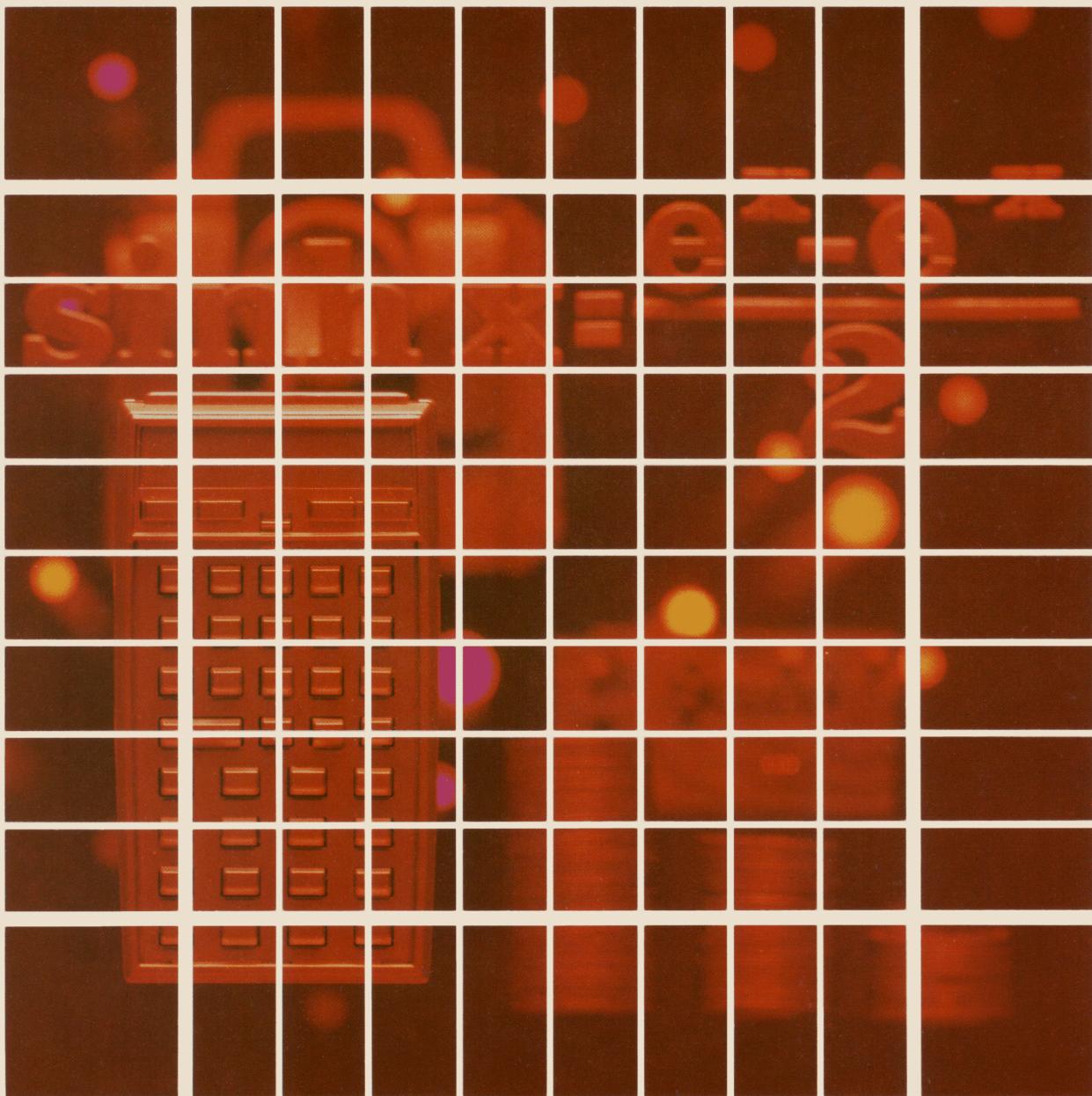


**HEWLETT-PACKARD**

**HP-41**

**USERS' LIBRARY SOLUTIONS  
High Level Math**

Includes barcode for easy software entry.



## **NOTICE**

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## INTRODUCTION

This HP-41C Solutions book was written to help you get the most from your calculator. The programs were chosen to provide useful calculations for many of the common problems encountered.

They will provide you with immediate capabilities in your everyday calculations and you will find them useful as guides to programming techniques for writing your own customized software. The comments on each program listing describe the approach used to reach the solution and help you follow the programmer's logic as you become an expert on your HP calculator.

### KEYING A PROGRAM INTO THE HP-41C

There are several things that you should keep in mind while you are keying in programs from the program listings provided in this book. The output from the HP 82143A printer provides a convenient way of listing and an easily understood method of keying in programs without showing every keystroke. This type of output is what appears in this handbook. Once you understand the procedure for keying programs in from the printed listings, you will find this method simple and fast. Here is the procedure:

1. At the end of each program listing is a listing of status information required to properly execute that program. Included is the SIZE allocation required. Before you begin keying in the program, press **XEQ ALPHA SIZE ALPHA** and specify the allocation (three digits; e.g., 10 should be specified as 010).

Also included in the status information is the display format and status of flags important to the program. To ensure proper execution, check to see that the display status of the HP-41C is set as specified and check to see that all applicable flags are set or clear as specified.

2. Set the HP-41C to PRGM mode (press the **PRGM** key) and press **■ GTO • •** to prepare the calculator for the new program.
3. Begin keying in the program. Following is a list of hints that will help you when you key in your programs from the program listings in this handbook.

- a. When you see " (quote marks) around a character or group of characters in the program listing, those characters are ALPHA. To key them in, simply press **ALPHA**, key in the characters, then press **ALPHA** again. So "SAMPLE" would be keyed in as **ALPHA "SAMPLE" ALPHA**.
- b. The diamond in front of each LBL instruction is only a visual aid to help you locate labels in the program listings. When you key in a program, ignore the diamond.
- c. The printer indication of divide sign is /. When you see / in the program listing, press **+**.
- d. The printer indication of the multiply sign is ×. When you see × in the program listing, press **×**.
- e. The † character in the program listing is an indication of the **APPEND** function. When you see †, press **■ APPEND** in ALPHA mode (press **■** and the K key).
- f. All operations requiring register addresses accept those addresses in these forms:

nn (a two-digit number)

IND nn (INDIRECT: **■**, followed by a two-digit number)

X, Y, Z, T, or L (a STACK address: **•** followed by X, Y, Z, T, or L)

IND X, Y, Z, T or L (INDIRECT stack: **■ •** followed by X, Y, Z, T, or L)

Indirect addresses are specified by pressing **■** and then the indirect address. Stack addresses are specified by pressing **•** followed by X, Y, Z, T, or L. Indirect stack addresses are specified by pressing **■ •** and X, Y, Z, T, or L.

#### Printer Listing

```
01♦LBL "SAM
PLE"
02 "THIS IS
A"
03 "†SAMPLE
"
04 AVIEW
05 6
06 ENTER↑
07 -2
08 /
09 ABS
10 STO IND
L
11 "R3="
12 ARCL 03
13 AVIEW
14 RTN
```

#### Keystrokes

```
■ LBL ALPHA SAMPLE ALPHA
ALPHA THIS IS A ALPHA
ALPHA ■ APPEND SAMPLE
■ AVIEW ALPHA
6
ENTER↑
2 CHS
+
XEQ ALPHA ABS ALPHA
STO ■ • L
ALPHA R3= ■ ARCL 03
■ AVIEW
ALPHA
■ RTN
```

#### Display

```
01 LBLT SAMPLE
02T THIS IS A
03T † SAMPLE
04 AVIEW
05 6
06 ENTER ↑
07 -2
08 /
09 ABS
10 STO IND L
11T R3=
12 ARCL 03
13 AVIEW
14 RTN
```

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|      | Computes determinant and inverse of a 4x4 matrix, solves four simultaneous equations and four unknowns, by Gaussian elimination.  |    |

\* This program requires 1 extra memory module

\*\* This program requires 2 extra memory modules

## SINE, COSINE, EXPONENTIAL INTEGRALS

This program will calculate the following integrals:

### Sine Integral

$$\begin{aligned} \text{Si}(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1) \cdot (2n+1)!} \\ &= \int_0^x \frac{\sin t}{t} dt \\ \text{Si}(-x) &= \text{Si}(x) \end{aligned}$$

### Cosine Integral

$$\begin{aligned} \text{Ci}(x) &= \gamma + \ln x + \int_0^x \frac{\cos t - 1}{t} dt \\ &= \gamma + \ln x + \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2n(2n)!} \\ \text{Ci}(-x) &= \text{Ci}(x) - i\pi \text{ for } x > 0 \end{aligned}$$

NOTE: For  $\text{Si}(x)$  and  $\text{Ci}(x)$  the accuracy of the answer decreases as  $x$  increases. For  $x=10$ , answer is accurate to the seventh decimal place. For  $x$  around 20, answers are accurate to about the second decimal place.

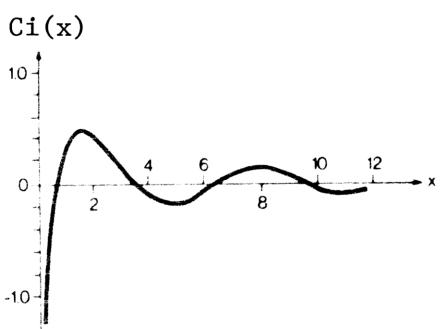
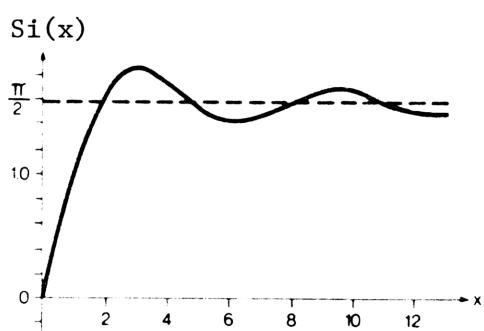
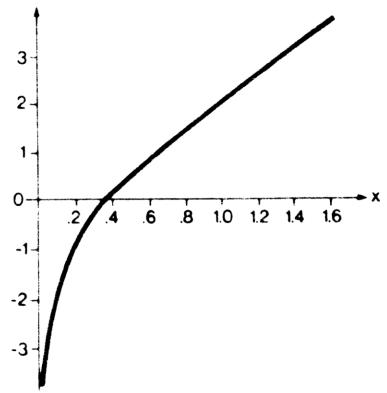
### Exponential Integral

$$\text{Ei}(x) = \int_{-\infty}^x \frac{e^t}{t} dt = \gamma + \ln x + \sum_{n=1}^{\infty} \frac{x^n}{n n!}$$

where  $x > 0$ , and  $\gamma = 0.5772156649$  is Euler's constant.

NOTE: For  $\text{Ei}(x)$  when  $x$  is too large, computing a new term of the series might cause an overflow. In that case, display shows "OUT OF RANGE" and the program halts.

The program computes successive partial sums of the series. When two consecutive partial sums are equal, the value is used as the sum of the series.

 $Ei(x)$ 

Reference: Abramowitz, *Handbook of Mathematical Functions*, National Bureau of Standards, 1968.

#### Examples:

1. Find  $Si(0.69)$
2. Find  $Si(9.8)$
3. Find  $Ci(1.38)$
4. Find  $Ci(5)$
5. Find  $Ei(1.59)$
6. Find  $Ei(0.61)$

Keystrokes:

```
[XEQ] [ALPHA] SIZE [ALPHA] 004
[XEQ] [ALPHA] SI [ALPHA]
```

1. .69 [A]
2. 9.8 [A]
3. 1.38 [B]
4. 5 [B]
5. 1.59 [C]
6. .61 [C]

Display:

|                  |
|------------------|
| $SI(0.69)=0.67$  |
| $SI(9.80)=1.67$  |
| $CI(1.38)=0.46$  |
| $CI(5.00)=-0.19$ |
| $EI(1.59)=3.57$  |
| $EI(0.61)=0.80$  |

# User Instructions

# Program Listings

```

01♦LBL "SI"
02 SF 27
03 STOP
04♦LBL A
05 STO 02
06 STO 03
07 X↑2
08 CHS
09 STO 00
10 1
11 STO 01
12 RCL 02
13 "S"
14♦LBL 00
15 RCL 00
16 RCL 01
17 1
18 +
19 /
20 LASTX
21 XEQ 02
22 X=Y?
23 GTO 00
24 GTO 04
25♦LBL 02
26 1
27 +
28 STO 01
29 /
30 RCL 02
31 *
32 STO 02
33 RCL 01
34 /
35 +
36 RTN
37♦LBL B
38 STO 03
39 X↑2
40 CHS
41 STO 00
42 1
43 STO 02
44 0
45 STO 01
46 LASTX
47 XEQ 01
48 "C"
49 GTO 00
50♦LBL 01
51 LN

```

|    |                   |  |
|----|-------------------|--|
|    | Initialize        |  |
| 01 | Si(x)             |  |
| 14 | Loop to add terms |  |
| 25 | Common subroutine |  |
| 37 | Ci(x)             |  |
| 80 |                   |  |
| 90 |                   |  |
| 00 |                   |  |

|             |  |                 |
|-------------|--|-----------------|
| 52 .5772156 |  |                 |
| 649         |  |                 |
| 53 +        |  | Ei(x)           |
| 54 RTN      |  |                 |
| 55♦LBL C    |  |                 |
| 56 STO 03   |  |                 |
| 57 STO 00   |  |                 |
| 58 1        |  |                 |
| 59 STO 02   |  |                 |
| 60 0        |  |                 |
| 61 STO 01   |  |                 |
| 62 RCL 00   |  |                 |
| 63 XEQ 01   |  |                 |
| 64♦LBL 03   |  |                 |
| 65 RCL 00   |  |                 |
| 66 RCL 01   |  |                 |
| 67 XEQ 02   |  |                 |
| 68 X=Y?     |  |                 |
| 69 GTO 03   |  |                 |
| 70 "E"      |  |                 |
| 71♦LBL 04   |  | Display routine |
| 72 FIX 2    |  |                 |
| 73 "H<"     |  |                 |
| 74 ARCL 03  |  |                 |
| 75 "H>="    |  |                 |
| 76 ARCL X   |  |                 |
| 77 AVIEW    |  |                 |
| 78 END      |  |                 |
| 80          |  |                 |
| 90          |  |                 |
| 00          |  |                 |

# REGISTERS, STATUS, FLAGS, ASSIGNMENTS

| DATA REGISTERS |               |    | STATUS      |           |                 |
|----------------|---------------|----|-------------|-----------|-----------------|
|                |               |    | SIZE        | TOT. REG. | USER MODE       |
|                |               |    | ENG         | FIX       | ON X OFF        |
|                |               |    | DEG         | RAD       | GRAD            |
| 00             | $-x^2$ , x    | 50 | FLAGS       |           |                 |
| 01             | temp. storage |    | #           | INIT S/C  | SET INDICATES   |
| 02             | temp. storage |    | 27          |           | CLEAR INDICATES |
| 03             | x             |    |             | User On   | User Off        |
| 05             |               | 55 |             |           |                 |
| 10             |               | 60 |             |           |                 |
| 15             |               | 65 |             |           |                 |
| 20             |               | 70 |             |           |                 |
| 25             |               | 75 |             |           |                 |
| 30             |               | 80 |             |           |                 |
| 35             |               | 85 |             |           |                 |
| 40             |               | 90 | ASSIGNMENTS |           |                 |
| 45             |               | 95 | FUNCTION    | KEY       | FUNCTION        |
|                |               |    | Si(x)       | A         |                 |
|                |               |    | Ci(x)       | B         |                 |
|                |               |    | Ei(x)       | C         |                 |

SINE COSINE

EXPONENTIAL INTEGRALS

PROGRAM REGISTERS NEEDED: 18

ROW 1 (1 : 6)



ROW 2 (7 : 18)



ROW 3 (19 : 27)



ROW 4 (28 : 39)



ROW 5 (40 : 49)



ROW 6 (49 : 52)



ROW 7 (52 : 63)



ROW 8 (63 : 70)



ROW 9 (71 : 75)



ROW 10 (76 : 78)



EIGENVALUES/VECTORS OF 3RD-ORDER  
SYSTEMS W/DISTINCT REAL EIGENVALUES  
(This program requires 1 memory module)

For a system matrix A, the eigenvalues are found from  $Ax = \lambda x$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad v_n = \begin{bmatrix} v_{n,1} \\ v_{n,2} \\ v_{n,3} \end{bmatrix}$$

(a) Power method

Assume the eigenvalues of A are  $\lambda_1, \lambda_2$  and  $\lambda_3$  where  $|\lambda_1| > |\lambda_2| \geq |\lambda_3|$ .

Now let A operate repeatedly on a vector v, which we express as a linear combination of the eigenvectors  $v = c_1 v_1 + c_2 v_2 + c_3 v_3$  then

$$Av = c_1 Av_1 + c_2 Av_2 + c_3 Av_3 = \lambda_1 (c_1 v_1 + c_2 \frac{\lambda_2}{\lambda_1} v_2 + c_3 \frac{\lambda_3}{\lambda_1} v_3)$$

$$A^P v = \lambda_1^P [c_1 v_1 + c_2 (\frac{\lambda_2}{\lambda_1})^P v_2 + c_3 (\frac{\lambda_3}{\lambda_1})^P v_3]$$

$$\text{therefore } \lambda_1 = \lim_{P \rightarrow \infty} \frac{(A^{P+1} v_1)}{(A^P v_1)}$$

(b) For deflation method, refer to reference (2).

