E.Z. Math™

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SMI

MANUAL

E.Z. Math™

by

Raymond La Barbera

Software for the HP48SX Calculator

Program and Manual Written and Conceived
by
Raymond La Barbera and E.Z. Software™

E.Z. Math™ Package Marketed and Distributed
by
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The author wishes to dedicate E.Z. Math to his wife *Stevie Lai La Barbera* who so patiently and lovingly tolerated his many late night programming and writing sessions and to his friend *Tom O'Brien* who has been such a steadfast and loyal friend during the last three decades.

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We welcome comments, suggestions for improvement, criticisms, bug reports and correspondence about the E.Z. Math program software and manual. We will consider all such correspondence in preparing possible future versions of E.Z. Math. Please address all letters to:

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Please fill out the registration card that was enclosed in your E.Z. Math package. This will enable us to keep you posted about possible future versions and upgrades. Please send all registration cards to:

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Chapter One

Basics

WHAT'S IN THIS CHAPTER

This chapter is designed to familiarize you with the features of the HP48SX calculator you'll need to make full use of all the features of E.Z. Math.

The Hewlett Packard 48SX calculator is without doubt the most powerful calculator ever produced. With 256K of built-in ROM, expansability to nearly 300K of RAM, the capability of accepting plug-in ROM cards [such as E.Z. Math] and the ability to do symbolic algebra and calculus as well as to solve a bewildering array of mathematical, scientific, statistical and financial problems, the HP48SX is more powerful than such early computers as the ATARI 800, Commodore 64 and Apple IIe, and is almost as powerful as an IBM XT computer. The HP48SX is actually a fully programmable scientific calculator with the power of a computer.

But all this power comes with a price. Just as learning how to operate an IBM computer can be a long, drawn out process, learning to make full use of the HP48SX requires quit a bit of time and study. Most people find the thought of wading through 850 pages of User Manual and learning how to program to be a most daunting prospect. This is especially true of people who have a fear of computers and electronic gadgets, bad experiences with math or simply a very short supply of time, patience and energy. Such people would rather simply type in a few numbers and immediately get the correct answer.

These are the people for whom E.Z. Math was created. With E.Z. Math, you can do a wide range of problems involving graphs, loans, savings and numbers without ever having to open the HP Owner's Manual. We've designed a simple, very easy-to-use, logically organized system of menus that enable you to quickly zip from one feature of E.Z. Math to another.

Within the next few pages, we'll tell you everything you need to know about the HP48SX to make full use of E.Z. Math. We'll show you how to locate any key you need to press. We'll explain everything you need to know about menus and display screens and how to get from one to another. We'll teach you how to enter, edit and delete numbers.

Before we proceed with these basics, we'd like to encourage you to look through the HP Owner's Manual from time to time. As we said above, you don't need to do so to use E.Z. Math. However, you may later on find learning about the stack, custom menus and various other goodies a fascinating adventure, especially if you browse through the manual a bit at a time. It's just possible that you may find that creating a program, even a little, tiny one, is a lot of fun as well as a source of great satisfaction.

Now, let's begin our adventure!!!

LOCATING KEYS

The HP48SX has a keyboard consisting of 49 keys arranged in nine rows. There are six keys in each of the four upper rows and five keys in each of the five lower rows.

Row 1 consists of six white-topped keys closest to the display screen. The next row, beginning on the left with the key labelled MTH and ending on the right with the key labelled NXT, is row 2. The last row, all the way down at the bottom, beginning with the key labelled ON and ending on the right with the key labelled + is row 9. A good way to remember this is that the numbering of the rows is exactly the same as the order in which the lines of a book are read – from top to bottom.

In each row, key 1 is the key at the left side of the keyboard and key 5 or 6 is the key at the right side. So, the key labelled ON is key 1 in row 9. The key labelled + is also in row 9 but it is key 5. The key labelled NXT is key 6 in row 2. A good way to remember this is that the order of the keys in a row is exactly the same as the order in which the words of a line in a book are read – from left to right.

So, when you are asked to press the ENTER key [*row 5, key 1*], you'll need to go down to the fifth row and look for the first key. There, you'll find the key labelled ENTER. This is the one you need to press. If you are asked to press the 6 key [*row 7, key 4*], you'll need to go down to the seventh row and look for the fourth key from the left. There you'll find the one labelled 6. This is the one you need to press.

We will consistently use this system throughout the E.Z. Math Manual to help you find any key that needs to be pressed when using E.Z. Math to do a problem.

E.Z. MATH MENU SCREENS

```
       GRAPH MENU
1 Polynomial 6 Systems
2 Inequality 7 Conic
3 Hyperbolic 8 Polar
4 Rational   9 Log
5 Parametric 0 Trig
  Pick a number
NEXT PREV KILL STAK MAIN OFF
```

E.Z. Math makes extensive use of menu screens such as the one shown above to help you get quickly and easily from one type of problem to another. Each menu screen makes use of two types of menus: the menu bar and the menu options.

The Menu Bar or Menu Line

The *menu bar* or *menu line* is the row of small blue rectangles at the bottom of the display screen. Let's refer to each of these rectangles as menu buttons. Most menu screens have six menu buttons in the menu bar, but quite a few have fewer than six and some even have none. Each menu button has a label printed in white giving some indication of the button's function.

The Graph Menu Screen shown above has six menu buttons in the menu bar. They are labelled: NEXT, PREV, KILL, STAK, MAIN and OFF. We'll find out in Chapter 3 what these keys actually do.

The menu buttons are controlled by the top row of white-topped keys nearest to the screen display. Each menu button is controlled by the white-topped key directly below it. To press or activate the menu button labelled NEXT, press the first, leftmost key in the first row. Later, we'll simply ask you to press the NEXT key [*row 1, key 1*]. To press or activate the menu button labelled STAK, press the fourth key in the first row. Later, we'll simply ask you to press the STAK key [*row 1, key 4*].

We'll use this system consistently throughout the E.Z. Math Manual any time you need to press or activate a menu button on the menu bar.

The Menu Options

The *menu options* are the choices listed on the menu screen under the title and above the menu bar. Each option is preceded by a digit, the first option being numbered 1 and each succeeding option being numbered 2, 3, 4, 5, 6, 7, 8, 9 and 0 as needed. Choosing an option leads to another menu screen, to a number input screen, to an answer screen or to a graphics screen. No matter which option you pick, E.Z. Math will always take you to the correct successor screen.

The Graph Menu Screen shown on the previous page has ten options beginning with "1 Polynomial" and ending with "0 Trig". To select option 1, the Polynomial option, you need to locate the key labelled 1. Go down to the eighth row and then look for the second key. There, you'll find the key labelled 1. Press this key and whatever option 1 is supposed to do will happen. Later, we'll simply ask you to press the 1 key [*row 8, key 2*]. To select option 6, the Systems option, go down to the seventh row and look for the fourth key. This is where you'll find the key labelled 6. Press this key to have option 6 do whatever it is supposed to do. Later, we'll simply ask you to press the 6 key [*row 7, key 4*].

We'll use this system consistently throughout the E.Z. Math Manual anytime you need to select an option on a menu screen.

Answer Screens and Option Screens

Some E.Z. Math screens are *answer screens* designed to remind you what problem you wanted solved, to redisplay the numbers you entered and to reveal the final answer. These screens have menu bars but no options. To leave an answer screen, press the desired menu button on the menu bar by pressing the white-topped key in row 1 just below it.

Some E.Z. Math screens are *option screens* having an option menu but no menu bar. To leave one of these screens, select the option you desire by pressing the key whose label is the same as the number of the desired option.

Printing an E.Z. Math Screen

An E.Z. Math menu screen, answer screen and option screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.

2. Press the orange LEFT-SHIFT key [*row 7, key 1*].

3. Press the MTH key [*row 2, key 1*].

4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.

5. Press the ENTER key [*row 5, key 1*] to begin printing.

6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

Other Screens

The following other screens are also used by E.Z. Math:

Some E.Z. Math screens are *delay screens* having neither menu bar nor options. These screens are designed to ask you to be patient while E.Z. Math is busy at work taking care of the last task you gave it to do. All you need do when you see one of these screens is to wait for E.Z. Math to get the job done and take you to another screen. These screens cannot be printed on the HP82240 infrared printer.

Some E.Z. Math screens are *graphics screens* having a menu bar but no options. These screens are designed to display the graphs of equations, inequalities, functions, systems of equations and systems of inequalities. We'll learn how to work with, leave and print these screens in Chapter Three of the E.Z. Math Manual.

Some E.Z. Math screens are *number input screens* having neither menu bar nor options. These screens appear when it is time for you to enter numbers for E.Z. Math to use in solving a problem. These screens, which cannot be printed on the HP82240 infrared printer, are fully discussed in the next section which is coming up on the next page.

Some Final Comments

To sum up, E.Z. Math makes use of the following types of screens:

Type of Screen	Menu Bar	Options	What to do
Menu	YES	YES	Make a selection.
Answer	YES	NO	Look at answer and make a selection.
Option	NO	YES	Make a selection.
Delay	NO	NO	Wait for next screen.
Graphics	YES	NO	Look at graph and make a selection.
Number Input	NO	NO	Enter a number.

All this information about various types of screens is presented for your information. Please bear in mind that, while this information may be helpful and interesting, it's not essential for using E.Z. Math. The most important thing to remember about any screen is to read what it says and act accordingly. If the screen presents choices, read what choices are available and then press the key corresponding to your selection. If the screen asks you to enter a number, select your number, type it and press the ENTER key [*row 5, key 1*]. If the screen displays a message indicating that your last selection is being acted upon, just wait until the next screen appears. Should you press a key in error, you'll never end up more than a few keypresses away from where you wanted to be. No matter what appears on the screen, the main thing is to read what is displayed and act accordingly.

E.Z. MATH NUMBER INPUT SCREENS

We now explain everything you need to know about entering, editing and deleting numbers on an E.Z. Math number input screen.

Number Input Screen Basics

When you get to a number input screen, you'll see a blinking or flashing arrow-shaped mark near the bottom right corner of the display screen. This flashing mark is called the *number input cursor* or simply the *cursor*. Each time you type a digit, decimal point or something else, it will appear at the cursor position on the display screen and the cursor will then move one place to the right. The cursor always marks the position at which what you next type will appear.

Each number input screen has a message in the middle of the screen asking you to enter a number. If you are not asked to type a whole number, a positive number or some other kind of number, you may type any kind of number you wish. If you type the wrong kind of number or type something that is not a number, E.Z. Math will tell you that you made an error and ask you to try again.

If you get to a number input screen by mistake or if you change your mind, you can press the ON key [*row 9, key 1*] to abort the number entry process. You'll be taken back to the screen you left in coming to the number input screen.

Entering Numbers

No matter what problem you call upon E.Z. Math to solve, you'll need at some point to enter one or more numbers. This process, which is similar to the way in which numbers are entered on most calculators and computers, involves three steps.

Step 1. You need to type the number.
Step 2. You need to check the number for errors.
Step 3. You need to press the ENTER key [*row 5, key 1*].

In order to successfully type numbers in an E.Z. Math number input screen, you need to be familiar with 12 keys: the ten digit keys, the DECIMAL POINT key and the +/- key.

First, you'll need to know how to type digits. Here are the ten digit keys and their locations:

1	*[row 8, key 2]*
2	*[row 8, key 3]*
3	*[row 8, key 4]*
4	*[row 7, key 2]*
5	*[row 7, key 3]*
6	*[row 7, key 4]*
7	*[row 6, key 2]*
8	*[row 6, key 3]*
9	*[row 6, key 4]*
0	*[row 9, key 2]*

Next, you'll need to know how to type a decimal point. For this, you'll need to use the DECIMAL POINT key *[row 9, key 3]*. To type a decimal number, type the digits before the decimal point, press the DECIMAL POINT key and type the digits after the decimal point. This is similar to the way you'd type a decimal on another calculator, on a computer or on a typewriter.

Finally, you'll need to know how to type a negative number. For this, you must use the +/- key *[row 5, key 2]*. In E.Z. Math, never, NEVER, use the SUBTRACTION key *[row 8, key 5]* to type a negative sign. On the HP48SX, the SUBTRACTION key is supposed to be used for subtracting two numbers, not for typing a negative sign. To type a negative number, just type the number as if it were not a negative number. In other words, just type the digits and the decimal point, if any. Then, press the +/- key to turn the number negative. If you made a number negative by mistake, another press of the +/- key will make the number positive. Remember, type your number BEFORE pressing the +/- key and do NOT use the SUBTRACTION key.

Hereafter, we'll use the word *character* to refer to a digit, a decimal point, a negative sign, or a positive sign. The number -5126.37 has eight characters: the six digits 5, 1, 2, 6, 3 and 7, the negative sign and the decimal point. [In computer lingo, the word *character* also refers to a letter, a space, a punctuation mark and just about any other single mark or symbol appearing on the display screen.]

Editing and Deleting Numbers

You've just learned how to type numbers. But what if you've noticed a mistake in one or more of the digits or in the placement of the decimal point. How do you correct any such error before pressing the ENTER key? And what do you do if you've already pressed the ENTER key?

To successfully edit or delete a number, you need to be familiar with five keys: the ON key, the BACKSPACE key, the LEFT CURSOR key, the RIGHT CURSOR key and the DEL key.

Press the ON key [*row 9, key 1*] to delete the entire number you've typed so that you can type a different number. Be careful not to press the ON key a second time because doing so will abort the .entire number input process and return you to the screen from which you started.

Use the BACKSPACE key [*row 5, key 5*] to delete the last part of the number. Each press of the BACKSPACE key moves the cursor one place to the left, erasing each time it moves the rightmost character of the number, the one that was immediately to the left of the cursor. Press the BACKSPACE key five times to erase the last five characters of the number. If you typed 879.6324, six presses of the BACKSPACE key will erase the 9.6324 and leave 87 displayed on the screen with the cursor immediately to the right of the 7. Once you've backspaced enough to remove all the unwanted characters, you may then add whatever new characters you desire to the end of your number.

Press the LEFT CURSOR key [*row 3, key 4*] to move the cursor to the left without erasing any characters it passes over. Each press of the LEFT CURSOR key moves the cursor one place to the left leaving the number you typed intact.

Press the RIGHT CURSOR key [*row 3, key 6*] to move the cursor to the right without erasing any characters it passes over. Each press of the RIGHT CURSOR key moves the cursor one place to the right leaving the number you typed intact.

Press the DEL key [*row 5, key 4*] to delete the character under the cursor. Each press of the DEL key will delete the character under the cursor and cause all the digits that were to the right of the cursor to move one place to the left.

To remove a character from your number, move the LEFT CURSOR and RIGHT CURSOR keys as many times as necessary to place the cursor over the character you want deleted. Then press the DEL key to erase the character under the cursor. You may then use the LEFT CURSOR, RIGHT CURSOR and DEL keys repeatedly to remove any other undesired characters from the number.

To insert a character in your number, use the LEFT CURSOR and RIGHT CURSOR keys to move the cursor to the character just before which you would like to place the new character. Then, type the new character. The number will split open and make a space to accept the newly typed character.

If you suddenly realize that the number you've just typed had an error but you've already pressed the ENTER key, then press the ON key [*row 9, key 1*] to abort the number input process. You'll be taken back to the screen from which you selected your problem and have the opportunity to start over again.

Some Notes on Sleep and Death

Due to various technical aspects of the HP48SX, there are occasions during which E.Z. Math will seem to be taking a nap. In most such cases, you'll see a suitable message displayed on the screen asking you to be patient while the HP48SX carries out your last instructions.

There are a few cases in which you'll see no such message screen. It takes almost ten seconds from the time you press a key while looking at the E.Z. Math Title Screen to get to the Main Menu Screen. This is the amount of time needed to set things up so that E.Z. Math can do its work efficiently. It usually takes between five and ten seconds from the time you press the **KILL** key [*row 1, key 3*] on any screen whose menu bar contains this button to leave E.Z. Math and restore the HP48SX to its original state before you started the program. However, if you've had E.Z. Math running for a long time or used it to do many problems, it might well take longer to leave the program. If so, please be patient.

By the way, the HP48SX displays a little hourglass figure at the top of the screen near the right corner to let you know that it's busy carrying out your last instructions. When the hourglass figure disappears, you'll then be able to make another selection or do another problem.

Many people do not realize that all computers, calculators and programs have defects or *bugs*, usually very subtle ones that the user will never encounter. It's practically impossible to build a computer or calculator or to write a program which is 100% perfect. The more power and complexity which are involved, the greater the likelihood that there will be bugs.

While the HP48SX and E.Z. Math work perfectly 99.99% of the time, it's possible that some strange sequence of key presses could trigger one of these subtle bugs and cause the calculator to go into a coma. We're not referring to to situations involving problems with large numbers, such as listing all the factors of 10,000,000 which will take a very long time (We're talking about hours!) to compute.

If your HP48SX remains asleep for more than, say, a half hour and it's not busy with a problem involving very large numbers, you can assume it's in a coma. Here's how to bring it out of its coma. Press the ON key [*row 9, key 1*] and keep it depressed. While the ON key continues to be depressed, press the key [*row 1, key 3*] just to the left of the letter "C". Release both keys and the HP48SX will wake up ready for business. You don't even need to give it coffee! To return to the E.Z. Math program, follow the start-up procedure described on Page 2-2 of Chapter 2.

Now that we've learned the HP48SX basics necessary to make full of E.Z. Math, let's turn to Chapter Two to learn how to plug in your E.Z. Math ROM card and to begin using E.Z. Math.

Have fun!!

Chapter Two

Start-up

WHAT'S IN THIS CHAPTER

This chapter describes how to get your E.Z. Math ROM card up and running on your HP48SX.

You'll first need to get your E.Z. Math ROM card inside your HP48SX. The procedure for installing and removing cards is fully described on pages 635 to 638 in your HP Owner's Manual. Here's a quick summary:

Make sure that your HP48SX is turned off before you begin. Holding it screen side up as if you were about to do a few calculations, turn it over so that you are now looking at its back side.

Down at the bottom, just below the tiny panel containing the words "Made in USA" and "© Hewlett Packard", there is a small cover under which is the battery compartment. Since we are not about to change the batteries, let's leave this cover alone.

Up at the top, just above the numbers 1 and 2, is another smaller cover under which are the two HP48SX ports. This is where you'll put the E.Z. Math ROM card once you've removed the port cover.

To remove the cover, put your thumb on the five-ridge grooved rectangular area near the bottom of the port cover. While gently applying pressure with your thumb, simultaneously push forward until the port cover slides off.

Inside you'll see two slots or ports. Holding the E.Z. Math ROM card label side up, carefully insert it into one of the two ports, making sure that the card doesn't end up half in one port and half in the other. When you first feel resistance as you are sliding the card in, you'll know that you've got just one quarter of an inch left to go before your card is properly in place.

Carefully slide the port cover back on so that it is attached as snugly to the HP48SX as it was originally.

Now turn on your HP48SX by pressing the ON key [*row 9, key 1*]. When a display appears on the screen, press the ALPHA key [*row 6, key 1*] twice, then press the E key [*row 1, key 5*] and finally press the Z key [*row 5, key 3*]. You should now see EZ displayed near the lower left corner of the screen. Press the ENTER key [*row 5, key 1*] to see the E.Z. Math Title Screen. If you look at the top of the Title Screen, just above the "t" in "E.Z. Math", you'll notice a little figure in the shape of an hourglass. You'll always see this little hourglass figure when the HP48SX is busy carrying out your last instructions which, in this case, are setting up E.Z. Math to run smoothly and without problems on your calculator. Finally, in about nine seconds, the Title Screen will be replaced by the Main Menu Screen from where you can access all the features of E.Z. Math. You'll find a full discussion of the Main Menu Screen on the next two pages.

THE MAIN MENU SCREEN

```
        MAIN MENU

1. Graphs    4. Numbers
2. Loans     5. Game
3. Savings   6. Music

     Pick a number
       KILL STAK      OFF
```

Getting to the Main Menu Screen

There are two ways to get to the Main Menu Screen:

- Wait nine seconds while viewing the E.Z. Math Title Screen.
- Press MAIN [*row 1, key 5*] on any screen whose menu bar has this button.

The Main Menu Screen Menu Bar

There are three keys active on the Main Menu Screen Menu Bar:

KILL Press this key [*row 1, key 3*] to terminate E.Z. Math. When you again want to use E.Z. Math, repeat the start-up sequence described in Chapter 2 which is as follows: Press the ALPHA key [*row 6, key 1*] twice, press E [*row 1, key 5*] and Z [*row 5, key 3*], press the ENTER key [*row 5, key 1*] to get to the E.Z. Math Title Screen which, after about nine seconds, will be replaced by the E.Z. Math Main Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the E.Z. Math Main Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Main Menu Screen.

The Main Menu Screen Options

There are six options available on the Main Menu Screen:

[1] **Graphs.** Press this key [*row 8, key 2*] to go to the Graph Menu Screen from which you can select any of 182 families of equations, inequalities, functions, systems of equations and systems of inequalities in rectangular, polar or parametric form for graphic analysis.

[2] **Loans.** Press this key [*row 8, key 3*] to go to the Loan Menu Screen from which you can select various problems involving fixed rate mortgages on houses, condos, co-ops, property and other such investments.

[3] **Savings.** Press this key [*row 8, key 4*] to go to the Save Menu Screen from which you can select various problems involving a single deposit or repeated deposits to a savings account, certificate of deposit [C.D.], term deposit [T.D.], money market account or other such investment.

[4] **Numbers.** Press this key [*row 7, key 2*] to go to the Number Menu Screen from which you can select various problems involving natural numbers, sequences of natural numbers, rational numbers and complex numbers.

[5] **Game.** Press this key [*row 7, key 3*] to go to the Game Menu Screen from which you can select a number guessing game in which either you or the HP48SX has to guess a number picked by the other.

[6] **Music.** Press this key [*row 7, key 4*] to go to the Music Menu Screen from which you can hear the HP48SX play various scales and compose original electronic music and see note value tables to assist you if you wish to compose your own music.

Chapter Three

Graphs

WHAT'S IN THIS CHAPTER

This chapter describes the E.Z. Math Graph Module. The graph Module allows you to select from 188 families of equations, inequalities, functions, systems of of equations and systems of inequalities in rectangular, polar or parametric form, all laid out and arranged in an easy-to-use, logically organized system of menus making it very easy to make a selection and to graphically analyze it.

Actually, equations, inequalities and systems are all examples of open sentences. In addition, functions are defined by using open sentences. Thus, we can say that the E.Z. Math Graph Module allows you to select from 188 families or types of open sentences.

A graph is a very good example of the "picture is worth a thousand words" adage. Much more information about an open sentence can be gained when it is converted to and displayed as a picture than when it is displayed simply as a bunch of numbers, variables and other mathematical symbols. By careful study of the picture or graph, including its shape, its high and low points and its intersection, if any, with the X-axis and Y-axis, it is possible to gain a deep understanding of the open sentence that was graphed.

E.Z. Math makes it easy to graph an open sentence. You simply go to the menu displaying the type or family to which your open sentence belongs, type the numbers defining the particular open sentence you wanted to graph and watch while the graph is drawn. Even though E.Z. Math guides you through every step of the process, you'll find complete details later in this chapter about what keys need to be pressed.

Unfortunately, many people come to algebra, trigonometry and calculus with a very poor background in the theory and practice of graphing open sentences. Having very little understanding of and experience with graphing can seriously impede the progress of someone trying to learn math.

To help remedy this, the next several pages are devoted to a thorough discussion of the basic concepts involved in graphing open sentences. Since every open sentence involves numbers and ordered pairs of numbers, you'll become acquainted with sets in general and both sets of numbers and sets of ordered pairs of numbers in particular. You'll begin to understand the concept and use of variables, so basic to mathematics and yet misunderstood by so many people. You'll find out about the real number line and the real number plane and how they are used in graphing sets of numbers and sets of ordered pairs of numbers. You'll be informed about numerical sentences and open sentences and learn how both to use and to speak about them. Finally, you'll discover exactly what it means to graph an open sentence and a system of open sentences.

We invite you to turn the page and let the adventure begin!

SETS

Sets are probably the most basic objects used in mathematics. In all types and branches of mathematics, we constantly study and use sets. In arithmetic, algebra and calculus, we study sets of numbers. In geometry, we study sets of points and sets of geometric figures. In probability, we study sets of events. In logic, we study sets of statements. To understand mathematics in general and graphs in particular, we must first understand sets.

The Meaning of a Set

A *set* may be defined as any definite collection, bunch or assortment of objects, taken all together and considered as a single, brand-new object. Let's examine this definition word by word.

An *object* may be thought of as anything which has or could be given a name. An *object* may be thought of as anything which could be classified as a noun. An *object* may be thought of as anything which could be the object of one's study. Here are some examples of objects which could be part of a set: you, me, your house, New York City, the United States, the world, any number such as 8, 26, 9.2, 0 or -6.325, any geometric figure such as a triangle, square or circle, air, a dream or any concept such as truth, beauty, justice or freedom.

Collection, bunch and *assortment* all pretty much have the same meaning. You are probably familiar with record collections, coin collections, jewelry collections and baseball card collections. You've probably bought a bunch of grapes or a bunch of bananas, or have had a bunch of people pay you a visit. You've doubtless eaten a piece or two from a candy assortment. Keep these examples in mind to help you understand the concept of set.

A collection is *definite* when we know exactly which objects are in the collection and which objects are not in the collection. Consider the collection which contains some numbers. This collection is not definite because we do not know whether it contains the number 5. On the other hand, the collection containing numbers with which you count starting with 1 and ending with 5 is definite since we know exactly which objects are part of the collection: 1, 2, 3, 4 and 5.

On last condition must be met in order for a collection to be a set. It's not enough to have a definite collection of objects scattered all over the place. They must all be gathered together and put into a single container or package. This package with all its enclosed objects is a brand-new object different from the individual objects we gathered together and is the set we've been trying to construct.

You, as is true of all living creatures, are a collection of cells, billions of cells. Not one cell in your body looks like you, talks like you, thinks like you or even resembles you in any way. Yet, if these cells are taken all together as a single object, we have you, a living, breathing person. Your body is a collection containing billions of cell as objects and yet you are a single, brand-new object that is completely different from the individual cells of your body. So, when thinking about any set of objects, you can always picture yourself as the set and the cells of your body as the objects in the set.

All sets used in E.Z. Math are sets containing numbers and sets containing points.

Picturing and Notating Sets

Whenever you think about a set of objects, picture it as a box which contains those objects. Actually let a picture of a box appear in your mind and actually visualize the objects of that set within the box. If you imagine a set in this manner, you'll make few, if any, mistakes when dealing with sets.

A set is written by listing all the objects in the set separated by commas and enclosed within braces as follows:

You read or write:	$\{1, 2, 3, 4, 5\}$
You say:	"The set containing 1, 2, 3, 4 and 5"
You picture:	A box with 1, 2, 3, 4 and 5 inside.

Types of Sets

There are many types of sets. Here are a few of the sets with which we need to be familiar in E.Z. Math:

A *finite set* is a set containing a finite number of objects. In other words, a set is finite if it contains a definite number of objects. The set of all Presidents of the United States, the set containing all living people and the set containing all grains of sand on all the beaches of the world are all finite sets. Obviously, any set whose objects we can write down or use with a calculator must be a finite set.

An *infinite set* is a set containing infinitely many objects. In other words, a set is infinite if the number of objects it contains is greater than EVERY number. The set containing all the counting numbers and the set containing all triangles are both infinite sets. While we can think about infinite sets as much as we wish, it is not possible to write one down or to enter one into a calculator.

An *ordered pair* is a set containing two objects in which the order of the objects makes a difference. With regard to sets in general, the number of objects in the set and the order in which the objects are written does not matter. But, an ordered pair must contain exactly two objects in a specific order. Change the order and you have a different ordered pair.

An ordered pair is written by listing the two objects separated by a comma and enclosed within parentheses as follows:

You read or write:	(5, -3)
You say:	"The ordered pair 5, -3"

Obviously, (5, -3) and (-3, 5) are different ordered pairs.

An *ordered n-tuple* is a set containing n objects in which the order of the objects makes a difference. (3, 9, -2) is read "the ordered triple 3, 9, -2" and is different from (9, 3, -2) and (-2, 9, 3). "The ordered quadruple 8, 1, 4, 7" is written (8, 1, 4, 7) and is different from (4, 1, 7, 8) and (7, 4, 1, 8). Likewise, we can have ordered quintuples like (5, 67, 23, -2, 2.4), ordered sextuples like (3, 4, 1, 34, 6, -5) ordered septuples, ordered octuples and so forth.

The *empty set* is the set which contains no objects. The *null set* is another name for the empty set. The empty set may be symbolized as { } or with the Greek letter Ø. When you think about the empty or null set, picture it as an empty box. Some examples of the empty set are the set of all female Presidents of the United States and the set of all five-sided triangles.

Equality of Sets

Two sets are *equal* if they contain exactly the same objects. Here are some examples:

Example 1. $\{1, 2, 3\} = \{3, 2, 1\}$
Remember that this is read "The set containing 1, 2 and 3 equals the set containing 3, 2 and 1." These two sets are equal because they contain exactly the same objects.

Example 2. $\{1, 2, 3\} \neq \{A, B, C\}$
Remember that this is read "The set containing 1, 2 and 3 is not equal to the set containing A, B and C." These two sets are unequal because they do not contain the same objects.

Example 3. $\{1, 2, 3\} = \{3, 2, 1, 2, 3, 1, 3, 2, 1\}$
These two sets are equal because they contain exactly the same objects. Neither set contains an object that is not in the other.

Example 4. $\{5\} \neq 5$
These two sets are unequal because 5 is not a set. In order for

two sets to be equal, both must actually be sets.

Example 4. $\{0\} \neq 0$
These are unequal because 0 is not a set.

Example 5. $\{0\} \neq \{\ \}$
Remember that this is read "The set containing 0 equals the empty set." These two sets are unequal because they don't contain exactly the same objects.

Example 6. $\{\ \} \neq 0$
These two sets are unequal because the 0 is not a set.

Example 7. $\{\ \} = \emptyset$
Remember that this is read "The empty set equals the empty set." These two sets are certainly equal.

Example 8. $\{\emptyset\} \neq \emptyset$
Remember that this is read "The set containing the empty set equals the empty set." These two sets are unequal because they don't contain exactly the same objects. If picture $\{\emptyset\}$ as a box which contains an empty box and $\emptyset$ as an empty box, you can see that they are unequal.

The Intersection of Sets

The intersection of two sets is the set containing all objects common to both sets. An upside down letter "U" is used to symbolize the intersection of two sets. Here are some examples:

Example 1. $\{1, 2, 3, 4\} \cap \{3, 4, 5, 6\} = \{3, 4\}$
Remember, this is read "The intersection of $\{1, 2, 3, 4\}$ and $\{3, 4, 5, 6\}$ is $\{3, 4\}$. The objects common to the two sets are 3 and 4. Hence, the intersection is $\{3, 4\}$.

Example 2. $\{1, 2, 3, 4\} \cap \{2, 3, 4\} = \{2, 3, 4\}$
Since the objects common to the two sets are 2, 3 and 4, the intersection is $\{2, 3, 4\}$.

Example 3. $\{1, 2, 3, 4\} \cap \emptyset = \emptyset$
Since there are no objects common to these two sets, the intersection is the empty set.

Example 4. $\{1, 2, 3, 4\} \cap \{7, 8, 9\} = \emptyset$
Since there are no objects common to these two sets, the intersection is the empty set.

VARIABLES ON SETS OF NUMBERS

A *variable* on a set of numbers is any symbol used to represent or hold a place for any number in the given set of numbers. Let's examine this definition word by word.

A *symbol* is any kind of written mark or spoken sound to which some kind of meaning can be attached. In the case of variables, we usually use letters such as X, a, n, T, µ or Ø from our alphabet or from a foreign alphabet as symbols. However, we may also use other symbols such as a pair of parentheses, a blank such as the one used in a fill in the blank test question, a word, a pronoun such as "it" and various other suitable symbols.

