Average cost of the 500th thru 1000th unit

Queuing and Waiting Theory

Waiting lines, or queues, cause problems in many marketing situations. Customer goodwill, business efficiency, labor and space considerations are only some of the problems which may be minimized by proper application of queuing theory.

Although queuing theory can be a complex and complicated subject, handheld calculators can be used to arrive at helpful decisions.

One common situation that we can analyze involves the case of several identical stations serving customers, where the customers arrive randomly in unlimited numbers. Suppose there are n (1 or more) identical stations serving the customers. λ is the arrival rate (Poisson input) and μ is the service rate for random (exponential) service completions. We will assume that all customers are served on a first-come, first-served basis and wait in a single line (queue) then are directed to whichever station is available. We also will assume that no customers are lost from the queue. This situation, for instance, would be closely approximated by customers at some banking operations.

The formulas for calculating some of the necessary probabilities are too complex for simple keystroke solution. However, tables listing these probabilities are available and can be used to aid in quick solutions. Using the assumptions outlined above and a suitable table giving mean waiting time as a multiple of mean service time (see page 512 of the Reference) the following keystroke solutions may be obtained:

1) Key in the arrival rate of customers, λ; press [ENTER↑] .
2) Key in the service rate, μ; press [÷] to calculate ρ, the **intensity factor.** (Note ρ must be less than n for valid results, otherwise the queue will lengthen without limit).
3) Key in n, the number of servers; press [÷] to calculate ρ/n.
4) For a given n and ρ/n find the mean waiting time as a multiple of mean service time from the table. Key it in; press [ENTER↑] .
5) Calculate the **average waiting time** in the queue by keying in the service rate, υ, and pressing [STO] 1 [÷] [STO] 2.
6) Calculate the **average waiting time** in the system by pressing [RCL] 1 [1/x] [+] .

7) Key in λ, the original rate; press [RCL] 2 [×] to calculate the **average queue length**.

8) Key in ρ, the intensity factor (from step 2 above) and press [+] to calculate the **average number of customers in the system**.

REFERENCE:

Richard E. Trueman, "An Introduction to Quantitative Methods for Decision Making," Holt, Rinehart and Winston, New York, 1977.

Example 1: Bank customers arrive at a bank on an average of 1.2 customers per minute. They join a common queue for three tellers. Each teller completes a transaction at the rate of one customer every 2 minutes (0.5 customers per minute). What is the average waiting time in the queue? In the system? What is the average number of customers in the queue? In the system?

Keystrokes:	**Display:**	
1.2 [ENTER↑]	*1.20*	
.5 [÷]	*2.40*	ρ, intensity factor
3 [÷]	*0.80*	ρ/n

From table 12.2, page 512 of the reference the mean waiting time as a multiple of mean service time for n = 3, ρ/n = 0.8 is 1.079. (Note S is used instead of n in the book's notation)

1.079 [ENTER↑]	*1.08*	
.5 [STO] 1 [÷] [STO] 2	*2.16*	Average wait in queue (min.)
[RCL] 1 [1/x] [+]	*4.16*	Average wait in system (min.)
1.2 [RCL] 2 [×]	*2.59*	Average queue length
2.4 [+]	*4.99*	Average # customers in system

If the number of servers is limited to one, with other conditions remaining the same (unlimited queue, Poisson arrival, exponential service), the average queue length can be readily calculated without reference to tables:

1) Key in the arrival rate, λ; press [STO] 1.

2) Key in the service rate, μ; press [STO] 2 [÷] [ENTER↑] [ENTER↑] 2 [y^x] [x≷y] 1 [x≷y] [−] [÷] to calculate the average number of customers waiting in queue at any one time.

3) Press [RCL] 1 [÷] to calculate the average waiting time.

4) Press [RCL] 2 [1/x] [+] to calculate the average total time the customer spends in the system.

5) Press [RCL] 1 [×] to calculate the average number of customers in the system.

