The HP 38G Graphic Calculator Beginner's Guide

Discover It For Yourself Series

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There are many additional features not mentioned in this manual due to limitations on size and also the fact that as a beginner's manual too much information could prove more confusing for the beginner. As you become more familiar with the HP38G you will discover a wide variety of features not mentioned in this manual. (eg If you do Calculus, open the FUNCTION ApLet & enter F1(X) = $x^3 - 5x^2$ then open the SOLVE ApLet & enter E1 ∂x(F1(X)) = K; then press NUM solve for various values of x to determine the resulting function. You could also input values for k and solve for x. Interpret your results!)

A wide variety of such extensions are possible on your HP 38G. You should experiment.

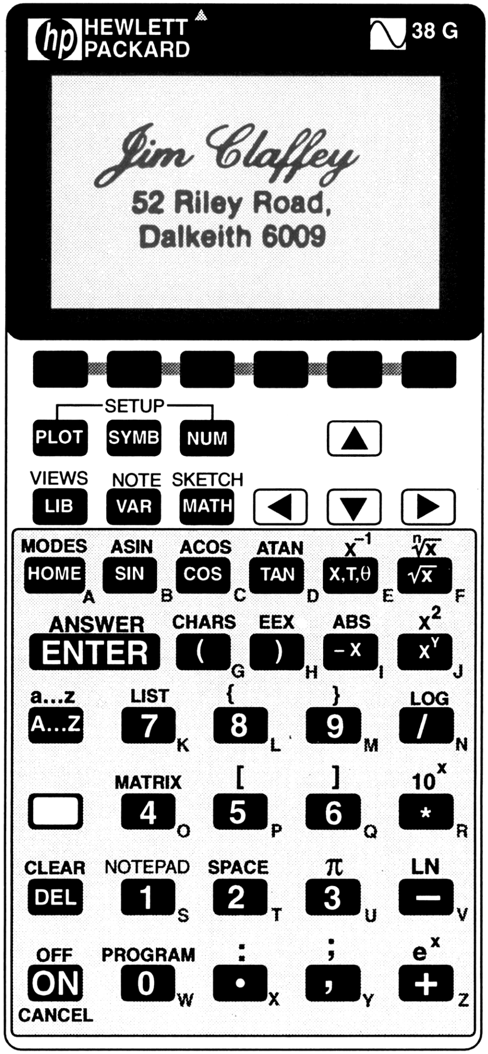
For example *How you can plot piecewise functions?* (Yes it is possible)

This beginners manual is just a start! Feel free to utilise the functions listed at the back of the User's Guide that came with your HP 38G and try out as many possibilities as you like.

Jim Claffey

THE HP 38G

GRAPHIC CALCULATOR



BEGINNER'S GUIDE

by

Jim CLAFFEY

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FOREWORD

The HP 38G graphic calculator opens a veritable Pandora's Box to the inquisitive mathematics student. The power is there, all it requires is an inquiring mind and an imaginative approach to the exploring of ideas

This manual is not a text book, nor is it a book of exercises. It is a beginner's guide. It simply attempts to explain *some* of the power available to students who use the HP 38G calculator. As with any power tool, unless you are familiar with the tool and feel both confident and comfortable in its use then all the power in the world will not enable you to use that tool effectively and efficiently.

This is a beginner's guide. It does not claim to develop the full potential of the HP 38G. One would not be so presumptuous. In writing this manual, even at this basic level, I have come to better understand the task that the writers of the original manual had in trying to encompass and explain the range of features available. Once you have familiarised yourself with the contents of this guide, the original manual that came with your calculator will provide many more ideas. I can only hope in some small way that the style in which this guide was written enabled you to come to grips with the tool a lot sooner. In outlining the use of this graphing calculator I have adopted a graphic approach.
A picture is worth a thousand words. A graph is worth a million numbers.

FREE AT LAST!

In my humble opinion the HP 38G marks the real beginning of freedom in the mathematics classroom. The freedom to think, conjecture and then test beyond more than just simple ideas. The power of computers and associated technologies is now within the grasp of all students. The HP 38G is itself a *micro-micro (m^2) computer* Its use of ApLets is a novel feature that enables one to store not just work and assignments, but *ideas* still in their gestation period.

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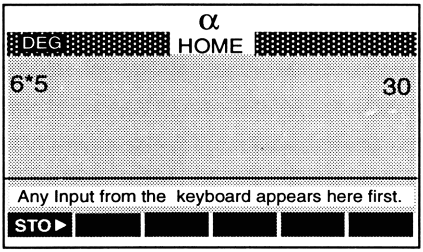
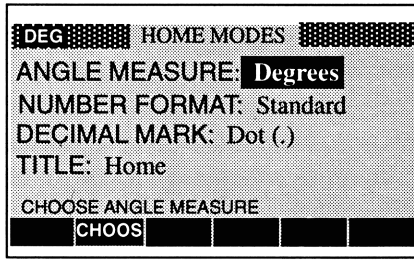
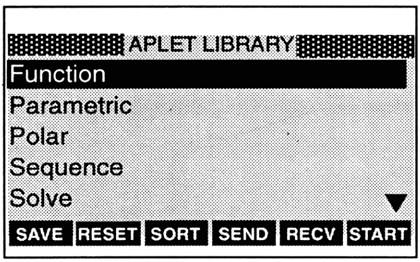
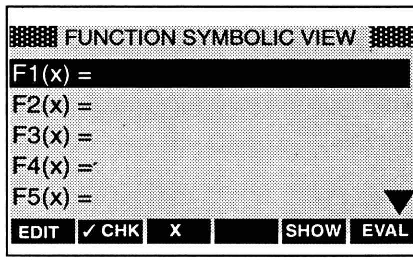
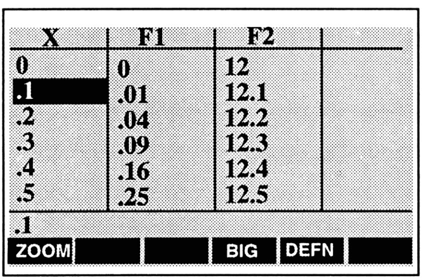
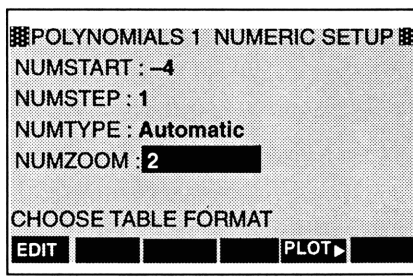


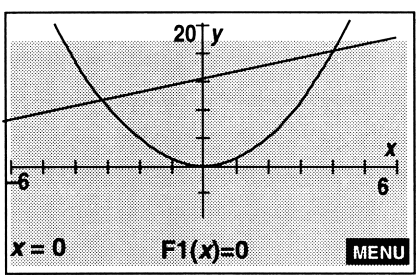


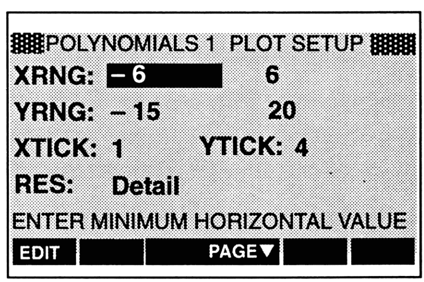












1.1 Some key features of the HP 38G

The HP 38G graphic calculator :

- can display algebraic expressions **as you would normally write them**
- generate and display **tables of values** associated with plotted functions.
- gives you the option of **viewing input** in any one of **four ways**:

  (i) **algebraically:** (**Sym**bolically) The SYMB key

  (ii) **Graphically: Plot**ting the graph over the range of values set by you. PLOT

  (iii) **Numerically:** showing a table of **Num**eric values. The NUM key

  (iv) Using a **Split Screen** showing both the *Graphic* and *numeric* views on the same display, or even two separate graphic views - one a zoom in on a particular feature of the original graph which is plotted alongside. VIEWS LIB

- Enables you to **zoom in on areas of interest** This zoom feature applies to both the graphic and numeric views.
- Plots graphs of **Cartesian, parametric, and polar functions** and includes a facility that enables you to show scales on the axes
- **Solves** equations and inequalities
- Computes, organises and **displays graphs of statistical data**
- Operates with both **Real** and **Complex** numbers in the HOME screen.
- Enables you to **draw diagrams**, place notes on the diagram, then store the diagram. These can be included with ApLets to help understand concepts.
- Enables you to **create and save ApLets** -
  Units of work can be stored this way. (Within the limits of memory available).
- Enables you to **share ApLets and machine settings** with other HP-38G calculators, and *download* stored work *to and from* normal PC computers
- Enables you to **retrieve and edit** previous *entries* and *answers* and reuse them in later work. Within limits, a history of entries is kept in the display.
- Enables you to work with **sequences** whether defined by *rule* or defined *recursively* , and graphs cobweb and step plots associated with these sequences
- Offers **programming capabilities.** This manual will not elaborate on this.

1.2 Conventions used throughout this manual

- When the cursor is to be used the appropriate key ◀ ▲ ▲ or ▼ will be indicated.

- A single key stroke is required to enter the information shown on the top of the key. The key used will be shown in white on a black button background

  Thus MATH means press the key marked MATH;

  ENTER means press the key marked ENTER;

  9 means press the key marked 9; and so on ...

- A...Z The colour of the A...Z on the **top** of this *alpha* button matches the colour of the alphabetic character (shown on the keyboard at the bottom right of the lowest six rows of keys). When this key is pressed an α symbol appears at the top of the screen. This means that if you press a key containing one of the letters of the alphabet that alphabetic character will be input into the screen display. In this manual the convention used to input an alpha character such as M will be A...Z 9 M . This convention is adopted for two reasons

  (i) It is easier to locate the 9 key than to search for the alpha character M.

  (ii) The 9 M is to remind you that it is a two-key operation to input M.

  ie A...Z followed by the key with the *alpha character* M located to the right.

- ▒ The turquoise coloured key must be pressed first to get to those functions *above* each key and written in the same turquoise colour as this key.

  Thus the combination ▒ then SPACE 2 is used to insert a space in text.

  ▒ MODES HOME gives the screen display showing the **setup** of the calculator in the HOME MODE, where most calculations are done.

  ▒ MATRIX 4 displays the MATRIX CATALOGUE. and so on ...

- [ ] [A...Z] (a...z) followed by [9 M] is used to type in the lower case letter **m.**

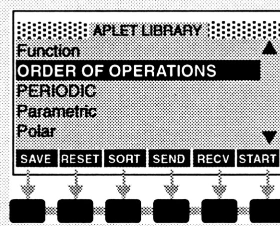
  The lower-case [a...z] above the [A...Z] indicates that the two keys [ ] [A...Z] must be pressed if you wish to type a lower-case letter.

- Screen displays will be shown exactly as they should appear on *your* calculator

---

- On your keyboard the top row of six BLANK KEYS [ ] which I shall call the ***screen menu keys*** are associated with the six ***screen menu labels*** which appear at the bottom of some of the display screens. At times some of these *screen menu labels* are blank indicating that if the associated *screen menu key* is pressed no action will be taken.

To activate a **screen menu** press the blank menu key immediately below the screen menu.Thus [SAVE] means press the blank button immediately below [SAVE]



Menu screen labels & the keys associated with these Menus

Press [LIB] to get a screen similar to the one above.

figure 1.1

- Press [LIB] to get the display shown in *figure 1.1*. This shows the library of stored ApLets. The ApLet templates built into the HP 38G are displayed in *lower case.*

- Any ApLets displayed in UPPER CASE are *user designed* or are available from another source such as another HP 38G, or through the ***Internet***.
  This *convention* for naming ApLets has been adopted throughout this manual.

- Where instructions are in boxes these are **not** keys but are choices made from selections offered in screen or menu displays. If you are asked to load the ApLet "ORDER OF OPERATIONS" as shown above in *figure 1.1*, the steps to be carried out will be written as "*Press* [LIB] *then select* [ORDER OF OPERATIONS]"
  That is, when a choice of menus is offered in the display screen anything in a box is a selection that ***you*** make from the choices offered on the screen.

1.3. Calculator Basics

The protective swivel cover

The protective swivel cover has non-slip grips for desk top use. You should not separate this cover from the main body of calculator - One of its functions is to protect the calculator from accidental damage. This cannot be done if this cover is removed.

The ON CANCEL KEY.

The ON CANCEL button also acts as a CANCEL key *while the calculator is in use.*

Press ON and then the HOME key which is located just above ENTER. This is the home base of the HP 38G. If ever you get lost or confused press the HOME key.

The Contrast control

To lighten or darken the screen display hold down the ON key and press either the + or the − key until the screen display is at the desired level of contrast.

The power source

The HP 38G uses 3 AAA batteries. These will last several months under normal use. To save batteries there is an automatic shutdown after a few minutes of non-use. To enable you to change batteries without losing your stored programs and ApLets, the contents of the calculator's memory are maintained *for a few minutes* while the old batteries are removed. Be mindful of this especially if you have ApLets that you wish to keep stored on your calculator. You must replace the batteries within this short period otherwise your data will be lost.

It would be a wise precaution to download your ApLets to another HP 38G before removing your batteries, or save them to a PC if you have the proper kit to do so.

1.4 The Turquoise Key ...also called the *shift key*

Whenever you press this key the symbol will appear at the top left of the display screen to show that the shift key has been pressed.

If you wish to access those functions written above the various keys of the calculator keyboard in this same turquoise colour then you must press this turquoise -coloured "shift" key first, before you press the other key.

Thus if you press then OFF ON this will turn the calculator **OFF** since OFF is the turquoise function written above the ON key. If you press then ABS - X this will insert **ABS(** into the edit line for you to enter a function involving the absolute value For example **ABS(** <u>3*x* – 12)</u> you would input the <u>underlined section.</u>

1.5 The a...z A...Z key

This key is just below the ENTER key and enables you to type letters, words or notes into the calculator display. An α symbol appears at the top of the display screen when the A...Z key is pressed. . The orange-brown colour of the A–Z on the face of this key indicates that the letters of the same colour to the bottom right of the lowest six rows of keys can only be accessed by first pressing the A...Z key. Whenever the alpha symbol α appears at the top of the display, pressing one of these keys will input that letter into the EDIT line of the display.

The A...Z key on its own will give the UPPER CASE letters shown in the same brown colour at the bottom right side of the keys. *Hold the key down if you wish to type several letters.* When the screen menu key A...Z is available, pressing this menu key has the same effect as holding down the A...Z key

You must use the SHIFT key combination a...z A...Z to get the **lower case** letters.

To get the word **Fred** into the display press the following sequence of keys.

A...Z √x F [shift] a...z A...Z * R [shift] a...z A...Z X,T,θ E [shift] a...z A...Z TAN D

If you want all UPPER CASE letters, to get **FRED** hold down the A...Z and key press the four keys √x F * R X,T,θ E TAN D, then press ENTER

Exercise 1: Press the HOME key then type the word **DEMO** as follows,

A...Z TAN D A...Z X,T,θ E A...Z 9 M A...Z 4 O then press ENTER

Now watch the display.

This demonstration ApLet runs through *some* of the features of the HP 38G

Exercise 2: *Personalise your Calculator*

Press [shift] then MODES HOME

The display should look like *figure 1.2*

Use the cursor key ▼ to move down to the word **TITLE** then press DEL or over-type *your name* next to the heading **TITLE**. using the A...Z key.

Press the HOME key when you are finished.

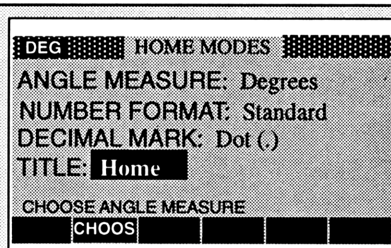


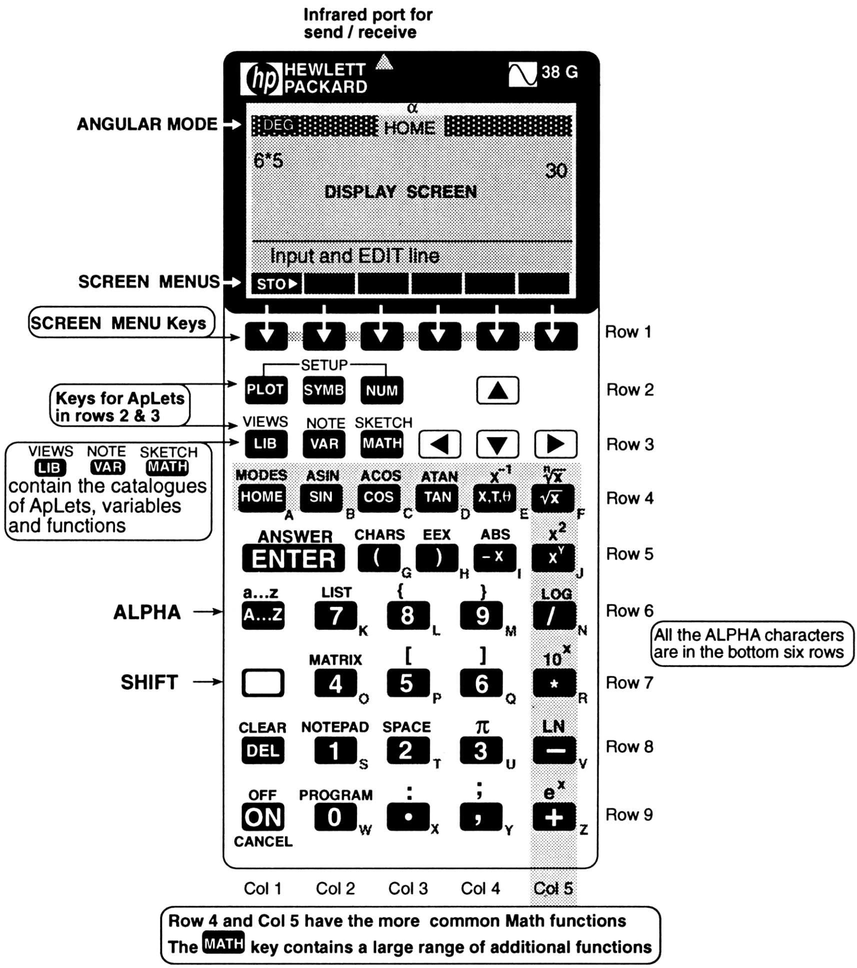
FIGURE 1.2

If you press DEL when the title is highlighted this will restore the title HOME

This will *personalise* your calculator so that whenever you enter the HOME mode your name will appear at the top of the screen. This is a useful feature, if you misplace your calculator or mix it up with others working with you, as you can quickly determine the owner of a calculator by turning it on.

For greater security you could *carefully* engrave your name on the top surface of the calculator keyboard (above the display screen?) and also along the front edge.

1.6 The Face of the HP 38G - The Keyboard Layout



The keyboard layout of the *HP* 38G

Row 1, Row 2 and Row 3 contain the heart of the *ApLet* setup.

Column 5 and Row 4 contain the more *common* Mathematics functions.

Column 2 contains the main lists and *programming* facilities.

1.7 A Summary of the Main Command Keys

Command keys in the left column & top two rows

Key	What it does
ON	Press this key to turn the calculator **ON**
[SHIFT] OFF/ON	This is a *two-key command* used to turn the calculator **OFF**
ON CANCEL	*While the calculator is turned on* this key acts as a **CANCEL** key. It clears any entry in the EDIT line prior to ENTER being pressed.
DEL	Used to delete entries, or make changes/corrections in the EDIT line. It behaves in much the same way as the DELETE key on a computer
[SHIFT] CLEAR/DEL	This is a two-key command that will clear the entire screen display *including the history of all calculations done* since the last time this same two-key command was used to clear the screen display
[SHIFT]	This turquoise coloured key, referred to as a SHIFT key, is used to gain access to those functions marked in the same turquoise colour *above* most of the keys on the keyboard. It is used with most of the two-key functions or commands .
A...Z	The ALPHA key. When this key is pressed an α symbol appears above the screen title. This indicates that if an alpha key is pressed the input will be that alphabetic character at the lower right of a key (in the bottom six rows) and in the same colour as the A...Z on the top face of the A...Z key
[SHIFT] a...z/A...Z	This two-key command gives the lower case version of the alpha characters mentioned above. It is used mostly in ApLet Notes, Titles

Key	What it does
ENTER	This key serves the same function as [=] on most other calculators. When ENTER is pressed, any pending commands or calculations are carried out and the *question* **and** *answer* placed in the display screen.
[ ] ANSWER ENTER	When a calculation is completed by your pressing of the ENTER key it is kept in a calculator memory location named [ANS]. The two-key command [ ] ANSWER ENTER will use the answer to the last calculation by inserting [ANS] in the EDIT line. [ANS] is treated like any other number and can be used as such in further calculations.
HOME	This is a **MAJOR KEY** It is the **HOME-BASE** of your calculator and will return you to the ***home screen where most calculations are done***. If you strike trouble or get lost within the screens and menus, press HOME You may need to press CANCEL first then HOME.
[ ] MODES HOME	This two-key command enables you to set up the format of your ***home screen.*** Angle mode; number format; the type of decimal separator to be used (DOT . or COMMA ,) and also to enter a TITLE for your *home-screen* (See *figure 1.2*)

ApLets	
SETUP PLOT SYMB NUM and VIEWS LIB NOTE VAR SKETCH MATH	The keys in rows 2 and 3 are used when working with ApLets ApLets will be explained in more detail in a later section of this manual

Remember: The HOME key will often be your main [HELP] key in emergencies.

The HOME screen

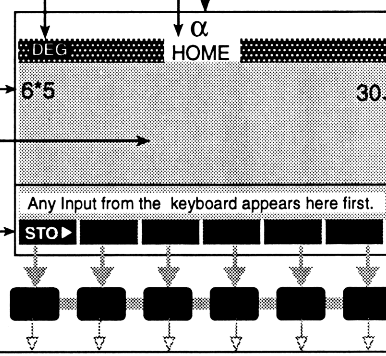
Angle mode indicator. To change press [ ] MODES HOME

Title bar indicates which screen is active

The [α] symbol at the top of the screen shows that the A...Z key has been pressed and an **alpha character** will be put into the EDIT line if an alpha key (from the bottom six rows) is pressed. To disable this press A...Z again. (*It acts as a toggle switch*)

What you **INPUT** via the keyboard appears on the left side after you press ENTER

This section of the display contains the **history of all entries and answers** from the last time [ ] CLEAR DEL was pressed.



After ENTER is pressed **Answers** appear on the right side of the screen

Menu labels are linked to the row of blank keys immediately below these menus. These menus change with the action being taken

The **EDIT LINE** Whatever you key into the calculator appears on this line. You can EDIT or correct your entry **before** you press ENTER

The **Menu label keys** carry out an action shown in a *menu label*. Press the blank key immediately below that menu label. Here the leftmost blank key will carry out the STO▶ (store) instruction. The other blank keys serve no purpose in this particular display.

1.8 A Tour of The HOME Screen

The HOME screen is the main display window for carrying out computations in the manner traditionally associated with scientific calculators.

The structure and components of the HOME SCREEN are outlined in the diagram on the previous page This is ***the HOME BASE of the HP 38G calculator***. You can easily return to this window at any time by pressing the **HOME** button.

If this does not put you into the HOME screen check the screen menus as there may be a pending action such as the need to press the screen menu key **CANCL** before pressing the **HOME** button.

- Press [ ] then **MODES HOME**
- Your screen display should look like *figure 1.3*
- With [Degrees] highlighted press **CHOOS**

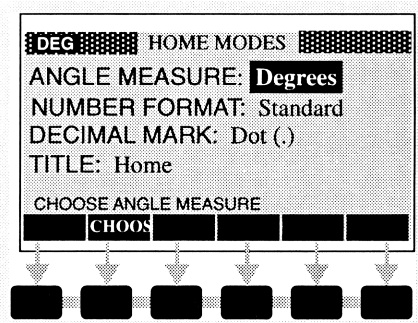


FIGURE 1.3

Figure 1.4 shows the choices offered

Use the cursor key [▼] to move down to the **Radians** then either press **ENTER** or **OK** (Both actions do the same thing)

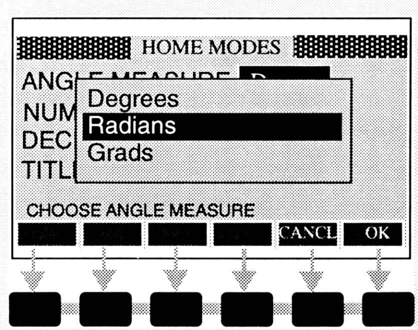


FIGURE 1.4

When you now press the **HOME** key the angle mode in the top left bar of the HOME SCREEN as shown in *figure 1.3* should change from [DEG] to [RAD]

- Press [shift] MODES HOME again.

  Your screen display should look like *figure 1.3*

- Highlight the **Number Format :**

  Standard then press CHOOS

Select the format you wish to use for your numbers, (see *figure 1.5*). Standard is the normal option.

- Press ENTER or OK when you have made your choice.

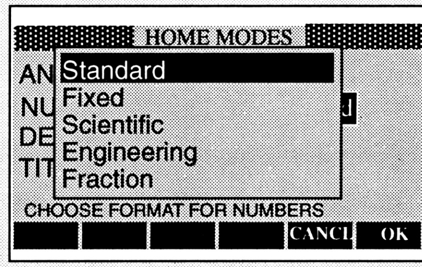


FIGURE 1.5

Standard - Uses floating point to 12 digits

Fixed- Fixes the number of decimal places shown

Scientific - Expresses numbers in Scientific Notation

Engineering - Expresses numbers in Engineering Notation

Fraction - Expresses results in fraction form

Note! If you press [shift] CLEAR DEL while you are in the HOME MODES screen (*figure 1.3*) all the settings revert to the factory default settings.

If you selected **fraction** format for the number format then input into the HOME SCREEN 2·5/10 ENTER the answer is given as 1/4.

- Now use the cursor key ▲ to move to the display screen, highlight the 2·5/10 then press the screen menu key SHOW The fraction form is displayed. as $\frac{\frac{5}{2}}{10}$ Click OK

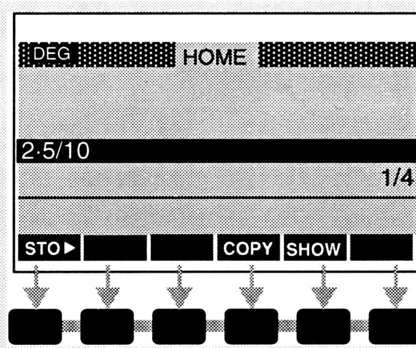


FIGURE 1.6

You should keep a record of techniques and short cuts as you learn how to use the HP 38G. This page is left blank below this line for this purpose.

CHAPTER 2
USING YOUR HP 38G
TO DO CALCULATIONS

Mathematical Computations are usually carried out in the HOME screen

Press the HOME key to get to the HOME screen.

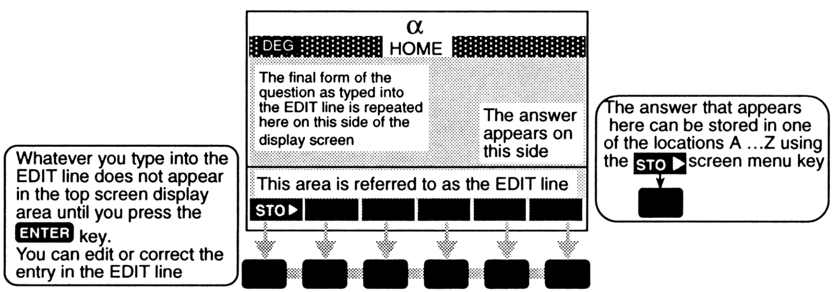


Figure 2.1

2.1 Mathematical Computations

- All the calculations normally done on a Scientific Calculator are carried out on the HP 38G in the HOME SCREEN. There are however some differences in the way in which this is done on the HP 38G. You should refer to the diagrams in the introductory section and also do the examples that follow in this section.

- Notice that the HP 38G has no [=] key.

  This function is carried out when you press the ENTER key.

- When you key in numbers, functions, characters or operations they first appear in an EDIT line at the bottom of the screen. See figure 2.1

The EDIT line enables you to check your entry and if necessary EDIT the input using the cursor keys ◀ ▶ and the DEL key. Once you press the ENTER key the calculator to carries out the calculation or instruction in the EDIT line.

- Pressing ENTER has the same effect as pressing [=] on a normal calculator. Any pending computations are carried out. Whatever you keyed into the EDIT line reappears on the **left side** in the main display screen and the *answer* or *result* of the calculation is placed on the **right side** of the screen.

- The calculations are carried out using direct algebraic logic (DAL) This means that the *convention for the rule of order of operations* is adhered to.If you wish to edit your entry **after** you have pressed the ENTER key, use the ▲ cursor to highlight the entry in the display screen, press the screen menu key COPY and the entry is copied into the EDIT line.

2.2 Calculations in the Real number system

Calculation to be done	Keystrokes	Answer Display
• 12 − 2 x 5 + 6	[1] [2] [−] [2] [*] [5] [+] [6] [ENTER]	8
• (⁻5) x 4	[-x] [5] [*] [4] [ENTER] Note use of the [-x] key to enter negative numbers. Do **not** use the [−] operation key.	–20
• 14^3	[1] [4] [x^y] [3] [ENTER]	2744
• $\frac{1}{7}$ (the reciprocal of 7)	[7] [ ] [X,T,θ] (x^{-1}) [ENTER] or [1] [/] [7] [ENTER]	.142857...
• 89·73 x 10^{15}	[8] [9] [.] [7] [3] [*] [ ] [*] (10^x) [1] [5] [ENTER] or use [8] [9] [.] [7] [3] [ ] [)] (EEX) [1] [5]	8·973E16
• √56	[√x] [5] [6] [ENTER]	7·483314...
• $36\sin(\sqrt{\frac{7\pi}{9}}) + 4$	[3] [6] [SIN(] [√x] [(] [7] [ ] [3] (π) [/] [9] [)] [)] [+] [4]	4·982038...

Now suppose that you made an error in entering this last caculation and the 7 should have been a 5. Rather than retype the whole entry use the [▲] cursor to highlight the question in the display screen. Press [COPY] to get the entry into the EDIT line then use the [◄] [►] and the [DEL] key to delete the 7, type 5 then press [ENTER]

Calculation to be done	Keystrokes	Answer Display
$\sqrt[3]{47^2} - \sqrt[5]{-256}$	3 [ ] $\sqrt{x}$ ($\sqrt[n]{x}$) 4 7 [ ] x^y (x^2) − 5 [ ] $\sqrt{x}$ ($\sqrt[n]{x}$) -x 2 5 6 ENTER	16·0550588...
To see how the typed expression actually looks and ensure that you have entered it correctly. You should see in the display $\sqrt[3]{47^2} - \sqrt[5]{-256}$	When the answer appears in the display screen use ▲ to select the *question* in the *display* then press SHOW to check the correct format. Press OK to get back to the EDIT line to complete the calculation.	

2.3 Real Variables - Storing values in memory locations

In calculation mode the HP 38G has 27 memory locations (A through to Z and X,T,θ) These memory locations are referred to as *home variables*. The default value stored in all of these memory locations is zero. Any subsequent value stored in a memory stays in that location until it is overwritten by storing another value into the same location. They are variables in HOME SCREEN calculations.To store 27 in memory location K proceed as follows:

Input the keystrokes 2 7 STO ▶ A...Z 7 K ENTER

The display shows 27▶K on the left side of the main screen and 27 on the right side of the display screen. K now has the value 27 assigned to it and in any calculation where K is used it will have the value of 27.

To calculate **|K – 12|,** assuming that K has the value 27 stored in this location

• **|K – 12|** You could type ABS(K – 12)	[ ] [ABS -x] ([A...Z] [7 K] [–] [1] [2] [ENTER]	15

Now do the calculation 34tan(68°) The result displayed is 84·1529

If you now press [STO ▶] [A...Z] [7 K] [ENTER] this will overwrite the old content of memory K (27) and replaces it with the answer (94·0533…) to this last calculation. You **do not** need to clear the existing contents of the memory before storing a new value in the same memory location.

Calculate **10sin56° – 4cos152°** and store the answer in memory location L

Warning: Check that you are in degree mode!

When the answer is obtained press the menu key [STO ▶] then [A...Z] [8 L] [ENTER]

This assigns the value of **(Ans)** to **L**

Your display screen should look like that shown in figure 2.2.

Now type the letter L into the EDIT line and press [ENTER]

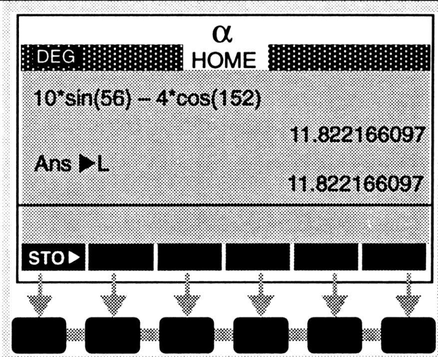


figure 2.2

2.4 The HOME SCREEN history

The HP 38G keeps a continuous screen record of all the calculations done, within memory limitations, even though they may have scrolled off the screen display. To check this use the ▼ ▲ cursor keys. When you highlight a previous entry two actions are possible.

(i) If you press ENTER this places the highlighted section just above the EDIT line. ie it is brought forward as the last entry.

(ii) If you press COPY the highlighted portion is *copied* into the EDIT line.

If you press CLEAR DEL the whole history of work done to date in the current session and stored in the screen memory is deleted.

If in later work you get the message Out of Memory you can free up memory by clearing this screen history.

If you have been following the examples above, *and have NOT yet pressed the* CLEAR DEL combination of keys, experiment with this idea of copying previous entries into the EDIT line using the existing screen history. For example go back through the previous entries and **copy** 10sin56° – 4cos152° into the EDIT line then assuming that you meant to enter 10sin**36**° – 4cos152° use the ◀ ▶ cursor keys to highlight the 5 in the **56**° then press (DEL), type in a **3** while the cursor is still in the position where you deleted, then press ENTER. Note that it is not necessary to be at the end of the entry in the EDIT line before you press ENTER

If at any time if you press the key combination ANSWER ENTER the letters Ans appear in the EDIT line. If the EDIT line is blank when you do this and you then press ENTER the answer to the last calculation done is repeated in the **display**.

[Ans] is a kind of *temporary memory.* Its contents always change to give the value of the last calculation done.

If [Ans] is used within a calculation (and it can be used more than once this way in the EDIT line) then [Ans] is treated as a home variable and has the value of the last calculation that was carried out.

2.5 Working with fractions

- To work with fractions in the normal manner press [ ] MODES HOME. You should get the display shown in figure 2.3.
- Select [Standard] then press the screen menu key CHOOS
- You are now given a choice of number formats. (figure 2.4).

[Standard] The default setting of the number format for the HP 38G. In the standard number format answers have the floating decimal point.

- Highlight [Fraction] then press ENTER Your display should now look like figure 2.5.

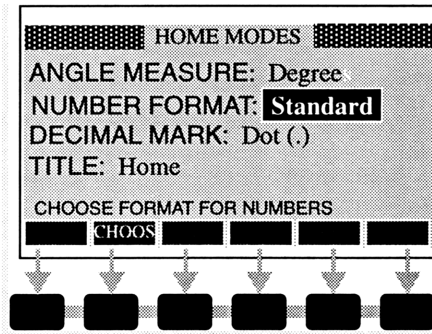


figure 2.3

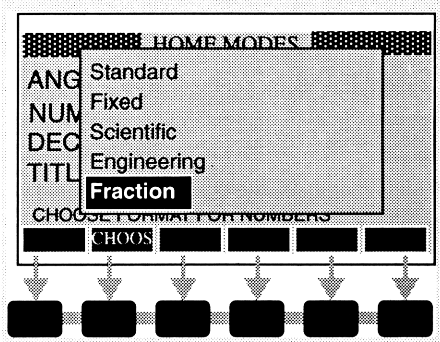


figure 2.4

The number that appears next to the word **Fraction** in figure 2.5 will be explained in the examples that follow.

Now press the HOME key to get back to the HOME SCREEN

Key in 4 / 3 + 1 / 7 ENTER

The answer is given as 31/21.

- Use the ▼ ▲ cursor keys to highlight the question in the display.
- Press SHOW to show the normal form of the question. Your display (figure 2.6) shows the way you would normally write and see this question.
- Press OK to get back to the HOME screen.

Try some calculations of your own to familiarise yourself with doing operations using fractions.

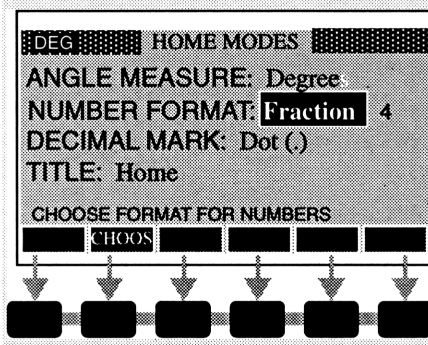


figure 2.5

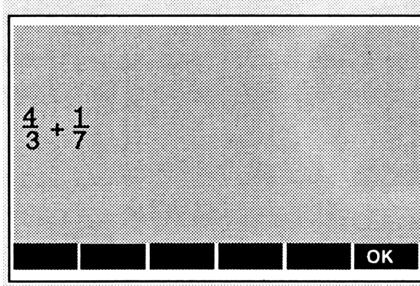


figure 2.6

With the calculator still set in the fraction mode as shown in figure 2.5 proceed on to section 2.6

2.6 The purpose of the [n] in the [Fraction] [n] mode

Key in 0·78958777 then ENTER. The answer given is $\frac{364}{461}$

Check the decimal form of $\frac{364}{461}$ and compare it to 0·78958777

(A quick way to do this would be to put the calculator back into **standard** number format, see figures 2.3 - 2.4 – 2.5 above, then in the HOME screen select the fraction $\frac{364}{461}$ in the screen display then press ENTER

This gives 0.789587852495.)

Now put the calculator back into fraction number format as shown in figures 2.3 - 2.4 – 2.5 above. This time while you are still at the stage shown in figure 2.5 use the cursor key ▶ to select the [4] . Change it to a [2] ..

Use the ▲ to highlight the decimal ·78958777 from the screen display history then press ENTER. This displays the answer $\frac{15}{19}$ which is not the same as $\frac{364}{461}$!

If you check the decimal form of $\frac{15}{19}$ you get 0·7894736842....

What has happened is that in the first case where you set [fraction] to [4] this was in effect an instruction to the calculator to give the smallest possible fraction that has its decimal form *the same as* <u>at least the first four decimal digits</u> of the given decimal number ·78958777.

You can check that $\frac{15}{19}$ agrees with ·78958777 to at least 2 digits.

Had we set [fraction] to [8] (see figure 2.5) this would have given the fractional equivalent for ·78958777 as $\frac{20763}{26296}$

If you check the decimal form for this fraction you get ·789587770003.

For you to try!

> Input the decimal approximation for π ([shift] 3 π)
>
> Using the feature just described determine a fraction that will agree with the value of π to 4, then 5 , then 6 decimal places. What is the most accurate fraction equivalent for π that can be given on this calculator?

You should be aware that while the calculator is in the fraction mode, any calculations entered, even if they are in decimal form, will have the output expressed in the set fraction format.

You can interchange between the standard decimal form and the fraction format for numbers. Suppose you have just done a calculation in the [Standard] decimal format and you reset the number format to [Fraction] If you return to the HOME screen and select that calculation from the screen display history then press ENTER the results will be displayed above the line in the chosen equivalent fraction format.

Non-Real Numbers – The Complex Numbers

When you input a calculation such as $\sqrt{-16}$ the output is displayed in the form (0,4). Unlike most of the scientific calculators, you do not have to change the mode of the calculator when dealing with such numbers. The HP 38G automatically recognises Complex Numbers when they arise in the context of a calculation. The results of any calculations that involve Complex Numbers will be displayed in the ordered pair format (a,b). For more information on these numbers read Chapter 11.

CHAPTER 3

THE SOLVE APLET

Designing and working with APLETS

3.1 What is an ApLet?

In computer jargon an Aplet is the name given to a small **appl**ication. However within the context of the HP 38G ApLets are slightly different and the following definition will suffice:

An ApLet is a small self contained Mathematics topic or investigation. It is a form of an electronic handout or assignment that can contain problems, variables, graphs, pictures and explanatory notes designed and saved on the calculator. The ApLet can be transferred from one calculator to another calculator. It can be stored on an external device such as a computer hard disk or floppy disk as long as you have the right equipment (either the *HP Graphic Calculator PC Connectivity Kit* or the *HP Graphic Calculator Macintosh Connectivity Kit*). Any HP 38G will link to the overhead projection unit.

One useful and powerful feature with the HP 38G is that an ApLet can be *loaded to* and *downloaded from* the Internet as well as to other calculators. The work done in setting up an ApLet is not lost once you remove it from the calculator as it can be *stored on disc* for later use. Another powerful feature is that the actual screen displays on the calculator can be captured and incorporated easily into any word processing/DTP document for Macintosh or PC computers.

Complete units of work, assignments or investigations can be created or designed, saved if need be, and the work transferred to other HP 38G calculators without the necessity for students to type in all the necessary information. The method of transfer, using the inbuilt infra-red transfer facility on the HP 38G, means no cables are necessary and students are not wasting time entering details. This also avoids the problem of potential errors in the entry of data as all students will have the same data in their calculator. This dissemination of data between calculators can take less than 4 minutes in a class of 30 to 60 students once the process is mastered. The process is explained later.

3.2 The Inbuilt ApLet *Templates*

The HP 38G graphic calculator has six inbuilt environments called ApLets.

Press the LIB key to obtain a list of the ApLets currently available. (*figure 3.1*) There are six **inbuilt** ApLets and these appear in the display.

Below is a brief indication of the function or purpose of each of these ApLets.

- **Function**: used when working with functions of the form $y = f(x)$
- **Parametric**: used when working with parametric functions of the form $x = f(t)$, $y = g(t)$
- **Polar**: used when working with polar functions of the form $r = f(\theta)$
- **Sequence** used when working with sequences $\{u_n\}$ for n= 1,2,3,...
- **Solve** used when working with equations in one or more variables.
- **Statistics** used when working with numerical data.

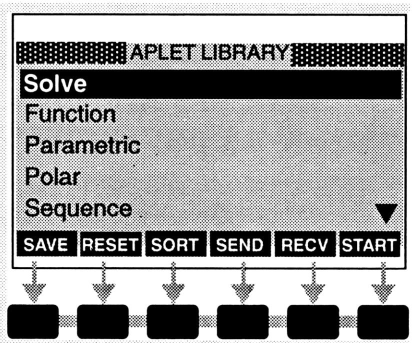


Figure 3.1

These ApLets are inbuilt and cannot be deleted You could consider them as the **templates** or *foundations upon which you build your own ApLets*. All settings and definitions that you incorporate into an ApLet <u>when you create it</u> are maintained and stored with your ApLet. As will be shown later this means that you can include explanatory notes, sketches, questions, lists, graphs and their settings, etc.

An ApLet can have several components which stay attached to it when the ApLet is saved. For the Function ApLets these components include.

◊ **Listed Functions or Data**	◊ **Notes**	◊ **Sketches**
◊ **Programs**	◊ **Views**	

3.3 Screen Menus in the ApLet Library

Press the LIB key to get the list of ApLets – *figure 3.1* Notice the **Screen Menus** that appear at the bottom of the screen displaying the ApLet library. Each of these menus is accessed by pressing the **blank key** immediately below that menu on the keyboard. (Remember the convention used in this manual.

SAVE means press the blank key below the SAVE screen menu These screen menus are not FIXED but vary according to the current status of the calculator. The screen menus shown in *figure 3.1* are explained below

Screen Menu Key	The function of this screen menu.
SAVE	If you press this menu key while in the ApLet Library window you can save the current ApLet under a new name (change name).
RESET	Resets the default values and settings of the selected *Template* ApLet to the default values and clears any entries in that ApLet.
SORT	Sorts the listing of ApLets in the Library placing them in either (i) Alphabetic order or (ii) Chronologically, by order of last use
SEND	Used to send or download an ApLet from your calculator to either another HP 38G or to a PC computer (*If you have the connectivity kit*)
RECV	Used to receive or download an ApLet from either another HP 38G or a PC computer to your calculator. (*If you have the connectivity kit*)
START	This opens the selected ApLet from within the ApLet Library. Pressing ENTER has the same effect as this screen menu key. It will open the ApLet, usually in Symbolic view.

3.4 Building an ApLet based on the *Solve ApLet*

- Press the **LIB** key to get the list of ApLets – see *figure 3.1*.
- Use the cursor keys **▼** **▲** to select the Solve ApLet as shown.
- Press either **START** or **ENTER** to open up the chosen ApLet.
- You should now have a screen like *figure 3.2* This screen view is referred to as the ***Symbolic view*** and is named this way at the top of the screen. This title names the ApLet *type* and the current *view*.

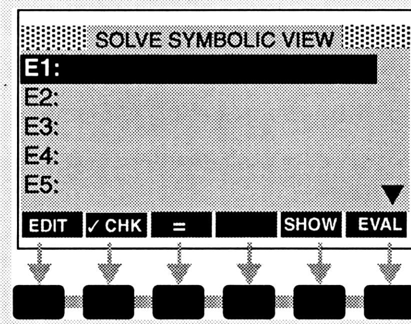


figure 3.2

You can get directly to the **SYMB**olic view by pressing **SYMB** when the required **Solve** ApLet is highlighted in the display screen *(figure 3.1)*.

At this stage this is the blank template for the Solve ApLet and there are no entries after E1, E2, … E0.. If you scroll through the display you will note that allowance is made for the entry of up to ten EQUATIONS or EXPRESSIONS from E1 to E0. These entries will be defined in the next section where you will create a *Solve ApLet* called MATHS FORMULAE

Select E1 if it is not already selected

Now input the following equation | 3A + 4B = 36 | as follows

3 **A...Z** **HOME** A **+** **4** **A...Z** **SIN** B **=** **3** **6** then press **ENTER**

The equal sign, **=**, can be entered one of two ways:

(i) either by using the screen menu key **=** in the Solve ApLet display

or (ii) by using **CHARS (** and then selecting the equal sign from the list of characters displayed on the screen and pressing **ENTER**

Notice that a check-mark appears next to the equation that you have just entered. (*figure 3.3*)

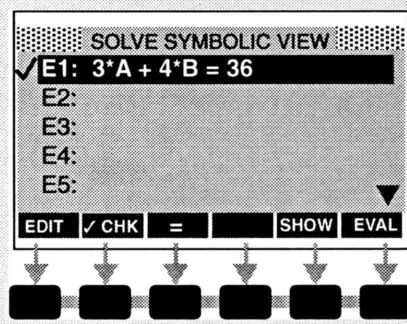


figure 3.3

- Press the **NUM** key.

Your screen display should look something like *figure 3.4*. The numbers next to the A and B may not be the same. The values that appear here are those currently stored in the memory locations for A and B.

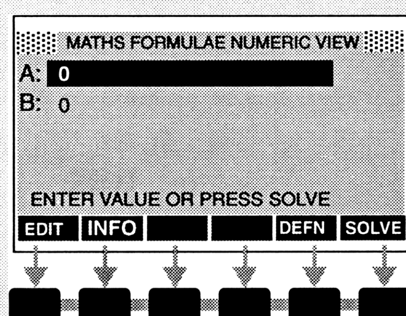


figure 3.4

If you wish to solve the equation for B when A =2, enter the value **2** for A then press **ENTER**. The cursor will move down to B. As we wish to solve for B press **SOLVE** while B is selected.

The value 7·5 is returned.

You can enter values for either A or B and solve for the other. You can only solve for one *unknown* variable.

The variable that is highlighted when the SOLVE menu key is pressed is treated as the unknown in the equation.

The equation to be solved can contain up to 27 variables, but to solve the equation the values of <u>all but one</u> of the variables must be provided by you.

A procedure to create a simple ApLet, to be named **MATHS FORMULAE** is outlined in the following section. It will contain some common mathematics formulae.

Using the same ApLet as in *figure 3.3*, press SYMB to put the view of the present ApLet back to the **SYMB**olic view shown in *figure 3.3* then input the following equations pressing ENTER at the end of each formula.

If you need to start over again press LIB, select SOLVE then press ENTER

Select E2 then input C = 2πR

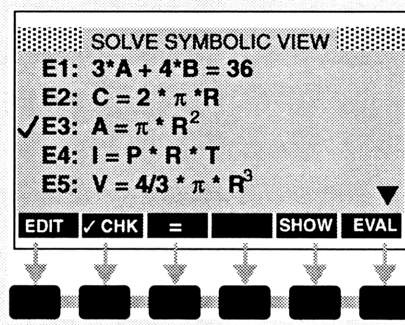
Press ENTER, the cursor moves to E3

For E3 input A = πR²

Press ENTER, the cursor moves to E4.

For E4 input I = PRT

Press ENTER,the cursor moves to E5



<u>figure 3.5</u>

Notice that a check-mark appears alongside each new entry.
The entry that has the check-mark next to it will be the one that is solved when SOLVE is pressed

For E5 input $V = \frac{4}{3}\pi R^3$

For E6 input $V = \frac{1}{3}\pi R^2 H$

For E7 input

$S = \pi R^2 + \pi R\sqrt{(R^2 + H^2)}$

Highlight E1 and over-type the entry

For E1 input $A = \sqrt{(B^2 + C^2)}$

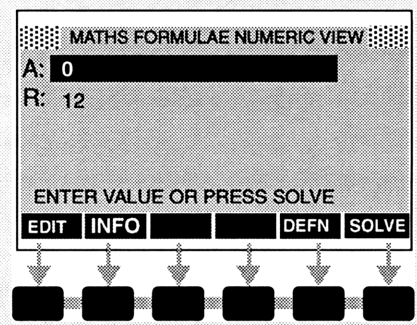


figure 3.6

- **Example**

  Select E3 $A = \pi R^2$ and press [✓CHK] This makes E3 the active equation.

  Note:

  In the Solve ApLet up to ten equations and expressions can be entered into the ApLet , ***but only one equation can be worked on at any one time.***
  If you wish to work on a particular equation you must first make sure that it is checked [✓]

- Press the [NUM] key. Your screen should look like (*figure 3.6*)

- Input 12 for R. Select A then determine the area A by pressing [SOLVE]

- Input several other values for R and note the area each time.

- Now input the value 220 for the area A. Select the R then press [SOLVE]

  You do not need to clear the value next to the R, nor do you need to rearrange the formula. The value for R when A = 220 is given and overwrites any numbers currently in R

  Input several more values for A and in each case determine the Radius R.

3.6 To save the equations as an ApLet

- Press LIB. The screen looks much the same as in *figure 3.1* Select the Solve ApLet if it is not highlighted. (It should already be selected).

- Press the screen menu SAVE

  You are then prompted for a name under which to save the ApLet. You type MATHS FORMULAE , or any other name, then press OK

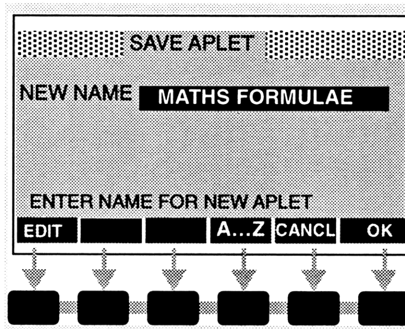


figure 3.7

> **Remember:**
> You can hold down the A...Z key while you type in the letters for the New Name.

This is the most basic form of an ApLet.
The name of your new ApLet should now appear in the LIB list of ApLets.

At this stage the equations E1 to E7 exist in both

(i) the *Template ApLet* called **Solve** as well as in

(ii) the newly named ApLet called **MATHS FORMULAE** .

Check this for yourself by opening both of these ApLets.