NOTE: Program only works for systems with distinct real eigenvalues and a "good" guess of the initial eigenvector  $v_1$ . If a first component of the eigenvectors is zero, then it is necessary to do similarity transformations in order to use this program.

Reference: Charles Cullen, *Matrices and Linear Transformations*.  
Addison-Wesley Pub. Company, March 1967

Carl-Erik Froberg, *Intro. to Numerical Analysis*.  
Addison-Wesley Pub. Company, 1969

Examples:

Given  $A = \begin{bmatrix} -3 & 1 & 0 \\ 2 & -3 & 2 \\ 0 & 1 & -3 \end{bmatrix}$  and  $v_1 \text{guess} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ , find  $\lambda_1, v_1, \lambda_2, v_2, \lambda_3, \text{ and } v_3$ .

**Keystrokes:** **Display:**

|                                |                 |
|--------------------------------|-----------------|
| [XEQ] [ALPHA] SIZE [ALPHA] 028 |                 |
| [///] [FIX] 4                  |                 |
| [XEQ] [ALPHA] EVV [ALPHA]      | a1,1=?          |
| 3 [CHS] [R/S]                  | a2,1=?          |
| 2 [R/S]                        | a3,1=?          |
| 0 [R/S]                        | a1,2=?          |
| 1 [R/S]                        | a2,2=?          |
| 3 [CHS] [R/S]                  | a3,2=?          |
| 1 [R/S]                        | a1,3=?          |
| 0 [R/S]                        | a2,3=?          |
| 2 [R/S]                        | a3,3=?          |
| 3 [CHS] [R/S]                  | READY           |
| [C]                            | V1,1=?          |
| 1 [R/S]                        | V1,2=?          |
| 1 [R/S]                        | V1,3=?          |
| 1 [R/S]                        | READY           |
| [R/S]                          | LAM.1=-5.0000   |
| [R/S]*                         | V1,1=1.0000     |
| [R/S]*                         | V1,2=-2.0000    |
| [R/S]*                         | V1,3=1.0000     |
| [R/S]*                         | LAM.2=-3.0000   |
| [R/S]*                         | V2,1=1.0000     |
| [R/S]*                         | V2,2=-1.0000E-9 |
| [R/S]*                         | V2,3=-1.0000    |
| [R/S]*                         | LAM.3=-1.0000   |
| [R/S]*                         | V3,1=1.0000     |
| [R/S]*                         | V3,2=2.0000     |
| [R/S]*                         | V3,3=1.0000     |

\* [R/S] is omitted when the printer is present.

# User Instructions

| STEP | INSTRUCTIONS  | INPUT            | FUNCTION  | DISPLAY   |
|------|---|------------------|-----------|-----------|
| 1    | Load the program.   |                  |           |           |
| 2    | Input the matrix A.   |                  | [XEQ] EVV | a1,1=?    |
|      |   | a <sub>1,1</sub> | [R/S]     | a2,1=?    |
|      |   | a <sub>2,1</sub> | [R/S]     | a3,1=?    |
|      |   | ⋮                | ⋮         | ⋮         |
|      |   | a <sub>2,3</sub> | [R/S]     | a3,3=?    |
|      |   | a <sub>3,3</sub> | [R/S]     | READY     |
| 3    | View and/or correct matrix A. While viewing any element, if a number is entered before pressing [R/S], it will replace the currently displayed element. This mode may be exited at any point. |                  | [B]       | a1,1=( )  |
|      |   |                  | [R/S]     | a2,1=( )  |
|      |   |                  | ⋮         | ⋮         |
|      |   |                  | [R/S]     | a3,3=( )  |
|      |   |                  | [R/S]     | READY     |
| 4    | Print matrix A. If the printer is not attached, this function will act just as [B].   |                  | [///] [b] | a1,1=( )  |
|      |   |                  |           | ⋮         |
|      |   |                  |           | READY     |
| 5    | Input v <sub>1</sub> . This step must be performed regardless of whether or not v <sub>1</sub> is different from that of the previous calculation.  |                  | [C]       | v1,1=?    |
|      |   | v <sub>1,1</sub> | [R/S]     | v1,2=?    |
|      |   | v <sub>1,2</sub> | [R/S]     | v1,3=?    |
|      |   | v <sub>1,3</sub> | [R/S]     | READY     |
| 6    | Print v <sub>1</sub> .  |                  | [///] [C] | v1,1=( )  |
|      |   |                  |           | ⋮         |
|      |   |                  |           | READY     |
| 7    | Calculate v <sub>1</sub> , v <sub>2</sub> , v <sub>3</sub> , λ <sub>1</sub> , λ <sub>2</sub> , λ <sub>3</sub> . This step may only be performed if "READY" is in the display.                 |                  | [R/S]     | LAM.1=( ) |
|      |   |                  | [R/S]*    | v1,1=( )  |
|      |   |                  | [R/S]*    | v1,2=( )  |
|      |   |                  | [R/S]*    | v1,3=( )  |
|      |   |                  | [R/S]*    | LAM.2=( ) |

# User Instructions

# Program Listings

|   |   |  |   |
|---|---|--|---|
| <pre> 01♦LBL "EVV" " 02 SF 21 03 SF 27 04♦LBL A 05 CF 05 06 CF 06 07 CF 07 08 GTO 01 09♦LBL b 10 ADV 11 SF 05 12 CF 06 13 SF 07 14 GTO 01 15♦LBL C 16 CF 05 17 SF 06 18 CF 07 19 GTO 01 20♦LBL c 21 ADV 22 SF 05 23 SF 06 24 SF 07 25 GTO 01 26♦LBL B 27 SF 05 28 CF 06 29 CF 07 30♦LBL 01 31 1.003 32 STO 10 33 STO 11 34 1 35 FS? 06 36 20 37 STO 00 38♦LBL 00 39 FIX 0 40 CF 29 41 "a" 42 FS? 06 43 "V1," 44 ARCL 11 45 FC? 06 46 "F," 47 FC? 06 48 ARCL 10 49 "F=" 50 FC? 05 </pre> | <pre> Input A matrix Print A matrix Input v1 Print v1 View A matrix Initialize Input/View routine Common Input/ View routine </pre> | <pre> 51 "F?" 52 FIX 4 53 SF 29 54 RCL IND 00 55 FS? 05 56 ARCL X 57 FC? 07 58 PROMPT 59 FS? 07 60 AVIEW 61 STO IND 00 62 ISG 00 63 CLD 64 ISG 11 65 GTO 00 66 FS? 07 67 ADV 68 FS? 06 69 GTO 02 70 1.003 71 STO 11 72 ISG 10 73 GTO 00 74♦LBL 02 75 "READY" 76 PROMPT 77 "A" 78 ASTO 26 79♦LBL 03 80 RCL 01 81 RCL 04 82 RCL 07 83 XEQ 10 84 STO 23 85 RCL 02 86 RCL 05 87 RCL 08 88 XEQ 10 89 STO 24 90 RCL 03 91 RCL 06 92 RCL 09 93 XEQ 10 94 STO 25 95 RCL 23 96 RCL 20 97 / 98 RCL 26 99 X=Y? </pre> | <p>"READY" prompt</p> <p>Iterative routine to find <math>\lambda_1</math></p> |
|---|---|--|---|

# Program Listings

|            |                          |   |
|------------|--------------------------|---|
| 100 GTO 04 |                          | 151 RCL 08  |
| 101 X<>Y   |                          | 152 RCL 21  |
| 102 STO 26 |                          | 153 RCL 07  |
| 103 RCL 23 |                          | 154 *   |
| 104 STO 20 |                          | 155 -   |
| 105 RCL 24 |                          | 156 STO 18  |
| 106 STO 21 |                          | 157 RCL 09  |
| 107 RCL 25 |                          | 158 RCL 22  |
| 108 STO 22 |                          | 159 RCL 07  |
| 109 GTO 03 |                          | 160 *   |
| 110+LBL 10 |                          | 161 -   |
| 111 RCL 22 | Iteration<br>subroutine  | 162 STO 19  |
| 112 *      |                          | 163 RCL 16  |
| 113 RCL 21 |                          | 164 +   |
| 114 ST* Z  |                          | 165 STO 25  |
| 115 RDN    |                          | 166 X↑2   |
| 116 RCL 20 |                          | 167 RCL 16  |
| 117 ST* T  |                          | 168 RCL 19  |
| 118 RDN    |                          | 169 *   |
| 119 +      |                          | 170 RCL 18  |
| 120 +      |                          | 171 RCL 17  |
| 121 RTN    |                          | 172 *   |
| 122+LBL 04 | Calculate v <sub>1</sub> | 173 -   |
| 123 CF 09  |                          | 174 4   |
| 124 1      |                          | 175 *   |
| 125 STO 00 |                          | 176 -   |
| 126 STO 27 |                          | 177 SQRT  |
| 127 STO 20 |                          | 178 2   |
| 128 RCL 24 |                          | 179 /   |
| 129 RCL 23 |                          | 180 STO 23  |
| 130 /      |                          | 181 RCL 25  |
| 131 STO 21 |                          | 182 2   |
| 132 RCL 25 |                          | 183 /   |
| 133 RCL 23 |                          | 184 STO 24  |
| 134 /      |                          | 185 RCL 23  |
| 135 STO 22 |                          | 186 -   |
| 136 RCL 26 |                          | 187 STO 10  |
| 137 RDN    |                          | 188 RCL 24  |
| 138 XEQ 06 |                          | 189 RCL 23  |
| 139 RCL 05 |                          | 190 +   |
| 140 RCL 21 | Calculate λ <sub>2</sub> | 191 STO 13  |
| 141 RCL 04 |                          | 192+LBL 08  |
| 142 *      |                          | 193 RCL 10  |
| 143 -      |                          | 194 RCL 16  |
| 144 STO 16 |                          | 195 -   |
| 145 RCL 06 |                          | 196 STO 12  |
| 146 RCL 22 |                          | 197 RCL 18  |
| 147 RCL 04 |                          | 198 STO 11  |
| 148 *      |                          | 199 X<>Y  |
| 149 -      |                          | 200 RCL 04  |
| 150 STO 17 |                          | 201 RCL 07  |
|            |                          | Common routine<br>to calculate<br>v <sub>2</sub> and v <sub>3</sub> |

# Program Listings

|            |                |             |            |
|------------|----------------|-------------|------------|
| 202 RDN    |                | 253 AVIEW   |            |
| 203 X<>Y   |                | 254 "V"     |            |
| 204 R↑     |                | 255 XEQ 07  |            |
| 205 *      |                | 256 ARCL Y  |            |
| 206 RDN    |                | 257 AVIEW   |            |
| 207 *      |                | 258 "V"     |            |
| 208 R↑     |                | 259 XEQ 07  |            |
| 209 +      |                | 260 ARCL X  |            |
| 210 RCL 10 |                | 261 AVIEW   |            |
| 211 RCL 26 |                | 262 ISG 00  |            |
| 212 -      |                | 263 CLD     |            |
| 213 /      |                | 264 1       |            |
| 214 ST/ 11 |                | 265 STO 27  |            |
| 215 ST/ 12 |                | 266 ADV     |            |
| 216 RCL 21 |                | 267 RTN     |            |
| 217 ST+ 11 |                | 268♦LBL 07  |            |
| 218 RCL 22 |                | 269 FIX 0   |            |
| 219 ST+ 12 |                | 270 CF 29   |            |
| 220 RCL 10 |                | 271 ARCL 00 | Output     |
| 221 RCL 20 |                | 272 FC? 08  | subroutine |
| 222 RCL 11 |                | 273 GTO 07  |            |
| 223 RCL 12 |                | 274 "T,"    |            |
| 224 FS? 09 |                | 275 ARCL 27 |            |
| 225 GTO 06 |                | 276 ISG 27  |            |
| 226 XEQ 06 |                | 277♦LBL 07  |            |
| 227 SF 09  |                | 278 FIX 4   |            |
| 228 RCL 10 |                | 279 SF 29   |            |
| 229 X<> 13 |                | 280 "T="    |            |
| 230 STO 10 |                | 281 .END.   |            |
| 231 RCL 11 |                | 80          |            |
| 232 RCL 12 |                |             |            |
| 233 RCL 14 |                |             |            |
| 234 RCL 15 |                |             |            |
| 235 STO 12 |                |             |            |
| 236 RDN    |                |             |            |
| 237 STO 11 |                |             |            |
| 238 RDN    |                |             |            |
| 239 STO 15 |                |             |            |
| 240 RDN    |                |             |            |
| 241 STO 14 |                | 90          |            |
| 242 GTO 08 |                |             |            |
| 243♦LBL 06 |                |             |            |
| 244 CF 08  |                |             |            |
| 245 "LAM." |                |             |            |
| 246 XEQ 07 |                |             |            |
| 247 ARCL T |                |             |            |
| 248 AVIEW  |                |             |            |
| 249 SF 08  |                |             |            |
| 250 "V"    |                |             |            |
| 251 XEQ 07 |                |             |            |
| 252 ARCL Z |                | 00          |            |
|            | Output routine |             |            |

## **REGISTERS, STATUS, FLAGS, ASSIGNMENTS**

| DATA REGISTERS |                  |    | STATUS |                      |          |                      |                   |           |
|----------------|------------------|----|--------|----------------------|----------|----------------------|-------------------|-----------|
| 00             | Index/Temp.      | 50 |        | SIZE                 | 28       | TOT. REG.            | 97                | USER MODE |
|                | a <sub>1,1</sub> |    |        | ENG                  |          | FIX                  | 4                 | SCI       |
|                | a <sub>2,1</sub> |    |        | DEG                  |          | RAD                  |                   | ON X OFF  |
|                | a <sub>3,1</sub> |    |        |                      |          | GRAD                 |                   |           |
|                | a <sub>1,2</sub> |    |        |                      |          |                      |                   |           |
| 05             | a <sub>2,2</sub> | 55 |        | FLAGS                |          |                      |                   |           |
|                | a <sub>3,2</sub> |    |        | #                    | INIT S/C | SET INDICATES        | CLEAR INDICATES   |           |
|                | a <sub>1,3</sub> |    |        | 05                   |          |                      |                   |           |
|                | a <sub>2,3</sub> |    |        | 06                   |          |                      |                   |           |
|                | a <sub>3,3</sub> |    |        | 07                   |          |                      |                   |           |
| 10             | Index/Temp.      | 60 |        | 08                   |          |                      |                   |           |
|                | Index/Temp.      |    |        | 21                   |          | Printer Enabled      | Printer Disabled  |           |
|                | Temp.            |    |        | 27                   |          | User Mode On         | User Mode Off     |           |
|                | Temp.            |    |        | 29                   |          | Digit Grouping       | No Digit Grouping |           |
|                | Temp.            |    |        |                      |          |                      |                   |           |
| 15             | Temp.            | 65 |        |                      |          |                      |                   |           |
|                | Temp.            |    |        |                      |          |                      |                   |           |
|                | Temp.            |    |        |                      |          |                      |                   |           |
|                | Temp.            |    |        |                      |          |                      |                   |           |
|                | Temp.            |    |        |                      |          |                      |                   |           |
| 20             | Temp.            | 70 |        |                      |          |                      |                   |           |
|                | Temp.            |    |        |                      |          |                      |                   |           |
|                | Temp.            |    |        |                      |          |                      |                   |           |
|                | Temp.            |    |        |                      |          |                      |                   |           |
|                | Temp.            |    |        |                      |          |                      |                   |           |
| 25             | Temp.            | 75 |        |                      |          |                      |                   |           |
|                | Temp.            |    |        |                      |          |                      |                   |           |
|                | Temp.            |    |        |                      |          |                      |                   |           |
|                | Temp.            |    |        |                      |          |                      |                   |           |
|                | Temp.            |    |        |                      |          |                      |                   |           |
| 30             |                  | 80 |        |                      |          |                      |                   |           |
|                |                  |    |        |                      |          |                      |                   |           |
|                |                  |    |        |                      |          |                      |                   |           |
|                |                  |    |        |                      |          |                      |                   |           |
|                |                  |    |        |                      |          |                      |                   |           |
| 35             |                  | 85 |        | ASSIGNMENTS          |          |                      |                   |           |
|                |                  |    |        | FUNCTION             | KEY      | FUNCTION             | KEY               |           |
|                |                  |    |        | Input A              | A        | Print A              | b                 |           |
|                |                  |    |        | View A               | B        | Print V <sub>1</sub> | c                 |           |
|                |                  |    |        | Input V <sub>1</sub> | C        |                      |                   |           |
| 40             |                  | 90 |        |                      |          |                      |                   |           |
|                |                  |    |        |                      |          |                      |                   |           |
|                |                  |    |        |                      |          |                      |                   |           |
|                |                  |    |        |                      |          |                      |                   |           |
|                |                  |    |        |                      |          |                      |                   |           |
| 45             |                  | 95 |        |                      |          |                      |                   |           |
|                |                  |    |        |                      |          |                      |                   |           |
|                |                  |    |        |                      |          |                      |                   |           |
|                |                  |    |        |                      |          |                      |                   |           |
|                |                  |    |        |                      |          |                      |                   |           |

EIGENVALUES/VECTORS OF  
3RD-ORDER SYSTEMS  
PROGRAM REGISTERS NEEDED: 69

ROW 1 (1 : 4)



ROW 2 (5 : 11)



ROW 3 (12 : 18)



ROW 4 (18 : 25)



ROW 5 (25 : 31)



ROW 6 (31 : 39)



ROW 7 (40 : 45)



ROW 8 (45 : 50)



ROW 9 (51 : 56)



ROW 10 (57 : 64)



ROW 11 (65 : 70)



ROW 12 (70 : 75)



ROW 13 (76 : 84)



ROW 14 (84 : 93)



ROW 15 (93 : 100)



ROW 16 (100 : 107)



ROW 17 (107 : 114)



ROW 18 (115 : 124)



## EIGENVALUES/VECTORS OF 3RD-ORDER SYSTEMS

ROW 19 (125 : 132)



ROW 20 (132 : 139)



ROW 21 (140 : 149)



ROW 22 (150 : 158)



ROW 23 (159 : 167)



ROW 24 (168 : 177)



ROW 25 (178 : 186)



ROW 26 (187 : 196)



ROW 27 (197 : 208)



ROW 28 (209 : 217)



ROW 29 (217 : 225)



ROW 30 (225 : 233)



ROW 31 (234 : 244)



ROW 32 (245 : 249)



ROW 33 (250 : 255)



ROW 34 (256 : 262)



ROW 35 (263 : 271)



ROW 36 (272 : 278)



EIGENVALUES/VECTORS OF  
3RD-ORDER SYSTEMS

ROW 37 (278 : 281)



## EIGENVALUES FOR 3RD ORDER SYSTEM

(Requires 1 memory module)

This program determines the eigenvalues of a 3rd order system described by  $\mathbf{Ax} = \lambda\mathbf{x}$ , i.e.,

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Eigenvalues  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  are solved from

$$\det(\lambda I - A) = 0$$

Roots for the cubic equation are solved by using the exact formula.

