HEWLETT hp PACKARD

HP-65

STAT PAC 2

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CONTENTS

INTRODUCTION

Programs for your HP-65 Stat Pac 2 have been selected from the areas of general statistics, distribution functions, curve fitting, analysis of variance, test statistics, probability, quality control and queueing theory.

Each program includes a general description, formulas used in the program solution, user instructions, and numerical examples. Program listing and register allocations are given in the back of the Pac.

Several programs included in this Pac were written based on programs submitted to the HP-65 Users' Library. We wish to acknowledge the following contributors:

Vernon J. Poehls for *Moving Averages (Order 2 to 8),*
Thomas L. Ward for *Erlang Distribution,*
Miles C. Collier for *Geometric Curve Fit,*
Robert S. Savage for *Gompertz Curve Fit,*
Robert E. Sherman for *3 x K Contingency Table,*
A. Oscar H. Roberts for *Fisher's Exact Test for a 2 x 2 Contingency Table.*

Special thanks are also due Norbert S. Jagodzinski, William M. Kolb and A. Oscar H. Roberts for their helpful discussions and suggestions. In addition, we wish to extend our appreciation to Prof. Thomas J. Boardman and Dr. Nancy L. Ferguson for reviewing this Pac and making valuable comments.

We hope you find the HP-65 Stat Pac 2 a useful tool for your computational work, and welcome your comments, requests and suggestions–these are our most important source of future user-oriented programs.

FORMAT OF USER INSTRUCTIONS

The completed User Instructions form is your guide to operating the programs in this Pac.

The form is composed of five columns. Reading from left to right, the STEP column gives the instruction step number.

The INSTRUCTIONS column gives instructions and comments concerning the operations to be performed. Steps are executed in sequential order except where the INSTRUCTIONS column directs otherwise.

Normally, the first instruction is "Enter program", which means load the prerecorded magnetic card (for instructions of loading a card, see "Entering A Program" on P. 7).

Repeated processes, used in most cases for a long string of input/output data, are outlined with a bold border together with a "Perform" instruction.

The INPUT DATA/UNITS column specified the input data to be supplied, and the units of data if applicable.

The KEYS column specifies the keys to be pressed. [↑] is the symbol used to denote the [ENTER↑] key. All other key designations are identical to those appearing on the HP-65. Ignore any blank positions in the KEYS column.

The OUTPUT DATA/UNITS column may show counters, intermediate or final results.

The following is an example of User Instructions (for the *Weibull Distribution Parameter Calculation* program).

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program		[ ]	[ ]	
2	Input sample size n	n	[A]	[ ]	
3	Perform 3 for i = 1, 2,..., n	x_i	[B]	[ ]	i
4	Compute a and b (any order)		[C]	[ ]	a
			[D]	[ ]	b
5	Compute r^2		[E]	[ ]	r^2
6	For a new case, go to 2		[ ]	[ ]	

STEP 1: The first step in all programs is to enter the program into the calculator.

STEP 2: Input the sample size n and press the **A** key.

STEP 3: This is a loop for input data. The first time through the loop, the dummy variable i takes the value 1; the second time, i takes the value 2; etc.

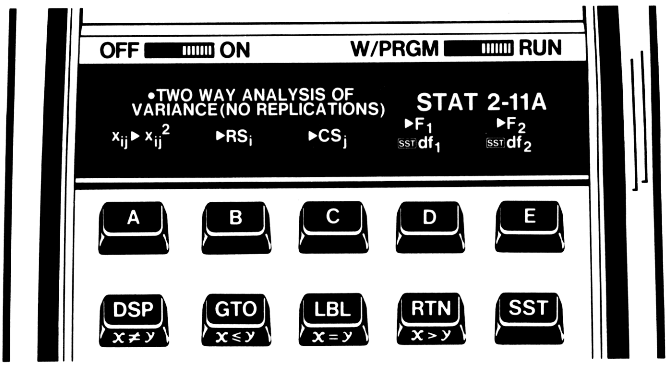
STEP 4: After all the data have been entered, press the **C** key to compute a and press the **D** key to compute b. The order of using these two keys is immaterial.

STEP 5: r^2 will be displayed after pressing the **E** key.

STEP 6: This step gives instructions for starting a new case. In this example, return to step 2.

THE PRERECORDED MAGNETIC CARDS

The prerecorded magnetic cards supplied with this Pac incorporate a shorthand set of operating instructions. In some cases, it is possible to run the program without referencing the manual after the program has been used once. The following is a typical prerecorded magnetic card shown in the window slot of an HP-65.



The dot ● in front of the program title indicates that in order to run this program, you have to refer to the manual for more user instructions. If the dot is not presented, then the card itself contains all the information.

Above the **A** key are the input data x_{ij} and the output x_{ij}^2 separated by a symbol ▸, which means "calculate". Therefore key in x_{ij}, press **A** and x_{ij}^2 will be computed and shown in the display. After entering all the data through the **A** key (refer to the manual for detailed user instructions), press **B** to calculate RS_i. The **C** key works in the same manner. There are two outputs associated with the **D** key. Pressing **D** will initiate the calculation of F_1, then press **SST** key to get the second output df_1. The **E** key works in the same manner as the **D** key.

In general, execution of user definable keys is from left to right.

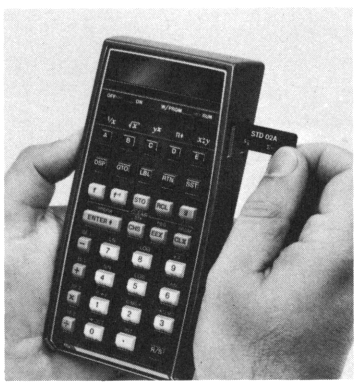
ENTERING A PROGRAM

From the card case supplied with this application pac, select a program card.

Set W/PRGM-RUN switch to RUN.

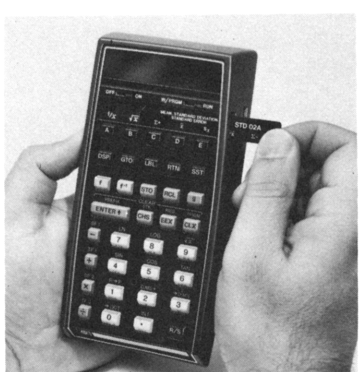
Turn the calculator ON. You should see 0.00

Gently insert the card (printed side up) in the right, lower slot as shown. When the card is part way in, the motor engages it and passes it out the left side of the calculator. Sometimes the motor engages but does not pull the card in. If this happens, push the card a little farther into the machine. Do not impede or force the card; let it move freely. (The display will flash if the card reads improperly. In this case, press [CLX] and reinsert the card.)

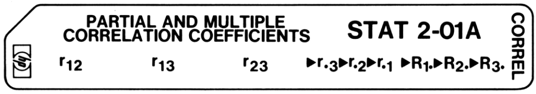


When the motor stops, remove the card from the left side of the calculator and insert it in the upper "window slot" on the right side of the calculator.

The program is now stored in the calculator. It remains stored until another program is entered or the calculator is turned off.



PARTIAL AND MULTIPLE CORRELATION COEFFICIENTS



The partial correlation coefficient measures the relationship between any two of the variables when all others are kept constant. Although this program uses the notations for the case of three variables, other cases of more than three variables can be handled as well.

Any higher order partial correlation coefficient can be computed by using this program (it might be necessary to run the program more than once), if correlation coefficients r_{12}, r_{13}, r_{23}, - - - are given.

The multiple correlation coefficient measures the relationship between three or more variables. A multiple correlation coefficient lies between 0 and 1. The closer it is to 1 the better the linear relationship between the variables. Zero indicates no linear relationship but a non-linear relationship may exist.

Equations:

1. For the case of three variables, the partial correlation coefficient between X_1 and X_2 keeping X_3 constant is

$$r_{12\cdot 3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}}$$

where r_{ij} denotes the correlation coefficient of X_i and X_j.

$r_{13\cdot 2}$ and $r_{23\cdot 1}$ can be found by appropriately changing the subscripts in the above formula.

2. For the case of three variables, the multiple correlation coefficient between X_1 and X_2, X_3 is

$$R_{1\cdot 23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}r_{23}}{1 - r_{23}^2}}$$

$R_{2\cdot 13}$ and $R_{3\cdot 12}$ can be found by changing the subscripts in the formula.

Notations used on magnetic card:

$$r_{\cdot 3} = r_{12\cdot 3},\ r_{\cdot 2} = r_{13\cdot 2},\ r_{\cdot 1} = r_{23\cdot 1}$$

$$R_{1\cdot} = R_{1\cdot 23},\ R_{2\cdot} = R_{2\cdot 13},\ R_{3\cdot} = R_{3\cdot 12}$$

References:

1. S. Wilks, *Mathematical Statistics,* John Wiley and Sons, 1962.
2. D. Morrison, *Multivariate Statistical Methods,* McGraw-Hill, 1967.
3. M. Spiegel, *Theory and Problems of Statistics,* Schaum's Outline, McGraw-Hill, 1961.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Input the data in any order	r_{12}	A		
		r_{13}	B		
		r_{23}	C		
3	Compute partial coefficients		D		$r_{12\cdot 3}$
			D		$r_{13\cdot 2}$
			D		$r_{23\cdot 1}$
4	Compute multiple coefficients		E		$R_{1\cdot 23}$
			E		$R_{2\cdot 13}$
			E		$R_{3\cdot 12}$
5	For a new case, go to 2 and				
	change appropriate data.				

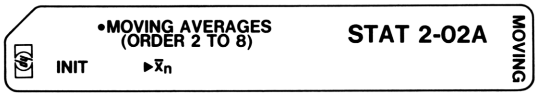
Example:

Find the partial and multiple correlation coefficients if $r_{12} = -0.96$, $r_{13} = -0.1$, $r_{23} = 0.12$.

Keystrokes:

.96 [CHS] [A] .1 [CHS] [B] .12 [C] [D] ——→ $-0.96\ (r_{12\cdot 3})$
[D] ——→ $0.05\ (r_{13\cdot 2})$
[D] ——→ $0.09\ (r_{23\cdot 1})$
[E] ——→ $0.96\ (R_{1\cdot 23})$
[E] ——→ $0.96\ (R_{2\cdot 13})$
[E] ——→ $0.13\ (R_{3\cdot 12})$

MOVING AVERAGES (ORDER 2 TO 8)



Given a set of numbers $\{x_1, x_2, ..., x_n, x_{n+1}, ...\}$, this program finds the moving averages $\{\overline{x}_n, \overline{x}_{n+1}, ...\}$ of order n $(2 \leqslant n \leqslant 8)$.

Equations:

$$\overline{x}_n = \frac{x_1 + x_2 + ... + x_n}{n}$$

$$\overline{x}_{n+1} = \frac{x_2 + x_3 + ... + x_n + x_{n+1}}{n}$$

etc.

Remark:

n is an integer and $2 \leqslant n \leqslant 8$.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Switch to W/PRGM mode				00 00
3	Enter STO n* into program		STO	n*	
4	Switch to RUN mode				
5	Initialize		A		0.
6	Perform 6 for i = 1, 2,..., n	x_i	R/S		i
7	Compute the average		B		$\overline{x}_n$
8	Perform 8 for i = n + 1, n + 2,...	x_i	R/S		$\overline{x}_i$
9	For a new case, go to 1				
	* n should be replaced by an				
	integer between 2 and 8.				

Example:

Find the moving averages of order 5 for the following numbers:

$\{11.1, 12.3, 13.0, 14.9, 15.5, 16.2, 17.5, 18.1, 19.8\}$

Keystrokes:

Enter program

Switch to W/PRGM mode

Press [STO] [5]

Switch to RUN mode

[A] ——————→ 0.

11.1 [R/S] 12.3 [R/S] 13 [R/S] 14.9 [R/S] 15.5 [R/S] → 5.

[B] ——————→ 13.36 ($\bar{x}_5$)

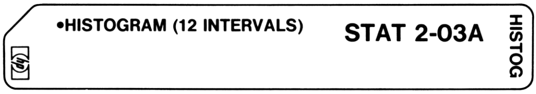
16.2 [R/S] ——————→ 14.38 ($\bar{x}_6$)

17.5 [R/S] ——————→ 15.42 ($\bar{x}_7$)

18.1 [R/S] ——————→ 16.44 ($\bar{x}_8$)

19.8 [R/S] ——————→ 17.42 ($\bar{x}_9$)

HISTOGRAM (12 INTERVALS)



This program sorts input data into twelve intervals of equal width between specified upper and lower limits. The input data may be supplied manually or generated by a subroutine.

The twelve intervals are displayed three at a time by recalling the contents of registers R_1, R_2, R_3 and R_4. Every three digits to the right of the decimal point represents the number of counts in the corresponding interval. For example, if $0.b_1b_2b_3$ represents the contents of register R_1, then each of b_1, b_2, b_3 is a three-digit number and b_1, b_2, b_3 are numbers of counts in intervals 1, 2, 3.

The main program calls subroutine E to obtain data. The code following **LBL** **E** may be as simple as **R/S** **RTN** which would accept user inputs or it may be a random number generator to be tested. Registers R_7, R_8 and 20 program steps are available for the number generator.

Equation:

The program maps the domain $x_{min} \leqslant x < x_{max}$ (values outside this domain will be ignored) onto the range $1 \leqslant y \leqslant 12$. The mapping is defined by

$$y = 1 + \text{Int}\left[12\,\frac{x - x_{min}}{x_{max} - x_{min}}\right]$$

where

y = interval number

x = input data

x_{min} = lower limit of histogram

x_{max} = upper limit of histogram

Int = integer part of

Remark:

Because each interval is represented by only three digits, overflow from one interval to the next lower interval will occur if there are more than 999 counts in that interval.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Change display format		DSP	•	
			9		
3	Initialize		f	REG	
4	Input lower limit	x_{min}	STO	5	
	upper limit	x_{max}	STO	6	
5	For user input data, go to 12				
6	Modify subroutine E		GTO	E	
	Switch to W/PRGM mode		SST		84
			g	DEL	15
	Key in the number generator				
	Switch to RUN mode				
7	Compute histogram		RTN	R/S	
8	Stop whenever desired		R/S		
9	Display histogram				
	intervals 1 to 3		RCL	1	0. b_1 b_2 b_3
	intervals 4 to 6		RCL	2	0. b_4 b_5 b_6
	intervals 7 to 9		RCL	3	0. b_7 b_8 b_9
	intervals 10 to 12		RCL	4	0. b_{10} b_{11} b_{12}
10	Optional—continue computa-				
	tion,* go to 7 or 12 (user inputs)				
11	For a new case, go to 1				
12	Start computation for user				
	inputs		RTN	R/S	
13	Perform 13 for all data x	x	R/S		
14	Go to 9				
	* One piece of data might be				
	lost or counted twice.				

Example 1:

Compute a histogram of the following data with $x_{min} = 0$, $x_{max} = 24$.
$\{18.1, 14.3, 8.4, 0.7, 20.2, 26, 14, 17.2, 24, 8.8, 5.7, 13.2, 22.1, 15.7, 18.9, 23\}$

Keystokes:

DSP [•] [9] [f] [REG] 0 [STO] [5] 24 [STO] [6]

RTN R/S 18.1 R/S 14.3 R/S 8.4 R/S ...

23 R/S ⟶ 0.000000001

[RCL] [1] ⟶ 0.001000001

($b_1 = 1, b_2 = 0, b_3 = 1$)

[RCL] [2] ⟶ 0.000002000

($b_4 = 0, b_5 = 2, b_6 = 0$)

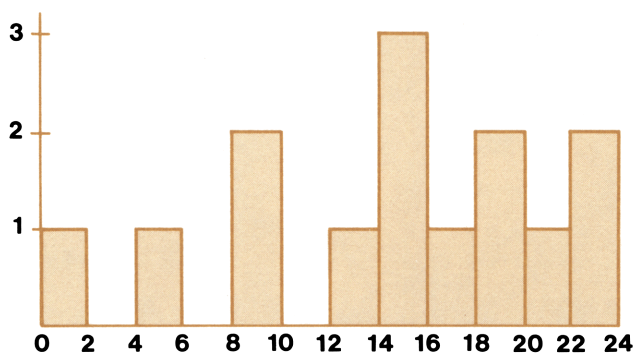
[RCL] [3] ⟶ 0.001003001

($b_7 = 1, b_8 = 3, b_9 = 1$)

[RCL] [4] ⟶ 0.002001002

($b_{10} = 2, b_{11} = 1, b_{12} = 2$)

HISTOGRAM



Example 2:

Test the uniform (0, 1) random number generator

$$x_{n+1} = \text{Frac}\ [29\ x_n]$$

using the starting value $x_0 = 0.2510637948$ with $x_{min} = 0, x_{max} = 1.2$. (Use the register R_7 to store the random number.)

Note:

$x_{max} = 1.2$ instead of $x_{max} = 1$ is used in order to have 10 intervals between 0 and 1. Frac = fractional part of.

Keystrokes:

DSP • 9 f REG 0 STO 5 1.2 STO 6 GTO E

Switch to W/PRGM mode

SST 9 DEL RCL 7 29 × f⁻¹ INT STO 7

Switch to RUN mode

0.2510637948 STO 7 RTN R/S

Let the program run for about 10 minutes.

(Note: you probably will not get the following answers, values depend upon the amount of time the program runs.)

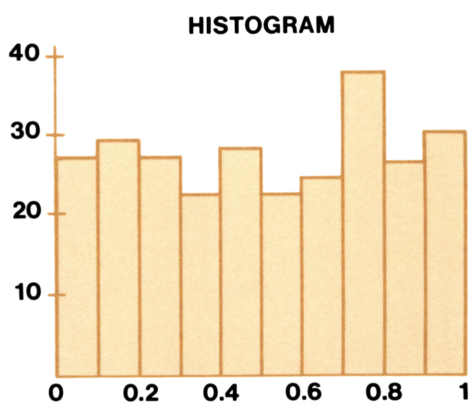
R/S

RCL 1 ——→ 0.027029027
($b_1 = 27, b_2 = 29, b_3 = 27$)

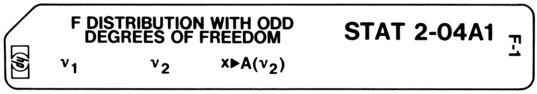
RCL 2 ——→ 0.022028022
($b_4 = 22, b_5 = 28, b_6 = 22$)

RCL 3 ——→ 0.024037026
($b_7 = 24, b_8 = 37, b_9 = 26$)

RCL 4 ——→ 0.030000000
($b_{10} = 30, b_{11} = 0, b_{12} = 0$)



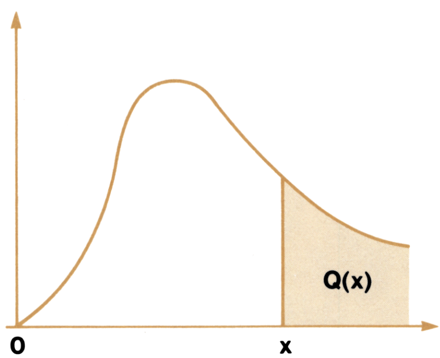
F DISTRIBUTION WITH ODD DEGREES OF FREEDOM



This program evaluates the integral of the F distribution

$$Q(x) = \int_x^{\infty} \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right) y^{\frac{\nu_1}{2}-1} \left(\frac{\nu_1}{\nu_2}\right)^{\frac{\nu_1}{2}}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)\left(1+\frac{\nu_1}{\nu_2}y\right)^{\frac{\nu_1+\nu_2}{2}}} dy$$

for given values of x (>0) and degrees of freedom ν_1, ν_2, provided that both ν_1, ν_2 are odd integers.



Equations:

$$Q(x) = 1 - A(\nu_2) + B(\nu_1, \nu_2)$$

$$A(\nu_2) = \begin{cases} \frac{2}{\pi}\left\{\theta + \sin\theta\cos\theta\left[1 + \frac{2}{3}\cos^2\theta + \ldots + \frac{2 \cdot 4 \ldots (\nu_2 - 3)}{3 \cdot 5 \ldots (\nu_2 - 2)}\cos^{\nu_2 - 3}\theta\right]\right\} & \text{for } \nu_2 > 1 \\ \frac{2\theta}{\pi} & \text{for } \nu_2 = 1 \end{cases}$$

$$B(\nu_1, \nu_2) = \begin{cases} \dfrac{2\left(\dfrac{\nu_2 - 1}{2}\right)!}{\sqrt{\pi}\ \Gamma\left(\dfrac{\nu_2}{2}\right)} \sin\theta \cos^{\nu_2}\theta \left[1 + \dfrac{\nu_2 + 1}{3} \sin^2\theta + \ldots \right. \\ \qquad \left. + \dfrac{(\nu_2 + 1)(\nu_2 + 3)\ldots(\nu_1 + \nu_2 - 4)\sin^{\nu_1 - 3}\theta}{3 \cdot 5 \ldots (\nu_1 - 2)} \right] \\ \qquad \text{for } \nu_1 > 1 \\ 0 \quad \text{for } \nu_1 = 1 \end{cases}$$

where

$$\theta = \tan^{-1}\left(\sqrt{\frac{\nu_1 x}{\nu_2}}\right)$$

$$\Gamma\left(\frac{\nu_2}{2}\right) = \left(\frac{\nu_2}{2} - 1\right)\left(\frac{\nu_2}{2} - 2\right) \cdots \left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right)$$

$$\text{and } \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}\ .$$

Remark:

After execution of the program, the calculator is in RAD mode.

References:

1. Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1970.

2. For ν_1 even or ν_2 even (or both), see *HP-65 Stat Pac 1,* STAT 1—14A, *F Distribution.*

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program on card 1				
2	Input numerator degrees of				
	freedom	ν_1	A		ν_1
	denominator degrees of				
	freedom	ν_2	B		ν_2
3	Input x	x	C		$A(\nu_2)$
4	Enter program on card 2				
5	Continue calculations		A		Q(x)
6	For a different x, enter				
	program on card 1 and go to 3.				
7	For a new case, go to 1				

Example 1:

If $\nu_1 = 3, \nu_2 = 5$, find Q(x) for x = 0.907 and x = 5.

Keystrokes:

Enter program on card 1

3 **A** 5 **B** .907 **C** ⟶ 0.84 ($A(\nu_2)$)

Enter program on card 2

A ⟶ 0.50 (Q(x))

Enter program on card 1

5 **C** ⟶ 0.99 ($A(\nu_2)$)

Enter program on card 2

A ⟶ 0.06 (Q(x))

Example 2:

If $\nu_1 = 1, \nu_2 = 15$ find Q(x) for x = 3.07.

Keystrokes:

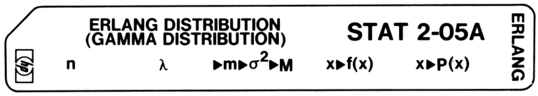
Enter program on card 1

1 **A** 15 **B** 3.07 **C** ⟶ 0.90 ($A(\nu_2)$)

Enter program on card 2

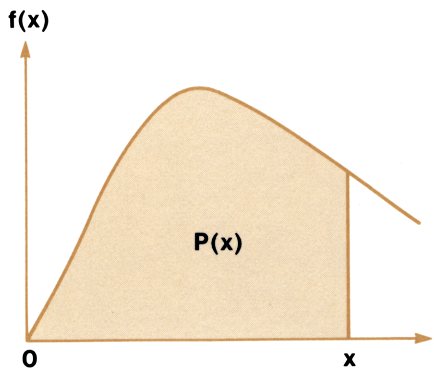
A ⟶ 0.10 (Q(x))

ERLANG DISTRIBUTION (GAMMA DISTRIBUTION)



This program evaluates the n-phase Erlang density function f(x) and the cumulative distribution function P(x). The mean, variance and mode are also computed.

The n-phase Erlang distribution is a special case of the gamma distribution with the shape factor being an integer n. If n = 1, the exponential distribution is obtained.



Equations:

$$f(x) = \frac{\lambda^n x^{n-1}}{(n-1)!} e^{-\lambda x}$$

where $x > 0$, $\lambda > 0$ and n is a positive interger.

$$P(x) = \int_0^x f(t)\,dt = 1 - e^{-\lambda x} \sum_{i=0}^{n-1} \frac{(\lambda x)^i}{i!}$$

mean	$m = n/\lambda$
variance	$\sigma^2 = n/\lambda^2$
mode	$M = (n-1)/\lambda$

Remarks:

1. $n \leqslant 70$ for f(x)

2. $(\lambda x)^{n-1} < 10^{100}$ for f(x) and P(x).

Reference:

R. B. Cooper, *Introduction to Queueing Theory,* Macmillan, 1972.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Input n	n	A		
3	Input λ	λ	B		
4	Compute the mean		C		m
	the variance		C		σ^2
	the mode		C		M
5	Compute f(x)	x	D		f(x)
6	Compute P(x)	x	E		P(x)
7	For a new x, go to 5 or 6				
8	For a new case, go to 2 or 3				

Example:

Compute f(x) and P(x) for n = 2, λ = 0.5 and x = 10.

