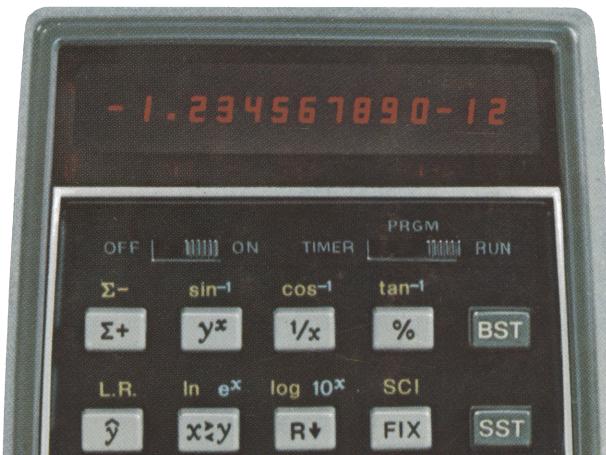
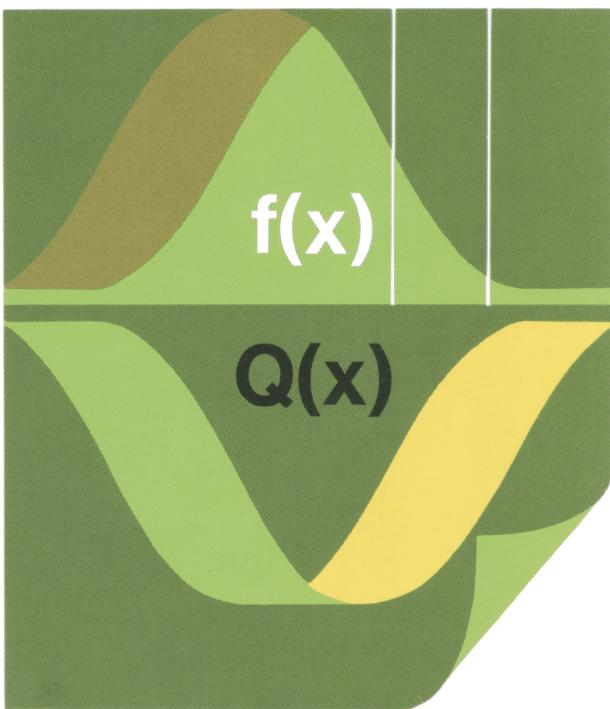




HP-55 statistics programs

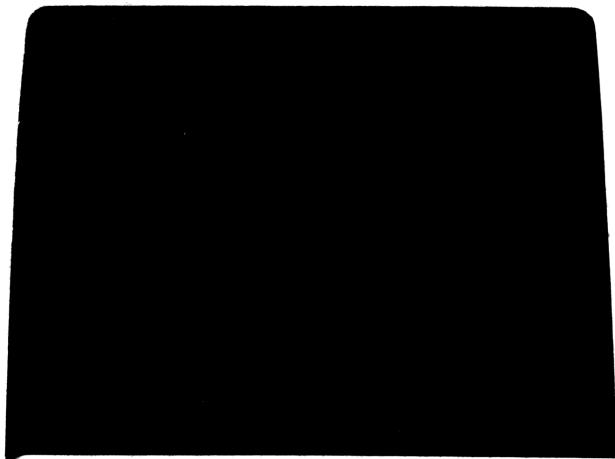


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HP-55 statistics programs



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INTRODUCTION

Material in *HP-55 Statistics Programs* has been selected from the areas of probability, general statistics, distribution functions, curve fitting, and test statistics.

Each program includes a general description, formulas used in the program solution, numerical examples, and user instructions. Program listings and register allocations are also given. The body of the book is arranged logically according to subject matter. The back cover contains an index.

We suggest that you first read the material explaining the Format of User Instructions, then use the programs. An understanding of the *HP-55 Owner's Handbook* is also required if, in addition, you wish to track the changes in the storage registers and stack registers on a step-by-step basis.

We hope you find *HP-55 Statistics Programs* a useful tool for your statistical work and welcome your comments, requests, and suggestions—these are our most important source of future user-oriented programs.

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4 Format of User Instructions

FORMAT OF USER INSTRUCTIONS

The completed User Instructions form is your guide to operating the programs in this book.

The form is composed of five columns. Reading from left to right, the STEP column gives the instruction step number. A step number with the symbol “prime” (') placed to its upper right indicates that step is optional or alternate to the step with the same number.

The INSTRUCTIONS column gives instructions and comments concerning the operations to be performed. Steps are executed in sequential order except where the INSTRUCTIONS column directs otherwise.

Normally, the first instruction is “Enter program”, which means to store the keystrokes of the program into memory (press **BST** in RUN mode, switch to PRGM mode, key in the program, then switch back to RUN mode).

Repeated processes, used in most cases for a long string of input/output data, are outlined with a bold border together with a “Perform” instruction.

The INPUT DATA/UNITS column specified the input data to be supplied, and the units of data if applicable.

The KEYS column specifies the keys to be pressed. **↑** is the symbol used to denote the **ENTER↑** key. All other key designations are identical to those appearing on the HP-55. Ignore any blank positions in the KEYS column.

Some programs are sufficiently complex that users have to press additional keys (other than program-control keys) in order to get the answers. Those keys will also be shown in the KEYS column.

The following is an example of User Instructions (for the Behrens-Fisher Statistic program).

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL·R			0.00
3	Perform 3 for $i = 1, 2, \dots, n_1$	x_i	$\Sigma+$				i
3'	Delete erroneous data x_k	x_k	f	$\Sigma-$			
4	Compute \bar{x} and $s_1/\sqrt{n_1}$		f	\bar{x}	STO	0	\bar{x}
			g	s	RCL	\cdot	
			0	f	\sqrt{x}	\div	$s_1/\sqrt{n_1}$
			STO	1	g	CL·R	0.00
5	Perform 5 for $i = 1, 2, \dots, n_2$	y_i	$\Sigma+$				i
5'	Delete erroneous data y_h	y_h	f	$\Sigma-$			
6	Input D and compute d and θ	D	BST	R/S			d
			R/S				θ
7	For a new case, go to 2						

- Step 1:** The first step in all programs is to enter the program into the calculator.
- Step 2:** The initialization step clears the stack and registers $R_{\bullet 0}$ through $R_{\bullet 9}$.
- Step 3:** This is a loop which accumulates sums for input data x_i 's. The first time through the loop the dummy variable i takes the value 1; the second time, i takes the value 2; etc.
- Step 3':** Only executed when you want to remove data entered in step 3.
- Step 4:** User has to press additional keystrokes to compute intermediate results and reinitialize registers. \bar{x} and $s_1/\sqrt{n_1}$ are computed and displayed.
- Step 5:** This is a loop which accumulates sums for input data y_i 's.
- Step 5':** Only executed when you want to remove data entered in step 5.
- Step 6:** D is an input. Answers d and θ are computed.
- Step 7:** This step gives instructions for starting a new case. In this example, return to step 2.

6 Permutation

PERMUTATION

A permutation is an ordered subset of a set of distinct objects. The number of possible permutations, each containing n objects, that can be formed from a collection of m distinct objects is given by

$${}_m P_n = \frac{m!}{(m-n)!} = m(m-1) \dots (m-n+1)$$

where m, n are integers and $0 \leq n \leq m$.

Notes:

1. ${}_m P_n$ can also be denoted by P_n^m , $P(m,n)$ or $(m)_n$.
2. ${}_m P_0 = 1$, ${}_m P_1 = m$, ${}_m P_m = m!$

DISPLAY		KEY ENTRY	DISPLAY	KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE	
00.			25.	00	$R_0 \leftarrow m$
01.	41	\uparrow	26.	01	R_1
02.	33	STO	27.	51	R_2
03.	00	0	28.	32	R_3
04.	84	R/S	29.	-32	R_4
05.	32	g	30.	23	R_5
06.	-35	$x=y$ 35	31.	-19	R_6
07.	31	f	32.	23	R_7
08.	-11	$x \leq y$ 11	33.	23	R_8
09.	00	0	34.	-00	R_9
10.	81	\div	35.	31	$R_{\bullet 0}$
11.	01	1	36.	43	$R_{\bullet 1}$
12.	32	g	37.	-00	$R_{\bullet 2}$
13.	-32	$x=y$ 32	38.	01	$R_{\bullet 3}$
14.	44	CLX	39.	-00	$R_{\bullet 4}$
15.	32	g	40.	41	$R_{\bullet 5}$
16.	-38	$x=y$ 38	41.	31	$R_{\bullet 6}$
17.	61	+	42.	43	$R_{\bullet 7}$
18.	51	-	43.	22	$R_{\bullet 8}$
19.	01	1	44.	84	$R_{\bullet 9}$
20.	61	+	45.	51	
21.	71	x	46.	31	
22.	31	f	47.	43	
23.	34	LAST X	48.	81	
24.	34	RCL	49.	-00	GTO 00

Examples:

1. ${}_2 {}_7 P_5 = 9687600.00$
2. ${}_7 {}_3 P_4 = 26122320.00$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input m, n	m	BST	R/S			m
		n	R/S				m^P_n
2'	If $m \leq 69$, for a faster execution		GTO	4	0		
		m	R/S				m
		n	R/S				m^P_n
3	For a new case, go to 2						

8 Combination

COMBINATION

A combination is a selection of one or more of a set of distinct objects without regard to order. The number of possible combinations, each containing n objects, that can be formed from a collection of m distinct objects is given by

$${}_m C_n = \frac{m!}{(m-n)! n!} = \frac{m(m-1) \dots (m-n+1)}{1 \cdot 2 \cdot \dots \cdot n}$$

where m, n are integers and $0 \leq n \leq m$.

This program computes ${}_m C_n$ using the following algorithm:

1. If $n \leq m - n$

$${}_m C_n = \frac{m-n+1}{1} \cdot \frac{m-n+2}{2} \cdot \dots \cdot \frac{m}{n} .$$

2. If $n > m - n$, program computes ${}_m C_{m-n}$.

Notes:

1. ${}_m C_n$, which is also called the binomial coefficient, can be denoted by C_n^m , $C(m,n)$, or $(\binom{m}{n})$.
2. ${}_m C_n = {}_m C_{m-n}$
3. ${}_m C_0 = {}_m C_m = 1$
4. ${}_m C_1 = {}_m C_{m-1} = m$

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE			
00.			25.	-29	x≤y 29	R₀	max(n, m-n)
01.	51	-	26.	34	RCL	R₁	Used
02.	31	f	27.	02	2	R₂	Used
03.	34	LAST X	28.	-00	GTO 00	R₃	
04.	31	f	29.	34	RCL	R₄	
05.	-42	x≤y 42	30.	00	0	R₅	
06.	33	STO	31.	22	x↔y	R₆	
07.	00	0	32.	61	+	R₇	
08.	01	1	33.	31	f	R₈	
09.	33	STO	34.	34	LAST X	R₉	
10.	01	1	35.	81	÷	R_{•0}	
11.	61	+	36.	34	RCL	R_{•1}	
12.	33	STO	37.	02	2	R_{•2}	
13.	02	2	38.	71	x	R_{•3}	
14.	44	CLX	39.	33	S10	R_{•4}	
15.	32	g	40.	02	2	R_{•5}	
16.	-44	x=y 44	41.	-17	GTO 17	R_{•6}	
17.	23	R↓	42.	22	x↔y	R_{•7}	
18.	01	1	43.	-06	GTO 06	R_{•8}	
19.	34	RCL	44.	01	1	R_{•9}	
20.	01	1	45.	-00	GTO 00		
21.	61	+	46.				
22.	33	STO	47.				
23.	01	1	48.				
24.	31	f	49.				

Examples:

- ${}_{73}C_4 = 1088430.00$
 - ${}_{27}C_5 = 80730.00$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input m, n	m	↑				
		n	BST	R/S			mC_n
3	For a new case, go to 2						

10 Bayes' Formula

BAYES' FORMULA

Suppose E_1, E_2, \dots, E_n are n mutually exclusive and exhaustive events, and A is an event for which the conditional probabilities, $P[A/E_i]$ of A given E_i , are known. If $P[E_i]$ are given, then the conditional probability $P[E_k/A]$ of any one event E_k given A is

$$P[E_k/A] = \frac{P[E_k] P[A/E_k]}{\sum_{i=1}^n P[E_i] P[A/E_i]}$$

where k can be 1, 2, ..., or n .

Reference:

E. Parzen, *Modern Probability Theory and its Applications*, John Wiley and Sons, 1960.

DISPLAY		KEY ENTRY	DISPLAY	KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE	
00.			25.	51	$R_0 \Sigma P[A/E_i] P[E_i]$
01.	00	0	26.	33	$R_1 n$
02.	33	STO	27.	01	R_2
03.	00	0	28.	-06	R_3
04.	33	STO	29.	71	R_4
05.	01	1	30.	34	R_5
06.	84	R/S	31.	00	R_6
07.	71	x	32.	81	R_7
08.	33	STO	33.	-00	R_8
09.	61	+	34.		R_9
10.	00	0	35.		R_{e0}
11.	34	RCL	36.		R_{e1}
12.	01	1	37.		R_{e2}
13.	01	1	38.		R_{e3}
14.	61	+	39.		R_{e4}
15.	33	STO	40.		R_{e5}
16.	01	1	41.		R_{e6}
17.	-06	GTO 06	42.		R_{e7}
18.	71	x	43.		R_{e8}
19.	33	STO	44.		R_{e9}
20.	51	-	45.		
21.	00	0	46.		
22.	34	RCL	47.		
23.	01	1	48.		
24.	01	1	49.		

Example:

If $P[E_1] = 0.95$
 $P[A/E_1] = 0.005$
 $P[E_2] = 0.05$
 $P[A/E_2] = 0.995$
then $P[E_1/A] = .09$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS					OUTPUT DATA/UNITS
1	Enter program							
2	Initialize		BST	R/S				0.00
3	Perform 3 for $i = 1, 2, \dots, n$	$P[E_i]$	\uparrow					
		$P[A/E_i]$	R/S					i
3'	Delete erroneous data $P[E_m]$,							
	$P[A/E_m]$	$P[E_m]$	\uparrow					
		$P[A/E_m]$	GTO	1	8	R/S		
4	Compute $P[E_k/A]$	$P[E_k]$	\uparrow					
		$P[A/E_k]$	GTO	2	9	R/S		$P[E_k/A]$
5	For a different k , go to 4							
6	For a new case, go to 2							

12 Probability of No Repetitions in a Sample

PROBABILITY OF NO REPETITIONS IN A SAMPLE

Suppose a sample of size n is drawn with replacement from a population containing m different objects. Let P be the probability that there are no repetitions in the sample, then

$$P = \left(1 - \frac{1}{m}\right) \left(1 - \frac{2}{m}\right) \cdots \left(1 - \frac{n-1}{m}\right).$$

Given integers m , n such that $m \geq n \geq 1$, this program finds the probability P .

Note:

The execution time of the program depends on n ; the larger n is, the longer it takes.

Reference:

E. Parzen, *Modern Probability Theory and its Applications*, John Wiley and Sons, 1960.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE		
00.			25.	00	0	R ₀ Used
01.	33	STO	26.	-06	GTO 06	R ₁ m
02.	02	2	27.	34	RCL	R ₂ Used
03.	01	1	28.	00	0	R ₃
04.	33	STO	29.	-00	GTO 00	R ₄
05.	00	0	30.			R ₅
06.	34	RCL	31.			R ₆
07.	01	1	32.			R ₇
08.	34	RCL	33.			R ₈
09.	02	2	34.			R ₉
10.	01	1	35.			R ₀
11.	51	-	36.			R _{•1}
12.	33	STO	37.			R _{•2}
13.	02	2	38.			R _{•3}
14.	00	0	39.			R _{•4}
15.	32	g	40.			R _{•5}
16.	-27	x=y 27	41.			R _{•6}
17.	23	R↓	42.			R _{•7}
18.	22	x↔y	43.			R _{•8}
19.	81	÷	44.			R _{•9}
20.	01	1	45.			
21.	22	x↔y	46.			
22.	51	-	47.			
23.	33	STO	48.			
24.	71	x	49.			

Example:

In a room containing n persons, what is the probability that no two or more persons have the same birthday for $n = 4, 23, 48$?

(Note: $m = 365$)

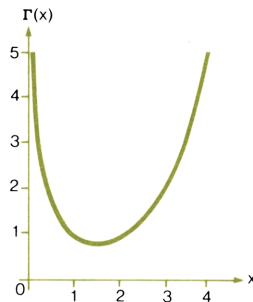
1. $n = 4, P = .98$
2. $n = 23, P = .49$
3. $n = 48, P = .04$

(That is, in a room having 48 persons, the probability that at least two of them will have the same birthday is as high as $1 - .04 = 0.96$.)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input m	m	STO	1	BST		
3	Input n	n	R/S				P
4	For different n , go to 3						
5	For a new case, go to 2						

GAMMA FUNCTION

This program approximates the value of the gamma function $\Gamma(x)$ for $1 \leq x \leq 64$.



$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$$

$$\cong \sqrt{2\pi/x} \cdot x^x e^{-\left(x - \frac{1}{12x} + \frac{1}{360x^3}\right)}$$

Suppose ϵ is the error, then

$$\frac{\epsilon}{\Gamma(x)} < 2 \times 10^{-7}$$

This approximation is good for large x . In order to increase the accuracy (especially for small values of x), the program computes $\Gamma(x+5)$, then $\Gamma(x)$ is calculated using the following formula

$$\Gamma(x) = \frac{\Gamma(x+5)}{(x+4)(x+3)(x+2)(x+1)x}.$$

Note:

This program can be used to find the generalized factorial $x!$ for $0 \leq x \leq 63$.

$$x! = \Gamma(x+1)$$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1968.

DISPLAY		KEY ENTRY	DISPLAY	KEY ENTRY	REGISTERS																			
LINE	CODE		LINE	CODE	R ₀	R ₁	R ₂	R ₃	R ₄	R ₅	R ₆	R ₇	R ₈	R ₉	R _{*0}	R _{*1}	R _{*2}	R _{*3}	R _{*4}	R _{*5}	R _{*6}	R _{*7}	R _{*8}	R _{*9}
00.			25.	32	g																			
01.	05	5	26.	22	e ^x																			
02.	61	+	27.	22	x \leftrightarrow y																			
03.	41	↑	28.	41	↑																			
04.	13	1/x	29.	61	+																			
05.	41	↑	30.	31	f																			
06.	71	x	31.	83	π																			
07.	41	↑	32.	71	x																			
08.	41	↑	33.	31	f																			
09.	03	3	34.	42	\sqrt{x}																			
10.	00	0	35.	71	x																			
11.	81	÷	36.	33	STO																			
12.	01	1	37.	00	0																			
13.	51	—	38.	44	CLX																			
14.	71	x	39.	05	5																			
15.	01	1	40.	51	—																			
16.	02	2	41.	33	STO																			
17.	81	÷	42.	81	÷																			
18.	22	x \leftrightarrow y	43.	00	0																			
19.	31	f	44.	01	1																			
20.	22	ln	45.	61	+																			
21.	51	—	46.	31	f																			
22.	71	x	47.	-41	x \leqslant y 41																			
23.	61	+	48.	34	RCL																			
24.	42	CHS	49.	00	0																			

Examples:

1. $\Gamma(5.25) = 35.21$
2. $7! = \Gamma(8) = 5040.00$
3. $2.34! = \Gamma(3.34) = 2.80$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Initialize		BST	
3	Input x	x	R/S	$\Gamma(x)$
4	For a new case, go to 3			

INCOMPLETE GAMMA FUNCTION

$$\begin{aligned}\gamma(a, x) &= \int_0^x e^{-t} t^{a-1} dt \\ &= x^a e^{-x} \sum_{n=0}^{\infty} \frac{x^n}{a(a+1)\dots(a+n)}\end{aligned}$$

where $a > 0, x > 0$.

This program computes successive partial sums of the above series. The program stops when two consecutive partial sums are equal and displays the last partial sum as the answer.

Note:

When x is too large, computing a new term of the series might cause an overflow. In that case, display shows all 9's and the program stops.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1968.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE		
00.			25.	02	2	R ₀ x
01.	33	STO	26.	61	+	R ₁ Used
02.	00	0	27.	32	g	R ₂ Used
03.	22	x↔y	28.	-30	x=y 30	R ₃
04.	33	STO	29.	-12	GTO 12	R ₄
05.	01	1	30.	34	RCL	R ₅
06.	12	y ^x	31.	00	0	R ₆
07.	34	RCL	32.	32	g	R ₇
08.	01	1	33.	22	e ^x	R ₈
09.	81	÷	34.	81	÷	R ₉
10.	33	STO	35.	-00	GTO 00	R _{•0}
11.	02	2	36.			R _{•1}
12.	34	RCL	37.			R _{•2}
13.	00	0	38.			R _{•3}
14.	34	RCL	39.			R _{•4}
15.	01	1	40.			R _{•5}
16.	01	1	41.			R _{•6}
17.	61	+	42.			R _{•7}
18.	33	STO	43.			R _{•8}
19.	01	1	44.			R _{•9}
20.	81	÷	45.			
21.	34	RCL	46.			
22.	02	2	47.			
23.	71	x	48.			
24.	33	STO	49.			