Example 2: A small grocery store has but a single check-out counter. Customers arrive at a rate of 1 every 2 minutes (λ = .5) and, on the average, customers can be checked out at a rate of .9 per minute (μ). What is the average number of customers in the waiting line at any time? The average waiting time? What is the average total time for a customer to wait and be checked out? The average number of customers in the system?

Keystrokes:	**Display:**	
.5 [STO] 1	*0.50*	
.9 [STO] 2 [÷] [ENTER↑]		
[ENTER↑] 2 [y^x] [x≷y]	*.56*	
1 [x≷y] [−] [÷]	*.69*	Average # customers waiting in queue
[RCL] 1 [÷]	*1.39*	Average waiting time
[RCL] 2 [1/x] [+]	*2.50*	Average total time in the system
[RCL] 1 [×]	*1.25*	Average # customers in the system

With an HP-38E/38C program one can readily calculate the necessary probabilities for this type of problem (dispensing with the use of tables) and perform additional calculations as well.

KEY ENTRY	DISPLAY
g P/R g CL P	00-
1	01- 1
STO − 0	02- 21 41 0
RCL . 0	03- 22 73 0
RCL 0	04- 22 0
0	05- 0
g x≤y	06- 25 5
g GTO 09	07- 25 7 09
g GTO 16	08- 25 7 16
+	09- 51
g y^x	10- 25 21
g LAST x	11- 25 31
g n!	12- 25 3
÷	13- 71
f Σ+	14- 24 74
g GTO 01	15- 25 7 01
RCL . 0	16- 22 73 0
RCL 7	17- 22 7
g y^x	18- 25 21
1	19- 1
RCL . 0	20- 22 73 0
RCL 7	21- 22 7
÷	22- 71
−	23- 41
÷	24- 71
RCL 7	25- 22 7
g n!	26- 25 3
÷	27- 71

KEY ENTRY	DISPLAY
STO 6	28- 21 6
RCL 2	29- 22 2
+	30- 51
f 1/x	31- 24 71
STO 1	32- 21 1
RCL 6	33- 22 6
×	34- 61
STO 2	35- 21 2
RCL . 0	36- 22 73 0
×	37- 61
RCL 7	38- 22 7
RCL . 0	39- 22 73 0
−	40- 41
÷	41- 71
STO 3	42- 21 3
RCL . 0	43- 22 73 0
+	44- 51
STO 4	45- 21 4
RCL 8	46- 22 8
÷	47- 71
STO 5	48- 21 5
RCL 3	49- 22 3
RCL 8	50- 22 8
÷	51- 71
STO 6	52- 21 6
R/S	53- 74
RCL 8	54- 22 8
RCL 7	55- 22 7

KEY ENTRY	DISPLAY
RCL 9	56- 22 9
×	57- 61
−	58- 41
×	59- 61
g e^x	60- 25 22

KEY ENTRY	DISPLAY
RCL 2	61- 22 2
×	62- 61
g GTO 53	63- 25 7 53
g P/R	

REGISTERS			
R_0 K	R_1 P_0	R_2 P_b	R_3 L_q
R_4 L	R_5 T	R_6 used, T_q	R_7 n
R_8 λ	R_9 μ	$R_{.0}$ ρ	$R_{.1}$

1) Key in the program; press f ALL .
2) Key in the number of servers, n; press STO 0 STO 7.
3) Key in the arrival rate of customers, λ; press STO 8.
4) Key in the service rate of each server, μ; press STO 9.
5) Press ÷ STO .0 to calculate and store ρ, the intensity factor.
6) Press R/S and see T_q, the average waiting time in the queue. Display P_0, probability that all servers, are idle, by pressing RCL 1. Display P_b, probability that all servers are busy, by pressing RCL 2. Display L_q, average number waiting in the queue, by pressing RCL 3. Display L, average number in the system (waiting and being served), by pressing RCL 4.

