HEWLETT PACKARD

HP 48G Series
Examples in Math Education - Part 1

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Last Revision 10/13/93

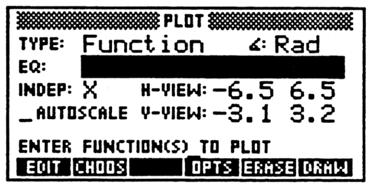
1

General Graphing

Example 1: Inequalities

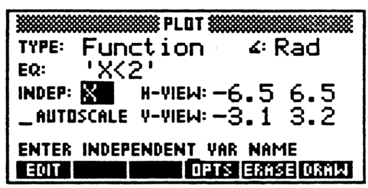
Enter the Plot environment by pressing

[→] [PLOT]



Enter the inequality by pressing

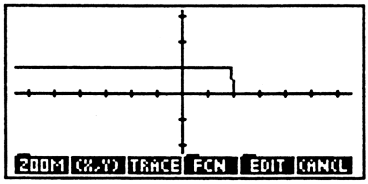
[DEL] OK EDIT X [→] [CHARS]
-64 -64 [◄] *(4 times)* ECHO
[ON] 2 OK



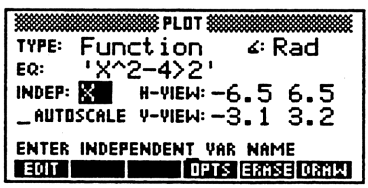
Note

Short-cut keystrokes are given in the lower left corner of the screen in the Chars environment. For example, [α] [←] 2 writes "<".

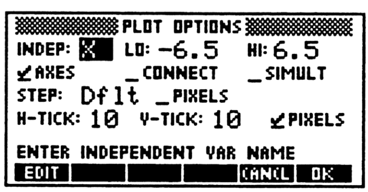
If CONNECT is checked in the Plot Options screen, then pressing ERASE DRAW plots:



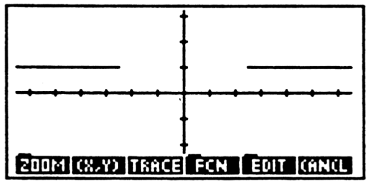
Trace this graph and recognize that the inequality function "less than" has a value of 1 when true and 0 when false. All inequalities, weak and strong, will have this characteristic. A better picture can be obtained by removing the check mark from CONNECT in the Plot Options screen. Thus when graphing an expression such as $x^2-4>2$, set the Plot screen to read:



and the Plot Options screen to show:



Pressing ERASE DRAW produces:

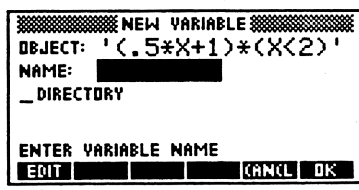


The truth set is the projected image of the plotted portion of the line y = 1 on the x-axis.

Example 2: Piece-Wise Defined Functions

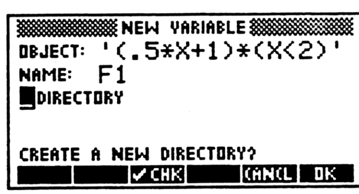
We take advantage of the inequality function values and resulting graphs to plot "piece-wise defined" functions. To plot f(x) = .5x + 1 for x < 2, enter the Plot environment, move the highlight to EQ:, and type the expression by pressing

CHOOS NEW ['] [◄] [()] .5 [×] [α] X [+] 1 [►] [×] [◄] [()] [α] X [►] [CHARS] -64 -64 [◄] *(4 times)* ECHO [ON] 2 OK

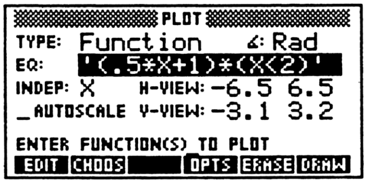


Give the expression a name by pressing

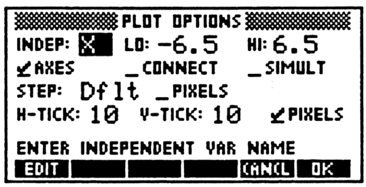
[α] F 1 OK



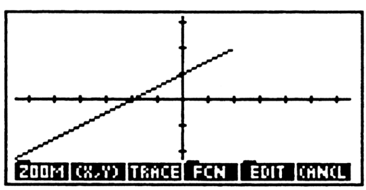
Move the highlight to F1 and press OK OK to see this screen:



Press OPTS and set the Plot Options screen to look like this:



Press OK to return to the main Plot screen, then press ERASE DRAW to view the graph.



Trace this graph. You may want to replot it with the CONNECT option checked (in the Plot Options screen) to note the difference. Note that by multiplying by $(x < 2)$, one is multiplying by 1 when x is less than 2 and by multiplying by 0 when x is not less than 2.

Example 3: Piece-Wise Defined Functions

Plot:

$$f(x) = \begin{cases} (x+2)^2, & x < -1 \\ 3 - x, & x \geq 2 \end{cases}$$

Enter these two functions in the variable menu. See example 2 for keystroke details.

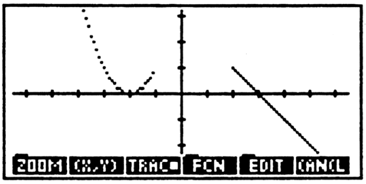
$'(x + 2)^2 \times (x < -1)'$ as F1

and

$'(3 - x) \times (x \geq 2)'$ as F2

Then enter the third function 'F1 + F2' as F3.

Return to the main Plot screen and move the highlight to EQ:. Press CHOOS, select F3, and press OK. Pressing ERASE DRAW plots:



Tracing this graph illustrates what is going on.

Extension

1. Using graphic techniques, solve: $(x-3)^2 < .5x + 2$
2. Plot:

$$f(x) = \begin{cases} \dfrac{(x^3 - 5)}{3x}, & x < 3 \\ 2, & -3 \leq x \leq 2 \\ 8x - x^2 - 10, & x > 2 \end{cases}$$

3. Plot:

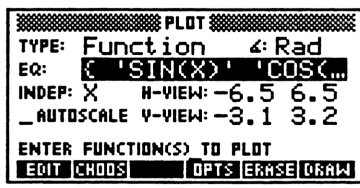
$$f(x) = \begin{cases} x^2 + 5x + 4, & x \leq -1 \\ .5x + .5, & -1 < x \leq 3 \\ -x^2 + 9x - 18, & x > 3 \end{cases}$$

Example 4: Justifying Derivatives

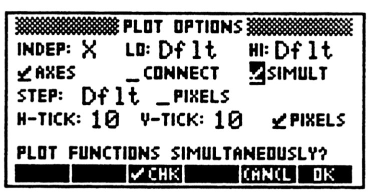
Enter the Plot environment and enter {'sin(x)' 'cos(x)'} as the two functions to plot. In the Plot Options screen, check SIMULT for simultaneous plotting. Use all other default settings.

Move the highlight to EQ: and enter the equation by pressing the following:

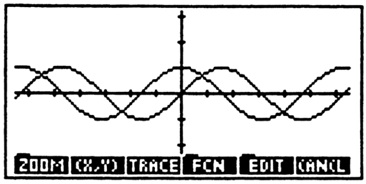
DEL OK ← {} ' SIN α
X ▶ ▶ SPC ' COS α X
OK



Press OPTS and the the Plot Options screen to look like the following:



Press OK ERASE DRAW to draw the graph:



One of these curves is the graph of f(x) and the other is the graph of f'(x). Which curve is the graph of the original function? With knowledge of the derivative only, give as many reasons as you can for your decision.

