HEWLETT PACKARD

HP 48G Series
Examples in Math Education - Part 2

Preface

This resource was developed through a joint effort by two high school math teachers during a summer internship at Hewlett-Packard. Because of individual idiosyncrasies, there will be some slight differences in style and language between Topics 1-5 and Topics 6-9. It is intended to be used as supplementary material to illustrate the usage of the HP 48G scientific calculator.

The topics covered are very diverse and do not focus on any particular mathematics course with concepts ranging from the Algebra 1 level to beginning Calculus. The order of presentation progesses from the lower to higher levels of mathematical difficulty with emphasis being placed on exploring the functionality of the calculator and the different aspects of its use. Anyone working through the activities in this manual will become comfortable enough with the 48G to be considered an "advanced user".

Each activity is divided into two parts – student exercises and the instructional section. The instructional section is intended for teacher use and includes keystroke documentation, illustrations, and correct solutions. Because of the level of diversity, it is not assumed that all the topics will be covered in the order presented. Each topic is self-contained enough to be done individually. However, it is assumed that the activities within a topic will done in order with respect to the documentation in the instructional section. In other words, the documentation in the instructional section for activity 2 might refer to a procedure described in the instructional section for activity 1 of that particular topic.

Because of time constraints placed on the authors, only a smattering of mathematical concepts are covered. It was decided that the best thing to do was to concentrate on a few important topics and to investigate them thoroughly. The main criteria used in selecting the topics came from the NCTM's *Curriculum and Evaluation Standards for School Mathematics*. Included in the Overview for the Curriculum Standards in grades 9-12, is a "Summary of Changes in Content and Emphases in 9-12 Mathematics". The concepts on which we chose to focus are reflective of those changes.

Acknowledgements

The authors wish to thank the Linn-Benton Business Education Compact for providing the Teacher Internship Program and Hewlett-Packard for making the effort to "bridge the span" between business and education.

Many thanks to Clain Anderson, Dennis York, John Loux, and many others at Hewlett-Packard who cheerfully provided resources and assistance over the duration of our stay.

CONTENTS

ACTIVITY 1...TRANSFORMATIONS WITH LINEAR FUNCTIONS

INSTRUCTIONAL SECTION

The purpose of this activity is to inductively discover the various effects transformations have on a reference linear function. Ultimately, you will be able to recognize the graph of any linear function as a transformation of the graph of the reference function y = x. Use the **"Basic Description"** blanks to describe the graph of the reference function. Use the **"Transformation"** blanks for writing the changes between the reference function and the transformed functions. Use the **"Prediction"** blanks to predict the changes between the reference function and the transformed functions. Then use the calculator to check your prediction. Use terminology such as vertical shift up or down, horizontal shift left or right, and by how many units, increased or decreased slope, reflected over the x-axis, etc. to illustrate the relative changes. The **"Conjecture"** blanks can be used for writing the general transformation that will occur relative to the reference function. Consider all possible real number substitutions for the variables in the **"Conjecture"** sections.

DIRECTIONS FOR USING THE HP 48G:

1. Let's begin by purging unwanted objects from the HOME directory. Pressing the **VAR** key will display the objects in the HOME directory on the menu labels. The fast way to purge is to create a list of unwanted objects and then purge the list. Press **PURPLE {** then press the white menu keys under the menu labels of those objects you want to purge, press **ENTER** then **PURPLE PURGE.**
2. Use the Equation Writer to key in the functions next to Reference, Transformation, and Prediction in an Investigation. Example: Enter y = 2x. **PURPLE EQUATION α Y PURPLE = 2 α X**, fig. 1, now press **ENTER**, fig. 2
3. Name and store each function using the corresponding names, Y, Y1, Y2, etc. given in the Investigations. Press **'αα XMPL α**, fig. 3, then press the **STO** key. Notice that your equation and the name you stored it under have disappeared from the stack. If you press **VAR**, you will see XMPL on the menu, fig. 4.
4. Plot the reference function and one of the transformed functions. Press **GREEN PLOT**, fig. 5, then press **CHOOS** and **CHK** the functions you want graphed on the same set of axes, fig. 6. Now press **OK**, fig. 7.
5. Press the menu keys under the **ERASE** and **DRAW** menu labels, then analyze and note the differences and similarities between the two graphs, fig. 8.
6. Repeat steps 4 and 5, always including the reference function and another transformed function until all pairs have been analyzed.
7. Repeat steps 1 through 6 after each Investigation is completed.

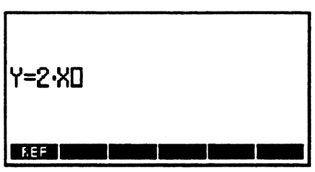


Fig. 1

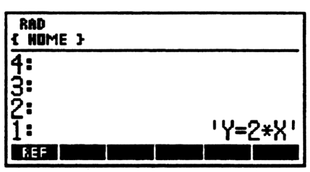


Fig. 2

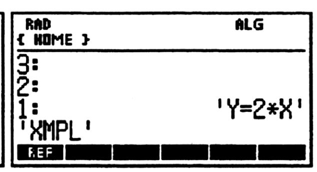
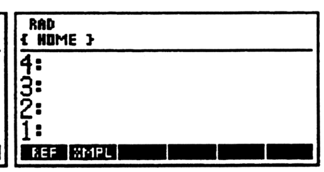


Fig. 3



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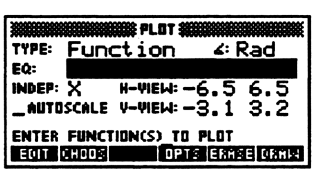
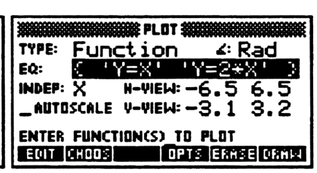


Fig. 5

Fig. 6



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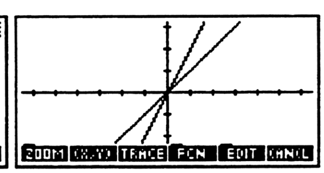

Fig. 8

Transformation The XMPL function is twice as steep as the REFERENCE function, or the slope of the XMPL function is twice that of the REFERENCE function's.

ACTIVITY 1...TRANSFORMATIONS WITH LINEAR FUNCTIONS

APPLICATIONS SECTION

You will be analyzing linear functions in this activity. The most basic linear function is $y = x$. We will call this the reference function. Any mathematical changes we make on this reference function will produce a graph that is different from that of the reference function's. These changes will result in what is known as a transformation of the reference function. Ultimately, you should be able to predict the placement and shape of the graph of any linear function relative to the reference function in the Cartesian plane. See the Instructional Section for more details.

INVESTIGATION 1:

Reference:	$y = x$	Basic description__________
	$y1 = x + 2$	Transformation__________
	$y2 = x - 3$	Transformation__________
	$y3 = x + 1.5$	Transformation__________
	$y4 = x + 10$	Prediction__________
	$y5 = x + c$	Conjecure__________

INVESTIGATION 2:

Reference:	$y = x$	Basic description__________
	$y1 = 2x$	Transformation__________
	$y2 = 5x$	Transformation__________
	$y3 = \frac{1}{4}x$	Transformation__________
	$y4 = \frac{5}{2}x$	Transformation__________
	$y5 = 4x$	Prediction__________
	$y6 = \frac{1}{5}x$	Prediction__________
	$y7 = a\cdot x$	Conjecture __________

ACTIVITY 1... APPLICATIONS SECTION CONTINUED

INVESTIGATION 3:

Reference: $y = x$ Basic description____

$y1 = -1x$ Transformation____

$y2 = -3x$ Transformation____

$y3 = \frac{-1}{2}x$ Transformation____

$y4 = \frac{-3}{2}x$ Transformation____

$y5 = -6x$ Transformation____

$y6 = \frac{-5}{3}x$ Prediction____

$y7 = \frac{-2}{7}x$ Prediction____

$y8 = a \cdot x$ Conjecture ____

INVESTIGATION 4:

Reference: $y = x$ Basic description____

$y1 = 2x + 1$ Transformation____

$y2 = 3x - 2$ Transformation____

$y3 = -4x + 3$ Transformation____

$y4 = \frac{5}{2}x + 3$ Prediction____

$y5 = \frac{-2}{3}x - 5$ Prediction____

$y6 = a \cdot x + b$ Conjecture____

ACTIVITY 2...TRANSFORMATIONS WITH ABSOLUTE VALUE FUNCTIONS

INSTRUCTIONAL SECTION

The purpose of this activity is to inductively discover the various effects Transformations have on a reference absolute value function. Ultimately, you will be able to recognize the graph of any absolute value function as a Transformation of the graph of the reference function $y = |x|$. Use the **"Basic Description"** blanks to describe the graph of the reference function. Use the **"Transformation"** blanks for writing the changes between the reference function and the transformed functions. Use the **"Prediction"** blanks to predict the changes between the reference function and the transformed functions. Then use the calculator to check your prediction. Use terminology such as vertical shift up or down, horizontal shift left or right, and by how many units, increased or decreased slope, reflected over the x-axis, etc. to illustrate the relative changes. The **"Conjecture"** blanks can be used for writing the general transformation that will occur relative to the reference function. Consider all possible real number substitutions for the variables in the **"Conjecture"** sections.

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2. Use the Equation Writer to key in the functions next to Reference, Transformation, and Prediction in an Investigation. Example: Enter $y = |2x|$. Press **PURPLE EQUATION α Y PURPLE = MTH,** menu label **REAL, NXT,** menu label **ABS, 2 α X,** fig. 1, now press **ENTER,** fig. 2
3. Name and store each function using the corresponding names, Y, Y1, Y2, etc. given in the Investigations. Press **'αα XMPL α**, fig. 3, then press the **STO** key. Notice that your equation and the name you stored it under have disappeared from the stack. If you press **VAR,** you will see XMPL on the menu, fig. 4.
4. Plot the reference function and one of the transformed functions. Press **GREEN PLOT**, fig. 5, then press **CHOOS** and **CHK** the functions you want graphed on the same set of axes, fig. 6. Now press **OK,** fig. 7.
5. Press the menu keys under the **ERASE** and **DRAW** menu labels, then analyze and note the differences and similarities between the two graphs, fig. 8.
6. Repeat steps 4 and 5, always including the reference function and another transformed function until all pairs have been analyzed.
7. Repeat steps 1 through 6 after each Investigation is completed.

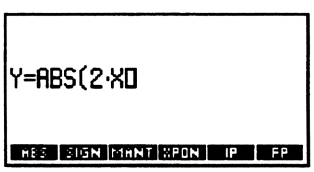


Fig. 1

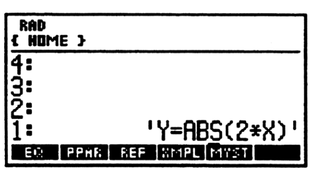


Fig. 2

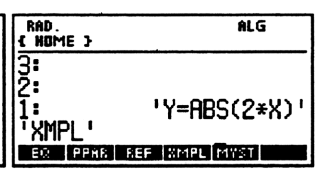
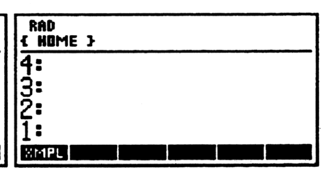


Fig. 3



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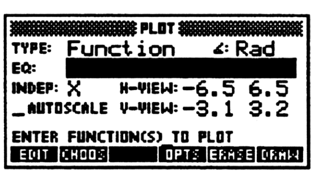
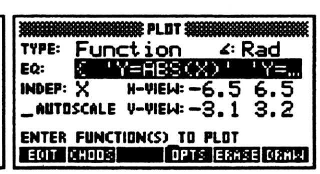


Fig. 5

Fig. 6



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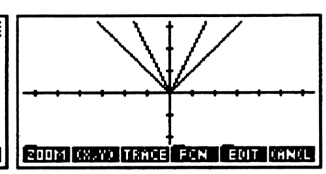
Fig. 8

Transformation <u>Function y1 is a vertical shift of two units up relative to function y; y and y1 are the same shape and size.</u>

ACTIVITY 2...TRANSFORMATIONS WITH ABSOLUTE VALUE FUNCTIONS

APPLICATIONS SECTION

You will be analyzing graphs of absolute value functions in this activity. The most basic absolute value function is $y = |x|$. We will call this the **reference function**. Any mathematical changes we make on this reference function will produce a graph that is different from that of the reference function's. These changes will result in what is known as a Transformation of the reference function. Ultimately, you should be able to predict the placement and shape of the graph of any absolute value function relative to the reference function in the Cartesian plane. See the Instructional Section for more details.

INVESTIGATION 1:

Reference: $y = | x |$ Basic description________________________________

$y1 = | x | + 2$ Transformation________________________________

$y2 = | x | - 3$ Transformation________________________________

$y3 = | x | + 1$ Transformation________________________________

$y4 = | x | + 4$ Prediction________________________________

$y5 = | x | + c$ Conjecure________________________________

INVESTIGATION 2:

Reference: $y = | x |$ Basic description________________________________

$y1 = | x + 2|$ Transformation________________________________

$y2 = | x - 4 |$ Transformation________________________________

$y3 = | x + 1 |$ Prediction________________________________

$y4 = | x - 3 |$ Prediction________________________________

$y5 = | x - b |$ Conjecture ________________________________

ACTIVITY 2... APPLICATIONS SECTION CONTINUED

INVESTIGATION 3:

Reference: $y = |x|$ Basic description__________

$y2 = |x - 2| + 3$ Transformation__________

$y3 = |x + 1| - 2$ Transformation__________

$y4 = |x + 5| + 2$ Prediction__________

$y5 = |x - 7| - 4$ Prediction__________

$y6 = |x - b| + c$ Conjecture__________

INVESTIGATION 4:

Reference: $y = |x|$ Basic description__________

$y1 = 2|x|$ Transformation__________

$y2 = 4|x|$ Transformation__________

$y3 = -1|x|$ Transformation__________

$y4 = -3|x|$ Transformation__________

$y5 = 5|x|$ Prediction__________

$y6 = -2|x|$ Prediction__________

$y7 = a \cdot |x|$ Conjecture __________

INVESTIGATION 5:

Reference: $y = |x|$ Basic description__________

$y1 = 2|x| + 1$ Transformation__________

$y2 = 3|x| - 2$ Transformation__________

$y3 = -4|x| + 3$ Transformation__________

$y4 = 2|x + 1| - 3$ Transformation__________

$y5 = 2|x + 4| + 3$ Prediction__________

$y6 = -2|x - 3| - 5$ Prediction__________

$y7 = a \cdot |x - b| + c$ Conjecture __________

ACTIVITY 3...TRANSFORMATIONS WITH QUADRATIC FUNCTIONS

INSTRUCTIONAL SECTION

The purpose of this lesson is to allow students to inductively discover the various effects transformations have on a reference quadratic function. Ultimately, students will be able to recognize the graph of any quadratic function as a transformation of the graph of the reference function $y = x^2$. Use the **"Basic Description"** blanks to describe the graph of the reference function. Use the **"Transformation"** blanks for writing the changes between the reference function and the transformed functions. Use the **"Prediction"** blanks to predict the changes between the reference function and the transformed functions. Then use the calculator to check your prediction. Use terminology such as vertical shift up or down, horizontal shift left or right, and by how many units, increased or decreased slope, reflected over the x-axis, etc. to illustrate the relative changes. The **"Conjecture"** blanks can be used for writing the general transformation that will occur relative to the reference function. Consider all possible real number substitutions for the variables in the **"Conjecture"** sections.

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2. Use the Equation Writer to key in the functions next to Reference, Transformation, and Prediction in an Investigation. Example: Enter $y = x^2 + 1$. Press **PURPLE EQUATION α Y PURPLE = α X y^x 2 ∇ + 1**, fig. 1, now press **ENTER**, fig. 2
3. Name and store each function using the corresponding names, Y, Y1, Y2, etc. given in the Investigations. Press **'$\alpha\alpha$ XMPL α**, fig. 3, then press the **STO** key. Notice that your equation and the name you stored it under have disappeared from the stack. If you press **VAR**, you will see XMPL on the menu, fig. 4.
4. Plot the reference function and one of the transformed functions. Press **GREEN PLOT**, fig. 5, then press **CHOOS** and **CHK** the functions you want graphed on the same set of axes, fig. 6. Now press **OK**, fig. 7.
5. Press the menu keys under the **ERASE** and **DRAW** menu labels, then analyze and note the differences and similarities between the two graphs, fig. 8.
6. Repeat steps 4 and 5, always including the reference function and another transformed function until all pairs have been analyzed.
7. Repeat steps 1 through 6 **after** each Investigation is completed.

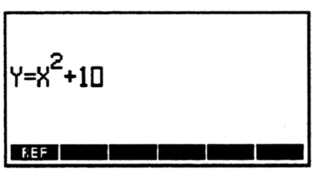


Fig. 1

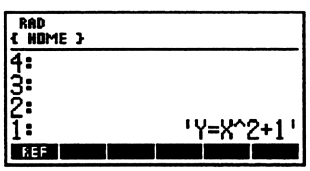


Fig. 2

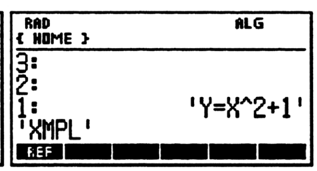
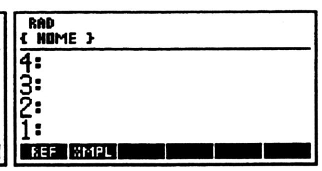


Fig. 3



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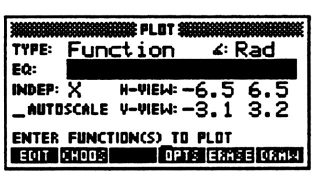

Fig. 5

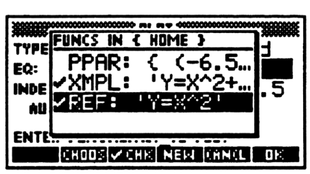

Fig. 6

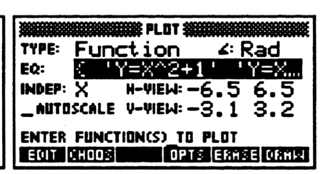

Fig. 7

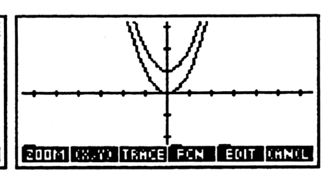

Fig. 8

Trans <u>Function XMPL represents a vertical shift up of one unit relative to the reference function REF; both graphs have the same shape and size.</u>

ACTIVITY 3...TRANSFORMATIONS WITH QUADRATIC FUNCTIONS

APPLICATIONS SECTION

You will be analyzing quadratic functions in this activity. The most basic quadratic function is $y = x^2$. We will call this the reference function. Any mathematical changes we make on this reference function will produce a graph that is different from that of the reference function's. These changes will result in what is known as a transformation of the reference function. Ultimately, you should be able to predict the placement and shape of the graph of any quadratic function relative to the reference function in the Cartesian plane. See the Instructional Section for more details.

INVESTIGATION 1:

Reference:	$y = x^2$	Basic description____________________
	$y1 = x^2 + 2$	Transformation____________________
	$y2 = x^2 - 3$	Transformation____________________
	$y3 = x^2 + 4$	Prediction____________________
	$y4 = x^2 + k$	Conjecture____________________

INVESTIGATION 2:

Reference:	$y = x^2$	Basic description____________________
	$y1 = (x-3)^2$	Transformation____________________
	$y3 = (x+1)^2$	Transformation____________________
	$y4 = (x-2)^2$	Prediction____________________
	$y5 = (x-h)^2$	Conjecture____________________

INVESTIGATION 3:

Reference:	$y = x^2$	Basic description____________________
	$y1 = (x-3)^2+2$	Transformation____________________
	$y2 = (x+1)^2-4$	Transformation____________________
	$y3 = (x+2)^2-3$	Prediction____________________
	$y4 = (x-h)^2+k$	Conjecture____________________

ACTIVITY 3... APPLICATIONS SECTION CONTINUED

INVESTIGATION 4:

Reference: $y = x^2$ Basic description________________________________

$y1 = 2x^2$ Transformation________________________________

$y2 = \frac{1}{2}x^2$ Transformation________________________________

$y3 = -3x^2$ Transformation________________________________

$y4 = 4x^2$ Prediction________________________________

$y5 = \frac{-3}{4}x^2$ Prediction________________________________

$y6 = a \cdot x^2$ Conjecture________________________________

INVESTIGATION 5:

Reference: $y = x^2$ Basic description________________________________

$y1 = 2(x+1)^2+3$ Transformation________________________________

$y2 = -3(x+2)^2-4$ Transformation________________________________

$y3 = \frac{1}{2}(x-3)^2-2$ Transformation________________________________

$y4 = 2(x-4)^2+3$ Transformation________________________________

$y5 = a \cdot (x-h)^2+k$ Conjecture________________________________

ACTIVITY 4...TRANSFORMATIONS WITH EXPONENTIAL FUNCTIONS

INSTRUCTIONAL SECTION

The purpose of this lesson is to allow students to inductively discover the various effects transformations have on a reference linear function. Ultimately, students will be able to recognize the graph of any exponential function as a transformation of the graph of the reference function $y = e^x$. Use the **"Basic Description"** blanks to describe the graph of the reference function. Use the **"Transformation"** blanks for writing the changes between the reference function and the transformed functions. Use the **"Prediction"** blanks to predict the changes between the reference function and the transformed functions. Then use the calculator to check your prediction. Use terminology such as vertical shift up or down, horizontal shift left or right, and by how many units, increased or decreased slope, reflected over the x-axis, etc. to illustrate the relative changes. The **"Conjecture"** blanks can be used for writing the general transformation that will occur relative to the reference function.

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2. Use the Equation Writer to key in the functions next to <u>Reference</u>, <u>Transformation</u>, and <u>Prediction</u> in an Investigation. Example: Enter $y = e^x - 2$. **PURPLE EQUATION α Y PURPLE = PURPLE e^x α X**, fig. 1, now press **ENTER**, fig. 2
3. Name and store each function using the corresponding names, Y, Y1, Y2, etc. given in the Investigations. Press **'αα XMPL α**, fig. 3, then press the **STO** key. Notice that your equation and the name you stored it under have disappeared from the stack. If you press **VAR**, you will see XMPL on the menu, fig. 4.
4. Plot the reference function and one of the transformed functions. Press **GREEN PLOT**, fig. 5, then press **CHOOS** and **CHK** the functions you want graphed on the same set of axes, fig. 6. Now press **OK**, fig. 7.
5. Press the menu keys under the **ERASE** and **DRAW** menu labels, then analyze and note the differences and similarities between the two graphs, fig. 8.
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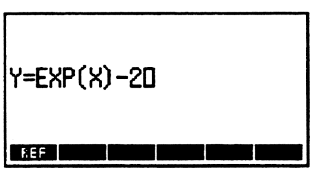


Fig. 1

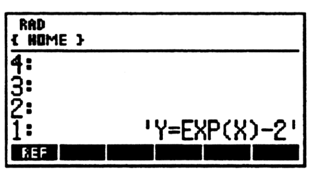


Fig. 2

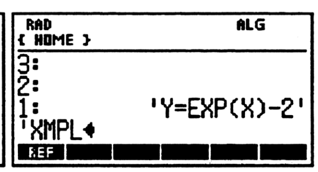


Fig. 3

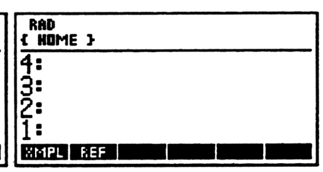


Fig. 4

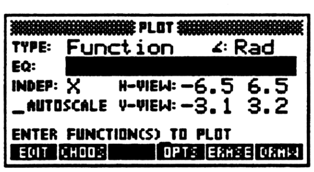


Fig. 5

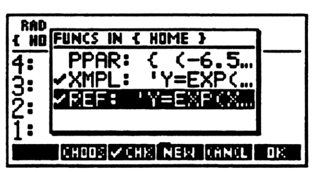


Fig. 6

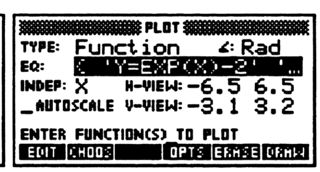


Fig. 7

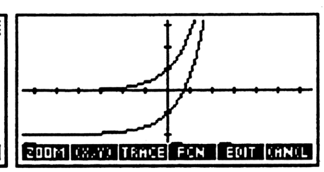


Fig. 8

Transformation <u>Function XMPL represents a vertical shift down of two units relative to the reference function REF; both have the same shape and size.</u>

ACTIVITY 4...TRANSFORMATIONS WITH EXPONENTIAL FUNCTIONS

APPLICATIONS SECTION

You will be analyzing exponential functions in this activity. The most basic exponential function is $y = e^x$. We will call this the reference function. Any mathematical changes we make on this reference function will produce a graph that is different from that of the reference function's. These changes will result in what is known as a transformation of the reference function. Ultimately, you should be able to predict the placement and shape of the graph of any exponential function relative to the reference function in the Cartesian plane. See the Instructional Section for more details.

INVESTIGATION 1:

Reference: $y = e^x$ Basic description____

$y1 = e^x + 2$ Transformation____

$y2 = e^x - 3$ Transformation____

$y3 = e^x + 1$ Transformation____

$y4 = e^x + 10$ Prediction____

$y5 = e^x + d$ Conjecture____

INVESTIGATION 2:

Reference: $y = e^x$ Basic description____

$y1 = e^{x+2}$ Transformation____

$y2 = e^{x-1}$ Transformation____

$y3 = e^{x-3}$ Transformation____

$y4 = e^{x+4}$ Prediction____

$y5 = e^{x+c}$ Conjecture____

ACTIVITY 4... APPLICATIONS SECTION CONTINUED

INVESTIGATION 3:

Reference: $y = e^x$ Basic description____________________

$y1 = e^{2x}$ Transformation____________________

$y2 = e^{-x}$ Transformation____________________

$y3 = e^{.5x}$ Transformation____________________

$y4 = e^{-5x}$ Transformation____________________

$y5 = e^{3x}$ Prediction____________________

$y6 = e^{-2x}$ Prediction____________________

$y7 = e^{.25x}$ Prediction____________________

$y8 = e^{bx}$ Conjecture____________________

INVESTIGATION 4:

Reference $y = e^x$ Basic Description____________________

$y1 = 2e^x$ Transformation____________________

$y2 = \frac{1}{2}e^x$ Transformation____________________

$y3 = -1e^x$ Transformation____________________

$y5 = -3e^x$ Transformation____________________

$y6 = 5e^{-x}$ Transformation____________________

$y7 = -2e^{-x}$ Prediction____________________

$y8 = \frac{1}{4}e^x$ Prediction____________________

$y9 = ae^x$ Conjecture____________________

ACTIVITY 4... APPLICATIONS SECTION CONTINUED

INVESTIGATION 5:

Reference	$y = e^{x}$	Basic Description______________________
	$y1 = e^{x+2}-3$	Transformation______________________
	$y2 = e^{2(x-1)}+4$	Transformation______________________
	$y3 = 2e^{3(x+1)}-4$	Transformation______________________
	$y4 = 4e^{2(x-3)}+5$	Prediction______________________
	$y5 = -3e^{-(x+2)}-1$	Prediction______________________
	$y6 = ae^{b(x-c)}+d$	Conjecture______________________

ACTIVITY 5...TRANSLATIONS OF CONICS

INSTRUCTIONAL SECTION

The purpose of this activity is to investigate translations of graphs of conic equations. We will use the standard form for conic equations with the reference equations having centers, or in the case of parabolas, vertices, at the origin. The analysis will include questions about the location of a center or vertex point after a translation has occurred. When analyzing the graphs of circles, ellipses, and hyperbolas, it is often difficult to locate their centers on the screen since their centers are not actually points on their graphs. To determine the translation, it is best to find the ordered pairs of two corresponding points from the graphs, then analyze the translation by studying the two ordered pairs. A conclusion can then be made about the location of the center point following a translation. A similar analysis can be conducted with parabolas, however, finding and using their vertices as the pair of corresponding points can be done directly by using the crosshairs and the **(x,y)** menu key command in the Plot environment. In the Applications Section, record your analysis in the blanks provided. Use blanks following **Translation** for recording the number of units and the direction, horizontally and vertically, the translation represents. Record the coordinates of the center or vertex of each translated conic at the right of the **Translation** blanks.

DIRECTIONS FOR USING THE HP 48G:

1. Let's begin by purging unwanted objects from the HOME directory. Pressing the **VAR** key will display the objects in the HOME directory on the menu labels. The fast way to purge is to create a list of unwanted objects and then purge the list. Press **PURPLE {** then press the white menu keys under the menu labels of those objects you want to purge, press **ENTER** then **PURPLE PURGE.**
2. Use the Equation Writer to key in the equations next to Reference, Translation, and Predict Translation in an Investigation. Example: Enter $x^2 + y^2 = 4$. **PURPLE EQUATION** α **x** y^x **2** ∇ **+** α **y** y^x **2** ∇ **PURPLE =** **4**, fig. 1. Now press **ENTER**, fig. 2. Note: Moving the cursor to the right of a parenthesis can be done by pressing the right arrow key.
3. Name and store each equation using the corresponding names, C, C1, C2, etc. given in the Investigations. Press **'**$\alpha\alpha$ **C** α, fig. 3, then press the **STO** key. Notice that your equation and the name you stored it under have disappeared from the stack. If you press **VAR**, you will see C on the menu, fig. 4.
4. Plot the reference equation and one of the transformed equations. Press **GREEN PLOT**, move the highlight to TYPE: in the PLOT menu, press **CHOOS**, highlight **CONIC** from the pull down, menu, fig. 5, then press **OK**. Highlight EQ: press **CHOOS** and **CHK** the functions you want graphed on the same set of axes, fig. 6. Now press **OK**, fig. 7.
5. Press the menu keys under the **ERASE** and **DRAW** menu labels, then analyze the two graphs, fig. 8.
6. Repeat steps 4 and 5, always including the reference function and another transformed function until all pairs have been analyzed.
7. Repeat steps 1 through 6 **after** each Investigation is completed.

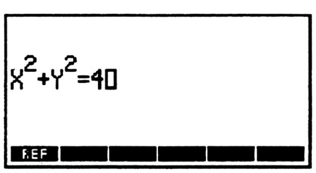


Fig. 1

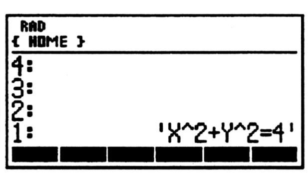


Fig. 2

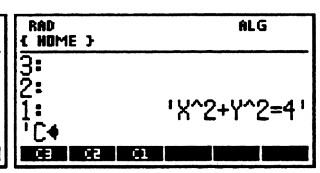


Fig. 3

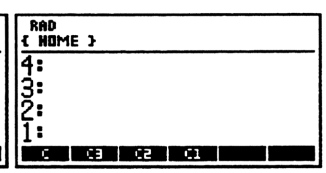


Fig. 4

Fig. 5

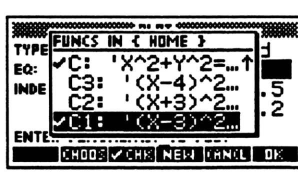


Fig. 6

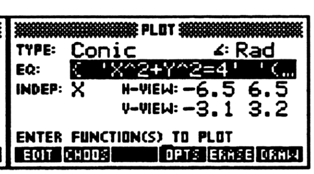


Fig. 7

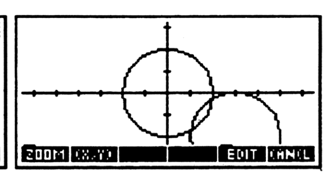


Fig. 8

ACTIVITY 5...TRANSLATIONS OF CONICS

APPLICATIONS SECTION

INVESTIGATION 1: TRANSLATIONS OF CIRCLES

Reference Circle **C**: $x^2 + y^2 = 4$
Coordinates of center of C are (,)

C1: $(x-3)^2 + (y+2)^2 = 4$
Translation____________________________________Coordinates of center of C1 are (,)

C2: $(x+3)^2 + (y-2)^2 = 4$
Translation____________________________________Coordinates of center of C2 are (,)

C3: $(x-4)^2 + (y-5)^2 = 4$
Translation____________________________________Coordinates of center of C3 are (,)

C4: $(x+3)^2 + (y+5)^2 = 4$
Predict Translation________________________________Coordinates of center of C4 are (,)

C5: $(x-h)^2 + (y-k)^2 = 4$
General Translation________________________________Coordinates of center of C5 are (,)

INVESTIGATION 2: TRANSLATIONS OF ELLIPSES

Reference Ellipse **E**: $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$
Coordinates of center of E are (,)

E1: $\dfrac{(x-1)^2}{9} + \dfrac{(y-1)^2}{4} = 1$
Translation____________________________________Coordinates of center of E1 are (,)

E2: $\dfrac{(x+2)^2}{9} + \dfrac{(y+3)^2}{4} = 1$
Translation____________________________________Coordinates of center of E2 are (,)

E3: $\dfrac{(x+3)^2}{9} + \dfrac{(y-4)^2}{4} = 1$
Predict Translation________________________________Coordinates of center of E3 are (,)

E4: $\dfrac{(x-h)^2}{9} + \dfrac{(y-k)^2}{4} = 1$
General Translation________________________________Coordinates of center of E4 are (,)

ACTIVITY 5... APPLICATIONS SECTION CONTINUED

INVESTIGATION 3: TRANSLATIONS OF PARABOLAS

Reference Parabola **P**: $x^2 = 1 \cdot y$
Coordinates of vertex of P are (,)

P1: $(x-2)^2 = 1 \cdot (y-2)$
Translation____Coordinates of vertex of P1 are (,)

P2: $(x+3)^2 = 1 \cdot (y+2)$
Translation____Coordinates of vertex of P2 are (,)

P3: $(x-3)^2 = 1 \cdot (y+1)$
Prediction____Coordinates of vertex of P3 are (,)

P4: $(x-h)^2 = 1 \cdot (y-k)$
General Translation____Coordinates of vertex of P4 are (,)

INVESTIGATION 4: TRANSLATIONS OF PARABOLAS CONTINUED

Reference Parabola **P**: $y^2 = 1 \cdot x$
Coordinates of vertex of P are (,)

P1: $(y-2)^2 = 1 \cdot (x-2)$
Translation____Coordinates of vertex of P1 are (,)

P2: $(y+3)^2 = 1 \cdot (x+2)$
Translation____Coordinates of vertex of P2 are (,)

P3: $(y-3)^2 = 1 \cdot (x+1)$
Prediction____Coordinates of vertex of P3 are (,)

P4: $(y-k)^2 = 1 \cdot (x-h)$
General Translation____Coordinates of vertex of P4 are (,)

ACTIVITY 5... APPLICATIONS SECTION CONTINUED

INVESTIGATION 5: TRANSLATIONS OF HYPERBOLAS

Reference Hyperbola: **H**: $\frac{x^2}{9} - \frac{y^2}{4} = 1$

Coordinates of center of H are (,)

H1: $\frac{(x-1)^2}{9} - \frac{(y-1)^2}{4} = 1$

Translation____Coordinates of center of H1 are (,)

H2: $\frac{(x+2)^2}{9} - \frac{(y+3)^2}{4} = 1$

Translation____Coordinates of center of H2 are (,)

H3: $\frac{(x+3)^2}{9} - \frac{(y-4)^2}{4} = 1$

Predict Translation____________________________________Coordinates of center of H3 are (,)

H4: $\frac{(x-h)^2}{9} - \frac{(y-k)^2}{4} = 1$

General Translation____________________________________Coordinates of center of H4 are (,)

ACTIVITY 6...TRANSFORMATIONS WITH TRIGONOMETRIC FUNCTIONS

INSTRUCTIONAL SECTION

This activity explores transformations of the function y = sin(x). We will look at translations, stretching and shrinking, and reflections. Use the **"Basic Description"** blanks to describe the graph of the reference function. Use the **"Transformation"** blanks for writing the changes between the reference function and the transformed functions. Use the **"Prediction"** blanks to predict the changes between the reference function and the transformed functions. Then use the calculator to check your prediction. Use terminology such as vertical shift up or down, horizontal shift left or right, and by how many units, increased or decreased slope, reflected over the x-axis, etc. to illustrate the relative changes. The **"Conjecture"** blanks can be used for writing the general transformation that will occur relative to the reference function. Students can challenge each other by writing their own function, plotting it, and giving it to another student for a prediction of the actual equation that produced the graph.

DIRECTIONS FOR USING THE HP 48G:

1. Let's begin by purging unwanted objects from the HOME directory. Pressing the **VAR** key will display the objects in the HOME directory on the menu labels. The fast way to purge is to create a list of unwanted objects and then purge the list. Press **PURPLE {** then press the white menu keys under the menu labels of those objects you want to purge, press **ENTER** then **PURPLE PURGE.**
2. Use the Equation Writer to key in the functions next to <u>Reference</u>, <u>Transformation</u>, and <u>Prediction</u> in an Investigation. Example: Enter y = 3sin(x). **PURPLE EQUATION α Y PURPLE = 3 SIN α X**, fig. 1, now press **ENTER**, fig. 2
3. Name and store each function using the corresponding names, Y, Y1, Y2, etc. given in the Investigations. Press **'αα XMPL α**, fig. 3, then press the **STO** key. Notice that your equation and the name you stored it under have disappeared from the stack. If you press **VAR**, you will see XMPL on the menu, fig. 4.
4. Plot the reference function and one of the transformed functions. Press **GREEN PLOT**, set the top of the screen to FUNCTION and RADIAN mode, fig. 5, then press **CHOOS** and **CHK** the functions you want graphed on the same set of axes, fig. 6. Now press **OK**, fig. 7.
5. Press the menu keys under the **ERASE** and **DRAW** menu labels, then analyze and note the differences and similarities between the two graphs, fig. 8.
6. Repeat steps 4 and 5, always including the reference function and another transformed function until all pairs have been analyzed.
7. Repeat steps 1 through 6 after each Investigation is completed.

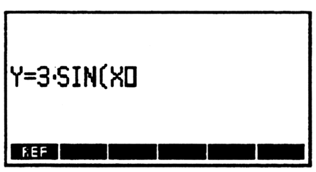


Fig. 1

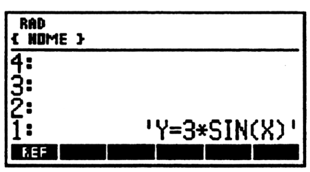


Fig. 2

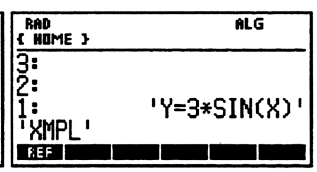


Fig. 3

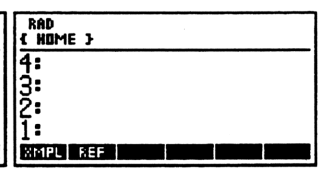


Fig. 4

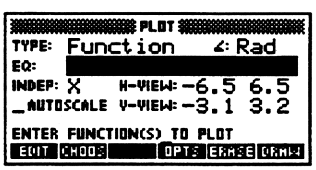


Fig. 5

Fig. 6

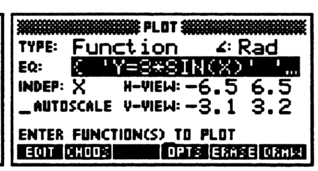


Fig. 7

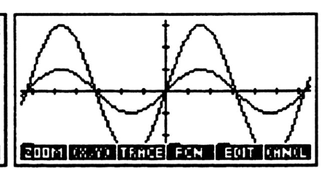


Fig. 8

Transformation <u>The graph of XMPL is vertically stretched by a factor of three relative to the reference function.</u>

ACTIVITY 6...TRANSFORMATIONS WITH TRIGONOMETRIC FUNCTIONS

APPLICATIONS SECTION

You will be analyzing the trigonometric function y = sin(x) in this activity. We will call this the reference function. Any mathematical changes we make on this reference function will produce a graph that is different from that of the reference function's. These changes will result in what is known as a transformation of the reference function. Ultimately, you should be able to predict the shape and placement of the graph of any transformed sin(x) function. These transformations generalize to the graphs of the functions y = cos(x), y = sec(x), y = csc(x), and some transformations of the functions y = tan(x) and y = cot(x).

INVESTIGATION 1:

Reference: $y = \sin(x)$ Basic description__________

$y1 = 2\sin(x)$ Transformation__________

$y2 = -1\sin(x)$ Transformation__________

$y3 = \frac{1}{3}\sin(x)$ Transformation__________

$y4 = -3\sin(x)$ Prediction__________

$y5 = 4\sin(x)$ Prediction__________

$y6 = \frac{-1}{2}\sin(x)$ Prediction__________

$y7 = a\cdot\sin(x)$ Conjecture__________

INVESTIGATION 2:

Reference: $y = \sin(x)$ Basic Description__________

$y1 = \sin(x) + 1$ Transformation__________

$y2 = \sin(x) - 2$ Transformation__________

$y3 = \sin(x) + 3$ Transformation__________

$y4 = 2\sin(x) - 1$ Transformation__________

$y5 = -3\sin(x) + 1$ Prediction__________

$y6 = \frac{-1}{3}\sin(x) - 2$ Prediction__________

$y7 = a\cdot\sin(x) + d$ Conjecture__________

ACTIVITY 6...APPLICATIONS SECTION CONTINUED

INVESTIGATION 3:

Reference:	$y = \sin(x)$	Basic Description____________________
	$y1 = \sin(x - 1)$	Transformation____________________
	$y2 = \sin(x + 3)$	Transformation____________________
	$y3 = \sin(x - 2)$	Transformation____________________
	$y4 = \sin(x + 2) - 1$	Transformation____________________
	$y5 = 2\sin(x - 3) + 2$	Prediction____________________
	$y6 = -1\sin(x + 4) - 3$	Prediction____________________
	$y7 = a\cdot\sin(x - c) + d$	Conjecture____________________

INVESTIGATION 4:

Reference:	$y = \sin(x)$	Basic Description____________________
	$y1 = \sin(2x)$	Transformation____________________
	$y2 = \sin(\frac{1}{2}x)$	Transformation____________________
	$y3 = \sin(4x)$	Transformation____________________
	$y4 = \sin(-1x)$	Transformation____________________
	$y5 = \sin(-2x)$	Transformation____________________
	$y6 = \sin(bx)$	Conjecture____________________
	$y7 = 3\sin(2x) + 1$	Transformation____________________
	$y8 = \sin[2(x - 1)] + 3$	Transformation____________________
	$y9 = 2\sin[4(x + 3)] - 5$	Prediction____________________
	$y10 = a\cdot\sin[b(x - c)] + d$	Conjecture____________________

ACTIVITY 1...ROOT FINDING OF POLYNOMIAL FUNCTIONS

INSTRUCTIONAL SECTION

This activity explores finding roots of polynomial functions using several different methods. The HP 48G can be used in several different ways to find the zeros of polynomial functions. We will use the TRACE and (X,Y) features to locate and view a zero, the FUNCTION and ROOT features, and the SOLVE environment. Functions will progress from real roots to mixed complex and real roots.

DIRECTIONS FOR USING THE HP 48G:

1. Use the Equation Writer to key in a function. **PURPLE EQUATON α Y PURPLE = α X y^x 2 ∇ - 4 α x + 3 ENTER,** Fig. 1.
2. Name and store the equation with the name suggested in 'ticks' next to each equation. **' $\alpha\alpha$ XMPL α STO,** fig. 2.
3. Plot the function. **GREEN PLOT CHOOS** , **CHK** (check) the function , then press**OK**, **ERASE**, and **DRAW**, fig. 3.
4. Use the **TRACE** and **(x,y)** features to find the roots. Press **Trace** from the menu bar, use left or right arrows to trace along the graph, when crosshairs are on an x-intercept, press **(X,Y)**, the ordered pair is displayed. Fig. 4 Press any white key to return the menu labels.
5. Use the **FUNCTION** and **ROOT** features to find the roots. Plot the function, press menu label **FCN**, then press **ROOT**, the crosshairs should be on the root it was nearest when you pressed **ROOT.**
   Activate the menu labels by pressing just about any key, arrow the crosshairs *close* to another root, and press **ROOT**. Fig. 5
6. Use the **SOLVE** environment to find the roots. **GREEN SOLVE**, arrow to **solve poly...**, **OK** , enter coefficients of the polynomial in square brackets. **PURPLE [1 SPC 4 $\pm$ SPC 3 OK** . Fig. 6. Press menu label **SOLVE**, the roots are displayed, fig. 7. Press menu label **SYMB** , you won't see anything new on the current screen, but the factored version and the roots will be on levels 1 and 2 of the stack respectively.
   You can view them by either pressing **NXT** then menu label **CALC**, or by pressing **ON** twice. Fig. 8.