Examples:

- $$1. \quad \gamma(1, 2) = .86$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input a, x	a					
		x	BST	R/S			$\gamma(a, x)$
3	For a new case, go to 2						

18 Error Function and Complementary Error Function

ERROR FUNCTION AND COMPLEMENTARY ERROR FUNCTION

$$\text{Error function } \operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$= \frac{2}{\sqrt{\pi}} e^{-x^2} \sum_{n=0}^{\infty} \frac{2^n}{1 \cdot 3 \cdot \dots \cdot (2n+1)} x^{2n+1}$$

Complementary error function

$$\operatorname{erfc} x = 1 - \operatorname{erf} x$$

where $x > 0$.

This program computes successive partial sums of the series. The program stops when two consecutive partial sums are equal and displays the last partial sum as the answer.

Notes:

1. When x is too large, computing a new term of the series might cause an overflow. In that case, display shows all 9's and the program stops.
2. The execution time of the program depends on x ; the larger x is, the longer it takes.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1968.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS										
LINE	CODE		LINE	CODE												
00.			25.	71	x	R ₀	Used									
01.	33	STO	26.	33	STO	R ₁	2x ²									
02.	00	0	27.	00	0	R ₂	Used									
03.	41	↑	28.	61	+	R ₃										
04.	71	x	29.	32	g	R ₄										
05.	02	2	30.	-32	x=y 32	R ₅										
06.	71	x	31.	-14	GTO 14	R ₆										
07.	33	STO	32.	02	2	R ₇										
08.	01	1	33.	71	x	R ₈										
09.	01	1	34.	31	f	R ₉										
10.	33	STO	35.	83	π	R _{•0}										
11.	02	2	36.	31	f	R _{•1}										
12.	34	RCL	37.	42	√x	R _{•2}										
13.	00	0	38.	34	RCL	R _{•3}										
14.	34	RCL	39.	01	1	R _{•4}										
15.	01	1	40.	02	2	R _{•5}										
16.	34	RCL	41.	81	÷	R _{•6}										
17.	02	2	42.	32	g	R _{•7}										
18.	02	2	43.	22	e ^x	R _{•8}										
19.	61	+	44.	71	x	R _{•9}										
20.	33	STO	45.	81	÷											
21.	02	2	46.	84	R/S											
22.	81	÷	47.	01	1											
23.	34	RCL	48.	22	x ⁻ y											
24.	00	0	49.	51	-											

Example:

$$\text{erf } 1.34 = .94$$

$$\text{erfc } 1.34 = .06$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Compute erf x and erfc x	x	BST	R/S			erf x
			R/S				erfc x
3	For a new case, go to 2						

RANDOM NUMBER GENERATOR

This program calculates uniformly distributed pseudo random numbers u_i in the range

$$0 \leq u_i \leq 1$$

using the following formula:

$$u_i = \text{Fractional part of } [(\pi + u_{i-1})^5].$$

The user has to specify the starting value u_0 such that

$$0 \leq u_0 \leq 1.$$

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE		
00.			25.	23	R↓	R ₀ u _i
01.	33	STO	26.	33	STO	R ₁
02.	00	0	27.	00	0	R ₂
03.	84	R/S	28.	-03	GTO 03	R ₃
04.	31	f	29.	51	—	R ₄
05.	83	π	30.	-26	GTO 26	R ₅
06.	34	RCL	31.			R ₆
07.	00	0	32.			R ₇
08.	61	+	33.			R ₈
09.	05	5	34.			H ₉
10.	12	y ^x	35.			R _{•0}
11.	41	↑	36.			R _{•1}
12.	41	↑	37.			R _{•2}
13.	43	EEX	38.			R _{•3}
14.	09	9	39.			R _{•4}
15.	61	+	40.			R _{•5}
16.	43	EEX	41.			R _{•6}
17.	09	9	42.			R _{•7}
18.	51	—	43.			R _{•8}
19.	01	1	44.			R _{•9}
20.	51	—	45.			
21.	51	—	46.			
22.	01	1	47.			
23.	31	f	48.			
24.	-29	x≤y 29	49.			

Example:

The following uniformly distributed pseudo random numbers are generated for $u_0 = 0$: .02, .73, .70, .31, .58, .85, .86, .43, .33, .60, .67, .93, .22, .32, .45, .50,

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS		
1	Enter program								
2	Input u_0	u_0	BST	R/S			u_0		
3	Perform 3 for $i = 1, 2, 3, \dots$		R/S				u_i		
4	For a new case, go to 2								

22 Mean, Standard Deviation, Standard Error for Grouped Data

MEAN, STANDARD DEVIATION, STANDARD ERROR FOR GROUPED DATA

Given a set of data points

$$x_1, x_2, \dots, x_n$$

with respective frequencies

$$f_1, f_2, \dots, f_n$$

the program computes the following statistics:

$$\text{mean } \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\text{standard deviation } s = \sqrt{\frac{\sum f_i x_i^2 - (\sum f_i) \bar{x}^2}{\sum f_i - 1}}$$

$$\text{standard error } s_{\bar{x}} = \frac{s_x}{\sqrt{\sum f_i}} .$$

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		R ₀	Σf_i
00.			25.	34	RCL	R ₁	
01.	33	STO	26.	00	0	R ₂	
02.	61	+	27.	33	STO	R ₃	
03.	00	0	28.	83	.	R ₄	
04.	22	x \leftrightarrow y	29.	00	0	R ₅	
05.	71	x	30.	34	RCL	R ₆	
06.	31	f	31.	83	.	R ₇	
07.	34	LAST X	32.	03	3	R ₈	
08.	22	x \leftrightarrow y	33.	33	STO	R ₉	
09.	71	x	34.	83	.	R ₀	$n, \Sigma f_i$
10.	31	f	35.	02	2	R ₁	$\Sigma f_i x_i$
11.	34	LAST X	36.	31	f	R ₂	$(\Sigma f_i x_i)^2, \Sigma f_i x_i^2$
12.	11	$\Sigma +$	37.	33	\bar{x}	R ₃	$\Sigma f_i x_i^2$
13.	-00	GTO 00	38.	84	R/S	R ₄	$(\Sigma f_i x_i^2)^2$
14.	42	CHS	39.	32	g	R ₅	$\Sigma f_i^2 x_i^3$
15.	34	RCL	40.	33	s	R ₆	0
16.	83	.	41.	84	R/S	R ₇	0
17.	00	0	42.	34	RCL	R ₈	0
18.	02	2	43.	00	0	R ₉	0
19.	51	-	44.	31	f		
20.	33	STO	45.	42	\sqrt{x}		
21.	83	.	46.	81	\div		
22.	00	0	47.	-00	GTO 00		
23.	23	R↓	48.				
24.	-01	GTO 01	49.				

Example:

x_i	2	3.4	7	11	23	3.41
f_i	5	3	4	2	3	1

$$\bar{x} = 7.92$$

$$s = 7.52$$

$$s_{\bar{x}} = 1.77$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL-R	STO	0	
			BST				0.00
3	Perform 3 for $i = 1, 2, \dots, n$	x_i	\uparrow				
		f_i	R/S				i
3'	Delete erroneous data x_k, f_k	x_k	\uparrow				
		f_k	GTO	1	4	R/S	
4	Compute \bar{x}, s and $s_{\bar{x}}$		GTO	2	5	R/S	\bar{x}
			R/S				s
			R/S				$s_{\bar{x}}$
5	For a new case, go to 2						

24 Geometric Mean

GEOMETRIC MEAN

For a set of n positive numbers $\{a_1, a_2, \dots, a_n\}$, the geometric mean is defined by

$$G = (a_1 a_2 \dots a_n)^{\frac{1}{n}}.$$

DISPLAY		KEY ENTRY	DISPLAY	KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE	
00.			25.	12	y^x
01.	01	1	26.	-00	GTO 00
02.	33	STO	27.	33	STO
03.	01	1	28.	81	\div
04.	00	0	29.	01	1
05.	33	STO	30.	34	RCL
06.	00	0	31.	00	0
07.	84	R/S	32.	01	1
08.	34	RCL	33.	51	-
09.	01	1	34.	33	STO
10.	71	x	35.	00	0
11.	33	STO	36.	-07	GTO 07
12.	01	1	37.		
13.	34	RCL	38.		
14.	00	0	39.		
15.	01	1	40.		
16.	61	+	41.		
17.	33	STO	42.		
18.	00	0	43.		
19.	-07	GTO 07	44.		
20.	34	RCL	45.		
21.	01	1	46.		
22.	34	RCL	47.		
23.	00	0	48.		
24.	13	${}^1/x$	49.		

Example:

The set of numbers $\{2, 3.4, 3.41, 7, 11, 23\}$ has the geometric mean $G = 5.87$.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Perform 3 for $i = 1, 2, \dots, n$	a_i	R/S				i
3'	Delete erroneous data a_k	a_k	GTO	2	7	R/S	
4	Compute the mean G		GTO	2	0	R/S	G
5	For a new case, go to 2						

26 Harmonic Mean

HARMONIC MEAN

For a set of n positive numbers $\{a_1, a_2, \dots, a_n\}$, the harmonic mean is defined by

$$H = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}} .$$

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE		
00.			25.	-00	GTO 00	$R_0 \cdot n$
01.	00	0	26.	13	$1/x$	$R_1 \cdot \sum 1/a_i$
02.	33	STO	27.	33	STO	R_2
03.	00	0	28.	51	-	R_3
04.	33	STO	29.	01	1	R_4
05.	01	1	30.	34	RCL	R_5
06.	84	R/S	31.	00	0	R_6
07.	13	$1/x$	32.	01	1	R_7
08.	34	RCL	33.	51	-	R_8
09.	01	1	34.	33	STO	R_9
10.	61	+	35.	00	0	$R_{\bullet 0}$
11.	33	STO	36.	-06	GTO 06	$R_{\bullet 1}$
12.	01	1	37.			$R_{\bullet 2}$
13.	34	RCL	38.			$R_{\bullet 3}$
14.	00	0	39.			$R_{\bullet 4}$
15.	01	1	40.			$R_{\bullet 5}$
16.	61	+	41.			$R_{\bullet 6}$
17.	33	STO	42.			$R_{\bullet 7}$
18.	00	0	43.			$R_{\bullet 8}$
19.	-06	GTO 06	44.			$R_{\bullet 9}$
20.	34	RCL	45.			
21.	00	0	46.			
22.	34	RCL	47.			
23.	01	1	48.			
24.	81	\div	49.			

Example:

The harmonic mean for the set of numbers $\{2, 3.4, 3.41, 7, 11, 23\}$ is $H = 4.40$.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Perform 3 for $i = 1, 2, \dots, n$	a_i	R/S				i
3'	Delete erroneous data a_k	a_k	GTO	2	6	R/S	
4	Compute the mean H		GTO	2	0	R/S	H
5	For a new case, go to 2						

GENERALIZED MEAN

For a set of n positive numbers $\{a_1, a_2, \dots, a_n\}$, the generalized mean is defined by

$$M(t) = \left(\frac{1}{n} \sum_{k=1}^n a_k^t \right)^{\frac{1}{t}}$$

where t is any desired number.

Notes:

1. If $t = 1$, the generalized mean M (1) is the same as the arithmetic mean.
2. If $t = -1$, the generalized mean M (-1) is the same as the harmonic mean.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE			
00.			25.	34	RCL	R_0	n
01.	00	0	26.	01	1	R_1	Σa_k^t
02.	33	STO	27.	34	RCL	R_2	t
03.	00	0	28.	00	0	R_3	
04.	33	STO	29.	81	\div	R_4	
05.	01	1	30.	34	RCL	R_5	
06.	84	R/S	31.	02	2	R_6	
07.	33	STO	32.	13	${}^1/x$	R_7	
08.	02	2	33.	12	y^x	R_8	
09.	84	R/S	34.	-00	GTO 00	R_9	
10.	34	RCL	35.	34	RCL	$R_{\bullet 0}$	
11.	02	2	36.	02	2	$R_{\bullet 1}$	
12.	12	y^x	37.	12	y^x	$R_{\bullet 2}$	
13.	34	RCL	38.	33	STO	$R_{\bullet 3}$	
14.	01	1	39.	51	-	$R_{\bullet 4}$	
15.	61	+	40.	01	1	$R_{\bullet 5}$	
16.	33	STO	41.	34	RCL	$R_{\bullet 6}$	
17.	01	1	42.	00	0	$R_{\bullet 7}$	
18.	34	RCL	43.	01	1	$R_{\bullet 8}$	
19.	00	0	44.	51	-	$R_{\bullet 9}$	
20.	01	1	45.	33	STO		
21.	61	+	46.	00	0		
22.	33	STO	47.	-09	GTO 09		
23.	00	0	48.				
24.	-09	GTO 09	49.				

Example:

The set of numbers $\{2, 3.4, 3.41, 7, 11, 23\}$ has the generalized mean $M(2) = 11.00$.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Input t	t	R/S				t
4	Perfrom 4 for i = 1, 2,..., n	a _i	R/S				i
4'	Delete erroneous data a _k	a _k	GTO	3	5	R/S	
5	Compute mean M(t)		GTO	2	5	R/S	M(t)
6	For a new case, go to 2						

MOVING AVERAGE

Given a set of numbers $\{x_1, x_2, x_3, \dots\}$, this program finds the moving averages of order n (n can be 2, 3, ..., or 9) given by the following sequence of arithmetic means:

$$\frac{x_1 + x_2 + \dots + x_n}{n}, \frac{x_2 + x_3 + \dots + x_{n+1}}{n}, \frac{x_3 + x_4 + \dots + x_{n+2}}{n}, \dots$$

The numerators are the moving totals of order n.

Note:

The program computes the total and the average of the first n numbers. Then x_{n+1} is added to and x_1 is removed from the total. A new average is computed. Similar procedure goes on until all answers are found. This program is written in such a way that the value that needed to be removed is stored in register R_n (where n is the order). In the following example, the order is 6, hence register R_6 contains the value.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		REGISTER	CONTENT
00.			25.	03	3	R_0	Used
01.	33	STO	26.	33	STO	R_1	Used
02.	83	.	27.	04	4	R_2	Used
03.	06	6	28.	34	RCL	R_3	Used
04.	34	RCL	29.	02	2	R_4	Used
05.	08	8	30.	33	STO	R_5	Used
06.	33	STO	31.	03	3	R_6	Used
07.	09	9	32.	34	RCL	R_7	Used
08.	34	RCL	33.	01	1	R_8	Used
09.	07	7	34.	33	STO	R_9	Used
10.	33	STO	35.	02	2	R_{00}	Used
11.	08	8	36.	34	RCL	R_{01}	Used
12.	34	RCL	37.	00	0	R_{02}	Used
13.	06	6	38.	33	STO	R_{03}	Used
14.	33	STO	39.	01	1	R_{04}	Used
15.	07	7	40.	34	RCL	R_{05}	Used
16.	34	RCL	41.	83	.	R_{06}	Used
17.	05	5	42.	06	6	R_{07}	0
18.	33	STO	43.	33	STO	R_{08}	0
19.	06	6	44.	00	0	R_{09}	0
20.	34	RCL	45.	11	$\Sigma+$		
21.	04	4	46.	-00	GTO 00		
22.	33	STO	47.				
23.	05	5	48.				
24.	34	RCL	49.				

Example:

For the following set of data $\{105, 121, 124, 97, 86, 134, 105, 81, 127, 132, 114, 121\}$, the moving averages of order 6 are 111.17, 111.17, 104.50, 105.00, 110.83, 115.50, 113.33.

The moving totals of order 6 are 667.00, 667.00, 627.00, 630.00, 665.00, 693.00, 680.00.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS			OUTPUT DATA/UNITS
1	Enter program					
2	Initialize		g	CL·R	BST	0.00
3	Perform 3 for $i = 1, 2, \dots, n$	x_i	R/S			i
4	Compute the moving average of order n		f	\bar{x}		average
5	(optional) Compute the moving total of order n		RCL	$\Sigma+$		total
6	Input next value	x_k	R/S			$n + 1$
7	Remove one old value		RCL			
	n^*	f	$\Sigma-$			n
8	Go to 4					
9	For a new case, go to 2					
	* n can be one of the values 2, 3, ..., 9.					

32 Covariance and Correlation Coefficient

COVARIANCE AND CORRELATION COEFFICIENT

For a set of given data points $\{(x_i, y_i), i = 1, 2, \dots, n\}$, the covariance and the correlation coefficient are defined as:

$$\text{covariance } s_{xy} = \frac{1}{n-1} \left(\sum x_i y_i - \frac{1}{n} \sum x_i \sum y_i \right)$$

$$\text{or } s_{xy}' = \frac{1}{n} \left(\sum x_i y_i - \frac{1}{n} \sum x_i \sum y_i \right)$$

$$\text{correlation coefficient } r = \frac{s_{xy}}{s_x s_y}$$

where s_x and s_y are standard deviations

$$s_x = \sqrt{\frac{\sum x_i^2 - (\sum x_i)^2/n}{n-1}}$$

$$s_y = \sqrt{\frac{\sum y_i^2 - (\sum y_i)^2/n}{n-1}}$$

Note:

$$-1 \leq r \leq 1$$

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE			
00.			25.	51	-	R ₀	
01.	32	g	26.	81	÷	R ₁	
02.	44	CL·R	27.	81	÷	R ₂	
03.	84	R/S	28.	-00	GTO 00	R ₃	
04.	31	f	29.			R ₄	
05.	21	L.R.	30.			R ₅	
06.	41	↑	31.			R ₆	
07.	32	g	32.			R ₇	
08.	33	s	33.			R ₈	
09.	81	÷	34.			R ₉	
10.	81	÷	35.			R _{0..n}	
11.	84	R/S	36.			R _{1..1} Σx _i	
12.	41	↑	37.			R _{2..2} Σx _i ²	
13.	41	↑	38.			R _{3..3} Σy _i	
14.	32	g	39.			R _{4..4} Σy _i ²	
15.	33	s	40.			R _{5..5} Σx _i y _i	
16.	71	x	41.			R _{6..6} 0	
17.	71	x	42.			R _{7..7} 0	
18.	84	R/S	43.			R _{8..8} 0	
19.	34	RCL	44.			R _{9..9} 0	
20.	83	•	45.				
21.	00	0	46.				
22.	41	↑	47.				
23.	41	↑	48.				
24.	01	1	49.				

Example:

y_i	92	85	78	81	54	51	40
x_i	26	30	44	50	62	68	74

$r = -0.96$

$s_{xy} = -354.14$

$s_{xy}' = -303.55$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2*	Initialize		BST R/S	0.00
3	Perform 3 for $i = 1, 2, \dots, n$	y_i	↑	
		x_i	Σ+	i
3'	Delete erroneous data x_k, y_k	y_k	↑	
		x_k	f Σ-	
4	Compute correlation coefficient			r
	r		R/S	
5	Compute covariance s_{xy}		R/S	s_{xy}
	(optional) Compute s_{xy}'		R/S	s_{xy}'
6	For a new case, go to 2			
	*Note: If sums are already accumulated in proper registers,			
	skip steps 2, 3 and 3'.			

