## HEWLETT-PACKARD

HP.41C

## REAL ESTATE PAC

(1)

## NOTICE

Hewlett-Packard Company makes no express or implied warranty with regard to the keystroke procedures and program material offered or their merchantability or their fitness for any particular purpose. The keystroke procedures and program material are made available solely on an "as is" basis, and the entire risk as to their quality and performance is with the user. Should the keystroke procedures or program material prove defective, the user (and not Hewlett-Packard Company nor any other party) shall bear the entire cost of all necessary correction and all incidental or consequential damages. HewlettPackard Company shall not be liable for any incidental or consequential damages in connection with or arising out of the furnishing, use, or performance of the keystroke procedures or program material.

## HEWLETT-PACKARD LISTENS

To provide better calculator support for you, the Application Engineering group needs your help. Your timely inputs enable us to provide higher quality software and improve the existing application pacs for your calculator. Your reply will be extremely helpful in this effort.

1. Pac name $\qquad$
2. How important was the availability of this pac in making your decision to buy a HewlettPackard calculator?
$\square$ Would not buy without it.
$\square$ Important
$\square$ Not important
3. What is the major application area for which you purchased the pac?
4. In the list below, please rate the usefulness of the programs in this pac.

|  |  |  |  |  |  |  |  |  | $\begin{aligned} & \stackrel{\rightharpoonup}{w} \\ & \stackrel{y}{\sim} \\ & \stackrel{\sim}{w} \\ & \stackrel{\sim}{\sim} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  | 9 |  |  |  |  |
| 2 |  |  |  |  | 10 |  |  |  |  |
| 3 |  |  |  |  | 11 |  |  |  |  |
| 4 |  |  |  |  | 12 |  |  |  |  |
| 5 |  |  |  |  | 13 |  |  |  |  |
| 6 |  |  |  |  | 14 |  |  |  |  |
| 7 |  |  |  |  | 15 |  |  |  |  |
| 8 |  |  |  |  | 16 |  |  |  |  |
| 5. Did you purchase a printer? $\square$ YES $\square$ NO <br> If you did, is the printing format in this pac useful? |  |  |  |  |  |  | ES | NO |  |

6. What programs would you add to this pac?
7. What additional application pacs would you like to see developed?

THANK YOU FOR YOUR TIME AND COOPERATION.

| Name | Position |
| :--- | :--- |
| Company |  |
| Address | State |
| City | Phone |


:słuəumoう ןeuo!!!pp*


NO POSTAGE NECESSARY IF MAILED IN THE
UNITED STATES

## BUSINESS REPLY MAIL

FIRST CLASS PERMIT NO. 33 CORVALLIS, OREGON

Postage will be paid by
Hewlett-Packard 1000 N.E. Circle Blvd.
Corvallis, Oregon 97330

## INTRODUCTION

Each program in this Pac is represented by one program in the Application Module and a section in this manual. The manual provides a description of the program with relevant equations, a set of instructions for using the program, and one or more example problems, each of which includes a list of the keystrokes required for its solution.

Before plugging in your Application Module, turn your calculator off, and be sure you understand the section Inserting and Removing Application Modules. And before using a particular program, take a few minutes to read Format of User Instructions and A Word About Program Usage.

You should first familiarize yourself with a program by running it once or twice while following the complete User Instructions in this manual. Thereafter, the program's prompting or the mnemonics on the overlay should provide the necessary instructions, including which variables are to be input, which keys are to be pressed, and which values will be output. A quick-reference card with a brief description of each program's operating instructions has been provided for your convenience.
We hope that the Real Estate Pac will assist you in the solution of numerous problems in your discipline. We would appreciate knowing your reactions to the programs in this Pac, and to this end we have provided a questionnaire inside the front cover of this manual. Would you please take a few minutes to give us your comments on these programs? It is from your comments that we learn how to increase the usefulness of our programs.

Acknowledgements:
Hewlett-Packard wishes to thank Gary M. Tenzer for his contributions to the definition of this Pac.

## TABLE OF CONTENTS

Introduction ..... 1
Contents ..... 2
Inserting and Removing Application Modules ..... 4
Format of User Instructions ..... 6
A Word About Program Usage ..... 7
Compound Interest Solutions:
The Cash Flow Diagram and Sign Convention ..... 10
Compound Interest and Loan Amortization ..... 14Solves for any of the five standard compound interest variablesN, I, PV, PMT, FV with either 3 or 4 knowns. Options includeBEGIN (annuity due) or END (ordinary annuity) payments, LISTvariables, CLFIN (clear financial registers), and amortizationschedules (the interest and principal portion of each periodicpayment and the remaining balance after the payment has beenmade). Amortization schedules may be done in specifiedincrements, i.e., yearly or monthly.
Internal Rate of Return ..... 23
Calculates the Internal Rate of Return of a series of up to 38 unequal cash flows or up to 19 groups of equal cash flows. Each additional Memory Module can accommodate an additional 64 individual cash flows or $\mathbf{3 2}$ groups of cash flows.
Modified Internal Rate of Return ..... 27
Solves for the Internal Rate of Return, using a modified IRR technique, of a series of up to 38 unequal cash flows or up to 19 groups of equal cash flows. Expandable with additional Memory Modules.
Net Present Value ..... 30
Discounts a series of up to 38 unequal cash flows or up to 19 groups of equal cash flows to a single net value. Expandable with additional Memory Modules.
Depreciation Schedules ..... 33
Produces an annual schedule for straight line, declining balance, and sum-of-the-years'-digits depreciation with consid- eration for partial first year. For declining balance depreciation, automatic crossover to straight line depreciation is optional.
Income Property Analysis ..... 37
Analyzes the investment potential (on both a pre- and post-tax basis) of income properties. An annualized summary of the property's performance is given, as well as a complete evaluation of the sales proceeds. With this, the project's overall equity yield is determined. A printer is most beneficial when using this program.
Graduated Payment Mortgage ..... 46
Computes the schedule of payments necessary to repay a graduated payment mortgage. The remaining balance at the end of each year is also calculated.
Wrap-Around Mortgage ..... 48
Calculates the periodic yield to the lender of a wrap-around mortgage. Then, if this yield is not acceptable, a desired yield can be input and the necessary payment and quoted interest on the wrap-around is calculated.
Home Owner's Equity Analysis ..... 52
Provides the home owner with approximate monthly payments, accumulated equity, anticipated market value, and anticipated tax deductions on a year-by-year basis.
The Rent or Buy Decision ..... 55
An analysis of the financial aspects of a rent-versus-buy decision on a home is provided.
Price and Yield of a Mortgage Traded at a Discount/Premium ..... 59
Annual Percentage Rate of a Loan With Fees ..... 62
Present Value of an Increasing/Decreasing Annuity ..... 64
Financial Formulas ..... 67
Program Data ..... 73
Subroutines ..... 75

## INSERTING AND REMOVING APPLICATION MODULES

Before you insert an Application Module for the first time, familiarize yourself with the following information.

Up to four Application Modules can be plugged into the ports on the HP-41C. While plugged in, the names of all programs contained in the Module can be displayed by pressing CATALOG 2.

## CAUTION

Always turn the HP-41C off before inserting or removing any plug-in extension or accessories. Failure to turn the HP-41C off could damage both the calculator and the accessory.

To insert Application Modules:

1. Turn the HP-41C off! Failure to turn the calculator off could damage both the Module and the calculator.
2. Remove the port covers. Remember to save the port covers; they should be inserted into the empty ports when no extensions are inserted.
3. Insert the Application Module with the label facing downward as shown, into any port after the last Memory Module. For example, if you have a Memory Module inserted in port 1, you can insert an Application Module in any of ports 2,3 , or 4 . (The port numbers are shown on the back of the calculator.) Never insert an
 Application Module into a lower numbered port than a Memory Module.
4. If you have additional Application Modules to insert, plug them into any port after the last Memory Module. Be sure to place port covers over unused ports.
5. Turn the calculator on and follow the instructions given in this book for the desired application functions.

To remove Application Modules:

1. Turn the HP-41C off! Failure to do so could damage both the calculator and the Module.
2. Grasp the desired Module handle and pull it out as shown.
3. Place a port cap into the empty ports.

## Mixing Memory Modules and Application Modules

Any optional accessories (such as the HP 82104A Card Reader, or the HP 82143A Printer) should be treated in the same manner as Application Modules. That is, they can be plugged into any port after the last Memory Module. Also, the HP-41C should be turned off prior to insertion or removal of these extensions.

The HP-41C allows you to leave gaps in the port sequence when mixing Memory and Application Modules. For example, you can plug a Memory Module into port 1 and an Application Module into port 4, leaving ports 2 and 3 empty.

## FORMAT OF USER INSTRUCTIONS

The User Instruction form is composed of five labeled columns. Reading from left to right, the first column, labeled STEP, gives the instruction step number.
The INSTRUCTIONS column gives instructions and comments concerning the operations to be performed.
The INPUT column specifies the input data, the units of data if applicable, or the appropriate alpha response to a prompted question. Data input keys consist of 0 to 9 and the decimal point (the numeric keys), EEX (enter exponent), and CHS (change sign).

The FUNCTION column specifies the keys to be pressed after keying in the corresponding input data.
The DISPLAY column specifies prompts, intermediate and final answers, and their units, where applicable.

Above the DISPLAY column is a box which specifies the minimum number of data storage registers necessary to execute the program. Refer to the Owner's Handbook for information on how the SIZE function affects storage configuration.
Several of the programs in this Pac use entirely prompted operation, eliminating this form in their explanation.

## A WORD ABOUT PROGRAM USAGE

## Catalog

When an Application Module is plugged into a port of the HP-41C, the contents of the Module can be reviewed by pressing CATALOG 2 (the Extension Catalog). Executing the CATALOG function lists the name of each program or function in the Module, as well as functions of any other extensions which might be plugged in.

## Overlays

An overlay has been included for one of the programs in this Pac. The mnemonics on the overlay are provided to help you run the program. The program's name is given vertically on the left side. When the calculator is in USER mode, a blue mnemonic identifies the key directly above it. Gold mnemonics are similar to blue mnemonics, except that they are above the appropriate key and the shift (gold) key must be pressed before the re-defined key. Once again, USER mode must be set.

## ALPHA and USER Mode Notation

This manual uses a special notation to signify ALPHA mode. Whenever a statement on the User Instruction Form is printed in gold, the ALPHA key must be pressed before the statement can be keyed in. After the statement is input, press ALPHA again to return the calculator to its normal operating mode, or to begin program execution. For example, XEO \$ means press the following keys: XEO ALPHA P ALPHA.
When the calculator is in USER mode, this manual will use the symbols A-J and A- E to refer to the reassigned keys in the top two rows. These key designations will appear on the User Instruction Form and in the keystroke solutions to sample problems.

## Prompted Operation

Prompts are requests for information represented by a word or abbreviated word followed by a question mark. The information requested is either an answer to a question, called an alpha prompt, or a request for numeric data, called a data prompt.

An alpha prompt is denoted by a question mark immediately following the last letter of a word in the display. GROUPS? is an example of an alpha prompt. Many alpha prompts in this Pac are requests for a yes or no response. A yes response is signified by pressing the letter Y (for yes), followed by a $R / S$ (a simple $R / S$ is also sufficient). A no is signified by pressing $N$ followed by a R/S.

A data prompt is identified by an equal sign (=) following a word or group of words in the display. The equal sign is followed immediately by either a question mark or a number and a question mark. CF 1=? and CF $1=-1,000.00$ ? are two examples of data prompts.
The response to a data prompt without an associated value (CF 1=?) is to key in the number requested followed by a $R / S$. The program then proceeds to the next prompt.
A data prompt with a value attached indicates that the memory associated with the prompt contains the value displayed. If the value is appropriate for the problem currently being solved, press $R / S$ to acknowledge that the value is correct and await the next prompt. If the value is not correct, key in the appropriate value and press R/S. The program then proceeds to the next prompt.

## Optional HP 82143A Printer

When the optional printer is plugged into the HP-41C along with the Real Estate Application Module, all results will be printed automatically. You may also want to keep a permanent record of the value input to a certain program. A convenient way to do this is to set the Print Mode switch to NORMAL before running the program. In this mode, all input values and the corresponding keystrokes will be listed on the printer, thus providing a record of the entire operation of the program.

## Downloading Module Programs

If you wish to trace execution, to modify, or to record on magnetic cards a program in this Application Module, it must first be copied into the HP-41C's program memory. For information concerning the HP-41C's COPY function, see the Owner's Handbook. It is not necessary to copy a program in order to run it.

## Program Interruption

These programs have been designed to operate properly when run from beginning to end, without turning the calculator off (remember, the calculator may turn itself off). If the HP-41C is turned off, it may be necessary to set flag 21 (SF 21) to continue proper execution.

## Use of Labels

You should generally avoid writing programs into the calculator memory that use program labels identical to those in your Application Module. In case of a label conflict, the label within program memory has priority over the label within the Application Pac program.

## Label Conflicts With Other Application Pacs

Several labels used in the Real Estate Module have the same name as those used in other modules. If you have this module and another
module plugged into your HP-41C at the same time, you should make sure that the module whose programs you want to use is in the lowestnumbered port to avoid conflicting use of these labels.

| Label |  |  | Pac |
| :---: | :---: | :---: | :---: |
| *AMORT | *N | *SOYD |  |
| *DB | *NPV | AMORT |  |
| *FV | *PMT | IRR | Financial Decisions |
| *IRR | *PV | MIRR |  |
| *MIRR | *SL | NPV |  |
|  | * |  | Financial Decisions, Home Management |
|  | N |  |  |
|  | PMT |  | Home Management |
|  | FV |  |  |
|  | DB |  | Standard |
|  | PV |  | Circuit Analysis, Home Management |
|  | SIZE? |  | Stress Analysis, Games, Structural Analysis |
|  | OUT |  | Petroleum Fluids |

In addition, the Real Estate label LIST is also a printer function. The Real Estate LIST option is automatically assigned to $\square$ when $\$$ is executed. However, when attempting to execute LIST with keystrokes ( XEO ALPHA LIST ALPHA) the printer function will prevail when it is attached.

