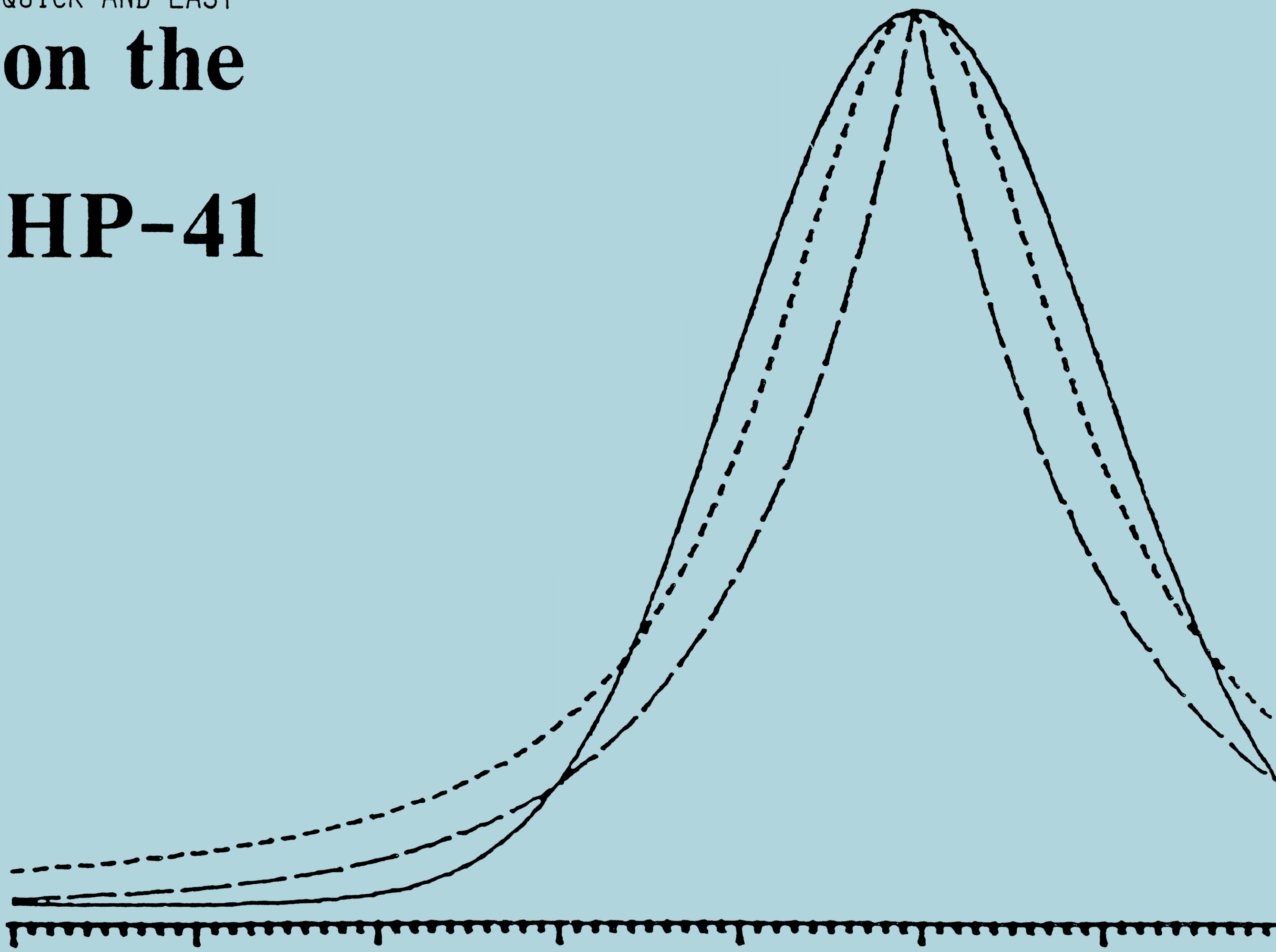
NONPARAMETRIC STATISTICS

QUICK AND EASY

on the

HP-41



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Donald L Vargo

NONPARAMETRIC STATISTICS
QUICK AND EASY
ON THE HP-41
BY DONALD L. VARGO

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"It is unworthy of excellent men to lose hours like slaves in the labor of calculation."

Gottfried Wilhelm Leibniz (1646-1716)

To my Parents

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PREFACE

This book is intended to make several widely used nonparametric tests immediately available to anyone with a background in inferential statistics. Although the importance of nonparametric methods has long been recognized by statisticians and behavioral scientists, these methods are still little known by other scientists. It requires no gift of prophesy, however, to foresee a growing and widening interest in nonparametric procedures due to their applicability and general utility.

The first section of this book contains an introduction to nonparametric statistics and measurement scales. Also included is a brief review of basic statistical terms and concepts.

The section is followed by the nonparametric tests themselves. The procedures are grouped according to what they test for: location, scale, goodness of fit and independence. Except for goodness of fit, each group is divided according to the type of data or number of treatments that the tests are designed to analyze.

Each test, along with any aliases, is presented with its use, data requirements, assumptions, equations and procedures for testing hypotheses. In addition, the three-letter program name, any subroutine requirement, and a size formula for determining data register space are presented. Limits and warnings are given for the test, program and table along with a large-sample approximation equation and hypothesis test for some procedures. When applicable, a comparison of the test with a competing nonparametric test is also included. Following this are instructions for using the program and sample problems. Fictitious problems, rather than examples from journals, are used to allow the general reader easier comprehension of the procedure's applicability. Run times denoted in the examples are given for compiled programs. (First-time runs through examples may take considerably longer than the listed times). For owners of the HP-41C, the maximum size limit given with each size formula presumes the calculator is equipped with a Quad Memory Module.

Appendix A lists programming steps for entering the programs and subroutines into the calculator. The programs are presented in the order their corresponding tests appear in the preceding section, with subroutines following the last program listing. Procedures for entering all steps can be found in the HP-41 "OWNER'S HANDBOOK AND PROGRAMMING GUIDE". No use has been made of synthetic programming steps.

Appendix B lists tables (and their sources) of distribution probabilities and selected critical values of test statistics.

I wish to express my appreciation to Mike Hale for permission to use a modified version of his sorting routine, Quicksort (QS); Bill Kolb for permission to incorporate his block duplicating routine, DUP, in several of my programs; Fred Ruland of the Research Computing Center at The Ohio State University for assistance in generating the distribution curves appearing on the cover and in the text using a VERSATEC™ Model 1200 plotter; and my wife, Agnes, for suggestions which greatly improved this book.

June, 1984 Donald L. Vargo

INTRODUCTION

Analyzing and testing data are skills essential in nearly every scientific discipline. Yet researchers are sometimes confused with regard to the interpretation of their data. This confusion is compounded when the data do not adhere to a basic assumption of classical statistics--that the observations are sampled from a normally distributed population.

Paradoxically, never before has the researcher's task of analyzing and interpreting results been made easier than it is today. The advent of the personal computer brought a convenient means of processing complex quantitative information literally to one's fingertips. In addition, methods have been developed that replace the intuitive approach to decision-making with objective procedures when certain assumptions are suspect. These methods belong to the realm of "nonparametric", or "distribution-free" statistics.

Nonparametric methods are valid even when the traditional assumption --that the underlying population distribution of the sample is normal-- is not met. They are only slightly less efficient than their parametric counterparts when the distribution is normal, and may be greatly more efficient when the distribution is not normal. Nonparametric tests are widely applicable and their concepts easily grasped. Though the arithmetic manipulation of most nonparametric tests is easy to perform, it is often tedious, laborious and, therefore, prone to error.

This collection of nonparametric programs is no substitute for a study of nonparametric methods. Rather, it is unabashedly a "cookbook" approach to aid those familiar with elementary statistical procedures to quickly and easily apply nonparametric procedures to the data at hand, particularly when the data are of a form not amenable to parametric procedures or the assumption of normality cannot be substantiated. It is designed to provide reliable, straightforward access to the more popular nonparametric tests. Every test is accompanied by a concise format for using the program and by representative examples. Quite simply, it merges two powerful modern scientific tools--nonparametric statistical methods and the HP-41 programmable calculator-- so that today's investigators can reach unbiased conclusions from facts and make decisions based on them.

WHAT IS NONPARAMETRIC STATISTICS?

There is no general agreement regarding the meaning of the adjective, "nonparametric". Strictly speaking, nonparametric procedures are unconcerned with population parameters. But this is misleading because nonparametric procedures do indeed deal with parameters such as the median and the variance. Also, through popular usage, the expression "distribution-free" is often used interchangeably with "nonparametric", though they are not synonymous. Distribution-free, in which no assumption is made about the precise form of the sampled population, better describes the attribute that makes these tests appealing.

Parametric statistics, in contrast, rests on the assumption that the sampled population follows a normal, or Gaussian, distribution. This assumption may be unjustified when the sample size is small and no prior knowledge exists as a basis for supposing a normal distribution for the population.

The population assumption most frequently encountered for distribution-free tests is that all observations are continuously distributed. Theoretically, samples from continuous populations will not have tied observations. The value of each observation in a sample should be unique. In practice, though, ties do occur and are taken to be due to limitations of our measuring instruments. There are several remedies for treating tied observations and the programs herein deal with ties in an apparent and appropriate manner.

ADVANTAGES AND DISADVANTAGES OF NONPARAMETRIC STATISTICS

For any subject or discipline where objects or methods may compete, there are advantages and disadvantages to be weighed. So, too, when choosing between a parametric and a nonparametric procedure. At times the circumstances are such that one method is obviously better suited than the other. But often the choice relies on subtler reasoning.

If a strong argument exists that the population under study is normally distributed, then parametric procedures are generally more efficient and make better use of the information conveyed by the data. If, however, the form of the underlying distribution is questionable or unknown, nonparametric tests offer a safe alternative.

Aside from the fact that nonparametric procedures can waste information if a parametric procedure is suitable (and that, as yet, there is no nonparametric method for testing interactions in the ANOVA model), there are several advantages inherent in nonparametric procedures.

1. Since the assumptions are generally weaker than those of parametric tests, there is less likelihood to misuse the tests by disregarding the assumptions.

2. When the sample size is small, nonparametric tests are only slightly less ef-

ficient than their parametric counterparts, even if all assumptions of the parametric test are met. Also, unless the nature of the population distribution is known exactly, only a nonparametric test is justifiable.

3. When the sample size is large, some nonparametric tests still compare well with their parametric counterparts.

4. Nonparametric tests may be applied when the data are measured on a measurement scale inappropriate for parametric procedures. (See the following section).

5. The concepts and mechanisms of most nonparametric tests are easy to understand. Their derivation requires little sophisticated mathematics. As a result, comprehension of their validity and appropriateness renders them truly scientific to the investigator.

MEASUREMENT SCALES

We usually think of measurements as numbers indicating quantities of some object or trait we wish to assess. But this is not always the case. There are different levels of measurement which constitute the measurement scales.

1. Nominal Scale

   This is the weakest of the four measurement scales. As its name implies, this scale distinguishes one object or event from another on the basis of a name, e.g., male/female, heads/tails. Usually we use this scale when we are interested in the number of objects or events falling into each of various mutually exclusive and collectively exhaustive categories. Data of this type are frequently referred to as "count" or "frequency" data.

2. Ordinal Scale

   The ordinal scale is distinguished from the nominal scale by the additional property of order among the categories. Only the comparisons "greater", "less" or "equal" are relevant. The numeric value serves only to arrange the data being ordered from, say, smallest to largest, or worst to best. These type of data are frequently called "rank" data.

3. Interval Scale

This scale is one step above the ordinal scale in that the differences between measurements also have meaning. This scale involves the concept of a unit distance (and also a zero point) and the distance between any two measurements may be expressed as some number of units. Such measurements may be treated with the common arithmetic operations of addition, subtraction, multiplication and division. For example, an exam score of 80 is midway between scores of 70 and 90. We may also state that a student whose score is 80 has scored 40 points more than another student whose score is 40. But this may not necessarily imply the first student is twice as intelligent as the second, or infinately more intelligent than a student who scored zero. Comparisons of this latter type fall into the last measurement scale.

4. Ratio Scale

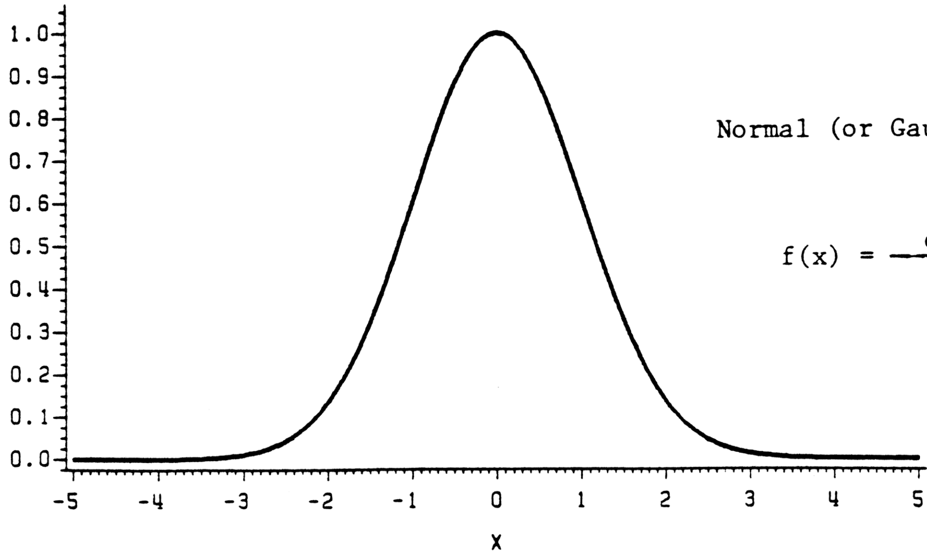
The ratio scale encompasses all the properties of the three lesser scales with the additional property of a true, or natural, zero. It is so named because the ratio of two measurements has meaning. Age, height, weight, yield and income are but a few examples of traits subject to this type of measurement.

While the data from a higher measurement scale may be analyzed using procedures designed for a lower measurement scale, the converse is not true. It is usually more profitable to take advantage of the higher level of measurement when it is available. Parametric procedures specifically require data obtained at the interval or the ratio scale of measurement, whereas most nonparametric tests require only an ordinal measurement scale.

POPULATIONS, SAMPLES AND DISTRIBUTIONS

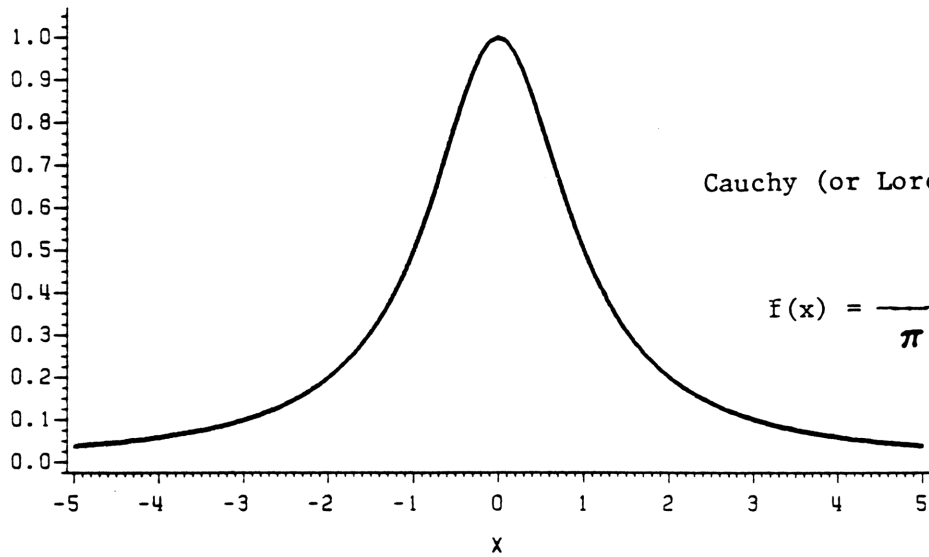
A <u>population</u> is all conceivable observations of a particular type, while a <u>sample</u> is a limited number of observations, or <u>data</u>, from a population. A <u>parameter</u> is a number used to describe a population, while a <u>statistic</u> is a number computed from the sample data. A <u>random sample</u> is chosen in a way that allows every observation an <u>equal chance</u> of occurring, while <u>independence</u>, another desirable and often necessary trait, implies the selection of one observation will not influence the selection of subsequent observations.

A <u>distribution</u> is the arrangement of observations according to their frequency of occurance. Some commonly encountered distributions are shown below.



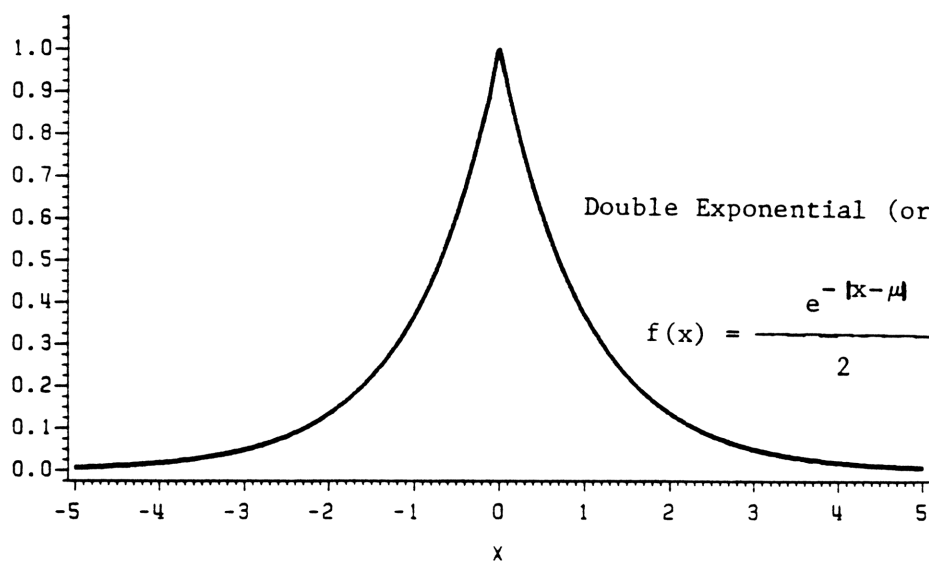
Normal (or Gaussian) Distribution

$$f(x) = \frac{e^{-(x-\mu)/2\sigma^2}}{(2\pi\sigma)^{1/2}}$$



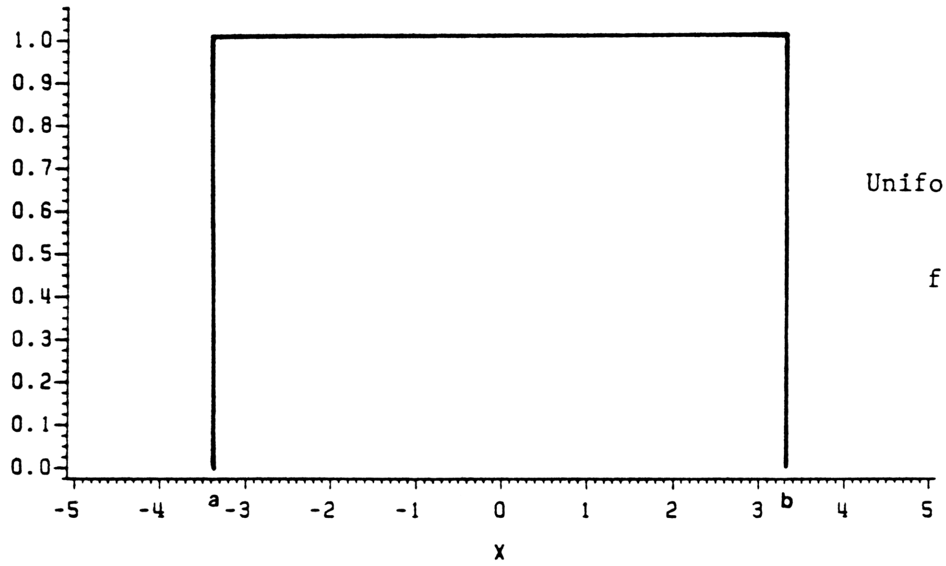
Cauchy (or Lorentzian) Distribution

$$f(x) = \frac{1}{\pi(1 + (x-\mu)^2)}$$



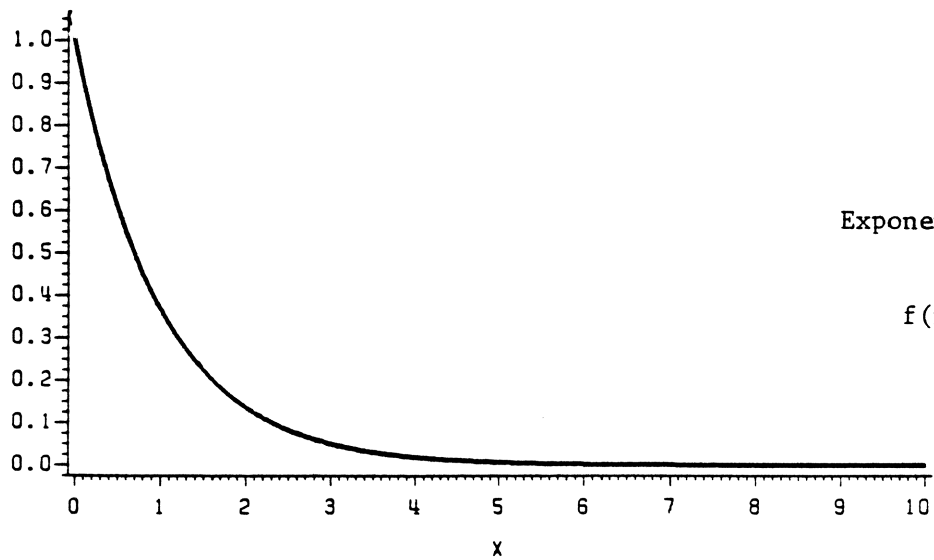
Double Exponential (or Laplace) Distribution

$$f(x) = \frac{e^{-|x-\mu|}}{2}$$



Uniform Distribution

$$f(x) = \frac{1}{(b-a)}$$



Exponential Distribution

$$f(x) = e^{-x}$$

CENTRAL TENDENCY AND VARIABILITY

A measure of central tendency is a figure typical or representative of the central value of the magnitudes of all observations in a distribution. The mean, median and mode are each measures of central tendency and are referred to as "location" parameters. The mean is the location parameter with which most parametric tests are concerned. Nonparametric tests substitute the median for the mean. This tends to attenuate errors associated with wild, extreme observations. For symmetrical distributions, the mean and median are equivalent.

A measure of variability indicates the dispersion of a set of observations. It is often used as a measure of precision. Synonyms for the term "variability" include dispersion, scale, scatter and spread. In nonparametric tests for the scale parameter, the symbols sigma (σ) and sigma squared (σ^2) should be interpreted as general measures of dispersion.

HYPOTHESIS TESTING

After planning and conducting an experiment, the data are analyzed according to a statistical test from which conclusions may be drawn. These tests consist of hypotheses, a test statistic, a critical region and a rejection rule.

There always exists the possibility of chance variation occurring when a population is sampled. The null hypothesis (H_0) is the assumption the results are due to chance alone. Results unlikely to have occurred by chance are called significant. The alternative hypothesis (H_1) is a statement or theory that an investigator concludes when the results are significant, i.e., when the null hypothesis is rejected.

A test statistic is used to help make the decision between accepting or rejecting the null hypothesis. It is nothing more than a random variable with a known distribution. Its distribution is usually discrete for nonparametric tests. Since its distribution is known, the range of values it may take when the null hypothesis is true is also known. This range is called the acceptance region. Values of the test statistic beyond the acceptance region constitute the critical, or rejection region, where the results are significant. We use the rejection rule as a criterion for deciding when the null hypothesis should be rejected.

The level of significance (α) is decided upon when the null and alternative hypotheses are formulated. It is the risk that the investigator is willing to accept in making a wrong decision when the null hypothesis is in fact true. Viewed another way, the investigator accepts the results of an experiment at a confidence level of $1-\alpha$.

Selection of a level of significance is an arbitrary choice of the investigator. However, the following guidelines are helpful in choosing a level of significance:

> If you suspect the null hypothesis is true, or if rejecting a true null hypothesis would be serious or costly, set $\alpha = 0.01$, i.e., the probability of rejecting a true null hypothesis is 1% or less.
>
> If you suspect the null hypothesis is false, or if accepting a false null hypothesis would have serious or costly consequences, set $0.25 < \alpha < 0.10$.
>
> If you have no conviction either way, select a moderate level of significance, say, 0.05.

As an example, a plant breeder hoping to develop a higher yielding hybrid corn may initially set the level of significance at 0.25 in selecting plants for study. This level is choosen to help ensure that the genetically superior crop is not inadvertently excluded from the study should its desirable characteristics happen to be momentarily suppressed because of environmental reasons. As breeding studies continue time and space constraints require that continued effort be expended only on the more promising candidates. So the level of significance may decrease with successive generations until a marketable product is obtained. Then, in an effort to maintain a firm's reputation, the marketed hybrid may be selected at a level of, say, 0.001.

As mentioned earlier, the distribution of the test statistic when the null hypothesis is true is usually discrete for nonparametric tests. For classical, or parametric tests, the distribution of the test statistic is generally continuous. This means that for some significance level α, a value of the classical test statistic can be found whose cumulative probability is exactly α, whereas for the distribution-free test such a value of the test statistic usually does not exist. As a result, we often must settle on a value for α only approximately equal to those given in the above guideline.

Obviously, two kinds of error are possible in hypothesis testing: We may reject the null hypothesis when it is true (Type I error) or accept it when it is false (Type II error). The probability of a Type I error is equal to our level of significance. The probability of a Type II error is designated beta (β). Another term, <u>power</u>, is the probability of rejecting a false null hypothesis. It may be defined as $1 - \alpha$. Typically, the power (and β) is not known in the practical research setting. It is a function, in part, of sample size and the level of significance used. In general, high power is a desirable characteristic in a test since it better enables us to reject the null hypothesis if our theory is true. But as power increases, so does the probability of a Type I error. This is why we must settle for, say, a 99% confidence level instead of 100%. The lower the value we assign to our level of significance, (or the higher the value we assign as our confidence level) the more we increase the risk of a Type II error.

The possible outcomes of hypothesis testing are tabulated below.

H_0		Accepted	Rejected
	True	*Confidence level* $1-\alpha$	*Type I error* α
	False	*Type II error* β	*Power* $1-\beta$

ESTIMATION

At times we may want to know more than whether the null hypothesis can be accepted or rejected. We may wish to generalize from the sample to the population, i.e., we wish to estimate the numerical value of the location or the scale parameter.

There are two types of estimation: Point estimation and interval estimation. In point estimation we obtain a single value from the sample data and make inferences regarding a population parameter. In interval estimation we obtain a lower value and an upper value. We can express with some degree of confidence that the parameter lies within these bounds. An interval estimate is frequently called a confidence interval.

We express our degree of confidence in this interval using a confidence coefficient. A 95% confidence coefficient, for instance, means we are 95% confident the interval contains the parameter being estimated. Many nonparametric procedures allow us to obtain both point and confidence interval estimates.

ASYMPTOTIC RELATIVE EFFICIENCY

The relative efficiency of two competitive tests is simply the ratio of their sample sizes needed to achieve the "same performance". For example, if Test A has an efficiency of 0.75 relative to Test B, then Test A requires 100 observations to achieve the same performance as Test B based on only 75 observations. A problem arises in specifying just exactly what criterion is used to measure performance, since relative efficiency generally depends on many factors, including sample size itself.

This problem tends to disappear by considering, instead, the asymptotic relative efficiency (A.R.E.). The A.R.E. is the limit of the relative efficiency as sample size approaches infinity. For this reason the most frequently encountered measure of efficiency for nonparametric tests is the A.R.E., even though it is a large-sample property.

ASYMPTOTIC RELATIVE EFFICIENCY

Test	Normal	Double Exponential	Uniform
Fisher Sign	0.637	2.000	0.333
Wilcoxon Signed Ranks	0.955	1.500	1.000
Mann-Whitney-Wilcoxon	0.955	1.500	1.000
Kruskal-Wallis	0.955	1.500	1.000
Friedman Ranked Sums	0.955k/(k+1)	1.500k/(k+1)	k/(k+1)
Freund-Ansari-Bradley†	0.608	0.94	0.60
Mood's†	0.760	1.08	1.00
Moses'†	0.304 (k=2)	0.591	0.133
	0.500 (k=3)		
	0.608 (k=4)		
Miller's Jackknife†‡	0.827	0.961	0.468
Kendall's	0.912	1.266	1.000
Spearman's	0.912	1.266	1.000

†Relative to the Box-Andersen modified F-test: Box, G.E.P. and S.L. Andersen. (1955), Permutation theory in the derivation of robust criteria and the study of departures from assumption. J. R. Statist. Soc.B, 17:1-26.

‡Miller, R.G., Jr. (1968), Jackknifing variances. Ann. Math. Statist. 39: 567-582. At a 0.05 level of significance.

NONPARAMETRIC TESTS
and their APPLICATIONS

A. One-Sample and Matched Observations Tests for Location

The Fisher Sign Test and Wilcoxon Signed Rank Test are useful for making an inference about a population's location parameter, (i.e., the median), based on a set of observations from a single random sample or paired observations that are reduced to a single sample by considering differences.

Where paired, or matched observations are sampled, "X" may represent a pretreatment observation and "Y" a posttreatment observation. Or X and Y may represent two treatments, both performed on the same subject. The subject acts as its own control. Treatments may be applied at the same time or at different times, assuming the subject's responsiveness to the second treatment is not altered by lingering effects of the first treatment or by the passage of time between treatments. Such assumptions are rarely fully justified, but "carry over" effects may be mitigated by judiciously counterbalancing the sequence treatments are administered.

Pairing reduces the variability of differences between observations from two populations while leaving the average difference, or treatment effect, unchanged. Experiments with human twins, animal littermates or clones attempt to reduce the variability in differences by controlling more background variables than would be possible with independent samples. But such matching is not restricted to situations where there exists a natural pairing of subjects. Pairing can be realized through careful matching of subjects that are alike in several respects which may influence the treatment response. For example, in studying the effectiveness of a new medication, patients may be paired with respect to age, sex and severity of illness. The idea is that both members of a matched pair tend to give similar responses, thus enabling the effect of the treatment to stand out more clearly.

I. Fisher Sign Test. a.k.a. Sign Test, Sign Test for the Median.

A. Use

To make inferences concerning the median of a certain population, (one-sample data), or to test for a shift in median due to a treatment effect using dependent samples, (paired-sample data).

B. Data

1. One-sample data: consist of n observations, $N_1, N_2, \ldots, N_n$, measured on at least an ordinal scale such that

$$D_i = N_i - M_0 \qquad i=1,\ldots,n$$

where M_0 is a hypothesized population median.

2. Paired-sample data: consist of 2n observations, $X_1, Y_1; X_2, Y_2; \ldots; X_n, Y_n$, measured on at least an ordinal scale such that

$$D_i = y_i - x_i = \Delta + e_i \qquad i=1,\ldots,n$$

where Δ is a shift in median due to a treatment effect, and
e's are unobservable random variables.

C. Assumptions

1. One-sample data

a. The N's are randomly drawn.
b. The N's are independent.
c. The probability of selecting a N_i greater than the true median, M, is equal to the probability of selecting a N_i less than M, i.e.,

$$P(N_i > M) = P(N_i < M) = 0.5$$

which implies $P(N_i = M) = 0$, or $D_i = 0$.

2. Paired-sample data

a. The e's are independent.
b. The probability of an e_i greater than zero is equal to the probability of an e_i less than zero, i.e.,

$$P(e_i > 0) = P(e_i < 0) = 0.5$$

which implies $P(e_i = 0) = 0$, or $D_i = 0$.

D. Equations

1. Test statistic

Let $$\psi_i = \begin{cases} 1 \text{ if } D_i > 0 \\ 0 \text{ if } D_i < 0 \end{cases}$$

and $$B = \sum_{i=1}^{n} \psi_i$$

then $$a = P(x \geq \beta) = 0.5^n \sum_{x=\beta}^{n} \frac{n!}{x!(n-x)!}$$

where β is the larger of $[B, n-B]$

2. Point estimator

$\bar{\Delta} =$ Median of the Di's

3. Confidence interval

Using Table B,..

$$n+1-\xi = b(\alpha/2, n, 0.5)$$

Both the lower and upper confidence intervals can then be found using the following equations:

The lower confidence interval boundary... $\Delta_L = D^{\xi}$

The upper confidence interval boundary... $\Delta_U = D^{(n+1-\xi)}$

for a confidence coefficient of $(1-\alpha)$.

E. Hypotheses

Select a level of significance, α, and compare it to a computed probability, a.

1. One-sample data

H_0	H_1	Accept H_0 if...
$M \leq M_0$	$M > M_0$	$a > \alpha$
$M \geq M_0$	$M < M_0$	$a > \alpha$
$M = M_0$	$M \neq M_0$	$a > \alpha/2$

2. Paired-sample data

H_o	H_1	Accept H_o if...
$\Delta \le 0$	$\Delta > 0$	$a > \alpha$
$\Delta \ge 0$	$\Delta < 0$	$a > \alpha$
$\Delta = 0$	$\Delta \ne 0$	$a > \alpha/2$

F. Program/Subroutine

FST and QS.

G. Size formula

SIZE = n + 12 + INT[ln n/ln 2] · (max. size = 260).

H. Limits and Warnings

Though n may theoretically be as large as 87, the program calculates a cumulative binomial distribution using the factorial (!) function, FACT, which restricts n to a value no greater than 69.

When using Table B for determining confidence intervals, be aware the table is limited to n no greater than 25.

TEST: Fisher Sign Test	PGM: FST	MAX SIZE: 260

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Set calculator to USER mode.			
1	Set SIZE according to the size formula		XEQ SIZE ___	
2	Load program FST and subroutine QS.			
3	Begin program.		XEQ FST	X/Y R/S
4	Enter the X and Y observations. For one-sample data, assign M_o as the X observations. It need only be entered once.	Xi Yi	ENTER R/S	Xi† i
	Repeat Step 4 until all X,Y pairs have been entered.			
	If an error occurs, erase the last data pair entered by simply pressing "E", then enter the correct data pair according to Step 4. If an error occurs upon entering X_1 or Y_1, return to Step 3 instead of pressing "E".		E	i-1
5	After all observation pairs have been entered, calculate the test statistic, "a".		C	a=________
6	If the null hypothesis is rejected, calculate Delta, the difference between the two populations. The Delta display is preceded by the word "SORT", the duration of which is a function of the sample size.		R/S	SORT DELTA=_______
7	If a confidence interval is desired, begin by selecting a confidence coefficient. (Because of the discrete nature of the test statistic distribution, the exact coefficient will not necessarily be equivalent to this selected value). Obtain $b(\alpha/2,n,\frac{1}{2})$ from TableB to calculate ξ. With this value of ξ and $(n + 1 - \xi)$, the confidence interval bounds may be determined.		R/S	D?

† Rounded to the nearest whole number.

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
8	Calculate the lower confidence value.	ξ	R/S	$DELTA_L$
9	Calculate the upper confidence value.		R/S	D?
		$(n+1-\xi)$	R/S	$DELTA_U$
	Steps 8 and 9 may be repeated any number of times.			

SAMPLE PROBLEM No. 1 **TEST FST**

Ball bearings produced on a certain machine should have a median diameter of 1.00 cm. A shop foreman selects 18 bearings at random and finds them to have the diameters listed below. He suspects the diameters are too large, but stopping production is costly. Does the machine require adjustment?

Bearing	Diameter (cm)
1	1.06
2	1.02
3	1.00
4	0.99
5	0.97
6	1.04
7	0.96
8	0.95
9	1.04
10	1.02
11	0.99
12	1.02
13	1.03
14	1.05
15	0.98
16	1.04
17	1.03
18	1.03

Note: In the case of one-sample data let the actual observations represent the Y population and the expected observation represent the X population.

H_0: M = 1.00
H_1: M > 1.00
α : 0.01

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program FST and subroutine QS and set SIZE = 034.
	XEQ FST	X/Y R/S	Enter the expected observation, M_o, and the first actual observation.
1.00	ENTER	1	Rounds off M_o. M_o need only be entered here.
1.06	R/S	1	i = 1.
1.02	R/S	2	i = 2.
1.00	R/S	2	Since the actual observation is equal to M_o, this observation is ignored. (See assumption 3 of one-sample data).
0.99	R/S	3	
0.79	R/S	4	An incorrect value was inputted for i = 4, so simply erase it with "E".
	E	3	i = i-1.
0.97	R/S	4	
.	.	.	
.	.	.	
.	.	.	

