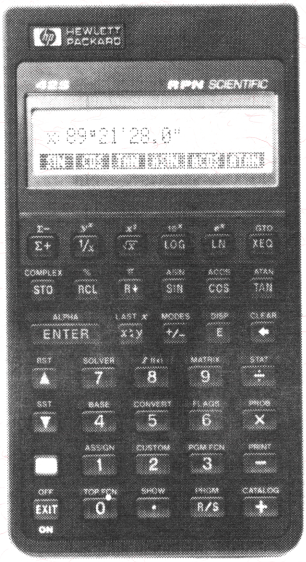

HP42S TRIANGLE SOLUTIONS

D'Zign

NOTICE

HP42S TRIANGLE SOLUTIONS

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TECHNICAL ASSISTANCE

The program material, instructions and procedures contained in this book assume that the user has a working knowledge of both surveying and the general operation of the HP-42S calculator.

Technical assistance is limited to verification of the results shown in the various examples used in the book.

If you have any questions or suggestions regarding this book or other **D'Zign** publications, please feel free to call us. The number is (818) 507-7408, and someone is available to answer technical questions from 3:00 A.M. to 5:30 A.M. and from 4:30 P.M to 6:30 P.M. (Pacific time), as a service to users from other time zones.

Hewlett-Packard has produced a really powerful calculator at a very good price, the HP-42 Scientific Calculator, which lends itself nicely to solving surveying problems. It can not be programmed by insertion of a module, or with a card reader, like the HP-41 series, but it has a really simple system for typing in a program.

the operations index

To find a function for the first time, HP has provided an "Operations Index" on pages 310 through 335 of the instruction manual, which tells you exactly what keystrokes to use to type in the function you want.

Even better, this index gives you the page number that you can refer to if you want to know more about the function you are using. If, while typing in a program, you aren't sure how to input a particular function, simply refer to the Operations Index.

the softkey menus

All of the programs in this booklet take advantage of the "softkey" menu system built into this calculator. When you want to start a program you stroke **XEQ** and then the softkey corresponding to the program you want, from the menu displayed in the bottom half of the screen.

the programs

The purpose of this booklet is to provide the user with the most flexible system for solving triangles possible.

The **use** of this program is simplicity itself. You need three known parts of the triangle being solved, and there are only three keys used for input. Because of the way the HP-42 works, those keys are labeled "SIDE", "ANGLE" and "AREA" by the menu. Another key is used, but only to return quickly to the program to solve another triangle.

learning while programming

We have tried to write this series of booklets in such a way that you quickly become aquainted with the calculator and its functions while you are programming it. Because the program is long, you might not want to do the whole job at one sitting.

The use of a printer is not required, but does make the whole job easier, Hewlett-Packard has the InfraRed Printer available for the 42S, and one feature of this calculator is that it already has the InfraRed transmitter built in.

subroutines

Because of the way the calculator works, we will start by input of some subroutines.

Once the subroutine has been input, its name appears in the menu when you stroke XEQ , and all you have to do to add it as a step in the program you are typing in is stroke GTO or XEQ followed by keystroking the key corresponding to the subroutine to input the program step GTO XXX or XEQ XXX.

```
01▶LBL "CL"
02 ΣREG 00
03 CLΣ
04 ΣREG 11
05 CLΣ
06 0
07 STO 24
08 RTN
09▶LBL "FCL"
10 0.013
11▶LBL "FN"
12 CF IND ST X
13 ISG ST X
14 GTO "FN"
15 ENTER
16 END
```

getting started

Begin by stroking the shift key, then the **XEQ** key. The display will show a menu which will be blank (if you haven't yet input any programs) except for **.END.** on the left. The keys just below each of the menu portions will correspond to the menu instruction above it. Stroke the key just below the **.END.** in the display.

Next, go into **program mode** by stroking the shifted R/S key. Scroll up once with the ▲ key.

```
00▶{ 0-Byte Prgm }
01 .END.
```

Your display should be similar to the one shown to the left. Begin typing in the program steps shown above.

