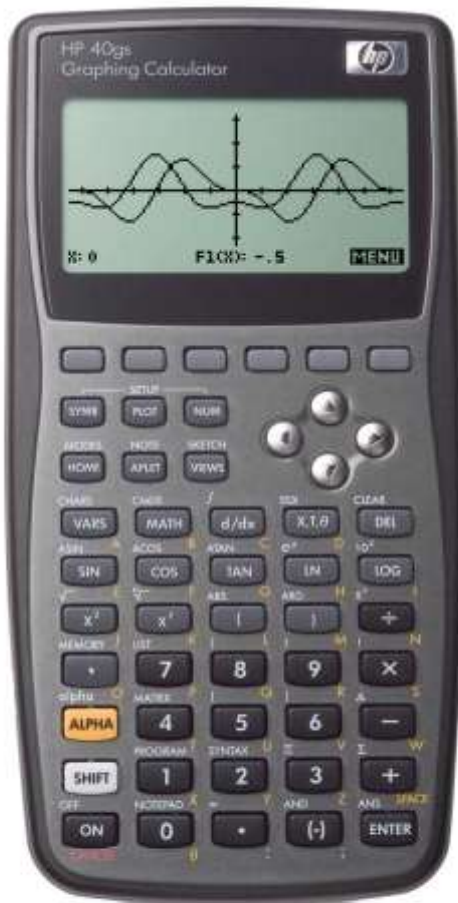


The Maths Activities Pack

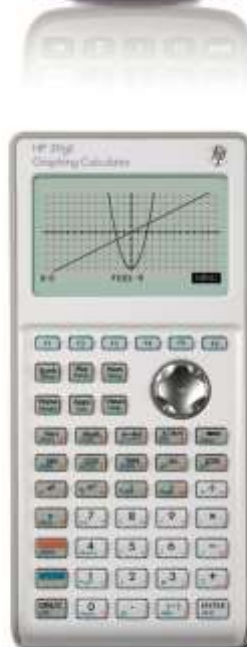
Using an HP39gII/40gs Graphing Calculator

Chris Olley



Contents

Introduction	3
Getting Started	5
APPS: There's nothing this machine will not do!	6
1. Number explorations: (HP39gII and HP40gs)	8
• Activities for exploring properties of number	
2. Exploring Factors (HP39gII and HP40gs)	11
3. Algebra explorations: (HP39gII and HP40gs)	12
• Exploring algebraic expressions	
• Place Value and the algebraic symbol system	
4. Exploring Calculus with CAS (HP40gs only)	15
5. Exploring Graphs (HP39gII and HP40gs)	17
• Graphing quadratics	
• Investigating quadratics	
6. Projectiles (HP39gII and HP40gs)	22
7. Pythagorean Triples (HP39gII and HP40gs)	23
8. Statistics (HP39gII and HP40gs)	25
• One variable (box-and-whisker)	
• Two variable (scatter plot)	
9. Data Streaming (HP39gII and HP40gs)	28
• Setting up	
• Walking the line	



Introduction

Graphing calculators have been around since the late 1980s. They are easily carried, turn on instantly, are extremely reliable, yet have software as powerful as anything available on a full computer for doing mathematics. In the classroom, ICT use is often difficult to arrange, booking the room, finding enough working machines and keeping the students off the internet are ever present problems. With a class set of graphing calculators in a small, easily carried box and a software emulator on the classroom computer, the teacher can have proper technology rich lessons with minimal set up time and maximal reliability. It just works!

Hewlett Packard have been in the forefront of calculator technology since its earliest days. They are renowned for the very high physical quality of the machines together with the most thoughtfully developed software. Sometimes, teachers are worried about the complexity of graphing calculators, but HP's designs work the way maths teachers think. Ease of teacher use is built in to the design from the start.

The activities in this booklet are designed for use with the HP39gII and HP40gs with additional activities also incorporating the StreamSmart data logging device. Both of the calculators have extremely comprehensive scientific functions, including matrices and complex numbers. However, their power is seen in the range of function types for which multiple representations can be set up. There is considerable research evidence showing the improvements in students' understanding when they have access to symbolic, graphical and table representations. The HP39gII and HP40gs make this a central feature and can quickly show all three, for functions plots, polar graphs, parametric graphs and many more. The same structure even applies to one variable and two variable statistics. Sets of side by side box-and-whisker plots can be drawn as soon as the data is entered, together with the summary statistics. Students don't need to laboriously draw the charts themselves, instead they can get straight to the statistical analysis.

The principle difference between the two machines is that the HP40gs additionally has a computer algebra system (CAS). This means that it can operate on algebraic statements, working symbolically. It will factorise symbolic expressions, it will integrate and differentiate symbolically and very much more. However it cannot be used in public examinations.

The HP39gII is a major development of this technology. Designed specifically to be acceptable for use in public exams, it takes the design of the HP39gs, which it has replaced and develops it. There is now a very high resolution screen able to show grey scales for beautiful, smooth graphs, coupled with a fast processor and more than enough memory. Also, a sophisticated battery management system means that standard AAA batteries can last for months, meaning you can always be sure of a complete set of working machines. There are very many ease-of-use improvements which the user will see in operation. Options will come up when they are needed. There are six function keys F1 to F6, which make navigation much clearer. Also, there is a comprehensive notes

system for developing classroom activities and a detailed, context sensitive help system.

The StreamSmart device together with data logging probes (available together as the HP Mobile Calculating Laboratory) plug directly into the calculators and allows very fast, real time logging of a variety of data types. For example, students can log their displacement from the device, creating real-time distance-time graphs. There is a thermometer, a microphone, a pressure probe and a light intensity probe in the kit, allowing for a range of experiments for STEM lessons.

The activities in this booklet are not set out as lesson plans. We assume that teachers will want to adapt them for different parts of lessons and different timings. Instead we have given details of the mathematics and instructions for conducting the activity, together with commentary on classroom use. All of these have been used by the author in perfectly ordinary secondary school classrooms.

The emphasis is on exploratory and investigative working. The power of the calculator lies in its ability to give instant feedback to ideas and to generate data very quickly. Students can generate data, make and test hypotheses about how mathematical operations and ideas work. They then test and develop their hypotheses to reach a conclusion. At this stage, the technology must be set aside and the student can practice their ideas away from the technology.

We would recommend a standard class set of one between two students, to maximise collaboration and discussion. However, there is strong evidence to suggest that students owning their own machine, personalising it and developing their understandings over time, brings the maximum benefit.

We would welcome your feedback and suggestions, whenever you try any of these activities with your students, to help us develop future editions of this booklet. Hewlett Packard is proud of the support that it offers and you should be confident to ask for any support you may need. A variety of manuals, guides and support is available at www.hp.com/calculators. Please also visit www.hpgraphingcalc.org for new downloads and details of training and support opportunities.