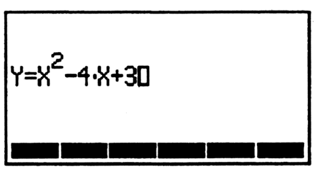


Figure 1

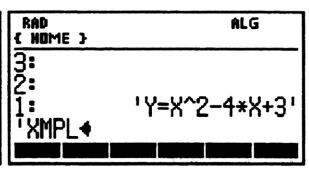


Figure 2

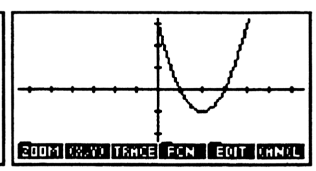


Figure 3

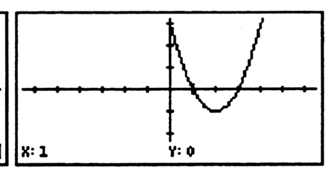


Figure 4

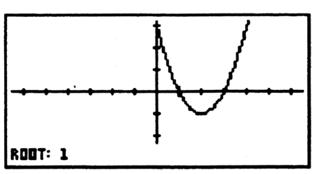


Figure 5

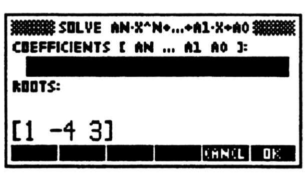


Figure 6

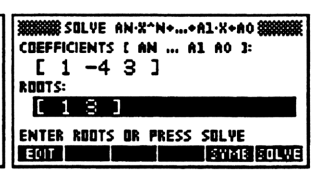


Figure 7

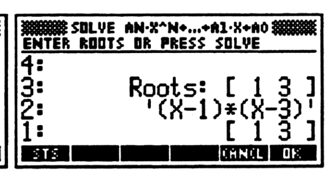


Figure 8

ACTIVITY 1...ROOT FINDING OF POLYNOMIAL FUNCTIONS

APPLICATIONS SECTION

1. P1: $y = x^2 + 3x$

Roots are: x = _____ and x = ______
P1 factored is y = (   )(   )

2. P2: $y = x^2 - x - 2$

Roots are: x = _____ and x = ______
P2 factored is y = (   )(   )

3. P3: $y = -x^2 - 2x + 15$

Roots are: x = _____ and x = ______
P3 factored is y = (   )(   )

4. P4: $y = x^2 - 2$

Roots are: x = _____ and x = ______
P4 factored is y = (   )(   )

5. P5: $y = x^3 - 2x^2 - 5x + 6$

Roots are: x = ___and x = ___and x = ___
P5 factored is y = (   )(   )(   )

6. P6: $y = x^2 + 4$

Roots are: x = _____ and x = ______
P6 factored is y = (   )(   )

7. P7: $y = x^2 - 4x + 5$

Roots are: x = ___and x = ___
P7 factored is y = (   )(   )

8. P8: $y = x^2 - 2x - 1$

Roots are: x = _____ and x = ______
P8 factored is y = (   )(   )

9. P9: $y = x^3 - 7x + 6$

Roots are: x = ___and x = ___
P9 factored is y = (   )(   )(   )

10. P10: $y = 6x^4 + 67x^3 + 118x^2 + 7x - 18$

Roots are: x = ___, ___, ___, and ___
P10 factored is y = (   )(   )(   )(   )

ACTIVITY 2...POLYNOMIAL MAX/MIN FINDING

INSTRUCTIONAL SECTION

The emphasis on this activity is to introduce the concepts of maximum and minimum points of a polynomial function. We will present a few theoretical functions and find their maxima and minima, which will then be followed by several problems with connections to the 'real' world. Use the TRACE feature to get the crosshairs on what *appears* to be a maximum or a minimum, select the (X,Y) feature to note the ordered pair, then check your hypothesis by selecting the FUNCTION and EXTREMA features.

DIRECTIONS FOR USING THE HP 48G:

1. Use the Equation Writer to key in a function. **PURPLE EQUATION α Y PURPLE = α X y^x 3 ∇ - 7 α X + 6 ENTER.**
2. Name and store the equation with the name suggested in 'ticks' next to each equation. **'αα XMPL αSTO** Figure 2. Note: after pressing **STO**, the function and the name you give it will disappear from the stack.
3. Plot the function. **GREEN PLOT,** from the menu labels press **CHOOS** then **CHK** (check) the fucntion, fig. 3. From the menu labels, press **OK**, **ERASE**, then **DRAW**. Fig. 4.
4. Use the **TRACE** and **(X,Y)** features on the menu bar to find the maximum and minimum points; these are the highest and lowest points respectively, along a turn of a graph. Fig. 5
5. A greater degree of accuracy can be found using the FUNCTION EXTREMA features. Get the crosshairs near a max. or min. point using the **arrow** keys, from the menu labels, press **FCN**, then **EXTR**. The crosshair will jump to the the point the calculator finds as the max. or min. point. Fig. 6. The data is automatically copied to the stack, fig. 7, which you can access by pressing **ON** twice. Use the **VIEW** key to see the ordered pair with better detail, fig. 8.

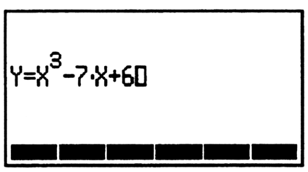


Figure 1

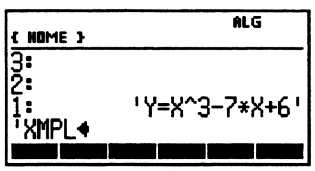


Figure 2

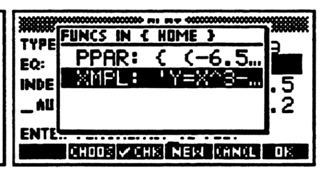
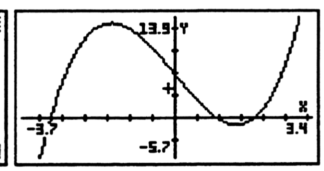


Figure 3



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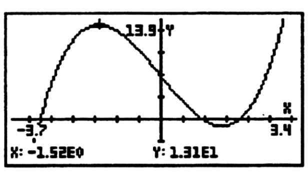


Figure 5

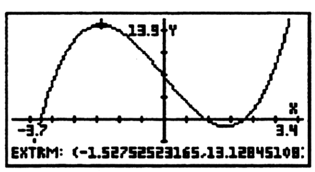


Figure 6

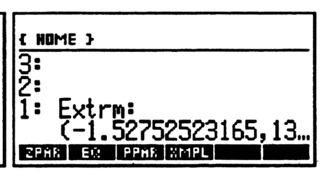


Figure 7

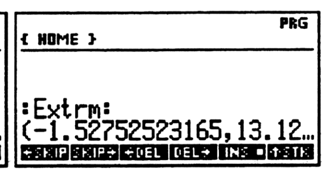


Figure 8

ACTIVITY 2...POLYNOMIAL MAX/MIN FINDING

APPLICATIONS SECTION

Polynomial Functions: Locating maximum and minimum points.

1. 'P1': $y = 2x^3 - x + 2$

Minimum:_________
Maximum:_________

2. 'P2': $y = x^3 - 3x^2 + 3$

Minimum:_________
Maximum:_________

3. 'P3': $y = -x^2 - 2x + 15$

Minimum:_________
Maximum:_________

4. 'P4': $y = x^4 - 5x^2 + 4$

Minimum:_________
Maximum:_________

When a projectile is fired straight up, its <u>height</u> equation is given by $y = -4.9t^2 + v_0t + y_0$ where y is measured in meters, t is time in seconds, v_0 is initial velocity, and y_0 is initial height. This equation will yield ordered pairs (t,y); remember v_0 and y_0 are constants, not variables. If you enter the functions on the calculator using t instead of x, be sure to change the independent variable selection to t instead of x in the PLOT menu. Finally, the graph this function produces <u>is not the path</u> the projectile takes, rather, it represents time vs distance from the ground.

5. 'P5': You shoot a rock from a sling shot straight up with an initial velocity of 10m/s, and let's say you fired the the rock exactly 2 meters from the ground.

A) Write the height equation from this given.

B) Use the HP 48G to graph the function.

C) Use the HP 48G to find the maximum height the rock reaches during its flight.

D) At what time does the rock hit the ground?

E) Describe the real world interpretation of the y - intercept for this problem.

ACTIVITY 2...APPLICATIONS SECTION CONTINUED

6. 'P6': Suppose you fire a model rocket straight up. At the end of the burn of the rocket engine, we will start counting time. At the **end** of the burn it has an upward velocity of 250m/s and is 37 m high.

   A) Write the height equation from this given.

   B) Use the HP 48G to graph this time-height function.

   C) What is the maximum height the rocket attained?

   D) At what time was maximum height attained?

   E) How much time is required for the rocket to come back down and hit the ground?

7. 'P7': Your job is to build a rectangular pen using extactly 120m of fencing. There are infinitely many different dimensions you could use, however there is exactly one set of length and width dimensions that will yield maximum area in the pen.

   A) Write an area function in terms of length and width. (hint: use one variable to express length and width)

   B) Use the HP 48G to graph the function.

   C) Use the graph to interpret the maximum area and the corresponding length and width dimensions. Length = _____ Width = _____

   D) What is the common geometric name for this rectangle of maximum area?

   E) The x-intercepts represent zero area, why?

8. 'P8': An open box is to be made from a square piece of material by cutting 3 inch squares from each corner and turning up the sides. You want the finished box to have a volume of 300 in.^{3}

   A) Write a volume function in terms of the length, width, and height of the box.

   B) Use the HP 48G to graph the function.

   C) Use the graph, along with the trace feature to find a volume of 300 in.^{3} and the corresponding length of the original piece of square material. Original length including the three inch squares on the ends is ________.

   D) If you want the finished box to have a volume of 192 in.^{3}, what must the length of the original piece of square material have been?

ACTIVITY 3...USING SLOPE OF LINES AND CURVES AS RATES

INSTRUCTIONAL SECTION

This activity explores finding rates of change in quantities using the slope of the curve at a given point. The emphasis on this activity is to develop the concept of rates of change as they relate to the slope of a curve at a point on the curve. We will start with lines and progress to non-linear functions. Problems are written to create connections in the 'real' world. Slope of a line is defined as the change in vertical distance divided by the change in the horizontal distance between two points on the line. Suppose we have a graph showing time -vs- distance, i.e. time on the horizontal axis, and distance on the vertical axis. Now, if we include the units when we calculate slope, the result is a ratio of distance to time which is a **rate** of speed quantity. Hence, slope is valuable for determining rates of changing quantities. To determine the slope of a curve at a given point on the curve, we must do so by considering the slope of the *tangent* line to the curve at the given point. The HP 48G has a very handy feature for determining the slope of a curve at a given point in the FUNCTION sub-menu.

DIRECTIONS FOR USING THE HP 48G:

1. Use the Equation Writer to key in a function. **PURPLE EQUATION α D PURPLE = 2.6 α T ENTER,** fig. 1.
2. Name and store the equation with the name suggested in 'ticks' next to each equation. **' ααXMPL α STO**, fig. 2.
3. Plot the function. **GREEN PLOT,** from the menu labels, press **CHOOS**, then **CHK** (check) the function, fig. 3. Press **OK**, then **arrow** down to **INDEP: X** and change to the independent variable used in the function, fig. 4. Now press **ERASE**, then **DRAW** from the menu labels. Zoom appropriately. Figure 5
4. From the menu labels, press **TRACE** to lock the crosshairs on to the graph, then **FCN** then **SLOPE**, fig. 7
6. If we wanted to know the slope of curve at a particular point on the curve, press **TRACE, (X,Y), left or right Arrow** until the ordered pair shows the coordinates you want, press **FCN**, then **SLOPE** from the menu labels, fig. 8.

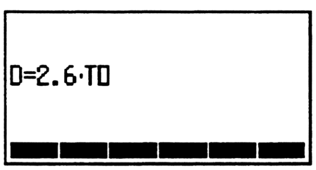


Figure 1

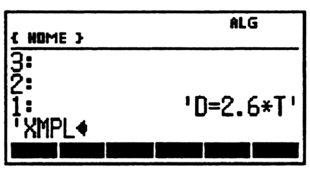


Figure 2

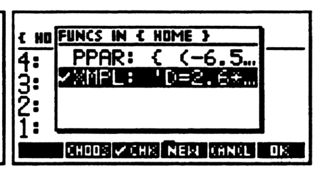


Figure 3

Figure 4

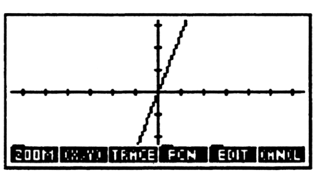


Figure 5

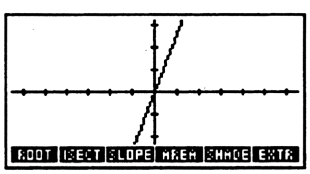


Figure 6

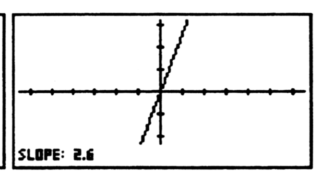


Figure 7

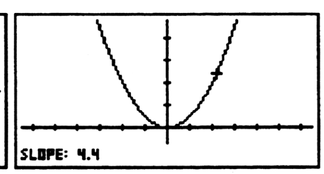


Figure 8

ACTIVITY 3...USING SLOPE OF LINES AND CURVES AS RATES

APPLICATIONS SECTION

1. 'P1' You have started to sell some software you recently wrote, and you sell each application for \$49. It costs you \$3 to produce each application, and the original time required to develop the software cost you \$4,000. An income equation for your business is $I = 46A - 4000$, where I is income and A is number of applications sold.

   A) Graph this function on the HP 48 G, zoom appropriately.

   B) Find the slope of the line you have graphed.

   C) What is your rate of earnings according to this model?

   D) How many copies must you sell to break even?

   E) Does your <u>rate of earnings</u> ever increase in this model? Why or Why not?

When a projectile is fired straight up, its <u>height</u> equation is given by $h = -4.9t^2 + v_0t + h_0$ where h is measured in meters, t is time in seconds, v_0 is initial velocity, and h_0 is initial height. This equation will yield ordered pairs (t,h); remember v_0 and h_0 are constants, not variables. If you enter the functions on the calculator using t instead of x, be sure to change the independent variable selection to t instead of x in the Plot menu. Finally, the graph this function produces **<u>is not the path</u>** the projectile takes, rather it represents **time -vs- height from the ground**. Notice that the slope calculation, $\Delta h/\Delta t$, has units in meters / seconds, a speed or velocity quantity. Hence, again this simple slope calculation (on the calculator) can provide powerful information about the object's speed at any time during its flight.

2. 'P2' You shoot a rock from a sling shot straight up with an initial velocity of 10m/s, and let's say you fired the rock exactly 2 meters from the ground.

   A) Write the height equation from this given.

   B) Use the HP 48 G to graph the function.

   C) What is the speed of the rock at one second into its flight?

   D) What is the speed of the rock at 1.6 seconds into its flight. Why is this quantity negative?

   E) The speed of the rock is approximately 7 m/s at what time during its flight?

ACTIVITY 3...APPLICATIONS SECTION CONTINUED

3. 'P3': Suppose you fire a model rocket straight up. At the end of the burn of the rocket engine, we will start counting time. At the end of the burn it has an upward velocity of 250m/s and is 37 m high.

   A) Write the height equation from this given.

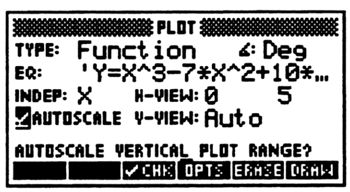
   B) Use the HP 48 G to graph this time-height function.

   C) What is the speed at approximately 10 seconds into the flight?

   D) What is the speed when the rocket is approximately 3000 m in the air?

   E) What is the speed of the rocket at the instant it hits the ground?

4. 'P4' A particle moves about the Cartesian plane according to the equation $y = x^3 - 7x + 6$. Enter and graph this motion function with the H-VIEW set to 0 on the left and 5 on the right, see figure below. Check AUTOSCALE, then press ERASE and DRAW.



   A) What is the speed of the particle at (.5, 3.375)?

   B) What is the speed when the particle intersects the x-axis at x = 2?

   C) What is the speed of the particle at y = -5.625?

   D) What are the coordinates on the points where the particle comes to rest for an instant of time?

ACTIVITY 4...CONICS AS COMBINATIONS OF FUNCTIONS

INSTRUCTIONAL SECTION

This acitivity will guide students through breaking conic equations down into <u>functions</u> and graphing the two functions simultaneously to complete the graph. The HP 48G does have a conic mode for graphing conics directly, however, the emphasis of this activity is to understand how conic equations implicity define two, (or more), functions. The calculator will be used to find the *principal* function y, in terms of x, the user will negate a copy of function y. For example, a circle can be thought of as a combination of two semi-circles; one function can represent the upper semi-circle, and the other can represent the lower semi-circle. Finally, the calculator will be used to graph both fucntions y and -y simultaneously to complete the graph of the original conic equation. It should be noted that *in general*, conic equations are not functions in and of themselves.

DIRECTIONS FOR USING THE HP 48G:

1. Use the Equation Writer to key in the equation of each conic section. **PURPLE EQUATION α X y^x 2 ∇+α Y y^x 2 ∇PURPLE = 4 ENTER**, fig. 1.
2. Name and store the equation with the name suggested in 'ticks' next to each equation. **' ααCIRCL α STO,** fig. 2.
3. Get into the Symbolic Environment, **GREEN SYMBOLIC**.
4. Select **Isolate var...** from the menu using the down arrow, then press **OK** from the menu label, fig. 3.
5. When the highlight is on EXPR:, select **CHOOS** from the menu label.
6. Highlight desired equation, then press **OK** from the menu label, fig.4.
7. Enter the variable you wish to isolate in the VAR: line on the menu, move the highlight to VAR: and press **α Y,** then **OK** from the menu label, fig. 5.
8. Highlight and **CHK** (check) PRINCIPAL, fig. 6.
9. Press **OK** from the menu label.
10. The variable should now be isolated; the result is transfered to the first level of the Stack, fig. 7.
11. Make a second copy of this function by pressing the **ENTER** key.
12. Name and store the function on level 1 of the stack, **' ααSMCUP α STO** {Abbreviation for semi-cirlce up}
13. Negate the right hand member of the remaining function on the stack . **PURPLE EDIT right arrow three times $-1 \times$ ENTER.**
14. Name and store the function on level 1 of the stack, 'SMCDN', {Abbreviation for semi-circle down}, see instruction #12.
15. Plot the two <u>fucntions</u> 'SMCUP' and 'SMCDN' simultaneously. **GREEN PLOT,** set the PLOT TYPE to Function; from the menu labels press: **CHOOS** , **CHK** (check) the functions 'SMCUP' and 'SMCDN' , press **OK**, then **ERASE**, then **DRAW**, fig. 8.

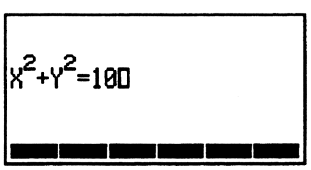


Figure 1

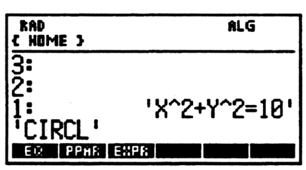


Figure 2

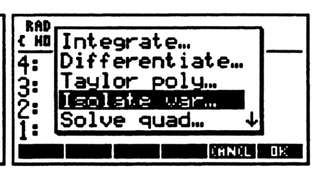


Figure 3

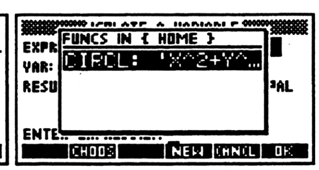


Figure 4

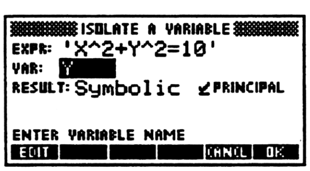


Figure 5

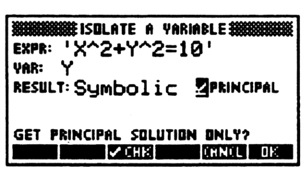


Figure 6

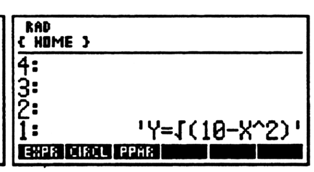


Figure 7

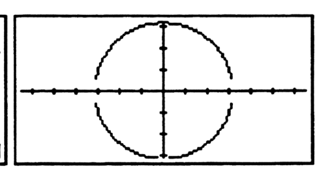


Figure 8

ACTIVITY 4...CONICS AS COMBINATIONS OF FUNCTIONS

APPLICATIONS SECTION

INVESTIGATION WITH CIRCLES.

Equation 1: $x^2 + y^2 = 9$

Solve the equation for y either by hand or using the calculator (see directions 1-14).

y = ___________ and y = ____________

Using the calculator, graph the two functions you found above to varify that taken together, the functions' graphs create that of the circle (see direction #15).

Equation 2: $x^2 + y^2 = 16$

Solve the equation for y either by hand or using the calculator (see directions 1-14).

y = ___________ and y = ____________

Using the calculator, graph the two functions you found above to varify that taken together, the functions' graphs create that of the circle (see direction #15).

INVESTIGATION WITH ELLIPSES.

Equation1: $\frac{x^2}{16} + \frac{y^2}{4} = 1$

Solve the equation for y either by hand or using the calculator (see directions 1-14).

y = ___________ and y = ____________

Using the calculator, graph the two functions you found above to varify that taken together, the functions' graphs create that of the ellipse (see direction #15).

Equation 2: $\frac{x^2}{9} + \frac{y^2}{16} = 1$

Solve the equation for y either by hand or using the calculator (see directions 1-14).

y = ___________ and y = ____________

Using the calculator, graph the two functions you found above to varify that taken together, the functions' graphs create that of the ellipse (see direction #15).

ACTIVITY 4...INSTRUCTIONAL SECTION CONTINUED

INVESTIGATION WITH PARABOLAS.

Equation 1: $y^2 = 2x$

Solve the equation for y either by hand or using the calculator (see directions 1-14).

y = ____________ and y = _____________

Using the calculator, graph the two functions you found above to varify that taken together, the functions' graphs create that of the parabola's (see direction #15).

Equation 2: $(y - 3)^2 = -1(x + 2)$

Solve the equation for y either by hand or using the calculator (see directions 1-14).

y = ____________ and y = _____________

Using the calculator, graph the two functions you found above to varify that taken together, the functions' graphs create that of the parabola's (see direction #15).

INVESTIGATION WITH HYPERBOLAS.

Equation 1: $\frac{x^2}{9} - \frac{y^2}{4} = 1$

Solve the equation for y either by hand or using the calculator (see directions 1-14).

y = ____________ and y = _____________

Using the calculator, graph the two functions you found above to varify that taken together, the functions' graphs create that of the hyperbola's (see direction #15).

Equation 2: $\frac{(x - 1)^2}{12} - \frac{(y - 1)^2}{3} = 1$

Solve the equation for y either by hand or using the calculator (see directions 1-14).

y = ____________ and y = _____________

Using the calculator, graph the two functions you found above to varify that taken together, the functions' graphs create that of the hyperbola's (see direction #15).

ACTIVITY 1...GRAPHICAL SOLUTIONS TO LINEAR SYSTEMS

INSTRUCTIONAL SECTION

This activity explores the graphing environment for finding solutions to systems of linear equations. We will keep this activity limited to systems of two equations in two variables. The graphical solution method is the most concrete of the many methods for solving two equations in two unknowns. While the graphical method is the most time consuming of all the methods, it is well worth the time invested; students will have a firm grasp of the concept of "solution to a system". The Applications Section will begin with four theoretical systems of equations so students can get used to using the calculator as well as the general set up for systems of equations. The Applications Section will then finish with 'one real world' analysis problem.

DIRECTIONS FOR USING THE HP 48G:

1. Convert all linear equations to functions of the form y = function of x. This way you can graph the equations in the function mode, thus giving you access to all its features.
2. Enter the first equation using the Equation Writer. **PURPLE EQUATION α Y PURPLE = - α X + 4,** fig. 1, **ENTER.**
3. Name and store this linear function. **'α Y1,** fig. 2, **STO.** Note: The object and variable you stored the object under should disappear from the screen.
4. Enter, name and store the second linear function: y = x - 2 following directions 2 and 3 above.
5. Plot functions Y1 and Y2. **GREEN PLOT,** from the menu labels press **CHOOS,** then **CHK** (check) the functions Y1 and Y2 from the menu, fig. 3, now press **OK,** then **ERASE** and **DRAW**, fig. 4.
6. When zoomed so that the intersection is visible, find the coordinates of the intersection between the two functions. Press **FCN**, then **ISECT** from the menu labels, fig. 5. You can bring the menu labels back by pressing any of the white keys. To return to the main menu labels for the 'PICTURE', press **NXT**, then from the menu labels press **PICT**, fig. 6.

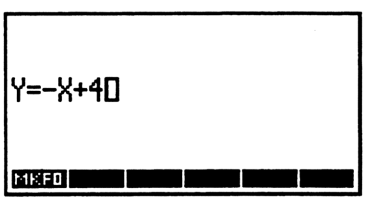


Fig. 1

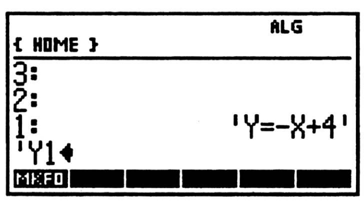


Fig. 2

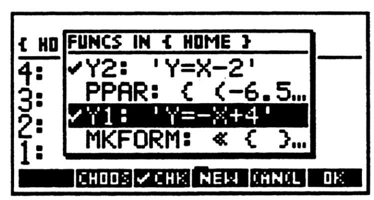


Fig. 3

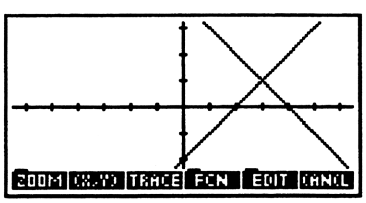


Fig. 4

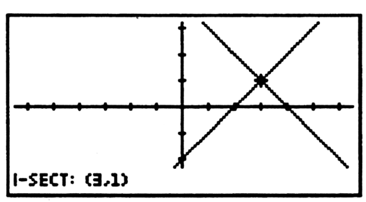


Fig. 5

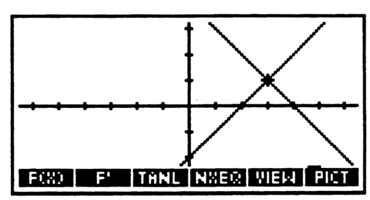


Fig. 6

ACTIVITY 1...GRAPHICAL SOLUTIONS TO LINEAR SYSTEMS

APPLICATIONS SECTION

System 1: $y = 2x + 1$
$y = -x - 2$

Solution: $(x, y) = (\_,\_)$

System 2: $2x - y = 4$
$5x - y = 13$

Solution: $(x, y) = (\_,\_)$

System 3: $y = -\frac{1}{3}x - 1$
$y = \frac{4}{3}x - 6$

Solution: $(x, y) = (\_,\_)$

System 4: $y = \frac{67}{9}x + 235$
$y = \frac{-3}{25}x + 300$

Solution: $(x, y) = (\_,\_)$

System 5: Car Purchase Problem.

You are buying a new car. You have narrowed your choices to two.

Car 1: Price $4,700
Mileage: 23 miles per gallon.
Current price gasoline: $1.22 per gallon.

Car 2: Price $5,995
Mileage: 35 miles per gallon.
Current price gasoline: $1.22 per gallon.

Car 1 costs less than car 2 initially, but gets worse gas mileage. Car 2 costs more than car 1 initially, but gets better gas mileage. Before buying either car, you would like to analyze this information so you can make an educated decision about which to buy. It seems reasonable given the price relationships for costs of both autos and their respective gas mileage that after driving a certain number of miles both vehicles will cost you the same amount of money overall. After that point, you will spend more on car 1 than on car 2 since its gas mileage is less than that of car 2.

A) Write an equation relating current <u>total costs</u> vs <u>miles</u> driven for each car. Assume the price of gasoline will remain constant for this analysis.

B) Graph each of these equations on the HP 48G. Note: The AUTOSCALE feature of the HP 48G is very helpful in these situations where zooming out by factors of hundreds or even thousands becomes a hit-and-miss game. The key to successful use of the autoscaler is entering a good estimate for the lower and upper bounds of the **horizontal** axis. Once chosen, the autoscaler determines the **vertical** range that will produce the best graph. You know you will drive a car more, but not less than zero miles, and you might assume you can drive either car 100,000 miles, so these may be good upper and lower bounds to try in H-VIEW in the PLOT menu. Then check AUTOSCALE before pressing ERASE and DRAW.

C) How many miles must you drive either car to pay the same amount in total costs?

D) Assuming you will drive either car less than 70,000 miles, which car would you buy assuming you want to keep costs at a minimum?

ACTIVITY 2...CRAMER'S RULE

INSTRUCTIONAL SECTION

This activity explores Cramer's Rule for finding solutions to systems of linear equations on the HP 48G. Cramer's Rule uses determinants of matrices to arrive at solutions. The process can be extremely time consuming if done with pencil and paper only, especially if the system is larger than a 3x3. The benefit in using the calculator for this application is in its time saving capacity. We encourage the **set up** of each determinant on paper before transfering them to the calculator. Storing and naming each determinant with a descriptive name will help with the organization of this process.

DIRECTIONS FOR USING THE HP 48G:

1. Cramer's Rule: If a system of n linear equations in n variables has a coefficient matrix with a nonzero determinant $|A|$, then the solution to the system is given by

$$x_1 = \frac{|A_1|}{|A|}, \qquad x_2 = \frac{|A_2|}{|A|}, \; \ldots, \; x_3 = \frac{|A_3|}{|A|}$$

where the ith column of A_i is the column of constants in the system of equations.

Example: System:

$$2x_1 - x_2 = 4$$
$$5x_1 - x_2 = 13$$

Matrix form: $\begin{bmatrix} 2 & -1 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$ $\quad |A| = \begin{vmatrix} 2 & -1 \\ 5 & -1 \end{vmatrix}$, $\quad |A_1| = \begin{vmatrix} 4 & -1 \\ 13 & -1 \end{vmatrix}$, $\quad |A_2| = \begin{vmatrix} 2 & 4 \\ 5 & 13 \end{vmatrix}$

Cramer's Rule yields: $x_1 = \dfrac{|A_1|}{|A|} = \dfrac{9}{3} = 3, \qquad x_2 = \dfrac{|A_2|}{|A|} = \dfrac{6}{3} = 2$

2. Enter matrix A in the Matrix Writer. **GREEN MATRIX 2 ENTER 1 +/- ENTER,** arrow to cell 2,1 by pushing the **down arrow** one time. In the Matrix Writer, the new location of the highlight after pushing ENTER is determined by **GO→** and **GO↓** menu label items. The directions above apply when the **GO→** menu label has the white bullet next to it, fig. 1. Now press **5 ENTER 1 +/- ENTER ENTER**, fig. 2.
3. Calculate, name, and store the determinant of this matrix. Press **MTH**, menu labels **MATR** then **NORM**, **NXT**, and finally, menu label **DET** , fig. 3, **α A STO.**
4. Enter, calculate, name and store the determinants of matrices A_1 and A_2 following directions 2 and 3 above. Store the determinants with their corresponding names A1 and A_2.
5. Calculate the value for x_1. Press **VAR**, press A1 then A from the menu labels, press ÷ , fig. 4.

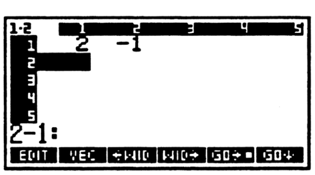


Fig. 1

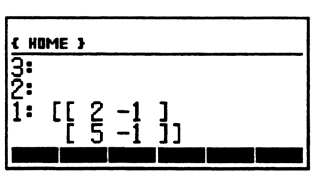


Fig. 2

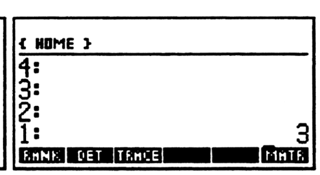


Fig. 3

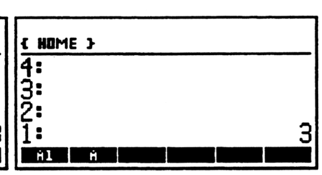


Fig. 4

ACTIVITY 2...CRAMER'S RULE

APPLICATIONS SECTION

System 1:
$$\begin{aligned} 5x_1 + 8x_2 &= 1 \\ 3x_1 + 7x_2 &= 5 \end{aligned}$$

System 2:
$$\begin{aligned} -2x_1 + 4x_2 &= 3 \\ 6x_1 + -14x_2 &= 2 \end{aligned}$$

System 3:
$$\begin{aligned} 2x_1 - 2x_2 + 4x_3 &= -6 \\ x_1 + 2x_2 + 3x_3 &= 4 \\ -2x_1 - x_2 - x_3 &= 3 \end{aligned}$$

System 4:
$$\begin{aligned} 3x_1 - x_2 + 2x_3 &= 1 \\ x_1 - x_2 + 2x_3 &= 3 \\ -2x_1 + 3x_2 + x_3 &= 1 \end{aligned}$$

System 5:
$$\begin{aligned} x_1 - 2x_2 - x_3 - 2x_4 &= 1 \\ 3x_1 - 5x_2 - 2x_3 - 3x_4 &= -2 \\ 2x_1 - 5x_2 - 2x_3 - 5x_4 &= 0 \\ -x_1 + 4x_2 + 4x_3 + 11x_4 &= -3 \end{aligned}$$

System 6:
$$\begin{aligned} x_1 - 2x_2 - x_3 - 2x_4 &= 1 \\ 3x_1 - 5x_2 - 2x_3 - 3x_4 &= -2 \\ 2x_1 - 5x_2 - 2x_3 - 5x_4 &= 0 \\ -x_1 + 4x_2 + 4x_3 + 11x_4 &= -3 \end{aligned}$$

ACTIVITY 3...SOLUTIONS USING INVERSE OF A MATRIX

INSTRUCTIONAL SECTION

This activity explores using inverses of matrices for finding solutions to systems of linear equations. The process of finding the solution to a linear system of n equations and n variables can be extremely time consuming if done with pencil and paper, especially if the system is equal to or larger than a 3x3. The benefit in using the HP 48G calculator for this application is in its time saving capacity. We encourage the **set up** of the matrix equation on paper before transferring the matrix of coefficients and constants to the calculator.

DIRECTIONS FOR USING THE HP 48G:

1. **Example:** System:

$$\begin{aligned} 2x_1 - x_2 &= 4 \\ 5x_1 - x_2 &= 13 \end{aligned}$$

Equivalent Matrix Equation:

$$\underset{A}{\begin{bmatrix} 2 & -1 \\ 5 & -1 \end{bmatrix}} \underset{\mathbf{x}}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 4 \\ 13 \end{bmatrix}} \quad \text{or} \quad A\cdot\mathbf{x} = B$$

The solution is found by *left* multiplying both sides of the above matrix equation by A^{-1}, read A inverse. That is: $A^{-1}\cdot A\cdot\mathbf{x} = A^{-1}\cdot B$, which yields: $I\cdot\mathbf{x} = A^{-1}\cdot B$, therefore: $\mathbf{x} = A^{-1}\cdot B$.

2. Enter, name, and store matrix A. **GREEN MATRIX 2 ENTER 1 +/- ENTER,** arrow to cell 2,1 by pushing the **down arrow** one time. In the Matrix Writer, the new location of the highlight after pushing ENTER is determined by the **GO→** and **GO↓** menu labels. The directions above apply when the **GO→** menu label has the white bullet next to it, fig. 1. Now press **5 ENTER1 +/- ENTER ENTER,** fig. 2. Finally, press **α A STO.**
3. Enter, name, and store matrix B by following the directions above.
4. Solve the system. Press **VAR,** menu label A, press **1/x,** fig. 3, (This is A^{-1}), press menu label B, now press **X,** (multiply), fig. 4. This is the solution matrix.

Fig. 1

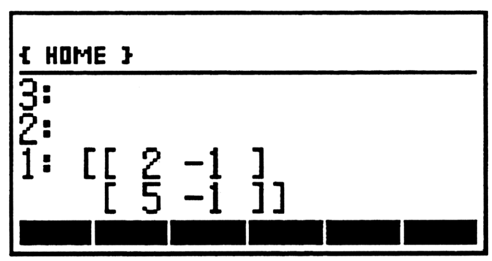


Fig. 2

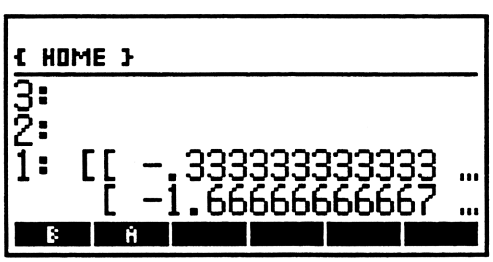


Fig. 3

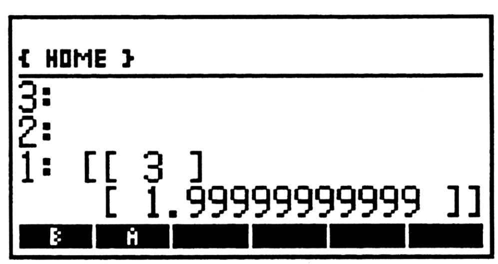


Fig. 4

ACTIVITY 3...SOLUTIONS USING INVERSE OF A MATRIX

APPLICATIONS SECTION

System 1:
$$\begin{aligned} 5x_1 + 8x_2 &= 1 \\ 3x_1 + 7x_2 &= 5 \end{aligned}$$

System 2:
$$\begin{aligned} -2x_1 + 4x_2 &= 3 \\ 6x_1 + -14x_2 &= 2 \end{aligned}$$

System 3:
$$\begin{aligned} 2x_1 - 2x_2 + 4x_3 &= -6 \\ x_1 + 2x_2 + 3x_3 &= 4 \\ -2x_1 - x_2 - x_3 &= 3 \end{aligned}$$

System 4:
$$\begin{aligned} 3x_1 - x_2 + 2x_3 &= 1 \\ x_1 - x_2 + 2x_3 &= 3 \\ -2x_1 + 3x_2 + x_3 &= 1 \end{aligned}$$

System 5:
$$\begin{aligned} x_1 - 2x_2 - x_3 - 2x_4 &= 1 \\ 3x_1 - 5x_2 - 2x_3 - 3x_4 &= -2 \\ 2x_1 - 5x_2 - 2x_3 - 5x_4 &= 0 \\ -x_1 + 4x_2 + 4x_3 + 11x_4 &= -3 \end{aligned}$$

System 6:
$$\begin{aligned} x_1 - 2x_2 - x_3 - 2x_4 &= 1 \\ 3x_1 - 5x_2 - 2x_3 - 3x_4 &= -2 \\ 2x_1 - 5x_2 - 2x_3 - 5x_4 &= 0 \\ -x_1 + 4x_2 + 4x_3 + 11x_4 &= -3 \end{aligned}$$

ACTIVITY 4...SOLUTIONS USING HP 48G SOLVE ENVIRONMENT

INSTRUCTIONAL SECTION

This activity uses the HP 48G's SOLVE Environment for solving systems of linear equations. The process of finding the solution to a linear system of n equations and n variables can be extremely time consuming if done with pencil and paper, especially if the system is equal to or larger than a 3x3. The benefit in using the calculator for this application is in its time saving capacity. We encourage the **set up** of the matrix equation on paper before transferring the matrix of coefficients and constants to the calculator.

DIRECTIONS FOR USING THE HP 48G:

1. **Example:** System:

$$\begin{aligned} 2x_1 - x_2 &= 4 \\ 5x_1 - x_2 &= 13 \end{aligned}$$

Equivalent Matrix Equation:

$$\underset{A}{\begin{bmatrix} 2 & -1 \\ 5 & -1 \end{bmatrix}} \underset{\mathbf{x}}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 4 \\ 13 \end{bmatrix}} \quad \text{or} \quad A \cdot \mathbf{x} = B$$

2. Enter, name, and store matrix A. **GREEN MATRIX 2 ENTER 1 +/- ENTER,** arrow to cell 2,1 by pushing the **down arrow** one time. In the Matrix Writer, the new location of the highlight after pushing ENTER is determined by the **GO→** and **GO↓** menu labels. The directions above apply when the **GO→** menu label has the white bullet next to it, fig. 1. Now press **5 ENTER1 +/- ENTER ENTER**, fig. 2. Finally, press **α A STO.**
3. Enter, name, and store matrix B by following the directions above.
4. Solve the system. **GREEN SOLVE**, highlight **Solve lin sys...** fig. 1**,** press menu label **OK**, fig. 2. Press menu label **CHOOS** , highlight **A:**, fig. 3, and press **OK**, fig. 4. Press the down arrow to highlight **B:** and press **CHOOS** , highlight **B:** and press **OK,** fig. 5. Highlight **x:** and press menu label **SOLVE** , fig. 6. This is the solution matrix.
5. Matrices A and B can be directly entered by pressing **EDIT** from the menu bar when in the **SOLVE SYSTEM** menu. This method is slightly faster, but matrices A and B will not be stored in memory.

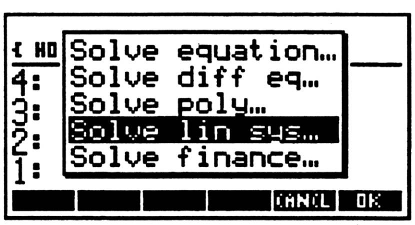


Fig. 1

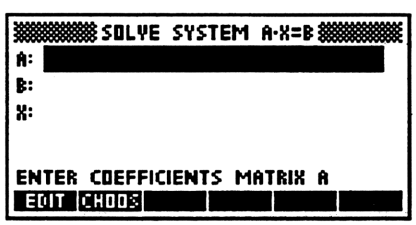


Fig. 2

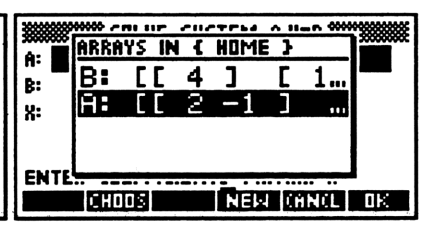


Fig. 3

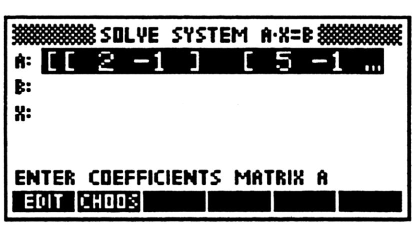


Fig. 4

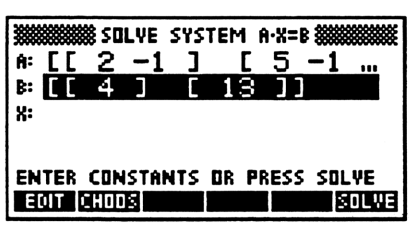


Fig. 5

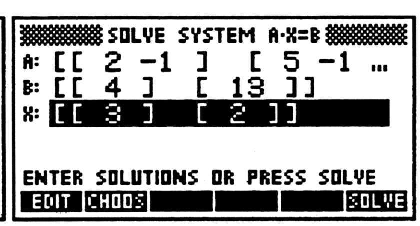


Fig. 6

ACTIVITY 4...SOLUTIONS USING HP 48G SOLVE ENVIRONMENT

APPLICATIONS SECTION

System 1:
$$\begin{aligned} 5x_1 + 8x_2 &= 1 \\ 3x_1 + 7x_2 &= 5 \end{aligned}$$

System 2:
$$\begin{aligned} -2x_1 + 4x_2 &= 3 \\ 6x_1 + -14x_2 &= 2 \end{aligned}$$

System 3:
$$\begin{aligned} 2x_1 - 2x_2 + 4x_3 &= -6 \\ x_1 + 2x_2 + 3x_3 &= 4 \\ -2x_1 - x_2 - x_3 &= 3 \end{aligned}$$

System 4:
$$\begin{aligned} 3x_1 - x_2 + 2x_3 &= 1 \\ x_1 - x_2 + 2x_3 &= 3 \\ -2x_1 + 3x_2 + x_3 &= 1 \end{aligned}$$

System 5:
$$\begin{aligned} x_1 - 2x_2 - x_3 - 2x_4 &= 1 \\ 3x_1 - 5x_2 - 2x_3 - 3x_4 &= -2 \\ 2x_1 - 5x_2 - 2x_3 - 5x_4 &= 0 \\ -x_1 + 4x_2 + 4x_3 + 11x_4 &= -3 \end{aligned}$$

System 6:
$$\begin{aligned} x_1 - 2x_2 - x_3 - 2x_4 &= 1 \\ 3x_1 - 5x_2 - 2x_3 - 3x_4 &= -2 \\ 2x_1 - 5x_2 - 2x_3 - 5x_4 &= 0 \\ -x_1 + 4x_2 + 4x_3 + 11x_4 &= -3 \end{aligned}$$

ACTIVITY 1...MATRIX ADDITION AND SCALAR MULTIPLICATION

INSTRUCTIONAL SECTION

This activity explores the use of matrices for organizing data, and for manipulating this data, with emphasis on addition of matrices and multiplication by scalars. Students will need to know common characteristics of matrices, especially the dimension naming system, entry naming system, and labeling the rows and columns of a matrix. Students are encouraged to **set up** each matrix and enter the values using pencil and paper, as well as formulating and writing descriptive row and column labels. The solutions will usually be in the form of a matrix, so we have included a solution matrix with blank entries. After students find solutions on the calculator, the blanks may be filled in. The HP 48G calculator will play the role of the 'number cruncher' in this activity, it should not be a substitute for the organization and presentation of the problems and solutions.

