

HEWLETT  PACKARD

**HP-65**

**STRESS ANALYSIS PAC 1**

MARCH, 1975

**SIGN CONVENTIONS FOR BEAMS**  
**(Programs 18 through 21)**

NAME	VARIABLE	SENSE	SIGN
DEFLECTION	$y$	$\uparrow$	+
SLOPE	$\theta$	$\uparrow$	+
INTERNAL MOMENT	$M_x$		+
SHEAR	$V$		+
EXTERNAL FORCE OR LOAD	$P$ or $W$	$\downarrow$	+
EXTERNAL MOMENT	$M$		+

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## 2 Using Stress Analysis Pac 1

# USING STRESS ANALYSIS PAC 1

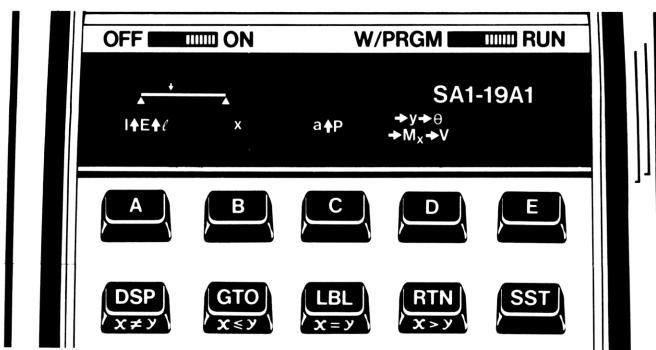
Stress Analysis Pac I is a collection of programs designed to aid the engineer in the calculation of the properties of structural elements. Each program includes a general description, formulas used in the program solution, general user instructions, example problems with keystroke solutions, and a program listing.

By using the keyboard functions of the HP-65 in combination with Stress Analysis Pac I complex problems can be solved in an easy, consistent manner. Very rarely will intermediate answers need to be written down for later use. Where possible, inputs are stored in consistent registers and remain unaltered from program card to program card. This allows similar programs to be linked with little or no reinput of data.

Any compatible system of units may be used with programs in this pac.

### PRERECORDED MAGNETIC CARDS

The prerecorded magnetic cards supplied with Stress Analysis Pac I incorporate a shorthand set of operating instructions. This should make it possible to run the programs without referencing the manual. A typical card inserted in the window slot of an HP-65 is shown below:



Above the **A** key are the input variables I, E, and  $\ell$  separated by  $\uparrow$ , which is the symbol for **ENTER**. This means key in I, press **ENTER**; key in E, press **ENTER**; key in  $\ell$ , then press **A**.

A variable by itself is input by keying in the value then pressing the corresponding user definable key. In this case, the value of x would be keyed in, then the **B** key would be pressed.

The input values associated with the **C** key are similar to those associated with the **A** key. First key in the value of a, then press **ENTER**. Then key in the value of P and press **C**.

The → symbol pointing at the variables associated with the **D** key means calculate. The first press of the **D** key would calculate y. The second press of **D** would calculate  $\theta$ , the third would calculate  $M_x$  and the fourth would calculate V.

Another symbol used throughout the pac is an arrow pointing down to a variable ( $\downarrow$ ). This indicates that the key may be used for both calculation and input. If a zero is displayed when the user definable key is pressed, the calculator calculates the value. Any other displayed value will be stored.

A modification of this technique was needed in *Soderberg's Equation For Fatigue*, SA1-14A. On this card the symbol is  $R\downarrow S$ . This means that values are input by pressing the associated user definable key but calculation is initiated by first pressing the associated user definable key, then **R/S**.

As you probably noticed in the example, execution was from left to right across the magnetic card. Left to right input is always safe. However, input order is generally immaterial to the program.

## 4 Format of User Instructions

### FORMAT OF USER INSTRUCTIONS

The completed User Instruction Form that accompanies each program is your guide to operating the programs in this Pac.

The form is composed of five labeled columns. Reading from left to right, the first column, labeled STEP, gives the instruction step number.

The INSTRUCTIONS column gives instructions and comments concerning the operations to be performed.

The INPUT-DATA/UNITS column specifies the input data. No units are specified in this pac since any consistent set of units can be used. Data input keys consist of **0** through **9** and the decimal point (the numeric keys), **EEX** (enter exponent), and **CHS** (change sign).

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SA1-19A1*			
2	Input moment of inertia	I	↑	I
	<i>then</i> modulus of elasticity	E	↑	E
	<i>then</i> length of beam	ℓ	A	EI
	Input distance from y axis to			
	point of interest	x	B	x
	Input distance from origin to			
	concentrated load	a	↑	a
	<i>then</i> concentrated load	P	C	a
3	Calculate deflection at x		D	y
4	Calculate slope at x		D	θ
5	Calculate moment at x		D	M <sub>x</sub>
6	Calculate shear at x		D	V
7	For new case go to step 2 and			
	change appropriate inputs.			
	* Register 8 is available for			
	intermediate storage.			

STEP 1: Step 1 of the example is “Enter SA1-19A1”. This calls for the entry of the prerecorded magnetic card into the HP-65 (Refer to Entering a Program, page 6).

STEP 2: This step specifies input of the variables necessary for program operation. The step may be subdivided into three separate parts. Each part is associated with a different program control key. The variables I, E, and  $\ell$  are associated with the **A** key as noted in the *KEYS* column. The variable x is associated with the **B** key. The variables a and P are associated with the **C** key. Any one of these three parts may be performed first. Order is not important since all three parts are under one step number. However, order is important within the parts. The word *then* is used to specify this order.

To perform step 2, key in the value of x according to the INSTRUCTIONS and INPUT-DATA/UNITS columns. When x is displayed, pressing **B** causes the value to be stored for later program use. After pressing **B** x is still in the display as noted in the OUTPUT-DATA/UNITS column. Now key in the value of a, press **ENTER↑**, *then* key in the value of P. With P displayed press **C** to initiate program execution. When execution halts, the value a is displayed. To finish step 2, key in I, press **ENTER↑**, *then* E and press **ENTER↑** again *then* key in  $\ell$  and press **A**. When program execution halts, EI is displayed and step 2 is completed.

STEP 3: The deflection y is calculated in step 3 by pressing **D**.

STEPS 4, 5 and 6: These steps are identical to step 3 except that  $\theta$  is calculated in step 4,  $M_x$  is calculated in step 5, and V is calculated in step 6.

STEP 7: This step gives information needed to start a new case or modify the existing problem. For this program, it is possible to change x, or the group a and P, or the group I, E and  $\ell$  independently. It is not possible to input only I or only P since the programs require all values of each group to be in place in the operational stack, before the associated program control key is pressed.

## 6 Entering a Program

### ENTERING A PROGRAM

Select a program card from the card case supplied with this application pac.

Set W/PRGM-RUN switch to RUN.

Turn the calculator ON. You should see 0.00.

Gently insert the card (printed side up) in the right, lower slot as shown. When the card is part way in, the motor engages it and passes it out the left side of the calculator. Sometimes the motor engages but does not pull the card in. If this happens, push the card a little farther into the machine. Do not impede or force the card; let it move freely. (The display will flash if the card reads improperly. In this case, press **CLX** and reinsert the card.)



When the motor stops, remove the card from the left side of the calculator and insert it in the upper "window slot" on the right side of the calculator.

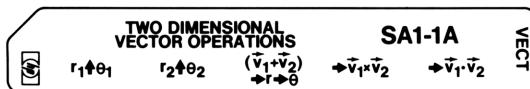
The program is now stored in the calculator. It remains stored until another program is entered or the calculator is turned off.



## ACKNOWLEDGEMENT

Stress Analysis Pac I has benefitted immeasurably from helpful suggestions of several mechanical, and structural engineers. We especially wish to thank Mr. Dean Lampman for reviewing this text and contributing his experience and expertise in the area of stress analysis.

## TWO DIMENSIONAL VECTOR OPERATIONS



This program performs the basic two dimensional vector operations of addition, cross product and dot, scalar, or inner product. The sum of a vector addition automatically replaces  $\vec{v}_1$  in storage so that sums of several vectors may be accumulated.

### Equations:

for addition:  $\vec{v}_1 + \vec{v}_2 = (x_1 + x_2)\vec{i} + (y_1 + y_2)\vec{j}$

for cross products:  $\vec{v}_1 \times \vec{v}_2 = (x_1 y_2 - x_2 y_1) \vec{k}$

for dot, scalar, or inner product:  $\vec{v}_1 \cdot \vec{v}_2 = x_1 x_2 + y_1 y_2$

where:

$x_1$  is the x component of  $\vec{v}_1$  ( $x_1 = r_1 \cos \theta_1$ );

$x_2$  is the x component of  $\vec{v}_2$  ( $x_2 = r_2 \cos \theta_2$ );

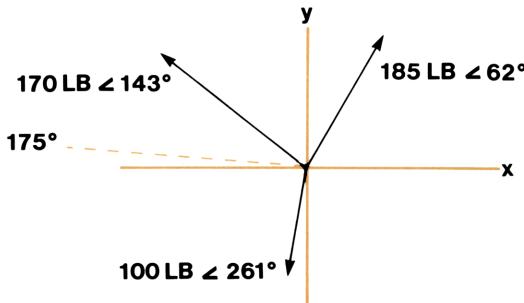
$y_1$  is the y component of  $\vec{v}_1$  ( $y_1 = r_1 \sin \theta_1$ );

$y_2$  is the y component of  $\vec{v}_2$  ( $y_2 = r_2 \sin \theta_2$ );

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input $\vec{v}_1$ and $\vec{v}_2$			
	radius of $\vec{v}_1$	$r_1$	$\uparrow$	
	then angle of $\vec{v}_1$	$\theta_1$	A	0.00
	radius of $\vec{v}_2$	$r_2$	$\uparrow$	
	then angle of $\vec{v}_2$	$\theta_2$	B	0.00
3	Perform vector operation			
	add vectors, display radius		C	$r$
	then display angle		C	$\theta$
	or			
	take cross product		D	$\vec{v}_1 \times \vec{v}_2$
	or			
	take dot product		E	$\vec{v}_1 \cdot \vec{v}_2$
4	For new case go to step 2. If			
	vector addition was performed			
	the sum of $v_1$ and $v_2$ is now			
	stored as $v_1$ .			

**Example 1:**

Resolve the following three loads along a 175 degree line.



Keystrokes:

First add the three vectors.

185  $\uparrow$  62 A 170  $\uparrow$  143 B C → 270.12 lb  
 100  $\uparrow$  261 B C → 178.94 lb  
 C → 111.15°

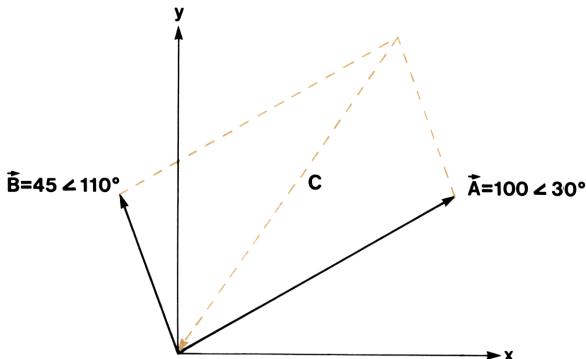
Then resolve the sum along a 175 degree line.

1  $\uparrow$  175 B E → 78.86 lb

## 10 SA1-01A

### Example 2:

Forces  $\vec{A}$  and  $\vec{B}$  are shown below. If static equilibrium exists, what is force  $\vec{C}$ ?



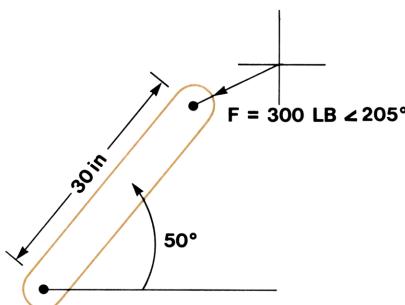
Keystrokes:

To obtain  $\vec{C}$ , add  $\vec{A}$  and  $\vec{B}$  using a negative radius for both

100 [CHS]  $\uparrow$  30 [A] 45 [CHS]  $\uparrow$  110 [B] [C]  $\longrightarrow$  116.57  
[C]  $\longrightarrow$  -127.66

### Example 3:

What is the moment at the bearing of the crank pictured below?  
What is the reaction force transmitted along the member?



Keystrokes:

Moment by cross product ( $\vec{r} \times \vec{F}$ ).

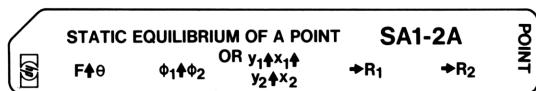
30  $\uparrow$  50 [A] 300  $\uparrow$  205 [B] [D]  $\longrightarrow$  3803.56 in-lb

Resolution along crank

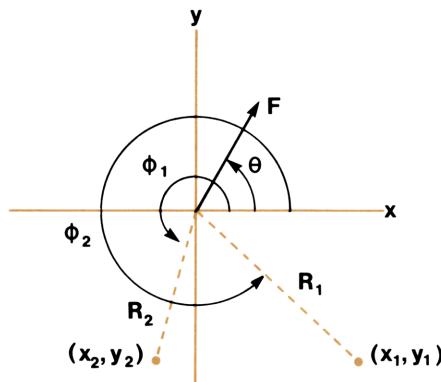
1  $\uparrow$  50 [A] [E]  $\longrightarrow$  -271.89 lb



## STATIC EQUILIBRIUM OF A POINT



By specifying a force vector and the direction of the two reaction forces, the magnitudes of the reaction forces necessary to achieve static equilibrium can be calculated. The direction of the reaction forces may be specified as an angle or by Cartesian coordinates using the point of force application as the origin.



### Equations:

$$R_1 \cos \phi_1 + R_2 \cos \phi_2 = F \cos \theta$$

$$R_1 \sin \phi_1 + R_2 \sin \phi_2 = F \sin \theta$$

where:

F is the known force;

$\theta$  is the direction of the known force;

$R_1$  is one reaction force;

$\phi_1$  is the direction of  $R_1$ ;

$R_2$  is the second reaction force;

$\phi_2$  is the direction of  $R_2$ ;

The coordinates  $x_1$  and  $y_1$  are referenced from the point where  $F$  is applied to the end of the member along which  $R_1$  acts;  $x_2$  and  $y_2$  are the coordinates referenced from the point where  $F$  is applied to the end of the member along which  $R_2$  acts.

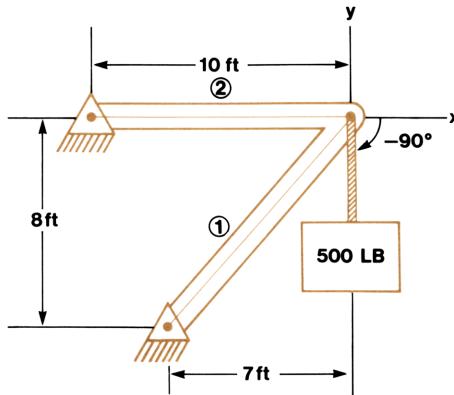
**Remarks:** Negative values for reaction forces indicate that the member is under compression.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input the known force	$F$	$\uparrow$	$F$
	<i>then the direction of the</i>			
	known force	$\theta$	A	$F$
	Input direction of reaction 1	$\phi_1$	$\uparrow$	$\phi_1$
	<i>then direction of reaction 2</i>	$\phi_2$	B	$\phi_1$
	<i>or</i>			
	Input y coordinate of reaction 1	$y_1$	$\uparrow$	$y_1$
	<i>then x coordinate of reaction</i>			
1		$x_1$	$\uparrow$	$x_1$
	<i>then y coordinate of reaction</i>			
2		$y_2$	$\uparrow$	$y_2$
	<i>then x coordinate of reaction</i>			
2		$x_2$	C	
3	Calculate reaction 1		D	$R_1$
	<i>and/or calculate reaction 2</i>		E	$R_2$
4	For new case go to step 2 and			
	change appropriate inputs.			

## 14 SA1-02A

### Example 1:

Find the reaction forces in the pin-jointed structure shown below.



Keystrokes:

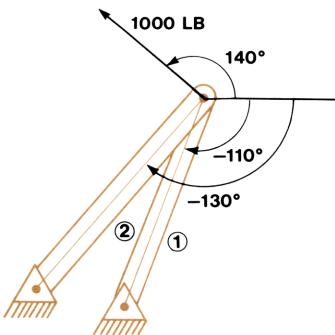
500  $\uparrow$  90 [CHS] [A] 8 [CHS]  $\uparrow$  7 [CHS]

$\uparrow$  0  $\uparrow$  10 [CHS] [C] [D]  $\longrightarrow$  -664.38 lb

[E]  $\longrightarrow$  437.50 lb

### Example 2:

Find the forces in the members 1 and 2 of the structure below.



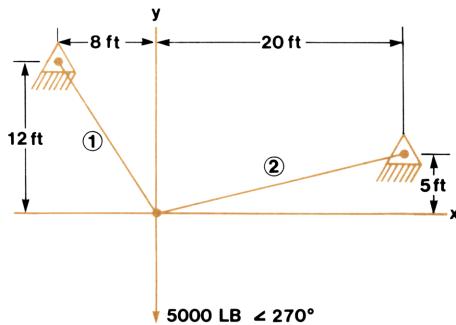
Keystrokes:

1000  $\uparrow$  140 [A] 110 [CHS]  $\uparrow$  130 [CHS] [B] [D]  $\longrightarrow$  2923.80 lb

[E]  $\longrightarrow$  -2747.48 lb

**Example 3:**

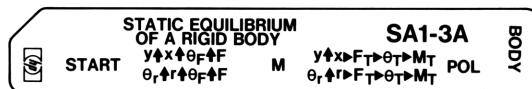
What forces are present in cables 1 and 2 below?



Keystrokes:

5000 **A** 12 **A** 8 **CHS** **A** 5 **A** 20 **C** **D** → 5150.79 lb  
**E** → 2945.08 lb

## STATIC EQUILIBRIUM OF A RIGID BODY



This program may be used to solve for an unknown force and couple for a specified point on a rigid body. Position of input force vectors may be defined using either Cartesian or polar coordinates. The position of the unknown force vector may also be specified using either coordinate system. Any number of forces and moments may be input.

**Equations:**

$$\vec{F}_T = -(\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n)$$

$$\vec{M}_T = - (\vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3 + \dots + \vec{r}_n \times \vec{F}_n) + \vec{r}_T \times \vec{F}_T$$

$$- (\vec{M}_1 + \vec{M}_2 + \vec{M}_3 + \dots + \vec{M}_n)$$

where:

$\vec{F}_1, \vec{F}_2, \vec{F}_3$ , etc. are the input force vectors;

$\vec{r}_1, \vec{r}_2, \vec{r}_3$ , etc. are the position vectors of those forces (i.e.,  $r_T$  is the position vector for  $F_T$ );

$\vec{M}_1, \vec{M}_2, \vec{M}_3$ , etc. are applied moments.

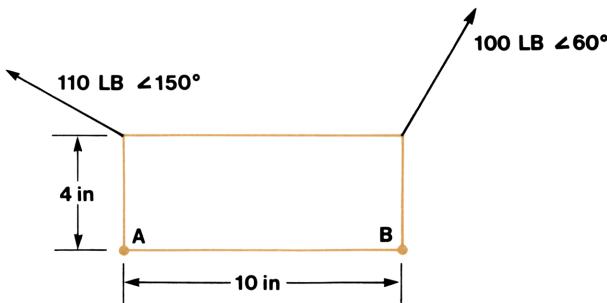
(Positive moments are counter-clockwise or come out of the plane of the paper when using the right hand rule.)

**Remarks:** Polar inputs with radius vectors less than 1 will cause an underflow halt when angles approach 90 degrees or 270 degrees.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Initialize		A	0.00
3	Input force vector located in Cartesian form			
	y displacement	y	↑	y
	then x displacement	x	↑	x
	then direction of force	$\theta_F$	↑	$\theta_F$
	then magnitude of force	F	B	0.00
	or in polar form:			
	direction of displacement	$\theta_r$	↑	$\theta_r$
	then magnitude of displacement			
	r		↑	r
	then direction of force	$\theta_F$	↑	$\theta_F$
	then magnitude of force	F	E B	0.00
	or input a moment	M	C	0.00
4	Calculate the magnitude of $\vec{F}_T$ by locating the point of action of $\vec{F}_T$ in Cartesian coor- dinates			
	y		↑	y
	x		D	$F_T$
	or in polar coordinates			
	$\theta_r$		↑	$\theta_r$
	r		E D	$F_T$
5	Calculate angle of $\vec{F}_T$		D	$\theta_T$
6	Calculate balancing moment		D	$M_T$
7	To modify this problem, go to step 3 and add appropriate forces or moments. For a new case go to step 2.			

**18 SA1-03A****Example 1:**

What force and moment applied at point A will yield equilibrium for the geometry shown? At point B?



Keystrokes for point A calculation:

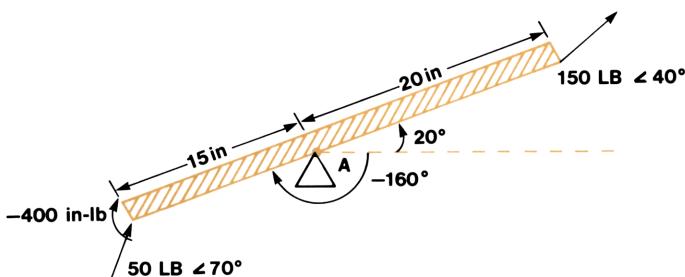
- A 4 [↑] 0 [↑] 150 [↑] 110 B 4 [↑] 10 [↑]  
60 [↑] 100 B 0 [↑] 0 D → 148.66 lb  
D → -72.27°  
D → -1047.08 in-lb

For point B calculation:

- 0 [↑] 10 D → 148.66 lb  
D → -72.27°  
D → 368.95 in-lb

**Example 2:**

What is the reaction at point A of the following lever?



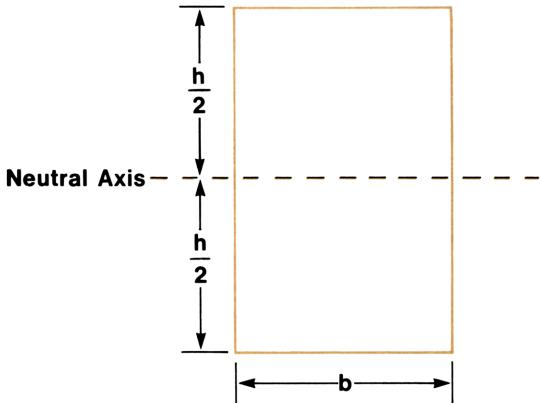
Keystrokes:

**A** 160 **CHS** **↑** 15 **↑** 70 **↑** 50 **E** **B** 20  
 20 **↑** 40 **↑** 150 **E** **B** 400 **CHS** **C** 0 **↑** 0 **D** → 194.91 lb  
**D** → -132.63°  
**D** → -51.53 in-lb

## PROPERTIES OF RECTANGULAR SECTIONS



This program performs an interchangeable solution for the moment of inertia  $I$ , the width  $b$  and the height  $h$  of a rectangular section. When  $b$  and  $h$  are known, the polar moment of inertia  $J$  and the section area can also be found.



**Equations:**

$$I = \frac{b h^3}{12}$$

$$J = \frac{b h (b^2 + h^2)}{12}$$

$$A = b h$$

**Remarks:** Values of polar moment of inertia  $J$  calculated by this program must not be used to calculate torsional stress and strain in rectangular members.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input two of the following			
	moment of inertia of section	I	A	0.00
	width of section	b	B	0.00
	height of section	h	C	0.00
3	Compute the remaining value			
	moment of inertia of section	0.00	A	I
	width of section	0.00	B	b
	height of section	0.00	C	h
4	Optional: Compute either or			
	both of the following			
	polar moment of inertia	D		J
	area of section	E		A
5	For new case go to step 2 and			
	change any or all of the inputs.			

**Example:**

What is the moment of inertia of a section with  $b = 3$  and  $h = 5$ ? What is the polar moment of inertia? What is the area? What would  $b$  have to be if  $I = 40$ ?

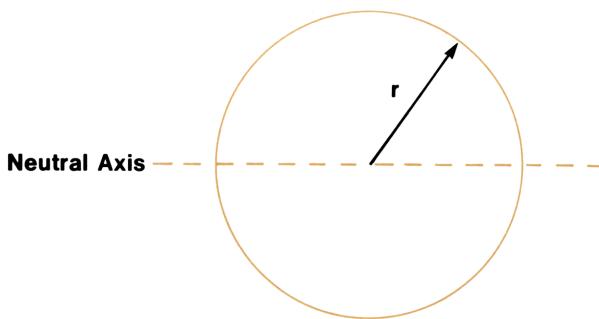
**Keystrokes:**

3 **B** 5 **C** **A** → 31.25 in<sup>4</sup>  
**D** → 42.50 in<sup>4</sup>  
**E** → 15 in<sup>2</sup>  
 40 **A** **B** → 3.84 in

## PROPERTIES OF CIRCULAR SECTIONS



This program performs an interchangeable solution for five properties of circular sections. Given either the moment of inertia  $I$ , diameter  $d$ , radius  $r$ , polar moment of inertia  $J$ , or area  $A$ , the four remaining properties can be calculated.



