KEYS TO CREATIVE FINANCING

BY EDRIC CANE

THE FINANCIAL CALCULATOR AND REAL ESTATE

HP–38 EDITION

KEYS TO CREATIVE FINANCING

**The Financial calculator
and Real Estate**

H.P.38 Edition

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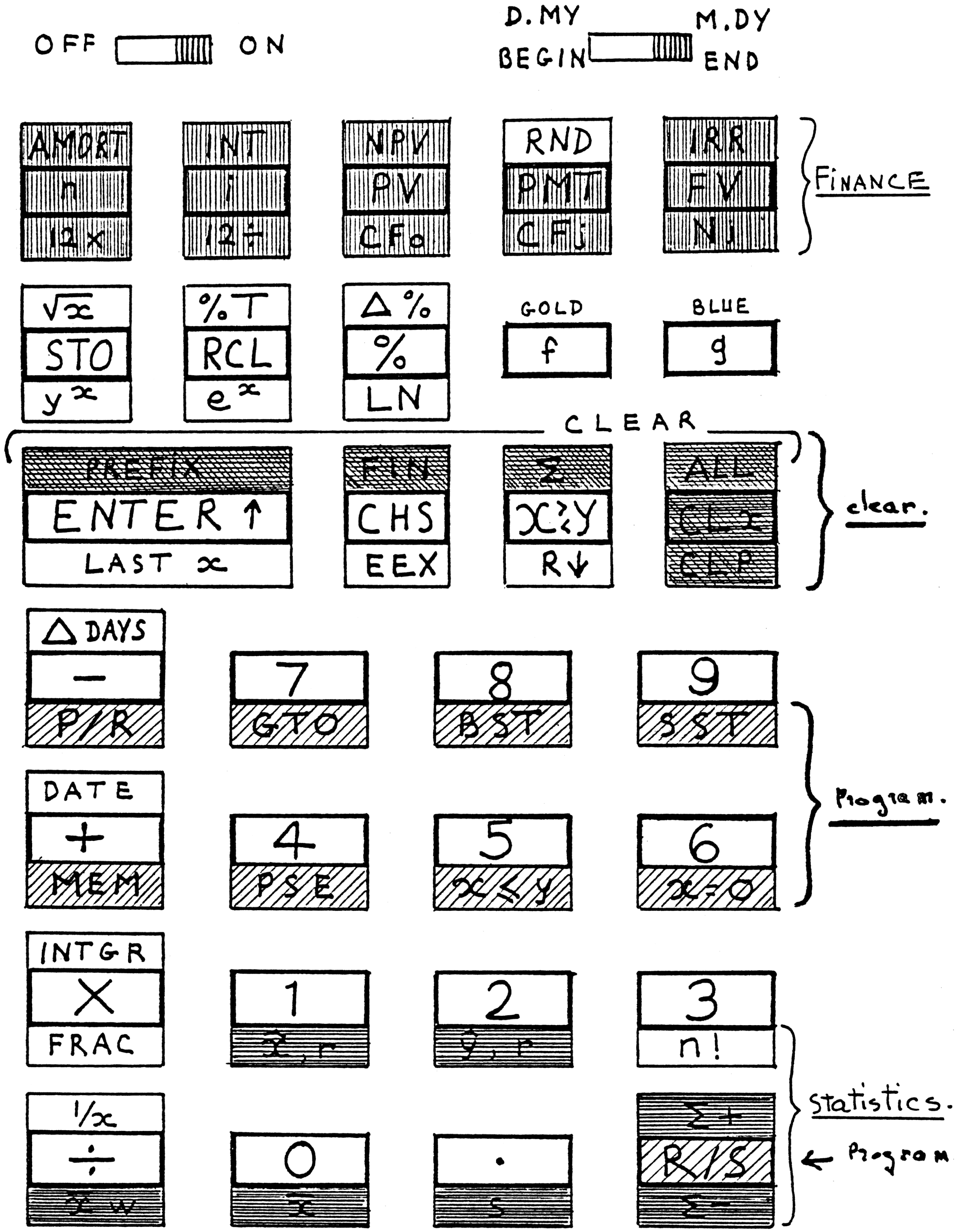


HEWLETT · PACKARD 38E or 38C

Preliminary.

The GOLD and BLUE keys.

They give a second and third meaning to all the other keys.
That second and third meaning is written in gold above the key, and in blue on the front face of the key.
For instance:

$\boxed{\text{CL x}}$ Clears x, which is the number shown on the display. Use CLx when the wrong number has been keyed in. No other data in the calculator is affected.

$\boxed{\text{GOLD}}\boxed{\text{CLEAR ALL}}$ Clears all the data in the calculator except the program memories. Use between problems for the time being until you learn when it is really needed.

$\boxed{\text{BLUE}}\boxed{\text{CL P}}$ Clears the program memories.

So the same key, without a prefix, and with the $\boxed{\text{GOLD}}$ and $\boxed{\text{BLUE}}$ keys as prefix, acquires three different meanings.

The two switches.

For the time being, operate with both switches to the right.

Trial and error.

You cannot harm the calculator by 'pushing the wrong button'.
So test your ideas; if you are not sure, try: the calculator will tell you whether you are right or wrong.

The Golden Rule

Note that the HP 38-E does not have an equal sign. This is because all operations follow a simple procedure, our Golden Rule:

$\boxed{\text{DATA}}$ then $\boxed{\text{QUESTION}}$

We first key in the data, then we ask the question, the answer appears directly in the display.

Illustrating the Golden Rule will provide us with an overview of some of the problems which the calculator can solve. This is meant only as a brief introduction. Most functions will be studied in detail later on.

Problem	DATA	+ QUESTION	Answer. (Display)
One number functions (The number showing in the display is 'x').			
1) $\sqrt{12}$	12	GOLD $\sqrt{x}$	(3.46)
2) $\frac{1}{12}$ (Reciprocal of 12)	12	GOLD 1/x	(0.08)
Two number functions			
3) 12^9	12 ENTER 9	BLUE y^x	(5159780352)
4) 9% of 12	12 ENTER 9	%	(1.08)
5) % difference between 12 and 9.	12 ENTER 9	GOLD Δ%	(-25)
6) 12 + 9	12 ENTER 9	+	(21)
7) 12 - 9	12 ENTER 9	-	(3)
8) 12 x 9	12 ENTER 9	x	(108)
9) How many days between 7/4/1776 and 7/4/1976	7.041776 ENTER 7.041976	GOLD ΔDAYS	(73,048)

Problem	DATA	QUESTION

Functions requiring more than two data input. (See later for details)

Problem	DATA	QUESTION
10) Monthly payments on a 30 year, $72,000 loan at 9% interest.	30 [BLUE] [n] 72000 [PV] 9 [BLUE] [i] (Keep data)	[PMT]
11) Balance of same loan after 10 years (120th payment): just key in new data:	10 [BLUE] [n]	[FV]
12) An investment of $35,000 yielded the following return: 1st year: - 1000 2nd year: 8000 3rd year: 12000 4th year (sold): 55,000 What is the Internal rate of return?	35000 [CHS] [BLUE] [CFo] 1000 [CHS] [BLUE] [CFj] 8000 [BLUE] [CFj] 12000 [BLUE] [CFj] 55000 [BLUE] [CFj]	[GOLD] [IRR]

In each of these instances
- We keyed in the DATA
- We asked the QUESTION
- The calculator provided the ANSWER directly.

Basic Arithmetic.

The systematic application of the Golden Rule (DATA then QUESTION) is perhaps most unusual for the very basic arithmetic calculations. For the 4 basic operations, instead of keying in the data as it is written out (12 + 9 = , or 12 x 9 =), we key in the data as the operations are done:

12	12	12
+ 9	x 9	- 9

Just as we ourselves could not add 12 before we are given the number to which we should add it, so the calculator cannot be told to add until we have provided it with two numbers. When the calculator has all the data, and when we tell it to add, then it does so immediately, and there is no need for us to press an = key.

Every time we key in two numbers in succession, we must tell the calculator where one number ends and the other begins. We do so by pressing the ENTER key. We do not press the ENTER key when the first of the two numbers we want to add (or mutliply, etc) is already the result of an operation performed by the calculator. We do so only when we key in the two numbers ourselves in direct succession. So for simple chain calculations only the very first number needs to be entered.

Operation	Keystroke		
29 + 35	29 $\boxed{\text{ENTER}}$ 35 $\boxed{+}$		(64)
29 + 35 + 83 + 172	29 $\boxed{\text{ENTER}}$ 35 $\boxed{+}$ 83 $\boxed{+}$ 172 $\boxed{+}$		(319)
34.5 x 12	34.5 $\boxed{\text{ENTER}}$ 12 $\boxed{x}$	(12 times)	(414)
$\frac{34.5 \times 12 \times 31}{18}$	34.5 $\boxed{\text{ENTER}}$ 12 $\boxed{x}$ 31 $\boxed{x}$ 18 $\boxed{\div}$		(713)

We do however press ENTER more than once if the problem requires us to key in two numbers in succession more than once:

(29 + 35)(72.5 - 16.7)	29 $\boxed{\text{ENTER}}$ 35 $\boxed{+}$	Calculates the number contained within the first parenthesis.
	72.5 $\boxed{\text{ENTER}}$ 16.7 $\boxed{-}$	Calculates number within the second parenthesis.
	$\boxed{x}$	Multiplies the two previous numbers to get 3,571.20

Note that the two partial answers are automatically remembered by the calculator. More on this feature later.

An interest rate of 11 5/8 (eleven and five eighth) can be keyed in by first calculating 5/8 (5 divided by 8) and then adding 11, as follows:

5	ENTER	Calculates 5/8	
8	÷		
11	+	Adds 11	63)

(Keep this data in the calculator)

Format

The number that appears on the display is rounded to 2 decimals (dollars and cents), but the unrounded number, with all hidden decimals, is used by the calculator in its operations. To see the hidden decimals, press GOLD and the number of decimals required. For instance, with the previous 11 5/8 still in, showing 11.63 in the display, explore the hidden decimals as follows:

GOLD	3	(11.625)
GOLD	9	(11.625000000)
GOLD	0	(12)
GOLD	2	(11.63) Re-establishes normal format.

The memory system.

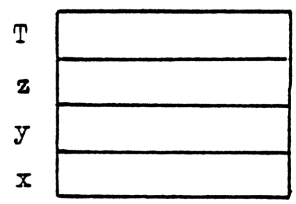
Data, of course, is stored in various memories. When we ask a question by pressing a key, we tell the calculator to perform specific operations on the data stored in specific memories. The only thing that matters when the question is asked is that the right data be in the right place. At any time any data input can be checked, modified, or cleared. Provided it ends up in the right place, the order in which it was keyed in is irrelevant. With a little bit of knowledge we have great control and flexibility. So knowing how to handle the memories is an essential part of this course.

There are three major groups of memories:

- The Stack
- The Register
- The financial memories.(See later)

The Stack   ENTER   x y

Four memories stacked on top of one another, with the bottom memory, x, showing in the display.



Display } Arithmetic calculations performed on content of x and y memories

With all 2 number operations performed on the content of the x and y memories of the stack, it is frequently useful to be able to check the data stored in the y memory. This can be done by pressing the $x \rightleftarrows y$ key.
Example: We want to divide 78 by **6** :

78 ENTER		Pushes 78 up into y memory.
6		
$x \rightleftarrows y$	(78)	Checks that 78 is indeed in y memory.
$x \rightleftarrows y$		Re-establishes correct order (78 in y, 6 in x)
÷	(13)	Performs the division.

Data floats up and down the **stack** which serves as an automatic memory for partial answers. The full power and flexibility of the stack will be studied later.

The Register   STO   RCL

The Register is a group of 20 memories in which we can store data for future use. They are accessed with STO (Store) and RCL (Recall) keys.

789 STO 1		Stores 789 in memory 1
456 STO 0		Stores 456 in memory 0
1981 STO 9		Stores 1981 in memory 9
RCL 1	(789)	Duplicates content of memory 1 in the display. This does <u>not</u> erase 789 from memory 1.
1776 STO 0		Replaces data in memory 0 with new data. 456 is now lost.
RCL 9	(1981)	
RCL 0	(1776)	
RCL 1	(789)	

Practice: basic arithmetic.

The answers are provided in parentheses.

1)

56.92	9,324	38.75	158.98	357
+ 37.27	+ 1,207.56	+ 61.25	35.29	987
(94.19)	(10,531.56)	(100.00)	69.50	159
			18.73	238
			+ 139.12	+ 74
			(421.62)	(1,815)

2)

39	15.36	108,750	2.05	789 / 456
x 74	- 11.45	- 6,525	x 12	(1.73)
(2,886)	(3.91)	(102,225)	(24.60)	

3) Calculate the running balance on this checking account

Withdrawal	Deposit	Balance	Keystroke		
		378.95	378.95	ENTER	
	1,582.75		1582.75	+	(1961.70)
	345.25		345.25	+	(2306.95)
175.50			175.50	-	(2131.45)
1,030.			1030	-	(1101.45)
45.80			45.80	-	(1055.65)

4) Percentage calculations:

80% of 117.500 (94,000)

6% of 98,250 (5,895)

30% of 324 (97.20)

5) Calculate the 6% sales tax, and add to the selling price:

\$ 56.75	(3.41, 60.16)	56.75 ENTER 6 % +
\$ 79.45	(4.77, 84.22)	
\$ 125.80	(7.55, 133.35)	
\$ 17.30	(1.04, 18.34)	

6) Calculate, and explore the hidden decimals. Restore the 2 decimal format after each calculation.

$\frac{80}{81}$ (.99, .987654321) 9 7/8 (9.88, 9.875)

7) Calculate:

327 + 592 + 879 - 117 - 222 + 73 - 532 (1000)

17.50 + 13.89 + 22 - 89 + 105 - 19.39 (50)

8) Practice freely adding, subtracting, multiplying, dividing numbers in succession, calculating percentages, storing and recalling, and changing the display format.

PART II

The Top Row Financial Keys.

[n] [i] [PV] [PMT] [FV]

I Three essential forms of money

- MONEY NOW (or at the beginning of the time span considered)
- MONEY in the FUTURE (or at the end of the time span considered)
- REGULAR PAYMENTS of EQUAL SUMS from NOW to FUTURE.

Borrowing and lending or investing are most often an exchange of one of these forms of money for another:

When I take a mortgage I get MONEY NOW and I pay it back by making REGULAR EQUAL PAYMENTS from NOW to FUTURE date. From the lender's point of view it is the opposite: He gives money NOW and is paid back by regular PAYMENTS. The problem of calculating the payments that will pay off the loan is, of course, the same for both parties.

When I deposit money in a savings account, I give MONEY NOW, and I can be paid back by withdrawing a lump sum in the FUTURE.

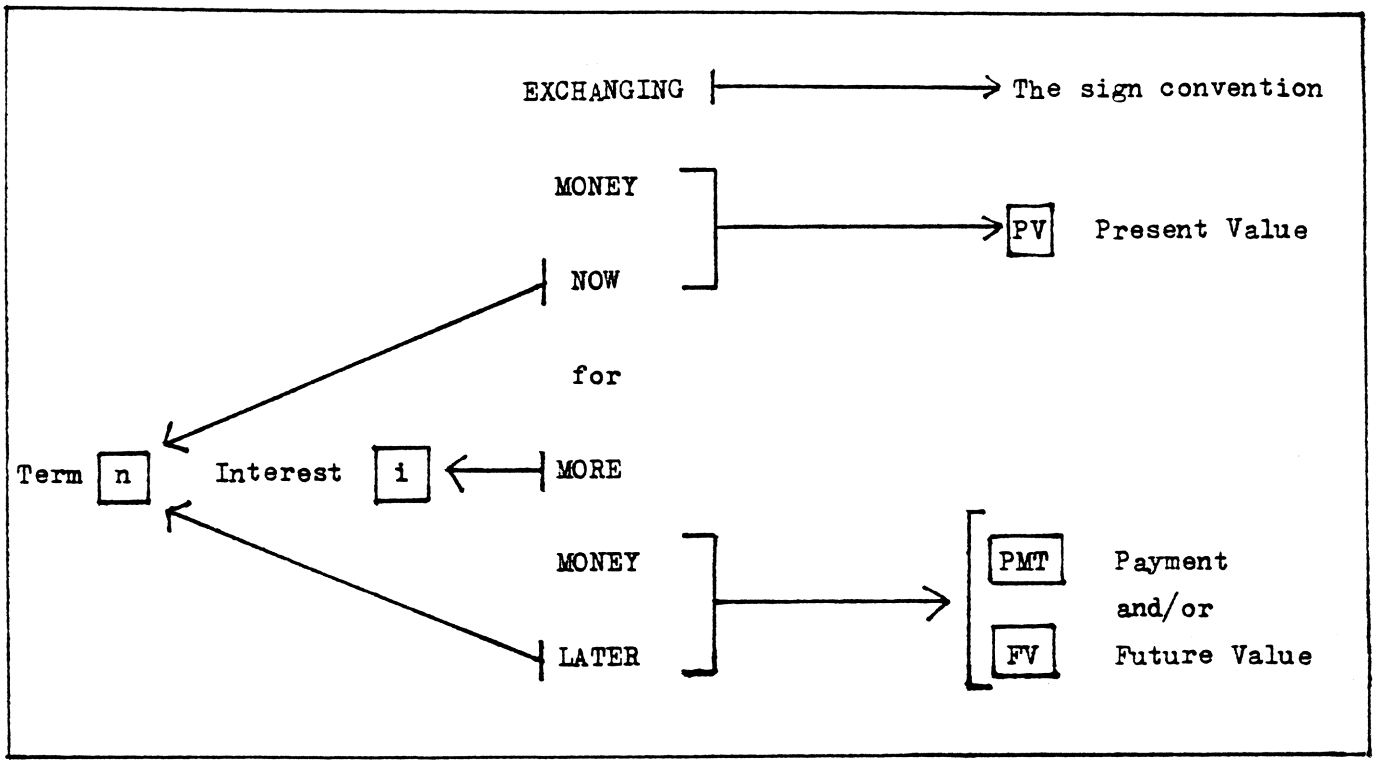
I can also borrow money NOW, and pay back with regular monthly PAYMENTS for a while, plus a larger balloon payment at the end. In this case one form of money is paid off by payments belonging to the two other forms of money.

The relationship between the money given out and the money received in exchange depends on two elements:

- TIME.
- INTEREST RATE.

It also depends on the way in which the interest is calculated. The most frequent and most complex process is compound interest, where interest starts earning interest as soon as it is earned. We will be concerned here with compound interest.

Investing and borrowing is
EXCHANGING MONEY NOW for MORE MONEY LATER.



If I get MONEY NOW I have to give back MORE MONEY LATER.

If I give MONEY NOW I expect to get back MORE MONEY LATER.

In many cases, among which the typical real estate loan, the MONEY LATER part of the exchange consists of regular equal payments or of one lump sum in the future--or a combination of both. The five top row keys of the calculator-- [n] [i] [PV] [PMT] [FV] -- represent the various elements of such transactions.

The 5 financial keys [n] [i] [PV] [PMT] [FV]

They represent the three 'forms of money' plus TIME and INTEREST.

[n] TIME measured by the Number of payments and/or of compounding periods.

[i] INTEREST as a percentage rate per period.

Three 'forms' of money.

[PV] PRESENT VALUE or money given or received NOW, at the beginning of the time span considered.

[PMT] PAYMENTS of equal amounts once every period.

[FV] FUTURE VALUE or money received or given at a FUTURE date, at the end of the time span considered.

These keys allow us to answer a number of questions involving a great variety of financial situations. Let us examine first two questions concerning the traditional real estate loan: calculating the payments and the balance of the loan. It is essential that considerable fluency be acquired in these two basic procedures as they constitute the initial step for many other calculations.

II Payments on an amortized loan.

What are the payments on a $ 78,000 loan, amortized over 30 years, with interest at 12% per year?

Let us turn the calculator on -- both switches to the right -- , and apply our GOLDEN RULE to this problem.

The problem, as formulated above, twice has the word 'year'. If we key in the data exactly as above, the answer provided would be the yearly payment amount that would pay off our loan in 30 years. To get the monthly payment--which is what we normally mean when we do not specify otherwise--we need to key in the monthly equivalent of 30 years (360 months) and of 12% per year (1% per month). This is the time convention: the data for the term of the loan, the interest, and the payments must be on the same time scale.

With our data now conforming to the time convention, let us apply our GOLDEN RULE: $\boxed{\text{DATA}}$ then $\boxed{\text{QUESTION}}$

78000	$\boxed{\text{PV}}$	Keys in the $\boxed{\text{DATA}}$
1	$\boxed{\text{i}}$	
360	$\boxed{\text{n}}$	
$\boxed{\text{PMT}}$		Asks the $\boxed{\text{QUESTION}}$. The answer (802.32) appears on the display.

The answer appears as a negative number because it is money we are going to have to pay. In most instances, when reading an answer provided by the calculator, we may ignore the sign and just consider the number. More on this important <u>Sign Convention</u> later.

Let us change the data:

> What are the monthly payments on a $ 112,000 loan amortized over 27 years at 13% interest?

We do not immediately know the number of months in 27 years or the monthly interest corresponding to 13% yearly interest. We can of course mutliply 27 by 12 and divide 13 by 12. But this double procedure is such a frequent occurence that the calculator provides a simpler means. <u>The $\boxed{\text{BLUE}}$ key automatically transforms yearly data into its monthly equivalent</u> when used in conjunction with the $\boxed{\text{n}}$ and $\boxed{\text{i}}$ keys.

112000	$\boxed{\text{PV}}$	
13	$\boxed{\text{BLUE}}$ $\boxed{\text{i}}$	Enters 1.08, the monthly interest, into $\boxed{\text{i}}$
27	$\boxed{\text{BLUE}}$ $\boxed{\text{n}}$	Enters 324, the term of the loan in months, in $\boxed{\text{n}}$.
$\boxed{\text{PMT}}$		(1251.46) Amount of the monthly payments.

This is the basic procedure to calculate the monthly payments on a loan. It is important to practice it with varied data until it becomes almost automatic. It is the most fundamental building bloc for any number of financial calculations.

Practice

If your answers differ:

- Check that the BEGIN - END switch is on END, and that it stays there. (This tells the calculator that the payments are made at the end of each period).
- Press GOLD Clear ALL or GOLD Clear FIN (Clear Financial memories), or 0 FV , and start again.
- Did you press the BLUE key before entering the data in n and i ?

Calculate the monthly payments:

1) $ 135,000 loan, 13.5% interest, 30 years.	(1,546.31)
2) $ 25,000, 9% interest, 20 years.	(224.93)
3) $ 75,500 loan, 12.75% interest, 30 years.	(820.45)
4) $ 96,000 loan, 14.625% interest, 30 years.	(1,185.13)
5) $ 235,000 loan, 15% interest, 25 years.	(3,009.95)

Practice with your own data, real or imagined, until the keystrokes come easily.

Recalling the data

The 5 financial memories can be looked upon as 5 boxes into which we put numbers. At any point the data stored in those boxes can be examined. We can use the RCL key to explore each one of those memories, as follows.

Let us first key in and solve practice problem n° 1, above:

135000 PV		Keys in data
13.5 BLUE i		
30 BLUE n		
PMT		The Question calculates the answer
RCL n	(360)	Recalls the data, and the answer, from financial memories.
RCL i	(1.13)	
RCL PV	(135000)	
RCL PMT	(1546.31)	

Here again we can use the BLUE key. This time it transforms our monthly data into its yearly equivalent:

RCL BLUE n	(30)	For 30 year term
RCL BLUE i	(13.5)	For 13.5% yearly interest.

(Despite the blue symbols on the face of the n and i keys, here BLUE n divides by 12, and BLUE i multiplies by 12)

Yearly data for term and interest ⇄ BLUE ⇄ monthly data in n and i

Full accessibility to the content of every memory is a feature of the HP 38 calculator.

It is important to build confidence in oneself and in the calculator. To do so, please make some effort to find the cause of the errors that inevitably occur as you practice various problems in this book. Checking the data, as above, will reveal a major cause of mistakes. It will show that you forgot the BLUE key and have yearly data in n or in i, that there is no data for PV -- perhaps you keyed it into i and then overwrote it with the i data --, that your loan amount has one zero too many or too few, etc.

Changing the data

At any point we may overwrite any piece of data and see how this affects the payments. This can be used to correct erroneous data or to test another hypothesis. For instance:

What are the payments on a $ 63,000, 11%, 30 year loan?
How are the payments affected if the interest increases to 11.5%

63000 PV 11 BLUE i 30 BLUE n PMT		(599.96)
11.5 BLUE i	PMT	(623.88)
-		(23.92)

When keyed in in this way, we may even press the - key and calculate the difference between the two payments, as done above.

III Balance of the loan

"Balancing the books"

Let us key in again, or retain, our previous loan of $ 63,000, at the new rate of 11.5%, amortized over 30 years. We find that the payments are $ 623.88. At 11.5% interest, 360 payments of $ 623.88 exactly pay off my $ 63,000. By questioning the calculator for the payments, I instructed it to calculate the payments that would "balance the books" on this particular exchange of money.

Let me now tell the calculator that I have no intention of making those payments for 30 years. I intend to make them only for 10 years:

10 [BLUE | n] (Overwrites 360 in [n] with 120, for 120 months)

I no longer have a balanced situation in my financial keys. The lender is not going to be too happy unless I recognize my obligation to him beyond those 120 payments. The amount I still owe him after 10 years, the amount that paid at that time balances the books once again on the transaction, is calculated by simply pressing the [FV] key:

[FV] (58,502.09) This is the balance of the loan after 10 years.

Illustration

$ 145,000 loan, 13.75% interest, amortized over 30 years.
What is the balance of the loan after 10 years, 15 years, 20 years, 1 year?

(Note that we must first calculate the payments as required by the 30 year amortization period)

As we move to a new problem here, let us clear the financial keys by pressing [GOLD | Clear FIN]

145000	[PV]	
13.75	[BLUE | i]	
30	[BLUE | n]	
[PMT]		(1689.41)
10	[BLUE | n]	
[FV]		(137,865.66) The balance of the loan after 10 years

15	BLUE n	FV	(128,473.73) Balance after 15 years.
20	BLUE n	FV	(109,868.52) Balance after 20 years.
1	BLUE n	FV	(144,642.57) Balance after 1 year.

Clearing between calculations

Now that we are using the 5 top row financial keys it is important to make sure that no data left over from previous calculations interfers with new calculations. This can be done in a number of ways.

GOLD Clear ALL — This clears all the memories of the calculator, except program memories. It clears the financial memories, but at the same time it may clear much useful data stored in the stack or the register. There are more selective ways of doing the job.

GOLD Clear FIN — This clears the 5 financial memories. It is the standard clearing procedure between financial calculations. Notice that it does not clear the number in the display (part of the stack).

0 FV — As we move to a second problem, we provide data for 3 of the 5 top row keys and ask the calculator to calculate the data for a 4th. So any data left over in those 4 memories from previous problems would be of no consequence as it would be overwritten by the new numbers. There is only one memory that could retain data and interfere with the new calculation, generally FV . If we so choose, we can decide to clear that memory alone by keying 0 into it: 0 FV

The advantage is that we can do this at any time before the question is asked; it can be done even after the new data has been keyed in.

Suggestion: We need to keep control at all times over the data in all of the 5 financial keys. Before questioning for the payments, we need to ask ourselves: "Do I have the correct data in all those memories?" If you remember having cleared the financial memories, you do. If you did not, or do not remember whether you did, then 0 FV is all that needs to be done. After all, 'fully amortized' means 0 in FV , in the same way as ' no payments, all due in 5 years', means 0 in PMT .

Note: - Data can be keyed in in any order. It is what is in those memories when the question is asked that matters.

- Because of the time convention, the data in $\boxed{n}$ represents both the duration of the transaction (360 months) and the number of payments (360 payments).

- With the BEGIN -- END switch on END, as in just about all cases here, the last PMT and the FV occur at the same point in time. The Balloon Payment is the sum of those two amounts which are both due at the same time. To calculate the balloon, we may calculate the PMT, key in the due date and calculate the Future Value, and then just press $\boxed{+}$, or $\boxed{\text{RCL}}\boxed{\text{PMT}}\boxed{+}$ (The latter is always possible).

- If data has been checked with the RCL key before the question is asked, then it is necessary to press the question key twice before it can be interpreted by the calculator as a question. The first time the question key is pressed, the calculator interprets the keystroke as an effort to key in the data showing on the display.

Practice: balance of a loan

Calculating the balance of a loan provides useful information of itself. It is also an essential first step in a number of financial problems. It is an essential building bloc that needs to be practiced until it becomes almost automatic. Note that we first need to calculate the payments: that amount is shown here with the answers.

1) $ 50,000 loan, 12% interest, fully amortized in 30 years. Balance after 5, 15, and 25 years. (514.31) (48,831.61; 42,852.86; 23,120.66)

2) $ 109,000 loan, 13%, 30 years. Balance after 10 years. (1205.76) (102,917.59)

3) $ 88,500 loan, 14.25%, 25 years. Balance after 5, 10, 15, 20 years. (1082.29) (85,779.08; 80, 254.13; 69,035.49; 46,255.58)

4) $ 135,000 loan, 30 years, 13.75%. Balance after 6 years. (1572.90)(132,112.54

5) $ 25,000 loan, 9% interest, 30 years. Balance after 12, 18, 24 , and 3 years. (201.16) (21,480.78; 17,675.77; 11,159.48 ; 24,438.03)

6) Practice with loans of your own choosing.

Solving for the term, the interest, the amount of the loan.

We have now examined two procedures involving the 5 top row financial keys: how to calculate the Payments, and how to calculate the Balance on the traditional Real Estate loan. We now have to broaden our understanding of those five keys to a realization that, with them, we have the FREEDOM to answer any question that makes sense involving the five elements represented by those keys. We shall gradually discover that this freedom represents a considerable amount of very useful information about a number of financial situations.

Because the PMT key represents a number of payments that are all equal, the 5 keys [n] [i] [PV] [PMT] [FV] will frequently be referred to as the regular cash flow keys. At a later stage, we will see that a different set of HP 38 functions provide us with almost equal freedom when seeking to answer questions involving irregular cash flow data, data where the payments vary from month to month or year to year.

The sign convention

At the heart of the freedom we have of asking the calculator any question that makes sense involving regular cash flow data is the sign convention. Each transaction must involve money received and money given in exchange. The sign convention specifies that amount of money that are on opposite sides of the transaction must be of different sign: one side positive and the other negative.