DIRECTIONS FOR USING THE HP 48G:

1. ENTER a matrix using the Matrix Writer. **GREEN MATRIX 2 ENTER 3 ENTER 4 ENTER down arrow 7 ENTER 14 ENTER 25 ENTER 14 +/- ENTER 0 ENTER 30 ENTER**, fig. 1.
2. Enter this 3x3 matrix from the Equation Writer to the stack by pressing **ENTER**, fig. 2.
3. Name and store this 3x3 matrix. **' α M STO**, fig. 3. After pressing STO, the object you stored will disappear.
4. Multiply matrix M by the scalar 5 by getting a copy of M on the first level of the stack, entering 5, and pressing X. **VAR**, press menu label **M**, press **5**, fig. 4, now press **X**, fig. 5.
5. Name and store this new matrix under the name 'N', **' α N STO.**
6. Add matrices M and N. Two ways to do this:
   A. Press **VAR**, then from the menu labels press **M** and then **N**, fig. 6, now press **+**, fig. 7.
   B. Press **'M + N ENTER**, fig. 8, then press **EVAL**, fig. 7.

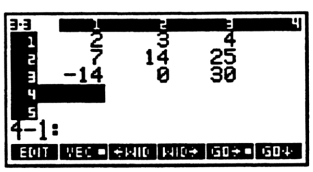


Figure 1

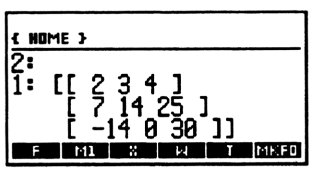


Figure 2

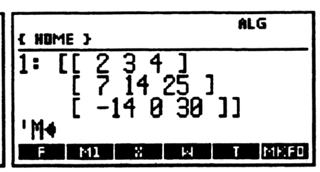


Figure 3

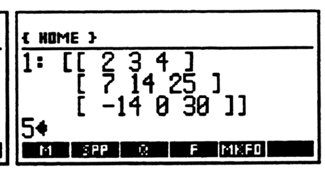


Figure 4

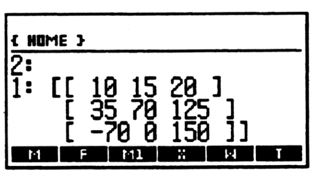


Figure 5

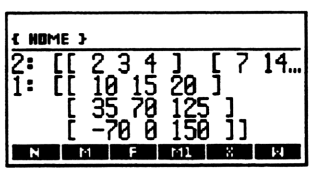
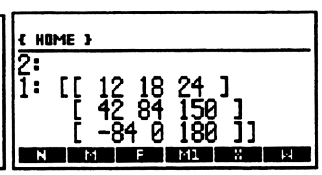


Figure 6



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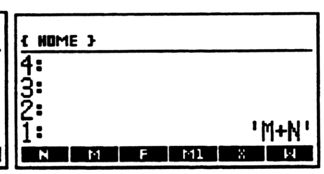

Figure 8

ACTIVITY 1...MATRIX ADDITION AND SCALAR MULTIPLICATION

APPLICATIONS SECTION

Problem 1: The recycling program

Your school is running a recycling program. Recycled items are white paper (W), colored paper (C), and printer paper (P). Each quarter the weight of each recycled paper type is recorded individually for the Math / Science (M), Business (B), and Language Arts (L) departments. Below is the data for the first quarter of the school year.

Math / Science: white - 312 lbs; colored - 96 lbs; computer - 17 lbs.
Business: white - 275 lbs; colored - 71 lbs; computer -73 lbs.
Language Arts: white - 417 lbs; colored - 63 lbs; computer - 27 lbs.

A) Represent the recycled products in matrix Q below.

$$Q = \begin{array}{c} \\ M \\ B \\ L \end{array} \begin{array}{c} \begin{array}{ccc} W & C & P \end{array} \\ \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix} \end{array}$$

B) ENTER, name, and store Q on the HP 48G

C) If the rate of recycling remains the same all year, use matrix multiplication by a scalar on your calculator to predict the total weight of each recycled item for each department at the end of the fourth quarter. Name and store this matrix of totals as 'Y'. Write the predicted totals in matrix Y below. Use labeling that is consistent with matrix Q above.

$$Y = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

D) If the rate of recycling remains about the same for the four years you spend in high school, use matrix multiplication by a scalar on your calculator to determine the predicted weight of recycled items after four years for each department. Write the expected grand totals for four years in matrix F below.

$$F = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

ACTIVITY 1...APPLICATIONS SECTION CONTINUED

Problem 2: School sports win - loss records.
Three sports that both boys and girls participate in are soccer, basketball, and track. Below is a listing of the win - loss records for both boys' and girls' programs.

Girls soccer: won - 12; lost - 2
Girls basketball: won-12; lost - 10
Girls track: won - 3; lost - 5

Boys soccer: won - 8; lost - 6
Boys basketball: won - 14; lost - 8
Boys track: won - 4; lost 4

A) Represent both sets of data with matrices. Let G be the girls' and B be the boys'.

$$G = \begin{bmatrix} -- & -- & -- \\ -- & -- & -- \\ -- & -- & -- \end{bmatrix} \qquad B = \begin{bmatrix} -- & -- & -- \\ -- & -- & -- \\ -- & -- & -- \end{bmatrix}$$

B) ENTER G and B on the calculator, name and store them.

C) Calculate the matrix representing the records of the girls and boys together. Call this matrix T.

$$T = \begin{bmatrix} -- & -- & -- \\ -- & -- & -- \\ -- & -- & -- \end{bmatrix}$$

D) In which sport did the school do the best overall? Explain the criterion you used to determine 'the best' overall record.

Problem 3: The Bookstore Problem[1]
The Campus Bookstore's inventory of books consists of the following quantities:

Hard cover: textbooks - 5280; fiction - 1680; nonfiction - 2320; reference - 1890
Paperback: textbooks - 1940; fiction - 2810; nonfiction - 1490; reference - 2070

The College Bookstore's inventory of books consists of the following quantities:

Hard cover: textbooks - 6340; fiction - 2220; nonfiction - 1790; reference - 1980
Paperback: textbooks - 2050; fiction - 3100; nonfiction - 1720; reference - 2710

A) Represent the inventory of the Campus Bookstore as a matrix. ENTER, name and store S on the calculator.

$$S = \begin{bmatrix} -- & -- & -- \\ -- & -- & -- \\ -- & -- & -- \end{bmatrix}$$

B) Represent the inventory of the College Bookstore as a matrix. ENTER, name and store E on the calculator.

$$E = \begin{bmatrix} -- & -- & -- \\ -- & -- & -- \\ -- & -- & -- \end{bmatrix}$$

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[1] New Topics for Secondary School Mathematics, Material and Software: Matrices. 2.1 Exercise 1, page 5. NCTM, 1988.

ACTIVITY 1...APPLICATIONS SECTION CONTINUED

C) Use matrix algebra on the calculator to determine the total inventory of a new company formed by the merger of the College Bookstore and the Campus Bookstore.

$$M = \begin{bmatrix} -- & -- & -- \\ -- & -- & -- \\ -- & -- & -- \end{bmatrix}$$

Problem 4: The Lucrative Bank problem[2]

The Lucrative Bank has three branches in Durham: Northgate (N), Downtown (D), and South Square (S). Matrix A shows the number of accounts of each type--checking (c), savings (s), and market (m) -- at each branch office on January 1.

$$A = \begin{array}{c} \\ N \\ D \\ S \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} 40039 & 10135 & 512 \\ 15231 & 8751 & 105 \\ 25612 & 12187 & 97 \end{bmatrix} \end{array}$$

Matrix B shows the number of accounts of each type at each branch that were opened during the first quarter, and Matrix C shows the number of accounts closed during the first quarter.

$$B = \begin{array}{c} \\ N \\ D \\ S \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} 5209 & 2506 & 48 \\ 1224 & 405 & 17 \\ 2055 & 771 & 21 \end{bmatrix} \end{array} \qquad C = \begin{array}{c} \\ N \\ D \\ S \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} 2780 & 1100 & 32 \\ 565 & 189 & 25 \\ 824 & 235 & 14 \end{bmatrix} \end{array}$$

A) Calculate the matrix representing the number of accounts of each type at each location at the end of the first quarter. Let's call this matrix F.

$$F = \begin{array}{c} \\ N \\ D \\ S \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} -- & -- & -- \\ -- & -- & -- \\ -- & -- & -- \end{bmatrix} \end{array}$$

B) The sudden closing of a large textile plant has led bank analysts to estimate that all accounts will decline in number by 7% during the second quarter. Calculate a matrix that represents the anticipated number of each type of account at each branch at the end of the second quarter. Assume that fractions of accounts are rounded to integer values. Use matrix F for the end of the second quarter <u>before</u> the 7% decline. Call the new matrix S.

$$S = \begin{array}{c} \\ N \\ D \\ S \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} -- & -- & -- \\ -- & -- & -- \\ -- & -- & -- \end{bmatrix} \end{array}$$

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[2] New Topics for Secondary School Mathematics, Material and Software: Matrices. 2.1 Exercise 2, page 5. NCTM, 1988.

ACTIVITY 1...APPLICATIONS SECTION CONTINUED

C) The bank president announces that the Lucrative Bank will merge with the Me. D. Okra Bank, which has branches in the same locations as those of the Lucrative Bank. The accounts at each branch of the Me. D. Okra Bank on January 1 are:

$$M = \begin{array}{c} \\ N \\ D \\ S \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} 1345 & 2531 & 52 \\ 783 & 1987 & 137 \\ 2106 & 3765 & 813 \end{bmatrix} \end{array}$$

Calculate the total number of accounts of each type at each branch of the bank formed by the merger of the two banks. Use the January 1 figures and assume that the accounts stay at their current branch offices. Call this matrix T.

$$T = \begin{array}{c} \\ N \\ D \\ S \end{array} \begin{array}{c} \begin{array}{ccc} c & s & m \end{array} \\ \begin{bmatrix} \text{--} & \text{--} & \text{--} \\ \text{--} & \text{--} & \text{--} \\ \text{--} & \text{--} & \text{--} \end{bmatrix} \end{array}$$

ACTIVITY 2...MATRIX MULTIPLICATION

INSTRUCTIONAL SECTION

This activity explores the use of matrices for organizing data and for manipulating this data, with emphasis on multiplication of matrices. Students will need to know common characteristics of matrices, especially the dimension naming system, entry naming system, and labeling the rows and columns of a matrix. It would be optimal if students already had some experience multiplying matrices. This activity helps to make the matrix multiplication process applicable to 'real world' situations. The HP 48G calculator will primarily be used for 'number crunching' in this activity; it should not be a substitute for the organization and presentation of the problems and solutions.

DIRECTIONS FOR USING THE HP 48G:

1. ENTER a matrix using the Matrix Writer. **GREEN MATRIX 2 ENTER 3 ENTER 4 ENTER down arrow 7 ENTER 14 ENTER 25 ENTER 14 +/- ENTER 0 ENTER 30 ENTER**, fig. 1.
2. ENTER this 3x3 matrix on the stack. **ENTER**, fig. 2.
3. Name and store this 3x3 matrix. **' α M STO**, fig. 3. After pressing STO, the object you stored will disappear.
4. ENTER, name, and store matrix N below following directions 1 and 2 from above.

$$N = \begin{bmatrix} 32 \\ -8 \\ 47 \end{bmatrix}$$

5. Multiply Matrices M and N. Press **VAR**, press **M,** then **N** from the menu labels, press multiply, **X**. fig. 4.
6. Try multiplying N by M, in this order. What happened and why?

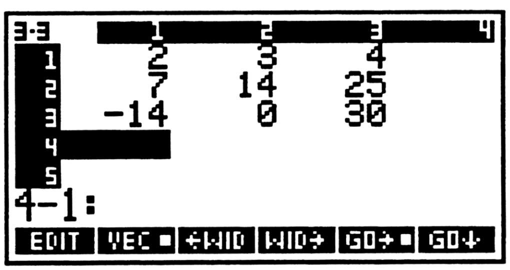


fig. 1

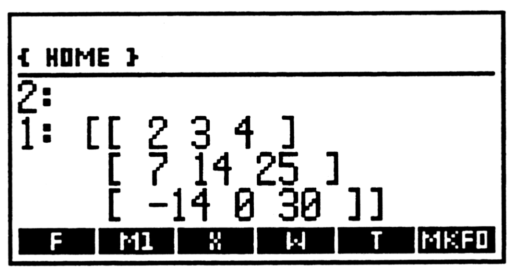


fig. 2

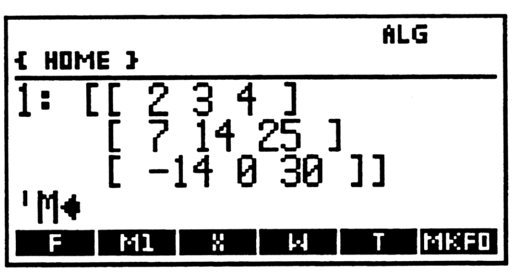


fig. 3

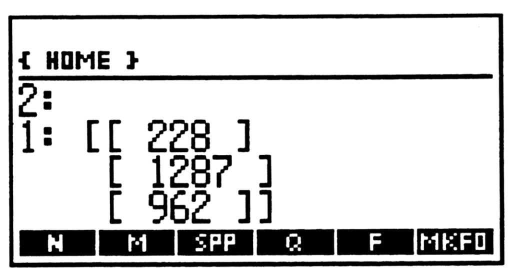


fig. 4

ACTIVITY 2...MATRIX MULTIPLICATION

APPLICATIONS SECTION

Problem 1: The recycling program: Estimating the number of recycled sheets of paper.

Your school is running a recycling program. Recycled items are white paper (W), colored paper (C), and printer paper (P). Each quarter the weight of each recycled paper type is recorded individually for the Math / Science (M), Business (B), and Language Arts (L) departments. Below is the data for the <u>first quarter</u> of the school year.

Math / Science: white - 312 lbs; colored - 96 lbs; computer - 17 lbs.
Business: white - 275 lbs; colored - 71 lbs; computer -73 lbs.
Language Arts: white - 417 lbs; colored - 63 lbs; computer - 27 lbs.

White paper: 120 sheets/lb.
Colored paper: 110 sheets/lb.
Printer paper: 90 sheets/lb.

Matrix R represents the recycled products in pounds for quarter 1. Notice that the labeling system of matrix R shows department by paper type. Matrix S represents the number of sheets per pound for each paper type. Matrix S labels are paper type by sheets per pound. ENTER, name, and store R and S on the HP 48G.

$$R = \begin{array}{c} \\ M \\ B \\ L \end{array}\!\!\begin{array}{c} \begin{array}{ccc} \text{W} & \text{C} & \text{P} \end{array} \\ \begin{bmatrix} 312 & 96 & 17 \\ 275 & 71 & 73 \\ 417 & 63 & 27 \end{bmatrix} \end{array} \qquad \text{S} = \begin{array}{c} \\ W \\ C \\ P \end{array}\!\!\begin{array}{c} \text{sheets/lb.} \\ \begin{bmatrix} 120 \\ 110 \\ 90 \end{bmatrix} \end{array}$$

Using the information from matrices R and S, we can find the number of recycled sheets manually. For example, the number of sheets recycled by the Math/Science departments is:
(312 lbs. x 120 sheets/lb.) + (96 lbs. x 110 sheets/lb.) + (17 lbs. x 90 sheets/lb.) = 49,530 sheets.
A similar operation can be done to find the number of recycled sheets for the other two departments, however, matrix multiplication lends itself to this very sort of calculation. Note that if the row and column labels are lined out such as (department x <u>paper type</u>) X (<u>paper type</u> x sheets/lb), the inside labels are the same, and the outside labels are the row and column labels respectively for the solution matrix. These facts will always hold for matrix multiplication, so in future problems with matrices let the row and column labels be a problem solving source for you. This issue is especially important because the order of multiplication does make a difference, i.e. matrix multiplication is not commutative. From the menu bar after pressing VAR, press R and S, perform this multiplication operation by simply pressing the multiplication key (X). Your solution matrix should now be displayed.

A) Fill in the solution matrix below on the right.

$$\begin{array}{c} \\ M \\ B \\ L \end{array}\!\!\begin{array}{c} \begin{array}{ccc} \text{W} & \text{C} & \text{P} \end{array} \\ \begin{bmatrix} 312 & 96 & 17 \\ 275 & 71 & 73 \\ 417 & 63 & 27 \end{bmatrix} \end{array} \text{ x } \begin{array}{c} \\ W \\ C \\ P \end{array}\!\!\begin{array}{c} \text{sheets/lb.} \\ \begin{bmatrix} 120 \\ 110 \\ 90 \end{bmatrix} \end{array} = \begin{array}{c} \\ M \\ B \\ L \end{array}\!\!\begin{array}{c} \text{sheets} \\ \begin{bmatrix} \ldots\ldots \\ \ldots\ldots. \\ \ldots\ldots. \end{bmatrix} \end{array}$$

B) Adjust the above matrix equation so the solution matrix reflects all four quarters of recycling. Assume the quarterly amounts are the same for this problem.

C) Adjust the above matrix equation so the solution matrix reflects four years, (your stay in high school), of recycling. Again, you may assume the recycling rates remain constant for the four years.

ACTIVITY 2...APPLICATIONS SECTION CONTINUED

You may want to follow the discussion below using your calculator to verify the results. **Note: the HP 48G will do matrix algebra on matrix objects only. The matrix [1 1 1] will not be interpreted as a matrix on the calculator, it will be interpreted as a vector instead. To make the calculator interpret this as a matrix, get into the Matrix Writer, and from the menu labels press VEC □. That menu label should now read VEC. Matrices other than single row matrices will not require clearing the bullet next to VEC.**

Suppose we are interested in adding all the entries in each row or each column of a matrix. Let A be an m x n matrix; if we right multiply A by matrix B such that B is an m x 1 matrix whose entries are all ones, we will have a new matrix, C, whose entries are the sums of the entries in each row of matrix A. Example:

$$\underset{A}{\begin{bmatrix} 2 & 3 & 4 \\ 7 & 14 & 25 \\ -14 & 0 & 30 \end{bmatrix}} \times \underset{B}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}} = \underset{C}{\begin{bmatrix} 9 \\ 46 \\ 16 \end{bmatrix}}$$

A similar procedure can produce the sums of the all the entries in each column of an m x n matrix. Let A be an m x n matrix; if we left multiply A by matrix B such that B is a 1 x m matrix whose entries are all ones, we will have a new matrix, C, whose entries are the sums of the entries in each column of matrix A. Example:

$$\underset{B}{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}} \times \underset{A}{\begin{bmatrix} 2 & 3 & 4 \\ 7 & 14 & 25 \\ -14 & 0 & 30 \end{bmatrix}} = \underset{C}{\begin{bmatrix} -5 & 17 & 59 \end{bmatrix}}$$

Relating this to our recycling problem; we can now find facts using matrix algebra such as the total number of pounds of white paper all three departments recycle, or the total number of pounds of all three types of paper each department recycles. We can also extend these facts using matrix algebra for finding the **total** estimated number of sheets of recycled paper there is for each quarter.

D) Use matrix R and the information from above to calculate the total number of pounds of each type of paper that was recycled for the first quarter. Write the resulting matrix below, label the rows and columns with descriptive labels.

E) Use matrix R and the information from above to determine the total number of pounds of paper each department recycled in the first quarter. Write the resulting matrix below, label the rows and columns with descriptive labels.

F) Use the results from parts D and E, add the entries in each matrix either with common arithmetic, or by the matrix operations discussed above. Your results should be scalars. Are they equal? Briefly explain why or why not.

G) Repeat exercises D, E and F for a year (four quarters), and for four years (your duration in high school).

ACTIVITY 3...POLYGONS IN THE CARTESIAN PLANE

INSTRUCTIONAL SECTION

In this activity, you will be using a matrix to store the coordinates of the vertices of a polygon in the Cartesian Plane; we will call this a vertex matrix.. For example, let A = $\begin{bmatrix} -5 & 1 \\ -3 & 3 \\ -1 & 2 \end{bmatrix}$. Matrix A contains the coordinates of the triangle with vertices (-5,1), (-3,3), and (-1,2) , see figure 1. Now, let B = $\begin{bmatrix} 5 & 1 \\ 3 & 3 \\ 1 & 2 \end{bmatrix}$. Figure 2 shows the graph of matrix B. Notice in figure 3 that when the triangles are graphed on the same set of axes, each is a reflection of the other over the y-axis.

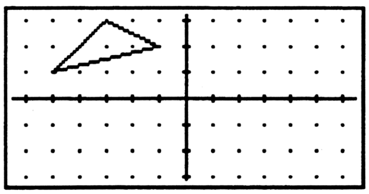


fig. 1

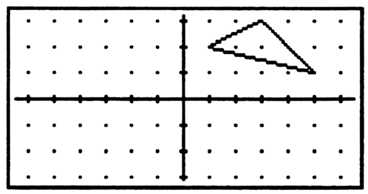


fig. 2

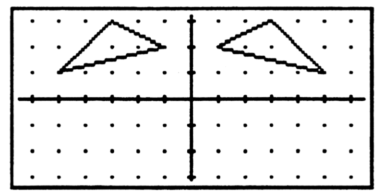


fig. 3

<u>Matrix B was found by right multiplying matrix A by a matrix</u>. i.e. $A \cdot M = B$ where M is the mystery matrix. The **challenge** in this activity is finding the mystery matrix that will produce a given transformation. Once discovered, you can enter the coordinates of any polygon, multiply by matrix M and the result will be matrix B. Finally, both matrices A and B can be graphed to verify the transformation.

The calculator will aide you in doing the matrix algebra, and graphing the solution, or non-solution for visual verification. This activity requires some calculator preparation prior to working the problems. There is a short draw program that must be entered and stored on the calculator, and there is an optional dot drawing program that you can enter and store on your calculator; figures 1-3 show the dots, figure 6 shows a graph without the dots.

DIRECTIONS FOR USING THE HP 48G:

Enter the Draw Polygon program below on the command line of the calculator.

<< DRAX →COL DROP R→C OBJ→ EVAL →LIST DUP HEAD + {#0d #0d} PVIEW 2 << LINE >> DOSUBS 7 FREEZE >>

Put this on the first line of the stack by pressing **ENTER**. Name and store this program. Press **' αα DRPG α STO**. DRPG should now be in your variable list. Let's test DRPG. Enter the vertex matrix found in figure 4: Press **GREEN MATRIX,** enter the values and press **ENTER** to copy the matrix to the stack, see fig. 5. Note: DRPG does not contain an ERASE command, so the graphics screen may not be clear prior to running DRPG. Clear the draw screen by pressing: **PURPLE PLOT**, then press menu label **ERASE**. Run DRPG by pressing **VAR** and **NXT** if necessary to find the page **DRPG** is on, from the menu labels press **DRPG**, see fig. 6.

ACTIVITY 3...INSTRUCTIONAL SECTION CONTINUED

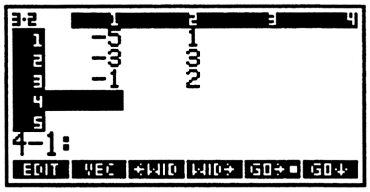


fig. 4

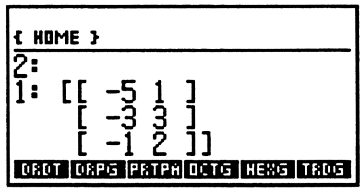


fig. 5

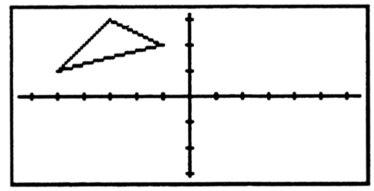


fig. 6

Optional: Enter the Draw Dots program below on the command line of the calculator

> **<< ERASE DRAX {#0d #0d} PVIEW 7 FREEZE PPAR 1 2 SUB C→R EVAL → xmn ymn xmx ymx << xmn CEIL xmx FLOOR FOR i ymn CEIL ymx FLOOR FOR j i j R→C PIXON NEXT NEXT >> >>**

Put this on the first line of the stack by pressing **ENTER**. Name and store this program. Press ' αα **DRDT** α **STO.** DRDT should now be in your variable list. Try running it by pressing **DRDT** from the variable menu. Your screen should look like figure 7. Note: DRDT **does** contain an ERASE command, so it will clear the draw screen when it is run. If you want to graph more than one polygon on the same set of axes, run DRDT before running the DRPG program. <u>Any subsequent run of DRDT will erase the current draw screen</u>.

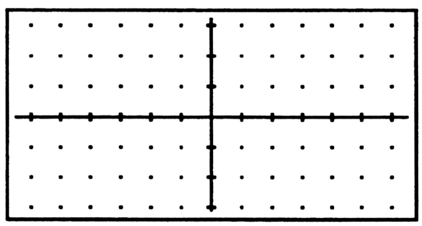


fig. 7

Pressing ON will return you to the STACK, but the PICTURE screen is not erased; you can verify this by pressing **PURPLE PICTURE**.

We are nearly ready to solve some problems, however, first we will review the transformations this activity will use. Figures 8 and 9 show reflections over the axes, figure 10 shows a reflection through the origin, and finally, figure 11 shows a horizontal translation of five units to the right.

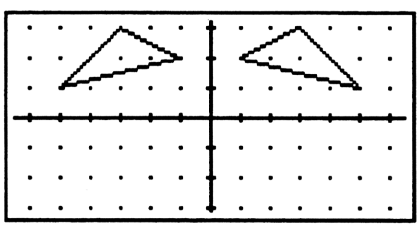


fig. 8

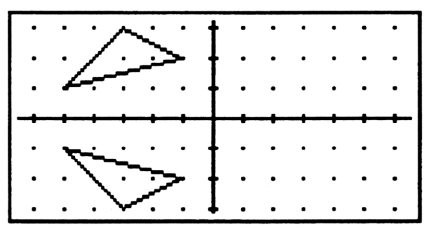


fig. 9

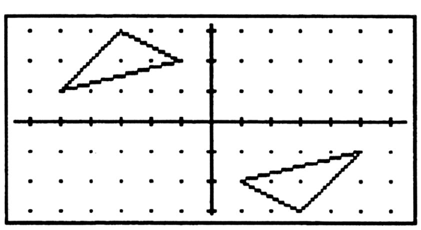


fig. 10

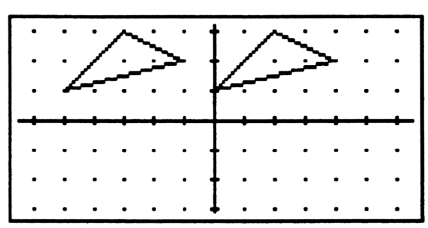


fig. 11.

ACTIVITY 3...POLYGONS IN THE CARTESIAN PLANE

APPLICATIONS SECTION

REFLECTIONS THROUGH THE Y-AXIS.

PROBLEM 1

Let's return to matrices A, M, and B from page 1 of the Instructional Section. We have asserted that A·M = B, we know the elements of matrix A, and we know the elements of matrix B. Our job now is to figure out what the elements of matrix M must be such that A·M = B. To solve this problem, you must know how to multiply matrices, and you must know how the dimensions of matrices relate for matrix multiplication to be defined. Below is the matrix equation A·M = B. Determine the dimensions of M by reasoning through what m and n must be.

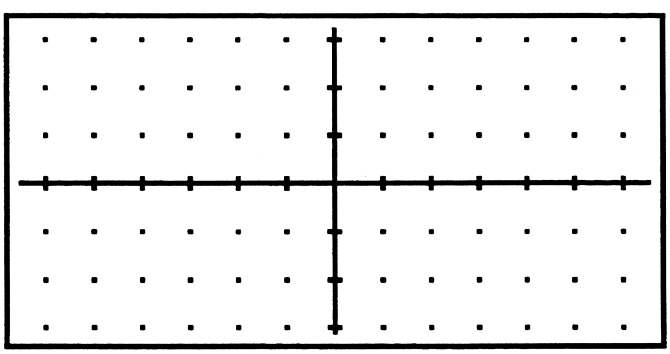
$$\underset{A}{\begin{bmatrix}-5 & 1\\ -3 & 3\\ -1 & 2\end{bmatrix}}\cdot\underset{M}{\begin{bmatrix}\quad & \quad\\ \quad & \quad\end{bmatrix}}=\underset{B}{\begin{bmatrix}5 & 1\\ 3 & 3\\ 1 & 2\end{bmatrix}}$$

Dimensions: (3 x 2) (m x n) (3 x 2)

Next, determine the elements of matrix M and fill them in the equation above. Test your solution: Enter and <u>store</u> matrices A and M. Put A and M on the stack; A in level 2 and M in level 1, press multiply. Did you get matrix B? Finally, let's graph A and B. If you typed in the Draw Dots program, run it, otherwise erase the draw screen. Put A on level 1 of the stack, run the Draw Polygon program, calculate matrix B and run the Draw Polygon program again. Are the triangles reflections of each other over the y-axis?

PROBLEM 2

Use the grid below to choose three points, preferably all in one quadrant. Connect the points to form a triangle. Use blank matrix C to write the coordinates of the three points you chose. Fill in matrix M from the work you did in problem 1. <u>Calculate</u> matrix D and fill it in below. Finally, use the calculator to graph both matrices C and D to verify that D is a reflection of C over the y-axis.

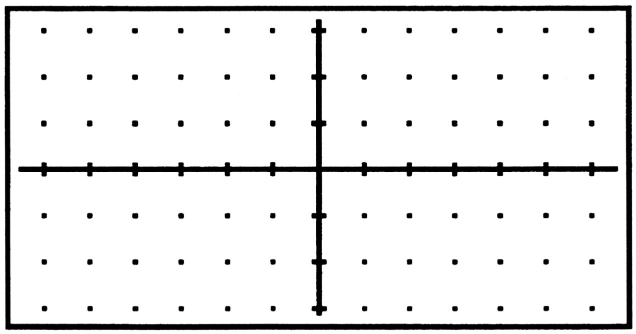


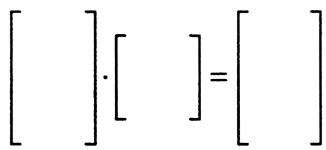
$$\underset{C}{\begin{bmatrix}\quad & \quad\\ \quad & \quad\\ \quad & \quad\end{bmatrix}}\cdot\underset{M}{\begin{bmatrix}\quad & \quad\\ \quad & \quad\end{bmatrix}}=\underset{D}{\begin{bmatrix}\quad & \quad\\ \quad & \quad\\ \quad & \quad\end{bmatrix}}$$

ACTIVITY 3...APPLICATIONS SECTION CONTINUED

PROBLEM 3

Use the grid below to choose four or more points, preferably all in one quadrant. Connect the points to form a polygon. Use blank matrix E to write the consecutive coordinates of the polygon. Fill in matrix M from the work you did in problem 1. Calculate matrix F and fill it in below. Finally, use the calculator to graph both matrices E and F to verify that F is a reflection of E over the y-axis.





E *M* *F*

EXPERIMENT 1

Thus far, you have probably been using the default H-VIEW and V-VIEW values, see figure 12. You can change these settings if you wish to have a larger selection of points to choose from when drawing polygons. Figure 13 shows a triangle on a screen with the default settings. Figure 14 shows the VIEW settings used for the same triangle which is displayed in figure 15. Change the VIEW settings by pressing **GREEN PLOT**, highlighting and changing the H-VIEW and V-VIEW settings and pressing **ENTER**. If you are using the DRAW DOTS program, you will notice that it is sensitive to the changes you make in the VIEW settings. Anytime VIEW settings are changed, DRDT will need to be rerun.

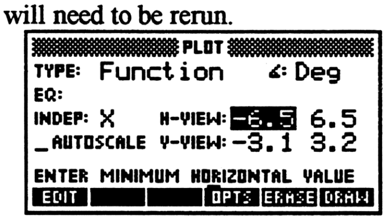


figure 12

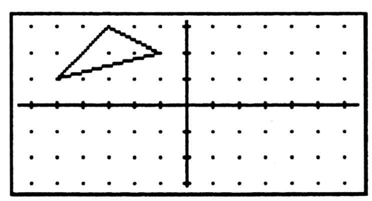


figure 13

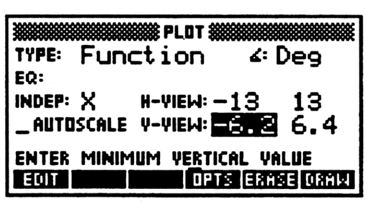


figure 14

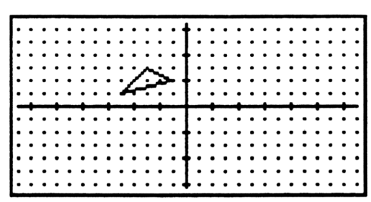


figure 15

Try changing the VIEW settings and graphing a new polygon and its reflection over the y-axis.

ACTIVITY 3...APPLICATIONS SECTION CONTINUED

REFLECTIONS THROUGH THE X-AXIS

PROBLEM 4

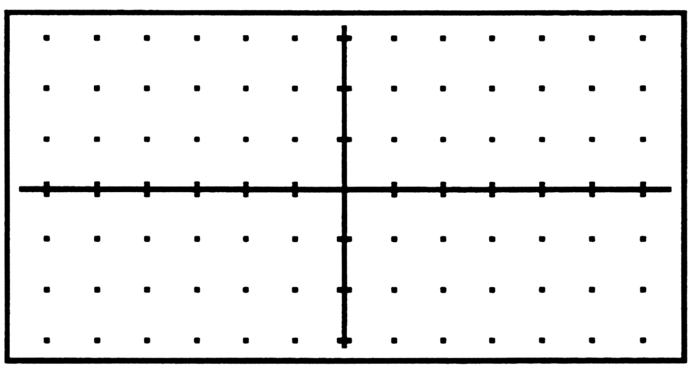
Examine figure 9 and notice that the triangles are *reflections over the x-axis*. Let's continue to call the vertex matrix representing the triangle in quadrant two matrix A, but let's name the vertex matrix representing its reflection over the x-axis matrix G, and let's name the 'mystery' matrix P. We then have the matrix equation:

$$\underset{A}{\begin{bmatrix} -5 & 1 \\ -3 & 3 \\ -1 & 2 \end{bmatrix}} \cdot \underset{P}{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}} = \underset{G}{\begin{bmatrix} -5 & -1 \\ -3 & -3 \\ -1 & -2 \end{bmatrix}}$$

Use a similar problem solving approach used in problem 1 to determine the elements of matrix P. Enter and store matrix P on the calculator. Put A on level 2 and P on level 1 of the stack, press multiply and verify that this product gives matrix G. Finally, graph matrices A and G to visually verify the reflection.

PROBLEM 5

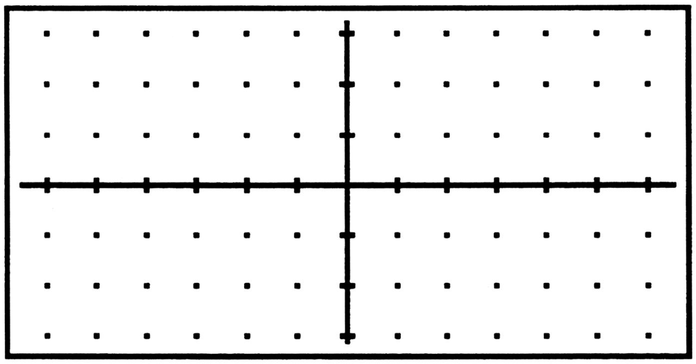
Use the grid below to choose three points, preferably all in one quadrant. Connect the points to form a triangle. Use blank matrix H to write the coordinates of the three points you chose. Fill in matrix P from the work you did in problem 4. Calculate matrix J and fill it in below. Finally, use the calculator to graph both matrices H and J to verify that J is a reflection of H over the x-axis.



$$\underset{H}{\begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix}} \cdot \underset{P}{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}} = \underset{J}{\begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix}}$$

ACTIVITY 3...APPLICATIONS SECTION CONTINUED

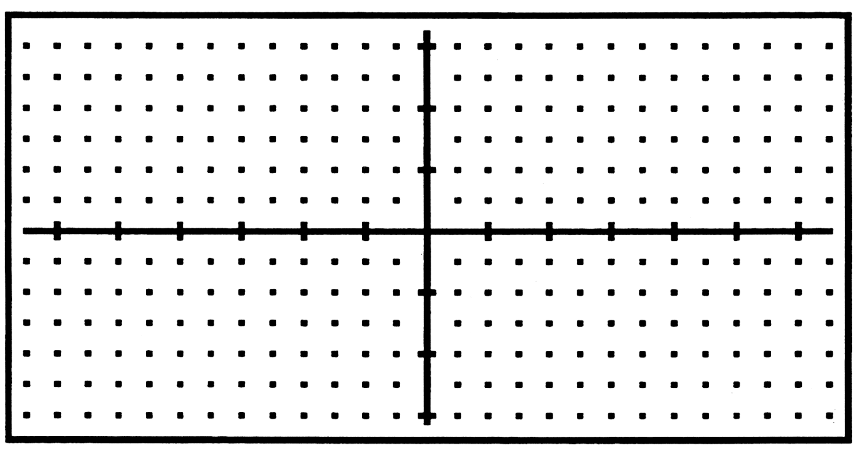
PROBLEM 6
Use the grid below to choose four or more points, preferably all in one quadrant. Connect the points to form a polygon. Use blank matrix K to write the consecutive coordinates of the polygon. Fill in matrix P from the work you did in problem 4. Calculate matrix L and fill it in below. Finally, use the calculator to graph both matrices K and L to verify that L is a reflection of K over the x-axis.



$$\begin{bmatrix} \quad & \quad \\ & \\ & \\ & \end{bmatrix} \cdot \begin{bmatrix} \quad & \quad \\ & \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ & \\ & \\ & \end{bmatrix}$$

$K \qquad P \qquad L$

EXPERIMENT 2
Apply the concept and directions of Experiment 1 for reflecting a polygon of your choice over the x -axis.



$$\begin{bmatrix} \quad & \quad \\ & \\ & \\ & \end{bmatrix} \cdot \begin{bmatrix} \quad & \quad \\ & \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ & \\ & \\ & \end{bmatrix}$$

$N \qquad P \qquad O$

ACTIVITY 3...APPLICATIONS SECTION CONTINUED

REFLECTIONS THROUGH THE ORIGIN.

PROBLEM 7

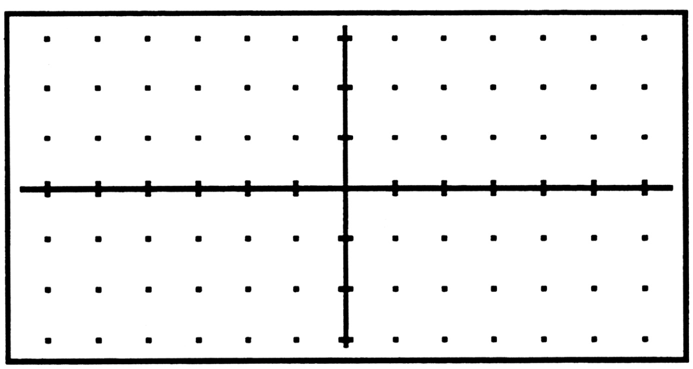
Examine figure 10 and notice that the triangles are *reflections through the origin*. Let's continue to call the vertex matrix representing the triangle in quadrant two matrix A, but let's name the vertex matrix representing its reflection through the origin matrix A, and let's name the 'mystery' matrix Q. We then have the matrix equation:

$$\underset{A}{\begin{bmatrix} -5 & 1 \\ -3 & 3 \\ -1 & 2 \end{bmatrix}} \cdot \underset{Q}{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}} = \underset{R}{\begin{bmatrix} -5 & -1 \\ -3 & -3 \\ -1 & -2 \end{bmatrix}}$$

Use a similar problem solving approach used in problems 1 and 4 to determine the elements of matrix Q. Enter and store matrix Q on the calculator. Put A on level 2 and Q on level 1 of the stack, press multiply and verify that this product gives matrix R. Finally, graph matrices A and R to visually verify the reflection.

PROBLEM 8

Use the grid below to choose three points, preferably all in one quadrant. Connect the points to form a triangle. Use blank matrix S to write the coordinates of the three points you chose. Fill in matrix Q from the work you did in problem 7. <u>Calculate</u> matrix T and fill it in below. Finally, use the calculator to graph both matrices S and T to verify that T is a reflection of S through the origin.

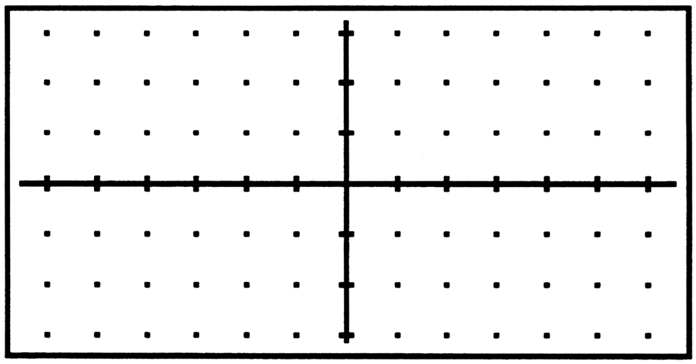


$$\underset{S}{\begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix}} \cdot \underset{Q}{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}} = \underset{T}{\begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix}}$$

ACTIVITY 3...APPLICATIONS SECTION CONTINUED

PROBLEM 9

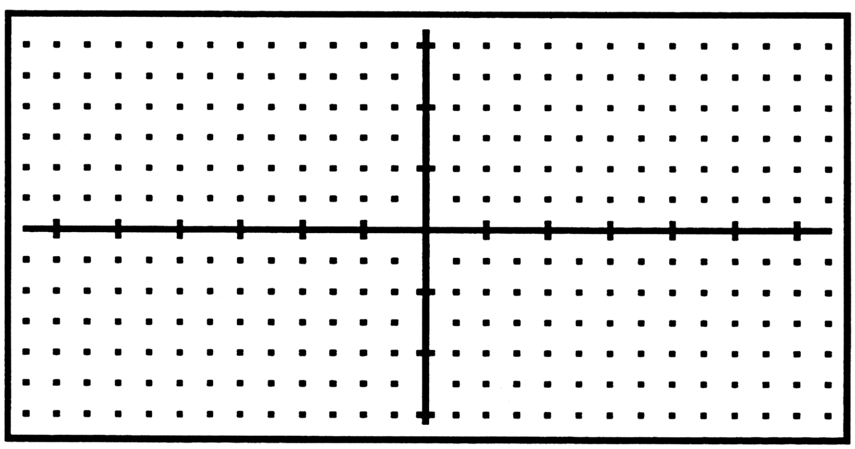
Use the grid below to choose four or more points, preferably all in one quadrant. Connect the points to form a polygon. Use blank matrix U to write the consecutive coordinates of the polygon. Fill in matrix Q from the work you did in problem 7 Calculate matrix V and fill it in below. Finally, use the calculator to graph both matrices U and V to verify that V is a reflection of U through the origin.



$$\underset{U}{\begin{bmatrix} \quad & \quad \\ & \\ & \\ & \end{bmatrix}} \cdot \underset{Q}{\begin{bmatrix} \quad & \quad \\ & \end{bmatrix}} = \underset{V}{\begin{bmatrix} \quad & \quad \\ & \\ & \\ & \end{bmatrix}}$$

EXPERIMENT 3

Apply the concept and directions of Experiments 1 and 2 for reflecting a polygon of your choice through the origin.



$$\underset{W}{\begin{bmatrix} \quad & \quad \\ & \\ & \\ & \end{bmatrix}} \cdot \underset{Q}{\begin{bmatrix} \quad & \quad \\ & \end{bmatrix}} = \underset{X}{\begin{bmatrix} \quad & \quad \\ & \\ & \\ & \end{bmatrix}}$$

ACTIVITY 3...APPLICATIONS SECTION CONTINUED

TRANSLATIONS OF POLYGONS

PROBLEM 10

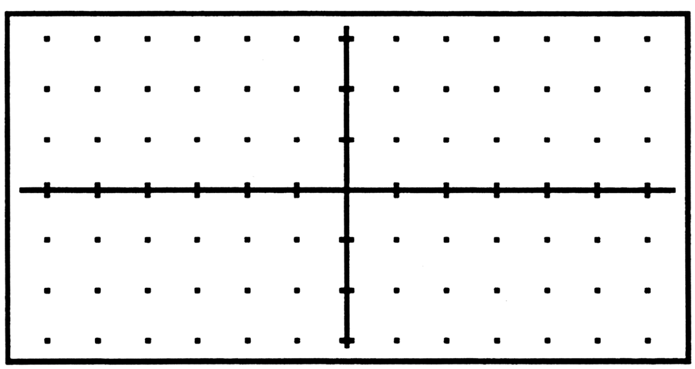
Examine figure 11 and notice that the triangle on the right is a *horizontal translation* of the triangle on the left. Let's continue to call the vertex matrix representing the triangle in quadrant two matrix A, but let's name the vertex matrix representing the translation matrix B, and let's name the 'mystery' matrix Z. We then have the matrix equation:

$$\underset{A}{\begin{bmatrix}-5 & 1\\ -3 & 3\\ -1 & 2\end{bmatrix}} ? \underset{Z}{\begin{bmatrix} \quad & \quad \\ & \\ & \end{bmatrix}} = \underset{B}{\begin{bmatrix}-5 & -1\\ -3 & -3\\ -1 & -2\end{bmatrix}}$$

Compare matrices A and B and determine what *matrix operation* is required between matrices A and Z, and finally, determine the elements of matrix Z. Enter and store matrix Z on the calculator. Put A on level 2 and Z on level 1 of the stack, press the proper arithmetic operation and verify that this gives matrix B. Finally, graph matrices A and B to visually verify the translation.