To make this clear, here are some sentences having essentially the same meaning, differing only in the symbol used as the variable:

Example 1. Find a number which when added to 5 gives 7.
Example 2. $X + 5 = 7$
Example 3. $\_\_\_ + 5 = 7$
Example 4. $(\ ) + 5 = 7$
Example 5. Number + 5 = 7
Example 6. It + 5 = 7

The set of numbers used with a variable is called the *replacement set* or *domain* of that variable. Any individual number in the replacement set of a variable is called a value of the variable. Suppose {1, 2, 3, 4, 5} is the replacement set for the variable X. Then 2 is a value of X. The other values of X are 1, 3, 4 and 5. No other number is a value of X. Thus, 2.34, 3 1/2, 72, -4, -2, 0 and 4.4 are not values of X.

Every variable must have a replacement set. Many, if not most, problems involving variables, however, do not explicitly state specific replacement sets. This does not mean that these variables have no replacement sets. Rather, it is understood that, unless otherwise stated, the replacement set for all variables in algebra, trigonometry and calculus is the set of all real numbers. This is the understanding followed in E.Z. Math.

Whenever you use a variable and its replacement set, think about a candy machine in the movies. Picture the variable as the coin slot of the candy machine. Picture the replacement set as the little sign on the candy machine which tells you what coins you are allowed to put into the coin slot.

Another good way to think about a variable and its replacement set is to imagine a flattened-out mailbox that can only hold one piece of mail at a time. The mail box is the variable and the set containing all pieces of mail is the replacement set. Any single piece of mail may be put in the mail box, but that piece of mail must then be taken out before another piece of mail can be inserted.

GRAPHING SETS OF NUMBERS

The Real Number Line

The *number line* is a partnership between two of the most important sets in mathematics: the set of all points on a line and the set of all real numbers.

A *straight line* is a geometric figure that is absolutely straight and has no beginning and no end. You can think of a straight line as one of those endless highways in the Midwest that seem to go on forever. A straight line has infinitely many points. [A point is any place, position or location on a line, and a line has infinitely many of them.]

The *set of real numbers* is the set of all numbers which can be written as decimals. The set of real numbers includes such numbers as 1, 5, 0, -3, 7/8, 1.35, -41.73 and 102.61385. All natural numbers, whole numbers, integers and rational numbers can be written as decimals and are therefore real numbers. [Please see Chapter Six for more information on these sets of numbers.]

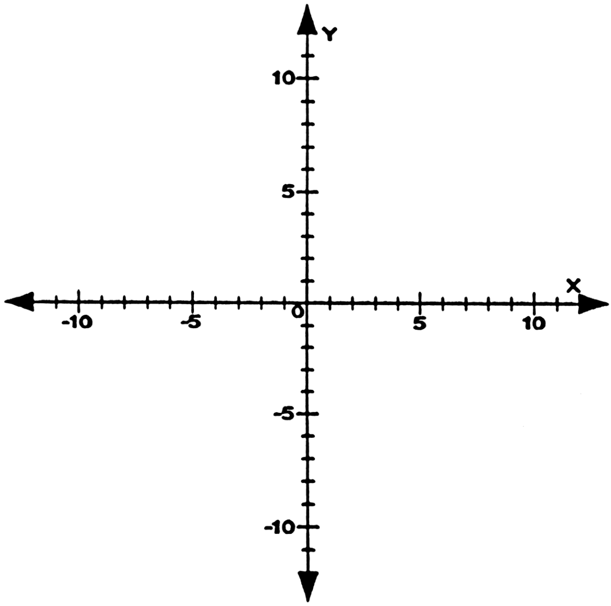
The *real number line*, or more simply the *number line*, is a partnership between the set of all points on a line and the set of all real numbers. This partnership is set up so that every point of the line is paired off with exactly one real number and every real number is paired off with exactly one point of the line. It's like the ideal high school dance in which the number of boys and the number of girls turn out to be the same. With the real number line, the set of all points on a straight line and the set of all real numbers exactly match and form, so to speak, infinitely many married couples. In math lingo, we say that there is a *one-to-one correspondence* between the set of all real numbers and the set of all points on a straight line.

The point of the number line that is paired off with a certain real number is called the *graph* of that number. The real number that is paired off with a certain point on the number line is called the *coordinate* of that point.

A good way to picture the number line is as follows. Think of the straight line as a very long street. Think of the points on that line as the houses on that street. Think of the set of real numbers as the addresses of all those houses. Picture the coordinate of each point as the address of each house. Picture the graph of each real number as the house which has that number as its address.

Remember, the number line assigns to every point of a straight line a real number as its address or coordinate and to every real number a point on the straight line as its house or graph. This, of course, means that there are as many points on a straight line as there are real numbers.

The Real Number Plane



The number line was a very good way to give addresses to all the houses on a street. Now, we'll see how to give addresses to all the houses in a city.

A *plane* is an absolutely flat surface, such as a table top, a piece of paper, a blackboard or a movie screen, except that a plane has no boundaries. You can picture a plane as a movie screen that has no top, no bottom, no left side and no right side. You can also picture a plane as a piece of paper with no edges that goes on forever. A plane has infinitely many points.

To set up a number plane, you must first draw a horizontal number line on a plane. We'll call this the *X-axis* or *horizontal* axis. Next, draw a vertical number line on the same plane so that the zero points or origins of the two number lines are the same. We'll call this the *Y-axis* or *vertical axis*.

The *real number plane*, or more simply the *number plane*, is a partnership between between the set of all points of a plane and the set of all ordered pairs of real numbers. This partnership is set up so that every point of the plane is paired off with exactly one ordered pair of real numbers and every ordered pair of real numbers is paired off with exactly one point of the plane. We then have what is called a *one-to-one correspondence* between the set of all points on a plane and the set of all ordered pairs of real numbers.

The point of the real number plane that is paired off with an ordered pair of real numbers is called the *graph* of that ordered pair. The ordered pair of real numbers that is paired off with a point of the real number planes is called the *coordinates* of the point.

To graph an ordered pair of real numbers, proceed as follows. With a finger from one hand, touch the point of the X-axis which is the graph of the first or X-coordinate of the ordered pair. With a finger from the other hand, touch the point of the Y-axis which is the graph of the second or Y-coordinate of the ordered pair. Move your two fingers toward each other by moving the finger on the X-axis straight up or down and the finger on the Y-axis straight left or right. The point at which your two fingers meet is the graph of the ordered pair.

To find the coordinates of a point on the number plane, move a finger placed on the point straight up or down to the X-axis. The coordinate of the point at which you arrived on the X-axis is the X-coordinate of the given point. Next, move a finger placed on the given point straight left or right to the Y-axis. The coordinate of the point at which you arrived on the Y-axis is the Y-coordinate of the given point.

A good way to visualize the number plane is as follows. Picture the plane as a city, say the city in which you live. Picture a point of the plane as a house in your city. Picture the coordinates of the point as the address of the house, imagining the first number or X-coordinate of the ordered pair as the name of the street in which the house is located and the second number or Y-coordinate of the ordered pair as the number on the door of the house. Picture the graph of an ordered pair of real numbers as the house which matches a given address.

Graphing Sets

It is very important to be able to graph a set because, by so doing, it becomes possible to discover patterns, relations and other information about the set's objects that we might otherwise not be able to uncover.

In algebra, trigonometry and calculus, we need to be able to graph sets of real numbers and sets of ordered pairs of real numbers. The procedure is very simple. To graph a set of real numbers:

Step 1. Draw a real number line.
Step 2. Graph or plot each real number from the set.

To graph a set of ordered pairs of real numbers:

Step 1. Draw a real number plane.
Step 2. Graph or plot each ordered pair of real numbers from the set.

OPEN SENTENCES

What Is a Numerical Sentence?

A *numerical sentence* is a sentence which contains no variables. Let's see what types of mathematical objects can be part of a numerical sentence.

As is true for any sentence, a numerical sentence must contain a *verb*. While there are thousands of verbs available for English sentences, there are just three available for numerical sentences. They are :

1. $=$ [You say: "is equal to"]
2. $>$ [You say: "is greater than"]
3. $<$ [You say: "is less than"]

These verbs can be negated as follows:

1. $\neq$ [You say "is not equal to"]
2. $\not>$ [You say "is not greater than"]
3. $\not<$ [You say "is not less than"]

These verbs may be combined to form compound numerical sentences as follows:

1. $\geq$ [You say: "is greater than or equal to"]
2. $\leq$ [You say: "is less than or equal to"]

A verb splits a numerical sentence into two parts: the *left side* of the numerical sentence, which consists of everything before the verb, and the *right side* of the numerical sentence, which consists of everything after the verb.

A numerical sentence must contain *numbers*, at least one on each side. For numerical sentences encountered in algebra, trigonometry and calculus, we may select numbers from the set of natural numbers, the set of whole numbers, the set of integers, the set of rational numbers and the set of real numbers. We generally select numbers from the set of complex numbers only in higher level math courses. [Please see Chapter Six for a full explanation of all these sets of numbers.]

If either side of a numerical sentence involves two or more numbers, then we must use *operations* to combine these numbers. For numerical sentences encountered in algebra, trigonometry and calculus, we may select any of the six important operations which are addition, subtraction, multiplication, division, power and root.

If either side of a numerical sentence involves three or more operations, then we may use *grouping symbols* to control the order in which the operations are to be carried out. The four available grouping symbols are:

1. ( ) [These are called *parentheses.*]
2. [ ] [These are called *brackets.*]
3. { } [These are called *braces.*]
4. — [This is called the *vinculum*]

The vinculum is used mostly for fractions to group what's in the numerator, to group what's in the denominator and, finally, to divide the two results.

In addition, a numerical sentence may contain such mathematical functions as: SIN, COS, TAN, SINH, LOG, LN, ABS and various others.

Here are some examples of numerical sentences:

Example 1. $3=7$

Example 2. $4+5<6$

Example 3. $9-2\neq 3+4$

Example 4. $2\cdot 5^2+3\cdot 9-6\cdot 8>40^2+(12+7)^2$

Example 5. $80-[40-[20-(10-2)]\}=42+10$

Example 6. $(9-2)(8^2-54)=70$

Example 7. $SIN^2(55^\circ)+COS^2(55^\circ)=1$

Example 8. $ABS[5\cdot TAN(37^\circ)+8\cdot COS(23^\circ)]>0$

Every numerical sentence has a *truth value.* The truth value for a numerical sentence is *true* if the numerical sentence is true. The truth value of a numerical sentence is *false* if the numerical sentence is false. Since a numerical sentence is always either true or false, and not both true and false at the same time, a numerical sentence always has exactly one particular truth value. In the eight examples of numerical sentences given above, the first four have a truth value of false and the last four have a truth value of true. In other words, the first four are false and the last four are true.

What is an Open Sentence?

An *open sentence* is a sentence which contains one or more variables. An open sentence contains exactly the same mathematical objects as a numerical sentence. The only difference is that an open sentence must have at least one variable while a numerical sentence can never have any variables.

Here are some examples of open sentences:

Example 1. $X=5$

Example 2. $X+2Y>2$

Example 3. $2X+3Y^2<4X^2+2X-3$

Example 4. $3X-7Y+9Z=45$

Example 5. $3(X+2)-4(X-7)=5X-3(2X-5)$

Example 6. $4X^2-3X+7=0$

Example 7. $X^2+7Y^2-2XZ^2<4X-7Y$

Example 8. $3X^2-\{2X+5[3Y-2(Z^2-7)]\}>5$

Example 9. $SIN^2(X)+COS(X)=1$

Example 10. $ABS[5\cdot TAN(X)+8\cdot COS(X)]=30$

An open sentence can be changed into a numerical sentence if each variable is replaced by one of its values. If we replace the variable X with the value 5, the open sentence $X+9=11$ becomes the numerical sentence $5+9=11$. Likewise, any numerical sentence can be changed into an open sentence if we replace at least one number with a value. If we replace the numbers 5 and 12 with the variables X and Y, the numerical sentence $5+12=16$ becomes the open sentence $X+Y=16$.

We learned above that numerical sentences always have a definite truth value: either true or false. What about an open sentence? Is $X+5=9$, for instance, true or false? It's not definitely true because there is a value of X, 8 for example, which when substituted for X turns the open sentence $X+5=9$ into the false numerical sentence $8+5=9$. On the other hand, $X+5=9$ is not definitely false because there is a value of X, 4 for example, which when substituted for X turns the open sentence $X+5=9$ into the true numerical sentence $4+5=9$. Thus, an open sentence is neither true nor false; it has no specific truth value. We can accordingly say that an open sentence has a changeable or variable truth value.

Classifying Open Sentences

An open sentence is classified in each of the following three ways:

An open sentence is classified according to the number of different variables in the open sentence. Here are some examples:

Example 1. $3X+7Y-2Z^2<4X^2-7Y+6$
This is an open sentence in three variables.

Example 2. $4X^2-3Y=2X-3$
This is an open sentence in two variables.

All open sentences and systems of open sentences graphed in the E.Z. Math Graph Module are in two variables.

An open sentence is classified according to the verb used. An *equation* is an open sentence using the verb "=". An *inequality* is an open sentence using the verb ">" or "<". Here are some examples:

Example 1. $3X+7Y-2Z^2<4X^3-7Y+6$
This is an inequality in three variables.

Example 2. $4X^2-3Y=2X-3$
This is an equation in two variables.

Example 3. $4X-3Y^2>2X^3-10Y+7$
This is an inequality in two variables.

Example 4. $9X^2-8X+5=0$
This is an equation in one variable.

An open sentence is classified according to the degree of the term of highest degree. Here are some examples:

Example 1. $3X^2+5X-2Y+11XY=4X+7$
This is a second degree equation in two variables. This equation has the terms $3X^2$, $+5X$, $-2Y$, $+11XY$, $4X$ and $+7$. The degree of a term is found by adding the exponents of the variables of that term. This makes $3X^2$ and $+11XY$ each second degree terms, $+5X$, $-2Y$ and $4X$ each first degree terms and $+7$ a zero degree term. Since the terms of highest degree are terms of the second degree, the equation is a second degree equation.

Example 2. $7X^3+3X^2Y^2-2Y<3Z^2-6$
This is a fourth degree inequality in three variables. The second term $+3X^2Y^2$ is a term of fourth degree since the sum of the exponents is 4. Since this is the term of highest degree, the inequality is of fourth degree.

Example 3. $6X-3=13X+7$
This is a first degree equation in one variable.

Some special words are used in referring to the degree of an open sentence. Here are the first eight such special words:

Degree of Open Sentence	Special Word
1	Linear
2	Quadratic
3	Cubic
4	Quartic
5	Quintic
6	Sextic
7	Septic
8	Octic

Here are some examples of open sentences likely to be encountered in a typical high school or college math courses:

Example 1. $5X+3=4X-2$
This is a linear equation in one variable.

Example 2. $5X^2-2X+3=0$
This is a quadratic equation in one variable.

Example 3. $4X-2Y=6$
This is a linear equation in two variables.

Example 4. $5X-9<3X-4$
This is a linear inequality in one variable.

Example 5. $7X^2-8XY-7Y^2+3X-5Y=12$
This is a quadratic equation in two variables.

Example 6. $8X-5Y+7Z=9$
This is a linear equation in three variables.

Example 7. $4X-3Y>16$
This is a linear inequality in two variables.

Example 8. $3X^3-2X^2+5X-2=6$
This is a cubic equation in one variable.

Solution of an Open Sentence in One Variable

A *solution* or *root* of an open sentence in one variable is any value of the variable which when substituted for the variable turns the open sentence into a true numerical sentence. Here are some examples:

Example 1. Find a solution of $X+5=9$.
4 is a solution because when 4 is substituted for X, the open sentence $X+5=9$ becomes the true numerical sentence $4+5=9$. There is no other solution.

Example 2. Find a root of $(X-2)(X-5)=0$.
2 is a root because when 2 is substituted for X, the open sentence $(X-2)(X-5)=0$ becomes the true numerical sentence $(2-2)(2-5)=0$. Likewise, 5 is a root. There is no other root.

Example 3. Find a solution of $X+5>3$.
8 is a solution because when 8 is substituted for X, the open sentence $X+5>3$ becomes the true numerical sentence $8+5>3$. In fact, every real number greater than 2 is a solution.

Example 4. Find a root of $X+5=5+X$.
12 is a root because when 12 is substituted for X, the open

sentence $X+5=5+X$ becomes the true numerical sentence $12+5=5+12$. In fact, every real number is a root.

Example 5. Find a solution of $X+3=X$.
There is no real number which when substituted for X turns the open sentence $X+3=X$ into a true numerical sentence. Therefore, there is no solution.

Example 6. Find a root of $X+7\neq 9$.
8 is a root because when 8 is substituted for X, the open sentence $X+7\neq 9$ becomes the true numerical sentence $8+7\neq 9$. Likewise, 5 is a root. In fact, every real number except 2 is a root.

Solution of an Open Sentence in Two Variables

A *solution* or *root* of an open sentence in two variables is any ordered pair of values of the variables which when substituted for the variables turns the open sentence into a true numerical sentence. Here are some examples:

Example 1. Find a solution of $X+2Y=10$.
(2, 4) is a solution because when 2 and 4 are substituted for X and Y, the open sentence $X+2Y=10$ is turned into the true numerical sentence $2+2\cdot 4=10$. Other solutions of $X+2Y=10$ are (1, 4.5), (0, 5), (-30, 20), (100, -45), (1.4, 4.3), (10, 0), (-100, 55) and (5.996, 2.002). In fact, this open sentence has infinitely many solutions. By the way, there are also infinitely many ordered pairs of real numbers which are not solutions of this open sentence.

Example 2. Find a root of $Y=2X^2$.
Some solutions are (1, 2), (0, 0), (-10, 200), (2, 8), (-3, 18), (10, 200), (-4, 32) and (0.2, 0.08). In fact, there are infinitely many solutions. There are also infinitely many ordered pairs of real numbers which are not solutions of this open sentence.

In general, almost all open sentences in two variables have infinitely many ordered pairs of real numbers which are solutions and infinitely many which are not solutions.

Solution Set of an Open Sentence

The *solution set* or *truth set* of an open sentence is the set containing all solutions. Here are some examples:

Example 1. Find the solution set of $X+5=9$.

The only solution of this open sentence is 4. Therefore, the solution set is { 4 }.

Example 2. Find the truth set of (X-3)(X+6)=0.
The only solutions of this open sentence are 3 and -6. Therefore, the solution set is { -6, 3 }.

Example 3. Find the solution set of X+5=X.
Since this open sentence has no solution, the solution set is Ø or { } [Remember, Ø is read "the empty set" or "the null set"].

Example 4. Find the truth set of X+3=3+X.
Since every real number is a solution of this open sentence, the solution set is the set of all real numbers.

Example 5. Find the solution set of X+3>5.
Since every real number greater than 2 is a solution of this open sentence, the solution set is the set of all real numbers greater than 2.

Example 6. Find the solution set of X+2Y=10.
This example is a but more complicated. We have learned that each solution of an open sentence in two variables is an ordered pair of real numbers. We've also learned that, for most open sentences in two variables, there are infinitely many ordered pairs which are solutions and infinitely many which are not solutions. Since writing infinitely many solutions would take forever, it is not possible to write the complete solution set. So the best we can hope for is to write a partial solution set. Here is a partial solution set: { (-2, 6), (0, 5), (2, 4), (4, 3), (6, 2), (8, 1), (10, 0), (12, -1) }. Another perhaps more familiar way to write this partial solution set is:

X	-2	0	2	4	6	8	10	12
Y	6	5	4	3	2	1	0	-1

You can easily see that a hundred different people working with an open sentence in two variables could come up with a hundred different partial solution sets. But unfortunately there is no way around this problem.

Solving an Open Sentence

To *solve* an open sentence means to find the solution set of the open sentence. This means that we are using the word "solve" in place of the phrase "find the solution set of". Since we learned all about solution sets in the last section, solving an open sentence should be a snap. Here are some examples:

Example 1. Solve X+3=9.
The solution set is { 6 }.

Example 2. Solve X(X-2)(X+3)(X-5)=0.
The solution set is { -3, 0, 2, 5 }.

Example 3. Solve X+5>9.
The solution set is { All real numbers greater than 4 }.

Example 4. Solve X+3=X.
The solution set is { } or Ø.

Example 5. Solve X+5=5+X.
The solution set is { All real numbers }.

Example 6. Solve X+7≠12.
The solution set is { All real numbers except 5 }.

Example 7. Solve X+2Y=10.
A partial solution set is { (-2, 6), (0, 5), (2, 4), (4, 3), (6, 2), (8, 1), (10, 0), (12, -1) }.

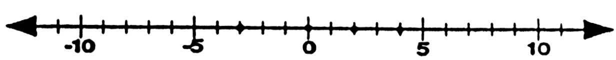
Graphing an Open Sentence

To *graph* an open sentence means to graph the solution set of that open sentence. Here are some examples:

Example 1. Graph X+3=9.
The solution set is { 6 }. The graph of this set is:

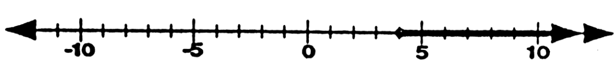
Example 2. Graph X(X-2)(X+3)(X-5)=0.

The solution set is { -3, 0, 2, 5 }. The graph of this set is:



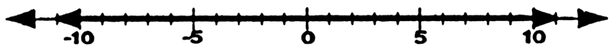
Example 3. Graph X+5>9.

The solution set is { All real numbers greater than 4 }. The graph of this set is:

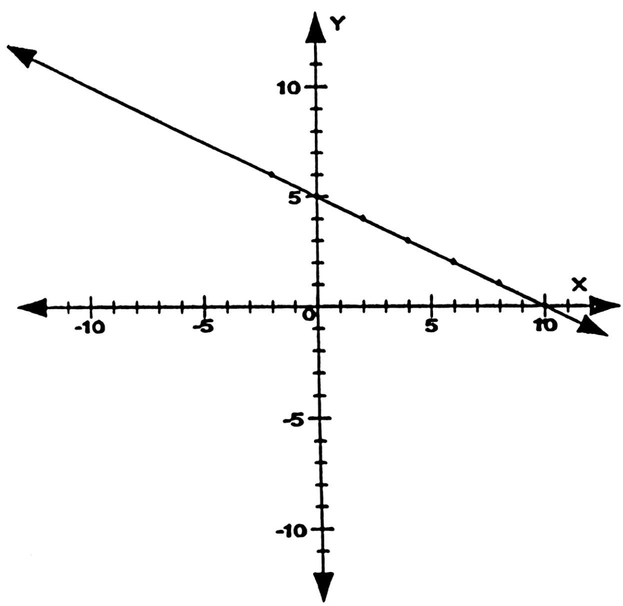


Example 4. Graph X+3=X.

Example 5. Graph X+5=5+X.
The solution set is { All real numbers }. The graph of this set is:



Example 6. Graph X+2Y=10.
A partial solution set is { (-2, 6), (0, 5), (2, 4), (4, 3), (6, 2), (8, 1), (10, 0), (12, -1) }. The graph of this partial solution set consists of the eight points shown below as dots on the real number plane. Observing that these eight points seem to all be part of one straight line, we conclude that the graph of every other solution of X+2Y=10 will also be a point on the same straight line. The amazing thing is that, even though we cannot write the complete solution set, we can nevertheless draw the straight line as the complete graph. In general, the HP48SX draws a graph exactly as you would. It first computes a partial solution set, then graphs each solution in the partial solution set and finally draws the complete graph suggested by the points of the partial graph.



Systems of Open Sentences

A *system of open sentences* is a set containing two or more open sentences all connected by the word "and". Here are some examples:

Example 1. $5X+2Y=7$ and $3X-4Y=19$.

Example 2. $5X-1>3$ and $2X+5<84$.

Example 3. $X>2$ and $X<8$ and $Y>0$ and $Y<5X^2-2X-1$.

The *solution set* or *truth set* of a system of open sentences is the intersection of the solution sets of the individual open sentences. In other words, finding the solution set of a system of open sentences involves the following three steps:

Step 1. Find the solution set of each open sentence in the system.

Step 2. Make note of each object which is common to each of the individual solution sets found in Step 1.

Step 3. Form a set containing all the common solutions found in step 2.

To *solve* a system of open sentences means to find the solution set of the system. Here are some examples:

Example 1. Find the solution set of $X(X-2)(X+3)=0$ and $X(X+2)(X+3)=0$.
The solution set of $X(X-2)(X+3)=0$ is $\{-3, 0, 2\}$. The solution set of $X(X+2)(X+3)=0$ is $\{-3, -2, 0\}$. Since these two sets have only -3 and 0 in common, their intersection is $\{-3, 0\}$. This means that $\{-3, 0\}$ is the solution set for $X(X-2)(X+3)=0$ and $X(X+2)(X+3)=0$.

Example 2. Solve $X+5>9$ and $X-2<15$.
The solution set of $X+5>9$ is the set containing all real numbers greater than 4. The solution set of $X-2<15$ is the set containing all real numbers less than 17. Since these two solution sets have all real numbers greater than 4 and less than 17 in common, the intersection of these two solution sets and, therefore, the solution set of the system is the set containing all real numbers greater than 4 and less than 17.

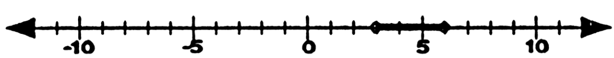
Example 3. Solve $X+2Y=10$ and $4X-3Y=7$.
The solution set of $X+2Y=10$ contains infinitely many ordered pairs of real numbers some of which are (8, 1), (6, 2), (4, 3), (2, 4), (0, 5) and (-2, 6). The solution set of $4X-3Y=7$ contains infinitely many ordered pairs of real numbers some of which

are (-2, -5), (1, -1), (4, 3), (7, 7), (10, 11) and (13, 15). If you were to examine the complete solution set for each open sentence, you'd find (4, 3) to be the only common solution, This means that the intersection of these two solutions sets and, therefore, the solution set of the system is { (4, 3) }. [This is read "The set containing the ordered pair 4, 3"]. Please note that even though we can't write the complete solution set for either one of the two open sentences in the system, we can write the complete solution set of the system. In most cases, we can solve a system of open sentences in two variables even though we can't solve either of the individual open sentences.

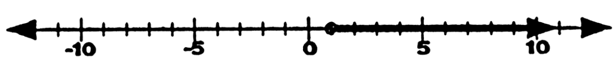
Example 4. Solve X+2=7 and X-3=8.
The solution set of X+2=7 is { 5 }. The solution set of X-3=8 is { 11 }. Since these two sets have no number in common, the intersection of these two sets and, therefore, the solution set of the system is Ø. [This is read "the empty set"]

To *graph* a system of open sentences means to graph the solution set of the system of open sentences. Here are some examples:

Example 1. Graph X+3<11 and X+2>1.
The solution set of X+3<11 is the set of all real numbers less than 8. The solution set of X+2>1 is the set of all real numbers greater than 3. The intersection of these two sets and, therefore, the solution set of the system is the set containing all real numbers greater than 3 and less than 6. Therefore the graph of the solution set of the system is:

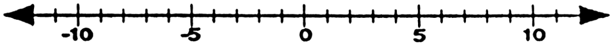


Example 2. Graph X+1>2 and X+4>1.
The solution set of X+1>2 is the set of all real numbers greater than 1. The solution set of X+4>1 is the set of all real numbers greater than -3. The intersection of these two sets and, therefore, the solution set of the system is the set containing all real numbers greater than 1. Therefore the graph of the solution set of the system is:

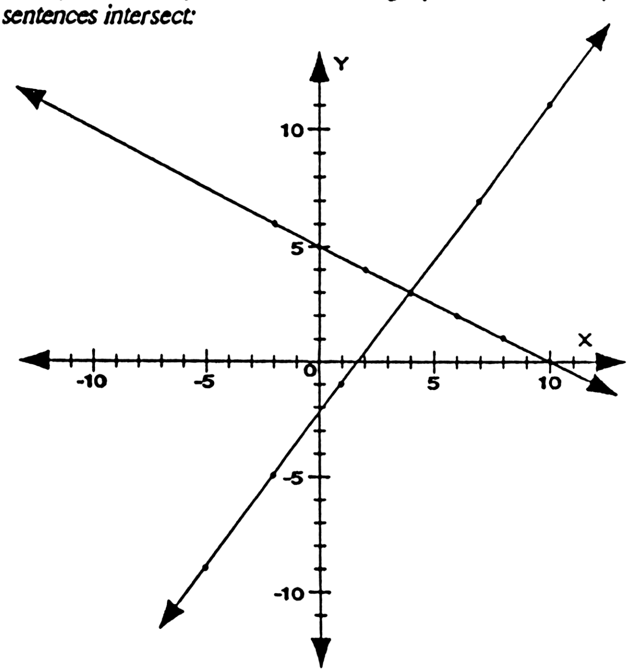


Example 3. Graph X+3>4 and X+2<1.
The solution set of X+3>4 is the set of all real numbers greater than 1. The solution set of X+2<1 is the set of all real numbers less than -1. Since these two set have no numbers in common, the intersection of these two solution sets and, therefore the

solution set of the system is Ø. Thus, the graph of this system is a number line with no points selected:



Example 4. Graph X+2Y=10 and 4X-3Y+7.
A partial solution set for X+2Y=10 is { (-2, 6), (0, 5), (2, 4), (4, 3), (6, 2), (8, 1), (10, 0), }. The graph of this partial solution set consists of the seven points shown below as dots on the real number plane. Observing that these seven points seem to all be part of one straight line, we conclude that the graph of every other solution of X+2Y=10 will also be a point on the same straight line. Likewise, a partial solution set for 4X-3Y=7 is { (-5, -9), (-2, -5), (1, -1), (4, 3), (7, 7), (10, 11) }. The graph of this partial solution set consists of the six points shown below as dots on the real number plane. Observing that these six points seem to all be part of one straight line, we conclude that the graph of every other solution of 4X-3Y=7 will also be a point on the same straight line. The intersection of the solution sets of the two open sentences in the system and, therefore, the solution set of the system is { (4, 3) }. The graph of this set is exactly the same point at which the graphs of the two open sentences intersect:



THE GRAPH MENU SCREEN

```
      GRAPH MENU
1 Polynomial 6 Systems
2 Inequality 7 Conic
3 Hyperbolic 8 Polar
4 Rational   9 Log
5 Parametric 0 Trig
  Pick a number
NEXT PREV KILL STAK MAIN OFF
```

Getting to the Graph Menu Screen

There are three ways to get to the Graph Menu Screen:

- Press 1 [*row 8, key 2*] on the Main Menu Screen.
- Press NEXT [*row 1, key 1*] on the Music Menu Screen.
- Press PREV [*row 1, key 2*] on the Loan Menu Screen.

The Graph Menu Screen Menu Bar

There are six keys active on the Graph Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the Loan Menu Screen.

PREV Press this key [*row 1, key 2*] to go to the Music Menu Screen.

KILL Press this key [*row 1, key 3*] to terminate E.Z. Math. When you again want to use E.Z. Math, repeat the start-up sequence described in Chapter 2 which is as follows: Press the ALPHA key [*row 6, key 1*] twice, press E [*row 1, key 5*] and Z [*row 5, key 3*], press the ENTER key [*row 5, key 1*] to get to the E.Z. Math Title Screen which, after about nine seconds, will be replaced by the E.Z. Math Main Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the E.Z. Math Graph Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Graph Menu Screen.