For example:

1. x-intercepts at local maximums and minimums of f(x)
2. increasing function where derivative is positive
3. decreasing function where derivative is negative
4. derivative of sine is cosine
5. local maximum/minimum of derivative when f(x) is steepest

2

Conics

Example 1: Matching Conic Types from Graphs to Algebraic Forms

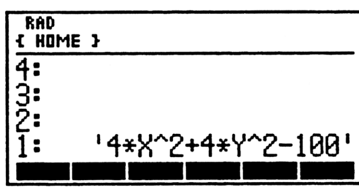
Via the EquationWriter or command line, put the following biquadratics on stack lines 1 through 5, in any order:

$$4x^2 + 4y^2 - 100 \qquad x^2 + 4y^2 - 100$$
$$x^2 + y^2 - 100 \qquad x^2 - y^2 - 9$$
$$x^2 - y^2 - 9$$

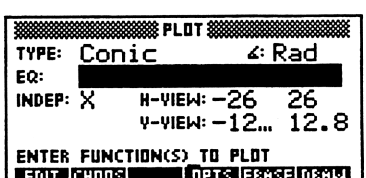
Command line entry example:

In the stack environment, press

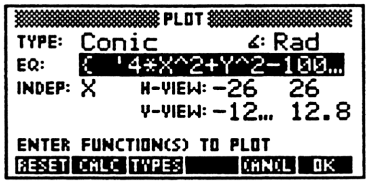
['] 4 [×] [α] X [y^x] 2 [+] 4 [×] [α]
Y [y^x] 2 [−] 100 [ENTER]



Repeat the above steps to enter the remaining four biquadratics. Then put a 5 on the command line and press [PRG] LIST →LIST. The five expressions should be in a list on line 1. Enter the Plot environment and set the screen to look like this:



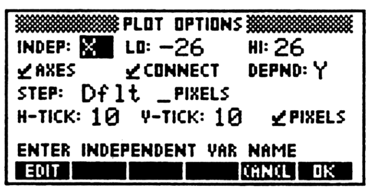
Move the highlight to EQ: and press (NXT) CALC OK. The set of equations should be entered in EQ:.



Note

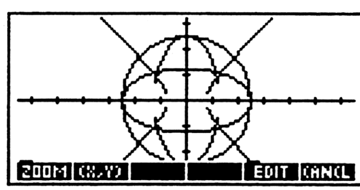
Alternate methods of entering the equations would include defining each of them with a name, e.g. F1, F2, F3, F4, F5, then selecting them in the variable menu, or forming a set {F1, F2, F3, F4, F5} and naming it S, then selecting S from the variable menu for EQ:.

Press (NXT) set the Plot Options screen to be:



Draw the graph by pressing

OK ERASE DRAW



The question is then to match the graphic form with the algebraic form.

Example 2: Piercing the Ellipse

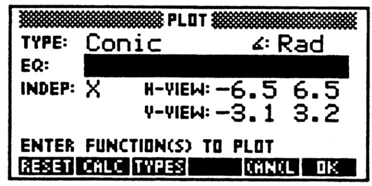
Via the EquationWriter or command line, enter the expression:

$'11(x+3)^2 + 4(y-1)^2 - 16 - 5xy'$

on stack line 1.

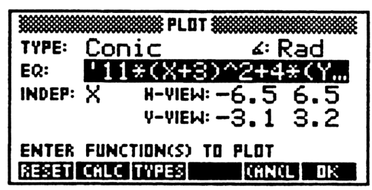
Reset the Plot screen.

NXT RESET ▼ OK



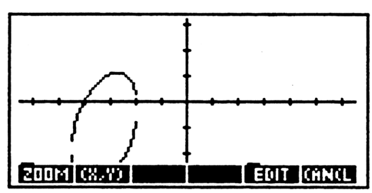
Move the highlight to EQ: and enter the expression from stack line 1 into EQ: by pressing

DEL OK NXT CALC OK



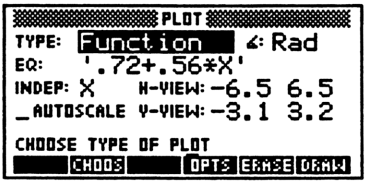
Draw the graph.

NXT ERASE DRAW

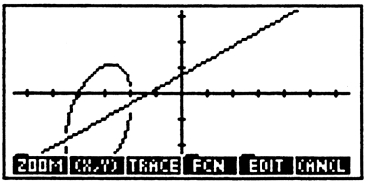


The challenge is to write the equation of a straight line that will pierce the ellipse through the "openings". (Solution: Find the coordinates of the centers of the openings then use the two-point form of a straight line.)

Press CANCL to return to the Plot screen, Change it to read:



Then press DRAW. (Do not press ERASE.)



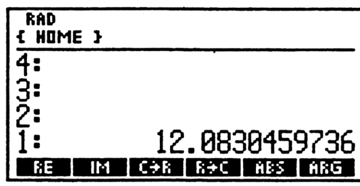
3

Distance Formula

The Reverse Polish Notation (RPN), complex number arithmetic, and stack architecture permits this procedure for finding the distance between two points on a rectangular coordinate system.

Find the distance between (1, -3) and (-4, 8):

(1, -3) [ENTER] (-4, 8) [−] [MTH]
[NXT] COMPL ABS



The subtraction of the two ordered pairs finds:

(the difference in x coordinates, the difference in y coordinates)

and the absolute value (ABS) finds the distance to the origin of the complex number represented by the ordered pair or *modulus* of the complex number. (ABS can also be found by [MTH] REAL [NXT] ABS.)

Note

Ordered pairs can be entered on the stack with a space between the coordinates instead of a comma.

4

Linear Systems

In the interactive computer games *Green Globs*, *Algebra Arcade*, and some forms of *Battleship*, the goal is to write functions which "shoot down" globs or ghosts on a rectangular grid, thus the need is to write functions that pass through specific points.

Find the cubic polynomial that passes through the points (-3, 12), (-1,4), (1, 12), and (2, -8). That is, solve:

$ax^3 + bx^2 + cx + d = y$

for the (x, y) values of the points. *(See the note at the end of this example.)*

$-27a + 9b - 3c + d = 12$

$-a + b - c + d = 4$

$a + b + c + d = 12$

$8a + 4b + 2c + d = -8$

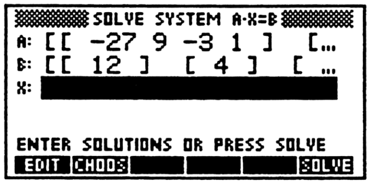
In the Solve environment, select `Solve lin sys...`, then enter the MatrixWriter and enter the coefficient matrix above as A, and the column of constants as B.

Enter the first row by pressing the following:

[→] [SOLVE] [▼] [▼] [▼] `OK` [→] [MATRIX] 27 [+/-] [ENTER] 9 [ENTER] 3 [+/-] [ENTER] 1 [ENTER] [▼].

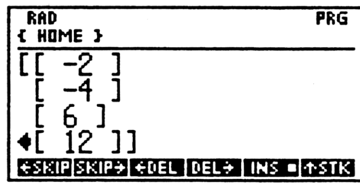
Enter the next three rows of coefficients just as you entered the first row. Press [ENTER] to return to the Solve System screen. Highlight `B:`. Enter the column of constants in the same manner you entered the matrix of coefficients.