### Equations:

$$I = \frac{\pi d^4}{64}$$

$$J = \frac{\pi d^4}{32}$$

$$A = \frac{\pi d^2}{4}$$

$$r = \frac{d}{2}$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input one of the following			
	moment of inertia	I	A	0.00
	diameter of section	d	B	0.00
	radius of section	r	C	0.00
	polar moment of inertia	J	D	0.00
	area of section	A	E	0.00
3	Compute one of the following			
	moment of inertia	0.00	A	I
	diameter of section	0.00	B	d
	radius of section	0.00	C	r
	polar moment of inertia	0.00	D	J
	area of section	0.00	E	A
4	For another calculation go to			
	step 3. For a new case go to			
	step 2.			

**Example 1:**

If the moment of inertia of a section must be  $60 \text{ in}^4$ , what is the necessary radius? What is the polar moment of inertia? What is the area?

Keystrokes:

$$\begin{array}{lcl} 60 \boxed{A} \boxed{C} & \longrightarrow & 2.96 \text{ in} \\ 0 \boxed{D} & \longrightarrow & 120.00 \text{ in}^4 \\ 0 \boxed{E} & \longrightarrow & 27.46 \text{ in}^2 \end{array}$$

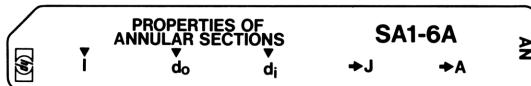
**Example 2:**

The diameter of a section is 10 centimeters. What is the moment of inertia? What is the polar moment of inertia? What is the area?

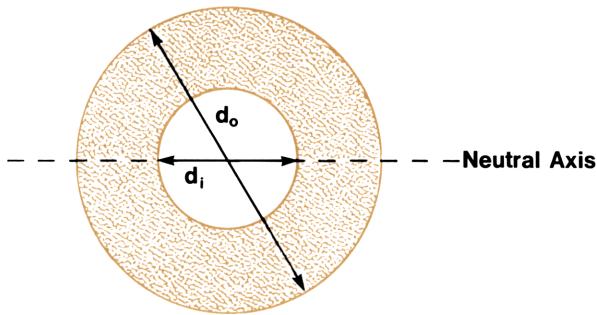
Keystrokes:

$$\begin{array}{lcl} 10 \boxed{B} \boxed{A} & \longrightarrow & 490.87 \text{ cm}^4 \\ 0 \boxed{D} & \longrightarrow & 981.75 \text{ cm}^4 \\ 0 \boxed{E} & \longrightarrow & 78.54 \text{ cm}^2 \end{array}$$

## PROPERTIES OF ANNULAR SECTIONS



This program provides an interchangeable solution for the moment of inertia  $I$ , the outside diameter  $d_o$ , and the inside diameter  $d_i$  of an annular section. Once  $d_o$  and  $d_i$  are known, the polar moment of inertia  $J$  and the area of the section can be calculated.



**Equations:**

$$I = \frac{\pi (d_o^4 - d_i^4)}{64}$$

$$J = \frac{\pi (d_o^4 - d_i^4)}{32}$$

$$A = \frac{\pi(d_o^2 - d_i^2)}{4}$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input two of the following			
	moment of inertia I	I	A	0.00
	outside diameter $d_o$	$d_o$	B	0.00
	inside diameter $d_i$	$d_i$	C	0.00
3	Calculate the remaining value			
	moment of inertia I	0.00	A	I
	outside diameter $d_o$	0.00	B	$d_o$
	inside diameter $d_i$	0.00	C	$d_i$
4	Optional: Compute either or both of the following			
	polar moment of inertia		D	J
	area of section		E	A
5	For new case go to step 2 and change appropriate inputs.			

**Example :**

If  $d_i$  equals 3 inches and I equals  $10 \text{ in}^4$ , what is  $d_o$ ? What is A?

Keystrokes:

3 **C** 10 **A** **B** → 4.11 in  
**D** →  $20.00 \text{ in}^4$   
**E** →  $6.18 \text{ in}^2$

What would I be if  $d_o$  equals 4.5 inches?

4.5 **B** **A** →  $16.15 \text{ in}^4$

## COMPOSITE SECTION PROPERTIES

 $(f, \text{REG})$ $y_{0i} \Delta x_i$	<b>COMPOSITE SECTION PROPERTIES 1</b> $\Delta y_i \Delta x_i A_{si}$ OR $A_{si}$	<b>SA1-7A1</b> $\rightarrow A \rightarrow \bar{y} \rightarrow \bar{x}$ $\rightarrow l_y \rightarrow l_x \rightarrow J$ $(RCL \rightarrow l_{xy})$
---	---	--

 $\rightarrow l_y \rightarrow l_x$ $(RCL \rightarrow l_{xy})$	<b>COMPOSITE SECTION PROPERTIES 2</b> $\rightarrow \phi$	<b>SA1-7A2</b> $\rightarrow l_y \phi \rightarrow l_x \phi$ $\rightarrow J_\phi$
--	---	---

The properties of arbitrarily shaped sections can be evaluated using this two card program. Exact solutions are obtained when the section is broken into a finite number of rectangles. Approximate solutions can be achieved by assuming that finite areas are concentrated at their centers.

The program calculates the area of the section, the centroid of the area, the moments of inertia about any specified set of axes, the polar moment of inertia about the specified axis, the moments of inertia about an axis translated to the centroid, the moments of inertia of the principal axis, the rotation angle between the translated axis and the principal axis, and the polar moment of inertia about the principal axis.

### Equations:

$$A_{si} = \Delta x_i \Delta y_i$$

$$A = A_{s1} + A_{s2} + A_{s3} + \dots + A_{sn}$$

$$\bar{x} = \frac{\sum_{i=1}^n x_{0i} A_{si}}{A}$$

$$\bar{y} = \frac{\sum_{i=1}^n y_{0i} A_{si}}{A}$$

$$I_x = \sum_{i=1}^n \left( y_{0i}^2 + \frac{\Delta y_i^2}{12} \right) A_{si}$$

$$I_y = \sum_{i=1}^n \left( x_{0i}^2 + \frac{\Delta x_i^2}{12} \right) A_{si}$$

$$J = I_x + I_y$$

$$I_{xy} = \sum_{i=1}^n x_{0i} y_{0i} A_{si}$$

$$I_{\bar{x}} = I_x - A_{\bar{y}}^2$$

$$I_{\bar{y}} = I_y - A_{\bar{x}}^2$$

$$I_{\bar{x}\bar{y}} = I_{xy} - A_{\bar{x}\bar{y}}$$

$$\phi = \frac{1}{2} \tan^{-1} \left( \frac{2 I_{\bar{x}\bar{y}}}{I_{\bar{x}} - I_{\bar{y}}} \right)$$

$$I_{y\phi} = I_{\bar{y}} \cos^2 \phi + I_{\bar{x}} \sin^2 \phi + I_{\bar{x}\bar{y}} \sin 2\phi$$

$$I_{x\phi} = I_{\bar{x}} \cos^2 \phi + I_{\bar{y}} \sin^2 \phi - I_{\bar{x}\bar{y}} \sin 2\phi$$

$$J_\phi = I_{x\phi} + I_{y\phi}$$

where:

$\Delta x_i$  is the width of a rectangular element;

$\Delta y_i$  is the height of a rectangular element;

$A_{si}$  is the area of an element;

$A$  is the total area of the section;

$\bar{x}$  is the x coordinate of the centroid;

$\bar{y}$  is the y coordinate of the centroid;

$x_{0i}$  is the x coordinate of the centroid of an element;

$y_{0i}$  is the y coordinate of the centroid of an element;

$I_x$  is the moment of inertia about the x-axis;

$I_y$  is the moment of inertia about the y-axis;

$J$  is the moment of inertia about the origin;

## 28 SA1-07A

$I_{xy}$  is the product of inertia;

$I_{\bar{x}}$  is the moment of inertia about the x-axis translated to the centroid;

$I_{\bar{y}}$  is the moment of inertia about the y-axis translated to the centroid;

$I_{\bar{x}\bar{y}}$  is the product of inertia about the translated axis;

$\phi$  is the angle between the translated axis and the principal axis;

$I_{x\phi}$  is the moment of inertia about the translated, rotated, principal x-axis;

$I_{y\phi}$  is the moment of inertia about the translated, rotated, principal y-axis;

$J_\phi$  is the polar moment of inertia about the origin of the translated, rotated, principal axes.

**Remarks:** Values of polar moment of inertia  $J$  should not be used in torsional stress or strain analysis.

### References:

Crandall, S.H., Dahl, N.C.

*An Introduction to the Mechanics of Solids*, McGraw-Hill, 1959

Rhodes, G.F.

*Section Properties*, HP-65 Users' Library, Number 262

STEP	INSTRUCTIONS	INPUT ATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter <i>Composite Section</i>			
	<i>Properties, SA1-7A1</i>			
2	Clear registers		f REG	
3	Input y coordinate of centroid of element	$y_{oi}$	$\uparrow$	$y_{oi}$
4	Input x coordinate of centroid of element	$x_{oi}$	A	$y_{oi}$
5	Input height of rectangular element	$\Delta y_i$	$\uparrow$	$\Delta y_i$
	and width of rectangular element			
	$\Delta x_i$	B		$A_{si}$
	or input area of element for approximate solution	$A_{si}$	C	$A_{si}$
6	Go to step 3 for each element			
7	Optional: Display area of section		D	A
	then display y coordinate of centroid		D	$\bar{y}$
	then display x coordinate of centroid		D	$\bar{x}$
8	Optional: Display moment of inertia about y axis		E	$I_y$
	then display moment of inertia about x axis		E	$I_x$
	then display polar moment of inertia about the origin		E	J
9	Optional: Display product of inertia	RCL 9		$I_{xy}$
10	Enter <i>Composite Section</i>			
	<i>Properties SA1-7A2</i>			

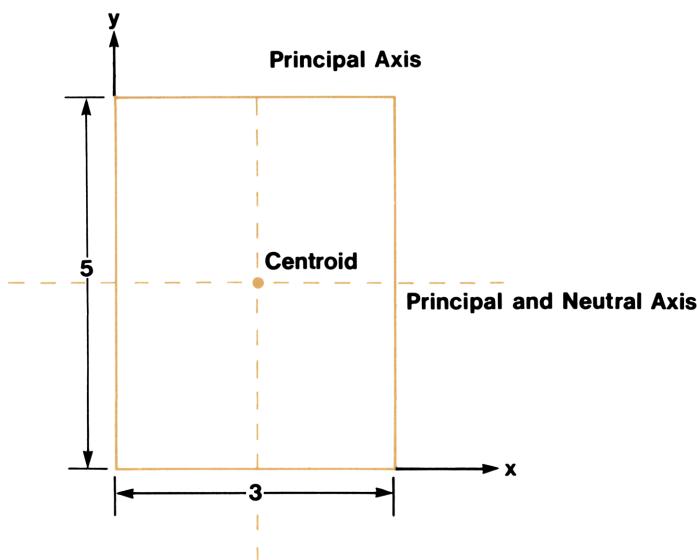
Continued

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STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
11	Calculate moment of inertia about translated y axis			
12	Calculate moment of inertia about translated x axis		A	$I_{\bar{y}}$
13	Display product of inertia for translated axis			$I_{x\bar{y}}$
14	Calculate axis rotation angle		B	$\phi$
15	Calculate moment of inertia about principal y axis		C	$I_{\bar{y}\phi}$
16	Calculate moment of inertia about principal x axis		C	$I_{\bar{x}\phi}$
17	Calculate polar moment of inertia about principal axis		D	$J_\phi$
18	For new case, go to step 2			

### Example 1:

What is the moment of inertia about the x-axis ( $I_x$ ) for the rectangular section shown? What is the moment of inertia about the neutral axis through the centroid of the section ( $I_{\bar{x}\phi}$ )?



## TABLE OF INPUTS

Section	$y_0$	$x_0$	$\Delta y$	$\Delta x$
1	2.5	1.5	5	3

Keystrokes:

Using card SA 1–07A1.

f [REG] 2.5 ↑ 1.5 A 5 ↑ 3 B → 15.00 ( $A_{si}$ )  
 D → 15.00 (A)  
 D → 2.50 ( $\bar{y}$ )  
 D → 1.50 ( $\bar{x}$ )  
 E → 45.00 ( $I_y$ )  
 E → 125.00 ( $I_x$ )

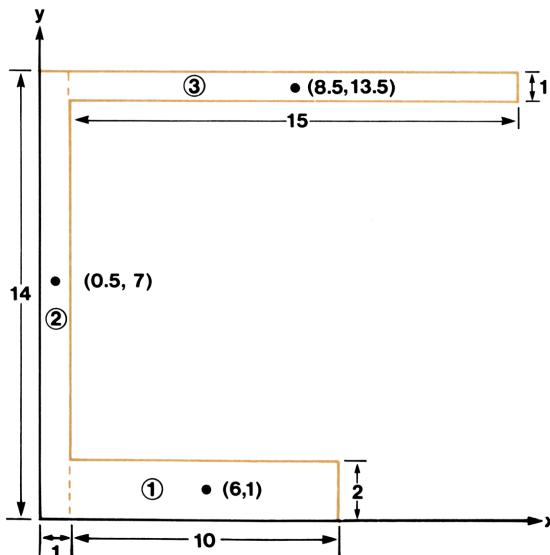
Using card SA1–07A2.

A → 11.25 ( $I_{\bar{y}}$ )  
 A → 31.25 ( $I_{\bar{x}}$ )  
 B → 0.00 ( $\phi$ )  
 C → 11.25 ( $I_{\bar{y}\phi}$ )  
 C → 31.25 ( $I_{\bar{x}\phi}$ )

Note that  $I_{\bar{x}\phi}$  and  $I_{\bar{x}}$  are equal since  $\phi = 0$ . Also note that  $I_{\bar{x}\phi}$  agrees with the moment of inertia calculated in example 1 of *Properties of Rectangular Sections*, SA1-4A.

**Example 2:**

Calculate the section properties for the beam shown below.

**TABLE OF INPUTS**

Section	$y_{oi}$	$x_{oi}$	$\Delta y$	$\Delta x$
1	1	6	2	10
2	7	0.5	14	1
3	13.5	8.5	1	15

Keystrokes:

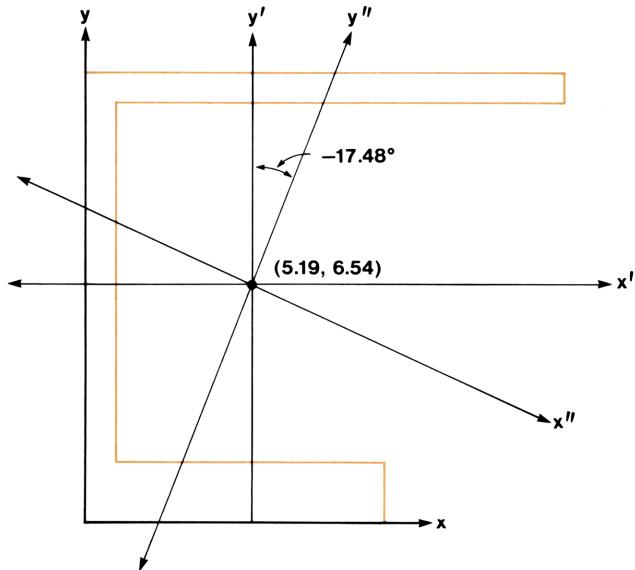
Using card SA 1–07A1.

- f** **REG** 1 **↑** 6 **A** 2 **↑** 10 **B** 7 **↑** .5  
**A** 14 **↑** 1 **B** 13.5 **↑** 8.5 **A** 1 **↑** 15 **B** **D** → 49.00 (A)  
**D** → 6.54 ( $\bar{y}$ )  
**D** → 5.19 ( $\bar{x}$ )  
**E** → 2256.33 ( $I_y$ )  
**E** → 3676.33 ( $I_x$ )  
**RCL** 9 → 1890.25 ( $I_{xy}$ )  
**E** → 5932.67 (J)

Using card SA 1-07A2.

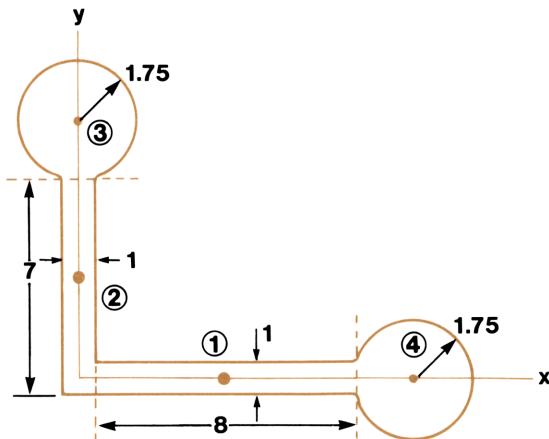
- A → 934.49 ( $I_{\bar{y}}$ )
- A → 1580.00 ( $I_{\bar{x}}$ )
- RCL 3 → 225.61 ( $I_{\bar{x}\bar{y}}$ )
- B →  $-17.48^\circ$  ( $\phi$ )
- C → 863.46 ( $I_{\bar{y}\phi}$ )
- C → 1651.04 ( $I_{\bar{x}\phi}$ )
- D → 2514.49 ( $J_\phi$ )

Below is a figure showing the translated axis and the rotated principal axis of example 2.



**Example 3:**

Find the approximate section properties of the member below by assuming that the circular parts are concentrated at their centers. Also assume that the boundaries between the circular and rectangular parts are straight lines.

**TABLE OF INPUTS**

Section	$y_{oi}$	$x_{oi}$	$\Delta y$	$\Delta x$	A
1	0.0	4.5	1	8	
2	3.0	0.0	7	1	
3	0.0	10.25			9.62
4	8.25	0.0			9.62

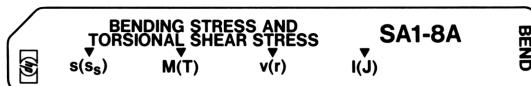
Using card SA 1–07A1.

- f REG** 0 ↑ 4.5 A 1 ↑ 8 B 3 ↑ 0 A 7 ↑ 1 B  
0 ↑ 10.25 A 9.62 C 8.25 ↑ 0 A 9.62 C D ➤ 34.24 (A)
- D → 2.93 ( $\bar{y}$ )  
D → 3.93 ( $\bar{x}$ )  
E → 1215.95 ( $I_y$ )  
E → 747.01 ( $I_x$ )  
**RCL** 9 → 0.00 ( $I_{xy}$ )  
E → 1962.96 (J)

Using card SA 1–07A2.

- A → 686.79 ( $I_{\bar{y}}$ )  
A → 452.82 ( $I_{\bar{x}}$ )  
**RCL** 3 → -394.56 ( $I_{\bar{x}\bar{y}}$ )  
B → -36.74° ( $\phi$ )  
C → 981.34 ( $I_{\bar{y}\phi}$ )  
C → 158.57 ( $I_{\bar{x}\phi}$ )  
D → 1139.61 (J $_{\phi}$ )

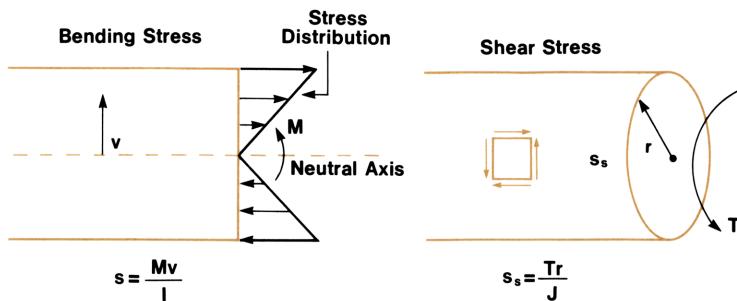
## BENDING STRESS IN BEAMS OR TORSIONAL SHEAR STRESS IN CIRCULAR SHAFTS



This card solves either the bending stress equation or the analogous torsional shear stress equation. Using an interchangeable solution. Given three known values, the remaining unknown value is calculated.

Variables involved in torsional shear stress calculations are shown in parenthesis on the magnetic card.

Equations:



where:

$s$  is the normal stress at  $v$ ;

$M$  is the moment applied to the beam;

$v$  is the distance from the neutral axis of the beam;

$I$  is the moment of inertia of the beam;

$s_s$  is the shear stress at  $r$ ;

$T$  is the applied torque;

$r$  is the distance from the shaft center to the point of interest;

$J$  is the polar moment of inertia.

**Remarks:** This program is not applicable for non-elastic media or elastic media where stresses exceed the elastic limit. Materials must be isotropic. Zero is an invalid input. Values of  $J$  and  $T$  are stored in appropriate registers for use by *Linear or Angular Deformation of a Shaft*, SA 1-09A.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input three of the following:			
	Bending stress (or shear stress)	s ( $s_s$ )	A	0.00
	Bending moment (or torque)	M (T)	B	0.00
	Distance from neutral axis (or radius)	v (r)	C	0.00
	Moment of inertia (or polar moment)	I (J)	D	0.00
3	Calculate the remaining value			
	Bending stress (or shear stress)	0.00	A	$s_s$
	Bending moment (or torque)	0.00	B	M (T)
	Distance from neutral axis (or radius)	0.00	C	v (r)
	Moment of inertia (or polar moment)	0.00	D	I (J)
4	For new case, go to step 2 and change appropriate inputs.			

**Example 1:**

If the maximum stress allowed in a beam is 10,000 pounds per square inch, the moment of inertia is  $4.8 \text{ in}^4$ , and the maximum distance from the neutral axis to the surface is 2 inches, what is the maximum applied moment?

Keystrokes:

10000 **A** 4.8 **D** 2 **C** **B** → 24000.00 in-lb

**38 SA1-08A****Example 2:**

What torque will result in a stress of 12,000 pounds per square inch at a radius of 1 inch for a 2 inch diameter shaft?

Keystrokes using card SA 1-05A:

2 **B** **D** → 1.57 (J)

Keystrokes using card SA 1-08A:

**D** 1 **C** 12000 **A** **B** → 18849.56 in-lb (T)

12 **÷** → 1570.80 ft-lb (T)

**Example 3:**

A moment of 30,000 inch-pounds is applied to a beam with a moment of inertia of  $3.8 \text{ in}^4$ . If the neutral axis is 1 inch from the surface, what is the stress at the surface?

Keystrokes:

30000 **B** 3.8 **D** 1 **C** **A** → 7894.74 psi



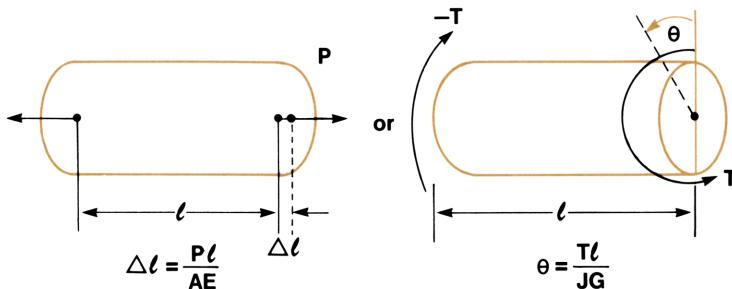
## LINEAR OR ANGULAR DEFORMATION OF A SHAFT



This card solves for linear deflection under tensile load or the analogous angular deflection under torque using an interchangeable solution. Given four of the five variables, the unknown is calculated.

Variables for circular shafts in torsion are shown in parenthesis on the magnetic card.

**Equations:**



where:

$\Delta\ell$  is the change in length;

$P$  is the applied load;

$\ell$  is the length;

$A$  is the cross sectional area;

$E$  is the modulus or elasticity;

$\theta$  is the deflection angle in radians;

$T$  is the applied torque;

$J$  is the polar moment of the section;

$G$  is the modulus of elasticity in shear.

**Remarks:** This program is not applicable for non elastic media or elastic media where stress exceeds the elastic limit. Materials must be isotropic. The equation for angular deflection is not valid in the neighborhood of the applied torque. Values of J and T are stored in appropriate registers for use by *Bending Stress in Beams and Torsional Shear Stress in Circular Shafts*, SA 1–08A.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input four of the following:			
	Linear deflection (or torsional)	$\Delta\ell(\theta)$	A	0.00
	Applied load (or torque)	P (T)	B	0.00
	Length of shaft	$\ell (\ell)$	C	0.00
	Area (or polar moment of inertia)	A (J)	D	0.00
	Modulus of elasticity (in shear)	E (G)	E	0.00
3	Calculate one of the following:			
	Linear deflection (angle of deformation)	0.00	A	$\Delta\ell(\theta)$
	Applied load (or torque)	0.00	B	P (T)
	Length of shaft	0.00	C	$\ell (\ell)$
	Area (or polar moment of inertia)	0.00	D	A (J)
	Modulus of elasticity (in shear)	0.00	E	E (G)
4	For new case go to step 2 and change appropriate inputs.			