PROBLEM 11

Use the grid below to draw the triangle produced by vertex matrix A. Now draw a second triangle, call it triangle C so that it is a *vertical translation* of triangle A. Use matrix C to write the coordinates of triangle C. Determine the elements of matrix Y so that this vertical translation will be produced by the equation A + Y = C. Enter matrices A and Y, press + and verify that C is produced. Finally, use the calculator to graph both matrices A and C to visually verify C is a vertical translation of A.



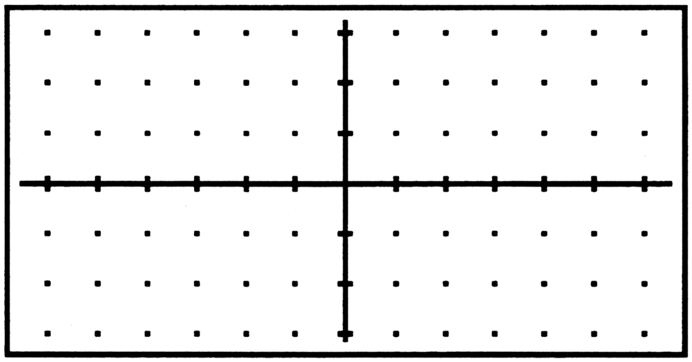
$$\underset{A}{\begin{bmatrix} \quad & \quad \\ & \\ & \end{bmatrix}} \cdot \underset{Y}{\begin{bmatrix} \quad & \quad \\ & \end{bmatrix}} = \underset{C}{\begin{bmatrix} \quad & \quad \\ & \\ & \end{bmatrix}}$$

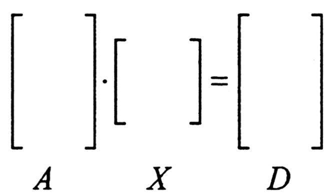
ACTIVITY 3...APPLICATIONS SECTION CONTINUED

TRANSLATIONS OF POLYGONS CONTINUED

PROBLEM 12

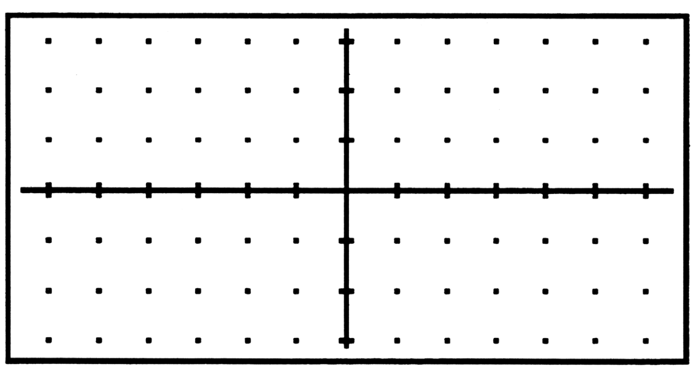
Use the grid below to draw the triangle produced by vertex matrix A. Now draw a second triangle, call it triangle D so that it is a *diagonal translation* of triangle A. Use matrix D to write the coordinates of triangle D. Determine the elements of matrix X so that this diagonal translation will be produced by the equation A + X = D. Enter matrices A and X, press + and verify that D is produced. Finally, use the calculator to graph both matrices A and D to visually verify D is a vertical translation of A.





EXPERIMENT 4

Use the grid below to draw a polygon of your choice. Enter, name, and store the consecutive vertices of your polygon on the calculator, let's call it matrix P. If you haven't already done so, enter, name, and store matrices X, Y, and Z from problems 10 through 12. Now place matrix P on the stack and experiment with different translations using X, Y, and Z. You may want to make changes on matrices X, Y, and Z to produce translations of greater or lesser distance. Another idea to experiment with is adding combinations of transformation matrices like X, Y, and Z to vertex matrices. You many even experiment with adding transformation matrices to vertex matrices, then multiplying by a matrix that produces a reflection.



ACTIVITY 1...FORMULA EVALUATION

INSTRUCTIONAL SECTION

This activity investigates the use of the HP 48G for evaluating formulas. This activity is best used as a demonstration of the 'number crunching' abilities the HP 48G has. Students should understand that the calculator is used as tool for speed in this case. The list of formulas below can be extended to accommodate any formula or list of formulas students are ready to handle.

DIRECTIONS FOR USING THE HP 48G:

1. Use the Equation Writer to key in the list of formulas presented below. Press **PURPLE EQUATION.** Variables are accessed by pressing the α key prior to selecting desired letter of alphabet. Lower case, use α **PURPLE** letter. To lock the α key, press it twice; to unlock, press the α key again.Press **α A PURPLE = PURPLE π α R y^x 2**, figure 1.
2. Name and store each formula with the name inside 'ticks' listed next to each below. **'αα ACIRC α**, fig. 2, **STO**. Remember, pressing STO will cause the object and its name to disappear from the stack.
3. Get into the Solve Environment, **GREEN SOLVE.**
4. Select **Solve equation...**by pressing menu label **OK**, figure 3.
5. Press menu label **CHOOS.**
6. Highlight desired formula by pressing up or down arrows, and press menu label **OK**, figure 4.
7. Move highlight to variables of know quantities, and enter these quantities, figure 5.
8. Move highlight to unknown variable, and press menu label **SOLVE,** figure 6.
9. Move highlight to **EQ**: ________ when finished with current formula, and press **CHOOS** to select another formula.

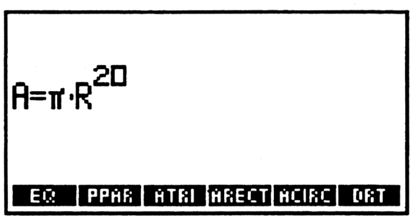


Figure 1

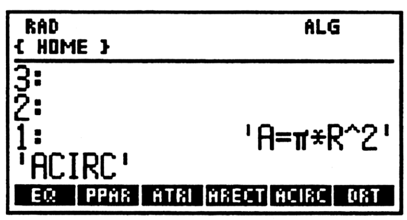


Figure 2

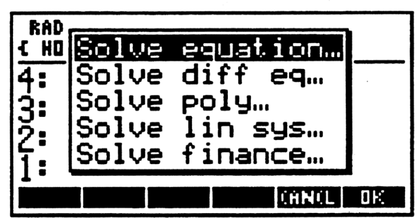


Figure 3

Figure 4

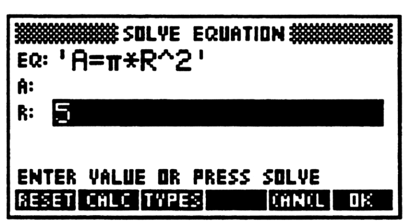


Figure 5

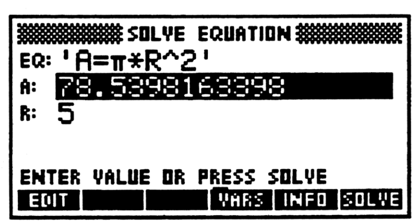


Figure 6

ACTIVITY 1...FORMULA EVALUATION

APPLICATIONS SECTION

Formulas to enter on the calculator and values for the variables. Solve for the unknown variable.

1. $A = \pi r^2$ 'ACIRC' [Area formula for circles when radius is known]
   r = 2 units. A = _____
   r = 4 units. A = _____
   r = 10 units. A = _____

2. $d = r \cdot t$ 'DRT' [Distance formula for known rate and time]
   r = 20, t = 2 d = ____
   r = 32, t = 4.4 d = ____
   r = 3.2 t = .5 d = ____

3. $A = L \cdot W$ 'ARECT' [Area formula for rectangles]
   L = 13, W = 21 A = ____
   L = 31.4, W = 47 A = ____
   L = 3,017, W = .912 A = ____

4. $C = \frac{5}{9}(F - 32)$ 'CTEMP' [Temperature conversion from Fahrenheit to Centigrade]
   F = 212, C = ____
   F = 96, C = ____
   F = 32 C = ____
   F = 82, C = ____
   F = -23 C = ____
   F = 3,429 C = ____

5. $A = \frac{1}{2} b \cdot h$ 'ATRI' [Area formula for triangles]
   b = 13, h = 3.045 A = ____
   b = 3.67 h = 21.72 A = ____
   b = 1,760 h = 4,081 A = ____

6. $V = w \cdot l \cdot h$ 'VPRSM' [Volume formula for a rectangular prism]
   w = 41 l = 13 h = 23 V = ____
   w = .23 l = 17.09 h = 21.77 V = ____

7. $V = \frac{1}{3} B \cdot h$ 'VPRMD' [Volume formula for a pyramid.]
   B = 100 h = 6q V = ____
   B = 213 h = 1.7 V = ____
   B = 1,029 h = 54.917 V = ____

8. $A = \frac{1}{2} h(b + c)$ 'ATRAP' [Area formula for a trapezoid.]
   h = 16 b = 4 c = 10 A = ____
   h = 8.2 b = 2.1 c = 18.9 A = ____
   h = 7.8 b = .45 c = 2.23 A = ____

ACTIVITY 2...EQUATION MANIPULATION

INSTRUCTIONAL SECTION

This activity investigates the use of the HP 48G for manipulating variables in equaitons. This activity is best used as a demonstration of the 'equation solving' abilities the HP 48G has. The list of equations below is only a sample of all the possibilites. Note: some equations use upper and or lower case variables; the calculator interprets an upper case letter such as B, as a different variable from lower case letter b.

DIRECTIONS FOR USING THE HP 48G:

1. Use the Equation Writer to key in the list of formulas presented below. Press **PURPLE EQUATION.** Variables are accessed by pressing the α key prior to selecting desired letter of alphabet. Lower case, use α **PURPLE** letter. To lock the α key, press it twice; to unlock, press the α key again. Press α **D PURPLE =** α **R x** (multiplication) α **T**, figure 1.
2. Name and store each formula with the name inside 'ticks' listed next to each below. 'αα **DRT** α, fig. 2, **STO**. Remember, pressing STO will cause the object and its name to disappear from the stack.
3. Get into the Symbolic Environment, **GREEN SYMBOLIC**.
4. Select **Isolate var...** from the menu using up or down arrow keys, then press menu label **OK**, figure 3.
5. Press menu label **CHOOS**.
6. Highlight desired equation by pressing up or down arrows, figure 4.
7. Enter the variable you wish to isolate in the **VAR**: line on the menu, figure 5.
8. Highlight Symbolic on the menu, and press menu label **OK**, figure 6.
9. The variable should now be isolated; the result is transfered to the first level of the Stack, figure 7.

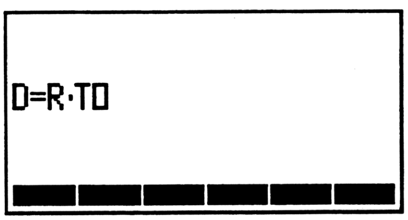


Figure 1

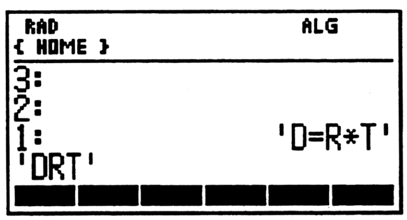


Figure 2

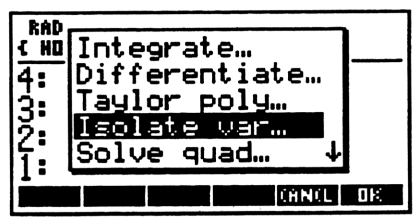


Figure 3

Figure 4

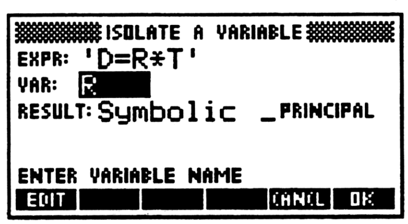


Figure 5

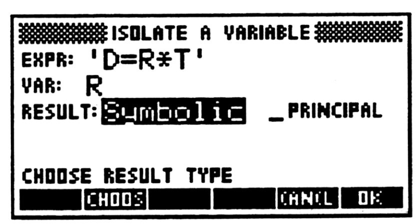


Figure 6

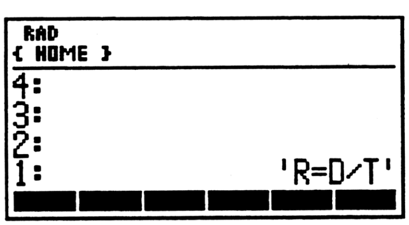


Figure 7

ACTIVITY 2...EQUATION MANIPULATION

APPLICATIONS SECTION

Formulas to enter on the calculator. Solve for the unknown variable.

1. $D=RT$ 'DRT' [Distance formula for known rate and time]

   Solve for R: R = _______ Solve for T: T = _______

2. $A = L\cdot W$ 'ARECT' [Area formula for rectangles]

   Solve for L: L = _____ Solve for W: W = _______

3. $C = \frac{5}{9}(F - 32)$ 'CENT' [Temperature conversion from Fahrenheit to Centigrade]

   Solve for F: F = ___________

4. $A = \frac{1}{2}b\cdot h$ 'ATRI' [Area formula for triangles]

   Solve for b: b = _______ Solve for h: h = _______

5. $V = w\cdot l\cdot h$ 'VPRSM' [Volume formula for a rectangular prism]

   Solve for w: w = _______ Solve for l: l = _______ Solve for h: h = _______

6. $V = \frac{1}{3}B\cdot h$ 'VPYMD' [Volume formula for a pyramid.]

   Solve for B: B = _______ Solve for h: h = ______

7. $A = \frac{1}{2}h(b + c)$ 'ATRAP' [Area formula for a trapezoid.]

   Solve for h: h = _______ Solve for b: b = _______ Solve for c: c = _______

ACTIVITY 1....TWO-DIMENSIONAL MOTION

In order for a graph to simulate the motion of an object, parametric equations must be used. Parametric equations consist of an x-component and a y-component each expressed in terms of the same independent variable t. Using parametric equations, we can graph the horizontal and vertical position of an object at any time t if we know the initial velocity, V_0, and the angle of elevaton, Θ.

We will be using the following generic parametric equations in this activity:

The horizontal component described by the equation:

$$X(t) = V_0 t(\cos\Theta) + c$$

The vertical component described by the equation:

$$Y(t) = V_0 t(\sin\Theta) - 16T^2 + d$$

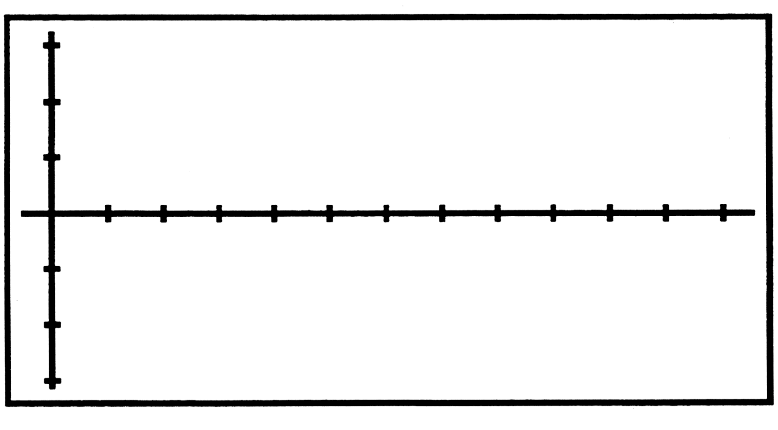
The second equation is only appropriate when the velocity is measured in ft/sec since the $-16t^2$ term of the vertical component represents the force due to gravity. *c* and *d* are constants that represent any other forces that might be acting in the horizontal or vertical direction.

Exercise 1

An object is launched with an initial velocity of 100 ft/sec at an angle of elevation of 60° with the positive x-axis. Assume that the only force acting on the object is due to gravity.

a) Find parametric equations that model the motion of the object.

b) Sketch a complete graphical representation of the path of the object. Use $\{0 \le t \le 6\}$
$\{-20 \le x \le 500\}$
$\{-125 \le y \le 125\}$



c) How far does the projectile travel in the horizontal direction?

d) Find the maximum height of the projectile and when it is attained.

e) How long is the projectile in flight?

Exercise 2

An NFL kicker kicks off at the 35 yard-line which means the end-zone is 65 yards away. If he can consistently kick the ball so that it lands one yard deep in the end-zone, the other team will take a touchback and the kicker can keep his job. The kicker kicks the ball with an initial velocity of 83 ft/sec at an angle of elevation of 56° and there is a 5 mph wind blowing directly in his face.

a) Find parametric equations that model the motion of the object.

b) Will the kick be a touchback? Would it be a touchback if there were no wind?

c) What is the kicker's hang time?

d) When kicking a field goal, the kicker can generate an initial velocity of 75 ft/sec. If the crossbar on the goal post is 10 feet high, at what angle of elevation should the kicker kick the ball to make the longest field goal possible? What is the length of that field goal?

Exercise 3

The Tigers are playing the Red Sox at Fenway. Cecil Fielder steps up to the plate and hits a shot towards the left field fence, also known as the "Green Monster", which is 315 feet from home plate and 37 feet high. If the hit clears the fence, it will be a home run. If it hits the wall above the 9 foot level (the Red Sox left fielder doesn't jump very high), it will be a double off the wall. If it hits below the 9 foot level, it will be an out. What will be the result if the hit:

a) Has initial velocity 113 ft/sec and an angle of elevation of 36° ?

b) Has initial velocity 110 ft/sec and the angle of elevation is 43° ?

c) Has initial velocity 103 ft/sec and the angle of elevation is 49° ?

d) Has initial velocity 114 ft/sec, the angle of elevation is 45°, and there is a wind of 22 ft/sec blowing in the horizontal direction of the ball?

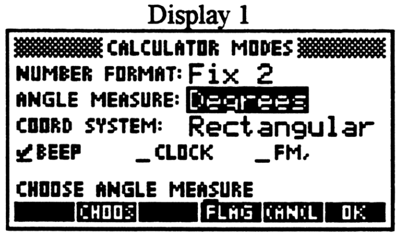
e) Has initial velocity 112 ft/sec, the angle of elevation is 47°, and there is a wind of 10 ft/sec blowing in the opposite direction as the horizontal path of the ball with an angle of depression of 12° ?

INSTRUCTIONAL SECTION

Exercise 1

We are going to be making some estimates in this activity so it will be more practical to have the calculator display numbers that are rounded to the nearest hundredth. To do this hit [green shift] ' MODES '. With NUMBER FORMAT highlighted, select ' CHOOS ' from the onscreen menu and use the arrow to highlight "Fixed". Select ' OK '. Change the number of decimal places from 0 to 2 so that we are viewing Display 1.

Display 1



At this point, it is very important to select ' OK ' from the onscreen menu and not just hit [ON] or [ENTER] if we want the changes we made to be implemented.

a)

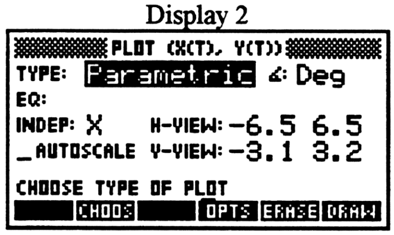
$$x(t) = 100t\cos 60°$$

and

$$y(t) = 100t\sin 60° - 16t^2$$

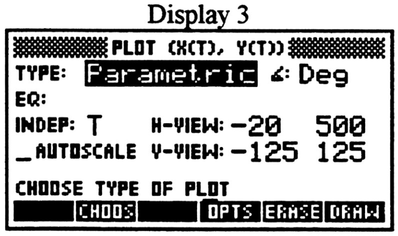
b) From the home screen hit [green shift] ' PLOT '. With TYPE highlighted, select ' CHOOS ' from the onscreen menu then use the arrows to highlight "Parametric". Select ' OK ' to view Display 2.

Display 2



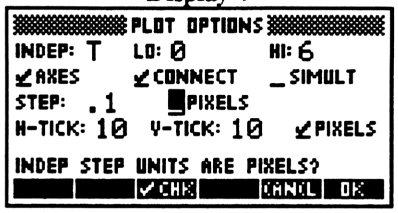
Change INDEP to T, H–VIEW to -20 500, and V–VIEW to -125 125 as shown in Display 3.

Display 3



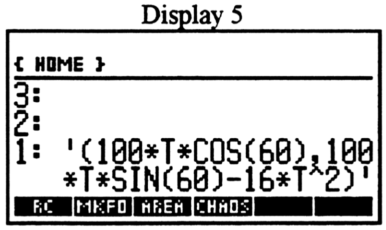
From the onscreen menu shown in Display 3, select ' OPTS '. In the PLOT OPTIONS menu, change LO to 0, HI to 6, and STEP to .1 as shown in Display 4.

Display 4



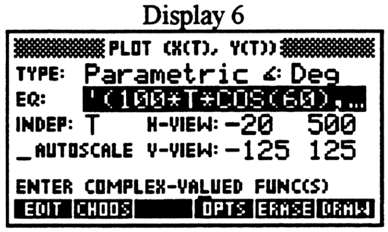
Select ' OK ' from the onscreen menu. We are now set up to graph our parametric equations. We want to store them in a variable called 'PROJ', short for projectile, so hit [ON] to return to the stack. Enter the two equations just like an ordered pair with parentheses and a comma as shown in line 1 of the stack in Display 5.

Display 5



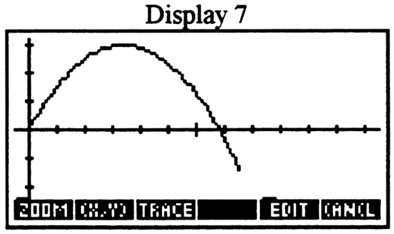
Enter 'PROJ' on line 1and hit [STO]. Hit [green shift] ' PLOT ' and, with EQ highlighted, select ' CHOOS ' from the onscreen menu. Highlight PROJ and select ' OK ' to store the equations in EQ as shown in Display 6.

Display 6



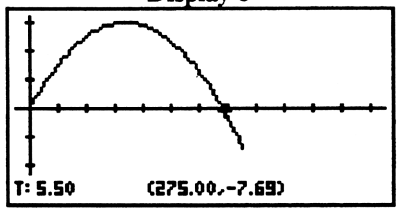
From the onscreen menu select ' ERASE ' and ' DRAW ' to view the graph shown in Display 7.

Display 7



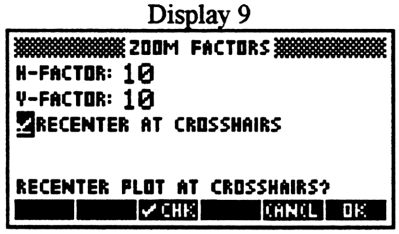
c) When using parametric equations in this manner, the x-axis represents the ground. In reality, the projectile will stop when it hits rather than continuing below the axis a shown in the graph. We are going to assume the projectile sticks right where it lands. To find the horizontal distance traveled we must find the x-intercept. To do this, select ' TRACE ' and then ' (X,Y) ' from the onscreen menu. Use the right arrow to move the crosshairs along the graph until they are as close as possible to the x-intercept. The value for T and the position of the crosshair will be displayed in place of the onscreen menu as shown in Display 8.

Display 8



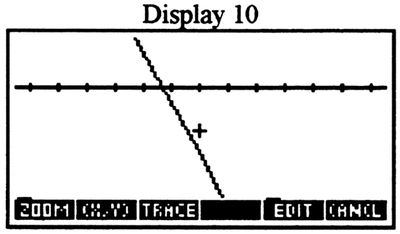
Display 8 shows that the crosshairs are over 7 feet below the x-axis. We can get much closer than this using the zoom feature. Hit the [+] key to display the onscreen menu and select ' ZOOM ' and then ' ZFACT ' from the onscreen menus to set our zoom factors. Set H–FACTOR to 10, Y–FACTOR to 10, and put a √ beside the RECENTER AT CROSSHAIRS option to let the calculator know where to center the zoom.

Display 9



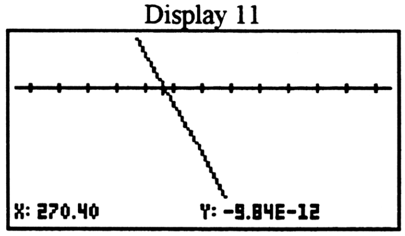
Select ' OK ' and then ' ZIN ' to display the magnified graph shown in Display 10.

Display 10



We don't really want to trace this graph since the trace mode will continue display T in one tenth of a second increments. Therefore, select ' (X,Y) ' and use the arrows to place the crosshairs directly on the x-intercept.

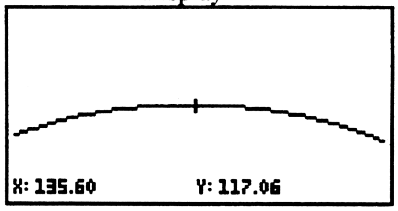
Display 11



The y-coordinate tells us we are $9.84x10^{-12}$ feet below the x-axis which is plenty close enough for our purposes. It appears that the horizontal distance traveled is ≈ 270.4 feet.

d) To do this we need to zoom in on the vertex of the graph using the same method shown in part c. Using the same zoom factors as before, we should be able to obtain a graph similar to Display 12.

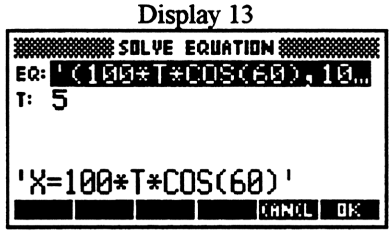
Display 12



The y-coordinate will represent the height attained by the projectile. It looks like it will attain a height of approximatley 117 feet.

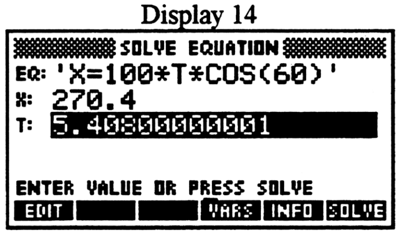
e) When using ' (X,Y) ' without the trace feature, T is not displayed. However, we do know the horizontal distance traveled from part c and we know the equation for the horizontal component so we can use the solve feature to compute T. Get out of the graphics environment by hitting [ON] then hit [green shift] ' SOLVE '. Choose "Solve equation..." We only want to do one computation so we will just enter the equation directly and not bother to save it under a variable name. On the SOLVE EQUATION screen enter the equation $X = 100 * T * \cos(60)$ as shown in Display 13. Note: We don't need to hit the ['] first, it will be inserted automatically.

Display 13



Select ' OK ', enter 270.4 for X, highlight T and select ' SOLVE '. You should obtain display 14.

Display 14

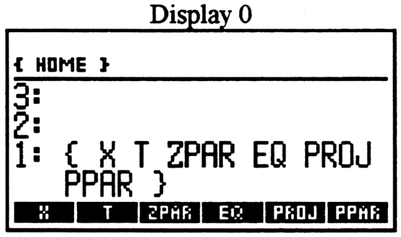


The calculator doesn't fix decimals in the solving environment so we can get a little practice at rounding mentally. It appears that the projectile is in flight for ≈ 5.41 seconds.

Exercise 2

Before we tackle this problem, lets get rid of some of the junk that has been accumulating in our variables menu. Go to the home screen and hit [VAR]. Also do a [purple shift] ' CLEAR ' to clear the stack. To delete any unwanted variables quickly, create a list by hitting [purple shift] '{ }'. We don't need to type in every variable name. Instead use the white keys under each undesired variable to create a list like the one shown in Display 0.

Display 0

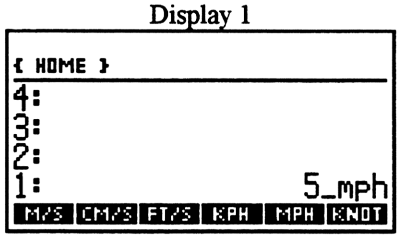


These are all things that we have no further use for. They can all be deleted by hitting [purple shift] ' PURGE '. It is a good idea to do this at the beginning of every class or after each problem so that a lot of junk doesn't pile up in memory. However, be careful not to purge anything you wish to keep!

The decimals should remain fixed at two places for this exercise.

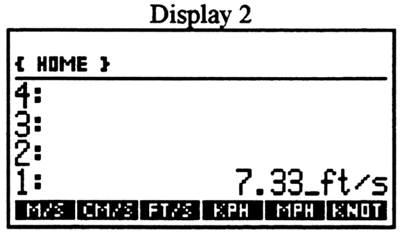
a) First we should convert miles per hour to feet per second. From the home screen hit [green shift] ' UNITS '. Select ' SPEED ' from the onscreen menu. Key in 5 and instead of entering it select ' MPH ' from the onscreen menu to get Display 1.

Display 1



To convert this to feet per second hit [purple shift] ' FT/S '.

Display 2



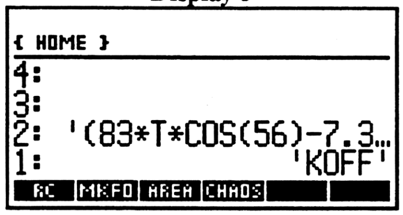
The kicker has an additional force of –7.33 ft/s acting in the horizontal direction of the ball. Therefore the parametric equations that simulate the motion of the kick are:

$$x(t) = 83t\cos 56° - 7.33t$$
$$y(t) = 83t\sin 56° - 16t^2$$

The wind is acting only in the horizontal direction which means that it is blowing with a 0° angle with respect to the positive x-axis. In reality the force of the wind should be multiplied by $\cos 0°$ in the first equation and by $\sin 0°$ in the second. However, for obvious reasons this is unnecessary

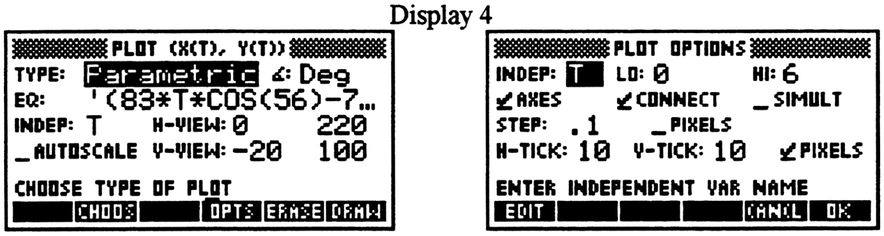
b) Enter the equations as they are shown in part a and store them in "KOFF" as shown in Display 3.

Display 3



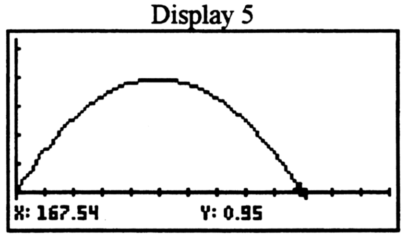
Remember to hit [STO]. Go to the plot menu and make sure that TYPE is "Parametric" and INDEP is T. Let students decide for themselves what the dimensions of the viewing screen and the range of T will be. We will be using $\{0 \le t \le 6\}$, $\{0 \le x \le 220\}$, and $\{-20 \le y \le 100\}$. The reason we are using a negative value for the low boundary on the vertical axis is so that the x-axis can be seen above the onscreen menu when viewing the graph. Enter "KOFF" into EQ and the PLOT and PLOT OPTIONS screens should be similar to Display 4.

Display 4



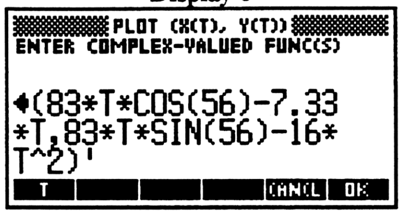
Select ' ERASE ' and ' DRAW ' from the onscreen menu to graph the equation then select ' (X,Y) ' and center the crosshairs as close to the x-intercept as possible.

Display 5



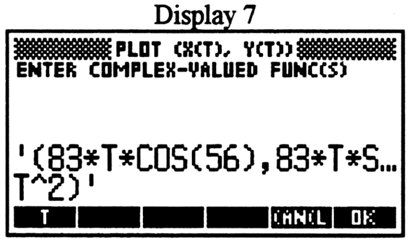
It should be apparent that the kick will fall way short of the end zone without having to zoom in since 66 yards is equivalent to 198 feet and the x-value is less than 170. To eliminate the wind factor we will need to edit the horizontal component. Hit [ON] to return to the PLOT menu, highlight EQ and select ' EDIT ' from the onscreen menu to edit the equation as shown in Display 6.

Display 6



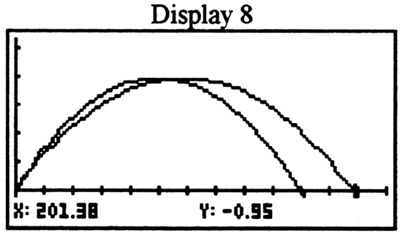
Use the right arrow to place the cursor on the comma and use the delete key to eliminate the unwanted portion of the equation. We should be left with the equation in Display 7.

Display 7



Select ' OK ' and then do a ' DRAW ' without erasing so that we can view the distance the ball travels with no wind in comparison to against the wind. Finding this distance might be a good exploration problem. Select (X,Y) and place the crosshairs on the x-intercept as shown in Display 8.

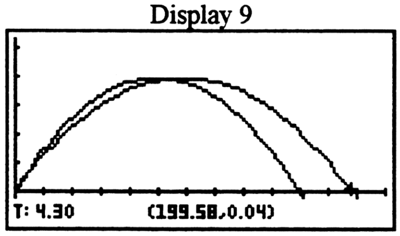
Display 8



It appears that the kick travels approximately 67 yards so it would be a touchback if there were no wind.

c) Hit the [+] key while Display 8 is pictured to get the onscreen menu. Trace the function and display the values as in Display 9.

Display 9



The ball is in the air approximately 4.3 seconds which is referred to as the "hang time" of a kick. For a closer approximation the zoom function can be used as in part c of exercise 1.

d) To do this we shall graph two pairs of parametric equations simultaneously. Enter these two sets of equations using the proper form and store them in the specified variables:

$$x(t) = 75t\cos 65^\circ \qquad x(t) = 40t$$

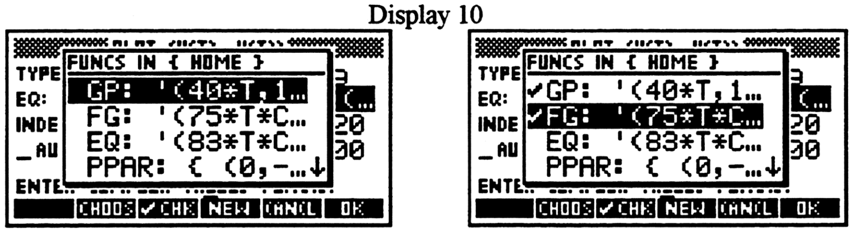
$$y(t) = 75t\sin 65^\circ - 16t^2 \qquad y(t) = 10$$

$$"FG" \qquad "GP"$$

The first pair of equations will simulate the motion of the ball and the second pair will draw a horizontal line at 10 feet. "FG" stands for "Field Goal" and "GP" stands for "Goalpost". We have chosen 65° as a starting point since it is a rather severe angle for a kick and doubtful that anything greater would be likely to maximize distance. Once we have graphed the equations this supposition will be obvious.

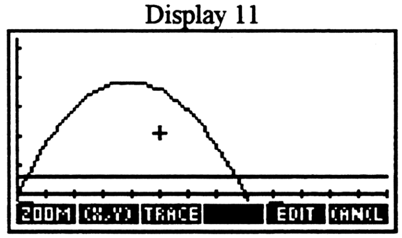
Return to the PLOT environment. We will be using the same parameters for T, X, and Y as before but they may be changed if the students so desire. Highlight EQ and select ' CHOOS '. "GP" and "FG" should be the first two on the list. Use the ' √ CHK ' from the onscreen menu to select both variables as shown in Display 10.

Display 10



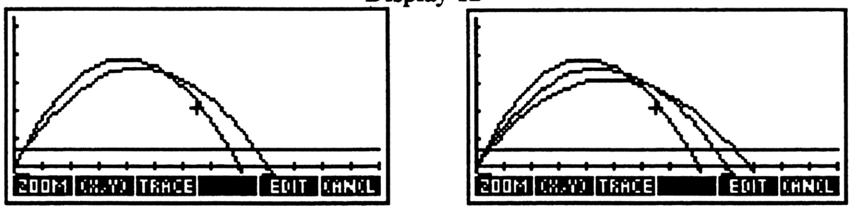
Select ' OK ' and then ' ERASE ' and ' DRAW '. The line representing the height of the goalpost will be drawn first and then the graph representing the path of the football.

Display 11



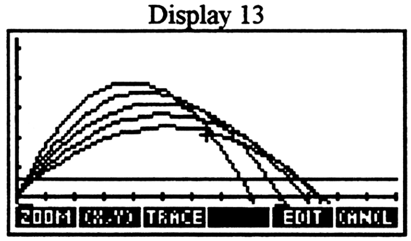
Now let's take a look at how far a 60° and then a 55° boot will travel by editing the equation the same way we did in part b of this exercise. <u>Do not use the ' ERASE '</u> feature when drawing the new graphs since we wish to compare distances. Display 12 shows two and then three kicks.

Display 12



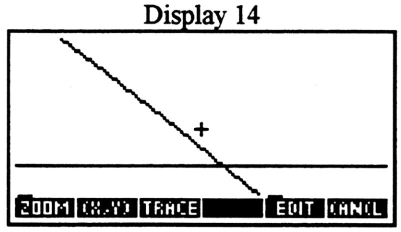
Display 13 adds 50° and 45° efforts.

Display 13



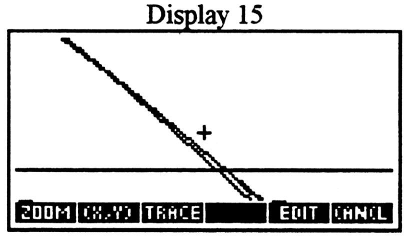
The last two kicks appear to travel about the same distance so our guess should be somewhere between the two. If students need to be convinced, go ahead and see where a 40° kick would cross. It is apparent that it is short of the previous two. Erase the graph and plot a 47° kick and then zoom in on the intersection as shown in Display 14. A zoom factor of 10 should be sufficient and do not forget to check the RECENTER AT CROSSHAIRS option in ' ZFACT '.

Display 14



Edit the equation and plot 48° and 49° kicks using only the ' DRAW ' option.

Display 15



It is difficult to tell what happens from Display 15 but if we watch the graphs as they are being plotted we can tell that the 47° and 48° kicks are virtually on top of each other while the 49° kick falls just a little bit shorter. If students wish to zoom further to try and distinguish which kick is actully closest then by all means let them do so.

Exercise 3

We are going to be altering the initial velocity , angle of elevation, and wind factor quite frequently in this excercise so we want the equations to be a generic as possible. Enter these two sets of equations using the proper form and store them in the specified variables:

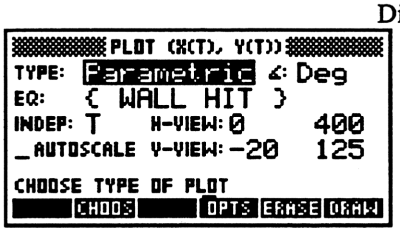
$$x(t) = vt\cos a + c$$
$$y(t) = vt\sin a - 16t^2 + d$$
$$"HIT"$$

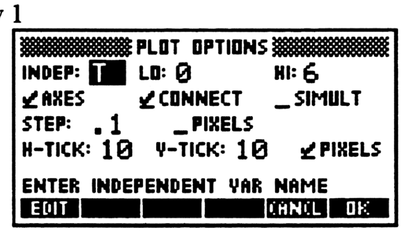
$$x(t) = 315$$
$$y(t) = \left(\frac{37}{6}\right)t$$
$$"WALL"$$

"HIT" simulates the motion of the baseball and "WALL" will draw the wall on the screen. A good exploratory activity would be to the let students try to figure out how to draw a 37 foot wall before giving them the equation. Before diving in to this excercise, check your variable list, [VAR], and make sure that there are no values stores in v,a,c, or d. If there are...PURGE THEM.

a) From the home screen go to the plot environment and highlight EQ. Instead of storing the equations using the ' CHOOS ' lets take a little shortcut. Hit [purple shift] [{}] and then [VAR] if your variable list isn't already on the onscreen menu. Select WALL and then HIT using the soft keys and then hit [ENTER]. This will store them in EQ. In this exercise we will be using $\{0 \le t \le 6\}$, $\{0 \le x \le 400\}$, and $\{-20 \le y \le 125\}$ since baseballs go a little farther than footballs. Set the PLOT and PLOT OPTIONS screens as shown in Display 1.

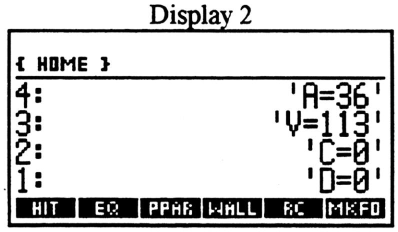
Display 1





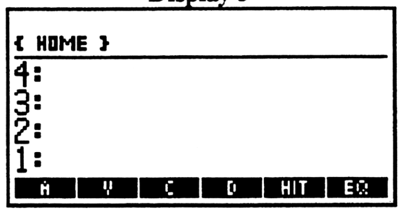
Hit [ON] twice to get the the home screen. In part a the only forces acting on the ball are the initial velocity and gravity so we will set them equal to 0. Enter the values on the stack as shown in Display 2.

Display 2



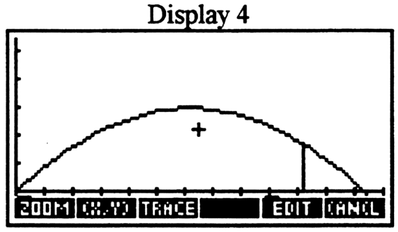
We want to define these four variables to be the indicated values so hit [purple shift] ' DEF ' four times. These should now be the first four variables on your list as seen in Display 3.

Display 3



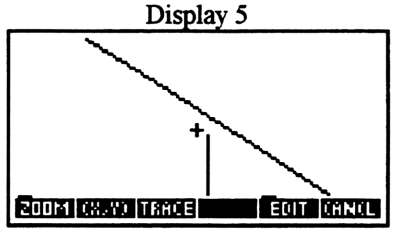
Everything is now all set up to graph. Return the the plotting environment and select 'ERASE ' and ' DRAW '. Display 4 simulates the motion of the baseball in flight.

Display 4



It appears that the hit will just barely hit the top of the fence. Check it out by zooming in. Display 5 is a close-up of the ball and the top of the fence with a zoom factor of 10.

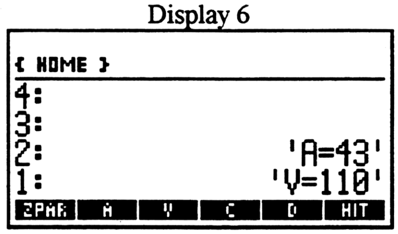
Display 5



It looks like Cecil got just enough on it to clear the Green Monster! HOME RUN!

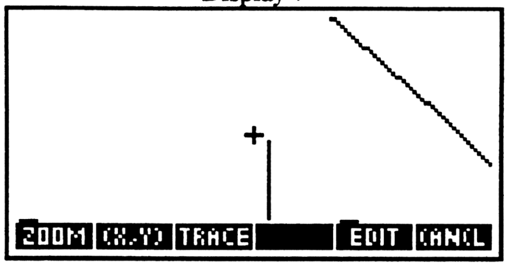
b) Now is when using the variables in the equations will come in handy. Hit [ON] twice to go to the home screen and hit [VAR] to put your variable list in the onscreen menu if it isn't already there. Enter the values for A and V on the stack as shown in Display 6.

Display 6



Hit [purple shift] ' DEF ' twice to store the values, and return to the plot environment. Note that the ' H–VIEW ' and ' V–VIEW ' values are not what we originally designated. The current values replaced the old ones when we zoomed. Since we know these values give us a good view of the top of the fence, it is not really necessary to change them unless, for some reason, we want to see the entire graph. Display 7 shows the graph without changing the parameters of the viewing rectangle.

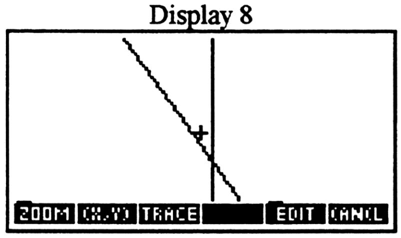
Display 7



Cecil had plenty of distance on this one. HOME RUN!

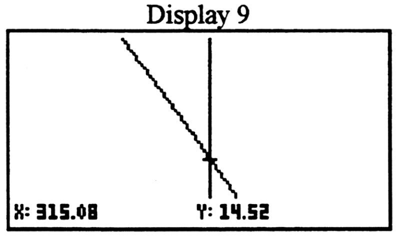
c) Go back to the original viewing rectangle and we will find that the ball strikes the wall. Display 8 is obtained by zooming in on the intersection of the WALL and the HIT.