Your screen should look like the following:



Highlight X: and solve the system and send the answer to line 1 of the stack by pressing the following:

SOLVE (ON) (←) (EDIT) (▼) (▼) (▼) (▼) (to view all four entries in the column matrix)



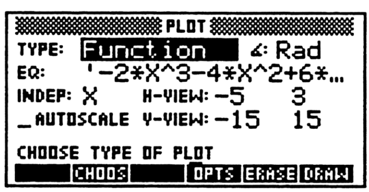
This permits the solution cubic polynomial to be written as:

$f(x) = -2x^3 - 4x^2 + 6x + 12.$

To play the game, proceed as follows:

Press (→) (PLOT). Highlight EQ: and press (DEL) (OK) (') (-) 2 (×) (α) X (y^x) 3 (-) 4 (×) (α) X (y^x) 2 (+) 6 (×) (α) X (+) 12 OK.

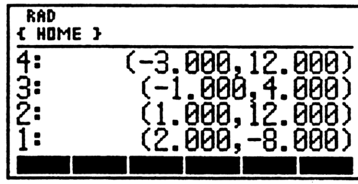
In the Plot environment, press ERASE and set the Plot screen to be:



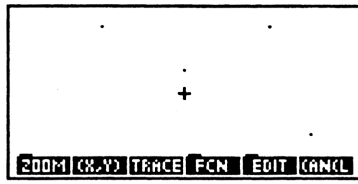
To save what you have stored in the Plot screen before returning to the stack, press OPTS OK.

Do not draw the plot at this time! Instead, press (ON) to return to the stack and enter the coordinates of the four points on the stack:

[⬑] [( )] 3 [+/-] [SPC] 12 [ENTER]
(Repeat these steps for the next 3 points.)

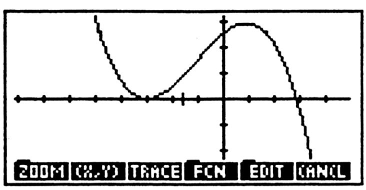


[PRG] PICT [NXT] PIXON PIXON PIXON PIXON [⬑] [PICTURE]



Return to the Plot environment, make sure the Plot screen still matches the previous screen, and press DRAW (Do not press ERASE!)

[ON] [⬏] [PLOT] DRAW



The resulting graph hits each lighted pixel, i.e. *GREEN GLOB*.

Note

These equations can be constructed on the HP 48 as follows:

Enter the general cubic polynomial through the EquationWriter (or on the command line if you know the syntax) in the following format:

$X^3*A + X^2*B + X*C + D = Y$.

(You must enter the "*" between X and C, otherwise you will have a new variable XC. This format will produce the coefficients and variables in the correct order without expanding and collecting terms.)

Store the expression as 'P' for ready recovery. Type: -3'X [STO], and 12'Y [STO]. Then, with P on line one, press [EVAL].

Follow with P again on line one, type -1'X[STO], and 4'Y[STO] [EVAL]. Follow with P again on line one, type 1'X[STO], and 12'Y[STO] [EVAL]. Follow with P again on line one, type 2'X[STO], and 8'Y[STO] [EVAL].

Your final screen should be:

```
RAD
{ HOME }
4: '-(27*A)+9*B-3*C+D…
3:           '-A+B-C+D=4'
2:           'A+B+C+D=12'
1:     '8*A+4*B+2*C+D=8'
```

5

Logistics Curve

Example 1

The equation for logistic growth has the general form:

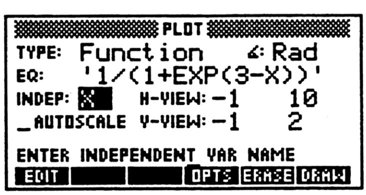
$$\text{Population} = \frac{M}{1 + Be^{-Mkt}}$$

where M is the maximum population possible and B and k are biological characteristics of reproduction and survivability of animals involved.

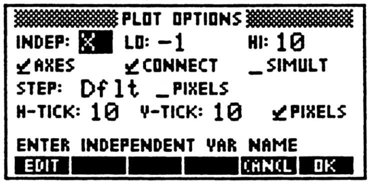
The graph of this equation can be analyzed using a simplified version, namely:

$$f(x) = \frac{1}{1 + e^{(3-x)}}$$

Set the Plot screen to:

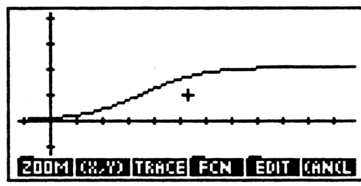


and the Plot Options screen to be:



then draw the graph by pressing

OK ERASE DRAW



Extension

Suppose that a lake is stocked with 100 fish. After 3 months there are 250 fish and after 12 months there are 900 fish in the lake. How many fish will the lake support? Find a formula for the number of fish in the lake t months after it has been stocked. Plot a graph of the fish population at time t months.

Solution: Using the known data (0, 100), (3, 250), and (12, 900) the three values of M, B, and k can be determined.

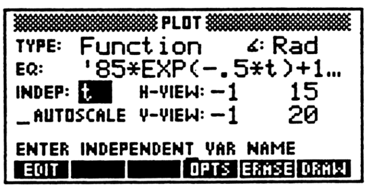
Example 2

Suppose that a student learns a certain amount of material for some class. Let f(t) denote the percentage of the material that the student can recall t weeks later. The psychologist Ebbinghaus has found that this percent retention can be modeled by a function of the form

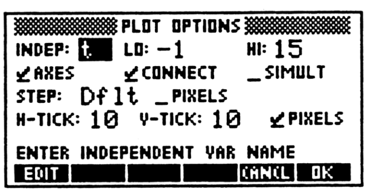
$$f(x) = (100\text{-}a)e^{-kt} + a$$

where k and a are positive constants and $0 < a < 100$. Sketch a graph of the function when $a = 15$ and $k = 0.5$.

Enter the Plot environment and set the Plot screen to:



and the Plot Options screen to be:



then draw the graph by pressing

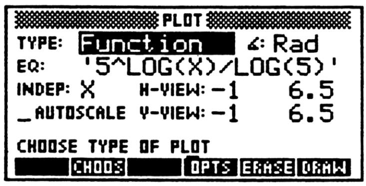
OK ERASE DRAW

Extension

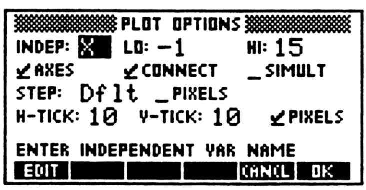
Discuss the meaning of the several regions of this graph.

Example 3: Logarithmic Identities

Enter the Plot environment and set the Plot screen to be:

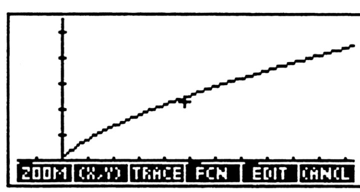


and the Plot Options screen to be:



then draw the graph by pressing

OK ERASE DRAW



Trace this curve and write another name for it.

