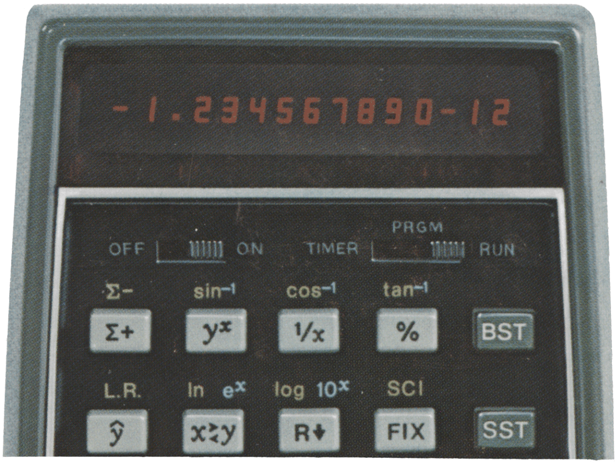
HEWLETT PACKARD

HP-55
mathematics programs



a comprehensive guidebook:
74 common programs
in such areas as

complex arithmetic and functions, linear algebra, trigonometry, geometry, business, and others

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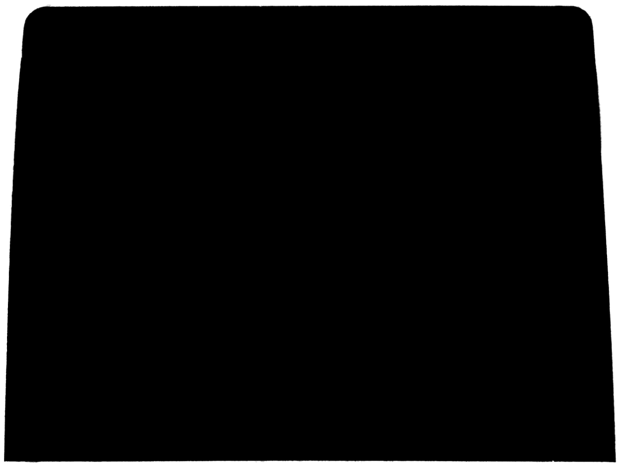

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HP-55
mathematics programs

Shown actual size.

PRINTED IN U.S.A.

INTRODUCTION

Material in *HP-55 Mathematics Programs* has been selected from the areas of complex variables, business, linear algebra, integration, interpolation, number theory, algebra, trigonometry, and analytical geometry.

Each program includes a general description, formulas used in the program solution, numerical examples, user instructions, program listings, and register allocations. The body of the book is arranged logically according to subject matter.

Some related individual programs were combined into one program when it seemed they might be useful together. In this way more programs could be included in the book. For individual program use it is possible for you to use only the portion of the combined program needed for your particular application.

In most cases the programs do not destroy stored data. Therefore, in order to run a different example only the data that is changed for the new example need be reentered.

We suggest that you first read the material explaining the Format of User Instructions, then use the programs. An understanding of the HP-55 Owner's Handbook is also required if, in addition, you wish to track the changes in the storage registers and stack registers on a step-by-step basis.

We hope you find *HP-55 Mathematics Programs* a useful tool for your mathematical work, and welcome your comments, requests and suggestions– these are our most important source of future user-oriented programs.

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FORMAT OF USER INSTRUCTIONS

The completed User Instructions form is your guide to operating the programs in this book.

The form is composed of five columns. Reading from left to right, the first column, labeled STEP, gives the instruction step number.

The INSTRUCTIONS column gives instructions and comments concerning the operations to be performed. Steps are executed in sequential order except where the INSTRUCTIONS column directs otherwise.

Normally, the first instruction is "Enter program", which means to store the keystrokes of the program into memory (press BST in RUN mode, switch to PRGM mode, then key in the program, switch back to RUN mode).

Repeated processes–used in most cases for a long string of input/output data–are outlined with a bold border together with a "Perform" instruction.

The INPUT DATA/UNITS column specifies the input data to be supplied, and the units of data if applicable.

The KEYS column specifies the keys to be pressed. [↑] is the symbol used to denote the [ENTER↑] key. All other key designations are identical to those appearing on the HP-55. Ignore any blank positions in the KEYS column.

Some programs are sufficiently complex that they cannot be done in the 49 programming steps. However, they were sufficiently important to be included in this pac. In these cases the users must press additional keystrokes (other than program control keys) in order to get the answer. Those keys will also be shown in the KEYS column.

COMPLEX ARITHMETIC, +, −, ×, ÷

Let $a_1 + ib_1$ and $a_2 + ib_2$ be two complex numbers. The arithmetic operations +, −, x, ÷ are defined as follows:

1. +, addition

$$(a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + (b_1 + b_2)i$$

2. −, subtraction

$$(a_1 + ib_1) - (a_2 + ib_2) = (a_1 - a_2) + (b_1 - b_2)i$$

3. x, multiplication

$$(a_1 + ib_1) \times (a_2 + ib_2) = r_1 \, r_2 \, e^{i(\theta_1 + \theta_2)}$$

4. ÷, division

$$\frac{(a_1 + ib_1)}{(a_2 + ib_2)} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}, \quad a_2 + ib_2 \neq 0$$

where $r_1 e^{i\theta_1}$ is the polar representation of $a_1 + ib_1$ and $r_2 e^{i\theta_2}$ is the polar representation of $a_2 + ib_2$. In each case let the answer be $x + iy$.

After a calculation is finished x is stored in R_1 as well as the X-register and y is stored in R_2 as well as the Y-register. In this way arithmetic operations can be chained together.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	42	CHS
02.	22	x⇄y
03.	42	CHS
04.	22	x⇄y
05.	34	RCL
06.	01	1
07.	61	+
08.	22	x⇄y
09.	34	RCL
10.	02	2
11.	61	+
12.	-43	GTO 43
13.	32	g
14.	00	R→P
15.	13	1/x
16.	22	x⇄y
17.	42	CHS
18.	22	x⇄y
19.	-22	GTO 22
20.	32	g
21.	00	R→P
22.	33	STO
23.	03	3
24.	23	R↓

DISPLAY		KEY ENTRY
LINE	CODE	
25.	34	RCL
26.	02	2
27.	34	RCL
28.	01	1
29.	32	g
30.	00	R→P
31.	34	RCL
32.	03	3
33.	71	x
34.	33	STO
35.	03	3
36.	23	R↓
37.	61	+
38.	34	RCL
39.	03	3
40.	31	f
41.	00	R←P
42.	22	x⇄y
43.	33	STO
44.	02	2
45.	22	x⇄y
46.	33	STO
47.	01	1
48.	-00	GTO 00
49.		

REGISTERS	
R_0	
R_1	a_1, x
R_2	b_1, y
R_3	Used
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. $(3+4i) + (7.4 - 5.6i) = 10.40 - 1.60i$

2. $(3+4i) - (7.4 - 5.6i) = -4.40 + 9.60i$

3. $(3.1 + 4.6i) \times (5 - 12i) = 70.70 - 14.20i$

4. $\dfrac{(3 + 4i)}{7 - 2i} = .25 + .64i$

5. $\left[\dfrac{(3+4i) + (7.4 - 5.6i)}{7 - 2i}\right]\left[3.1 + 4.6i\right] = 3.61 + 7.16i$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	For first calculation in chain						
	enter $a_1 + ib_1$	b_1	STO	2			
		a_1	STO	1			
3	Enter $a_2 + ib_2$	b_2	↑				
		a_2					
4	For Addition		GTO	0	5	R/S	x
	or						
	Subtraction		BST	R/S			x
	or						
	Multiplication		GTO	2	0	R/S	x
	or						
	Division		GTO	1	3	R/S	x
5	For imaginary part		x⇄y				y
6	For next calculation in chain go						
	to step 3						
	or						
	For a new calculation, go to						
	step 2						

COMPLEX FUNCTIONS ln z, $\log_z w$, $\log_c z$

Let $z = a_1 + ib_1$ and $w = a_2 + ib_2$ be complex numbers with polar representations $r_1 e^{i\theta_1}$ and $r_2 e^{i\theta_2}$ respectively. Also, let c be a positive real number. The formulas used to evaluate ln z, $\log_z$ w, and $\log_c$ z are as follows:

1. $\ln z = \ln r_1 + i\theta_1$

2. $\log_z w = \dfrac{\ln w}{\ln z} = \dfrac{r_3}{r_4} e^{i(\theta_3 - \theta_4)} \qquad z \neq 0$

   where $\ln r_2 + i\theta_2 = r_3 e^{i\theta_3}$

   and $\ln r_1 + i\theta_1 = r_4 e^{i\theta_4}$

3. $\log_c z = \dfrac{\ln z}{\ln c} = \dfrac{\ln r_1}{\ln c} + \dfrac{\theta_1}{\ln c} i \qquad c > 0$

In each case let the solution be x + iy. The calculator must be in RADIAN mode for all three functions.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	32	g
02.	00	R→P
03.	31	f
04.	22	ln
05.	-00	GTO 00
06.	32	g
07.	00	R→P
08.	31	f
09.	22	ln
10.	32	g
11.	00	R→P
12.	34	RCL
13.	02	2
14.	34	RCL
15.	01	1
16.	32	g
17.	00	R→P
18.	31	f
19.	22	ln
20.	32	g
21.	00	R→P
22.	22	x⇄y
23.	23	R↓
24.	81	÷
25.	33	STO
26.	03	3
27.	23	R↓
28.	51	–
29.	42	CHS
30.	34	RCL
31.	03	3
32.	31	f
33.	00	R←P
34.	-00	GTO 00
35.	32	g
36.	00	R→P
37.	31	f
38.	22	ln
39.	34	RCL
40.	01	1
41.	31	f
42.	22	ln
43.	81	÷
44.	22	x⇄y
45.	31	f
46.	34	LAST X
47.	81	÷
48.	22	x⇄y
49.	-00	GTO 00

REGISTERS	
R_0	
R_1	a_1, c
R_2	b_1
R_3	r_3/r_4
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. $\ln(1 + i) = .35 + .79i$

2. $\log_{(1+i)}(1.49 + 4.13i) = 2.00 - 1.00i$

3. $\log_2(-7.46 + 2.89i) = 3.00 + 4.00i$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set in radian mode		f	RAD			
3	For ln z, enter z	b_1	↑				
		a_1	BST	R/S			x
			x⇄y				y
	or						
3	For $\log_z$ w, store z	b_1	STO	2			
		a_1	STO	1			
4	Enter w	b_2	↑				
		a_2	GTO	0	6	R/S	x
			x⇄y				y
	or						
3	For $\log_c$ z, store c	c	STO	1			
4	Enter z^c	b_1	↑				
		a_1	GTO	3	5	R/S	x
			x⇄y				y

COMPLEX FUNCTIONS $|z|$, z^2, $1/z$, e^z, $\sqrt{z}$

A complex number $z = a + ib$ has polar representation $re^{i\theta}$. The formulas used to evaluate the given functions are as follows:

1. $|z| = r$

2. $z^2 = r^2 e^{i2\theta}$

3. $^1/z = \frac{1}{r} e^{-i\theta}, z \neq 0$

4. $e^z = e^a e^{ib}$

5. $\sqrt{z} = \pm(\sqrt{r}\, e^{i\theta/2}) = \pm(x + iy)$

The answer is represented by $x + iy$. For e^z the calculator must be in RADIAN mode.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	22	e^x
01.	32	g	26.	31	f
02.	00	R→P	27.	00	R←P
03.	-00	GTO 00	28.	-00	GTO 00
04.	32	g	29.	32	g
05.	00	R→P	30.	00	R→P
06.	41	↑	31.	31	f
07.	71	×	32.	42	$\sqrt{x}$
08.	22	x⇄y	33.	22	x⇄y
09.	41	↑	34.	02	2
10.	61	+	35.	81	÷
11.	22	x⇄y	36.	22	x⇄y
12.	31	f	37.	31	f
13.	00	R←P	38.	00	R←P
14.	-00	GTO 00	39.	-00	GTO 00
15.	32	g	40.		
16.	00	R→P	41.		
17.	13	$^1/x$	42.		
18.	22	x⇄y	43.		
19.	42	CHS	44.		
20.	22	x⇄y	45.		
21.	31	f	46.		
22.	00	R←P	47.		
23.	-00	GTO 00	48.		
24.	32	g	49.		

REGISTERS	
R_0	
R_1	
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. $|3 + 4i| = 5.00$

2. $(7 - 2i)^2 = 45.00 - 28.00i$

3. $\frac{1}{2 + 3i} = .15 - .23i$

4. $e^{(3+4i)} = -13.13 - 15.20i$

5. $\sqrt{7 + 6i} = \pm(2.85 + 1.05i)$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set to radian mode (only neces-						
	sary for e^z)		f	RAD			
3	Enter z	b	↑				
		a					
4	For $\|z\|$		BST	R/S			$\|z\|$
	or						
	z^2		GTO	0	4	R/S	x
			x⇄y				y
	or						
	1/z		GTO	1	5	R/S	x
			x⇄y				y
	or						
	e^z		GTO	2	4	R/S	x
			x⇄y				y
	or						
	$\sqrt{z}$ (one root only)		GTO	2	9	R/S	x
			x⇄y				y

COMPLEX FUNCTIONS z^n, $z^{1/n}$

A complex number z = a + ib has polar representation $re^{i\theta}$. The formulas used to evaluate z^n and $z^{1/n}$ where n is a positive integer are:

1. $z^n = r^n e^{in\theta}$

2. $z^{1/n} = r^{1/n} \left[\cos\left(\frac{\theta + ck}{n}\right) + i \sin\left(\frac{\theta + ck}{n}\right)\right]$

where $c = 4 \sin^{-1} 1 = 360° = 2\pi$ radians = 400 grads

and $k = 0, 1, ..., n - 1$

The solution to z^n is represented by x + iy and the n solutions to $z^{1/n}$ are represented by $x_k + iy_k$.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	71	x
01.	34	RCL	26.	33	STO
02.	01	1	27.	04	4
03.	13	$^1/x$	28.	22	x⇄y
04.	33	STO	29.	31	f
05.	01	1	30.	00	R←P
06.	01	1	31.	84	R/S
07.	32	g	32.	34	RCL
08.	12	$\sin^{-1}$	33.	04	4
09.	71	x	34.	34	RCL
10.	04	4	35.	02	2
11.	71	x	36.	61	+
12.	33	STO	37.	33	STO
13.	02	2	38.	04	4
14.	23	R↓	39.	34	RCL
15.	32	g	40.	03	3
16.	00	R→P	41.	31	f
17.	34	RCL	42.	00	R←P
18.	01	1	43.	-31	GTO 31
19.	12	y^x	44.		
20.	33	STO	45.		
21.	03	3	46.		
22.	22	x⇄y	47.		
23.	34	RCL	48.		
24.	01	1	49.		

REGISTERS	
R_0	
R_1	n, $^1/n$
R_2	c/n
R_3	$R^{1/n}$
R_4	θ/n
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. $(3 + 4.5i)^5 = 926.44 - 4533.47i$

2. $$(5+3i)^{1/3} = \begin{cases} 1.77 + .32i \\ -1.16 + 1.37i \\ -.61 - 1.69i \end{cases}$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store n	n	STO	1			
3	Enter z	b	↑				
		a					
4	For z^n		GTO	1	5	R/S	x
			x⇄y				y
	or						
	$(a + ib)^{1/n}$ (primary root)		BST	R/S			x_0
			x⇄y				y_0
5	Then for other n^{th} roots		R/S				x_i
			x⇄y				y_i

COMPLEX FUNCTIONS z^w, $z^{1/w}$, c^z

Let $z = a_1 + ib_1$ and $w = a_2 + ib_2$ be complex numbers with polar representations $r_1 e^{i\theta_1}$ and $r_2 e^{i\theta_2}$ respectively. Also, let c be a positive real number. The formulas used to evaluate z^w, $z^{1/w}$, and c^z are as follows:

1. $z^w = e^{w \ln z} = e^{(r_2 e^{i\theta_2})(\ln r_1 + i\theta_1)} = e^{r_2 r_3 e^{i(\theta_2+\theta_3)}} = e^{a_4} e^{ib_4}$

   where $\ln z = \ln r_1 + i\theta_1 = r_3 e^{i\theta_3}$

   and $a_4 + ib_4 = r_2 r_3 e^{i(\theta_2+\theta_3)}$

2. $z^{1/w} = z^{w'}$

   where $w' = \frac{1}{r_2} e^{-i\theta_2}$

3. $c^z = e^{z \ln c} = e^{a_1 \ln c} e^{ib_1 \ln c}$

Let the solution in each case be x + iy. The calculator must be set in RADIAN mode for all three cases.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	32	g
02.	00	R→P
03.	13	1/x
04.	22	x⇄y
05.	42	CHS
06.	22	x⇄y
07.	-10	GTO 10
08.	32	g
09.	00	R→P
10.	34	RCL
11.	02	2
12.	34	RCL
13.	01	1
14.	32	g
15.	00	R→P
16.	31	f
17.	22	ln
18.	32	g
19.	00	R→P
20.	22	x⇄y
21.	23	R↓
22.	71	x
23.	33	STO
24.	03	3

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	23	R↓
26.	61	+
27.	34	RCL
28.	03	3
29.	31	f
30.	00	R←P
31.	32	g
32.	22	e^x
33.	31	f
34.	00	R←P
35.	-00	GTO 00
36.	22	x⇄y
37.	34	RCL
38.	01	1
39.	31	f
40.	22	ln
41.	71	x
42.	22	x⇄y
43.	31	f
44.	34	LAST X
45.	71	x
46.	32	g
47.	22	e^x
48.	31	f
49.	00	R←P

REGISTERS	
R_0	
R_1	a_1, c
R_2	b_1
R_3	r_1 r_3
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. $(1 + i)^{(2-i)} = 1.49 + 4.13i$

2. $(1.49 + 4.13i)^{1/(2-i)} = 1.00 + 1.00i$

3. $2^{(3+4i)} = -7.46 + 2.89i$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set to radian mode		f	RAD			
3	For z^w, store z	b_1	STO	2			
		a_1	STO	1			
4	Enter w	b_2	↑				
		a_2	GTO	0	8	R/S	x
			x⇄y				y
	or						
3	For $z^{1/w}$, store z	b_1	STO	2			
		a_1	STO	1			
4	Enter w	b_2	↑				
		a_2	BST	R/S			x
			x⇄y				y
	or						
3	For c^z, store c	c	STO	1			
4	Enter z	b_1	↑				
		a_1	GTO	3	6	R/S	x
			x⇄y				y

COMPLEX TRIGONOMETRIC sin z, csc z

Let z = a + ib be a complex number. The functions sin z and csc z are evaluated by the following formulas:

1. $\sin z = \sin a \cosh b + i \cos a \sinh b$

2. $\csc z = \dfrac{1}{\sin z} \qquad z \neq 0, \pm \pi, \pm 2\pi, \ldots$

Let the solutions be $x_1 + iy_1$ and $x_2 + iy_2$ respectively.

The calculator must be set in RADIAN mode.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	13	1/x
01.	33	STO	26.	51	−
02.	01	1	27.	02	2
03.	31	f	28.	81	÷
04.	12	sin	29.	71	x
05.	22	x⇄y	30.	34	RCL
06.	32	g	31.	03	3
07.	22	e^x	32.	84	R/S
08.	33	STO	33.	32	g
09.	02	2	34.	00	R→P
10.	41	↑	35.	13	1/x
11.	13	1/x	36.	22	x⇄y
12.	61	+	37.	42	CHS
13.	02	2	38.	22	x⇄y
14.	81	÷	39.	31	f
15.	71	x	40.	00	R←P
16.	33	STO	41.	-00	GTO 00
17.	03	3	42.		
18.	34	RCL	43.		
19.	01	1	44.		
20.	31	f	45.		
21.	13	cos	46.		
22.	34	RCL	47.		
23.	02	2	48.		
24.	41	↑	49.		

REGISTERS
R_0
R_1 a
R_2 e^b
R_3 x_1
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Examples:

1. sin (2 + 3i) = 9.15 – 4.17i

2. csc (2 + 3i) = .09 + .04i

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set to radian mode		f	RAD			
3	For sin z, enter z	b	↑				
		a	BST	R/S			x_1
			x⇄y				y_1
	or						
3	For csc z, enter z	b	↑				
		a	BST	R/S			x_1
4	(Do not change the contents of						
	the X and Y registers at this						
	point.)		R/S				x_2
			x⇄y				y_2

COMPLEX TRIGONOMETRIC cos z, sec z

Let z = a + ib be a complex number. The functions cos z and sec z are evaluated by the following formulas:

1. $\cos z = \cos a \cosh b - i \sin a \sinh b$

2. $\sec z = \dfrac{1}{\cos z} \qquad z \neq \pm \dfrac{\pi}{2}, \pm \dfrac{3\pi}{2}, \pm \dfrac{5\pi}{2}, \ldots$

Let the solutions be $x_1 + iy_1$ and $x_2 + iy_2$ respectively.

The calculator must be set in RADIAN mode.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	13	1/x
01.	33	STO	26.	51	–
02.	01	1	27.	02	2
03.	31	f	28.	81	÷
04.	13	cos	29.	71	x
05.	22	x⇄y	30.	42	CHS
06.	32	g	31.	34	RCL
07.	22	e^x	32.	03	3
08.	33	STO	33.	84	R/S
09.	02	2	34.	32	g
10.	41	↑	35.	00	R→P
11.	13	1/x	36.	13	1/x
12.	61	+	37.	22	x⇄y
13.	02	2	38.	42	CHS
14.	81	÷	39.	22	x⇄y
15.	71	x	40.	31	f
16.	33	STO	41.	00	R←P
17.	03	3	42.	-00	GTO 00
18.	34	RCL	43.		
19.	01	1	44.		
20.	31	f	45.		
21.	12	sin	46.		
22.	34	RCL	47.		
23.	02	2	48.		
24.	41	↑	49.		

