HEWLETT hp PACKARD

HP-65

MATH PAC 1

SEPTEMBER, 1974

PRINTED IN U.S.A.

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INTRODUCTION

Programs for your HP-65 Math Pac 1 have been selected from the areas of algebra, trigonometry, geometry and numerical analysis.

Each program includes a general description, formulas used in the program solution, numerical examples, and user instructions. Program listings and register allocations are given in the back of the Pac.

Some related individual programs were combined on one card when it seemed they might be useful together. In this way more programs could be included in the Pac.

We hope you find the HP-65 Math Pac 1 a useful tool for your computational work, and welcome your comments, requests and suggestions—these are our most important source of future user-oriented programs.

FORMAT OF USER INSTRUCTIONS

The following is an example of a set of User Instructions.

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Clear registers		A		
3	Perform 3–4 for i=1, ..., n	a_i	↑		
4		b_i	B		
5			C		Answer
	(To run a new case, go to 2)				

To follow the instructions, start with line 1 and read from left to right, performing indicated operations as you proceed. Lines having no numbers contain special notes to the user and are inside parentheses in the INSTRUCTIONS column. The message "To run a new case, go to 2" following line 5 in the above example is a special note.

Lines are read in sequential order except where the INSTRUCTIONS column directs otherwise. For example, "go to 2" means to jump to line 2. Repeated processes–used in most cases for a long string of input/output data–are outlined with a bold border together with a "Perform" instruction. In the above example. "Perform 3–4 for i = 1, ..., n" means to execute the loop (line 3 and line 4) n times. The first time, the dummy variable i takes the value 1; the second time i takes the value 2; etc.

Normally, as in the above example, the first instruction is "Enter program" which means load the preprogrammed magnetic card (for instructions of loading a card, see "Entering A Program" on P. 7). Some instructions are self-contained and can be carried out by just reading the INSTRUCTIONS column alone, e.g., "Enter program". But some instructions depend on the information supplied by the DATA and/or KEYS columns. In line 2 of the example above, "Clear registers" appears in the INSTRUCTIONS column and **A** appears in the KEYS column, which means you have to clear the working registers by pressing the **A** key.

The DATA column specifies the input data to be supplied. Invalid arguments which result in division by zero, finding square root of a negative number, etc. will result in flashing zeros. Arguments out of the designated program range will result in incorrect answers or flashing zeros. When a computed value exceeds the calculator range, an overflow or underflow occurs and halts the program.

The KEYS column specifies the keys to be pressed. [↑] is the symbol used to denote the [ENTER↑] key. All other key designations are identical to those appearing on the HP-65. Ignore any blank positions in the KEYS column.

The DISPLAY column may show counters, intermediate or final results. In line 5 of the example, the answer will be displayed after pressing the [C] key.

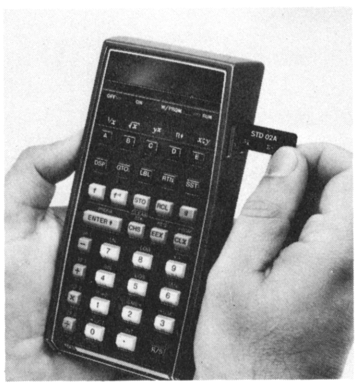
ENTERING A PROGRAM

From the card case supplied with this application pac, select a program card.

Set W/PRGM-RUN switch to RUN.

Turn the calculator ON. You should see 0.00

Gently insert the card (printed side up) in the right, lower slot as shown. When the card is part way in, the motor engages it and passes it out the left side of the calculator. Sometimes the motor engages but does not pull the card in. If this happens, push the card a little farther into the machine. Do not impede or force the card; let it move freely. (The display will flash if the card reads improperly. In this case, press [CLX] and reinsert the card.)



When the motor stops, remove the card from the left side of the calculator and insert it in the upper "window slot" on the right side of the calculator.

The program is now stored in the calculator. It remains stored until another program is entered or the calculator is turned off.

FACTORS OF AN INTEGER

FACTORS OF AN INTEGER MATH 1-01A

This program finds all prime factors—p_1, p_2, p_3, ...—of a positive integer n ($\leqslant 2 \times 10^9$) and hence, determines if n is a prime number.

Note: Flashing zeros indicate that either

(1) n is not an integer

or (2) n is not positive

or (3) $n > 2 \times 10^9$

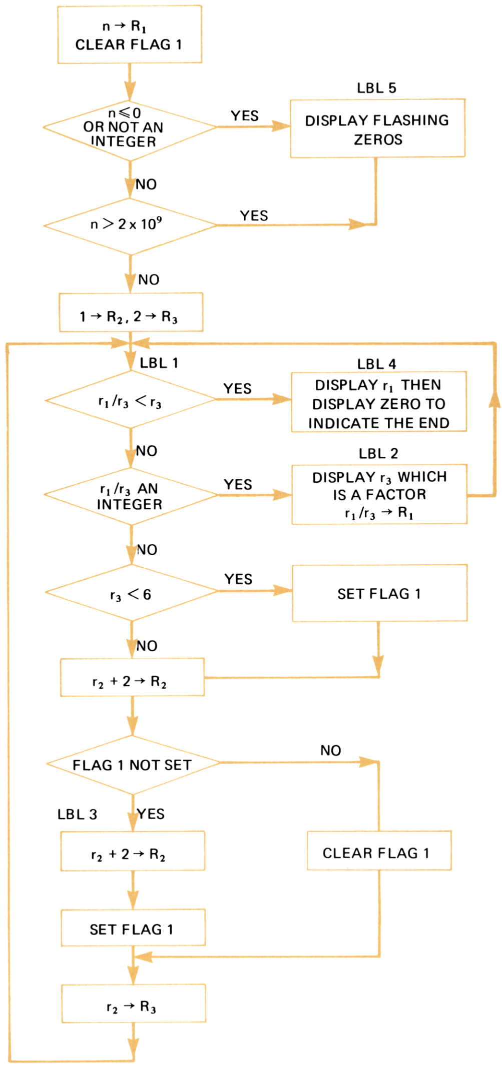
Examples:

1. 124 = 2 x 2 x 31

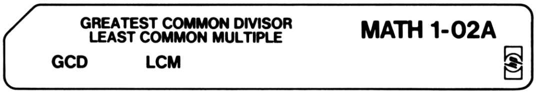
2. 523 is a prime.

3. 4807 = 11 x 19 x 23

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2		n	A		p_1
3	Perform 3 until $p_i = 0$		R/S		p_i
	($p_i = 0$ is an indication that all				
	factors have been found.)				


n → R1
CLEAR FLAG 1
LBL 5
DISPLAY FLASHING ZEROS
n ≤ 0 OR NOT AN INTEGER
YES
NO
n > 2 x 10^9
YES
NO
1 → R2, 2 → R3
LBL 1
r1/r3 < r3
YES
LBL 4
DISPLAY r1 THEN DISPLAY ZERO TO INDICATE THE END
NO
r1/r3 AN INTEGER
YES
LBL 2
DISPLAY r3 WHICH IS A FACTOR r1/r3 → R1
NO
r3 < 6
YES
SET FLAG 1
NO
r2 + 2 → R2
FLAG 1 NOT SET
NO
LBL 3
YES
r2 + 2 → R2
CLEAR FLAG 1
SET FLAG 1
r2 → R3

GREATEST COMMON DIVISOR
LEAST COMMON MULTIPLE



This program computes the greatest common divisor (GCD) and the least common multiple (LCM) for integers a, b. The program also finds integral coefficients s and t such that

$$\mathrm{GCD}\,(a, b) = sa + tb$$

Note:

$$\mathrm{LCM}\,(a, b) = \frac{ab}{\mathrm{GCD}\,(a, b)}$$

The program does not use this relationship so that the two subprograms for finding GCD and LCM can be used independently in other main programs. Subroutine E is used for both subprograms.

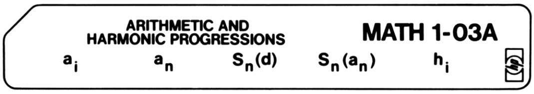
Examples:

1. a = 240, b = 1144
   GCD (a, b) = 8.00
   s = 62.00
   t = –13.00

2. a = 240, b = 1144
   LCM (a, b) = 34320.00

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Obtain GCD (a, b)	a	↑		
3		b	A		GCD
4			R/S		s
5			R/S		t
6	Obtain LCM (a, b)	a	↑		
7		b	B		LCM

ARITHMETIC AND HARMONIC PROGRESSIONS



This program can be used for the following:

1. To display an arithmetic progression a_i $(i = 1, 2, 3, ...)$

$$a_i = a_1 + (i - 1)\, d$$

2. To find the n^{th} term a_n of the progression

$$a_n = a_1 + (n - 1)\, d$$

3. To find the sum $S_n(d)$ of the first n terms of an arithmetic progression if the first term a_1 and the difference d are known

$$S_n(d) = na_1 + \frac{1}{2}\, n\,(n-1)\, d$$

4. To find the sum $S_n(a_n)$ if the first and the n^{th} terms are known

$$S_n(a_n) = \frac{n}{2}\,(a_1 + a_n)$$

5. To display the terms h_i $(i = 1, 2, 3, ...)$ of a harmonic progression

$$h_i = \frac{a}{b + (i - 1)\, c}$$

where $b + (i - 1)\, c \neq 0$ for $i = 1, 2, 3, ...$

Examples:

1. $a_1 = 0$, $d = 17$, the arithmetic progression will be displayed as 0.00, 17.00, 34.00, 51.00, ...

2. The 25^{th} term of the arithmetic progression with $a_1 = 2$, $d = 3.14$ is 77.36. ($S_{25} = 992.00$)

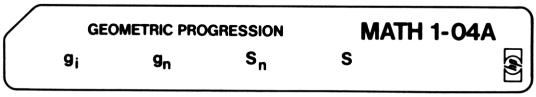
3. If $a_1 = 3.5$, $d = 2.15$ and $n = 11$, then $S_{11} = 156.75$

4. If $a_1 = 3.5$, $a_{11} = 25$, $n = 11$, then $S_{11} = 156.75$.

5. The harmonic progression with a = 1, b = 2, c = 3 is 0.50, 0.20, 0.13, 0.09, ...

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Display a_i	a_1	↑		
3		d	A		a_1
4	Perform 4 for i = 2, 3, ...		R/S		a_i
5	Calculate a_n (and S_n)	a_1	↑		
6		d	↑		
7		n	B		a_n
8			R/S		S_n
9	Calculate sum (given a_1, d, n)	a_1	↑		
10		d	↑		
11		n	C		S_n(d)
12	Calculate sum (given a_1, a_n, n)	a_1	↑		
13		a_n	↑		
14		n	D		$S_n(a_n)$
15	Display h_i	a	↑		
16		b	↑		
17		c	E		h_1
18	Perform 18 for i = 2, 3, ...		R/S		h_i

GEOMETRIC PROGRESSION



This program can be used for the following:

1. To display a geometric progression g_i $(i = 1, 2, 3, ...)$

$$g_i = ar^{i-1} \qquad (a = g_1 \text{ is the first term})$$

2. To find the n^{th} term g_n of the progression

$$g_n = ar^{n-1}$$

3. To find the sum S_n of the first n terms of the progression

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

4. To find the infinite sum S if the ratio r of two successive terms has absolute value less than 1.

$$S = \frac{a}{1 - r} \quad \text{if} \quad -1 < r < 1$$

Note: If $|r| \geqslant 1$, use of the [D] key will cause flashing zeros in the display.

Examples:

1. Powers of 8 are 8.00, 64.00, 512.00, 4096.00, ...

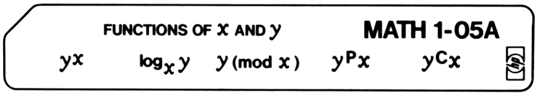
2. The 14th term of the geometric progression with a = 2, r = –3.14 is –5769197.69.

3. If a = 1, r = –2.1, n = 6, then S_6 = –27.34.

4. If a = 2, r = 0.5, then S = 4.00.

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Display the progression	a	↑		
3		r	A		g_1
4	Perform 4 for i = 2, 3, ...		R/S		g_i
5	Obtain the nth term	a	↑		
6		r	↑		
7		n	B		g_n
8	Obtain the sum S_n	a	↑		
9		r	↑		
10		n	C		S_n
11	Obtain the infinite sum	a	↑		
12		r	D		S

FUNCTIONS OF x AND y



The program can be used to find:

1. y^x for any real y and x. If y is negative then x must be an integer.

2. Logarithm of y (base x)

$$\log_x y = \frac{\ln y}{\ln x}$$

3. $y \pmod x) = y - x \left[\text{integer part of } \left(\frac{y}{x}\right)\right]$

4. Permutation $_yP_x$

$${}_yP_x = \frac{y!}{(y-x)!}$$

   where x, y are positive integers and $x \leqslant y$.

   Program requires $y \leqslant 69$.

5. Combination $_yC_x$ (binomial coefficient)

$${}_yC_x = \frac{y!}{x!\,(y-x)!}$$

   where x, y are positive integers and $x \leqslant y$.

   Program requires $y \leqslant 69$.

Examples:

1. $(-32.24)^3 = -33510.82$

2. $\log_7 5 = 0.83$

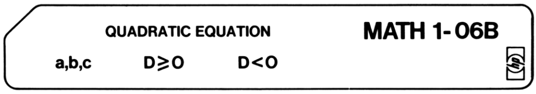
3. 52 (mod 12) = 4.00

4. ${}_7P_5 = 2520.00$

5. ${}_7C_5 = 21.00$

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Compute y^x	y	↑		
3		x	A		y^x
4	Compute $\log_x y$	y	↑		
5		x	B		$\log_x y$
6	Compute y (mod x)	y	↑		
7		x	C		y (mod x)
8	Compute ${}_yP_x$	y	↑		
9		x	D		${}_yP_x$
10	Compute ${}_yC_x$	y	↑		
11		x	E		${}_yC_x$

QUADRATIC EQUATION



The roots x_1, x_2 of $ax^2 + bx + c = 0$

are given by

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

However, better significance can be obtained in some cases by first computing the root with the largest absolute value using the following formula

$$x_1 = \frac{-ab}{|ab|}\left(\left|\frac{b}{2a}\right| + \sqrt{\frac{b^2 - 4ac}{4a^2}}\right)$$

then the smaller root by $x_2 = \dfrac{c}{x_1\, a}$

If $D = (b^2 - 4ac)/4a^2$

is positive or zero, the roots are real. Otherwise, they are complex, being

$$u \pm iv = \frac{-b}{2a} \pm \frac{\sqrt{4ac - b^2}}{2a}\, i\ .$$

Note: Subroutines D and E which are not used in this program will be used in conjunction with *Math 1–08A, Fourth Degree Polynomial Equation.*

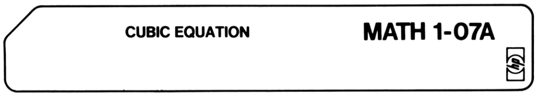
Examples:

1. $2x^2 + 5x + 3 = 0$
   $(D = 0.06 > 0)$
   $x_1 = -1.50$
   $x_2 = -1.00$

2. $2x^2 + 3x + 4 = 0$
   $(D = -1.44 < 0)$
   $x_1 = -0.75 + 1.20i$
   $x_2 = -0.75 - 1.20i$

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2		a	↑		
3		b	↑		
4		c	A		D
5	If $D \geq 0$, roots are real		B		root 1
6			R/S		root 2
7	If $D < 0$, roots are complex		C		u
8			R/S		v

CUBIC EQUATION

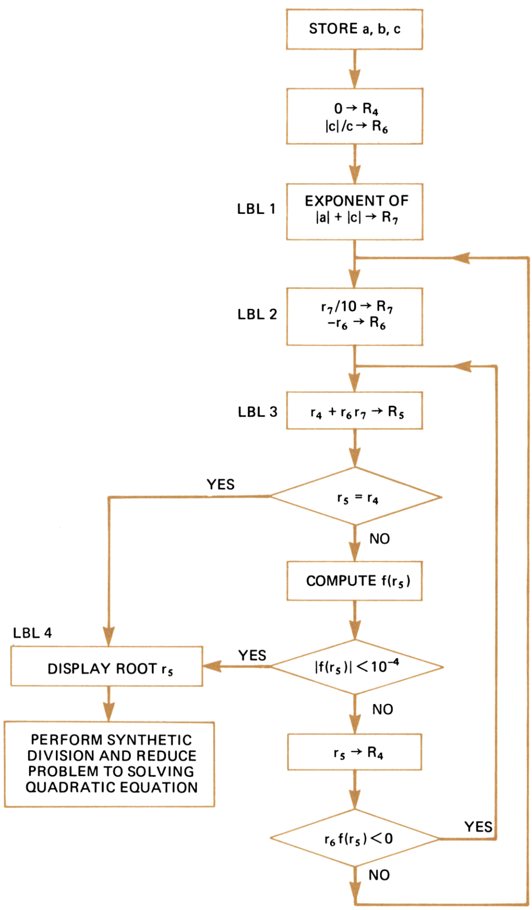


This program finds a real root of the cubic equation

$$f(x) = x^3 + ax^2 + bx + c = 0$$

where a, b, c are real.

Then by synthetic division, the problem is reduced to solving a quadratic equation.



Note: Program requires $c \neq 0$.

For if $c = 0$, zero is a real root and by factoring out x, we can use the quadratic formula to find the other two roots.