Keystrokes:

2 A .5 B C ——→ 4.00 (mean)

C ——→ 8.00 (variance)

C ——→ 2.00 (mode)

10 D ——→ 0.02 (f(x))

10 E ——→ 0.96 (P(x))

GEOMETRIC CURVE FIT

This program fits a geometric curve

$$y = ab^x$$

to a set of n data points

$$\left\{(x_i, y_i), i = 1, 2, ..., n\right\}$$

where

$$y_i > 0.$$

By writing the equation as

$$\ln y = \ln a + x \ln b$$

the problem can be solved as a linear regression problem.

Equations:

1. Coefficients

$$b = \exp\left[\frac{\Sigma x_i \ln y_i - \dfrac{\Sigma x_i \, \Sigma \ln y_i}{n}}{\Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n}}\right]$$

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - \ln b \, \frac{\Sigma x_i}{n}\right]$$

2. Coefficient of determination

$$r^2 = \frac{\left[\Sigma x_i \ln y_i - \frac{\Sigma x_i \, \Sigma \ln y_i}{n}\right]^2}{\left[\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right]\left[\Sigma (\ln y_i)^2 - \frac{(\Sigma \ln y_i)^2}{n}\right]}$$

3. Estimated value $\hat{y}$ for given x

$$\hat{y} = ab^x$$

Remark:

n is a positive integer and $n \neq 1$.

Reference:

S. B. Richmond, *Statistical Analysis,* Roland Press Co., 1964.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Initialize		A		
3	Perform 3 for i = 1, 2,..., n	x_i	↑		
		y_i	B		i
4	Compute the coefficients		C		a
			C		b
5	Compute r^2		D		r^2
6	Compute the estimated value $\hat{y}$	x	E		$\hat{y}$
7	For a different x, go to 6				
8	For a new case, go to 2				

Example:

Find the geometric curve fit to the following data and compute the estimated values at x = 7 and x = 20.

x_i	2	3	5	8	9	10	15
y_i	101	120	138	173	180	187	209

Keystrokes:

A 2 [↑] 101 **B** 3 [↑] 120 **B** 5 [↑] 138 **B** 8 [↑] 173 **B** 9 [↑] 180 **B** 10 [↑] 187 **B** 15 [↑] 209 **B** ⟶ 7.00

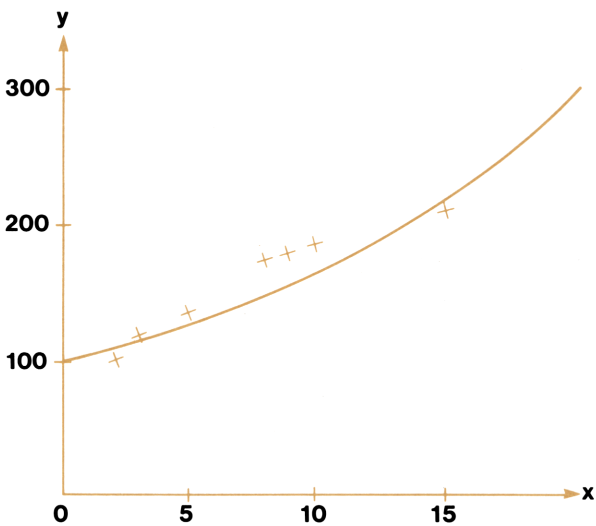
C ⟶ 101.66 (a)

C ⟶ 1.06 (b)

D ⟶ 0.90 (r^2)

7 **E** ⟶ 150.17 ($\hat{y}$ at x = 7)

20 **E** ⟶ 309.94 ($\hat{y}$ at x = 20)



GOMPERTZ CURVE FIT

This program fits a Gompertz curve to a set of data points $\{(i, y_i), i = 1, 2, ..., n, ..., 3n\}$. Data points are divided into 3 groups, each having n observations. The x's should be equally spaced and $y_i > 0$.

Equations:

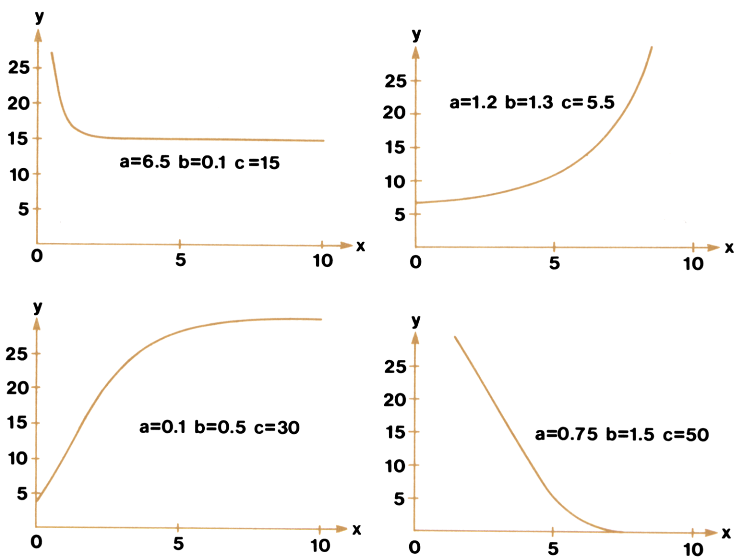
The Gompertz curve is given by the equation

$$y = c\, a^{(b^x)}$$

or

$$\ln y = \ln c + b^x \ln a$$

where x, y, a, b, c are positive.



Let

$$S_1 = \sum_{i=1}^{n} \ln y_i = n \ln c + b(\ln a)\, \frac{b^n - 1}{b - 1}$$

$$S_2 = \sum_{i=n+1}^{2n} \ln y_i = n \ln c + b^{n+1} (\ln a)\, \frac{b^n - 1}{b - 1}$$

$$S_3 = \sum_{i=2n+1}^{3n} \ln y_i = n \log c + b^{2n+1} (\ln a)\, \frac{b^n - 1}{b - 1}$$

then a, b, c can be determined by solving the three equations above simultaneously.

$$b = \left(\frac{S_3 - S_2}{S_2 - S_1} \right)^{1/n}$$

$$c = \exp \left[\frac{1}{n} \left(\frac{S_1 S_3 - S_2{}^2}{S_1 + S_3 - 2S_2} \right) \right]$$

$$a = \exp \left[\frac{(b - 1)(S_2 - S_1)}{b\,(b^n - 1)^2} \right]$$

The estimated value $\hat{y} = c\, a^{(b^x)}$ is also computed.

Remark:

This is only one of the many ways to fit a Gompertz curve.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Initialize		f	REG	
3	Perform 3 for i = 1, 2,..., n	y_i	↑		
		y_{n+i}	↑		
		y_{2n+i}	A		i
4	Compute coefficients		B		a
			C		b
			D		c
5	Compute estimated value $\hat{y}$	x	E		$\hat{y}$
6	For a different x, go to 5				
7	For a new case, go to 2				

Example:

Fit a Gompertz curve to the following table of data points and find the predicted values for 1975 and 1977.

Year	x	y	
1960	1	226	Group 1
1961	2	244	
1962	3	265	
1963	4	287	
1964	5	290	
1965	6	317	Group 2
1966	7	316	
1967	8	362	
1968	9	378	
1969	10	417	
1970	11	442	Group 3
1971	12	461	
1972	13	526	
1973	14	566	
1974	15	649	

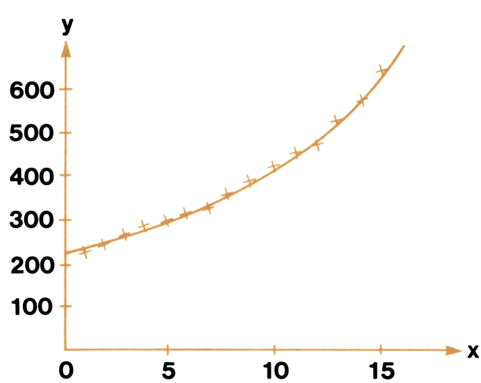
(n = 5)

Rearrange the data as follows (for the convenience of data entry):

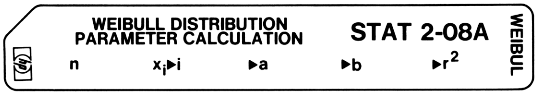
Group 1	Group 2	Group 3
226	317	442
244	316	461
265	362	526
287	378	566
290	417	649

Keystrokes:

[f] [REG] 226 [↑] 317 [↑] 442 [A] →	1.00
244 [↑] 316 [↑] 461 [A] →	2.00
265 [↑] 362 [↑] 526 [A] →	3.00
287 [↑] 378 [↑] 566 [A] →	4.00
290 [↑] 417 [↑] 649 [A] →	5.00
[B] →	2.99 (a)
[C] →	1.05 (b)
[D] →	74.63 (c)
16 [E] →	687.06 (Predicted value for 1975)
18 [E] →	843.35 (Predicted value for 1977)



WEIBULL DISTRIBUTION PARAMETER CALCULATION



For a set of data $\{x_1, x_2, ..., x_n\}$, this program calculates the estimates of the Weibull parameters a and b.

A common application is to use Weibull analysis for failure data where all samples are tested to failure. To use the program, list the items in order of increasing time to failure.

Equations:

$$f(x) = abx^{b-1} e^{-ax^b}, a > 0, b > 0, x > 0$$

$$F(x) = 1 - e^{-ax^b}$$

$$M_i = \frac{R_i - 0.3}{n + 0.4}$$

where

$f(x)$ is the Weibull density function,

$F(x)$ is the cumulative distribution function,

M_i is the median rank of failure data x_i,

and R_i is the rank of x_i $(i = 1, 2, ..., n)$.

Using the median rank M_i as an approximation of $F(x_i)$, a least squares fit is performed to the linearized form of the cumulative distribution function

$$\ln[-\ln(1 - F(x))] = b \ln x + \ln a$$

The solution is similar to the linear regression problem, and estimates of a and b are obtained. The coefficient of determination r^2 is also computed.

Remark:

This is only one of the many ways to estimate the Weibull parameters.

Reference:

HP-65 Stat Pac 1, program Stat 1–22A. *Linear Regression.*

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Input sample size n	n	A		
3	Perform 3 for i = 1, 2,..., n	x_i	B		i
4	Compute a and b (any order)		C		a
			D		b
5	Compute r^2		E		r^2
6	For a new case, go to 2				

Example:

Compute the Weibull parameters for x_i: 34, 60, 75, 95, 119, 158 (hours to failure). (x_i's must be entered in increasing order.)

Keystrokes:

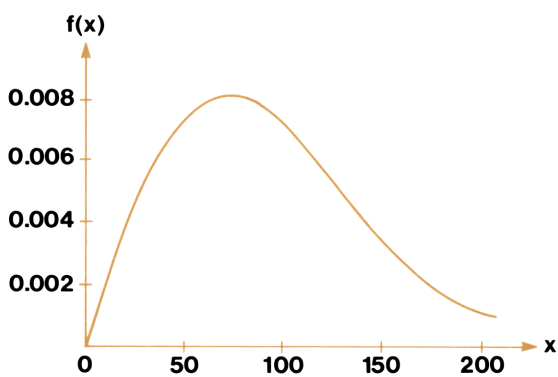
6 **A** 34 **B** 60 **B** 75 **B** 95 **B** 119 **B** 158 **B** ⟶ 6.00

C ⟶ 0.00 (a small number)

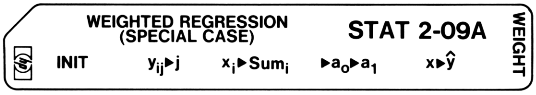
DSP **.** **4** ⟶ 0.0001 (a)

D ⟶ 1.9531 (b)

E ⟶ 0.9975 (r^2)



WEIGHTED REGRESSION (SPECIAL CASE)



In the case of simple linear regression, certain assumptions are made. Suppose the model is

$$y_{ij} = a_0 + a_1 x_i + \epsilon_{ij}$$

where

$i = 1, ..., k$

$j = 1, ..., n_i$

n_i = number of Y values associated with the i^{th} X value.

One assumption is that ϵ_{ij} are normally and independently distributed with mean zero and standard deviation σ. That is, the variance of Y given X, $\sigma_{Y|X}{}^2$, is the same for each X and, therefore, can be denoted by σ^2.

Suppose now the assumption of homogeneous variances is no longer justified. That is, suppose we can not assume that $\sigma_{Y|X}{}^2$ is the same for all X, but that we must assume

$$\sigma_{Y/X_i}{}^2 = \sigma_i{}^2 = \frac{\sigma^2}{w_i}$$

where w_i are known weighting constants. In order to get the least squares fit, we have to use the weighted regression. One particular case in certain areas of experimentation is that $\sigma_i{}^2$ is proportional to x_i, i.e.

$$\sigma_i{}^2 = \sigma^2/w_i = \sigma^2 x_i$$

This program finds the weighted regression line for this particular case.

Equations:

$$\text{Sum}_i = \sum_{j=1}^{n_i} y_{ij}$$

$$a_1 = \frac{\left(\sum_{i=1}^{k} \frac{n_i}{x_i}\right)\left(\sum_{i=1}^{k}\sum_{j=1}^{n_i} y_{ij}\right) - \left(\sum_{i=1}^{k} n_i\right)\left(\sum_{i=1}^{k}\sum_{j=1}^{n_i} \frac{y_{ij}}{x_i}\right)}{\left(\sum_{i=1}^{k} \frac{n_i}{x_i}\right)\left(\sum_{i=1}^{k} n_i x_i\right) - \left(\sum_{i=1}^{k} n_i\right)^2}$$

$$a_0 = \frac{\left(\sum_{i=1}^{k}\sum_{j=1}^{n_i} \frac{y_{ij}}{x_i}\right) - a_1 \sum_{i=1}^{k} n_i}{\sum_{i=1}^{k} \frac{n_i}{x_i}}$$

Predicted value $\hat{y} = a_0 + a_1 x$

Remark:

$x_i \neq 0$

Reference:

B. Ostle, *Statistics in Research,* Iowa State University Press, 1972.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program		[ ] [ ]	
2	Initialize		[A] [ ]	0.00
3	Perform 3-5 for i = 1, 2,..., k		[ ] [ ]	
4	Perform 4 for j = 1, 2,..., n_i	y_{ij}	[B] [ ]	j
5	Input x_i and compute Sum_i	x_i	[C] [ ]	Sum_i
6	Compute regression coefficients		[D] [ ]	a_0
			[D] [ ]	a_1
7	Compute the estimated value $\hat{y}$	x	[E] [ ]	$\hat{y}$
8	For a different x, go to 7		[ ] [ ]	
9	For a new case, go to 2		[ ] [ ]	

Example:

x \ y				
25	12	11	13.2	
28	13.6			
31	12.5	16		
35	16.4	18	13.8	15
37	15.8	14		
45	17	18.2	15	
48	18	14.7		
52	23	20		
53	21			

Suppose the variance of Y is proportional to X, find the weighted regression line and the estimated values for x = 40 and 50.

Keystrokes:

A 12 B 11 B 13.2 B 25 C ⟶ 36.20 (Sum_1)

13.6 B 28 C ⟶ 13.60 (Sum_2)

12.5 B 16 B 31 C ⟶ 28.50 (Sum_3)

16.4 B 18 B 13.8 B 15 B 35 C ⟶ 63.20 (Sum_4)

15.8 B 14 B 37 C ⟶ 29.80 (Sum_5)

17 B 18.2 B 15 B 45 C ⟶ 50.20 (Sum_6)

18 B 14.7 B 48 C ⟶ 32.70 (Sum_7)

23 B 20 B 52 C ⟶ 43.00 (Sum_8)

21 B 53 C ⟶ 21.00 (Sum_9)

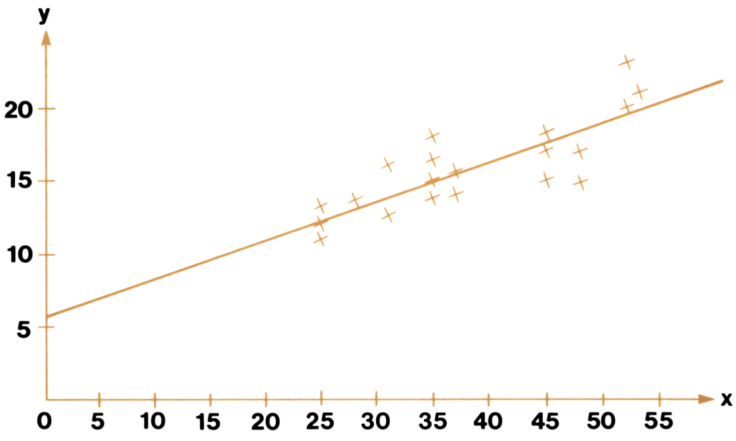
D ⟶ 5.71 (a_0)

D ⟶ 0.27 (a_1)

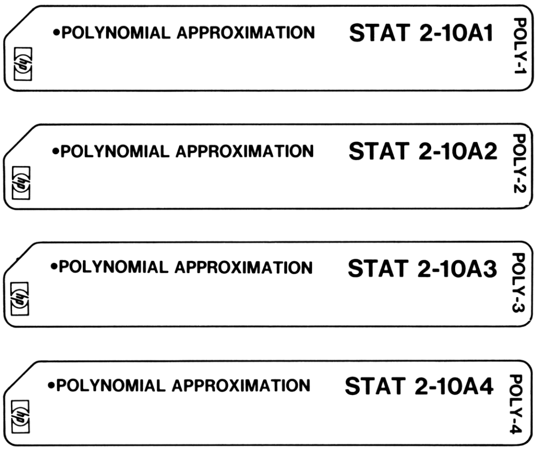
40 E ⟶ 16.35 ($\hat{y}$ for x = 40)

50 E ⟶ 19.01 ($\hat{y}$ for x = 50)

The weighted regression is y = 5.71 + 0.27x


y
20
15
10
5
0
5
10
15
20
25
30
35
40
45
50
55
x

POLYNOMIAL APPROXIMATION



Suppose $x_0, x_1, \ldots, x_N$ are equally spaced points ($x_0 < x_N$) at which the corresponding values $f(x_0), f(x_1), \ldots, f(x_N)$ of a function $f(x)$ are known.

This program approximates in the least squares sense the function $f(x)$ by a polynomial of degree m, where $2 \leqslant m \leqslant 4$. The special Chebyshev polynomials for discrete intervals are used.

Equations:

Let $f_n(x)$ be the orthogonal polynomials ($x = 0, 1, 2, \ldots, N$) such that

$$f_0(x) = 1$$

$$f_1(x) = 1 - \frac{2x}{N} \quad \text{and}$$

$$(n+1)(N-n)\, f_{n+1}(x) = (2n+1)(N-2x)\, f_n(x) - n\,(N+n+1)\, f_{n-1}(x)$$

where

$$n = 1, 2, ..., m - 1.$$

Then let

$$(f_n, f_n) = \frac{(N + n + 1)!\,(N - n)!}{(2n + 1)\,(N!)^2}$$

$$(f, f_n) = \sum_{j=0}^{n} f_n(j)\, f(x_j)$$

and

$$a_n = \frac{(f, f_n)}{(f_n, f_n)} .$$

This program computes all values of (f, f_n) for $n = 0, 1, 2, 3, 4$. If the degree $m = 4$, all terms are used; if $m = 3$, (f, f_4) is replaced by zero in later calculations; and if $m = 2$, (f, f_4) and (f, f_3) are both replaced by zero.

Let $g_n(u)$ be the symmetrical form of the orthogonal polynomial in the domain $-1 < u < 1$ such that

$$g_0(u) = 1 \qquad g_1(u) = u$$

and

$$g_{n+1}(u) = \frac{(2n + 1)\,N}{(n + 1)\,(N - n)}\, u g_n(u) - \frac{n(N + n + 1)}{(n + 1)\,(N - n)}\, g_{n-1}(u)$$

where

$$n = 1, 2, ..., m - 1.$$

The program computes the coefficients of the polynomial

$$\sum_{n=0}^{N} a_n\, g_n(u) = b_0 + b_1\, u + b_2\, u^2 + b_3\, u^3 + b_4\, u^4 . \qquad (1)$$

Then $g_n(u)$ is shifted to a proper interval between x_0 and x_N by letting

$$u = \beta + \alpha x$$

where

$$\alpha = -\frac{2}{x_N - x_0}$$

$$\beta = \frac{x_N + x_0}{x_N - x_0}$$

The transformation is done in two steps. First, let $z = u - \beta$, thus (1) becomes

$$c_0 + c_1 z + c_2 z^2 + c_3 z^3 + c_4 z^4 \qquad (2)$$

where

$$c_0 = b_0 + b_1 \beta + b_2 \beta^2 + b_3 \beta^3 + b_4 \beta^4$$

$$c_1 = b_1 + 2b_2 \beta + 3b_3 \beta^2 + 4b_4 \beta^3$$

$$c_2 = b_2 + 3b_3 \beta + 6b_4 \beta^2$$

$$c_3 = b_3 + 4 b_4 \beta$$

$$c_4 = b_4 .$$

Then set $x = \alpha z$ and (2) becomes

$$d_0 + d_1 x + d_2 x^2 + d_3 x^3 + d_4 x^4 \qquad (3)$$

where

$$d_i = \alpha^i c_i \; (i = 0, 1, 2, 3, 4).$$

(3) is the polynomial approximation for the function f(x).

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1970.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program on card 1				
2	Input N*	N	R/S		N
3	Perform 3 for i = 0, 1,..., N	$f(x_i)$	R/S		i + 1
4	For 2nd order fit		GTO	2	
			R/S		0.00
	or				
	For 3rd order fit		GTO	3	
			R/S		0.00
	or				
	For 4th order fit		GTO	4	
			R/S		N + 1
5	Enter program on card 2		R/S		0.00
6	Enter program on card 3		R/S		1.00
7	Input x_N	x_N	↑		
	and x_0	x_0	R/S		$-\alpha$
8	Enter program on card 4				
9	Compute coefficients		R/S		d_0
			SST		d_1
			SST		d_2
			SST		d_3
			SST		d_4
10	For a new case, go to 1				
	* N = number of data points –1				

Example:

Find a third order polynomial approximation for the following data.

x	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
f(x)	2.72	3.49	4.48	5.75	7.39	9.49	12.18	15.64	20.09

(Note: $f(x) = e^x$)

Keystrokes:

Enter program on card 1

8 **R/S** → 8.00

2.72 **R/S** 3.49 **R/S** ... 20.09 **R/S** → 9.00

GTO [3] **R/S** → 0.00

Enter program on card 2

R/S → 0.00

Enter program on card 3

R/S → 1.00

3 [↑] 1 **R/S** → 1.00 (–α)

Enter program on card 4

R/S → –1.79 (d_0)

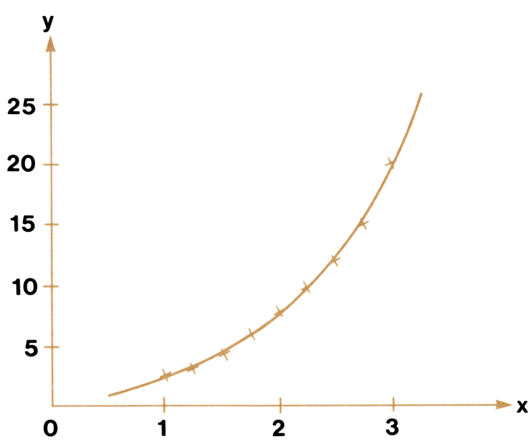
SST → 7.03 (d_1)

SST → –3.85 (d_2)

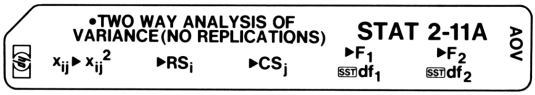
SST → 1.31 (d_3)

SST → 0.00 (d_4)

The polynomial is $-1.79 + 7.03\ x - 3.85\ x^2 + 1.31\ x^3$.



TWO WAY ANALYSIS OF VARIANCE (NO REPLICATIONS)



The two way analysis of variance tests the row effects and the column effects independently. This program will generate the ANOVA table for the case such that (1) each cell only has one observation and (2) the row and column effects do not interact.