Example: Find the eigenvalues for:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & -0.5 \end{bmatrix}$$

Keystrokes:

[XEQ] [ALPHA] SIZE [ALPHA] 022

[XEQ] [ALPHA] EV [ALPHA]

1 [R/S]

0 [R/S]

0 [R/S]

0 [R/S]

0 [R/S]

1 [R/S]

0 [R/S]

1 [CHS] [R/S]

.5 [CHS] [R/S]

[C]

[R/S]\*

[R/S]\*

Display:

a1,1=?

a2,1=?

a3,1=?

a1,2=?

a2,2=?

a3,2=?

a1,3=?

a2,3=?

a3,3=?

READY

RLAM.1=-0.2500

ILAM.1=-0.9682

RLAM.2=-0.2500

**Keystrokes:**

[R/S]\*

[R/S]\*

**Display:**

ILAM.2=0.9682

LAM.3=1.0000

\*[R/S] is omitted when the printer is present

# User Instructions

SIZE: 028

| STEP | INSTRUCTIONS   | INPUT            | FUNCTION  | DISPLAY               |
|------|--|------------------|-----------|-----------------------|
| 1    | Load the program.  |                  |           |                       |
| 2    | Input the matrix A.  |                  | [XEQ] EV  | a <sub>1,1</sub> =?   |
|      |  | a <sub>1,1</sub> | [R/S]     | a <sub>2,1</sub> =?   |
|      |  | a <sub>2,1</sub> | [R/S]     | a <sub>3,1</sub> =?   |
|      |  | :                | :         | :                     |
|      |  | a <sub>3,1</sub> | [R/S]     | READY                 |
| 3    | View and/or correct the matrix A. While viewing any element, if a number is entered before pressing [R/S] it will replace the currently displayed element. |                  | [B]       | a <sub>1,1</sub> =( ) |
|      |  |                  | [R/S]     | a <sub>2,1</sub> =( ) |
|      |  |                  | :         | :                     |
|      |  |                  | [R/S]     | READY                 |
| 4    | Print the matrix A. If the printer is not attached, this function will act as [B].   |                  | [///] [b] | a <sub>1,1</sub> =( ) |
|      |  |                  |           | :                     |
|      |  |                  |           | READY                 |
| 5    | Calculate the Eigenvalues.   |                  | [C]       | RLAM.1=( )            |
|      | RLAM. ≡ the real portion of $\lambda$ .  |                  | [R/S]*    | ILAM.1=( )            |
|      | ILAM. ≡ the imaginary portion of $\lambda$ .   |                  | [R/S]*    | RLAM.2=( )            |
|      |  |                  | [R/S]*    | ILAM.2=( )            |
|      |  |                  | [R/S]*    | LAM.3=( )             |
| 6    | For a new problem:   |                  | [A]       | a <sub>1,1</sub> =?   |
|      | This is the same as step 2. Note that the value of each element from the previous matrix A may be reused by just pressing [R/S].                           |                  |           |                       |
|      | *[R/S] is omitted if the printer is attached.  |                  |           |                       |
|      |  |                  |           |                       |
|      |  |                  |           |                       |
|      |  |                  |           |                       |

# Program Listings

|             |               |            |              |
|-------------|---------------|------------|--------------|
| 01♦LBL "EV" |               | 50 ISG 10  |              |
| 02 SF 21    |               | 51 GTO 00  |              |
| 03 SF 27    |               | 52♦LBL 02  |              |
| 04♦LBL A    |               | 53 "READY" |              |
| 05 CF 05    | Input A       | 54 PROMPT  |              |
| 06 CF 07    |               | 55♦LBL C   |              |
| 07 GTO 01   |               | 56 RCL 08  |              |
| 08♦LBL b    |               | 57 RCL 06  |              |
| 09 ADV      |               | 58 RCL 05  |              |
| 10 SF 05    | Print A       | 59 RCL 09  |              |
| 11 SF 07    |               | 60 XEQ 09  | Calculate    |
| 12 GTO 01   |               | 61 RCL 01  | coefficients |
| 13♦LBL B    |               | 62 *       | of 3rd order |
| 14 SF 05    | View A        | 63 STO 10  | polynomial   |
| 15 CF 07    |               | 64 RCL 02  |              |
| 16♦LBL 01   |               | 65 RCL 09  |              |
| 17 1.003    |               | 66 RCL 08  |              |
| 18 STO 10   | Initialize    | 67 RCL 03  |              |
| 19 STO 11   | Input Routine | 68 XEQ 09  |              |
| 20 1        |               | 69 RCL 04  |              |
| 21 STO 00   |               | 70 *       |              |
| 22♦LBL 00   |               | 71 ST+ 10  |              |
| 23 FIX 0    |               | 72 RCL 03  |              |
| 24 CF 29    |               | 73 RCL 05  |              |
| 25 "a"      |               | 74 RCL 06  |              |
| 26 ARCL 11  | Input/View/   | 75 RCL 02  |              |
| 27 "F,"     | Print Routine | 76 XEQ 09  |              |
| 28 ARCL 10  |               | 77 RCL 07  |              |
| 29 "F="     |               | 78 *       |              |
| 30 FC? 05   |               | 79 ST+ 10  |              |
| 31 "F?"     |               | 80 RCL 01  |              |
| 32 FIX 4    |               | 81 RCL 05  |              |
| 33 SF 29    |               | 82 RCL 07  |              |
| 34 RCL IND  |               | 83 RCL 03  |              |
| 00          |               | 84 XEQ 09  |              |
| 35 FS? 05   |               | 85 STO 11  |              |
| 36 ARCL X   |               | 86 RCL 01  |              |
| 37 FC? 07   |               | 87 RCL 09  |              |
| 38 PROMPT   |               | 88 RCL 08  |              |
| 39 FS? 07   |               | 89 RCL 06  |              |
| 40 AVIEW    |               | 90 XEQ 09  |              |
| 41 STO IND  |               | 91 ST+ 11  |              |
| 00          |               | 92 RCL 05  |              |
| 42 ISG 00   |               | 93 RCL 09  |              |
| 43 CLD      |               | 94 RCL 04  |              |
| 44 ISG 11   |               | 95 RCL 02  |              |
| 45 GTO 00   |               | 96 XEQ 09  |              |
| 46 FS? 07   |               | 97 ST+ 11  |              |
| 47 ADV      |               | 98 RCL 01  |              |
| 48 1.003    |               | 99 RCL 05  |              |
| 49 STO 11   |               | 100 RCL 09 |              |

# Program Listings

|            |                                    |            |                                  |
|------------|------------------------------------|------------|----------------------------------|
| 101 +      |                                    | 153 XEQ 05 | S+T                              |
| 102 +      |                                    | 154 RCL 14 |                                  |
| 103 CHS    |                                    | 155 RCL 19 |                                  |
| 104 STO 12 |                                    | 156 -      |                                  |
| 105 CF 06  |                                    | 157 XEQ 05 |                                  |
| 106 RCL 10 |                                    | 158 +      |                                  |
| 107 X#0?   |                                    | 159 GTO 04 |                                  |
| 108 GTO 10 | Test if constant<br>is zero        | 160♦LBL 03 |                                  |
| 109 STO 21 |                                    | 161 RCL 14 |                                  |
| 110 RCL 11 |                                    | 162 RCL 13 |                                  |
| 111 STO 17 |                                    | 163 3      |                                  |
| 112 RCL 12 |                                    | 164 Y↑X    | $\lambda_3$ for<br>$Q^3+R^2 < 0$ |
| 113 STO 18 |                                    | 165 CHS    |                                  |
| 114 GTO 07 |                                    | 166 SQRT   |                                  |
| 115♦LBL 10 |                                    | 167 /      |                                  |
| 116 RCL 11 |                                    | 168 ACOS   |                                  |
| 117 3      |                                    | 169 3      |                                  |
| 118 *      |                                    | 170 /      |                                  |
| 119 RCL 12 |                                    | 171 COS    |                                  |
| 120 X↑2    |                                    | 172 RCL 13 |                                  |
| 121 -      |                                    | 173 CHS    |                                  |
| 122 9      |                                    | 174 SQRT   |                                  |
| 123 /      |                                    | 175 *      |                                  |
| 124 STO 13 |                                    | 176 ST+ X  |                                  |
| 125 RCL 11 |                                    | 177♦LBL 04 |                                  |
| 126 RCL 12 |                                    | 178 RCL 12 |                                  |
| 127 *      |                                    | 179 3      |                                  |
| 128 9      | Solve for<br>Real Root $\lambda_3$ | 180 /      | $\lambda_3$                      |
| 129 *      |                                    | 181 -      |                                  |
| 130 RCL 10 |                                    | 182 STO 19 |                                  |
| 131 27     |                                    | 183 STO 21 |                                  |
| 132 *      |                                    | 184 RCL 12 |                                  |
| 133 -      |                                    | 185 +      |                                  |
| 134 RCL 12 |                                    | 186 STO 18 | Reduce to<br>second order        |
| 135 3      |                                    | 187 RCL 10 | and calculate                    |
| 136 Y↑X    |                                    | 188 RCL 19 | $\lambda_1, \lambda_2$           |
| 137 ST+ X  |                                    | 189 /      |                                  |
| 138 -      |                                    | 190 CHS    |                                  |
| 139 54     |                                    | 191 STO 17 |                                  |
| 140 /      |                                    | 192♦LBL 07 |                                  |
| 141 STO 14 |                                    | 193 RCL 18 |                                  |
| 142 X↑2    |                                    | 194 X↑2    |                                  |
| 143 RCL 13 |                                    | 195 RCL 17 |                                  |
| 144 3      |                                    | 196 4      |                                  |
| 145 Y↑X    |                                    | 197 *      |                                  |
| 146 +      |                                    | 198 -      |                                  |
| 147 X<0?   |                                    | 199 CHS    |                                  |
| 148 GTO 03 |                                    | 200 X>0?   |                                  |
| 149 SQRT   |                                    | 201 SF 06  |                                  |
| 150 STO 19 |                                    | 202 ABS    |                                  |
| 151 RCL 14 |                                    | 203 SQRT   |                                  |
| 152 +      |                                    | ..         |                                  |

# Program Listings

|             |                |             |
|-------------|----------------|-------------|
| 204 2       |                | 255 "I"     |
| 205 /       |                | 256 SF 05   |
| 206 STO 20  |                | 257 XEQ 08  |
| 207 RCL 18  |                | 258 "R"     |
| 208 CHS     |                | 259 SF 07   |
| 209 2       |                | 260 XEQ 08  |
| 210 /       |                | 261 "I"     |
| 211 STO 00  |                | 262 SF 07   |
| 212 FS? 06  |                | 263 XEQ 08  |
| 213 GTO 11  |                | 264 CLA     |
| 214 RCL 20  |                | 265 SF 09   |
| 215 +       |                | 266 RCL 21  |
| 216 0       |                | 267♦LBL 08  |
| 217 X<>Y    |                | 268 "FLAM." |
| 218 RCL 00  |                | 269 FS?C 05 |
| 219 RCL 20  |                | 270 "F1"    |
| 220 -       |                | 271 FS?C 07 |
| 221 0       |                | 272 "F2"    |
| 222 X<>Y    |                | 273 FS?C 09 |
| 223 GTO 06  |                | 274 "F3"    |
| 224♦LBL 11  |                | 275 "F="    |
| 225 RCL 20  |                | 276 ARCL X  |
| 226 X<>Y    |                | 277 AVIEW   |
| 227 RCL 20  |                | 278 RDN     |
| 228 CHS     |                | 279 END     |
| 229 RCL 00  |                |             |
| 230 GTO 06  |                |             |
| 231♦LBL 09  |                |             |
| 232 *       |                |             |
| 233 RDN     | ZT-XY          |             |
| 234 *       |                | 80          |
| 235 RT      |                |             |
| 236 -       |                |             |
| 237 RTN     |                |             |
| 238♦LBL 05  |                |             |
| 239 CF 08   |                |             |
| 240 X<0?    |                |             |
| 241 SF 08   | ± 3            |             |
| 242 ABS     |                |             |
| 243 3       |                |             |
| 244 1/X     |                | 90          |
| 245 Y↑X     |                |             |
| 246 FS?C 08 |                |             |
| 247 CHS     |                |             |
| 248 RTN     |                |             |
| 249♦LBL 06  |                |             |
| 250 CF 09   |                |             |
| 251 SF 05   |                |             |
| 252 CF 07   |                |             |
| 253 "R"     |                |             |
| 254 XEQ 08  | Output routine | 00          |

# REGISTERS, STATUS, FLAGS, ASSIGNMENTS

| DATA REGISTERS |                  |    |  | STATUS                         |          |                  |                 |
|----------------|------------------|----|--|--------------------------------|----------|------------------|-----------------|
| 00             | -6 (or b b/2)    | 50 |  | SIZE 22 TOT. REG. 86 USER MODE |          |                  |                 |
|                | a <sub>1</sub> 1 |    |  | ENG FIX 4 SCI ON X OFF         |          |                  |                 |
|                | a <sub>2</sub> 1 |    |  | DEG RAD GRAD                   |          |                  |                 |
|                | a <sub>3</sub> 1 |    |  |                                |          |                  |                 |
| 05             | a <sub>1</sub> 2 |    |  | FLAGS                          |          |                  |                 |
|                | a <sub>2</sub> 2 | 55 |  | #                              | INIT S/C | SET INDICATES    | CLEAR INDICATES |
|                | a <sub>3</sub> 2 |    |  | 05                             |          | Display Elements | Input Elements  |
|                | a <sub>1</sub> 3 |    |  | 07                             |          | Print Elements   | Any other mode. |
| 10             | a <sub>2</sub> 3 |    |  |                                |          |                  |                 |
|                | a <sub>3</sub> 3 |    |  |                                |          |                  |                 |
|                | λ <sub>3</sub>   | 60 |  |                                |          |                  |                 |
|                | used             |    |  |                                |          |                  |                 |
| 15             | used             |    |  |                                |          |                  |                 |
|                | used             |    |  |                                |          |                  |                 |
|                | used             |    |  |                                |          |                  |                 |
|                | used             |    |  |                                |          |                  |                 |
| 20             | used             |    |  |                                |          |                  |                 |
|                | used             | 70 |  |                                |          |                  |                 |
|                | λ <sub>3</sub>   |    |  |                                |          |                  |                 |
|                |                  |    |  |                                |          |                  |                 |
| 25             |                  | 75 |  |                                |          |                  |                 |
|                |                  |    |  |                                |          |                  |                 |
|                |                  |    |  |                                |          |                  |                 |
|                |                  |    |  |                                |          |                  |                 |
| 30             |                  | 80 |  |                                |          |                  |                 |
|                |                  |    |  |                                |          |                  |                 |
|                |                  |    |  |                                |          |                  |                 |
|                |                  |    |  |                                |          |                  |                 |
| 35             |                  | 85 |  | ASSIGNMENTS                    |          |                  |                 |
|                |                  |    |  |                                |          |                  |                 |
|                |                  |    |  | FUNCTION                       |          | FUNCTION         |                 |
|                |                  |    |  | Input A                        | A        | Print A          | b               |
| 40             |                  | 90 |  | View A                         | B        | Compute λ's      | c               |
|                |                  |    |  |                                |          |                  |                 |
|                |                  |    |  |                                |          |                  |                 |
|                |                  |    |  |                                |          |                  |                 |
| 45             |                  | 95 |  |                                |          |                  |                 |
|                |                  |    |  |                                |          |                  |                 |
|                |                  |    |  |                                |          |                  |                 |
|                |                  |    |  |                                |          |                  |                 |

EIGENVALUES FOR 3RD-  
ORDER SYSTEM

PROGRAM REGISTERS NEEDED: 62

ROW 1 (1 : 5)



ROW 2 (5 : 12)



ROW 3 (12 : 17)



ROW 4 (18 : 26)



ROW 5 (27 : 31)



ROW 6 (32 : 38)



ROW 7 (39 : 46)



ROW 8 (46 : 52)



ROW 9 (53 : 59)



ROW 10 (60 : 68)



ROW 11 (69 : 78)



ROW 12 (79 : 88)



ROW 13 (89 : 96)



ROW 14 (97 : 107)



ROW 15 (108 : 115)



ROW 16 (116 : 128)



ROW 17 (129 : 139)



ROW 18 (139 : 150)



## EIGENVALUES FOR 3RD- ORDER SYSTEM

ROW 19 (150 : 157)



ROW 20 (158 : 169)



ROW 21 (170 : 181)



ROW 22 (182 : 190)



ROW 23 (191 : 200)



ROW 24 (201 : 210)



ROW 25 (211 : 219)



ROW 26 (220 : 229)



ROW 27 (230 : 240)



ROW 28 (241 : 250)



ROW 29 (251 : 256)



ROW 30 (257 : 262)



ROW 31 (262 : 268)



ROW 32 (268 : 272)



ROW 33 (273 : 279)



ROW 34 (279 : 279)



## CHEBYSHEV, LEGENDRE, HERMITE AND LAGUERRE POLYNOMIALS

Label T computes the value of the Chebyshev polynomial  $T_n(x)$  by using the recurrence equation

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$

where starting values are  $T_0(x) = 1$ ,  $T_1(x) = x$  and  $n$  is a positive integer.