   Display T, average total time through the system, by pressing RCL 5. T_q, the average waiting time in the queue, may again be displayed by pressing RCL 6.
7) If desired, calculate P(t), the probability of waiting longer than a given time, by keying in the time and pressing R/S .
8) Repeat step 7 for other times of interest.

Example 3: Using the data from Example 1 of the keystroke solutions verify the data obtained. In addition, obtain P_o, the probability that none of the tellers are busy, and P_b, the probability that all the tellers are busy. What is the probability that a customer will have to wait 2 minutes or more?

Keystrokes:	**Display:**	
f ALL	*0.00*	
3 STO 0 STO 7	*3.00*	n
1.2 STO 8	*1.20*	λ
.5 STO 9	*0.50*	μ
÷ STO .0	*2.40*	ρ
R/S	*2.16*	T_q, average waiting time in queue
RCL 1	*0.06*	P_0, probability all servers are idle
RCL 2	*0.65*	P_b, probability all servers are busy
RCL 3	*2.59*	L_q, average # waiting in queue
RCL 4	*4.99*	L, average # waiting in system
RCL 5	*4.16*	T, average total time in the system
2 R/S	*0.36*	Probability of having to wait 2 minutes or more

Cash Flow Loader

The discounted cash flow analysis capabilities of the HP-38E/38C are extremely powerful tools. Since the calculator is able to accept a large number of cash flows, an HP-38E/38C program can be a valuable aid in storing, altering and reviewing the cash flow entries. Routines of this type illustrate the value of programmability in making your already powerful calculator even easier and more convenient to use.

This program allows you to conveniently enter an initial investment and up to 15 groups of cash flows. In addition, it aids the user in altering one or more of the cash flow groups. Reviewing each of the cash flow groups and the initial investment can be accomplished automatically by another portion of the program.

KEY ENTRY	DISPLAY
g P/R g CL P	00–
g CF_0	01– 25 13
R/S	02– 74
x≷y	03– 33
g CF_j	04– 25 14
x≷y	05– 33
g N_j	06– 25 15
RCL n	07– 22 11
g GTO 02	08– 25 7 02
RCL n	09– 22 11
x≷y	10– 33
n	11– 11
g R↓	12– 25 33
x≷y	13– 33
g N_j	14– 25 15
g GTO 34	15– 25 7 34

KEY ENTRY	DISPLAY
RCL n	16– 22 11
ENTER↑	17– 31
R/S	18– 74
g R↓	19– 25 33
RCL g N_j	20– 22,25 15
RCL g CF_j	21– 22,25 14
g PSE	22– 25 4
g R↓	23– 25 33
g PSE	24– 25 4
g R↓	25– 25 33
RCL n	26– 22 11
g x=0	27– 25 6
g GTO 30	28– 25 7 30
g GTO 18	29– 25 7 18
g PSE	30– 25 4
g R↓	31– 25 33

KEY ENTRY	DISPLAY
RCL 0	32- 22 0
g PSE	33- 25 4
g R↓	34- 25 33

KEY ENTRY	DISPLAY
n	35- 11
g P/R	

REGISTERS			
R_0 CF_0	R_1 CF_1	R_2 CF_2	R_3 CF_3
R_4 CF_4	R_5 CF_5	R_6 CF_6	R_7 CF_7
R_8 CF_8	R_9 CF_9	$R_{.0}$ CF_{10}	$R_{.1}$ CF_{11}
$R_{.2}$ CF_{12}	$R_{.3}$ CF_{13}	$R_{.4}$ CF_{14}	$R_{.5}$ CF_{15}

1) Key in the program and press f ALL .

2) Key in the amount of the initial investment (CF_0). If there is no initial investment key in zero for the amount. Press R/S .

3) Key in the amount of the first cash flow; press ENTER↑ .

4) Key in the number of times that cash flow occurs (if the cash flow occurs only once, be sure to key in 1); press R/S .

5) Repeat steps 3 and 4 for each cash flow. After each entry the number of cash flows entered will be displayed.