Equations:

1. Sums

$$\text{Row } RS_i = \sum_j x_{ij} \qquad i = 1, 2, ..., r$$

$$\text{Column } CS_j = \sum_i x_{ij} \qquad j = 1, 2, ..., c$$

2. Sums of squares

$$\text{Total TSS} = \Sigma\Sigma x_{ij}^2 - (\Sigma\Sigma x_{ij})^2/rc$$

$$\text{Row RSS} = \sum_i \left(\sum_j x_{ij}\right)^2/c - (\Sigma\Sigma x_{ij})^2/rc$$

$$\text{Column CSS} = \sum_j \left(\sum_i x_{ij}\right)^2/r - (\Sigma\Sigma x_{ij})^2/rc$$

$$\text{Error ESS} = \text{TSS} - \text{RSS} - \text{CSS}$$

3. Degrees of freedom

$$\text{Row } df_1 = r - 1$$

$$\text{Column } df_2 = c - 1$$

$$\text{Error } df_3 = (r - 1)(c - 1)$$

4. F ratios

$$\text{Row } F_1 = \frac{RSS}{df_1} \Big/ \frac{ESS}{df_3}$$

$$\text{Column } F_2 = \frac{CSS}{df_2} \Big/ \frac{ESS}{df_3}$$

Reference:

Dixon and Massey, *Introduction to Statistical Analysis,* McGraw-Hill, 1969.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Initialize for rows		f	REG	
3	Input number of rows r	r	STO	5	r
	number of columns c	c	STO	6	c
4	Perform 4-6 for i = 1, 2,..., r				
5	Perform 5 for j = 1, 2,..., c	x_{ij}	A		x_{ij}^2
6	Compute row sums		B		RS_i
7	Initialize for columns		R/S		0.00
8	Perform 8-10 for j = 1, 2,..., c				
9	Perform 9 for i = 1, 2,..., r	x_{ij}	A		x_{ij}^2
10	Compute column sums		C		CS_j
11	Compute F ratio for rows		D		F_1
	row degrees of freedom		SST		df_1
12	Compute F ratio for columns		E		F_2
	column degrees of freedom		SST		df_2
13	Recall error degrees of freedom		RCL	7	df_3
14	Recall total sum of squares		RCL	1	TSS
	row sum of squares		RCL	2	RSS
	column sum of squares		RCL	3	CSS
	error sum of squares		RCL	4	ESS
15	For a new case, go to 2				

Example:

Row \ Column (i \ j)	1	2	3	4
1	7	6	8	7
2	2	4	4	4
3	4	6	5	3

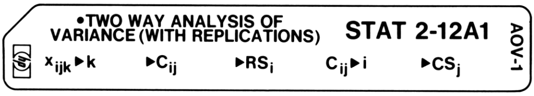
Keystrokes:

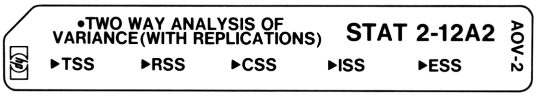
[f] [REG] 3 [STO] [5] 4 [STO] [6]	4.00
7 [A] 6 [A] 8 [A] 7 [A] [B]	28.00 (RS_1)
2 [A] 4 [A] 4 [A] 4 [A] [B]	14.00 (RS_2)
4 [A] 6 [A] 5 [A] 3 [A] [B]	18.00 (RS_3)
[R/S]	0.00
7 [A] 2 [A] 4 [A] [C]	13.00 (CS_1)
6 [A] 4 [A] 6 [A] [C]	16.00 (CS_2)
8 [A] 4 [A] 5 [A] [C]	17.00 (CS_3)
7 [A] 4 [A] 3 [A] [C]	14.00 (CS_4)
[D]	11.70 (F_1)
[SST]	2.00 (df_1)
[E]	1.00 (F_2)
[SST]	3.00 (df_2)
[RCL] [7]	6.00 (df_3)
[RCL] [1]	36.00 (TSS)
[RCL] [2]	26.00 (RSS)
[RCL] [3]	3.33 (CSS)
[RCL] [4]	6.67 (ESS)

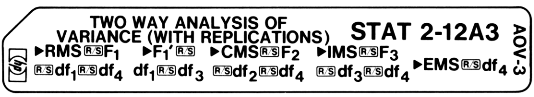
ANOVA

	SS	df	F ratio
Row	26.00	2	11.70
Column	3.33	3	1.00
Error	6.67	6	
Total	36.00		

TWO WAY ANALYSIS OF VARIANCE (WITH REPLICATIONS)







This program generates the complete two way ANOVA table for the fixed or mixed model with interactions. The number of observations per cell must be equal. Let r be the number of rows, c be the number of columns, and n be the cell size. The mixed model covered by this program is "fixed row, random column effects". The case with "random row, fixed column effects" can also be handled by writing the table with the fixed treatments shown in rows. Note that only the statistics for row effects are different for the two models (fixed or mixed).

Equations:

1. Sums

Cell $$C_{ij} = \sum_k x_{ijk}$$

Row $$RS_i = \sum_j \sum_k x_{ijk}$$

Column $$CS_j = \sum_i \sum_k x_{ijk}$$

where

$i = 1, 2, ..., r$

$j = 1, 2, ..., c$

$k = 1, 2, ..., n$

2. Sums of squares

Total $$TSS = \sum_i \sum_j \sum_k x_{ijk}^2 - \left(\sum_i \sum_j \sum_k x_{ijk}\right)^2 / nrc$$

Row $$RSS = \sum_i \left(\sum_j \sum_k x_{ijk}\right)^2 / nc - \left(\sum_i \sum_j \sum_k x_{ijk}\right)^2 / nrc$$

Column $$CSS = \sum_j \left(\sum_i \sum_k x_{ijk}\right)^2 / nr - \left(\sum_i \sum_j \sum_k x_{ijk}\right)^2 / nrc$$

Interaction $$ISS = \sum_i \sum_j \left(\sum_k x_{ijk}\right)^2 / n - \sum_i \left(\sum_j \sum_k x_{ijk}\right)^2 / nc$$

$$- \sum_j \left(\sum_i \sum_k x_{ijk}\right)^2 / nr + \left(\sum_i \sum_j \sum_k x_{ijk}\right)^2 \; nrc$$

$$= TSS - RSS - CSS - ESS$$

Error $$ESS = \sum_i \sum_j \sum_k x_{ijk}^2 - \sum_i \sum_j \left(\sum_k x_{ijk}\right)^2 / n$$

3. Degrees of freedom

$df_1 = r - 1$

$df_2 = c - 1$

$df_3 = (r - 1)(c - 1)$

$df_4 = rc(n - 1)$

4. Mean squares

Row $$RMS = \frac{RSS}{df_1}$$

Column $$CMS = \frac{CSS}{df_2}$$

Interaction $$IMS = \frac{ISS}{df_3}$$

Error $$EMS = \frac{ESS}{df_4}$$

5. F ratios

Row
- fixed model $F_1 = \dfrac{RMS}{EMS}$
- mixed model $F_1' = \dfrac{RMS}{IMS}$

Column $F_2 = \dfrac{CMS}{EMS}$

Interaction $F_3 = \dfrac{IMS}{EMS}$

References:

1. Dixon and Massey, *Introduction to Statistical Analysis*, McGraw-Hill, 1969.

2. I. N. Gibra, *Probability and Statistical Inference for Scientists and Engineers*, Prentice-Hall, 1973.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program on card 1				
2	Initialize		f	REG	
3	Perform 3-8 for i = 1, 2,..., r				
4	Perform 4-7 for j = 1, 2,..., c				
5	Perform 5 for k = 1, 2,..., n	x_{ijk}	A		k
6	Optional—correct erroneous				
	x_{ijm}	x_{ijm}	GTO	1	
			R/S		
7	Compute and record cell total		B		C_{ij}
8	Compute the row sum		C		RS_i
9	Perform 9-11 for j = 1, 2,..., c				
10	Perform 10 for i = 1, 2,..., r	C_{ij}	D		i
11	Compute the column sum		E		CS_j
12	Enter program on card 2				
13	Input number of rows	r	R/S		r
	number of columns	c	R/S		c
	cell size	n	R/S		n

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
14	Compute total sum of squares		A		TSS
	row sum of squares		B		RSS
	column sum of squares		C		CSS
	interaction sum of squares		D		ISS
	error sum of squares		E		ESS
15	Enter program on card 3				
16	Compute row mean square		A		RMS
17	Fixed model—row F ratio		R/S		F_1
	row degrees of freedom		R/S		df_1
	error degrees of freedom		R/S		df_4
	or				
18	Mixed model—row F ratio		B		F_1'
	row degrees of freedom		R/S		df_1
	interaction degrees of				
	freedom		R/S		df_3
19	Compute column mean square		C		CMS
	column F ratio		R/S		F_2
	column degrees of freedom		R/S		df_2
	error degrees of freedom		R/S		df_4
20	Compute interaction mean				
	square		D		IMS
	interaction F ratio		R/S		F_3
	interaction degrees of				
	freedom		R/S		df_3
	error degrees of freedom		R/S		df_4
21	Compute error mean square		E		EMS
	error degrees of freedom		R/S		df_4
22	For a new case, go to 1				

Example:

i \ j	1	2	3
1	4, 7, 5	2, 3, 2	5, 6, 4
2	9, 8, 8	8, 7, 5	10, 8, 7

Keystrokes:

Enter program on card 1

f REG 4 A 7 A 5 A B ⟶ 16.00 (C_{11})
2 A 3 A 2 A B ⟶ 7.00 (C_{12})
5 A 6 A 4 A B ⟶ 15.00 (C_{13})
C ⟶ 38.00 (RS_1)
9 A 8 A 8 A B ⟶ 25.00 (C_{21})
8 A 7 A 5 A B ⟶ 20.00 (C_{22})
10 A 8 A 7 A B ⟶ 25.00 (C_{23})
C ⟶ 70.00 (RS_2)
16 D 25 D E ⟶ 41.00 (CS_1)
7 D 20 D E ⟶ 27.00 (CS_2)
15 D 25 D E ⟶ 40.00 (CS_3)

Enter program on card 2

2 R/S 3 R/S 3 R/S A ⟶ 96.00 (TSS)
B ⟶ 56.89 (RSS)
C ⟶ 20.33 (CSS)
D ⟶ 1.44 (ISS)
E ⟶ 17.33 (ESS)

Enter program on card 3

A —— 56.89 (RMS)

For fixed model:

R/S ⟶ 39.38 (F_1)
R/S ⟶ 1.00 (df_1)
R/S ⟶ 12.00 (df_4)

or

For mixed model:

B ⟶ 78.77 (F_1')
R/S ⟶ 1.00 (df_1)
R/S ⟶ 2.00 (df_3)

Key		Display
C	→	10.17 (CMS)
R/S	→	7.04 (F_2)
R/S	→	2.00 (df_2)
R/S	→	12.00 (df_4)
D	→	0.72 (IMS)
R/S	→	0.50 (F_3)
R/S	→	2.00 (df_3)
R/S	→	12.00 (df_4)
E	→	1.44 (EMS)
R/S	→	12.00 (df_4)

Cell Totals

i \ j	1	2	3
1	16	7	15
2	25	20	25

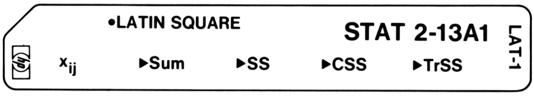
ANOVA for fixed row, fixed column effects

	SS	MS	df	F
Row	56.89	56.89	1	39.38
Column	20.33	10.17	2	7.04
Interaction	1.44	0.72	2	0.50
Error	17.33	1.44	12	
Total	96.00			

ANOVA for fixed row, random column effects

	SS	MS	df	F
Row	56.89	56.89	1	78.77
Column	20.33	10.17	2	7.04
Interaction	1.44	0.72	2	0.50
Error	17.33	1.44	12	
Total	96.00			

LATIN SQUARE



A Latin square design is an incomplete three-way layout in which all the three factors are at the same number n of levels and observations are taken on only n^2 of the n^3 possible combinations. The program generates the complete ANOVA table for the Latin square design. The analysis tests the effects of the rows, the columns and the treatments.

Example of a 3 x 3 Latin square

	j \ i	Column 1	2	3
Row	1	A	B	C
	2	C	A	B
	3	B	C	A

A, B, C are the three different treatments. x_{ij} represents the value in the i^{th} row and the j^{th} column. In the above example, x_{31} which belongs to the second treatment B, represents the value in the 3^{rd} row and the 1^{st} column.

Equations:

1. Row sums

$$RS_i = \sum_j x_{ij} \qquad i = 1, 2, ..., n$$

2. Total sum of squares

$$TSS = \Sigma\Sigma x_{ij}^2 - \frac{(\Sigma\Sigma x_{ij})^2}{n^2}$$

3. Row sum of squares

$$RSS = \frac{1}{n} \sum_i \left(\sum_j x_{ij} \right)^2 - \frac{(\Sigma\Sigma x_{ij})^2}{n^2}$$

Similar formulas can be obtained for column sums CS_i, column sum of squares CSS, treatment sums TrS_i and treatment sum of squares TrSS.

4. Residual sum of squares

$$ReSS = TSS - RSS - CSS - TrSS$$

5. Degrees of freedom

$$df_1 = n - 1$$

$$df_2 = n^2 - 3n + 2$$

6. Mean squares

$$RMS = \frac{RSS}{df_1}$$

$$CMS = \frac{CSS}{df_1}$$

$$TrMS = \frac{TrSS}{df_1}$$

$$ReMS = \frac{ReSS}{df_2}$$

7. F ratios

$$F_1 = \frac{RMS}{ReMS}$$

$$F_2 = \frac{CMS}{ReMS}$$

$$F_3 = \frac{TrMS}{ReMS}$$

References:

1. Dixon and Massey, *Introduction to Statistical Analysis,* McGraw-Hill, 1969.

2. H. Scheffé, *The Analysis of Variance,* John Wiley and Sons, 1970.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program on card 1				
2	Initialize for row sums		f	REG	
3	Perform 3-5 for i = 1, 2,..., n				
4	Perform 4 for j = 1, 2,..., n	x_{ij}	A		j
5	Compute row sums		B		RS_i
6	Compute total sum of squares		C		TSS
	row sum of squares		C		RSS
7	Initialize for column sums		R/S		0.00
8	Perform 8-10 for j = 1, 2,..., n				
9	Perform 9 for i = 1, 2,..., n	x_{ij}	A		i
10	Compute column sums		B		CS_j
11	Compute total sum of squares		C		TSS*
	column sum of squares		D		CSS
12	Initialize for treatment sums		R/S		0.00
13	Perform 13-15 for k = 1, 2,..., n				
14	Perform 14 for x_{ij} in treatment k	x_{ij}	A		
15	Compute the treatment sums		B		TrS_k
16	Compute total sum of squares		C		TSS*
	treatment sum of squares		E		TrSS
	residual sum of squares		R/S		ReSS
17	Enter program on card 2				
18	Compute row mean square		A		RMS
	F ratio for rows		A		F_1
19	Compute column mean square		B		CMS
	F ratio for columns		B		F_2
20	Compute treatment mean				
	square		C		TrMS
	F ratio for treatments		C		F_3
21	Compute residual mean square		D		ReMS

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
22	Compute degrees of freedom		E		df_1
			E		df_2
23	For a new case, go to 1				
	* If this TSS does not agree				
	with the one computed in step				
	6, go to step 2 and restart the				
	calculations.				

Example:

i \ j	1	2	3
1	.194 A	.73 B	1.187 C
2	.758 C	.311 A	.589 B
3	.369 B	.558 C	.311 A

Keystrokes:

Enter program on card 1

f REG .194 A .73 A 1.187 A B ——→ 2.11 (RS_1)

.758 A .311 A .589 A B ——→ 1.66 (RS_2)

.369 A .558 A .311 A B ——→ 1.24 (RS_3)

C ——→ 0.76 (TSS)

C ——→ 0.13 (RSS)

R/S → 0.00

.194 A .758 A .369 A B → 1.32 (CS_1)

.73 A .311 A .558 A B → 1.60 (CS_2)

1.187 A .589 A .311 A B → 2.09 (CS_3)

C → 0.76 (TSS)

D → 0.10 (CSS)

R/S → 0.00

.194 A .311 A .311 A B → 0.82 (TrS_1)

.73 A .589 A .369 A B → 1.69 (TrS_2)

1.187 A .758 A .558 A B → 2.50 (TrS_3)

C → 0.76 (TSS)

E → 0.47 (TrSS)

R/S → 0.05 (ReSS)

Enter program on card 2

A → 0.06 (RMS)

A → 2.33 (F_1)

B → 0.05 (CMS)

B → 1.84 (F_2)

C → 0.24 (TrMS)

C → 8.70 (F_3)

D → 0.03 (ReMS)

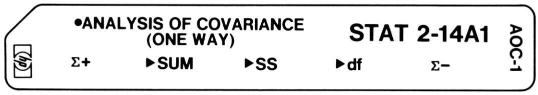
E → 2.00 (df_1)

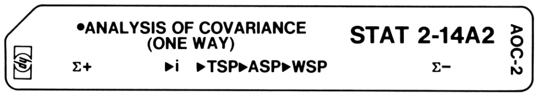
E → 2.00 (df_2)

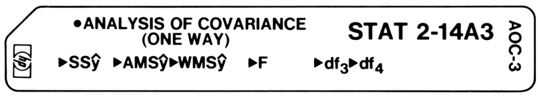
ANOVA Table

	SS	df	MS	F
Row	0.13	2	0.06	2.33
Column	0.10	2	0.05	1.84
Treatment	0.47	2	0.24	8.70
Residual	0.05	2	0.03	
Total	0.76			

ANALYSIS OF COVARIANCE (ONE WAY)







The one way analysis of covariance tests the effect of one variable separately from the effect of the second variable if the second variable represents an actual measurement for each individual (rather than a category).

Suppose (x_{ij}, y_{ij}) represents the j^{th} observation from the i^{th} population $(i = 1, 2, ..., k, j = 1, 2, ..., n_i)$. Note that samples may have equal or unequal number of observations. The analysis of covariance tests for a difference in means of residuals. The residuals are the differences of the observations and a regression quantity based on the associated second variable. The analysis of covariance procedure is based on the separations of the sums of squares and the sums of products into several portions. This program will generate the complete ANOCOV table.

Equations:

1. Sums and sums of squares

$$Sx_i = \sum_j x_{ij} \qquad (i = 1, 2, ..., k)$$

$$TSSx = \Sigma\, \Sigma x_{ij}^2 - \frac{(\Sigma\Sigma x_{ij})^2}{\sum_i n_i}$$

$$ASSx = \sum_i \frac{\left(\sum_j x_{ij}\right)^2}{n_i} - \frac{(\Sigma\Sigma x_{ij})^2}{\sum_i n_i}$$

$$WSSx = TSSx - ASSx$$

2. Degrees of freedom

$$df_1 = k - 1$$

$$df_2 = \sum_i n_i - k$$

3. Mean squares and F statistic

$$AMSx = \frac{ASSx}{df_1}$$

$$WMSx = \frac{WSSx}{df_2}$$

$$F_x = \frac{AMSx}{WMSx} \text{ with degrees of freedom } df_1, df_2$$

By changing x_{ij} to y_{ij}, similar formulas for y_{ij} can be obtained.

4. Sums of products

$$TSP = \Sigma\Sigma x_{ij}\, y_{ij} - \frac{(\Sigma\Sigma x_{ij})\,(\Sigma\Sigma y_{ij})}{\sum_i n_i}$$

$$ASP = \sum_i \frac{\left(\sum_j x_{ij}\right)\left(\sum_j y_{ij}\right)}{n_i} - \frac{(\Sigma\Sigma x_{ij})\,(\Sigma\Sigma y_{ij})}{\sum_i n_i}$$

$$WSP = TSP - ASP$$

5. Residual sums of squares

$$TSS\hat{y} = TSSy - \frac{(TSP)^2}{TSSx}$$

$$WSS\hat{y} = WSSy - \frac{(WSP)^2}{WSSx}$$

$$ASS\hat{y} = TSS\hat{y} - WSS\hat{y}$$

6. Residual degrees of freedom

$$df_3 = k - 1$$

$$df_4 = \sum_i n_i - k - 1$$

7. Residual mean squares and F statistic

$$AMS\hat{y} = \frac{ASS\hat{y}}{df_3}$$

$$WMS\hat{y} = \frac{WSS\hat{y}}{df_4}$$

$$F = \frac{AMS\hat{y}}{WMS\hat{y}} \text{ with degrees of freedom } df_3, df_4$$

ANOCOV Table

	degrees of freedom	SSx	SP	SSy	degrees of freedom	Residuals SS$\hat{y}$	MS$\hat{y}$	F statistic
Among means	df_1	ASSx	ASP	ASSy	df_3	ASS$\hat{y}$	AMS$\hat{y}$	F
Within groups	df_2	WSSx	WSP	WSSy	df_4	WSS$\hat{y}$	WMS$\hat{y}$	
Total		TSSx	TSP	TSSy		TSS$\hat{y}$		

Remarks:

1. F_x can be used to test if the X means are equal (ANOVA for X).

   F_y can be used to test if the Y means (not making use of the X values) are equal (ANOVA for unadjusted Y).

2. In order to get more accurate answers, intermediate results (TSSx, WSSx, TSSy, WSSy) should be recorded and entered to as many decimal places as possible.

Reference:

Dixon and Massey, *Introduction to Statistical Analysis,* McGraw-Hill, 1969.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program on card 1				
2	Initialize		f	REG	
3	Perform 3-6 for i = 1, 2,..., k				
4	Perform 4 for j = 1, 2,..., n_i	x_{ij}	A		j
5	Optional—correct erroneous x_{im}	x_{im}	E		
6	Compute the sum		B		Sx_i
7	Compute the total sum of				
	squares		C		TSSx
	among means sum of squares		C		ASSx
	within groups sum of squares		C		WSSx
8	Compute degrees of freedom		D		df_1
			D		df_2
	and F_x		D		F_x
9	Initialize		f	REG	
10	Perform 10-13 for i = 1, 2,..., k				
11	Perform 11 for j = 1, 2,..., n_i	y_{ij}	A		j
12	Optional—correct erroneous y_{ih}	y_{ih}	E		
13	Compute the sum		B		Sy_i
14	Compute the total sum of				
	squares		C		TSSy
	among means sum of squares		C		ASSy
	within groups sum of squares		C		WSSy

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
15	Compute degrees of freedom		D		df_1
			D		df_2
	and F_y		D		F_y
16	Enter program on card 2				
17	Initialize		f	REG	
18	Perform 18–21 for i = 1, 2,..., k				
19	Perform 19 for j = 1, 2,..., n_i	x_{ij}	↑		
		y_{ij}	A		j
20	Optional—correct erroneous				
	x_{im}, y_{im}	x_{im}	↑		
		y_{im}	E		
21	Store sums for the ith popula-				
	tion		B		i
22	Compute the total sum of				
	products (SP)		C		TSP
	among means SP		C		ASP
	within groups SP		C		WSP
23	Enter program on card 3				
24	Input TSSx, TSSy and com-				
	pute $TSS\hat{y}$	TSSx	↑		
		TSSy	A		$TSS\hat{y}$
25	Input WSSx, WSSy and com-				
	pute $WSS\hat{y}$	WSSx	↑		
		WSSy	A		$WSS\hat{y}$
26	Compute $ASS\hat{y}$		A		$ASS\hat{y}$
27	Compute residual mean squares		B		$AMS\hat{y}$
			B		$WMS\hat{y}$
28	Compute the F statistic		C		F
29	Compute the degrees of				
	freedom		D		df_3
			D		df_4
30	For a new case, go to 1				

Example:

			j			
			1	2	3	4
		x	3	2	1	2
	1	y	10	8	8	11
		x	4	3	3	5
i	2	y	12	12	10	13
		x	1	2	3	1
	3	y	6	5	8	7

($k = 3, n_1 = n_2 = n_3 = 4$)

Keystrokes:

Enter program on card 1

f REG 3 A 2 A 1 A 2 A B → 8.00 (Sx_1)

4 A 3 A 3 A 5 A B → 15.00 (Sx_2)

1 A 2 A 3 A 1 A B → 7.00 (Sx_3)

C → 17.00 (TSSx)

C → 9.50 (ASSx)

C → 7.50 (WSSx)

D → 2.00 (df_1)

D → 9.00 (df_2)

D → 5.70 (F_x)

f REG 10 A 8 A 8 A 11 A B → 37.00 (Sy_1)

12 A 12 A 10 A 13 A B → 47.00 (Sy_2)

6 A 5 A 8 A 7 A B → 26.00 (Sy_3)

C → 71.67 (TSSy)

C → 55.17 (ASSy)

C → 16.50 (WSSy)

D → 2.00 (df_1)

D → 9.00 (df_2)

D → 15.05 (F_y)

Enter program on card 2

[f] [REG] 3 [↑] 10 [A] 2 [↑] 8 [A] 1 [↑] 8 [A] 2 [↑] 11 [A] [B] → 1.00

4 [↑] 12 [A] . . . 5 [↑] 13 [A] [B] → 2.00

1 [↑] 6 [A] . . . 1 [↑] 7 [A] [B] → 3.00

[C] → 27.00 (TSP)

[C] → 20.75 (ASP)

[C] → 6.25 (WSP)

Enter program on card 3

17 [↑] 71.67 [A] → 28.79 (TSS$\hat{y}$)

7.5 [↑] 16.5 [A] → 11.29 (WSS$\hat{y}$)

[A] → 17.50 (ASS$\hat{y}$)

[B] → 8.75 (AMS$\hat{y}$)

[B] → 1.41 (WMS$\hat{y}$)

[C] → 6.20 (F)

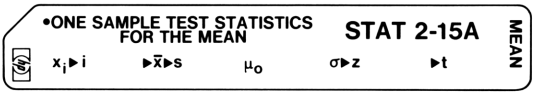
[D] → 2.00 (df_3)

[D] → 8.00 (df_4)

ANOCOV Table

					Residuals			
	df	**SSx**	**SP**	**SSy**	**df**	**SS$\hat{y}$**	**MS$\hat{y}$**	**F**
Among means	2	9.50	20.75	55.17	2	17.50	8.75	6.20
Within groups	9	7.50	6.25	16.50	8	11.29	1.41	
Total		17.00	27.00	71.67		28.79		

ONE SAMPLE TEST STATISTICS FOR THE MEAN



Suppose $\{x_1, x_2, ..., x_n\}$ is a sample from a normal population with a known variance σ^2 and unknown mean μ. A test of the null hypothesis

$$H_0: \mu = \mu_0$$

is based on the z statistic which has a standard normal distribution.