## Assigning Program Names

When the Compound Interest and Loan Amortization program is executed (XEO ALPHA \$ ALPHA), 11 global labels within the Real Estate Application Module and 4 main-frame functions are automatically assigned to keys $A-J$ and $\square A-\square$. These assignments take priority over any previous user assignments to the keys. And, since key assignments are stored in program memory, registers $55-63$ are unavailable for data storage.
To accommodate register $55-63$ (without additional Memory Modules), or to use keys $A-J$ and $\square A-\square E$ when the Real Estate Module is not in the calculator, the 15 functions must be de-assigned. This is accomplished by executing CLK (XEQ ALPHA CLK ALPHA). CLK clears only those assignments made in the \$program. Other user assignments are preserved and must be cleared to obtain the maximum number of storage registers. Refer to Page 63 of the HP-41C Owner's Handbook for a description of how to re-assign a key to its original normal mode function.

## COMPOUND INTEREST SOLUTIONS: The Cash Flow Diagram and Sign Convention

The most universal financial calculations involving money, time and interest rates are the compound interest functions. Although the functions have been known for hundreds of years, their use has been restricted by the need for complicated tables until the advent of highspeed digital computers and hand-held financial calculators.

The five variables which have become standard for formatting and describing most compound interest problems can best be explained by referring to a pictorial representation called the cash flow diagram.

The diagram begins with a horizontal line called the time line. It represents the duration of a financial problem and is divided into a number ( N ) of compounding periods of equal duration (length).


Exchange of cash is represented with vertical arrows. Cash received is represented by an arrow pointing up (positive) from the point on the time line where the transaction occurred, and cash paid out is represented by an arrow pointing down (negative).

Payments (PMT) represent a series of cash exchanges of the same direction and amount. In the standard cash flow diagram, the payments occur coincidental with the compounding periods and are equal to the number of periods. The payments can either occur at the beginning of the period (BEGIN) or at the end of the period (END).


END Payment


It is always necessary when working compound interest problems involving payments (PMT) to specify which of the two possible payment streams is applicable, BEGIN or END. In the parlance of various industries BEGIN payments are often referred to as annuity due, or first payment in advance. END payments are referred to as ordinary annuity or payment in arrears.

A single cash flow at the start of the time line is called the present value (PV). A similar single cash flow at the end of the time line is called the future value (FV).


The fifth variable is I, the compound interest rate per period.

When using the cash flow diagram and the cash flow sign convention to format compound interest problems the following rules always apply.

- N and I must correspond to the same period of time.
- Both N and I must be present in a problem. Either both values are known, or one is known and the other is to be computed.
- A valid financial transaction must always include at least one positive cash flow and one negative cash flow.

The cash flow diagram provides a means of describing financial problems without using terminology specific to a particular segment of the business community. It is in a sense, a universal financial language.

The following examples demonstrate the five variables N, I, PV, PMT and FV and their use in the framework of the cash flow diagram to depict common compound interest problems.

## 12 The Cash Flow Diagram and Sign Convention

## Example:

Draw a cash flow diagram to depict the following transaction.
A payment amount of $\$ 402.31$ is necessary to amortize a mortgage loan of $\$ 50,000$ over 30 years. Payments are made monthly; interest is compounded monthly at $.75 \%$ ( $9 \%$ annually) and the first payment is made 1 month after the exchange of the initial loan amount (END).
$P V=\$ 50,000$


Note: PV is positive (arrow pointing up) because it represents cash received. PMT is negative (arrow pointing down) because it represents cash paid out. The use of positive and negative signs to represent the direction in which cash is exchanged is called the cash flow sign convention. It is important in eliminating ambiguity in analyzing various transactions.

## Example:

What will be the balance in a savings account (FV) at the end of 4 years if an initial deposit of $\$ 1000$ is made followed by 4 annual deposits of $\$ 300$ (END)? Interest is compounded yearly at $5 \%$ (I).

| $I=5 \%$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | $\downarrow$ |
| PMT $=$ \$-300 |  |  |  |  |

$$
P V=\$-1,000
$$

Six variations of the basic diagram are presented below. Under each diagram are listed a number of the more common industry terminology used to describe the represented cash exchange. Note that diagrams involving payments may be represented with payments at the beginning (BEGIN) or end (END) of the period.


Some of the terms you see listed above may be common to your industry and some may not. There also may be diagrams represented that correspond to familiar transactions, but which do not bear familiar names. The important point to remember is that it is the magnitude and timing of the cash exchanges, represented by the cash flow diagram that are important, not the industry dependent terminology.

## COMPOUND INTEREST AND LOAN AMORTIZATION



This program will allow you to solve a variety of problems involving money, time and interest. The following variables may be inputs or outputs:

N the number of compounding periods. (For a 30 year loan with monthly payments, $\mathrm{N}=12 \times 30=360$.)
I the periodic interest rate expressed as a percent. (For other than annual compounding, divide the annual percentage rate, or APR, by the number of compounding periods per year, i.e., $9 \%$ APR compounded monthly equals $9 / 12$ or $0.75 \%$.)
PV the present value; the initial cash flow or the discounted value of a series of future cash flows.
PMT the periodic payment amount.
FV the future value; the final cash flow or the compounded value of a series of cash flows.

The program accommodates payments which are made at the end of compounding periods (END payment) or at the beginning (BEGIN payment). Payments made at the end of compounding periods (ordinary annuity) are common in direct reduction loans and mortgages while payments at the beginning of compounding periods (annuity due) are common in leasing.

When this program is executed (XEO \$), global labels within the program are automatically assigned to keys $A-J$ and $\square A-\square$ E. These assignments take priority over any previous user assignments. For a complete explanation, refer to Assigning Program Names on page 9 of this manual.

## Compound Interest

A cash flow diagram enables you to describe a compound interest problem in terms that the calculator can understand. Once you draw and label your diagram, you simply key in the known data when requested and solve for the unknown value.


(For more information on cash flow diagrams, see Compound Interest Solutions: the Cash Flow Diagram and Sign Convention).

Instead of "What is my problem?", ask yourself, "What are the cash flows?"

Solving for any of the values (N, I, PV, PMT, or FV) is easy with your calculator. There are four simple rules to remember-rules that are the same for all compound interest calculations.

1. Given three or four of the financial values (N, I, PV, PMT, or FV), you can solve for the fourth and/or fifth values, as long as N and/or I are known.* Both N and I are involved in all financial calculations. You can enter the values in any order.
2. Use the cash flow sign convention throughout all compound interest calculations (including amortization): Cash received (arrow pointing up) is represented by a positive value ( + ). Cash paid out (arrow pointing down) is represented by a negative value (-).
3. Whenever payments (PMT) are involved, it is always necessary to specify whether the payments are made at the beginning of the payment period (BEGIN) or at the end of the payment period (END). The label BEG/END denotes a toggled computation option. Pressing $\quad C$ causes the HP-41C to display an alpha prompt showing the current status. If you wish to change the status, press C C again. Flag 0 is displayed (set) in BEGIN mode, and cleared in END mode.
4. Remember that $N$ and I must correspond to the same time frame, i.e., the number of compounding periods.

## Loan Amortization

Most mortgages and installment loans are called direct reduction loans. The debt is discharged by equal periodic payments paid at equal intervals. As each payment is received by the note holder, interest is calculated on the outstanding balance since the last payment, subtracted from the payment amount and the remainder applied to the balance. As the balance drops with each payment, so does the interest. With a smaller portion of each payment being deducted for interest, the amount remaining to pay off the balance increases. The breakdown of each payment into the principal reduction portion and interest portion over the life of a loan is called an amortization schedule.
The program calculates the principal and interest portion of each periodic payment and the remaining balance after the payment has been made, for any series of equal periodic amortization payments from any starting period to any ending period using a specified periodic increment (the increment must be less than 1000).
Periodic payments may be made at the end of the compounding period (END payment) or at the beginning (BEGIN payment). Most loans are END payment and most leases are BEGIN payment. Press $\square$ to select the payment mode. Flag 0 indicates current status. Flag 0 set indicates BEGIN mode, and Flag 0 cleared indicates END mode.

[^0]The periodic installment schedule generated is valid for loans that have a single large last payment (balloon) as well as for loans which are arranged to be fully amortized. For a loan with a balloon payment, the remaining balance of the last payment period is the balloon payment due. For BEGIN payment problems (annuity due), any remaining balance or balloon must be compounded for one period since it is due at the end of the last payment period.
When using a calculated payment amount in the amortization routine, it is often necessary to round the payment amount to dollars and cents. This is because the calculated amount usually contains fractional cents, even though the display may show only two decimal places. For the greatest accuracy, the calculated payment amount should be appropriately rounded to the actual payment made.


| STEP | INSTRUCTIONS | INPUT | FUNCTION | dISPLAY |
| :---: | :---: | :---: | :---: | :---: |
| 4. | Review stored values. |  | RCL A - E |  |
| 5. | To compute an amortization schedule (I, PV, and PMT must already be stored). |  | J | START PER=? |
| 6. | Key in starting period of schedule. | Start PER | R/S | END PER=? |
| 7. | Key in ending period of schedule. | END PER | R/S | INCREMENT=? |
| 8. | Key in increment (the number of periods to be amortized before principal and interest are displayed/printed). | INCREMENT | R/S |  |
| 9. | Generate period number, interest, principal and remaining balance. |  | $\begin{aligned} & \mathrm{R} / \mathrm{S}^{*} \\ & \hline \mathrm{R} / \mathrm{S} \\ & \hline \mathrm{R} / \mathrm{S} \\ & \end{aligned}$ | $\begin{aligned} & \text { PERIOD }(n) \\ & \text { INT }= \\ & \text { PRN }= \\ & \text { BAL= } \end{aligned}$ |
| 10. | Repeat step 9 until entire schedule is calculated. Then calculate total principal. |  | R/S * | SINT= |
| 11. | Calculate total interest. |  | R/S * | EPRN= |
| 12. | Calculate remaining balance. <br> * Not necessary when the HP 82143A printer is used. |  | R/S ${ }^{\text {* }}$ | RBAL $=$ |

## Example 1:

A borrower can afford a $\$ 450.00$ monthly principal and interest payment on a 30 year, $11 \frac{1}{4} \%$ mortgage. What is the largest such mortgage he can obtain?

## Keystrokes

XEQ ALPHA SIZE ALPHA 013
XEQ ALPHA $\$$ ALPHA


450 CHS D

30 A
11.25 B

Display

| 0.00 | Minimum Size Initialize and clear financial registers |
| :---: | :---: |
| END | Set END mode |
| $P M T=-450.00$ | Monthly payment |
| $N=360.00$ | Total term |
| $\mathrm{f}=0.94$ | Monthly interest rate |

If the mortgage required is only $\$ 45,500$, what is the monthly payment? (Change the PV and solve for PMT.)

Keystrokes
45500 C
D

Display
$P V=45,500.00$
$P M T=-441.92$

Loan amount
Monthly payment

## Example 2:

A house which was purchased 3 years ago for $\$ 48,000$ is currently listed for $\$ 65,500$. What yearly appreciation rate does this represent?

Keystrokes

| E | 0.00 |
| :---: | :---: |
| C | END |
| 3 A | $N=3.00$ |
| 48000 CHS | $P V=-48,000.00$ |
| 65500 E | $F V=65,500.00$ |
| B | $t=10.92$ |

Clear financial registers
Set END mode Term
Original purchase price Today's value Annual appreciation rate

## Example 3:

What payment amount is necessary to fully amortize a 30 year, $\$ 70,000$ mortgage paid monthly with an annual interest rate of $11 \%$ ?

| Keystrokes | Display |  |
| :---: | :---: | :---: |
| - E | 0.00 | Clear financial registers |
| - $\square^{\text {a }}$ | END | Set END mode |
| 70000 C | $P V=70,000.00$ | Loan amount |
| 30 A | $N=360.00$ | Total term |
| 11 B | $\boldsymbol{I}=0.92$ | Monthly interest rate |
| D | $P M T=-666.63$ | Monthly payment |

What is the balance on the mortgage at the end of year 5 ?

Keystrokes
XEQ ALPHA RND ALPHA STO D

5 A

E

Display
$\left.\left.\begin{array}{l|l}\text {-666.63 } & \begin{array}{l}\text { Round the } \\ \text { payment to two } \\ \text { decimal places } \\ \text { and re-store the }\end{array} \\ \text { value }\end{array}\right\} \begin{array}{l}\text { Payments until } \\ \text { balloon occurs }\end{array}\right\}$

## Example 4:

How much money must be set aside in a savings account each month in order to accumulate $\$ 4,000$ in three years if the account compounds monthly at $6 \%$ per year? The deposits "begin" immediately.

| Keystrokes | Display |  |
| :---: | :---: | :---: |
| E | 0.00 | Clear financial registers |
| C | BEGIN | Set BEGIN mode |
| $3 \square$ | $N=36.00$ | Total number of deposits |
| $6 \square$ B | $f=0.50$ | Monthly interest rate |
| 4000 E | $F V=4,000.00$ | Total to accumulate |
| D | $P M T=-101.18$ | Amount to deposit each month |

What interest rate did the bank pay if the actual amount at the end of the 3 years was $\$ 4,025.50$ ?

| Keystrokes | Display |  |
| :--- | :--- | :--- |
| $4025.50 \boxed{E}$ | $F V=4.025 .50$ | Total <br> accumulated |
| $B$ | $I=0.53$ | Monthly <br> interest rate |
| $12 \boxed{x}$ | 6.40 | Annual interest <br> rate |

## Example 5:

Generate an amortization schedule for the first two payments of an $\$ 80,000$ loan with monthly ordinary annuity payments of $\$ 658.14$ at $9.25 \%$ annual interest rate.

Keystrokes
$\left.\begin{array}{l|l|l}\hline X E Q & \text { ALPHA } \$ \text { ALPHA } & \text { O.OO }\end{array} \begin{array}{l}\text { Initialize and } \\ \text { clear financial } \\ \text { registers }\end{array}\right)$

## Example 6:

Generate a yearly amortization schedule for the first two years of a $\$ 25,000$ loan at $10 \%$, monthly payments, 10 -year term. (End of period payments.)