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June 2012

Getting Started

[Note: the photo shows an HP39gII, the key layout for the HP40gs is very similar]

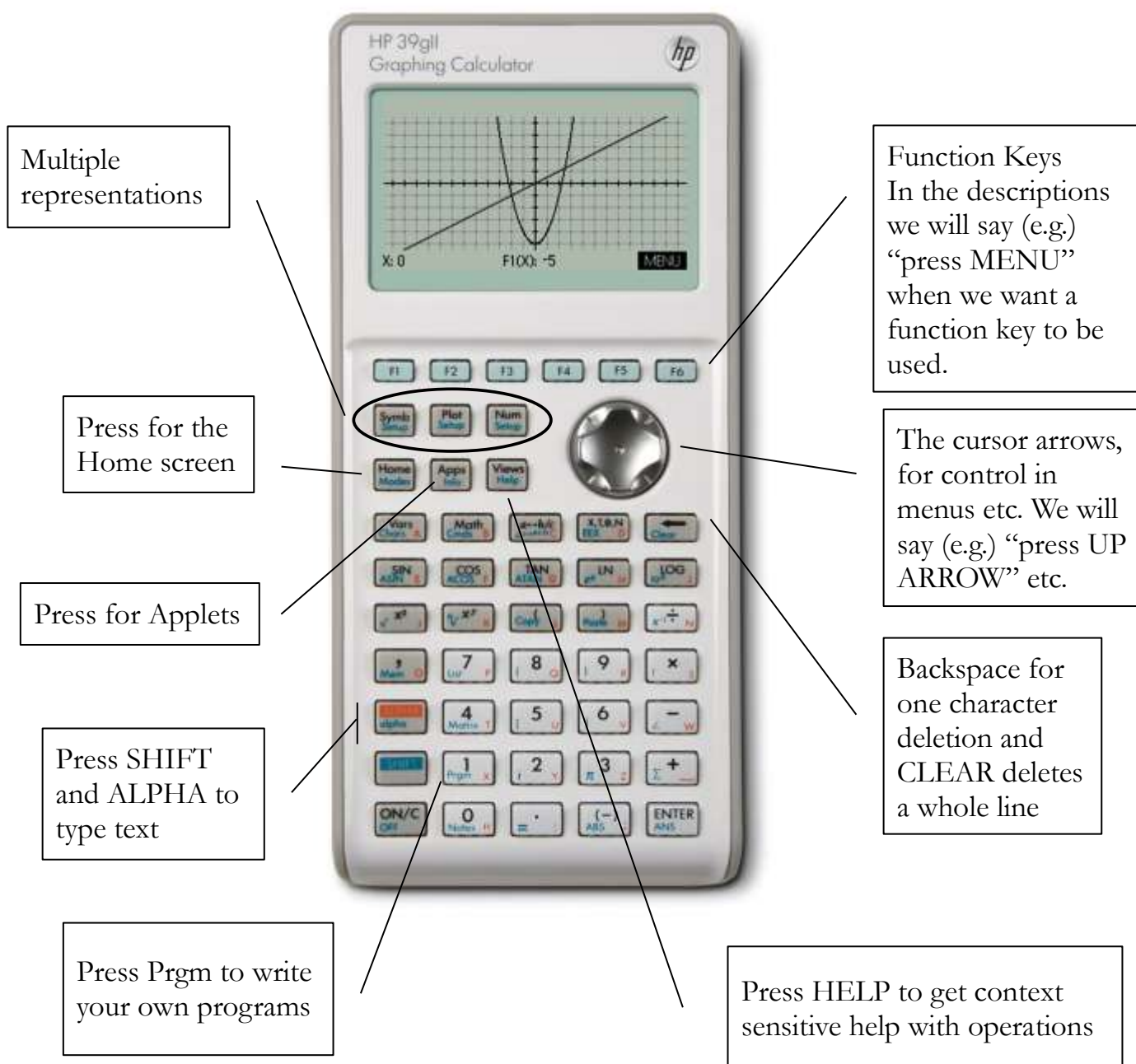
First, notice there is no = key. Use the ENTER key to evaluate.

Now find the HOME key. If you ever get stuck, press the home key to get back to the home screen.

Now find the Apps/APLET key. Press this to find all of the built in applications. This is the central place for advanced functions and graphing.

Find the SYMB, PLOT and NUM keys. These allow you to instantly switch between the three representations: symbolic, graph and table.

The top row of 5 (HP40gs)/6 (HP39gII) keys are function keys. Look at the bottom of the display. Small black rectangles above each function key show what the function key will do. In the photo, pressing the right most function key will show the graph menu. On the screen shown to access the menu press F6 ... we would say “press MENU”.



APPS: There's nothing this machine will not do!

As soon as they see a new piece of technology, students (i.e. all young people) are keen to see what it will do. Equally, teachers of mathematics have that really important activity that is a personal favourite and they will search to see if the machine will do the thing they need for it. The HP39gII and HP40gs are fully programmable machines with a sophisticated programming language. So, it is perfectly possible to develop applications to do anything (pretty much!) The manual contains the core of the information you will need, however you should download the excellent 'Mastering the hp 39gs & hp 40gs' by Colin Croft and read the chapters on programming. For up-to-date information and downloads go to my support site at www.hpgraphingcalc.org

For HP40gs users Colin maintains a web site at www.hphomeview.com. Your students will already have found this site and the vast array of games that these calculators can run, so best tell them about it first! Watch out for developments as new users develop Apps and classroom activities for the new HP39gII on www.hpgraphingcalc.org

Downloading Applets

You will need to install the connectivity software that comes on the CD supplied with the calculator. You may also need to download and install the USB driver (also on the CD).

Now follow these instructions:

- Find an applet you would like to install. Make sure it works with your machine (HP39gII or HP40gs), some older applets will only work with older machines. Choose to save the folder somewhere sensible on your computer. Unzip the folder and launch the connectivity software.
- Plug in your calculator with the mini USB cable supplied.
- In the left hand pane of the connectivity software, find the folder with the app you have downloaded and click on it.
- On an HP39gII, that's it. The new Applet will appear in your Apps menu.
- On the HP40gs press the Apps/APLET button. Press the function key for RECV (to receive a new applet). Choose USB Disk drive and press the function key for OK.
- The calculator will find any applets in this folder and show them as a list. Click the OK function key to confirm and the applet will install. The new applet will now appear in the list whenever you press the Apps/APLET button.

Using the Applets

- For instructions on using the applet, it is always worth looking at the More Information link on the web site.
- Press the Apps/APLET key, find the applet you want to run and press the function key for START. When you start the applet it should support you with suitable options.
- Once started the options for the applet can be found by pressing the VIEWS key on the calculator.

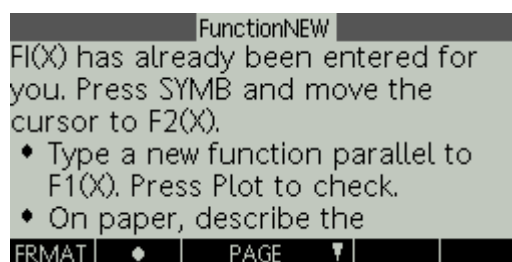
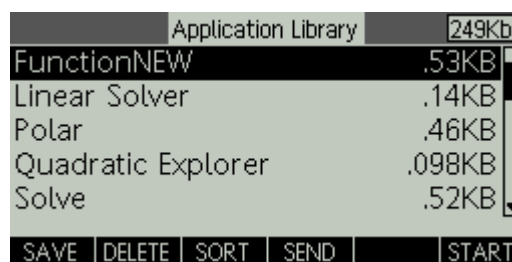
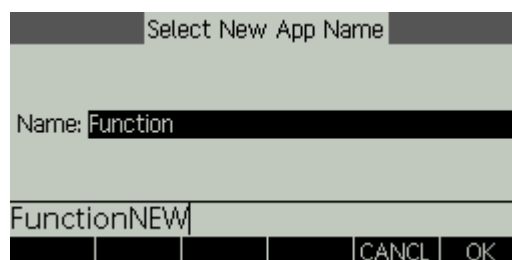
Saving Applets as Classroom Activities

You can save an applet with any settings or data that you have included. When you are ready, having used the applet, press Apps/APLET and press the function key for SAVE. You will be prompted to give the applet a new name. Press the function key for EDIT and use the ALPHA key to modify the name. Press OK, then OK again.