Extension

1. Plot the same function, but with LN instead of LOG.
2. Plot f(x) = e^Ln(x) and g(x) = Ln(e^x) on a viewing rectangle:

```
INDEP: X       H-VIEW: -2    6
_AUTOSCALE     V-VIEW: -20   60
```

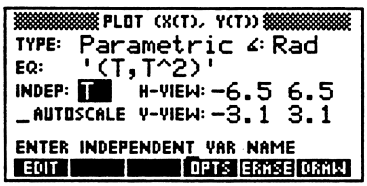
3. Then try f(x) = e^Ln(x) and g(x) = Ln(e^x) + 4 on the same viewing rectangle.
4. Plot {'2^(x+1)' '2*2^x'} on the same viewing rectangle as above.
5. Plot {'2^(x+3)' '8*2^x'} on the same viewing rectangle as above.

6

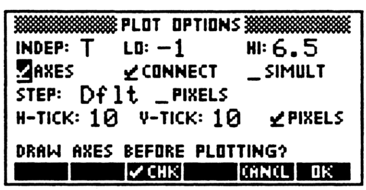
Parametrics

Example 1

Enter the Plot environment and set the Plot screen to:

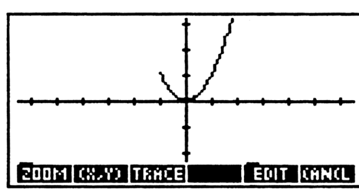


and the Plot Options screen to be:



then draw the graph by pressing

OK ERASE DRAW



Example 2

A baseball is hit when the ball is 3 ft above the ground and leaves the bat with initial velocity of 130 ft/sec at an angle of elevation of 30 degrees. A 7.5-mph wind is blowing in the horizontal direction directly against the batter from center field. A 24-ft-high fence is 410 ft from home plate. Is the hit a home run over the fence?

First, convert miles per hour to feet per second.

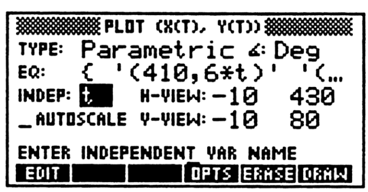
(→) (UNITS) SPEED 7.5 MPH
(←) FT/S

The parameterization of the fence is: $x(t) = 410$, $y(t) = 6t$, $0 \leq t \leq 4$. The parameterization of the ball's path is: $x(t) = 130\cos(30)t - 11t$, $y(t) = 130\sin(30)t - 16t^2 + 3$.

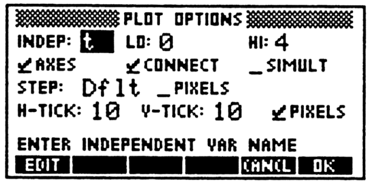
Enter the plot environment, highlight EQ: and enter the following expression:

{'(410,6*t)' '(130*COS(30)*t - 11*t,130*SIN(30)*t - 16*t^2 + 3)'}

Set the Plot screen to:

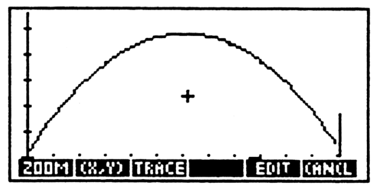


and the Plot Options screen to be:



then draw the graph by pressing

OK ERASE DRAW



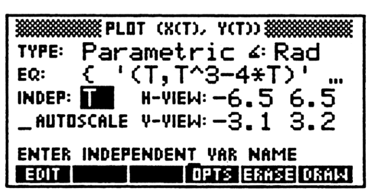
Example 3: Inverses

For plotting the inverse relation, take advantage of the reverse coordinate possibility in defining the function parametrically.

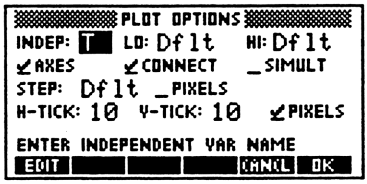
Enter the Plot environment, move the highlight to EQ:, and enter the following expression:

{'(T,T^3−4*T)' '(T^3−4*T,T)'}

Set the Plot screen to:

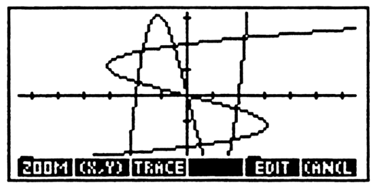


and the Plot Options screen to be:



then draw the graph by pressing

OK ERASE DRAW



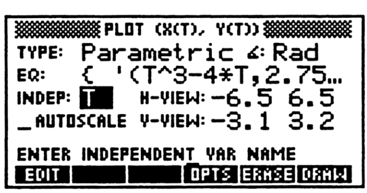
The first plotted is the original function f(x) = x^3 − 4x, and the second is its inverse relation. This clearly illustrates the coordinate reversal nature of inverse relationships.

Example 4: Linear Motion Simulations

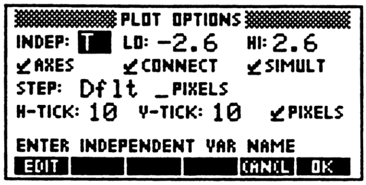
In the Plot environment, highlight EQ: and enter the following:

{'(T^3−4*T,2.75)' '(T^3−4*T,T)'}

Set the Plot screen to:

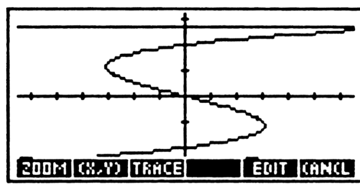


and the Plot Options screen to be:



then draw the graph by pressing

OK ERASE DRAW



Tracing the two curves and jumping between them illustrates the relation between the linear motion of a particle and its position function. Including the derivative communicates further relationships.

7

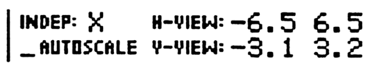
Polar Plotting

Example 1

Lets examine the graph of the polar equation r = 3cos3Θ.

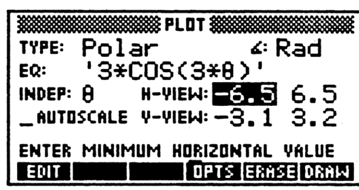
Reset the Plot screen.

(NXT) RESET (▼) OK

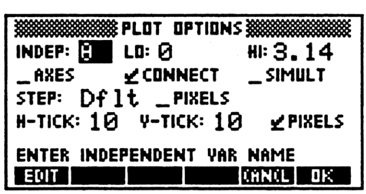


Set the Plot screen to:

(→) (CHARS) *Highlight θ* ECHO
(ON) *Highlight INDEP* (ENTER)

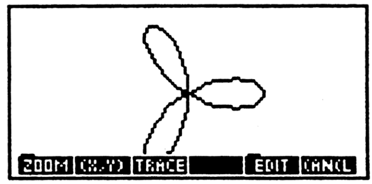


and the Plot Options screen to be:



then draw the graph by pressing

OK ERASE DRAW



Extensions

1. r = 3sin$n\Theta$, for n a natural number (roses of n or n/2 leaves)

2. r = 1 − sinΘ (cardioid)

3. r = e^{Θ} (logarithmic spiral)

4. r = (3cosΘ sinΘ)/($\cos^3\Theta + \sin^3\Theta$) (folium of Descartes)
Enter $\cos^3\Theta$ as $\cos(\Theta)^3$, etc.