SAMPLE PROBLEM No 1 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
1.03	R/S	17	
	C	a=0.1662	Calculates "a". Since 0.1662 is greater than 0.01, accept H_o.
	R/S	SORT	Approximately 30 seconds, then...
		DELTA=0.0200	Calculates the difference, Delta, between M_o and the sample median.
			Though the sample median is 1.00+0.02, or 1.02 cm, the machine does not require adjustment at the level of significance selected.

SAMPLE PROBLEM No. 2 **TEST** FST

A bicycle manufacturer pays a premium price for alloy tubing weighing under 0.42 kg/m. Ten 1 meter lengths of tubing were randomly selected from a shipment. Based on their weights, should the manufacturer pay the higher price?

Tube	Weight (kg)
1	0.41
2	0.40
3	0.39
4	0.41
5	0.40
6	0.40
7	0.41
8	0.40
9	0.43
10	0.41

Note: In the case of one-sample data let the actual observations represent the Y population and the expected observation represent the X population.

H_0: M = 0.42
H_1: M < 0.42
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program FST and subroutine QS and set SIZE = 025.
	XEQ FST	X/Y R/S	Enter the expected observation, M_o, and the first actual observation.
0.42	ENTER	4 -01	Rounds off M_o. M_o need only be entered here.
0.41	R/S	1	i = 1.
0.40	R/S	2	i = 2.
.	.	.	
.	.	.	
.	.	.	
0.41	R/S	10	i = 10.
	C	a=0.0107	Calculates "a". Since 0.0107 is less than 0.05, reject H_o.
	R/S	SORT	Approximately 11 seconds, then...
		DELTA=-0.015	Calculates the difference, Delta, between M_o and the sample median, i.e., the tubes in the sample weigh, on "average", 0.015 kg less than 0.42 kg/m.

SAMPLE PROBLEM No 2 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
			Choosing a confidence coefficient of about 90%, from Table B we see that b=8 when n=10 and $\alpha/2$=0.0547 (i.e., α=2(0.0547)=0.1094 and the exact confidence coefficient is 100(1 - 0.1094) or 89.06%.
			Since $n + 1 - \xi = b(\alpha/2,n,\frac{1}{2})$ $10 + 1 - \xi = 8$ $\xi = 3$
			So $Delta_L = D^3$ and $Delta_U = D^8$
		DELTA=-0.015	
	R/S	D?	Prompt for confidence interval bounds.
3	R/S	-0.0200	$Delta_L$
	R/S	D?	
8	R/S	-0.0100	$Delta_U$
			So we may feel 89% confident that a meter length of tubing in this shipment weighs between 0.01 and 0.02 kg less than 0.42 kg.

SAMPLE PROBLEM No. 3 **TEST** FST

Weights of all twin babies born at a local hospital during the month of July were compared to determine if the firstborn tended to be the heavier twin.

Set	Firstborn	Secondborn
1	2.2	2.6
2	3.9	3.9
3	3.7	2.4
4	3.5	3.7
5	4.0	4.1
6	5.7	3.6
7	4.3	2.9
8	4.1	3.3
9	3.3	2.6
10	2.3	2.6
11	2.9	4.8
12	4.2	3.1
13	4.4	2.9

Values represent weight in Kg.

H_0: Delta = 0
H_1: Delta < 0
α : 0.05

We always choose our alternative hypothesis on the basis of the Y sample. Allowing the secondborn twins to represent the Y sample, we are asking, "Is the secondborn the lighter twin?"

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program FST and subroutine QS and set SIZE = 028.
	XEQ FST	X≠Y R/S	Enter the X,Y pairs.
2.2	ENTER	2	Rounds off Xi.
2.6	R/S	1	i = 1.
3.9	ENTER	4	
3.9	R/S	1	Since the X and Y observations are equal, these observations are ignored. (See assumption 2 of paired-sample data).
3.7	ENTER	4	
4.2	R/S	2	An incorrect value was inputted for Y_2, so simply erase the observation pair with "E".
	E	1	i = i-1.
3.7	ENTER	4	
2.4	R/S	2	i = 2.
.	.	.	
.	.	.	
.	.	.	

SAMPLE PROBLEM No 3 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
4.4	ENTER	4	
2.9	R/S	12	
	C	a=0.3872	Calculates "a". Since 0.3872 is greater than 0.05, accept H_o.
	R/S	SORT	Approximately 85 seconds, then...
		DELTA=-0.750	For the sample tested, the secondborn twins do weigh, on the "average", 0.75 kg less than the firstborn, but this is not significant at the level chosen.

SAMPLE PROBLEM No. 4 **TEST** FST

Two airport taxi drivers were secretly tested to see if either driver took longer routes than the other, as determined by the farebox odometers. The odometers were standardized immediately prior to the test, and the drivers drove identical cars. Do the drivers tend to differ in distances covered?

Run	Driver A	Driver B
1	6.8	6.6
2	6.4	5.4
3	10.5	8.9
4	3.8	3.4
5	8.1	6.8
6	5.2	4.3
7	12.6	11.4

Values represent miles covered.

H_0: DELTA = 0
H_1: DELTA $\neq$ 0
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program FST and subroutine QS and set SIZE = 021.
	XEQ FST	X/Y R/S	Enter the X,Y pairs.
6.8	ENTER	7	Rounds off Xi.
6.6	R/S	1	i = 1.
.	.	.	
.	.	.	
.	.	.	
12.6	ENTER	13	
11.4	R/S	7	
	C	a=0.0078	Calculates "a". Since 0.0078 is less than 0.025 (i.e., $\alpha/2$), reject H_0.
	R/S	SORT	Approximately 10 seconds, then...
		DELTA=-1.000	For the seven test runs Driver B "averaged" 1 mile less than Driver A.
			Choosing a confidence coefficient of about 90%, from Table B we see that b=6 when n=7 and $\alpha/2$= 0.0625 (i.e., α =2(0.0625)=0.125 and the exact

INPUT	FUNCTION	DISPLAY	COMMENTS
			confidence coefficient is 100(1 - 0.125) or 87.5%.
			Since $n + 1 - \xi = b(\alpha/2, 7, \frac{1}{2})$ $7 + 1 - \xi = 6$ $\xi = 2$
			So $\text{Delta}_L = D^2$ and $\text{Delta}_U = D^6$
		DELTA=-1.000	
	R/S	D?	Prompt for confidence interval bounds.
2	R/S	-1.3000	Delta_L
	R/S	D?	
6	R/S	-0.4000	Delta_U
			So we may feel 87.5% confident that Driver B drives between 0.4 and 1.3 less miles than Driver A along the routes traveled during the test.

SAMPLE PROBLEM No. 5 **TEST** FST

Two couples were competing as finalists in a dance contest at a popular discothèque with 5 other couples and the emcee serving as judges. Arbitrarily assigning a "1" to the couple selected by each judge as the better dancers and a "0" to the other couple, determine if the winners are judged significantly better than the runners up.

Judge	Couple #1	Couple #2
1	1	0
2	1	0
3	1	0
4	0	1
5	1	0
6	1	0
7	0	1
8	1	0
9	0	1
10	0	1
11	1	0

H_0: Delta = 0
H_1: Delta < 0
α : 0.25

Since Couple #2, the losers, were assigned as the Y sample, we wish to know if they were judged significantly worse. Thus the form of our alternative hypothesis.

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program FST and subroutine QS and set SIZE = 026.
	XEQ FST	X/Y R/S	Enter the X,Y pairs.
1	ENTER	1	
0	R/S	1	i = 1.
.	.	.	
.	.	.	
.	.	.	
1	ENTER	1	
0	R/S	11	i = 11.
	C	a=0.2745	Calculates "a". Since 0.2745 is greater than 0.25, accept H_o.
			At a 100(1 - 0.25) or 75% confidence level, the winners are not necessarily deemed significantly better dancers.

II. Wilcoxon Signed Rank Test.

A. Use

To make inferences concerning the median of a certain population, (one-sample data) or to test for a shift in median due to a treatment effect, (paired-sample data).

B. Data

1. One-sample data: consist of n observations, $N_1, N_2, \ldots, N_n$, measured on at least an interval scale such that

$$D_i = N_i - M_0 \quad i=1,\ldots,n$$

where M_0 is a hypothesized population median.

2. Paired-sample data: consist of 2n observations, $X_1, Y_1; X_2, Y_2; \ldots; X_n, Y_n$, measured on at least an interval scale such that

$$D_i = y_i - x_i = \Delta + e_i \quad i=1,\ldots,n$$

where Δ is a shift in median due to a treatment effect, and
e's are unobservable random variables.

C. Assumptions

1. One-sample data

a. The N's are randomly drawn.
b. The N's are independent.
c. Each N belongs to a continuous population that is symmetrical about M, the population median.
d. $P(N_i = M) = 0$.

2. Paired-sample data

a. The e's are independent.
b. Each e belongs to a continuous population that is symmetrical about zero.
c. $P(e_i = 0) = 0$.

D. Equations

1. Test statistic

$$\text{Let} \quad \psi_i = \begin{cases} 1 \text{ if } D_i > 0 \\ 0 \text{ if } D_i < 0 \end{cases}$$

$$\text{then} \quad T = \sum_{i=1}^{n} r_i \psi_i$$

where r_i denotes the average rank of the absolute value of D_i in the joint ranking of $|D_i|$ from least to greatest.

2. Point estimator

$$\text{Let} \quad \theta = \frac{n(n+1)}{2}$$

$$\text{and} \quad W^k = \left[\frac{D_i + D_j}{2}, i \le j \right]_{k=1,\ldots,\theta}$$

then $\tilde{\Delta}$ = Median of the W^k's

3. Confidence interval

Using Table C...

$$\theta + 1 - \xi = t(\alpha/2, n)$$

Both the lower and upper confidence intervals can then be found using the following equations:

The lower confidence interval boundary... $\Delta_L = W^{\xi}$
The upper confidence interval boundary... $\Delta_U = W^{(\theta+1-\xi)}$

for a confidence coefficient of $(1-\alpha)$.

E. Hypotheses

Select an approximate level of significance, α', and compare the test statistic, T, to $t(\alpha,n)$ from Table C for an α closest to α'.

1. One-sample data

H_0	H_1	Accept H_0 if...
$M \le M_0$	$M > M_0$	$T < t(\alpha,n)$
$M \ge M_0$	$M < M_0$	$-t(\alpha,n) < T$
$M = M_0$	$M \ne M_0$	$-t(\alpha/2,n) < T < t(\alpha/2,n)$

2. Paired-sample data

H_0	H_1	Accept H_0 if...
$\Delta \leq 0$	$\Delta > 0$	$T < t(\alpha, n)$
$\Delta \geq 0$	$\Delta < 0$	$-t(\alpha, n) < T$
$\Delta = 0$	$\Delta \neq 0$	$-t(\alpha/2, n) < T < t(\alpha/2, n)$

F. Program/Subroutine

WSR and QS.

G. Size formula

SIZE = $(n^2 + 5n + 16)/2$ (max. size = 226).

H. Limits and Warnings

Though the program has room to accommodate a value of n as large as 18, Table C lists the test statistic probabilities for n less than or equal to 15. If there are no ties among the D_i's, the following large sample approximation may be used for hypothesis testing:

$$T^* = \frac{T - [n(n+1)/4]}{\sqrt{n(n+1)(2n+1)/24}}$$

Using Table A for Z_α ...

H_1: $M > M_0$ or $\Delta > 0$ Accept H_0 if $T^* < Z_\alpha$

For H_1: $M < M_0$ or $\Delta < 0$ Substitute $(T - \theta)$ for T in the above equation and accept H_0 if $-Z_\alpha < T$.

For H_1: $M \neq M_0$ or $\Delta \neq 0$ Solve the above equation twice, once using T and again using $(T - \theta)$. Accept H_0 if

$$T^*(T) < Z_{\alpha/2}$$

$$-Z_{\alpha/2} < T^*(T - \theta)$$

In the event of ties, refer to a standard nonparametric textbook for the appropriate adjustments.

If paired-sample data are tested, it is advantageous to record the D_i's after each X,Y entry for use in Gupta's Symmetry Test in the following section.

I. Comparison with Fisher Sign Test

1. For the Fisher Sign Test the data may be ordinal, but the data must be at least on an interval scale for the Wilcoxon Signed Rank Test.

2. The Wilcoxon Signed Rank Test requires the sampled population to be symmetric. Though this assumption places a certain restriction on the data, conclusions about the population median equally apply to the mean in a symmetric distribution. To test for symmetry, use Gupta's Symmetry Test in the following section.

3. Generally, the Wilcoxon Test is more efficient.

4. For the programs FST and WSR, n cannot be greater than 69 and 18, respectively.

TEST: Wilcoxon Signed Ranks	PGM: WSR	MAX SIZE: 226

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Set calculator to USER mode.			
1	Set SIZE according to the size formula.		XEQ SIZE___	
2	Load program WSR and subroutine QS.			
3	Begin program.		XEQ WSR	X/Y R/S
4	Enter the X and Y observations. For one-sample data, assign M_0 as the X observations. It need only be entered once.	Xi Yi	ENTER R/S	Xi† Di i
	Repeat Step 4 until all X,Y pairs have been entered.			
	If an error occurs, erase the last data pair entered by simply pressing "E", then enter the correct data pair according to Step 4. If an error occurs upon entering X_1 or Y_1, return to Step 3 instead of pressing "E".		E	i-1
5	After all observation pairs have been entered, begin calculations. Following two "SORT"'s, the length of which are functions of sample size, the absolute difference between each X,Y pair is displayed along with its average ranking.		C	SORT SORT |Di| R |Di|
	After the last absolute Di and its ranking, the test statistic, T, is displayed accompanied by a tone.			. . . T=_____
6	If the null hypothesis is rejected, calculate DELTA, the difference between the two populations. The DELTA display is preceded by an accumulating Walsh Average counter and the word "SORT".		R/S	W _._ SORT DELTA=_____
7	If a condifence interval is desired, begin by selecting a cofidence coefficient. (Because of the discrete nature of the text statistic distribution, the exact coefficient will not			

† Rounded to the nearest whole number.

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	necessarily be equivalent to this selected value). Obtain the value for $t(\alpha/2,n)$ from Table C to solve for ξ. With this value of ξ and $(n + 1 - \xi)$, the confidence interval bounds may be determined.		R/S	W?
8	Calculate the lower confidence limit.	ξ	R/S	$DELTA_L$
9	Calculate the upper confidence limit.	 $(n+1-\xi)$	R/S R/S	W? $DELTA_U$
	Steps 8 and 9 may be repeated any number of times.			

SAMPLE PROBLEM No. 1 **TEST** WSR

Ball bearings produced on a certain machine should have a median diameter of 1.00 cm. A shop foreman selects 18 bearings at random and finds them to have the diameters listed below. He suspects the diameters are too large, but stopping production is costly. Does the machine require adjustment?

Bearing	Diameter (cm)
1	1.06
2	1.02
3	1.00
4	0.99
5	0.97
6	1.04
7	0.96
8	0.95
9	1.04
10	1.02
11	0.99
12	1.02
13	1.03
14	1.05
15	0.98
16	1.04
17	1.03
18	1.03

Note: In the case of one-sample data let the actual observations represent the Y population and the expected observation represent the X population.

H_0: M = 1.00
H_1: M > 1.00
α : 0.01

INPUT	FUNCTION	DISPLAY	COMMENTS
			Since n>15 and there are ties among the $\lvert Di \rvert$, program WSR is inappropriate for this problem. Note: This is not to say that the Wilcoxon Signed Ranks test is inappropriate for this problem. But calculation of the Large Sample Approximation, and particularly determination of confidence intervals, without aid of a calculating device is tedious.

SAMPLE PROBLEM No. 2 **TEST** WSR

A bicycle manufacturer pays a premium price for alloy tubing weighing under 0.42 kg/m. Ten 1 meter lengths of tubing were randomly selected from a shipment. Based on their weights, should the manufacturer pay the higher price?

Tube	Weight (kg)
1	0.41
2	0.40
3	0.39
4	0.41
5	0.40
6	0.40
7	0.41
8	0.40
9	0.43
10	0.41

Note: In the case of one-sample data let the actual observations represent the Y population and the expected observation represent the X population.

Ho: $M = 0.42$
H1: $M < 0.42$
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program WSR and subroutine QS and set SIZE = 166.
	XEQ WSR	X/Y R/S	Enter the expected observation, M_o, and the first actual observation.
0.42	ENTER	4. -01	Rounds off M_o. M_o need only be entered here.
0.41	R/S	-0.0100 1	|Di| i, where i = 1.
0.40	R/S	-0.0200 2	
0.93	R/S	0.5100 3	An incorrect value was inputted for i = 3, so simply erase it with "E".
	E	2	Displays last correct "i".
0.39	R/S	-0.0300 3	
0.42	R/S	-0.0300 3 3	Since this error involves entering a value of Y that is equal to X, the observation is ignored. (See assumption 4 of one-sample data). The display of "i" twice can also be ignored.
0.41	R/S	-0.0100 4	
.	.	. .	
.	.	. .	
.	.	. .	

SAMPLE PROBLEM No 2 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
0.41	R/S	-0.0100 10	
	C	SORT	Approximately 10 seconds, then...
		SORT	Approximately 20 seconds, then...
		0.01 3.00	D_1 R_{D_1}
		0.02 7.50	. .
		0.03 10.00	D_3 R_{D_3} then...
		T=3.00	Calculates the test statistic, T. From Table C, t(0.053,10)† = 44. $\theta = n(n+1)/2$ or $10(11)/2$ or 55. θ - t(0.053,10) = 55 - 44 = 11. Since 11 is not less than 3.00, reject H_o.
	R/S	W 1.00	
		. .	Calculate and accumulate the θ Walsh Averages.
		. .	
		. .	
		W 55.00	
		SORT	Approximately 3 minutes, then...
		DELTA=-0.015	Calculates the difference, Delta, between M_o and the sample median.
			Choosing a confidence coefficient of about 90%, from Table C we see that t(0.053,10) = 44 (i.e. $\alpha = 2(0.053) = 0.106$, and the exact confidence coefficient is 100(1 - 0.106) or 89.4%.)
			Since $\theta + 1 - \xi = t(\alpha,n)$ $55 + 1 - \xi = 44$ $\xi = 12$
			So $Delta_L = W^{12}$ and $Delta_U = W^{44}$
	R/S	W?	Prompt for confidence interval bounds.
12	R/S	-0.0200	$Delta_L$
	R/S	W?	
44	R/S	-0.0100	$Delta_U$
			Compare these results of the Wilcoxon Signed Ranks test with those of the Fisher Sign Test, Sample Problem 2.
			† Because of the discrete nature of the test statistic distribution, an exact value for t(0.05,10) is not possible. Therefore, select an appropriate approximation for α.

SAMPLE PROBLEM No. 3 **TEST** WSR

Weights of all twin babies born at a local hospital during the month of July were compared to determine if the firstborn tended to be the heavier twin.

Set	Firstborn	Secondborn
1	2.2	2.6
2	3.9	3.9
3	3.7	2.4
4	3.5	3.7
5	4.0	4.1
6	5.7	3.6
7	4.3	2.9
8	4.1	3.3
9	3.3	2.6
10	2.3	2.6
11	2.9	4.8
12	4.2	3.1
13	4.4	2.9

Values represent weight in Kg.

H_0: Delta = 0
H_1: Delta < 0
α : 0.05

We always choose our alternative hypothesis on the basis of the Y sample. Allowing the secondborn twins to represent the Y sample, we are asking, "Is the secondborn the lighter twin?"

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program WSR and subroutine QS and set SIZE = 125.
	XEQ WSR	X/Y R/S	Enter the X,Y pairs.
2.2	ENTER	2	Rounds off Xi.
2.6	R/S	0.4000 1	D_1 1
3.9	ENTER	4	
3.9	R/S	0.4000 1 1	Since the X and Y observations are equal, these observations are ignored. (See assumption 4 of paired-sample data). The display of "i" twice can also be ignored.
3.7	ENTER	4	
4.2	R/S	0.5000 2	An incorrect value was inputted for Y_2, so simply erase the observation pair with "E".
	E	1	Last correct data pair "i".
3.7	ENTER	4	
2.4	R/S	-1.3000 2	
.	.	. .	
.	.	. .	
.	.	. .	

SAMPLE PROBLEM No 3 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
4.4	ENTER	4	
2.9	R/S	-1.5000 12	
	C	SORT	Approximately 20 seconds, then...
		SORT	Approximately 25 seconds, then...
		0.1000 1.0	$\lvert Di \rvert$ $R_{\lvert Di \rvert}$
		. .	
		. .	
		. .	
		2.1000 12.0	
		T=21.00	Calculates the test statistic, T. From Table C, t(0.046,12) = 61. Θ = 12(13)/2 or 78, and Θ - t(0.046,12) = 78 - 61 = 17. Since 17 is less than 21.00, accept H_o.

SAMPLE PROBLEM No. 4 **TEST** WSR

Two airport taxi drivers were secretly tested to see if either driver took longer routes than the other, as determined by the farebox odometers. The odometers were standardized immediately prior to the test, and the drivers drove identical cars. Do the drivers tend to differ in distances covered?

Run	Driver A	Driver B
1	6.8	6.6
2	6.4	5.4
3	10.5	8.9
4	3.8	3.4
5	8.1	6.8
6	5.2	4.3
7	12.6	11.4

Values represent miles covered.

Ho: DELTA = 0
H1: DELTA ≠ 0
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program WSR and subroutine QS and set SIZE = 050.
	XEQ WSR	X/Y R/S	Enter the X,Y pairs.
6.8	ENTER	7	Rounds off Xi.
6.6	R/S	-0.2000 1	Di i
.	.	. .	
.	.	. .	
.	.	. .	
12.6	ENTER	13	
11.4	R/S	-1.2000 7	
	C	SORT	Approximately 10 seconds, then...
		SORT	Approximately 15 seconds, then...
		0.2000 1.0	$\lvert Di \rvert$ $R_{\lvert Di \rvert}$
		. .	. .
		. .	. .
		. .	. .
		1.6000 7.0	. .
		T=0.00	Calculates the test statistic, T. From Table C, t(0.023,7) = 26. (α = 2(0.023) = 0.046).

SAMPLE PROBLEM No 4 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
			$\theta = 7(8)/2 = 28$. Since $\theta - t(\alpha/2,n)$ is equal to 28 - 26 = 2, and the test statistic value of 0.00 does not lie between 2 and 26, reject H_o.
	R/S	W 1.00 . . . . . . W 28.00	Calculate and accumulate the θ Walsh Averages.
		SORT	Approximately 1 minute, then...
		DELTA=-0.975	Calculates the difference, Delta, between the two samples. Choosing a confidence coefficient of about 90%, from Table C we see that t(0.055,7) = 24. (i.e., $\alpha = 2(0.055) = 0.11$, and the exact confidence coefficient is 100(1 - 0.11) or 89%). Since $\theta + 1 - \xi = t(\alpha,n)$ $28 + 1 - \xi = 24$ $\xi = 5$ So $Delta_L = W^5$ and $Delta_U = W^{24}$
	R/S	W?	Prompt for confidence interval bounds.
5	R/S	-1.3000	$Delta_L$
	R/S	W?	
24	R/S	-0.6000	$Delta_U$ Compare these results of the Wilcoxon Signed Ranks test with those of the Fisher Sign Test, Sample Problem 4.

SAMPLE PROBLEM No. 5 **TEST** WSR

For Sample Problem No. 5 of the FST Test, the data are presented on an ordinal scale, i.e., "1" better than "0", and not at the interval scale as required for the Wilcoxon Signed Ranks test. If instead of simply choosing one dancing couple over the other, the judges were asked to score the contestants on a scale of 1 to 10 (1 being awful and 10 being superb), the resulting data would be amenable to analysis by the Wilcoxon test.

Supposing this were the case and the judges' decisions tabulated below were used to decide if the winning couple danced significantly better, do the results agree with those obtained by the Fisher Sign Test?

Judge	Couple #1	Couple #2
1	8	7
2	7	5
3	9	8
4	6	7
5	8	5
6	8	5
7	4	7
8	5	4
9	8	9
10	7	9
11	10	8

H_0: Delta = 0
H_1: Delta < 0
α : 0.25

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program WSR and subroutine QS and set SIZE = 096.
	XEQ WSR	X/Y R/S	Enter the X,Y pairs.
8	ENTER	8	Xi
7	R/S	-1.0000 1	Di i
.	.	. .	. .
.	.	. .	. .
.	.	. .	. .
10	ENTER	10	
8	R/S	-2.0000 11	
	C	SORT	Approximately 20 seconds, then...
		SORT	Approximately 15 seconds, then...
		1.0000 3.0	|Di| $R_{\|Di\|}$
		. .	
		. .	
		. .	
		3.0000 10.0	
		T=23.00	Calculates the test statistic, T. From Table C, t(0.26,11) = 41. θ = 11(12)/2 = 66. Since

SAMPLE PROBLEM No 5 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
			θ - t(0.26,11), or 66 - 41 = 25 is not less than 23.00, reject H_o at the 100(1 - 0.26) or 74% confidence level. Note: At the 75% confidence level, we did not reject H_o during the Fisher Sign Test. If FST were run on the interval data of this problem, we would obtain the same result as with the ordinal data.

III. Gupta's Symmetry Test

A. Use

To test if a population distribution is symmetric about an unknown location parameter.

B. Data

Consist of n observations, $D_1, D_2, \ldots, D_n$, measured at least on an interval scale.

C. Assumptions

1. The D's are independent.
2. The D's are from the same continuous population having unknown median M.

D. Equations

Let
$$\delta_{ij} = \begin{cases} 1 \text{ if } [D_i + D_j] > 2\tilde{M} \\ 0 \text{ otherwise} \end{cases} \qquad \begin{matrix} i = 1, \ldots, j-1 \\ j = 2, \ldots, n \end{matrix}$$

where $\tilde{M} = M + \Delta$, i.e., the sample median (calculated by the program).

Define
$$\zeta_a = \sum_{i=1}^{j-1} \sum_{j=2}^{n} \delta_{ij}$$

and
$$\zeta_b = \frac{n[n-2]}{8}$$

so that...
$$\Gamma = \frac{2[\zeta_a - \zeta_b]}{n[n-1]}$$

Let
$$\phi_i = \begin{cases} 1 \text{ if } [\tilde{M} - n^{-1/5}] \leq D_i \leq [\tilde{M} + n^{-1/5}] \\ 0 \text{ otherwise} \end{cases} \qquad i = 1, \ldots, n$$

and
$$\Omega = \frac{\nu}{2n^{4/5}}$$

where ν is the larger of $[1, \sum_{1}^{n} \phi_i]$

If $$t = \frac{Z_{|\alpha/2|}}{\sqrt{3n}\,(M_U - M_L)}$$

$$\left.\begin{array}{l} M_U = \tilde{M} + \Delta_U \\ M_L = \tilde{M} + \Delta_L \end{array}\right\}$$ Are calculated by the program.

and $$V = \frac{1}{12} + \frac{(1 - [2t/\ell])^2}{4}$$

then $$J = \frac{r - 0.25}{\sqrt{V/n}}$$

E. Hypotheses

Select a level of significance, α, and compare the test statistic, J, to the upper tail probability for the standard normal distribution of Table A.

H_0	H_1	Accept H_0 if...
Not skewed right	Skewed right	$J < Z_\alpha$
Not skewed left	Skewed left	$-Z_\alpha < J$
Symmetric	Asymmetric	$-Z_{\alpha/2} < J < Z_{\alpha/2}$

F. Program/Subroutine

GST and QS.

G. Size formula

SIZE = n + 10 + INT[ln n/ln 2] (max. size = 243).

H. Limits and Warnings

Program GST is useful in checking for population symmetry AFTER performing the Wilcoxon Signed Rank Test since values for $Z_{\alpha/2}$, Δ_L and Δ_U, which are generated during the execution of WSR, must be entered at the start of GST.

TEST: Gupta's Symmetry Test	PGM: GST	MAX SIZE: 243

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Set calculator to USER mode.			
1	Set SIZE according to the size formula.		XEQ SIZE___	
2	Load program GST and subroutine QS.			
3	Begin program.		XEQ GST	$Z_{\alpha/2}$?
4	Enter the value of Z for the $\alpha/2$ used in determining the confidence coefficient for the WSR test.	$Z_{\alpha/2}$	R/S	DLT-L?
5	Enter the lower confidence interval boundary from the WSR test.	$Delta_L$	R/S	DLT-U?
6	Enter the upper confidence interval boundary from the WSR test.	$Delta_U$	R/S	DATA: R/S
7	Enter the observations of the Y sample for one-data sample, and Di, the difference between the Y and the X samples for paired-data samples.	Yi or Di	R/S	i Yi or Di
8	Repeat Step 7 until all the observations have been entered.			
9	For input errors, erase the last observation entered by pressing "E" and then entering the correct observation. If an input error occurs on the first entry, return to Step 3 instead of pressing "E".		E	i-1 D_{i-1}
10	After all data have been entered, begin calculation of the test statistic, J.		C	SORT J=_____

SAMPLE PROBLEM No. 1 **TEST** GST

For the bicycle tube problem of the Wilcoxon Signed Ranks test (WSR, Sample Problem No. 2), it was assumed that the data came from a symmetrical distribution. Given that same data below, test if this assumption is valid.

Tube	Weight (kg)
1	0.41
2	0.40
3	0.39
4	0.41
5	0.40
6	0.40
7	0.41
8	0.40
9	0.43
10	0.41

M_o was 0.42 kg/m.

From Sample Problem No. 2 of WSR, $\alpha/2$ for the confidence interval was 0.053, $Delta_L$ was -0.0200 and $Delta_U$ was -0.0100.

Ho: The sample came from a symmetrical distribution.
H1: The sample came from an asymmetrical distribution.
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program GST and subroutine QS and set SIZE = 023.
	XEQ GST	$Z\alpha/2$	Enter $Z_{\alpha/2}$ at the same level of significance used in determining the confidence interval in Sample Problem No. 2 of WSR, i.e., since $\alpha/2$ was 0.053 for the 89.4% confidence coefficient, using Table A with interpolarion gives $Z_{0.053}$ at about 1.616.
1.616	R/S	DLT-L?	Enter the lower confidence interval boundary.
-0.02	R/S	DLT-U?.	Enter the upper confidence interval boundary.
-0.01	R/S	DATA: R/S	Enter the actual observations listed above.
0.41	R/S	1 0.410	i Di
0.40	R/S	2 0.400	
0.93	R/S	3 0.930	An incorrect value was inputted for D_3, so simply erase it by pressing "E".
	E	2 0.400	i-1 D_{i-1}
0.39	R/S	3 0.390	
.	.	. .	
.	.	. .	

SAMPLE PROBLEM No 1 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
0.41	R/S	10 0.410	
	C	SORT	Approximately 10 seconds, then after about 40 seconds more...
		J=-0.012	Calculates the test statistic, J. From Table A, $Z_{0.05/2} = 1.96$. Since -0.012 lies between $\pm$ 1.96, accept H_o.

SAMPLE PROBLEM No. 2 **TEST** GST

For the taxi driver problem of the Wilcoxon Signed Ranks test (WSR, Sample Problem No. 4), it was assumed the data came from a symmetrical distribution. Given that same data below, test if this assumption is valid.

Run	Driver A	Driver B
1	6.8	6.6
2	6.4	5.4
3	10.5	8.9
4	3.8	3.4
5	8.1	6.8
6	5.2	4.3
7	12.6	11.4

From Sample Problem No. 4 of WSR, $\alpha/2$ for the confidence interval was 0.0550, $Delta_L$ was -1.30 and $Delta_U$ was -0.60.

H_0: The sample came from a symmetrical distribution.
H_1: The sample came from an asymmetrical distribution.
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program GST and subroutine QS and set SIZE = 022.
	XEQ GST	$Z\alpha/2$	Enter $Z_{\alpha/2}$ at the same level of significance used in determining the confidence interval in Sample Problem No. 4 of WSR. Using Table A gives $Z_{0.055}$ at about 1.60.
1.60	R/S	DLT-L	Enter the lower confidence interval boundary.
-1.30	R/S	DLT-U	Enter the upper confidence interval boundary.
-0.60	R/S	DATA: R/S	Enter Di, i.e., the difference between the Y and X paired observations. (These were displayed during data entry in WSR).
-0.2	R/S	1 -0.200	i Di
.	.	. .	
.	.	. .	
.	.	. .	
-1.2	R/S	7 -1.200	
	C	SORT	Approximately 10 seconds, then after about 20 seconds more...
		J=0.424	Calculates the test statistic, J. From Table A.

SAMPLE PROBLEM No 2 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
			$Z_{0.05/2} = 1.96$. Since 0.424 lies between $\pm$ 1.96, accept H_o.

B. Two-Sample Test for Location of Independent Observations.

Just as the Fisher Sign Test and the Wilcoxon Signed Rank Test were useful and appropriate for making an inference about a population's median when paired, or dependent, observations were considered, the Mann-Whitney-Wilcoxon Test is useful and appropriate for making a similar inference when independent samples are being considered. Though the vicissitudes of variability are likely to be larger because of the absence of pairing, the test does require the two population distributions to be similar in dispersion and shape. (This requirement is also the case for the classical two-sample t-test).

I. Mann-Whitney-Wilcoxon Test. a.k.a. Mann-Whitney Test, Mann-Whitney U-Test, Wilcoxon-Mann-Whitney Test, Wilcoxon Rank Sum Test, Wilcoxon Test.

A. Use

To test for a shift in median due to a treatment effect for independent samples.

B. Data

Consist of m observations from one random sample and n observations from an independent random sample. The samples differ (if they differ at all) only with respect to their medians. The N = m + n observations, measured at least on an ordinal scale, are such that

$$X_i = e_i \qquad i = 1,\ldots,m$$

and $$Y_j = \Delta + e_{m+j} \qquad j = 1,\ldots,n$$

where Δ is a shift in median due to a treatment effect. e's are unobservable random variables.

C. Assumptions

1. The e's are independent.
2. The e's are from the same continuous population.

D. Equations

1. Test statistic

Let $$W = U + \left[n(n+1)/2\right]$$

where $$U = \sum_{i=1}^{m} \sum_{j=1}^{n} f(X_i, Y_j)$$

and $$f(X_i, Y_j) = \begin{cases} 1 & \text{if } X<Y \\ 0.5 & \text{if } X=Y \\ 0 & \text{if } X>Y \end{cases}$$

2. Point estimator

Let $D^k = Y_j - X_i \quad k = 1,\ldots,mn$

$i = 1,\ldots,m$

$j = 1,\ldots,n$

then $\tilde{\Delta} =$ Median of the D^k's.

3. Confidence interval

Using Table D...

Let $\theta + 1 - \xi = \omega(\alpha/2, m, n)$

where $\theta = n(2m+n+1)/2$

The lower confidence interval boundary.. $\Delta_L = D^{\xi}$

The upper confidence interval boundary.. $\Delta_U = D^{mn+1-\xi}$

for a confidence coefficient of $(1-\alpha)$.

E. Hypotheses

Select an approximate level of significance, α', and compare the test statistic, W, against $w(\alpha,m,n)$ from Table D for an α closest to α'.

H_0	H_1	Accept H_0 if...
$\Delta \leq 0$	$\Delta > 0$	$W < w(\alpha,m,n)$
$\Delta \geq 0$	$\Delta < 0$	$n(m+n+1) - w(\alpha,m,n) < W$
$\Delta = 0$	$\Delta \neq 0$	$n(m+n+1) - w(\alpha/2,m,n) < W < w(\alpha/2,m,n)$

F. Program/Subroutine

MWW and QS.

G. Size formula

SIZE = mn + 10 + larger of $\{m,n\}$ (max. size = 240).

H. Limits and Warnings

For easier use of Table D, allow $m \geq n$.

Table D lists test statistic probabilities for $m,n < 10$. If there are no ties among the N observations, the following large sample approximation may be used for hypothesis testing:

$$W^* = \frac{W - (n[m+n+1]/2)}{\sqrt{mn[m+n+1]/12}}$$

Using Table A for Z_α ...

For H_1: $\Delta > 0$ accept H_o if $W^* < Z_\alpha$.

For H_1: $\Delta < 0$ substitute $[W - n(m+n+1)]$ for W in the above equation and accept H_o if $-Z_\alpha < W^*$.

For H_1: $\Delta \neq 0$ solve the above equation twice: once using W and again using $[W - n(m+n+1)]$. Accept H_o if

$$W^*_W < Z_{\alpha/2} \text{ and}$$

$$-Z_{\alpha/2} < W^*_{[W - n(m+n+1)]}$$

In the event of ties, refer to a standard nonparametric textbook for the appropriate adjustments.