The Graph Menu Screen Options

There are ten options available on the Graph Menu Screen:

[1] **Polynomial.** Press this key [*row 8, key 2*] to go to the Polynomial Menu Screens from which you can select polynomial equations and functions, with and without absolute values for graphical analysis

[2] **Inequality.** Press this key [*row 8, key 3*] to go to the Inequality Menu Screens from which you can select inequalities and systems of inequalities for graphical analysis

[3] **Hyperbolic.** Press this key [*row 8, key 4*] to go to the Hyperbolic Menu Screens from which you can select equations and functions involving hyperbolic functions for graphical analysis

[4] **Rational.** Press this key [*row 7, key 2*] to go to the Rational Menu Screens from which you can select equations and functions involving rational functions for graphical analysis

[5] **Parametric.** Press this key [*row 7, key 3*] to go to the Parametric Menu Screens from which you can select equations expressed in parametric form for graphical analysis

[6] **Systems.** Press this key [*row 7, key 4*] to go to the Systems Menu Screens from which you can select systems of equations for graphical analysis

[7] **Conic.** Press this key [*row 6, key 2*] to go to the Conic Menu Screens from which you can select equations and functions involving the conic sections for graphical analysis

[8] **Polar.** Press this key [*row 6, key 3*] to go to the Polar Menu Screens from which you can select equations and functions expressed in polar form for graphical analysis

[9] **Log.** Press this key [*row 6, key 4*] to go to the Log Menu Screens from which you can select equations and functions involving logarithmic and exponential functions for graphical analysis

[0] **Trig.** Press this key [*row 9, key 2*] to go to the Trig Menu Screens from which you can select equations and functions involving trigonometric functions for graphical analysis

THE POLYNOMIAL MENU SCREENS

POLYNOMIAL MENU 1
1 Y=aX+b
2 Y=aX^2+bX+c
3 Y=aX^3+bX^2+cX+d
4 Y=aX^4+bX^3+cX^2+dX+e
5 Y=aX^5+. . .+dx^2+eX+f
Pick a number
NEXT | PREV | EXIT | STAK | MAIN | OFF

POLYNOMIAL MENU 2
1 Y=aX^6+bX^5+. . .+fX+g
2 Y=aX^7+bX^6+. . .+gX+h
3 Y=aX^8+bX^7+. . .+hX+i
4 Y=aX^9+bX^8+. . .+iX+j
5 Y=aX^10+bX^9+. . .+k
Pick a number
NEXT | PREV | EXIT | STAK | MAIN | OFF

POLYNOMIAL MENU 3
1 Y=(aX+b)(cX+d)
2 Y=(aX+b)(cX+d)(eX+f)
3 Y=(aX+b)·. . . .·(gX+h)
4 Y=(aX+b)·. . . .·(iX+j)
5 Y=(aX+b)·. . . .·(kX+l)
Pick a number
NEXT | PREV | EXIT | STAK | MAIN | OFF

POLYNOMIAL MENU 4
1 Y=ABS(aX+b)
2 Y=ABS(aX^2+bX+c)
3 Y=ABS(aX^3+bX^2+cX+d)
4 Y=ABS(aX^4+. . .+dX+e)
5 Y=ABS(aX^5+. . .+eX+f)
Pick a number
NEXT | PREV | EXIT | STAK | MAIN | OFF

```
   POLYNOMIAL MENU 5
1 Y=ABS(aX^6+...+fX+g)
2 Y=ABS(aX^7+...+gX+h)
3 Y=ABS(aX^8+...+hX+i)
4 Y=ABS(aX^9+...+iX+j)
5 Y=ABS(aX^10+..+jX+k)
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
   POLYNOMIAL MENU 6
1 Y=ABS[(aX+b)(cX+d)]
2 Y=ABS[(aX+b)(cX+d)
       ·(eX+f)]
3 Y=ABS[(aX+b)(cX+d)
       ·(eX+f)(gX+h)]
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

Getting to the Polynomial Menu Screens

Proceed as follows to get to any Polynomial Menu Screen:

- Press 1 [*row 8, key 2*] on the Graph Menu Screen to go to the Polynomial Menu 1 Screen.
- Press NEXT [*row 1, key 1*] to go to the Polynomial Menu 2 Screen and, from there, to the subsequent Polynomial Menu Screens.
- Press PREV [*row 1, key 2*] to go to the Polynomial Menu 6 Screen and, from there, to the previous Polynomial Menu Screens.

The Polynomial Menu Screen Menu Bar

There are six keys active on the Polynomial Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the next Polynomial Menu Screen. When you reach the Polynomial Menu 6 Screen, another press will take you to the Polynomial Menu 1 Screen.

PREV Press this key [*row 1, key 2*] to go to the previous Polynomial Menu Screen. When you reach the Polynomial Menu 1 Screen, another press will take you to the Polynomial Menu 6 Screen.

EXIT Press this key [*row 1, key 3*] to go to the Graph Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the same E.Z. Math Polynomial Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Polynomial Menu Screen.

Printing a Polynomial Menu Screen

Any Polynomial Menu Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.
2. Press the orange LEFT-SHIFT key [*row 7, key 1*].
3. Press the MTH key [*row 2, key 1*].
4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.
5. Press the ENTER key [*row 5, key 1*] to begin printing.
6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

The Polynomial Menu Screen Options

Each Polynomial Menu Screen displays several polynomial equation families, each of which contains two variables [X and Y] and several coefficients [a, b, c, and so on].

When you select one of the families, you'll be asked to enter a number to replace each coefficient. For each coefficient, type the desired number and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Graphics Screen 1 where the graph of the family member you selected will be drawn. Since there are infinitely many numbers from which to choose for each coefficient, each polynomial equation family has infinitely many family members.

Here's an example to make this clear. Suppose you press 2 [*row 8, key 3*] on the Polynomial Menu 3 Screen to select the polynomial equation family Y=(aX+b)(cX+d)(eX+f). After a short pause, you'll be asked to enter numbers for the coefficients a, b, c, d, e and f. In each case, type the desired numbers and press the ENTER key. Let's say you selected 5, 0, 1, -2, 1.6 and -47.3 to replace a, b, c, d, e and f. After a short pause, you'll be taken to the Graphics Screen 1 to see the graph of Y=5X(X-2)(1.6X-47.3) drawn.

As you can see, you need to make two choices before your graph can be drawn:

> **Step 1:** You must select a polynomial equation family from one of the Polynomial Menu Screens. In the above example, we selected Y=(aX+b)(cX+d)(eX+f) from the Polynomial Menu 3 Screen.
>
> **Step 2:** You must select a particular member of the family by entering the appropriate numbers to substitute for the coefficients when you are prompted to do so. In the above example, we selected Y=5X(X-2)(1.6X-47.3).

Once these two steps are done, you'll be then taken to the Graphics Screen 1 where you can see, study and analyze your graph.

Had you selected other numbers to replace the coefficients, you'd have gotten a different family member and therefore a different graph. It is very interesting to graph several members of the same family to see what features the various family members have in common.

Please note that pressing the ON key [*row 9, key 1*] instead of entering a number for one of the coefficients to abort the graphing process and return to the Polynomial Menu Screen from which you started. Also, pressing the ENTER key without selecting a number will take you on a side trip to the Decimal Place Menu from which you can specify the number of decimal places of accuracy you'd like displayed on the Graphics Screens, after which you'll be able to continue the number entering process.

THE INEQUALITY MENU SCREENS

```
INEQUALITY MENU 1
1 Y<aX+b
2 Y>aX+b
3 aX+bY<c
4 aX+bY>c
5 Y<aX^2+bX+c
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
INEQUALITY MENU 2
1 Y>aX^2+bX+c
2 Y<aX+b Y<cX=d
3 Y<aX+b Y>cX+d
4 Y>aX+b Y>cX+d
5 Y<aX+b Y<cX+d Y<eX+f
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
INEQUALITY MENU 3
1 Y<aX+b Y<cX+d Y>eX+f
2 Y<aX+b Y>cX+d Y>eX+f
3 Y>aX+b Y>cX+d Y>eX+f
4 Y<aX+b Y<cX+d
  Y<eX+f Y<gX+h
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
   INEQUALITY MENU 4
1 Y<aX+b Y<cX+d
  Y<eX+f Y>gX+h
2 Y<aX+b Y<cX+d
  Y>eX+f Y>gX+h
3 Y<aX+b Y>cX+d
  Y>eX+f Y>gX+h
NEXT PREV EXIT STAK MAIN OFF
```

```
INEQUALITY MENU 5
1 Y>aX+b Y>cX+d
  Y>eX+f Y>gX+h
2 (X-a)^2+(Y-b)^2<c^2
3 Y<aSIN(bX)
  Y>cCOS(dX)
4 aABS(bX)+cABS(dY)<e
NEXT PREV EXIT STAK MAIN OFF
```

Getting to the Inequality Menu Screens

Proceed as follows to get to any Inequality Menu Screen:

- Press [2] [*row 8, key 3*] on the Graph Menu Screen to go to the Inequality Menu 1 Screen.
- Press **NEXT** [*row 1, key 1*] to go to the Inequality Menu 2 Screen and, from there, to the subsequent Inequality Menu Screens.
- Press **PREV** [*row 1, key 2*] to go to the Inequality Menu 5 Screen and, from there, to the previous Inequality Menu Screens.

The Inequality Menu Screen Menu Bar

There are six keys active on the Inequality Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the next Inequality Menu Screen. When you reach the Inequality Menu 5 Screen, another press will take you to the Inequality Menu 1 Screen.

PREV Press this key [*row 1, key 2*] to go to the previous Inequality Menu Screen. When you reach the Inequality Menu 1 Screen, another press will take you to the Inequality Menu 5 Screen.

EXIT Press this key [*row 1, key 3*] to go to the Graph Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the same E.Z. Math Inequality Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Inequality Menu Screen.

Printing an Inequality Menu Screen

Any Inequality Menu Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.

2. Press the orange LEFT-SHIFT key [*row 7, key 1*].

3. Press the MTH key [*row 2, key 1*].

4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.

5. Press the ENTER key [*row 5, key 1*] to begin printing.

6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

The Inequality Menu Screen Options

Each Inequality Menu Screen displays several inequality families and inequality system families, each of which contains two variables [X and Y] and several coefficients [a, b, c, and so on].

When you select one of the families, you'll be asked to enter a number to replace each coefficient. For each coefficient, type the desired number and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Graphics Screen 1 where the graph of the family member you selected will be drawn. Since there are infinitely many numbers from which to choose for each coefficient, each inequality family has infinitely many family members.

Here's an example to make this clear. Suppose you press 5 [*row 7, key 3*] on the Inequality Menu 2 Screen to select the inequality system family Y<aX+b and Y<cX+d and Y<eX+f. After a short pause, you'll be asked to enter numbers for the coefficients a, b, c, d, e and f. In each case, type the desired numbers and press the ENTER key. Let's say you selected 4, -5, 1, 0, 3.7 and -9.2 to replace a, b, c, d, e and f. After a short pause, you'll be taken to the Graphics Screen 1 to see the graph of Y<4X-5 and Y<X and Y<3.7X-9.2 drawn.

As you can see, you need to make two choices before your graph can be drawn:

> **Step 1:** You must select an inequality family from one of the Inequality Menu Screens. In the above example, we selected Y<aX+b and Y<cX+d and Y<eX+f from the Inequality Menu 2 Screen.
>
> **Step 2:** You must select a particular member of the family by entering the appropriate numbers to substitute for the coefficients when you are prompted to do so. In the above example, we selected Y<4X-5 and Y<X and Y<3.7X-9.2.

Once these two steps are done, you'll be then taken to the Graphics Screen 1 where you can see, study and analyze your graph.

Had you selected other numbers to replace the coefficients, you'd have gotten a different family member and therefore a different graph. It is very interesting to graph several members of the same family to see what features the various family members have in common.

Please note that pressing the ON key [*row 9, key 1*] instead of entering a number for one of the coefficients to abort the graphing process and return to the Inequality Menu Screen from which you started. Also, pressing the ENTER key without selecting a number will take you on a side trip to the Decimal Place Menu from which you can specify the number of decimal places of accuracy you'd like displayed on the Graphics Screens, after which you'll be able to continue the number entering process.

THE HYPERBOLIC MENU SCREENS

```
   HYPERBOLIC MENU 1
1 Y=aSINH(bX)
2 Y=aCOSH(bX)
3 Y=aTANH(bX)
4 Y=aCOTH(bX)
5 Y=aSECH(bX)
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
   HYPERBOLIC MENU 2
1 Y=aCSCH(bX)
2 Y=aASINH(bX)
3 Y=aACOSH(bX)
4 Y=aATANH(bX)
5 Y=aACOTH(bX)
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
   HYPERBOLIC MENU 3
1 Y=aASECH(bX)
2 Y=aACSCH(bX)
3 Y=aSINH(bX)
    +cCOSH(dX)
4 Y=aSINH(bX²+cX+d)
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

Getting to the Hyperbolic Menu Screens

Proceed as follows to get to any Hyperbolic Menu Screen:

- Press 3 [*row 8, key 4*] on the Graph Menu Screen to go to the Hyperbolic Menu 1 Screen.
- Press NEXT [*row 1, key 1*] to go to the Hyperbolic Menu 2 Screen and, from there, to the subsequent Hyperbolic Menu Screens.
- Press PREV [*row 1, key 2*] to go to the Hyperbolic Menu 3 Screen and, from there, to the previous Hyperbolic Menu Screens.

The Hyperbolic Menu Screen Menu Bar

There are six keys active on each Hyperbolic Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the next Hyperbolic Menu Screen. When you reach the Hyperbolic Menu 3 Screen, another press will take you to the Hyperbolic Menu 1 Screen.

PREV Press this key [*row 1, key 2*] to go to the previous Hyperbolic Menu Screen. When you reach the Hyperbolic Menu 1 Screen, another press will take you to the Hyperbolic Menu 3 Screen.

EXIT Press this key [*row 1, key 3*] to go to the Graph Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the same E.Z. Math Hyperbolic Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Hyperbolic Menu Screen.

Printing a Hyperbolic Menu Screen

Any Hyperbolic Menu Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.
2. Press the orange LEFT-SHIFT key [*row 7, key 1*].
3. Press the MTH key [*row 2, key 1*].
4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.
5. Press the ENTER key [*row 5, key 1*] to begin printing.
6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

The Hyperbolic Menu Screen Options

Each Hyperbolic Menu Screen displays several hyperbolic equation families, each of which contains two variables [X and Y] and several coefficients [a, b, c, and so on].

When you select one of the families, you'll be asked to enter a number to replace each coefficient. For each coefficient, type the desired number and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Graphics Screen 1 where the graph of the family member you selected will be drawn. Since there are infinitely many numbers from which to choose for each coefficient, each hyperbolic equation family has infinitely many family members.

Here's an example to make this clear. Suppose you press 2 [*row 8, key 3*] on the Hyperbolic Menu 1 Screen to select the hyperbolic equation family Y=aCOSH(bX). After a short pause, you'll be asked to enter numbers for the coefficients a and b. In each case, type the desired numbers and press the ENTER key. Let's say you selected 2 and 3 to replace a and b. After a short pause, you'll be taken to the Graphics Screen 1 to see the graph of Y=2COSH(3X) drawn.

As you can see, you need to make two choices before your graph can be drawn:

> **Step 1:** You must select a polynomial equation family from one of the Hyperbolic Menu Screens. In the above example, we selected Y=aCOSH(bX) from the Hyperbolic Menu 1 Screen.
>
> **Step 2:** You must select a particular member of the family by entering the appropriate numbers to substitute for the coefficients when you are prompted to do so. In the above example, we selected Y=2COSH(3X).

Once these two steps are done, you'll be then taken to the Graphics Screen 1 where you can see, study and analyze your graph.

Had you selected other numbers to replace the coefficients, you'd have gotten a different family member and therefore a different graph. It is very interesting to graph several members of the same family to see what features the various family members have in common.

Please note that pressing the ON key [*row 9, key 1*] instead of entering a number for one of the coefficients to abort the graphing process and return to the same Hyperbolic Menu Screen from which you started. Also, pressing the ENTER key without selecting a number will take you on a side trip to the Decimal Place Menu from which you can specify the number of decimal places of accuracy you'd like displayed on the Graphics Screens, after which you'll be able to continue the number entering process.

THE RATIONAL MENU SCREENS

```
    RATIONAL MENU 1
1 Y=(aX+b)/(cX+d)
2 Y=(aX²+bX+c)
     /(dX²+eX+f)
3 Y=[(aX+b)(cX+d)]
     /[(eX+f)(gX+h)]
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
    RATIONAL MENU 2
1 Y=(aX2+bX+c)/(dX+e)
2 Y=[(aX+b)(cX2+dX+e)]
   /[(fX+g)(hX2+gX+h)]
3 Y=(aX+b)^c.(dX+e)^f
  /[(gX+h)^i.(jX+k)^l]
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
    RATIONAL MENU 3
1 Y=(aX+b)/(cX²+dX+e)
2 Y=[(aX²+bX+c)^d
        ·(eX²+fX+g)^h]
    /[(iX²+jX+k)^l
        ·(mX²+nX+o)^p]
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
    RATIONAL MENU 4
1 Y=(aX+b)^c/(dX+e)^f
2 Y=(aX²+bX+c)^d
     /(eX²+fX+g)^h
3 Y=(aX+b)(cX+d)(eX+f)
 /[(gX+h)(iX+j)(kX+l)]
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

Getting to the Rational Menu Screens

Proceed as follows to get to any Polynomial Menu Screen:

- Press [4] [*row 7, key 2*] on the Graph Menu Screen to go to the Rational Menu 1 Screen.
- Press NEXT [*row 1, key 1*] to go to the Rational Menu 2 Screen and, from there, to the subsequent Rational Menu Screens.
- Press PREV [*row 1, key 2*] to go to the Rational Menu 4 Screen and, from there, to the previous Rational Menu Screens.

The Rational Menu Screen Menu Bar

There are six keys active on the Rational Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the next Rational Menu Screen. When you reach the Rational Menu 4 Screen, another press will take you to the Rational Menu 1 Screen.

PREV Press this key [*row 1, key 2*] to go to the previous Rational Menu Screen. When you reach the Rational Menu 1 Screen, another press will take you to the Rational Menu 4 Screen.

EXIT Press this key [*row 1, key 3*] to go to the Graph Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the same E.Z. Math Rational Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Rational Menu Screen.

Printing a Rational Menu Screen

Any Rational Menu Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.
2. Press the orange LEFT-SHIFT key [*row 7, key 1*].
3. Press the MTH key [*row 2, key 1*].
4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.
5. Press the ENTER key [*row 5, key 1*] to begin printing.
6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

The Rational Menu Screen Options

Each Rational Menu Screen displays several rational equation families, each of which contains two variables [X and Y] and several coefficients [a, b, c, and so on].

When you select one of the families, you'll be asked to enter a number to replace each coefficient. For each coefficient, type the desired number and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Graphics Screen 1 where the graph of the family member you selected will be drawn. Since there are infinitely many numbers from which to choose for each coefficient, each rational equation family has infinitely many family members.

Here's an example to make this clear. Suppose you press 1 [*row 8, key 2*] on the Rational Menu 1 Screen to select the rational equation family Y=(aX+b)/(cX+d). After a short pause, you'll be asked to enter numbers for the coefficients a, b, c and d. In each case, type the desired numbers and press the ENTER key. Let's say you selected 8, -3, 4 and 1 to replace a, b, c and d. After a short pause, you'll be taken to the Graphics Screen 1 to see the graph of Y=(8X-3)/(4X-1) drawn.

As you can see, you need to make two choices before your graph can be drawn.

> **Step 1:** You must select a rational equation family from one of the Rational Menu Screens. In the above example, we selected Y=(aX+b)/(cX+d) from the Rational Menu 1 Screen.
>
> **Step 2:** You must select a particular member of the family by entering the appropriate numbers to substitute for the coefficients when you are prompted to do so. In the above example, we selected Y=(8X-3)/(4X-1).

Once these two steps are done, you'll be then taken to the Graphics Screen 1 where you can see, study and analyze your graph.

Had you selected other numbers to replace the coefficients, you'd have gotten a different family member and therefore a different graph. It is very interesting to graph several members of the same family to see what features the various family members have in common.

Please note that pressing the ON key [*row 9, key 1*] instead of entering a number for one of the coefficients to abort the graphing process and return to the Rational Menu Screen from which you started. Also, pressing the ENTER key without selecting a number will take you on a side trip to the Decimal Place Menu from which you can specify the number of decimal places of accuracy you'd like displayed on the Graphics Screens, after which you'll be able to continue the number entering process.

THE PARAMETRIC MENU SCREENS

```
   PARAMETRIC MENU 1
1 X=aT+b   Y=cT+d
2 X=aT²+bT+c
  Y=dT²+eT+f
3 X=aT³+bT²+cT+d
  Y=eT³+fT²+gT+h
  Pick a number
NEXT | PREV | EXIT | STAK | MAIN | OFF
```

```
   PARAMETRIC MENU 2
1 X=(aT+b)/(cT+d)
2 Y=(eT+f)/(gT+h)
3 X=[(aT+b)(cT+d)]
   /[(eT+f)(gT+h)]
  Y=[(iT+j)(kT+l)]
   /[(mT+n)(oT+p)]
NEXT | PREV. | EXIT | STAK | MAIN | OFF
```

```
   PARAMETRIC MENU 3
1 X=aSIN(T)   Y=bCOS(T)
2 X=aSIN(T)   Y=bTAN(T)
3 X=a+bSIN(cT)
  Y=d+eCOS(fT)
4 X=aTSIN(bT)
  Y=cTCOS(dT)
NEXT | PREV | EXIT | STAK | MAIN | OFF
```

```
   PARAMETRIC MENU 4
1 X=aSIN(bT)
  Y=cCOS(dT)
2 X=aSIN(T)+bCOS(T)
  Y=cSIN(T)+dCOS(T)
3 X=aSIN(bT)+cCOS(dT)
  Y=eSIN(fT)+gCOS(hT)
NEXT | PREV | EXIT | STAK | MAIN | OFF
```

```
   PARAMETRIC MENU 5
1 X=a[SIN(bT)]^c
  Y=d[COS(eT)]^f
2 X=a[T-SIN(T)]
  Y=a[1-COS(T)]
3 X=(aT²+bT+c)SIN(dT)
  Y=(eT²+fT+g)COS(hT)
NEXT PREV EXIT STAK MAIN OFF
```

```
   PARAMETRIC MENU 6
1 X=(aT³+bT²+cT+d)
   /(eT³+fT²+gT+h)
  Y=iT³+jT²+kT+l
2 X=(aT³+bT²+cT+d)
   /(eT³+fT²+gT+h)
  Y=i[SIN(jT)]^k
NEXT PREV EXIT STAK MAIN OFF
```

```
   PARAMETRIC MENU 7
1 X=aASIN(bT)
  Y=cACOS(dT)
2 X=(aX+b)/(cX+d)
     ·SIN(eT)
  Y=(fT+g)/(hT+i)
     ·COS(jT)
NEXT PREV EXIT STAK MAIN OFF
```

Getting to the Parametric Menu 1 Screen

There are three ways to get to the Parametric Menu 1 Screen:

- Press [5] [*row 7, key 3*] on the Graph Menu Screen to go to the Parametric Menu 1 Screen.
- Press NEXT [*row 1, key 1*] to go to the Parametric Menu 2 Screen and, from there, to the subsequent Parametric Menu Screens.
- Press PREV [*row 1, key 2*] to go to the Parametric Menu 7 Screen and, from there, to the previous Parametric Menu Screens.

The Parametric Menu Screen Menu Bar

There are six keys active on the Parametric Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the next Parametric Menu Screen. When you reach the Parametric Menu 7 Screen, another press will take you to the Parametric Menu 1 Screen.

PREV Press this key [*row 1, key 2*] to go to the previous Parametric Menu Screen. When you reach the Parametric Menu 1 Screen, another press will take you to the Parametric Menu 7 Screen.

EXIT Press this key [*row 1, key 3*] to go to the Graph Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the same E.Z. Math Parametric Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Parametric Menu Screen.

Printing a Polynomial Menu Screen

Any Polynomial Menu Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.

2. Press the orange LEFT-SHIFT key [*row 7, key 1*].

3. Press the MTH key [*row 2, key 1*].

4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.

5. Press the ENTER key [*row 5, key 1*] to begin printing.

6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

The Parametric Menu Screen Options

Each Parametric Menu Screen displays several parametric equation families, each of which contains three variables [T, X and Y] and several coefficients [a, b, c, and so on].

When you select one of the families, you'll be asked to enter a number to replace each coefficient. For each coefficient, type the desired number and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Graphics Screen 1 where the graph of the family member you selected will be drawn. Since there are infinitely many numbers from which to choose for each coefficient, each parametric equation family has infinitely many family members.

Here's an example to make this clear. Suppose you press 1 [*row 8, key 2*] on the Parametric Menu 4 Screen to select the parametric equation family X=aSIN(bT) and Y=cCOS(dT). After a short pause, you'll be asked to enter numbers for the coefficients a, b, c and d. In each case, type the desired numbers and press the ENTER key. Let's say you selected 3, 5, 4 and 0.2 to replace a, b, c and d. After a short pause, you'll be taken to the Graphics Screen 1 to see the graph of X=3SIN(5T) and Y=4COS(0.2T) drawn.

As you can see, you need to make two choices before your graph can be drawn:

> **Step 1:** You must select a parametric equation family from one of the Parametric Menu Screens. In the above example, we selected X=aSIN(bT) and Y=cCOS(dT) from the Parametric Menu 4 Screen.
>
> **Step 2:** You must select a particular member of the family by entering the appropriate numbers to substitute for the coefficients when you are prompted to do so. In the above example, we selected X=3SIN(5T) and Y=4COS(0.2T).

Once these two steps are done, you'll be then taken to the Graphics Screen 1 where you can see, study and analyze your graph.

Had you selected other numbers to replace the coefficients, you'd have gotten a different family member and therefore a different graph. It is very interesting to graph several members of the same family to see what features the various family members have in common.

Please note that pressing the ON key [*row 9, key 1*] instead of entering a number for one of the coefficients to abort the graphing process and return to the Parametric Menu Screen from which you started. Also, pressing the ENTER key without selecting a number will take you on a side trip to the Decimal Place Menu from which you can specify the number of decimal places of accuracy you'd like displayed on the Graphics Screens, after which you'll be able to continue the number entering process.

THE SYSTEMS MENU SCREENS

```
    SYSTEMS MENU 1
1 Y=aX+b    Y=cX+d
2 aX+bY=c   dX+eY=f
3 Y=aX+b    cX+dY=e
4 Y=aX²+bX+c Y=dX+e
5 Y=aX²+bX+c dX+eY=f
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
    SYSTEMS MENU 2
1 Y=aX²+bX+c
  Y=dX²+eX+f
2 Y=aX+b  (X-c)(Y-d)=e
3 aX+bY=c (X-d)(Y-e)=f
4 Y=aSINbX Y=cCOSdX
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
    SYSTEMS MENU 3
1 Y=aX³+bX²+cX+d
  Y=eX³+fX²+gX+h
2 Y=aTANbX   Y=cCOTdX
3 Y=aSINbX+cCOSdX
  Y=eSINfX+gCOShX
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
    SYSTEMS MENU 4
1 Y=aX+b
  Y=cSINdX+eCOSfX
2 Y=aX²+bX+c
  Y=dSINeX+fCOSgX
3 Y=aX+b   Y=cASINdX
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

Getting to the Systems Menu Screens

Proceed as follows to get to any Systems Menu Screen:

- Press [6] [*row 7, key 3*] on the Graph Menu Screen to go to the Systems Menu 1 Screen.
- Press NEXT [*row 1, key 1*] to go to the Systems Menu 2 Screen and, from there, to the subsequent Systems Menu Screens.
- Press PREV [*row 1, key 2*] to go to the Systems Menu 4 Screen and, from there, to the previous Systems Menu Screens.

The Systems Menu Screen Menu Bar

There are six keys active on the Systems Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the next Systems Menu Screen. When you reach the Systems Menu 4 Screen, another press will take you to the Systems Menu 1 Screen.

PREV Press this key [*row 1, key 2*] to go to the previous Systems Menu Screen. When you reach the Systems Menu 1 Screen, another press will take you to the Systems Menu 4 Screen.

EXIT Press this key [*row 1, key 3*] to go to the Graph Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the same E.Z. Math Systems Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Systems Menu Screen.

Printing a Systems Menu Screen

Any Systems Menu Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.

2. Press the orange LEFT-SHIFT key [*row 7, key 1*].

3. Press the MTH key [*row 2, key 1*].

4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.

5. Press the ENTER key [*row 5, key 1*] to begin printing.

6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

The Systems Menu Screen Options

Each Systems Menu Screen displays several equation system families, each of which contains two variables [X and Y] and several coefficients [a, b, c, and so on].

When you select one of the families, you'll be asked to enter a number to replace each coefficient. For each coefficient, type the desired number and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Graphics Screen 1 where the graph of the family member you selected will be drawn. Since there are infinitely many numbers from which to choose for each coefficient, each equation system family has infinitely many family members.

Here's an example to make this clear. Suppose you press 2 [row 8, key 3] on the Systems Menu 1 Screen to select the equation system family aX+bY=c and dX+eY=f. After a short pause, you'll be asked to enter numbers for the coefficients a, b, c, d, e and f. In each case, type the desired numbers and press the ENTER key. Let's say you selected 2, -3, 5, -4, 6 and 1 to replace a, b, c, d, e and f. After a short pause, you'll be taken to the Graphics Screen 1 to see the graph of 2X-3Y=5 and -4X+6Y=1 drawn.

As you can see, you need to make two choices before your graph can be drawn:

> **Step 1:** You must select an equation system family from one of the Systems Menu Screens. In the above example, we selected aX+bY=c and dX+eY=f from the Systems Menu 1 Screen.
>
> **Step 2:** You must select a particular member of the family by entering the appropriate numbers to substitute for the coefficients when you are prompted to do so. In the above example, we selected 2X-3Y=5 and -4X+6Y=1.

Once these two steps are done, you'll be then taken to the Graphics Screen 1 where you can see, study and analyze your graph.

Had you selected other numbers to replace the coefficients, you'd have gotten a different family member and therefore a different graph. It is very interesting to graph several members of the same family to see what features the various family members have in common.

Please note that pressing the ON key [*row 9, key 1*] instead of entering a number for one of the coefficients to abort the graphing process and return to the Systems Menu Screen from which you started. Also, pressing the ENTER key without selecting a number will take you on a side trip to the Decimal Place Menu from which you can specify the number of decimal places of accuracy you'd like displayed on the Graphics Screen 1, after which you'll be able to continue the number entering process.

THE CONIC MENU SCREENS

CONIC MENU 1

1 $X^2+Y^2=a^2$

2 $(X-a)^2+(Y-b)^2=c^2$

3 $X^2/a^2+Y^2/b^2=1$

4 $(X-a)^2/c^2+(Y-b)^2/d^2$
$=1$

Pick a number

NEXT	PREV	EXIT	STAK	MAIN	OFF

CONIC MENU 2

1 $(X-a)^2=2c(Y-b)$

2 $(Y-a)^2=2c(X-b)$

3 $Y=aX^2+bX+c$

4 $X=aY^2+bY+c$

5 $X^2/a^2-Y^2/b^2=1$

Pick a number

NEXT	PREV	EXIT	STAK	MAIN	OFF

CONIC MENU 3

1 $Y^2/a^2-X^2/b^2=1$

2 $(X-a)^2/c^2-(Y-b)^2/d^2$
$=1$

3 $(Y-a)^2/c^2-(X-b)^2/d^2$
$=1$

Pick a number

NEXT	PREV	EXIT	STAK	MAIN	OFF

CONIC MENU 4

1 $XY=a$

2 $(X-a)(Y-b)=c$

3 $aX^2+bXY+cY^2+dX+eY$
$+f=0$

Pick a number

NEXT	PREV	EXIT	STAK	MAIN	OFF

Getting to the Conic Menu Screens

Proceed as follows to get to any Conic Menu Screen:

- Press [7] [*row 6, key 2*] on the Graph Menu Screen to go to the Conic Menu 1 Screen.
- Press NEXT [*row 1, key 1*] to go to the Conic Menu 2 Screen and, from there, to the subsequent Conic Menu Screens.
- Press PREV [*row 1, key 2*] to go to the Conic Menu 4 Screen and, from there, to the previous Conic Menu Screens.