REGISTERS	
R_0	
R_1	a
R_2	e^b
R_3	x_1
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. $\cos(2 + 3i) = -4.19 - 9.11i$

2. $\sec(2 + 3i) = -.04 + .09i$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set to radian mode		f	RAD			
3	For cos z, enter z	b	↑				
		a	BST	R/S			x_1
			x⇄y				y_1
	or						
3	For sec z, enter z	b	↑				
		a	BST	R/S			x_1
	(Do not change the contents of						
	the X and Y registers at this						
	point.)		R/S				x_2
			x⇄y				y_2

COMPLEX TRIGONOMETRIC tan z, cot z

Let z = a + ib be a complex number. The functions tan z and cot z are evaluated by the following formulas:

1. $\tan z = \dfrac{\sin 2a + i \sinh 2b}{\cos 2a + \cosh 2b} \qquad z \neq \pm \dfrac{\pi}{2}, \pm \dfrac{3\pi}{2}, \ldots$

2. $\cot z = \dfrac{1}{\tan z} \qquad z \neq 0, \pm \dfrac{\pi}{2}, \pm \pi, \pm \dfrac{3\pi}{2}, \ldots$

Let the solutions be $x_1 + iy_1$ and $x_2 + iy_2$ respectively.

The calculator must be set in RADIAN mode.

If $z = \pm \dfrac{\pi}{2}, \pm \dfrac{3\pi}{2}, \ldots$ then cot z = 0.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY	REGISTERS
00.			25.	13	1/x	R_0
01.	02	2	26.	51	–	R_1 2a
02.	71	x	27.	02	2	R_2 e^{2b}
03.	33	STO	28.	81	÷	R_3 cos 2a + cosh 2b
04.	01	1	29.	22	x⇄y	R_4
05.	31	f	30.	81	÷	R_5
06.	13	cos	31.	34	RCL	R_6
07.	22	x⇄y	32.	01	1	R_7
08.	02	2	33.	31	f	R_8
09.	71	x	34.	12	sin	R_9
10.	32	g	35.	34	RCL	$R_{.0}$
11.	22	e^x	36.	03	3	$R_{.1}$
12.	33	STO	37.	81	÷	$R_{.2}$
13.	02	2	38.	84	R/S	$R_{.3}$
14.	41	↑	39.	32	g	$R_{.4}$
15.	13	1/x	40.	00	R→P	$R_{.5}$
16.	61	+	41.	13	1/x	$R_{.6}$
17.	02	2	42.	22	x⇄y	$R_{.7}$
18.	81	÷	43.	42	CHS	$R_{.8}$
19.	61	+	44.	22	x⇄y	$R_{.9}$
20.	33	STO	45.	31	f	
21.	03	3	46.	00	R←P	
22.	34	RCL	47.	-00	GTO 00	
23.	02	2	48.			
24.	41	↑	49.			

Examples:

1. tan (4 + .01i) = 1.16 + .02i

2. cot (4 + .01i) = .86 − .02i

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set to radian mode		f	RAD			
3	For tan z, enter z	b	↑				
		a	BST	R/S			x_1
			x⇄y				y_1
	or						
3	For cot z, enter z	b	↑				
		a	BST	R/S			x_1
	(Do not change the contents of						
	the X and Y registers at this						
	point.)		R/S				x_2
			x⇄y				y_2

COMPLEX HYPERBOLIC sinh z, csch z

Let z = a + ib be a complex number. The functions sinh z and csch z are evaluated by the following formulas:

1. $\sinh z = -i \sin i z = \cos b \sinh a + i \sin b \cosh a$

2. $\text{csch } z = \dfrac{1}{\sinh z} \qquad z \neq 0, \pm i\pi, \pm 2i\pi, \ldots$

Let the solutions be $x_1 + iy_1$ and $x_2 + iy_2$ respectively.

The calculator must be set in RADIAN mode.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	31	f
01.	32	g	26.	13	cos
02.	22	e^x	27.	71	x
03.	41	↑	28.	84	R/S
04.	33	STO	29.	32	g
05.	01	1	30.	00	R→P
06.	13	1/x	31.	13	1/x
07.	61	+	32.	22	x⇄y
08.	02	2	33.	42	CHS
09.	81	÷	34.	22	x⇄y
10.	22	x⇄y	35.	31	f
11.	33	STO	36.	00	R←P
12.	02	2	37.	-00	GTO 00
13.	31	f	38.		
14.	12	sin	39.		
15.	71	x	40.		
16.	34	RCL	41.		
17.	01	1	42.		
18.	41	↑	43.		
19.	13	1/x	44.		
20.	51	–	45.		
21.	02	2	46.		
22.	81	÷	47.		
23.	34	RCL	48.		
24.	02	2	49.		

REGISTERS	
R_0	
R_1	e^a
R_2	b
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. sinh (3 – 2i) = –4.17 – 9.15i

2. csch (1 + 2i) = –.22 – .64i

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set to radian mode		f	RAD			
3	For sinh z, enter z	b	↑				
		a	BST	R/S			x_1
			x⇄y				y_1
	or						
3	For csch z, enter z	b	↑				
		a	BST	R/S			x_1
	(Do not change the contents of						
	the X and Y registers at this						
	point.)		R/S				x_2
			x⇄y				y_2

COMPLEX HYPERBOLIC cosh z, sech z

Let z = a + ib be a complex number. The functions cosh z and sech z are evaluated by the following formulas:

1. $\cosh z = \cos iz = \cos b \cosh a + i \sin b \sinh a$

2. $\text{sech}\, z = \dfrac{1}{\cosh z} \qquad z \neq \pm \dfrac{i\pi}{2}, \pm \dfrac{3i\pi}{2}, \pm \dfrac{5i\pi}{2}, \ldots$

Let the solutions be $x_1 + iy_1$ and $x_2 + iy_2$ respectively.

The calculator must be set in RADIAN mode.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	31	f
01.	32	g	26.	13	cos
02.	22	e^x	27.	71	x
03.	33	STO	28.	84	R/S
04.	01	1	29.	32	g
05.	41	↑	30.	00	R→P
06.	13	1/x	31.	13	1/x
07.	51	–	32.	22	x⇄y
08.	02	2	33.	42	CHS
09.	81	÷	34.	22	x⇄y
10.	22	x⇄y	35.	31	f
11.	33	STO	36.	00	R←P
12.	02	2	37.	-00	GTO 00
13.	31	f	38.		
14.	12	sin	39.		
15.	71	x	40.		
16.	34	RCL	41.		
17.	01	1	42.		
18.	41	↑	43.		
19.	13	1/x	44.		
20.	61	+	45.		
21.	02	2	46.		
22.	81	÷	47.		
23.	34	RCL	48.		
24.	02	2	49.		

REGISTERS	
R_0	
R_1	e^a
R_2	b
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. cosh (1 + 2i) = –.64 + 1.07i

2. sech (1 + 2i) = –.41 –.69i

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set to radian mode		f	RAD			
3	For cosh z, enter z	b	↑				
		a	BST	R/S			x_1
			x⇄y				y_1
	or						
3	For sech z, enter z	b	↑				
		a	BST	R/S			x_1
	(Do not change the contents of						
	the X and Y registers at this						
	point.)		R/S				x_2
			x⇄y				y_2

COMPLEX HYPERBOLIC tanh z, coth z

Let z = a + ib be a complex number. The functions tanh z and coth z are evaluated by the following formulas:

1. $\tanh z = -i \tan iz = \dfrac{\sinh 2a + i \sin 2b}{\cosh 2a + \cos 2b}$ $\qquad z \neq \pm \dfrac{i\pi}{2}, \pm \dfrac{3i\pi}{2}, \ldots$

2. $\coth z = \dfrac{1}{\tanh z}$ $\qquad z \neq \pm \dfrac{i\pi}{2}, \pm \dfrac{3i\pi}{2}, \ldots$ or $z \neq 0, \pm i\pi, \pm 2i\pi$

Let the solutions be $x_1 + iy_1$ and $x_2 + iy_2$ respectively.

The calculator must be set in RADIAN mode.

If $z = \dfrac{i\pi}{2}, \dfrac{3i\pi}{2}, \ldots \coth z = 0$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	02	2
02.	71	x
03.	32	g
04.	22	e^x
05.	41	↑
06.	33	STO
07.	01	1
08.	13	1/x
09.	61	+
10.	02	2
11.	81	÷
12.	22	x⇄y
13.	02	2
14.	71	x
15.	33	STO
16.	02	2
17.	31	f
18.	13	cos
19.	61	+
20.	33	STO
21.	03	3
22.	34	RCL
23.	02	2
24.	31	f
25.	12	sin
26.	22	x⇄y
27.	81	÷
28.	34	RCL
29.	01	1
30.	41	↑
31.	13	1/x
32.	51	–
33.	02	2
34.	81	÷
35.	34	RCL
36.	03	3
37.	81	÷
38.	84	R/S
39.	32	g
40.	00	R→P
41.	13	1/x
42.	22	x⇄y
43.	42	CHS
44.	22	x⇄y
45.	31	f
46.	00	R←P
47.	-00	GTO 00
48.		
49.		

REGISTERS	
R_0	
R_1	e^{2a}
R_2	2b
R_3	cosh 2a + cos 2b
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. tanh (1 + 2i) = 1.17 – .24i

2. coth (1 + 2i) = .82 + .17i

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set to radian mode		f	RAD			
3	For tanh z, enter z	b	↑				
		a	BST	R/S			x_1
			x⇄y				y_1
	or						
3	For coth z, enter z	b	↑				
		a	BST	R/S			x_1
	(Do not change the contents of						
	the X and Y registers at this						
	point.)		R/S				x_2
			x⇄y				y_2

COMPLEX INVERSE TRIGONOMETRIC $\sin^{-1} z$, $\csc^{-1} z$

Let $z=a+ib$ be a complex number. The functions $\sin^{-1} z$ (arc sin) and $\csc^{-1} z$ (arc csc) are evaluated by the following formulas:

1. $\sin^{-1} z = k\pi + (-1)^k \sin^{-1}\beta + (-1)^k\, i\, \mathrm{sgn}(b) \ln\left[\alpha + (\alpha^2 - 1)^{1/2}\right]$

   where $\alpha = \frac{1}{2}\sqrt{(a+1)^2 + b^2} + \frac{1}{2}\sqrt{(a-1)^2 + b^2}$

   $\beta = \frac{1}{2}\sqrt{(a+1)^2 + b^2} - \frac{1}{2}\sqrt{(a-1)^2 + b^2}$

   $\mathrm{sgn}(b) = b/\sqrt{b^2} = \begin{cases} 1 & \text{if } b > 0 \\ -1 & \text{if } b < 0 \end{cases}$

   $k = 0, \pm 1, \pm 2, \ldots$

2. $\csc^{-1} z = \sin^{-1}\left[\frac{1}{z}\right] \quad z \neq 0$

k is assumed to be zero for this program.

Program does not work for $b = 0$ but the calculator functions can be used instead.

Let the solutions be $x_1 + iy_1$ and $x_2 + iy_2$ respectively.

The calculator must be in RADIAN mode and all angles are assumed to be in radians.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	71	x
01.	22	x⇄y	26.	01	1
02.	33	STO	27.	51	–
03.	02	2	28.	31	f
04.	22	x⇄y	29.	42	$\sqrt{x}$
05.	01	1	30.	61	+
06.	51	–	31.	31	f
07.	32	g	32.	22	ln
08.	00	R→P	33.	34	RCL
09.	33	STO	34.	02	2
10.	03	3	35.	41	↑
11.	31	f	36.	32	g
12.	00	R←P	37.	42	x^2
13.	02	2	38.	31	f
14.	61	+	39.	42	$\sqrt{x}$
15.	32	g	40.	81	÷
16.	00	R→P	41.	71	x
17.	34	RCL	42.	22	x⇄y
18.	03	3	43.	34	RCL
19.	61	+	44.	03	3
20.	02	2	45.	51	–
21.	81	÷	46.	32	g
22.	41	↑	47.	13	$\sin^{-1}$
23.	41	↑	48.	-00	GTO 00
24.	41	↑	49.		

REGISTERS	
R_0	
R_1	
R_2	b
R_3	$\sqrt{(a-1)^2+b^2}$
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. $\sin^{-1}(5 + 8i) = .56 + 2.94i$

2. $\csc^{-1}(5 + 8i) = .06 - .09i$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set to radian mode		f	RAD			
3	For $\sin^{-1} z$, enter z	b	↑				
		a	BST	R/S			x_1
			x⇄y				y_1
	or						
3	For $\csc^{-1} z$, enter z						
	and calculate 1/z	b	↑				
		a	g	R→P	1/x	x⇄y	
			CHS	x⇄y	f	R←P	
			BST	R/S			x_2
			x⇄y				y_2

COMPLEX INVERSE TRIGONOMETRIC $\cos^{-1} z$, $\sec^{-1} z$

Let $z = a + ib$ be a complex number. The functions $\cos^{-1} z$ (arc cos) and $\sec^{-1} z$ (arc sec) are evaluated by the following formulas:

1. $\cos^{-1} z = 2k\pi \pm \left\{\cos^{-1} \beta - i \operatorname{sgn}(b) \ln \left[\alpha + (\alpha^2 - 1)^{1/2}\right]\right\}$

   where $\alpha = \frac{1}{2}\sqrt{(a+1)^2 + b^2} + \frac{1}{2}\sqrt{(a-1)^2 + b^2}$

   $\beta = \frac{1}{2}\sqrt{(a+1)^2 + b^2} - \frac{1}{2}\sqrt{(a-1)^2 + b^2}$

   $\operatorname{sgn}(b) = b/\sqrt{b^2} = \begin{cases} 1 \text{ if } b > 0 \\ -1 \text{ if } b < 0 \end{cases}$

   $k = 0, \pm 1, \pm 2, \ldots$

2. $\sec^{-1} z = \cos^{-1}\left(\frac{1}{z}\right), \quad z \neq 0$

k is assumed to be zero for this program.

Program does not work for $b = 0$ but the inbuilt calculator functions can be used instead.

Let the solutions be $x_1 + iy_1$ and $x_2 + iy_2$ respectively.

The calculator must be in RADIAN mode and all angles are assumed to be in radians.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	71	x
01.	22	x⇄y	26.	01	1
02.	33	STO	27.	51	–
03.	02	2	28.	31	f
04.	22	x⇄y	29.	42	$\sqrt{x}$
05.	01	1	30.	61	+
06.	51	–	31.	31	f
07.	32	g	32.	22	ln
08.	00	R→P	33.	42	CHS
09.	33	STO	34.	34	RCL
10.	03	3	35.	02	2
11.	31	f	36.	41	↑
12.	00	R←P	37.	32	g
13.	02	2	38.	42	x^2
14.	61	+	39.	31	f
15.	32	g	40.	42	$\sqrt{x}$
16.	00	R→P	41.	81	÷
17.	34	RCL	42.	71	x
18.	03	3	43.	22	x⇄y
19.	61	+	44.	34	RCL
20.	02	2	45.	03	3
21.	81	÷	46.	51	–
22.	41	↑	47.	32	g
23.	41	↑	48.	13	$\cos^{-1}$
24.	41	↑	49.	-00	GTO 00

REGISTERS
R_0
R_1
R_2 b
R_3 $\sqrt{(a-1)^2 + b^2}$
R_4
R_5
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Examples:

1. $\cos^{-1}(0.9 + 3i) = 1.29 - 1.86i$

2. $\sec^{-1}(0.9 + 3i) = 1.48 + .30i$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set to radian mode		f	RAD			
3	For $\cos^{-1} z$, enter z	b	↑				
		a	BST	R/S			x_1
			x⇄y				y_1
	or						
3	For $\sec^{-1} z$, enter z						
	and calculate 1/z	b	↑				
		a	g	R→P	1/x	x⇄y	
			CHS	x⇄y	f	R←P	
			BST	R/S			x_2
			x⇄y				y_2

COMPLEX INVERSE TRIGONOMETRIC $\tan^{-1} z$, $\cot^{-1} z$

Let $z = a + ib$ be a complex number. The functions $\tan^{-1}$ (arc tan) and $\cot^{-1} z$ (arc cot) are evaluated by the following formulas:

1. $$\tan^{-1} z = \tfrac{1}{2}\left[(2k+1)\pi - \tan^{-1}\left(\frac{1+b}{a}\right) - \tan^{-1}\left(\frac{1-b}{a}\right)\right] + \frac{i}{2}\ln\left(\frac{[(1+b)^2+a^2]^{1/2}}{[(1-b)^2+a^2]^{1/2}}\right)$$

   where $k = 0, \pm 1, \pm 2, \ldots \quad (z^2 \neq -1)$

2. $$\cot^{-1} z = \tan^{-1}\left(\frac{1}{z}\right) \quad (z^2 \neq -1 \text{ and } z \neq 0)$$

The rectangular to polar routine is used so the division by zero problem is avoided in the evaluation of $\tan^{-1}\left(\frac{1+b}{a}\right)$ and $\tan^{-1}\left(\frac{1-b}{a}\right)$.

k is assumed to be zero.

Let the solutions be $x_1 + iy_1$ and $x_2 + iy_2$ respectively.

The calculator must be in RADIAN mode and all angles are assumed to be in radians.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	33	STO
02.	01	1
03.	22	x⇄y
04.	33	STO
05.	02	2
06.	01	1
07.	61	+
08.	22	x⇄y
09.	32	g
10.	00	R→P
11.	01	1
12.	34	RCL
13.	02	2
14.	51	–
15.	34	RCL
16.	01	1
17.	32	g
18.	00	R→P
19.	22	x⇄y
20.	23	R↓
21.	81	÷
22.	31	f
23.	22	ln
24.	02	2

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	81	÷
26.	31	f
27.	83	π
28.	23	R↓
29.	23	R↓
30.	61	+
31.	51	–
32.	02	2
33.	81	÷
34.	-00	GTO 00
35.	32	g
36.	00	R→P
37.	13	1/x
38.	22	x⇄y
39.	42	CHS
40.	22	x⇄y
41.	31	f
42.	00	R←P
43.	-01	GTO 01
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	a
R_2	b
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. $\tan^{-1}(5 + 8i) = 1.51 + .09i$

2. $\cot^{-1}(5 + 8i) = .06 - .09i$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set to radian mode		f	RAD			
3	For $\tan^{-1} z$, enter z	b	↑				
		a	BST	R/S			x_1
			x⇄y				y_1
	or						
3	For $\cot^{-1} z$, enter z	b	↑				
		a	GTO	3	5	R/S	x_2
			x⇄y				y_2

COMPLEX INVERSE HYPERBOLIC $\sinh^{-1} z$, $\operatorname{csch}^{-1} z$

Let $z = a + ib$ be a complex number. The functions $\sinh^{-1} z$ (arc hyperbolic sine) and $\operatorname{csch}^{-1} z$ (arc hyperbolic cosecant) are evaluated by the following formulas:

1. $\sinh^{-1} z = -i \sin^{-1} iz = (-1)^k \operatorname{sgn}(a) \ln [\alpha + (\alpha^2 - 1)^{1/2}]$

$$+ [(k\pi + (-1)^k \sin^{-1}(-\beta)]\, i$$

where $\alpha = \tfrac{1}{2}\sqrt{(1-b)^2 + a^2} + \tfrac{1}{2}\sqrt{(1+b)^2 + a^2}$

$\beta = \tfrac{1}{2}\sqrt{(1-b)^2 + a^2} - \tfrac{1}{2}\sqrt{(1+b)^2 + a^2}$

$\operatorname{sgn}(a) = a/\sqrt{a^2}$

$k = 0, \pm 1, \pm 2, \ldots$

2. $\operatorname{csch}^{-1} z = \sinh^{-1}\left(\frac{1}{z}\right) \quad z \neq 0$

k is assumed to be zero.

Let the solutions be $x_1 + iy_1$ and $x_2 + iy_2$ respectively.

The calculator must be in RADIAN mode and all angles are assumed to be in radians.

Note:

If the calculator flashes zero because $a = 0$ in the division at line 39, the result can still be found by rolling down twice, [R↓], [R↓], switching to PRGM, single stepping twice, **SST** , **SST** , switching to RUN, and pressing **R/S** .

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	01	1
01.	33	STO	26.	51	–
02.	02	2	27.	31	f
03.	22	x⇄y	28.	42	√x
04.	01	1	29.	61	+
05.	51	–	30.	31	f
06.	32	g	31.	22	ln
07.	00	R→P	32.	34	RCL
08.	33	STO	33.	02	2
09.	03	3	34.	41	↑
10.	31	f	35.	32	g
11.	00	R←P	36.	42	x^2
12.	02	2	37.	31	f
13.	61	+	38.	42	√x
14.	32	g	39.	81	÷
15.	00	R→P	40.	71	x
16.	34	RCL	41.	22	x⇄y
17.	03	3	42.	34	RCL
18.	61	+	43.	03	3
19.	02	2	44.	51	–
20.	81	÷	45.	32	g
21.	41	↑	46.	12	sin^{-1}
22.	41	↑	47.	22	x⇄y
23.	41	↑	48.	-00	GTO 00
24.	71	x	49.		

REGISTERS	
R_0	
R_1	
R_2	a
R_3	$\sqrt{(1-b)^2+a^2}$
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. $\sinh^{-1}(3.14 + 10.3i) = 3.07 + 1.27i$

2. $\operatorname{csch}^{-1}(3.14 + 10.3i) = .03 - .09i$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set to radian mode		f	RAD			
3	For $sinh^{-1}$ z, enter z	b	↑				
		a	BST	R/S			x_1
			x⇄y				y_1
	or						
3	For $csch^{-1}$ z, enter z						
	and calculate 1/z	b	↑				
		a	g	R→P	1/x	x⇄y	
			CHS	x⇄y	f	R←P	
			BST	R/S			x_2
			x⇄y				y_2

COMPLEX INVERSE HYPERBOLIC $\cosh^{-1}$ z, sech^{-1} z

Let z = a + ib be a complex number. The functions $\cosh^{-1}$ z (arc hyperbolic cosine) and sech^{-1} z (arc hyperbolic secant) are evaluated by the following formulas:

1. $\cosh^{-1} z = 2k\pi i \pm \left\{\text{sgn}(b) \ln \left[\alpha + (\alpha^2 - 1)^{1/2}\right] + i \cos^{-1} \beta\right\}$

   where $\alpha = \tfrac{1}{2}\sqrt{(a+1)^2 + b^2} + \tfrac{1}{2}\sqrt{(a-1)^2 + b^2}$

   $\beta = \tfrac{1}{2}\sqrt{(a+1)^2 + b^2} - \tfrac{1}{2}\sqrt{(a-1)^2 + b^2}$

   $\text{sgn}(b) = b/\sqrt{b^2}$

   $k = 0, \pm 1, \pm 2, \ldots$

2. $\text{sech}^{-1} z = \cosh^{-1}\left(\frac{1}{z}\right) \qquad z \neq 0$

Let the solutions be $x_1 + iy_1$ and $x_2 + iy_2$ respectively.

