

Model

46

Electronic Computations

The Hewlett-Packard Company Calculator Products Division

INTRODUCTION

The HP-46 is a desk-top printing calculator power packed for scientists and engineers. With its 9 data storage registers and up to 10-digit accuracy (depending upon the calculation), it is a truly versatile machine. This booklet contains examples you can do on the HP-46 to show yourself how this calculator will solve your problems.



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DECIBEL CONVERSION

This program converts voltage or power ratios to decibels. In turn, decibels may be converted to voltage or power ratios.

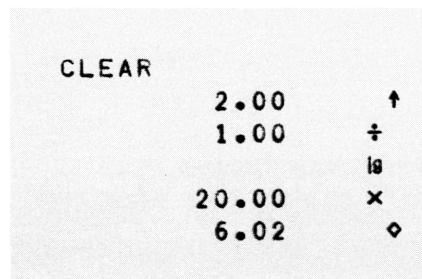
$$dB = 10 \log \frac{P_2}{P_1} = 20 \log \frac{V_2}{V_1}$$

$$\frac{P_2}{P_1} = 10^{\frac{dB}{20}}$$

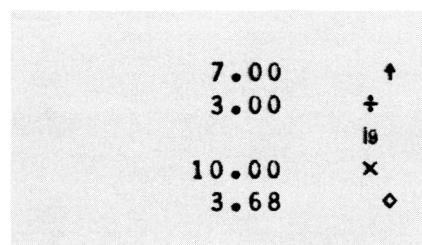
$$\frac{V_2}{V_1} = 10^{\frac{dB}{20}}$$

Examples:

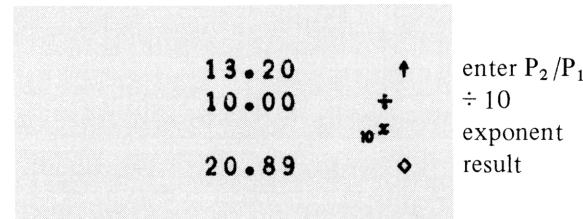
1. $V_1 = 1$ volt
 $V_2 = 2$ volt Calculate $20 \log \frac{V_2}{V_1} = 6.02$ dB



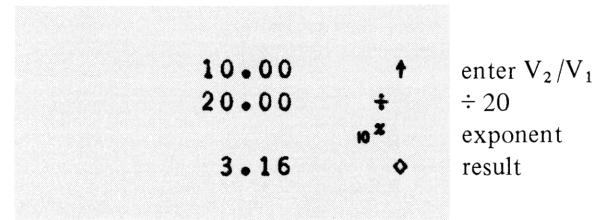
2. $P_1 = 3$ mW
 $P_2 = 7$ mW Calculate $10 \log \frac{P_2}{P_1} = 3.68$ dB



3. $\frac{P_2}{P_1} = 13.2$ dB Calculate $\frac{P_2}{P_1} = 20.89$

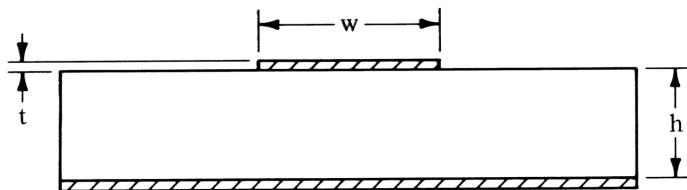


4. $\frac{V_2}{V_1} = 10$ dB Calculate $\frac{V_2}{V_1} = 3.16$



MICROSTRIP TRANSMISSION LINE

This program computes the characteristic impedance and propagation delay of microstrip line using the formulas from p. 39 of Blood, William R., *MECL System Design Handbook*, Motorola, Inc., 1971.



The characteristic impedance of the line shown is:

$$Z_0 = \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln \left(\frac{5.98h}{0.8w+t} \right)$$

and the propagation delay is:

$$t_{pd} = 1.017 \sqrt{0.475 \epsilon_r + 0.67} \text{ ns}$$

NOTE: The dimensions of w, h, and t may be anything as long as they are alike.