You can add entries into an ApLet or change the definitions within the ApLet. *Explanatory notes can also be included with the ApLet.* You can then save the altered ApLet either under a new name or keep the old name. In either case the old ApLet is replaced by the changed form.

Remember: You are unable to delete any of the original template ApLets.

Open the newly created ApLet [MATHS FORMULAE]

Scroll to E8 which is empty. If there is already an entry in E8 you can simply over-type the entry or you could clear the entry by selecting it (using the [▼][▲] cursor keys) and then pressing the DEL key.

For E8 input $3x + 5y - 4z = 120$

and for E9 $3x + 5y - 4z$

Notice that E8 like all the previous entries E1 to E7 is an EQUATION while E9 is simply an EXPRESSION.

To solve an equation you need to have the values of all the variables except the one for which the equation is to be solved. That is you are *solving* for this unknown variable.

Select E8 and press ✓ CHK then press the NUM key.

This takes you from the **SYMB**olic view to the **NUM**eric view*(figure 3.8)* It is in this **NUM**eric view in the Solve ApLets that equations are solved.

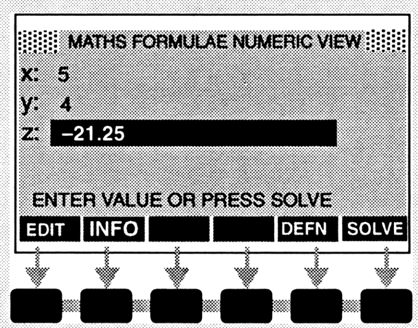


figure 3.8

In the problem shown above *X* and *Y* are the known values. Highlight *Z*, ignore any value listed alongside the *Z*. Press the **solve key**. The solution overwrites any previous value.

The Letters entered into equations as variables are all upper case.

Remember to press the A...Z key to input the letter.

You can now solve the equation for one of the unknown variables by giving values to the other two variables as shown in *figure 3.8*

3.5 Expression Vs Equation

If you now try the same action with the expression E9 the value given as the solution for Z is 8·75. This solution is the value of the unknown variable (in this case Z) that makes the value of the whole expression E9 equal to zero.

You do not SOLVE an expression, you merely provide values of the variables and EVALUATE the expression for those values. Different values of the variables will usually result in different values for the expression.

Since you are asking for a solution (this is after all the ***Solve*** ApLet) then the expression is treated as Expression = 0 and the appropriate solution is given to this equation.

You are in effect determining the *roots of the equation* Expression = 0

You may wish to save this altered ApLet. Press LIB then SAVE You can save the ApLet under a new name or, to keep the old name, just press ENTER at the prompt.

Putting these ideas to work!

Any ApLet that was created by you using the SOLVE ApLet is itself a SOLVE ApLet. You can modify your ApLet and change the entries to suit your current requirements and the ApLet will behave the same way as the original template ApLet called SOLVE. With the ApLet that you create you can save explanatory notes and the SETUP settings that you worked with in SETUP PLOT SETUP SYMB SETUP NUM

Any problem which has a formula associated with it can be worked on by placing the formula into a SOLVE ApLet. You then work on the formula as shown in this chapter. Some further examples of this are provided in the next sections.

3.6 Some additional routines using the Solve ApLet

The Cosine Rule:

Determine the size of angle Z in the given triangle given A=12; B=17; and C=10

Input E0 $\cos(Z) = (A^2 + B^2 - C^2)/(2AB)$

Note the angle opposite side C must be given a different name to the side C.

Press NUM to get to the NUMeric view

Input the known values, highlight the unknown value then press SOLVE

This gives the solution as [Z≈ 35·296°]

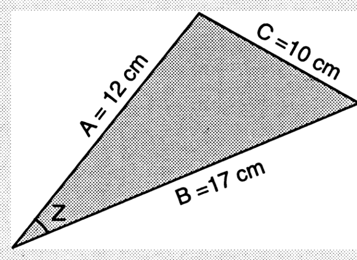


figure 3.9

This one equation can be used to solve problems involving the cosine rule.
This does not however remove the need to interpret the answers in light of thegiven information.

Note In the NUMeric view, if at Z you enter an initial guess (or *Guesstimate*), highlight this value of Z and then press SOLVE you will be given the answer that is nearest to your guesstimate. This may not necessarily be the correct answer - it could be the supplement of the correct answer or a co-terminal angle.

With trigonometric functions you will need to be careful and check that you do indeed have the correct solution and not its supplement or a related value
The solver ApLet gives the solution that is nearest to the value assigned to Z.

You must interpret the solutions offered.

Solving Trigonometric equations

To solve $\sin x°(2\cos x°-1)=0$

Select a solve ApLet. For E9: input $\sin(x)(2\cos(x) -1)=0$

Press the NUM key to get the NUMeric view. Put x= 0 then press SOLVE

Change the value of x to 90, then 180, then 270 and see what SOLVE yields in each case. (Any set of values will serve the same purpose.)

To check the angle mode, press SETUP SYMB Rather than using the key CHOOS to cycle through the DEGREES - RADIANS - RADS use the + to cycle through the choices and press enter when the appropriate choice appears in the display.

A wide range of routine mathematics problems are open to solution using the Solve ApLet. Many of these may be better analysed in the Function ApLet if they involve equations in one or two variables as such equations are better suited to a graphical analysis. If there is a formula then in the majority of cases you can include it in the Solve ApLet in its general form. (eg try the formula for the solution of quadratic equations, although one could now question the need to do this any more).

There is a PLOT VIEW but this is more versatile in the **Function ApLet** and will not be developed at this stage. Its main purpose in this ApLet is to provide a guide to your Guesstimates and possibly show why some guesses lead to no answers.

At this stage it is assumed that you have saved your first ApLet under the name MATHS FORMULAE .

Growth and Decay. Finance Appreciation and Depreciation.

(i) Simple Interest and Amount at Simple Interest

(ii) Compound Interest and the Amount at Compound Interest

- Press LIB and select the SOLVE ApLet ENTER (you should get *figure 3·2*)

- If there are entries currently in the ApLet use CLEAR DEL to clear the entries.

Input the following :

For E1 input I = PRT then press ENTER ,the cursor moves to E2

For E2 input A = P(1 +RT) then press ENTER ,the cursor moves to E3

For E3 input $A = P(1 + R)^T$ then press ENTER ,the cursor moves to E4

For E4 input $A = P(1 + \frac{R}{N})^{NT}$ Use the following keys. (As you become more familiar with your HP 38G you will do this more efficiently).

A...Z HOME A = A...Z 5 P * (1 + A...Z -X I / A...Z / N) X^Y (A...Z / N * A...Z 2 T)

then press ENTER , the cursor moves to E5

To save these formulae in an ApLet under the title FINANCE proceed as follows.

- Press LIB . The screen looks much the same as in *figure 3.1* Select the Solve ApLet if it is not highlighted.

- Press the screen menu SAVE You are then prompted for a name under which to save the ApLet. You type the name FINANCE then press OK

- The name of your new ApLet should now appear in the LIB list of ApLets.

Example: *If $8000 is invested at 7% per annum for a period of 6 years, determine (i) the amount at Simple interest.*

(ii) the amount at compound interest

(iii) the amount at compound interest if it is compounded monthly.

- Part (i) Press LIB and select the ApLet FINANCE then press ENTER
  The ApLet will open in the **SYMBolic view** showing the formulae.

- Select **E2** A = P(1 +RT) the formula for amount at Simple Interest.

- press the check key ✓CHK then the NUM key to get the **NUM**eric view *figure 3.4*

- Input the values P= 8000 ENTER; I = 0·07 ENTER; T = 6 ENTER
  For A input any value roughly the same as the Principal P.
  When you press ENTER the cursor moves off this guesstimate for A

- Use the cursor keys to select the A then press SOLVE The X at the top of the display indicates busy time - The calculator is busy doing the necessary calculations. After a short time interval the answer for A is given as **11 360**

If you get an error message regarding AN INVALID USER FUNCTION check the entry of your formula. A = P*(1 + R*T) The first * is usually the problem

Press SYMB to get back to the **SYMBolic view** then Select **E2** A = P(1 +RT) .

Use the Edit facility. Press EDIT then insert the * between **P** and **(**

Repeat the solve process above.

- Part (ii) Press SYMB to get back to the **SYMBolic view**

  Select **E3** $A = P(1 + R)^T$ the formula for amount at Compound Interest.

- press the check key ✓CHK then the NUM key to get the **NUMeric view**

  Note that the values for P; R; and T are as before so simply press SOLVE

  The answer is given as **12005·84**

- Part (iii) Press SYMB to get back to the **SYMBolic view**

  Select **E4** $A = P(1 + \frac{R}{N})^{NT}$ the formula for amount at Compound Interest where N is the number of payment periods per year

- Press the check key ✓CHK then the NUM key to get the **NUMeric view**

  Note that the values for P; R; and T are as before but you must now input a value for N. Here N = 12

- Be sure to select A before you press SOLVE

  The answer is given as **12160·84**

If you made corrections, you may wish to save this altered ApLet. Press LIB then SAVE You can save the ApLet under a new name.

To keep the old name, just press ENTER at the prompt.

Almost any Growth and decay problem involving functions of the form

Y=K*B^X ($y = k\,b^x$) will work in the solve ApLet

3.7 Clearing ApLets and entries within an ApLet

To clear individual entries within an ApLet

(eg to delete the entry E9 in the MATHS FORMULAE ApLet)

first open the ApLet; select the entry E9, then press DEL

To clear an entire ApLet from the ApLet Library listing, press LIB,

select the ApLet to be deleted, then press DEL

As a safety check you will be prompted with the question:

Delete the ApLet **name of ApLet selected**

If this is the ApLet that you want deleted press the screen menu key YES

If you made an error or you do not wish to delete that particular ApLet press

NO and repeat the above procedure with the right ApLet.

To clear All *user designed* ApLets press the key combination CLEAR DEL

This will delete ALL ApLets and leave just the six default *template ApLets.*

It does not empty any content in these standard templates.

Note: RESET will not clear the contents of an ApLet unless it is one of the six *template ApLets* provided as default ApLets described at the start of this chapter.

Practice exercise: To create a Solve ApLet and name it SCIENCE

Below are some common formulae used in science.

Science ApLet 1 **PROJECTILE-MOTION**

E1 input $S = UT + \frac{1}{2}AT^2$

E2 input $V = U + AT$

E3 input $V^2 = U^2 + 2AS$

E4 input $S = \frac{(V + U)T}{2}$

Another ApLet could contain science formulae such as

E1 input $C = \frac{5}{9}(F - 32)$

E2 input $\frac{1}{F} = \frac{1}{U} + \frac{1}{V}$

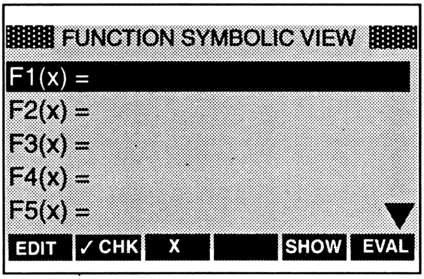
E3 input $T = 2\pi\sqrt{\frac{\ell}{g}}$ etc.

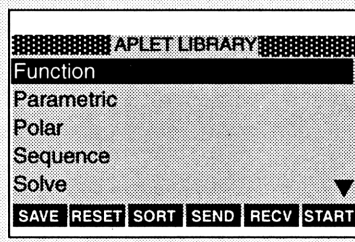
Add some of your own to this list. (Simple Harmonic Motion, Hooke's Law, work - power- energy, projectile motion, circular motion etc). Test the formulae then save them in an ApLet with a suitable name. One function that you may find useful is **ISOLATE**. (Press HOME MATH Symbolic then select this function from the menu).

CHAPTER 4

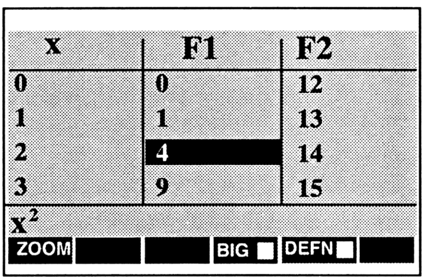
THE FUNCTION APLET

SYMB The ***SYMBolic*** view

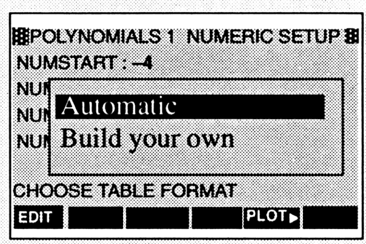




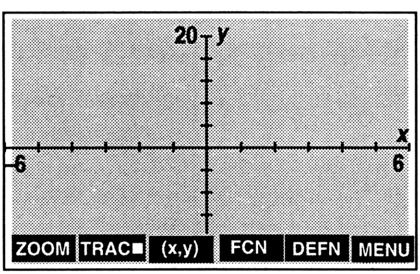
NUM The ***NUMeric*** view



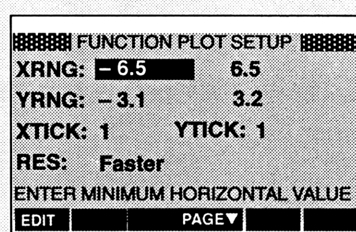
SETUP NUM



PLOT The *graphing* view



SETUP PLOT



4.1 ApLets and their views.

The keys in the second row of the calculator keyboard PLOT SYMB NUM (SETUP) apply to all ApLets. These keys determine which view you display. Using the **shift key** ▢ with these same three keys enables you to set up the parameters (Axes labels, x-range, y-range etc) that control the setup of the views.

Each of the views will be demonstrated in the examples that follow.

Each ApLet in the HP 38G operates within a three level environment.
The ApLet can be viewed interchangeably in any of these environments by pressing the appropriate key.

- PLOT The *graphing* environment for plotting graphs. *figure 4.6*
- SYMB The ***SYMBolic*** view for entering formulae and functions. *figure 4.2-4.3*
  ALL functions entered use *X* as the independent variable.
- NUM The ***NUMeric*** view for viewing tables of values *figure 4.17*

▢ SETUP SYMB	If you are in the Function ApLet, this enables you to change the angular mode for the ApLet. This is of concern only when you are dealing with Trigonometric functions.
▢ SETUP PLOT	This enables you to set up the conditions for viewing your graph. You determine the range of values for both the *x*-axis and the *y*-axis, whether the axes will be labelled, the interval for tick marks along each axis etc. see figures 4.4 & 4.5
▢ SETUP NUM	Gives a Numeric display of the *x* and the *y* values in a table or spreadsheet form. The values are given for each function that is checked from the ten included in the SYMBolic view of the ApLet.

4.2 The Function ApLet

- Press LIB to get the screen display shown in *figure 4.1*. This list shows the ApLets currently stored in your calculator.

- Use the cursor keys ▼ ▲ to move up and down the list to select an ApLet. *Figure 4.1* shows that the Function ApLet has been selected.

- Press either START or ENTER to open up the selected ApLet.

- ApLets will normally open in the **SYMB**olic view. *Figure 4.2* shows the symbolic view for the Function ApLet. NOTE: You can also get to the **SYMB**olic view by pressing SYMB when the ApLet is selected in the LIB display screen. (fig.4.1)

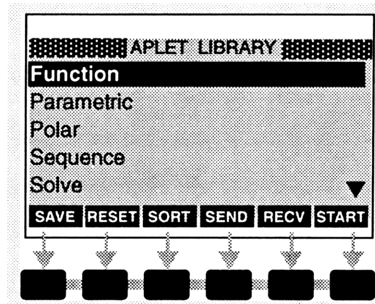


figure 4.1

Where applicable, the following keys carry out the same action:

START or ENTER to start an ApLet.	OK or ENTER to begin an action

Note that all of the *in-built ApLets* are named using *lower-case*.

It would be a good idea if you got into the habit of naming all of the ApLets that ***you design or import*** using UPPER-CASE. That way the original ApLet templates or temporary ApLets that you may be working with currently can be easily identified. Its also easier to type the name this way.

4.3 A Comment

Notice the **Screen Menus** that appear at the bottom of the screen displays.

Each of these menus is accessed by pressing the **blank key** immediately below that menu on the keyboard. (Remember the convention used in this manual.

SAVE means press the blank key below the SAVE screen menu

These screen menus are not FIXED but vary according to the current status of the calculator, the ApLet type, which View the ApLet is in PLOT SYMB or NUM. The screen menus in *figure 4.1* are those associated with the ApLet **LIB**rary and the display is obtained by pressing the LIB key. The menu labels at the bottom of the **LIB**rary View were explained at the start of the chapter on the SOLVE ApLet.

As you get to understand the HP 38 you will find there are better ways to work through many of the ideas that are explained below. This manual simply aims to get you comfortable with using and moving around the HP 38G. Once this is achieved the alternative procedures will make more sense.

Yes! There are ways of designing the ApLets so that they are interactive.
Yes! There are ways of down-loading ApLets designed by others more expert at doing this task. For example the **Internet** provides a rich source of such ApLets which can be easily down-loaded into your own calculator and saved on your own computer. Most of the ApLets available on the **Internet** also contain work-sheets related to the concepts being developed by the ApLet. These features, and more, can be tackled once you become familiar with moving around the HP 38G calculator.

4.4 Designing an Elementary Function ApLet (i)

Example 1

To design an ApLet on the linear function $y = mx + b$

Step 1 Select the Function ApLet from the LIBrary list

- Press LIB Use ▼ ▲ to select the Function ApLet (ie you highlight it).
- Press either START or ENTER to open up the chosen Function ApLet.
- ApLets open in the ***SYMBolic view***. You should have a screen like *figure 4.2*

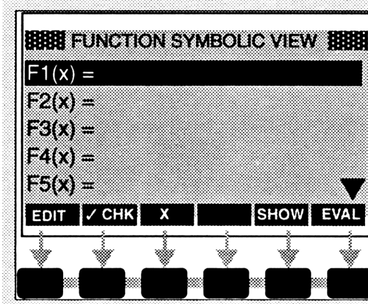


figure 4.2

You can get directly to the SYMBolic view for any ApLet by pressing the SYMB key when the required ApLet is selected in the display screen. (*figure 4.1*)

Step 2 SYMBolic view. Enter the definitions of the functions

- If your screen shows functions already entered you can clear these individually Simply select a function then press DEL
- To clear **all** previously defined functions press ▒ CLEAR DEL.

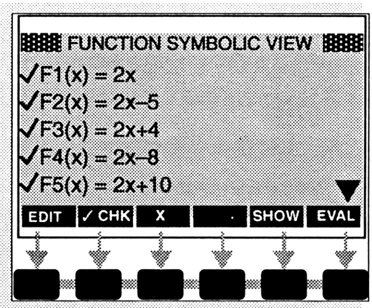


figure 4.3

For F1(x) input 2 X,T,θ then press ENTER

Once a function has been entered if you need to make a correction press EDIT

Make the correction then press ENTER

Note: you could use X in place of X,T,θ

Input F2(x) = **2x – 5**;

F3(x) = **2x + 4;**

F4(x) = **2x – 8**;

F5(x) = **2x + 10**

Press ENTER after you input each function.

> The black triangle ▼ above the EVAL means that you can scroll down the screen using the cursor keys.
> You can input up to 10 functions.

Notice that a check-mark(✓) appears alongside each function after you press ENTER

Step 3 The PLOT SETUP *Set up the conditions for plotting the graphs.*

- Press SETUP PLOT to set up the constraints on the graph plotting. You should get *figure 4.4*
- For XRNG: input **–10** ENTER then **10** ENTER

*****Remember** to use -X for the negative sign

- For YRNG: input –15 ENTER then 15 ENTER
- For XTICK: input 2 ENTER

  For YTICK: input 2 ENTER

- Notice that when Faster is selected in fig. 4.4 the screen menu label changes to CHOOS
- Press CHOOS and select More Detail
- then press OK or ENTER to get *figure 4.5*

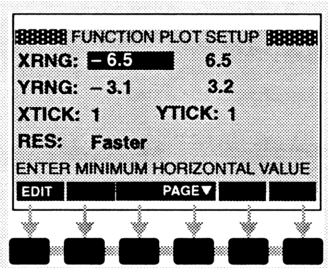


figure 4.4

> Use the cursors ◀ ▶ ▼ ▲ to move around this screen. If you make any changes you must press ENTER

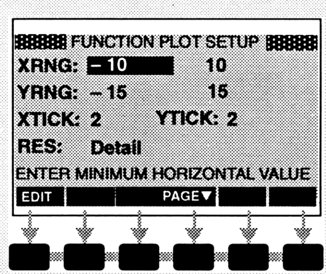


figure 4.5

- Now press PAGE▼ (the screen menu key)

  Your screen should look like *figure 4.6*

- Use the cursor keys ◀ ▶ ▼ ▲ and press ✓ CHK to check (or un-check) the items shown in *figure 4.6*

- When all this setup is completed press PLOT

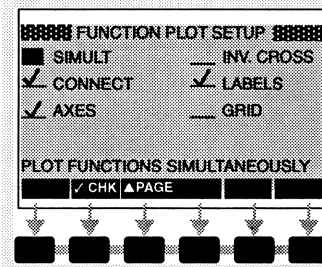


figure 4.6

I recommend in these early stages that for other than the scaling and tick marks you keep the setup as shown in *figure 4.5* and *figure 4.6*

- The PLOT view graphs all the functions listed in *figure 4.3* that have a check mark (✓) in front of the definition.

- You can un-check a function by selecting it and pressing ✓ CHK This does not remove the definition from the ApLet, it simply removes it from the list to be graphed. It is a *toggle key*.

- The functions are plotted in the order in which they are listed in the ApLet. Your screen display should look like that in *figure 4.7*.

- The coordinates of the cursor are shown at the bottom of the screen. In this case the cursor is on F1(x) at (–1,–2)

  Use the ▼ ▲ cursors to move from one function to another. Use the ◀ ▶ cursors to move along a chosen function.

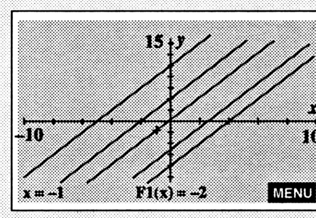


figure 4.7

As you move the *cross hairline cursor* on the screen, using the cursor keys ◀ ▶ ▼ ▲, the coordinates of the position of the screen cursor are given at the bottom of the display screen

- Press the **MENU** key and the screen menus change to those shown in *figure 4.8.*

- Press **DEFN** to get the definition of the function currently being traced by the cursor.

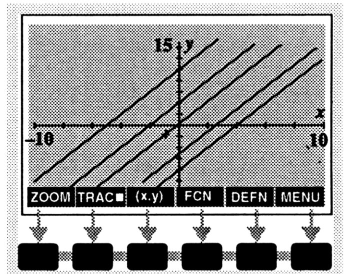


figure 4.8

The **ZOOM** feature will be considered later.

- **TRAC■** If the trace screen menu label **does not contain a small white square** the cursor can be moved to any position on the screen using ◀ ▶ ▼ ▲ but the coordinates of the cursor will not be shown. Press the menu key and a small white square covers the E in TRACE The cursor is now limited to tracing the functions and its coordinates are given in the form (*x*, **Fn**(*x*)) where **n** gives the number of the function being traced.

- Press **LIB** to get back to the view showing the **LIB**rary of stored ApLets (*figure 4.1*)

- Press **SAVE** then press **A...Z** and type the name LINEAR FUNCTIONS 1.(*figure 4.9*)

- then press **ENTER**

Your display should now be back in the ApLet **LIB**rary view with your newly saved ApLet listed amongst the other ApLets

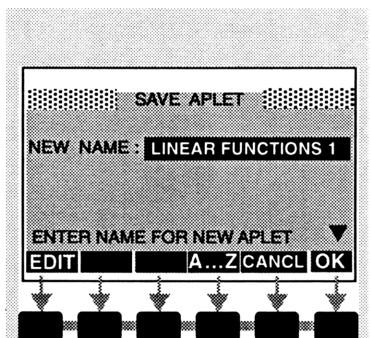


figure 4.9

4.5 Managing Memory

- This ApLet is now stored in the Memory of the calculator. It can be recalled at any time, the functions can be edited (changed) and the ApLet saved under a new name.

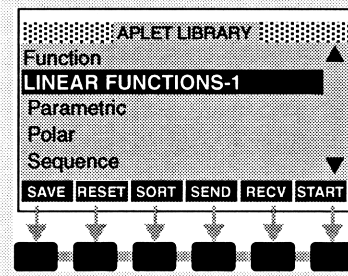


figure 4.10

(This will replace the old ApLet in the APLET LIBRARY unless you used one of the six template ApLets. These six ApLets cannot be removed or renamed)

To delete an ApLet, select it then press DEL.

- The number of ApLets that can be stored depends upon the amount of memory available. The size of the ApLet is also a factor. An ApLet, such as [QUADRATIC], down-loaded from the Internet, will require almost all of the available memory.

To give yourself the maximum possible memory you should clear unwanted ApLets from the **LIB**rary and also clear the screen history in the HOME screen. The HP 38G keeps a record of ALL calculations done in the HOME screen. This handy feature enables you to use the cursor keys to scroll through the history, copy and if necessary EDIT a previous entry for re-use.
This screen history takes up some memory and can be cleared by pressing the keys [shift] CLEAR DEL.
This will clear the current history and a new screen history begins from this step.

4.6 Attaching a note to an ApLet

You may wish to attach a note to an ApLet explaining the purpose of the ApLet, or wish to include questions about the functions done as homework, or to remind you of key aspects of the content of the ApLet.

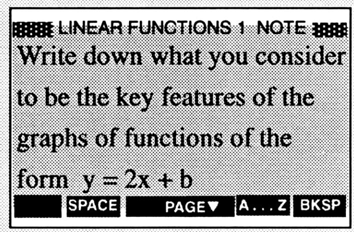


figure 4.11

Note the Screen menus at the bottom of the screen display. Page▼ indicates more text can be seen by pressing this key.

To include a note with an ApLet first select the ApLet (*figure 4.10*), then press [shift] NOTE VAR

This will bring up a blank screen with the title of the ApLet across the top of the screen.

Type in your note. Use A...Z to fix the keyboard to type alpha-characters. This does not require you to hold down the A...Z when typing alpha-characters. When this key is pressed the keyboard stays locked in the Alphabet mode.

This can be seen by the appearance of α at the top of the screen display.

If you wish to include a number with the text press A...Z and the alpha keyboard will use the normal numeric keypad for the next character, after which it will revert back to the alpha locked keyboard.

Press the A...Z again to return the numeric keyboard to normal setting.

Use SPACE to insert spaces into your text

Use BKSP to edit mistakes and backspace the cursor in your note's text.

4.7 Designing Elementary ApLets (ii)

Polynomial Functions using the Function ApLet

1. SYMB view for Defining the functions.

- Press LIB Use ▼ ▲ to select the Function ApLet.
- Press either START or ENTER to open up the Function ApLet.
- To clear **all** previously defined functions within this ApLet press CLEAR DEL

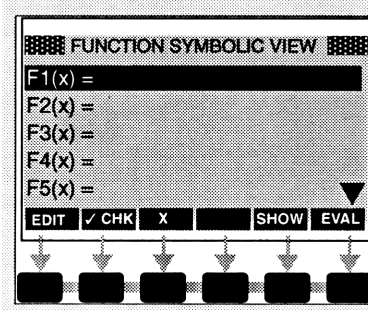


figure 4.12

You should now have a clear screen in the **SYMBolic view** *figure 4.12*

You can get directly to the SYMBolic view by pressing SYMB when the required ApLet is highlighted in the display screen.

- Remember: To input x use X,T,θ or X

To input x^2 press X,T,θ x^2

- Input the functions shown in *figure 4.13*

Press ENTER after you input each function.

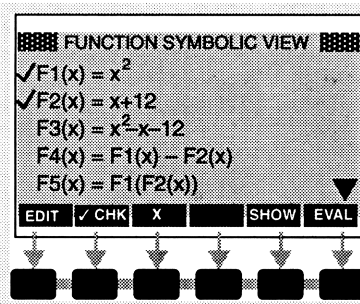


figure 4.13

- Enter the following additional functions. The screen display will automatically scroll to the next position after you press ENTER.

  Add **F6(x)** = F2(F1(x))

  F7(x) = ABS(x – 5) + 6

  F8(x) = $2x^3 + 7x^2 - 4x - 6$

Note: to enter the *absolute value function* you could either type the ABS from the keyboard or use the keystrokes [shift] ABS [-x] to insert ABS into the EDIT line.

To enter F8(x) use | 2 X,T,θ x^Y 3 + 7 X,T,θ [shift] x² – 4 X,T,θ – 6 |

- Press LIB to get back to the ApLet Library view.
- Press SAVE then press A...Z and type the name POLYNOMIALS 1 ENTER

  The ApLet **LIB**rary screen now shows your new ApLet included in the library

2. PLOT view Setting up the plotting conditions.

- With your new ApLet selected press ENTER (If you press SYMB this has the same effect).
- Use the cursor keys ▼ ▲ and ✓CHK to uncheck all functions except F1(x) and F2(x)

ie Leave only F1(x) and F2(x) checked(✓) See Fig 4.13

Press [shift] SETUP PLOT and set up the constraints as shown in *figure 4.14*

Use ◀ ▶ ▼ ▲ to move through the values. If you make any changes press ENTER

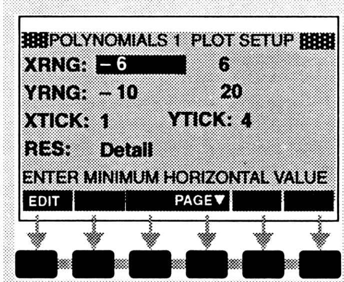


figure 4.14

- Press PAGE▼ ,

  then use the cursor keys ◀ ▶ ▼ ▲ to move to any one of the six positions shown in *figure 4.15*.

  Use ✓CHK to set up the checks as shown in *figure 4.15*

  Note: SIMULT is **not** checked.

  This Setup is simply a personal preference.

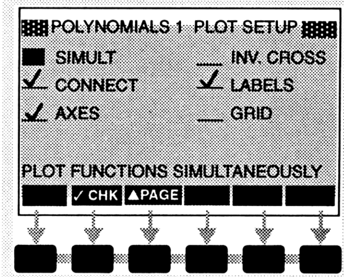


figure 4.15

Other than when a GRID may be useful you should use this setup for all of your plots.

- Press PLOT to get the graph **PLOT** view shown in *figure 4.16*

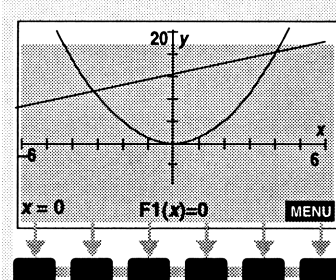


figure 4.16

Experiment with the SETUP PLOT

Set up the constraints in different ways to familiarise yourself with this feature.

Try different *x* and *y* ranges,
Try different values for the TICK marks.
Although LABELS is not a default setting you should maintain this in your settings as at helps to keep the graph in some form of perspective.

3. [NUM] view Working in the Table format.

- Press NUM to get to the NUMeric view shown in *figure 4.17.*

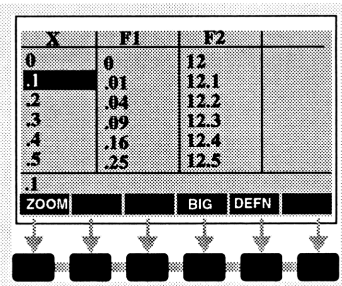


figure 4.17

- Move around the screen using the cursor ◄ ► ▼ ▲ keys.
  At this stage a grey cursor box will indicate your position in the table. The value selected within this box is repeated at the bottom of the table.

- The first, leftmost column, gives values of the independent variable which, in the Function ApLets is x .

The size of the screen numbers.

- All functions that are checked in the **SYM**bolic view of the ApLet (*figure 4.13*) will have a column in this table

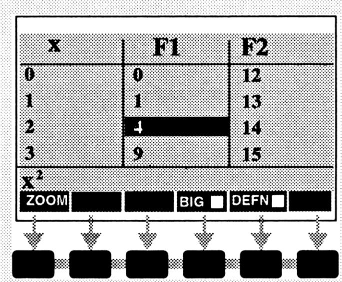


figure 4.18

- The screen menu BIG ■ makes the numbers in the screen display larger. It is a *toggle key.*

- When BIG is activated a white square appears in the menu label BIG ■.
  Less data is now shown in the display since the figures and letters are larger. (*figure 4.18)*

4.9 Tables centred on specific values.

- Move to the left column (x) and type a number (eg **1** **2** **ENTER**) Notice the effect! The value that you enter becomes the middle value of the column and the rest of the table adjusts around this value. (fig 4.19)
  This feature can be done in this LEFT column only.

- If **BIG** is active then the input value may be the *last value* displayed in the column.

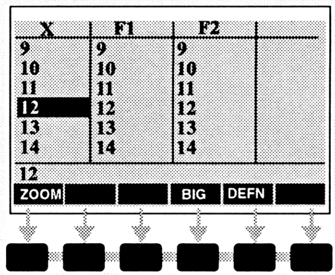


figure 4.19

4.10 Setting up the number table format.

Press SETUP **NUM** to set the options for the table layout . (*figure 4.20*)

These settings are independent of the graph shown in the PLOT view

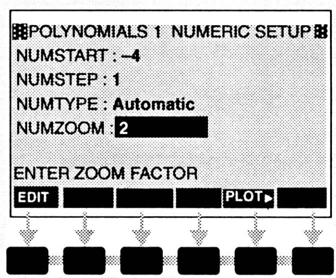


figure 4.20

- **NUMSTART** input here the value of x at which you wish to start the table whenever this ApLet is opened. **ENTER**

- **NUMSTEP** input here the value for the increment of x. If you wish to go in steps of 0·1 then input 0·1 then press **ENTER**

If you do not need to change an existing value you will need to use the **cursor key** to move to the next field as there is nothing to **ENTER** and the selector will stay on this field if you do press **ENTER** instead of ▼

- **NUMTYPE:** the default value is Automatic

  Note the change in the screen menus

  Press CHOOS You are given the choice of

(i) **Automatic** The table is built for you.

The table will be based on the settings in *figure 4.20* or

(ii) **Build your own** table.

With this choice the table in *figure 4.19* is blank. You input a value for *x* ; the values for the other columns are inserted (*figure 4.21*)

- from the two choices offered highlight

  Build Your Own

- Press ENTER then NUM to get back to the numeric view.
- Move to the first column (X). Type in the x value 10 ENTER

  The values for both F1(x) and F2(x) are listed in the appropriate column (*figure 4.22*) Enter several more values as shown in the figure. You can build your own table of values within this window.

Now experiment with the two new screen menus INS (insert) and SORT (sort table in order). (*figures 4.22 and 4.23*)

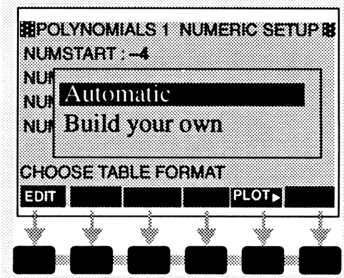


figure 4.21

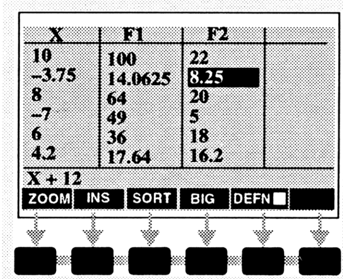


figure 4.22

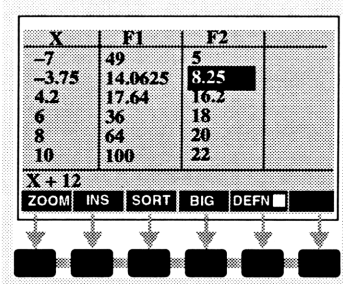


figure 4.23

SORT will rearrange the values in the table in either (i) Ascending order or (ii) Descending order

When DEFN is activated a white square appears in the menu label DEFN■.

Activate the screen menu by pressing DEFN (It is a toggle key)

Now move the cursor across the columns of the table using the ◀ ▶ keys. Instead of the value of the selected cell appearing at the bottom of the screen the defining rule of the function for that column of data is displayed.
(*figures 4.18, 4.22, 4.23)*

4.11 Zoom In the NUMeric View

Press the ZOOM menu key. A choice menu is displayed (*figure 4.24*).

Try each option in turn.

The zoom-in, zoom-out factor is based on the choice made in *figure 4.20* In this case the zoom factor is set to 2. (*figure 4.20*)
Un-zoom each time before going to the next item in the menu.

Un-zoom will only reverse the last zoom action. If you do three successive zoom outs then unzoom will only undo the last one. If you wish to get back to the original table of values you must either **zoom in** two more times or reset the table format as shown in *figure 4.20*

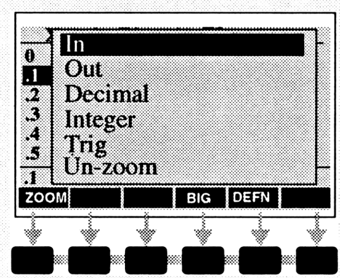


figure 4.24

Press PLOT . Note that the table setup has not affected the PLOT SETUP. Your screen should look like *figure 4.25*. (No change from *fig4.16)*

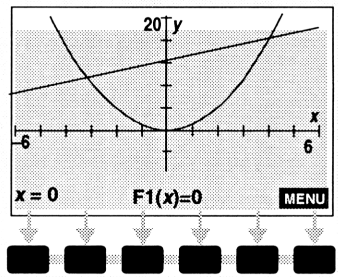


figure 4.25

The conditions that you set up in *figures14 &15* using SETUP PLOT should still be the same as when you initially set them up. The work and the setup in the NUM view has no effect on the PLOT SETUP.

When you save (or re-save) this ApLet both settings for NUM view and PLOT view are saved with the ApLet

- Press MENU then the screen menu key FCN

  This should give you *figure 4.26*.

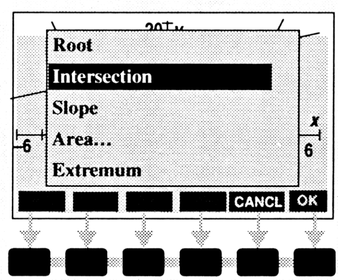


figure 4.26

- Select Intersection ENTER. You are given the choice of the intersection of F1(x) with either F2(x) or the x-axis. (*figure 4.27*)

- Select F2(x) ENTER The cursor will move to the first point of intersection *to the right of its current position*. The coordinates of the cursor are given at the bottom of the screen. Move the cursor if necessary using the cursor keys ◀ ▶. Repeat the procedure to determine the second point of intersection.

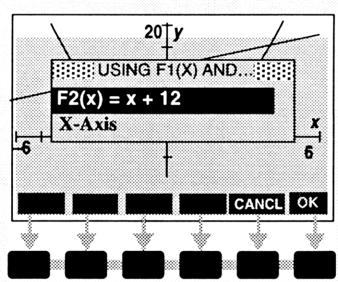


figure 4.27

Go back to the SYMBolic VIEW. (Press SYMB).

- Use ✓CHK to uncheck both F1(x) and F2(x) and to check (✓) F3(x)
- Use SETUP PLOT to set

  XRNG to –6, 6; YRNG to –20,20,

  XTICK = 2 YTICK = 5

  Then press PLOT to get the graph **PLOT** view shown in *figure 4.28*

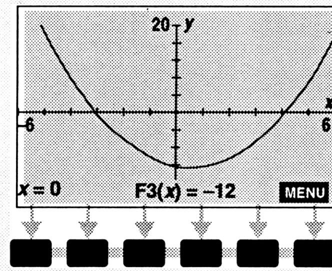


Figure 4.28

- Press MENU to show the other screen menus available in plot view. (*figure 4.29*)
- Press FCN This time from the choices offered (*figure 4.26*) choose Root

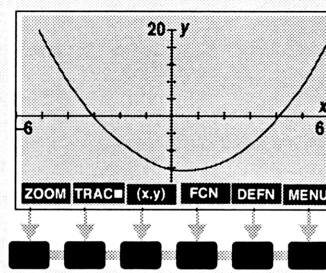


Figure 4.29

That root closest to the current position of the cursor is given in the bottom left of the display screen. You will need to manoeuvre the cursor before repeating this procedure to determine the second root.

- Compare the roots of F3(x) with the solution to the system F1(x) = F2(x).

4.12 Locating Extremum - Turning points

- Press [MENU] so that the screen menus are again visible and press [FCN] (*figure 4.26*)

Select [Extremum] The cursor moves to the turning point and states its coordinates (*x*,*y*) in the bottom left of the display. (*figure 4.30*)

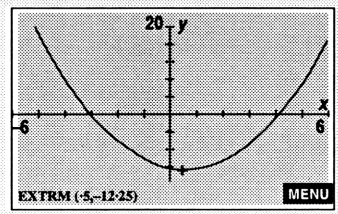


figure 4.30

4.13 Composite functions

Return to the SYMBolic VIEW. (Press [SYMB]).

- Use [✓ CHK] to check in turn F5(*x*) and F6(*x*)
- Note that the graph of F5(*x*) does not appear on the screen. Press [NUM] to get a NUMeric or table view, this should explain the problem! (*figure 4.31*)

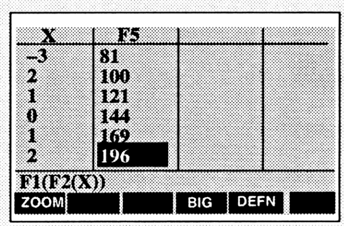


Figure 4.31

- Press [SYMB] to get back to the function definitions (*SYMBolic*) view (*figure 4.13*)
- Highlight the composite function

  F5(*x*) = F1(F2(*x*)) and press [EVAL]

  This **defines** the composite function as

  F5(*x*) = (*x* + 12)²

  Repeat this process for F4(*x*) and F6(*x*)

CHAPTER 5

POLYNOMIAL FUNCTIONS

Working with polynomial functions

5.1 The Algebra of Polynomial Functions

(i) using the Function ApLet

- Press SYMB to get to the SYMBolic VIEW.
- Use ✓CHK to check (✓) F8(x) only,

  where F8(x) = $2x^3 + 7x^2 - 4x - 6$

Un-check any other functions in the ApLet list.

- Press ⬜ SETUP PLOT and use the setup as shown in *figure 5.1*

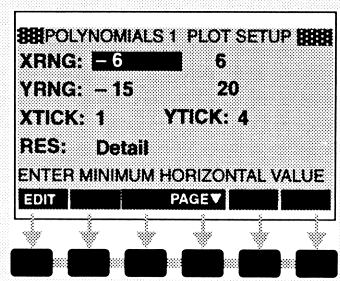


figure 5.1

- Press PLOT to get the graph **PLOT** view (*figure 5.2*)
- Press MENU to show the screen menus *fig 5.2*

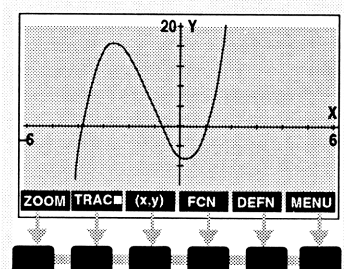


figure 5.2

- Use the cursor keys ◀ ▶ and move the on-screen cross cursor so that it is near the left side of the screen.

  The cross cursor should trace along the graph. Stop anywhere along the curve that is *near* the left-most intercept on the x-axis.

HINT: ⬜ ◀ will move the cursor to the bottom left side of the display screen

If there is more than one function checked then, in PLOT view, the ApLet works with the function that is currently marked with the cursor.

To determine the Roots of a Polynomial

- Press 

- From the choices offered choose Root (*figure 5.3*)
  The cursor moves to the root that is closest to the current position of the cursor. It's value is given in the bottom left of the display screen. You will need to manoeuvre the cursor before repeating this procedure to determine the second and further roots.

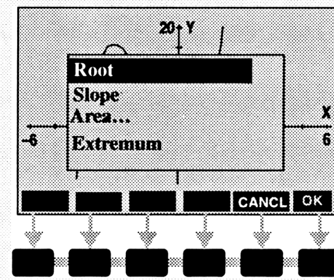


figure 5.3

Alternatively:

- After the first root from the left side has been found press 

- From the choices offered select Extremum *figure 5.3*
  This will give the coordinates of the turning point next to the root just found.

- Repeat the above procedure for the root and the turning point in succession until all the values A, B, C, D, and E have been determined. *figure 5.5*

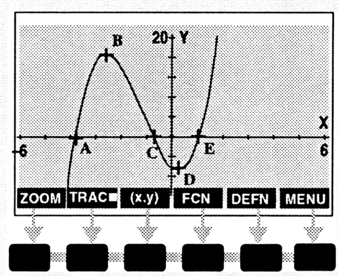


figure 5.4

The solutions provided are
A = –3·818...
B = (–2·590...,16·568...)
C = –0·741...
D = (0·257...,–6·531...)
E = 1·059...
Hint: Set the number format to **Fixed 2**

5.2 The Algebra of Polynomial Functions

(ii) using the MATH Menus

Four very useful functions relating to polynomials are available in the catalogued list of functions obtained by pressing the MATH key.

Each of these functions has a specific **syntax** associated with it.
(ie The way in which the function is expressed together with its arguments.)
This should not be of major concern here as each function, its purpose and use, will be carefully explained in the examples that follow.
A summary of the *main* MATH functions of interest in most high school mathematics courses is included in Chapter 14.

To get to these four functions for working with polynomials proceed as follows.

- Press HOME.
- Press MATH and a screen view similar to figure 5.5 is displayed.

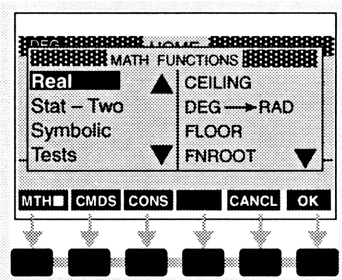


figure 5.5

Explanation of the screen view in figure 5.5:
The left side categorises the functions into types. The listing is in alphabetic order.
Using the ▼ ▲ cursor keys scroll down the left side . As you scroll you will notice that the functions on the right side change to list those functions available in each of the categories as they are highlighted.

Since we are interested in the Polynomials scroll up or down until Polynom. is selected.

HINT: If you know the category of function that you wish to scroll to, simply press the key that contains the alpha character for the first letter of the *category*.
Thus to get to the **Polynom.** *category* quickly, just press 5 P . You **do not** need to press the A...Z key first

5.3 To obtain the roots of a Polynomial Function.

- The display screen should look like *figure 5.6*
- Now use the ◀ ▶ cursor keys to move to the right side of the display menus. Scroll down to select POLYROOT then press OK or ENTER This puts you back in the HOME screen with the EDIT line containing the entry **POLYROOT (**

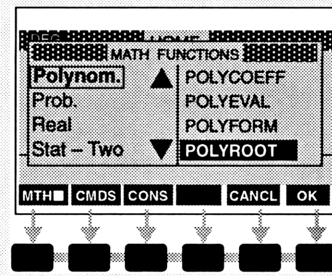


figure 5.6

Once you become familiar with the functions, instead of going through the menu to get POLYROOT, as shown in figure 5.6 above, you would simply type POLYROOT(etc directly into the EDIT line of the HOME screen using the A...Z key

- The function POLYROOT will determine the n roots of any polynomial function of the form

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0$$

- The Syntax (ie what you MUST key in) is

POLYROOT($[a_n, a_{n-1}, a_{n-2}, \ldots a_3, a_2, a_1, a_0]$)

This general case will return the roots of the n^{th} degree polynomial. Put briefly you enter

POLYROOT([coefficients separated by a comma])

- For the polynomial

  $P(x) = 2x^3 + 7x^2 - 4x - 6$

  at the EDIT line in the HOME screen input POLYROOT([2, 7, –4, –6]) then press ENTER

  This returns [–0·74, 1·06, –3·82] *figure 5.7*

  Thus the roots are –3·82, –0·74, 1·06.

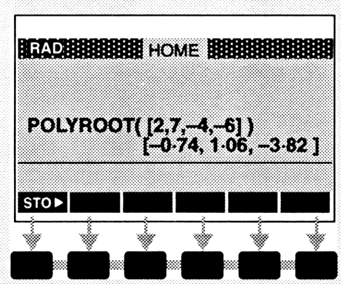


figure 5.7

Use MODES HOME and set the NUMBER FORMAT to FIXED 2

Example 2:

Determine the roots of the polynomial function

$$P(x) = 6x^3 - 32x^2 - 18x + 140$$

- At the EDIT line input

  POLYROOT([6, –32, –18, 140]) ENTER

Note the use of (and [brackets in the entry

- This returns the result [2·33, –2, 5]

ie the roots of the polynomial

$P(x) = 6x^3 - 32x^2 - 18x + 140$

are 2·33, –2, and 5 *figure 5.8*

- Use [ ] MODES HOME and set the NUMBER FORMAT to *Fraction 2* to get answers in fraction form

- Use the cursor keys ▼ ▲ to highlight **POLYROOT**([6, –32, –18, 140]) in the display then press ENTER

- The roots of $P(x) = 6x^3 - 32x^2 - 18x + 140$

are then given as $[\frac{7}{3}, -2, 5]$ *figure 5.9*

It is not necessary to copy the question back into the EDIT line before you press ENTER

The fraction number format could prove useful when dealing with the Factor Theorem.

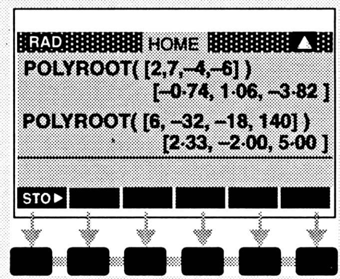


figure 5.8

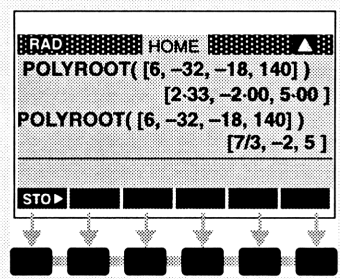


figure 5.9

It will often be quicker to copy the POLYROOT function into the EDIT line from a previous calculation and then edit by deleting the old coefficients and inputting those of the next polynomial being solved.

It is also possible to select the original question in the display screen, change the number format to a more suitable form, then press HOME then ENTER

Now consider the roots of the polynomial

$$P(x) = x^4 - 4x^3 + 3x^2 + 5x - 9$$

Enter this function into the function ApLet (SYMB) and then Plot the graph (PLOT)

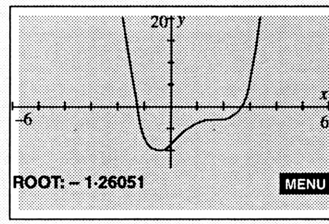


figure 5.10

The plot is shown in *figure 5.10*
The Fundamental Theorem of Algebra states that this fourth degree polynomial in x should have four roots. Two Real Roots can be observed from the graph.

- Press HOME, change the number format to Fraction 2 then input **POLYROOT**([1, –4, 3, 5, –9]) ENTER

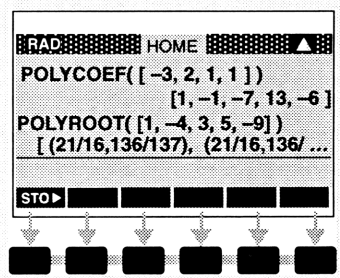


figure 5.11

- The solutions offered are *figure 5.11*

$(\frac{21}{16},\frac{136}{137})$; $(\frac{21}{16},-\frac{136}{137})$; $(-\frac{29}{23},0)$; $(\frac{21}{8},0)$

That is there are two Real roots $-\frac{29}{23}$ and $\frac{21}{8}$

and two complex roots

$(\frac{21}{16} + \frac{136}{137}\,i)$ and $(\frac{21}{16} - \frac{136}{137}\,i)$

Also note that the complex roots are conjugates.
To see the full solution on screen, use the cursor keys ▼ ▲ to highlight the answer

then press SHOW   Press OK when

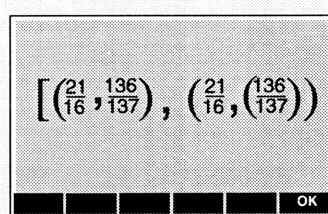


figure 5.12

done. *figure 5.12*
This shows the solutions in their normal form.