Label P computes the value of the Legendre polynomial  $P_n(x)$  by using the recurrence equation

$$P_{n+1}(x) = \frac{(2n+1)xP_n(x) - nP_{n-1}(x)}{n+1}$$

where starting values are  $P_0(x)=1$ ,  $P_1(x)=x$  and  $n$  is a positive integer.

Label H computes the value of the Hermite polynomial  $H_n(x)$  by using the recurrence equation

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$$

where the starting values are  $H_0(x) = 1$ ,  $H_1(x) = 2x$  and  $n$  is a positive integer.

Label L computes the value of the Laguerre polynomial  $L_n(x)$  by using the recurrence equation

$$L_{n+1}(x) = \frac{(2n+1-x)L_n(x) - nL_{n-1}(x)}{n+1}$$

Note that all four functions leave  $f(x)$  in the  $x$  register if you wish to see more accuracy (see ex. 4).

Examples:

1. Find  $T_3(0.4)$
2. Find  $P_{10}(0.98)$
3. Find  $H_5(3)$
4. Find  $L_6(3)$

**Keystrokes:****Display:**

|                            |                   |
|----------------------------|-------------------|
| [XEQ] [ALPHA] SIZE [ALPHA] |                   |
| [XEQ] [ALPHA] T [ALPHA]    | N?                |
| 3 [R/S]                    | X?                |
| .4 [R/S]                   | T3(0.40)=-0.94    |
| [XEQ] [ALPHA] P [ALPHA]    | N?                |
| 10 [R/S]                   | X?                |
| 0.98 [R/S]                 | P10(0.98)=0.16    |
| [XEQ] [ALPHA] H [ALPHA]    | N?                |
| 5 [R/S]                    | X?                |
| 3 [R/S]                    | H5(3.00)=3,816.00 |
| [XEQ] [ALPHA] L [ALPHA]    | N?                |
| 6 [R/S]                    | X?                |
| 3 [R/S]                    | L6(3.00)=-0.01    |
| ■ [FIX] 4                  | -0.0125           |

# User Instructions

# Program Listings

|  |   |   |   |
|--|---|---|---|
| <pre> 01 *LBL "T" 02 XEQ A 03 STO 00 04 2 05 STO 01 06 * 07 STO 02 08 CLX 09 X&lt;&gt;Y 10 X&lt;=Y? 11 GTO 00 12 1 13 STO 03 14 X=Y? 15 GTO 01 16 *LBL 02 17 CLX 18 RCL 02 19 RCL 00 20 * 21 RCL 03 22 LASTX 23 STO 03 24 RDN 25 - 26 STO 00 27 CLX 28 RCL 01 29 1 30 + 31 STO 01 32 X&lt;=Y? 33 GTO 02 34 *LBL 01 35 RCL 00 36 "T" 37 GTO d 38 *LBL 00 39 1 40 "T" 41 GTO d 42 *LBL "P" 43 XEQ A 44 STO 01 45 STO 04 46 CLX 47 X&lt;&gt;Y 48 STO 00 49 X&lt;=Y? 50 GTO 00 51 1 </pre> | <p>Chebyshev polynomial</p> <p>Loop for recurrence equation</p> <p>Test, <math>r_1 \leq n_1</math><br/>so back to loop</p> <p>Legendre polynomial</p> | <pre> 52 STO 03 53 X&lt;&gt;Y 54 X&lt;=Y? 55 GTO 04 56 2 57 STO 02 58 *LBL 03 59 RCL 04 60 RCL 01 61 * 62 ENTER↑ 63 ENTER↑ 64 RCL 03 65 - 66 + 67 LASTX 68 RCL 04 69 STO 03 70 CLX 71 RCL 02 72 / 73 - 74 STO 04 75 RCL 00 76 RCL 02 77 1 78 + 79 STO 02 80 X&lt;=Y? 81 GTO 03 82 RCL 04 83 "P" 84 GTO d 85 *LBL 04 86 RCL 01 87 RTN 88 *LBL H 89 XEQ A 90 STO 01 91 2 92 * 93 STO 03 94 CLX 95 X&lt;&gt;Y 96 X&lt;=Y? 97 GTO 00 98 1 99 STO 00 100 STO 02 101 X&lt;&gt;Y 102 X&lt;=Y? </pre> | <p>Loop for recurrence equation</p> <p>Test</p> <p>Hermite polynomial</p> |
|--|---|---|---|

# Program Listings

|             |                              |             |         |
|-------------|------------------------------|-------------|---------|
| 103 GTO 05  |                              | 154 RCL 03  |         |
| 104+LBL 06  | - - - - -                    | 155 -       |         |
| 105 RCL 00  | Test for recurrence equation | 156 RCL 04  |         |
| 106 RCL 03  |                              | 157 +       |         |
| 107 STO 00  |                              | 158 RCL 01  |         |
| 108 RCL 01  |                              | 159 RCL 04  |         |
| 109 *       |                              | 160 *       |         |
| 110 X<>Y    |                              | 161 RCL 03  |         |
| 111 RCL 02  |                              | 162 -       |         |
| 112 *       |                              | 163 RCL 02  |         |
| 113 -       |                              | 164 /       |         |
| 114 2       |                              | 165 RCL 04  |         |
| 115 *       |                              | 166 STO 03  |         |
| 116 STO 03  |                              | 167 RDN     |         |
| 117 CLX     |                              | 168 -       |         |
| 118 RCL 02  |                              | 169 STO 04  |         |
| 119 1       |                              | 170 RCL 00  |         |
| 120 +       |                              | 171 RCL 02  |         |
| 121 STO 02  |                              | 172 1       |         |
| 122 X<>Y    |                              | 173 +       |         |
| 123 X=Y?    | - - - - -                    | 174 STO 02  |         |
| 124 GTO 06  | Test                         | 175 X<=Y?   |         |
| 125 RCL 03  |                              | 176 GTO 08  | Test    |
| 126 "H"     |                              | 177+LBL 07  |         |
| 127 GTO d   |                              | 178 RCL 04  |         |
| 128+LBL 05  |                              | 179 "L"     |         |
| 129 RCL 03  |                              | 180 GTO d   |         |
| 130 RTN     |                              | 181 STOP    |         |
| 131+LBL "L" | - - - - -                    | 182+LBL A   |         |
| 132 XEQ A   | Laguerre polynomial          | 183 "H?"    |         |
| 133 ENTER†  |                              | 184 PROMPT  |         |
| 134 1       |                              | 185 STO 05  |         |
| 135 STO 03  |                              | 186 "X?"    |         |
| 136 +       |                              | 187 PROMPT  |         |
| 137 STO 01  |                              | 188 STO 06  |         |
| 138 2       |                              | 189 RTN     |         |
| 139 STO 02  |                              | 190+LBL d   |         |
| 140 X<>Y    |                              | 191 CF 29   | Display |
| 141 -       |                              | 192 FIX 0   |         |
| 142 STO 04  |                              | 193 ARCL 05 |         |
| 143 CLX     |                              | 194 "F<"    |         |
| 144 X<>Y    |                              | 195 SF 29   |         |
| 145 X<=Y?   |                              | 196 FIX 2   |         |
| 146 GTO 00  |                              | 197 ARCL 06 |         |
| 147 STO 00  |                              | 198 "F>="   |         |
| 148 1       |                              | 199 ARCL X  |         |
| 149 X<>Y    |                              | 200 AVIEW   |         |
| 150 X<=Y?   |                              | 201 STOP    |         |
| 151 GTO 07  | - - - - -                    | 202 .END.   |         |
| 152+LBL 08  | Loop for recurrence equation |             |         |
| 153 RCL 04  |                              |             |         |

## **REGISTERS, STATUS, FLAGS, ASSIGNMENTS**

| DATA REGISTERS |              |    | STATUS                             |                      |               |                 |  |     |
|----------------|--------------|----|------------------------------------|----------------------|---------------|-----------------|--|-----|
| 00             | temp. change | 50 | SIZE 007<br>ENG _____<br>DEG _____ | TOT. REG. 45         | USER MODE     |                 |  |     |
|                | temp. change |    |                                    | FIX 2 SCI _____      | ON            | OFF X           |  |     |
|                | temp. change |    |                                    | RAD _____ GRAD _____ |               |                 |  |     |
|                | temp. change |    |                                    |                      |               |                 |  |     |
| 05             | n            | 55 | FLAGS                              |                      |               |                 |  |     |
|                | x            |    | #                                  | INIT S/C             | SET INDICATES | CLEAR INDICATES |  |     |
| 10             |              | 60 |                                    |                      | NONE          |                 |  |     |
| 15             |              | 65 |                                    |                      |               |                 |  |     |
| 20             |              | 70 |                                    |                      |               |                 |  |     |
| 25             |              | 75 |                                    |                      |               |                 |  |     |
| 30             |              | 80 |                                    |                      |               |                 |  |     |
| 35             |              | 85 |                                    |                      |               |                 |  |     |
| ASSIGNMENTS    |              |    |                                    |                      |               |                 |  |     |
| 40             |              | 90 | FUNCTION                           |                      | KEY           | FUNCTION        |  | KEY |
|                |              |    | NONE                               |                      |               |                 |  |     |
| 45             |              | 95 |                                    |                      |               |                 |  |     |

CHEBYSHEV LEGENDRE HERMITE  
AND LAGUERRE POLYNOMIALS  
PROGRAM REGISTERS NEEDED: 39

ROW 1 (1 - 7)



ROW 2 (8 - 18)



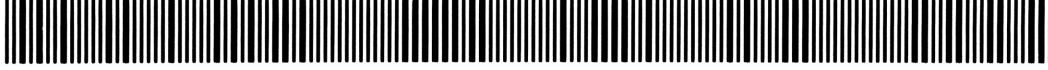
ROW 3 (19 - 31)



ROW 4 (32 - 40)



ROW 5 (40 - 44)



ROW 6 (45 - 55)



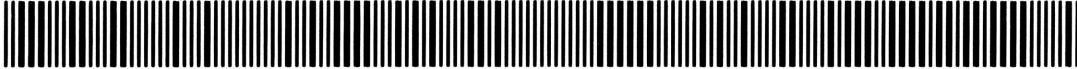
ROW 7 (56 - 68)



ROW 8 (69 - 81)



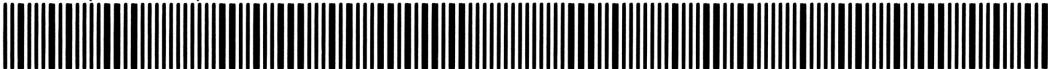
ROW 9 (81 - 89)



ROW 10 (89 - 99)



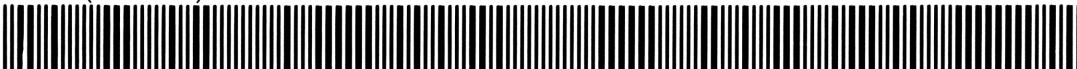
ROW 11 (100 - 111)



ROW 12 (112 - 124)



ROW 13 (124 - 131)



ROW 14 (131 - 140)



ROW 15 (141 - 151)



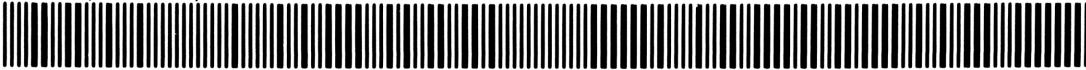
ROW 16 (152 - 164)



ROW 17 (165 - 176)



ROW 18 (177 - 183)



CHEBYSHEV LEGENDRE HERMITE  
AND LAGUERRE POLYNOMIALS

ROW 19 (184 - 192)



ROW 20 (192 - 198)



ROW 21 (198 - 202)



## SIXTEEN-POINT GAUSSIAN QUADRATURE

This program will compute approximations for integrals over finite or infinite intervals by the sixteen-point Gauss-Legendre quadrature method. If  $f(x)$  is the function to be integrated, then either

$$\int_a^b f(x) \, dx \quad \text{or} \quad \int_a^\infty f(x) \, dx \quad \text{may be found.}$$

The function  $f(x)$  must be explicitly known and keyed into program memory under a separate program label. This function assumes the value of  $x$  will be in the X-register. Registers greater than 20 and the stack are available to the user to define  $f(x)$ .

$$\int_a^b f(x) \, dx = \frac{b-a}{2} \sum_{i=1}^{16} w_i f\left(\frac{z_i(b-a) + b + a}{2}\right)$$

$$\int_a^\infty f(x) \, dx = 2 \sum_{i=1}^{16} \frac{w_i}{(1+z_i)^2} f\left(\frac{2}{1+z_i} + a-1\right)$$

The constants ( $w_i$ 's and  $z_i$ 's) can be stored on a data card; their values and memory locations are given on the following page.

### NOTE:

1. The Trig mode should be set to radians by your routine to avoid any oversights in computing integrals involving trig functions.
2. The total space required to execute this program is 46 registers plus the program space used to define  $f(x)$ .

**Examples:**

1. Find  $\int_1^4 \frac{1}{x^2} dx$

2. Find  $\int_1^\infty \frac{1}{x^2} dx$

**Keystrokes:**

[XEQ] [ALPHA] SIZE [ALPHA] 021  
 [///] [FIX] 4  
 2.715245941 [EEX] 2 [CHS] [STO] 01  
 9.894009350 [EEX] 1 [CHS] [STO] 02  
 6.225352394 [EEX] 2 [CHS] [STO] 03  
 9.445750231 [EEX] 1 [CHS] [STO] 04  
 9.515851168 [EEX] 2 [CHS] [STO] 05  
 8.656312024 [EEX] 1 [CHS] [STO] 06  
 1.246289713 [EEX] 1 [CHS] [STO] 07  
 7.554044084 [EEX] 1 [CHS] [STO] 08  
 1.495959888 [EEX] 1 [CHS] [STO] 09  
 6.178762444 [EEX] 1 [CHS] [STO] 10  
 1.691565194 [EEX] 1 [CHS] [STO] 11  
 4.580167777 [EEX] 1 [CHS] [STO] 12  
 1.826034150 [EEX] 1 [CHS] [STO] 13  
 2.816035508 [EEX] 1 [CHS] [STO] 14  
 1.894506105 [EEX] 1 [CHS] [STO] 15  
 9.501250984 [EEX] 2 [CHS] [STO] 16  
 [///] [GTO] ..

[PRGM]

[///] [LBL] [ALPHA] X [ALPHA]  
 [1/x]  
 [///] [x<sub>2</sub>]  
 [///] [GTO] ..