Example 2

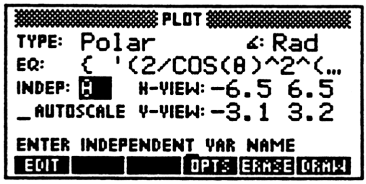
Plot the locus of a point on the circumference of a circle rolling around the inside of a circle which is four times as large (for example, a circle of radius 3/4 inches inside a circle of radius of 3 inches). The resulting curve is called an hypocycloid and its polar equation is given by:

$$r = \left[\frac{2}{\cos^{(2/3)}\theta + \sin^{(2/3)}\theta}\right]^{\frac{3}{2}}$$

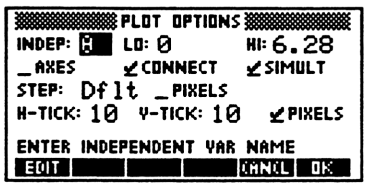
Highlight EQ: and enter the expression

{'(2/COS(Θ)^2^(1/3) + SIN(Θ)^2^(1/3))^(3/2)' 3 }

Set the Plot screen to:

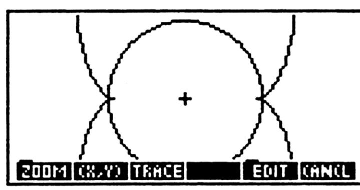


and the Plot Options screen to be:



then draw the graph by pressing

OK ERASE DRAW



Example 3: Plotting Straight Lines in Polar Form

The general form for a straight line in polar form is:

$$r = \frac{A}{B\cos\theta - C\sin\theta}$$

The parameters A, B, and C determine “slope” and “y-intercept”.

For the line

$$y = \frac{3}{2}x\text{-}2$$

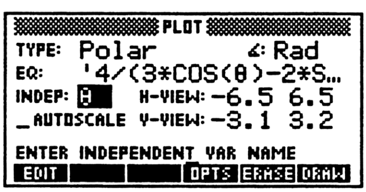
the polar form is

$$r = \frac{4}{3\cos\theta - 2\sin\theta}$$

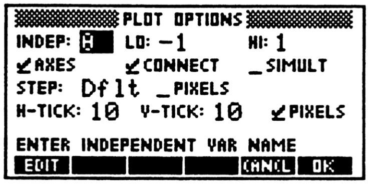
Highlight EQ: and enter the following expression:

’4/(3*COS(Θ)−2*SIN(Θ))’

Press OK and set the Plot screen to:

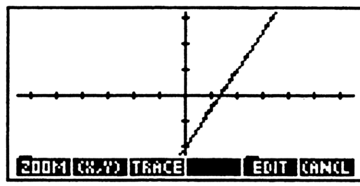


and the Plot Options screen to be:



then draw the graph by pressing

OK ERASE DRAW



Extension

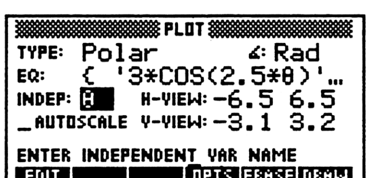
Vary the parameters A, B, and C to discover their effects on slope and intercepts.

Example 4: Designing a Rose Window

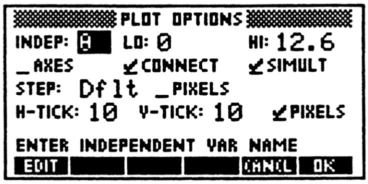
Highlight EQ: and enter the expression

{'(3*COS(2.5*Θ))' 3 3.5}

Press OK and set the Plot screen to:

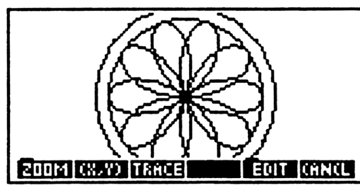


and the Plot Options screen to be:



then draw the graph by pressing

OK ERASE DRAW



Extensions

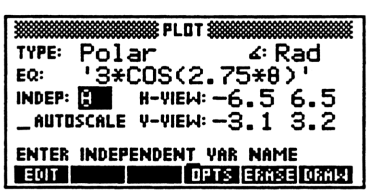
1. Use sine instead of cosine.
2. Use other rational numbers for multiples of Θ.

Example 5: Drawing a Chrysanthemum

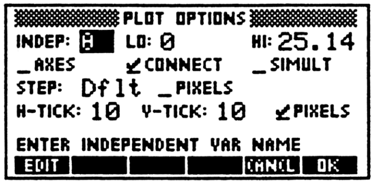
Move the highlight to EQ:, press EDIT, and enter the expression

'3*COS(2.75*Θ)'

Press OK and set the Plot screen to:

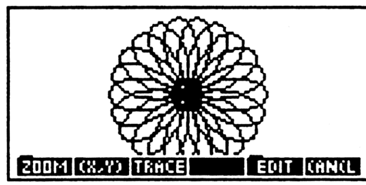


and the Plot Options screen to be:



then draw the graph by pressing·

OK ERASE DRAW



8

The Solver

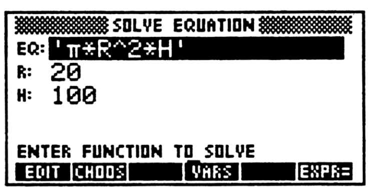
Example 1

The volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder. A cylinder is measured and found to have a radius of 20 millimeters and a height of 100 millimeters. Find the error in calculating the volume of the cylinder if each measurement is in error by at most 1 millimeter. How many teaspoons is this?

Press (→) (SOLVE), move the highlight to Solve equation..., and press OK. Enter the following expression:

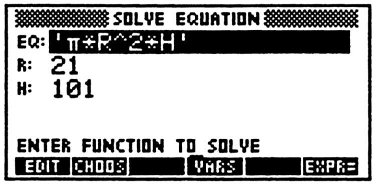
'π*R^2*H'

Set the Solve Equation screen to:

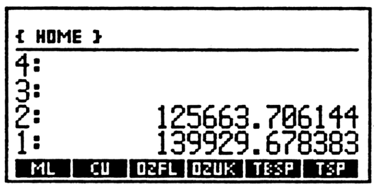


Highlight EQ: and press EXPR=. The value is sent to the stack.

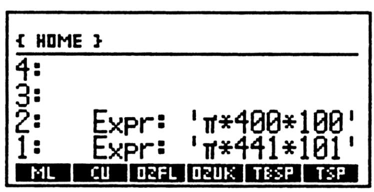
Change the Solve Equation screen to:



Highlight EQ: and press EXPR=. Press (ON) to return to the stack. If your HP48 is not in symbolic mode, the stack will look like:



If your HP48 is in symbolic mode, the stack will look like:



If your HP48 *is* in symbolic mode, press (◄) (►NUM) (◄) (SWAP) (◄) (►NUM) (◄) (SWAP) to convert the symbolic forms to numeric forms.

Press (−) (+/−) to display the error.

Enter the result in cubic millimeters by pressing

[→] [UNITS] LENG MM [←]
[EDIT] [▶] (*16 times*) [y^x] 3
[ENTER]

```
{ HOME }
4:
3:
2:
1:     14265.972239_mm^3
 M | CM | MM | YD | FT | IN
```

Convert the answer to teaspoons by pressing

[→] [UNITS] VOL [NXT] [NXT]
[←] TSP

```
{ HOME }
4:
3:
2:
1:     2.89433945492_tsp
 ML | CU | OZFL | OZUK | TBSP | TSP
```

Extension

If S denotes the sales of a certain product, p the unit price, and a the number of dollars spent per unit in advertising, studies have shown that:

$$S = 100{,}000 + 20{,}000\left(1 - \frac{1}{a+1}\right) e^{-p}$$

a) Calculate the total sales when $a = \$1$, and $p = \$3$.

b) Approximate the effect of reducing a to \$.95 and increasing p to \$3.10.