Display 8



Select ' (X,Y) ' and center the crosshairs on the intersection.

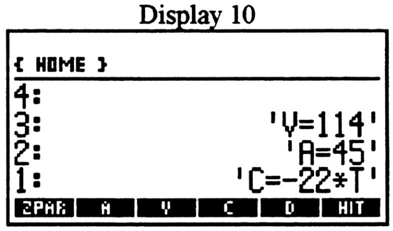
Display 9



It appears the ball will hit the wall about 14.5 feet above the ground. DOUBLE!

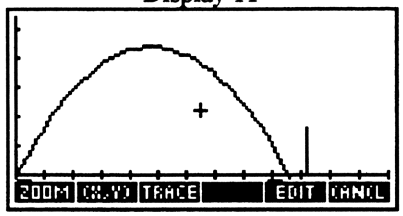
d) The wind is only affecting the horizontal component. The value C needs to be changed as well as V and A as shown in Display 10.

Display 10



In display 11, the graph is redrawn with the original viewing rectangle.

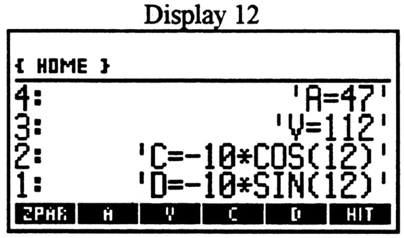
Display 11



The hit is obviously well short of the fence. Cecil is OUT!

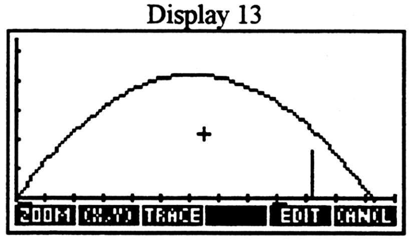
e) All four variables need to be changed in this instance. Display 12 shows the necessary values for A, V, C, and D. The numbers input for C and D should prompt a discussion of vectors and forces which is a nice tie-in to Physics class.

Display 12



Display 13 shows the result.

Display 13



ANOTHER HOMER FOR CECIL!

ACTIVITY 2....ONE-DIMENSIONAL MOTION

Parametric equations can also be used to simulate the linear motion of an object moving along the ground. In such cases, there wouldn't be an angle of elevation or a gravitational force that would affect the motion as far as our purposes are concerned.

We will be using the following generic parametric equations in this activity:

$$x(t) = rt \quad \textit{and} \quad y(t) = c$$

The x-component will be representing the distance traveled and the y-component will represent the position on the graph. In this activity, we will be solving old problems in new and more interesting ways.

Exercise 1

A train leaves a station and travels north at 75 km/h. Two hours later a train leaves on a parallel track and travels north at 125 km/h.

a) Which is the first train to arrive at Otisburg which is 400 km from the station?

b) At what distance will the second train overtake the first train?

Exercise 2

Two cars leave town traveling in opposite directions. One is going 80 km/h and the other is going 96 km/h. In how many hours will they be 528 km apart?

Exercise 3

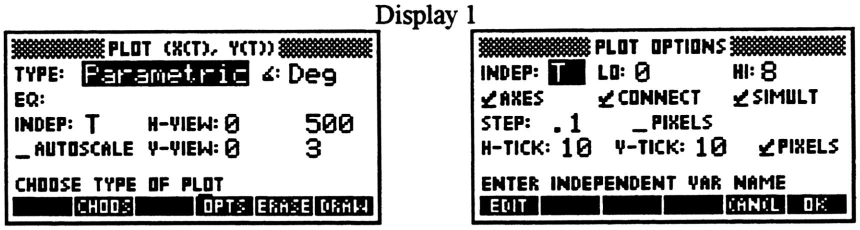
Two motorcycles travel toward each other from Chicago and Indianapolis at rates of 110 km/h and 90 km/h respectively. If Chicago is 350 km from Indianapolis and the two motorcycles started at the same time, in how many hours will they meet?

INSTRUCTIONAL SECTION

Exercise 1

Remember to purge any variables from memory that we will no longer be using. Set up the PLOT and PLOT OPTIONS screens to look like Display 1. Make sure to put a √ by SIMULT in the PLOT OPTIONS menu so that the graphs will be plotted simultaneously.

Display 1

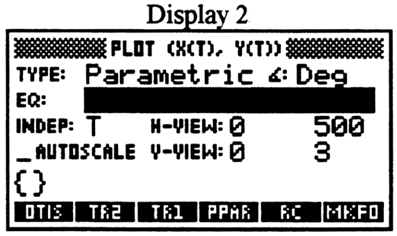


a) Enter the following parametric equations and store them in the indicated variables.

$x(t) = 75t$	$x(t) = 125(t-2)$	$x(t) = 400$
$y(t) = 1$	$y(t) = 2$	$y(t) = t$
"$TR1$"	"$TR2$"	"$OTIS$"

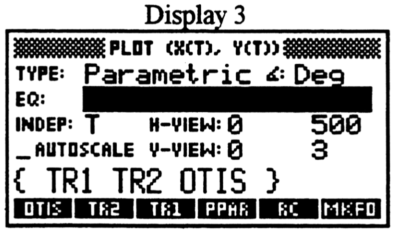
TR1 will plot the path of the train leaving first on "track one". TR2 will plot the path of the train leaving second on "track two" and OTIS will draw a vertical line 400 "miles" from the starting point. All three of these equations aren't going to fit into EQ so that they can be seen. When this is the case, it is better to store them in EQ in a different manner that also happens to be a lot quicker. Hit [green shift] ' PLOT ' to go to the plotting environment. Hit [purple shift] '{}' [VAR] to view Display 2.

Display 2

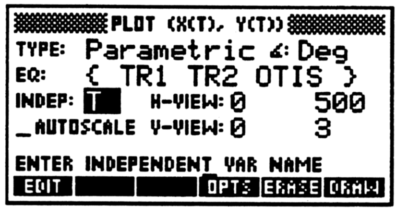


Now select the soft keys under TR1, TR2, and OTIS (in doesn't matter what order since they will be graphed simultaneously), to create a list as shown in Display 3.

Display 3

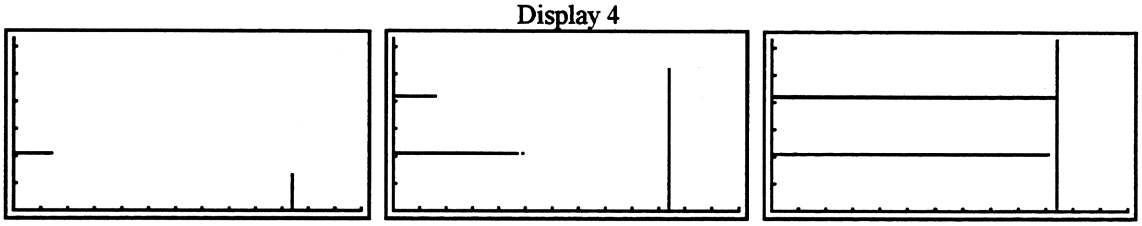


Hit [ENTER] to store the variables in EQ as shown in Display 4. Now we can tell exactly what EQ is plotting just by looking at the PLOT menu. This is one of the big reasons to use meaningful variable names instead of single letters even though it takes a few extra keystrokes to key them in.



Return to the PLOT menu and select ' ERASE ' and ' DRAW ' to watch the trains race to Otisburg. Display 4 illustrates this extremely dynamic problem.

Display 4

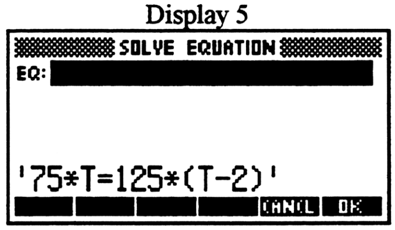


It appears that TR2 barely nips TR1 in the race to Otisburg!

b) This is a Distance = Rate x Time problem that is commonly found in texts when applying the concept of solving systems of two linear equations. After students have practiced a few of the more rudimentary problems using paper and pencil, they should be allowed to use the solving feature of the calculator to explore more advanced applications. The NCTM standards call for increased attention to be given to "computer-based methods such as successive approximations and graphing utilities for solving equations and inequalities" and decreased attention given to "paper and pencil evaluation". Students will be able to do more problems at a more advanced level with less chance of error using technology. Therefore, don't be alarmed when you show them how to solve this problem with a calculator rather than making a D = RT chart, writing two equations, and solving for T simultaneously.

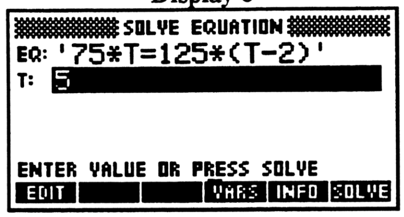
Hit [ON] twice to return to the home screen and then hit [green shift] ' SOLVE ' to go to the solving environment. We want to "Solve equation...". With EQ highlight enter ' ' on the command line as shown in Display 5.

Display 5



Hit [ENTER] and a T will since it is the only variable to appear in the equation. Highlight T and select ' SOLVE '. The calculator will show the correct value for T as shown in Display 6.

Display 6



The calculator tells us that the second train will catch the first in 5 hours. However that is not the answer to the question. We want to know the distance. Students need to know to plug 5 into either equation and come up with a value for distance. 5 x 75 = 350 so the correct answer is 350 miles.

Exercise 2

This is an example of a problem that needs to be analyzed correctly in order to be able to solve it regardless of calculator assistance. Students need to use D = RT to write the equations:

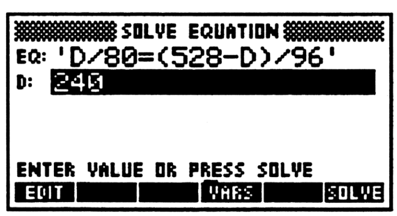
and

They need to identify the variable that has the same value in both equations, namely T, and solve both equations for that variable to obtain:

and

Now they can set the two equations equal to each other:

and then plug the equations into the solver and let the calculator do the tedious algebraic manipulations.

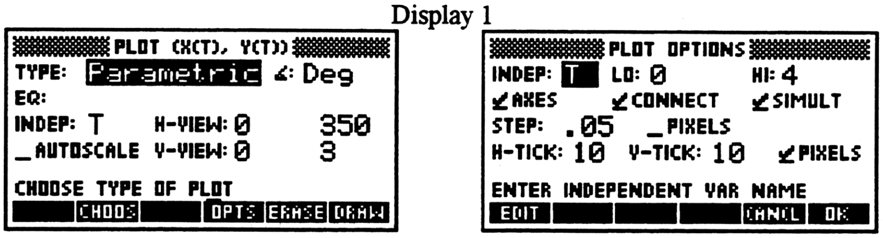


Again, this is not the answer to the problem. The student needs to use the problem solving skill of looking back to discover that we are looking for the time, not the distance. Plugging the distance into either equation will yield a time t = 6 hours.

Exercise 3

This problem can be solved similarly to the one in exercise 2. However, lets try an graph some parametric equations that will simulate the motion of the motorcycles. Set up the PLOT and PLOT OPTIONS menus to look like Display 1. Again, be sure SIMULT is checked.

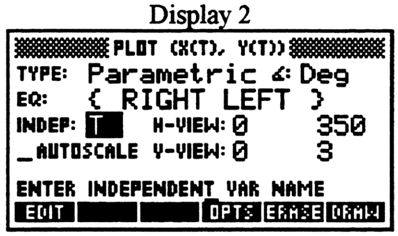
Display 1



Enter the following parametric equations and store them in the indicated variables:

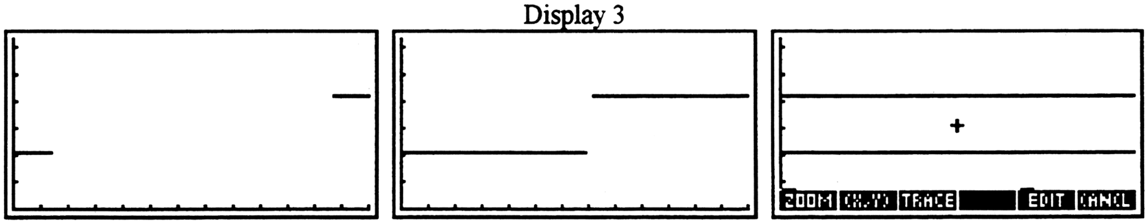
The first equation will plot the course of the motorcycle from Chicago on one side of the freeway and the second equation will plot the motorycyle from Indianapolis on the other side. Store the equations in EQ as shown in exercise 1 to get Display 2. Remember, EQ can be {RIGHT LEFT} or {LEFT RIGHT} since they are going to be graphed simultaneously.

Display 2



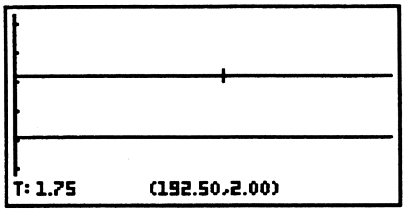
Select ' ERASE ' and ' DRAW ' to watch them go.

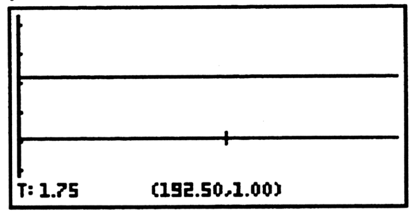
Display 3



The finished graph will be two parallel line running horizontally across the screen as illustrated in Display 3. Select ' TRACE ' and ' (X,Y) ' and use the right arrow key to trace one of the graphs. The up and down arrow keys will skip the cursor back and forth between the two graphs. The trick is to place the cursor at a point where the T value remains the same when the up and down arrow keys are used to go between the graphs as shown in Display 4.

Display 4





They will pass each other in 1.75 hours. The more sadistic students in the class will want to see what happens when both motorcycles are put in the same lane so that the motorcycles actually "meet" as stated in the problem.

ACTIVITY 1.....CONTINUOUS POPULATION GROWTH

In this activity we will be using the following equation as a model for continuous change in population:

$$P(t) = P_0 e^{rt}$$

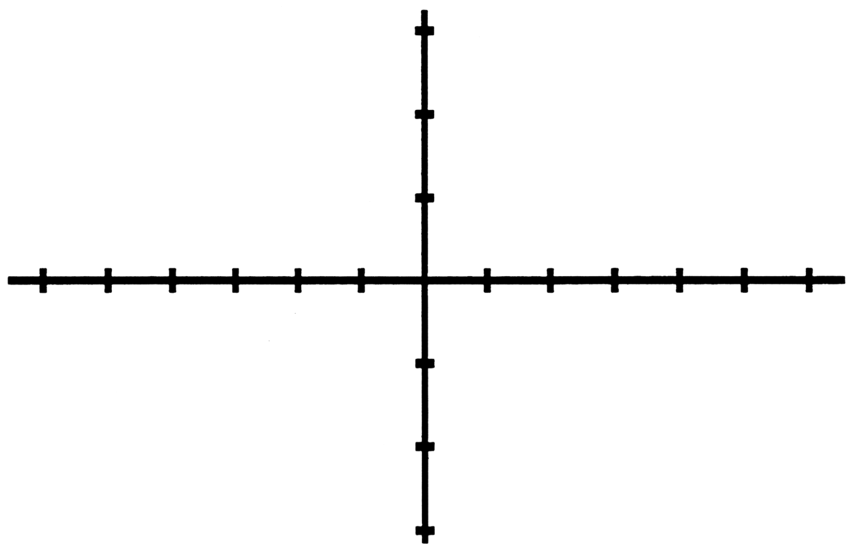
where $P(t)$ = Population at any time t
P_0 = Initial population
r = annual rate of growth
t = time

Exercise 1

The population of a town is 50,000 and is increasing at the rate of 2.5% each year.

a) Write the population as a function of time.

b) Create a complete graph of the function on your calculator and sketch it. For the Domain and Range use {x: $-20 \le x \le 20$} and {y: $-10{,}000 \le y \le 100{,}000$}



c) What portion of the graph represents the problem situation? Why?

d) Use your calculator to estimate when the population of the town will be 75,000.

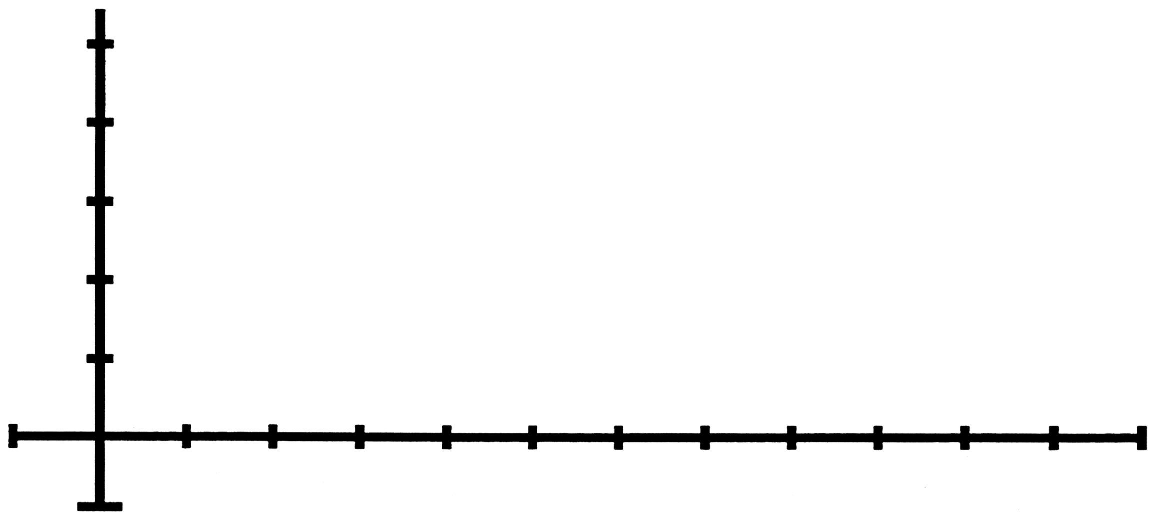
e) Use your calculator to find the exact population of the town in 10 years, rounded to the nearest whole person.

f) If this population model holds true to form, what will the population of the town be in 200 years? Do you think that this will really be the population in 200 years? Why or why not?

Exercise 2

Assume that the number of bacteria in a certain culture is continually increasing as a rate of 100% and that 100 are present initially.

a) Draw a graph that shows the number of bacteria present during the first 12 hours.



b) How many bacteria are present after 7 hours and 15 minutes? 9 hours and 23 minutes?

c) Determine when the number of bacteria present will be 350,000.

d) Explain why this exponential growth model is not a good one for bacterial population over a long period of time, such as years. What other factors influence population growth?

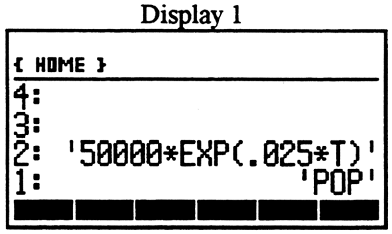
INSTRUCTIONAL SECTION

<u>Exercise 1</u>

a) $P = 50{,}000\, e^{.025t}$.

b) We want to graph the function . On the stack, key in the function. It is <u>not</u> necessary to include the 'P = '. Place the word ' POP ' on line1 of the stack. This will be the variable name used to store the equation for future use.

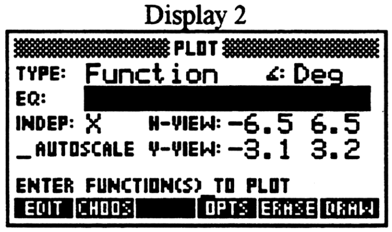
Display 1



Now hit the [STO] key to store the equation in POP. Both lines of the stack will disappear from the screen.

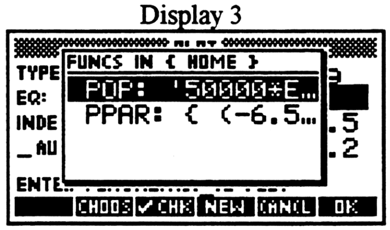
The green shift of the [8] key will take us to the graphing environment. Use the arrows to highlight the field to the right of EQ.

Display 2



From the onscreen menu select ' CHOOS ' using the white keys. If there is more than one function in your home directory, use the arrows to highlight "POP". It should be the first one on the list since we just stored it.

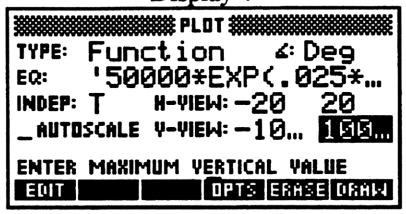
Display 3



From the onscreen menu select ' OK '. The equation is now stored in EQ. Now we will probably have to change the independent variable to T. This can be accomplished by using the arrows to highlight INDEP. Now type in ' T ' and select ' OK ' from the onscreen menu.

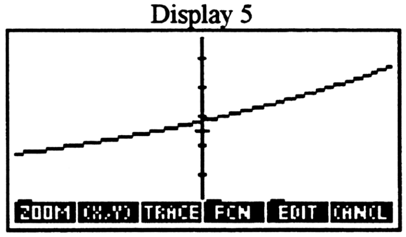
The domain is entered into H–VIEW . Type in the value ' –20 ' by keying in 20 and then hitting the [/] key. Hit [ENTER] or ' OK ', either one will enter the value . Repeat the process for the maximum domain value. Repeat the procedure to enter the proper range values in V–VIEW.

Display 4



We are now ready to graph! From the onscreen menu select ' ERASE ' (this will delete anything previously graphed and then ' DRAW '.

Display 5

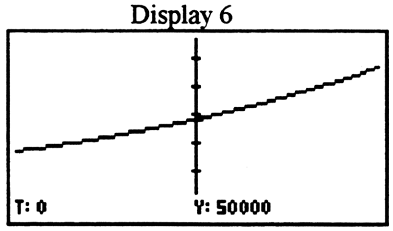


Note: We can delete the onscreen menu by hitting the minus key. In this case it would reveal the x-axis.

c) The x-axis represents time and the y-axis represents population. Since it is not practical to think of either of these two concepts in terms of being negative, we should only concentrate on the part of the graph in the 1st quadrant.

d) We will use the last screen we viewed to estimate the how long it will take for the town to grow to 75,000. If students have wandered elsewhere and no longer have the graph onscreen, it can be viewed by hitting [purple shift] of the left arrow key, [ ], labeled ' PICTURE '. Hit the plus key to reveal the onscreen menu and we should be viewing Display 5. From the onscreen menu, select the ' TRACE ' option. ' TRACE ' should become ' TRAC□ ' indicating the option is activated. Now select the ' (X,Y) ' option from the onscreen menu.

Display 6

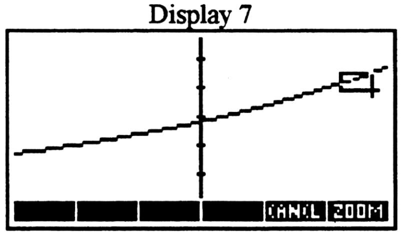


It is difficult to see but the crosshairs are now illuminated and are centered on the point where the function crosses the y-axis. Now may be a good time to discuss y-intercepts and why this particular one is at (0,50,000). Move the crosshairs by hitting the right arrow key. The function will be traced and the values for T and Y will be displayed at the bottom. Sometimes the values are in scientific notation. Move the crosshairs until you obtain T: 1.63E1 and Y: 7.52E4.

These values mean that when T is equal to 16.3, the Y-coordinate is 75,200. If we hit the left arrow and go back a little, we can see that when T is equal to 16, the Y-coordinate is approximately 74,591. Therefore, a good estimate of the time it would take for the town to reach a population of 75,000 would be ≈ 16.2 years using a little mental interpolation.

If we want a more accurate estimate, we can hit the plus key to return to the onscreen menu. Remove the trace feature by selecting ' TRAC□ ' from the onscreen menu and the box hiding the E should disappear. Now select the ' ZOOM ' feature from the onscreen menu. There are many zooming options to choose from but the best one to use in this situation would be the box zoom. We want to draw a box around the area where the crosshairs are now centered. Move the crosshairs with the arrow keys up and to the left until reaching a point that would correspond to the upper left corner of the box we wish to draw (hitting the up arrow twice and the left arrow twice should be sufficient). From the onscreen menu select ' BOXZ '. Use the down and right arrow to draw the box as shown in Display 7.

Display 7

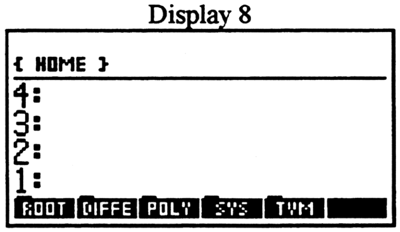


From the onscreen menu select ' ZOOM ' and repeat the tracing process. Notice that the T value increments are much smaller.

e) There are a couple of ways to do this. We can hit the ' ON ' key and then go back to the plotting environment, reset your ' H–VIEW ' and ' V–VIEW ' values, regraph the function and use the trace feature to determine the Y value when T = 10.

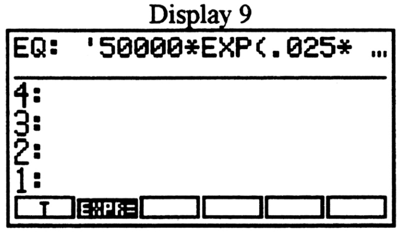
Or an even better way would be to use the ' SOLVE ' feature of the calculator. From the home screen hit [purple shift] and the [7] key. This takes you to an environment of the calculator known as the solver. Using the purple shift instead of the green shift allows us to go directly to the numeric root finder, also known as the "number cruncher". This hidden feature as well as many others on the "dark side" of the calculator come in very handy at times.

Display 8



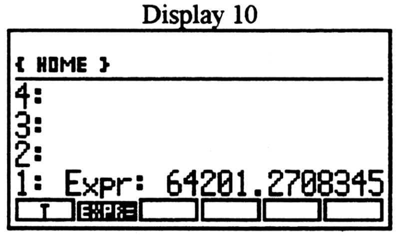
If the stack isn't empty , clear the screen by hitting [purple shift] ' CLEAR '. From the onscreen menu shown in Display 8, select the ' ROOT ' directory and from the new onscreen menu select ' SOLVR '. Display 9 shows the "number crunching" environment.

Display 9



Since we want to know what the population (represented by EXPR) is equal to when the time (T) is 10 years, enter the number 10 on the stack <u>or</u> the command line and select ' T ' from the onscreen menu. 10 is now stored as the T value. To find out what the expression is select ' EXPR ' from the onscreen menu.

Display 10



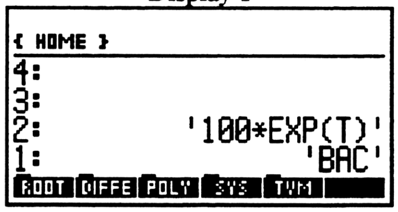
Display 10 shows that in ten years the population will be ≈ 64, 200. It should be apparent that the ' SOLVE ' feature can render a much more accurate answer than ' ZOOM ' and it is also much quicker!

f) Talk about factors that limit population such as space, disease, war, etc.. A good example is Hawaii. If you look at the population in 1960 compared to the current population, one might predict a population sometime in the near future that would force people to take up residence in active volcanoes!

Exercise 2

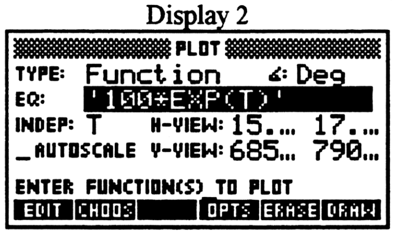
a) Use the equation P = 100 e^t since the rate is 100% (r = 1). Store it under the variable 'BAC ' as shown in Display 1.

Display 1



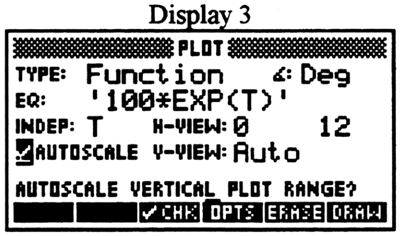
As before, hit the [STO] key. Go to the graphing environment by hitting the [green shift] ' PLOT ' and use the same method as described in exercise 1 to enter the expression in EQ.

Display 2



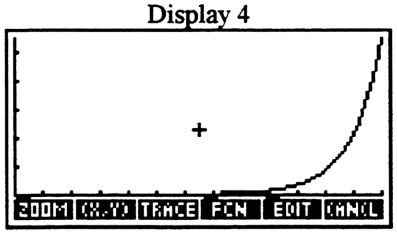
When the domain of the function to be graphed is known, it is sometimes handy to use the autoscaling feature of the 48G. We want to graph the function over the interval $0 \le t \le 12$ so input 0 into the minimum and 12 into the maximum H–VIEW values using the same method described in exercise 1. After you enter the maximum value, the highlight will appear next to the word AUTOSCALE on your screen. From the onscreen menu select ' √ CHK ' and a check mark should appear on the screen as shown in Display 3.

Display 3



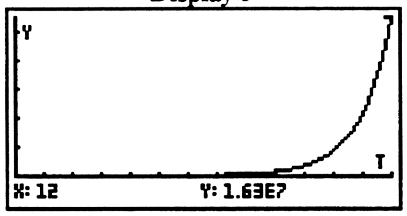
The calculator will automatically choose V–VIEW values that place as much of the graph on the screen as possible. From the onscreen menu select ' ERASE ' to delete the old picture and then ' DRAW ' to create the graph shown in Display 4.

Display 4



Just for fun, let's see how many bacteria are present at the end of the 12 hours. To do this we need to move the crosshairs to the upper right-hand corner of the viewing rectangle. This can be accomplished easily by using the green shift and the arrow keys. Hit [green shift] [Δ] to jump the crosshairs to the top of the screen and then [green shift] [ ] to move the crosshairs to the far right of the screen. Select ' (X,Y) ' from the onscreen menu to display the corresponding T and Y values.

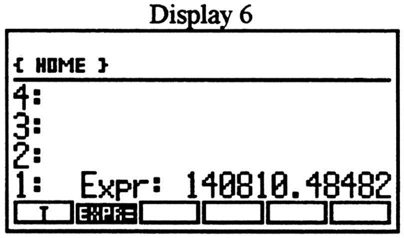
Display 5



Display 5 shows that after 12 hours there will be 16,300,000 bacteria present. Now would be a good time to discuss the behavior of exponential functions and why the graph seems to make such a drastic jump at t = 10. Additionally, because of the vastness of the range, the graph is very deceptive from 0 ≤ t ≤ 7as it appears to be on top of the x-axis.

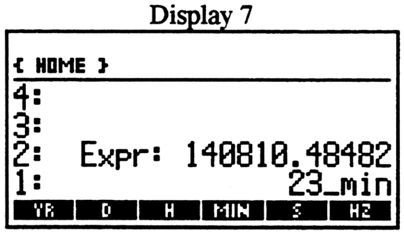
b) Let's use the solver to handle this one. From the home screen hit [purple shift] [7] to go to the solver and [purple shift] ' CLEAR ' to clear the home screen. As in exercise 1, select ' ROOT ' from the onscreen menu and then ' SOLVR '. 7 hours and 15 minutes represents 7.25 hours so enter 7.25 onto the stack, store it in T and select ' EXPR= '.

Display 6



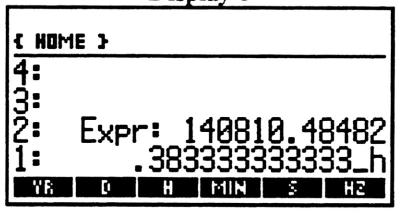
It looks like ≈ 141,000 of the little buggers will be present after 7¼ hours. 9 hours and 23 minutes is going to be a bit more of a challenge. It is not difficult to figure the decimal representation for 23/60 but let's take this opportunity to illustrate another feature of the 48G that is extremely useful in the science classroom. Hit [green shift] ' UNITS ' and select ' TIME ' from the onscreen menu. Enter 23 on the command line and choose ' MIN ' from the onscreen menu.

Display 7



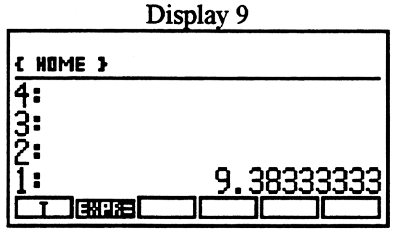
As Display 7 illustrates, the 48G keeps track of units. To convert the minutes to hours hit [purple shift] and select ' H ' from the onscreen menu.

Display 8



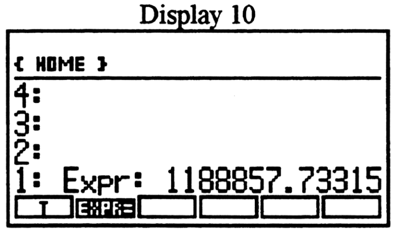
23 minutes is converted to ≈.3833 hours. Any like units can be converted in this manner. Since we want 9 hours and 23 minutes, place 9.38333333... on the command line and enter it, and go back to the solver.

Display 9



Store the value in T using the soft key directly below the T and select ' EXPR= '. <u>Note</u>: make sure that there is no label on the value when it is stored in T.

Display 10



There should be ≈ 1,118,860 bacteria present after 9 hours 23 minutes according to Display 10.

c) Once again, this can be done graphically or numerically. If we want to use the solver we can start making guesses as to the value of t until we home in on an expression value of 350,000. Using the "number crunching" capacity of the calculator the students will be able to approximate the t value to the nearest hundredth in less than a minute. The correct value should be approximately 8.16. This can also be done more quickly and precisely using the "Solve equation...." environment which will be illustrated in Activity 2.

d) By looking at the exponential curve over the first 12 hours it would seem that the bacteria would be taking over the world in a few days. The population over long periods of time would be a number too big for Cray supercomputers to grind out. Again, this is a good time to talk about limiting factors.

ACTIVITY 2....COMPOUND INTEREST

In this activity we will be using the following equation to compute compound interest:

— where = Amount at any time t

= Principal investment
= rate of return
= time in years
= number of times the quantity is increased per year

Exercise 1

Suppose $100 is invested for three years at 4.75% compounded semi-annually and another $100 is invested at a different bank at the same rate but is compounded quarterly. Find the ending account balances.

Exercise 2

Tasha's bank offers a 5.5% return on investments of $3000 dollars or more compounded annually. She wants to buy a $10,000 car when she graduates in two years and her parents have offered to pay half if she can come up with the other half by the time she graduates. Tasha already has her share but would like to go to California next summer with some friends instead of working.

a) If Tasha goes to the bank today, how much will she have to deposit so that she has exactly enough money to buy the car when she graduates?

b) If Tasha decides she can't afford to tie up more than $4300 of her money in the bank, what interest rate would she need to negotiate to reach her goal?

c) Tasha's friend Angela tells her that at 2nd National Bank, they pay 5.25% interest on deposits of $3000 or more but they compound interest quarterly. Should Tasha stick with her bank or switch to 2nd National?

d) Tasha sees Sally driving around in a snazzy little car that she likes even more than the one she is planning to buy. She goes to the dealership and discovers the sticker price on the car is $10,800. If she puts $4600 in her bank account, how long will it be before she, with her parents help, can afford a car like Sally's? (We are going to have to assume the cost of the car will remain constant for awhile.)

Exercise 3

Donovan's parents aren't quite as benevolent as Tasha's and have told him that they will give him $1500 for a down payment on a car upon his graduation but he will be responsible for the payments.

a) At the Puff City used car lot, Donovan finds an '89 Chevy IROC Z28 that the salesman assures him is in fantastic condition for a car with 120,000 miles on it. The selling price is $8500 and the salesman is sure they can carry him for four years if he finances through them at a 16.5% interest rate. Donovan figures that if he lives at home while going to the local community college, he can afford a payment of $225 a month. Can Donovan by the IROC? Should he?

b) Much to the chagrin of the Puff City salesman, Donovan's insurance agent made sure the car was out of Donovan's price range. He urged Donovan to find a car with a little less "spunk" and his father suggested he finance through the Credit Union which was currently offering a 10.5% rate on 3 year used car loans and an 11.5% rate for 4 years. As luck would have it, Donovan's friend Walker is interested in selling his '89 Volkswagen Cabriolet. Walker wants $7000 for the car. If Donovan's insurance agent says full coverage will run $300 every 6 months and his budget remains $225/month for both payment and insurance, can he take out a 3 year loan or will he need to use the 4 year payment plan? What will his monthly payment be for the loan he chooses?

c) Donovan ends up buying the VW from Walker and goes to the local Community College for two years. When he is a sophomore in college he hits a growth spurt and becomes one of the best JUCO basketball players in the state. When he is offered a scholarship at the local University, his parents are so pleased that they give him $3000 that they had been saving for his education and Donovan decides to use it to pay off his car loan. Donovan looks at his payment book and discovers he has 23 payments left on his 4 year loan. Did Donovan's parents give him enough to cover the loan balance?

d) How much interest did Donovan pay over the life of his loan? How much did he save in interest by paying the loan off early?

Exercise 4

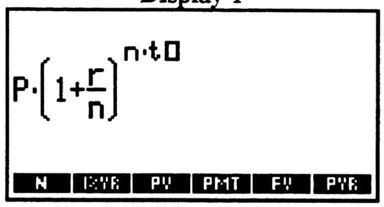
At the Great Northwestern Bank, they pay an interest rate of 5% on savings accounts and it is compounded daily. First Security Bank pays the same rate, but they compound the interest continually. Can we use the equation, $A(t) = P(1 + {}^{r}/_{n})^{nt}$, to figure interest compounded continually? If Heidi puts $500 into Great Northwestern and Gary puts the same amount in First Security, how much more interest will Gary earn in one year?

INSTRUCTIONAL SECTION

Exercise 1

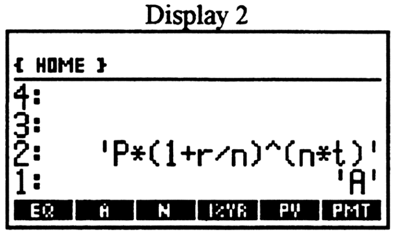
Use the Equation Writer to enter the compound interest expression exactly as it looks. Do not substitute values for P, r, n, and t.

Display 1



To obtain the lower case letters for r, n, and t, hit the purple shift key in between the alpha key and the letter. e.g. to enter the ' r ' hit [α] [purple shift] [▷]. After entering the expression, store it in a variable called ' A '. Lower case letters aren't necessary, simply more aesthetically pleasing.

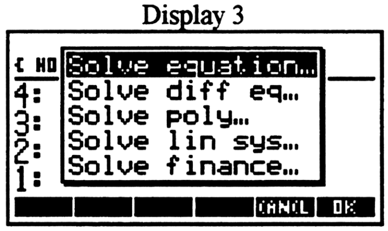
Display 2



Now hit [STO] to store the expression.

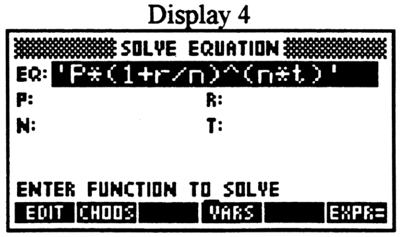
In this type of problem, when there are a number of choices for the independent variable, it is best to use the ' SOLVE ' feature of the HP48. The green shift of the [7] key will take us to the solving environment.

Display 3



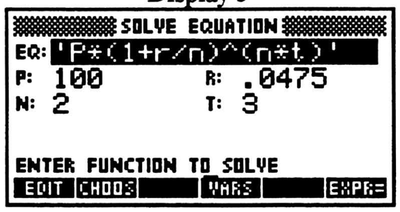
We want to choose the highlighted field, "Solve equation...", so select ' OK ' from the onscreen menu. The field to the right of EQ will be highlighted. Select ' CHOOS ' from the onscreen menu and highlight the appropriate expression. Select ' OK ' to store it in EQ as shown in Display 4.

Display 4



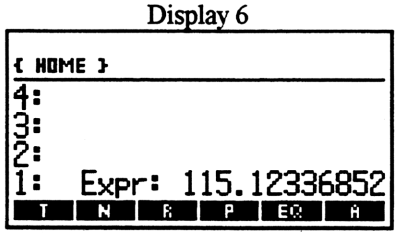
Any variables used in the expression will appear on the screen directly below it. We can enter values for the variables and the calculator will crunch the numbers. In this exercise we want to compare values when N=2 and N=4. Use the arrow keys to highlight the variables and enter the values to solve for N=2. Even if the numbers were entered as lower case, they will always be capitalized in the solver.

Display 5



With the expression highlighted, from the onscreen menu select ' EXPR= '. This will print the amount on the stack. To view the value of the expression, hit [ON] once.

Display 6

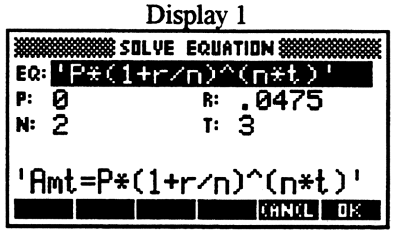


Display 6 shows that the yield will be about $15.12 in 3 years. Go back to the solving environment and change the value of N to 4 and the yield should be approximately $15.21.

Exercise 2

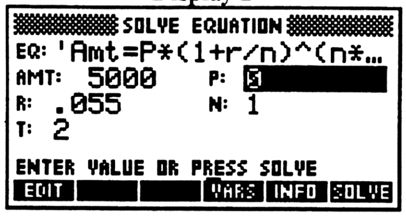
a) We will be using the expression from Exercise 1 so go back to the "Solve equation...." environment. The first thing we want to do is to edit the expression so that it becomes an equation. With the expression highlighted, from the onscreen menu select ' EDIT '. Type in "Amt=" as shown in Display 1. Do not use "A" since that is what the expression is stored in!

Display 1



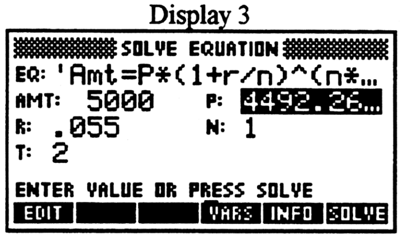
Select ' OK ' from the onscreen menu to store the new equation. We want to know how much principal needs to be invested so enter the values for AMT, R, N, and T into the appropriate spots so that we are viewing Display 2.

Display 2



Highlight P as shown in Display 2 and select ' SOLVE ' from the onscreen menu.

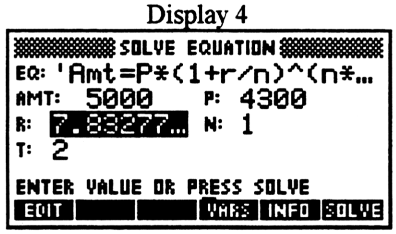
Display 3



It appears Tasha will need to deposit ≈ \$4492 to be able to buy her car.

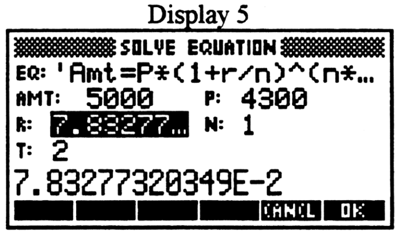
b) This is easily accomplished by entering \$4300 for P, highlighting R, and selecting ' SOLVE ' from the onscreen menu to obtain Display 4.

Display 4



There is a slight problem here since the rate should be expressed as a decimal. The current value would indicate that she needs a 783% interest rate. What happened is that the calculator has truncated the end of the number. To view the entire number, select ' EDIT ' from the onscreen menu.

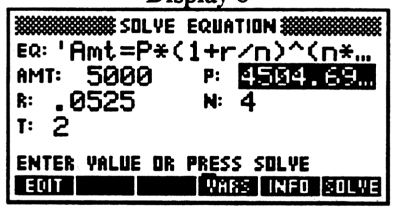
Display 5



The "E–2" at the end of the number indicates the number is displayed using scientific notation which means that the actual rate can be obtained by moving the decimal two places. Therefore Tasha will need an interest rate ≈ 7.83%.

c) Tasha would have needed to deposit ≈ \$4492 in her bank to reach her goal. To figure the amount she would have to deposit in 2nd National, change the values for R and N and highlight P as shown in Display 6. Select ' SOLVE '.

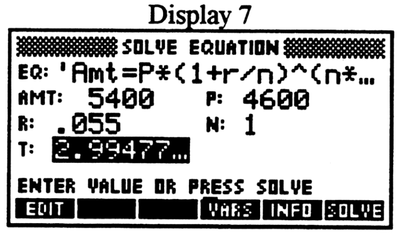
Display 6



It looks like her bank is a better deal.

d) Insert the appropriate amounts in AMT, P, R, and N and solve for T as shown in Display 7.

Display 7

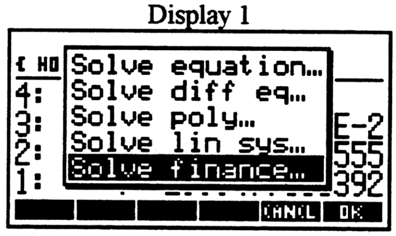


If she wants a car like Sally's, she will have to wait an extra year.

Exercise 3

In this problem we will be still be using the ' SOLVE ' menu, but instead of choosing "Solve equation..." we want to choose "Solve finance...." from the main menu.

