HP42S
TRIANGLE
SOLUTIONS
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TECHNICAL ASSISTANCE

The program material, instructions and procedures contained in this book assume that the user has a working knowledge of both surveying and the general operation of the HP-42S calculator.

Technical assistance is limited to verification of the results shown in the various examples used in the book.

If you have any questions or suggestions regarding this book or other D'Zign publications, please feel free to call us. The number is (818) 507-7408, and someone is available to answer technical questions from 3:00 A.M. to 5:30 A.M. and from 4:30 P.M to 6:30 P.M. (Pacific time), as a service to users from other time zones.

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Hewlett-Packard has produced a really powerful calculator at a very good price, the HP-42 Scientific Calculator, which lends itself nicely to solving surveying problems. It can not be programmed by insertion of a module, or with a card reader, like the HP-41 series, but it has a really simple system for typing in a program.

the operations index

To find a function for the first time, HP has provided an "Operations Index" on pages 310 through 335 of the instruction manual, which tells you exactly what keystrokes to use to type in the function you want.

Even better, this index gives you the page number that you can refer to if you want to know more about the function you are using. If, while typing in a program, you aren't sure how to input a particular function, simply refer to the Operations Index.

the softkey menus

All of the programs in this booklet take advantage of the "softkey" menu system built into this calculator. When you want to start a program you stroke XEQ and then the softkey corresponding to the program you want, from the menu displayed in the bottom half of the screen.

the programs

The purpose of this booklet is to provide the user with the most flexible system for solving triangles possible.

The use of this program is simplicity itself. You need three known parts of the triangle being solved, and there are only three keys used for input. Because of the way the HP-42 works, those keys are labeled "SIDE", "ANGLE" and "AREA" by the menu. Another key is used, but only to return quickly to the program to solve another triangle.

learning while programming

We have tried to write this series of booklets in such a way that you quickly become acquainted with the calculator and its functions while you are programming it. Because the program is long, you might not want to do the whole job at one sitting.
The use of a printer is not required, but does make the whole job easier. Hewlett-Packard has the InfraRed Printer available for the 42S, and one feature of this calculator is that it already has the InfraRed transmitter built in.

**subroutines**

Because of the way the calculator works, we will start by input of some subroutines.

Once the subroutine has been input, its name appears in the menu when you stroke $\text{REC}$, and all you have to do to add it as a step in the program you are typing in is stroke GTO or XEQ followed by keystroking the key corresponding to the subroutine to input the program step GTO XXX or XEQ XXX.

**getting started**

Begin by stroking the shift key, then the XEQ key. The display will show a menu which will be blank (if you haven't yet input any programs) except for $.END.$ on the left. The keys just below each of the menu portions will correspond to the menu instruction above it. Stroke the key just below the $.END.$ in the display.

Next, go into **program mode** by stroking the shifted key. Scroll up once with the $\text{UP}$ key. Your display should be similar to the one shown to the left. Begin typing in the program steps shown above.
Proof read the program. If you scroll to 00 it should now say "45-Byte Prgm" ... the byte count is one quick check on the program, but you still need to check every step. Look, in particular, for steps that are alpha (with " " marks) but shouldn't be. Or should be but aren't.

"DMS"

This program puts the output of angles into the form ° ' " , and gives all answers to the nearest tenth of a second. Repeat the procedure of going to the permanent .END., scroll up to 00, and input the program below.

```
01  LBL "DMS"  14  ABS  27  XEQ 01
02  FS?C 19  15  STO 18  28  ARCL ST X
03  CLA  16  IP  29  f""
04  ENTER  17  XEQ 01  30  CLX
05  STO 19  18  ARCL ST X  31  FIX 04
06  IP  19  f""
07  CF 29  20  RCL 18  32  SF 29
08  FIX 00  21  FP  33  RCL 19
09  ARCL ST X  22  100  34  LBL 01
10  f""
11  -  23  X  36  X<>Y
12  100  24  FIX 02  37  X<>Y?
13  ×  25  RND  38  f"0"
14  26  FIX 01  39  END
```

The byte count on this one should be 78 bytes, as a check that everything is alright. Read through it to check for typos, and as a final check, input a number (try 25.25252) and execute the function. Your display should show 25°25'25.2" if the program is working correctly. If it doesn't, check it again.

the main program

The next two pages contain the main program. Take your time typing it in, to avoid errors.

Don't type an END on this one, the permanent end will work just as well, and it removes it from your XEQ/GTO menu.
The triangle shown to the right will be used for the examples. It should be noted that the output will vary slightly, depending on the number of places input, particularly in the input of the angles.