Examples:

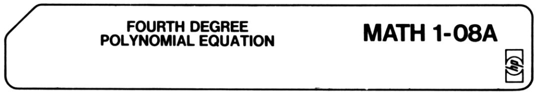
1. $x^3 - 6x^2 + 11x - 6 = 0$
   $x_1 = 3.00, x_2 = 2.00, x_3 = 1.00$ (D = 0.25)

2. $x^3 - 4x^2 + 8x - 8 = 0$
   $x_1 = 2.00, x_2 = 1.00 + 1.73i, x_3 = 1.00 - 1.73i$ (D = –3.00)

3. $x^3 - 10x^2 - 2.25x + 22.5 = 0$
   $x_1 = -1.50, x_2 = 10.00, x_3 = 1.50$ (D = 18.06)

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program *Math 1–07A*				
2		a	STO	1	
3		b	STO	2	
4		c	STO	3	
5			A		root 1
6			R/S		
7	Enter program *Math 1–06B*		A		D
8	If $D < 0$, go to 11				
9	$D \geqslant 0$, roots are real		B		root 2
10			R/S		root 3
11	$D < 0$, roots $u \pm iv$ are complex		C		u
12			R/S		v

FOURTH DEGREE POLYNOMIAL EQUATION



This program can be used in conjunction with *Math 1–07A, Cubic Equation,* and *Math 1–06B, Quadratic Equation,* to find the roots (real and/or imaginary) of a fourth degree polynomial equation of the form:

$$x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0$$

where a_i (i = 0, 1, 2, 3) are real.

Brown's Method is used. First, solve the cubic equation:

$$f(y) = y^3 + b_2y^2 + b_1y + b_0 = 0$$

where $b_2 = -a_2$

$b_1 = a_3a_1 - 4a_0$

$b_0 = a_0(4a_2 - a_3^2) - a_1^2$

Suppose f(y) has roots y_1, y_2, y_3 and let y_0 be the largest real root of f(y).

Then the fourth degree equation is reduced to two quadratic equations:

$$x^2 + (A + C)x + (B + D) = 0$$

$$x^2 + (A - C)x + (B - D) = 0$$

where $A = \frac{a_3}{2}$, $B = \frac{y_0}{2}$

$D = \sqrt{B^2 - a_0}$

$$C = \begin{cases} \left(AB - \frac{a_1}{2}\right) \Big/ D & \text{if } D \neq 0 \\ \sqrt{A^2 - a_2 + y_0} & \text{if } D = 0 \end{cases}$$

Roots can be found by solving the quadratic equations.

Note: In order to get more accurate answers, intermediate results (e.g., y_0, a_3, a_2, a_1, a_0) should be recorded and entered to as many decimal places as possible.

Reference: Numerical methods in Engineering, Salvadori and Baron, Prentice-Hall, 1961

Examples:

1. $x^4 - 2x^3 + 3x^2 - 2x + 2 = 0$

   $b_0 = 12.00$ $y_1 = -2.00$
   $D_2 = 0.25$ $y_2 = 3.00$
   $y_3 = 2.00$ (hence $y_0 = y_2 = 3$)
   $D_3 = -1.00$
   (a) Root 1 = i
   (b) Root 2 = $-i$
   $D_4 = -1.00$
   (c) Root 3 = $1.00 + 1.00i$
   (d) Root 4 = $1.00 - 1.00i$

2. $4x^4 - 8x^3 - 13x^2 - 10x + 22 = 0$

   Rewrite equation as: $x^4 - 2x^3 - \frac{13}{4}x^2 - 2.5x + \frac{22}{4} = 0$

   $b_0 = -99.75$
   $y_1 = 4.75$
   $D_2 = -5.00$ (hence $y_0 = y_1 = 4.75$)
   $D_3 = -1.00$
   (a) Root 1 = $-1.00 + 1.00i$
   (b) Root 2 = $-1.00 - 1.00i$
   $D_4 = 1.25$
   (c) Root 3 = 3.12
   (d) Root 4 = 0.88

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program *Math 1–08A*				
2		a_3	R/S		
3		a_2	R/S		
4		a_1	R/S		
5		a_0	R/S		b_0
6	If $b_0 \neq 0$, go to 17				
7	$b_0 = 0$ hence $y_1 = 0$				
8	Enter program *Math 1–06B*				
9			D	A	D_1
10	If $D_1 < 0$, $y_0 = 0$, go to 13				
11	$D_1 \geq 0$, y_2, y_3 are real		B		y_2
12	Record y_2, y_3		R/S		y_3
13	y_0 = Max. of real roots	y_0	STO	4	
14	Enter program *Math 1–08A*				

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
15	Compute A ± C, B ± D		B		
16	Go to 32				
17	Enter program *Math 1–07A*				
18			A		y_1
19	Record y_1		R/S		
20	Enter program *Math 1–06B*				
21			A		D_2
22	If $D_2 < 0$, $y_0 = y_1$, go to 25				
23	$D_2 \geqslant 0$, y_2, y_3 are real		B		y_2
24	Record y_2, y_3		R/S		y_3
25	y_0 = Max. of real roots	y_0	STO	4	
26	Enter program *Math 1–08A*				
27		a_3	R/S		
28		a_2	R/S		
29		a_1	R/S		
30		a_0	R/S		
31	Compute A ± C, B ± D		B		
32	Enter program *Math 1–06B*				
33			A		D_3
34	If $D_3 < 0$, go to 38				
35	$D_3 \geqslant 0$, roots are real		B		root
36			R/S		root
37	Go to 40				
38	$D_3 < 0$, roots are $u \pm iv$		C		u
39			R/S		v
40	Solve the 2nd quadratic		E	A	D_4
41	If $D_4 < 0$, go to 44				
42	$D_4 \geqslant 0$, roots are real		B		root
43			R/S		root
	(All roots are found)				
44	$D_4 < 0$, roots are $s \pm it$		C		s
45			R/S		t

FIFTH DEGREE POLYNOMIAL EQUATION

This program finds one real root of

$$f(x) = x^5 + \alpha_4 x^4 + \alpha_3 x^3 + \alpha_2 x^2 + \alpha_1 x + \alpha_0 = 0$$

where α_i (i = 0, 1, 2, 3, 4) are real.

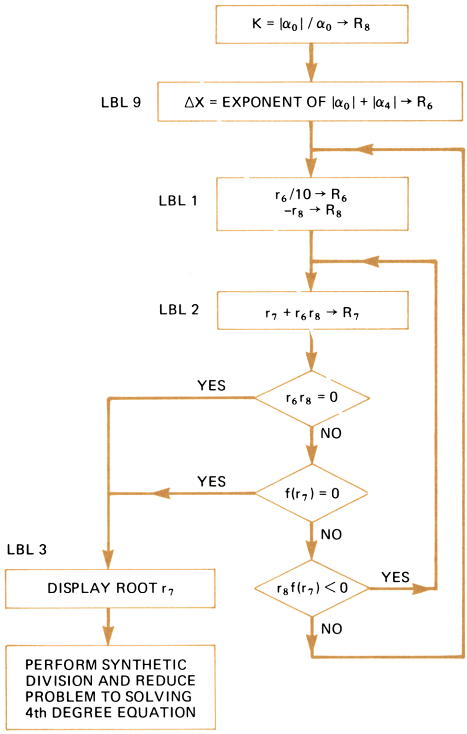
The real root is found by an iterative process, then the fifth degree equation is reduced to a fourth degree equation

$$x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0$$

by synthetic division.

Math 1–08A, Fourth Degree Polynomial Equation then can be used to find the other four roots (real and/or imaginary) of f(x).

Note: Program requires $\alpha_0 \neq 0$. For if $\alpha_0 = 0$, zero is a root, problem is reduced to solving fourth degree polynomial equation.


K = |α0| / α0 → R8
LBL 9
ΔX = EXPONENT OF |α0| + |α4| → R6
LBL 1
r6/10 → R6
−r8 → R8
LBL 2
r7 + r6r8 → R7
YES
r6r8 = 0
NO
YES
f(r7) = 0
NO
LBL 3
DISPLAY ROOT r7
r8f(r7) < 0
YES
NO
PERFORM SYNTHETIC DIVISION AND REDUCE PROBLEM TO SOLVING 4th DEGREE EQUATION

Examples:

1. $x^5 - x^4 - 101x^3 + 101x^2 + 100x - 100 = 0$

   (a) Root 1 = 10.00

   $a_3 = 9.00$, $a_2 = -11.00$, $a_1 = -9.00$, $a_0 = 10.00$

   $b_0 = -1331.00$, $y_1 = 11.00$, $D_2 = 0.00$, $y_2 = -11.00$,
   $y_3 = -11.00$ (hence $y_0 = y_1 = 11$)

   $D_3 = 0.00$

   (b) Root 2 = 1.00

   (c) Root 3 = 1.00

   $D_4 = 20.25$

   (d) Root 4 = −10.00

   (e) Root 5 = −1.00

2. $x^5 - 23x^4 + 12x^2 + 13x + 69 = 0$ (Note: $\alpha_3 = 0$)

   (a) Root 1 = −1.31474954 (Press **DSP** **[•]** **[8]**)

   $a_3 = -24.31474954$

   $a_2 = 31.96780575$

   $a_1 = -30.02965787$

   $a_0 = 52.48147884$

   $b_0 = -2.521832950\ \ 04$

   $y_1 = 36.58750070$

   $D_2 < 0$ (hence $y_0 = y_1 = 36.5875007$)

   $D_3 < 0$

   (b) Root 2 = −0.09426686 + 1.21928470i

   (c) Root 3 = −0.09426686 − 1.21928470i

   $D_4 > 0$

   (d) Root 4 = 22.97594875

   (e) Root 5 = 1.52733450

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program *Math 1–09A*				
2	Clear registers		f	REG	
3	Store coefficients	α_4	STO	1	
4		α_3	STO	2	
5		α_2	STO	3	
6		α_1	STO	4	
7	($\alpha_0 \neq 0$)	α_0	STO	5	
8			R/S		root
9	Perform synthetic division		R/S		
10	Recall and record coefficients		RCL	8	a_3
11			RCL	7	a_2
12			RCL	6	a_1
13			RCL	5	a_0
14	Enter program *Math 1–08A*				
15			A		b_0
	(If b_0 = 0 go to line 7 of the				
	Instructions for *Math 1–08A,*				
	Fourth Degree Polynomial				
	Equation. If $b_0 \neq 0$ go to line				
	17 of that program.)				

SIMULTANEOUS EQUATIONS IN TWO UNKNOWNS

$$\begin{cases} ax + by = e \\ cx + dy = f \end{cases}$$

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{D}$$

$$y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{D}$$

where determinant $D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \neq 0$

Note: Flashing zeros indicate $D = 0$.

Example:

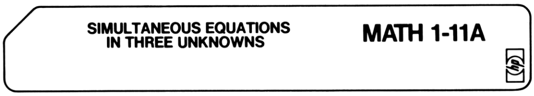
$$\begin{cases} 7.32x - 9.08y = 3.14 \\ 12.39x + 7y = 0.05 \end{cases}$$

$$D = 163.74$$

$$\begin{cases} x = 0.14 \\ y = -0.24 \end{cases}$$

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2		a	↑		
3		b	↑		
4		e	A		
5		c	↑		
6		d	↑		
7		f	B		D
8	If $D \neq 0$		R/S		x
9			R/S		y

SIMULTANEOUS EQUATIONS IN THREE UNKNOWNS



$$\begin{cases} a_1 x + b_1 y + c_1 z = d_1 & (1) \\ a_2 x + b_2 y + c_2 z = d_2 & (2) \\ a_3 x + b_3 y + c_3 z = d_3 & (3) \end{cases}$$

Renumber the equations if necessary such that $a_1 \neq 0$.

This program uses the following algorithm:

$$x + \frac{b_1}{a_1} y + \frac{c_1}{a_1} z = \frac{d_1}{a_1} \qquad (1)'$$

$$\left(\frac{b_1 a_2}{a_1} - b_2\right) y + \left(\frac{c_1 a_2}{a_1} - c_2\right) z = \frac{d_1 a_2}{a_1} - d_2 \qquad (2)'$$

$$\left(\frac{b_1 a_3}{a_1} - b_3\right) y + \left(\frac{c_1 a_3}{a_1} - c_3\right) z = \frac{d_1 a_3}{a_1} - d_3 \qquad (3)'$$

Solve $(2)'$ and $(3)'$ for y, z if $D_2 \neq 0$, where

$$D_2 = \begin{vmatrix} \dfrac{b_1 a_2}{a_1} - b_2 & \dfrac{c_1 a_2}{a_1} - c_2 \\ \dfrac{b_1 a_3}{a_1} - b_3 & \dfrac{c_1 a_3}{a_1} - c_3 \end{vmatrix}$$

Then solve for x from $(1)'$.

Example:

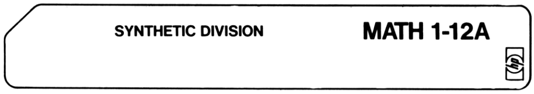
$$\begin{cases} 3.14x + 10.02y - 7z = 1 \\ 0.25x + 30.3y - 9.1z = 2 \\ -3.5x + 27.4y + 8z = 3 \end{cases}$$

$$D_2 = 335.31$$

$$\begin{cases} x = 0.29 \\ y = 0.11 \\ z = 0.14 \end{cases}$$

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2		a_1	RTN		
3		b_1	↑		
4		c_1	↑		
5		d_1	R/S		
6		a_2	↑		
7		b_2	↑		
8		c_2	R/S		
9		a_3	↑		
10		b_3	↑		
11		c_3	R/S		
12		d_2	↑		
13		d_3	R/S		D_2
14	If $D_2 \neq 0$		R/S		x
15			R/S		y
16			R/S		z

SYNTHETIC DIVISION



This program performs synthetic division on a polynomial of degree n (with real coefficients)

$$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

by $x - x_0$ so that

$$a_n x^n + \ldots + a_1 x + a_0$$

$$= (x - x_0)(b_{n-1} x^{n-1} + b_{n-2} x^{n-2} + \ldots + b_1 x + b_0) + R$$

where $b_{n-1} = a_n$

$b_k = b_{k+1} x_0 + a_{k+1}$ for $k = n-2, \ldots, 0$

$R = b_0 x_0 + a_0$

Note: Program requires $n \leqslant 7$.

If $n < 7$, let

$a_7 = \ldots = a_{n+1} = 0$

Examples:

1. $x^5 - 4x^4 + 7x^3 - 10x^2 + 8 = (x-2)(x^4 - 2x^3 + 3x^2 - 4x - 8) - 8$

2. $x^5 - 4x^4 + 7x^3 - 10x^2 + 8$

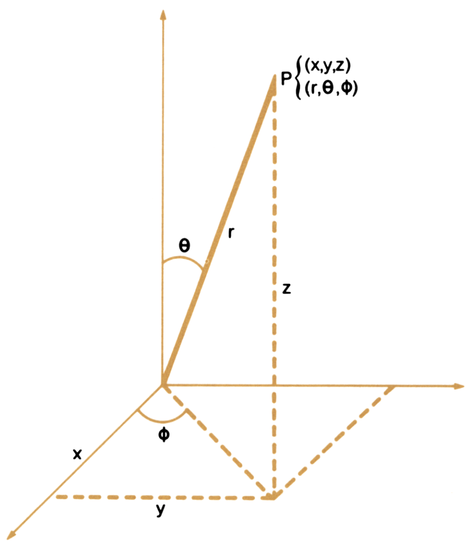
$$= (x - 2.65)(x^4 - 1.35x^3 + 3.42x^2 - 0.93x - 2.47) + 1.47$$

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	If $n < 7$, let $a_7 = \ldots = a_{n+1} = 0$	a_7	↑		
3		a_6	↑		
4		a_5	↑		
5		a_4	A		
6		a_3	↑		
7		a_2	↑		
8		a_1	↑		
9		a_0	R/S		
10		x_0	B		b_6
11	Perform 11 for i = 5, 4, ..., 0		R/S		b_i
12			R/S		R
	(For a new x_0, go to 10)				

RECTANGULAR, SPHERICAL CONVERSIONS

$$\begin{cases} x = r \sin\theta \cos\phi \\ y = r \sin\theta \sin\phi \\ z = r \cos\theta \end{cases}$$

$$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \phi = \tan^{-1}\left(\dfrac{y}{x}\right) \\ \theta = \cos^{-1}\left(z / \sqrt{x^2 + y^2 + z^2}\right) \end{cases}$$



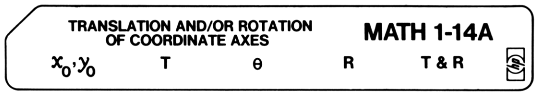
Example:

$$\begin{cases} x = 3 \\ y = 4 \\ z = 5 \end{cases}$$

$$\begin{cases} r = 7.07 \\ \phi = 53.13^\circ = 0.93 \text{ radians} = 59.03 \text{ grads} \\ \theta = 45.00^\circ = 0.79 \text{ radians} = 50.00 \text{ grads} \end{cases}$$

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Convert R→S	z	↑		
3		y	↑		
4		x	A		r
5			R/S		ϕ
6			R/S		θ
7	Put r, ϕ, θ in X, Y, Z registers		R/S		r
8	Convert S→R	θ	↑		
9		ϕ	↑		
10		r	B		x
11			R/S		y
12			R/S		z
13	Put x, y, z in X, Y, Z registers		R/S		x

TRANSLATION AND/OR ROTATION OF COORDINATE AXES



1. Suppose point P has coordinates (x, y) with respect to the coordinate system having x, y axes. If we translate the system to a new system with origin at (x_0, y_0), then point P will have coordinates (x', y') with respect to the new system.

$$\begin{cases} x' = x - x_0 \\ y' = y - y_0 \end{cases}$$

2. Suppose point P has coordinates (x, y) with respect to the coordinate system having x, y axes. If we rotate the axes with an angle θ, point P will have new coordinates (x'', y'').

$$\begin{cases} x'' = x \cos\theta + y \sin\theta \\ y'' = -x \sin\theta + y \cos\theta \end{cases}$$

3. Suppose point P has coordinates (x, y) with respect to the coordinate system having x, y axes. If we translate the system to a new system with origin at (x_0, y_0), then rotate the axes with an angle θ, point P will have coordinates (x''', y''') with respect to the new system.

$$\begin{cases} x''' = (x - x_0) \cos\theta + (y - y_0) \sin\theta \\ y''' = -(x - x_0) \sin\theta + (y - y_0) \cos\theta \end{cases}$$

Examples:

1. P = (5, –10)
   (x_0, y_0) = (2, 3)
   After translation, P = (3, –13).

2. P_1 = (5, –10), P_2 = (3, –13)
   θ = 45°
   After rotation, P_1 = (–3.54, –10.61), P_2 = (–7.07, –11.31).

3. P = (5, –10)

   After a translation to a new origin (2, 3) and a rotation of 45°, P = (–7.07, –11.31).