You will see your new applet in the applet library. Notice that you can easily delete it again.

On the HP39gII you have a really important new feature. Tied to each applet is an info/notes page. Press SHIFT + Apps for Info then the function key for EDIT. You have comprehensive formatting options and bullets available. You can now type instructions for students to tell them how to engage with the activity on the applet that you have set up for them.

We would anticipate that you would type up your notes on a computer using the emulator and then transfer them to your handheld. From there, it can be transferred to students using the cable machine-to-machine connection. Simply connect the two machines. On the first machine select the applet and press SEND. (On the HP40gs you will also need to press RCV for receive).



1: Number explorations: (HP39gII and HP40gs)

It is important to get students started with an activity which will make them comfortable with the machine. For any new group I have always started with a lesson asking students to explore the different functions that they can find. They can be guided to focus on the two rows of keys starting with SIN and X^2 and those below. I have found that if the teacher continues to prompt students simply to keep trying out new numbers and new functions, then interesting discoveries are made. Students can present their findings at the end of the lesson. In my experience in very ordinary schools, key stage 3 students will say things like “that SIN button, doesn’t matter which number you try, the answer is always less than 1”. Some students will explore lots of features and the teacher may need to keep them focussed on one interesting idea or may be prepared to let them try out anything!



Activities for Exploring Properties of Number

A very powerful use of this technology is to quickly generate lots of data from which students can draw inference. For me, it is much better if students can work out how mathematical processes work and then report their thinking to their teacher and their peers, rather than the teacher tell them their way of doing things. The learning is then much more secure and expressed in terms the student understands because they did it themselves. I use a set of simple activities (which nonetheless can generate some very thoughtful mathematics), to help students get started with this way of working. These activities could all be done just as well on a scientific calculator, but the aim here is to develop a way of working which can then take advantage of the advanced possibilities of the graphing calculator.

Comparing accuracy to different decimal places

Make sure the number mode is set to standard:
 shift MODES ↓ Choose Standard OK HOME

Now pick a suitable division to work with:

$3 \div 7$ ENTER

Start by setting the number mode to fixed:

shift MODES ↓ Choose Fixed → 1 OK Home (on the HP39gII you will need to copy the full decimal ... click UP to highlight it, then COPY) ENTER

shift MODES ↓ → 2 OK Home ENTER

shift MODES ↓ → 3 OK Home ENTER

shift MODES ↓ → 4 OK Home ENTER

etc.

RAD	FunctionNEW	
$\overline{7}$		0.428571428571
0.4		0.4
0.43		0.43
STO ▶		

Students should make conjectures as to what the calculator is doing before each change. They may need to develop their conjectures as they see different outcomes to what they expected. When they think they have a good conjecture using $3 \div 7$, the teacher can offer a different division to try (or students choose their own), then another. The task is only complete when the student can explain to their peers (and the teacher) how to write any number to a given number of decimal places (in terms that make sense to them) and can give sufficient examples of their own.

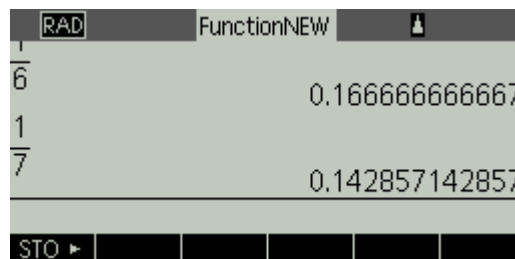
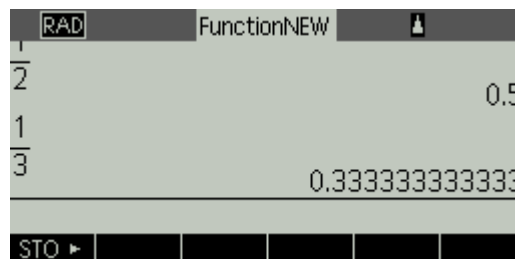
Exploring decimal representations of fractions

Make sure the number mode is set to standard:
 shift MODES ↓ Choose Standard OK HOME

Now enter fractions to explore e.g.

- 1 ÷ 2 ENTER
- 1 ÷ 3 ENTER
- 1 ÷ 4 ENTER
- 1 ÷ 5 ENTER
- 1 ÷ 6 ENTER
- etc.

Students can be asked to conjecture in each case. The order given is amusing because it suggests a pattern which does not emerge. Students can look for relationships e.g. between 0.5 and 0.25 and 0.33333... and 0.1666... Important details such as the suppression of trailing zeros and final rounding ($1 \div 6 = 0.1666\dots67$) become issues for discussion. Then students can explore with the calculator to find families of fractions and relationships between their decimal representations.



Exploring addition of fractions

(HP39gII) use the $a \leftrightarrow b/c$ key to show the result as a fraction.

(HP40gs) Make sure the number mode is set to fraction:

shift MODES ↓ Choose Fraction

Now enter added fractions to test

- $2 \div 7 + 3 \div 7$
- $4 \div 13 + 7 \div 13$
- $1 \div 3 + 1 \div 5$
- $2 \div 3 + 1 \div 5$
- $2 \div 5 + 3 \div 7$



(HP40gs) the calculator will show the results correctly formatted by using the up arrow to highlight the fraction and pressing SHOW.

Students should write the correct form of the output into their notebooks. $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$

The examples given are chosen to support students to explore each type thoroughly. They should try different examples of each type, make and then test conjectures about how the fractions are being added. They can then move on to the next type to see if their conjecture still holds or if they need to develop it further. In the end they should have a secure mechanism for adding fractions which they can explain and exemplify. Note that the calculator will always simplify fractions, so it may be necessary to guide students to examples which allow them to investigate the effect of this.

Exploring standard form

Make sure the number mode is set to scientific and fixed to (say) 3 d.p.

shift MODES ↓ Choose Scientific → 3 OK Home
 e.g. 56000 ENTER
 0.37 ENTER
 0.00000458 ENTER

RAD	Function
5.600E4	5.600E4
3.700E-1	3.700E-1
4.580E-6	4.580E-6
STO ▶	

Write the results in standard form:
 The calculator reports 56000 as 5.600E4
 Students should write $56000 = 5.6 \times 10^4$

then: 370 shift EEX 4 ENTER

Again, students should generate sufficient data to be able to describe their process and give examples in each case (convert into standard form, convert from standard form, numbers greater than 1 and numbers between 1 and -1 and finally arithmetic on numbers in standard form). As always it is important for the students to write their data, correctly formatted into notebooks. This allows a clear focus on accurate written mathematics.

RAD	Function
2.700E3*5.200E4	1.404E8
8.400E3/2.100E7	4.000E-4
STO ▶	

2: Exploring Factors

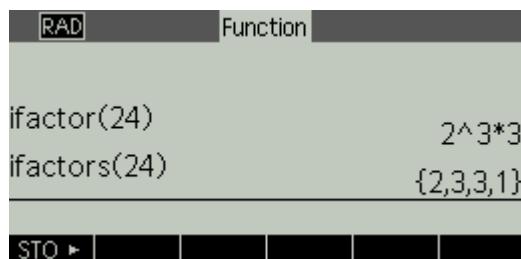
By generating lots of data from prime factorisations students will develop an idea of prime numbers as those with exactly two distinct factors and how only these numbers can be used to make up all other numbers. They can develop this work by investigating the effect of the Factor command on algebraic expressions.

The objective is to develop an understanding of prime numbers, methods for prime decomposition and the process of factorisation in general.