Proof read the program. If you scroll to 00 it should now say "45-Byte Prgm" . . . the byte count is one quick check on the program, but you still need to check every step. Look, in particular, for steps that are **alpha** (with " " marks) but **shouldn't** be. Or should be but aren't.

"DMS"

This program puts the output of angles into the form ° ' ", and gives all answers to the nearest tenth of a second. Repeat the procedure of going to the permanent **.END.**, scroll up to 00, and input the program below.

```
01▸LBL "DMS"
02 FS?C 19
03 CLA
04 ENTER
05 STO 19
06 IP
07 CF 29
08 FIX 00
09 ARCL ST X
10 ⊢"°"
11 -
12 100
13 ×
14 ABS
15 STO 18
16 IP
17 XEQ 01
18 ARCL ST X
19 ⊢"'"
20 RCL 18
21 FP
22 100
23 ×
24 FIX 02
25 RND
26 FIX 01
27 XEQ 01
28 ARCL ST X
29 ⊢"""
30 CLX
31 FIX 04
32 SF 29
33 RCL 19
34▸LBL 01
35 10
36 X<>Y
37 X<Y?
38 ⊢"0"
39 END
```

The byte count on this one should be 78 bytes, as a check that everything is alright. Read through it to check for typos, and as a final check, input a number (try 25.25252) and execute the function. Your display **should** show 25°25'25.2" if the program is working correctly. If it doesn't, check it again.

the main program

The next two pages contain the main program. Take your time typing it in, to avoid errors.

Don't type an END on this one, the permanent end will work just as well, and it removes it from your XEQ/GTO menu.

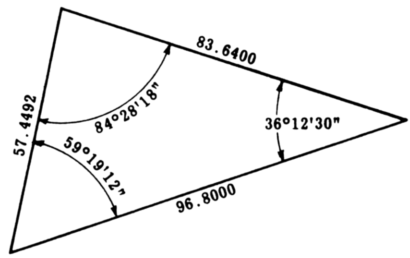
```
▶LBL "TRI<"
XEQ "CL"
XEQ "FCL"
0.002
STO 00
CLX
XEQ 20
RTN
▶LBL 21
FS? 09
SF 03
FS? 04
SF 01
SF 04
ISG 00
RTN
GTO 10
RTN
▶LBL 22
FS? 01
SF 08
FS? 04
SF 06
FS? 01
CF 06
FS? 09
SF 05
SF 09
ISG 00
RTN
GTO 10
▶LBL 23
SF 07
ISG 00
RTN
▶LBL 10
FS? 07
GTO 06
FS? 08
GTO 15
FS? 06
GTO 07
FS? 03
GTO 12
GTO 11
RTN
▶LBL 06
FS? 05
GTO 18
FS? 01
GTO 17
GTO 16
RTN
▶LBL 07
FS? 01
GTO 14
FS? 03
GTO 12
GTO 13
RTN
▶LBL 11
STO 04
R↓
STO 02
R↓
STO 09
R↓
R↓
+
+
2
÷
STO 07
X↑2
LASTX
RCLX 02
-
RCL 09
RCLX 04
÷
SQRT
ACOS
2
X
STO 05
SIN
RCLX 09
STO 08
RCL 07
X↑2
LASTX
RCLX 09
-
RCL÷ 02
RCL÷ 04
SQRT
ACOS
2
X
STO 03
RCL 05
XEQ 00
STO 01
GTO 01
RTN
▶LBL 12
" 2nd Solution"
FS? 20
AVIEW
FS?C 20
STOP
→HR
STO 01
R↓
STO 09
R↓
→HR
STO 05
▶LBL 03
RCL 01
XEQ 00
STO 03
RCL 05
RCL 09
→REC
X<>Y
STO 08
RCL 03
1
→REC
R↓
÷
STO 02
R↑
X
+
STO 04
GTO 01
▶LBL 13
→HR
STO 03
R↓
→HR
STO 01
R↓
STO 09
RCL 03
RCL 01
XEQ 00
RCL 09
RCL 01
XEQ 04
GTO 03
▶LBL 14
STO 02
R↓
→HR
STO 01
R↓
STO 09
RCL 01
RCL 02
→REC
RCL- 09
→POL
STO 04
RCL 09
RCL 02
RCL 04
GTO 11
▶LBL 15
→HR
STO 03
R↓
STO 02
R↓
STO 09
```

```
RCL 03
SIN
RCL× 02
RCL÷ 09
ASIN
STO 05
RCL 03
XEQ 00
STO 01
RCL 05
RCL 09
RCL 01
XEQ 04
XEQ 03
180
RCL- 05
RCL+ 03
+/-
180
+
→HMS
RCL 02
RCL 03
→HMS
SF 20
GTO 12
▶LBL 00
+
COS
+/-
ACOS
RTN
▶LBL 01
FS? 10
CF 02
FS? 20
CF 02
FS? 02
CLA
"Side 1 = "
ARCL 09
⊢"␊"
RCL 01
→HMS
⊢"Angle 1 = "
XEQ "DMS"
AVIEW
ADV
"Side 2 = "
ARCL 02
⊢"␊"
RCL 03
→HMS
⊢"Angle 2 = "
XEQ "DMS"
AVIEW
ADV
"Side 3 = "
ARCL 04
```