Keystrokes


XEQ ALPHA RND ALPHA STO D

## J

1 R/S
$24 R / S$
$12 \mathrm{R} / \mathrm{S}$
R/S
$R / S$
R/S
$R / S$
R/S
R/S
R/S
R/S
R/S
R/S

Display
0.00
END
$N=120.00$
$I=0.83$
$P V=25,000.00$
$P M T=-330.38$
$-330.38$

START PER=?
END PER=?
INCREMENT=?
PERIOD 12
INT $=-2,430.97$
$P R N=-1,533.59$
$B A L=23,466.41$
PERIOD 24
INT=-2,270.39
PRN=-1,694.17
$B A L=21,772.24$
$\Sigma I N T=-4.701 .36$
$\Sigma P R N=-3.227 .76$
RBAL=21,772.24

Set END mode Total term Monthly interest rate Loan amount Monthly payment

Round the payment to two decimal places and re-store the value

## INTERNAL RATE OF RETURN

An Internal Rate of Return is that discount rate at which the net present value of all present and future cash flows equals zero (see Net Present Value). This program calculates the IRR of a series of unequal cash flows or groups of equal cash flows. The number of cash flows which can be accommodated depends upon the memory space available. The basic HP-41C without any Memory Modules can accommodate up to 38 individual cash flows or 19 groups of cash flows. With each additional Memory Module installed, the program can accommodate an additional 64 individual cash flows or 32 groups of cash flows up to a maximum of 230 individual or 115 groups.

One of the important features of the program is the ability to make changes in one or more of the cash flows after the entire series has been entered. This allows the user to ask numerous "what if" questions by altering particular values and noting the effect on the outcome.
The cash flow sign convention is used to enter all cash flows. From the point of view of the user, a negative sign represents cash paid out and positive represents cash received. Each time the cash flow changes from a positive value to a negative value or vice versa, it is called a sign change. It is necessary to have at least one sign change in a cash flow series for the series to have an IRR solution. Cash flows with more than one sign change, however, can lead to more than one answer and are not recommended for use with this program. While the program may find one of the answers, it has no way of finding or indicating other possibilities. For an alternative method of solution for cash flows with multiple sign changes, see Modified Internal Rate of Return.

This program was designed for optimum operation when the interest rate being solved for is between 0 and $100 \%$. The program will often solve for interest rates outside this range, but occasionally may halt prematurely with DATA ERROR in the display. This is an error condition generated by an intermediate calculation, and indicates that the program cannot solve that particular problem.

## Prompted Operation

(Minimum size 025)*
After executing IRR, the following user prompts will be displayed. After each response, press $R / S$ to continue.

[^1]GROUPS? Are the cash flows to be entered as groups of equal cash flows? Press R/S to enter the cash flow in groups, or press $N$ R/S if the cash flows are not in groups.

CF (\#)=? What is the amount of the cash flow or group of cash flows in period $0,1,2$, etc.? Use the cash flow sign convention. If all of the cash flows have been entered, press $R / S$ to continue to the next prompt.

NO. CFS (\#)=? How many cash flows are in period 0, 1, 2, etc?
CF CHANGES? Do you wish to review or change any cash flows? Press R/S to make changes, or press $N R / S$ if no changes need to be made.

When the result has been calculated and displayed, changes can easily be made in the cash flows or number of cash flows simply by pressing R/S. The program responds by prompting for the number of the group (GROUP NO.=?) or cash flow (CF NO.=?) to be changed. After responding, the program displays current values. Values are either changed by entering the new value followed by a $R / S$ or not changed by responding with a lone R/S.

After the last group or cash flow change, the correction routine is terminated by pressing $R / S$ with no data entry in response to the prompt GROUP NO. $=$ ? or CF NO. $=$ ?. The computation of the new IRR is then computed and displayed.

## Example 1:

An investment proposal calls for an increasing outlay of cash for each of 5 years and then a substantial payoff. If the annual cash flows are as follows, what is the return on the investment?

| Year | Cash Flow (\$) |
| :---: | :---: |
| 0 | -2000 |
| 1 | -2500 |
| 2 | -3000 |
| 3 | -3500 |
| 4 | -4000 |
| 5 | 25000 |

## Keystrokes

## Display

```
XEQ ALPHA SIZE ALPHA 031
XEQ ALPHA IRR ALPHA
N R/S
```

GROUPS?
CF $0=$ ?
Keystrokes
$2000 \mathrm{CHS} \mathrm{R} / \mathrm{S}$
$2500 \mathrm{CHS} \mathrm{R} / \mathrm{S}$
$3000 \mathrm{CHS} \mathrm{R} / \mathrm{S}$
$3500 \mathrm{CHS} \mathrm{R} / \mathrm{S}$
$4000 \mathrm{CHS} \mathrm{R} / \mathrm{S}$
$25000 \mathrm{R} / \mathrm{S}$
$\mathrm{R} / \mathrm{S}$
$N \mathrm{R} / \mathrm{S}$

Display
CF $1=$ ?
CF 2=?
CF $3=$ ?
CF 4=?
CF 5=?
CF 6=?
CF CHANGES?
IRR=19.71 Annual yield

If the payoff were only $\$ 20,000$, what would the IRR be?

## Keystrokes

R/S
$5 \mathrm{R} / \mathrm{S}$
20000 R/S
R/S

Display
CF NO. =?
CF 5=25,000.00?
CF NO. $=$ ?
IRR=10.96

Annual yield

## Example 2:

A graduated-payment mortgage calls for the following schedule of increasing monthly mortgage payments on a 30-year, \$40,000 loan. What is the actual yearly interest rate (APR)?

| Group | Year | Payment |
| :---: | :---: | ---: |
| 0 | 0 | $-40,000.00$ |
| 1 | 1 | 252.01 |
| 2 | 2 | 264.61 |
| 3 | 3 | 277.84 |
| 4 | 4 | 291.73 |
| 5 | 5 | 306.32 |
| 6 | $6-30$ | 321.64 |

Keystrokes

## Display

XEQ ALPHA SIZE ALPHA 039

XEQ ALPHA IRR ALPHA
R/S
$40000 \mathrm{CHS} \mathrm{R} / \mathrm{S}$
1 R/S
$252.01 \mathrm{R} / \mathrm{S}$
$12 \mathrm{R} / \mathrm{S}$
$264.61 \mathrm{R} / \mathrm{S}$

GROUPS?
CF $0=$ ?
NO. CFS $0=$ ?
CF 1 =?
NO. CFS $1=$ ?
CF 2=?
NO. CFS 2=?

26 Internal Rate of Return

| Keystrokes | Display |  |
| :---: | :---: | :---: |
| $12 \mathrm{R} / \mathrm{S}$ | CF 3=? |  |
| $277.84 \mathrm{R} / \mathrm{S}$ | NO. CFS 3=? |  |
| $12 \mathrm{R} / \mathrm{S}$ | CF 4=? |  |
| $291.73 \mathrm{R} / \mathrm{S}$ | NO. CFS 4=? |  |
| $12 \mathrm{R} / \mathrm{S}$ | CF 5=? |  |
| $306.32 \mathrm{R} / \mathrm{S}$ | NO. CFS 5=? |  |
| $12 \mathrm{R} / \mathrm{S}$ | CF 6=? |  |
| $321.64 \mathrm{R} / \mathrm{S}$ | NO. CFS 6=? |  |
| $300 \mathrm{R} / \mathrm{S}$ | CF $7=$ ? |  |
| R/S | CF CHANGES? |  |
| $N$ R/S | IRR $=0.70$ | Monthly yield |
| $12 \times$ | 8.40 | Annual nominal yield |

Note: The effective annual yield may be obtained by doing the following:
$1200 \div 1 \boxed{+} 12 \boxed{y^{x}} 1-100 \boxed{x}-8.73$

## MODIFIED INTERNAL RATE OF RETURN

The traditional IRR technique has several drawbacks which hamper its usefulness in some investment applications. The technique implicitly assumes that all cash flows are either reinvested or discounted at the computed yield rate. This assumption is financially reasonable as long as the rate is within a realistic borrowing and lending range (e.g., 10$15 \%$ ). When the IRR becomes significantly greater or smaller, the assumption becomes less valid and the resulting value less sound as an investment measure.

IRR also is limited by the number of times the sign of the cash flow changes (positive to negative or vice versa). For every change of sign, the IRR solution has the potential for an additional answer. The cash flow sequence in the following example has three sign changes and hence up to three potential internal rates of return. This particular example has three positive real answers: $1.86,14.35$ and 29 . Although mathematically sound, the multiple answers are meaningless as an investment measure.

The Modified Internal Rate of Return program is one of several IRR alternatives which avoids the drawbacks of the traditional IRR technique. The program eliminates the sign change problem and the reinvestment (or discounting) assumption by utilizing user stipulated reinvestment and borrowing rates.

Negative cash flows are discounted at a rate (SAFE RATE) that reflects the return on an investment in a liquid account. The figure generally used is a short-term security (T-Bill) or bank passbook rate.
Positive cash flows are reinvested at a rate which reflects the return on an investment of comparable risk (RISK RATE). An average return rate on recent market investments might be used.
The basic HP-41C without any Memory Modules can accommodate up to 38 individual cash flows or 19 groups of cash flows. With each additional Memory Module installed, the program can accommodate an additional 64 individual cash flows or 32 groups of cash flows up to a maximum of 230 individual or 115 groups.

## Prompted Operation

(Minimum size 025)*
After executing MIRR, the following user prompts will be displayed. After each response, press $\mathrm{R} / \mathrm{S}$ to continue.

[^2]GROUPS? Are the cash flows to be entered as groups of equal cash flows? Press R/S to enter the cash flows in groups, or press $N R / S$ if the cash flows are not in groups.

CF (\#)=? What is the amount of the cash flow or group of cash flows in period $0,1,2$, etc.? Use the cash flow sign convention. If all of the cash flows have been entered, press $R / S$ to continue to the next prompt.

NO. CFS (\#)=? How many cash flows are in period 0, 1, 2, etc.?
CF CHANGES? Do you wish to review or change any cash flows? Press R/S to make changes, or press $N$ R/S if no changes need to be made.

SAFE RATE=? What is the liquid rate of return, in percent?
RISK RATE=? What is the "risky" rate of return, in percent?

When the result has been calculated and displayed, changes can easily be made in the cash flows, number of cash flows and rates, simply by pressing R/S. The program responds by prompting for the number of the group (GROUP NO=?) or cash flow (CF NO.=?) to be changed. After responding, the program displays the current values. Values are either changed by entering the new value followed by a/S or not changed by responding with a lone R/S.

After the last group or cash flow change, the correction routine is terminated by pressing R/S with no data entry in response to the prompt GROUP=? or CF=?. The safe rate (SAFE RATE) and risk rate (RISK RATE) are then displayed and any changes made. The computation of a new MIRR then proceeds automatically.

## Example:

An investor has the following unconventional investment opportunity. The cash flows are:

| Group | \# of Months | Cash Flow (\$) |
| :---: | :---: | :---: |
| 0 | 1 | $-180,000$ |
| 1 | 5 | 100,000 |
| 2 | 5 | $-100,000$ |
| 3 | 9 | 0 |
| 4 | 1 | 200,000 |

Calculate the MIRR using a safe rate of $6 \%$ and a reinvestment (risk) rate of $10 \%$.

| Keystrokes | Display |  |
| :---: | :---: | :---: |
| XEQ ALPHA SIZE ALPHA 035 |  |  |
| XEO ALPHA MIRR ALPHA | GROUPS? |  |
| R/S | CF $0=$ ? |  |
| $180000 \mathrm{CHS} \mathrm{R} / \mathrm{S}$ | NO. CFS 0=? |  |
| 1 R/S | CF 1-? |  |
| $100000 \mathrm{R} / \mathrm{S}$ | NO. CFS 1=? |  |
| 5 R/S | CF 2=? |  |
| $100000 \mathrm{CHS} \mathrm{R} / \mathrm{S}$ | NO. CFS 2=? |  |
| $5 \mathrm{R} / \mathrm{S}$ | CF 3=? |  |
| $0 \mathrm{R} / \mathrm{S}$ | NO. CFS 3=? |  |
| $9 \mathrm{R} / \mathrm{S}$ | CF 4=? |  |
| $200000 \mathrm{R} / \mathrm{S}$ | NO. CFS 4=? |  |
| 1 R/S | CF 5=? |  |
| R/S | CF CHANGES? |  |
| $N$ R/S | SAFE RATE=? |  |
| 6 ENTERT $12 \div \mathrm{R} / \mathrm{S}$ | RISK RATE=? |  |
| 10 ENTERT $12 \div \mathrm{B} / \mathrm{S}$ | MIRR $=0.81$ | Monthly yield |
| $12 \times$ | 9.70 | Annual nominal yield |

Note: The effective yield may be obtained by doing the following:


## NET PRESENT VALUE (Discounted Cash Flow Analysis)

The Net Present Value procedure reduces a series of cash flows which occur at different times in the future to a single net value at one point in time, the present. The technique that makes this transformation possible is called discounting. If one dollar were invested at the present in an investment which yielded $10 \%$ a year, it would grow to $\$ 1.10$ in one year. The $\$ 1.10$ is referred to as the future value ( FV ) of $\$ 1$ invested at $10 \%$ for one year. Conversely, the opportunity to realize $\$ 1.10$ a year from now would be equivalent to having one dollar now (PV) with a $10 \%$ investment opportunity. The technique of comparing a future value to its equivalent present value is called discounting, and the investment rate is called the discount rate.

This program produces a net present value (NPV) for a number of future cash flows or groups of equal future cash flows when given a discounting rate (DSCNT RATE). The cash flows, which must occur at equal intervals, are represented using the cash flow sign convention. Negative cash flows represent invested money and positive cash flows represent the return from the investment.

The first cash flow is assumed to occur at time period zero (the present). The discount rate must coincide with the period between cash flows. For example, a $9 \%$ yearly discount rate applied to monthly cash flows must be entered as $9 / 12$ or 0.75 .

One of the important features of the program is the ability to make changes in one or more of the cash flows or the discount rate after the entire series has been entered. This allows the user to ask numerous "what if" questions by altering particular values and noting the effect on the outcome.

If the Net Present Value of all future positive and negative cash flows has a positive sign, it means that the net return rate of the investment exceeds the discounting rate. If the final Net Present Value is negative, the investment did not return the discounting rate. If the Net Present Value is zero, the discounting rate and the investment return rate are equal.

The basic HP-41C without any Memory Modules can accommodate up to 38 individual cash flows or 19 groups of cash flows. With each additional Memory Module installed, the program can accommodate an additional 64 individual cash flows or 32 groups of cash flows up to a maximum of 230 individual or 115 groups.

## Prompted Operation

(Minimum size 025)*

After executing NPV, the following user prompts will be displayed. After each response, press $\mathrm{R} / \mathrm{S}$ to continue.

GROUPS? Are the cash flows to be entered as groups of equal cash flows? Press $R / \mathrm{S}$ to enter the cash flows in groups, or press $N R / S$ if the cash flows are not in groups.