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Perform translation	y_0	↑		
3		x_0	A		
4		y	↑		
5		x	B		x′
6			g	x⇄y	y′
	(For a different point, go to 4.)				
7	Perform rotation	θ	C		
8		y	↑		
9		x	D		x″
10			g	x⇄y	y″
	(For a different point, go to 8.)				
11	Perform translation and rotation	y_0	↑		
12		x_0	A		
13		θ	C		
14		y	↑		
15		x	E		x‴
16			g	x⇄y	y‴
	(For a different point, go to 14.)				

ANGLE CONVERSIONS

ANGLE CONVERSIONS			MATH 1-15A
DEG	RAD	GRD	MIL

This program can be used to convert an angle in one angular unit to any other unit. Angles can be expressed in degrees, radians, grads or mils.

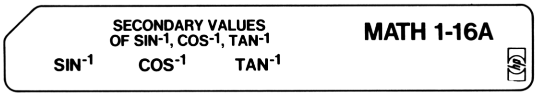
$$360^\circ = 2\pi \text{ radians} = 400 \text{ grads} = 6400 \text{ mils}$$

Examples:

1. 270° = 300 grads
2. 1600 mils = 90°
3. 2 radians = 127.32 grads
4. 360° = 6.28 radians

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Clear flag 1		f^{-1}	SF 1	
3	Convert x from degrees to grads	x	A	C	
4	Convert x from mils to degrees	x	D	A	
5	Convert x from radians to grads	x	B	C	
6	Convert x from degrees to				
7	radians	x	A	B	
	etc.				

SECONDARY VALUES OF SIN^{-1}, COS^{-1}, TAN^{-1}



Inverse trigonometric functions are multiple-valued functions, the HP-65 gives principal values as answers.

This program computes the secondary values of arc sine, arc cosine and arc tangent.

Secondary value for arc sin = 180° – principal value

Secondary value for arc cos = 360° – principal value

Secondary value for arc tan = 180° + principal value

Examples:

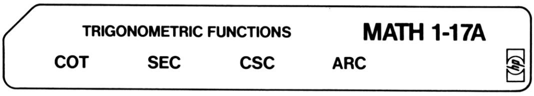
1. In DEG mode: $\sin^{-1}(.7) = 135.57°$
   $\cos^{-1}(.7) = 314.43°$

2. In RAD mode: $\cos^{-1}(.7) = 5.49$
   $\tan^{-1}(.7) = 3.75$

3. In GRD mode: $\sin^{-1}(-0.87) = 267.18$
   $\tan^{-1}(-0.87) = 154.42$

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Clear flags		E		
3	If machine is in RAD mode		f	SF1	
4	If machine is in GRD mode		f	SF2	
5	Obtain $\sin^{-1}$ x	x	A		$\sin^{-1}$ x
6	Obtain $\cos^{-1}$ x	x	B		$\cos^{-1}$ x
7	Obtain $\tan^{-1}$ x	x	C		$\tan^{-1}$ x
	(For a new case in different				
	mode, go to 2.)				

TRIGONOMETRIC FUNCTIONS



$$\cot x = \frac{1}{\tan x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\text{arc} \cot x = \text{arc} \tan \frac{1}{x}$$

$$\text{arc} \sec x = \text{arc} \cos \frac{1}{x}$$

$$\text{arc} \csc x = \text{arc} \sin \frac{1}{x}$$

Restriction: x can not be a discontinuous point of the function or flashing zeros will result.

Examples:

1. In DEG mode: $\cot 37° = 1.33$ $\sec 45° = 1.41$
2. In RAD mode: $\cot 2 = -0.46$ $\text{arc} \sec 2 = 1.05$
3. In GRAD mode: $\csc 100 = 1.00$ $\text{arc} \csc 1 = 100.00$

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Clear flag 1		E		
3	Obtain cot x	x	A		cot x
4	Obtain sec x	x	B		sec x
5	Obtain csc x	x	C		csc x
6	Obtain arc cot x	x	D	A	arc cot x
7	Obtain arc sec x	x	D	B	arc sec x
8	Obtain arc csc x	x	D	C	arc csc x

HYPERBOLIC FUNCTIONS

HYPERBOLIC FUNCTIONS			MATH 1-18A
SINH	COSH	TANH	1/x

$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} \qquad (x \neq 0)$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\coth x = \frac{1}{\tanh x} \qquad (x \neq 0)$$

Examples:

1. sinh 1.5 = 2.13
2. cosh 5.9 = 182.52
3. tanh 1.3 = 0.86
4. csch 0.95 = 0.91
5. sech (–3) = 0.10
6. coth (–1.99) = –1.04

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Obtain sinh x	x	A		sinh x
3	Obtain cosh x	x	B		cosh x
4	Obtain tanh x	x	C		tanh x
5	Obtain csch x	x	A	D	csch x
6	Obtain sech x	x	B	D	sech x
7	Obtain coth x	x	C	D	coth x

INVERSE HYPERBOLIC FUNCTIONS

$$\sinh^{-1} x = \ln [x + (x^2 + 1)^{1/2}]$$

$$\cosh^{-1} x = \ln [x + (x^2 - 1)^{1/2}] \qquad (x \geqslant 1)$$

$$\tanh^{-1} x = \frac{1}{2} \ln \frac{1 + x}{1 - x} \qquad (x^2 < 1)$$

$$\operatorname{csch}^{-1} x = \sinh^{-1} \frac{1}{x} \qquad (x \neq 0)$$

$$\operatorname{sech}^{-1} x = \cosh^{-1} \frac{1}{x} \qquad (0 < x \leqslant 1)$$

$$\coth^{-1} x = \tanh^{-1} \frac{1}{x} \qquad (x^2 > 1)$$

Examples:

1. $\sinh^{-1} 3.5 = 1.97$

2. $\cosh^{-1} 100 = 5.30$

3. $\tanh^{-1} (-0.7) = -0.87$

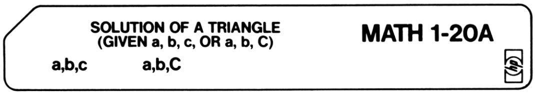
4. $\text{csch}^{-1} 3 = 0.33$

5. $\text{sech}^{-1} 0.5 = 1.32$

6. $\coth^{-1} 5.4 = 0.19$

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Obtain $\sinh^{-1}$ x	x	A		$\sinh^{-1}$ x
3	Obtain $\cosh^{-1}$ x	x	B		$\cosh^{-1}$ x
4	Obtain $\tanh^{-1}$ x	x	C		$\tanh^{-1}$ x
5	Obtain csch^{-1} x	x	D	A	csch^{-1} x
6	Obtain sech^{-1} x	x	D	B	sech^{-1} x
7	Obtain $\coth^{-1}$ x	x	D	C	$\coth^{-1}$ x

SOLUTION OF A TRIANGLE (GIVEN a, b, c, OR a, b, C)



The following formulas are used to solve an oblique triangle in this program and also in *Math 1–21A, Math 1–22A.*

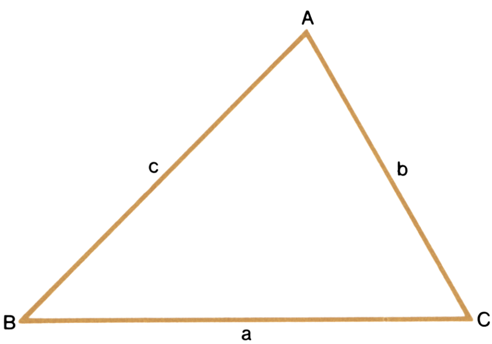
Law of sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosine

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$A + B + C = 180^\circ$$



Notes: 1. In some cases of obtuse triangles, this program generates one incorrect angle such that the sum of the three angles does not add up to 180°. Draw a sketch of the given triangle to locate the incorrect angle. Adding the difference between 180° and the sum of the three angles to this angle will yield the correct answer. This is due to the fact that the program uses inverse trigonometric functions of the HP-65 which always returns the principal values. For cases mentioned above, secondary values are required (see *Math 1–16A, Secondary Values of* sin^{-1}, cos^{-1}, tan^{-1}).

2. If machine is in DEG mode, all angles are assumed to be in decimal degrees.

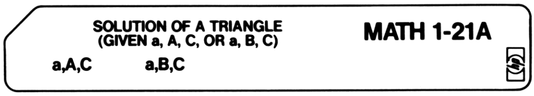
Examples:

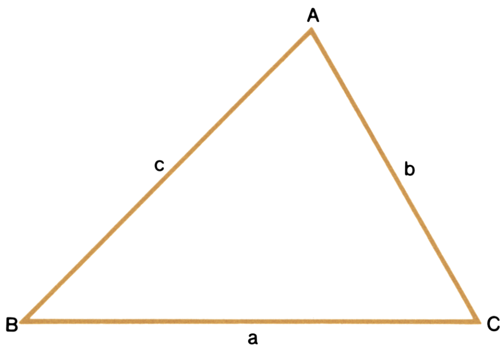
1. Given a = 30.3, b = 40.4, c = 62.6 then
   C = 123.99°
   B = 32.35°
   A = 23.66°

2. Given a = 132, b = 224, C = 28°40′ (convert angle C to decimal degrees first), then
   c = 125.35
   A = 30.34°
   B = 120.99° (secondary value, display shows 59.01)

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	If a, b, c are given	a	↑		
3		b	↑		
4		c	A		C
5			R/S		B
6			R/S		A
7	If a, b, C are given	a	↑		
8		b	↑		
9		C	B		c
10			R/S		A
11			R/S		B

SOLUTION OF A TRIANGLE
(GIVEN a, A, C, OR a, B, C)





Note: All angles are assumed to be in decimal degrees.

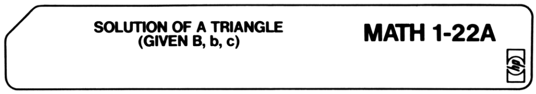
Examples:

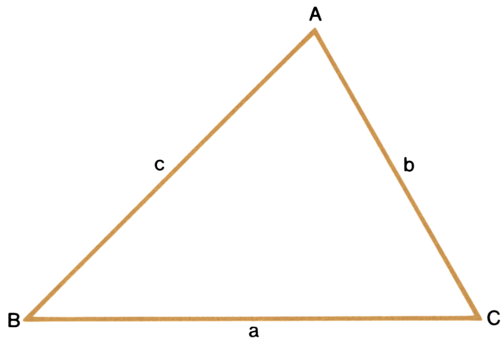
1. Given a = 17.5, A = 41°14′, C = 62°12′ (convert A, C to decimal degrees first) then
   B = 76.57°
   b = 25.82
   c = 23.49

2. Given a = 25.2, B = 35°20′, C = 68°30′, (convert B, C to decimal degrees first) then
   A = 76.17°
   c = 24.15
   b = 15.01

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	If a, A, C are given	a	↑		
3		A	↑		
4		C	A		B
5			R/S		b
6			R/S		c
7	If a, B, C are given	a	↑		
8		B	↑		
9		C	B		A
10			R/S		c
11			R/S		b
	(Machine now is in DEG mode.)				

SOLUTION OF A TRIANGLE (GIVEN B, b, c)





Notes:

1. In some cases of obtuse triangles, this program generates one incorrect angle such that the sum of the three angles does not add up to 180°. Draw a sketch of the given triangle to locate the incorrect angle. Adding the difference between 180° and the sum of the three angles to this angle will yield the correct answer. This is due to the fact that the program uses inverse trigonometric functions of the HP-65 which always returns the principal values. For cases mentioned above, secondary values are required (see *Math 1–16A, Secondary Values of* sin^{-1} cos^{-1}, tan^{-1}).

2. If machine is in DEG mode, all angles are assumed to be in decimal degrees.

3. If B is acute $(<90^\circ)$ and $b < c$, two sets of solutions exist.

4. Flashing zeros for angle C indicates no solution exists.

Example:

Given B = 33°40′, b = 31.5, c = 51.8, then

$$\begin{cases} C = 65.73^\circ \\ A = 80.60^\circ \\ a = 56.06 \end{cases} \qquad \begin{cases} C_1 = 114.27^\circ \\ A_1 = 32.06^\circ \\ a_1 = 30.16 \end{cases}$$

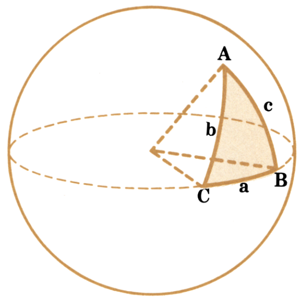
LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Clear flags		A		
3	If machine is in RAD mode		f	SF 1	
4	If machine is in GRD mode		f	SF 2	
5		B	↑		
6		b	↑		
7		c	B		C
8			R/S		A
9			R/S		a
10	Obtain second set of solutions		R/S		C_1
11			R/S		A_1
12			R/S		a_1

SPHERICAL TRIANGLES

SPHERICAL TRIANGLES — MATH 1-23B

A,b,c | a,b,c | ANGLES

If A,B,C are the three angles of a spherical triangle and a,b,c the opposite sides, this program solves the triangle for any one of the cases:



1. A,b,c are given
2. a,b,c are given
3. a,B,C are given
4. A,B,C are given .

The laws of cosines are used:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$

Notes: 1. Area may be determined from the equation:

$$\text{Area} = r^2 (A + B + C - \pi)$$

where r is the radius of the sphere and A,B,C are in radians.

2. Program works in any angular mode. If the calculator is in DEG mode, all angles are in decimal degrees.

Examples:

1. If A = 30°, b = 50.5°, c = 47.3°

   then a = 22.71°, B = 87.88°, C = 72.13°.

2. If a = 0.2 radians, b = 0.91 radians, c = 0.93 radians

   then A = 0.25 radians, B = 1.40 radians, C = 1.59 radians.

3. If a = 1.12°, B = 21.63°, C = 158.05°

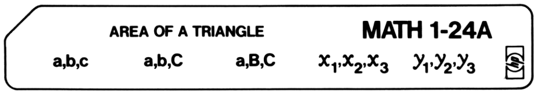
   then A = 0.52°, b = 51.90°, c = 52.94°.

4. If A = 47 grads, B = 160 grads, C = 60 grads

   then a = 62.51 grads, b = 148.25 grads, c = 101.70 grads.

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Initialize		RTN	R/S	0.00
3	Case 1: given A, b, c	A	↑		
		b	↑		
		c	R/S		0.00
			A		a
	Go to 5				
4	Case 2: given a, b, c	a	↑		
		b	↑		
		c	R/S		0.00
			B		a
5	Compute solution		R/S		b
			R/S		c
			R/S		A
			R/S		B
			R/S		C
6	For a new case, go to 2				
7	Case 3: given a, B, C	a	↑		
		B	↑		
		C	R/S		0.00
			C	A	A
	Go to 9				
8	Case 4: given A, B, C	A	↑		
		B	↑		
		C	R/S		0.00
			C	B	A
9	Compute solution		R/S		B
			R/S		C
			R/S		a
			R/S		b
			R/S		c
10	For a new case, go to 2				

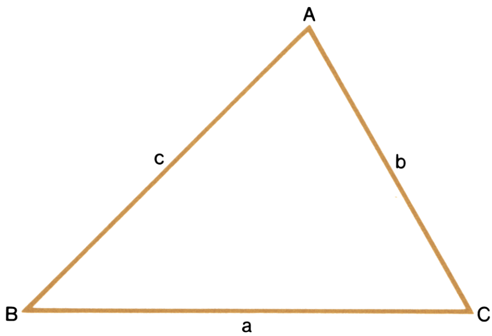
AREA OF A TRIANGLE



Case 1. Three sides a, b, c are given.

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{1}{2}(a+b+c).$$



Case 2. a, b and C are given.

$$\text{area} = \frac{1}{2}\, a\, b \sin C$$

Case 3. Side a and adjacent angles B, C are known.

$$\text{area} = \frac{a^2}{2}\,\frac{\sin B \sin C}{\sin(B+C)}$$

Case 4. Three vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) are given.

$$\text{area} = |D|$$

$$\text{where} \quad D = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\,[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

Note: If calculator is in DEG mode, angles are assumed to be in decimal degrees.

Examples:

1. Given a = 5.31, b = 7.09, c = 8.86 then
   area = 18.82

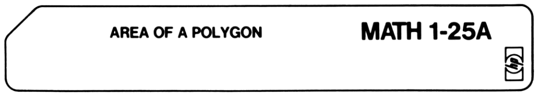
2. Given a = 5.3174, b = 7.0898, C = 45° then
   area = 13.33

3. Given a = 14.625, B = 70.54°, C = 62.96° then
   area = 123.82

4. If (0, 0), (4, 0), (4, 3) are vertices of a triangle, then
   area = 6.00

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Case 1	a	↑		
3		b	↑		
4		c	A		area
5	Case 2	a	↑		
6		b	↑		
7		C	B		area
8	Case 3	a	↑		
9		B	↑		
10		C	C		area
11	Case 4	x_1	↑		
12		x_2	↑		
13		x_3	D		
14		y_1	↑		
15		y_2	↑		
16		y_3	E		area

AREA OF A POLYGON

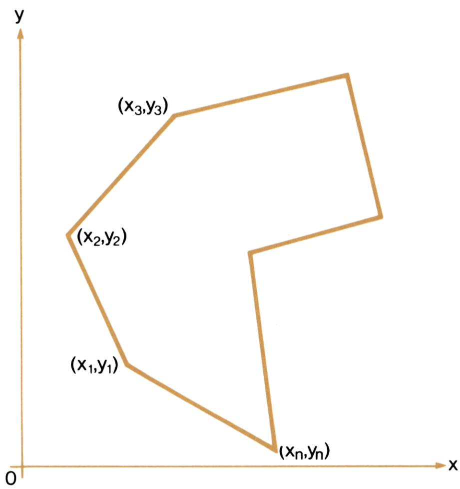


This program calculates the area of a polygon of n sides with $n \geqslant 3$.