One way of handling this convention is to decide that money received will be positive, and money given out negative. The disadvantage of this procedure is that the payments are the same whether we are the lender or the borrower: why should there be two different ways of calculating the payments for the one and for the other? Or again, why should a neutral party have to take sides? There are times when taking sides might well be the easiest way, and we can always revert to it when the need arises. I suggest another point of view.

I suggest putting positive data into [PV] and negative data into [PMT] and [FV]. This will take care of over 90% of our calculations. The few cases where this could not be applied will be unusual enough that we will have little difficulty knowing that something is different and adjusting accordingly. In the mean time we can acquire a habit instead of having to think out each problem individually. We remain free to switch to the 'money received positive, money paid out negative' procedure whenever that suits our fancy or the circumstances.

Let us illustrate this freedom we have, still within the compass of the traditional real estate loan, by 'balancing the books' from a variety of perspectives.

Fully amortized loan: calculating the term, the amount of the loan, the interest ([n] [i] [PV]).

1) A private lender agrees to lend $ 15,000, and wants to be repaid with monthly payments of $ 232 for 9 years. What interest is he charging?

The solution consists in keying in the DATA, while respecting the sign convention and the time convention, and Questioning for the answer.

15000 [PV]	Amount of the loan keyed in
232 [CHS] [PMT]	PMT keyed in as negative amount (sign convention)
9 [BLUE] [n]	Term keyed in in monthly format (time convention
i	(1.04) Interest per month (time convention)
12 [x]	(12.50%) Annual rate calculated. ([RCL] [BLUE] [i] is an alternative keystroke applicable only after 1.04 has been calculated)

2) A seller has an assumable loan on his property. The balance is for $ 46,666, his payments are $ 450, his interest is 10%. What is the remaining term.

Here we must consider the present indebtedness ($ 46,666) as the PV of the situation we want to analyze.

46666 [PV]	
450 [CHS] [PMT]	Sign convention !
10 [BLUE] [i]	Time convention !
[n]	Questions for term (240.57 month)
[RCL] [BLUE] [n]	Converts to 20.05 years.

3) A Real Estate broker's client advises the broker that he can only afford monthly payments of $ 800. The broker knows that his client can get a 30 year loan at 13.75%. What loan can he afford?

800	CHS | PMT	Data, with sign and time conventions.
30	BLUE | n	
13.75	BLUE | i	
PV		(68,662.90)

Note that the CHS could be omitted here. The calculator would then provide a negative PV for the same amount. In the two previous examples CHS had to be used as dollar amounts from both sides of the transaction were being keyed in.

Partially amortized loans: asking various questions

1) A $ 55,000 loan at 14% interest with payments of $ 680 has a 6 year due date. What is the balance after 6 years of payments?

55000	PV	
680	CHS | PMT	Data, with sign and time conventions.
14	BLUE | i	
6	BLUE | n	
FV		(50,711.71)

What would be the balloon payment at that time?

RCL | PMT | + — Adds payment amount to balance: (51.391.71).

2) A client purchases a property with a $ 100,000 loan (30 years, 12% int.) The client intends to sell the property when he retires 10 years from now. What balance will he owe? (This is our traditional FV problem)

100000	PV	
30	BLUE | n	
12	BLUE | i	
PMT		(1,028.61)
10	BLUE | n	
FV		(93,418.00)

The client is surprised at how high the balance is, and asks what monthly payments he would have to make to bring the balance down to $ 45,000.

45000 [CHS] [FV]	Keys in new data. Watch for sign convention!
[PMT]	(1,239.09)

3) A client has just borrowed $ 80,000 which he has agreed to pay back with payments of $ 1,300 for 4 and 1/2 years, and a balance of $57 ,060. What inerest is he being charged?

80000 [PV]		
1300 [CHS] [PMT]		
4.5 [BLUE] [n]		
57060 [CHS] [FV]		
[i]	(1.25)	Monthly interest
12 [x]	(15)	Annual interest.

In each of the previous problems we have '<u>balanced the books</u>' on a financial transaction. We provided the calculator with the DATA, and questioned it for the missing piece of information. A little practice will provide us with the ability to do this fast, keying in data while it is being presented to us, in person or on the phone, perhaps even before we know what the question is going to be. When the question comes we just about immediately have the answer.

The ability to 'balance the books' on a regular cash flow transaction, however unusual the formulation of the data or the question, is an essential step in learning how to get the most out of the HP 38. It provides useful information of itself, and is a first step in solving more complex problems. We will move to these problems after some practice.

Practice

1) A private lender agrees to lend $ 10,000 and be paid back with monthly payments of $ 250 for 5 years. What interest is he charging? (1.44% per month, 17.27% per annum)

2) The borrower on the previous transaction proposes to pay back with payments of $ 200 for 5 years, plus an extra $ 5,000 at the end of the 5 years. What interest is he offering? (1.48/month, 17.71 yearly)

3) Outstanding balance: $ 29,519. 9% interest, payments of $ 325 per month. What is the remaining term of the loan? (153 months, 12 3/4 years)

4) $ 88,000 loan, 14% interest, 30 years. How long will it take to amortize this loan to 1/2 of its original amount? (301.56 months, just over 25 years. Same answer with those terms whatever the loan amount.)

5) $ 93,000 loan, 13.75%, 30 years. How long would it take to amortize the loan if the borrower decide to increase his payments by 10% ? (16.42 yrs) (Calculate PMT, then: 10 $\boxed{\%}\,\boxed{+}\,\boxed{\text{PMT}}$ and question for $\boxed{n}$)

6) $ 25,000 loan, 10% interest, payments of 1% of original loan amount per month ($ 250), all due in 6 years. What is the balance, what is the balloon payment? (20,912.03; 21,162.03)

7) After lengthy discussions a borrower and a private lender have agreed on the following repayment: $ 500 per month for 6 years, plus $ 25,000 for the remaining balance. They also agree on a 12.5% interest rate. In the process, they have lost sight of the amount being financed. Can you help? (37,093.34)
If the interest is increased to 13.5%, what is the amount of the **loan?** (35,755.22) (13.5 $\boxed{\text{BLUE}}\,\boxed{i}\,\boxed{\text{PV}}$)

8) An investor pays $ 20,000 for a Trust Deed note with a 6 year due date. He will receive monthly payments of 285 dollars plus the remaining balance of $ 29,000. What rate of return is he getting on his investment? (20.91%). (Note that the face value of the discounted second is irrelevant if we are given the PMT and FV).

9) A buyer is willing to pay $ 750 per month. What **30 year** loan can he obtain with interest at 12% and at 14.5% ? (72,913.75; 61,246.69)

Looking at financial transactions from two points of view.

We are going to consider three very different problems which nevertheless have very similar solutions:
- Annual Percentage Rate (APR) calculations,
- Yield on discounted note,
- Cost equivalent pricing.

In each case we
- first balance the books on a traditional real estate loan situation,
- then look at the transaction from a second point of view, and modify one piece of data accordingly,
- finally we balance the books again on the modified data, which provides us with the answer we were seeking.

A The Annual Percentage Rate (APR)

The two points of view are those of the lender: "This is an $ 80,000 loan ", and of the borrower who answers: "But I only got $ 78,000 ".

Let us imagine an $ 80,000 loan (12%, 30 years), where the cost to the borrower is 2 points and $ 400, leaving him $ 78,000 net.

The Annual Percentage Rate (APR) is the rate that corresponds to the net amounts received and paid out at each point in time by the borrower. The $ 80,000, which the borrower did not receive in full, is needed only in so far as it enables us to calculate what the borrower must pay out and what he has received.

1st step: Balancing the books on traditional loan.	80,000 $\boxed{\text{PV}}$ 12 $\boxed{\text{BLUE}}\boxed{\text{i}}$ 30 $\boxed{\text{BLUE}}\boxed{\text{n}}$ $\boxed{\text{PMT}}$	(822.89)
2nd step: Second point of view: "I only got $ 78,000"	78000 $\boxed{\text{PV}}$	
3rd step: Balancing the books on the modified data:	$\boxed{\text{i}}$ 12 $\boxed{\text{x}}$	(1.03; 12.34)

In terms of the $ 78,000 he actually received, the borrower is being charged an Annual Percentage Rate of 12.34%.

In most cases we would have to calculate the net amount received ($ 78,000) from the knowledge of the face value ($ 80,000), and of the cost of the loan (2 points and $ 400). The keystroke would then be modified as follows:

80000 [PV]	
12 [BLUE] [i]	
30 [BLUE] [n]	
[PMT]	
[RCL] [PV]	Recalls 80,000
2 [%] [-]	Calculates points and deducts from 80,000
400 [-]	Deducts $ 400 to calculate $ 78,000.
[PV]	Keys 78,000 into PV memory.
[i] 12 [x]	

Instead of just keying in 78000 [PV] we must here calculate the 78000 dollar amount and key it into the [PV] memory. (If we just calculate the 78,000 and forget to key the amount back into [PV], when we finally calculate [i] we find our nominal rate of 12%. Not surprisingly, as nothing has changed in the financial memories since they were first balanced

If there is a due date on the loan, then the first step (balancing the books on the traditional loan) must include calculating the FV . It makes a significant difference in the APR if the initial costs must be absorbed in 3 years instead of being spread out over 30 years.

So the initial step by which we balance the books on the loan requires that we end up in [n] [PMT] and [FV] with the number of payments, the amount of the payments, and the amount of the remaining balance that represent the agreed upon repayment for the loan as per the loan contract.

<u>Problem:</u> $ 25,000 loan, 14% interest, amortized over 30 years, all due in 3 years. What is the APR if the loan costs are 10 points and $350 ?

<u>1st step:</u>	25000 [PV]	
Balancing	14 [BLUE] [i]	
the books	30 [BLUE] [n]	
	[PMT]	Calculates PMT (296.22).
	3 [BLUE] [n]	
	[FV]	Calculates balance (24797.82)
<u>2nd step:</u>	[RCL] [PV]	
calculating	10 [%] [-]	
and keying		
in 2nd point	350 [-]	Net amount received (22,150)
of view	[PV]	Net amount keyed into PV
<u>3rd step:</u>	[i] 12 [x]	APR is 19.03%

As the rate that corresponds to the <u>net amount received by the borrower</u> the APR makes it easier for the borrower to choose between loans that have different rates, different loan costs, and possibly different terms. We saw that investing and borrowing is exchanging money <u>in time.</u> It does not make sense to borrowe money now and pay back, even in part, with money now. This is however what takes place from the borrower's point of view when a lender tells him: "I will lend you $80,000 if you give me $ 2,000". The APR is the borrower's way of saying: "Why don't you just lend me $ 78,000 and charge me a higher interest on that amount if you have to".

Discosing the APR is a lender's legal obligation. The law (Regulation Z, Truth in Lending) also specifies which costs must be taken into account. The object is to further consumer protection by making it easier to choose between loans and not be misled by low advertised rates that are in reality offset by high loan costs.

Precisely because the APR conforms to legal norms, and is valid for a loan whoever the borrower may be, it does not always fully take into account the specific circumstances affecting the borrower.

If the borrower or his agent have the ability to calculate the APR, he might as well use the approach to the full and calculate what I like to call the Personalized APR, a rate that takes into account the specific costs and circumstances affecting the borrower.

For instance, the borrower may intend to keep the loan only for 5 years instead of for the full 30 year term. This affects the personalized APR in the same way as if there were a 5 year due date on the loan, and the personalized APR would be calculated accordingly. Similarly, choosing one loan rather than another may imply some costs which are not included in the legal APR calculation: a prepayment penalty that will apply only because the borrower does not intend to keep the loan for its full term--this amount should be added to the FV--; a prepayment penalty on the existing loan that will have to be paid if the new loan is not with the lender of record--this amount would be added to the loan costs; more extensive upgrading requirements on the property such as a new foundation on an old garage not required by other lenders and not adding significantly to the value of the property in the borrower's opinion. These and other costs can be taken into account when calculating a personalized APR that will more clearly reflect the advantages or disadvantages of one loan over another than would the legal APR.

Calculating the personalized APR is in no way different than calculating the APR: only the term and costs included better reflect the circumstances of a particular borrower.

Practice:

APR and personalized APR calculations.

1) Calculate the APR on a 2nd T.D. loan of $ 35,000 amortized over 20 years, with an interest rate of 16%. Loan costs: 12 points and $350. (PMT 486.94; net PV 30,450; APR 18.72%, 12 times monthly rate of

2) Same loan, same costs, but 3 year due date. What is the APR? (PMT 486.94; FV 34,071.06; Net PV 30,450; APR 22.02%. To solve problem the best is to start from scratch)

3) $ 75,000 loan, 30 years, 14.75% interest. APR if loan costs are 2 points and $ 400. (15.15%)

4) Same loan, but the buyer knows that he will keep the loan for only 2 years. What is the personalized APR? (16.24%)
(As in problem n° 2, it is best to start from scratch)

5) Same loan, loan to be paid off in two years as above. What is the personalized APR if there is a $ 4,400 prepayment penalty? (18.73%)
(Calculate 2 year FV, then: 4400 [CHS] [+] [FV] so as to end up with following cash flow: 73,100 in PV, -933.36 in PMT, -79,081.74 in FV)

6) $ 17,250 loan, 15% interest, payable 1.5% per month, balance due in 3 years. Loan costs are 10% plus $250. What is the APR? (20.37%)

7) $ 115,000 loan, 13.50%, 30 years. Loan costs are 2.5 points and $ 400. What is the APR? (13.93%)

8) $ 6,000 loan, 16% interest, payments of $ 90 per month, all due in 1 year. Loan costs are 6% plus $ 200. What is the APR? (26.84%)

B Discounted Second Trust Deeds

Calculating the yield on a discounted 2nd Trust Deed is very similar to calculating the Annual Percentage Rate on a loan. In both cases we are dealing with loans with a face value, nominal rate, and a term which allow us to calculate the cash flow that will pay off the loan. In both cases we then look at the data from a second point of view, and in both cases, from that second point of view, the cash flow that pays off the loan does not change. But in both cases, from the second point of view, the face value of the loan becomes meaningless and needs to be replaced by a different amount: I did not actually get $ 60,000 when I signed up for my $ 60,000 loan, and of course I did not pay $ 15,000 when I bought that $ 15,000 2nd T.D. at a 30% discount. In both cases we just need to adjust PV to whatever we actually received or paid, and calculate the rate i that will make sense of the new data.

As with APR calculations, the first step consists in fully balancing the transaction from the original point of view. We then modify PV and see how it affects i .

Example.

2nd T.D. note for $ 15,000, 10% interest, payable 1% a month, all due in 4 years.

Keystrokes	
15000 PV	Keys in initial data.
150 CHS PMT	
4 BLUE n	
10 BLUE i	
FV	Balances initial transaction. (We may not be interested in the FV but the calculator needs to know as it remains one benefit of the discounted 2nd T.D.)

1st Question: What is the yield (the return on my investment) if I buy at a 30% discount?

Keystrokes	
RCL PV	
30 % −	Calculates discounted price.
PV	Keys price back into financial memory
i RCL BLUE i	Calculates yield and retrieves annual rate.

2nd Question: What is the yield if I buy the T.D. for $ 11,500?

This is basically the same question, with the selling price given directly in dollars instead of relative to the face value of the loan. With previous data still in:

11500 PV			
i	RCL BLUE i		(18.66)

3rd Question: What must I pay for the 2nd T.D. if I want a 25% return?

This time we impose $\boxed{i}$ and question for $\boxed{PV}$

25 BLUE i	
PV	(9553.45)

Subsidiary Question: What discount does a selling price of $ 9553.45 represent?

15000		This puts 15000 in the y memory of the
x⇄y		stack, and 9553.45 in x.
Gold Δ%	(36.31)	Calculates the % difference between y and x.

Seasoned 2nd T.D.

So far, our 2nd T.D. was discounted and sold at the time it was created. If the sale occurs later we need to let the calculator know by adjusting the value in n so as to express the remaining term of the loan.

We do not need to know the remaining balance of the 2nd T.D. note at the time the purchase is made, as the only thing that matters to us is what we pay for the note, and what we get out of it. In fact, the lower balance of the loan, when there is an early due date, does not result in most cases in a lower yield: on the opposite, as we have less time to wait for the large balloon payment the yield will generally be higher.

(We need to know the balance only if the selling price is expressed as a percentage discount of the balance at the time of sale).

<u>Example of seasoned loan</u>

Same 2nd T.D. loan as in previous example. I buy it after 14 months.

$\boxed{15000}\boxed{\text{PV}}$

$\boxed{150}\boxed{\text{CHS}}\boxed{\text{PMT}}$

$\boxed{4}\boxed{\text{BLUE}}\boxed{n}$

$\boxed{10}\boxed{\text{BLUE}}\boxed{i}$

$\boxed{\text{FV}}$

Keys in data and calculates FV as previously.

$\boxed{\text{RCL}}\boxed{n}\boxed{14}\boxed{-}\boxed{n}$

Deducts 14 months from n, and keys back remaining term into n.

We may now ask the same questions and solve in exactly the same way as before. Note the higher yields resulting from the shorter wait for the FV money.

<u>1st Question:</u> What is the yield if I buy at 30% discount from initial loan.

$\boxed{\text{RCL}}\boxed{\text{PV}}$

$\boxed{30}\boxed{\%}\boxed{-}$

$\boxed{\text{PV}}$

$\boxed{i}\boxed{\text{RCL}}\boxed{\text{BLUE}}\boxed{i}$ (24.32% instead of 21.73%)

<u>2nd Question:</u> What is the yield if I buy for \$ 11,500?

$\boxed{11500}\boxed{\text{PV}}$

$\boxed{i}\boxed{\text{RCL}}\boxed{\text{BLUE}}\boxed{i}$ (20.32% instead of 18.66%)

<u>3rd Question:</u> What must I pay if I want a 25% return?

$\boxed{25}\boxed{\text{BLUE}}\boxed{i}$

$\boxed{\text{PV}}$ (\$ 10,341.03 instead of 9553.45, more expensive because it is a better deal).

Practice

1) $ 18,000 2nd T.D. note, 10% interest, payable 1% a month, all due in 6 years.
   Yield if sold at 20% discount? (15.47%)
   Yield if sold at 25% discount? (17.11%)
   Yield if sold for $13,000? (18.09%)
   Price to obtain 19% return? (12,558.57)
   What discount is that? (See bottom of page)* (30.23%)

2) $ 25,000 2nd T.D. note, amortized over 20 years, all due in 8 years.
   10 % interest.
   Yield if sold at a 30% discount? (17.44%)
   Selling price to obtain 20% yield? (15,643.88)
   What discount is that? (37.42%)

3) $ 6,000 2nd T.D. note, 10% interest payable interest only. All due in 5 years.
   Yield if sold at 22% disoount? (16.49%)
   Selling price to obtain 18% return? (4,424.79)
   What discount is that? (26.25%)

4) $ 9,000 note, no monthly payments, 10% interest with monthly
   compounding, all due in 4 years.
   Yield if sold at 20% discount? (15.64%)
   Yield if sold for $ 6,500? (18.23%)
   Selling price to obtain 20% return? (6,062.73)
   (Monthly compounding assumed throughout)

5) $ 45,000 note, 13% interest, monthly payments of $ 500,
   balance due in 5 years.
   Yield if sold for $ 35,000? (20.15%)
   Yield if sold for $ 35,000 a year after creation
   of the note? (21.25%)

* (To calculate discount put face value in y memory, discount price in x and press $\boxed{\text{GOLD} \mid \Delta\%}$. In first problem, with $ 12,558.57 showing in display, press: 18000 $\boxed{x \gtrless y}$ $\boxed{\text{GOLD} \mid \Delta\%}$)

Price and Cost

People do not generally pay cash for a house. They pay for it with a downpayment, a number of regular monthly payments, and possibly a balloon payment. So what matters to the buyer is not so much the sales price, as the amount of the downpayment and of the other payments he will have to make. What matters is not so much the price, but the cost.

Clearly, it is possible for two houses to sell for the same price, and yet to require very different payments if the interest rates on the loans are different. The prices are the same, but the cost to each buyer is different.

It is also possible for the cost of two properties to be the same--same down payment, same monthly payments for the same period of time, and same balloon payment--and yet for the prices to be significantly different.

This is important as real estators frequently have to deal with properties that are financed in unusual ways--either because they suggest such financing to their clients, or because they have to advise a client on the purchase of a property that is being offered with such financing. For instance, there may be an assumable low interest loan, or an assumable FHA or VA loan. Or the seller may be willing to finance the property himself at a rate that is below the market rate for new loans.

Common sense tells us that the availability of a low interest rate loan on a property tends to make it a good buy, and may even justify paying a price that is higher than the market price for comparable properties. So the adds stress: "Owner will carry at 9%", or "Assume FHA loan at 8%". As with many other subjects in this book, the calculator makes it possible to transform a vague feeling into solid facts: precisely how much better is it? At what point would a higher selling price offset the advantages of the more generous financing? To what extent would special circumstances adversely affect the benefits of low interest financing? These are some of the questions we will consider here.

Case study: the Cost-Equivalent Price.

A property can be bought for $ 120,000. The seller is offering to carry 80% of the price, at 10% interest, amortized over 30 years, balance due in 10 years. The current market rate for 80% loans is 12%. As the prospective buyer, or as the buyer's agent, you appreciate the financing but your judgment is that similar properties have been selling for $ 115,000. Does the advantageous financing outweigh the seemingly high selling price? Your decision concerning that property depends on your answer to that question.

The question becomes easy to solve if, instead of focusing on the price, we consider the cost. However we cannot just compare the various elements of the cost between our property and a $ 115,000 property financed with a conventional loan: on our property, the payments would be lower, but the downpayment and the amount owed after 10 years would be higher which makes it difficult to form a clear picture of where the advantage lies. So we can ask ourselves a question: what would be the price of a property that would cost as much as the one offered for sale--same downpayment, same payments, same balloon payment--when financed by a conventional loan at market rates?

We were bold enough to ask the question because the calculator provides us with a simple means to get an answer. The answer, in this instance, is $ 109,172.10. This means that we can purchase our property for $ 120,000, and keep it for 10 years, and not pay more than if we had bought that property at the bargain price of $ 109,172.10. Should we buy that $ 120,000 property, or wait until a similar one comes on the market for $ 115.000? We certainly have a clear incentive to buy, and for the simple reason that it will cost us less.

The $ 109,172.10 is the cost equivalent price at market rates of our $ 120,000 owner financed property.

Keystrokes.

The question is solved with the 5 top row financial keys in the same two-step approach used with APR and discounted 2nd T.D. calculations. We provide the calculator with the loan amount, the payments, and the balance of the loan after 10 years: we have balanced the situation from the owner-financed point of view. We change the interest to the market rate and ask the calculator for the loan amount that will require the same payments and the same balance. By adding back the original downpayment we obtain the cost equivalent price.

120000 ENTER 20 %	Calculates downpayment
STO 0	Stores for future use
-	Calculates loan amount
PV	Enters loan amount in Present Value.
10 BLUE i 30 BLUE n	Enters rest of financial data.
PMT	Calculates monthly payments.($ 842.47)
10 BLUE n	Enters due date
FV	Calculates balance of loan ($ 87,300.50)
12 BLUE i	Modifies interest rate to 12%.
PV	Calculates 12% loan that could be paid off with same payments and same balance.
RCL 0 +	Adds back downpayment to get cost equivalent price of $ 109,172.10.

<u>Comments.</u>

In this example, as in most realistic situations, the two situations compared (here a $ 109,000 price with conventional financing and the $ 120,000 price with owner carried financing) are not perfectly equivalent in all other respects. A $ 109,000, even a $ 115,000 property, with conventional financing would have a lower downpayment than our $ 120,000 purchase. Can the buyer afford the higher downpayment? Is the inconvenience of the extra initial expense compensated by the lower cost equivalent price? Also, the 10 year due date on the owner carried loan can be a major inconvenience if the buyer keeps the property for a longer period of time. On the other hand, if the buyer does not keep the property for ten years he does not fully benefit from the low interest rate loan. The same keystrokes with $\boxed{10}\ \boxed{\text{BLUE}}\ \boxed{n}$ replaced by $\boxed{1}\ \boxed{\text{BLUE}}\ \boxed{n}$ will calculate the cost equivalent price if the buyer intends to pay off the loan in 1 year instead of 10. The cost equivalent price now climbs to $ 118,203.59, and the property is no longer a good buy. So the decision to buy a property where unusual financing is offered cannot be made solely on the basis of the cost equivalent price, any more than a loan can be chosen solely on the basis of the Annual Percentage Rate. But, like the APR, it contributes invaluable data to the decision-making process.

Problems

1) "Assume VA loan at 8%". A property is advertised for sale at $ 59,000 with an assumable $ 43,500 VA loan at 8% interest and monthly payments of $330. You are advising a client who likes the house, likes the price, can afford the somewhat high downpayment, but would prefer to pay the customary 20% down ($ 11,800 instead of the required $ 15,500 if the loan is assumed). You feel the financing makes it an exceptionally good buy, as current rates for new loans are 12%. Your clients intention is to live in that property until he retires 15 years from now. How can you convince him that the property is an excellent buy?

(The cost-equivalent price is $ 47,942. Ask your client if he would be willing to put $ 15,500 down if he could get the property for $ 48,000 instead of $ 59,000).

2) Two equivalent properties in the same tract can be bought for $ 130,000 (property A), and $ 135,000 (property B). Property A will require 80% conventional financing at 12.5%, amortized over 30 years. Property B is offered with an 80% owner carried All-Inclusive Trust Deed at 11% interest, amortized over 20 years. Compare the costs from the point of view of a buyer who intends to keep the property for 12 years.

Answer:	Down pmt.	Payment	Ballance after 12 years
Property A	$ 26,000	$ 1109.95	$ 95,192.73
Property B	$ 27,000	$ 1114.76	$ 70,965.96
Difference:	- 1,000	- 4.81	+ 24,226.77

Cost-equivalent price of property B at market rate used with property A: $ 125,910.50. One can also consider the difference in downpayment and payment as an investment that returns $ 24,226.77, a 25.18% annual rate.

Special feature:

Programming: an introduction

When instructing someone on how to prepare a gourmet dish, we can give an instruction, see it executed, and then give the next instruction. Or we can say: "Now listen ", give all the instructions, and then say: "Now do it". In this second case, we may even be tempted to say: "That was good. Do it again".

The first situation is what we normally do with the calculator: we punch keys, and the calculator performs each instruction as we give it. The second situation is what happens when we program the calculator. We first tell the calculator not to perform the various instructions, but just to remember them. This is done by pressing | BLUE | P/R | the program/run key. This switches the calculator into programming mode. We then give the calculator all the instructions we want it to remember: this is writing our program. We then need to switch out of programming: the same | BLUE | P/R | keystroke that switched us in switches us out of programming, like a light switch that turns the light on and off.

With the calculator out of programming mode, and a program safely recorded in the calculator, we can use the calculator in the normal way to solve any problem we choose. At any time, we can tell the calculator: "Now do it", and the calculator performs all the instructions in the program. The "Now do it" instruction is the Run/Stop key | R/S |. Pressing it a second time while the program is being executed interrupts the program, and pressing it again gets the program going again.

There are two majors uses for programming:

1) To take care of repetitions. Any time a series of keystrokes needs to be repeated over and over, using a program enables us to key in the keystrokes only once, and to tell the calculator "Now do it" any number of times by just pressing the | R/S | key.

2) Because the program 'does it' at the punch of a key, we can key in a complex program that we do not understand, and then use it in the same way as we use all the pre-programmed functions of the calculator. Even if we initially wrote the program ourselves, it is good to be able to use it without having to think out the 'why' of every keystroke every time.

Example:

A businessman announces a 15% discount on all his merchandise. When customers bring various items to the cash register, how much should he charge? The solution is simple: marked price, minus 15% discount, plus 6% sales tax. For an $89 item:

89 [ENTER] 15 [%] [−] 6 [%] [+] (80.19)

As this sequence has to be repeated for every item, with only the initial data changed, he decides to program the calculator. He can do so while he is solving for the 89 dollar item.

	Previous keystrokes	Same keystrokes + programming.	Note change between programming and regular keystrokes:
Changing data.	89	89	
		[BLUE] [P/R]	Switch into Programming mode.
Non-Changing Key-strokes.	[ENTER] 15 [%] [−] 6 [%] [+]	[ENTER] 15 [%] [−] 6 [%] [+]	Record the program
		[BLUE] [P/R]	Switch out of Programming mode.
		[R/S]	Solve for initial data

To solve for other items:

New item price:	120	14.56	23.99	71.25
Solve:	[R/S]	[R/S]	[R/S]	[R/S]
Cost:	108.12	13.12	21.61	64.20

The inconvenience of the previous program is that, though it calculates the discounted price and the sales tax, it does not show us those numbers. We can tell the calculator to show us by including one of two instructions:

BLUE PSE	Instructs the calculator to PAUSE for a second.
R/S	The calculations STOP when the RUN/STOP instruction is included in a program. The calculations resume when R/S is pressed.

Let us modify the previous program twice, with each of these keys:

With PAUSE instruction	With STOP instruction	
BLUE P/R	BLUE P/R	Switches to programming
BLUE CLP	BLUE CLP	Clears previous Program
ENTER	ENTER	
15	15	
%	%	
-	-	
BLUE PSE	R/S	Displays discount price
6	6	
%	%	
BLUE PSE	R/S	Displays tax
+	+	
BLUE P/R	BLUE P/R	Switches out of programming

To solve for the discount price, the tax amount, and the total cost, key in the price and press R/S once with the PAUSE program; 3 times in a row with the STOP program.

Note that these programs are nothing else than the keystrokes leading from 89 to 80.19, plus two PAUSE or two STOP instructions. Either one can be used over and over again to perform the same calculations on any price we choose to submit.

Making the calculator count.

This very simple program introduces another key used exclusively in programming

BLUE GTO	The GO TO instruction which must be followed by the two digit (00 to 99) address of a program instruction. The calculator immediately switches to that new instruction. This keystroke can be used to produce a loop.

BLUE P/R	Switches into programming mode.
1 +	Adds one.
BLUE PSE	Displays total.
BLUE GTO 01	Tells calculator to start again at line 1 of program.
Blue P/R	Switches out of programming mode.
R/S	Runs program.

Programs, that can include all the pre-programmed functions of this calculator, add a powerful dimension to the HP 38. Programming was introduced at this stage because we will need to apply the features we have just seen to the next real estate problem on our list.

Amortization

We have been so far answering questions involving the traditional real estate loan, and using the 5 top row regular cash flow keys to do so. There is still some very basic data we need to obtain concerning the traditional real estate loan: we need the ability to obtain a complete amortization schedule. This could be done using the 5 top row keys, but the procedure would be cumbersome. A simpler approach is built into the calculator. It uses some of those 5 financial memories, but only as convenient places to store data: it does not use the balancing logic of our 5 top row financial memories.