**42 SA1-09A****Example 1:**

A torque of 47 foot-pounds is applied to a circular drive bar 15 inches long with a diameter of 3/8 inches.

If  $G = 11,500,000$  pounds per square inch, what is the angular deflection in degrees?

Keystrokes using card SA 1-05A:

3 **↑** 8 **÷** **B** **D** **DSP** **•** **4** → 0.0019 in<sup>4</sup> (J)

Keystrokes using card SA 1-09A:

**D** 47 **↑** 12 **X** **B** 15 **C** 11,500,000 **E** **A** → 0.3789 radians

180 **X** **g** **T** **÷** → 21.7105°

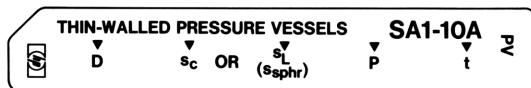
**Example 2:**

A load of 9000 pounds is applied to a coupler with a cross sectional area of 1.73 square inches. The coupler is 40 inches long and the modulus of elasticity is  $30 \times 10^6$  pounds per square inch. What is the increase in length of the coupler?

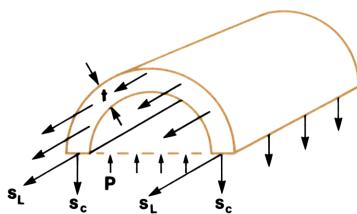
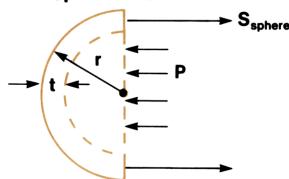
Keystrokes:

9000 **B** 1.73 **D** 40 **C** 30 **EEX** 6 **E** **A** → 0.01 in



**THIN-WALLED PRESSURE VESSELS**

This program can be used to correlate diameter, stress, pressure and thickness for cylindrical and spherical pressure vessels. Either the hoop stress  $s_c$  or the longitudinal stress  $s_L$  may be input for cylinders. For spheres, only the hoop stress  $s_{\text{sphere}}$  is applicable.

**Cut Cylindrical Section****Spherical Section**

**Equations:**

$$\text{for hoop stress in cylinders: } s_c = \frac{Pr}{t}$$

$$\text{for longitudinal stress in cylinders: } s_L = \frac{Pr}{2t}$$

$$\text{for hoop stress in spheres: } s_{\text{sphere}} = \frac{Pr}{2t}$$

where:

P is internal pressure;

D is diameter of vessel ( $r = D/2$ );

t is thickness of vessel.

**Remarks:** The thickness of the walls must be negligible with respect to the value of the radius. The equations are not valid in the neighborhood of end closures for cylindrical vessels.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input three of the following			
	vessel diameter	D	A	0.00
	hoop stress for cylinder	$s_c$	B	0.00
	or longitudinal stress for cylinder			
	or hoop stress for sphere	$s_{\text{sphere}}$	C	0.00
	pressure	P	D	0.00
	wall thickness	t	E	0.00
3	Calculate one of the following			
	vessel diameter	0.00	A	D
	hoop stress for cylinder	0.00	B	$s_c$
	or longitudinal stress for cylinder			
	or hoop stress for sphere	0.00	C	$s_L$
	pressure	0.00	D	P
	wall thickness	0.00	E	t
4	For new case go to step 2 and change appropriate inputs.			

**46 SA1-10A****Example 1:**

A basketball has a diameter of 9.3 inches. The thickness of the cord layer which resists virtually all of the internal pressure is 1/32 inch. The recommended pressure is 9 pounds per square inch. What is the stress in the cord layer?

Keystrokes:

9.3 **A** 9 **D** 32 **g** **1/x** **E** **C** → 669.60 psi

**Example 2:**

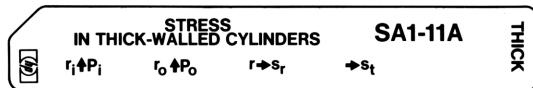
A four inch diameter pipe contains steam at 1000 pounds per square inch. What thickness is required if hoop stress is not to exceed 15000 pounds per square inch?

Keystrokes:

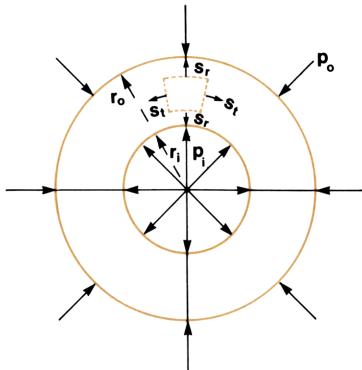
4 **A** 1000 **D** 15000 **B** **E** → 0.13 in



## STRESS IN THICK-WALLED CYLINDERS



This program calculates the radial and tangential components of normal stress for thick-walled, cylindrical, pressure vessels.



**Equations:** 
$$s_r = \frac{r_i^2 P_i - r_o^2 P_o}{r_o^2 - r_i^2} - \frac{r_i^2 r_o^2 (P_i - P_o)}{r^2 (r_o^2 - r_i^2)}$$

$$s_t = \frac{r_i^2 P_i - r_o^2 P_o}{r_o^2 - r_i^2} + \frac{r_i^2 r_o^2 (P_i - P_o)}{r^2 (r_o^2 - r_i^2)}$$

where:

- $s_r$  is the radial component of stress;
- $s_t$  is the tangential component of stress;
- $r_i$  is the internal radius;
- $r_o$  is the outer radius;
- $r$  is the radius where calculated stresses occur;
- $P_i$  is the internal pressure;
- $P_o$  is the outside pressure.

**Remarks:** A negative stress indicates compression.

**Reference:**

J. E. Shigley

*Mechanical Engineering Design*, McGraw Hill, 1963.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input the following			
	Inside radius	$r_i$	$\uparrow$ [ ]	$r_i$
	then inside pressure	$P_i$	[ A ] [ ]	$r_i$
	outside radius	$r_o$	$\uparrow$ [ ]	$r_o$
	then outside pressure	$P_o$	[ B ] [ ]	$r_o$
3	Input radius of interest and			
	calculate radial component of			
	stress	$r$	[ C ] [ ]	$s_r$
4	Calculate tangential stress		[ D ] [ ]	$s_t$
5	For a new radius of interest go			
	to step 3. For a new case go to			
	step 2.			

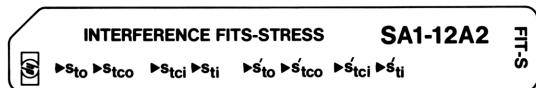
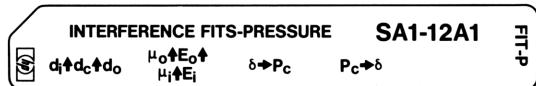
**Example:**

A cylinder has an inner radius of 1.00 inch and an outer radius of 2.00 inches. The inner pressure is 10,000 pounds per square inch and the outer pressure is 150 pounds per square inch. What are the values of radial and tangential stresses for radii of 1.00, 1.25, 1.75 and 2.00 inches?

**Keystrokes**

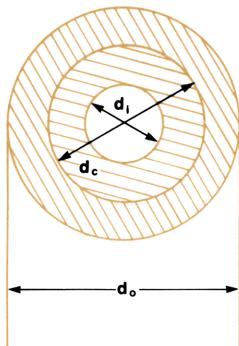
- 1 [  $\uparrow$  ] 10000 [ A ] 2 [  $\uparrow$  ] 150 [ B ] 1.00 [ C ]  $\longrightarrow$  -10,000 psi ( $s_r$ )  
 [ D ]  $\longrightarrow$  16,266.67 psi ( $s_t$ )
- 1.25 [ C ]  $\longrightarrow$  -5272.00 psi ( $s_r$ )  
 [ D ]  $\longrightarrow$  11538.67 psi  $s_t$
- 1.75 [ C ]  $\longrightarrow$  -1155.10 psi ( $s_r$ )  
 [ D ]  $\longrightarrow$  7421.77 psi ( $s_t$ )
- 2.00 [ C ]  $\longrightarrow$  -150.00 psi ( $s_r$ )  
 [ D ]  $\longrightarrow$  6416.67 psi ( $s_t$ )

## INTERFERENCE FITS



The first card of this two card set can be used to determine contact pressure or interference for concentric cylinders. Once the contact pressure has been determined, the second card may be used to determine the actual tangential stresses at the surfaces of the cylinders. These stresses may be used in maximum shear theory of failure analysis. Modified tangential stresses for use in maximum strain theory of failure analysis can also be computed.

**Concentric Cylinders**



**Equations:**

for contact pressure:

$$\delta = d_c P_c \left[ \frac{d_c^2 + d_i^2}{E_i(d_c^2 - d_i^2)} + \frac{d_o^2 + d_c^2}{E_o(d_o^2 - d_c^2)} - \frac{\mu_i}{E_i} + \frac{\mu_o}{E_o} \right]$$

for actual tangential stresses:

$$s_{to} = \frac{2P_c d_c^2}{d_o^2 - d_c^2}$$

$$s_{tco} = P_c \left( \frac{d_o^2 + d_c^2}{d_o^2 - d_c^2} \right)$$

$$s_{tci} = -P_c \left( \frac{d_c^2 + d_i^2}{d_c^2 - d_i^2} \right)$$

$$s_{ti} = \frac{-2P_c d_c^2}{d_c^2 - d_i^2}$$

for modified tangential stresses:

$$s_{to}' = \frac{2 P_c d_c^2}{d_o^2 - d_c^2}$$

$$s_{tco}' = P_c \left( \frac{d_o^2 + d_c^2}{d_o^2 - d_c^2} + \mu_o \right)$$

$$s_{tci}' = -P_c \left( \frac{d_c^2 + d_i^2}{d_c^2 - d_i^2} - \mu_i \right)$$

$$s_{ti}' = \frac{-2 P_c d_c^2}{d_c^2 - d_i^2}$$

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where:

- $\delta$  is the total interference;
- $P_c$  is the contact pressure;
- $d_i$  is the inside diameter;
- $d_c$  is the contact surface diameter;
- $d_o$  is the outside diameter;
- $\mu_o$  is Poisson's ratio for the outside material;
- $\mu_i$  is Poisson's ratio for the inside material;
- $E_o$  is the modulus of elasticity for the outside material;
- $E_i$  is the modulus of elasticity for the inside material;
- $s_{to}$  is the tangential stress of the outer surface;
- $s_{tco}$  is the tangential stress in the outer material at the contact surface;
- $s_{tci}$  is the tangential stress in the inner material at the contact surface;
- $s_{ti}$  is the stress at the inner surface of the inner cylinder;
- $s_{to}'$ ,  $s_{tco}'$ ,  $s_{tci}'$ , and  $s_{ti}'$  are the modified tangential stresses corresponding to the actual stresses just described.

### Reference:

Hall, Holowenko, Laughlin

*Theory and Problems of Machine Design*, Schaum's Outline Series, McGraw Hill Co., 1961.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SA1-12A1			
2	Input inside diameter	$d_i$	$\uparrow$	$d_i$
3	Input contact diameter	$d_c$	$\uparrow$	$d_c$
4	Input outside diameter	$d_o$	A	$d_i$
5	Input Poisson's ratio for outer cylinder	$\mu_o$	$\uparrow$	$\mu_o$
6	Input modulus of elasticity for outer cylinder	$E_o$	$\uparrow$	$E_o$
7	Input Poisson's ratio for inner cylinder	$\mu_i$	$\uparrow$	$\mu_i$
8	Input modulus of elasticity for inner cylinder	$E_i$	B	$\mu_o$
9	Input the total interference and calculate the contact pressure	$\delta$	C	$P_c$
	<i>or</i>			
	input the contact pressure and calculate the total interference	$P_c$	D	$\delta$
10	Enter SA1-12A2			
11	Calculate the following			
	Stress at outer surface		A	$s_{to}$
	<i>then</i> stress of outer material			
	at contact surface		A	$s_{tco}$
	Stress of inner material at contact surface			
	contact surface		B	$s_{tci}$
	<i>then</i> stress at inner surface		B	$s_{ti}$
	Modified stress at outer surface			
	face		C	$s'_{to}$
	<i>then</i> modified stress of outer material at contact surface		C	$s'_{tco}$

Continued

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STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	Modified stress of inner material at contact surface		D	$s'_{tci}$
	<i>then</i> modified stress at inner surface		D	$s'_{ti}$
12	For new case enter SA1-12A1 and go to step 9 for new interference or contact pressure or execute steps 2-4 or 5-8 to change inputs.			

### Example 1:

The choke at the end of a shotgun barrel is to be attached using an interference fit. If 5000 pounds per square inch must be developed to hold the choke in place, what is the minimum allowable interference? What are the values of actual stress.

$$d_i = 0.75 \text{ in}$$

$$d_c = 0.9375 \text{ in}$$

$$d_o = 1.125 \text{ in}$$

$$\mu_o = \mu_i = 0.3$$

$$E_o = E_i = 30 \times 10^6 \text{ psi}$$

Keystrokes using card SA 1-12A1:

.75  $\uparrow$  .9375  $\uparrow$  1.125 A .3  $\uparrow$  30 EEX 6  $\uparrow$  .3  
 $\uparrow$  30 EEX 6 B 5000 D DSP  $\bullet$  5  $\longrightarrow$  0.00158 in

Keystrokes using card SA 1-12A2:

A DSP  $\bullet$  2  $\longrightarrow$  22727.27 psi ( $s_{to}$ )  
A  $\longrightarrow$  27727.27 psi ( $s_{tco}$ )  
B  $\longrightarrow$  -22777.78 psi ( $s_{tci}$ )  
B  $\longrightarrow$  -27777.78 psi ( $s_{tc}$ )

**Example 2:**

Two concentric cylinders have an interference of 0.005 inches. What is the contact pressure? What are the stresses?

$$d_i = 3 \text{ in}$$

$$d_c = 5 \text{ in}$$

$$d_o = 6.5 \text{ in}$$

$$\mu_o = 0.3$$

$$E_o = 30 \times 10^6 \text{ psi}$$

$$\mu_i = 0.28$$

$$E_i = 15 \times 10^6$$

Keystrokes using card SA 1-12A1:

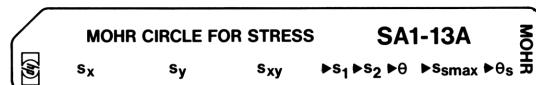
3 **↑** 5 **↑** 6.5 **A** .3 **↑** 30 **EEX** 6 **↑** .28 **↑**

15 **EEX** 6 **B** .005 **C** → 3802.98 psi

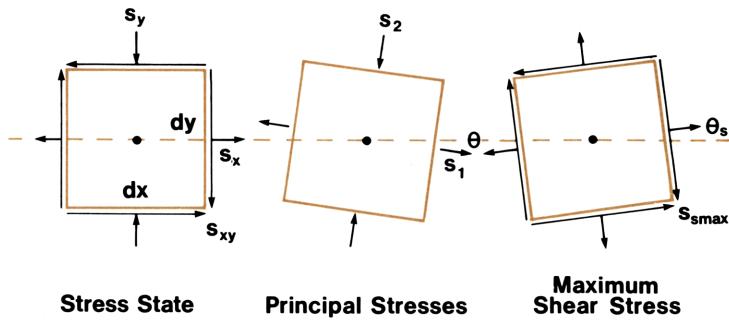
Keystrokes using card SA 1-12A1:

<b>A</b>	→ 11023.13 psi ( $s_{to}$ )
<b>A</b>	→ 14826.11 psi ( $s_{tco}$ )
<b>B</b>	→ -8081.33 psi ( $s_{tci}$ )
<b>B</b>	→ -11884.31 psi ( $s_{ti}$ )
<b>C</b>	→ 11023.13 psi ( $s_{to}'$ )
<b>C</b>	→ 15967.00 psi ( $s_{tci}'$ )
<b>D</b>	→ -7016.50 psi ( $s_{tci}'$ )
<b>D</b>	→ -11884.31 psi ( $s_{ti}'$ )

## MOHR CIRCLE FOR STRESS



Given the state of stress on an element, the principal stresses and their orientation can be found. The maximum shear stress and its orientation can also be found.



**Equations:**

$$s_{smax} = \sqrt{\left(\frac{s_x - s_y}{2}\right)^2 + s_{xy}^2}$$

$$s_1 = \frac{s_x + s_y}{2} + s_{smax}$$

$$s_2 = \frac{s_x + s_y}{2} - s_{smax}$$

$$\theta = \frac{1}{2} \tan^{-1} \left( \frac{2s_{xy}}{s_x - s_y} \right)$$

$$\theta_s = \frac{1}{2} \tan^{-1} \left( \frac{s_x - s_y}{2s_{xy}} \right)$$

where:

$s_{\text{max}}$  is the maximum shear stress;

$s_1$  and  $s_2$  are the principal normal stresses;

$\theta$  is the angle of rotation from the principal axis to the original axis;

$\theta_s$  is the angle of rotation from the axis of maximum shear stress to the original axis;

$s_x$  is the stress in the x direction;

$s_y$  is the stress in the y direction;

$s_{xy}$  is the shear stress on the element.

### Reference:

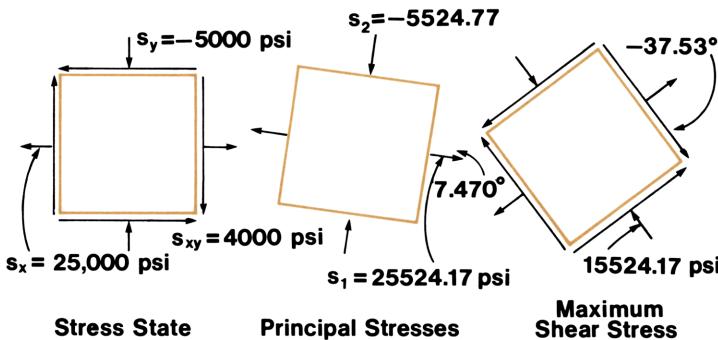
Spotts, M. F.

*Design of Machine Elements*, Prentice-Hall, 1971.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program		<input type="text"/> <input type="text"/>	
2	Input the following values  stress in x direction (negative for compression)	$s_x$	<input type="text"/> A <input type="text"/>	$s_x$
	stress in y direction (negative for compression)	$s_y$	<input type="text"/> B <input type="text"/>	$s_y$
	shear stress	$s_{xy}$	<input type="text"/> C <input type="text"/>	$s_{xy}$
3	Compute first principal stress:  <i>then</i> compute second principal stress		<input type="text"/> D <input type="text"/>	$s_1$
	<i>then</i> compute angle of rotation from principal stress		<input type="text"/> D <input type="text"/>	$s_2$
	$s_1$ to original axis		<input type="text"/> D <input type="text"/>	$\theta$
4	Compute maximum shear stress  <i>then</i> compute angle of rotation from axis of maximum shear to original axis		<input type="text"/> E <input type="text"/>	$s_{\text{max}}$
			<input type="text"/> E <input type="text"/>	$\theta_s$
5	For new case change appropriate inputs in step 2.		<input type="text"/> <input type="text"/>	

**Example :**

If  $s_x = 25000$  psi,  $s_y = -5000$  psi, and  $s_{xy} = 4000$  psi, compute the principal stresses and the maximum shear stress.



Keystrokes:

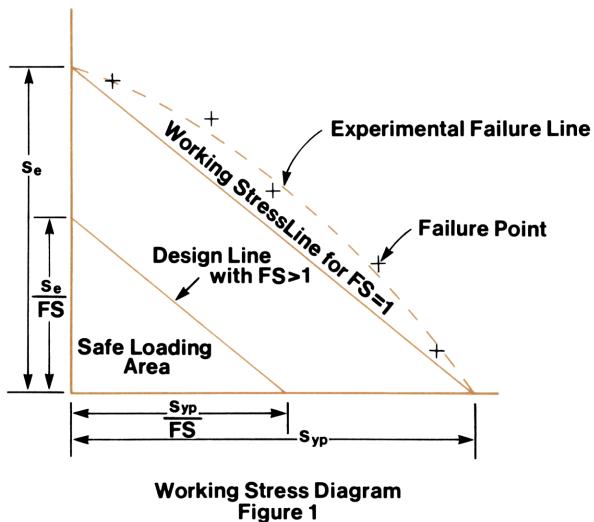
- 25000 **A** 5000 **CHS** **B** 4000 **C** **D** → 25524.17 psi ( $s_1$ )
- D** → -5524.17 psi ( $s_2$ )
- D** → 7.47 degrees ( $\theta$ )
- E** → 15524.17 psi ( $s_{\max}$ )
- E** → -37.53 degrees ( $\theta_s$ )



## SODERBERG'S EQUATION FOR FATIGUE



This program may be used to estimate maximum safe cyclic loads for a given size part or the minimum cross sectional area needed to sustain a cyclic loading. The program uses Soderberg's equation which is graphically represented in Figure 1.



$$\frac{s_{yp}}{FS} = \frac{s_{max} + s_{min}}{2} + K \left( \frac{s_{yp}}{s_e} \right) \left( \frac{(s_{max} - s_{min})}{2} \right)$$

$$\frac{s_{max} + s_{min}}{2} = \frac{P_{max} + P_{min}}{2A}$$

$$\frac{s_{max} - s_{min}}{2} = \frac{P_{max} - P_{min}}{2A}$$

where:

$s_{yp}$  is the yield point stress of the material;

$s_e$  is the material endurance stress from reversed bending tests;

K is the stress concentration factor for the part;

FS is the factor of safety ( $FS \geq 1.00$ )

$s_{max}$  is the maximum stress;

$s_{min}$  is the minimum stress;

$P_{max}$  is the maximum load;

$P_{min}$  is the minimum load;

A is the cross sectional area of the part.

#### Reference:

Spotts, M. F.

*Design of Machine Elements*; Prentice-Hall, Inc., 1971.

Baumeister, T.

*Marks Standard Handbook for Mechanical Engineers*, McGraw-Hill Book Company, 1967.

**Remarks:** This implementation of Soderberg's equation is for ductile materials only. Values of stress concentration factors and material endurance limits may be found in the referenced sources. In the presence of corrosive media, or for rough surfaces, fatigue effects may be much more significant than predicted by this program.

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STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input yield point stress	$s_{yp}$	↑ [ ]	$s_{yp}$
3	Input endurance stress	$s_e$	↑ [ ]	$s_e$
4	Input stress concentration			
	factor	K	↑ [ ]	K
5	Input factor of safety	FS	A [ ]	$s_{yp}$
6	Input two of the following:			
	maximum load	$P_{max}$	B [ ]	$P_{max}$
	minimum load	$P_{min}$	C [ ]	$P_{min}$
	area of cross section	A	D [ ]	A
7	Calculate the unknown value			
	maximum load		B R/S	$P_{max}$
	minimum load		C R/S	$P_{min}$
	area of cross section		D R/S	A
8	For new loading or area go to			
	step 6. For new case go to step			
	2.			

### Example :

What is the maximum permissible cyclic load for a part if the minimum load is 2000 pounds and the area is 0.5 square inches?

$$s_{yp} = 70000 \text{ psi}$$

$$s_e = 25000 \text{ psi}$$

$$K = 1.25$$

$$FS = 2.0$$

### Keystrokes:

70000 [↑] 25000 [↑] 1.25 [↑] 2.0 [A] 2000 [C]

.5 [D] [B] [R/S] → 8888.89 lb



## CIRCULAR PLATES WITH SIMPLY SUPPORTED EDGES



This program can be used to calculate the deflection and stress at the center of a simply supported circular plate with uniformly distributed or concentrated central loads.

### Equations:

for a concentrated central load:

$$y_{\max} = \frac{(3 + \mu) P r^2}{16\pi (1 + \mu) D}$$

$$s_{\max} = \frac{P}{h^2} \left[ (1 + \mu) \left( 0.485 \ln \frac{r}{h} + 0.52 \right) + 0.48 \right]$$

for a uniformly distributed load:

$$y_{\max} = \frac{(5 + \mu) W r^4}{64D (1 + \mu)}$$

$$s_{\max} = \frac{3(3 + \mu) W r^2}{8h^2}$$

where:

$$D = \frac{E h^3}{12(1 - \mu^2)}$$

$y_{\max}$  is the maximum deflection;

$s_{\max}$  is the maximum stress;

$\mu$  is Poisson's ratio;

$E$  is the modulus of elasticity;

$h$  is the thickness of the plate;

$r$  is the radius of the plate;

$W$  is the uniformly distributed load;

$P$  is the concentrated central load.

**Reference:**

Spotts, M. F.

*Design of Machine Elements*, Prentice-Hall, Inc., 1971**Remarks:** Deflections must be small compared to thickness of plate.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input modulus of elasticity	E	↑	E
3	Input thickness of plate	h	↑	h
4	Input Poisson's ratio	μ	↑	μ
5	Input radius of plate	r	A	D
6	If the load is distributed go to step 10			
7	Input concentrated load and calculate deflection	P	B	Y <sub>max</sub>
8	Calculate maximum stress		C	S <sub>max</sub>
9	For new load go to step 7. For new case go to step 2.			
10	Input distributed load and calculate deflection	W	D	Y <sub>max</sub>
11	Calculate maximum stress		E	S <sub>max</sub>
12	For new load go to step 10. For new case go to step 2.			