This is a very simple one on the HP39gII. We use the ifactor (for integer factorisation) command in the Math menu.

Press Math and navigate to Integer then ifactor type a number, say 24 and close the bracket. Press ENTER.

You can also see a list of the factors using the ifactors command.



On an HP40gs, this works very differently. Here, students will use the Factor command in the HP40's Computer Algebra System (CAS).

Press HOME to make sure you are on the home screen.

Note that on the HOME screen the CAS option is always available by pressing the right hand function key. Note that this option is NOT available on the HP39gII.

Press CAS.

Try out the function key to see the algebra menus. There is a lot to explore here, but save the temptation for later! For now press ALGB



We want to choose FACTOR. So press the DOWN ARROW three times, then press OK (on the function key) or press ENTER

Now we can choose a number to find the factors of. 24 is an interesting choice to start with. So type 24 and press ENTER

This yields the result $24 = 2^3 \times 3$



(The cursor keys allow us to select different parts of the expression to work on. At this stage it is not important, but it would be useful to experiment using up and down arrows, then left and right arrows to see how this works).

3: Algebra explorations: (HP39gII and HP40gs)

Exploring algebraic expressions

[Note STO is the left function key on the home screen]

The idea here is to get students to experiment with the values of expressions. They choose values for (say) 3 different letter variables.

Set some values for A, B, and C e.g.

5 STO ALPHA A ENTER

2 STO ALPHA B ENTER

(-) 1 STO ALPHA C ENTER



Students can then type different expressions to test. This works well as a paired activity. Students are allowed a maximum of (say) 5 characters to type an expression. Their neighbour has to say what the value of the expression is, before the first student presses the ENTER key to evaluate.

Now test different expressions (for example)

5 ALPHA A + 4 ALPHA B ENTER

(ALPHA A + ALPHA B) X² ENTER

2 ALPHA C X² ENTER

ALPHA B ^ ALPHA C ENTER

(ALPHA A + ALPHA C) ÷ ALPHA B ENTER



(HP39gII) To show the algebra correctly
SHIFT/Modes/Page down/Textbook display



(HP40gs) As before, to show the algebra correctly use the up arrow to highlight the expression and press SHOW.

Students should use this facility to copy examples into their notebooks.



Place Value and the Algebraic Symbol System

This activity gets students to explore place value by setting up expressions. First, the letter variables A to F are defined as a suitable set of powers of 10.

Press 1000 STO ALPHA A ENTER
 Press 100 STO ALPHA B ENTER
 Press 10 STO ALPHA C ENTER
 Press 1 STO ALPHA D ENTER
 Press .1 STO ALPHA E ENTER
 Press .01 STO ALPHA F ENTER

A is 1000
 B is 100
 C is 10
 D is 1
 E is 0.1
 F is 0.01

We suggest that the teacher does this on the board before students have access to the calculators themselves. They can then have a worksheet to fill in like the one on the next page. Students can be allocated a calculator when they have filled in the ‘My Idea’ column for every question. They then follow the instructions to set up the calculator and check, filling the calculator response into the end column.

After this students working in pairs can set and test challenges for each other as before.



Place Value and the Algebraic Symbol System Worksheet

Press 1000 STO ALPHA A ENTER

Press 100 STO ALPHA B ENTER

Press 10 STO ALPHA C ENTER

Press 1 STO ALPHA D ENTER

Press .1 STO ALPHA E ENTER

Press .01 STO ALPHA F ENTER

A is 1000

B is 100

C is 10

D is 1

E is 0.1

F is 0.01

1.		My Idea	Calculator check
	3A		
	5C		
	2B		
	8E		
	A + B		
	2B + C		
	5C + D		

2.		My Idea	Calculator check
	D + E		
	3C+2E		
	A+F		
	B+2C+3D		
	5B+D+3E		
	6A+3D+F		
	3C+D+E+7F		
	6A+E+F		

3.		My Idea	Calculator check
	700		
	0.8		
	15		
	5020		
	307.6		
	5004		
	12.83		
	5070.09		

4: Exploring Calculus with CAS (HP40gs only)

Having a machine which can find do the algebra for you is very powerful. Clearly, if an exercise question (or exam) requires a student to just do the algebra, then the machine cannot be used. So, we have two principal opportunities:

- Use the calculator to provide plentiful data to explore how the algebra is being done.
- Support more complex tasks by giving confidence in the algebra, to free up thinking time on the mathematics.

We will be using the CAS functions in this section. So, on your HP40gs press the CAS function key on the home screen.

Start by exploring the sums of series. Use shift and + to enter a sum.

Use the left and right cursor keys to navigate to each entry point in turn, to input a suitable sum. Use ALPHA to choose your variable.

Press enter to generate the solution (55)



TOOL | ALGE | DIFF | REWRI | SOLV | TRIG



TOOL | ALGE | DIFF | REWRI | SOLV | TRIG

This also works in general. But, we must use the up arrow to ensure the whole expression is selected. (The CAS acts on whatever is selected when ENTER is pressed).



TOOL | ALGE | DIFF | REWRI | SOLV | TRIG

Unsimplified solutions are always displayed first, so keep pressing ENTER to simplify further.



TOOL | ALGE | DIFF | REWRI | SOLV | TRIG



TOOL | ALGE | DIFF | REWRI | SOLV | TRIG

If you press the Σ key on the home screen, then you can only find numeric solutions in calculator format. For example enter $\Sigma(I=1,10,I)$ and ENTER.



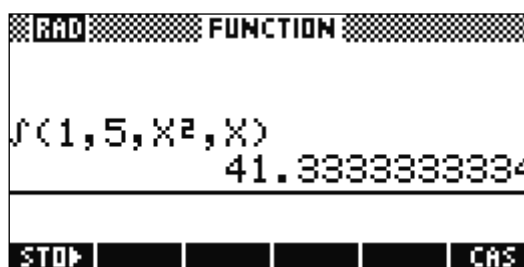
You will also have seen the calculus button.

The same is true of this button. On the home screen it works numerically e.g. press d/dx then X then the function. It will evaluate the differential at the current value of X. Use STO to change the value of X. In this screen I have used STO to set X=4 then entered the differential.



For numeric integrals, for example type in

$$\int_1^5 x^2 dx \text{ as } \int (1,5,X^2,X)$$



Press CAS to return to the CAS screen and explore the calculus commands again.

Pressing d/dx gives the template for differentiation. Type in suitable elements using the right arrow, then use the up arrow to select the whole expression. Then press ENTER.

(Press ENTER again to simplify if needs be).

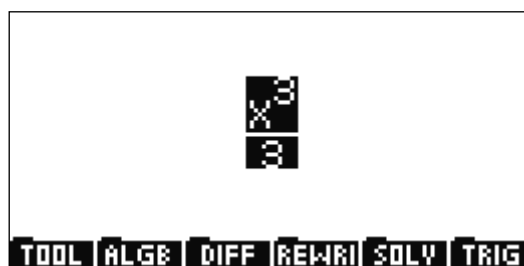
When you are done press CLEAR or the next operation will make use of whatever was left on screen.



Here is an example of integration. Experiment with different settings for the limits, to handle the constant of integration. Don't forget to use the up arrow to select the whole expression before pressing ENTER.



Students can now explore how the sums, differentials and integrals change with changes of function and with changes of values and limits. They can categorise calculus methods. How open or how structured their exploration, is down to the teacher's judgement, but the calculator provides any amount of data needed.



5: Exploring Graphs (HP39gII and HP40gs)

The graphing calculator naturally offers very quick and easy to use graphing software. Most graphing is controlled from the top row of labelled keys. These give quick access to the *multiple representations* of functions: algebra, graph, table. These are the SYMB, PLOT and NUM keys.