If the variance σ^2 is unknown then the t statistic, which has the t distribution with n – 1 degrees of freedom, is used instead.

Equations:

$$z = \frac{\sqrt{n}\,(\bar{x} - \mu_0)}{\sigma}$$

$$t = \frac{\sqrt{n}\,(\bar{x} - \mu_0)}{s}$$

where $\bar{x}$ and s are sample mean and sample standard deviation.

Remark:

$n > 1$.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	If $\bar{x}$ and s are known, go to 8				
3	Initialize		RTN	R/S	0.00
4	Perform 4 for i = 1, 2,..., n	x_i	A		i
5	Optional—delete erroneous				
	data x_k ($k \neq 1$)	x_k	GTO	1	
			R/S		
6	Compute $\bar{x}$ and s		B		$\bar{x}$
			B		s
7	Go to 9				
8	Store $\bar{x}$ and s	$\bar{x}$	STO	2	
		s	STO	5	
9	Input μ_0	μ_0	C		
10	Input σ and compute z	σ	D		z
	or				
	Compute t		E		t
11	For a new case, go to 2				

Example:

Compute the z and the t statistics for the following set of data if $\mu_0 = 2$ and $\sigma = 1$.

{2.73, 0.45, 2.52, 1.19, 3.51, 2.75, 1.79, 1.83, 1, 0.87, 1.9, 1.62, 1.74, 1.92, 1.24, 2.68}

Keystrokes:

RTN R/S ——→ 0.00
2.73 A .45 A . . . 2.68 A ——→ 16.00
B ——→ 1.86 ($\bar{x}$)
B ——→ 0.82 (s)
2 C ——→ 2.00
1 D ——→ –0.57 (z)
E ——→ –0.69 (t)

TEST STATISTICS FOR THE CORRELATION COEFFICIENT

Under the assumptions of normal correlation analysis, the t statistic, which has the t distribution with n – 2 degrees of freedom, can be used to test the null hypothesis that the true correlation coefficient $\rho = 0$.

To test the null hypothesis $\rho = \rho_0$, where ρ_0 is a given number, the z statistic is used. z has approximately the standard normal distribution.

Equations:

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

$$z = \frac{\sqrt{n-3}}{2} \ln\left[\frac{(1+r)(1-\rho_0)}{(1-r)(1+\rho_0)}\right]$$

where r is an estimate (based on a sample of size n) of the correlation coefficient ρ.

Remarks:

1. This program requires that $n > 3$, $|r| < 1$ and $|\rho_0| < 1$; otherwise, flashing zeros will result.
2. Usually, the z statistic is used when the sample size is large.

References:

1. Hogg and Craig, *Introduction to Mathematical Statistics,* Macmillan Co., 1970.
2. J. Freund, *Mathematical Statistics,* Prentice-Hall, 1971.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Input r and n in any order	r	A		
		n	B		
3	Compute t		C		t
	or				
	Input ρ_0 and compute z	ρ_0	D		
			E		z
4	For a new case, go to 2				

Example:

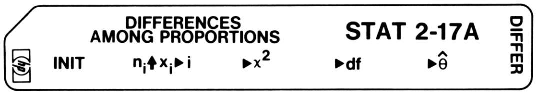
Given r = 0.12, n = 31, and ρ_0 = 0, find t and z.

Keystrokes:

.12 **A** 31 **B** **C** ⟶ 0.65 (t)

0 **D** **E** ⟶ 0.64 (z)

DIFFERENCES AMONG PROPORTIONS



Suppose $x_1, x_2, \ldots, x_k$ are observed values of a set of independent random variables having binomial distributions with parameters n_i and θ_i $(i = 1, 2, \ldots, k)$.

A chi-square statistic χ^2 can be used to test the null hypothesis $\theta_1 = \theta_2 = \ldots = \theta_k$. The χ^2 statistic has the chi-square distribution with $k - 1$ degrees of freedom.

Equation:

$$\chi^2 = \sum_{i=1}^{k} \frac{(x_i - n_i \hat{\theta})^2}{n_i \hat{\theta}(1 - \hat{\theta})} = \sum_{i=1}^{k} n_i \left[\frac{1}{\sum_{i=1}^{k} x_i} \sum_{i=1}^{k} \frac{x_i^2}{n_i} + \frac{1}{\sum_{i=1}^{k} (n_i - x_i)} \sum_{i=1}^{k} \frac{(n_i - x_i)^2}{n_i} - 1 \right]$$

where

$$\hat{\theta} = \sum_{i=1}^{k} x_i \Big/ \sum_{i=1}^{k} n_i$$

Reference:

J. Freund, *Mathematical Statistics,* Prentice-Hall, 1971.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Initialize		A		0.00
3	Perform 3 for i = 1, 2,..., k	n_i	↑		
		x_i	B		i
4	Compute χ^2 statistic		C		χ^2
5	Compute df		D		df
6	Compute $\hat{\theta}$		E		$\hat{\theta}$
7	For a new case, go to 2				

Example:

	n_i	x_i
Sample 1	400	232
Sample 2	500	260
Sample 3	400	197

Keystrokes:

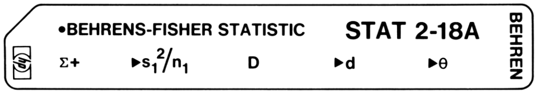
A 400 ↑ 232 B 500 ↑ 260 B 400 ↑ 197 B → 3.00 (k)

C → 6.47 (χ^2)

D → 2.00 (df)

E → 0.53 ($\hat{\theta}$)

BEHRENS-FISHER STATISTIC



Suppose $\{x_1, x_2, ..., x_{n_1}\}$ and $\{y_1, y_2, ..., y_{n_2}\}$ are independent random samples from two normal populations having means μ_1, μ_2 (unknown). If the variances σ_1^2, σ_2^2 cannot be assumed equal, then the Behrens-Fisher statistic d is used instead of the t statistic to test the null hypothesis

$$H_0: \mu_1 - \mu_2 = D.$$

Equation:

$$d = \frac{\bar{x} - \bar{y} - D}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

where $\bar{x}, \bar{y}$ and s_1^2, s_2^2 are sample means and variances.

Critical values of this test are tabulated in the Fisher-Yates Tables for various values of n_1, n_2, α and θ, where α is the level of significance and

$$\theta = \tan^{-1}\left(\frac{s_1}{s_2}\sqrt{\frac{n_2}{n_1}}\right).$$

Remark:

$n_1 > 1, n_2 > 1.$

Reference:

Fisher and Yates, *Statistical Tables for Biological, Agricultural and Medical Research,* Hafner Publishing Co., 1970.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	If $\bar{x}$, $\bar{y}$ and s_1, s_2 are known, go				
	to 11				
3	Initialize		RTN	R/S	0.00
4	Perform 4 for i = 1, 2,..., n_1	x_i	A		i
5	Optional—delete erroneous x_k	x_k	GTO	1	
	$(k \neq 1)$		R/S		
6	Compute and store $\bar{x}$, $s_1{}^2/n_1$		B		$s_1{}^2/n_1$
7	Initialize		RTN	R/S	0.00
8	Perform 8 for i = 1, 2,..., n_2	y_i	A		i
9	Optional—delete erroneous y_h	y_h	GTO	1	
	$(h \neq 1)$		R/S		
10	Go to 12				
11	Store $\bar{x}$, $\bar{y}$ and $s_1{}^2/n_1$, $s_2{}^2/n_2$				
	in any order	$\bar{x}$	STO	5	
		$s_1{}^2/n_1$	STO	6	
		$\bar{y}$	STO	2	
		$s_2{}^2/n_2$	STO	3	
12	Input D	D	C		
13	Compute d and θ		D		d
			E		θ
14	Optional—recall means		RCL	5	$\bar{x}$
			RCL	2	$\bar{y}$
15	For a different D, go to 12				
16	For a new case, go to 2				

Example:

Compute the Behrens-Fisher statistic for D = 0.

x:	79,	84,	108,	114,	120,	103,	122,	120		
y:	91,	103,	90,	113,	108,	87,	100,	80,	99,	54

Keystrokes:

RTN R/S 79 A 84 A ... 120 A ⟶ 8.00 (n_1)

B ⟶ 34.60 ($s_1{}^2/n_1$)

RTN R/S 91 A 103 A ... 54 A ⟶ 10.00 (n_2)

0 C D ⟶ 1.73 (d)

E ⟶ 47.88° (θ)

or 0.84 radians

or 53.20 grads

KRUSKAL-WALLIS STATISTIC

Suppose we want to test the null hypothesis that k independent random samples of sizes $n_1, n_2, \ldots, n_k$ come from identical continuous populations.

Arrange all values from k samples jointly (as if they were one sample) in an increasing order of magnitude. Let R_{ij} $(i = 1, 2, \ldots, k, j = 1, 2, \ldots, n_i)$ be the rank of the j^{th} value in the i^{th} sample.

The Kruskal-Wallis statistic H can be used to test the null hypothesis.

When all sample sizes are large (> 5), H is distributed approximately as the chi-square with k − 1 degrees of freedom. For small samples, the test is based on special tables.

Equation:

$$H = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{\left(\sum_{j=1}^{n_i} R_{ij} \right)^2}{n_i} - 3(N+1)$$

where

$$N = \sum_{i=1}^{k} n_i$$

Reference:

W. J. Conover, *Practical Nonparametric Statistics,* John Wiley and Sons, 1971.

Table for small samples (k = 3):

Alexander and Quade, *On the Kruskal-Wallis Three Sample H-statistic,* University of North Carolina, Department of Biostatistics, Inst. Statistics Mimeo Ser. 602, 1968.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Initialize		A		0.00
3	Perform 3-6 for i = 1, 2,..., k				
4	Perform 4 for j = 1, 2,..., n_i	R_{ij}	B		j
5	Optional—delete erroneous R_{ih}	R_{ih}	GTO	1	
			R/S		
6	End of the ith sample		C		i
7	Compute H statistic		D		H
8	Compute df		E		df
9	Optional—recall N		RCL	5	N
10	For a new case, go to 2				

Example:

i \ j	Ranks R_{ij}: 1	2	3	4	5	6	7	8	9	10
1	29	5	26	10	33	30				
2	11	12	9	7	20	18	19	21		
3	14	28	8	25	17	15	32	4	2	
4	6	27	3	16	24	13	1	31	22	23

Keystrokes:

A 29 B 5 B ... 30 B ——→ 6.00
C ——→ 1.00
11 B 12 B ... 21 B C ——→ 2.00
14 B 28 B ... 2 B C ——→ 3.00
6 B 27 B ... 23 B C ——→ 4.00
D ——→ 2.29 (H)
E ——→ 3.00 (df)

MEAN-SQUARE SUCCESSIVE DIFFERENCE

When test and estimation techniques are used, the method of drawing the sample from the population is specified to be random in most cases. If observations are chosen in sequence $x_1, x_2, ..., x_n$, the mean-square successive difference η can be used to test for randomness.

If the sample size n is large (say, greater than 20) and the population is normal, then a z statistic has approximately the standard normal distribution. Long trends are associated with large positive values of z and short oscillations with large negative values.

Equations:

$$\eta = \sum_{i=2}^{n} (x_i - x_{i-1})^2 \Big/ \sum_{i=1}^{n} (x_i - \overline{x})^2$$

$$= \sum_{i=2}^{n} (x_i - x_{i-1})^2 \Bigg/ \left[\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n} \right]$$

$$z = \frac{1 - \eta/2}{\sqrt{\dfrac{n-2}{n^2-1}}}$$

Reference:

Dixon and Massey, *Introduction to Statistical Analysis,* McGraw-Hill, 1969.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program		[ ]	[ ]	
2	Initialize		[A]	[ ]	0.00
3	Input x_1	x_1	[B]	[ ]	1.00
4	Perform 4 for i = 2, 3,..., n	x_i	[C]	[ ]	i
5	Compute η		[D]	[ ]	η
6	Compute z		[E]	[ ]	z
7	For a new case, go to 2		[ ]	[ ]	

Example:

Find the mean-square successive difference for the following set of data:

{0.53, 0.52, 0.39, 0.49, 0.97, 0.29, 0.65, 0.30, 0.40, 0.06, 0.14, 0.16, 0.68, 0.22, 0.68, 0.08, 0.52, 0.50, 0.63, 0.20, 0.67, 0.44, 0.64, 0.40, 0.97, 0.03, 0.73, 0.24, 0.57, 0.35}

Keystrokes:

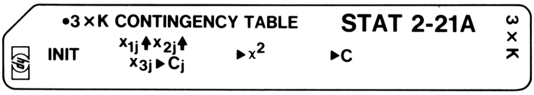
A .53 **B** ──────► 1.00

.52 **C** .39 **C** ... .35 **C** ──────► 30.00

D ──────► 2.81 (η)

E ──────► –2.29 (z)

3 x k CONTINGENCY TABLE



Contingency tables can be used to test the null hypothesis that two variables are independent.

i \ j	1	2	. . .	k	Totals
1	x_{11}	x_{12}	. . .	x_{1k}	R_1
2	x_{21}	x_{22}	. . .	x_{2k}	R_2
3	x_{31}	x_{32}	. . .	x_{3k}	R_3
Totals	C_1	C_2	. . .	C_k	N

This program computes the χ^2 statistic (with 2(k – 1) degrees of freedom) for testing the independence of the two variables. Also Pearson's coefficient of contingency C, which measures the degree of association between the two variables, is calculated.

Equations:

Row sum $$R_i = \sum_{j=1}^{k} x_{ij} \qquad i = 1, 2, 3$$

Column sum $$C_j = \sum_{i=1}^{3} x_{ij} \qquad j = 1, 2, ..., k$$

Total $$N = \sum_{i=1}^{3} \sum_{j=1}^{k} x_{ij}$$

Chi-square statistic

$$\chi^2 = \sum_{i=1}^{3} \sum_{j=1}^{k} \frac{(x_{ij} - E_{ij})^2}{E_{ij}}$$

$$= N\left(\sum_{i=1}^{3} \sum_{j=1}^{k} \frac{x_{ij}^2}{R_i C_j}\right) - N$$

where expected frequency

$$E_{ij} = \frac{R_i C_j}{N}$$

Contingency coefficient

$$C = \sqrt{\frac{\chi^2}{N + \chi^2}}$$

Reference:

B. Ostle, *Statistics in Research,* Iowa State University Press, 1972.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Initialize		A		0.00
3	Perform 3 for j = 1, 2,..., k	x_{1j}	↑		
		x_{2j}	↑		
		x_{3j}	B		C_j
4	Compute χ^2		C		χ^2
5	Compute contingency coeffi-				
	cient		D		C
6	Optional—recall sums		SST		R_1
			SST		R_2
			SST		R_3
			SST		N
7	For a new case, go to 2				

Example:

i \ j	1	2	3	4
1	36	67	49	58
2	31	60	49	54
3	58	87	80	68

Keystrokes:

A 36 [↑] 31 [↑] 58 **B** ⟶ 125.00 (C_1)

67 [↑] 60 [↑] 87 **B** ⟶ 214.00 (C_2)

49 [↑] 49 [↑] 80 **B** ⟶ 178.00 (C_3)

58 [↑] 54 [↑] 68 **B** ⟶ 180.00 (C_4)

C ⟶ 3.36 (χ^2)

D ⟶ 0.07 (C)

SST ⟶ 210.00 (R_1)

SST ⟶ 194.00 (R_2)

SST ⟶ 293.00 (R_3)

SST ⟶ 697.00 (N)

THE RUN TEST FOR RANDOMNESS

Consider a sequence of symbols such that the symbols are of two types only. A run is a continuous string of identical symbols preceded and followed by a different symbol or no symbol at all. For example, the sequence 1110100011 has five runs.

Let the total number of runs in a given sequence be u, and let n_1 and n_2 represent the number of symbols of type 1 and type 2 respectively. If the sample sizes are large (say, n_1 and n_2 are both greater than 10), then the randomness of the sequence may be tested using a z statistic which has the standard normal distribution.

Equations:

The sample distribution of the run has the mean μ and the standard deviation σ.

$$\mu = \frac{2 n_1 n_2}{n_1 + n_2} + 1$$

$$\sigma = \sqrt{\frac{2 n_1 n_2 (2 n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}}$$

The test is based on the statistic

$$z = \frac{u - \mu}{\sigma}$$

Remarks:

1. For small samples, the test is based on special tables.

2. This program can also be used for other tests involving runs. For example, one might want to test runs of scores above and below the median based on the order in which the scores were obtained. In this case, a sequence could be constructed in which each score would be replaced by a 1 if it was above the median or a 0, if below the median.

   The run test for randomness can then be applied to the sequence of 0's and 1's.

   Another use might be for Wald-Wolfowitz run test, which tests the null hypothesis that two random samples have been drawn from identical populations. The data from both groups are combined into one sequence according to magnitude. Each value may be assigned a 0 or 1 depending on which population it came from, and the run test for randomness then performed on the resulting sequence.

Reference:

Freund and Williams, *Dictionary/Outline of Basic Statistics,* McGraw-Hill, 1966.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Input				
	number of symbols of type 1	n_1	↑		
	number of symbols of type 2	n_2	A		n_1
3	Input number of runs	u	B		u
4	Compute the mean		C		μ
5	Compute the standard de-				
	viation		D		σ
6	Compute the z statistic		E		z
7	For a new case, go to 2				

Example:

A statistician sits by the roulette table one night in a Las Vegas casino, suspiciously watching the house rake in stake upon stake. To test the null hypothesis that the sequence of numbers is random, the statistician observes the following sequence of red (R) and black (B) numbers (ignoring 0 and 00):

RRRR B RRR BBBBB RR BBB RR BB RRR

In the sequence are 14 R's, 11B's, and a total of 9 runs. Find the mean and standard deviation of the sampling distribution and the z statistic.

Keystrokes:

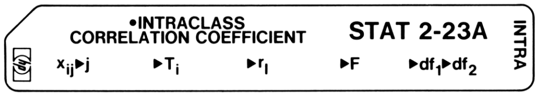
14 [↑] 11 [A] 9 [B] [C] ————→ 13.32 (μ)

[D] ————→ 2.41 (σ)

[E] ————→ –1.79 (z)

(His suspicion is not entirely unjustified.)

INTRACLASS CORRELATION COEFFICIENT



The intraclass correlation coefficient r_I measures the degree of association among individuals within classes or groups.

		Observations			
	1	x_{11}	x_{12}	...	x_{1n}
	2	x_{21}	x_{22}	...	x_{2n}
	.	.	.		.
Groups	.	.	.		.
	.	.	.		.
	k	x_{k1}	x_{k2}	...	x_{kn}

The coefficient is most easily calculated using the analysis of variance techniques. r_I is the sample estimate of the population intraclass correlation coefficient ρ_I. If we can assume that the individuals within groups are random samples from normal populations with the same variance, then the hypothesis $\rho_I = 0$ can be tested using the F statistic.

Equations:

1. Sums

Group
$$T_i = \sum_{j=1}^{n} x_{ij} \qquad i = 1, 2, ..., k$$

Total
$$T = \sum_{i=1}^{k} T_i$$

2. Sums of squares

Mean

$$MSS = T^2/k\,n$$

Among groups

$$ASS = \sum_{i=1}^{k} T_i^2/n - MSS$$

Within groups

$$WSS = \sum_{i=1}^{k} \sum_{j=1}^{n} x_{ij}^2 - MSS - ASS$$

3. Intraclass correlation coefficient

$$r_I = \left(\frac{ASS}{k-1} - \frac{WSS}{k(n-1)}\right) \bigg/ \left(\frac{ASS}{k-1} + \frac{WSS}{k}\right)$$

4. F statistic

$$F = \frac{ASS}{k-1} \bigg/ \frac{WSS}{k(n-1)}$$

with $df_1 = k - 1$ and $df_2 = k(n-1)$ degrees of freedom.

Reference:

B. Ostle, *Statistics in Research,* Iowa State University Press, 1972.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Initialize		f	REG	
3	Perform 3-5 for i = 1, 2,..., k				
4	Perform 4 for j = 1, 2,..., n	x_{ij}	A		j
5	Compute the group mean		B		T_i
6	Compute the coefficient		C		r_I
7	Compute the F statistic		D		F
8	Compute the degrees of freedom		E		df_1
			E		df_2
9	For a new case, go to 2				

Example:

		Observations	
	1	71	71
	2	69	72
	3	59	65
Groups	4	65	64
	5	66	60
	6	73	72
	7	68	67
	8	70	68

Keystrokes:

[f] [REG] 71 [A] 71 [A] [B] ——→ 142.00 (T_1)

69 [A] 72 [A] [B] ——→ 141.00 (T_2)

.
.
.

70 [A] 68 [A] [B] ——→ 138.00 (T_8)

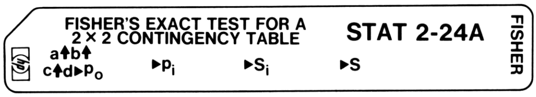
[C] ——→ 0.70 (r_I)

[D] ——→ 5.61 (F)

[E] ——→ 7.00 (df_1)

[E] ——→ 8.00 (df_2)

FISHER'S EXACT TEST FOR A 2 x 2 CONTINGENCY TABLE



Fisher's exact probability test is used for analyzing a 2 x 2 contingency table when the two independent samples are small in size.

a	b
c	d

Suppose a, b, c, d are the frequencies and **a is the smallest frequency**, this program computes the following:

1. The exact probability p_0 of observing the given frequencies in a 2 x 2 table, when the marginal totals are regarded as fixed.

2. The exact probability p_i (i = 1, 2, ..., a) of each more extreme table having the same marginal totals.

3. The sum S_i of the probabilities of the first i + 1 tables.

4. The sum S of the probabilities of all tables with the same margins (i.e., $S = S_a$).

Equations:

1.

$$p_0 = \frac{(a+b)!\,(c+d)!\,(a+c)!\,(b+d)!}{N!\,a!\,b!\,c!\,d!}$$

where

$$N = a + b + c + d.$$

2. For the more extreme table (with the same margins)

a – i	b + i
c + i	d – i

$$p_i = \frac{(a+b)!\,(c+d)!\,(a+c)!\,(b+d)!}{N!\,(a-i)!\,(b+i)!\,(c+i)!\,(d-i)!}$$

where

$$i \text{ can be } 1, 2, \ldots \text{ or } a.$$

3.

$$S_n = \sum_{i=0}^{n} p_i$$

where

$$n \text{ can be } 1, 2, \ldots, a.$$

4.

$$S = \sum_{i=0}^{a} p_i$$

Remarks:

1. a must be the smallest among the frequencies. Rearrange the table if necessary.

2. This program requires $N \leqslant 69$. However, Fisher's exact test is normally used for $N \leqslant 30$.

References:

1. S. Siegel, *Nonparametric Statistics,* McGraw-Hill, 1956.

2. Sir R. A. Fisher, *Statistical Methods for Research Workers,* Oliver and Boyd, 1950.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Enter frequencies and compute				
	p_0	a	↑		
		b	↑		
		c	↑		
		d	A		p_0
3*	Optional—perform 3 or 3-4 for				
	i = 1, 2,..., a		B		p_i
4	Optional—recall current S_i		C		S_i
5	Compute the sum of all				
	probabilities		D		S
6	For a new case, go to 2				
	* It is not necessary to com-				
	plete the loop of 3 and 4. Go to				
	5 for S when desired.				