34 Moments, Skewness and Kurtosis

MOMENTS, SKEWNESS AND KURTOSIS

This program computes the following statistics for a set of given data $\{x_1, x_2, \dots, x_n\}$:

$$1^{\text{st}} \text{ moment} \quad \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$2^{\text{nd}} \text{ moment} \quad m_2 = \frac{1}{n} \sum x_i^2 - \bar{x}^2$$

$$3^{\text{rd}} \text{ moment} \quad m_3 = \frac{1}{n} \sum x_i^3 - \frac{3}{n} \bar{x} \sum x_i^2 + 2\bar{x}^3$$

$$4^{\text{th}} \text{ moment} \quad m_4 = \frac{1}{n} \sum x_i^4 - \frac{4}{n} \bar{x} \sum x_i^3 + \frac{6}{n} \bar{x}^2 \sum x_i^2 - 3\bar{x}^4$$

moment coefficient of skewness

$$\gamma_1 = \frac{m_3}{m_2^{3/2}}$$

moment coefficient of kurtosis

$$\gamma_2 = \frac{m_4}{m_2^2}$$

Reference:

M. R. Spiegel, *Theory and Problems of Statistics*, Schaum's Outline, McGraw-Hill, 1961.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		R ₀	\bar{x}
00.			25.	04	4	R ₁	n
01.	71	x	26.	71	x	R ₂	m_2
02.	03	3	27.	51	-	R ₃	m_3
03.	71	x	28.	34	RCL	R ₄	m_4
04.	51	-	29.	83	.	R ₅	
05.	34	RCL	30.	04	4	R ₆	
06.	01	1	31.	34	RCL	R ₇	
07.	81	\div	32.	00	0	R ₈	
08.	34	RCL	33.	32	g	R ₉	
09.	00	0	34.	42	x^2	R _{•0}	n
10.	03	3	35.	71	x	R _{•1}	Σx_i^2
11.	12	y^x	36.	06	6	R _{•2}	Σx_i^4
12.	02	2	37.	71	x	R _{•3}	Σx_i
13.	71	x	38.	61	+	R _{•4}	Σx_i^2
14.	61	+	39.	34	RCL	R _{•5}	Σx_i^3
15.	84	R/S	40.	01	1	R _{•6}	0
16.	34	RCL	41.	81	\div	R _{•7}	0
17.	83	.	42.	34	RCL	R _{•8}	0
18.	02	2	43.	00	0	R _{•9}	0
19.	34	RCL	44.	04	4		
20.	00	0	45.	12	y^x		
21.	34	RCL	46.	03	3		
22.	83	.	47.	71	x		
23.	05	5	48.	51	-		
24.	71	x	49.	-00	GTO 00		

36 Moments, Skewness and Kurtosis

Example:

i	1	2	3	4	5	6	7	8	9
x_i	2.1	3.5	4.2	6.5	4.1	3.6	5.3	3.7	4.9

$$\bar{x} = 4.21, m_2 = 1.39, m_3 = .39, m_4 = 5.49$$

$$\gamma_1 = .24, \gamma_2 = 2.84$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g CL-R BST				0.00
3	Perform 3 for $i = 1, 2, \dots, n$	x_i	↑	↑	x	$\Sigma+$	i
3'	Delete erroneous data x_k	x_k	↑	↑	x	f	
			$\Sigma-$				
4	Compute the mean \bar{x}		f	\bar{x}	$x\bar{x}y$	STO	
			0				\bar{x}
5	Compute 2 nd moment m_2		RCL	.	1	RCL	
			.	0	STO	1	
			÷	$x\bar{x}y$	g	x^2	
			–	STO	2		m_2
6	Compute 3 rd moment m_3		RCL	.	5	RCL	
			0	RCL	–	1	
			R/S	STO	3		m_3
7	Compute 4 th moment m_4		R/S	STO	4		m_4
8	(optional) Compute γ_1, γ_2		RCL	3	RCL	2	
			1	–	5	y^x	
			÷				γ_1
			RCL	4	RCL	2	
			g	x^2	÷		γ_2
9	For a new case, go to 2						

STANDARD ERRORS FOR LINEAR REGRESSION

Suppose $y = a_0 + a_1x$ is the least squares fit to a set of data points $\{(x_i, y_i), i = 1, 2, \dots, n\}$ and \hat{y} is the estimated value on the line for a given x value.

The program computes:

1. Standard error of estimate (of y on x)

$$\begin{aligned}s_{y \cdot x} &= \sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{n - 2}} \\ &= \sqrt{\frac{\sum y_i^2 - a_0 \sum y_i - a_1 \sum x_i y_i}{n - 2}}\end{aligned}$$

2. Standard error of the regression coefficient a_0

$$s_0 = s_{y \cdot x} \sqrt{\frac{\sum x_i^2}{n \left[\sum x_i^2 - \frac{(\sum x_i)^2}{n} \right]}}$$

3. Standard error of the regression coefficient a_1

$$s_1 = \frac{s_{y \cdot x}}{\sqrt{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}}$$

Note:

n is a positive integer and $n \neq 1$ or 2 .

Reference:

Draper and Smith, *Applied Regression Analysis*, John Wiley and Sons, 1966.

38 Standard Errors For Linear Regression

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		R ₀	a ₀
00.			25.	32	g	R ₁	a ₁
01.	71	x	26.	42	x ²	R ₂	
02.	51	-	27.	34	RCL	R ₃	
03.	34	RCL	28.	83	.	R ₄	
04.	83	.	29.	00	0	R ₅	
05.	05	5	30.	81	÷	R ₆	
06.	34	RCL	31.	51	-	R ₇	
07.	01	1	32.	31	f	R ₈	
08.	71	x	33.	42	√x	R ₉	
09.	51	-	34.	81	÷	R _{•0}	n
10.	34	RCL	35.	34	RCL	R _{•1}	Σx _i
11.	83	.	36.	83	.	R _{•2}	Σx _i ²
12.	00	0	37.	02	2	R _{•3}	Σy _i
13.	02	2	38.	34	RCL	R _{•4}	Σy _i ²
14.	51	-	39.	83	.	R _{•5}	Σx _i y _i
15.	81	÷	40.	00	0	R _{•6}	0
16.	31	f	41.	81	÷	R _{•7}	0
17.	42	√x	42.	31	f	R _{•8}	0
18.	84	R/S	43.	42	√x	R _{•9}	0
19.	34	RCL	44.	22	x \leftrightarrow y		
20.	83	.	45.	71	x		
21.	02	2	46.	84	R/S		
22.	34	RCL	47.	31	f		
23.	83	.	48.	34	LAST X		
24.	01	1	49.	-00	GTO 00		

Example:

y_i	92	85	78	81	54	51	40
x_i	26	30	44	50	62	68	74

$$a_0 = 121.04$$

$$a_1 = -1.03$$

Regression line is $y = 121.04 - 1.03x$

$$s_{y \cdot x} = 6.34$$

$$s_0 = 7.47$$

$$s_1 = .14$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2*	Initialize		g	CL · R			0.00
3	Perform 3 for $i = 1, 2, \dots, n$	y_i	\uparrow				
		x_i	$\Sigma +$				i
3'	Delete erroneous data x_k, y_k	y_k	\uparrow				
		x_k	f	$\Sigma -$			
4	Compute a_0, a_1		f	L. R.	STO	0	a_0
			$x \rightarrow y$	STO	1		a_1
5	Compute standard errors		RCL	.	4	RCL	
			0	RCL	.	3	
			BST	R/S			$s_{y \cdot x}$
			R/S				s_0
			R/S				s_1
6	For a new case, go to 2						
	*Note: If sums are already in proper registers, skip steps 2, 3 and 3'.						

40 Partial Correlation Coefficients

PARTIAL CORRELATION COEFFICIENTS

The partial correlation coefficient measures the relationship between any two of the variables when all others are kept constant.

For the case of 3 variables, the partial correlation coefficient between X_1 and X_2 keeping X_3 constant is

$$r_{12 \cdot 3} = \frac{r_{12} - r_{13} r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}}$$

where r_{ij} denotes the correlation coefficient of X_i and X_j .

Similarly, for the case of 4 variables, the partial correlation coefficient between X_1 and X_2 keeping X_3 and X_4 constant is

$$r_{12 \cdot 34} = \frac{r_{12 \cdot 4} - r_{13 \cdot 4} r_{23 \cdot 4}}{\sqrt{(1 - r_{13 \cdot 4}^2)(1 - r_{23 \cdot 4}^2)}} = \frac{r_{12 \cdot 3} - r_{14 \cdot 3} r_{24 \cdot 3}}{\sqrt{(1 - r_{14 \cdot 3}^2)(1 - r_{24 \cdot 3}^2)}} .$$

Any partial correlation coefficient can be computed by means of these formulas (using this program) if correlation coefficients $r_{12}, r_{13}, r_{23}, \dots$ are given.

Note:

This program finds $r_{13 \cdot 2}, r_{23 \cdot 1}$ by similar formulas.

Reference:

S. Wilks, *Mathematical Statistics*, John Wiley and Sons, 1962.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE			
00.			25.	51	-	R ₀	r ₁₂ , r ₁₃ , r ₂₃
01.	33	STO	26.	22	x \leftrightarrow y	R ₁	r ₁₃ , r ₂₃ , r ₁₂
02.	02	2	27.	81	\div	R ₂	r ₂₃ , r ₁₂ , r ₁₃
03.	32	g	28.	84	R/S	R ₃	
04.	42	x ²	29.	34	RCL	R ₄	
05.	01	1	30.	01	1	R ₅	
06.	51	-	31.	34	RCL	R ₆	
07.	22	x \leftrightarrow y	32.	02	2	R ₇	
08.	33	STO	33.	34	RCL	R ₈	
09.	01	1	34.	00	0	R ₉	
10.	32	g	35.	-01	GTO 01	R _{e0}	
11.	42	x ²	36.			R _{e1}	
12.	01	1	37.			R _{e2}	
13.	51	-	38.			R _{e3}	
14.	71	x	39.			R _{e4}	
15.	31	f	40.			R _{e5}	
16.	42	\sqrt{x}	41.			R _{e6}	
17.	22	x \leftrightarrow y	42.			R _{e7}	
18.	33	STO	43.			R _{e8}	
19.	00	0	44.			R _{e9}	
20.	34	RCL	45.				
21.	01	1	46.				
22.	34	RCL	47.				
23.	02	2	48.				
24.	71	x	49.				

Example:

Suppose $r_{12} = -0.96$, $r_{13} = -0.1$, $r_{23} = 0.12$, then the partial correlation coefficients are

$$r_{12 \cdot 3} = -0.96$$

$$r_{13 \cdot 2} = .05$$

$$r_{23 \cdot 1} = .09.$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input data and compute						
	correlation coefficients	r_{12}	↑				
		r_{13}	↑				
		r_{23}	BST	R/S			$r_{12 \cdot 3}$
			R/S				$r_{13 \cdot 2}$
			R/S				$r_{23 \cdot 1}$
3	For a new case, go to 2						

STANDARDIZED SCORES

Given a set of data $\{x_1, x_2, \dots, x_n\}$, this program finds $\{y_1, y_2, \dots, y_n\}$ such that

$$y_i = \frac{x_i - \bar{x}}{s}$$

for $i = 1, 2, \dots, n$

where \bar{x} and s are sample mean and standard deviation of $\{x_1, x_2, \dots, x_n\}$. $\{y_1, y_2, \dots, y_n\}$ has mean zero and its standard deviation is 1.

This program can also transform y_i 's to z_i 's such that $\{z_1, z_2, \dots, z_n\}$ has mean μ and standard deviation σ (μ and σ are given).

$$z_i = \sigma y_i + \mu$$

for $i = 1, 2, \dots, n$

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	34	RCL
02.	02	2
03.	51	-
04.	34	RCL
05.	03	3
06.	81	÷
07.	84	R/S
08.	34	RCL
09.	01	1
10.	71	x
11.	34	RCL
12.	00	0
13.	61	+
14.	-00	GTO 00
15.	31	f
16.	33	\bar{x}
17.	33	STO
18.	02	2
19.	32	g
20.	33	s
21.	33	STO
22.	03	3
23.	-00	GTO 00
24.		

DISPLAY		KEY ENTRY
LINE	CODE	
25.		
26.		
27.		
28.		
29.		
30.		
31.		
32.		
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS
$R_0 \ \mu$
$R_1 \ \sigma$
$R_2 \ \bar{x}$
$R_3 \ s$
R_4
R_5
R_6
R_7
R_8
R_9
$R_{e0} \ n$
$R_{e1} \ \Sigma x_i$
$R_{e2} \ \Sigma x_i^2$
$R_{e3} \ \text{Used}$
$R_{e4} \ \text{Used}$
$R_{e5} \ \text{Used}$
$R_{e6} \ 0$
$R_{e7} \ 0$
$R_{e8} \ 0$
$R_{e9} \ 0$

Example:

$$\mu = 75, \sigma = 10, s = 10.54$$

i	1	2	3	4	5	6	7	8	9	10	11
x_i	57	62	73	48	78	54	59	75	67	81	66
y_i	-.80	-.33	.72	-1.66	1.19	-1.09	-.61	.91	.15	1.48	.05
z_i	66.98	71.72	82.16	58.44	86.90	64.13	68.88	84.06	76.47	89.75	75.52

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS					OUTPUT DATA/UNITS
1	Enter program							
2	Initialize		g	CL+R				0.00
3	Input μ, σ if z_i 's are desired	μ	STO	0				
		σ	STO	1				
4	Perform 4 for $i = 1, 2, \dots, n$	x_i	$\Sigma+$					i
4'	Delete erroneous data x_k	x_k	f	$\Sigma-$				
5	Compute and store \bar{x}, s		GTO	1	5	R/S		s
6	Perform 6 for $i = 1, 2, \dots, n$	x_i	BST	R/S				y_i
	(optional) Compute z_i		R/S					z_i
7	For a new case, go to 2							

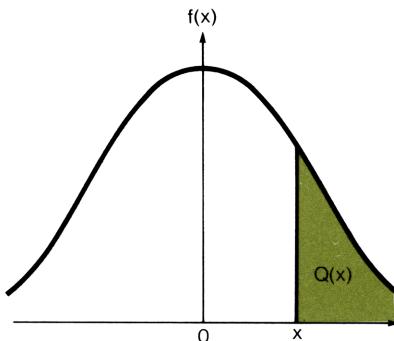
NORMAL DISTRIBUTION

The density function for a standard normal variable is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} .$$

The upper tail area is

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt .$$



For $x \geq 0$, polynomial approximation is used to compute $Q(x)$:

$$Q(x) = f(x) (b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) + \epsilon(x)$$

where $|\epsilon(x)| < 7.5 \times 10^{-8}$

$$t = \frac{1}{1 + rx}, r = 0.2316419$$

$$b_1 = .31938153, \quad b_2 = -.356563782$$

$$b_3 = 1.781477937, \quad b_4 = -1.821255978$$

$$b_5 = 1.330274429$$

Note:

The program only works for $x \geq 0$. Equations $f(-x) = f(x)$, $Q(-x) = 1 - Q(x)$, where $x \geq 0$, can be used to find f and Q for negative numbers.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1968.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS										
LINE	CODE		LINE	CODE												
00.			25.	41	↑	R ₀ r										
01.	71	x	26.	41	↑	R ₁ b ₁										
02.	02	2	27.	41	↑	R ₂ b ₂										
03.	81	÷	28.	34	RCL	R ₃ b ₃										
04.	42	CHS	29.	05	5	R ₄ b ₄										
05.	32	q	30.	71	x	R ₅ b ₅										
06.	22	e ^x	31.	34	RCL	R ₆ x										
07.	31	f	32.	04	4	R ₇ f(x)										
08.	83	π	33.	61	+	R ₈										
09.	02	2	34.	71	x	R ₉										
10.	71	x	35.	34	RCL	R _{•0}										
11.	31	f	36.	03	3	R _{•1}										
12.	42	√x	37.	61	+	R _{•2}										
13.	81	÷	38.	71	x	R _{•3}										
14.	33	STO	39.	34	RCL	R _{•4}										
15.	07	7	40.	02	2	R _{•5}										
16.	84	R/S	41.	61	+	R _{•6}										
17.	34	RCL	42.	71	x	R _{•7}										
18.	00	0	43.	34	RCL	R _{•8}										
19.	34	RCL	44.	01	1	R _{•9}										
20.	06	6	45.	61	+											
21.	71	x	46.	71	x											
22.	01	1	47.	34	RCL											
23.	61	+	48.	07	7											
24.	13	¹/x	49.	71	x											

Examples:

1. $x = 1.18$
 $f(x) = .20$
 $Q(x) = .12$
2. $x = 2.28$
 $f(x) = .03$
 $Q(x) = .01$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS					OUTPUT DATA/UNITS
1	Enter program							
2	Store constants	r	STO	0				
		b ₁	STO	1				
		b ₂	STO	2				
		b ₃	STO	3				
		b ₄	STO	4				
		b ₅	STO	5	BST			
3	Input x and compute f(x)	x	↑	STO	6	R/S		f(x)
4	Compute Q(x)		R/S					Q(x)
5	For a new case, go to 3							

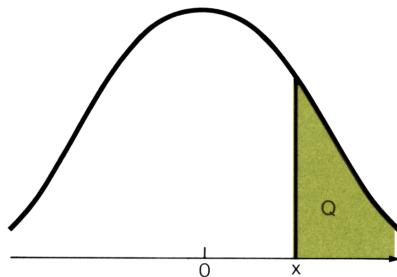
46 Inverse Normal Integral

INVERSE NORMAL INTEGRAL

This program determines the value of x such that

$$Q = \int_x^{\infty} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}} dt$$

where Q is given and $0 < Q \leq 0.5$.



The following rational approximation is used:

$$x = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(Q)$$

where $|\epsilon(Q)| < 4.5 \times 10^{-4}$

$$t = \sqrt{\ln \frac{1}{Q^2}}$$

$$c_0 = 2.515517 \quad d_1 = 1.432788$$

$$c_1 = 0.802853 \quad d_2 = 0.189269$$

$$c_2 = 0.010328 \quad d_3 = 0.001308$$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1968.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE			
00.			25.	61	+	R ₀ c ₀	
01.	41	↑	26.	33	STO	R ₁ c ₁	
02.	71	x	27.	07	7	R ₂ c ₂	
03.	13	1/x	28.	44	CLX	R ₃ d ₁	
04.	31	f	29.	34	RCL	R ₄ d ₂	
05.	22	ln	30.	02	2	R ₅ d ₃	
06.	31	f	31.	71	x	R ₆ t	
07.	42	√x	32.	34	RCL	R ₇ 1+d ₁ t+d ₂ t ² +d ₃ t ³	
08.	33	STO	33.	01	1	R ₈	
09.	06	6	34.	61	+	R ₉	
10.	41	↑	35.	71	x	R _{•0}	
11.	41	↑	36.	34	RCL	R _{•1}	
12.	41	↑	37.	00	0	R _{•2}	
13.	34	RCL	38.	61	+	R _{•3}	
14.	05	5	39.	34	RCL	R _{•4}	
15.	71	x	40.	07	7	R _{•5}	
16.	34	RCL	41.	81	÷	R _{•6}	
17.	04	4	42.	51	-	R _{•7}	
18.	61	+	43.	-00	GTO 00	R _{•8}	
19.	71	x	44.			R _{•9}	
20.	34	RCL	45.				
21.	03	3	46.				
22.	61	+	47.				
23.	71	x	48.				
24.	01	1	49.				

Examples:

1. Q = 0.12

x = 1.18

2. Q = 0.05

x = 1.65

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store constants	c ₀	STO	0			
		c ₁	STO	1			
		c ₂	STO	2			
		d ₁	STO	3			
		d ₂	STO	4			
		d ₃	STO	5	BST		
3	Input Q	Q	R/S				x
4	For a new case, go to 3						

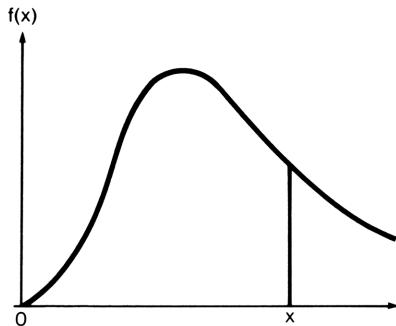
48 Chi-Square Density Function

CHI-SQUARE DENSITY FUNCTION

This program evaluates the chi-square density function

$$f(x) = \frac{\frac{\nu}{2} - 1}{x^{\frac{\nu}{2}}} \cdot \frac{e^{-\frac{x}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}$$

where $x \geq 0$ and ν is the degrees of freedom.



Notes:

1. The program requires that $\nu \leq 141$. If $\nu > 141$ and ν is even, then the display shows all 9's for $\Gamma(\nu/2)$; if $\nu > 141$ and ν is odd, no warnings are given, but the answers are incorrect.
2. If both x and ν are large, $f(x)$ may overflow the machine.
3. If ν is even,

$$\Gamma\left(\frac{\nu}{2}\right) = \left(\frac{\nu}{2} - 1\right) !$$

If ν is odd,

$$\Gamma\left(\frac{\nu}{2}\right) = \left(\frac{\nu}{2} - 1\right) \left(\frac{\nu}{2} - 2\right) \dots \left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right).$$

4. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
5. $f(x)$ may be used as an input for *Chi-Square Distribution* program to find the cumulative distribution. In that case, record $f(x)$ to as many digits as possible for reentry.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1968.