Notice that settings are available for each view by pressing shift plus the appropriate key.

You will want to put a variable in when you define a function. You can use the ALPHA key to choose any letter. However, there is an X,T,θ (HP40gs) or X,T,θ,N (HP39gII) key, which chooses the correct variable for any given graph type.

By default, the calculator graphs function plots $y = f(x)$. However, there are many other types of function which can be plotted. Press the Apps/APLET key to see what is available:

- Function
- Parametric
- Polar
- Sequence

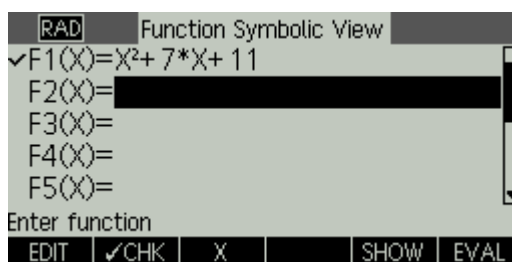
You can store a number of functions of each type and keep them in the calculator's memory, switching between the different types using the Apps/APLET key. We suggest that whenever you start a new activity, you RESET the applet before starting. This way all of the settings will be returned to normal and any graphs from an earlier user will be deleted.

Try this out to get started:

Press Apps/APLET navigate to Function and press RESET then OK/YES then START

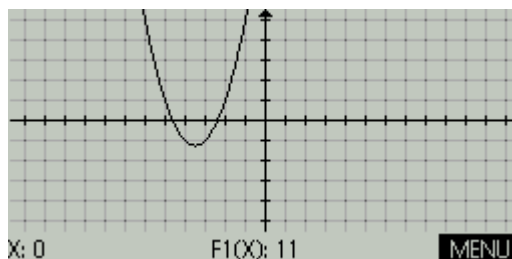
[This achieves the same effect as pressing the SYMB button on a new calculator]

Type a function (say) $X^2+7X+11$ and OK



Press PLOT

Notice the function key MENU. This gives access to all of the graphing facilities. You can ZOOM in on parts of the graph. You can TRACE to see values of points on the graph. The FCN key allows roots, turning points, slope, etc. to be found.



Press NUM

You can navigate up and down the table to look for (say) turning points or points of intersection.

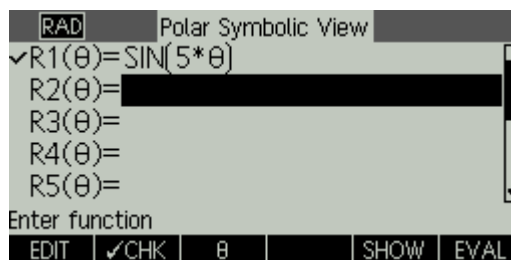
You can even zoom in the table, to find (say) roots to greater levels of accuracy.

X	F1		
0	11		
0.1	11.71		
0.2	12.44		
0.3	13.19		
0.4	13.96		
0			
ZOOM		BIG•	DEFN WIDTH3

Polar Graphs

Now press Apps/APLET navigate to Polar and press RESET then OK/YES then START

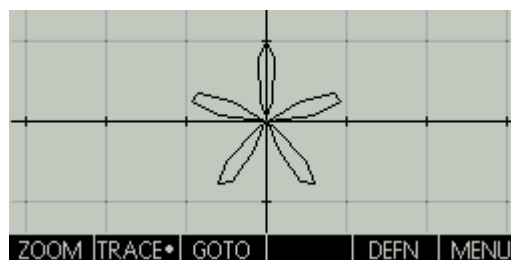
Notice the function definitions are now in polar form. Type $\sin(5\theta)$. The X,T,θ button now inserts a θ instead of an X. Press OK.



Press PLOT to see a tiny graph.

(HP39gII) Press the + or – keys to get a good view.

(HP40gs) Let's fill the screen by zooming. Press MENU then ZOOM. Choose Autoscale and OK.



Press Apps/APLET navigate to Function and press START (**Do not** press RESET!) You will see that your previous graph is still available as you left it.

You should experiment with all of the different graph and function types. Try out different ZOOM's. See what SETUP options are available in each applet and on each representation. Your students now have an extremely flexible tool to explore functions of almost any type choosing the most suitable representation to solve the problem at hand.

The next two pages contain a worksheet you can use which structures a student centred investigation into the effect of changing the coefficients on the graph of a quadratic function.

Investigating Quadratics Worksheet

A Quadratic Equation has the form $ax^2 + bx + c$
 These are all quadratic equations

$$y = x^2$$

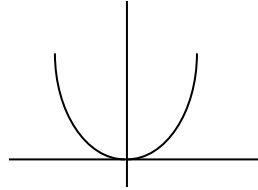
$$y = x^2 + 3x$$

$$y = x^2 - 4$$

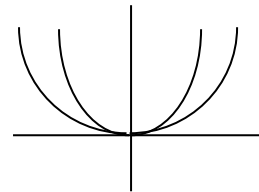
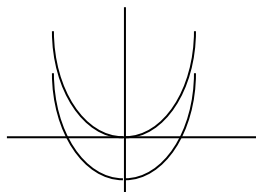
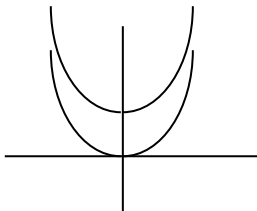
$$y = x^2 + 2x + 1$$



1. Graph $y = x^2$

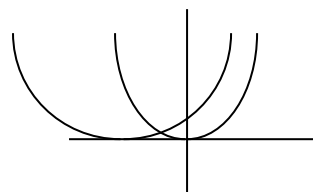
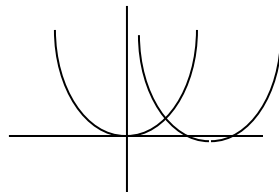
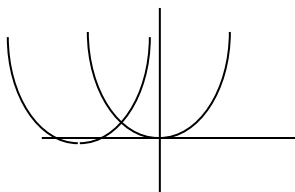


2. Make graphs similar to these.
Write down the equations you have used.



Make as many pairs as you can for each graph.

3. Make graphs similar to these.
Write down the equations you have used.



Make as many pairs as you can for each graph.

4. Write notes to explain how a graph of $y = x^2$ can be changed:
- to move up or down the y axis.
 - to move left or right on the x axis.
 - to get wider or thinner.

Hint: Try $y = (x - 1)^2$

5. Draw graphs of $y = x^2 + 2x$ and $y = x^2 - 4x$.

Draw more graphs, changing only the coefficient of x .

Explain the effect of changing the coefficient of x .

*Note: the number that we multiply the x by is the called the *coefficient* of x .*

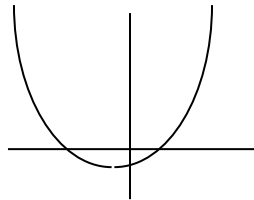
6. We can write a quadratic equation like this: $y = (x - 3)(x + 5)$

Multiply out the brackets and write down the expanded form of the equation.