TEST: Mann-Whitney-Wilcoxon	PGM: MWW	MAX SIZE: 240

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Set calculator to USER mode.			
1	Set SIZE according to the size formula.		XEQ SIZE ___	
2	Load program MWW and subroutine QS.			
3	Begin program.		XEQ MWW	X: R/S
4	Enter the X sample observations.	Xi	R/S	"Xi" Xi
	Repeat Step 4 until all of the X observations have been entered.			
5	If an error occurs, erase the last observation by pressing "E" and entering the correct observation according to Step 4, (or Step 7 for Y data).		E	"Xi-1" Xi-1
6	After all the X observations have been entered, prompt for the Y entry display.		A	Y: R/S
7	Enter the Y sample observations.	Yj	R/S	"Yj" Yj
	If an error occurs in entering a Y observation, refer to Step 5.			
	Repeat Step 7 until all of the Y observations have been entered.			
8	After all the Y observations have been entered, begin calculation of the test statistic, W.		C	W ___
	Display shows the accumulating test statistic, W. When the summation of W is complete, a tone sounds and the test statistic value is displayed.			W=___
9	If the null hypothesis is rejected, calculate Delta, the difference between the Y and X samples.		R/S	SORT DELTA=______
10	If a confidence interval for Delta is desired...		R/S	U?
11	Determine the lower confidence limit.	ξ	R/S	$Delta_L$

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
12	Determine the upper confidence limit.		R/S	U?
		mn+1- ξ	R/S	$Delta_U$
	Steps 11 and 12 may be repeated any number of times for difference confidence coefficients.			

SAMPLE PROBLEM No. 1 **TEST** MWW

A researcher suspects fish in Lake A are contaminated. He catches 8 fish from Lake A and 6 fish from Lake B, an isolated mountain lake thought to be free of pollution. In the laboratory he tests their tissue for a certain industrial chemical. Do the results support the researcher's suspicions?

Fish	Lake A	Lake B
1	0.092	0.050
2	0.085	0.051
3	0.101	0.048
4	0.069	0.050
5	0.067	0.049
6	0.147	0.047
7	0.110	
8	0.081	

The values indicate the concentration of chemical in fish tissue, given in mg/kg.

Ho: Delta = 0
H1: Delta < 0
α : 0.05

From the form of the alternative hypothesis, the question is, "Is there less chemical in Lake B fish".

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program MWW and subroutine QS and set SIZE = 066.
	XEQ MWW	X: R/S	Enter the X observations.
0.092	R/S	X1 0.092	"Xi" Xi
0.085	R/S	X2 0.085	
0.110	R/S	X3 0.110	An error has occurred inputting X3, so simply erase it by pressing "E".
	E	X2 0.085	"X(i-1)" X(i-1)
0.101	R/S	X3 0.101	
.	.	.	
.	.	.	
.	.	.	
0.081	R/S	X8 0.081	
	A	Y: R/S	Enter the Y observations.
0.050	R/S	Y1 0.050	"Yj" Yj
.	.	.	
.	.	.	
.	.	.	

INPUT	FUNCTION	DISPLAY	COMMENTS
0.047	R/S	Y6 0.047	
	C	W 21.0	Calculates the test statistic, W, blinking as display shows the accumulating value for W. (In the particular problem, the value for W remains unchanged throughout the accumulation process.)
		W=21.0	The test statistic.
			From Table D, w(0.054, 8,6) = 58. In order to test the null hypothesis, the above value must first be subtracted from m(m + n + 1), or 8(8 + 6 + 1) to give 120 - 58 = 62. Since 62 is not less than 21.0, reject H_o.
	R/S	SORT	Approximately 2 minutes, then...
		DELTA=-0.0395	Calculates the difference, Delta, between the Y and the X observations, i.e., fish from Lake B have, on "average", 0.0395 mg less chemical per kg of tissue than fish from Lake A. If fish from Lake B are indeed pollution-free, the concentration of chemical discovered in their tissue may actually reflect baseline "noise" or error in the assay procedure. In that case, it would be more appropriate to state that fish from Lake A contain, on "average", about 0.04 mg of chemical per kg of tissue. For a 90% confidence interval, we begin by calculating θ. $\theta = 6(2(8) + 6 + 1)/2 = 69$ Then from Table D... $\theta + 1 - \xi = w(\alpha/2,8,6)$ $69 + 1 - \xi = 58$ $\xi = 12$ i.e., letting $\alpha/2 = 0.054$, then $\alpha = 2(0.054)$ or 0.108, so the confidence coefficient is actually $100(1 - \alpha) = 89.2\%$. $mn + 1 - \xi = 8(6) + 1 - 12 = 37$
	R/S	U?	Prompt for confidence interval bounds.
12	R/S	-0.0590	Delta_L
	R/S	U?	
37	R/S	-0.0220	Delta_U

C. Analysis of Variance.

The Kruskal-Wallis Test and the Friedman Rank Sums Test can be thought of as nonparametric versions of the one- and two-way analysis of variance, respectively, where ranks replace the original measurements.

The Kruskal-Wallis Test is an extension of the Mann-Whitney-Wilcoxon Test applied to analyzing the treatment effects of two or more independent samples. (When only two samples are being considered, the Kruskal-Wallis Test is equivalent to the Mann-Whitney-Wilcoxon Test).

Likewise, the Friedman Rank Sums Test can be thought of as an extension of the Wilcoxon Signed Rank Test in that it analyzes the treatment effects of two or more dependent, or related, samples. This matching of data into groups on the basis of some natural or obvious criterion enhances the ability to discern differences in the variable of interest. When considering more than two samples, these groups are often referred to as "blocks", and the samples themselves as "experimental units".

A block may consist of plots, littermates or any group of subjects carefully matched in age, sex, weight, etc. An individual subject may even serve as a block. For instance, in studying the effects of drugs or in taste-testing, each of several subjects (blocks) may be tested at the same time or at different times, assuming there is no "carry over" effect.

As for the classical F-test, both the Kruskal-Wallis and the Friedman Rank Sums Test operate under the assumption that population variances are equal.

I. Kruskal-Wallis Test. a.k.a. One-way layout.

A. Use

Analyzes the treatment effects of two or more independent samples set up on the Completely Randomized Design.

B. Data

Consist of k random samples of sizes $n_1, n_2, \ldots, n_k$, with observations measured at least on an ordinal scale such that

$$X_{ij} = M + \tau_j + e_{ij} \qquad \begin{matrix} i = 1, \ldots, n \\ j = 1, \ldots, k \end{matrix}$$

where X_{ij} is the ith observation in treatment j.
M is the unknown overall mean.
τ_j is the unknown treatment j effect.
e_{ij} are unobservable random variables.

C. Assumptions

1. The e's are independent.
2. Each e comes from the same continuous population.
3. $\sum_{j=1}^{k} \tau_j = 0$

D. Equations

1. Test statistic

Let
$$H = \frac{\left(\frac{12}{N(N+1)} \sum_{j=1}^{k} \frac{R_j^2}{n_j}\right) - 3(N+1)}{1 - \left(\sum_{j=1}^{g} (t_j^3 - t_j)/(N^3 - N)\right)}$$

where $N = \sum_{j=1}^{k} n_j$ i.e., the total observations.

$R_j = \sum_{i=1}^{n_j} r_{ij}$

where r_{ij} is the rank of X_{ij} in the overall ranking of the X's from least to greatest.

g is the number of tied groups.

t_j is the size of tied group j.

2. Multiple comparisons

$\tau_u \neq \tau_v$ if...

$$|R_{.u} - R_{.v}| \geq z_{\left(\frac{\alpha}{k(k-1)}\right)} \left[\frac{N(N+1)}{12}\right]^{1/2} \left(\frac{1}{n_u} + \frac{1}{n_v}\right)^{1/2}$$

where $R_{.j} = \sum_{i=1}^{n_j} r_{ij}/n_j$ for j=u and v, where u < v.

E. Hypotheses

Select a level of significance, α , and compare the test statistic, H, to a chi-square value from Table H based on α and (k-1) degrees of freedom.

H_0: $\tau_1 = ... = \tau_k$

H_1: Not all treatment effects are equal.

Accept H_0 if $H < \chi^2_{(k-1,\alpha)}$

F. Program/Subroutine

KWT and QS.

G. Size formula

SIZE = 2N + 14 + INT[ln N/ln 2] (max. size = 197).

H. Limits and Warnings

For program KWT, k must be no greater than 8.

When H_0 is true the test statistic, H, has an asymptotic chi-square distribution. For small samples, tables exist for exact comparison with H. But because of the large number of possible combinations of sample sizes and observations within samples, such tables are necessarily limited in scope, (usually i < 6 and j = 3). Because of its limited appeal and large space requirement, no such table is included in this publication.

TEST: Kruskal-Wallis Test	PGM: KWT	MAX SIZE: 197

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Set calculator to USER mode.			
1	Set SIZE according to the size formula.		XEQ SIZE___	
2	Load program KWT and subroutine QS.			
3	Begin program.		XEQ KWT	DATA: R/S
4	Enter observations according to treatments.	X_{ij}	R/S	X_{ij} i
	Continue entering observations for a given treatment according to Step 4.			
5	If an error occurs, erase it by pressing "E" and entering the correct value according to Step 4.		E	$X_{i-1,j}$ i-1
6	When all observations in one treatment have been entered, press "A". Enter the observations in the next treatment according to Step 4.		A	DATA: R/S
7	After all observations for all treatments have been entered and "A" has been pressed after the last treatment, calculate the test statistic, H.		C	SORT X_{ij} R_{ij}
	Program lists the observations and their average ranking, from least to greatest, for the pooled observations before displaying the test statistic, H, with a tone.			. . . H=_____
8	For a multiple comparisons at an experimentwise error rate of α...	$Z_{(\alpha/(k^2-k))}$	R/S R/S	Z α/KK-K? (See equations section)
	If two treatments are significantly different, a tone sounds.		R/S . .	. . .
9	For the sum of the rankings in any treatment j...	j	R/S	SUM? $R_{.j}$
	For next treatment, j', simply enter j' without summoning the "SUM?" display.	j'	R/S	$R_{.j'}$

SAMPLE PROBLEM No. 1 **TEST** KWT

Random samples from each of 5 brands of batteries were tested in a toy to determine median battery life. Is there a difference in battery life among the brands tested?

#	A	B	C	D	E
1	80	77	78	82	70
2	77	80	88	71	73
3	76	81		79	62
4	79	84		75	67
5	90				64

(Brands: A–E)

Battery life is given in minutes.

Ho: All battery lives are equal, independent of brand.
H1: The median battery life of at least one brand differs from the others.
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program KWT and subroutine QS and set SIZE = 058.
	XEQ KWT	DATA: R/S	Enter observations by treatment, i.e., brand.
80	R/S	80.0000 1	X_{ij} i
77	R/S	77.0000 2	
67	R/S	67.0000 3	An incorrect value was inputted for $X_{3,1}$, so erase it by pressing "E".
	E	77.0000 2	$X_{i-1,j}$ i-1
76	R/S	76.0000 3	
79	R/S	79.0000 4	
90	R/S	90.0000 5	
	A	DATA: R/S	Enter observations in next treatment.
77	R/S	77.0000 1	Continue entering all observations, pressing "A" between treatments.
.	.	.	
.	.	.	
.	.	.	
64	R/S	64.0000 5	
	A	DATA: R/S	
	C	SORT	Approximately 45 seconds, then...

SAMPLE PROBLEM No 1 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
		62.00 1.00	Lists X_{ij} from least to greatest and assigns them average ranks, R_{ij}.
		. . .	
		90.00 20.00	
		H=11.449	Calculates the test statistic, H. From Table H, $\chi^2_{(4,0.05)} = 9.488$. Since the test statistic, H, is not less than this chi-square value, reject H_0.
			For a multiple comparisons test...
	R/S	Z α/KK-K?	Enter $Z_{(\alpha/(k(k-1))}$.
			If $\alpha = 0.05$, $Z_{(0.05/5(4))} = Z_{0.0025}$ which, from Table A gives 2.81.
2.81	R/S	1,2 1.6 11.15	u,v R.u - R.v $Z_{(\alpha/k(k-1))}\left(N(N+1)/12\right)^{\frac{1}{2}}$ etc.
	. . . .	. . . .	(See equations section).
			Tone sounds if comparison is significant.
	R/S	SUM?	Prompt for R.j
2	R/S	58.00	$R_{.2}$
5	R/S	16.00	$R_{.5}$
			Treatment 2 (Brand B) is significantly different than treatment 5 (Brand E) at the 95% confidence level.
			$R_{.2}/n_2 = 58/4 = 14.5$
			$R_{.5}/n_5 = 16/5 = 3.2$
			The average ranking in treatment 2 is significantly higher than the average ranking in treatment 5, suggesting Brand B has a longer battery life in the toy than Brand E.

II. Friedman Rank Sums Test. a.k.a. Two-way layout.

A. Use

Analyzes the treatment effects of two or more dependent samples set up on the Randomized Complete Block Design.

B. Data

Consist of k x n observations with one observation from each of k treatments in each of n blocks measured, blockwise, at least on an ordinal scale such that

$$X_{ij} = M + B_i + \tau_j + e_{ij} \qquad \begin{matrix} i = 1, \ldots, n \\ j = 1, \ldots, k \end{matrix}$$

where X_{ij} is the observation in block i, treatment j.
M is the unknown overall mean.
B_i is the unknown block i effect.
τ_j is the unknown treatment j effect.
e_{ij} are unobservable random variables.

C. Assumptions

1. The e's are independent.
2. Each e comes from the same continuous population.
3. $\sum_{i=1}^{n} B_i = \sum_{j=1}^{k} \tau_j = 0$

D. Equations

1. Test statistic

Let $$S = \frac{12 \sum_{j=1}^{k} \left(R_j - \frac{n(k+1)}{2} \right)}{(k+1) - \left\{ \frac{1}{k-1} \sum_{i=1}^{n} \left[\left(\sum_{j=1}^{g_i} t_{ij}^3 \right) - k \right] \right\}}$$

$$R_j = \sum_{i=1}^{n_j} r_{ij}$$

where r_{ij} is the rank of X_{ij} in the ranking of X's in each block from least to greatest.

g_i is the number of tied groups in block i.

t_{ij} is the size of the jth tied group in block i.

2. Multiple comparisons

$$\tau_u \neq \tau_v \text{ if...}$$

$$\left|R_u - R_v\right| \geq q(\alpha,k,\infty)\sqrt{\frac{nk(k+1)}{12}}$$

where $q(\alpha,k,\infty)$ is obtained from Table E.

R_u and R_v are defined above for R_j

(letting $u < v$).

E. Hypotheses

Select a level of significance, α , and compare the test statistic, S, to a chi-square value from Table H based on α and (k-1) degrees of freedom.

H_0: $\tau_1 = ... = \tau_k$

H_1: Not all treatment effects are equal.

Accept H_0 if $S < \chi^2_{(k-1,\alpha)}$

F. Program/Subroutine

FSR and QS.

G. Size formula

SIZE = k(n+1) + 11 + INT[ln k/ln 2] (max. size = 196).

H. Limits and Warnings

When H_0 is true the test statistic, S, has an asymptotic chi-square distribution. For small samples, tables exist for exact comparison with S. But because of the large number of possible combinations of sample sizes and observations within samples, such tables are necessarily limited in scope, (usually with k between 3 and 5 and n between 2 and 8 (or 13, depending on k)). Because of its limited appeal and large space requirement, no such table is included in this publication.

It is not possible to detect interactions between the blocks and the treatments using this test.

TEST: Friedman Rank Sums	PGM: FRS	MAX SIZE: 196

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Set calculator to USER mode.			
1	Set SIZE according to the size formula.		XEQ SIZE___	
2	Load program FRS and subroutine QS.			
3	Begin program.		XEQ FRS	DATA: R/S
4	Enter observations by Blocks.	X_{ij}	R/S	i,j= X_{ij}
	Repeat Step 4 until all observations in a particular block have been entered.			
5	If an error occurs, erase the incorrect entry by pressing "E" and entering the correct observation according to Step 4.		E	i,j?
6	When all observations in a block have been entered, press "A". Go to Step 4.		A	i+1,1?
	If the number of observations entered in a particular block are not equal to the number of observations entered for the first block, a warning is given and the block must be reentered.			TRTS UNEQUAL i+1,1?
7	After all the observations from all the blocks have been entered, follow Step 6 by pressing "C".		C	SORT
	SORT appears on and off as many times as there are blocks.			. . . SORT
	SORT is then followed by listing of the summation of the average rank for each treatment.			R1=_____ . . Rj=_____
	After the last summation listing, the test statistic, S, is displayed.			S=_____
8	For a multiple comparisons test at an experimentwise error rate of α....		R/S	$Q\alpha$,K?
9	Enter the appropriate value for q from Table E.	$q(\alpha,k,\infty)$	R/S	(See equations section)
	A tone sounds for each significant comparison.		. . .	. .

SAMPLE PROBLEM No. I **TEST** FRS

A group of seven crossword puzzle enthusiasts (Blocks) were given 4 crossword puzzles (Treatments) in random sequences to determine if any one puzzle was more difficult than the others, as measured by the time needed for completion. Given the data below, did one puzzle require significantly more time to complete as compared to the others?

	Puzzles			
Subject	A	B	C	D
1	15.3	18.7	21.6	19.1
2	14.5	14.6	16.2	12.7
3	13.6	14.0	16.2	18.0
4	14.8	16.3	15.5	15.3
5	16.8	23.2	18.7	17.9
6	17.3	21.0	19.4	18.7
7	16.6	16.9	16.6	17.0

Values are minutes to complete the puzzles.

H_0: The puzzles are of equal difficulty.
H_1: The puzzles differ in their degree of difficulty.
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program FRS and subroutine QS and set SIZE = 045.
	XEQ FRS	DATA: R/S	Enter observations by Blocks.
15.3	R/S	1,1=15.300	i,j=X_{ij}
18.7	R/S	1,2=18.700	
26.1	R/S	1,3=26.100	An incorrect value was inputted for $X_{1,3}$, so erase it by pressing "E".
	E	1,3?	Prompts for correct value for $X_{1,3}$.
21.6	R/S	1,3=21.600	
19.1	R/S	1,4=19.100	
	A	2,1?	Prompts for next block entry.
14.5	R/S	2,1=14.500	
14.6	R/S	2,2=14.600	
16.2	R/S	2,3=16.200	
	A	TRTS UNEQUAL 2,1?	"A" had been pressed before the block was completely entered, and the programs prompts for the block to be loaded again. (The same response occurs if the number of treatments entered exceeds the number of treatments in the

SAMPLE PROBLEM No 1 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
			first block.)
14.5	R/S	2,1=14.500	
.	.	.	
.	.	.	
.	.	.	
17.0	R/S	7,4=17.000	
	A	8,1?	
	C	SORT	After all blocks have been entered, calculate the test statistic, S, by pressing "C". SORT will appear off and on as many times as there are blocks.
		.	
		.	
		.	
		R1=8.500	The sum of the rankings in treatment 1.
		.	. . . .
		.	. . . .
		R4=18.000	. . . 4.
		S=10.217	Test statistic, S. (Approximate run time is $3\frac{1}{4}$ minutes.)
			From Table H, $\chi^2_{(3,0.05)} = 7.815$. Since "S" is greater than 7.815, reject H_o.
			For a multiple comparison test at an experimentwise error rate of 0.05...
	R/S	Qα,K?	Prompt for q(α,k,∞) from Table E.
			q(0.05,4,∞) = 3.633
3.633	R/S	1,2 13.5 12.4	(See equations section).
	.	.	Tone sounds if the comparison is significant.
	.	.	
	.	.	
	.	22.00	End of program.
			From the multiple comparisons, treatments 1&2, and 1&3 are significantly different, i.e., puzzle A differs in difficulty from puzzles B and C. From their average rankings, R_j, or simply taking the average time the seven subjects needed to complete each puzzle we find the puzzle A took less time to complete than any of the other puzzles. This is most easily interpreted by stating puzzle A is easiest to complete. Therefore, no puzzle required significantly more time to complete.

D. Two-Sample Dispersion.

Frequently, the researcher is interested in the reproducibility, or consistency, of data. Attention then shifts from the median to the dispersion, or scale, parameter as a measure of precision.

In parametric statistics the F-test is used to test if two population dispersion parameters are equal. The F-test, however, is not a very safe test to use unless the populations of interest are truly normal. For instance, the double exponential distribution is symmetric and resembles somewhat the normal distribution. Yet the true level of significance may be 2 or 3 times larger than it should be if the F-test were applied to this distribution. Thus the acute sensitivity of the F-test to the assumption of normality plus its lack of power in some reasonable non-normal situations encourages the consideration of a nonparametric test for dispersion. In short, choose a nonparametric scale parameter test over the F-test for any of the following reasons:

1. The sample sizes are small.
2. The data are measured on an ordinal scale.
3. The population distribution is unknown, or is known not to be normal.

The first step in selecting a nonparametric procedure for dispersion is to consider the populations' medians. If they are known to be equal, or unknown but assumed equal, then either of the first two tests in this section--the Freund-Ansari-Bradley Test or the Mood Test--may be applied to the data. These tests may also be used if the two population medians are known and unequal. In this event, the observations of one population may be adjusted so that the populations have a common median.

Some tests (not included here) substitute the sample median or mean for the population parameter. These tests, though, are not generally distribution-free.

Oftentimes, the population medians are unknown and must be assumed unequal. The last two tests in this section--Moses' Test and Miller's Jackknife Test--are specifically designed for such a circumstance. Because these tests do not require the two underlying populations to have equal medians, they are more broadly applicable.

I. Freund-Ansari-Bradley Test. a.k.a. Freund-Ansari Test, Ansari-Bradley Test.

A. Use

Tests for the equality of dispersion parameters when two population medians are equal.

B. Data

Consist of two random samples, $X_1, X_2, \ldots, X_m$, and $Y_1, Y_2, \ldots, Y_n$, from different populations, measured at least on an ordinal scale.

C. Assumptions

1. The two samples are independent.
2. The population distributions are continuous.
3. The populations differ (if they differ at all) only in dispersion.

D. Equations

1. Test statistic

$$W = \sum_{a=1}^{p} r_a$$

where r_a is the rank of X_a in the joint ranking of all X and Y, whereby the smallest and largest observations are assigned rank 1; rank 2 is assigned to the second smallest and second largest observation; etc.

E. Hypotheses

Select an approximate level of significance, α', and compare the test statistic, W, to $w(\alpha, m, n)$ from Table F for an α closest to α'.

H_0	H_1	Accept H_0 if...
$\gamma \leq 1$	$\gamma > 1$	$W < w(\alpha, m, n)$
$\gamma \geq 1$	$\gamma < 1$	$w([1-\alpha], m, n) < W$
$\gamma = 1$	$\gamma \neq 1$	$w([1-\alpha/2], m, n) < W < w(\alpha/2, m, n)$

where $\gamma = \frac{\sigma_y}{\sigma_x}$

F. Program/Subroutine

FAB and QS.

G. Size formula

SIZE = 2m + n + 11 + INT[ln (m+n)/ln 2] (max. size = 221).

H. Limits and Warnings

To facilitate use of Table F, allow $m \leq n$.

For $m > 10$, Table F cannot be used and we must refer to the large sample approximation test statistic, W^*.

If m + n is even...

$$W^* = \frac{W - \left[m\left(m+n+2\right)/4\right]}{\left\{\dfrac{mn\left(16\sum_{b=1}^{g} t_b r_b^2 - \left[m+n\right]\left[m+n+2\right]^2\right)}{16\left[m+n\right]\left[m+n-1\right]}\right\}^{\frac{1}{2}}}$$

If m + n is odd...

$$W^* = \frac{W - \left[m\left(m+n+1\right)^2/4(m+n)\right]}{\left\{\dfrac{mn\left(16\left[m+n\right]\sum_{b=1}^{g} t_b r_b^2 - \left[m+n+1\right]^4\right)}{16\left[m+n\right]^2\left[m+n-1\right]}\right\}^{\frac{1}{2}}}$$

where g is the number of tied groups.

t_b is the size of tied group j.

Select a level of significance, α, and compare W^* to Z_α from Table A.

H_1	Accept H_0 if...
$\gamma > 1$	$W^* < Z_\alpha$
$\gamma < 1$	$-Z_\alpha < W^*$
$\gamma \neq 1$	$-Z_{\alpha/2} < W^* < Z_{\alpha/2}$

TEST: Freund-Ansari-Bradley	PGM: FAB	MAX SIZE: 221

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Set calculator to USER mode.			
1	Set SIZE according to the size formula.		XEQ SIZE___	
2	Load program FAB and subroutine QS.			
3	Begin program.		XEQ FAB	X: R/S
4	Enter the X observations.	X_i	R/S	i X_i
	Repeat Step 4 until all X observations have been entered.			
5	If an error occurs, simply erase it by pressing "E" and enter the correct value for X according to Step 4.		E	i-1 X_{i-1}
6	After all X observations have been entered, prompt for entering the Y observations.		A	Y: R/S
7	Enter the Y observations.	Y_j	R/S	j Y_j
	Repeat Step 7 until all Y observations have been entered.			
8	If an error occurs, simply erase it by pressing "E" and enter the correct value for Y according to Step 7.		E	j-1 Y_{j-1}
9	After all Y observations have been entered, calculate the test statistic, W.		C	SORT SORT
	After two SORT's, the X and Y observations are displayed from least to greatest and their average rankings, beginning at a minimum, passing through a maximum and returning to a minimum, also displayed.			X_i or Y_j Rank . . . W=_______
10	For the large sample approximation test statistic...		R/S	W*=_______

SAMPLE PROBLEM No. 1 **TEST** FAB

Two vintners, both anxious to have their best bordeaux sold exclusively at a famous restaurant, approached the restaurant's sommelier. Knowing the reputations of both wines were rated "first growth", the wines were undoubtably comparable in quality. The sommelier was also aware that, after level of quality, consistency was of utmost importance.

The sommelier selected 4 bottles of wine from vineyard X and 5 bottles from vineyard Y. After rating their quality on a scale of 1 to 10 (worst to best), does he have reason to believe quality from one vineyard is more uniform than from the other?

Wine	Vineyard X	Vineyard Y
1	9.3	9.9
2	9.7	9.1
3	8.8	9.4
4	8.6	9.6
5		8.9

Ho: $\gamma = 1$

H1: $\gamma \neq 1$

α : 0.10

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program FAB and subroutine QS and set SIZE = 024.
	XEQ FAB	X: R/S	Enter the X observations.
9.3	R/S	1 9.300	i X_i
9.7	R/S	2 9.700	
9.8	R/S	3 9.800	An error occurred inputting X_3, so simply erase it by pressing "E" and entering the correct value for X_3.
	E	2 9.700	i-1 X_{i-1}
8.8	R/S	3 8.800	
8.6	R/S	4 8.600	
	A	Y: R/S	After all X observations have been entered, prompt for the Y observations.
9.9	R/S	1 9.900	j Y_j
.	.	.	
.	.	.	
.	.	.	
8.9	R/S	5 8.900	

SAMPLE PROBLEM No 1 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
	C	SORT	Approximately 15 seconds, then...
		SORT	Approximately 5 seconds, then...
		8.600 1.00 . . . . . . 9.900 1.00	(Xi or Yj) and its average ranking among the combined observations. The observations are displayed from least to greatest, while the ranking proceeds from a minimum through a maximum back to a minimum.
		W=10.00	Test statistic. From Table F, w(0.0476,4,5) = 15, and w(0.9603,4,5) = 8. Since the test statistic lies between 8 and 15, accept H_o. i.e., $8<10<15$ Accept H_o Since the wines from both vineyards have equal medians and equal dispersion, the sommelier is free to choose either vineyard with equal assurance.

SAMPLE PROBLEM No. 2 **TEST** FAB

Two EPA laboratories evaluated separate procedures designed to measure the concentration of ethylene dibromide (EDB) in muffin mixes. Fifteen replicates were conducted using one procedure on a sample of muffin mix known to contain 8 parts per million (ppm) EDB. And 13 replicates were conducted at the other laboratory using a new procedure on a sample which, because of a typical snafu, contained 18 ppm EDB. Does the new procedure afford less scattered results than the old procedure? (There are several tacit assumptions being made here, most notably 1) the samples are entirely homogeneous and 2) the two operators are equally competent.)

Replicate	Old Procedure	New Procedure	
1	8.21	18.06	Values are ppm EDB.
2	8.59	18.06	
3	8.07	17.89	
4	8.14	18.14	
5	8.09	17.93	
6	8.20	17.92	
7	8.16	18.04	
8	7.68	17.92	
9	7.14	17.82	
10	8.06	17.80	
11	8.27	17.95	
12	7.88	17.69	
13	8.56	17.64	
14	7.95		
15	7.94		

H_0: $\gamma = 1$
H_1: $\gamma < 1$
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program FAB and subroutine QS and set SIZE = 058.
	XEQ FAB	X: R/S	Enter the X observations.
			Since the two population medians are known and unequal, we can adjust the observations of, say, sample X by adding 10.00 ppm so the two samples share a common median.
18.21	R/S	1 18.210	i X_i
18.59	R/S	2 18.590	
8.07	R/S	3 8.070	An error occurred inputting X_3, so simply erase it by pressing "E" and entering the correct value for X_3.
	E	2 18.590	i-1 X_{i-1}
18.07	R/S	3 18.070	
.	.	.	
.	.	.	
.	.	.	
17.94	R/S	15 17.940	

SAMPLE PROBLEM No 2 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
	A	Y: R/S	After all X observations have been entered, prompt for the Y observations.
18.06	R/S	1 18.060	
.	.	.	
.	.	.	
.	.	.	
17.64	R/S	13 17.640	
	C	SORT	Approximately 70 seconds, then...
		SORT	Approximately 25 seconds, then...
		17.14 1.00	(X_i or Y_j) and its average ranking among the combined observations. The observations are
		. .	displayed from least to greatest, while the
		. .	ranking proceeds from a minimum through a max-
		. .	
		18.59 1.00	imum back to a minimum.
		W=96.00	Test statistic.
			Since the values for m and n exceed the limits of Table F, the large sample approximation for the test statistic must be calculated.
	R/S	W*=-1.529	The large sample approximation test statistic.
			From Table A, $Z_{0.05}$ = 1.645. Since W* is greater than $-Z_{0.05}$, accept H_o.
			The newer procedure does not necessarily give more uniform results than the old method.

II. Mood's Dispersion Test.

A. Use

Tests for the equality of dispersion parameters when the two population medians are equal.

B. Data

Consist of two random samples, $X_1, X_2, \ldots, X_p$, and $Y_1, Y_2, \ldots, Y_q$, from different populations, measured at least on an ordinal scale.

C. Assumptions

1. The two samples are independent.
2. The population distributions are continuous.
3. The populations differ (if they differ at all) only in dispersion.

D. Equations

1. Test statistic

$$M = \sum_{a=1}^{p} \left(r_a - \frac{N+1}{2} \right)^2$$

where r_a is the rank of X_a in the joint ranking, from least to greatest, of all X and Y.

$N = p + q$

E. Hypotheses

Select a level of significance, α, and compare the test statistic, M, to M' and/or M" from Table G using the following guidelines:

H_0	H_1	Accept H_0 if...
$\gamma \leq 1$	$\gamma > 1$	M' (or M") < M for α' (or α'') closest to α under the column labeled α.
$\gamma \geq 1$	$\gamma < 1$	M < M' (or M") for α' (or α'') closest to α under the column labeled $1-\alpha$.
$\gamma = 1$	$\gamma \neq 1$	M" (or M') < M < M' (or M") for α' and α'' closest to $\alpha/2$ under columns labeled α and $1-\alpha$, respectively.

where $\gamma = \frac{\sigma_y}{\sigma_x}$

For a chosen level of significance, α, and given values of p and q, Table G gives several values. Each cell under the columns labeled α and $1-\alpha$ usually contains four entries:

M' α' M" α''	Choose M' or M", whichever's probability (α' or α'', respectively) is closer to α, under the appropriate column (i.e., α or $1-\alpha$) to compare with the test statistic.

F. Program/Subroutine

MOD and QS.

G. Size formula

SIZE = 2p + q + 11 + INT[ln(p+q)/ln 2] (max. size = 230)

H. Limits and Warnings

To facilitate the use of Table G, allow $p \le q$.

For $p > 10$, Table G cannot be used and we must refer to the large sample approximation test statistic, M^*, calculated by MOD.

$$M^* = \frac{M - Em}{\sqrt{Vm}} + \frac{1}{\sqrt{4\,Vm}}$$

where $Em = p(N^2-1)/12$

$$Vm = \frac{pq}{180}\left\{(N+1)(N^2-4) - \frac{1}{N(N-1)}\sum_{a=1}^{g} t_a(t_a^2-1)\left[t_a^2-4+15\left(N-S_a-S_{a-1}\right)^2\right]\right\}$$

where g is the number of tied groups.

t_a is the size of tied group a.

$S_a = \sum_{h=1}^{a} t_h$

Select a level of significance, α, and compare the large sample approximation test statistic, M^*, to Z_α from Table A.

H_1	Accept H_0 if...
$\gamma > 1$	$M^* < Z_\alpha$
$\gamma < 1$	$M^* < Z_\alpha$
$\gamma \ne 1$	$M^* < Z_{\alpha/2}$

Note: M and M^* are always positive.

I. Comparison with the Freund-Ansari-Bradley Test[†].

The Freund-Ansari-Bradley Test is preferable when the central rankings give more dispersion information than do the extreme rankings. The Mood Test gives more weight to the extreme ranks and is more efficient than the Freund-Ansari-Bradley Test under normal alternatives. So the Freund-Ansari-Bradley Test is recommended over the Mood Test if the distribution under study is known to have "heavy" tails (e.g., double exponential, Cauchy). Otherwise, Mood's Dispersion Test is the preferable test.

[†]Duran, Benjamin S., "A Survey of Nonparametric Tests for Scale", Commun. Statist.-Theor. Meth., A5(14), 1287-1312 (1976).

TEST: Mood's Dispersion Test	PGM: MOD	MAX SIZE: 230

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Set calculator to USER mode.			
1	Set SIZE according to the size formula.		XEQ SIZE___	
2	Load program MOD and subroutine QS.			
3	Begin program.		XEQ MOD	X: R/S
4	Enter the X observations.	X_i	R/S	i X_i
	Repeat Step 4 until all X observations have been entered.			
5	If an error occurs, simply erase it by pressing "E" and enter the correct value of X according to Step 4.		E	i-1 X_{i-1}
6	After all X observations have been entered, prompt for entering the Y observations.		A	Y: R/S
7	Enter the Y observations.	Y_j	R/S	j Y_j
	Repeat Step 7 until all Y observations have been entered.			
8	If an error occurs, simply erase it by pressing "E" and enter the correct value for Y according to Step 7.		E	j-1 Y_{j-1}
9	After all Y observations have been entered, calculate the test statistic, M.		C	SORT M=________
10	For the large sample approximation test statistic...		R/S	M*=______

SAMPLE PROBLEM No. 1 **TEST** MOD

Two vintners, both anxious to have their best bordeaux sold exclusively at a famous restaurant, approached the restaurant's sommelier. Knowing the reputations of both wines were rated "first growth", the wines were undoubtably comparable in quality. The sommelier was also aware that, after level of quality, consistency was of utmost importance.

The sommelier selected 4 bottles of wine from vineyard X and 5 bottles from vineyard Y. After rating their quality on a scale of 1 to 10 (worst to best), does he have reason to believe quality from one vineyard is more uniform than from the other?

Wine	Vineyard X	Vineyard Y
1	9.3	9.9
2	9.7	9.1
3	8.8	9.4
4	8.6	9.6
5		8.9

Ho: $\gamma = 1$
H1: $\gamma \neq 1$
α : 0.10

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program MOD and subroutine QS and set SIZE = 027.
	XEQ MOD	X: R/S	Enter the X observations.
9.3	R/S	1 9.300	i X_i
9.7	R/S	2 9.700	
9.8	R/S	3 9.800	An error occurred inputting X_3, so simply erase it by pressing "E" and entering the correct value for X_3.
	E	2 9.700	i-1 X_{i-1}
8.8	R/S	3 8.800	
8.6	R/S	4 8.600	
	A	Y: R/S	After all X observations have been entered, prompt for the Y observations.
9.9	R/S	1 9.900	j Y_j
.	.	.	
.	.	.	
.	.	.	
8.9	R/S	5 8.900	

INPUT	FUNCTION	DISPLAY	COMMENTS
	C	SORT M=34.00	Approximately 15 seconds, then... Test statistic. (Run time approximately 40 seconds). From Table G, $w(\alpha/2,p,q) = (0.05,4,5)$ and $M', \alpha' = 10,0.0397$ $M'', \alpha'' = 11,0.0556$ Since α'' is closer to $\alpha/2$, or 0.05, M'' is selected as the lower limit of our test statistic acceptance region. Consequently, M' under $(1 - \alpha/2)$, or 0.95, (i.e., $w(0.95,4,5) = 41$) must serve as the upper limit. Since $11 < 34 < 41$, accept H_o.