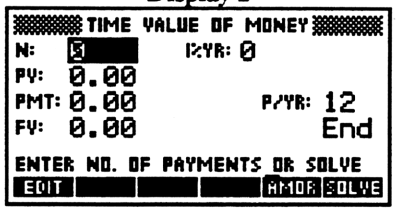
Display 1



Select ' OK ' from the onscreen menu in Display 1.

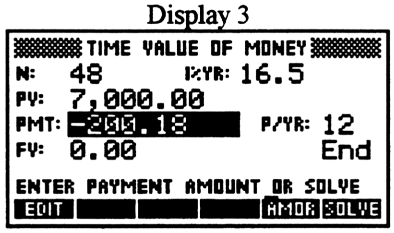
a) The "Solve finance..." feature of the HP 48G is very useful in figuring loans. We are still using exponential equations, its just that the calculator has this one built in. After choosing the finance solver, we should be viewing Display 2.

Display 2



Enter the values N= 48 (12 months a year for 4 years), %YR= 16.5 (do not change to a decimal), PV= 8500 (PV stands for Present Value). If, in place of the word "END" in Display 3, there is the word "BEGIN", change it! The default value for P/YR is 12 and should remain that way for loans that require monthly payments. Use the arrows to highlight PMT and select ' SOLVE ' from the onscreen menu.

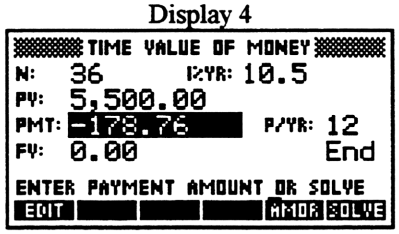
Display 3



The reason the value is negative is that the financial solver is showing the loan from the borrower's point of view. Money that is received is a positive value and money that is paid out is a negative value. It looks like Donovan can purchase the IROC. The reason he shouldn't is answered in part b.

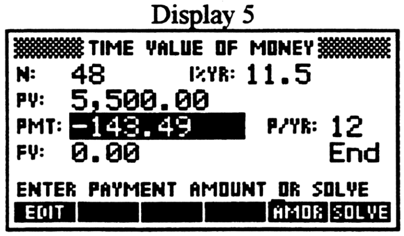
b) Some quick division tells us that insurance will run $60 per month. Let's check out the 3 year loan first. Enter the appropriate numbers as shown in Display 4 and select ' SOLVE '.

Display 4



It looks like payment+insurance is going to be beyond Donovan's means so enter N=48 instead of 36 and %YR=11.5, as shown in Display 5, to see what payments on a 4 year loan will be.

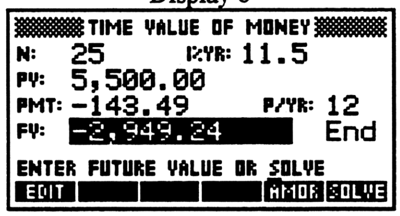
Display 5



His monthly payment will be $143.49 on a 4 year loan. A nice feature about the finance solver is that dollar amounts are automatically rounded to the nearest cent so number won't be truncated as the were in the equation solver.

c) Donovan has made 25 payments, so that becomes the value for N. Now we are going to use the FV portion of the financial solver which stands for Future Value. Highlight FV and select ' SOLVE '.

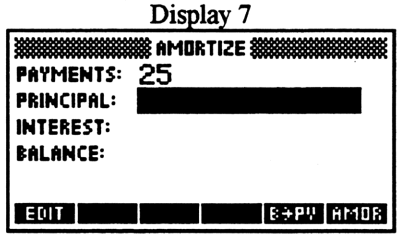
Display 6



The loan balance after 25 payments will be $2949.24 so he has just enough with a little left over to go out for pizza!

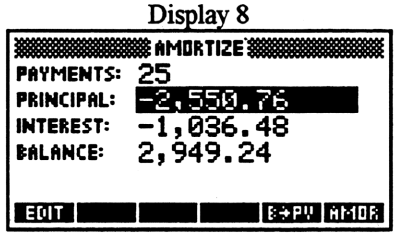
d) The best way to do this is to us the amortization feature of the finance solver. With the proper values for %YR, PV, and PMT entered, select the ' AMOR ' directory from the onscreen menu. The only information required is the number of payments made so enter 25 for N.

Display 7



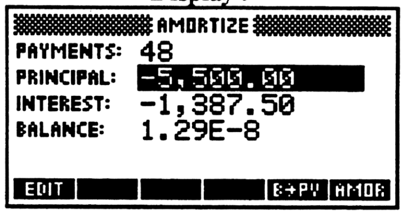
From the onscreen menu shown in Display 7, select ' AMOR '.

Display 8



As seen in Display 8, the principal paid, interest paid and remaining loan balance are computed. We wanted interest paid so far which is $1,036.48. To find out how much he saved by paying the loan off early, set the number of payments equal to 48 and select ' AMOR ' to see what the interest would have been had he not paid off the loan early.

Display 9



The interest over the life of the loan would have been $1387.50 so he saved $351.02 in interest. Because of some rounding that was done during the amortization, there is an extremely small number in BALANCE instead of the expected "0". Now would be a good time for a discussion on why so much interest was paid during the first 25 months compared to the last 23 months of the loan.

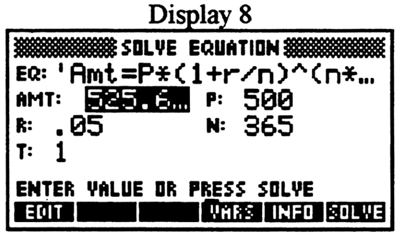
Exercise 4

In this exercise we need to use the fact that = *e* where *e* is the irrational base 2.71828182... to change from our current equation to the continuous change model that was used in Activity 1. If a principal of P is invested at an annual rate of r, then the amount A(t) after t years is given by the formula

— = — which approached Pe^{rt} as

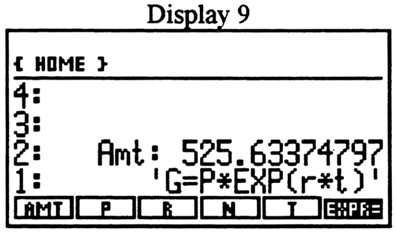
Go to the "Solve equation..." environment and use the current equation with the values as shown in Display 8 to figure how much Heidi earns in interest by solving for "Amt".

Display 8



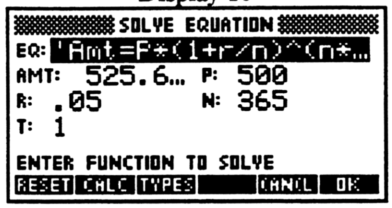
By hitting [ON] and returning to the stack we can see that she earns $25.63 in one year. While at the stack enter the equation shown on line 1of Display 9.

Display 9



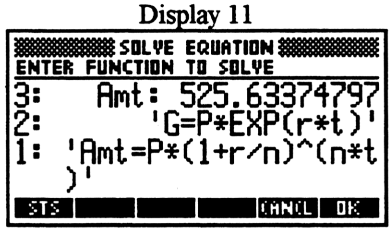
This is the equation we will need to figure Gary's interest income. Since we are going to use this briefly, we won't bother to store it. Instead, go to the equation solving environment and hit the [NXT] key to view Display 10.

Display 10



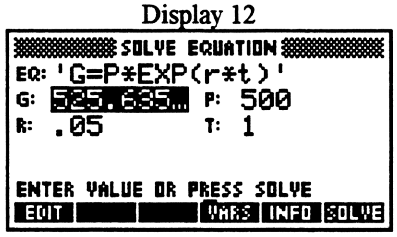
From the onscreen menu select ' CALC ' to obtain Display 11.

Display 11



We want the equation that is on line 2 of the stack so we need to swap it with line 1 by hitting [purple shift] ' SWAP '. Now select ' OK ' from the onscreen menu and the equation will be stored in EQ. The values for P, R, and T should be the same so highlight G and select ' SOLVE '.

Display 12



It appears that Gary will earn $.01 more than Heidi and only because the bank would round up.

ACTIVITY 3....RADIOACTIVE DECAY

In this activity we will be using the following equation as a model for continuous radioactive decay:

$$A(t) = Be^{rt}$$

where

$A(t)$ = Amount present after t years
$B = A(0)$
r = annual rate of decay
t = time in years

Exercise 1

One of the best examples of continuous decrease is radioactive decay. Carbon 14, also known as radiocarbon, is the radioactive isotope of carbon used in the dating of organic materials. It disintegrates at a very slow rate r ≈ .000124.

a) 23,000 years ago, an animal died. At that time, 4.3 mg of Carbon 14 located in its bones started to decay. If archaeologists dig up the remains of the animal today, how much Carbon 14 would remain?

b) Sometime in the distant future, aliens stumble upon earth. It is uninhabited but they find signs of a civilization that existed long ago. They find an ancient burial ground with stones bearing strange inscriptions marking the graves. They are able to decipher the inscriptions and find out that one of the beings died in what was know as the year 2245. They examine a bone that they know had approximately 7.5 mg of radiocarbon when the being died and it now has 2.9 mg. What year is it?

Exercise 2

Radiocarbon has a half-life of approximately 5730 years. This means that if B grams is the initial amount, $\frac{B}{2}$ will be the amount remaining after 5730 years.

a) Using this information, solve the radioactive decay equation to verify that the rate of decay of radiocarbon is ≈ .000124.

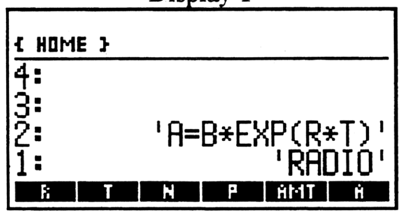
b) The skull of a prehistoric creature is found to have only 10% of the carbon 14 that bones of living creatures do. How old is the skull?

INSTRUCTIONAL SECTION

<u>Exercise 1</u>

First, input the equation and store it in "RADIO" as shown in Display 1.

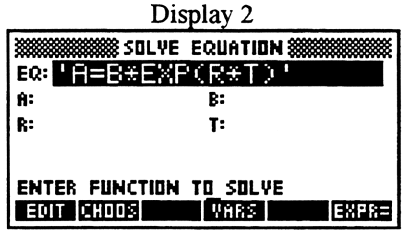
Display 1



Hit the [STO] key to store the equation.

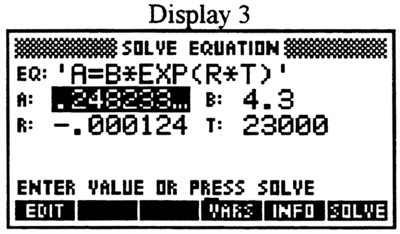
a) This is best done using the solving environment. Hit [green shift] ' SOLVE ' and select "Solve equation....". If there is already an equation there, input "RADIO" by selecting ' CHOOS ' from the onscreen menu.

Display 2



In this problem we want to find A. Since it is a decay problem, <u>the rate should be entered as a negative number</u>. When R, B, And T have been entered select ' CHOOS ' from the onscreen menu.

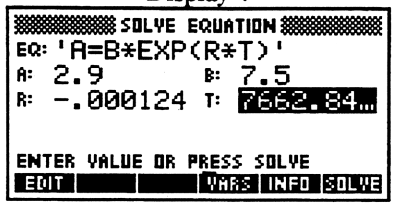
Display 3



It appears that ≈ .25 mg of carbon 14 remain.

b) When solving an equation of this type, the technology being used really shows its power. Without the solving capabilities of the HP 48G, this problem would have required the use of logarithms to find the solution. Now we simply input the values for A, R, and B and let the calculator perform the tedious computations.

Display 4



This problem is now solved with a much diminished chance for error. Students must still be able to analyze the problem, use the correct equation, identify the correct values for the variables, and then analyze the solution. A student who merely writes down 7662 instead of adding 7662 and 2245 isn't using the problem solving technique of "looking back" to make sure the solution provided is what they were looking for. Since a calculator was used to obtain the solution, this student should be evaluated far more critically than when technology wasn't being used to this extent.

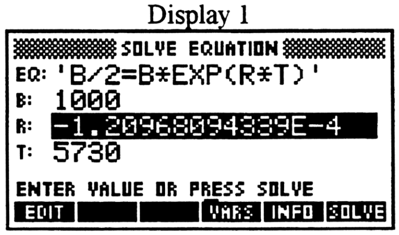
<u>Exercise 2</u>

a) The first thing we want to do is show that an arbitrary value for B can be chosen since it doesn't matter what the value is. So, using a little algebra and rewriting $\frac{B}{2}$ as $\frac{1}{2}B$ yields:

$$\frac{1}{2}B = Be^{rt} \rightarrow \frac{1}{2} = e^{rt} \rightarrow \ln\frac{1}{2} = rt \rightarrow -.6931 = 5730r \rightarrow r = -.0001209$$

Since the B cancels in step two, the rate will remain the same no matter what value is assigned to it. Now that the algebra has been done, verify our findings with the calculator. Input the equation and assign any value for B and 5730 for T and solve for R.

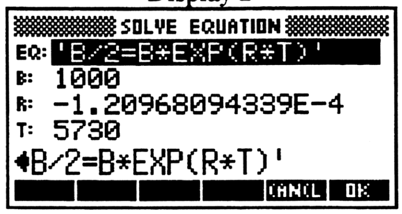
Display 1



Display 1 seems to verify the rate we are using is correct.

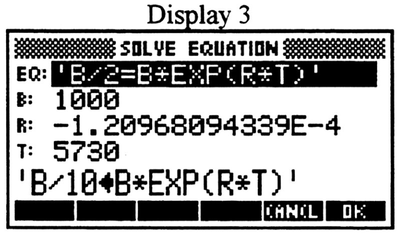
b) This problem is the same as the previous one except instead of $\frac{B}{2}$ we want to substitute $\frac{B}{10}$. This can be done easily by editing the old equation. With EQ highlighted select ' EDIT ' from the onscreen menu and we should be viewing Display 2.

Display 2



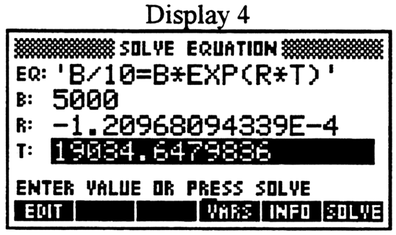
Use the right arrow [⊳] to put the blinking cursor on top the the "=" in the equation and then use the delete key [⇐] to delete the 2 and then key in 10.

Display 3



Select ' OK ' from the onscreen menu to return the edited equation to EQ. Pick any arbitrary value for B, leaving R as we solved for in the previous problem, and solve for T. It would be a good idea to have different students choose various values for B to help verify the results.

Display 4



This particular creatures remains are over 19,000 years old according to the T value shown in Display 4.

ACTIVITY 1....ANALYSIS OF SINGLE VARIABLES

The following chart shows the population and square mileage by county for the state of Oregon:

COUNTY	POPULATION	LAND AREA SQ. MILES	COUNTY	POPULATION	LAND AREA SQ. MILES
Baker	15317	3089	Lake	7186	8359
Benton	70811	679	Lane	282912	4620
Clackamas	278850	1870	Lincoln	38889	992
Clatsop	33301	873	Linn	91227	2296
Columbia	37557	687	Malheur	26038	9926
Coos	60273	1629	Marion	228483	1194
Crook	14111	2991	Morrow	7625	2094
Curry	19327	1648	Multnomah	583887	465
Deschutes	74958	3055	Polk	49541	741
Douglas	94649	5071	Sherman	1918	831
Gilliam	1717	1223	Tillamook	21570	1125
Grant	7853	4525	Umatilla	59249	3218
Harney	7060	10228	Union	23598	2038
Hood River	16903	533	Wallowa	6911	3150
Jackson	146389	2801	Wasco	21683	2396
Jefferson	13676	1791	Washington	311554	727
Josephine	62649	1640	Wheeler	1396	1713
Klamath	57702	6135	Yamhill	65551	718

Enter the data into a 36 x 2 matrix with the population in column 1 and the land area in column 2. Store it in OREG.

Exercise 1

a) What is the average population of a county in Oregon? What is the average area?

b) How many people live in Oregon? What is the total area?

c) What is the largest county in Oregon with respect to population? Area?

d) What is the smallest county in Oregon with respect to population? Area?

Exercise 2

On your calculator, create bar graphs showing the population and area of each county.

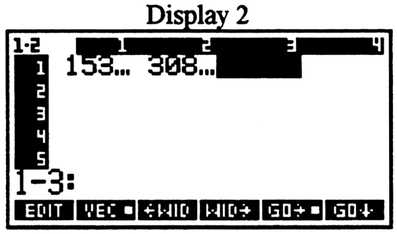
INSTRUCTIONAL SECTION

To enter the matrix hit [green shift] ' MATRIX '.

Display 1

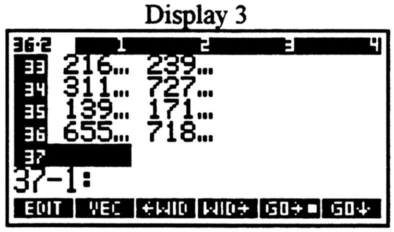
Make sure that the the selection ' $GO \rightarrow \square$ ' from the onscreen menu is activated as shown in Display 1. This tells the calculator to enter the data by rows rather than columns. Key in the population for Baker county and hit [ENTER] to store the value in cell 1-1. Key in the area and hit [ENTER] to store the value in cell 1-2.

Display 2



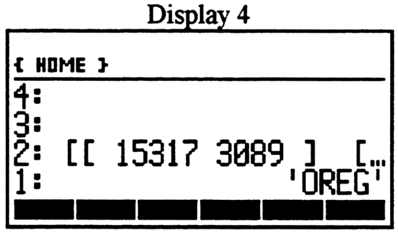
With cell 1-3 highlighted as shown, hit the down arrow key [∇] once to send the highlight to cell 2-1. The matrix writer now knows that this is going to be an n x 2 matrix so there will be no need to use the arrow keys. Key in the rest of the data hitting [ENTER] after each entry until all 36 rows have been completed.

Display 3



With cell 37-1 highlighted as shown, hit [ENTER] again to let the matrix writer know we are done entering data. The matrix will be put on line 1 of the stack. Type in 'OREG' and enter it on line 1as shown in Display 4 .

Display 4

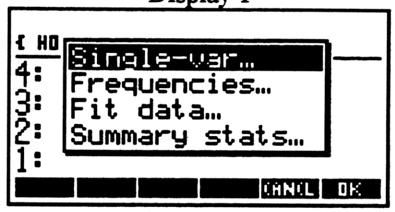


Hit [STO] to store the matrix in OREG.

Exercise 1

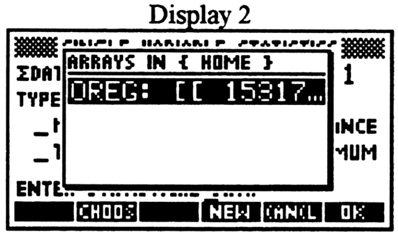
a) From the home screen, hit [green shift] ' STAT ' to obtain Display 1.

Display 1



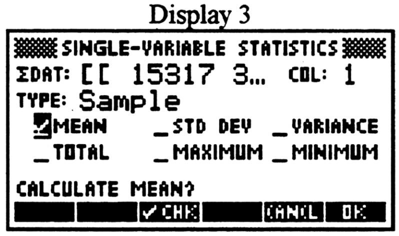
With "Single-var..." highlighted, select ' OK ' from the onscreen menu. We need to store OREG into the $\sum DAT$ variable, so with the $\sum DAT$ field highlighted, select ' CHOOS ' from the onscreen menu.

Display 2



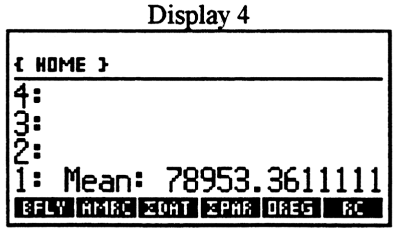
With OREG highlighted, select ' OK ' to store the matrix and return to the SINGLE-VARIABLE STATISTICS environment. Population is in column 1 so highlight COL and key in a 1 if necessary. We want the average so, with the cursor to the left of MEAN, select ' √ CHK ' from the onscreen menu. We should be viewing Display 3.

Display 3



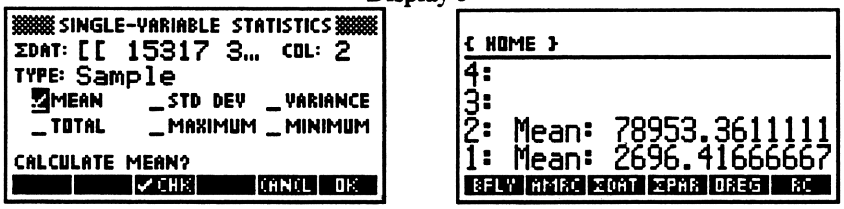
The TYPE is either "Sample" or "Population" and only influences the standard deviation and variance values and then only minimally. We will be using "Sample" for all three of the activities in this unit. Select ' OK ' from the onscreen menu in Display 3 to return the mean to the stack along with a label as shown in Display 4.

Display 4



To find the average area, hit [green shift] ' STAT ' and choose "Single-var..." to return to the SINGLE-VARIABLE STATISTICS environment. Change COL to 2, place a √ next to MEAN and select ' OK ' from the onscreen menu.

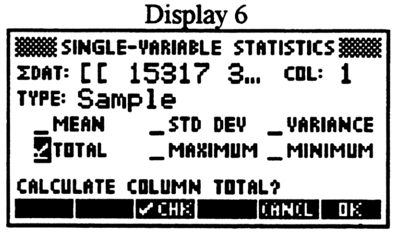
Display 5



The average population is now on line 2 and the average area on line 1 of the stack.

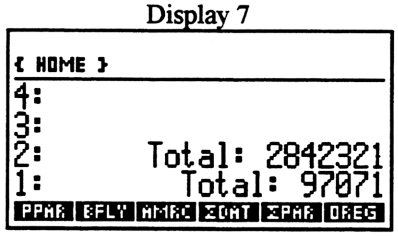
b) This is also done in the SINGLE-VARIABLE STATISTICS environment. Follow the procedure described in part a and set up the screen to look like Display 6.

Display 6



Select ' OK ' from the onscreen menu to send the total of column 1, which is the population, to the stack. Repeat the procedure to total column 2.

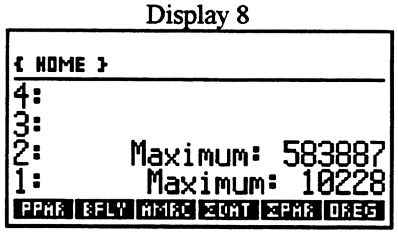
Display 7



The total population is shown on line 2 and the total area on line 1 of the stack.

c) This is done in the same manner as a and b except the √ mark will be to the left of MAXIMUM.

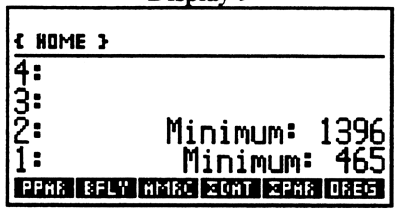
Display 8



The maximum population is on line 2 and the maximum area is on line 1.

d) Repeat the same procedure with the √ mark next to MINIMUM.

Display 9

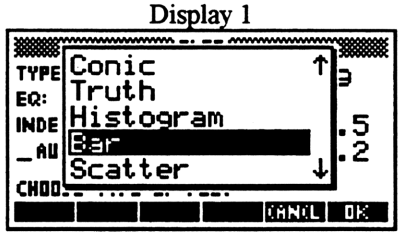


Once again, population is on line 2 and area on line 1.

Exercise 2

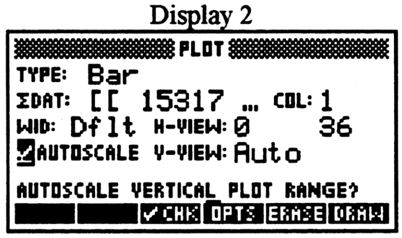
To create a bar graph, hit [green shift] ' PLOT ' to go to the plotting environment. With the TYPE field highlighted, select ' CHOOS ' from the onscreen menu and use the down arrow key to move the cursor down until "Bar" is highlighted as shown in Display 1.

Display 1



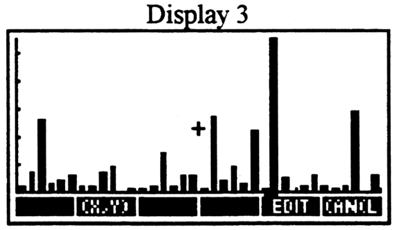
Select ' OK ' from the onscreen menu. Set the PLOT screen to look like Display 2.

Display 2



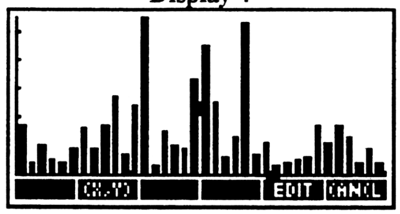
This menu tells us that we will be plotting a bar graph using the information in column 1 of the matrix stored in . With the AUTOSCALE option selected as shown, the value in H–VIEW is ignored and the number of rows in the matrix is automatically used, therefore it is not necessary to enter 0 and 36 as shown in Display 2. The V–VIEW is also automatically selected to plot the entire graph in the viewing window. From the onscreen menu in Display 2, select ' ERASE ' to delete any old pictures and then ' DRAW ' to plot the new graph.

Display 3



Hit [ON] to return to the PLOT environment and change COL to 2. We will need to re-select AUTOSCALE and then select ' ERASE ' and ' DRAW ' from the onscreen menu.

Display 4



ACTIVITY 2...STANDARD DEVIATION, FREQUENCIES, AND CORRELATION

The following chart shows some final team statistics for the American League in 1992:

TEAM	WINS	RUNS	HITS	HOMERS	RBI
Minnesota	90	747	1544	104	701
Milwaukee	92	740	1477	82	683
Cleveland	76	674	1495	127	637
Seattle	64	679	1466	149	638
Toronto	96	780	1458	163	737
New York	76	733	1462	163	703
Chicago	86	738	1434	110	686
Baltimore	89	705	1423	148	680
Oakland	96	745	1389	142	693
Kansas City	72	610	1411	75	568
Detroit	75	791	1411	182	746
Texas	77	682	1387	159	646
Boston	73	599	1343	84	567
California	72	579	1306	88	537

Enter the data into a 14 x 5 matrix and store it under AMRCN.

Exercise 1

Calculate the mean and standard deviation for the RBI column.

a) How many teams were within one standard deviation of the mean?

b) How many teams were below one standard deviation?

c) How many teams were above one standard deviation?

Exercise 2

On your calculator, make a histogram for the number of home runs for each team using bar widths of 25 homers. Use $\{70 \le x \le 195\}$ and $\{-1 \le y \le 6\}$.

a) How many teams hit between 95 and 120 homers?

b) What range do most of the teams fall into?

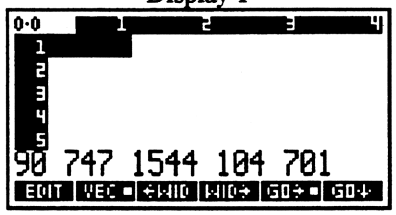
Exercise 3

Which hitting statistic has the most positive influence on winning? Which hitting statistic has the least to do with winning?

INSTRUCTIONAL SECTION

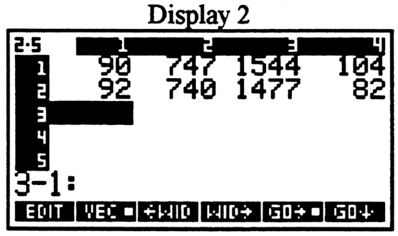
When entering matrices with many rows, the [SPC] key can be used to save some time. Hit [green shift] ' MATRIX ' to open the Matrix Writer. With cell 1-1 highlighted, enter the first row of the matrix on the command line as shown in Display 1.

Display 1

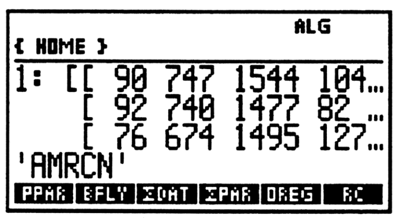


Hit [ENTER] and the entire row will be inserted into the matrix. Hit the down arrow to let the Matrix Writer know we are done with row 1 and the cursor will move to cell 2-1. Enter the values for row two in the same manner and hit [ENTER]. As seen in Display 2, the cursor will now move to cell 1 of the subsequent row.

Display 2



Large arrays of numbers can be entered quickly and accurately using this method. After entering all 14 rows, hit [ENTER] to send the matrix to the stack and type in 'AMRCN' on the command line as shown in Display 3.

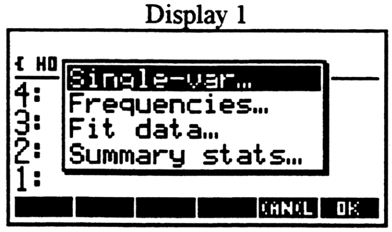


Hit [STO] to store the matrix.

Exercise 1

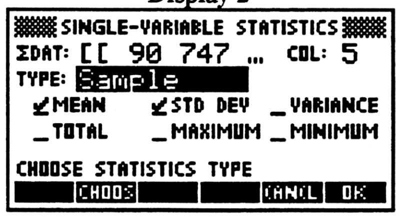
Hit [green shift] ' STAT ' to obtain Display 1.

Display 1



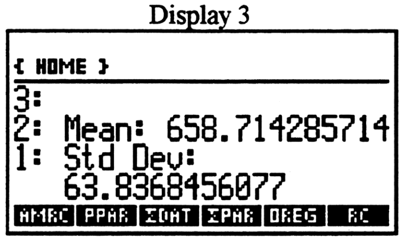
With "Single-var..." highlighted select ' OK ' from the onscreen menu to get to the SINGLE-VARIABLE STATISTICS environment. With the $\sum DAT$ field highlighted, select ' CHOOS ' from the onscreen menu. Highlight AMRCN and select ' OK ' from the onscreen menu to store the matrix in $\sum DAT$. Set the SINGLE-VARIABLE STATISTICS screen to look like Display 2.

Display 2



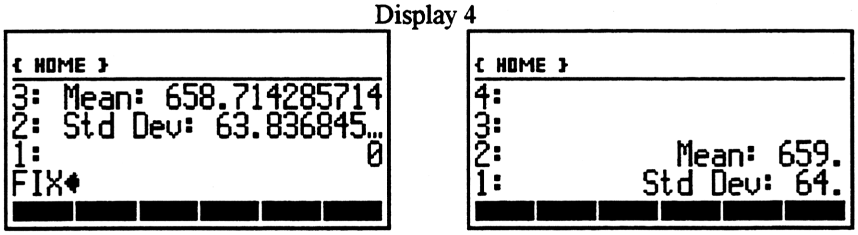
Select ' OK ' from the onscreen menu and both the mean and standard deviation for column 5 will be sent to the stack and labeled.

Display 3



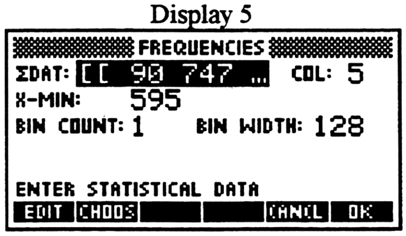
We only want to be dealing with whole numbers in this exercise so lets fix the number of decimal places to 0. This can be done in ' MODES ' or directly on the home screen. To quickly set the calculator so that all values are rounded to the nearest whole number, enter a 0 onto the stack and type in the word FIX <u>without tick marks</u> on the command line as shown in Display 4. Hit [ENTER] to set the number of decimal places at 0.

Display 4



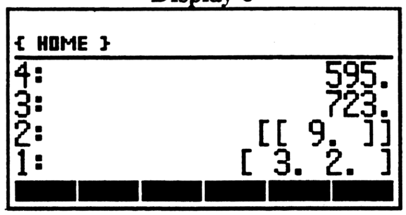
a) Hit [green shift] ' STAT ' and highlight "Frequencies". Select ' OK ' to go to the FREQUENCIES environment. $\sum DAT$ should still contain AMRCN. If it doesn't, use ' CHOOS ' to store it in the same manner as before. Set the FREQUENCIES screen to look like Display 5.

Display 5



This information tells the calculator to look at the elements in column five and find out how many are between 525 and 525+128 (128 = 2 standard deviations). The BIN COUNT is 1 because we only want one subdivision of the data points. Select ' OK ' to obtain Display 6.

Display 6



We now have all the information we need to answer parts a, b, and c! On line two of the stack is an array of integer elements that tell us how many data points are in each bin. We only had one bin and 9 points were in it so nine teams were within one standard deviation of the mean (a). Line 1 of the stack is a two-element vector that represents the number of outliers. The first value represents the number of outliers below the lowest bin and the second value represents the number of outliers above the highest bin. Therefore, there were 3 teams below one standard deviation (b) and two teams were above one standard deviation (c).

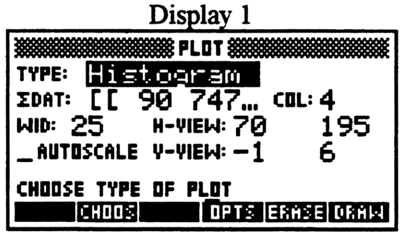
b) See above.

c) See above.

Exercise 2

Hit [green shift] ' PLOT ' to go to the PLOT environment. We want to do a histogram so we need to change TYPE. An alternative to using the ' CHOOS ' option from the onscreen menu is to highlight the TYPE field and hit [α] ' H '. Since histogram is the only plotting option that begins with the letter H, it will be selected and stored in TYPE. Set the PLOT screen to look like Display 1.

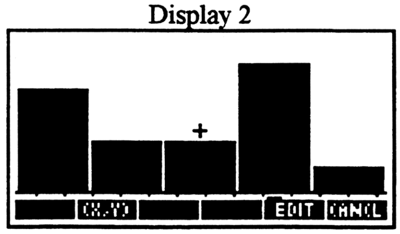
Display 1



Note: It is not wise to use the autoscaling option when making a histogram since the calculator will alter the H–VIEW value to match the lowest and highest data points in the column.

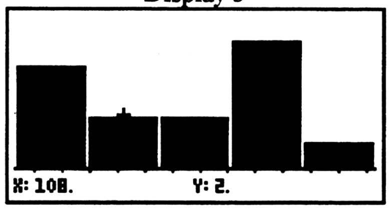
Select ' ERASE ' and ' DRAW ' from the onscreen menu to plot the histogram.

Display 2



a) To find the number of teams that hit between 95 and 120 homers we need to know which bar corresponds to the specified range. Select ' (X,Y) ' from the onscreen menu to view the coordinates of the crosshairs. Using the arrows, move the crosshairs until a bar with x-values between 95 and 120 is located. Place the crosshairs anyplace on top of the bar and read the y-value as shown in Display 3.

Display 3



It appears that 2 teams fall within this range.

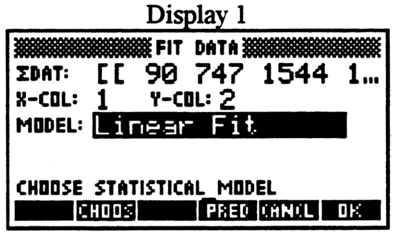
b) Move the crosshairs to the upper left-hand corner of the tallest bar and take a reading of the x-value. Move the crosshairs to the upper right-hand corner of the bar and take another reading. Most of the teams fall into the 145-170 range.

Exercise 3

The first order of business needs to be to change the number of decimal places from 0 to 3. Enter 3 on line 1 of the stack and type in FIX without the ticks. Hit [ENTER] to fix the decimal to three places.

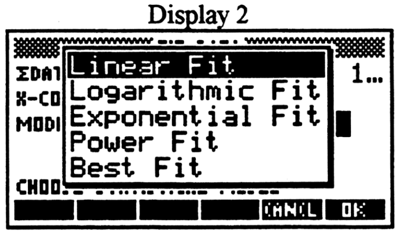
Hit [green shift] ' STAT ' and highlight "Fit data...". Select ' OK ' to go to the FIT DATA environment. We want to compare the win column to the other four and check out the various correlations. Set the FIT DATA screen to look like Display 1.

Display 1



This will allow us to find the correlation between wins (column 1) and runs (column 2) using a linear fit as a model. Display 2 shows the different models to choose from.

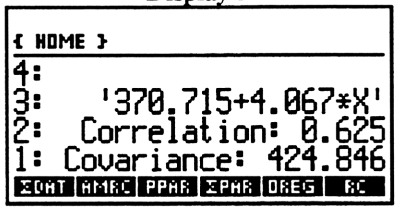
Display 2



In this case, using the linear fit makes the most sense since it is unlikely that wins would increase logarithmically or exponentially with runs. This can be verified by selecting the "Best Fit" option which will return a linear fit in this instance.

From the onscreen menu shown in Display 1, select ' OK '.

Display 3

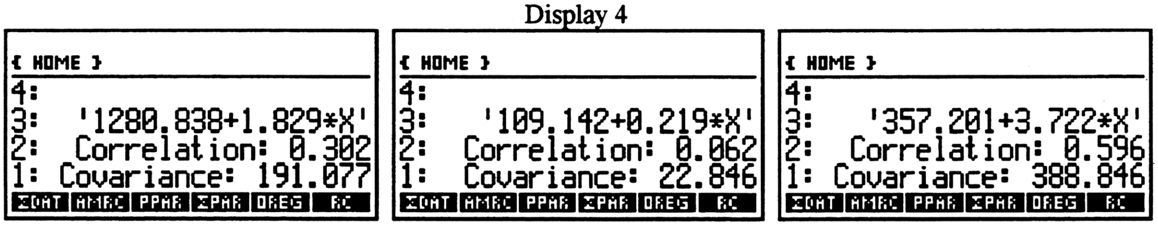


In Display 3, line 3 of the stack shows the equation of the line that represents the best "fit". We will be dealing more with this in Activity 3. Line 2 is the line we are interested in. It shows the correlation coefficient. The correlation is a number between -1 and 1 that gives us an idea of just how good of a "fit" we have obtained. The closer to 1 or -1, the better the fit. The closer to 0, the worse the fit. The correlation between wins and runs with respect to our data points is .625.

Line 1 of the stack shows the covariance which we won't be concerning ourselves with in this unit.

Display 4 shows the correlation coefficients for hits, homers and rbi's respectively. These are obtained by returning to the FIT DATA screen and changing Y–COL to the appropriate values.

Display 4



It appears, based on our data, that the number of runs scored has the most positive influence on winning since it has a correlation coefficient closest to 1.

Since the screen corresponding to homers in Display 4 shows a correlation very close to 0, we can conclude, based on our data, that home runs have almost nothing to do with the amount of games a team wins in a season.

ACTIVITY 3...SCATTER PLOTS AND LINEAR REGRESSION

The following chart shows the winning times for the Olympic women's 100 meter butterfly since the inception of the event in 1956:

YEAR	GOLD MEDAL WINNER	COUNTRY	WINNING TIME
1956	Shelley Mann	United States	1:11.0
1960	Carolyn Schuler	United States	1:09.5
1964	Sharon Stouder	United States	1:04.7
1968	Lynn McClements	Australia	1:05.5
1972	Mayumi Aoki	Japan	1:03.34
1976	Kornelia Ender	East Germany	1:00.13
1980	Caren Metschuck	East Germany	1:00.42
1984	Mary T. Meagher	United States	59.26
1988	Kristin Otto	East Germany	59.00
1992	Qian Hong	Japan	58.62

Enter the data into a 10 x 2 matrix with the year in column 1 and the winning time in column 2. Let 1956 correspond to 1, 1960 to 2, 1964 to 3, ..., and 1992 to 10. Enter the winning times in seconds. Store it in BFLY.

Exercise 1

On your calculator, make a scatter plot of the data.

a) Create separate graphs using linear, exponential, logarithmic, and power fits superimposed on top of the scatter plot.

b) Visually estimate which fit yields the best representation.

c) Use the calculator to find the "best fit". What is the correlation for this function?

d) Discuss why this particular model is the best fit of the four.

Exercise 2

Use the model that provided the "best fit" to answer the following questions.

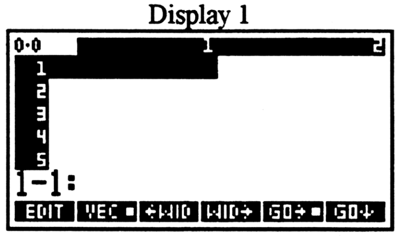
a) The world record in this event is 57.93 seconds by Mary T. Meagher. According to our model, will her record be broken in the next Olympiad?

b) According to our model, in what year will the 56 second barrier be broken?

INSTRUCTIONAL SECTION

Enter the matrix using the year and winning times as the two columns. For our purposes, it will be advantageous to enter the year in numerical order according to Olympiad (e.g., 1956 is 1, 1960 is 2, etc.), and the winning time in seconds. Fix the decimal to two places and hit [green shift] ' MATRIX ' to go to the Matrix Writer. Since we are only going to have two columns it would be nice to be able to see the entire number when it occupies a cell instead of having them truncated. From the onscreen menu select ' *WID* →' two times so that only two columns are seen as shown in Display 1.

Display 1

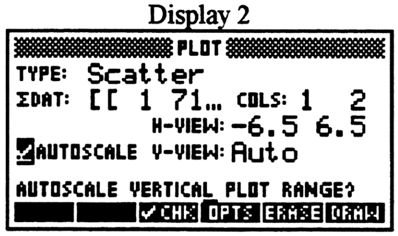


Enter the matrix, send it to the stack, and type in 'BFLY' on the command line. Hit [STO] to store the matrix. We will be using decimals rounded to the nearest thousandth in this activity so fix the number of decimal places to 3.

Exercise 1

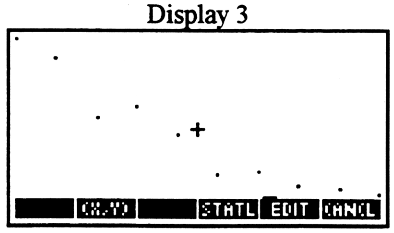
Make the scatter plot by hitting [green shift] ' PLOT ' to go to the plotting environment. Highlight TYPE and hit [α] ' S '. The "Scatter" PLOT screen is the only one that begins with "S" so it will be the one selected. We want to work with the BFLY matrix so it needs to be stored in $\sum DAT$ in the same manner shown in previous activities. Set the PLOT screen to look like Display 2.

Display 2



The AUTOSCALE feature is very handy when doing scattergrams. The screen dimensions will be chosen automatically to include all data points. Select ' ERASE ' and ' DRAW ' to plot the scattergram.

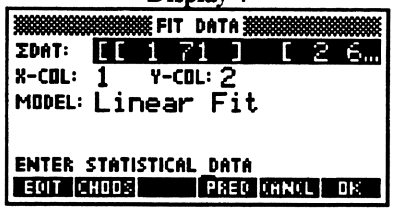
Display 3



As shown in Display 3, all 10 data points are plotted between the top of the screen and the onscreen menu with no wasted space.

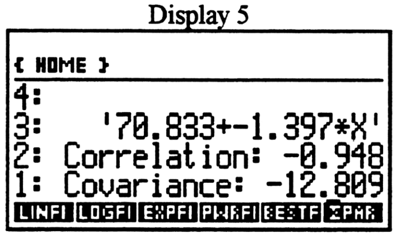
a) We need to return to the FIT DATA screen to specify which fit will be used by the calculator. Hit [ON] twice to return to the home screen and then [green shift] ' STAT '. Highlight "Fit data..." and select ' OK ' from the onscreen menu.

Display 4



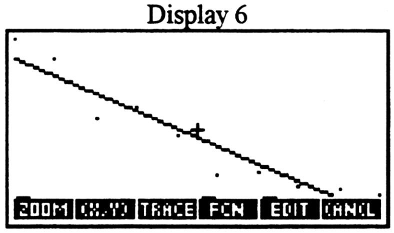
Linear Fit is the default choice for MODEL and is the one we will be using first. If it is necessary to change it, do so by highlighting MODEL and using the ' CHOOS ' option from the onscreen menu. From the onscreen menu shown in Display 4, select ' OK '.

Display 5



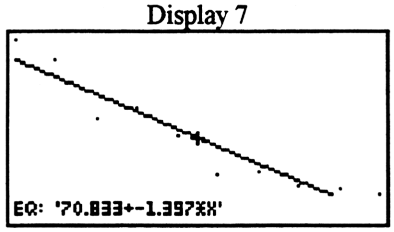
Unfortunately, the only way to get back to the plotting enviroment is through the home screen which is going to reveal the correlations. Ignore this screen and hit [purple shift] ' PICTURE '. Then select ' STATL ' from the onscreen menu to obtain Display 6. <u>NOTE</u>: If the statistics line isn't drawn, check the PLOT OPTIONS menu in the FUNCTION mode. CONNECTED needs to be checked and STEP needs to be at the default.

Display 6



To see the equation of the statistics line, select ' FCN ' from the onscreen menu, then hit the [NXT] key to scroll the onscreen menu, and then select ' NXEQ '.