To generate a complete amortization schedule:

Key in initial data for [i] [PV] [PMT] (with sign convention).

1 [GOLD] [AMORT] then does the following:

1) Shows interest portion of 1st payment in x .
2) Stores principal portion of 1st payment in y. Recall with [x⇄y] .
3) Deducts principal portion of first payment from PV. This leaves the balance of the loan in PV. Recall by pressing [RCL] [PV] .
4) Adds 1 to value in n (which should be 0 initially)

So complete data can be obtained by keying in data on the loan in i PV PMT and then pressing the following for each successive month to be amortized.

1 [GOLD] [AMORT]	Shows interest for the 1st period
[x⇄y]	Shows principal reduction
[RCL] [PV]	Shows remaining balance

12 GOLD AMORT used in the same way provides a yearly amortization schedule, with yearly interest in x, principal reduction for the year stored in y, and the balanced at the end of the year in PV .

8 GOLD AMORT in the same way calculates the data for an 8 months period, and can be used to bring a loan up to date to the end of the first calendar year before using the 12 GOLD AMORT keystroke.

Program

Amortization schedule

The following program will display

- the number of the month or year
- the interest paid during the stated month or year
- the principal reduction during that period
- the balance of the loan at end of period.

Monthly schedule	or Yearly schedule	
BLUE P/R	BLUE P/R	Initiates programming
RCL n	RCL BLUE n	Calculates number of next payment
1 +	1 +	
BLUE PSE	BLUE PSE	"Show me" instruction
1 GOLD AMORT	12 GOLD AMORT	Questions for data
BLUE PSE	BLUE PSE	"Show me" flashes interest paid.
x:y	x:y	Retrieves principal reduction
BLUE PSE	BLUE PSE	"Show me"
RCL PV	RCL PV	Retrieves new balance
BLUE PSE	BLUE PSE	"Show me"
BLUE GTO 01	BLUE GTO 01	Tells calculator to start again.
BLUE P/R	BLUE P/R	Stops writing of program.

To generate a full schedule for any loan

- key in the data (loan amount, payment as negative amount, interest)
- press R/S to run the program.

For instance:

For a $ 32,000 loan, at 9%, with monthly payments of $ 258:

32000	PV	Keys in data
258	CHS PMT	
9	BLUE i	
	R/S	Runs program

One of the major purpose of using a yearly amortization schedule program is to acquire data that will be used for tax purposes. If so, the yearly schedule needs to be provided on the basis of the calendar year. For most loans, the first calendar year corresponds to an incomplete year as far as the loan is concerned, and that incomplete calendar year is taken care of by amortizing the loan without the use of the program. For instance, for a loan that has only 8 payments during the first calendar year:

8 GOLD AMORT	(Shows total interest payment for partial year)
x⇄y	(Shows principle reduction for same period)
RCL PV	(Shows balance of loan at end of1st calendar year).

No problems so far.

But if we are interested in calendar years, why not have the calculator tell us what year we are in (1980, 1981, etc), instead of just year 1, year 2, etc.? With the program previously given (Yearly Schedule, on previous page), when we are ready to use the program--that is after we have brought the loan up to date to the end of a calendar year, as shown above,--if we then want 1981 to show, we can key in

1980	BLUE	n

And we can then start running our program by pressing R/S.

A simpler procedure consists in deleting from our program steps 2 and 3 (1 +)
We can then key into n , with the BLUE key, the actual year we want to see appearing on the display.

Revised Yearly Amortization Schedule program

```
RCL   BLUE   n
R/S
12    GOLD   AMORT
R/S
x⇄y
R/S
RCL   PV
R/S
BLUE  GTO    01
```

To use this program to amortize a loan, beginning with calendar year 1981, we simply need to key in

1981	BLUE	n		R/S

(We have used here the R/S alternative to the BLUE PSE of the previous program.

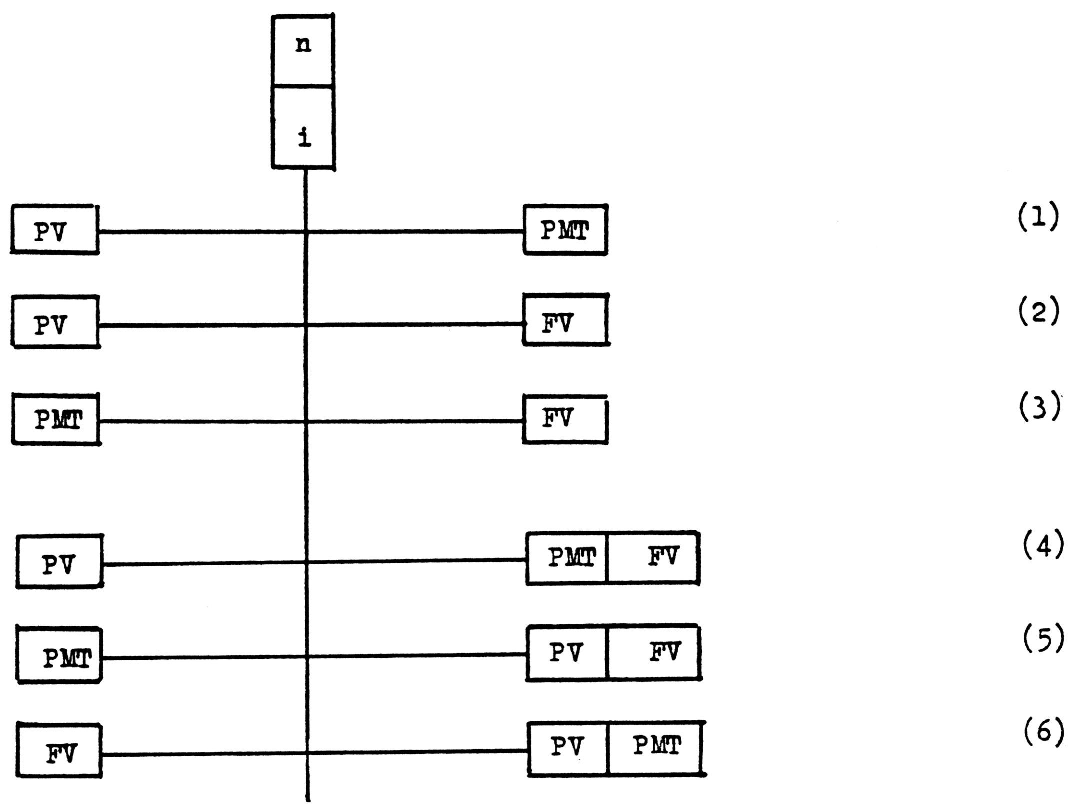
PART III

Regular Cash Flow Keys,
Beyond the traditional Real Estate loan.

So far we have used the 5 top row regular cash flow keys to obtain data on the traditional real estate loan. We now need to broaden our understanding of the full freedom these keys provide to include a variety of problems that go beyond the traditional real estate loan.

A fully balanced trasaction will necessarily include data for $\boxed{n}$ and $\boxed{i}$ and at least two amounts of money representing the two sides of the exchange. If we consider the three memories that store amounts of money -- $\boxed{PV}$ $\boxed{PMT}$ $\boxed{FV}$ - we can have one of these amounts balanced off by another one, or any one of these amounts balanced off by the remaining two. This leads to the following chart:

A The logic of the top row financial keys.



This illustrates the possible combinations of the 5 top row keys.

Every financial transaction must include the vertical line with $\boxed{n}$ and $\boxed{i}$ and one of the horizontal lines with the various combinations of the three different forms of money. For any transaction (represented by the vertical line and one of the horizontal line) the calculator solves for any one of the 4 or 5 values involved when fed data for the remaining 3 or 4 values.

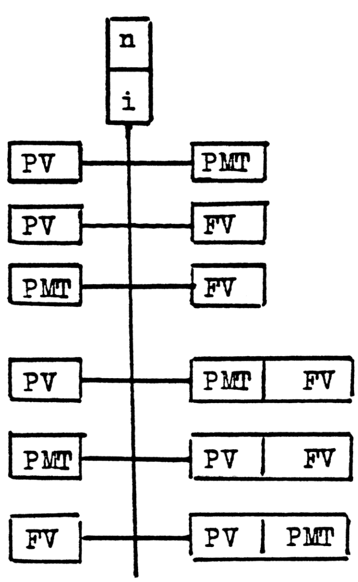
The sign convention

One side of the illustration represents money received, the other money given out in exchange. So the values will be negative on one side, positive on the other. Which side is which is immaterial.

Our practical approach to the sign convention suggested that we acquire the habit of keying in PV as positive, and PMT and FV as negative. The chart shows that this approach is valid for transactions represented by lines 1, 2, and 4 for which PV is on one side, PMT and/or FV on the other. Lines 1 and 4 represent fully amortized and partially amortized loan situations, line 2 represents the next most useful combination. Line 5 represents a combination that is hardly likely to occur. This leaves us with lines 3 and 6, two useful but not overly frequent occurences, where our practical approach to the sign convention would not be valid.

The various combinations

Let us examine some actual situations that correspond to the various lines:



(1) Amortized loan.

(2) Lump deposit, lump withdrawal.

(3) Sinking fund or Christmas Club account: regular deposits and lump withdrawal.

(4) Amortized loan paid off before term

(5) Regular payments pay off loan and then accumulate for lump withdrawal (unlikely).

(6) Lump sum deposit followed by regular deposits accumulate interest for lump withdrawal.

Note that only transactions 1 and 4 correspond to the traditional Real estate loan. Let us illustrate some of the other possibilities.

1) You deposit $ 14,700 in a Savings account that offers 6.5% interest, with monthly compounding. What amount can you withdraw after 2 years?

14700 $\boxed{\text{PV}}$
6.5 $\boxed{\text{BLUE} \mid \text{i}}$
2 $\boxed{\text{BLUE} \mid \text{n}}$
$\boxed{\text{FV}}$ (16,734.91)

The number entered into n must represent the number of compounding periodicity, and i must express the interest per compounding period. (See later for more detailed discussion).

2) You lend $ 6,000 to a friend who promises to pay you back $ 7,000 two years later. Assuming yearly compounding, what interest are you getting?

6000 $\boxed{\text{PV}}$
7000 $\boxed{\text{CHS} \mid \text{FV}}$ Sign convention !
2 $\boxed{\text{n}}$ Yearly periodicity for yearly compounding.
$\boxed{\text{i}}$ (8.01%)

3) A city has issued bonds to build a stadium and wishes to establish a sinking fund in order to accumulate the $ 1,200,000 that will become due in 15 years. Assuming an investment return of 8% a year (yearly compounding), what amount should the city budget each year for the sinking fund?

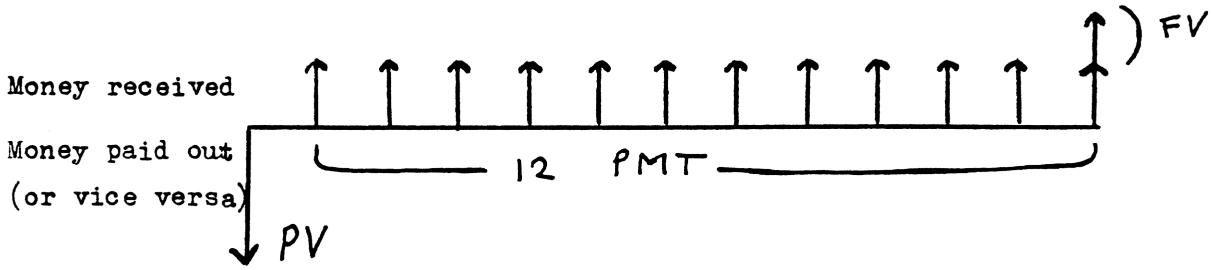
1200000 $\boxed{\text{CHS}}$ $\boxed{\text{FV}}$ (We could omit $\boxed{\text{CHS}}$ if we chose to)
8 $\boxed{\text{i}}$
15 $\boxed{\text{n}}$
$\boxed{\text{PMT}}$ ($ 44,195.45)

We balance the books here with 0 in PV.

B Cash Flows and the Cash Flow diagram.

I The cash flow diagram.

Cash flows can be represented by arrows pointing up or down from the horizontal time line.



Such diagrams help us visualize what exchange is really taking place in a way which is not affected by the terms in which we would describe the transaction. The Owner's Handbook suggests: " Instead of 'What is my problem?' ask yourself, 'What are the cash flows?' ". By examining the cash flow diagram you may realize that an apparently new and difficult problem is really a simple, routine problem under different terminology.

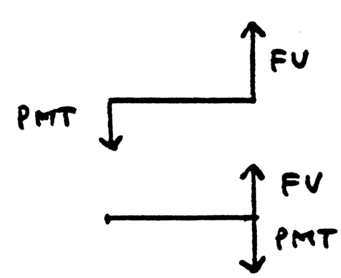
We are going to examine matters which are best understood in terms of cash flow diagrams.

II The BEGIN - END switch.

Payments entered with the PMT key can be made at the Beginning or at the End of a period. This affects how much interest is earned. The BEGIN - END switch adjusts calculations to one or the other situation.

Consider $ 100 yearly payments invested at 6% interest. What is the Future Value at the end of 1 period?

100 PMT	6 i	1 n	
Switch on BEGIN		FV	(106)
Switch on END		FV	(100)



The payment made at the end of the period has had no time in which to earn interest. (Note: we can switch to the right setting even after all the data has been keyed in, and even after a question was asked with the wrong setting).

Different kinds of financial transactions require different settings of the BEGIN - END switch. Mortgage payments are made at the end of each periods, payments on rents and rentals generally at the beginning of the period which they pay for. When dealing mostly with real estate loans, we should leave the key in the END position, switch to BEGIN only when the need arises, and immediately switch back to our standard setting.

The switch affects only entries made with the PMT key. Cash flows entered with the CFj key are assumed to be made at the end of the time period they are assigned to.

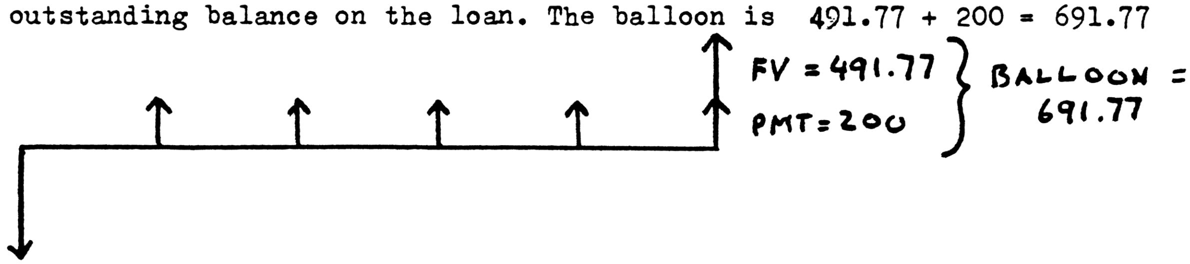
III Balloon payment and Future Value.

In most instances they are not identical. The cash flow diagram will show us why.

Problem: \$ 1,000 loan at 12% interest payable \$ 200 at the end of each year with a balloon payment at the end of the 5th year.

5 [n] 1000 [CHS PV] 200 [PMT] 12 [i] (Switch on [END])
[FV] (491.77)

This however is not the balloon. It is the balance of the loan after the 5th payment has been made. If the whole loan is paid off at the time the 5th payment is made, then that 5th payment is increased by the outstanding balance on the loan. The balloon is 491.77 + 200 = 691.77



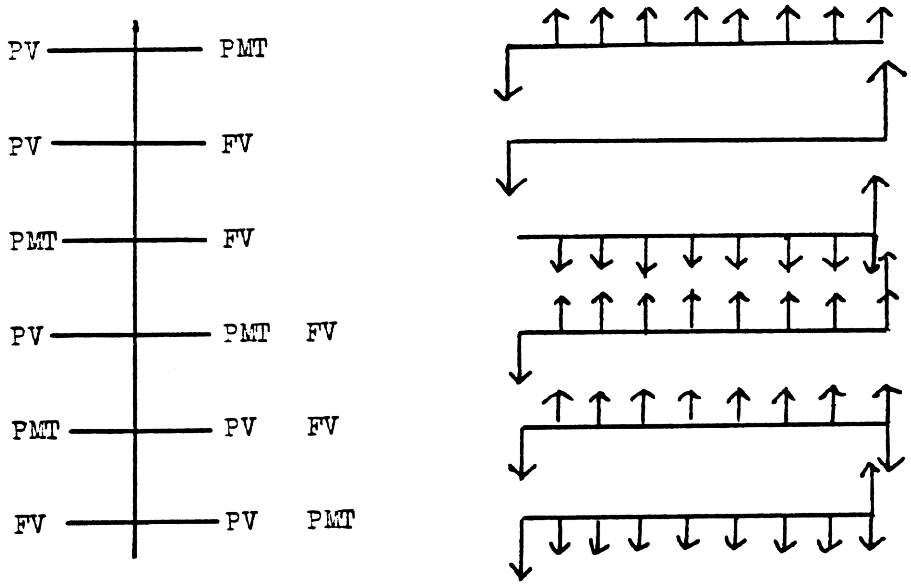
The balloon payment is equal to the Future Value plus one payment:

[BALLOON] = [FV] + [PMT] (When switch in [END] position)

With the Future Value visible in the display [RCL PMT] [+] adds 1 payment to the FV, and gives us the Balloon payment.

Possible combinations.

Each one of the horizontal lines of our earlier illustration corresponds to a different cash flow diagram.



We will now turn to some uses of the PV--FV combination (line 2).

Present Value---Future Value $\boxed{PV}$ $\boxed{FV}$

The PV--FV combination invites us to study two important topics: the notion of Present Value, and Inflation.

A The Notion of Present Value

The notion of a Present Value as equivalent or balancing off a future amount is not restricted to situations where the money is actually invested in an interest bearing account, or even to situations where an initial investment grows into future assets.

Let us suppose that you have a promise that $ 10,000 will be paid to you 5 years from now. A valid Present Value question is "How much is that promise worth to you now?".

That promise is obviously not worth $ 10,000, because if you had $ 10,000 now you could, among other things, put it in a Savings account at 5% and get about $12,800 5 years from now. In fact, if you had as little as $ 7,800 you could safely let it grow in the same account to $10,000 in 5 years.

So the promise cannot be worth more than $7,800 to you. It is probably worth less, as even a lesser sum might be invested at better rates and still produce $ 10,000 in five years, or might pay off a debt on which you are paying a higher rate, or might be spent right now for your present enjoyment. So if you were offered $ 7,800 now in exchange for the promised money, you would of course accept the offer, and choose among the many options open to you.

However, as the price offered goes down, there is a price below which you would no longer want to make the exchange. That price is the Present Value, to you, of those $ 10,000 in 5 years.

There is a rate which relates the PV to those $ 10,000 in 5 years. It is the rate that would allow the PV to grow into $ 10,000 in 5 years. That rate is higher than (or at least equal to) the interest rate offered by any safe investment available to you. The rate depends on your personal needs and desires at this moment. It is the rate at which you would be willing to discount the promised amount: it is called a discount rate.

Given a discount rate, it is possible to calculate the PV of any amount of money receivable in the future. The calculator handles a discount rate as it does any other compounded rate: interest rate, rate of return, rate of inflation, or other rates of increase. The discounted amount can be a single sum (FV), equal payments at regular intervals (PMT), or a combination of both PMT and FV. With the use of the irregular cash flow keys it can also be a wide variety of unequal payments.

<u>Examples:</u>

1) Given a discount rate of 15%, what is the Present Value of $ 20,000 in 12 years?

15 $\boxed{\text{i}}$
20000 $\boxed{\text{FV}}$
12 $\boxed{\text{n}}$
$\boxed{\text{PV}}$ ($ 3,738)

2) What discount rate would someone establish for himself if he did not want to sell his $ 20,000 in 12 years for less than $ 14,000?

20000 $\boxed{\text{PV}}$
12 $\boxed{\text{n}}$
14000 $\boxed{\text{CHS}}$ $\boxed{\text{FV}}$
$\boxed{\text{i}}$ (3.02%)

3) What is the Present Value of an income of $ 5,000 a year for 10 years, assuming a discount rate of 11% ?

10 $\boxed{\text{n}}$ 11 $\boxed{\text{i}}$ 5000 $\boxed{\text{PMT}}$ $\boxed{\text{PV}}$ ($ 29,446)

B Inflation and Real Estate.

Every decision we make is based on assumptions. A major assumption affecting Real Estate decisions, and buying decisions in particular, concerns inflation. Given our assumption concerning a likely rate of inflation, the calculator allows us to translate the effects of that rate into specific numerical data. It does so with the simplest possible keystrokes.

I 1) What would a $ 79,000 property be worth in 5 years assuming a 10% rate of inflation?

79000 [PV]		Present Value of property in [PV].
5 [n]		Number of years in [n].
10 [i]		Rate of inflation in [i].
[FV]	($ 127,230)	Calculates Future Value of property.

We may now change the initial data in any way we want. For instance:

2) Dollar value of the same property after 10 years:

10 [n]	
[FV]	($ 204,905)

3) Dollar value after 15 years:

15 [n]	
[FV]	($ 330,002)

4) Dollar value after 15 years assuming an 8% rate of inflation:

8 [i]	
[FV]	($ 250,601)

5) Dollar value after 10 years at 12%:

10 [n]	
12 [i]	
[FV]	($ 245,362)

The effect of these changes is cumulative. The last entry made in one of the financial memories remains valid until overwritten.

II A second effect of inflation is its <u>impact on the monthly payments</u>. As time goes by, the payments become easier and easier to make. They remain constant in dollar amount, but are paid in devalued dollars. If the owner's standard of living keeps up with inflation, his payments will represent a constantly smaller percentage of his income. To measure this major advantage of home ownership and traditional financing, it is possible to calculate the value of a future payment in <u>constant dollars</u>.

1) A buyer signs a note that commits him to making 360 monthly payments of $ 1000. Assuming an 8% rate of inflation, what is the value of the payment he will be making 10 years from now, in terms of today's dollars?

1000 [FV]
10 [n]
8 [i]
[PV] ($ 463.19)

Here again data entry and calculation could not be simpler.

So that 120th payment will be as painful to make as if the owner was paying $ 465 now. This measures the value of the 120th payment in terms of the goods and services (steaks, rent, etc.) which the owner could buy with the money.

2) We may here again modify the original data. For instance: What is the value in current dollars of the very last payment to be made on the property 30 years from now?

30 [n]
[PV] ($ 99.38)

With these two problems we have considered <u>a specific payment</u> made some time into the future. That payment is considered on its own, as a single lump sum, and is keyed into [FV]. Keying it into [PMT] would mean that a number of payments were considered together, with the value in [n] representing the number of these payments.

III Comments.

1) The effect of inflation can be calculated with the financial keys because we assume a constant rate of increase. The fact that we call that rate of increase inflation rate instead of interest rate has no effect on the calculations. Of course, here, the sign convention is even more arbitrary than with interest calculations.

2) The rate of inflation considered in the second group of calculations is the rate affecting the cost of living, not the rate affecting Real Estate values as in the first group of calculations. However we may point out that it is impossible for Real Estate Values to increase for a long period of time at a rate substantially higher than the rate affecting the cost of living. It may be wise, when making medium term and long term calculations of Real Estate values, to make a very conservative estimate based on our assumptions concerning the cost of living in general. Calculations based on the rate of inflation for Real Estate over the past few years could be very misleading if projected many years into the future. In making projections for a client, wisdom requires that we ask our client what his assumptions are concerning inflation, and that we adjust them downwards if they are not already very conservative.

3) All the calculations in this section overlook a number of circumstances which affect one way or another the benefits of home ownership. In particular the impact of obsolescence, maintenance costs, property tax, and the tax advantages of home financing are not being considered.

IV Let us consider two more basic keystrokes about inflation.

1) Equity increase due to inflation.

This is simply the Future Value of the property minus the Present Value. Changing the Future Value to a positive number makes the calculation easy.

Keystrokes	Display	
79000 $\boxed{\text{PV}}$		Calculates inflated value as previously.
5 $\boxed{\text{n}}$		
10 $\boxed{\text{i}}$		
$\boxed{\text{FV}}$	(127,230)	
$\boxed{\text{CHS}}$		Calculates equity increase due to inflation.
$\boxed{\text{RCL}}$ $\boxed{\text{PV}}$		
$\boxed{-}$	($48,230)	

2) Calculating a rate of inflation.

A property sold for $ 59,000 4 years ago and is now selling for $ 112,000. What rate of inflation has affected the property?

Here again, it is difficult to imagine how the calculations could be made more simple. We just need to remember the sign convention.

Keystrokes	Display
59000 $\boxed{\text{PV}}$	
4 $\boxed{\text{n}}$	
112000 $\boxed{\text{CHS}}$ $\boxed{\text{FV}}$	
$\boxed{\text{i}}$	(17.38%)

V Looking at inflation from various angles.

The full impact of inflation is best brought out when the simple inflation calculations which we have just seen are combined with other calculations in order to bring out various aspects of the situation.

Note:
In the following example we are going to proceed with a succession of calculations. As we do so, we will use a number of keystrokes which are meant to store, retrieve, position, and modify our data, but which are not in themselves separate calculations. Writing down answers and keying them in again when needed could eliminate those keystrokes. The major keys which will be used to Transfer and to Transform data --we will refer to them as T and T-- are $\boxed{\text{STO}}$, $\boxed{\text{RCL}}$, $\boxed{\text{ENTER}}$, $\boxed{x \rightleftarrows y}$, $\boxed{\text{CHS}}$, and the various clearing keystrokes. We should not let the use of these keys confuse our understanding of the basic calculations involved here.

Example:
Purchase price: \$ 96,000.
80% loan at 11.5% interest, amortized over 30 years.
What can we tell the buyer about his situation in 10 years assuming a rate of inflation for Real Estate and for the Cost of Living of 8%.

(Do not Clear unless specified)

1) Price of property in 10 years:

96000	$\boxed{\text{PV}}$	
10	$\boxed{\text{n}}$	
8	$\boxed{\text{i}}$	
$\boxed{\text{FV}}$		(\$ 207,256)

2) Equity increase due to inflation:

CHS	
RCL PV	
-	($ 111,256)

3) Loan data:

RCL PV	
80 %	(Loan amount: $ 76,800)
GOLD CLEAR FIN	
PV	
11.5 BLUE i	
30 BLUE n	
PMT	(Monthly payment: $760.54)
10 BLUE n	
FV	(Balance of loan after 10 years: $ 71,316)
CHS RCL PV	T and T
-	(Principal reduction over 10 years: $ 5,483)

4) Total equity increase.

CHS	
111256 +	($ 116,739)

5) Total equity:

96000 ENTER	
20 %	(Calculates Downpayment of $ 19,200)
x⇄y BLUE R↓	T and T to eliminate 96000.
+	(Total equity: $ 135,939)

6) Total equity as a percentage of property value

207256 x⇄y	
GOLD %T	(65.59%)

7) Equity as a percentage of property value without inflation:

96000 [ENTER]		Property value
20 [%]		Downpayment
5483 [+]		Adds principal reduction to downpmt.
[GOLD] [%T]	(25.71%)	

8) Total payments over 10 years:

[RCL] [PMT]	
120 [x]	($ 91,265)

9) Total payments including downpayment:

[CHS]	
19200 [+]	(110,465)

10) How painful are the payments after 10 years?

[RCL] [PMT]	
[GOLD] [CLEAR FIN]	
[FV]	
10 [n]	
8 [i]	
[PV]	($ 352.28)

Conclusion: With 8% inflation rate.

The $ 96,000 property is worth $ 207,256 ten years later. The owner's equity has increased by $ 111,256 because of inflation. His total equity of $ 135,939 represents 65.59% of the value of the property as opposed to an equity of 25.71% without inflation. The equity increase due to inflation is greater than the total amount of his payments and his downpayment. This implies that he is gaining more in equity increase each year than he is paying for his mortgage (This has been true ever since the fourth year). His payments of $ 760.54 per month are now paid in devalued dollars and are no more painful than if he had originally paid $ 352.28.

VI Successive increases.

There are a number of ways for obtaining the successive increases resulting from inflation (or from any other compounded rate of increase).

Let us take our former $ 96,000 property and the 8% rate of inflation. We want the projected value of the property after 1 year, 2 years, etc.

1) Adjusting [n].

96 000 [PV]				
8 [i]				
1 [n]	[FV]		(103,680)	Value after 1 year
2 [n]	[FV]		(111,974.40)	Value after 2 years
3 [n]	[FV]		(120,932.35)	Value after 3 years.

2) With [%] key.

96000	[ENTER]		
8 [%]	[+]		(103,680)
8 [%]	[+]		(111,974.40)
8 [%]	[+]		(120,932.35)

Advantage: this procedure also provides theaamount of each yearly increase. This may tell us, for instance, by what year the increase due to the projected inflation pays for the mortgage.

3) Use of constant.

Taking 8% is equivalent to multipying by .08.
Adding 8% is equivalent to multipying by 1.08. This will be our constant.

1.08	[ENTER] [ENTER] [ENTER]	Creates constant in stack.
96000		
[x]	(103,680)	
[x]	(111,974.40)	
[x]	(120,932.35)	

One keystroke provides each successive value.

Annual Rate, Annual Yield
The effect of the compounding periodicity.

Let us compare two \$ 10,000 one year investments.

Investment A is a straight note returning \$ 12,400 at the end of the year.

Investment B pays 12 monthly payments of \$ 100, plus an extra \$ 11,200 at the end of the year.

Rate of return on A:

10000 $\boxed{\text{PV}}$
12400 $\boxed{\text{CHS}}$ $\boxed{\text{FV}}$
1 $\boxed{\text{n}}$
i

(24% return)

Rate of return on B:

10000 $\boxed{\text{PV}}$
100 $\boxed{\text{CHS}}$ $\boxed{\text{PMT}}$
11200 $\boxed{\text{CHS}}$ $\boxed{\text{FV}}$
12 $\boxed{\text{n}}$
$\boxed{\text{i}}$ 12 $\boxed{\text{x}}$

(22.8% return)

On the basis of these numbers one would be tempted to prefer investment A over investment B. But clearly B is a better investment as both investments provide \$ 2,400 in interest, and B makes part of that interest available earlier. (The first 11 monthly payments of \$ 100 could be re-invested, thus providing an increase of more than \$ 2,400 at the end of the year).

The discrepancy occurs because of the different compounding periodicities of the two rates. We have been comparing with different yardsticks: investment A offers 24% with annual compounding, investment B 22.8% with monthly compounding and the two cannot be compared.