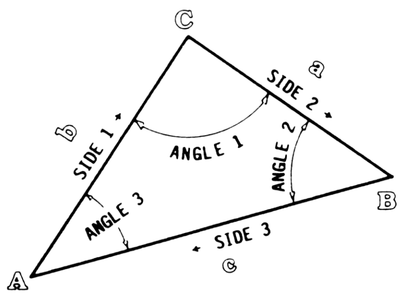
```
⊢"␊"
RCL 05
→HMS
⊢"Angle 3 = "
XEQ "DMS"
AVIEW
ADV
RCL 08
RCL× 04
2
÷
"AREA = "
ARCL ST X
AVIEW
ADV
FS? 10
XEQ 08
RTN
▶LBL 08
" 2nd Solution"
FS?C 10
AVIEW
STOP
180
RCL- 01
RCL 09
X<>Y
→HMS
RCL 02
GTO 14
▶LBL 09
R↓
R↓
RTN
▶LBL 04
STO 01
R↓
STO 09
R↓
STO 05
RTN
▶LBL 16
→HR
STO 01
SIN
XEQ 02
STO 02
XEQ 05
RTN
▶LBL 17
SF 10
STO 02
XEQ 02
ASIN
STO 01
XEQ 05
RTN
▶LBL 18
→HR
```

```
STO 03
X<>Y
→HR
STO 01
+
180
X<>Y
-
STO 05
SIN
2
×
×
RCL 01
SIN
RCL 03
SIN
×
÷
SQRT
STO 02
RCL 01
→HMS
X<>Y
RCL 03
→HMS
XEQ 12
RTN
▶LBL 02
X<>Y
STO 09
×
÷
2
×
RTN
▶LBL 05
RCL 09
RCL 01
→HMS
RCL 02
XEQ 14
RTN
▶LBL 20
CLA
CLMENU
"SIDE"
KEY 1 GTO 21
"ANGL"
KEY 2 GTO 22
"AREA"
KEY 3 GTO 23
"MORE"
KEY 6 GTO 24
MENU
RTN
▶LBL 24
GTO "TRI<"
.END.
```

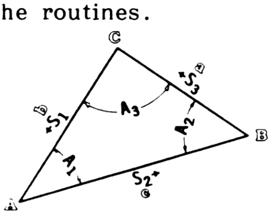
The triangle shown to the right will be used for the examples. It should be noted that the output will vary slightly, depending on the number of places input, particularly in the input of the angles.



The notations for angles and sides is familiar to HP users, but is not the standard, or 'textbook', notation which you have learned in trigonometry (side **a** opposite angle **A**, side **b** opposite angle **B** and side **c** opposite angle **C**). The sides and angles are numbered, in order, going around the triangle.



The example triangle (above) shows this style of labeling, compared to the standard notation for sides and angles. **Side 1** may be assigned to any side that is convenient to use, depending upon the available information about the triangle. It should be located at a side where the known information then falls into position for solution by one of the routines.