Example:

Compute p_0, p_1, p_2, S_4 and S for the following table

7	10
8	5

Note:

The table must be rearranged as

5	8
10	7

Keystrokes:

5 [↑] 8 [↑] 10 [↑] 7 [A] ⟶ 0.16 (p_0)

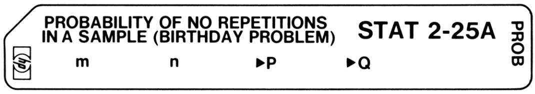
[B] ⟶ 0.06 (p_1)

[B] ⟶ 0.01 (p_2)

[B] [B] [C] ⟶ 0.23 (S_4)

[D] ⟶ 0.23 (S)

PROBABILITY OF NO REPETITIONS IN A SAMPLE (BIRTHDAY PROBLEM)



Suppose a sample of size n is drawn with replacement from a population containing m different objects. Let P be the probability that there are no repetitions in the sample and Q be the probability that there is at least one repetition in the sample. This program finds P and Q for given integers m, n such that $m \geqslant n \geqslant 1$.

Equations:

$$P = \left(1 - \frac{1}{m}\right)\left(1 - \frac{2}{m}\right) \cdots \left(1 - \frac{n-1}{m}\right)$$

$$Q = 1 - P$$

Remark:

m, n must be integers and $m \geqslant n \geqslant 1$; otherwise, flashing zeros will result.

Reference:

E. Parzen, *Modern Probability and its Applications,* John Wiley and Sons, 1960.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input m	m	A	
3	Input n	n	B	
4	Compute P		C	P
5	Optional—compute Q		D	Q
6	For a different n, go to 3			
7	For a new case, go to 2			

Example:

In a room containing n persons, what is the probability that no two or more persons have the same birthday for (1) $n = 4$, (2) $n = 23$, (3) $n = 48$? (Note: $m = 365$, ignoring leap years)

Keystrokes:

1. 365 **A** 4 **B** **C** ⟶ 0.98 (P)

   D ⟶ 0.02 (Q)

2. 23 **B** **C** ⟶ 0.49 (P)

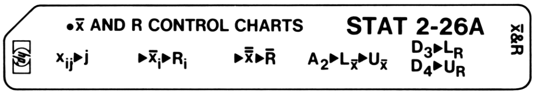
   D ⟶ 0.51 (Q)

3. 48 **B** **C** ⟶ 0.04 (P)

   D ⟶ 0.96 (Q)

That is, in a room having 48 persons, the probability that at least two of them will have the same birthday is as high as 0.96.

$\overline{x}$ AND R CONTROL CHARTS



Suppose x_{ij} represents the j^{th} data point from the i^{th} sample, i = 1, 2, ..., m and j = 1, 2, ..., n. This program computes (1) the sample mean $\overline{x}_i$ and the sample range R_i, (2) the over-all mean $\overline{\overline{x}}$ and the average range $\overline{R}$, (3) the upper control limit $U_{\overline{x}}$ and the lower control limit $L_{\overline{x}}$ for $\overline{x}$, and (4) the upper control limit U_R and the lower control limit L_R for R.

Equations:

1.
$$\overline{x}_i = \sum_{j=1}^{n} x_{ij}/n$$

$$R_i = x_{max} - x_{min}$$

where x_{max} is the maximum of the x values and x_{min} is the minimum of the x values in the i^{th} sample.

2.
$$\overline{\overline{x}} = \sum_{i=1}^{m} \overline{x}_i/m$$

$$\overline{R} = \sum_{i=1}^{m} R_i/m$$

3.
$$L_{\overline{x}} = \overline{\overline{x}} - A_2\overline{R}$$

$$U_{\overline{x}} = \overline{\overline{x}} + A_2\overline{R}$$

where A_2 is the factor for the $\overline{x}$ chart, which can be found in the following table.

4.
$$L_R = D_3\overline{R}$$

$$U_R = D_4\overline{R}$$

D_3 and D_4 are factors for the R chart, which can be found in the table.

Factors for determining from $\overline{R}$ the 3-sigma control limits for $\overline{x}$ and R charts.

Number of observations in subgroup n	Factor for $\bar{x}$ chart A_2	Factors for R chart	
		Lower limit D_3	Upper limit D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65
16	0.21	0.36	1.64
17	0.20	0.38	1.62
18	0.19	0.39	1.61
19	0.19	0.40	1.60
20	0.18	0.41	1.59

All factors are based on the normal distribution.

The table is reproduced from *Statistical Quality Control,* by Grant and Leavenworth, 1972, with permission of McGraw-Hill Book Company.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Initialize		RTN	R/S	1.00
3	Perform 3-6 for i = 1, 2,..., m				
4	Perform 4 for j = 1, 2,..., n	x_{ij}	A		j
5	Optional—recall x_{max}		RCL	4	x_{max}
	recall x_{min}		RCL	5	x_{min}
6	Compute the mean and range		B		$\bar{x}_i$
			B		R_i
7	Compute $\bar{\bar{x}}$ and $\bar{R}$		C		$\bar{\bar{x}}$
			C		$\bar{R}$
8	Compute the $\bar{x}$ limits	A_2	D		$L_{\bar{x}}$
			D		$U_{\bar{x}}$
9	Compute the R limits	D_3	E		L_R
		D_4	E		U_R
10	For a new case, go to 2				

Example:

	i \ j	1	2	3	4	5
Sample	1	10.04	10.00	10.02	10.01	10.02
	2	10.00	10.01	10.03	10.02	10.01
	3	10.02	10.02	10.02	10.04	10.01

Find the lower and upper control limits for $\bar{x}$ and R.

(Note: $n = 5, A_2 = 0.58, D_3 = 0, D_4 = 2.11$)

Keystrokes:

RTN R/S 10.04 A 10 A 10.02 A 10.01 A

10.02 A ──────────► 5.00

RCL 4 ──────────► 10.04 (x_{max})

RCL 5 ──────────► 10.00 (x_{min})

B ──────────► 10.02 ($\bar{x}_1$)

B ──────────► 0.04 (R_1)

10 **A** 10.01 **A** 10.03 **A** 10.02 **A** 10.01 **A** ⟶ 5.00

B ⟶ 10.01 ($\overline{x}_2$)

B ⟶ 0.03 (R_2)

10.02 **A** 10.02 **A** 10.02 **A** 10.04 **A** 10.01 **A** ⟶ 5.00

B ⟶ 10.02 ($\overline{x}_3$)

B ⟶ 0.03 (R_3)

C ⟶ 10.02 ($\overline{\overline{x}}$)

C ⟶ 0.03 ($\overline{R}$)

.58 **D** ⟶ 10.00 ($L_{\overline{x}}$)

D ⟶ 10.04 ($U_{\overline{x}}$)

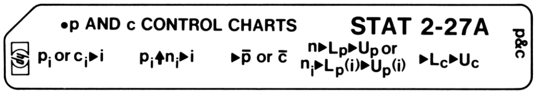
0 **E** ⟶ 0.00 (L_R)

2.11 **E** ⟶ 0.07 (U_R)

Reference:

Grant and Leavenworth, *Statistical Quality Control,* McGraw-Hill, 1972.

p AND c CONTROL CHARTS



This program constructs a control chart for fraction defective p and a control chart for defects c. A defective is an article that in some way fails to conform to one or more given specifications. Each instance of the article's lack of conformity to specifications is a defect. So every defective contains one or more defects.

Fraction defective p is defined as the ratio of the number of defective articles found in any inspection to the total number of articles actually inspected. c is the total number of defects in the sample.

Let p_i be the sample fraction defective and n_i be the size of the i^{th} sample (i = 1, 2, ..., m). Also let c_i be the sampled number of defects. The samples must be of constant size for the c chart.

Equations:

1.
$$\text{The mean } \bar{p} = \begin{cases} \sum_{i=1}^{m} p_i/m & \text{if } n_i = n \text{ for all } i \\ \sum_{i=1}^{m} n_i p_i \Big/ \sum_{i=1}^{m} n_i & \text{otherwise} \end{cases}$$

2. 3-sigma control limits on a p-chart

$$\text{Lower limit } L_p(i) = \begin{cases} p_o' - 3\sqrt{\dfrac{p_o'(1-p_o')}{n_i}} & \text{if } p_o' \text{ is used} \\ \bar{p} - 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n_i}} & \text{otherwise} \end{cases}$$

If $L_p(i) < 0$, then zero is used as the value for $L_p(i)$.

$$\text{Upper limit } U_p(i) = \begin{cases} p_0' + 3\sqrt{\dfrac{p_0'(1-p_0')}{n_i}} & \text{if } p_0' \text{ is used} \\ \overline{p} + 3\sqrt{\dfrac{\overline{p}(1-\overline{p})}{n_i}} & \text{otherwise} \end{cases}$$

where p_0' is the standard or aimed-at value chosen for control chart purposes.

If sample size n is constant, then replace n_i by n in the above formulas.

3. The mean

$$\overline{c} = \sum_{i=1}^{m} c_i/m$$

4. 3-sigma control limits on a c-chart

$$\text{Lower limit } L_c = \begin{cases} c_0' - 3\sqrt{c_0'} & \text{if } c_0' \text{ is used} \\ \overline{c} - 3\sqrt{\overline{c}} & \text{otherwise} \end{cases}$$

If $L_c < 0$, then zero is used as the value for L_c.

$$\text{Upper limit } U_c = \begin{cases} c_0' + 3\sqrt{c_0'} & \text{if } c_0' \text{ is used} \\ \overline{c} + 3\sqrt{\overline{c}} & \text{otherwise} \end{cases}$$

where c_0' is the standard value of average number of defects.

Reference:

Grant and Leavenworth, *Statistical Quality Control,* McGraw-Hill, 1972.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Initialize		RTN	R/S	0.00
3	For c chart, go to 8				
4	If p_0' is used, input p_0'	p_0'	STO	4	
	Go to 6				
5	If $\bar{p}$ is used and sample size is				
	constant,				
	input p_i for i = 1, 2,..., m	p_i	A		i
	Calculate $\bar{p}$		C		$\bar{p}$
	If $\bar{p}$ is used and sample size				
	varies,				
	input p_i and n_i for				
	i = 1, 2,..., m	p_i	↑		
		n_i	B		i
	calculate $\bar{p}$		C		$\bar{p}$
6	Compute control limits				
	for constant sample size,				
	input n	n	D		Lp
			D		Up
	for varying sample size,				
	input n_i for i = 1, 2,..., m	n_i	D		Lp (i)
			D		Up (i)
7	For a new case, go to 2				
8	If c_0' is used, input c_0'	c_0'	STO	4	
	Go to 10				
9	If $\bar{c}$ is used,				
	input c_i for i = 1, 2,..., m	c_i	A		i
	calculate $\bar{c}$		C		$\bar{c}$
10	Compute limits		E		L_c
			E		U_c
11	For a new case, go to 2				

Example 1:

i	p_i	n_i
1	0.0092	3350
2	0.0337	3354
3	0.0185	1509
4	0.0091	2190

Find the trial control limits for the p chart. (For trial control limits, the mean $\bar{p}$ is used.)

Keystrokes:

RTN R/S .0092 ↑ 3350 B ⟶ 1.00
.0337 ↑ 3354 B .0185 ↑ 1509 B
.0091 ↑ 2190 B ⟶ 4.00
C DSP • 4 ⟶ 0.0184 ($\bar{p}$)
3350 D ⟶ 0.0115 (L_p for sample 1)
D ⟶ 0.0254 (U_p for sample 1)
3354 D ⟶ 0.0115 (L_p for sample 2)
D ⟶ 0.0254 (U_p for sample 2)
1509 D ⟶ 0.0080 (L_p for sample 3)
D ⟶ 0.0288 (U_p for sample 3)
2190 D ⟶ 0.0098 (L_p for sample 4)
D ⟶ 0.0270 (U_p for sample 4)

Example 2:

i	1	2	3	4	5	6	7	8	9	10
c_i	7	6	6	7	4	7	8	12	9	9

Find the trial control limits for the c chart. (For trial control limits, the mean $\bar{c}$ is used.)

Keystrokes:

RTN R/S DSP • 2 ——→ 0.00
7 A 6 A 6 A 7 A 4 A 7 A 8 A
12 A 9 A 9 A ——→ 10.00
C ——→ 7.50 ($\bar{c}$)
E ——→ 0.00 (L_c)
E ——→ 15.72 (U_c)

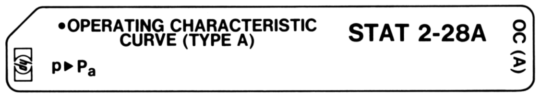
Example 3:

Find the control limits for the c chart if $c_0{}' = 7$.

Keystrokes:

RTN R/S 7 STO 4 E ——→ 0.00 (L_c)
E ——→ 14.94 (U_c)

OPERATING CHARACTERISTIC CURVE (TYPE A)



This program uses the hypergeometric distribution to evaluate the probability P_a of acceptance for a single sampling plan with finite lot size N. The sample size n and the acceptance number c (maximum allowable number of defectives in the sample) should also be given. Then P_a, which is the ordinate of the type A operating characteristic curve, can be computed for different values of the fraction defective p in the lot.

Equations:

$$P_a = \sum_{x=0}^{c} f(x)$$

$$f(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

where f(x) is the hypergeometric density function, M is the number of defectives in a lot which is calculated as the integer part of Np.

The recursive relation

$$f(x+1) = \frac{(x-M)(x-n)}{(x+1)(N-M-n+x+1)} f(x)$$

$$(x = 0, 1, 2, \ldots, n-1)$$

is used to find the probability

$$P_a = \sum_{x=0}^{c} f(x)$$

with starting value

$$f(0) = \frac{\binom{N-M}{n}}{\binom{N}{n}}$$

The binomial coefficient $\binom{N}{n}$ is computed by the formula

$$\binom{N}{n} = \frac{N(N-1)\ldots(N-n+1)}{1 \cdot 2 \cdot \ldots \cdot n}$$

Remarks:

1. The program requires that $0 \leqslant p < 1$.
2. If $c = 0, P_a = f(0)$.
3. The execution time of the program mainly depends on the sample size n and the acceptance number c; the larger they are, the longer it takes.
4. For certain combinations of N, n and c (usually when they are large), an overflow condition will occur. In that case, the program halts and the display shows all 9's.
5. The type A OC curve for finite lot sizes is really a set of discrete points, since defectives can occur only as whole numbers. For very large lot sizes, these points come very close together, giving a practically continuous curve.
6. The lot size N has a relatively small effect on the OC curve as long as n/N is not large. The absolute sample size n is a much more controlling factor in determining the type A OC curve.

References:

1. Dodge and Romig, *Sampling Inspection Tables,* John Wiley and Sons, 1959.
2. Grant and Leavenworth, *Statistical Quality Control,* McGraw-Hill, 1972.

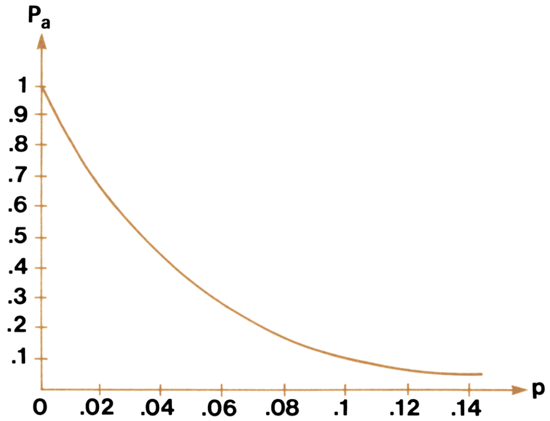
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Input lot size	N	STO	1	
	sample size	n	STO	2	
	acceptance number	c	STO	3	
3	Compute P_a for given p	p	A		P_a
4	For different value of p, go to 3				
5	For a new case, go to 2				

Example:

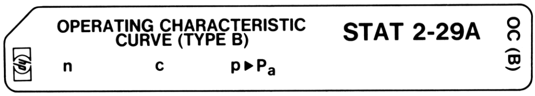
Find the OC curve for the sampling plan with N=200, n=20 and c=0 (compute P_a for p = 0, 0.02, 0.04, 0.06, 0.08, 0.1, 0.12, 0.14).

Keystrokes:

200 [STO] [1] 20 [STO] [2] 0 [STO] [3] 0 [A] → 1.00
.02 [A] → 0.65
.04 [A] → 0.42
.06 [A] → 0.27
.08 [A] → 0.17
.1 [A] → 0.11
.12 [A] → 0.07
.14 [A] → 0.04



OPERATING CHARACTERISTIC CURVE (TYPE B)



This program uses the binomial distribution to evaluate the probability P_a of acceptance for a single sampling plan with infinite lot size (i.e., sampling from a product). The sample size n and the acceptance number c should be given. Then P_a, which is the ordinate of the type B operating characteristic curve, can be computed for different values of the fraction defective p.

Equations:

$$P_a = \sum_{x=0}^{c} f(x)$$

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

where $0 \leqslant p < 1$.

The recursive relation

$$f(x+1) = \frac{p(n-x)}{(x+1)(1-p)} f(x)$$

$$(x = 0, 1, 2, \ldots, n-1)$$

is used to find the probability

$$P_a = \sum_{x=0}^{c} f(x)$$

with starting value

$$f(0) = (1-p)^n.$$

Remarks:

1. The program requires that $0 \leqslant p < 1$.
2. Type B OC curves can be considered as suitable approximation to type A OC curves, provided the sample size n is small compared with the lot size N (in general, if $n/N \leqslant 0.1$).
3. The acceptance number c affects the probability of acceptance drastically for any given fraction defective p.

Reference:

Dodge and Romig, *Sampling Inspection Tables,* John Wiley and Sons, 1959.

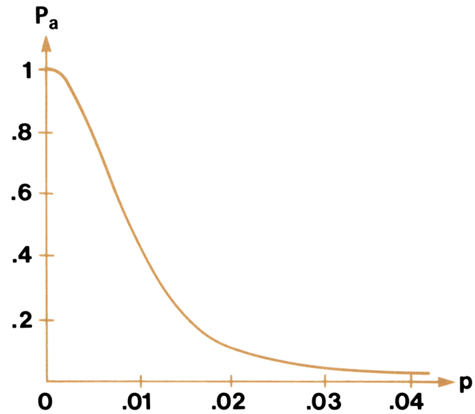
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Input sample size	n	A		
	acceptance number	c	B		
3	Input fraction defective	p	C		P_a
4	For a different p, go to 3				
5	For a new case, go to 2				

Example:

Find the type B OC curve for the sampling plan with n = 200, c = 1 (compute P_a for p = 0, 0.01, 0.02, 0.03 and 0.04).

Keystrokes:

200 A 1 B 0 C ——► 1.00
.01 C ——► 0.40
.02 C ——► 0.09
.03 C ——► 0.02
.04 C DSP • 4 ——► 0.0027



P_a
1
.8
.6
.4
.2
0
.01
.02
.03
.04
p

SINGLE- AND MULTI-SERVER QUEUES (INFINITE CUSTOMERS)

Suppose there are n ($n \geqslant 1$) identical stations available to service calls from an infinite number of customers. Let λ be the arrival rate of customers (poisson input), μ be the service rate of each server (exponential service), and let the service discipline be first-come, first-served. Assume all customers wait in a single line and are directed to whichever station is available. Assume further that, no customers are lost from the queue.

This program computes the following values for given n, λ and μ.

Equations:

1. The intensity factor

$$\rho = \frac{\lambda}{\mu}$$

(ρ must be less than n)

2. The probability that all servers are idle

$$P_0 = \left[\sum_{k=0}^{n-1} \frac{\rho^k}{k!} + \frac{\rho^n}{n!\left(1 - \frac{\rho}{n}\right)} \right]^{-1}$$

3. The probability that all servers are busy

$$P_b = \frac{\rho^n P_0}{n!\left(1 - \frac{\rho}{n}\right)}$$

4. The average number of customers in the queue

$$L_q = \frac{\rho P_b}{n - \rho}$$

5. The average number of customers in the system (waiting or being served)

$$L = L_q + \rho$$

6. The average waiting time in the queue

$$T_q = \frac{L_q}{\lambda}$$

7. The average flow time through the system

$$T = \frac{L}{\lambda}$$

8. The probability of waiting longer than time t

$$P(t) = P_b \, e^{-(n\mu-\lambda)\,t}$$

Remarks:

1. n must be an integer greater than or equal to 1.
2. $\rho < n$, otherwise the queue increases without bound.
3. λ and μ are rates, that is, numbers per unit time.

References:

1. H. M. Wagner, *Principles of Operations Research with Applications to Managerial Decisions,* Prentice-Hall, 1969.
2. James Martin, *Systems Analysis for Data Transmission,* Prentice-Hall, 1972.
3. Hillier and Lieberman, *Introduction to Operations Research,* Holden-Day, 1970.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Input data	μ	↑		
		λ	↑		
		n	A		ρ
3	Compute P_0 and P_b		B		P_0
			B		P_b
4	Compute L_q and L		C		L_q
			C		L
5	Compute T_q and T		D		T_q
			D		T
6	Compute P (t)	t	E		P (t)
7	For a different t, go to 6				
8	For a new case, go to 2				

Example:

Bank customers arrive at a bank on the average of 1.2 customers per minute. They join a common queue for 3 tellers, each teller serves at a rate of 30 customers per hour. Find ρ, P_0, P_b, L_q, L, T_q, T and the probability P(2) that a customer will have to wait for more than 2 minutes.

(Note: Service rate $\mu = \frac{30}{60} = 0.5$ customers per minute

Arrival rate $\lambda = 1.2$ customers per minute)

Keystrokes:

.5 [↑] 1.2 [↑] 3 [A] ⟶ 2.40 (ρ)

[B] ⟶ 0.06 (P_0)

[B] ⟶ 0.65 (P_b)

[C] ⟶ 2.59 (L_q)

[C] ⟶ 4.99 (L)

[D] ⟶ 2.16 (T_q)

[D] ⟶ 4.16 (T)

2 [E] ⟶ 0.36 (P(2))

SINGLE- AND MULTI-SERVER QUEUES (FINITE CUSTOMERS)

Suppose there are n ($n \geqslant 1$) identical stations available to service calls. This program handles the case in which demand arises from a finite rather than an infinite population of customers.

Let the number of customers m be fixed; let a be the mean time between service calls; and s be the mean time to serve one customer. Given m, n, s and a, this program computes the following values.

Equations:

1. The average number of customers in the system (waiting or being served)

$$L = \frac{\sum_{k=0}^{m} k\, Q_k}{\sum_{k=0}^{m} Q_k}$$

where

$$Q_0 = 1$$

$$(m - k + 1)\,\rho\, Q_{k-1} = \begin{cases} kQ_k & \text{if } 1 \leqslant k \leqslant n \\ nQ_k & \text{if } n < k \leqslant m \end{cases}$$

and

$$\rho = \frac{s}{a}\,.$$

2. The average flow time through the system

$$T = aL$$

3. The average number of customers in the queue

$$L_q = m \left[(\rho + 1) \left(\frac{L}{M} - 1 \right) + 1 \right]$$

4. The average waiting time in the queue

$$T_q = aL_q$$

5. The over-all efficiency factor of the system

$$F = -(\rho + 1) \left(\frac{L}{m} - 1 \right)$$

Remarks:

1. For large values of m and/or small values of ρ, the calculation of Q_k in the routine under LBL C may underflow. To avoid this, the program tests to see if $Q_k < 10^{-90}$. If it does, the program will halt its recursive solution for Q_k and go directly to the calculation of L. This should not affect the calculated value of L.

2. For certain combinations of m, n, s and a, an overflow condition will occur. In that case, the program halts and the display shows all 9's.

3. The execution time for L depends on m; the larger m is, the longer it takes. A rough estimate of the time for this routine (LBL C) is given by m/30 minutes.

4. Suppose instead of knowing s and a, the service rate μ of each server and the arrival rate λ are given. Then the following formulas can be used to compute s and a in order to run this program.

$$s = \frac{1}{\mu}$$

$$a = \frac{1}{\lambda}$$

Note that $$\rho = \frac{\lambda}{\mu} .$$

Reference:

Peak and Hazelwood, *Finite Queuing Tables,* John Wiley and Sons, 1958.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Input number of customers	m	↑		
	number of servers	n	A		m
3	Input service time	s	↑		
	arrival time	a	B		ρ
4	Compute customers in system		C		L
	time through system		C		T
5	Compute queue length		D		L_q
	waiting time in queue		D		T_q
6	Compute factor F		E		F
7	For a new case, go to 2				

Example:

A laundromat has 12 washers which require an average of 4 hours of service after every 60 hours of operation. If there is only one service person in the laundromat, find ρ, L, T, L_q, T_q and F.