CF (\#)=? What is the amount of the cash flow or group of cash flows in period $0,1,2$, etc.? Use the cash flow sign convention. If all the cash flows have been entered, press $\mathrm{R} / \mathrm{S}$ to continue to the next prompt.

NO. CFS (\#)=? How many cash flows are in period $0,1,2$, etc.?

CFCHANGES? Do you wish to review or change any cash flows? Press $R / \mathrm{S}$ to make changes, or press $N \mathrm{R} / \mathrm{S}$ if no changes need to be made.

DSCNT RATE=? Key in the discount rate (in percent). The rate must coincide with the period between cash flows.

When the result has been calculated and displayed, changes can easily be made in the cash flows, number of cash flows or the discount rate, simply by pressing $R / S$. The program responds by prompting for the number of the group (GROUP NO.=?) or cash flow (CF NO.=?) to be changed. After responding, the program displays the current values. Values are either changed by entering the new value followed by a R/S or not changed by responding with a lone $R / S$.

After the last group or cash flow change, the correction routine is terminated by pressing $R / S$ with no data entry in response to the prompt GROUP NO.=? or CF NO.=?. The discount rate (DSCNT=?) is then displayed and may be changed. After a response has been made, the new Net Present Value is calculated and displayed.

[^3]
## Example:

An investor has an opportunity to purchase a piece of property for $\$ 70,000$. If the after-tax cash flows are forecast as follows, should the investor purchase the property if he desires a $15 \%$ rate of return?

| Group | Cash Flow (\$) | Number of Months |
| :---: | :---: | :---: |
| 0 | $-70,000$ | 1 |
| 1 | $-1,000$ | 2 |
| 2 | 800 | 56 |
| 3 | 87,500 | 1 (property sold) |

Keystrokes

## Display

XEQ ALPHA SIZE ALPHA 033
XEQ ALPHA NPV ALPHA
R/S
$70000 \mathrm{CHS} \mathrm{R} / \mathrm{S}$
GROUPS?
CF $0=$ ?
NO. CFS O=?
1 R/S
CF $1=$ ?
$1000 \mathrm{CHS} \mathrm{R} / \mathrm{S}$
2 R/S
800 R/S
NO. CFS $1=$ ?

56 R/S
87500 R/S
1 R/S
R/S
$N$ R/S
CF 2=?
NO. CFS 2=?
CF 3=?
NO. CFS 3=?
CF 4=?
CF CHANGES?
DSCNT RATE=?
15 ENTER $12 \div \mathrm{R} / \mathrm{S}$

Since the final NPV is positive, the investment meets the $15 \%$ rate of return objective.
Would the $15 \%$ rate of return objective be met if the property were projected to sell for $\$ 84,000$ ?

Keystrokes

## R/S

3 R/S
$84000 \mathrm{R} / \mathrm{S}$
R/S
R/S
R/S

Display
GROUP NO. =?
GROUP $3=87.500 .00$ ?
NO. CFS 3=1.00?
GROUP NO. =?
DSCNT RATE=1.25?
$N P V=-308.01$

Since the NPV is negative, the $15 \%$ guideline would not have been met.

## DEPRECIATION SCHEDULES

In theory, tangible assets such as buildings, machines, tools, trucks, etc. gradually decline in value over time through usage, technological obsolescence, environmental deterioration or a combination of all of these. Depreciation is a method of periodically accounting for the declining value of an asset. Depreciation is an annual deduction from Net Operating Income, before taxable income and income tax liability are calculated, and is an accounting expense item.
The three most commonly used depreciation methods are: Straight Line (SL), Sum-of-the-Years'-Digits (SOYD), and Declining Balance (DB). SOYD and DB are methods of accelerated depreciation whereby higher annual amounts of depreciation are charged during the early years of an asset's life than with the straight line depreciation method, reflecting the fact that many assets decline in value most during the early part of their lives.
The program computes depreciation based on fiscal or calendar year. When the purchase date of an asset does not coincide with the beginning of a year-which is the rule rather than the exception-the amount of depreciation in the first and last years are computed as fractions of a full year's depreciation. The program recognizes this and prompts you to enter the number of months from purchase until the end of the first fiscal or calendar year (MONS YR 1=?).

Often a complete depreciation schedule for the asset's entire life is not needed. You are also given prompts to enter the starting and ending periods for which a depreciation schedule is desired.
Calculated results include the amount of depreciation (DEP), the remaining depreciable value (RDV), and the remaining book value (RBV).

## Straight Line Depreciation (SL) Method

The annual amount of depreciation using this method is determined by dividing the total depreciable amount (starting book value less salvage value) by the useful life expectancy. This amount is subtracted each year from the previous year's ending book value or ending depreciable value.

## Sum-of-the-Years'-Digits (SOYD) Method

The SOYD method is based on the sum of the digits from one year to the number of years in the asset's life. For instance an asset with a 6 year life would have an SOYD total of $21 .(6+5+4+3+2+1=21)$.

## 34 Depreciation Schedules

Theoretically, $6 / 21$ of the asset's life is used up during the first year, $5 / 21$ during the second year, etc. The annual amount of depreciation for each year is the asset's total depreciable amount multiplied by that year's use factor. Partial first year SOYD depreciation is correctly treated, based on formulas in the back of this book.

## Declining Balance (DB) Method

With the DB method, a constant percentage is applied each year to the remaining book value to find the annual amount of depreciation. The salvage value is not subtracted initially but the asset may not be depreciated below this salvage value.

Since income tax regulations currently allow for the use of more than one declining balance percentage rate, you are prompted for the percentage rate you wish to use. A rate of $150 \%$ means $150 \%$ declining balance, and $200 \%$ means double-declining balance. This percentage rate divided by the asset life, in years, gives the constant rate applied each year to the remaining book value. For instance, $150 \%$ declining balance applied to an asset with a 6 year life would give a constant rate of $150 / 6$ or $25 \%$ each year.

The annual amount of depreciation declines over time and the DB method frequently will not depreciate an asset fully in the asset's lifetime. In these situations, there is an optimum point in the asset's life where a change from the DB method to the straight line method should be used. This is the "crossover point", the first year in which the amount of depreciation by the SL method is greater than if depreciation were continued using the DB method. The annual SL amount of depreciation is determined by dividing the remaining depreciable value by the remaining asset life.

A program prompt will allow you to determine whether or not you want to crossover to the SL method. If you elect to "crossover", the calculated output will give the crossover point, if it occurs in any year up to and including the last year of the printed depreciation schedule.

|  |  |  |  | SIZE: 018 |
| :---: | :---: | :---: | :---: | :---: |
| STEP | INSTRUCTIONS | INPUT | FUNCTION | DISPLAY |
| 1 | Initialize the program. |  | XEQ DEPR | DEP AMT=? |
| 2. | Key in the amount to be depreciated. | DEP AMT | R/S | LIFE=? |
| 3. | Key in the expected useful life (in years). The life must be an integer. | LIFE | $R / S$ | SALVAGE=? |
| 4. | Key in the estimated salvage value. | SALVAGE | R/S | MONS YR 1=? |
| 5. | Key in the number of months from acquisition until the end of the first year. | MONS YR 1 | R/S | START PER=? |


| STEP | INSTRUCTIONS | INPUT | FUNCTION | dISPLAY |
| :---: | :---: | :---: | :---: | :---: |
| 6. | Key in the first year for which results are desired. | START PER | R/S | END PER=? |
| 7. | Key in the last year for which results are desired. | END PER | R/S | DEP TYPE? |
| 8. | Key in the depreciation type: <br> Straight line <br> Sum-of-the-Years'-Digits Declining Balance | $\begin{gathered} \text { SL } \\ \text { SOYD } \\ \text { DB } \end{gathered}$ | $\mathrm{R} / \mathrm{S}$ <br> $\mathrm{R} / \mathrm{S}$ <br> $\mathrm{R} / \mathrm{S}$ | (Go to 11) <br> (Go to 11) DB\%=? |
| 9. | Key in the declining balance rate factor (in percent) | DB\% | R/S | X-OVER? |
| 10. | Accept or decline crossover to straight line: <br> Crossover <br> No crossover | $N$ | $\begin{array}{\|l\|} \mathrm{R} / \mathrm{S} \\ \hline \mathrm{R} / \mathrm{S} \\ \hline \end{array}$ |  |
| 11. | Display the year for which depreciation is to be calculated |  |  | YEAR |
| 12. | Calculate the amount of depreciation |  | R/S * | DEP= |
| 13. | Calculate the remaining depreciable value. |  | R/S* | RDV= |
| 14. | Calculate the remaining book value. |  | R/S * | RBV= |
| 15. | Press R/S for the next year. Return to step 12. <br> * Press $\mathrm{R} / \mathrm{S}$ if you are not using a printer. |  | R/S* | YEAR |

## Example 1:

A property has been acquired for $\$ 250,000$. The purchase price is allocated to $\$ 50,000$ for land (non-depreciable) and $\$ 200,000$ for improvements (building, etc.). The remaining useful life of the building is determined to be 30 years with no estimated salvage value. If the property was acquired on September 1, calculate the $150 \%$ declining balance depreciation with crossover for years 11-13.

Keystrokes
Display
XEQ ALPHA SIZE ALPHA 018
XEQ ALPHA DEPR ALPHA
$200000 \mathrm{R} / \mathrm{S}$
$30 \mathrm{R} / \mathrm{S}$
$0 \mathrm{R} / \mathrm{S}$
$4 \mathrm{R} / \mathrm{S}$

DEP AMT=?
LIFE=?
SALVAGE=?
MONS YR $1=$ ?
START PER=?

| Keystrokes | Display |
| :---: | :---: |
| $11 \mathrm{R} / \mathrm{S}$ | END PER=? |
| $13 \mathrm{R} / \mathrm{S}$ | DEP TYPE? |
| DB R/S | DB\%=? |
| $150 \mathrm{R} / \mathrm{S}$ | X-OVER? |
| R/S | YEAR 11 |
| R/S | $D E P=6,197.45$ |
| R/S | $R D V=117.751 .61$ |
| R/S | $R B V=117.751 .61$ |
| R/S | YEAR 12 |
| R/S | $D E P=5,987.37$ |
| R/S | $R D V=111.764 .24$ |
| R/S | $R B V=111.764 .24$ |
| R/S | YEAR 13 |
| R/S | $D E P=5,987.37$ |
| R/S | $R D V=105.776 .87$ |
| R/S | $R B V=105,776.87$ |

Note that the depreciable amounts in years 12 and 13 are the same. Crossover from declining balance to straight line occurs in year 12.

## Example 2:

An electron beam welder which costs $\$ 50,000$ is purchased 4 months before the end of the accounting year. What will the depreciation be during the first full accounting year (year 2) if the welder has a 6 year depreciable life, a salvage value of $\$ 8,000$ and is depreciated using the sum-of-the-years'-digits method?

Keystrokes
XEO ALPHA DEPR ALPHA
$50000 \mathrm{R} / \mathrm{S}$
$6 \mathrm{R} / \mathrm{S}$
$8000 \mathrm{R} / \mathrm{S}$
$4 \mathrm{R} / \mathrm{S}$
$2 \mathrm{R} / \mathrm{S}$
$2 \mathrm{R} / \mathrm{S}$
$\mathrm{SOYD} \mathrm{R} / \mathrm{S}$
$\mathrm{R} / \mathrm{S}$
$\mathrm{R} / \mathrm{S}$
$\mathrm{R} / \mathrm{S}$

## Display

DEP AMT=?
LIFE=?
SALVAGE=?
MONS YR $1=$ ?
START PER=?
END PER=?
DEP TYPE?
YEAR 2
DEP=11,333.33
$R D V=26,666.67$
$R B V=34,666.67$

## INCOME PROPERTY ANALYSIS

This program analyzes the investment potential of income-producing properties, on both a before-tax and after-tax basis. With this program, the user is able to compare investments of similar lifetimes, and to evaluate the effects of alternative methods of financing, depreciation methods, capital improvements, and possible exchanges.

Variables used in this evaluation include: gross income, expenses, mortgage data (interest, term, payment), depreciation, capital expenditures, the investor's income tax and capital gains tax rates, yearly appreciation, yearly escalation (growth) in income, yearly increases in expenses, and the original basis.
The user has the option of obtaining a summary of the performance of the property, or a full schedule for each year of holding. An annual summary of the property's performance includes the following major items: net operating income, taxable income, tax liability, before-tax cash flow, and after-tax cash flow.

Then, at the end of the investment's holding period, a summary is output listing the property's adjusted tax basis, sale price, capital gain realized on sale, taxes (both capital gain taxes and the taxes due when accelerated depreciation has been used), and the after-tax proceeds realized from the sale. The investment's overall equity yield is then determined using an Internal Rate of Return routine.
A full schedule includes all of the above calculations, plus equity yield, for each year of holding. This allows the user to determine the property's optimum holding period.
In either case, when the schedule or summary is complete, the user is able to change the input data, and may re-do the evaluation for a new financing situation.
The program is based on RNMI Form 1-75-F612, and is designed to handle three monthly payment mortgages where the total terms, yearly interest rates, and amounts of each mortgage are known. During execution the mortgages must be entered in decreasing term order (that is, longest term mortgage first). The program prompts for the monthly payment. If the payment is not known, the program will calculate and display the monthly payment necessary to amortize the mortgage. Using this approach, the program can accommodate fully amortized loans, interest-only loans with a balloon payment, and reduced payment loans with a balloon payment.
Two depreciation options are also available-one for the building and one for personal property. The necessary inputs are the depreciable value, the depreciable life, and the depreciation factor.

The basic HP-41C without any Memory Modules can accommodate a projection period of 11 years.* With each additional Memory Module installed, the program can accommodate an additional 64 years.

## Prompted Operation

(Minimum size 052) $\dagger$
After executing IPA, the following user prompts will be displayed. After each response, press $R / \mathrm{S}$ to continue.
PROJ PER=? What is the projected number of holding periods in
APPREC\%=? What is the annual investment appreciation rate in percent?

GROWTH\%=? What is the annual underlying rate which is applied to the gross income (rents) in percent?
GROSS INC=? What is the annual gross income?
VACANCY\%=? What is the expected vacancy rate in percent?
INCREASE\%=? What is the annual underlying rate which is applied to the operating expenses and capital improvements in percent?

OP EXP=? What are the expected annual operating expenses in dollars?

1ST MTG=? What is the amount of the mortgage with the longest term? Enter a positive, non-zero value.