The Conic Menu Screen Menu Bar

There are six keys active on the Conic Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the next Conic Menu Screen. When you reach the Conic Menu 4 Screen, another press will take you to the Conic Menu 1 Screen.

PREV Press this key [*row 1, key 2*] to go to the previous Conic Menu Screen. When you reach the Conic Menu 1 Screen, another press will take you to the Conic Menu 4 Screen.

EXIT Press this key [*row 1, key 3*] to go to the Graph Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the same E.Z. Math Conic Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Conic Menu Screen.

Printing a Conic Menu Screen

Any Conic Menu Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.

2. Press the orange LEFT-SHIFT key [*row 7, key 1*].

3. Press the MTH key [*row 2, key 1*].

4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.

5. Press the ENTER key [*row 5, key 1*] to begin printing.

6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

The Conic Menu Screen Options

Each Conic Menu Screen displays several conic equation families, each of which contains two variables [X and Y] and several coefficients [a, b, c, and so on].

When you select one of the families, you'll be asked to enter a number to replace each coefficient. For each coefficient, type the desired number and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Graphics Screen 1 where the graph of the family member you selected will be drawn. Since there are infinitely many numbers from which to choose for each coefficient, each conic equation family has infinitely many family members.

Here's an example to make this clear. Suppose you press 2 [*row 8, key 3*] on the Conic Menu 4 Screen to select the conic equation family (X-a)(Y-b)=c. After a short pause, you'll be asked to enter numbers for the coefficients a, b and c. In each case, type the desired numbers and press the ENTER key. Let's say you selected 5, -3 and 2 to replace a, b and c. After a short pause, you'll be taken to the Graphics Screen 1 to see the graph of (X-5)(X+3)=2 drawn.

As you can see, you need to make two choices before your graph can be drawn:

> **Step 1:** You must select a conic equation family from one of the Conic Menu Screens. In the above example, we selected (X-a)(Y-b)=c from the Conic Menu 4 Screen.
>
> **Step 2:** You must select a particular member of the family by entering the appropriate numbers to substitute for the coefficients when you are prompted to do so. In the above example, we selected (X-5)(X+3)=2.

Once these two steps are done, you'll be then taken to the Graphics Screen 1 where you can see, study and analyze your graph.

Had you selected other numbers to replace the coefficients, you'd have gotten a different family member and therefore a different graph. It is very interesting to graph several members of the same family to see what features the various family members have in common.

Please note that pressing the ON key [*row 9, key 1*] instead of entering a number for one of the coefficients to abort the graphing process and return to the Conic Menu Screen from which you started. Also, pressing the ENTER key without selecting a number will take you on a side trip to the Decimal Place Menu from which you can specify the number of decimal places of accuracy you'd like displayed on the Graphics Screens, after which you'll be able to continue the number entering process.

THE POLAR MENU SCREENS

```
     POLAR MENU 1
1 R=a
2 R=2aSIN(θ)
3 R=2aCOS(θ)
4 R=a/[1-bSIN(θ)]
5 R=a/[1-bCOS(θ)]
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
     POLAR MENU 2
1 R=a/[bSIN(θ)-cCOS(θ)
2 R=aSIN(bθ)-cCOS(dθ)
3 R=aSIN(bθ)
4 R=aCOS(bθ)
5 R²=a²SIN(bθ)
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
     POLAR MENU 3
1 R=a[1-COS(θ)]
2 R=a-bCOS(θ)
3 R=a[1-COS(bθ)]
4 R=a-bSIN(cθ)
5 R=a-bCOS(cθ)
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
     POLAR MENU 4
1 R=aθ
2 R=e^(aθ)
3 R=a/θ
4 R=[aSIN(bθ)+cCOS(dθ)
  /[eSIN(fθ)+gCOS(hθ)]
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
POLAR MENU 5
1 R=aθ+b
2 R=aθ²+bθ+c
3 R=aθ³+bθ²+cθ+d
4 R=(aθ+b)(cθ+d)
5 R=(aθ+b)/(cθ+d)
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

Getting to the Polar Menu Screens

Proceed as follows to get to any Polar Menu Screen:

- Press [8] [*row 6, key 3*] on the Graph Menu Screen to go to the Polar Menu 1 Screen.
- Press NEXT [*row 1, key 1*] to go to the Polar Menu 2 Screen and, from there, to the subsequent Polar Menu Screens.
- Press PREV [*row 1, key 2*] to go to the Polar Menu 5 Screen and, from there, to the previous Polar Menu Screens.

The Polar Menu Screen Menu Bar

There are six keys active on the Polar Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the next Polar Menu Screen. When you reach the Polar Menu 5 Screen, another press will take you to the Polar Menu 1 Screen.

PREV Press this key [*row 1, key 2*] to go to the previous Polar Menu Screen. When you reach the Polar Menu 1 Screen, another press will take you to the Polar Menu 5 Screen.

EXIT Press this key [*row 1, key 3*] to go to the Graph Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the same E.Z. Math Polar Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Polar Menu Screen.

Printing a Polar Menu Screen

Any Polar Menu Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.
2. Press the orange LEFT-SHIFT key [*row 7, key 1*].
3. Press the MTH key [*row 2, key 1*].
4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.
5. Press the ENTER key [*row 5, key 1*] to begin printing.
6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

The Polar Menu Screen Options

Each Polar Menu Screen displays several polar equation families, each of which contains two variables [Θ and R] and several coefficients [a, b, c, and so on].

When you select one of the families, you'll be asked to enter a number to replace each coefficient. For each coefficient, type the desired number and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Graphics Screen 1 where the graph of the family member you selected will be drawn. Since there are infinitely many numbers from which to choose for each coefficient, each polar equation family has infinitely many family members.

Here's an example to make this clear. Suppose you press 2 [row 8, key 3] on the Polar Menu 3 Screen to select the polar equation family R=a-bCOS(Θ). After a short pause, you'll be asked to enter numbers for the coefficients a and b. In each case, type the desired numbers and press the ENTER key. Let's say you selected 5 and -2 to replace a and b. After a short pause, you'll be taken to the Graphics Screen 1 to see the graph of R=5-2COS(Θ) drawn.

As you can see, you need to make two choices before your graph can be drawn.

> **Step 1:** You must select a polynomial equation family from one of the Polar Menu Screens. In the above example, we selected R=a-bCOS(Θ) from the Polar Menu 3 Screen.
>
> **Step 2:** You must select a particular member of the family by entering the appropriate numbers to substitute for the coefficients when you are prompted to do so. In the above example, we selected R=5-2COS(Θ).

Once these two steps are done, you'll be then taken to the Graphics Screen 1 where you can see, study and analyze your graph.

Had you selected other numbers to replace the coefficients, you'd have gotten a different family. member and therefore a different graph. It is very interesting to graph several members of the same family to see what features the various family members have in common.

Please note that pressing the ON key [*row 9, key 1*] instead of entering a number for one of the coefficients to abort the graphing process and return to the Polar Menu Screen from which you started. Also, pressing the ENTER key without selecting a number will take you on a side trip to the Decimal Place Menu from which you can specify the number of decimal places of accuracy you'd like displayed on the Graphics Screens, after which you'll be able to continue the number entering process.

THE LOG MENU SCREENS

LOG MENU 1
1 Y=aLOG(bX)
2 Y=aLN(bX)
3 Y=aLOG(bX+c)
4 $Y=aLOG(bX^2+cX+d)$
5 $Y=aLOG(bX^3+cX^2+dX+e)$
Pick a number
NEXT | PREV | EXIT | STAK | MAIN | OFF

LOG MENU 2
1 Y=aLOG[SIN(bX)]
2 Y=aSIN[LOG(bX)]
3 Y=a^(bX+c)
4 Y=a^(bX^2+cX+d)
5 Y=a^(bX^3+cX^2+dX+e)
Pick a number
NEXT | PREV | EXIT | STAK | MAIN | OFF

LOG MENU 3
1 Y=X^(aX+b)
2 Y=X^(aX^2+bX+c)
3 Y=X^{aSINbX)
4 Y=X^{aSINbX+cCOSdX)
5 Y=X^[(aX+b)/(cX+d)]
Pick a number
NEXT | PREV | EXIT | STAK | MAIN | OFF

Getting to the Log Menu Screens

Proceed as follows to get to any Log Menu Screen:

- Press [9] [*row 6, key 4*] on the Graph Menu Screen to go to the Log Menu 1 Screen.
- Press NEXT [*row 1, key 1*] to go to the Log Menu 2 Screen and, from there, to the subsequent Log Menu Screens.
- Press PREV [*row 1, key 2*] to go to the Log Menu 3 Screen and, from there, to the previous Log Menu Screens.

The Log Menu Screen Menu Bar

There are six keys active on the Log Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the next Log Menu Screen. When you reach the Log Menu 3 Screen, another press will take you to the Log Menu 1 Screen.

PREV Press this key [*row 1, key 2*] to go to the previous Log Menu Screen. When you reach the Log Menu 1 Screen, another press will take you to the Log Menu 3 Screen.

EXIT Press this key [*row 1, key 3*] to go to the Graph Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the same E.Z. Math Log Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Log Menu Screen.

Printing a Log Menu Screen

Any Log Menu Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.
2. Press the orange LEFT-SHIFT key [*row 7, key 1*].
3. Press the MTH key [*row 2, key 1*].
4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.
5. Press the ENTER key [*row 5, key 1*] to begin printing.
6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

The Log Menu Screen Options

Each Log Menu Screen displays several logarithmic and exponential equation families, each of which contains two variables [X and Y] and several coefficients [a, b, c, and so on].

When you select one of the families, you'll be asked to enter a number to replace each coefficient. For each coefficient, type the desired number and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Graphics Screen 1 where the graph of the family member you selected will be drawn. Since there are infinitely many numbers from which to choose for each coefficient, each logarithmic and exponential equation family has infinitely many family members.

Here's an example to make this clear. Suppose you press 3 [*row 8, key 4*] on the Log Menu 1 Screen to select the logarithmic equation family Y=aLOG(bX+c). After a short pause, you'll be asked to enter numbers for the coefficients a, b and c. In each case, type the desired numbers and press the ENTER key. Let's say you selected 2, 5 and -6 to replace a, b and c. After a short pause, you'll be taken to the Graphics Screen 1 to see the graph of Y=2LOG(5X-6) drawn.

As you can see, you need to make two choices before your graph can be drawn:

> **Step 1:** You must select a logarithmic or exponential equation family from one of the Log Menu Screens. In the above example, we selected Y=aLOG(bX+c) from the Log Menu 1 Screen.
>
> **Step 2:** You must select a particular member of the family by entering the appropriate numbers to substitute for the coefficients when you are prompted to do so. In the above example, we selected Y=2LOG(5X-6).

Once these two steps are done, you'll be then taken to the Graphics Screen 1 where you can see, study and analyze your graph.

Had you selected other numbers to replace the coefficients, you'd have gotten a different family member and therefore a different graph. It is very interesting to graph several members of the same family to see what features the various family members have in common.

Please note that pressing the ON key [*row 9, key 1*] instead of entering a number for one of the coefficients to abort the graphing process and return to the Log Menu Screen from which you started. Also, pressing the ENTER key without selecting a number will take you on a side trip to the Decimal Place Menu from which you can specify the number of decimal places of accuracy you'd like displayed on the Graphics Screens, after which you'll be able to continue the number entering process.

THE TRIG MENU SCREENS

```
      TRIG MENU 1
1 Y=aSIN(bX)
2 Y=aCOS(bX)
3 Y=aTAN(bX)
4 Y=aCOT(bX)
5 Y=aSEC(bX)
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
      TRIG MENU 2
1 Y=aCSC(bX)
2 Y=aASIN(bX)
3 Y=aACOS(bX)
4 Y=aATAN(bX)
5 Y=aACOT(bX)
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
      TRIG MENU 3
1 Y=aSINbX+cCOSdX
2 Y=aTANbX+cCOTdX
3 Y=aSECbX+cCSCdX
4 Y=(aSINbX+cCOSdX)
   /(eSINfX+gCOShX)
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
      TRIG MENU 4
1 Y=bSINaX+c
2 Y=b(SINaX)²+cSINaX+d
3 Y=b(SINaX)³
   +c(SINaX)²+dSINaX+e
4 Y=SIN(aX+b)
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

```
       TRIG MENU 5
1  Y=SIN(aX²+bX+c)
2  Y=SIN(aX³+bX²+cX+d)
3  Y=a(SINdX)²+bSINdX
     ·COSeX+c(COSeX)²
4  Y=(aSINbX+cCOSdX)^e
   Pick a number
NEXT | PREV | EXIT | STAK | MAIN | OFF
```

```
       TRIG MENU 6
1  Y=aSIN(X²)+bSIN(X)+c
2  Y=aSIN(X³)+bSIN(X²)
     +cSIN(X)+d
3  Y=aSIN(X²)+bSIN(X)
     ·COS(X)+cCOS(X²)
   Pick a number
NEXT | PREV | EXIT | STAK | MAIN | OFF
```

Getting to the Trig Menu Screens

Proceed as follows to get to any Trig Menu Screen:

- Press [0] *[row 9, key 2]* on the Graph Menu Screen to go to the Trig Menu 1 Screen.
- Press **NEXT** *[row 1, key 1]* to go to the Trig Menu 2 Screen and, from there, to the subsequent Trig Menu Screens.
- Press **PREV** *[row 1, key 2]* to go to the Trig Menu 6 Screen and, from there, to the previous Trig Menu Screens.

The Trig Menu Screen Menu Bar

There are six keys active on the Trig Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the next Trig Menu Screen. When you reach the Trig Menu 6 Screen, another press will take you to the Trig Menu 1 Screen.

PREV Press this key [*row 1, key 2*] to go to the previous Trig Menu Screen. When you reach the Trig Menu 1 Screen, another press will take you to the Trig Menu 6 Screen.

EXIT Press this key [*row 1, key 3*] to go to the Graph Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the same E.Z. Math Trig Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Trig Menu Screen.

Printing a Trig Menu Screen

Any Trig Menu Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.
2. Press the orange LEFT-SHIFT key [*row 7, key 1*].
3. Press the MTH key [*row 2, key 1*].
4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.
5. Press the ENTER key [row 5, key 1] to begin printing.
6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

The Trig Menu Screen Options

Each Trig Menu Screen displays several trigonometric equation families, each of which contains two variables [X and Y] and several coefficients [a, b, c, and so on].

When you select one of the families, you'll be asked to enter a number to replace each coefficient. For each coefficient, type the desired number and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Graphic Screen 1 where the graph of the family member you selected will be drawn. Since there are infinitely many numbers from which to choose for each coefficient, each trigonometric equation family has infinitely many family members.

Here's an example to make this clear. Suppose you press 1 [*row 8, key 2*] on the Trig Menu 3 Screen to select the trigonometric equation family Y=aSINbX+cCOSdX. After a short pause, you'll be asked to enter numbers for the coefficients a, b, c and d. In each case, type the desired numbers and press the ENTER key. Let's say you selected 2, 4, -3 and 1.5 to replace a, b, c and d. After a short pause, you'll be taken to the Graphic Screen 1 to see the graph of Y=2SIN4X-3COS1.5X drawn.

As you can see, you need to make two choices before your graph can be drawn:

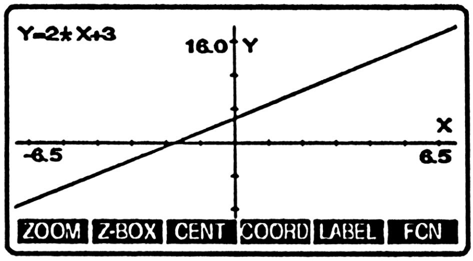
> **Step 1:** You must select a polynomial equation family from one of the Trig Menu Screens. In the above example, we selected Y=aSINbX+cCOSdX from the Trig Menu 3 Screen.
>
> **Step 2:** You must select a particular member of the family by entering the appropriate numbers to substitute for the coefficients when you are prompted to do so. In the above example, we selected Y=2SIN4X-3COS1.5X.

Once these two steps are done, you'll be then taken to the Graphics Screen 1 where you can see, study and analyze your graph.

Had you selected other numbers to replace the coefficients, you'd have gotten a different family member and therefore a different graph. It is very interesting to graph several members of the same family to see what features the various family members have in common.

Please note that pressing the ON key [*row 9, key 1*] instead of entering a number for one of the coefficients to abort the graphing process and return to the Trig Menu Screen from which you started. Also, pressing the ENTER key without selecting a number will take you on a side trip to the Decimal Place Menu from which you can specify the number of decimal places of accuracy you'd like displayed on the Graphics Screens, after which you'll be able to continue the number entering process.

THE GRAPHICS SCREEN 1



Getting to the Graphics Screen 1

There are three ways to get to the Graphics Screen 1:

- You'll get to this screen automatically anytime you select a graph to be drawn.
- Press NXT [*row 2, key 6*] on the Graphics Screen 3.
- Press BACK [*row 1, key 1*] on the Graphic Options Menu Screen.

The Graphics Screen 1 Menu Bar

There are six keys active on the Graphics Screen 1 Menu Bar:

ZOOM Press this key [*row 1, key 1*] to go to the Graphics Zoom Screen where you can enlarge or reduce the size of your graph.

Z-BOX Press this key [*row 1, key 2*] to enlarge a rectangular portion of the graphics screen. Move the cursor to one corner of the rectangle and press the MULTIPLICATION key [*row 7, key 5*] to set the mark. Move the cursor to the diagonally opposite corner of the rectangle and press this key. The portion of your graph within the selected rectangular region will be redrawn to fill the entire display screen allowing you to examine the selected portion of your graph with much greater detail.

CENT Press this key [*row 1, key 3*] to redraw the graph, making the current cursor position the center of the new screen display. Move the cursor to the desired center position before pressing this key.

COORD Press this key [*row 1, key 4*] to see the coordinates of the cursor. Pressing the ADDITION key [*row 9, key 5*] has exactly the same effect as pressing this key. The cursor coordinates will be displayed at the lower left corner of the screen display in place of the menu bar. As you move the cursor around the screen, the cursor coordinate display will be constantly updated to show the new coordinates. Press any key in row 1, the NXT key [*row 2, key 6*] or the ADDITION key to quit the cursor coordinates display and redisplay the menu bar.

LABEL Press this key [*row 1, key 5*] to label the two ends of each of the coordinate axes. This is especially useful after you've redrawn your graph with a new center or with a change in size.

FCN Press this key [*row 1, key 6*] to go to the Graphics Function Screens where you can further analyze your graph. Due to built-in limitations of the HP48SX, the Graphics Function Screens are not available for inequalities, polar equations, parametric equations and most conics.

Some Graphics Screen Basics

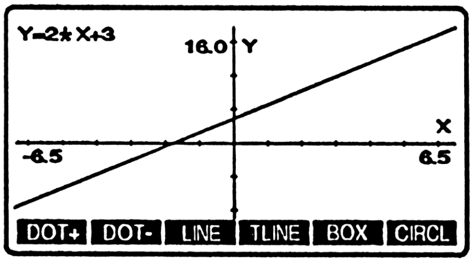
If you look closely at the HP48SX graphics screen, perhaps with the help of a magnifying glass, you'll see lots of tiny square dots, 8,384 dots to be exact, arranged in 64 rows with 131 dots per row. These dots are the basic units or elements which are used to construct every picture you see displayed on the graphics screen. For this reason, these tiny dots are called *picture elements* or *pixels* for short. For a picture, graph, word or open sentence to be visible, the appropriate pixels must be turned on, that is, changed from the normal white background color to blue.

It is frequently necessary to point to or to select various pixels. For this, we use the graphics cursor which looks like a small plus sign. You may think of the graphics cursor as a small finger pointing at the graphics screen, as the "X" on a treasure map or as a bull's eye.

The cursor keys are used to move the graphics cursor around the screen. Use the LEFT CURSOR key [*row 3, key 4*] to move the graphics cursor to the left. Use the RIGHT CURSOR key [*row 3, key 6*] to move the graphics cursor to the right. Use the UP CURSOR key [*row 2, key 5*] to move the graphics cursor up. Use the DOWN CURSOR key [*row 3, key 5*] to move the graphics cursor down.

Each press of a cursor key moves the graphics cursor one pixel in the direction indicated by the key. Keeping a cursor key depressed continually moves the cursor in the direction indicated by the key. If you press the blue RIGHT SHIFT key [*row 8, key 1*] before pressing a cursor key, the graphics cursor will move to the end of the screen in the direction indicated by the key.

THE GRAPHICS SCREEN 2



Getting to the Graphics Screen 2

There is one way to get to the Graphics Screen 2:

- Press NXT [*row 2, key 6*] on the Graphics Screen 1.

The Graphics Screen 2 Menu Bar

There are six keys active on the Graphics Screen 2 Menu Bar:

DOT+ Press this key [*row 1, key 1*] to activate line-drawing mode. In this mode, the cursor turns on each pixel it touches. This means that, as you move the cursor around the screen, each pixel touched will turn blue. By so moving the cursor, you can add lines and curves to your graph. Press this key a second time to turn off line-drawing mode.

DOT- Press this key [*row 1, key 2*] to activate line-erasing mode. In this mode, the cursor turns off each pixel it touches. This means that, as you move the cursor around the screen, each pixel touched will turn white. By so moving the cursor, various parts of your graph can be erased. Press this key a second time to turn off line-drawing mode.

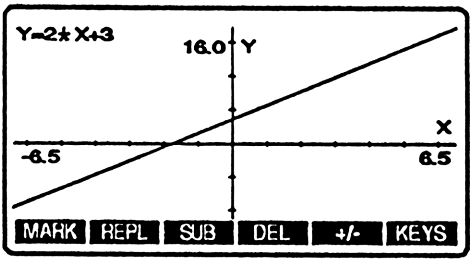
LINE Press this key [*row 1, key 3*] to draw a zigzag line. A *zigzag line* is a set of line segments in which the end of one line segment is the beginning of the next. Move the cursor to the beginning of the zigzag line and press the MULTIPLICATION key [*row 7, key 5*] to set the mark. Move the cursor to the end of the first line segment and press this key. The first line segment will be drawn and the mark will jump to the current cursor position which is also the end of the first line segment. Move the cursor to the end of the second line segment and press this key. The second line segment will be drawn and the mark will jump to the current cursor position which is also the end of the second line segment. Move the cursor to the end of the third line segment and press this key. The third line segment will be drawn and the mark will jump to the current cursor position which is also the end of the third line segment. Repeat until the entire zigzag line is drawn. Then press the MULTIPLICATION key one more time to erase the mark.

T-LINE Press this key [*row 1, key 4*] to draw a set of line segments all starting from the same point. Move the cursor to the point from which you want all the line segments to begin and press the MULTIPLICATION key [*row 7, key 5*] to set the mark. Move the cursor to the end of the first line segment and press this key. The first line segment will be drawn. Move the cursor to the end of the second line segment and press this key. The second line segment will be drawn. Repeat this procedure for each of the other line segments you wish to draw. When all the line segments are drawn, press the MULTIPLICATION key two more times to erase the mark.

BOX Press this key [*row 1, key 5*] to draw a rectangle. Move the cursor to one corner of the rectangle and press the MULTIPLICATION key [*row 7, key 5*] to set the mark. Move the cursor to the diagonally opposite corner of the rectangle and press this key to draw the rectangle. Press the MULTIPLICATION key two more times to erase the mark.

CIRCL Press this key [*row 1, key 6*] to draw a circle. Move the cursor to the point you want to be the center of the circle and press the MULTIPLICATION key [*row 7, key 5*] to set the mark. Then move the cursor to any point of the circle and press this key. The circle will be drawn through the current cursor position with the mark as the center. Press the MULTIPLICATION key two more times to erase the mark.

THE GRAPHICS SCREEN 3



Getting to the Graphics Screen 3

There is one way to get to the Graphics Screen 3:

- Press NXT [*row 2, key 6*] on the Graphics Screen 2.

The Graphics Screen 3 Menu Bar

There are six keys active on the Graphics Screen 3 Menu Bar:

MARK Press this key [*row 1, key 1*] to set the mark on the graphics screen. The mark, which looks like the letter X, is used in conjunction with several menu buttons which alter the graphics screen in various ways. If no mark exists on the graphics screen, press this key to create the mark at the current cursor position. If the mark already exists at the cursor position, press this key to erase it. If the mark already exists at a point other than at the current cursor position, use this key to move the mark to the current cursor position. The MULTIPLICATION key [*row 7, key 5*] functions in exactly the same way as this key.

REPL Do not press this key [*row 1, key 2*]. It has no function when used in E.Z. Math.

SUB Do not press this key [*row 1, key 3*]. It has no function when used in E.Z. Math..

DEL Press this key [*row 1, key 4*] to erase a rectangular portion of the graphics screen. Move the cursor to one corner of the rectangle and press the MULTIPLICATION key [*row 7, key 5*] to set the mark. Move the cursor to the diagonally opposite corner of the rectangle and press this key. Everything within the rectangle will be erased.

Press the MULTIPLICATION key two more times to erase the mark.

+/- Press this key [*row 1, key 5*] to change the graphics cursor style. Normally, the graphics cursor looks like a plus sign [+] which remains a solid dark blue color at all times. Press this key to change the cursor style into one in which the cursor is blue over a white background and white over a blue background. Pressing this key again changes the cursor style back to the one in which the cursor is always a solid blue. The +/- key [*row 5, key 2*] functions in exactly the same way as this key.

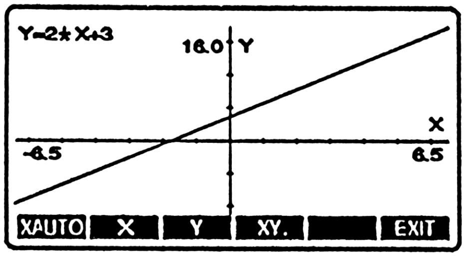
KEYS Press this key [*row 1, key 6*] to temporarily erase the menu bar so that you can see more of your graph. Press any key in row 1 or the NXT key [*row 2, key 6*] to make the menu bar reappear. The SUBTRACTION key [*row 8, key 5*] functions in exactly the same way as this key.

Some More Graphics Screen Basics

If you look at the upper left hand side of the graphics screen when a graph is being drawn, you'll see which equation, inequality, or system is being graphed. It may, however, look a bit strange because, while the HP48SX represents addition and subtraction in the usual way, it uses an asterisk [*] for multiplication, a slash [/] for division and fractions and a caret [^] for power. So, when you graph $Y=3X^3-8X^2+7X-2$, what you will see displayed on the upper left hand corner of the graphics screen is Y=3*X^3-8*X^2+7*X-2.

Should your equation, inequality or system be too long to fit within the width of the graphics screen, the graphics screen will show only the middle portion of a widened graphics display. To move the graphics screen to see the right side of the graphics display, move the graphics cursor to the right side of the screen by pressing the blue RIGHT SHIFT key [*row 8, key 1*] and the RIGHT CURSOR key [*row 3, key 6*]. Then hold down the RIGHT CURSOR key to allow the right side of the graphics display to scroll into view. To move the graphics screen to see the left side of the graphics display, move the graphics cursor to the left side of the screen by pressing the blue RIGHT SHIFT key [*row 8, key 1*] and the LEFT CURSOR key [*row 3, key 6*]. Then hold down the LEFT CURSOR key to allow the left side of the graphics display to scroll into view.

THE GRAPHICS ZOOM SCREEN



Getting to the Graphics Zoom Screen

There is one way to get to the Graphics Zoom Screen:

- Press ZOOM [*row 1, key 1*] on the Graphics Screen 1.

The Graphics Zoom Screen Menu Bar

There are six keys active on the Graphics Zoom Screen Menu Bar:

XAUTO Press this key [*row 1, key 1*] to alter the size of your graph in the X or horizontal direction while scaling the size in the Y or vertical direction to redraw the graph with the best possible proportions. You'll be taken to a number input screen. Type a number and press the ENTER key [*row 5, key 1*]. If you enter a number greater than 1, you graph will be reduced in size showing you more of the overall graph but with less detail. If you enter a number between 0 and 1 [such as .1 or .976 or .5623], your graph will be enlarged in size showing you less of the overall graph but with more detail. If you enter 0 or a negative number, the HP48SX will become totally confused and return you to the menu screen from which you selected the family whose member was graphed. If you press ON [*row 9, key 1*] without entering a number, you'll be returned to the Graphics Screen 1 where you'll find your graph displayed unchanged in any way.

X Press this key [*row 1, key 2*] to alter the size of your graph in the X or horizontal direction leaving the size in the Y or vertical direction unchanged. You'll be taken to a number input screen. Type a number and press the ENTER key [*row 5, key 1*]. If you enter a number greater than 1, you graph will be reduced in size showing you more of the overall graph but with less detail. If you enter a number between 0 and 1 [such as .1 or .976 or .5623], your graph will be enlarged in size showing you less of the overall graph but with more detail. If you enter 0 or a negative number, the HP48SX will become totally confused and return you to the menu screen from which you selected the family whose member was graphed. If you press ON [*row 9, key 1*] without entering a number, you'll be returned to the Graphics Screen 1 where you'll find your graph displayed unchanged in any way.

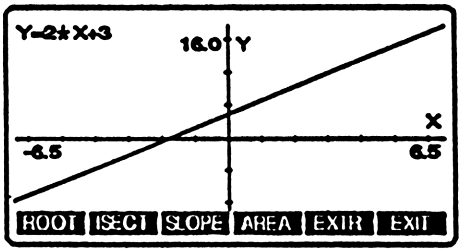
Y Press this key [*row 1, key 3*] to alter the size of your graph in the Y or vertical direction while leaving its size in the X or horizontal direction unchanged. You'll be taken to a number input screen. Type a number and press the ENTER key [*row 5, key 1*]. If you enter a number greater than 1, you graph will be reduced in size showing you more of the overall graph but with less detail. If you enter a number between 0 and 1 [such as .1 or .976 or .5623], your graph will be enlarged in size showing you less of the overall graph but with more detail. If you enter 0 or a negative number, the HP48SX will become totally confused and return you to the menu screen from which you selected the family whose member was graphed. If you press ON [*row 9, key 1*] without entering a number, you'll be returned to the Graphics Screen 1 where you'll find your graph displayed unchanged in any way.

XY Press this key [*row 1, key 4*] to alter the size of your graph in both the X or horizontal direction and Y or vertical direction. You'll be taken to a number input screen. Type a number and press the ENTER key [*row 5, key 1*]. If you enter a number greater than 1, you graph will be reduced in size showing you more of the overall graph but with less detail. If you enter a number between 0 and 1 [such as .1 or .976 or .5623], your graph will be enlarged in size showing you less of the overall graph but with more detail. If you enter 0 or a negative number, the HP48SX will become totally confused and return you to the menu screen from which you selected the family whose member was graphed. If you press ON [*row 9, key 1*] without entering a number, you'll be returned to the Graphics Screen 1 where you'll find your graph displayed unchanged in any way.

Don't bother pressing this key [*row 1, key 5*]. It doesn't do anything!

EXIT Press this key [*row 1, key 6*] to return to the Graphics Screen 1 where you'll find your graph displayed unchanged in any way.

THE GRAPHICS FUNCTION SCREEN 1



Getting to the Graphics Function Screen 1

There are two ways to get to the Graphics Function Screen 1:

- Press FCN [*row 1, key 2*] on the Graphics Screen 1.
- Press NXT [*row 2, key 6*] on the Graphics Function Screen 2.

The Graphics Function Screen 1 Menu Bar

There are six keys active on the Graph Menu Screen Menu Bar:

ROOT Press this key [*row 1, key 1*] to compute the X-coordinate of the point nearest the graphics cursor at which the graph crosses the X-axis. The graphics cursor will jump to the intersection point and the menu bar will disappear revealing the X-coordinate of the intersection point. Press any key to redisplay the menu bar.