[PRGM]

[XEQ] [ALPHA] GAUSS [ALPHA]  
 [A]  
 X [R/S]  
 1 [R/S]  
 4 [R/S]  
 [B]  
 X [R/S]  
 1 [R/S]

**Display:**

|        |                    |
|--------|--------------------|
| 01     | LBL <sup>T</sup> X |
| 02     | 1/X                |
| 03     | X <sup>1/2</sup>   |
| NAME?  |                    |
| a?     |                    |
| b?     |                    |
| 0.7500 |                    |
| NAME?  |                    |
| a?     |                    |
| 1.0000 |                    |

# User Instructions

# Program Listings

|   |  |  |
|---|--|--|
| <pre> 01+LBL "GAU SS" 02 SF 27 03 STOP 04+LBL A 05 CF 05 06 GTO 00 07+LBL B 08 SF 05 09+LBL 00 10 "NAME?" 11 RON 12 STOP 13 AOFF 14 ASTO 20 15 "a?" 16 PROMPT 17 STO 17 18 "b?" 19 FC? 05 20 PROMPT 21 FC? 05 22 STO 18 23 0 24 STO 00 25 SF 06 26+LBL 01 27 16 28 STO 19 29+LBL 11 30 FS? 05 31 XEQ 06 32 FC? 05 33 XEQ 05 34 DSE 19 35 GTO 11 36 FC?C 06 37 SF 06 38 FC? 06 39 GTO 01 40 FS? 05 41 GTO 04 42+LBL 02 43 RCL 18 44 RCL 17 45 - 46 2 47 / 48 RCL 00 49 * 50 RTN </pre> | <p>Set User Mode</p> <p>from a to b</p> <p>from a to <math>\infty</math></p> <p>Input</p> <p>Iterative loop</p> <p><math>b-a \Sigma</math></p> | <pre> 51+LBL 04 52 RCL 00 53 2 54 * 55 RTN 56+LBL 05 57 RCL IND 19 58 FS? 06 59 CHS 60 RCL 18 61 RCL 17 62 - 63 + 64 RCL 18 65 + 66 RCL 17 67 + 68 2 69 / 70 XEQ IND 20 71 DSE 19 72 RCL IND 19 73 * 74 ST+ 00 75 RTN 76+LBL 06 77 RCL IND 19 78 FS? 06 79 CHS 80 1 81 + 82 2 83 X&lt;&gt;Y 84 / 85 RCL 17 86 + 87 1 88 - 89 XEQ IND 20 90 RCL IND 19 91 FS? 06 92 CHS 93 1 94 + 95 X†2 </pre> |
|---|--|--|

# Program Listings

|     |        |     |  |    |  |
|-----|--------|-----|--|----|--|
| 96  | DSE    | 19  |  | 51 |  |
| 97  | RCL    | IND |  |    |  |
| 19  |        |     |  |    |  |
| 98  | X< >Y  |     |  |    |  |
| 99  | /      |     |  |    |  |
| 100 | *      |     |  |    |  |
| 101 | ST+ 00 |     |  |    |  |
| 102 | END    |     |  |    |  |
| 10  |        |     |  | 60 |  |
| 20  |        |     |  | 70 |  |
| 30  |        |     |  | 80 |  |
| 40  |        |     |  | 90 |  |
| 50  |        |     |  | 00 |  |

# REGISTERS, STATUS, FLAGS, ASSIGNMENTS

| DATA REGISTERS |                                    |       |  | STATUS                  |          |                                |                                |
|----------------|------------------------------------|-------|--|-------------------------|----------|--------------------------------|--------------------------------|
| #              | Registers                          | Value |  | Status Parameters       |          |                                |                                |
|                |                                    |       |  | SIZE                    | 21       | TOT. REG.                      | 46                             |
| 00             | Sum                                | 50    |  | ENG                     | _____    | FIX                            | _____                          |
|                | W <sub>15</sub> , W <sub>16</sub>  |       |  | DEG                     | _____    | SCI                            | _____                          |
| 05             | Z <sub>15</sub> , -Z <sub>16</sub> |       |  | ENG                     | _____    | ON X                           | OFF _____                      |
|                | W <sub>13</sub> , W <sub>14</sub>  |       |  | DEG                     | _____    | RAD                            | GRAD _____                     |
| 10             | Z <sub>13</sub> , -Z <sub>14</sub> |       |  | FLAGS                   |          |                                |                                |
|                | W <sub>11</sub> , W <sub>12</sub>  | 55    |  | #                       | INIT S/C | SET INDICATES                  | CLEAR INDICATES                |
| 15             | Z <sub>11</sub> , -Z <sub>12</sub> |       |  | 05                      |          | Integral from<br>a to b        | Integral from<br>a to $\infty$ |
|                | W <sub>9</sub> , W <sub>10</sub>   |       |  | 06                      |          | Negative loop                  | Positive loop                  |
| 20             | Z <sub>9</sub> , -Z <sub>10</sub>  |       |  |                         |          |                                |                                |
|                | W <sub>7</sub> , W <sub>8</sub>    |       |  |                         |          |                                |                                |
| 25             | Z <sub>7</sub> , -Z <sub>8</sub>   | 60    |  |                         |          |                                |                                |
|                | W <sub>5</sub> , W <sub>6</sub>    |       |  |                         |          |                                |                                |
| 30             | Z <sub>5</sub> , -Z <sub>6</sub>   |       |  |                         |          |                                |                                |
|                | W <sub>3</sub> , W <sub>4</sub>    |       |  |                         |          |                                |                                |
| 35             | Z <sub>3</sub> , -Z <sub>4</sub>   |       |  |                         |          |                                |                                |
|                | W <sub>1</sub> , W <sub>2</sub>    | 65    |  | ASSIGNMENTS             |          |                                |                                |
| 40             | Z <sub>1</sub> , -Z <sub>2</sub>   |       |  | FUNCTION                | KEY      | FUNCTION                       | KEY                            |
|                | b                                  |       |  | Integral from<br>a to b | A        | Integral from<br>a to $\infty$ | B                              |
| 45             | a                                  |       |  |                         |          |                                |                                |
|                | Index                              |       |  |                         |          |                                |                                |
| 50             | Function Name                      | 70    |  |                         |          |                                |                                |
|                |                                    |       |  |                         |          |                                |                                |
| 55             |                                    | 75    |  |                         |          |                                |                                |
|                |                                    |       |  |                         |          |                                |                                |
| 60             |                                    | 80    |  |                         |          |                                |                                |
|                |                                    |       |  |                         |          |                                |                                |
| 65             |                                    | 85    |  |                         |          |                                |                                |
|                |                                    |       |  |                         |          |                                |                                |
| 70             |                                    | 90    |  |                         |          |                                |                                |
|                |                                    |       |  |                         |          |                                |                                |
| 75             |                                    | 95    |  |                         |          |                                |                                |
|                |                                    |       |  |                         |          |                                |                                |

SIXTEEN-POINT GAUSSIAN  
QUADRATURE  
PROGRAM REGISTERS NEEDED: 24

ROW 1 (1 : 3)



ROW 2 (4 : 8)



ROW 3 (9 : 16)



ROW 4 (16 : 24)



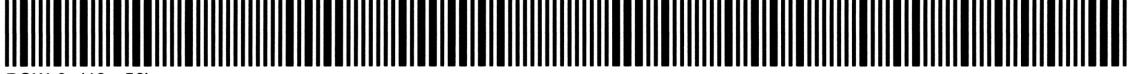
ROW 5 (25 : 31)



ROW 6 (31 : 37)



ROW 7 (37 : 45)



ROW 8 (46 : 56)



ROW 9 (57 : 65)



ROW 10 (66 : 74)



ROW 11 (75 : 84)



ROW 12 (85 : 94)



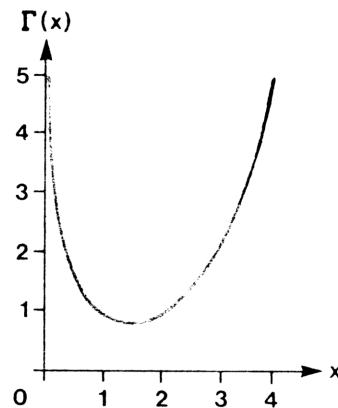
ROW 13 (94 : 100)



## GAMMA FUNCTION

This program approximates the value of the gamma function,  $\Gamma(x)$ , for  $1 \leq x \leq 70$ .

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$$



1.  $\Gamma(x) = (x - 1) \Gamma(x-1)$  if  $x > 2$

2. For  $1 \leq x \leq 2$ , polynomial approximation can be used.

$$\Gamma(x) \approx 1 + b_1(x - 1) + b_2(x - 1)^2 + \dots + b_8(x - 1)^8$$

where  $b_1 = -0.577191652$ ,  $b_2 = 0.988205891$

$b_3 = -0.897056937$ ,  $b_4 = 0.918206857$

$b_5 = -0.756704078$ ,  $b_6 = 0.482199394$

$b_7 = -0.193527818$ ,  $b_8 = 0.035868343$

Remarks:

1. This program can be used to find the generalized factorial  $x!$  for  $0 \leq x \leq 69$ . where  $x! = \Gamma(x + 1)$ .
2. When the value keyed in for  $x$  is an integer,  $\Gamma(x)$  is evaluated as the factorial of  $(x-1)$ .
3. If  $x < 1$ , the program will halt and display "ILLEGAL X".

References:

Handbook of Mathematical Functions, Abramowitz and Stegun, National Bureau of Standards, 1968.

Examples: Find the gamma function for the arguments

5.25, 8, and 3.34.

Keystrokes:

[XEQ] [ALPHA] SIZE [ALPHA] 001  
[///] [FIX] 2  
5.25 [XEQ] [ALPHA] GAMMA [ALPHA]  
8 [R/S]  
3.34 [R/S]

Display:

GAMMA=35.21  
GAMMA=5,040.00  
GAMMA=2.80

# User Instructions

# Program Listings

|  |  |  |
|--|--|--|
| 01+LBL "GAM<br>MA"<br>02 1<br>03 -<br>04 "ILLEGAL<br>X"<br>05 X<0?<br>06 GTO 01<br>07 INT<br>08 LASTX<br>09 X=Y?<br>10 GTO 02<br>11 1<br>12 STO 00<br>13 X<>Y<br>14+LBL 03<br>15 X<=Y?<br>16 GTO 00<br>17 ST* 00<br>18 1<br>19 -<br>20 GTO 03<br>21+LBL 00<br>22 ENTER↑<br>23 ENTER↑<br>24 ENTER↑<br>25 .0358683<br>43<br>26 *<br>27 .1935278<br>18<br>28 -<br>29 *<br>30 .4821993<br>94<br>31 +<br>32 *<br>33 .7567040<br>78<br>34 -<br>35 *<br>36 .9182068<br>57<br>37 +<br>38 *<br>39 .8970569<br>37<br>40 -<br>41 *<br>42 .9882058<br>91 | (x-1)<0, error<br><br>(x-1)(x-2)<br>(x-3) ...<br>until <1<br><br>polynomial<br>approximation | 43 +<br>44 *<br>45 .5771916<br>52<br>46 -<br>47 *<br>48 1<br>49 +<br>50 RCL 00<br>51 *<br>52 GTO 04<br>53+LBL 02<br>54 FACT<br>55+LBL 04<br>56 "GAMMA=" Output<br>57 ARCL X<br>58+LBL 01<br>59 AVIEW<br>60 .END.<br><br>70<br><br>80<br><br>90<br><br>00 |
|--|--|--|

## **REGISTERS, STATUS, FLAGS, ASSIGNMENTS**

| DATA REGISTERS |   |    | STATUS      |          |               |                 |           |
|----------------|---|----|-------------|----------|---------------|-----------------|-----------|
| 00             | Γ | 50 | SIZE        | 001      | TOT. REG.     | 29              | USER MODE |
|                |   |    | ENG         | —        | FIX           | —               | ON OFF    |
|                |   |    | DEG         | —        | RAD           | —               | GRAD      |
| 05             |   | 55 | FLAGS       |          |               |                 |           |
|                |   |    | #           | INIT S/C | SET INDICATES | CLEAR INDICATES |           |
| 10             |   | 60 |             |          |               |                 |           |
| 15             |   | 65 |             |          |               |                 |           |
| 20             |   | 70 |             |          |               |                 |           |
| 25             |   | 75 |             |          |               |                 |           |
| 30             |   | 80 |             |          |               |                 |           |
| 35             |   | 85 |             |          |               |                 |           |
| 40             |   | 90 | ASSIGNMENTS |          |               |                 |           |
|                |   |    | FUNCTION    | KEY      | FUNCTION      | KEY             |           |
| 45             |   | 95 |             |          |               |                 |           |

## GAMMA FUNCTION

PROGRAM REGISTERS NEEDED: 24

ROW 1 (1 : 4)



ROW 2 (4 : 8)



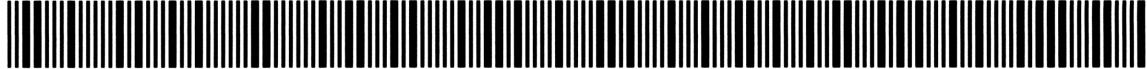
ROW 3 (9 : 18)



ROW 4 (19 : 25)



ROW 5 (25 : 27)



ROW 6 (27 : 30)



ROW 7 (30 : 33)



ROW 8 (34 : 37)



ROW 9 (38 : 41)



ROW 10 (42 : 45)



ROW 11 (45 : 49)



ROW 12 (50 : 56)



ROW 13 (56 : 60)



## BESSEL FUNCTIONS, ERROR FUNCTION

The first routine computes the Bessel functions  $J_n(x)$  and  $I_n(x)$ , where  $n$  is a positive integer and  $x > \emptyset$ . The second of the two routines finds the error function and complementary error function for positive arguments.

### Bessel Functions

The Bessel functions  $J_n(x)$  and  $I_n(x)$  are computed by generating trial values  $T_k$  through the use of recurrence relations. The recurrence is begun at an index  $m$  given by

$$m = 2 \text{ INT} \left[ \frac{6 + \max(n, z) + \frac{9z}{z+2}}{2} \right]$$

where

$$z = \frac{3x}{2} .$$

The initial values selected for recurrence are  $T_{m+1} = 10^{-9}$ ,  $T_{m+2} = 0$ .

For the functions  $J_n(x)$ , each term  $T_k$  ( $0 \leq k \leq m$ ) is computed by the relation

$$T_k(x) = \frac{2(k+1)}{x} T_{k+1}(x) - T_{k+2}(x)$$

beginning with  $k = m$ .

$J_n(x)$  is then found by dividing the term  $T_n(x)$  by the normalizing constant

$$K = T_0(x) + 2 \sum_{k=1}^{m/2} T_{2k}(x).$$

After calculating a  $J_n(x)$ , the values of  $J_0(x)$  and  $J_1(x)$  may also be found with very little additional computation.

For the functions  $I_n(x)$ , each  $T_k$  is calculated from the recurrence relation

$$T_k(x) = \frac{2(k+1)}{x} T_{k+1}(x) + T_{k+2}(x),$$

$0 \leq k \leq m$ , beginning with  $k = m$ .

$I_n(x)$  is then found from the equation:

$$I_n(x) = e^x \frac{T_n(x)}{T_0(x) + 2 \sum_{k=1}^m T_k(x)}$$

### Error Function

The error function is defined as

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

and the complementary error function as

$$\text{erfc}(x) = 1 - \text{erf}(x).$$

For large values of  $x (> 3)$ , the error function is very close to 1. If  $\text{erfc}(x)$  is computed as  $1 - \text{erf}(x)$ , most of the significant figures of  $\text{erfc}(x)$  will be lost for  $x > 3$ . Hence two different algorithms are employed in this program, one for  $x \leq 3$  and one for  $x > 3$ . For  $x \leq 3$ , the error function is computed by a series sum

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} e^{-x^2} \sum_{n=0}^{\infty} \frac{2^n}{1 \cdot 3 \dots (2n+1)} x^{2n+1}$$

and the complementary error function

$$\text{erfc}(x) = 1 - \text{erf}(x).$$

For  $x > 3$ , the complementary error function is computed first, by the asymptotic expansion

$$\text{erfc}(x) = \frac{1}{x \sqrt{\pi}} e^{-x^2} \left[ 1 + \sum_{n=1}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot \dots \cdot (2n-1)}{(2x^2)^n} \right]$$

and the error function by

$$\text{erf}(x) = 1 - \text{erfc}(x).$$

The accuracy of the calculation of  $\text{erf}(x)$  and  $\text{erfc}(x)$  from series sums may be controlled by the user. For  $x \leq 3$ , it is quite reasonable to specify 9 for maximum accuracy; for  $x > 3$ , the series may never converge with 9, and a safer specification would be 6 digits.

**Remarks:**

1. The range of values  $0 \leq x \leq 10^{-6}$  is out of bounds for the Bessel functions in this program. In this range, however, one may take  $J_0(x) = J_0(0) = I_0(x) = I_0(0) = 1$ , and  $J_n(x) = J_n(0) = I_n(x) = I_n(0) = 0$ ,  $n \neq 0$ .
2. The computation of  $\text{erfc}(x)$  will halt on overflow for  $x \geq 15$ .