5.4 To find a Polynomial given its roots.

POLYCOEF

The full syntax is **POLYCOEF([*root1*,*root2*, *root3*, … *rootn*])**

Example 3: *Determine the polynomial that has as its roots −3, 2, 1, 1*

- Press HOME then the MATH key.
- Press the key 5 P so the right side of the menu table scrolls down to the function categories starting with the letter P *figure 5.13*
- Use the ◀ ▶ cursor keys to move to the right side of the display menus. Scroll down and select POLYCOEF *figure 5.13*
- Press OK or ENTER This puts you back in the HOME screen with the EDIT line containing the entry **POLYCOEF(**
- Add to the EDIT line so it reads POLYCOEF([−3, 2, 1, 1]) then press ENTER
- The result given is [1,−1,−7,13,−6] *figure 5.14*

Thus the polynomial that has as its roots

−3, 2, 1, 1

is $P(x) = \mathbf{1}x^4 - \mathbf{1}x^3 - \mathbf{7}x^2 + \mathbf{13}x - \mathbf{6}$

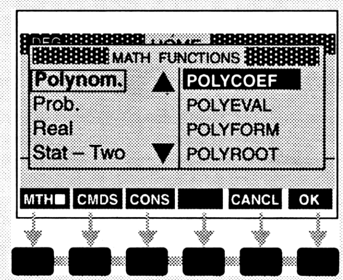


figure 5.13

You could simply type POLY

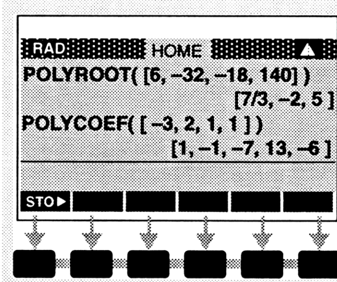


figure 5.14

The coefficients are given in descending order of powers of *x*

5.5 To determine P(x) for any Polynomial.

POLYEVAL

The full Syntax is **POLYEVAL**([coefficients], Variable's value)

Example 4

Determine P(–5) given

$$P(x) = 5x^4 - 9x^3 - 21x^2 + 12x - 10$$

- Press HOME then the MATH key.
- Press the key 5 P
  select Polynom. ▶ POLYEVAL ENTER
- In the EDIT line of the HOME screen input

POLYEVAL([5, –9, –21, 12,–10], –5) ENTER

The value 3655 is displayed *figure 5.15*

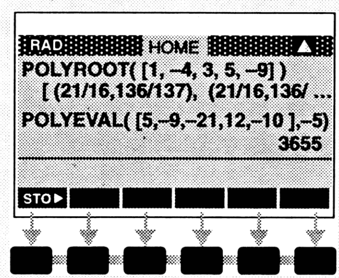


figure 5.15

This type of question is probably better done in the NUMeric view of the Function ApLet.

The function POLYEVAL is more useful within the programming routines.

5.6 To simplify the form of a Polynomial.

POLYFORM

The full Syntax is **POLYFORM** (expression, var name1, var name2 ...)

We shall use only **POLYFORM** (expression, var name1) to develop polynomials in one variable. Other forms are possible.

This generates a polynomial in *variable 1* from the expression

Example : POLYFORM($(2x - 3)^4$, x) ENTER will give

$16x^4 - 96x^3 + 216x^2 - 216x + 81$

Example : POLYFORM($(x^2 + 2x - 3)^3 - 5x + 17$, x) ENTER will give

$x^6 + 6x^5 + 3x^4 - 28x^3 - 9x^2 + 49x - 10$

Example : POLYFORM($(2w + 5)(w - 4)(3w - 2)$, w) ENTER will give

$6w^3 - 13w^2 - 54w + 40$

Note that the variable does not have to be *X, it can be any letter!*

General: ***Factorise the expression*** $6x^3 - 13x^2 - 54x + 40$

- Use [shift] MODES HOME and set the NUMBER FORMAT to *Fraction 2* then input **POLYROOT**([6, –13, –54, 40]) ENTER

The solutions offered are $\frac{2}{3}$; $\frac{-5}{2}$; 4.

From this it can be deduced that $(x - \frac{2}{3})(x + \frac{5}{2})(x - 4) = 0$

ie $6x^3 - 13x^2 - 54x + 40 \equiv (3x - 2)(2x + 5)(x - 4)$

5.7 Some ideas to investigative.

Another function dealing with polynomials that is available in the MATH menus is FNROOT. The full syntax is

FNROOT(**expression**, **variable name**, ***first guess***)

The function root finder **FNROOT** determines the values of the variable that make the value of the expression equal to zero.

eg Press HOME to get to the HOME SCREEN. Key in

FNROOT($x^2 - 7x + 6, x , 3$) ENTER where 3 is an *estimate of the root.*

This returns an answer of 1.

Now use the ▲ cursor to highlight FNROOT($x^2 - 7x + 6,x ,3$) Press COPY

to bring it to the EDIT line, then alter the 3 to another *estimate* or *guess* (Try 4, then try 5 as the *guesstimate.*) The technique used is the Newton-Raphson iterative process and the root provided is usually the one closest to your entered guess. The process is similar to that used by the **Solve ApLet.**

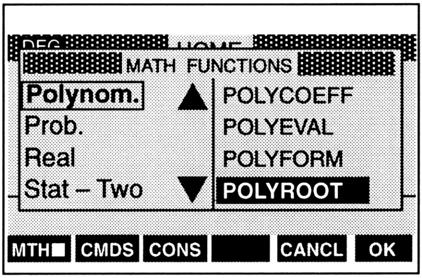
FNROOT serves a more useful role in the programming aspect of the HP 38G.

This leaves open a broader investigation into quadratics where the estimates for the roots will be values based either side of the axis of symmetry.

The next page shows some screen views that you might find useful.
You may need to locate some of the functions on your calculator when working with polynomials.

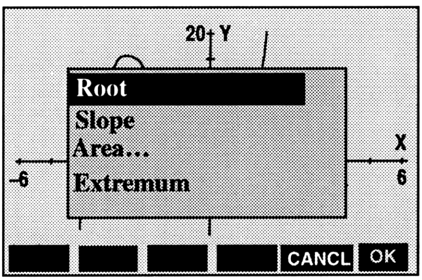
Location of functions used with polynomials

Press MATH then 5 P



Inbuilt polynomial functions that can be used in calculations in the HOME SCREEN.

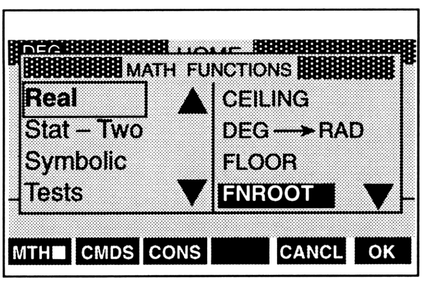
Select the required function then press ENTER



Determine roots and intersections graphically.

To get this screen use the FCN in the PLOT view.

Press MATH then * R



To get FNROOT into the EDIT line in the HOME SCREEN press MATH then scroll to the menu shown.

CHAPTER 6

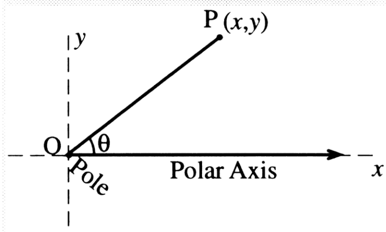
POLAR EQUATIONS

6.1 Polar Coordinates

Summary of the relationship between polar and rectangular coordinate systems.

In the Polar Coordinate system

The reference for point P is (R,θ) where θ is the angle between the Polar axis and OP. R is the *distance* from the pole O to the point P



Polar equations take the form

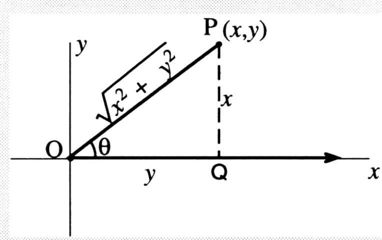
$\{(\mathbf{R},\theta) \mid \mathbf{r} = \mathbf{f}(\theta)\}$

In the Rectangular Coordinate system

The reference for the point P is P(x,y)
Equations in the rectangular coordinate system take the form

$\{(x,y) \mid y = \mathbf{f}(x)\}$

The relationship between equations in *Polar form* and *Rectangular form*



The relationship between the coordinates in both systems can be described using the figure to the right. These relationships are summarised below. Using these, one can convert a given equation from Polar form to Rectanular form and vice-versa

$\mathbf{R} = \sqrt{(x^2 + y^2)}$	$x = \mathrm{R}\cos\theta$	$y = \mathrm{R}\sin\theta$	$\theta = \arctan(\frac{y}{x})$

6.2 Plotting equations in Polar form

- Press LIB Use ▼ ▲ to select the ApLet **Polar**
- Press either START or ENTER to open up the Polar ApLet.
- To clear any previously defined functions within this ApLet press ⬜ CLEAR DEL

You should now have a clear screen in the Polar **SYMBolic view** .... *figure 6.1*

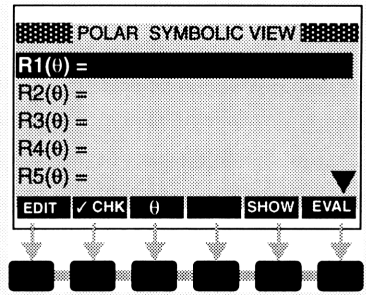


figure 6.1

Use ⬜ SETUP SYMB to set angle mode to Radians

6.3 Creating a simple Polar ApLet

- Up to ten Polar equations can be entered into an ApLet R1(θ) through to R0(θ)

The independent variable is θ.
To insert θ use either the screen menu key θ or the X.T.θ key.

In the Polar ApLet environment the X.T.θ key changes. Instead of entering *x* when it is pressed it enters θ.
The screen menu key has also changed from *x* to θ. ...... *figure 6.1*

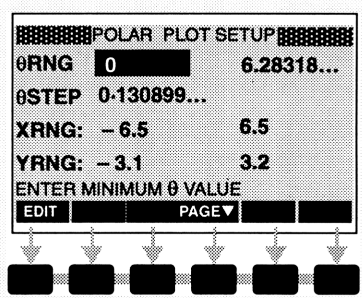


figure 6.2a

Polar plotting and the Polar screen setup.

- Input the following equation:

  R1(θ) = 2

- Press [SHIFT] SETUP PLOT to get to *figure 6.2a*

  If your setup screen does not look like this then press [SHIFT] CLEAR DEL while you are in the polar plot setup screen. This will change the settings to the **default values** as shown in *figures 6.2a and 6.2b*

- Press PLOT to get the graph in *figure 6.3*

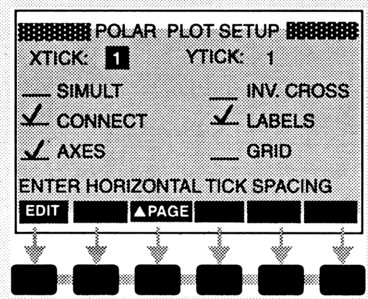


figure 6.2b

The Polar Plot Setup.

- **θ RNG** the default values are $0 \le \theta \le 2\pi$
- **θ STEP** is set at $\frac{\pi}{24}$ (≈ 0·130899)

  For more precise graphs you may need to set smaller steps eg θ step = 0·01
- **XRNG** $-6{\cdot}5 < x < 6{\cdot}5$;
- **YRNG** $-3{\cdot}1 < y < 3.2$;
- These settings maintain the aspect ratio of the screen at 2:1 (ie equal-distance scaling on both axes). This means that a plotted circle will appear properly as a circle without the distortion created by using different scales on each of the axes.

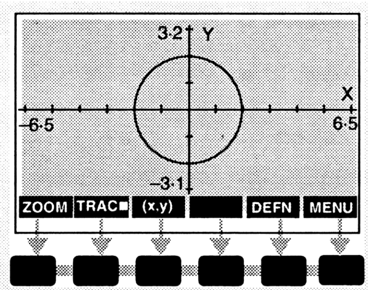


figure 6.3

Notice that the Cartesian axes are drawn rather than the polar axis!. It is possible to remove the axes (un-check axes in figure 6.2b)

- Now create an ApLet named **Polar-1** containing the following equations:

  $R1(\theta) = 2$

  $R2(\theta) = \theta$ … may need re-scaling

  $R3(\theta) = 2\sin\theta$ suggest θ step set to ·05 ?

  $R4(\theta) = 3\cos\theta$ suggest θ step set to ·02 ?

  $R5(\theta) = 3\sin(2\theta)$

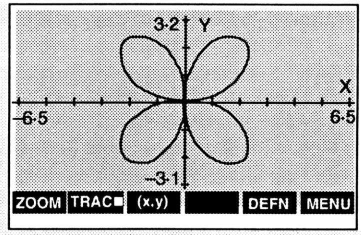
  $R6(\theta) = 3\sin(3\theta)$

  $R7(\theta) = 3\cos(2\theta)$

  $R8(\theta) = 3\cos(3\theta)$

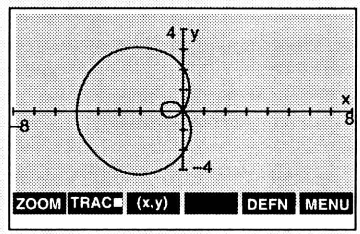
  $R9(\theta) = e\^\theta$

  $R0(\theta) = 2 - 3\cos(\theta)$



$R5(\theta) = 3\sin 2\theta$

figure 6.4



$R0(\theta) = 2 - 3\cos(\theta)$

figure 6.5

6.4 Solving polar equations

Example 1: Sketch the graph, then Solve:

$$\begin{cases} R = 3\sin(3\theta) \\ R = 2 \end{cases} \quad \text{for } 0 \le \theta \le 2\pi$$

- Use (i) the NUMeric view

  (ii) the **Solve ApLet**

  to determine the solutions of the system

- In the PLOT view use the ▼ ▲ cursor keys to move from one graph to the other. Use ◀ ▶ to trace the chosen graph.

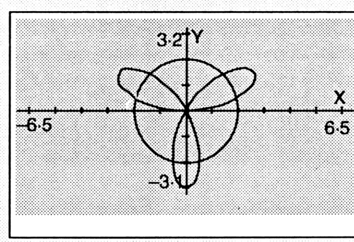


figure 6.6

This is equivalent to asking you to solve

$2 = 3\sin(3\theta)$ for $0 \le \theta \le 2\pi$

This can be solved using the Solve ApLet, but be aware of the many possible answers. See the next example for the technique!

Example 2: Solve the system

$$\begin{cases} R = 6\sin(3\theta) \\ R = \dfrac{6}{\theta} \end{cases} \quad \text{for } R > 0 \text{ and } 0 \le \theta \le 2\pi$$

Method 1: Tracing the graphs

- Open a **Polar** ApLet, enter the two Polar equations into the **SYMB**olic view.
- Use SETUP PLOT to set the XRNG and YRNG as shown by the axis in *figure 6.7*
- Press PLOT to get the graph in *figure 6.7*
- Press VIEWS LIB and select Plot-Table
- You should get a split display with the left side showing a graph, and the right side a table of values. *figure 6.8a or figure 6.8b*
- Press DEFN Use the ▼ ▲ cursor keys to move to the graph of $R(\theta) = 6\sin(3\theta)$.

The equation of the graph that the *cross cursor* is on, (tracing) in the PLOT view, is displayed at the bottom of the screen. In the table *figure 6.8a or figure 6.8b* the top of the second column shows which graph has its values displayed.

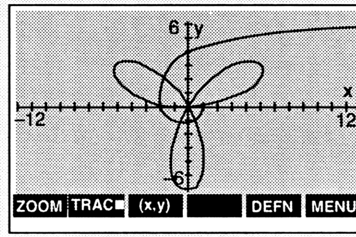


figure 6.7

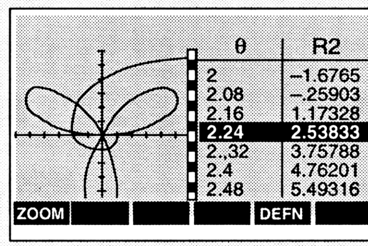


figure 6.8a

Your table may not be identical to that shown as it will depend on the setup used in SETUP NUM (The NUMSTEP used, the NUMSTART or starting value for θ etc may be different).

In the full numeric view (press the NUM key to get the **NUM**eric view), both R1 and R2 display their value for each value of θ. Check for each approximation to θ

Use ◀ ▶ to trace the chosen graph. The values are scrolled in the numeric display as the cursor moves along the graph. When you are near a point of intersection (in *figure 6.8a*) use ▼ ▲ to change to the other graph. The highlighted value of θ stays fixed and the value on the other graph for the set θ value can be compared. *figure 6.8b*

Note the approximate values for the intersections of the graphs as these will be used in the SOLVE method

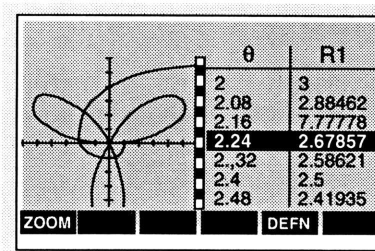


figure 6.8b

The approximate values for θ are:
1·14; **2·04**; 2·26; 3·02
4·28; 5·18; **5·4**; **6·16**.

Method 2 Using a SOLVE ApLet

- Press LIB select the SOLVE ApLet ENTER
- Input the following : (use *x* in place of θ)

  For E1 input $6/X = 6\sin(3X)$
- Press NUM to get the Solve **NUM**eric view *fig6.9*
- Enter *each approximation* for θ as given from the Polar PLOT or NUMeric view.
- Press SOLVE The solution that is nearest to the input estimate is given in the display.

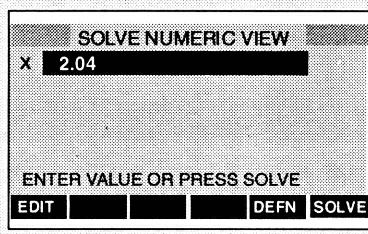


figure 6.9

The solutions for each *approximation* in the Solve Numeric view were
2·248056; 3·02946
4·26763 5·17112
Why are there not 8 solutions as seemingly suggested from tracing the graph?

Graph the system for (i) $0 < \theta \leq \pi$ (ii) $\pi < \theta \leq 2\pi$
This should help explain the correct solutions. Note the effect when $r<0$
Do further analysis of those approximate values given in **bold** below figure 6.8b

You should keep a record of techniques and short cuts as you learn how to use the HP 38G. This page is left blank below this line for this purpose.

CHAPTER 7

PARAMETRIC EQUATIONS

Working with Parametric Equations

7.1 Comments on the Parametric form of a line

The *symmetric form* of a line (equation ⑦) leads directly to the parametric form, a form which is often used in applications in physics.

Since the ratios $\dfrac{x - x_0}{q} = \dfrac{y - y_0}{m}$ ......⑦

are equal, let t be the value of this *ratio*.

ie $\boxed{\dfrac{x - x_0}{q} = t}$ and $\boxed{\dfrac{y - y_0}{m} = t}$

$\therefore x - x_0 = qt$ and $y - y_0 = mt$

$$\text{ie} \begin{cases} x = x_0 + qt \\ y = y_0 + mt \end{cases} \quad ....⑧$$

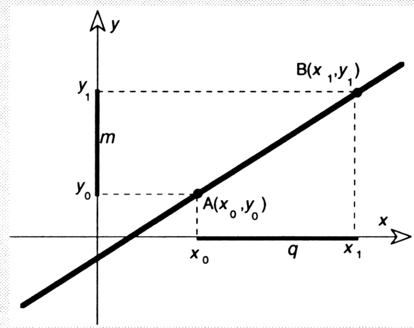


figure 7.1

Note that q and m are the projections on the x-axis and y-axis respectively.
That is, they represent the horizontal and vertical displacements **from** $A(x_0,y_0)$ **to** $B(x_1,y_1)$

In many physical situations it is often more convenient to express the horizontal and vertical displacements separately in terms of another variable. (t is often used as this auxiliary variable, as *time* is often the variable involved in many such problems).

The pair of equations in ⑧ is called a system of **parametric equations** for a line.

Note that for each *different point* (x,y) of this line the ratios $\dfrac{x - x_0}{q}$ and $\dfrac{y - y_0}{m}$,

although equal, form different pairs of values. That is t **is not** a constant but is a *variable*..

This auxiliary variable t is called a ***PARAMETER***

x and y are both separately expressed in terms of this variable t.. Here ⑧ is written in this form. Parametric equations are particularly suitable in describing the motion of a particle along a line in three-space. As with the symmetric form of a straight line, the generalisation from two-space ($\mathbb{R}^2$) to three-space ($\mathbb{R}^3$) or higher is very straight-forward.

It is common practice in Mathematics to express two related variables, say x and y, in terms of a third variable such as t or θ. Such functions are not confined to just linear functions.

The functions are written in the form $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ or $\begin{cases} x = f(\theta) \\ y = g(\theta) \end{cases}$

The equations are called *parametric equations* and the variables t, θ, are called *parameters*.
One example with which you should be familiar is the unit circle in trigonometry.

In this instance $x^2 + y^2 = 1$ where $\begin{cases} x = \cos(\theta) \\ y = \sin(\theta) \end{cases}$ and θ *is the parameter*

7.2 Entering equations in Parametric form

- Press **LIB**. Use ▼ ▲ to select the ApLet **Parametric** *figure 7.2*
- Press either START or **ENTER** to open up the Parametric ApLet.
- Use [shift] CLEAR DEL to clear any previously defined functions within this ApLet template.

You should now have a clear screen in the Parametric **SYMBolic view** *figure 7.3*

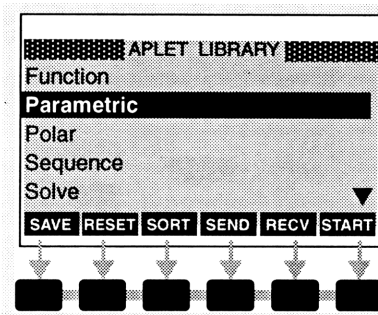


figure 7.2

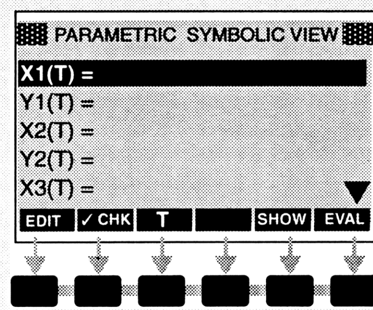


figure 7.3

On the HP 38G the parametric equations

$\begin{cases} x = \mathbf{f}(t) \\ y = \mathbf{g}(t) \end{cases}$ are expressed as $\begin{cases} \mathbf{X1(T)=} \\ \mathbf{Y1(T)=} \end{cases}$

The function must be entered in two parts.

You must input the definition for both parts *as functions of T.*

Highlight X1(T) then input f(*t*) **ENTER** then input Y1(T)

The brace on the left is a reminder that both components are necessary to define the function in parametric form.

> Use [shift] SETUP SYMB to set angle mode to Radians as trigonometric functions may be used in defining either X1(T) or Y1(T)
>
> Use the + key to scroll through the choices for Degrees - Radians - Grads

- Both X1(T) and Y1(T) are checked (✓)
  If you un-check one of these components the other is automatically unchecked.

- Up to ten Parametric equations can be entered into an ApLet

  $\begin{cases}X1(T)= \\ Y1(T)=\end{cases}$ through to $\begin{cases}X0(T)= \\ Y0(T)=\end{cases}$

- The independent variable is T.
  To insert T use either the screen menu key T or the X,T,θ key.

In the Parametric ApLet the X,T,θ key changes from entering *x* when it is pressed to entering T.
The screen menu key has also changed from *x* to T. ...... *figure 7.3*

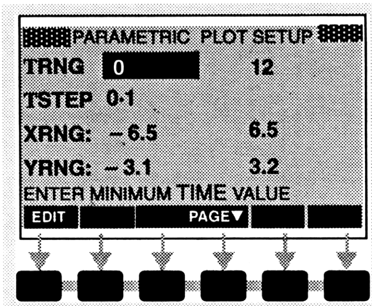


figure 7.4a

If your setup screen does not look like this, press CLEAR DEL while you are in the Parametric plot setup screen. This will change the settings to the **default values** as shown in *figure 7.4a*

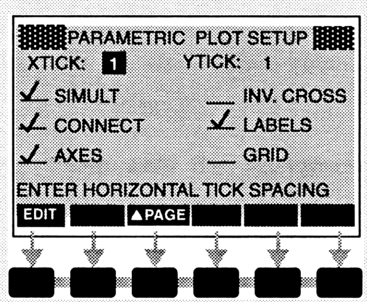


figure 7.4b

Parametric Plots and the screen setup.

- Input the following parametric equations:

  $\begin{cases}X1(T)= \cos(T) \\ Y1(T)= \sin(T)\end{cases}$

- Press SETUP PLOT to get to *figure 7.4a*
  If your setup screen does not look like this, press CLEAR DEL to get to the **default values** shown in *figures 7.4a*

- Press PLOT to get the graph in *figure 7.5*

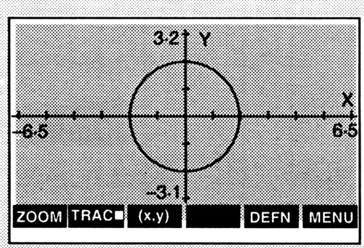


figure 7.5

7.3 Creating a simple Parametric ApLet

Input the following parametric equations and then save the set as an ApLet under the name **PARAM-ALGEBRAIC**

Function	T values	Range for x-Values	Range for y-values
2 X2(T)= T Y2(T)= 2	$-2 \le T \le 4$) Step 0·1	Default setting	Default setting
3 X3(T)= –3 Y3(T)= T	$-1 \le T \le 2$ Step 0·1	Default setting	Default setting
4 X4(T)= 3 + 4T Y4(T)= –2 + T	$-2 \le T \le 4$ Step 0·1	XRNG $-10 \le x \le 30$ X-Tick 5	YRNG $-12 \le y \le 12$ Y-Tick 5
5 X5(T)= 2T Y5(T)= $4T^2$	$-2 \le T \le 2$ Step 0·1	XRNG $-6 \le x \le 6$ X-Tick 2	YRNG $-5 \le y \le 20$ Y-Tick 5
6 X6(T)= 17+3T Y6(T)= –13+10T	$-2 \le T \le 4$ Step 0·1	XRNG $-5 \le x \le 40$ X-Tick 5	YRNG $-15 \le y \le 20$ Y-Tick 5
7 X7(T)= 7+7T Y7(T)= 17–2T	$-2 \le T \le 4$ Step 0·1	XRNG $-5 \le x \le 40$ X-Tick 5	YRNG $-15 \le y \le 20$ Y-Tick 5

The last three entries can be general linear equations or quadratic equations or a mix of either. These parametric equations can be used to investigate the impact of the values of A,B,C,D. In the Symbolic view highlight X8(T) then press [EDIT] Delete A and B and replace them with number values. Repeat this for Y8(T). By using the EDIT facility a whole class of parametric functions can be investigated within the one ApLet. Replace T with other functions of T.

	TRNG & STEP	XRNG & X-Tick	YRNG &Y-Tick
X8(T)= A+BT **Y8(T)= C+DT**	set to suit	set to suit	set to suit

7.4 Using a Parametric ApLet to solve problems.

Problem 1: *From a harbour H the position and velocity of two ships A and B are noted.*

*The position vector of A from H is (7**i** + 17**j**) and its velocity is (7**i** – 2**j**) kmh^{-1}.*

*The position vector of B from H is (17**i** – 13**j**) and its velocity is (3**i**+10**j**) kmh^{-1}.*

Assuming that the ships maintain their respective courses:

(i) For each ship, give the parametric equations that describes their course.

(ii) Determine whether a change in course will be needed to avoid a collision, and give the time frame within which such a course change will be necessary.

SOLUTION

Let $\mathbf{r}_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ be the postion vector of ship A at time T hours.

Then $\mathbf{r}_1 = (7\mathbf{i} + 17\mathbf{j}) + T(7\mathbf{i} - 2\mathbf{j})$

ie $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 7 \\ 17 \end{pmatrix} + T\begin{pmatrix} 7 \\ -2 \end{pmatrix}$

$\therefore \begin{cases} \mathbf{X1(T)= 7 + 7T} \\ \mathbf{Y1(T)= 17 - 2T} \end{cases}$

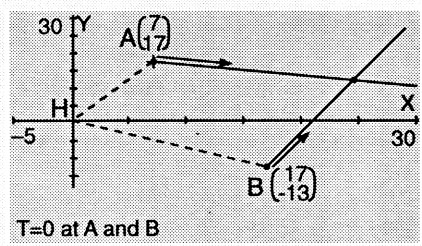


figure 7.6

Let $\mathbf{r}_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ be the postion vector of ship B at time T hours.

Then $\mathbf{r}_2 = (17\mathbf{i} - 13\mathbf{j}) + T(3\mathbf{i} + 10\mathbf{j})$

ie $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 17 \\ -13 \end{pmatrix} + T\begin{pmatrix} 3 \\ 10 \end{pmatrix}$

$\therefore \begin{cases} \mathbf{X2(T)= 17 + 3T} \\ \mathbf{Y2(T)= -13 + 10T} \end{cases}$

Go to the PLOT SETUP (Press SETUP PLOT)
For this work on Parametric Equations you should check the option SIMULT to enable the graphs to be drawn simultaneously. See figure 7.7b

The parametric plot on the HP38G provides a good model of the time and motion for this problem.

If there is going to be a collision then both A and B must be *at the same point for a given value of T.* ie 17 + 3T = 7 + 7T **AND** –13 + 10T = 17-2T. Here T= 2·5 satisfies both conditions, therefore collision will occur in 2·5 hours. One of the ships must change its course within 2·5 hours to avoid collision.

- Now do this problem on your calculator.
- Open a Parametric ApLet and input the parametric equations.

Note: These parametric equations have already been entered into the newly created ApLet **PARAM-ALGEBRAIC** (as Number 6 and 7 from the previous section). Either load this ApLet or open a new Parametric ApLet and enter these parametric equations.

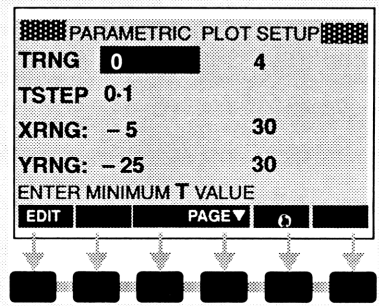


figure 7.7a

- Check the two sets of parametric equations (✓) and un-check any remaining equations in the ApLet
- Use SETUP PLOT to set up the display ranges as shown in *figures 7a & 7b*
- Press PLOT

  This means that any equations checked in the ApLet will be graphed simultaneously from the initial or starting value for T

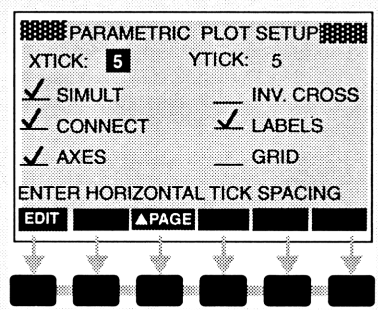


figure 7.7b

Note that SIMULT is checked.

If collision is suggested from the graph then this can be investigated and confirmed in the NUMeric view Table. (Press the NUM key), or use the Solve Aplet .

- Press [ ] VIEWS LIB and select Plot-Table

- You should get a split display with the left side showing a graph, and the right side a table of values.

  figure7.8a or figure 7.8b

- Use the ▼ ▲ keys to move from on parametric system to the other.

  The X1 and Y1 at the top of the table indicates which parametric equations you are tracing. As the cursor moves along the graph the table gives three items of information:

  The value of T......(Bottom of the table)

  The value of X1(T) and Y1(T) HIGHLIGHTED

- If you press the ◀ ▶ keys the cursor traces along the graph and the table gives the information about T, X1(T) and Y1(T)

- The ZOOM feature enables you to zoom in on the point of interest.

- The NUM key will give a fuller display of the table. In this fuller view you can quickly zero in on points of interest by entering a value for T. The table centres around this new value.

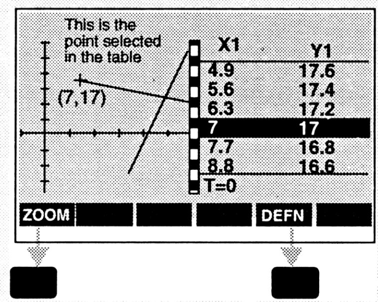


figure 7.8a

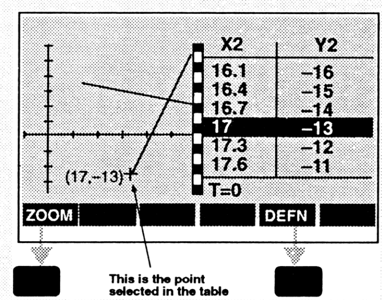


figure 7.8b

When you use the ZOOM feature the settings in [ ] SETUP NUM are changed. If you wish to redo the problem you will need to **reset the values.**

When you learn to create more advanced ApLets this feature can be designed into the ApLet.

Problem 2

In an 'Australian Rules' football game a player is awarded a 'free kick' 50 metres from the goal and directly in front of the goals. It is known that the player is a straight kicker. It is also known that if the ball is kicked below a height of 2·5 metres before it crosses the goal line the defence will prevent any goal being kicked. If the player taking the free kick kicks the ball with a velocity of 25m s^{-1} at an angle of 32° to the horizontal determine:
(i) The maximum height reached by the ball and the time taken to reach this height
(ii) If the player taking the kick scores with his kick.
(iii) The equation describing the flight path of the kicked ball.

Investigate the effect of kicking the ball at different angles and with different initial velocities. Summarise your conclusions.

Horizontal motion

u = 25cos32° m/s
s = *x* (horizontal displacement)
t = T the time interval
acceleration - None

Vertical Motion

u = 25sin32° m/s
s = h (vertical displacement)
v = 0 at maximum height
a = 9.8 m/s^2 ↑ or −9.8 m/s^2↓

Laws governing motion involving uniform acceleration

$s = ut + \frac{1}{2}at^2$ $\qquad s = \frac{(u+v)t}{2}$

$v = u + at$ $\qquad v^2 = u^2 + 2as$

- Open a Parametric ApLet and input the parametric equations.

$$\begin{cases} X1(T) = 25T\cos 32° \\ Y1(T) = 25T\sin 32° - 4{\cdot}9T^2 \end{cases}$$

and $\begin{cases} X2(T) = 50 \\ Y2(T) = 2{\cdot}5 - T \end{cases}$

- Check the two sets parametric equations (✓) and un-check any other equations in the ApLet

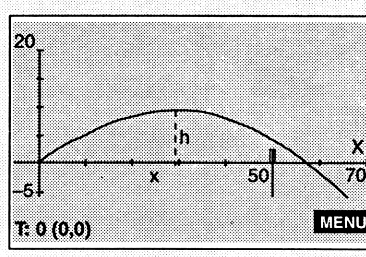


figure 7.9

The Parametric equations $\begin{cases} X2(T)=50 \\ Y2(T)= 2{\cdot}5 - T \end{cases}$ are used to draw a vertical line at x = 50 m with a height of 2.5 (for T > 0).
The graph can then show if the paths cross

- Use [SHIFT] SETUP PLOT to set up the display ranges as shown in *figures 7.10a & 7.10b*

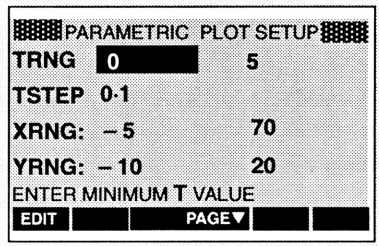


figure 7.10a

- Press PLOT
  The equations checked in the ApLet will be graphed simultaneously from the initial or starting value for T (Here T=0)

The graph suggests that a goal would be scored. This can be investigated and confirmed in the NUMeric view Table.

At the goal line when s = x = 50m
T≈ 2·358s; Height s = h = Y1 ≈ 3·99m
The horizontal distance travelled by the ball is when T ≈ 2·7, X1 ≈ 57·3 metres

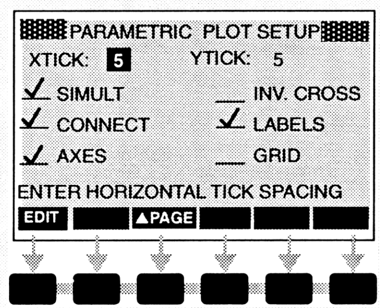


figure 7.10b

- Press [SHIFT] VIEWS LIB and select Plot-Table

You should get a split display with the left side showing a graph, and the right side a table of values. (see *figure7.8a or figure 7.8b*)

To investigate further press the NUM key.

Key in values for T to zoom in on points of interest.

The full solution to the problems posed are left for you to complete.

7.5 Parametric Equations involving Trig functions.

The following series of parametric equations are suggested as a basis for further investigation. They not intended as an exhaustive set. It is up to you to investigate and extend on any ideas, at your leisure. eg Vary the range of T; show why some like number 4 are parabolas with a special pattern of behaviour the distinguishes them from the similar algebraic function. (compare it with the parabola $y = 1 - 2x^2$; Are there any connections with Double angles? etc)

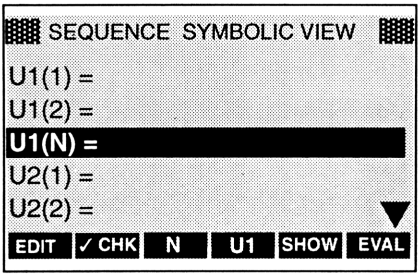
Parametric Equations		
1.	$\begin{cases} X1(T)= \sin T \\ Y1(T)= \sin(2T) \end{cases}$	Remember to change to Radian mode. Press [ ] SETUP SYMB
2	$\begin{cases} X2(T)= \cos^3 T \\ Y2(T)= \sin^3 T \end{cases}$	
3	$\begin{cases} X3(T)= \sin T \\ Y3(T)= \sin(3T) \end{cases}$	
4	$\begin{cases} X4(T)= \sin T \\ Y4(T)= \cos(2T) \end{cases}$	
5	$\begin{cases} X5(T)= \sin(2T) \\ Y5(T)= \sin(3T) \end{cases}$	
6	$\begin{cases} X6(T)= \sin(T) \\ Y6(T)=\cos(3T) \end{cases}$	**The ABC logo**
7	$\begin{cases} X7(T)= a\sin(bT) \\ Y7(T)= K\cos(dT) \end{cases}$	**Vary the values a,b,K,d**
8.	$\begin{cases} X8(T)= 6\cot(T) \\ Y8(T)= 6\sin^2(T) \end{cases}$	**The witch of Agnesi**
9.	$\begin{cases} X9(T)= A(T - \sin T) \\ Y9(T)= A(1 - \cos T) \end{cases}$	**(cycloid)** **Vary A**

You should keep a record of techniques and short cuts as you learn how to use the HP 38G. This page is left blank below this line for this purpose.

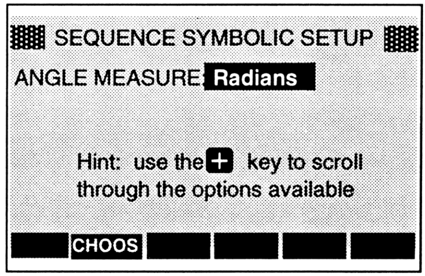
CHAPTER 8 SEQUENCES & SERIES

Screen views Associated with the Sequence ApLet

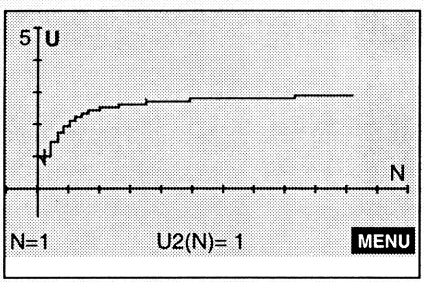
SYMBolic View Press LIB select SEQUENCE then press SYMB or ENTER
U1(1); U1(2); U1(N) define **one** sequence



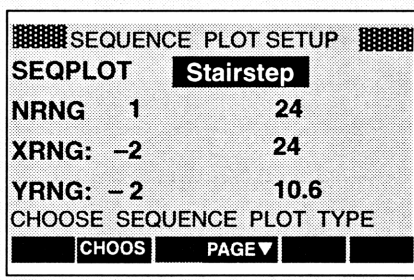
SYMBolic Setup Press SETUP SYMB



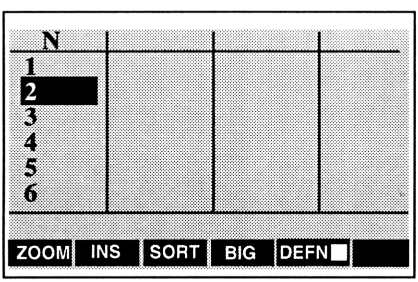
PLOT View Press PLOT



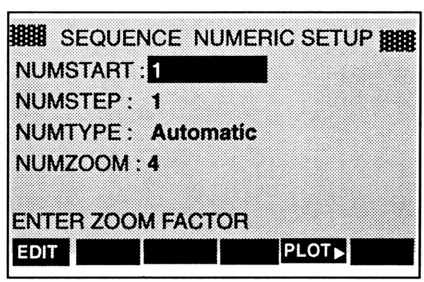
Default **PLOT** Setup Press SETUP PLOT



NUMeric or Table View Press NUM



NUMeric Setup Press SETUP NUM



8.1 A Comment on Sequences & Series

A sequence is a function that has for its domain the set of positive integers N.

$$N = \{ 1, 2, 3, 4, 5, \ \dots \ \}$$

Thus

N	1	2	3	4	5	6	7
Term N	**20·4**	**23·1**	**15·7**	**19·9**	**22·0**	**23·1**	**21·1**

is a sequence. The domain is a subset of the positive integers.

A convention adopted by mathematicians is to talk about

the sequence $\{a_n\}$ = *20·4, 23·1, 15·7, 19·9, 22·0, 23·1, 21·1*

where the domain is implied and taken to be a subset of the positive integers. The sequence described above was the maximum temperatures, in degrees Celsius, recorded in Perth over a one week period during winter. Sequences do not necessarily have a pattern to them nor do they need to have a defining rule. However most of the sequences you will be dealing with will have an underlying mathematical pattern from which the sequence can be generated.

In any sequence the general or n^{th} term is usually denoted by T_n or u_n.

T_3 or u_3 refers to the third term. u_{n+1} is the next term after the n^{th} term

u_{n-1} refers to the term immediately before the n^{th} term

u_{n-2} is the term immediately before the $(n-1)^{th}$ term, or two terms before u_n.

A Series is an indicted sum of the terms of a sequence.

A series is a *sequence of partial sums.*

For the series $T_1 + T_2 + T_3 + \dots T_n$ a sequence of partial sums can be formed

$\mathbf{S_1} = T_1$; $\mathbf{S_2} = T_1 + T_2$; $\mathbf{S_3} = T_1 + T_2 + T_3$; $\boxed{\mathbf{S_n} = T_1 + T_2 + T_3 + \dots T_n}$

$S_1, \ S_2, \ S_3, \ \dots \ S_n, \dots$ forms a *sequence of partial sums* called a **SERIES**

This can be written using the notation $\boxed{\mathbf{S_n} = \sum_{i=1}^{n} T_i = T_1 + T_2 + T_3 + \dots T_n}$

8.2 Entering sequences into the HP 38G

Sequences: the SYMBolic view SYMB

- Press LIB . then use ▼ ▲ to select the **Sequence** ApLet *figure 8.1*
- Press either START or ENTER to open up the **Sequence** ApLet.
- Use CLEAR DEL to clear any previously defined sequences.

For this section you must be in the **Sequence** ApLet.

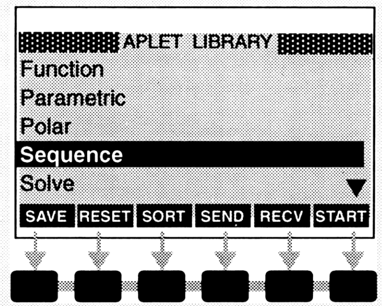


figure 8.1

You should now have a clear screen in the **SEQUENCE SYMBolic VIEW** *figure 8.2*

You could just over-type an existing sequence if need be. You do not need to start a new ApLet each time.

NOTE! If you wish to save any existing definitions press the SAVE screen menu key and save the work under a suitable name.

The HP 38G defines sequences using three components. For example, the sequence **{U1}** is defined by

U1(1)= **U1(2)=** **U1(N)=**	where	U1(1) is the first term U1(2) is the second term U1(N) is the n^{th} term

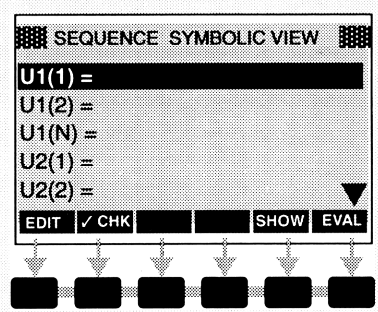


figure 8.2

You can input sequence U1 in one of two ways - either by

(i) entering the **General Term** U1(N) or

(ii) entering a **recursive formula**

8.3 Defining a sequence by entering the nth Term

Consider the sequence whose n^{th} or general term is given by $\mathbf{U_n = (2n + 3)}$

In the steps outlined below you should observe the changes that occur in the screen menus.

- Scroll past U1(1) and U1(2). *figure 8.3* Highlight U1(N) Notice the two screen menus that were added; **N** and **U1**
- In the EDIT line key in 2N + 3
- Press **ENTER** There will be a brief pause (⌛ appears at the top of the screen. This indicates that the calculator is busy computing.)
- Both U1(1) and U1(2) are calculated and inserted into the sequence, since your formula for the nth term has determined their value. A checkmark (✓)appears alongside **U1**(1), **U1**(2) and **U1**(N)

If you un-check one of these components the other two are automatically unchecked.

- Up to ten Sequences can be defined and entered into an ApLet in the form

U1(1)= **U1(2)=** **U1(N)=**	through to	**U0(1)=** **U0(2)=** **U0(N)=**

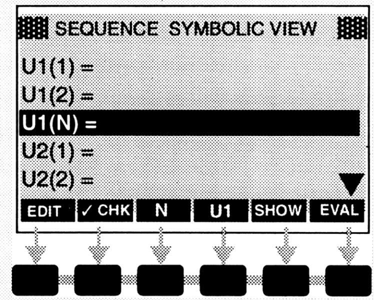


figure 8.3

Use **N** to put **N** into the EDIT line

U1 is used mainly to define U1(N) or T_n

Any checked sequences will be **graphed** simultaneously in the **PLOT** view.
Also
Any checked sequences will be **listed** simultaneously in the **NUM** view.

- The variable is N, a positive integer.

  To insert N you can use either the screen menu key N or the X.T.θ key or the alpha keys A...Z / N.

  In the Sequence ApLet the X.T.θ key changes from entering x when it is pressed to entering N.

  The screen menu key has also changed from X to N, *but only while the entry for U(N) is selected.* *figures 8.3 & 8.4*

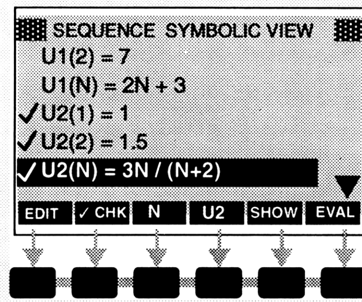


figure 8.4

Uncheck any other sequences – eg { UI }

- **Example:2** Select U2(N) and input the sequence:

U2(1)=
U2(2)=
$\mathbf{U2(N)=\dfrac{3N}{(N+2)}}$

  The first two terms will be automatically inserted when you press ENTER

Hint: Press [shift] CLEAR DEL while you are in the **Sequence plot setup** screen. This will change the settings to the **default values** as shown in *figure 8.5*

Uncheck sequence 1; Sequence 2 is already checked

Sequence PLOT View and [shift] SETUP PLOT .

Press [shift] SETUP PLOT to get to *figure 8·5*

If your setup screen does not look like this, press [shift] CLEAR DEL to get to the **default values** as shown in *figures 8.5*

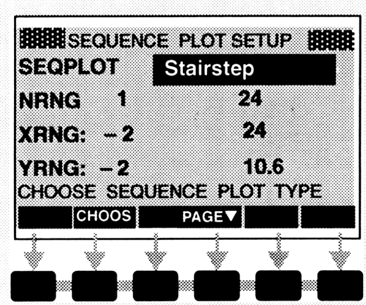


figure 8.5

- Change the **PLOT** setup to that shown in *figures 8.6a & 8.6b* . Use [◄][►] to move to the next value, press [ENTER] to input new values. To get to *figure 8.6b* press [PAGE▼]

- Press [PLOT] to get the graph in *figure 8.7*

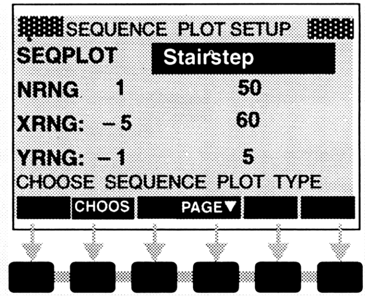


figure 8.6a

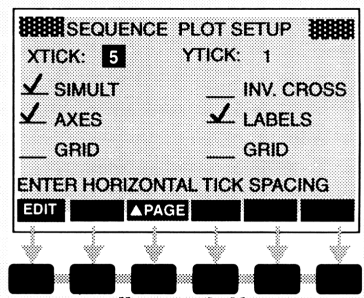


figure 8.6b

Some points to note about the PLOT

- Although the axes are drawn over the range $-5 \le X \le 60$ and $-1 \le Y \le 5$ the graph is drawn only for $1 \le N \le 50$ as this was the number of terms requested. (See NRNG in *figure 8.6a*)

- You can use the [◄][►] cursor keys to scroll the cross hairline through the sequence along the graph, and beyond. The value of both N and U(N) are given beneath the graph for the cursor position.

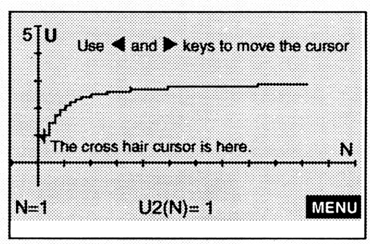


figure 8.7

- Press [MENU] Check what each of the menu keys [ZOOM] [TRAC■] [(x,y)] [DEFN] does

- To use the cursor *quick move feature* [ ] [►] or [ ] [◄] the trace key [TRAC■] must be *active*. Press [TRACE] to get this.

HINT Use [ ] [►] to move to the furthest point to the right on the graph. This is the maximum U(N) for the set values of N.

Use [ ] [◄] to get back to the leftmost term on the graph.

The NUMeric view NUM and SETUP NUM

- Press NUM to get the NUMeric (table) display as shown in *figure 8.8*

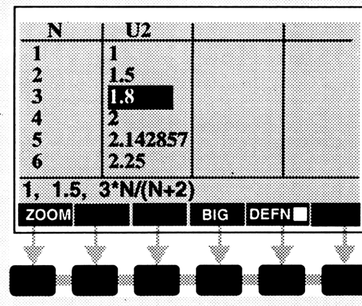


figure 8.8

- You can scroll through the sequence using the ▼ ▲ keys. Use the ◀ ▶ to move from one column to the next.