SAMPLE PROBLEM No. 2 **TEST** MOD

Two EPA laboratories evaluated separate procedures designed to measure the concentration of ethylene dibromide (EDB) in muffin mixes. Fifteen replicates were conducted using one procedure on a sample of muffin mix known to contain 8 parts per million (ppm) EDB. And 13 replicates were conducted at the other laboratory using a new procedure on a sample which, because of a typical snafu, contained 18 ppm EDB. Does the new procedure afford less scattered results than the old procedure? (There are several tacit assumptions being made here, most notably 1) the samples are entirely homogeneous and 2) the two operators are equally competent.)

Replicate	Old Procedure	New Procedure	
1	8.21	18.06	Values are ppm EDB.
2	8.59	18.06	
3	8.07	17.89	
4	8.14	18.14	
5	8.09	17.93	
6	8.20	17.92	
7	8.16	18.04	
8	7.68	17.92	
9	7.14	17.82	
10	8.06	17.80	
11	8.27	17.95	
12	7.88	17.69	
13	8.56	17.64	
14	7.95		
15	7.94		

Ho: $\gamma = 1$
H1: $\gamma < 1$
α: 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program MOD and subroutine QS and set SIZE = 058.
	XEQ MOD	X: R/S	Enter the X observations.
			Since the two population medians are known and unequal, we can adjust the observations of, say, sample X by adding 10.00 ppm so the two samples share a common median.
18.21	R/S	1 18.210	i X_i
18.59	R/S	2 18.590	
8.07	R/S	3 8.07	An error occurred inputting X_3, so simply erase it by pressing "E" and entering the correct value for X_3.
	E	2 18.590	i-1 X_{i-1}
18.07	R/S	3 18.070	
.	.	.	
.	.	.	
.	.	.	
17.94	R/S	15 17.940	

SAMPLE PROBLEM No 2 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
	A	Y: R/S	After all X observations have been entered, prompt for the Y observations.
18.06	R/S	1 18.060	j Y_j
.	.	.	
.	.	.	
.	.	.	
17.64	R/S	13 17.640	
	C	SORT	Approximately 1 minute, then...
		M=1227.250	Test statistic. (Run time approximately $3\frac{1}{4}$ minutes).
			Since Table G does not include M' and M" values for p = 15 and q = 13, we must use the large sample approximation test statistic, M*.
	R/S	M*=1.591	From Table A, $Z_{0.05} = 1.645$.
			Since $1.595 < 1.645$, accept H_o.

III. Moses' Dispersion Test. a.k.a. Moses' Distribution-Free Ranklike Test.

A. Use

Tests the equality of dispersion parameters when the two population medians are unknown and unequal.

B. Data

Consist of two random samples, $X_1, X_2, \ldots X_{m'}$, and $Y_1, Y_2, \ldots, Y_{n'}$, from different populations, measured at least on an ordinal scale. The X and Y observations are randomly assigned into m and n subgroups, respectively, of size k, ($k \geq 2$), discarding any leftover observations.

C. Assumptions

1. The two samples are independent.
2. The population distributions are continuous.

D. Equations

1. Test statistic

Let $$C_a = \left[\sum_{s=1}^{k} X_{as}^2 - \frac{(\sum_{s=1}^{k} X_{as})^2}{k}\right]$$

and $$D_b = \left[\sum_{t=1}^{k} Y_{bt}^2 - \frac{(\sum_{t=1}^{k} Y_{bt})^2}{k}\right] \qquad \begin{array}{l} a = 1,\ldots,m \\ b = 1,\ldots,n \end{array}$$

then $$W = \sum_{b=1}^{n} r_b$$

where r_b is the rank of D_b in the joint ranking, from least to greatest, of all C and D.

2. Point estimate

Let $$V^k = D_b/C_a \qquad k = 1,\ldots,mn$$

If mn is odd.. γ = the median of the V^k's.

If mn is even... $\gamma = \sqrt{V^r \cdot V^{r+1}}$

where $r = \frac{mn}{2}$

3. Confidence interval

Using Table D...

Let $\theta = n(2m+n+1)/2$

then $\theta + 1 - \xi = \omega(\alpha/2, m, n)$

The lower confidence interval boundary... $\gamma_L = V^{\xi}$
The upper confidence interval boundary... $\gamma_U = V^{mn+1-\xi}$

E. Hypotheses

Select an approximate level of significance, α', and com- compare the test statistic, W, to $w(\alpha,m,n)$ from Table D for an α closest to α'.

H_0	H_1	Accept H_0 if...
$\gamma \leq 1$	$\gamma > 1$	$W < w(\alpha,m,n)$
$\gamma \geq 1$	$\gamma < 1$	$n(m+n+1) - w(\alpha,m,n) < W$
$\gamma = 1$	$\gamma \neq 1$	$n(m+n+1) - w(\alpha/2,m,n) < W < w(\alpha/2,m,n)$

F. Program/Subroutine

MOS and QS.

G. Size formula

SIZE = mn + (m+n) + 10   (max. size = 230).

H. Limits and Warnings

For ease in using Table D, allow $m \geq n$.

In selecting the subgroup size, k, it should be as large as possible (but not greater than 10) without making m and n so small that the distribution of the appropriate test statistic cannot provide meaningful significance levels.

It is possible for two people to arrive at different conclusions based on the same data, depending upon the random grouping of the observations into subgroups.

TEST: Moses' Dispersion Test	PGM: MOS	MAX SIZE: 230

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Set calculator to USER mode.			
1	Set SIZE according to the size formula.		XEQ SIZE____	
2	Load program MOS and subroutine QS.			
3	Begin program.		XEQ MOS	DATA: R/S
4	Enter the X observations as subgroups of equal size, k.	$X_{s,a}$	R/S	s $X_{s,a}$
	Repeat Step 4 until the first X subgroup has been entered.			
5	If an error occurs, simply erase it by pressing "E" and enter the correct X observation according to Step 4.		E	s-1 $X_{s-1,a}$
6	After an X subgroup has been entered, press "A" for its sum of squares.		A	SS=________
7	Continue with Step 4 if another X subgroup must be entered. If all "a" X subgroups have been entered, press "B" to begin entering the Y subgroups.		B	DATA: R/S
8	Enter the Y observations as subgroups of equal size.	$Y_{t,b}$	R/S	t $Y_{t,b}$
	Repeat Step 8 until the Y subgroup has been entered.			
9	If an error occurs, simply erase it by pressing "E" and enter the correct Y observation according to Step 8.		E	t-1 $Y_{t-1,b}$
10	After a Y subgroup has been entered, press "A" for its sum of squares.		A	SS=________
11	Continue with Step 8 if another Y subgroup must be entered. If all "b" Y subgroups have been entered, press "C" to begin calculations.		C	SORT D_b R_b . .
	Test statistic, W.			W=______

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
12	Calculate the estimator of γ.		R/S	SORT GMA= ______
13	Determine the confidence interval for γ.		R/S	V?
14	For the lower confidence interval limit	ξ	R/S	$GAMMA_L$
15	For the upper confidence interval limit	 $mn+1-\xi$	R/S R/S	V? $GAMMA_U$
	Steps 14 and 15 may be repeated as often as desired for different confidence coefficients.			

SAMPLE PROBLEM No. 1 **TEST** MOS

An agronomist suspected that the second of two plots of equal area but dissimilar dimensions was more homogeneous in its soil properties. So he planted the same pure hybrid of corn on both plots and harvested it one row at a time (discarding end rows), recording the yield per row. Is Plot Y more homogeneous than Plot X?

Row	Plot X	Plot Y
1	54	60
2	42	58
3	58	63
4	57	64
5	70	55
6	56	67
7	52	56
8	71	56
9	43	65
10	42	57
11	49	69
12	52	63
13	28	56
14	51	69
15	60	67
16	39	65
17	49	55
18	54	
19	30	
20	45	

Values are yield of corn per row.

The data at left were randomly divided into subgroups of size k = 4, listed below.

X Subgroups

58	52	42	39
71	51	30	56
28	45	70	52
54	57	54	49
49	42	60	43

Y Subgroups

63	58	56	56
55	69	69	64
67	60	55	67
56	65	65	63

The extra observation, i.e. 57, is discarded.

Ho: $\gamma = 1$
H1: $\gamma < 1$
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program MOS and subroutine QS and set SIZE = 039.
	XEQ MOS	DATA: R/S	Enter the X observations by subgroup.
58	R/S	1 58.0000	s X_s
52	R/S	2 52.0000	
24	R/S	3 24.0000	An incorrect value was inputted for X_3, so simply erase it by pressing "E".
	E	2 52.0000	s-1 X_{s-1}
42	R/S	3 42.0000	
39	R/S	4 39.0000	
	A	SS=232.75	Pressing "A" gives the sum of squares, SS, for the subgroup. (The sum of squares may or may not be of interest to the user.)
71	R/S	1 71.0000	Continue entering the remaining X subgroups, pressing "A" between subgroups.
.	.	.	
.	.	.	
.	.	.	
43	R/S	4 43.0000	
	A	SS=205.00	After all X subgroups have been entered and

SAMPLE PROBLEM No 1 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
			and sum of squares displayed, press "B" to enter the Y subgroups.
	B	DATA: R/S	Enter the Y observations by subgroup.
63	R/S	1 63.0000	t Y_t
58	R/S	2 58.0000	
.	.	.	Continue entering the Y observations by subgroups, pressing "A" between subgroups as was done for entering the X data.
.	.	.	
.	.	.	
63	R/S	4 63.0000	
	A	SS=54.750	
	C	SORT	Approximately 15 seconds, then...
		32.75 1.00	Displays the sums of squares and their average ranking.
		. .	
		. .	
		. .	
		906.75 9.00	
		W=13.00	The test statistic, W. (Run time approximately 45 seconds.)
			From Table D, $w(0.056,5,4) = 27$
			$n(m + n + 1) = 40$ So $40 - 27 = 13$
			Since W does not lie between 13 and 27, reject H_o.
	R/S	SORT	Approximately 45 seconds, then...
		GMA=0.19	GMA is the estimate of gamma, or γ. (Run time approximately 1 minute.)
			For a 95% confidence interval around γ:
			$\theta = n(2m + n + 1)/2 = 30$
			$\theta + 1 - \xi = w(\alpha/2,m,n)$
			$30 + 1 - \xi = w(0.032,5,4)$
			$31 - \xi = 28$
			$\xi = 3$
			and $mn + 1 - \xi = 20 + 1 - 3 = 17$
			(i.e., $\alpha = 2(0.032) = 0.064$ for an exact confidence interval of $100(1 - 0.064)$ or 93.6%.)
	R/S	V?	Determine the lower confidence interval limit.
3	R/S	0.06	
	R/S	V?	Determine the upper confidence interval limit.
17	R/S	0.99	

IV. Miller's Jackknife Test.

A. Use

Tests for the equality of dispersion parameters when the two population medians are unknown and unequal.

B. Data

Consist of two random samples, $X_1, X_2, \ldots, X_m$, and $Y_1, Y_2, \ldots, Y_n$, from separate populations, measured at least on an ordinal scale.

C. Assumption

1. The two samples are independent.
2. The population distributions are continuous.

D. Equations

1. Test statistic

Let $$S_0 = \ln\left[\frac{\sum_{i=1}^{m} X_i^2 - \left(\left(\sum_{i=1}^{m} X_i\right)^2/m\right)}{m-1}\right]$$

and $$S_i = \ln\left[\frac{\sum_{a=1}^{m-1} X_{ia}^2 - \left(\left(\sum_{a=1}^{m-1} X_{ia}\right)^2/(m-1)\right)}{m-2}\right]$$

where X_{ia}'s denote the $m-1$ observations obtained by deleting the ith of the X observations.

Set $$A_i = (m-1)S_0 - (m-2)S_i \qquad i=1,\ldots,m-1$$

and $$\bar{A}_x = \sum_{i=1}^{m-1} A_i/(m-1)$$

so that $$V_x = \left[\frac{\sum_{i=1}^{m-1} A_i^2 - \left(\left(\sum_{i=1}^{m} A_i\right)^2/(m-1)\right)}{(m-1)(m-2)}\right]$$

When analyzing the Y observations, substitute Y for X, n for m and j for i in the above equations.

Then ... $$Q = \frac{\bar{A}_Y - \bar{A}_X}{\sqrt{V_X + V_Y}}$$

2. Point estimate

$$\gamma = e^{|\bar{A}_y - \bar{A}_x|}$$

3. Confidence interval

Using Table A...

Both confidence interval boundaries may be found by...

$$\gamma_L = \text{antiln}\left\{(\bar{A}_Y - \bar{A}_X) - Z_{\alpha/2}\sqrt{V_X + V_Y}\right\} \quad \text{(lower boundary)}$$

$$\gamma_U = \text{antiln}\left\{(\bar{A}_Y - \bar{A}_X) + Z_{\alpha/2}\sqrt{V_X + V_Y}\right\} \quad \text{(upper boundary)}$$

E. Hypotheses

Select a level of significance, α, and compare the test statistic, Q, to Z_α from Table A.

H_0	H_1	Accept H_0 if...
$\gamma \leq 1$	$\gamma > 1$	$Q < Z_\alpha$
$\gamma \geq 1$	$\gamma < 1$	$-Z_\alpha < Q$
$\gamma = 1$	$\gamma \neq 1$	$-Z_{\alpha/2} < Q < Z_{\alpha/2}$

F. Program

MJK.

G. Size formula

SIZE = 9 + larger of $\{m,n\}$ (max. size = 274).

H. Limits and Warnings

The X and Y observations are not divided into subgroups, (though they may be). Besides being more efficient by making use of all the data, dispensing with subgroups avoids the ambiguity associated with Moses' Test.

When m and n are small and equal, the hypothesis test may be improved by replacing Z_α by $t_{\{[(m-1) + (n-1)] - 2, \alpha\}}$ from a t-distribution table.

I. Comparison with Moses' Dispersion Test

The following is a comparison of Moses' test with Miller's Jackknife Test. Neither, it should be pointed out, is highly desirable in very small samples.

1. Moses' Test is exactly distribution-free, while Miller's Test is asymptotically distribution-

free. Therefore, use the former when an exact Type-I error probability is desired.

2. Moses' Test is not very efficient, whereas Miller's Test is generally more efficient and more powerful, especially as sample size increases.

3. Different researchers may reach different conclusions on the same data with Moses' Test, depending upon how the observations are arranged into subgroups. By avoiding the partitioning of the data into subgroups in the Miller Jackknife Test, the results become unambiguous.

TEST: Miller Jackknife	PGM: MJK	MAX SIZE: 274

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Set calculator to USER mode.			
1	Set SIZE according to the size formula.		XEQ SIZE___	
2	Load program MJK.			
3	Begin program.		XEQ MJK	X: R/S
4	Enter X observations.	X_i	R/S	i X_i
	Repeat Step 4 until all X observations have been entered.			
5	If an error occurs, simply erase it by pressing "E" and enter the correct value of X according to Step 4.		E	i-1 X_{i-1}
6	After all X observations have been entered, begin calculations.		C	
	After a tone, calculator prompts for the Y observations.			Y: R/S
7	Enter Y observations.	Y_j	R/S	j Y_j
	Repeat Step 7 until all Y observations have been entered.			
8	If an error occurs, simply erase it by pressing "E" and enter the correct value of Y according to Step 7.		E	j-1 Y_{j-1}
9	After all Y observations have been entered, begin calculations.		C	
	After a tone, calculator displays the test statistic, Q.			Q=______
10	For an estimate of γ....		R/S	GMA=______
11	To establish a $100(1-\alpha)$ confidence interval around γ....		R/S	$Z\alpha/2$?
12	For the lower confidence interval limit.		R/S	LO=______
13	For the upper confidence interval limit.		R/S	UP=______
	Steps 12 and 13 may be repeated any number of times for different α.		R/S	$Z\alpha/2$?

SAMPLE PROBLEM No. 1 **TEST** MJK

An agronomist suspected that the second of two plots of equal area but dissimilar dimensions was more homogeneous in its soil properties. So he planted the same pure hybrid of corn on both plots and harvested it one row at a time (discarding end rows), recording the yield per row. Is Plot Y more homogeneous than Plot X?

Row	Plot X	Plot Y
1	54	60
2	42	58
3	58	63
4	57	64
5	70	55
6	56	67
7	52	56
8	71	56
9	43	65
10	42	57
11	49	69
12	52	63
13	28	56
14	51	69
15	60	67
16	39	65
17	49	55
18	54	
19	30	
20	45	

Values are yield of corn per row.

H_0: $\gamma = 1$
H_1: $\gamma < 1$
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program MJK and set SIZE = 029.
	XEQ MJK	X: R/S	Enter the X observations.
54	R/S	1 54.0000	i X_i
42	R/S	2 42.0000	
85	R/S	3 85.0000	An incorrect value was inputted for X_3, so simply erase it by pressing "E".
	E	2 42.0000	i-1 X_{i-1}
58	R/S	3 58.0000	
.	.	.	
.	.	.	
.	.	.	
45	R/S	20 45.0000	
	C	Y: R/S	Performs calculations on the X observations then prompts for the Y observations. (Run time approximately 70 seconds.)
60	R/S	1 60.0000	
.	.	.	
.	.	.	
.	.	.	

SAMPLE PROBLEM No 1 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
55	R/S	17 55.0000	
	C	Q=-3.962	Performs calculations on the Y observations then displays the test statistic, Q. (Run time approximately 60 seconds.)
			From Table A, $Z_{0.05} = 1.645$.
			Since Q is less than -1.645, reject H_o.
	R/S	GMA=0.203	GMA is the estimate of gamma, or γ.
			For, say, a 95% confidence interval around γ:
	R/S	Z α/2?	Enter $Z_{\alpha/2}$ from Table A.
			$Z_{(0.05/2)} = 1.96$
1.96	R/S	LO=0.093	The lower confidence interval limit.
	R/S	UP=0.447	The upper confidence interval limit.

E. Goodness of Fit.

The validity of parametric procedures depends on the form of the population distribution from which the sample had been drawn, i.e., that it is normal. The chi-square goodness of fit procedure allows us to test the assumption that a sample was drawn from a population with a given distribution. In asking how good a "fit" the sample is, we are asking how close the observed data approach the expected data for a given theoretical distribution. (Strictly speaking, the chi-square test is applicable for testing discrete distributions. Its use on continuous distributions is often justified by viewing the continuous population distribution as grouped into a finite number of nonoverlapping categories).

In order to provide an adequate approximation of a distribution, fairly large samples are required. For instance, use of the chi-square test to test for normality is not recommended for samples smaller than 50. A good approximation is achieved when at least 80% of the categories have an expected frequency equal to or greater than 5, and no category has an expected frequency of zero. Adjacent categories may be combined to increase the expected frequency if it falls short of 5 (or 1), provided there is some logical basis for interpretting the combination.

Use of the chi-square test requires only a nominal level of measurement. It is useful not only for situations where the data naturally form counts and categories, but also when the assumptions of higher powered tests are not met. For example, both the t-test and analysis of variance are parametric procedures which assume that the populations are normally distributed and have equal variances. If the assumption of normality is not met, appropriate nonparametric tests may be used. However, these also require the populations to have equal variances. If this assumption cannot also be met, the chi-square test may redeem the experiment. Instead of using the measurements to calculate location parameters, simply use them to categorize individual observations (as, say, "high", "median" or "low", or "above" and "below" the average).

I. Chi-Square Goodness of Fit Test. a.k.a. Chi-Square One-Sample Test, Chi-Square Test of a Distribution.

A. Use

Tests if sample data come from a population with a certain distribution.

B. Data

Consist of n observations, measured on a nominal scale, classified into R distinct mutually exclusive, collectively exhaustive categories.

C. Assumptions

1. The observations are randomly drawn.
2. The observations are independent.

D. Equations

1. Test statistic

Let $$CHI = \sum_{1}^{R} \frac{(O_i - E_i)^2}{E_i}$$

where O_i is the actual number of observations in category i.

E_i is the expected number of observations.

$df = R - 1$ i.e. the degrees of freedom.

E. Hypotheses

Select a level of significance, α, and compare the test statistic, CHI, to $\chi_{(d.f., \alpha)}$ from Table H, or if subroutine CP is used, compare α directly against the probability of obtaining the value of CHI at the given d.f.

H_0: The sample is drawn from a population with the suspected distribution.
H_1: The sample is drawn from a different population.

Accept H_0 if: $CHI < \chi^2_{(k-1, \alpha)}$

or $P(CHI)_{df} > \alpha$

F. Program/Subroutine

CGF and CP (Optional).

G. Size formula

SIZE = 002 (or 010 if CP is used).

H. Limits and Warnings

No more than 20% of the expected frequencies should be smaller than 5 and none should have an expected frequency less than 1. If adjacent categories are combined, be sure to adjust the degrees of freedom based on the revised number of categories.

The Kolmogorov-Smirnov One-Sample Test is a popular competitor of the Chi-Square Goodness of Fit Test. The rationale and calculations behind the Kolmogorov-Smirnov Test are even simpler than those of the Chi-Square Test. Determining a hypothetical frequency distribution for either test may be a considerable task, though.

I. Comparison with the Kolmogorov-Smirnov Test

Below is listed a comparison of the Chi-Square Test with the Kolmogorov-Smirnov Test to aid the investigator in deciding which test is more appropriate for the data. (See any standard textbook on nonparametric statistics for information about the Kolmogorov-Smirnov Tests--both one-sample and two-sample).

1. The Chi-Square Test is designed for discrete distributions while Kolmogorov's Test is designed for continuous distributions. However, the former test may be used with continuous data, while the latter test will yield conservative, if inexact, results with discrete data with some loss of power.

2. The Chi-Square Test uses nominal data, and the Kolmogorov Test requires at least ordinal data.

3. The Chi-Square Test usually requires large samples. Kolmogorov's Test may be used with samples of any size, but is not recommended if there are many tied observations.

4. The Chi-Square Test does not distinguish the direction of discrepancies between the observed and the expected frequencies, whereas Kolmogorov's Test does.

5. The Chi-Square Test allows for estimation of population parameters (like μ and σ) from the sample data. No such provision for estimating parameters is provided in the Kolmogorov-Smirnov Test. The investigator must completely specify the parameters.

6. The test statistic for the Chi-Square Test is only approximately distributed as chi-square for infinate sample size. Kolmogorov's test statistic is known exactly.

TEST: Chi-Square Goodness of Fit	PGM: CGF	MAX SIZE: NA

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Set calculator to USER mode.			
1	Set SIZE 002 or, if using subroutine CP (optional), SIZE 010.		XEQ SIZE___	
2	Load program CGF (and subroutine CP).			
3	Begin program.		XEQ CGF	OBS 1?
4	Enter actual observation.	OBSi	R/S	EXP 1?
5	Enter expected observation.	EXPi	R/S	OBS 2?
	Repeat Steps 4 and 5 until all observation pairs have been entered.			
	If an error occurs, erase the OBS and EXP values and their chi-square contribution by simply pressing "E". If an error upon entering OBS/EXP 1, return to Step 3 instead of pressing "E".		E	OBS n-1?
6	After all observation pairs have been entered, calculate chi-square.		C	CHI=_____
7	If subroutine CP was loaded, the probability of obtaining the chi-square value at the given degrees of freedom can be determined upon entering the degrees of freedom.		R/S	D.F?
8	Enter the degrees of freedom.	d.f.	R/S	P=_______
	Note: If subroutine CP was not loaded the display in Step 8 would read "NONEXISTENT".			
	The degrees of freedom are usually one less than the number of categories, but exceptions do exist.			

SAMPLE PROBLEM No. 1 **TEST CGF**

The following algorithm was used to generate uniformly distributed random numbers between 0 and 1:

$$X_{n+1} = FRC(147*X_n)$$

where FRC means "the fraction portion of".

Using $\sqrt{\pi}$ as seed, i.e., as X_1, 1000 observations were collected and placed into ten categories. Were the numbers, indeed, uniformly distributed?

Categories	Oi	Ei
$0.0 < X \le 0.1$	122	100
0.1 " X " 0.2	107	100
0.2 " X " 0.3	96	100
0.3 " X " 0.4	98	100
0.4 " X " 0.5	97	100
0.5 " X " 0.6	80	100
0.6 " X " 0.7	92	100
0.7 " X " 0.8	102	100
0.8 " X " 0.9	89	100
$0.9 < X \le 1.0$	117	100

where Oi are actual observations

Ei are expected observations

H_0: The observations are uniformly distributed between 0 and 1.
H_1: The observations are not uniformly distributed.
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program CGF and subroutine CP (optional) and set SIZE = 010.
	XEQ CGF	OBS 1?	Enter actual observation for category 1.
122	R/S	EXP 1?	Enter expected observation for category 1.
100	R/S	OBS 2?	
107	R/S	EXP 2?	
100	R/S	OBS 3?	
50	R/S	EXP 3?	An error was made inputting OBS 3, but no calculation has yet occurred. So just enter the correct OBS 3, press "ENTER", enter EXP 3 and press "R/S" as usual.
96	ENTER	96	
100	R/S	OBS 4?	
89	R/S	EXP 4?	
100	R/S	OBS 5?	An error was made inputting OBS 4, but EXP 4 was also entered and a chi-square value was calculated. Erase both OBS 4 and EXP 4 and their contribution to the chi-square summation by simply pressing "E".

SAMPLE PROBLEM No 1 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
	E	OBS 4?	Prompts for correct value for OBS 4. This procedure is also followed if an error occurs in entering an incorrect EXP i value and pressing R/S.
98	R/S	EXP 4?	
100	R/S	OBS 5?	
.	.	.	
.	.	.	
.	.	.	
117	R/S	EXP 10?	
100	R/S	OBS 11?	
	C	CHI=14.4	The cumulative chi-square statistic.
			From Table H, $\chi^2_{(9,0.05)} = 16.9190$.
			Since 14.4 is less than 16.919, accept H_o.
			Alternatively, if CP was loaded, the exact probability of obtaining the chi-square value of 14.4 at 9 degrees of freedom can be determined.
	R/S	D.F?	Enter the degrees of freedom. This is usually one less than the number of categories. However, if you are testing your data to determine whether they might fit a normal distribution, you may use your sample data to estimate the mean and the standard deviation. In that case, a degree of freedom must be deducted for each parameter estimated.
9	R/S	P=0.1088	Since 0.1088 is not less than 0.05, accept H_o.

F. Independence

A question that frequently arises is whether two variables are associated. If there is no association, the variables are said to be independent. Two variables are independent if the distribution of one has no effect on the distribution of the other.

The chi-square test for independence makes use of a contingency table. All the information is displayed on one table, with one variable represented by rows and the other by columns, making the contingency table an extremely versatile and valuable research instrument. (The chi-square one-sample test in the preceding section may be thought of as based on a 1 x R contingency table). The rows and columns represent different levels for their respective variates. The observed number of subjects characterized by one level of each criterion is placed in the cell formed by the intersection of a particular row and column.

Kendall's test for independence not only tests for association, but also provides an estimate for the degree of association --the coefficient of correlation. Kendall's test is one of the most important distribution-free tests owing to its great versitility and general utility.

I. Chi-Square Test for Independence. a.k.a. Chi-Square Test for Two Variables.

A. Use

Investigates the possibility of association between two variables in a single sample.

B. Data

Consist of a sample of size n whose observations, measured on a nominal scale, are classified according to two criteria. One criterion is associated with the rows (R) and the other criterion with the columns (C) of a R by C contingency table. The number of observations associated with a particular row and column is entered in the table.

C. Assumptions

1. Sampling is random.
2. Each observation belongs to only one level of the R different rows and only one level of the C different columns.

D. Equations

1. Test statistic

Let $$CHI = \sum_{i=1}^{R}\sum_{j=1}^{C}\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$$

where O_{ij} is the actual number of observations in row i, column j.

E_{ij} is the expected number of observations.

$$E_{ij} = n_{i.}n_{.j}/n$$

where $n_{i.}$ is the sum of all j observations in row i.
$n_{.j}$ is the sum of all i observations in column j.
n is the sum of all observations.

$$DF = (R-1)(C-1)$$

E. Hypotheses

Select a level of significance, α, and compare the test statistic, CHI, with $\chi_{(d.f., \alpha)}$ from Table H, or if subroutine CP is used, compare α directly against the probability of obtaining the value of CHI at the given d.f.

H_0: The two criterion are independent.

H_1: The two criterion are not independent.

Accept H_o if $CHI < \chi^2_{((R-1)(C-1), \alpha)}$

or $P(CHI)_{df} > \alpha$

F. Program/Subroutine

CHI and CP (optional).

G. Size formula

SIZE = (R x C) + (R + C) + 10   (max. size = 283).

H. Limits and Warnings

No more than 20% of the expected frequencies should be smaller than 5 and none should have an expected frequency less than 1.

TEST: Chi-Square Test for Independence	PGM: CHI	MAX SIZE: 283†

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Set calculator to USER mode.			
1	Set SIZE according to the size formula.		XEQ SIZE___	
2	Load program CHI (and subroutine CP).			
3	Begin program.		XEQ CHI	DATA: R/S
4	Enter observations by row.	$OBS_{i,j}$	R/S	$i,j=OBS_{i,j}$
	Repeat Step 4 until all observations in row i have been entered.			
5	Erase error in last observation by pressing "E" and entering the correct observation according to Step 4.		E	i,(j-1)?
6	After all observations in row i have been entered, press "A" to prompt for next row.		A	i+1,1?
7	Enter next row of observations according to Step 4. If there are no more observations to be entered, press "C" instead of returning to Step 4.		C	CHI=________
8	The chi-square test statistic, CHI, may be compared to the value for chi-square at the given degrees of freedom and level of significance from Table H, or if subroutine CP is loaded, the exact probability of obtaining the value of CHI at the given degrees of freedom can be obtained.		R/S	P=________
	Note: If subroutine CP was not loaded the display in Step 8 would read "NONEXISTENT".			

†Without subroutine CP. With CP, maximum SIZE = 266.

SAMPLE PROBLEM No. 1 **TEST** CHI

A curious Justice of the Peace wished to test her theory that couples with the same hair color tend to marry more often than couples with different hair color. Assuming her record, tabulated below, represents a random sample of all marrying couples, is her theory correct?

		Husbands' Hair Color			
		Black	Brown	Blonde	Red
Wives' Hair Color	Black	32	24	6	2
	Brown	26	26	16	8
	Blonde	12	14	22	18
	Red	5	10	19	15

Ho: Hair color plays no role in choosing a spouse.
H1: Hair color influences selection of a spouse.
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program CHI and subroutine CP and set SIZE = 034.
	XEQ CHI	DATA: R/S	Enter the data by rows.
32	R/S	1,1=32.000	$r,c=D_{r,c}$
24	R/S	1,2=24.000	
8	R/S	1,3=8.000	An error occurred inputting $D_{1,3}$. Simply erase it by pressing "E" and enter the correct value for $D_{1,3}$.
	E	1,3?	Prompts for correct value of $D_{1,3}$.
6	R/S	1,3=6.000	
2	R/S	1,4=2.000	
	A	2,1?	Prompts for first entry in next row.
26	R/S	2,1=26.000	Enter rows 2, 3 and 4 similarly, pressing "A" to begin a new row.
.	.	.	
.	.	.	
.	.	.	
	A	5,1?	
	C	CHI=54.8461	The cumulative chi-square statistic. (Run time approximately 30 seconds).

SAMPLE PROBLEM No. 1 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
			From Table H $\chi^2_{(9,0,05)}$ = 16.9190.
			Since 54.8461 is greater than 16.9190, reject H_o.
			Alternatively, if CP was loaded, the exact probability of obtaining the chi-square value of 54.8461 at 9 degrees of freedom can be determined.
	R/S	P=1.470E-8	(Run time approximately 30 seconds). Since the probability is less than 0.05, reject H_o.
			While we may conclude that hair color has an effect on selecting a spouse, a look at the original data does not necessarily confirm that couples prefer a spouse with the same hair color. For instance, redheaded women seem to prefer blonde men over redheaded men, while brownhaired wives are equally split in preferring husbands with either brown or black hair. Also, redheaded men prefer blondes.

II. Kendall's Independence Test. a.k.a. Kendall's Test for Correlation, Kendall's Tau Test.

A. Use

Investigates the possibility of association between two variables in a single sample.

B. Data

Consist of an X and a Y observation, measured at least on an ordinal scale, on each of n units.

C. Assumption

1. Sampling is random.

D. Equations

1. Test statistic

$$\text{Let } K = \sum_{a=1}^{n-1} \sum_{b=a+1}^{n} f(X_a, X_b, Y_a, Y_b)$$

$$\text{where} \quad f(X_a, X_b, Y_a, Y_b) = \begin{cases} 1 & \text{if } (X_a - X_b)(Y_a - Y_b) > 0 \\ 0 & \text{if } (X_a - X_b)(Y_a - Y_b) = 0 \\ -1 & \text{if } (X_a - X_b)(Y_a - Y_b) < 0 \end{cases}$$

2. Correlation coefficient

$$\bar{\tau} = 2K/n(n-1)$$

E. Hypotheses

Select a level of significance, α, and compare the test statistic, K, to $k(\alpha, n)$ from Table I.

H_0	H_1	Accept H_0 if...
$\tau \leq 0$	$\tau > 0$	$K < k(\alpha, n)$
$\tau \geq 0$	$\tau < 0$	$-k(\alpha, n) < K$
$\tau = 0$	$\tau \neq 0$	$-k(\alpha/2, n) < K < k(\alpha/2, n)$

or, compare $\bar{\tau}$ to τ^*:

H_1	Accept H_0 if...
$\tau > 0$	$\bar{\tau} < \tau^*(\alpha, n)$
$\tau < 0$	$-\tau^*(\alpha, n) < \bar{\tau}$
$\tau \neq 0$	$-\tau^*(\alpha/2, n) < \bar{\tau} < \tau^*(\alpha/2, n)$

F. Program
KIT.

G. Size formula

SIZE = 2n + 16 (max. size = 267).

H. Limits and Warnings

Table I does not extend to values of n > 4Ø. In such cases the large sample approximation, calculated by KIT, may be used.

$$K^* = \frac{K}{\sqrt{\frac{n[n-1][2n+5]}{18}}}$$

Using Table A for Z ...

H_1	Accept H_0 if...
$\tau > 0$	$K^* < Z_\alpha$
$\tau < 0$	$-Z_\alpha < K^*$
$\tau \neq 0$	$-Z_{\alpha/2} < K^* < Z_{\alpha/2}$

I. Comparison with Spearman's Rank Correlation Test

A popular nonparametric competitor of Kendall's Rank Correlation is Spearman's Rank Correlation. Though some investigators suggest there is little reason for preferring one method over the other, the following comparison is worth considering:

1. Both have an ARE of Ø.912.

2. Both use ordinal scale data.

3. Exact tables for Kendall's test statistic usually extend to a much higher n than is the case for Spearman's test statistic.

4. At a given sample size, n, the distribution of Kendall's tau is generally smoother and approaches the normal distribution more rapidly than does Spearman's correlation coefficient. So when the normal approximation must be used, Kendall's Test provides a more reliable test statistic.

5. Kendall's tau provides an unbiased estimator of a population parameter (i.e., the difference between the probabilities of concordance and discordance), making it more useful in data analysis.

TEST: Kendall's Independence Test	PGM: KIT	MAX SIZE: 267

STEP	INSTRUCTIONS	INPUT	FUNCTION	DISPLAY
	Set calculator to USER mode.			
1	Set SIZE according to the size formula.		XEQ SIZE ___	
2	Load program KIT.			
3	Begin program.		XEQ KIT	X/Y R/S
4	Enter the X,Y paired observations.	Xi Yi	ENTER R/S	Xi† i
	Repeat Step 4 until all X,Y pairs have been entered.			
5	If an error occurs after pressing "R/S" for an X,Y pair, simply erase the error by pressing "E". If an input error occured upon entering X/Y_1, return to Step 3 instead of pressing "E".		E	i-1
6	After all the X,Y pairs have been entered, calculate the test statistic, K.		C	K=_______
7	For an estimate of τ...		R/S	T=_______
8	For the large sample approximation test statistic, K*...		R/S	K*=_______

†Rounded to nearest whole number.

SAMPLE PROBLEM No. 1 **TEST** KIT

Students at two universities were asked to list activities they would prefer to be engaged in during a weekend evening. The activities, in order of preference, are listed below. Are the two groups of students in agreement?