DISPLAY		KEY ENTRY	DISPLAY	KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE	R ₀ (ν/2) -1	
00.			25.	31	f	
01.	41	↑	26.	42	\sqrt{x}	
02.	02	2	27.	71	x	
03.	81	÷	28.	84	R/S	
04.	01	1	29.	33	STO	
05.	51	-	30.	02	2	
06.	33	STO	31.	34	RCL	
07.	00	0	32.	00	0	
08.	84	R/S	33.	12	y^x	
09.	83	•	34.	22	$x \leftarrow y$	
10.	05	5	35.	81	÷	
11.	32	g	36.	02	2	
12.	-20	$x=y$ 20	37.	34	RCL	
13.	23	R↓	38.	00	0	
14.	33	STO	39.	01	1'	
15.	71	x	40.	61	+	
16.	01	1	41.	12	y^x	
17.	01	1	42.	81	÷	
18.	51	-	43.	34	RCL	
19.	-09	GTO 09	44.	02	2	
20.	34	RCL	45.	02	2	
21.	01	1	46.	81	÷	
22.	71	x	47.	32	g	
23.	31	f	48.	22	e^x	
24.	83	π	49.	81	÷	

Examples:

1. $\nu = 20$,

$$\Gamma\left(\frac{\nu}{2}\right) = 362880.00$$

$$f(9.591) = .02$$

(Press [f] SCI [9] to see
1.527751934-02)

2. $\nu = 3$

$$\Gamma\left(\frac{\nu}{2}\right) = .89$$

$$f(7.82) = .02$$

(Press [f] SCI [9] to see
2.235743714-02)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Initialize	1	STO 1 BST	1.00
3	Input ν	ν	R/S	$(\nu/2)-1$
4	If ν is even, go to 6			
5	Compute $\Gamma(\nu/2)$ for odd ν		R/S	$\Gamma(\nu/2)$
	Go to 7			
6	Compute $\Gamma(\nu/2)$ for even ν		f nl GTO 2	
		9		
7	Input x and compute f(x)	x	R/S	$f(x)$
8	For a new case, go to 2			

CHI-SQUARE DISTRIBUTION

Given x , ν and $f(x)$, this program uses a series approximation to evaluate the chi-square cumulative distribution

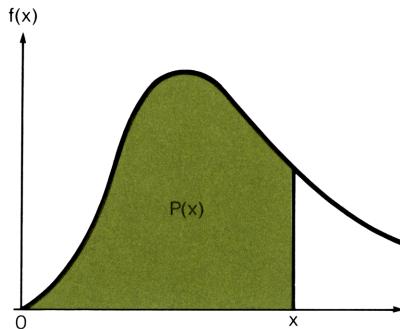
$$P(x) = \int_0^x f(t) dt$$

$$= \frac{2x}{\nu} f(x) \left[1 + \sum_{k=1}^{\infty} \frac{x^k}{(\nu+2)(\nu+4)\dots(\nu+2k)} \right]$$

where $x \geq 0$

ν is the degrees of freedom, and density function

$$f(x) = \frac{\frac{\nu}{2} - 1}{2^{\frac{\nu}{2}} \Gamma\left(\frac{\nu}{2}\right) e^{\frac{x}{2}}}.$$



The program computes successive partial sums of the series. When two consecutive partial sums are equal, the value is used as the sum of the series.

Note:

$f(x)$ may be computed using *Chi-square Density Function* program.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1968.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		R ₀ ν	
00.			25.	03	3	R ₁ 2xf(x)/ν	
01.	33	STO	26.	71	x	R ₂ x	
02.	02	2	27.	33	STO	R ₃ Used	
03.	84	R/S	28.	03	3	R ₄	
04.	33	STO	29.	61	+	R ₅	
05.	00	0	30.	32	g	R ₆	
06.	81	÷	31.	-33	x=y 33	R ₇	
07.	02	2	32.	-15	GTO 15	R ₈	
08.	71	x	33.	34	RCL	R ₉	
09.	71	x	34.	01	1	R ₁₀	
10.	33	STO	35.	71	x	R ₁₁	
11.	01	1	36.	-00	GTO 00	R ₁₂	
12.	01	1	37.			R ₁₃	
13.	33	STO	38.			R ₁₄	
14.	03	3	39.			R ₁₅	
15.	34	RCL	40.			R ₁₆	
16.	02	2	41.			R ₁₇	
17.	34	RCL	42.			R ₁₈	
18.	00	0	43.			R ₁₉	
19.	02	2	44.				
20.	61	+	45.				
21.	33	STO	46.				
22.	00	0	47.				
23.	81	÷	48.				
24.	34	RCL	49.				

Examples:

1. $f(x) = 1.527751934 \times 10^{-2}$

$x = 9.591$

$\nu = 20$

$P(x) = .03$

Note: For $f(x)$, see *Chi-square Density Function* program.

2. $f(x) = 2.235743714 \times 10^{-2}$

$x = 7.82$

$\nu = 3$

$P(x) = .95$

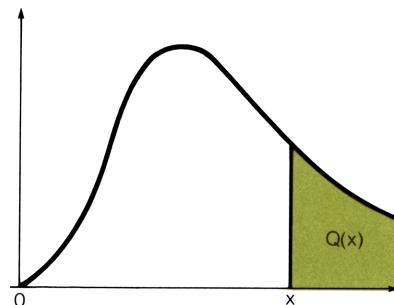
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input $f(x)$, x and ν	$f(x)$	↑				
		x	BST	R/S			x
		ν	R/S				$P(x)$
3	For a new case, go to 2						

F DISTRIBUTION

This program evaluates the integral of the F distribution

$$Q(x) = \int_x^{\infty} \frac{\Gamma\left(\frac{\nu_1 + \nu_2}{2}\right) y^{\frac{\nu_1}{2}-1} \left(\frac{\nu_1}{\nu_2}\right)^{\frac{\nu_1}{2}}}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right) \left(1 + \frac{\nu_1}{\nu_2} y\right)^{\frac{\nu_1 + \nu_2}{2}}} dy$$

for given values of $x (> 0)$, degrees of freedoms ν_1, ν_2 , provided either ν_1 or ν_2 is even.



The integral is evaluated by means of the following series:

1. ν_1 even

$$Q(x) = t^{\frac{\nu_2}{2}} \left[1 + \frac{\nu_2}{2}(1-t) + \dots + \frac{\nu_2(\nu_2+2)\dots(\nu_2+\nu_1-4)}{2\cdot4\dots(\nu_1-2)} (1-t)^{\frac{\nu_1-2}{2}} \right]$$

2. ν_2 even

$$Q(x) = 1 - (1-t)^{\frac{\nu_1}{2}} \left[1 + \frac{\nu_1}{2} t + \dots + \frac{\nu_1(\nu_1+2)\dots(\nu_2+\nu_1-4)}{2\cdot 4 \dots (\nu_2-2)} t^{\frac{\nu_2-2}{2}} \right]$$

where $t = \frac{\nu_2}{\nu_2 + \nu_1 x}$.

Note:

If both ν_1 , ν_2 are even, the two formulas would generate identical answers. Using the smaller of ν_1 , ν_2 could save computation time. For example, if $\nu_1 = 10$, $\nu_2 = 20$, then classify the problem as ν_1 is even to obtain the answer.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1968.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE		
00.			25.	34	RCL	$R_0 t, 1-t$
01.	61	+	26.	00	0	$R_1 \nu_1$
02.	81	\div	27.	71	x	$R_2 \nu_2$
03.	33	STO	28.	34	RCL	$R_3 t^{\nu_2} / 2$
04.	00	0	29.	04	4	$R_4 0, 2, \dots$
05.	34	RCL	30.	02	2	$R_5 \text{ Used}$
06.	02	2	31.	61	+	R_6
07.	02	2	32.	33	STO	R_7
08.	81	\div	33.	04	4	R_8
09.	12	y^x	34.	34	RCL	R_9
10.	33	STO	35.	01	1	$R_{\bullet 0}$
11.	03	3	36.	32	g	$R_{\bullet 1}$
12.	01	1	37.	-44	$x=y$	$44 R_{\bullet 2}$
13.	34	RCL	38.	23	$R \downarrow$	$R_{\bullet 3}$
14.	00	0	39.	81	\div	$R_{\bullet 4}$
15.	51	-	40.	33	STO	$R_{\bullet 5}$
16.	33	STO	41.	61	+	$R_{\bullet 6}$
17.	00	0	42.	05	5	$R_{\bullet 7}$
18.	01	1	43.	-19	GTO 19	$R_{\bullet 8}$
19.	34	RCL	44.	34	RCL	$R_{\bullet 9}$
20.	02	2	45.	05	5	
21.	34	RCL	46.	34	RCL	
22.	04	4	47.	03	3	
23.	61	+	48.	71	x	
24.	71	x	49.	-00	GTO 00	

54 F Distribution

Examples:

$$1. \quad \nu_1 = 7, \nu_2 = 6$$

$$Q(4.21) = .05$$

$$2. \quad \nu_1 = 4, \nu_2 = 20$$

$$Q(2.25) = .10$$

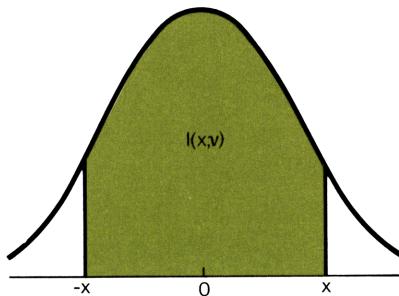
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS					OUTPUT DATA/UNITS
1	Enter program							
2	Initialize	0	STO	4				
		1	STO	5	BST			1.00
3	If ν_2 is even, go to 5							
4	Input ν_1, ν_2 and x	ν_1	STO	1				
		ν_2	STO	2				
		x	RCL	1	x	RCL		
			2	R/S				Q(x)
5	ν_2 even	ν_2	STO	1				
		ν_1	STO	2				
		x	1/x	RCL	1	x		
			RCL	2	R/S			1 - Q(x)
			1	x \leftrightarrow y	-			Q(x)
6	For a new case, go to 2							

t DISTRIBUTION

This program evaluates the integral for t distribution

$$I(x, \nu) = \int_{-x}^x \frac{\Gamma\left(\frac{\nu+1}{2}\right)\left(1+\frac{y^2}{\nu}\right)^{-\frac{\nu+1}{2}}}{\sqrt{\pi\nu} \Gamma\left(\frac{\nu}{2}\right)} dy$$

where $x > 0$,
 ν is the degrees of freedom.



Formulas used are:

1. ν even

$$I(x, \nu) = \sin \theta \left\{ 1 + \frac{1}{2} \cos^2 \theta + \frac{1 \cdot 3}{2 \cdot 4} \cos^4 \theta + \dots + \frac{1 \cdot 3 \cdot 5 \dots (\nu-3)}{2 \cdot 4 \cdot 6 \dots (\nu-2)} \cos^{\nu-2} \theta \right\}$$

56 t Distribution

2 ν odd

$$I(x, \nu) = \begin{cases} \frac{2\theta}{\pi} & \text{if } \nu = 1 \\ \frac{2\theta}{\pi} + \frac{2}{\pi} \cos \theta & \left\{ \sin \theta \left[1 + \frac{2}{3} \cos^2 \theta + \dots \right. \right. \\ & \left. \left. + \frac{2 \cdot 4 \dots (\nu - 3)}{1 \cdot 3 \dots (\nu - 2)} \cos^{\nu-3} \theta \right] \right\} \quad \text{if } \nu > 1 \end{cases}$$

where $\theta = \tan^{-1} \left(\frac{x}{\sqrt{v}} \right)$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1968.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		LINE	CODE
00.			25.	61	+	R ₀	1 + ($\cos^2 \theta$) / 2 + ...
01.	31	f	26.	33	STO	R ₁	ν
02.	42	\sqrt{x}	27.	03	3	R ₂	$\cos^2 \theta$
03.	81	\div	28.	34	RCL	R ₃	0, 2, 4,... or 1, 3, 5...
04.	32	g	29.	01	1	R ₄	θ
05.	14	\tan^{-1}	30.	32	g	R ₅	
06.	33	STO	31.	-41	x=y 41	R ₆	
07.	04	4	32.	23	R↓	R ₇	
08.	31	f	33.	81	\div	R ₈	
09.	13	cos	34.	34	RCL	R ₉	
10.	32	g	35.	02	2	R _{•0}	
11.	42	x^2	36.	71	x	R _{•1}	
12.	33	STO	37.	33	STO	R _{•2}	
13.	02	2	38.	61	+	R _{•3}	
14.	01	1	39.	00	0	R _{•4}	
15.	33	STO	40.	-17	GTO 17	R _{•5}	
16.	00	0	41.	34	RCL	R _{•6}	
17.	34	RCL	42.	00	0	R _{•7}	
18.	03	3	43.	34	RCL	R _{•8}	
19.	01	1	44.	04	4	R _{•9}	
20.	61	+	45.	31	f		
21.	71	x	46.	12	sin		
22.	34	RCL	47.	71	x		
23.	03	3	48.	-00	GTO 00		
24.	02	2	49.				

Examples:

1. $I(2.201, 11) = .95$
2. $I(2.75, 30) = .99$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Put machine in RAD mode		f	RAD	BST		
3	If ν is odd, go to 4'						
4	ν is even	0	STO	3			
		x	↑				
		ν	STO	1	R/S		$I(x, \nu)$
4'	If $\nu = 1$, go to 4''	1	STO	3			
		x	↑				
		ν	STO	1	f	\sqrt{x}	
			÷	g	\tan^{-1}	STO	
			4	GTO	0	8	
			R/S				
			RCL	4	f	cos	
			x	RCL	4	+	
			2	x	f	π	
			÷				
4''	$\nu = 1$	x	g	\tan^{-1}	2	x	
			f	π	÷		$I(x, 1)$
5	For a new case, go to 3						

58 Bivariate Normal Distribution

BIVARIATE NORMAL DISTRIBUTION

This program evaluates the joint probability density function

$$f(x, y) = \frac{1}{2\pi \sigma_1 \sigma_2 \sqrt{1 - \rho^2}} e^{-P(x, y)}$$

where

$$P(x, y) = \frac{1}{2(1 - \rho^2)} \left[\frac{(x - \mu_1)^2}{\sigma_1^2} - 2\rho \frac{(x - \mu_1)(y - \mu_2)}{\sigma_1 \sigma_2} + \frac{(y - \mu_2)^2}{\sigma_2^2} \right]$$

Notes:

1. $\sigma_1 \neq 0, \sigma_2 \neq 0$
2. The program requires that $\rho^2 < 1$.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		R ₀ μ_1	R ₁ σ_1
00.			25.	51	-	R ₂ μ_2	R ₃ σ_2
01.	32	g	26.	34	RCL	R ₄ ρ	R ₅ $1 - \rho^2$
02.	42	x ²	27.	05	5	R ₆ $(x - \mu_1)/\sigma_1$	R ₇ $(y - \mu_2)/\sigma_2$
03.	22	x ² -y	28.	02	2	R ₈	R ₉
04.	34	RCL	29.	71	x	R ₀	R ₁
05.	00	0	30.	81	÷	R ₂	R ₃
06.	51	-	31.	42	CHS	R ₄	R ₅
07.	34	RCL	32.	32	g	R ₆	R ₇
08.	01	1	33.	22	e ^x	R ₈	R ₉
09.	81	÷	34.	34	RCL	R ₀	R ₁
10.	33	STO	35.	05	5	R ₂	R ₃
11.	06	6	36.	31	f	R ₄	R ₅
12.	32	g	37.	42	\sqrt{x}	R ₆	R ₇
13.	42	x ²	38.	34	RCL	R ₈	R ₉
14.	61	+	39.	01	1	R ₀	R ₁
15.	34	RCL	40.	71	x	R ₂	R ₃
16.	06	6	41.	34	RCL	R ₄	R ₅
17.	34	RCL	42.	03	3	R ₆	R ₇
18.	07	7	43.	71	x	R ₈	R ₉
19.	71	x	44.	02	2		
20.	34	RCL	45.	71	x		
21.	04	4	46.	31	f		
22.	71	x	47.	83	π		
23.	02	2	48.	71	x		
24.	71	x	49.	81	÷		

Example:

$$\mu_1 = -1, \sigma_1 = 1.5$$

$$\mu_2 = 1, \sigma_2 = 0.5$$

$$\rho = 0.7$$

$$f(1, 2) = .04$$

$$f(-1, 1) = .30$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input $\mu_1, \sigma_1, \mu_2, \sigma_2, \rho$	μ_1	STO	0			
		σ_1	STO	1			
		μ_2	STO	2			
		σ_2	STO	3	1	\uparrow	
		ρ	STO	4	g	x^2	
			—	STO	5	BST	
3	Input x and y	x	\uparrow				
		y	RCL	2	—	RCL	
			3	\div	STO	7	
			R/S				f(x, y)
4	For different x, y, go to 3						
5	For a new case, go to 2						

60 Logarithmic Normal Distribution

LOGARITHMIC NORMAL DISTRIBUTION

If X is a random variable whose logarithm is normally distributed with mean m and variance σ^2 , then X has a logarithmic normal distribution with density function

$$f(x) = \frac{1}{x \sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2} (\ln x - m)^2}$$

where $x > 0$.

This program computes $f(x)$ and the following statistics for given m, σ^2 :

$$\text{median} = e^m$$

$$\text{mode} = e^{m-\sigma^2}$$

$$\text{mean} = e^{m+(\sigma^2/2)}$$

$$\text{variance} = e^{\sigma^2 + 2m} (e^{\sigma^2} - 1).$$

Note:

The program requires that $\sigma^2 \neq 0$.

Reference:

K. A. Brownlee, *Statistical Theory and Methodology in Science and Engineering*, John Wiley and Sons, 1965.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		R ₀ σ ²	R ₁ m
00.			25.	51	-	R ₂ x	R ₃
01.	34	RCL	26.	32	g	R ₄	R ₅
02.	01	1	27.	42	x ²	R ₆	R ₇
03.	34	RCL	28.	34	RCL	R ₈	R ₉
04.	00	0	29.	00	0	R _{•0}	R _{•1}
05.	02	2	30.	81	÷	R _{•2}	R _{•3}
06.	81	÷	31.	02	2	R _{•4}	R _{•5}
07.	61	+	32.	81	÷	R _{•6}	R _{•7}
08.	32	g	33.	42	CHS	R _{•8}	R _{•9}
09.	22	e ^x	34.	32	g		
10.	84	R/S	35.	22	e ^x		
11.	32	g	36.	31	f		
12.	42	x ²	37.	83	π		
13.	34	RCL	38.	02	2		
14.	00	0	39.	71	x		
15.	32	g	40.	34	RCL		
16.	22	e ^x	41.	00	0		
17.	01	1	42.	71	x		
18.	51	-	43.	31	f		
19.	71	x	44.	42	√x		
20.	84	R/S	45.	81	÷		
21.	31	f	46.	34	RCL		
22.	22	ln	47.	02	2		
23.	34	RCL	48.	81	÷		
24.	01	1	49.	-20	GTO 20		

Example: $\sigma^2 = 1$, $m = 1$ $f(.1) = .02$
 median = 2.72 $f(.6) = .21$
 mode = 1.00 $f(1) = .24$
 mean = 4.48
 variance = 34.51

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store m, σ ²	σ ²	STO	0			
		m	STO	1	BST		
3	Compute median and mode		g	e ^x			median
			RCL	1	RCL	0	
			-	g	e ^x		mode
4	Compute mean and variance		R/S				mean
			R/S				variance
5	Input x	x	STO	2	R/S		f(x)
6	For a new x, go to 5						

62 Weibull Distribution Parameter Calculation

WEIBULL DISTRIBUTION PARAMETER CALCULATION

The Weibull probability density function is given by

$$f(x) = \frac{bx^{(b-1)}}{\theta^b} e^{-\left(\frac{x}{\theta}\right)^b}$$

where $\theta > 0$, $b > 0$, $x > 0$.

The cumulative distribution function is

$$F(x) = 1 - e^{-\left(\frac{x}{\theta}\right)^b}$$

For a set of data $\{x_1, \dots, x_n\}$, the Weibull parameters b and θ are to be calculated for these functions.

A common application is to use Weibull analysis for failure data where all samples are tested to failure. To use the program, list the items in order of increasing time to failure.

The median rank (M. R.) is calculated by

$$\frac{R_i - 0.3}{n + 0.4}$$

where R_i is the rank of failure data x_i . Using this median rank as an approximation of $F(x_i)$, a least squares fit is performed to the linearized form of the cumulative distribution function

$$\ln \ln \left(\frac{1}{1 - F(x)} \right) = b \ln x - b \ln \theta.$$

The solution is similar to the linear regression problem, and estimates of b and θ are obtained.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		LINE	CODE
00.			25.	61	+	R_0	Used
01.	01	1	26.	00	0	R_1	n
02.	33	STO	27.	22	$x \leftrightarrow y$	R_2	
03.	00	0	28.	51	-	R_3	
04.	32	g	29.	13	$1/x$	R_4	
05.	44	CL·R	30.	31	f	R_5	
06.	84	R/S	31.	22	ln	R_6	
07.	33	STO	32.	31	f	R_7	
08.	01	1	33.	22	ln	R_8	
09.	84	R/S	34.	22	$x \leftrightarrow y$	R_9	
10.	31	f	35.	11	$\Sigma +$	$R_{\bullet 0}$	n
11.	22	ln	36.	-09	GTO 09	$R_{\bullet 1}$	Used
12.	34	RCL	37.	31	f	$R_{\bullet 2}$	Used
13.	00	0	38.	21	L. R.	$R_{\bullet 3}$	Used
14.	83	.	39.	22	$x \leftrightarrow y$	$R_{\bullet 4}$	Used
15.	03	3	40.	84	R/S	$R_{\bullet 5}$	Used
16.	51	-	41.	81	\div	$R_{\bullet 6}$	0
17.	34	RCL	42.	42	CHS	$R_{\bullet 7}$	0
18.	01	1	43.	32	g	$R_{\bullet 8}$	0
19.	83	.	44.	22	e^x	$R_{\bullet 9}$	0
20.	04	4	45.	-00	GTO 00		
21.	61	+	46.				
22.	81	\div	47.				
23.	01	1	48.				
24.	33	STO	49.				