If (x_i, y_i) are rectangular coordinates of the vertices of the polygon $(i = 1, 2, ..., n)$, then

area

$$= \frac{1}{2}\left[(x_1+x_2)(y_1-y_2)+(x_2+x_3)(y_2-y_3)+\ldots+(x_n+x_1)(y_n-y_1)\right]$$

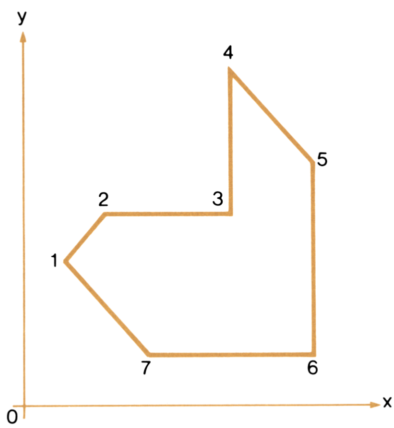


Note: When traversing counterclockwise, the area will be displayed as a negative number.

Example:

Point	Coordinates (x, y)
1	(1, 3)
2	(2, 4)
3	(5, 4)
4	(5, 7)
5	(7, 5)
6	(7, 1)
7	(3, 1)

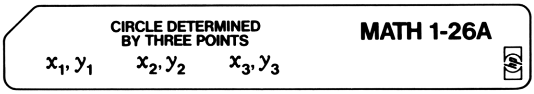
area = 19.50



LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2		x_1	↑		
3		y_1	↑		
4		x_2	↑		
5		y_2	A		
6*	Perform 6–7 for i = 3, 4, ..., n−1	x_i	↑		
7		y_i	R/S		
8*		x_n	↑		
9		y_n	B		area

*When entering new coordinates, be careful not to lose the values already in the X and Y registers.

CIRCLE DETERMINED BY THREE POINTS



Given three distinct points (x_1, y_1), (x_2, y_2), (x_3, y_3) on a circle, this program solves for the radius and center point of the circle using the following formulas:

$$y_0 = \frac{K_2 - K_1}{N_2 - N_1}, \quad x_0 = K_2 - N_2 y_0$$

$$r = \sqrt{(x_3 - x_0)^2 + (y_3 - y_0)^2}$$

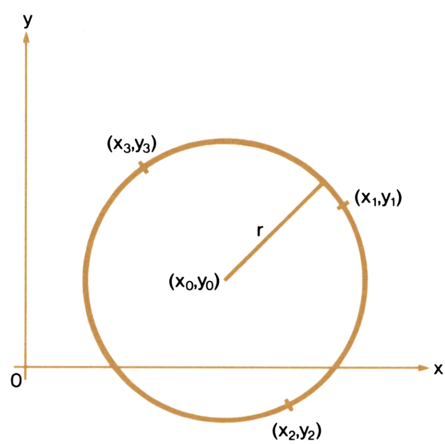
where

$$K_1 = \frac{(x_2 - x_1)(x_2 + x_1) + (y_2 - y_1)(y_2 + y_1)}{2(x_2 - x_1)}$$

$$K_2 = \frac{(x_3 - x_1)(x_3 + x_1) + (y_3 - y_1)(y_3 + y_1)}{2(x_3 - x_1)}$$

$$N_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$N_2 = \frac{y_3 - y_1}{x_3 - x_1}$$



Restrictions: $x_1 \neq x_2$, $x_1 \neq x_3$

$N_1 \neq N_2$ (three points can not be on a line)

Examples:

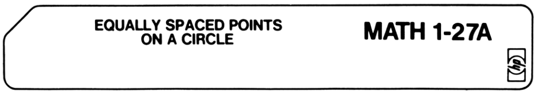
1. The three points (1, 1), (3.5, −7.6), (12, 0.8) determine a circle with center (6.45, −2.08) and radius 6.26.

2. Three points (0, 1), (−1, 0), (0, −1) determine the unit circle with center at the origin.

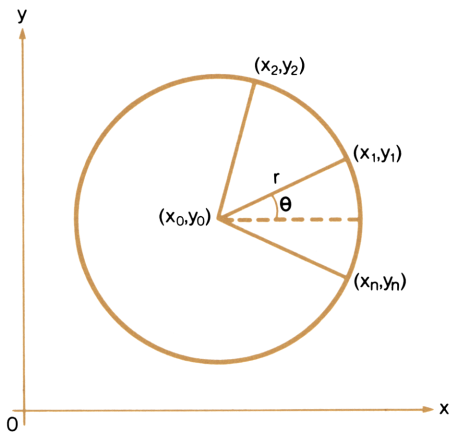
Note: (−1,0) must be chosen as (x_1, y_1).

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2		x_1	↑		
3		y_1	A		
4		x_2	↑		
5		y_2	B		
6		x_3	↑		
7		y_3	C		x_0
8			R/S		y_0
9			R/S		r

EQUALLY SPACED POINTS ON A CIRCLE



Given a circle with radius r and center (x_0, y_0), the program computes the rectangular coordinates of equally spaced points (x_i, y_i), $(i = 1, 2, ..., n)$ on the circle if angle θ and number of points n are known. The position of the first point (x_1, y_1) on the circle is determined by the angle θ.



$$\begin{cases} x_{k+1} = x_0 + r\cos(\theta + ck) \\ y_{k+1} = y_0 + r\sin(\theta + ck) \end{cases}$$

where $k = 0, 1, 2, ..., n-1$

$$c = \begin{cases} \dfrac{360}{n} & \text{if in DEG mode} \\ \dfrac{2\pi}{n} & \text{if in RAD mode} \\ \dfrac{400}{n} & \text{if in GRD mode} \end{cases}$$

Examples:

1. $\theta = \frac{\pi}{4}$, r = 1, n = 5, x_0 = 4.28, y_0 = 3.1

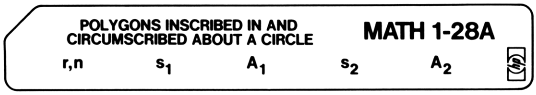
   $(x_1, y_1) = (4.99, 3.81)$
   $(x_2, y_2) = (3.83, 3.99)$
   $(x_3, y_3) = (3.29, 2.94)$
   $(x_4, y_4) = (4.12, 2.11)$
   $(x_5, y_5) = (5.17, 2.65)$

2. $\theta = 36^\circ$, r = 3.21, n = 3, x_0 = −3.4, y_0 = 1.8

   $(x_1, y_1) = (-0.80, 3.69)$
   $(x_2, y_2) = (-6.33, 3.11)$
   $(x_3, y_3) = (-3.06, -1.39)$

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2			A		
3		θ	↑		
4		r	↑		
5		n	R/S		
6		y_0	↑		
7		x_0	R/S		x_1
8			R/S		y_1
9	Perform 9–10 for i = 2, 3, ..., n		R/S		x_i
10			R/S		y_i

POLYGONS INSCRIBED IN AND CIRCUMSCRIBED ABOUT A CIRCLE



This program finds:

1. The side s_1 and area A_1 of an n-sided regular polygon inscribed in a circle of radius r

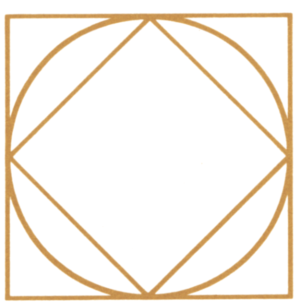
$$s_1 = 2r \sin\left(\frac{180°}{n}\right)$$

$$A_1 = \frac{1}{2} nr^2 \sin\left(\frac{360°}{n}\right)$$

2. The side s_2 and area A_2 of an n-sided regular polygon circumscribed about a circle of radius r

$$s_2 = 2r \tan\left(\frac{180°}{n}\right)$$

$$A_2 = nr^2 \tan\left(\frac{180°}{n}\right)$$



Example:

If r = 5, n = 6, then

s_1 = 5.00

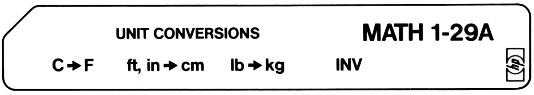
A_1 = 64.95

s_2 = 5.77

A_2 = 86.60

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2		r	↑		
3		n	A		
4	Compute s_1		B		s_1
5	Compute A_1		C		A_1
6	Compute s_2		D		s_2
7	Compute A_2		E		A_2
	(Subroutine A sets machine in				
	DEG mode)				

UNIT CONVERSIONS: C→F; ft, in→cm; lb→kg



This program can be used to perform unit conversions between:

1. Centigrade and Fahrenheit

   $$\text{Fahrenheit degrees} = \frac{9}{5}\ \text{Centigrade degrees} + 32$$

2. feet, inches and centimeters

   1 inch = 2.54 centimeters

3. pounds and kilograms

   1 pound = 0.45359237 kilograms

Examples:

1. 28°C = 82.40°F
2. 165 cm = 5′4.96″
3. 51.34 kg = 113.19 lb

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Clear flag 1		E		
3	Convert C→F	C	A		F
4	Convert ft, in→cm	ft	↑		
5		in	B		cm
6	Convert lb→kg	lb	C		kg
7	Convert F→C	F	D	A	C
8	Convert cm→ft, in	cm	D	B	ft
9			R/S		in
10	Convert kg→lb	kg	D	C	lb

UNIT CONVERSIONS: mi→km; gal→ltr; yd→m; ac→ft^2

UNIT CONVERSIONS MATH 1-30A

mi→km gal→ltr yd→m ac→ft^2 INV

This program can be used to perform unit conversions between:

1. miles and kilometers

   1 mile = 1.609344 kilometers

2. gallons and liters

   1 gallon = 3.785411784 liters

3. yards and meters

   1 yard = 0.9144 meters

4. acres and square feet

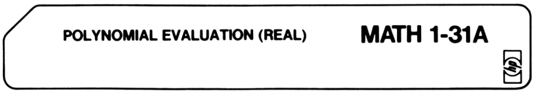
   1 acre = 43560 ft^2

Examples:

1. 12.34 mi = 19.86 km
2. 20 ltr = 5.28 gal
3. 1000 m = 1093.61 yd
4. 1.82 ac = 79279.20 ft^2

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	Clear flag 1		f^{-1}	SF 1	
3	Convert mi→km	mi	A		km
4	Convert gal→ltr	gal	B		ltr
5	Convert yd→m	yd	C		m
6	Convert ac→ft^2	ac	D		ft^2
7	Convert km→mi	km	E	A	mi
8	Convert ltr→gal	ltr	E	B	gal
9	Convert m→yd	m	E	C	yd
10	Convert ft^2→ac	ft^2	E	D	ac

POLYNOMIAL EVALUATION (REAL)



This program evaluates polynomials of the form

$$f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$$

for real coefficients a_k ($k = 0, 1, \ldots, n$) and real x_0, where $n \leqslant 8$.

Example:

$$f(x) = 11 - 7x - 3x^2 + 5x^4 + x^5 \qquad (\text{Note: } a_3 = 0)$$

$$f(2.5) = 267.72$$

$$f(-5) = -29.00$$

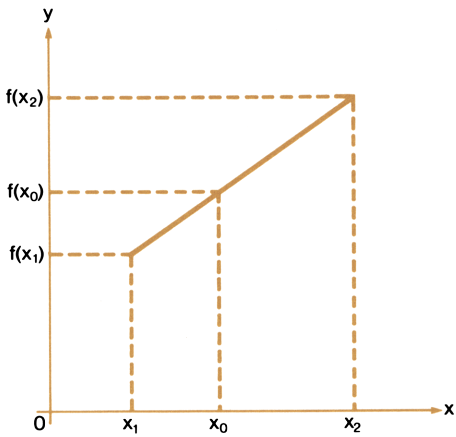
LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2		a_0	A		
3	Perform 3 for i = 1, ..., n	a_i	R/S		
4		x_0	B		$f(x_0)$
	(For new value of x_0, go to 4)				

LINEAR AND LAGRANGIAN INTERPOLATIONS

1. Linear Interpolation

   If f(x) is a function of x and $x_1 < x_0 < x_2$, $f(x_0)$ can be approximated by

$$f(x_0) \cong \frac{(x_2 - x_0)\, f(x_1) + (x_0 - x_1)\, f(x_2)}{x_2 - x_1}$$



2. This program also evaluates for interpolation argument x the Lagrangian interpolating polynomial $P_2(x)$ of degree two passing through the points (x_0, y_0), (x_1, y_1), (x_2, y_2).

$$P_2(x) = \sum_{i=0}^{2} L_i(x)\, y_i$$

where

$$L_i(x) = \prod_{\substack{j=0 \\ i \neq j}}^{2} \frac{(x - x_j)}{(x_i - x_j)}, \quad i = 0, 1, 2$$

Examples:

1.

i	1	2
x	1.2	1.3
f(x)	0.30119	0.27253

f(1.27) = 0.28113
f(1.29) = 0.27540

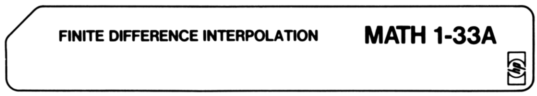
2.

i	0	1	2
x	1	3	10
y	−5	1	25

x = 1.7, y = −2.94
x = 9, y = 21.29

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	For linear interpolation	x_1	↑		
3		$f(x_1)$	↑		
4		x_2	↑		
5		$f(x_2)$	A		
6		x_0	B		$f(x_0)$
	(For a new value of x_0, go to 6.)				
7	For Lagrangian interpolation	x_0	↑		
8		x_1	↑		
9		x_2	C		
10		y_0	↑		
11		y_1	↑		
12		y_2	D		
13		x	E		y
	(For a new x, go to 13.)				

FINITE DIFFERENCE INTERPOLATION



This program interpolates for data points in the region of tabulated data for uniformly spaced abscissas, with spacing h. The equation used is the backward-interpolation formula of Gauss which uses four pairs of data points and sets up the polynomial for cubic interpolation.

The equation used is:

$$y = y_3 + u\delta y_{-1/2} + \frac{1}{2}u(u+1)\,\delta^2 y_0 + \frac{1}{3!}u(u+1)(u-1)\,\delta^3 y_{-1/2}$$

The difference table is:

u	x	y			
−2	x_1	y_1			
			$y_2 - y_1$		
−1	x_2	y_2		$y_3 - 2y_2 + y_1$	
			$y_3 - y_2$		$y_4 - 3y_3 + 3y_2 - y_1$
0	x_3	y_3		$y_4 - 2y_3 + y_2$	
			$y_4 - y_3$		
1	x_4	y_4			

where $\delta y_{-1/2} = y_3 - y_2$

$\delta^2 y_0 = y_4 - 2y_3 + y_2$

$\delta^3 y_{-1/2} = y_4 - 3y_3 + 3y_2 - y_1$

and $u = \dfrac{x - x_3}{h}$

Example:

i	1	2	3	4
x_i	−1	1	3	5
y_i	−1	2	9	30

(Note: h = 2)

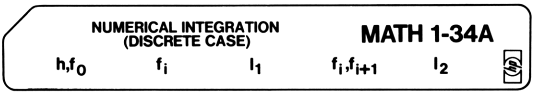
x = −0.5, y = −0.08

x = 2.567, y = 6.64

x = 4.8, y = 26.99

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2		y_1	↑		
3		y_2	↑		
4		y_3	↑		
5		y_4	A		
6		x_3	↑		
7		h	B		
8		x	C		y
	(For a new x, go to 8.)				

NUMERICAL INTEGRATION (DISCRETE CASE)



Let x_0, x_1, ..., x_n be equally spaced points, $x_i = x_0 + ih$ for i = 1, 2, ..., n, at which corresponding values $f(x_0)$, $f(x_1)$, ..., $f(x_n)$ of a function f(x) are known. Using only this information, with no explicit expression for f(x) itself,

$$\int_{x_0}^{x_n} f(x)\,dx$$

may be approximated using

(1) The trapezoidal rule:

$$\int_{x_0}^{x_n} f(x)\,dx \cong \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right] = I_1$$

(2) Simpson's rule:

$$\int_{x_0}^{x_n} f(x)\,dx \cong \frac{h}{3}\Big[f(x_0) + 4f(x_1) + 2f(x_2)$$

$$+ \ldots + 4f(x_{n-3}) + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\Big] = I_2$$

In order to apply Simpson's rule, n must be even.

Notation used on magnetic card: $f_i = f(x_i)$, i = 0, 1, 2, ..., n

Examples:

i	0	1	2	3	4	5	6	7	8
x_i	0	.25	.5	.75	1	1.25	1.5	1.75	2
$f(x_i)$	2	2.8	3.8	5.2	7	9.2	12.1	15.6	20

$h = 0.25$

1. Trapezoidal rule

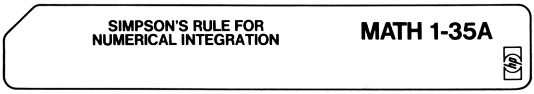
$$\int_0^2 f(x)\,dx \cong I_1 = 16.68$$

2. Simpson's rule

$$\int_0^2 f(x)\,dx \cong I_2 = 16.58$$

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2	For the trapezoidal rule	h	↑		
3		$f(x_0)$	A		
4	Perform 4 for i = 1, 2, ..., n – 1	$f(x_i)$	B		
5		$f(x_n)$	C		I_1
6	For Simpson's rule	h	↑		
7		$f(x_0)$	A		
8	Perform 8–9 for i = 1, 3, ..., n–3	$f(x_i)$	↑		
9		$f(x_{i+1})$	D		
10		$f(x_{n-1})$	↑		
11		$f(x_n)$	E		I_2

SIMPSON'S RULE FOR NUMERICAL INTEGRATION



The definite integral $\int_a^b f(x)\,dx$ can be approximated by Simpson's rule:

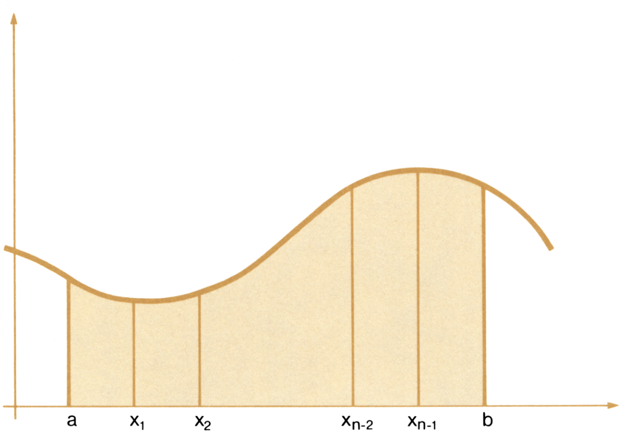
$$\int_a^b f(x)\,dx \cong \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \ldots + 4f(x_{n-3}) + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

where $x_0 = a,\ x_i = x_0 + ih,\ x_n = b$

$$h = \frac{b-a}{n} \qquad (i = 1, 2, \ldots, n-1)$$

This program approximates the integral $\int_a^b f(x)\,dx$ for given f(x) and finite a, b such that $a \leqslant b$.