The notations for angles and sides is familiar to HP users, but is not the standard, or 'textbook', notation which you have learned in trigonometry (side \( a \) opposite angle \( A \), side \( b \) opposite angle \( B \) and side \( c \) opposite angle \( C \)). The sides and angles are numbered, in order, going around the triangle.

The example triangle (above) shows this style of labeling, compared to the standard notation for sides and angles. **Side 1** may be assigned to any side that is convenient to use, depending upon the available information about the triangle. It should be located at a side where the known information then falls into position for solution by one of the routines.

In the example, the assigned designations go clockwise. If it will better fit the information available, it may go counterclockwise instead, as shown to the left.
THREE SIDES KNOWN is one of the most used solutions for triangles, particularly in recent years in surveying. The lower cost and higher accuracy of electronic distance measurement equipment has resulted in more trilateration being used, instead of time-consuming repetitions of the angles.

**keystroke example:**

```
8 3 • 6 4
SIDE
9 6 • 8
SIDE
5 7 • 4 4 9 2
SIDE
```

**output:**

- Side 1 = 83.6400
- Angle 1 = 36°12'30.0''
- Side 2 = 96.8000
- Angle 2 = 59°19'12.1''
- Side 3 = 57.4492
- Angle 3 = 84°28'18.0''
- AREA = 2,391.3506

This routine uses the equations

\[
A_3 = 2 \cos^{-1} \sqrt{\frac{P(P - S_2)}{S_1 S_3}}
\]

\[
A_2 = 2 \cos^{-1} \sqrt{\frac{P(P - S_1)}{S_2 S_3}}
\]

and

\[
A_1 = \cos^{-1}(-\cos(A_3 + A_2))
\]

where \( P = \frac{1}{2}(S_1 + S_2 + S_3) \)

You can return for a second solution by stroking...
TWO SIDES AND THE INCLUDED ANGLE KNOWN is resolved by finding the third side, and then solving the triangle as shown on the previous page. The third side is found through the use of the equation

\[ S_3 = \sqrt{S_1^2 + S_2^2 - 2 S_1 S_2 \cos A_1} \]

**keystrokes:**

```
5 7 . 4 4 9 2
SIDE
8 4 . 2 8 1 8
ANGLE
8 3 . 6 4
SIDE
```

**output:**

- Side 1 = 57.4492
- Angle 1 = 84°28'18"
- Side 2 = 83.6400
- Angle 2 = 36°12'30"
- Side 3 = 96.8000
- Angle 3 = 59°19'12"
- AREA = 2,391.3506

**NOTE:** areas are calculated by this program using the equation shown below.

\[ \text{Area} = \frac{1}{2} (S_1 S_3 \sin A_3) \]
Side 1, Angle 1, Angle 2

ONE SIDE AND THE TWO FOLLOWING ANGLES KNOWN. This solution first solves for the third angle with the equation

\[ A_3 = \cos^{-1} \left( -\cos \left( A_1 + A_2 \right) \right) \]

Once angle 3 has been found, the remainder of the triangle is solved as Angle, Side, Angle (see page 8 for equations) to determine the other missing sides.

output:
Side 1 = 57.4492
Angle 1 = 84°28'18"
Side 2 = 83.6400
Angle 2 = 36°12'30"
Side 3 = 96.8000
Angle 3 = 59°19'12".
AREA = 2,391.3500
TWO SIDES AND THE FOLLOWING ANGLE KNOWN has two possible solutions. When this configuration is used, both solutions are output. The second solution will not necessarily show the parts in the same order as the input.

The other two angles are calculated with the equations below, and the remaining side is calculated as an Angle, Side, Angle configuration.

\[ A_3 = \sin^{-1} \left( \frac{S_2}{S_1} \sin A_2 \right) \]
\[ A_1 = \cos^{-1} \left( -\cos (A_2 + A_3) \right) \]

Output:

- Side 1 = 57.4492
- Angle 1 = 34°23'17.9"
- Side 2 = 33.6400
- Angle 2 = 36°12'30"
- Side 3 = 96.5980
- Angle 3 = 59°19'12.1"

Area = 2,391.3505

2nd Solution:

- Side 1 = 83.6400
- Angle 1 = 84°28'17.9"
- Side 2 = 83.6400
- Angle 2 = 36°12'30.0"
- Side 3 = 38.1740
- Angle 3 = 120°40'47.9"

Area = 943.0510
Angle 3, Side 1, Angle 1

TWO ANGLES AND THE INCLUDED SIDE ARE KNOWN.

\[ S_2 = S_1 \frac{\sin A_3}{\sin A_2} \]

\[ S_3 = S_1 \cos A_3 + S_2 \cos A_2 \]

\[ A_2 = \cos^{-1}(-\cos(A_3 + A_1)) \]

This configuration is solved by using the equations shown to the left.