A comparison between rates becomes possible if we re-express the return on investment A with monthly compounding:

10000 $\boxed{\text{PV}}$ 12400 $\boxed{\text{CHS}}$ $\boxed{\text{FV}}$
12 $\boxed{\text{n}}$ $\boxed{\text{i}}$ 12 $\boxed{\text{x}}$

(21.71% return)

21.71% is clearly not as good as 22.8%.

Annual rate, annualized yield.

With the monthly equivalent of 21.71% still in the i memory, let us invest \$100 for one year, and see what it grows into:

100 [PV] [FV] (124)

So \$ 100 grows into \$ 124. That is a 24% increase over the year. The annual interest rate may be 21.71%, but the annualized yield is 24%. Considering the effect of investing \$ 100 for one year at a given rate and at a given compounding periodicity provides a simple way of calculating the equivalent annualized yield.

Example

Calculate the annualized yield that corresponds to 8% interest with monthly compounding and with daily compounding.

100 [PV] 8 [BLUE] [i] 1 [BLUE] [n] [FV] (108.30)	The annualized yield,with monthly compounding, is 8.30%.
8 [ENTER] 365 [÷] [i] 365 [n] [FV] (108.33)	With daily compounding, the annualized yield is 8.33%.

Annualized yield, annual rate.

What is the rate with monthly compounding that corresponds to a given annualized yield? The opposite of our previous problem is also easy to solve if we consider the effect of our requirements on a \$ 100 investment for one year.

Example.

At what monthly compounded interest do I need to invest my money if I want an annualized yield of 15%?

This implies that \$ 100 must grow into \$ 115 in one year.

$\boxed{\begin{array}{llll} 100 & \boxed{PV} & 115 & \boxed{CHS}\ \boxed{FV} \\ 1 & \boxed{BLUE \mid n} & & \\ \boxed{i} & & 12 & \boxed{x} \end{array}}$ (14.06%)

Provided the money is invested for an exact number of years, a rate of 15% with annual compounding has the same effect as a rate of 14.06% with monthly compounding. (Note that the 14.06% really has some hidden decimals to it. If we key it in ourselves rather than calculating it as above slight discrepancies will result from the rounding).

Problems

1) A person invests $ 9,000 at 10% interest amortized with payments of $ 200 per month. How much higher would the outstanding balance of the loan be if he invested $ 9,100 instead?

9,000	PV		
10	BLUE	i	
1	BLUE	n	
200	CHS	PMT	
FV			(7,429.30)
9,100	PV		
FV			(7,539.78)
-			(110.47)

Each hundred dollar so invested for 1 year returns $ 110.47, or 10.47 in interest.

So any amount invested for one year yields 10.47%; that is the annualized yield.

2) A 25,000 dollar 2nd T.D. at 12% interest, payable $ 300 per month is being sold at a discount. There is a 3 year due date on the note.
What selling price will result in a 20% rate of return (monthly compounding) for the buyer?
What selling price would result in a 20% annualized yield to the buyer?

100 $\boxed{PV}$ 120 $\boxed{CHS}$ $\boxed{FV}$ 0 $\boxed{PMT}$ 1 $\boxed{BLUE \mid n}$ The data
i (1.53% or 18.37% annual corresponds to 20% annualized yield)
$\boxed{STO \mid 7}$ 25000 $\boxed{PV}$ 300 $\boxed{CHS}$ $\boxed{PMT}$ 12 $\boxed{BLUE \mid i}$ 3 $\boxed{BLUE \mid n}$
$\boxed{FV}$ 20 $\boxed{BLUE \mid i}$ $\boxed{PV}$ (20,672.81) $\boxed{RCL \mid 7}$ $\boxed{i}$
$\boxed{RCL \mid 7}$ $\boxed{i}$ $\boxed{PV}$ (21,476.76) $\boxed{-}$ (803.95 difference between prices)

Double the interest, double the rate? Generally not.

It is frequently assumed that if we double the interest received for an investment, we have doubledthe rate of return on that investment. This is forgetting the effect of compounding on the amount of interest that is earned. In fact, the assumption is correct only in those circumstances when no compounding is allowed to occur: investments that last for a single compounding periodicity, or interest only situations.

Example:

Let us invest \$ 10,000 for 2 years at 10% interest, annual compounding. Let us then invest the same amount for 2 years at 20% interest.

10000 $\boxed{\text{PV}}$
2 $\boxed{\text{n}}$
10 $\boxed{\text{i}}$
$\boxed{\text{FV}}$ (\$ 12,100) or \$ 2,100 in interest.

20 $\boxed{\text{i}}$
$\boxed{\text{FV}}$ (\$ 14,400) or \$ 4,400 in interest, more than double the interest for 10%.

This implies that if we had earned only twice the amount of interest, our interest rate would be less than twice the initial rate-- 19.16% in fact.

Why this occurs, how the compounding of interest is the cause, may be seen by examining year after year what happens to our two previous investments.

Interest rate (annual compounding)	Investment	First year	2nd year
10%	10,000	11,000 (1,000)	12,100 (1,100)
20%	10,000	12,000 (2,000) 11,000 1,000 12,000	14,400 (2,400) 13,200 (2,200) 1,200 (200) 14,400 (2,400)

Each number is followed in parenthesis by the previous year's interest. It is clear that the extra interest earned at 20% during the first year (\$ 1,000) is what causes the 2nd year interest to be \$ 200 more than double the interest earned at 10% during the 2nd year.

If double the amount of interest is not equivalent to double the rate of interest, it follows that adding the interest amounts earned from two different rates does not provide a rate of return that is the sum of those two rates.

Example:

$ 10,000 invested at 10% for 10 years (annual compounding) grows into $ 25,937.42, which means $ 15,937.42 in interest.
The same $ 10,000 amount invested for 10 years at 6% grows into $ 17,908.48, which means $ 7,908.48 in interest.
Let us now consider an investment of $ 10,000 for 10 years that earns as much interest as the two previous investments, or $ 23,845.90, and returns principal and interest of $ 33,845.90. The rate of return provided by such an investment can be calculated as follows:

10,000 [PV]
33,845.90 [CHS] [FV]
10 [n]
[i] (12.97%)

The rate of 12.97% is a far cry from the sum of the two rates, 16% !

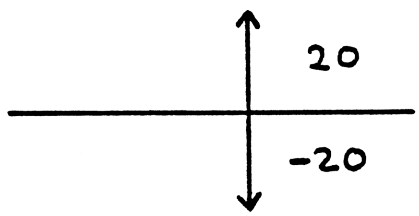
Three theoretical considerations (Skip on first reading)

We are now going to examine three techniques which consist in interpreting or adjusting cash flows. They can be used to make it possible, or easier, for problems to be solved with the calculator. The techniques are:

- The Net Amount approach.
- The Time adjustment approach.
- The Arbitrary value approach.

1) The Net Amount approach.

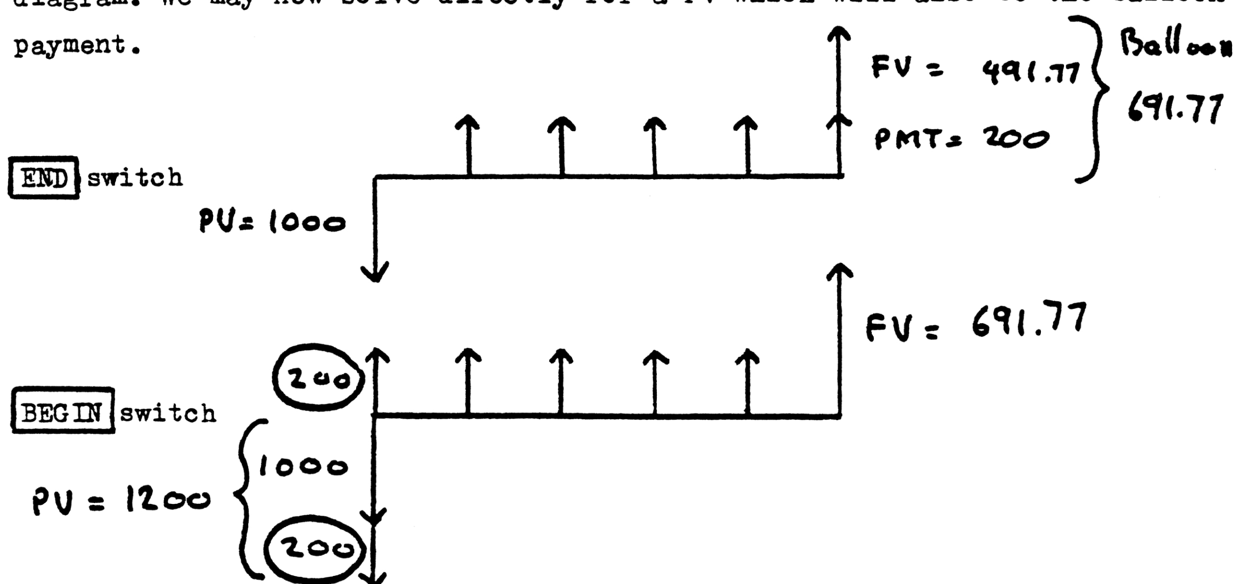
What matters in a cash flow diagram is the Net Amount of cash changing hands at any one time. The net amount is not affected if we add or subtract the same amount at the same point in time to both sides of the time line.



These two opposing cash flows could be added without affecting the financial transaction: the net amount added at that point in time is zero.

Application: transform an END switch situation into a BEGIN switch situation , or vise versa.

Let us go back to our last Balloon payment problem. Let us then create another diagram with 1 payment added to both sides of the time line at the very beginning of the line . The second diagram can be interpreted as a BEGIN switch situation with the same net amounts as the first diagram. We may now solve directly for a FV which will also be the balloon payment.



(12 [i])

2) Time adjustment approach.

A cash flow can always be changed to another time slot, without affecting any other aspect of the financial transaction, provided that the value of the cash flow is adjusted for the time change at the given rate.

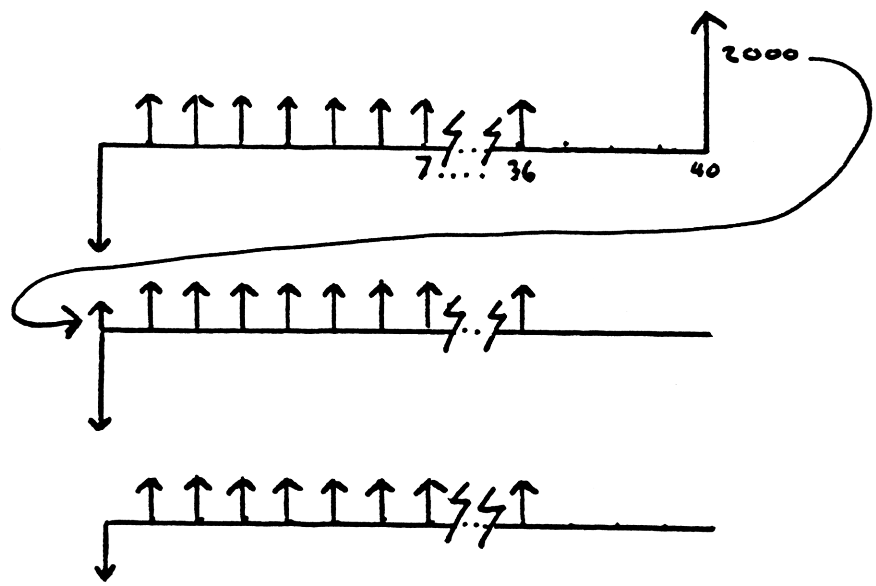
Example: A $ 5,000 car loan is to be paid off with 36 monthly payments, paid at the end of each month, plus a balloon payment of $ 2,000 to be paid at the end of the 40th month. The interest is 12%. What are the monthly payments?

The easiest solution consists in eliminating the troublesome balloon payment by deducting its Present Value from the loan amount itself.

Original situation

Time adjustment

Net amount



We are now in a position to solve for the payments.

Keystroke suggestion:

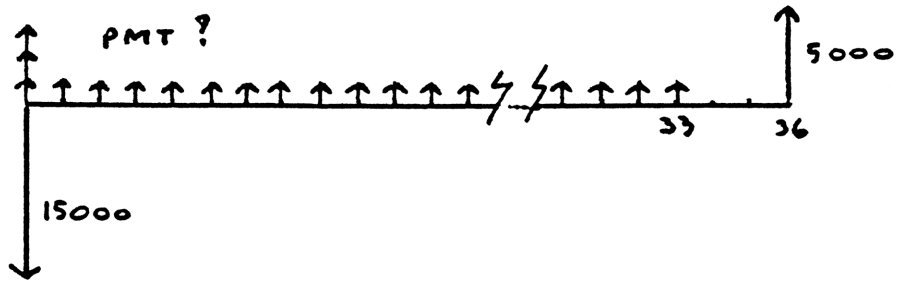
5000 [CHS] [ENTER] 40 [n] 12 [BLUE] [i] 2000 [FV]

[PV] [-] [PV] 36 [n] 0 [FV] [PMT] (121.45)

It is as if two loans had been made initially: one which was paid off with $ 2,000 after 40 months, the other paid off with the 36 monthly payments. The time adjustment approach helped us to eliminate the first loan, and solve the second.

3) <u>The arbitrary value approach</u>

<u>Example:</u> A rental firm rents a tractor for which they paid $ 15,000. When it is returned in 3 years it will have a residual value of 1/3 of it original amount, or $ 5,000. There are 36 monthly payments, paid at the end of each month, except that the last three payments are paid at the time of purchase. What should those payments be to assure an 18% return?



As we do not know the amount of each payment, we cannot calculate the net amount invested by the firm at the beginning of the rental. Because we do not know this net amount we cannot calculate the payments. We are caught in a vicious circle.

<u>Solution.</u>
Let us give an arbitrary value to the payments, for instance $ 1. Let us calculate the **cost** of the equipment which would allow 18% return for such payments. As the actual net cost of the equipment was 418.11 times greater, the payments must be that much greater, or $ 418.11.
(We first need to calculate the net cost of the equipment by deducting from its actual cost the PV of the residual value: $ 5,000 discounted for 3 years at 18%, monthly compounding. This is using the time adjustment approach to take care of the residual value).

<u>Keystroke:</u>

36 [n] 18 [BLUE] [i] 5000 [FV] [PV]	(-2,925.45: PV of residual)
15000 [+] [STO] [1]	(12,074.55: net cost)
33 [n] 1 [PMT] 0 [FV] [PV]	(-25.88 : PV of 33 payments
[CHS] 3 [+]	(28.88: PV of the 36 pmts.
[RCL] 1 [x≷y] [÷]	(418.11: ratio cost /PMT.)

That ratio, multiplied by the arbitrary value chosen for the payments, gives us the value of the payments which we sought: 418.11 x 1 = 418.11.

Now that we have an answer, we may want to check with the irregular cash flow keys that these payments, and the $ 5,000 residual value, do provide an 18% return, calculated this time with no adjustments.

The six functions of the dollar.

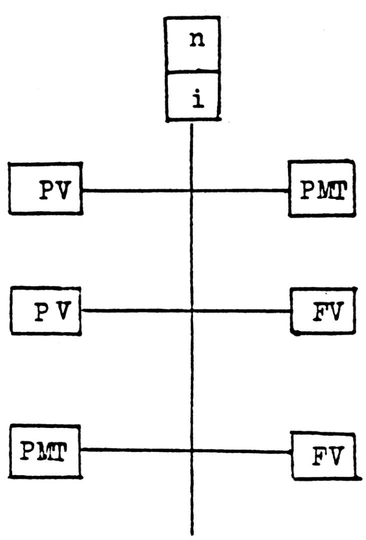
What monthly payments would amortize a 30 year, 8.5% interest loan of $ 1 ? We know how to get the answer:

30	BLUE	n
8.5	BLUE	i
1	PV	
PMT	(.007689135)	

Is the question absurd? Not really. If we did not have a financial calculator it would be very useful to know that number. We could multiply it by whatever the loan amount really was, and we would have the monthly payments.

Six functions of the dollar tables provide answers to this and similar questions, all concerning ONE dollar. They do so for all likely combinations of interest rate, term, and periodicity of compounding and payments. Needless to say, these tables can be extensive, and expensive.

The six functions correspond to six different questions. We saw earlier that there are 3 different ways in which the three forms of money (PV, PMT and FV) could be paired, each pair corresponding to a different financial trasaction. For any given value for n and i two questions can be asked according to which element of the pair is known, and which is being sought. TWO questions for each of THREE financial trasactions gives us the SIX questions which the tables answer for most combinations of n and i .



PV	PMT	FV	Name of function
1	?		Partial payment (amortized PMT)
?	1		Present worth of one per period
1		?	(Future) amount of one.
?		1	Present worth of one (in future)
	1	?	(Future) amount of one per period
	?	1	Sinking fund factor

The easiest way of visualizing what the six functions of the dollar mean is to express them in terms of the three 'forms' of money represented by the PV, PMT and FV keys. For instance:

A sinking fund factor situation is when a FV is known, and we seek to know what PMT will build up to that FV. The sinking fund factor itself is the value of the PMT which, under the conditions described by $\boxed{n}$ and $\boxed{i}$, will build up to a FV of ONE DOLLAR.

It is clear that:

1) The calculator can find <u>any</u> function of the dollar for <u>any</u> combination of $\boxed{n}$ and $\boxed{i}$.

2) With the calculator we hardly ever need the functions of the dollar: we can calculate directly the value which balances the full amount of the loan, of the payments, or of the future value.

3) The HP 38 allows us to answer directly questions involving all various combinations of the three forms of money, not just those which pair two of these forms.

4) For all of these various combinations, the calculator allows us to solve for $\boxed{i}$ or for $\boxed{n}$.

Financial calculators have done to functions of the dollar tables what scientific calculators have done to the slide rule.

PART IV

Arithmetic

See Part I for a basic presentation of the most essential features.

The storage register.

There is little to add to our preliminary presentation. We will see later on that these memories can also be used by the calculator itself for programming, to store irregular cash flows, and for statistical data.

The 20 register memories are numbered from 0 to 19: R 0, R 1, R 2, etc. To store data into memories 10 and above, and to recall that data, we use the decimal point instead of the initial 1 . The reason for this is simple: As soon as we press STO 1 the number in the display is stored in memory 1, and erases any previous number stored in R 1 . This would take place even if our intention was to add a 5 and store in memory 15 !

Store 789 in memory 5	789	STO	5	
Store 53 in memory 15	53	STO	.5	
Recall R 5		RCL	5	(789)
Recall R 15		RCL	5	(53)

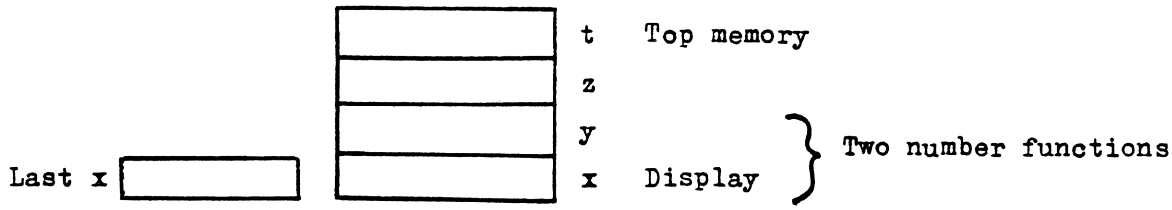
Storage register arithmetic.

With register memories 0 to 6 only, arithmetic operations can be performed directly on the content of the register memories. This is done by inserting the operation sign (+ - x ÷) between the STO key and the memory address:

Store 789 in memory 1	789	STO	1		
Add 123 to content of memory 1	123	STO	+	1	
Mutliply result by 12	12	STO	x	1	
Recall final result		RCL	1		(10944)

<u>The stack</u>

A stack of 4 memories, plus one.



Data floats up and down from one memory to another forming an automatic memory bank for partial answers. The bottom memory, x, is visible on the display. It is through memory x that all communication takes place between the calculator and the user.

One number functions are performed on the content of memory x.
Two number functions are performed on the content of x and y.

[CLx] The <u>Clear x</u> key. Use when wrong data has been entered in x.

[ENTER↑] This key <u>separates</u> two numbers keyed in in succession. It does so by pushing the first number into the y memory. (At the same time it pushes data from y into z, and from z to t. Any data in t would be pushed over the top and lost).

[BLUE] [R↓] The <u>Roll down</u> key makes it possible to review the content of the stack. The content of each memory falls down to the memory below, with x moving up to t. By pressing 4 times the whole content of the stack has been displayed, and is back in its original position.

[x⇄y] This key exchanges the content of memories x and y. This can be used to put the numbers in the correct position for a two number operation. Pressing the key twice in succession provides a rapid check on the content of the y memory.

[BLUE] [LAST x] Provides access to to the adjunct <u>Last x</u> memory. This is a fail-safe feature. By making it possible to retrieve the previous value of x, an operation performed by mistake can be cancelled.

Example: the stack at work.

	Key in	x (Display)	y	z	t
1) I go shopping with \$100.	100	100			
2)	$\boxed{\text{ENTER}}$	100.00	100		
3) I see a gift costing \$12	12	12	100		
4)	$\boxed{\text{ENTER}}$	12.00	12	100	
5) I buy two.	2	2	12	100	
6) For a total of	$\boxed{\text{X}}$	24.00	100		
7) I see another item at \$17	17	17	24	100	
8)	$\boxed{\text{ENTER}}$	17.00	17	24	100
9) I buy three	3	3	17	24	100
10) For a total of	$\boxed{\text{X}}$	51.00	24	100	(100)
11) My total purchase is	$\boxed{+}$	75.00	100	(100)	(100)
12) I get a 10 % discount	10	10	75	100	(100)
13)	$\boxed{\%}$	7.5	75	100	(100)
14)	$\boxed{-}$	67.5	100	(100)	(100)
15) 6 % sales tax is added.	6	6	67.5	100	(100)
16)	$\boxed{\%}$	4.05	67.5	100	(100)
17)	$\boxed{+}$	71.54	100	(100)	(100)
18) Which leaves me with	$\boxed{-}$	28.45	(100)	(100)	(100)

Notes on the previous example.

- Note how the \$100 keyed in initially floats 'up and down' the stack and becomes available when it is needed. The same is true of partial answers such as 24 (keystrokes 6 to 11).

- When the initial 100 is ENTERed (2),it is pushed into the y memory but remains visible in the display. It remains in x for convenience, and will be erased whenever another number is keyed in (3) or recalled.

- When a number appearing on the display is the result of a calculation there is no need to ENTER it before keying in a new number. In (6)and(7) 24 automatically moves up into y when 17 is keyed in. This also applies to numbers which have been recalled in any way. Note that numbers which have become immune are displayed with the standard two decimals.

- When a two number arithmetic operation is performed on the contents of x and y, the two numbers are replaced by the result of the operation, and the whole stack slides down again (Keystrokes 6,10,11, 14,17,18). This is not the case when a percentage is calculated (13 and 16). With a percentage, the original calculation remains in the y memory for immediate calculation of a percentage discount or percentage added (Keystrokes 14 and 17).

- When a number has been entered into the top memory and is made to float down again, it is not erased from the higher memories which it has once occupied (See the 100 in parenthesis from (9) to the end). This feature makes it possible to use the stack as a constant.

Illustration: the stack as a constant.

The first year payments on a FHA Graduated Payment Mortgage, plan II are $ 371.23. Five years in succession these payments will increase by 5% each year. Calculate the payments for each year. Adding 5 % is equivalent to multiplying by 1.05. This will be our constant.

	Key in	x	y	z	t
Establish constant	1.05	1.05			
	ENTER	1.05	1.05		
	ENTER	1.05	1.05	1.05	
	ENTER	1.05	1.05	1.05	1.05
Initial payments	371.23	371.23	1.05	1.05	1.05
1st increase	X	389.79	1.05	1.05	1.05
2nd increase	X	409.28	1.05	1.05	1.05
3rd increase	X	429.75	1.05	1.05	1.05
4th increase	X	451.23	1.05	1.05	1.05
5th increase	X	473.79	1.05	1.05	1.05

- Note that there is no need to clear between operations performed with the stack. Useless data previously keyed in is pushed up, and perhaps even out, but does not interfere with the new calculations.

Last x

When an operation is performed in the stack (on 1 or 2 numbers stored in x and y) the original x number is always discarded and replaced by the result of the operation. However the value of x which has been discarded is retained in the Last x memory. It can be retrieved by pressing [BLUE | LAST x].

Last x is a fail safe feature. Using it is like retrieving a document from the waste basket. It is particularly useful when the wrong operation key has been pressed.

Example

You want to calculate 82.123 — 31.789 :

Last x memory	x (display.)	y	Your keystroke.
0	82.123	0	82.123
0	82.123	82.123	[ENTER]
0	31.789	82.123	31.789
31.789	2,610.61	0.	[X] (Mistake!!)
			To correct mistake:
31.789	31.789	2,610.61	[BLUE | Last x]
31.789	82.123	0.	[÷] (opposite operation as mistake)
31.789	31.789	82.789	[BLUE | LAST x]
31.789	50.334	0.	[—] (Correct operation!)

SO:

[BLUE | LAST x]
[ ] (Opposite operation as mistake)
[BLUE | LAST x]

This simple keystroke restores the calculator to the status quo ante, the situation before the mistake. The correct operation can be performed. ([BLUE | LAST x] alone will do with one number functions.)

FORMAT

I For the sake of convenience the number shown in the display is not always exactly the same as the internal number on which calculations are made. The following rules apply:

1) A number which has been entered (ENTER↑ , x⇄y twice, R↓ four times), which has been stored and recalled, or which is the result of a calculation is normally displayed with two decimals only: dollars and cents.

2) The display is automatically rounded to the nearest cent.

Keyed in	Display	
3.106	3.106	rounded to nearest 'cent'.
ENTER	3.11	
1.996	1.996	rounded to nearest 'cent'
ENTER	2.00	

3) Though the display is rounded, the calculations are performed with the full number of decimals. The internal number is not rounded.

4) To display more (or fewer) than 2 decimals, press GOLD and the number of decimals selected.
GOLD 9 will provide the maximum number of decimals.
GOLD 0 will round the display to the nearest integer (no decimals).

5) For a temporary glance at the full internal number, press and hold down GOLD ENTER (This is the Clear prefix function)

II Scientific notation

1) GOLD • Switches the display format to scientific notation.

(In scientific notation a number is expressed by the product of a number from 1 to 9 followed by decimals and by 10 to a given power.
For instance: $320 = 3.2 \times 10^2$ $(3.2 \times 10 \times 10)$
$56{,}971 = 5.6971 \times 10^4$ $(5.6971 \times 10 \times 10 \times 10 \times 10)$
A negative exponent means that the number from 1 to 9 and its decimals are divided by 10 to the given power.
For instance: $0.065 = 6.5 \times 10^{-2}$ $(6.5 \div 100)$
Use the GOLD • keystroke to check your understanding of scientific notation).

In the display, 10 is left blank. Only its exponent is given:

320 will appear as 3.2000000 02 standing for 3.2×10^2.
The blank space indicates scientific notation.

2) The calculator automatically shifts to scientific notation if the result of an operation is too large or too small to appear significantly in ordinary notation. For instance:

Calculate 372^{21} (372 multiplied 21 times by itself)

Key in	Display
372	372
ENTER	372.00
21	21
BLUE y^x	9.5807992 53 (=9.5807992 x 10^{53})

3) The maximum exponent is 99.

9.999999999 99 or 9.999999999 -99 is a sign that the result of an operation is too large or too small to figure in the calculator.

4) A number can be keyed in directly in scientific notation by using the ENTER EXPONENT key: BLUE | EEX

7.32×10^{39} will be keyed in as follows: 7.32 BLUE | EEX 39

III Number alteration.

1) GOLD | RND — Rounds the internal value of a number to the value shown on the display.

2) GOLD | INTGR — Rounds the internal value of a number to the integer portion of the number displayed: the decimals are dropped.

3) BLUE | FRAC — Retains only the fractional or decimal part of a number. The integer is dropped. 5.372 becomes .372

When these these three alterations are performed the original number is lost, except that it is temporarily retained in the Last x memory.

The Percentage Keys

You are buying a $ 135,000 property with a 29% downpayment of $ 39,150.

In this and a multitude of other instances, we are dealing with
a total amount: the selling price, $ 135,000,
a part of that amount: the downpayment, $ 39,150,
a rate that expresses as a percent the ratio of the part to the whole, 29%.

Though speaking in terms of total, part, and rate does not do justice to all circumstances--such as when the percentage rate is over 100--, using these terms will allow a simple presentage of various percentage calculations.

	Data in y and x memories		Question	Answer in x memory of stack.	
1)	y	TOTAL	%	y	TOTAL
	x	RATE		x	PART
2)	y	TOTAL	GOLD %T	y	TOTAL
	x	PART		x	RATE

These two keystrokes are the opposite of one another. One transforms the rate into the corresponding value of the part, the other transforms the part into its value as a percentage of the total $\boxed{\%T}$.

The 2nd question can be used to perform a different, less orthodox, function:

3)	y	RATE	GOLD %T	y	RATE
	x	PART		x	TOTAL

(See The 3-Step solution for another approach to solving the same problem)

Illustration:

Knowing any two elements from the initial example, calculate the third.

-	135000	ENTER	29	%		(39150)
-	135000	ENTER	39150	GOLD	%T	(29)
-	29	ENTER	39150	GOLD	%T	(135000)

4) Another key allows us to calculate the percentage difference (Δ %) between two number, for instance the increase or decrease expressed as a percent of the first amount as one moves from the first amount to the second: (The Greek letter Delta Δ is a mathematical symbol for the difference).

y	First amount	GOLD Δ %	y	First amount
x	Second amount		x	Percentage difference

<u>Illustration</u>

- A property previously purchased for $ 85,000 is now selling for $ 142,000. What is the percentage increase?

  85000 [ENTER] 142000 [GOLD | Δ %] (67.06%)

- A 2nd T.D. note with a face value of $ 18,000 is selling for $ 14,400. What is the discount?

  18000 [ENTER] 14400 [GOLD | Δ %] (20%)

<u>Comments</u>

-(1) retains the <u>total</u> in the y memory. So adding the <u>part</u>, or subtracting it, can be done immediately with the [+] or the [-] key.

Ex.: Selling price of a $ 789 refrigerator after 15% discount and 6% sales tax:

789 [ENTER] 15 [% | -] 6 [% | +] (710.89)

- Instead of (1), if one just wants to calculate the value of the part, and not add it back or deduct it as in the previous comment, it is possible to key in the initial data in the opposite order:

y	RATE	%	y	RATE
x	TOTAL		x	PART

This is so because 15% of 789 is the same amount as 789% of 15. So there is frequently no need to press [x⇄y] if the data happens to be keyed in in the wrong order. This may also save a step in some programs.