Example: x_i : 34, 60, 75, 95, 119, 158 (hours to failure)

(math>x_i's must be entered in increasing order.)

 $n = 6$ $b = 1.95$ $\theta = 104.09$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Input n	n	R/S				
4	Perform 4 for $i = 1, 2, \dots, n$	x_i	R/S				i
5	Compute b and θ		GTO	3	7	R/S	b
			R/S				θ
6	For a new case, go to 2						

64 Binomial Distribution

BINOMIAL DISTRIBUTION

This program evaluates the binomial density function for given p and n:

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

where n is a positive integer

$0 < p < 1$ and

$x = 0, 1, 2, \dots, n$.

The recursive relation

$$f(x+1) = \frac{p(n-x)}{(x+1)(1-p)} f(x)$$

$$(x = 0, 1, 2, \dots, n-1)$$

is used to find the cumulative distribution

$$P(x) = \sum_{k=0}^x f(k).$$

Notes:

1. $f(0) = P(0)$
2. When x is large, due to round-off error, the computed value for $P(x)$ might be slightly greater than one. In that case, let $P(x) = 1$.
3. The execution time of the program depends on x; the larger x is, the longer it takes.
4. The mean m and the variance σ^2 are given by

$$m = np$$

$$\sigma^2 = np(1-p).$$

Reference:

E. Parzen, *Modern Probability Theory and its Applications*, John Wiley and Sons, 1960.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE		
00.			25.	34	RCL	R_0 counter
01.	33	STO	26.	04	4	R_1 n
02.	06	6	27.	71	x	R_2 p, $p/(1-p)$
03.	00	0	28.	33	STO	R_3 f(0)
04.	33	STO	29.	04	4	R_4 Used
05.	00	0	30.	33	STO	R_5 Used
06.	34	RCL	31.	61	+	R_6 x
07.	03	3	32.	05	5	R_7
08.	33	STO	33.	34	RCL	R_8
09.	04	4	34.	00	0	R_9
10.	33	STO	35.	01	1	$R_{\bullet 0}$
11.	05	5	36.	61	+	$R_{\bullet 1}$
12.	34	RCL	37.	33	STO	$R_{\bullet 2}$
13.	01	1	38.	00	0	$R_{\bullet 3}$
14.	34	RCL	39.	34	RCL	$R_{\bullet 4}$
15.	00	0	40.	06	6	$R_{\bullet 5}$
16.	51	-	41.	32	g	$R_{\bullet 6}$
17.	34	RCL	42.	-44	x=y 44	$R_{\bullet 7}$
18.	00	0	43.	-12	GTO 12	$R_{\bullet 8}$
19.	01	1	44.	34	RCL	$R_{\bullet 9}$
20.	61	+	45.	04	4	
21.	81	÷	46.	84	R/S	
22.	34	RCL	47.	34	RCL	
23.	02	2	48.	05	5	
24.	71	x	49.	-00	GTO 00	

Example:

n = 6, p = 0.49

$$f(0) = .02$$

$$f(4) = .22$$

$$P(4) = .90$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input n and p	n	STO	1			
		p	STO	2	STO	4	
			1	-	CHS	RCL	
			1	y ^x	STO	3	f(0)
			RCL	2	1	RCL	
			2	-	÷	STO	
			2	BST			
3	For $x \geq 1$	x	R/S				f(x)
			R/S				P(x)
4	For a new x, go to 3						
5	For a new case, go to 2						

66 Poisson Distribution

POISSON DISTRIBUTION

Density function

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

where $\lambda > 0$

and $x = 0, 1, 2, \dots$.

Cumulative distribution is

$$P(x) = \sum_{k=0}^x f(k).$$

This program evaluates $f(x)$ and $P(x)$ for a given λ using the recursive relation

$$f(x + 1) = \frac{\lambda}{x + 1} f(x).$$

Notes:

1. $f(0) = P(0)$
2. When x is large, due to round-off error, the computed value for $P(x)$ might be slightly greater than one. In that case, let $P(x) = 1$.
3. The execution time of the program depends on x ; the larger x is, the longer it takes.
4. Mean = variance = λ

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS																	
LINE	CODE		LINE	CODE		R ₀	R ₁ , λ	R ₂ , f(0)	R ₃ Used	R ₄ Used	R ₅ x	R ₆	R ₇	R ₈	R ₉	R ₁₀	R ₁₁	R ₁₂	R ₁₃	R ₁₄	R ₁₅	R ₁₆	R ₁₇
00.			25.	34	RCL																		
01.	42	CHS	26.	03	3																		
02.	32	g	27.	71	x																		
03.	22	e ^x	28.	33	STO																		
04.	33	STO	29.	03	3																		
05.	02	2	30.	33	STO																		
06.	84	R/S	31.	61	+																		
07.	33	STO	32.	04	4																		
08.	05	5	33.	34	RCL																		
09.	00	0	34.	00	0																		
10.	33	STO	35.	01	1																		
11.	00	0	36.	61	+																		
12.	34	RCL	37.	33	STO																		
13.	02	2	38.	00	0																		
14.	33	STO	39.	34	RCL																		
15.	03	3	40.	05	5																		
16.	33	STO	41.	32	g																		
17.	04	4	42.	-44	x=y 44																		
18.	34	RCL	43.	-18	GTO 18																		
19.	01	1	44.	34	RCL																		
20.	34	RCL	45.	03	3																		
21.	00	0	46.	84	R/S																		
22.	01	1	47.	34	RCL																		
23.	61	+	48.	04	4																		
24.	81	÷	49.	-06	GTO 06																		

Example:

$$\lambda = 3.2$$

$$f(0) = .04$$

$$f(7) = .03$$

$$P(7) = .98$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input λ	λ	STO	1	BST	R/S	f(0)
3	For $x \geq 1$	x	R/S				f(x)
			R/S				P(x)
4	For a new x, go to 3						
5	For a new case, go to 2						

NEGATIVE BINOMIAL DISTRIBUTION

This program evaluates the negative binomial density function for given p and r:

$$f(x) = \binom{x + r - 1}{r - 1} p^r (1 - p)^x$$

where r is a positive integer

$0 < p < 1$ and

$x = 0, 1, 2, \dots$.

The recursive relation

$$f(x + 1) = \frac{(1 - p)(x + r)}{x + 1} f(x)$$

is used to find the cumulative distribution

$$P(x) = \sum_{k=0}^x f(k).$$

Notes:

1. $f(0) = P(0)$
2. When x is large, due to round-off error, the computed value for P(x) might be slightly greater than one. In that case, let $P(x) = 1$.
3. The execution time of the program depends on x; the larger x is, the longer it takes.
4. The mean m and the variance σ^2 are given by

$$m = \frac{r(1 - p)}{p}$$

$$\sigma^2 = \frac{r(1 - p)}{p^2} .$$

5. If we interpret p as the probability of success of a given event, then $f(x)$ is the probability that exactly $x + r$ trials will be required to get r successes.

Reference:

E. Parzen, *Modern Probability Theory and its Applications*, John Wiley and Sons, 1960.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		R ₀	R ₁
00.			25.	61	+		
01.	33	STO	26.	33	STO	R ₀ counter	
02.	06	6	27.	00	0	R ₁ p	
03.	00	0	28.	81	÷	R ₂ r	
04.	33	STO	29.	34	RCL	R ₃ f(0)	
05.	00	0	30.	04	4	R ₄ Used	
06.	34	RCL	31.	71	x	R ₅ Used	
07.	03	3	32.	33	STO	R ₆ x	
08.	33	STO	33.	04	4	R ₇	
09.	04	4	34.	33	STO	R ₈	
10.	33	STO	35.	61	+	R ₉	
11.	05	5	36.	05	5	R ₀	
12.	01	1	37.	34	RCL	R ₁	
13.	34	RCL	38.	00	0	R ₂	
14.	01	1	39.	34	RCL	R ₃	
15.	51	-	40.	06	6	R ₄	
16.	34	RCL	41.	32	g	R ₅	
17.	00	0	42.	-44	x=y 44	R ₆	
18.	34	RCL	43.	-12	GTO 12	R ₇	
19.	02	2	44.	34	RCL	R ₈	
20.	61	+	45.	04	4	R ₉	
21.	71	x	46.	84	R/S		
22.	34	RCL	47.	34	RCL		
23.	00	0	48.	05	5		
24.	01	1	49.	-00	GTO 00		

Examples:

$$p = .9, r = 4$$

$$f(0) = .66$$

$$f(1) = .26$$

$$P(1) = .92$$

$$f(2) = .07$$

$$P(2) = .98$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input p and r	p	STO	1			
		r	STO	2	y ^x	STO	
			3	BST			f(0)
3	For $x \geq 1$	x	R/S				f(x)
			R/S				P(x)
4	For a new x, go to 3						
5	For a new case, go to 2						

HYPERGEOMETRIC DISTRIBUTION

This program evaluates the hypergeometric density function for given a, b and n:

$$f(x) = \frac{\binom{a}{x} \binom{b}{n-x}}{\binom{a+b}{n}}$$

where a, b, n are positive integers

$x \leq a, n - x \leq b$ and

$x = 0, 1, 2, \dots, n$.

The recursive relation

$$f(x+1) = \frac{(x-a)(x-n)}{(x+1)(b-n+x+1)} f(x)$$

$$(x = 0, 1, 2, \dots, n-1)$$

is used to find the cumulative distribution

$$P(x) = \sum_{k=0}^x f(k).$$

Notes:

1. The program requires that $n \leq 69$.
2. $f(0) = P(0)$
3. The execution time of the program depends on x; the larger x is, the longer it takes.
4. When x is large, due to round-off error, the computed value for $P(x)$ might be slightly greater than one. In that case, let $P(x) = 1$.
5. The mean m and the variance σ^2 are given by

$$m = \frac{an}{a+b}$$

$$\sigma^2 = \frac{abn(a+b-n)}{(a+b)^2(a+b-1)} .$$

Reference:

J. E. Freund, *Mathematical Statistics*, Prentice-Hall, 1971.

DISPLAY		KEY ENTRY	DISPLAY	KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE	
00.			25.	81	\div
01.	33	STO	26.	34	RCL
02.	00	0	27.	05	5
03.	34	RCL	28.	71	x
04.	01	1	29.	33	STO
05.	51	-	30.	05	5
06.	34	RCL	31.	33	STO
07.	00	0	32.	61	+
08.	34	RCL	33.	06	6
09.	03	3	34.	34	RCL
10.	51	-	35.	07	7
11.	71	x	36.	01	1
12.	34	RCL	37.	34	RCL
13.	00	0	38.	00	0
14.	01	1	39.	61	+
15.	61	+	40.	33	STO
16.	81	\div	41.	00	0
17.	31	f	42.	32	g
18.	34	LAST X	43.	-45	$x=y$ 45
19.	34	RCL	44.	-03	GTO 03
20.	02	2	45.	34	RCL
21.	34	RCL	46.	05	5
22.	03	3	47.	84	R/S
23.	51	-	48.	34	RCL
24.	61	+	49.	06	6

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Example:

Given $a = 8$, $b = 12$, $n = 6$, then

$$f(0) = .02$$

$$f(3) = .32$$

$$P(3) = .86$$

$$f(5) = .02$$

$$P(5) = 1.00$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input a, b, n	a	STO	1			
		b	STO	2			
		n	STO	3	RCL	2	
			f	$n!$	f	LAST x	
			RCL	3	-	f	
			$n!$	\div	RCL	1	
			RCL	2	+	f	
			$n!$	f	LAST x	RCL	
			3	-	f	$n!$	
			\div	\div	STO	4	$f(0)$
3	For $x \geq 1$	x	STO	7	RCL	4	
			STO	5	STO	6	
			0	BST	R/S		$f(x)$
			R/S				$P(x)$
4	For a new x , go to 3						
5	For a new case, go to 2						

MULTINOMIAL DISTRIBUTION

This program evaluates the joint probability function of k (k can be 2, 3, ..., or 8) random variables having the multinomial distribution

$$f(x_1, x_2, \dots, x_k) = \frac{n!}{x_1! x_2! \dots x_k!} \theta_1^{x_1} \theta_2^{x_2} \dots \theta_k^{x_k}$$

where

$$\sum_{i=1}^k \theta_i = 1, \quad \sum_{i=1}^k x_i = n, \quad \theta_i > 0 \text{ and}$$

$$x_i = 0, 1, 2, \dots, n \quad (i = 1, 2, \dots, k).$$

The parameters of this distribution are $n, \theta_1, \theta_2, \dots$ and θ_k .

Note:

The program requires that $n \leq 69$.

74 Multinomial Distribution

DISPLAY		KEY ENTRY	DISPLAY	KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE	
00.			25.	34	RCL
01.	31	f	26.	04	4
02.	43	n!	27.	33	STO
03.	34	RCL	28.	03	3
04.	01	1	29.	34	RCL
05.	31	f	30.	05	5
06.	34	LAST X	31.	33	STO
07.	12	y ^x	32.	04	4
08.	22	x \leftrightarrow y	33.	34	RCL
09.	81	\div	34.	06	6
10.	33	STO	35.	33	STO
11.	71	x	36.	05	5
12.	00	0	37.	34	RCL
13.	34	RCL	38.	07	7
14.	01	1	39.	33	STO
15.	33	STO	40.	06	6
16.	09	9	41.	34	RCL
17.	34	RCL	42.	08	8
18.	02	2	43.	33	STO
19.	33	STO	44.	07	7
20.	01	1	45.	34	RCL
21.	34	RCL	46.	09	9
22.	03	3	47.	33	STO
23.	33	STO	48.	08	8
24.	02	2	49.	-00	GTO 00

Example:

Given $\theta_1 = 0.2, \theta_2 = 0.1, \theta_3 = 0.2, \theta_4 = 0.15, \theta_5 = 0.17, \theta_6 = 0.18$ and $n = 20$,
then $f(1, 2, 3, 4, 5, 5) = 1.274857927 - 04$

$$f(2, 4, 0, 4, 2, 8) = 1.688980098 - 06$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Perform 2 for $i = 1, 2, \dots, k$	θ_i	STO				
		i					θ_i
3	If $k = 8$, go to 6						
4	Set all other $\theta_i = 1$	1					
5	Perform 5 for $i = k + 1, \dots, 8$		STO				
		i					1.00
6	Input n	n	f	$n!$	STO	0	
			STO	.	0	BST	
7	Perform 7 for $i = 1, 2, \dots, k$	x_i	R/S				θ_i
8	If $k = 8$, go to 11						
9	Set all other $x_i = 1$	1					
10	Perform 10 8 - k times		R/S				1.00
11	Compute $f(x_1, \dots, x_k)$		RCL	0			$f(x_1, \dots, x_k)$
12	For new x's		RCL	.	0	STO	
				0			
	Go to 7						
13	For a new case, go to 2						

EXPONENTIAL CURVE FIT

This program computes the least squares fit of n pairs of data points $\{(x_i, y_i), i = 1, 2, \dots, n\}$, where $y_i > 0$, for an exponential function of the form

$$y = a e^{bx} \quad (a > 0).$$

The equation is linearized into

$$\ln y = \ln a + bx.$$

The following statistics are computed:

1. Coefficients a, b

$$b = \frac{\sum x_i \ln y_i - \frac{1}{n} (\sum x_i)(\sum \ln y_i)}{\sum x_i^2 - \frac{1}{n} (\sum x_i)^2}$$

$$a = \exp \left[\frac{\sum \ln y_i}{n} - b \frac{\sum x_i}{n} \right]$$

2. Coefficient of determination

$$r^2 = \frac{\left[\sum x_i \ln y_i - \frac{1}{n} \sum x_i \sum \ln y_i \right]^2}{\left[\sum x_i^2 - \frac{(\sum x_i)^2}{n} \right] \left[\sum (\ln y_i)^2 - \frac{(\sum \ln y_i)^2}{n} \right]}$$

3. Estimated value \hat{y} for a given x

$$\hat{y} = a e^{bx}$$

Note:

n is a positive integer and n ≠ 1.

Reference:

K. A. Brownlee, *Statistical Theory and Methodology in Science and Engineering*, John Wiley and Sons, 1965.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE			
00.			25.	84	R/S	R_0 a	
01.	32	g	26.	32	g	R_1 b	
02.	44	CL·R	27.	42	x^2	R_2	
03.	84	R/S	28.	41	\uparrow	R_3	
04.	31	f	29.	41	\uparrow	R_4	
05.	22	ln	30.	32	g	R_5	
06.	22	$x \leftrightarrow y$	31.	33	s	R_6	
07.	11	$\Sigma+$	32.	22	$x \leftrightarrow y$	R_7	
08.	-03	GTO 03	33.	81	\div	R_8	
09.	31	f	34.	32	g	R_9	
10.	22	ln	35.	42	x^2	$R_{\bullet 0}$ n	
11.	22	$x \leftrightarrow y$	36.	71	x	$R_{\bullet 1}$ Σx_i	
12.	31	f	37.	84	R/S	$R_{\bullet 2}$ Σx_i^2	
13.	11	$\Sigma-$	38.	34	RCL	$R_{\bullet 3}$ $\Sigma \ln y_i$	
14.	-03	GTO 03	39.	01	1	$R_{\bullet 4}$ $\Sigma (\ln y_i)^2$	
15.	31	f	40.	71	x	$R_{\bullet 5}$ $\Sigma x_i \ln y_i$	
16.	21	L. R.	41.	32	g	$R_{\bullet 6}$ 0	
17.	32	g	42.	22	e^x	$R_{\bullet 7}$ 0	
18.	22	e^x	43.	34	RCL	$R_{\bullet 8}$ 0	
19.	33	STO	44.	00	0	$R_{\bullet 9}$ 0	
20.	00	0	45.	71	x		
21.	84	R/S	46.	-37	GTO 37		
22.	22	$x \leftrightarrow y$	47.				
23.	33	STO	48.				
24.	01	1	49.				

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Example:

x_i	.72	1.31	1.95	2.58	3.14
y_i	2.16	1.61	1.16	.85	0.5

1. $a = 3.45, b = -.58$
 $y = 3.45 e^{-0.58x}$
2. $r^2 = .98$
3. For $x = 1.5, \hat{y} = 1.44$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Perform 3 for $i = 1, 2, \dots, n$	x_i	\uparrow				
		y_i	R/S				i
3'	Delete erroneous data x_k, y_k	x_k	\uparrow				
		y_k	GTO	0	9	R/S	
4	Compute a, b and r^2		GTO	1	5	R/S	a
			R/S				b
			R/S				r^2
5	Compute estimated value \hat{y}	x	R/S				\hat{y}
6	For a new x , go to 5						
7	For a new case, go to 2						

LOGARITHMIC CURVE FIT

This program fits a logarithmic curve

$$y = a + b \ln x$$

to a set of data points

$$\{(x_i, y_i), i = 1, 2, \dots, n\}$$

where $x_i > 0$.

Program computes:

1. Regression coefficients

$$b = \frac{\sum y_i \ln x_i - \frac{1}{n} \sum \ln x_i \sum y_i}{\sum (\ln x_i)^2 - \frac{1}{n} (\sum \ln x_i)^2}$$

$$a = \frac{1}{n} (\sum y_i - b \sum \ln x_i)$$

2. Coefficient of determination

$$r^2 = \frac{\left[\sum y_i \ln x_i - \frac{1}{n} \sum \ln x_i \sum y_i \right]^2}{\left[\sum (\ln x_i)^2 - \frac{1}{n} (\sum \ln x_i)^2 \right] \left[\sum y_i^2 - \frac{1}{n} (\sum y_i)^2 \right]}$$

3. Estimated value \hat{y} for given x

$$\hat{y} = a + b \ln x$$

Note:

n is a positive integer and $n \neq 1$.

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DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		R ₀	a
00.			25.	42	x ²	R ₁	b
01.	32	g	26.	41	↑	R ₂	
02.	44	CL·R	27.	41	↑	R ₃	
03.	84	R/S	28.	32	g	R ₄	
04.	22	x↔y	29.	33	s	R ₅	
05.	31	f	30.	22	x↔y	R ₆	
06.	22	ln	31.	81	÷	R ₇	
07.	11	Σ+	32.	32	g	R ₈	
08.	-03	GTO 03	33.	42	x ²	R ₉	
09.	22	x↔y	34.	71	x	R _{•0}	n
10.	31	f	35.	84	R/S	R _{•1}	Σ ln x _i
11.	22	ln	36.	31	f	R _{•2}	Σ (ln x _i) ²
12.	31	f	37.	22	ln	R _{•3}	Σ y _i
13.	11	Σ-	38.	34	RCL	R _{•4}	Σ y _i ²
14.	-03	GTO 03	39.	01	1	R _{•5}	Σ y _i ln x _i
15.	31	f	40.	71	x	R _{•6}	0
16.	21	L. R.	41.	34	RCL	R _{•7}	0
17.	33	STO	42.	00	0	R _{•8}	0
18.	00	0	43.	61	+	R _{•9}	0
19.	84	R/S	44.	-35	GTO 35		
20.	22	x↔y	45.				
21.	33	STO	46.				
22.	01	1	47.				
23.	84	R/S	48.				
24.	32	g	49.				