The Angle, Side, Angle routine has also been used as a secondary solution to some of the other routines, after the problem has first been reduced to these three known parts.

keystrokes: \[ \text{KEQ} \text{ TRG} \]

\[
\begin{align*}
5 & 9 & : & 1 & 9 & 1 & 2 \\
5 & 7 & : & 4 & 4 & 9 & 2 \\
8 & 4 & : & 2 & 8 & 1 & 18
\end{align*}
\]

output:

\[
\begin{align*}
\text{Side 1} & = 57.4492 \\
\text{Angle 1} & = 84.2818' \\
\text{Side 2} & = 83.6400 \\
\text{Angle 2} & = 36.12'30.0" \\
\text{Side 3} & = 96.8000 \\
\text{Angle 3} & = 59.19'12.0" \\
\text{AREA} & = 2,391.3500
\end{align*}
\]
THE AREA, ONE SIDE AND ONE ANGLE KNOWN is the first of the three routines in this program which allow the area to be used as one of the known parts. Whenever the area is one of the parts, it is input first.

The equation

\[ S_2 = \frac{2 \text{ AREA}}{S_1 \sin A_1} \]

is used first to reduce the problem for solution as Side 1, Angle 1, Side 2.

**Output:**

- Side 1 = 96.8000
- Angle 1 = 59°19'12.0"
- Side 2 = 57.4492
- Angle 2 = 84°28'18.0"
- Side 3 = 83.6400
- Angle 3 = 36°12'30.0"

AREA = 2,391.3500
**Area, Angle 3, Angle 1**

**AREA AND TWO ANGLES KNOWN** is first solved for the included side, and then solved as Angle, Side, Angle. The first angle input is treated as Angle 3, the second as Angle 1. The equation used for finding side 1 is

\[ S_1 = \sqrt{\frac{2\sin A_2(AREA)}{\sin A_1 \sin A_3}} \text{ where } A_2 = \cos^{-1}(-\cos(A_1 + A_3)) \]

- **Area** = 2391.35 sq. ft.
- **Output:**
  - **Side 1** = 57.4492
  - **Angle 1** = 84°28'18.13"
  - **Side 2** = 23.6488
  - **Angle 2** = 36°12'24.8"
  - **Side 3** = 96.3484
  - **Angle 3** = 59°19'12.4"
  - **Area** = 2,391.3500
AREA AND TWO SIDES KNOWN is another problem which has two possible solutions.

We first find Angle 1 with the equation

$$\alpha_1 = \sin^{-1}\left(\frac{2\text{AREA}}{S_1 \cdot S_2}\right)$$

and then solve as Side, Angle, Side. The second solution is possible where angle 1 may also be equal to 180° - angle 1. This value is substituted, and the second solution is output.

The prompt, 2nd Solution appears in the display after output of the first solution. If you want output of the second solution, stroke BS.

If not, stroke EXIT to leave the program, or MORE to solve another triangle.
MORE HP42S PROGRAMS

Vertical Curves (booklet, $8.00)
Calculates CONTINUOUS vertical alignment without changing back and forth between Grade and Curve routines. Also calculates vertical intersections, symmetrical or asymmetrical vertical curves. Solves for station when the elevation is known, or the station can be given, to calculate the elevation.

Spiral Curves (booklet, $8.00)
Calculate the coordinates to any station, or offset to a station, within a spiral system. Options include coordinate output, auto-inverse, or both.
Calculates intersections of the entrance or exit spiral with a circular curve or straight line.

Topography (booklet, $8.00)
This one turns your 42S into a manual data collector, complete with a labeling system that you can customize to suit the type of topo work you do.
All shots are stored as finished data, by shot number, for later output. Choice of 3-D coordinates or Station-Offset-Elevation for the output.

EDM Slope Staking (booklet, $8.00)
Set up anywhere near an alignment and slope stake it. Sets slope stakes from the remote instrument location directly. Includes a three-point resection program for finding the instrument's location by either station-offset or coordinates. All data needed to mark the stake is output (or may be stored), and there is a subroutine for setting the reference stake.

Urban Survey Programs (book, $20.00)
Contains programs for stakeout (or design) of street intersections, returns, culs-de-sac, bulbs and knuckles. A special program calculates and prorates blocks from the street intersections (with auto-store and auto-inverse of the points) from the street intersection setups for A.L.T.A. surveys.
Also contains layout programs for curb and gutter, or storm/sanitary sewers, which can store the data and later print out cut sheets before leaving the job site.