The number n must be a positive even integer $\geqslant 4$. f(x) should not have any singularities in the integeration interval.



f(x) must be programmed in the calculator by the user. Assuming the value x is in the X register, 24 memory locations, stack registers and storage registers R_6, R_7 are available for f(x). Register R_9 is also available for temporary storage only.

Note: If n is odd, error will be indicated by flashing zeros.

Examples:

1.

$$\int_0^2 x^2\,dx \cong 2.67 \qquad (n = 6)$$

Keys for f(x): [↑] [×]

Correct answer is $\frac{8}{3}$

2.

$$\int_{-1}^8 x^2\,dx \cong 171.00 \qquad (n = 10)$$

Correct answer is 171

3.

$$\int_0^{2\pi} \frac{dx}{1 - \cos x + 0.25} \cong 8.22$$

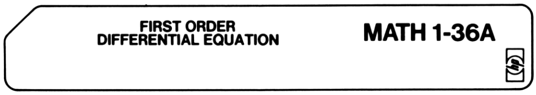
(n = 10, set machine to RAD mode)

Keys for f(x): [f] [COS] 1 [g] [x⇄y] [−] .25 [+] [g] [1/x]

Correct answer is $\frac{2\pi}{0.75}$

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2			GTO	A	
3	Switch to W/PRGM mode				
4	Enter f(x)		RTN		
5	Switch to RUN mode				
6		a	↑		
7		b	↑		
8	(n must be even)	n	RTN	R/S	
	(For different values of a, b or				
	n go to 6. For different				
	function f(x), go to 2.)				

FIRST ORDER DIFFERENTIAL EQUATION



This program may be used to solve a wide variety of first order differential equations of the form

$$y' = f(x, y)$$

with initial values x_0, y_0.

The solution is a numerical solution which calculates y_i for $x_i = x_0 + ih$ ($i = 1, 2, 3, \ldots$). h is an increment specified by the user.

The program uses the third-order Runge-Kutta method:

$$y_{i+1} = y_i + \frac{h}{6}(k_1 + 4k_2 + k_3)$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{h}{2}, y_i + \frac{h}{2} k_1\right)$$

$$k_3 = f(x_i + h, y_i + 2hk_2 - hk_1)$$

f(x, y) must be programmed in the calculator by the user. Assuming x, y are in X and Y registers, 37 memory locations, the stack registers and registers R_6, R_7, R_8, R_9 are available for f(x, y).

Example:

$$y' = \frac{x+1}{x} + \frac{2y}{x}$$

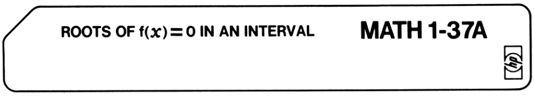
$$x_0 = 1, \quad y_0 = -\frac{1}{2}, \quad h = 0.5$$

Keys for f(x, y): STO 6 1 + g x⇄y 2 × + RCL 6 ÷

x	1.5	2	2.5	3	3.5	4	4.5	5
y	0.23	1.46	3.18	5.40	8.11	11.32	15.02	19.21

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2			GTO	B	
3	Switch to W/PRGM mode				
4	Enter f(x, y)		RTN		
5	Switch to RUN mode				
6		h	↑		
7		y_0	↑		
8		x_0	A		y_1
9	Perform 9 for i = 2, 3, ...		R/S		y_i
	(For a new set of initial condi-				
	tions, go to 6. For a new case,				
	go to 2.)				

ROOTS OF f(x)=0 IN AN INTERVAL



This program uses the principle of interval-halving to find real roots of an equation f(x) = 0 in a closed interval [a, b] where the equation may be algebraic (e.g., $5x^4 - 3x + 1 = 0$), rational (e.g., $x^{3/2} + \sqrt{x} - 2 = 0$), or transcendental (e.g., 3 cos x − 4x = 0).

The user specifies the continuous, real function f, an interval [a, b], an accuracy tolerance ϵ, and a search increment Δx. The program then begins at the left of the interval and compares the functional values at the ends of the interval $[a, a + \Delta x]$. If f(a) and $f(a + \Delta x)$ are of opposite sign, this interval will be searched for a root. Otherwise, or even after a root is found, the program proceeds in the same manner with the interval $[a + \Delta x, a + 2\Delta x]$, etc. At most one root will be found by the program for each of these small intervals.

Key in and store the function f(x) in the calculator assuming the value x is in the X register. 17 memory locations and the stack registers are available for f(x). Register R_9 is also available for temporary storage only.

Examples:

1. The real roots of $x^3 - 8x^2 + 5x + 14 = 0$ in the interval [–10, 10] using $\Delta x = 1$ and $\epsilon = 10^{-6}$ are –1.00, 2.00 and 7.00.

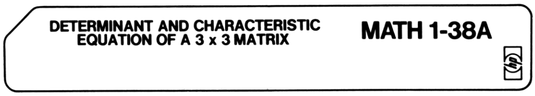
   Keys for f(x): [↑] [↑] [↑] 8 [–] [×] 5 [+] [×] 14 [+]

2. The real root of $x^{5/2} - 2\sqrt{x} = 0$ in the interval [1, 10] using $\Delta x = 1$ and $\epsilon = 10^{-6}$ is 1.41.

   Keys for f(x): [f] [√x] [↑] [↑] 5 [g] [y^x] [g] [x⇄y] 2 [×] [–]

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2			GTO	A	
3	Switch to W/PRGM mode				
4	Enter f(x)		RTN		
5	Switch to RUN mode	ϵ	STO	5	
6		Δx	STO	6	
7		a	↑		
8		b	RTN	R/S	root
9	Perform 9 until display = b + Δx		R/S		root
	(Display = b + Δx is an indication				
	of the end of search, b + Δx is				
	not necessarily a root.)				

DETERMINANT AND CHARACTERISTIC EQUATION OF A 3x3 MATRIX



$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

Matrix A has characteristic equation

$$|A - \lambda I| = -\lambda^3 + d_1\lambda^2 + d_2\lambda + d_3 = 0$$

where

$$d_1 = a_1 + b_2 + c_3$$

$$d_2 = a_3c_1 + a_2b_1 + b_3c_2 - a_1b_2 - a_1c_3 - b_2c_3$$

$$d_3 = |A|$$

$$= a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 - a_2b_1c_3 - a_1b_3c_2$$

Notes:
1. d_3 is the determinant of matrix A.
2. *Math 1–07A, Cubic Equation* can be used to find the eigenvalues.

Example:

$$\text{Matrix A} = \begin{pmatrix} 8 & 4 & 1 \\ 0 & 3 & 0 \\ 1 & 5 & -1 \end{pmatrix}$$

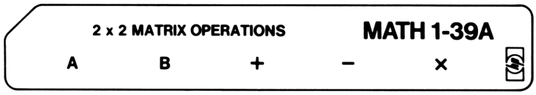
has characteristic equation

$$-\lambda^3 + 10\lambda^2 - 12\lambda - 27 = 0$$

Determinant of A = d_3 = –27.00

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2		a_1	↑		
3		a_2	↑		
4		a_3	A		
5		b_1	↑		
6		b_2	↑		
7		b_3	B		
8		c_1	↑		
9		c_2	↑		
10		c_3	C	R/S	d_1
11			R/S		d_2
12			R/S		d_3

2 x 2 MATRIX OPERATIONS



Suppose

$$A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}, \quad B = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$$

then

$$A + B = \begin{pmatrix} a_1 + b_1 & a_2 + b_2 \\ a_3 + b_3 & a_4 + b_4 \end{pmatrix} = \begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix}$$

$$A - B = \begin{pmatrix} a_1 - b_1 & a_2 - b_2 \\ a_3 - b_3 & a_4 - b_4 \end{pmatrix} = \begin{pmatrix} d_1 & d_2 \\ d_3 & d_4 \end{pmatrix}$$

$$AB = \begin{pmatrix} a_1 b_1 + a_2 b_3 & a_1 b_2 + a_2 b_4 \\ a_3 b_1 + a_4 b_3 & a_3 b_2 + a_4 b_4 \end{pmatrix} = \begin{pmatrix} e_1 & e_2 \\ e_3 & e_4 \end{pmatrix}$$

Example:

$$A = \begin{pmatrix} -1.2 & 5.8 \\ 7.31 & -4.39 \end{pmatrix} \qquad B = \begin{pmatrix} 10.21 & 15.8 \\ -9.33 & 7.24 \end{pmatrix}$$

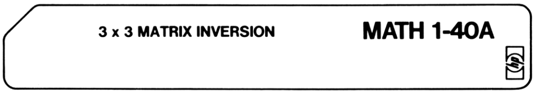
$$A + B = \begin{pmatrix} 9.01 & 21.60 \\ -2.02 & 2.85 \end{pmatrix}$$

$$A - B = \begin{pmatrix} -11.41 & -10.00 \\ 16.64 & -11.63 \end{pmatrix}$$

$$AB = \begin{pmatrix} -66.37 & 23.03 \\ 115.59 & 83.71 \end{pmatrix}$$

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program				
2		a_1	↑		
3		a_2	↑		
4		a_3	↑		
5		a_4	A		
6		b_1	↑		
7		b_2	↑		
8		b_3	↑		
9		b_4	B		
10	Compute A + B		C		c_1
11	Perform 11 for i = 2, 3, 4		R/S		c_i
12	Compute A – B		D		d_1
13	Perform 13 for i = 2, 3, 4		R/S		d_i
14	Compute AB		E		e_1
15	Perform 15 for i = 2, 3, 4		R/S		e_i

3 x 3 MATRIX INVERSION



$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

has an inverse

$$A^{-1} = \begin{pmatrix} \alpha_1 & \alpha_4 & \alpha_7 \\ \alpha_2 & \alpha_5 & \alpha_8 \\ \alpha_3 & \alpha_6 & \alpha_9 \end{pmatrix}$$

where
$$\begin{aligned}
\alpha_1 &= (b_2 c_3 - b_3 c_2) / \text{Det} \\
\alpha_2 &= (a_3 c_2 - a_2 c_3) / \text{Det} \\
\alpha_3 &= (a_2 b_3 - a_3 b_2) / \text{Det} \\
\alpha_4 &= (b_3 c_1 - b_1 c_3) / \text{Det} \\
\alpha_5 &= (a_1 c_3 - a_3 c_1) / \text{Det} \\
\alpha_6 &= (a_3 b_1 - a_1 b_3) / \text{Det} \\
\alpha_7 &= (b_1 c_2 - b_2 c_1) / \text{Det} \\
\alpha_8 &= (a_2 c_1 - a_1 c_2) / \text{Det} \\
\alpha_9 &= (a_1 b_2 - a_2 b_1) / \text{Det}
\end{aligned}$$

if determinant Det of A is non-zero.

Note: This program must be used in conjunction with *Math 1–38A, Determinant and Characteristic Equation of a 3 x 3 Matrix.*

Example:

$$A = \begin{pmatrix} -1 & 0 & 3 \\ 7 & 1 & -1 \\ 2 & 3 & 0 \end{pmatrix}$$

Det = 54.00

$$A^{-1} = \begin{pmatrix} 0.06 & 0.17 & -0.06 \\ -0.04 & -0.11 & 0.37 \\ 0.35 & 0.06 & -0.02 \end{pmatrix}$$

LINE	INSTRUCTIONS	DATA	KEYS		DISPLAY
1	Enter program *Math 1–38A*				
2		a_1	↑		
3		a_2	↑		
4		a_3	A		
5		b_1	↑		
6		b_2	↑		
7		b_3	B		
8		c_1	↑		
9		c_2	↑		
10		c_3	C	D	Det
11	Enter program *Math 1–40A*				
12			R/S		α_1
13	Perform 13 for i = 2, ..., 9		R/S		α_i

PROGRAM LISTINGS

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FACTORS OF AN INTEGER

CODE	KEYS
23	LBL
11	A
32	f^{-1}
51	SF1
33 01	STO 1
00	0
35 07	g x⇄y
35 22	g x≤y
22	GTO
05	5
41	↑
31	f
83	INT
35 21	g x≠y
22	GTO
05	5
02	2
43	EEX
09	9
34 01	RCL 1
35 24	g x>y
22	GTO
05	5
01	1
33 02	STO 2
02	2
33 03	STO 3
23	LBL
01	1
34 01	RCL 1
34 03	RCL 3
81	÷
34 03	RCL 3
35 24	g x>y
22	GTO
04	4
35 08	g R↓
31	f
83	INT
35 00	g LST X
35 23	g x=y
22	GTO
02	2
34 03	RCL 3
06	6
35 24	g x>y
31	f
51	SF1
02	2
34 02	RCL 2
61	+
33 02	STO 2
02	2
35 07	g x⇄y
32	f^{-1}
61	TF1
22	GTO
03	3
32	f^{-1}
51	SF1
33 03	STO 3
22	GTO
01	1
23	LBL
02	2
34 03	RCL 3
84	R/S
35 08	g R↓
33 01	STO 1
22	GTO
01	1
23	LBL
04	4
34 01	RCL 1
84	R/S
00	0
84	R/S
23	LBL
05	5
00	0
81	÷
23	LBL
03	3
61	+
33 02	STO 2
31	f
51	SF1
33 03	STO 3
22	GTO
01	1
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	Used	R_4		R_7	
R_2	Used	R_5		R_8	
R_3	Used	R_6		R_9	Used

GREATEST COMMON DIVISOR
LEAST COMMON MULTIPLE

CODE	KEYS
23	LBL
11	A
35 07	g x⇄y
33 01	STO 1
35 07	g x⇄y
33 02	STO 2
01	1
33 03	STO 3
44	CLX
33 04	STO 4
35 23	g x=y
22	GTO
02	2
33 03	STO 3
33 05	STO 5
01	1
33 04	STO 4
33 06	STO 6
23	LBL
01	1
15	E
35 23	g x=y
22	GTO
02	2
34 06	RCL 6
34 03	RCL 3
33 06	STO 6
34 07	RCL 7
71	x
61	+
33 03	STO 3
34 05	RCL 5
34 04	RCL 4
33 05	STO 5
34 07	RCL 7

CODE	KEYS
71	x
61	+
33 04	STO 4
22	GTO
01	1
23	LBL
02	2
34 01	RCL 1
35 24	g x>y
22	GTO
04	4
42	CHS
84	R/S
34 03	RCL 3
42	CHS
84	R/S
34 04	RCL 4
42	CHS
24	RTN
23	LBL
04	4
84	R/S
34 03	RCL 3
84	R/S
34 04	RCL 4
24	RTN
23	LBL
12	B
33 02	STO 2
35 07	g x⇄y
33 01	STO 1
71	x
33 03	STO 3
00	0
35 23	g x=y

CODE	KEYS
24	RTN
23	LBL
03	3
15	E
35 21	g x≠y
22	GTO
03	3
34 03	RCL 3
34 01	RCL 1
81	÷
35	g
06	ABS
24	RTN
23	LBL
15	E
34 01	RCL 1
34 01	RCL 1
34 02	RCL 2
33 01	STO 1
81	÷
31	f
83	INT
42	CHS
33 07	STO 7
34 02	RCL 2
71	x
61	+
33 02	STO 2
00	0
24	RTN

R_1	Used	R_4	Used	R_7	Used
R_2	Used	R_5	Used	R_8	
R_3	Used	R_6	Used	R_9	Used

ARITHMETIC AND HARMONIC PROGRESSIONS

CODE	KEYS
23	LBL
11	A
35 07	g x⇄y
33 01	STO 1
35 08	g R↓
41	↑
41	↑
41	↑
34 01	RCL 1
23	LBL
01	1
84	R/S
61	+
22	GTO
01	1
23	LBL
12	B
33 04	STO 4
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 01	STO 1
35 08	g R↓
35 08	g R↓
01	1
51	–
71	x
61	+
84	R/S
34 01	RCL 1
34 02	RCL 2
34 04	RCL 4
13	C
24	RTN
23	LBL

CODE	KEYS
13	C
33 01	STO 1
71	x
34 01	RCL 1
01	1
51	–
71	x
02	2
81	÷
35 07	g x⇄y
34 01	RCL 1
71	x
61	+
24	RTN
23	LBL
14	D
33 01	STO 1
35 08	g R↓
61	+
34 01	RCL 1
71	x
02	2
81	÷
24	RTN
23	LBL
15	E
33 03	STO 3
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 01	STO 1
34 03	RCL 3
34 02	RCL 2
34 01	RCL 1
34 02	RCL 2

CODE	KEYS
81	÷
84	R/S
23	LBL
02	2
44	CLX
61	+
61	+
41	↑
35	g
04	1/x
34 01	RCL 1
71	x
84	R/S
22	GTO
02	2
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	Used	R_4	Used	R_7	
R_2	Used	R_5		R_8	
R_3	Used	R_6		R_9	