The calculator must be in RADIAN mode.

If the calculator flashes zero because b = 0 in the division at line 40, the result can still be found by rolling down twice, [R↓], [R↓], switching to PRGM, single stepping twice, [SST], [SST], switching to RUN, and pressing [R/S].

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	71	x
01.	22	x⇄y	26.	01	1
02.	33	STO	27.	51	–
03.	02	2	28.	31	f
04.	22	x⇄y	29.	42	$\sqrt{x}$
05.	01	1	30.	61	+
06.	51	–	31.	31	f
07.	32	g	32.	22	ln
08.	00	R→P	33.	34	RCL
09.	33	STO	34.	02	2
10.	03	3	35.	41	↑
11.	31	f	36.	32	g
12.	00	R←P	37.	42	x^2
13.	02	2	38.	31	f
14.	61	+	39.	42	$\sqrt{x}$
15.	32	g	40.	81	÷
16.	00	R→P	41.	71	x
17.	34	RCL	42.	22	x⇄y
18.	03	3	43.	34	RCL
19.	61	+	44.	03	3
20.	02	2	45.	51	–
21.	81	÷	46.	32	g
22.	41	↑	47.	13	$\cos^{-1}$
23.	41	↑	48.	22	x⇄y
24.	41	↑	49.	-00	GTO 00

REGISTERS	
R_0	
R_1	
R_2	b
R_3	$\sqrt{(a-1)^2+b^2}$
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. $\cosh^{-1}(5 + 8i) = 2.94 + 1.01i$
2. $\text{sech}^{-1}(5 + 8i) = -.09 + 1.51i$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set to radian mode		f	RAD			
3	For $\cosh^{-1} z$, enter z	b	↑				
		a	BST	R/S			x_1
			x⇄y				y_1
	or						
3	For $\text{sech}^{-1} z$, enter z						
	and calculate 1/z	b	↑				
		a	g	R→P	1/x	x⇄y	
			CHS	x⇄y	f	R←P	
			BST	R/S			x_2
			x⇄y				y_2

COMPLEX INVERSE HYPERBOLIC $\tanh^{-1} z$, $\coth^{-1} z$

Let $z = a + ib$ be a complex number. The functions $\tanh^{-1} z$ (arc hyperbolic tangent) and $\coth^{-1} z$ (arc hyperbolic cotangent) are evaluated by the following formulas:

1. $$\tanh^{-1} z = -i \tan^{-1} i z = \tfrac{1}{2} \ln \left[\frac{[(1 + a)^2 + b^2]^{1/2}}{[(1 - a)^2 + b^2]^{1/2}} \right]$$

$$+ \left(\frac{i}{2}\right) \left[-(2k + 1)\pi + \tan^{-1}\left(\frac{1 + a}{-b}\right) + \tan^{-1}\left(\frac{1 - a}{-b}\right) \right] \quad (z^2 \neq 1)$$

2. $$\coth^{-1} z = \tanh^{-1}\left(\frac{1}{z}\right) \quad (z^2 \neq 1,\ z \neq 0)$$

The rectangular to polar routine is used so the division by zero problem is avoided in the evaluation of $\tan^{-1}\left(\frac{1 + a}{-b}\right)$ and $\tan^{-1}\left(\frac{1 - a}{-b}\right)$.

k is assumed to be zero.

Let the solutions be $x_1 + iy_1$ and $x_2 + iy_2$ respectively.

The calculator must be in RADIAN mode.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	33	STO
02.	02	2
03.	01	1
04.	61	+
05.	22	x⇄y
06.	42	CHS
07.	33	STO
08.	01	1
09.	32	g
10.	00	R→P
11.	01	1
12.	34	RCL
13.	02	2
14.	51	–
15.	34	RCL
16.	01	1
17.	32	g
18.	00	R→P
19.	22	x⇄y
20.	23	R↓
21.	81	÷
22.	31	f
23.	22	ln
24.	02	2
25.	81	÷
26.	31	f
27.	83	π
28.	23	R↓
29.	23	R↓
30.	61	+
31.	51	–
32.	02	2
33.	81	÷
34.	42	CHS
35.	22	x⇄y
36.	-00	GTO 00
37.	32	g
38.	00	R→P
39.	13	1/x
40.	22	x⇄y
41.	42	CHS
42.	22	x⇄y
43.	31	f
44.	00	R←P
45.	-01	GTO 01
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	–b
R_2	a
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. $\tanh^{-1}(8 - 5i) = .09 - 1.51i$

2. $\coth^{-1}(-7i) = .14i$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Set to radian mode		f	RAD			
3	For $\tanh^{-1} z$, enter z	b	↑				
		a	BST	R/S			x_1
			x⇄y				y_1
	or						
3	For $\coth^{-1} z$, enter z	b	↑				
		a	GTO	3	7	R/S	x_2
			x⇄y				y_2

COMPLEX POLYNOMIAL EVALUATION

Given a polynomial (with complex coefficients) of the form

$$f(z) = (a_1 + ib_1) z^n + (a_2 + ib_2) z^{n-1} + \ldots + (a_n + ib_n) z + (a_{n+1} + ib_{n+1})$$

This program evaluates f(z) for a complex number $z_o = c + di$. Let the solution be x + iy.

If a coefficient is zero it still must be entered.

The polynomial is evaluated in the form

$$(\ldots((a_1 + ib_1) z + (a_2 + ib_2)) z + \ldots) + (a_{n+1} + ib_{n+1})$$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	34	RCL
01.	33	STO	26.	07	7
02.	05	5	27.	31	f
03.	23	R↓	28.	00	R←P
04.	33	STO	29.	34	RCL
05.	06	6	30.	05	5
06.	34	RCL	31.	61	+
07.	04	4	32.	33	STO
08.	34	RCL	33.	03	3
09.	03	3	34.	22	x⇄y
10.	32	g	35.	34	RCL
11.	00	R→P	36.	06	6
12.	34	RCL	37.	61	+
13.	02	2	38.	33	STO
14.	34	RCL	39.	04	4
15.	01	1	40.	22	x⇄y
16.	32	g	41.	-00	GTO 00
17.	00	R→P	42.		
18.	22	x⇄y	43.		
19.	23	R↓	44.		
20.	71	x	45.		
21.	33	STO	46.		
22.	07	7	47.		
23.	23	R↓	48.		
24.	61	+	49.		

REGISTERS	
R_0	
R_1	c
R_2	d
R_3	a_1
R_4	b_1
R_5	Used
R_6	Used
R_7	Used
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Evaluate the complex polynomial $f(z) = (3 + 4i)\ z^4 + 18\ z^3 + (-2 + i)\ z^2 - 10\ z + (5 - 7i)$ at the complex number $z_o = 2 + i$.

Solution:

$f(z_o) = -106.00 + 220.00i$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store z_0	d	STO	2			
		c	STO	1			
3	Store $a_1 + b_1 i$	b_1	STO	4			
		a_1	STO	3	BST		
4	Repeat this step for k = 2, 3,..., n						
	Enter $a_k + b_k i$	b_k	↑				
		a_k	R/S				Temp
5	Enter $a_{n+1} + ib_{n+1}$	b_{n+1}	↑				
		a_{n+1}	R/S				x
			x⇄y				y

COMPOUNDED AMOUNT

Let n = number of time periods

i = periodic interest rate expressed as a decimal, e.g., 6% is represented as .06

PV = present value or principal

FV = future value or amount

I = interest amount

Each value can be calculated from the others by the following formulas:

1. $FV = PV\,(1 + i)^n$

2. $PV = FV\,(1 + i)^{-n}$

3. $n = \dfrac{\ln (FV/PV)}{\ln (1 + i)}$

4. $i = \left(\dfrac{FV}{PV}\right)^{1/n} - 1$

5. $I = PV\,[(1 + i)^n - 1]$

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	34	RCL
02.	02	2
03.	01	1
04.	61	+
05.	34	RCL
06.	01	1
07.	12	y^x
08.	34	RCL
09.	03	3
10.	71	x
11.	-00	GTO 00
12.	34	RCL
13.	04	4
14.	31	f
15.	22	ln
16.	34	RCL
17.	02	2
18.	01	1
19.	61	+
20.	31	f
21.	22	ln
22.	81	÷
23.	-00	GTO 00
24.	34	RCL

DISPLAY		KEY ENTRY
LINE	CODE	
25.	04	4
26.	34	RCL
27.	01	1
28.	13	$1/x$
29.	12	y^x
30.	01	1
31.	51	–
32.	-00	GTO 00
33.	34	RCL
34.	02	2
35.	01	1
36.	61	+
37.	34	RCL
38.	01	1
39.	12	y^x
40.	01	1
41.	51	–
42.	34	RCL
43.	03	3
44.	71	x
45.	-00	GTO 00
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	n, –n
R_2	i
R_3	PV, FV
R_4	FV/PV
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. Find the future amount of $1500 invested at 6% (.06) compounded annually for 5 years.

2. What sum invested today at 6% compounded annually will amount to $2007.34 in 5 years?

3. How long does it take $100 to double if it is invested at 6% annual interest compounded quarterly?

4. Find the rate of return on $2000 compounded monthly if it amounts to $3000 at the end of 5 years?

5. Find the compound interest on $1500 for 5 years if interest at 6% is compounded annually.

Solutions:

1. $2007.34

2. $1500.00

3. 46.56 quarters = 11.64 years (i = .06/4)

4. .0068 monthly = 8.14% annually (n = 60)

5. $507.34

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	For FV	n	STO	1			
		i	STO	2			
		PV	STO	3			
			BST	R/S			FV
2	For PV	n	CHS	STO	1		
		i	STO	2			
		FV	STO	3			
			BST	R/S			PV
2	For n	FV	↑				
		PV	÷	STO	4		
		i	STO	2			
			GTO	1	2	R/S	n
2	For i	FV	↑				
		PV	÷	STO	4		
		n	STO	1			
			GTO	2	4	R/S	i
2	For I	n	STO	1			
		i	STO	2			
		PV	STO	3			
			GTO	3	3	R/S	I

DIRECT REDUCTION LOAN INTEREST RATE

This program calculates the interest rate on a mortgage where payments are made at the end of period.

Let n = number of payments

i = periodic interest rate expressed as a decimal, e.g., 6% is represented as .06

PMT = payment

PV = present value or principal

This routine solves the equation f(i) by an iteration for i using Newton's method:

$$i_{k+1} = i_k - \frac{f(i)}{f'(i)}$$

where $f(i) = \frac{1-(1+i)^{-n}}{i} - \frac{PV}{PMT}$

and $f'(i)$ is the first derivative of f(i)

First an initial guess i_0 is stored in R_2. For i_0 either the suggested routine or a rough guess can be used. However, i_0 cannot be zero. If the suggested routine produces zero, then zero is the solution.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	34	RCL
02.	03	3
03.	34	RCL
04.	02	2
05.	71	x
06.	01	1
07.	34	RCL
08.	02	2
09.	01	1
10.	61	+
11.	34	RCL
12.	01	1
13.	42	CHS
14.	12	y^x
15.	33	STO
16.	05	5
17.	51	–
18.	51	–
19.	34	RCL
20.	01	1
21.	34	RCL
22.	02	2
23.	13	$1/x$
24.	01	1

DISPLAY		KEY ENTRY
LINE	CODE	
25.	61	+
26.	81	÷
27.	01	1
28.	61	+
29.	34	RCL
30.	05	5
31.	71	x
32.	01	1
33.	51	–
34.	34	RCL
35.	02	2
36.	81	÷
37.	81	÷
38.	33	STO
39.	61	+
40.	02	2
41.	41	↑
42.	71	x
43.	43	EEX
44.	01	1
45.	02	2
46.	42	CHS
47.	31	f
48.	-01	x≤y 01
49.	-00	GTO 00

REGISTERS	
R_0	
R_1	n
R_2	i
R_3	PV/PMT
R_4	
R_5	$(1 + i)^{-n}$
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Find the monthly interest rate on a mortgage of $30,000 if the loan requires 360 monthly payments of $179.86 to be payed off.

Answer:

.0050 (0.5%) [FIX] [4] (15 seconds iteration)

The suggested routine supplies an initial guess of .0047 (.47%). The user can notice the change in iteration time if he tries different i_0's such as .001 or .01 (25 seconds iteration).

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store PV and PMT	PV	↑				
		PMT	÷	STO	3		
3	Store n	n	STO	1			
4	Store a rough guess	i_0	STO	2			
	or						
4	Calculate an initial guess		RCL	3	1/x	RCL	
			1	↑	x	1/x	
			RCL	3	x	–	
			STO	2			i_0
5	Iterate for i		BST	R/S			1.0000000–12
6	When iteration stops		RCL	2			i

DIRECT REDUCTION LOAN PAYMENT, PRESENT VALUE, NUMBER OF TIME PERIODS

Calculates payment, present value, and number of time periods of a mortgage given two of the three and the interest rate.

Let n = number of payment periods

PV = present value or principal

PMT = payment

i = periodic interest rate expressed as a decimal, e.g., 6% is represented as .06.

Then, PMT, PV, and n can be calculated from the other three by the following formulas:

1. $$PMT = PV\left[\frac{i}{1-(1+i)^{-n}}\right]$$

2. $$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

3. $$n = -\frac{\ln(1 - iPV/PMT)}{\ln(1+i)}$$

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	34	RCL
02.	02	2
03.	01	1
04.	34	RCL
05.	02	2
06.	01	1
07.	61	+
08.	34	RCL
09.	01	1
10.	42	CHS
11.	12	y^x
12.	51	–
13.	81	÷
14.	-28	GTO 28
15.	01	1
16.	34	RCL
17.	02	2
18.	01	1
19.	61	+
20.	34	RCL
21.	01	1
22.	42	CHS
23.	12	y^x
24.	51	–

DISPLAY		KEY ENTRY
LINE	CODE	
25.	34	RCL
26.	02	2
27.	81	÷
28.	34	RCL
29.	03	3
30.	71	x
31.	-00	GTO 00
32.	01	1
33.	34	RCL
34.	03	3
35.	34	RCL
36.	02	2
37.	71	x
38.	51	–
39.	31	f
40.	22	ln
41.	34	RCL
42.	02	2
43.	01	1
44.	61	+
45.	31	f
46.	22	ln
47.	81	÷
48.	42	CHS
49.	-00	GTO 00

REGISTERS	
R_0	
R_1	n
R_2	i
R_3	PMT, PV, PV/PMT
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. To pay off a loan of $4000 at 9.5% (.095) in 30 months, what monthly payment is required?

2. A person is willing to pay $150 per month for 30 months for a loan at 9.5%. How much can be borrowed?

3. How many monthly payments of $100 must be made to pay off a loan of $4000 at 9.5% annual interest?

Note:

Divide 0.095 by 12 to find the monthly interest rate expressed as a decimal.

Answers:

1. $150.32

2. $3991.55

3. 48.29 (4.02 years)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	For payment	PV	STO	3			
		i	STO	2			
		n	STO	1			
			BST	R/S			PMT
	or						
2	For present value	PMT	STO	3			
		i	STO	2			
		n	STO	1			
			GTO	1	5	R/S	PV
	or						
3	For number of payments	PV	↑				
		PMT	÷	STO	3		
		i	STO	2			
			GTO	3	2	R/S	n

DIRECT REDUCTION LOAN ACCUMULATED INTEREST, REMAINING BALANCE

This program finds the accumulated interest and remaining balance of a mortgage.

Let I_{c-k} = the accumulated interest paid by payments c through k

PV_k = the remaining balance after payment k.

n = number of payments

i = periodic interest rate expressed as a decimal, e.g., 6% is expressed as .06

j = c − 1

Then, I_{c-k} and PV_k can be calculated by the following formulas:

1. $$I_{c-k} = PMT\left[k - j - \frac{(1+i)^{k-n}}{i}(1 - (1+i)^{j-k})\right]$$

2. $$PV_k = \frac{PMT}{i}\left[1 - (1+i)^{k-n}\right]$$

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	34	RCL
02.	02	2
03.	01	1
04.	61	+
05.	34	RCL
06.	04	4
07.	34	RCL
08.	01	1
09.	00	0
10.	61	+
11.	51	–
12.	12	y^x
13.	22	x⇄y
14.	34	RCL
15.	05	5
16.	34	RCL
17.	04	4
18.	51	–
19.	33	STO
20.	06	6
21.	12	y^x
22.	01	1
23.	51	–
24.	71	x

DISPLAY		KEY ENTRY
LINE	CODE	
25.	34	RCL
26.	02	2
27.	81	÷
28.	34	RCL
29.	06	6
30.	51	–
31.	–47	GTO 47
32.	01	1
33.	34	RCL
34.	02	2
35.	01	1
36.	61	+
37.	34	RCL
38.	04	4
39.	34	RCL
40.	01	1
41.	51	–
42.	12	y^x
43.	51	–
44.	34	RCL
45.	02	2
46.	81	÷
47.	34	RCL
48.	03	3
49.	71	x

REGISTERS
R_0
R_1 n
R_2 i
R_3 PMT
R_4 k
R_5 j
R_6 j –k
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Examples:

1. A house costs $30,000 with a mortgage life of 30 years at 8% (.08) yearly interest. Find the interest paid on the first 36 monthly payments. The payment can be calculated from the program, Direct Reduction Loan Payment but is $220.13.

2. Using the above example calculate the remaining balance after the 36th payment.

Solutions:

1. $7108.72

2. $29,184.13

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	For accumulated						
	interest of c^{th}						
	through k^{th} payment	n	STO	1			
		i	STO	2			
		PMT	STO	3			
		k	STO	4			
		c−1	STO	5			
			BST	R/S			Ic–k
	or						
2	For remaining balance						
	after k^{th} payment	n	STO	1			
		i	STO	2			
		PMT	STO	3			
		k	STO	4			
			GTO	3	2	R/S	PV_k

DIRECT REDUCTION LOAN AMORTIZATION SCHEDULES

This program calculates a table of interest paid, payment to principal, and present value of a mortgage. It also can be used to find accumulated interest.

Let I_k = interest paid in k^{th} payment

PMT = payment

PP_k = payment to principal of k^{th} payment

PV_k = remaining balance after k^{th} payment

PV_0 = amount of loan

i = periodic interest rate expressed as a decimal, e.g., 6% is expressed as .06

An amortization schedule consists of the interest paid, the payment to principal, and the remaining balance for each payment k = 1, 2, ...

These quantities are calculated by the following formulas:

1. $I_k = i\ PV_{k-1}$
2. $PP_k = PMT - I_k$
3. $PV_k = PV_{k-1} - PP_k$

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	00	0
02.	33	STO
03.	01	1
04.	34	RCL
05.	04	4
06.	34	RCL
07.	02	2
08.	71	x
09.	33	STO
10.	05	5
11.	33	STO
12.	61	+
13.	01	1
14.	84	R/S
15.	34	RCL
16.	03	3
17.	34	RCL
18.	05	5
19.	51	–
20.	33	STO
21.	06	6
22.	84	R/S
23.	34	RCL
24.	04	4

DISPLAY		KEY ENTRY
LINE	CODE	
25.	34	RCL
26.	06	6
27.	51	–
28.	33	STO
29.	04	4
30.	84	R/S
31.	–04	GTO 04
32.		
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS
R_0
R_1 ΣI_K
R_2 i
R_3 PMT
R_4 PV_K
R_5 I_K
R_6 PP_K
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

Find the amortization schedule for a loan of \$30,000 at 7% (.07) annual interest with payments of \$200 monthly.

Note:

Be sure to enter monthly interest rate by dividing 0.07 by 12.

Solution:

Payment No.	I_k	PP_k	PV_k	ΣI
1	175.00	25.00	29,975.00	175.00
2	174.85	25.15	29,949.85	349.85
3	174.71	25.29	29,924.56	524.56

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store periodic interest rate	i	STO	2			
3	Store payment	PMT	STO	3			
4	Store initial loan amount	PV_0	STO	4	BST		
5	Calculate interest		R/S				I_k
6	Calculate payment to principal		R/S				PP_k
7	Calculate remaining balance		R/S				PV_k
8	Repeat steps 5, 6, and 7						
	For k = 2, 3, 4,...						
	Note: At anytime after steps						
	5, 6, or 7 have been executed the						
	accumulated interest can be						
	found.		RCL	1			ΣI

SINKING FUND INTEREST RATE

This program calculates the interest rate on a savings program where payments are assumed to be made at the end of compounding period.

Let n = number of payments

i = periodic interest rate expressed as a decimal, e.g., 6% is expressed as .06

PMT = payment

FV = future value or amount

This routine solves the equation f(i) by an interation for i using Newton's method:

$$i_{k+1} = i_k - \frac{f(i)}{f'(i)}$$

where $$f(i) = \frac{(1+i)^n - 1}{i} - \frac{FV}{PMT}$$

and $f'(i)$ is the first derivative of $f(i)$. First an initial guess i_0 is stored in R_2. Either the suggested routine or a rough guess can be used. However, i_0 cannot be zero. If the suggested routine produces zero, then zero is the solution.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	34	RCL
02.	03	3
03.	34	RCL
04.	02	2
05.	71	x
06.	01	1
07.	61	+
08.	34	RCL
09.	02	2
10.	01	1
11.	61	+
12.	34	RCL
13.	01	1
14.	12	y^x
15.	33	STO
16.	04	4
17.	51	–
18.	34	RCL
19.	01	1
20.	01	1
21.	34	RCL
22.	02	2
23.	13	$^1/x$
24.	61	+

DISPLAY		KEY ENTRY
LINE	CODE	
25.	81	÷
26.	01	1
27.	51	–
28.	34	RCL
29.	04	4
30.	71	x
31.	01	1
32.	61	+
33.	34	RCL
34.	02	2
35.	81	÷
36.	81	÷
37.	33	STO
38.	61	+
39.	02	2
40.	41	↑
41.	71	x
42.	43	EEX
43.	42	CHS
44.	01	1
45.	02	2
46.	31	f
47.	-01	x≤y 01
48.	34	RCL
49.	02	2

REGISTERS
R_0
R_1 n
R_2 i
R_3 FV/PMT
R_4 $(1 + i)^n$
R_5
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

What annual rate of interest must be obtained to amass a total of $10,000 in 10 years on an annual investment of $600.