- If DEFN is active it will have a white square next to it. When this is active the defining rule for each sequence will be given at the bottom of the display as you scroll across the columns. *figure 8.8*

> If you scroll to values of N that are less than 1 the term value is given as **UNDEFINED**. This is because N must be a **positive integer**

- Scroll to the *leftmost* column. This is reserved for the values of N. Enter any integer - say 100. The whole NUMeric display re-centres around the 100th term. **You can only input values for N.**

> Press SETUP NUM to get the default values for the numeric display.

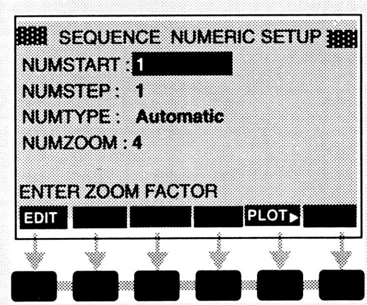


figure 8.9

- If the numbers are too small for you to read press BIG. You will get bigger numbers in the display but there will be a smaller set of values displayed. This is a toggle key so pressing BIG again will restore the smaller numerical values.

- If you wanted only every third term then in SETUP NUM (*figure 8.9*) set NUMSTEP to 3

Auto Scaling [VIEWS/LIB]

When a [PLOT] is done for a sequence it is sometimes difficult to get the scaling on the vertical axis right for the chosen domain X. If the plot is not scaled suitably, press

[VIEWS/LIB] Select **Auto Scale** from the options offered, then press [ENTER]. *figure 8.10*
The graph is redrawn with the vertical scale rescaled to display the graph for the chosen X values (as entered in figure 8.6a).
This action can be carried out while you are in the Plot view. Be prepared for very large values.

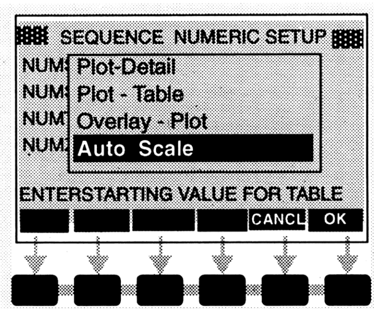


figure 8.10

Split view: Graph + Table [VIEWS/LIB]

- Sometimes it is useful and instructive to have **BOTH** the graph and the numeric display showing at the same time.

  Press [VIEWS/LIB] and select Plot - Table

- As you move the cursor along the graph its position is matched by the highlighted values in the Table.

  The quick move feature [ ] [▶] or [ ] [◀] is also available in this view.

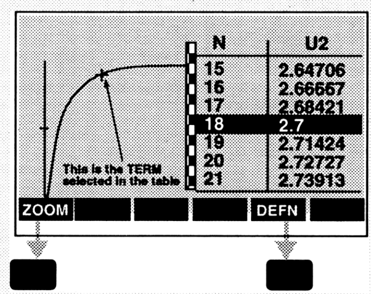


figure 8.11

If more than one sequence is plotted: use the [▼] [▲] keys to move from one sequence to the other. The table also changes to match the chosen sequence.

8.4 Working with Series.

Example 3:

The Sum to N terms of an Arithmetic Series.

- Press SYMB to get to the SYMBolic view.
- Scroll down to the sequence $\{U_3\}$ key in

U3(1)=5
U3(2)=5+7
U3(N)= Σ(J=1,N,U1(J))

Note the use of brackets and commas!

U1(J) refers to the sequence defined by **{U1}**

[Instead of U1(J) you could put **2*J+3**
The letter used for the running index must be used in defining the N^{th} term in the summation.]

Check sequences**{U1}**&**{U3}** and Uncheck **{U2}**

The NUMeric view NUM *figure 8.12*

In this view you can see both

$\mathbf{T_n}$, the n^{th} term of the *Sequence* {U1} and

$\mathbf{S_n}$, the n^{th} term of the Series {U3}

You can move across the columns, check the defining conditions for either sequence or, by moving to the leftmost column, enter a value for N.

For example put N = 100 ENTER *figure 8.13*
The on-screen table moves down to the terms that are centred around N=100.

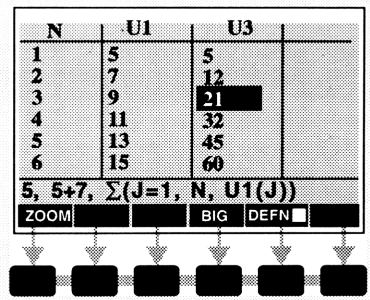


figure 8.12

In figure 8.12 the cursor is in the third column. This shows the sequence {U3}.

Since DEFN has been made active then the defining conditions for the sequence {U3} are stated at the bottom of the table.

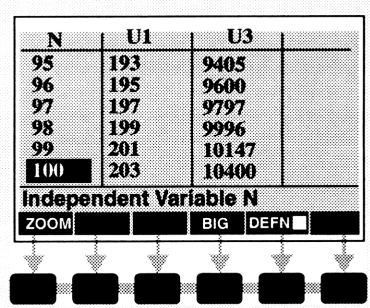


figure 8.13

This shows (i) $T_{100} = 203$ and

(ii) $S_{100} = 10\,400$

The PLOT view PLOT *figure 8.14*

- Press SETUP PLOT and set the default plot values by pressing CLEAR DEL *see figure 8.5*
- Press PLOT If the graph plot is not a very good one. Press VIEWS LIB and select Auto Scale. (see *figure 8.10*)

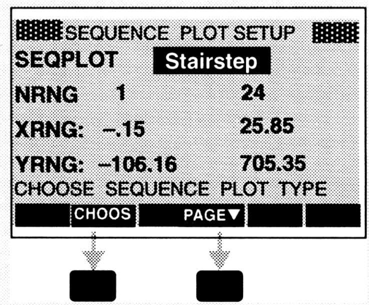


figure 8.14a

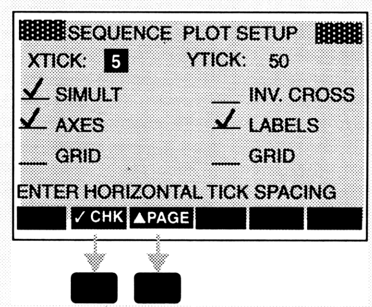


figure 8.14b

> ***As a matter of interest*** :
> Check the scaling that was carried out by the Auto Scaling process. To do this press SETUP PLOT

The thick vertical axis is due to the large number of tick marks. Change this to something like that suggested in *figure 8.14b*

The graph plot should be more distinct. The *nature* of the plots of $\mathbf{T_n}$ and $\mathbf{S_n}$ should be worthy of further investigation.

EXERCISE:

Plot ***both** the sequence* and *the series* associated with the following and determine what happens to both T_n and S_n as n gets larger (ie as n increases without bound).

In each case give the value of T_{50} and S_{50}

1. 12, 3, 0·75, ... $12 \times (0{\cdot}25)^{n-1}$, ...

2. 18, 15, 12, ... (15 – 3n) ...

3. 5, 10, 20, ... $5 \times 2^{n-1}$, ...

4. 4, 4·5, 5, 5·5, ... $\frac{n+7}{2}$, ...

8.5 Iterative processes.

What is an Iterative Process

- The set of instructions provided in the flow chart to the right (*figure 8.15)* produces a sequence of ten numbers.
- As each number is generated, it is used to produce the *next* number in the sequence.
- Such a *repetitive process* is called an iterative process, and the rule describing the repetition is referred to as a *recurrence relation.*
- In an iterative process, each *output* or answer becomes the *input* for the *next calculation.* The steps in the calculation are identical each time; all that differs are the value(s) used in each calculation Such processes are tedious and best done on a calculator or on a computer.

This process could have been *defined recursively* as

- $$\begin{cases} T_1 = 3 \\ T_{n+1} = T_n + 7 \end{cases}$$

- Here $T_{n+1} = T_n + 7$ gives the **recursive** relationship where each term is defined in terms of its predecessor. However on its own this does not give a specific sequence. Stating $T_1 = 3$ enables a *particular sequence* to be formed. The Arithmetic sequence :

$$\begin{cases} T_1 = 3 \\ T_{n+1} = T_n + 7 \end{cases}$$ is said to be *defined recursively.*

An Iterative Process

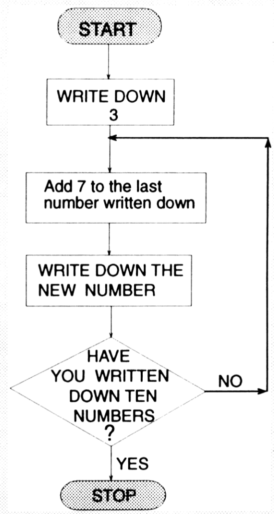


figure 8.15

To enter this sequence into the HP 38g using this iterative process, open the Sequence ApLet - SYMB viewand input–

U4(1)= 3
U4(2)= 10
U4(N)= U4(N−1) + 7

Use the Autoscale feature discussed in the previous section to plot the graph, then view it in the numeric mode.

8.6 Defining a sequence recursively.

Here is another example of a sequence defined *recursively* $\begin{cases} t_1 = 4 \\ t_n = \frac{1}{2} t_{n-1} + 3 \end{cases}$

In a recursive definition the rule relating a term to the term(s) immediately before it is given. Usually u_{n+1} is defined in terms of u_n **or** u_n is defined in terms of u_{n-1} All that is required one term of the sequence. The term usually given is the first term u_1. Once this starting value is given then the *recursive relationship* is applied and the process used to generate the sequence of results is referred to as an *iterative process.*

To enter a sequence defined recursively you must enter the *first two terms* as well as the recurrence relation. Note the difference between the sequences **{U1}** defined by $U_n = (2n + 3)$ and the sequences **{U5}** to **{U8}** as defined below. **{U1}** is defined by T_n while **{U5}** to **{U8}** are defined recursively.

- **Some examples:** Input the following four sequences into your HP 38G

1. **{U5}** $= \begin{cases} T_1 = 9 \\ T_2 = 12 \\ T_n = T_{n-1} + 3 \end{cases}$ enter this as

U5(1)= 9
U5(2)= 12
U5(N)= U5(N−1)+3

2. **{U6}** $= \begin{cases} T_1 = 3 \\ T_2 = 1\cdot5 \\ T_n = \frac{1}{2}T_{n-1} \end{cases}$ enter this as

U6(1)=3
U6(2)=1.5
U6(N)=U6(N−1)*(0.5)

3. {U7} $= \begin{cases} t_1 = 4 \\ t_n = 2t_{n-1} + 3 \end{cases}$ and {U8} $= \begin{cases} t_1 = 4 \\ t_n = \frac{1}{2} t_{n-1} + 3 \end{cases}$

For {U7} and U8} compare both the *sequence* and its associated *series* as n increases. Include a plot of the sequence *and* the series and determine t_{100} and S_{100} in each case.

Study each sequence & series in the PLOT view and in the NUM view.

8.7 Stairstep Plots and Cobweb Plots.

Up to this stage the graphs of the sequences have been *Stairstep* plots. In these plots a step function similar to the *Greatest Integer function* is drawn. Here the values of ($[\![N]\!]$,U_N) are graphed. Mathematically there is no vertical join from one term to the next. This is a limitation on the calculator's graphing capability and screen resolution.

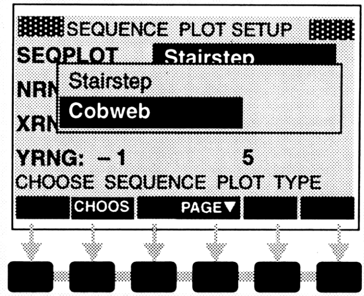


figure 8.16

The cobweb plot is useful when dealing with sequences that are defined recursively.

> Hint:
> With **SEQPLOT** Stairstep selected use the + key to scroll through the options offered.

- Press [SETUP PLOT] , See *figure 8·6a.* highlight Stairstep then press [CHOOS] .
- From the two choices offered for **SEQPLOT** choose Cobweb *figure 8.16*

  Press either [OK] or [ENTER]

Use [CLEAR DEL] to put the plot setup to its default values and then start experimenting. A Plot Setup is suggested for{U7} in *fig 8.17* You could use the Autoscale facility outlined in the previous section but be careful! The values can get extremely large. *figure 8.18*

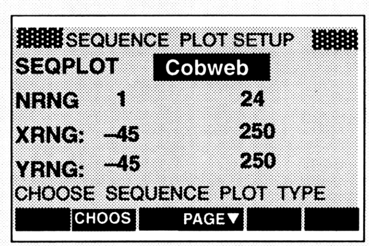


figure 8.17

- Compare the plot of the sequence with that of its associated series.
  The plot can be scrolled beyond the confines of the set axes.
- Now do the Cobweb plot for **{U8}**.
- It will soon be obvious that a rescale from figure 8.18 will be necessary Compare the Cobweb plots in *figures 8.18 and 8.19 b*

Note the scale on this plot!

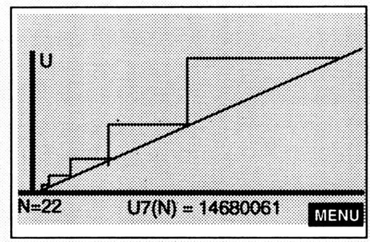


figure 8.18

Make one minor modification to {U8} so that

$$U8\} = \begin{cases} t_1 = 4 \\ t_n = \frac{1}{2} t_{n-1} \end{cases}$$

Study and explain the difference between *figures 8.19a and 8.19 c*

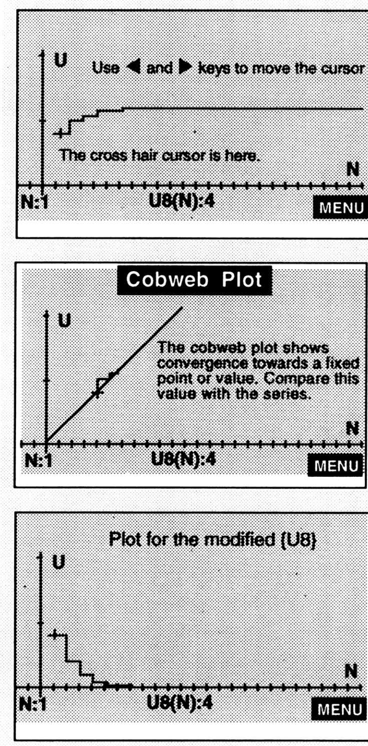


figure 8.19 a,b,c

Save the sequences studied to this stage in an ApLet under the name Sequences-SERIES1

I would suggest that any Title you use should keep the word **Sequence** at the front of it.

8.8 FIBONACCI NUMBERS

Some interesting properties. Can you Prove or Disprove them?

Use the NUMeric view to view the sequences. Press NUM

In each case test the assumption for several cases to see if the claim is possibly true. The HP 38G makes this straight forward. Try to prove the hypothesis. Several recursive definitions have been provided at the end of this section

1. No two adjacent Fibonacci numbers have any common factors (other than 1).

2. a. The ratio of $\frac{T_{n+1}}{T_n}$ approaches a fixed value. What is this value?

   b. The ratio of $\frac{T_n}{T_{n+1}}$ also approaches a fixed value. What is this value?

   c. Now consider the relationship between the answers to part **a** and part **b**.

The answer to (b) = $\frac{1}{\text{the answer to (a)}}$ ie $\frac{T_{n+1}}{T_n} = \frac{1}{\frac{T_n}{T_{n+1}}}$

3 a. The twelfth Fibonacci Number is $144 = 12^2$ Other than T_1 ($T_1 = 1$) are there any other Fibonacci numbers with this property?

3b. Are there any other terms in the Fibonacci sequence that are perfect squares?

4. Confirm the following observations / conjectures in the numeric mode:

   that every 3^{rd} Fibonacci Number is divisible by 2.

   that every 4^{th} Fibonacci Number is divisible by 3.

   that every 5^{th} Fibonacci Number is divisible by 5.

   that every 6^{th} Fibonacci Number is divisible by 8.

Investigate, extend and generalise this. (Hint: note the divisors!)

5. For *any three consecutive* Fibonacci numbers

[the square of the third – square of the first] is a Fibonacci number

eg. **5,** 8, **13** $13^2 - 5^2 = 169 - 25 = 144$

6. For any set of four consecutive Fibonacci numbers :

the 1st Number in the set = 2 x (the 3rd number) – (the 4th number)

eg **5** 8 **13** **21** 5 = (2 x 13) – 21

7. For any 4 consecutive Fibonacci numbers $(T_3)^2 - (T_2)^2 = T_1 \times T_4$

eg 5 8 13 21 $13^2 - 8^2 = 105$ also $5 \times 21 = 105$

8. The sum of any set of ten consecutive Fibonacci numbers is divisible by 11, and the result appears to be the seventh number in the sequence of the ten consecutive terms,

thus 1 + 1 + 2 + 3 + 5 + 8 + **13** + 21 + 34 + 55 = 143

and 143 ÷ 11 = 13 (the 7th term)

For the investigation into Fibonacci numbers the following recursive definitions could prove helpful for use with the HP 38G. Try different *seed values* for u_1 and u_2 in the second sequence {U2}. Which of the above conjectures still hold?

Which, if any, are altered by the use of different initial values?

How do the ratios mentioned in the properties (2 a,b,c) above change in the second sequence {U2}. Use the Numeric view to investigate the ratios.

1.
```
U1(1)=1
U1(2)=1
U1(N)=U1(N-1) + U1(N-2)
```

2.
```
U2(1)= 4
U2(2)= 7
U2(N)= U2(N-1) + U2(N-2)
```

3.
```
U3(1)= 1
U3(2)= 1
U3(N)= U1(N-1) / U1(N)
```

4.
```
U4(1)= 1
U4(2)= 1
U4(N)= U1(N) / U1(N-1)
```

8.9 The nth root of a Real number.

Heron's square root algorithm to obtain √m

$$A_{n+1} = \left[\frac{m}{A_n} + A_n\right] \div 2$$

where A_n is the initial estimate

Example: Use *Heron's Method* to determine the value of $\sqrt{29}$.

Step 1 Make a *rough* estimate of the value of $\sqrt{29}$; for example choose 5

Step 2 The First Approximation

$$A_1 = \left[\frac{\text{number}}{\text{estimate}} + \text{estimate}\right] \div 2$$

Step 3 To get the next approximation A_2, repeat the process but this time, instead of using your first estimate, *use* A_1 *the answer just obtained.*

Step 4 To get the next approximation A_3 repeat the process but this time, instead of using A_1, *use* A_2 *the answer obtained* in the last calculation.

Step 5 To get the next approximation A_4 repeat the process but this time, instead of using A_2, *use* A_3 *the answer obtained* in the calculation just completed.

In the HOME SCREEN, store the initial estimate in A, then into the EDIT line put (29/A+A)/2 **STO ▶** A . Press **ENTER**

$$A_1 = \left[\frac{29}{5} + 5\right] \div 2$$

$= \mathbf{5{\cdot}4}$ press **ENTER**

$$A_2 = \left[\frac{29}{5{\cdot}4} + 5{\cdot}4\right] \div 2$$

$= \mathbf{5{\cdot}385185}$ press **ENTER**

$$A_3 = \left[\frac{29}{5{\cdot}385\ldots} + 5{\cdot}385\ldots\right] \div 2$$

$= \mathbf{5{\cdot}385164}$ press **ENTER**

$$A_4 = \left[\frac{29}{5{\cdot}385\ldots} + 5{\cdot}385\ldots\right] \div 2$$

$= \mathbf{5{\cdot}385164\ldots}$

One more iterationgives agreement to more than 12 decimal places

Since these last two iterations agree to 6 decimal places we can confidently state

$\sqrt{29} = 5{\cdot}3852$ *correct to 4 decimal places.*

This iterative process converges rapidly towards the correct answer even if your first estimate was an extremely crude one. (If you check the actual result for $\sqrt{29}$ on your calculator it can be seen that it took just the three iterations above to obtain $\sqrt{29}$ to nine decimal places.)

Heron's Method can be shown to be a special case of the Newton Raphson process.

To calculate the square root of any non negative number m.

The algorithm is $A_{n+1} = \left[\frac{m}{A_n} + A_n\right] \div 2$ where A_n is the initial estimate

Start with the equation $x^2 - m = 0$ (from which $x = \pm\sqrt{m}$)

To find the zeros of the function f(x)) solve $\begin{cases} f(x) = x^2 - m \\ f(x) = 0 \end{cases}$

Using the Newton-Raphson iteration $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

If x_0 is the initial estimate then

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = x_0 - \frac{x_0^2 - m}{2x_0}$$

$$= \frac{2x_0^2 - x_0^2 + m}{2x_0}$$

$$= \frac{x_0^2}{2x_0} + \frac{m}{2x_0}$$

$$= \frac{x_0}{2} + \frac{m}{2x_0}$$

$$\text{ie} \quad = \frac{1}{2}\left(x_0 + \frac{m}{x_0}\right)$$

This is Heron's original iterative procedure for the square root of a number!

This recursive definition can be entered into the HP 38G as follows:

U1(1)=m
U1(2)=m_0
U1(N)=(U1(N−1) + U1(1)/U1(N−1))/2

Where m is the number whose square root is required and m_0 is the initial estimate.

Select U1(N) and use SHOW to check the correctness of the formula you entered.

To calculate the cube root of any number m.

EXAMPLE **:** To find $\sqrt[3]{100}$

Solve $x^3 - 100 = 0$ ie. Find the zeros of $f(x) = x^3 - 100$

Let our first guess be x_0. Using Newton-Raphson $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

then $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$ ie. $x_1 = x_0 - \dfrac{x_0^3 - 100}{3x_0^2}$

$$x_1 = \frac{3x_0^3 - x_0^3 + 100}{3x_0^2}$$

From this you get the iterative algorithm $x_1 = \dfrac{1}{3}\left(2x_0 + \dfrac{100}{x_0^2}\right)$

Operation (For m > 0)	**Iterative process** (Recurrence relation)
1. The Square root of a number **m**	$x_n = \frac{1}{2}\left(x_{n-1} + \frac{m}{x_{n-1}}\right)$
2. The Cube root of a number **m**	$x_n = \frac{1}{3}\left(2x_{n-1} + \frac{m}{(x_{n-1})^2}\right)$
3. The fourth root of a number **m**	$x_n = \frac{1}{4}\left(3x_{n-1} + \frac{m}{(x_{n-1})^3}\right)$
General p^{th} root recurrence relation	$x_n = ?$

Is there a pattern? If so does it hold true for n ≥ 4? Check it out!

You should keep a record of techniques and short cuts as you learn how to use the HP 38G. This page is left blank below this line for this purpose.

CHAPTER 9

WORKING WITH LISTS

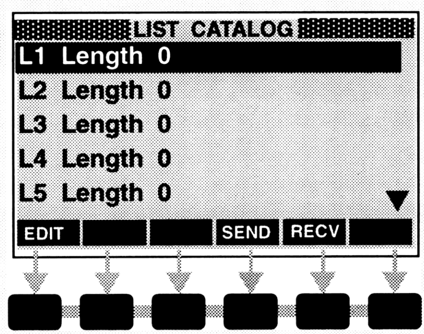
Creating, Storing & working with LISTS

Default Screen views associated with LISTS

Press [ ] LIST 7 to get this view.

In the default setting all lists are empty (ie Length 0)

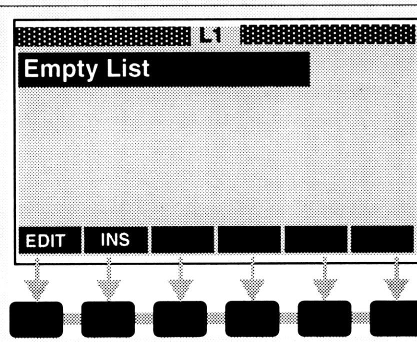
To clear all lists and start with this default setting press [ ] CLEAR DEL when you are in this screen view



To view any List, select the list then press EDIT to open the list.

The list can then be edited. This is the entry screen where you key in the elements for a List.

Press ENTER after each entry



9.1 Introducing Lists names and the conventions used.

The HP 38G permits you to store up to ten lists in the LIST CATALOG.
The lists are named **L1, L2, L3, L4, L5, L6, L7, L8, L9, L0**
The List naming convention does not permit the use of a single letter such as **L** for a list as the alpha characters have already been reserved for use as *Home Variables* and usually contain a stored numerical value. Lists **MUST** be named using **L1, L2 ... L0.**

***Home Variables* What are they?.** Go to the HOME SCREEN, input [L] then press ENTER You will get a number in the display. This number will usually be a zero unless you have previously assigned another value to this memory location. All twenty seven memories (each alpha character and θ) have the default value of 0. This can only be changed by assigning another value to the alpha character. To do this go to the HOME SCREEN, input the number to be stored, then store it.

Example: To store 43 in memory location L

- input 43, press STO▶ then A...Z 8 L.

  This process has now assigned 43 to the memory location L. This stored value can be used in calculations. *figure 9.1*
  For example input into the EDIT line [17 + 5L] ENTER. the display shows the answer 232. The calculation has been interpreted as [17 + 5 x L } where L has the value 43.

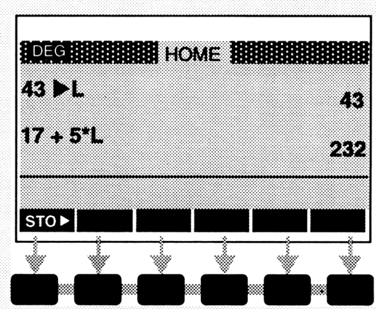


figure 9.1

Whenever the letter L is used in an input, whether in a calculation or in a program the number 43 will be assigned to L, *unless you change the stored value of L.* You can change stored values while you are in the SOLVE ApLet.

9.2 Creating LISTS in the List Catalog

Example 1: *To create a List* L1 *containing the first TEN prime numbers.* **2,3,5,7,...,29**

NOTE! Lists are **NOT** ApLets!

- Press [shift] [7] (LIST) to get the view in *figure 9.2*
  The list default is Length 0
  (ie all lists are empty at default settings)
- Since the list is to be stored as L1
  (i) select L1 as shown in *figure 9.2*
  (ii) press EDIT to open the list L1.

  This is the entry screen for inputting, or editing, the elements of the List. The "*Empty List*" message is a reminder that this list currently contains no members.
- Key in the elements of the list.
  2 ENTER ... 3 ENTER ... 5 ENTER etc
  Press ENTER **after each entry** *figure 9.3*
  If you make an error in an entry you can edit it before you press ENTER.
- When you have entered all elements of the list, press [shift] [7] (LIST) to get back to the List Catalog view *figure 9.2*
  Notice L1 now has a length of 10
- You can now input another list, or edit an existing list by repeating the above steps.

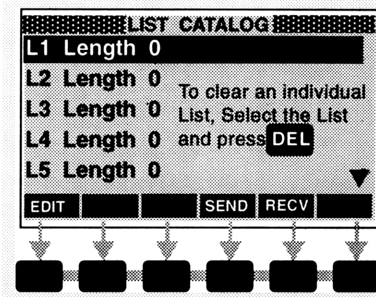


figure 9.2

If the **List Catalog** shows several unwanted Lists when you open it,
press [shift] DEL (CLEAR) to **clear all** the Lists.

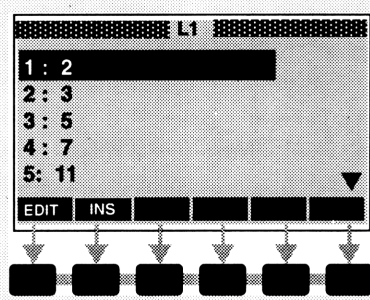


figure 9.3

To move out of this List Edit view
(i) press [shift] [7] (LIST) to get the view in *figure 9.2* **or**
(ii) press LIB to change to an ApLet **or**
(iii) press HOME

9.3 To Edit LISTS in the List Catalog

- To make corrections to an existing list or add elements to it, you must first load the list into the display screen. To EDIT, use the List Catalog (LIST 7), select the list to be edited, then press EDIT

- **To correct an error** in a list.
  Highlight the element containing the error, type in the correct element then press ENTER
  If the element to be corrected is long you could select the element then press EDIT This will copy the element into the EDIT line where you can carry out any necessary editing rather than retyping the whole element again. Press ENTER when the editing has been completed.

- **To insert a value** or element into an existing list,
  (i) highlight the term that is currently in the position where you wish to insert the new element,
  (ii) press INS first, then type in the value to be inserted.
  (iii) press ENTER. The elements of the list are automatically renumbered

- **To delete an element from the List:**
  Select the element to be deleted then press DEL

- **Viewing the contents of a List**
  To move down the List one screen at a time press ▶
  To move up the List one screen at a time press ◀
  To scroll up and down the list use ▼ ▲

9.4 Working with Lists in the HOME screen

- The entries/elements of a List can be
  - (i) A Real Number
  - (ii) A Complex Number
  - (iii) An expression

- Lists in the List Catalog can be viewed in the HOME SCREEN. To view list the L1 go to the HOME SCREEN, input L1 into the Edit line then press ENTER *figure 9.4*

  To view the whole List use the ▲ key to select the list, then press SHOW

  You can now scroll through the list using the ◀ ▶ keys. Press OK when done.

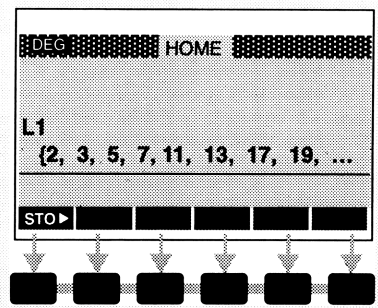


figure 9.4

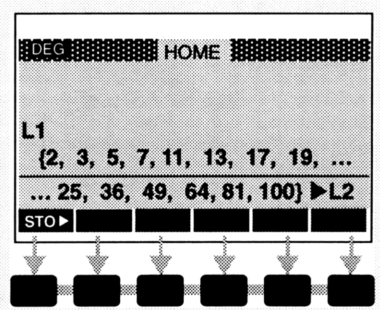


figure 9.5

- Lists can be input directly into the HOME SCREEN and stored in the List Catalog.

Example 2: *To create list L2 consisting of the first ten square numbers.*

- Press HOME to get to the HOME SCREEN input the following:

  {1, 4, 9, 16, 25, 36, 49, 64, 81,100}

*The braces {} **must** be included and each element of the List **must** be separated by a comma.*

- Press STO ▶ and type in L2 after the ▶ cursor appears in the EDIT line. *figure 9.5.*

Check the *List Catalog* - Press LIST 7
You will see that L2 contains the list entered in the HOME SCREEN.

LISTS stay stored in the List Catalog until you clear them or overwrite them.
If you store a list using the HOME screen as described here it will overwrite any previous list stored in the List Catalog in that location.

- Lists can also be **edited** from within the HOME SCREEN.

  Example 3: In the HOME SCREEN input {12, 18, 24, 35, 36} and store it as L6

- To Edit List L6 from the HOME SCREEN and change the 4th element from 35 to 30 key into the EDIT line:

3 0 STO▶ A...Z 8 L 6 (4)

ENTER

- Your EDIT line should look like that shown in *figure 9.6*

  This stores 30 in L6 as the 4^{th} element. It replaces the previous 4^{th} element 35 with the new value 30.

  To check the new list, type in **L6** ENTER. This should give the revised list L6 in the HOME SCREEN display.

- If you are in the HOME SCREEN and you need to know the 5th element of L2:

  Just key in L2(5) ENTER The value of the 5th element is given in the display. This applies to any lists in the List Catalog.

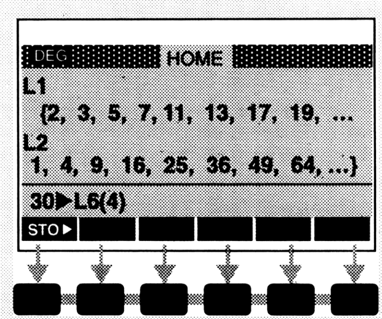


figure 9.6

In the HOME SCREEN

A...Z L2 ENTER will give a listing of List 2

A...Z L8(23) ENTER will give the 23^{rd} element in list 8

(returns a 0 if there is no 23rd term)

54 STO▶ A...Z L7(15) ENTER will **replace** the 15^{th} element in **list 7** with 54

9.5 Entering and Storing LISTS.

> Remember:
> Only **ten lists** can be stored , and accessed from, the List Catalog

- If you have worked through section 9.2 to 9.4 then the List Catalog should contain three Lists.

  [L1] *containing the first TEN prime numbers.* **2,3,5,7,...,29**

  [L2] *the first TEN square numbers.*

  {1, 4, 9, 16, 25, 36, 49, 64, 81,100}

  [L6] {12, 18, 24, 30, 36}

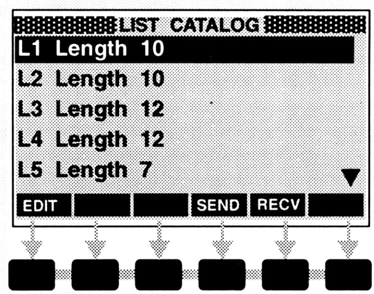


figure 9.7

- Delete L6. (Press [ ] [7] (LIST), select L6 in the List Catalog and press DEL)

- Store the following additional Lists into your calculator using either of the methods explained in sections 9.2 and 9.4.

> Since lists are not stored by names that would make it easy to remember what they contain, it would be a wise precaution to include a **notepad** note called **My Lists** in the Note Catalog where a title or Comment for each list is kept. Establish good calculator habits from the start.
>
> To write such a note press [ ] [1] (NOTEPAD)
> New note – Give it the name **My Lists**
> **L1** = set of first 10 prime numbers
> **L2** = The first ten square numbers.
> and so on

[L3] **The first 12 odd numbers**

{1, 3, 5, 7, 9, 11, 13, 15, 17,19, 21, 23}

[L4] **The first 10 even numbers**

{2, 4, 6, 8, 10, 12, 14, 16, 18, 20 }

[L5] **The first seven Cubic numbers**

{1, 8, 27, 64, 125, 216, 343}

[L6] **The first 12 Triangular numbers**

{1, 3, 6, 10, 15, 21, 28, 36, 45,55, 66, 78 }

Check the List Catalog - Press [ ] [7] (LIST)

> Remember!
> LISTS stay stored in the List Catalog until you clear them or overwrite them

9.6 Sending and Receiving LISTS.

In the List Catalog the two screen menus SEND and RECV are used to transfer selected lists between HP 38G calculators. Line up your calculator with another calculator so that the infra-red ports are facing one another. Align the small black triangles at the end of the name **Hewlett Packard™** above the display screen before any transmissions are attempted. *Figure 9.8*

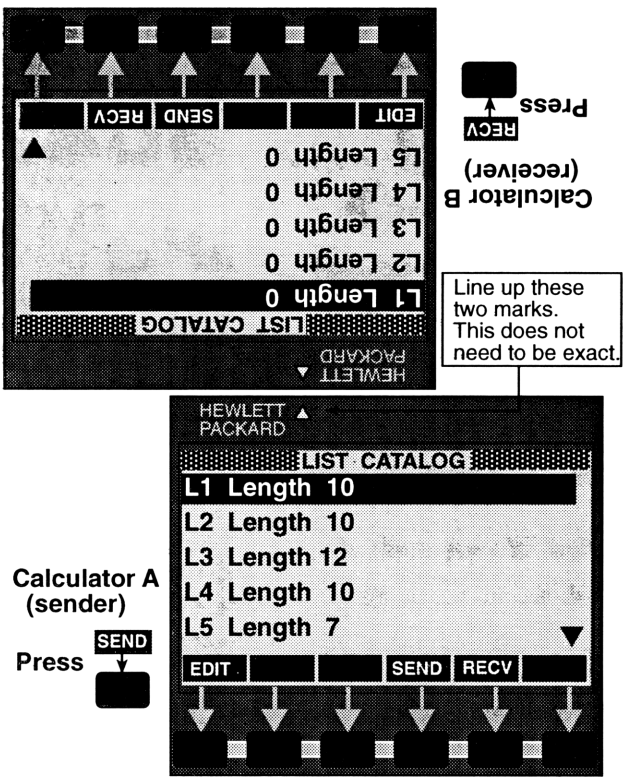


figure 9.8

To transmit **lists** both calculators must be in the **List Catalog** display.

9.7 Calculations & Operations using LISTS

Lists can be included in normal calculations in the HOME SCREEN Lists can also be added together, subtracted from each other, multiplied or divided, raised to a power etc.. However where such operations involve two or more lists, the lists must have the same number of elements.. (ie the lists must have the same dimension or length).

The length of lists can be checked by pressing [ ] [7] (LIST) to get the List Catalog.

Example 1:

L1 + 3*L1 ENTER

The result is given as
{8, 12, 20, 28, 44, 52, 68,... *figure 9.9*

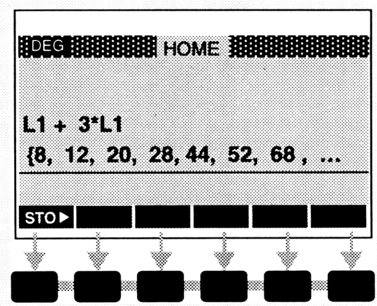


figure 9.9

- Use [▲] to highlight the resulting list which appears above the EDIT line.
- Two additional screen menus now appear. *figure 9.10*
- Press SHOW .
- Use [◀][▶] to scroll through the list given on the display screen.
- Press OK when you have finished.

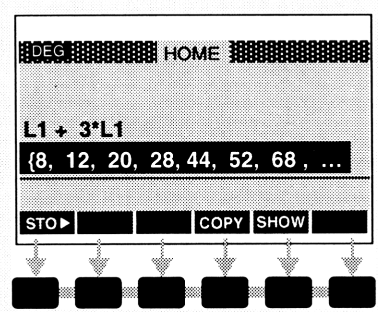


figure 9.10

Example 2: L1 + 2*L3 ENTER

L1 has 10 elements in it. (Length L1= 10)
L3 has 12 elements in it. (Length L3= 12)

- This operation gives the **Invalid Dimension** message shown in *figure 9.11*

- Press OK, then clear the EDIT line –

  (To do this use ON CANCEL).

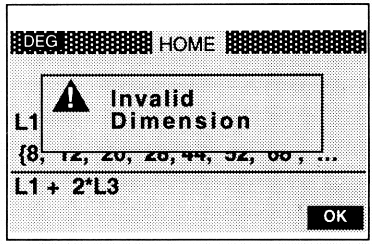


figure 9.11

The results of an operation on lists produces another List.
This new list can be stored under one of the List names L1, L2, ...L0

Example 3: –7*L4 ENTER

Returns **{–7, –28, –63, –112, –175,...}**

Example 4: L1^2 ENTER

Returns **{4, 9, 25, 49, 121, 169, ...}**

The results of any operation is itself a list
This list can be stored in the List Catalog.
The new list is formed by corresponding elements in each list acting upon each other according to the operation. *see below fig 9.12*

Example 5: L1*L4 ENTER *figure 9.12*

Returns **{2, 12, 45, 112, 275, 486, ...}**

To store this result in the List Catalog as L9

press STO ▶ A...Z 8 L 9 ENTER

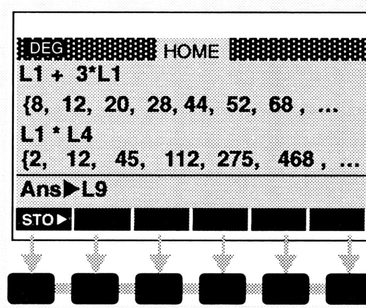


figure 9.12

L1 = { 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 }
L4 = { 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 }
L1*L4= {2, 12, 45, 112, 275, 468, ... 2900}

In L1*L4 the corresponding elements are multiplied together. This is one reason why the Lists must be the **same length**.
Experiment with L1^L1; L1^(2*L1) ; L2+5;...

Example 6: To add or subtract a constant

2*L4 – 8 [ENTER]

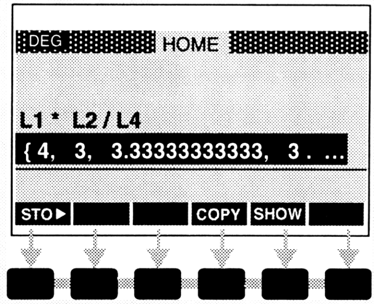


figure 9.13a

Example 7: L1 * L2 ÷ L4 [ENTER]

In example 7, if the number format is in standard form the answer is given as {4, 3, 3·33333333333, 3·5, 4·4, 4·333…}

To view this list use [▲] to highlight the list then press [SHOW]. The list in the display screen is no different.

It may be more informative, especially if you are looking for patterns, to put the number format in fraction form, *figure 9.13b*

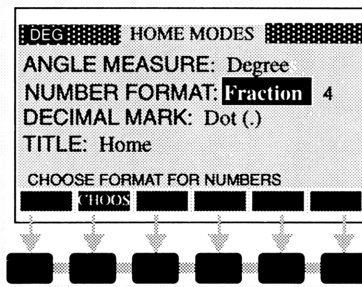


figure 9.13b

To do this:…With the list still highlighted, press [ ] [MODES / HOME], select [NUMBER FORMAT]

press [CHOOS] select [Fraction] [OK] [HOME]

When you now press [SHOW] the list display is in fraction form *figure 9.13c*

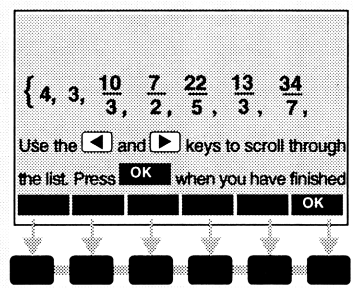


figure 9.13c

Example 8 Investigating Mersenne Primes.

Study L1 ^p – 1 [ENTER] where p is prime. Use the [▲] [COPY] feature to EDIT **p**.

HINT using [ ] [▶] will move to the extreme right of the list,

and [ ] [◀] will move to the extreme left

9.8 LISTS and Statistics

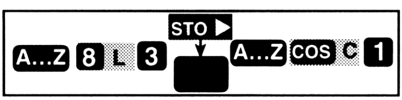
It is possible to transfer the data from a List into the Statistics ApLet environment. Using this facility it is then possible to obtain a quick statistical summary of the contents of a List. Section 9 will outline several features connected with lists on the HP 38G. One of these MAKELIST enables you to generate sequences of any desired length (within memory limitations) and enter them as a list into the List Catalog. Without attempting to fully outline the statistical feature, as a more in-depth treatment of Statistics will be developed in a later chapter, a simple example will suffice.

Press [ ] [7] (LIST), select [L3] (press DEL to clear any existing contents.).

Create List 3 (L3) containing 20 entries, by entering each element followed by ENTER,where L3 = {2, 3, 3, 3, 4, 5, 6, 6, 7, 7, 7, 7, 7, 8, 11, 11, 14, 14, 14, 14}

Press HOME when this has been completed.

In the HOME SCREEN input (L3 STO▶ C1) as



Press ENTER

figure 9.14

This places the contents of L3 into C1, the first data column of the Statistics ApLet.

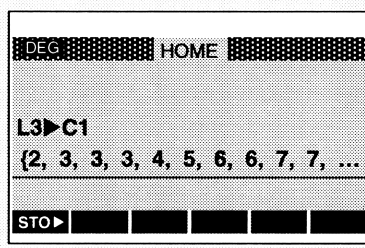


figure 9.14

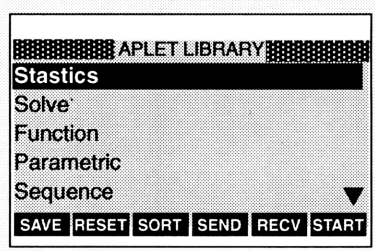


figure 9.15

To check this data transfer:

- Press LIB, select the Statistics ApLet, *figure 9.15,* then press ENTER
- This opens the Statistics ApLet environment.

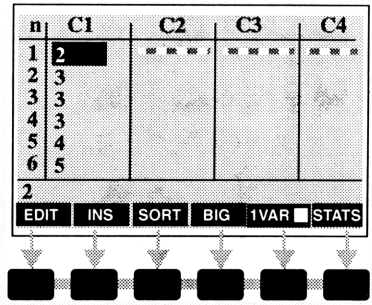


figure 9.16

- The default view when you press ENTER to load the Statistics ApLet is the NUMeric view. *figure 9.16*

  If this view does not appear press NUM to get to the NUMeric view.

If the screen menu 1VAR shows 2VAR press that screen menu key so that 1VAR is displayed.
(Its a toggle switch)

- Column 1 contains **n** the number of the element. Column 2 is labelled C1. It contains the list you have just transferred across to the statistics environment.
- List L3 *still exists* in the List Catalog. It has not been removed by the transfer of data.

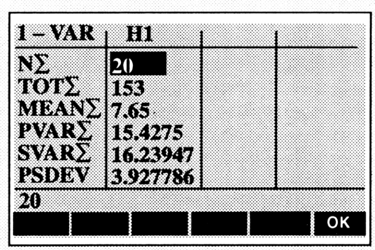


figure 9.17a

Press 

to display the statistics for the data in the selected column C1.
(L3 is now referred to as C1 in the statistics environment).

figures 9.17 a and 9.17b

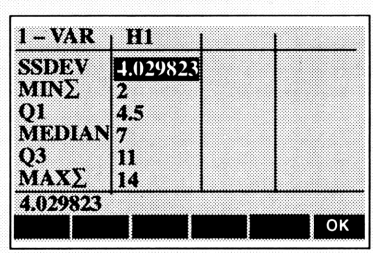


figure 9.17b

9.9 The MATH Menu of the List Functions.

A special set of **LIST** functions are available in the MATH Menu. These functions can be used in both the home screen calculations and in programs. Lists can be used as arguments with any of the normal operators + – * ÷ √ etc. Where more than one list is used in the arguments the lists must be of the same length. If ✪ indicates an operator then

List1 ✪ *List2* (lists must be of the same length)

forms a new list which pairs the values under the operation ✪

- Press the MATH key on the calculator keyboard.
- You should have a screen display that looks something like *figure 9.18*

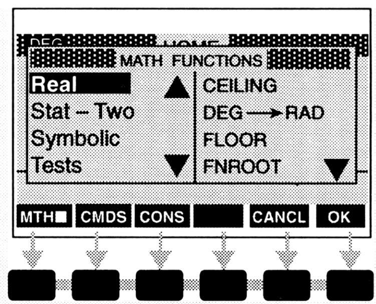


figure 9.18

HINT Press the key 8 L to move to the first menu item starting with L.

- Use the ▼ ▲ to scroll up and down the choices offered on the left side of the MATHS FUNCTIONS window. Notice that the menu choices on the right side change according to the function that you select on the left side. See the hint!
- Highlight the left side function called List *figure 9.19*

  With **List** selected use the ▶ key to move from the left side to the right side.

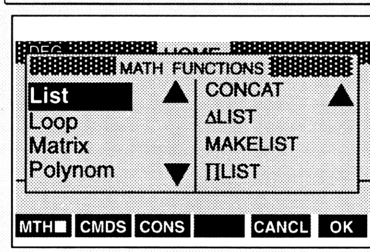


figure 9.19

Use the ◀ ▶ keys to move from the left to the right sides and back again.
The quick move HINT applies to both sides.

- A description of each of the sub–functions on the right side, associated with the LIST function is given below.

CONCAT

If L1 = {2,4,6,8} and L2 = {3,5,6,7,8,9} .

- Press **HOME** to go to the HOME SCREEN
- Press **MATH** and press **8 L** to get directly to the [List] Menu *figure 9.19*

Use [▶] to cross to the right side .

Select [CONCAT]

CONCAT (will appear in the EDIT line

You type in **L1,L2)** **ENTER**

- The result of this concatenation appears above the EDIT line as shown in *figure 9.20*

CONCAT Check the dictionary definition of **concatenate**

Syntax

CONCAT ([LIST1], [LIST2])

This function concatenates, connects (or chains together) the two lists into one list

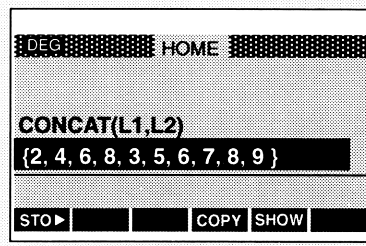


figure 9.20

ΔLIST

Example

Use the **ΔLIST** *function on the list of the Triangular numbers in* L6

L6={1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78}

- ΔLIST(L6) will form a new list of the first differences of the sequence of numbers in List 6.
- ΔLIST(Ans) will give a list of second differences,
- ΔLIST(Ans) will give a list of third differences and so on. *figure 9.21*

Use [▲] and copy ΔLIST(Ans) to the EDIT line and repeat as often as required to obtain second and higher differences.

Syntax

ΔLIST([**LIST1**])

Press **MATH** then press **8 L** to get directly to the [List] Menu *figure 9.19*

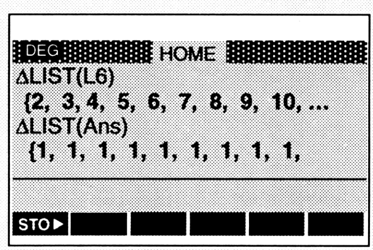


figure 9.21

MAKELIST is a useful function that allows Lists to be built.

The syntax is **MAKELIST(** [expression], [var name], [Start Val], [End Val], [step])

MAKELIST

- Press HOME to go to the HOME SCREEN
- Press MATH and press 8 L to get directly to the [List] Menu *figure 9.19*
- Use ▶ to cross to the right side . Select [**MAKELIST**] then press ENTER
- **MAKELIST(** appears in the EDIT line. Type in the formula to be used for generating the list. For the Triangular Numbers this is N/2(N+1)
- An explanation of the syntax

MAKELIST(N/2(N+1), N, 1,10,1)

N/2(N+1) is the *list generating formula*
N is the *variable*, with *starting value* **1**, *finishing value* **10**, going up *in steps of* **1**
A comma separator must be inserted at the end of each part as shown above.

- This will give the list *figure 6.22*
  {1, 3, 6, 10, 15, 21, 28, 36, 45, 55}

- **MAKELIST**(2N–1, N, 1,20,2)

- Will form a list 20 odd numbers starting at N = 1 going up to N=20 in steps of 2

This very useful feature could have been used at the start of this chapter to generate most of the lists stored in the List Catalog

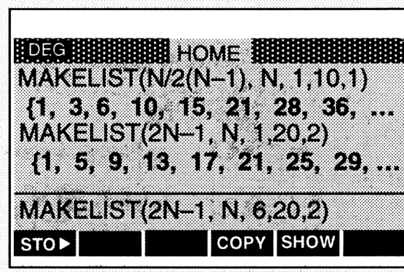


FIGURE 6.22

MAKELIST has been used to generate three separate lists in the above screen display.
Two of the outcomes are shown.
The third list is currently in the EDIT line waiting to be generated prior to your pressing ENTER
What will be the output of this MAKELIST?

Answer { 11, 15, 19, 23, 27, 31, 35, 39}

You can store any list generated into the LIST Catalogue as follows:

Press STO▶ , when [Ans ▶] appears in the EDIT line you type L3 (or any other available list name from L1, ...L0), then press ENTER
If L3 already contains a stored list this will overwrite the old list.

Other List functions available through the MATH menu, together with the associated syntax include:

Function	Syntax (Press ENTER at the end of each input)
ΠLIST	**ΠLIST(** LIST1 **)** **ΠLIST(L2)** will give the product of all the elements in List 2 If List 2 is as above then **ΠLIST(L2)** = 3 715 891 200
ΣLIST	**ΣLIST(** LIST1 **)** **ΣLIST(L2)** sums all the elements of in List 2. If List 2 is as above then ΣLIST(L2) = 110 ΣLIST({3,5,7}) returns 15 NOTE! It is not essential that you use a list from the List catalogue.
POS	**POS(** LIST, **any number** **)** POS(L1,23) returns **9**, indicating that 23 is the 9th element in the list of Primes L1 If the chosen number is not an element of the list a **0** is returned.
REVERSE	**REVERSE(** LIST **)** REVERSE(L3) lists the elements of List 3 in reverse order.
SIZE	**SIZE(** LIST **)** This will give the number of elements in a list. eg **SIZE**(L4) = 10
SORT	**SORT(** LIST **)** This will rearrange the elements of the list in ascending order. If you require the list in descending order, first use **SORT**(LIST) then follow this with **REVERSE**(LIST) on the answer.