Example:

w=50 mils
t=1.5 mils
h=30 mils
 $\epsilon_r=4.7$

Compute
 $Z_0=51.52\Omega$
 $t_{pd}=1.73 \frac{\text{ns}}{\text{ft.}}$

0 . 8 0	↑	enter 0.8
5 0 . 0 0	×	Xw
1 . 5 0	+	+t
	✖	inverse
5 . 9 8	×	X 5.98
3 0 . 0 0	×	X h
	ln	natural log
8 7 . 0 0	×	X 87
4 . 7 0	↑	store in 1
1 . 4 1	+	enter ϵ_r
	✓	+ 1.41
	✖	square root
	↑	inverse
1 2 7 . 3 6	↔ 1	recall 1
	×	contents of 1
5 1 . 5 2	◊	multiply
		Z_0

CLEAR		
0 . 4 8	↑	enter .475
4 . 7 0	×	X ϵ_r
0 . 6 7	+	+ 0.67
	✓	square root
1 . 0 2	×	X 1.017
1 . 7 3	◊	t

IMPEDANCE OF TRANSMISSION LINES

This program computes high frequency characteristic impedance for coaxial transmission lines.

The characteristic impedance of a coaxial line is:

$$Z_o = \frac{K}{\sqrt{\epsilon_r}} \log \frac{D}{d}$$

where:

D = inner diameter of outer conductor

d = outer diameter of inner conductor

ϵ_r = relative permittivity of dielectric medium

$$K = \frac{\sqrt{\mu_0}}{2\pi\sqrt{\epsilon_0} \log e} \approx 138.06$$

Example:

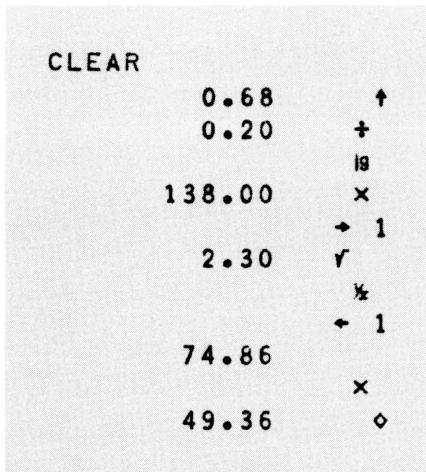
D = .68 in. RG-218/U coaxial cable

d = .195 in.

ϵ_r = 2.3 (polyethylene)

Compute

$$Z \odot = 49.36 \Omega$$



INDUCTANCE OF A SINGLE-LAYER CLOSE-WOUND COIL

The inductance of a single-layer coil is given approximately by Wheeler's formula:

$$L = \frac{N^2 R^2}{9R + 10ND}$$

where:

L = inductance in μ H

N = number of turns

R = inside radius of coil in inches

D = turn spacing in inches

NOTE: This formula is accurate to about 1% when $ND/R > 0.8$.

Example:

R = 1 inch

D = 0.086 inch

N = 30 turns

Calculate

L = 25.86 μ H expect 26 μ H

CLEAR	
0.68	↑
0.20	÷
138.00	×
2.30	→ 1
74.86	×
49.36	◊
	enter D
	÷ d
	evaluate log
	× K
	store in 1
	enter ϵ_r , square root
	inverse
	recall 1
	contents of 1
	× $1/\sqrt{\epsilon_r}$
	Z ⊙
9.00	↑
1.00	×
10.00	→ 1
30.00	×
0.09	×
9.00	→ 1
30.00	+
0.09	%
9.00	+
30.00	*
1.00	*
25.86	◊
	enter 9.0
	X R
	store in 1
	enter 10.0
	X N
	X D
	recall 1
	contents of 1
	add
	inverse
	N^2
	product
	R^2
	product
	result, L

CAPACITANCE OF PARALLEL PLATES

The capacitance of parallel plates and thin strips is given approximately by:

$$C = 0.0885419 \frac{\epsilon_r LW}{d} [1 + P]$$

where:

$$P = 0, 100 \quad W \geq L$$

ϵ_r = relative permittivity of medium between plates

d = distance between plates in cm

L = length of plates in cm

W = width of plates in cm

C = capacitance in picofarads

The formula given is accurate only when $L \gg d$ and $W \gg d$, however the error is only -4% for $W/d=2$.

Example:

$$\epsilon_r = 1$$

$$d = .01 \text{ cm}$$

$$L = 10 \text{ cm}$$

$$W = 1 \text{ cm}$$

$$C = 88.5 \text{ pF}$$

CLEAR

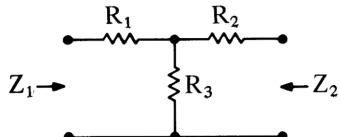
0.01	x
0.09	x
1.00	x
10.00	x
1.00	x
88.50	◊

enter d, 1/x
enter constant, X
enter ϵ_r , X
enter L, X
enter W, X
result, C = 88.5 pF

T ATTENUATOR

The T attenuator can be used to match between two impedances, Z_1 and Z_2 .