Keystrokes:

12 [↑] 1 [A] ——→ 12.00
4 [↑] 60 [B] ——→ 0.07 (ρ)
[C] ——→ 1.64 (L)
[C] ——→ 98.66 (T)
[D] ——→ 0.95 (L_q)
[D] ——→ 57.24 (T_q)
[E] ——→ 0.92 (F)

PROGRAM LISTINGS

PARTIAL AND MULTIPLE CORRELATION COEFFICIENTS

KEYS	CODE
LBL	23
E	15
RCL 1	34 01
STO 4	33 04
RCL 2	34 02
STO 1	33 01
RCL 3	34 03
STO 2	33 02
RCL 4	34 04
STO 3	33 03
RTN	24
LBL	23
A	11
STO 1	33 01
RTN	24
LBL	23
B	12
STO 2	33 02
RTN	24
LBL	23
C	13
STO 3	33 03
RTN	24
LBL	23
D	14
RCL 1	34 01
RCL 2	34 02
RCL 3	34 03
x	71
–	51
1	01
RCL 2	34 02
f^{-1}	32
$\sqrt{x}$	09
–	51

KEYS	CODE
1	01
RCL 3	34 03
f^{-1}	32
$\sqrt{x}$	09
–	51
x	71
f	31
$\sqrt{x}$	09
÷	81
STO 5	33 05
LBL	23
0	00
E	15
RCL 5	34 05
RTN	24
LBL	23
E	15
RCL 1	34 01
RCL 2	34 02
f	31
R→P	01
f^{-1}	32
$\sqrt{x}$	09
RCL 1	34 01
RCL 2	34 02
x	71
2	02
x	71
RCL 3	34 03
x	71
–	51
1	01
RCL 3	34 03
f^{-1}	32
$\sqrt{x}$	09

KEYS	CODE
–	51
÷	81
f	31
$\sqrt{x}$	09
STO 5	33 05
E	15
GTO	22
0	00
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1 Used	R_4 Used	R_7
R_2 Used	R_5 Used	R_8
R_3 Used	R_6	R_9 Scratch

MOVING AVERAGES (ORDER 2 TO 8)

KEYS	CODE
GTO	22
1	01
LBL	23
A	11
f	31
REG	43
DSP	21
•	83
0	00
0	00
R/S	84
STO 1	33 01
1	01
R/S	84
STO 2	33 02
2	02
R/S	84
STO 3	33 03
3	03
R/S	84
STO 4	33 04
4	04
R/S	84
STO 5	33 05
5	05
R/S	84
STO 6	33 06
6	06
R/S	84
STO 7	33 07
7	07
R/S	84
STO 8	33 08
8	08
R/S	84

KEYS	CODE
8	08
LBL	23
B	12
STO	33
9	09
LBL	23
1	01
RCL 1	34 01
RCL 2	34 02
STO 1	33 01
+	61
RCL 3	34 03
STO 2	33 02
+	61
RCL 4	34 04
STO 3	33 03
+	61
RCL 5	34 05
STO 4	33 04
+	61
RCL 6	34 06
STO 5	33 05
+	61
RCL 7	34 07
STO 6	33 06
+	61
RCL 8	34 08
STO 7	33 07
+	61
RCL	34
9	09
÷	81
DSP	21
•	83
2	02

KEYS	CODE
R/S	84
GTO	22
C	13
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1 x_1, Used	R_4 x_4, Used	R_7 x_7, Used
R_2 x_2, Used	R_5 x_5, Used	R_8 x_8, Used
R_3 x_3, Used	R_6 x_6, Used	R_9 n

HISTOGRAM (12 INTERVALS)

KEYS	CODE
E	15
RCL 6	34 06
g x≤y	35 22
GTO	22
0	00
g x⇄y	35 07
RCL 5	34 05
g x>y	35 24
GTO	22
0	00
–	51
g x⇄y	35 07
RCL 5	34 05
–	51
÷	81
1	01
2	02
x	71
f	31
INT	83
1	01
+	61
3	03
g x⇄y	35 07
g x≤y	35 22
GTO	22
1	01
3	03
–	51
g x≤y	35 22
GTO	22
2	02
3	03
–	51
g x≤y	35 22

KEYS	CODE
GTO	22
3	03
3	03
–	51
D	14
STO	33
+	61
4	04
GTO	22
0	00
LBL	23
1	01
D	14
STO	33
+	61
1	01
GTO	22
0	00
LBL	23
2	02
D	14
STO	33
+	61
2	02
GTO	22
0	00
LBL	23
3	03
D	14
STO	33
+	61
3	03
GTO	22
0	00
LBL	23

KEYS	CODE
D	14
3	03
CHS	42
x	71
f^{-1}	32
LOG	08
RTN	24
LBL	23
E	15
R/S	84
RTN	24
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1 $0.b_1 b_2 b_3$	R_4 $0.b_{10} b_{11} b_{12}$	R_7 0
R_2 $0.b_4 b_5 b_6$	R_5 x_{min}	R_8 0
R_3 $0.b_7 b_8 b_9$	R_6 x_{max}	R_9 Used

F DISTRIBUTION WITH ODD DEGREES OF FREEDOM (CARD 1)

KEYS	CODE
STO 1	33 01
R/S	84
LBL	23
B	12
STO 2	33 02
R/S	84
LBL	23
C	13
g	35
RAD	42
RCL 1	34 01
x	71
RCL 2	34 02
÷	81
f	31
$\sqrt{x}$	09
f^{-1}	32
TAN	06
STO 3	33 03
2	02
x	71
g	35
π	02
÷	81
STO 4	33 04
RCL 2	34 02
1	01
STO 6	33 06
g x=y	35 23
RCL 4	34 04
R/S	84
D	14
RCL 3	34 03
f	31
COS	05

KEYS	CODE
x	71
2	02
x	71
g	35
π	02
÷	81
RCL 4	34 04
+	61
R/S	84
LBL	23
D	14
RCL 3	34 03
f	31
COS	05
f^{-1}	32
$\sqrt{x}$	09
STO 7	33 07
RCL 3	34 03
f	31
SIN	04
STO 8	33 08
RCL 2	34 02
3	03
g x=y	35 23
RCL 8	34 08
RTN	24
—	51
2	02
÷	81
STO 8	33 08
1	01
STO 5	33 05
LBL	23
1	01
RCL 7	34 07

KEYS	CODE
x	71
RCL 6	34 06
1	01
+	61
x	71
g LST X	35 00
1	01
+	61
STO 6	33 06
÷	81
STO	33
+	61
5	05
g	35
DSZ	83
GTO	22
1	01
RCL 5	34 05
RCL 3	34 03
f	31
SIN	04
x	71
RTN	24
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1 ν_1	R_4 $2\theta/\pi$	R_7 $\cos^2\theta$
R_2 ν_2	R_5 Used	R_8 $\sin\theta$, Used
R_3 θ	R_6 Used	R_9 Used

F DISTRIBUTION WITH ODD DEGREES OF FREEDOM (CARD 2)

KEYS	CODE
CHS	42
1	01
STO 6	33 06
+	61
STO 5	33 05
1	01
STO 4	33 04
RCL 1	34 01
g x=y	35 23
RCL 5	34 05
R/S	84
RCL 2	34 02
2	02
÷	81
LBL	23
C	13
•	83
5	05
g x=y	35 23
GTO	22
2	02
g x⇄y	35 07
1	01
–	51
STO	33
x	71
6	06
C	13
LBL	23
2	02
g	35
π	02
RCL 6	34 06
x	71
RCL 2	34 02

KEYS	CODE
1	01
–	51
2	02
÷	81
g	35
n!	03
2	02
x	71
g x⇄y	35 07
÷	81
RCL 3	34 03
1	01
f^{-1}	32
R→P	01
RCL 2	34 02
g	35
y^x	05
x	71
x	71
STO 6	33 06
STO	33
+	61
5	05
RCL 1	34 01
3	03
g x=y	35 23
RCL 5	34 05
R/S	84
–	51
2	02
÷	81
STO 8	33 08
0	00
STO 7	33 07
1	01

KEYS	CODE
LBL	23
B	12
RCL 3	34 03
f	31
SIN	04
f^{-1}	32
$\sqrt{x}$	09
x	71
RCL 4	34 04
RCL 2	34 02
+	61
x	71
RCL 4	34 04
2	02
+	61
STO 4	33 04
÷	81
STO	33
+	61
7	07
g	35
DSZ	83
GTO	22
B	12
RCL 6	34 06
RCL 7	34 07
x	71
RCL 5	34 05
+	61
R/S	84

R_1 ν_1	R_4 Used	R_7 Used
R_2 ν_2	R_5 Used	R_8 Used
R_3 θ	R_6 Used	R_9 Used

ERLANG DISTRIBUTION (GAMMA DISTRIBUTION)

KEYS	CODE
STO 1	33 01
R/S	84
LBL	23
B	12
STO 2	33 02
R/S	84
LBL	23
C	13
RCL 1	34 01
RCL 2	34 02
÷	81
R/S	84
LBL	23
C	13
RCL 2	34 02
÷	81
R/S	84
LBL	23
C	13
RCL 1	34 01
1	01
–	51
RCL 2	34 02
÷	81
R/S	84
LBL	23
D	14
↑	41
CLX	44
g x=y	35 23
GTO	22
0	00
g R↓	35 08
RCL 2	34 02
x	71

KEYS	CODE
CHS	42
f^{-1}	32
LN	07
g LST X	35 00
CHS	42
RCL 1	34 01
1	01
–	51
g	35
y^x	05
g LST X	35 00
g	35
n!	03
÷	81
x	71
RCL 2	34 02
x	71
R/S	84
LBL	23
0	00
1	01
RCL 1	34 01
g x=y	35 23
RCL 2	34 02
R/S	84
0	00
R/S	84
LBL	23
E	15
RCL 2	34 02
x	71
↑	41
↑	41
↑	41
RCL 1	34 01

KEYS	CODE
STO 8	33 08
CLX	44
1	01
LBL	23
3	03
g	35
DSZ	83
GTO	22
1	01
LBL	23
2	02
g x⇄y	35 07
CHS	42
f^{-1}	32
LN	07
x	71
CHS	42
1	01
+	61
R/S	84
LBL	23
1	01
x	71
RCL 8	34 08
÷	81
1	01
+	61
GTO	22
3	03
g NOP	35 01

R_1 n	R_4	R_7
R_2 λ	R_5	R_8 i
R_3	R_6	R_9 Used

GEOMETRIC CURVE FIT

KEYS	CODE
f	31
REG	43
R/S	84
LBL	23
B	12
f	31
LN	07
STO 7	33 07
STO	33
+	61
4	04
f^{-1}	32
$\sqrt{x}$	09
STO	33
+	61
5	05
g $x \rightleftarrows y$	35 07
STO	33
+	61
2	02
f^{-1}	32
$\sqrt{x}$	09
STO	33
+	61
3	03
g LST X	35 00
RCL 7	34 07
x	71
STO	33
+	61
6	06
RCL 1	34 01
1	01
+	61
STO 1	33 01

KEYS	CODE
R/S	84
LBL	23
C	13
RCL 6	34 06
RCL 2	34 02
RCL 4	34 04
x	71
RCL 1	34 01
÷	81
–	51
STO	33
9	09
RCL 3	34 03
RCL 2	34 02
f^{-1}	32
$\sqrt{x}$	09
RCL 1	34 01
÷	81
–	51
÷	81
f^{-1}	32
LN	07
STO 7	33 07
RCL 4	34 04
g LST X	35 00
RCL 2	34 02
x	71
–	51
RCL 1	34 01
÷	81
f^{-1}	32
LN	07
STO 8	33 08
R/S	84
LBL	23

KEYS	CODE
C	13
RCL 7	34 07
R/S	84
LBL	23
D	14
RCL 7	34 07
f	31
LN	07
RCL	34
9	09
x	71
RCL 5	34 05
RCL 4	34 04
f^{-1}	32
$\sqrt{x}$	09
RCL 1	34 01
÷	81
–	51
÷	81
R/S	84
LBL	23
E	15
RCL 7	34 07
g $x \rightleftarrows y$	35 07
g	35
y^x	05
RCL 8	34 08
x	71
R/S	84
g NOP	35 01

R_1 n	R_4 $\Sigma \ln y_i$	R_7 $\ln y_i$, b
R_2 Σx_i	R_5 $\Sigma (\ln y_i)^2$	R_8 a
R_3 Σx_i^2	R_6 $\Sigma x_i \ln y_i$	R_9 Used

GOMPERTZ CURVE FIT

KEYS	CODE
f	31
LN	07
STO	33
+	61
3	03
g R↓	35 08
f	31
LN	07
STO	33
+	61
2	02
g R↓	35 08
f	31
LN	07
STO	33
+	61
1	01
RCL 4	34 04
1	01
+	61
STO 4	33 04
R/S	84
LBL	23
B	12
RCL 3	34 03
RCL 2	34 02
–	51
RCL 2	34 02
RCL 1	34 01
–	51
÷	81
RCL 4	34 04
g	35
1/x	04
g	35

KEYS	CODE
y^x	05
STO 6	33 06
RCL 1	34 01
RCL 3	34 03
x	71
RCL 2	34 02
↑	41
x	71
–	51
RCL 1	34 01
RCL 3	34 03
+	61
RCL 2	34 02
2	02
x	71
–	51
÷	81
RCL 4	34 04
÷	81
f^{-1}	32
LN	07
STO 7	33 07
RCL 6	34 06
1	01
–	51
RCL 6	34 06
RCL 4	34 04
g	35
y^x	05
1	01
–	51
↑	41
x	71
÷	81
RCL 6	34 06

KEYS	CODE
÷	81
RCL 2	34 02
RCL 1	34 01
–	51
x	71
f^{-1}	32
LN	07
STO 5	33 05
R/S	84
LBL	23
C	13
RCL 6	34 06
R/S	84
LBL	23
D	14
RCL 7	34 07
R/S	84
LBL	23
E	15
RCL 6	34 06
g x⇄y	35 07
g	35
y^x	05
RCL 5	34 05
g x⇄y	35 07
g	35
y^x	05
RCL 7	34 07
x	71
R/S	84

R_1 S_1	R_4 n	R_7 c
R_2 S_2	R_5 a	R_8 0
R_3 S_3	R_6 b	R_9 0

WEIBULL DISTRIBUTION PARAMETER CALCULATION

KEYS	CODE
f	31
REG	43
STO 1	33 01
R/S	84
LBL	23
B	12
f	31
LN	07
STO	33
+	61
2	02
STO 8	33 08
f^{-1}	32
$\sqrt{x}$	09
STO	33
+	61
3	03
1	01
RCL 7	34 07
1	01
+	61
STO 7	33 07
•	83
3	03
–	51
RCL 1	34 01
•	83
4	04
+	61
÷	81
–	51
f	31
LN	07
CHS	42
f	31

KEYS	CODE
LN	07
STO	33
+	61
4	04
f^{-1}	32
$\sqrt{x}$	09
STO	33
+	61
5	05
g LST X	35 00
RCL 8	34 08
x	71
STO	33
+	61
6	06
RCL 7	34 07
RTN	24
LBL	23
C	13
D	14
RCL 4	34 04
RCL 7	34 07
RCL 2	34 02
x	71
–	51
RCL 1	34 01
÷	81
f^{-1}	32
LN	07
RTN	24
LBL	23
D	14
RCL 6	34 06
RCL 2	34 02
RCL 4	34 04

KEYS	CODE
x	71
RCL 1	34 01
÷	81
–	51
STO 8	33 08
RCL 3	34 03
RCL 2	34 02
f^{-1}	32
$\sqrt{x}$	09
RCL 1	34 01
÷	81
–	51
÷	81
STO 7	33 07
RTN	24
LBL	23
E	15
RCL 7	34 07
RCL 8	34 08
x	71
RCL 5	34 05
RCL 4	34 04
f^{-1}	32
$\sqrt{x}$	09
RCL 1	34 01
÷	81
–	51
÷	81
RTN	24
g NOP	35 01

R_1	n	R_4	Used	R_7	R_i
R_2	$\Sigma \ln x_i$	R_5	Used	R_8	$\ln x_i$, Used
R_3	$\Sigma (\ln x_i)^2$	R_6	Used	R_9	

WEIGHTED REGRESSION (SPECIAL CASE)

KEYS	CODE
f	31
REG	43
0	00
R/S	84
LBL	23
B	12
STO	33
+	61
5	05
STO	33
+	61
7	07
↑	41
x	71
STO	33
+	61
9	09
1	01
STO	33
+	61
1	01
RCL 8	34 08
+	61
STO 8	33 08
R/S	84
LBL	23
C	13
RCL 8	34 08
g x⇄y	35 07
x	71
STO	33
+	61
3	03
RCL 8	34 08
g LST X	35 00

KEYS	CODE
÷	81
STO	33
+	61
2	02
RCL 7	34 07
g LST X	35 00
÷	81
STO	33
+	61
4	04
RCL	34
9	09
g LST X	35 00
÷	81
STO	33
+	61
6	06
RCL 7	34 07
0	00
STO 7	33 07
STO 8	33 08
STO	33
9	09
g x⇄y	35 07
R/S	84
LBL	23
D	14
RCL 5	34 05
RCL 2	34 02
x	71
RCL 1	34 01
RCL 4	34 04
x	71
–	51
RCL 2	34 02

KEYS	CODE
RCL 3	34 03
x	71
RCL 1	34 01
↑	41
x	71
–	51
÷	81
STO 8	33 08
RCL 1	34 01
x	71
CHS	42
RCL 4	34 04
+	61
RCL 2	34 02
÷	81
STO 7	33 07
R/S	84
LBL	23
D	14
RCL 8	34 08
R/S	84
LBL	23
E	15
RCL 8	34 08
x	71
RCL 7	34 07
+	61
R/S	84
g NOP	35 01
g NOP	35 01

R_1	Σn_i	R_4	$\Sigma\Sigma y_{ij}/x_i$	R_7	$\Sigma y_{ij}, a_0$
R_2	$\Sigma n_i/x_i$	R_5	$\Sigma\Sigma y_{ij}$	R_8	n_i, a_1
R_3	$\Sigma n_i x_i$	R_6	$\Sigma\Sigma y_{ij}^2/x_i$	R_9	Σy_{ij}^2

POLYNOMIAL APPROXIMATION (CARD 1)

KEYS	CODE
STO 7	33 07
R/S	84
STO 1	33 01
STO 2	33 02
STO 3	33 03
STO 4	33 04
STO 5	33 05
2	02
STO 6	33 06
LBL	23
0	00
1	01
STO 8	33 08
RCL 6	34 06
2	02
÷	81
R/S	84
STO	33
+	61
1	01
1	01
RCL 6	34 06
RCL 7	34 07
÷	81
—	51
x	71
STO	33
+	61
2	02
STO	33
9	09
E	15
STO	33
+	61
3	03

KEYS	CODE
E	15
STO	33
+	61
4	04
E	15
STO	33
+	61
5	05
2	02
STO	33
+	61
6	06
GTO	22
0	00
LBL	23
E	15
RCL 8	34 08
↑	41
+	61
1	01
+	61
x	71
RCL 7	34 07
RCL 6	34 06
—	51
x	71
g x⇄y	35 07
RCL 7	34 07
RCL 8	34 08
+	61
1	01
+	61
x	71
RCL 8	34 08
x	71

KEYS	CODE
—	51
RCL 7	34 07
RCL 8	34 08
—	51
÷	81
RCL 8	34 08
1	01
+	61
STO 8	33 08
÷	81
RCL	34
9	09
g x⇄y	35 07
STO	33
9	09
RTN	24
LBL	23
2	02
CLX	44
STO 4	33 04
LBL	23
3	03
CLX	44
STO 5	33 05
LBL	23
4	04
R/S	84
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1	Used, (f, f_0)	**R_4**	Used, (f, f_3) or 0	**R_7**	N
R_2	Used, (f, f_1)	**R_5**	Used, (f, f_4) or 0	**R_8**	n
R_3	Used, (f, f_2)	**R_6**	2j	**R_9**	$f_n(i)\ f(x_i)$

POLYNOMIAL APPROXIMATION (CARD 2)

KEYS	CODE
RCL 1	34 01
RCL 7	34 07
1	01
STO 8	33 08
+	61
÷	81
STO 1	33 01
A	11
STO	33
÷	81
2	02
A	11
STO	33
÷	81
3	03
A	11
STO	33
÷	81
4	04
A	11
STO	33
÷	81
5	05
RCL 7	34 07
RCL 7	34 07
RCL 7	34 07
1	01
—	51
2	02
x	71
STO 6	33 06
÷	81
3	03
x	71
STO 8	33 08

KEYS	CODE
g R↓	35 08
2	02
+	61
RCL 6	34 06
÷	81
STO 6	33 06
RCL 3	34 03
x	71
STO	33
—	51
1	01
RCL 8	34 08
RCL 3	34 03
x	71
STO 3	33 03
RCL 7	34 07
RCL 7	34 07
RCL 7	34 07
2	02
—	51
÷	81
5	05
x	71
3	03
÷	81
RCL 8	34 08
g x⇄y	35 07
x	71
STO	33
9	09
g LST X	35 00
RCL 6	34 06
x	71
STO 7	33 07
CLX	44

KEYS	CODE
R/S	84
LBL	23
A	11
RCL 7	34 07
RCL 8	34 08
1	01
+	61
+	61
g	35
n!	03
RCL 7	34 07
RCL 8	34 08
—	51
g	35
n!	03
x	71
RCL 8	34 08
RCL 8	34 08
1	01
+	61
STO 8	33 08
+	61
÷	81
RCL 7	34 07
g	35
n!	03
↑	41
x	71
÷	81
RTN	24

R_1	(f, f_0), a_0	R_4	(f, f_3), a_3	R_7	Used
R_2	(f, f_1), a_1	R_5	(f, f_4), a_4	R_8	Used
R_3	(f, f_2), a_2	R_6	Used	R_9	Used

POLYNOMIAL APPROXIMATION (CARD 3)

KEYS	CODE
g R↑	35 09
g R↑	35 09
3	03
+	61
↑	41
↑	41
5	05
—	51
÷	81
2	02
x	71
3	03
÷	81
RCL 7	34 07
+	61
STO 7	33 07
RCL 4	34 04
x	71
STO	33
—	51
2	02
g LST X	35 00
RCL	34
9	09
x	71
STO 4	33 04
g R↑	35 09
g R↑	35 09
4	04
+	61
↑	41
↑	41
7	07
—	51
÷	81

KEYS	CODE
3	03
x	71
4	04
÷	81
RCL 6	34 06
g x⇄y	35 07
x	71
STO 6	33 06
g LST X	35 00
RCL 8	34 08
x	71
STO 8	33 08
g R↑	35 09
↑	41
↑	41
↑	41
3	03
—	51
÷	81
7	07
x	71
4	04
÷	81
RCL 7	34 07
g x⇄y	35 07
x	71
STO	33
+	61
8	08
g LST X	35 00
RCL	34
9	09
x	71
RCL 5	34 05
x	71

KEYS	CODE
STO 5	33 05
g LST X	35 00
STO	33
x	71
8	08
RCL 6	34 06
x	71
STO	33
+	61
1	01
RCL 8	34 08
STO	33
—	51
3	03
1	01
R/S	84
—	51
STO 8	33 08
g LST X	35 00
2	02
x	71
+	61
RCL 8	34 08
÷	81
STO 6	33 06
2	02
RCL 8	34 08
÷	81
STO 8	33 08
R/S	84

R_1	Used, b_0	R_4	Used, b_3	R_7	Used
R_2	Used, b_1	R_5	Used, b_4	R_8	Used, $-\alpha$
R_3	Used, b_2	R_6	Used, β	R_9	Used

POLYNOMIAL APPROXIMATION (CARD 4)

KEYS	CODE
RCL 1	34 01
RCL 6	34 06
RCL 2	34 02
x	71
+	61
RCL 6	34 06
↑	41
x	71
RCL 3	34 03
x	71
+	61
STO 1	33 01
RCL 2	34 02
RCL 6	34 06
RCL 3	34 03
x	71
2	02
x	71
+	61
STO 2	33 02
RCL 4	34 04
RCL 6	34 06
x	71
3	03
x	71
STO	33
+	61
3	03
RCL 6	34 06
x	71
STO	33
+	61
2	02
3	03
÷	81

KEYS	CODE
RCL 6	34 06
x	71
STO	33
+	61
1	01
RCL 5	34 05
RCL 6	34 06
x	71
4	04
x	71
STO	33
+	61
4	04
RCL 6	34 06
x	71
1	01
•	83
5	05
x	71
STO	33
+	61
3	03
g LST X	35 00
÷	81
RCL 6	34 06
x	71
STO	33
+	61
2	02
4	04
÷	81
RCL 6	34 06
x	71
STO	33
+	61