- Keystroke (3) clearly was not meant by the calculator to provide the data indicated here, but it does so in a convenient way that may save programming steps.

The Percentage Keys, Practice

There are of course more than one way to solve these problems. Rather than the arithmetic calculations you may know, I am suggesting that you use here the three percentage keys. The number in brackets refers to the four cases shown in the previous explanations.

a) What is 75% of $ 117,000 ? (1) (87,750)

b) A property sells for $ 235,000. The agent gets a commission of $ 12,000. What percentage of the selling price is that? (2) (5.11%)

c) A client gets a 65% loan of $ 92,000. What is the selling price of the property he is referring to? (3) (141,538.46)

d) A note with a face value of $ 25,000 is being offered for $ 17,000. What discount is that? (4) (32%)

e) A 2nd T.D. lender charges 13 points. What amount must you borrow in order to net $ 17,000? ($ 17,000 = 87% of loan) (3) (19,540.23)

f) A seller estimates that his selling costs are 8% of his selling price. How much must he sell his property to net $ 100,000? ($ 100,000 = 92%) (3) (108,695.65)

g) A property that was bought for $ 73,000 is on the market for $ 128,000. What percentage increase is that? (4) (75.34%)

The 3 step solution

The following very simple arithmetic reasoning can be put to many uses, and is an alternative to the unorthodox use of GOLD %T in previous pages.

1) A broker is setting up a new office. He calls an office furniture store and is told that 7 desks will cost him $ 903. Having thought things over, he decides to order 9 desks. How much will these cost?

3 step solution:

Step one states the facts:

7 desks cost $ 903.

Step two calculates the cost of one desk:

1 desk costs 903 ÷ 7

Step three draws the conclusion:

9 desks cost 9 times more than one desk, or $\frac{903}{7} \times 9 = \$\ 1161.$

903 [ENTER] 7 [÷]	Calculates cost of one desk
9 [x]	Multiplies by 9 for cost of 9 desks.

2) Your client can afford a $ 92,000 loan. It is an 80% loan. What is the price of the property?

Step one: 80% is $ 92,000

Step two: 1% is 92,000 ÷ 80

Step three: 100% is $\frac{92,000}{80} \times 100$ (115,000)

92000 [ENTER] 80 [÷]	Calculates one percent of selling price.
100 [x]	Multiplies by 100 for full selling price.

Dividing by 80 and multiplying by 100 is the same thing as dividing by .8 (point 8). The three step solution allows us to forget the formula, and can be used in cases where the formula to be applied is not obvious.

3) A client wants to net \$ 112,000 from the sale of a property he owns free and clear. Commission and other selling costs will approximate 8% of selling price. What should the property be sold for?

After having paid the 8% costs, the seller will net 92% of the selling price. (100% - 8% = 92%). So \$ 112,000 represents 92% of selling price. We can now proceed with the three step solution.

112000 $\boxed{\text{Enter}}$ 92 $\boxed{\div}$	Calculates 1%
100 $\boxed{\text{x}}$	Calculates 100% or full selling price

4) A client wants to net \$ 56,000 after 8% selling costs and \$ 42,000 loan pay-off. What should the property be sold for?

The 92% of the selling price which will remain after the selling costs have been paid represent the sellers loan obligations plus his cash return, so:

92% of Selling Price = \$ 56,000 + \$ 42,000. With three step solution:

56000 $\boxed{\text{ENTER}}$ 42000 $\boxed{+}$	Calculates 92% of selling price.
92 $\boxed{\div}$	Calculates 1% of selling price.
100 $\boxed{\text{x}}$	(\$ 106,521.74) Full selling price.

5) A client needs to net \$ 18,000 from a 2nd T.D. loan. The lender is charging 15 points and \$ 350 for the loan. How much must your client borrow to net \$ 18,000?

Here 85% of selling price (100% - 15%) is equal to \$ 18,000 + \$ 350, as the 15% for the points are taken from the full loan amount, before the \$ 350 are paid. Three step solution:

<u>Step one:</u> 85% = 18350

<u>Step two:</u> 1% = $18350 \div 85$

<u>Step three:</u> 100% = $\frac{18350}{85} \times 100$

18000 $\boxed{\text{ENTER}}$ 350 $\boxed{+}$	
85 $\boxed{\div}$ 100 $\boxed{\text{x}}$	(\$ 21,588.24)

6) Three investors finance a property (the institutional lender, the seller who carries an 2nd T.D., and the buyer who invests his down payment). All together, they require 11.43% of the selling price as the yearly cash return on their investment. That cash return is to be paid to them with the Net Operating Income (NOI) of $ 52,500. What is the selling price of the property that satisfies their requirement? (See section on Income Stream appraisal).

<u>Step one:</u> 11.43% of Selling Price equals $ 52,500.

<u>Step two:</u> 1% of selling price equals $\frac{52,500}{11.43}$

<u>Step three:</u> Full 100% of selling price equals $\frac{52500}{11.43} \times 100 = \$ 459,317.$

<u>More problems</u> requiring the three step solution:

1) What must a property sell for if the seller is to receive 112,000 dollars after he has paid the 6% commission? ($ 119,148.94)

2) What must a property sell for if the seller is to receive $ 45,000 after having paid for 7% selling costs and $ 36,000 in outstanding loans? ($ 87096.77)

3) A lender charges 1.5 points and $ 200 as loan costs. What loan will net $ 80,000 to the borrower? ($ 81,421.32)

4) What is the price of the property that can be financed with an 80% loan of $ 115,000? ($ 143,750)

5) A Net Operating Income of $ 100,000 must satisfy the investors' cash requirement of 9.17% of the selling price. What selling price will achieve this? (9.17 is the Cap. rate). ($ 1,090,512)

6) An office pays 6% of its commissions to the Franchisor. You get 65% of the amount of the commission that remains in the office. Your office gets 1/2 (50%) of the 5% commission on a sale. What selling price will bring you $ 2,000?

5 [ENTER] 2 [÷] 6 [%] [-] 65 [%] Calculates your share as %

2000 [x≷y] [÷] 100 [x] (130,932.90) Three step solution

Calculations with percents.

It is frequently useful to have answers as a percentage of another number. This can be done by solving specific problems and then transforming the answers into the desired percentage. It can also be done by doing the whole calculation with percents instead of actual amounts.

As an illustration, let us ask various questions concerning an 80% loan at 13% interest amortized over 30 years. The answers, expressed as a percentage of the selling price, will apply to all properties financed with such a loan.

1) What percentage of the price of the house goes to monthly payments?

Because we want all the answers to be a percentage of the selling price, the selling price must be 100 (that means 100% of itself). So loan is 80. Let us key it in, and question for the payments:

80 [PV]	
13 [BLUE | i]	
30 [BLUE | n]	
[PMT]	(.88) The buyer must pay .88% of the selling price per month.

2) With the previous data still in, find what percentage of the selling price will go to interest and to principle reduction during the first year.

A special precaution must be taken before we use the [GOLD | AMORT] procedure. The procedure automatically rounds the internal numbers to the amounts visible on the display. With all the numbers now expressed as a percentage, the automatic rounding which normally eliminates only fractions of a cent could now chop off the equivalent of a few dollars. To prevent this from happening, we need to switch to a 5 or 6 decimal format:

[GOLD | 6]	Switches to 6 decimal format.
12 [GOLD | AMORT]	10.386% of selling price is the interest for year 1.
[x⇄y]	.233 %, principal reduction for the first year.
[RCL | PV]	79.76% is the balance of the loan at the end of yr.1.

We can repeat for the 2nd year data, or use the amortization program.

3) What interest on a 30 year, 80% loan-to-value ratio loan will result in payments of 1% of the selling price per month?

(After having used the AMORT procedure, we need to start with new data. We also no longer need the 6 decimal format)

[GOLD] [2]	Re-establishes 2 decimal format.
80 [PV]	
30 [BLUE] [n]	
1 [CHS] [PMT]	
[i] 12 x	(14.82%)

4) For tax purposes, a seller who agrees to finance the sale, wants to be paid less than 30% of his selling price during the first calendar year of his sale. What is the maximum downpayment that he can accept, as a percentage of the selling price, knowing that he will receive 8 monthly payments during that first year, and that he is charging 13% interest, amortized over 30 years on his loan?

The problem can easily be solved by a trial and error procedure, using percents instead of actual numbers.

72 [PV]	First test: 72% loan or 28 % downpayment.
13 [BLUE] [i]	
30 [BLUE] [n]	
[PMT]	Calculates payment as % of selling price.
8 [n]	
[FV]	Calculates balance of loan at the end of the first calendar year. That number must be as close as 70 as possible, but not equal to or lower than 70. A value of 70 would mean that the seller has been paid 30% of his selling price. The answer here (71.86) is too high.

70.5 [PV]	Second test. 29.5% downpayment.
30 [BLUE] [n]	Re-sets 30 year amortization data.
0 [FV] [PMT]	Clears FV and calculates payment.
8 [n] [FV]	70.37% loan, still too much. Further trial would show

70.13 too low, and 70.14, for a downpayment of 29.86% just above the limit.

The advantage of working with percentages instead of actual amounts is that the answers remains valid whatever the actual selling price turns out to be.

5) An investor wants to buy a $ 25,000 2nd T.D (interest only, 3 year due date) in order to get a 25% return on his investment. What discount should he obtain?

In this case, the discount would be the same whatever the loan amount. By doing the calculations on a $ 100 loan instead of the actual amount of $ 25,000 we calculate a discounted amount that immediately reveals the discount as a percentage.

100 [PV]	
13 [BLUE | i]	
20 [BLUE | n]	
[PMT]	
3 [BLUE | n]	
[FV]	
25 [BLUE | i]	
[PV]	(75.23)
100 [-]	(24.77% discount)

So it is clear that many calculations where the final answers are percentages can be done throughout with percents instead of with actual dollar amounts. Our calculation of the loan constant (Part V, page 1) was a slightly modified example of that approach.

DATES.

1) Date format.

With top right hand switch on END/M.DY (Month.Day Year), a date is keyed in in that order, with the decimal point after the number of the month, and two digit (01, 31, 15) for the day of the month.

July 2nd, 1982 becomes 7.021982

December 15, 1990 becomes 12.151990

2) GOLD | Δ DAYS gives actual number of days between two dates keyed in in x and y register. The number will be negative if the first date keyed in is in fact the last date in time.

The number of days based on a 30 day month and 360 day year is stored in y. Press x⇄y to retrieve.

3) GOLD | DATE calculates the date that is a specific number of days before or after a given date. Key in that date, ENTER, key in the number of days between the two days (negative number if you are moving back in time). GOLD | DATE calculates the date. The single number to the right gives the day of the week, with Monday, the first day of the working week, as 1.

4) To find the day of the week for a specific date, question the calculator for the date that is 0 days away from that date.

Examples

How many days between March 31, 1962 and December 2, 1972?

3.311962 ENTER 12.021972 GOLD | ΔDAYS (3.899)

How many days on a 30 day month, 360 year basis?

x⇄y (3842)

Day of the week for Jan. 1st, 2000?

1.012000 ENTER 0 GOLD | DATE (1,01,2000 6 or Saturday)

Date 220 days from February 9, 1980?

2.091980 ENTER 220 GOLD | DATE (Tuesday Sept 16, 1980)

One approach to
Income property appraisal.

I Loan constant.

The loan constant measures the yearly payment on a loan as a proportion of the original amount of that loan. Expressed as a percentage of the loan it is a number slightly higher than the interest rate as it represents the payment of interest and principal. It would be the same as the interest rate for a loan payable interest only.

It can be calculated by dividing the loan amount by the corresponding yearly payments.(Multiply by 100 to get the constant as a percentage). As it depends entirely on the interest rate and the term of the loan, not the loan amount, the loan constant can be calculated without referring to a specific loan. The simplest procedure is to calculate the monthly payment corresponding to a $ 1,200 loan. (12 times the monthly payment on a $ 100 loan).

Example: Loan constant for 9%, 30 year loans:

9 [BLUE | i] 30 [BLUE | n] 1200 [PV]
[PMT] (-9.66) The loan constant is 9.66%

This means that 9.66% of the original amount of the loan has to be paid each year. That amount is constant throughout the life of the loan.

The loan constant can be more important to investors than the interest rate as it bears directly on the cash return to the investment for a considerable period of time.

Practice: Calculate the loan constant for

9.75% 20 year loan (11.38%)
10.25% 30 year loan (10.75%)
Which could an investor prefer?

II Loan factor.

Some financial institutions calculate payments on a loan by means of a loan factor. It is the monthly payment corresponding to a $ 1000 loan.

9 [BLUE | i] 30 [BLUE | n] 1000 [PV] [PMT] (8.05)

Payments on a 75,000 dollar loan will be 75 x 8.05 = 603.75

Income stream appraisal.

The problem.

An income property with Net Operating Income (NOI) of $ 31,250 will be financed by a 75% loan at 9.75% amortized over 25 years and a purchase money 2nd T.D. note at 9% interest, payable 1% a month, all due in 5 years for 10% of the selling price. The buyer expects a 6% return on his 15% downpayment. What should be the selling price for these various requirements to be met?

Understanding the problem.

There are three investors: the institutional lender, the seller and the buyer. We know what proportion of the sales price each one is going to finance (75%, 10%, 15%). We know what proportion of his investment each one wants as a yearly cash return: the institutional lender wants interest and principal on his loan, or, as a percentage, he wants the loan constant (10.69%). The seller wants 1% a month, or 12% a year; and the buyer wants his 6% return.
By putting these two series of data together we can calculate the requirement of each lender as a percentage of the sales price:

Investor	Investment as a % of sales price	Yearly return on investment as % of investment	Yearly return on investment as % of sales price.
Lending institution (9.75%, 25 year.)	75%	10.69% (loan constant)	8.02%
Seller (1%/mo)	10%	12%	1.2%
Buyer (6%)	15%	6%	.9%
			$\boxed{10.12\%}$

To satisfy these requirements, the Net Operating Income must be equal to 10.12% of the selling price. So 1% of the selling price will be equal to NOI ÷ 10.12, and the full selling price to 100 times that.

$$\text{Selling Price} = \frac{31{,}250}{10.12} \times 100 = 308{,}794$$

Keystrokes.

9.75 [BLUE] [i]	25 [BLUE] [n]	1200 [CHS] [PV] [PMT]		(10.69, constant)
75 [%]		[GOLD] [Σ+]		
12 [ENTER]	10 [%]	[GOLD] [Σ+]		(Stores results in memory 2)
6 [ENTER]	15 [%]	[GOLD] [Σ+]		
31250 [RCL] 2	[÷]	100 [x]		(308,787)

(Note slight discrepancy due to rounding of loan constant in first calculation)

Packaging' the sale

Let us calculate what the sale price would be in the previous example if the seller offered a 2nd T.D. at 8% interest, interest only, all due in 5 years. The seller would now require only .8% of the selling price every year instead of 1.2%. The Capitalization rate would be 9.72% instead of 10.12, and the selling price would be $ 321,502. This represents an increase of $ 12,715 , 90% of which ($ 11,443) would represent the immediate cash increase received by the seller, and the remaining 10% the increase in the amount of the 2nd T.D. That sale would quite obviously be a much better deal for the seller.

This brings out the following:

- By being'greedy' on the cash return for the 2nd T.D., the seller is really using the leverage of the situation against himself.

- By being generous on the terms of the 2nd T.D., the seller is giving the buyer what he most needs while not sacrificing what he most wants. When sellers and buyers cannot agree on a price, compromising on the terms of the 2nd T.D. may provide the least painful means of reaching an agreement.

Program

Band of Investment Appraisal.

I Objective:

To provide an easy means of checking the effect of various financing options on the selling price of an income property, or on the buyer's cash on cash return.

Given the projected financing on an income property, the program
- calculates the selling price that provides a specific cash on cash return to the buyer,
- calculates the return to the buyer that results from a given selling price.

This program is best understood if it is considered as two separate programs, with the first 22 steps common to both. The program calculates a return on the downpayment when the data provides a selling price. When no selling price is provided, this indicates to the calculator that it must switch to the second option, and calculate a selling price.

II The variables

Memory	Data	Example
0	Selling price. This memory must be cleared (0 STO 0) if one wants to calculate a selling price. It acts as the switch between the two parts of the program.	0 STO 0
1	Net Operating Income (NOI)	50000 STO 1
2	Downpayment, as percentage.	20 STO 2
3	Cash on cash return to buyer, as %.	5 STO 3
4	2nd T.D. loan ratio.	10 STO 4
5	Monthly payment on 2nd T.D. as % of loan amount (= monthly payment on $ 100 loan)	1 STO 5
n	Term in months for full amortization of 1st T.D. loan.	25 BLUE n
i	Monthly interest rate on 1st T.D. loan	13 BLUE i

The example corresponds to an income property with a Net Operating Income of $ 50,000, purchased with a 20% downpayment by a buyer who requires a 5% cash on cash return on his investment. The seller carries a 10% 2nd T.D. payable 1% per month. The 1st T.D. loan (70%) is at 13% interest, amortized over 25 years. The program is set to finding the selling price.

III The program

Step	Keys	Comment
	BLUE P/R BLUE CLP	Switches into programming and clears programs
	1200 CHS PV	
7-	PMT	Calculates loan constant
	BLUE EEX 2	Keys in 100 in 2 steps.
10-	RCL 4 RCL 2	
	+ −	
14-	BLUE PSE	<u>Displays</u> briefly 1st T.D. loan ratio
	%	
	STO 6	
17-	RCL 5	
	12 x	
	RCL 4 %	Cash return on 2nd T.D. as % of selling price.
23-	STO + 6	
	RCL 0	Switching routine
25-	BLUE x=0	(x=0 is key 6)
	BLUE GTO 35	
	RCL 1	
28-	GOLD %T	Calculates Cap. rate.
	RCL 6	
	−	
31-	RCL 2	
	x⇄y	
	GOLD %T	Calculates cash return to buyer as % of Dn.PMT.
34-	BLUE GTO 00	
	RCL 3 RCL 2	
	%	
	STO + 6 RCL 6	
41-	RCL 1 GOLD %T	Calculates selling price.
	BLUE P/R	Switches out of programming mode.

IV Testing and modifying.

Test with variables provided in these pages, and with data solved through other means.

Any modification will require careful adjustment of the GTO instructions (steps 25- and 26-), and thorough testing and debugging. Possible minor changes might include BLUE PSE instructions after steps 13- and 31- to display 1st T.D. loan ratio and Cap. rate. Also, STO 3 after step 42- to automatically store in memory 3 the latest cash return rate on buyer's downpayment.

V Using the program (Note: All changes are cumulative!)

With the program and the data for the variables keyed in:

1) What selling price meets these requirements?
   [R/S] (\$ 428,308)
2) What selling price if seller agrees to 8% interest, interest only 2nd?
   8 [ENTER] 12 [÷] [STO 5] [R/S] (\$ 443,505)
3) What selling price with break-even (no cash return to buyer)?
   0 [STO 3] [R/S] (\$ 486,674)
4) What return if property sells for \$ 500,000?
   500000 [STO 0] [R/S] (-1.37%)
5) What return if a 30 year loan can be obtained?
   30 [BLUE n] [R/S] (-0.46%)
6) What return if seller carries a 15% 2nd T.D. ?
   15 [STO 4] [R/S] (0.86%)
7) What return with a 30% downpayment?
   30 [STO 2] [R/S] (5%)
8) Under those conditions, what selling price for a breakeven return?
   0 [STO 0] [STO 3] [R/S] (\$ 588,171)
9) What Selling price with a 5% negative return?
   5 [CHS] [STO 3] [R/S] (\$ 714,192)
10) What selling price with 5% negative return with all other conditions as in the very first question?
   20 [STO 2] 10 [STO 4] 1 [STO 5] 25 [BLUE n]
   [R/S] (\$ 516,859)

   This last question requires that all the changes that had accumulated be cancelled, and the original data restored.
   When non-cumulative changes are desired, the original situation must be restored for all input other than the one being changed.

Part V

IRREGULAR CASH FLOWS

So far we have considered exchanges of money in time involving only three dollar amounts: PV , PMT , and FV. This was possible because the Payments, however numerous, were all equal. Of course, money can be exchanged in less regular packages. The irregular cash flow capabilities of the HP-38 offer a rich source of valuable information by providing us with almost as much flexibility to answer any question that makes sense involving irregular cash flows as the 5 top row keys provided for regular cash flow data.

Here again, the basic rule is DATA then QUESTION.

We need to be familiar with the mechanics for

- Keying in the data,
- Checking and changing the data,
- Asking the questions.

We need to understand the meaning of the answers provided by the calculator, and the limitations of those answers. We need to examine a number of practical applications. In fact, we may want to consider practical applications before we have fully mastered the full range of options for checking and changing the data and before we have fully considered the theoretical implications and possible limitations. So we will proceed gradually more than systematically, introducing concepts, procedures and applications, and more concepts, more procedures, and more applications. The reader may want to practice the more basic features for some time, and use the more simple applications, before he ventures into more elaborate procedures.

A <u>Irregular Cash Flows</u>: <u>The Basics</u>.

In attempting to obtain full information concerning an exchange of money in time, we may find ourselves in two situations:

- Either we know all about the Cash Flows (their amount, their sign, their timing), and we want to know the rate that balances the books on the transaction,
- Or we know the rate and some of the Cash Flows, and we want to calculate the missing cash flow or cash flows, the amounts that added back at the right points in time will balance the books on our transaction.

The irregular cash flow keys of the HP-38 provide us with two questions that allow us to respond to these two situations. In the first case we key in all the cash flows and question for the rate with | GOLD | IRR |. This gives us the <u>Internal Rate of Return</u>.

In the second case we key in the cash flows that are known and the rate, and we question for the Present Value of the missing cash flows by pressing | GOLD | NPV |. In so doing, we obtain the Present Value of the missing cash flow with the opposite sign (negative if the cash flow is positive, positive if the cash flow is negative): we obtain the <u>Negative Present Value</u> of the missing cash flow or cash flows. Knowing the Negative Present Value of the missing cash flow, we will find it possible to calculate the missing amount itself by projecting the NPV back to the cash flow's rightful place in time. We will do so by using the regular cash flow keys which automatically readjust the amount <u>and the sign</u> to their correct value.

Of course, if the missing amount is the initial cash flow, then no time adjustment is needed. The NPV amount is the amount of our initial cash flow,and we just need to adjust the sign. So, if we key in the income (positive and negative cash flows) that will result from an initial investment, and we also key in the rate of return that we require, the NPV key directly gives us the amount of the initial investment that will result in the desired return.

Examples will clarify these procedures.

In summary, we may ask for two possible questions:

- If we need a rate, we question for the Internal Rate of Return by pressing $\boxed{\text{GOLD}}\boxed{\text{IRR}}$.

- If we need a missing cash flow, we calculate the Negative Present Value of the missing cash flow by pressing $\boxed{\text{GOLD}}\boxed{\text{NPV}}$. Knowledge of the Negative Present Value of the missing cash flow allows us to calculate the missing cash flow itself.

(As we shall see later, the 'official' name for the NPV function is Net Present Value. But thinking of it as the Negative Present Value of any missing cash flow is the key to a rich harvest of practical applications).

What is the Internal Rate of Return?

Let us consider two investments, and the returns they provide:

- I deposit \$ 2,000 in a savings account, and I withdraw as the needs arise various amounts at various points in time.
- I invest \$ 2,000 in a business venture, and receive returns of different sums at different intervals.

My rate of return on the savings account investment is the interest rate agreed upon when I opened the account. If the cash flows provided by the business venture are the same in amount and in timing as those obtained from the savings account, the Internal Rate of Return on the business venture is identical to the rate on the savings account. The internal Rate of Return (IRR) on a series of positive and negative cash flows is the rate that would have to be offered by a savings account that would provide the same mixture of money disbursed (deposits) and money received (withdrawals) at the same points in time. It is called internal rate of return because it is concerned only with amounts actually invested for the time that these amounts are invested. Before a deposit is made, no interest is earned on that amount. After money is withdrawn, the amount is no longer invested and ceases to be taken into consideration. The interest rate received on a savings account is an Internal Rate of Return, as are the interest received by a lender on a real estate loan and the rate of return secured by the buyer of a discounted note.

(We will consider later the more unusual case of irregular cash flows that could not be provided by a savings account, and for which, therefore, the comparison no longer holds true).

Entering the Data.

Observe the strict sign convention: positive for money received, negative for money paid out.

Three keys allow us to enter the data: [BLUE | CFo]
[BLUE | CFj]
[BLUE | Nj]

[BLUE | CFo] Cash Flow zero. This key enters the initial cash flow. The amount is automatically stored in Register memory 0. At the same time [BLUE | CFo] sets the stage for irregular cash flow analysis by clearing what-ever needs to be cleared. If the initial cash flow is not known, we should still key in 0 [BLUE | CFo], though [GOLD | CLEAR ALL] would achieve the same purpose.

[BLUE | CFj] Cash Flow j, where j stands for 1, 2, 3 etc. All the cash flows following CFo are keyed into CFj. They are automatically stored in Register memories 1 to 19. (A 20th CFj entry will automatically be stored in the FV memory).

[BLUE | Nj] When a cash flow occurs more than once in succession, it can be keyed in once: the calculator is then told how many times it occurs with the [BLUE | Nj] keystroke. For instance:

2000 [BLUE | CFj] 12 [BLUE | Nj] This keystroke enters 12 successive cash flows of $ 2,000. The maximum number that can be stored with [BLUE | Nj] is 99 corresponding to 99 equal successive cash flows.

Examples

1) An investor buys a property, keeps it for 11 years, and sells it. Calculate the IRR on his investment given the following cash flows:

Initial investment: $ 20,000	20000 [CHS] [BLUE | CFo]
End of first 3 years: negative $ 1000	1000 [CHS] [BLUE | CFj]
	3 [BLUE | Nj]
End of Year 4: breakeven	0 [BLUE | CFj]
EOY 5 to 10: $ 2,000 return each year.	2000 [BLUE | CFj]
	6 [BLUE | Nj]
EOY 11: Net reversion and income $ 85,000	85000 [BLUE | CFj]

Now we have keyed in the data, we can ask the question:

What is the IRR? | GOLD | IRR | (15.16%)

Note - | CHS | was used to key in amounts invested (disbursed) as negative. The calculator needs to be told whether the amounts keyed in are money received or money disbursed. When we question for the rate, the cash flows submitted to the calculator must represent an exchange of money in time: there must be at least one positive and one negative cash flow.

- No income was receive for year 4. The calculator needs to be informed of that fact, as otherwise it would interpret the following entries as being for years 4 to 10 instead of years 5 to 11. So we key in 0 as if it was any other dollar amount.

- When a cash flow occurs only once, we do not need to key in 1 | BLUE | Nj. The calculator makes that assumption unless we overwrite the 1 with a different number.

- Let us explore some of the data stored in the calculator:

RCL | 0	(-20,000)	The cash flow amounts are automatically
RCL | 1	(-1,000)	stored in register memories 0, 1, 2 etc.
RCL | 2	(0.00)	These dollar amounts can be modified directly
RCL | 3	(2,000)	in those memories, either because we want
RCL | 4	(85,000)	to change the data, or because on checking
		we realize that we keyed in the wrong amount.

For instance, if on pressing RCL 1 I had found 1,000 without the negative sign, I could correct the omission by keying in the correct amount: 1000 | CHS | | STO | 1 | . The new amount (- 1,000) would still be affected by the coefficient 3 previously stored with the Nj key.

| RCL | n | (4) The number of CFj entries is stored in n (not the number of years, not the CFo entry, only the number of times the CFj key was pressed). It is important to have the correct number stored in n: if we keyed in 3 n and pressed | GOLD | IRR | once again, the calculator would not take the 4th CFj entry ($ 85,000) into account. Checking the amount in n is a fast way of verifying that all the CFj entries have been made.

| RCL | i | (15.16) The rate is automatically stored in the i memory. The calculator has no knowledge of time. I interpret the answer as being a yearly rate because the data I keyed in was known to me to be yearly data. If it were quarterly data, then the answer would be a rate per quarter, and should be multiplied by 4 to obtain a yearly rate.

2) A note promises the following return:

$ 500 per month for 5 years.

$ 800 per month for the following 10 years.

How much should I pay for it if I want a 20% annual rate of return?

This is a case where we know the rate and some of the cash flows, and we want to calculate a missing cash flow. We need to key in the data and question for NPV . As in this case the missing cash flow occurs at time 0, the Negative Present Value is the amount we seek, with only the sign needing to be adjusted.

Initial amount unknown:	0	BLUE CFo		
$ 500 (60)	500	BLUE CFj	60	BLUE Nj
$ 800 (99) (120)	800	BLUE CFj	99	BLUE Nj
$ 800 (21)	800	BLUE CFj	21	BLUE Nj
20% annual rate of return	20	BLUE i		
Negative PV of missing cash flow:		GOLD NPV	(34,227.03)	

We will get a 20% return if our intial investment is $ 34,227.03.

<u>Note:</u> - We cannot key in more than 99 cash flows with any one Nj entry. The 120 cash flows of $ 800 must be keyed in as two separate CFj entries that just happen to be of the same amount.(Instead of pressing 800 a second time, we could just press x≷y BLUE CFj).

- As we key in monthly data, we need to store a monthly rate in i .

- The NPV answer is stored in the PV memory. Press RCL PV to retrieve after other numbers have replaced it in the display.

- There would be no need to press 0 BLUE CFo if we had chosen to press GOLD Clear FIN or GOLD Clear ALL.

- With the previous data still in, we may want to ask another question: How much should I pay in order to get a 25% return?

25 BLUE i	
GOLD NPV	(27,240.44)

or again: What rate of return will I get if I pay $ 30,000?

	30000 CHS STO 0	
Calculates monthly rate	GOLD IRR	(1.90%)
Yearly rate:	12 x	(22.80%)

3) A property can be bought with $ 50,000 down. Income projections show the following returns:

End of year 1:	- 12,000	
EOY 2	0	
EOY 3, 4 and 5:	10,000	
EOY 6:	135,000	includes income for that year and net reversion on sale of property.

- Key in the data:

50000	CHS	BLUE CFo		
12000	CHS	BLUE CFj		
0		BLUE CFj		
10000		BLUE CFj	3	BLUE Nj
135000		BLUE CFj		

- Check the number of CFj entries, and the cash flow amounts:

RCL n	(4)
RCL 0	(-50,000)
RCL 1	(-12,000)
RCL 2	(0.00)
RCL 3	(10,000)
RCL 4	(135,000)

- Calculate the Internal Rate of Return.