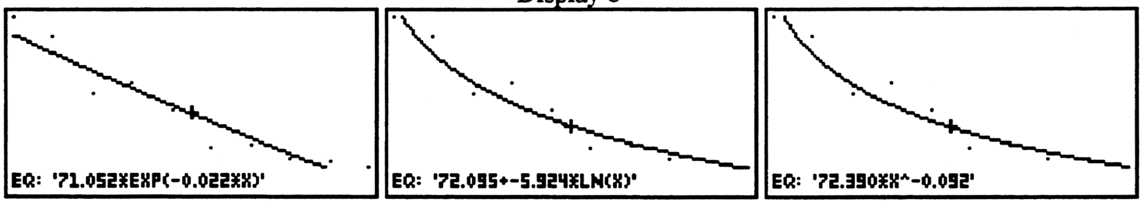
Display 7



Now go back to the FIT DATA screen and repeat the procedure for exponential, logarithmic, and power fits. We will need to replot the scattergram using [green shift] ' PLOT ' in order to erase the linear fit rather than using the [purple shift] ' PICTURE ' method to go to the graph. However, this method can still be used when the fits are to be drawn on the same screen.

Display 8 shows exponential, logarithmic, and power fits as well as their equations.

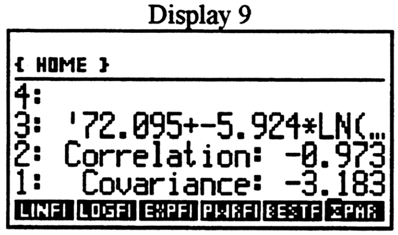
Display 8



b) It's too close to call between logarithmic and power. If students were paying attention to the correlations they already know what the "best fit" is going to be.

c) Go to the FIT DATA screen and choose "Best Fit" for MODEL. Select ' OK ' to send the data to the stack.

Display 9



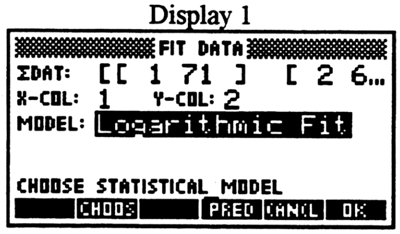
It appears the logarithmic fit is the best one with a correlation of –.973. The correlation for the power fit turns out to be –.971 which is why the two were so difficult to separate visually.

d) Now is a good time for a class discussion as to why linear fits are good regression models for some sets of data but not others.

Exercise 2

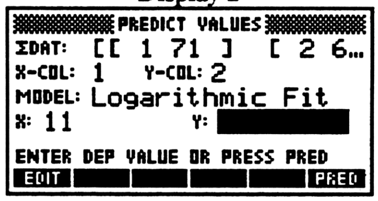
a) Go to the FIT DATA screen in the ' STAT ' environment and set the screen to look like Display 1.

Display 1



Select ' PRED ' from the onscreen menu to view the PREDICT VALUE screen. As shown in Display 2, enter 11 for X and move the highlight to Y.

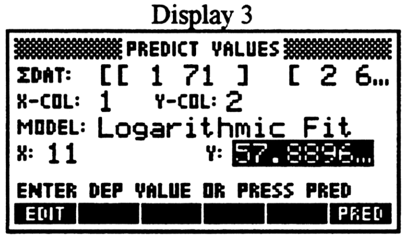
Display 2



Since we input the corresponding Olympiads in numerical order into column 1 and the last number entered was 10, 11 will correspond to the next one. Display 2 shows that X-COL is 1 which means the value in X represents data points in that column.

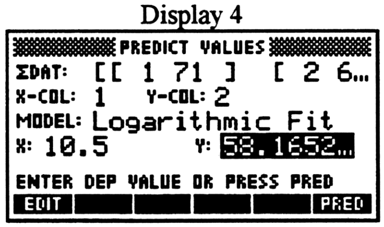
We want to know, based on our model, what the point in column 2 should be when the point in column 1 is 11. Highlight Y as shown in Display 2 and select ' PRED ' from the onscreen menu.

Display 3



Our model predicts that the record will fall in 1996. BUT HOLD IT!! The olympic format is changing so that the summer and winter olympics alternate every two years. The next summer olympics will be held in 1994! We really need to use 10.5 for X to predict what is going to happen in the <u>next</u> summer olympics.

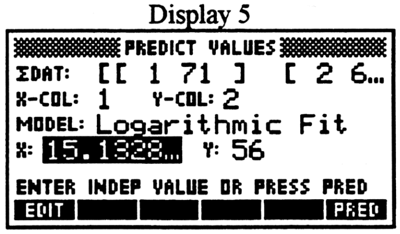
Display 4



Our model predicts it will be broken, but not by much!

b) Enter 56 for Y, highlight X and select ' PRED '.

Display 5



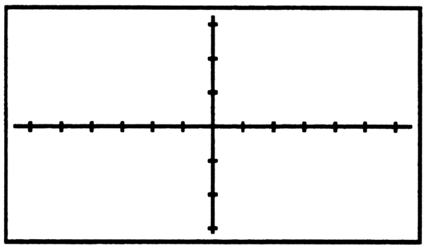
Display 5 tells us that when the data point in column 2 is 56, column 1 should be ≈ 15.1. Be careful not to use this value as a solution. Each increment of 1 represent 4 years so the 56 second barrier should be broken a little over 20 years from 1992 in the year 2012.

ACTIVITY 1...LIMITS AND CONTINUITY

Exercise 1

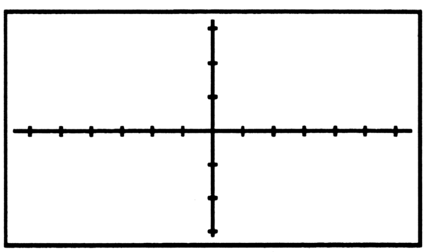
For each function, graph over the intervals $\{-6.5 \le x \le 6.5\}$ and $\{-3.1 \le y \le 3.2\}$ and visually estimate the left and right-hand limits as $x \to 0$. Create a table of values to justify your answer numerically.

a) $f(x) = \dfrac{1-\cos x}{x}$



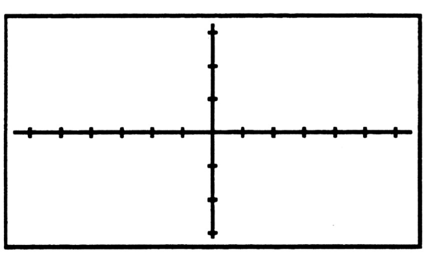
x	f(x)	x	f(x)

b) $g(x) = \dfrac{|x|}{x}$



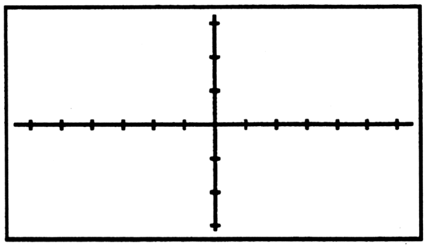
x	g(x)	x	g(x)

c) $h(x) = \sqrt{x}$



x	h(x)	x	h(x)

d) $j(x) = \arctan\frac{1}{x}$



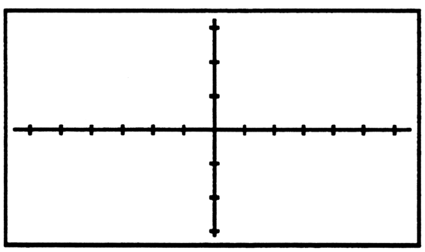
x	j(x)

x	j(x)

Exercise 2

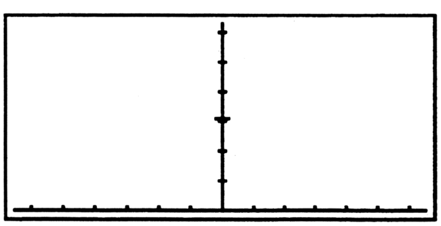
Graph each function using the designated screen parameters and identify the intervals over which the graph is continuous. At all points of discontinuity, state whether it is a hole, jump, pole, or oscillator.

a) $f(x) = \frac{1}{x^2 + 2x - 3}$ $\{-6.5 \leq x \leq 6.5\}$ $\{-3.1 \leq y \leq 3.2\}$



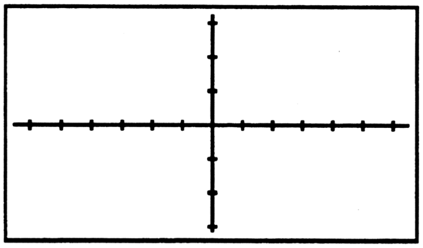
x	discontinuity

b) $g(x) = 1 - \cos\frac{1}{x}$ $\{-.5 \leq x \leq .5\}$ $\{0 \leq y \leq 1\}$



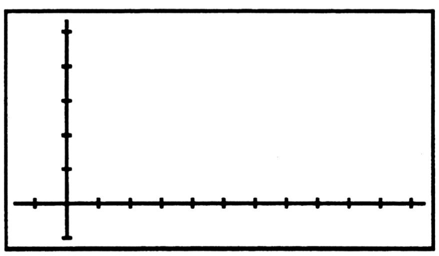
x	discontinuity

c) $h(x)=\begin{cases} |x| & \text{if } x \le 1 \\ \dfrac{1}{x^2-16} & \text{if } x > 1 \end{cases}$ $\{-6.5 \le x \le 6.5\}$ $\{-3.1 \le y \le 3.2\}$



x	discontinuity

d) $j(x)=\dfrac{x^2-5x+6}{x^2-6x+8}$ $\{-6.5 \le x \le 6.5\}$ $\{-3.1 \le y \le 3.2\}$



x	discontinuity

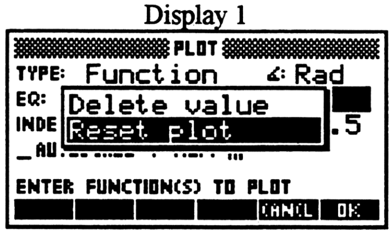
INSTRUCTIONAL SECTION

<u>Exercise 1</u>

In this activity we want the decimal places to be displayed in standard form without any rounding. To make sure this is the case, type STD <u>without</u> tick marks on the command line and hit [ENTER]. The calculator will now display all decimals in standard form.

a) Hit [green shift] ' PLOT ' to go to the plotting environment. Hit [NXT] and then select ' RESET ' from the onscreen menu.

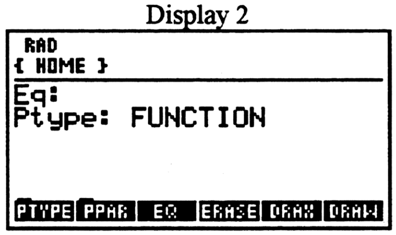
Display 1



Use the down arrow to highlight "Reset plot", as shown in Display 1, and select ' OK ' from the onscreen menu. This will reset all of the plot options to the default values. Since we are going to be using the same viewing window and plot options for all of the exercises, we will be using a "hidden" feature of the calculator that allows us to plot graphs more quickly.

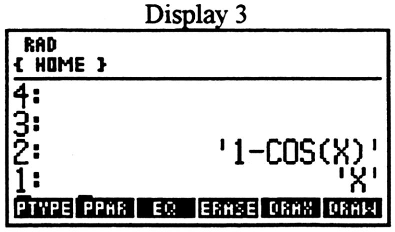
Hit [ON] to return to the home screen and then hit [purple shift] ' PLOT ' to go to the "dark side" of the plotting environment as it is affectionately known.

Display 2



Instead of fiddling around with parentheses, we can use the stack to our advantage. Type in the numerator and hit [ENTER] then type in the denominator and hit [ENTER] to obtain Display 3.

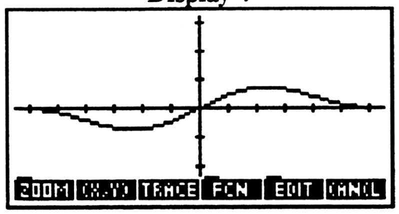
Display 3



Hit [÷] and $f(x)$ will be on line 1 of the stack.

Hit ['] and select ' EQ ' from the onscreen menu and 'EQ' will appear on the command line. Now hit [STO] to store $f(x)$ in EQ. From the onscreen menu select 'ERASE ' , to delete any old graphs, ' DRAX ' to draw in the axes, and then ' DRAW ' to plot $f(x)$.

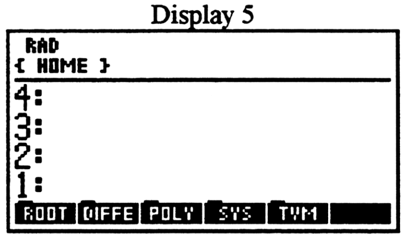
Display 4



From Display 4, it appears that $f(x)$ is approaching 0 as x approaches 0.

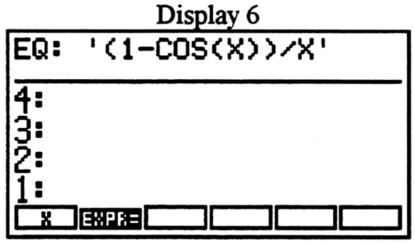
To create the table of values, we will using a hidden feature of the calculator called the numeric root finder. Hit [ON] twice to return to the home screen and hit [purple shift] ' SOLVE ' to obtain Display 5.

Display 5



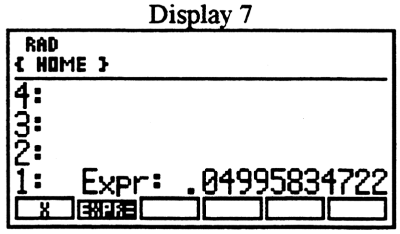
From the onscreen menu, select ' ROOT ' and then ' SOLVR ' to get to the numeric root finder shown in Display 6.

Display 6



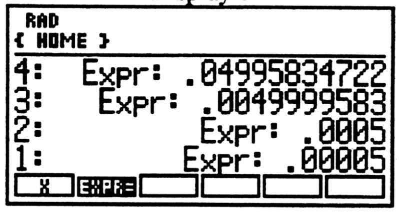
The equation which we stored in EQ appears at the top of the screen. We want to see what happens as x gets close to 0 from the positive side. Key in .1 on the command line and select ' X ' from the onscreen menu to store the value. Select ' EXPR= ' to evaluate EQ as shown in Display 7.

Display 7



Repeat the procedure using .01, .001, and .0001 for X. Leave all four outputs on the stack as shown in Display 8.

Display 8

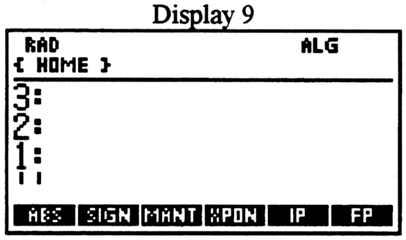


The x-values along with the corresponding $f(x)$ values should be entered into one of the tables provided. Repeat the procedure using negative values of X that approach 0 and enter the information in the other table.

It should be apparent that as x approaches 0 from the right and left side, $f(x)$ also approaches 0.

b) Hit [purple shift] ' CLEAR ' to clear the screen and then [purple shift] ' PLOT '. Hit ['] to start the string and then [MATH] ' REAL ' [NXT] to obtain Display 9.

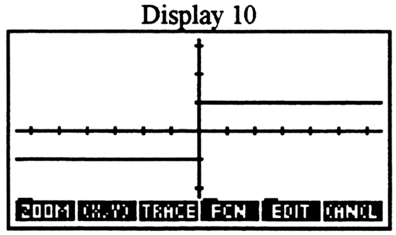
Display 9



Select ' ABS ' from the onscreen menu and then [α] ' X '. Hit [ENTER] to place it on the stack. Key in an X and enter it. Once again, the numerator is now on line 2 and the denominator on line 1. Hit [÷] to obtain $g(x)$.

To store the equation, hit [purple shift] ' PLOT ' to get the correct onscreen menu and then [ON] to view the stack. Store the equation in EQ. From the onscreen menu, select ' ERASE ', ' DRAX ' and ' DRAW '.

Display 10

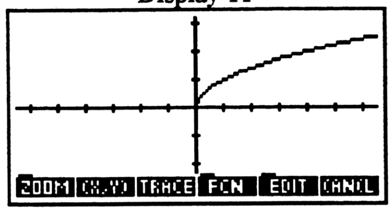


Display 10 shows that the limit appears to be −1 as X approaches 0 from the left, and 1 as it approaches 0 from the right.

Use the numeric root finder as we did in part a to make tables that verify this visual estimate.

c) Enter the equation on the stack by putting an X on line 1 and hitting [$\sqrt{x}$]. Store the equation and graph it as we did in parts a and b. Display 11 shows the graph.

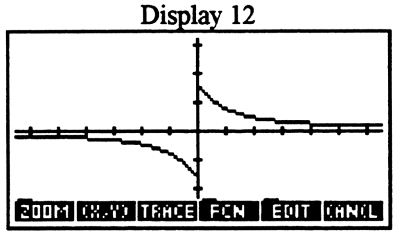
Display 11



The function is only defined for positive values of X so the left-hand limit does not exist and the right-hand limit is 0.

d) Enter the equation on the stack by entering '1/X' and then hitting [purple shift] ' ATAN '. Display 12 shows the graph.

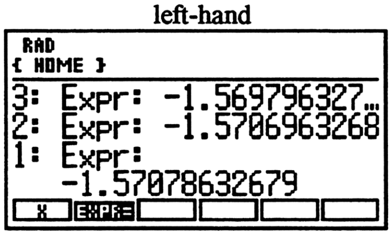
Display 12



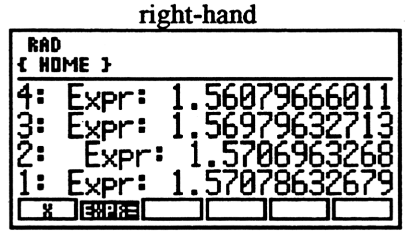
It appears the right and left-hand limits are approximately 1.5 and −1.5 respectively. The numeric root finder becomes very useful in instances like this. Display 13 shows the numeric result

Display 13

left-hand



right-hand

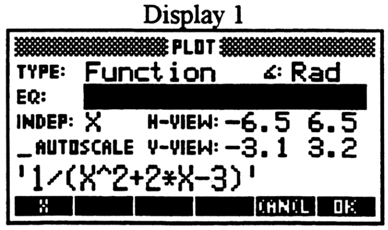


It appears that our visual estimate was close, but not quite as exact as the numerical justification.

Exercise 2

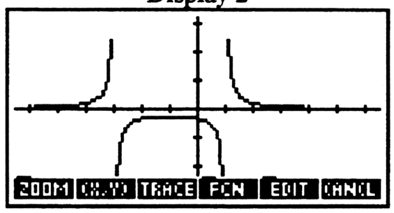
a) The screen parameters will be changing in this exercise so it will be best to do our graphing in the regular PLOT environment. Hit [green shift] ' PLOT ' to go to the PLOT screen. With EQ highlighted type in $f(x)$. Do not use the tick marks, they will be inserted automatically.

Display 1



From the onscreen menu shown in Display 1, select ' OK ' to enter the equation in EQ. Make sure TYPE, ∠, INDEP, H–VIEW, and V–VIEW are as shown. Select ' ERASE ' and ' DRAW ' from the onscreen menu to plot the graph.

Display 2

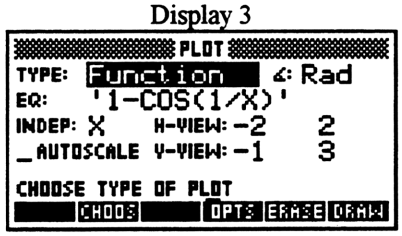


By factoring the denominator we can see that –3 and 1 cannot be members of the domain. Therefore we have two pole discontinuities at these values. The graph is continuous over the intervals:

$$(-\infty,-3) \cup (-3,1) \cup (1,\infty)$$

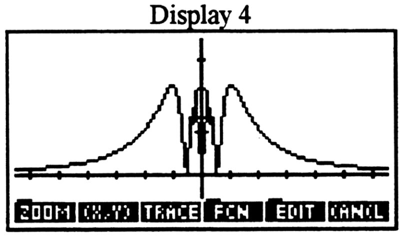
b) Return to the PLOT screen and set it to look like Display 3. Make sure that ∠ is in radians!

Display 3



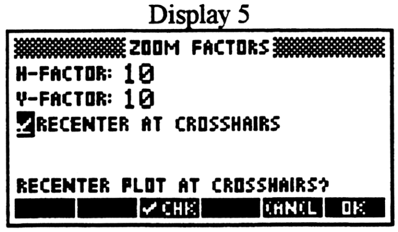
Select ' ERASE ' and ' DRAW ' from the onscreen menu.

Display 4



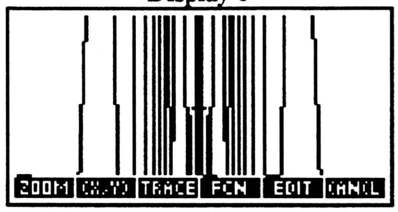
This is a rather strange looking function. The discontinuity occurs as X approaches 0 from the left and right and is known as an oscillating discontinuity. The behavior of the function becomes more apparent if we zoom in. To do so, select ' ZOOM ' from the onscreen menu and ' ZFACT ' from the subsequent onscreen menu to obtain the ZOOM FACTORS screen.

Display 5



Set the parameters to those shown in Display 5 and put a √ next to the RECENTER AT CROSSHAIRS option. From the onscreen menu select ' OK ' and then ' ZIN '.

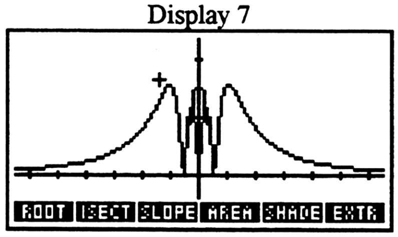
Display 6



Display 6 illustrates the erratic behavior of the function around 0. Going to the numeric root finder and investigating various values that approach 0 will substantiate the visual evidence.

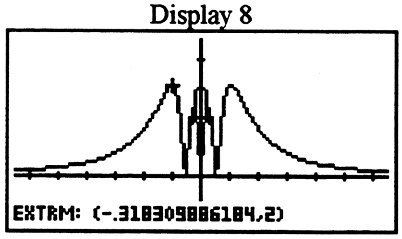
The graph shown in Display 4 would suggest that the function is continuous up until a certain point. That point seems to correspond to the local maximums to the left and right of the y-axis. Replot the graph to get Display 4 and select ' FCN ' from the onscreen menu. Use the arrow keys to move the crosshairs close the the local maximum on the left as shown in Display 7.

Display 7



Select ' EXTR ' from the onscreen menu and the crosshairs will move to the local extremum and the coordinates will be displayed.

Display 8



The X-value where the graph starts to behave erratically is approximately −.318. Using the same procedure for the local maximum on the right gives us a value of .318. It appears that there is some symmetry here with respect to the y-axis. The intervals over which the graph is continuous are:

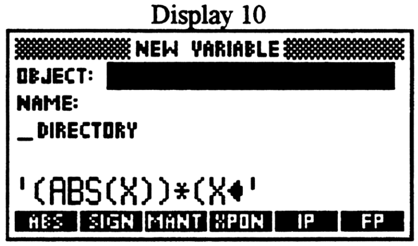
$$(-\infty, -3.18] \cup [3.18, \infty)$$

c) This is a piecewise function and it can be graphed on the 48G. Go to the PLOT screen and, with EQ highlighted, select ' CHOOSE ' from the onscreen menu. Instead of storing a function in HOME, we want to write a new equation, so select ' NEW ' from the onscreen menu.

Display 9

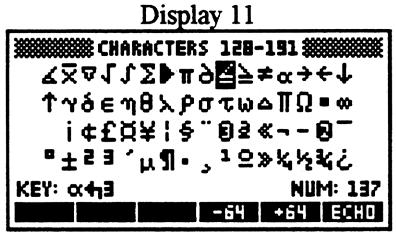
Using the ['], enter the top equation by hitting [( )] [MTH] ' REAL ' [NXT] 'ABS ' [α] X. Now hit the right arrow twice to move the cursor outside both sets of parentheses. Now hit [X] [( )] [α] X to obtain Display 10.

Display 10



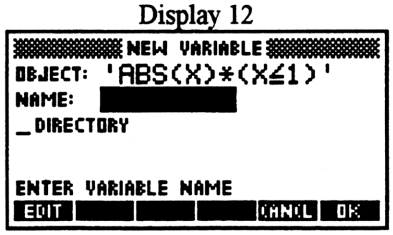
We are now ready to tell the function the domain over which $|x|$ is to be graphed. To do this hit [green shift] ' CHARS ' and use the right arrow to highlight the ≤ sign as shown in Display 11.

Display 11



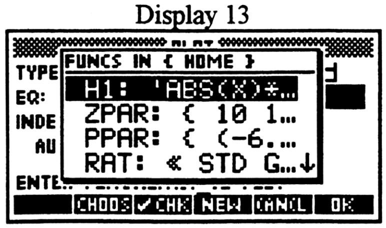
Select ' ECHO ' from the onscreen menu and then hit [ON] to insert the inequality symbol. key in 1 to finish the equation and hit [ENTER].

Display 12



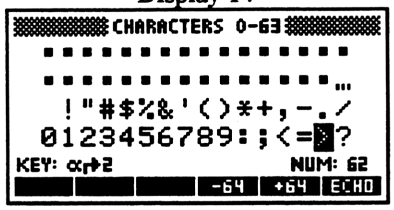
Name the equation H1 by hitting [α] H 1 ' OK '. Select ' OK ' and H1 will appear at the top of the variable list shown in Display 13.

Display 13



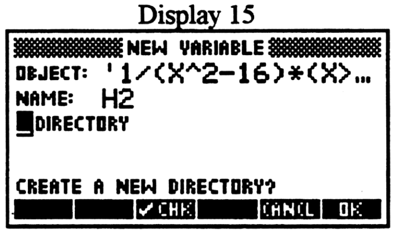
Select ' NEW ' again to enter the other half of the piecewise function in the same manner. The > sign is in the ' CHARS ' menu but won't be on the first screen that comes up. Hit ' –64 ' twice and then the left arrow twice to highlight the correct inequality symbol.

Display 14



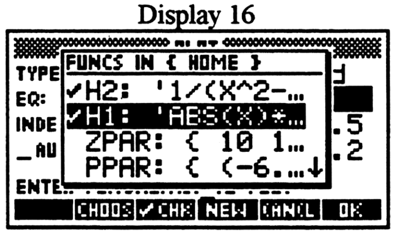
After the equation has been entered, name it H2.

Display 15



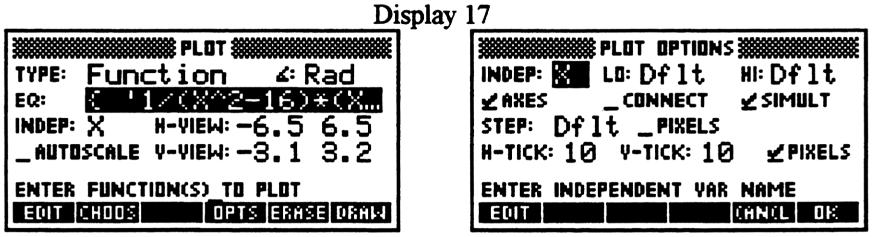
Select ' OK ' to go back to the variable list and use the cursor and the ' √CHK ' soft key to place check marks next to H1 and H2.

Display 16



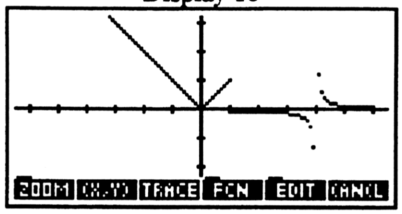
Select ' OK ' to store $h(x)$ in EQ and return to the PLOT screen. Select ' OPTS ' from the onscreen menu to go to the PLOT OPTIONS screen. CONNECT should not have a check mark and SIMULT should be checked. The correct PLOT and PLOT OPTIONS screen are shown in Display 17.

Display 17



Select ' OK ' and then ' ERASE ' and ' DRAW ' to plot the piecewise function.

Display 18

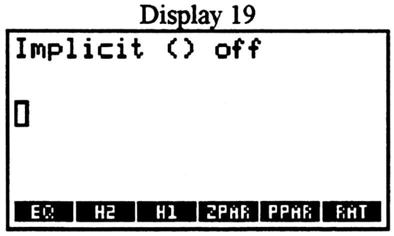


There is a jump discontinuity at 1, where the function switched rules and also a pole discontinuity at 4, where the denominator of the lower function would be 0. $h(x)$ is continuous over the interval:

$$(-\infty,1] \cup (1,4) \cup (4,\infty)$$

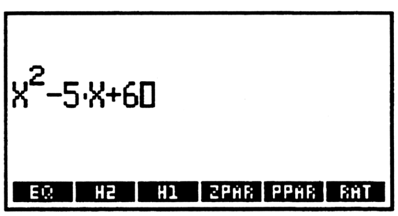
d) The best way to enter $j(x)$ is with the Equation Writer and the stack. From the home screen, hit [purple shift] ' EQUATION '. When entering polynomials it is more convenient to turn the implicit parentheses option off. To do this hit [purple shift] [{ }] and the message shown in Display 19 will appear for a few seconds and then disappear.

Display 19

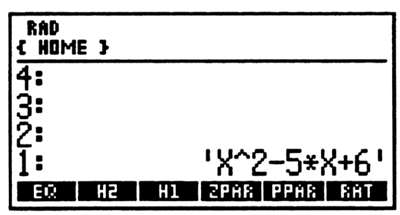


Now the calculator will automatically begin a new term when a + or – is entered. In other words, the polynomial can be typed as it appears without using the right arrow to let the calculator know we are done entering an exponent. The implicit parentheses should be left on when entering most exponential functions. Key in the numerator and hit [ENTER] to send it to the stack.

Display 20

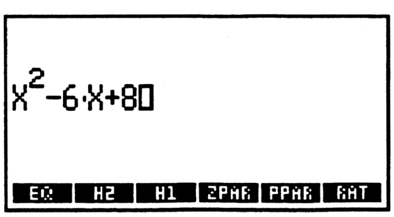


$[ENTER] \rightarrow$

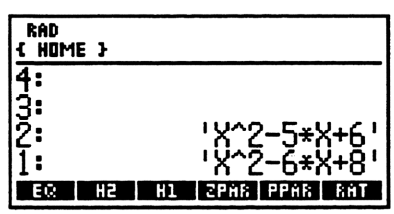


Hit [purple shift] ' EQUATION ' again and key in the denominatior. Hit [ENTER] to send it to the stack.

Display 21

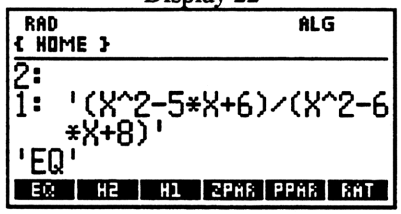


$[ENTER] \rightarrow$



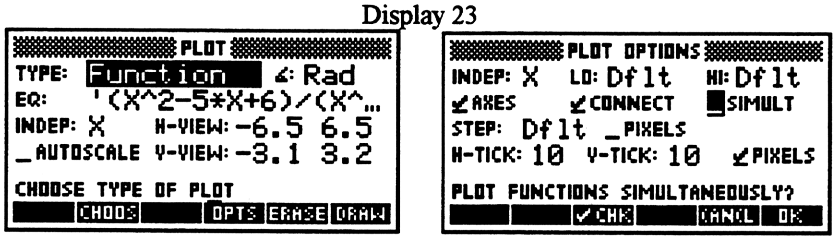
Now that the numerator is on line 2 and the denominator is on line 1, hit [÷] to create $j(x)$. If the onscreen menu is the same as the one shown in Display 21, hit ['] ' EQ ' to put 'EQ' on the command line. If there is no EQ in the variables menu, simply type it in. Use either method to obtain Display 22.

Display 22



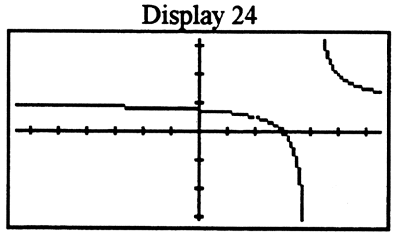
Hit [STO] to store $j(x)$ in EQ. Now when we hit [green shift] ' PLOT ', EQ will contain $j(x)$.

Display 23



Set the PLOT and PLOT OPTIONS screens to the ones shown in Display 23 and select ' ERASE ' and ' DRAW '.

Display 24



Close inspection will reveal a small hole at 2 where no pixel was darkened. There is also a pole discontinuity at 4. $j(x)$ is continuous over the interval:

$$(-\infty,2) \cup (2,4) \cup (4,\infty)$$

ACTIVITY 2....ESTIMATING DERIVATIVES

In this activity we will be using the definition of the derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

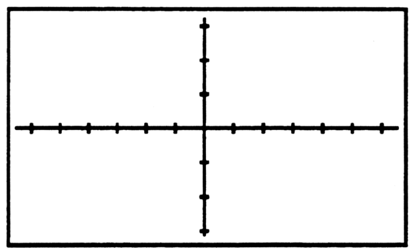
Exercise

For each of the following functions: 1) Use the definition of the derivative to estimate the numerical derivative at the specified f' value to the nearest thousandth; 2) Sketch the graph of the function and the tangent line at the specified point to verify the numerical estimate.

a) $f(x) = x^3$
$\mathit{find}\ f'(-1)$

Numerical estimate:______________

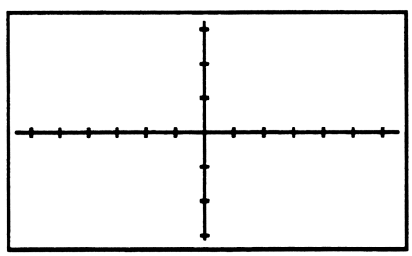
Tangent line equation:______________



b) $f(x) = \sin(2x)$
$\mathit{find}\ f'(2)$

Numerical estimate:______________

Tangent line equation:______________

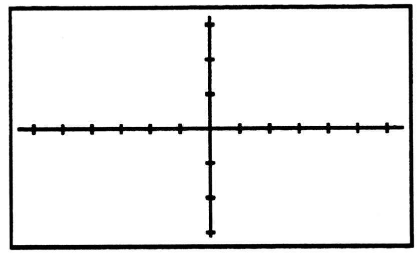


c) $f(x) = 2\ \ln x$

$\mathit{find}\ f'(3)$

Numerical estimate:______________

Tangent line equation:______________

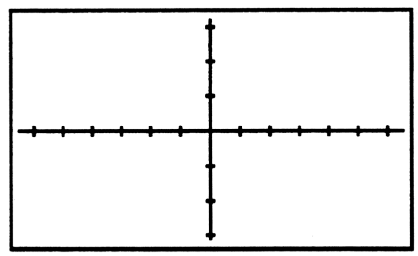


d) $f(x) = \frac{1}{e^x}$

$\mathit{find}\ f'(0)$

Numerical estimate:______________

Tangent line equation:______________

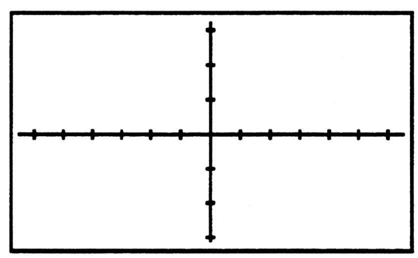


e) $f(x) = |x-2|+1$

$\mathit{find}\ f'(2)$

Numerical estimate:______________

Tangent line equation:______________

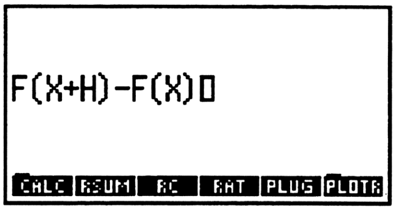


INSTRUCTIONAL SECTION

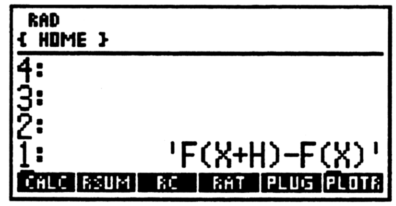
Exercise

The first thing we need to do is store the definition of derivative in a variable called FPRM. Hit [purple shift] ' EQUATION ' to go to the Equation Writer. Enter the numerator and hit [ENTER] to send it to the stack as shown in Display 1.

Display 1

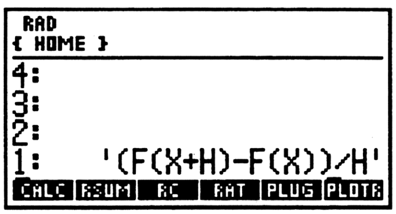


$[ENTER] \rightarrow$

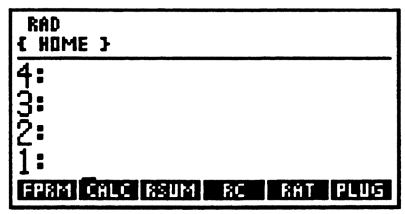


Now key in H and enter it on line one of the stack and hit [÷] to complete the difference quotient. Store the result in FPRM as shown in Display 2.

Display 2



'FPRM' [STO] →

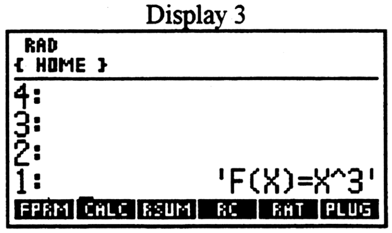


FPRM will now be the first listing in the [VAR] menu.

Exercise 1

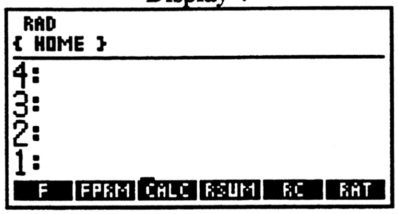
a) We need to define $f(x)$ to be x^3 so hit ['] and enter the equation as shown in Display 3.

Display 3



Now hit [purple shift] ' DEF ' to define the function and F will become the first listing in the [VAR] menu.

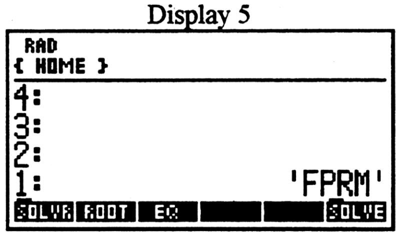
Display 4



Note: ' DEF ' must be used instead of [STO] when defining functions!

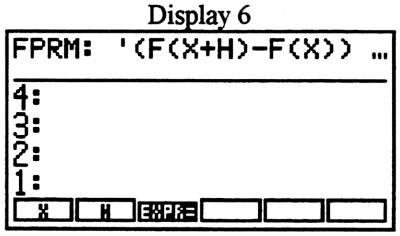
From the onscreen menu in Display 4, hit ['] and then select ' FPRM ' from the onscreen menu. Hit [ENTER] to place it on the stack. Now hit [purple shift] ' SOLVE ' and then select ' ROOT ' from the onscreen menu.

Display 5



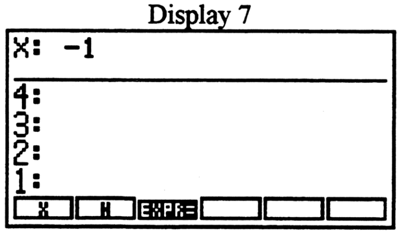
Hit ['] and select ' EQ ' from the onscreen menu to place EQ on the command line and then hit [STO] to store the difference quotient in EQ. Select ' SOLVR ' from the onscreen menu to go to the numeric root finder environment as shown in Display 6.

Display 6



We want to estimate the derivative at −1 so key it in and select ' X ' from the onscreen menu to obtain Display 7.

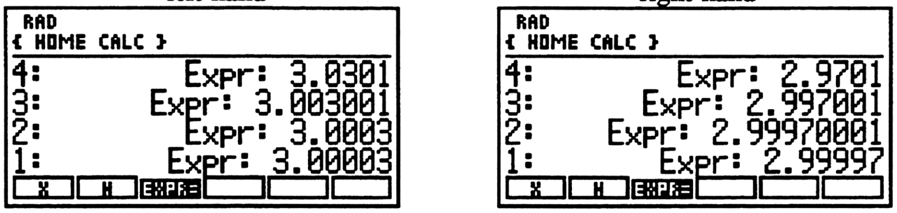
Display 7



Now we need to select values of h that get progressively closer to 0. Key in .01 and select ' H ' and then ' EXPR= ' from the onscreen menu. Repeat the procedure for .001. .0001, and .00001. We also need to check very small negative values of H to see what happens to the difference quotiont as the x-value approaches −1 from the left.

Display 8

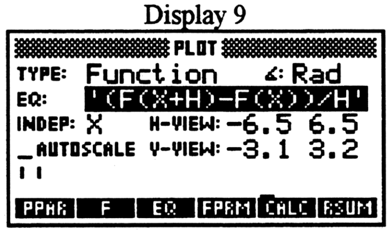
left-hand   right-hand



Both screens shown in Display 8 show the difference quotient getting very close to 3. When rounding to the nearest thousandth, both sides become 3 so this will be the numerical estimate.

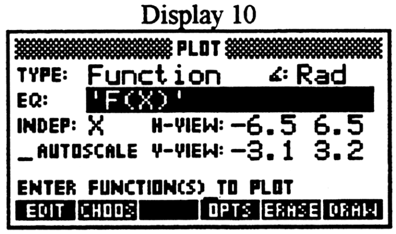
To find the equation of the tangent line at −1 we first need to graph the function. From the home screen, hit [green shift] ' PLOT '. With EQ highlighted, hit ['] and then [VAR] to call up the variables menu.

Display 9



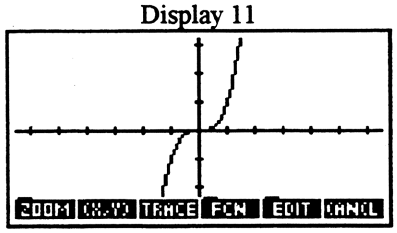
We have already stored the function in $F(x)$ so select ' F ' from the onscreen menu in Display 9 and then hit [purple shift] ' ( ) ' [α] ' X ' [ENTER] to store $F(x)$ in EQ.

Display 10



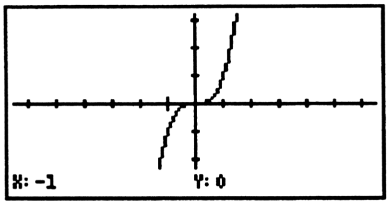
Select ' ERASE ' and ' DRAW ' from the onscreen menu to plot the function.

Display 11

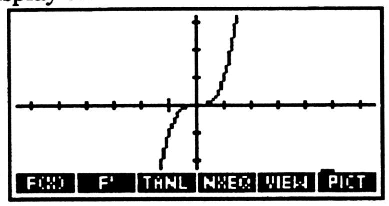


From the onscreen menu shown in Display 11, select ' FCN ' and then hit [NXT]. We want to draw the tangent line at −1 so hit [+] to display the coordinates of the crosshairs and use the left arrow until X is −1. The Y coordinate doesn't really matter so just leave it at 0. Hit [+] again to reveal the onscreen menu and select ' TANL '.

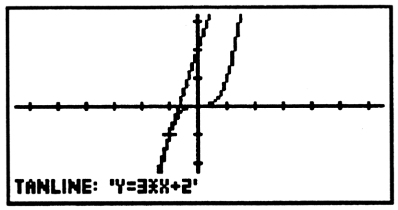
Display 12



[+]→



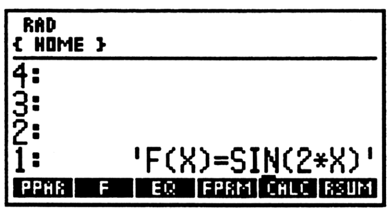
'TANL'→



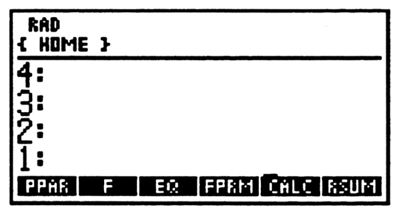
As shown in Display 12, the calculator will draw the tangent line at the specified point and also display the equation.

b) Call up the variables menu and repeat the procedure for $f(x) = \sin(2x)$ by redefining $F(x)$ as shown in Display 13.

Display 13

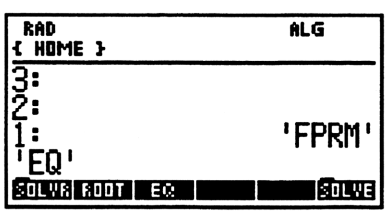


'DEF'→

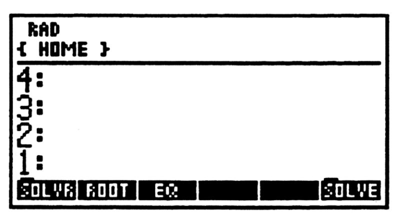


We need to restore FPRM in EQ so hit ['] and select ' FPRM ' from the onscreen menu shown in Display 13. Hit [purple shift] ' SOLVE ' and select ' ROOT '. Hit ['] ' EQ ' to obtain the left screen in Display 14.

Display 14



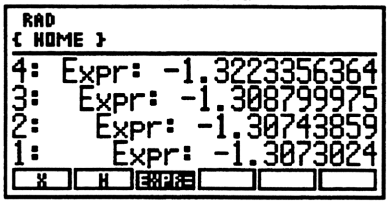
[*STO*]→



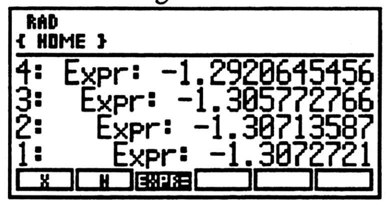
From the onscreen menu, select ' SOLVR ' and store 2 in X. Repeat the procedure described in part a. Display 15 shows the left and right hand numerical estimates.