Remember the quick-move short cut

–Press the key of the *first Letter* of the menu item.

You should keep a record of techniques and short cuts as you learn how to use the HP 38G. This page is left blank below this line for this purpose.

CHAPTER 10

VECTORS & MATRICES

Default Screen views associated with VECTORS

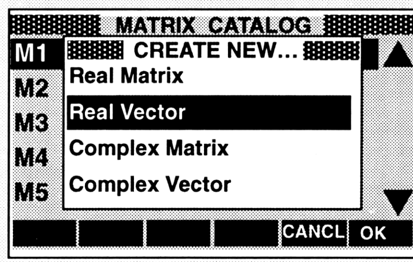
Press [MATRIX 4] to get this view.

The default setting has all ten matrices as **1 x 1** containing only one element **0**

To clear all matrices and start with this setting press [CLEAR DEL] in this view.

To create a **new vector** in the Matrix Catalog , first select the name (eg M1) then press [NEW] to get to this menu

then select your choice of Real vector

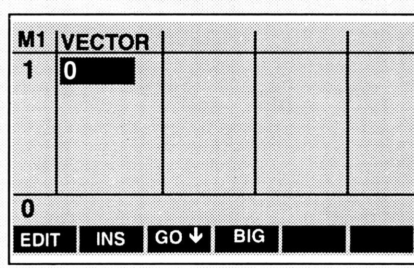


To edit, or view any vector, *select the vector* then press [EDIT] to open the vector. The vector can then be edited and new vectors input and stored.

This is the **entry screen** where you key in the elements for a vector. If you asked for a new vector as in figure 2 you will get this screen.

Press [ENTER] after each element entry.

You select your vector by highlighting its name (See in the top diagram).



Vectors – figures 1 -2 - 3

10.1 VECTORS and the conventions used.

The HP 38G permits you to store up to ten matrices or vectors in the MATRIX CATALOG.

The matrices are named **M1, M2, M3, M 4, M5, M6, M7, M8, M9, M0**
The Matrix naming convention does not permit the use of a single letter such as **M** for a matrix as the alpha characters have already been reserved for use as *Home Variables* and usually contain a stored Real Number value. This was explained earlier. Matrices **MUST** be named using **M1, M2 ... M0.**

In the HOME SCREEN, a Vector is input as a row matrix using square brackets.

eg In two dimensions ($\mathbb{R}^2$) a typical vector would be input as **[4,5]**

In three dimensions ($\mathbb{R}^3$) a typical vector would be input as **[3,2,5]**

Example 1: *To enter the vector* **[3,2,5,]** *while in the* HOME SCREEN *and then store it in the* Matrix Catalog *under the name* **M1**

- Press HOME to get to the HOME SCREEN
  Key in the elements of the vector.

Press ENTER. ......*figure 10.1*

- To store this in the Matrix Catalog as M1 press  .
- When **ANS▶** appears in the EDIT line type in **M1** after the ▶ symbol. *figure10.1*
- Press ENTER.
  This will store the vector [3,2,5] in M1.

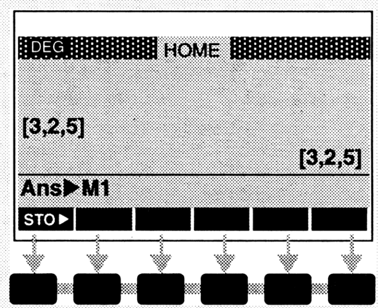


figure 10.1

> Alternatively: to directly store the vector M1 without pressing ENTER
> Input [3,2,5] in the EDIT line,
> Press  then
> type in M1 after the ▶ symbol

- To check this press MATRIX 4 to get to the Matrix Catalog view in. *figure 10.2*

  Notice that M1 is recorded as a **3-element real vector** .

  (ie it is a vector with three components, each of which is a Real Number).

- Select M1 if it is not already selected then press EDIT to open the contents of M1 for editing. You should have a screen like *figure 10.3*
- At this stage you can EDIT the vector. You can extend the vector by inputting additional components; or you can make corrections to existing elements.
- To move out of the EDIT screen as shown in figure 10.3 choose from

(i) LIB to move into an ApLet environment. **or**

(ii) HOME to enter another vector, or to carry on with calculations, **or**

(iii) Press MATRIX 4 to select another vector to EDIT in the Matrix Catalog

Press NEW to get the choice menu to enter a real vector. *figures 10.2 & 4*

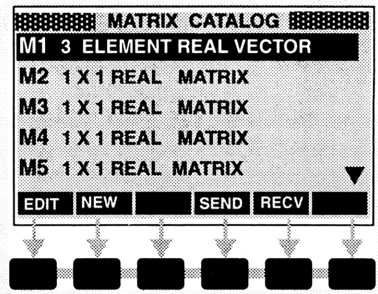


figure 10.2

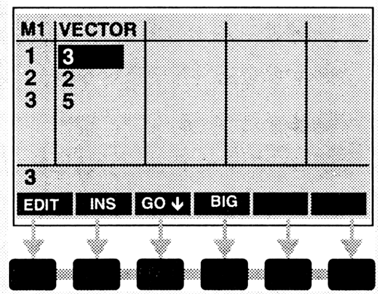


figure 10.3

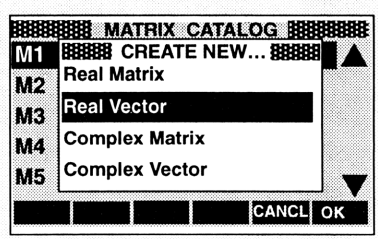


figure 10.4

Highlight the name of the new vector (Here it is M2) The **new** entry will clear any previous entries in M2 and open M2 with one element 0.

10.2 Entering vectors into the HP 38G.

- Enter the vector **M2**=[4,–5,7];
- The entry could be done in the HOME SCREEN as shown in section 10.1. **or**

  Press [ ] MATRIX 4 , select M2 in the Matrix Catalog, then press NEW to get the menu shown in *figure 10.4* . To enter a new vector select Real Vector ENTER

When figure 10.5 is displayed input each element followed by ENTER,

Figure 10.6 shows an attempt to enter the vector M2 into the Matrix Catalog using the EDIT key. As a consequence the input was accepted as a **matrix** and the word *vector* did not appear at the top of the column. You *must see the name vector* if you are entering a **vector**. You will not be able to combine the vectors M1 and M2 under the operations of vector addition or subtraction if M2 was entered as a Matrix as shown in *figures 10.6.*

If you attempt to do the operation M1 + M2 where M2 was entered as a matrix , you will get the error message:

Invalid Dimension

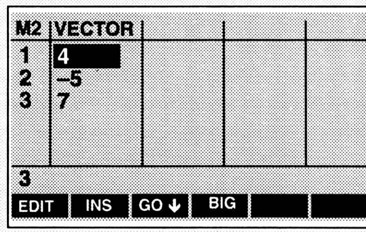


figure 10.5

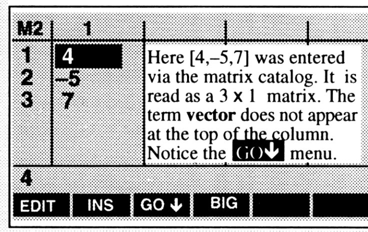


figure 10.6

CAREFUL! Warning!

If you enter **M2** = [4,–5,7] using [ ] MATRIX 4 select M2 press EDIT ,

GO ↓ 4 ENTER –5 ENTER 7 ENTER

This is NOT a vector.

This will give the 3 x 1 matrix M2,

You should enter your vectors in a consistent format either

(i) in the HOME SCREEN or

(ii) using the Matrix Catalog key NEW. This give the menu choice shown in *figure 10.4*

Now input the *vector* **M3**= [9,0,–1]

Press MATRIX 4 to see the catalog of vectors

10.3 Operations with vectors

(a) Vector Addition, subtraction, and Scalar Multiplication

Vector operations are usually carried out in the **HOME SCREEN**

Determine 5[4,–5,7] + 4 [9,0,–1]

- Press HOME to get to the HOME SCREEN
- Input into the EDIT line

  5[4,–5,7] + 4 [9,0,–1] ENTER
- The answer displayed is [56,–25,31]. This could be stored as M4 if you intend using the result in further calculations.

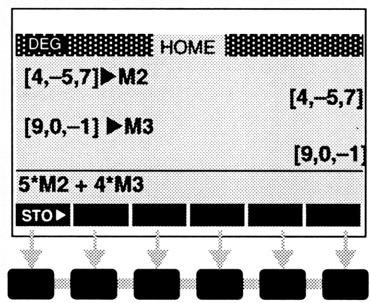


figure 10.7

If you have previously stored the vectors **M2**=[4,–5,7] and **M3**= [9,0,–1] you could have done the same calculation using

5M2+4M3 ENTER   *figure10.7* .

> If you get the error message
>
> Invalid Dimension
>
> check that the vectors were input correctly. Do not enter new vectors using the Matrix Catalog Editor

(b) The Magnitude of a vector.

The magnitude of vector **a** is denoted by |a|
On the HP 38G

$$|M1| = ABS(M1)$$

In the HOME SCREEN input ABS(M1) using

[SHIFT] [ABS –x] [(] [A...Z] [9 M] 1 [)] [ENTER]

This gives ABS(M1) ≈ 6·1644

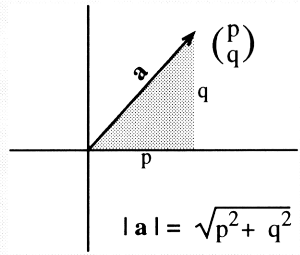


figure 10.8

(c) The DOT Product

*The dot product of two vectors **a** and **b** is defined to be*

$$a \cdot b = |a| \times |b| \times \cos\theta$$

where θ is the angle between the two vectors.

figure 10.9

$$a \cdot b = |a| \times |b| \times \cos\theta$$

The Dot Product of two vectors

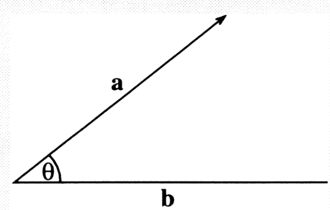


figure 10.9

To obtain the dot product of two vectors on the HP 38G.

The *syntax* for inputting the Dot Product is

DOT([Vector1],[Vector2])

Example Using the vectors

M1= [3,2,5] and **M2** = [4,–5,7]

determine the dot product **M1•M2**

- **Method 1** Hold down the [A...Z] and type in the word **DOT** The full input on the EDIT line should read

  DOT(M1,M2) then press [ENTER]

Note the brackets and the comma see figure 10.10

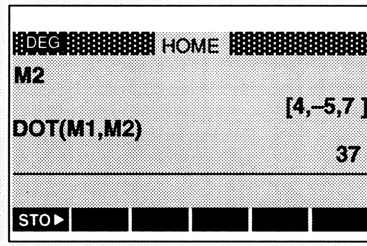


figure 10.10

You could input the vectors directly.
They do not have to be named from the Matrix Catalog.

eg **DOT**([3,2,5], [4,–5,7])

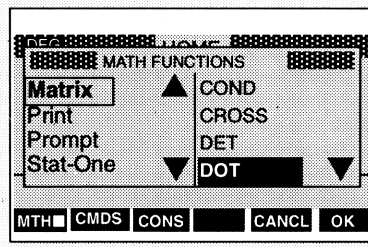


figure 10.11

- **Method 2** Using the [MATH] function key.

- Press [MATH] then *either* press [9 M], or scroll using [▼][▲] until the Matrix Menu appears highlighted on the left side. Use [▶] to move to the right side menu, scroll down to highlight **DOT** then press [ENTER] *figure 10.11*

- DOT(will be inserted into the EDIT line in the HOME SCREEN. (It may be quicker to type the word DOT) You now complete the entry

  DOT(**M1,M2**) [ENTER]

HINT When you are in the [MATH] function display screen (figure 10.11) you can move through the menus in one of three ways.

(i) Use the [▼][▲] keys to scroll up & down. Use [◀][▶] to move left - right

(ii) Use [ ][◀] or [ ] [▶] to move up and down one screen at a time and use [ ][▲] or [ ][▼] to move from the first to the last item in the menu list.

(iii) If you know the name of the function required press the alpha key containing the first letter of the function name

eg press [9 M] to go to **Matrix**

(d) The CROSS Product

- The procedure for obtaining the CROSS product of two vectors is similar to that for obtaining the DOT product.
  The only difference is that you either type **CROSS** into the EDIT line or select **CROSS** from the choices in *figure 10.11* then press **ENTER**

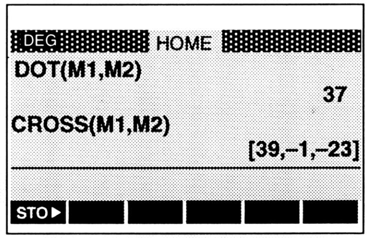


figure 10.12

The syntax for the CROSS PRODUCT is

CROSS(Vector1 , Vector2 **)**

A model of a three dimensional coordinate system would do much to enhance the understanding of the CROSS PRODUCT. Obtain the result of the cross product of two vectors in $\mathbb{R}^3$ then use straws and the model to demonstrate the three position vectors.

- **Example 3**

- **CROSS(M1,M2)** = [39,–1,–23]

see figure 10.12

The example that follows is not *the definitive way* of solving the problem posed. The chosen method was used to demonstrate the use of vector methods on the

HP 38G. There are several alternative ways of solving this problem. However *vector methods* do prove to be more effective in problems that could be difficult, especially in dimensions higher than $\mathbb{R}^2$. The ease by which the vector methods generalise from one dimension to the next make their use worthwhile.

Example 4

Given a triangle ABC where the position vectors of the vertices are given by

$\overrightarrow{OA}$[3,2,5]; $\overrightarrow{OB}$ {4,–5,7] *and* $\overrightarrow{OC}$[9,0,–1] .

Determine (i) *The length of each side.*

(ii) *The size of each angle*

(iii) *The area of the triangle ABC*

Press HOME and store the given matrices into the Matrix Catalog.

$\overrightarrow{OA}$[3,2,5] = M1; $\overrightarrow{OB}$ {4,–5,7] = M2 and $\overrightarrow{OC}$[9,0,–1] = M3

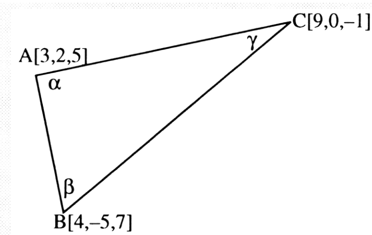


figure 10.13

- Magnitude $\overrightarrow{AC}$: Input into the EDIT line

  [shift] ABS -X (M3 – M1) ENTER 8·71779...

  Store this answer in B (ie STO▶ A...Z B)

- Magnitude $\overrightarrow{BC}$: Input into the EDIT line

  [shift] ABS -X (M3 – M2) ENTER 10·677...

  Store this answer in A (ie STO▶ A...Z A)

- Magnitude $\overrightarrow{BA}$: Input into the EDIT line

  [shift] ABS -X (M1 – M2) ENTER 7·348...

  Store this answer in C (ie STO▶ A...Z C)

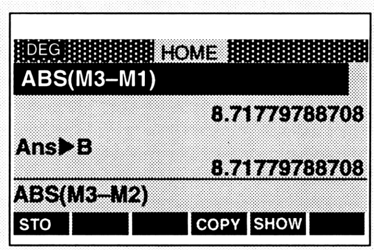


figure 10.14

HINT: Use the COPY feature to copy ABS(M3–M1) to the EDIT line then change the M1 to M2

(i) Magnitudes of the sides are

AC =8·71779... BC =10·677... AC =7·348...

By definition $\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos\beta$. $\therefore$ $\beta = \text{ACOS}\left(\frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}| \times |\mathbf{b}|}\right)$

To get angle β key in [ ] ACOS [COS] (DOT((M1 – M2),(M3-M2)) / (A*C)) [ENTER]

this gives β ≈ 54·106°. Use the sine rule to determine the other two angles. To determine the area several methods are open to you.

(i) **Heron's formula** Area = $\sqrt{s.(s-A)(s-B)(s-C)}$ where $s = \frac{A+B+C}{2}$

Store the value for s in memory location S, then input into the EDIT line

[√x] ([A...Z] S([A...Z] S – [A...Z] A)([A...Z] S – [A...Z] B)([A...Z] S [A...Z] C)) [ENTER]

(ii) Using the **SINE RULE** Area = $\frac{1}{2}$ A x C sinβ

Input into the EDIT line: 1/2* [A...Z] A* [A...Z] C* [SIN] β

(iii) Using the **CROSS PRODUCT**

Area = $\frac{1}{2}\,|\,\overrightarrow{BC} \times \overrightarrow{BA}\,|$

To use the cross product method input the following:

0·5* [ ] ABS [–X] (CROSS(M3–M2,M1–M2)) [ENTER]

This gives Area ≈31·780 units² *figure 10.15*

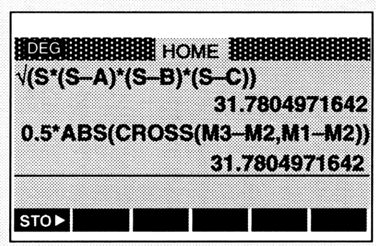


figure 10.15

The examples above are only a sample of some of the work on vectors that can be done on the HP 38G. You will discover many other techniques that can be used with vectors as you become more familiar with the calculator. In this respect the *note* facility is useful feature as you can keep notes filed on techniques as you develop them. When you create an ApLet, any notes are saved with the ApLet.

10.4 General Comment.

To distinguish between a vector and a Matrix on the HP 38G

Vectors are considered as one-dimensional arrays. They consist of n elements arranged either in 1 row or 1 column.

Consider the vector $M = \begin{pmatrix} 9 \\ 12 \\ 14 \end{pmatrix}$ or M = (9, 12, 14)

(i) To enter M1 in the HOME SCREEN as a ***Real Vector***

Input M1 into the EDIT line in square brackets in the form: [9,12,4] ENTER. This will automatically enter the array as a *vector*. To check this press

MATRIX 4. The Matrix Catalog should show that M1 is a 3 element vector.

Press EDIT to view and confirm the vector entry.

(ii) To enter *M1 as a vector* using the Matrix Catalog Press MATRIX 4 NEW

then select **Real Vector** and proceed as outlined in section 10·2.

Note! The GO screen menu key has only two choices in vector mode, GO or GO↓ The HP 38G will only permit vectors to be entered in the Matrix Catalog as *column vectors*. Enter the elements. Press ENTER after each element.

(iii) To enter M1 in the HOME SCREEN as a ***Real Matrix***

Input M1 into the EDIT line in square brackets in the form:[[9,12,4]] ENTER. This will enter the matrix as one row and three columns, as a 1 by 3 matrix). To enter a column matrix (3 by 1 matrix) input M1 in the form [[9],[12],[4]] The double set of square brackets is used to distinguish between the *vector* and the *matrix*.

(iv) To enter M1 as a Matrix using the Matrix Catolog proceed as in section 10.5

Matrices

Default Screen views associated with MATRICES

Press [ ] MATRIX [4] to get this view.

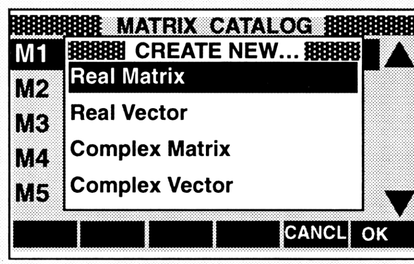
The default setting has all ten matrices as **1 x 1** containing only one element **0**

To clear all matrices and start with this setting press [ ] CLEAR [DEL] while you are in this screen view.

To create a **new matrix M1** in the Matrix Catalog , first select the matrix M1 then press NEW [ ] to get to this menu

Select Real matrix,

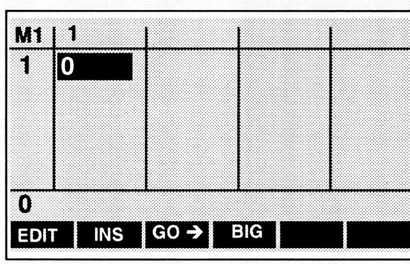
then press OK [ ] or ENTER



To edit, or view any matrix, *select the matrix* then press EDIT [ ] to open the matrix. The matrix can then be edited and **new** matrices input and stored. This is the **entry screen** where you key in the elements for a matrix.

Press ENTER after each element entry.

You select your matrix by highlighting its name (in the top diagram).



10.5 Entering & storing MATRICES

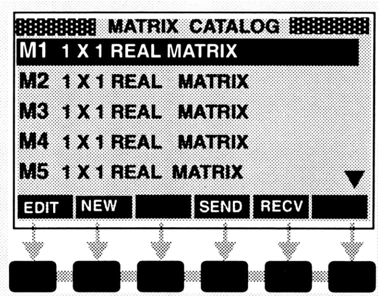
To enter and store a matrix using the Matrix Editor.

To open the Matrix Editor

- First open the Matrix Catalog.

  Press [ ] [MATRIX 4]

- Clear all existing entries press [ ] [CLEAR DEL]
  This resets the Matrix Catalog to the default setting.
- Select a matrix location (M1, M2,...M0) Here we have selected M1. *figure 10.16*



10.16

- To enter a new matrix press [NEW]

  The choice menu *figure 10.17* is displayed.

  Select Real Matrix then press [ENTER]

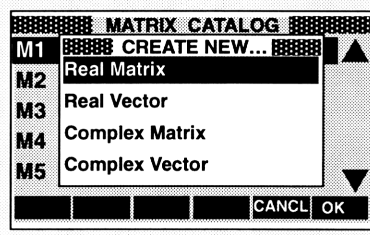


figure 10.17

- Your display should be as shown in *figure 10.18*

Notice the four screen menus at the bottom of the display screen. [BIG] is, as explained previously, a toggle switch used to make the display numbers bigger. [BIG ■] with the white square indicates that this feature is ON

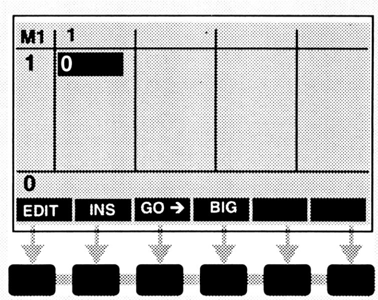


figure 10.18

Notice that the screen menu GO → has an arrow pointing to the right. This indicates the direction in which any elements that you key in will be entered into the matrix when you press ENTER.

A special note on the screen menu GO →. GO is a *toggle key*, that goes through a three stage cycle (As opposed to a two stage cycle for vectors.)

GO ↓ The entry bar automatically moves vertically **down** as you enter the elements of the matrix. The elements are entered by column.

GO → The entry bar automatically moves horizontally **to the right** as you enter the elements of the matrix. The elements are entered by rows.

GO without the directional arrow. The black entry bar must be located manually by the user. It will stay in the same location after ENTER is pressed.

Experiment by entering some elements. Try all three possibilities for the directional entry bar. When you are satisfied that you have mastered this press CLEAR DEL.
In response to the on-screen message

Clear matrix?

press YES see *figures 10.19 & 10.20*

Your display screen should now look like that shown in *figure 10.18*.

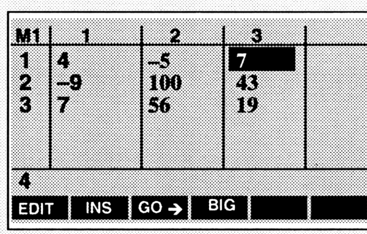


figure 10.19

Now enter the Matrix

$$M1 = \begin{pmatrix} 1 & 2 & 3 \\ -4 & -5 & -6 \\ 7 & 8 & 9 \end{pmatrix}$$

1 ENTER, 2 ENTER, 3 ENTER If you carry on and press 4 ENTER the entries will keep moving to the right.

After entering the third element 3 you must use the ▼ key to position the black entry bar at row 2 column 1.

Now complete the entries. The black entry bar will not proceed past the third column and will automatically move to row 3 column 1 for the 7 to be entered.

figure 10.21

When all the elements have been entered press [shift] MATRIX 4 to get back to the Matrix Catalog and press either

(i) EDIT to open a selected matrix to EDITED or

(ii) Select another matrix (say M2) and press NEW to enter a new matrix..

If you press keys that take you into a different environment, such as HOME , or LIB this will also EXIT the matrix entry screen.

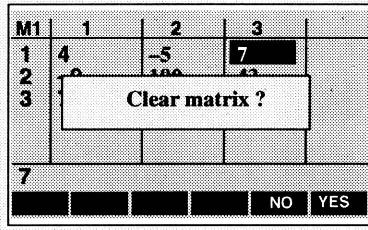


figure 10.20

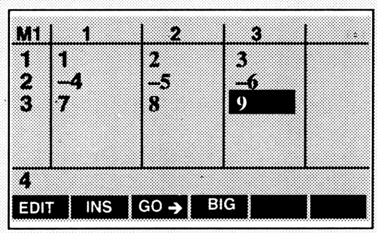


figure 10.21

Use the Matrix Editor to enter the following matrices in the storage location indicated.

The **2 x 3** Matrix **M2** = $\begin{pmatrix} 3 & -2 & -12 \\ -5 & 3 & -1 \end{pmatrix}$

The **2 x 3** Matrix **M1** = $\begin{pmatrix} 3 & -2 & 7 \\ 5 & 2 & 4 \end{pmatrix}$

This will overwrite the matrix that you had previously stored in M1.

The **3 x 4** Matrix **M3** = $\begin{pmatrix} 1 & 3 & -2 & 7 \\ 2 & -1 & -1 & 4 \\ 5 & -2 & 3 & -3 \end{pmatrix}$

The **2 x 2** Matrix **M5** = $\begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}$

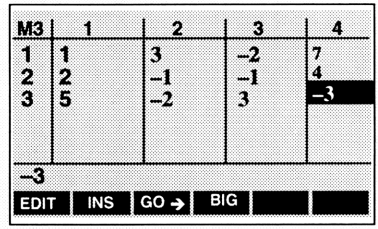


figure 10.22

To enter and store a matrix while in the HOME SCREEN.

In the HOME SCREEN a matrix is entered as a ***vector*** whose elements are ROW VECTORS.

- If M6 = $\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 7 & 9 & 8 \end{pmatrix}$ and if this matrix is to be input while you are in the HOME SCREEN proceed as follows
- Key in to the EDIT line

[[1,3,5], [2,4,6], [7,9,8]] STO ▶ A...Z 9 M 6

- Press ENTER when done. see *figure 10.23*

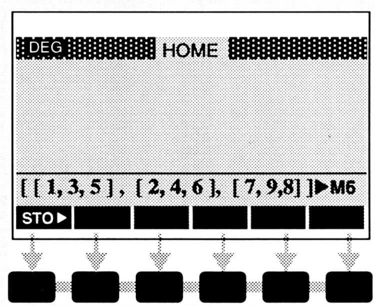


figure 10.23

Note the square brackets used for vectors.

10.6 To INSERT a row or a column into a matrix

Suppose Matrix M6 is to be altered to read

$$M6 = \begin{pmatrix} 1 & 3 & 5 \\ -3 & -2 & -4 \\ 2 & 4 & 6 \\ 7 & 9 & 8 \end{pmatrix}$$

Here the second row has been *inserted.*

There are two ways to do this insertion.

(i) To insert using the HOME SCREEN

Insert the cursor in the EDIT line immediately after the comma following the first row vector. Key in [–2, –3, –4], then press ENTER *figure 10.24*

Don't forget to store the new matrix in M6

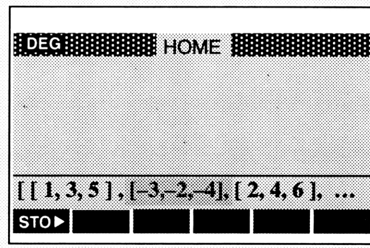


figure 10.24

(ii) To insert using the Matrix Editor

figures 10.25 & 10.26

Press [ ] MATRIX 4 to get to the Matrix Catalog. Select M6 then press EDIT to open the matrix. Place the black entry bar in the second row, then press INS

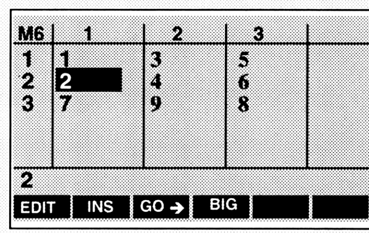


figure 10.25

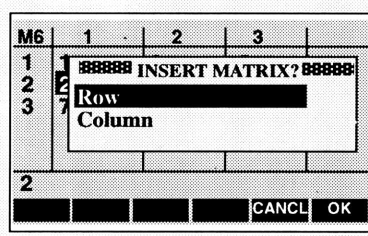


figure 10.26

In answer to the screen question select **Row** then press ENTER

A row of zeros is inserted. You now overtype the zeros with the new elements.

To delete a row select an element in that row and press DEL then follow the prompts.

10.7 Operations with matrices.

Operations with matrices are usually carried out in the HOME SCREEN.

(i) Addition, Subtraction & Scalar Multiplication

If two matrices are to be added or subtracted then the operation cannot be defined unless both matrices *have the same dimension.*

If $\mathbf{M1} = \begin{pmatrix} 3 & -2 & 7 \\ 5 & 2 & 4 \end{pmatrix}$ and $\mathbf{M2} = \begin{pmatrix} 3 & -2 & -12 \\ -5 & 3 & -1 \end{pmatrix}$

To add M1 and M2 the input into the EDIT line of the HOME SCREEN is

A...Z 9 M 1 + A...Z 9 M 2 ENTER

The result is displayed as [6,–4,–5], [0,5,3]

figure 10.27

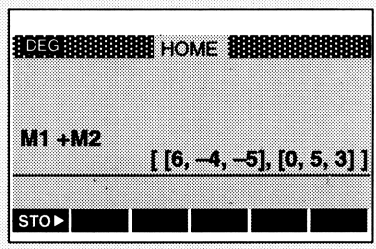


figure 10.27

$$\begin{pmatrix} 3 & -2 & 7 \\ 5 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 3 & -2 & -12 \\ -5 & 3 & -1 \end{pmatrix} = \begin{pmatrix} 6 & -4 & -5 \\ 0 & 5 & 3 \end{pmatrix}$$

Each row is written as a vector and is separated from the other rows by a comma. The whole matrix is then enclosed in square brackets.

If you attempt to add or subtract two matrices that do not have the same dimensions you will get the error message as shown in *figure 10.28*

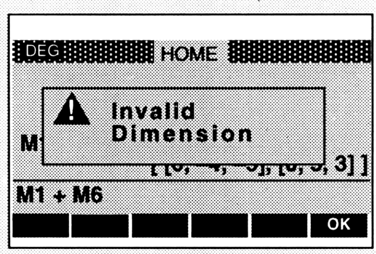


figure 10.28

Try each of the following:

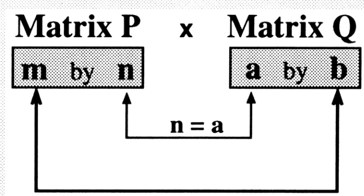
(i)	**5M1**	(ii)	**M1** ÷ 5
(iii)	**M1** $\times \frac{1}{5}$	(iv)	**M2** – **M1**
(v)	**M1** – **M2**	(vi)	4**M3** + 2**M4**
(vii)	5**M1** + 5**M2**	(viii)	5(**M1** + **M2**)

When doing operations with matrices it is often more convenient to first enter the matrices into the Matrix Catalog. The results can also be stored in this Catalog.

(ii) Matrix Multiplication

When two matrices are to be multiplied together there is a constraint on the dimensions. Given matrix P is an **m by n matrix** (m rows n columns) and matrix Q is an **a by b matrix** (a rows b columns)

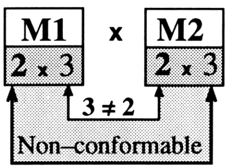
For matrix product P x Q to be defined **n** must have the same value as **a**



The result is an **m by b** matrix

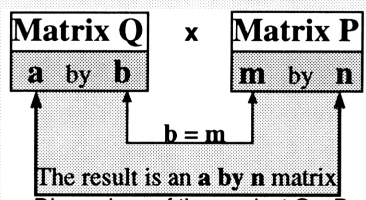
If the two inside dimensions n and a are not the same, the two matrices are said to be non–conformable

$$\mathbf{M1} = \begin{pmatrix} 3 & -2 & 7 \\ 5 & 2 & 4 \end{pmatrix} \text{and } \mathbf{M2} = \begin{pmatrix} 3 & -2 & -12 \\ -5 & 3 & -1 \end{pmatrix}$$



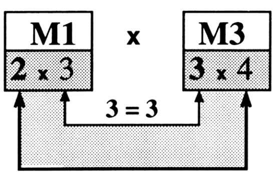
M1 x M2 cannot be evaluated.

M1 and M3 shown below *are* conformable



Dimensions of the product Q x P

$$\mathbf{M1} = \begin{pmatrix} 3 & -2 & 7 \\ 5 & 2 & 4 \end{pmatrix} \text{and } \mathbf{M3} = \begin{pmatrix} 1 & 3 & -2 & 7 \\ 2 & -1 & -1 & 4 \\ 5 & -2 & 3 & -3 \end{pmatrix}$$



Result is 2 x 4 Matrix

M1 * M3

$$= \begin{pmatrix} 3 & -2 & 7 \\ 5 & 2 & 4 \end{pmatrix} * \begin{pmatrix} 1 & 3 & -2 & 7 \\ 2 & -1 & -1 & 4 \\ 5 & -2 & 3 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 34 & -3 & 17 & -8 \\ 29 & 5 & 0 & 31 \end{pmatrix}$$

On your HP 38G:

In the EDIT line of the HOME SCREEN input

A...Z 9 M 1 * A...Z 9 M 3 ENTER

The result is given as

[[34, –3, 17, –8], [29, 5, 0, 31]]

(iii) The INVERSE of a Matrix

The inverse of a matrix A is denoted A^{-1} and is defined such that

$$A \times A^{-1} = I \quad \text{where I is the Identity Matrix}$$

The Identity Matrix is a square matrix and consequently both A and A^{-1} are also square matrices.

When working with Inverse Matrices it is helpful if the calculator number format is set to Fraction (Remember how to do this? Press [shift] [MODES / HOME] and change number format)

If $M6 = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 7 & 9 & 8 \end{pmatrix}$ to obtain $M6^{-1}$

- In the HOME SCREEN key in

  [A...Z] [9 M] 6 [shift] [x^{-1} / X,T,θ] [ENTER]

- The result appears as shown in *figure 10.29*
- This answer is made clearer if you use the [▲] cursor key to highlight the result then press the menu label key [SHOW]
- Try $M6 \times M6^{-1} = I$

  Can you explain this?.

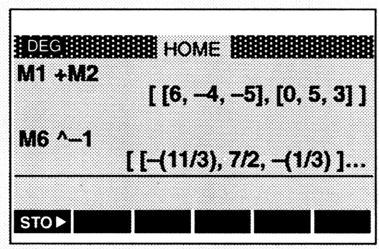


figure 10.29

You should learn to read numbers such as those that appear in $M6^{-1}$ and interpret them for what they attempt to convey. Thus 1/33333333... should be interpreted as 0

10.8 Solving Systems of Equations in $\mathbb{R}^2$ and $\mathbb{R}^3$

(i) Using the inverse of the coefficient matrix.

A system of equations in three variables can be expressed in (symbolic) algebraic form or in matrix form.

ALGEBRAIC FORM **MATRIX FORM**

$$\begin{cases} ax + by + cz = d \\ ex + fy + gz = h \\ kx + \ell y + mz = n \end{cases} \Rightarrow \begin{pmatrix} a & b & c \\ e & f & g \\ k & \ell & m \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} d \\ h \\ n \end{pmatrix}$$

$$\therefore \quad \begin{pmatrix} a & b & c \\ e & f & g \\ k & \ell & m \end{pmatrix}^{-1} \begin{pmatrix} a & b & c \\ e & f & g \\ k & \ell & m \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a & b & c \\ e & f & g \\ k & \ell & m \end{pmatrix}^{-1} \begin{pmatrix} d \\ h \\ n \end{pmatrix}$$

from which

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a & b & c \\ e & f & g \\ k & \ell & m \end{pmatrix}^{-1} \begin{pmatrix} d \\ h \\ n \end{pmatrix}$$

Example 1 *Using the idea outlined above use matrices to solve the system. Note the order of the matrices!*

$$\begin{cases} 2x + 5y - 3z = 2 \\ 3x - 2y + 4z = 1 \\ -5x - 3y + 2z = 3 \end{cases}$$

SOLUTION: Key the matrix of coefficients and the matrix of the constants into the matrix catalog.

The coefficient matrix **M1** $= \begin{pmatrix} 2 & 5 & -3 \\ 3 & -2 & 4 \\ -5 & -3 & 2 \end{pmatrix}$; the constants matrix **M2** $= \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

In the HOME SCREEN input | A...Z 9 M 1 [shift] x^{-1} X,T,θ * A...Z 9 M 2 ENTER |

This gives $M1^{-1}M2 = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$ This appears in the display as **[[–1]. [2], [2]]**

The solution: $x = -1$, $y = 2$, $z = 2$

The geometric interpretation is: *The system represents three planes that intersect in 1 point.*

Example 2: Solve the system

$$\begin{cases} x + 3y - 2z = 4 \\ 4x - y + z = -1 \\ 3x - 4y + 3z = -5 \end{cases}$$

- Use $\mathbf{M3} = \begin{pmatrix} 1 & 3 & -2 \\ 4 & -1 & 1 \\ 3 & -4 & 3 \end{pmatrix}$; $\mathbf{M4} = \begin{pmatrix} 4 \\ -1 \\ -5 \end{pmatrix}$

A...Z 9 M 3 [shift] x^{-1} X,T,θ A...Z 9 M 4 ENTER

displays the response **INFINITE RESULT**

Warning: This merely indicates that the solving process has involved division by zero at some stage. It is NOT stating that the solution set is infinite.

figure 10.30

Solution offered on HP 38G

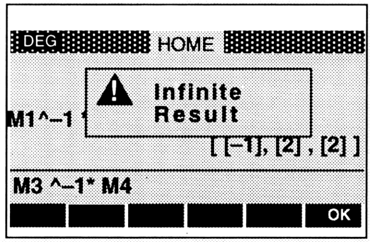


figure 10.30

The system given is actually equivalent to two non-parallel planes. (Notice that the third equation is the difference of the other two!) As such they intersect in a line and therefore there are an infinite number of solutions

---

Example 3: Solve the system

$$\begin{cases} 2x - y + z = 3 \\ 3x + 2y - z = 5 \\ x - 4y + 3z = -2 \end{cases}$$

- Use $\mathbf{M5} = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 1 & -4 & 3 \end{pmatrix}$; $\mathbf{M6} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$

A...Z 9 M 5 [shift] x^{-1} X,T,θ A...Z 9 M 6 ENTER

displays the response **INFINITE RESULT**

Warning: This merely indicates that the solving process has involved division by zero at some stage. It is NOT stating that the solution set is infinite

figure 10.31

Solution offered

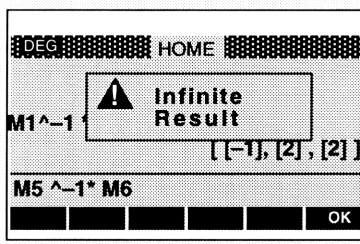


figure 10.31

The given system can be described as equivalent to three planes with no points common to all three planes. As such the system is said to be inconsistent. There are no solutions even though the display states **Infinite result.**

An alternative method which does not use the inverse of the coefficient matrix, and which avoids this problem of the message **Infinite Results,** is to find the reduced echelon form of the augmented matrix. This approach is more illuminating as it enables a more suitable interpretation of the answer to be made.

(ii) Reduced Row Echelon Form of the augmented matrix.

ALGEBRAIC FORM		AUGMENTED MATRIX	Row reduced Echelon Form
$\begin{cases} ax + by + cz = d \\ ex + fy + gz = h \\ kx + \ell y + mz = n \end{cases}$	$\Rightarrow$	$\left(\begin{array}{ccc\|c} a & b & c & d \\ e & f & g & h \\ k & \ell & m & n \end{array}\right)$	$\left(\begin{array}{ccc\|c} 1 & 0 & 0 & k_1 \\ 0 & 1 & 0 & k_2 \\ 0 & 0 & 1 & k_3 \end{array}\right)$

This method enables you to draw conclusions about the system being solved. The same three problems set out above will be repeated here using this method of row reducing the augmented matrix . The function used is named **RREF** The function **RREF** can be typed directly into the EDIT LINE of the HOME SCREEN. The syntax is

RREF(*Matrix name*)

This same function can be loaded into the EDIT LINE of the HOME SCREEN by using the MATH function key. Choose the Matrix menu

(HINT: *Moving quickly around the keyboard.* Press 9 M to get to the matrix sub-menu, then ▶ * R or scroll down to **RREF**, then press ENTER
This will input **RREF(** into the EDIT LINE of the HOME SCREEN without the need to type it.

Example 1.1 Solve $\begin{cases} 2x + 5y - 3z = 2 \\ 3x - 2y + 4z = 1 \\ -5x - 3y + 2z = 3 \end{cases}$

Alter M1, the coefficient matrix, by adding the extra column of constants.

$$\mathbf{M1} = \begin{pmatrix} 2 & 5 & -3 \\ 2 & 3 & -2 \\ 4 & 1 & -5 \\ -3 & 2 & 3 \end{pmatrix}$$

Input into the EDIT LINE of the HOME screen

RREF(M1) ENTER   *figure 10.32 & 10.33*

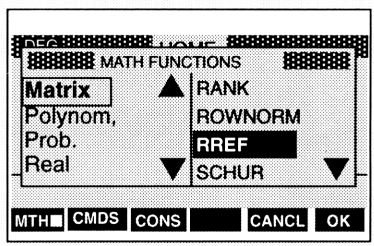


figure 10.32

The row reduced echelon form

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

Example 2.2 Solve $\begin{cases} x + 3y - 2z = 4 \\ 4x - y + z = -1 \\ 3x - 4y + 3z = -5 \end{cases}$

Use **M3** = $\begin{pmatrix} 1 & 3 & -2 & 4 \\ 4 & -1 & 1 & -1 \\ 3 & -4 & 3 & -2 \end{pmatrix}$;

Press [ ] MODES HOME and use the number format

FRACTION then press HOME

Input into the EDIT LINE of the HOME screen

RREF(M3) ENTER

The result **M0** = $\begin{pmatrix} 1 & 0 & \frac{1}{13} & \frac{1}{13} \\ 0 & 1 & -\frac{9}{13} & \frac{13}{10} \\ 0 & 0 & 0 & 0 \end{pmatrix}$

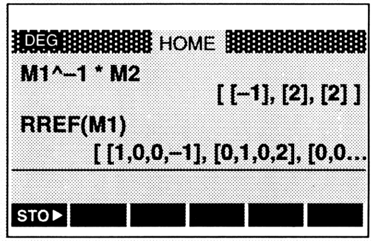


figure 10.33

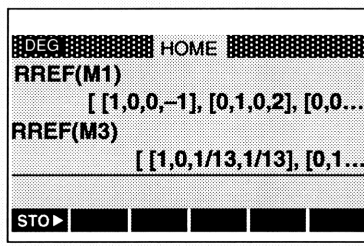


figure 10.34

This indicates that one equation was a linear combination of the other two and consequently the system really consists of two equations in three unknowns. There are many solutions. (These lie on the line of intersection of the two planes)

Example 3: Solve $\begin{cases} 2x - y + z = 3 \\ 3x + 2y - z = 5 \\ x - 4y + 3z = -2 \end{cases}$

Use **M5** = $\begin{pmatrix} 2 & -1 & 1 & 3 \\ 3 & 2 & -1 & 5 \\ 1 & -4 & 3 & -2 \end{pmatrix}$

Input into the EDIT LINE of the HOME screen

RREF(M5) ENTER

The result **M9** = $\begin{pmatrix} 1 & 0 & \frac{1}{7} & 0 \\ 0 & 1 & -\frac{5}{7} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ *figure 10.34*

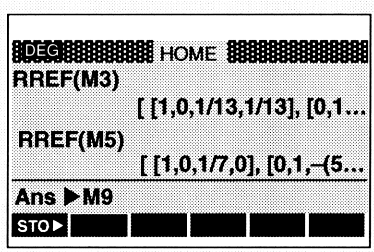


figure 10.34

The system is said to be ***inconsistent***. It has no solutions. (Three planes with no common points.)

The same rules will hold in $\mathbb{R}^2$ as in $\mathbb{R}^3$

10.9 Matrix functions available in the MATH menu.

Below is a brief summary of the main matrix functions of importance in a high school mathematics course. The HP38G User's Guide outlines many other matrix functions, some of which are not in these Mathematics Courses.

The MATRIX Functions in MATH

Matrix operations are divided into two sections.

Section 1 *Matrix functions* These are accessed through the MATH key *figure 10.35*

Section 2. *Matrix Commands* these are used in programming and are accessed using the screen menu CMDS■ in *figure10.35*

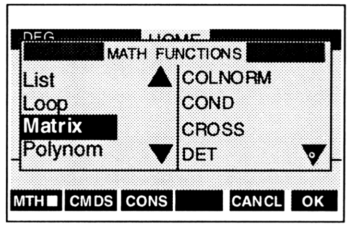


figure 10.35

Press the MATH key, then 9 M to get quickly to the menus beginning with the letter M

Use ▶ to move to the right side and scroll through these using the ▲ ▼ keys. When one of these submenu items is selected followed by ENTER the selected item or function appears in the EDIT LINE in the HOME SCREEN. You simply type in the necessary arguments or remaining parts of the syntax as outlined below.

Function	Syntax
CROSS	**CROSS(** Vector1 , Vector2)
	This give the ***cross product*** of two vectors. Remember that you enter vectors with the elements enclosed in square brackets with a comma used as a separator between the elements.

Function	Syntax
DET	**DET(** Matrix name) **DET** gives the ***determinant*** of a *square matrix*. eg **DET(M5)** where M5 is a matrix stored in the matrix catalogue.
DOT	**DOT(** Matrix1 name, Matrix2 name) Gives the ***dot product*** of two arrays eg **DOT(**Vector1, Vector2)
IDENMAT	**IDENMAT(** Positive integerN) N gives the size of the matrix IDENMAT(3) creates a 3 x 3 *Identity matrix* $I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
INVERSE	**INVERSE(** Matrix name) Creates the Inverse of a *square matrix* M, denoted as M^{-1} eg **INVERSE(M1)** ENTER. *or* Home screen input M1 [shift] x^{-1} (X,T,θ).
RREF	**RREF(** Matrix name) **RREF** gives the **R**educed **R**ow-**E**chelon **F**orm of matrix M Given the system $\begin{cases} 2x + 3y = 8 \ldots\ldots ① \\ 3x - y = 1 \ldots\ldots ② \end{cases}$ the augmented matrix is $\left[\begin{array}{cc:c} 2 & 3 & 8 \\ 3 & -1 & 1 \end{array}\right]$ The **RREF** = $\left[\begin{array}{cc:c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$
TRN	**TRN(**Matrix name) **TRN(M4)** will give the transpose of matrix M4

CHAPTER 11 COMPLEX NUMBERS

With complex numbers the following are available using the MATH key.

ARG is the argument θ of a Complex Number Z expressed as (r, θ)

CONJ The conjugate of a Complex Number Z=(a + b*i*) is $\overline{Z}$ = (a – b*i*)

IM The Imaginary part **b** of a Complex Number (a + **b***i*)

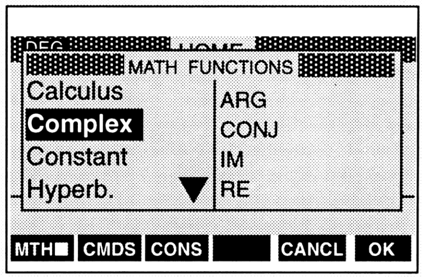
RE The Real part **a** of a Complex Number (**a** + b*i*)

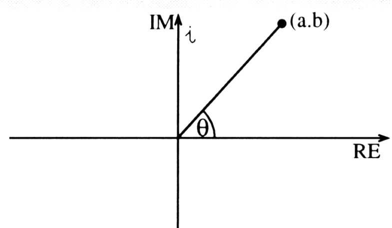
When you press ENTER the selection is placed in the EDIT line. You input the rest of the necessary information.

This screen was obtained by pressing the VAR key.
This gives a catalog of all the home variables that can be used in ApLets, calculations, programs and commands.

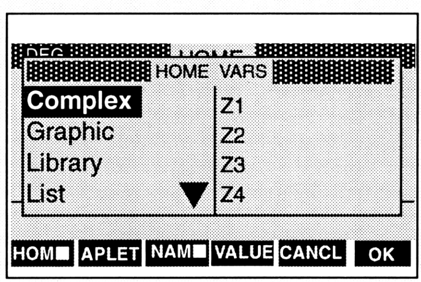
When you select a variable from the right side list and then press ENTER the selection is placed in the EDIT line.

Press MATH key to get to this screen.
Use the ◀ ▶ ▼ ▲ to make your selection.





Arg Z = θ
Modulus Z = ABS(Z) = | **Z** | = $\sqrt{a^2 + b^2}$



11.1 How to enter a Complex Number

A complex number of the form a + bi can be input into the HP 38G either

(i) Directly, in the form a + bi or

(ii) As an ordered pair (a,b)

Example 1. To enter the complex number

$Z = 3 - 5i$

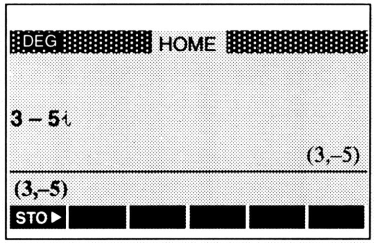


figure 11.1

- Press HOME to get to the HOME SCREEN

  (i) Key in 3 – 5i using

This displays (3,–5) as the entry

To store the selected complex number press

and type in the name to be assigned to that complex number (this must be one of the ten possible from the set Z1 to Z0) then press ENTER

(ii) For the second method of inputting a complex number, simply enter the ordered pair **(3,–5)** into the EDIT line then press ENTER *figure 11.1*

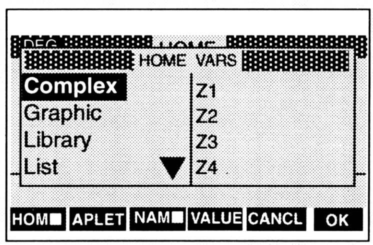


figure 11.2

- Up to ten complex numbers can be stored into memory locations on the HP 38G. However, since the Real numbers use as *home variables* the letters A, B, C, ... Z and θ these cannot be used for the storage of complex numbers.

  Instead, much like *Lists* and *Matrices*, *complex numbers*. use a combination letter and number. All ten stored complex numbers must have their name chosen from the list **Z1, Z2,...Z0.** *figure 11.2*

11.2 Storing Complex Numbers.

- To store the Complex Number 4 + 5*i* into the location Z1.

  Input 4 + 5*i* into the EDIT line of the HOME SCREEN.

- Press STO ▶ then type [Z1] ENTER

- As a check, input **Z1** into the EDIT line then press ENTER *figure 11.3*

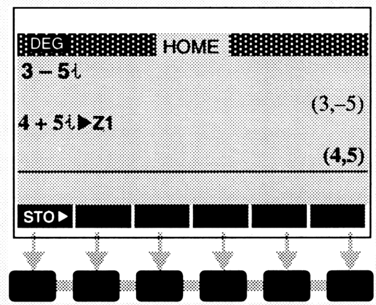


figure 11.3

- For the purpose of demonstrating some of the operations in this section you should store the following complex variables.