GEOMETRIC PROGRESSION

CODE	KEYS
23	LBL
11	A
41	↑
41	↑
35 09	g R↑
23	LBL
01	1
84	R/S
71	x
22	GTO
01	1
23	LBL
12	B
35 08	g R↓
35 08	g R↓
33 03	STO 3
35 08	g R↓
35 08	g R↓
01	1
51	–
15	E
34 03	RCL 3
71	x
24	RTN
23	LBL
13	C
35 08	g R↓
33 04	STO 4
35 08	g R↓
33 03	STO 3
35 08	g R↓
35 08	g R↓
15	E
01	1
51	–

CODE	KEYS
34 03	RCL 3
71	x
34 04	RCL 4
01	1
51	–
81	÷
24	RTN
23	LBL
14	D
35	g
06	ABS
01	1
35 22	g x≤y
00	0
81	÷
35 00	g LST X
51	–
35 07	g x⇄y
35 08	g R↓
81	÷
24	RTN
23	LBL
15	E
33 01	STO 1
35 07	g x⇄y
33 02	STO 2
00	0
35 07	g x⇄y
35 24	g x>y
22	GTO
09	9
42	CHS
33 02	STO 2
34 01	RCL 1
02	2

CODE	KEYS
81	÷
32	f^{-1}
83	INT
00	0
35 23	g x=y
22	GTO
09	9
34 02	RCL 2
34 01	RCL 1
35	g
05	y^x
42	CHS
24	RTN
23	LBL
09	9
34 02	RCL 2
34 01	RCL 1
35	g
05	y^x
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	n	R_4	r	R_7	
R_2	Used	R_5		R_8	
R_3	a	R_6		R_9	Used

FUNCTIONS OF x AND y

CODE	KEYS
23	LBL
11	A
33 01	STO 1
35 07	g x⇄y
33 02	STO 2
00	0
35 07	g x⇄y
35 23	g x=y
35 08	g R↓
24	RTN
35 24	g x>y
22	GTO
01	1
42	CHS
33 02	STO 2
34 01	RCL 1
32	f^{-1}
83	INT
00	0
35 21	g x≠y
81	÷
24	RTN
34 01	RCL 1
02	2
81	÷
32	f^{-1}
83	INT
00	0
35 23	g x=y
22	GTO
01	1
34 02	RCL 2
34 01	RCL 1
35	g
05	y^x

CODE	KEYS
42	CHS
24	RTN
23	LBL
01	1
34 02	RCL 2
34 01	RCL 1
35	g
05	y^x
24	RTN
23	LBL
12	B
31	f
07	LN
35 07	g x⇄y
31	f
07	LN
35 07	g x⇄y
81	÷
24	RTN
23	LBL
13	C
33 01	STO 1
35 07	g x⇄y
33 02	STO 2
35 07	g x⇄y
81	÷
31	f
83	INT
34 01	RCL 1
71	x
34 02	RCL 2
35 07	g x⇄y
51	–
24	RTN
23	LBL

CODE	KEYS
14	D
33 01	STO 1
35 07	g x⇄y
35	g
03	n!
35 00	g LST X
34 01	RCL 1
51	–
35	g
03	n!
81	÷
24	RTN
23	LBL
15	E
14	D
34 01	RCL 1
35	g
03	n!
81	÷
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	x	R_4		R_7	
R_2	y	R_5		R_8	
R_3		R_6		R_9	Used

QUADRATIC EQUATION

CODE	KEYS
23	LBL
11	A
33 03	STO 3
35 08	g R↓
42	CHS
33 02	STO 2
35 07	g $x \rightleftarrows y$
41	↑
33 01	STO 1
61	+
81	÷
41	↑
32	f^{-1}
09	$\sqrt{x}$
34 03	RCL 3
34 01	RCL 1
81	÷
33 03	STO 3
51	–
24	RTN
23	LBL
12	B
31	f
09	$\sqrt{x}$
35 07	g $x \rightleftarrows y$
35	g
06	ABS
61	+
34 02	RCL 2
34 01	RCL 1
71	x
41	↑
35	g
06	ABS
81	÷

CODE	KEYS
71	x
84	R/S
34 03	RCL 3
35 07	g $x \rightleftarrows y$
81	÷
24	RTN
23	LBL
13	C
42	CHS
31	f
09	$\sqrt{x}$
35 07	g $x \rightleftarrows y$
84	R/S
35 07	g $x \rightleftarrows y$
24	RTN
23	LBL
14	D
01	1
34 01	RCL 1
34 02	RCL 2
24	RTN
23	LBL
15	E
01	1
34 07	RCL 7
34	RCL
09	9
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	a	R_4		R_7	
R_2	–b	R_5		R_8	
R_3	c, c/a	R_6		R_9	

CUBIC EQUATION

CODE	KEYS
23	LBL
11	A
00	0
33 04	STO 4
34 03	RCL 3
35	g
06	ABS
34 03	RCL 3
81	÷
33 06	STO 6
34 03	RCL 3
35	g
06	ABS
34 01	RCL 1
35	g
06	ABS
61	+
43	EEX
42	CHS
02	2
23	LBL
01	1
01	1
00	0
71	x
35 22	g x≤y
22	GTO
01	1
33 07	STO 7
23	LBL
02	2
34 07	RCL 7
01	1
00	0
81	÷

CODE	KEYS
33 07	STO 7
34 06	RCL 6
42	CHS
33 06	STO 6
23	LBL
03	3
34 07	RCL 7
34 06	RCL 6
71	x
34 04	RCL 4
61	+
33 05	STO 5
34 04	RCL 4
35 23	g x=y
22	GTO
04	4
34 05	RCL 5
34 01	RCL 1
61	+
34 05	RCL 5
71	x
34 02	RCL 2
61	+
34 05	RCL 5
71	x
34 03	RCL 3
61	+
33 08	STO 8
35	g
06	ABS
43	EEX
42	CHS
04	4
35 24	g x>y
22	GTO

CODE	KEYS
04	4
34 05	RCL 5
33 04	STO 4
34 08	RCL 8
34 06	RCL 6
71	x
00	0
35 24	g x>y
22	GTO
03	3
22	GTO
02	2
23	LBL
04	4
34 05	RCL 5
84	R/S
34 01	RCL 1
61	+
33 08	STO 8
34 05	RCL 5
71	x
34 02	RCL 2
61	+
33 03	STO 3
01	1
34 08	RCL 8
34 03	RCL 3
84	R/S
35 01	g NOP
35 01	g NOP

R_1	Used	R_4	Used	R_7	Used
R_2	Used	R_5	Used	R_8	Used
R_3	Used	R_6	Used	R_9	Used

FOURTH DEGREE POLYNOMIAL EQUATION

CODE	KEYS
33 08	STO 8
84	R/S
33 07	STO 7
84	R/S
33 06	STO 6
84	R/S
33 05	STO 5
23	LBL
11	A
34 07	RCL 7
42	CHS
33 01	STO 1
34 08	RCL 8
34 06	RCL 6
71	x
34 05	RCL 5
04	4
71	x
51	–
33 02	STO 2
34 07	RCL 7
04	4
71	x
34 08	RCL 8
32	f^{-1}
09	$\sqrt{x}$
51	–
34 05	RCL 5
71	x
34 06	RCL 6
32	f^{-1}
09	$\sqrt{x}$
51	–
33 03	STO 3
84	R/S

CODE	KEYS
23	LBL
12	B
02	2
33	STO
81	÷
08	8
34 04	RCL 4
02	2
81	÷
33 04	STO 4
32	f^{-1}
09	$\sqrt{x}$
34 05	RCL 5
51	–
31	f
09	$\sqrt{x}$
33 05	STO 5
00	0
35 23	g x=y
22	GTO
01	1
34 08	RCL 8
34 04	RCL 4
71	x
34 06	RCL 6
02	2
81	÷
51	–
34 05	RCL 5
81	÷
33 06	STO 6
22	GTO
02	2
23	LBL
01	1

CODE	KEYS
34 08	RCL 8
32	f^{-1}
09	$\sqrt{x}$
34 07	RCL 7
51	–
34 04	RCL 4
02	2
71	x
61	+
31	f
09	$\sqrt{x}$
33 06	STO 6
23	LBL
02	2
34 08	RCL 8
61	+
33 07	STO 7
34 04	RCL 4
34 05	RCL 5
61	+
33	STO
09	9
01	1
34 08	RCL 8
34 06	RCL 6
51	–
34 04	RCL 4
34 05	RCL 5
51	–
24	RTN

R_1	b_2	R_4	y_0, B	R_7	a_2, A + C
R_2	b_1	R_5	a_0, D	R_8	a_3, A
R_3	b_0	R_6	a_1, C	R_9	B + D

FIFTH DEGREE POLYNOMIAL EQUATION

CODE	KEYS
35	g
06	ABS
33 06	STO 6
35 00	g LST X
81	÷
33 08	STO 8
34 01	RCL 1
35	g
06	ABS
34 06	RCL 6
61	+
83	•
01	1
23	LBL
09	9
01	1
00	0
71	x
35 22	g x≤y
22	GTO
09	9
33 06	STO 6
23	LBL
01	1
34 06	RCL 6
01	1
00	0
81	÷
33 06	STO 6
34 08	RCL 8
42	CHS
33 08	STO 8
23	LBL
02	2
34 06	RCL 6

CODE	KEYS
34 08	RCL 8
71	x
34 07	RCL 7
61	+
33 07	STO 7
35 00	g LST X
35 23	g x=y
22	GTO
03	3
35 07	g x⇄y
41	↑
41	↑
41	↑
34 01	RCL 1
61	+
71	x
34 02	RCL 2
61	+
71	x
34 03	RCL 3
61	+
71	x
34 04	RCL 4
61	+
71	x
34 05	RCL 5
61	+
00	0
35 23	g x=y
22	GTO
03	3
44	CLX
34 08	RCL 8
71	x
00	0

CODE	KEYS
35 24	g x>y
22	GTO
02	2
22	GTO
01	1
23	LBL
03	3
34 07	RCL 7
84	R/S
34 01	RCL 1
61	+
33 08	STO 8
34 07	RCL 7
71	x
34 02	RCL 2
61	+
33 01	STO 1
34 07	RCL 7
71	x
34 03	RCL 3
61	+
33 06	STO 6
34 07	RCL 7
71	x
34 04	RCL 4
61	+
33 05	STO 5
34 01	RCL 1
33 07	STO 7
84	R/S

R_1	α_4	R_4	α_1	R_7	x_i
R_2	α_3	R_5	α_0	R_8	k
R_3	α_2	R_6	$\|\alpha_0\|, \Delta x$	R_9	Used

SIMULTANEOUS EQUATIONS IN TWO UNKNOWNS

CODE	KEYS
23	LBL
11	A
33 01	STO 1
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 03	STO 3
24	RTN
23	LBL
12	B
33 04	STO 4
35 08	g R↓
33 05	STO 5
35 08	g R↓
33 06	STO 6
34 03	RCL 3
35 09	g R↑
71	x
34 06	RCL 6
34 01	RCL 1
71	x
51	–
34 03	RCL 3
34 05	RCL 5
71	x
34 02	RCL 2
34 06	RCL 6
71	x
51	–
84	R/S
33 07	STO 7
81	÷
34 01	RCL 1
34 05	RCL 5
71	x

CODE	KEYS
34 02	RCL 2
34 04	RCL 4
71	x
51	–
34 07	RCL 7
81	÷
84	R/S
35 07	g x⇄y
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	e	R_4	f	R_7	D
R_2	b	R_5	d	R_8	
R_3	a	R_6	c	R_9	

SIMULTANEOUS EQUATIONS IN THREE UNKNOWNS

CODE	KEYS
35 09	g R↑
81	÷
33 03	STO 3
44	CLX
35 00	g LST X
81	÷
33 02	STO 2
44	CLX
35 00	g LST X
81	÷
33 01	STO 1
84	R/S
11	A
33	STO
09	9
35 07	g x⇄y
33 08	STO 8
34 04	RCL 4
33 07	STO 7
84	R/S
11	A
33 06	STO 6
35 07	g x⇄y
33 05	STO 5
84	R/S
41	↑
34 03	RCL 3
34 04	RCL 4
71	x
35 07	g x⇄y
51	–
33 04	STO 4
34 03	RCL 3
34 07	RCL 7
71	x

CODE	KEYS
35 09	g R↑
51	–
33 07	STO 7
34 08	RCL 8
34 06	RCL 6
71	x
34	RCL
09	9
34 05	RCL 5
71	x
51	–
84	R/S
34 07	RCL 7
34 06	RCL 6
71	x
34	RCL
09	9
34 04	RCL 4
71	x
51	–
35 07	g x⇄y
81	÷
33 06	STO 6
44	CLX
34 08	RCL 8
34 04	RCL 4
71	x
34 07	RCL 7
34 05	RCL 5
71	x
51	–
35 07	g x⇄y
81	÷
33 08	STO 8
34 02	RCL 2

CODE	KEYS
71	x
34 01	RCL 1
34 06	RCL 6
71	x
61	+
34 03	RCL 3
35 07	g x⇄y
51	–
84	R/S
34 06	RCL 6
84	R/S
34 08	RCL 8
84	R/S
23	LBL
11	A
33 06	STO 6
35 08	g R↓
33 05	STO 5
35 08	g R↓
33 04	STO 4
34 01	RCL 1
71	x
34 05	RCL 5
51	–
34 04	RCL 4
34 02	RCL 2
71	x
34 06	RCL 6
51	–
24	RTN

R_1	b_1/a_1	R_4	Used	R_7	Used
R_2	c_1/a_1	R_5	Used	R_8	a, z
R_3	d_1/a_1	R_6	Used	R_9	b

SYNTHETIC DIVISION

CODE	KEYS
23	LBL
11	A
33 04	STO 4
35 08	g R↓
33 03	STO 3
35 08	g R↓
33 02	STO 2
35 08	g R↓
33	STO
09	9
84	R/S
33 08	STO 8
35 08	g R↓
33 07	STO 7
35 08	g R↓
33 06	STO 6
35 08	g R↓
33 05	STO 5
24	RTN
23	LBL
12	B
33 01	STO 1
34	RCL
09	9
84	R/S
71	x
34 02	RCL 2
61	+
84	R/S
34 01	RCL 1
71	x
34 03	RCL 3
61	+
84	R/S
34 01	RCL 1

CODE	KEYS
71	x
34 04	RCL 4
61	+
84	R/S
34 01	RCL 1
71	x
34 05	RCL 5
61	+
84	R/S
34 01	RCL 1
71	x
34 06	RCL 6
61	+
84	R/S
34 01	RCL 1
71	x
34 07	RCL 7
61	+
84	R/S
34 01	RCL 1
71	x
34 08	RCL 8
61	+
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	x_0	R_4	a_4	R_7	a_1
R_2	a_6	R_5	a_3	R_8	a_0
R_3	a_5	R_6	a_2	R_9	a_7

RECTANGULAR, SPHERICAL CONVERSIONS

CODE	KEYS
23	LBL
11	A
33 01	STO 1
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 03	STO 3
32	f^{-1}
09	$\sqrt{x}$
35 09	g R↑
32	f^{-1}
09	$\sqrt{x}$
61	+
34 01	RCL 1
32	f^{-1}
09	$\sqrt{x}$
61	+
31	f
09	$\sqrt{x}$
33 04	STO 4
34 03	RCL 3
35 07	g x⇄y
81	÷
32	f^{-1}
05	COS
34 02	RCL 2
34 01	RCL 1
31	f
01	R→P
35 08	g R↓
34 04	RCL 4
84	R/S
35 08	g R↓
84	R/S
35 08	g R↓

CODE	KEYS
84	R/S
35 08	g R↓
35 08	g R↓
24	RTN
23	LBL
12	B
33 01	STO 1
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 03	STO 3
31	f
05	COS
34 01	RCL 1
71	x
34 02	RCL 2
31	f
04	SIN
34 03	RCL 3
31	f
04	SIN
34 01	RCL 1
71	x
71	x
35 00	g LST X
34 02	RCL 2
31	f
05	COS
71	x
84	R/S
35 08	g R↓
84	R/S
35 08	g R↓
84	R/S
35 08	g R↓

CODE	KEYS
35 08	g R↓
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	x, r	R_4	r	R_7	
R_2	y, ϕ	R_5		R_8	
R_3	z, θ	R_6		R_9	Used

TRANSLATION AND/OR ROTATION OF COORDINATE AXES

CODE	KEYS
23	LBL
11	A
33 01	STO 1
35 07	g x⇄y
33 02	STO 2
24	RTN
23	LBL
12	B
35 07	g x⇄y
34 02	RCL 2
51	–
35 07	g x⇄y
34 01	RCL 1
51	–
24	RTN
23	LBL
13	C
33 03	STO 3
24	RTN
23	LBL
14	D
33 04	STO 4
35 07	g x⇄y
33 05	STO 5
34 03	RCL 3
31	f
05	COS
71	x
35 07	g x⇄y
42	CHS
34 03	RCL 3
31	f
04	SIN
71	x
61	+

CODE	KEYS
34 03	RCL 3
31	f
05	COS
34 04	RCL 4
71	x
34 03	RCL 3
31	f
04	SIN
34 05	RCL 5
71	x
61	+
24	RTN
23	LBL
15	E
12	B
14	D
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	x_0	R_4	x	R_7	
R_2	y_0	R_5	y	R_8	
R_3	θ	R_6		R_9	Used

ANGLE CONVERSIONS

CODE	KEYS
23	LBL
11	A
33 01	STO 1
31	f
61	TF 1
22	GTO
00	0
15	E
34 01	RCL 1
24	RTN
23	LBL
00	0
35 07	g x⇄y
32	f^{-1}
51	SF1
24	RTN
23	LBL
12	B
33 01	STO 1
01	1
08	8
00	0
35	g
02	π
81	÷
31	f
61	TF1
22	GTO
01	1
71	x
15	E
34 01	RCL 1
24	RTN
23	LBL
01	1