**Reference:** Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1968.

**Examples:**

1. Find  $J_5$  (9.2)
2. Find  $J_0$  (9.2)
3. Find  $J_1$  (9.2)
4. Find  $I_3$  (4.7)
5. Find  $\text{erf}$  and  $\text{erfc}$  4.55 to 6 places

**Keystrokes:****Display:**

|                                |                   |
|--------------------------------|-------------------|
| [USER]                         | (set USER mode)   |
| [XEQ] [ALPHA] SIZE [ALPHA] 007 |                   |
| [XEQ] [ALPHA] INIT [ALPHA]     |                   |
| [J]                            | N?                |
| 5 [R/S]                        | X?                |
| 1) 9.2 [R/S]                   | J=-0.1005         |
| 2) [R/S]                       | J0=-0.1367        |
| 3) [R/S]                       | J1=0.2174         |
| [I]                            | N?                |
| 3 [R/S]                        | X?                |
| 4) 4.7 [R/S]                   | I=7.4195          |
| [E]                            | ACCURACY?         |
| 6 [R/S]                        | X?                |
| 5) 4.55 [R/S]                  | ERF=1.000000      |
| [R/S]                          | ERFC=1.237405E-10 |

# User Instructions

# Program Listings

|   |   |   |
|---|---|---|
| <pre> 01♦LBL "INI T" 02 CLRG 03 RTN 04♦LBL J 05 XEQ a 06 SF 00 07♦LBL 09 08 XEQ b 09 CF 02 10 ST+ 00 11 XEQ b 12 FS?C 02 13 GTO 09 14 RCL 03 15 RCL 00 16 ENTER↑ 17 + 18 RCL 05 19 - 20 / 21 "J=" 22 XEQ d 23 GTO C 24♦LBL a 25 FIX 4 26 "N?" 27 PROMPT 28 STO 01 29 "X?" 30 PROMPT 31 1.5 32 * 33 STO 03 34 RCL 01 35 X&lt;=Y? 36 X&lt;&gt;Y 37 6 38 + 39 RCL 03 40 9 41 * 42 RCL 03 43 2 44 + 45 / 46 + 47 2 48 / 49 INT 50 ENTER↑ </pre> | <pre> "J" -----<br/> input n + x<br/> -----<br/> initialization<br/> for Bessel<br/> (J<sub>n</sub> + I<sub>n</sub>) </pre> | <pre> 51 + 52 2 53 + 54 STO 06 55 3 56 RCL 03 57 / 58 STO 02 59 0 60 STO 05 61 STO 00 62 E-9 63 STO 04 64 RTN 65♦LBL b 66 DSE 06 67 SF 02 68 RCL 06 69 RCL 01 70 X#Y? 71 GTO 00 72 RCL 04 73 STO 03 74♦LBL 00 75 RDN 76 RCL 05 77 FS? 00 78 CHS 79 X&lt;&gt;Y 80 RCL 02 81 * 82 RCL 04 83 STO 05 84 * 85 + 86 STO 04 87 RTN 88♦LBL C 89 RCL 05 90 RCL 00 91 ENTER↑ 92 + 93 RCL 05 94 - 95 / 96 "J0=" 97 XEQ d 98 RCL 04 99 CHS 100 RCL 00 101 ENTER↑ 102 + </pre> <p>-----<br/> Compute one term<br/> FZ set except<br/> for F = 0</p> <p>-----<br/> Compute J<sub>0</sub>(x) +<br/> J<sub>1</sub>(x)</p> |
|---|---|---|

# Program Listings

|              |               |            |
|--------------|---------------|------------|
| 103 RCL 05   |               | 152 X>Y?   |
| 104 -        |               | 153 GTO 03 |
| 105 /        |               | 154♦LBL 07 |
| 106 "J1="    |               | 155 RCL 02 |
| 107 GTO d    |               | 156 RCL 03 |
| 108♦LBL I    | "I"           | 157 2      |
| 109 CF 00    |               | 158 +      |
| 110 XEQ a    |               | 159 STO 03 |
| 111♦LBL 08   |               | 160 /      |
| 112 ST+ 00   |               | 161 RCL 01 |
| 113 XEQ b    |               | 162 *      |
| 114 FS?C 02  |               | 163 STO 01 |
| 115 GTO 08   |               | 164 +      |
| 116 RCL 03   |               | 165 X<>Y   |
| 117 RCL 00   |               | 166 RND    |
| 118 ENTER†   |               | 167 X<>Y   |
| 119 +        |               | 168 RND    |
| 120 RCL 05   |               | 169 X=Y?   |
| 121 -        |               | 170 GTO 00 |
| 122 /        |               | 171 LASTX  |
| 123 2        |               | 172 GTO 07 |
| 124 RCL 02   |               | 173♦LBL 00 |
| 125 /        |               | 174 LASTX  |
| 126 E↑X      |               | 175 RCL 04 |
| 127 *        |               | 176 /      |
| 128 " I="    |               | 177 2      |
| 129 GTO d    |               | 178 *      |
| 130♦LBL E    |               | 179 1      |
| 131 "ACCURAC | Compute error | 180 X<>Y   |
| Y?"          |               | 181 -      |
| 132 PROMPT   |               | 182 LASTX  |
| 133 FIX IND  |               | 183 GTO 02 |
| X            |               | 184♦LBL 03 |
| 134 "X?"     |               | 185 RCL 02 |
| 135 PROMPT   |               | 186 1/X    |
| 136 STO 01   |               | 187 STO 02 |
| 137 X↑2      |               | 188 RCL 01 |
| 138 STO 04   |               | 189 1/X    |
| 139 2        |               | 190 STO 01 |
| 140 *        |               | 191♦LBL 06 |
| 141 STO 02   |               | 192 RCL 02 |
| 142 1        |               | 193 RCL 03 |
| 143 STO 03   |               | 194 2      |
| 144 RCL 04   |               | 195 -      |
| 145 E↑X      |               | 196 STO 03 |
| 146 PI       |               | 197 *      |
| 147 SQRT     |               | 198 RCL 01 |
| 148 *        |               | 199 *      |
| 149 STO 04   |               | 200 STO 01 |
| 150 3        |               | 201 +      |
| 151 RCL 01   |               | 202 X<>Y   |
|              |               | 203 RND    |

# Program Listings

|             |         |    |  |
|-------------|---------|----|--|
| 204 X<>Y    |         | 51 |  |
| 205 RND     |         |    |  |
| 206 X=Y?    |         |    |  |
| 207 GTO 00  |         |    |  |
| 208 LASTX   |         |    |  |
| 209 GTO 06  |         |    |  |
| 210♦LBL 00  | -----   |    |  |
| 211 LASTX   | erfc(x) |    |  |
| 212 RCL 04  |         |    |  |
| 213 /       |         | 60 |  |
| 214 1       |         |    |  |
| 215 X<>Y    |         |    |  |
| 216 -       | -----   |    |  |
| 217 LASTX   | erf(x)  |    |  |
| 218 X<>Y    |         |    |  |
| 219♦LBL 02  |         |    |  |
| 220 "ERF="  |         |    |  |
| 221 XEQ d   |         |    |  |
| 222 X<>Y    |         |    |  |
| 223 "ERFC=" | -----   | 70 |  |
| 224♦LBL d   | display |    |  |
| 225 ARCL X  |         |    |  |
| 226 AVIEW   |         |    |  |
| 227 STOP    |         |    |  |
| 228 .END.   |         |    |  |
| 30          |         | 80 |  |
| 40          |         | 90 |  |
| 50          |         | 00 |  |

## **REGISTERS, STATUS, FLAGS, ASSIGNMENTS**

| DATA REGISTERS |   |                |               | STATUS |      |           |           |
|----------------|---|----------------|---------------|--------|------|-----------|-----------|
| #              | NAME  | DECIMAL PLACES | INITIAL VALUE | SIZE   |      | TOT. REG. | USER MODE |
|                |   |                |               | ENG    | DEG  | 007       | 53        |
|                |   |                |               | FIX    | SCI  | 09        | ON X OFF  |
|                |   |                |               | RAD    | GRAD |           |           |
| 00             | $\sum T_k$<br>n; erf term<br>$2/x$<br>$1.5x, T_n$<br>$T_k; (e^{x^2} \sqrt{\pi})^{-1}$ | 50             |               |        |      |           |           |
| 05             | $T_{k+1}$<br>K; places  | 55             |               |        |      |           |           |
| 10             |   | 60             |               |        |      |           |           |
| 15             |   | 65             |               |        |      |           |           |
| 20             |   | 70             |               |        |      |           |           |
| 25             |   | 75             |               |        |      |           |           |
| 30             |   | 80             |               |        |      |           |           |
| 35             |   | 85             |               |        |      |           |           |
| 40             |   | 90             |               |        |      |           |           |
| 45             |   | 95             |               |        |      |           |           |

## BESSEL FUNCTIONS

## ERROR FUNCTION

PROGRAM REGISTERS NEEDED: 47

ROW 1 (1 - 5)



ROW 2 (5 - 11)



ROW 3 (11 - 20)



ROW 4 (21 - 25)



ROW 5 (26 - 32)



ROW 6 (33 - 45)



ROW 7 (46 - 58)



ROW 8 (59 - 67)



ROW 9 (67 - 77)



ROW 10 (78 - 89)



ROW 11 (90 - 97)



ROW 12 (98 - 107)



ROW 13 (107 - 113)



ROW 14 (113 - 122)



ROW 15 (123 - 130)



ROW 16 (131 - 133)



ROW 17 (134 - 144)



ROW 18 (145 - 156)

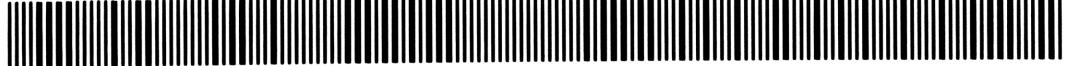


BESSEL FUNCTIONS  
ERROR FUNCTION

ROW 19 (157 - 169)



ROW 20 (170 - 180)



ROW 21 (181 - 192)



ROW 22 (193 - 205)



ROW 23 (206 - 216)



ROW 24 (217 - 223)



ROW 25 (223 - 228)



ROW 26 (228 - 228)



## CHARACTERISTIC EQUATION OF A $4 \times 4$ MATRIX

(THIS PROGRAM REQUIRES ONE ADDITIONAL MEMORY MODULE)

Given

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}$$

The characteristic equation is  $\lambda^4 + r_1\lambda^3 + r_2\lambda^2 + r_3\lambda + r_4 = 0$

where

$$r_1 = -(a_{1,1} + a_{2,2} + a_{3,3} + a_{4,4})$$

$$r_2 = (a_{1,1} + a_{3,3})a_{2,2} + (a_{1,1} + a_{4,4})a_{3,3} + (a_{1,1} + a_{2,2})a_{4,4} - a_{2,4}a_{4,2} - a_{3,4}a_{4,3} - a_{2,3}a_{3,2} - a_{1,2}a_{2,1} - a_{1,3}a_{3,1} - a_{1,4}a_{4,1}$$

$$\begin{aligned} r_3 = & -\det(A_1) - a_{1,1}(a_{2,2}a_{3,3} + a_{2,2}a_{4,4} + a_{3,3}a_{4,4} - a_{2,4}a_{4,2} - a_{3,4}a_{4,3} - a_{2,3}a_{3,2}) + a_{1,2}[a_{2,1}(a_{3,3} + a_{4,4}) - a_{2,4}a_{4,1} - a_{2,3}a_{3,1}] - a_{1,3}[-a_{3,1}(a_{2,2} + a_{4,4}) + a_{2,1}a_{3,2} + a_{3,4}a_{4,1}] + a_{1,4}[a_{4,1}(a_{2,2} + a_{3,3}) - a_{3,1}a_{4,3} - a_{2,1}a_{4,2}] \end{aligned}$$

$$r_4 = a_{1,1}\det(A_1) - a_{1,2}\det(A_2) + a_{1,3}\det(A_3) - a_{1,4}\det(A_4)$$

and

$$A_1 = \begin{pmatrix} a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix} \quad A_2 = \begin{pmatrix} a_{2,1} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,3} & a_{4,4} \end{pmatrix} \quad A_3 = \begin{pmatrix} a_{2,1} & a_{2,2} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,4} \end{pmatrix}$$

and

$$A_4 = \begin{pmatrix} a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \\ a_{4,1} & a_{4,2} & a_{4,3} \end{pmatrix}$$

NOTE: Trace (A) =  $r_1$ ,  $\det (A) = -r_4$ .

Example: Find the characteristic equation of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 2 & -1 \\ 3 & -1 & 0 & 2 \\ -2 & -1 & -1 & 0 \end{pmatrix} \quad (\text{Ans. } \lambda^4 - \lambda^3 + 7\lambda + 2 = 0)$$

Keystrokes:

|                                |                     |
|--------------------------------|---------------------|
| [XEQ] [ALPHA] SIZE [ALPHA] 026 |                     |
| [XEQ] [ALPHA] CEM [ALPHA]      | a <sub>1,1</sub> =? |
| 1 [R/S]                        | a <sub>1,2</sub> =? |
| 0 [R/S]                        | a <sub>1,3</sub> =? |
| 1 [R/S]                        | a <sub>1,4</sub> =? |
| 0 [R/S]                        | a <sub>2,1</sub> =? |
| 1 [R/S]                        | a <sub>2,2</sub> =? |
| 0 [R/S]                        | a <sub>2,3</sub> =? |
| 2 [R/S]                        | a <sub>2,4</sub> =? |
| 1 [CHS] [R/S]                  | a <sub>3,1</sub> =? |
| 3 [R/S]                        | a <sub>3,2</sub> =? |
| 1 [CHS] [R/S]                  | a <sub>3,3</sub> =? |
| 0 [R/S]                        | a <sub>3,4</sub> =? |
| 2 [R/S]                        | a <sub>4,1</sub> =? |
| 2 [CHS] [R/S]                  | a <sub>4,2</sub> =? |
| 1 [CHS] [R/S]                  | a <sub>4,3</sub> =? |
| 1 [CHS] [R/S]                  | a <sub>4,4</sub> =? |
| 0 [R/S]                        | READY               |
| [R/S]                          | R1=-1.000           |
| [R/S]*                         | R2=0.000            |
| [R/S]*                         | R3=7.000            |
| [R/S]*                         | R4=2.000            |

Display:

\*[R/S] is omitted if printer is attached.

# User Instructions

SIZE: 026

# Program Listings

|             |               |            |
|-------------|---------------|------------|
| 01♦LBL "CEM |               | 49 STO 02  |
| "           | Initialize    | 50 ISG 01  |
| 02 SF 21    |               | 51 GTO 04  |
| 03 SF 27    |               | 52 "READY" |
| 04♦LBL A    |               | 53 PROMPT  |
| 05 CF 05    |               | 54♦LBL C   |
| 06 CF 07    | Input matrix  | 55 RCL 10  |
| 07 GTO 03   |               | 56 STO 00  |
| 08♦LBL b    |               | 57 RCL 15  |
| 09 ADV      |               | 58 +       |
| 10 SF 05    | Print matrix  | 59 RCL 20  |
| 11 SF 07    |               | 60 +       |
| 12 GTO 03   |               | 61 RCL 25  |
| 13♦LBL B    |               | 62 +       |
| 14 SF 05    | View matrix   | 63 CHS     |
| 15 CF 07    |               | 64 "R1"    |
| 16♦LBL 03   |               | 65 XEQ 02  |
| 17 1.004    |               | 66 RCL 10  |
| 18 STO 01   | Initialize    | 67 RCL 20  |
| 19 STO 02   | Input Routine | 68 +       |
| 20 10       |               | 69 RCL 15  |
| 21 STO 00   |               | 70 *       |
| 22♦LBL 04   |               | 71 RCL 10  |
| 23 FIX 0    |               | 72 RCL 25  |
| 24 CF 29    |               | 73 +       |
| 25 "a"      |               | 74 RCL 20  |
| 26 ARCL 01  | Common Input  | 75 *       |
| 27 "L,"     | Routine       | 76 +       |
| 28 ARCL 02  |               | 77 RCL 10  |
| 29 "L="     |               | 78 RCL 15  |
| 30 FC? 05   |               | 79 +       |
| 31 "L?"     |               | 80 RCL 25  |
| 32 FIX 4    |               | 81 *       |
| 33 SF 29    |               | 82 +       |
| 34 RCL IND  |               | 83 RCL 17  |
| 00          |               | 84 RCL 23  |
| 35 FS? 05   |               | 85 *       |
| 36 ARCL X   |               | 86 -       |
| 37 FC? 07   |               | 87 RCL 21  |
| 38 PROMPT   |               | 88 RCL 24  |
| 39 FS? 07   |               | 89 *       |
| 40 AVIEW    |               | 90 -       |
| 41 STO IND  |               | 91 RCL 16  |
| 00          |               | 92 RCL 19  |
| 42 ISG 00   |               | 93 *       |
| 43 CLD      |               | 94 -       |
| 44 ISG 02   |               | 95 RCL 11  |
| 45 GTO 04   |               | 96 RCL 14  |
| 46 FS? 07   |               | 97 *       |
| 47 ADV      |               | 98 -       |
| 48 1.004    |               | 99 RCL 12  |
| --          |               |            |