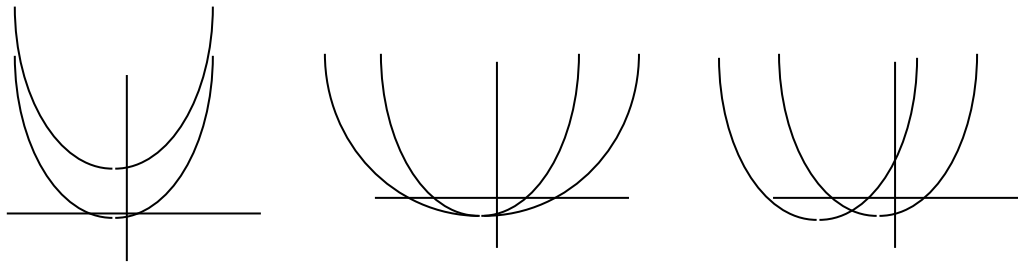
Try out equations of this type. Change the numbers. Change the $+$ and $-$.

Explain what effect the numbers 3 and 5 have on the position of the graph.

7. Draw a graph of $y = (x - 3)(x + 5)$



Make graphs similar to these. Write down the equations you have used.



Make as many pairs as you can for each graph.

8. Write notes to explain how a graph of $y = (x - 3)(x + 5)$ can be changed:
- (a) to move up or down the y axis.
 - (b) to move left or right on the x axis.
 - (c) to get wider or thinner.

Quadratic Roots

Try this activity yourself to consider the advantages of different methods for finding roots of a quadratic.

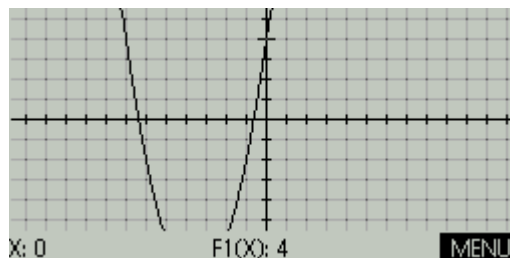
Method 1

Press Apps/APLET navigate to Function and press RESET then OK/YES then START

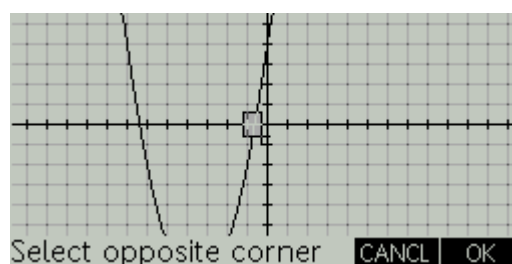
Type $X^2 + 7X + 4$ then press OK (Use X,T,θ,N for the X then X^2)

Press PLOT then MENU then ZOOM
Choose BOX... then OK

Move the cursor so that it is a little to the left and a little above one of the roots. Press OK.

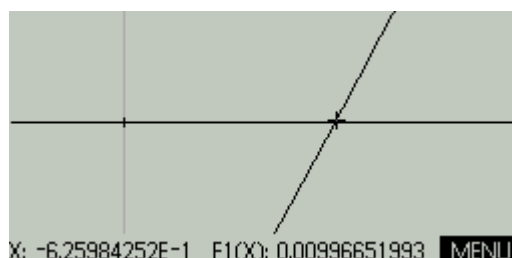


Now move the cursor a little to the right and a little below the root. You will see a box formed around the root. You should aim to make a very small box with the root in the middle. If it all goes wrong you can press ZOOM again and choose Un-zoom! Now press OK to zoom in so the contents of the box fill the screen.



Now you can press TRACE and move the cursor as close as possible to the root, giving its value.

You can now use Un-zoom to go back to the original graph and find the second root.



Method 2

Now find the solutions from a table of values for this function.

- Press NUM and use the up/down cursor to explore the table of values.
- Find which two numbers one of the solutions of $x^2 + 4x + 7 = 0$ falls between.
- Press SHIFT NUM and you can change the intervals in the table.
- Select NUMSTEP and type 0.01 then OK then NUM
- Repeat this to find both solutions correct to 2 decimal places.

Method 3

Now get the calculator to find the roots.

- Press PLOT MENU FCN ROOTS
- This will show the root closest to the cursor
- Move the cursor with the arrow keys closer to the other root
- Press MENU FCN ROOTS
- This will show the other root

6: Projectiles (HP39gII and HP40gs)

We can set up graphs with variable coefficients. In this way we can explore the effect of changing the coefficients. A good example of this is to graph a general projectile.

We know the horizontal equation of motion is $x = ut \cos A$

The vertical equation is $y = ut \sin A - 0.5gt^2$

Where u is the initial velocity, A the angle of projection, g acceleration due to gravity and t the time taken.

So, we have two parametric equations with parameter t .

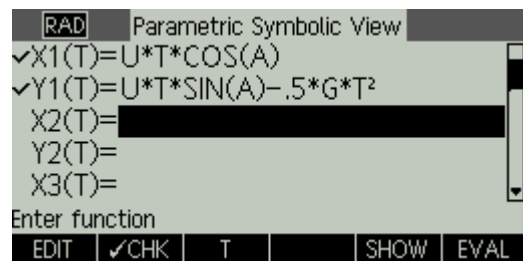
First we enter the equations: (note that T is the parameter so use the X,T,θ,N key)

Press Apps/APLET, find Parametric, press RESET then OK/YES, then START

In the first line type ALPHA U \times T \times COS(ALPHA A) then OK

Type ALPHA U \times T \times SIN(ALPHA A) $- 0.5 \times$ ALPHA G \times T² then OK

If you press PLOT now, you will see nothing, since the value of all of the variables defaults to zero. So, now we should assign values to the variables. First we will take $g = 9.8$



Press HOME, type 9.8 press STO ALPHA G ENTER

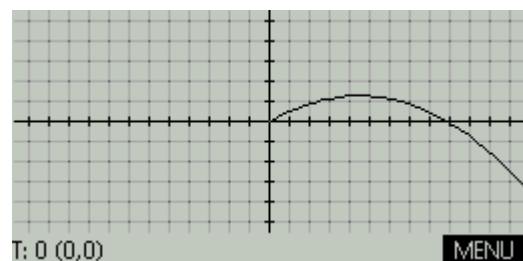
Now choose some sensible values for the other variables, (say):

30 STO ALPHA A ENTER

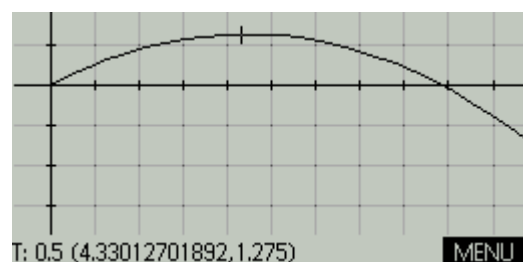
5 STO ALPHA U ENTER

Press SHIFT MODES and set the Angle Measure to degrees.

Now press PLOT again.



We can use ZOOM Box... to zoom in on the motion and use TRACE to find key points.



7: Pythagorean Triples (HP39gII and HP40gs)

This activity is based on an article by Bill Richardson which is highly recommended and will provide all you need to extend this introduction (last accessed 15.05.11):
www.math.wichita.edu/~richardson/pythagoreantriples.html

The idea is that if we know the first 4 Pythagorean triples in a sequence connected by some rule, then we can find a generating matrix to produce the others. Students can generate triples using a spreadsheet to find sets in this way. The most well-known would be where the last two values are consecutive integers. Now, students could find an algebraic generator for this set, but this is very hard for some other sets, so see how the matrix method works.

The first 4 triples are:

[3 4 5], [5 12 13], [7 24 25], [9 40 41]

Now we can set up a matrix statement showing the first three triples being generated by the 2nd, 3rd and 4th.

$$\begin{pmatrix} 3 & 4 & 5 \\ 5 & 12 & 13 \\ 7 & 24 & 25 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} 5 & 12 & 13 \\ 7 & 24 & 25 \\ 9 & 40 & 41 \end{pmatrix}$$

Which we can solve by pre-multiplying by the first matrix i.e.