The minimum loss in decibels is given by:



$$\text{Min Loss} = 10 \log \left(\sqrt{\frac{Z_1}{Z_2}} + \sqrt{\frac{Z_1}{Z_2} - 1} \right)^2$$

where: $Z_1 \geq Z_2$

If N is the desired loss of the attenuator expressed as a ratio (loss in dB = $10 \log N$), then:

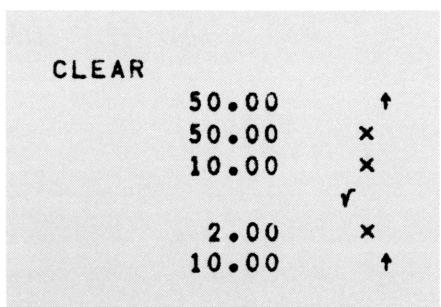
$$R_3 = \frac{2\sqrt{NZ_1Z_2}}{N-1}$$

$$R_1 = Z_1 \left(\frac{N+1}{N-1} \right) - R_3$$

$$R_2 = Z_2 \left(\frac{N+1}{N-1} \right) - R_3$$

NOTE: If the Desired Loss is less than the Minimum Loss, R_2 will be negative.

Example: $Z_1 = 50$ Compute Min Loss=0
 $Z_2 = 50$ $R_1 = 25.975$
 $\text{Loss} = 10 \text{ dB}$ $R_2 = 25.975$
 $\qquad\qquad\qquad R_3 = 35.136$



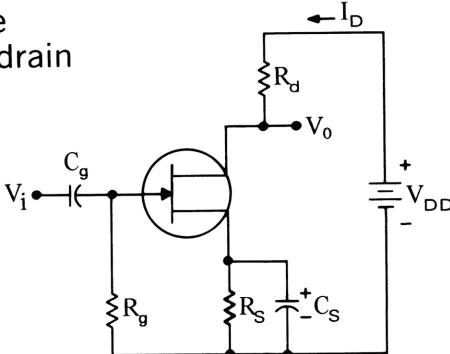
1.00	-	subtract 1
	%	inverse
	×	product
	→ 3	store in 3
10.00	↑	enter N
1.00	-	subtract 1
	%	inverse
10.00	↑	enter N
1.00	+	add 1
	×	product
50.00	×	$\times Z_1$
	← 3	recall 3
35.14	-	value of R_3
25.97	◊	minus R_3
	→ 1	value of R_1
10.00	↑	store in 1
1.00	-	enter N
	%	subtract 1
10.00	↑	inverse
1.00	+	enter N
	×	plus 1
50.00	×	product
	← 3	$\times Z_2$
35.14	-	recall 3
25.97	◊	value of R_3
50.00	↑	minus R_3
50.00	+	value R_2
1.00	-	enter Z_1
	✓	$\div Z_2$
50.00	↑	subtract 1
50.00	+	square root
	✓	enter Z_1
	÷	$\div Z_2$
	+	square root
	↖	sum
	↖	square
	lg	log
10.00	×	$\times 10$
0.00	◊	minimum loss

J-FET BIAS AND TRANSCONDUCTANCE

Given the FET parameters, V_p and I_{DSS} , and the desired drain current and voltage gain for the circuit shown, this program computes V_{GS} , g_m , and values for R_d and R_s .

The gate-source voltage necessary for a desired drain current is :

$$V_{GS} = V_p \left[1 - \left(\frac{I_D}{I_{DSS}} \right)^{1/2} \right]$$



where:

I_D =drain current ($I_D > 0$ for n-channel FET)

I_{DSS} =saturation drain current with gate shorted to source

V_{GS} =gate to source voltage ($V_{GS} < 0$ for n-channel FET)

V_p =pinch-off voltage

Knowing V_{GS} , we can compute the transconductance and the source and drain resistors.

$$g_m = -\frac{2I_{DSS}}{V_p} \left(1 - \frac{V_{GS}}{V_p} \right)$$

$$R_s = -\frac{V_{GS}}{I_D}$$

$$R_d = \frac{|A_V|}{g_m}$$

where:

g_m =transconductance in siemens

$|A_V|$ =magnitude of voltage gain

Example:

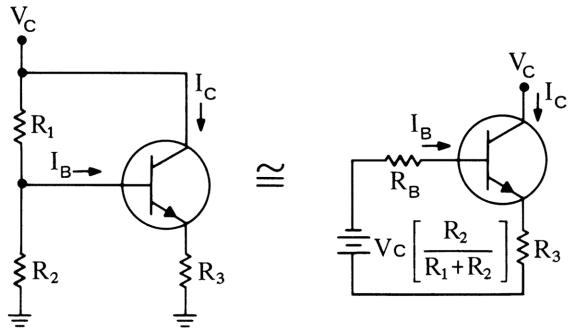
$$\begin{aligned} V_p &= -2V \\ I_{DSS} &= 1.5 \text{ mA} \\ I_D &= .7 \text{ mA} \\ A_V &= 10 \end{aligned}$$

$$\begin{aligned} \text{Compute } V_{GS} &= -0.63V \\ g_m &= 1.025 \text{ mS} \\ R_s &= 905 \Omega \\ R_d &= 9759 \Omega \end{aligned}$$