CODE	KEYS
35 07	g x⇄y
35 08	g R↓
81	÷
32	f^{-1}
51	SF1
24	RTN
23	LBL
13	C
33 01	STO 1
83	•
09	9
31	f
61	TF1
22	GTO
02	2
71	x
15	E
34 01	RCL 1
24	RTN
23	LBL
02	2
35 07	g x⇄y
35 08	g R↓
81	÷
32	f^{-1}
51	SF1
24	RTN
23	LBL
14	D
33 01	STO 1
09	9
41	↑
01	1
06	6
00	0

CODE	KEYS
81	÷
31	f
61	TF1
22	GTO
03	3
71	x
15	E
34 01	RCL 1
24	RTN
23	LBL
03	3
35 07	g x⇄y
35 08	g R↓
81	÷
32	f^{-1}
51	SF1
24	RTN
23	LBL
15	E
31	f
51	SF1
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1 x	R_4	R_7
R_2	R_5	R_8
R_3	R_6	R_9

SECONDARY VALUES OF SIN^{-1}, COS^{-1}, TAN^{-1}

CODE	KEYS
23	LBL
11	A
35	g
41	DEG
32	f^{-1}
04	SIN
01	1
08	8
00	0
35 07	g x⇄y
51	–
14	D
24	RTN
23	LBL
12	B
35	g
41	DEG
32	f^{-1}
05	COS
03	3
06	6
00	0
35 07	g x⇄y
51	–
14	D
24	RTN
23	LBL
13	C
35	g
41	DEG
32	f^{-1}
06	TAN
01	1
08	8
00	0

CODE	KEYS
61	+
14	D
24	RTN
23	LBL
14	D
31	f
61	TF1
22	GTO
03	3
31	f
81	TF2
22	GTO
02	2
24	RTN
23	LBL
02	2
83	•
09	9
81	÷
35	g
43	GRD
24	RTN
23	LBL
03	3
35	g
02	π
71	x
01	1
08	8
00	0
81	÷
35	g
42	RAD
24	RTN
23	LBL

CODE	KEYS
15	E
32	f^{-1}
51	SF1
32	f^{-1}
71	SF2
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	R_4	R_7
R_2	R_5	R_8
R_3	R_6	R_9 Used

TRIGONOMETRIC FUNCTIONS

CODE	KEYS
23	LBL
11	A
31	f
61	TF1
22	GTO
01	1
31	f
06	TAN
35	g
04	$1/x$
24	RTN
23	LBL
12	B
31	f
61	TF1
22	GTO
02	2
31	f
05	COS
35	g
04	$1/x$
24	RTN
23	LBL
13	C
31	f
61	TF1
22	GTO
03	3
31	f
04	SIN
35	g
04	$1/x$
24	RTN
23	LBL
01	1

CODE	KEYS
35	g
04	$1/x$
32	f^{-1}
06	TAN
15	E
24	RTN
23	LBL
02	2
35	g
04	$1/x$
32	f^{-1}
05	COS
15	E
24	RTN
23	LBL
03	3
35	g
04	$1/x$
32	f^{-1}
04	SIN
15	E
24	RTN
23	LBL
14	D
31	f
51	SF1
24	RTN
23	LBL
15	E
32	f^{-1}
51	SF1
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	R_4	R_7
R_2	R_5	R_8
R_3	R_6	R_9 Used

HYPERBOLIC FUNCTIONS

CODE	KEYS
23	LBL
11	A
32	f^{-1}
07	LN
41	↑
35	g
04	$1/x$
51	−
02	2
81	÷
24	RTN
23	LBL
12	B
32	f^{-1}
07	LN
41	↑
35	g
04	$1/x$
61	+
02	2
81	÷
24	RTN
23	LBL
13	C
32	f^{-1}
07	LN
33	STO
09	9
41	↑
35	g
04	$1/x$
51	−
34	RCL
09	9
35 00	g LST X

CODE	KEYS
61	+
81	÷
24	RTN
23	LBL
14	D
35	g
04	$1/x$
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	R_4	R_7
R_2	R_5	R_8
R_3	R_6	R_9 Used

INVERSE HYPERBOLIC FUNCTIONS

CODE	KEYS
23	LBL
11	A
41	↑
32	f^{-1}
09	$\sqrt{x}$
01	1
61	+
31	f
09	$\sqrt{x}$
61	+
31	f
07	LN
24	RTN
23	LBL
12	B
41	↑
32	f^{-1}
09	$\sqrt{x}$
01	1
51	–
31	f
09	$\sqrt{x}$
61	+
31	f
07	LN
24	RTN
23	LBL
13	C
41	↑
01	1
35 07	g x⇄y
61	+
01	1
35 00	g LST X
51	–

CODE	KEYS
81	÷
31	f
07	LN
02	2
81	÷
24	RTN
23	LBL
14	D
35	g
04	$^1/x$
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	R_4	R_7
R_2	R_5	R_8
R_3	R_6	R_9

SOLUTION OF A TRIANGLE
(GIVEN a, b, c, or a, b, C)

CODE	KEYS
23	LBL
11	A
33 03	STO 3
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 01	STO 1
32	f^{-1}
09	$\sqrt{x}$
34 02	RCL 2
32	f^{-1}
09	$\sqrt{x}$
61	+
34 03	RCL 3
32	f^{-1}
09	$\sqrt{x}$
51	–
02	2
81	÷
34 01	RCL 1
34 02	RCL 2
71	x
81	÷
32	f^{-1}
05	COS
84	R/S
31	f
04	SIN
34 03	RCL 3
81	÷
33 03	STO 3
34 02	RCL 2
71	x
32	f^{-1}
04	SIN

CODE	KEYS
84	R/S
34 03	RCL 3
34 01	RCL 1
71	x
32	f^{-1}
04	SIN
24	RTN
23	LBL
12	B
33 03	STO 3
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 01	STO 1
32	f^{-1}
09	$\sqrt{x}$
34 02	RCL 2
32	f^{-1}
09	$\sqrt{x}$
61	+
34 01	RCL 1
34 02	RCL 2
71	x
34 03	RCL 3
31	f
05	COS
71	x
02	2
71	x
51	–
31	f
09	$\sqrt{x}$
84	R/S
34 03	RCL 3
31	f

CODE	KEYS
04	SIN
35 07	g x⇄y
81	÷
33 03	STO 3
34 01	RCL 1
71	x
32	f^{-1}
04	SIN
84	R/S
34 03	RCL 3
34 02	RCL 2
71	x
32	f^{-1}
04	SIN
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	a	R_4		R_7	
R_2	b	R_5		R_8	
R_3	c or C	R_6		R_9	Used

SOLUTION OF A TRIANGLE
(GIVEN a, A, C or a, B, C)

CODE	KEYS
23	LBL
11	A
35	g
41	DEG
33 01	STO 1
35 08	g R↓
33 02	STO 2
35 09	g R↑
61	+
01	1
08	8
00	0
35 07	g x⇄y
51	−
84	R/S
33 03	STO 3
31	f
04	SIN
71	x
34 02	RCL 2
31	f
04	SIN
81	÷
84	R/S
34 01	RCL 1
31	f
04	SIN
71	x
34 03	RCL 3
31	f
04	SIN
81	÷
24	RTN
23	LBL
12	B

CODE	KEYS
35	g
41	DEG
33 02	STO 2
35 08	g R↓
33 01	STO 1
35 08	g R↓
33 03	STO 3
34 02	RCL 2
31	f
04	SIN
71	x
01	1
08	8
00	0
34 02	RCL 2
34 01	RCL 1
61	+
51	−
84	R/S
31	f
04	SIN
33 02	STO 2
81	÷
84	R/S
34 01	RCL 1
31	f
04	SIN
34 03	RCL 3
71	x
34 02	RCL 2
81	÷
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	Used	R_4		R_7	
R_2	Used	R_5		R_8	
R_3	Used	R_6		R_9	Used

SOLUTION OF A TRIANGLE (GIVEN B, b, c)

CODE	KEYS
23	LBL
11	A
32	f^{-1}
51	SF1
32	f^{-1}
71	SF2
24	RTN
23	LBL
12	B
33 01	STO 1
35 08	g R↓
33 04	STO 4
35 08	g R↓
33 02	STO 2
31	f
04	SIN
34 01	RCL 1
71	x
34 04	RCL 4
81	÷
32	f^{-1}
04	SIN
84	R/S
33 03	STO 3
23	LBL
05	5
02	2
00	0
00	0
41	↑
01	1
08	8
00	0
31	f
61	TF1

CODE	KEYS
35	g
02	π
31	f
81	TF2
35 08	g R↓
35 01	g NOP
34 02	RCL 2
34 03	RCL 3
61	+
35 07	g x⇄y
51	−
42	CHS
84	R/S
35 00	g LST X
35 07	g x⇄y
31	f
04	SIN
34 01	RCL 1
71	x
34 03	RCL 3
31	f
04	SIN
81	÷
84	R/S
44	CLX
34 03	RCL 3
51	−
84	R/S
33 03	STO 3
22	GTO
05	5
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	c	R_4	b	R_7	
R_2	B	R_5		R_8	
R_3	C	R_6		R_9	Used

SPHERICAL TRIANGLES

CODE	KEYS
44	CLX
32	f^{-1}
51	SF 1
84	R/S
33 03	STO 3
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 01	STO 1
44	CLX
84	R/S
23	LBL
13	C
31	f
51	SF 1
84	R/S
23	LBL
11	A
34 01	RCL 1
34 02	RCL 2
31	f
04	SIN
32	f^{-1}
01	R→P
34 03	RCL 3
31	f
04	SIN
71	x
34 02	RCL 2
31	f
05	COS
34 03	RCL 3
31	f
05	COS
71	x

CODE	KEYS
31	f
61	TF 1
42	CHS
35 01	g NOP
61	+
32	f^{-1}
05	COS
33 01	STO 1
23	LBL
12	B
14	D
33 04	STO 4
14	D
33 05	STO 5
14	D
33 06	STO 6
34 01	RCL 1
84	R/S
34 02	RCL 2
84	R/S
34 03	RCL 3
84	R/S
34 04	RCL 4
84	R/S
34 05	RCL 5
84	R/S
34 06	RCL 6
24	RTN
23	LBL
14	D
34 01	RCL 1
34 02	RCL 2
33 01	STO 1
34 03	RCL 3
33 02	STO 2

CODE	KEYS
35 09	g R↑
35 09	g R↑
33 03	STO 3
31	f
05	COS
34 01	RCL 1
31	f
05	COS
34 02	RCL 2
31	f
05	COS
71	x
31	f
61	TF 1
42	CHS
35 01	g NOP
51	–
34 01	RCL 1
31	f
04	SIN
81	÷
34 02	RCL 2
31	f
04	SIN
81	÷
32	f^{-1}
05	COS
24	RTN
35 01	g NOP
35 01	g NOP

R_1	Used	R_4	A or a	R_7	
R_2	Used	R_5	B or b	R_8	
R_3	Used	R_6	C or c	R_9	Used

AREA OF A TRIANGLE

CODE	KEYS
23	LBL
11	A
14	D
35 07	g x⇄y
35 08	g R↓
61	+
61	+
02	2
81	÷
41	↑
41	↑
41	↑
34 01	RCL 1
51	−
71	x
35 07	g x⇄y
34 02	RCL 2
51	−
71	x
35 07	g x⇄y
34 03	RCL 3
51	−
71	x
31	f
09	$\sqrt{x}$
24	RTN
23	LBL
12	B
31	f
04	SIN
71	x
71	x
02	2
81	÷
24	RTN

CODE	KEYS
23	LBL
13	C
33 01	STO 1
35 07	g x⇄y
31	f
04	SIN
35 00	g LST X
35 08	g R↓
35 07	g x⇄y
31	f
04	SIN
71	x
35 07	g x⇄y
32	f^{-1}
09	$\sqrt{x}$
71	x
02	2
81	÷
35 07	g x⇄y
34 01	RCL 1
61	+
31	f
04	SIN
81	÷
24	RTN
23	LBL
14	D
33 03	STO 3
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 01	STO 1
24	RTN
23	LBL
15	E

CODE	KEYS
33 06	STO 6
35 08	g R↓
33 05	STO 5
35 08	g R↓
33 04	STO 4
34 06	RCL 6
51	−
34 02	RCL 2
71	x
34 06	RCL 6
34 05	RCL 5
51	−
34 01	RCL 1
71	x
61	+
34 05	RCL 5
34 04	RCL 4
51	−
34 03	RCL 3
71	x
61	+
02	2
81	÷
35	g
06	ABS
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	Used	R_4	Used	R_7	
R_2	Used	R_5	Used	R_8	
R_3	Used	R_6	Used	R_9	Used

AREA OF A POLYGON

CODE	KEYS
23	LBL
11	A
33 04	STO 4
35 08	g R↓
33 03	STO 3
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 01	STO 1
34 03	RCL 3
61	+
34 02	RCL 2
34 04	RCL 4
51	−
71	x
33 05	STO 5
34 04	RCL 4
34 03	RCL 3
23	LBL
01	1
84	R/S
33 04	STO 4
35 07	g x⇄y
33 03	STO 3
15	E
22	GTO
01	1
23	LBL
12	B
33 04	STO 4
35 07	g x⇄y
33 03	STO 3
15	E
34 02	RCL 2
34 01	RCL 1

CODE	KEYS
15	E
34 05	RCL 5
02	2
81	÷
24	RTN
23	LBL
15	E
35 07	g x⇄y
35 08	g R↓
61	+
33 06	STO 6
35 08	g R↓
35 07	g x⇄y
51	−
34 06	RCL 6
71	x
33	STO
61	+
05	5
34 04	RCL 4
34 03	RCL 3
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	x_1	R_4	Used	R_7	
R_2	y_1	R_5	Used	R_8	
R_3	Used	R_6	Used	R_9	

CIRCLE DETERMINED BY THREE POINTS

CODE	KEYS
23	LBL
11	A
33 02	STO 2
35 07	g x⇄y
33 01	STO 1
24	RTN
23	LBL
12	B
33 04	STO 4
35 07	g x⇄y
33 03	STO 3
15	E
33 07	STO 7
14	D
33 08	STO 8
24	RTN
23	LBL
13	C
33 04	STO 4
35 07	g x⇄y
33 03	STO 3
15	E
33 05	STO 5
14	D
33 06	STO 6
34 08	RCL 8
51	–
35 07	g x⇄y
34 07	RCL 7
51	–
35 07	g x⇄y
81	÷
33 02	STO 2
34 06	RCL 6
71	x

CODE	KEYS
34 05	RCL 5
35 07	g x⇄y
51	–
84	R/S
33 01	STO 1
34 02	RCL 2
84	R/S
34 04	RCL 4
51	–
35 07	g x⇄y
34 03	RCL 3
51	–
31	f
01	R→P
84	R/S
23	LBL
14	D
34 04	RCL 4
34 02	RCL 2
51	–
34 03	RCL 3
34 01	RCL 1
51	–
81	÷
24	RTN
23	LBL
15	E
34 03	RCL 3
34 01	RCL 1
51	–
34 03	RCL 3
34 01	RCL 1
61	+
71	x
34 04	RCL 4

CODE	KEYS
34 02	RCL 2
51	–
34 04	RCL 4
34 02	RCL 2
61	+
71	x
61	+
34 03	RCL 3
34 01	RCL 1
51	–
02	2
71	x
81	÷
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	x_1, x_0	R_4	y_2, y_3	R_7	K_1
R_2	y_1, y_0	R_5	K_2	R_8	N_1
R_3	x_2, x_3	R_6	N_2	R_9	Used

EQUALLY SPACED POINTS ON A CIRCLE

CODE	KEYS
23	LBL
11	A
35	g
02	π
02	2
71	x
33 01	STO 1
04	4
00	0
00	0
33 02	STO 2
03	3
06	6
00	0
33 05	STO 5
43	EEX
02	2
31	f
04	SIN
01	1
35 23	g x=y
34 02	RCL 2
33 05	STO 5
44	CLX
35 24	g x>y
34 01	RCL 1
33 05	STO 5
84	R/S
41	↑
34 05	RCL 5
35 07	g x⇄y
81	÷
33 05	STO 5
35 08	g R↓
33 02	STO 2

CODE	KEYS
35 08	g R↓
33 01	STO 1
84	R/S
33 03	STO 3
35 07	g x⇄y
33 04	STO 4
34 01	RCL 1
23	LBL
01	1
34 02	RCL 2
32	f^{-1}
01	R→P
34 03	RCL 3
61	+
84	R/S
35 07	g x⇄y
34 04	RCL 4
61	+
84	R/S
34 01	RCL 1
34 05	RCL 5
61	+
33 01	STO 1
22	GTO
01	1
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	$2\pi, \theta$	R_4	y_0	R_7	
R_2	400, r	R_5	360, c	R_8	
R_3	x_0	R_6		R_9	Used

POLYGONS INSCRIBED IN AND CIRCUMSCRIBED ABOUT A CIRCLE

CODE	KEYS
23	LBL
11	A
33 03	STO 3
01	1
08	8
00	0
35 07	g x⇄y
81	÷
33 01	STO 1
35 07	g x⇄y
33 02	STO 2
35	g
41	DEG
24	RTN
23	LBL
12	B
34 01	RCL 1
31	f
04	SIN
34 02	RCL 2
71	x
02	2
71	x
24	RTN
23	LBL
13	C
34 01	RCL 1
02	2
71	x
31	f
04	SIN
34 02	RCL 2
32	f^{-1}
09	$\sqrt{x}$
71	x

CODE	KEYS
34 03	RCL 3
71	x
02	2
81	÷
24	RTN
23	LBL
14	D
34 01	RCL 1
31	f
06	TAN
34 02	RCL 2
71	x
02	2
71	x
24	RTN
23	LBL
15	E
34 01	RCL 1
31	f
06	TAN
34 02	RCL 2
32	f^{-1}
09	$\sqrt{x}$
71	x
34 03	RCL 3
71	x
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	180/n	R_4		R_7	
R_2	r	R_5		R_8	
R_3	n	R_6		R_9	Used