Solution:

.1093 (10.93%) [FIX] [4] (20 seconds iteration)

The suggested routine supplies an initial guess of .1365 (13.65%). The user can try other initial guesses such as .10 or .20 and notice the change in iteration time. (15 and 25 seconds iteration time respectively.)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store FV and PMT	FV	↑				
		PMT	÷	STO	3		
3	Store n	n	STO	1			
4	Enter a rough guess for i	i_0	STO	2			
	or						
4	Calculate an initial guess		RCL	3	RCL	1	
			–	2	x	RCL	
			1	1	–	↑	
			x	RCL	3	+	
			÷	STO	2		i_0
5	Calculate i		BST	R/S			i

SINKING FUND PAYMENT, FUTURE VALUE, NUMBER OF TIME PERIODS

This program calculates payment, future value, or number of time periods given two of the three and the interest rate.

Let n = number of payments

i = periodic interest rate expressed as a decimal, e.g., 6% is expressed as .06

PMT = payment

FV = future value

Then, FV, PMT, or n can be calculated from the other three by the following formulas:

1. $$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

2. $$PMT = FV\left[\frac{i}{(1+i)^n - 1}\right]$$

3. $$n = \frac{\ln\left(i\,\frac{FV}{PMT} + 1\right)}{\ln(1+i)}$$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	61	+
01.	34	RCL	26.	34	RCL
02.	02	2	27.	01	1
03.	01	1	28.	12	y^x
04.	61	+	29.	01	1
05.	34	RCL	30.	51	–
06.	01	1	31.	81	÷
07.	12	y^x	32.	-00	GTO 00
08.	01	1	33.	34	RCL
09.	51	–	34.	03	3
10.	34	RCL	35.	34	RCL
11.	02	2	36.	02	2
12.	81	÷	37.	71	x
13.	34	RCL	38.	01	1
14.	03	3	39.	61	+
15.	71	x	40.	31	f
16.	-00	GTO 00	41.	22	ln
17.	34	RCL	42.	34	RCL
18.	02	2	43.	02	2
19.	34	RCL	44.	01	1
20.	03	3	45.	61	+
21.	71	x	46.	31	f
22.	34	RCL	47.	22	ln
23.	02	2	48.	81	÷
24.	01	1	49.	-00	GTO 00

REGISTERS	
R_0	
R_1	n
R_2	i
R_3	PMT, FV, FV/PMT
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. How much money will a person have if he saves $100 a month for 5 years at 7% (.07) annual interest?

2. How big a monthly payment does a person have to make to save $10,000 at the end of 5 years? Assume an annual interest rate of 7%.

3. How long will it take to save $10,000 making monthly payments of $100 at 7% annual interest?

Note:

Remember to enter interest rate as 0.07 divided by 12.

Solutions:

1. $7,159.29

2. $139.68

3. 79.01 months or 6.58 years

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	For FV	n	STO	1			
		i	STO	2			
		PMT	STO	3			
			BST	R/S			FV
	or						
2	For PMT	n	STO	1			
		i	STO	2			
		FV	STO	3			
			GTO	1	7	R/S	PMT
	or						
2	For n	FV	↑				
		PMT	÷	STO	3		
		i	STO	2			
			GTO	3	3	R/S	n

DISCOUNTED CASH FLOW ANALYSIS

Let PV_0 = original investment

PV_k = cash flow of k^{th} period

i = discount rate per period as a decimal, e.g., 6% is expressed as .06

C_k = net present value at period k

Then

$$C_k = -PV_0 + \sum_{k=1}^{n} \frac{PV_k}{(1+i)^k}$$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	34	RCL
02.	02	2
03.	01	1
04.	61	+
05.	81	÷
06.	01	1
07.	33	STO
08.	03	3
09.	23	R↓
10.	34	RCL
11.	01	1
12.	51	–
13.	33	STO
14.	04	4
15.	84	R/S
16.	41	↑
17.	01	1
18.	34	RCL
19.	02	2
20.	61	+
21.	34	RCL
22.	03	3
23.	01	1
24.	61	+
25.	33	STO
26.	03	3
27.	12	y^x
28.	81	÷
29.	34	RCL
30.	04	4
31.	61	+
32.	-13	GTO 13
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	PV_0
R_2	i
R_3	k
R_4	C_k
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

You are offered an investment opportunity for \$100,000 at a capital cost of 10% after taxes. Will this investment be profitable based on the following cash flows?

Year	Cash Flow
1	\$34,000
2	\$27,500
3	\$59,700
4	\$ 7,800

Solution:

C_1 = \$−69,090.91

C_2 = \$−46,363.64

C_3 = \$−1,510.14

C_4 = \$3817.36

Since C_4 is positive the cash flow is profitable to the extent that the cost of capital is 10%.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store PV_0 and i	PV_0	STO	1			
		i	STO	2			
3	Enter PV_1	PV_1	BST	R/S			C_1
4	Perform for k = 2,..., n	PV_k	R/S				C_k

DEPRECIATION SCHEDULES
STRAIGHT LINE

Let PV = original value of asset (less salvage value)

n = lifetime number of periods of asset

B_k = book value at time period K

D = each year's depreciation

k = number of time period, i.e., 1, 2, 3, ..., or n

Then, B_k and D can be calculated by the following formulas:

1. $D = PV/n$
2. $B_k = PV - kD$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.		
01.	34	RCL	26.		
02.	03	3	27.		
03.	34	RCL	28.		
04.	01	1	29.		
05.	81	÷	30.		
06.	33	STO	31.		
07.	00	0	32.		
08.	84	R/S	33.		
09.	34	RCL	34.		
10.	03	3	35.		
11.	34	RCL	36.		
12.	04	4	37.		
13.	34	RCL	38.		
14.	00	0	39.		
15.	71	x	40.		
16.	51	–	41.		
17.	84	R/S	42.		
18.	01	1	43.		
19.	33	STO	44.		
20.	61	+	45.		
21.	04	4	46.		
22.	-01	GTO 01	47.		
23.			48.		
24.			49.		

REGISTERS	
R_0	D
R_1	n
R_2	
R_3	PV
R_4	k
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

A fleet car has a value (not including salvage value) of $2100 and a life expectancy of six years. Using the straight line method, what is the amount of depreciation and what is the book value after two years?

Solutions:

D = $350.00

B_2 = $1400.00

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store n, PV, and k	n	STO	1			
		PV	STO	3			
		k	STO	4	BST		
3	To calculate D and B_k		R/S				D
			R/S				B_k
4	For the next year go to step 3						

DEPRECIATION SCHEDULES SUM-OF-THE-YEAR'S DIGITS

Let n = life time number of periods of asset

S = salvage value

D_k = depreciaton over time period k

B_k = book value at time period k

PV = original value of asset (less salvage value)

k = number of time period, i.e., 1, 2, 3, ..., or n

Then, D_k and B_k can be calculated by the following formulas:

1. $D_k = \frac{2(n - k + 1)}{n(n + 1)} PV$

2. $B_k = S + \frac{(n - k)\, D_k}{2}$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	84	R/S
01.	34	RCL	26.	34	RCL
02.	01	1	27.	02	2
03.	34	RCL	28.	02	2
04.	04	4	29.	81	÷
05.	51	–	30.	34	RCL
06.	33	STO	31.	00	0
07.	02	2	32.	71	x
08.	01	1	33.	34	RCL
09.	61	+	34.	05	5
10.	02	2	35.	61	+
11.	71	x	36.	84	R/S
12.	34	RCL	37.	01	1
13.	03	3	38.	33	STO
14.	71	x	39.	61	+
15.	34	RCL	40.	04	4
16.	01	1	41.	-01	GTO 01
17.	01	1	42.		
18.	61	+	43.		
19.	34	RCL	44.		
20.	01	1	45.		
21.	71	x	46.		
22.	81	÷	47.		
23.	33	STO	48.		
24.	00	0	49.		

REGISTERS	
R_0	D_k
R_1	n
R_2	n – k
R_3	PV
R_4	k
R_5	S
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

A car has a value (not including the salvage value of $800) of $2100 and a life expectancy of 6 years. Using the Sum-of-Year's Digits method what is the amount of depreciation and what is the book value after 2 years?

Solutions:

D_2 = $500.00

B_2 = $1800.00

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store n, PV, k, and S	n	STO	1			
		PV	STO	3			
		k	STO	4			
		S	STO	5	BST		
3	To calculate B_k and D_k		R/S				D_k
			R/S				B_k
4	For the next year go to step 3						

DEPRECIATION SCHEDULES
VARIABLE RATE DECLINING BALANCE

Let PV = original value of asset (less salvage value)

n = lifetime periods of asset

R = depreciation rate (given by user)

D_k = depreciation at time period k

B_k = book value at time period k

k = number of time period, i.e., 1, 2, 3, ..., or n

Then, D_k and B_k can be calculated by the following formulas:

1. $$D_k = PV \frac{R}{n}\left(1 - \frac{R}{n}\right)^{k-1}$$

2. $$B_k = PV\left(1 - \frac{R}{n}\right)^{k}$$

If R = 2 the program gives the double declining balance method. If R = 1.5 the program gives the 150% declining balance method.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	01	1
02.	34	RCL
03.	01	1
04.	51	–
05.	34	RCL
06.	04	4
07.	01	1
08.	51	–
09.	12	y^x
10.	34	RCL
11.	01	1
12.	71	x
13.	34	RCL
14.	03	3
15.	71	x
16.	84	R/S
17.	01	1
18.	34	RCL
19.	01	1
20.	51	–
21.	34	RCL
22.	04	4
23.	12	y^x
24.	34	RCL

DISPLAY		KEY ENTRY
LINE	CODE	
25.	03	3
26.	71	x
27.	84	R/S
28.	01	1
29.	33	STO
30.	61	+
31.	04	4
32.	–01	GTO 01
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	R/n
R_2	
R_3	PV
R_4	k
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

A fleet car has a value of $2500 and a life expectancy of six years. Use the double declining balance method (R = 2) to find the amount of depreciation and book value after four years.

Solutions:

D_4 = $246.91

B_4 = $493.83

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store R/n, PV, k	R	↑				
		n	÷	STO	1		
		PV	STO	3			
		k	STO	4	BST		
3	Calculate D_k and B_k		R/S				D_k
			R/S				B_k
4	For next year go to step 3						

CALENDAR ROUTINES
DAY OF THE WEEK
DAYS BETWEEN TWO DATES

This program calls March 1, 1700, day 1 and gives every succeeding day a corresponding number. The program works for days to and including February 28, 2100. However, for days from March 1, 1700, to February 28, 1800, 2 days must be added to the answer and for days from March 1, 1800, to February 18, 1900, 1 day must be added.

Let M = month, D = day, Y = year, W = day of the week (0 = Sunday, 1 = Monday, etc.)

The day's number is calculated from the following formula:

$$N(M, D, Y) = [365.25\, g(y,m)] + [30.6\, f(m)] + D - 621049$$

where

$$g(y, m) = \begin{cases} y - 1 \text{ if } m = 1 \text{ or } 2 \\ y \text{ if } m > 2 \end{cases} \quad \text{and} \quad f(m) = \begin{cases} m + 13 \text{ if } m = 1 \text{ or } 2 \\ m + 1 \text{ if } m > 2 \end{cases}$$

[m] represents the integer part of a number, i.e., if n = 7.2 then [7.2] = 7. This must be put in by user.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	02	2
02.	34	RCL
03.	01	1
04.	31	f
05.	-11	x≤y 11
06.	01	1
07.	61	+
08.	34	RCL
09.	03	3
10.	-18	GTO 18
11.	01	1
12.	03	3
13.	61	+
14.	34	RCL
15.	03	3
16.	01	1
17.	51	–
18.	03	3
19.	06	6
20.	05	5
21.	83	•
22.	02	2
23.	05	5
24.	71	x
25.	84	R/S
26.	22	x⇄y
27.	23	R↓
28.	22	x⇄y
29.	03	3
30.	00	0
31.	83	•
32.	06	6
33.	71	x
34.	84	R/S
35.	22	x⇄y
36.	23	R↓
37.	61	+
38.	34	RCL
39.	02	2
40.	61	+
41.	06	6
42.	02	2
43.	01	1
44.	00	0
45.	04	4
46.	09	9
47.	51	–
48.	-00	GTO 00
49.		

REGISTERS	
R_0	
R_1	Month
R_2	Day
R_3	Year
R_4	
R_5	
R_6	
R_7	
R_8	Temporary
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. What day of the week was Pearl Harbor? (December 7, 1941)

2. How many days between February 28, 1972, and March 1, 1972?

Solutions:

1. 0, Sunday

2. 2 (1972 is a Leap Year)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store M, D, Y	M	STO	1			
		D	STO	2			
		Y	STO	3			
3	Calculate N(M, D, Y)		BST	R/S			N_1
4	Enter integer part of N_1	$[N_1]$ **	R/S				N_2
5	Enter integer part of N_2	$[N_2]$ **	R/S				N(M, D, Y)*
6	For days between two dates		STO	8			
7	Repeat steps 2 thru 5 for second						
	date, then,		RCL	8	–		Days
	or						
6	For day of the week		7	÷			N_3
7	Enter integer part of N_3	$[N_3]$ **	–	7	x		W
	* Add 2 for days between						
	March 1, 1700 and February 28,						
	1800.						
	* Add 1 for days between March						
	1, 1800 and February 28, 1900.						
	** The value is put in the X						
	register, the ENTER key must not						
	be pushed, and the stack must						
	be maintained. Decide on [n]						
	with the calculator in Fix 9 .						

DETERMINANT AND INVERSE OF A 2 × 2 MATRIX

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a 2 x 2 matrix.

The determinant of A denoted by Det A or |A| is evaluated by the following formula:

$$\text{Det } A = a_{22}\, a_{11} - a_{12}\, a_{21}$$

Also, the program evaluates the multiplicative inverse A^{-1} of A. The following formulas is used:

$$A^{-1} = \begin{bmatrix} a_{22}/\text{Det } A & -a_{12}/\text{Det } A \\ -a_{21}/\text{Det } A & a_{11}/\text{Det } A \end{bmatrix}$$

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE		
00.			25.	09	9	R_0 Det A
01.	34	RCL	26.	81	÷	R_1 a_{11}
02.	04	4	27.	33	STO	R_2 a_{12}
03.	34	RCL	28.	08	8	R_3 a_{21}
04.	01	1	29.	34	RCL	R_4 a_{22}
05.	71	x	30.	02	2	R_5 a_{11}^{-1}
06.	34	RCL	31.	34	RCL	R_6 a_{12}^{-1}
07.	02	2	32.	09	9	R_7 a_{21}^{-1}
08.	34	RCL	33.	81	÷	R_8 a_{22}^{-1}
09.	03	3	34.	42	CHS	R_9 Det A
10.	71	x	35.	33	STO	$R_{.0}$
11.	51	–	36.	06	6	$R_{.1}$
12.	33	STO	37.	34	RCL	$R_{.2}$
13.	09	9	38.	03	3	$R_{.3}$
14.	84	R/S	39.	34	RCL	$R_{.4}$
15.	34	RCL	40.	09	9	$R_{.5}$
16.	04	4	41.	81	÷	$R_{.6}$
17.	34	RCL	42.	42	CHS	$R_{.7}$
18.	09	9	43.	33	STO	$R_{.8}$
19.	81	÷	44.	07	7	$R_{.9}$
20.	33	STO	45.	-00	GTO 00	
21.	05	5	46.			
22.	34	RCL	47.			
23.	01	1	48.			
24.	34	RCL	49.			

Example:

Find the determinant and inverse of the matrix

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

Solution:

Det A = –10

$$A^{-1} = \begin{bmatrix} -.20 & .40 \\ .30 & -.10 \end{bmatrix}$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	For Det A	a_{11}	STO	1			
		a_{12}	STO	2			
		a_{21}	STO	3			
		a_{22}	STO	4			
			BST	R/S			Det A
3	Then for A^{-1}		R/S				
			RCL	5			a_{11}^{-1}
			RCL	6			a_{12}^{-1}
			RCL	7			a_{21}^{-1}
			RCL	8			a_{22}^{-1}

DETERMINANT OF A 3 × 3 MATRIX

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a 3 x 3 matrix.

The determinant of A denoted by |A| or Det A, is calculated by expanding A by minors about the first column. The formula is:

$$\text{Det } A = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$= a_{11} [a_{22} a_{33} - a_{23} a_{32}] - a_{21} [a_{33} a_{12} - a_{32} a_{13}]$$

$$+ a_{31} [a_{23} a_{12} - a_{13} a_{22}]$$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	34	RCL
02.	05	5
03.	34	RCL
04.	09	9
05.	71	x
06.	34	RCL
07.	06	6
08.	34	RCL
09.	08	8
10.	71	x
11.	51	–
12.	34	RCL
13.	01	1
14.	71	x
15.	34	RCL
16.	09	9
17.	34	RCL
18.	02	2
19.	71	x
20.	34	RCL
21.	08	8
22.	34	RCL
23.	03	3
24.	71	x
25.	51	–
26.	34	RCL
27.	04	4
28.	71	x
29.	51	–
30.	34	RCL
31.	06	6
32.	34	RCL
33.	02	2
34.	71	x
35.	34	RCL
36.	03	3
37.	34	RCL
38.	05	5
39.	71	x
40.	51	–
41.	34	RCL
42.	07	7
43.	71	x
44.	61	+
45.	33	STO
46.	00	0
47.	–00	GTO 00
48.		
49.		

REGISTERS	
R_0	Det A
R_1	a_{11}
R_2	a_{12}
R_3	a_{13}
R_4	a_{21}
R_5	a_{22}
R_6	a_{23}
R_7	a_{31}
R_8	a_{32}
R_9	a_{33}
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Find the Determinant of

$$A = \begin{bmatrix} -1 & 0 & 3 \\ 7 & 1 & -1 \\ 2 & 3 & 0 \end{bmatrix}$$

Solution:

Det A = 54

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store A	a_{11}	STO	1			
		a_{12}	STO	2			
		a_{13}	STO	3			
		a_{21}	STO	4			
		a_{22}	STO	5			
		a_{23}	STO	6			
		a_{31}	STO	7			
		a_{32}	STO	8			
		a_{33}	STO	9			
3	Calculate Det A		BST	R/S			Det A

3 × 3 MATRIX INVERSION

If a_{ij} indicates a number in the i^{th} row, j^{th} column then a 3 x 3 matrix A can be represented as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

then the multiplicative inverse of A is denoted by A^{-1} and is calculated as follows:

$$A^{-1} = \begin{bmatrix} \dfrac{\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}}{\text{Det A}} & -\dfrac{\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}}{\text{Det A}} & \dfrac{\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}}{\text{Det A}} \\ -\dfrac{\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}}{\text{Det A}} & \dfrac{\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}}{\text{Det A}} & -\dfrac{\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}}{\text{Det A}} \\ \dfrac{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}{\text{Det A}} & -\dfrac{\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}}{\text{Det A}} & \dfrac{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}{\text{Det A}} \end{bmatrix}$$

For the i^{th}, j^{th} position of A^{-1} use the minor of the j^{th}, i^{th} position of the original matrix. The minor is the two by two matrix left after crossing out the i^{th} row and j^{th} column of A.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	23	R↓
02.	71	x
03.	41	↑
04.	23	R↓
05.	23	R↓
06.	71	x
07.	51	–
08.	42	CHS
09.	34	RCL
10.	00	0
11.	81	÷
12.	-00	GTO 00
13.		
14.		
15.		
16.		
17.		
18.		
19.		
20.		
21.		
22.		
23.		
24.		

DISPLAY		KEY ENTRY
LINE	CODE	
25.		
26.		
27.		
28.		
29.		
30.		
31.		
32.		
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	Det A
R_1	a_{11}
R_2	a_{12}
R_3	a_{13}
R_4	a_{21}
R_5	a_{22}
R_6	a_{23}
R_7	a_{31}
R_8	a_{32}
R_9	a_{33}
$R_{\bullet 0}$	
$R_{\bullet 1}$	
$R_{\bullet 2}$	
$R_{\bullet 3}$	
$R_{\bullet 4}$	
$R_{\bullet 5}$	
$R_{\bullet 6}$	
$R_{\bullet 7}$	
$R_{\bullet 8}$	
$R_{\bullet 9}$	

Example:

Find the inverse of the matrix

$$A = \begin{bmatrix} -1 & 0 & 3 \\ 7 & 1 & -1 \\ 2 & 3 & 0 \end{bmatrix}$$

Solution:

Det A = 54

$$A^{-1} = \begin{bmatrix} .056 & .167 & -.056 \\ -.037 & -.111 & .370 \\ .352 & .056 & -.019 \end{bmatrix}$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Follow instruction of Determi-						
	nant of 3 x 3 Matrix program						
2	Enter this program (do not						
	change contents of registers.)						
3	To find A^{-1}		RCL	5	RCL	6	
	(Note: coefficients must be		RCL	8	RCL	9	
	calculated in order shown.)		BST	R/S			a_{11}^{-1}
			RCL	8	RCL	9	
			RCL	2	RCL	3	
			R/S				a_{12}^{-1}
			RCL	2	RCL	3	
			RCL	5	RCL	6	
			R/S				a_{13}^{-1}
			RCL	7	RCL	9	
			RCL	4	RCL	6	
			R/S				a_{21}^{-1}
			RCL	1	RCL	3	
			RCL	7	RCL	9	
			R/S				a_{22}^{-1}
			RCL	4	RCL	6	
			RCL	1	RCL	3	
			R/S				a_{23}^{-1}
			RCL	4	RCL	5	
			RCL	7	RCL	8	
			R/S				a_{31}^{-1}
			RCL	7	RCL	8	
			RCL	1	RCL	2	
			R/S				a_{32}^{-1}
			RCL	1	RCL	2	
			RCL	4	RCL	5	
			R/S				a_{33}^{-1}

VECTOR CROSS PRODUCT

If A = (a_1, a_2, a_3) and B = (b_1, b_2, b_3) are two three dimensional vectors then the cross product of A and B is denoted by A x B and is calculated as follows:

$$A \times B = \left(\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}, - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}, \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \right) = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1)$$

Let the solution be represented by (c_1, c_2, c_3).