Z1 = 4 + 5*i*	or	Z1 = (4,5)
Z2 = 4 – 5*i*	or	Z2 = (4,–5)
Z3 = 3 – 4*i*	or	Z3 = (3,–4)
Z4 = 3 + 4*i*	or	Z4 = (3,4)
Z5 = –4 – 5*i*	or	Z5 = (–4,–5)

To recall the stored complex number simply type its name (from the set Z1 to Z0) or use the VAR menu key , select from the list then press ENTER (see figure 11.2)

Like the *Real Home Variables* the complex variables **Z1, Z2,...Z0.**can be used when doing calculations that involve complex numbers

11.3 Operations with Complex Numbers

- If you operate with a mixture of real and complex numbers the result is given as a complex number.

Example 2 **7 + Z1**

- In the Home screen EDIT line input

  7 + Z1 ENTER

  The result displayed is (11,5) ie 11 + 5i

  The real number 7 has been interpreted in its complex form as 7 + 0i and the calculation carried out as

  (7 + 0i) + (4 + 5i) = 11 + 5i or (11,5)

- For the complex number Z = a + bi

  mod Z = ABS(Z) = |Z| = $\sqrt{a^2 + b^2}$

  arg Z = θ where θ = ATAN($\frac{b}{a}$) *figure 11.4*

If Z = a + bi, the conjugate $\overline{Z}$ = a – bi

- **Example 3** For Z3 = 3 – 4i

  Mod(Z3) = **ABS(Z3)** = 5

  RE(Z3) = 3

  IM(Z3) = –4

  ARG(Z3) = Arg(3 – 4i) = 53·1301°

  CONJ(Z3) = $\overline{\mathbf{Z3}}$ = 3 + 4i

These can be typed directly into the EDIT line, or inserted when you press the MATH key and select the function from the Complex menu and press ENTER. *figure 11.6*

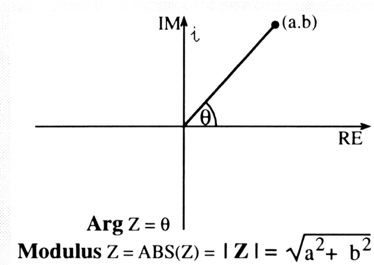


figure 11.4

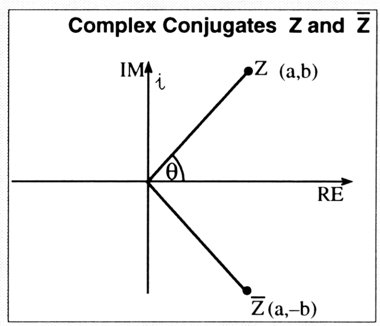


figure 11.5

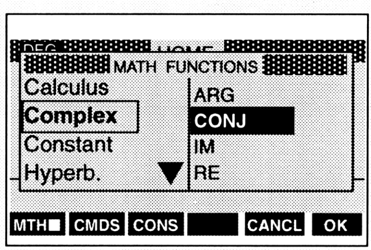


figure 11.6

Press ENTER for **CONJ(** to be input into the EDIT line of the HOME SCREEN

Example 4 3(4 + 5i) – 2(3 + 4i)

Input into the EDIT line 3**Z1** – 2**Z4** ENTER

Result displayed (6,7)

Example 5 4(4 – 5i)(3 – 4i)

Input into the EDIT line 4**Z2*****Z3** ENTER

Result displayed (–32,–124) *figure 11.7*

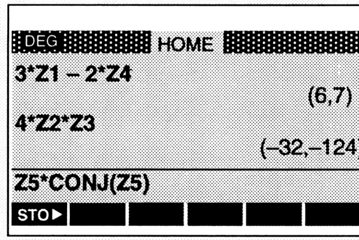


figure 11.7

Example 6 (–4 – 5i)(–4 + 5i)

Input into the EDIT line

Z5*CONJ(**Z5**) ENTER

Result displayed (41,0)

Example 7 $\frac{4 + 5i}{3 - 4i}$

Input into the EDIT line **Z1/Z3** ENTER

Result displayed (–0·32,1·24) *figure 11.8*

It may be more meaningful to view the result in fraction number format.

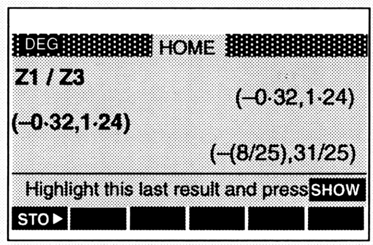


figure 11.8

- Press ▢ MODES HOME .

  Select NUMBER FORMAT

- Press the + key until the Fraction format is displayed. Use ◀ ▶ to select the number that is to the right of the word *Fraction* and key in the value 5.

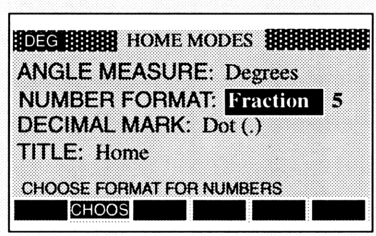


figure 11.9

Press HOME, then use ▲ to highlight the answer (–0·32,1·24) then press ENTER The answer is displayed in fraction form. *fig 11.8*

Use the complex numbers currently stored to *show*

(i) **(Z3)** = $\overline{\mathbf{Z3}}$ = $|Z|^2$ (ii) **CONJ(Z1*Z2)** = $\overline{\mathbf{Z1}}*\overline{\mathbf{Z2}}$

(iii) **CONJ(Z1+Z2)** = $\overline{\mathbf{Z1}} + \overline{\mathbf{Z2}}$

Example 8 $(3 - 4i)^2$

Enter this as **A…Z** Z3 ☐ **x^Y** (x^2) **ENTER** Result displayed (–7,–24)

11.4 Roots of complex numbers

Example 9

Determine the three cube roots of 1.

This is equivalent to asking you to solve

$$Z^3 = 1$$

or $Z^3 - 1 = 0$

This is a polynomial with Real coefficients, therefore we shall solve it using POLYROOT. This technique was explained in the chapter on polynomials. You could type in

POLYROOT ([1, 0, 0, 1]) **ENTER** or proceed as follows

- Press **HOME** then the **MATH** key.
- Press the key **5** P to get to *figure 11.10*

Select [Polynom.] ▶ [POLYROOT] **ENTER**

Recall the form of entry is

POLYROOT([[coeffs separated by a comma]])

- In the EDIT line of the HOME screen input

POLYROOT([1,0,0,1]) **ENTER** *figure 11.12*

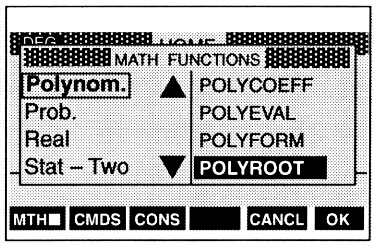


figure 11.10

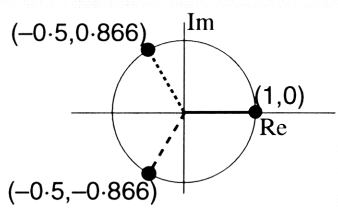


figure 11.11

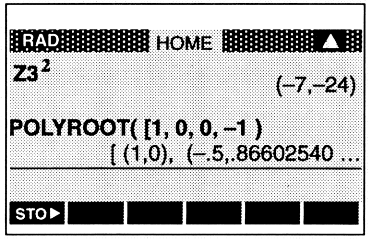


figure 11.12

If you wish to get a clear view of the roots displayed you could select the answer that is displayed then

(i) Press [SHOW], use [◀][▶] to scroll through the display.

Press [OK] when done. ...... *figure 11.13*

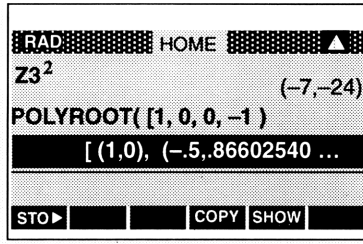


figure 11.13

or

(ii) Since the roots are shown as a vector with complex elements you could store the result in the Matrix Catalog as a vector. To store the roots vector as M0 press

[STO ▶] [A...Z] [9] M [0] [ENTER]

To view the roots press [ ] [4] (MATRIX) select M0 then press [EDIT] ...... *figure 11.14*

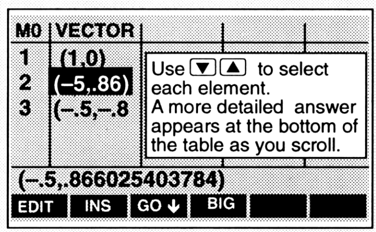


figure 11.14

One for you to try:

Find the fifth roots of $(1 + i)$
Hint Solve $Z^5 = (1 + i)$ watch your signs!
POLYROOT([1,0,0,0,0,−1−i])

You should obtain the solutions shown in figures 11.15 and 11.6

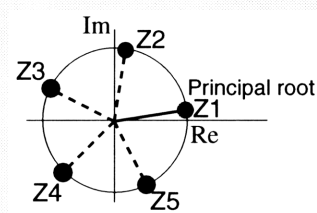


figure 11.15

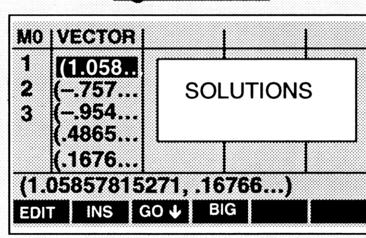


figure 11.16

11.5 Extension ideas with Complex Numbers

Consider the exponential form of a complex number a + bi

Example: Z = 1 + $i\sqrt{3}$ Mod Z = ___ Arg Z = _____

Thus the polar form of 1 + $i\sqrt{3} = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}) = 2e^{i\frac{\pi}{3}}$

$\cos\theta + i\sin\theta = e^{i\theta}$ **($\cos\theta + i\sin\theta$)** is abbreviated to **$\text{cis}\theta$**

Use these ideas to find a value for

(i) $e^{i\pi}$ and $e^{-i\pi}$ Compare this with the result obtained on your

calculator which gives something like $e^{i\pi}$ = (–1,–2·067E-13).

(iii) Use the equations derived in parts (i) & (ii) to find the exponential form for $\cos\theta$ and $\sin\theta$

(iv) Use the MacLaurin series expansions for $\sin x$, $\cos x$ and e^x to prove that

a. $e^{i\theta} = \text{cis}\theta$ b. $e^{i\pi} = -1$

(v) Use any of the above ideas to determine (prove?) the value of $\ell n(-1)$

Compare this with the value given on your calculator.

(vi) Consider the three sequences and notice the general pattern.

$\ell n(-n)$ n ∈ Integers	**$\ell n(-e^n)$**	**$\ell n(-n)$ n ∈ Integers**
$\ell n(-1)$	$\ell n(-e^1)$	$\ell n(-1e)$
$\ell n(-2)$	$\ell n(-e^2)$	$\ell n(-2e)$
$\ell n(-3)$	$\ell n(-e^3)$	$\ell n(-3e)$
$\ell n(-4)$	$\ell n(-e^4)$	$\ell n(-4e)$
and so on		

CHAPTER 12

DIFFERENTIAL & INTEGRAL CALCULUS

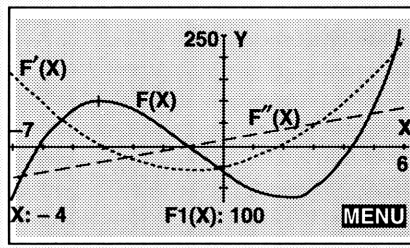
The PLOT view

can be used to show the graph of a function $y = f(x)$ and the successive derivatives

$y = f'(x)$;

$y = f''(x)$

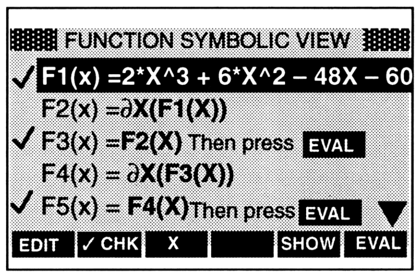
$y = f'''(x)$ $y = f''''(x)$ and so on



The SYMB view

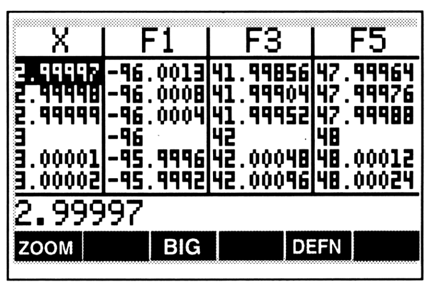
To graph functions and their derivatives note the process used.

With F3(X) EVAL draws the graph of the derivative quicker than if it is in the form F2(X)



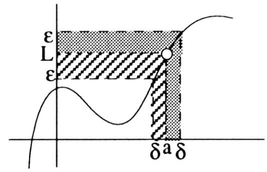
The NUM view

The **NUM**eric view can be used to determine the slope of a curve at a point. ie the value of $\left.\frac{dy}{dx}\right|_{x=a}$



12.1 Limits

Limits of Functions



Consider a function f and a number a.

What happens to f(x) as x gets closer to a ?

Note: This question asks about the BEHAVIOUR of f(x) as x approaches a.
It **does not** ask about the value of f(x) when x is **equal** to a..
In many functions of interest to us f may not even be defined at $x = a$.
We often answer the above question in terms of a number L that f(x) is getting closer to L as x gets closer and closer to ***a***. Several alternative ways of describing this *limiting behaviour* may be used - each assertion bringing out a slightly different aspect of the behaviour of f(x) but all essentially having the same meaning.

(a) The ***number*** f(x) approaches the value L as x approaches ***a*** **.**

(b) The *distance* from f(x) to the value L gets closer to zero as the distance from ***a*** to x approaches 0. (ie δ x in the diagram approaches 0).

(c) The number f(x) becomes an arbitrarily good approximation to L whenever x is sufficiently close to ***a***

We say the limit of f as x approaches a is L.

This is written symbolically as $\lim_{x \to a} f(x) = L$ or $f(x) \to L$ as $x \to a$

Example 1 $\lim_{x \to 1} (2x + 3) = 5$

Example 2 $\lim_{x \to 2} (x^2 + 2) = 6$

Example 3 $\lim_{x \to 0} |x| = 0$

Example 4 $\lim_{x \to 0} \frac{1}{x} = ??$

Example 5 $\lim_{x \to 3} \frac{1}{x} =$

Example 6 $\lim_{x \to 8} \frac{|x|}{x} =$

Example 7 $\lim_{x \to 0} \frac{|x|}{x} = ?$

Example 8 $\lim_{x \to \infty} \frac{|x|}{x} =$

Use your calculator to verify the answers obtained. Would a graph help?

An approach to limits using the HP 38G

Determine (i) $\lim_{x \to 2} \left(\frac{x^2 - 9}{x - 3} \right)$

and (ii) $\lim_{x \to 3} \left(\frac{x^2 - 9}{x - 3} \right)$

The answer to (i) is easily determined by putting $x = 2$ into the expression to get 5.

However since $\left(\frac{x^2 - 9}{x - 3} \right)$ is undefined at $x = 3$ this second limit cannot be evaluated the same way.

- Open the Function ApLet *figure 12.1*
- Press LIB select Function ENTER
- Press ▢ CLEAR DEL to clear any contents
- Input F1(x) = $(x^2 - 9)/(x - 3)$ *fig 12·2*

The problem can now be tackled in one of many ways.

Method 1: In the HOME SCREEN

- *Approach 3 **from the left*** (2·9999...)

Input F1(2.99) ENTER Result 5·99

Input F1(2.999) ENTER Result 5·999

Input F1(2.9999) ENTER Result 5·9999

Input F1(2.99999) ENTER Result 5·99999

Input F1(2.999999) ENTER Result 6

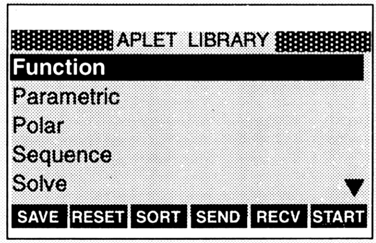


figure 12.1

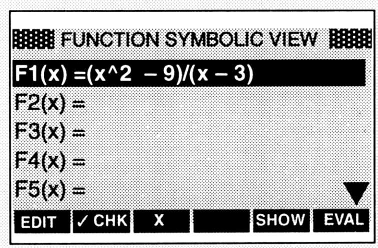


figure 12.2

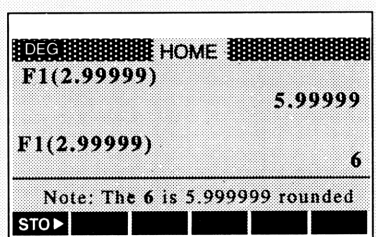


figure 12.3

Use ▲ to select the previous FI(2.999) COPY it to the EDIT line, use ◀ to move to the end of the x value and type in another 9 each time, then press enter

- *Approach 3 from the right* (3·0001...)

Input F1(3·01) ENTER Result 6·01

Input F1(3·001) ENTER Result 6·001

Input F1(3·0001) ENTER Result 6·0001

Input F1(3·00001) ENTER Result 6·00001

Input F1(3·000001) ENTER Result 6

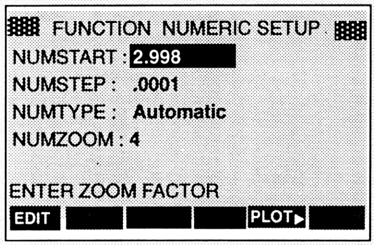


figure 12.4

Method 2:
The NUMeric view. Press NUM

- Press SETUP NUM Put NUMSTART = 2·998

  Put NUMSTEP = 0·0001 *figure 12.4*

- Press NUM then scroll to 3 *figure 12.5*

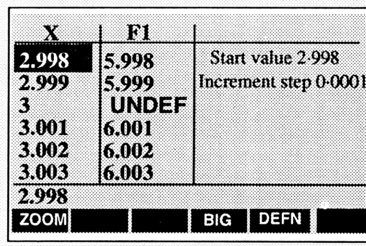


figure 12.5

Notice the values either side of 3 and also the fact that the value of the function is UNDEFINED *at* $x = 3$

- Now refine this further. Press SETUP NUM

  Put NUMSTART = 2·99998;

  NUMSTEP = 0·000001

  Press NUM then scroll to 3 *figure 12.6*

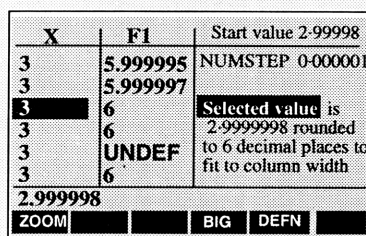


figure 12.6

The values shown in the left-most column are rounded values to 6 decimal places. This enables the numbers to fit into the column. The **actual value** for a *selected value* is given below the that column, at the bottom of the table. In figure 12.6 the x-value selected shows a 3 but its actual value is 2·999998

- The process can be refined further, still but by now it can be seen that as you get closer to 3 the value of the function gets closer to **6**

- ie $\lim_{x \to 3} \left(\frac{x^2 - 9}{x - 3} \right) = 6$

This is not a proof. It merely demonstrates the plausibility of the claim that the limiting value of F(x) as one gets closer to 3 is 6

Method 3:
The graphic view. Press PLOT

- Make sure that F1(x) is checked (✓)

  Press PLOT You will get a graph similar to that in *figure 12.7* The scale may be different

- Since you are interested in values **near 3** press SETUP PLOT and set the XRNG 2 to 4.

- Press VIEWS LIB and select [Auto Scale]

  An appropriate y-scale to suit the chosen XRNG is set. The graph is re-drawn and will now look like *figure 12.8*

- Press the MENU key . Make sure that the cross cursor is *on the line* **NEAR** x = 3

  Press ZOOM select [In 4x4] OK *fig 12.9*

- This Zoom in process can be carried on to a greater degree if required. What should be clear from both this graphical approach and the **NUM**eric view is that the closer x gets to 3, the closer the value of F(x) gets to 6

6 is called the limiting value of $\left(\dfrac{x^2 - 9}{x - 3}\right)$ as x approaches 3. In mathematical notation

this is written as $$\lim_{x \to 3}\left(\frac{x^2 - 9}{x - 3}\right) = 6$$

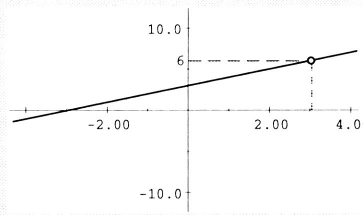


figure 12.7

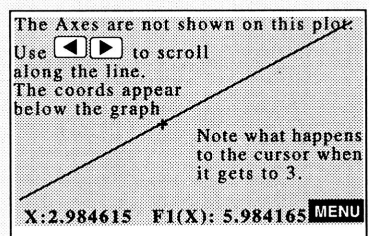


figure 12.8

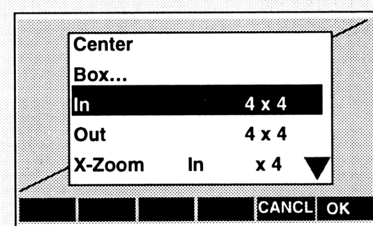


figure 12.9

If your cursor was not on the line near x = 3 when the zoom occurred you may have zoomed in on a section of the screen where the line did not appear. In this case un-zoom

To do this press ZOOM scroll down the menu

in *figure 12.9*, select [Un-zoom] then OK

Further examples on Limits

Use **SYMB**, **NUM** and **PLOT** to find the value of the following limits, if they exist.

1. $\lim_{x\to 0}\left(\dfrac{3x^2 - 2x^2 + 5x}{x^2 - x}\right)$

2. $\lim_{x\to 0}\left(\dfrac{(2 + x)^2 - 4}{x}\right)$

3. $\lim_{x\to 4}\left(\dfrac{\sqrt{x - 9} - 1}{x - 4}\right)$

4. $\lim_{x\to -1}\left(\dfrac{\sqrt{x + 2} - 1}{x + 1}\right)$

5. $\lim_{m\to 0}\left(\dfrac{(x + m)^2 - x^2}{m}\right)$

6. $\lim_{x\to 0}\left(\dfrac{|x|}{x}\right)$

Challenge: Investigate the following limit

$$\lim_{x\to 0}\left(x \sin\left(\frac{1}{x}\right)\right)$$

Figures 12.10a and 12.10b show the graphs of this function. Note the scale on the axes Treat this using the same three approaches **SYMB**, **NUM** and **PLOT** as outlined above. For **NUM** start at -0·001, with steps 0·0001

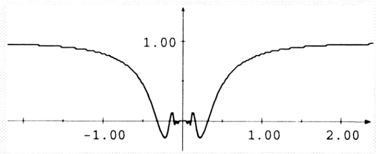


figure 12.10a

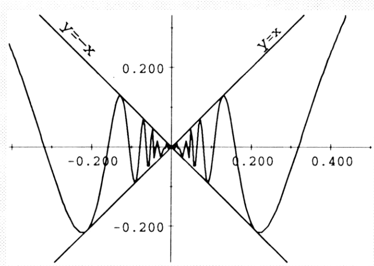


figure 12.10b

These graphs were drawn using a computer graphing package to give greater clarity than that provided by the HP 38G

Now show $\lim_{x\to 0}\left(\dfrac{\sin x}{x}\right) = 1$

Also determine $\lim_{x\to 0}\left(\dfrac{\sin 2x}{2x}\right)$

$\lim_{x\to 0}\left(\dfrac{\sin 5x}{5x}\right)$ and $\lim_{\theta\to 0}\left(\dfrac{\sin 3\theta}{\theta}\right)$

What conclusion(s) could be drawn?

Hence determine

(i) $\lim_{x\to 0}\left(\dfrac{\sin(mx)}{mx}\right)$ and

(ii) $\lim_{x\to 0}\left(\dfrac{\sin(mx)}{x}\right)$ where $m \in \mathbb{R}$eals $m \neq 0$

Now consider the following limit

$$\lim_{x\to 0} \sin\frac{1}{x}$$

Figures 12.11a and 12.11b are two views of the function $y = \sin\frac{1}{x}$ centred on zero.

The second view (*figure 12.11b*) was obtained by using the [X-Zoom In] rather than the more general [Zoom]. See *figure 12.9*

As x approaches 0

$\sin\frac{1}{x}$ oscillates between ± 1

$\therefore \quad \lim_{x\to 0} \sin\frac{1}{x}$ does not exist.

What does it actually mean to make the statement that $\lim_{x\to a} f(x) = L$?

Other interesting limits worthy of further investigation are

(i) $\lim_{x\to\infty}\left(1 + \left(\frac{1}{x}\right)\right)^x$

(ii) $\lim_{x\to 0}(1 + x)^{(1/x)}$

(iii) $\lim_{x\to 0}\left(\frac{1 - \cos x}{x}\right)$

(iv) $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)$

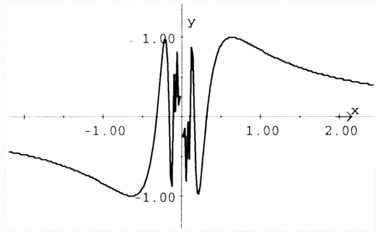


figure 12.11a

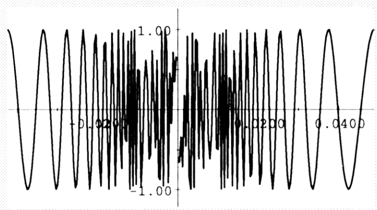


figure 12.11b

These graphs were drawn using a computer graphing package to give greater clarity than that provided by the HP 38G

Determine $\lim_{x\to 0}\left(\frac{x^2 \cos x}{x^2 + x}\right)$

12.2 Gradient functions and the slope of a curve

Slopes of Secants and Tangents

The slope of the secant PQ $=\dfrac{F(X+H) - F(X)}{H}$

As H approaches zero, Q will get closer and closer to P, and the Secant PQ will come closer to coinciding with the tangent to the curve y = f(x) at P.

The tangent is the limiting position of the secant as H approaches zero. ie the slope of

tangent at P = $\lim\limits_{H \to 0} \left[\dfrac{F(X+H) - F(X)}{H}\right]$

figure 12.12

The slope of the curve at P is defined to be the slope of the tangent to the curve at P.

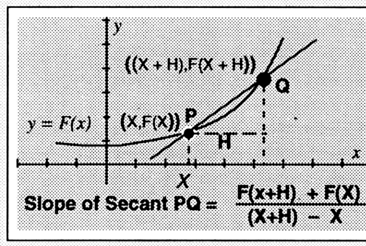


figure 12.12

As H approaches zero, the slope of the secant PQ will take on forms such as

$$\frac{F1(X + 0{\cdot}00001) - F1(X)}{0{\cdot}00001}$$

The Gradient Function on the HP 38G:

- Press LIB select Function then press ENTER or SYMB
- Define F1(X) = 3X – 4

  F2(X) = $\left(\dfrac{F1(X+H) - F1(X)}{H}\right)$

  F3(X) = F2(X)
- With F3(X) highlighted press EVAL This defines F3(X)= F2(X) directly in terms of X

Check ✓ F1(x) and F3(X). Uncheck F2(X)

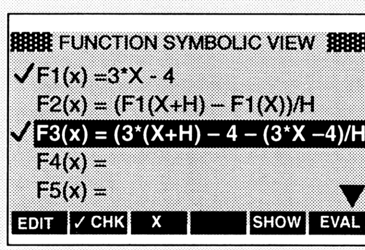


figure 12.13

The plotting is much faster for F3(x) than it is for F2(X)

- Press HOME
- Store the value 0·00001 for H in memory location H. ie (0·00001 STO▶ H ENTER)
- Go to SETUP PLOT and set up the PLOT screen as shown in *figures 12.14a and 12.14b*
- Press PLOT The graph of $y = 3x - 4$ is drawn and also the graph of the gradient function $F2(X) = (\frac{F1(X+H) - F1(X)}{H})$ for each point on F1(X)
- Press NUM to get the **NUM**eric view of the plotted functions. *figure 12.15*
  The definition of the gradient function can easily be seen to be $y = 3$
- This approach will now be generalised for several types of polynomial functions.

(i) **LINEAR FUNCTIONS** $Y = MX + B$

- Press SYMB to get back to the **SYMB**olic view (*figure 12.13*)
  Uncheck F1(x) to F3(X) then input
  F4(X) = MX + B
  $F5(X) = \frac{F4(X+H) - F4(X)}{H}$
  F6(X) = F5(X)

With F6(X) highlighted press EVAL *fig 12.16*

Check F4(X) and F6(X); uncheck F5(X)

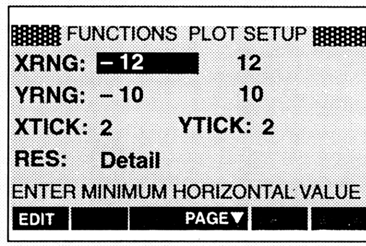


figure 12.14a

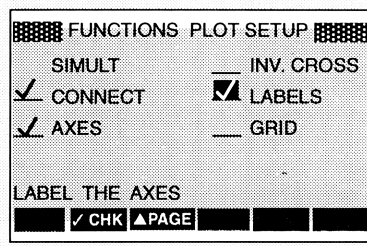


figure 12.14b

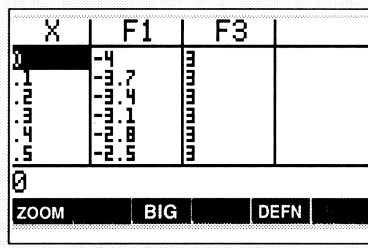


figure 12.15

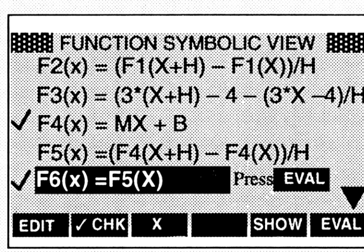


figure 12.16

Press HOME For different functions of the form $Y = MX + B$ store the values for M and B. H already has the value 0·00001 from the last example. Build up a table of your actions and what occurs.

You should use both the **NUM**eric view (NUM) and the PLOT view in this exercise.

Linear function	Value stored in M	Value stored in B	Value stored in H	Gradient Function
$y = 2x - 9$	2	–9	0·00001	$y' = f'(x) = 2$
$y = 7x + 4$	7	4	0·00001	$y' = f'(x) = 7$
$y = -3x - 5$	–3	–5	0·00001	$y' = f'(x) = -3$
$y = -8x - 2$	–8	–2	0·00001	$y' = f'(x) = -8$

Try several more then note any conclusions that may be drawn.

(i) **QUADRATIC FUNCTIONS** $Y = AX^2 + BX + C$

- Press SYMB to get back to the **SYMB**olic view (*figure 12.13*)

  Input $F7(X) = AX^2 + BX + C$

  $$F8(X) = \frac{F7(X+H) - F7(X)}{H}$$

  $F9(X) = F8(X)$

With F9(X) highlighted press EVAL *fig 12.17*

Check F7(X) and F9(X); uncheck all the other functions in this ApLet list..

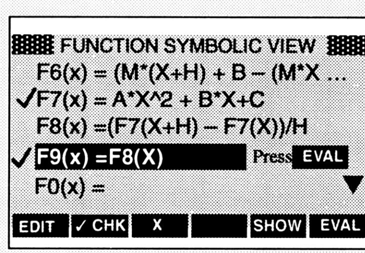


figure 12.17

SAVE this as an ApLet under the name Functions-GRADIENT1 Include an explanatory note.

Press **HOME** For each function of the form $Y = AX^2 + BX + C$ store the values for A and B and C into those memory locations. Store 0·0000001 as the value for H . Each time you store the values for A, B and C press **PLOT** Build up a table of your actions and what occurs. A sample is set out below.

Press ⬜ **SETUP NUM** and set the NUMSTART value at 0 and the NUMSTEP at 1

Use both the **PLOT** view and the **NU**Meric view (**NUM**).

You may need to reset the scale in the **PLOT** SETUP

Quadratic function	Value stored in A	Value stored in B	Value stored in C	Value stored in H	Gradient Function
$y = x^2$	1	0	0	0·0000001	$y' = f'(x) = 2x$
$y = 3x^2 + 4$	3	0	4	0·0000001	$y' = f'(x) = 6x$
$y = -2x^2 + 5$	−2	0	5	0·0000001	$y' = f'(x) = -4x$
$y = \frac{1}{2}x^2 - 8$	−8	−2		0·0000001	$y' = f'(x) = x$

- The plot for $y = \frac{1}{2}x^2 - 8$ is shown in *figure 12.18*
- The equation of the gradient function in these cases is reasonably straight forward to obtain. A study of the table in **NU**Meric view (**NUM**) will yield the answer $y' = f'(x) = x$.

But what of equations of the of the form

$y = 3x^2 + 4x - 5$?

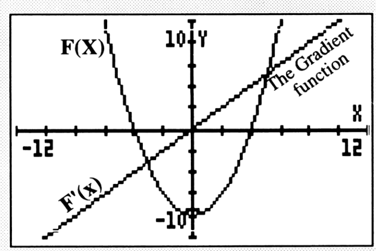


figure 12.18

©jc

- In the HOME SCREEN
  store 3 =A; 4=B and –5 =C
- The plot for this function is shown in *figure 12.19*
- It can be seen that the gradient function is Linear. To determine the rule for $f'(x)$ go to the **NUM**eric view (Press **NUM**) *figure 12.20*
- Some knowledge of Finite differences and difference patterns will yield the rule for the gradient function F9 as

  $f'(x) = 6x + 4$

Warning: The functions arrived at by this method are results obtained as H approaches zero. They do not give the LIMIT but can be made as close to within the restrictions imposed by the calculator.

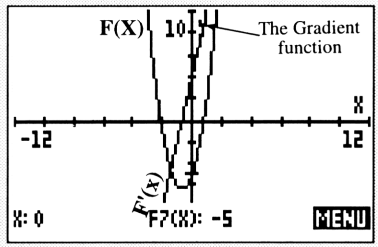
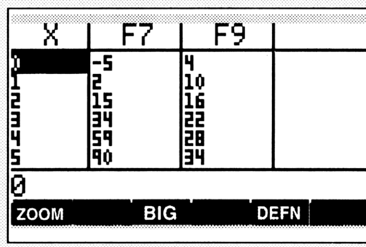


figure 12.19



X	F7	F9
0	-5	4
1	2	10
2	15	16
3	34	22
4	59	28
5	90	34

figure 12.20

If you have some knowledge of Finite differences and difference patterns this approach to *gradient functions* is worth pursuing with polynomials as well as with other kinds of functions (Logarithmic, exponential, etc)

The limit $\lim_{H \to 0} \left(\frac{f(x + H) - F(x)}{H} \right)$ yields a *function* . Since it is based on the gradient of the secant as it approaches the tangent, it is based on the slope of the curve. This function is sometimes referred to as the *gradient function.*

DEFINITION

$\lim_{H \to 0} \left(\frac{f(x + H) - F(x)}{H} \right) = \frac{dy}{dx}$ this is called the derivative of y with respect to x

A better approximation of the derivative or gradient function can be gained using the symmetric difference quotient $\left(\frac{f(x + H) - F(x - H)}{2H} \right)$

12.3 The use of special characters.

Special characters obtained using [shift] [CHARS (] keys.

Press [shift] [CHARS (] Figure 12.21a will display a screen of characters that can be selected and placed into the EDIT line of the HOME SCREEN or in ApLets.

Use [PAGE▼] to see other screens of special characters that can be accessed.

Use [PAGE▼] and [PAGE▲] to move from one screen of characters to the other.

see figures 12.21a & 21b

Two symbols of interest in this section, as they are used in the calculus, are [∂] and [∫]

Use the [◀] [▶] [▼] [▲] cursor keys to move around the screen of characters.

- *To insert a character into the EDIT line of the HOME SCREEN select the character then press* ENTER

For example ∂ will be inserted if you press ENTER on the first screen *fig12.21a* The Greek alpha character α will be inserted if you press ENTER on the second screen. *figure 12.21b.*

- A second way of inserting certain special characters is outlined below.

figure 2.21a

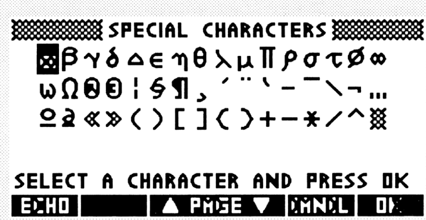


figure 12.21b

12.4 DIFFERENTIATION

The [∂] symbol used as the differentiation operator

The derivative operator on the HP 38G graphic calculator is denoted by ∂

There are two ways of inserting the operator [∂] into the display.

Method 1 was outlined in the previous section using the special characters screen.

A second way of entering special characters such as [∂] is to use the MATH key

- Press MATH This gives a list of Math functions by category. *figure 12.22*
- The left side of the split screen gives the *category of function*, the left side lists what functions are available within this category. Use ▼ ▲ to scroll up or down the left side until the **Calculus** is highlighted. Use the ▶ cursor key to move to the right side sub menu.
- Use the ▼ ▲ cursor keys to select the item or symbol that you wish to insert into your EDIT line. In this case the [∂] was already selected as it was the first item.
- Press ENTER. [∂] is now placed on the EDIT line of the HOME SCREEN

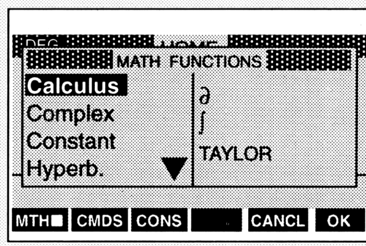


figure 12.22a

To quickly select a category without scrolling just type the first letter of the category name. You do not need to press the A...Z key.

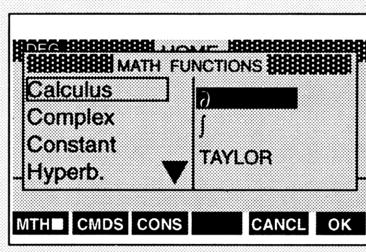


figure 12.22b

Note that the choices on the right side change according to which function category is selected on the left

To determine the value of the derivative of F(X) at a point

Example 1 Determine $\frac{dy}{dx}$ at $x = 3$ given $y = F(x) = x^2$

In the HOME SCREEN First store the value 3 in the X memory location: (3 STO▶ A...Z X)

- Press MATH

  Select the category **Calculus**

  Use ▶ to move to the right side menu,

  Highlight ∂ then press ENTER

- Next to the ∂ that appears in the EDIT line, input X(X^2) ENTER *see figure 12.23*

  The result **6** is displayed

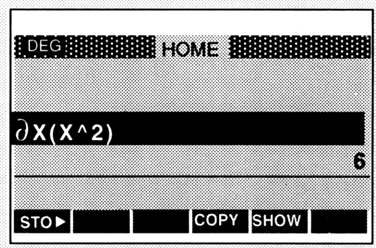


figure 12.23

The syntax for numeric differentiation is
∂X(F(X))

Example 2 Determine $\frac{dy}{dx}$ at $x = 3$ given $y = 2x^3 - 4x^2 - 14$

- Use the ▲ cursor key to select the previous question **∂X(X^2)** above the EDIT line. *figure 12.23*

  Note the change in the screen menus.

- Press COPY This makes a copy of the **∂X(X^2)** in the EDIT line. Edit the x^2 to read $2x^3 - 4x^2 - 14$ then press ENTER.

- Since 3 is already stored as the value of X

  The answer to $\frac{dy}{dx}\Big|_{x=2}$ is displayed as 30

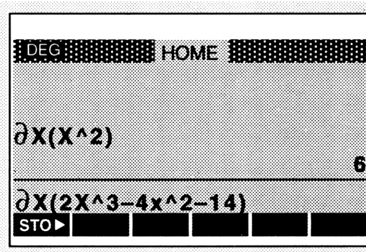


figure 12.24

12.5 Symbolic differentiation (i) In the HOME SCREEN

Recall that in the HOME SCREEN there are 27 *home variables* A, B, ... Z and θ. These **always** have a number assigned to them. The default value is 0 until you overwrite this value with another one. The *home variables* are shared throughout all the environments (The HOME SCREEN and the ApLets). When these letters are used in an expression they are interpreted as having the value assigned or stored in that memory location. Thus if 5 is stored in A and 9 is stored in B then A + B ENTER will return the result 14. A Solve ApLet involving A and B will also have these same values for A and B.

Any expression involving A, B, ... Z, will return a *numeric result*.

If you wish to use a *symbolic variable as a place-holder* you will need to use one of the six *formal names* provided for this purpose.

The HP 38G uses the set **s0; s1; s2; s3; s4; s5** as its formal variables.

(The Upper case S0, S1, S2, S3, S4 and S5 are also accepted. These are not be confused with the data sets used in the bivariate section of the statistics ApLet).

Here s1 or S1 does not represent a value; it *is just a symbol*

Whereas $4(A*B)^2$ will return a result of 8100 (ie $4*(5*9)^2$, an input of $5(S1*S1)^2$ will return $5*(S1*S1)^2$ and not a numeric result. The appropriate variable can then be assigned when you write down your answers.

As shown above **∂X(X^2)** will return a *number*, the value of which will depend on the value stored in X. However

(i) **∂s1(s1²)** will return the expression 2*s1

(ii) **∂s1(5s1^3)** will return the expression 5*(3*s1^2) *see figure 12.25*

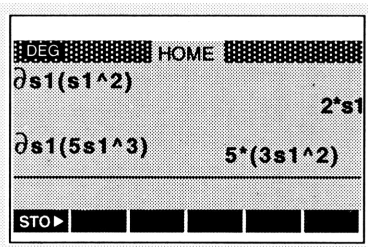


figure 12.25

These results can be interpreted as

$$\frac{dx^2}{dx} = 2x \quad \text{and} \quad \frac{d(5x^3)}{dx} = 5*(3*x^2) = 15x^2$$

Further examples of Symbolic differentiation

Exploratory example: Determine $\frac{d}{dx}\left(\frac{7}{x^{3}}\right)$

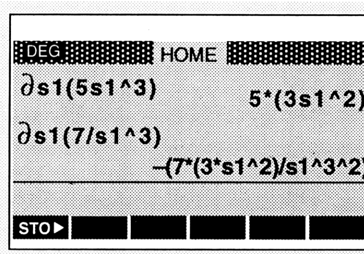


figure 12.26

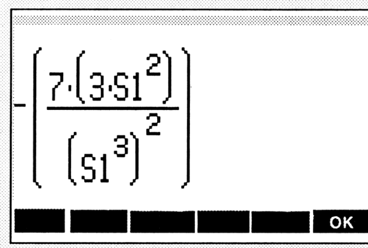


figure 12.27

- Press HOME to get to the home screen.
- Press MATH select [Calculus] then [∂] ENTER. ∂ is now placed on the EDIT line of the HOME SCREEN
- In the EDIT line put ∂S1(7/S1^3) ENTER The result –(7*(3*S1^2)/S1^3^2) is displayed. *see figure 12.26*
- This is better viewed if you use [▲] to select the result then press SHOW *fig 12.27*

 If you substitute x for S1 you should recognise: $\frac{d}{dx}\left(\frac{7}{x^{3}}\right) = -\left(\frac{7*3x^{2}}{(x^{3})^{2}}\right) = -\left(\frac{21}{x^{4}}\right)$

The HP 38G *"knows and can apply"* the normal rules of differentiation. The *Power* rule; the *Product* rule; the *Quotient* rule; the *Chain* rule. Furthermore it can apply these rules to all the in-built functions such as sin$x$., cos$x$, tan$x$, $\ell n(x)$, e^{x}, a^{x} and so on. As a consequence, the derivative of any function composed of these functions can be also differentiated on the HP 38G. In the example above the quotient rule was used

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = -\left(\frac{f'(x)g(x) - f(x)g'(x)}{(g(x)^{2})}\right) = \left(\frac{0.x^{3} - 7(3x^{2})}{(x^{3})^{2}}\right) = \left(\frac{-7(3x^{2})}{(x^{3})^{2}}\right) = -\left(\frac{21}{x^{4}}\right)$$

This should explain the form of the answer given when SHOW was pressed.

- Now try the same example but this time enter it in the form $\frac{d}{dx}(7x^{-3})$

- Input ∂S1(7S1^–3) ENTER
  The answer given 7*–(3*S1^–4) indicates that the power rule has been used.
  fig 12.28

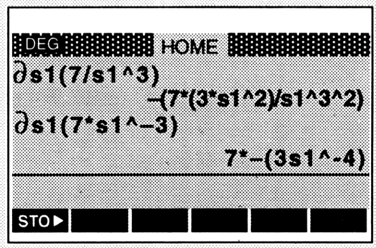


figure 12.28

Example 3a - The Chain Rule

- Determine $\frac{d}{dx}(3x^4 - 7x)^5$

- Input ∂S1((3S1^4–7S1)^5).

- The result is shown in both the HOME SCREEN format *figure 12.29a* and also using

figure 12.29b

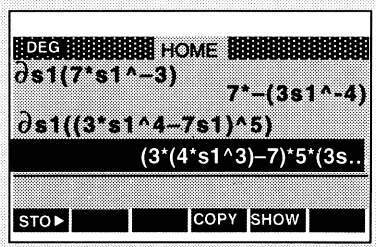


figure 12.29a

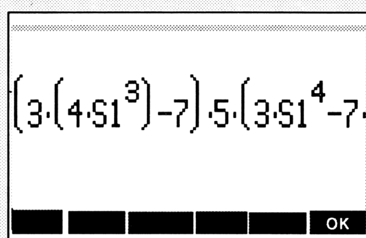


figure 12.29b

Example 3b: The Chain Rule

- Determine $\frac{d}{dx}(\sin^3 7x)$

  Input ∂S1(sin(7S1^3) ENTER
  Result: cos(7*s1)*7*3*sin(7*s1)^2

- ie $\frac{d}{dx}(\sin^3 7x) = 21\cos(7x)\sin^2(7x)$

WARNING
Make sure that you are in Radian Mode

Example 4: The Product Rule

- Determine $\frac{d}{dx}[\ell n(5x^2) \times (7x-6)]$

  Input ∂S1(ℓn(5S1^2) · (7S1– 6)) **ENTER**

  Result: 5*(2*S1)/(5*s1^2) *7*...

  SHOW gives the result as

  $\frac{5(2*S1)}{5.S1^2}.(7S1-6) + Ln(5S1^2).7$

ie $\frac{d}{dx}[\ell n(5x^2) \times (7x-6)]$

$= \frac{2}{x}(7x - 6) + 7\ \ell n(5x^2)$

Example 5: The Quotient Rule

Determine $\frac{d}{dx}\left(\frac{5x^3-4}{2x+7}\right)$

Input ∂S1((5S1^3 – 4)/(2S1+ 7)) **ENTER**

Result: 5*(3*S1^2)/(2*s1+7)–...

SHOW gives the result as shown in *figure 12.30*

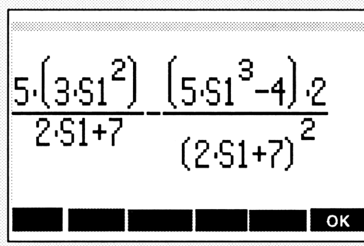


figure 12.30

The work done to date in this section shows how to obtain the symbolic derivative while working in the HOME SCREEN.

There is another approach which combines both the Symbolic and Graphical approach which tends to be far more informative and permits you to obtain and graph the original function and its first, second. third,... derivatives. This is a powerful tool in the analysis of relationships between these various derivatives.

12.6 Symbolic differentiation (ii) In the **Function ApLet**

First enter the Function ApLet

- Press LIB then select the Function ApLet then ENTER
- Press [shift] CLEAR DEL to clear any previous entries (Press YES to clear)
- Highlight F1(x) and input

  F1(*x*) = 2*x*³ + 6*x*² – 48*x* – 60
- Highlight F2(*x*) and input

  F2(*x*) = ∂x(F1(x))
- Highlight F3(*x*) and input

  F3(*x*) = F2(x) *figure 12.31*
- Select F3(x) and press EVAL then

  Select F3(X) and press SHOW your screen should look like *figure 12.33*
- Check F1(X) and its derivative F3(X)
- Press [shift] SETUP PLOT and set the plot features as shown in *figures 12.34a & 12.34b*

You may need to check the number format

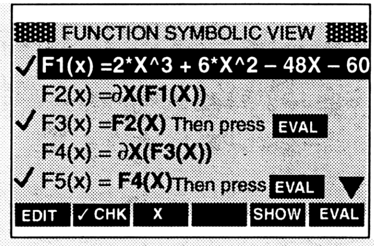


figure 12.31

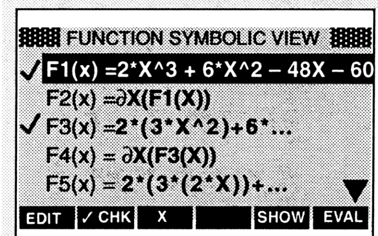


figure 12.32

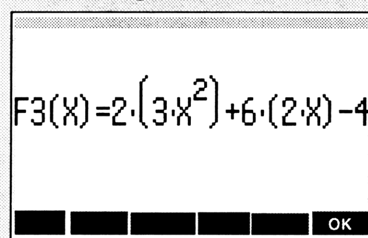


figure 12.33

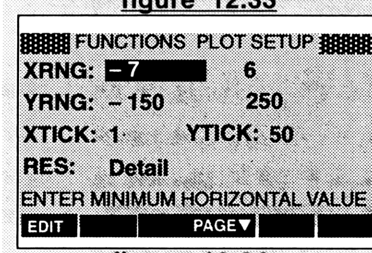


figure 12.34a

- Press **PLOT** . Note the intercepts and turning points of each of the graphs F1(x) and F3(X). *figure 12.35*

- Press **NUM** to get to the **NUM**eric view. From this table, which shows the values for both F1(x) and its derivative function F3(X), the slope of the curve F1(x) can be determined at any point. Thus at the point (1,–100) the slope of the curve is –30, at the point (2,–116) the slope of the curve is 0 (ie the slope of the tangent at that point is 0) *figure 12.36*

- F3 gives the *rate of change of function F1(X) with respect to X.* As X increases from 0 to 5 the value of F1(x) decreases until it reaches 2 it then increases. This same information is given in a slightly different way by the values of F3(X)

- Press **SYMB** to get to the **SYMB**olic view. Input **F4(*x*) = ∂x(F3(x))**
  F5(*x*) = F4(x) *figures 12.31 & 32*

- Select F5(x); press EVAL then SHOW to see the second derivative defined.

- Check F1(X); F3(X); F5(X); then **PLOT** The graphs of f(*x*); f′(*x*); f″(*x*); are drawn. *figure 12.37*

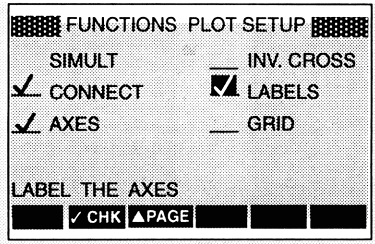


figure 12.34b

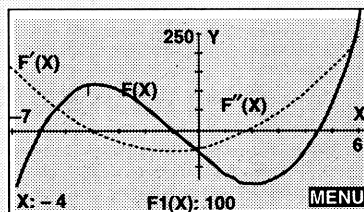
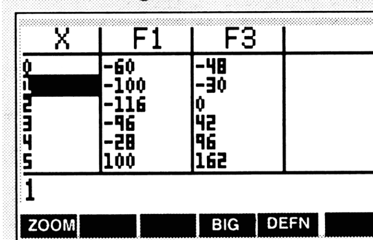


figure 12.35



X	F1	F3
0	-60	-48
1	-100	-30
2	-116	0
3	-96	42
4	-28	96
5	100	162

figure 12.36

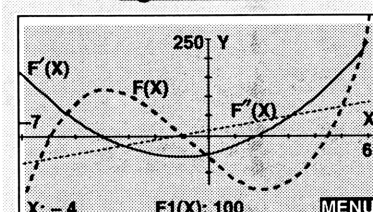


figure 12.37

Inputting and defining the functions in the **SYMB**olic view in the form

F1(x) = $2x^3 + 6x^2 - 48x - 60$

F2(x) = **∂x(F1(x))**

F3(x) = **F2(x)** and then evaluating F3(X)

was deliberately done this way for several reasons, The first reason is easy to demonstrate if you check F1(X) and F2(X) and then plot the graphs. Notice how much slower the graph of F2(X) is compared to the drawing of F3(X)? Check this same comparative speed problem in the **NUM**eric view.