GOLD IRR (19.94%)

- On the basis of a different assumption about inflation the End Of Year 6 return is changed to $ 155,000. How does that affect the Internal Rate of Return?

155000 STO 4 (Changes the data in memory 4)

GOLD IRR (22.34%)

- Leaving $ 155,000 for the 6th year projection, what initial investment would result in a 25% return?

0 STO 0	Erases the initial CF that is no longer valid.
25 i	Stores the required return
GOLD NPV	(43,525.12) Amount of required initial investment.

Note: Cash flow amounts can be changed directly in the Register memories (provided the Nj value is not changed).

Practice.

The immediate objective should be to acquire some fluency in keying in irregular cash flow data, checking and changing that data, and asking the basic questions as in the previous examples.

1) A note promises the following income:
$ 200 a month for 3 years, then $ 500 a month for 5 years.
What is the IRR if you buy it for $ 17,000? (18.08%)
How much should you pay for it if you want a 20% return? (15,790.28)

2) A $ 200,000 investment is expected to provide the following returns.

End Of Year 1:	0
End of Year 2:	20,000
End of Year 3:	50,000
End of Year 4:	70,000
EOY 5 to 10:	85,000

(6 years here !)

Recall n and the various dollar amounts before you calculate the IRR (21.34)
What rate of return if you have to pay $ 250,000 instead of $ 200,000?(16.67)
What initial investment will result in a 20% return? ($212,899.48)

(Keystrokes for 2nd and 3rd questions:
250000 CHS STO 0 GOLD IRR (16.67)
0 STO 0 20 i GOLD NPV (212,899.48))

3) Carrying an All Inclusive Trust Deed (AITD) should result in the following net cash flows for the seller of a property. What rate of return is he getting on his investment?

Initial net investment (AITD equity):	$ 40,000	(Negative!)
Monthly return for first 35 months:	$ 210	
36th month (a balloon is due!)	Negative $ 13,000	
Then for 204 months:	$ 700	
Then for 120 months:	$ 1000	
		(13.30%)

($ 700 a month for 204 months can be keyed in as follows:
700 BLUE CFj 99 BLUE Nj
x y BLUE CFj x y BLUE Nj
x y BLUE CFj 6 BLUE Nj)

4) Similar problem. Find the IRR for the following cash flows.

Initial investment:	- 25,000	
Monthly return, for 3 years	150	(36)
Monthly return, for 5 years:	350	(60)
Monthly return, for 10 years:	800	(120)

(1.40 per month, 16.80 per annum)

How much should you pay at a discount if you want a 20% return?

(19,790.92)

(Note that this last question implies that our initial $ 25,000 is no longer valid. We erase it by keying in 0 [STO | 0]. We then key in the new piece of information, our requirement that we get a 20% return: 20 [BLUE | i]. We then question for a dollar amount: [GOLD | NPV]).

5) A $ 30,000 note is to be paid off as follows:

First 2 years: no payments

The 5 years that follow: $ 778 per month.

What interest is being charged ? (10%)

How much should you pay for it at a discount to get a 20% return?

($ 19,749.13)

(Key in the first two years on a monthly basis: 24 months at $ 0 per month).

6) A developer is projecting the following cash flows on a subdivision:

Initial investment:	- 120,000
EOY 1	- 250,000
EOY 2	- 80,000
EOY 3	400,000
EOY 4	600,000

What is the Internal Rate of Return? (34.17%)

How is the rate of return affected if delays postpone by 1 year the two positive cash flows, with a breakeven (0) situation for year 3 ?

(24.02%)

(One way to correct the data: RCL 4 BLUE CFj RCL 3 STO 4 0 STO 3)

Application:
All Inclusive Trust Deeds (AITD)

The decision to create an AITD (or wrap around loan) is affected by the yield it will provide. That yield can be calculated using the IRR key of the HP 38 calculator. The procedure requires two steps:
1) Establishing the cash flows that will result from the AITD. This implies getting the fully balanced data on the AITD and on each of the underlying loans. One should calculate the remaining balance, the remaining term, the payments, and the balloon payment, where applicable. This data makes it possible to establish the net amounts that are to be received or to be paid by the AITD lender at each point in time. There is no alternative to the careful, sometimes fastidious procedure that establishes these net amounts. But then, who would want to create an AITD without knowing what cash flows will result from it?
2) Analyzing these cash flows with the irregular cash flow keys. The rate of return on the AITD is the annual IRR corresponding to the cash flow data.

I What is an All Inclusive Trust Deed loan? (A reminder)

Let us imagine a house that sells for $ 100,000. There is an 8% loan on it it with a balance of $ 30,000. The current rate on new loans is 12%. The traditional way of financing the purchase is for the buyer to get an $ 80,000 loan. With that money, escrow will pay off the $ 30,000 outstanding loan. The remaining $ 50,000, along with the buyer's $ 20,000 down payment, will pay for the seller's $ 70,000 equity in the property.

Very little needs to change with an AITD created by a third party:
- The buyer still provides his $ 20,000 down payment and borrows $ 80,000 for the difference.
- The seller still gets his $ 70,000 for his equity in the property, and the original loan is taken off his hands.
- Things may be different only for the lender of the $ 80,000. He commits himself to pay off the $ 30,000 outstanding loan, and gives the seller the balance of $ 50,000, which is what the seller would get anyway from the new loan. But instead of paying off immediately the $ 30,000 loan, the AITD lender pays it off as that loan was originally scheduled to be paid off: with regular monthly payments stretched out over the remaining term of the loan. So his only out of pocket expenditure are the $ 50,000 he pays to the

seller. His later return on that investment is the difference between the monthly payments he receives from the buyer on his $ 80,000 loan, and the payments he has to make on the original $ 30,000 loan, now called the underlying loan.

The advantage to the AITD lender comes from the different rate of interest between the loan he assumes and the loan he makes. If the underlying loan is at 8% and the new AITD at 12%, the lender is in effect borrowing $ 30,000 at 8% and lending it at 12%. On top of the 12% interest he gets for the $ 50,000 he actually lends, he receives an additional 4% interest on $ 30,000 which he does not have to provide.

In many instances, the lender of the AITD loan is not a third party but the seller himself. His $ 50,000 investment in the AITD is not an out of pocket expenditure, but his agreement not to receive the $ 50,000 which are owed him as the seller of the property. In practical terms, analyzing the AITD from his point of view is made easier if we consider him as a third party in his capacity as a lender as distinct from his role as the seller of the property.

II Calculating the rate of return on an AITD loan.

Data:
- $ 80,000 AITD loan, 12% interest, amortized over 30 years.
- $ 30,000 underlying loan (balance of a $ 37,000 loan), 8% interest, payments of $ 272 per month.

What is the yield to the lender of the AITD?

Solution:

1 a: Calculate the cash flows for each loan.

AITD:

80000	[PV]
12	[BLUE] [i]
30	[BLUE] [n]
	[PMT]

($ 822.89, amount of monthly payment)

Underlying loan:

30000	PV
8	BLUE i
272	CHS PMT
n	

(200.03 months, the remaining term on the original loan. We will round to 200).

1 b: Net amounts paid out and received.

We saw how the lender's net cash outlay was $ 50,000. For the first 200 months he receives 822.89 dollars but pays out $ 272, leaving a net cash flow of $ 550.89 per month. For the remaining 160 months of the 30 year AITD the lender receives $822.89 and retains the full amount as he has paid off the only underlying loan.

In summary:

- 80,000	822.89	for 360 months.
30,000	-272	for 200 months.
- 50,000	550.89	for 200 months and 822.89 for 160 months.

The bottom line represents the net amounts paid out and received.

2) Calculating the annual rate of return

50000	CHS	BLUE	CFo			
550.89	BLUE	CFj	99	BLUE	Nj	For a total of 200 months
x⇄y	BLUE	CFj	99	BLUE	Nj	
x⇄y	BLUE	CFj	2	BLUE	Nj	
822.89	BLUE	CFj	99	BLUE	Nj	For a total of 160 months
x⇄y	BLUE	CFj	61	BLUE	Nj	
Gold	IRR					(1.13% per month)
12	x					(13.57% annual rate of return)

Note the use of the x⇄y key as a means of retrieving the last CFj entry. Of course, we could instead key in 550.89 or 822.89 once again.

III Multiple underlying loans

The existence of more than one underlying loan in no way changes the procedure leading to the rate of return on the AITD loan. It just makes establishing the net amounts paid out and received somewhat more complex.

Example

Same data as with the previous example, but there is in addition to the $ 30,000 first Trust deed an existing 2nd T.D. loan with a balance of $ 18,000. The interest is 10%, and the payments are $ 190 per month. It is all due 40 months from the date of the new sale.

Solution

2nd T.D.

18000	PV			
190	CHS	PMT		
10	BLUE	i		
40	n			
FV				(16,110.31, balance of the loan)
RCL	PMT	+		(16,300.31, balloon amount of 40th payment. We will ignore the cents)

So this loan requires 39 payments of $ 190, and 1 payment of $ 16,300.
It also allows an initial cash outlay of 50,000 - 18,000 = 32,000 dollars.

Along with the rest of the data, this leads to the following cash flows:

$ amount	N° of pmts	Calculator key strokes					
- 32,000	Initial invest.	32000	CHS	BLUE	CFo		
360.89	39	360.89	BLUE	CFj	39	BLUE	Nj
- 15749.42	1	15749.42	CHS	BLUE	CFj		
550.89	160	550.89	BLUE	CFj	99	BLUE	Nj
		x⇄y	BLUE	CFj	61	BLUE	Nj
822.89	160	822.89	BLUE	CFj	99	BLUE	Nj
		x⇄y	BLUE	CFj	61	BLUE	Nj
		GOLD	IRR	(wait)	12	x	

(14.08% return)

Note the large negative cash flow which the seller will have to make 40 months after the sale of his house. This is information the seller would want to be aware of before agreeing to an AITD. It is also an inconvenience he might want to avoid by having the buyer pay for the $ 16,110.31 balance 40 months after his purchase. (We are assuming here that the buyer will pay for the balance, not for the balloon). If the buyer agrees, it will significantly increase the seller's return. With the previous data still in the calculator, a simple correction can change the data and provide for the new return.
- 15749.42 stored in memory 2 of the stack needs to be corrected to a positive 360.89 as follows:

360.89 [STO] [2]
[GOLD] [IRR] (wait) [12] [x] (The return is now 17.86%!)

(See next example for a procedure that indemnifies the buyer for his payment)

Notes:

1) In cases were the AITD and all the underlying loans have the same due date the net cash flows that result form such a simple pattern that they can be analyzed with the top-row, regular cash flow financial keys.

2) Because of the monthly payments, the rates calculated imply monthly compounding whenever compounding is allowed to occur.

3) Most sellers who carry an AITD would not want the negative that occurred 40 months down the line in our previous example. This could be taken care of by agreeing that the buyer will pay the balance of the underlying second T.D. when the note is due, with that payment offset by decreasing the balance of the AITD by the same amount at the same point in time. Similarly, most AITD lenders will want a due date on their loan, at which time any remaining underlying loan could be either paid off or, which is the same thing as far as the calculations are concerned, taken over by the buyer if feasible. This features lead to the following modifications of our previous problem.

AITD: a more realistic situation

Data: $ 80,000 AITD loan, 12% interest, payments of $ 822.89, due in 10 years. A large lump payment will reduce the outstanding balance after 40 months (See 2nd T.D. underlying note).

$ 30,000 underlying 1st T.D., 8% interest, payments of $ 272 per month. This loan will be paid off when the AITD becomes due.

$ 18,000 2nd T.D., 10% interest, Payable $190 per month, balance due in 40 months. The holder of the AITD will make the 40 payments, but the balance is to be paid by the borrower of the AITD (the buyer). When that lump payment is made, the outstanding balance of the AITD will be reduced by the same amount as a means of compensating the buyer for making that payment. This lump reduction will not affect the payments on the AITD, only the balance when it becomes due.

Questions:

1) Establish the cash flows that will result from this transaction for the lender of the AITD (net amounts).

2) What is the Internal Rate of Return on his investment ?

Solution

1) Balance the books on each loans:

2nd T.D.: Calculate balance at 40 months, $ 16,110.31.

1st T.D.: Calculate 10 year balance: $ 16,827.89.

AITD: Calculate 10 year balance after early pay down:

Keystrokes	
80000 [PV]	
12 [BLUE] [i]	
822.89 [CHS] [PMT]	
40 [n]	
[FV] [CHS]	(78,880.99, balance at end of month 40)
16110.31 [-]	(62,770.68, balance after lump reduction to balance 2nd T.D. pay off).
[PV]	Stores new AITD balance in PV.
80 [n] [FV]	(39,022.45, balance of AITD after 10 years-- 120 months - 40 months = 80 months).

2) Establishing the net cash flows for the lender.

Initial investment 80,000 - 30,000 - 18,000 = 32,000

Payments received first 40 months: 822.89 - 272 - 190 = 360.89

Payments received next 79 months: 822.89 - 272 = 550.89

Amount received end of 10th year:

550.89 + 39,022.45 - 16,827.89 = 22,745.45

This data can be expressed as follows:

- 32,000	
360.89	(40)
550.89	(79)
22,745.45	

This information answers our first question.

3) We may now key this data in and question for the IRR (1.31 x 12 = 15.72%)

Clearly, establishing the net cash flows for the lender is the more delicate part of the calculation. It requires careful reading of the terms of the AITD agreement.

(Do not erase the data in your calculator. We are going to use it)

Exploring and changing the data. (Previous data retained).

Review: We have already seen how we could explore the cash flow amounts, the number of CFj entries, and the IRR as follows:

RCL 0	(-32,000)	Cash flow amounts.
RCL 1	(360.89)	
RCL 2	(550.89)	
RCL 3	(22,745.45)	
RCL n	(3)	3 CFj entries.
RCL i	(1.31)	Periodic rate, or
RCL BLUE i	(15.72)	Annual IRR.

We also saw that we could modify a cash flow amount directly in the register memories, here in memories 0 to 3.

Let us now explore the Nj values, and the amount they apply to. We will do so by exploring the data in the opposite order in which it was initially stored:

RCL BLUE Nj RCL BLUE CFj	(Shows 1, and 22,745.45)
RCL BLUE Nj RCL BLUE CFj	(Shows 79, and 550.89)
RCL BLUE Nj RCL BLUE CFj	(Shows 40, and 360.89)
RCL BLUE Nj RCL BLUE CFj	(Shows 1, and - 32,000)
RCL BLUE Nj	(Error sign, there is no Cash Flow before - 32,000)

(Note: as we explore our data backwards in this way, we will recall the CFo entry by pressing RCL BLUE CFj)

(Press any key to eliminate the ERROR sign. That key will not operate).

By recalling the data backwards as above we have modified the data stored in n (3, for the three CFj entries). To calculate the IRR after having explored the data in this way, we need to key the number of CFj entries back again into the n memory. In this case, we press:

3 n

Controlling the number stored in $\boxed{n}$.

What has happened as we explored our CFj and Nj values backwards? Understanding the process will provide us with great flexibility in dealing with irregular cash flow data. The key is an understanding of what happens to the data stored in n .

- When we enter our initial CF into CFo, we put 0 in n and store our initial amount in Register memory 0.

- Every time we enter a Cash Flow amount with CFj, we increase the value stored in n by 1 , and we store the new Cash Flow amount in the Register memory that corresponds to the new n value: 1, 2, 3, etc. ('j' means 1, 2, 3, etc).

- When we key in an Nj value, the factor keyed in affects the Cash Flow stored in the Register memory that corresponds to the value stored in n . With 2 in n , 79 | BLUE | Nj | tells the calculator that we want 79 Cash Flows of the amount stored in Register memory 2.

- Every time we press | RCL | BLUE | CFj |, we not only recall the Cash Flow amount corresponding to the value stored in n , we also decrease the value stored in n by 1 (the opposite of keying in a number with BLUE CFj). This allows us to automatically explore the previous Nj and CFj values with the RCL procedure. It also allows us to key back in a corrected Nj value as soon as we recall an Nj value that we want to change, or to key back in a CFj and the corresponding Nj value as soon as we have recalled a CFj value that we want to change. (Keying in a CFj value brings the corresponding Nj value back to 1, so we need to key in the desired Nj value if it is not 1).

- We may explore with the Recall key any Nj and CFj combination (in that order) by merely storing in n the number of that CFj entry.

- We may overwrite any CFj and Nj combination (in that order) by merely storing in n the value of that combination minus 1.

- We may tell the calculator to ignore any Cash Flow data above a certain j value by storing that value in n . With 2 in n the calculator will take into account only Cash Flows 0, 1, and 2 when calculating the IRR or the NPV.

- We may also, if we so choose, completely eliminate a CFj entry, in amount and in the time delay that it represents by affecting it with the Nj factor 0 (zero).

These various features are easily understood if we focus on the crucial role played by the value stored in n , how it can be changed by us, how it is automatically increased by one every time we press CFj to add an entry, and reduced by one every time we press CFj to recall data.

Also of importance in many of the problems that follow is our ability to consider only some of the cash flows keyed in by keying in n the number of the last cash flow we want to consider.

Examples: expanded uses of the irregular cash flow procedure.

The following examples illustrate possible uses of the NPV key in combination with the regular cash flow procedure.

1) Future Value of Irregular Cash Flow.

Year	Deposit	Withdrawal	Balance
0	10,000		10,000
1		3,000	7,800
2		2,000	6,424
3	7,000		13,937.92
4		1,000	14,052.95
5		5,000	10,177.19
6		2,000	8,991,37
7			9,710.67

(Data: Year, Deposit, Withdrawal; Answers: Balance)

The account bears interest at 8%, with annual compounding.

a) What is the balance of the account at the end of the 6th year— immediately after the last $ 2,000 transaction has been made?

Solution

- With a Clear calculator, key in the irregular cash flow data and the periodic interest rate (Negative for deposits, positive for money withdrawn, or vice-versa).

10000	CHS	BLUE	CFo
3000		BLUE	CFj
2000		BLUE	CFj
7000	CHS	BLUE	CFj
1000		BLUE	CFj
5000		BLUE	CFj
2000		BLUE	CFj
8	i		

Keys in data.

- Calculate the **Net** Present Value and press $\boxed{FV}$ twice.

$\boxed{GOLD}\boxed{NPV}$ $\boxed{FV}\boxed{FV}$ (8991.37)

With NPV calculated, and automatically stored in the $\boxed{PV}$ memory, $\boxed{n}$, $\boxed{i}$, and $\boxed{PV}$ have the correct data to project the NPV amount 6 years into the future using the regular cash flow logic. The answer is the balance that was sought.

Note.

The first time $\boxed{FV}$ is pressed, it keys the NPV amount into the $\boxed{FV}$ memory--which is not what we wanted. The second time around, the calculator ignores the number in the display and in the FV memory, and calculates the desired Future Value.

If the financial keys had not been cleared at the outset, $\boxed{0}\boxed{PMT}$ would clear the only memory **that** could contain unwanted data.

b) Establish the running balance of the account to the end of the seventh year. (There is no transaction in year 7).

Solution

With the data kept in as previously keyed in, adjusting n to the desired period provides the running balance:

1	n	GOLD	NPV	FV	FV	(7800)
2	n	GOLD	NPV	FV	FV	(6424)
Etc.						
7	n	GOLD	NPV	FV	FV	(9710.67)

(The PMT memory and register memory 7 are the only two memories that could contain unwanted data. If the calculator had not been cleared intially, these two memories should be cleared: $\boxed{0}\boxed{PMT}\boxed{STO}\boxed{7}$).

The same procedures will allow us to answer specific Real Estate questions.

2) Reversion

The problem: Given a cash-flow projection on an income property, what net reversion do we need on the sale of the property in order to obtain a desire return on investment.

As far as the calculations are concerned, there is no difference between the balance on the account in the previous problems, and the reversion in this problem.

Data:

Initial investment	-30,000
Year 1	-5,000
Year 2	-2,000
Year 3	0
Year 4	1,000
Year 5	7,000

What reversion do we need at the end of year 5 in order to have a 15% internal rate of return on the investment?

Solution

30000	CHS	BLUE	CF•	Keys in data.
5000	CHS	BLUE	CFj	
2000	CHS	BLUE	CFj	
0		BLUE	CFj	
1000		BLUE	CFj	
7000		BLUE	CFj	
15	i			
GOLD	NPV	FV	FV	($ 63,977.50)

What reversion would be needed to obtain the same return if the property was sold at the end of the 4th year?

4	n	GOLD	NPV	FV	FV	($ 61,719.55)

In all the previous examples, because the BLUE Nj keys were not used in keying in the irregular cash flow data, the value in n is the same for regular and irregular cash flow calculations.

3) Balance on loan with changing payments.

The problem:

A private party lender makes a $ 145,000 loan at 13% interest. He is to be paid back with monthly payments of $ 1,000 that are to increase by $ 100 every year. The balance of the loan is due at the end of the 6th year. What is that balance?

145000	CHS	BLUE	CFo		
1000	BLUE	CFj	12	BLUE	Nj
1100	BLUE	CFj	12	BLUE	Nj
1200	BLUE	CFj	12	BLUE	Nj
1300	BLUE	CFj	12	BLUE	Nj
1400	BLUE	CFj	12	BLUE	Nj
1500	BLUE	CFj	12	BLUE	Nj
13	BLUE	i			
GOLD	NPV				
6	BLUE	n	Adjusts value in $\boxed{n}$ to regular C.F. logic.		
FV			($ 183, 759.97)		

As we used the Nj key the value in n had to be adjusted from 6 to 72 as we switched from irregular cash flow logic to regular cash flow logic. Also note that in this instance we had negative amortization.

The same procedure can be used if we know the amount of the 'missing' Cash Flow, but not its position in time. When switching to regular Cash Flow logic, we then know i PV and FV , and question for n , as in the following example:

4) Position in time of a known Cash Flow.

A lender agrees to lend $ 16,180 at 12% interest. He also agrees to be paid back as follows:

1st year: no payments

Years 2 and 3: $ 300 per month.

Years 4 and 5: $ 500 per month.

Plus an extra lump sum of $ 5,000 sometime during those years. There is some disagreement between lender and borrower as to when that lump sum is due. Can you help?

16180	CHS	BLUE	CFo				Keys in Cash Flow for which both amount and timing are known.
0		BLUE	CFj	12	BLUE	Nj	
300		BLUE	CFj	24	BLUE	Nj	
500		BLUE	CFj	24	BLUE	Nj	
12	BLUE	i					
GOLD	NPV		(3,100.51, which is the PV of the missing Cash Flow)				
5000	FV		Keys in amount of missing Cash Flow, as a positive amount because the NPV was negative and we need a change of sign to analyse regular Cash Flow data.				
n			(48.03: the $ 5,000 needs to be added to the 48th payment, at the end of the 4th year).				

The problems on the following page are all applications of the previous procedures.

Problems

1) An income property is bought with **an initial** investment of $ 65,000 (Downpayment and buying costs). The projected income is as follows:

Initial investment:	- 65,000
1st year	0
2nd year	5,000
3rd year	10,000
4th year	15,000

What reversion (net cash return as a result of the sale) will result in a 20% return on the investment if the property is sold at the end of the fourth year? ($ 100,584)

What net cash return (income for the year + reversion) is needed for the 5th year, if the sale of the property is **postponed** 1 year?

($ 120,700)

(5 n GOLD NPV FV FV)

What net reversion would be required at the end of the 3rd year, if the property was to be sold then, and still produce a 20% return?

($ 96,320)

(3 n GOLD NPV FV FV)

2) A private lender agrees to be paid back as follows on a $ 50,000 loan:

1st year	no payments.
2nd year	$ 500 per month.
3rd year	$ 1,000 per month.
4th year	$ 1,500 per month.

The rate of interest is 14%. Beginning with year 4, what is the balance of the loan at the end of each one of these 4 years?

($ 44,881.98; 55,756.34; 59,648.98; 57,467.10)

(Key in cash flow , then $\boxed{\text{14 BLUE i}}$ then

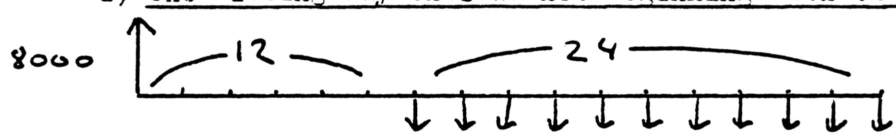
		GOLD	NPV	4	BLUE	n	FV
3	n	GOLD	NPV	3	BLUE	n	FV
2	n	GOLD	NPV	2	BLUE	n	FV
1	n	GOLD	NPV	1	BLUE	n	FV

Notice how the value in n is adjusted back and forth for regular and irregular Cash Flow uses).

Transforming the NPV into the missing cash flows.

We have seen many instances where knowledge of the Present Value--or the Negative Present Value--of some missing cash flows, and of the rate at which the discounting occurred, leads very simply to the value of the missing cash flows: the FV key, or the PMT key of the calculator immediately provides the answer. Let us examine a few instances where calculating the answer is not quite as simple.

1) The missing Payments do not beginning with the first period.



The missing cash flows are twenty four equal payments occuring at the end of months 13 to 36. Their NPV, discounted at 15%, is $ 8000.

To calculate these missing cash flows, we first transfer the NPV from time 0 to the end of the 12th months (the beginning ofthe 13th month) We then proceed as with a normal PV -- PMT transformation:

8000 PV				
15 BLUE				
12 n	FV	(9,286.04)		
CHS PV				The FV becomes the new PV
24 n				
0 FV	PMT	(450.25)		

2) The missing cash flows are a combination of PMT and FV .

Without further qualifications there would be an infinite number of possible answers, as any value for PMT provides a possible valid answer for FV , or vice-versa. Any qualifying requirement that allows us to find the amount of FV or of the Payments, immediately leads to an answer concerning the one truly missing element, and the problem is easy to solve. The solution isless obvious when the requirements imposed on PMT and FV are relative to one another. The most frequent instance is the requirement that PMT and FV be the PMT and the balance of a loan with an interest rate other than the rate that resulted in the calculation of their Present Value.

Example:

Calculate the PMT and balance after 5 years of a 13% loan, amortized over 30 years, and with balance due at the end of 5 years, knowing that the Present Value at a 20% discount of these PMTs and that FV is equal to $6,000.

A trial and error procedure is required. Though less convenient than a direct calculation, this need not be very time consuming.

First step: By examining <u>any</u> 13% loan, amortized over 30 years, balance due in 5 years, calculate the PMTs as a percentage of the FV.

100,000 PV	(Or 6000 if we so choose)
13 BLUE i	
30 BLUE n PMT	(1,106.20)
5 BLUE n FV	(98,081.65)
x y	or any other procedure that puts the FV in y and the PMT amount in x
GOLD %T	(1.13)
STO 7	Note that storing will retain all the hidden decimals.

Second step: By trial and error, find the PMTs that result in a FV 1.13% of which is equal to our PMT amount.

6000 CHS PV	(A negative here will allow us to key in positive PMTs for our various trials)
20 BLUE i	(5 BLUE n is already keyed in)
70 PMT	Our first guess
FV (9052.75)	Corresponding FV
RCL 7 % (102.10)	The ratio is not appropriate. The correct value for PMT is between 70 and 102.

90 PMT FV	
RCL 7 % (79.15)	Our second guess is too high, the answer once again is between our guess (90) and the percentage calculated.
85 PMT FV	
RCL 7 % (84.89)	We are almost there!
84.95 PMT FV	(7532.48)
RCL 7 % (84.94)	Short of looking at hidden decimals, it may not be possible to improve on these numbers.

If these are indeed the PMT and 5 year balance of a 13% loan, what is the amount of that loan?

13 BLUE i	
PV	(7,679.19)

As a check, we may want to calculate what payments would amortize that loan in 30 years:

30 BLUE n	
0 FV PMT	(84.95)

Problem:

A $ 6000 commission is due to me. I accept to take it in paper by creating a note (amortized over 30 years, balance due in 5 years, 13% interest) of an amount such that I can sell it at a discount for $ 6000, providing the buyer with a 20% return. What should be the amount of the note?

It is, of course, this problem that we have solved in the previous keystrokes.

Note: Different circumstances might suggest modified procedures. With a 12% interest only note, the PMTs are 1% of the PV and of the FV. So we may search for a Payment amount that results in a Future Value that is 100 times greater than itself. Once found, that Future Value is our loan amount.

AITD: Finding the rate that will provide a given yield.

We saw earlier how to calculate the yield on an AITD when we knew the rate. Since then we have considered new procedures involving irregular cash flows and acquired some practice in their use, and we are better able to consider the reverse problem: how to calculate the rate that will result in a given yield.

According to the requirements imposed on the AOTD loan the procedure can be quite simple, or require a more complex trial and error approach. In all cases the procedure consists in the following:

1) Keying in the cash flows that are fully known (amount, sign, and timing), or the known components of these cash flows. In our first example the known components are the payments that the lender has to make in order to service the underlying notes: we key in only negative amounts.

2) Keying in the desired yield in the i memory, and calculating the NPV: this provides us with the Negative Present Value of the cash flows or the components that are missing.

3) Transforming the NPV into the missing cash flows or components. This is were varying requirements concerning the AITD may result in simple calculations or lead to more cumbersome trial and error procedures.

4) Using the knowledge of the previously missing cash flows or cash flow components to calculate the payments, and the balance if any, on the AITD, and using this information to calculate the rate we are seeking.

Illustration:

What rate on the following wraparound loan will provide an 18% yield?

Data: $ 100,000 AITD fully amortized in 30 years. Rate unknown.

$ 55,000 underlying 1st T.D. at 8% interest, payments of $ 430, (and therefore a remaining term of 288 months).

$ 15,000 underlying 2nd T.D., 10% interest, payments of $ 150, 5 years due date (and therefore a balance of $ 13,064.

The data provides us with the following cash flows:

Cash flows:	Keystrokes
- 30,000	30000 CHS BLUE CFo
-580 for 59 months.	580 CHS BLUE CFj
	59 BLUE Nj
- 13,644	13644 CHS BLUE CFj
- 430 for 228 months.	430 CHS BLUE CFj
	99 BLUE Nj
	x≷y BLUE CFj
	99 BLUE Nj
	x≷y BLUE CFj
	30 BLUE Nj
Required yield: 18%	18 BLUE i
Negative Present Value of missing data:	GOLD NPV (- 69527.06)
We now need to switch to regular cash flow logic in order to spread our NPV over the 30 years of the AITD:	30 BLUE n
In case of doubt, clear FV	0 FV
Calculate PMT on AITD	PMT (1047.83)
These are the payments on a 100,000 loan, as they are already keyed in as positive, let me key in a negative 100,000 for the loan amount:	100000 CHS PV
Pressing i calculates the monthly interest:	i 12 x (12.25%)

Note that the lengthiest part of these calculations consists in the routine keying in of our initial irregular cash flow data.