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} 3 & 4 & 5 \\ 5 & 12 & 13 \\ 7 & 24 & 25 \end{pmatrix}^{-1} \begin{pmatrix} 5 & 12 & 13 \\ 7 & 24 & 25 \\ 9 & 40 & 41 \end{pmatrix}$$

On the calculator we can find the generator matrix using the MATRIX facilities. First we create the two matrices:

Press MATRIX then EDIT. Enter the values one at a time pressing OK. When you reach the end of the first row, press the down arrow. It will now carry on, knowing that the matrix has 3 columns.

The matrix M1 is now defined, so press MATRIX again and repeat for M2

M1	1	2	3
1	3	4	5
2	5	12	13
3	7	24	25

INS BIG* GO→ WIDTH4

M2	1	2	3
1	5	12	13
2	7	24	25
3	9	40	41

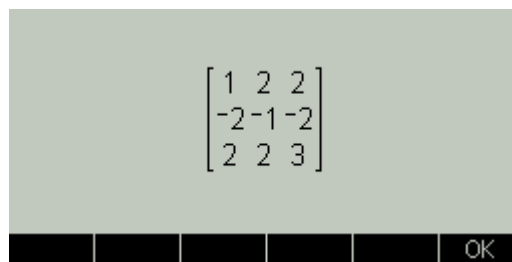
INS BIG* GO→ WIDTH4

We can now find the generator matrix. So, press HOME and type the statement $M1^{-1} * M2$

DEG	Parametric
M1 ⁻¹ *M2	[[1,2,2],[-2,-1,-2],[2,2,3]]

STO ▶

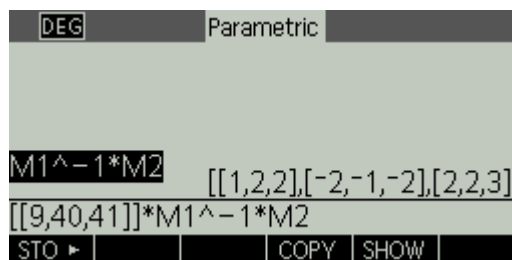
You can see the matrix clearly by using the up arrow to highlight it then choosing show.
Choose OK to return to the home screen.



To enter matrices directly on the home screen we use the square bracket, one bracket for the matrix, then another for each new row. Everything separated by commas.

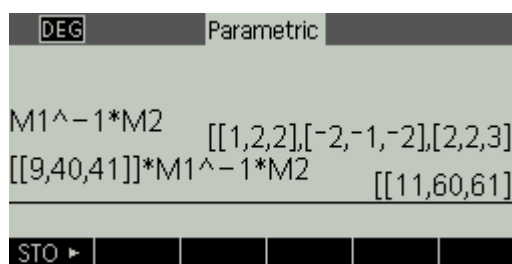
We can now generate triples:

Enter the last triple we know as $[[9,40,41]]$ then press \times then use the up arrow to highlight the statement we used to get the generator matrix $M1^{-1} * M2$ and choose COPY

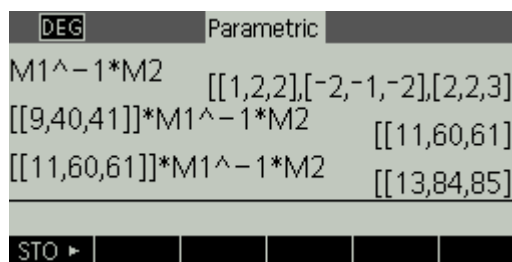


Then press ENTER

The next triple is $[11,60,61]$



Now use up arrow and COPY then \times then the up arrow to find the $M1^{-1} * M2$ and choose COPY again, then ENTER to find the next, and the next, ...



I hope this is sufficiently compelling an idea for students to want to work out how it works. The powerful matrix tools make the mathematics possible. Students need now to go away and work out how matrix multiplication and matrix inverses work. Then they can use this knowledge to work out how the method works and hopefully go on and prove it.

To practice, here are two further sets of triples. Firstly, where the first and last numbers are consecutive odd numbers:

$[3 \ 4 \ 5]$, $[15 \ 8 \ 17]$, $[35 \ 12 \ 37]$, $[63 \ 16 \ 65]$

Secondly, where the first two numbers are consecutive whole numbers:

$[3 \ 4 \ 5]$, $[20 \ 21 \ 29]$, $[119 \ 120 \ 169]$, $[696 \ 697 \ 985]$

8: Statistics (HP39gII and HP40gs)

A very powerful feature of the graphing calculator is the ability to generate statistical charts and graphs and summary statistics for data sets, very quickly. This allows students to see the multiple representations: the data set, the chart, the summary statistics. They can see the relationships between these representations.

If we have single variable data, then we are likely to be comparing two or more data sets. We want to see the differences in distribution and we want to find summary statistics to make comparisons. Box-and-whisker plots are widely seen to be excellent visual tools for comparing single variable data sets. They clearly show the inter-quartile range together with the outlying quartiles and visually showing the average (median), and the range and inter-quartile range. Two such plots side by side allow powerful and immediate inference to be drawn. The HP39gII and HP40gs allow up to three box-and-whisker plots to be drawn side-by-side.

Step-by-step instructions for this work are on the following pages.

Students could collect data on (say) the age profile of visitors to three restaurants in town to support an investigation into their marketing requirements.

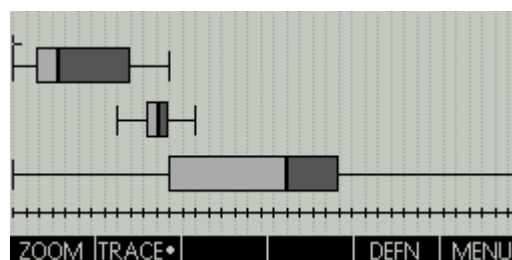
We can enter the data:

	D1	D2	D3	D4
1	3	14	3	
2	15	15	12	
3	6	13	42	
4	5	13.5	13	
5	9	14.5	15	

3

EDIT INS SORT BIG• MAKE STATS

Draw the plots:



See detailed summary statistics:

X	H1	H2	H3
n	11	12	19
Min	3	11	3
Q1	4.8	13.25	15
Med	6.5	14.15	24
Q3	12	14.9	28

11

BIG• WIDTH3 OK

This is achieved very quickly and now allows a detailed statistical analysis of the data, to support the project at hand.

Similarly 2 variable data can be handled equally efficiently, generating scatter plots, with regression and correlations calculated and different regression analysis available.

Step-by-step instructions start on page 27.

Drawing Three Side-by-Side Box and Whisker Plots

Choose the Statistics Applet:

Press Apps/APLET and use the arrow keys to find Statistics 1VAR (HP39gII) or Statistics (HP40gs)

Use the function keys for RESET and OK/YES and START

Enter your Data:

Type data in column D1 (HP39gII) or C1 (HP40GS) and press ENTER repeat until finished

(Use the right and left arrow keys to move between the lists)

- Under D1/C1 enter the data for the first set
- Under D2/C2 enter the data for the second set
- Under D3/C3 enter the data for the second set

Choose the Plots:

Press SYMB

Plot H1 will already be set to D1/C1

Press the down arrow to set plot H2 use the function key then 2 then OK

Repeat for H3.