Rank	University A	University B
1	sex	sex
2	tavern	sports
3	movies	tavern
4	hang around	TV
5	sports	movies
6	TV	hang around
7	job	sleep
8	sleep	job
9	study	study

By assigning the left column ranks to the activities selected by University A students, i.e., sex = 1, tavern = 2,..., study = 9, we can assign rank values to the activities selected by University B students for comparison.

University A: 1, 2, 3, 4, 5, 6, 7, 8, 9.

University B: 1, 5, 2, 6, 3, 4, 8, 7, 9.

H_0: $\tau = 0$
H_1: $\tau \neq 0$
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load KIT and set SIZE = 034.
	XEQ KIT	X/Y R/S	Enter the X,Y observation pairs.
1	ENTER	1	X_i
1	R/S	1	i
2	ENTER	2	
5	R/S	2	
3	ENTER	3	
6	R/S	3	An incorrect value was inputted for Y_3. Erase it by pressing "E" and enter the correct value.
	E	2	i-1
3	ENTER	3	
2	R/S	3	
.	.	.	
.	.	.	
.	.	.	
9	ENTER	9	
9	R/S	9	
	C	K=24	Test statistic, K. (Run time approximately 1½ minutes).

SAMPLE PROBLEM No 1 CONTINUED

INPUT	FUNCTION	DISPLAY	COMMENTS
			From Table I , k(0.025,9) = 20. i.e., α=0.05 or 2(0.025).
			Since K does not lie between $\pm$ 20, reject H_0.
	R/S	T=0.667	There seems to be a positive correlation between the students from the two universities as far as preference for weekend evening activities is concerned.

SAMPLE PROBLEM No. 2 **TEST** KIT

On vacation in the Caribbean with his wife and friends, Harry Angstrom shared a room with an unwelcomed intruder--a cricket--whose nightly serenades provided the data for an experiment Harry's son, Nelson, once suggested. Nelson told his father that the rate of a cricket's chirping is proportional to the temperature. Out of curiosity, Harry kept the following record during his weeklong stay. Was Nelson correct or merely probing his father's naiveté?

Day of Week	Ave. chirps/15 seconds	°F
Monday	50	84
Tuesday	53	86
Wednesday	40	79
Thursday	46	83
Friday	47	82
Saturday	55	83
Sunday	42	80

H_0: $\tau = 0$
H_1: $\tau > 0$
α : 0.05

INPUT	FUNCTION	DISPLAY	COMMENTS
			Load program KIT and set SIZE = 030.
	XEQ KIT	X↑Y R/S	Enter the X,Y observation pairs.
50	ENTER	50	Xi
84	R/S	1	i
53	ENTER	53	
86	R/S	2	
.	.	.	
.	.	.	
.	.	.	
42	ENTER	42	
80	R/S	7	
	C	K=14	Test statistic. (Approximate run time of 1 minute).
			From Table I, k(0.050,7) = 13. Since K is not less than 13, reject H_0 at the 0.050 level of significance.
	R/S	T=0.667	The estimator of tau, τ.

APPENDIX A

Program Listings

TEST FST

```
01♦LBL "FST"
02 SF 01
03 .
04 STO 00
05 STO 01
06 STO 05
07 10
08 STO 04
09 SF 21
10 FIX 0
11 "X↑Y R/S"
12♦LBL 00
13 RCL 03
14 AVIEW
15 X<>Y
16 FS?C 01
17 STO 03
18 -
19 X=0?
20 GTO 01
21 STO IND 04
22 SIGN
23 X>0?
24 ST+ 00
25 STO 02
26 ISG 01
27 0
28 ISG 04
29♦LBL 01
30 CLA
31 ARCL 01
32 GTO 00
33♦LBL E
34 RCL 02
35 X>0?
36 ST- 00
37 DSE 04
38 DSE 01
39 GTO 01
40♦LBL C
41 RCL 01
42 RCL 00
43 -
44 LASTX
45 X<=Y?
46 X<>Y
47 STO 00
48 RCL 01
49 RCL 00
50♦LBL 02
51 X<>Y
52 FACT
53 LASTX
54 RCL Z
55 -
56 FACT
57 /
58 X<>Y
59 FACT
60 /
61 ST+ 05
62 RCL 01
63 ISG 00
64 .
65 RCL 00
66 X<=Y?
67 GTO 02
68 .5
69 RCL 01
70 Y↑X
71 ST* 05
72 FIX 4
73 "a="
74 ARCL 05
75 AVIEW
76 RCL 01
77 STO 00
78 XEQ "QS"
79 RCL 01
80 2
81 /
82 FRC
83 X=0?
84 GTO 03
85 LASTX
86 10
87 +
88 RCL IND X
89 GTO 04
90♦LBL 03
91 LASTX
92 9
93 +
94 STO Y
95 1
96 +
97 RCL IND Y
98 RCL IND Y
99 +
100 2
101 /
102♦LBL 04
103 SF 21
104 "DELTA="
105 ARCL X
106 AVIEW
107♦LBL 05
108 "D?"
109 AVIEW
110 9
111 +
112 RCL IND X
113 RTN
114 GTO 05
115 .END.
```

TEST WSR

```
♦LBL "WSR"
SF 01
SF 02
CF 03
FIX 0
10
STO 00
.
STO 01
STO 09
"X↑Y R/S"
♦LBL 00
RCL 02
PROMPT
X<>Y
FS?C 02
STO 02
-
X=0?
GTO 01
STO IND 00
CLA
FIX 4
ARCL X
ISG 01
.
ISG 00
♦LBL 01
"├ "
FIX 0
ARCL 01
GTO 00
♦LBL E
DSE 00
DSE 01
CLA
FIX 4
GTO 01
♦LBL C
RCL 00
10
-
STO 00
STO 01
XEQ "QS"
RCL 01
ENTER↑
ENTER↑
1
+
*
2
/
STO 02
RCL 01
8
+
+
STO 03
RCL 01
9
+
.1
%
10
+
R↑
♦LBL 02
RCL IND Y
STO IND Y
RDN
1
+
ISG Y
GTO 02
RCL 01
9
+
9 E-3
+
STO 00
♦LBL 03
RCL IND 00
ABS
STO IND 00
DSE 00
GTO 03
RCL 01
STO 00
XEQ "QS"
9
STO 00
RCL 01
RCL 03
+
1
-
.1
%
RCL 03
+
STO 03
STO 06
RCL 01
11
+
STO 07
STO 08
XEQ 10
CF 01
♦LBL 12
10
STO 00
RCL 08
STO 07
♦LBL 13
RCL IND 06
X<=0?
GTO 15
RCL IND 00
X=Y?
GTO 14
ISG 00
.
ISG 07
.
GTO 13
♦LBL 14
RCL IND 07
ST+ 09
♦LBL 15
ISG 06
GTO 12
TONE 4
SF 21
"T="
ARCL 09
AVIEW
RCL 03
INT
STO 02
STO 04
STO 05
9
STO 00
.
♦LBL 05
CF 21
RCL IND 05
RCL IND 02
+
2
/
ISG 00
.
STO IND 00
ISG Y
.
"W "
ARCL Y
AVIEW
ISG 05
.
RCL 02
RCL 05
X>Y?
GTO 07
R↑
GTO 05
♦LBL 07
R↑
RCL 04
STO 05
ISG 02
.
RDN
ISG 03
GTO 05
X<> 01
ENTER↑
ENTER↑
1
+
*
2
/
STO 00
XEQ "QS"
SF 21
RCL 01
2
/
FRC
X=0?
GTO 06
LASTX
10
+
RCL IND X
GTO 08
♦LBL 06
LASTX
9
+
STO Y
1
+
RCL IND Y
RCL IND Y
+
2
/
♦LBL 08
FIX 4
"DELTA="
ARCL X
AVIEW
♦LBL 09
"W?"
AVIEW
9
+
RCL IND X
RTN
GTO 09
♦LBL 10
.
STO 02
STO 04
STO 05
10
STO 00
♦LBL 04
ISG 02
.
RCL 02
ST+ 04
ISG 05
.
RCL IND 00
ISG 00
.
RCL IND 00
X=Y?
GTO 04
CLA
FIX 4
ARCL Y
"├ "
FIX 2
```

TEST WSR

```
RCL 04
RCL 05
/
ARCL X
AVIEW
FS? 01
XEQ 11
.
STO 04
STO 05
RCL 02
RCL 01
X>Y?
GTO 04
RTN
♦LBL 11
STO IND 07
ISG 07
.
DSE 05
GTO 11
END
```

TEST GST

```
01♦LBL "GST"
02 .
03 STO 02
04 STO 03
05 STO 06
06 1
07 STO 00
08 10
09 STO 01
10 SF 21
11 "Za/2?"
12 PROMPT
13 STO 07
14 "DLT-L?"
15 PROMPT
16 STO 08
17 "DLT-U?"
18 PROMPT
19 STO 09
20 "DATA: R/S"
21 PROMPT
22♦LBL 01
23 STO IND 01
24 FIX 0
25 CLA
26 ARCL 00
27 "├ "
28 FIX 3
29 ARCL IND 01
30 AVIEW
31 ISG 00
32 .
33 ISG 01
34 .
35 GTO 01
36♦LBL E
37 1
38 ST- 00
39 ST- 01
40 RCL IND 01
41 GTO 01
42♦LBL C
43 RCL 00
44 STO 01
45 XEQ "QS"
46 10
47 STO 00
48 RCL 01
49 FIX 3
50 2
51 /
52 STO Y
53 FRC
54 X=0?
55 GTO 02
56 X<>Y
57 10
58 +
59 RCL IND X
60 2
61 *
62 STO 02
63 GTO 03
64♦LBL 02
65 X<>Y
66 9
67 +
68 STO Y
69 1
70 +
71 RCL IND Y
72 RCL IND Y
73 +
74 STO 02
75♦LBL 03
76 RCL 00
77 1
78 +
79 STO 05
80♦LBL 04
81 RCL 02
82 RCL IND 00
83 RCL IND 05
84 +
85 X>Y?
86 ISG 03
87 .
88 ISG 05
89 .
90 RCL 05
91 RCL 01
92 10
93 +
94 X>Y?
95 GTO 04
96 1
97 -
98 ISG 00
99 .
100 RCL 00
101 X<Y?
102 GTO 03
103 RCL 01
104 STO Y
105 2
106 -
107 *
108 8
109 /
110 RCL 03
111 -
112 -2
113 *
114 RCL 01
115 RCL 01
116 1
117 -
118 *
119 /
120 STO 04
121 2
122 ST/ 02
123 RCL 02
124 ST+ 08
125 ST+ 09
126 RCL 01
127 -.2
128 Y↑X
129 ST- 02
130 +
131 STO 03
132♦LBL 05
133 RCL 03
134 RCL IND 00
135 X>Y?
136 GTO 06
137 RCL 02
138 X<=Y?
139 ISG 06
140♦LBL 06
141 DSE 00
142 RCL 00
143 10
144 X<=Y?
145 GTO 05
146 RCL 06
147 X=0?
148 1
149 ENTER↑
150 2
151 /
152 RCL 01
153 .8
154 Y↑X
155 /
156 STO 05
157 RCL 09
158 RCL 08
159 -
160 RCL 01
161 3
162 *
163 SQRT
164 *
165 ST/ 07
166 RCL 07
167 2
168 *
169 RCL 05
170 /
171 1
172 -
173 X↑2
174 4
175 /
176 12
177 1/X
178 +
179 RCL 01
180 /
181 SQRT
182 1/X
183 RCL 04
184 4
185 1/X
186 -
187 *
188 TONE 9
189 "J="
190 ARCL X
191 AVIEW
192 END
```

TEST _MWW_

```
01♦LBL "MWW"
02 SF 01
03 SF 21
04 CLRG
05 10
06 STO 00
07 "X: R/S"
08 PROMPT
09♦LBL 01
10 STO IND 00
11 FS? 01
12 ISG 01
13 .
14 FC? 01
15 ISG 02
16 .
17♦LBL 00
18 FIX 0
19 "X"
20 FC? 01
21 "Y"
22 FS? 01
23 ARCL 01
24 FC? 01
25 ARCL 02
26 FIX 3
27 "⊢ "
28 ARCL IND 00
29 AVIEW
30 ISG 00
31 .
32 GTO 01
33♦LBL A
34 ISG 00
35 .
36 CF 01
37 "Y: R/S"
38 PROMPT
39 GTO 01
40♦LBL E
41 1
42 ST- 00
43 FS? 01
44 ST- 01
45 FC? 01
46 ST- 02
47 GTO 00
48♦LBL 05
49 RCL 05
50 STO 04
51 RDN
52 GTO 04
53♦LBL 07
54 RCL 02
55 STO Y
56 1
57 +
58 ∗
59 2
60 /
61 RTN
62♦LBL C
63 RCL 00
64 ENTER↑
65 ENTER↑
66 RCL 02
67 -
68 STO Z
69 1 E3
70 /
71 +
72 STO 00
73 RDN
74 ENTER↑
75 RCL 01
76 -
77 1 E3
78 /
79 +
80 STO 04
81 STO 05
82 RCL 01
83 RCL 02
84 ∗
85 RCL 01
86 10
87 +
88 +
89 STO 03
90 XEQ 07
91♦LBL 04
92 CF 21
93 1
94 ST- 03
95 RCL IND 00
96 RCL IND 04
97 -
98 STO IND 03
99 X>0?
100 ISG Z
101 STO X
102 X=0?
103 XEQ 11
104 X<> Z
105 FIX 1
106 "W "
107 ARCL X
108 AVIEW
109 DSE 04
110 GTO 04
111 DSE 00
112 GTO 05
113 TONE 9
114 SF 21
115 "W="
116 ARCL X
117 AVIEW
118 RCL 01
119 RCL 02
120 ∗
121 STO 07
122 1
123 -
124 RCL 03
125 +
126 .1
127 %
128 RCL 03
129 +
130 10
131♦LBL 06
132 RCL IND Y
133 STO IND Y
134 ISG Y
135 .
136 RDN
137 ISG Y
138 GTO 06
139 RCL 07
140 STO 00
141 XEQ "GS"
142 SF 21
143 RCL 07
144 2
145 /
146 FRC
147 X=0?
148 GTO 09
149 LASTX
150 10
151 +
152 RCL IND X
153 GTO 10
154♦LBL 09
155 LASTX
156 9
157 +
158 STO Y
159 1
160 +
161 RCL IND Y
162 RCL IND Y
163 +
164 2
165 /
166♦LBL 10
167 TONE 2
168 FIX 4
169 "DELTA="
170 ARCL X
171 AVIEW
172♦LBL 02
173 RCL 07
174 "U?"
175 AVIEW
176 X>Y?
177 GTO 02
178 9
179 +
180 RCL IND X
181 RTN
182 GTO 02
183♦LBL 11
184 .5
185 ST+ T
186 RDN
187 END
```

TEST KWT

```
01♦LBL "KWT"
02 CLRG
03 SF 01
04 CF 29
05 9
06 STO 01
07 6
08 STO 02
09♦LBL 00
10 ISG 00
11 .
12 ISG 01
13 .
14 "DATA: R/S"
15 PROMPT
16♦LBL 01
17 CLA
18 STO IND 01
19 FIX 3
20 ARCL IND 01
21 FIX 0
22 "├ "
23 ARCL 00
24 PROMPT
25 ISG 00
26 .
27 ISG 01
28 .
29 GTO 01
30♦LBL E
31 1
32 ST- 00
33 ST- 01
34 RCL 00
35 X=0?
36 GTO 00
37 RCL IND 01
38 GTO 01
39♦LBL A
40 SF 02
41 RCL IND 02
42 X=0?
43 CF 02
44 1
45 RCL 00
46 FS? 02
47 %
48 ST÷ IND 02
49 FS? 02
50 ISG 02
51 .
52 .
53 STO 00
54 GTO 00
55♦LBL C
56 DSE 01
57 RCL 01
58 9
59 -
60 STO 00
61 RCL 01
62 .1
63 %
64 10
65 +
66 RCL 01
67 RCL 00
68 LN
69 2
70 LN
71 /
72 ÷
73 4
74 +
75 INT
76 STO 05
77♦LBL 06
78 RCL IND Y
79 STO IND Y
80 RDN
81 1
82 +
83 ISG Y
84 GTO 06
85 STO 04
86 XEQ "QS"
87 RCL 04
88 1
89 -
90 .1
91 %
92 RCL 05
93 +
94 .
95 STO 02
96 STO 03
97 STO 04
98 STO 05
99 10
100 STO 00
101 1
102 -
103 ST- 01
104♦LBL 03
105 R↑
106 ISG 02
107 .
108 RCL 02
109 ST+ 04
110 ISG 03
111 .
112 RCL IND 00
113 ISG 00
114 .
115 RCL IND 00
116 X=Y?
117 GTO 03
118♦LBL 04
119 X<> T
120 CLA
121 FIX 4
122 ARCL Y
123 "├ "
124 FIX 2
125 RCL 04
126 RCL 03
127 /
128 X<>Y
129 ARCL Y
130 STO Y
131 AVIEW
132 X<> Z
133♦LBL 12
134 RCL IND Z
135 X=Y?
136 XEQ 15
137 RDN
138 ISG Z
139 GTO 12
140 RCL 04
141 RCL 03
142 /
143 FRC
144 X≠0?
145 XEQ 18
146 RDN
147 .
148 STO 03
149 STO 04
150 RDN
151 RCL 02
152 RCL 01
153 X>Y?
154 GTO 03
155 18.01
156 STO 01
157♦LBL 10
158 .
159 STO IND 01
160 DSE 01
161 GTO 10
162 6
163 STO 10
164 8
165 STO 04
166 11
167 STO 01
168 RCL 00
169 1
170 -
171 RCL 00
172 10
173 -
174 LN
175 2
176 LN
177 /
178 +
179 4
180 +
181 INT
182 STO 00
183 SF 02
184 RCL IND 10
185♦LBL 14
186 FC?C 02
187 XEQ 17
188 INT
189♦LBL 02
190 RCL IND 00
191 1 E50
192 /
193 ST+ IND 01
194 ISG 00
195 .
196 RDN
197 DSE X
198 GTO 02
199 ISG 01
200 .
201 DSE 04
202 RCL IND 10
203 X≠0?
204 GTO 14
205 6
206 STO 10
207 11
208 STO 00
209 SF 02
210♦LBL 07
211 RCL IND 00
212 X↑2
213 RCL IND 10
214 FC?C 02
215 XEQ 17
216 INT
217 X=0?
218 GTO 09
219 /
220 ST+ 03
221 1 E-3
222 ST+ 00
223 ISG 00
224 .
225 GTO 07
226♦LBL 09
227 RCL 03
228 12
229 *
230 RCL 02
231 1
232 +
233 STO 01
234 RCL 02
235 *
236 /
237 RCL 01
238 3
239 *
240 -
241 RCL 05
242 RCL 02
243 3
244 Y↑X
245 RCL 02
246 -
247 /
248 CHS
249 1
250 +
```

TEST KWT

```
251 /
252 SF 21
253 "H="
254 FIX 3
255 ARCL X
256 TONE 9
257 AVIEW
258 RCL 01
259 RCL 02
260 *
261 12
262 /
263 SQRT
264 X<> 00
265 FRC
266 1 E-3
267 -
268 STO 01
269 11
270 STO 02
271 CF 03
272 "Za/KK-K?"
273 PROMPT
274 ST* 00
275 6
276 STO 10
277 SF 02
278•LBL 16
279 RCL IND 02
280 RCL IND 10
281 FC?C 02
282 XEQ 17
283 INT
284 STO Z
285 /
286 RCL 02
287 10
288 -
289 FC? 03
290 STO 05
291 FS? 03
292 GTO 11
293 RDN
294 STO 04
295 X<>Y
296 1/X
297 STO 03
298 RCL 01
299 FRC
300 1 E-3
301 -
302 STO 01
303 X=0?
304 CF 01
305 ISG 02
306 .
307 SF 03
308 GTO 16
309•LBL 11
310 CLA
311 FIX 0
312 ARCL 05
313 "⊢ "
314 ARCL X
315 "⊢ "
316 RDN
317 FIX 2
318 RCL 04
319 -
320 ABS
321 ARCL X
322 "⊢ "
323 X<>Y
324 1/X
325 RCL 03
326 +
327 SQRT
328 RCL 00
329 *
330 ARCL X
331 X<=Y?
332 TONE 7
333 AVIEW
334 ISG 02
335 .
336 ISG 01
337 GTO 16
338 "SUM?"
339 FC? 01
340 GTO 08
341 SF 02
342 CF 03
343 RCL 05
344 11
345 +
346 STO 02
347 2
348 /
349 STO Y
350 INT
351 X<>Y
352 FRC
353 X≠0?
354 ISG Y
355 CF 02
356 X<>Y
357 STO 10
358 GTO 16
359•LBL 18
360 RDN
361 RCL 03
362 STO Y
363 3
364 Y↑X
365 -
366 CHS
367 ST+ 05
368 RTN
369•LBL 15
370 RDN
371 RDN
372 RCL 04
373 RCL 03
374 /
375 1 E50
376 *
377 X<> IND Z
378 R↑
379 RTN
380•LBL 17
381 FRC
382 1 E2
383 *
384 SF 02
385 ISG 10
386 .
387 RTN
388•LBL 08
389 SF 21
390 AVIEW
391 10
392 +
393 CLA
394 ARCL IND X
395 GTO 08
396 .END.
```

TEST FRS

```
01♦LBL "FRS"
02 CLRG
03 SF 01
04 SF 21
05 CF 29
06 10
07 STO 01
08 STO 07
09 1
10 STO 03
11 "DATA. R/S"
12♦LBL 01
13 AVIEW
14 CLA
15 FIX 0
16 ISG 04
17 .
18 STO IND 01
19 ARCL 03
20 "⊦,"
21 ARCL 04
22 FIX 3
23 "⊦="
24 ARCL X
25 ISG 01
26 .
27 GTO 01
28♦LBL A
29 FS?C 01
30 XEQ 00
31 RCL 08
32 RCL 04
33 X≠Y?
34 GTO 12
35 1
36 STO 04
37 ISG 03
38 .
39 ISG 06
40 .
41 RCL 08
42 ST+ 07
43 GTO 05
44♦LBL 00
45 RCL 04
46 STO 08
47 RTN
48♦LBL 12
49 CF 21
50 "TRTS UNEQUAL"
51 AVIEW
52 TONE 9
53 SF 21
54 RCL 07
55 STO 01
56 1
57 STO 04
58 GTO 05
59♦LBL E
60 DSE 01
61♦LBL 05
62 CLA
63 FIX 0
64 ARCL 03
65 "⊦,"
66 ARCL 04
67 "⊦?"
68 DSE 04
69 .
70 GTO 01
71♦LBL C
72 RCL 06
73 RCL 08
74 *
75 LASTX
76 ENTER↑
77 LN
78 2
79 LN
80 /
81 1
82 ST- 01
83 +
84 ÷
85 +
86 9
87 +
88 INT
89 STO 07
90 RCL 01
91 X<>Y
92♦LBL 02
93 RCL IND Y
94 STO IND Y
95 10
96 R↑
97 X=Y?
98 GTO 06
99 R↑
100 DSE Y
101 DSE Y
102 GTO 02
103♦LBL 00
104 RCL 06
105 STO 01
106♦LBL 09
107 RCL 08
108 STO 00
109 10
110 STO 03
111♦LBL 04
112 RCL IND 07
113 STO IND 03
114 ISG 03
115 .
116 DSE 07
117 DSE 00
118 GTO 04
119 RCL 08
120 STO 00
121 XEQ "QS"
122 10
123 STO 00
124 RCL 08
125 STO 03
126 .
127 STO 02
128 STO 04
129 STO 05
130 RCL 07
131 1
132 +
133 STO 09
134 ENTER↑
135 RDN
136 RDN
137♦LBL 06
138 R↑
139 R↑
140 ISG 02
141 .
142 RCL 02
143 ST+ 04
144 RDN
145 ISG 05
146 .
147 RCL IND 00
148 ISG 00
149 .
150 RCL IND 00
151 X=Y?
152 GTO 06
153 RDN
154 RCL 05
155 ST/ 04
156 RDN
157 1 E50
158 ST* 04
159 RDN
160♦LBL 07
161 RCL IND Z
162 X=Y?
163 XEQ 15
164 RDN
165 ISG Z
166 .
167 DSE 03
168 GTO 07
169 .
170 STO 04
171 STO 05
172 RCL 08
173 STO 03
174 RDN
175 RCL 09
176 RDN
177 RDN
178 RDN
179 ISG X
180 .
181 RCL 08
182 RCL 02
183 X≠Y?
184 GTO 06
185 DSE 01
186 GTO 08
187 RCL 09
188 RCL 06
189 RCL 08
190 STO 03
191 *
192 +
193 1
194 -
195 RCL 08
196 1 E2
197 /
198 +
199 1 E3
200 /
201 RCL 09
202 +
203 STO 04
204 STO 05
205 11
206 STO 10
207♦LBL 17
208 .
209 STO IND 10
210♦LBL 16
211 RCL IND 05
212 ST+ IND 10
213 ISG 05
214 GTO 16
215 1
216 ST+ 04
217 RCL 04
218 STO 05
219 1 E50
220 ST/ IND 10
221 FIX 0
222 RCL 10
223 10
224 -
225 CLA
226 "R"
227 ARCL X
228 "⊦="
229 FIX 3
230 ARCL IND 10
231 AVIEW
232 ISG 10
233 .
234 DSE 03
235 GTO 17
236 .
237 STO 04
238 STO 05
239 STO 10
240 RCL 06
241 STO 01
242♦LBL 14
243 RCL 09
244 RCL 04
245 +
246 STO Y
247 RCL 08
248 1
249 RCL 04
250 +
```

TEST FRS

```
251 -
252 +
253 1 E3
254 /
255 +
256 STO 03
257 1
258 STO 02
259 RCL IND 03
260 X=0?
261 GTO 13
262 STO 00
263♦LBL 10
264 ISG 03
265 FS? 55
266 GTO 00
267 RCL 00
268 RCL IND 03
269 X≠Y?
270 GTO 10
271 ISG 02
272 .
273 .
274 STO IND 03
275 GTO 10
276♦LBL 00
277 RCL 02
278 3
279 Y↑X
280 ST+ 05
281♦LBL 13
282 ISG 04
283 .
284 RCL 04
285 RCL 08
286 X≠Y?
287 GTO 14
288 RCL 05
289 RCL 08
290 -
291 ST+ 10
292 RCL 08
293 ST+ 09
294 .
295 STO 04
296 STO 05
297 DSE 01
298 GTO 14
299 1
300 RCL 08
301 -
302 ST/ 10
303 1
304 RCL 08
305 +
306 STO 00
307 LASTX
308 *
309 RCL 06
310 *
311 STO 05
312 ST+ 10
313 .
314 STO 03
315 RCL 00
316 2
317 /
318 RCL 06
319 *
320 STO 00
321 11
322 STO 01
323 RCL 08
324 STO 02
325♦LBL 09
326 RCL IND 01
327 RCL 00
328 -
329 X↑2
330 ST+ 03
331 ISG 01
332 .
333 DSE 02
334 GTO 09
335 RCL 03
336 12
337 *
338 RCL 10
339 /
340 FIX 3
341 SF 21
342 "S="
343 ARCL X
344 TONE 2
345 AVIEW
346 RCL 05
347 12
348 /
349 SQRT
350 STO 00
351 "Ga,K?"
352 PROMPT
353 ST* 00
354 11
355 STO 05
356 STO 10
357 RCL 08
358 STO 03
359♦LBL 03
360 DSE 03
361 RCL 03
362 STO 04
363♦LBL 11
364 CLA
365 ISG 10
366 .
367 FIX 0
368 RCL 05
369 10
370 -
371 RCL 10
372 10
373 -
374 ARCL Y
375 "⊦,"
376 ARCL X
377 RCL IND 05
378 RCL IND 10
379 -
380 ABS
381 FIX 2
382 "⊦ "
383 ARCL X
384 "⊦ "
385 RCL 00
386 ARCL X
387 X<=Y?
388 TONE 7
389 PROMPT
390 DSE 03
391 GTO 11
392 ISG 05
393 .
394 RCL 05
395 STO 10
396 RCL 04
397 STO 03
398 1
399 X≠Y?
400 GTO 03
401♦LBL 15
402 RDN
403 RCL 04
404 X<> IND T
405 .END.
```

TEST FAB

```
01♦LBL "FAB"
02 CF 00
03 CF 02
04 CF 03
05 SF 21
06 CLRG
07 10
08 STO 00
09 "X: R/S"
10 PROMPT
11♦LBL 01
12 STO IND 00
13 ISG 02
14 .
15♦LBL 00
16 CLA
17 FIX 0
18 ARCL 02
19 FIX 3
20 "⊢ "
21 ARCL IND 00
22 AVIEW
23 ISG 00
24 .
25 GTO 01
26♦LBL A
27 ISG 00
28 .
29 RCL 02
30 STO 03
31 STO 05
32 .
33 STO 02
34 "Y: R/S"
35 PROMPT
36 GTO 01
37♦LBL E
38 DSE 00
39 DSE 02
40 .
41 GTO 00
42♦LBL C
43 RCL 05
44 ST+ 02
45 1
46 %
47 RCL 02
48 STO 00
49 +
50 STO 09
51 RCL 02
52 ENTER↑
53 LN
54 2
55 LN
56 /
57 +
58 1
59 +
60 INT
61 10
62 +
63 LASTX
64 9
65 RCL 03
66 +
67 1 E3
68 /
69 +
70 X<>Y
71♦LBL 02
72 RCL IND Y
73 STO IND Y
74 RDN
75 1
76 +
77 ISG Y
78 GTO 02
79 STO 08
80 XEQ "QS"
81 RCL 08
82 STO 05
83 10
84 -
85 2
86 /
87 INT
88 STO 04
89 DSE 05
90 16
91 STO 00
92♦LBL 03
93 RCL IND 00
94 X<> IND 05
95 STO IND 00
96 1
97 ST+ 00
98 ST- 05
99 DSE 04
100 GTO 03
101 RCL 03
102 STO 00
103 XEQ "QS"
104 DSE 08
105 RCL 08
106 RCL 02
107 -
108 1 E-3
109 *
110 ST+ 08
111 LASTX
112 ST- 08
113 RCL 02
114 2
115 /
116 FRC
117 X≠0?
118 SF 04
119 LASTX
120 INT
121 STO 02
122 .
123 STO 01
124 STO 03
125 STO 04
126 10
127 STO 00
128 CLST
129♦LBL 04
130 RCL Z
131 1
132 FS? 02
133 CHS
134 ST+ 01
135 X<>Y
136 RCL 01
137 ST÷ Y
138 ISG 04
139 .
140 RCL IND 08
141 DSE 08
142 SF 01
143 RCL IND 08
144 FC?C 01
145 GTO 10
146 X=Y?
147 GTO 06
148 CLA
149 FIX 4
150 ARCL Y
151 "⊢ "
152 FIX 2
153 RCL T
154 RCL 04
155 /
156 ARCL X
157 AVIEW
158 STO 06
159 X↑2
160 X<>Y
161 RDN
162 RCL 04
163 *
164 ST+ 07
165 RDN
166 RCL Z
167 RCL IND 00
168 X=Y?
169 XEQ 05
170 .
171 STO 04
172 STO T
173 RDN
174♦LBL 06
175 RDN
176 FS? 03
177 GTO 04
178 RDN
179 RDN
180 RCL 02
181 RCL 01
182 X≠Y?
183 GTO 04
184 SF 02
185 1
186 ST+ 01
187 FS? 04
188 ST+ 01
189 SF 03
190 RDN
191 GTO 04
192♦LBL 05
193 RCL 06
194 ST+ 03
195 RCL IND 00
196 ISG 00
197 .
198 RCL IND 00
199 X≠Y?
200 RTN
201 GTO 05
202♦LBL 10
203 SF 21
204 TONE 7
205 "W="
206 ARCL 03
207 AVIEW
208 RCL 09
209 INT
210 2
211 /
212 FRC
213 X=0?
214 SF 00
215 RCL 09
216 FRC
217 1 E2
218 *
219 STO 02
220 RCL 09
221 INT
222 STO 09
223 -
224 CHS
225 STO 01
226 RCL 02
227 *
228 STO 00
229 16
230 ST* 07
231 RCL 09
232 FC? 00
233 X↑2
234 *
235 RCL 09
236 1
237 -
238 *
239 STO 04
240 RCL 09
241 FS? 00
242 2
243 FC? 00
244 1
245 +
246 FC? 00
247 X↑2
248 STO 05
249 RCL 02
250 *
```

TEST FAB

```
251 4
252 /
253 FC? 06
254 RCL 09
255 FC? 00
256 /
257 ST- 03
258 RCL 05
259 ST* 05
260 RCL 07
261 RCL 09
262 FS? 00
263 RCL 05
264 *
265 FC? 00
266 RCL 05
267 -
268 RCL 00
269 *
270 RCL 04
271 /
272 SQRT
273 ST/ 03
274 FIX 4
275 "W*="
276 ARCL 03
277 AVIEW
278 .END.
```

TEST MOD

```
01♦LBL "MOD"
02 SF 00
03 CF 01
04 SF 21
05 CLRG
06 10
07 STO 01
08 "X: R/S"
09 PROMPT
10♦LBL 00
11 1 E10
12 *
13 STO IND 01
14 ISG 02
15 .
16♦LBL 01
17 FIX 0
18 CLA
19 ARCL 02
20 "⊢ "
21 FIX 3
22 RCL IND 01
23 1 E10
24 /
25 ARCL X
26 AVIEW
27 ISG 01
28 .
29 GTO 00
30♦LBL A
31 ISG 01
32 .
33 RCL 02
34 STO 00
35 STO 03
36 ST- 02
37 "Y: R/S"
38 PROMPT
39 GTO 00
40♦LBL E
41 DSE 01
42 DSE 02
43 GTO 01
44♦LBL C
45 RCL 01
46 RCL 02
47 ST+ 00
48 -
49 1 E3
50 /
51 10
52 +
53 RCL 00
54 ENTER↑
55 LN
56 2
57 LN
58 /
59 +
60 11
61 +
62 INT
63 STO 04
64♦LBL 02
65 RCL IND Y
66 STO IND Y
67 RDN
68 1
69 +
70 ISG Y
71 GTO 02
72 XEQ "QS"
73 RCL 03
74 RCL 04
75 1
76 STO 05
77 -
78 +
79 1 E3
80 /
81 ST+ 04
82 .
83 STO 08
84 10
85♦LBL 03
86 RCL IND X
87 ISG Y
88 .
89 ISG 08
90 .
91 RCL IND Y
92 X=Y?
93 GTO 04
94 X<> Z
95 RCL 08
96 FS? 01
97 ST+ 00
98 FS? 01
99 RDN
100 FS?C 01
101 RCL 00
102 RCL 05
103 /
104 X<> Z
105 RCL 04
106 FC? 00
107 STO 10
108 FS? 00
109 STO 09
110♦LBL 05
111 RDN
112 RCL IND 04
113 X=Y?
114 XEQ 06
115 ISG 04
116 GTO 05
117 FC? 00
118 RCL 10
119 FS? 00
120 RCL 09
121 FS?C 00
122 ST- 09
123 STO 04
124 R↑
125 RCL 01
126 X<>Y
127 X>Y?
128 GTO 07
129 RCL 05
130 ST+ 07
131 RCL 07
132 RCL 06
133 +
134 RCL 02
135 -
136 RCL 03
137 -
138 -15
139 *
140 4
141 -
142 RCL 05
143 X↑2
144 +
145 *
146 RCL 05
147 X↑2
148 1
149 -
150 *
151 ST+ 09
152 RCL 07
153 STO 06
154 1
155 STO 05
156 R↑
157 .
158 STO 08
159 RDN
160 GTO 03
161♦LBL 04
162 SF 01
163 RCL 08
164 ST+ 00
165 ISG 05
166 .
167 R↑
168 GTO 03
169♦LBL 06
170 R↑
171 STO IND 04
172 RDN
173 RTN
174♦LBL 07
175 .
176 STO 06
177 RCL 02
178 RCL 03
179 +
180 STO 07
181 1
182 +
183 2
184 /
185 STO 01
186♦LBL 08
187 RCL IND 04
188 RCL 01
189 -
190 X↑2
191 ST+ 06
192 ISG 04
193 GTO 08
194 SF 21
195 TONE 6
196 "M="
197 ARCL 06
198 AVIEW
199 RCL 09
200 RCL 07
201 1
202 -
203 RCL 07
204 *
205 /
206 CHS
207 1
208 RCL 07
209 +
210 LASTX
211 X↑2
212 4
213 -
214 *
215 +
216 RCL 02
217 RCL 03
218 *
219 *
220 180
221 /
222 SQRT
223 STO 05
224 RCL 07
225 X↑2
226 1
227 -
228 RCL 03
229 *
230 12
231 /
232 CHS
233 RCL 06
234 +
235 RCL 05
236 /
237 LASTX
238 2
239 *
240 1/X
241 +
242 "M*="
243 ARCL X
244 AVIEW
245 .END.
```