In the example, the assigned designations go clockwise. If it will better fit the information available, it may go counter-clockwise instead, as shown to the left.

Side 1, Side 2, Side 3

THREE SIDES KNOWN is one of the most used solutions for triangles, particularly in recent years in surveying.

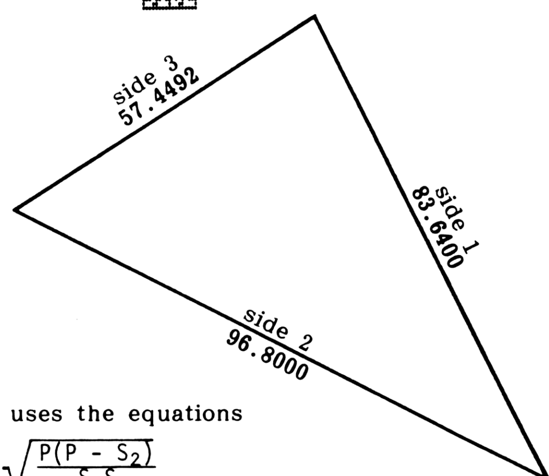
The lower cost and higher accuracy of electronic distance measurement equipment has resulted in more trilateration being used, instead of time-consuming repetitions of the angles.

keystroke example: XEQ [illegible]

8 3 . 6 4
SIDE
9 6 . 8
SIDE
5 7 . 4 4 9 2
SIDE

output:

```
Side 1 = 83.6400
Angle 1 = 36°12'30.0"

Side 2 = 96.8000
Angle 2 = 59°19'12.1"

Side 3 = 57.4492
Angle 3 = 84°28'18.0"

AREA = 2,391.3506
```



This routine uses the equations

$$A_3 = 2\cos^{-1}\sqrt{\frac{P(P - S_2)}{S_1 S_3}}$$

$$A_2 = 2\cos^{-1}\sqrt{\frac{P(P - S_1)}{S_2 S_3}}$$

and

$$A_1 = \cos^{-1}\left(-\cos(A_3 + A_2)\right)$$

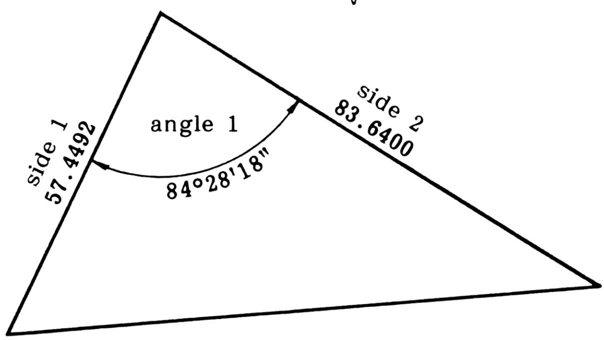
where $P = \frac{1}{2}(S_1 + S_2 + S_3)$

You can return for a second solution by stroking MORE

Side 1, Angle 1, Side 2

TWO SIDES AND THE INCLUDED ANGLE KNOWN is resolved by finding the third side, and then solving the triangle as shown on the previous page. The third side is found through the use of the equation

$$S_3 = \sqrt{S_1^2 + S_2^2 - 2\,S_1 S_2 \cos A_1}$$



keystrokes:

5 7 . 4 4 9 2 SIDE

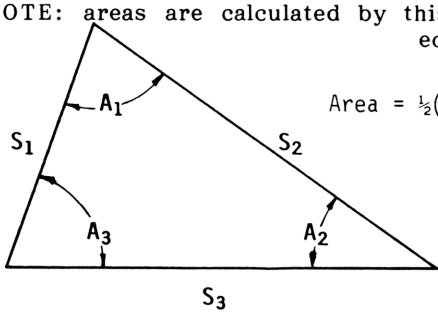
8 4 . 2 8 1 8 ANGLE

8 3 . 6 4 SIDE

output:

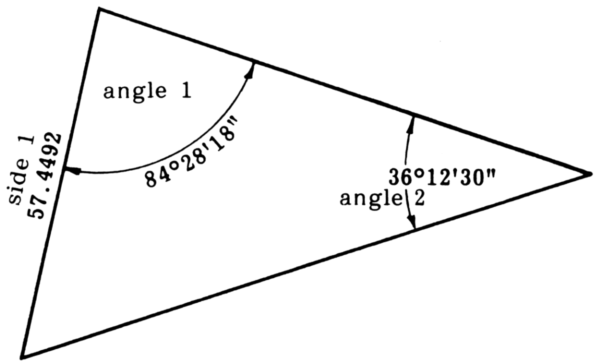
```
Side 1 = 57.4492
Angle 1 = 84°28'18.0"

Side 2 = 83.6400
Angle 2 = 36°12'30.0"

Side 3 = 96.8000
Angle 3 = 59°19'12.0"

AREA = 2,391.3506
```

NOTE: areas are calculated by this program using the equation shown below.



$$\text{Area} = \tfrac{1}{2}(\,S_1\,S_3 \sin A_3\,)$$

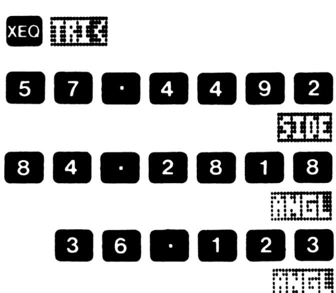
Side 1, Angle 1, Angle 2



ONE SIDE AND THE TWO FOLLOWING ANGLES KNOWN. This solution first solves for the third angle with the equation

$$A_3 = \cos^{-1}\left(-\cos\left(A_1 + A_2\right)\right)$$

Once angle 3 has been found, the remainder of the triangle is solved as Angle, Side, Angle (see page 8 for equations) to determine the other missing sides.



output:

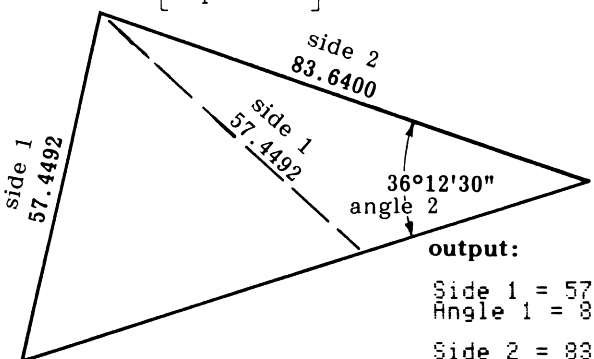
```
Side 1 = 57.4492
Angle 1 = 84°28'18.0"

Side 2 = 83.6400
Angle 2 = 36°12'30.0"

Side 3 = 96.8000
Angle 3 = 59°19'12.0"

AREA = 2,391.3500
```

Side 1, Side 2, Angle 2

TWO SIDES AND THE FOLLOWING ANGLE KNOWN has two possible solutions. When this configuration is used, both solutions are output. The second solution will not necessarily show the parts in the same order as the input.

The other two angles are calculated with the equations below, and the remaining side is calculated as an Angle, Side, Angle configuration.

$$A_3 = \sin^{-1}\left[\frac{S_2}{S_1}\sin A_2\right] \qquad A_1 = \cos^{-1}\left[-\cos\left(A_2 + A_3\right)\right]$$



output:

Side 1 = 57.4492
Angle 1 = 84°28'17.9"

Side 2 = 83.6400
Angle 2 = 36°12'30.0"

Side 3 = 96.8000
Angle 3 = 59°19'12.1"

AREA = 2,391.3505

2nd Solution
Side 1 = 83.6400
Angle 1 = 36°12'30.0"

Side 2 = 38.1740
Angle 2 = 120°40'47.9"

Side 3 = 57.4492
Angle 3 = 23°06'42.1"

AREA = 943.0510

The prompt, 2nd Solution appears in the display after output of the first solution. If you want output of the second solution, stroke R/S.

If not, stroke EXIT to leave the program, or MORE to solve another triangle.