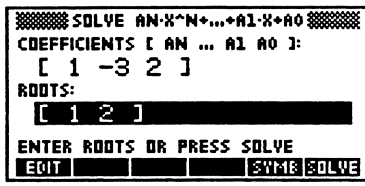
Example 2: Roots of Polynomials

Find the zeros of:

- a) $x^2 - 3x + 2$
- b) $x^3 - x^2 + x - 1$

a) Press [→] [SOLVE], highlight `Solve poly...`, and press `OK`. Highlight `COEFFICIENTS:` and solve for the roots of the expression by typing

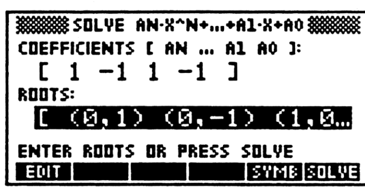
[←] [[ ]] 1 [SPC] 3 [+/-] [SPC] 2
`OK` `SOLVE`



The roots, 1 and 2, are displayed in vector notation. Press [ON] to return to the stack where roots are also displayed with labels. The labels are transparent to the calculator for computation purposes, thus the entries can be used for further computations without reentry.

b) Press [→] [SOLVE], highlight `Solve poly...`, and press `OK`. Highlight `EQ:` and solve for the roots of the expression by typing

[←] [[ ]] 1 [SPC] 1 [+/-] [SPC] 1
[SPC] 1 [+/-] `OK` `SOLVE`



This time the roots are given as [(0,1) (0,-1) (1,0)], which indicates three complex zeros, {i, -i, 1}, i.e. {0 + i, 0 - I, 1 +0i}. Whenever one of the zeros has an imaginary part, all will be displayed as complex numbers. Press [ON] [←] [EDIT] to show them completely.

Extensions

1. Solve the equation $x^3 - 15x^2 - 16x + 420 = 0$.
2. Solve the equation $x^3 - 11x^2 - 43x - 65 = 0$.

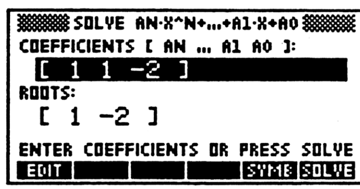
Example 3: Finding Polynomials with Given Roots

Find the polynomial that has roots:

- {1, 2}
- {-5, 6, 23, 41}

a) Press (→) (SOLVE), highlight Solve poly..., and press OK. Highlight ROOTS: and solve for the polynomial by typing

(←) ([]) 1 (SPC) 2(+/-) OK SOLVE

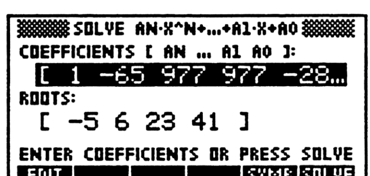


Thus the coefficients are 1, 1, and -2 for a resultant polynomial of $x^2 + x - 2$.

Press (ON) to display the coefficients on the stack or press EDIT to display them in the MatrixWriter.

b) Reenter the Solve environment and select Solve poly.... Highlight ROOTS: and solve for the polynomial by typing

(←) ([]) 5 (+/-) (SPC) 6 (SPC) 23 (SPC) 41 OK SOLVE



Press EDIT ▶ *(4 times)* and read the coefficients as {1, -65, 977, 977, -28290} for a resultant polynomial of $x^4 - 65x^3 + 977x^2 + 977x - 28290$.

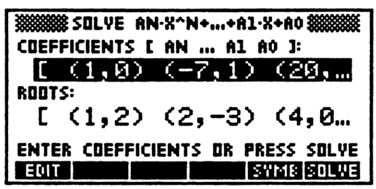
Example 4: Finding Polynomials with Given Complex Roots

Find the polynomial that has roots: {(1, 2) (2, -3) (4, 0)}.

Enter the Solve environment and select Solve poly.... Highlight ROOTS: and solve for the polynomial by entering the roots

[(1, 2) (2, -3) (4, 0)]

and then pressing OK SOLVE. Your screen should look like the following:



Press (ON) to display the coefficients on the stack. Pressing (↱) (STACK) VIEW will allow you to scan over the coefficients: {[(1, 0) (-7, 1) (20, -3) (-32, -4)]}. The resultant polynomial, with complex coefficients, is $x^3 - (7 - i)x^2 + (20 - 3i)x - (32 + 4i)$.

Extensions

1. Find the polynomial with roots: { -1, 2 + 3i, 2 − 3i }.
2. Find the polynomial with roots: { 2, 3 − 6i, -3i }.

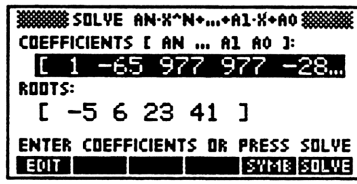
Example 5

Use the Solver to find the 7th term in the 10th row of Pascal's triangle.

Solution: Expand $(x + 1)^{10}$ and find the coefficient of the 7th term.

Press [↱] [SOLVE], move the highlight to Solve poly..., and press OK. Highlight ROOTS: and solve for the polynomial by entering 10 "-1"s as roots.

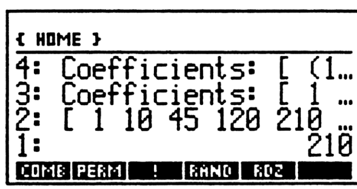
[↰] [[]] 1 [+/-] [SPC] 1 [+/-] [SPC] 1 [+/-] [SPC] 1 [+/-] [SPC] 1 [+/-] [SPC] 1 [+/-] [SPC] 1 [+/-] [SPC] 1 [+/-] [SPC] 1 [+/-] [SPC] 1 [+/-] OK SOLVE



Press EDIT to view the MatrixWriter with the expansion of coefficients as row one. Press [▶] six times to deliver the numbered seventh column as 210.

To check: Press the following:

[ON] [ON] 10 [ENTER] ['] 7-1 [ENTER] [MTH] [NXT] PROB COMB [→NUM]



Note

The roots could be entered as positive 1's. The coefficients will then appear with alternating signs. Consideration must then be made of the absolute values thereof.

Extension

Find the n^{th} term of the m^{th} row of Pascal's triangle two different ways.

9

Examples for Statistics

Example 1

Enter the following matrix in the MatrixWriter and store it under a convenient name, e.g. SM (Statistics Matrix).

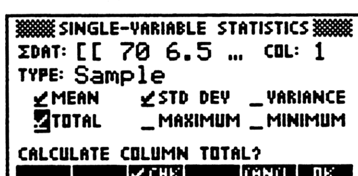
Height	Hat Size	Shoe Size	I.Q.
70	6.5	12	120
68	5.5	8.5	112
61	6.5	8	145
71	7	10	101
76	7.5	15	90
71.5	8.5	10.5	105
74	5.5	13	100
64	8	9.5	130
75	7	13.5	86
72.5	6.5	11	112

Single Variable Statistics:

Enter the Statistics environment by pressing (⮫) (STAT). Highlight `Single-var...` and press `OK`.

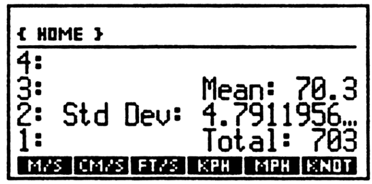
Choose SM as the current statistical matrix and choose the statistical values you wish to calculate.