ISECT Press this key [*row 1, key 2*] to compute the coordinates of the point nearest the graphics cursor at which the graphs of a system intersect. The graphics cursor will jump to the intersection point and the menu bar will disappear revealing the coordinates of the intersection point. Press any key to redisplay the menu bar. If your graph is not the graph of a system, this key functions exactly like the ROOT key.

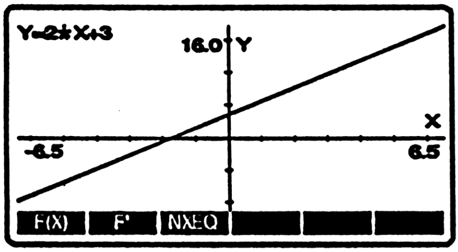
SLOPE Press this key [*row 1, key 3*] to compute the slope of the graph at the point whose X-coordinate is the same as the X-coordinate of the graphics cursor. The graphics cursor will jump to the specified point of the graph and the menu bar will disappear revealing the slope of the point. Press any key to redisplay the menu bar.

AREA Press this key [*row 1, key 4*] to compute the area under the graph. Move the graphics cursor to a point having the same X-coordinate as the vertical line which is to be the left boundary of the area. Press the MULTIPLICATION key [*row 7, key 5*] to set the mark. Move the graphics cursor to a point having the same X-coordinate as the vertical line which is to be the right boundary of the area. Press this key to make the menu bar disappear and reveal the desired area. Press any key to redisplay the menu bar.

EXTR Press this key [*row 1, key 5*] to compute the coordinates of the relative maximum point, relative minimum point or inflection point of the graph nearest the graphics cursor. The graphics cursor will jump to the specified point and the menu bar will disappear revealing the coordinates of the point. Press any key to redisplay the menu bar.

EXIT Press this key [*row 1, key 6*] to return to the Graphics Screen 1.

THE GRAPHICS FUNCTION SCREEN 2



Getting to the Graphics Function Screen 2

There is one way to get to the Graphics Function Screen 2:

• Press NXT [*row 2, key 5*] on the Graphics Function Screen 1.

The Graphics Function Screen 2 Menu Bar

There are six keys active on the Graphics Function Screen 2 Menu Bar:

F(X) Press this key [*row 1, key 1*] to compute the Y-coordinate of the point of the graph whose X-coordinate is the same as the X-coordinate of the graphics cursor. The graphics cursor will jump to the specified point of the graph and the menu bar will disappear revealing the Y-coordinate of the point Press any key to redisplay the menu bar..

F' Press this key [*row 1, key 2*] to graph the first derivative of the open sentence whose graph is now displayed on the graphics screen. The graphics screen will be erased, the first derivative will be drawn and the original graph will be redrawn. Press this key again to erase the graphics screen, draw the second derivative and redraw both the first derivative and the original graph. Each additional press of this key will cause the graphics screen to be erased, the next derivative to be graphed and all previous derivatives to be redrawn. Please note that, if you press this key and later choose Option 5, the Redraw Graph option, on the Graph Option Menu Screen, your original graph will be redrawn along with all the derivatives that were drawn by pressing this key

NXEQ Just before a graph is drawn, the particular open sentence being graphed is displayed on the upper left corner of the graphics screen.

Press this key [*row 1, key 3*] to display the family of which the particular open sentence being graphed is a member. The menu bar will disappear revealing the family in the lower left corner of the graphics screen. Press any key to redisplay the menu bar. If you've graphed a system or used the **F'** key [*row 1, key 2*] to graph derivatives, each press of this key will display in turn each of the family members whose graph is displayed on the graphics screen.

Don't bother pressing this key [*row 1, key 4*]. It doesn't do anything.

Don't bother pressing this key [*row 1, key 5*]. It doesn't do anything.

Don't bother pressing this key [*row 1, key 6*]. It doesn't do anything.

THE GRAPH OPTIONS MENU SCREEN

```
   GRAPH OPTIONS MENU
1  Print last graph
2  Set X-Range
3  Set Y-Range
4  Set decimal places
5  Redraw graph
   Pick a number
BACK | AGAIN | OTHER | EXIT | MAIN | OFF
```

Getting to the Graph Options Menu Screen

There is one way to get to the Graph Options Menu Screen:

• Press ON [*row 9, key 1*] from any Graphics Screen.

The Graph Options Menu Screen Menu Bar

There are six keys active on the Graph Options Screen Menu Bar:

BACK Press this key [*row 1, key 1*] to go back to the Graphics Screen 1 to take another look at your graph.

AGAIN Press this key [*row 1, key 2*] to enter new numbers to graph another member of the family whose member you just graphed.

OTHER Press this key [*row 1, key 3*] to return to the screen from which you selected the family whose member you just graphed.

EXIT Press this key [*row 1, key 4*] to return to the Graph Menu Screen.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Graph Options Menu Screen.

The Graph Options Screen Options

There are five options available on the Graph Options Menu Screen:

1 **Print last graph.** Press this key [*row 8, key 2*] to print the last graph drawn on the Graphics Screen on the HP82240 infrared printer. Turn on the printer and place it on a flat surface. Place the HP48SX on the same surface with the upper screen end of the calculator facing the bottom end of the printer and press the ENTER key [*row 5, key 1*] to start the printing. When the printing is done, you'll return to the Graph Options Menu Screen. Press the ON key [*row 9, key 1*] at any time before or during printing to abort the printing and return immediately to the Graph Options Menu Screen.

2 **Set X-Range.** Press this key [*row 8, key 3*] to set a new range for the horizontal or X-Axis of your graph. You'll be asked to enter the lower and higher X-Range values. In each case, type the number and press the ENTER key [*row 5, key 1*], after which you'll be taken back to the Graph Options Menu Screen. Had you pressed the ON key [*row 9, key 1*] without entering a number, you would have kept the original X-Range and returned immediately to the Graph Options Menu Screen.

3 **Set Y-Range.** Press this key [*row 8, key 4*] to set a new range for the vertical or Y-Axis of your graph. You'll be asked to enter the lower and higher Y-Range values. In each case, type the number and press the ENTER key [*row 5, key 1*], after which you'll be taken back to the Graph Options Menu Screen. Had you pressed the ON key [*row 9, key 1*] without entering a number, you would have kept the original Y-Range and returned immediately to the Graph Options Menu Screen.

4 **Set decimal places.** Press this key [*row 7, key 2*] to go to the Decimal Place Menu Screen where you can select the number of decimal places to display on the Graphics Screen.

5 **Redraw graph.** Press this key [*row 7, key 3*] to redraw your graph. Any change you made in the X-Range, Y-Range and number of displayed decimal places will be reflected in the new graph.

THE DECIMAL PLACE MENU SCREEN

```
   DECIMAL PLACE MENU
1. Fix 1         6. Fix 6
2. Fix 2         7. Fix 7
3. Fix 3         8. Fix 8
4. Fix 4         9. Fix 9
5. Fix 5         0. Fix 0
Press ENTER for Normal
```

Getting to the Decimal Place Menu Screen

There are two ways to get to the Decimal Place Menu Screen:

- Press $\boxed{4}$ [*row 7, key 2*] on the Graph Options Screen.
- Press the ENTER key [*row 5, key 1*] without typing a number when asked to type a number to serve as a coefficient.

The Decimal Place Menu Screen Options

There are ten options available on the Decimal Place Menu Screen:

[1] **Fix 1.** Press this key [*row 8, key 2*] to round off all numbers displayed on the Graphics Screens to 1 decimal place.

[2] **Fix 2.** Press this key [*row 8, key 3*] to round off all numbers displayed on the Graphics Screens to 2 decimal places.

[3] **Fix 3.** Press this key [*row 8, key 4*] to round off all numbers displayed on the Graphics Screens to 3 decimal places.

[4] **Fix 4.** Press this key [*row 7, key 2*] to round off all numbers displayed on the Graphics Screens to 4 decimal places.

[5] **Fix 5.** Press this key [*row 7, key 3*] to round off all numbers displayed on the Graphics Screens to 5 decimal places.

[6] **Fix 6.** Press this key [*row 7, key 4*] to round off all numbers displayed on the Graphics Screens to 6 decimal places.

[7] **Fix 7.** Press this key [*row 6, key 2*] to round off all numbers displayed on the Graphics Screens to 7 decimal places.

[8] **Fix 8.** Press this key [*row 6, key 3*] to round off all numbers displayed on the Graphics Screens to 8 decimal places.

[9] **Fix 9.** Press this key [*row 6, key 4*] to round off all numbers displayed on the Graphics Screens to 9 decimal places.

[0] **Fix 0.** Press this key [*row 9, key 2*] to round off all numbers displayed on the Graphics Screens to 0 decimal places.

> **Note:** Press the ENTER key [*row 5, key 1*] to display all numbers on the Graphics Screen as they were entered without rounding off. Press the ON key [*row 9, key 1*] to leave the Decimal Place Menu Screen without changing the number of places to which decimals are rounded off on the Graphics Screens.

Chapter Four

Loans

WHAT'S IN THIS CHAPTER

This chapter describes the E.Z. Math Loan Module. The Loan Module will enable you to do various calculations involving fixed rate mortgages on houses, condos, co-ops, property and other such investments

Here are some examples of the types of problems the E.Z. Math Loan Module will help you to solve:

> **Example 1.** You want to borrow $50,000 to purchase a beautiful condo in Florida.. You can get a 30 year, fixed rate mortgage. How much will you have to pay each month to pay off the mortgage.
>
> **Example 2.** You have found the house of your dreams but you are not sure you can afford it. You'd like your monthly mortgage payment to be the same as your current monthly rent of $793.18. If the best deal you can find is a 15 year, 10.25% fixed rate mortgage, how much can you afford to borrow?
>
> **Example 3.** You need to borrow $125,000 to buy a luxury coop on the East Side. You need to make monthly payments of $1987.56 on an 8.375% fixed rate mortgage. How long will it take to pay off this loan?
>
> **Example 4.** You want to buy a house. The current owner is paying off a $75,000 loan with monthly payments of $1250 on a 25 year fixed rate assumable mortgage. You are interested in assuming the mortgage provided that the interest rate is not too high. What is the interest rate being paid by the current owner?
>
> **Example 5.** It's income tax time and you need to know how much mortgage interest you can claim on your income tax statement. In December, you made payment number 16 on a 30 year, 12% fixed rate mortgage. If your monthly payment is $514.31, how much mortgage interest on your home did you pay last year?

If you'd like to be able to solve problems such as these, welcome to the E.Z. Math Loan Module.

E.Z. LOANS

It is often said that the single biggest purchase made by the typical American family is the home in which they live.

For most families, a house costs much more than all the money they've managed to save. Unless they have very generous and well-off relations or win the lottery, there are just two alternatives in purchasing a house: save until enough money has been accumulated or borrow the necessary money.

While saving would most likely eventually enable a family to buy a house, the troublesome word here is "eventually". It could take ten, twenty or more years to save enough to make the purchase. Buying a family home when the kids have grown up and moved away would for most people be an exercise in futility.

For this reason, the second alternative, borrowing the money, is for most families the only real alternative. With borrowed money, a family can buy a home right away or in the near future and enjoy it while the parents are young and the children are growing up. Also, owning a house brings various tax advantages. Of course, the borrowed money must be repaid, but the payments are spread out over a period of years to make them relatively easy to manage.

Let's take a look at what's involved in borrowing money to purchase a house, condo, co-op or other piece of property.

When you borrow money to purchase a piece of property, you sign a *mortgage agreement* with whoever lends you the money. We'll use the word *lender* to refer to the person, persons or institution lending you the money. The mortgage agreement acknowledges that you have borrowed a certain amount of money, that you must repay the lender within a specified period of time, that you must pay the lender a fee for having the use of his money and that you must make regular payments until the mortgage is paid off.

The amount you borrow is called the *principal* or *loan amount*. Usually, you can't borrow enough to cover the full purchase price of the property. You are required to pay part of the purchase price using money you already have. This is called the *down payment*.

Suppose you want to buy a house costing $150,000 and the lender requires a 20% down payment. This means that you must come up with 20% of $150,000 or $30,000 and the lender will let you borrow the remaining $120,000 of the purchase price. The purchase price is $150,000, the down payment is $30,000 and the principal or loan amount is $120,000, the amount upon which the mortgage will be based. All problems in the E.Z. Math Loan Module involve the loan amount, not the purchase price.

In your mortgage agreement, you agree to repay the lender within a certain period of time. This is called the *time* or *term* of the mortgage. Typical mortgages involve terms of 30 years, 15 years or 20 years, but other terms are also possible. Most mortgages involve monthly payments during the agreed-upon term, but some mortgages require payment every two weeks. All problems in the E.Z. Math Loan Module involve monthly payments.

Naturally, the lender is not letting you borrow his money just to be a nice guy. In effect, he is letting you rent his money. Just as a landlord in renting his apartment expects eventually to get back his apartment and meanwhile be paid rent each month, the lender also expects to get his money back and meanwhile be paid rent. The rent or fee paid for the use of borrowed money is called *interest*. Interest is always computed as a certain percent of the amount owed. This percent is called the *interest rate*.

A *fixed rate* mortgage is one in which the annual interest rate remains the same during the entire term of the mortgage. An *adjustable rate* or *variable rate* mortgage is one in which the annual interest rate may change from time to time according to a certain formula. All problems in the E.Z. Math Loan Module involve fixed rate mortgages.

Suppose you borrow $100,000 at an interest rate of 10% per year and keep the money for a full 30 years. Using the interest formula, *Interest = Principal x Interest Rate x Time*, the interest you'd have to pay would be $100,000 x 10% x 30 which comes to $300,000. This means that at the end of 30 years, you'd have to repay the $100,000 you borrowed plus the $300,000 in interest, making a total of $400,000.

If a mortgage worked in this manner, there'd probably be lots of heart attacks just before repayment time. To avoid this, you are required to pay some principal and some interest each month. A formula is used to compute the total that must be paid each month to make sure that the lender gets all the principal and interest within the agreed-upon term.

Each monthly payment has a principal part, which reduces the *loan balance*, that is, the amount you still owe the lender. If you borrowed $100,000 and the principal part of your first payment is $1,000, your loan balance would be $99,000 after the first payment.

Each month's payment has an interest part computed based on the current loan balance. Since paying principal reduces the loan balance, which in turn reduces the amount of interest due, the interest part becomes smaller with each payment. Since the total payment is the same each month, the principal part becomes larger with each payment. That's why you pay mostly interest during the early years of a mortgage and mostly principal during the last years.

A table or chart showing the principal part, interest part and remaining balance for every payment during the term of a mortgage is called the *amortization* table for that mortgage.

When using the E.Z. Math Loan Module to compute mortgage problems, you always have the opportunity to view the entire amortization table or the payment details for any particular payment.

THE LOAN MENU SCREEN

```
        LOAN MENU
1 Monthly payment
2 Amount of loan
3 Years
4 Interest rate %
5 Payment detail
6 Amortization table
NEXT | PREV | KILL | STAK | MAIN | OFF
```

Getting to the Loan Menu Screen

There are three ways to get to the Loan Menu Screen:

- Press 2 [*row 8, key 3*] on the Main Menu Screen.
- Press NEXT [*row 1, key 1*] on the Graph Menu Screen.
- Press PREV [*row 1, key 2*] on the Save Menu Screen.

The Loan Menu Screen Menu Bar

There are six keys active on the Loan Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the Save Menu Screen.

PREV Press this key [*row 1, key 2*] to go to the Graph Menu Screen.

KILL Press this key [*row 1, key 3*] to terminate E.Z. Math. When you again want to use E.Z. Math, repeat the start-up sequence described in Chapter 2 which is as follows: Press the ALPHA key [*row 6, key 1*] twice, press E [*row 1, key 5*] and Z [*row 5, key 3*], press the ENTER key [*row 5, key 1*] to get to the E.Z. Math Title Screen which, after about nine seconds, will be replaced by the E.Z. Math Main Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the E.Z. Math Loan Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Loan Menu Screen.

The Loan Menu Screen Options

There are six options available on the Loan Menu Screen:

[1] **Monthly payment.** Press this key [*row 8, key 2*] to compute the monthly payment on a fixed rate loan. You'll be asked to enter the loan amount, number of years and interest rate. In each case, type the requested numbers without dollar and percent signs and press the ENTER key [*row 5, key 1*].

[2] **Amount of loan.** Press this key [*row 8, key 3*] to compute the amount of money you can borrow. You'll be asked to enter the number of years, interest rate and monthly payment. In each case, type the requested numbers without dollar and percent signs and press the ENTER key [*row 5, key 1*].

[3] **Years.** Press this key [*row 8, key 4*] to compute the number of years needed to pay off a fixed rate loan. You'll be asked to enter the loan amount, interest rate and monthly payment. In each case, type the requested numbers without dollar and percent signs and press the ENTER key [*row 5, key 1*].

[4] **Interest rate %.** Press this key [*row 7, key 2*] to compute the interest rate on a fixed rate mortgage. You'll be asked to enter the loan amount, number of years and monthly payment. In each case, type the requested numbers without dollar and percent signs and press the ENTER key [*row 5, key 1*].

[5] **Payment detail.** Press this key [*row 7, key 3*] to compute all the details for a specific monthly payment on a fixed rate mortgage. You'll be asked to enter the loan amount, number of years and interest rate. In each case, type the requested numbers without dollar and percent signs and press the ENTER key [*row 5, key 1*].

[6] **Amortization table.** Press this key [*row 7, key 4*] to compute the Amortization Table for a fixed rate mortgage. You'll be asked to enter the loan amount, number of years and interest rate. In each case, type the requested numbers without dollar and percent signs and press the ENTER key [*row 5, key 1*].

Note: After any of the above six options, you'll then be taken to the Loan Information Screen to check your entered data for accuracy. You may press the ON key [*row 9, key 1*] at any time during data entry to return to the Loan Menu Screen without solving the problem.

THE LOAN INFORMATION SCREEN

```
    LOAN INFORMATION
Int rate: 12%
Years: 30 years
Loan amt: $50000.00


   Is this correct?
 O.K.  REDO     EXIT  MAIN  OFF
```

```
    LOAN INFORMATION
Payment: $793.18
Int rate: 10.25%
Years: 15 years


   Is this correct?
 O.K.  REDO     EXIT  MAIN  OFF
```

```
    LOAN INFORMATION
Payment: $1987.56
Int rate: 8.375%
Loan amt: $125000.00


   Is this correct?
 O.K.  REDO     EXIT  MAIN  OFF
```

```
    LOAN INFORMATION
Payment: $1250.00
Years: 25 years
Loan amt: $78000.00


   Is this correct?
 O.K.  REDO     EXIT  MAIN  OFF
```

Getting to the Loan Information Screen

There is one way to get to the Loan Information Screen

- Select any of the six options available on the Loan Menu Screen.

The Loan Information Screen Menu Bar

There are six keys active on the Savings Information Screen Menu Bar:

O.K. Press this key [*row 1, key 1*] to accept the information displayed on the Loan Information Screen. If you selected options 1, 2, 3 or 4 on the Loan Menu Screen, you'll be taken to the Loan Computation Screen where you'll find your answer. If you selected option 5 on the Loan Menu Screen, you'll be asked to enter the number of the payment for which you want information. Type the number and press the ENTER key [*row 5, key 1*] to go to the requested Loan Payment Detail Screen. If you selected option 6 on the Loan Menu Screen, you'll be taken to the first Loan Payment Detail Screen of the requested Amortization Table.

REDO Press this key [*row 1, key 2*] to reject the information displayed on the Loan Information Screen and have the opportunity to enter different loan information.

EXIT Press this key [*row 1, key 4*] to go to the Loan Menu Screen.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Loan Information Screen.

Printing the Loan Information Screen

Any Loan Information Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.
2. Press the orange LEFT-SHIFT key [*row 7, key 1*].
3. Press the MTH key [*row 2, key 1*].
4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.
5. Press the ENTER key [*row 5, key 1*] to begin printing.
6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

THE LOAN COMPUTATION SCREEN

```
   LOAN COMPUTATION
Int rate: 12%
Years: 30 years
Loan amt: $50000.00


Payment: $514.31
AMORT AGAIN DTAIL EXIT MAIN OFF
```

```
   LOAN COMPUTATION
Payment: $793.18
Int rate: 10.25%
Years: 15 years


Loan amt: $72772.08
AMORT AGAIN DTAIL EXIT MAIN OFF
```

```
   LOAN COMPUTATION
Payment: $1987.56
Int rate: 8.375%
Loan amt: $125000.00


Years: 6.92 years
AMORT AGAIN DTAIL EXIT MAIN OFF
```

```
   LOAN COMPUTATION
Payment: $1250.00
Years: 25 years
Loan amt: $78000.00


Int rate: 19.06%
AMORT AGAIN DTAIL EXIT MAIN OFF
```

Getting to the Loan Computation Screen

There is one way to get to the Loan Computation Screen:

- Press O.K. [*row 1, key 1*] on the Loan Information Screen. This will work if you selected option 1, 2, 3 or 4 on the Loan Menu Screen. If you selected option 5 or 6, you'll bypass the Loan Computation Screen and go directly to a Loan Payment Detail Screen.

The Loan Computation Screen Menu Bar

There are six keys active on the Loan Computation Screen Menu Bar:

AMORT Press this key [*row 1, key 1*] to go to the first Loan Payment Detail Screen of the Amortization Table displaying all details about each monthly payment of your mortgage.

AGAIN Press this key [*row 1, key 2*] to redo the same problem with different loan information.

DTAIL Press this key [*row 1, key 3*] to go to a Loan Payment Detail Screen. You'll be asked to enter the number of the payment for which you want information. Type the number and press ENTER [*row 5, key 1*] to be taken to the requested Loan Payment Detail Screen.

EXIT Press this key [*row 1, key 4*] to go to the Loan Menu Screen.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Loan Computation Screen.

Printing the Loan Computation Screen

Any Loan Computation Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.

2. Press the orange LEFT-SHIFT key [*row 7, key 1*].

3. Press the MTH key [*row 2, key 1*].

4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.

5. Press the ENTER key [*row 5, key 1*] to begin printing.

6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

THE LOAN PAYMENT DETAIL SCREEN

```
Payment no: 15
Payment: $514.31
Prn part: $16.44
Int part: $497.86
Tot prn: $230,29
Tot int: $7484,31
Balance: $49769.71
NEXT AGAIN DTAIL EXIT MAIN OFF
```

```
        ONE YEAR
  LOAN PAYMENT SUMMARY
Payments 4 to 15
Tot prn: $186.94
Tot int: $5984.74
Tot pd: $6171.68
Balance: $49769.71
NEXT AGAIN DTAIL EXIT MAIN OFF
```

```
Payment no: 16
Payment: $514.31
Prn part: $16.61
Int part: $497.70
Tot prn: $246.90
Tot int: $7982.00
Balance: $49753.10
NEXT AGAIN DTAIL EXIT MAIN OFF
```

```
        ONE YEAR
  LOAN PAYMENT SUMMARY
Payments 5 to 16
Tot prn: $188.81
Tot int: $5982.87
Tot pd: $6171.68
Balance: $49753.10
NEXT AGAIN DTAIL EXIT MAIN OFF
```

Getting to the Loan Payment Detail Screen

There is one way to get to the Loan Payment Detail Screen:

- Press AMORT [*row 1, key 1*] on the Loan Computation Screen.
- Press DTAIL [*row 1, key 3*] on the Loan Computation Screen
- Select option 5 or 6 on the Loan Menu Screen.

The Loan Payment Detail Screen Menu Bar

There are six keys active on the Loan Payment Detail Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the next Loan Payment Detail Screen. Each Payment Detail Screen has 2 pages. Page 1 displays details for the current payment as well as totals since the start of the loan. Page 2, The One Year Loan Payment Summary, displays totals for the current and previous 11 payments. Press the ALPHA key [*row 6, key 1*] prior to pressing NEXT to go from Page 1 to Page 2 or from Page 2 to Page 1 of the current Loan Payment Detail Screen. If you press the ALPHA key and then press the NEXT key while looking at Loan Payment Detail Screen 15, you'll be taken to the One Year Loan Payment Summary Screen for payments 4 to 15. Press these two keys again to return to Page 1.

AGAIN Press this key [*row 1, key 2*] to redo the same problem with different loan information.

DTAIL Press this key [*row 1, key 3*] to go to a Loan Payment Detail Screen. You'll be asked to enter the number of the payment for which you want information. Type the number and press ENTER [*row 5, key 1*] to be taken to the requested Loan Payment Detail Screen.

EXIT Press this key [*row 1, key 4*] to go to the Loan Menu Screen.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Loan Payment Detail Screen.

Printing the Loan Payment Detail Screen

Any Loan Payment Detail Screen can be printed on an HP82240 infrared printer as follows: Turn on the printer, put it on a flat surface, place the HP48SX on the same surface with the upper screen end pointing to the bottom end of the printer, and press the ENTER key [*row 5, key 1*] to begin printing. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

Chapter Five

Savings

WHAT'S IN THIS CHAPTER

This chapter describes the E.Z. Math Save Module. The Save Module will enable you to do various calculations involving a single, one-time deposit or repeated deposits to a savings or money market account, certificate of deposit [C.D.], term deposit [T.D.] or any other type of savings plan paying compound interest.

Here are some examples of the types of problems the E.Z. Save Module will help you to solve:

> **Example 1.** If you can deposit $200 in an account paying 12% interest compounded once a year, how much will you accumulate if you leave it on deposit for 30 years.
>
> **Example 2.** You want to deposit $5000 in a 5-year C.D. paying 8.25% interest compounded quarterly. How much money will you have when you cash in the C.D. after the five years have passed?
>
> **Example 3.** You estimate that you'll need $40,000 to put a newborn child through college 17 years from now. You have found an investment paying 7.5% interest compounded daily over the next 17 years. How much will you need to deposit now to accumulate the $40,000 in time for college?
>
> **Example 4.** You can save $300 per month in an account paying 6.25% interest compounded monthly. How many years will it take for you to accumulate a million dollars?
>
> **Example 5.** You have $10,000 available to deposit into a C.D. paying interest compounded quarterly. What interest rate must the C.D. pay in order for you to double your money in 10 years?
>
> **Example 6.** How much must you deposit each month in an account paying 7.5% interest compounded monthly to accumulate $100,000 in 20 years?

If you'd like to be able to solve problems such as these, welcome to the E.Z. Math Save Module.

E.Z. SAVINGS

When a person agrees to let you have exclusive use of his property for a certain period of time, you are *renting* or *leasing* his property for which you must pay him a certain amount of money, usually monthly or weekly. The fee you pay for having exclusive use of his property is called *rent.*

Let's say you live in an apartment owned by Ms. Smith. You are Ms. Smith's *tenant* and she is your *landlord.* While you remain her tenant, you have the exclusive right to use her property almost as if it belonged to you. In return, you must pay her the agreed-upon rent.

In some cases, the landlord agrees to let the tenant have use of his property for a certain period of time, say two years, in which case they sign a contract which is called a *lease.* In the case of a two year lease, the landlord is obligated to allow the tenant full use of his property for two years in return for which the tenant is obligated to pay the landlord the agreed-upon rent during the same two-year period.

In other cases, the tenant and landlord have no lease. In this week-to-week or month-to-month arrangement, the tenant may leave at any time and the landlord may reclaim his property at any time.

So what does all this have to do with savings? Simple! Depositing money in a savings account is just like renting an apartment, except that you are the landlord and the bank is the tenant. You give the bank the exclusive right to use your property, in this case your money, for a period of time in return for which the bank must pay you rent. Depending upon the bank and the type of account, you'll be paid interest each day, each week, each month, each year or at some other interval.

The amount of money you deposit, that is, the amount you rent to the bank, is called the *principal.* The bank has exclusive use of your money until you withdraw it. With most accounts, you may ask for your money back at any time. In the case of C.D.'s [Certificates of Deposit] and T.D.'s [Term Deposits], you cannot get your money back until an agreed-upon period of time has passed. These two situations are just like those described above of the month-by-month tenant and the tenant with a lease.

The rent paid for the use of your money is called *interest.* If a deposit of $100 has grown to $115 one year later, the bank rented $100 from you and in return for having had exclusive use of your property for one year has paid you $15 rent.

Interest is always computed as a percent of the principal. This percent is called the *interest rate.* Suppose you deposit $200 for one year in an account paying 10% per year interest. The principal is $200, the interest rate is 10% per year and the time is 1 year. The bank is renting your money for one year and will pay you 10% of the amount you rented to them as interest.

Let's see now... 10% of $200 is $20. So, at the end of one year, you can reclaim your principal of $200 plus your interest of $20, making a total of $220.

There are two types of interest; *simple* and *compound*. With simple interest, interest is paid directly to you or deposited into some other account. With compound interest, each interest payment is added to your account becoming part of the money being rented from you, with the next interest payment being computed on the total amount.

If you have $100 in an account paying 10% per year simple interest, at the end of each year you'd be sent $10 in interest with $100 still remaining on deposit in your account. The following year, you'd again be sent $10 in interest and still have $100 on deposit in your account. As long as that $100 remains on deposit in your account, you'd continue to receive $10 interest each year.

With compound interest, the story would be quite different. At the end of the first year, the $10 in interest, instead of being sent to you, would be added to your account bringing it up to $110. The following year, your account would be increased by $11 in interest [10% of $110] bringing it up to $121. At the end of the third year, your account would go up by $12.10 in interest [10% of $121] bringing your account up to $133.10.

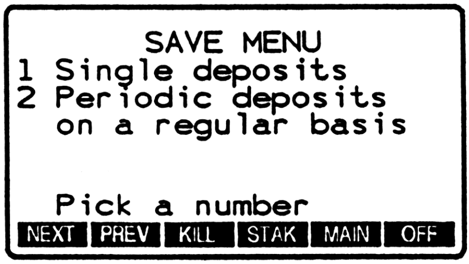
In this last example, interest was computed and added to your account once a year. In bank lingo, interest was *compounded* annually. If interest is compounded daily, interest is computed and added to your account each day, 365 days per year. If interest is compounded quarterly, interest is computed and added to your account four times a year, or every three months.

Let's see what happens to your $100 at 10% per year interest compounded quarterly. At the end of the first three months [the first quarter], your $100 would yield $2.50 interest bringing your account up to $102.50. [A full year's interest at 10% comes to $10, but for three months you only get one quarter of $10, or $2.50]. At the end of the second three months [the second quarter], your $102.50 would yield $2.56 interest bringing your account up to $105.06. [A full year's interest at 10% comes to $10.25, but for three months you only get one quarter of $10.25, or $2.56]. At the end of the third quarter, your $105.06 would yield $2.63 interest bringing your account up to $107.69. At the end of the fourth quarter, your $107.69 would yield $2.69 interest bringing your account up to $110.38.

At first glance, there seems to be little difference in yield between simple and compound interest. But appearances can be deceiving. Over a 30 year period at 10% per year interest, your $100 would yield $300 in simple interest, $1,644.94 in interest compounded annually and $1,835.81 in interest compounded quarterly. As you can see, compound interest pays much more than simple interest, and the more frequently interest is compounded, the better you'll make out.

All savings problems solved by E.Z. Math involve compound interest.

THE SAVE MENU SCREEN



Getting to the Save Menu Screen

There are three ways to get to the Save Menu Screen:

- Press 3 [*row 8, key 4*] on the Main Menu Screen.
- Press NEXT [*row 1, key 1*] on the Loan Menu Screen.
- Press PREV [*row 1, key 2*] on the Number Menu Screen.

The Save Menu Screen Menu Bar

There are six keys active on the Save Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the Number Menu Screen.

PREV Press this key [*row 1, key 2*] to go to the Loan Menu Screen.

KILL Press this key [*row 1, key 3*] to terminate E.Z. Math. When you again want to use E.Z. Math, repeat the start-up sequence described in Chapter 2 which is as follows: Press the ALPHA key [*row 6, key 1*] twice, press E [*row 1, key 5*] and Z [*row 5, key 3*], press the ENTER key [*row 5, key 1*] to get to the E.Z. Math Title Screen which, after about nine seconds, will be replaced by the E.Z. Math Main Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the E.Z. Math Save Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Save Menu Screen.