5 7 . 4 4 9 2 SIDE

8 3 . 6 4 SIDE

3 6 . 1 2 3 ANGLE

Angle 3, Side 1, Angle 1

TWO ANGLES AND THE INCLUDED SIDE ARE KNOWN.

$$S_2 = S_1 \frac{\sin A_3}{\sin A_2}$$

$$S_3 = S_1 \cos A_3 + S_2 \cos A_2$$

$$A_2 = \cos^{-1}\left(-\cos(A_3 + A_1)\right)$$

This configuration is solved by using the equations shown to the left.

The Angle, Side, Angle routine has also been used as a secondary solution to some of the other routines, after the problem has first been reduced to these three known parts.

keystrokes: XEQ ASA

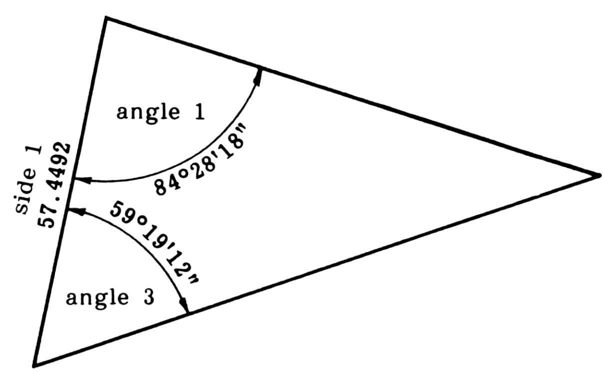
5 9 . 1 9 1 2 ANGL

5 7 . 4 4 9 2 SIDE

8 4 . 2 8 1 8 ANGL

output:

```
Side 1 = 57.4492
Angle 1 = 84°28'18.0"

Side 2 = 83.6400
Angle 2 = 36°12'30.0"

Side 3 = 96.8000
Angle 3 = 59°19'12.0"

AREA = 2,391.3500
```



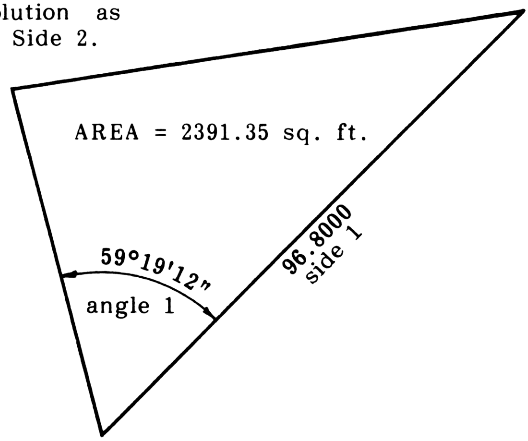
Area, Side 1, Angle 1

THE AREA, ONE SIDE AND ONE ANGLE KNOWN is the first of the three routines in this program which allow the area to be used as one of the known parts. Whenever the area is one of the parts, it is input first.

The equation

$$S_2 = \frac{2\ \text{AREA}}{S_1 \sin A_1}$$

is used first to reduce the problem for solution as Side 1, Angle 1, Side 2.



XEQ TRG

2 3 9 1 . 3 5

AREA

9 6 . 8

SIDE

5 9 . 1 9 1 2

ANGL

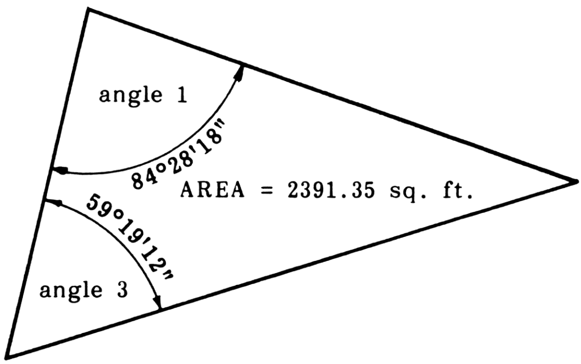
output:

```
Side 1 = 96.8000
Angle 1 = 59°19'12.0"

Side 2 = 57.4492
Angle 2 = 84°28'18.0"

Side 3 = 83.6400
Angle 3 = 36°12'30.0"

AREA = 2,391.3500
```