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	34	RCL
02.	02	2
03.	34	RCL
04.	06	6
05.	71	x
06.	34	RCL
07.	03	3
08.	34	RCL
09.	05	5
10.	71	x
11.	51	–
12.	84	R/S
13.	34	RCL
14.	03	3
15.	34	RCL
16.	04	4
17.	71	x
18.	34	RCL
19.	01	1
20.	34	RCL
21.	06	6
22.	71	x
23.	51	–
24.	84	R/S
25.	34	RCL
26.	01	1
27.	34	RCL
28.	05	5
29.	71	x
30.	34	RCL
31.	02	2
32.	34	RCL
33.	04	4
34.	71	x
35.	51	–
36.	-00	GTO 00
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	a_1
R_2	a_2
R_3	a_3
R_4	b_1
R_5	b_2
R_6	b_3
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Find the cross product of the two vectors A = (2.34, 5.17, 7.43) and B = (.072, .231, .409).

Solution:

A x B = (.40, −.42, .17)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store A	a_1	STO	1			
		a_2	STO	2			
		a_3	STO	3			
3	Store B	b_1	STO	4			
		b_2	STO	5			
		b_3	STO	6			
4	Calculate A x B		BST	R/S			c_1
			R/S				c_2
			R/S				c_3

SIMULTANEOUS EQUATIONS IN TWO UNKNOWNS

Let $ax + by = e$

and $cx + dy = f$

be a system of two equations in two unknowns. Cramer's Rule is used to find the solution.

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{ed - bf}{ad - bc}$$

$$y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{af - ec}{ad - bc}$$

If $ad - bc = 0$ the calculator flashes 0. In this case no solution or no unique solution exists.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	34	RCL
02.	03	3
03.	34	RCL
04.	05	5
05.	71	x
06.	34	RCL
07.	02	2
08.	34	RCL
09.	06	6
10.	71	x
11.	51	–
12.	34	RCL
13.	01	1
14.	34	RCL
15.	05	5
16.	71	x
17.	34	RCL
18.	02	2
19.	34	RCL
20.	04	4
21.	71	x
22.	51	–
23.	33	STO
24.	00	0

DISPLAY		KEY ENTRY
LINE	CODE	
25.	81	÷
26.	84	R/S
27.	34	RCL
28.	01	1
29.	34	RCL
30.	06	6
31.	71	x
32.	34	RCL
33.	03	3
34.	34	RCL
35.	04	4
36.	71	x
37.	51	–
38.	34	RCL
39.	00	0
40.	81	÷
41.	-00	GTO 00
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	ad – bc
R_1	a
R_2	b
R_3	e
R_4	c
R_5	d
R_6	f
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Solve for x and y the following system of equations.

$$7.32\,x - 9.08\,y = 3.14$$
$$12.39\,x + 7.00\,y = .05$$

Solution:

x = .14

y = –.24

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store coefficients	a	STO	1			
		b	STO	2			
		e	STO	3			
		c	STO	4			
		d	STO	5			
		f	STO	6			
3	Find x and y		BST	R/S			x
			R/S				y

SIMULTANEOUS EQUATIONS IN THREE UNKNOWNS

Let $a_1 x + b_1 y + c_1 z = d_1$

$a_2 x + b_2 y + c_2 z = d_2$

$a_3 x + b_3 y + c_3 z = d_3$

be a system of three equations in three unknowns. Cramer's Rule is used to find the solution.

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\text{Det A}} \qquad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\text{Det A}} \qquad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\text{Det A}}$$

where | | and Det represent the determinant and

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

If Det A = 0 in step 3 the system has no solution or no unique solution. Continuing the program will cause the calculator to flash zero.

Note:

The program for *"Determinant of a 3 x 3 Matrix"* on p. 80 is also used here.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
	"Determinant of 3 x 3 Matrix"						
2	Store coefficients	a_1	STO	1			
		b_1	STO	2			
		c_1	STO	3			
		a_2	STO	4			
		b_2	STO	5			
		c_2	STO	6			
		a_3	STO	7			
		b_3	STO	8			
		c_3	STO	9			
3	Calculate Det A		BST	R/S			Det A
4	Store Det A		STO	•	1		
5	Store constants	d_1	STO	1			
		d_2	STO	4			
		d_3	STO	7			
6	Calculate x		R/S	RCL	•	1	
			÷				x
7	Calculate y	a_1	STO	2			
		a_2	STO	5			
		a_3	STO	8			
			R/S	RCL	•	1	
			÷	CHS			y
8	Calculate z	b_1	STO	3			
		b_2	STO	6			
		b_3	STO	9			
			R/S	RCL	•	1	
			÷				z
	(If the coefficients are quite						
	long it may be useful at step 2						
	to also store a_1, b_1, a_2, b_2, a_3,						
	b_3 in $R_{\bullet 2}$, $R_{\bullet 3}$,..., $R_{\bullet 7}$ respec-						
	tively and then recall them in						
	steps 7 and 8 as needed.)						

Example:

Solve the following system for x, y, and z.

$$x + y + z = 6$$

$$3x + 5y + 4z = 23$$

$$6x - 7y - 2z = 2$$

Solution:

$$\text{Det } A = -3$$

$$x = 3, \quad y = 2, \quad z = 1$$

2 × 2 MATRIX MULTIPLICATION

Let

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

be two 2 x 2 matrices. The matrix product of A and B is calculated as follows:

$$AB = \begin{bmatrix} a_{11}\,b_{11} + a_{12}\,b_{21} & a_{11}\,b_{12} + a_{12}\,b_{22} \\ a_{21}\,b_{11} + a_{22}\,b_{21} & a_{21}\,b_{12} + a_{22}\,b_{22} \end{bmatrix}$$

Let the answer be denoted by:

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY
LINE	CODE		LINE	CODE	
00.			25.	34	RCL
01.	34	RCL	26.	03	3
02.	01	1	27.	34	RCL
03.	34	RCL	28.	05	5
04.	05	5	29.	71	x
05.	71	x	30.	34	RCL
06.	34	RCL	31.	04	4
07.	02	2	32.	34	RCL
08.	34	RCL	33.	07	7
09.	07	7	34.	71	x
10.	71	x	35.	61	+
11.	61	+	36.	84	R/S
12.	84	R/S	37.	34	RCL
13.	34	RCL	38.	03	3
14.	01	1	39.	34	RCL
15.	34	RCL	40.	06	6
16.	06	6	41.	71	x
17.	71	x	42.	34	RCL
18.	34	RCL	43.	04	4
19.	02	2	44.	34	RCL
20.	34	RCL	45.	08	8
21.	08	8	46.	71	x
22.	71	x	47.	61	+
23.	61	+	48.	-00	GTO 00
24.	84	R/S	49.		

REGISTERS	
R_0	
R_1	a_{11}
R_2	a_{12}
R_3	a_{21}
R_4	a_{22}
R_5	b_{11}
R_6	b_{12}
R_7	b_{21}
R_8	b_{22}
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Find the product of the two matrices

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$$

Solution:

$$C = \begin{bmatrix} 5 & 7 \\ 5 & 13 \end{bmatrix}$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store A	a_{11}	STO	1			
		a_{12}	STO	2			
		a_{21}	STO	3			
		a_{22}	STO	4			
3	Store B	b_{11}	STO	5			
		b_{12}	STO	6			
		b_{21}	STO	7			
		b_{22}	STO	8			
4	Calculate C		BST	R/S			c_{11}
			R/S				c_{12}
			R/S				c_{21}
			R/S				c_{22}

ANGLE BETWEEN, NORM, AND DOT PRODUCT OF VECTORS

Let $\vec{a} = (a_1, a_2, ..., a_n)$ and $\vec{b} = (b_1, b_2, ..., b_n)$ be two vectors.

The norm of $\vec{a}$ is denoted by $|\vec{a}|$ and is calculated by the following formula:

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + ... + a_n^2}$$

similarly,

$$|\vec{b}| = \sqrt{b_1^2 + b_2^2 + ... + b_n^2}$$

The dot product of $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} \cdot \vec{b}$ and is calculated by the following formula:

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + ... + a_n b_n$$

The angle between a and b is denoted by θ and is calculated by the following formula:

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}\right)$$

The angle is calculated in any mode the calculator is set. However, if in degrees, decimal degrees are assumed.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	34	RCL
02.	83	•
03.	02	2
04.	31	f
05.	42	$\sqrt{x}$
06.	-00	GTO 00
07.	34	RCL
08.	83	•
09.	04	4
10.	31	f
11.	42	$\sqrt{x}$
12.	-00	GTO 00
13.	34	RCL
14.	83	•
15.	05	5
16.	34	RCL
17.	83	•
18.	02	2
19.	34	RCL
20.	83	•
21.	04	4
22.	71	x
23.	31	f
24.	42	$\sqrt{x}$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	81	÷
26.	32	g
27.	13	$\cos^{-1}$
28.	-00	GTO 00
29.		
30.		
31.		
32.		
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	n
$R_{.1}$	Σa_i
$R_{.2}$	Σa_i^2
$R_{.3}$	Σb_i
$R_{.4}$	Σb_i^2
$R_{.5}$	$\Sigma a_i b_i$
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Let $\vec{a} = (2.34, 5.17, 7.43)$ and $\vec{b} = (.07, .23, .41)$, find the norms of $\vec{a}$ and $\vec{b}$, the dot product of $\vec{a}$ and $\vec{b}$, and the angle between $\vec{a}$ and $\vec{b}$.

Solution:

$$|\vec{a}| = 9.35$$

$$|\vec{b}| = .48$$

$$\vec{a} \cdot \vec{b} = 4.40$$

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}\right) = 8.11^\circ = .14 \text{ radians} = 9.01 \text{ grads}$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL•R			
3	Perform for i = 1, 2,..., n	b_i	↑				
		a_i	Σ+				i
4	For $\|\vec{a}\|$		BST	R/S			$\|\vec{a}\|$
	or						
	$\|\vec{b}\|$		GTO	0	7	R/S	$\|\vec{b}\|$
	or						
	$\vec{a} \cdot \vec{b}$		RCL	•	5		$\vec{a} \cdot \vec{b}$
	or						
	θ		GTO	1	3	R/S	θ

SINE INTEGRAL

The sine integral is denoted by Si (x) and is defined as follows:

$$\mathrm{Si}\,(x) = \int_{o}^{x} \frac{\sin t}{t}\, dt$$

where x is a real number. Also, a Taylor's series expansion of Si (x) yields

$$\mathrm{Si}\,(x) = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+1}}{(2n+1)\,(2n+1)!}$$

This program computes successive partial sums of the series. It stops when two consecutive partial sums are equal, and displays the last partial sum as the answer.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY
LINE	CODE		LINE	CODE	
00.			25.	02	2
01.	33	STO	26.	81	÷
02.	03	3	27.	34	RCL
03.	41	↑	28.	03	3
04.	71	x	29.	71	x
05.	42	CHS	30.	33	STO
06.	33	STO	31.	03	3
07.	01	1	32.	34	RCL
08.	01	1	33.	02	2
09.	33	STO	34.	81	÷
10.	02	2	35.	61	+
11.	34	RCL	36.	32	g
12.	03	3	37.	-00	x=y 00
13.	34	RCL	38.	-13	GTO 13
14.	01	1	39.		
15.	34	RCL	40.		
16.	02	2	41.		
17.	01	1	42.		
18.	61	+	43.		
19.	81	÷	44.		
20.	31	f	45.		
21.	34	LAST X	46.		
22.	01	1	47.		
23.	61	+	48.		
24.	33	STO	49.		

REGISTERS	
R_0	
R_1	$-x^2$
R_2	2n + 1
R_3	Used
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. Si (.69) = .67 (15 seconds iteration)

2. Si (9.8) = 1.67 (50 seconds iteration)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Calculate Si(x)	x	BST	R/S			Si(x)

COSINE INTEGRAL

The cosine integral is denoted by Ci (x) and is defined as follows:

$$\mathrm{Ci}\,(x) = \gamma + \ln x + \int_0^x \frac{\cos t - 1}{t}\,dt$$

where $x > 0$, and $\gamma = 0.5772156649$ is Euler's constant.

Also, a Taylor series expansions yields

$$\mathrm{Ci}\,(x) = \gamma + \ln x + \sum_{n=1}^{\infty} \frac{(-1)^n\, x^{2n}}{2n\,(2n)!}$$

This program computes successive partial sums of the series. When two consecutive partial sums are equal, the value is used as the sum of the series.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	41	↑
02.	71	x
03.	42	CHS
04.	33	STO
05.	01	1
06.	01	1
07.	33	STO
08.	03	3
09.	00	0
10.	33	STO
11.	02	2
12.	31	f
13.	34	LAST X
14.	31	f
15.	22	ln
16.	34	RCL
17.	04	4
18.	61	+
19.	34	RCL
20.	01	1
21.	34	RCL
22.	02	2
23.	01	1
24.	61	+
25.	81	÷
26.	31	f
27.	34	LAST X
28.	01	1
29.	61	+
30.	33	STO
31.	02	2
32.	81	÷
33.	34	RCL
34.	03	3
35.	71	x
36.	33	STO
37.	03	3
38.	34	RCL
39.	02	2
40.	81	÷
41.	61	+
42.	32	g
43.	-00	x=y 00
44.	-19	GTO 19
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	$-x^2$
R_2	2n
R_3	Used
R_4	γ
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. Ci (1.38) = .46 (20 seconds iteration)

2. Ci (5) = –.19 (40 seconds iteration)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store γ = .5772156649		•	5	7	7	
			2	1	5	6	
			6	4	9		
			STO	4			
3	Calculate Ci(x)	x	BST	R/S			Ci(x)

EXPONENTIAL INTEGRAL

The exponential integral is denoted by Ei (x) and is defined as follows:

$$\text{Ei}(x) = \int_{-\infty}^{x} \frac{e^t}{t}\,dt$$

where x > 0.

Using a Taylor's series expansion and letting γ = 0.5772156649 be Euler's constant:

$$\text{Ei}(x) = \gamma + \ln x + \sum_{n=1}^{\infty} \frac{x^n}{n(n!)}$$

This program computes successive partial sums of the series. When two consecutive partial sums are equal, the value is used as the sum of the series.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY	REGISTERS
00.			25.	34	RCL	R_0
01.	33	STO	26.	01	1	R_1 x
02.	01	1	27.	34	RCL	R_2 Used
03.	01	1	28.	02	2	R_3 Used
04.	33	STO	29.	01	1	R_4
05.	03	3	30.	61	+	R_5
06.	00	0	31.	33	STO	R_6
07.	33	STO	32.	02	2	R_7
08.	02	2	33.	81	÷	R_8
09.	34	RCL	34.	34	RCL	R_9
10.	01	1	35.	03	3	$R_{.0}$
11.	31	f	36.	71	x	$R_{.1}$
12.	22	ln	37.	33	STO	$R_{.2}$
13.	83	•	38.	03	3	$R_{.3}$
14.	05	5	39.	34	RCL	$R_{.4}$
15.	07	7	40.	02	2	$R_{.5}$
16.	07	7	41.	81	÷	$R_{.6}$
17.	02	2	42.	61	+	$R_{.7}$
18.	01	1	43.	32	g	$R_{.8}$
19.	05	5	44.	-00	x=y 00	$R_{.9}$
20.	06	6	45.	-25	GTO 25	
21.	06	6	46.			
22.	04	4	47.			
23.	09	9	48.			
24.	61	+	49.			

Examples:

1. Ei (1.59) = 3.57 (35 seconds iteration)

2. Ei (.61) = .80 (25 seconds iteration)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Compute Ei(x)	x	BST	R/S			Ei(x)

NUMERICAL INTEGRATION, TRAPEZOIDAL RULE

Let x_0, x_1, ..., x_n be equally spaced points such that $x_i = x_0 + ih$ for i = 0, 1, 2, ..., n, at which corresponding values $f(x_0)$, $f(x_1)$, ..., $f(x_n)$ of a function f(x) are known. The function need not be known explicity but if it is, these values can be found previously by writing the function into memory and evaluating at the various points. R_2 through $R_{\bullet 9}$ could be used to store these values.

The Trapezoidal Rule is:

$$\int_{x_0}^{x_n} f(x)\,dx \cong \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right]$$

Let the answer be represented by I.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	34	RCL
01.	02	2	26.	00	0
02.	81	÷	27.	71	x
03.	33	STO	28.	34	RCL
04.	00	0	29.	01	1
05.	84	R/S	30.	61	+
06.	34	RCL	31.	33	STO
07.	00	0	32.	01	1
08.	71	x	33.	-24	GTO 24
09.	33	STO	34.		
10.	01	1	35.		
11.	84	R/S	36.		
12.	34	RCL	37.		
13.	00	0	38.		
14.	71	x	39.		
15.	33	STO	40.		
16.	61	+	41.		
17.	01	1	42.		
18.	02	2	43.		
19.	33	STO	44.		
20.	71	x	45.		
21.	00	0	46.		
22.	34	RCL	47.		
23.	01	1	48.		
24.	84	R/S	49.		

REGISTERS	
R_0	h/2
R_1	Σ
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{\bullet 0}$	
$R_{\bullet 1}$	
$R_{\bullet 2}$	
$R_{\bullet 3}$	
$R_{\bullet 4}$	
$R_{\bullet 5}$	
$R_{\bullet 6}$	
$R_{\bullet 7}$	
$R_{\bullet 8}$	
$R_{\bullet 9}$	

Example:

Find the $\int_0^2 x^3\,dx$ using h = .25.

The following data must be found first:

i	0	1	2	3	4	5	6	7	8
x_i	0	.25	.50	.75	1.00	1.25	1.50	1.75	2.00
$f(x_i)$	0	.0156	.1250	.4219	1.0000	1.9531	3.3750	5.3594	8.0000

Solution:

$$\int_0^2 x^3\,dx \cong 4.0625$$

Actual solution is 4.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Enter h	h	BST	R/S			h/2
3	Enter $f(x_0)$	$f(x_0)$	R/S				Partial Sum
4	Enter $f(x_n)$	$f(x_n)$	R/S				Partial Sum
5	Perform step 5 for i = 1, 2, ..., n−2	$f(x_i)$	R/S				Partial Sum
6	Enter $f(x_{n-1})$	$f(x_{n-1})$	R/S				I

NUMERICAL INTEGRATION, SIMPSON'S RULE

Let $x_0, x_1, ..., x_n$ be equally spaced points such that $x_i = x_0 + ih$ for $i = 0, 1, 2, ..., n$ at which corresponding values $f(x_0), f(x_1), ..., f(x_n)$ of a function f(x) are known. This function need not be known explicitly but if it is, these values can be found previously by writing the function into memory and evaluating at the various points. R_2 through $R_{\bullet 9}$ could be used to store these values. n must be an even positive integer.

Simpson's Rule is:

$$\int_{x_0}^{x_n} f(x)\,dx \cong \frac{h}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + ... + 4f(x_{n-3}) + 2f(x_{n-2})$$

$$+ 4f(x_{n-1}) + f(x_n)].$$

Let the solution be indicated by I.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	71	x
01.	03	3	26.	34	RCL
02.	81	÷	27.	01	1
03.	33	STO	28.	61	+
04.	00	0	29.	33	STO
05.	84	R/S	30.	01	1
06.	34	RCL	31.	84	R/S
07.	00	0	32.	34	RCL
08.	71	x	33.	00	0
09.	33	STO	34.	71	x
10.	01	1	35.	02	2
11.	84	R/S	36.	71	x
12.	34	RCL	37.	34	RCL
13.	00	0	38.	01	1
14.	71	x	39.	61	+
15.	34	RCL	40.	33	STO
16.	01	1	41.	01	1
17.	61	+	42.	-20	GTO 20
18.	33	STO	43.		
19.	01	1	44.		
20.	84	R/S	45.		
21.	34	RCL	46.		
22.	00	0	47.		
23.	71	x	48.		
24.	04	4	49.		

REGISTERS	
R_0	h/3
R_1	Σ
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{\bullet 0}$	
$R_{\bullet 1}$	
$R_{\bullet 2}$	
$R_{\bullet 3}$	
$R_{\bullet 4}$	
$R_{\bullet 5}$	
$R_{\bullet 6}$	
$R_{\bullet 7}$	
$R_{\bullet 8}$	
$R_{\bullet 9}$	

Example:

Compute $\int_0^2 x^3\,dx$ using Simpson's Rule with h = .25.

The following data must be found first:

i	0	1	2	3	4	5	6	7	8
x_i	0	.25	.50	.75	1.00	1.25	1.50	1.75	2.00
$f(x_i)$	0	.0156	.1250	.4219	1.0000	1.9531	3.3750	5.3594	8.0000

Solution:

$$\int_0^2 x^3\,dx \cong 4.0000$$

The exact solution is 4.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Enter h	h	BST	R/S			h/3
3	Enter $f(x_0)$	$f(x_0)$	R/S				Partial Sum
4	Enter $f(x_n)$	$f(x_n)$	R/S				Partial Sum
5	Perform for i = 1, 2,..., n−1	$f(x_i)$	R/S				Partial Sum
6	Enter $f(x_{n-1})$	$f(x_{n-1})$	R/S				I

NUMERICAL SOLUTION TO DIFFERENTIAL EQUATIONS

This program may be used to solve a wide variety of first order differential equations of the form

$$y' = f(x, y)$$

with initial values x_0, y_0.

The solution is a numerical solution which calculates y_i for $x_i = x_0 + ih$ $(i = 1, 2, 3, \ldots)$. h is an increment specified by the user.