TEST MOS

```
01♦LBL "MOS"
02 CF 00
03 SF 21
04 CLRG
05 10
06 STO 05
07♦LBL 03
08 "DATA: R/S"
09 PROMPT
10♦LBL 00
11 ISG 00
12 .
13 STO Y
14 ST+ 03
15 X↑2
16 ST+ 04
17♦LBL 01
18 FIX 0
19 CLA
20 ARCL 00
21 "├ "
22 FIX 4
23 ARCL Y
24 AVIEW
25 GTO 00
26♦LBL E
27 X<>Y
28 ST- 03
29 X↑2
30 ST- 04
31 RDN
32 DSE 00
33 GTO 01
34 GTO 03
35♦LBL B
36 RCL 07
37 STO 02
38 .
39 STO 07
40 STO 00
41 STO 03
42 STO 04
43 ISG 05
44 .
45 GTO 03
46♦LBL 10
47 X<>Y
48 RCL IND 06
49 X=Y?
50 XEQ 11
51 RDN
52 X<>Y
53 DSE 06
54 GTO 10
55 RCL 08
56 STO 06
57 RTN
58♦LBL 11
59 RCL Z
60 ST+ 09
61 RDN
62 RTN
63♦LBL A
64 ISG 07
65 .
66 RCL 04
67 RCL 03
68 X↑2
69 RCL 00
70 /
71 -
72 STO IND 05
73 "SS="
74 ARCL X
75 AVIEW
76 ISG 05
77 .
78 .
79 STO 00
80 STO 03
81 STO 04
82 RDN
83 GTO 00
84♦LBL C
85 RCL 07
86 ST+ 02
87 STO 01
88 10
89 ENTER↑
90 9
91 RCL 02
92 +
93 1 E3
94 /
95 +
96 RCL 02
97 RCL 01
98 -
99 RCL 01
100 *
101 10
102 +
103♦LBL 05
104 RCL IND Y
105 STO IND Y
106 RDN
107 1
108 +
109 ISG Y
110 GTO 05
111 DSE X
112 STO Y
113 RCL 07
114 -
115 1 E3
116 /
117 +
118 STO 06
119 STO 08
120 RCL 02
121 STO 00
122 XEQ "QS"
123 XEQ 06
124 .
125 STO 05
126 RCL 02
127 RCL 01
128 -
129 STO 02
130 RCL 01
131 *
132 10
133 STO 00
134 +
135 STO 03
136 RCL 02
137 +
138 STO 04
139♦LBL 07
140 RCL IND 04
141 RCL IND 03
142 /
143 STO IND 00
144 ISG 00
145 .
146 ISG 04
147 .
148 ISG 05
149 .
150 RCL 05
151 RCL 01
152 X>Y?
153 GTO 07
154 RCL 05
155 ST- 04
156 .
157 STO 05
158 ISG 03
159 .
160 DSE 02
161 GTO 07
162 RCL 00
163 10
164 -
165 STO 00
166 STO 02
167 XEQ "QS"
168 SF 00
169 SF 21
170 RCL 02
171 2
172 /
173 FRC
174 X=0?
175 GTO 12
176 LASTX
177 10
178 +
179 RCL IND X
180 GTO 13
181♦LBL 12
182 LASTX
183 9
184 +
185 STO Y
186 1
187 +
188 RCL IND Y
189 RCL IND Y
190 *
191 SQRT
192♦LBL 13
193 "GSG="
194 ARCL X
195 AVIEW
196♦LBL 02
197 "V?"
198 AVIEW
199 9
200 +
201 CLA
202 ARCL IND X
203 AVIEW
204 GTO 02
205♦LBL 06
206 .
207 STO 03
208 STO 04
209 STO 05
210 10
211 STO 00
212♦LBL 08
213 ISG 03
214 .
215 RCL 03
216 ST+ 04
217 ISG 05
218 .
219 RCL IND 00
220 ISG 00
221 .
222 RCL IND 00
223 FS? 00
224 GTO 09
225 X=Y?
226 GTO 08
227♦LBL 09
228 CLA
229 FIX 2
230 X<>Y
231 ARCL X
232 "├ "
233 RCL 04
234 RCL 05
235 /
236 ARCL X
237 AVIEW
238 FC? 00
239 XEQ 10
240 .
241 STO 04
242 STO 05
243 RCL 03
244 RCL 02
245 X>Y?
246 GTO 08
247 SF 21
248 "W="
249 ARCL 09
250 AVIEW
251 END
```

TEST MJK

```
01♦LBL "MJK"
02 SF 00
03 .
04 STO 05
05♦LBL 00
06 SF 01
07 SF 02
08 CF 21
09 .
10 STO 01
11 STO 02
12 STO 04
13 1
14 STO 00
15 9
16 STO 03
17 "Y"
18 FS? 00
19 "X"
20 "⊢: R/S"
21 PROMPT
22♦LBL 01
23 STO IND 03
24 FIX 0
25 CLA
26 ARCL 00
27 "⊢ "
28 FIX 3
29 ARCL IND 03
30 ISG 00
31 .
32 ISG 03
33 .
34 ST+ 01
35 X↑2
36 ST+ 02
37 FS? 02
38 PROMPT
39 FS? 02
40 GTO 01
41 RTN
42♦LBL E
43 1
44 ST- 00
45 ST- 03
46 RCL IND 03
47 ST- 01
48 X↑2
49 ST- 02
50 FS? 02
51 RTN
52 DSE 03
53 RCL IND 03
54 ST- 01
55 X↑2
56 ST- 02
57 SQRT
58 DSE 00
59 STO X
60 GTO 01
61♦LBL C
62 DSE 00
63 CF 02
64 10
65 STO 03
66♦LBL 02
67 RCL 02
68 RCL 01
69 X↑2
70 RCL 00
71 /
72 -
73 RCL 00
74 1
75 -
76 /
77 LN
78 "S"
79 XEQ 08
80 ISG 04
81 .
82 RCL 04
83 6
84 +
85 X<>Y
86 STO IND Y
87 RCL 00
88 1
89 +
90 RCL 04
91 X>Y?
92 GTO 04
93 FC?C 01
94 XEQ 03
95 XEQ E
96 GTO 02
97♦LBL 08
98 FIX 0
99 ARCL 04
100 "⊢="
101 FIX 3
102 ARCL X
103 AVIEW
104 RTN
105♦LBL 03
106 RCL IND 03
107 XEQ 01
108 ISG 03
109 .
110 RTN
111♦LBL 04
112 RCL 07
113 RCL 00
114 1
115 STO 04
116 +
117 *
118 STO 03
119 8
120 STO 07
121♦LBL 05
122 RCL 03
123 RCL 00
124 RCL IND 07
125 *
126 -
127 "A"
128 XEQ 08
129 STO IND 07
130 ISG 04
131 .
132 ISG 07
133 .
134 RCL 00
135 1
136 +
137 RCL 04
138 X<=Y?
139 GTO 05
140 .
141 STO 01
142 STO 02
143♦LBL 07
144 DSE 07
145 RCL IND 07
146 ST+ 01
147 X↑2
148 ST+ 02
149 8
150 RCL 07
151 X>Y?
152 GTO 07
153 RCL 02
154 RCL 01
155 FS? 00
156 STO 06
157 STO 03
158 X↑2
159 RCL 00
160 1
161 +
162 /
163 -
164 RCL 00
165 RCL 00
166 1
167 +
168 *
169 /
170 SF 21
171 "VY="
172 FS? 00
173 "VX="
174 ARCL X
175 AVIEW
176 ST+ 05
177 RCL 00
178 1
179 +
180 FS? 00
181 ST/ 06
182 ST/ 03
183 FS?C 00
184 GTO 00
185 "ABAR="
186 ARCL 06
187 AVIEW
188 "BBAR="
189 ARCL 03
190 AVIEW
191 RCL 03
192 RCL 06
193 -
194 RCL 05
195 SQRT
196 /
197 SF 21
198 "Q="
199 ARCL X
200 AVIEW
201 RCL 06
202 ST- 03
203 RCL 03
204 E↑X
205 "GSQ="
206 ARCL X
207 AVIEW
208 .END.
```

TEST CGF

```
01♦LBL "CGF"
02 .
03 STO 00
04 1
05 STO 01
06 FIX 0
07♦LBL 00
08 "OBS "
09 ARCL 01
10 "⊢?"
11 PROMPT
12 "EXP "
13 ARCL 01
14 "⊢?"
15 PROMPT
16 STO Z
17 -
18 X↑2
19 X<>Y
20 X≠0?
21 /
22 ST+ 00
23 ISG 01
24 .
25 GTO 00
26♦LBL E
27 ST- 00
28 DSE 01
29 GTO 00
30♦LBL C
31 SF 21
32 FIX 4
33 "CHI="
34 ARCL 00
35 AVIEW
36 "D.F.?"
37 PROMPT
38 STO 01
39 GTO "CP"
40 END
```

TEST CHI

```
01♦LBL "CHI"
02 CLRG
03 CF 01
04 SF 21
05 CF 29
06 10
07 STO 00
08 STO 05
09 1
10 STO 01
11 .
12 STO 03
13 STO 04
14 "DATA: R/S"
15♦LBL 01
16 AVIEW
17 FIX 0
18 ISG 03
19 .
20 STO IND 00
21 ST+ 04
22 CLA
23 ARCL 01
24 "├,"
25 ARCL 03
26 "├="
27 FIX 3
28 ARCL X
29 ISG 00
30 .
31 GTO 01
32♦LBL A
33 SF 01
34 RCL 03
35 STO 02
36 1
37 STO 03
38 ISG 01
39 .
40♦LBL E
41 FC? 01
42 DSE 00
43 FIX 0
44 CLA
45 ARCL 01
46 "├,"
47 ARCL 03
48 "├?"
49 FC?C 01
50 ST- 04
51 DSE 03
52 .
53 GTO 01
54♦LBL C
55 RCL 00
56 STO 03
57 DSE 01
58♦LBL 03
59 RCL 01
60♦LBL 00
61 RCL 02
62♦LBL 02
63 RCL IND 05
64 ST+ IND 00
65 ISG 05
66 .
67 RDN
68 DSE X
69 GTO 02
70 ISG 00
71 .
72 RDN
73 FS? 01
74 XEQ 04
75 DSE X
76 GTO 00
77 FS?C 01
78 GTO 05
79 RCL 01
80 X<> 02
81 X<> 01
82 RCL 00
83 STO 06
84 STO 07
85 10
86 STO Y
87 RCL 01
88 1 E5
89 /
90 +
91 STO 05
92 SF 01
93 RDN
94 GTO 03
95♦LBL 04
96 ISG Y
97 .
98 RCL 05
99 FRC
100 RCL Z
101 ÷
102 STO 05
103 RDN
104 RTN
105♦LBL 05
106 10
107 STO 05
108 .
109 STO 08
110♦LBL 06
111 RCL IND 06
112 RCL IND 03
113 RCL 04
114 /
115 *
116 1/X
117 LASTX
118 RCL IND 05
119 -
120 X↑2
121 *
122 ST+ 08
123 1
124 ST+ 05
125 ST+ 06
126 ST+ 08
127 RCL 08
128 RCL 01
129 X>Y?
130 GTO 06
131 .
132 STO 08
133 RCL 07
134 STO 06
135 1
136 ST+ 03
137 ST+ 09
138 RCL 09
139 RCL 02
140 X>Y?
141 GTO 06
142 FIX 4
143 "CHI="
144 ARCL 08
145 AVIEW
146 DSE 01
147 DSE 02
148 RCL 02
149 ST* 01
150 GTO "CP"
151 END
```

TEST KIT

```
01♦LBL "KIT"
02 SF 04
03 SF 21
04 CLRG
05 14
06 STO 00
07 2 E-5
08 +
09 STO 02
10 8.00002
11 STO 07
12 FIX 0
13 "X↑Y R/S"
14 GTO 01
15♦LBL E
16 2
17 ST- 00
18 DSE 01
19 CLA
20 ARCL 01
21♦LBL 01
22 PROMPT
23 X<>Y
24 STO IND 00
25 ISG 00
26 .
27 X<>Y
28 STO IND 00
29 ISG 00
30 .
31 ISG 01
32 .
33 CLA
34 ARCL 01
35 GTO 01
36♦LBL C
37 DSE 00
38♦LBL 05
39 RCL 02
40 1
41 +
42 STO 03
43 1
44 +
45 STO 04
46 1
47 +
48 STO 05
49♦LBL 02
50 RCL IND 02
51 RCL IND 04
52 -
53 RCL IND 03
54 RCL IND 05
55 -
56 *
57 X≠0?
58 SIGN
59 ST+ 06
60 ISG 04
61 .
62 ISG 05
63 .
64 RCL 00
65 RCL 05
66 INT
67 X<=Y?
68 GTO 02
69 ISG 02
70 .
71 RCL 02
72 RCL 00
73 1
74 -
75 X>Y?
76 GTO 05
77 1 E99
78 RCL 00
79 ISG X
80 .
81 X<>Y
82 STO IND Y
83 ISG Y
84 .
85 STO IND Y
86 14
87 RCL 00
88 .02
89 +
90 1 E-3
91 *
92 +
93 STO 02
94 1
95 STO 00
96 +
97 STO 05
98 RCL 07
99 STO 04
100♦LBL 08
101 RCL 02
102 2
103 +
104 STO 03
105♦LBL 06
106 1 E99
107 RCL IND 02
108 X=Y?
109 GTO 07
110 RCL IND 03
111 X=Y?
112 XEQ 03
113 ISG 03
114 GTO 06
115♦LBL 07
116 -1
117 RCL 00
118 +
119 LASTX
120 *
121 STO Y
122 ST+ IND 04
123 ISG 04
124 ENTER↑
125 -2
126 LASTX
127 +
128 LASTX
129 RDN
130 *
131 ST+ IND 04
132 ISG 04
133 .
134 X<>Y
135 R↑
136 2
137 *
138 5
139 +
140 *
141 ST+ IND 04
142 RCL 07
143 STO 04
144 1
145 STO 00
146 ISG 02
147 GTO 08
148 FS? 04
149 RCL 05
150 STO 02
151 1
152 ST+ 04
153 ST+ 07
154 FS?C 04
155 GTO 08
156 "K="
157 ARCL 06
158 TONE 3
159 AVIEW
160 2
161 RCL 06
162 *
163 -1
164 RCL 01
165 +
166 STO 02
167 LASTX
168 *
169 /
170 FIX 3
171 "T="
172 ARCL X
173 AVIEW
174 13
175 STO 04
176 RCL 02
177 RCL 01
178 *
179 LASTX
180 2
181 *
182 5
183 +
184 *
185 RCL IND 04
186 -
187 DSE 04
188 RCL IND 04
189 -
190 18
191 /
192 STO 07
193 DSE 04
194 RCL IND 04
195 DSE 04
196 RCL IND 04
197 *
198 RCL 01
199 RCL 01
200 1
201 -
202 STO 00
203 *
204 RCL 01
205 2
206 -
207 *
208 9
209 *
210 /
211 ST+ 07
212 DSE 04
213 RCL IND 04
214 DSE 04
215 RCL IND 04
216 *
217 RCL 01
218 2
219 *
220 RCL 00
221 *
222 /
223 RCL 07
224 +
225 SQRT
226 1/X
227 RCL 06
228 *
229 "K*="
230 ARCL X
231 AVIEW
232♦LBL 03
233 ISG 00
234 .
235 1 E99
236 STO IND 03
237 .END.
```

TEST QS

```
♦LBL "QS"
CF 21
"SORT"
AVIEW
CF 04
RCL 00
9
+
1 E3
/
10
+
ST+ 00
STO IND 00
♦LBL 15
RCL IND 00
INT
LASTX
FRC
1 E3
*
STO Z
X<>Y
-
15
X>Y?
XEQ 28
FS?C 04
GTO 24
RCL IND Z
R↑
RCL L
X<>Y
♦LBL 11
R↑
CLX
RCL IND Z
R↑
X<=Y?
GTO 12
♦LBL 13
ISG T
STOP
RDN
RDN
X<=Y?
GTO 14
GTO 11
♦LBL 12
STO IND T
X<>Y
STO IND Z
GTO 20
♦LBL 19
R↑
RCL IND Y
X<=Y?
GTO 21
♦LBL 20
DSE Z
CLA
RDN
RDN
X<=Y?
GTO 14
GTO 19
♦LBL 21
STO IND T
X<>Y
STO IND Z
GTO 13
♦LBL 22
1 E3
/
+
X<> IND 00
ISG 00
STOP
STO IND 00
ENTER↑
RTN
♦LBL 14
1
-
RCL IND 00
INT
X<>Y
X>Y?
XEQ 22
RDN
RDN
1
+
RCL IND 00
FRC
1 E3
*
X>Y?
XEQ 22
♦LBL 24
DSE 00
GTO 15
CLD
RTN
♦LBL 28
SF 04
ISG IND 00
♦LBL 25
RCL IND 00
RCL IND X
X<>Y
1
-
INT
R↑
♦LBL 26
RDN
X<>Y
RCL IND Y
X<=Y?
GTO 27
ISG Z
STO X
STO IND Z
RDN
X<>Y
2
-
9
X≠Y?
GTO 26
RDN
STO Z
♦LBL 27
CLX
1
ST+ Z
RDN
STO IND Y
ISG IND 00
GTO 25
END
```

TEST CP

```
01♦LBL "CP"
02 .
03 STO 09
04 RCL 01
05 2
06 /
07 STO 02
08 STO 05
09 .5
10 +
11 STO 06
12 FRC
13 X=0?
14 GTO 01
15 PI
16 SQRT
17 2
18 /
19 STO 03
20 1
21 STO 04
22 GTO 02
23♦LBL 01
24 PI
25 SQRT
26 STO 04
27 1
28 STO 03
29♦LBL 02
30 RCL 05
31 X<=Y?
32 GTO 03
33 1
34 ST- 05
35 ST- 06
36 RCL 05
37 ST* 04
38 RCL 06
39 ST* 03
40 1
41 GTO 02
42♦LBL 03
43 RCL 04
44 ST* 02
45 RCL 00
46 2
47 /
48 RCL 01
49 Y↑X
50 RCL 00
51 E↑X
52 /
53 SQRT
54 RCL 00
55 /
56 RCL 04
57 /
58 2
59 *
60 RCL 01
61 STO 05
62 /
63 RCL 00
64 *
65 STO 08
66 1
67 STO 06
68 STO 07
69♦LBL 04
70 X<> 09
71 2
72 ST+ 05
73 RCL 00
74 RCL 05
75 /
76 ST* 06
77 RCL 06
78 ST+ 07
79 RCL 09
80 RCL 07
81 X≠Y?
82 GTO 04
83 RCL 08
84 *
85 CHS
86 1
87 +
88 "P="
89 ARCL X
90 AVIEW
91 .END.
```

APPENDIX B

Tables

Table A: Upper Tail Probabilities for the Standard Normal Distribution.

Form: $Z_{(\alpha)} = x$

Example: $Z_{(0.0039)} = 2.66$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002

Source: Table A.1 of Nonparametric Statistical Methods, M. Hollander and D. A. Wolfe, John Wiley & Sons, NY, 1973.

Table B: Upper Tail Probabilities for the Binomial Distribution for $p=\frac{1}{2}$.

Form: $b(\alpha,n,\frac{1}{2}) = x$

Example: $b(0.0037,15,\frac{1}{2}) = 13$

	n							
x	2	3	4	5	6	7	8	9
1	.7500							
2	.2500	.5000	.6875					
3		.1250	.3125	.5000	.6563			
4			.0625	.1875	.3438	.5000	.6367	
5				.0313	.1094	.2266	.3633	.5000
6					.0156	.0625	.1445	.2539
7						.0078	.0352	.0898
8							.0039	.0195
9								.0020

	n							
x	10	11	12	13	14	15	16	17
5	.6230							
6	.3770	.5000	.6128					
7	.1719	.2744	.3872	.5000	.6047			
8	.0547	.1133	.1938	.2905	.3953	.5000	.5982	
9	.0107	.0327	.0730	.1334	.2120	.3036	.4018	.5000
10	.0010	.0059	.0193	.0461	.0898	.1509	.2272	.3145
11		.0005	.0032	.0112	.0287	.0592	.1051	.1662
12			.0002	.0017	.0065	.0176	.0384	.0717
13				.0001	.0009	.0037	.0106	.0245
14					.0001	.0005	.0021	.0064
15						.0000	.0003	.0012
16							.0000	.0001
17								.0000

	n							
x	18	19	20	21	22	23	24	25
9	.5927							
10	.4073	.5000	.5881					
11	.2403	.3238	.4119	.5000	.5841			
12	.1189	.1796	.2517	.3318	.4159	.5000	.5806	
13	.0481	.0835	.1316	.1917	.2617	.3388	.4194	.5000
14	.0154	.0318	.0577	.0946	.1431	.2024	.2706	.3450
15	.0038	.0096	.0207	.0392	.0669	.1050	.1537	.2122
16	.0007	.0022	.0059	.0133	.0262	.0466	.0758	.1148
17	.0001	.0004	.0013	.0036	.0085	.0173	.0320	.0539
18	.0000	.0000	.0002	.0007	.0022	.0053	.0113	.0216
19		.0000	.0000	.0001	.0004	.0013	.0033	.0073
20			.0000	.0000	.0001	.0002	.0008	.0020
21				.0000	.0000	.0000	.0001	.0005
22					.0000	.0000	.0000	.0001
23						.0000	.0000	.0000
24							.0000	.0000
25								.0000

Source: Portion of Table A.2 of Nonparametric Statistical Methods, M. Hollander and D.A. Wolfe, John Wiley & Sons, NY, 1973.

Table C: Upper Tail Probabilities for the Null Distribution of Wilcoxon's Signed Rank Test Statistic.

Form: $t(\alpha,n) = x$

Example: $t(0.109,6) = 17$

	n						
x	3	4	5	6	7	8	9
3	.625						
4	.375						
5	.250	.562					
6	.125	.438					
7		.312					
8		.188	.500				
9		.125	.406				
10		.062	.312				
11			.219	.500			
12			.156	.422			
13			.094	.344			
14			.062	.281	.531		
15			.031	.219	.469		
16				.156	.406		
17				.109	.344		
18				.078	.289	.527	
19				.047	.234	.473	
20				.031	.188	.422	
21				.016	.148	.371	
22					.109	.320	
23					.078	.273	.500
24					.055	.230	.455
25					.039	.191	.410
26					.023	.156	.367
27					.016	.125	.326
28					.008	.098	.285
29						.074	.248
30						.055	.213
31						.039	.180
32						.027	.150
33						.020	.125
34						.012	.102
35						.008	.082
36						.004	.064
37							.049
38							.037
39							.027
40							.020
41							.014
42							.010
43							.006
44							.004
45							.002

Source: Adapted from Table C of A Nonparametric Introduction to Statistics, C.H. Kraft and C. van Eeden, Macmillan, NY, 1968, and Table A.4 of Nonparametric Statistical Methods, M. Hollander and D.A. Wolfe, John Wiley & Sons, NY, 1973.

Table C
continued

	n					
x	10	11	12	13	14	15
28	.500					
29	.461					
30	.423					
31	.385					
32	.348					
33	.312	.517				
34	.278	.483				
35	.246	.449				
36	.216	.416				
37	.188	.382				
38	.161	.350				
39	.138	.319	.515			
40	.116	.289	.485			
41	.097	.260	.455			
42	.080	.232	.425			
43	.065	.207	.396			
44	.053	.183	.367			
45	.042	.160	.339			
46	.032	.139	.311	.500		
47	.024	.120	.285	.473		
48	.019	.103	.259	.446		
49	.014	.087	.235	.420		
50	.010	.074	.212	.393		
51	.007	.062	.190	.368		
52	.005	.051	.170	.342		
53	.003	.042	.151	.318	.500	
54	.002	.034	.133	.294	.476	
55	.001	.027	.117	.271	.452	
56		.021	.102	.249	.428	
57		.016	.088	.227	.404	
58		.012	.076	.207	.380	
59		.009	.065	.188	.357	
60		.007	.055	.170	.335	.511
61		.005	.046	.153	.313	.489
62		.003	.039	.137	.292	.467
63		.002	.032	.122	.271	.445
64		.001	.026	.108	.251	.423
65		.001	.021	.095	.232	.402
66		.000	.017	.084	.213	.381
67			.013	.073	.196	.360
68			.010	.064	.179	.339
69			.008	.055	.163	.319
70			.006	.047	.148	.300
71			.005	.040	.134	.281
72			.003	.034	.121	.262
73			.002	.029	.108	.244
74			.002	.024	.097	.227
75			.001	.020	.086	.211
76			.001	.016	.077	.195
77			.000	.013	.068	.180
78				.011	.059	.165
79				.009	.052	.151
80				.007	.045	.138
81				.005	.039	.126
82				.004	.034	.115
83				.003	.029	.104
84				.002	.025	.094
85				.002	.021	.084
86				.001	.018	.076
87				.001	.015	.068
88				.001	.012	.060
89				.000	.010	.053
90					.008	.047
91					.007	.042
92					.005	.036
93					.004	.032
94					.003	.028
95					.003	.024
96					.002	.021
97					.002	.018
98					.001	.015
99					.001	.013
100					.001	.011
101					.000	.009
102						.008
103						.006
104						.005
105						.004
106						.003
107						.003
108						.002
109						.002
110						.001

Table D: Upper Tail Probabilities for the Null Distribution of the Mann-Whitney-Wilcoxon Test Statistic.

Form: w(α,m,n) = x

Example: w(0.333,8,1) = 7

n = 1

x	*m* = 3	*m* = 4	*m* = 5	*m* = 6	*m* = 7	*m* = 8	*m* = 9	*m* = 10	*m* = 11
3	.500	.600							
4	.250	.400	.500	.571					
5		.200	.333	.429	.500	.556			
6			.167	.286	.375	.444	.500	.545	
7				.143	.250	.333	.400	.455	.500
8					.125	.222	.300	.364	.417
9						.111	.200	.273	.333
10							.100	.182	.250
11								.091	.167
12									.083

n = 1

x	*m* = 12	*m* = 13	*m* = 14	*m* = 15	*m* = 16	*m* = 17	*m* = 18	*m* = 19	*m* = 20
7	.538								
8	.462	.500	.533						
9	.385	.429	.467	.500	.529				
10	.308	.357	.400	.438	.471	.500	.526		
11	.231	.286	.333	.375	.412	.444	.474	.500	.524
12	.154	.214	.267	.312	.353	.389	.421	.450	.476
13	.077	.143	.200	.250	.294	.333	.368	.400	.429
14		.071	.133	.188	.235	.278	.316	.350	.381
15			.067	.125	.176	.222	.263	.300	.333
16				.062	.118	.167	.211	.250	.286
17					.059	.111	.158	.200	.238
18						.056	.105	.150	.190
19							.053	.100	.143
20								.050	.095
21									.048

Source: Table B of A Nonparametric Introduction to Statistics, C.H. Kraft and C. van Eeden, Macmillan, NY, 1968, and Table A.5 of Nonparametric Statistical Methods, M. Hollander and D. A. Wolfe, John Wiley & Sons, NY, 1973.

Table D continued.

n = 2

x	*m* = 3	*m* = 4	*m* = 5	*m* = 6	*m* = 7	*m* = 8	*m* = 9	*m* = 10	*m* = 11
6	.600								
7	.400	.600							
8	.200	.400	.571						
9	.100	.267	.429	.571					
10		.133	.286	.429	.556				
11		.067	.190	.321	.444	.556			
12			.095	.214	.333	.444	.545		
13			.048	.143	.250	.356	.455	.545	
14				.071	.167	.267	.364	.455	.538
15				.036	.111	.200	.291	.379	.462
16					.056	.133	.218	.303	.385
17					.028	.089	.164	.242	.321
18						.044	.109	.182	.256
19						.022	.073	.136	.205
20							.036	.091	.154
21							.018	.061	.115
22								.030	.077
23								.015	.051
24									.026
25									.013

n = 2

x	*m* = 12	*m* = 13	*m* = 14	*m* = 15	*m* = 16	*m* = 17	*m* = 18	*m* = 19	*m* = 20
15	.538								
16	.462	.533							
17	.396	.467	.533						
18	.330	.400	.467	.529					
19	.275	.343	.408	.471	.529				
20	.220	.286	.350	.412	.471	.526			
21	.176	.238	.300	.360	.418	.474	.526		
22	.132	.190	.250	.309	.366	.421	.474	.524	
23	.099	.152	.208	.265	.320	.374	.426	.476	.524
24	.066	.114	.167	.221	.275	.327	.379	.429	.476
25	.044	.086	.133	.184	.235	.287	.337	.386	.433
26	.022	.057	.100	.147	.196	.246	.295	.343	.390
27	.011	.038	.075	.118	.163	.211	.258	.305	.351
28		.019	.050	.088	.131	.175	.221	.267	.312
29		.010	.033	.066	.105	.146	.189	.233	.277
30			.017	.044	.078	.117	.158	.200	.242
31			.008	.029	.059	.094	.132	.171	.212
32				.015	.039	.070	.105	.143	.182
33				.007	.026	.053	.084	.119	.156
34					.013	.035	.063	.095	.130
35					.007	.023	.047	.076	.108
36						.012	.032	.057	.087
37						.006	.021	.043	.069
38							.011	.029	.052
39							.005	.019	.039
40								.010	.026
41								.005	.017
42									.009
43									.004

Table D continued

$n = 3$

x	$m = 3$	$m = 4$	$m = 5$	$m = 6$	$m = 7$	$m = 8$	$m = 9$	$m = 10$	$m = 11$
11	.500								
12	.350	.571							
13	.200	.429							
14	.100	.314	.500						
15	.050	.200	.393	.548					
16		.114	.286	.452					
17		.057	.196	.357	.500				
18		.029	.125	.274	.417	.539			
19			.071	.190	.333	.461			
20			.036	.131	.258	.388	.500		
21			.018	.083	.192	.315	.432	.531	
22				.048	.133	.248	.364	.469	
23				.024	.092	.188	.300	.406	.500
24				.012	.058	.139	.241	.346	.442
25					.033	.097	.186	.287	.385
26					.017	.067	.141	.234	.330
27					.008	.042	.105	.185	.277
28						.024	.073	.143	.228
29						.012	.050	.108	.184
30						.006	.032	.080	.146
31							.018	.056	.113
32							.009	.038	.085
33							.005	.024	.063
34								.014	.044
35								.007	.030
36								.003	.019
37									.011
38									.005
39									.003

$n = 3$

x	$m = 12$	$m = 13$	$m = 14$	$m = 15$	$m = 16$	$m = 17$	$m = 18$	$m = 19$	$m = 20$
24	.527								
25	.473								
26	.420	.500							
27	.367	.450	.524						
28	.316	.400	.476						
29	.268	.352	.429	.500					
30	.224	.305	.384	.456	.521				
31	.182	.261	.338	.412	.479				
32	.147	.220	.296	.369	.438	.500			
33	.116	.182	.254	.327	.396	.461	.519		
34	.090	.148	.216	.287	.356	.421	.481		
35	.068	.120	.181	.249	.317	.382	.444	.500	
36	.051	.095	.150	.213	.280	.345	.407	.464	.517
37	.035	.073	.122	.180	.244	.308	.370	.429	.483
38	.024	.055	.099	.151	.211	.273	.335	.394	.449
39	.015	.041	.078	.125	.180	.239	.300	.359	.415
40	.009	.029	.060	.102	.152	.208	.267	.325	.382
41	.004	.020	.046	.082	.127	.179	.235	.293	.349
42	.002	.012	.034	.065	.105	.153	.206	.262	.317
43		.007	.024	.050	.086	.129	.178	.232	.286
44		.004	.016	.038	.069	.108	.153	.204	.257
45		.002	.010	.028	.055	.089	.131	.178	.229
46			.006	.020	.042	.073	.111	.154	.202
47			.003	.013	.032	.059	.092	.132	.177
48			.001	.009	.024	.046	.077	.113	.155
49				.005	.017	.036	.062	.095	.134
50				.002	.011	.027	.050	.080	.115
51				.001	.007	.020	.040	.066	.098
52					.004	.014	.031	.054	.083
53					.002	.010	.023	.044	.069
54					.001	.006	.017	.034	.058
55						.004	.012	.027	.047
56						.002	.008	.020	.038
57						.001	.005	.015	.030
58							.003	.010	.023
59							.002	.007	.018
60							.001	.005	.013
61								.003	.009
62								.001	.006

Table D
continued.

n = 4

x	*m* = 4	*m* = 5	*m* = 6	*m* = 7	*m* = 8	*m* = 9	*m* = 10	*m* = 11
18	.557							
19	.443							
20	.343	.548						
21	.243	.452						
22	.171	.365	.543					
23	.100	.278	.457					
24	.057	.206	.381	.536				
25	.029	.143	.305	.464				
26	.014	.095	.238	.394	.533			
27		.056	.176	.324	.467			
28		.032	.129	.264	.404	.530		
29		.016	.086	.206	.341	.470		
30		.008	.057	.158	.285	.413	.527	
31			.033	.115	.230	.355	.473	
32			.019	.082	.184	.302	.420	.525
33			.010	.055	.141	.252	.367	.475
34			.005	.036	.107	.207	.318	.426
35				.021	.077	.165	.270	.377
36				.012	.055	.130	.227	.330
37				.006	.036	.099	.187	.286
38				.003	.024	.074	.152	.245
39					.014	.053	.120	.206
40					.008	.038	.094	.171
41					.004	.025	.071	.140
42					.002	.017	.053	.113
43						.010	.038	.089
44						.006	.027	.069
45						.003	.018	.052
46						.001	.012	.039
47							.007	.028
48							.004	.020
49							.002	.013
50							.001	.009
51								.005
52								.003
53								.001
54								.001

n = 4

x	*m* = 12	*m* = 13	*m* = 14	*m* = 15	*m* = 16	*m* = 17	*m* = 18	*m* = 19	*m* = 20
34	.524								
35	.476								
36	.431	.522							
37	.385	.478							
38	.342	.435	.521						
39	.299	.392	.479						
40	.260	.352	.439	.519					
41	.223	.312	.399	.481					
42	.190	.274	.360	.443	.518				
43	.158	.239	.323	.405	.482				
44	.131	.206	.287	.368	.446	.517			
45	.106	.175	.253	.332	.410	.483			
46	.085	.148	.221	.298	.375	.449	.516		
47	.066	.123	.191	.265	.341	.415	.484		
48	.052	.101	.164	.235	.308	.381	.451	.516	
49	.039	.082	.139	.205	.277	.349	.419	.484	
50	.029	.065	.116	.179	.247	.318	.387	.453	.515

Table D
continued.

n = 4

x	*m* = 12	*m* = 13	*m* = 14	*m* = 15	*m* = 16	*m* = 17	*m* = 18	*m* = 19	*m* = 20
51	.021	.051	.096	.154	.219	.287	.356	.422	.485
52	.015	.039	.079	.131	.192	.258	.326	.392	.455
53	.010	.030	.063	.110	.168	.231	.297	.363	.426
54	.007	.022	.051	.092	.145	.205	.269	.334	.397
55	.004	.016	.040	.076	.124	.181	.242	.306	.368
56	.002	.011	.031	.062	.106	.158	.217	.279	.341
57	.001	.008	.023	.050	.089	.138	.193	.253	.314
58	.001	.005	.017	.040	.074	.119	.171	.228	.288
59		.003	.012	.031	.061	.101	.150	.205	.262
60		.002	.009	.024	.050	.086	.131	.183	.239
61		.001	.006	.018	.040	.072	.113	.162	.216
62		.000	.004	.014	.032	.060	.098	.143	.194
63			.002	.010	.025	.049	.083	.125	.174
64			.001	.007	.019	.040	.070	.109	.155
65			.001	.005	.015	.032	.059	.094	.137
66			.000	.003	.011	.026	.049	.081	.120
67				.002	.008	.020	.040	.069	.105
68				.001	.006	.016	.033	.058	.091
69				.001	.004	.012	.027	.049	.079
70				.000	.002	.009	.021	.041	.067
71					.001	.006	.017	.033	.057
72					.001	.005	.013	.027	.048
73					.000	.003	.010	.022	.041
74					.000	.002	.007	.018	.034
75						.001	.005	.014	.028
76						.001	.004	.011	.023
77						.000	.002	.008	.018
78						.000	.002	.006	.015
79							.001	.004	.011
80							.001	.003	.009
81							.000	.002	.007
82							.000	.001	.005
83								.001	.004
84								.000	.003
85								.000	.002
86								.000	.001
87									.001
88									.000
89									.000
90									.000