**66 SA1-15A****Example 1:**

Assuming that a manhole cover with an automobile tire at its center may be modeled as a simply supported flat plate with concentrated central load, what is the deflection at the center of the plate? What is the stress?

$$E = 30 \times 10^6 \text{ psi}$$

$$h = 0.75 \text{ in}$$

$$\mu = 0.3$$

$$r = 15 \text{ in}$$

$$P = 1500 \text{ lb}$$

Keystrokes:

30 [EEX] 6 [↑] .75 [↑] .3 [↑] 15 [A] 1500 [B] → 0.01 in

[C] → 8119.49 psi

**Example 2:**

A simply supported  $\frac{1}{4}$  inch thick steel plate ( $E = 30 \times 10^6$ ,  $\mu = 0.3$ ) withstands 50 pounds per square inch. If the radius is 5 inches, what is the deflection and what is the stress at the center of the plate?

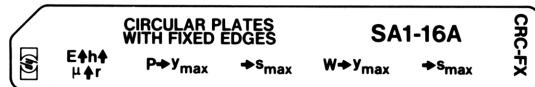
Keystrokes:

30 [EEX] 6 [↑] .25 [↑] .3 [↑] 5 [A] 50 [D] DSP [•] 4 → 0.0464 in

[E] → 24750.00 psi



## CIRCULAR PLATES WITH FIXED EDGES



This program can be used to calculate the maximum deflection and stress for a circular plate with fixed edges. Either central concentrated loads or distributed loads may be input.

### **Equations:**

for concentrated central load:

$$y_{max} = \frac{Pr^2}{16\pi D}$$

$$s_{max} = \frac{P}{h^2} (1 + \mu) \left( 0.485 \ln \frac{r}{h} + 0.52 \right)$$

for distributed loads:

$$y_{max} = \frac{Wr^4}{64D}$$

$$s_{max} = \frac{3Wr^2}{4h^2} \quad (\text{at edge of plate})$$

where:

$$D = \frac{Eh^3}{12(1 - \mu^2)}$$

$y_{max}$  is the maximum deflection;

$s_{max}$  is the maximum stress;

$P$  is the concentrated load;

$W$  is the distributed load;

$r$  is the radius of the plate;

$h$  is the thickness of the plate;

$\mu$  is Poisson's ratio;

$E$  is the modulus of elasticity.

**Reference:**

Spotts, M.F.,

*Design of Machine Elements*, Prentice-Hall, Inc., 1971

**Remarks:** Deflections must be small compared to the thickness of plate.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input modulus of elasticity	E	↑	E
3	Input thickness of plate	h	↑	h
4	Input Poisson's ratio	μ	↑	μ
5	Input radius of plate	r	A	D
6	If the load is distributed go to step 10			
7	Input concentrated load and calculate deflection	P	B	Y <sub>max</sub>
8	Calculate maximum stress		C	S <sub>max</sub>
9	For new load go to step 7. For new case go to step 2			
10	Input distributed load and cal- culate deflection	W	D	Y <sub>max</sub>
11	Calculate maximum stress		E	S <sub>max</sub>
12	For new load go to step 10. For new case go to step 2.			

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### Example 1:

The cap on a pressure vessel is a  $\frac{1}{4}$  inch thick steel plate ( $E = 30 \times 10^6$  psi,  $\mu = 0.3$ ) with a 6 inch radius. It is clamped to the opening of the pressure vessel by a ring of bolts. What are the maximum and minimum deflections and stresses in the plate if pressure cycles from 50 to 60 psi?

Keystrokes for minimum pressure:

30 **EEX** 6 **↑** .25 **↑** .3 **↑**

6 **A** 50 **D** → 0.02 in

**E** → 21600.00 psi

Keystrokes for maximum pressure:

60 **D** → 0.03 in

**E** → 25920.00 psi

### Example 2:

An adjustable focal length mirror is to derive its concaved shape due to a variable force applied at its center. The mirror is chrome plated steel ( $E = 30 \times 10^6$  psi,  $\mu = 0.3$ ), 0.1 inches thick and has a radius of 12 inches. What is the deflection of the center for a force of 6.0 pounds. The edges are held securely.

Keystrokes:

30 **EEX** 6 **↑** .1 **↑** .3 **↑** 12 **A**

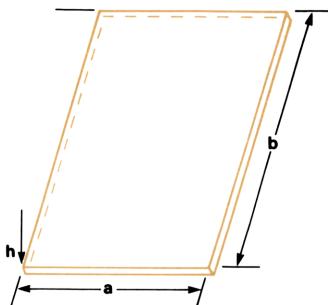
6 **B** DSP **•** **5** → 0.00626 in



## RECTANGULAR PLATES

	<b>RECTANGULAR PLATES-INPUT</b> $E \nabla h$ $b \nabla a$	<b>SA1-17A1</b> REC-IN
	<b>RECTANGULAR PLATES- SIMPLY SUPPORTED EDGES</b> $P \rightarrow y_{max}$ $r_0 \nabla s_{max}$ $W \rightarrow y_{max}$ $\nabla s_{max}$	<b>SA1-17A2</b> REC-S
	<b>RECTANGULAR PLATES- FIXED EDGES</b> $P \rightarrow y_{max}$ $\nabla s_{max}$ $W \rightarrow y_{max}$ $\nabla s_{max}$	<b>SA1-17A3</b> REC-F

This three card set can be used to calculate the deflection and stress at the centroids of rectangular plates. Loadings may be concentrated at the centroid of the plate or distributed uniformly over the surface. Card one is for data input and preliminary calculation. Card two calculates deflections and stresses for simply supported plates. Card three calculates deflections and stresses for plates with fixed edges.



**Equations:**

for simply supported edges and concentrated central load:

$$y_{\max} = \frac{\alpha Pa^2}{Eh^3}$$

$$s_{\max} = \frac{1.5P}{\pi h^2} \left[ (1 + \mu) \ln \frac{2a}{\pi r_0} - \gamma_1' \right]$$

$$\alpha = 0.0807 - \frac{1}{9.6} \left[ 8.0714 e^{-2.6249 \frac{b}{a}} - 1 \right]$$

$$\gamma_1' = 8.0714 e^{-2.6249 \frac{b}{a}} - 1$$

for simply supported edges with uniformly distributed load:

$$y_{\max} = \frac{0.142 Wa^4}{Eh^3 \left[ 2.21 \left( \frac{a}{b} \right)^3 + 1 \right]}$$

$$s_{\max} = \frac{0.75 Wa^2}{h^2 \left[ 1.61 \left( \frac{a}{b} \right)^3 + 1 \right]}$$

for clamped edges and concentrated central loads:

$$y_{\max} = \frac{\alpha' Pa^2}{Eh^3}$$

$$s_{\max} = \frac{\beta P}{h^2}$$

$$\alpha' = 0.0791 - 0.018 e^{-4 \frac{b}{a} + 4}$$

$$\beta = 1.009 - 0.255 e^{-4 \frac{b}{a} + 4}$$

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for clamped edges and distributed loads:

$$y_{\max} = \frac{0.0284 Wa^4}{Eh^3 [1.056 (a/b)^5 + 1]}$$

$$s_{\max} = \frac{Wa^2}{2 h^2 [0.623 (a/b)^6 + 1]} \quad (\text{at centers of long edges})$$

where:

$y_{\max}$  is the maximum deflection;

$s_{\max}$  is the maximum stress;

$P$  is the concentrated central load;

$W$  is the uniformly distributed load;

$E$  is the modulus of elasticity;

$\mu$  is Poisson's ratio;

$h$  is the thickness of the plate;

$b$  is the length of the plate;

$a$  is the width of the plate;

$r_0$  is the radius of application of the concentrated load for simply supported rectangular plates.

### Reference:

Spotts, M.F.

*Design of Machine Elements*, Prentice-Hall, 1971

**Remarks:** Deflections must be small in comparison to the thickness of the plate.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter <i>Rectangular Plates I,</i>			
	SA1-17A1			
2	Input modulus of elasticity	E	↑ A	E
3	Input thickness of plate	h	A	0.00
4	Input length of plate	b	↑	b
5	Input width of plate	a	B	0.00
6	For rectangular plates with clamped edges go to step 14. For rectangular plates with simply supported edges enter			
	SA1-17A2			
7	For a distributed load go to step 11			
8	Input concentrated load and calculate deflection	P	A	$\gamma_{max}$
9	Input $r_0$	$r_0$	↑	$r_0$
	then $\mu$ and calculate	$\mu$	B	$s_{max}$
10	For new load go to step 8. For new case go to step 1.			
11	Input uniformly distributed load and calculate deflection	W	C	$\gamma_{max}$
12	Calculate stress		D	$s_{max}$
13	For new load go to step 11. For new case go to step 7.			
14	Enter SA1-17A3			
15	For a distributed load go to step 19			
16	Input concentrated load and calculate deflection	P	A	$\gamma_{max}$
17	Calculate stress		B	$s_{max}$

Continued

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STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
18	For new load go to step 16. For new case go to step 1.			
19	Input uniformly distributed load and calculate deflection	W	C	$\gamma_{max}$
20	Calculate stress		D	$s_{max}$
21	For new load go to step 19. For new case go to step 1.			

### Example 1:

A simply supported rectangular plate has a concentrated load ( $r_0 = 1.0$  in) of 150 pounds at the center. The plate is 0.25 inch thick steel ( $E = 30 \times 10^6$ ,  $\mu = 0.3$ ). It is 3 feet long by 2 feet wide. What is the deflection and the stress at the centroid?

Keystrokes using card SA1-17A1:

30 [EEX] 6 [↑] .25 [A] 36 [↑] 24 [B] → 0.00

Keystrokes using card SA1-17A2:

150 [A] [DSP] [•] [4] → 0.0311 in

1 [↑] 0.3 [B] [DSP] [•] [2] → 5027.16 psi

### Example 2:

A pressure-sensitive switch is activated by the deformation of a rectangular plate that is welded at its perimeter. The plate is 4 inches by 5 inches and 0.01 inches thick. What deformation occurs for a pressure of 0.01 pounds per square inch?

$$E = 15 \times 10^6$$

Keystrokes using card SA1-17A1:

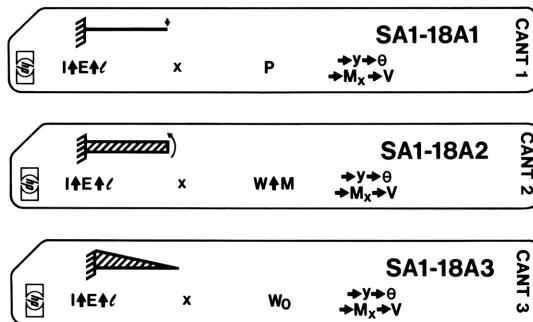
15 [EEX] 6 [↑] .01 [A] 5 [↑] 4 [B] → 0.00

Keystrokes using card SA1-17A3:

.01 [C] [DSP] [•] 4 → 0.0036 in



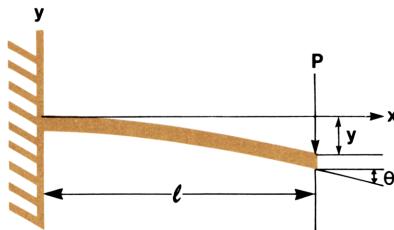
## CANTILEVER BEAMS



This three card set solves for deflection, angle, moment and shear at specified points along cantilever beams. By using the principle of superposition (summing calculated values in registers 7, 8 and 9) it is possible to solve for the effects of complicated loadings.

### Equations:

for a cantilever beam with a concentrated load at the end:



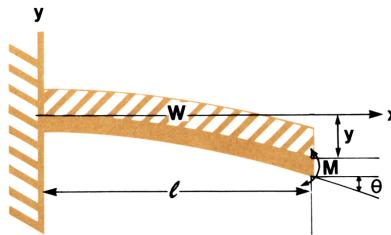
$$y = \frac{Px^2}{6EI} (x - 3l) \quad (x \leq l)$$

$$\theta = \frac{Px}{2EI} (x - 2l) \quad (x \leq l)$$

$$M_x = P(x - l) \quad (x \leq l)$$

$$V = P \quad (x < l)$$

for a cantilever beam with a uniformly distributed load and/or an applied moment at the end:



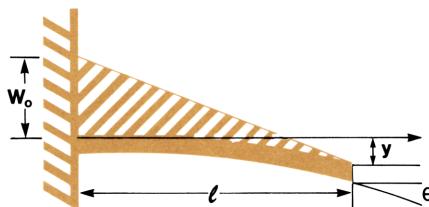
$$y = \frac{-x^2}{EI} \left[ \frac{W}{24} (x(x - 4\ell) + 6\ell^2) - \frac{M}{2} \right] \quad (x \leq \ell)$$

$$\theta = \frac{-x}{EI} \left[ W \left( x \left( \frac{x}{6} - \frac{\ell}{2} \right) + \frac{\ell^2}{2} \right) - M \right] \quad (x \leq \ell)$$

$$M_x = -W \left( x \left( \frac{x}{2} - \ell \right) + \frac{\ell^2}{2} \right) - M \quad (x \leq \ell)$$

$$V = W(\ell - x) \quad (x \leq \ell)$$

for a cantilever beam with a linear load distribution:



$$y = \frac{W_o}{24EI} \left[ \ell^3 \left( \frac{\ell}{5} - x \right) - \frac{(\ell - x)^5}{5\ell} \right] \quad (x \leq \ell)$$

$$\theta = \frac{W_o}{24EI} \left[ \frac{(\ell - x)^4}{\ell} - \ell^3 \right] \quad (x \leq \ell)$$

$$M_x = \frac{W_o (\ell - x)^3}{6\ell} \quad (x \leq \ell)$$

$$V = \frac{W_o (\ell - x)^2}{2\ell} \quad (x \leq \ell)$$

where:

$y$  is the deflection at a distance  $x$  from the wall;

$\theta$  is the slope at  $x$ ;

$M_x$  is the moment at  $x$ ;

$V$  is the shear at  $x$ ;

$I$  is the moment of inertia of the beam;

$E$  is the modulus of elasticity of the beam;

$\ell$  is the length of the beam;

$P$  is a concentrated load;

$W$  is a uniformly distributed load;

$M$  is an applied moment;

$W_o$  is the maximum value of a linearly distributed load.

**Remarks:** Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.  $x$  must be in the range  $0 \leq x \leq l$ .

### Cantilever 1

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SA1-18A1*			
2	Input moment of inertia	I	↑	I
	<i>then</i> modulus of elasticity	E	↑	E
	<i>then</i> length of beam	l	A	El
	Input distance from y axis to			
	point of interest	x	B	x
	Input concentrated load	P	C	P
3	Calculate deflection at x		D	y
4	Calculate slope at x		D	θ
5	Calculate moment at x		D	M <sub>x</sub>
6	Calculate shear at x		D	V
7	For new case go to step 2.			
	* Registers 7, 8 and 9 are available for intermediate storage.			

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### Cantilever 2

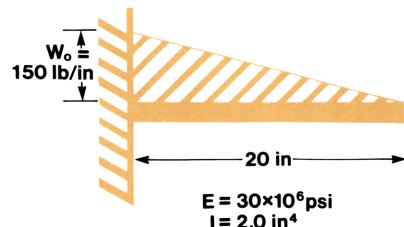
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SA1-18A2*		<input type="text"/> <input type="text"/>	
2	Input moment of inertia	I	<input type="text"/> <input type="text"/>	I
	<i>then</i> modulus of elasticity	E	<input type="text"/> <input type="text"/>	E
	<i>then</i> length of beam	l	<input type="text"/> <input type="text"/>	EI
	Input distance from y axis to		<input type="text"/> <input type="text"/>	
	point of interest	x	<input type="text"/> <input type="text"/>	x
	Input uniformly distributed		<input type="text"/> <input type="text"/>	
	load	w	<input type="text"/> <input type="text"/>	w
	<i>then</i> applied moment	M	<input type="text"/> <input type="text"/>	w
3	Calculate deflection at x		<input type="text"/> <input type="text"/>	y
4	Calculate slope at x		<input type="text"/> <input type="text"/>	$\theta$
5	Calculate moment at x		<input type="text"/> <input type="text"/>	$M_x$
6	Calculate shear at x		<input type="text"/> <input type="text"/>	v
7	For new case go to step 2.		<input type="text"/> <input type="text"/>	
			<input type="text"/> <input type="text"/>	
	* Registers 7, 8 and 9 are available for intermediate storage.		<input type="text"/> <input type="text"/>	

**Cantilever 3**

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SA1-18A3*			
2	Input moment of inertia	I	↑	I
	then modulus of elasticity	E	↑	E
	then length of beam	ℓ	A	EI
	Input distance from y axis to			
	point of interest	x	B	x
	Input maximum value of			
	linearly distributed load	W <sub>o</sub>	C	W <sub>o</sub>
3	Calculate deflection at x		D	y
4	Calculate slope at x		D	θ
5	Calculate moment at x		D	M <sub>x</sub>
6	Calculate shear at x		D	V
7	For new case go to step 2.			
	* Registers 7, 8 and 9 are available for intermediate storage.			

**Example 1:**

Find the deflection, slope, moment, and shear for the beam below at distances of 5, 10, 15 and 20 inches. Neglect the weight of the beam.

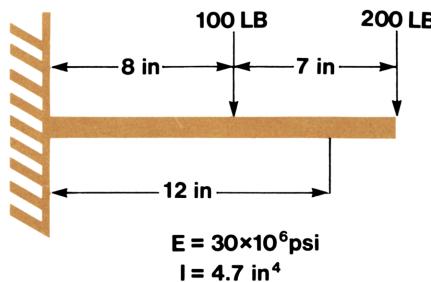


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Keystrokes using card SA1-18A3:

2 **↑** 30 **EEX** 6 **↑** 20 **A** 5 **B**

150	<b>C</b>	<b>D</b>	<b>DSP</b>	<b>•</b>	<b>5</b>	→ -0.00162 in
	<b>D</b>					→ -0.00057 in/in
	<b>D</b>					→ -4218.75000 in-lb
	<b>D</b>					→ 843.75000 lb
10	<b>B</b>	<b>D</b>				→ -0.00510 in
	<b>D</b>					→ -0.00078 in/in
	<b>D</b>					→ -1250.00000 in-lb
	<b>D</b>					→ 375.00000 lb
15	<b>B</b>	<b>D</b>				→ -0.00917 in
	<b>D</b>					→ -0.00083 in/in
	<b>D</b>					→ -156.25000 in-lb
	<b>D</b>					→ 93.75000 lb
20	<b>B</b>	<b>D</b>				→ -0.01333 in
	<b>D</b>					→ -0.00083 in/in
	<b>D</b>					→ 0.00000 in-lb
	<b>D</b>					→ 0.00000 lb

**Example 2:**What is the deflection at  $x = 12$ ? Neglect the weight of the beam.

Keystrokes using card SA1-18A1:

First compute the deflection at  $x = 8$  due to the 100 pound force and store the result in register 8.

4.7  $\uparrow$  30 [EEX] 6  $\uparrow$  8 [A] 8 [B]  
 100 [C] [D] [STO] 8 [DSP]  $\bullet$  5  $\longrightarrow$  -0.00012 in

Now compute the slope at 8 inches, multiply it by the distance from 8 inches to 12 inches, and add it to the value in register 8.

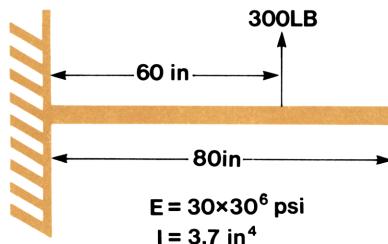
[D] 12  $\uparrow$  8  $-$  [X] [STO]  $+$  8  $\longrightarrow$  -0.00009

Now compute the deflection at 12 inches due to the force of 200 pounds at the end of the beam and add the contents of register 8 to it. This yields the final result. Note from the register allocation chart on page 124 that the new value of  $\ell$  (15 inches) may be manually stored in register 2 without reinput of I and E.

15 [STO] 2 12 [B] 200 [C] [D]  $\longrightarrow$  -0.00112 in  
 [RCL] 8  $+$   $\longrightarrow$  -0.00134 in

### Example 3:

Compute the deflection 25 inches from the wall for the beam below. Consider the weight of the beam to be 8 pounds per inch (applied moment is zero).

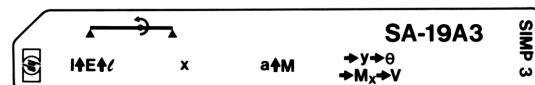
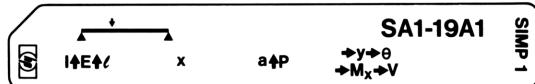


Keystrokes using card SA1-18A1:

3.7  $\uparrow$  30 [EEX] 6  $\uparrow$  60 [A] 25 [B]  
 300 [CHS] [C] [D] [STO] 8 [DSP]  $\bullet$  5  $\longrightarrow$  0.04364 in

Keystrokes using card SA1-18A2:

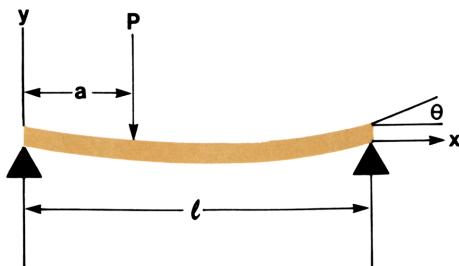
80 [STO] 2 8  $\uparrow$  0 [C] [D]  $\longrightarrow$  -0.05823 in  
 [RCL] 8  $+$   $\longrightarrow$  -0.01459 in

**SIMPLY SUPPORTED BEAMS**

This three card set solves for deflection, slope, moment and shear at specified points along simply supported beams. By using the principle of super-position (summing intermediate values in register 8), it is possible to solve for the effects of complicated loadings.

**Equations:**

for a simply supported beam with a concentrated load:



$$y = \frac{P(\ell - a)x}{6 EI\ell} (x^2 + (\ell - a)^2 - \ell^2)$$

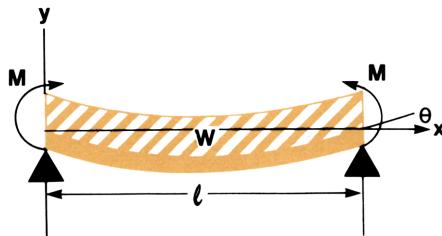
$$\theta = \frac{P(\ell - a)}{6 EI\ell} (3x^2 + (\ell - a)^2 - \ell^2)$$

$$M_x = \frac{P(\ell - a)x}{\ell}$$

$$V = \frac{P(\ell - a)}{\ell}$$

If  $x$  is greater than  $a$ ,  $(\ell - a)$  is replaced by  $-a$  and  $x$  is replaced by  $x - \ell$ .

for a simply supported beam with a uniformly distributed load and a couple applied at the end:



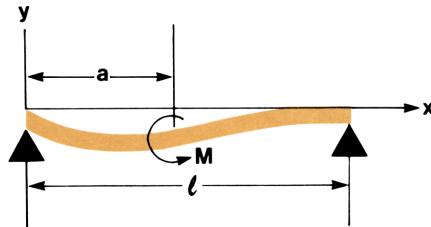
$$y = \frac{x}{EI} \left\{ \frac{M}{2} (x - \ell) - \frac{w}{24} [ \ell^3 + x^2 (x - 2\ell) ] \right\}$$

$$\theta = \frac{1}{EI} \left\{ M \left( x - \frac{\ell}{2} \right) - \frac{w}{24} [ \ell^3 + x^2 (4x - 6\ell) ] \right\}$$

$$M_x = M - \frac{w}{2} x [x - \ell]$$

$$V = w \left( \frac{\ell}{2} - x \right)$$

for a moment applied at a point along a simply supported beam:



$$y = \frac{-Mx}{EI} \left[ a - \frac{x^2}{6\ell} - \frac{\ell}{3} - \frac{a^2}{2\ell} \right]$$

$$\theta = \frac{-M}{EI} \left[ a - \frac{x^2}{2\ell} - \frac{\ell}{3} - \frac{a^2}{2\ell} \right]$$

$$M_x = \frac{Mx}{\ell}$$

$$V = \frac{M}{\ell}$$

If  $x$  is greater than  $a$ ,  $x$  is replaced by  $\ell - x$  and  $a$  is replaced by  $\ell - a$ .

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where:

$y$  is the deflection at a distance  $x$  from the wall;

$\theta$  is the slope at  $x$ ;

$M_x$  is the moment at  $x$ ;

$V$  is the shear at  $x$ ;

$I$  is the moment of inertia of the beam;

$E$  is the modulus of elasticity of the beam;

$\ell$  is the length of the beam;

$P$  is the concentrated load;

$W$  is a uniformly distributed load;

$M$  is an applied moment.