INT\%=? What is the annual interest rate (in percent) of the first mortgage?

TERM=?
PMT=?

2ND MTG=? What is the amount of the mortgage with the second longest term? If there is no second mortgage, key in 0 ; the next seven prompts will then be skipped.

[^4]| INT\%=? | What is the annual interest rate (in percent) of the <br> second mortgage? |
| :--- | :--- |
| TERM=? | What is the total term (in years) of the second <br> mortgage? |
| PMT=? | What is the level monthly payment on the second <br> mortgage? Pressing R/S, with no value entered, |
| results in calculation of a level monthly payment with |  |
| full amortization. This value is displayed. Press R/S |  |,

CAP IMP=? What are the expected annual capital improvements in dollars?

TAX\%=? What is the investor's marginal income tax rate, as a percent?
CG TAX\%=? What is the investor's capital gains tax rate, as a percent?

FULL SCHED? Do you wish a full schedule including equity yield for each year? Press $R / S$ for a full schedule, or press $N$ R/S for a yearly summary.

LIST?

CHANGE?
Do you wish to list the values which have been keyed in? Press $R / S$ to list the values, or press $N R / S$ to proceed.

Do you wish to review or change any values which have been keyed in? Press $R / S$ to make changes, or press $N R / S$ if no changes need to be made.

TRANS CST\%=? What are the transaction costs as a percent of the selling price?

When the last result has been calculated (EQ YLD:), changes can easily be made in the input data simply by pressing $R / S$. The program responds with CHANGE? If you wish to make changes, press $R / S$. The program will then display the values currently being used. Values are either changed by entering the new value followed by a $R / S$ or not changed by responding with a lone $R / S$. After the last data input, the computation of a new summary then proceeds automatically.

If you do not wish to make changes, press $N$ R/S. The previous computation is repeated.

## Example:

You are considering the purchase of a $\$ 179,500$ multi-use apartment/ office space/warehouse complex consisting of nine distinct rentable units. The property is currently generating yearly rental income of $\$ 17,580$, with a $2 \%$ vacancy rate and total yearly operating expenses of $\$ 3,620$. As conservative estimates, the value of the complex will appreciate $6 \%$ per year, the annual rents will grow $5 \%$ per year, and expenses (including improvements) will increase $8 \%$ per year.
The property is currently financed with an assumable $9.75 \%$ mortgage which has a remaining balance of $\$ 62,667.17$ and a remaining term of 25 years. If this mortgage were assumed, the lender would require a two point increase in the interest rate, thus increasing the rate to $11.75 \%$.

The seller of the property is willing to carry the balance as a second mortgage, with the stipulation that he receive $20 \%$ of the asking price as a down payment. The second $\$ 80,932.83$ would be written at $11 \%$, with monthly payments of $\$ 750.00$ and a balloon payment occurring at the end of the fourth year for the outstanding balance.

The building is valued at $\$ 139,500$ and can be depreciated using a $125 \%$ declining balance method for a useful life of 20 years. Personal property, valued at $\$ 5000$, has an expected useful life of 5 years and will be depreciated using the straight line method.
Capital improvements are projected to be $\$ 1,500$ the first year. Your income tax rate is $50 \%$, and your capital gains tax rate is $30 \%$. When you sell the property, transaction costs are estimated to be $7 \%$ of the selling price. The tax basis is $\$ 179,500$.
Based on this data, analyze the property using an expected holding (projection) period of 5 years. For this analysis, a yearly summary is sufficient.

Keystrokes
XEQ ALPHA SIZE ALPHA 057 $\dagger$

| XEQ ALPHA IPA ALPHA | PROJ PER=? |  |
| :--- | :--- | :--- |
| $5 \mathrm{R} / \mathrm{S}$ | APPREC\%=? |  |
| $6 \mathrm{R} / \mathrm{S}$ | GROWTH\%=? |  |
| $5 \mathrm{R} / \mathrm{S}$ | GROSS INC=? |  |
| $17580 \mathrm{R} / \mathrm{S}$ | VACANCY\%=? |  |
| $2 \mathrm{R} / \mathrm{S}$ | INCREASE\%=? |  |
| $8 \mathrm{R} / \mathrm{S}$ | OP EXP=? |  |
| $3620 \mathrm{R} / \mathrm{S}$ | 1ST MTG=? |  |
| $62667.17 \mathrm{R} / \mathrm{S}$ | INT\%=? |  |
| $11.75 \mathrm{R} / \mathrm{S}$ | TERM=? |  |
| $25 \mathrm{R} / \mathrm{S}$ | PMT=? |  |
| $\mathrm{R} / \mathrm{S}$ | PMT:648.48 |  |
| $\mathrm{R} / \mathrm{S}$ |  | 2ND $\mathbf{M T G}=?$ |
| $80932.83 \mathrm{R} / \mathrm{S}$ | INT\%=? |  |
| $11 \mathrm{R} / \mathrm{S}$ | TERM=? |  |
| $4 \mathrm{R} / \mathrm{S}$ | PMT=? |  |
| $750 \mathrm{R} / \mathrm{S}$ | 3RD MTG=? |  |

## Display

It may be necessary to execute CLK.

[^5]
## Keystrokes

0 R／S
$139500 \mathrm{R} / \mathrm{S}$
20 R／S
$125 \mathrm{R} / \mathrm{S}$
5000 R／S
5 R／S
100 R／S
$179500 \mathrm{R} / \mathrm{S}$
$179500 \mathrm{R} / \mathrm{S}$
1500 R／S
50 R／S
$30 \mathrm{R} / \mathrm{S}$
$N R / S$
$N R / S$
$N R / S$
7 R／S

## Display

BLD DEP＝？
LIFE＝？
FACTOR＝？
PER PROP＝？
LIFE＝？
FACTOR＝？
PRICE＝？
BASIS＝？
CAP IMP＝？
TAX\％＝？
CG TAX\％＝？
FULL SCHED？
LIST？
CHANGE？
TRANS CST\％＝？

| IHIT IHY：35，900．60 | Initial investment（down payment） |
| :---: | :---: |
| YEAF： 1 |  |
| GROSS INC：17，586，日6 | Gross income |
| － 4 AC － 351.60 | Vacancy |
| －0F EXP：3，626．60 | Operating expenses |
| ＝ HOL ：13．688．46 | Net operating income |
| －IHT LST：7，340．11 | Interest on $1^{\text {st }}$ mortgage |
| －INT 2HIT：8．897．55 | Interest on $2^{\text {nd }}$ mortgage |
| －ELI IEF：8，718．75 | Building depreciation |
| －PEE PROP：1，001． 00 | Personal property depreciation |
| $=T \mathrm{~F}$ INC：$-12,343.81$ | Taxable income |
| ＊ HTR S可． 10 | Marginal tax rate |
| ＝TAX LIAE：－6，174，日1 | Tax liability |
| H01 13．698． 48 | Net operating income |
| －RHP FMTS：16，781．76 | Total annual mortgage payments |
| －CAP IMF：1，506． 60 | Capital improvements |
| ＝ETCF -4.673 .36 | Before－tax cash flow |
| －THX LIAE：－6．174．01 | Tax liability |
| ＝ATCF： $1,506.65$ | After－tax cash flow |

```
YEARE: Z
GR0SS INC:18:459.60
-VHC 369.18
-0P EyP:7.964.60
=NOI:14:160.22
-IHT 1ST:7.285.73
-IHT 2HI:8.885.69
-ELT IEF:8.173.83
-PER FROF:1, 600.,00
=TH员 IHC:-11,164,63
*HTE:50.06
=TH:LIHE:-5,502.72
H01:14,160.22
-AHH PMTS:16.78L.76
-GP IMF:1.626.00
=ETGF:-4.221.54
-THRLLIAE:-5,502.72
=ATCF:1,360.76
YEAF: F
GROSS INC:19,381.95
-पПC: 307.64
-0P EXF:4,222.37
=k01:14,771.94
-IHT 1ST:7,223.75
-IHT 2HD:8.872.47
-BLT IEP:7,662.96
-PER PROP:1, 601.日6
=THX IHC:-9,987.24
*MTR:50.60
=THX LIHB:-4,993.62
H01:14:771.94
-RHN PMTS:16,781.76
-CAP IMP:1,749.69
=BTCF:-3.759.42
-TAR LIME:-4,993.62
=ATCF:1,234.20
```

```
YEAR:4
CROSS INC:20,351,65
-HOC:407.02
-0F EXF:4,566.16
=N0I:15,303.87
-INT 1ST:7,154.53
-INT 2HI:8.857.70
-ELII DEF:7:184.03
-PER FROF:1, पी0. 90
=THE IHC:-8,812.39
*MTE:50.60
=TAY LIAE:-4:466.20
H01: 15, 383.87
-RHH FMTS:97,228.00
-GF IMF : 1,809.57
=BTCF :-83.733.70
-TAX LIAE:-4,466.20
=ATCF : -79.327.50
YEAR:5
GR0SS INO:21,360.60
-HAC:427.37
-OP EXF:4.924.97
=N0I :16.016.26
-IHT 1ST:7.676.74
-ELI DEF:6:735,0%
-FER FROF:1,珀,0]
=TAR IHG:1,204.49
*|TR:5й. 昭
=THY LIHE:6日2.25
H01:16.016.26
-AHH FMTS:7.781.76
-GF IMP:2,日4घ.75
=ETCF:6.19].77
-THP LIHE:642.25
=ATCF:5,591.52
```

Includes balloon payment on second mortgage

Second mortgage is paid off

| EASIS:179, 506.06 | Tax basis |
| :---: | :---: |
| + SCAF IMF:8.799.90 | Total capital improvements |
| +TRAHS COST: 16.814.80 | Transaction costs |
| -EELI IEF : 38.474 .60 | Total building depreciation |
| -EPER IEP : $5,000.60$ | Total personal property depreciation |
| =TAF ERSIS:161:640.10 | Tax basis |
| CELI IEP:36,474,66 | Total building depreciation |
| + EPER IEF $5,060.60$ | Total personal property depreciation |
| -EELT SL:34,875,00 | Building depreciation using straight line depreciation |
| -EPER SL: 5, 006.60 | Personal property depreciation using straight line depreciation |
| =EXCESS $7,599.60$ | Total excess depreciation |
|  | Marginal tax rate |
| =THL:1.799.80 | Tax on excess depreciation |
| SHLE:240,211.49 | Selling price |
| -TAL EASIS 161.640.10 | Tax basis |
| =GALH 78.571.39 | Gain |
| -EXCESS:3,599.60 | Excess depreciation |
| =CAF GMIH:74:971.79 | Total capital gain |
| *CTO\% 30. $\mathrm{B0}$ | Capital gain tax rate |
| =TAX:22,491.54 | Tax on capital gain |
| ShLE 240.211.49 | Selling price |
| -TRAHS COST: 16,814:80 | Transaction costs |
| -REH EAL:59,838,83 | Remaining balance on mortgage(s) |
| =PROCEEIS:163,557.96 | Proceeds |
| -ETAR:24.291.34 | Total tax (from excess depreciation and capital gain) |
| = ATPR:139.266.52 | After-tax proceeds |
| +HTCF:5,591.52 | After-tax cash flow (this year) |
| = EATCF : 1444858.64 | Total after-tax cash flow upon sale |
| EETY MLILIL.50 | Equity yield (as a percent) |

## GRADUATED PAYMENT MORTGAGE

The Graduated Payment Mortgage is designed to meet the needs of young homebuyers who currently cannot afford high mortgage payments, but who have the potential of increased earnings in the years to come.

Under the GPM plan, the payments increase by a fixed percentage at the end of each year for a specified number of years. Thereafter, the payment amount remains constant for the remaining life of the mortgage.

The result is that the borrower pays a reduced payment (a payment which is less than a traditional mortgage payment) in the early years, and in the later years makes larger payments than he would with a traditional loan. Over the entire term of the mortgage, the borrower would pay more than he would with conventional financing.

Given the term of the mortgage (in years), the annual interest rate, the loan amount, the number of payments per year, the percentage that the payments increase, and the number of years that the payments increase, this program determines the periodic payments and the remaining balance at the end of each year. The cash flow sign convention is followed.

## Prompted Operation

(Minimum size 022)
After executing GPMT, the following user prompts will be displayed. After each response, press R/S to continue.

TERM=? What is the total term of the mortgage (in years)?
INTEREST\%=? What is the annual interest rate (in percent)?
LOAN=? What is the amount of the loan?
NO. PMT/YR=? How many payments are made each year?
\% INCR=? How much do the payments increase each year (in percent)?

YRS GRAD=? How many years do the payments graduate (increase)?

## Example 1:

A $\$ 45,000,30$-year mortgage at $11 \frac{1}{4} \%$ interest has monthly payments increasing $5 \%$ each year for 5 years after the first year, with a constant payment thereafter. What is the amount of each payment and the remaining balance at the end of each year?

## Keystrokes

XEQ ALPHA SIZE ALPHA 022
XEQ ALPHA GPMT ALPHA
30 R/S
$11.25 \mathrm{R} / \mathrm{S}$
45000 R/S
$12 R / S$
$5 \mathrm{R} / \mathrm{S}$
$5 \mathrm{R} / \mathrm{S}$
$R / S$
R/S
R/S
$R / S$
R/S
R/S
$R / S$
R/S
R/S
R/S
R/S

## Display

TERM=?
INTEREST\%=?
LOAN=?
NO. $P M T / Y R=$ ?
\% INCR=?
YRS GRAD=?
PMT1 $=-365.76$
BAL1 $=-45,709.21$
PMT2=-384.05
BAL2 $=-46.271 .29$
PMT3=-403.25
BAL3 $=-46,657.31$
PMT4=-423.41
BAL4 $=-46,834.28$
PMT5=-444.58
BAL5 $=-46.764 .66$
PMT6=-466.81
BAL6 $=-46.405 .84$

## WRAP-AROUND MORTGAGE

A wrap-around mortgage is essentially the same as a refinancing mortgage, except that the new mortgage is granted by a different lender, who assumes the payments on the existing mortgage, which remains in full force. The new (second) mortgage is thus "wrapped around" the existing mortgage. The "wrap-around" lender advances the net difference between the new (second) mortgage and the balance on the existing mortgage in cash to the borrower, and receives as net cash flow the difference between the debt service on the new (second) mortgage and the debt service on the existing mortgage.