With all the data now in, we may ask the same question for a different yield: for instance: what interest on the AITD will result in a 20% yield.

Switch back to irregular cash flow logic by adjusting n to the 5 CFj entries:	5 n
Impose the required yield:	20 BLUE i
Calculate Negative Present Value of missing cash flows:	GOLD NPV (-66,086.54)
Switch back to regular cash flow logic,	30 BLUE n
Calculate the PMT,	PMT (1104.32)
Key in the AITD loan amount (as negative)	100000 CHS PV
Calculate interest rate	i 12 x (12.98%)

So we have calculated that a rate of 12.25% on the AITD would result in an 18% yield, and a rate of 12.98 a yield of 20%.

Remarks:

- Slight adjustments need to be made according to how the AITD payments are defined. There is no problem if they are defined as above (fully amortized), or as 'interest only, all due in 10 years'. If they are defined as 'amortized over 30 years, all due in 10', then an exact solution would require a cumbersome iterative procedure.The calculations would be simplified if the parties could agree on the amount of the payments, allowing the yield to affect the balance only, or agree on the balance, with theyield affecting the payments only.

- If a yield has been calculated on the basis of a given rate (say 12%, fully amortized in 30 years), and the yield is then judged to be insufficient, there is no reason to start from scratch in order to calculate the desired yield (say 18%). 18 BLUE | i GOLD | NPV still calculates the Negative Present Value of the missing amounts. In this case the missing amounts are the extra amounts that need to be added to each payment of the AITD loan. Knowing that extra amount, it is easy to calculate the new payments, and the interest that it implies. Let us illustrate this with our previous AITD data, but we now begin by imposing an 11% interest on the AITD (fully amortized in 30 years), and by setting out to calculated the net cash flow, and the resulting yield.

  The payments on the AITD are now set at \$ 1,028.61 (\$ 100,000 loan at 12% amortized in 30 years), and the net cash flows for the lender are as follows:

The net cash flows to the lender would be as follows:

- 30,000	
448.61	(59 months)
- 12,615.39	
598.61	(228 months)
1,028.61	(72 months)

With this data keyed in, we caclulate the IRR and find 17.33%.

This is found insufficient by our seller-lender, who asks what interest he should charge on the AITD in order to get a yield of 20%. We may proceed as follows:

Negative Present Value of missing cash flows to yield 20%:	20	BLUE	i	GOLD	NPV	
Amount of missing CF spread over 30 years:	30	BLUE	n	PMT		(75.71)
Add to previous payments (Keyed in as CF 6)	RCL	6	+			(1104.32)

Of course, we have rediscovered the payment amount already calculated in our previous example. The same keystrokes will lead us to our final answer, though here we might as well key in PMT as negative and PV as positive:

CHS	PMT		
100000	PV		
i	12	x	(12.98%)

The Notion of Net Present Value.

As originally conceived, the NPV function calculates the Present Value of all the cash flows submitted discounted at the given rate. It is the sum of the Present Values of all the cash flows discounted at the given rate, including any negative cash flows, the Net Present Value.

For example: Let us find the NPV of the following financial transaction assuming a 9% discount rate. (Follow sign convention)

Cash flows	Cash flows	Present Values
Initial cost (no discount).	- 10,000	- 10,000
1st year: extra $ 2,000 invested	- 2,000	- 1,834.86
2nd year return	4,000	3,366.72
3rd year	6,000	4,633.10
4th and last year	8,000	5,667.40
The total is the Net Present Value :		1,832.36

In this case we have discounted separately each cash flow using the PV and FV keys. A simpler procedure would be to key in the data with the irregular cash flow keys (CFo, CFj and Nj if needed), enter the same 9% rate in i , and question the calculator with GOLD NPV . Please do so to verify that the result is the same, and keep the data in the calculator.

What is the meaning of those $ 1,832 NPV ? It simply means that I could pay $ 1,832 more initially and still get a 9% return. With the present initial cost of only $ 10,000, my return is higher.

A negative NPV of the same amount would mean that I would have to pay $ 1,832 less initially to get a 9% return. With the present initial cost, my return is lower.

A NPV equal to zero would mean that the data entered (cash flows and rate) forms a balanced financial transaction: the rate keyed in would be the Internal Rate of Return.

If we did not have a special IRR key, we could find the Internal Rate of Return by a process of trial and error: 9% is too low, let us try 12 %; a positive NPV shows that this too is too low, let us try 15%, etc.

Let us in fact do this with the data from the previous example still in the calculator:

12	i	GOLD	NPV	(757.89) The rate is still too low
15	i	GOLD	NPV	(-195.43) The rate is too high
14	i	GOLD	NPV	(109.96) Too low
14.5	i	GOLD	NPV	(-44.22) Too high
14.3	i	GOLD	NPV	(17.09) Too low, but closer.
14.35	i	GOLD	NPV	(1.71) low, but close enough.

We may also find the answer directly with the IRR key:

| GOLD | IRR | | (14.36, or 14.35558089) |

With that rate now entered in the calculator we can check the NPV:

| GOLD | NPV | | (0.0000002, or a very small fraction of a cent) |

The calculator itself, when it calculates the IRR, must proceed by trial and error. This can be time consuming. The calculator also sometimes misleads itself into pursuing irrelevant negative values for the IRR. In both cases, the NPV approach may help in finding the correct IRR. (See Owner's handbook)

So the Net Present Value is the Present Value of <u>all</u> the cash flows keyed in discounted at the rate stored in i .

If the cash flows keyed in represent the complete transaction, and if the rate stored in i is the IRR, then we have a fully balanced situation; the returns have been purchased for what they are worth--given the IRR--, and the Net Present Value is equal to 0 .

If the IRR is still stored in i , but one cash flow is missing from the data, then the NPV will be deficient by the Present Value of that missing cash flow. This is why the NPV also provides us with the Negative Present Value of any missing Cash flow. That interpretation of the NPV function is frequently more immediately useful than the more abstract notion of Net Present Value.

The Internal Rate of Return (IRR): a closer look.

'The internal Rate of Return on a series of positive and negative Cash Flows is the rate that would have to be offered by a savings account to balance off the same mixture of money received and money disbursed at the same points in time'.

This definition provides a valuable point of comparison, a valid one, but one that does not cover all possible circumstances. In the most frequent investment situations the investor begins with a large investment and the major returns come towards the end of the transaction: we have a situation where our make-believe depositor never withdraws more money than his deposits and the accumulated interest allow. But nothing prevents us from submitting to an IRR calculation a series of cash flows that could not be duplicated by a savings account. When that occurs, a number of problems arise, both in the concept of the IRR, and in the calculations themselves. Let us consider such a situation and key it into the irregular cash flow keys:

Cash Flow		Keystrokes				
Year 0	- 30,000	30000	CHS	BLUE	CFo	
End Year 1	50,000	50000		BLUE	CFj	
End Year 2	- 17,000	17000	CHS	BLUE	CFj	
Calculating IRR shows a rate of 19.08%.		GOLD	IRR			(19.08%)

With the rate of return stored in the i memory, let us use the regular cash flow keys to examine in more detail what occurs in our imaginary savings account (Negative Cash Flows are deposits, positive CFs are withdrawals)

The initial deposit grows into $ 35,723.81 **over** the first year. With that amount in my account, I withdraw $ 50,000, leaving me with a negative balance of 14,276.19 dollars. I am in effect borrowing that amount. I am also being charged 19.08% interest for that loan, which leaves me with a debt of $ 17,000 by the end of the second year. It is that amount which I deposit in order to pay my debt and close the account at that time.

With the previous data still in, and the rate of return already stored in i , we may use the regular cash flow keys as follows to bring out the data just presented:

Keystrokes	Display	Meaning
1 n RCL 0		
PV FV	(35,723.81)	Amount of $30,000 after 1 year at 19.08%
RCL 1 -	(-14,276.19)	Amount 'overdrawn' in account
PV FV	(17,000.)	Amount of $ 14,276.19 borrowed for 1 year at 19.08%.

So rather than 2 years of investment at 19.08%, our investment is equivalent to $30,000 invested for 1 year at 19.08, followed by 1 year of borrowing $ 14,276.19 at the same rate of 19.08%.

Clearly, the assumption that I could borrow at the same rate, or that I would want to if I could, is highly unjustified. Yet the obligation to do so is built into the mathematics of the IRR whenever the cash flows analyzed are such that they could not possibly be produced by an ordinary savings account. When such is the case, then a cloud is cast over the very meaning of Internal Rate of Return.

<u>When do we have a problem with the IRR</u>?

From the previous example it might appear that the problem arises because too large an amount is withdrawn at the end of the first year. This is not really so, as our initial investment could grow to astronomical figures, resulting in astronomically high rates of return, without creating any problem. The problem occurs because the negative cash flow that follows reduces the rate of return below the rate that would result from the previous cash flows alone. That situation can only occur when there is more than one sign change in the cash flow sequence, though it does not necessarily result from that circumstance. (For a fast check on a suspiciously large negative cash flow at end of year 6, for instance, key in 5 [n] [GOLD NPV] . If the number is positive, then the rate that would result from the previous cash flows alone would be higher than the IRR calculated for the whole sequence of Cash Flows, and we have a problem. This check is valid only for data keyed in without using the Nj key).

Multiple solutions

When the IRR calculations require the investor to be also a borrower, then, as we have seen, the validity of the notion of IRR itself is put in question. How seriously the notion itself is clouded is underlined by the fact that, on occasion, it is possible to have two rates that both fully explain the irregular cash flows, both equally valid though far apart.

Inability to calculate the IRR.

In circumstances where there is more than one change of sign among the cash flows, but not necessarily only in those instances where this results in our tainted borrower-investor situation, the HP-38 may find it impossible to make a valid first guess in its trial an error search for the IRR, and signifies this by displaying an error sign. (We may imagine a procedure where the first guess would be the rate that balances offthe initial investment and the last cash flow. If they are both negative, then there is no exchange, and no rate to balance them off). When this occurs, it is possible to suggest the first guess to the calculator by keying in a rate in i and pressing | RCL | BLUE | R/S | to get the IRR calculation going.

It is also not impossible for the trial and error procedure to lock itself in the trial and error quest, without ever making up its mind on a final answer. When this appears to occur, we may take over the trial and error procedure ourselves. To do this we may interrupt the calculator's quest by pressing R/S, the calculator will display after a while the latest guess that it made. We then check how accurate that guess was by pressing GOLD NPV . If the value is zero or sufficiently close to zero we may take the rate as our answer. If the answer is not accurate enough, we may key into i a better guess, and question GOLD NPV . We may repeat the procedure improving our guesses until we reach one that we judge satisfactory. Of course, the guess is the IRR when the corresponding NPV is equal to zero.

One approach that lifts the cloud that may exist over the IRR, and that circumvents the possible mathematical ambiguity, is to seek not the Internal Rate of Return, but the Financial Manager's Rate of Return, the FMRR.

Financial Manager's Rate of Return (FMRR)

Let us consider the following cash flows:

0	- 30,000
1	- 5,000
2	3,000
3	10,000
4	50,000

We may key this data in and calculate an Internal Rate of Return of 17.80%.. This rate, as we have seen, is the interest rate we would be getting on a savings account that could provide us with the same cash flows. Of course, I would not be getting interest on the $5,000 deposit until I made it at the end of the first year, and I would cease getting interest on the $ 3,000 and 10,000 withdrawals as soon as I had received the money. So we are not talking here of a $ 35,000 investment that 4 years later has produced $ 63,000. The rate on such an investment can easily be calculated:

35000 [PV]
63000 [CHS] [FV]
4 [n]
i (15.83%) This number is not our IRR.

However, the idea behind this last calculation makes sense. Perhaps I want to have at time 0 all the money I will need to cover any future negative cash flows, and perhaps I want to retain all the returns on my investment until all the money is in. If I should do this, I would not keep the money that is not presently invested idle. I would invest it at a safe rate in an instrument that would provide sufficient liquidity. Let me imagine that I could get 10% interest on such an investment. The question becomes: How much money do I need at time 0 to cover all my negative cash flows? How much money will I have accumulated at the end of the 4 years? What is the rate of return that transform my initial amount into my future amount?

To cover my $ 5,000 at the end of year 1 I need:

5000 [CHS] [FV] 10 [i] 1 [n] [PV] ($4,545.45)

So I need a total of $ 34,545.45 at time 0 to cover all my negative cash flows.

My $ 3,000, invested at 10%, will grow into the following amount in 2 years:

2 [n] 10 [i] 3000 [PV] [FV] ($ 3,630)

My $ 10,000, invested at 10%, will grow as follows in 1 year:

1 [n] 10000 [PV] [FV] ($ 11,000)

So at the end of year 4 I will have accumulated a total of

3,630 + 11,000 + 50,000 = 64,630.

Under the conditions we have set for ourselves, $ 34,545.45 has grown in 4 years into $ 64,630.

34545.45 [PV]

64630 [CHS] [FV]

4 [n]

i

(16.95%) That rate is a Financial Manager's Rate of Return (FMRR)

So theFMRR is really the average of two rates, both of which are Internal Rates of Return. One is the higher rate of the particular investment we are considering, the other is a readily available rate on a safe, liquid investment where amounts needed or produced by my major investment are deposited while they are not being used for the major transaction.

The FMRR can frequently be a more realistic rate to consider than the IRR, even if I do not actually intent to acquire at time 0 all the money I will subsequently need, and may not keep all the money earned from my investment until the end of the investment period. The very fact that there are negative cash flows to take care of, and that the returns are earned whether or not they are needed or can be re-invested at a high rate guarantees that, because of my major investment, some amounts of money will not be used to their highest and best potential. The safe rate used in FMRR calculations is a recognition of that fact.

The FMRR provides the added advantage that, by providing all the money that will subsequently be needed, it dispels the cloud that exists on those IRR calculations that would require the investor to borrow as well as invest.

How to calculate the FMRR.

Let us summarize what we did with the previous cash flows:

0	-30,000	-4,545.45		=	- 34,545.45	16.95%
1	-5,000					
2	3,000					
3	10,000					
4	50,000	+ 11,000	+ 3,630	=	64,630	

The calculations can be done, as we have done them above, by considering each cash flow separately and transferring it to time 0 or to the very last year of the transaction. However, there is no need to consider each cash flow individually. Let us key in the 5 cash flows as we did to calculate the IRR, and let us in fact calculate the IRR. Then we may proceed as follows:

Consider the first group of negative cash flows and calculate **the Net Present Value** with our safe rate as the discount rate:

1 $\boxed{\text{n}}$ 10 $\boxed{\text{i}}$ $\boxed{\text{GOLD}}\boxed{\text{NPV}}$ (-34,545.45, the same number as above)

Let us now eliminate these two cash flows from our irregular cash flow data

0 $\boxed{\text{STO}}\boxed{\text{0}}$ $\boxed{\text{STO}}\boxed{\text{1}}$

Let us now calculate the FV of our positive cash flows, invested at 10%:

4 $\boxed{\text{n}}$ $\boxed{\text{GOLD}}\boxed{\text{NPV}}$ $\boxed{\text{FV}}$ $\boxed{\text{FV}}$ (-64,630: we only need to correct the sign discrepancy brought about by the use of the regular cash flow keys to find the amount we were looking for)

We may even correct the sign discrepancy by merely keying back into PV a positive 34,545.45, and calculating the FMRR by pressing i :

34545.45 $\boxed{\text{PV}}$ $\boxed{\text{i}}$ (16.95%)

We have used the irregular cash flow capabilities of the calculator to calculate, in a single procedure, first the Net Present Value of the initial negative cash flows, then the future value of the positive cash flows that follow. In both calculations the rate used was our safe rate for investing funds not used for our major investment. We may illustrate the procedure as follows:

0	- 30,000	34545.45	PV	
1	- 5,000			
2	3,000			FMRR: 16.95%
3	10,000			
4	50,000	-64,630	FV	

Slight care must be used to assure a sign difference between our PV and FV.

The procedure is easily understood if we remember that we want to address ourselves first to the initial group of negative cash flows. With all the cash flows keyed in, we do so by keying in 1 n (in this particular example). This instructs the calculator to ignore any cash flow data beyond the one stored in register memory 1, our end-of-year 1 cash flow of -5,000 dollars. We then address ourselves to all the remaining cash flows by clearing from our register memories those negative cash flows that have already been replaced by their Net Present Value. We do so here by keying 0 into memories 0 and 1. It is then a simple matter of calculating the NPV for our positive cash flows, and then transferring this amount to where it belongs, the end of year 4 in our previous example.

Our treatment of the three positive cash flows can be illustrated as follows:

0	0	NPV	44,143.16		
1	0				
2	3,000				
3	10,000				
4	50,000			FV	-64,630

The rate that brings about these shifts in amount is, of course, our safe reinvestment rate, here 10%.

Special circumstances.

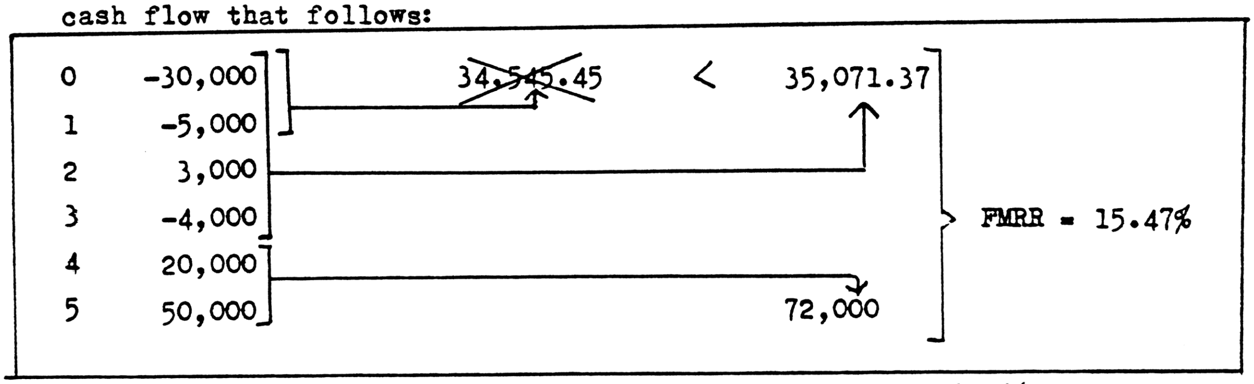
We have so far considered the case of an irregular cash flow sequence which had only 1 sign change. What if there is more than one sign change?

When this occurs, our aim still remains to have just the right amount of money at time 0 in order to cover all future negative cash flows, except that any positive cash flow that occurs before a negative cash flow should be used, along with what interest it has earned, to offset in part or in full the negative cash flows that follow. When this occurs, dealing with each cash flow separately can be a tedious and delicate business. Working with the irregular cash flow keys greatly simplifies the procedure:

1) Key in irregular cash flow data (and calculate IRR if desired).
2) Isolate the first group of negative cash flows and calculate their NPV, discounted at the chosen safe re-investment rate. Repeat to include the first and second group of negative cash flows, the first, second and third group or negative cash flows, etc. Retain the largest NPV value calculated, and clear all the cash flows that contributed to the calculation of that specific NPV value.
3) Calculate the FV of all the remaining cash flows, including any remaining negative cash flows that did not contribute to a higher initial NPV requirement.
4) We are left with a PV and a FV. The rate that relates these two numbers is our Financial Manager's Rate of Return.

Some examples will clarify the procedure: (10% re-investment rate throughout

First example: a positive cash flow only partially offsets the negative cash flow that follows:



We first key in the irregular cash flow data: IRR = 16.16%

Then:

10 [i]		Keys in re-investment rate.
1 [n]		Isolates cash flows 0 and 1.
[Gold] [NPV]	(34,545.45)	Calculates initial amount required to cover the first group of negative cash flows.
3 [n]		Isolates cash flows 0, 1, 2, and 3.
[Gold] [NPV]	(35,071.37)	Calculates amount required to cover the first two groups of negative cash flows. As this amount is larger than the amount required by the first group alone, this is the amount required at time 0.
0 [STO] [0] [STO] [1] [STO] [2] [STO] [3]		Clears all the cash flows, including the 3000 dollar positive amount, that contributed to the calculation of our initial requirement.
5 [n]		Instructs the calculator to consider all remaining irregular cash flows, that is cash flows 4 and 5.
[GOLD] [NPV]	(44706.34)	Calculates Net Present Value of remaining cash flows.
[FV] [FV]	(72,000)	Calculates Future Value of remainig cash flows.

We now have the Present Value and Future Value amounts we were seeking. We just need to key our required $ 35,071.37 back into PV, with the opposite sign of the FV amount already stored in FV, and to calculate the rate:

35071.37 [PV]		
[i]	(15.47%)	This is our FMRR.

<u>Second example</u>: A positive cash flow more than offsets the negative cash flow that follows. We will consider the same series of cash flows, except that cash flow 2 has been increased from $ 3,000 to 6,000. It is quite clear that this $ 6,000 cash flow can more than offset the negative $4,000 that follows. We will proceed as if we did not know whether it did or not.

0	-30,000
1	-5,000
2	6,000
3	-4,000
4	20,000
5	50,000

34,545.45 > ~~32,592.04~~

75,146

FMRR = 16.82%

Key in irregular cash flows and calculate the IRR, if desired,(17.85%).

Then:

10 $\boxed{\text{i}}$

1 $\boxed{\text{n}}$ $\boxed{\text{GOLD}}\boxed{\text{NPV}}$ (34,545.45)

3 $\boxed{\text{n}}$ $\boxed{\text{GOLD}}\boxed{\text{NPV}}$ (32,592.04) This second NPV amount, lower than the previous NPV value (press x y to compare), shows that we do not need to increase our initial investment because of the negative $ 4,000 cash flow—it confirms that the preceding $ 6,000 cash flow and the interest it earns fully covers the negative cash flow.

Because cash flows 2 and 3 have a positive balance, we need to include them in our calculation of our FV:

0 $\boxed{\text{STO}}\boxed{\text{0}}$ This keystroke enables us to consider separately

$\boxed{\text{STO}}\boxed{\text{1}}$ cash flows 2,3,4, and 5.

5 $\boxed{\text{n}}$

$\boxed{\text{GOLD}}\boxed{\text{NPV}}$ $\boxed{\text{FV}}$ $\boxed{\text{FV}}$ (75,146) This is our desired Future Value amount.

34545.45 $\boxed{\text{PV}}$ $\boxed{\text{i}}$ (16.82%) The Fanancial Manager's Rate of Return.

The same problem, solved by handling each cash flow individually with the PV - FV keys, requires that we calculate numbers that can be summarized as follows:

0	-30,000	+	(-4,545.45)	=			-34,545.45		
1	-5,000								
2	6,000							FMRR = 16.82%	
3	-4,000	+	6,600	=	2,600				
4	20,000								
5	50,000	+	22,000	+	3,146	=	75,146		

With $ 3,000 as Cash Flow 2 (see first example), line 3 would become:

3 -4,000 + 3,300 = - 700

The -700 would then be thrown back to time 0 as a negative 525.92: the $3,000 positive cash flow was not enough to offset the -4,000 that followed, and an extra amount of $ 525.92 was required as initial investment to cover the deficit.

Problems

1) Consider the following cash flows:

0	- 95,000
1	10,000
2	12,000
3	16,000
4	20,000
5	30,000
6	45,000
7	100,000

- Calculate the IRR. (18.83%)
- Given a re-investment rate of 9%, what is the FMRR? (16.00%)

FMRR solution, with cash flows keyed in:

0	STO	0	9	i	
GOLD	NPV	FV	FV		
95000	PV	i			(16.00%)

2) Consider the following cash flows:

Initial investment: $ 45,000

Return, each year, first 5 years: $ 5,000

Return, each year, next 10 years: $ 10,000

Return, each year, last 10 years: $ 15,000

- Calculate the IRR (16.65%)
- Given a re-investment rate of 10%, what is the FMRR? (12.51%)

FMRR solution, with cash flows keyed in:

0	STO	0	10	i	
GOLD	NPV	25	n	FV	
45000	PV	i			(12.51%)

3) Consider the following cash flows:

0	-135,000
1	-25,000
2	20,000
3	20,000
4	-45,000
5	350,000

- What is the IRR ? (18.02%)
- Considering a re-investment rate of 10%, what is the FMRR?(17.37%)

FMRR solution, with data keyed in without using Nj function:

10 i 1 n	
1 n GOLD NPV (157,727.27)	
4 n GOLD NPV (156,907.66)	Ignore this amount as it is smaller than the previous amount.
x⇄y STO 7	Retrieves and stores 157,727.27
0 STO 0 STO 1	Clears Cash flows 0 and 1.
5 n GOLD NPV FV FV	(351,320)
RCL 7 CHS PV i	(17.37%)

4) Consider the following cash flows:

0	-400,000
1	20,000
2	-100,000
3	200,000
4	1,000,000

- Calculate the IRR (29.78%)
- Given a 12% re-investment rate, what is the FMRR? (27.59%)

FMRR solution, with cash flows already keyed in:

12 i 2 n GOLD NPV	(461,862.24, obviously higher than our initial $ 400,000)
STO 7	
0 STO 0	
STO 1	
STO 2	
5 n GOLD NPV FV FV	(1224,000)
RCL 7 CHS PV i	(27.59%)

FMRR and minimum reinvestment requirement.

The FMRR can be subjected to various refinements. In particular, rather than reinvesting all positive cash flows at the safe rate it is possible to specify that amounts permanently earned will be reinvested at a higher reinvestment rate each time a specified minimum reinvestment amount has been accumulated.

Let us consider the following example:

0	-30,000
1	-5,000
2	3,000
3	10,000
4	12,000
5	12,000
6	12,000
7	50,000

Our safe rate is 6%.
Our reinvestment rate is 12%
Our minimum reinvestment requirement to benefit from the higher reinvestment rate is $ 15,000.
Calculate the IRR and the FMRR.

Let us key in the data without using the Nj key (this will make things easier for us), and calculate the IRR. (21.54%)

We now turn to the FMRR calculation:

6 [i]		
1 [n]		
[GOLD] [NPV]	(-34,716.98)	Amount needed at time 0 to cover the negative cash flows (CF_0 and CF_1).
0 [STO] [0]		
[STO] [1]		Erases the two negative cash flows.

We are now going to test the balance of the investment at the safe rate of 6% for every time period where the minimum reinvestment requirement may have been met:

Keystrokes	Display	Comment
3 $\boxed{n}$ $\boxed{GOLD}$ $\boxed{NPV}$ $\boxed{FV}$ $\boxed{FV}$	(13,180)	Balance at end of year 3 not sufficient to meet 12% reinvestment requirement.
4 $\boxed{n}$ $\boxed{GOLD}$ $\boxed{NPV}$ $\boxed{FV}$ $\boxed{FV}$	(25,970.80)	This amount is above minimum reinvestment requirement so the full amount must be reinvested at 12% for the remaining 3 years.

Let us immediately calculate what it will grow into in the remaining 3 years:

Keystrokes	Display	Comment
$\boxed{CHS}$ $\boxed{PV}$		
3 $\boxed{n}$ 12 $\boxed{i}$ $\boxed{FV}$	(36,487.10)	This is the amount that cash flows 2, 3, and 4 will contribute to the final FV. Let us clear these cash flows:
0 $\boxed{STO}$ $\boxed{2}$ $\boxed{STO}$ $\boxed{3}$ $\boxed{STO}$ $\boxed{4}$		

CF 5 of itself is not enough to meet our minimum reinvestment requirement, and is therefor reinvested at the safe rate of 6%.

The balance at the end of year 6 can be calculated as follows:

Keystrokes	Display	Comment
6 $\boxed{n}$ 6 $\boxed{i}$		
$\boxed{GOLD}$ $\boxed{NPV}$ $\boxed{FV}$ $\boxed{FV}$	(24,720)	This is above our reinvestment requirement and can be reinvested at 12% for the remaining year:
$\boxed{CHS}$ $\boxed{PV}$		
12 $\boxed{i}$ 1 $\boxed{n}$ $\boxed{FV}$	(27,686.40)	Amount contribute by CF 5 and 6 to the final FV amount.

So the total amount accumulated at the end of year 7 is as follows:

50,000 + 36,487.10 + 27,686.40 = 114173.50

Our FMRR is the rate that allows 34,716.98 to grow into $ 114,173.50 in 7 years:

Keystrokes	Display
114,173.50 (already in display) $\boxed{CHS}$ $\boxed{FV}$	
34,716.98 $\boxed{PV}$ 7 $\boxed{n}$	
$\boxed{i}$	(18.54%)

The procedure can be illustrated as follows:

0	-30,000		6%		-34,716.98	
1	-5,000					
2	3,000					
3	10,000		6%			
4	12,000			25,970.80		18.54%
5	12,000	6%				
6	12,000	24,720	12%	12%		
7	50,000 +	27,686.40	+	36,487.10	= 114,173.50	

Of course, the keystrokes used here are not the only ones that could be used. In particular it would be possible to deal with each individual cash flow separately and use only the PV and FV keys. There are circumstances where dealing with each cash flow separately, one step at a time, may well be the simplest way to proceed. One instance is the following:

2) <u>Safe rate</u> for all amounts that are kept in reserve to meet future negative cash flow requirments: 8%.
<u>Reinvestment rate</u> for all amounts, however small, that are not needed until the end of year 5: 15%.
Use of the PV and FV keys leads to the following passage from the data to the final Financial Manager's Rate of Return:

0	-30,000				-30,000	
1	10,000	- 7,407.41	= 2,592.59			
2	-8,000	8%				
3	15,000	- 9,259.26	= 5,740.74	12%		25.16%
4	-10,000	8%	12%			
5	80,000	+	7,592.13	+ 4,534.46	= 92,126.59	

Keystrokes could be as follows:

1	n	8	i	8000	FV	PV	10000	+	(2,592.59)	STO	7
				10000	FV	PV	15000	+	(5,740.74)	CHS	PV
2	n	15	i	FV	(7,592.13)	STO	6				
RCL	7	CHS	PV	4	n	FV	RCL	6	+	80000	+ FV
30000	CHS	PV	5	n	i	(25.16%)					

Statistical functions

I DATA.