UNIT CONVERSIONS:
C→F; ft, in→cm; lb→kg

CODE	KEYS
23	LBL
11	A
31	f
61	TF 1
22	GTO
01	1
41	↑
01	1
83	•
08	8
71	x
03	3
02	2
61	+
24	RTN
23	LBL
01	1
41	↑
03	3
02	2
51	—
01	1
83	•
08	8
81	÷
15	E
24	RTN
23	LBL
12	B
31	f
61	TF 1
22	GTO
02	2
35 07	g x⇄y
01	1

CODE	KEYS
02	2
71	x
61	+
02	2
83	•
05	5
04	4
71	x
24	RTN
23	LBL
02	2
41	↑
03	3
00	0
83	•
04	4
08	8
81	÷
41	↑
31	f
83	INT
33 01	STO 1
51	—
01	1
02	2
71	x
34 01	RCL 1
15	E
84	R/S
35 07	g x⇄y
24	RTN
23	LBL
13	C
41	↑
83	•

CODE	KEYS
04	4
05	5
03	3
05	5
09	9
02	2
03	3
07	7
31	f
61	TF 1
22	GTO
03	3
71	x
24	RTN
23	LBL
03	3
81	÷
15	E
24	RTN
23	LBL
14	D
31	f
51	SF 1
24	RTN
23	LBL
15	E
32	f^{-1}
51	SF 1
24	RTN
35 01	g NOP

R_1	Used	R_4		R_7	
R_2		R_5		R_8	
R_3		R_6		R_9	

UNIT CONVERSIONS:
mi→km; gal→ltr; yd→m; ac→ft^2

CODE	KEYS
23	LBL
11	A
41	↑
01	1
83	•
06	6
00	0
09	9
03	3
04	4
04	4
22	GTO
02	2
23	LBL
12	B
41	↑
03	3
83	•
07	7
08	8
05	5
04	4
01	1
01	1
07	7
08	8
04	4
22	GTO
02	2
23	LBL
13	C
41	↑
83	•
09	9
01	1

CODE	KEYS
04	4
04	4
22	GTO
02	2
23	LBL
14	D
41	↑
04	4
03	3
05	5
06	6
00	0
23	LBL
02	2
31	f
61	TF1
22	GTO
01	1
71	x
24	RTN
23	LBL
15	E
31	f
51	SF1
24	RTN
23	LBL
01	1
81	÷
32	f^{-1}
51	SF1
84	R/S
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	R_4	R_7
R_2	R_5	R_8
R_3	R_6	R_9

POLYNOMIAL EVALUATION (REAL)

CODE	KEYS
23	LBL
11	A
31	f
43	REG
33 01	STO 1
84	R/S
33 02	STO 2
84	R/S
33 03	STO 3
84	R/S
33 04	STO 4
84	R/S
33 05	STO 5
84	R/S
33 06	STO 6
84	R/S
33 07	STO 7
84	R/S
33 08	STO 8
84	R/S
33	STO
09	9
84	R/S
23	LBL
12	B
41	↑
41	↑
41	↑
34	RCL
09	9
71	x
34 08	RCL 8
61	+
71	x
34 07	RCL 7

CODE	KEYS
61	+
71	x
34 06	RCL 6
61	+
71	x
34 05	RCL 5
61	+
71	x
34 04	RCL 4
61	+
71	x
34 03	RCL 3
61	+
71	x
34 02	RCL 2
61	+
71	x
34 01	RCL 1
61	+
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	a_0	R_4	a_3	R_7	a_6
R_2	a_1	R_5	a_4	R_8	a_7
R_3	a_2	R_6	a_5	R_9	a_8

LINEAR AND LAGRANGIAN INTERPOLATIONS

CODE	KEYS
33 04	STO 4
35 08	g R↓
33 03	STO 3
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 01	STO 1
84	R/S
23	LBL
12	B
33 05	STO 5
34 01	RCL 1
51	—
34 04	RCL 4
71	x
34 03	RCL 3
34 05	RCL 5
51	—
34 02	RCL 2
71	x
61	+
34 03	RCL 3
34 01	RCL 1
51	—
81	÷
24	RTN
23	LBL
13	C
33 03	STO 3
35 07	g x⇄y
33 02	STO 2
51	—
35 07	g x⇄y
33 01	STO 1
34 03	RCL 3

CODE	KEYS
35 07	g x⇄y
51	—
71	x
33 06	STO 6
24	RTN
23	LBL
14	D
41	↑
34 06	RCL 6
81	÷
33 06	STO 6
44	CLX
34 02	RCL 2
34 01	RCL 1
51	—
81	÷
34 02	RCL 2
34 03	RCL 3
51	—
81	÷
33 05	STO 5
44	CLX
34 01	RCL 1
34 02	RCL 2
51	—
81	÷
34 01	RCL 1
34 03	RCL 3
51	—
81	÷
33 04	STO 4
24	RTN
23	LBL
15	E
33 07	STO 7

CODE	KEYS
34 02	RCL 2
51	—
34 07	RCL 7
34 03	RCL 3
51	—
71	x
34 04	RCL 4
71	x
34 07	RCL 7
34 01	RCL 1
51	—
34 07	RCL 7
34 03	RCL 3
51	—
71	x
34 05	RCL 5
71	x
61	+
34 07	RCL 7
34 01	RCL 1
51	—
34 07	RCL 7
34 02	RCL 2
51	—
71	x
34 06	RCL 6
71	x
61	+
24	RTN
35 01	g NOP

R_1	Used	R_4	Used	R_7	Used
R_2	Used	R_5	Used	R_8	
R_3	Used	R_6	Used	R_9	

FINITE DIFFERENCE INTERPOLATION

CODE	KEYS
23	LBL
11	A
33 04	STO 4
35 08	g R↓
33 03	STO 3
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 01	STO 1
24	RTN
23	LBL
12	B
33 08	STO 8
35 08	g R↓
33	STO
09	9
34 03	RCL 3
34 02	RCL 2
51	—
33 05	STO 5
34 03	RCL 3
61	+
34 04	RCL 4
35 07	g x⇄y
51	—
33 06	STO 6
34 03	RCL 3
51	—
34 02	RCL 2
02	2
71	x
61	+
34 01	RCL 1
51	—
33 07	STO 7

CODE	KEYS
24	RTN
23	LBL
13	C
41	↑
34	RCL
09	9
51	—
34 08	RCL 8
81	÷
33 01	STO 1
34 05	RCL 5
71	x
34 03	RCL 3
61	+
34 01	RCL 1
34 01	RCL 1
01	1
61	+
71	x
33 02	STO 2
02	2
81	÷
34 06	RCL 6
71	x
61	+
34 02	RCL 2
34 01	RCL 1
01	1
51	—
71	x
06	6
81	÷
34 07	RCL 7
71	x
61	+

CODE	KEYS
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	y_1, u	R_4	y_4	R_7	$\delta^3 y_{-1/2}$
R_2	y_2, $(u+1)u$	R_5	$\delta y_{-1/2}$	R_8	h
R_3	y_3	R_6	$\delta^2 y_0$	R_9	x_3

NUMERICAL INTEGRATION (DISCRETE CASE)

CODE	KEYS
23	LBL
11	A
33 02	STO 2
35 07	g x⇄y
33 01	STO 1
24	RTN
23	LBL
12	B
41	↑
61	+
33	STO
61	+
02	2
24	RTN
23	LBL
13	C
41	↑
34 02	RCL 2
61	+
34 01	RCL 1
71	x
02	2
81	÷
24	RTN
23	LBL
14	D
41	↑
61	+
35 07	g x⇄y
04	4
71	x
61	+
33	STO
61	+
02	2

CODE	KEYS
24	RTN
23	LBL
15	E
35 07	g x⇄y
04	4
71	x
61	+
34 02	RCL 2
61	+
34 01	RCL 1
71	x
03	3
81	÷
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	h	R_4		R_7	
R_2	Used	R_5		R_8	
R_3		R_6		R_9	

SIMPSON'S RULE FOR NUMERICAL INTEGRATION

CODE	KEYS
33 03	STO 3
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 01	STO 1
35 09	g R↑
35 07	g x⇄y
51	−
34 03	RCL 3
81	÷
33 04	STO 4
34 01	RCL 1
11	A
33 05	STO 5
34 03	RCL 3
02	2
81	÷
01	1
51	−
33 08	STO 8
41	↑
31	f
83	INT
35 23	g x=y
22	GTO
01	1
00	0
81	÷
84	R/S
23	LBL
01	1
34 01	RCL 1
34 04	RCL 4
61	+
33 01	STO 1

CODE	KEYS
11	A
04	4
71	x
33	STO
61	+
05	5
34 01	RCL 1
34 04	RCL 4
61	+
33 01	STO 1
11	A
02	2
71	x
33	STO
61	+
05	5
35	g
83	DSZ
22	GTO
01	1
34 01	RCL 1
34 04	RCL 4
61	+
11	A
04	4
71	x
33	STO
61	+
05	5
34 02	RCL 2
11	A
34 05	RCL 5
61	+
34 04	RCL 4
71	x

CODE	KEYS
03	3
81	÷
84	R/S
23	LBL
11	A
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	a	R_4	h	R_7	
R_2	b	R_5	$\int f$	R_8	Used
R_3	n	R_6		R_9	Used

FIRST ORDER DIFFERENTIAL EQUATION

CODE	KEYS
23	LBL
11	A
33 01	STO 1
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 03	STO 3
35 08	g R↓
35 08	g R↓
23	LBL
01	1
12	B
33 04	STO 4
34 03	RCL 3
71	x
02	2
81	÷
34 02	RCL 2
61	+
34 03	RCL 3
02	2
81	÷
34 01	RCL 1
61	+
12	B
33 05	STO 5
34 03	RCL 3
71	x
02	2
71	x
34 02	RCL 2
61	+
34 04	RCL 4
34 03	RCL 3
71	x

CODE	KEYS
51	–
34 01	RCL 1
34 03	RCL 3
61	+
12	B
34 05	RCL 5
04	4
71	x
61	+
34 04	RCL 4
61	+
06	6
81	÷
34 03	RCL 3
71	x
34 02	RCL 2
61	+
84	R/S
33 02	STO 2
34 01	RCL 1
34 03	RCL 3
61	+
33 01	STO 1
22	GTO
01	1
23	LBL
12	B
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

CODE	KEYS
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	Used	R_4	Used	R_7	
R_2	Used	R_5	Used	R_8	
R_3	Used	R_6		R_9	

ROOTS OF f(x) = 0 IN AN INTERVAL

CODE	KEYS
33 07	STO 7
35 07	g x⇄y
33 01	STO 1
23	LBL
02	2
11	A
33 03	STO 3
00	0
35 23	g x=y
34 01	RCL 1
84	R/S
34 01	RCL 1
34 06	RCL 6
61	+
33 02	STO 2
33 08	STO 8
11	A
34 03	RCL 3
71	x
00	0
35 24	g x>y
22	GTO
00	0
34 02	RCL 2
33 01	STO 1
34 06	RCL 6
61	+
33 02	STO 2
34 07	RCL 7
35 07	g x⇄y
35 24	g x>y
84	R/S
35 01	g NOP
34 01	RCL 1
22	GTO

CODE	KEYS
02	2
23	LBL
00	0
34 01	RCL 1
34 02	RCL 2
61	+
02	2
81	÷
33 04	STO 4
11	A
35	g
06	ABS
34 05	RCL 5
35 24	g x>y
22	GTO
03	3
34 01	RCL 1
11	A
33 03	STO 3
34 04	RCL 4
11	A
34 03	RCL 3
71	x
00	0
35 24	g x>y
22	GTO
04	4
34 04	RCL 4
33 01	STO 1
22	GTO
00	0
23	LBL
04	4
34 04	RCL 4
33 02	STO 2

CODE	KEYS
22	GTO
00	0
23	LBL
03	3
34 04	RCL 4
84	R/S
34 08	RCL 8
33 01	STO 1
22	GTO
02	2
23	LBL
11	A
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	Used	R_4	Used	R_7	b
R_2	Used	R_5	ϵ	R_8	Used
R_3	Used	R_6	Δx	R_9	Used

DETERMINANT AND CHARACTERISTIC EQUATION OF A 3 x 3 MATRIX

CODE	KEYS
23	LBL
11	A
33 06	STO 6
35 08	g R↓
33 08	STO 8
35 08	g R↓
33 01	STO 1
24	RTN
23	LBL
12	B
33	STO
09	9
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 04	STO 4
24	RTN
23	LBL
13	C
33 03	STO 3
35 08	g R↓
33 05	STO 5
35 08	g R↓
33 07	STO 7
84	R/S
34 01	RCL 1
34 02	RCL 2
61	+
34 03	RCL 3
61	+
84	R/S
34 06	RCL 6
34 07	RCL 7
71	x
34 08	RCL 8

CODE	KEYS
34 04	RCL 4
71	x
61	+
34	RCL
09	9
34 05	RCL 5
71	x
61	+
34 01	RCL 1
34 02	RCL 2
71	x
51	–
34 01	RCL 1
34 02	RCL 2
61	+
34 03	RCL 3
71	x
51	–
84	R/S
23	LBL
14	D
34 01	RCL 1
34 02	RCL 2
71	x
34 03	RCL 3
71	x
34 08	RCL 8
34	RCL
09	9
71	x
34 07	RCL 7
71	x
61	+
34 06	RCL 6
34 04	RCL 4

CODE	KEYS
71	x
34 05	RCL 5
71	x
61	+
34 06	RCL 6
34 02	RCL 2
71	x
34 07	RCL 7
71	x
51	–
34 08	RCL 8
34 04	RCL 4
71	x
34 03	RCL 3
71	x
51	–
34 01	RCL 1
34	RCL
09	9
71	x
34 05	RCL 5
71	x
51	–
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	a_1	R_4	b_1	R_7	c_1
R_2	b_2	R_5	c_2	R_8	a_2
R_3	c_3	R_6	a_3	R_9	b_3

2 x 2 MATRIX OPERATIONS

CODE	KEYS
23	LBL
11	A
33 04	STO 4
35 08	g R↓
33 03	STO 3
35 08	g R↓
33 02	STO 2
35 08	g R↓
33 01	STO 1
24	RTN
23	LBL
12	B
33 08	STO 8
35 08	g R↓
33 07	STO 7
35 08	g R↓
33 06	STO 6
35 08	g R↓
33 05	STO 5
24	RTN
23	LBL
13	C
34 01	RCL 1
34 05	RCL 5
61	+
84	R/S
34 02	RCL 2
34 06	RCL 6
61	+
84	R/S
34 03	RCL 3
34 07	RCL 7
61	+
84	R/S
34 04	RCL 4

CODE	KEYS
34 08	RCL 8
61	+
24	RTN
23	LBL
14	D
34 01	RCL 1
34 05	RCL 5
51	–
84	R/S
34 02	RCL 2
34 06	RCL 6
51	–
84	R/S
34 03	RCL 3
34 07	RCL 7
51	–
84	R/S
34 04	RCL 4
34 08	RCL 8
51	–
24	RTN
23	LBL
15	E
34 05	RCL 5
34 01	RCL 1
71	x
34 02	RCL 2
34 07	RCL 7
71	x
61	+
84	R/S
34 01	RCL 1
34 06	RCL 6
71	x
34 02	RCL 2

CODE	KEYS
34 08	RCL 8
71	x
61	+
84	R/S
34 03	RCL 3
34 05	RCL 5
71	x
34 04	RCL 4
34 07	RCL 7
71	x
61	+
84	R/S
34 03	RCL 3
34 06	RCL 6
71	x
34 04	RCL 4
34 08	RCL 8
71	x
61	+
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	a_1	R_4	a_4	R_7	b_3
R_2	a_2	R_5	b_1	R_8	b_4
R_3	a_3	R_6	b_2	R_9	

3 x 3 MATRIX INVERSION

CODE	KEYS
34	RCL
09	9
34 03	RCL 3
34 02	RCL 2
11	A
84	R/S
35 08	g R↓
34 06	RCL 6
34 03	RCL 3
34 08	RCL 8
11	A
42	CHS
84	R/S
35 08	g R↓
34 05	RCL 5
34 02	RCL 2
33 05	STO 5
35 08	g R↓
33 02	STO 2
35 08	g R↓
34 06	RCL 6
34	RCL
09	9
34 08	RCL 8
11	A
84	R/S
35 08	g R↓
34 05	RCL 5
34 07	RCL 7
33 05	STO 5
35 08	g R↓
33 07	STO 7
35 08	g R↓
34	RCL
09	9

CODE	KEYS
34 03	RCL 3
34 04	RCL 4
11	A
42	CHS
84	R/S
35 08	g R↓
34 06	RCL 6
34 03	RCL 3
34 01	RCL 1
11	A
84	R/S
12	B
34 06	RCL 6
34	RCL
09	9
34 01	RCL 1
11	A
42	CHS
84	R/S
12	B
34 07	RCL 7
34 02	RCL 2
34 04	RCL 4
11	A
84	R/S
35 08	g R↓
34 08	RCL 8
34 02	RCL 2
34 01	RCL 1
11	A
42	CHS
84	R/S
12	B
34 08	RCL 8
34 07	RCL 7

CODE	KEYS
34 01	RCL 1
11	A
84	R/S
23	LBL
11	A
71	x
35 07	g x⇄y
34 05	RCL 5
71	x
51	–
35 07	g x⇄y
81	÷
24	RTN
23	LBL
12	B
35 08	g R↓
34 05	RCL 5
34 04	RCL 4
33 05	STO 5
35 08	g R↓
33 04	STO 4
35 08	g R↓
24	RTN
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP
35 01	g NOP

R_1	Used	R_4	Used	R_7	Used
R_2	Used	R_5	Used	R_8	Used
R_3	Used	R_6	Used	R_9	Used

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