**Remarks:** Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.  $x$  must be in the range  $0 \leq x \leq \ell$ .

### Simple 1

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SA1-19A1*			
2	Input moment of inertia	I	↑	I
	then modulus of elasticity	E	↑	E
	then length of beam	ℓ	A	EI
	Input distance from y axis to			
	point of interest	x	B	x
	Input distance from origin to			
	concentrated load	a	↑	a
	then concentrated load	P	C	a
3	Calculate deflection at x		D	y
4	Calculate slope at x		D	θ
5	Calculate moment at x		D	$M_x$
6	Calculate shear at x		D	V
7	For new case go to step 2.			
	* Register 8 is available for			
	intermediate storage.			

## Simple 2

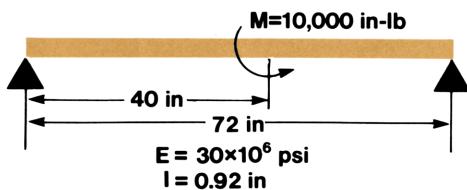
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SA1-19A2*			
2	Input moment of inertia	I	↑	I
	<i>then</i> modulus of elasticity	E	↑	E
	<i>then</i> length of beam	l	A	EI
	Input distance from y axis to			
	point of interest	x	B	x
	Input couple applied at end of			
	beam	M	↑	M
	<i>then</i> distributed load	w	C	M
3	Calculate deflection at x		D	y
4	Calculate slope at x		D	θ
5	Calculate moment at x		D	M <sub>x</sub>
6	Calculate shear at x		D	V
7	For new case go to step 2.			
	* Register 8 is available for			
	intermediate storage.			

## Simple 3

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SA1-19A3*			
2	Input moment of inertia	I	↑	I
	then modulus of elasticity	E	↑	E
	then length of beam	ℓ	A	EI
	Input distance from y axis to point of interest	x	B	x
	Input distance from origin to applied moment	a	↑	a
	then input applied moment	M	C	a
3	Calculate deflection at x		D	y
4	Calculate slope at x		D	θ
5	Calculate moment at x		D	M <sub>x</sub>
6	Calculate shear at x		D	V
7	For new case go to step 2.			
	* Register 8 is available for intermediate storage.			

## Example 1:

Find the deflection, slope, internal moment and shear at distances of 0, 24, 48 and 60 inches for the beam below. Neglect the weight of the beam.



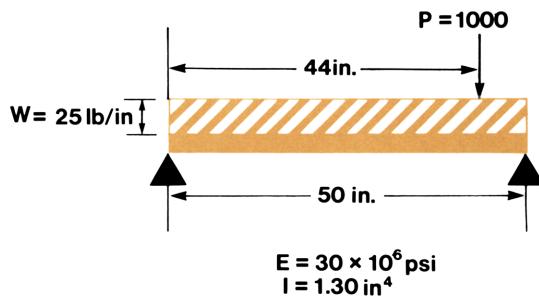
Keystrokes using card SA1-19A3:

92 ↑ 30 EEX 6 ↑ 72 A 0 B

40 ↑ 10000 C D DSP ⚡ 5 → 0.00000 in  
 D → -0.00177 in/in  
 D → 0.00000 in-lb  
 D → 138.88889 lb  
 24 B D → -0.03092 in  
 D → -0.00032 in/in  
 D → 3333.33333 in-lb  
 D → 138.88889 lb  
 48 B D → -0.00386 in  
 D → 0.00113 in/in  
 D → -3333.33333 in-lb  
 D → 138.88889 lb  
 60 B D → 0.00242  
 D → 0.00004 in/in  
 D → -1666.66667 in-lb  
 D → 138.88889 lb

### Example 2:

What is the slope of the beam below at  $x = 38$  inches?



Keystrokes:

First solve for slope due to the concentrated load using card SA1-19A1.

1.30  $\uparrow$  30 [EEX] 6  $\uparrow$  50 [A] 38 [B] 44  $\uparrow$

1000 [C] [D] [D] [STO] [8]  $\longrightarrow$  0.00096 in/in

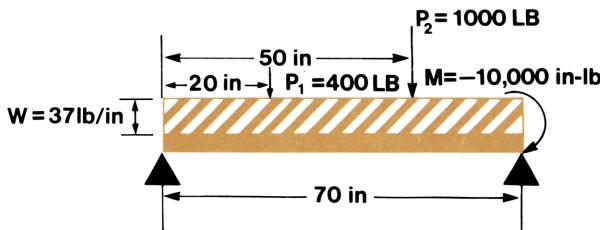
Now solve for slope due to distributed load using card SA1-19A2. Add this result to the contents of register 8 to obtain the desired result.

0  $\uparrow$  25 [C] [D] [D]  $\longrightarrow$  0.00237 in/in

[RCL] [8] [+]  $\longrightarrow$  0.00333 in/in

### Example 3:

What is the total moment at the center of the beam below? (It is not necessary to know the values of E or I to solve the problem. However, set them equal to 1.0 to avoid division by zero.)



Keystrokes:

Using card SA1-19A1 find the moment due to the concentrated loads and store the sum in register 8.

1  $\uparrow$  1  $\uparrow$  70 [A] 35 [B] 20  $\uparrow$

400 [C] [D] [D] [D] [STO] [8]  $\longrightarrow$  4000.00 in-lb

50  $\uparrow$  1000 [C] [D] [D] [D] [STO] [+] [8]  $\longrightarrow$  10000.00 in-lb

Using card SA1-19A2 find the moment due to the uniformly distributed load and add it to register 8.

0  $\uparrow$  37 [C] [D] [D] [D] [STO] [+] [8]  $\longrightarrow$  22662.50

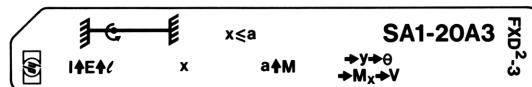
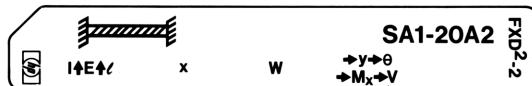
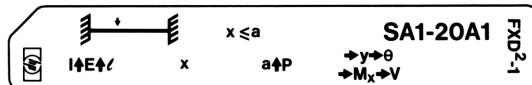
Using card SA1-19A3 find the internal moment due to the applied moment and add it to the contents of register 8.

70  $\uparrow$  10000 [CHS] [C] [D] [D] [D]  $\longrightarrow$  -5000.00 in-lb

[RCL] [8] [+]  $\longrightarrow$  31662.5 in-lb



## BEAMS FIXED AT BOTH ENDS



This three card set solves for deflection, slope, moment and shear at specified points along beams rigidly clamped at both ends. By using the principle of superposition (summing intermediate results in registers 8 and 9) it is possible to solve for the effects of complicated loadings.

It should be noted that  $x$  must be less than or equal to  $a$  in cards SA1-20A1 and SA1-20A3. That is, the point of interest,  $x$ , must be between the point at which the load is applied,  $a$ , and the left support. However, this does not limit the use of the cards. If  $x > a$ , simply view the beam from the opposite side and treat the angle and shear as negative values. Figures 1 and 2 below and the first example problem should make this clear.

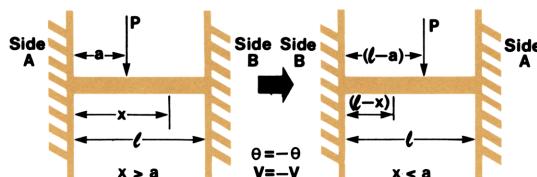


Figure 1—Transformation of Beam Geometry for Concentrated Load

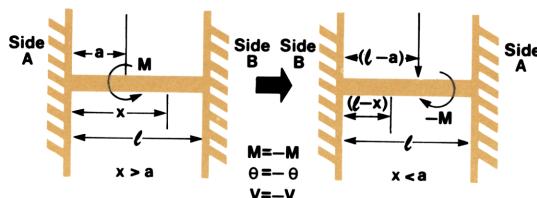
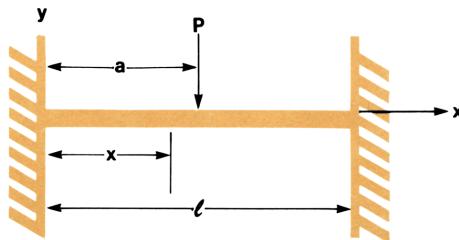


Figure 2—Transformation of Beam Geometry for Applied Moments

for concentrated loads:



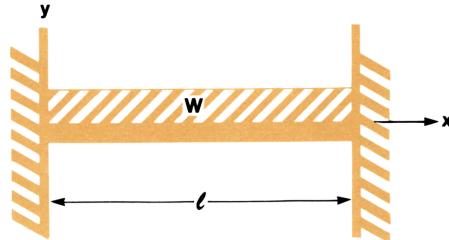
$$y = \frac{P(\ell - a)^2 x^2}{6EI\ell^3} [x(\ell + 2a) - 3a\ell] \quad x \leq a$$

$$\theta = \frac{P(\ell - a)^2 x}{2EI\ell^3} [x(\ell + 2a) - 2a\ell] \quad x \leq a$$

$$M_x = \frac{P(\ell - a)^2}{\ell^3} [x(\ell + 2a) - a\ell] \quad x \leq a$$

$$V = \frac{P(\ell - a)^2}{\ell^3} (\ell + 2a) \quad x \leq a$$

for distributed loads:



$$y = \frac{Wx^2}{24 EI} [x(2\ell - x) - \ell^2]$$

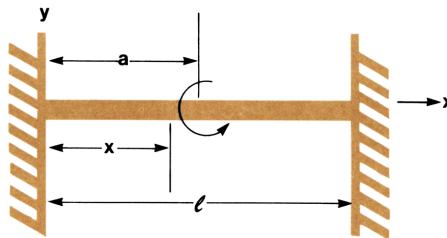
$$\theta = \frac{Wx}{12 EI} [x(3\ell - 2x) - \ell^2]$$

$$M_x = \frac{W}{12} [6x(\ell - x) - \ell^2]$$

$$V = \frac{-W}{2} (2x - \ell)$$

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for applied moments:



$$y = \frac{M(\ell - a)x^2}{\ell^2 EI} \left[ \frac{ax}{\ell} + \frac{\ell - 3a}{2} \right] \quad x \leq a$$

$$\theta = \frac{M(\ell - a)x}{\ell^2 EI} \left[ \frac{3ax}{\ell} + \frac{\ell - 3a}{2} \right] \quad x \leq a$$

$$M_x = \frac{M(\ell - a)}{\ell^2} \left[ \frac{6ax}{\ell} + \frac{\ell - 3a}{2} \right] \quad x \leq a$$

$$V = \frac{-6M(\ell - a)a}{\ell^3} \quad x \leq a$$

where:

$y$  is the deflection at a distance  $x$  from the wall;

$\theta$  is the slope at  $x$ ;

$M_x$  is the internal moment at  $x$ ;

$V$  is the shear at  $x$ ;

$I$  is the moment of inertia of the beam;

$E$  is the modulus of elasticity of the beam;

$\ell$  is the length of the beam;

$P$  is a concentrated load;

$W$  is a uniformly distributed load;

$M$  is an applied moment.

**Remarks:** Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.  $x$  must be in the range  $0 \leq x \leq a$ .

### Fixed-Fixed 1

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SA1-20A1*		<input type="text"/> <input type="text"/>	
2	Input moment of inertia	I	<input type="text"/> ↑ <input type="text"/>	I
	then modulus of elasticity	E	<input type="text"/> ↑ <input type="text"/>	E
	then length of beam	ℓ	<input type="text"/> A <input type="text"/>	EI
	Input distance from y axis to		<input type="text"/> <input type="text"/>	
	point of interest ( $x \leq a$ )	x	<input type="text"/> B <input type="text"/>	x
	Input distance from origin to		<input type="text"/> <input type="text"/>	
	concentrated load	a	<input type="text"/> ↑ <input type="text"/>	a
	then concentrated load	P	<input type="text"/> C <input type="text"/>	a
3	Calculate deflection at x		<input type="text"/> D <input type="text"/>	y
4	Calculate slope at x		<input type="text"/> D <input type="text"/>	θ
5	Calculate moment at x		<input type="text"/> D <input type="text"/>	$M_x$
6	Calculate shear at x		<input type="text"/> D <input type="text"/>	V
7	For new case go to step 2.		<input type="text"/> <input type="text"/>	
			<input type="text"/> <input type="text"/>	
	* Register 8 is available for		<input type="text"/> <input type="text"/>	
	intermediate storage.		<input type="text"/> <input type="text"/>	

**Fixed-Fixed 2**

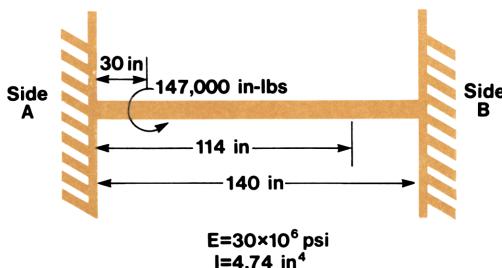
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SA1-20A2*			
2	Input moment of inertia	I	↑	I
	then modulus of elasticity	E	↑	E
	then length of beam	l	A	EI
	Input distance from y axis to			
	point of interest	x	B	x
	Input distributed load	W	C	W
3	Calculate deflection at x		D	y
4	Calculate slope at x		D	θ
5	Calculate moment at x		D	M <sub>x</sub>
6	Calculate shear at x		D	V
7	For new case go to step 2.			
	* Register 8 is available for			
	intermediate storage.			

**Fixed-Fixed 3**

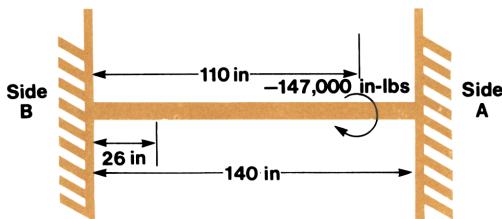
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SA1-20A3*			
2	Input moment of inertia	I	↑	I
	then modulus of elasticity	E	↑	E
	then length of beam	l	A	EI
	Input distance from y axis to			
	point of interest (x ≤ a)	x	B	x
	Input distance from origin to			
	applied moment	a	↑	a
	then input applied moment	M	C	a
3	Calculate deflection at x		D	y
4	Calculate slope at x		D	θ
5	Calculate moment at x		D	M <sub>x</sub>
6	Calculate shear at x		D	V
7	For new case go to step 2.			
	* Register 8 is available for			
	intermediate storage.			

**Example 1:**

For the beam below, what are the values of deflection, slope, moment, and shear at an  $x$  of 114 inches?



Transform the problem so that  $x < a$ .



Keystrokes using card SA1-20A3:

4.74 **[↑** 30 **[EEX]** 6 **[↑** 140 **[A** 26 **[B**

110 **[↑** 147000 **[CHS** **[C** **[D** → 0.08 in

**[D** → 0.01 in/in\*

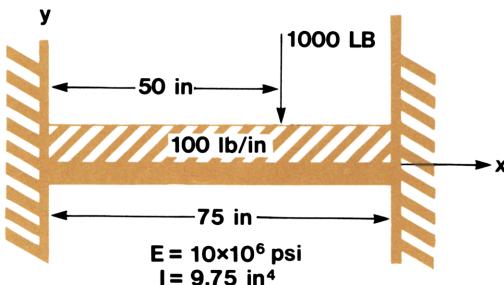
**[D** → 15171.43 in-lbs

**[D** → -1060.71\*

\*To return to the original coordinate system, change the sign of these values as shown in Figure 2.

**Example 2:**

Find the internal moment at  $x = 0$  in for the configuration below.



Keystrokes using card SA1-20A1:

9.75 **[↑]** 10 **EEX** 6 **[↑]** 75 **A**

0 **B** 50 **[↑]** 1000 **C**

**D D D STO 8** → -5555.56 in-lbs

Keystrokes using card SA1-20A2:

100 **C D D D** → -46875.00 in-lbs

**RCL 8 +** → -52430.56 in-lbs

Find the deflection at  $x = 40$  inches for the same beam.

Keystrokes with SA1-20A2 still in program memory:

40 **B D STO 8** → -0.08 in

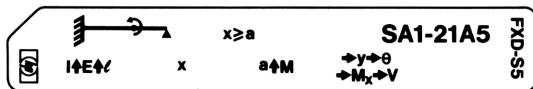
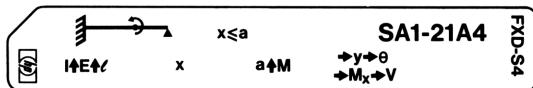
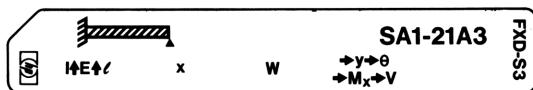
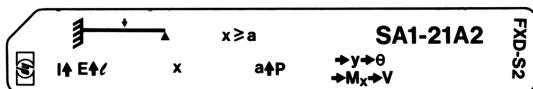
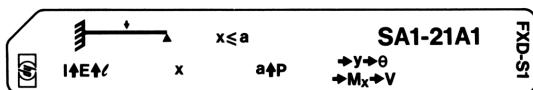
Keystrokes using card SA1-20A1:

50 **[↑]** 1000 **C D** → -0.02 in

**RCL 8 +** → -0.10 in



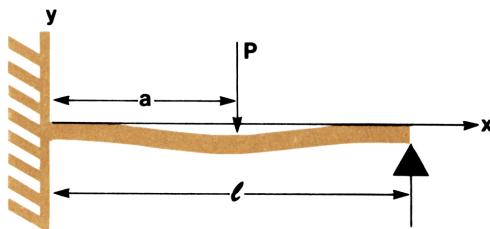
## BEAMS FIXED AT ONE END AND SIMPLY SUPPORTED AT THE OTHER



This five card set solves for deflection, slope, moment, and shear at specified points along beams that are rigidly fixed at one end and simply supported at the other end. By using the principle of superposition (summing intermediate values in registers 8 and 9), it is possible to solve for the effects of complicated loadings.

### Equations:

for concentrated loads:



$$y = \frac{P(\ell - a)x^2}{12\ell^2 EI} \begin{bmatrix} x & \left(2(\ell + a) - \frac{a^2}{\ell}\right) & -3a(2\ell - a) \end{bmatrix} \quad x \leq a$$

$$y = \frac{Pa^2(\ell - x)}{12\ell^2 EI} \begin{bmatrix} (\ell - x)^2 & \left(3 - \frac{a}{\ell}\right) & -3\ell(\ell - a) \end{bmatrix} \quad x \geq a$$

$$\theta = \frac{P(\ell - a)x}{12\ell^2 EI} \begin{bmatrix} 3x & \left(2(\ell + a) - \frac{a^2}{\ell}\right) & -6a(2\ell - a) \end{bmatrix} \quad x \leq a$$

$$\theta = \frac{Pa^2}{12\ell^2 EI} \begin{bmatrix} 3\ell(\ell - a) & -3(\ell - x)^2 & \left(3 - \frac{a}{\ell}\right) \end{bmatrix} \quad x \geq a$$

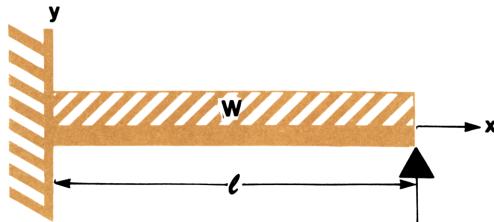
$$M_x = \frac{6P(\ell - a)}{12\ell^2} \begin{bmatrix} x & \left(2(\ell + a) - \frac{a^2}{\ell}\right) & -a(2\ell - a) \end{bmatrix} \quad x \leq a$$

$$M_x = -\frac{Pa^2}{12\ell^2} \begin{bmatrix} 6 & \left(3 - \frac{a}{\ell}\right) & (\ell - x) \end{bmatrix} \quad x \geq a$$

$$V = \frac{6P(\ell - a)}{12\ell^2} \begin{bmatrix} 2(\ell + a) - \frac{a^2}{\ell} \end{bmatrix} \quad x \leq a$$

$$V = -\frac{Pa^2}{12\ell^2} \begin{bmatrix} 6 & \left(3 - \frac{a}{\ell}\right) \end{bmatrix} \quad x \geq a$$

for distributed loads:



$$y = \frac{Wu}{48 EI} \left( 3\ell u^2 - 2u^3 - \ell^3 \right)$$

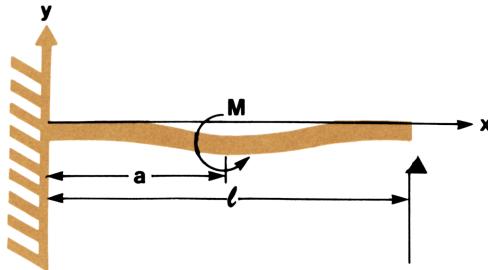
$$\theta = \frac{W}{48 EI} \left( 9\ell u^2 - 8u^3 - \ell^3 \right)$$

$$M_x = \frac{Wu}{8} (3\ell - 4u)$$

$$V = \frac{W}{8} (3\ell - 8u)$$

$$u = (\ell - x)$$

for applied moments:



$$y = \frac{M}{EI} \left[ \frac{a\ell - \frac{a^2}{2}}{2\ell^3} \left( 2\ell^3 - x^2(3\ell - x) \right) + \frac{x^2}{2} - \left( \ell a - \frac{a^2}{2} \right) \right] \quad x \leq a$$

$$y = \frac{M}{EI} \left[ \frac{2a\ell - a^2}{4\ell^3} \left( 2\ell^3 - x^2(3\ell - x) \right) - a(\ell - x) \right] \quad x \geq a$$

$$\theta = \frac{M}{EI} \left[ \frac{a\ell - \frac{a^2}{2}}{4\ell^3} (3x^2 - 6\ell x) + x \right] \quad x \leq a$$

$$\theta = \frac{M}{EI} \left[ \frac{2a\ell - a^2}{4\ell^3} (3x^2 - 6\ell x) + a \right] \quad x \geq a$$

$$M_x = M \left[ \frac{a\ell - \frac{a^2}{2}}{2\ell^3} (6x - 6\ell) + 1 \right] \quad x \leq a$$

$$M_x = M \left[ \frac{2a\ell - a^2}{4\ell^3} (6x - 6\ell) \right] \quad x \geq a$$

$$V = 6M \frac{2a\ell - a^2}{4\ell^3}$$

where:

$y$  is the deflection at a distance  $x$  from the wall;

$\theta$  is the slope at  $x$ ;

$M_x$  is the moment at  $x$ ;

$V$  is the shear at  $x$ ;

$I$  is the moment of inertia of the beam;

$E$  is the modulus of elasticity of the beam;

$\ell$  is the length of the beam;

$P$  is a concentrated load;

$W$  is a uniformly distributed load;

$M$  is an applied moment.

**Remarks:** Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.  $x$  must be in the range  $0 \leq x \leq \ell$ .

**Fixed-Simple 1 & 2**

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SA1-21A1 if $x \leq a$ , or enter SA1-21A2 if $x \geq a^*$		<input type="text"/> <input type="text"/>	
2	Input moment of inertia	I	<input type="text"/> <input type="uparrow"/> <input type="text"/>	I
	<i>then</i> modulus of elasticity	E	<input type="text"/> <input type="uparrow"/> <input type="text"/>	E
	<i>then</i> length of beam	l	<input type="text"/> A <input type="text"/>	EI
	Input distance from y axis to		<input type="text"/> <input type="text"/>	
	point of interest	x	<input type="text"/> B <input type="text"/>	x
	Input distance from y to		<input type="text"/> <input type="text"/>	
	concentrated load	a	<input type="text"/> <input type="uparrow"/> <input type="text"/>	a
	<i>then</i> input concentrated load	P	<input type="text"/> C <input type="text"/>	a
3	Calculate deflection at x		<input type="text"/> D <input type="text"/>	y
4	Calculate slope at x		<input type="text"/> D <input type="text"/>	$\theta$
5	Calculate moment at x		<input type="text"/> D <input type="text"/>	$M_x$
6	Calculate shear at x		<input type="text"/> D <input type="text"/>	V
7	For new case go to step 2.		<input type="text"/> <input type="text"/>	
			<input type="text"/> <input type="text"/>	
	* Registers 8 and 9 may be used		<input type="text"/> <input type="text"/>	
	for storage.		<input type="text"/> <input type="text"/>	

**Fixed-Simple 3**

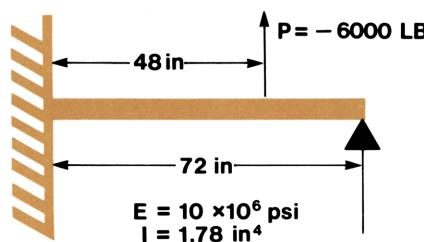
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SA1-21A3*			
2	Input moment of inertia	I	↑	I
	<i>then</i> modulus of elasticity	E	↑	E
	<i>then</i> length of beam	ℓ	A	EI
	Input distance from y axis to			
	point of interest	x	B	x
	Input uniformly distributed			
	load	W	C	W
3	Calculate deflection at x		D	y
4	Calculate slope at x		D	θ
5	Calculate moment at x		D	M <sub>x</sub>
6	Calculate shear at x		D	V
7	For new case go to step 2.			
	* Registers 8 and 9 may be used for storage.			