The program uses Euler's method:

$$y_{i+1} = y_i + f(x_i, y_i)\,h$$

f (x, y) is keyed into memory starting at line 20. The user has 30 program steps and all registers except R_4, R_5, and R_6 available to write f (x, y). The user can assume x to be in the X-register and in R_6 and can assume y to be in the Y-register and in R_5. The routine should return the value of f (x, y) in the X-register and should end with GTO 08. The accuracy of this method is low unless a small step size is used.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.		
01.	22	x⇄y	26.		
02.	33	STO	27.		
03.	05	5	28.		
04.	22	x⇄y	29.		
05.	33	STO	30.		
06.	06	6	31.		
07.	-20	GTO 20	32.		
08.	34	RCL	33.		
09.	04	4	34.		
10.	71	x	35.		
11.	34	RCL	36.		
12.	05	5	37.		
13.	61	+	38.		
14.	34	RCL	39.		
15.	06	6	40.		
16.	34	RCL	41.		
17.	04	4	42.		
18.	61	+	43.		
19.	-00	GTO 00	44.		
20.			45.		
21.			46.		
22.			47.		
23.			48.		
24.			49.		

REGISTERS	
R_0	
R_1	
R_2	
R_3	
R_4	h
R_5	y
R_6	x
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Solve numerically the differential equation

$$y' = \frac{x + 1 + 2y}{x}$$

with the initial conditions $x_0 = 1$, $y_0 = -.5$. Use a step size of $h = .1$.

Solutions:

The keystrokes for f (x, y) are: [1] [+] [x⇄y] [2] [×] [+] [RCL] [6] [÷]

x	1	1.1	1.2	1.3	1.4	1.5
y	–.5	–.4	–.28	–.15	.01	.18

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Switch to RUN mode		GTO	1	9		
3	Switch to PRGM						
4	Key in function						
5	Key in		GTO	0	8		
6	Switch to RUN						
7	Store step size	h	STO	4			
8	Put in initial values	y_0	↑				
		x_0	BST	R/S			x_i
9	If y value desired		x⇄y				y_i
10	If y value has been displayed		x⇄y				x_i
11	For next x value		R/S				x_{i+1}
12	Go to step 9						

LINEAR INTERPOLATION

If $(x_1, f(x_1))$ and $(x_2, f(x_2))$ are two points of a function $f(x)$, then the function at x_0 can be approximated by the following formula:

$$f(x_0) \cong \frac{(x_2 - x_0)\,f(x_1) + (x_0 - x_1)\,f(x_2)}{(x_2 - x_1)}$$

This is called the linear interpolation formula. Of course, x_2 cannot equal x_1.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY
LINE	CODE		LINE	CODE	
00.			25.	-00	GTO 00
01.	33	STO	26.		
02.	05	5	27.		
03.	34	RCL	28.		
04.	03	3	29.		
05.	22	x⇄y	30.		
06.	51	−	31.		
07.	34	RCL	32.		
08.	02	2	33.		
09.	71	x	34.		
10.	34	RCL	35.		
11.	05	5	36.		
12.	34	RCL	37.		
13.	01	1	38.		
14.	51	−	39.		
15.	34	RCL	40.		
16.	04	4	41.		
17.	71	x	42.		
18.	61	+	43.		
19.	34	RCL	44.		
20.	03	3	45.		
21.	34	RCL	46.		
22.	01	1	47.		
23.	51	−	48.		
24.	81	÷	49.		

REGISTERS
R_0
R_1 x_1
R_2 $f(x_1)$
R_3 x_2
R_4 $f(x_2)$
R_5 x_0
R_6
R_7
R_8
R_9
$R_{.0}$
$R_{.1}$
$R_{.2}$
$R_{.3}$
$R_{.4}$
$R_{.5}$
$R_{.6}$
$R_{.7}$
$R_{.8}$
$R_{.9}$

Example:

If (1.2, .30119) and (1.3, .27253) are two points of a function, find f(1.27) and f(1.29).

Solution:

1. f(1.27) = .28113 [FIX] [5]

2. f(1.29) = .27540 [FIX] [5]

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store (x_1, $f(x_1)$)	x_1	STO	1			
		$f(x_1)$	STO	2			
3	Store (x_2, $f(x_2)$)	x_2	STO	3			
		$f(x_2)$	STO	4			
4	Find $f(x_0)$	x_0	BST	R/S			$f(x_0)$

QUADRATIC EQUATIONS

A general quadratic equation is of the form $a x^2 + b x + c = 0$

The equation has two roots x_1 and x_2. Let $D = \dfrac{b^2 - 4ac}{4a^2}$

If $D \geqslant 0$ then $x_1 = \begin{cases} -\dfrac{b}{2a} + \sqrt{D} & \text{if } -\dfrac{b}{2a} \geqslant 0 \\ -\dfrac{b}{2a} - \sqrt{D} & \text{if } -\dfrac{b}{2a} < 0 \end{cases}$

and $x_2 = \dfrac{c}{a x_1}$

These formulas compute the larger root (in absolute value) first. Better significance can be obtained by this method.

If $D < 0$ then $x_1, x_2 = -\dfrac{b}{2a} \pm \sqrt{-D}\, i = u \pm iv$

The coefficient a cannot be zero.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY
LINE	CODE		LINE	CODE	
00.			25.	42	CHS
01.	34	RCL	26.	31	f
02.	02	2	27.	42	$\sqrt{x}$
03.	34	RCL	28.	22	x⇄y
04.	01	1	29.	84	R/S
05.	02	2	30.	22	x⇄y
06.	71	×	31.	-00	GTO 00
07.	81	÷	32.	23	R↓
08.	42	CHS	33.	31	f
09.	41	↑	34.	42	$\sqrt{x}$
10.	32	g	35.	22	x⇄y
11.	42	x^2	36.	00	0
12.	34	RCL	37.	31	f
13.	03	3	38.	-43	x≤y 43
14.	34	RCL	39.	23	R↓
15.	01	1	40.	22	x⇄y
16.	81	÷	41.	51	−
17.	33	STO	42.	-45	GTO 45
18.	00	0	43.	23	R↓
19.	51	−	44.	61	+
20.	84	R/S	45.	84	R/S
21.	00	0	46.	34	RCL
22.	31	f	47.	00	0
23.	-32	x≤y 32	48.	22	x⇄y
24.	23	R↓	49.	81	÷

REGISTERS	
R_0	c/a
R_1	a
R_2	b
R_3	c
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

Find the solution to the following three equations:

1. $x^2 - 3x - 4 = 0$
2. $2x^2 + 5x + 3 = 0$
3. $2x^2 + 3x + 4 = 0$

Solutions:

1. D = 6.25 $x_1 = 4, x_2 = -1$
2. D = .06 $x_1 = -1.50, x_2 = -1.00$
3. D = -1.44 $x_1, x_2 = -.75 \pm 1.20i$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store coefficients	a	STO	1			
		b	STO	2			
		c	STO	3			
3	Calculate D		BST	R/S			D*
4	If D ⩾ 0 roots are real		R/S				x_1*
			R/S				x_2
	or						
4	If D < 0 roots are complex of the						
	form u ± i v		R/S				u*
			R/S				v
	* The stack must be maintained						
	at these positions.						

SYNTHETIC DIVISION

This program divides a polynomial of the form

$$a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_n$$

by a binomial of the form

$$x - x_0$$

The positive integer n must be less than or equal to 8. The answer is of the form

$$b_0 x^{n-1} + b_1 x^{n-2} + \ldots + b_{n-1} + \frac{b_n}{x - x_0}$$

Be sure to input the x_0 and not the $-x_0$ of $x - x_0$.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	41	↑
02.	41	↑
03.	41	↑
04.	34	RCL
05.	00	0
06.	84	R/S
07.	71	x
08.	34	RCL
09.	01	1
10.	61	+
11.	84	R/S
12.	71	x
13.	34	RCL
14.	02	2
15.	61	+
16.	84	R/S
17.	71	x
18.	34	RCL
19.	03	3
20.	61	+
21.	84	R/S
22.	71	x
23.	34	RCL
24.	04	4

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	61	+
26.	84	R/S
27.	71	x
28.	34	RCL
29.	05	5
30.	61	+
31.	84	R/S
32.	71	x
33.	34	RCL
34.	06	6
35.	61	+
36.	84	R/S
37.	71	x
38.	34	RCL
39.	07	7
40.	61	+
41.	84	R/S
42.	71	x
43.	34	RCL
44.	08	8
45.	61	+
46.	-00	GTO 00
47.		
48.		
49.		

REGISTERS	
R_0	a_0
R_1	a_1
R_2	a_2
R_3	a_3
R_4	a_4
R_5	a_5
R_6	a_6
R_7	a_7
R_8	a_8
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Divide $x^5 - 4x^4 + 7x^3 - 10x^2 + 8$ by $x - 2$. (Note the coefficient of x is 0.)

Solution:

$$\frac{x^5 - 4x^4 + 7x^3 - 10x^2 + 8}{x - 2} = x^4 - 2x^3 + 3x^2 - 4x - 8 - \frac{8}{x - 2}$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store a_0	a_0	STO	0			
3	Store a_i for i = 1, 2,..., n	a_i	STO				
	($n \leqslant 8$)**	i					
4	Enter x_0	x_0	BST	R/S			b_0
5	Perform for i = 1, 2,..., n		R/S				b_i
	* The stack must be maintained						
	at these positions.						
	** a_i for $i \leqslant n$ must be stored						
	even if it is zero.						

FACTORING INTEGERS AND DETERMINING PRIMES

With the following list, numbers up to 40,000 can be factored. Of course, if a number x is prime it has no factors. Let p be a prime from the list below and let max = $\sqrt{x}$. The only prime numbers that need be checked as factors are those such that $p \leqslant \sqrt{x}$.

2	13	31	53	73	101	127	151	179
3	17	37	59	79	103	131	157	181
5	19	41	61	83	107	137	163	191
7	23	43	67	89	109	139	167	193
11	29	47	71	97	113	149	173	197
								199

Examples:

1. Find the factors of 823.

2. Find the factors of 221.

Solutions:

1. 823 is prime. Since $\sqrt{823} = 28.69$ only 2, 3, 5, 7, 11, 13, 17, 19, 23 need be checked.

2. 13 and 17

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Let p = 2 if x is even, p = 3						
	otherwise	x	STO	1			
2	Let max = $\sqrt{x}$		f	$\sqrt{x}$			max
3			RCL	1			
4		p	÷				x/p
5	If x/p is an integer p is a factor	x/p	STO	1			
6	Go to step 4						
7	If x/p is prime then x/p is the						
	only remaining factor. Stop.						
	or						
5	If x/p is not an integer let p be						
	the next prime and go to step 3.						
	Note: Only those p's less than						
	or equal to max need be						
	checked.						

POLYNOMIAL EVALUATION

A polynomial of the form

$$f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1} x + a_n$$

is evaluated by writing it in the form

$$x(\dots x(x(a_0 x + a_1) + a_2) + \dots + a_{n-1}) + a_n$$

n can be any positive integer.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	33	STO
02.	02	2
03.	34	RCL
04.	01	1
05.	41	↑
06.	41	↑
07.	41	↑
08.	34	RCL
09.	02	2
10.	84	R/S
11.	33	STO
12.	02	2
13.	44	CLX
14.	61	+
15.	71	x
16.	34	RCL
17.	02	2
18.	61	+
19.	-10	GTO 10
20.		
21.		
22.		
23.		
24.		

DISPLAY		KEY ENTRY
LINE	CODE	
25.		
26.		
27.		
28.		
29.		
30.		
31.		
32.		
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	x
R_2	Temporary
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Evaluate $f(x) = x^2 + 2x + 3$ for $x = 2$.

Solution:

$f(2) = 11$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store x	x	STO	1			
3	Input a_0	a_0	BST	R/S			
4	Perform for i = 1, 2,..., n−1	a_i*	R/S				Partial
5	Input a_n		R/S				f(x)
	* The stack must be maintained						
	at this position.						

NUMBER IN BASE b TO A NUMBER IN BASE 10

This program consists of two subprograms. The first changes the integer part of a number in base b to a number in base 10.

$$I_{10} = i_n i_{n-1} \dots i_2 i_1 = i_n b^{n-1} + i_{n-1} b^{n-2} + \dots + i_2 b + i_1$$

This is evaluated in the form

$$b(\dots(b(b(i_n b + i_{n-1}) + i_{n-2}) + \dots) + i_2) + i_1$$

The second subprogram changes the fraction part of a number in base b to a number in base 10.

$$F_{10} = f_1 f_2 \dots f_m = f_1 b^{-1} + f_2 b^{-2} + \dots + f_m b^{-m}$$

The two programs together can then convert any number in base b to a number in base 10. Zeros must be entered in their proper place.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	41	↑
01.	33	STO	26.	41	↑
02.	02	2	27.	41	↑
03.	34	RCL	28.	34	RCL
04.	01	1	29.	02	2
05.	41	↑	30.	71	x
06.	41	↑	31.	84	R/S
07.	41	↑	32.	33	STO
08.	34	RCL	33.	02	2
09.	02	2	34.	44	CLX
10.	84	R/S	35.	61	+
11.	33	STO	36.	33	STO
12.	02	2	37.	03	3
13.	44	CLX	38.	44	CLX
14.	61	+	39.	61	+
15.	71	x	40.	71	x
16.	34	RCL	41.	41	↑
17.	02	2	42.	41	↑
18.	61	+	43.	34	RCL
19.	-10	GTO 10	44.	02	2
20.	33	STO	45.	71	x
21.	02	2	46.	34	RCL
22.	34	RCL	47.	03	3
23.	01	1	48.	61	+
24.	13	1/x	49.	-31	GTO 31

REGISTERS	
R_0	
R_1	b
R_2	Temporary
R_3	Temporary
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. Convert 101.0101_2 to a decimal number.
2. Convert $A1_{16}$ to a decimal number (in base 16, A = 10, B = 11, C = 12, D = 13, E = 14, and F = 15).

Solutions:

1. 5.31
2. 161

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store b	b	STO	1			
3	For integer part	i_n	BST	R/S			
4	Perform for j = n−1, n−2, ..., 2	i_j*	R/S				Partial
5	Input i_1	i_1	R/S				I_{10}
	or						
3	For fractional part	f_1	GTO	2	0	R/S	
4	Perform for j = 2, 3, ..., m−1	f_j*	R/S				Partial
5	Input f_m	f_m	R/S				F_{10}
	*The stack must be maintained						
	at these positions.						

NUMBER IN BASE 10 TO NUMBER IN BASE b

This routine converts any number in base 10 to a number in base b, N_b. The user must input the greatest integer of the number displayed after the R/S . The greatest integer of a number is the largest integer less than or equal to the number; i.e., if

$$x = 1.5 \text{ then } GI[1.5] = 1$$

and if

$$x = -1.5 \text{ then } GI[-1.5] = -2$$

The user ends the routine when the display flashes or when the accuracy of the machine is exceeded. The variable c must be input as 1 to allow one display position per digit for $2 \leqslant b \leqslant 10$ or as 10 to allow 2 display positions for $11 \leqslant b \leqslant 100$.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	33	STO
02.	02	2
03.	00	0
04.	33	STO
05.	03	3
06.	23	R↓
07.	31	f
08.	22	ln
09.	34	RCL
10.	01	1
11.	31	f
12.	22	ln
13.	81	÷
14.	84	R/S
15.	33	STO
16.	05	5
17.	01	1
18.	00	0
19.	34	RCL
20.	04	4
21.	71	x
22.	22	x⇄y
23.	12	y^x
24.	33	STO

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	61	+
26.	03	3
27.	34	RCL
28.	02	2
29.	34	RCL
30.	01	1
31.	34	RCL
32.	05	5
33.	12	y^x
34.	51	−
35.	33	STO
36.	02	2
37.	-07	GTO 07
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	b
R_2	N_{10}
R_3	N_b
R_4	c
R_5	Temporary
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. Convert 18_{10} to base 17.
2. Convert 2.333333333 to base 2.

Solutions:

1. 101; i.e., 11_{17}
2. 10.01010101_2

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store b	b	STO	1			
3	Store c	c	STO	4			
	(c = 1 if 2 ⩽ b ⩽ 10 or						
	c = 10 if 11 ⩽ b ⩽ 100)						
3	Input n	n	BST	R/S			x_1
4	Repeat this step until display						
	flashes or until x_i is more than						
	10 less than x_1 if c = 1 or more						
	than 5 less than x_1 if c = 10.						
	Input greatest integer less than						
	or equal to x_i	$[x_i]$ *	R/S				x_{i+1}
5	To obtain answer		RCL	3			N_b
	* Decide on $[x_i]$ displaying x_i						
	in Fix 9.						

NEWTON'S METHOD SOLUTION TO f(x)=0

Newton's method is an iterative solution technique that uses an initial guess x_0 and calculates succeeding x's by the formula

$$x_{k+1} = x_k - \frac{f(x)}{f'(x)}$$

where $f'(x)$ is the first derivative of f(x).

The user must write in $f(x)/f'(x)$ to be solved starting at line 18. He can assume x is in the X-register and R_1. He has 31 program memory locations, the stack registers, and all storage registers except R_1 available for this evaluation. The user must end his code with GTO 04.

The code as written gives 5 place accuracy to the right of the decimal. For more or less accuracy change the constant (10^{-12}) in program memory locations 9 through 12. The constant 10^{-12} assures that the square or the change in x is less than 10^{-12}, i.e., that x is off no more than one count in the sixth decimal place, assuring that the fifth decimal place is correct.

Warning:

Newton's method may not converge and will only find one solution. A new solution may be found or convergence may be improved with a different initial guess.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	34	RCL
02.	01	1
03.	-18	GTO 18
04.	33	STO
05.	51	−
06.	01	1
07.	41	↑
08.	71	x
09.	43	EEX
10.	01	1
11.	02	2
12.	42	CHS
13.	31	f
14.	-01	x≤y 01
15.	34	RCL
16.	01	1
17.	-00	GTO 00
18.		
19.		
20.		
21.		
22.		
23.		
24.		

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.		
26.		
27.		
28.		
29.		
30.		
31.		
32.		
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	x_k
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

1. Find the solution to the function

$$f(x) = x^3 - 2x - 4$$

Solution:

$$\frac{f(x)}{f'(x)} = \frac{x^3 - 2x - 4}{3x^2 - 2}$$

Keystrokes:

[↑] [↑] [↑] [×] [×] [x⇄y]   [2] [×] [−] [4] [−] [x⇄y] [↑] [×]

[3] [×] [2] [−] [÷]

$x = 2.00$

With an initial guess of 10, iteration time is about 30 seconds.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Switch to RUN		GTO	1	7		
3	Switch to PRGM						
4	Enter f(x)/f′(x)						
5	Key in		GTO	0	4		
6	Switch to RUN						
7	Enter initial guess	x_0	STO	1			
8	Find solution		BST	R/S			x

TRIGONOMETRIC FUNCTIONS

(cot, sec, csc, $\cot^{-1}$, $\sec^{-1}$, $\csc^{-1}$)

The above functions can be evaluated by the following formulas:

1. $\cot x = \dfrac{1}{\tan x}$

2. $\sec x = \dfrac{1}{\cos x}$

3. $\csc x = \dfrac{1}{\sin x}$

4. $\cot^{-1} x = \tan^{-1} \left(\dfrac{1}{x}\right)$

5. $\sec^{-1} x = \cos^{-1} \left(\dfrac{1}{x}\right)$

6. $\csc^{-1} x = \sin^{-1} \left(\dfrac{1}{x}\right)$

Examples:

1. $\cot(30^\circ) = 1.73$
2. $\sec\left(\frac{\pi}{4}\right) = 1.41$
3. csc (100 grads) = 1.00
4. $\cot^{-1}(5) = 11.31^\circ$
5. $\sec^{-1}(2) = 60.00^\circ$
6. $\csc^{-1}(4) = 14.48^\circ$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	For cot x	x	f	tan	1/x		cot x
	or						
	sec x	x	f	cos	1/x		sec x
	or						
	csc x	x	f	sin	1/x		csc x
	or						
	cot^{-1} x	x	1/x	g	tan^{-1}		cot^{-1} x
	or						
	sec^{-1} x	x	1/x	g	cos^{-1}		sec^{-1} x
	or						
	csc^{-1} x	x	1/x	g	sin^{-1}		csc^{-1} x

VERSINE, COVERSINE, HAVERSINE, EXSECANT

The above are calculated by the following formulas:

1. versine (versed sine)

$$\text{vers}\ \theta = 1 - \cos\theta$$

2. coversine (covered sine)

$$\text{cov}\ \theta = 1 - \sin\theta$$

3. haversine

$$\text{hav}\ \theta = \tfrac{1}{2}\ \text{vers}\ \theta = \sin^2 \tfrac{1}{2}\theta$$

4. exsecant

$$\text{exsec}\ \theta = \sec\theta - 1$$

The program works in any angular mode. However, if in degrees decimal degrees are assumed.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	31	f
02.	13	cos
03.	42	CHS
04.	01	1
05.	61	+
06.	-00	GTO 00
07.	31	f
08.	12	sin
09.	42	CHS
10.	01	1
11.	61	+
12.	-00	GTO 00
13.	31	f
14.	13	cos
15.	42	CHS
16.	01	1
17.	61	+
18.	02	2
19.	81	÷
20.	-00	GTO 00
21.	31	f
22.	13	cos
23.	13	1/x
24.	01	1

DISPLAY		KEY ENTRY
LINE	CODE	
25.	51	–
26.	-00	GTO 00
27.		
28.		
29.		
30.		
31.		
32.		
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

vers 100° = 1.1736

cov 100° = .0152

hav 100° = .5868

exsec 100° = –6.7588

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Versine	θ	BST	R/S			Vers θ
	or						
	Coversine	θ	GTO	0	7	R/S	Cov θ
	or						
	Haversine	θ	GTO	1	3	R/S	Hav θ
	or						
	Exsecant	θ	GTO	2	1	R/S	Exsec θ

HYPERBOLIC FUNCTIONS

This program evaluates the six hyperbolic functions by the following formulas:

1. $\sinh x = \dfrac{e^x - e^{-x}}{2}$

2. $\cosh x = \dfrac{e^x + e^{-x}}{2}$

3. $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

4. $\operatorname{csch} x = \dfrac{1}{\sinh x}$ $(x \neq 0)$

5. $\operatorname{sech} x = \dfrac{1}{\cosh x}$

6. $\coth x = \dfrac{1}{\tanh x}$ $(x \neq 0)$

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	32	g
02.	22	e^x
03.	41	↑
04.	13	$^1/x$
05.	51	–
06.	02	2
07.	81	÷
08.	-00	GTO 00
09.	32	g
10.	22	e^x
11.	41	↑
12.	13	$^1/x$
13.	61	+
14.	-06	GTO 06
15.	32	g
16.	22	e^x
17.	41	↑
18.	13	$^1/x$
19.	51	–
20.	41	↑
21.	41	↑
22.	31	f
23.	34	LAST X
24.	02	2

DISPLAY		KEY ENTRY
LINE	CODE	
25.	71	×
26.	61	+
27.	81	÷
28.	-00	GTO 00
29.		
30.		
31.		
32.		
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	
R_2	
R_3	
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. sinh 1.5 = 2.13
2. cosh 5.9 = 182.52
3. tanh 1.3 = .86
4. csch 0.95 = .91
5. sech (−3) = .10
6. coth (−1.99) = −1.04

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
	sinh x	x	BST	R/S			sinh x
	or						
	cosh x	x	GTO	0	9	R/S	cosh x
	or						
	tanh x	x	GTO	1	5	R/S	tanh x
	or						
	csch x	x	BST	R/S	1/x		csch x
	or						
	sech x	x	GTO	0	9	R/S	
			1/x				sech x
	or						
	coth x	x	GTO	1	5	R/S	coth x
			1/x				

INVERSE HYPERBOLIC FUNCTIONS

This program evaluates the inverse hyperbolic functions by the following formulas:

1. $\sinh^{-1} x = \ln\,[x + (x^2 + 1)^{1/2}]$
2. $\cosh^{-1} x = \ln\,[x + (x^2 - 1)^{1/2}]$ $\qquad x \geqslant 1$
3. $\tanh^{-1} x = \frac{1}{2} \ln \left[\frac{1+x}{1-x}\right]$ $\qquad x^2 < 1$
4. $\operatorname{csch}^{-1} x = \sinh^{-1} \left[\frac{1}{x}\right]$ $\qquad x \neq 0$
5. $\operatorname{sech}^{-1} x = \cosh^{-1} \left[\frac{1}{x}\right]$ $\qquad 0 < x \leqslant 1$
6. $\coth^{-1} x = \tanh^{-1} \left[\frac{1}{x}\right]$ $\qquad x^2 > 1$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY	REGISTERS
00.			25.	01	1	R_0
01.	41	↑	26.	61	+	R_1
02.	41	↑	27.	22	x⇄y	R_2
03.	71	x	28.	42	CHS	R_3
04.	01	1	29.	01	1	R_4
05.	61	+	30.	61	+	R_5
06.	31	f	31.	81	÷	R_6
07.	42	√x	32.	31	f	R_7
08.	61	+	33.	22	ln	R_8
09.	31	f	34.	02	2	R_9
10.	22	ln	35.	81	÷	$R_{.0}$
11.	-00	GTO 00	36.	-00	GTO 00	$R_{.1}$
12.	41	↑	37.			$R_{.2}$
13.	41	↑	38.			$R_{.3}$
14.	71	x	39.			$R_{.4}$
15.	01	1	40.			$R_{.5}$
16.	51	–	41.			$R_{.6}$
17.	31	f	42.			$R_{.7}$
18.	42	√x	43.			$R_{.8}$
19.	61	+	44.			$R_{.9}$
20.	31	f	45.			
21.	22	ln	46.			
22.	-00	GTO 00	47.			
23.	41	↑	48.			
24.	41	↑	49.			

Examples:

1. $\sinh^{-1}(3.5) = 1.97$
2. $\cosh^{-1}(100) = 5.30$
3. $\tanh^{-1}(-.7) = -.87$
4. $\text{csch}^{-1}(3) = .33$
5. $\text{sech}^{-1}(.5) = 1.32$
6. $\coth^{-1}(5.4) = .19$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
	$\sinh^{-1} x$	x	BST	R/S			$\sinh^{-1} x$
	or						
	$\cosh^{-1} x$	x	GTO	1	2	R/S	$\cosh^{-1} x$
	or						
	$\tanh^{-1} x$	x	GTO	2	3	R/S	$\tanh^{-1} x$
	or						
	$\text{csch}^{-1} x$	x	1/x	BST	R/S		$\text{csch}^{-1} x$
	or						
	$\text{sech}^{-1} x$	x	1/x	GTO	1	2	
			R/S				$\text{sech}^{-1} x$
	or						
	$\coth^{-1} x$	x	1/x	GTO	2	3	
			R/S				$\coth^{-1} x$

POLYGONS INSCRIBED IN A CIRCLE

Given the radius of a circle this program calculates the length S_1 of a side and the area A_1 of a polygon of n sides inscribed in the circle. The formulas used are:

1. $S_1 = 2r \sin\left(\frac{c}{n}\right)$

2. $A_1 = \frac{1}{2} n\, r^2 \sin\left(\frac{2c}{n}\right)$

where

$c = 2 \sin^{-1} 1 = \pi$ radians $= 180° = 200$ grads

n = number of sides

r = radius of the circle

It must be true that n is an integer greater than 2.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	34	RCL
01.	01	1	26.	02	2
02.	32	g	27.	41	↑
03.	12	$\sin^{-1}$	28.	71	x
04.	02	2	29.	71	x
05.	71	x	30.	34	RCL
06.	34	RCL	31.	01	1
07.	01	1	32.	71	x
08.	81	÷	33.	02	2
09.	33	STO	34.	81	÷
10.	03	3	35.	-00	GTO 00
11.	31	f	36.		
12.	12	sin	37.		
13.	02	2	38.		
14.	71	x	39.		
15.	34	RCL	40.		
16.	02	2	41.		
17.	71	x	42.		
18.	84	R/S	43.		
19.	34	RCL	44.		
20.	03	3	45.		
21.	02	2	46.		
22.	71	x	47.		
23.	31	f	48.		
24.	12	sin	49.		

REGISTERS	
R_0	
R_1	n
R_2	r
R_3	c/n
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Given a circle of radius 5 find the length of a side and area of a polygon of 6 sides inscribed in the circle.

Solution:

$S_1 = 5.00$

$A_1 = 64.95$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store n and r	n	STO	1			
		r	STO	2			
3	Find S_1		BST	R/S			S_1
4	Find A_1		R/S				A_1

POLYGONS CIRCUMSCRIBED ABOUT A CIRCLE

Given the radius of a circle this program calculates the length S_2 of a side and the area A_2 of a polygon of n sides circumscribed about a circle. The formulas used are:

1. $S_2 = 2r \tan (c/n)$
2. $A_2 = n\, r^2 \tan (c/n)$

where

$c = 2 \sin^{-1} 1 = \pi$ radians $= 180° = 200$ grads

n = number of sides

r = radius of the circle

It must be true that n is an integer greater than 2.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY
LINE	CODE		LINE	CODE	
00.			25.	01	1
01.	01	1	26.	71	x
02.	32	g	27.	-00	GTO 00
03.	12	$\sin^{-1}$	28.		
04.	02	2	29.		
05.	71	x	30.		
06.	34	RCL	31.		
07.	01	1	32.		
08.	81	÷	33.		
09.	31	f	34.		
10.	14	tan	35.		
11.	34	RCL	36.		
12.	02	2	37.		
13.	71	x	38.		
14.	33	STO	39.		
15.	03	3	40.		
16.	02	2	41.		
17.	71	x	42.		
18.	84	R/S	43.		
19.	34	RCL	44.		
20.	03	3	45.		
21.	34	RCL	46.		
22.	02	2	47.		
23.	71	x	48.		
24.	34	RCL	49.		

REGISTERS	
R_0	
R_1	n
R_2	r
R_3	r tan (c/n)
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Find the length of a side and the area of a polygon of 6 sides circumscribed about a circle of radius 5.

Solution:

$S_2 = 5.77$

$A_2 = 86.60$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store n and r	n	STO	1			
		r	STO	2			
3	Calculate S_2		BST	R/S			S_2
4	Calculate A_2		R/S				A_2

CIRCLE DETERMINED BY THREE POINTS

Let (x_1, y_1) (x_2, y_2) (x_3, y_3) be three points such that $x_1 \neq x_2$ and $x_1 \neq x_3$. If the points cannot be renumbered to satisfy this condition, the points cannot be on a circle. Let the center of the circle be (x_0, y_0) and the radius of the circle be r. Then

$$y_0 = \frac{k_2 - k_1}{n_2 - n_1}, \qquad x_0 = k_2 - n_2 y_0, \qquad \text{and } r = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

where

$$k_1 = \frac{1}{2}[(x_1 + x_2) + n_1(y_1 + y_2)], \qquad k_2 = \frac{1}{2}[(x_1 + x_3) + n_2(y_1 + y_3)]$$

$$n_1 = \frac{y_1 - y_2}{x_1 - x_2}, \qquad \text{and } n_2 = \frac{y_1 - y_3}{x_1 - x_3}$$

If $n_1 = n_2$ the points cannot form a circle.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY	REGISTERS
00.			25.	02	2	R_0
01.	34	RCL	26.	81	÷	R_1 x_1
02.	02	2	27.	-00	GTO 00	R_2 y_1
03.	34	RCL	28.	34	RCL	R_3 x_2, x_3
04.	04	4	29.	08	8	R_4 y_2, y_3
05.	51	–	30.	34	RCL	R_5 n_1
06.	34	RCL	31.	06	6	R_6 k_1
07.	01	1	32.	51	–	R_7 n_2
08.	34	RCL	33.	34	RCL	R_8 k_2
09.	03	3	34.	07	7	R_9 y_0
10.	51	–	35.	34	RCL	$R_{.0}$
11.	81	÷	36.	05	5	$R_{.1}$
12.	84	R/S	37.	51	–	$R_{.2}$
13.	34	RCL	38.	81	÷	$R_{.3}$
14.	02	2	39.	33	STO	$R_{.4}$
15.	34	RCL	40.	09	9	$R_{.5}$
16.	04	4	41.	84	R/S	$R_{.6}$
17.	61	+	42.	34	RCL	$R_{.7}$
18.	71	x	43.	07	7	$R_{.8}$
19.	34	RCL	44.	71	x	$R_{.9}$
20.	01	1	45.	34	RCL	
21.	61	+	46.	08	8	
22.	34	RCL	47.	22	x⇄y	
23.	03	3	48.	51	–	
24.	61	+	49.	-00	GTO 00	

Examples:

1. Find the equation of the circle that goes through the three points (1, 1), (3.5, −7.6), (12, 0.8).

2. Find the equation of the circle that passes through the three points (0, 1), (−1, 0), (0, −1).

Solutions:

1. $n_1 = -3.44$, $k_1 = 13.60$, $n_2 = -.02$, $k_2 = 6.48$
   Center = (6.45, −2.08), r = 6.26
   Equation: $(x - 6.45)^2 + (y + 2.08)^2 = (6.26)^2$

2. $n_1 = 1.00$, $k_1 = 0.00$, $n_2 = -1.00$, $k_2 = 0.00$
   Center = (0, 0), r = 1
   Equation: $x^2 + y^2 = 1$
   Note: (−1, 0) must be (x_1, y_1)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store (x_1, y_1)	x_1	STO	1			
		y_1	STO	2			
3	Store (x_2, y_2)	x_2	STO	3			
		y_2	STO	4			
4	Calculate and store n_1		BST	R/S			n_1 *
			STO	5			
5	Calculate and store k_1		R/S				k_1
			STO	6			
6	Store (x_3, y_3)	x_3	STO	3			
		y_3	STO	4			
7	Calculate and store n_2		R/S				n_2 *
			STO	7			
8	Calculate and store k_2		R/S				k_2
			STO	8			
9	Find y_0		GTO	2	8	R/S	y_0 *
10	Find x_0		R/S				x_0 *
11	Calculate r		RCL	1	−	RCL	
			9	RCL	2	−	
			g	R→P			r
	* The stack should be maintain-						
	ed at these points.						

EQUALLY SPACED POINTS ON A CIRCLE

Given a circle with center (x_0, y_0) and radius r, this program calculates the coordinates of equally spaced points on the circle. The user inputs the coordinates of the center, the radius, the number of points n to be spaced on the circle, and an angle θ (measured from the positive x-axis) which describes the position of the first point on the circle.

The formulas used are:

$$x_{k+1} = x_0 + r \cos (\theta + ck)$$

$$y_{k+1} = y_0 + r \sin (\theta + ck)$$

where

$$k = 0, 1, 2, ..., n - 1$$

and

$$c = \frac{4 \sin^{-1} 1}{n} = \frac{2\pi \text{ radians}}{n} = \frac{360^\circ}{n} = \frac{400 \text{ grads}}{n}$$

The program works in any angular mode but if in degrees decimal degrees are assumed.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	13	1/x
02.	01	1
03.	32	g
04.	12	$\sin^{-1}$
05.	04	4
06.	71	x
07.	71	x
08.	33	STO
09.	05	5
10.	01	1
11.	42	CHS
12.	33	STO
13.	06	6
14.	01	1
15.	33	STO
16.	61	+
17.	06	6
18.	34	RCL
19.	03	3
20.	34	RCL
21.	05	5
22.	34	RCL
23.	06	6
24.	71	x

DISPLAY		KEY ENTRY
LINE	CODE	
25.	61	+
26.	34	RCL
27.	04	4
28.	31	f
29.	00	R←P
30.	34	RCL
31.	01	1
32.	61	+
33.	84	R/S
34.	22	x⇄y
35.	34	RCL
36.	02	2
37.	61	+
38.	84	R/S
39.	-14	GTO 14
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	x_0
R_2	y_0
R_3	θ
R_4	r
R_5	c
R_6	k
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. Find five points equally spaced around a circle with center at (4.28, 3.10) with radius 1. Start the first point at $\pi/4$ radians around the circle.

2. Find three points equally spaced around a circle with center at (–3.4, 1.8) with radius 3.21. Start the first point 36° around the circle.

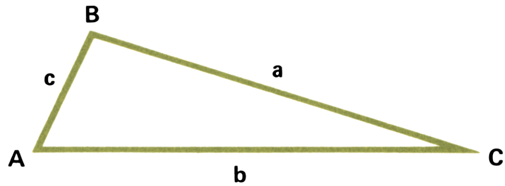
Solutions:

1. (4.99, 3.81), (3.83, 3.99), (3.29, 2.94), (4.12, 2.11), (5.17, 2.65) (Set calculator in radian mode.)

2. (–.80, 3.69), (–6.33, 3.11), (–3.06, –1.39)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store (x_0, y_0)	x_0	STO	1			
		y_0	STO	2			
3	Store θ	θ	STO	3			
4	Store r	r	STO	4			
5	Enter n	n	BST	R/S			x_1 *
			R/S				y_1
6	Perform for i = 2,..., n		R/S				x_i *
			R/S				y_i
	* Stack must be maintained at						
	these positions.						

TRIANGLE SOLUTION B, b, c



Given two sides and a non-included angle, this program solves the triangle for the remaining parameters by the following formulas:

1. $C = \sin^{-1}\left(\frac{c \sin B}{b}\right)$

2. $A = 2 \sin^{-1} 1 - (B + C) = \pi \text{ radians} - (B + C) = 180^\circ - (B + C)$

   $= 200 \text{ grads} - (B + C)$

3. $a = \frac{b \sin A}{\sin B}$

If B is acute ($< 90^\circ$) and $b < c$, a second set of solutions exists and is calculated by the following formulas:

4. $C' = 2 \sin^{-1} 1 - C$

5. $A' = 2 \sin^{-1} 1 - (B + C')$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	34	RCL
02.	03	3
03.	34	RCL
04.	01	1
05.	31	f
06.	12	sin
07.	71	x
08.	34	RCL
09.	02	2
10.	81	÷
11.	32	g
12.	12	$\sin^{-1}$
13.	33	STO
14.	05	5
15.	84	R/S
16.	34	RCL
17.	01	1
18.	61	+
19.	01	1
20.	32	g
21.	12	$\sin^{-1}$
22.	02	2
23.	71	x
24.	33	STO

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.	04	4
26.	22	x⇄y
27.	51	−
28.	84	R/S
29.	31	f
30.	12	sin
31.	34	RCL
32.	02	2
33.	71	x
34.	34	RCL
35.	01	1
36.	31	f
37.	12	sin
38.	81	÷
39.	84	R/S
40.	34	RCL
41.	04	4
42.	34	RCL
43.	05	5
44.	51	−
45.	84	R/S
46.	−16	GTO 16
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	B
R_2	b
R_3	c
R_4	$2 \sin^{-1} 1$
R_5	C
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Given the following two sides and non-included angle:

$B = 33°40'$ (convert to decimal degrees)

$b = 31.5$

$c = 51.8$

Solve the triangle.

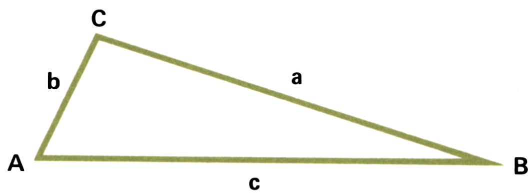
Solution:

Since B is less than 90° and $b < c$, two sets of solutions exist.

$C = 65.73$	$C' = 114.27°$
$A = 80.60$	$A' = 32.06°$
$a = 56.06$	$a' = 30.16$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store B, b, and c	B	STO	1			
		b	STO	2			
		c	STO	3			
3	Solve triangle		BST	R/S			C*
			R/S				A*
			R/S				a*
4	If B < 90° and b < c						
	find alternate solution		R/S				C'*
			R/S				A'*
			R/S				a'
	* The stack must be maintained						
	at these positions.						

TRIANGLE SOLUTION a, b, c



Given three sides of a triangle this program solves the triangle for the remaining parameters by the following formulas:

$$C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$$

$$B = \sin^{-1}\left(\frac{b \sin C}{c}\right) \qquad A = \sin^{-1}\left(\frac{a \sin C}{c}\right)$$

Reletter if necessary to make c the largest side. The program works in any angular mode. However, if in degree mode decimal degrees are assumed.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY
LINE	CODE		LINE	CODE	
00.			25.	84	R/S
01.	34	RCL	26.	31	f
02.	01	1	27.	12	sin
03.	41	↑	28.	34	RCL
04.	71	x	29.	03	3
05.	34	RCL	30.	81	÷
06.	02	2	31.	34	RCL
07.	41	↑	32.	02	2
08.	71	x	33.	22	x⇄y
09.	61	+	34.	71	x
10.	34	RCL	35.	31	f
11.	03	3	36.	34	LAST X
12.	41	↑	37.	22	x⇄y
13.	71	x	38.	32	g
14.	51	−	39.	12	$\sin^{-1}$
15.	34	RCL	40.	84	R/S
16.	01	1	41.	23	R↓
17.	34	RCL	42.	34	RCL
18.	02	2	43.	01	1
19.	71	x	44.	71	x
20.	02	2	45.	32	g
21.	71	x	46.	12	$\sin^{-1}$
22.	81	÷	47.	-00	GTO 00
23.	32	g	48.		
24.	13	$\cos^{-1}$	49.		

REGISTERS	
R_0	
R_1	a
R_2	b
R_3	c
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Given the following three sides:

a = 30.3

b = 40.4

c = 62.6

Solve the triangle.

Solution:

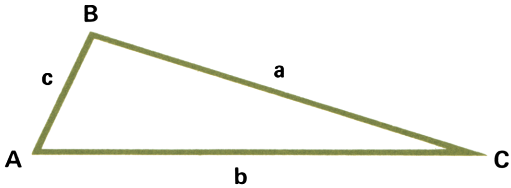
C = 123.99°

B = 32.35°

A = 23.66°

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store a, b, and c (c is the largest)	a	STO	1			
		b	STO	2			
		c	STO	3			
3	Find the solution		BST	R/S			C*
			R/S				B*
			R/S				A
	* The stack must be maintained						
	at these positions.						

TRIANGLE SOLUTION a, A, C



Given two angles and an opposite side this program solves the triangle for the remaining parameters by the following formulas:

$$B = 2\sin^{-1} 1 - (A + C) = \pi \text{ radians} - (A + C) = 180^\circ - (A + C)$$
$$= 200 \text{ grads} - (A + C)$$

$$b = \frac{a \sin B}{\sin A}$$

$$c = \frac{a \sin C}{\sin A}$$

The program works in any angular mode. However, if in degree mode all angles are assumed to be in decimal degrees.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY
LINE	CODE		LINE	CODE	
00.			25.	01	1
01.	01	1	26.	31	f
02.	32	g	27.	34	LAST X
03.	12	$\sin^{-1}$	28.	81	÷
04.	02	2	29.	34	RCL
05.	71	x	30.	03	3
06.	34	RCL	31.	31	f
07.	02	2	32.	12	sin
08.	34	RCL	33.	71	x
09.	03	3	34.	-00	GTO 00
10.	61	+	35.		
11.	51	–	36.		
12.	84	R/S	37.		
13.	31	f	38.		
14.	12	sin	39.		
15.	34	RCL	40.		
16.	01	1	41.		
17.	71	x	42.		
18.	34	RCL	43.		
19.	02	2	44.		
20.	31	f	45.		
21.	12	sin	46.		
22.	81	÷	47.		
23.	84	R/S	48.		
24.	34	RCL	49.		

REGISTERS	
R_0	
R_1	a
R_2	A
R_3	C
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Given the following two angles and opposite side:

$a = 17.5$

$A = 41.23°$

$C = 62.20°$

Solve the triangle.

Solution:

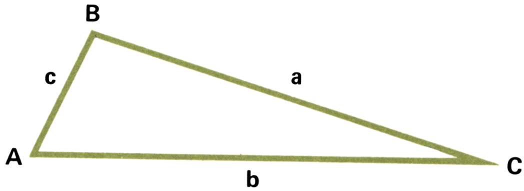
$B = 76.57°$

$b = 25.83$

$c = 23.49$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store a, A, and C	a	STO	1			
		A	STO	2			
		C	STO	3			
3	Find the solution		BST	R/S			B*
			R/S				b*
			R/S				c
	* The stack must be maintained						
	at these positions.						