Display 15

left-hand



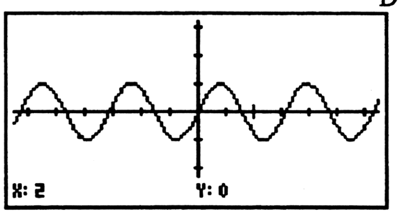
right-hand

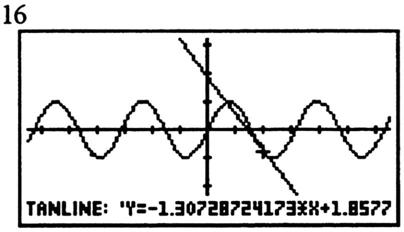


Both sides are −1.307 when rounding to the nearest thousandth.

Repeat the graphing procedure from part a. Display 16 shows the graph and then the graph with the tangent line.

Display 16

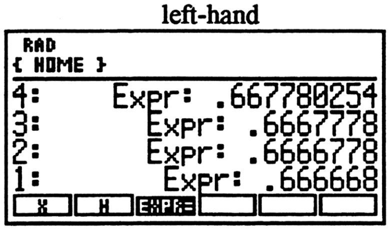




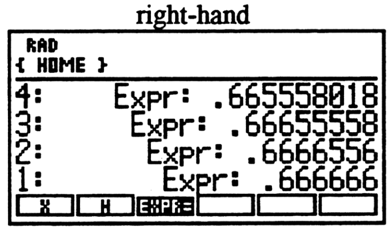
c) Repeat the process to obtain the numerical estimates and graphs shown in Displays 17 and 18.

Display 17

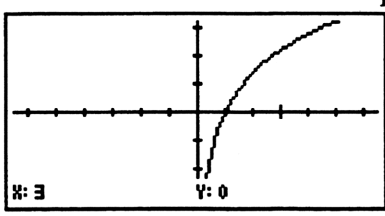
left-hand

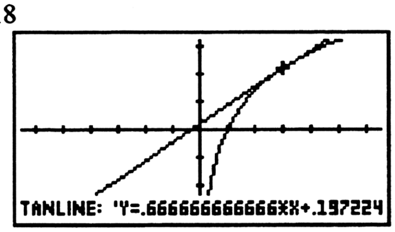


right-hand



Display 18

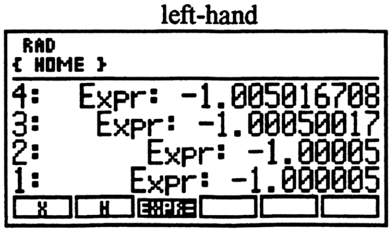




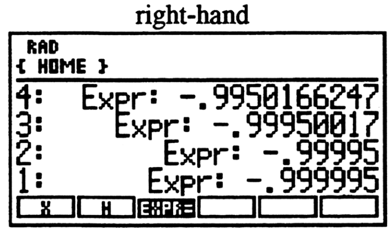
d) Repeat the process to obtain the numerical estimates and graphs shown in Displays 19 and 20.

Display 19

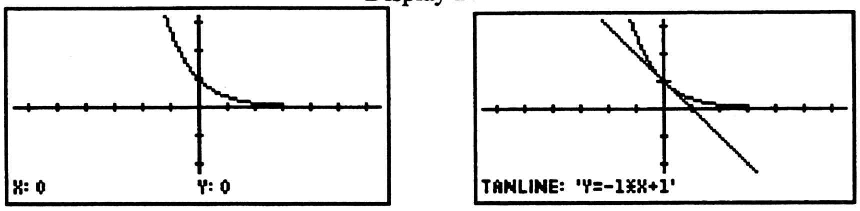
left-hand



right-hand

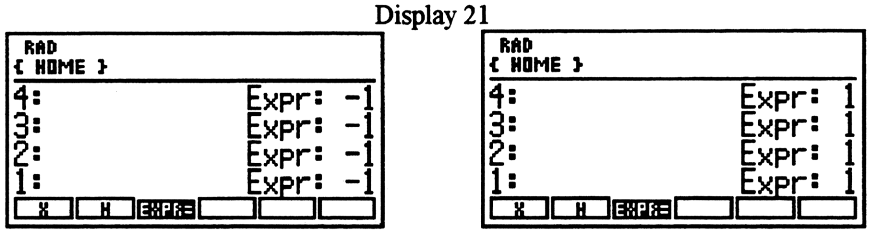


Display 20

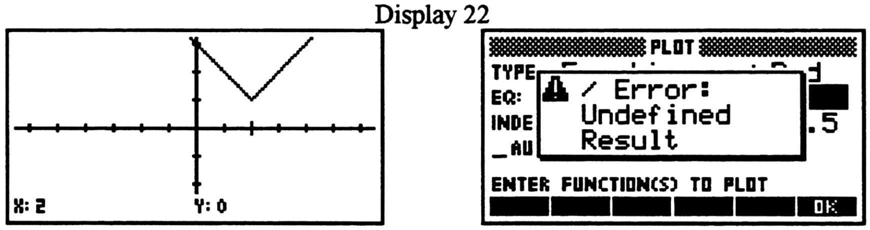


e) This function is not differentiable at 2 so the left and right-hand numerical estimates are not the same. Displays 21 and 22 show the results of the investigation. To enter this particular function, key in ' $F(x) = ABS(x-2)+1$ ' and hit [purple shift] ' DEF '.

Display 21



Display 22



When we try to graph the tangent line we get and undefined result since tangents can only be drawn at points where the curve is smooth.

ACTIVITY 3....COMPUTING DERIVATIVES

Exercise 1

For each function f, find f'.

a) $f(x) = 3x^2 - 5x + 3$

b) $f(x) = x^3 \sin x$

c) $f(x) = \dfrac{x^2 - 1}{e^x}$

d) $f(x) = \sqrt[3]{3x^2 - 4x}$

e) $f(x) = \sin^3 x$

Exercise 2

For each of the following functions, determine the numerical value of the specified derivative, if it exists. Round answers to the nearest thousandth if necessary.

a) $f(x) = \sqrt{2 + \sqrt{1 + x}}$
find $f'(1)$

b) $g(x) = \cos(x^2)$
find $g'\left(\dfrac{\pi}{4}\right)$

c) $h(x) = x^{-3}$
find $h'(0)$

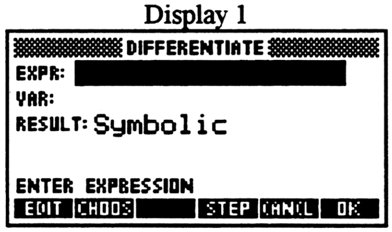
INSTRUCTIONAL SECTION

Exercise 1

The problems in this exercise use the 5 basic derivative rules: linearity properties, product rule, quotient rule, chain rule, and power rule. The 48G will compute the derivative directly or go through the steps so students can see what is happening. We will be taking this set of functions step by step. We will be finding the derivative of all of these functions with respect to X. Since we want only symbolic answers, make sure that no value is stored in X in the [VAR] menu. If there is an X in this menu, purge it.

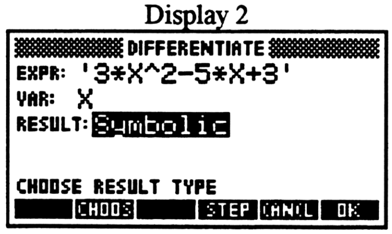
a) Hit [green shift] ' SYMBOLIC ' to get the symbolic menu. Use the down arrow to highlight "Differentiate..." and select ' OK ' from the onscreen menu.

Display 1



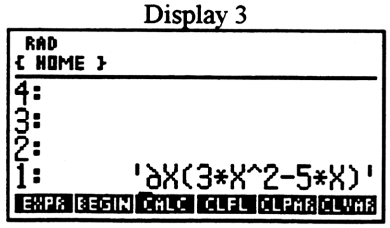
Key in the expression and select ' OK ' from the onscreen menu to store it in EXPR. With VAR highlighted, key in X and select ' OK '. The screen should now look like Display 2.

Display 2



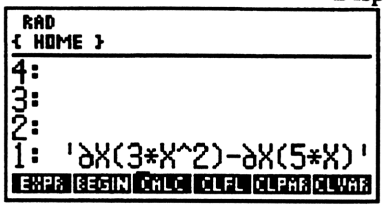
We want to see the steps involved in the differentiation, so select ' STEP ' from the onscreen menu.

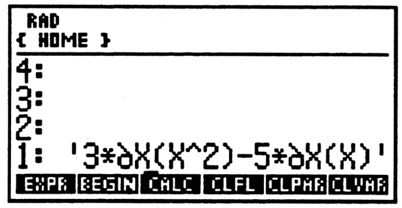
Display 3

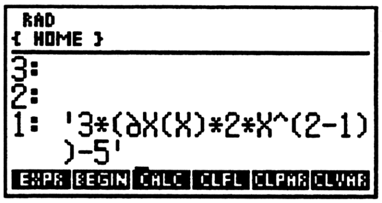


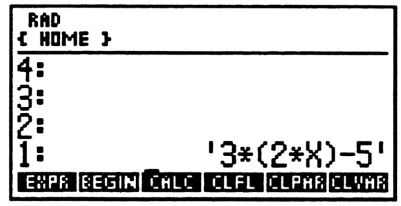
The expression is sent to the stack and the first step is executed. We will use the [EVAL] key to complete the step by step differentiation as shown in Display 4.

Display 4

[*EVAL*]→


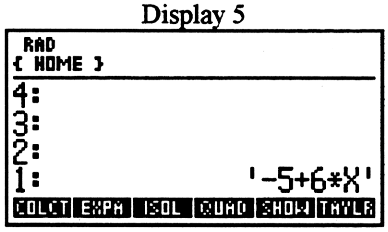
[*EVAL*]→


[*EVAL*]→


[*EVAL*]→


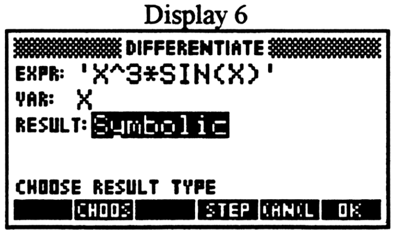
From here it should be apparent that $f'(x) = 6x - 5$. If we want to do the multiplication in the first term, we need to hit [purple shift] ' SYMBOLIC ' and then select ' EXPA ' from the onscreen menu.

Display 5



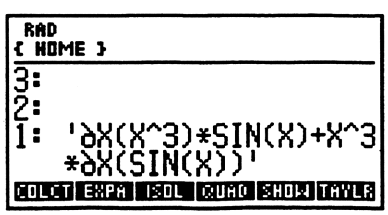
b) This differentiation will use the product rule. Hit [purple shift] ' CLEAR ' to clear the home screen and then go to the "Differentiate.." environment. Enter the function as we did in part a.

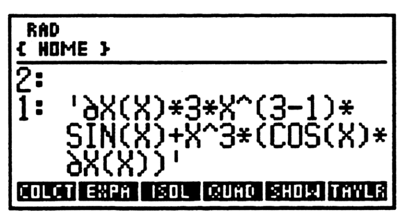
Display 6



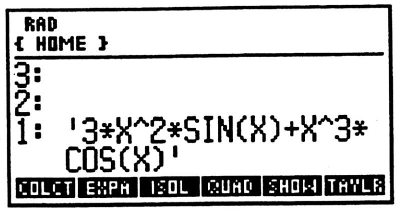
Display 7 shows the step by step differentiation.

Display 7

'*STEP* '→


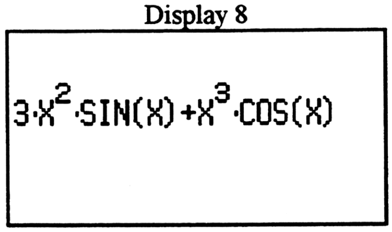
[*EVAL*]→


[*EVAL*]→



The final result can be a little difficult to read when on the stack. To view what is on line 1 of the stack in Equation Writer mode, hit the down arrow ([∇]) once.

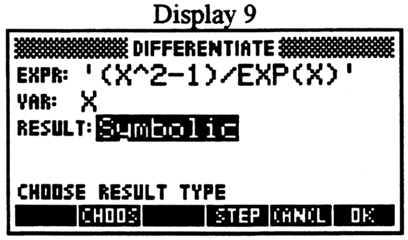
Display 8



$f'(x) = 3x^2 \sin x + x^3 \cos x$.

c) This differentiation will use the quotient rule. Clear the home screen and enter the function.

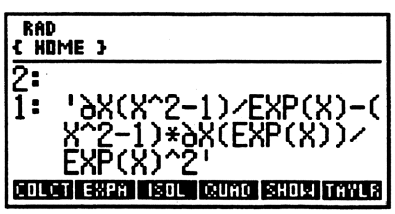
Display 9



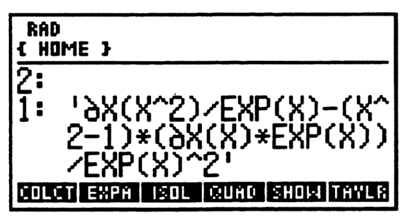
Display 10 shows the step by step differentiation.

Display 10

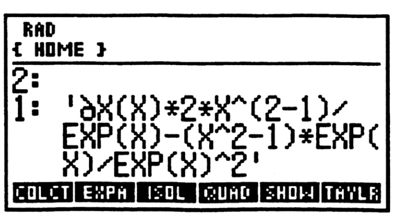
'*STEP* '→



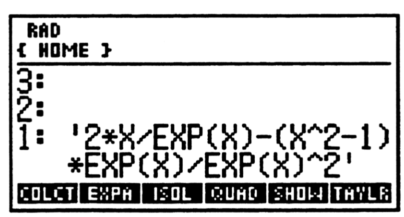
[*EVAL*]→



[*EVAL*]→

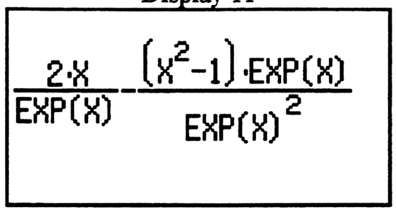


[*EVAL*]→



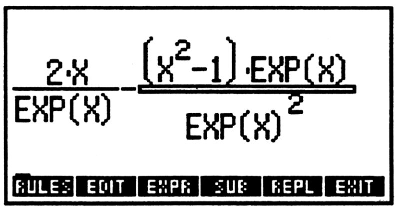
The calculator separates the quotient into two terms from the start of the differentiation which is probably not the way it would have been done on the board. Hit the down arrow to view the completed differentiation.

Display 11

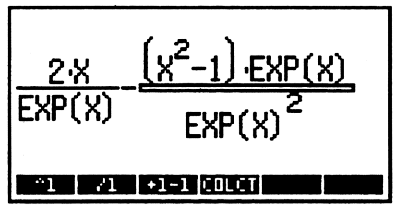


The second term is not reduced completely. To have the calculator perform the reduction, hit [ON] to bring up the onscreen menu and use the down arrow to highlight the division bar of the second term.

Display 12

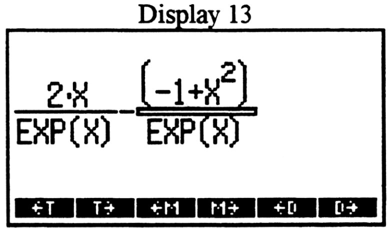


'*RULES*'[*NXT*] [*NXT*][*NXT*]→



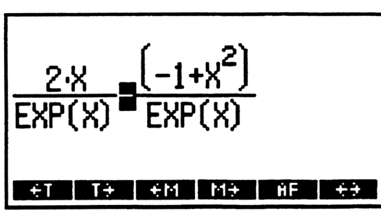
Use the keystrokes documented in Display 12 to obtain the screen on the right. From the onscreen menu, select ' COLCT '.

Display 13

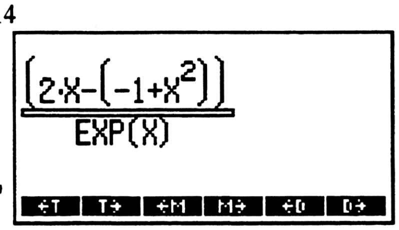


Now we would like to combine the terms into a single term. To do this, use the left arrow to highlight the "–" sign between the terms and select ' RULES ' from the onscreen menu.

Display 14

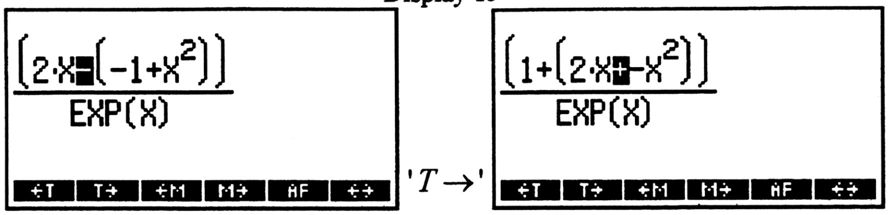


'*M*→'



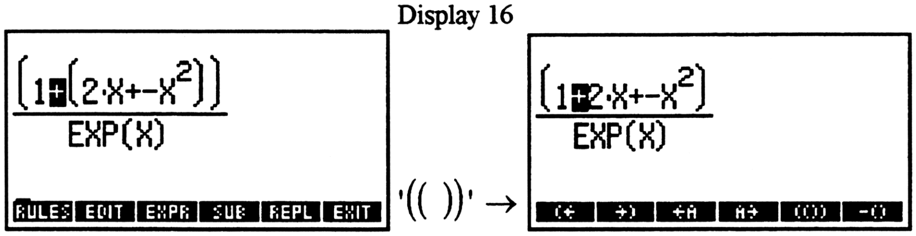
The onscreen menu selection in Display 14 will combine the terms. Hit the up arrow once to highlight the "–" sign outside the parentheses and select ' RULES ' again.

Display 15



To continue simplifying, hit the left arrow four times to highlight the "+" outside of the parentheses and select ' RULES ' once again.

Display 16

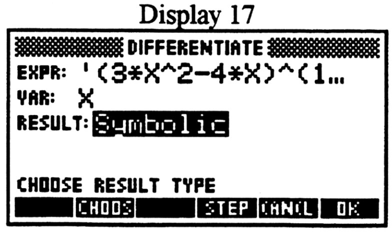


It is also possible to arrange the polynomial in the numerator in descending order with respect to the exponent, however students should be able to perform this task much more quickly than the calculator can.

$$f'(x) = \frac{-x^2 + 2x + 1}{e^x}$$

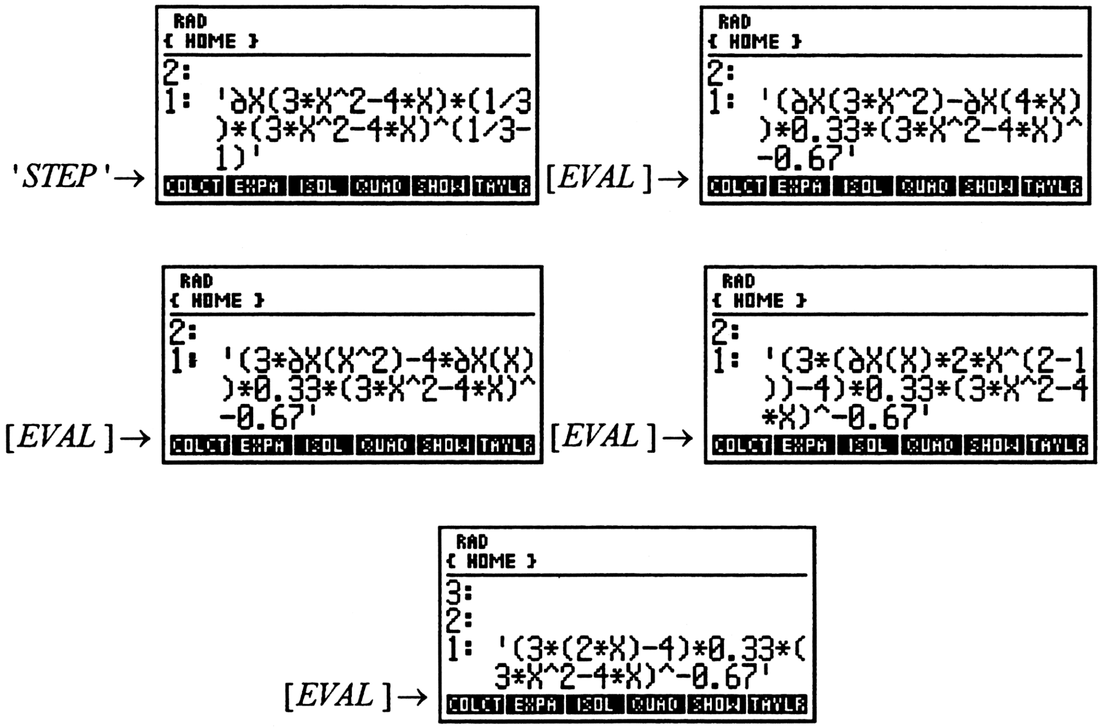
d) $f(x)$ needs to be written with a rational exponent in order to do the differentiation using the chain rule. The DIFFERENTIATE environment should look like Display 17. We also want to fix the number of decimal places to 2. Key in a 2 on line 1 of the stack and type in FIX without tick marks. Hit [ENTER] to fix the decimal.

Display 17



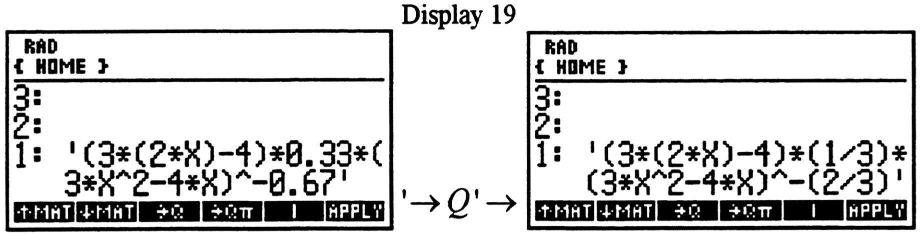
Display 18 shows the step by step differentiation.

Display 18



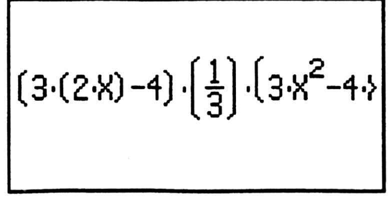
The calculator has evaluated the derivative completely when there are no more δx symbols in the equation. To change the decimals to fractions, hit [purple shift] ' SYMBOLIC ' if the onscreen menu is different from the one shown in Display 18, and then hit [NXT] to obtain the onscreen menu in Display 19.

Display 19



The selection in Display 19 changes decimals to quotients. Now hit the down arrow to send the derivative to the Equation Writer.

Display 20

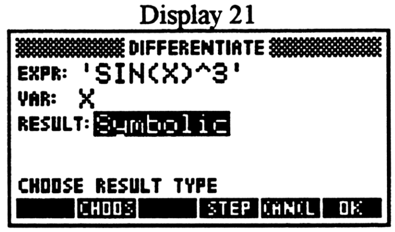


The best thing to do from this point is to use the right arrow to scroll through the entire equation, write it down, and manipulate it into whatever form is desired.

$$f'(x) = \frac{6x-4}{3\sqrt[3]{\left(3x^2-4x\right)^2}}$$

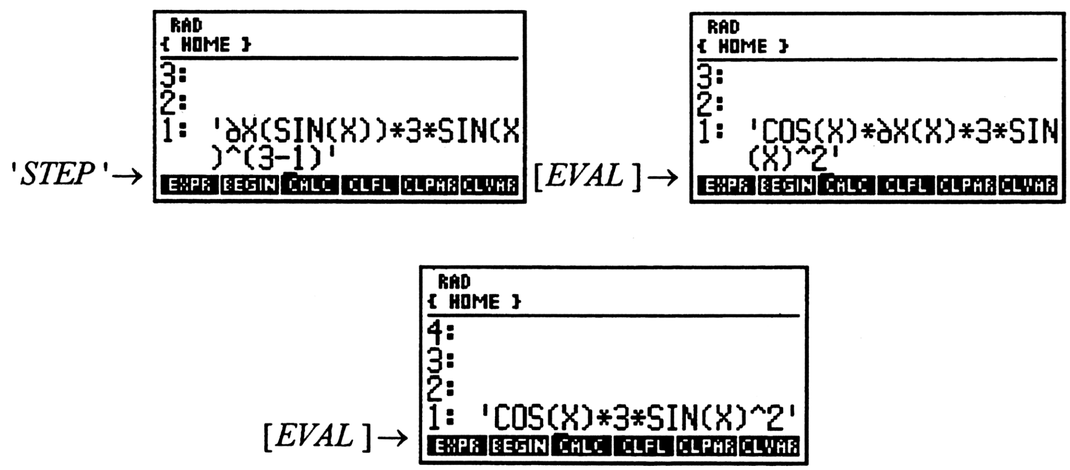
e) This differentiation will use the power rule. The DIFFERENTIATE environment should look like Display 21.

Display 21



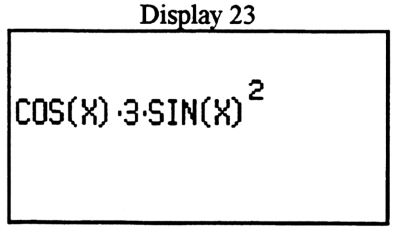
Display 22 shows the step by step differentiation.

Display 22



Hit the down arrow to view the equation in Equation Writer.

Display 23



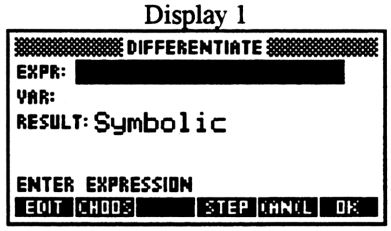
$f'(x) = 3\cos x\ \sin^2 x.$

Exercise 2

In this exercise, we will be computing derivatives quickly without using the STEP method.

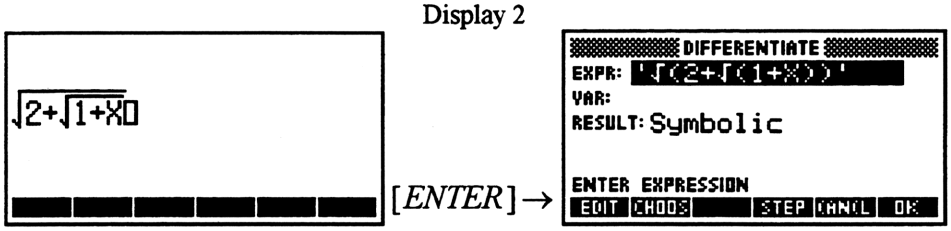
a) Let's use Equation Writer to enter the function into EXPR. Hit [green shift] ' SYMBOLIC ', highlight "Differentiate..." and select ' OK ' from the onscreen menu.

Display 1



With EXPR highlighted as shown in Display 1, hit [purple shift] ' EQUATION ' to go to Equation Writer mode. Key in the equation and hit [ENTER] as shown in Display 2.

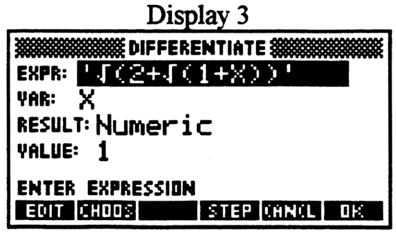
Display 2



Use the down arrow to highlight VAR and type in X. Select ' OK ' from the onscreen menu to store X in VAR and move the highlight to RESULT. We need to change RESULT from "Symbolic" to "Numeric". The easiest way to do this is to hit [α] N. It can also be accomplished using the ' CHOOS ' option demonstrated previously.

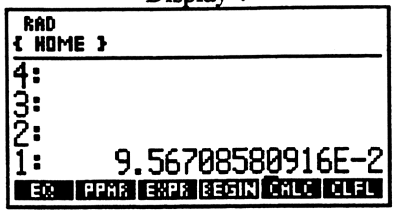
When the change has been made from symbolic to numeric, a new field appears on the DIFFERENTIATE screen called VALUE. We want to input the x-value where we want the derivative to be evaluated. With VALUE highlighted key in 1and select ' OK '. The DIFFERENTIATE screen should now look like Display 3.

Display 3



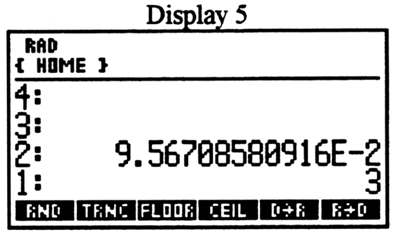
To evaluate the derivative directly, select ' OK ' from the onscreen menu instead of ' STEP '.

Display 4



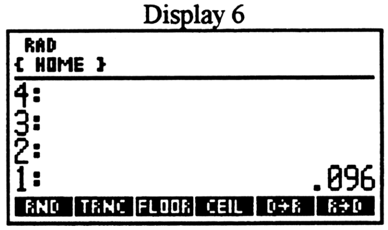
To round the answer to the nearest thousandth, enter a 3 (3 decimal places) on the stack and then hit [MTH] ' REAL ' [NXT] [NXT] to obtain Display 5.

Display 5



From the onscreen menu, select ' RND '.

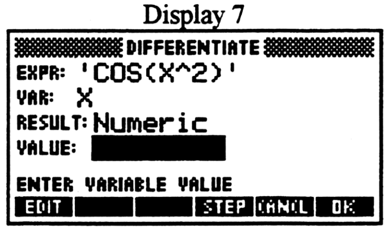
Display 6



$f'(1) = .096$.

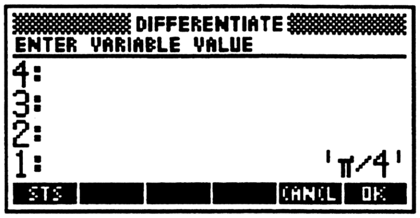
b) Set up the DIFFERENTIATE screen to look like Display 7.

Display 7

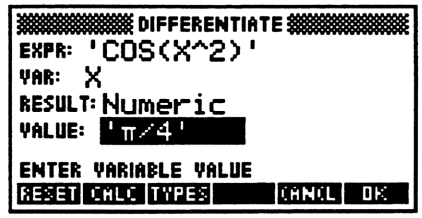


We want to enter $\frac{\pi}{4}$ into VALUE. To do this hit [NXT] and select ' CALC ' from the onscreen menu. This will take us to the stack while still in the DIFFERENTIATE environment. Hit ['] and then [purple shift] ' π ' [÷] [4] and the [ENTER] to put it on line 1.

Diplay 8



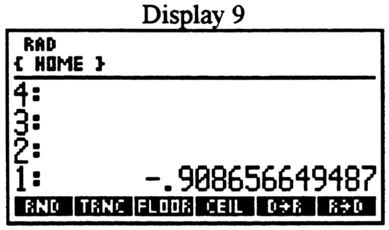
$'OK' \rightarrow$



The ' OK ' command will store whatever is on line 1 in VALUE as illustrated in Display 8.

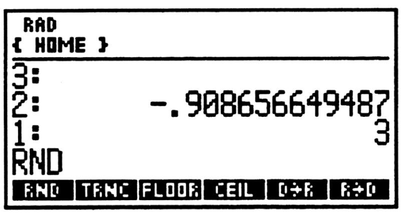
Select ' OK ' to evaluate the derivative.

Display 9

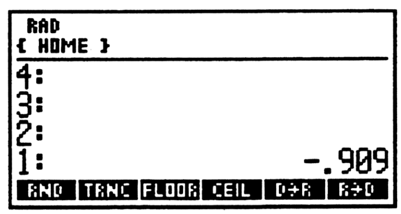


Instead of digging into the [MATH] menu to do the rounding, enter a 3 on the stack and type in RND <u>without</u> tick marks on the command line as shown in Display 10.

Display 10



$[ENTER] \rightarrow$

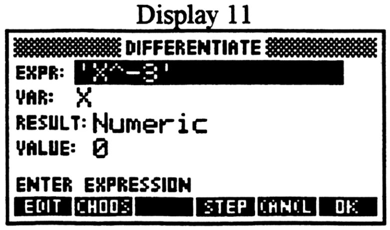


It is often preferable to use the command line in this manner to perform functions rather than bring them up on the onscreen menu and using the soft keys.

$$f'\left(\frac{\pi}{4}\right) = -.909 .$$

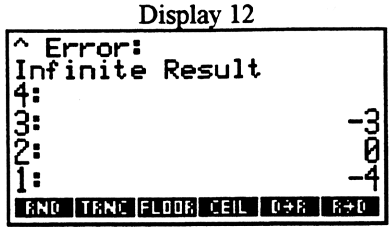
c) Set up the DIFFERENTIATE screen to look like Display 11.

Display 11



Select ' OK ' from the onscreen menu.

Display 12



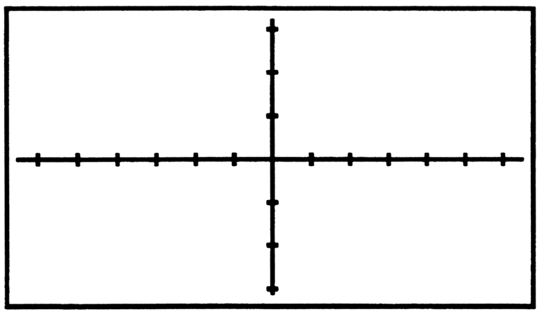
It appears that $f'(0)$ does not exist.

ACTIVITY 4....FUNCTION BEHAVIOR AND CRITICAL POINTS

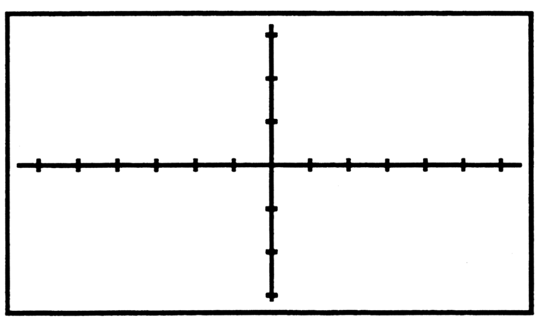
Exercise

For each function in this exercise: 1) Graph the function and its derivative within the given viewing rectangle; 2) Find all critical points and classify them as a local maximums or minimums; 3) Identify the interval over which the function is increasing and decreasing.

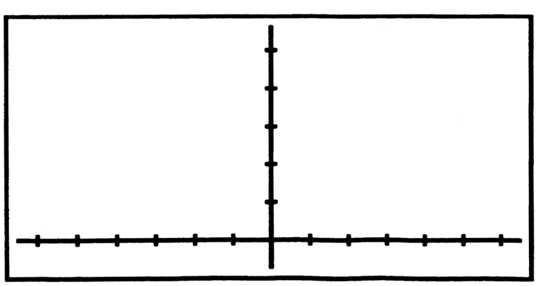
a) $f(x) = \frac{x^3}{2} - 3x; \quad \{-8 \le x \le 8\} \{-15 \le y \le 15\}$



b) $f(x) = \cos(\sin x^2); \quad \{-2 \le x \le 2\} \{-2 \le y \le 2\}$



c) $f(x) = |3x^2 - 5|; \quad \{-3 \le x \le 3\} \{-2 \le y \le 8\}$

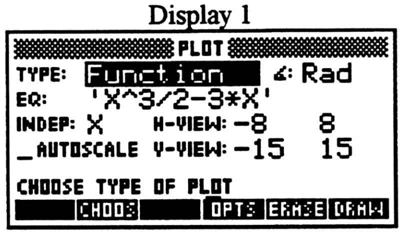


INSTRUCTIONAL SECTION

Exercise

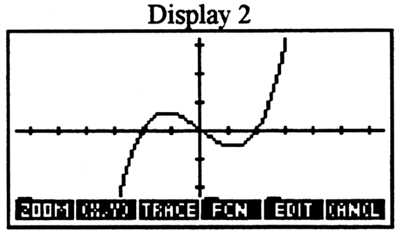
a) Hit [green shift] ' PLOT ' to go to the PLOT environment. Set up the screen to look like Display 1.

Display 1



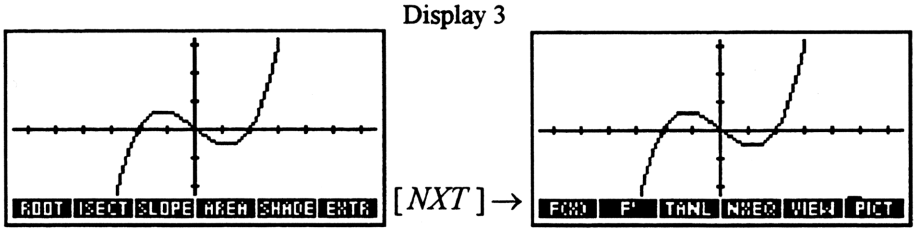
From the onscreen menu, select ' ERASE ' and ' DRAW ' to graph the function.

Display 2



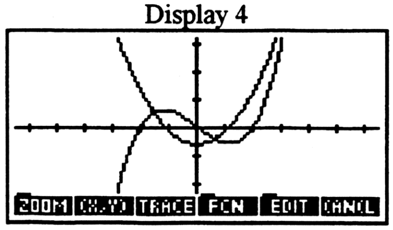
From the onscreen menu in Display 2, select ' FCN '.

Display 3



The onscreen menus in Display 3 contain all of the options in the FCN directory. We want to graph the derivative so select ' F' '.

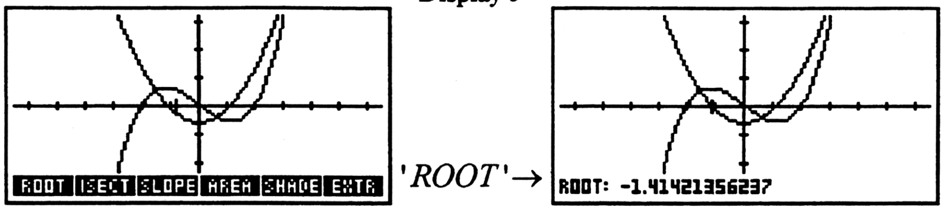
Display 4



The derivative and the original function are both plotted on the same set of axes.

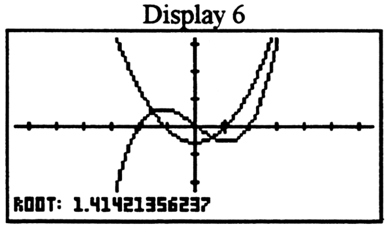
Critical points will occur when the derivative is equal to 0. To find the values of X where this occurs, select ' FCN ' again. Use the arrow keys to move the crosshairs into the proximity of the intersection of the derivative and the negative x-axis.

Display 5



We have a local maximum at –1.414 a.k.a $-\sqrt{2}$. Move the crosshairs to the positive x-axis and hit [+] to bring up the onscreen menu. Select ' ROOT ' again.

Display 6



We have a local minimum at 1.414 a.k.a $\sqrt{2}$.

From this information we can ascertain that the function is increasing over the interval:

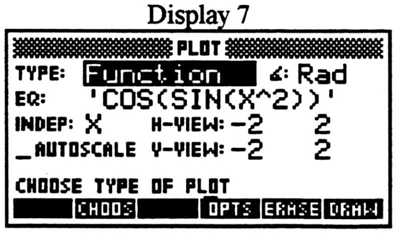
$$(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$$

and decreasing over the interval:

$$(-\sqrt{2},\sqrt{2})$$

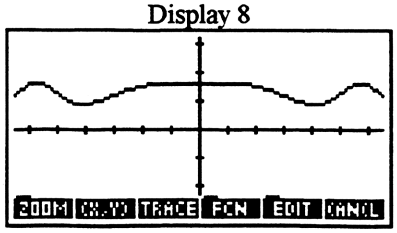
b) Go to the PLOT environment and set the screen to look like Display 7.

Display 7



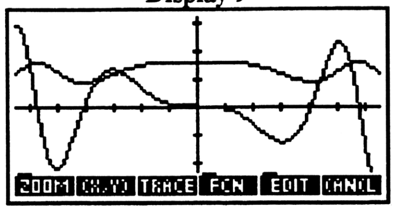
Select ' ERASE ' and ' DRAW ' from the onscreen menu.

Display 8



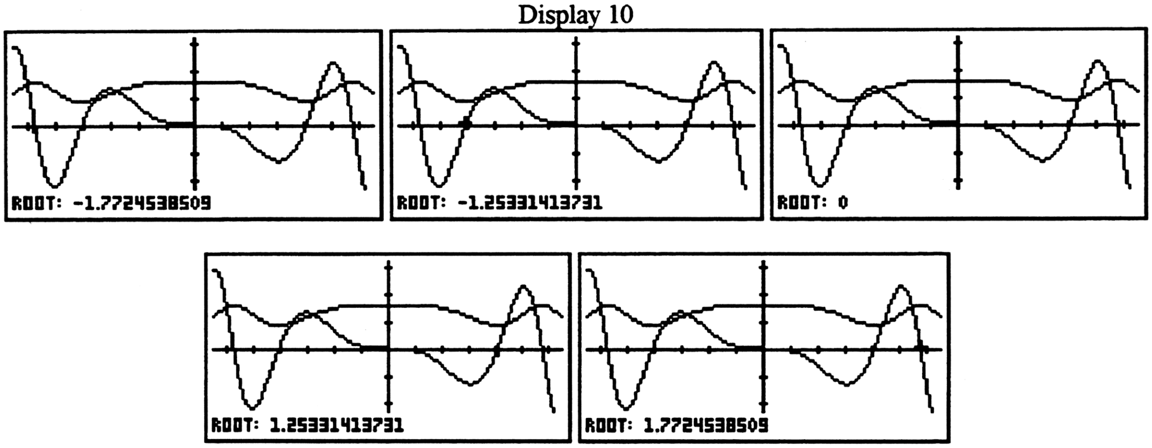
From the onscreen menu select ' FCN ', then hit [NXT] and select ' F' '.

Display 9



Use the ' ROOT ' option as in part a to find all five points where the derivative intersects the x-axis.

Display 10



There is a critical point at each one of these roots. From Display 10 we can see that local maximums occur at -1.77, 0, and 1.77. Local minimums occur at -1.25 and 1.25.

The function is increasing over the interval:

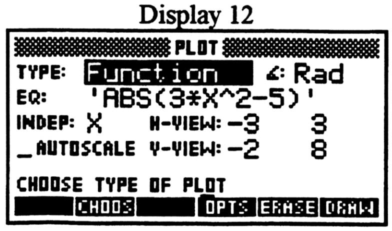
$$[-2,1.77) \cup (-1.25,0) \cup (1.25,1.77)$$

and decreasing over the interval:

$$(-1.77,1.25) \cup (0,1.25) \cup (1.77,2]$$

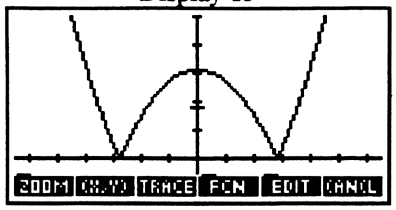
c) Go to the PLOT environment and set the screen to look like Display 12.

Display 12



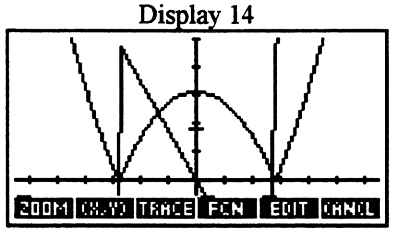
Select ' ERASE ' and ' DRAW ' from the onscreen menu.

Display 13



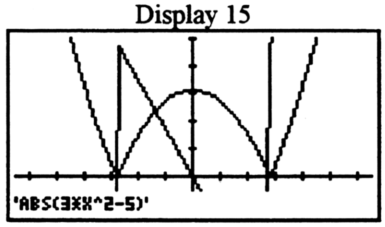
From the onscreen menu select ' FCN ', then hit [NXT] and select ' F' '.

Display 14



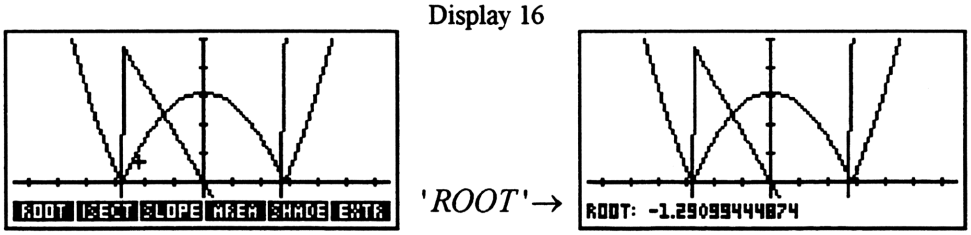
Since the function is not differentiable at the two cusps, the ROOT technique will not work to find the local minimums when using the graph of the derivative. To find these values, select ' FCN ', hit [NXT] and select ' NXEQ ' to switch from the graph of the derivative to the graph of $f(x)$.

Display 15



Hit [+] to bring up the onscreen menu and hit next to obtain the onscreen menu in Display 16. Move the crosshairs into the proximinity of the local minimum on the left.

Display 16



Local minimums occur at -1.29 and, because of symmetry, 1.29. There is a local maximum at 0.

The function is increasing over the interval:

$$(-1.29,0) \cup (1.29,3]$$

and decreasing over the interval:

$$[-2,-1.29) \cup (0,1.29)$$