# Program Listings

|            |                          |            |
|------------|--------------------------|------------|
| 100 RCL 18 |                          | 151 RCL 10 |
| 101 *      |                          | 152 *      |
| 102 -      |                          | 153 -      |
| 103 RCL 13 |                          | 154 RCL 20 |
| 104 RCL 22 |                          | 155 RCL 25 |
| 105 *      |                          | 156 +      |
| 106 -      |                          | 157 RCL 14 |
| 107 "R2"   |                          | 158 *      |
| 108 XEQ 02 | Calculate r <sub>3</sub> | 159 RCL 17 |
| 109 RCL 15 |                          | 160 RCL 22 |
| 110 STO 01 |                          | 161 *      |
| 111 RCL 16 |                          | 162 -      |
| 112 STO 02 |                          | 163 RCL 16 |
| 113 RCL 17 |                          | 164 RCL 18 |
| 114 STO 03 |                          | 165 *      |
| 115 RCL 19 |                          | 166 -      |
| 116 STO 04 |                          | 167 RCL 11 |
| 117 RCL 20 |                          | 168 *      |
| 118 STO 05 |                          | 169 +      |
| 119 RCL 21 |                          | 170 RCL 15 |
| 120 STO 06 |                          | 171 RCL 25 |
| 121 RCL 23 |                          | 172 +      |
| 122 STO 07 |                          | 173 RCL 18 |
| 123 RCL 24 |                          | 174 *      |
| 124 STO 08 |                          | 175 RCL 19 |
| 125 RCL 25 |                          | 176 RCL 14 |
| 126 STO 09 |                          | 177 *      |
| 127 XEQ 00 |                          | 178 -      |
| 128 ST* 00 |                          | 179 RCL 21 |
| 129 CHS    |                          | 180 RCL 22 |
| 130 RCL 25 |                          | 181 *      |
| 131 RCL 20 |                          | 182 -      |
| 132 +      |                          | 183 RCL 12 |
| 133 RCL 15 |                          | 184 *      |
| 134 *      |                          | 185 +      |
| 135 LASTX  |                          | 186 RCL 15 |
| 136 RCL 25 |                          | 187 RCL 20 |
| 137 *      |                          | 188 +      |
| 138 +      |                          | 189 RCL 22 |
| 139 RCL 17 |                          | 190 *      |
| 140 RCL 23 |                          | 191 RCL 18 |
| 141 *      |                          | 192 RCL 24 |
| 142 -      |                          | 193 *      |
| 143 RCL 21 |                          | 194 -      |
| 144 RCL 24 |                          | 195 RCL 14 |
| 145 *      |                          | 196 RCL 23 |
| 146 -      |                          | 197 *      |
| 147 RCL 16 |                          | 198 -      |
| 148 RCL 19 |                          | 199 RCL 13 |
| 149 *      |                          | 200 *      |
| 150 -      |                          | 201 +      |

# Program Listings

|            |  |            |  |
|------------|--|------------|--|
| 202 "R3"   |  | 253 *      |  |
| 203 XEQ 02 |  | 254 RCL 04 |  |
| 204 RCL 14 | Calculate r <sub>4</sub>                     | 255 RCL 09 |  |
| 205 STO 01 |  | 256 *      |  |
| 206 RCL 18 |  | 257 -      |  |
| 207 STO 04 |  | 258 RCL 02 |  |
| 208 RCL 22 |  | 259 *      |  |
| 209 STO 07 |  | 260 +      |  |
| 210 XEQ 00 |  | 261 RCL 04 |  |
| 211 RCL 11 |  | 262 RCL 08 |  |
| 212 *      |  | 263 *      |  |
| 213 ST- 00 |  | 264 RCL 05 |  |
| 214 RCL 15 |  | 265 RCL 07 |  |
| 215 STO 02 |  | 266 *      |  |
| 216 RCL 19 |  | 267 -      |  |
| 217 STO 05 |  | 268 RCL 03 |  |
| 218 RCL 23 |  | 269 *      |  |
| 219 STO 08 |  | 270 +      |  |
| 220 XEQ 00 |  | 271 .END.  |  |
| 221 RCL 12 |  | 70         |  |
| 222 *      |  |            |  |
| 223 ST+ 00 |  |            |  |
| 224 RCL 16 |  |            |  |
| 225 STO 03 |  |            |  |
| 226 RCL 20 |  |            |  |
| 227 STO 06 |  |            |  |
| 228 RCL 24 |  |            |  |
| 229 STO 09 |  |            |  |
| 230 XEQ 00 |  |            |  |
| 231 RCL 13 |  |            |  |
| 232 *      |  | 80         |  |
| 233 ST- 00 |  |            |  |
| 234 RCL 00 |  |            |  |
| 235 "R4"   | Output routine                               |            |  |
| 236♦LBL 02 |  |            |  |
| 237 "T="   |  |            |  |
| 238 ARCL X |  |            |  |
| 239 AVIEW  |  |            |  |
| 240 RTN    |  |            |  |
| 241♦LBL 00 |  |            |  |
| 242 RCL 05 |  | 90         |  |
| 243 RCL 09 | Find   |            |  |
| 244 *      |  |            |  |
| 245 RCL 06 | R <sub>1</sub> R <sub>2</sub> R <sub>3</sub> |            |  |
| 246 RCL 08 | R <sub>4</sub> R <sub>5</sub> R <sub>6</sub> |            |  |
| 247 *      |  |            |  |
| 248 -      | R <sub>7</sub> R <sub>8</sub> R <sub>9</sub> |            |  |
| 249 RCL 01 |  |            |  |
| 250 *      |  |            |  |
| 251 RCL 06 |  |            |  |
| 252 RCL 07 |  | 00         |  |

## **REGISTERS, STATUS, FLAGS, ASSIGNMENTS**

CHARACTERISTIC EQUATION  
OF A 4 X 4 MATRIX  
PROGRAM REGISTERS NEEDED: 60

ROW 1 (1 : 4)



ROW 2 (5 : 11)



ROW 3 (12 : 17)



ROW 4 (17 : 25)



ROW 5 (26 : 31)



ROW 6 (31 : 37)



ROW 7 (37 : 45)



ROW 8 (45 : 51)



ROW 9 (51 : 57)



ROW 10 (58 : 65)



ROW 11 (65 : 74)



ROW 12 (75 : 84)



ROW 13 (85 : 93)



ROW 14 (94 : 104)



ROW 15 (105 : 112)



ROW 16 (113 : 121)



ROW 17 (121 : 128)



ROW 18 (129 : 138)



change

CHARACTERISTIC EQUATION  
OF A 4 X 4 MATRIX

ROW 19 (139 : 147)



ROW 20 (147 : 156)



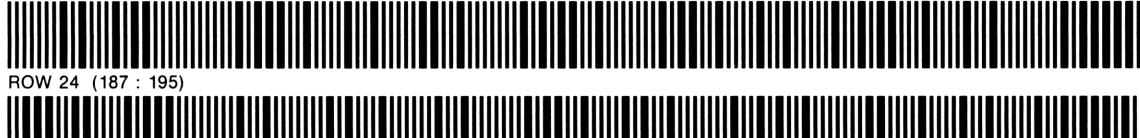
ROW 21 (157 : 165)



ROW 22 (166 : 175)



ROW 23 (176 : 186)



ROW 24 (187 : 195)



ROW 25 (196 : 203)



ROW 26 (204 : 212)



ROW 27 (213 : 220)



ROW 28 (221 : 229)



ROW 29 (230 : 237)



ROW 30 (237 : 247)



ROW 31 (248 : 260)



ROW 32 (261 : 271)



## 4 X 4 MATRIX OPERATIONS

(THIS PROGRAM REQUIRES TWO ADDITIONAL MEMORY MODULES)

This program allows the calculations of the determinant, and inverse of a 4 x 4 matrix, and the solution of a system of simultaneous equations in 4 unknowns.

The method used in this program is that of Gaussian elimination with partial pivoting. Space does not allow a full treatment of the pertinent equations; however, the Comments section of the program listing shows the operations in detail, step by step.

Basically, the program allows for input of the matrix A and transforms A into an upper triangular matrix U, assuming A is nonsingular. The multipliers used to accomplish this transformation form a lower triangular matrix, L, which has 1's along its diagonal. If we disregard pivoting (a technique of row interchanges which may improve accuracy and which may introduce one or more permutation matrices) then the relationship among these matrices is  $U = LA$ . The original matrix A will be lost. The initial elements  $a_{ij}$  have been replaced by the elements of U ( $i \leq j$ ) and of L ( $i > j$ ). (The elements of U will still be referred to as  $a_{ij}$ ; those of L will be called  $m_{ij}$  in the program listing comments). The second part of the program uses the transformed matrices U and L to compute the determinant and inverse of A, and to solve systems of simultaneous equations.

Equations:

Let  $A =$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

The determinant of A,  $\text{Det } A$ , is found after its transformation to U by the product of the diagonal elements:

$$\text{Det } A = (-1)^k a_{11} a_{22} a_{33} a_{44},$$

where k is the number of row interchanges required by pivoting.

A set of 4 simultaneous equations in 4 unknowns may be written as

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = b_3$$

$$a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4$$

where the  $\{x_i\}$  are unknowns and the  $\{b_i\}$  constants.

In matrix notation, this becomes  $A x = b$ , where  $x$  and  $b$  are the column

vectors  $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$  and  $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$  respectively.

This problem is solved (neglecting pivoting) as  $Ux = Lb$ .

Let  $C$  be the inverse of  $A$ , i.e., the  $4 \times 4$  matrix such that  $AC = CA = I$ , where  $I$  is the  $4 \times 4$  identity matrix.

$C$  is computed a column at a time in the following way:

let  $c^{(j)}$  be the  $j^{\text{th}}$  column vector of  $C$ , i.e.,

$$c^{(j)} = \begin{bmatrix} c_{1j} \\ c_{2j} \\ c_{3j} \\ c_{4j} \end{bmatrix}, \quad j = 1, 2, 3, 4.$$

Then  $c^{(j)}$  is found by the solution of the equation

$$Ac^{(j)} = I^{(j)}.$$

For example,  $c^{(1)}$  is found by solution of

$$A c^{(1)} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

If operations are to be carried out on the same matrix over a period of time, it might be convenient to record the elements of the matrix on a magnetic card for rapid input at a later date.

#### References:

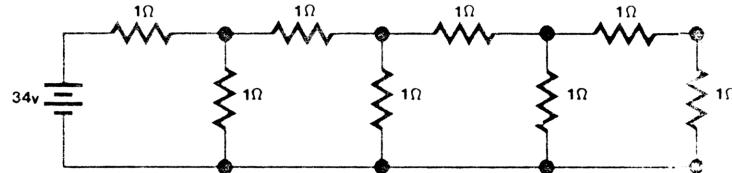
George E. Forsythe, Michael A. Malcolm, and Cleve B. Moler, Computer Methods in Mathematical Computation, Computer Science Department, Stanford University, 1972.

G. Forsythe and C. Moler, Computer Solution of Linear Algebraic Systems, Prentice-Hall, 1967.

C. Moler, "Matrix Computations with Fortran and Paging," Comm, ACM, vol. 15, no. 4, pp. 268-270 (April, 1972).

#### Example 1:

By applying the technique of loop currents to the circuit below, find the currents  $I_1$ ,  $I_2$ ,  $I_3$ , and  $I_4$ .



The equations to be solved are

$$\begin{array}{rcl} 2I_1 & -I_2 & = 34 \\ -I_1 & +3I_2 & = 0 \\ & -I_2 & +3I_3 = 0 \\ & & -I_3 +3I_4 = 0 \end{array}$$

In matrix form,

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 34 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

## Keystrokes:

|                            |                                |
|----------------------------|--------------------------------|
| [XEQ] [ALPHA] SIZE [ALPHA] |                                |
| [XEQ] [ALPHA] 4*4 [ALPHA]  | a1,1=?                         |
| 2 [R/S]                    | a2,1=?                         |
| 1 [CHS] [R/S]              | a3,1=?                         |
| 0 [R/S]                    | a4,1=?                         |
| 0 [R/S]                    | a1,2=?                         |
| 1 [CHS] [R/S]              | a2,2=?                         |
| 3 [R/S]                    | a3,2=?                         |
| 1 [CHS] [R/S]              | a4,2=?                         |
| 0 [R/S]                    | a1,3=?                         |
| 0 [R/S]                    | a2,3=?                         |
| 1 [CHS] [R/S]              | a3,3=?                         |
| 3 [R/S]                    | a4,3=?                         |
| 1 [CHS] [R/S]              | a1,4=?                         |
| 0 [R/S]                    | a2,4=?                         |
| 0 [R/S]                    | a3,4=?                         |
| 1 [CHS] [R/S]              | a4,4=?                         |
| 3 [R/S]                    | READY                          |
| [R/S]                      | READY                          |
| [C]                        | b1,1=?                         |
| 34 [R/S]                   | b2,1=?                         |
| 0 [R/S]                    | b3,1=?                         |
| 0 [R/S]                    | b4,1=?                         |
| 0 [R/S]                    | READY                          |
| [R/S]                      | X1,1=21.0000 (I <sub>1</sub> ) |
| [R/S]*                     | X2,1=8.0000 (I <sub>2</sub> )  |
| [R/S]*                     | X3,1=3.0000 (I <sub>3</sub> )  |
| [R/S]*                     | X4,1=1.0000 (I <sub>4</sub> )  |

## Display:

\*[R/S] is omitted if printer is present.

Example 2:

Find the determinant and inverse of the 4x4 matrix in the previous problem.

Continuing from the previous page:

Keystrokes:

|       |             |
|-------|-------------|
| [D]   | DET=34.0000 |
| [E]   | c1,1=0.6176 |
| [R/S] | c2,1=0.2353 |
| [R/S] | c3,1=0.0882 |
| [R/S] | c4,1=0.0294 |
| [R/S] | c1,2=0.2353 |
| [R/S] | c2,2=0.4706 |
| [R/S] | c3,2=0.1765 |
| [R/S] | c4,2=0.0588 |
| [R/S] | c1,3=0.0882 |
| [R/S] | c2,3=0.1765 |
| [R/S] | c3,3=0.4412 |
| [R/S] | c4,3=0.1471 |
| [R/S] | c1,4=0.0294 |
| [R/S] | c2,4=0.0588 |
| [R/S] | c3,4=0.1471 |
| [R/S] | c4,4=0.3824 |

# User Instructions

|      |   |                  |           | SIZE: 029 |
|------|---|------------------|-----------|-----------|
| STEP | INSTRUCTIONS                              | INPUT            | FUNCTION  | DISPLAY   |
| 1    | Load the program.                         |                  |           |           |
| 2    | Enter matrix A.                           |                  | [XEQ] 4*4 | a1,1=?    |
|      |   | a <sub>1,1</sub> | [R/S]     | a2,1=?    |
|      |   | :                | :         | :         |
|      |   | a <sub>4,4</sub> | [R/S]     | READY     |
| 3    | To review matrix A.                       |                  | [B]       | a1,1=( )  |
|      | Corrections may be made by keying in the  |                  | [R/S]     | a2,1=( )  |
|      | correct number when the erroneous element |                  | :         | :         |
|      | element is displayed.                     |                  | [R/S]     | READY     |
| 4    | To print matrix A (if printer exists)     |                  | [///] [b] | a1,1=( )  |
|      |   |                  |           | :         |
|      |   |                  |           | READY     |
| 5    | Initialize the calculation routines.      |                  | [R/S]     | READY     |
|      | NOTE: This step must be done before       |                  |           |           |
|      | you continue!                             |                  |           |           |
|      | For Simultaneous Equations                |                  |           |           |
| 6    | Input matrix b.                           |                  | [C]       | b1,1=?    |
|      |   | b <sub>1,1</sub> | [R/S]     | b2,1=?    |
|      |   | :                | :         | :         |
|      |   | b <sub>4,4</sub> | [R/S]     | READY     |
| 7    | To print matrix b.                        |                  | [///] [c] | b1,1=( )  |
|      | NOTE: matrix b may be reviewed by         |                  | :         | :         |
|      | pressing [C] and then [←] when the        |                  |           | READY     |
|      | prompt is displayed to view the           |                  |           |           |
|      | numerical value of the element.           |                  |           |           |
| 8    | Calculate x.                              |                  | [R/S]*    | x1,1=( )  |