## Fixed-Simple 4 &amp; 5

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SA1-21A4 if $x \leq a^*$ or SA1-21A5 if $x \geq a$ .		<input type="text"/> <input type="text"/>	
2	Input moment of inertia	I	<input type="text"/> ↑ <input type="text"/>	I
	then modulus of elasticity	E	<input type="text"/> ↑ <input type="text"/>	E
	then length of beam	l	<input type="text"/> A <input type="text"/>	EI
	Input distance from y axis to point of interest	x	<input type="text"/> B <input type="text"/>	x
	Input distance from y axis to application of moment	a	<input type="text"/> ↑ <input type="text"/>	a
	then applied moment	M	<input type="text"/> C <input type="text"/>	a
3	Calculate deflection at x		<input type="text"/> D <input type="text"/>	y
4	Calculate slope at x		<input type="text"/> D <input type="text"/>	θ
5	Calculate moment at x		<input type="text"/> D <input type="text"/>	M <sub>x</sub>
6	Calculate shear at x		<input type="text"/> D <input type="text"/>	V
7	For new case go to step 2.			
	* Registers 8 and 9 may be used for storage.			

## Example 1:

Find the deflection, slope, moment, and shear along the beam below at x values of 0, 24, 48, and 72 inches. Neglect the weight of the beam.



Keystrokes using card SA1-21A1:

1.78  $\uparrow$  10 EEX 6  $\uparrow$  72 A

0 B 48  $\uparrow$  6000 CHS C D  $\longrightarrow$  0.00 in

D  $\longrightarrow$  0.00 in/in

D  $\longrightarrow$  64000.00 in-lb

D  $\longrightarrow$  -2888.89 lb

24 B D  $\longrightarrow$  0.66 in

D  $\longrightarrow$  0.04 in/in

D  $\longrightarrow$  -5333.33 in-lb

D  $\longrightarrow$  -2888.89 lb

48 B D  $\longrightarrow$  1.15 in

D  $\longrightarrow$  -0.01 in/in

D  $\longrightarrow$  -74666.67 in-lb

D  $\longrightarrow$  -2888.89 lb

Keystrokes using card SA1-21A2 since  $x > a$ :

72 B D  $\longrightarrow$  0.00 in

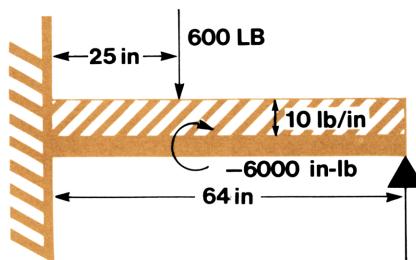
D  $\longrightarrow$  -0.06 in/in

D  $\longrightarrow$  0.00 in-lb

D  $\longrightarrow$  3111.11 lb

### Example 2:

Find the moment and shear at the wall for the beam below.



Since deflection and slope are not of interest, assume the arbitrary value of 1.0 for I and E. Store the sum of the moments in register 8 and the sum of the shear in register 9.

## 110 SA1-21A

Keystrokes using card SA1-21A1:

1  $\uparrow$  1  $\uparrow$  64 A 0 B 25  $\uparrow$

600 C  $\longrightarrow$  25 in

D D D STO 8  $\longrightarrow$  -7355.35 in-lb

D STO 9  $\longrightarrow$  480.55 lb

Keystrokes using card SA1-21A3:

10 C D D D STO + 8  $\longrightarrow$  -5120.00 in-lb

D STO + 9  $\longrightarrow$  400.00 lb

Keystrokes using card SA1-21A4:

25  $\uparrow$  6000 CHS C D D D  $\longrightarrow$  -342.04

RCL 8 +  $\longrightarrow$  -12817.39 in-lb

D  $\longrightarrow$  -88.41 lb

RCL 9 +  $\longrightarrow$  792.15 lb



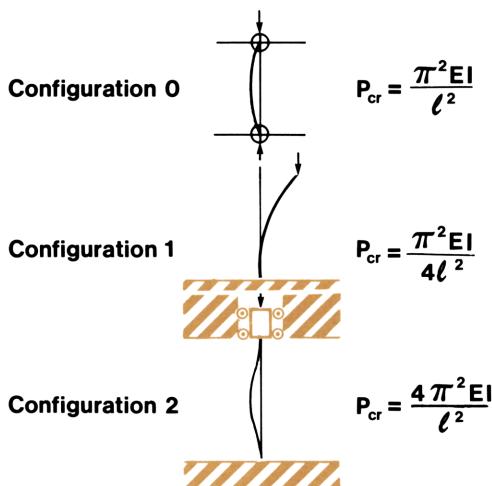
## COMPRESSIVE BUCKLING



This program performs an interchangeable solution for the four properties of slender compression members or columns:  $P_{cr}$ , the critical buckling load;  $E$ , the modulus of elasticity;  $I$ , the minimum moment of inertia; and  $l$ , the length of the member.

### Equations:

Three configurations are possible, identified by the number of fixed ends on the member: 0, both ends hinged; 1, one end free and one fixed; 2, both ends fixed.



**Remarks:** Uncertainties such as the amount of restraint at the ends, eccentricity of the load, initial warp, nonhomogeneity of the material and deflection caused by lateral loads, can cause very significant changes in the behavior of a compressive member.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Select column geometry by inputting number of fixed ends	0,1, or 2	A	0,1, or 2
3	Input three of the following			
	vertical load	P	B	0.00
	modulus of elasticity	E	C	0.00
	moment of inertia	I	D	0.00
	length of column	l	E	0.00
4	Compute remaining value			
	vertical load	0.00	B	P
	modulus of elasticity	0.00	C	E
	moment of inertia	0.00	D	I
	length of column	0.00	E	l
5	For new case with same type of column go to step 3. For a new type of column go to step 2.			

**Example 1:**

If an 8 inch steel ( $E = 30 \times 10^6$  psi) piston rod (a piston rod has zero fixed ends) must withstand a load of 15000 pounds without buckling, what moment of inertia must it have?

Keystrokes:

0 **A** 15000 **B** 30 **EEX** 6 **C** 8 **E** → 0.00  
**D** **DSP** 3 →  $3.242 \times 10^{-3}$

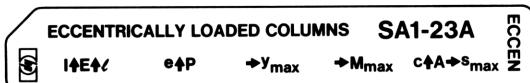
**Example 2:**

Steel columns 40 feet long are used to support a bridge. What is the maximum load that the column can withstand without buckling? Assume 1 fixed end.  $E = 30 \times 10^6$  psi,  $I = 700 \text{ in}^4$ .

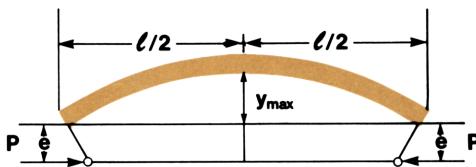
Keystrokes:

1 **A** 30 **EEX** 6 **C** 700 **D** 40 **↑** 12 **X** **E** →  $0.000 \times 10^0$   
**B** →  $2.249 \times 10^5$

## ECCENTRICALLY LOADED COLUMNS



This program calculates the maximum deflection, the maximum moment, and the maximum stress in an eccentrically loaded column under compressive stress.



**Equations:**

$$y_{\max} = e \left[ \sec \frac{\lambda}{2} \sqrt{\frac{P}{EI}} - 1 \right]$$

$$M_{\max} = P [e + y_{\max}]$$

$$s_{\max} = \frac{P}{A} \left[ 1 + \frac{ecA}{I} \sec \frac{\lambda}{2} \sqrt{\frac{P}{EI}} \right]$$

where:

$y_{\max}$  is the maximum deflection;

$e$  is the eccentricity;

$\lambda$  is the column length;

$P$  is the compressive load;

$E$  is the modulus of elasticity;

$I$  is the moment of inertia;

$M_{\max}$  is the maximum internal moment;

$s_{\max}$  is the maximum normal stress in the column;

$c$  is the distance from the neutral axis of the column to the outer surface;

$A$  is the area of the cross section;

**Remarks:** Columns must be of constant cross section. Stresses may not exceed the elastic limit of the material.

### Reference:

Spotts, M.F.

*Design of Machine Elements*, Prentice-Hall, 1971

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input moment of inertia	I	↑	I
	then input modulus of elasticity	E	↑	E
	then input length of column	ℓ	A	ℓ
	Input eccentricity	e	↑	e
	then input load	P	B	e
3	Calculate maximum deflection		C	y <sub>max</sub>
	or calculate maximum moment			
	or input distance from neutral axis	c	D	M <sub>max</sub>
	then section area and calculate maximum stress	A	E	s <sub>max</sub>
4	For new case go to step 2 and change inputs.			

### Example:

A column 50 feet long is to support 8000 pounds. The load is to be offset 6 inches. What are the maximum values of deflection, moment, and stress in the member?

$$E = 30 \times 10^6$$

$$I = 107 \text{ in}^4$$

$$A = 7 \text{ in}^2$$

$$c = 2 \text{ in}$$

Keystrokes:

107 ↑ 30 EX 6 ↑ 50 ↑

12 X A 6 ↑ 8000 B C → 0.74

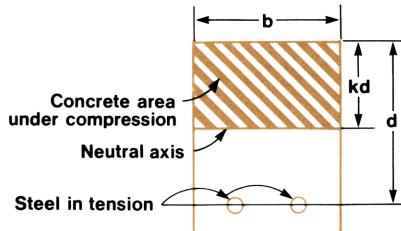
D → 53936.76 in-lb

2 ↑ 7 E → 2151.02 psi

## RECTANGULAR, REINFORCED CONCRETE SECTIONS



While concrete has good strength in compression, it is very weak in tension. In order to take advantage of the strength in compression, steel reinforcing members are often imbedded in concrete beams to carry the tension load. This program calculates the fraction of the total sectional area that must be steel in order to reach limiting values of load in both steel and concrete simultaneously. The ratio of the depth of the neutral axis to the depth of the section can also be calculated. Finally, given a section width, the depth can be calculated or, given the depth, the width can be calculated.



**Equations:**

$$k = \sqrt{2 pn + (pn)^2} - pn$$

$$p = \frac{1/2}{\frac{f_s}{f_c} \left( \frac{f_s}{nf_c} + 1 \right)} = \frac{A_s}{A_c}$$

$$bd^2 = \frac{6M \left( \frac{f_s}{f_c} + n \right)^2}{nf_c \left( \frac{3f_s}{f_c} + 2n \right)}$$

where:

- p is the ratio of steel area ( $A_s$ ) to concrete area ( $A_c$ );
- k is the ratio of the depth of the neutral axis to the depth of the section;
- n is the ratio of the modulus of elasticity of steel to that of concrete ( $E_s/E_c$ );
- $f_s$  is the maximum tensile stress allowed in the steel;
- $f_c$  is the maximum compressive stress allowed in the concrete;
- M is the total bending moment resisted by the section;
- b is the width of the section;
- d is the distance from the outer compressive fiber of the section to the centroid of the steel.

**Reference:**

Maurer, E.R. and Withey, M.O.

*Strength of Materials*, John Wiley and Sons, Inc. 1940.

**Remarks:** It is assumed that the concrete supports none of the tensile load. The value of d is the minimum possible section depth. It does not include the concrete encasing the steel below the centroid of the steel.

## 118 SA1-24A

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input elasticity ratio (Es/Ec)	n	↑ [ ] [ ]	n
3	Input maximum tensile stress for steel	$f_s$	[ ] [ ]	$f_s$
4	Input maximum compressive stress for concrete	$f_c$	↑ [ ] [ ] A [ ]	n
5	Optional: calculate steel ratio <i>then</i> neutral axis ratio		B [ ] [ ]	p
6	Input moment	M	C [ ] [ ]	M
7	If you know the depth calculate the width	d	D [ ] [ ]	b
	If you know the width calculate the depth	b	E [ ] [ ]	d
8	For a new beam size go to step 7. For a new moment go to step 6. For a new case go to step 2.			

**Example:**

A beam withstands a bending moment of 200,000 inch-pounds. The allowable stress in the concrete is 750 pounds per square inch and the allowable stress in the steel is 10000 pounds per square inch. The modulus of elasticity of the steel is  $30 \times 10^6$  pounds per square inch and the modulus of elasticity of the concrete is  $2.7 \times 10^6$  pounds per square inch yielding an n of 11.11 ( $30/2.7$ ). If the member is limited to a width of 8 inches, what depth is necessary to support the load? What is the necessary area of steel rods? Where is the neutral axis of the section?

Keystrokes:

11.11 ↑ 10000 ↑ 750 A B DSP

• 4 → 0.0170

B → 0.4545 (k)

200000 C 8 E → 13.1478 in (d)

From the formula  $A_s = P A_c$ :

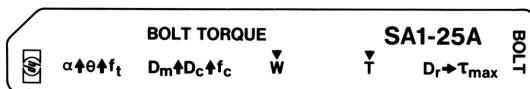
8 × .0170 × → 1.7881 in<sup>2</sup> (A<sub>s</sub>)

and since the distance to the neutral axis is kd:

13.1478 ↑ 0.4545 × → 5.9757 in



## BOLT TORQUE



This program may be used to calculate either the torque that will yield a specified bolt load or the load resulting from a specified torque. The maximum shear stress in the body of the screw may also be calculated.

**Equations:**

$$T = W \frac{D_m}{2} \left[ \frac{\tan \alpha + f_t / \cos \theta}{1 - f_t \tan \alpha / \cos \theta} \right] + W f_c \frac{D_c}{2}$$

$$\tau_{\max} = \sqrt{(W/2 A_r)^2 + (16 T_t / \pi D_r^3)^2}$$

$$T_t = T - W f_c \frac{D_c}{2}$$

where:

$T$  is the applied torque;

$W$  is the bolt load;

$D_m$  is the mean thread diameter;

$\alpha$  is the helix angle of the thread;

$f_t$  is the coefficient of thread friction;

$\theta$  is one-half of the thread angle;

$f_c$  is the collar coefficient of friction;

$D_c$  is the collar diameter;

$\tau_{\max}$  is the maximum shear stress in the body of the screw;

$A_r$  is the root area;

$D_r$  is the diameter at the root of the thread.

**Remarks:** The accuracy with which  $f_t$  and  $f_c$  are approximated has a significant effect on the applicability of the resulting computations.

**Reference:**

Hall, Holowenko, Laughlin

*Machine Design*, Schaum's Outline Series, McGraw-Hill Co., 1961.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program			
2	Input helix angle of thread	$\alpha$	$\uparrow$	$\alpha$
	<i>then</i> one-half of thread angle	$\theta$	$\uparrow$	$\theta$
	<i>then</i> coefficient of thread			
	friction	$f_t$	A	0.00
	Input mean thread diameter	$D_m$	$\uparrow$	$D_m$
	<i>then</i> collar diameter	$D_c$	$\uparrow$	$D_c$
	<i>then</i> collar coefficient of			
	friction	$f_c$	B	0.00
3	Input one of the following			
	bolt load	W	C	0.00
	bolt torque	T	D	0.00
4	Calculate one of the following			
	bolt load	0.00	C	W
	bolt torque	0.00	D	T
5	Optional: Input diameter of			
	the root of the thread and com-			
	pute shear stress	$D_r$	E	$\tau_{max}$
6	For a new load or torque go to			
	step 3. For a new case go to			
	step 2.			

**Example:**

The head bolts on an engine must exert a force of 11,000 pounds each. What torque is necessary to achieve this load assuming the following specifications? What is the shear stress in the bolt?

$$D_m = 0.3344 \text{ in}$$

$$\alpha = 3.40^\circ$$

$$f_t = 0.15$$

$$\theta = 30^\circ$$

$$f_c = 0.30$$

$$D_c = 0.8750$$

$$D_r = 0.2983$$

**122 SA1-25A**

Keystrokes:

3.40 **↑** 30 **↑** .15 **A** .3344 **↑**.8750 **↑** .3 **B** 11000 **C** **D** → 1876.03 in-lb12 **÷** → 156.34 ft-lbs.2983 **E** → 114335.98 psi

If the torque were set at 140 foot-pounds (1680 inch-pounds), what would be the bolt load?

1680 **D** **C** → 9850.61 lbs



# REGISTER ALLOCATION STRESS ANALYSIS PAC 1

CARD	#'s	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>4</sub>	R <sub>5</sub>	R <sub>6</sub>	R <sub>7</sub>	R <sub>8</sub>	R <sub>9</sub>
Vect	SA1-1A	x <sub>1</sub>	y <sub>1</sub>	x <sub>2</sub>	y <sub>2</sub>					Used
Point	SA1-2A	Fcosθ	Fsinθ	cos φ <sub>1</sub>	sin φ <sub>1</sub>	cos φ <sub>2</sub>	sin φ <sub>2</sub>			Used
Body	SA1-3A	M <sub>T</sub>	ΣF <sub>X</sub>	ΣF <sub>Y</sub>	xΣF <sub>Y</sub>	yΣF <sub>X</sub>				Used
Rec	SA1-4A	I	b	h						Used
Circ	SA1-5A	d								Used
An	SA1-6A	d <sub>o</sub>	d <sub>i</sub>	I						Used
Comp 1	SA1-7A1	x <sub>0i</sub>	y <sub>0i</sub>	Δx <sub>i</sub>	Σx <sub>0</sub> A	Σy <sub>0</sub> A	ΣA	ΣI <sub>x</sub>	ΣI <sub>y</sub>	
Comp 2	SA1-7A2	I <sub>̄x</sub>	I <sub>̄y</sub>	I <sub>̄xy</sub>	φ	I <sub>̄yφ</sub>	I <sub>̄xφ</sub> cos <sup>2</sup> φ	sin <sup>2</sup> φ	I <sub>̄xy</sub> sin φ	Used
Bend	SA1-8A	I(J)	M(T)	v(r)	s(s <sub>g</sub> )					Used
Def	SA1-9A	A(J)	P(T)	ℓ	ΔΩ(θ)	E(G)				Used
PV	SA1-10A	r	s <sub>c</sub>	P	t					Used
Thick	SA1-11A	P <sub>i</sub>	r <sub>i</sub>	P <sub>o</sub>	r <sub>o</sub>	r	s <sub>r</sub>	s <sub>t</sub>		
Fit P	SA1-12A1	d <sub>i</sub>	d <sub>c</sub>	d <sub>o</sub>	μ <sub>o</sub>	E <sub>o</sub>	μ <sub>i</sub>	E <sub>i</sub>	P <sub>c, δ</sub>	
Fit S	SA1-12A2	d <sub>i</sub>	d <sub>c</sub>	d <sub>o</sub>	μ <sub>o</sub>	E <sub>o</sub>	μ <sub>i</sub>	E <sub>i</sub>	P <sub>c</sub>	
Mohr	SA1-13A	s <sub>x</sub>	s <sub>y</sub>	s <sub>xy</sub>	(s <sub>x</sub> - s <sub>y</sub> )/2	θ <sub>s</sub>	s <sub>smax</sub>	s <sub>2</sub>		Used
Sodr	SA1-14A	k	s <sub>e</sub>	s <sub>yp</sub>	P <sub>max</sub>	P <sub>min</sub>	2A	FS		
CRC-SM	SA1-15A	r	μ	h	D	P	W			
CRC-FX	SA1-16A	r	μ	h	D	P	W			

Rec-In	SA1-17A1	b/a	a <sup>2</sup>	h <sup>2</sup>	Eh	$\beta$			$\gamma_1'$	$\alpha'$
Rec-S	SA1-17A2	b/a	a <sup>2</sup>	h <sup>2</sup>	Eh	$\beta$	P	W	$\gamma_1'$	$\alpha'$
Rec-F	SA1-17A3	b/a	a <sup>2</sup>	h <sup>2</sup>	Eh	$\beta$	P	W	$\gamma_1'$	$\alpha'$
Cant 1	SA1-18A1	EI	$\varrho$	P	x					
Cant 2	SA1-18A2	EI	$\varrho$	x	M	W				
Cant 3	SA1-18A3	EI	$\varrho$	( $\varrho - x$ )	x					
Simp 1	SA1-19A1	EI	$\varrho$	P	x	a	$\varrho - a$ or a	x or $x - \varrho$		Used
Simp 2	SA1-19A2	EI	$\varrho$	x	M	W				
Simb 3	SA1-19A3	EI	$\varrho$	x, $x - \varrho$	x	a	-M			
FXD <sup>2</sup> 1	SA1-20A1	EI	$\varrho$	P	x	a	( $\varrho + 2a$ )	$a, \varrho - a$		Used
FXD <sup>2</sup> 2	SA1-20A2	EI	$\varrho$	x	W					
FXD <sup>2</sup> 3	SA1-20A3	EI	$\varrho$	( $\varrho - 3a$ )	x	a	M	$M(\varrho - a)/\varrho^2$		
FXD-S1	SA1-21A1	EI	$\varrho$	P	x	a			$a(2\varrho - a)$	
FXD-S2	SA1-21A2	EI	$\varrho$	P	x	a	$3 - a/\varrho$		$3\varrho(\varrho - a)$	
FXD-S3	SA1-21A3	EI	$\varrho$	x	W		$\varrho - x$			
FXD-S4	SA1-21A4	EI	$\varrho$	( $a\varrho - a^2/2)/2\varrho^3$	x	a	M	M/EI		
FXD-S5	SA1-21A5	EI	$\varrho$	( $2a\varrho^2 - a^2)/4\varrho^3$	x	a	M	M/EI		
Buck	SA1-22A	C	P	E	I	$\varrho^2$				Used
Eccen	SA1-23A	I	E	$\varrho$	e	P	c	A		Used
Conc	SA1-24A	n	M	f <sub>c</sub>	f <sub>s/f<sub>c</sub></sub>	P				
Bolt	SA1-25A	f <sub>t, used</sub>	$\cos \theta$	$\tan \alpha$	f <sub>c</sub>	D <sub>c/2</sub>	D <sub>m/2</sub>	W	T	Used



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## TWO DIMENSIONAL VECTOR OPERATIONS

KEYS	CODE	KEYS	CODE	KEYS	CODE
LBL	23	STO 2	33 02	g NOP	35 01
A	11	RCL 1	34 01	g NOP	35 01
1	01	RCL 3	34 03	g NOP	35 01
f <sup>-1</sup>	32	+	61	g NOP	35 01
R→P	01	STO 1	33 01	g NOP	35 01
g R↓	35 08	f	31	g NOP	35 01
g x↔y	35 07	R→P	01	g NOP	35 01
x	71	RTN	24	g NOP	35 01
STO 2	33 02	LBL	23	g NOP	35 01
g LST X	35 00	C	13	g NOP	35 01
g R↑	35 09	g x↔y	35 07	g NOP	35 01
x	71	RTN	24	g NOP	35 01
STO 1	33 01	LBL	23	g NOP	35 01
0	00	D	14	g NOP	35 01
RTN	24	RCL 1	34 01	g NOP	35 01
LBL	23	RCL 4	34 04	g NOP	35 01
B	12	x	71	g NOP	35 01
1	01	RCL 2	34 02	g NOP	35 01
f <sup>-1</sup>	32	RCL 3	34 03	g NOP	35 01
R→P	01	x	71	g NOP	35 01
g R↓	35 08	—	51	g NOP	35 01
g x↔y	35 07	RTN	24	g NOP	35 01
x	71	LBL	23	g NOP	35 01
STO 4	33 04	E	15	g NOP	35 01
g LST X	35 00	RCL 1	34 01	g NOP	35 01
g R↑	35 09	RCL 3	34 03	g NOP	35 01
x	71	x	71	g NOP	35 01
STO 3	33 03	RCL 4	34 04	g NOP	35 01
0	00	RCL 2	34 02	g NOP	35 01
RTN	24	x	71	g NOP	35 01
LBL	23	+	61	g NOP	35 01
C	13	RTN	24	g NOP	35 01
RCL 2	34 02	g NOP	35 01	g NOP	35 01
RCL 4	34 04	g NOP	35 01	g NOP	35 01
+	61	g NOP	35 01	g NOP	35 01

R <sub>1</sub>	x <sub>1</sub>	R <sub>4</sub>	y <sub>2</sub>	R <sub>7</sub>
R <sub>2</sub>	y <sub>1</sub>	R <sub>5</sub>		R <sub>8</sub>
R <sub>3</sub>	x <sub>2</sub>	R <sub>6</sub>		R <sub>9</sub> Used