It is also more convenient to enter a new function at F1(X). To obtain its derivative you only need to retype F3(x) = F2(X) and then press EVAL

The definition of the derivative is given in F3(X) without the necessity of inserting the special character ∂ . The function F3(x) is no longer tied to F1(x). From the plot of f(x); f′(x); f″(x); certain conclusions may be drawn. These can be verified in the **NUM**eric view. You should investigate these features. Note the special relationship between local maximum and minimum of F1(X) and its derivative function. Comment of the relationship between zeros of the various functions, relative extrema, points of inflection and so on.

Further derivatives may be defined by extending beyond F5(X) in the same manner, ie F6(X) = ∂F5(X) and F7(X) = F6(X)

Try this same approach, SYMB PLOT and NUM with a wider range of functions.

For work involving Trigonometric functions make sure that the angle mode is set to RADIANS.

Try F1(X) = sinX; F2(X) = ∂X(F1(X); F3(X) = F2(X) and then EVAL F3(X).

Use SETUP PLOT CLEAR DEL VIEWS LIB Auto Scale to set the scale.

Try F4(X) = $\cos^2$X; F5(X) = ∂X(F4(X); F6(X) = F5(X) and then EVAL F6(X).

Try exponential and logarithmic functions. With the log functions try base 10 logarithms as well as natural logarithms and find the connection between the two.

12.7 Numeric Integration and Symbolic Integration.

Like differentiation the HP 38G enables you to do both **NUM**eric and **SYMB**olic integration. For symbolic integration the formal variables S0, S1, S2, S3, S4, S5 must be used in much the same manner as they were used in differentiation.

Areas bounded by curves can be determined as can volumes of solids of revolution.

The Definite Integral in the HOME SCREEN

- Like its counterpart [∂], the integral operator [∫] can be input into the EDIT line of the HOME SCREEN using one of two methods;
  either using the special characters key (press [ ] [CHARS (])
  or by pressing [MATH] and choosing the [∫] symbol from the [Calculus] menu.

figure 12.38

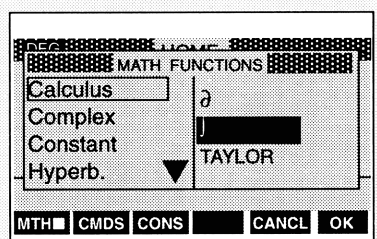


figure 12.38

The definite integral $\int_a^b f(x)\,dx$ *is a number.*

The number $\int_a^b \mathbf{f(x)\,dx} = F(b) - F(a)$

where F is *any* anti-derivative of f

The value of a *definite integral* can be determined on the HP 38 G as shown in the next example

Example: Determine the value of $\int_1^5 x^2 dx$

- Press HOME MATH [Calculus] select [∫] then press ENTER. This will put the integral sign into the EDIT line.

- The input into the EDIT line should read as

  [∫(1,5,X²,X)] ENTER Answer 41·33

- Alternatively store the value for the lower limit in A and the upper limit in B

  1 STO▶ A; 5 STO▶ B then input

  [∫(A,B,X²,X)] ENTER Answer 41·33

  figure 12.39

Several values can be determined using the MAKELIST feature as follows.

Press HOME MATH, select [List] [MAKELIST] then press ENTER

This should put MAKELIST(into the EDIT line Input so the edit line reads

[MAKELIST(∫(0, N, X², X), N, 0, 10, 1)]

STO▶ L1, to store this in the LIST CATALOG

For this section of the work set the NUMBER FORMAT to [Fraction] [4]

(Remember to do this use [ ] MODES HOME)

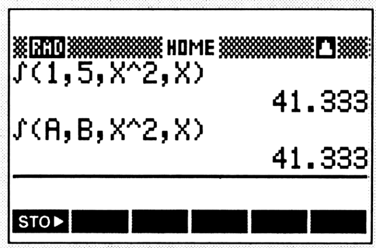


figure 12.39

NEED a REMINDER?

For this section of the work set the NUMBER FORMAT to [Fraction] [4]

The reason will become more obvious as you work on

12.8 Investigating the Integral Function using Difference patterns.

Use the **MAKELIST** and the **ΔList** functions to work with the following.

Original function $f(x)$	from 0 to x $\int_0^x f$	$\int_0^1 f$	$\int_0^2 f$	$\int_0^3 f$	$\int_0^4 f$	$\int_0^5 f$	$\int_0^6 f$	The rule of Function $\int_0^x f \rightarrow F(x)$
1	x	1	2	3	4	5	6	
$y = 3x + 1$	A(x)	2·5						
2	x	1	2	3	4	5	6	
$y = x - 3$	A(x)							
3	x	1	2	3	4	5	6	
$y = -5x + 7$	A(x)							
4	x	1	2	3	4	5	6	
$y = -\frac{1}{2}x - 6$	A(x)							
5	x	1	2	3	4	5	6	
$y = x^2$	A(x)							
6	x	1	2	3	4	5	6	
$y = -2x^2$	A(x)							
7	x	1	2	3	4	5	6	
$y = x^2 - 4x + 6$	A(x)							
8	x	1	2	3	4	5	6	
$y = 2x^3$	A(x)							
9	x	1	2	3	4	5	6	
$y = x^3 - 9x^2 + 5x - 2$	A(x)							

Example Use the **MAKELIST** and the **ΔList** functions to determine the defining rule for the indefinite integral $\int_a^x f(t)dt$ where f(t) = 6t – 2.

- Press HOME MATH, select List MAKELIST then press ENTER
- This should put MAKELIST(into the EDIT line. Input so the edit line reads

MAKELIST(∫(0,X,6T – 2,T),X,0,10,1)

What this is instructing the calculator to do is **Make a List** of those definite integrals of (6T – 2) with the lower limit 0 and the upper limit X where X starts at 0 and goes to 10 in steps of 1

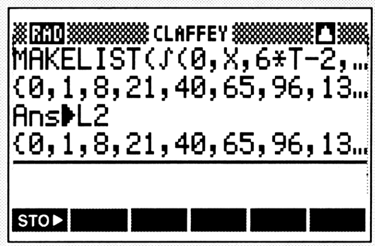


figure 12.40

- STO▶ L2, this will store the list just created in the LIST CATALOG as LIST(L2) *figure 12.40*
- Press MATH, select List ΔLIST then press ENTER then complete the line by inputting **Ans** so the EDIT line reads ΔLIST(Ans) ENTER
- Use ▲ to select ΔLIST(Ans) then press ENTER
- Repeat this process until the differences are constant. *figure 12.41*

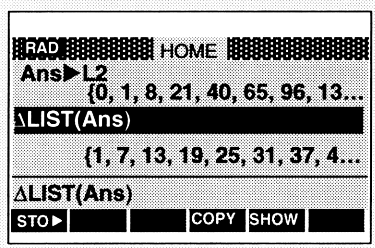


figure 12.41

The next press of ENTER will give 6,6,6,...

If you are familiar with difference patterns use the Appendix to see the generalisation for polynomials of degree 1; degree 2; degree 3

If you are not familiar with finite differences and difference patterns you may wish to skip over this section. You will unfortunately miss out on a pearl of a discovery if you do skip this section.

- *Now determine a defining rule for* $\int_a^x (6t - 2)dt$

The Indefinite Integral in the HOME SCREEN

Whereas a definite integral $\int_a^b f(x)\,dx$ *is a number*, (Here a and b are any constants)

the INDEFINITE INTEGRAL $\int_a^x f(t)\,dt$ *is a function.* (a is any constant, x is a variable)

To determine an indefinite integral you must use one of the six formal variables s0; s1; s2; s3; s4; s5; as one of the limits of integration.

Example: Determine $\int_a^x 3t^2 dt$ This is usually written as $\int 3t^2 dt$ without any specific limits being stated.

Input into the EDIT line of the HOME SCREEN

∫(0,S1,6T – 2,T) ENTER *figure 12.42*

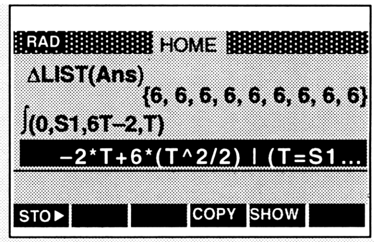


figure 12.42

Select the result *figure 12.42* and press SHOW

This shows the interpretation using the Fundamental Theorem; in unsimplified form. *figure 12.43a*

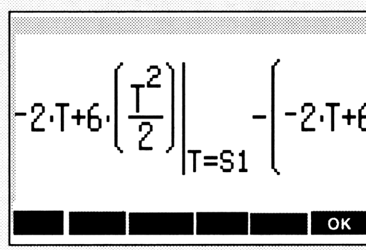


figure 12.43a

Press OK to get back to *figure 12.42*

With the result still highlighted press ENTER

This gives the final symbolic result in terms of S1. Select the result and press SHOW to see the answer in symbolic form.*figure 12.43b*

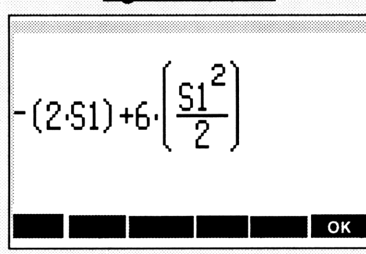


figure 12.43b

12.9 Applications of integration.

Plot of an anti-derivative

To plot an integral function

- Press LIB Select [Function] ENTER
- Input F1(x) = ∫(0,X,T-2,T) ENTER
- SETUP PLOT then CLEAR DEL to reset to the default plot options. then press PLOT
  figure 12.24
- Input F2(x) = ∫(–2,X,T-2,T) ENTER
- F3(x) = ∫(–5,X,T-2,T) ENTER
- F4(x) = ∫(3,X,T-2,T) ENTER
- F5(x) = ∫(7,X,T-2,T) ENTER

Plot all 5 functions. Refer back to section 12.8 to see how all these antiderivatives differ from each other. Use an *area function approach* to the questions in section 12.8 and you should be able to show why any two antiderivatives of a function f(x) can only differ by a constant.

Figure 12.46 shows the graphs of the five requested antiderivatives of (T – 2). What conclusions could you draw?

For this section of the work set the
NUMBER FORMAT to [Fixed] [2]

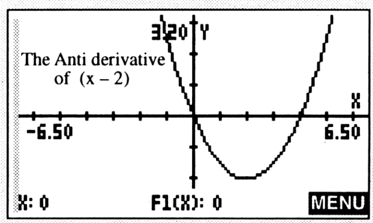


figure 12.44

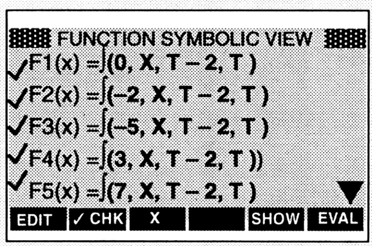


Figure 12.45

WARNING The plotting is very slow.

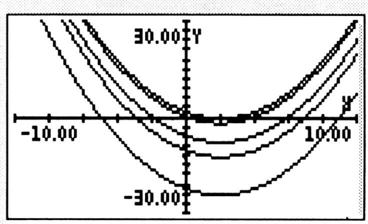


figure 12.46

Areas bounded by curves

Determine (i) any local max–min of function f

(ii) The roots of f(x)

(iii) The points of intersection of f and g

(iv) The area bounded by the curves f(x) and g(x)

where f(x) $= 2x^3 + 7x^2 - 4x - 6$ and g(x) $= x + 5$

- Press LIB Select Function ENTER
- Input F1(x) = $x^3 + 7x^2 - 4x - 6$ ENTER
- Input F2(x) = $x + 5$ ENTER
- SETUP PLOT then CLEAR DEL Reset to the plot options in *figure 12.47* then press PLOT

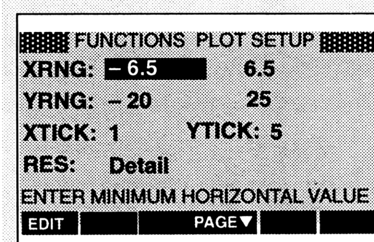


figure 12.47

Move the cursor to the left side of the screen.

(i) Locating the ROOTS

- Press MENU until the screen menus show,

then press FCN . Select Root OK

Root1 is –3·818;

- Move the cursor to somewhere near the second root and repeat the process. This gives root 2 = –0·742 . Do the same for the third root to get root 3 = 1·060

Roots are –3·818; –0·742; and 1·060

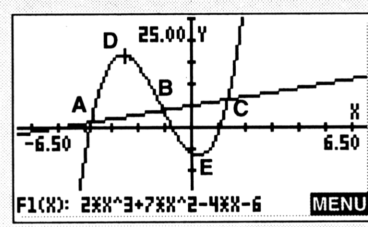


figure 12.48

(ii) Locating the points of intersection

- Press MENU FCN [Intersection] OK

  Choose [F2(X)= X+5] OK

  Intersection Point A = (–3·776,1·224)

  Move the cursor near the next point of intersection and repeat the previous steps. This gives point B = (–1·077,3·923)

  Repeat the process for point C

  This gives C= (1·353,6·353)

  Press HOME and store the *x*–values for A, B, and C into the memory locations A, B, and C, then return to PLOT *fig 12.48*

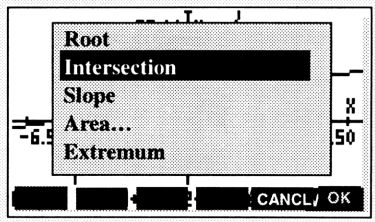


figure 12.49

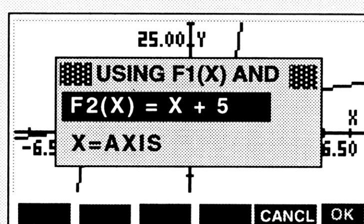


figure 12.50

(iii) Locating the Extremum

- Press MENU FCN [Extremum] OK

  This gives Point D = (–2.591,16.569)

  Move the cursor near the other extremum and repeat the steps to get Point E (0.257,–6·532)

figure 12.48

(iii) The Area bounded by f(x) and g(x)

- Relocate the cursor at point A.
- Press MENU FCN [Area] OK

figure 12.50

- Select [F2(X)=X+5] then press OK

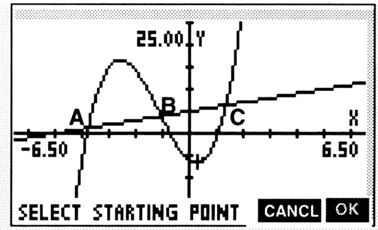


figure 12.51

Your screen should look like *figure 2.51.* With the cursor starting at A, press the ▶ cursor key to move to point B. The area shades as you progress.

When you reach B press OK *figure 12.52*

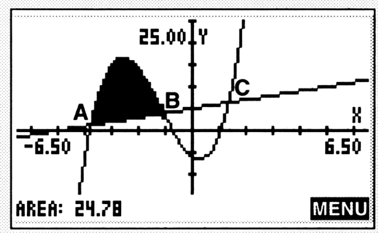


figure 12.52

- Leave the cursor at B and repeat the process for the right side of the graph. Notice that the area is given as –18.67 for this section. If you want the total area you should add the absolute value of the two quantities obtained in *figures 12.52.53*

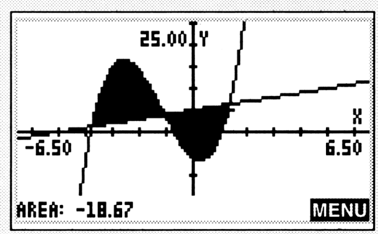


figure 12.53

- Area = 24·78 + |–18·67| = 43·45 units^2

- To obtain a more precise answer go to the HOME SCREEN and evaluate

  ABS([∫(A,B,F1(X)–F2(X),X)]) +

  ABS[∫(B,C,F1(X)–F2(X),X)] ENTER

  Answer (24·77 + 18·71) = 43·48 units^2

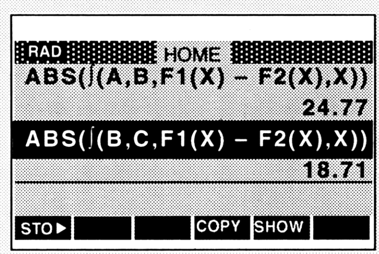


figure 12.54

Volumes of Solids of Revolution

Example 1 *Determine the volume of the cone generated by rotating the line $y = x$ about the x-axis, over the interval $x = 0$ to 10*

- For rotation of a function $y = x$ about the x-axis over the interval $a \le x \le b$ the volume of the solid generated is given by $${}_aV_b = \int_a^b \pi * f(x)^2\,dx$$

For this example set the NUMBER FORMAT to either [Fraction] [2] or [Fixed] [2]

In this problem the volume is given by $${}_0V_{10} = \int_0^{10} \pi\, x^2\,dx$$

- Store values for the lower limit in A and the upper limit in B

0 STO▶ A; 10 STO▶ B

Press LIB Select [Function] ENTER Input F3(X)= X ENTER then input

F4(x)= ∫(A,B,π*F3(X)^2,X) ENTER

At this stage you could either (i) Select F4(X) and press EVAL

This will give the result F4(X) =1047·1879...but the definition of F4(X) is lost.

or (my preferred choice) (ii) Go to the HOME SCREEN(press HOME)

Type into the EDIT line F4(X) ENTER

This gives the result as 1047. F4(X) is still defined and can be reused. If the limits A and B are altered all you do is store the new limits into A and B. This problem could also have been done completely in the HOME SCREEN.

- HOME MATH [Calculus] select [∫] ENTER. The EDIT line should contain the as input [∫(A,B,πF3(X)^2,X)] ENTER The result given is 1047.

Example 2 *Determine the volume of the torus generated by rotating the circle* $x^2 + (y-3)^2 = 4$ *about the x-axis.*

To get the required volume first determine the solid of revolution formed by rotating area 1 (shaded) about the x-axis then subtract the volume generated by rotating area 2 (hatched) about the x-axis *figure 12.39*

The functions involved must be expressed explicitly in the form $y = f(x)$

- Use the ISOLATE function to get y on its own.
  (Done here just to demonstrate a useful feature)

Press **HOME** **MATH** select Symbolic then select ISOLATE **ENTER** This puts ISOLATE (into the EDIT line. Complete the line to read:

ISOLATE($X^2 + (Y-3)^2 - 4$,Y) **ENTER**

The result of the rearrangement is s1*√(4–X²)+3

(s1 is used in place of ±) This is therefore read as

$y = \pm\sqrt{(4-X^2)}+3$

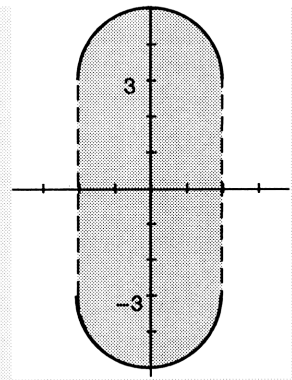


figure 12.55

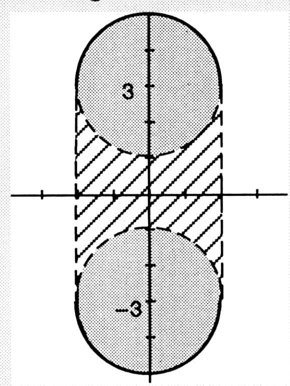


figure 12.56

- Press **LIB** Select Function **ENTER**
- Input F5(X)= 3 + √(4 – x²) **ENTER** then input
  F6(X)= 3 – √(4 – x²) **ENTER**
  F7(X)= π∫(–2,2, F5(X)^2– F5(X)^2,X)
  ENTER
- Go to the HOME SCREEN and input
  F7(X) **ENTER**
- The result is 236 units^3

The traditional approach involves using the trig substitution $x = 2\sin\theta$ to get the result $24\pi^2$!

You should keep a record of techniques and short cuts as you learn how to use the HP 38G. This page is left blank below this line for this purpose.

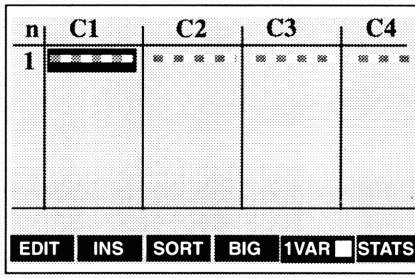
CHAPTER 13

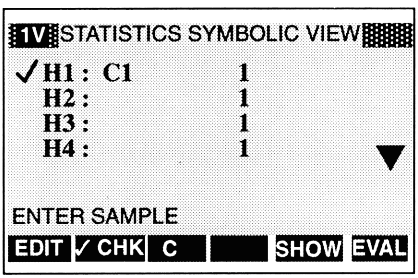
STATISTICS

Default screen views in the Statistics ApLet – One Variable Data

The NUMeric view Press NUM

This is the screen used for data entry. Up to ten data sets can be stored in columns C1, C2,…C9,C0. For data in one variable check that 1VAR■ is showing.





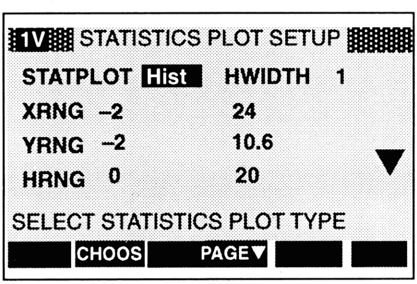
Statistics:

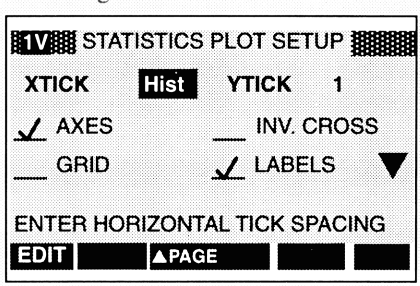
The SYMBolic view SYMB

Use this view to define which of the ten data sets C1, C2,…C9,C0 are to be analysed.. They can contain expressions such as H2: C2=3*C1 – 5. If you do not get this view check 1VAR■ in NUM view.

Statistics SETUP PLOT View

These views are used to set up the type of graph, (histogram or Box&Whisker plot), the scales and the positioning of the Histogram.





13.1 Statistical Data – Univariate Data

The Statistics ApLet is used for the analysis of data in both one and two variables.

For univariate data (ie *statistics of one variable*) the analyses involves

- Entering the data
- Developing the summary statistics associated with one variable data.
- Plotting the distributions associated with this data
  Histograms; or **Box & Whisker Plots**
- Analysing and interpreting the descriptive statistics using both the summary statistics and the associated Plots
- For explanation purposes, data for the following example will be developed.
  A survey of the TV watching habits of a class of 20 students was conducted over a 10 week period . The average number of hours per week that this group spent watching TV over this time period was as follows:

$N^{o.}$ Hours	2	3	4	5	6	7	8	11	14
$N^{o.}$ Students	1	3	1	1	2	5	1	2	4

To start the statistics ApLet and enter this data proceed as follows:

- Press **LIB** Select the Statistics ApLet then press **ENTER** or **START** *figure 13.1*
- The Statistics ApLet, unlike the other ApLet templates, opens in the **NUM**eric view with the selection bar at the top of column C1 It is into this column that the data will be entered.

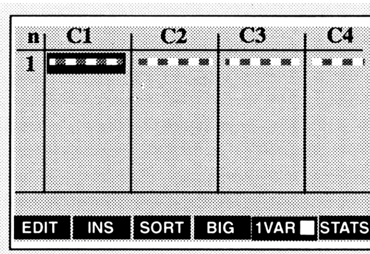


figure 13.1

To start with a clear ApLet as shown in figure 13.1 press ⬜ CLEAR DEL to clear **all** data.

- Make sure that the 1VAR■ is showing.

  This is a toggle key that switches between 2VAR■ and 1VAR■

- Press 2 ENTER 3 ENTER 3 ENTER 3 ENTER … 14 ENTER 14 ENTER See *figure 13.2*

- If a score was omitted (say only one 11 was entered instead of two) press INS .

  This inserts a 0 at the current position of the cursor. Over-type this with the value 11 that is to be inserted. If you want the elements in the list to be in order press SORT . A display like *figure 13.3* will ask you to CHOOS between having the data sorted in ascending or descending order. (Press CHOOS to see the selection choice).

- You should now have 20 data points in the data set C1. A grey bar appears under the last data item.

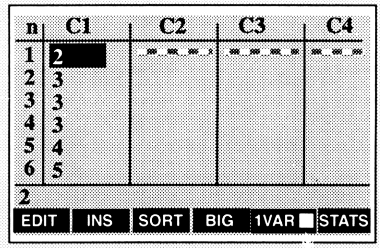


figure 13.2

HINT You can move quickly from the top to the bottom of the data list by pressing ▼. To get back to the top press ▲.

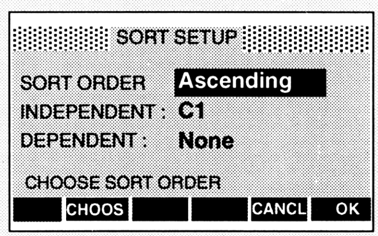


figure 13.3

Up to ten data sets can be stored in columns C1, C2,...C9,C0. For data in one variable check that 1VAR■ is active. To delete a column of data select the column then press DEL

13.2 Statistics in one variable

Once data has been entered you can select any of the three views

PLOT **SYMB** or **NUM** to carry out the appropriate analysis.

The SYMBolic view. Press SYMB

figure 13.4

- Although up to ten columns of data can be entered in the **NUM**eric view only five *data sets* can be defined for analysis These sets are defined as
  H1; H2; H3; H4; H5.
- *For any analysis of data to occur the data set must be defined and **checked** here.*

Figure 13.4 shows the data set **H1** defined as C1. This means that the Data set H1, on which any statistical analysis will be performed, will use only the data that exists in column 1 (C1) This data is entered in the **NUM**eric view and, in this example, is the data that you have just entered. *figure 13.2*

To *work* with this data the H1 must be

checked ✓ (Press ✓ CHK) *figure 13.4*

Now return to the **NUM**eric view to obtain the summary statistics associated with one variable data.

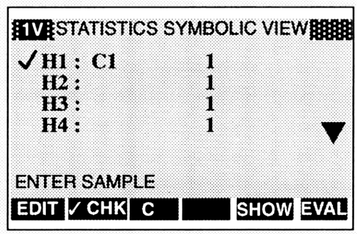


figure 13.4

The **left** column indicates which data set is to be analysed. The **right** column indicates frequency associated with this data.
If there is no column for the frequency this defaults to the value 1.
See the next example for clarification

The NUMeric view Press NUM

- There are two screen displays associated with the **NUM**eric view.
  The first of these you have already used when you entered the data, see *figure 13.2*
- The second of these views contains the **summary statistics** for the data set H1.
- Press STATS If you forgot to check a defined data set in the **SYMB**olic view a warning is given. *figure 13.5*
- If this happens press OK then enter the **SYMB**olic view, check the H1 data set as described in the last section then go back to the **NUM**eric view and press STATS again. The screen display should look like *figure 13.6a*

Twelve items (**Summary Statistics**) are listed for any **active data sets**. In this case the summary statistics are for H1 = C1. These are shown in *figures 13.6a and 13.6b*

Press OK to get back to the **NUM**eric view

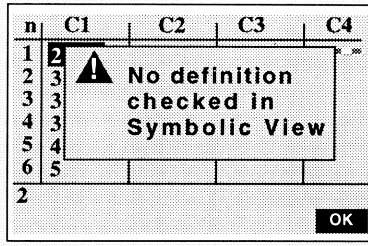


figure 13.5

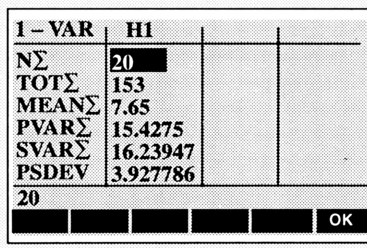


figure 13.6a

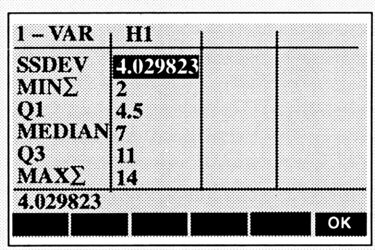


figure 13.6b

HINT You can move quickly from the top screen to the bottom screen by pressing ▼. To get back to the top screen press ▲.

Statistics PLOT View

- When PLOT is pressed you should get a display similar to that shown in *figure 13.7*
- The data chosen in this first example fits the default plot setup nicely.
- Press SETUP PLOT STATPLOT should already be selected. If it isn't select it now.
- Press CHOOS
- Select BoxWhisker OK *figure 13.8 & 9*
- Press PLOT again to get a Box &Whisker plot as shown in *figure 13.10*

  Use the ◀ ▶ keys to move the screen cursor to the five points that characterise any Box & Whisker plot.
  Min; Q1; Median; Q3; Max

The value shown at the cursor position is stated at the bottom of the screen.
H1: MED:7 means that the plot is for the data set defined by H1 and the cursor is currently located at the median which has a value of 7.

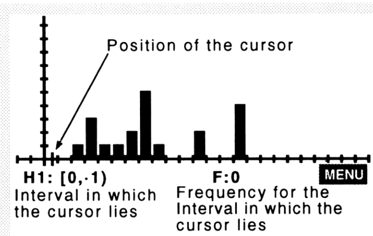


figure 13.7

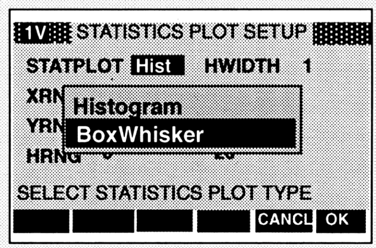


figure 13.8

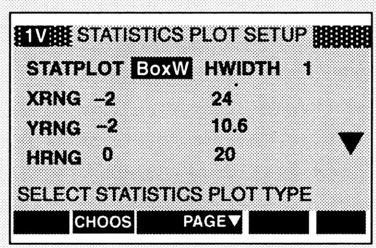


figure 13.9

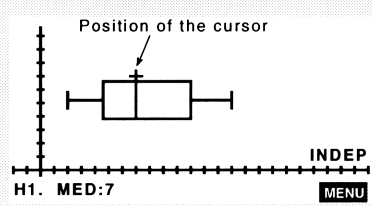


figure 13.10

Example 2 The same data as the first example, but entered using frequencies.

$N^{o.}$ Hours	2	3	4	5	6	7	8	11	14
$N^{o.}$ Students	1	3	1	1	2	5	1	2	4

To enter the data use two columns. For this example use columns C2 and C3.

- Press NUM and move the entry box to C2
- Place the data value in C2 and its frequency in C3 *figure 13.11*
- Press SYMB to get back to the view shown in *figure 13.4*. Select H2 and enter C2 on the left side next to H2: and C3 on the right column.

Notice that a **C** can be entered by using the menu label key C *figure 13.12*

Notice also that **H2** has been checked as an active *defined Data set* based on the data in C2 with C3 giving the frequencies for each data item in C2. H2 is a 2-column data set.

- Press NUM to get back to the **NUM**eric view then STATS to get the summary statistics. Note that the heading at the top of the columns refer to the *defined Data sets* H1 and H2

The statistics for both sets are identical.

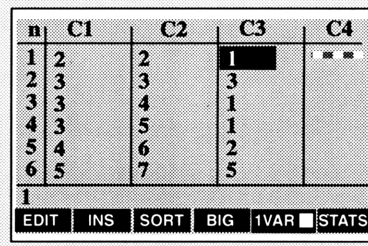


figure 13.11

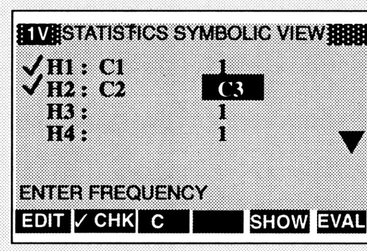


figure 13.12

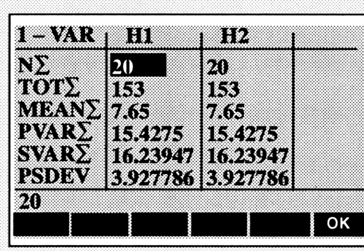


figure 13.13

- Press PLOT to obtain the Box & Whisker plot for both sets H1 and H2 as both of these were checked ✓. *figure 13.12*
  Both the plot and summary statistics show that the same data set has been plotted.

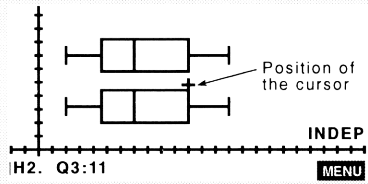


figure 13.14

If you un-check any of the data sets in *figure 13.12* then neither the *graph* nor the *summary statistics* will be displayed for the unchecked data set.

13.3 Change of Scale & Change of Origin

- Press SYMB to get *figure 13.12*. Select H3 and enter 2*C1 on the left next to H3:

Un-check H2 as it is the same as H1. *fig 13.15*

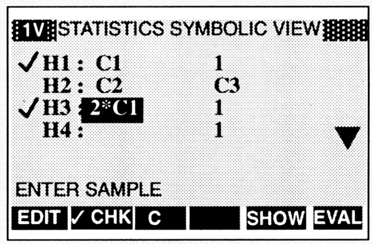


figure 13.15

- Press PLOT to get the Box & Whisker plot for both sets H1 and H3.

You will need to rescale your graph using either SETUP PLOT. Change the XRNG to read from –2 to 40 then press PLOT again, or alternatively use VIEWS LIB [Auto Scale]

This process of multiplying a data set by a constant is referred to as **CHANGE OF SCALE.**

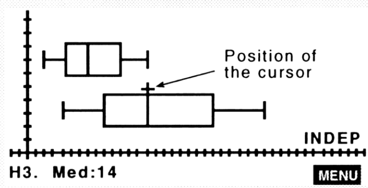


figure 13.16

- Compare the five key data points for each plot **Min; Q1; Median; Q3; Max**

What conclusion could you draw? *figure 13.16*

- Press NUM then STATS to compare the summary statistics for both sets. *fig 13.17*

1 – VAR	H1	H3
NΣ	20	20
TOTΣ	153	306
MEANΣ	7.65	15.3
PVARΣ	15.4275	61.71
SVARΣ	16.23947	64.95789
PSDEV	3.927786	7.855571

20

OK

figure 13.17

Repeat the process in all three views but this time input **H4 = C1 +10**

Study the effect of *adding a constant to each data item* in a data set. This process whereby every item in a data set is increased by adding a fixed value is referred to as **CHANGE OF ORIGIN.** You should be able to explain why statistics indicating *location* are affected by CHANGE OF ORIGIN while those measuring *spread* are not affected.

Finally combine both multiplying all values of a data set by a value K and adding a constant D to each new score generated. For example define H5 = 3*C1 + 10

Will the order in which you do these two operations make any difference? Explain!

13.4 Centring the Histogram

- Press SYMB to get back to the SYMBolic view *figure 13.15*
- Press SETUP PLOT Change the STATPLOT to Histogram , then press CLEAR DEL to reset the scaling back to the default values.
- Press PLOT
- Move the cursor to the first bar of the Histogram. Note that the histogram gives the interval as [2,3). To centre this on 2 the whole histogram needs to be moved to the left 0·5 units. *figures 13.18-19*

To do this Press SETUP PLOT and change the HRNG from (0,20) to (–0·5,19·5) then press PLOT again. Move the cursor along the Histogram and notice the interval widths.

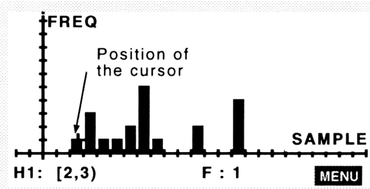


figure 13.18

Before centring the Histogram

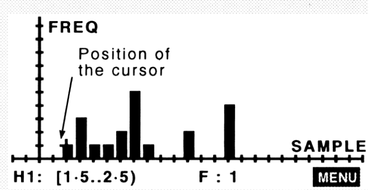


figure 13.19

After centring the Histogram note the interval boundaries. Also note that the mid point of each interval is now the same as the stated value in the problem.

To alter the width of the intervals change the value of HWIDTH See figure 13.8b

13.5 Summary Statistics for One Variable

Statistic Computed	Description or purpose
• NΣ	The **number of data points** in the set
• TOTΣ	The **sum** of the data points $\sum x$ or $\sum f \bullet x$ if applicable.
• NΣ	The **Mean** of the data set $\overline{x} = \frac{\sum x}{n}$ or $\overline{x} = \frac{\sum f.x}{n}$
• PVARΣ	The population variance of the data set $\sigma_x^2 = \frac{\sum x^2}{n} - \overline{x}^2$
• SVARΣ	The sample variance $s_x^2 = \frac{n}{n-1}\left[\frac{\sum x^2}{n} - \overline{x}^2\right]$
• PSDEV	The population Standard Deviation
• SSDEV	The sample standard deviation
• MINΣ	The minimum value in a data set
• Q1	The first Quartile (25th percentile)
• Median	The median value of a set of data. (50th percentile)
• Q3	The third Quartile (75th percentile)
• MAXΣ	The maximum value in a data set (100th percentile)

13.6 Standardising scores.

To express the data in C1 as standardised scores and store the scores in C5 check that the data has been defined as a data set (H1 in this case.) Uncheck all the other defined data sets so that when STATS is pressed the statistics for **H1 only** are displayed. Press HOME to get the HOME SCREEN then input into the EDIT line

(C1−MEANΣ)/PSDEV STO▶ **C5** ENTER	C5 will now contain the standardised scores

13.7 Two Variable Statistics – Bivariate Data

Statistics in two variable data requires *paired data*. This is entered into, and stored in, two columns selected from C1, C2, ... C0.

- Press LIB. Select the Statistics ApLet *figure 13.20*

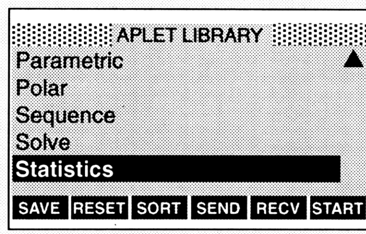


figure 13.20

- Press ENTER or START

The Statistics ApLet opens in the **NUM**eric view.. The view displayed is the last one used, which in this case is *figure 13.21*

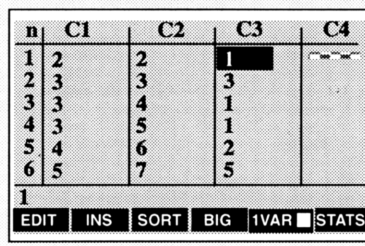


figure 13.21

- Press MODES HOME and set the NUMBER FORMAT to Fixed 2 then press NUM to return to the **NUM**eric view

- To start with a clear ApLet

  press CLEAR DEL to clear **all** data currently in columns C1, C2, ...C0. This will completely clear any data currently in the **NUM**eric view and give a screen display as shown in figure 13.22 It is into the selected column that the data will be entered.

- Make sure that the 2VAR is showing in the screen menus. This is a toggle key that switches between 2VAR and 1VAR

If you wish to save this data as an ApLet before deleting it read this note.

With any ApLet that you save you should include a note as a reminder of what the contents of the ApLet cover. To do this press NOTE VAR type a note indicating what the ApLet is about and what the data sets represent. Press HOME when this is done.

Press LIB then SAVE and save it under an appropriate name.

For the purpose of explanation, the following problem will be used.

In a research project into the relationship between weight and exercise a sample of ten people was randomly chosen. Their present weight was noted. Each of the ten was then given an exercise program that they were to adhere to over the following two months. The exercise program was allocated randomly to the ten and involved vigorous exercise over a set number of hours each week for the two month period. At the end of this period the participants were weighed again and any weight loss in kilograms noted. The results were as follows:

Hours Exercise /week ***H***	4	2	6	3	7	5	2	9	4	8
Weight loss(kg) ***W***	3	2	4	5	8	8	4	8	5	10

Do a statistical analysis of this data. What conclusions, if any, could be drawn?

The data can be *entered in one of two ways.*

(i) Enter each set into a column, for example

Enter hours of exercise H into column C1

Press 4 ENTER 2 ENTER 6 ENTER 3 ENTER

and so on up to 8 ENTER

When this is done move the cursor to C2

Enter weight loss W into column C2

The columns do not necessarily have to be adjacent, although in most cases this would be preferable. *figure 13.23*

(ii) The second way of entering data is to enter the paired data *as an ordered pair* (T,W)

Type (4,3) ENTER ; (2,2) ENTER , (6,4) ENTER and so on. Both the parentheses and comma must be included.

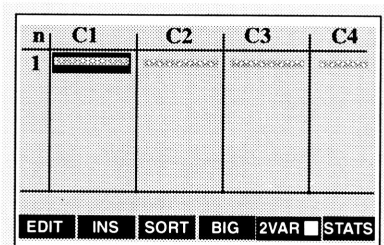


figure 13.22

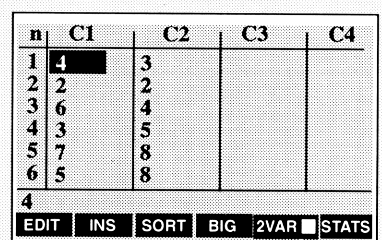


figure 13.23

13.8 Sorting Data

The paired values can be *ranked* on either of the two variables. You will need to nominate which of the two, T or W is the independent or ***explanatory*** variable on which the ranking will be based, and which variable is the dependent or ***response*** variable.

To demonstrate this, without affecting the data that has already been entered, a copy of C1 (T) and C2 (W) will be placed in columns C3 and C4. However the SORT procedure outlined below can be done directly on C1 and C2.

- Press **HOME** to open the HOME SCREEN
- Input C1 press **STO ▶** type C3 **ENTER**

  This now stores the values currently in C1 into C3.

The column of values in C1 remains as it is and a *duplicate* set is placed into C3.

- Input C2 press **STO ▶** type C4 **ENTER**

  This now stores the values currently in C2 into C4. The column of values in C2 remains as it is. *figure 13.24*

  Press **NUM** to return to **NUM**eric view

- To sort the data press **SORT**
- You **CHOOS** between having the data sorted in *ascending* or *descending* order.

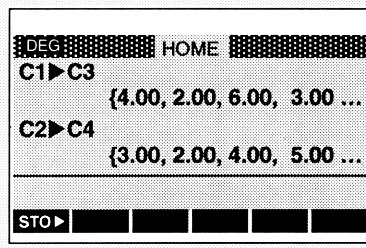


figure 13.24

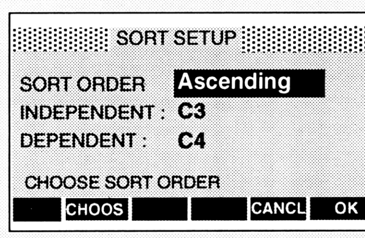


figure 13.25

- Determine which data column C3 or C4 is to be used to sort or rank the data.
  If you wish to rank the data based on the value of T, which is now repeated in C3, select INDEPENDENT then press **CHOOS** and select C3 from the list to be the independent variable. (*figure 13.25*)
  Move down to DEPENDENT Press **CHOOS** and select C4 as the dependent variable. This second set choice is necessary to keep the *paired scores* together as the ranking occurs.

- Press **OK** The sort is done.
  Your display should look like *figure 13.26*

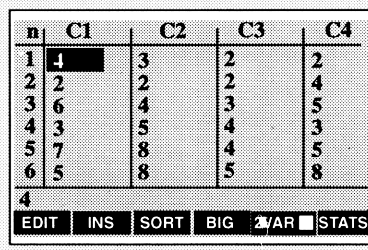
A grey bar appears under the last data item in each column.

Up to ten data sets can be stored in columns C1, C2,…C9,C0.
These are used to define the data sets S1 S2, S3, S4, or S5 in the **SYMB**olic view

For data in two-variables check that **2VAR■** is active. To delete a column of data select the column then press **CLEAR DEL**, then select from the on screen menu.

Remember: You can scroll through the choices offered in a **CHOOS** menu by repeatedly pressing the **+** key

Note C1 and C2 could have been sorted directly by this method without first copying to C3 and C4
This was done here to enable you to compare the result of the sort and to check later if any of the summary statistics were affected



n	C1	C2	C3	C4
1	4	3	2	2
2	2	2	2	4
3	6	4	3	5
4	3	5	4	3
5	7	8	4	5
6	5	8	5	8

figure 13.26

HINT
You can move quickly from the top to the bottom of the data list by pressing [shift] [▼].
To get back to the top press [shift] [▲].

13.9 Analysing Bivariate Data

Once your data has been entered you can select any of the three views

PLOT SYMB or NUM to carry out the appropriate analysis.

First you will need to define the data sets that will be used in the analysis. Remember C1, C2, … C9, C0 provide the *source* of the raw data . They are **NOT** the defined data sets. Whereas in the statistics of one variable the defined data sets were named H1, H2, … H5 *in the analysis of two variable data the defined sets are named* **S1, S2; S3; S4; S5.**

The SYMBolic view. Press SYMB

- Although up to ten sets of data can be stored in the NUMeric view only five data sets can be defined for analysis in the SYMBolic view These defined sets are named **S1, S2; S3; S4; S5**

- *For any analysis of data to occur the data set **must be defined** and **checked** here in the SYMBolic view.*

- Figure 13.27 shows the default setting for the SYMBolic view. Here the defined data set S1 has (C1,C2) as its data source.

- The default curve for best fit chosen for S1 is Linear denoted by Fit1: m*X + b

- Select Fit1: m*X + b and press SETUP SYMB . *figure 13.28* is displayed.

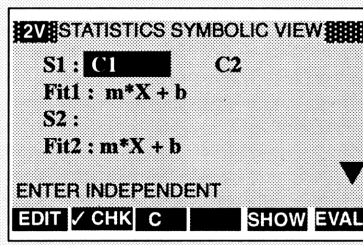


figure 13.27

The **left** value C1 indicates which column of data is to be taken as the independent variable.
The **right** value C2 indicates which column of data is to be used as the dependent variable.
Any measures of relationship or curves of best fit will use this setup.

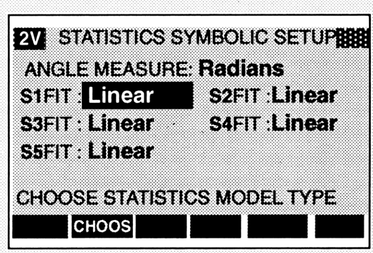
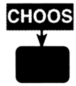


figure 13.28

- Select [S1Fit: Linear] and press 

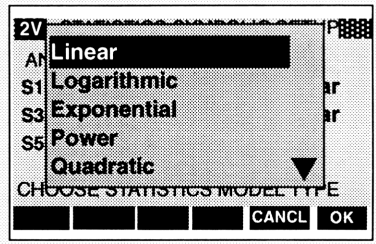


figure 13.29a

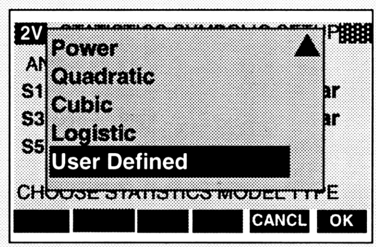
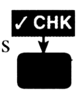


figure 13.29b

- The resulting displays, *figures 13.29a & 13.29b*, show the different regression models that may be used to fit the data sets **S1, S2; S3; S4; S5**
- You have already defined the data set **S1** as (C1,C2). This means that the Data set S1, on which any statistical analysis will be done, will use only the data that exists in column 1 (C1) and column 2 (C2) in the NUMeric view. C1 has been chosen as the independent variable and C2 the dependent variable. This is the data that you have previously entered (*figure 13.23*)
- To work with this defined data set the **S1** must be *checked* ✓

  To do this select S1 then press 

- The second line is automatically checked and reads [Fit1:.85185185…] *figure 13.30*

  Since the regression model chosen for the fit was [S1Fit: **Linear**] (*figure 13.29*)

  [Fit1:.85185185…] gives the defining rule of the equation for the line that **best fits** this data.

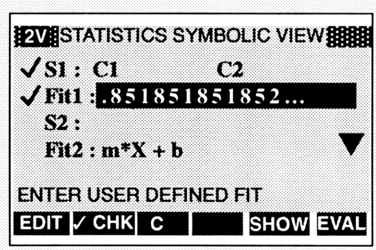


figure 13.30

- To understand what Fit1:.85185185... is telling you about the line of best fit, select this line *figure 13.30*

  then press SHOW to see this equation in better detail.

  Remember earlier you set the NUMBER FORMAT to Fixed 2 This is why the equation of best fit is as shown in *fig 13.31* to two decimal places.

Now return to the **NUM**eric view

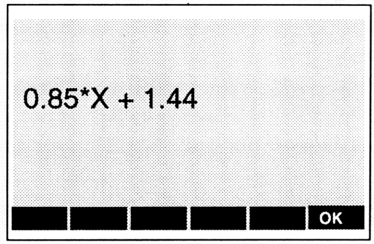


figure 13.31

Without the restriction of Fixed 2 the equation given for the line of best fit is Y = 0·851851852*X + 1·4407407...

The NUMeric view Press NUM

- There are two windows associated with the **NUM**eric view.
  The first of these you have already used when you entered the data. *figure 13.23*

- The second of these views contains the **summary statistics** for the data set **S1.**

- Press STATS

  If you forgot to check a defined data set from **S1 - S5** in the **SYMB**olic view a warning is given. *figure 13.32*

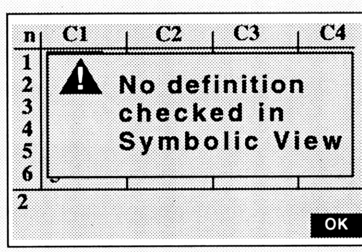


figure 13.32

If you un-check any of the data sets in *figure 13.30* then neither the **graph** nor the **summary statistics** for that data set will be displayed or given.

- If this happens press **OK** then enter the **SYMB**olic view,(press **SYMB**), check the S1 data set as shown in *figure 13.30*, then go back to the **NUM**eric view and press **STATS** again. The screen display should look like *figure 13.33a*

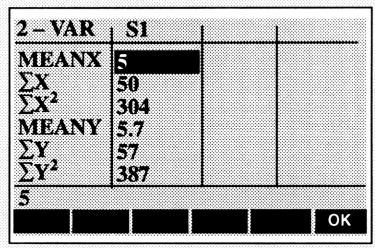


figure 13.33a

- Nine items (**Two-Variable Statistics**) are listed for any checked or active data sets. In this case this is for S1 only.

These are shown in *figures 13.33a and 13.33b*

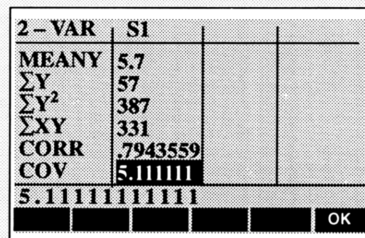


figure 13.33b

- Move to the left-most column (2–VAR) and scroll up and down the list of the statistics offered. A fuller description of the selected statistic appears in the EDIT Line at the bottom of the table.

- Press **OK** to get back to the **NUM**eric view

- Now go to the PLOT view

HINT You can scroll quickly from the top of the list to the bottom of the list by pressing [shift] [▼].
To get back to the top screen press [shift] [▲].