Table D
continued.

n = 5

x	*m* = 5	*m* = 6	*m* = 7	*m* = 8	*m* = 9	*m* = 10
28	.500					
29	.421					
30	.345	.535				
31	.274	.465				
32	.210	.396				
33	.155	.331	.500			
34	.111	.268	.438			
35	.075	.214	.378	.528		
36	.048	.165	.319	.472		
37	.028	.123	.265	.416		
38	.016	.089	.216	.362	.500	
39	.008	.063	.172	.311	.449	
40	.004	.041	.134	.262	.399	.523
41		.026	.101	.218	.350	.477
42		.015	.074	.177	.303	.430
43		.009	.053	.142	.259	.384
44		.004	.037	.111	.219	.339
45		.002	.024	.085	.182	.297
46			.015	.064	.149	.257
47			.009	.047	.120	.220
48			.005	.033	.095	.185
49			.003	.023	.073	.155
50			.001	.015	.056	.127
51				.009	.041	.103
52				.005	.030	.082
53				.003	.021	.065
54				.002	.014	.050
55				.001	.009	.038
56					.006	.028
57					.003	.020
58					.002	.014
59					.001	.010
60					.000	.006
61						.004
62						.002
63						.001
64						.001
65						.000

n = 6

x	*m* = 6	*m* = 7	*m* = 8	*m* = 9	*m* = 10
39	.531				
40	.469				
41	.409				
42	.350	.527			
43	.294	.473			
44	.242	.418			
45	.197	.365	.525		
46	.155	.314	.475		
47	.120	.267	.426		
48	.090	.223	.377	.523	
49	.066	.183	.331	.477	
50	.047	.147	.286	.432	
51	.032	.117	.245	.388	.521
52	.021	.090	.207	.344	.479
53	.013	.069	.172	.303	.437
54	.008	.051	.141	.264	.396
55	.004	.037	.114	.228	.356
56	.002	.026	.091	.194	.318
57	.001	.017	.071	.164	.281
58		.011	.054	.136	.246
59		.007	.041	.112	.214
60		.004	.030	.091	.184
61		.002	.021	.072	.157
62		.001	.015	.057	.132
63		.001	.010	.044	.110
64			.006	.033	.090
65			.004	.025	.074
66			.002	.018	.059
67			.001	.013	.047
68			.001	.009	.036
69			.000	.006	.028
70				.004	.021
71				.002	.016
72				.001	.011
73				.001	.008
74				.000	.005
75				.000	.004
76					.002
77					.001
78					.001
79					.000
80					.000
81					.000

Table D
continued.

n = 7

x	m = 7	m = 8	m = 9	m = 10
53	.500			
54	.451			
55	.402			
56	.355	.522		
57	.310	.478		
58	.267	.433		
59	.228	.389		
60	.191	.347	.500	
61	.159	.306	.459	
62	.130	.268	.419	
63	.104	.232	.379	.519
64	.082	.198	.340	.481
65	.064	.168	.303	.443
66	.049	.140	.268	.406
67	.036	.116	.235	.370
68	.027	.095	.204	.335
69	.019	.076	.176	.300
70	.013	.060	.150	.268
71	.009	.047	.126	.237
72	.006	.036	.105	.209
73	.003	.027	.087	.182
74	.002	.020	.071	.157
75	.001	.014	.057	.135
76	.001	.010	.045	.115
77	.000	.007	.036	.097
78		.005	.027	.081
79		.003	.021	.067
80		.002	.016	.054
81		.001	.011	.044
82		.001	.008	.035
83		.000	.006	.028
84		.000	.004	.022
85			.003	.017
86			.002	.012
87			.001	.009
88			.001	.007
89			.000	.005
90			.000	.003
91			.000	.002
92				.002
93				.001
94				.001
95				.000
96				.000
97				.000
98				.000

n = 8

x	m = 8	m = 9	m = 10
68	.520		
69	.480		
70	.439		
71	.399		
72	.360	.519	
73	.323	.481	
74	.287	.444	
75	.253	.407	
76	.221	.371	.517
77	.191	.336	.483
78	.164	.303	.448
79	.139	.271	.414
80	.117	.240	.381
81	.097	.212	.348
82	.080	.185	.317
83	.065	.161	.286
84	.052	.138	.257
85	.041	.118	.230
86	.032	.100	.204
87	.025	.084	.180
88	.019	.069	.158
89	.014	.057	.137
90	.010	.046	.118
91	.007	.037	.102
92	.005	.030	.086
93	.003	.023	.073
94	.002	.018	.061
95	.001	.014	.051
96	.001	.010	.042
97	.001	.008	.034
98	.000	.006	.027
99	.000	.004	.022
100	.000	.003	.017
101		.002	.013
102		.001	.010
103		.001	.008
104		.000	.006
105		.000	.004
106		.000	.003
107		.000	.002
108		.000	.002
109			.001
110			.001
111			.000
112			.000
113			.000
114			.000
115			.000
116			.000

Table D
continued.

n = 9		
x	*m* = 9	*m* = 10
86	.500	
87	.466	
88	.432	
89	.398	
90	.365	.516
91	.333	.484
92	.302	.452
93	.273	.421
94	.245	.390
95	.218	.360
96	.193	.330
97	.170	.302
98	.149	.274
99	.129	.248
100	.111	.223
101	.095	.200
102	.081	.178
103	.068	.158
104	.057	.139
105	.047	.121
106	.039	.106
107	.031	.091
108	.025	.078
109	.020	.067
110	.016	.056
111	.012	.047
112	.009	.039
113	.007	.033
114	.005	.027
115	.004	.022
116	.003	.017
117	.002	.014
118	.001	.011
119	.001	.009
120	.001	.007
121	.000	.005
122	.000	.004
123	.000	.003
124	.000	.002
125	.000	.001
126	.000	.001
127		.001
128		.000
129		.000
130		.000
131		.000
132		.000
133		.000
134		.000
135		.000

n = 10	
x	*m* = 10
105	.515
106	.485
107	.456
108	.427
109	.398
110	.370
111	.342
112	.315
113	.289
114	.264
115	.241
116	.218
117	.197
118	.176
119	.157
120	.140
121	.124
122	.109
123	.095
124	.083
125	.072
126	.062
127	.053
128	.045
129	.038
130	.032
131	.026
132	.022
133	.018
134	.014
135	.012
136	.009
137	.007
138	.006
139	.004
140	.003
141	.003
142	.002
143	.001
144	.001
145	.001
146	.001
147	.000
148	.000
149	.000
150	.000
151	.000
152	.000
153	.000
154	.000
155	.000

Table E: Critical Values for the Range of k Independent N(0,1) Variables.

Form: $q(\alpha, k, \infty)$

Example: $q(0.01, 9, \infty) = 5.078$

	α								
k	.0001	.0005	.001	.005	.01	.025	.05	.10	.20
2	5.502	4.923	4.654	3.970	3.643	3.170	2.772	2.326	1.812
3	5.864	5.316	5.063	4.424	4.120	3.682	3.314	2.902	2.424
4	6.083	5.553	5.309	4.694	4.403	3.984	3.633	3.240	2.784
5	6.240	5.722	5.484	4.886	4.603	4.197	3.858	3.478	3.037
6	6.362	5.853	5.619	5.033	4.757	4.361	4.030	3.661	3.232
7	6.461	5.960	5.730	5.154	4.882	4.494	4.170	3.808	3.389
8	6.546	6.050	5.823	5.255	4.987	4.605	4.286	3.931	3.520
9	6.618	6.127	5.903	5.341	5.078	4.700	4.387	4.037	3.632
10	6.682	6.196	5.973	5.418	5.157	4.784	4.474	4.129	3.730
11	6.739	6.257	6.036	5.485	5.227	4.858	4.552	4.211	3.817
12	6.791	6.311	6.092	5.546	5.290	4.925	4.622	4.285	3.895
13	6.837	6.361	6.144	5.602	5.348	4.985	4.685	4.351	3.966
14	6.880	6.407	6.191	5.652	5.400	5.041	4.743	4.412	4.030
15	6.920	6.449	6.234	5.699	5.448	5.092	4.796	4.468	4.089
16	6.957	6.488	6.274	5.742	5.493	5.139	4.845	4.519	4.144
17	6.991	6.525	6.312	5.783	5.535	5.183	4.891	4.568	4.195
18	7.023	6.559	6.347	5.820	5.574	5.224	4.934	4.612	4.242
19	7.054	6.591	6.380	5.856	5.611	5.262	4.974	4.654	4.287
20	7.082	6.621	6.411	5.889	5.645	5.299	5.012	4.694	4.329
22	7.135	6.677	6.469	5.951	5.709	5.365	5.081	4.767	4.405
24	7.183	6.727	6.520	6.006	5.766	5.425	5.144	4.832	4.475
26	7.226	6.773	6.568	6.057	5.818	5.480	5.201	4.892	4.537
28	7.266	6.816	6.611	6.103	5.866	5.530	5.253	4.947	4.595
30	7.303	6.855	6.651	6.146	5.911	5.577	5.301	4.997	4.648
32	7.337	6.891	6.689	6.186	5.952	5.620	5.346	5.044	4.697
34	7.370	6.925	6.723	6.223	5.990	5.660	5.388	5.087	4.743
36	7.400	6.957	6.756	6.258	6.026	5.698	5.427	5.128	4.786
38	7.428	6.987	6.787	6.291	6.060	5.733	5.463	5.166	4.826
40	7.455	7.015	6.816	6.322	6.092	5.766	5.498	5.202	4.864
50	7.571	7.137	6.941	6.454	6.228	5.909	5.646	5.357	5.026
60	7.664	7.235	7.041	6.561	6.338	6.023	5.764	5.480	5.155
70	7.741	7.317	7.124	6.649	6.429	6.118	5.863	5.582	5.262
80	7.808	7.387	7.196	6.725	6.507	6.199	5.947	5.669	5.353
90	7.866	7.448	7.259	6.792	6.575	6.270	6.020	5.745	5.433
100	7.918	7.502	7.314	6.850	6.636	6.333	6.085	5.812	5.503

Source: Adapted from Harter, H.L., Tables of range and studentized range, Ann. Math. Statist., (1960), 31: 1122-1147, and Table A.10 of Nonparametric Statistical Methods, M. Hollander and D. A. Wolfe, John Wiley & Sons, NY, 1973.

Table F: Upper Tail Probabilities for the Null Distribution of the Freund-Ansari-Bradley Test Statistic.

Form: $w(\alpha,m,n) = x$

Example: $w(0.7222,2,7) = 5$

$m = 2$

x	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$	$n = 10$
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	.8333	.9000	.9333	.9524	.9643	.9722	.9778	.9818	.9848
4	.1667	.5000	.6667	.7619	.8214	.8611	.8889	.9091	.9242
5		.2000	.3333	.5238	.6429	.7222	.7778	.8182	.8485
6			.0667	.2381	.3571	.5000	.6000	.6727	.7273
7				.0952	.1786	.3056	.4000	.5091	.5909
8					.0357	.1389	.2222	.3273	.4091
9						.0556	.1111	.2000	.2727
10							.0222	.0909	.1515
11								.0364	.0758
12									.0152

$m = 2$

x	$n = 11$	$n = 12$	$n = 13$	$n = 14$	$n = 15$	$n = 16$	$n = 17$	$n = 18$
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	.9872	.9890	.9905	.9917	.9926	.9935	.9942	.9947
4	.9359	.9451	.9524	.9583	.9632	.9673	.9708	.9737
5	.8718	.8901	.9048	.9167	.9265	.9346	.9415	.9474
6	.7692	.8022	.8286	.8500	.8676	.8824	.8947	.9053
7	.6538	.7033	.7429	.7750	.8015	.8235	.8421	.8579
8	.5000	.5714	.6286	.6750	.7132	.7451	.7719	.7947
9	.3590	.4286	.5048	.5667	.6176	.6601	.6959	.7263
10	.2308	.2967	.3714	.4333	.5000	.5556	.6023	.6421
11	.1410	.1978	.2667	.3250	.3897	.4444	.5029	.5526
12	.0641	.1099	.1714	.2250	.2868	.3399	.3977	.4474
13	.0256	.0549	.1048	.1500	.2059	.2549	.3099	.3579
14		.0110	.0476	.0833	.1324	.1765	.2281	.2737
15			.0190	.0417	.0809	.1176	.1637	.2053
16				.0083	.0368	.0654	.1053	.1421
17					.0147	.0327	.0643	.0947
18						.0065	.0292	.0526
19							.0117	.0263
20								.0053

Source: Table A.6 of Nonparametric Statistical Methods, M. Hollander and D. A. Wolfe, John Wiley & Sons, NY, 1973.

Table F
continued.

m = 3

x	*n* = 3	*n* = 4	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9	*n* = 10	*n* = 11
4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	.9000	.9429	.9643	.9762	.9833	.9879	.9909	.9930	.9945
6	.7000	.8286	.8929	.9286	.9500	.9636	.9727	.9790	.9835
7	.3000	.5714	.7143	.8095	.8667	.9030	.9273	.9441	.9560
8	.1000	.3429	.5000	.6548	.7500	.8182	.8636	.8951	.9176
9		.1429	.2857	.4643	.5833	.6909	.7636	.8182	.8571
10		.0286	.1071	.2857	.4167	.5455	.6364	.7168	.7747
11			.0357	.1429	.2500	.3939	.5000	.5979	.6703
12				.0595	.1333	.2606	.3636	.4755	.5604
13				.0119	.0500	.1455	.2364	.3497	.4396
14					.0167	.0727	.1364	.2413	.3297
15						.0303	.0727	.1503	.2253
16						.0061	.0273	.0839	.1429
17							.0091	.0420	.0824
18								.0175	.0440
19								.0035	.0165
20									.0055

m = 3

x	*n* = 12	*n* = 13	*n* = 14	*n* = 15	*n* = 16	*n* = 17
4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	.9956	.9964	.9971	.9975	.9979	.9982
6	.9868	.9893	.9912	.9926	.9938	.9947
7	.9648	.9714	.9765	.9804	.9835	.9860
8	.9341	.9464	.9559	.9632	.9690	.9737
9	.8857	.9071	.9235	.9363	.9463	.9544
10	.8198	.8536	.8794	.8995	.9154	.9281
11	.7341	.7821	.8206	.8505	.8741	.8930
12	.6374	.6964	.7485	.7892	.8225	.8491
13	.5297	.6000	.6632	.7132	.7575	.7930
14	.4242	.5000	.5735	.6324	.6852	.7281
15	.3209	.4000	.4794	.5441	.6058	.6561
16	.2286	.3036	.3868	.4559	.5232	.5789
17	.1516	.2179	.2985	.3676	.4396	.5000
18	.0945	.1464	.2206	.2868	.3591	.4211
19	.0527	.0929	.1529	.2108	.2817	.3439
20	.0264	.0536	.1015	.1495	.2136	.2719
21	.0110	.0286	.0632	.1005	.1548	.2070
22	.0022	.0107	.0353	.0637	.1073	.1509
23		.0036	.0176	.0368	.0712	.1070
24			.0074	.0196	.0444	.0719
25			.0015	.0074	.0248	.0456
26				.0025	.0124	.0263
27					.0052	.0140
28					.0010	.0053
29						.0018

Table F continued.

$m = 4$

x	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$	$n = 10$	$n = 11$	$n = 12$
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	.9857	.9921	.9952	.9970	.9980	.9986	.9990	.9993	.9995
8	.9286	.9603	.9762	.9848	.9899	.9930	.9950	.9963	.9973
9	.8000	.8889	.9333	.9576	.9717	.9804	.9860	.9897	.9923
10	.6286	.7778	.8571	.9091	.9394	.9580	.9700	.9780	.9835
11	.3714	.6032	.7333	.8242	.8788	.9161	.9401	.9560	.9670
12	.2000	.4286	.5810	.7152	.7980	.8573	.8961	.9238	.9429
13	.0714	.2619	.4190	.5818	.6889	.7762	.8342	.8769	.9066
14	.0143	.1349	.2667	.4424	.5677	.6783	.7542	.8154	.8582
15		.0476	.1429	.3030	.4323	.5650	.6593	.7385	.7951
16		.0159	.0667	.1939	.3111	.4503	.5554	.6520	.7225
17			.0238	.1061	.2020	.3357	.4446	.5546	.6374
18			.0048	.0515	.1212	.2378	.3407	.4564	.5473
19				.0182	.0606	.1538	.2458	.3590	.4527
20				.0061	.0283	.0923	.1658	.2711	.3626
21					.0101	.0490	.1039	.1934	.2775
22					.0020	.0238	.0599	.1319	.2049
23						.0084	.0300	.0821	.1418
24						.0028	.0140	.0484	.0934
25							.0050	.0256	.0571
26							.0010	.0125	.0330
27								.0044	.0165
28								.0015	.0077
29									.0027
30									.0005

$m = 4$

x	$n = 13$	$n = 14$	$n = 15$	$n = 16$
6	1.0000	1.0000	1.0000	1.0000
7	.9996	.9997	.9997	.9998
8	.9979	.9984	.9987	.9990
9	.9941	.9954	.9964	.9971
10	.9874	.9902	.9923	.9938
11	.9748	.9804	.9845	.9876
12	.9563	.9660	.9732	.9785
13	.9286	.9444	.9561	.9649
14	.8908	.9144	.9324	.9459
15	.8408	.8742	.9002	.9197
16	.7811	.8245	.8599	.8867
17	.7101	.7647	.8101	.8448
18	.6319	.6967	.7528	.7961
19	.5471	.6209	.6873	.7391
20	.4613	.5412	.6166	.6764
21	.3761	.4588	.5413	.6078
22	.2979	.3791	.4654	.5368
23	.2261	.3033	.3896	.4632
24	.1655	.2353	.3189	.3922
25	.1151	.1755	.2531	.3236
26	.0765	.1258	.1953	.2609
27	.0471	.0856	.1450	.2039
28	.0277	.0556	.1042	.1552
29	.0147	.0340	.0712	.1133
30	.0071	.0196	.0470	.0803
31	.0025	.0098	.0289	.0541
32	.0008	.0046	.0170	.0351
33		.0016	.0090	.0215
34		.0003	.0044	.0124
35			.0015	.0062
36			.0005	.0029
37				.0010
38				.0002

Table F
continued.

m = 5

x	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9	*n* = 10	*n* = 11	*n* = 12	*n* = 13	*n* = 14	*n* = 15
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	.9921	.9957	.9975	.9984	.9990	.9993	.9995	.9997	.9998	.9998	.9999
11	.9762	.9870	.9924	.9953	.9970	.9980	.9986	.9990	.9993	.9995	.9996
12	.9286	.9610	.9773	.9860	.9910	.9940	.9959	.9971	.9979	.9985	.9988
13	.8492	.9156	.9495	.9689	.9800	.9867	.9908	.9935	.9953	.9966	.9974
14	.7302	.8420	.9015	.9386	.9600	.9734	.9817	.9871	.9907	.9931	.9948
15	.5873	.7446	.8333	.8936	.9291	.9524	.9670	.9767	.9832	.9876	.9907
16	.4127	.6147	.7374	.8275	.8821	.9197	.9437	.9601	.9711	.9787	.9840
17	.2698	.4805	.6237	.7451	.8212	.8761	.9116	.9368	.9538	.9659	.9743
18	.1508	.3463	.5000	.6457	.7423	.8182	.8681	.9047	.9295	.9476	.9604
19	.0714	.2294	.3763	.5385	.6523	.7483	.8132	.8633	.8978	.9235	.9417
20	.0238	.1342	.2626	.4266	.5514	.6663	.7468	.8116	.8569	.8920	.9171
21	.0079	.0693	.1667	.3209	.4486	.5771	.6708	.7508	.8079	.8533	.8861
22		.0303	.0985	.2269	.3477	.4832	.5870	.6810	.7498	.8067	.8483
23		.0108	.0505	.1507	.2577	.3916	.5000	.6054	.6846	.7530	.8038
24		.0022	.0227	.0917	.1788	.3044	.4130	.5254	.6130	.6923	.7523
25			.0076	.0513	.1179	.2268	.3292	.4449	.5383	.6267	.6950
26			.0025	.0249	.0709	.1608	.2532	.3662	.4617	.5572	.6329
27				.0109	.0400	.1086	.1868	.2928	.3870	.4864	.5673
28				.0039	.0200	.0686	.1319	.2262	.3154	.4157	.5000
29				.0008	.0090	.0406	.0884	.1690	.2502	.3478	.4327
30					.0030	.0220	.0563	.1214	.1921	.2840	.3671
31					.0010	.0107	.0330	.0835	.1431	.2262	.3050
32						.0047	.0183	.0546	.1022	.1751	.2477
33						.0017	.0092	.0339	.0705	.1318	.1962
34						.0003	.0041	.0197	.0462	.0960	.1517
35							.0014	.0107	.0289	.0675	.1139
36							.0005	.0052	.0168	.0455	.0829
								.0023	.0093	.0294	.0583
								.0008	.0047	.0181	.0396
								.0002	.0021	.0105	.0257
									.0007	.0057	.0160
									.0002	.0028	.0093
										.0012	.0052
										.0004	.0026
										.0001	.0012
											.0004
											.0001

Table F
continued.

m = 6

x	*n* = 6	*n* = 7	*n* = 8	*n* = 9	*n* = 10	*n* = 11	*n* = 12	*n* = 13	*n* = 14
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	.9989	.9994	.9997	.9998	.9999	.9999	.9999	1.0000	1.0000
14	.9946	.9971	.9983	.9990	.9994	.9996	.9997	.9998	.9999
15	.9848	.9918	.9953	.9972	.9983	.9989	.9992	.9995	.9996
16	.9632	.9802	.9887	.9932	.9958	.9973	.9982	.9987	.9991
17	.9264	.9592	.9760	.9856	.9910	.9942	.9961	.9973	.9981
18	.8658	.9242	.9547	.9724	.9825	.9887	.9925	.9948	.9964
19	.7846	.8735	.9217	.9518	.9692	.9799	.9865	.9907	.9935
20	.6807	.8048	.8751	.9215	.9487	.9663	.9772	.9843	.9890
21	.5649	.7203	.8139	.8803	.9202	.9469	.9636	.9749	.9823
22	.4351	.6189	.7366	.8260	.8812	.9199	.9445	.9613	.9725
23	.3193	.5122	.6474	.7600	.8322	.8849	.9190	.9431	.9591
24	.2154	.4038	.5501	.6829	.7717	.8407	.8860	.9191	.9413
25	.1342	.3030	.4499	.5984	.7025	.7877	.8451	.8887	.9184
26	.0736	.2133	.3526	.5085	.6246	.7259	.7962	.8514	.8896
27	.0368	.1410	.2634	.4190	.5425	.6574	.7398	.8074	.8549
28	.0152	.0851	.1861	.3323	.4575	.5831	.6765	.7564	.8138
29	.0054	.0484	.1249	.2543	.3754	.5065	.6082	.6996	.7668
30	.0011	.0239	.0783	.1860	.2975	.4292	.5364	.6376	.7139
31		.0105	.0453	.1303	.2283	.3549	.4636	.5723	.6566
32		.0035	.0240	.0859	.1678	.2851	.3918	.5049	.5954
33		.0012	.0113	.0539	.1188	.2226	.3235	.4376	.5322
34			.0047	.0312	.0798	.1678	.2602	.3716	.4678
35			.0017	.0170	.0513	.1226	.2038	.3094	.4046
36			.0003	.0082	.0308	.0859	.1549	.2518	.3434
37				.0036	.0175	.0579	.1140	.2002	.2861
38				.0012	.0090	.0370	.0810	.1550	.2332
39				.0004	.0042	.0226	.0555	.1170	.1862
40					.0017	.0128	.0364	.0855	.1451
41					.0006	.0069	.0228	.0608	.1104
42					.0001	.0033	.0135	.0415	.0816
43						.0015	.0075	.0274	.0587
44						.0005	.0039	.0172	.0409
45						.0002	.0018	.0104	.0275
46							.0008	.0058	.0177
47							.0003	.0031	.0110
48							.0001	.0015	.0065
49								.0007	.0036
50								.0002	.0019
51								.0001	.0009
52									.0004
53									.0001
54									.0000

Table F
continued.

$m = 7$							
x	$n = 7$	$n = 8$	$n = 9$	$n = 10$	$n = 11$	$n = 12$	$n = 13$
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	.9994	.9997	.9998	.9999	1.0000	1.0000	1.0000
18	.9983	.9991	.9995	.9997	.9998	.9999	.9999
19	.9948	.9972	.9984	.9991	.9994	.9996	.9998
20	.9878	.9935	.9963	.9978	.9987	.9992	.9995
21	.9744	.9862	.9921	.9954	.9972	.9982	.9988
22	.9534	.9744	.9851	.9912	.9946	.9966	.9978
23	.9196	.9549	.9734	.9841	.9901	.9937	.9959
24	.8730	.9270	.9559	.9734	.9833	.9893	.9930
25	.8106	.8878	.9306	.9574	.9729	.9826	.9885
26	.7348	.8375	.8965	.9354	.9583	.9730	.9820
27	.6463	.7748	.8523	.9059	.9381	.9595	.9727
28	.5507	.7021	.7981	.8685	.9118	.9415	.9602
29	.4493	.6194	.7336	.8221	.8782	.9181	.9435
30	.3537	.5324	.6608	.7676	.8374	.8889	.9223
31	.2652	.4435	.5820	.7052	.7887	.8532	.8958
32	.1894	.3577	.5000	.6368	.7333	.8111	.8637
33	.1270	.2777	.4180	.5637	.6714	.7626	.8258
34	.0804	.2075	.3392	.4888	.6050	.7085	.7822
35	.0466	.1478	.2664	.4139	.5353	.6494	.7332
36	.0256	.1005	.2019	.3421	.4647	.5869	.6795
37	.0122	.0648	.1477	.2753	.3950	.5220	.6219
38	.0052	.0393	.1035	.2154	.3286	.4568	.5616
39	.0017	.0221	.0694	.1633	.2667	.3925	.5000
40	.0006	.0115	.0441	.1199	.2113	.3311	.4384
41		.0053	.0266	.0847	.1626	.2735	.3781
42		.0022	.0149	.0576	.1218	.2213	.3205
43		.0008	.0079	.0375	.0882	.1749	.2668
44		.0002	.0037	.0233	.0619	.1350	.2178
45			.0016	.0136	.0417	.1014	.1742
46			.0005	.0075	.0271	.0742	.1363
47			.0002	.0038	.0167	.0526	.1042
48				.0017	.0099	.0361	.0777
49				.0007	.0054	.0239	.0565
50				.0003	.0028	.0152	.0398
51				.0001	.0013	.0092	.0273
52					.0006	.0053	.0180
53					.0002	.0029	.0115
54					.0001	.0015	.0070
55						.0007	.0041
56						.0003	.0022
57						.0001	.0012
58						.0000	.0005
59							.0002
60							.0001
61							.0000

Table F
continued.

m = 8

x	n = 8	n = 9	n = 10	n = 11	n = 12
20	1.0000	1.0000	1.0000	1.0000	1.0000
21	.9999	1.0000	1.0000	1.0000	1.0000
22	.9996	.9998	.9999	.9999	1.0000
23	.9989	.9994	.9997	.9998	.9999
24	.9974	.9986	.9992	.9996	.9997
25	.9941	.9969	.9983	.9990	.9994
26	.9885	.9938	.9965	.9980	.9988
27	.9789	.9886	.9935	.9962	.9977
28	.9643	.9804	.9887	.9934	.9960
29	.9428	.9680	.9813	.9889	.9932
30	.9133	.9504	.9704	.9823	.9890
31	.8737	.9262	.9551	.9728	.9830
32	.8246	.8947	.9344	.9598	.9745
33	.7650	.8549	.9075	.9423	.9629
34	.6970	.8069	.8738	.9199	.9477
35	.6212	.7508	.8328	.8918	.9281
36	.5413	.6877	.7847	.8578	.9038
37	.4587	.6184	.7296	.8174	.8742
38	.3788	.5457	.6686	.7710	.8392
39	.3030	.4714	.6031	.7189	.7986
40	.2350	.3983	.5347	.6621	.7528
41	.1754	.3281	.4653	.6015	.7022
42	.1263	.2636	.3969	.5386	.6476
43	.0867	.2055	.3314	.4746	.5898
44	.0572	.1557	.2704	.4113	.5302
45	.0357	.1139	.2153	.3500	.4698
46	.0211	.0807	.1672	.2925	.4102
47	.0115	.0548	.1262	.2394	.3524
48	.0059	.0358	.0925	.1919	.2978
49	.0026	.0221	.0656	.1503	.2472
50	.0011	.0131	.0449	.1150	.2014
51	.0004	.0072	.0296	.0856	.1608
52	.0001	.0037	.0187	.0621	.1258
53		.0017	.0113	.0437	.0962
54		.0007	.0065	.0298	.0719
55		.0002	.0035	.0196	.0523
56		.0001	.0017	.0124	.0371
57			.0008	.0075	.0255
58			.0003	.0043	.0170
59			.0001	.0023	.0110
60			.0000	.0012	.0068
61				.0006	.0040
62				.0002	.0023
63				.0001	.0012
64				.0000	.0006
65					.0003
66					.0001
67					.0000
68					.0000

m = 9

x	n = 9	n = 10	n = 11
25	1.0000	1.0000	1.0000
26	1.0000	1.0000	1.0000
27	.9999	.9999	1.0000
28	.9996	.9998	.9999
29	.9991	.9995	.9997
30	.9981	.9990	.9995
31	.9963	.9980	.9989
32	.9932	.9964	.9980
33	.9882	.9937	.9964
34	.9805	.9894	.9940
35	.9695	.9831	.9903
36	.9540	.9741	.9849
37	.9332	.9618	.9773
38	.9062	.9453	.9669
39	.8724	.9240	.9532
40	.8313	.8972	.9355
41	.7833	.8646	.9133
42	.7283	.8259	.8862
43	.6677	.7813	.8538
44	.6025	.7310	.8160
45	.5346	.6759	.7731
46	.4654	.6166	.7251
47	.3975	.5548	.6729
48	.3323	.4916	.6173
49	.2717	.4287	.5593
50	.2167	.3673	.5000
51	.1687	.3092	.4407
52	.1276	.2552	.3827
53	.0938	.2064	.3271
54	.0668	.1632	.2749
55	.0460	.1262	.2269
56	.0305	.0952	.1840
57	.0195	.0700	.1462
58	.0118	.0500	.1138
59	.0068	.0347	.0867
60	.0037	.0232	.0645
61	.0019	.0150	.0468
62	.0009	.0093	.0331
63	.0004	.0056	.0227
64	.0001	.0031	.0151
65	.0000	.0017	.0097
66		.0008	.0060
67		.0004	.0036
68		.0002	.0020
69		.0001	.0011
70		.0000	.0005
71			.0003
72			.0001
73			.0000
74			.0000

m = 10

x	n = 10
30	1.0000
31	1.0000
32	1.0000
33	.9999
34	.9998
35	.9996
36	.9992
37	.9984
38	.9971
39	.9951
40	.9920
41	.9874
42	.9808
43	.9718
44	.9597
45	.9440
46	.9239
47	.8993
48	.8694
49	.8344
50	.7940
51	.7486
52	.6986
53	.6449
54	.5881
55	.5296
56	.4704
57	.4119
58	.3551
59	.3014
60	.2514
61	.2060
62	.1656
63	.1306
64	.1007
65	.0761
66	.0560
67	.0403
68	.0282
69	.0192
70	.0126
71	.0080
72	.0049
73	.0029
74	.0016
75	.0008
76	.0004
77	.0002
78	.0001
79	.0000
80	.0000

Table G: Critical Values of Mood's Test Statistic.

Form: $w(\alpha,p,q) = x$

Example: For $w(0.05,2,15)$...

$M' = 4.00$ and $\alpha' = 0.0368$
$M'' = 5.00$ and $\alpha'' = 0.0662$

Sample sizes p	q	Nominal significance levels α: α 0.005	0.010	0.025	0.050	0.100	1−α 0.900	0.950	0.975	0.990	0.995
2	2						2.50	2.50	2.50	2.50	2.50
							0.8333	0.8333	0.8333	0.8333	0.8333
		0.50	0.50	0.50	0.50	0.50	4.50	4.50	4.50	4.50	4.50
		0.1667	0.1667	0.1667	0.1667	0.1667	1.0000	1.0000	1.0000	1.0000	1.0000
2	3						4.00	5.00	5.00	5.00	5.00
							0.5000	0.9000	0.9000	0.9000	0.9000
		1.00	1.00	1.00	1.00	1.00	5.00	8.00	8.00	8.00	8.00
		0.2000	0.2000	0.2000	0.2000	0.2000	0.9000	1.0000	1.0000	1.0000	1.0000
2	4					0.50	6.50	8.50	8.50	8.50	8.50
						0.0667	0.6667	0.9333	0.9333	0.9333	0.9333
		0.50	0.50	0.50	0.50	2.50	8.50	12.50	12.50	12.50	12.50
		0.0667	0.0667	0.0667	0.0667	0.3333	0.9333	1.0000	1.0000	1.0000	1.0000
2	5					1.00	10.00	10.00	13.00	13.00	13.00
						0.0952	0.7619	0.7619	0.9524	0.9524	0.9524
		1.00	1.00	1.00	1.00	2.00	13.00	13.00	18.00	18.00	18.00
		0.0952	0.0952	0.0952	0.0952	0.1429	0.9524	0.9524	1.0000	1.0000	1.0000
2	6				0.50	0.50	14.50	14.50	18.50	18.50	18.50
					0.0357	0.0357	0.8214	0.8214	0.9643	0.9643	0.9643
		0.50	0.50	0.50	2.50	2.50	18.50	18.50	24.50	24.50	24.50
		0.0357	0.0357	0.0357	0.1786	0.1786	0.9643	0.9643	1.0000	1.0000	1.0000
2	7					2.00	20.00	20.00	25.00	25.00	25.00
						0.0833	0.8611	0.8611	0.9722	0.9722	0.9722
		1.00	1.00	1.00	1.00	4.00	25.00	25.00	32.00	32.00	32.00
		0.0556	0.5566	0.0556	0.0556	0.1389	0.9722	0.9722	1.0000	1.0000	1.0000
2	8			0.50	0.50	0.50	26.50	26.50	26.50	32.50	32.50
				0.0222	0.0222	0.0222	0.8889	0.8889	0.8889	0.9778	0.9778
		0.50	0.50	2.50	2.50	2.50	32.50	32.50	32.50	40.50	40.50
		0.0222	0.0222	0.1111	0.1111	0.1111	0.9778	0.9778	0.9778	1.0000	1.0000
2	9				1.00	4.00	32.00	34.00	34.00	41.00	41.00
					0.0364	0.0909	0.8364	0.9091	0.9091	0.9818	0.9818
		1.00	1.00	1.00	2.00	5.00	34.00	41.00	41.00	50.00	50.00
		0.0364	0.0364	0.0364	0.0545	0.1636	0.9091	0.9818	0.9818	1.0000	1.0000
2	10			0.50	0.50	4.50	40.50	42.50	42.50	50.00	50.50
				0.0152	0.0152	0.0909	0.8636	0.9242	0.9242	0.9848	0.9848
		0.50	0.50	2.50	2.50	6.50	42.50	50.50	50.50	60.50	60.50
		0.0152	0.0152	0.0758	0.0758	0.1515	0.9242	0.9848	0.9848	1.0000	1.0000
2	11				2.00	4.00	50.00	52.00	52.00	61.00	61.00
					0.0385	0.0641	0.8846	0.9359	0.9359	0.9872	0.9872
		1.00	1.00	1.00	4.00	5.00	52.00	61.00	61.00	72.00	72.00
		0.0256	0.0256	0.0256	0.0641	0.1154	0.9359	0.9872	0.9872	1.0000	1.0000
2	12			0.50	0.50	4.50	54.50	62.50	62.50	72.50	72.50
				0.0110	0.0110	0.0659	0.8901	0.9451	0.9451	0.9890	0.9890
		0.50	0.50	2.50	2.50	6.50	60.50	72.50	72.50	84.50	84.50
		0.0110	0.0110	0.0549	0.0549	0.1099	0.9011	0.9890	0.9890	1.0000	1.0000
2	13			1.00	4.00	8.00	61.00	72.00	74.00	74.00	85.00
				0.0190	0.0476	0.0952	0.8667	0.9143	0.9524	0.9524	0.9905
		1.00	1.00	2.00	5.00	9.00	65.00	74.00	85.00	85.00	98.00
		0.0190	0.0190	0.0286	0.0857	0.1143	0.9048	0.9524	0.9905	0.9905	1.0000
2	14		0.50	0.50	4.50	6.50	72.50	84.50	86.50	86.50	98.50
			0.0083	0.0083	0.0500	0.0833	0.8833	0.9250	0.9583	0.9583	0.9917
		0.50	2.50	2.50	6.50	8.50	76.50	86.50	98.50	98.50	112.50
		0.0083	0.0417	0.0417	0.0833	0.1167	0.9167	0.9583	0.9917	0.9917	1.0000
2	15			2.00	4.00	9.00	85.00	98.00	100.00	100.00	113.00
				0.0221	0.0368	0.0882	0.8971	0.9338	0.9632	0.9632	0.9926
		1.00	1.00	4.00	5.00	10.00	89.00	100.00	113.00	113.00	128.00
		0.0147	0.0147	0.0368	0.0662	0.1176	0.9265	0.9632	0.9926	0.9926	1.0000
2	16		0.50	0.50	4.50	8.50	92.50	112.50	114.50	114.50	128.50
			0.0065	0.0065	0.0392	0.0915	0.8824	0.9412	0.9673	0.9673	0.9935
		0.50	2.50	2.50	6.50	12.50	98.50	114.50	128.50	128.50	144.50
		0.0065	0.0327	0.0327	0.0654	0.1242	0.9085	0.9673	0.9935	0.9935	1.0000
2	17			2.00	4.00	10.00	106.00	128.00	130.00	130.00	145.00
				0.0175	0.0292	0.0936	0.8947	0.9474	0.9708	0.9708	0.9942
		1.00	1.00	4.00	5.00	13.00	113.00	130.00	145.00	145.00	162.00
		0.0117	0.0117	0.0292	0.0526	0.1170	0.9181	0.9708	0.9942	0.9942	1.0000
2	18		0.50	0.50	4.50	12.50	114.50	132.50	146.50	146.50	162.50
			0.0053	0.0053	0.0316	0.1000	0.8842	0.9474	0.9737	0.9737	0.9947
		0.50	2.50	2.50	6.50	14.50	120.50	144.50	162.50	162.50	180.50
		0.0053	0.0263	0.0263	0.0526	0.1211	0.9053	0.9526	0.9947	0.9947	1.0000

Table G
continued.