The Save Menu Screen Options

There are two options available on the Save Menu Screen:

[1] **Single deposits.** Press this key [*row 8, key 2*] to do a problem involving a single deposit to a savings account, certificate of deposit [C.D.], term deposit [T.D.] or other such investment. This will take you to the Save Menu 1 Screen.

[2] **Periodic deposits on a regular basis.** Press this key [*row 8, key 3*] to do a problem involving a regular deposit on a regular basis, say $125.00 each week or month, to a savings account, certificate of deposit [C.D.], term deposit [T.D.] or other such investment. This will take you to the Save Menu 2 Screen.

THE SAVE MENU 1 SCREEN

```
      SAVE MENU 1
    SINGLE DEPOSITS
1 Amount saved
2 Amount of deposit
3 Years on deposit
4 Annual interest rate
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

Getting to the Save Menu 1 Screen

There are three ways to get to the Save Menu 1 Screen:

- Press 1 [*row 8, key 2*] on the Save Menu Screen.
- Press NEXT [*row 1, key 1*] on the Save Menu 2 Screen.
- Press PREV [*row 1, key 2*] on the Save Menu 2 Screen.

The Save Menu 1 Screen Menu Bar

There are six keys active on the Save Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the Save 2 Menu Screen.

PREV Press this key [*row 1, key 2*] to go to the Save 2 Menu Screen.

EXIT Press this key [*row 1, key 3*] to to go to the Save Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the E.Z. Math Save Menu 1 Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Save Menu 1 Screen.

The Save Menu 1 Screen Options

There are four options available on the Save Menu 1 Screen:

1. **Amount saved.** Press this key [*row 8, key 2*] to learn how much you'll accumulate from a single, one-time deposit. You'll be asked to enter the amount of deposit, the number of years on deposit, the annual interest rate and the number of times per year interest is compounded. In each case, type the requested numbers without dollar and percent signs and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Savings Information Screen to check your entered data for accuracy. You may press the ON key [*row 9, key 1*] at any time during data entry to return to the Save Menu 1 Screen without solving the problem.

2. **Amount of deposit.** Press this key [*row 8, key 3*] to learn how much must be deposited to accumulate a certain mount of money. You'll be asked to enter the amount to accumulate, the number of years on deposit, the annual interest rate and the number of times per year interest is compounded. In each case, type the requested numbers without dollar and percent signs and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Savings Information Screen to check your entered data for accuracy. You may press the ON key [*row 9, key 1*] at any time during data entry to return to the Save Menu 1 Screen without solving the problem.

3. **Years on deposit.** Press this key [*row 8, key 4*] to learn in how many years a single, one-time deposit will grow to a certain amount of money. You'll be asked to enter the amount to accumulate, the amount of deposit, the annual interest rate and the number of times per year interest is compounded. In each case, type the requested numbers without dollar and percent signs and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Savings Information Screen to check your entered data for accuracy. You may press the ON key [*row 9, key 1*] at any time during data entry to return to the Save Menu 1 Screen without solving the problem.

4. **Annual interest rate.** Press this key [*row 7, key 2*] to learn the annual interest rate needed for a single, one-time deposit to grow to a certain amount of money. You'll be asked to enter the amount to accumulate, the amount of deposit, the number of years on deposit and the number of times per year interest is compounded. In each case, type the requested numbers without dollar and percent signs and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Savings Information Screen to check your entered data for accuracy. You may press the ON key [*row 9, key 1*] at any time during data entry to return to the Save Menu 1 Screen without solving the problem.

THE SAVE MENU 2 SCREEN

```
      SAVE MENU 2
   PERIODIC DEPOSITS
1 Total saved
2 Amt of each deposit
3 Years saved
4 Annual interest rate
  Pick a number
NEXT | PREV | EXIT | STAK | MAIN | OFF
```

Getting to the Save Menu 2 Screen

There are three ways to get to the Save Menu 2 Screen:

- Press 2 [*row 8, key 3*] on the Save Menu 2 Screen.
- Press NEXT [*row 1, key 1*] on the Save Menu 1 Screen.
- Press PREV [*row 1, key 2*] on the Save Menu 1 Screen.

The Save Menu 2 Screen Menu Bar

There are six keys active on the Save Menu 2 Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the Save Menu 1 Screen.

PREV Press this key [*row 1, key 2*] to go to the Save Menu 1 Screen.

EXIT Press this key [*row 1, key 3*] to go to the Save Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the E.Z. Math Save Menu 2 Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Save Menu 2 Screen.

The Save Menu 2 Screen Options

There are four options available on the Save Menu 2 Screen:

[1] **Total saved.** Press this key [*row 8, key 2*] to learn how much you'll accumulate from repeated deposits. You'll be asked to enter the number of years on deposit, the annual interest rate, the amount of each deposit, the number of deposits per year and the number of times per year interest is compounded. In each case, type the requested numbers without dollar and percent signs and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Savings Information Screen to check your data. You may press the ON key [*row 9, key 1*] at any time during data entry to return to the Save Menu 2 Screen without solving the problem.

[2] **Amt of each deposit.** Press this key [*row 8, key 3*] to learn how much must be repeatedly deposited to accumulate a certain mount of money. You'll be asked to enter the amount to accumulate, the number of years on deposit, the annual interest rate, the number of deposits per year and the number of times per year interest is compounded. In each case, type the requested numbers without dollar and percent signs and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Savings Information Screen to check your data. You may press the ON key [*row 9, key 1*] at any time during data entry to return to the Save Menu 2 Screen without solving the problem.

[3] **Years saved.** Press this key [*row 8, key 4*] to learn how many years of repeated deposits are needed to accumulate a certain amount of money. You'll be asked to enter the amount to accumulate, the annual interest rate, the amount of each deposit, the number of deposits per year and the number of times per year interest is compounded. In each case, type the requested numbers without dollar and percent signs and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Savings Information Screen to check your data . You may press the ON key [*row 9, key 1*] at any time during data entry to return to the Save Menu 2 Screen without solving the problem.

[4] **Annual interest rate.** Press this key [*row 7, key 2*] to learn the annual interest rate needed for repeated deposits to grow to a certain amount of money. You'll be asked to enter the amount to accumulate, the number of years on deposit, the amount of each deposit, the number of deposits per year and the number of times per year interest is compounded. In each case, type the requested numbers without dollar and percent signs and press the ENTER key [*row 5, key 1*]. You'll then be taken to the Savings Information Screen to check your data. You may press the ON key [*row 9, key 1*] at any time during data entry to return to the Save Menu 2 Screen without solving the problem.

THE SAVINGS INFORMATION SCREEN

```
  SAVINGS INFORMATION
Times comp: 1
Int rate: 12 %
Years: 30 years
Dep: $200.00

   Is this correct?
O.K. REDO OTHER EXIT MAIN OFF
```

```
  SAVINGS INFORMATION
Times comp: 4
Int rate: 8.25%
Years: 5 years
Dep: $5000.00

   Is this correct?
O.K. REDO OTHER EXIT MAIN OFF
```

```
  SAVINGS INFORMATION
Times comp: 365
Int rate: 7.50%
Years: 17 years
End amt: $40000.00

   Is this correct?
O.K. REDO OTHER EXIT MAIN OFF
```

```
  SAVINGS INFORMATION
Times comp: 12
Dep per year: 12
Each dep: $300.00
Int rate: 6.25%
End amt: $1000000.00
   Is this correct?
O.K. REDO OTHER EXIT MAIN OFF
```

Getting to the Savings Information Screen

There are two ways to get to the Savings Information Screen

- Select any of the four options available on the Save Menu 1 Screen.
- Select any of the four options available on the Save Menu 2 Screen.

The Savings Information Screen Menu Bar

There are six keys active on the Savings Information Screen Menu Bar:

O.K. Press this key [*row 1, key 1*] to accept the information displayed on the Savings Information Screen and go to the Savings Computation Screen to get your answer.

REDO Press this key [*row 1, key 2*] to reject the information displayed on the Savings Information Screen and have the opportunity to enter different numbers.

OTHER Press this key [*row 1, key 3*] to return to the Save Menu 1 Screen or to the Save Menu 2 Screen, whichever one you originally came from.

EXIT Press this key [*row 1, key 4*] to go to the Save Menu Screen.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Savings Information Screen.

Printing the Savings Information Screen

Any Savings Information Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.
2. Press the orange LEFT-SHIFT key [*row 7, key 1*].
3. Press the MTH key [*row 2, key 1*].
4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.
5. Press the ENTER key [*row 5, key 1*] to begin printing.
6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

THE SAVINGS COMPUTATION SCREEN

```
  SAVINGS COMPUTATION
Times comp: 1
Int rate: 12 %
Years: 30 years
Dep: $200.00

Total: $5991.98
      AGAIN OTHER EXIT MAIN OFF
```

```
  SAVINGS COMPUTATION
Times comp: 4
Int rate: 8.25%
Years: 5 years
Dep: $5000.00

Total: $7521.32
      AGAIN OTHER EXIT MAIN OFF
```

```
  SAVINGS COMPUTATION
Times comp: 365
Int rate: 7.50%
Years: 17 years
End amt: $40000.00