Statistical data is entered by pressing GOLD Σ+

Each entry can consist of one number keyed into the x memory of the stack (the display): 3 GOLD Σ+

Each entry can also consist of a pair of numbers keyed into the x and y memories: 2 ENTER 3 GOLD Σ+ (Here, 2 is y and 3 is x)

GOLD Σ+ (Sigma +, or the summation key) transfers the data to memories 1 to 6 of the register. It adds in those memories the data provided by each entry. This creates a powerful data bank from which the various statistical question keys will retrieve precious information.

Use RCL to explore and review the content of memories 1 to 6(R 1 to R 6)
GOLD CLEARΣ clears content of memories 1 to 6.

Key in 2 ENTER 3 GOLD Σ+ and explore R 1 to R 6.
Now key in 100 ENTER 200 GOLD Σ+ and explore R 1 to R 6
What do we find? (Σ means sum, Σx means the sum of various x entries)

Memory 1		Counts the number of entries (that number appears on display
2	Σx	The sum of each value of x keyed in.
3	Σx^2	The sum of the squares of each x.
4	Σy	The sum of each y entry.
5	Σy^2	The sum of the square of each y entry.
6	Σxy	The sum of the product of each x and y entry.

BLUE Σ- does the opposite of GOLD Σ+ . It subtracts one from R 1, x from R 2, etc. It is used to erase an x, or an x and y entry.

II QUESTIONS

The mathematical processing of the data bank, which the statistical question keys initiate, retrieves precious information concerning the entries.
For instance the mean, or arithmetic average of all the x entries is given by pressing BLUE $\bar{x}$. This divides the sum of x entries (Σx in memory 2) by the number of entries (memory 1). Three keys ask questions concerning averages, two keys are related to linear estimates.

III Mean, or arithmetic average.

1) $\boxed{\text{BLUE} \mid \bar{x}}$ gives average of x entries.
It also stores the average of the y entries in the y memory. Retrieve by pressing $\boxed{x \leftrightarrow y}$.

Example.
A survey of two bedroom apartments shows the following rents:
$ 260, $ 310, $ 215, $ 245, $ 330, $ 275, $ 220, $ 245.
What is the average rent?

260	GOLD	Σ+
310	GOLD	Σ+
215	GOLD	Σ+
245	GOLD	Σ+
330	GOLD	Σ+
275	GOLD	Σ+
220	GOLD	Σ+
245	GOLD	Σ+

Note: The order of entry is not important.
If survey shows two equal data, they should both be entered.

$\boxed{\text{BLUE} \mid \bar{x}}$ (Average rent: 262.50) (Do not clear)

2) $\boxed{\text{BLUE} \mid s}$ gives standard deviation of x entries.
It stores standard deviation of y entries in y.

Standard deviation for the previous entries?

$\boxed{\text{BLUE} \mid s}$ (40.80)

In practical terms this means that, on the basis of the sample data keyed into the calculator, we can expect about 2/3 of two bedroom apartments in the area surveyed to have rents of 262.50 ± 40.80. Two thirds of the rents should be between $ 221.70 and $303.30.

It is important to know what standard deviation corresponds to a given average. It would make a difference if the same average of $ 262.50 was the average between four entries of $ 50 and four more of $ 475. In that case the standard deviation would be 227.17, and we would know that many rents differ greatly from the average rent.

3) $\boxed{\text{BLUE} \mid \bar{x}\ \text{w}}$ provides us with the <u>weighted average.</u>

Consider the following situation:

Over a period of time I purchased	10 lb of potatoes	at 30 c/lb.
	5 lb	at 40 c/lb
	8 lb	at 42 c/lb
	15 lb	at 31 c/lb.

Two questions can be asked on the basis of that data:

- What was the average price of potatoes over that period of time?
- What was the average cost to me of a pound of potatoes?

The second question is affected by the fact that I bought more potatoes when they were cheap than when they were more expensive. The second question asks the <u>weighted average</u> of a pound of potatoes. The average which the first question asks is not affected by the amount I chose to buy. The simple average of 30, 40, 42 and 31 gives the answer.

<u>Keying in the data</u>.

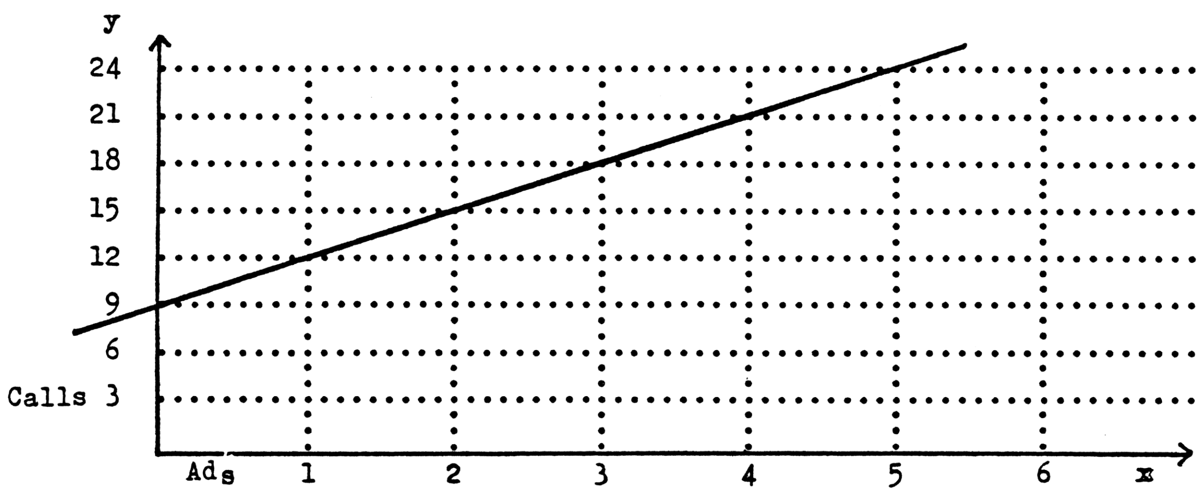
Item (price) first, entered in the y memory, then the weight:

30	ENTER	10	GOLD $\Sigma+$		
40	ENTER	5	GOLD $\Sigma+$		
42	ENTER	8	GOLD $\Sigma+$		
31	ENTER	15	GOLD $\Sigma+$		
BLUE $\bar{x}$ w				(34.24)	<u>Weighted average</u>. (Average cost to me
BLUE $\bar{x}$				(9.5)	Average weight purchased
$x \rightleftarrows y$				(35.75)	<u>Average price</u> (average of y entries)

IV Linear estimates.

1) Linear calculation.

Let us imagine a situation where a broker gets 9 calls a day without any advertising, and gets 3 extra call a day for every house advertised in the paper. There is a linear relationship between the number of ads and the number of calls. In a graph plotting the number of calls in relation to the number of ads a straight line would represent that relationship.



If one ad, we have 12 calls, if 2 ads 15 calls, if 3 ads 18 calls etc. Any two points on the line can define the whole line. Let us key in two points in the calculator, with the number of ads as x, and the number of calls as y. (Key in y first, $\boxed{\text{Enter}}$ then key in x).

12	$\boxed{\text{ENTER}}$	1	$\boxed{\text{GOLD}}\boxed{\Sigma+}$	Any two points on the line, defined by x and y, keyed in in any order.
18	$\boxed{\text{ENTER}}$	3	$\boxed{\text{GOLD}}\boxed{\Sigma+}$	

The calculator can now tell us how many ads (x) are needed to get a given number of calls (y), or how many calls will be received if a given number of ads are published.

10	$\boxed{\text{BLUE}}\boxed{\hat{y}, r}$	(39)	There will be 39 calls if we have 10 ads We keyed in an x and questioned for y.
60	$\boxed{\text{BLUE}}\boxed{\hat{x}, r}$	(17)	We need 17 ads to get 60 calls. Here we keyed in a y, and questioned for x

(Check for other values, including those where we already know the answer)

2) Linear estimates

It is unlikely that the broker in our previous example could really know that he gets 3 extra calls for every ad, and that he would get 9 calls without any ads. It is more likely that he would know what actually happened over a number of weeks. His records might show the following correlation between calls and ads:

Number of calls (y)	Number of ads (x)
30	6
30	8
36	10
38	10
12	2
46	11
21	3

Let us key that data in as statistical information:

30 $\boxed{\text{ENTER}}$ 6 $\boxed{\text{GOLD}}$ $\boxed{\Sigma+}$
30 $\boxed{\text{ENTER}}$ 8 $\boxed{\text{GOLD}}$ $\boxed{\Sigma+}$
Etc.

The calculator then establishes a line which best fits that data (or acts as if it had established such a line). It can then calculate y for any given x, or x for any given y as in the previous example. The predictions will be statistically valid: they will provide the most likely value for x or for y according to the data given.

With the data above keyed in, calculate the number of calls for the following number of ads: 0, 1, 2, 3, 10.

0	BLUE	$\hat{y}$,r	(8.91)	N° of calls to expect with no ads.
1	BLUE	$\hat{y}$,r	(11.92)	
2	BLUE	$\hat{y}$,r	(14.93)	
3	BLUE	$\hat{y}$,r	(17.95)	
10	BLUE	$\hat{y}$,r	(39.04)	

The accuracy of the predictions, or estimates, will be all the greater as the relationship between the two variables is more strictly linear. The calculator provides us with a measurement of the degree to which the data fits the linear relationship expressed by the best fit line. That measurement is stored in the y memory whenever a linear estimate is calculated

That correlation coefficient, r, is retrieved by pressing $\boxed{x \rightleftarrows y}$. It is a number between 1 and −1. It is negative when the line slopes down as x increases. The closer the coefficient is to 1 or to −1, the greater the correlation between the data and the best fit line.

With the previous data, the coefficient is .96, which promises a reasonable measure of accuracy. In our very first example where the line was established with two points only, we would of course have the perfect fit: r = 1 .

3) Equation of the line.

The equation is on the model of $y = Ax + B$.

Clearly, if $x = 0$, then $y = B$. B is the value of y when $x = 0$. It is calculated by finding the value of y for $x = 0$. (It is the y intercept).

It is clear also that when x increases by 1, then y increases by A.

A is called the slope of the line. It is negative when y decreases as x increases. To calculate it, find out by how much y increases (or decreases) when x increases by 1.

With the previous data keyed in:

Keys	Display	Comment
0 [BLUE] [$\hat{y}$,r]	(8.91)	Calculates B.
[STO] [0]		
1 [CHS] [BLUE] [$\hat{y}$,r]	(5.89)	Value of y for x = − 1
[STO] [−] [0]		Calculates A = 8.91 − 5.89
[RCL] [0]	(3.01)	Recalls value of A

The equation is $y = 3.01\,x + 8.91$

(Store in memory 0, as memories 1 to 6 are stored with statistical data).

We calculated here the increase in y when x increases from − 1 to 0.

Any other increase of x by 1 would yield the same result. For instance, we could calculate x for y = 0, add 1 to x : the corresponding value for y would be equal to A

Keys	Display	Comment
[0] [BLUE] [$\hat{y}$,r]	(8.91)	Calculates B.
[0] [BLUE] [$\hat{x}$,r]	(−2.95)	Calculates x for y = 0
[1] [+]	(−1.95)	Adds 1 to previous x value.
[BLUE] [$\hat{y}$,r]	(3.01)	Calculates A.

V Other uses of Σ+ key.

1) Keeping count.

There are a number of situations where it is important, or simply convenient, to keep track of the number of times something has been done. This can be achieved by pressing GOLD Σ+ every time the event occurs. All the features of the calculator remain available, except statistical functions and register memories 1 to 6.

This keeping count feature of the Σ+ key can be used when we have a long list of numbers to add--rents in a large apartment building, entries in a checkbook, etc. Pressing GOLD Σ+ adds entries in memory 2 at the same time as it displays the number of entries. If we are interrupted and lose track of where we left off, the number of entries will let us know where we need to continue. When we have finished keying in the data, we obtain the total by recalling R 2 (RCL 2).

2) Σ+ as a single key function.

If we have a number of entries to perform with the Σ+ key, having it as a gold key function is an inconvenience. This can be remedied by programming so that R/S will perform GOLD Σ+ . As Σ+ and R/S are the same key, this mini-program has in fact simply eliminated the need to press the gold key.

BLUE	P/R	Mini-program transforms R/S into equivalent of GOLD Σ+ keystroke.
GOLD	Σ+	
BLUE	P/R	

Example

Program as above, then add 125, 135, 250, 175, 175, 150.

BLUE P/R GOLD Σ+ BLUE P/R

125 R/S 135 R/S 250 R/S 175 R/S 175 R/S 150 R/S

(Display shows 6. We know that we have not skipped one figure.)

RCL 2 (1010) Retrieves total

VI Practice.

1) A survey shows that 120' deep commercial lots have been selling for the following prices:

Frontage	Price.
30'	40,000
80'	150,000
50'	70,000
25'	30,000
90'	180,000

a) What is the average price per lot? (94,000)

b) What is the average frontage? (55)

c) What is the average price per frontage foot? (1707.09)
(Divide average price by average frontage)

d) On the basis of average price per frontage foot, what would be the likely price for a lot with 35' frontage and one with 100' frontage? (59,818 and 170,909)

e) Answer the same question on the basis of the best fit line. (47,538 and 198,538)

f) What is the correlation coefficient for the best fit line? (0.99)

g) Which answer is likely to be more accurate, d) or e) ?
(e): the correlation is good, and the best fit line approach takes into account the premium paid per frontage foot for larger lots).

2) Over the years you have been buying stock in SOS corporation. Despite the ups and downs of the market, you feel the basic trend corresponds to a linear increase.

a) Test the correlation (0.94)

b) Give best estimate of stock price in 1980 and 1985. (51.41; 60.82)

c) Best estimate of when the stock will be worth $ 90. (Year 2000)

DATA:				
	1960	$ 15	1975	$ 34
	1964	$ 26	1977	$ 48
	1971	$ 34	1978	$ 49

(The full dates can be keyed in. With large number of entries it might be necessary to key in only the last two digits: 60, 64, 71, etc.

PART VII

Multiple programs

With the introduction of the HP 38-C, which retains programs when the calculator is turned off, it becomes all the more important to be able to record more than one program. This section discusses some specific requirements and some optional keystrokes that may lead to greater convenience.

I Changes in the programs themselves.

All GTO instructions written for a program expected to begin on programming step 01 must be adjusted if the program does not begin on step 01. For instance a GTO 01 instruction will need to be modified to GTO 05 if the program begins on step 05. This represents a 4 step displacement. Other GTO instructions (except GTO 00) similarly need to be adjusted by an increment of 4 (Please note: 4, not 5).

II Between two programs.

Note that single programs naturally end with a GTO instruction:

- either the GTO instruction we key in ourselves (to create a loop for instance),
- or the GTO 00 instruction which is pre-recorded in every programming memory before it is overwritten by our own program instructions. With a program that does not end with a GTO instruction, when the last step of the program is reached by the calculator, it automatically moves on to the pre-recorded GTO 00 instruction, and shifts back to step 00 where it stops.

Similarly, when two programs are keyed in in succession the first of the two programs must end with a GTO instruction. This can be either the GTO instruction which is naturally part of that program(for instance to create a loop), or a GTO 00 instruction which we will want to key in ourselves between the two programs. In other words, if the first program ends on step 15 with a GTO 01 instruction, we can start the second program on step 16. If that same 15 step first program does not end with a GTO instruction, we should key in GTO 00 as step 16 and begin the second program on step 17.

III **Running a program** (other than the first)

To run a program starting on step 17 we just need to punch

BLUE	GTO	17		R/S

This will need to be done each time we want to run this program, and requires that we remember on what program step the program begins. If we are willing to spend some time and some programming steps on the matter, these two inconveniences can be alleviated.

IV **Programs that select our programs.**

Let us imagine that we want to key in for long and frequent use three programs that, keyed in as above, would start on steps 01, 12, and 30. We might want to key in at the outset three 1 step programs as follows:

01-	BLUE	GTO	04
02-	BLUE	GTO	15
03-	BLUE	GTO	33

Our first program would then begin on step 04, our second program on step 15, and our third program on step 33, each program having been shifted three steps (and adjusted accordingly).

To select program 1, 2, or 3, we simply need to key in

BLUE	GTO	01	R/S	or
BLUE	GTO	02	R/S	or
BLUE	GTO	03	R/S	

R/S alone is enough for the first program if the calculator has been switched back to step 00 the last time a program was used.

V Running a program repetitively.

We frequently need to run a program a number of times. If it is our first program, this is done with R/S every time we want to run the program. If it is not the first program, we need to key in each time a BLUE GTO instruction switching to the first step of our program. This can become inconvenient. To avoid the repetition, programs other than the first can be sandwiched between a R/S instruction at the beginning and a BLUE GTO instruction at the end that would send back the program to the R/S instruction.

For instance, let us imagine that we have a 10 step program that would normally begin on step 17 and end on step 26. We would modify it as follows:

17-	R/S
18-	Our 10 step program.
⋮	
27-	
28-	BLUE GTO 17

After we run the program once, just pressing R/S will allow us to run this program again.

Remarks:

- To run the program, we now have to switch to step 18, not 17. Similarly, a programming instruction to select this program would be BLUE GTO 18 (See § IV).

- The GTO 17 instruction that now ends our program means that we can start a new program on step 18. The GTO 17 instruction replaces the GTO 00 instruction that would otherwise have been required.

- Any GTO 00 instruction within the program would also be replaced with a GTO 17, sending the program back to the holding pattern of the R/S instruction. (See Band of Investment program for an example of GTO 00 instruction within a program).

A program

<u>Depreciation schedule</u>, <u>declining balance</u>. (Not for straight line).

This program will provide the <u>year number</u>, the <u>depreciation</u> for the year, and the remaining <u>depreciable value</u>. Before running the program initial data is keyed in and followed by R/S . Key in, in order:

- Initial depreciable value (not land value!) R/S
- Life of building R/S
- Depreciation rate (125,150 or 200) R/S

Each successive piece of data establishing the schedule is then obtained by pressing R/S .

Blue P/R

	STO 0	Stores initial depreciable value.
	100	
	R/S	Allows for life to be keyed in.
	÷	Calculates straight line depreciation rate
	R/S	Allows for depreciation rate to be keyed in.
	%	Calculates declining balance rate.
	ENTER ENTER ENTER	Establishes rate as a constant.
12-	GOLD Σ+ R/S	Keeps count of year number
	CLx	Clears year number
	RCL 0	Calculates depreciation for the year.
	x⇄y	
	%	
	R/S	
	− R/S	Calculates remaining depreciable value.
	STO 0	
22-	Blue GTO 12	

BLUE P/R

Check: 100000 R/S 25 R/S 125 R/S should give 4th year depreciation: $ 4,286.88, depreciable value: $ 81,450.63

Program

Square footage.

This program allows measurements expressed in feet and inches to be keyed in using decimals:

10 feet 6 inches as 10.06

9 feet 11 inches as 9.11

(This does not mean here 10.06 feet, just 10' 6". The program will interpret this as such, and process ft. and in. separately.)

The program converts these measurements into inches.

With this program, the area of a room 17' 8" by 11' 7" would be found as follows:

17.08	R/S	(212)	Length in inches.
11.07	R/S	(139)	Width in inches.
	x	(29,468)	Product of length by width gives area in square inches
144	÷	(204.64)	Area in square feet obtained by dividing by the number of square inches in a square foot (12 x 12 = 144)

BLUE P/R — BLUE CLP

BLUE FRAC	
STO 7	Stores "decimal inches" into memory 7
BLUE LAST x GOLD INTGR 12 x STO 0	Transforms feet into inches and stores in memory 0
RCL 7 100 x STO + 0	Transforms "decimal inches" into inches, and adds to inches already stored in memory 0
RCL 8 RCL 0 STO 8	Positions the previous measurement into the y memory of the stack for easy multiplication.

BLUE P/R

Use Gold Σ+ to add up partial answers in square inches or square feet.

A program

Income projection and inflation

In its simplest usage, the following program aims at providing a 5-year projection based on data such as this:

An income property is purchased with an $ 80,000 investment.
It is scheduled to provide a first year gross income of $ 32,000 affect in subsequent years by a 9% rate of increase.
First year operating expenses are $ 15,000, affected by a 12% rate of increase.
The debt service is $ 38,000 per year.
First question: What reversion will result in a 15% return on investment ?
2nd question: What is the (internal) rate of return if there is a reversion of $ 350,000 when the property is sold at the end of year 5 ?

The program

	Keystrokes	Comment
	BLUE P/R	
01-	STO 6	Stores income or expense in memory 6
	5 n	
04-	R/S	Allows rate of increase to be keyed in.
	BLUE EEX 2 ÷	Divides rate of increase by 100
	1 +	Adds 1 to establish factor
	ENTER ENTER ENTER	Creates constant with factor
13-	RCL 6	Recalls income or expense amount.
	STO + 1	**Adds to memory 1.**
	x STO + 2	Multiplies by increase factor and adds to
	x STO + 3	to memories 2, 3, 4 and 5.
	x STO + 4	
	x STO + 5	
23-	BLUE GTO 00	Omit if program memories were cleared
	BLUE P/R	

With the program recorded, clear GOLD Clear ALL, and key data in:

DATA		KEYSTROKE	
Year 0	-80,000	80000 CHS STO 0	(Can be keyed in at end)
G.Inc.	32,000	32000 R/S	
Increase	9%	9 R/S	
Op. Exp.	-15,000	15000 CHS R/S	(Negative because expense)
Increase	12%	12 R/S	
Debt	-38,000	38000 CHS R/S	
Increase	0%	0 R/S	(Debt service is constant)

15% return	15 i	
Calculate NPV	GOLD NPV	
Project to end of year 5	FV FV	(A reversion of $ 289,502 will provide 15% return)

The second question, which could be the only question and replace the 3 lines on this page, may follow:

Reversion: $ 350,000	35000 STO + 5	Added to memory 5
Question for IRR	GOLD IRR	(20.73% return)

This program does not do all the work for us. To use it properly, it is important to understand what occurs.

With a calculator that has been cleared (GOLD Clear ALL), it is possible to enter irregular cash flow data directly in the register memories. This is what occurs here. When doing so, it is not possible to use the Nj key, but that is not required here as each yearly cashflow is a separate CF entry. When given an amount ($ 32,000), and a rate of increase (9%), the program enters 32000 in memory 1, 32000 plus 9% increase in memory 2 ($34,880), 34880 increased by 9% in memory 3, etc. When a second pair of numbers(Minus 15,000 dollars, and 12% increase)is keyed in with the program, the same calculations occur, and the results are added (or subtracted) from the contents of memories 1 to 5. A constant number ($38,000) can be added or subtracted from each of these memories by pairing it off with a 0% increase. We may then key the intial investment into memory 0 and question for the reversion that will provide a given return, or key the initial investment in memory 0 and add the reversion to the content of memory 5, and question for the resulting rate of return.

We could also calculate separately the various tax consequences of the operation, add or subtract these numbers separately from the contents of memories 1 to 5, and ask the same questions on an after tax basis.

If the Gross Income and Operating Expenses for each of the years needed to be recorded, a R/S instruction included in the program before each one of the x instructions would allow us to see these numbers.

It is important not to forget the sign convention for each entry. Writing down the data, with the correct sign, as in our DATA column. If the minus sign has been forgotten, this can easily be corrected before the rate is keyed in by simply storing the correct number directly into memory 6. That temporary storing of the first year data was introduced in the program in order to maintain what seemed to be a more natural order ($ 32,000 and then 9%, or minus 15.000 and then 12) rather than having to key in the rate of increase first.

Step 23-, the | BLUE | GTO | 00 | instruction, can be omitted if the program memories were cleared (| BLUE | CLP | while in programming mode) before the program was recorded. I add it here as a reminder that all programs end with a GTO instruction, either the one we have written in, or the automatic GTO 00 that follows the last step we have keyed in. The disadvantage of keying it in ourselves is that it may on occasion--and it is the case here--cost us one more register memory.

Graduated Payment Mortgage program.

At 15% interest on a 30 year loan of the right amount, the payments are $100. If the payments were interest only, the amount would go down to $ 98.86, a saving of 1.14%. Increasing the term of our initial loan above 30 years could at best, for a term approaching infinity, decrease the payments by 1.4% . The solution is to allow for initial payments that do not fully pay for the interest, with the interest owed but not paid accruing to the principal. This is negative amortization. If the payments are allowed to increase in time, the loan can still be fully amortized. This creates a graduated payment mortgage.

FHA loans, which pioneered 30 year loans many decades age, now offer a Graduated Payment Mortgage option. The following program calculates the payments on these loans, at least to within a cent or two, as there is no way the arcane rounding specifications of the FHA can be duplicated by a rational instrument such as the HP 38. More importantly, the program puts in the hands of private parties the means of creating their own graduated payment mortgages, loans with low initial payments that increase by a given percentage for a set number of years.

Let us consider a $ 100,000 loan at 13% interest, amortized over 15 years. Level payments are $ 1265.24. (Amortized over 30 years, with a 15 year due date the monthly payments would be $ 1106.20, and the balance $ 87,429.87). How convenient would it be for the borrower to start with payments of $ 837.36? If the payments were kept level, this would correspond to an interest rate of 5.88% ! But of course they are not going to remain level, they are going to increase by 8% per year. In other words, if we have an 8% rate of inflation, they are going to keep up with inflation, if the inflation rate is higher, the amount of the payment will increase from year to year, but the value of the payment will decrease somewhat. In any case, only in the 7th year will the borrower begin to pay more than he would have with level payments, and only after 5 years does he pay more than for a level payment 30 year loan.

From the lender's point of view, he is certainly getting less at the beginning.
But:
- He now has a buyer that can afford his house,
- He is making 13% on his loan,
- He has established for himself an income that increases by a given percentage every year and has a better chance of keeping up with inflation. In the 15th year, he will be receiving $ 2,453.49 per month.

The program that follows at least provides the option of choosing a graduated payment loan.

The program originates with Hewlett-Packard. I modified it slightly in order to shorten it by 5 steps and to standardize the way in which the interest and the term are keyed in-- the usual 15 BLUE n and 13 BLUE i for the previous problem instead of 15 n and 13 i required by the Hewlett-Packard program.

I hope the program will add a new option to your creative financing bag of tricks.

Graduated Payment Mortgage: the program.

BLUE P/R Switches into programming

	Display		
STO 1	01-	21	1
STO 2	02-	21	2
x⇄y	03-		33
1 %	05-		23
1 +	07-		51
STO 0	08-	21	0
RCL BLUE n	09-	22,25	11
RCL 2	10-	22	2
-	11-		41
BLUE n	12-	25	11
RCL PV	13-	22	13
STO 3	14-	21	3
1 CHS PMT	17-		14
PV	18-		13
CHS FV	20-		15
1 BLUE n	22-	25	11
RCL PMT	23-	22	14
RCL 0	24-	22	0
÷	25-		71
PMT PV	27-		13
CHS FV	29-		15
1	30-		1
STO - 1	31- 21	41	1
RCL 1	32-	22	1
BLUE x=0	33-	25	6
BLUE GTO 36	34- 25	7	36
BLUE GTO 23	35- 25	7	23
RCL 3	36-	22	3
RCL PV	37-	22	13
÷	38-		71
STO 4	39-	21	4
RCL 4	40-	22	4
RCL 0	41-	22	0
RCL 2	42-	22	2

(Continued)

(Continued)			
BLUE y^x	43-	25	21
÷	44-		71
R/S	45-		74
1	46-		1
STO - 2	47- 21	41	2
RCL 2	48-	22	2
BLUE x= 0	49-	25	6
BLUE GTO 52	50- 25	7	52
BLUE GTO 40	51- 25	7	40
RCL 4	52-	22	4
R/S	53-		74
BLUE GTO 53	54- 25	7	53

BLUE P/R Ends programming

This program originates with Hewlett-Packard. It was slightly shortened and revised by Edric Cane.

Keying in the variables

With the program keyed in, the Begin-End switch on End as usual, and the calculator cleared (GOLD Clear ALL):

1) Key in the term, interest, and loan amount in the usual way.

15 BLUE n	Loan amortized in 15 years
13 BLUE i	13 % interest
100000 PV	100,000 dollar loan.

(At this stage you may if you wish press PMT and calculate the level payments that would amortize the loan. No need to clear that number.)

2) Key in the yearly rate of increase in y and the number of increases in x .

8 ENTER	8% yearly increase in payment amount.
14	14 yearly increases. For a 15 year loan with payments that increase every year, there will be 14 increases.

3) Press R/S to calculate each of the successive payment amounts:

R/S	(837.36)
R/S	(904.35)
R/S	(976.70)
Etc.	

When the payment amount just repeats itself the required increases have been made, and the amount of the payments remains level until the loan is fully paid off.

To key in a new loan, press GOLD Clear ALL and start over.

(Note that GOLD Clear ALL not only clears your non program memories, it also switches back your program to step 00-)

Testing your program

30 year $50,000 loan at 14% interest, with 5 increases of 7.5% :

30 BLUE n 50000 PV 14 BLUE i

7.5 ENTER 5

R/S (461.26) R/S (495.86) R/S (533.04) R/S (573.02)

R/S (616.00) R/S (662.20) R/S (662.20 level payment reappears)

Amortization schedule for graduated payment loan.

The regular GOLD AMORT procedure can be used to write a complete amortization schedule of a graduated payment loan. Care should be taken to modify the payment amount each time the payments change.

For instance, with the previous $ 50,000 loan at 14% and the graduated payments already calculated, a yearly schedule is obtained as follows:

50000 PV 14 BLUE i		Loan amount and interest
461.26 CHS PMT		1st year payment
12 Gold AMORT	(-7097.76)	Interest owed, 1st 12 months.
x⇄y	(1562.64)	Principal increase, first year.
RCL PV	(51,562.64)	Balance of loan end of first year.
495.86 CHS PMT		Payment amount for second year.
12 GOLD AMORT	(-7303.40)	Interest, 2nd year.
x⇄y	(1,353.08)	Principal increase, 2nd year.
RCL PV	(52,915.72)	Balance of loan, end 2nd year.
533.04 CHS PMT		3rd year -payment
12 GOLD AMORT	(-7,475.71)	-interest
x⇄y	(1,079.23)	-principal increase
RCL PV	(53,994.95)	-balance
573.02 CHS PMT		4th year -payment
12 GOLD AMORT	(7604.88)	-interest
x⇄y	(728.64)	-principal increase
RCL PV	(54,723.59)	-balance

Etc. In time, the principal reduction begins. The loan is fully paid off in 30 years.

For tax purposes, on a cash basis the full amount of the payments can be deducted from income until the loan amount is brought back down to its initial amount. On an accrual basis, the full amount of interest would be deductible, even if it is more than the actual payments made.