## STATIC EQUILIBRIUM OF A POINT

KEYS	CODE	KEYS	CODE	KEYS	CODE
LBL	23	RTN	24	x	71
A	11	LBL	23	RCL 4	34 04
1	01	C	13	RCL 1	34 01
f <sup>-1</sup>	32	f	31	x	71
R→P	01	R→P	01	—	51
STO 1	33 01	g R↓	35 08	RCL 4	34 04
g R↓	35 08	g R↓	35 08	RCL 5	34 05
STO 2	33 02	f	31	x	71
g R↓	35 08	R→P	01	RCL 3	34 03
STO	33	g x↔y	35 07	RCL 6	34 06
x	71	g R↑	35 09	x	71
1	01	B	12	—	51
STO	33	RTN	24	÷	81
x	71	LBL	23	RTN	24
2	02	D	14	g NOP	35 01
RTN	24	RCL 1	34 01	g NOP	35 01
LBL	23	RCL 6	34 06	g NOP	35 01
B	12	x	71	g NOP	35 01
f	31	RCL 2	34 02	g NOP	35 01
COS	05	RCL 5	34 05	g NOP	35 01
STO 5	33 05	x	71	g NOP	35 01
CLX	44	—	51	g NOP	35 01
g LST X	35 00	RCL 4	34 04	g NOP	35 01
f	31	RCL 5	34 05	g NOP	35 01
SIN	04	x	71	g NOP	35 01
STO 6	33 06	RCL 3	34 03	g NOP	35 01
g R↓	35 08	RCL 6	34 06	g NOP	35 01
f	31	x	71	g NOP	35 01
COS	05	—	51	g NOP	35 01
STO 3	33 03	÷	81	g NOP	35 01
g LST X	35 00	RTN	24	g NOP	35 01
f	31	LBL	23	g NOP	35 01
SIN	04	E	15	g NOP	35 01
STO 4	33 04	RCL 3	34 03	g NOP	35 01
g LST X	35 00	RCL 2	34 02	g NOP	35 01

<b>R<sub>1</sub></b>	F cos θ	<b>R<sub>4</sub></b>	sin φ <sub>1</sub>	<b>R<sub>7</sub></b>
<b>R<sub>2</sub></b>	F sin θ	<b>R<sub>5</sub></b>	cos φ <sub>2</sub>	<b>R<sub>8</sub></b>
<b>R<sub>3</sub></b>	cos φ <sub>1</sub>	<b>R<sub>6</sub></b>	sin φ <sub>2</sub>	<b>R<sub>9</sub></b> Used

## STATIC EQUILIBRIUM OF A RIGID BODY

KEYS	CODE	KEYS	CODE	KEYS	CODE
0	00	SF1	51	g x↔y	35 07
STO 1	33 01	RTN	24	RTN	24
STO 2	33 02	LBL	23	LBL	23
STO 3	33 03	C	13	D	14
f <sup>-1</sup>	32	STO	33	RCL 4	34 04
SF1	51	+	61	RCL 5	34 05
RTN	24	1	01	—	51
LBL	23	CLX	44	RCL 1	34 01
B	12	RTN	24	+	61
f <sup>-1</sup>	32	LBL	23	CHS	42
R→P	01	D	14	RTN	24
STO	33	f	31	LBL	23
+	61	TF1	61	E	15
2	02	f <sup>-1</sup>	32	f	31
g R↓	35 08	R→P	01	SF1	51
STO	33	STO 4	33 04	RTN	24
+	61	g x↔y	35 07	g NOP	35 01
3	03	STO 5	33 05	g NOP	35 01
g R↓	35 08	RCL 3	34 03	g NOP	35 01
f	31	CHS	42	g NOP	35 01
TF1	61	STO	33	g NOP	35 01
f <sup>-1</sup>	32	x	71	g NOP	35 01
R→P	01	4	04	g NOP	35 01
g R↓	35 08	RCL 2	34 02	g NOP	35 01
x	71	CHS	42	g NOP	35 01
STO 4	33 04	STO	33	g NOP	35 01
g R↓	35 08	x	71	g NOP	35 01
x	71	5	05	g NOP	35 01
RCL 4	34 04	f	31	g NOP	35 01
—	51	R→P	01	g NOP	35 01
STO	33	f <sup>-1</sup>	32		
+	61	SF1	51		
1	01	RTN	24		
CLX	44	LBL	23		
f <sup>-1</sup>	32	D	14		

<b>R<sub>1</sub></b>	<b>M<sub>T</sub></b>	<b>R<sub>4</sub></b>	<b>xΣF<sub>y</sub></b>	<b>R<sub>7</sub></b>
<b>R<sub>2</sub></b>	<b>ΣF<sub>x</sub></b>	<b>R<sub>5</sub></b>	<b>yΣF<sub>x</sub></b>	<b>R<sub>8</sub></b>
<b>R<sub>3</sub></b>	<b>ΣF<sub>y</sub></b>	<b>R<sub>6</sub></b>		<b>R<sub>9</sub></b> Used

## PROPERTIES OF RECTANGULAR SECTIONS

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 1	33 01	C	13	RTN	24
0	00	STO 3	33 03	LBL	23
g x $\neq$ y	35 21	0	00	E	15
RTN	24	g x $\neq$ y	35 21	RCL 2	34 02
RTN	24	RTN	24	RCL 3	34 03
RCL 3	34 03	RTN	24	x	71
3	03	RCL 1	34 01	RTN	24
g	35	1	01	g NOP	35 01
y $^x$	05	2	02	g NOP	35 01
RCL 2	34 02	x	71	g NOP	35 01
x	71	RCL 2	34 02	g NOP	35 01
1	01	$\div$	81	g NOP	35 01
2	02	3	03	g NOP	35 01
$\div$	81	g	35	g NOP	35 01
STO 1	33 01	$^1/x$	04	g NOP	35 01
RTN	24	g	35	g NOP	35 01
LBL	23	y $^x$	05	g NOP	35 01
B	12	STO 3	33 03	g NOP	35 01
STO 2	33 02	RTN	24	g NOP	35 01
0	00	LBL	23	g NOP	35 01
g x $\neq$ y	35 21	D	14	g NOP	35 01
RTN	24	RCL 2	34 02	g NOP	35 01
RTN	24	$\uparrow$	41	g NOP	35 01
RCL 1	34 01	x	71	g NOP	35 01
1	01	RCL 3	34 03	g NOP	35 01
2	02	$\uparrow$	41	g NOP	35 01
x	71	x	71	g NOP	35 01
RCL 3	34 03	+	61	g NOP	35 01
3	03	RCL 2	34 02	g NOP	35 01
g	35	x	71	g NOP	35 01
y $^x$	05	RCL 3	34 03	g NOP	35 01
$\div$	81	x	71	g NOP	35 01
STO 2	33 02	1	01	g NOP	35 01
RTN	24	2	02	g NOP	35 01
LBL	23	$\div$	81	g NOP	35 01

<b>R<sub>1</sub></b>	I	<b>R<sub>4</sub></b>	<b>R<sub>7</sub></b>
<b>R<sub>2</sub></b>	b	<b>R<sub>5</sub></b>	<b>R<sub>8</sub></b>
<b>R<sub>3</sub></b>	h	<b>R<sub>6</sub></b>	<b>R<sub>9</sub></b> Used

## PROPERTIES OF CIRCULAR SECTIONS

KEYS	CODE	KEYS	CODE	KEYS	CODE
LBL	23	C	13	x	71
A	11	2	02	RTN	24
0	00	x	71	LBL	23
g x=y	35 23	0	00	E	15
GTO	22	g x $\neq$ y	35 07	0	00
1	01	g x $\neq$ y	35 21	g x=y	35 23
g R $\downarrow$	35 08	B	12	GTO	22
2	02	RTN	24	3	03
x	71	RCL 1	34 01	g R $\downarrow$	35 08
D	14	2	02	4	04
RTN	24	$\div$	81	x	71
LBL	23	RTN	24	g	35
1	01	LBL	23	$\pi$	02
RCL 1	34 01	D	14	$\div$	81
4	04	0	00	f	31
g	35	g x=y	35 23	$\sqrt{x}$	09
y <sup>x</sup>	05	GTO	22	B	12
6	06	2	02	RTN	24
4	04	g R $\downarrow$	35 08	LBL	23
$\div$	81	3	03	3	03
g	35	2	02	RCL 1	34 01
$\pi$	02	x	71	$\uparrow$	41
x	71	g	35	x	71
RTN	24	$\pi$	02	g	35
LBL	23	$\div$	81	$\pi$	02
B	12	f	31	x	71
0	00	$\sqrt{x}$	09	4	04
g x=y	35 23	f	31	$\div$	81
RCL 1	34 01	$\sqrt{x}$	09	RTN	24
RTN	24	B	12	g NOP	35 01
g R $\downarrow$	35 08	RTN	24		
STO 1	33 01	LBL	23		
0	00	2	02		
RTN	24	A	11		
LBL	23	2	02		

$R_1$	d	$R_4$	$R_7$
$R_2$		$R_5$	$R_8$
$R_3$		$R_6$	$R_9$ Used

## PROPERTIES OF ANNULAR SECTIONS

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 3	33 03	LBL	23	-	51
0	00	C	13	g	35
g x $\neq$ y	35 21	STO 2	33 02	$\pi$	02
RTN	24	0	00	x	71
RTN	24	g x $\neq$ y	35 21	3	03
D	14	RTN	24	2	02
2	02	RTN	24	$\div$	81
$\div$	81	RCL 1	34 01	RTN	24
STO 3	33 03	4	04	LBL	23
RTN	24	g	35	E	15
LBL	23	$y^x$	05	RCL 1	34 01
B	12	RCL 3	34 03	$\uparrow$	41
STO 1	33 01	6	06	x	71
0	00	4	04	RCL 2	34 02
g x $\neq$ y	35 21	x	71	$\uparrow$	41
RTN	24	g	35	x	71
RTN	24	$\pi$	02	-	51
RCL 3	34 03	$\div$	81	g	35
6	06	-	51	$\pi$	02
4	04	f	31	x	71
x	71	$\sqrt{x}$	09	4	04
g	35	f	31	$\div$	81
$\pi$	02	$\sqrt{x}$	09	RTN	24
$\div$	81	STO 2	33 02	g NOP	35 01
RCL 2	34 02	RTN	24	g NOP	35 01
4	04	LBL	23	g NOP	35 01
g	35	D	14	g NOP	35 01
$y^x$	05	RCL 1	34 01	g NOP	35 01
+	61	4	04	g NOP	35 01
f	31	g	35	g NOP	35 01
$\sqrt{x}$	09	$y^x$	05	g NOP	35 01
f	31	RCL 2	34 02		
$\sqrt{x}$	09	4	04		
STO 1	33 01	g	35		
RTN	24	$y^x$	05		

$R_1$ d <sub>o</sub>	$R_4$	$R_7$
$R_2$ d <sub>i</sub>	$R_5$	$R_8$
$R_3$ I	$R_6$	$R_9$ Used

## COMPOSITE SECTION PROPERTIES I

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 1	33 01	+	61	D	14
g R↓	35 08	x	71	RCL 6	34 06
STO 2	33 02	STO	33	RTN	24
RTN	24	+	61	LBL	23
LBL	23	8	08	D	14
B	12	CLX	44	RCL 5	34 05
STO 3	33 03	RCL 1	34 01	RCL 6	34 06
g x↔y	35 07	x	71	÷	81
x	71	STO	33	RTN	24
g LST X	35 00	+	61	LBL	23
LBL	23	4	04	D	14
1	01	RCL 2	34 02	RCL 4	34 04
↑	41	g R↑	35 09	RCL 6	34 06
x	71	x	71	÷	81
1	01	STO	33	RTN	24
2	02	+	61	LBL	23
÷	81	5	05	E	15
RCL 2	34 02	x	71	RCL 8	34 08
↑	41	g R↑	35 09	RTN	24
x	71	÷	81	LBL	23
+	61	STO	33	E	15
x	71	9	09	RCL 7	34 07
STO	33	g R↑	35 09	RTN	24
+	61	STO	33	LBL	23
7	07	+	61	E	15
CLX	44	9	09	RCL 7	34 07
RCL 3	34 03	g R↑	35 09	RCL 8	34 08
↑	41	STO	33	+	61
x	71	+	61	RTN	24
1	01	6	06	g NOP	35 01
2	02	RTN	24		
÷	81	LBL	23		
RCL 1	34 01	C	13		
↑	41	0	00		
x	71	STO 3	33 03		
		GTO	22		
		1	01		
		LBL	23		

<b>R<sub>1</sub></b>	$x_{0i}$	<b>R<sub>4</sub></b>	$\sum x_{0i}A$	<b>R<sub>7</sub></b>	$\sum I_x$
<b>R<sub>2</sub></b>	$y_{0i}$	<b>R<sub>5</sub></b>	$\sum y_{0i}A$	<b>R<sub>8</sub></b>	$\sum I_y$
<b>R<sub>3</sub></b>	$\Delta x_i$	<b>R<sub>6</sub></b>	$\sum A$	<b>R<sub>9</sub></b>	$I_{xy}$

## COMPOSITE SECTION PROPERTIES II

KEYS	CODE	KEYS	CODE	KEYS	CODE
RCL 3	34 03	x	71	f	31
2	02	RCL 6	34 06	SIN	04
x	71	÷	81	RCL 3	34 03
RCL 2	34 02	-	51	x	71
RCL 1	34 01	STO 3	33 03	STO 8	33 08
-	51	RCL 7	34 07	+	61
0	00	RCL 5	34 05	STO 5	33 05
g x=y	35 23	↑	41	RTN	24
STO 4	33 04	x	71	LBL	23
RTN	24	RCL 6	34 06	C	13
g R↓	35 08	÷	81	RCL 6	34 06
÷	81	-	51	RCL 1	34 01
f <sup>-1</sup>	32	STO 1	33 01	x	71
TAN	06	RTN	24	RCL 7	34 07
2	02	LBL	23	RCL 2	34 02
÷	81	C	13	x	71
STO 4	33 04	RCL 4	34 04	+	61
RTN	24	1	01	RCL 8	34 08
LBL	23	f <sup>-1</sup>	32	-	51
A	11	R→P	01	STO 6	33 06
RCL 8	34 08	↑	41	RTN	24
RCL 4	34 04	x	71	LBL	23
↑	41	STO 6	33 06	D	14
x	71	RCL 2	34 02	RCL 5	34 05
RCL 6	34 06	x	71	RCL 6	34 06
÷	81	g x↔y	35 07	+	61
-	51	↑	41	STO 7	33 07
STO 2	33 02	x	71	RTN	24
RTN	24	STO 7	33 07	g NOP	35 01
LBL	23	RCL 1	34 01	g NOP	35 01
A	11	x	71		
RCL	34	+	61		
9	09	RCL 4	34 04		
RCL 4	34 04	2	02		
RCL 5	34 05	x	71		

<b>R<sub>1</sub></b>	I <sub><math>\bar{x}</math></sub>	<b>R<sub>4</sub></b>	$\phi$	<b>R<sub>7</sub></b>	$\sin^2 \phi$
<b>R<sub>2</sub></b>	I <sub><math>\bar{y}</math></sub>	<b>R<sub>5</sub></b>	I <sub><math>\bar{y}\phi</math></sub>	<b>R<sub>8</sub></b>	$I_{\bar{x}\bar{y}} \sin 2\phi$
<b>R<sub>3</sub></b>	I <sub><math>\bar{x}\bar{y}</math></sub>	<b>R<sub>6</sub></b>	$I_{\bar{x}\phi}, \cos^2 \phi$	<b>R<sub>9</sub></b>	Used

**BENDING STRESS IN BEAMS OR TORSIONAL  
SHEAR STRESS IN CIRCULAR SHAFTS**

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 7	33 07	x	71	g NOP	35 01
0	00	RCL 2	34 02	g NOP	35 01
g x≠y	35 21	÷	81	g NOP	35 01
RTN	24	STO 6	33 06	g NOP	35 01
RTN	24	RTN	24	g NOP	35 01
RCL 2	34 02	LBL	23	g NOP	35 01
RCL 6	34 06	D	14	g NOP	35 01
x	71	STO 1	33 01	g NOP	35 01
RCL 1	34 01	0	00	g NOP	35 01
÷	81	g x≠y	35 21	g NOP	35 01
STO 7	33 07	RTN	24	g NOP	35 01
RTN	24	RTN	24	g NOP	35 01
LBL	23	RCL 2	34 02	g NOP	35 01
B	12	RCL 6	34 06	g NOP	35 01
STO 2	33 02	x	71	g NOP	35 01
0	00	RCL 7	34 07	g NOP	35 01
g x≠y	35 21	÷	81	g NOP	35 01
RTN	24	STO 1	33 01	g NOP	35 01
RTN	24	RTN	24	g NOP	35 01
RCL 7	34 07	g NOP	35 01	g NOP	35 01
RCL 1	34 01	g NOP	35 01	g NOP	35 01
x	71	g NOP	35 01	g NOP	35 01
RCL 6	34 06	g NOP	35 01	g NOP	35 01
÷	81	g NOP	35 01	g NOP	35 01
STO 2	33 02	g NOP	35 01	g NOP	35 01
RTN	24	g NOP	35 01	g NOP	35 01
LBL	23	g NOP	35 01	g NOP	35 01
C	13	g NOP	35 01	g NOP	35 01
STO 6	33 06	g NOP	35 01	g NOP	35 01
0	00	g NOP	35 01	g NOP	35 01
g x≠y	35 21	g NOP	35 01	g NOP	35 01
RTN	24	g NOP	35 01		
RTN	24	g NOP	35 01		
RCL 7	34 07	g NOP	35 01		
RCL 1	34 01	g NOP	35 01		

<b>R<sub>1</sub></b>	I(J)	<b>R<sub>4</sub></b>		<b>R<sub>7</sub></b>	s(s <sub>s</sub> )
<b>R<sub>2</sub></b>	M(T)	<b>R<sub>5</sub></b>		<b>R<sub>8</sub></b>	
<b>R<sub>3</sub></b>		<b>R<sub>6</sub></b>	v(r)	<b>R<sub>9</sub></b>	Used

**LINEAR OR ANGULAR DEFORMATION  
OF A SHAFT**

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 4	33 04	RTN	24	RCL 3	34 03
0	00	RTN	24	x	71
g x≠y	35 21	RCL 4	34 04	RCL 4	34 04
RTN	24	RCL 1	34 01	RCL 1	34 01
RTN	24	x	71	x	71
RCL 2	34 02	RCL 5	34 05	÷	81
RCL 3	34 03	x	71	STO 5	33 05
x	71	RCL 2	34 02	RTN	24
RCL 1	34 01	÷	81	g NOP	35 01
÷	81	STO 3	33 03	g NOP	35 01
RCL 5	34 05	RTN	24	g NOP	35 01
÷	81	LBL	23	g NOP	35 01
STO 4	33 04	D	14	g NOP	35 01
RTN	24	STO 1	33 01	g NOP	35 01
LBL	23	0	00	g NOP	35 01
B	12	g x≠y	35 21	g NOP	35 01
STO 2	33 02	RTN	24	g NOP	35 01
0	00	RTN	24	g NOP	35 01
g x≠y	35 21	RCL 2	34 02	g NOP	35 01
RTN	24	RCL 3	34 03	g NOP	35 01
RTN	24	x	71	g NOP	35 01
RCL 4	34 04	RCL 4	34 04	g NOP	35 01
RCL 1	34 01	RCL 5	34 05	g NOP	35 01
x	71	x	71	g NOP	35 01
RCL 5	34 05	÷	81	g NOP	35 01
x	71	STO 1	33 01	g NOP	35 01
RCL 3	34 03	RTN	24	g NOP	35 01
÷	81	LBL	23	g NOP	35 01
STO 2	33 02	E	15	g NOP	35 01
RTN	24	STO 5	33 05	g NOP	35 01
LBL	23	0	00	g NOP	35 01
C	13	g x≠y	35 21	g NOP	35 01
STO 3	33 03	RTN	24	g NOP	35 01
0	00	RTN	24	g NOP	35 01
g x≠y	35 21	RCL 2	34 02	g NOP	35 01

<b>R<sub>1</sub></b>	A(J)	<b>R<sub>4</sub></b>	$\Delta\ell(\theta)$	<b>R<sub>7</sub></b>
<b>R<sub>2</sub></b>	P(T)	<b>R<sub>5</sub></b>	E(G)	<b>R<sub>8</sub></b>
<b>R<sub>3</sub></b>	$\ell$	<b>R<sub>6</sub></b>		<b>R<sub>9</sub></b> Used

## THIN-WALLED PRESSURE VESSELS

KEYS	CODE	KEYS	CODE	KEYS	CODE
LBL	23	x	71	g NOP	35 01
A	11	B	12	g NOP	35 01
2	02	2	02	g NOP	35 01
÷	81	÷	81	g NOP	35 01
STO 1	33 01	RTN	24	g NOP	35 01
0	00	LBL	23	g NOP	35 01
g x≠y	35 21	D	14	g NOP	35 01
0	00	STO 3	33 03	g NOP	35 01
RTN	24	0	00	g NOP	35 01
RCL 2	34 02	g x≠y	35 21	g NOP	35 01
RCL 4	34 04	0	00	g NOP	35 01
x	71	RTN	24	g NOP	35 01
RCL 3	34 03	RCL 2	34 02	g NOP	35 01
÷	81	RCL 4	34 04	g NOP	35 01
STO 1	33 01	x	71	g NOP	35 01
2	02	RCL 1	34 01	g NOP	35 01
x	71	÷	81	g NOP	35 01
RTN	24	STO 3	33 03	g NOP	35 01
LBL	23	RTN	24	g NOP	35 01
B	12	LBL	23	g NOP	35 01
STO 2	33 02	E	15	g NOP	35 01
0	00	STO 4	33 04	g NOP	35 01
g x≠y	35 21	0	00	g NOP	35 01
0	00	g x≠y	35 21	g NOP	35 01
RTN	24	0	00	g NOP	35 01
RCL 1	34 01	RTN	24	g NOP	35 01
RCL 3	34 03	RCL 3	34 03	g NOP	35 01
x	71	RCL 1	34 01	g NOP	35 01
RCL 4	34 04	x	71	g NOP	35 01
÷	81	RCL 2	34 02	g NOP	35 01
STO 2	33 02	÷	81	g NOP	35 01
RTN	24	STO 4	33 04	g NOP	35 01
LBL	23	RTN	24	g NOP	35 01
C	13	g NOP	35 01	g NOP	35 01
2	02	g NOP	35 01	g NOP	35 01

<b>R<sub>1</sub></b>	r	<b>R<sub>4</sub></b>	t	<b>R<sub>7</sub></b>
<b>R<sub>2</sub></b>	s <sub>c</sub>	<b>R<sub>5</sub></b>		<b>R<sub>8</sub></b>
<b>R<sub>3</sub></b>	P	<b>R<sub>6</sub></b>		<b>R<sub>9</sub></b> Used

## STRESS IN THICK-WALLED CYLINDERS

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 1	33 01	RCL 4	34 04	g NOP	35 01
g R↓	35 08	↑	41	g NOP	35 01
STO 2	33 02	x	71	g NOP	35 01
RTN	24	x	71	g NOP	35 01
LBL	23	RCL 1	34 01	g NOP	35 01
B	12	RCL 3	34 03	g NOP	35 01
STO 3	33 03	—	51	g NOP	35 01
g R↓	35 08	x	71	g NOP	35 01
STO 4	33 04	RCL 5	34 05	g NOP	35 01
RTN	24	↑	41	g NOP	35 01
LBL	23	x	71	g NOP	35 01
C	13	÷	81	g NOP	35 01
STO 5	33 05	RCL 4	34 04	g NOP	35 01
RCL 2	34 02	↑	41	g NOP	35 01
↑	41	x	71	g NOP	35 01
x	71	RCL 2	34 02	g NOP	35 01
RCL 1	34 01	f <sup>-1</sup>	32	g NOP	35 01
x	71	√x	09	g NOP	35 01
RCL 4	34 04	—	51	g NOP	35 01
↑	41	÷	81	g NOP	35 01
x	71	—	51	g NOP	35 01
RCL 3	34 03	STO 6	33 06	g NOP	35 01
x	71	g LST X	35 00	g NOP	35 01
—	51	2	02	g NOP	35 01
RCL 4	34 04	x	71	g NOP	35 01
↑	41	+	61	g NOP	35 01
x	71	STO 7	33 07	g NOP	35 01
RCL 2	34 02	RCL 6	34 06	g NOP	35 01
↑	41	RTN	24	g NOP	35 01
x	71	LBL	23	g NOP	35 01
—	51	D	14	g NOP	35 01
÷	81	RCL 7	34 07	g NOP	35 01
RCL 2	34 02	RTN	24	g NOP	35 01
↑	41	g NOP	35 01	g NOP	35 01
x	71	g NOP	35 01	g NOP	35 01

<b>R<sub>1</sub></b>	<b>P<sub>i</sub></b>	<b>R<sub>4</sub></b>	<b>r<sub>o</sub></b>	<b>R<sub>7</sub></b>	<b>s<sub>t</sub></b>
<b>R<sub>2</sub></b>	<b>r<sub>i</sub></b>	<b>R<sub>5</sub></b>	<b>r</b>	<b>R<sub>8</sub></b>	
<b>R<sub>3</sub></b>	<b>P<sub>o</sub></b>	<b>R<sub>6</sub></b>	<b>s<sub>r</sub></b>	<b>R<sub>9</sub></b>	