CLEAR	-01 ↑	enter I_D
7.00	+00 +	$\div I_{DSS}$
1.50	✓	square root
	✖	change sign
1.00	+00 +	plus 1
-2.00	+00 ×	$\times V_p$
-5.34	-01 ◇	result, V_{GS}
CLEAR	-01 ↑	enter V_{GS}
6.30	✖	change sign
2.00	+00 +	$\div V_p$
	✖	change sign
1.00	+00 +	+ 1
2.00	+00 ×	\times constant
1.50	-03 ×	$\times I_{DSS}$
	✖	change sign
-2.00	+00 +	$\div V_p$
1.03	-03 ◇	result, g_m
CLEAR	-01 ↑	enter V_{GS}
-6.30	-04 ÷	$\div I_D$
-7.00	+02 ◇	result, R_s
9.00		
CLEAR	+01 ↑	enter $ A_V $
1.00	-03 +	$\div g_m $
1.03	+03 ◇	result, R_d
9.71		

TRANSISTOR BIAS

This program computes the dc collector current of the bipolar transistor circuit shown below.



It is assumed that $I_B \ll$ current through R_1 and R_2 .

Given R_1 , R_2 , R_3 , β_{dc} , and V_C , we have:

$$I_C = \beta \left[\frac{KV_C - V_{BE}}{R_B + (\beta + 1)R_3} \right] = \beta \left[\frac{KV_C - .6}{R_B + (\beta + 1)R_3} \right]$$

where:

$$K = \frac{R_2}{R_1 + R_2}$$

$\beta = h_{FE}$ = dc current gain

$$R_B = \frac{R_1 R_2}{R_1 + R_2} = \text{parallel combination of } R_1 \text{ and } R_2$$

$V_{BE} = 0.6$ volts = Base-emitter voltage drop for silicon transistor

Example: $R_1 = 1000 \Omega$
 $R_2 = 5000 \Omega$
 $R_3 = 1000 \Omega$
 $V_C = 10$ volts
 $\beta = 100$

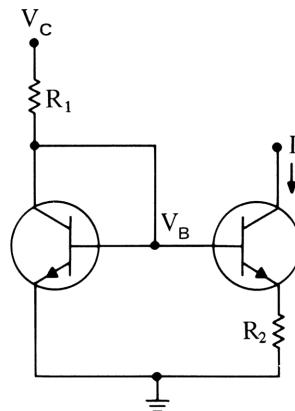
Compute
 $I_C = 7.6$ mA

CLEAR	
1.00	+03 ↑
5.00	+03 ×
	↑ 1
1.00	+03 ↑
5.00	+03 +
	%
	↑ 1
5.00	+06 ×
8.33	+02 ◊
	↑ 1
1.00	+02 ↑
1.00	+00 +
1.00	+03 ×
	↑ 1
8.33	+02 +
	%
1.00	+02 ×
	↑ 2
	↑ 1
8.33	+02
1.00	+03 ÷
1.00	+01 ×
6.00	-01 -
	↑ 2
9.82	-04 ×
7.59	-03 ◊

enter R_1
 $\times R_2$
store in register 1
enter R_1
 $+ R_2$
inverse
recall register 1
contents register 1
multiply
value of R_B
store in register 1
enter β
 $\beta + 1$
 $\times R_3$
recall register 1
contents of register 1
add: $R_B + (1 + \beta) R_3$
inverse
 $\times B$
store in register 2
recall register 1
contents register 1
 $R_B \div R_1$
 $K \times V_C$
 $- V_{BE}$
recall register 2
contents of register 2
 $\times (K V_C - .6)$
result, I_C

INTEGRATED CIRCUIT CURRENT SOURCE

For this common IC bias circuit shown below, the resistance R_2 can be found from:



$$R_2 = \frac{kT}{qI} \ln \left[\frac{V_C - V_B}{R_1 I} \right]$$

where:

$$k = 1.38 \times 10^{-23} \frac{\text{J}}{\text{K}}, \text{ Boltzman's constant}$$