   (NOTE: Use the cash flow sign convention; positive values for cash received and negative values for cash paid out.)

6) After all cash flows have been entered proceed with your f IRR or f NPV calculations, or go to step 7) to alter cash flow entries or step 9) to review the cash flow entries.

7) **To change the cash flow entries;** key in the new cash flow value and press STO j, where j is 0 thru 9 or .0 thru .5 (i.e., the number of the register containing the cash flow to be altered).

8) Key in the number of times the new cash flow appears and press ENTER↑. Then key in the number of the cash flow being changed (0 thru 15) and press g GTO 09 R/S. The final display shows the total number of cash flow entries. Now proceed with your calculation.

9) **To review the cash flow entries;** press [g] [GTO] 16 [R/S]. The calculator will display the number of the last cash flow entry. Press [R/S], the calculator pauses to display the amount of the cash flow, then the number of times the cash flow appears, then the number of the next preceding cash flow. Pressing [R/S] continues display of this data for each preceding cash flow, then displays 0.00, followed by the value of the initial investment. Operation then halts with the total number of cash flows displayed. Proceed with your calculations as desired.

Example: Let's do the problem on page 55 thru 57 of the HP-38 Owner's Handbook:

An income property is available for $50,000. The annual income over a 23-year projection period (all payments received at the end of the year) may be grouped as follows:

CASH FLOW (CF_j)	NUMBER OF YEARS (N_j)
$9,000	First 5 years
$7,500	Next 4 years
$6,000	Next 4 years
$7,500	Next 3 years
$5,000	Last 7 years

What is the annual rate of return?

Keystrokes:	**Display:**	
[f] [ALL]	*0.00*	
50000 [CHS] [R/S]	*-50,000.00*	Initial investment
9000 [ENTER↑] 5 [R/S]	*1.00*	First cash flow
7500 [ENTER↑] 4 [R/S]	*2.00*	
6000 [ENTER↑] 4 [R/S]	*3.00*	
7500 [ENTER↑] 3 [R/S]	*4.00*	
5000 [ENTER↑] 7 [R/S]	*5.00*	Five cash flows entered
[f] [IRR]	*15.27*	Annual IRR of 15.27%
[f] [NPV]	*0.00001*	[NPV] is close to zero, IRR is correct.

With the data intact from our last example, change the third cash flow entry (6,000) to 7,000 and change the corresponding N_j value (4) to 5. Then find the new IRR.

7000 STO 3	*7,000.00*	
5 ENTER↑ 3	*3.*	Number of cash flows being
g GTO 09 R/S	*5.00*	changed.

Now review the cash flow entries to verify the change has been properly made.

g GTO 16 R/S	*5.00*	
R/S	*5000.00*	
	7.00	
	4.00	
R/S	*7,500.00*	
	3.00	
	3.00	
R/S	*7,000.00*	Altered cash flow entry
	5.00	Number of times it appears
	etc.	etc.
	0.00	
	–50,000.00	Initial investment
	5.00	Total number of cash flows

Find the new IRR

f IRR	*15.69*	New annual IRR is 15.69%

Percentage Tabulator

One of the most common arithmetic tasks is calculating the percentage of total represented by each number in a column of numbers. Done manually, each number must be keyed into the calculator twice; once to find the total and a second time to find each percent.

This program saves computation time and decreases the chances of error by requiring each number to be keyed in only once.

KEY ENTRY	DISPLAY
g P/R g CL P	00–
g CFi	01– 25 14
STO + 0	02– 21 51 0
RCL n	03– 22 11
g GTO 00	04– 25 7 00
RCL 0	05– 22 0

KEY ENTRY	DISPLAY
RCL g CFi	06– 22, 25 14
f %T	07– 24 22
R/S	08– 74
CLx	09– 34
g GTO 06	10– 25 7 06
g P/R	

REGISTERS			
R_0 total	R_1 x_1	R_2 x_2	R_3 x_3
R_4 x_4	R_5 x_5	R_6 x_6	R_7 x_7, etc.