The wrap-around mortgage is implemented when one lender (often a seller) uses the positive leverage from an existing loan by another lender, to create a higher yielding loan. It is also used when it is impractical to alter existing financing, or when the seller does not want a second or third mortgage at current interest rates. Using a wraparound mortgage can provide measurable benefit to both the buyer and the seller.

Given the particulars of the original mortgage (remaining balance, remaining term, periodic payment or periodic interest, and balloon payment), and the particulars of the wrap-around mortgage (amount of the wrap-around, term, periodic interest, balloon payment, and any points (fees) which may be charged), this program calculates the periodic yield to the lender of a wrap-around mortgage. Then, if this yield is not acceptable, a desired periodic yield can be input, and the necessary payment to achieve this yield is calculated.* The quoted interest rate on the wrap-around mortgage (the interest rate which the borrower is charged) is then displayed. An option is also available to summarize the net cash flows in the entire transaction (XEO SUMMARY).

This program assumes that the term of the original mortgage does not exceed the term of the wrap-around mortgage. The sign convention is not used.

[^6]
## Prompted Operation

(Minimum size 035)
After executing WRAP the following user prompts will be displayed. After each response, press R/S to continue.

PV1ST=? What is the remaining balance of the underlying mortgage?

TERM 1ST=? What is the total remaining term of the underlying mortgage?

PMT1ST=? What is the periodic payment of the underlying mortgage? Press R/S if the payment is not known.

INT 1ST=? What is the periodic interest rate of the underlying mortgage? This prompt occurs if a payment amount was not keyed in.

| BAL $1 S T=?$ | What is the balloon payment of the underlying <br> mortgage? |
| :--- | :--- |
| WRAP AMT=? | What is the amount of the wrap-around mortgage? |

WRAP TERM=? What is the term of the wrap-around mortgage?
INT WRAP=? What is the periodic interest rate of the wrap-around mortgage?
BAL WRAP=? What is the balloon payment of the wrap-around mortgage?
NO. POINTS=? How many points are charged on the wrap-around mortgage?
YIELD=? What is your desired yield on the entire transaction?

## Example 1:

A mortgage loan on an income property currently has a remaining balance of $\$ 150,681.54$. When the loan originated 8 years ago, it had a 20-year term with full amortization in level monthly payments at $8 \%$ interest.

A lender has agreed to "wrap" a $\$ 250,000$ second mortgage at $11 \%$, with full amortization in level monthly payments over 12 years. What is the yield (IRR) to the lender on net cash advanced, assuming that no points are charged?

Keystrokes
XEQ ALPHA SIZE ALPHA 035

Display

PV1ST=?
TERM 1ST=?
PMT 1ST=?

Minimum size
Keystrokes
(R/S

8 ENTERT $12 \square \mathrm{R} / \mathrm{S}$
$0 \mathrm{R} / \mathrm{S}$
$250000 \mathrm{R} / \mathrm{S}$
$144 \mathrm{R} / \mathrm{S}$
$11 \mathrm{ENTERT} 12 \square \mathrm{R} / \mathrm{S}$
$0 \mathrm{R} / \mathrm{S}$
$0 \mathrm{R} / \mathrm{S}$
12 X
$\left.\begin{array}{l|l}\begin{array}{l}\text { Display } \\ \text { INT } 1 S T=?\end{array} & \begin{array}{l}\text { Since the } \\ \text { payment wasn't } \\ \text { entered, what is } \\ \text { the periodic }\end{array} \\ \text { interest rate? }\end{array}\right\}$

What monthly payment must be received to achieve a $17 \%$ overall yield? What interest rate is the borrower paying?
Keystrokes
R/S
17 ENTERT $12 \square R / S$
$R / S$
$12 \square$

| Display |  |
| :--- | :--- |
| YIELD $=?$ <br> WPMT $=3.251 .86$ | Payment on <br> wrap-around |
| INT WRAP $=0.98$ | Periodic interest <br> on wrap-around |
| 11.79 | Annual wrap- <br> around interest <br> rate |

Summarize the entire transaction.

## Keystrokes

XEQ ALPHA SUMMARY ALPHA

R/S
R/S

R/S
R/S

## Display

| NEWS=99,318.46 | Net cash <br> advanced |
| :--- | :--- |
| TERM1=144.00 | Net cash flow |
| PMT1=1.620.80 | received |
| BAL1=0.00 | Annual yield is |
| YIELD=1.42 | $17 \%$ |

## Example 2:

A customer has an existing mortgage with a balance of $\$ 64,531.63$, a remaining term of 10 years, and a $\$ 716.43$ monthly payment. He wishes to obtain a $\$ 125,000,11 \frac{1}{2} \%$ wrap-around with monthly payments and a balloon payment at the end of the $180^{\text {th }}$ month of $\$ 75,000$. If you, as a lender, accept the proposal and charge 2 points on the wrap-around mortgage, what is your rate of return?

Keystrokes

XEQ ALPHA WRAP ALPHA
$64531.63 \mathrm{R} / \mathrm{S}$
10 ENTERT $12 \times R / S$
$716.43 \mathrm{R} / \mathrm{S}$
0 R/S
125000 R/S
180 R/S
11.5 ENTER $12 \div \mathrm{R} / \mathrm{S}$
$75000 \mathrm{R} / \mathrm{S}$
2 R/S
$12 x$

Display

PV 1ST=? TERM 1 ST=?
PMT 1ST=?
BAL 1 ST=?
WRAP AMT=?
WRAP TERM=?
INT WRAP=?
BAL WRAP=?
NO. POINTS=?
YIELD=1.22 Monthly yield
14.69

Annual yield

## HOME OWNER'S EQUITY ANALYSIS

This program is designed to provide the home owner or purchaser of real estate with useful information on future monthly payments, accumulated equity, and tax deductions. The user keys in the purchase price, the down payment, the interest rate and the term of the mortgage. The program calculates and displays the basic monthly mortgage payment.

Further inputs include the expected percent appreciation or depreciation in value per year, the number of months remaining in the first year of analysis, the amount of property taxes (in dollars), and the anticipated yearly percentage increase in taxes.

The program then calculates, for each year of ownership, the estimated monthly payment (covering mortgage payments and taxes), the total accumulated equity in the property, and the tax deduction due to interest and property taxes for the year.

|  |  |  |  | SIZE: 021 |
| :---: | :---: | :---: | :---: | :---: |
| STEP | INSTRUCTIONS | INPUT | FUNCTION | display |
| 1. | Initialize the program. |  | XEQ EQ | PRICE=? |
| 2. | Key in the price of the home. | Price | R/S | DOWN PMT=? |
| 3. | Key in the amount of the down payment (in dollars). | Down Payment | $\mathrm{R} / \mathrm{S}$ | \% INT=? |
| 4. | Key in the mortgage interest rate (\% per year). | Int (\%) | $R / S$ | TERM=? |
| 5. | Key in the term of the mortgage (in years). The monthly payment to amortize the mortgage is displayed. | Term | R/S | PMT= |
| 6. | Key in the anticipated yearly appreciation in the value of the house (as a \%). | Apprec (\%) | R/S** | \% APPREC=? MONS YR $1=$ ? |
| 7. | Key in the number of months remaining in the first tax year. | \# Months | R/S | TAXES=? |
| 8. | Key in the current property taxes. | Taxes | R/S | \% INCREASE=? |
| 9. | Key in the expected percent increase or decrease in taxes each year. | Increase (\%) |  |  |
|  | - Press R/S if you are not using a printer. |  |  |  |


| STEP | INSTRUCTIONS | INPUT | FUNCTION | DISPLAY |
| :---: | :---: | :---: | :---: | :---: |
| 10. <br> 11. <br> 12. | Calculate the following information for each year： <br> Monthly payment Total equity Total tax deductions Appreciated value of the property <br> For succeeding years，go to step 10. <br> For a new case，go to step 1. <br> －Press R／S if you are not using a printer． |  | $\mathrm{R} / \mathrm{S}$ <br> $\mathrm{R} / \mathrm{S}{ }^{*}$ <br> $\mathrm{R} / \mathrm{S}^{*}$ <br> $\mathrm{R} / \mathrm{S}^{*}$ <br> $\mathrm{R} / \mathrm{S}^{*}$ | YEAR 1 <br> 之PMT＝ <br> 之EO＝ <br> 之DED＝ <br> VAL＝ |

## Example：

A family is contemplating purchase of a new home priced at $\$ 62,500$ ． They will be able to pay $\$ 12,500$ as a down payment and can obtain a 30 year mortgage for the balance at $13.25 \%$ ．Property values in the area have been appreciating at about $10 \%$ per year．Property taxes for the current year are $\$ 1437.50$ and have been increasing at the rate of about $8 \%$ per year．The buyer will take possession of the house with 10 months remaining in the tax year．What is the basic mortgage payment？ Calculate the expected total monthly payments，total equity in the property and income tax deductions for years 1 through 3 of the mortgage．

Keystrokes
XEQ ALPHA SIZE ALPHA 021
XEQ ALPHA EQ ALPHA

62500 R／S
$12500 \mathrm{R} / \mathrm{S}$
$13.25 \mathrm{R} / \mathrm{S}$
$30 \mathrm{R} / \mathrm{S}$
R／S
$10 R / S$
$10 \mathrm{R} / \mathrm{S}$
$1437.50 \mathrm{R} / \mathrm{S}$
8 R／S
R／S
R／S
R／S

## Display

## PRICE＝？

DOWN PMT＝？
\％INT＝？
TERM＝？
PMT＝562．89
\％APPREC＝？
MONS YR 1 ＝？
TAXES＝？
\％INCREASE＝？
YEAR 1
इPMT＝682．68
$\Sigma E Q=17.821 .93$
$\Sigma D E D=6,713.22$

54 Home Owner's Equity Analysis

Keystrokes

| $\mathrm{R} / \mathrm{S}$ |
| :--- |
| $\mathrm{R} / \mathrm{S}$ |
| $\mathrm{R} / \mathrm{S}$ |
| $\mathrm{R} / \mathrm{S}$ |
| $\mathrm{R} / \mathrm{S}$ |
| $\mathrm{R} / \mathrm{S}$ |
| $\mathrm{R} / \mathrm{S}$ |
| $\mathrm{R} / \mathrm{S}$ |
| $\mathrm{R} / \mathrm{S}$ |
| $\mathrm{R} / \mathrm{S}$ |
| $\mathrm{R} / \mathrm{S}$ |

Display
$V A L=67,708.33$
YEAR 2
$\Sigma P M T=692.27$
$\Sigma E Q=24,746.62$
$\Sigma D E D=8,153.33$
$V A L=74,479.17$
YEAR 3
$\Sigma P M T=702.62$
$\Sigma E Q=32,370.06$
$\Sigma D E D=8,255.86$
$V A L=81,927.08$

## THE RENT OR BUY DECISION

The question of whether to rent or purchase a residence is not always easy to answer, especially when the time period over which you would own or rent the house is short. This program performs an analysis which could be helpful in reaching a decision. In essence, it calculates a yield or rate of return on the proposed investment and its estimated resale value, and compares this yield with that which could be obtained by renting a residence and investing the down payment and monthly payment differences in a savings account or other investment opportunity. The program takes into account the tax advantages obtained by a home owner on property taxes and mortgage interest.

The user keys in the price of the house, the down payment, the mortgage data, the holding period, the expected rate of appreciation of the house, the real estate costs, the total marginal income tax rate,* the monthly property taxes, the estimated monthly maintenance expenses, the rent which would have to be paid for a satisfactory alternative residence, and the percentage interest which could be obtained on an alternative investment (in a bank).

The program then calculates the monthly mortgage payment, the anticipated market value, the Net Cash Proceeds upon Resale (NCPR), ${ }^{\dagger}$ the annual yield on the investment in the house, and the actual cash gain or loss on the investment in the house. In addition, the user may calculate the actual market value, the annual appreciation which would be necessary to equal the proceeds of the alternate investment, and the amount of rent to "break-even". At the "break-even" rent, the yield on the investment is equal to the bank's interest rate, and the gain is zero.
At the end of the calculation, the user is able to change the input data, and may re-do the computation for a different situation. The sign convention is not used.

[^7]|  |  |  |  | SIZE: 029 |
| :---: | :---: | :---: | :---: | :---: |
| STEP | INSTRUCTIONS | INPUT | FUNCTION | DISPLAY |
| 1. | Initialize the program. |  | XEQ RENT | PRICE=? |
| 2. | Key in the price of the house. | Price | R/S | DOWN PMT=? |
| 3. | Key in the amount of the down payment. | Down Payment | $\mathrm{R} / \mathrm{S}$ | \% INT=? |
| 4. | Key in the mortgage interest rate (\% per year). | Int (\%) | R/S | TERM=? |
| 5. | Key in the term of the mortgage (in years). | Term | R/S | NO. YEARS=? |
| 6. | Key in the number of years you intend to occupy the house. | Years | R/S | \% APPREC=? |
| 7. | Key in the expected appreciation in value of the house (\% annually). | Apprec (\%) | R/S | \% COMM $=$ ? |
| 8. | Key in the percentage commission charged to sell the house. | Comm (\%) | R/S | CLOSE COST=? |
| 9. | Key in the closing costs paid on the purchase of the house. | Costs | R/S | TAX RATE=? |
| 10. | Key in the marginal income tax rate (in percent). | \% Tax | R/S | TAXES=? |
| 11. | Key in the anticipated monthly property taxes. | Taxes | R/S | MAINT=? |
| 12. | Key in the anticipated monthly cost of maintenance and insurance. | Maint. | R/S | RENT=? |
| 13. | Key in the monthly rent for an acceptable alternative residence. | Rent | R/S | BANK \% INT=? |
| 14. | Key in the annual rate of interest (as \%) available if you were to invest the money in some form of savings. | Bank \% Int | R/S | CHANGE? |
| 15. | To change data, press $R / S$. Otherwise, press $N$ R/S to calculate the monthly payment to amortize the mortgage. |  |  | PMT $=$ |
| 16. | Calculate the anticipated resale value at the end of occupancy. |  | R/S* | VAL $=$ |
| 17. | Calculate the total real estate commission. |  | R/S * | -COMM: |
| 18. | Calculate the mortgage balance. |  | R/S* | -RBAL: |
| 19. | Calculate the Net Cash Proceeds upon Resale. |  | R/S * | NCPR $=$ |
| 20. | Calculate the annual rate of return on the investment in the house. |  | $\mathrm{R} / \mathrm{S}$ | YLD $=$ |
|  | *Press R/S if you are not using a printer. |  |  |  |


| STEP | INSTRUCTIONS | INPUT | FUNCTION | dISPLAY |
| :---: | :---: | :---: | :---: | :---: |
| 21. | Calculate the total dollar gain (if positive) or loss (if negative) from buying a house as compared to renting. |  | R/S* | \$GAIN= |
| 22. | Calculate the market value of the house at which you would obtain the same annual return as on the investment at step 14. |  | R/S | VAL $=$ |
| 23. | Calculate the yearly percent appreciation at which you would obtain this value. |  | R/S* | \% APPREC= |
| 24. | Calculate the monthly rent to breakeven. |  | $\mathrm{R} / \mathrm{S}$ | B-E RENT $=$ |
| 25. | To change data and repeat the calculation, respond to the prompt by pressing R/S. <br> Otherwise, press $N$ R/S. <br> - Press R/S if you are not using a printer. |  | $\begin{array}{\|l\|} \hline \mathrm{R} / \mathrm{S} \\ \hline \mathrm{R} / \mathrm{S} \\ \hline \end{array}$ | CHANGE? PRICE $=$ |

## Example:

You are being transferred for 2 years to a distant city and are faced with the decision of whether to rent or buy a house. A quick survey of the housing market indicates that you can purchase an acceptable house for $\$ 65,000$ with a $\$ 13,000$ down payment on a 30 year mortgage at $13 \%$ interest. The closing costs would be about $\$ 950$ and a real estate agency would charge a $6 \%$ commission on resale. Houses in the area are appreciating at $10 \%$ per year. Property taxes would be about $\$ 110$ per month, and you estimate that maintenance would cost an additional \$70 per month.
An alternative would be to rent a similar dwelling at $\$ 550$ per month and invest the difference between purchase costs and rent at $7.75 \%$ interest.