(HP39gII) For each plot select the Plot type and press CHOOS and select BoxWhisker and OK

(HP40gs) Press shift PLOT CHOOSE then STATPLOT and select BoxWhisker and OK

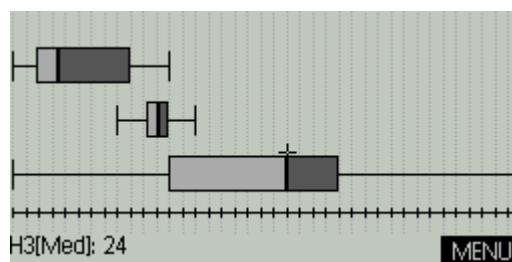
Now press PLOT and press MENU/ZOOM/Autoscale to optimise the view.

Get the Statistics:

Use the cursor keys to find the summary statistics.

(Press MENU to see the values)

- Minimum value
- Lower Quartile (Q1)
- Median
- Upper Quartile (Q3)
- Maximum value



Students can work out and write down the interquartile range.

Remembering: Interquartile range = upper quartile – lower quartile

Find the Statistics for the Data

Press NUM and function key STATS for summary statistics

Press function key OK when finished

Two Variable Statistics: Correlation and Regression

Choose the Statistics Applet:

Press Apps/APLET and use the arrow keys to find Statistics 2VAR (HP39gII) or Statistics (HP40gs)

Use the function keys for RESET and OK/YES and START

Enter your Data:

Type data in column C1 and press ENTER repeat until finished

(Use the right and left arrow keys to move between the lists)

- Under C1 enter the data for the first set
- Under C2 enter the data for the second set

Make sure both lists are the same length.

(HP40gs) Use the function key 1VAR to change to 2VAR

	C1	C2	C3	C4
1	12	29		
2	5	21		
3	6	20		
4	5.5	23		
5	4	18		

12

EDIT	INS	SORT	BIG•	MAKE	STATS
------	-----	------	------	------	-------

Choose the Plots:

Press SYMB

Plot S1 should already be set to lists C1 and C2

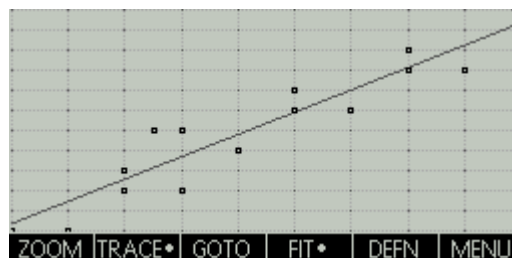
Now press PLOT

Press MENU ZOOM and use the arrow keys to choose Autoscale

Press MENU FIT to show the line of best fit. Press again to remove it.

DEG Statistics 2Var Symbolic View	
✓S1:	C1 C2
✓Type1:	Linear
✓Fit1:	M*X+B
S2:	
Type2:	Linear
Enter Independent Column	

EDIT	✓CHK	C	FIT•	SHOW	EVAL
------	------	---	------	------	------



Find the Statistics for the Data:

Press NUM and function key STATS for summary statistics

Press function key OK when finished

(Use the down arrow to see all the statistics)

Press SYMB and the regression line will now be showing as Fit1

Press function key SHOW to see it.

X	S1		
n	19		
r	9.3068E-1		
R ²	8.6616E-1		
sCOV	6.896199		
σCOV	6.533241		

19

STATS•	X	Y	BIG•	WIDTH3	OK
--------	---	---	------	--------	----

DEG Statistics 2Var Symbolic View	
✓S1:	C1 C2
✓Type1:	Linear
✓Fit1:	1.10909945921*X+15.037738
S2:	
Type2:	Linear
Enter Independent Column	

EDIT	✓CHK	C	FIT•	SHOW	EVAL
------	------	---	------	------	------

9: Data Streaming

Setting Up

You will need an HP39gII or an HP40G and a StreamSmart 400 or 410 interface unit.

You will also need suitable probes for the experiment.

Check that the probe you are using is connected to the CH1 input on the StreamSmart unit and that this is connected via its attached lead to the calculator. Also check that the StreamSmart unit is switched on and the green light is showing that the battery is OK.

On the calculator:

Press Apps/APLET and use the arrow keys to find StreamSmart. When it is highlighted press the function keys RESET then START

```
APLET LIBRARY 1998
Polar          0KB ▲
Sequence       0KB
Solve          0KB
StreamSmart    0KB
Linear Solver  0KB ▼
SAVE RESET SORT SEND RECV START
```

There are a variety of probes which can be used.

A Distance Probe can be used for matching distance time graphs. The following activity uses this feature. Also, mechanics experiments can be measured or pendula for simple harmonic motion etc.

A temperature probe can be used for analysing cooling (or heating) curves.

In a standard kit there are also probes to measure pressure and light intensity and even a microphone.

Walking the Line

Follow the set up instructions above with the Distance Probe attached to the StreamSmart unit.

Ch1: Dist. (m) Win:18.0s



x:21.42s y:6.008 m
 PAN SCOPE STOP



In this activity, students will walk along a line closer to and further away from the distance probe.

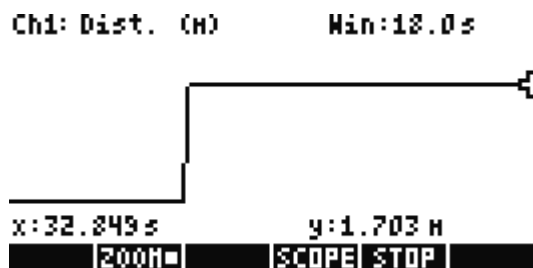
Put the probe on a table, facing a space at least 1 metre wide and 3 metres long. The centre of a classroom is good if the desks are moved apart a little. Find a volunteer and ask them to stand facing the probe, about 2 metres away.

We want to set up the screen so that we can see the motion.

Press the function key PAN and it will change to ZOOM. Press it again and it changes back. We will use PAN and ZOOM to set up the screen. PAN moves the image up and down. ZOOM makes the image expand in scale.

Choose ZOOM and press the up arrow about three times to increase the scale.

Then choose PAN and put your hand in front of the sensor to measure near to zero metres. Now use the up arrow to move the line to just above the numbers on screen. Take your hand away to measure the person 2 metres away again. Your aim is to look like this:



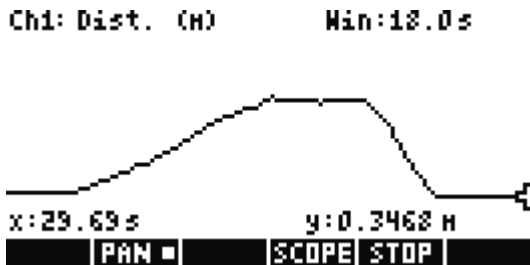
So the 2 metre point is about 2/3 of the way up the screen (so full screen will show 3 metres) and the bottom of the screen shows zero metres (i.e. with the hand in front).

This is a little fiddly the first time, but you will get better with practice.

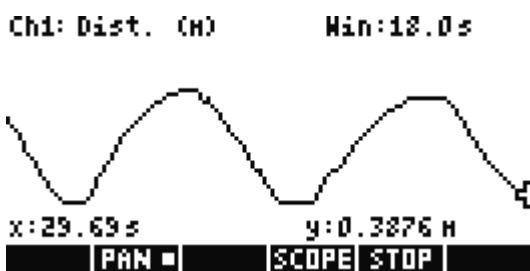
The Activity

You will need a collection of drawings of standard distance time graphs from a text book. The aim is for students to match their motion to the graph.

The student on the line should move backwards and forwards (always facing the probe) until they have created the required graph.



Teachers can be inventive with which graphs they use!



The author has prepared a presentation sequence with a variety of graphs which can be printed or displayed to support this activity. It is available on request.

Notes

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