Example:

x_i	3	4	6	10	12
y_i	1.5	9.3	23.4	45.8	60.1

1. $a = -47.02, b = 41.39$
 $y = -47.02 + 41.39 \ln x$
2. $r^2 = .98$
3. For $x = 8, \hat{y} = 39.06$
For $x = 14.5, \hat{y} = 63.67$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Perform 3 for $i = 1, 2, \dots, n$	x_i	\uparrow				
		y_i	R/S				i
3'	Delete erroneous data x_k, y_k	x_k	\uparrow				
		y_k	GTO	0	9	R/S	
4	Compute a, b , and r^2		GTO	1	5	R/S	a
			R/S				b
			R/S				r^2
5	Compute estimated value \hat{y}	x	R/S				\hat{y}
6	For a new x , go to 5						
7	For a new case, go to 2						

POWER CURVE FIT

This program fits a power curve

$$y = ax^b \quad (a > 0)$$

to a set of data points

$$\{(x_i, y_i), i = 1, 2, \dots, n\}$$

where $x_i > 0, y_i > 0$.

By writing this equation as

$$\ln y = b \ln x + \ln a$$

the problem can be solved as a linear regression problem.

Output statistics are:

1. Regression coefficients

$$b = \frac{\sum (\ln x_i)(\ln y_i) - \frac{(\sum \ln x_i)(\sum \ln y_i)}{n}}{\sum (\ln x_i)^2 - \frac{(\sum \ln x_i)^2}{n}}$$

$$a = \exp \left[\frac{\sum \ln y_i}{n} - b \frac{\sum \ln x_i}{n} \right]$$

2. Coefficient of determination

$$r^2 = \frac{\left[\sum (\ln x_i)(\ln y_i) - \frac{(\sum \ln x_i)(\sum \ln y_i)}{n} \right]^2}{\left[\sum (\ln x_i)^2 - \frac{(\sum \ln x_i)^2}{n} \right] \left[\sum (\ln y_i)^2 - \frac{(\sum \ln y_i)^2}{n} \right]}$$

3. Estimated value \hat{y} for given x

$$\hat{y} = ax^b$$

Note:

n is a positive integer and n ≠ 1.

Reference:

K. A. Brownlee, *Statistical Theory and Methodology in Science and Engineering*, John Wiley and Sons, 1965.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		R ₀ a	R ₁ b
00.			25.	84	R/S	R ₂	R ₃
01.	32	g	26.	22	x ⁻² y	R ₄	R ₅
02.	44	CL+R	27.	33	STO	R ₆	R ₇
03.	84	R/S	28.	01	1	R ₈	R ₉
04.	31	f	29.	84	R/S	R ₀ n	R ₁ Σ ln x _i
05.	22	ln	30.	32	g	R ₂ Σ (ln x _i) ²	R ₃ Σ ln y _i
06.	22	x ⁻² y	31.	42	x ²	R ₄ Σ (ln y _i) ²	R ₅ Σ ln x _i ln y _i
07.	31	f	32.	41	↑	R ₆ 0	R ₇ 0
08..	22	ln	33.	41	↑	R ₈ 0	R ₉ 0
09.	11	Σ+	34.	32	g		
10.	-03	GTO 03	35.	33	s		
11.	31	f	36.	22	x ⁻² y		
12.	22	ln	37.	81	÷		
13.	22	x ⁻² y	38.	32	g		
14.	31	f	39.	42	x ²		
15.	22	ln	40.	71	x		
16.	31	f	41.	84	R/S		
17.	11	Σ-	42.	34	RCL		
18.	-03	GTO 03	43.	01	1		
19.	31	f	44.	12	y ^x		
20.	21	L. R.	45.	34	RCL		
21.	32	g	46.	00	0		
22.	22	e ^x	47.	71	x		
23.	33	STO	48.	-41	GTO 41		
24.	00	0	49.				

84 Power Curve Fit

Example:

x_i	10	12	15	17	20	22	25	27	30	32	35
y_i	0.95	1.05	1.25	1.41	1.73	2.00	2.53	2.98	3.85	4.59	6.02

1. $a = .03, b = 1.46$
 $y = .03x^{1.46}$
2. $r^2 = .94$
3. For $x = 18, \hat{y} = 1.76$
 $x = 23, \hat{y} = 2.52$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST	R/S			0.00
3	Perform 3 for $i = 1, 2, \dots, n$	x_i	\uparrow				
		y_i	R/S				
3'	Delete erroneous data x_k, y_k	x_k	\uparrow				
		y_k	GTO	1	1	R/S	
4	Compute a, b , and r^2		GTO	1	9	R/S	a
			R/S				b
			R/S				r^2
5	Compute estimated value \hat{y}	x	R/S				\hat{y}
6	For a new x , go to 5						
7	For a new case, go to 2						

ANALYSIS OF VARIANCE

The one-way analysis of variance tests the differences between the population means of k treatment groups. Group i ($i = 1, 2, \dots, k$) has n_i observations (treatment group may have equal or unequal number of observations).

Sum_i = sum of observations in treatment group i

$$= \sum_{j=1}^{n_i} x_{ij}$$

$$\text{Total SS} = \sum_{i=1}^k \sum_{j=1}^{n_i} x_{ij}^2 - \frac{\left(\sum_{i=1}^k \sum_{j=1}^{n_i} x_{ij} \right)^2}{\sum_{i=1}^k n_i}$$

$$\text{Treat SS} = \sum_{i=1}^k \frac{\left(\sum_{j=1}^{n_i} x_{ij} \right)^2}{n_i} - \frac{\left(\sum_{i=1}^k \sum_{j=1}^{n_i} x_{ij} \right)^2}{\sum_{i=1}^k n_i}$$

$$\text{Error SS} = \text{Total SS} - \text{Treat SS}$$

$$df_1 = \text{Treat df} = k - 1$$

$$df_2 = \text{Error df} = \sum_{i=1}^k n_i - k$$

$$\text{Treat MS} = \frac{\text{Treat SS}}{\text{Treat df}}$$

$$\text{Error MS} = \frac{\text{Error SS}}{\text{Error df}}$$

$$F = \frac{\text{Treat MS}}{\text{Error MS}} \quad \left(\text{with } k - 1 \text{ and } \sum_{i=1}^k n_i - k \text{ degrees of freedom} \right)$$

86 Analysis of Variance

Reference:

J. E. Freund, *Mathematical Statistics*, Prentice-Hall, 1962.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE		
00.			25.	01	1	R₀ Used
01.	33	STO	26.	81	÷	R₁ Used
02.	61	+	27.	33	STO	R₂ Used
03.	00	0	28.	61	+	R₃ Used
04.	32	g	29.	02	2	R₄ Σn _i
05.	42	x ²	30.	34	RCL	R₅ ΣΣx _{ij} ²
06.	33	STO	31.	00	0	R₆ ΣΣx _{ij}
07.	61	+	32.	33	STO	R₇ Sum _i
08.	05	5	33.	07	7	R₈ 0
09.	01	1	34.	33	STO	R₉ 0
10.	34	RCL	35.	61	+	R_{*0}
11.	01	1	36.	06	6	R_{*1}
12.	61	+	37.	34	RCL	R_{*2}
13.	33	STO	38.	01	1	R_{*3}
14.	01	1	39.	33	STO	R_{*4}
15.	-00	GTO 00	40.	61	+	R_{*5}
16.	01	1	41.	04	4	R_{*6}
17.	33	STO	42.	00	0	R_{*7}
18.	61	+	43.	33	STO	R_{*8}
19.	03	3	44.	00	0	R_{*9}
20.	34	RCL	45.	33	STO	
21.	00	0	46.	01	1	
22.	32	g	47.	34	RCL	
23.	42	x ²	48.	07	7	
24.	34	RCL	49.	-00	GTO 00	

Example:

		j	1	2	3	4	5	6
		i						
Treatment	1	10	8	5	12	14	11	
	2	6	9	8	13			
	3	14	13	10	17	16		

$$\text{Sum}_1 = 60.00$$

$$\text{Sum}_2 = 36.00$$

$$\text{Sum}_3 = 70.00$$

$$\text{Total SS} = 172.93$$

$$\text{Treat SS} = 66.93$$

$$\text{Error SS} = 106.00$$

$$F = 3.79$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		f CLR BST				0.00
3	Perform 3-5 for $i = 1, 2, \dots, k$						
4	Perform 4 for $j = 1, 2, \dots, n_i$	x_{ij}	R/S				j
5			GTO	1	6	R/S	Sum _i
6	Compute the F statistic		RCL	5	RCL	6	
			g	x^2	RCL	4	
			\div	-			Total SS
			RCL	2	RCL	6	
			g	x^2	RCL	4	
			\div	-			Treat SS
			-				Error SS
			f	LAST x	RCL	3	
			1	-	\div	$x \leftrightarrow y$	
			RCL	4	RCL	3	
			-	\div	\div		F
7	For a new case, go to 2						

PAIRED t STATISTIC

Given a set of paired observations from two normal populations with means μ_1, μ_2 (unknown)

x_i	x_1	x_2	\dots	x_n
y_i	y_1	y_2	\dots	y_n

let

$$D_i = x_i - y_i$$

$$\bar{D} = \frac{1}{n} \sum_{i=1}^n D_i$$

$$s_D = \sqrt{\frac{\sum D_i^2 - \frac{1}{n} (\sum D_i)^2}{n - 1}}$$

$$s_{\bar{D}} = \frac{s_D}{\sqrt{n}}$$

The test statistic

$$t = \frac{\bar{D}}{s_{\bar{D}}} ,$$

which has $n - 1$ degrees of freedom (df), can be used to test the null hypothesis

$$H_0: \mu_1 = \mu_2 .$$

Reference:

B. Ostle, *Statistics in Research*, Iowa State University Press, 1963.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		R ₀	sD
00.			25.	81	÷	R ₁	
01.	32	g	26.	84	R/S	R ₂	
02.	44	CL·R	27.	34	RCL	R ₃	
03.	84	R/S	28.	83	·	R ₄	
04.	51	-	29.	00	0	R ₅	
05.	11	Σ+	30.	01	1	R ₆	
06.	-03	GTO 03	31.	51	-	R ₇	
07.	51	-	32.	-00	GTO 00	R ₈	
08.	31	f	33.			R ₉	
09.	11	Σ-	34.			R ₀ n	
10.	-03	GTO 03	35.			R _{•1} ΣD _i	
11.	32	g	36.			R _{•2} ΣD _i ²	
12.	33	s	37.			R _{•3} Used	
13.	34	RCL	38.			R _{•4} Used	
14.	83	·	39.			R _{•5} Used	
15.	00	0	40.			R _{•6} 0	
16.	31	f	41.			R _{•7} 0	
17.	42	√x	42.			R _{•8} 0	
18.	81	÷	43.			R _{•9} 0	
19.	33	STO	44.				
20.	00	0	45.				
21.	31	f	46.				
22.	33	Ȑx	47.				
23.	34	RCL	48.				
24.	00	0	49.				

Example:

x _i	14	17.5	17	17.5	15.4
y _i	17	20.7	21.6	20.9	17.2

$$t = -7.16$$

$$df = 4.00$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		BST R/S				0.00
3	Perform 3 for i = 1, 2, ..., n	x _i	↑				
		y _i	R/S				i
3'	Delete erroneous data x _k , y _k	x _k	↑				
		y _k	GTO 0 7 R/S				
4	Compute t and df		GTO 1 1 R/S				t
			R/S				df
5	For a new case, go to 2						

t STATISTIC FOR TWO MEANS

Suppose $\{x_1, x_2, \dots, x_{n_1}\}$ and $\{y_1, y_2, \dots, y_{n_2}\}$ are independent random samples from two normal populations having means μ_1, μ_2 (unknown) and the same unknown variance σ^2 .

We want to test the null hypothesis

$$H_0: \mu_1 - \mu_2 = D$$

where D is a given number.

Define

$$\bar{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

$$\bar{y} = \frac{1}{n_2} \sum_{i=1}^{n_2} y_i$$

$$t = \frac{\bar{x} - \bar{y} - D}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sqrt{\frac{\sum x_i^2 - n_1 \bar{x}^2 + \sum y_i^2 - n_2 \bar{y}^2}{n_1 + n_2 - 2}} .$$

We can use this t statistic, which has the t distribution with $n_1 + n_2 - 2$ degrees of freedom, to test the null hypothesis H_0 .

Reference:

K. A. Brownlee, *Statistical Theory and Methodology in Science and Engineering*, John Wiley and Sons, 1965.

DISPLAY		KEY ENTRY	DISPLAY	KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		
00.			25.	83	.	R ₀ n ₁
01.	22	x \leftrightarrow y	26.	02	2	R ₁ Σx_i
02.	51	-	27.	61	+	R ₂ Σx_i^2
03.	34	RCL	28.	34	RCL	R ₃ \bar{x}
04.	00	0	29.	04	4	R ₄ \bar{y}
05.	13	$1/x$	30.	32	g	R ₅
06.	34	RCL	31.	42	x^2	R ₆
07.	83	.	32.	34	RCL	R ₇
08.	00	0	33.	83	.	R ₈
09.	13	$1/x$	34.	00	0	R ₉
10.	61	+	35.	71	x	R ₁₀ n ₂
11.	31	f	36.	51	-	R ₁₁ Σy_i
12.	42	\sqrt{x}	37.	34	RCL	R ₁₂ Σy_i^2
13.	81	\div	38.	00	0	R ₁₃ Used
14.	34	RCL	39.	34	RCL	R ₁₄ Used
15.	02	2	40.	83	.	R ₁₅ Used
16.	34	RCL	41.	00	0	R ₁₆ 0
17.	03	3	42.	61	+	R ₁₇ 0
18.	32	g	43.	02	2	R ₁₈ 0
19.	42	x^2	44.	51	-	R ₁₉ 0
20.	34	RCL	45.	81	\div	
21.	00	0	46.	31	f	
22.	71	x	47.	42	\sqrt{x}	
23.	51	-	48.	81	\div	
24.	34	RCL	49.	-00	GTO 00	

92 t Statistic for Two Means

Example:

x: 79, 84, 108, 114, 120, 103, 122, 120

y: 91, 103, 90, 113, 108, 87, 100, 80, 99, 54

$n_1 = 8$

$n_2 = 10$

If D = 0 (i.e., $H_0: \mu_1 = \mu_2$)

then $\bar{x} = 106.25$

$\bar{y} = 92.5$

t = 1.73

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL+R			0.00
3	Perform 3 for i = 1, 2,..., n ₁	x _i	Σ+				i
3'	Delete erroneous data x _k	x _k	f	Σ-			
4	Store sums and compute \bar{x}		RCL	.	0	STO	
			0	RCL	.	1	
			STO	1	RCL	.	
			2	STO	2	f	
			⋮	⋮	⋮	⋮	
			⋮	⋮	⋮	⋮	
			⋮	⋮	⋮	⋮	
5	Initialize for y's		g	CL+R			0.00
6	Perform 6 for j = 1, 2,..., n ₂	y _j	Σ+				j
6'	Delete erroneous data y _h	y _h	f	Σ-			
7	Compute \bar{y}		f	⋮	⋮	⋮	⋮
8	Input D and compute t	D	RCL	4	+	RCL	
			3	BST	R/S		
			⋮	⋮	⋮	⋮	
9	For a different D, go to 8		⋮	⋮	⋮	⋮	
10	For a new case, go to 2		⋮	⋮	⋮	⋮	

ONE SAMPLE TEST STATISTICS FOR THE MEAN

For a normal population (x_1, x_2, \dots, x_n) with a known variance σ^2 , a test of the null hypothesis

$$H_0: \text{mean } \mu = \mu_0$$

is based on the z statistic (which has a standard normal distribution)

$$z = \frac{\sqrt{n} (\bar{x} - \mu_0)}{\sigma} .$$

If the variance σ^2 is unknown, then

$$t = \frac{\sqrt{n} (\bar{x} - \mu_0)}{s}$$

is used instead. This t statistic has the t distribution with $n - 1$ degrees of freedom. \bar{x} and s are the sample mean and standard deviation.

94 One Sample Test Statistics for the Mean

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	33	STO
02.	00	0
03.	84	R/S
04.	31	f
05.	33	\bar{x}
06.	34	RCL
07.	00	0
08.	51	-
09.	34	RCL
10.	83	*
11.	00	0
12.	31	f
13.	42	\sqrt{x}
14.	71	x
15.	33	STO
16.	01	1
17.	32	g
18.	33	s
19.	34	RCL
20.	01	1
21.	22	$x \leftrightarrow y$
22.	81	\div
23.	84	R/S
24.	34	RCL

DISPLAY		KEY ENTRY
LINE	CODE	
25.	01	1
26.	22	$x \leftrightarrow y$
27.	81	\div
28.	-00	GTO 00
29.		
30.		
31.		
32.		
33.		
34.		
35.		
36.		
37.		
38.		
39.		
40.		
41.		
42.		
43.		
44.		
45.		
46.		
47.		
48.		
49.		

REGISTERS
$R_0 \mu_0$
$R_1 \sqrt{n} (\bar{x} - \mu_0)$
R_2
R_3
R_4
R_5
R_6
R_7
R_8
R_9
$R_{\#0} n$
$R_{\#1} \sum x_i$
$R_{\#2} \sum x_i^2$
$R_{\#3}$ Used
$R_{\#4}$ Used
$R_{\#5}$ Used
$R_{\#6} 0$
$R_{\#7} 0$
$R_{\#8} 0$
$R_{\#9} 0$

Example:

Suppose $\mu_0 = 2$, for the following set of data

$$\{2.73, 0.45, 2.52, 1.19, 3.51, 2.75, 1.79, 1.83, 1, 0.87, 1.9, 1.62, 1.74, 1.92, 1.24, 2.68,\}$$

test statistic $t = -0.69$

and $z = -0.57$ if $\sigma = 1$.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL+R			0.00
3	Perform 3 for $i = 1, 2, \dots, n$	x_i	$\Sigma+$				i
4	Input μ_0	μ_0	BST	R/S			μ_0
5	Compute t		R/S				t
6	Input σ (if known)	σ	R/S				z
7	For a new case, go to 2						

TEST STATISTICS FOR CORRELATION COEFFICIENT

Under the assumptions of normal correlation analysis, the following t statistic can be used to test the null hypothesis $\rho = 0$,

$$t = \frac{r \sqrt{n-2}}{\sqrt{1 - r^2}}$$

where r is an estimate (based on a sample of size n) of the true correlation coefficient ρ . This t statistic has the t distribution with $n - 2$ degrees of freedom.

To test the null hypothesis $\rho = \rho_0$, the z statistic is used.

$$z = \frac{\sqrt{n-3}}{2} \ln \frac{(1+r)(1-\rho_0)}{(1-r)(1+\rho_0)}$$

where z has approximately the standard normal distribution.

References:

1. Hogg and Craig, *Introduction to Mathematical Statistics*, Macmillan Co., 1970.
2. J. Freund, *Mathematical Statistics*, Prentice-Hall, 1971.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS																	
LINE	CODE		LINE	CODE		R ₀	R ₁	R ₂	R ₃	R ₄	R ₅	R ₆	R ₇	R ₈	R ₉	R ₁₀	R ₁₁	R ₁₂	R ₁₃	R ₁₄	R ₁₅	R ₁₆	R ₁₇
00.			25.	34	RCL																		
01.	34	RCL	26.	00	0																		
02.	01	1	27.	51	-																		
03.	02	2	28.	81	÷																		
04.	51	-	29.	01	1																		
05.	31	f	30.	34	RCL																		
06.	42	\sqrt{x}	31.	02	2																		
07.	34	RCL	32.	51	-																		
08.	00	0	33.	71	x																		
09.	71	x	34.	01	1																		
10.	01	1	35.	34	RCL																		
11.	34	RCL	36.	02	2																		
12.	00	0	37.	61	+																		
13.	41	↑	38.	81	÷																		
14.	71	x	39.	31	f																		
15.	51	-	40.	22	ln																		
16.	31	f	41.	34	RCL																		
17.	42	\sqrt{x}	42.	01	1																		
18.	81	÷	43.	03	3																		
19.	84	R/S	44.	51	-																		
20.	34	RCL	45.	31	f																		
21.	00	0	46.	42	\sqrt{x}																		
22.	01	1	47.	71	x																		
23.	61	+	48.	02	2																		
24.	01	1	49.	81	÷																		

Example:

Suppose $r = 0.12$ and $n = 31$, then
 $t = .65$ and
 $z = .64$ (for $\rho_0 = 0$).