`CHOOS` `OK` (▼) (▶) `✓CHK` (▶) `✓CHK` (▶) (▶) `✓CHK`



Calculate the values.

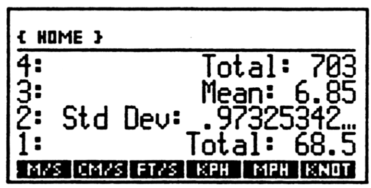
OK



These statistical values are displayed on the stack with their respective labels. As usual, the labels are transparent to the calculator for computation purposes, thus the stack lines may be used as desired for further data manipulation.

Calculate the values for column 2.

(→) (STAT) (▶) OK 2 OK (▶)
✓CHK (▶) ✓CHK (▶) (▶) ✓CHK
OK

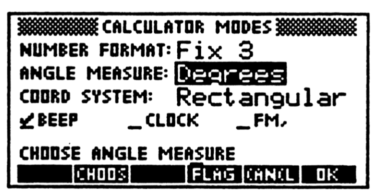


Repeat the procedure for the other columns.

Bivariate Statistics

Set the number format to Fixed.

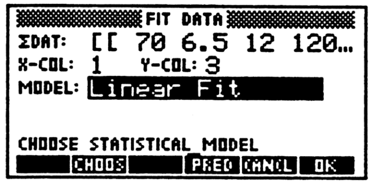
(→) (MODES) CHOOS (▼) OK (▶)
3 OK



Press OK, enter the Statistics environment, move the highlight to Fit data..., and press OK.

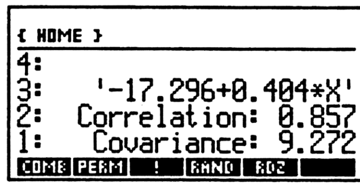
Choose the appropriate fields.

▼ 1 OK 3 OK



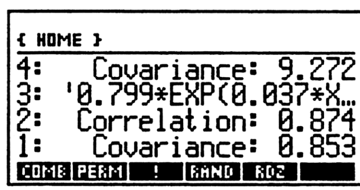
Compute the regression line, correlation coefficient, and covariance coefficient and send them to the stack.

OK



Return to the Fit Data screen, highlight MODEL:, and recalculate using a Best Fit model by pressing

CHOOS ▲ OK OK



Extension

1. Explore bivariate relations between other columns in the statistical matrix, SM.

2. Graph the several regression equations on the same axes along with a scatter graph of the data.

Example 2

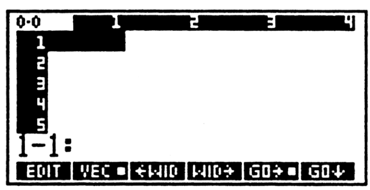
Press [↱] [MODES] and set NUMBER FORMAT to FIX 3.

The data here represent the number of newly diagnosed cases of AIDS in the United States every six months from January, 1981, through January, 1986.

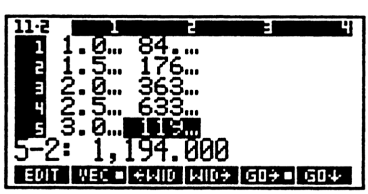
Year	No. of Cases
1981	84
1981.5	176
1982	363
1982.5	633
1983	1194
1983.5	1567
1984	2392
1984.5	3141
1985	4232
1985.5	5199
1986	5789

Enter the Statistics environment and choose a new matrix.

[↱] [STAT] OK CHOOS NEW
[↱] [MATRIX]

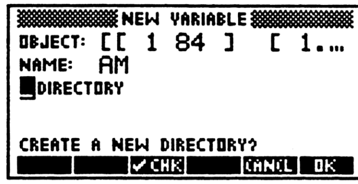


Use 1980 as year zero and enter an 11×2 matrix in the MatrixWriter using the units of each year as the first column.



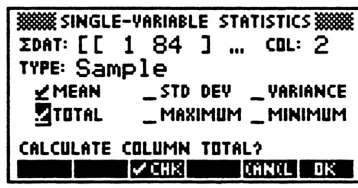
Return to the New Variable screen and name the matrix AM (Aids Matrix).

[ENTER] [▼] [α] A [α] M OK

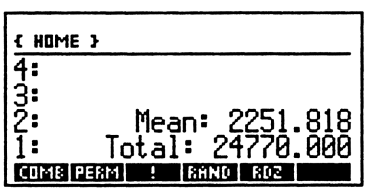


Press OK OK to return to the Single-Variable Statistics screen. Set the screen to match the following screen by pressing the following keys:

[▶] 2 OK [▶] ✓CHK [▼] ✓CHK

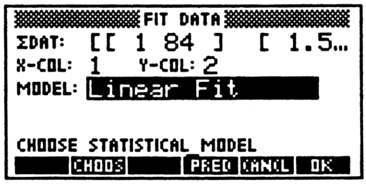


Press OK to compute the mean and total and send them to the stack. As usual, the labels shown are transparent to the calculator for computation purposes, thus the stack may be used as desired for further data manipulation.

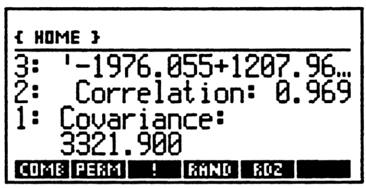


Reenter the Statistics environment and highlight Fit data.... Press OK

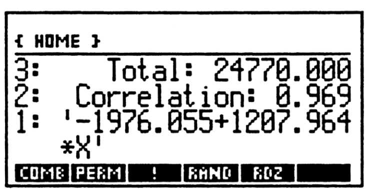
Set the Fit Data screen to:



Press OK to display the regression equation correlation coefficient, and covariance coefficient on the stack.

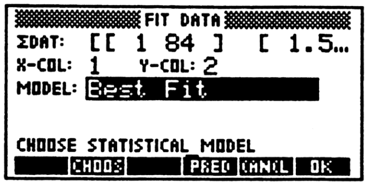


Press (←) (DROP) (←) (SWAP) to see the entire regression equation.

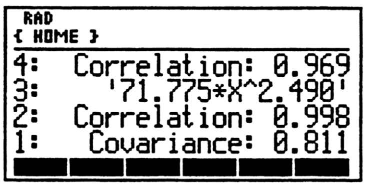


It is convenient at this time to store the equation under the name LIN by typing LIN (STO).

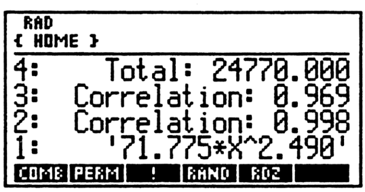
Reenter the Statistics environment and set the Fit Data screen to be:



Press OK to display the regression equation, correlation coefficient, and covariance coefficient on the stack.

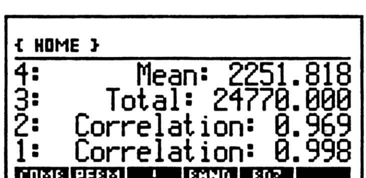


Press (⬅) (DROP) (⬅) (SWAP) to put the power regression equation on the first line.

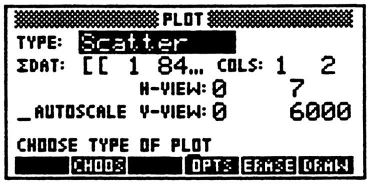


It is convenient at this time to store the equation under the name POW by typing POW (STO).