Dep: $11178.70
      AGAIN OTHER EXIT MAIN OFF
```

```
  SAVINGS COMPUTATION
Times comp: 12
Dep per year: 12
Each dep: $300.00
Int rate: 6.25%
End amt: $1000000.00
Years:46.61 years
      AGAIN OTHER EXIT MAIN OFF
```

Getting to the Savings Computation Screen

There is one way to get to the Savings Computation Screen

- Press O.K. [*row 1, key 1*] on the Savings Information Screen..

The Savings Computation Screen Menu Bar

There are five keys active on the Savings Computation Screen Menu Bar:

AGAIN Press this key [*row 1, key 2*] to redo the same problem using different numbers.

OTHER Press this key [*row 1, key 3*] to return to the Save Menu 1 Screen or to the Save Menu 2 Screen, whichever one you originally came from.

EXIT Press this key [*row 1, key 4*] to go to the Save Menu Screen.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Savings Computation Screen.

Printing the Savings Computation Screen

Any Savings Computation Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.
2. Press the orange LEFT-SHIFT key [*row 7, key 1*].
3. Press the MTH key [*row 2, key 1*].
4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.
5. Press the ENTER key [*row 5, key 1*] to begin printing.
6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.

Chapter Six

Numbers

WHAT'S IN THIS CHAPTER

This chapter describes the E.Z. Math Number Module. The Number Module will enable you to do various calculations involving natural numbers, sequences of natural numbers, rational numbers and complex numbers.

Here are some examples of the types of problems the E.Z. Math Number Module will help you to solve:

Example 1. What is the smallest natural number that can be divided evenly by 2, 3, 4, 5, 6, 8 and 12?

Example 2. What is the largest natural number which can be divided evenly into 200, 100, 250, 750, 450, 600, 900, 1000, 250 and 700?

Example 3. List all the natural numbers which divide evenly into 12,000. What is the prime factorization of 800?

Example 4. What is the average (arithmetic mean) of 45, 58, 36, 48, 62, 74, 68 and 48?

Example 5. List the first 12 perfect third powers.

Example 6. List all the 10th power binomial coefficients.

Example 7. List the first 15 triangular numbers, the first 11 Fibonacci numbers and the first 27 multiples of 9.

Example 8. How do you add, subtract, multiply and divide fractions and mixed numbers on the HP48SX and have your answer come out as a fraction or mixed number in lowest terms, rather than as a decimal.

Example 9. How do you add, subtract, multiple, and divide complex, real and imaginary numbers? How do you raise a complex number to a complex number power?

If you want to solve problems such as these, then welcome to the E.Z. Math Number Module.

THE SET OF NATURAL NUMBERS

What is the Set of Natural Numbers?

{ 1, 2, 3, 4, 5, 6, 7, 8, . . . }

This funny looking thing is read, "The set containing 1, 2, 3, 4, 5, 6, 7, 8 and so forth." This is the set of *natural numbers*. It is also called the set of *counting numbers*, since these are the numbers we use for counting.

The first natural number is 1. There is no last natural number since, no matter which counting number we might select, say 1,000,000,000,000,000, there is always a higher natural number. There are infinitely many natural numbers.

A set of numbers is *dense* if between any two numbers in the set there is always another number in the set. The set of natural numbers is not dense because there is no natural number between 1 and 2, between 2 and 3, and so on.

Factors of Natural Numbers

One natural number *divides evenly* into another natural number if, when the second is divided by the first, the remainder is zero. 10 divides evenly into 40 because when 40 is divided by 10, the remainder is zero. On the other hand, 10 does not divide evenly into 53 because, when 53 is divided by 10, the remainder is 3 and not zero.

One natural number is a *factor* or *divisor* of another natural number if the first one divides evenly into the second one. 10 is a factor of 40 because 10 divides evenly into 40 whereas 10 is not a divisor of 53 because 10 does not divide evenly into 53.

It's easy to see that every number has 1 and itself as factors. 50 has 1 and 50 among its factors.

It is always possible to find all the factors of a natural number. The factors of 10 are 1, 2, 5 and 10. The factors of 13 are 1 and 13. The divisors of 9 are 1, 3 and 9. The only divisor of 1 is 1. As you can see, natural numbers have various amounts of factors: some have 1 factor, some have 2, some have 3, and so on.

Prime and Composite Numbers

A *prime* number is a natural number which has exactly two different factors. 17 is a prime number because 1 and 17 are its only two factors. 2 is also a prime number because 1 and 2 are its only two factors. 9 is not a prime number because it has more than two factors [1, 3 and 9]. The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29. There are infinitely many prime numbers.

A *conposite* number is a natural number with more than two different numbers. 25 is a composite number because it has the three factors 1, 5 and 25. Likewise, 12 is a composite number because it has the six factors 1, 2, 3, 4, 6 and 12. The first ten composite numbers are 4, 6, 8, 9, 10, 12, 14, 15, 16 and 18. There are infinitely many composite numbers.

What about 1? The natural number 1 is in a special category since it has only the single factor 1. It's not a prime number because it doesn't have just two different factors. It's not a composite number since it doesn't have more than two different factors. Therefore, 1 is neither a prime number nor a composite number.

Prime Factorization

A factorization of a natural number is a multiplication of natural numbers which is equal to the original natural number. A factorization of 120 is 40 x 3. Another factorization of 120 is 2 x 5 x 12. Still other factorizations of 120 are 10 x 12 and 3 x 4 x 5. As you can see, a natural number can have several different factorizations.

The *prime factorization* of a natural number is a factorization composed entirely of prime numbers. The prime factorization of 120 is 2 x 2 x 3 x 5. The prime factorization of 81 is 3 x 3 x 3 x 3. The prime factorization of 25 is 5 x 5. The prime factorization of 7 is 7. Every natural number except 1 has exactly one prime factorization.

The Greatest Common Factor

The *Greatest Common Factor [G.C.F.]* or *Greatest Common Divisor [G.C.D.]* of a set of natural numbers is the largest natural number which is a factor of all the numbers in the set.

Let's compute the G.C.F. of 80, 100 and 120:

Factors of 80: 1, 2, 4, 5, 8, 10, 16, 20, 40 and 80.
Factors of 100: 1, 2, 4, 5, 10, 20, 25, 50 and 100.
Factors of 140: 1, 2, 4, 5, 7, 10, 14, 20, 28, 35, 70 and 140.

The common factors of 80, 100 and 140 are 1, 2, 4, 5, 10 and 20. Therefore, the Greatest Common Factor of 80, 100 and 120 is 20. Likewise, the G.C.F. of 4 and 7 is 1. Every set of natural numbers has a unique Greatest Common Factor.

Multiples of Natural Numbers

One natural number is a *multiple* of another natural number if the second natural number divides evenly into the first. In other words, one natural number is a multiple of another natural number if the second is a factor of the first. 40 is a multiple of 10 because 10 divides evenly into 40. Likewise, 14 and 21 are both multiples of 7 because 7 is a factor of both 14 and 21. On the other hand, 53 is not a multiple of 10 because 10 does not divide evenly into 53.

When we talk about all the multiples of a natural number, we are really talking about the times table for that number. The multiples of 6 are simply the numbers in the 6 times table: 6, 12, 18, 24, 30, 36, 42 and so on. The multiples of 30 are the numbers in the 30 times table: 30, 60, 90, 120, 150, 180 and so on. Every natural number has infinitely many multiples, the first being the number itself. There is no highest or last multiple for any natural number.

By the way, the set of all multiples of 2 is also called the set of *even* numbers. These are 2, 4, 6, 8, 10, 12, 14, 16, 18 and so on. The rest of the natural numbers are called the *odd* numbers. These are 1, 3, 5, 7, 9, 11, 13, 15, 17 and so on.

The Least Common Multiple

The *Least Common Multiple [L.C.M.]* of a set of natural numbers is the smallest natural number which is a multiple of all the numbers in the set.

Let's compute the L.C.M. of 6, 8 and 12:

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54 and so on.
Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72 and so on.
Multiples of 12: 12, 24, 36, 48, 60, 72 and so on.

The common multiples of 6, 8 and 12 are 24, 48, 72, 96 and so on. Therefore, the Least Common Multiple of 6, 8 and 12 is 24. Likewise, the L.C.M. of 4 and 7 is 28. Every set of natural numbers has a unique Least Common Multiple.

Sequences of Natural Numbers

A *sequence* of natural numbers is any set of natural numbers in which the numbers are arranged in a specific order. Here are some examples:

Example 1. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, . . .
Example 2. 2, 1, 4, 3, 6, 5, 8, 7, 10, 9, 12, 11, . . .
Example 3. 3, 2, 1, 6, 5, 4, 9, 8, 7, 12, 11, 10, . . .

Since all three sequences are composed of the same numbers, they would, if considered simply as sets, all be equal. However, considered as sequences, they are all different since the numbers in each sequence are arranged in different orders.

The individual numbers in a sequence are called the *terms* of the sequence. You can think of a sequence as a list and each term of the sequence as an item in the list. The terms are numbered exactly as common sense would lead you to believe. The first term in Example 2 on the previous page is 2 and the seventh term in Example 3 is 9.

There are infinitely many possible examples of sequences of natural numbers of which quite a few are mathematically important. Here are the ones included in the E.Z. Math Number Module:

Perfect n th Powers. The sequence of perfect n th powers is produced by raising each natural number to the n th power. Some examples include the sequence of perfect squares or perfect second powers, the first seven terms of which are 1, 4, 9, 16, 25, 36 and 49, and the sequence of perfect cubes or perfect third powers, the first five terms of which are 1, 8, 27, 64 and 125.

Binomial coefficients. The sequence of binomial coefficients is the sequence of numerical coefficients obtained when a binomial, such as X+Y, is raised to a power. Since $(X+Y)^3=X^3+3X^2Y+3XY^2+Y^3$, then 1, 3, 3, 1 would be the third power binomial coefficients.

Triangular Numbers. The sequence of triangular numbers is formed as follows: The first term is 1. The second term is 2 plus the first term. The third term is 3 plus the second term. Any other term is the number of that term plus the preceding term. The first seven triangular numbers are 1, 3, 6, 10, 15, 21 and 28.

Fibonacci numbers. The sequence of Fibonacci numbers is formed as follows: The first and second terms are both 1. The third term is the sum of the first and second terms. Any other term is the sum of the two previous terms. The first eight terms are 1, 1, 2, 3, 5, 8, 13 and 21.

Multiples. The sequence of multiples of a number is formed as follows: The first term is 1 times that number. The second term is 2 times that number. Every other term is the term number times that number. What we really have is the times table for that number. The first six multiples of 7 are 7, 14, 21, 28, 35 and 42.

THE SET OF WHOLE NUMBERS

Why the Set of Whole Numbers?

So far, we've learned about the set of natural numbers which is used for counting. Now, we'd like to see if the set of natural numbers contains all possible answers to the important question "How many . . . ?" How many people were in Times Square on New Year's Eve? How many cars were manufactured last year by General Motors? How many countries are in the United Nations? How many stars are in the sky? How many women have served as Presidents of the United States? How many grains of sand are on all the beaches in the world?

If we have only the set of natural numbers, we can answer most but not all of these questions. There is simply no natural number which can answer the question "How many five-sided triangles are there?" Likewise, there is no natural number answer to the question "How many men are over ten feet tall?"

What we need is a new set of numbers which contains all possible answers to the question "How many . . . ?" This set obviously must contain all the natural numbers since, besides being used for counting, they all can answer the question "How many . . . ?" In order to answer the above questions about female United States Presidents and five-sided triangles, this new set must also contain the new number zero.

What is the Set of Whole Numbers?

We now have a brand-new set of numbers containing all answers to the question "How many . . . ?" This is called *the set of whole numbers* and looks like this:

$$\{ 0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots \}$$

To read this, you say: "The set containing 1, 2, 3, 4, 5, 6, 7, 8 and so forth.

As you can see, the first whole number is zero. There is no last whole number since, no matter which whole number you might select, there is always another whole number after it. There are infinitely many whole numbers. The set of whole numbers is not dense because it has gaps, such as the one between 0 and 1, which contain no whole numbers. Every natural number is a whole number, but not every whole number is a natural number.

We have taken the set of natural numbers, perfect for counting but not for answering the question "How many . . . ?" and expanded it into the set of whole numbers by including the number zero.

A Mathematician's View of the Whole Numbers

A mathematician views the process of expanding one set of numbers into another differently. One of a mathematician's fondest goals is to have a set of numbers in which all operations can be done. An *operation on a set of numbers* is any definite rule or procedure by which we can combine any two numbers in the set and get a single, unique number as the answer. Although there are infinitely many operations on sets of numbers, in most branches of mathematics, such as arithmetic, algebra and calculus, there are six important operations for combining numbers. They are addition, subtraction, multiplication, division, power and root. A mathematician would love to have a set of numbers in which, anytime he uses one of these operations to combine any two numbers from the set, the answer will also be a number from the same set.

If he restricted his attention only to the operations of addition and multiplication, he would need no set of numbers other than the set of natural numbers. The set of natural numbers is *closed with respect to* addition and multiplication This means that, when any two natural numbers are added or multiplied, the answer is always a natural number.

However, once subtraction enters the picture, there's trouble. The problem is that, although many subtractions involving natural numbers do yield a natural number answer, many don't. 7-2, 6-3 and 597-480 all have natural number answers but 5-5, 3-6 and 92-673 don't. The set of whole numbers is not closed with respect to subtraction.

Expanding the set of natural numbers to the set of whole numbers partially solves this problem. Whole number subtractions like 9-9, 28-28 and 159-159, impossible with the set of natural numbers, are now possible within the set of whole numbers. However, many whole number subtractions such as 9-87, 526-697 and 2-100 still do not yield answers within the set of whole numbers.

To find out if we can create a set of numbers closed with respect to subtraction, you'll have to turn the page and read about the set of integers.

THE SET OF INTEGERS

Why the Set of Integers?

So far, we've learned about the set of natural numbers which is used for counting and the set of whole numbers which contains all answers to the question "How many . . . ?" Now, we'd like to be able to find two whole numbers, one representing a gain of ten pounds and the other representing a loss of ten pounds. Since gaining ten pounds is very different from losing ten pounds, we need two different whole numbers.

Unfortunately, we can't find two different whole numbers to represent a gain of ten pounds and a loss of ten pounds. Of course, we could use the whole number 10 to express a gain of ten pounds. But what whole number could we possibly use to represent a loss of ten pounds. If we have only the set of whole numbers, we simple have no way to solve this problem.

What we need is a new set of numbers which can represent opposites. A loss of ten pounds is the opposite of a gain of ten pounds. 30 years in the future is the opposite of 30 years in the past. Depositing $100 in a bank is the opposite of withdrawing $100 from a bank. 15 miles North is the opposite of 15 miles South. Walking up two flights of stairs is the opposite of walking down two flights of stairs.

To represent all these opposite situations, we need a pair of words, one representing a certain, particular direction, such as North or up, and the other representing the opposite direction, such as South or down. We should be able to use these two words anytime we need to talk about a pair of opposite situations. Lucky for us, we do indeed have such a pair of words.

The word used to refer to any particular direction is the word *positive*. The word used to refer to the opposite of that direction is *negative*. We use "+" to symbolize "positive" and "-" to symbolize "negative". With this system, a gain of ten pounds would be represented by +10 [Say "positive ten"; never say "plus ten"] and a loss of ten pounds would be represented by -10 [Say "negative ten"; never say "minus ten"]. Likewise 30 miles North would be written as +30 and 30 miles South as -30. If +15 means depositing $15 in the bank, then -23 means withdrawing $23 from the bank.

What is the Set of Integers

We now have a brand new set of numbers which can be used to represent all situations involving opposites. This is called the set of integers and looks like this:

$$\{\ldots, -4, -3, -2, -1, 0, +1, +2, +3, +4, \ldots\}$$

To read this, you say: "The set containing and so forth, -4, -3, -2, -1, 0, +1, +2, +3, +4 and so forth"

As you can see, there is no last integer, since, no matter which integer you might select, there is always another integer after it. Likewise, there is no first integer since, no matter which integer you select, there is always an integer before it. There are infinitely many integers. The set of integers is not dense because it has gaps, such as the one between 0 and +1, which contain no integers. Integers get bigger as you move to the right and smaller as you move to the left. So, +3 is greater than -10, 0 is greater than -20 and -2 is greater than -978. [Remember, +3 and -10 are read "positive 3" and "negative 10" respectively.]

It may appear at first glance that in going to the set of integers, we have discarded the set of whole numbers to create a totally new set with all new numbers. Such, however, is not the case. The positive integers, rather than being totally new numbers, are really the natural numbers in disguise. Zero is the same zero we first in the set of whole numbers. Only the negative integers are brand-new numbers which did not exist before.

It is then easy to see that every whole number and every natural number is an integer, but that not every integer is a natural or whole number.

We have now taken the set of whole numbers, which contains all answers to the question "How many . . . ?" and, by including the negative integers, expanded it to the set of integers which is perfect for representing opposite situations.

A Mathematician's View of the Integers

Let's rejoin our mathematician friend in his quest for a set of numbers in which all operations are possible. With the set of whole numbers, we had a set of numbers closed with respect to addition and multiplication, but not with respect to subtraction. Anytime two whole numbers are added or multiplied, the answer is always a whole number. But, unfortunately, such is not the case for subtraction.

Now, with the set of integers, we finally have a set of numbers closed, not only with respect to addition and multiplication, but also with respect to subtraction. If you add, subtract or multiply any two integers, the answer is always an integer.

However, bring in division and trouble returns. The set of integers is not closed with respect to division. Divisions such as +10 divided by -47 or -2 divided by +87 cannot be done if all you have is the set of integers.

To find out if we can create a set of numbers closed, not only with respect to addition, subtraction and multiplication, but also with respect to division, you'll have to turn the page and read about the set of rational numbers.

THE SET OF RATIONAL NUMBERS

Why the Set of Rational Numbers

So far, we've learned about the set of natural numbers which is used for counting, the set of whole numbers which contains all answers to the question "How many . . . ?" and the set of integers which can represent all situations involving opposites. We've also learned that the set of integers contains the set of natural numbers and the set of whole numbers.

Now, we'd like to find a number within the set of integers to represent the action of eating three pieces of an apple cut into eight equal pieces. Using the integer 8 would not work because no mention is made of the three pieces eaten. Likewise, using the integer 3 would not work because no mention is made of the eight equal pieces. In fact, it appears that no single integer is capable of representing the action of eating three pieces of an apple cut into eight equal pieces. If only there were a way to make a number which somehow blended 3 and 8 together . . .

But wait, we learned a way to do this back in elementary school. Just use the fraction 3/8 [which is read "three eighths"]. The bottom or second number of the fraction, which is called the *denominator*, tells us into how many equal pieces something should be cut. In 3/8, the denominator 8 tells us to cut something into eight equal pieces. The top or first number of the fraction, which is called the *numerator*, tells us how many of the equal pieces we are supposed to take. In 3/8, the numerator 3 tells us to take three of the equal pieces. So, with the fraction 3/8, we have solved the problem of eating three pieces of an apple cut into eight equal pieces.

If we now create a set which contains all possible fractions, we would have a brand-new set of numbers which can represent any situation in which objects are cut into equal parts, some of which we take. This set of numbers is called the set of rational numbers.

What Is the Set of Rational Numbers?

The *set of rational numbers* is defined as the set containing all numbers which can be expressed as fractions in which the numerators and denominators are integers and in which the denominator is not zero. Put another way, the set of rational numbers is the set containing all positive fractions, all negative fractions and zero, the positive fractions being the ones we learned about in elementary school. In fact, the rational number section of the E.Z. Math Number Module deals exclusively with the positive fractions.

There are two types of fractions: proper and improper. A *proper* *fraction* is a fraction in which the numerator is less than the denominator. Some examples of proper fractions are: 3/4 [Say "three fourths"], 9/27 [Say "nine twenty-sevenths"], 1/50 [Say "one fiftieth"], 87/300 [Say "eighty-seven three-hundreths"] and 951/3793. An *improper fraction* is a fraction in which the numerator is equal to or greater than the denominator. Some examples of improper fractions are 5/5 [Say "five fifths"], 22/7 [Say "twenty-two sevenths"], 8/1 [Say "eight firsts"], 900/43 [Say "nine-hundred forty-thirds"] and 85/85.

While proper fractions are easy for most people to understand, improper fractions are not. How can you take 17 slices from a pizza cut into eight equal slices and thereby get 17/8 of a pizza? Actually, it's very simple. Cut a pizza into eight equal slices and take all eight. Now you've got eight slices and need nine more. Cut another pizza into eight equal slices and again take all eight. Now you've got a total of 16 slices and need just one more. Finally, cut one more pizza into eight equal slices and take one. Now you've got all 17 of the slices that were due to you. So you can picture 17/8 of a pizza by imagining two whole pizzas plus one slice from a third. We could represent 17/8 as two whole pizzas plus 1/8 of another pizza.

This brings us to the concept of a mixed number which blends an integer with a fraction. A *mixed number* is an integer plus a proper fraction. Some examples of mixed numbers are: 8 3/4 [Say "eight and three-fourths"], 23 4/5 [Say "twenty-three and four-fifths"] and -5 7/11 [Say "negative five and seven-elevenths"].

Every mixed number can be expressed as an improper fraction by using the elementary school method which directs us to multiply the whole number by the denominator, add the numerator and put the result over the denominator. Since $11 \cdot 12+5=137$, we can convert the mixed number 11 5/12 into the improper fraction 137/12. Likewise, every improper fraction can be converted into a whole number or mixed number by using the elementary school method which directs us to divide the numerator by the denominator and, if the remainder is not zero, put it over the denominator. Since when 59 is divided by 8 the quotient is 7 and the remainder is 3, we can convert the improper fraction 59/8 into the mixed number 7 3/8. Since when 50 is divided by 5 the quotient is 19 and the remainder is zero, we can convert the improper fraction 50/5 into the whole number 10.

It may appear at first glance that in creating the set of rational numbers we have gone away from the set of integers and created a set of totally new numbers. Such, however, is not the case. Remember, the set of rational numbers is not the set of all fractions. The set of rational numbers is the set of all numbers which can be expressed as fractions. While 7 is not a fraction, it is a rational number since it can be expressed as the fraction 7/1. Although 0 is not a fraction, it is a rational number because it can be written as the fraction 0/1. Even though -4 is not a fraction, it is a rational number since it can be converted to the fraction -4/1. It's easy to see that every natural number, whole number and integer can likewise be expressed as a fraction with integer numerator and nonzero integer denominator and is therefore a rational number. So once again in creating a new set of numbers to serve a new purpose, in this case the set of rational numbers, we've kept the previous set of numbers, in this case the set of integers, into which we've included brand-new numbers, in this case the set of noninteger fractions.

As is the case for the set of integers, the set of rational numbers has no first or smallest number and no last or largest number. Unlike the set of integers, the set of rational numbers is dense since, between any two rational numbers, there is always another rational number. One way to get a rational number between two others is to compute their average or arithmetic mean, that is, add them up and divide by two. This is a process that will always yield a rational number halfway between the two original rational numbers. If we apply this to 7/8 and 8/9, we get 127/144 as a rational number in between.

What's mind-boggling about this is that, if there is always a rational number between any two rational numbers, then there are infinitely many rational numbers between them. In other words, between any two rational numbers, no matter how close, there are infinitely many rational numbers. This means that the set of rational numbers has no gaps devoid of rational numbers. It would now appear that we now have in the set of rational numbers all the numbers we could ever possibly need for any conceivable purpose.

A Mathematician's View of the Rational Numbers

Let's again rejoin our mathematician friend in his quest for a set of numbers in which all operations are possible. When we last met him, we had learned that the set of integers was closed with respect to addition, subtraction and multiplication, but not with respect to division. Anytime you add, subtract or multiply two integers, your answer will also be an integer. But, unfortunately, such is not the case with division.

Going to the set of rational numbers solves this problem. You may recall from your elementary school fraction rules that anytime you divide two fractions, the result is always a fraction. This seems to imply that, when you divide any two rational numbers, the result is a rational number. This should make the set of rational numbers closed with respect to division. It appears that we have finally created a set of numbers in which all operations are possible.

This conclusion, however, is just a bit premature. First, there is the matter of division by zero. The answer to 100 divided by 20 is 5 because 100 divided by 20 means: "What number multiplied by 20 equals 100?" Likewise, 10 divided by 1/2 means: "What number multiplied by 1/2 equals 10?" The answer is 20. To compute 30 divided by zero, we'd need to find a number which multiplied by zero produces 30. Since every number multiplied by zero is zero, there is no number which multiplied by zero produces 30. Consequently, division by zero simply cannot be done. In math lingo, division by zero is undefined or meaningless. So, the set of rational numbers is closed with respect to division, provided division by zero is excluded.

Because of this, anytime a problem in E.Z. Math encounters a division by zero, the problem will be aborted and you'll be returned to the menu screen from which you came so that you may select another problem. Also because of this problem with division by zero, zero is never allowed as the denominator of a fraction. [A fraction is considered to be a single number, but if you decide to view the numerator and denominator as two separate numbers, then the numerator is divided by the denominator; thus, the denominator can't be zero].

The other problem with our premature conclusion that we had found a set of numbers closed with respect to all operations has to do with the operations of power and root. The Greeks proved over two thousand years ago that the square root of 2, which comes out to a number between 1 and 2, is not a rational number. This means that the set of rational numbers is not closed with respect to power and our mathematician friend must continue his quest for a set of numbers in which all operations, except division by zero, are possible.

To find out if we can create a set of numbers closed with respect to all six important operations, you'll have to turn the page and read about the set of real numbers.

THE SET OF REAL NUMBERS

Why the Set of Real Numbers

So far, we've learned about the set of natural numbers which is used for counting, the set of whole numbers which contains all answers to the question "How many . . . ?", the set of integers which can represent all situations involving opposites and the set of rational numbers which can represent all situations in which objects are cut into equal pieces, some of which we take. We also learned that the set of rational numbers includes the set of natural numbers, the set of whole numbers and the set of integers. Finally, we learned that the set of rational numbers is dense, meaning that between any two rational numbers, there is always another rational number and, therefore, infinitely many rational numbers.

Now, we'd like to find out whether or not the set of rational numbers contains all numbers needed to express every possible measurement. In everyday life, we measure all kinds of things including length, area, time, pressure, temperature, speed and volume. Surely, every problem involving measurements can be handled using the set of rational numbers.

The amazing fact is that the set of rational numbers does not contain all the numbers needed to express every possible measurement. Suppose you walk one mile East from point A to point B and one mile North from point B to point C. It would be very natural to wonder how far you'd walk if you went directly from point A to point C. Solving this problem would be equivalent to finding the length of the diagonal of a square each of whose sides is one mile long.

There must obviously be an answer since, in walking from point A to point C, you do go a certain distance. In fact, the famous Pythagorean Theorem from high school math tells us that we can compute the length of the diagonal of the square as follows:

> **Step 1.** Square the length of a side of the square. Since the length of each side is one mile and squaring a number means to multiply the number by itself, we have 1^2 or 1 x 1, which yields 1 as the result.
>
> **Step 2.** Double the answer obtained in Step 1. If we double 1, we get 2.
>
> **Step 3.** Compute the square root of the result from Step 2. This means tat we must compute the square root of 2 or $\sqrt{2}$.

So, the diagonal of a square each side of which is one mile in length is $\sqrt{2}$ miles in length. In order to find the square root of a number, we must find a number which when squared equals the original number. According to this, $\sqrt{2}$ means: "What number squared equals 2?" The square root of 4 or $\sqrt{4}$ is 2 because $2^2 = 2 \times 2 = 4$. The square root of 1 or $\sqrt{1}$ is 1 because $1^2 = 1 \times 1 = 1$1. Therefore, $\sqrt{2}$ is between 1 and 2. So the length of the diagonal of a square, each of whose sides measure one mile, is a number between 1 and 2 which is equal to $\sqrt{2}$.

It is, therefore, totally amazing and unexpected to learn that $\sqrt{2}$ is not a rational number! Over two thousand years ago, the Greeks proved that there is no rational number that yields 2 when squared. No matter which rational number you pick between 1 and 2, and there are infinitely many of them, not one of them when squared will yield 2!

Yet, we can easily find decimals which are very close in value to $\sqrt{2}$. In fact, we can actually construct a sequence of decimals in which each decimal is closer in value to $\sqrt{2}$ than the decimal which came before. Squaring the decimals 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8 and 1.9 reveals that $\sqrt{2}$ is between 1.4 and 1.5, but closer to 1.4, because $1.4^2=1.96$ and $1.5^2=2.25$. Squaring the decimals 1.41, 1.42, 1.43, 1.44, 1.45, 1.46, 1.47, 1.48 and 1.49 reveals that $\sqrt{2}$ is between 1.41 and 1.42 because $1.41^2=1.9881$ and $1.42^2=2.0164$. Several more such trial and error experiments would eventually reveal that $\sqrt{2}$ is between 1.4142135623 and 1.4142135624, but closer to 1.4142135624. This process can be continued on and on producing decimals ever closer in value to $\sqrt{2}$.

If it were possible to continue this process forever, the result would be an infinitely long decimal whose value would be exactly equal to $\sqrt{2}$. Naturally, infinitely long decimals do not exist in the real world because they would take forever to compute and write. But is no such limit to what we can do in our imagination.

If we now create a set which contains all possible decimals, including those which are infinitely long and therefore exist only in the imagination, we would have a set of numbers which can represent all possible measurements.

What Is the Set of Real Numbers?

The set of *real numbers* is defined as the set of all numbers which can be expressed as decimals. Like the set of rational numbers, the set of real numbers has no first or smallest number and no last or largest number, and has infinitely many numbers. Also like the set of rational numbers, the set of real numbers is dense because, between any two real numbers there is always another real number and, therefore, infinitely many real numbers.

Unlike the set of rational numbers, the set of real numbers can join with the set of all points on a line to form the partnership between algebra and geometry known as the *real number line*. In this partnership, each real number is paired off with exactly one point on the line and each point on the line is paired off with exactly one real number. Please see Chapter Three for more information about the real number line.

Why can this not be done with the set of rational numbers? Simple! Remember the square whose diagonal was $\sqrt{2}$ miles long? Move this diagonal over to the number line, placing one end of the diagonal on the point of the number line which is paired off with zero. Then the other end of the diagonal would have to be on a point of the number line paired off with $\sqrt{2}$, which is not a rational number. This means that there are not enough rational numbers to be partners with all the points on a line.

This leads to another mind-boggling fact about numbers. The set of rational numbers and the set of real numbers are both dense; yet, even though there are infinitely many rational numbers between any two rational numbers and infinitely many real numbers between any two real numbers, only the set of real numbers is *complete*, that is, contains exactly the right amount of numbers necessary to provide a partner for each point of a line. What's mind-boggling about this is the fact that the amount of real numbers is, therefore, a much bigger infinity than the amount of rational numbers!

Does the set of real numbers contain the set of natural numbers, the set of whole numbers, the set of integers and the set of rational numbers. or is the set of real numbers composed entirely of brand-new numbers? While 7 is not a decimal, it is a real number since it can be expressed as the decimal 7.0. Although 0 is not a decimal, it is a real number since it can be expressed as the decimal 0.0. Even though -4 is not a decimal, it is a real number since it can be expressed as the decimal -4.0. It's easy to see from this that every natural number, every whole number and every integer is a real number.

But what about the set of rational numbers? Is it also contained within the set of real numbers? To answer this, we must remember the elementary school method for changing a fraction to a decimal by dividing the numerator by the denominator. To do this, a decimal point and, after the decimal point, some zeros must be added to the numerator. Since there is no limit to the number of zeros which can be added to a decimal, the division can be continued as long as necessary until a remainder of zero is obtained.

In the case of some fractions, a remainder of zero is obtained after a certain number of steps producing what is called a *terminating decimal*. Here are some examples:

Example 1. 3/4 = .75

Example 2. 1/2 = .5

Example 3. 9/16 = .5625

Example 4. 7/8 = .875

Example 5. $3/125 = .024$

Example 6. $4/5 = .8$

Example 7. $7/25 = .28$

Example 8. $6/625 = .0096$

In other cases, the remainder is never zero, in which case the division process never terminates and the result is an infinite decimal. The interesting thing about these infinite decimals is that, except possibly for the first few digits, they consist exclusively of a group of digits which repeats forever. They are called *infinitely repeating decimals* and are written showing just one occurrence of the repeating group over which is placed a bar line to symbolize the infinitely many repetitions. Here are some examples:

Example 1. $1/3 = .3333333\ldots = .\overline{3}$

Example 2. $3/11 = .272727\ldots = .\overline{27}$

Example 3. $5/6 = .8333333\ldots = .8\overline{3}$

Example 4. $3/7 = .428571428571\ldots = .\overline{428571}$

Example 5. $7/36 = .194444444\ldots = .19\overline{4}$

Example 6. $37/72 = .513888888\ldots = .513\overline{8}$

Example 7. $7/101 = .06930693\ldots = .\overline{0693}$

So every fraction can be converted into either a terminating decimal or an infinitely repeating decimal. In other words, the set of rational numbers is represented within the set of real numbers by the terminating and infinitely repeating decimals. This means that those decimals which neither terminate nor infinitely repeat represent real numbers which are not rational numbers. These nonrepeating, nonterminating decimals are called the *irrational real numbers* or simply the *irrational numbers*. Here are some examples:

Example 1. $\pi = 3.1415926535\ldots$

Example 2. $e = 2.7182818284\ldots$

Example 3. $\sqrt{2} = 1.4142136523\ldots$

Example 4. $\text{SIN}(23^\circ) = .39073112848\ldots$

Example 5. $1.2121121112111121111\ldots$

Example 6. $\text{LOG}(234) = 2.3692158574\ldots$

Example 7. $\sqrt[3]{7} = 1.9129311827\ldots$

Example 8. $45.37377377737777377777\ldots$

The set of irrational real numbers can be further subdivided into types. The irrational numbers which result from roots of rational numbers which do not yield rational number answers are called ***algebraic irrational numbers*** or simply ***algebraic numbers***. All irrational numbers which are not algebraic numbers are called ***transcendental irrational numbers*** or simply ***transcendental numbers***. With regard to the examples of irrational numbers at the bottom of Page 6-18, the numbers in Examples 3 and 7 are algebraic numbers and the rest are transcendental numbers.

A Mathematician's View of the Real Numbers

Let's once again rejoin our mathematician friend in his quest for a set of numbers in which all operations are possible. When we last met him, we had learned that the set of rational numbers, while closed with respect to addition, subtraction, multiplication and division (except division by zero), was not closed with respect to root. Any time you add, subtract or multiply any two rational numbers, or divide any rational number by a nonzero rational number, the result is a rational number. But, unfortunately, such is not the case for root.

Going to the set of real numbers appears to solve this problem. It seems that all the roots which could not be computed within the set of rational numbers can now be calculated within the set of real numbers. Now that we have decimals, roots like $\sqrt{2}$, $\sqrt{5}$, $\sqrt{345}$ and $\sqrt{257}$ all have real number answers.

Unfortunately, even though we can compute all these roots, the set of real numbers turns out not to be closed with respect to root. Consider, for instance, $\sqrt{-4}$ which is asking us to find a number which when squared gives -4. A basic fact about the set of real numbers is that every real number is either positive or negative or zero. Since the square of every positive real number is positive, $\sqrt{-4}$ is not a positive number. Since the square of every negative real number is a positive real number, $\sqrt{-4}$ is not a negative real number. Since $0^2=0$, $\sqrt{-4}$ is not zero. There is, therefore, no real number whose square is -4. This means that $\sqrt{-4}$ is not a real number. Likewise, the square root of any negative real number is never a real number. In fact, the even root [fourth root, sixth root, eighth root, and so on] of any negative real number is not a real number. The set of real numbers is therefore not closed with respect to root.

To find out if we can create a set of numbers which is closed with respect to all six important operations, that is, addition, subtraction, multiplication, division, power and root, you'll need to turn to the next page and read about the set of complex numbers.

THE SET OF COMPLEX NUMBERS

Why the Set of Complex Numbers

So far, we've learned about the set of natural numbers which is used for counting, the set of whole numbers which contains all answers to the question "How many . . . ?", the set of integers which can represent all situations involving opposites, the set of rational numbers which can represent all situations in which objects are cut into equal pieces, some of which we take and the set of real numbers which is the set of all numbers needed to represent every possible measurement. We also learned that the set of real numbers includes the set of natural numbers, the set of whole numbers, the set of integers and the set of rational numbers. In addition, we learned that the set of real numbers is dense, meaning that between any two real numbers, there is always another real number and, therefore, infinitely many real numbers. Finally, we learned that, unlike with the set of rational numbers, it's possible to pair off the set of all points on a straight line with the set of all real numbers in the one-to-one correspondence known as the real number line. [The real number line was discussed on Page 3-8 in Chapter 3.]

Now, we'd like to have a set of numbers which can similarly be paired off in a one-to-one correspondence with the set of all points on a plane.

If you look at Pages 3-9 and 3-10 in Chapter 3, you'll find a discussion of the real number plane in which we set up a one-to-one correspondence between the set of all points in a plane and the set of all real numbers. What we did was to pair off the set of all points of a plane with the set of all ordered pairs of real numbers so that for every ordered pair of real numbers there is exactly one point of the plane and for every point of the plane there is exactly one ordered pair of real numbers. The only problem is that ordered pairs of real numbers, such as (5, 3) and (-2, 8), are not numbers; they are, rather, sets of numbers each containing two numbers in a specific order. If only we had a set of numbers which could behave exactly like the set of all ordered pairs of real numbers.

If you think back to how the set of rational numbers was created from the set of integers, you'll recall that every rational number could be expressed as a fraction composed of two integers, the first called the numerator and the second called the denominator. Since the order of the two integers in a fraction does indeed make a difference, it's clear that a fraction is really nothing more than an ordered pair of integers written in a new way. What we did in creating the set of rational numbers was to take ordered pairs of integers, such as (3, 5) and (-4, 7), and write them as fractions, such as 3/5 and -4/7.

Perhaps we can use ordered pairs of real numbers in some similar way to create a new set of numbers. In doing this, we'd need to follow a number of guidelines. This new set of numbers should be an extension of the set of real numbers and therefore include the set of real numbers in some form. There should be a precise way to decide whether or not two of these numbers are equal. Finally, there should be some definite method by which we can combine any two of these new numbers using any of the six important operations.

Happily, some very clever people developed a set of numbers meeting all these requirements. This set of numbers is called the set of complex numbers.

What Is the Set of Complex Numbers?

The set of *complex numbers* is the set of all ordered pairs of real numbers in which equality, addition and multiplication are defined as explained in the following paragraphs:

Two complex numbers are *equal* if their first components are equal and their second components are equal. Here are some examples:

Example 1. $(5, 2) = (5, 2)$
These two complex numbers are equal because the first components, 5 and 5, are equal and the second components, 2 and 2, are equal.

Example 2. $(7, 3) \neq (3, 7)$
These two complex numbers are not equal because the first components, 7 and 3, are not equal and the second components, 3 and 7, are not equal.

Example 3. $(8, 1) \neq (8, 3)$
These two complex numbers are not equal because the second components, 3 and 1, are not equal.

To *add* two complex numbers, proceed as follows. Compute the first component of the sum by adding the first components of the two complex numbers. Compute the second component of the sum by adding the second components of the two complex numbers. Here are some examples:

Example 1. $(4, 2) + (7, 8) = (11, 10)$
Example 2. $(3, -7) + (-9, 2) = (-6, -5)$
Example 3. $(2, 0) + (7, 0) = (9, 0)$
Example 4. $(0, 2) + (0, 5) = (0, 7)$
Example 5. $(5, 0) + (0, 2) = (5, 2)$

To *multiply* two complex numbers, proceed as follows. Compute the first component of the product by multiplying the first components of the two complex numbers, then by multiplying the second components of the two complex numbers and finally by subtracting the second result from the first. For example, in (5, 8) · (6, 2), we would compute 5·6=30, then 8·2=16 and finally 30-16=14. The final result, in this case 14, is the first component of the product. Compute the second component of the product by multiplying the first component of the first complex number by the second component of the second complex number, then by multiplying the second component of the first complex number by the first component of the second complex number and finally by adding the results. For example, in (5, 8).· (6, 2), we would compute 5·2=10, then 8·6=48 and finally 10+48=58. The final result, in this case 58, is the second component of the product. Here are some examples:

Example 1.	$(5, 8) \cdot (6, 2) = (14, 58)$
Example 2.	$(2, -3) \cdot (-3, 2) = (0, 13)$
Example 3.	$(2, 3) \cdot (2, -3) = (13, 0)$
Example 4.	$(-4, 3) \cdot (5, -6) = (-2, 39)$
Example 5.	$(2, 0) \cdot (5, 0) = (10, 0)$
Example 6.	$(3, 0)^2 = (3, 0) \cdot (3, 0) = (9, 0)$
Example 7.	$(0, 4) \cdot (0, 5) = (-20, 0)$
Example 8.	$(0, 2)^2 = (0, 2) \cdot (0, 2) = (-4, 0)$
Example 9.	$(0, 1)^2 = (0, 1) \cdot (0, 1) = (-1, 0)$
Example 10.	$(0, 1)^3 = (0, 1) \cdot (0, 1) \cdot (0, 1)$ $= (-1, 0) \cdot (0, 1) = (0, -1)$

The interesting thing about the rules for adding and multiplying complex numbers is that they are so similar to the rule for multiplying and adding rational numbers. When we multiply two fractions, we find the first component or numerator of the product by multiplying the first components or numerators of the two fractions. We find the second component or denominator of the product by multiplying the second components or denominators of the two fractions. We add complex numbers in exactly the same way except that we add the components instead of multiplying them. The rules for multiplying complex numbers and adding fractions, while not as close as identical twins, are more like siblings displaying a close family resemblance. The point to keep in mind is that the rules for adding and multiplying complex numbers are no more complicated, strange and unreasonable than those for multiplying and adding rational numbers.

We've defined equality, addition and multiplication of complex numbers. Now, we'd like to discover what part of the set of complex numbers represents the set of real numbers. Additions such as (3, 0) + (5, 0) = (8, 0) and (25, 0) + (-3, 0) = (22, 0) and multiplications such as (2, 0) · (7, 0) = (14, 0) and (-5, 0) · (-3, 0) = (15, 0) suggest that complex numbers which have zero as the second component act just like real numbers. This means that complex numbers such as (5, 0) and (-8, 0) are the representatives in the set of complex numbers of the real numbers 5 and -8 and that, from now on, we'll regard (5, 0) and (-8, 0) as being interchangeable with 5 and -8. Since the first component of a complex number is computed just like the first component of an ordered pair of real numbers by looking straight up or down at the X-axis, we can now refer to the X-axis as the *real axis*.

Additions such as (3, 0) + (0, 2) = (3, 2) and (-5, 0) + (0, 11) = (-5, 11) suggest that every complex number can be split into two complex numbers, the first being a real number and the second being a complex number whose first component is zero. For instance, the complex number (3, 2) can be split into (3, 0) which is the complex number representing the real number 3 plus the complex number (0, 2). This suggests that the complex numbers whose first component are zero are the brand-new, nonreal numbers we've just created.

Unfortunately, those who first investigated complex numbers referred to these new numbers as *imaginary* numbers. This means that we are stuck with having to refer to complex numbers such as (0, 5) and (0, -8) as imaginary numbers even though they are just as "real" as real numbers. Since the second component of a complex number is computed just like the second component of an ordered pair of real numbers by looking straight left or right at the Y-axis, we can now refer to the Y-axis as the *imaginary axis*.

Since any complex number can now be split into a real number plus an imaginary number, we can now refer to the first component of a complex number as the *real part* of the complex number and the second component as the *imaginary part* of the complex number. Since (7, -9) = (7, 0) + (0, -9), we can refer to 7 as the real part and to -9 as the imaginary part of the complex number (7, -9). Likewise, since (0, 0) = (0, 0) + (0, 0), we can refer to 0 as the real part and to 0 as the imaginary part of the complex number (0, 0).

As we learned above, we can use the real number 1 in referring to the complex number (1, 0). We will now use the lower case letter "i" to refer to the imaginary number (0, 1). Since (5, 7) = (5, 0) + (0, 7) and since (7, 0) · (0, 1) = (0, 7), we can write (5, 7) = (5, 0) + (7, 0) · (0, 1) = 5+7i. Just as we previously learned that the rational number (5, 7) can be written as the fraction 5/7, now we have learned that the complex number (5, 7) can be written as 5+7i. In general, the complex number (a, b) can be written as a+bi.

Let's now take a look at how complex number additions and multiplications look when written in the "a+bi" form. Here are the addition examples we did on Page 6-21 above:

Example 1.	(4+2i) + (7+8i) = 11+10i
Example 2.	(3-7i) + (-9+2i) = -6-5i
Example 3.	2 + 7 = 9
Example 4.	2i + 5i = 7i
Example 5.	5 + 2i = 5+2i

Here are the multiplication examples we did on Page 6-22 above:

Example 1. $(5+8i) \cdot (6+2i) = 14+58i$
Example 2. $(2-3i) \cdot (-3+2i) = 13i$
Example 3. $(2+3i) \cdot (2-3i) = 13$
Example 4. $(-4+3i) \cdot (5-6i) = -2+39i$
Example 5. $2 \cdot 5 = 10$
Example 6. $3^2 = 9$
Example 7. $(4i) \cdot (5i) = -20$
Example 8. $(2i)^2 = -4$
Example 9. $i^2 = -1$
Example 10. $i^3 = -i$

A Mathematician's View of the Complex Numbers

Let's once again rejoin our mathematician friend in his quest for a set of numbers in which all operations are possible. When we last met him, we had learned that the set of real numbers was closed with respect to addition, subtraction, multiplication and division (except division by zero). Any time you add, subtract or multiply two real numbers, or divide a real number by a nonzero real number, the result is a real number. Unfortunately, even though all roots of positive real numbers yield real number answers, even roots of negative real numbers are never real numbers. The set of real numbers is not closed with respect to root.

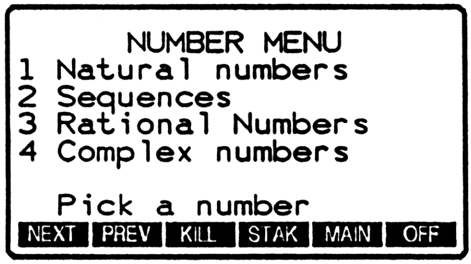
What about the set of complex numbers? Does every root of a complex number yield a complex number answer? In particular, will roots such as $\sqrt{-4}$ and $\sqrt{-1}$, impossible with the set of real numbers, now have answers within the set of complex numbers?

A glance at the complex number multiplication examples above brings us good news. Recalling that $\sqrt{-4}$ asks us to find a number whose square is -4, we see in Example 8 that the square of 2i is -4. Thus, $\sqrt{-4}$ yields the complex number 2i. Likewise, Example 9 reveals that $\sqrt{-1}$ yields the complex number i because $i^2=-1$. It seems that we now have in the set of complex numbers a set in which all roots can be computed.

In fact, a careful study of the six important operations applied to the set of complex numbers would reveal that whenever we add, subtract or multiply any two complex numbers, whenever we divide a complex number by a nonzero complex number, whenever we raise a complex number to a complex number power and whenever we compute a complex root of a complex number, the answer is always a complex number.

We have now found that the set of complex numbers is closed with respect to addition, subtraction, multiplication, division (except by zero), power and root. At last, our mathematician friend has found a set of numbers in which all six important operations can always be done. His quest has finally ended!

THE NUMBER MENU SCREEN



Getting to the Number Menu Screen

There are three ways to get to the Number Menu Screen:

- Press 4 [*row 7, key 2*] on the Main Menu Screen.
- Press NEXT [*row 1, key 1*] on the Save Menu Screen.
- Press PREV [*row 1, key 2*] on the Game Menu Screen.

The Number Menu Screen Menu Bar

There are six keys active on the Number Menu Screen menu bar:

NEXT Press this key [*row 1, key 1*] to go to the Game Menu Screen.

PREV Press this key [*row 1, key 2*] to go to the Save Menu Screen.

KILL Press this key [*row 1, key 3*] to terminate E.Z. Math. When you again want to use E.Z. Math, repeat the start-up sequence described in Chapter 2 which is as follows: Press the ALPHA key [*row 6, key 1*] twice, press E [*row 1 key 5*] and Z [*row 5, key 3*], press the ENTER key [*row 5, key 1*] to get to the E.Z. Math Title Screen which, after about nine seconds, will be replaced by the E.Z. Math Main Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use the HP48SX for other tasks. To return to the E.Z. Math Number Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Number Menu Screen.

The Number Menu Screen Options

There are four options available on the Number Menu Screen:

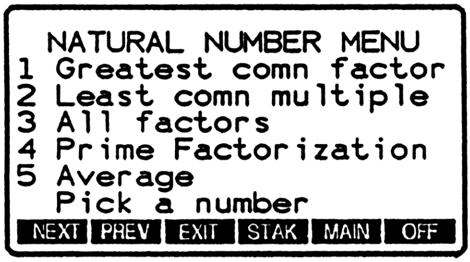
[1] **Natural numbers.** Press this key [*row 8, key 2*] to go to the Natural Number Menu Screen where you can do problems involving factors, prime factorizations, least common multiples, greatest common factors and averages of natural numbers.

[2] **Sequences.** Press this key [*row 8, key 3*] to go to the Sequence Menu Screen where you can compute sequences of perfect nth powers, binomial coefficients, triangular numbers, Fibonacci numbers and multiples.

[3] **Rational numbers.** Press this key [*row 8, key 4*] to get to the Rational Number Menu Screen where you can add, subtract, multiply and divide any set of positive rational numbers.

[4] **Complex numbers.** Press this key [*row 7, key 2*] to get to the Complex Number Menu Screen where you can add, subtract, multiply, divide and raise to powers any set of complex numbers.

THE NATURAL NUMBER MENU SCREEN



Getting to the Natural Number Menu Screen

There are three ways to get to the Natural Number Menu Screen:

- Press $\boxed{1}$ [*row 8, key 2*] on the Number Menu Screen.
- Press NEXT [*row 1, key 1*] on the Complex Number Menu Screen.
- Press PREV [*row 1, key 2*] on the Sequence Menu Screen.

The Natural Number Menu Screen Menu Bar

There are six keys on the Natural Number Menu Screen menu bar:

NEXT Press this key [*row 1, key 1*] to go to the Sequence Menu Screen,

PREV Press this key [*row 1, key 2*] to go to the Complex Number Menu Screen.

EXIT Press this key [*row 1, key 3*] to go to the Number Menu Screen.

STAK Press this key [row 1, key 4] to temporarily leave E.Z. Math to use the HP48SX for other tasks. To return to the E.Z. Math Natural Number Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Natural Number Menu Screen.

The Natural Number Menu Screen Options

There are five options available on the Natural Number Menu Screen:

[1] **Greatest common factor.** Press this key [*row 8, key 2*] to compute the greatest common factor [G.C.F.] of a set of natural numbers. You'll be repeatedly prompted to enter a natural number. Type the first number on your list and press the ENTER key [*row 5, key 1*]. Likewise, enter each of the other numbers on your list. When you are done, press the ENTER key one more time and you'll be taken to the Natural Number Answer Screen where you'll find the G.C.F. of the numbers you entered.

[2] **Least common multiple.** Press this key [*row 8, key 3*] to compute the least common multiple [L.C.M.] of a set of natural numbers. You'll be repeatedly prompted to enter a natural number. Type the first number on your list and press the ENTER key [*row 5, key 1*]. Likewise, enter each of the other numbers on your list. When you are done, press the ENTER key one more time and you'll be taken to the Natural Number Answer Screen where you'll find the L.C.M. of the numbers you entered.

[3] **All factors.** Press this key [*row 8, key 4*] to compute all the factors of a natural number. You'll be prompted to enter a natural number. Type the number and press the ENTER key [*row 5, key 1*]. After a pause, the duration of which will depend upon the number you entered, you'll be taken to the Natural Number Answer Screen where you'll find the complete list of factors you requested.

[4] **Prime factorization.** Press this key [*row 7, key 2*] to compute the prime factorization of a natural number. You'll be prompted to enter a natural number. Type the number and press the ENTER key [*row 5, key 1*]. After a pause, the duration of which will depend upon the number you entered, you'll be taken to the Natural Number Answer Screen where you'll find the prime factorization of the number you entered.

[5] **Average.** Press this key [*row 7, key 3*] to compute the average of a set of natural numbers. You'll be repeatedly prompted to enter a natural number. Type the first number on your list and press the ENTER key [*row 5, key 1*]. Likewise, enter each of the other numbers on your list. When you are done, press the ENTER key one more time and you'll be taken to the Natural Number Answer Screen where you'll find the average of the numbers you entered.

NATURAL NUMBER ANSWER SCREENS

```
The least common
multiple [L.C.M.] of
2, 3, 4, 5, 6, 8 and
12 is: 120

     AGAIN OTHER EXIT MAIN OFF
```

```
The greatest common
factor [G.C.F.] of
200, 100, 250, 750,
450, 600, 900, 1000,
250 and 700 is: 50

     AGAIN OTHER EXIT MAIN OFF
```

```
All factors of
12000 are:
1, 2, 3, 4, 5, 6, 8,
10, 12, 15, 16, 20,
24, 25, 30, 32, 40,
48, 50, 60, 75, 80,
96, 100, 120, 125,
MORE AGAIN OTHER EXIT MAIN OFF
```

```
150, 160, 200, 240,
250, 300, 375, 400,
480, 500, 600, 750,
800, 1000, 1200.
1500, 2000, 2400,
3000, 4000, 6000 and
12000
MORE AGAIN OTHER EXIT      OFF
```

```
Prime factorization of
800 is:
2x2x2x2x2x5x5

    AGAIN OTHER EXIT MAIN OFF
```

```
The average or
arithmetic mean of
45, 58, 36, 48, 62,
74, 68 and 48 is:
54.88