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Input r and n	r	STO	0			
		n	STO	1			
3	(If z is desired) input ρ_0	ρ_0	STO	2			
4	Go to 6 if only z is needed						
5	Compute t		BST	R/S			t
6	Compute z		GTO	2	0	R/S	z
7	For a new case, go to 2						

98 Chi-Square Evaluation (expected values equal)

CHI-SQUARE EVALUATION (EXPECTED VALUES EQUAL)

This program calculates the value of the χ^2 statistic for the goodness of fit test when the expected frequencies are equal.

$$\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i} = \frac{n \sum O_i^2}{\sum O_i} - \sum O_i$$

where O_i = observed frequency

$$E = \text{expected frequency} = \frac{\sum O_i}{n}$$

DISPLAY		KEY ENTRY	DISPLAY	KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE	
00.			25.		R ₀
01.	34	RCL	26.		R ₁
02.	83	.	27.		R ₂
03.	02	2	28.		R ₃
04.	34	RCL	29.		R ₄
05.	83	.	30.		R ₅
06.	00	0	31.		R ₆
07.	71	x	32.		R ₇
08.	34	RCL	33.		R ₈
09.	83	.	34.		R ₉
10.	01	1	35.		R ₀₀ n
11.	81	:	36.		R ₀₁ ΣO_i
12.	31	f	37.		R ₀₂ ΣO_i^2
13.	34	LAST X	38.		R ₀₃ Used
14.	51	-	39.		R ₀₄ Used
15.	-00	GTO 00	40.		R ₀₅ Used
16.			41.		R ₀₆ 0
17.			42.		R ₀₇ 0
18.			43.		R ₀₈ 0
19.			44.		R ₀₉ 0
20.			45.		
21.			46.		
22.			47.		
23.			48.		
24.			49.		

Example:

The following table shows the observed frequencies in tossing a die 120 times. Assume that the expected frequencies are equal ($E = 20$), χ^2 can be used to test if the die is fair.

number	1	2	3	4	5	6
frequency O_i	25	17	15	23	24	16

$$\chi^2 = 5.00$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Initialize		g CL+R	0.00
3	Perform 3 for $i = 1, 2, \dots, n$	O_i	$\Sigma+$	i
3'	Delete erroneous data O_k	O_k	f $\Sigma-$	
4	Compute χ^2		BST R/S	χ^2
5	For a new case, go to 2			

CHI-SQUARE EVALUATION (EXPECTED VALUES UNEQUAL)

This program calculates the value of the χ^2 statistic for the goodness of fit test by the equation

$$\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

where O_i = observed frequency

E_i = expected frequency.

The χ^2 statistic measures the closeness of the agreement between the observed frequencies and expected frequencies.

Note:

In order to apply the goodness of fit test to a set of given data, combining some classes may be necessary to make sure that each expected frequency is not too small (say, not less than 5).

Reference:

J. E. Freund, *Mathematical Statistics*, Prentice-Hall, 1962.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE			
00.			25.	51	-	R ₀	n
01.	00	0	26.	31	f	R ₁	$\sum(O_i - E_i)^2 / E_i$
02.	33	STO	27.	34	LAST X	R ₂	
03.	00	0	28.	22	x \leftrightarrow y	R ₃	
04.	33	STO	29.	32	g	R ₄	
05.	01	1	30.	42	x ²	R ₅	
06.	84	R/S	31.	22	x \leftrightarrow y	R ₆	
07.	51	-	32.	81	\div	R ₇	
08.	31	f	33.	33	STO	R ₈	
09.	34	LAST X	34.	51	-	R ₉	
10.	22	x \leftrightarrow y	35.	01	1	R _{•0}	
11.	32	g	36.	34	RCL	R _{•1}	
12.	42	x ²	37.	00	0	R _{•2}	
13.	22	x \leftrightarrow y	38.	01	1	R _{•3}	
14.	81	\div	39.	51	-	R _{•4}	
15.	33	STO	40.	33	STO	R _{•5}	
16.	61	+	41.	00	0	R _{•6}	
17.	01	1	42.	-06	GTO 06	R _{•7}	
18.	34	RCL	43.			R _{•8}	
19.	00	0	44.			R _{•9}	
20.	01	1	45.				
21.	61	+	46.				
22.	33	STO	47.				
23.	00	0	48.				
24.	-06	GTO 06	49.				

Example:

1.	O _i	8	50	47	56	5	14
	E _i	9.6	46.75	51.85	54.4	8.25	9.15

$$\chi^2 = 4.84$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Initialize		BST R/S	0.00
3	Perform 3 for i = 1, 2, ..., n	O _i	↑	
		E _i	R/S	i
3'	Delete erroneous data O _k , E _k	O _k	↑	
		E _k	GTO 2 5 R/S	
4	Recall χ^2 from register R ₁		RCL 1	χ^2
5	For a new case, go to 2			

102 2 x k Contingency Table

2 x k CONTINGENCY TABLE

Contingency tables can be used to test the null hypothesis that two variables are independent.

	1	2	3	...	k	Totals
A	a ₁	a ₂	a ₃	...	a _k	N _A
B	b ₁	b ₂	b ₃	...	b _k	N _B
Totals	N ₁	N ₂	N ₃	...	N _k	N

Test statistic χ^2 has $k - 1$ degrees of freedom.

$$\chi^2 = \frac{N}{N_A} \sum_{i=1}^k \frac{a_i^2}{N_i} + \frac{N}{N_B} \sum_{i=1}^k \frac{b_i^2}{N_i} - N$$

Pearson's coefficient of contingency C measures the degree of association between the two variables.

$$C = \sqrt{\frac{\chi^2}{N + \chi^2}}$$

Reference:

B. Ostle, *Statistics in Research*, Iowa State University Press, 1963.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		R ₀ N _A	R ₁ N _B
00.			25.	-00	GTO 00	R ₂ a _i	
01.	33	STO	26.	34	RCL	R ₃ b _i	
02.	03	3	27.	83	.	R ₄	
03.	33	STO	28.	02	2	R ₅	
04.	61	+	29.	34	RCL	R ₆	
05.	01	1	30.	00	0	R ₇	
06.	22	x \leftrightarrow y	31.	81	\div	R ₈	
07.	33	STO	32.	34	RCL	R ₉	
08.	02	2	33.	83	.	R _{•0} k	
09.	33	STO	34.	04	4	R _{•1} $\sum a_i / \sqrt{N_i}$	
10.	61	+	35.	34	RCL	R _{•2} $\sum a_i^2 / N_i$	
11.	00	0	36.	01	1	R _{•3} $\sum b_i / \sqrt{N_i}$	
12.	61	+	37.	81	\div	R _{•4} $\sum b_i^2 / N_i$	
13.	31	f	38.	61	+	R _{•5} $\sum a_i b_i / N_i$	
14.	42	\sqrt{x}	39.	01	1	R _{•6} 0	
15.	34	RCL	40.	51	-	R _{•7} 0	
16.	03	3	41.	34	RCL	R _{•8} 0	
17.	22	x \leftrightarrow y	42.	00	0	R _{•9} 0	
18.	81	\div	43.	34	RCL		
19.	34	RCL	44.	01	1		
20.	02	2	45.	61	+		
21.	31	f	46.	71	x		
22.	34	LAST X	47.	-00	GTO 00		
23.	81	\div	48.				
24.	11	$\Sigma +$	49.				

Example:

	1	2	3
A	2	5	4
B	3	8	7

$$\chi^2 = .02 \quad C = .03$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL-R	STO	0	
			STO	1	BST		0.00
3	Perform 3 for i = 1, 2, ..., k	a _i	\uparrow				
		b _i	R/S				i
4	Compute χ^2		GTO	2	6	R/S	χ^2
5	Compute C		\uparrow	\uparrow	RCL	0	
			RCL	1	+	+	
			\div	f	\sqrt{x}		C
6	For a new case, go to 2						

2 x 2 CONTINGENCY TABLE WITH YATES CORRECTION

This program calculates χ^2 for a 2 x 2 contingency table containing observed frequencies. Yates correction for continuity is used.

		1	2
Group A		a	b
Group B		c	d

$$\chi^2 = \frac{(a + b + c + d) [|ad - bc| - \frac{1}{2} (a + b + c + d)]^2}{(a + b) (a + c) (c + d) (b + d)}$$

DISPLAY		KEY ENTRY
LINE	CODE	
00.		
01.	61	+
02.	33	STO
03.	05	5
04.	61	+
05.	61	+
06.	33	STO
07.	04	4
08.	22	$x \leftrightarrow y$
09.	34	RCL
10.	03	3
11.	71	x
12.	34	RCL
13.	01	1
14.	34	RCL
15.	02	2
16.	71	x
17.	51	-
18.	32	g
19.	42	x^2
20.	31	f
21.	42	\sqrt{x}
22.	22	$x \leftrightarrow y$
23.	02	2
24.	81	\div

DISPLAY		KEY ENTRY
LINE	CODE	
25.	51	-
26.	32	g
27.	42	x^2
28.	34	RCL
29.	00	0
30.	34	RCL
31.	01	1
32.	61	+
33.	81	\div
34.	34	RCL
35.	00	0
36.	34	RCL
37.	02	2
38.	61	+
39.	81	\div
40.	34	RCL
41.	05	5
42.	81	\div
43.	34	RCL
44.	01	1
45.	34	RCL
46.	03	3
47.	61	+
48.	81	\div
49.	71	x

REGISTERS
R ₀ a
R ₁ b
R ₂ c
R ₃ d
R ₄ a + b + c + d
R ₅ c + d
R ₆
R ₇
R ₈
R ₉
R _{•0}
R _{•1}
R _{•2}
R _{•3}
R _{•4}
R _{•5}
R _{•6}
R _{•7}
R _{•8}
R _{•9}

Example:

	1	2
A	9	21
B	17	13

$$\chi^2 = 3.33$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Store data	a	STO	0			
		b	STO	1			
		c	STO	2			
		d	STO	3			
3	Compute χ^2		BST	R/S			χ^2
4	For a new case, go to 2						

BARTLETT'S CHI -SQUARE STATISTIC

$$\chi^2 = \frac{f \ln s^2 - \sum_{i=1}^k f_i \ln s_i^2}{1 + \frac{1}{3(k-1)} \left[\left(\sum_{i=1}^k \frac{1}{f_i} \right) - \frac{1}{f} \right]}$$

where s_i^2 = sample variance of the i^{th} sample

f_i = degrees of freedom associated with s_i^2

$i = 1, 2, \dots, k$

k = number of samples

$$s^2 = \frac{\sum_{i=1}^k f_i s_i^2}{f}$$

$$f = \sum_{i=1}^k f_i .$$

This χ^2 has a chi-square distribution (approximately) with $k - 1$ degrees of freedom, which can be used to test the null hypothesis that $s_1^2, s_2^2, \dots, s_k^2$ are all estimates of the same population variance σ^2 (H_0 : Each of $s_1^2, s_2^2, \dots, s_k^2$ is an estimate of σ^2).

Reference:

A. Hald, *Statistical Theory with Engineering Applications*, John Wiley and Sons, 1960.

DISPLAY		KEY ENTRY		DISPLAY		KEY ENTRY		REGISTERS	
LINE	CODE			LINE	CODE				
00.				25.	22	In		$R_0 s_i^2$	
01.	33	STO		26.	34	RCL		$R_1 f_i$	
02.	61	+		27.	03	3		$R_2 \sum f_i$	
03.	03	3		28.	71	x		$R_3 \sum f_i$	
04.	13	$\frac{1}{x}$		29.	34	RCL		R_4	
05.	33	STO		30.	83	.		R_5	
06.	61	+		31.	01	1		R_6	
07.	02	2		32.	51	-		R_7	
08.	81	\div		33.	34	RCL		R_8	
09.	34	RCL		34.	02	2		R_9	
10.	00	0		35.	34	RCL		$R_{\bullet 0} k$	
11.	31	f		36.	03	3		$R_{\bullet 1} \sum f_i \ln s_i^2$	
12.	22	In		37.	13	$\frac{1}{x}$		$R_{\bullet 2} \sum (f_i \ln s_i^2)^2$	
13.	34	RCL		38.	51	-		$R_{\bullet 3} \sum f_i s_i^2$	
14.	01	1		39.	34	RCL		$R_{\bullet 4} \sum (f_i s_i^2)^2$	
15.	71	x		40.	83	.		$R_{\bullet 5} \sum f_i^2 s_i^2 \ln s_i^2$	
16.	11	$\Sigma +$		41.	00	0		$R_{\bullet 6} 0$	
17.	-00	GTO 00		42.	01	1		$R_{\bullet 7} 0$	
18.	34	RCL		43.	51	-		$R_{\bullet 8} 0$	
19.	83	.		44.	03	3		$R_{\bullet 9} 0$	
20.	03	3		45.	71	x			
21.	34	RCL		46.	81	\div			
22.	03	3		47.	01	1			
23.	81	\div		48.	61	+			
24.	31	f		49.	81	\div			

Example:

i	1	2	3	4	5	6
s_i^2	5.5	5.1	5.2	4.7	4.8	4.3
f_i	10	20	17	18	8	15

$$\chi^2 = .25$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		<input type="button" value="g"/> <input type="button" value="CL-R"/> <input type="button" value="STO"/> <input type="button" value="2"/>				
			<input type="button" value="STO"/>	<input type="button" value="3"/>	<input type="button" value="BST"/>		0.00
3	Perform 3 for i = 1, 2,..., k	s_i^2	<input type="button" value="STO"/>	<input type="button" value="0"/>			
		f_i	<input type="button" value="STO"/>	<input type="button" value="1"/>	<input type="button" value="R/S"/>		i
4	Compute χ^2		<input type="button" value="GTO"/>	<input type="button" value="1"/>	<input type="button" value="8"/>	<input type="button" value="R/S"/>	χ^2
5	For a new case, go to 2						

BEHRENS-FISHER STATISTIC

Suppose $\{x_1, x_2, \dots, x_{n_1}\}$ and $\{y_1, y_2, \dots, y_{n_2}\}$ are independent random samples from two normal populations with means μ_1, μ_2 (unknown). If the variances σ_1^2, σ_2^2 can not be assumed equal, then the Behrens-Fisher statistic

$$d = \frac{\bar{x} - \bar{y} - D}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

is used instead of the t statistic to test the null hypothesis

$$H_0: \mu_1 - \mu_2 = D$$

Critical values of this test are tabulated in the Fisher-Yates Tables for various values of n_1, n_2, α and θ , where α is the level of significance and

$$\theta = \tan^{-1} \left(\frac{s_1}{s_2} \sqrt{\frac{n_2}{n_1}} \right).$$

Notation:

$$\bar{x} = \frac{\sum x_i}{n_1} \quad s_1^2 = \frac{\sum x_i^2 - [(\sum x_i)^2 / n_1]}{n_1 - 1}$$

$$\bar{y} = \frac{\sum y_i}{n_2} \quad s_2^2 = \frac{\sum y_i^2 - [(\sum y_i)^2 / n_2]}{n_2 - 1}$$

Reference:

Fisher and Yates, *Statistical Tables for Biological, Agricultural and Medical Research*, Hafner Publishing Co., 1970.

DISPLAY		KEY ENTRY	DISPLAY	KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		
00.			25.	23	R↓	$R_0 \bar{x}$
01.	41	↑	26.	32	g	$R_1 s_1 / \sqrt{n_1}$
02.	41	↑	27.	42	x^2	$R_2 s_2 / \sqrt{n_2}$
03.	31	f	28.	34	RCL	R_3
04.	33	\bar{x}	29.	01	1	R_4
05.	22	$x \leftrightarrow y$	30.	32	g	R_5
06.	23	R↓	31.	42	x^2	R_6
07.	61	+	32.	61	+	R_7
08.	34	RCL	33.	31	f	R_8
09.	00	0	34.	42	\sqrt{x}	R_9
10.	22	$x \leftrightarrow y$	35.	81	÷	$R_{\bullet 0}$ Used
11.	51	-	36.	84	R/S	$R_{\bullet 1}$ Used
12.	41	↑	37.	34	RCL	$R_{\bullet 2}$ Used
13.	41	↑	38.	01	1	$R_{\bullet 3}$ Used
14.	32	g	39.	34	RCL	$R_{\bullet 4}$ Used
15.	33	s	40.	02	2	$R_{\bullet 5}$ Used
16.	34	RCL	41.	81	÷	$R_{\bullet 6} 0$
17.	83	.	42.	32	g	$R_{\bullet 7} 0$
18.	00	0	43.	14	\tan^{-1}	$R_{\bullet 8} 0$
19.	31	f	44.	-00	GTO 00	$R_{\bullet 9} 0$
20.	42	\sqrt{x}	45.			
21.	81	÷	46.			
22.	33	STO	47.			
23.	02	2	48.			
24.	22	$x \leftrightarrow y$	49.			

Example: $x: 79, 84, 108, 114, 120, 103, 122, 120$

$y: 91, 103, 90, 113, 108, 87, 100, 80, 99, 54$

$H_0: \mu_1 = \mu_2$ ($D = 0$), $n_1 = 8$, $n_2 = 10$, $\bar{x} = 106.25$

$s_1 / \sqrt{n_1} = 5.88$, $d = 1.73$, $\theta = 47.88^\circ$ ($= .84$ radians $= 53.20$ grads)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL·R			0.00
3	Perform 3 for $i = 1, 2, \dots, n_1$	x_i	$\Sigma +$				i
3'	Delete erroneous data x_k	x_k	f	$\Sigma -$			
4	Compute \bar{x} and $s_1 / \sqrt{n_1}$		f	\bar{x}	STO	0	\bar{x}
			g	s	RCL	*	
			0	f	\sqrt{x}	÷	$s_1 / \sqrt{n_1}$
			STO	1	g	CL·R	0.00
5	Perform 5 for $i = 1, 2, \dots, n_2$	y_i	$\Sigma +$				i
5'	Delete erroneous data y_h	y_h	f	$\Sigma -$			
6	Input D and compute d, θ	D	BST	R/S			d
			R/S				θ
7	For a new case, go to 2						

BISERIAL CORRELATION COEFFICIENT

The biserial correlation coefficient r_b is used where one variable Y is quantitatively measured while the other continuous variable X is artificially dichotomized (that is, artificially defined by two groups). It measures the degree of linear association between X and Y.

$$r_b = \frac{n (\Sigma' y_i) - n_1 \Sigma y_i}{na \sqrt{n \Sigma y_i^2 - (\Sigma y_i)^2}}$$

Suppose X takes the value 0 or 1.

Define n_1 = number of x's such that $x = 1$

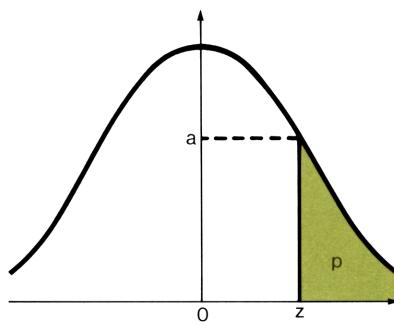
n = total number of data points

$\Sigma' y_i$ = sum of the y's for which $x = 1$

Σy_i = sum of all y's

a = ordinate of the standard normal curve at point z cutting off a

tail of that distribution with area equal to $p = \frac{n_1}{n}$.



Note:

Among the necessary assumptions for a meaningful interpretation of r_b are:

1. Y is normally distributed
2. The true distribution of X should be of normal form.

Reference:

B. Ostle, *Statistics in Research*, Iowa State University Press, 1963.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		R ₀	a
00.			25.	83	.	R ₁	n ₁
01.	34	RCL	26.	02	2	R ₂	Σy_i
02.	83	.	27.	34	RCL	R ₃	
03.	03	3	28.	83	.	R ₄	
04.	34	RCL	29.	04	4	R ₅	
05.	83	.	30.	61	+	R ₆	
06.	01	1	31.	71	x	R ₇	
07.	61	+	32.	34	RCL	R ₈	
08.	33	STO	33.	02	2	R ₉	
09.	02	2	34.	32	g	R ₀₀	n
10.	31	f	35.	42	x^2	R ₀₁	$\sum' y_i$
11.	34	LAST X	36.	51	-	R ₀₂	$\sum' y_i^2$
12.	34	RCL	37.	31	f	R ₀₃	$\Sigma y_i - \sum' y_i$
13.	83	.	38.	42	\sqrt{x}	R ₀₄	$\Sigma y_i^2 - \sum' y_i^2$
14.	00	0	39.	34	RCL	R ₀₅	0
15.	71	x	40.	00	0	R ₀₆	0
16.	22	$x \leftrightarrow y$	41.	34	RCL	R ₀₇	0
17.	34	RCL	42.	83	.	R ₀₈	0
18.	01	1	43.	00	0	R ₀₉	0
19.	71	x	44.	71	x		
20.	51	-	45.	71	x		
21.	34	RCL	46.	81	\div		
22.	83	.	47.	-00	GTO 00		
23.	00	0	48.				
24.	34	RCL	49.				