KEYS	CODE
1	01
RCL 8	34 08
CHS	42
↑	41
↑	41
↑	41
STO	33
x	71
2	02
x	71
STO	33
x	71
3	03
x	71
STO	33
x	71
4	04
x	71
STO	33
x	71
5	05
RCL 1	34 01
R/S	84
RCL 2	34 02
RCL 3	34 03
RCL 4	34 04
RCL 5	34 05
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1	b_0, c_0, d_0	R_4	b_3, c_3, d_3	R_7	
R_2	b_1, c_1, d_1	R_5	b_4, c_4, d_4	R_8	$-\alpha$
R_3	b_2, c_2, d_2	R_6	β	R_9	

TWO WAY ANALYSIS OF VARIANCE (NO REPLICATIONS)

KEYS	CODE
STO	33
+	61
7	07
f^{-1}	32
$\sqrt{x}$	09
STO	33
+	61
2	02
R/S	84
LBL	23
B	12
RCL 7	34 07
STO	33
+	61
1	01
f^{-1}	32
$\sqrt{x}$	09
STO	33
+	61
3	03
LBL	23
0	00
RCL 7	34 07
0	00
STO 7	33 07
g x⇄y	35 07
R/S	84
0	00
STO 2	33 02
R/S	84
LBL	23
C	13
RCL 7	34 07
f^{-1}	32
$\sqrt{x}$	09

KEYS	CODE
STO	33
+	61
4	04
GTO	22
0	00
LBL	23
D	14
RCL 1	34 01
f^{-1}	32
$\sqrt{x}$	09
RCL 5	34 05
RCL 6	34 06
x	71
÷	81
STO 7	33 07
CHS	42
RCL 2	34 02
+	61
STO 1	33 01
RCL 3	34 03
RCL 6	34 06
÷	81
RCL 7	34 07
–	51
STO 2	33 02
RCL 4	34 04
RCL 5	34 05
÷	81
RCL 7	34 07
–	51
STO 3	33 03
RCL 2	34 02
+	61
CHS	42
RCL 1	34 01

KEYS	CODE
+	61
STO 4	33 04
RCL 5	34 05
1	01
–	51
STO 5	33 05
RCL 6	34 06
1	01
–	51
STO 6	33 06
x	71
STO 7	33 07
÷	81
STO 8	33 08
RCL 2	34 02
RCL 5	34 05
÷	81
RCL 8	34 08
÷	81
R/S	84
RCL 5	34 05
LBL	23
E	15
RCL 3	34 03
RCL 6	34 06
÷	81
RCL 8	34 08
÷	81
R/S	84
RCL 6	34 06

R_1	$\Sigma\Sigma x_{ij}$, TSS	R_4	$\Sigma(\Sigma x_{ij})^2$, ESS	R_7	Used
R_2	$\Sigma\Sigma x_{ij}^2$, RSS	R_5	r, r–1	R_8	ESS/(r–1) (c–1)
R_3	$\Sigma(\Sigma x_{ij})^2$, CSS	R_6	c, c–1	R_9	0

TWO WAY ANALYSIS OF VARIANCE (WITH REPLICATIONS) CARD 1

KEYS	CODE
LBL	23
A	11
STO	33
+	61
1	01
STO	33
+	61
7	07
f^{-1}	32
$\sqrt{x}$	09
STO	33
+	61
2	02
1	01
RCL 4	34 04
+	61
STO 4	33 04
R/S	84
LBL	23
B	12
RCL 7	34 07
STO	33
+	61
6	06
f^{-1}	32
$\sqrt{x}$	09
STO	33
+	61
8	08
RCL 7	34 07
0	00
STO 4	33 04
STO 7	33 07
g x⇄y	35 07
R/S	84

KEYS	CODE
LBL	23
C	13
RCL 6	34 06
f^{-1}	32
$\sqrt{x}$	09
STO	33
+	61
9	09
RCL 8	34 08
STO	33
+	61
5	05
RCL 6	34 06
0	00
STO 8	33 08
STO 6	33 06
g x⇄y	35 07
R/S	84
LBL	23
D	14
STO	33
+	61
6	06
1	01
RCL 8	34 08
+	61
STO 8	33 08
R/S	84
LBL	23
E	15
RCL 6	34 06
f^{-1}	32
$\sqrt{x}$	09
STO	33
+	61

KEYS	CODE
3	03
0	00
STO 6	33 06
STO 8	33 08
g LST X	35 00
R/S	84
LBL	23
1	01
STO	33
—	51
1	01
STO	33
—	51
7	07
f^{-1}	32
$\sqrt{x}$	09
STO	33
—	51
2	02
RCL 4	34 04
1	01
—	51
STO 4	33 04
R/S	84
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1 $\Sigma\Sigma\Sigma\, x$	R_4 n	R_7 $\Sigma\, x$
R_2 $\Sigma\Sigma\Sigma\, x^2$	R_5 $\Sigma\Sigma\,(\Sigma\, x)^2$	R_8 $\Sigma\,(\Sigma\, x)^2$
R_3 $\Sigma\,(\Sigma\Sigma\, x)^2$	R_6 $\Sigma\Sigma\, x, \Sigma\Sigma\, x$	R_9 $\Sigma\,(\Sigma\Sigma\, x)^2$

TWO WAY ANALYSIS OF VARIANCE (WITH REPLICATIONS) CARD 2

KEYS	CODE
STO 8	33 08
R/S	84
STO 7	33 07
R/S	84
STO 6	33 06
R/S	84
LBL	23
A	11
RCL 1	34 01
f^{-1}	32
$\sqrt{x}$	09
RCL 6	34 06
÷	81
RCL 7	34 07
÷	81
RCL 8	34 08
÷	81
STO 1	33 01
RCL 3	34 03
RCL 6	34 06
÷	81
RCL 8	34 08
÷	81
STO 3	33 03
RCL	34
9	09
RCL 6	34 06
÷	81
RCL 7	34 07
÷	81
STO 4	33 04
RCL 5	34 05
RCL 6	34 06
÷	81
STO 5	33 05

KEYS	CODE
STO	33
9	09
RCL 4	34 04
–	51
RCL 3	34 03
–	51
RCL 1	34 01
+	61
STO 5	33 05
RCL 2	34 02
RCL	34
9	09
–	51
RCL 4	34 04
STO	33
9	09
g R↓	35 08
STO 4	33 04
RCL 3	34 03
RCL 1	34 01
–	51
STO 3	33 03
RCL	34
9	09
RCL 1	34 01
–	51
RCL 2	34 02
STO	33
9	09
g R↓	35 08
STO 2	33 02
RCL	34
9	09
RCL 1	34 01
–	51

KEYS	CODE
STO 1	33 01
R/S	84
LBL	23
B	12
RCL 2	34 02
R/S	84
LBL	23
C	13
RCL 3	34 03
R/S	84
LBL	23
D	14
RCL 5	34 05
R/S	84
LBL	23
E	15
RCL 4	34 04
R/S	84
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1 $\Sigma\Sigma\Sigma$ x, Used	R_4 Used, ESS	R_7 c
R_2 $\Sigma\Sigma\Sigma$ x^2, RSS	R_5 Used, ISS	R_8 r
R_3 Used, CSS	R_6 n	R_9 Used

TWO WAY ANALYSIS OF VARIANCE (WITH REPLICATIONS) CARD 3

KEYS	CODE	KEYS	CODE	KEYS	CODE
LBL	23	R/S	84	RCL 6	34 06
A	11	RCL 6	34 06	R/S	84
RCL 4	34 04	R/S	84	LBL	23
RCL 7	34 07	LBL	23	E	15
RCL 8	34 08	B	12	RCL 4	34 04
x	71	RCL 2	34 02	R/S	84
RCL 6	34 06	RCL 5	34 05	RCL 6	34 06
1	01	÷	81	R/S	84
—	51	R/S	84	g NOP	35 01
x	71	RCL 8	34 08	g NOP	35 01
STO 6	33 06	R/S	84	g NOP	35 01
÷	81	RCL 1	34 01	g NOP	35 01
STO 4	33 04	R/S	84	g NOP	35 01
RCL 5	34 05	LBL	23	g NOP	35 01
RCL 7	34 07	C	13	g NOP	35 01
1	01	RCL 3	34 03	g NOP	35 01
—	51	RCL 7	34 07	g NOP	35 01
STO 7	33 07	÷	81	g NOP	35 01
RCL 8	34 08	R/S	84	g NOP	35 01
1	01	RCL 4	34 04	g NOP	35 01
—	51	÷	81	g NOP	35 01
STO 8	33 08	R/S	84	g NOP	35 01
x	71	RCL 7	34 07	g NOP	35 01
STO 1	33 01	R/S	84	g NOP	35 01
÷	81	RCL 6	34 06	g NOP	35 01
STO 5	33 05	R/S	84	g NOP	35 01
RCL 2	34 02	LBL	23	g NOP	35 01
RCL 8	34 08	D	14	g NOP	35 01
÷	81	RCL 5	34 05	g NOP	35 01
STO 2	33 02	R/S	84	g NOP	35 01
R/S	84	RCL 4	34 04		
RCL 4	34 04	÷	81		
÷	81	R/S	84		
R/S	84	RCL 1	34 01		
RCL 8	34 08	R/S	84		

R_1	Used	R_4	ESS, EMS	R_7	c, c–1
R_2	RSS, RMS	R_5	ISS, IMS	R_8	r, r–1
R_3	CSS	R_6	n, rc(n–1)	R_9	

LATIN SQUARE (CARD 1)

KEYS	CODE
R/S	84
0	00
STO 1	33 01
STO 2	33 02
STO 3	33 03
STO 5	33 05
STO 6	33 06
R/S	84
LBL	23
A	11
STO	33
+	61
1	01
f^{-1}	32
$\sqrt{x}$	09
STO	33
+	61
6	06
1	01
RCL 2	34 02
+	61
STO 2	33 02
R/S	84
LBL	23
B	12
RCL 1	34 01
f^{-1}	32
$\sqrt{x}$	09
RCL 2	34 02
÷	81
STO	33
+	61
3	03
RCL 1	34 01
STO 4	33 04

KEYS	CODE
STO	33
+	61
5	05
RCL 2	34 02
0	00
STO 1	33 01
STO 2	33 02
CLX	44
RCL 4	34 04
R/S	84
LBL	23
C	13
RCL 6	34 06
RCL 5	34 05
f^{-1}	32
$\sqrt{x}$	09
g R↑	35 09
STO 2	33 02
↑	41
x	71
÷	81
STO 1	33 01
—	51
R/S	84
LBL	23
C	13
STO 7	33 07
E	15
STO 8	33 08
GTO	22
0	00
LBL	23
D	14
E	15
STO	33

KEYS	CODE
9	09
GTO	22
0	00
LBL	23
E	15
RCL 3	34 03
RCL 1	34 01
—	51
STO	33
—	51
7	07
RTN	24
STO 1	33 01
RCL 7	34 07
R/S	84
LBL	23
1	01
STO	33
—	51
1	01
f^{-1}	32
$\sqrt{x}$	09
STO	33
—	51
6	06
RCL 2	34 02
1	01
—	51
STO 2	33 02
R/S	84

R_1	Used, TrSS	R_4	Used	R_7	TSS, Used, ReSS
R_2	n	R_5	$\Sigma\Sigma\ x_{ij}$	R_8	RSS
R_3	Used	R_6	$\Sigma\Sigma\ x_{ij}^2$	R_9	CSS

LATIN SQUARE (CARD 2)

KEYS	CODE
LBL	23
A	11
RCL 2	34 02
3	03
−	51
RCL 2	34 02
x	71
2	02
+	61
STO 3	33 03
RCL 8	34 08
RCL 2	34 02
1	01
−	51
STO 2	33 02
÷	81
R/S	84
LBL	23
A	11
RCL 7	34 07
RCL 3	34 03
÷	81
STO 7	33 07
÷	81
R/S	84
LBL	23
B	12
RCL	34
9	09
RCL 2	34 02
÷	81
R/S	84
LBL	23
B	12
RCL 7	34 07

KEYS	CODE
÷	81
R/S	84
LBL	23
C	13
RCL 1	34 01
RCL 2	34 02
÷	81
R/S	84
LBL	23
C	13
RCL 7	34 07
÷	81
R/S	84
LBL	23
D	14
RCL 7	34 07
R/S	84
LBL	23
E	15
RCL 2	34 02
R/S	84
LBL	23
E	15
RCL 3	34 03
R/S	84
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

KEYS	CODE
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1	TrSS	R_4	Used	R_7	ReSS
R_2	n, n−1	R_5	$\Sigma\Sigma x_{ij}$	R_8	RSS
R_3	Used, df_2	R_6	$\Sigma\Sigma x_{ij}^2$	R_9	CSS

ANALYSIS OF COVARIANCE (ONE WAY) CARD 1

KEYS	CODE
STO	33
+	61
1	01
f^{-1}	32
$\sqrt{x}$	09
STO	33
+	61
6	06
1	01
RCL 2	34 02
+	61
STO 2	33 02
R/S	84
LBL	23
B	12
1	01
STO	33
+	61
4	04
RCL 1	34 01
f^{-1}	32
$\sqrt{x}$	09
RCL 2	34 02
÷	81
STO	33
+	61
3	03
RCL 1	34 01
STO 8	33 08
STO	33
+	61
7	07
RCL 2	34 02
STO	33
+	61

KEYS	CODE
5	05
0	00
STO 1	33 01
STO 2	33 02
RCL 8	34 08
R/S	84
LBL	23
C	13
RCL 6	34 06
RCL 7	34 07
f^{-1}	32
$\sqrt{x}$	09
RCL 5	34 05
÷	81
—	51
R/S	84
LBL	23
C	13
RCL 3	34 03
RCL 7	34 07
f^{-1}	32
$\sqrt{x}$	09
RCL 5	34 05
÷	81
—	51
R/S	84
LBL	23
C	13
—	51
R/S	84
LBL	23
D	14
g LST X	35 00
RCL 4	34 04
1	01

KEYS	CODE
—	51
R/S	84
LBL	23
D	14
÷	81
g x⇄y	35 07
RCL 5	34 05
RCL 4	34 04
—	51
R/S	84
LBL	23
D	14
÷	81
÷	81
R/S	84
LBL	23
E	15
STO	33
—	51
1	01
f^{-1}	32
$\sqrt{x}$	09
STO	33
—	51
6	06
RCL 2	34 02
1	01
—	51
STO 2	33 02
R/S	84

R_1 Used	R_4 Used	R_7 Used
R_2 Used	R_5 Σn_i	R_8 Used
R_3 Used	R_6 Used	R_9

ANALYSIS OF COVARIANCE (ONE WAY) CARD 2

KEYS	CODE
LBL	23
A	11
STO	33
+	61
8	08
g x⇄y	35 07
STO	33
+	61
7	07
x	71
STO	33
+	61
3	03
1	01
RCL 5	34 05
+	61
STO 5	33 05
RTN	24
LBL	23
B	12
RCL 7	34 07
STO	33
+	61
1	01
RCL 8	34 08
STO	33
+	61
2	02
x	71
RCL 5	34 05
÷	81
STO	33
+	61
4	04
RCL 5	34 05

KEYS	CODE
STO	33
+	61
9	09
0	00
STO 5	33 05
STO 7	33 07
STO 8	33 08
1	01
RCL 6	34 06
+	61
STO 6	33 06
RTN	24
LBL	23
C	13
RCL 3	34 03
RCL 1	34 01
RCL 2	34 02
x	71
RCL	34
9	09
STO 5	33 05
÷	81
STO 7	33 07
–	51
STO 8	33 08
RTN	24
LBL	23
C	13
RCL 4	34 04
RCL 7	34 07
–	51
STO	33
9	09
RTN	24
LBL	23

KEYS	CODE
C	13
RCL 8	34 08
RCL	34
9	09
–	51
STO 7	33 07
RTN	24
LBL	23
E	15
STO	33
–	51
8	08
g x⇄y	35 07
STO	33
–	51
7	07
x	71
STO	33
–	51
3	03
RCL 5	34 05
1	01
–	51
STO 5	33 05
R/S	84
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1 $\Sigma\Sigma\, x_{ij}$	R_4 Used	R_7 $\Sigma\, x_{ij}$, WSP
R_2 $\Sigma\Sigma\, y_{ij}$	R_5 j, $\Sigma\, n_i$	R_8 $\Sigma\, y_{ij}$, TSP
R_3 $\Sigma\Sigma\, x_{ij}y_{ij}$	R_6 k	R_9 $\Sigma\, n_i$, ASP

ANALYSIS OF COVARIANCE (ONE WAY) CARD 3

KEYS	CODE	KEYS	CODE	KEYS	CODE
LBL	23	STO 1	33 01	g NOP	35 01
A	11	RTN	24	g NOP	35 01
RCL 8	34 08	LBL	23	g NOP	35 01
E	15	B	12	g NOP	35 01
STO 3	33 03	RCL 4	34 04	g NOP	35 01
RTN	24	RCL 5	34 05	g NOP	35 01
LBL	23	RCL 6	34 06	g NOP	35 01
A	11	—	51	g NOP	35 01
RCL 7	34 07	2	02	g NOP	35 01
E	15	—	51	g NOP	35 01
STO 4	33 04	STO 5	33 05	g NOP	35 01
RTN	24	÷	81	g NOP	35 01
LBL	23	STO 2	33 02	g NOP	35 01
A	11	RTN	24	g NOP	35 01
RCL 3	34 03	LBL	23	g NOP	35 01
RCL 4	34 04	C	13	g NOP	35 01
—	51	RCL 1	34 01	g NOP	35 01
STO 3	33 03	RCL 2	34 02	g NOP	35 01
RTN	24	÷	81	g NOP	35 01
LBL	23	RTN	24	g NOP	35 01
E	15	LBL	23	g NOP	35 01
↑	41	D	14	g NOP	35 01
x	71	RCL 6	34 06	g NOP	35 01
g R↑	35 09	RTN	24	g NOP	35 01
÷	81	LBL	23	g NOP	35 01
—	51	D	14	g NOP	35 01
RTN	24	RCL 5	34 05	g NOP	35 01
LBL	23	RTN	24	g NOP	35 01
B	12	g NOP	35 01	g NOP	35 01
RCL 3	34 03	g NOP	35 01	g NOP	35 01
RCL 6	34 06	g NOP	35 01		
1	01	g NOP	35 01		
—	51	g NOP	35 01		
STO 6	33 06	g NOP	35 01		
÷	81	g NOP	35 01		

R_1 $\Sigma\Sigma x_{ij}$, AMS$\hat{y}$	R_4 Used, WSS$\hat{y}$	R_7 WSP
R_2 $\Sigma\Sigma y_{ij}$, WMS$\hat{y}$	R_5 Σn_i, Σn_i-k-1	R_8 TSP
R_3 Used	R_6 k, k−1	R_9 ASP

ONE SAMPLE TEST STATISTICS FOR THE MEAN

KEYS	CODE
0	00
STO 1	33 01
STO 2	33 02
STO 3	33 03
STO 4	33 04
R/S	84
LBL	23
A	11
RCL 2	34 02
—	51
RCL 4	34 04
—	51
RCL 1	34 01
1	01
+	61
÷	81
↑	41
↑	41
RCL 4	34 04
+	61
↑	41
↑	41
RCL 2	34 02
+	61
STO 2	33 02
g LST X	35 00
—	51
—	51
STO 4	33 04
g R↓	35 08
x	71
RCL 1	34 01
x	71
1	01
g LST X	35 00

KEYS	CODE
+	61
g	35
ABS	06
STO 1	33 01
x	71
STO	33
+	61
3	03
RCL 1	34 01
RTN	24
LBL	23
B	12
RCL 2	34 02
R/S	84
LBL	23
B	12
RCL 3	34 03
RCL 1	34 01
1	01
—	51
÷	81
f	31
$\sqrt{x}$	09
STO 5	33 05
RTN	24
LBL	23
C	13
STO 6	33 06
RTN	24
LBL	23
D	14
RCL 2	34 02
RCL 6	34 06
—	51
g x⇄y	35 07

KEYS	CODE
÷	81
RCL 1	34 01
f	31
$\sqrt{x}$	09
x	71
RTN	24
LBL	23
E	15
RCL 2	34 02
RCL 6	34 06
—	51
RCL 5	34 05
÷	81
RCL 1	34 01
f	31
$\sqrt{x}$	09
x	71
RTN	24
LBL	23
1	01
RCL 1	34 01
CHS	42
STO 1	33 01
g R↓	35 08
A	11
R/S	84
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1 n (or −n)	R_4 Used	R_7
R_2 running mean	R_5 s	R_8
R_3 Used	R_6 μ_0	R_9

TEST STATISTICS FOR THE CORRELATION COEFFICIENT

KEYS	CODE
LBL	23
A	11
STO 1	33 01
LBL	23
0	00
g	35
ABS	06
1	01
g $x \rightleftarrows y$	35 07
g $x > y$	35 24
0	00
÷	81
g LST X	35 00
RTN	24
LBL	23
B	12
STO 2	33 02
3	03
g $x \rightleftarrows y$	35 07
g $x \leq y$	35 22
0	00
÷	81
RTN	24
LBL	23
C	13
RCL 2	34 02
2	02
–	51
1	01
RCL 1	34 01
f^{-1}	32
$\sqrt{x}$	09
–	51
÷	81
f	31

KEYS	CODE
$\sqrt{x}$	09
RCL 1	34 01
x	71
RTN	24
LBL	23
D	14
STO 3	33 03
GTO	22
0	00
LBL	23
E	15
RCL 1	34 01
1	01
+	61
1	01
RCL 1	34 01
–	51
÷	81
1	01
RCL 3	34 03
–	51
x	71
1	01
RCL 3	34 03
+	61
÷	81
f	31
LN	07
RCL 2	34 02
3	03
–	51
f	31
$\sqrt{x}$	09
x	71
2	02

KEYS	CODE
÷	81
RTN	24
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1 r	R_4	R_7
R_2 n	R_5	R_8
R_3 ρ_0	R_6	R_9 Scratch

DIFFERENCES AMONG PROPORTIONS

KEYS	CODE
LBL	23
A	11
0	00
STO 1	33 01
STO 2	33 02
STO 3	33 03
STO 5	33 05
STO 6	33 06
RTN	24
LBL	23
B	12
STO	33
+	61
1	01
–	51
STO 4	33 04
STO	33
+	61
2	02
g LST X	35 00
+	61
g LST X	35 00
↑	41
x	71
g x⇄y	35 07
÷	81
STO	33
+	61
5	05
g LST X	35 00
RCL 4	34 04
↑	41
x	71
g x⇄y	35 07
÷	81

KEYS	CODE
STO	33
+	61
6	06
1	01
RCL 3	34 03
+	61
STO 3	33 03
RTN	24
LBL	23
C	13
RCL 5	34 05
RCL 1	34 01
÷	81
RCL 6	34 06
RCL 2	34 02
÷	81
+	61
1	01
–	51
RCL 1	34 01
RCL 2	34 02
+	61
x	71
RTN	24
LBL	23
D	14
RCL 3	34 03
1	01
–	51
RTN	24
LBL	23
E	15
RCL 1	34 01
RCL 1	34 01
RCL 2	34 02

KEYS	CODE
+	61
÷	81
RTN	24
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1 Σx_i	R_4 $n_i - x_i$	R_7
R_2 $\Sigma (n_i - x_i)$	R_5 $\Sigma (x_i^2 / n_i)$	R_8
R_3 k	R_6 $\Sigma (n_i - x_i)^2 / n_i$	R_9

BEHRENS-FISHER STATISTIC

KEYS	CODE
0	00
STO 1	33 01
STO 2	33 02
STO 3	33 03
STO 4	33 04
R/S	84
LBL	23
A	11
RCL 2	34 02
–	51
RCL 4	34 04
–	51
RCL 1	34 01
1	01
+	61
÷	81
↑	41
↑	41
RCL 4	34 04
+	61
↑	41
↑	41
RCL 2	34 02
+	61
STO 2	33 02
g LST X	35 00
–	51
–	51
STO 4	33 04
g R↓	35 08
x	71
RCL 1	34 01
x	71
1	01
g LST X	35 00

KEYS	CODE
+	61
g	35
ABS	06
STO 1	33 01
x	71
STO	33
+	61
3	03
RCL 1	34 01
R/S	84
LBL	23
B	12
RCL 2	34 02
STO 5	33 05
RCL 3	34 03
RCL 1	34 01
1	01
–	51
÷	81
RCL 1	34 01
÷	81
STO 6	33 06
RTN	24
LBL	23
C	13
STO 7	33 07
RTN	24
LBL	23
D	14
RCL 5	34 05
RCL 2	34 02
–	51
RCL 7	34 07
–	51
RCL 3	34 03

KEYS	CODE
RCL 1	34 01
1	01
–	51
÷	81
RCL 1	34 01
÷	81
STO 8	33 08
RCL 6	34 06
+	61
f	31
$\sqrt{x}$	09
÷	81
RTN	24
LBL	23
E	15
RCL 6	34 06
RCL 8	34 08
÷	81
f	31
$\sqrt{x}$	09
f^{-1}	32
TAN	06
RTN	24
LBL	23
1	01
RCL 1	34 01
CHS	42
STO 1	33 01
g R↓	35 08
A	11