Your marginal income tax rate is $25 \%$ federal and $5 \%$ state. Which alternative is more financially attractive?

## Keystrokes

XEQ ALPHA SIZE ALPHA 029
XEQ ALPHA RENT ALPHA
65000 R/S
13000 R/S
$13 \mathrm{R} / \mathrm{S}$

Display

PRICE=?
DOWN PMT=?
\% INT=?
TERM=?

Keystrokes
$30 \mathrm{R} / \mathrm{S}$
2 R/S
$10 \mathrm{R} / \mathrm{S}$
6 R/S
950 R/S
30 R/S
$110 \mathrm{R} / \mathrm{S}$
70 R/S
550 R/S
$7.75 \mathrm{R} / \mathrm{S}$
$N$ R/S
R/S
R/S
$R / S$
R/S
$R / S$
R/S

Display
NO. YEARS=?
\% APPREC=?
\% COMM=?
CLOSE COST=?
TAX RATE=?
TAXES=?
MAINT=?
RENT=?
BANK \% INT=?
CHANGE?
PMT=575.22
$V A L=78,650.00$
-COMM:4,719.00
-RBAL:51,676.19
NCPR $=22,254.81$
$Y L D=18.63$
$\$ G A I N=4,119.73$

By purchasing the house, you would gain $\$ 4,119.73$ over an alternate investment at $7.75 \%$ interest.

| Keystrokes | Display |
| :--- | :--- |
| $R / S$ | VAL $=74,267.31$ |
| $R / S$ | S APPREC $=6.89$ |
| $R / S$ | $B-E$ RENT $=390.75$ |

To obtain the same return as the alternate investment, the house would have to appreciate $6.89 \%$ or obtain a market value of $\$ 74,267.31$. The amount of rent to "break-even" is $\$ 390.75$.

## PRICE AND YIELD OF A MORTGAGE TRADED AT A DISCOUNT/PREMIUM



Mortgages may be bought and/or sold at prices lower (discounted) or higher (at a premium) than the remaining balance of the loan at the time of purchase. Given the remaining term of the mortgage, the periodic payment, the amount of the balloon payment, and either the desired yield or the purchase price, the remaining variable may be calculated using the Compound Interest program.

There are many possible combinations of discounted mortgage problems, so it is important for you to ask yourself "What are the cash flows?" You might wish to review the section Compound Interest Solutions: The Cash Flow Diagram and Sign Convention.

## Example 1:

A lender wants to induce the borrower to prepay a low interest rate loan. The interest rate is $5 \%$ with 6 years ( 72 payments) remaining of $\$ 137.17$ and a balloon payment at the end of the sixth year of $\$ 2000$. If the lender is willing to discount the future payments $7 \frac{1}{2} \%$, how much would the borrower need to prepay the note?

Keystrokes
XEQ ALPHA SIZE ALPHA 013 XEQ ALPHA $\$$ ALPHA

72 A
7.5 B

## Display

| 0.00 | Clear |
| :---: | :---: |
|  | "financial" registers |
| END | Optional step to clear flag 0 and set END mode |
| $N=72.00$ | Months |
| $\mathrm{f}=0.63$ | Monthly |
|  | interest |

60 Discounted Mortgages

Keystrokes
137.17 D

2000 E

C

Display
$P M T=137.17 \quad$ Monthly payment received by lender Balloon payment received Amount necessary to prepay the note

## Example 2:

A $91 / 2 \%$ mortgage with 28 years remaining may be acquired which has a remaining balance of $\$ 49,350$. Determine the price to pay for this mortgage if the desired annual yield is $15 \%$. (Since the payment amount is not given, it must be calculated.)

| Keystrokes | Display |  |
| :---: | :---: | :---: |
| E | 0.00 | Clear |
|  |  | "financial" registers |
| C | END | Set END mode |
| 28 A | $N=336.00$ | Months |
| 9.5 B | $\boldsymbol{I}=0.79$ | Monthly |
|  |  | interest rate (\%) |
| 49350 CHS C | $P V=-49,350.00$ |  |
| D | $P M T=420.40$ | Monthly |
|  |  | payment to be received |
| 15 B | $f=1.25$ | Desired |
|  |  | monthly |
|  |  | interest rate (\%) |
| C | $P V=-33.114 .65$ | Purchase price |
|  |  | to achieve a $15 \%$ |
|  |  | annual yield |

## Example 3:

Find the annual yield of a $7 \%, 20$ year, $\$ 95,000$ mortgage purchased for $\$ 75,000$. Monthly payments are being made.

| Keystrokes | Display |  |
| :---: | :---: | :---: |
| E | 0.00 | Clear |
|  |  | "financial" |
|  |  | registers. |
| C | END | Set END mode |
| 20 A | $N=240.00$ | Total term |
| 7 B | $\mathrm{l}=0.58$ |  |
| 95000 CHS | $P V=-95,000.00$ | Loan amount |
| D | $P M T=736.53$ | Monthly payment |
| 75000 CHS C | $P V=-75,000.00$ | Purchase price |
| B | $\mathrm{f}=0.85$ | Monthly yield (\%) |
| $12 \times$ | 10.26 | Annual yield (\%) |

## APR OF A LOAN WITH FEES



Borrowers are often charged fees in connection with the issuance of a mortgage, which effectively raises the effective yield on the mortgage to the lender. The actual amount received by the borrower ( PV ) is reduced, while the periodic payments remain the same. Given the term or life of the mortgage, the interest rate, the mortgage amount, and the basis of the fee charge (how the fee is calculated), the true Annual Percentage Rate may be calculated using the Compound Interest program.

Since there are several possible combinations of mortgages with fees, it is important to understand what the cash flows are, and when they occur. You might wish to review the section Compound Interest Solutions: The Cash Flow Sign Convention.

## Example 1:

A borrower is charged 2 points for the issuance of his mortgage. If the mortgage amount is $\$ 50,000$ for 30 years, and the interest rate is $11.5 \%$ per year, with monthly payments, what annual percentage rate is the borrower paying? ( 1 point is equal to $1 \%$ of the mortgage amount.)

Keystrokes
XEQ ALPHA SIZE ALPHA 013 XEQ ALPHA $\$$ ALPHA $\square$
$\square C$
30 A
11.5 B

## Display

| 0.00 | Clear <br> "financial" <br> registers |
| :---: | :---: |
| END | Set END mode |
| $N=360.00$ | Total term (months) |
| $\boldsymbol{I}=0.96$ | Monthly interest rate (\%) |
| $P V=50,000.00$ | Loan amount |


| Keystrokes | Display |  |
| :---: | :---: | :---: |
| D | $P M T=-495.15$ | Monthly payment |
| RCL C 2 \% * - C | $P V=49,000.00$ | Actual amount received by borrower |
| B | $1=0.98$ | Monthly interest rate (\%) |
| $12 \times$ | 11.76 | Annual percentage rate |

## Example 2:

Assuming Example 1 has just been done, calculate the APR if the mortgage fee is $\$ 750$ instead of a percentage.


* \% is calculated by pressing $H$.


## PRESENT VALUE OF AN INCREASING/DECREASING ANNUITY



The majority of real estate financing arrangements deal with equal periodic payments. It is possible however to consider a payment stream where the payment amounts change, such as income, repair costs or price changes reflecting inflation or deflation. The present worth of such a payment stream may easily be calculated by summing the present value of each individual payment. A quicker and easier way to sum the payments is possible using the Compound Interest program and the following keystrokes:

1. Clear the "financial" registers ( $\square$ ) and set to $E N D$ mode ( $\square$ ).
2. Key in the total number of payment periods; press A.
3. Key in the payment's percentage increase per period expressed as one plus the decimal interest rate; press ENTERT. If there is a percentage decrease, express it as one minus the decimal interest rate.
4. Key in the discount (interest) rate per period expressed as one plus the decimal interest rate; press $\% \mathrm{CH}$ B.
5. Press CLx/A.
6. Key in the starting payment; press $x=y \square \square$.*
7. Press $C$ to obtain the present value of the payment stream.*

Note: This procedure assumes that payments occur at the end of the period.

[^8]
## Example 1:

You are appraising a piece of income property which is providing increasing rents. Assuming a $7 \%$ rate of increase over the next 5 years, what is the present value of the income stream? Your discount rate is $12 \%$, and the rent for the first year is expected to be $\$ 8,500$.

Keystrokes


C

Display
0.00

END
$N=5.00$
1.07
$1=4.67$
$P M T=7,943.93$
$P V=-34,706.26$

Clear "financial" registers Set END mode Years Periodic increase factor (1+.07)
Adjustifor interest rate
Adjusted
starting payment
Present value of payments

When the payment amounts change by a constant amount instead of by a constant ratio, as in the previous problem, the following keystrokes will give the present value:

1. Clear the "financial" registers ( $\square$ ( $E$ ) and set to $E N D$ mode ( $\square C$ ).
2. Key in the periodic discount (interest) rate as a percent; press B.
3. Key in the starting payment; press ENTERT.
4. Key in the amount that the payment increases each period; press ENTERT.
5. Key in the periodic discount (interest) rate as a decimal; press $\div$ STO $10 \square$ D.
6. Key in the total number of payment periods; press ENTERT $\triangle$ RCL $10 \triangle$ CHS STO E.
7. Press $C$ to obtain the present value of the payments.

## Example 2:

If the rents in the previous example increase by $\$ 500$ each year, what is the present value of the payment stream?

| Keystrokes | Display |  |
| :---: | :---: | :---: |
| - E | 0.00 | Clear <br> "financial" <br> registers |
| $\square$ | END | Set END mode |
| 12 B | $1=12.00$ | Periodic discount (interest rate) |
| 8500 ENTER | 8,500.00 | Starting payment |
| 500 ENTERT | 500.00 | Periodic payment increase |
| $\begin{aligned} & .12 \square \text { STO } 10 \\ & \square D \end{aligned}$ | $P M T=12,666.67$ | Adjusted payment |
| 5 ENTERT A RCL 10 |  |  |
| $\triangle$ CHS STO E | -20,833.33 | Adjusted FV |
| C | $P V=-33.839 .11$ | Present value of the payments |

## FINANCIAL FORMULAS

## Compound Interest Solutions

$$
0=\mathrm{PV}+(1+\mathrm{Is}) \mathrm{PMT}\left[\frac{1-(1+\mathrm{I})^{-\mathrm{n}}}{\mathrm{I}}\right]+\mathrm{FV}(1+\mathrm{I})^{-\mathrm{n}}
$$

where
$\mathrm{s}=1$ given BEGIN
$\mathrm{s}=0$ given END

## Amortization Schedules

$$
\begin{aligned}
& \mathrm{INT}_{\mathrm{j}}=\mathrm{RND}^{\left[\mathrm{BAL}_{\mathrm{j}-1} \cdot \mathrm{i}\right] \times(\text { sign of } P M T)} \\
& \mathrm{PRN}_{\mathrm{j}}=\mathrm{PMT}_{\mathrm{j}}-\mathrm{INT}_{\mathrm{j}} \\
& \mathrm{BAL}_{\mathrm{j}}=\mathrm{BAL}_{\mathrm{j}-1}-\mathrm{PRN}_{\mathrm{j}}
\end{aligned}
$$

$\Sigma I N T=\sum_{\mathrm{j}=\mathrm{P} 1}^{\mathrm{P} 2} \mathrm{INT}_{\mathrm{j}}$
$\Sigma P R N=\sum_{\mathrm{j}=\mathrm{P} 1}^{\mathrm{P} 2} \mathrm{PRN}_{\mathrm{j}}$
For BEGIN payments
$\mathrm{INT}_{1}=0$
where:
$\mathrm{INT}_{\mathrm{j}}=$ interest portion of $\mathrm{j}^{\text {th }}$ payment
PRN ${ }_{j}=$ principal portion of $\mathrm{j}^{\text {th }}$ payment
$B A L_{j}=$ remaining balance after the $j^{\text {th }}$ payment
PMT = payment amount
$\mathrm{i}=$ interest rate (as a decimal)
IINT = accumulated interest from P1 to P2 inclusive
$\Sigma \mathrm{PRN}=$ accumulated principal from P1 to P2 inclusive

## Internal Rate of Return

Solve for IRR

$$
0=\sum_{\mathrm{j}=1}^{\mathrm{k}} \mathrm{CF}_{\mathrm{j}}\left[\frac{1-(1+\mathrm{IRR})^{-\mathrm{n}_{\mathrm{j}}}}{\text { IRR }}\right]\left[(1+\mathrm{IRR})^{-\sum_{<\mathrm{j}} \mathrm{n}}\right]-\mathrm{CF}_{0}
$$

where:
$\mathrm{n}=$ number of cash flows
$\mathrm{CF}_{\mathrm{j}}=\mathrm{j}^{\text {th }}$ cash flow