    AGAIN OTHER EXIT MAIN OFF
```

The Natural Number Answer Screen Menu Bar

MORE Press this key [*row 1, key 1*] to see more of the answer. This key is visible and operational when your answer is too long to be displayed on one screen. Each time you press this key, you'll see the next answer screen. When you're at the last answer screen, another press will return you to the first answer screen. Any answer screen can be printed on an HP82240 infrared Printer by pressing first the ALPHA key [*row 6, key 1*] and then the MTH key [*row 2, key 1*].

AGAIN Press this key [*row 1, key 2*] to redo the same problem with different numbers.

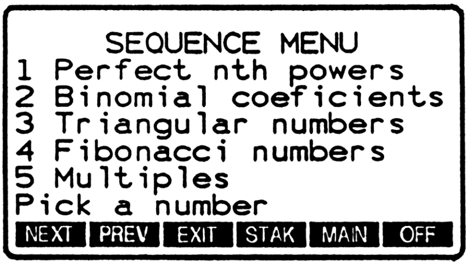
OTHER Press this key [*row 1, key 3*] to return to the Natural Number Menu Screen.

EXIT Press this key [*row 1, key 4*] to return to the Number Menu Screen.

MAIN Press this key [*row 1, key 5*] to return to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Natural Number Answer Screen.

THE SEQUENCE MENU SCREEN



Getting to the Sequence Menu Screen

There are three ways to get to the Sequence Menu Screen:

- Press 2 [*row 8, key 3*] on the Number Menu Screen.
- Press NEXT [*row 1, key 1*] on the Natural Number Menu Screen.
- Press PREV [*row 1, key 2*] on the Rational Number Menu Screen.

The Sequence Menu Screen Menu Bar

There are six keys on the Sequence Menu Screen menu bar:

NEXT Press this key [*row 1, key 1*] to go to the Rational Number Menu Screen.

PREV Press this key [*row 1, key 2*] to go to the Natural Number Menu Screen.

EXIT Press this key [*row 1, key 3*] to go to the Number Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use the HP48SX for other tasks. To return to the E.Z. Math Sequence Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Sequence Menu Screen.

The Sequence Menu Screen Options

There are five options available on the Sequence Menu Screen:

[1] **Perfect n th powers.** Press this key [*row 8, key 2*] to compile a list of perfect n th powers. You'll be asked how many sequence terms you want and what power you want for each term. In each case, type the number and press the ENTER key [*row 5, key 1*]. After a pause, the length of which will depend upon how many sequence terms you requested, you'll be taken to the Sequence Answer Screen where you'll find the requested sequence terms.

[2] **Binomial coefficients.** Press this key [*row 8, key 3*] to compile the list of coefficients obtained when a binomial is raised to a power.. You'll be prompted to enter the power to which the binomial is to be raised. Type the number and press the ENTER key [*row 5, key 1*]. After a pause, the length of which will depend upon how many sequence terms you requested, you'll be taken to the Sequence Answer Screen where you'll find the requested sequence terms.

[3] **Triangular numbers.** Press this key [*row 8, key 4*] to compile a list of triangular numbers. You'll be asked how many sequence terms you want. Type the number and press the ENTER key [*row 5, key 1*]. After a pause, the length of which will depend upon how many sequence terms you requested, you'll be taken to the Sequence Answer Screen where you'll find the requested sequence terms.

[4] **Fibonacci numbers.** Press this key [*row 7, key 2*] to compile a list of Fibonacci numbers, You'll be asked how many sequence terms you want. Type the number and press the ENTER key [*row 5, key 1*]. After a pause, the length of which will depend upon how many sequence terms you requested, you'll be taken to the Sequence Answer Screen where you'll find the requested sequence terms.

[5] **Multiples.** Press this key [*row 7, key 3*] to compile a list of multiples of a natural number. You'll be asked how many sequence terms you want and what number you want multiples of. In each case, type the number and press the ENTER key [*row 5, key 1*]. After a pause, the length of which will depend upon how many sequence terms you requested, you'll be taken to the Sequence Answer Screen where you'll find the requested sequence terms.

SEQUENCE ANSWER SCREENS

```
The first 12 per-
fect 3rd powers are:
1, 8, 27, 64, 125,
216, 343, 512, 729,
1000. 1331 and 1728

AGAIN OTHER EXIT MAIN OFF
```

```
The 10th power bino-
mial coefficients are:
1, 10, 45, 120, 210,
252, 210, 120, 45, 10
and 1

AGAIN OTHER EXIT MAIN OFF
```

```
The first 15 tri-
angular numbers are:
1, 3, 6, 10, 15, 21,
28, 36, 45, 55, 66,
78, 91, 105 and 120

AGAIN OTHER EXIT MAIN OFF
```

```
The first 11 Fibo-
nacchi numbers are:
1, 1, 2, 3, 5, 8, 13,
21, 34, 55 and 89

AGAIN OTHER EXIT MAIN OFF
```

```
The first 27 mul-
tiples of 9 are:
9, 18, 27, 36, 45,
54, 63, 72, 81, 90,
99, 108, 117, 126,
135, 144, 153, 162,
171, 180, 189, 198,
MORE AGAIN OTHER EXIT MAIN OFF
```

```
207, 216, 225, 234
and 243






MORE AGAIN OTHER EXIT MAIN OFF
```

The Sequence Answer Screen Menu Bar

MORE Press this key [*row 1, key 1*] to see more of the answer. This key is visible and operational when your answer is too long to be displayed on one screen. Each time you press this key, you'll see the next answer screen. When you're at the last answer screen, another press will return you to the first answer screen. Any answer screen can be printed on an HP82240 infrared Printer by pressing first the ALPHA key [*row 6, key 1*] and then the MTH key [*row 2, key 1*].

AGAIN Press this key [*row 1, key 2*] to redo the same problem with different numbers.

OTHER Press this key [*row 1, key 3*] to return to the Sequence Menu Screen.

EXIT Press this key [*row 1, key 4*] to return to the Number Menu Screen.

MAIN Press this key [*row 1, key 5*] to return to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Sequence Answer Screen.

THE RATIONAL NUMBER MENU SCREEN

```
RATIONAL NUMBER MENU
1 Add mixed numbers
2 Subtract mxd numbers
3 Multiply mxd numbers
4 Divide mixed numbers

  Pick a number
NEXT | PREV | EXIT | STAK | MAIN | OFF
```

Getting to the Rational Number Menu Screen

There are three ways to get to the Rational Number Menu Screen:

- Press 3 [*row 8, key 4*] on the Number Menu Screen.
- Press NEXT [*row 1, key 1*] on the Sequence Menu Screen.
- Press PREV [*row 1, key 2*] on the Complex Number Menu Screen.

The Rational Number Menu Screen Menu Bar

There are six keys on the Rational Number Menu Screen menu bar:

NEXT Press this key [*row 1, key 1*] to go to the Complex Number Menu Screen.

PREV Press this key [*row 1, key 2*] to go to the Sequence Menu Screen.

EXIT Press this key [*row 1, key 3*] to go to the Number Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use the HP48SX for other tasks. To return to the E.Z. Math Rational Number Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Rational Number Menu Screen.

The Rational Menu Screen Options

There are four options available on the Rational Number Menu Screen:

[1] **Add mixed numbers.** Press this key [*row 8, key 2*] to add a set of mixed numbers. You'll be repeatedly prompted to enter the whole number, numerator and denominator of each mixed number. In each case, type the number and press the ENTER key [*row 5, key 1*]. To enter a whole number, enter it as the whole number, zero as the numerator and any number as the denominator. To enter a fraction, enter zero as the whole number, your fraction's numerator as the numerator and your fraction's denominator as the denominator. When you are done, press the ENTER key one more time to go to the Rational Number Answer Screen for your answer.

[2] **Subtract mxd numbers.** Press this key [*row 8, key 3*] to subtract a set of mixed numbers. You'll be repeatedly prompted to enter the whole number, numerator and denominator of each mixed number. In each case, type the number and press the ENTER key [*row 5, key 1*]. To enter a whole number, enter it as the whole number, zero as the numerator and any number as the denominator. To enter a fraction, enter zero as the whole number, your fraction's numerator as the numerator and your fraction's denominator as the denominator. When you are done, press the ENTER key one more time to go to the Rational Number Answer Screen for your answer.

[3] **Multiply mxd numbers.** Press this key [*row 8, key 4*] to multiply a set of mixed numbers. You'll be repeatedly prompted to enter the whole number, numerator and denominator of each mixed number. In each case, type the number and press the ENTER key [*row 5, key 1*]. To enter a whole number, enter it as the whole number, zero as the numerator and any number as the denominator. To enter a fraction, enter zero as the whole number, your fraction's numerator as the numerator and your fraction's denominator as the denominator. When you are done, press the ENTER key one more time to go to the Rational Number Answer Screen for your answer.

[4] **Divide mixed numbers.** Press this key [*row 7, key 2*] to divide as set of mixed numbers. You'll be repeatedly prompted to enter the whole number, numerator and denominator of each mixed number. In each case, type the number and press the ENTER key [*row 5, key 1*]. To enter a whole number, enter it as the whole number, zero as the numerator and any number as the denominator. To enter a fraction, enter zero as the whole number, your fraction's numerator as the numerator and your fraction's denominator as the denominator. When you are done, press the ENTER key one more time to go to the rational Number Answer Screen for your answer.

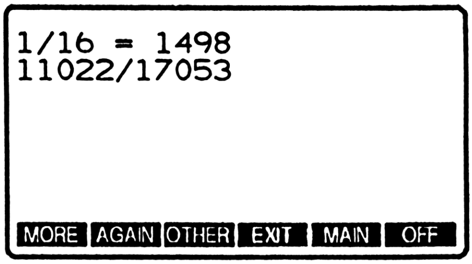
RATIONAL NUMBER ANSWER SCREENS

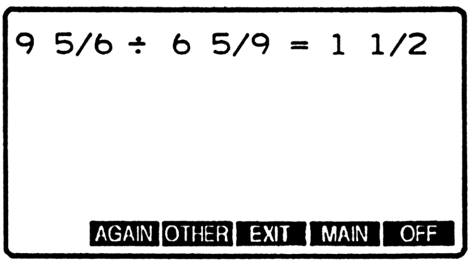
```
12 3/4 + 5 2/3 + 8
1/2 + 4/5 + 15 + 2
7/10 + 3 5/6 + 4 7/8
+ 4 5/6 + 2 3/5 + 14
+ 5/6 + 7 2/3 + 3 2/5
+ 8 1/6 + 6 1/4 + 12
+ 15 + 4/5 + 4 5/8 =
MORE AGAIN OTHER EXIT MAIN OFF
```

```
134 3/10

MORE AGAIN OTHER EXIT MAIN OFF
```

```
5 1/7 - 2 1/6 = 2
41/42

AGAIN OTHER EXIT MAIN OFF
```

```
1 1/2 x 2 2/3 x 3 3/4
x 4 4/5 x 5 5/6 x 6
6/7 x 7 7/8 x 1/2 x
1/4 x 1/80 x 1 2/3 x
1 2/3 x 2 3/4 x 3 4/5
x 4 5/8 x 2 4/5 x 3/8
x 2/7 x 4/5 x 21 x
MORE AGAIN OTHER EXIT MAIN OFF
```





The Rational Number Answer Screen Menu Bar

MORE Press this key [*row 1, key 1*] to see more of the answer. This key is visible and operational when your answer is too long to be displayed on one screen. Each time you press this key, you'll see the next answer screen. When you're at the last answer screen, another press will return you to the first answer screen. Any answer screen can be printed on an HP82240 infrared Printer by pressing first the ALPHA key [*row 6, key 1*] and then the MTH key [*row 2, key 1*].

AGAIN Press this key [*row 1, key 2*] to redo the same problem with different numbers.

OTHER Press this key [*row 1, key 3*] to return to the Rational Number Menu Screen.

EXIT Press this key [*row 1, key 4*] to return to the Number Menu Screen.

MAIN Press this key [*row 1, key 5*] to return to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Rational Number Answer Screen.

THE COMPLEX NUMBER MENU SCREEN

```
  COMPLEX NUMBER MENU
1 Addition
2 Subtraction
3 Multiplication
4 Division
5 Power
  Pick a number
NEXT PREV EXIT STAK MAIN OFF
```

Getting to the Complex Number Menu Screen

There are three ways to get to the Complex Number Menu Screen:

- Press 4 [*row 7, key 2*] on the Number Menu Screen.
- Press NEXT [*row 1, key 1*] on the Rational Number Menu Screen.
- Press PREV [*row 1, key 2*] on the Natural Number Menu Screen.

The Complex Number Menu Screen Menu Bar

There are six keys on the Complex Number Menu Screen menu bar:

NEXT Press this key [*row 1, key 1*] to go to the Natural Number Menu Screen.

PREV Press this key [*row 1, key 2*] to go to the Rational Number Menu Screen.

EXIT Press this key [*row 1, key 3*] to go to the Number Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use the HP48SX for other tasks. To return to the E.Z. Math Complex Number Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Complex Number Menu Screen.

The Complex Number Menu Screen Options

There are five options on the Complex Number Menu Screen:

[1] **Addition.** Press this key [*row 8, key 2*] to add complex numbers. You'll be repeatedly prompted to enter the real and imaginary part of each complex number. In each case, type the number and press the ENTER key [*row 5, key 1*]. To enter a real number, enter it as the real part and zero as the imaginary part. To enter an imaginary number, enter zero as the real part. When you are done, press the ENTER key one more time to go to the Complex Number Answer Screen for your answer.

[2] **Subtraction.** Press this key [*row 8, key 3*] to subtract complex numbers. You'll be repeatedly prompted to enter the real and imaginary part of each complex number. In each case, type the number and press the ENTER key [*row 5, key 1*]. To enter a real number, enter it as the real part and zero as the imaginary part. To enter an imaginary number, enter zero as the real part. When you are done, press the ENTER key one more time to go to the Complex Number Answer Screen for your answer.

[3] **Multiplication.** Press this key [*row 8, key 4*] to multiply complex numbers. You'll be repeatedly prompted to enter the real and imaginary part of each complex number. In each case, type the number and press the ENTER key [*row 5, key 1*]. To enter a real number, enter it as the real part and zero as the imaginary part. To enter an imaginary number, enter zero as the real part. When you are done, press the ENTER key one more time to go to the Complex Number Answer Screen for your answer.

[4] **Division.** Press this key [*row 7, key 2*] to divide complex numbers. You'll be repeatedly prompted to enter the real and imaginary part of each complex number. In each case, type the number and press the ENTER key [*row 5, key 1*]. To enter a real number, enter it as the real part and zero as the imaginary part. To enter an imaginary number, enter zero as the real part. When you are done, press the ENTER key one more time to go to the Complex Number Answer Screen for your answer.

[5] **Power.** Press this key [*row 7, key 3*] to raise complex numbers to powers. You'll be repeatedly prompted to enter the real and imaginary part of each complex number. In each case, type the number and press the ENTER key [*row 5, key 1*]. To enter a real number, enter it as the real part and zero as the imaginary part. To enter an imaginary number, enter zero as the real part. When you are done, press the ENTER key one more time to go to the Complex Number Answer Screen for your answer.

COMPLEX NUMBER ANSWER SCREENS

```
(2+3i)+(5+6i)+(-8+9i)
+(-4+7i)+(1+i)+(2-4i)
+(-6+3i)+(8i)+(-2+5i)
+(1)+(0)+(4+8i)
+(12+32i)+(-14-56i)
+(74+2i)+(3+8i)+(5-8i)
+(-9-6i)+(-2+5i)
MORE AGAIN OTHER EXIT MAIN OFF
```

```
+(6-3i)+(-5-4i)+(-8)
+(-9-12i)+(8+3i)=56+7i
MORE AGAIN OTHER EXIT MAIN OFF
```

```
(5+9i)-(2+3i)-(5+i)
-(1+9i)-(8i)-(2+8i)
=-5-20i
AGAIN OTHER EXIT MAIN OFF
```

```
(5+3i)x(2i)x(8i)x(3)
x(6)x(4-i)x(1+i)
=-4608-8640i
AGAIN OTHER EXIT MAIN OFF
```

```
(2+3i)÷(5-6i)
=-0.131+0.443i

AGAIN OTHER EXIT MAIN OFF
```

```
(4+5i)^(2+3i)
=1.316+2.458i

AGAIN OTHER EXIT MAIN OFF
```

The Complex Number Answer Screen Menu Bar

MORE Press this key [*row 1, key 1*] to see more of the answer. This key is visible and operational when your answer is too long to be displayed on one screen. Each time you press this key, you'll see the next answer screen. When you're at the last answer screen, another press will return you to the first answer screen. Any answer screen can be printed on an HP82240 infrared Printer by pressing first the ALPHA key [*row 6, key 1*] and then the MTH key [*row 2, key 1*].

AGAIN Press this key [*row 1, key 2*] to redo the same problem with different numbers.

OTHER Press this key [*row 1, key 3*] to return to the Complex Number Menu Screen.

EXIT Press this key [*row 1, key 4*] to return to the Number Menu Screen.

MAIN Press this key [*row 1, key 5*] to return to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Complex Number Answer Screen.

Chapter Seven

Game

WHAT'S IN THIS CHAPTER

This chapter describes the E.Z. Math Game Module. The Game Module consists of two number guessing games which you may find both enjoyable and stimulating.

In the first game, the HP48SX randomly selects a natural number. Depending upon which option you select, the number can be anywhere from between 1 and 100 up to between 1 and 1,000,000. When asked to do so, enter the number you believe was selected. If you guess incorrectly, you'll be told whether your guess is too high or too low. You repeatedly make guesses until you finally get the right number, at which point you'll be told how many guesses it took.

In the second game, the tables are turned. This time, you pick a natural number anywhere from between 1 and 100 up to between 1 and 1,000,000, depending upon which option you selected. The HP48SX tells you what number it believes you selected. If the guess is incorrect, you tell it whether its guess was too high or too low. The HP48SX will repeatedly make guesses until it finally guesses the correct number, at which point you'll be told how many guesses it took.

These two games are simple strategy games. Although it's hoped you'll enjoy them as pleasant recreations, the reason for their inclusion in E.Z. Math is mostly educational.

If you simply pick numbers at random, finding the correct number becomes merely a matter of luck, and can take a long time. If you discover and use a logical strategy, you can find the correct number with fewer guesses. See if you can find the best system for guessing the selected number with the fewest guesses. See if you can discover what system the HP48SX uses as it tries to guess the number you selected.

Despite the fact that using E.Z. Math to solve problems requires almost no technical knowledge of the HP48SX, we do hope that you'll use spare moments from time to time to browse through the Owner's Manual that came with your calculator, and eventually try your hand at programming. You may well find making a program to be a most enjoyable, stimulating and satisfying experience, as well as much easier than you thought it would be.

Should you try your hand at programming the HP48SX, you may find it very interesting to try to program a number guessing game such as those included in the E.Z. Math Game Module. The truth is that a number guessing game involves only beginning level programming techniques and is therefore a very good project for someone just learning to program.

Now let's turn the page and read more details about the E.Z. Math Game Module.

THE GAME MENU SCREEN

```
        GAME MENU
1 You guess number
2 HP guesses number


  Pick a number
NEXT PREV KILL STAK MAIN OFF
```

Getting to the Game Menu Screen

There are three ways to get to the Game Menu Screen:

- Press 5 [*row 7, key 3*] on the Main Menu Screen.
- Press NEXT [*row 1, key 1*] on the Number Menu Screen.
- Press PREV [*row 1, key 2*] on the Music Menu Screen.

The Game Menu Screen Menu Bar

There are six keys active on the Game Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the Music Menu Screen.

PREV Press this key [*row 1, key 2*] to go to the Number Menu Screen.

KILL Press this key [*row 1, key 3*] to terminate E.Z. Math. When you again want to use E.Z. Math, repeat the start-up sequence described in Chapter 2 which is as follows: Press the ALPHA key [*row 6, key 1*] twice, press E [*row 1, key 5*] and Z [*row 5, key 3*], press the ENTER key [*row 5, key 1*] to get to the E.Z. Math Title Screen which, after about nine seconds, will be replaced by the E.Z. Math Main Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the E.Z. Math Game Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Game Menu Screen.

The Game Menu Screen Options

There are two options available on the Game Menu Screen:

[1] **You guess number.** Press this key [*row 8, key 2*] to play a number guessing game in which you guess a number picked by the HP48SX. You will be taken to the Guess Number Screen to select the bounds within which the HP48SX will choose the number to be guessed.

[2] **HP guesses number.** Press this key [*row 8, key 3*] to play a number guessing game in which the HP48SX will guess a number you have picked. You will be taken to the Guess Number Screen to select the bounds within which you will choose the number to be guessed.

THE GUESS NUMBER MENU SCREEN

```
   GUESS NUMBER MENU
1 From 1 to 100
2 From 1 to 1,000
3 From 1 to 10,000
4 From 1 to 100,000
5 From 1 to 1,000,000
  Pick a number
                 EXIT  MAIN  OFF
```

Getting to the Guess Number Menu Screen

There are two ways to get to the Guess Number Menu Screen:

- Press [1] [*row 8, key 2*] on the Game Menu Screen.
- Press [2] [*row 8, key 3*] on the Game Menu Screen.

The Guess Number Menu Screen Menu Bar

There are three keys active on the Guess Number Menu Screen Menu Bar:

EXIT Press this key [*row 1, key 4*] to go to the Game Menu Screen.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

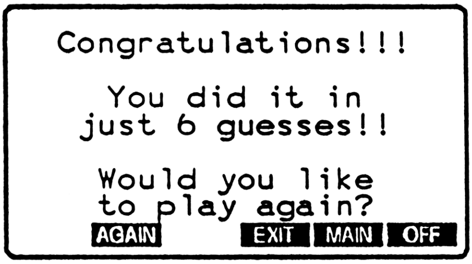
OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Guess Number Menu Screen.

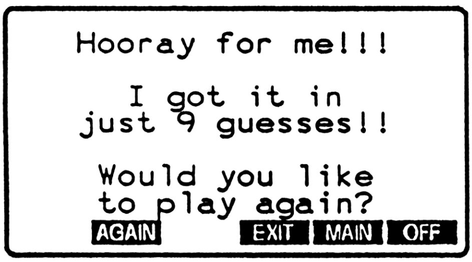
The Guess Number Menu Screen Options

There are five options available on the Guess Number Menu Screen:

1. **From 1 to 100.** Press this key [*row 8, key 2*] to limit the number to be guessed to a natural number from 1 to 100.
2. **From 1 to 1,000.** Press this key [*row 8, key 3*] to limit the number to be guessed to a natural number from 1 to 1,000.
3. **From 1 to 10,000.** Press this key [*row 8, key 4*] to limit the number to be guessed to a natural number from 1 to 10,000.
4. **From 1 to 100,000.** Press this key [*row 7, key 2*] to limit the number to be guessed to a natural number from 1 to 100,000.
5. **From 1 to 1,000,000.** Press this key [*row 7, key 3*] to limit the number to be guessed to a natural number from 1 to 1,000,000.

THE GAME FINISH SCREEN





Getting to the Game Finish Screen

There are two ways to get to the Game Finish Screen:

- Press 1 [*row 8, key 2*] on the Guess Number Menu Screen.
- Press 2 [*row 8, key 3*] on the Guess Number Menu Screen.

The Game Finish Screen Menu Bar

There are four keys active on the Game Finish Screen Menu Bar:

AGAIN Press this key [*row 1, key 2*] to go to the Guess Number Menu Screen to play the game again with a different number.

EXIT Press this key [*row 1, key 4*] to go to the Game Menu Screen.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Game Menu Screen.

Chapter Eight

Music

WHAT'S IN THIS CHAPTER

This chapter describes the E.Z. Math Music Module. The Music Module is designed to play musical scales and original electronic music and to display a series of note value tables.

Having a scientific calculator with a music making ability is really quite remarkable. The HP48SX has this ability and the E.Z. Math Music Module is designed to help you to begin exploring this capability.

The Music Module's first option let's you hear how musical scales sound when played on the HP48SX. You may choose to hear the major scale which is frequently used to compose brighter, happier melodies, the minor scale which is frequently used to compose darker, sadder melodies, and the chromatic scale which is frequently used to compose more sensuous, unsettled melodies. You may listen to each of these scales in any of ten different speeds, ranging from extremely slow to extremely fast.

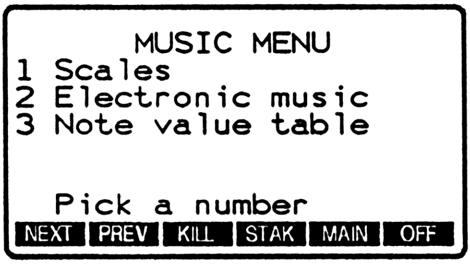
The Music Module's second option let's you hear original electronic music composed by the HP48SX. Just decide which of the three scales and ten speeds described in the last paragraph you want used in the composition. The HP48SX will then randomly pick and play notes from the scale you selected at the speed you selected. Listen carefully to hear if a melody is ever repeated.

The Music Module's third option let's you see note value tables showing all 8 octaves playable by the HP48SX's built-in tone generator which produces tones by causing the surrounding air to vibrate. The speed at which the air vibrates is called the *frequency* of the tone and is measured in units called *Hertz* [abbreviated Hz.]. A low tone has a low frequency and a high tone has a high frequency. The note value tables list the frequencies of the tones generated by the HP48SX which correspond to the notes played on a piano or other instrument.

If you decide to explore more of the capabilities of the HP48SX by browsing from time to time through the Owner's Manual you received when you purchased your calculator, you might eventually feel like trying your hand at a bit of simple programming. One of the most enjoyable ways to take the plunge is by making a simple music program. Using the note value tables to translate notes into frequencies, it's not very difficult to put some of your favorite melodies into a program.

Now let's turn the page and read some more details about the E.Z. Math Music Module.

THE MUSIC MENU SCREEN



Getting to the Music Menu Screen

There are three ways to get to the Music Menu Screen:

- Press **6** [*row 7, key 4*] on the Main Menu Screen.
- Press **NEXT** [*row 1, key 1*] on the Game Menu Screen.
- Press **PREV** [*row 1, key 2*] on the Graph Menu Screen.

The Music Menu Screen Menu Bar

There are six keys active on the Music Menu Screen Menu Bar:

NEXT Press this key [*row 1, key 1*] to go to the Graph Menu Screen.

PREV Press this key [*row 1, key 2*] to go to the Game Menu Screen.

KILL Press this key [*row 1, key 3*] to terminate E.Z. Math. When you again want to use E.Z. Math, repeat the start-up sequence described in Chapter 2 which is as follows: Press the ALPHA key [*row 6, key 1*] twice, press E [*row 1, key 5*] and Z [*row 5, key 3*], press the ENTER key [*row 5, key 1*] to get to the E.Z. Math Title Screen which, after about nine seconds, will be replaced by the E.Z. Math Main Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the E.Z. Math Music Menu Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Music Menu Screen.

The Music Menu Screen Options

There are three options available on the Music Menu Screen:

1. **Scales.** Press this key [*row 8, key 2*] to hear scales. You'll be taken to the Scale Menu Screen where you can select the scale you'd like to hear.
2. **Electronic Music.** Press this key [*row 8, key 3*] to hear an original electronic piece composed on the spot by your HP48SX. You'll be taken to the Scale Menu Screen to select the scale in which you'd like to piece to be composed.
3. **Note value table.** Press this key [*row 8, key 4*] to go to the Note Value Tables which can assist you if you want to use the HP48SX "beep" command to make your own music and sound effects.

THE SCALE MENU SCREEN

```
        SCALE MENU
1 Major scale
2 Minor scale
3 Chromatic Scale

  Pick a number
              EXIT MAIN OFF
```

Getting to the Scale Menu Screen

There are two ways to get to the Scale Menu Screen:

- Press 1 [*row 8, key 2*] on the Music Menu Screen.
- Press 2 [*row 8, key 3*] on the Music Menu Screen.

The Scale Menu Screen Menu Bar

There are three keys active on the Scale Menu Screen Menu Bar:

EXIT Press this key [*row 1, key 4*] to go to the Music Menu Screen.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Music Menu Screen.

The Scale Menu Screen Options

There are three options available on the Scale Menu Screen:

1. **Major scale.** Press this key [*row 8, key 2*] to play using the major scale.
2. **Minor scale.** Press this key [*row 8, key 3*] to play using the minor scale.
3. **Chromatic scale.** Press this key [*row 8, key 4*] to play using the chromatic scale.

THE TEMPO MENU SCREEN

```
        TEMPO MENU
1 Largo       6 Moderato
2 Adagio      7 Allegro
3 Adagietto   8 Presto
4 Andante     9 Vivace
5 Andantino   0 Wow!!
  Pick a number
             EXIT MAIN OFF
```

Getting to the Tempo Menu Screen

There are three ways to get to the Tempo Menu Screen:

- Press 1 [*row 8, key 2*] on the Scale Menu Screen.
- Press 2 [*row 8, key 3*] on the Scale Menu Screen.
- Press 3 [*row 8, key 4*] on the Scale Menu Screen.

The Tempo Menu Screen Menu Bar

There are three keys active on the Tempo Menu Screen Menu Bar:

EXIT Press this key [*row 1, key 4*] to go to the Music Menu Screen.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the E.Z. Math Music Menu Screen.

The Tempo Menu Screen Options

There are ten options available on the Tempo Menu Screen:

1 **Largo.** Press this key [*row 8, key 2*] to play a scale or music at an extremely slow speed.

2 **Adagio.** Press this key [*row 8, key 3*] to play a scale or music at a very slow speed.

3 **Adagietto.** Press this key [*row 8, key 4*] to play a scale or music at a slower speed.

4 **Andante.** Press this key [*row 7, key 2*] to play a scale or music at a slow speed.

5 **Andantino.** Press this key [*row 7 key 3*] to play a scale or music at a moderately slow speed.

6 **Moderato.** Press this key [*row 7, key 4*] to play a scale or music at a moderately fast speed.

7 **Allegro.** Press this key [*row 6, key 2*] to play a scale or music at a fast speed.

8 **Presto.** Press this key [*row 6, key 3*] to play a scale or music at a faster speed.

9 **Vivace.** Press this key [*row 6, key 4*] to play a scale or music at a very fast speed.

0 **Wow!!.** Press this key [*row 9, key 2*] to play a scale or music at an extremely fast speed.

THE NOTE VALUE TABLE SCREEN

OCTAVE 1

C	16Hz	F#	23Hz
C#	17Hz	G	24Hz
D	18Hz	G#	25Hz
D#	19Hz	A	27Hz
E	21Hz	A#	29Hz
F	22Hz	B	31Hz

MORE | BACK | OTHER | STAK | MAIN | OFF

Getting to the Note Value Table Screen

There is one way to get to the Note Value Table Screen:

• Press [3] [*row 8, key 4*] on the Music Menu Screen.

The Note Value Table Screen Menu Bar

There are six keys active on the Note Value Table Screen Menu Bar:

MORE Press this key [*row 1, key 1*] to go to the next octave Note Value Table Screen. Press this key while looking at the Octave 8 Note Value Table Screen to go to the Octave 1 Note Value Table Screen.

BACK Press this key [*row 1, key 2*] to go to the previous octave Note Value Table Screen. Press this key while looking at the Octave 1 Note Value Table Screen to go to the Octave 8 Note Value Table Screen.

OTHER Press this key [*row 1, key 3*] to go to the Music Menu Screen.

STAK Press this key [*row 1, key 4*] to temporarily leave E.Z. Math to use your HP48SX for other tasks. To return to the same E.Z. Math Note Value Table Screen, press the CST key [*row 2, key 3*] and then press the CONT key [*row 1, key 1*] on the menu bar.

MAIN Press this key [*row 1, key 5*] to go to the Main Menu Screen.

OFF Press this key [*row 1, key 6*] to turn off your HP48SX. To turn it back on, press the ON key [*row 9, key 1*] and you will find yourself back in the same E.Z. Math Note Value Table Screen.

Printing a Note Value Table Screen

Any Note Value Table Screen can be printed on an HP82240 infrared printer as follows:

1. Turn on the printer and set it down on a flat surface.

2. Press the orange LEFT-SHIFT key [*row 7, key 1*].

3. Press the MTH key [*row 2, key 1*].

4. Set the HP48SX down so that its upper screen end is facing the bottom end of the printer.

5. Press the ENTER key [*row 5, key 1*] to begin printing.

6. Press the ON [*row 9, key 1*] key to cancel printing before commencement or to stop printing before completion.