R_1 n	R_4 Used	R_7 D
R_2 running mean	R_5 $\bar{x}$	R_8 $s_2{}^2/n_2$
R_3 Used	R_6 $s_1{}^2/n_1$	R_9

KRUSKAL-WALLIS STATISTIC

KEYS	CODE	KEYS	CODE	KEYS	CODE
LBL	23	+	61	1	01
A	11	STO 4	33 04	–	51
0	00	0	00	STO 1	33 01
STO 1	33 01	STO 1	33 01	R/S	84
STO 2	33 02	STO 2	33 02	g NOP	35 01
STO 3	33 03	RCL 4	34 04	g NOP	35 01
STO 4	33 04	RTN	24	g NOP	35 01
STO 5	33 05	LBL	23	g NOP	35 01
RTN	24	D	14	g NOP	35 01
LBL	23	RCL 3	34 03	g NOP	35 01
B	12	4	04	g NOP	35 01
STO	33	x	71	g NOP	35 01
+	61	RCL 5	34 05	g NOP	35 01
2	02	÷	81	g NOP	35 01
RCL 1	34 01	RCL 5	34 05	g NOP	35 01
1	01	1	01	g NOP	35 01
+	61	+	61	g NOP	35 01
STO 1	33 01	÷	81	g NOP	35 01
RTN	24	g LST X	35 00	g NOP	35 01
LBL	23	–	51	g NOP	35 01
C	13	3	03	g NOP	35 01
RCL 1	34 01	x	71	g NOP	35 01
STO	33	RTN	24	g NOP	35 01
+	61	LBL	23	g NOP	35 01
5	05	E	15	g NOP	35 01
RCL 2	34 02	RCL 4	34 04	g NOP	35 01
f^{-1}	32	1	01	g NOP	35 01
$\sqrt{x}$	09	–	51	g NOP	35 01
g x⇄y	35 07	RTN	24	g NOP	35 01
÷	81	LBL	23	g NOP	35 01
STO	33	1	01		
+	61	STO	33		
3	03	–	51		
RCL 4	34 04	2	02		
1	01	RCL 1	34 01		

R_1 n_i	R_4 k	R_7
R_2 ΣR_{ij}	R_5 N	R_8
R_3 Used	R_6	R_9

MEAN-SQUARE SUCCESSIVE DIFFERENCE

KEYS	CODE
LBL	23
A	11
0	00
STO 1	33 01
STO 2	33 02
STO 3	33 03
STO 4	33 04
RTN	24
LBL	23
B	12
STO 5	33 05
STO	33
+	61
2	02
↑	41
x	71
STO	33
+	61
3	03
RCL 1	34 01
1	01
+	61
STO 1	33 01
RTN	24
LBL	23
C	13
RCL 5	34 05
g x⇄y	35 07
STO 5	33 05
—	51
↑	41
x	71
STO	33
+	61
4	04

KEYS	CODE
RCL 5	34 05
GTO	22
B	12
LBL	23
D	14
RCL 4	34 04
RCL 3	34 03
RCL 2	34 02
↑	41
x	71
RCL 1	34 01
÷	81
—	51
÷	81
STO 5	33 05
RTN	24
LBL	23
E	15
1	01
RCL 5	34 05
2	02
÷	81
—	51
RCL 1	34 01
2	02
—	51
RCL 1	34 01
↑	41
x	71
1	01
—	51
÷	81
f	31
$\sqrt{x}$	09
÷	81

KEYS	CODE
RTN	24
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1 n	R_4 $\Sigma (x_i - x_{i-1})^2$	R_7
R_2 Σx_i	R_5 x_i, η	R_8
R_3 Σx_i^2	R_6	R_9

3XK CONTINGENCY TABLE

KEYS	CODE
f	31
REG	43
0	00
R/S	84
LBL	23
B	12
STO	33
+	61
3	03
STO 7	33 07
↑	41
x	71
STO 8	33 08
g R↓	35 08
STO	33
+	61
2	02
STO	33
+	61
7	07
↑	41
x	71
STO	33
9	09
g R↓	35 08
STO	33
+	61
1	01
STO	33
+	61
7	07
↑	41
x	71
RCL 7	34 07
÷	81

KEYS	CODE
STO	33
+	61
4	04
RCL	34
9	09
RCL 7	34 07
÷	81
STO	33
+	61
5	05
RCL 8	34 08
RCL 7	34 07
÷	81
STO	33
+	61
6	06
RCL 7	34 07
R/S	84
LBL	23
C	13
RCL 1	34 01
RCL 2	34 02
RCL 3	34 03
+	61
+	61
STO 8	33 08
RCL 4	34 04
RCL 1	34 01
÷	81
STO	33
9	09
RCL 5	34 05
RCL 2	34 02
÷	81
STO	33

KEYS	CODE
+	61
9	09
RCL 6	34 06
RCL 3	34 03
÷	81
STO	33
+	61
9	09
RCL	34
9	09
1	01
–	51
RCL 8	34 08
x	71
R/S	84
LBL	23
D	14
↑	41
↑	41
RCL 8	34 08
+	61
÷	81
f	31
$\sqrt{x}$	09
R/S	84
RCL 1	34 01
RCL 2	34 02
RCL 3	34 03
RCL 8	34 08
g NOP	35 01

R_1	R_1	**R_4**	$\Sigma\, x_{1j}^2/C_j$	**R_7**	C_j
R_2	R_2	**R_5**	$\Sigma\, x_{2j}^2/C_j$	**R_8**	x_{3j}^2, N
R_3	R_3	**R_6**	$\Sigma\, x_{3j}^2/C_j$	**R_9**	x_{2j}^2, Used

THE RUN TEST FOR RANDOMNESS

KEYS	CODE	KEYS	CODE	KEYS	CODE
LBL	23	↑	41	g NOP	35 01
A	11	x	71	g NOP	35 01
STO 2	33 02	RCL 8	34 08	g NOP	35 01
g R↓	35 08	1	01	g NOP	35 01
STO 1	33 01	–	51	g NOP	35 01
R/S	84	x	71	g NOP	35 01
LBL	23	÷	81	g NOP	35 01
B	12	f	31	g NOP	35 01
STO 3	33 03	$\sqrt{x}$	09	g NOP	35 01
R/S	84	STO 5	33 05	g NOP	35 01
LBL	23	R/S	84	g NOP	35 01
C	13	LBL	23	g NOP	35 01
RCL 1	34 01	E	15	g NOP	35 01
RCL 2	34 02	RCL 3	34 03	g NOP	35 01
x	71	RCL 4	34 04	g NOP	35 01
2	02	–	51	g NOP	35 01
x	71	RCL 5	34 05	g NOP	35 01
STO 7	33 07	÷	81	g NOP	35 01
RCL 1	34 01	STO 6	33 06	g NOP	35 01
RCL 2	34 02	R/S	84	g NOP	35 01
+	61	g NOP	35 01	g NOP	35 01
STO 8	33 08	g NOP	35 01	g NOP	35 01
÷	81	g NOP	35 01	g NOP	35 01
1	01	g NOP	35 01	g NOP	35 01
+	61	g NOP	35 01	g NOP	35 01
STO 4	33 04	g NOP	35 01	g NOP	35 01
R/S	84	g NOP	35 01	g NOP	35 01
LBL	23	g NOP	35 01	g NOP	35 01
D	14	g NOP	35 01	g NOP	35 01
RCL 7	34 07	g NOP	35 01	g NOP	35 01
RCL 8	34 08	g NOP	35 01		
–	51	g NOP	35 01		
RCL 7	34 07	g NOP	35 01		
x	71	g NOP	35 01		
RCL 8	34 08	g NOP	35 01		

R_1 n_1	R_4 μ	R_7 $2\,n_1\,n_2$
R_2 n_2	R_5 σ	R_8 n_1+n_2
R_3 u	R_6 z	R_9

INTRACLASS CORRELATION COEFFICIENT

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO	33	R/S	84	÷	81
+	61	LBL	23	R/S	84
6	06	C	13	LBL	23
f^{-1}	32	RCL 4	34 04	D	14
$\sqrt{x}$	09	RCL 3	34 03	RCL 7	34 07
STO	33	f^{-1}	32	RCL 8	34 08
+	61	$\sqrt{x}$	09	RCL 1	34 01
5	05	RCL 2	34 02	÷	81
1	01	÷	81	÷	81
RCL 1	34 01	–	51	R/S	84
+	61	RCL 7	34 07	LBL	23
STO 1	33 01	STO 1	33 01	E	15
R/S	84	÷	81	RCL 2	34 02
LBL	23	RCL 2	34 02	1	01
B	12	1	01	–	51
RCL 6	34 06	–	51	R/S	84
STO 8	33 08	÷	81	LBL	23
STO	33	STO 7	33 07	E	15
+	61	RCL 5	34 05	RCL 1	34 01
3	03	RCL 4	34 04	RCL 2	34 02
f^{-1}	32	RCL 1	34 01	x	71
$\sqrt{x}$	09	÷	81	R/S	84
STO	33	–	51	g NOP	35 01
+	61	RCL 2	34 02	g NOP	35 01
4	04	÷	81	g NOP	35 01
RCL 1	34 01	STO 8	33 08	g NOP	35 01
STO 7	33 07	RCL 1	34 01	g NOP	35 01
0	00	1	01	g NOP	35 01
STO 1	33 01	–	51	g NOP	35 01
STO 6	33 06	STO 1	33 01	g NOP	35 01
1	01	÷	81		
RCL 2	34 02	–	51		
+	61	RCL 7	34 07		
STO 2	33 02	RCL 8	34 08		
RCL 8	34 08	+	61		

R_1 n, n–1	R_4 ΣT_i^2	R_7 n, ASS/k–1
R_2 k	R_5 Σx_{ij}^2	R_8 T_i, WSS/k
R_3 ΣT_i	R_6 T_i	R_9 0

FISHER'S EXACT TEST FOR A 2x2 CONTINGENCY TABLE

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 4	33 04	n!	03	+	61
g R↓	35 08	x	71	2	02
STO 3	33 03	STO 7	33 07	STO	33
g R↓	35 08	0	00	+	61
STO 2	33 02	STO 5	33 05	3	03
g x⇄y	35 07	g R↓	35 08	STO	33
STO 1	33 01	LBL	23	—	51
STO 8	33 08	0	00	4	04
+	61	RCL 1	34 01	STO	33
STO 5	33 05	g	35	—	51
g R↓	35 08	n!	03	8	08
+	61	÷	81	RCL 7	34 07
STO 6	33 06	RCL 2	34 02	GTO	22
g	35	g	35	0	00
n!	03	n!	03	LBL	23
RCL 5	34 05	÷	81	C	13
g	35	RCL 3	34 03	RCL 5	34 05
n!	03	g	35	R/S	84
x	71	n!	03	LBL	23
RCL 5	34 05	÷	81	D	14
RCL 6	34 06	RCL 4	34 04	RCL 8	34 08
+	61	g	35	0	00
g	35	n!	03	g x=y	35 23
n!	03	÷	81	RCL 5	34 05
÷	81	STO	33	R/S	84
RCL 1	34 01	+	61	B	12
RCL 3	34 03	5	05	GTO	22
+	61	RTN	24	D	14
g	35	LBL	23	g NOP	35 01
n!	03	B	12	g NOP	35 01
x	71	1	01		
RCL 2	34 02	STO	33		
RCL 4	34 04	—	51		
+	61	1	01		
g	35	STO	33		

R_1 a	R_4 d	R_7 Used
R_2 b	R_5 a+b, Used	R_8 Used
R_3 c	R_6 c+d	R_9

PROBABILITY OF NO REPETITIONS IN A SAMPLE (BIRTHDAY PROBLEM)

KEYS	CODE	KEYS	CODE	KEYS	CODE
LBL	23	LBL	23	g NOP	35 01
A	11	1	01	g NOP	35 01
STO 1	33 01	RCL 3	34 03	g NOP	35 01
RTN	24	1	01	g NOP	35 01
LBL	23	–	51	g NOP	35 01
B	12	STO 3	33 03	g NOP	35 01
STO 2	33 02	1	01	g NOP	35 01
STO 3	33 03	g x>y	35 24	g NOP	35 01
RTN	24	RCL 4	34 04	g NOP	35 01
LBL	23	RTN	24	g NOP	35 01
C	13	g x⇄y	35 07	g NOP	35 01
RCL 2	34 02	RCL 1	34 01	g NOP	35 01
↑	41	÷	81	g NOP	35 01
f	31	–	51	g NOP	35 01
INT	83	RCL 4	34 04	g NOP	35 01
g x≠y	35 21	x	71	g NOP	35 01
0	00	STO 4	33 04	g NOP	35 01
÷	81	GTO	22	g NOP	35 01
RCL 1	34 01	1	01	g NOP	35 01
↑	41	LBL	23	g NOP	35 01
f	31	D	14	g NOP	35 01
INT	83	1	01	g NOP	35 01
g x≠y	35 21	RCL 4	34 04	g NOP	35 01
0	00	–	51	g NOP	35 01
÷	81	RTN	24	g NOP	35 01
RCL 2	34 02	g NOP	35 01	g NOP	35 01
g x>y	35 24	g NOP	35 01	g NOP	35 01
0	00	g NOP	35 01	g NOP	35 01
÷	81	g NOP	35 01	g NOP	35 01
1	01	g NOP	35 01	g NOP	35 01
g x>y	35 24	g NOP	35 01		
0	00	g NOP	35 01		
÷	81	g NOP	35 01		
1	01	g NOP	35 01		
STO 4	33 04	g NOP	35 01		

R_1 m	R_4 1, Used	R_7
R_2 n	R_5	R_8
R_3 n, Used	R_6	R_9 Scratch

$\bar{x}$ AND R CONTROL CHARTS

KEYS	CODE
f^{-1}	32
TF1	61
GTO	22
1	01
0	00
STO 1	33 01
STO 2	33 02
STO 3	33 03
g x⇄y	35 07
STO 4	33 04
STO 5	33 05
f^{-1}	32
SF1	51
LBL	23
1	01
RCL 4	34 04
g x⇄y	35 07
g x>y	35 24
g NOP	35 01
STO 4	33 04
RCL 5	34 05
g x⇄y	35 07
g x≤y	35 22
g NOP	35 01
STO 5	33 05
STO	33
+	61
2	02
f^{-1}	32
$\sqrt{x}$	09
STO	33
+	61
3	03
RCL 1	34 01
1	01

KEYS	CODE
+	61
STO 1	33 01
RTN	24
f	31
REG	43
f	31
SF1	51
R/S	84
LBL	23
B	12
f	31
SF1	51
RCL 6	34 06
1	01
+	61
STO 6	33 06
RCL 2	34 02
RCL 1	34 01
÷	81
STO	33
+	61
7	07
R/S	84
LBL	23
B	12
RCL 4	34 04
RCL 5	34 05
–	51
STO	33
+	61
8	08
R/S	84
LBL	23
C	13
RCL 7	34 07

KEYS	CODE
RCL 6	34 06
÷	81
RTN	24
LBL	23
C	13
RCL 8	34 08
RCL 6	34 06
÷	81
STO 3	33 03
R/S	84
LBL	23
D	14
RCL 3	34 03
x	71
C	13
g x⇄y	35 07
–	51
R/S	84
LBL	23
D	14
g LST X	35 00
2	02
x	71
+	61
R/S	84
LBL	23
E	15
RCL 3	34 03
x	71
R/S	84

R_1	n	R_4	x_{max}	R_7	$\Sigma \bar{x}_i$
R_2	Σx_{ij}	R_5	x_{min}	R_8	ΣR_i
R_3	$\Sigma x_{ij}^2, \bar{R}$	R_6	m	R_9	Scratch

p AND c CONTROL CHARTS

KEYS	CODE
0	00
STO 1	33 01
STO 2	33 02
STO 3	33 03
R/S	84
LBL	23
A	11
STO	33
+	61
2	02
RCL 1	34 01
1	01
+	61
STO 1	33 01
R/S	84
LBL	23
B	12
STO	33
+	61
1	01
x	71
STO	33
+	61
2	02
RCL 3	34 03
1	01
+	61
STO 3	33 03
R/S	84
LBL	23
C	13
RCL 2	34 02
RCL 1	34 01
÷	81
STO 4	33 04

KEYS	CODE
R/S	84
LBL	23
D	14
1	01
RCL 4	34 04
–	51
RCL 4	34 04
x	71
g x⇄y	35 07
÷	81
f	31
$\sqrt{x}$	09
3	03
x	71
STO 3	33 03
RCL 4	34 04
g x⇄y	35 07
–	51
0	00
g x≤y	35 22
g x⇄y	35 07
g NOP	35 01
R/S	84
LBL	23
D	14
RCL 4	34 04
RCL 3	34 03
+	61
R/S	84
LBL	23
E	15
RCL 4	34 04
RCL 4	34 04
f	31
$\sqrt{x}$	09

KEYS	CODE
3	03
x	71
STO 3	33 03
–	51
0	00
g x≤y	35 22
g x⇄y	35 07
g NOP	35 01
R/S	84
LBL	23
E	15
RCL 4	34 04
RCL 3	34 03
+	61
R/S	84
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1	m or Σn_i	R_4	$\bar{p}$ or $p_0{}'$ or $\bar{c}$	R_7	
R_2	Used	R_5		R_8	
R_3	m, Used	R_6		R_9	Scratch

OPERATING CHARACTERISTIC CURVE (TYPE A)

KEYS	CODE
RCL 1	34 01
x	71
f	31
INT	83
STO 4	33 04
RCL 1	34 01
RCL 2	34 02
E	15
RCL 1	34 01
RCL 4	34 04
–	51
RCL 2	34 02
E	15
g R↑	35 09
÷	81
STO 5	33 05
STO 6	33 06
STO 7	33 07
RCL 3	34 03
0	00
STO 8	33 08
g x=y	35 23
RCL 5	34 05
R/S	84
LBL	23
0	00
RCL 4	34 04
–	51
RCL 8	34 08
RCL 2	34 02
–	51
x	71
RCL 8	34 08
1	01
+	61

KEYS	CODE
÷	81
g LST X	35 00
RCL 1	34 01
RCL 4	34 04
–	51
RCL 2	34 02
–	51
+	61
÷	81
RCL 6	34 06
x	71
STO 6	33 06
STO	33
+	61
7	07
RCL 3	34 03
1	01
RCL 8	34 08
+	61
STO 8	33 08
g x≠y	35 21
GTO	22
0	00
1	01
RCL 7	34 07
g x>y	35 24
g x⇄y	35 07
g NOP	35 01
R/S	84
LBL	23
E	15
–	51
g LST X	35 00
g x≤y	35 22
STO 6	33 06

KEYS	CODE
g x⇄y	35 07
STO 5	33 05
1	01
STO 7	33 07
+	61
STO 6	33 06
CLX	44
g x=y	35 23
1	01
RTN	24
LBL	23
1	01
g R↓	35 08
1	01
RCL 7	34 07
+	61
STO 7	33 07
g x>y	35 24
RCL 6	34 06
RTN	24
RCL 5	34 05
g x⇄y	35 07
+	61
g LST X	35 00
÷	81
RCL 6	34 06
x	71
STO 6	33 06
GTO	22
1	01

R_1	N	**R_4**	M	**R_7**	Used
R_2	n	**R_5**	f(0)	**R_8**	Counter x
R_3	c	**R_6**	Used	**R_9**	Scratch

OPERATING CHARACTERISTIC CURVE (TYPE B)

KEYS	CODE
LBL	23
A	11
STO 1	33 01
R/S	84
LBL	23
B	12
STO 6	33 06
R/S	84
LBL	23
C	13
STO 2	33 02
RCL 2	34 02
1	01
–	51
CHS	42
÷	81
STO 8	33 08
g LST X	35 00
RCL 1	34 01
g	35
y^x	05
STO 3	33 03
RCL 6	34 06
0	00
STO 7	33 07
g x=y	35 23
RCL 3	34 03
R/S	84
CLX	44
RCL 1	34 01
g x⇄y	35 07
g x>y	35 24
0	00
÷	81
RCL 3	34 03

KEYS	CODE
STO 4	33 04
STO 5	33 05
LBL	23
1	01
RCL 1	34 01
RCL 7	34 07
–	51
RCL 7	34 07
1	01
+	61
÷	81
RCL 8	34 08
x	71
RCL 4	34 04
x	71
STO 4	33 04
STO	33
+	61
5	05
RCL 7	34 07
1	01
+	61
STO 7	33 07
RCL 6	34 06
g x≠y	35 21
GTO	22
1	01
1	01
RCL 5	34 05
g x>y	35 24
g x⇄y	35 07
g NOP	35 01
R/S	84
g NOP	35 01
g NOP	35 01

KEYS	CODE
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1 n	R_4 Used	R_7 Counter
R_2 p	R_5 Used	R_8 p/(1–p)
R_3 f(0)	R_6 c	R_9 Scratch

SINGLE- AND MULTI-SERVER QUEUES (INFINITE CUSTOMERS)

KEYS	CODE
STO 1	33 01
STO 8	33 08
g R↓	35 08
STO 2	33 02
g x⇄y	35 07
STO 5	33 05
÷	81
STO 3	33 03
R/S	84
LBL	23
B	12
1	01
STO 4	33 04
0	00
LBL	23
1	01
RCL 4	34 04
+	61
g LST X	35 00
RCL 3	34 03
x	71
RCL 1	34 01
RCL 8	34 08
–	51
1	01
+	61
÷	81
STO 4	33 04
g R↓	35 08
g	35
DSZ	83
GTO	22
1	01
1	01
RCL 3	34 03

KEYS	CODE
RCL 1	34 01
÷	81
–	51
RCL 4	34 04
g x⇄y	35 07
÷	81
STO 8	33 08
+	61
g	35
$^1/x$	04
R/S	84
LBL	23
B	12
RCL 8	34 08
x	71
STO 8	33 08
R/S	84
LBL	23
C	13
RCL 8	34 08
RCL 3	34 03
x	71
RCL 1	34 01
RCL 3	34 03
–	51
÷	81
STO 4	33 04
R/S	84
LBL	23
C	13
RCL 3	34 03
+	61
STO 6	33 06
R/S	84
LBL	23

KEYS	CODE
D	14
RCL 4	34 04
RCL 2	34 02
÷	81
R/S	84
LBL	23
D	14
RCL 6	34 06
RCL 2	34 02
÷	81
R/S	84
LBL	23
E	15
RCL 1	34 01
RCL 5	34 05
x	71
RCL 2	34 02
–	51
x	71
CHS	42
f^{-1}	32
LN	07
RCL 8	34 08
x	71
R/S	84
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01
g NOP	35 01

R_1 n	R_4 Used, L_q	R_7
R_2 λ	R_5 μ	R_8 Used, P_b
R_3 ρ	R_6 L	R_9

SINGLE- AND MULTI-SERVER QUEUES (FINITE CUSTOMERS)

KEYS	CODE
STO 2	33 02
g R↓	35 08
STO 1	33 01
R/S	84
LBL	23
B	12
STO 8	33 08
÷	81
STO 3	33 03
R/S	84
LBL	23
C	13
CLX	44
STO 7	33 07
1	01
STO 4	33 04
STO 5	33 05
STO 6	33 06
LBL	23
1	01
RCL 2	34 02
RCL 4	34 04
g x>y	35 24
g x⇄y	35 07
g NOP	35 01
RCL 3	34 03
g x⇄y	35 07
÷	81
RCL 1	34 01
RCL 4	34 04
–	51
1	01
+	61
x	71
RCL 5	34 05

KEYS	CODE
x	71
STO 5	33 05
EEX	43
CHS	42
9	09
0	00
g x>y	35 24
GTO	22
2	02
g R↓	35 08
STO	33
+	61
6	06
RCL 4	34 04
x	71
STO	33
+	61
7	07
RCL 1	34 01
RCL 4	34 04
1	01
+	61
STO 4	33 04
g x≤y	35 22
GTO	22
1	01
LBL	23
2	02
RCL 7	34 07
RCL 6	34 06
÷	81
STO 5	33 05
R/S	84
LBL	23
C	13

KEYS	CODE
RCL 8	34 08
x	71
R/S	84
LBL	23
D	14
RCL 5	34 05
RCL 1	34 01
÷	81
1	01
–	51
RCL 3	34 03
1	01
+	61
x	71
STO 7	33 07
1	01
+	61
RCL 1	34 01
x	71
R/S	84
LBL	23
D	14
RCL 8	34 08
x	71
R/S	84
LBL	23
E	15
RCL 7	34 07
CHS	42
R/S	84

R_1 m	R_4 k	R_7 ΣkQ_k, –F
R_2 n	R_5 Q_k, L	R_8 a
R_3 ρ	R_6 ΣQ_k	R_9 Scratch

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