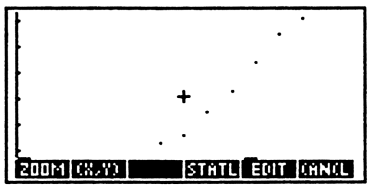
A comparison of the correlation coefficients is convenient at this time:



Enter the Plot environment and set the Plot screen to show:



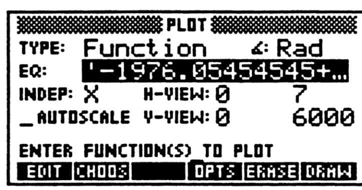
Press ERASE DRAW to display the scatter diagram of the AIDS data.



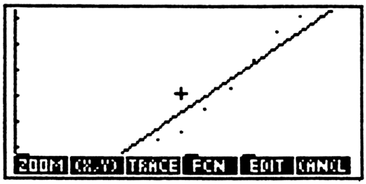
Return to the Plot screen by pressing (ON).

Change the plot type to Function and enter LIN as the equation to plot.

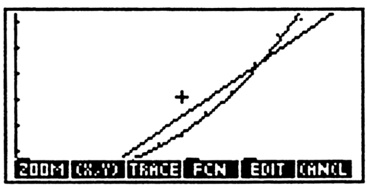
(α) F (▶) CHOOS (▼) OK (▶)
CHOOS (▼) OK



Press DRAW (Do Not press ERASE) to plot the linear regression line through the scattergram.



Press (ON) to return to the Plot screen. Change EQ: to POW and press DRAW again to plot the power regression equation through the other graph and show its better fit.



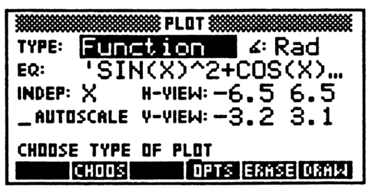
10

Trigonometric Identities

Example 1

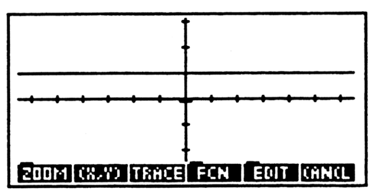
Show that $\sin^2 x + \cos^2 x = 1$.

Enter the Plot environment, highlight EQ:, enter the expression 'SIN(X)^2 + COS(X)^2'. Set the Plot screen to



Draw the graph by pressing

ERASE DRAW



Of course this does not prove the identity, but it makes it very believable.

Example 2

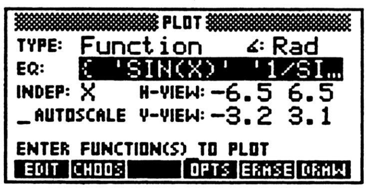
Is the expression csc x − cos x cot x = sin x an identity or a conditional equation?

By entering csc x - cos x cot x and sin x as functions in the Plot environment and plotting them simultaneously, we should be able to determine if it is an identity or not. Therefore, enter them as two functions in the variable menu and check them both to be graphed.

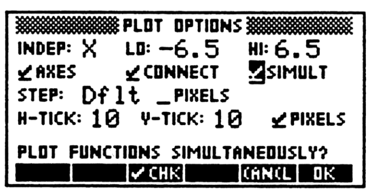
csc x − cos x cot x must be entered as

'1/SIN(X) − COS(X)*(1/TAN(X))'

The Plot screen should be:

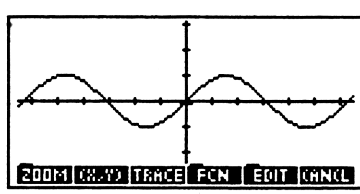


The Plot Options screen should read:

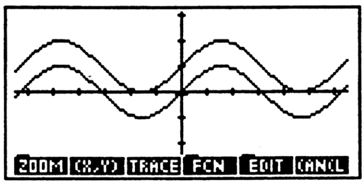


Graph the functions by pressing:

OK ERASE DRAW



TRACE ▶ and ◀ will show that both curves are on the screen, however a standard device to illustrate the effect of a vertical transformation on the concept of these being identical is to EDIT one of the functions by adding a constant, e.g. + 1. By doing so and graphing simultaneously, then tracing and shifting between the two curves and observing the y coordinate.



Example 3

Is the expression

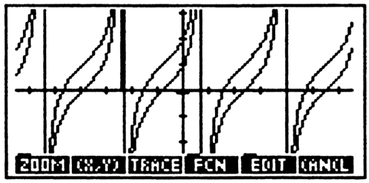
$$\frac{1+\tan x}{1-\tan x}+\frac{1+\cot x}{1-\cot x}=0$$

an identity or a conditional expression?

By entering the expression as the function in the plotting environment and plotting, the graph is the x-axis. This leads to the need to adjust the function in some way without changing the problem, e.g. transposing one of the fractional expressions to the other side:

$$\frac{1+\tan x}{1-\tan x}=-\frac{1+\cot x}{1-\cot x}$$

and plotting two functions, to one of which a constant is added. Cot(x) must be entered as "1/tan(x)" unless it has been defined as a customized function.

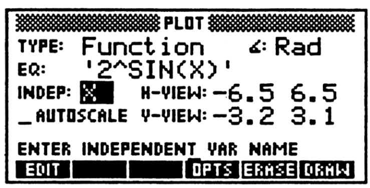


Example 4

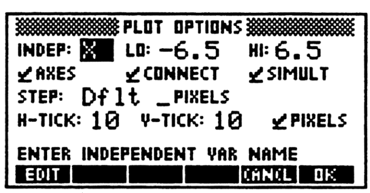
Given $f(x) = 2^{\sin(x)}$

What are the a) domain, b)range, c)period, and d) axis intercepts, if any of f. Sketch a graph of f.

Enter the Plot environment and set the Plot screen to be:

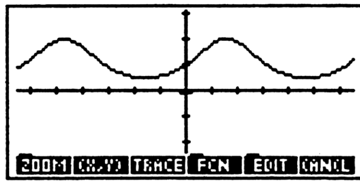


The Plot Options screen should be:



Plot the function by pressing

OK ERASE DRAW

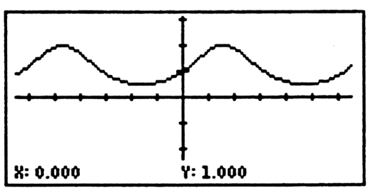


a) The domain is obviously {all real numbers}.

b) Press FCN and move the cross-hair to the left of the y-axis. Press EXTR to display ($-\pi/2$, .5). ($-\pi/2$ is in numeric form.) Move the cross-hair close to the subsequent local extremum. Press + EXTR to display ($\pi/2$, 2). ($\pi/2$ is in numeric form.) Therefore, the range is { y: $0.5 \leq y \leq 2$}.

c) Move the cross-hair over a local minimum and press EXTR. Move the cross-hair over a subsequent local minimum and press EXTR. Press ON − to display ($\pm$6.28318 ... , 0), the sign depending on the order of local minimums selected. The result shows the period to be 2π.

d) Press PICTURE TRACE + to display the y-intercept as (0, 1).



Extension

Given $f(x) = 2000\pi(\sin 9\pi x))(\cos^2 x)$

Find the a) domain, b) range, c) period, and d) axis intercepts, if any, of f. Sketch a graph of f.