Area, Angle 3, Angle 1

AREA AND TWO ANGLES KNOWN is first solved for the included side, and then solved as Angle, Side, Angle. The first angle input is treated as Angle 3, the second as Angle 1. The equation used for finding side 1 is

$$S_1 = \sqrt{\frac{2\sin A_2(\text{AREA})}{\sin A_1 \sin A_3}} \qquad \text{where } A_2 = \cos^{-1}\left(-\cos(A_1 + A_3)\right)$$



XEQ TRG

2 3 9 1 . 3 5 AREA

5 9 . 1 9 1 2 ANGL

8 4 . 2 8 1 8 ANGL

output:

```
Side 1 = 57.4492
Angle 1 = 84°28'18.0"

Side 2 = 83.6400
Angle 2 = 36°12'30.0"

Side 3 = 96.8000
Angle 3 = 59°19'12.0"

AREA = 2,391.3500
```

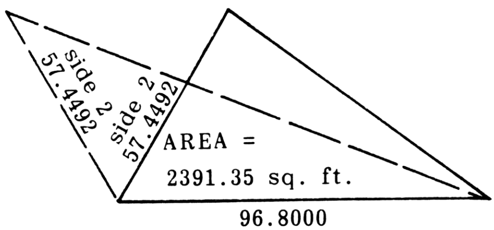
Area, Side 1, Side 2

AREA AND TWO SIDES KNOWN is another problem which has two possible solutions.

We first find Angle 1 with the equation

$$A_1 = \sin^{-1}\left[\frac{2AREA}{S_1\ S_2}\right]$$

and then solve as Side, Angle, Side. The second solution is possible where angle 1 may also be equal to 180° - angle 1. This value is substituted, and the second solution is output.



keystrokes:

XEQ [illegible]

2 3 9 1 . 3 5

AREA

9 6 . 8

SIDE

5 7 . 4 4 9 2

SIDE

The prompt, 2nd Solution appears in the display after output of the first solution. If you want output of the second solution, stroke R/S.

If not, stroke EXIT to leave the program, or MORE to solve another triangle.

output:

```
Side 1 = 96.8000
Angle 1 = 59°19'12.0"

Side 2 = 57.4492
Angle 2 = 84°28'18.0"

Side 3 = 83.6400
Angle 3 = 36°12'30.0"

AREA = 2,391.3500

 2nd Solution
Side 1 = 96.8000
Angle 1 = 120°40'48.0"

Side 2 = 57.4492
Angle 2 = 37°55'33.4"

Side 3 = 135.4461
Angle 3 = 21°23'38.6"

AREA = 2,391.3500
```

MORE HP42S PROGRAMS

Vertical Curves (booklet, $8.00)

Calculates CONTINUOUS vertical alignment without changing back and forth between Grade and Curve routines. Also calculates vertical intersections, symmetrical or asymmetrical vertical curves. Solves for station when the elevation is known, or the station can be given, to calculate the elevation.

Spiral Curves (booklet, $8.00)

Calculate the coordinates to any station, or offset to a station, within a spiral system. Options include coordinate output, auto-inverse, or both.

Calculates intersections of the entrance or exit spiral with a circular curve or straight line.

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This one turns your 42S into a manual data collector, complete with a labeling system that you can customize to suit the type of topo work you do.

All shots are stored as finished data, by shot number, for later output. Choice of 3-D coordinates or Station-Offset-Elevation for the output.

EDM Slope Staking (booklet, $8.00)

Set up anywhere near an alignment and slope stake it. Sets slope stakes from the remote instrument location directly. Includes a three-point resection program for finding the instrument's location by either station-offset or coordinates. All data needed to mark the stake is output (or may be stored), and there is a subroutine for setting the reference stake.

Urban Survey Programs (book, $20.00)

Contains programs for stakeout (or design) of street intersections, returns, culs-de-sac, bulbs and knuckles. A special program calculates and prorates blocks from the street intersections (with auto-store and auto-inverse of the points) from the street intersection setups for A.L.T.A. surveys.

Also contains layout programs for curb and gutter, or storm/sanitary sewers, which can store the data and later print out cut sheets before leaving the job site.

$8.00

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