Sample sizes p	q	α 0.005	0.010	0.025	0.050	0.100	1 − α 0.900	0.950	0.975	0.990	0.995
3	3					2.75 0.1000	10.75 0.8000	12.75 0.9000	12.75 0.9000	12.75 0.9000	12.75 0.9000
		2.75 0.1000	2.75 0.1000	2.75 0.1000	2.75 0.1000	4.75 0.2000	12.75 0.9000	14.75 1.0000	14.75 1.0000	14.75 1.0000	14.75 1.0000
3	4				2.00 0.0286	2.00 0.0286	18.00 0.8857	19.00 0.9429	19.00 0.9429	19.00 0.9429	19.00 0.9429
		2.00 0.0286	2.00 0.0286	2.00 0.0286	5.00 0.1429	5.00 0.1429	19.00 0.9429	22.00 1.0000	22.00 1.0000	22.00 1.0000	22.00 1.0000
3	5				2.75 0.0357	4.75 0.0714	20.75 0.8571	24.75 0.9286	26.75 0.9643	26.75 0.9643	26.75 0.9643
		2.75 0.0357	2.75 0.0357	2.75 0.0357	4.75 0.0714	6.75 0.1071	24.75 0.9286	26.75 0.9643	30.75 1.0000	30.75 1.0000	30.75 1.0000
3	6			2.00 0.0119	2.00 0.0119	8.00 0.0952	29.00 0.8929	33.00 0.9286	34.00 0.9524	36.00 0.9762	36.00 0.9762
		2.00 0.0119	2.00 0.0119	5.00 0.0595	5.00 0.0595	9.00 0.1190	32.00 0.9048	34.00 0.9524	36.00 0.9762	41.00 1.0000	41.00 1.0000
3	7			2.75 0.0167	6.75 0.5000	6.75 0.0500	34.75 0.8500	40.75 0.9333	44.75 0.9667	46.75 0.9833	46.75 0.9833
		2.75 0.0167	2.75 0.0167	4.75 0.0333	8.75 0.1167	8.75 0.1167	38.75 0.9167	42.75 0.9500	46.75 0.9833	52.75 1.0000	52.75 1.0000
3	8		2.00 0.0061	2.00 0.0061	8.00 0.0485	11.00 0.0970	45.00 0.8848	50.00 0.9394	54.00 0.9636	59.00 0.9879	59.00 0.9879
		2.00 0.0061	5.00 0.0303	5.00 0.0303	9.00 0.0606	13.00 0.1212	50.00 0.9394	51.00 0.9515	57.00 0.9758	66.00 1.0000	66.00 1.0000
3	9		2.75 0.0091	4.75 0.0182	6.75 0.0273	12.75 0.0909	54.75 0.8727	60.75 0.9182	66.75 0.9727	70.75 0.9818	72.75 0.9909
		2.75 0.0091	4.75 0.0182	6.75 0.0273	8.75 0.0636	14.75 0.1364	56.75 0.9091	62.75 0.9636	70.75 0.9818	72.75 0.9909	80.75 1.0000
3	10	2.00 0.0035	2.00 0.0035	6.00 0.0245	10.00 0.0490	14.00 0.0979	68.00 0.8986	76.00 0.9441	77.00 0.9720	86.00 0.9860	88.00 0.9930
		5.00 0.0175	5.00 0.0175	8.00 0.0280	11.00 0.0559	17.00 0.1189	70.00 0.9266	77.00 0.9720	81.00 0.9790	88.00 0.9930	97.00 1.0000
3	11		2.75 0.0055	6.75 0.0165	10.75 0.0440	16.75 0.0879	74.75 0.8846	84.75 0.9451	90.75 0.9560	102.75 0.9890	104.75 0.9945
		2.75 0.0055	4.75 0.0110	8.75 0.0385	12.75 0.0549	18.75 0.1099	78.75 0.9066	86.75 0.9505	92.75 0.9780	104.75 0.9945	114.75 1.0000
3	12	2.00 0.0022	2.00 0.0022	9.00 0.0220	13.00 0.0440	20.00 0.0945	89.00 0.8879	99.00 0.9385	107.00 0.9648	114.00 0.9868	121.00 0.9912
		5.00 0.0110	5.00 0.0110	10.00 0.0308	14.00 0.0615	21.00 0.1121	90.00 0.9055	101.00 0.9560	110.00 0.9824	121.00 0.9912	123.00 0.9956
3	13	2.75 0.0036	4.75 0.0071	8.75 0.0250	12.75 0.0357	20.75 0.0893	102.75 0.8893	114.75 0.9464	124.75 0.9714	132.75 0.9893	140.75 0.9929
		4.75 0.0071	6.75 0.0107	10.75 0.0286	14.75 0.0536	22.75 0.1036	104.75 0.9071	116.75 0.9500	128.75 0.9857	140.75 0.9929	142.75 0.9964
3	14	2.00 0.0015	5.00 0.0074	11.00 0.0235	17.00 0.0500	25.00 0.0868	116.00 0.8926	128.00 0.9353	138.00 0.9735	149.00 0.9882	162.00 0.9941
		5.00 0.0074	6.00 0.0103	13.00 0.0294	18.00 0.0544	26.00 0.1044	117.00 0.9044	129.00 0.9500	144.00 0.9765	153.00 0.9912	164.00 0.9971
3	15	4.75 0.0049	6.75 0.0074	12.75 0.0245	18.75 0.0490	26.75 0.0907	132.75 0.8995	146.75 0.9485	156.75 0.9681	164.75 0.9804	174.75 0.9926
		6.75 0.0074	8.75 0.0172	14.75 0.0368	20.75 0.0613	28.75 0.1005	134.75 0.9191	148.75 0.9583	158.75 0.9779	170.75 0.9902	184.75 0.9951
3	16	2.80 0.0010	8.00 0.0083	13.00 0.0206	20.00 0.0444	32.00 0.0970	146.00 0.8937	164.00 0.9463	179.00 0.9732	187.00 0.9835	198.00 0.9938
		5.00 0.0052	9.00 0.0103	14.00 0.0289	21.00 0.0526	33.00 0.1011	149.00 0.9102	166.00 0.9567	181.00 0.9814	194.00 0.9917	209.00 0.9959
3	17	4.75 0.0035	6.75 0.0053	12.75 0.0175	20.75 0.0439	34.75 0.1000	162.75 0.8930	180.75 0.9421	192.75 0.9719	210.75 0.9860	222.75 0.9947
		6.75 0.0053	8.75 0.0123	14.75 0.0263	22.75 0.0509	36.75 0.1070	164.75 0.9018	182.75 0.9509	200.75 0.9754	218.75 0.9930	234.75 0.9965

Column heading: Nominal significance levels α

Source: Nico, F. Laubscher, F.E. Steffens and E.M. Delange, "Exact Critical Values for Mood's Distribution-Free Test Statistic for Dispersion and Its Normal Approximation", Technometrics, (1968), 10: 497-507, and Table A.9 of Applied Nonparametric Statistics, W.W. Daniel, Houghton Mifflin Company, Boston, 1978.

Table G
continued.

Sample sizes p	q	α			Nominal significance levels α				1−α		
		0.005	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990	0.995
4	4			5.00	5.00	9.00	29.00	31.00	31.00	33.00	33.00
				0.0143	0.0143	0.0714	0.8714	0.9286	0.9286	0.9857	0.9857
		5.00	5.00	9.00	9.00	11.00	31.00	33.00	33.00	37.00	37.00
		0.0143	0.0143	0.0714	0.0714	0.1286	0.9286	0.9857	0.9857	1.0000	1.0000
4	5			6.00	10.00	11.00	37.00	41.00	42.00	42.00	45.00
				0.0159	0.0397	0.0556	0.8730	0.9286	0.9603	0.9603	0.9921
		6.00	6.00	9.00	11.00	14.00	38.00	42.00	45.00	45.00	50.00
		0.0159	0.0159	0.0317	0.0556	0.1190	0.9048	0.9603	0.9921	0.9921	1.0000
4	6	5.00	5.00	9.00	13.00	15.00	47.00	51.00	53.00	55.00	55.00
		0.0048	0.0048	0.0238	0.0476	0.0857	0.8952	0.9333	0.9571	0.9762	0.9762
		9.00	9.00	11.00	15.00	17.00	49.00	53.00	55.00	59.00	59.00
		0.0238	0.0238	0.0429	0.0857	0.1095	0.9143	0.9571	0.9762	0.9952	0.9952
4	7		6.00	11.00	14.00	20.00	58.00	63.00	68.00	70.00	70.00
			0.0061	0.0212	0.0455	0.0909	0.8848	0.9394	0.9727	0.9848	0.9848
		6.00	9.00	14.00	15.00	21.00	59.00	66.00	70.00	75.00	75.00
		0.0061	0.0121	0.0455	0.0576	0.1152	0.9030	0.9576	0.9848	0.9970	0.9970
4	8	5.00	5.00	13.00	17.00	21.00	69.00	77.00	81.00	87.00	87.00
		0.0020	0.0020	0.0202	0.0465	0.0869	0.8970	0.9475	0.9636	0.9899	0.9899
		9.00	9.00	15.00	19.00	23.00	71.00	79.00	83.00	93.00	93.00
		0.0101	0.0101	0.0364	0.0545	0.1030	0.9051	0.9556	0.9798	0.9980	0.9980
4	9	6.00	11.00	14.00	20.00	27.00	85.00	92.00	98.00	104.00	106.00
		0.0028	0.0098	0.0210	0.0420	0.0965	0.8979	0.9497	0.9748	0.9874	0.9930
		9.00	14.00	15.00	21.00	29.00	86.00	93.00	101.00	106.00	113.00
		0.0056	0.0210	0.0266	0.0531	0.1077	0.9231	0.9552	0.9804	0.9930	0.9986
4	10	9.00	13.00	17.00	21.00	31.00	97.00	105.00	115.00	121.00	125.00
		0.0050	0.0100	0.0230	0.0430	0.0969	0.8961	0.9491	0.9740	0.9860	0.9910
		11.00	15.00	19.00	23.00	33.00	99.00	107.00	117.00	123.00	127.00
		0.0090	0.0180	0.0270	0.0509	0.1129	0.9161	0.9530	0.9820	0.9900	0.9950
4	11	10.00	11.00	20.00	26.00	35.00	113.00	125.00	134.00	143.00	148.00
		0.0037	0.0051	0.0220	0.0462	0.0967	0.8967	0.9495	0.9722	0.9897	0.9934
		11.00	14.00	21.00	27.00	36.00	114.00	126.00	135.00	146.00	150.00
		0.0051	0.0110	0.0278	0.0505	0.1011	0.9099	0.9612	0.9780	0.9927	0.9963
4	12	11.00	15.00	21.00	29.00	39.00	129.00	141.00	153.00	161.00	171.00
		0.0049	0.0099	0.0236	0.0489	0.0978	0.8962	0.9495	0.9747	0.9879	0.9945
		13.00	17.00	23.00	31.00	41.00	131.00	143.00	155.00	163.00	173.00
		0.0055	0.0126	0.0280	0.0533	0.1093	0.9159	0.9538	0.9791	0.9901	0.9951
4	13	11.00	17.00	25.00	33.00	45.00	146.00	162.00	173.00	186.00	193.00
		0.0029	0.0088	0.0227	0.0475	0.0971	0.8933	0.9496	0.9710	0.9891	0.9941
		14.00	18.00	26.00	34.00	46.00	147.00	163.00	174.00	187.00	198.00
		0.0063	0.0113	0.0265	0.0504	0.1071	0.9000	0.9529	0.9777	0.9908	0.9958
4	14	13.00	19.00	27.00	37.00	49.00	163.00	181.00	195.00	207.00	217.00
		0.0033	0.0088	0.0235	0.0477	0.0928	0.8931	0.9487	0.9739	0.9889	0.9941
		15.00	21.00	29.00	39.00	51.00	165.00	183.00	197.00	213.00	221.00
		0.0059	0.0141	0.0291	0.0582	0.1059	0.9049	0.9539	0.9755	0.9915	0.9954
4	15	15.00	21.00	29.00	41.00	56.00	183.00	202.00	218.00	234.00	245.00
		0.0049	0.0098	0.0199	0.0472	0.0993	0.8965	0.9466	0.9727	0.9892	0.9943
		17.00	22.00	30.00	42.00	57.00	185.00	203.00	219.00	235.00	247.00
		0.0054	0.0114	0.0261	0.0524	0.1045	0.9017	0.9518	0.9768	0.9902	0.9954
4	16	17.00	21.00	33.00	43.00	61.00	203.00	223.00	241.00	259.00	275.00
		0.0047	0.0089	0.0233	0.0436	0.0962	0.8933	0.9451	0.9728	0.9870	0.9946
		19.00	23.00	35.00	45.00	63.00	205.00	225.00	243.00	261.00	277.00
		0.0056	0.0105	0.0283	0.0504	0.1061	0.9028	0.9525	0.9752	0.9903	0.9955

Table G
continued.

Sample sizes		α			Nominal significance levels α			$1-\alpha$			
p	q	0.005	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990	0.995
5	5		11.25	15.25	17.25	23.25	55.25	59.25	61.25	65.25	67.25
			0.0079	0.0159	0.0317	0.0952	0.8889	0.9365	0.9683	0.9841	0.9921
		11.25	15.25	17.25	21.25	25.25	57.25	61.25	65.25	67.25	71.25
		0.0079	0.0159	0.0317	0.0635	0.1111	0.9048	0.9683	0.9841	0.9921	1.0000
5	6	10.00	10.00	19.00	24.00	27.00	69.00	75.00	76.00	83.00	84.00
		0.0022	0.0022	0.0238	0.0476	0.0758	0.8810	0.9459	0.9632	0.9870	0.9913
		15.00	15.00	20.00	25.00	30.00	70.00	76.00	79.00	84.00	86.00
		0.0108	0.0108	0.0260	0.0563	0.1104	0.9069	0.9632	0.9805	0.9913	0.9957
5	7	11.25	15.25	21.25	27.25	33.25	83.25	89.25	93.25	101.25	105.25
		0.0025	0.0051	0.0202	0.0480	0.0884	0.8990	0.9495	0.9646	0.9899	0.9949
		15.25	17.25	23.25	29.25	35.25	85.25	91.25	95.25	103.25	107.25
		0.0051	0.0101	0.0303	0.0631	0.1136	0.9167	0.9520	0.9773	0.9924	0.9975
5	8	15.00	20.00	26.00	31.00	39.00	99.00	106.00	113.00	118.00	123.00
		0.0039	0.0093	0.0225	0.0490	0.0979	0.8974	0.9448	0.9697	0.9852	0.9938
		18.00	22.00	27.00	33.00	40.00	101.00	107.00	114.00	122.00	126.00
		0.0070	0.0124	0.0272	0.0521	0.1049	0.9068	0.9510	0.9759	0.9922	0.9953
5	9	17.25	21.25	29.25	35.25	45.25	115.25	123.25	133.25	141.25	145.25
		0.0040	0.0080	0.0250	0.0450	0.0999	0.8951	0.9411	0.9710	0.9890	0.9910
		21.25	23.25	31.25	37.25	47.25	117.25	125.25	135.25	143.25	147.25
		0.0080	0.0120	0.0300	0.0509	0.1149	0.9121	0.9500	0.9790	0.9900	0.9960
5	10	20.00	26.00	33.00	41.00	52.00	134.00	146.00	154.00	166.00	174.00
		0.0040	0.0097	0.0223	0.0456	0.0989	0.8934	0.9494	0.9724	0.9897	0.9947
		22.00	27.00	34.00	42.00	53.00	135.00	147.00	155.00	168.00	175.00
		0.0053	0.0117	0.0266	0.0503	0.1002	0.9068	0.9547	0.9757	0.9923	0.9973
5	11	21.25	27.25	37.25	45.25	57.25	153.25	165.25	177.25	187.25	197.25
		0.0037	0.0087	0.0234	0.0458	0.0934	0.8997	0.9473	0.9748	0.9881	0.9950
		23.25	29.25	39.25	47.25	59.25	155.25	167.25	179.25	191.25	199.25
		0.0055	0.0114	0.0275	0.0527	0.1053	0.9125	0.9519	0.9776	0.9918	0.9954
5	12	26.00	30.00	42.00	53.00	65.00	174.00	189.00	202.00	216.00	226.00
		0.0047	0.0082	0.0244	0.0486	0.0931	0.8993	0.9473	0.9746	0.9888	0.9945
		27.00	31.00	43.00	54.00	66.00	175.00	190.00	203.00	217.00	227.00
		0.0057	0.0102	0.0267	0.0535	0.1021	0.9071	0.9551	0.9772	0.9901	0.9952
5	13	27.25	33.25	45.25	57.25	73.25	195.25	211.25	227.25	243.25	255.25
		0.0044	0.0082	0.0233	0.0476	0.0997	0.8985	0.9444	0.9741	0.9893	0.9946
		29.25	35.25	47.25	59.25	75.25	197.25	213.25	229.25	245.25	257.25
		0.0058	0.0105	0.0268	0.0537	0.1076	0.9059	0.9512	0.9762	0.9904	0.9958
5	14	30.00	38.00	51.00	65.00	81.00	219.00	238.00	254.00	275.00	285.00
		0.0044	0.0088	0.0248	0.0495	0.0978	0.8999	0.9479	0.9720	0.9896	0.9946
		31.00	39.00	52.00	66.00	82.00	220.00	239.00	255.00	276.00	287.00
		0.0054	0.0108	0.0255	0.0544	0.1034	0.9037	0.9520	0.9754	0.9906	0.9953
5	15	33.25	39.25	55.25	69.25	89.25	241.25	265.25	283.25	305.25	319.25
		0.0045	0.0077	0.0235	0.0470	0.0988	0.8951	0.9494	0.9739	0.9896	0.9946
		35.25	41.25	57.25	71.25	91.25	243.25	267.25	285.25	307.25	321.25
		0.0058	0.0103	0.0263	0.0526	0.1053	0.9005	0.9542	0.9763	0.9906	0.9957
6	6	17.50	27.50	33.50	39.50	45.50	93.50	99.50	105.50	111.50	115.50
		0.0011	0.0097	0.0238	0.0465	0.0963	0.8734	0.9307	0.9675	0.9848	0.9946
		23.50	29.50	35.50	41.50	47.50	95.50	101.50	107.50	113.50	119.50
		0.0054	0.0152	0.0325	0.0693	0.1266	0.9037	0.9535	0.9762	0.9903	0.9989
6	7	27.00	31.00	38.00	45.00	54.00	114.00	122.00	129.00	135.00	140.00
		0.0047	0.0099	0.0204	0.0466	0.0973	0.8980	0.9476	0.9749	0.9883	0.9948
		28.00	34.00	39.00	46.00	55.00	115.00	123.00	130.00	138.00	142.00
		0.0052	0.0146	0.0251	0.0524	0.1206	0.9108	0.9580	0.9779	0.9918	0.9971
6	8	29.50	35.50	41.50	49.50	59.50	131.50	141.50	149.50	157.50	165.50
		0.0047	0.0100	0.0213	0.0430	0.0942	0.8924	0.9461	0.9737	0.9873	0.9940
		31.50	37.50	43.50	51.50	61.50	133.50	143.50	151.50	159.50	167.50
		0.0060	0.0130	0.0266	0.0509	0.1062	0.9004	0.9540	0.9750	0.9900	0.9967
6	9	34.00	39.00	49.00	58.00	69.00	154.00	165.00	175.00	186.00	193.00
		0.0050	0.0086	0.0232	0.0488	0.0969	0.8973	0.9467	0.9734	0.9894	0.9944
		35.00	40.00	50.00	59.00	70.00	155.00	166.00	176.00	187.00	195.00
		0.0062	0.0110	0.0256	0.0547	0.1039	0.9065	0.9504	0.9766	0.9910	0.9956
6	10	37.50	43.50	53.50	63.50	75.50	175.50	189.50	201.50	213.50	221.50
		0.0049	0.0100	0.0237	0.0448	0.0888	0.8976	0.9476	0.9734	0.9891	0.9948
		39.50	45.50	55.50	65.50	77.50	177.50	191.50	203.50	215.50	223.50
		0.0054	0.0111	0.0262	0.0521	0.1010	0.9063	0.9540	0.9784	0.9901	0.9953
6	11	42.00	49.00	61.00	73.00	87.00	200.00	216.00	229.00	244.00	253.00
		0.0048	0.0094	0.0243	0.0490	0.0977	0.8998	0.9491	0.9737	0.9898	0.9941
		43.00	50.00	62.00	74.00	88.00	201.00	217.00	230.00	245.00	254.00
		0.0060	0.0103	0.0255	0.0512	0.1037	0.9009	0.9504	0.9758	0.9901	0.9954

Table G
continued.

Sample sizes		α		Nominal significance levels α				$1-\alpha$			
p	q	0.005	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990	0.995
6	12	45.50	51.50	67.50	79.50	95.50	223.50	243.50	257.50	273.50	285.50
		0.0048	0.0082	0.0248	0.0470	0.0950	0.8954	0.9494	0.9733	0.9879	0.9944
		47.50	53.50	69.50	81.50	97.50	225.50	245.50	259.50	275.50	287.50
		0.0063	0.0102	0.0273	0.0513	0.1033	0.9004	0.9542	0.9757	0.9900	0.9950
6	13	50.00	58.00	74.00	89.00	107.00	252.00	273.00	290.00	310.00	323.00
		0.0047	0.0090	0.0234	0.0483	0.0985	0.8979	0.9499	0.9736	0.9898	0.9949
		51.00	59.00	75.00	90.00	108.00	253.00	274.00	291.00	311.00	324.00
		0.0053	0.0101	0.0256	0.0503	0.1008	0.9001	0.9510	0.9751	0.9902	0.9951
6	14	53.50	63.50	81.50	97.50	117.50	279.50	301.50	321.50	343.50	357.50
		0.0049	0.0093	0.0246	0.0495	0.0974	0.8972	0.9459	0.9730	0.9888	0.9944
		55.50	65.50	83.50	99.50	119.50	281.50	303.50	323.50	345.50	359.50
		0.0054	0.0108	0.0281	0.0527	0.1043	0.9040	0.9501	0.9754	0.9901	0.9950
7	7	41.75	47.75	57.75	65.75	75.75	147.75	157.75	165.75	175.75	179.75
		0.0029	0.0082	0.0233	0.0466	0.0950	0.8869	0.9452	0.9709	0.9889	0.9948
		43.75	49.75	59.75	67.75	77.75	149.75	159.75	167.75	177.75	183.75
		0.0052	0.0111	0.0291	0.0548	0.1131	0.9050	0.9534	0.9767	0.9918	0.9971
7	8	50.00	55.00	66.00	75.00	87.00	173.00	184.00	195.00	204.00	211.00
		0.0050	0.0082	0.0238	0.0479	0.0977	0.8988	0.9455	0.9745	0.9890	0.9939
		51.00	56.00	67.00	76.00	88.00	174.00	185.00	196.00	205.00	212.00
		0.0059	0.0110	0.0272	0.0533	0.1052	0.9004	0.9510	0.9776	0.9902	0.9952
7	9	53.75	59.75	71.75	83.75	95.75	197.75	211.75	221.75	235.75	245.75
		0.0049	0.0087	0.0224	0.0495	0.0920	0.8970	0.9495	0.9706	0.9895	0.9949
		55.75	61.75	73.75	85.75	97.75	199.75	213.75	223.75	237.75	247.75
		0.0058	0.0103	0.0267	0.0556	0.1016	0.9073	0.9549	0.9764	0.9911	0.9963
7	10	59.00	67.00	82.00	94.00	109.00	226.00	242.00	254.00	270.00	279.00
		0.0046	0.0090	0.0243	0.0478	0.0975	0.8978	0.9499	0.9726	0.9896	0.9949
		60.00	68.00	83.00	95.00	110.00	227.00	243.00	255.00	271.00	280.00
		0.0053	0.0100	0.0268	0.0521	0.1009	0.9051	0.9544	0.9753	0.9902	0.9951
7	11	63.75	73.75	89.75	103.75	119.75	253.75	271.75	287.75	303.75	315.75
		0.0042	0.0096	0.0246	0.0495	0.0946	0.8991	0.9483	0.9742	0.9882	0.9943
		65.75	75.75	91.75	105.75	121.75	255.75	273.75	289.75	305.75	317.75
		0.0050	0.0103	0.0272	0.0526	0.1012	0.9053	0.9506	0.9767	0.9904	0.9952
7	12	71.00	82.00	99.00	115.00	135.00	285.00	306.00	323.00	343.00	357.00
		0.0048	0.0094	0.0241	0.0489	0.0996	0.8997	0.9491	0.9738	0.9893	0.9950
		72.00	83.00	100.00	116.00	136.00	286.00	307.00	324.00	344.00	358.00
		0.0051	0.0104	0.0258	0.0519	0.1044	0.9020	0.9515	0.9754	0.9900	0.9952
7	13	75.75	87.75	107.75	125.75	147.75	315.75	339.75	359.75	381.75	397.75
		0.0042	0.0089	0.0239	0.0487	0.0983	0.8972	0.9487	0.9745	0.9889	0.9949
		77.75	89.75	109.75	127.75	149.75	317.75	341.75	361.75	383.75	399.75
		0.0050	0.0101	0.0261	0.0528	0.1054	0.9039	0.9523	0.9758	0.9905	0.9953
8	8	72.00	78.00	92.00	104.00	118.00	218.00	232.00	244.00	258.00	264.00
		0.0043	0.0078	0.0239	0.0496	0.0984	0.8908	0.9457	0.9740	0.9900	0.9942
		74.00	80.00	94.00	106.00	120.00	220.00	234.00	246.00	260.00	266.00
		0.0058	0.0100	0.0260	0.0543	0.1092	0.9016	0.9504	0.9761	0.9922	0.9957
8	9	79.00	90.00	103.00	116.00	132.00	250.00	266.00	279.00	294.00	303.00
		0.0042	0.0096	0.0229	0.0487	0.0988	0.8959	0.9477	0.9742	0.9896	0.9945
		80.00	91.00	104.00	117.00	133.00	251.00	267.00	280.00	295.00	304.00
		0.0050	0.0102	0.0253	0.0510	0.1016	0.9005	0.9520	0.9760	0.9901	0.9952
8	10	88.00	98.00	114.00	128.00	146.00	280.00	300.00	316.00	332.00	344.00
		0.0050	0.100	0.0245	0.0481	0.0980	0.8917	0.9487	0.9744	0.9891	0.9948
		90.00	100.00	116.00	130.00	148.00	282.00	302.00	318.00	334.00	346.00
		0.0059	0.0112	0.0280	0.0525	0.1033	0.9001	0.9532	0.9768	0.9900	0.9950
8	11	95.00	107.00	126.00	143.00	163.00	316.00	337.00	355.00	376.00	388.00
		0.0047	0.0095	0.0247	0.500	0.0988	0.8984	0.9489	0.0739	0.9900	0.9948
		96.00	108.00	127.00	144.00	164.00	317.00	338.00	356.00	377.00	389.00
		0.0051	0.0105	0.0256	0.0530	0.1039	0.9021	0.9501	0.9759	0.9909	0.9953
8	12	102.00	116.00	136.00	156.00	178.00	352.00	376.00	396.00	418.00	434.00
		0.0044	0.0097	0.0234	0.0496	0.0970	0.8995	0.9497	0.9749	0.9894	0.9949
		104.00	118.00	138.00	158.00	180.00	354.00	378.00	398.00	420.00	436.00
		0.0051	0.0103	0.0252	0.0533	0.1031	0.9056	0.9531	0.9763	0.9903	0.9953
9	9	110.25	120.25	138.25	154.25	172.25	308.25	326.25	342.25	360.25	370.25
		0.0045	0.0085	0.0230	0.0481	0.0973	0.8975	0.9476	0.9742	0.9899	0.9949
		112.25	122.25	140.25	156.25	174.25	310.25	328.25	344.25	362.25	372.25
		0.0051	0.0101	0.0258	0.0524	0.1025	0.9027	0.9519	0.9770	0.9915	0.9955
9	10	122.00	134.00	154.00	171.00	191.00	347.00	368.00	385.00	404.00	419.00
		0.0049	0.0096	0.0250	0.0492	0.0963	0.8987	0.9489	0.9738	0.9890	0.9950
		123.00	135.00	155.00	172.00	192.00	348.00	369.00	386.00	405.00	420.00
		0.0050	0.0101	0.0256	0.0514	0.1003	0.9021	0.9515	0.9751	0.9900	0.9955
9	11	132.25	144.25	166.25	186.25	210.25	384.25	408.25	430.25	452.25	468.25
		0.0049	0.0089	0.0235	0.0484	0.0984	0.8942	0.9465	0.9744	0.9896	0.9950
		134.25	146.25	168.25	188.25	212.25	386.25	410.25	432.25	454.25	470.25
		0.0056	0.0102	0.0251	0.0519	0.1049	0.9005	0.9500	0.9765	0.9900	0.9955
10	10	162.50	176.50	198.50	218.50	242.50	418.50	442.50	462.50	484.50	498.50
		0.0050	0.0098	0.0241	0.0489	0.0982	0.8966	0.9479	0.9740	0.9891	0.9944
		164.50	178.50	200.50	220.50	244.50	420.50	444.50	464.50	486.50	500.50
		0.0056	0.0109	0.0260	0.0521	0.1034	0.9018	0.9511	0.9759	0.9902	0.9950

Table H: Critical Values of the Chi-Square Distribution.

Form: $\chi^2_{(\alpha,df)}$

Example: $\chi^2_{(0.01,14)} = 29.1413$

df \ α	0.050	0.025	0.010	0.005
1	3.84146	5.02389	6.63490	7.87944
2	5.99147	7.37776	9.21034	10.5966
3	7.81473	9.34840	11.3449	12.8381
4	9.48773	11.1433	13.2767	14.8602
5	11.0705	12.8325	15.0863	16.7496
6	12.5916	14.4494	16.8119	18.5476
7	14.0671	16.0128	18.4753	20.2777
8	15.5073	17.5346	20.0902	21.9550
9	16.9190	19.0228	21.6660	23.5893
10	18.3070	20.4831	23.2093	25.1882
11	19.6751	21.9200	24.7250	26.7569
12	21.0261	23.3367	26.2170	28.2995
13	22.3621	24.7356	27.6883	29.8194
14	23.6848	26.1190	29.1413	31.3193
15	24.9958	27.4884	30.5779	32.8013
16	26.2962	28.8454	31.9999	34.2672
17	27.5871	30.1910	33.4087	35.7185
18	28.8693	31.5264	34.8053	37.1564
19	30.1435	32.8523	36.1908	38.5822
20	31.4104	34.1696	37.5662	39.9968
21	32.6705	35.4789	38.9321	41.4010
22	33.9244	36.7807	40.2894	42.7956
23	35.1725	38.0757	41.6384	44.1813
24	36.4151	39.3641	42.9798	45.5585
25	37.6525	40.6465	44.3141	46.9278
26	38.8852	41.9232	45.6417	48.2899
27	40.1133	43.1944	46.9630	49.6449
28	41.3372	44.4607	48.2782	50.9933
29	42.5569	45.7222	49.5879	52.3356
30	43.7729	46.9792	50.8922	53.6720
40	55.7585	59.3417	63.6907	66.7659
50	67.5048	71.4202	76.1539	79.4900
60	79.0819	83.2976	88.3794	91.9517
70	90.5312	95.0231	100.425	104.215
80	101.879	106.629	112.329	116.321
90	113.145	118.136	124.116	128.299
100	124.342	129.561	135.807	140.169

Source: Table VIII of Statistics, D.J. Koosis, John Wiley & Sons, Inc., NY, 1972.

Table I: Critical Values for Kendall's Tau Test Statistic.

Form: $k(\alpha, n)$

Example: $k(0.025,10) = 23$

n \ α	0.005 k	0.005 τ^*	0.010 k	0.010 τ^*	0.025 k	0.025 τ^*	0.050 k	0.050 τ^*	0.100 k	0.100 τ^*
4	8	1.000	8	1.000	8	1.000	6	1.000	6	1.000
5	12	1.000	10	1.000	10	1.000	8	.800	8	.800
6	15	1.000	13	.867	13	.867	11	.733	9	.600
7	19	.905	17	.810	15	.714	13	.619	11	.524
8	22	.786	20	.714	18	.643	16	.571	12	.429
9	26	.722	24	.667	20	.556	18	.500	14	.389
10	29	.644	27	.600	23	.511	21	.467	17	.378
11	33	.600	31	.564	27	.491	23	.418	19	.345
12	38	.576	36	.545	30	.455	26	.394	20	.303
13	44	.564	40	.513	34	.436	28	.359	24	.308
14	47	.516	43	.473	37	.407	33	.363	25	.275
15	53	.505	49	.467	41	.390	35	.333	29	.276
16	58	.483	52	.433	46	.383	38	.317	30	.250
17	64	.471	58	.426	50	.368	42	.309	34	.250
18	69	.451	63	.412	53	.346	45	.294	37	.242
19	75	.439	67	.392	57	.333	49	.287	39	.228
20	80	.421	72	.379	62	.326	52	.274	42	.221
21	86	.410	78	.371	66	.314	56	.267	44	.210
22	91	.394	83	.359	71	.307	61	.264	47	.203
23	99	.391	89	.352	75	.296	65	.257	51	.202
24	104	.377	94	.341	80	.290	68	.246	54	.196
25	110	.367	100	.333	86	.287	72	.240	58	.193
26	117	.360	107	.329	91	.280	77	.237	61	.188
27	125	.356	113	.322	95	.271	81	.231	63	.179
28	130	.344	118	.312	100	.265	86	.228	68	.180
29	138	.340	126	.310	106	.261	90	.222	70	.172
30	145	.333	131	.301	111	.255	95	.218	75	.172
31	151	.325	137	.295	117	.252	99	.213	77	.166
32	160	.323	144	.290	122	.246	104	.210	82	.165
33	166	.314	152	.288	128	.242	108	.205	86	.163
34	175	.312	157	.280	133	.237	113	.201	89	.159
35	181	.304	165	.277	139	.234	117	.197	93	.156
36	190	.302	172	.273	146	.232	122	.194	96	.152
37	198	.297	178	.267	152	.228	128	.192	100	.150
38	205	.292	185	.263	157	.223	133	.189	105	.149
39	213	.287	193	.260	163	.220	139	.188	109	.147
40	222	.285	200	.256	170	.218	144	.185	112	.144

Source: L. Kaarsemaker and A. van Wijngaarden, "Table for Use in Rank Correlation", Statistica Neerlandica, (1953), 7: 41-54, and Table A.20 of Applied Nonparametric Statistics, W. W. Daniel, Houghton Mifflin Company, Boston, 1978.

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