1) Key in the program.
2) Clear extraneous data; press f ALL.
3) Key in all numbers in the list (up to 19) following each entry with R/S.
4) Optional: The running total may be viewed at any point in step 3) by pressing RCL 0.
5) After all the numbers have been keyed in, press g GTO 05.
6) Calculate and display each percent (in the opposite order from which they were keyed in) by successively pressing R/S.

Example: Calculate each salesman's unit sales as a percent of total month's sales.

Salesman	Sales (units)	%
A	5	10
B	10	20
C	15	30
D	20	40
TOTAL		

Keystrokes:	Display:	
[f] [ALL]	*0.00*	
5 [R/S]	*1.00*	Number of entries
10 [R/S]	*2.00*	Number of entries
15 [R/S]	*3.00*	Number of entries
20 [R/S]	*4.00*	Number of entries
[RCL] 0	*50.00*	Total
[g] [GTO] 05 [R/S]	*40.00*	4th entry % of total
[R/S]	*30.00*	3rd entry % of total
[R/S]	*20.00*	2nd entry % of total
[R/S]	*10.00*	1st entry % of total
[R/S]	*100.00*	All results have been reviewed

Appendix

Formulas

Simple Moving Average

$\bar{x}$ = Moving Average
m = Number of elements in the moving average

$$\bar{x}_1 = \frac{x_1 + x_2 + x_3 + \ldots . x_m}{m}$$

$$\bar{x}_2 = \frac{x_2 + x_3 + x_4 + \ldots . x_{m+1}}{m}$$

Seasonal Variation Factors based on a Centered Moving Average

$\bar{x}_c$ = Centered moving average
m = number of elements in the centered moving average

$$\bar{x}_c = \frac{\frac{x_1}{2} + (x_2 + x_3 + \ldots . x_m) + \frac{x_{m+1}}{2}}{m}$$

SV = Seasonal variation factor
x_i = Value of the ith data point
$\bar{x}_i$ = Centered moving average of the ith data point

$$SV = \frac{x_i}{\bar{x}_i}$$

Exponential Curve Fit

$$y = Ae^{Bx}$$

$$B = \frac{\Sigma x_i \ln y_i - \frac{1}{n}(\Sigma x_i)(\Sigma \ln y_i)}{\Sigma x_i^2 - \frac{1}{n}(\Sigma x_i)^2}$$

$$A = \exp\left[\frac{\Sigma \ln y_i}{n} - B\frac{\Sigma x_i}{n}\right]$$

$$\hat{y} = Ae^{Bx}$$

Gompertz Curve Trend Analysis

$y = ca^{(b^x)}$ where x, y, a, b and c are positive

$$b = \left(\frac{S_3 - S_2}{S_2 - S_1}\right)^{1/n}$$

$$c = \exp\left[\frac{1}{n}\left(\frac{S_1 S_3 - S_2^2}{S_1 + S_3 - 2S_2}\right)\right]$$

$$a = \exp\left[\frac{(b-1)(S_2 - S_1)}{b(b^n - 1)^2}\right]$$

where S_1, S_2 and S_3 are

$$S_1 = \sum_{i=1}^{n} \ln y_i = n \ln c + b(\ln a)\frac{b^n - 1}{b - 1}$$

$$S_2 = \sum_{i=n+1}^{2n} \ln y_i = n \ln c + b^{n+1} (\ln a) \frac{b^n - 1}{b - 1}$$

$$S_3 = \sum_{i=2n+1}^{3n} \ln y_i = n \ln c + b^{2n+1} (\ln a) \frac{b^n - 1}{b - 1}$$

a, b and c are determined by solving the three equations above simultaneously

Forecasting with Exponential Smoothing

α = Smoothing constant ($0 < \alpha < 1$)