112 Biserial Correlation Coefficient

Example:

x_i	0	1	1	0	1	0	0	0	1
y_i	3.1	2.8	5.6	0.3	2.5	2.4	4.8	2.9	7.7

$$n_1 = 4$$

$$n = 9$$

$$a = 0.40$$

$$r_b = .59$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g CL+R BST				0.00
3	Perform 3 for $x_i = 1$	0	↑				
		y_i	$\Sigma+$				
3'	Delete erroneous data y_k	0	↑				
	$(x_k = 1)$	y_k	f	$\Sigma-$			
4	Perform 4 for $x_i = 0$	y_i	↑				
		0	$\Sigma+$				
4'	Delete erroneous data y_h	y_h	↑				
	$(x_h = 0)$	0	f	$\Sigma-$			
5	Input a and n_1	a	STO	0			
		n_1	STO	1			
6	Compute r_b		R/S				r_b
7	For a new case, go to 2						

SPEARMAN'S RANK CORRELATION COEFFICIENT

Spearman's rank correlation coefficient is defined by

$$r_s = 1 - \frac{6 \sum_{i=1}^n D_i^2}{n(n^2 - 1)}$$

where n = number of paired observations (x_i, y_i)

$$D_i = \text{rank } (x_i) - \text{rank } (y_i) = R_i - S_i.$$

If the X and Y random variables from which these n pairs of observations are derived are independent, then r_s has zero mean and a variance

$$\frac{1}{n - 1}.$$

A test for the null hypothesis

$$H_0: X, Y \text{ are independent}$$

is using

$$z = r_s \sqrt{n - 1}$$

which is approximately a standardized normal variable (for large n , say $n \geq 10$).

If the null hypothesis of independence is not rejected, we can infer that the population correlation coefficient $\rho(x, y) = 0$, but dependence between the variables does not necessarily imply that $\rho(x, y) \neq 0$.

Note:

$$-1 \leq r_s \leq 1$$

where $r_s = 1$ indicates complete agreement in order of the ranks and $r_s = -1$ indicates complete agreement in the opposite order of the ranks.

Reference:

J. D. Gibbons, *Nonparametric Statistical Inference*, McGraw Hill, 1971.

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DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE		
00.			25.	01	1	$R_0 \ n$
01.	51	-	26.	34	RCL	$R_1 \ \Sigma D_i^2$
02.	32	g	27.	01	1	R_2
03.	42	x^2	28.	06	6	R_3
04.	33	STO	29.	71	x	R_4
05.	61	+	30.	34	RCL	R_5
06.	01	1	31.	00	0	R_6
07.	34	RCL	32.	32	g	R_7
08.	00	0	33.	42	x^2	R_8
09.	01	1	34.	01	1	R_9
10.	61	+	35.	51	-	R_{e0}
11.	33	STO	36.	34	RCL	R_{e1}
12.	00	0	37.	00	0	R_{e2}
13.	-00	GTO 00	38.	71	x	R_{e3}
14.	51	-	39.	81	\div	R_{e4}
15.	32	g	40.	51	-	R_{e5}
16.	42	x^2	41.	84	R/S	R_{e6}
17.	33	STO	42.	34	RCL	R_{e7}
18.	51	-	43.	00	0	R_{e8}
19.	01	1	44.	01	1	R_{e9}
20.	34	RCL	45.	51	-	
21.	00	0	46.	31	f	
22.	01	1	47.	42	\sqrt{x}	
23.	51	-	48.	71	x	
24.	-11	GTO 11	49.	-00	GTO 00	

Example:(Note: Only the ranks R_i 's and S_i 's are used as the input data.)

Student	x_i Math Grade	y_i Stat Grade	R_i Rank of x_i	S_i Rank of y_i
1	82	81	6	7
2	67	75	14	11
3	91	85	3	4
4	98	90	1	2
5	74	80	11	8
6	52	60	15	15
7	86	94	4	1
8	95	78	2	9
9	79	83	9	6
10	78	76	10	10
11	84	84	5	5
12	80	69	8	13
13	69	72	13	12
14	81	88	7	3
15	73	61	12	14

$$r_s = .76$$

$$z = 2.85$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize	0	STO	0	STO	1	
			BST				0.00
3	Perform 3 for $i = 1, 2, \dots, n$	R_i	\uparrow				
		S_i	R/S				i
3'	Delete erroneous data R_k, S_k	R_k	\uparrow				
		S_k	GTO	1	4	R/S	
4	Compute r_s and z		GTO	2	5	R/S	r_s
			R/S				z

DIFFERENCES AMONG PROPORTIONS

Suppose x_1, x_2, \dots, x_k are observed values of a set of independent random variables having binomial distributions with parameters n_i and θ_i ($i = 1, 2, \dots, k$).

A chi-square statistic given by

$$\chi^2 = \sum_{i=1}^k \frac{(x_i - n_i \hat{\theta})^2}{n_i \hat{\theta} (1 - \hat{\theta})}$$

can be used to test the null hypothesis $\theta_1 = \theta_2 = \dots = \theta_k$, where

$$\hat{\theta} = \frac{\sum_{i=1}^k x_i}{\sum_{i=1}^k n_i}.$$

This χ^2 has the chi-square distribution with $k - 1$ degrees of freedom.

Reference:

J. Freund, *Mathematical Statistics*, Prentice-Hall, 1971.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		R ₀	Σx_i
00.			25.	81	÷	R ₁	$\Sigma(n_i - x_i)$
01.	51	-	26.	11	$\Sigma+$	R ₂	x_i
02.	33	STO	27.	-00	GTO 00	R ₃	$n_i - x_i$
03.	03	3	28.	34	RCL	R ₄	
04.	33	STO	29.	83	•	R ₅	
05.	61	+	30.	02	2	R ₆	
06.	01	1	31.	34	RCL	R ₇	
07.	31	f	32.	00	0	R ₈	
08.	34	LAST X	33.	81	÷	R ₉	
09.	33	STO	34.	34	RCL	R ₁₀ k	
10.	02	2	35.	83	•	R ₁₁ Used	
11.	33	STO	36.	04	4	R ₁₂ Used	
12.	61	+	37.	34	RCL	R ₁₃ Used	
13.	00	0	38.	01	1	R ₁₄ Used	
14.	61	+	39.	81	÷	R ₁₅ Used	
15.	31	f	40.	61	+	R ₁₆ 0	
16.	42	\sqrt{x}	41.	01	1	R ₁₇ 0	
17.	34	RCL	42.	51	-	R ₁₈ 0	
18.	03	3	43.	34	RCL	R ₁₉ 0	
19.	22	$x \leftrightarrow y$	44.	00	0		
20.	81	÷	45.	34	RCL		
21.	34	RCL	46.	01	1		
22.	02	2	47.	61	+		
23.	31	f	48.	71	x		
24.	34	LAST X	49.	-00	GTO 00		

Example:

	n_i	x_i
Sample 1	400	232
Sample 2	500	260
Sample 3	400	197

$$\chi^2 = 6.47$$

$$\hat{\theta} = .53$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
			↑	g	CL+R	STO	
1	Enter program					0	
2	Initialize			STO	1	BST	0.00
3	Perform 3 for $i = 1, 2, \dots, k$	n_i	↑				
		x_i	R/S				i
4	Compute χ^2			GTO	2	8	R/S
5	(optional) Compute $\hat{\theta}$			RCL	0	↑	↑
				RCL	1	+	÷
6	For a new case, go to 2						$\hat{\theta}$

KENDALL'S COEFFICIENT OF CONCORDANCE

Suppose n individuals are ranked from 1 to n according to some specified characteristic by k observers; the coefficient of concordance W measures the agreement between observers (or concordance between rankings).

$$W = \frac{12}{k^2 n(n^2 - 1)} \left(\sum_{i=1}^n \left(\sum_{j=1}^k R_{ij} \right)^2 - \frac{3(n+1)}{n-1} \right)$$

where R_{ij} is the rank assigned to the i^{th} individual by the j^{th} observer.

W varies from 0 (no community of preference) to 1 (perfect agreement). The null hypothesis that the observers have no community of preference may be tested using special tables or, if $n > 7$, by computing

$$\chi^2 = k(n-1)W$$

which has approximately the chi-square distribution with $n-1$ degrees of freedom.

Reference:

J. D. Gibbons, *Nonparametric Statistical Inference*, McGraw-Hill, 1971.

Table for small samples:

M. G. Kendall, *Rank Correlation Methods*, Hafner Publishing Co., 1962.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE		
00.			25.	61	+	$R_0 \ k$
01.	33	STO	26.	33	STO	$R_1 \ j$
02.	61	+	27.	04	4	$R_2 \ \sum R_{ij}$
03.	02	2	28.	00	0	$R_3 \ (\sum R_{ij})^2$
04.	34	RCL	29.	33	STO	$R_4 \ n$
05.	01	1	30.	01	1	R_5
06.	01	1	31.	33	STO	R_6
07.	61	+	32.	02	2	R_7
08.	33	STO	33.	34	RCL	R_8
09.	01	1	34.	04	4	R_9
10.	-00	GTO 00	35.	-00	GTO 00	$R_{\bullet 0}$
11.	34	RCL	36.	01	1	$R_{\bullet 1}$
12.	01	1	37.	61	+	$R_{\bullet 2}$
13.	33	STO	38.	81	\div	$R_{\bullet 3}$
14.	00	0	39.	31	f	$R_{\bullet 4}$
15.	34	RCL	40.	34	LAST X	$R_{\bullet 5}$
16.	02	2	41.	51	-	$R_{\bullet 6}$
17.	32	g	42.	34	RCL	$R_{\bullet 7}$
18.	42	x^2	43.	04	4	$R_{\bullet 8}$
19.	33	STO	44.	01	1	$R_{\bullet 9}$
20.	61	+	45.	51	-	
21.	03	3	46.	81	\div	
22.	34	RCL	47.	03	3	
23.	04	4	48.	71	x	
24.	01	1	49.	-00	GTO 00	

120 Kendall's Coefficient of Concordance

Example:

Table for R_{ij} ($n = 10, k = 3$)

i \ j	1	2	3
1	6	7	3
2	1	4	2
3	9	3	5
4	2	6	1
5	10	8	9
6	3	2	6
7	5	9	8
8	4	1	4
9	8	10	10
10	7	5	7

$$W = .69$$

$$\chi^2 = 18.64$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize	0	STO	1	STO	2	
			STO	3	STO	4	
			BST				0.00
3	Perform 3-5 for $i = 1, 2, \dots, n$						
4	Perform 4 for $j = 1, 2, \dots, k$	R_{ij}	R/S				j
5			GTO	1	1	R/S	i
6	Compute W		RCL	3	4	x	
			RCL	0	g	x^2	
			\div	RCL	4	\div	
			RCL	4	GTO	3	
			6	R/S			W
7	Compute χ^2		RCL	0	x	RCL	
			4	1	-	x	χ^2
8	For a new case, go to 2						

KRUSKAL-WALLIS STATISTIC

Suppose we want to test the null hypothesis that k independent random samples of sizes n_1, n_2, \dots , and n_k come from identical continuous populations.

Arrange all values from k samples jointly (as if they were one sample) in an increasing order of magnitude. Let R_{ij} ($i = 1, 2, \dots, k$, $j = 1, 2, \dots, n_i$) be the rank of the j^{th} value in the i^{th} sample.

The Kruskal-Wallis statistic H can be used to test the null hypothesis.

$$H = \frac{12}{N(N+1)} \sum_{i=1}^k \left(\sum_{j=1}^{n_i} R_{ij} \right)^2 - 3(N+1)$$

$$\text{where } N = \sum_{i=1}^k n_i.$$

When all sample sizes are large (> 5), H is distributed approximately as chi-square with $k - 1$ degrees of freedom. For small samples, the test is based on special tables.

Table for small samples ($k = 3$):

Alexander and Quade, *On the Kruskal-Wallis Three sample H-statistic*, University of North Carolina, Department of Biostatistics, Inst. Statistics Mimeo Ser. 602, 1968.

122 Kruskal-Wallis Statistic

DISPLAY		KEY ENTRY	DISPLAY	KEY ENTRY	REGISTERS
LINE	CODE		LINE	CODE	
00.			25.	34	RCL
01.	33	STO	26.	04	4
02.	61	+	27.	01	1
03.	02	2	28.	61	+
04.	34	RCL	29.	33	STO
05.	01	1	30.	04	4
06.	01	1	31.	00	0
07.	61	+	32.	33	STO
08.	33	STO	33.	01	1
09.	01	1	34.	33	STO
10.	-00	GTO 00	35.	02	2
11.	34	RCL	36.	34	RCL
12.	01	1	37.	04	4
13.	33	STO	38.	-00	GTO 00
14.	61	+	39.	81	÷
15.	00	0	40.	34	RCL
16.	34	RCL	41.	00	0
17.	02	2	42.	01	1
18.	32	g	43.	61	+
19.	42	x ²	44.	81	÷
20.	22	x↔y	45.	31	f
21.	81	÷	46.	34	LAST X
22.	33	STO	47.	51	-
23.	61	+	48.	03	3
24.	03	3	49.	71	x

Example:(Note: Only the ranks R_{ij} 's are used as the input data.)

Sample 1	2.73	0.45	2.52	1.19	3.51	2.75			
Ranks R_{1j}	29	5	26	10	33	30			
Sample 2	1.79	1.83	1	0.87	1.9	1.62	1.74	1.92	
Ranks R_{2j}	11	12	9	7	20	18	19	21	
Sample 3	1.24	2.68	0.88	2.5	1.61	1.55	3.03	0.38	0.22
Ranks R_{3j}	14	28	8	25	17	15	32	4	2
Sample 4	0.57	2.54	0.36	1.56	2.39	1.23	-0.1	2.98	2.15
Ranks R_{4j}	6	27	3	16	24	13	1	31	22
									23

$$N = 33.00$$

$$H = 2.29$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		f	CLR	BST		0.00
3	Perform 3-5 for $i = 1, 2, \dots, k$						
4	Perform 4 for $j = 1, 2, \dots, n_i$	R_{ij}	R/S				j
5			GTO	1	1	R/S	i
6	Compute H		RCL	3	4	x	
			RCL	0			N
			GTO	3	9	R/S	H
7	For a new case, go to 2						

MANN-WHITNEY STATISTIC

This program computes the Mann-Whitney test statistic on two independent samples of equal or unequal sizes. This test is designed for testing the null hypothesis of no difference between two populations.

Mann-Whitney test statistic is defined as

$$U = n_1 n_2 + \frac{n_1 (n_1 + 1)}{2} - \sum_{i=1}^{n_1} R_i$$

where n_1 and n_2 are the sizes of the two samples. Arrange all values from both samples jointly (as if they were one sample) in an increasing order of magnitude; let R_i ($i = 1, 2, \dots, n_1$) be the ranks assigned to the values of the first sample (it is immaterial which sample is referred to as the “first”).

When n_1 and n_2 are small, the Mann-Whitney test bases on the exact distribution of U and specially constructed tables. When n_1 and n_2 are both large (say, greater than 8) then

$$z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{n_1 n_2 (n_1 + n_2 + 1)/12}}$$

is approximately a random variable having the standard normal distribution.

Reference:

J. E. Freund, *Mathematical Statistics*, Prentice-Hall, 1962.

Table for small samples:

D. B. Owen, *Handbook of Statistical Tables*, Addison-Wesley, 1962.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS																																		
LINE	CODE		LINE	CODE		R ₀ Σ R _i	R ₁ n ₁	R ₂ n ₂	R ₃	R ₄	R ₅	R ₆	R ₇	R ₈	R ₉	R ₁₀	R ₁₁	R ₁₂	R ₁₃	R ₁₄	R ₁₅	R ₁₆	R ₁₇	R ₁₈	R ₁₉	R ₂₀	R ₂₁	R ₂₂	R ₂₃	R ₂₄	R ₂₅	R ₂₆	R ₂₇	R ₂₈	R ₂₉	R ₃₀				
00.			25.	22	x \leftrightarrow y																																			
01.	33	STO	26.	34	RCL																																			
02.	61	+	27.	02	2																																			
03.	00	0	28.	71	x																																			
04.	34	RCL	29.	02	2																																			
05.	01	1	30.	81	\div																																			
06.	01	1	31.	51	-																																			
07.	61	+	32.	22	x \leftrightarrow y																																			
08.	33	STO	33.	34	RCL																																			
09.	01	1	34.	02	2																																			
10.	-00	GTO 00	35.	61	+																																			
11.	34	RCL	36.	01	1																																			
12.	02	2	37.	61	+																																			
13.	34	RCL	38.	34	RCL																																			
14.	01	1	39.	01	1																																			
15.	01	1	40.	71	x																																			
16.	61	+	41.	34	RCL																																			
17.	02	2	42.	02	2																																			
18.	81	\div	43.	71	x																																			
19.	61	+	44.	01	1																																			
20.	71	x	45.	02	2																																			
21.	34	RCL	46.	81	\div																																			
22.	00	0	47.	31	f																																			
23.	51	-	48.	42	\sqrt{x}																																			
24.	84	R/S	49.	81	\div																																			

Example:(Note: Only the ranks R_i's for the first sample are used as the input data.)

Sample 1	14.9	11.3	13.2	16.6	17	14.1	15.4	13	16.9																															
Rank R _i	7	1	4	12	14	5	10	3	13																															
Sample 2	15.2	19.8	14.7	18.3	16.2	21.2	18.9	12.2	15.3	19.4																														
Rank	8	18	6	15	11	19	16	2	9	17																														

$$n_1 = 9, n_2 = 10, U = 66.00, z = 1.71$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize	0	STO	0	STO	1	
			BST				0.00
3	Store n ₂	n ₂	STO	2			
4	Perform 4 for i = 1, 2, ..., n ₁	R _i	R/S				i
5	Compute U and z		GTO	1	1	R/S	U
			R/S				z
6	For a new case, go to 2						

MEAN-SQUARE SUCCESSIVE DIFFERENCE

When test and estimation techniques are used, the method of drawing the sample from the population is specified to be random in most cases. If observations are chosen in a sequence x_1, x_2, \dots, x_n , the mean-square successive difference

$$\eta = \sum_{i=1}^{n-1} (x_i - x_{i+1})^2 \quad \left/ \sum_{i=1}^n (x_i - \bar{x})^2 \right.$$

can be used to test for randomness.

If n is large (say, greater than 20), and the population is normal, then

$$z = \frac{1 - \eta/2}{\sqrt{\frac{n-2}{n^2-1}}}$$

has approximately the standard normal distribution. Long trends are associated with large positive values of z and short oscillations with large negative values.

Reference:

Dixon and Massey, *Introduction to Statistical Analysis*, McGraw-Hill, 1969.

DISPLAY		KEY ENTRY	DISPLAY		KEY ENTRY	REGISTERS	
LINE	CODE		LINE	CODE		R ₀	
00.			25.	04	4	R ₁	
01.	34	RCL	26.	22	x \leftrightarrow y	R ₂	
02.	83	.	27.	81	\div	R ₃	
03.	06	6	28.	84	R/S	R ₄	
04.	22	x \leftrightarrow y	29.	02	2	R ₅	
05.	51	-	30.	81	\div	R ₆	
06.	31	f	31.	01	1	R ₇	
07.	34	LAST X	32.	22	x \leftrightarrow y	R ₈	
08.	33	STO	33.	51	-	R ₉	
09.	83	.	34.	34	RCL	R _{•0} n	
10.	06	6	35.	83	.	R _{•1} Σx_i	
11.	11	$\Sigma +$	36.	00	0	R _{•2} Σx_i^2	
12.	-00	GTO 00	37.	02	2	R _{•3} $\Sigma (x_i - x_{i+1})$	
13.	32	g	38.	51	-	R _{•4} $\Sigma (x_i - x_{i+1})^2$	
14.	33	s	39.	34	RCL	R _{•5} Used	
15.	32	g	40.	83	.	R _{•6} x _i	
16.	42	x ²	41.	00	0	R _{•7} 0	
17.	34	RCL	42.	32	g	R _{•8} 0	
18.	83	.	43.	42	x ²	R _{•9} 0	
19.	00	0	44.	01	1		
20.	01	1	45.	51	-		
21.	51	-	46.	81	\div		
22.	71	x	47.	31	f		
23.	34	RCL	48.	42	\sqrt{x}		
24.	83	.	49.	81	\div		

Example:

For the following set of data

$$\{0.53, 0.52, 0.39, 0.49, 0.97, 0.29, 0.65, 0.30, 0.40, \\0.06, 0.14, 0.16, 0.68, 0.22, 0.68, 0.08, 0.52, 0.50, \\0.63, 0.20, 0.67, 0.44, 0.64, 0.40, 0.97, 0.03, 0.73, \\0.24, 0.57, 0.35\}$$

$n = 30$

$\eta = 2.81$

$z = -2.29$.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Enter program						
2	Initialize		g	CL+R	BST		0.00
3	Input x ₁	x ₁	STO	.	6	$\Sigma +$	1.00
4	Perform 4 for i = 2, 3, ..., n	x _i	R/S				i
5	Compute η and z		GTO	1	3	R/S	η
			R/S				z
6	For a new case, go to 2						

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