# User Instructions

# Program Listings

|             |               |            |
|-------------|---------------|------------|
| 01♦LBL "4*4 | Initialize    | 51♦LBL 00  |
| "           |               | 52 FIX 0   |
| 02 SF 21    |               | 53 CF 29   |
| 03 SF 27    |               | 54 "a"     |
| 04♦LBL A    |               | 55 FS? 06  |
| 05 CF 05    |               | 56 "b"     |
| 06 CF 06    | Input A       | 57 FS? 08  |
| 07 CF 07    |               | 58 "X"     |
| 08 CF 08    |               | 59 FS? 09  |
| 09 CF 09    |               | 60 "c"     |
| 10 GTO 01   |               | 61 ARCL 27 |
| 11♦LBL C    |               | 62 "F,"    |
| 12 CF 05    | Input b       | 63 ARCL 26 |
| 13 SF 06    |               | 64 "F="    |
| 14 CF 07    |               | 65 FC? 05  |
| 15 CF 08    |               | 66 "F?"    |
| 16 CF 09    |               | 67 FIX 4   |
| 17 GTO 01   |               | 68 SF 29   |
| 18♦LBL b    |               | 69 RCL IND |
| 19 ADV      | Print A       | 28         |
| 20 SF 05    |               | 70 FS? 05  |
| 21 CF 06    |               | 71 ARCL X  |
| 22 SF 07    |               | 72 FC? 07  |
| 23 CF 08    |               | 73 PROMPT  |
| 24 CF 09    |               | 74 FS? 07  |
| 25 GTO 01   |               | 75 AVIEW   |
| 26♦LBL c    |               | 76 STO IND |
| 27 ADV      |               | 28         |
| 28 SF 05    | Print b       | 77 ISG 28  |
| 29 SF 06    |               | 78 CLD     |
| 30 SF 07    |               | 79 ISG 27  |
| 31 CF 08    |               | 80 GTO 00  |
| 32 CF 09    |               | 81 FS? 07  |
| 33 GTO 01   |               | 82 ADV     |
| 34♦LBL B    |               | 83 FS? 06  |
| 35 SF 05    | View A        | 84 GTO 02  |
| 36 CF 06    |               | 85 1.004   |
| 37 CF 07    |               | 86 STO 27  |
| 38 CF 08    |               | 87 FS? 09  |
| 39 CF 09    |               | 88 RTN     |
| 40♦LBL 01   |               | 89 ISG 26  |
| 41 1.004    |               | 90 GTO 00  |
| 42 FC? 09   |               | 91♦LBL 02  |
| 43 STO 26   | Initialize    | 92 "READY" |
| 44 STO 27   | Input Routine | 93 PROMPT  |
| 45 5        |               | 94 FS? 06  |
| 46 FS? 06   |               | 95 GTO 08  |
| 47 1        |               | 96 0       |
| 48 FS? 08   |               | 97 STO 00  |
| 49 1        |               | 98 1       |
| 50 STO 28   |               | 99 STO 21  |

# Program Listings

|     |      |    |               |  |     |         |
|-----|------|----|---------------|--|-----|---------|
| 100 | STO  | 24 |               |  | 151 | X=Y?    |
| 101 | RCL  | 05 | Pivot Routine |  | 152 | GTO 03  |
| 102 | ABS  |    |               |  | 153 | 10      |
| 103 | STO  | 22 |               |  | 154 | *       |
| 104 | 2    |    |               |  | 155 | XEQ 11  |
| 105 | RCL  | 06 |               |  | 156 | 2       |
| 106 | XEQ  | 10 |               |  | 157 | XEQ 12  |
| 107 | 3    |    |               |  | 158 | 3       |
| 108 | RCL  | 07 |               |  | 159 | XEQ 12  |
| 109 | XEQ  | 10 |               |  | 160 | 4       |
| 110 | 4    |    |               |  | 161 | XEQ 12  |
| 111 | RCL  | 08 |               |  | 162 | *LBL 03 |
| 112 | XEQ  | 10 |               |  | 163 | RCL 10  |
| 113 | 1    |    |               |  | 164 | CHS     |
| 114 | RCL  | 21 |               |  | 165 | ST/ 11  |
| 115 | X=Y? |    |               |  | 166 | ST/ 12  |
| 116 | GTO  | 03 |               |  | 167 | RCL 11  |
| 117 | XEQ  | 11 |               |  | 168 | RCL 14  |
| 118 | 1    |    |               |  | 169 | *       |
| 119 | XEQ  | 12 |               |  | 170 | ST+ 15  |
| 120 | 2    |    |               |  | 171 | RCL 12  |
| 121 | XEQ  | 12 |               |  | 172 | RCL 14  |
| 122 | 3    |    |               |  | 173 | *       |
| 123 | XEQ  | 12 |               |  | 174 | ST+ 16  |
| 124 | 4    |    |               |  | 175 | RCL 11  |
| 125 | XEQ  | 12 |               |  | 176 | RCL 18  |
| 126 | *LBL | 03 |               |  | 177 | *       |
| 127 | RCL  | 05 |               |  | 178 | ST+ 19  |
| 128 | CHS  |    |               |  | 179 | RCL 12  |
| 129 | ST/  | 06 |               |  | 180 | RCL 18  |
| 130 | ST/  | 07 |               |  | 181 | *       |
| 131 | ST/  | 08 |               |  | 182 | RCL 20  |
| 132 | 9    |    |               |  | 183 | +       |
| 133 | STO  | 25 |               |  | 184 | STO 20  |
| 134 | XEQ  | 13 |               |  | 185 | RCL 15  |
| 135 | XEQ  | 13 |               |  | 186 | ABS     |
| 136 | XEQ  | 13 |               |  | 187 | RCL 16  |
| 137 | 2    |    |               |  | 188 | ABS     |
| 138 | STO  | 21 |               |  | 189 | X<=Y?   |
| 139 | STO  | 23 |               |  | 190 | GTO 03  |
| 140 | RCL  | 10 |               |  | 191 | RCL 15  |
| 141 | ABS  |    |               |  | 192 | RCL 16  |
| 142 | STO  | 22 |               |  | 193 | X<> 15  |
| 143 | 3    |    |               |  | 194 | STO 16  |
| 144 | RCL  | 11 |               |  | 195 | RCL 19  |
| 145 | XEQ  | 10 |               |  | 196 | X<> 20  |
| 146 | 4    |    |               |  | 197 | STO 19  |
| 147 | RCL  | 12 |               |  | 198 | .4      |
| 148 | XEQ  | 10 |               |  | 199 | XEQ 11  |
| 149 | 2    |    |               |  | 200 | *LBL 03 |
| 150 | RCL  | 21 |               |  | 201 | RCL 15  |

# Program Listings

|             |    |                        |
|-------------|----|------------------------|
| 202 CHS     |    | 251 RCL IND            |
| 203 ST/ 16  |    | 25                     |
| 204 RCL 19  |    | 252 RTN                |
| 205 RCL 16  |    | 253♦LBL 13             |
| 206 *       |    | 254 RCL IND            |
| 207 ST+ 20  |    | 25                     |
| 208 GTO 02  |    | 255 STO 21             |
| 209♦LBL 10  |    | 256 ISG 25             |
| 210 ABS     |    | 257 CLD                |
| 211 RCL 22  |    | 258 RCL 06             |
| 212 X>Y?    |    | 259 XEQ 09             |
| 213 RTN     |    | 260 RCL 07             |
| 214 RDN     |    | 261 XEQ 09             |
| 215 STO 22  |    | 262 RCL 08             |
| 216 RDN     |    | 263♦LBL 09             |
| 217 STO 21  |    | 264 RCL 21             |
| 218 RTN     |    | 265 *                  |
| 219♦LBL 11  |    | 266 ST+ IND            |
| 220 ST+ 00  |    | 25                     |
| 221 RCL 24  |    | 267 ISG 25             |
| 222 CHS     |    | 268 RTN                |
| 223 STO 24  |    | 269 RTN                |
| 224 RTN     |    | 270♦LBL D              |
| 225♦LBL 12  |    | 271 ADV                |
| 226 STO 22  |    | 272 RCL 24             |
| 227 RCL 23  |    | 273 RCL 05             |
| 228 RCL 22  |    | Determinant<br>Routine |
| 229 XEQ 14  |    | 274 *                  |
| 230 RCL 21  |    | 275 RCL 10             |
| 231 RCL 22  |    | 276 *                  |
| 232 XEQ 14  |    | 277 RCL 15             |
| 233 X<>Y    |    | 278 *                  |
| 234 STO IND |    | 279 RCL 20             |
| 25          |    | 280 *                  |
| 235 X<>Y    |    | 281 "DET="             |
| 236 RCL 23  |    | 282 ARCL X             |
| 237 RCL 22  |    | 283 AVIEW              |
| 238 4       |    | 284 RTN                |
| 239 *       |    | 285♦LBL 08             |
| 240 +       |    | 286 RCL 00             |
| 241 STO 25  |    | 287 10                 |
| 242 RDN     |    | 288 STO 23             |
| 243 STO IND |    | 289 /                  |
| 25          |    | 290 FRC                |
| 244 RTN     |    | 291 RCL 23             |
| 245♦LBL 14  |    | 292 *                  |
| 246 4       |    | 293 INT                |
| 247 *       |    | 294 X=0?               |
| 248 +       |    | 295 GTO 03             |
| 249 STO 25  |    | 296 STO 25             |
| 250 CLX     |    | 297 RCL IND            |
|             | -- | 25                     |

# Program Listings

```

298 X<> 01
299 STO IND
25
300♦LBL 03
301 RCL 01
302 RCL 06
303 *
304 ST+ 02
305 RCL 01
306 RCL 07
307 *
308 ST+ 03
309 RCL 01
310 RCL 08
311 *
312 ST+ 04
313 RCL 00
314 RCL 23
315 /
316 INT
317 X=0?
318 GTO 03
319 STO 25
320 RCL IND
25
321 X<> 02
322 STO IND
25
323♦LBL 03
324 RCL 12
325 RCL 11
326 RCL 02
327 *
328 ST+ 03
329 CLX
330 RCL 02
331 *
332 ST+ 04
333 RCL 00
334 FRC
335 RCL 23
336 *
337 X=0?
338 GTO 03
339 STO 25
340 RCL IND
25
341 X<> 03
342 STO IND
25
343♦LBL 03

```

```

344 RCL 16
345 RCL 03
346 *
347 ST+ 04
348 RCL 20
349 ST/ 04
350 RCL 04
351 CHS
352 STO 21
353 RCL 15
354 STO 22
355 RCL 19
356 RCL 18
357 RCL 17
358 RCL 21
359 *
360 ST+ 01
361 CLX
362 RCL 21
363 *
364 ST+ 02
365 CLX
366 RCL 21
367 *
368 ST+ 03
369 RCL 22
370 ST/ 03
371 RCL 03
372 CHS
373 STO 21
374 RCL 10
375 STO 22
376 RCL 14
377 RCL 13
378 RCL 21
379 *
380 ST+ 01
381 CLX
382 RCL 21
383 *
384 ST+ 02
385 RCL 22
386 ST/ 02
387 RCL 09
388 RCL 02
389 CHS
390 *
391 ST+ 01
392 RCL 05
393 ST/ 01
394 SF 05

```

# Program Listings

```
395 SF 05
396 SF 06
397 FS? 09
398 CF 06
399 SF 07
400 SF 08
401 FC? 09
402 GTO 01
403 XEQ 01
404 ISG 26
405♦LBL 07
406 CLX
407 STO 01
408 STO 02
409 STO 03
410 STO 04
411 RTN
412♦LBL E
413 ADV
414 SF 09      Inverse Routine
415 XEQ 07
416 1
417 STO 01
418 XEQ 08
419 1
420 STO 02
421 XEQ 08
422 1
423 STO 03
424 XEQ 08
425 1
426 STO 04
427 XEQ 08
428 END
```



## 4 X 4 MATRIX OPERATIONS

PROGRAM REGISTERS NEEDED: 103

ROW 1 (1 : 4)



ROW 2 (5 : 11)



ROW 3 (11 : 17)



ROW 4 (18 : 24)



ROW 5 (25 : 31)



ROW 6 (32 : 38)



ROW 7 (38 : 43)



ROW 8 (44 : 52)



ROW 9 (52 : 58)



ROW 10 (59 : 64)



ROW 11 (64 : 70)



ROW 12 (70 : 77)



ROW 13 (78 : 85)



ROW 14 (85 : 90)



ROW 15 (91 : 96)



ROW 16 (97 : 105)



ROW 17 (106 : 113)



ROW 18 (113 : 120)



## 4 X 4 MATRIX OPERATIONS

ROW 19 (120 : 126)



ROW 20 (127 : 135)



ROW 21 (135 : 140)



ROW 22 (141 : 149)



ROW 23 (149 : 156)



ROW 24 (157 : 163)



ROW 25 (164 : 173)



ROW 26 (174 : 182)



ROW 27 (183 : 191)



ROW 28 (192 : 198)



ROW 29 (199 : 206)



ROW 30 (206 : 215)



ROW 31 (216 : 224)



ROW 32 (224 : 231)



ROW 33 (231 : 238)



ROW 34 (238 : 248)



ROW 35 (249 : 257)



ROW 36 (257 : 265)



## 4 X 4 MATRIX OPERATIONS

ROW 37 (265 : 273)



ROW 38 (274 : 282)



ROW 39 (282 : 291)



ROW 40 (292 : 299)



ROW 41 (300 : 309)



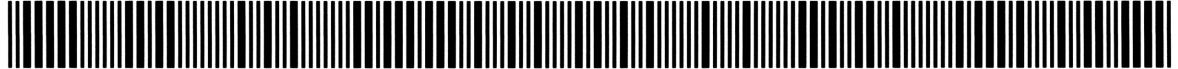
ROW 42 (310 : 319)



ROW 43 (320 : 328)



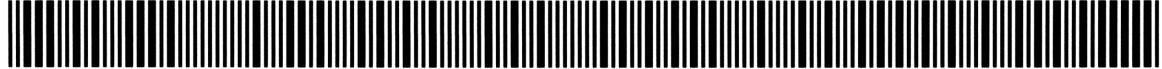
ROW 44 (329 : 338)



ROW 45 (339 : 345)



ROW 46 (346 : 354)



ROW 47 (355 : 361)



ROW 48 (362 : 370)



ROW 49 (370 : 379)



ROW 50 (379 : 387)



ROW 51 (387 : 396)



ROW 52 (396 : 402)



ROW 53 (403 : 412)



ROW 54 (412 : 419)



## 4 X 4 MATRIX OPERATIONS

ROW 55 (420 : 427)



ROW 56 (427 : 428)



**NOTES**

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## Hewlett-Packard Software

In terms of power and flexibility, the problem-solving potential of the HP-41 programmable calculator is nearly limitless. And in order to see the practical side of this potential, HP has different types of software to help save you time and programming effort. Every one of our software solutions has been carefully selected to effectively increase your problem-solving potential. Chances are, we already have the solutions you're looking for.

## Application Pacs

To increase the versatility of your HP-41, HP has an extensive library of "Application Pacs". These programs transform your HP-41 into a specialized calculator in seconds. Included in these pacs are detailed manuals with examples, miniature plug-in Application Modules, and keyboard overlays. Every Application Pac has been designed to extend the capabilities of the HP-41.

You can choose from:

|  |   |
|--|---|
| Aviation (Pre-Flight Only) 00041-15018 | Statistics 00041-15002                    |
| Clinical Lab 00041-15024               | Stress Analysis 00041-15027               |
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| Financial Decisions 00041-15004        | Home Management 00041-15023               |
| Mathematics 00041-15003                | Machine Design 00041-15020                |
| Structural Analysis 00041-15021        | Navigation 00041-15017                    |
| Surveying 00041-15005                  | Real Estate 00041-15016                   |
| Securities 00041-15026                 | Thermal and Transport Science 00041-15019 |
|  | Petroleum Fluids 00041-15039              |

## Users' Library

The Users' Library provides the best programs from contributors and makes them available to you. By subscribing to the HP-41 Users' Library you'll have at your fingertips literally hundreds of different programs from many different application areas.

## \*Users' Library Solutions Books

Hewlett-Packard offers a wide selection of Solutions Books complete with user instructions, examples, and listings. These solution books will complement our other software offerings and provide you with a valuable tool for program solutions.

You can choose from:

|   |   |
|---|---|
| Business Stat/Marketing/Sales 00041-90094 | Civil Engineering 00041-90089                       |
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| Lending, Saving and Leasing 00041-90086   | Mechanical Engineering 00041-90090                  |
| Real Estate 00041-90136                   | Solar Engineering 00041-90138                       |
| Small Business 00041-90137                | Calendars 00041-90145                               |
| Geometry 00041-90084                      | Cardiac/Pulmonary 00041-90097                       |
| High-Level Math 00041-90083               | Chemistry 00041-90102                               |
| Test Statistics 00041-90082               | Games 00041-90099                                   |
| Antennas 00041-90093                      | Optometry I (General) 00041-90143                   |
| Chemical Engineering 00041-90100          | Optometry II (Contact Lens) 00041-90144             |
| Control Systems 00041-90092               | Physics 00041-90142                                 |
| Electrical Engineering 00041-90088        | Surveying 00041-90141                               |
| Fluid Dynamics and Hydraulics 00041-90139 | Time Module Solutions 00041-90395                   |
| Games II 00041-90443                      |   |

\*Some books require additional memory modules to accomodate all programs.

## **HIGH-LEVEL MATH**

SINE, COSINE, EXPONENTIAL INTEGRALS  
EIGENVALUES/VECTORS OF 3RD - ORDER SYSTEMS  
EIGENVALUES FOR 3RD ORDER SYSTEMS  
CHEBYSHEV, LEGENDRE, HERMITE, AND LAGUERRE POLYNOMIALS  
SIXTEEN-POINT GAUSSIAN QUADRATURE  
GAMMA FUNCTION  
BESSEL FUNCTIONS, ERROR FUNCTION  
CHARACTERISTIC EQUATION OF 4 X 4 MATRIX  
4 X 4 MATRIX OPERATIONS