X_t = Actual current period usage

Smoothed average, $S_t = \alpha X_t + (1 - \alpha) S_{t-1}$

Change, $C_t = S_t - S_{t-1}$

Trend, $T_t = \alpha C_t + (1 - \alpha) T_{t-1}$

Current period expected usage, $D_t = S_t + \frac{(1 - \alpha)}{\alpha} T_t$

Forecast of next period expected usage, $\hat{D}_{t+1} = S_t + \left(\frac{1}{\alpha}\right) T_t$

Error, $e_t = \hat{D}_t - X_t$

Cumulative error $= \sum_{t=1}^{m} e_t^2$

Initial conditions: $S_{t-1} = X_{t-1}$

$T_{t-1} = 0$

Breakeven Analysis and Operating Leverage

GP = Gross profit

P = Price per unit

V = Variable costs per unit

F = Fixed costs

U = Number of units

OL = Operating leverage

$$GP = U(P - V) - F$$

$$OL = \frac{U(P - V)}{U(P - V) - F}$$

Profit and Loss Analysis

Net income = (1 − tax) (net sales price − mfg. − operating expense)

Net sales price = list price (1 − discount rate)

Where operating expense represents a percentage of net sales price. ot net sales price.

Markup and Margin Calculations

Ma = Margin (%)

Mu = Markup (%)

S = Selling Price

C = Cost

$$Ma = 100\ \frac{S - C}{S} \qquad Mu = 100\ \frac{S - C}{C}$$

$$S = \frac{C}{1 - \frac{Ma}{100}} \qquad S = C\left(1 + \frac{Mu}{100}\right)$$

$$C = S\left(1 - \frac{Ma}{100}\right) \qquad C = \frac{S}{1 + \frac{Mu}{100}}$$

$$Ma = \frac{Mu}{1 + \frac{Mu}{100}} \qquad Mu = \frac{Ma}{1 - \frac{Ma}{100}}$$

Calculation of List and Net Prices with Discounts

L = List Price
N = Net Price
D = Discount (%)

$$D' = 1 - \frac{D}{100}$$

$$L = \frac{N}{D'_1 \times D'_2 \times \ldots D'_x}$$

$$D_x = 100\left(1 - \frac{N}{L(D'_1 \times D'_2 \times \ldots D'_{x-1})}\right)$$

Learning Curve for Manufacturing Cost

C_n = Cost of n^{th} unit
C_1 = Cost of first unit
n = Number of units
r = Learning factor
K = ln r/ln 2
$Cn = C_1 n^k$

If $\bar{C}ij$ = the average cost of the i^{th} through j^{th} unit:

$$\bar{C}_{ij} = \frac{C_1}{j - i}\left[\frac{j^{k+1} - i^{k+1}}{k + 1}\right]$$

This formula is only approximate, may give appreciable error at small i.

Queuing and Waiting Line Theory

where

n = Number of servers

λ = Arrival rate of customers (Poisson input).

μ = Service rate of each server (exponential service).

ρ = Intensity factor = λ/μ ($\rho < n$ for valid results).

P = Probability that all servers are idle.

P_b = Probability that all servers are busy.

Lq = Average number of customers in queue.

L = Average number of customers in system (waiting and being served).

T = Average waiting time in queue.

T = Average flow time through the system.

P(t) = Probability of waiting longer than time t.

$$P_o = \left[\sum_{k=0}^{n-1} \frac{\rho^k}{k!} + \frac{\rho^n}{n!\left(1 - \frac{\rho}{n}\right)} \right]^{-1}$$

$$P_b = \frac{\rho^n P_o}{n!\left(1 - \frac{\rho}{n}\right)}$$

$$Lq = \frac{\rho P_o}{n - \rho}, \quad L = Lq + \rho, \quad T = L/\lambda, \quad Tq = Lq/\rho$$

$$P(t) = P_b e^{-(n\mu - \lambda) t}$$

Percentage Tabulator

$$\% = 100 \frac{x_i}{\sum_{i=1}^{n} x_i}$$

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Printed in U.S.A.