Net Present Value

$$
\mathrm{NPV}_{\mathrm{k}}=\mathrm{CF}_{0}+\sum_{\mathrm{k}=1}^{\mathrm{n}} \frac{\mathrm{CF}_{\mathrm{k}}}{(1+\mathrm{i})^{\mathrm{k}}}
$$

where:
$\mathrm{CF}_{\mathrm{k}}=\mathrm{k}^{\text {th }}$ cash flow
$i=$ discount rate (as a decimal)

## Modified Internal Rate of Return

$$
\operatorname{MIRR}=\left[\left(\frac{\mathrm{NFV}(\text { pos. CFs })}{-\mathrm{NPV}(\text { neg.CFs })}\right)^{1 / \mathrm{n}}-1\right] 100
$$

## Straight Line Depreciation

$$
\operatorname{Dep}_{\mathrm{n}}=\frac{\mathrm{A}-\mathrm{S}}{\mathrm{~N}}
$$

For Partial First Year

$$
\begin{aligned}
& \operatorname{Dep}_{1}=\frac{\mathrm{A}-\mathrm{S}}{\mathrm{~N}} \cdot\left(\frac{\mathrm{Y}_{1}}{12}\right) \\
& \operatorname{Dep}_{\mathrm{n}+1}=\mathrm{B}_{\mathrm{n}}
\end{aligned}
$$

Sum-of-the-Years'-Digits Depreciation

$$
\operatorname{Dep}_{\mathrm{n}}=\left(\mathrm{d}_{\mathrm{n}-1}-\mathrm{x}_{\mathrm{n}-1}\right)+\mathrm{d}_{\mathrm{n}}\left(\frac{\mathrm{Y}_{1}}{12}\right)
$$

where:

$$
\begin{aligned}
& \mathrm{x}_{\mathrm{n}}=\mathrm{d}_{\mathrm{n}}\left(\frac{\mathrm{Y}_{1}}{12}\right) \\
& \mathrm{d}_{\mathrm{n}}=\frac{\mathrm{N}-\mathrm{n}+1}{(\mathrm{INT}(\mathrm{~N})+1)\left(\mathrm{N}-\frac{\mathrm{INT}(\mathrm{~N})}{2}\right)} \cdot(\mathrm{A}-\mathrm{S})
\end{aligned}
$$

For Partial First Year

$$
\begin{aligned}
& \operatorname{Dep}_{1}=d_{1}\left(\frac{Y_{1}}{12}\right) \\
& \operatorname{Dep}_{n+1}=d_{n}-x_{n}
\end{aligned}
$$

## Declining Balance Depreciation

$$
\operatorname{Dep}_{\mathrm{n}}=\mathrm{B}_{\mathrm{n}-1}\left(\frac{\mathrm{R}}{\mathrm{~N}(100)}\right)
$$

For Partial First Year

$$
\operatorname{Dep}_{1}=A\left(\frac{\mathrm{R}}{\mathrm{~N}(100)}\right)\left(\frac{\mathrm{Y}_{1}}{12}\right)
$$

where:
n = year number
$\mathrm{N}=$ useful life
$S$ = salvage value
A = starting book value
$\mathrm{B}_{\mathrm{n}}=$ remaining depreciable amount
$\mathrm{Y}_{1}=$ number of months in partial first year
$\mathrm{R}=$ declining balance rate (in percent)

## Income Property Analysis

Net Operating Income $=$ Gross Income $(1+G)^{N-1}\left(1-\frac{\text { Vacancy }}{100}\right)$

- Operating Expenses $(1+G)^{\mathrm{N}-1}$

Taxable Income $=$ Net Operating Income - Yearly Mortgage Interest

- Yearly Depreciation

Tax Liability $=($ Taxable Income $)($ Marginal Tax Rate $)$

Before Tax Cash Flow $=$ Net Operating Income

- Total Yearly Mortgage Payments
- Capital Improvements $(1+G)^{\mathrm{N}-1}$

After Tax Cash Flow = Before Tax Cash Flow - Tax Liability
Tax Basis $=$ Basis + Capital Improvements $\left[\frac{(1+G)^{N-1}-1}{G}\right]$

+ Purchase Price $(1+A)^{N}\left(\frac{\text { Transaction Costs }}{100}\right)$
- Total Depreciation

Excess Depreciation $=$ Total Depreciation - Straight Line Depreciation
Capital Gain $=$ Purchase Price $(1+\mathbf{A})^{\mathrm{N}}-$ Tax Basis

- Excess Depreciation

Before Tax Proceeds $=$ Purchase Price $(1+A)^{N}\left[1-\frac{\text { Transaction Costs }}{100}\right]$

- Remaining Balance Mortgage(s)

After Tax Proceeds $=$ Before Tax Proceeds

$$
\begin{aligned}
& \text { - Excess Depreciation }\left(\frac{\text { Marginal Tax Rate }}{100}\right) \\
& \text { - Capital Gain }\left(\frac{\text { Capital Gain Tax }}{100}\right)
\end{aligned}
$$

where:
$\mathrm{G}=$ Growth rate (as decimal)
A = Appreciation rate (as decimal)
N = Current year

## Wrap-Around Mortgage

$$
\begin{aligned}
\mathrm{PV}_{2}-\mathrm{PV}_{1} & =\frac{\mathrm{PMT}_{2}\left[1-(1+\mathrm{I})^{-\mathrm{n}_{2}}\right]}{\mathrm{I}}-\frac{\mathrm{PMT}_{1}\left[1-(1+\mathrm{I})^{-\mathrm{n}_{1}}\right]}{\mathrm{I}} \\
& +\mathrm{FV}_{2}(1+\mathrm{I})^{-\mathrm{n}_{2}}-\mathrm{FV}_{1}(1+\mathrm{I})^{-\mathrm{n}_{1}}
\end{aligned}
$$

where:
$\mathrm{I}=$ yield (as decimal)

## Graduated Payment Mortgage

$$
\begin{aligned}
\mathrm{PV}=\mathrm{PMT}_{1} & \left\{\frac{1-(1+\mathrm{I})^{-\mathrm{A}}}{\mathrm{I}}\right]\left[\frac{(1+\mathrm{Q})^{\mathrm{B}}-1}{\mathrm{Q}}\right] \\
& \left.+\frac{(1+\mathrm{C})^{\mathrm{B}}\left[\frac{1-(1+\mathrm{I})^{-(\mathrm{n}-\mathrm{AB})}}{\mathrm{I}}\right]}{(1+\mathrm{I})^{\mathrm{AB}}}\right\}
\end{aligned}
$$

where:
$\mathrm{Q}=\frac{1+\mathrm{C}}{(1+\mathrm{I})^{\mathbf{A}}}-1$
$A=$ number of payments per year
$B=$ number of years that payments increase
$\mathrm{C}=$ percent increase in periodic payment (as decimal)
$\mathrm{PMT}_{1}=$ amount of first payment

## Home Owner's Equity Analysis

$\Sigma$ PMT $=$ PMT + Taxes $/$ month
$\Sigma E Q=$ DOWN PMT + Accumulated Principal + Appreciation
$\Sigma \mathrm{DED}=$ Interest + Taxes $/$ year
where:
$\Sigma P M T=$ monthly payment
PMT = mortgage payment
$\Sigma \mathrm{EQ}=$ total equity
$\Sigma \mathrm{DED}=$ total tax deductions
DOWN PMT = down payment

## The Rent or Buy Decision

Market Value $=\operatorname{PRICE}(1+i)^{n}$
where:
$\mathrm{i}=$ appreciation per year (as decimal)
$\mathrm{n}=$ number of years
Net Cash Proceeds on Resale $=$ Market Value - Mortgage Balance

- Commission

The interest rate is obtained by solving the financial (compound interest) equation for I using:
$\mathrm{N}=$ number of years house is owned
$\mathrm{PV}=$ down payment + closing costs
PMT = mortgage payment + taxes + maintenance + investment interest - rent - (\% tax) (interest + taxes + investment interest)
$\mathrm{FV}=$ net cash proceeds on resale
Annual interest rate $=12 \times \mathrm{I}$
At the break-even rent, $I=$ bank's interest rate.

|  |  | $N$ | $N$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 元 | N | － | － | $\sim$ | $N$ |
| ．9 ${ }^{\text {a }}$ | － | 0 | $\bigcirc$ | 즌 | － |
| － |  | $\times$ | $\times$ |  |  |

$\quad$ Flags
00－Begin
05－Used in ।＊
21－Printer
27－USER mode
00－BEGIN
05－Used
01－Groups
05－IRR
06－MIRR
07－Changes
08－2
21－Printer pass
01－Cross－over
06－Partial ${ }^{\text {st }}$ year
07－X－over occurred
21－Printer
01－List \＆change
02－Change
04－Bld．depr．

PROGRAM DATA


은든

| $N$ |
| :--- |
| 0 |
| 0 |

\＃Regs．to
COPY
111
ナ $\quad \odot へ \underset{\sim}{\infty}$
へ
옹

Program
＊AMORT
IRR，MIRR，NPV
＊IRR
＊MIRR
DEPR
IPA
CALC
SALE



Fix 0/Fix 2
$\stackrel{N}{\text { 즌 }}$
N
Display
Format

SUBROUTINES
This table provides the information necessary to use various portions of the Real Estate Application Module as subroutines.
Flags
SF OO-bEGIN
CF OO-END
05-used by ${ }^{*}$
은
SF OO-BEGIN
SF 05-BEGIN $\left(1^{\text {st }}\right.$
خ
$\quad$ Registers Used
$\mathrm{R}_{00}-\mathrm{R}_{09}$
$R_{\text {09-Increment/1000 }}$
$\mathrm{R}_{10}$-RBAL
$\mathrm{R}_{12}$-Start + End/1000
$\quad$ Initial Registers
$\mathrm{R}_{00} \mathrm{O}(\mathrm{END}) / 1$ (BEGIN)
$\mathrm{R}_{01} \mathrm{~N}$
$\mathrm{R}_{02} \mathrm{I}$ (as \%)
$\mathrm{R}_{03} \mathrm{PV}$
$\mathrm{R}_{04} \mathrm{PMT}$
$\mathrm{R}_{05} \mathrm{FV}$
$\mathrm{R}_{00}$ O(END)/1(BEGIN)
$\mathrm{R}_{\mathrm{O} 2} \mathrm{I}$ (as \%)
INIK-90y
INT
$\mathrm{R}_{10}-\mathrm{RBAL}$



| $n$ |
| :--- |
| 0 |
| 0 |
| $\vdots$ |
| $\vdots$ |
| $\vdots$ |
| $\vdots$ |


| $n$ |
| :--- |
| 0 |
| 0 |
| 0 |
| $\vdots$ |
| $\vdots$ |





Subroutine
*NPV

*MIRRFlags
SF 06-Partial $1^{\text {st }}$ year
SF 01-Cross-over to SL
SF 07-Cross-over occurred
Registers Used

$\quad$ Initial Registers

$\mathrm{R}_{12}$ Start + (End/1000)
$\mathrm{R}_{13}$ Dep. amount
$\mathrm{R}_{14}$ Life
$\mathrm{R}_{15}$ Salvage
$\mathrm{R}_{16}$ \# mons year 1
$\mathrm{R}_{17}$ Factor (as \%)
Description
Use program on next
page as an example of
how to use the
depreciation routines.
Subroutine
*SL, *DB, *SOYD




Straight line and Stop?


Displays machine size necessary to execute the program. To call this routine,
place the necessary SIZE in the X-register prior to the call. The calling sequence
must never be in a subroutine. The calling sequence for a SIZE of 20 is:
0120
$02 \times R O M \quad S I$
$2 E ? \cdot$
03 FC?C 2S
日4 PROMPT

Formats and prints, or displays, the value in the X-register. Flag 21 is set in this routine.

01 *LBL "OUT " 02
03
$0 F=?$
04
05
06 ARIEN
06
RTH

START is a micro-coded function which assigns 11 global labels and 4 mainframe functions to the top 2 rows of keys. This function cannot be copied into program memory. CLK is a micro-coded function which de-assigns the functions assigned by START. This function cannot be copied into program memory.

START
\%

Corvallis Division 1000 N.E. Circle Blvd., Corvallis, OR 97330, U.S.A.


[^0]:    * The calculator uses all four variables to solve for the fifth. Previously computed values will be used unless the financial registers are cleared.

[^1]:    * Actual size depends on the number of cash flows to be entered: for groups ( $2 \times$ (number of groups) +25 ); for individual cash flows (number of cash flows +25 ).

[^2]:    * Actual size depends on the number of cash flows to be entered: for groups ( $2 \times$ (number of groups) +25 ); for individual cash flows (number of cash flows +25 ).

[^3]:    * Actual size depends on the number of cash flows entered: for groups $(2 \times$ (number of groups) +25 ); for individual cash flows (number of cash flows +25 ).

[^4]:    * To accommodate 55 to 63 storage registers, be sure to de-assign the 15 functions of the Compound Interest program ( $\$$ ). This is accomplished by executing CLK (XEO ALPHA CLK (ALPHA). CLK clears only those assignments made in the $\$$ program. Other user assignments are preserved and may need to be cleared to obtain a sufficient number of storage registers. Refer to Page 63 of the HP-41C Owner's Handbook for a description of how to re-assign a key to its original normal mode funtion.
    $\dagger$ Actual size is the projection period +52 .

[^5]:    $\dagger$ Minimum size is the projection period +52 .

    * Press $R / S$ if you are not using a printer.

[^6]:    *There is more than one way to adjust the wrap-around mortgage to achieve a desired yield. In this program, the wrap-around balloon payment initially input remains intact, while the periodic payment changes. If this is not the case for your particular mortgage, then the calculated wrap-around payment and interest rate should be used only as a guideline. WRAP must be repeated, with new wrap-around mortgage data, until the desired yield is achieved.

[^7]:    * The user should key in the total marginal income tax rate-federal plus state-to obtain calculations which reflect the tax advantages of home ownership. Because of the complexities of tax laws and different financial and tax considerations for each individual, this program should only serve as a guide in considering an investment of this type. For more specific, detailed information, consult a tax accountant or qualified tax advisor.
    $\dagger$ The Net Cash Proceeds upon Resale (NCPR=sales price-commission-mortgage balance), is the pre-tax proceeds. The program assumes that the buyer reinvests in like property and is then not subject to capital gains tax.

[^8]:    * Positive for cash received; negative for cash paid out.