T = absolute temperature of junction in Kelvins

$$q = 1.6 \times 10^{-19} \text{ C}, \text{ the electronic charge}$$

$$V_B = 0.6 \text{ volts, the work function for Si}$$

This program evaluates the above equation given:

T , the junction temperature in $^{\circ}\text{C}$

I , the desired current in amperes

R_1 , the desired value for R_1 in ohms

V_C , the supply voltage in volts

Example:

1. $T = 50^{\circ}\text{C}$
- $I = 10 \mu\text{A}$
- $R_1 = 10 \text{ k}\Omega$
- $V_C = 10\text{V}$

Compute
 $R_2 = 12.7 \text{ k}\Omega$

CLEAR

1.00

6.00

1.00

1.00

1.38

1.60

1.00

5.00

2.73

3.92

1.27

+01 ↑

-01 -

+04 +

-05 +

ln

-23 ×

-19 +

-05 +

→ 1

+01 ↑

+02 +

← 1

+01

×

+04 ♦

enter V_C

subtract V_B

÷ R_1

÷ I

take natural log

× k

÷ q

÷ I

store in 1

enter T , in $^{\circ}\text{C}$

calculate T in $^{\circ}\text{K}$

recall 1

× T in $^{\circ}\text{K}$

R_2 in ohms

RESISTOR NOISE

The thermal noise of a resistor used in low-level amplifiers can be calculated from the expression:

$$i_n = \sqrt{\frac{4KTB}{R}}$$

where:

K=Boltzman's constant= 1.38×10^{-23} joules/ $^{\circ}\text{K}$

T=temperature in $^{\circ}\text{K}$

B=bandwidth of system

R=resistor value

Example:

R=10K

B=1K Hz

T=100 $^{\circ}\text{C}$

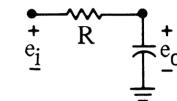
Compute $i_n = 44.5$ picoamps

2.73	+02	↑	enter 273 $^{\circ}\text{K}$
1.00	+02	+	add T
4.00	+00	×	$\times 4$
1.33	-23	×	$\times K$
1.00	+03	×	$\times B$
1.00	+04	%	enter R, inverse R
		x	product
		r	square root
4.45	-11	◊	result

INTEGRATOR RESPONSE

The response of a single pole integrator to a unit step function $U(t)$ can be evaluated for any time t.

Consider the RC network:



and its response expression for $e_o(t) = U(t)$.

$$\frac{e_o}{e_i} = \left[1 - e^{-\frac{t}{RC}} \right]$$

Example:

R=1K

C=1 μF

Calculate $e_o/e_i = .632$ at $t=1$ millisec.

CLEAR			
1.00	-03	↑	enter t
1.00	+03	%	enter R, 1/R
		x	$\times t$
1.00	-06	%	enter C, 1/R
		x	$\times t R$
	zS		change sign
	e ^x		e^x
	zS		change sign
1.00	+00	+	add 1
6.32	-01	◊	e_o/e_i

MOS-LSI DEVICE CURRENT

The drain-source current of a MOS-LSI device can be calculated for given terminal conditions. Using Sah's equation:

$$I_{DS} = K \left(\frac{W}{L} \right) [2(V_{GS} - V_t)V_{DS} - V_{DS}^2]$$

$$|V_{GS} - V_t| > |V_{DS}|$$

where:

K =process gain constant,

(assume $K = -1.6 \times 10^{-6}$ amps/v²)

W =device width

L =effective device length

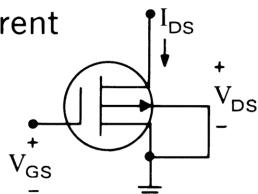
V_{GS} =gate-source voltage

V_T =device threshold, assume $V_T = -1$ V

V_{DS} =drain-source voltage

one can calculate the current

I_{DS} for the given circuit.



Example:

$$K = -1.6 \times 10^{-6} \text{ amps/volt}^2$$

$$W = 1.0 \text{ mils}$$

$$L = 0.26 \text{ mils}$$

$$V_{GS} = -4 \text{ volts}$$

$$V_{DS} = -1.5 \text{ volts}$$

$$V_T = -1.0 \text{ volt}$$

$$\text{Calculate } I_{DS} = 41.5 \mu \text{ amps}$$

CLEAR			
-4.00	+00	↑	enter V_{GS}
1.00	+00	+	subtract V_T
2.00	+00	×	$\times 2$
-1.50	+00	×	$\times V_{DS}$
1.50	+00	‡	V_{DS}^2
	-		$-V_{DS}^2$
1.00	+00	×	$\times W$
2.60	-01	÷	$\times 1/L$
-1.60	-06	×	$\times K$
-4.15	-05	◊	result