Note! ***The Correlation Coefficient provided is for a linear fit only.*** This disregards the Fit model chosen in the **SYMB**olic view. The value returned is in terms of the Linear relationship between the independent and dependent variables

Statistics PLOT View

- Press PLOT you should get a display similar to that shown in *figure 13.34*

- If you need to change the scale of your graph press SETUP PLOT *Figure 13.35a & b*

  If the marks that are used to represent the paired data points on your graph are not very clear select S1MARK and press CHOOS You can now select from the seven different icons that can be used to represent plotted points

- Press PAGE▼ to get to *figure 13.35b*

  Use the setup as shown

- **SUMMARY:** $\overline{T} = 5$ $\overline{W} = 5{\cdot}7$
- Covariance $S_{TW} = 5{\cdot}11111$
- Correlation Coefficient $r_{TW} = 0{\cdot}794$
- Equation of line of best fit

  $$W = 0{\cdot}85T + 1{\cdot}44$$

- **Remember**: The Correlation Coefficient is a measure of linear correlation only.

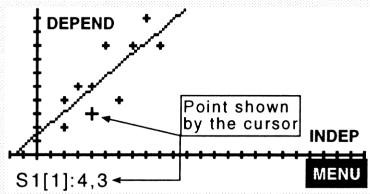


figure 13.34

In figure 13.34 **S1[1]: 4,3** indicates that the cursor has marked the location of the first data point (4,3) in the defined data set S1

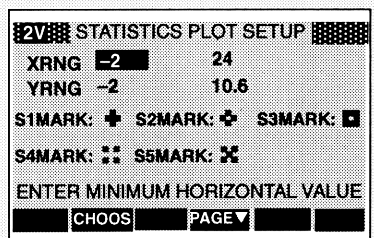


figure 13.35a

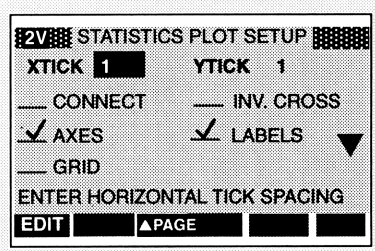


figure 13.35b

The ✓CHK screen menu will appear when you highlight those items that can be checked or unchecked.

13.10 Predicted values

You can now make some use of the statistics developed so far.

However you should recognise the potential problems that can occur with

(i) using samples that are small to *predict* further values.

(ii) working with data where the magnitude of the correlation coefficient gives the message that the best fit curve may not be very reliable.

(iii) extrapolated values based on the chosen model for the Best Fit curve where the model chosen is not appropriate to the situation. This is possible even in situations where the given data has a near perfect fit to the model over the given range of the data.

- *Using the model just determined, with the reservations as suggested above, predict the weight loss for a person from this population who does vigorous exercise for 15 hours per week*

- Press **HOME** to return to the HOME SCREEN

- Press **MATH** and select [Stat-Two] from the menu options on the left side. Use [▶] to move to the right side and select [PREDY]

  then press **OK** or **ENTER** *figure 13.36*

  This should place

  PREDY(

  into the EDIT line of the HOME SCREEN.

- Type in the value 15, close the brackets, then press **ENTER** *figure 13.37*

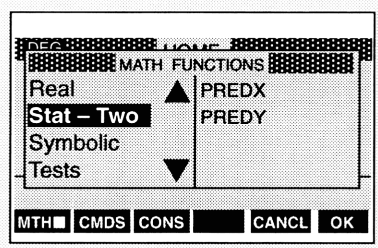


figure 13.36

To obtain the predicted scores for all values in C1

This uses the line of best fit to determine the predicted Y-score (W) for each X-score (W). To obtain these values:

- Press HOME to get to the HOME SCREEN

  The HOME SCREEN is used to transfer values from one column to another.

- Press MATH and select Stat-Two

  Use ▶ to move to the right side, select PREDY then press OK or ENTER.

  This should place PREDY(into the EDIT line of the HOME SCREEN.

- Type **C1**, close the brackets, press STO▶ then type C3 ENTER *figure 13.37*

- Press NUM to get back to the **NUM**eric view. Check C3 contains the predicted values for the Independent or explanatory variable in C1

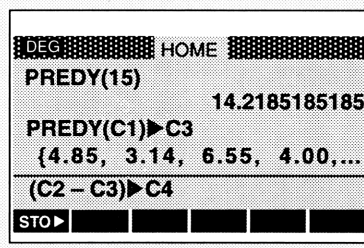


figure 13.37

Recall here that the *independent* variable used in our example was T. (The HP 38G uses X to represent this independent variable)
The *dependent* variable in our example was W. (The HP 38G uses Y to represent this dependent variable)

13.11 Residuals

The residual is given by

(Actual value Y – Predicted value Y)

- Press HOME to get to the HOME SCREEN
- Input (C2 – C3) STO ▶ then type C4

ENTER *figure 13.37*

- Press NUM to get back to the **NUM**eric view. The residuals should now be showing in column 4 (C4).

n	C1	C2	C3	C4
1	4	3	4.84815	-1.8481
2	2	2	3.14444	-1.1444
3	6	4	6.55185	2.5519
4	3	5	3.9963	1.0037
5	7	8	7.4037	.596296
6	5	8	5.7	2.3

4

EDIT | INS | SORT | BIG | 2VAR | STATS

figure 13.38

C1 is the independent variable
C2 is the dependent variable
C3 gives the predicted values of C2 for each value in C1, based upon the regression model used.
C4 gives the residuals, the difference between the actual values of the dependent variable C2 and the predicted values C3

Graphing a Residual plot.

To obtain a plot of the residuals go to the **SYMB**olic view (press SYMB) Define the Data set S2 Independent Variable **C2** Dependent Variable **C4**

For Fit2 enter 0 (ie y = 0)

Check S2 and un-check any other data sets.

Press SETUP PLOT and set the YRNG –4 to 4. You will need to set this according to the residuals obtained in your NUMeric view, or VIEWS LIB select Auto Scale could be used.

Press PLOT to get a plot of the Residuals.

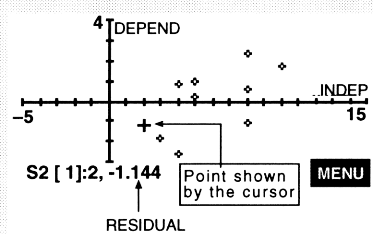


figure 13.38

This sample is too small for meaningful interpretations and conclusions to be drawn.

13.12 Change of Scale & Origin - Bivariate data

- Press SYMB to get back to the view shown in *figure 13.30.*
- Check S1 and un-check S2
- Select S3 and enter 2*C1 on the left side next to S3: (the Independent variable).
- Input C2 as the dependent variable
- For Fit3 select the linear model **m*X + b**
- Select S3 then press ✓CHK a *check* ✓ mark should appear alongside the **S3** and also Fit3. Your **SYMB**olic view should now look similar to *figure 13.39*
- Press NUM then STATS to compare the summary statistics for the data set S3 and compare it with the original Data set S1. What is the effect of multiplying one of the scores by 2? *figure 13.40*
- Press SETUP PLOT. Change the XRNG to read from –5 to 30 then press and the YRNG –5 to 20 then press PLOT.
- The scatter diagram shows the plot and lines of best fit for both Data sets S1 and S2. *figure 13.41*

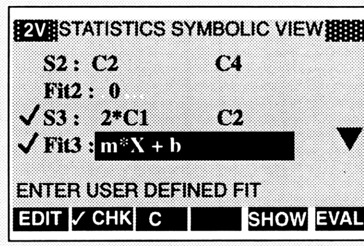


figure 13.39

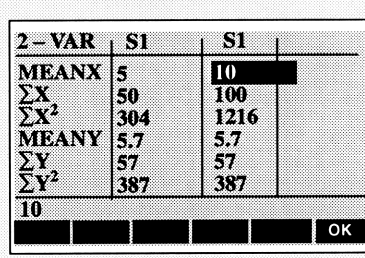


figure 13.40

Use ▼ to move to the remaining statistics.
Note the effect of multiplying by 2 on both the Covariance and the Correlation coefficient.

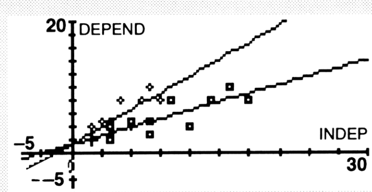


figure 13.41

Use all three views NUM SYMB and PLOT and the ideas developed above to study the effects of

(i) **adding or subtracting a constant**

to either one or both of the independent or dependent variable.

(ii) **Multiplying or dividing**

either one or both of the independent or dependent variables by :

(a) a positive Real number

(b) a negative Real number

You should explain the effects on measures of *location* ; on measures of *spread*; and on measures of relationship.
The graphing approach would prove useful here.

General comment on modelling, extrapolation & predictions

There are many other statistical functions and ideas that can be developed on the HP 38G. For example there is an excellent ApLet available on the Internet, called **RESIDUALS**, which enables you to do some investigative work on regression and residuals.

This manual only attempts to provide you with the starting points that should help you to move into the more expert areas that still remain to be explored on your HP 38G

In the section below I have outlined one example using the power of the HP 38G to consider a *best fit* problem drawn from actual experimental data.
It serves as a warning that predictions are only as good as the model upon which they are based. Many forecasts of future trends suffer from this very problem where the underlying model upon which the extrapolations were based were not necessarily the most appropriate models.

13.13 Experimental Data

Curve fitting - and a warning!

An experiment on Decay. This experiment was done in a normal classroom. The material used in the experiment consisted of 194 dice, distributed amongst five groups into which the class had been divided.

- The teacher rolled a die – the outcome was **5.** This number was used to represent DECAY. All 194 dice were then rolled. All 5's occurring on each roll were counted then removed, placed to the side, and not used in the remaining rolls. The total number of fives rolled by all the groups combined was recorded as was the number of dice left.
- The process of rolling the remaining dice was repeated for 10 sets of rolls.
- The results of two experiments in one particular class were as follows:

Experiment 1: n = 194

Roll	No. 5's	No. left
0	0	194
1	31	163
2	34	129
3	15	114
4	21	93
5	12	81
6	19	62
7	13	49
8	8	41
9	4	37
10	8	29
:	:	:
n	?	?

Experiment 2: n = 194

Roll	No. 5's	No. left
0	0	194
1	38	156
2	24	132
3	21	111
4	20	91
5	9	82
6	10	72
7	7	65
8	19	46
9	9	37
10	7	30
:	:	:
n	?	?

- At the end of each experiment, the number of dice left was counted.. Plot both sets of data. The shape of the plotted data ***suggests*** an *exponential function* , or possibly even a *reciprocal function*? or ??

Investigating the experimental decay.

(i) What ***function*** of the form $y = \mathbf{f}(x)$ would *you* plot that would model this situation as closely as possible? The graph of your function should be a likely candidate for the *best fit curve* for the results of the experiment.

(ii) In determining this defining rule for your function, outline those factors you considered in reaching your defining rule or model.

(iii) Enter the data into your calculator then, using a linear fit, get both the equation of the line of best fit and also the correlation coefficient r_{xy} for the plotted data.

For the decay model in experiment 1 (The Number of rolls Vs the number of dice LEFT) the **line of best fit** was $y = 15{\cdot}95x + 170$ and the correlation coefficient $r_{xy} = -0{\cdot}97$

Consider these readings (or those for your own data & graph) and comment on any possible interpretation!

(iv) Select the exponential model. Check how well this model *fits* your data.

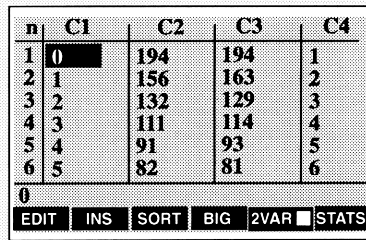
(v) Now try a best fit polynomial of order 2 (ie quadratic) ?
Then try a best fit polynomial of order 3 (cubic); then of order 4(Quartic)
What about order 5 (quintic). **WOW!!** The result is worth discussing.

(vi) WHAT IS THE MOST APPROPRIATE MODEL AND WHY IS IT THE MOST APPROPRIATE?

(vii) How well would the various models handle predictions involving extrapolation? Should this have any bearing on the *best fit model* that you finally choose?

- Press NUM Enter the results of both experiments as shown in *figure 13.42*
- The independent variable C1 is the number of rolls.
- The dependent variables in C2 and C3 represent the number of dice remaining after each roll. The data in C2 was obtained from experiment 2 while the data in C3 was from experiment 1.

Experimental results fed into the HP 38G



n	C1	C2	C3	C4
1	0	194	194	1
2	1	156	163	2
3	2	132	129	3
4	3	111	114	4
5	4	91	93	5
6	5	82	81	6

figure 13.42

- Press SYMB to get to the **SYMB**olic view. Define the data sets S1 and S2 as shown in *figure 13.43* (You may need to clear any existing definitions then press SETUP SYMB to select the linear fit model, then press SYMB to get back to *figure 13.43*

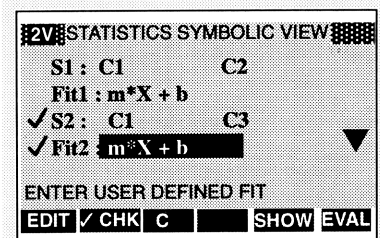


figure 13.43

- Press VIEWS LIB and select Auto Scale The plot and linear fit are shown in figure 13.44..

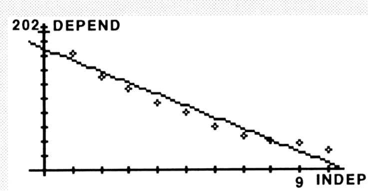


figure 13.44

- Go to SYMB select Fit2 then SHOW

  The **line of best fit** is given as

  y = 15·95x + 170 and the

  (linear) correlation coefficient r_{xy} =–0·97

- Select Fit2 press DEL then SETUP SYMB an for the S2FIT CHOOS Exponential

  This will fit a curve of the form

  f(x) = b*EXP(m*X) or $y = b(e^{mx})$

- Press PLOT to get the plot in *figure13.45*
- Press SYMB, select Fit2 then SHOW The best fit exponential curve is given as

  f(x) = 196·4EXP(–0·191*x*)

  ie $f(x) = 196{\cdot}4*e^{-0{\cdot}191x}$

- This is an excellent fit to the real probabilistic model! But is it the best fit?
- *Figure 13.46* shows a 5th degree polynomial fit for the same experimental data that gives a better fit than the exponential model. The sum of the squared residuals is **91·27**

  $f(x) = -0{\cdot}0056x^5 + 0{\cdot}158x^4 - 1{\cdot}6441x^3 + 8{\cdot}9168x^2 - 43{\cdot}213x + 195{\cdot}0018$

- There are many models that could fit given data. You should have sound reasons for accepting one model while rejecting another.

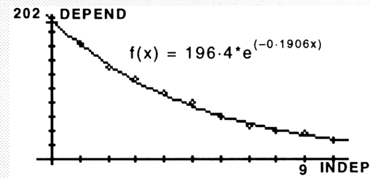


figure 13.45

The sum of the squared residuals for this model is $\Sigma R^2 = 93{\cdot}24$

Do you recognise this curve?
Can you give its definition in simpler terms.
$f(x) = 196*(0{\cdot}826^x)$ or $196*\left(\frac{5}{6}\right)^x$

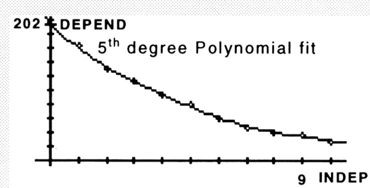


figure 13.46

For this data the polynomial model gives a better fit than the exponential model. On what basis would you justify using one model above the other?
Could you extrapolate with confidence with either model? There is a warning here that your predictions are only as reliable as the model you choose!

Treat the data provided in the other experiment in a similar way and come up with an appropriate model. Can you give the theoretic *mathematical model*?

CHAPTER 14

THE MENU OF MATH FUNCTIONS

To gain access to these functions

press the MATH key

The Menu of Math Functions available using the MATH key.

In addition to those functions displayed on the keyboard face of the HP38G there is available a wide range of further functions that can be used in normal calculations or within programs used in the design of ApLets.

Press the MATH key to obtain a display of these Functions and Commands. You should have a screen display similar to that shown in figure M1. Use the cursor keys ▼ ▲ to move up and down the list of the sub-menus. Note that as you move down the left side the list of functions and commands on the right side change.

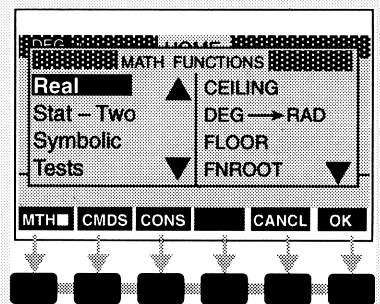
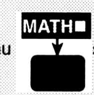


figure M1

The screen menu



should have a white box next to the word MATH to show that it is activated. If it is not active press the menu key and it will become active.

For example if Polynom. is selected on the left side, the math functions offered on the right side change to those shown in figure M2. Use ◀ ▶ to move across to the right side to access the functions or commands listed.

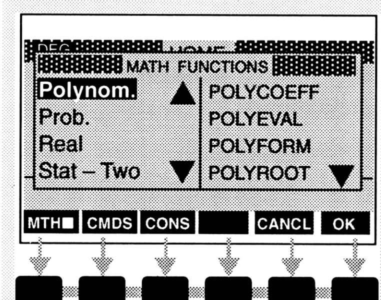


figure M2

Note: The lists on both sides of the split screen menus of the MATH Functions are given in alphabetical order. Real is chosen by default on the left side menu. Scrolling through the screen menus can be slow. If you know the first letter of the function that you wish to select *press the key of the first letter of that functions name*. Thus to choose Polynom. simply press 5 P (You do **not** need to press A...Z first). This shortcut applies to both sides of the split screen for MATHS FUNCTIONS menus.

A Summary and Description of the Main MATH FUNCTIONS

The outline below will cover and expand on those functions most *commonly* used in high school courses. For details on any additional functions not covered here refer to the users guide that comes with the HP38G.

The terminology for the syntax should be obvious from the context as set out over the next several pages. As a guide, the following conventions are adopted.

variable name refers to the name of the variable being used, (X or N or etc)

expression refers to any expression containing the variable (eg $N^2 + 5N$)

any number refers to a number, Real or Complex.

LIST2 refers to a list stored in the List Catalogue under the name L2.

Use [shift] 7 (LIST) then EDIT to check the Lists stored in the List Catalogue.

Characters such as this ∂ can be entered into the Calculator either by

(i) using the list of special characters obtained by pressing [shift] ((CHARS) or

(ii) pressing MATH and selecting the required function as described below.

Catalogue of the MATH Functions

Those for which the syntax is shown and examples provided are in **bold type**

TOPIC	FUNCTIONS AVAILABLE				
Calculus	∂	∫	TAYLOR		
Complex	**ARG**	**CONJ**	**IM**	**RE**	
Constant	e≈ 2·71828182846	**i** (=(0,1))	π ≈ 3·14159265359		
	MAXREAL =9·99999999999 x 10^{499}; **MINREAL**= 1·0 x 10^{-499}				
Hyperb.	ACOSH(x)	ASINH(x)	ATANH(x)	**ALOG**	**EXP**
	COSH(x)	SINH(x)	TANH(x)	**LNP1**	**EXPM1**
List	**CONCAT**	**ΔLIST**	**MAKELIST**		**∏LIST**
	POS	**REVERSE**	**SIZE**	**ΣLIST**	**SORT**
Loop	**ITERATE**	**RECURSE**	Σ		
Matrix	COLNORM	COND	**CROSS**	**DET**	**DOT**
	EIGENVAL	**EIGENVV**	**IDENMAT**	**INVERSE**	LQ
	MAKEMAT	QR	RANK	ROWNORM	
	RREF	SCHUR	SIZE	SPECNORM	
	SPECRAD	SVD	SVL	TRACE	**TRN**
Polynom.	**POLYCOEF**	**POLYEVAL**			
	POLYFORM		**POLYROOT**		
Prob.	**COMB**	**!**	**PERM**	**RANDOM**	
	UTPC	UTPF	**UTPN**	UTPT	
Stat–Two	**PREDX**	**PREDY**			

TOPIC	FUNCTIONS AVAILABLE IN MATH MENU			
Real	**CEILING**	**DEG→RAD**	**FLOOR**	**FNROOT**
	FRAC	**HMS→**	**→HMS**	**INT**
	MANT	**MAX**	**MIN**	**MOD**
	%	**%CHANGE**	**%TOTAL**	**RAD→DEG**
	ROUND	**SIGN**	TRUNC	XPON
Symbolic	=	ISOLATE	LINEAR?	QUAD
	QUOTE	|		
Tests	< ≤	==	≠ >	≥
	AND IFTE	NOT	OR XOR	
Trig.	ACOT	ACSC	ASEC	
	COT	CSC	SEC	

These functions can be accessed directly in the HOME screen by typing the function name directly into the EDIT line.

If **RREF(M2)** is typed directly into the EDIT line and ENTER is pressed this will give the **R**ow **R**educed **E**chelon **F**orm of the matrix stored as M2 in the matrix list.

Refer to the User's Guide that came with your HP38G if you wish to explore those functions which have not been developed in this Beginner's Guide.

1. The [Calculus] Functions

These can be entered into calculations in the EDIT line of the HOME screen.

Symbolic calculus can be carried out in the function ApLet. See chapter

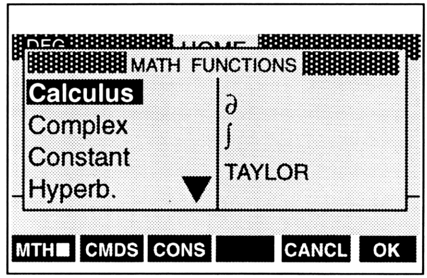


figure M3

Function	Syntax & an Example
∂	∂ [variable name] ([expression]) **Numeric differentiation:** To find the value of $\frac{dy}{dx}$ at $x = 4$ where $y = 5x^3$ In the HOME screen first store the value 4 in the variable name. [4] [STO▶] [A...Z] [•x] [ENTER] In this example *x* is the *variable name*. To get the [∂] into the EDIT line- use either [ ] [CHARS] and select ∂ or Press [MATH]; Select [Calculus]; then [▶] to select [∂] press [ENTER] then key in [A...Z] [•x] [(] [5] [X,T,θ] [x^y] [3] [)] [ENTER] ...... Answer 240 For Symbolic differentiation see the chapter on Calculus
∫	∫ ([Real No], [Real No], [expression], [variable name]) In the EDIT line you input ∫ (2,4,$3x^2$,x) ....Answer 56
TAYLOR	TAYLOR([expression], [variable name], [positive Integer n]) Gives the nth order Taylor polynomial of the expression at the point where the *variable* = 0

2. The Complex Number Functions

These simple concepts are used mainly in programming but can also be used in calculations in the HOME screen.

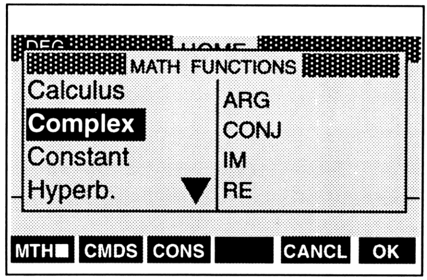


figure M4

Function	Syntax & an Example
ARG	**ARG**((x, y)) **Argument** Gives the angle defined by a complex number. The output depends on the setting of the angular mode. This is shown in the top left of the HOME screen. If the complex number is in the polar form (r,θ) then the argument is θ . eg In the EDIT line key in **ARG**((3,4)) ENTER … Ans 53·13
CONJ	**CONJ**((x,y)) **CONJUGATE** Gives the conjugate of a complex number eg In the EDIT line key in **CONJ**((3,4)) ENTER … Ans (3,–4)
IM	**IM**((x,y)) **IMAGINARY** Gives the imaginary component y of a complex number. (3,–4) ≡ (3 – 4i) Here the imaginary part is –4
RE	**RE**((x,y)) **REAL** Gives the Real component x of a complex number

3. The Constant Functions

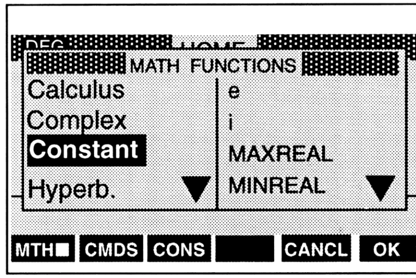


figure M5

These values take no arguments. They are simply numbers with special names.

These constants can be entered either into calculations in the EDIT line of the HOME screen, or into programs.

Function	Syntax & an Example
e	**e≈ 2·7182818284590...** The HP38G rounds this to 2·71828182846
i	The complex number (0,1). **i** is the principal square root of (–1)
MAXREAL	**MAXREAL = 9·99999999999 x 10^{499}** **MAXREAL** is the Maximum possible value for a Real number on the HP38G (If this is about k!, experiment using the EDIT line and COPY to determine the value of k. Use MATH Prob. menu to get the **!**).
MINREAL	**MINREAL = 1·0 x 10^{-499}** **MINREAL** is the Minimum possible value for a Real number on the HP38G.
π	Internally stored in rounded form on the HP38G the constant π ≈ **3·14159265359**

4. The Hyperbolic Functions

The Hyperbolic Trigonometric functions

ACOSH(x) ASINH(x) ATANH(x)
COSH(x) SINH(x) TANH(x)

will not be developed here.

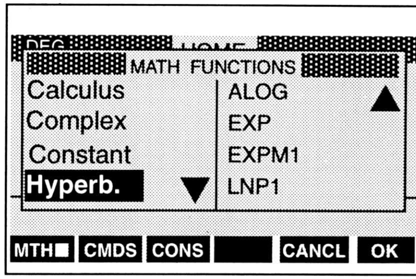


figure M6

Function	Syntax & an Example
ALOG	**ALOG(any number)** More accurate than 10^x (due to limitations of the power function)
EXP	**EXP(any number)** More accurate than e^x (due to limitations of the power function)
EXPM1	**EXP(any number) – 1** EXP(x) –1 is more accurate than e^x or EXP(x) for x close to zero. This uses the fact that *for x close to zero* $\mathbf{e^x \approx 1 + x}$ or $e^x - 1 \approx x$ eg $\mathbf{e}^{0{\cdot}000\ 000\ 000\ 003}$ gives 1 due to limitations of the power function. but EXPM1(0·000 000 000 003) gives 0·000 000 000 003
LNP1	**LNP1(any small number + 1)** Ln(x + 1). Uses the fact that *for x close to zero* $\mathbf{Ln(x + 1) \approx x}$ LN(1.000003) ≈ 2·9999955E–6; LNP1(0.000003) ≈ 2·9999955E–6 But Ln(1·000 000 000 003) gives 0 on the calculator while LnP1(1·000 000 000 003) gives 0·000 000 000 003

You can compare y = LN(x) and y = LN(x) +1 near zero graphically using PLOT and also numerically using NUM. Also compare y = EXP(x) and EXP(x) – 1

5. The LIST Functions

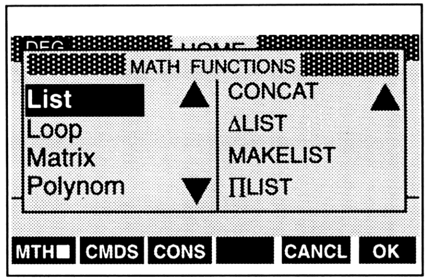


figure M7

These can be used in both the home screen and in programs.

Lists can be used as arguments with any of the normal operators (+ – * ÷ √ and so on.) Where more than one list is used in the arguments the lists must be of the same length. If ✪ indicates an operator then

List1 ✪ *List2* (lists must be of the same length) forms a new list which pairs the values under the operation ✪

Value ✪ *List* operation (value ✪) is done with each element in the list.

Function	Syntax & an Example
CONCAT	**CONCAT (LIST1, LIST2)** Concatenates, connects (chains together) the two lists into one list.
ΔLIST	**ΔLIST(LIST1)** ΔLIST(L1) will form a new list of the first differences of the sequence of numbers in List 1. ΔLIST(Ans) will give a list of second differences, and so on. eg if List 2 is {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41} then ΔLIST(L2) = {1, 2, 2, 4, 2, 4, 2, 4, 6, 2, 6, 4} and ΔLIST(Ans) = {1, 0, 2, –2, 2, –2, 2, 2, –4, 4, –2} Use ▲ and copy ΔLIST(Ans) to the EDIT line and repeat as often as required to obtain second and higher differences.
ΠLIST	**ΠLIST(LIST1)** ΠLIST(L2) gives the product of all the elements in List 2 If List 2 is as above then ΠLIST(L2) = 3·04250263527E14

Function	Syntax & an Example
ΣLIST	**ΣLIST([LIST1])** eg ΣLIST(L2) sums all the elements of in List 2. If List 2 is as above then ΣLIST(L2) = 238 Also ΣLIST({3,5,7}) returns 15 It is not essential that you use a list from the List catalogue.
MAKELIST	**MAKELIST(** [expression],[var name],[Start Val],[End Val],[step]) eg **MAKELIST(**N/2(N+1), N, 1,10,1) gives the list of the first ten triangular numbers {1, 3, 6, 10, 15, 21, 28, 36, 45, 55} You can store any list generated into the LIST Catalogue. Just press STO ▶ , when [Ans ▶] appears in the EDIT line you type L3 (or any other available list name from L1, …L0), then press ENTER If L3 already contains a stored list this will overwrite the old list.
POS	**POS([LIST], [any number])** POS(L3,9) returns **4**, indicating that 9 is the 4th element in List 3 If the chosen number is not an element of the list a **0** is given.
REVERSE	**REVERSE([LIST])** REVERSE(L3) lists the elements of List3 in reverse order.
SIZE	**SIZE([LIST])** This will give the number of elements in a list
SORT	**SORT([LIST])** This will rearrange the elements of the list in ascending order. If you require the list in descending order follow SORT([LIST]) with REVERSE([LIST])

6. The MATRIX Functions

Matrix operations are divided into two sections.
Section 1 *Matrix functions* These are accessed through the MATH key (Figure M9)

Section 2. *Matrix Commands* these are used in programming and are accessed

using the screen menu  in figure M9

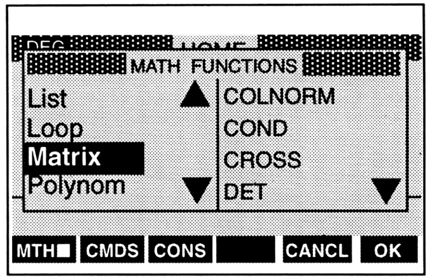


figure M9

Matrices are stored in the matrix catalogue and named M1, M2, ... M9, M0.
The listing below treats only those functions encountered in high school. Refer to the User's Guide that comes with the HP38G for information on other matrix functions.

Function	Syntax or function & an Example
CROSS	**CROSS(** Vector1 , Vector2) This give the ***cross product*** of two vectors
DET	**DET(** Matrix name) **DET** gives the ***determinant*** of a *square matrix.* eg **DET(M5)** where M5 is a matrix stored in the matrix catalogue.
DOT	**DOT(** Matrix1 name , Matrix2 name) Gives the ***dot product*** of two arrays eg **DOT(** Vector1 , Vector2)

Function	Syntax or function & an Example
IDENMAT	**IDENMAT(** Positive integerN) N gives the size of the matrix IDENMAT(4) creates a 4 x 4 *Identity matrix*
INVERSE	**INVERSE(** Matrix name) Creates the Inverse of a *square matrix* M, denoted as M^{-1} eg INVERSE(M1) ENTER. Note In the home screen M1 x^{-1} X,T,θ will give the same result.
RREF $M4=\begin{pmatrix} 1 & 2 & 5 & 8 \\ 3 & 5 & 8 & 1 \\ 5 & 8 & 9 & 4 \end{pmatrix}$ Try $M9=\begin{pmatrix} 1 & 2 & -1 & 5 \\ 3 & 6 & -9 & 7 \\ 2 & 4 & 4 & 5 \end{pmatrix}$ Interpret the solutions geometrically	**RREF(** Matrix name) **RREF** gives the **R**educed **R**ow-**E**chelon **F**orm of matrix M If M4 is the augmented matrix of a system of equations in 3 variables then **RREF**(M4) gives $\begin{pmatrix} 1 & 0 & 0 & -83 \\ 0 & 1 & 0 & 58 \\ 0 & 0 & 1 & -5 \end{pmatrix}$ The solution of the system can be interpreted from this matrix.. To store this new matrix in the matrix catalogue as M6 : press STO ▶, when Ans▶ appears in the EDIT line you type M6 and press ENTER. Check this by typing in the EDIT line M6 ENTER
TRN	**TRN(** Matrix name) **TRN(M4)** will give the transpose of matrix M

There are many other matrix functions that are outlined in the HP38G User's Guide which are not relevant to High School Secondary Mathematics Courses. Two of these are mentioned below as you may come across them in your research on the topic of matrices.

Function	Syntax & an Example
EIGENVAL	**EIGENVAL** (Matrix name) EIGENVAL(Matrix name) gives the eigenvalues in vector form of the Matrix
EIGENVV	**EIGENVV** (Matrix name) **EIGENVV** *Square* Matrix Gives two arrays (i) the eigenvectors; (ii) the eigenvalues

NOTE!

The screen menu key CMDS■ which appears in the screen menus (figure M10), enables you to move to another MATRIX sub-menu in the same manner as was done above with the MATH functions. All the features offered in this mode are ***commands*** that can be incorporated into programs.

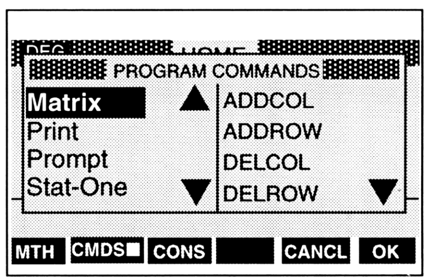


figure M10

7. The Polynomial Functions

A polynomial in x of degree 4 can be written as $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ or as $P(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

In either case it consists of powers of the variable x and constant coefficients

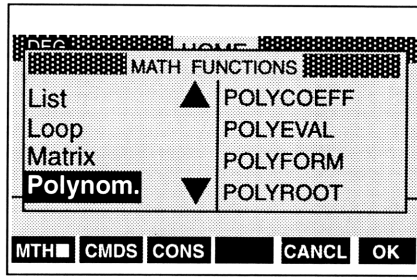


figure M11

Function	Syntax & an Example
POLYCOEF	**POLYCOEF** ([*root*1],[*root*2], [*root*3], ... [*root*n]]) Given the roots of a polynomial, determine the coefficients. eg **POLYCOEF** ([–3,–1,–1, 4]) ENTER returns the answer [1, 1, –13, –25, –12] Thus the polynomial with roots [1,1,–13,–25,–12] is $\mathbf{P(x) = 1x^4 + x^3 - 13x^2 - 25x - 12}$ WARNING THE COEFFICIENTS ARE ENTERED AS A VECTOR AND AS SUCH THE SQUARE BRACKETS ARE ESSENTIAL. Check that you enter both the parentheses and the square brackets as shown above.
POLYEVAL	**POLYEVAL**([coefficients], [Variable's value]) eg to evaluate P(**9**) given $P(x) = 1x^4 + x^3 - 13x^2 - 25x - 12$ POLYEVAL([–3,–1,–1, 4], **9**) ENTER This gives P(9) = 6000 Highlight **POLYEVAL**([–3,–1,–1, 4]**, 9)** and copy it to the EDIT line in the HOME screen. Alter the value 9 to other values. This work could also be done using the **function aplet** and the NUM view. Enter $F1(x) = 1x^4 + x^3 - 13x^2 - 25x - 12$ then press NUM. (You may need to scroll to the appropriate values of *x*., or you could choose the option to build your own table.)

Function	Syntax & an Example
POLYFORM	**POLYFORM** (expression , var name1 , var name2 ...) We shall use only **POLYFORM** (expression , var name1) to develop polynomials in one variable. Other forms are possible. This generates a polynomial in *variable 1* from the expression eg POLYFORM($(4x^2 - 1)^3 - 5x$, x) ENTER will give $64x^6 - 48x^4 + 12x^2 - 5x - 1$ eg POLYFORM($(2x + 4)(x - 5)(3x - 7)$, x) ENTER returns $6x^3 - 32x^2 - 18x + 140$ eg POLYFORM($(3x^2 + 4)(x^2 - 5x)$, x) ENTER gives the answer $3x^4 - 15x^3 + 4x^2 - 20x$
POLYROOT	**POLYROOT** ([coefficients]) eg For the polynomial $\mathbf{P}(x) = 6x^3 - 32x^2 - 18x + 140$ at the EDIT line input **POLYROOT**([6, –32, –18, 140]) ENTER This returns the result [2·3333333333, –2, 5] ie the roots of the polynomial $\mathbf{P}(x) = 6x^3 - 32x^2 - 18x + 140$ are 2·3333333333, –2, and 5 Use [shift] MODES HOME and set the NUMBER FORMAT to *Fraction* to get answers in fraction form [$\frac{7}{3}$ –2, 5] **POLYROOT**([a_n, a_{n-1}, a_{n-2}, ... a_3, a_2, a_1, a_0])the general case, will return the roots of the n^{th} degree polynomial.

8. The Prob. Functions

Functions associated with probability functions but not treated here include

(i) UTPC -Upper tail Chi-square Distribution.

(ii) UTPF -Upper tail F-Distribution

(iii) UTPT-Upper tail Student's T-Distribution

Refer to the Manual provided with the HP38g

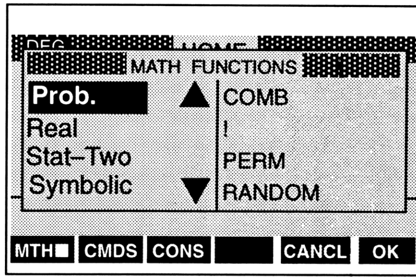
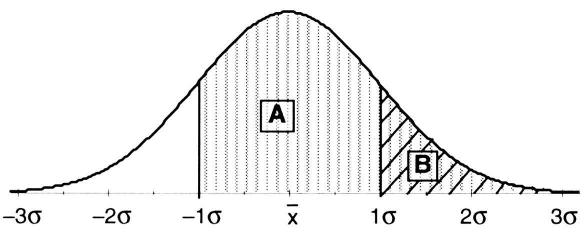


figure M12

Function	Syntax & an Example
COMB	**COMB (number *n* , number *r*)** COMB(n,r) The number of r-subsets in an n-set, is given by $\binom{n}{r}$ $\binom{n}{r} = \frac{n!}{(n-r)! \times r!}$ Read $\binom{n}{r}$ as *n choose r* Alternative notation for $\binom{n}{r}$ … $C_{(n,r)}$ or C^{n}_{r} or ${}_{n}C_{r}$ or ${}^{n}C_{r}$ eg To determine the value $\binom{12}{5}$, in the EDIT line of the HOME screen input COMB(12,5) ENTER The result 792 is displayed.
!	**(integer *n*) !** eg (12)! ENTER …… displays the result 479 001 600. The factorial symbol can be entered into the EDIT line by using (i) CHARS (Select the ! symbol then press ENTER **or** (ii) Press the MATH key. From the menus select Prob. ▶ !

Function	Syntax & an Example
PERM	**PERM(integer *n*, integer *r*)** PERM(n,r) gives the number of ordered arrangements (or Permutations) of *r objects* from a set of *n objects*. This is denoted by the symbol $^{n}\mathbf{P}_{r}$ where $^{n}P_{r} = \frac{n!}{(n-r)!}$
RAND	**RAND** eg **RAND** ENTER Note there are no arguments. Each time that you press ENTER after the initial entry a random number x is generated where $0 < x < 1$ Try to make a **List** of Random Numbers
UTPN The Normal Distribution does not need to be the Standard Normal Distribution **Any** Normal Distribution is accepted as long as the mean μ and variance σ^2 are known.	**UTPN(mean μ, variance σ^2, value)**  Gives the probability that a Normal Random Variable X is **greater** than the stated *value* for a Normal Distribution. eg To determine the probability that a Random Variable lies within ± 1σ of the mean μ in the Standard Normal Distribution Key in UTPN(0,1,−1) − UTPN(0,1, 1) ENTER The result for $P(-1 < X < 1) =$ 0·6826894492138 If a normal distribution has μ= 18 and σ^2 = 32·49 then to get P(X > 25) key in UTPN(18,32·49,25) ENTER Result 0·1097... To determine P(16 < X < 25) for this same distribution key in UTPN(18,32·49,16) − UTPN(18,32·49,25) ENTER = 0·5274...

9. The Real Functions

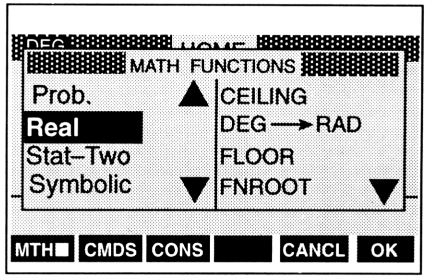


figure M13

Function	Syntax & an Example
CEILING	**CEILING(number x)** The smallest integer value that is greater than or equal to x. eg Ceiling(12·5) ENTER returns the value 13. In analysis this is referred to as the Least Upper Bound, or *lub*.
DEG→RAD & **RAD→DEG**	**DEG→RAD(number x)** Converts x° to its radian equivalent value eg DEG→RAD(47·65) ENTER displays the result 0·8316493… eg RAD →DEG (4·75) ENTER displays the result 272·14595°
FLOOR	**FLOOR(number x)** The biggest (or greatest) integer value that is less than or equal to x. (The function associated with this is named *the greatest integer function*) In analysis it is referred to as a Greatest Lower Bound or *glb*. eg FLOOR(12·5) ENTER returns the value 12.

Function	Syntax & an Example
FNROOT	**FNROOT(** expression, variable name, *first guess* **)** Function root finder. determines the values of the variable that make the value of the expression equal to zero. eg key in FNROOT($x^2 - 7x + 6, x$, 3) ENTER Returns an answer of 1. Now use the ▲ cursor to highlight FNROOT($x^2 - 7x + 6,x$,3) Press COPY to bring it to the EDIT line, then alter the 3 to another *estimate* or *guess* (Try 4, then try 5 as the *guesstimate.*) The technique used is the Newton-Raphson iterative process and the root provided is usually the one closest to your entered guess. The process is similar to that used by the **Solve ApLet.** The **Function ApLet** could also be used in problems of this kind.
FRAC	**FRAC(Any Real number *x*)** This will return just the fraction or decimal fraction part of a number. eg FRAC(54·65) ENTER will return **·65** the decimal fraction part of the number entered. If the number format is set to fraction (using MODES HOME), then the result for FRAC(54·65) is $\frac{13}{20}$

Function	Syntax & an Example
HMS→	**HMS→(hours.MinSec)** Converts the entered time in hours.MinutesSeconds or angle entered in(deg.MinSecs) into decimal time or decimal angle. For 14°30′45″ key in HMS→(14·3045) ENTER returns 14·5125
→HMS	**→HMS (Decimal number)** →HMS converts the entered decimal number for time or angle to hours-Minutes Seconds or (deg-min-secs) eg key in →HMS(14·5125) ENTER returns 14·3045 =14°30′45″
INT	**INT(Real Number *x*)** INT returns the integer part of the number *x* eg INT(47·352) displays the answer **47**
MAX	**MAX(number *1*, number *2*)** MAX gives the maximum of the two numbers. eg MAX(34·87,–45) displays the answer **34·87**
MIN	**MIN(number *1*, number *2*)** MIN gives the minimum of the two numbers. eg MIN(34·87,–45) displays the answer **–45**
MOD	**(number *1* MOD number *2*)** Gives the remainder when (number1) is divided by (number2). eg 43 MOD 8 gives the answer **3** (Note the spaces in the entry)

Function	Syntax & an Example
%	%(number *1* , number *2*) eg 5% of 80 Key in %(5,80) displays the answer **4**
%CHANGE	%CHANGE(number *1* , number *2*) Gives the percentage change when a number **c** is changed to number **d** % change $= \frac{d - c}{c} \times 100$ eg Give the % change when 40 is increased to 50 **%CHANGE(40,50)** displays the answer **25.** ∴ Change is 25%
%TOTAL	%TOTAL(number *c* , number *d*) What percentage of **c** is **d**? or Express **d** as a percentage of **c**. eg Express 35 as a percentage of 60 Key in %TOTAL(60,35) displays the answer **58·333**
SIGN	SIGN(number) The number can be Real or Complex. For a Real number x $\text{SIGN}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$ this is known as the signum function **sgn(*x*)** eg SIGN(5·6) = 1; SIGN(–8·2) = –1 For complex numbers **SIGN**$((x,y)) = \left(\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}\right)$ (This is the *unit vector* in the direction of the given number) eg SIGN((3,4)) = (0·6,0·8)

Function	Syntax & an Example
TRUNCATE	**TRUNCATE(Any Real number *x*, Positive integer *n*)** Truncates the number *x* to display n decimal places. eg TRUNCATE(78·987453,5) displays 78·98745

10. The Stat–Two Functions

These functions are used in statistics invoving bivariate data (ie data sets for two-variables)

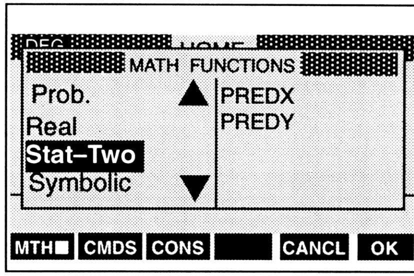


figure M14

Function	Syntax & an Example
PREDX	**PREDX(for this *y*-value)** After the data sets for the two variables *x* and *y* have been entered PREDX(for this y-value ENTER) will give a predicted value of x for the nominated *y*-value
PREDY	**PREDY(for this *x*-value)** After the data sets for the two variables *x* and *y* have been entered PREDY(for this x-value ENTER) will give a predicted value *y* for the nominated *x*-value

11. The Symbolic Functions

These functions are used for symbolic manipulation of expressions.
The = symbol serves an obvious purpose. It is available as a screen menu in the **Solve ApLet** It can also be inserted using CHARS (, choose = from the list of characters presented then press ENTER

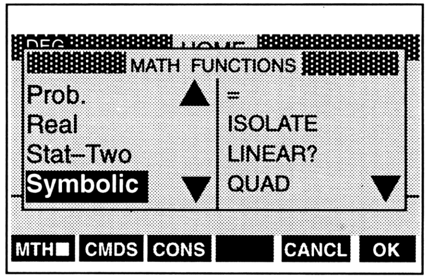


figure M15

I leave you to investigate this set on your own. The functions within this category certainly enable one to do much of the elementary traditional mathematics, but other functions available on this calculator do a better job in routine calculations.
However these functions do serve a purpose if you are interested in the programming capabilities of the HP 38G. This is beyond the brief of this introductory volume! Refer to your user guide for further details on many of the unlisted MATH functions. If you got this far with your calculator you are certainly ready to work through the HP 38G user guide with confidence.

APPENDIX

DIFFERENCE PATTERNS

DIFFERENCE PATTERNS - for POLYNOMIAL FUNCTIONS

Cases where the First Difference is constant.

Linear Functions of the form $y = mx + b$

x	$y = mx + b$	△1
0	b	
1	m + b	m
2	2m + b	m
3	3m + b	m
4	4m + b	m
5	5m + b	m
6	6m + b	m
	⋮	⋮

Example : *Determine an underlying rule that gives the linear relationship between v and n for this data.*

n	0	1	2	3	4	6	6
v	4	9	14	19	24	29	34

n	v	1stDiff
0	4	
		5
1	9	
		5
2	14	
3	19	5
4	24	5
5	29	5

In this example
the 1st difference is constant

∴ the Rule has the form:

$v = \square n + \triangle$

Here: $\square = 5$ and $\triangle = 4$.

The linear rule $v = 5n + 4$

Difference patterns that lead to QUADRATIC RELATIONSHIPS

A generalisation for cases where the first difference is not constant but the **second difference is constant**

x	$ax^2 + bx + c$	△1	△2
0	c	a + b	
1	a + b + c	3a + b	2a
2	4a + 2b + c	5a + b	2a
3	9a + 3b + c	7a + b	2a
4	16a + 4b + c	9a + b	2a
5	25a + 5b + c	11a + b	2a
6	36a + 6b + c	⋮	⋮
	⋮		

Example : Determine an underlying rule that gives the quadratic relationship between y and x for this data.

x	0	1	2	3	4	6	6
y	1	3	6	10	15	21	28

x	f(x)	△1	△2
0	1	2	
1	3	3	**1**
2	6	4	**1**
3	10	5	**1**
4	15	6	**1**
5	21	7	**1**
6	28	⋮	⋮
	⋮		

The relationship is quadratic

$2a = 1 \Rightarrow \quad a = \frac{1}{2}$

$a + b = 2 \Rightarrow \quad \frac{1}{2} + b = 2 \quad b = \frac{3}{2}$

$c = 1$

$\therefore\ y = \frac{1}{2}x^2 + \frac{3}{2}x + 1$

or $y = \frac{1}{2}[x^2 + 3x + 2] = \frac{1}{2}[(x + 2)(x + 1)]$

You may have been able to *guess* this pattern or rule without going through all this work. However this is not always as simple as in this example. The processes described here are easily carried out on the HP 38C using the **MAKELIST** and the **ΔList** functions.

An example using these functions is shown in the **Chapter 12 : Calculus Section 12.8**

DIFFERENCE PATTERNS THAT LEAD TO CUBIC RELATIONSHIPS

A generalisation for cases where the first and second differences are not constant but the **third difference is constant**

x	$ax^3 + bx^2 + cx + d$	△ 1	△ 2	△ 3
0	d	a + b + c		
1	a + b + c + d	7a + 3b + c	6a + 2b	**6a**
2	8a + 4b + 2c + d	19a + 5b + c	12a + 2b	**6a**
3	27a + 9b + 3c + d	37a + 7b + c	18a + 2b	**6a**
4	64a + 16b + 4c + d	61a + 9b + c	24a + 2b	
5	125a + 25b + 5c + d			
6				

Example : Determine an underlying rule that gives the cubic relationship between y and x for this data.

x	0	1	2	3	4	6	6
y	1	4	10	20	35	56	

x	f(x)	△ 1	△ 2	△ 3
0	1	3		
1	4	6	3	1
2	10	10	4	1
3	20	15	5	1
4	35	21	6	
5	56			
6				

The relationship is cubic

$6a = 1 \quad \therefore \quad a = \frac{1}{6}$

$6a + 2b = 3 \quad \therefore \quad b = 1$

$a + b + c = 3 \quad \therefore \quad c = \frac{11}{6}$

and $d = 1$

$\therefore \; y = \frac{1}{6}x^3 + 1x^2 + \frac{11}{6}x + 1$

$y = \frac{1}{6}\left[(x + 3)(x + 2)(x + 1)\right]$

You should now be able to extend this idea to higher power polynomials.

INDEX

Angle mode indicator. To change press MODES HOME

This section of the display contains the **history of all entries and answers** from the last time you pressed CLEAR DEL

What you **INPUT** via the keyboard appears on the screen's left side after you press **ENTER**

Menu labels are linked to the row of blank keys immediately below these menus. These menus change with the action you take.

The **Title bar** indicates which screen is active.

The α symbol at the top of the screen shows that you have pressed the **A...Z** key and an **alpha character** will appear in the EDIT line if you press an alpha key from the bottom six rows. To disable this press **A...Z** again. *The key acts as a toggle switch.*

After you press **ENTER** **answers** appear on the right side of the screen.

This is the **edit line.** Whatever you key into the calculator first appears on this line. You can **edit** or correct your entry **before** you press **ENTER**

The **screen menu label keys** carry out the action shown in a *menu label*. Press the blank key immediately below that menu label. Here the leftmost blank key will carry out the **STO ▶** (store) instruction. The other blank keys serve no purpose in **this** particular display.