TRIANGLE SOLUTION a, b, C



Given two sides and their included angle this program solves the triangle for the remaining parameters by the following formulas:

$$c = \sqrt{a^2 + b^2 - 2ab \cos C} \qquad A = \sin^{-1}\left(\frac{a \sin C}{c}\right)$$

$$B = 2 \sin^{-1} 1 - (A + C) = \pi \text{ radians} - (A + C) = 180^\circ - (A + C)$$
$$= 200 \text{ grads} - (A + C)$$

Reletter if necessary, to make a the smaller of a and b.

This program works in any angular mode. However, if in degrees decimal degrees are assumed.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.		
01.	34	RCL
02.	03	3
03.	34	RCL
04.	01	1
05.	31	f
06.	00	R←P
07.	42	CHS
08.	34	RCL
09.	02	2
10.	61	+
11.	32	g
12.	00	R→P
13.	84	R/S
14.	22	x⇄y
15.	84	R/S
16.	34	RCL
17.	03	3
18.	61	+
19.	31	f
20.	13	cos
21.	42	CHS
22.	32	g
23.	13	cos
24.	-00	GTO 00

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
25.		
26.		
27.		
28.		
29.		
30.		
31.		
32.		
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	a
R_2	b
R_3	C
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

Given the following two sides and included angle:

a = 132

b = 224

C = 28°40′ (convert to decimal degrees first)

Solve the triangle.

Solution:

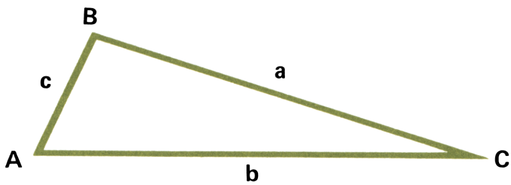
c = 125.35

A = 30.34°

B = 120.99°

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store a, b, and C						
	(a is smaller of a and b)	a	STO	1			
		b	STO	2			
		C	STO	3			
3	Solve triangle		BST	R/S			c*
			R/S				A*
			R/S				B
	* The stack must be maintained						
	at these positions.						

TRIANGLE SOLUTION a, B, C



Given two angles and their included side this program solves the triangle for the remaining parameters by the following formulas:

$$A = 2 \sin^{-1} 1 - (B + C) = \pi \text{ radians} - (B + C) = 180^\circ - (B + C)$$

$$= 200 \text{ grads} - (B + C)$$

$$b = \frac{a \sin B}{\sin A}$$

$$c = \frac{a \sin C}{\sin A}$$

The program works in any angular mode. However, if in degrees the program assumes decimal degrees.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	01	1
02.	32	g
03.	12	$\sin^{-1}$
04.	02	2
05.	71	x
06.	34	RCL
07.	02	2
08.	34	RCL
09.	03	3
10.	61	+
11.	51	–
12.	33	STO
13.	04	4
14.	84	R/S
15.	34	RCL
16.	01	1
17.	34	RCL
18.	04	4
19.	31	f
20.	12	sin
21.	81	÷
22.	33	STO
23.	04	4
24.	34	RCL

DISPLAY		KEY ENTRY
LINE	CODE	
25.	02	2
26.	31	f
27.	12	sin
28.	71	x
29.	84	R/S
30.	34	RCL
31.	04	4
32.	34	RCL
33.	03	3
34.	31	f
35.	12	sin
36.	71	x
37.	-00	GTO 00
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	a
R_2	B
R_3	C
R_4	A, (a/sin A)
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Given the following two angles and their included side:

$a = 25.2$

$B = 35°20'$ (convert B and C to decimal degrees first)

$C = 68°30'$

Solve the triangle.

Solution:

$A = 76.17°$

$b = 15.01$

$c = 24.15$

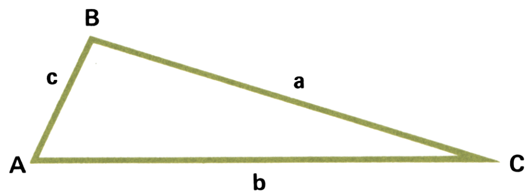
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store a, B, C	a	STO	1			
		B	STO	2			
		C	STO	3			
3	Solve triangle		BST	R/S			A*
			R/S				b*
			R/S				c
	* The stack must be maintained						
	at these positions.						

AREA OF A TRIANGLE a, b, c

Given three sides of a triangle this program computes the area by the following formula:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where s = ½ (a + b + c)



DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE		
00.			25.	03	3	R_0
01.	34	RCL	26.	51	–	R_1 a
02.	01	1	27.	71	x	R_2 b
03.	34	RCL	28.	31	f	R_3 c
04.	02	2	29.	42	$\sqrt{x}$	R_4
05.	61	+	30.	-00	GTO 00	R_5
06.	34	RCL	31.			R_6
07.	03	3	32.			R_7
08.	61	+	33.			R_8
09.	02	2	34.			R_9
10.	81	÷	35.			$R_{.0}$
11.	41	↑	36.			$R_{.1}$
12.	41	↑	37.			$R_{.2}$
13.	41	↑	38.			$R_{.3}$
14.	34	RCL	39.			$R_{.4}$
15.	01	1	40.			$R_{.5}$
16.	51	–	41.			$R_{.6}$
17.	71	x	42.			$R_{.7}$
18.	22	x⇄y	43.			$R_{.8}$
19.	34	RCL	44.			$R_{.9}$
20.	02	2	45.			
21.	51	–	46.			
22.	71	x	47.			
23.	22	x⇄y	48.			
24.	34	RCL	49.			

Example:

Find the area of a triangle with the following three sides:

a = 5.31

b = 7.09

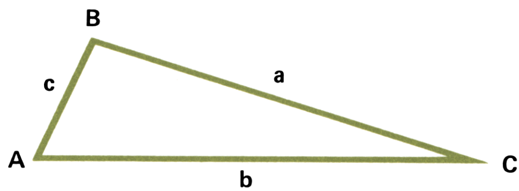
c = 8.86

Solution:

Area = 18.82

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store a, b, c	a	STO	1			
		b	STO	2			
		c	STO	3			
3	Find area		BST	R/S			Area

AREA OF A TRIANGLE a, b, C



Given two sides and an included angle of a triangle this program computes the area by the following formula:

$$\text{Area} = \tfrac{1}{2} ab \sin C$$

The angle C can be in any angular mode but if in degrees it is assumed to be in decimal degrees.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY
LINE	CODE		LINE	CODE	
00.			25.		
01.	34	RCL	26.		
02.	03	3	27.		
03.	31	f	28.		
04.	12	sin	29.		
05.	34	RCL	30.		
06.	01	1	31.		
07.	71	x	32.		
08.	34	RCL	33.		
09.	02	2	34.		
10.	71	x	35.		
11.	02	2	36.		
12.	81	÷	37.		
13.	-00	GTO 00	38.		
14.			39.		
15.			40.		
16.			41.		
17.			42.		
18.			43.		
19.			44.		
20.			45.		
21.			46.		
22.			47.		
23.			48.		
24.			49.		

REGISTERS	
R_0	
R_1	a
R_2	b
R_3	C
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Find the area of the triangle with the following two sides and included angle.

$a = 5.3174$

$b = 7.0898$

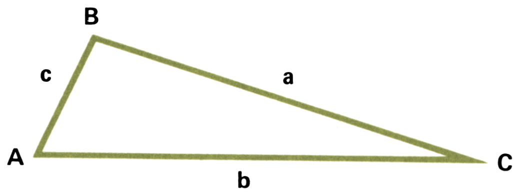
$C = 45°$

Solution:

Area = 13.33

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store data	a	STO	1			
		b	STO	2			
		C	STO	3			
3	Find area		BST	R/S			Area

AREA OF A TRIANGLE a, B, C



Given two angles and an included side of a triangle this program computes the area by the following formula:

$$\text{Area} = \frac{a^2 \sin B \sin C}{2 \sin (B + C)}$$

Angles B and C can be in any angular mode. If in degrees all angles are assumed to be decimal degrees.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY
LINE	CODE		LINE	CODE	
00.			25.	-00	GTO 00
01.	34	RCL	26.		
02.	01	1	27.		
03.	32	g	28.		
04.	42	x^2	29.		
05.	02	2	30.		
06.	81	÷	31.		
07.	34	RCL	32.		
08.	02	2	33.		
09.	31	f	34.		
10.	12	sin	35.		
11.	71	x	36.		
12.	34	RCL	37.		
13.	03	3	38.		
14.	31	f	39.		
15.	12	sin	40.		
16.	71	x	41.		
17.	34	RCL	42.		
18.	02	2	43.		
19.	34	RCL	44.		
20.	03	3	45.		
21.	61	+	46.		
22.	31	f	47.		
23.	12	sin	48.		
24.	81	÷	49.		

REGISTERS	
R_0	
R_1	a
R_2	B
R_3	C
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Given the following two angles and included side find the area of the triangle.

$a = 14.625$

$B = 70.54°$

$C = 62.96°$

Solution:

Area = 123.82

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input data	a	STO	1			
		B	STO	2			
		C	STO	3			
3	Find area		BST	R/S			Area

AREA OF A TRIANGLE [(x_1, y_1), (x_2, y_2), (x_3, y_3)]

Given the coordinates of the vertices of a triangle, the area is found by the following formulas:

$$\text{Area} = \tfrac{1}{2}\ \text{Determinant of D where}$$

$$D = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

Therefore,

$$\text{Area} = \tfrac{1}{2}\ [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00.			25.	71	x
01.	34	RCL	26.	61	+
02.	01	1	27.	02	2
03.	34	RCL	28.	81	÷
04.	04	4	29.	-00	GTO 00
05.	34	RCL	30.		
06.	06	6	31.		
07.	51	–	32.		
08.	71	x	33.		
09.	34	RCL	34.		
10.	03	3	35.		
11.	34	RCL	36.		
12.	06	6	37.		
13.	34	RCL	38.		
14.	02	2	39.		
15.	51	–	40.		
16.	71	x	41.		
17.	61	+	42.		
18.	34	RCL	43.		
19.	05	5	44.		
20.	34	RCL	45.		
21.	02	2	46.		
22.	34	RCL	47.		
23.	04	4	48.		
24.	51	–	49.		

REGISTERS	
R_0	
R_1	x_1
R_2	y_1
R_3	x_2
R_4	y_2
R_5	x_3
R_6	y_3
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Example:

Find the area of the triangle with the following x–y coordinate vertices.

(0, 0)

(4, 0)

(4, 3)

Solution:

Area = 6

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Enter coordinates of vertices	x_1	STO	1			
		y_1	STO	2			
		x_2	STO	3			
		y_2	STO	4			
		x_3	STO	5			
		y_3	STO	6			
3	Find area		BST	R/S			Area

AREA OF A POLYGON

If the x–y coordinates of the vertices of a polygon are known, the area can be found by the following formula:

$$\text{Area} = \tfrac{1}{2}\,[(x_1 + x_2)(y_1 - y_2) + (x_2 + x_3)(y_2 - y_3) + \ldots + (x_n + x_1)(y_n - y_1)]$$

Traverse the coordinates clockwise for a positive area.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY
LINE	CODE		LINE	CODE	
00.			25.	81	÷
01.	33	STO	26.	33	STO
02.	02	2	27.	61	+
03.	23	R↓	28.	05	5
04.	33	STO	29.	34	RCL
05.	01	1	30.	04	4
06.	00	0	31.	33	STO
07.	33	STO	32.	02	2
08.	05	5	33.	34	RCL
09.	84	R/S	34.	03	3
10.	33	STO	35.	33	STO
11.	04	4	36.	01	1
12.	23	R↓	37.	34	RCL
13.	33	STO	38.	05	5
14.	03	3	39.	-09	GTO 09
15.	34	RCL	40.		
16.	01	1	41.		
17.	61	+	42.		
18.	34	RCL	43.		
19.	02	2	44.		
20.	34	RCL	45.		
21.	04	4	46.		
22.	51	–	47.		
23.	71	x	48.		
24.	02	2	49.		

REGISTERS	
R_0	
R_1	x_{i-1}
R_2	y_{i-1}
R_3	x_i
R_4	y_i
R_5	Σ_{AREA}
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

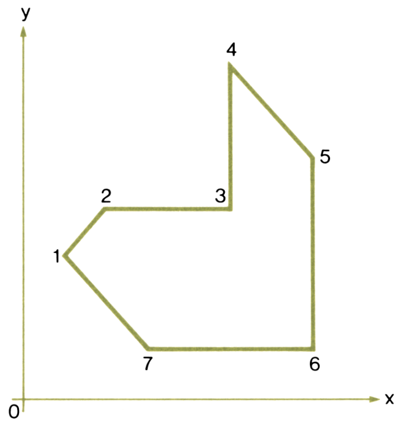
Example:

Find the area of the polygon with the following x–y coordinate vertices.

Point	Coordinates (x, y)
1	(1, 3)
2	(2, 4)
3	(5, 4)
4	(5, 7)
5	(7, 5)
6	(7, 1)
7	(3, 1)

Solution:

Area = 19.50



STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input (x_1, y_1)	x_1 *	↑				
		y_1 *	BST	R/S			0.00
3	Input (x_i, y_i) for i = 2, ..., n	x_i	↑				
		y_i	R/S				Intermediate
4	Input (x_1, y_1) again	x_1	↑				
		y_1	R/S				Area
	*If x_1 and y_1 are quite com-						
	plicated it may be convient to						
	store them in R_6 and R_7 and						
	recall them when needed.						

SPHERICAL TRIANGLE SOLUTION A, b, c

Given two sides and an included angle of a spherical triangle this program finds the other side by the following formula:

$$a = \cos^{-1} (\cos b \cos c + \sin b \sin c \cos A)$$

The program "Spherical Triangle Solution a, b, c" can then be used to find the other angles.

The program works in any angular mode. However, if in degree mode all angles are assumed to be decimal degrees.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE			
00.			25.	13	$\cos^{-1}$	R_0	
01.	34	RCL	26.	33	STO	R_1	a
02.	04	4	27.	01	1	R_2	b
03.	34	RCL	28.	-00	GTO 00	R_3	c
04.	03	3	29.			R_4	A
05.	31	f	30.			R_5	
06.	12	sin	31.			R_6	
07.	31	f	32.			R_7	
08.	00	R←P	33.			R_8	
09.	34	RCL	34.			R_9	
10.	02	2	35.			$R_{.0}$	
11.	31	f	36.			$R_{.1}$	
12.	12	sin	37.			$R_{.2}$	
13.	71	x	38.			$R_{.3}$	
14.	34	RCL	39.			$R_{.4}$	
15.	02	2	40.			$R_{.5}$	
16.	31	f	41.			$R_{.6}$	
17.	13	cos	42.			$R_{.7}$	
18.	34	RCL	43.			$R_{.8}$	
19.	03	3	44.			$R_{.9}$	
20.	31	f	45.				
21.	13	cos	46.				
22.	71	x	47.				
23.	61	+	48.				
24.	32	g	49.				

Example:

Given the following two sides and included angle, find the remaining parameters of a spherical triangle.

$$A = 30^\circ, \quad b = 50.5^\circ, \quad c = 47.3^\circ$$

Solution:

$$a = 22.71^\circ$$

After running the program Spherical Triangle Solution a, b, and c

$$B = 87.88^\circ$$

$$C = 72.13^\circ$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store A, b, c	b	STO	2			
		c	STO	3			
		A	STO	4			
3	Find solution		BST	R/S			a
4	To find B and C the program now						
	has a, b, and c in the correct						
	registers to run Spherical						
	Triangle Solution a, b, and c						

SPHERICAL TRIANGLE SOLUTION a, b, c

Given the three sides of a spherical triangle this program calculates the angles by the following formula:

$$A = \cos^{-1}\left(\frac{\cos a - \cos b \cos c}{\sin b \sin c}\right)$$

$$B = \cos^{-1}\left(\frac{\cos b - \cos a \cos c}{\sin a \sin b}\right)$$

$$C = \cos^{-1}\left(\frac{\cos c - \cos a \cos b}{\sin a \sin b}\right)$$

The program works in any angular mode. However, if in degree mode all angles are assumed to be in decimal degrees.

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	34	RCL
02.	01	1
03.	31	f
04.	13	cos
05.	34	RCL
06.	02	2
07.	31	f
08.	13	cos
09.	34	RCL
10.	03	3
11.	31	f
12.	13	cos
13.	71	x
14.	51	–
15.	34	RCL
16.	02	2
17.	31	f
18.	12	sin
19.	34	RCL
20.	03	3
21.	31	f
22.	12	sin
23.	71	x
24.	81	÷

DISPLAY		KEY ENTRY
LINE	CODE	
25.	32	g
26.	13	$\cos^{-1}$
27.	84	R/S
28.	34	RCL
29.	01	1
30.	34	RCL
31.	02	2
32.	34	RCL
33.	03	3
34.	33	STO
35.	02	2
36.	23	R↓
37.	33	STO
38.	01	1
39.	23	R↓
40.	33	STO
41.	03	3
42.	-01	GTO 01
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS	
R_0	
R_1	a
R_2	b
R_3	c
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

Given the following three sides of a spherical triangle calculate the three angles.

$$a = 1.12°, \quad b = 52.38°, \quad \text{and } c = 53.42°$$

Solution:

$$A = .52°, \quad B = 21.63°, \quad C = 158.05°$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store a, b, and c	a	STO	1			
		b	STO	2			
		c	STO	3			
3	Solve for A, B, and C		BST	R/S			A
			R/S				B
			R/S				C

SPHERICAL TRIANGLE SOLUTION A, B, C

Given the three angles of a spherical triangle this program calculates the sides by the following formulas:

$$a = \cos^{-1}\left(\frac{\cos A + \cos B \cos C}{\sin B \sin C}\right)$$

$$b = \cos^{-1}\left(\frac{\cos B + \cos A \cos C}{\sin A \sin C}\right)$$

$$c = \cos^{-1}\left(\frac{\cos C + \cos A \cos B}{\sin A \sin B}\right)$$

The program works in any angular mode. However, if in degree mode all angles are assumed to be in decimal degrees.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	DISPLAY LINE	DISPLAY CODE	KEY ENTRY	REGISTERS
00.			25.	32	g	R_0
01.	34	RCL	26.	13	$\cos^{-1}$	R_1 A
02.	01	1	27.	84	R/S	R_2 B
03.	31	f	28.	34	RCL	R_3 C
04.	13	cos	29.	03	3	R_4
05.	34	RCL	30.	34	RCL	R_5
06.	02	2	31.	02	2	R_6
07.	31	f	32.	34	RCL	R_7
08.	13	cos	33.	01	1	R_8
09.	34	RCL	34.	33	STO	R_9
10.	03	3	35.	03	3	$R_{.0}$
11.	31	f	36.	23	R↓	$R_{.1}$
12.	13	cos	37.	33	STO	$R_{.2}$
13.	71	x	38.	01	1	$R_{.3}$
14.	61	+	39.	23	R↓	$R_{.4}$
15.	34	RCL	40.	33	STO	$R_{.5}$
16.	02	2	41.	02	2	$R_{.6}$
17.	31	f	42.	-01	GTO 01	$R_{.7}$
18.	12	sin	43.			$R_{.8}$
19.	34	RCL	44.			$R_{.9}$
20.	03	3	45.			
21.	31	f	46.			
22.	12	sin	47.			
23.	71	x	48.			
24.	81	÷	49.			

Example:

Given the following three angles of a spherical triangle find the three sides.

$$A = .52°, \quad B = 21.63°, \quad C = 158.05°$$

Solution:

$$a = 1.10°, b = 51.51°, \quad c = 52.53°$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store A, B, and C	A	STO	1			
		B	STO	2			
		C	STO	3			
3	Calculate a, b, and c		BST	R/S			a
			R/S				b
			R/S				c

TRANSLATION AND/OR ROTATION OF COORDINATE AXIS

Let (x, y) be coordinates in the old system and let (x_0, y_0) be the center of a new coordinate system rotated through an angle of θ. The new coordinates are (x', y') and are calculated by the following formulas:

1. $x' = (x - x_0) \cos\theta + (y - y_0) \sin\theta$

2. $y' = -(x - x_0) \sin\theta + (y - y_0) \cos\theta$

For no rotation put in $\theta = 0$.

For no translation put in $(x_0, y_0) = (0, 0)$

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY
LINE	CODE		LINE	CODE	
00.			25.	51	–
01.	34	RCL	26.	22	x⇄y
02.	02	2	27.	34	RCL
03.	51	–	28.	04	4
04.	34	RCL	29.	61	+
05.	01	1	30.	-00	GTO 00
06.	22	x⇄y	31.		
07.	31	f	32.		
08.	00	R←P	33.		
09.	33	STO	34.		
10.	04	4	35.		
11.	23	R↓	36.		
12.	33	STO	37.		
13.	05	5	38.		
14.	23	R↓	39.		
15.	34	RCL	40.		
16.	03	3	41.		
17.	51	–	42.		
18.	34	RCL	43.		
19.	01	1	44.		
20.	22	x⇄y	45.		
21.	31	f	46.		
22.	00	R←P	47.		
23.	34	RCL	48.		
24.	05	5	49.		

REGISTERS	
R_0	
R_1	θ
R_2	x_0
R_3	y_0
R_4	$(x - x_0) \cos\theta$
R_5	$(x - x_0) \sin\theta$
R_6	
R_7	
R_8	
R_9	
$R_{.0}$	
$R_{.1}$	
$R_{.2}$	
$R_{.3}$	
$R_{.4}$	
$R_{.5}$	
$R_{.6}$	
$R_{.7}$	
$R_{.8}$	
$R_{.9}$	

Examples:

1. Translate the point (1, 1) to a new coordinate system with center at (1, 2). ($\theta = 0$)

2. Translate the point (1, 3) to a new coordinate system with center at (−1, 4) at 30° to the old system.

Solutions:

1. (0, − 1)

2. (1.23, −1.87)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store constants	θ	STO	1			
		x_0	STO	2			
		y_0	STO	3			
3	Enter coordinates	y	↑				
		x	BST	R/S			x′
			x⇄y				y′
4	For a new point go to step 3.						

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