## INTERFERENCE FITS – PRESSURE

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 3	33 03	↑	41	RCL 5	34 05
g R↓	35 08	x	71	÷	81
STO 2	33 02	RCL 2	34 02	+	61
g R↓	35 08	↑	41	RCL 2	34 02
STO 1	33 01	x	71	×	71
RTN	24	+	61	RTN	24
LBL	23	g LST X	35 00	g NOP	35 01
B	12	RCL 1	34 01	g NOP	35 01
STO 7	33 07	↑	41	g NOP	35 01
g R↓	35 08	x	71	g NOP	35 01
STO 6	33 06	—	51	g NOP	35 01
g R↓	35 08	÷	81	g NOP	35 01
STO 5	33 05	RCL 7	34 07	g NOP	35 01
g R↓	35 08	÷	81	g NOP	35 01
STO 4	33 04	RCL 2	34 02	g NOP	35 01
RTN	24	↑	41	g NOP	35 01
LBL	23	x	71	g NOP	35 01
C	13	RCL 3	34 03	g NOP	35 01
STO 8	33 08	↑	41	g NOP	35 01
E	15	x	71	g NOP	35 01
RCL 8	34 08	+	61	g NOP	35 01
g x↔y	35 07	g LST X	35 00	g NOP	35 01
÷	81	RCL 2	34 02	g NOP	35 01
STO 8	33 08	f <sup>-1</sup>	32	g NOP	35 01
RTN	24	√x	09	g NOP	35 01
LBL	23	—	51	g NOP	35 01
D	14	÷	81	g NOP	35 01
STO 8	33 08	RCL 5	34 05	g NOP	35 01
E	15	÷	81	g NOP	35 01
RCL 8	34 08	+	61	g NOP	35 01
x	71	RCL 6	34 06		
RTN	24	RCL 7	34 07		
LBL	23	÷	81		
E	15	—	51		
RCL 1	34 01	RCL 4	34 04		

$R_1$	$d_i$	$R_4$	$\mu_o$	$R_7$	$E_i$
$R_2$	$d_c$	$R_5$	$E_o$	$R_8$	$P_c, \delta$
$R_3$	$d_o$	$R_6$	$\mu_i$	$R_9$	

## INTERFERENCE FITS – STRESS

KEYS	CODE	KEYS	CODE	KEYS	CODE
LBL	23	RTN	24	E	15
A	11	LBL	23	RCL 8	34 08
0	00	C	13	x	71
RCL 3	34 03	0	00	2	02
E	15	RCL 3	34 03	x	71
RCL 8	34 08	E	15	RTN	24
x	71	RCL 8	34 08	LBL	23
2	02	x	71	E	15
x	71	2	02	$f^{-1}$	32
RTN	24	x	71	$\sqrt{x}$	09
LBL	23	RTN	24	$g \ x \leftrightarrow y$	35 07
A	11	LBL	23	$f^{-1}$	32
RCL 3	34 03	C	13	$\sqrt{x}$	09
RCL 3	34 03	RCL 3	34 03	RCL 2	34 02
E	15	RCL 3	34 03	$f^{-1}$	32
RCL 8	34 08	E	15	$\sqrt{x}$	09
x	71	RCL 4	34 04	+	61
RTN	24	+ RCL 8	34 08	$g \ x \leftrightarrow y$	35 07
LBL	23	x	71	$g \ LST \ X$	35 00
B	12	RTN	24	-	51
RCL 1	34 01	LBL	23	$\div$	81
RCL 1	34 01	D	14	RTN	24
E	15	RCL 1	34 01	$g \ NOP$	35 01
RCL 8	34 08	RCL 1	34 01	$g \ NOP$	35 01
x	71	E	15	$g \ NOP$	35 01
RTN	24	RCL 6	34 06	$g \ NOP$	35 01
LBL	23	+	61	$g \ NOP$	35 01
B	12	RCL 8	34 08	$g \ NOP$	35 01
0	00	x	71	$g \ NOP$	35 01
RCL 1	34 01	RTN	24	$g \ NOP$	35 01
E	15	LBL	23	$g \ NOP$	35 01
RCL 8	34 08	D	14		
x	71	0	00		
2	02	RCL 1	34 01		
x	71				

$R_1$	$d_i$	$R_4$	$\mu_o$	$R_7$	$E_i$
$R_2$	$d_c$	$R_5$	$E_o$	$R_8$	$P_c$
$R_3$	$d_o$	$R_6$	$\mu_i$	$R_9$	

## MOHR CIRCLE FOR STRESS

KEYS	CODE	KEYS	CODE	KEYS	CODE
LBL	23	+	61	RTN	24
A	11	STO 7	33 07	LBL	23
STO 1	33 01	RCL 1	34 01	E	15
RTN	24	RCL 2	34 02	RCL 5	34 05
LBL	23	+	61	RTN	24
B	12	-	51	g NOP	35 01
STO 2	33 02	CHS	42	g NOP	35 01
RTN	24	RTN	24	g NOP	35 01
LBL	23	LBL	23	g NOP	35 01
C	13	D	14	g NOP	35 01
STO 3	33 03	RCL 7	34 07	g NOP	35 01
RTN	24	RTN	24	g NOP	35 01
LBL	23	LBL	23	g NOP	35 01
D	14	D	14	g NOP	35 01
RCL 1	34 01	RCL 3	34 03	g NOP	35 01
RCL 2	34 02	RCL 4	34 04	g NOP	35 01
-	51	÷	81	g NOP	35 01
2	02	f <sup>-1</sup>	32	g NOP	35 01
÷	81	TAN	06	g NOP	35 01
STO 4	33 04	g LST X	35 00	g NOP	35 01
↑	41	g	35	g NOP	35 01
x	71	<sup>1</sup> /x	04	g NOP	35 01
RCL 3	34 03	CHS	42	g NOP	35 01
↑	41	f <sup>-1</sup>	32	g NOP	35 01
x	71	TAN	06	g NOP	35 01
+	61	2	02	g NOP	35 01
f	31	÷	81	g NOP	35 01
$\sqrt{x}$	09	STO 5	33 05	g NOP	35 01
STO 6	33 06	g R↓	35 08	g NOP	35 01
CHS	42	2	02	g NOP	35 01
RCL 1	34 01	÷	81	g NOP	35 01
RCL 2	34 02	RTN	24	g NOP	35 01
+	61	LBL	23	g NOP	35 01
2	02	E	15	g NOP	35 01
÷	81	RCL 6	34 06		

<b>R<sub>1</sub></b>	s <sub>x</sub>	<b>R<sub>4</sub></b>	(s <sub>x</sub> - s <sub>y</sub> )/2	<b>R<sub>7</sub></b>	s <sub>2</sub>
<b>R<sub>2</sub></b>	s <sub>y</sub>	<b>R<sub>5</sub></b>	$\theta_s$	<b>R<sub>8</sub></b>	
<b>R<sub>3</sub></b>	s <sub>xy</sub>	<b>R<sub>6</sub></b>	s <sub>smax</sub>	<b>R<sub>9</sub></b>	Used

## SODERBERG'S EQUATION FOR FATIGUE

KEYS	CODE	KEYS	CODE	KEYS	CODE
LBL	23	STO 4	33 04	STO 6	33 06
A	11	RTN	24	2	02
STO 7	33 07	LBL	23	÷	81
g R↓	35 08	C	13	R/S	84
STO 1	33 01	STO 5	33 05	RCL 4	34 04
g R↓	35 08	R/S	84	RCL 5	34 05
STO 2	33 02	RCL 3	34 03	+	61
g R↓	35 08	RCL 1	34 01	RCL 4	34 04
STO 3	33 03	x	71	RCL 5	34 05
RTN	24	RCL 2	34 02	—	51
LBL	23	÷	81	RCL 3	34 03
B	12	1	01	x	71
STO 4	33 04	+	61	RCL 1	34 01
R/S	84	RCL 4	34 04	x	71
RCL 6	34 06	x	71	RCL 2	34 02
RCL 3	34 03	RCL 6	34 06	÷	81
x	71	RCL 3	34 03	+	61
RCL 7	34 07	x	71	RCL 3	34 03
÷	81	RCL 7	34 07	RCL 7	34 07
RCL 5	34 05	÷	81	÷	81
—	51	—	51	÷	81
RCL 5	34 05	RCL 3	34 03	STO 6	33 06
RCL 3	34 03	RCL 1	34 01	2	02
RCL 1	34 01	x	71	÷	81
x	71	RCL 2	34 02	RTN	24
RCL 2	34 02	÷	81	g NOP	35 01
÷	81	1	01	g NOP	35 01
x	71	—	51	g NOP	35 01
g LST X	35 00	÷	81	g NOP	35 01
1	01	STO 5	33 05	g NOP	35 01
+	61	RTN	24		
g R↓	35 08	LBL	23		
+	61	D	14		
g R↑	35 09	2	02		
÷	81	x	71		

$R_1$	$k$	$R_4$	$P_{max}$	$R_7$	$FS$
$R_2$	$s_e$	$R_5$	$P_{min}$	$R_8$	
$R_3$	$s_{yp}$	$R_6$	$2A$	$R_9$	

**CIRCULAR PLATES WITH  
SIMPLY SUPPORTED EDGES**

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 1	33 01	RCL 4	34 04	D	14
g R↓	35 08	÷	81	STO 6	33 06
STO 2	33 02	RTN	24	RCL 1	34 01
g R↓	35 08	LBL	23	2	02
STO 3	33 03	C	13	÷	81
3	03	RCL 1	34 01	4	04
g	35	RCL 3	34 03	g	35
y <sup>x</sup>	05	÷	81	y <sup>x</sup>	05
x	71	f	31	x	71
3	03	LN	07	5	05
÷	81	•	83	GTO	22
1	01	4	04	1	01
RCL 2	34 02	8	08	LBL	23
—	51	5	05	E	15
÷	81	x	71	RCL 1	34 01
STO 4	33 04	•	83	RCL 3	34 03
RTN	24	5	05	÷	81
LBL	23	2	02	↑	41
B	12	+	61	x	71
STO 5	33 05	RCL 2	34 02	8	08
RCL 1	34 01	1	01	÷	81
↑	41	+	61	3	03
x	71	x	71	x	71
x	71	•	83	3	03
4	04	4	04	RCL 2	34 02
÷	81	8	08	+	61
g	35	+	61	x	71
π	02	RCL 5	34 05	RCL 6	34 06
÷	81	x	71	x	71
3	03	RCL 3	34 03	RTN	24
LBL	23	↑	41		
1	01	x	71		
RCL 2	34 02	÷	81		
+	61	RTN	24		
x	71	LBL	23		

<b>R<sub>1</sub></b>	r	<b>R<sub>4</sub></b>	D	<b>R<sub>7</sub></b>
<b>R<sub>2</sub></b>	μ	<b>R<sub>5</sub></b>	P	<b>R<sub>8</sub></b>
<b>R<sub>3</sub></b>	h	<b>R<sub>6</sub></b>	W	<b>R<sub>9</sub></b>

## CIRCULAR PLATES WITH FIXED EDGES

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 1	33 01	RTN	24	x	71
g R↓	35 08	LBL	23	6	06
STO 2	33 02	C	13	4	04
g R↓	35 08	RCL 1	34 01	÷	81
STO 3	33 03	RCL 3	34 03	RCL 4	34 04
3	03	÷	81	÷	81
g	35	f	31	RTN	24
y <sup>x</sup>	05	LN	07	LBL	23
x	71	·	83	E	15
1	01	4	04	RCL 1	34 01
2	02	8	08	↑	41
÷	81	5	05	x	71
1	01	x	71	3	03
RCL 2	34 02	·	83	x	71
↑	41	5	05	RCL 6	34 06
x	71	2	02	x	71
-	51	+	61	4	04
÷	81	1	01	÷	81
STO 4	33 04	RCL 2	34 02	RCL 3	34 03
RTN	24	+	61	↑	41
LBL	23	x	71	x	71
B	12	RCL 5	34 05	÷	81
STO 5	33 05	x	71	RTN	24
RCL 1	34 01	RCL 3	34 03	g NOP	35 01
↑	41	↑	41	g NOP	35 01
x	71	x	71	g NOP	35 01
x	71	÷	81	g NOP	35 01
1	01	RTN	24	g NOP	35 01
6	06	LBL	23	g NOP	35 01
÷	81	D	14	g NOP	35 01
RCL 4	34 04	STO 6	33 06	g NOP	35 01
÷	81	RCL 1	34 01	g NOP	35 01
g	35	4	04		
π	02	g	35		
÷	81	y <sup>x</sup>	05		

<b>R<sub>1</sub></b>	r	<b>R<sub>4</sub></b>	D	<b>R<sub>7</sub></b>
<b>R<sub>2</sub></b>	μ	<b>R<sub>5</sub></b>	P	<b>R<sub>8</sub></b>
<b>R<sub>3</sub></b>	h	<b>R<sub>6</sub></b>	W	<b>R<sub>9</sub></b>

## RECTANGULAR PLATES—INPUT

KEYS	CODE	KEYS	CODE	KEYS	CODE
LBL	23	CHS	42	.	83
A	11	.	83	0	00
x	71	0	00	7	07
STO 4	33 04	7	07	1	01
g LST X	35 00	9	09	4	04
↑	41	1	01	x	71
x	71	+	61	1	01
STO 3	33 03	STO	33	—	51
0	00	9	09	STO 8	33 08
RTN	24	RCL 5	34 05	0	00
LBL	23	.	83	RTN	24
B	12	2	02	g NOP	35 01
g x>y	35 24	5	05	g NOP	35 01
g x≤y	35 07	5	05	g NOP	35 01
g NOP	35 01	x	71	g NOP	35 01
÷	81	CHS	42	g NOP	35 01
STO 1	33 01	1	01	g NOP	35 01
g LST X	35 00	.	83	g NOP	35 01
↑	41	0	00	g NOP	35 01
x	71	0	00	g NOP	35 01
STO 2	33 02	9	09	g NOP	35 01
RCL 1	34 01	+	61	g NOP	35 01
4	04	STO 5	33 05	g NOP	35 01
x	71	RCL 1	34 01	g NOP	35 01
CHS	42	2	02	g NOP	35 01
4	04	.	83	g NOP	35 01
+	61	6	06	g NOP	35 01
f <sup>-1</sup>	32	2	02	g NOP	35 01
LN	07	4	04	g NOP	35 01
STO 5	33 05	9	09	g NOP	35 01
.	83	x	71	g NOP	35 01
0	00	CHS	42		
1	01	f <sup>-1</sup>	32		
8	08	LN	07		
x	71	8	08		

<b>R<sub>1</sub></b>	b/a	<b>R<sub>4</sub></b>	Eh	<b>R<sub>7</sub></b>	
<b>R<sub>2</sub></b>	a <sup>2</sup>	<b>R<sub>5</sub></b>	β	<b>R<sub>8</sub></b>	γ <sub>1</sub> '
<b>R<sub>3</sub></b>	h <sup>2</sup>	<b>R<sub>6</sub></b>		<b>R<sub>9</sub></b>	α'

**RECTANGULAR PLATE—SIMPLY SUPPORTED EDGES**

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 6	33 06	f	31	2	02
RCL 2	34 02	LN	07	1	01
x	71	x	71	LBL	23
RCL 4	34 04	RCL 8	34 08	1	01
÷	81	—	51	RCL 1	34 01
RCL 3	34 03	1	01	3	03
÷	81	•	83	g	35
.	83	5	05	y <sup>x</sup>	05
0	00	x	71	÷	81
8	08	RCL 6	34 06	1	01
0	00	x	71	+	61
7	07	g	35	GTO	22
RCL 8	34 08	π	02	2	02
9	09	LBL	23	LBL	23
.	83	2	02	D	14
6	06	÷	81	RCL 7	34 07
÷	81	RCL 3	34 03	.	83
—	51	÷	81	7	07
x	71	RTN	24	5	05
RTN	24	LBL	23	x	71
LBL	23	C	13	RCL 2	34 02
B	12	STO 7	33 07	x	71
1	01	•	83	1	01
+	61	1	01	•	83
g x↔y	35 07	4	04	6	06
2	02	2	02	1	01
g x↔y	35 07	x	71	GTO	22
÷	81	RCL 2	34 02	1	01
RCL 2	34 02	↑	41	g NOP	35 01
f	31	x	71	g NOP	35 01
√x	09	x	71		
x	71	RCL 4	34 04		
g	35	÷	81		
π	02	2	02		
÷	81	•	83		

<b>R<sub>1</sub></b>	b/a	<b>R<sub>4</sub></b>	Eh	<b>R<sub>7</sub></b>	W
<b>R<sub>2</sub></b>	a <sup>2</sup>	<b>R<sub>5</sub></b>	β	<b>R<sub>8</sub></b>	γ <sub>1</sub> '
<b>R<sub>3</sub></b>	h <sup>2</sup>	<b>R<sub>6</sub></b>	P	<b>R<sub>9</sub></b>	α'

## RECTANGULAR PLATES—FIXED EDGES

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 6	33 06	÷	81	÷	81
RCL 2	34 02	1	01	RTN	24
x	71	•	83	g NOP	35 01
RCL 3	34 03	0	00	g NOP	35 01
÷	81	5	05	g NOP	35 01
RCL 4	34 04	6	06	g NOP	35 01
÷	81	RCL 1	34 01	g NOP	35 01
RCL	34	5	05	g NOP	35 01
9	09	g	35	g NOP	35 01
x	71	y <sup>x</sup>	05	g NOP	35 01
RTN	24	÷	81	g NOP	35 01
LBL	23	1	01	g NOP	35 01
B	12	+	61	g NOP	35 01
RCL 6	34 06	÷	81	g NOP	35 01
RCL 5	34 05	RTN	24	g NOP	35 01
x	71	LBL	23	g NOP	35 01
RCL 3	34 03	D	14	g NOP	35 01
÷	81	RCL 7	34 07	g NOP	35 01
RTN	24	RCL 2	34 02	g NOP	35 01
LBL	23	x	71	g NOP	35 01
C	13	2	02	g NOP	35 01
STO 7	33 07	÷	81	g NOP	35 01
.	83	RCL 3	34 03	g NOP	35 01
0	00	÷	81	g NOP	35 01
2	02	•	83	g NOP	35 01
8	08	6	06	g NOP	35 01
4	04	2	02	g NOP	35 01
x	71	3	03	g NOP	35 01
RCL 2	34 02	RCL 1	34 01	g NOP	35 01
↑	41	6	06	g NOP	35 01
x	71	g	35		
x	71	y <sup>x</sup>	05		
RCL 4	34 04	÷	81		
÷	81	1	01		
RCL 3	34 03	+	61		

<b>R<sub>1</sub></b>	b/a	<b>R<sub>4</sub></b>	Eh	<b>R<sub>7</sub></b>	W
<b>R<sub>2</sub></b>	a <sup>2</sup>	<b>R<sub>5</sub></b>	β	<b>R<sub>8</sub></b>	γ <sub>1</sub> '
<b>R<sub>3</sub></b>	h <sup>2</sup>	<b>R<sub>6</sub></b>	P	<b>R<sub>9</sub></b>	α'

## CANTILEVER 1

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 2	33 02	2	02	g NOP	35 01
g R↓	35 08	x	71	g NOP	35 01
x	71	—	51	g NOP	35 01
STO 1	33 01	2	02	g NOP	35 01
RTN	24	÷	81	g NOP	35 01
LBL	23	RCL 1	34 01	g NOP	35 01
C	13	÷	81	g NOP	35 01
STO 3	33 03	RCL 3	34 03	g NOP	35 01
RTN	24	x	71	g NOP	35 01
LBL	23	RCL 4	34 04	g NOP	35 01
B	12	x	71	g NOP	35 01
STO 4	33 04	RTN	24	g NOP	35 01
RTN	24	LBL	23	g NOP	35 01
LBL	23	D	14	g NOP	35 01
D	14	RCL 4	34 04	g NOP	35 01
RCL 3	34 03	RCL 2	34 02	g NOP	35 01
RCL 4	34 04	—	51	g NOP	35 01
↑	41	RCL 3	34 03	g NOP	35 01
x	71	x	71	g NOP	35 01
x	71	RTN	24	g NOP	35 01
6	06	LBL	23	g NOP	35 01
÷	81	D	14	g NOP	35 01
RCL 1	34 01	RCL 3	34 03	g NOP	35 01
÷	81	RTN	24	g NOP	35 01
RCL 4	34 04	g NOP	35 01	g NOP	35 01
RCL 2	34 02	g NOP	35 01	g NOP	35 01
3	03	g NOP	35 01	g NOP	35 01
x	71	g NOP	35 01	g NOP	35 01
—	51	g NOP	35 01	g NOP	35 01
x	71	g NOP	35 01	g NOP	35 01
RTN	24	g NOP	35 01	g NOP	35 01
LBL	23	g NOP	35 01	g NOP	35 01
D	14	g NOP	35 01	g NOP	35 01
RCL 4	34 04	g NOP	35 01	g NOP	35 01
RCL 2	34 02	g NOP	35 01	g NOP	35 01

$R_1$	EI	$R_4$	x	$R_7$
$R_2$	$\ell$	$R_5$		$R_8$
$R_3$	P	$R_6$		$R_9$

## CANTILEVER 2

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 2	33 02	RCL 5	34 05	÷	81
g R↓	35 08	2	02	RTN	24
x	71	÷	81	LBL	23
STO 1	33 01	+	61	D	14
RTN	24	RCL 4	34 04	RCL 2	34 02
LBL	23	↑	41	RCL 4	34 04
C	13	x	71	2	02
STO 5	33 05	x	71	÷	81
g R↓	35 08	RCL 1	34 01	-	51
STO 6	33 06	÷	81	RCL 4	34 04
RTN	24	RTN	24	x	71
LBL	23	LBL	23	RCL 2	34 02
B	12	D	14	↑	41
STO 4	33 04	RCL 2	34 02	x	71
RTN	24	2	02	2	02
LBL	23	÷	81	÷	81
D	14	RCL 4	34 04	-	51
RCL 2	34 02	6	06	RCL 6	34 06
4	04	÷	81	x	71
x	71	-	51	RCL 5	34 05
RCL 4	34 04	RCL 4	34 04	+	61
-	51	RCL 2	34 02	RTN	24
RCL 4	34 04	↑	41	LBL	23
x	71	x	71	D	14
RCL 2	34 02	2	02	RCL 2	34 02
↑	41	÷	81	RCL 4	34 04
x	71	-	51	-	51
6	06	RCL 6	34 06	RCL 6	34 06
x	71	x	71	x	71
-	51	RCL 5	34 05	RTN	24
RCL 6	34 06	+	61		
x	71	RCL 4	34 04		
2	02	x	71		
4	04	RCL 1	34 01		
÷	81				

<b>R<sub>1</sub></b>	EI	<b>R<sub>4</sub></b>	x	<b>R<sub>7</sub></b>
<b>R<sub>2</sub></b>	l	<b>R<sub>5</sub></b>	M	<b>R<sub>8</sub></b>
<b>R<sub>3</sub></b>		<b>R<sub>6</sub></b>	W	<b>R<sub>9</sub></b>

## CANTILEVER 3

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 2	33 02	x	71	÷	81
g R↓	35 08	RCL 2	34 02	RTN	24
x	71	÷	81	LBL	23
STO 1	33 01	5	05	D	14
RTN	24	÷	81	RCL 3	34 03
LBL	23	—	51	↑	41
C	13	RCL 6	34 06	↑	41
STO 6	33 06	x	71	x	71
RTN	24	2	02	x	71
LBL	23	4	04	6	06
B	12	÷	81	÷	81
STO 4	33 04	RCL 1	34 01	RCL 2	34 02
RTN	24	÷	81	÷	81
LBL	23	RTN	24	RCL 6	34 06
D	14	LBL	23	x	71
RCL 2	34 02	D	14	CHS	42
5	05	RCL 3	34 03	RTN	24
÷	81	↑	41	LBL	23
RCL 4	34 04	x	71	D	14
—	51	↑	41	RCL 3	34 03
RCL 2	34 02	x	71	↑	41
↑	41	RCL 2	34 02	x	71
x	71	÷	81	2	02
g LST X	35 00	RCL 2	34 02	÷	81
x	71	↑	41	RCL 2	34 02
x	71	x	71	÷	81
RCL 2	34 02	g LST X	35 00	RCL 6	34 06
RCL 4	34 04	x	71	x	71
—	51	—	51	RTN	24
STO 3	33 03	RCL 6	34 06	g NOP	35 01
↑	41	x	71		
↑	41	2	02		
x	71	4	04		
↑	41	÷	81		
x	71	RCL 1	34 01		

<b>R<sub>1</sub></b>	EI	<b>R<sub>4</sub></b>	x	<b>R<sub>7</sub></b>
<b>R<sub>2</sub></b>	ℓ	<b>R<sub>5</sub></b>		<b>R<sub>8</sub></b>
<b>R<sub>3</sub></b>	(ℓ - x)	<b>R<sub>6</sub></b>	Wo	<b>R<sub>9</sub></b>

## SIMPLE 1

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 2	33 02	RCL 6	34 06	x	71
g R↓	35 08	↑	41	+	61
x	71	x	71	RCL 2	34 02
STO 1	33 01	RCL 2	34 02	↑	41
RTN	24	↑	41	x	71
LBL	23	x	71	—	51
B	12	—	51	x	71
STO 4	33 04	RCL 7	34 07	RTN	24
RTN	24	↑	41	LBL	23
LBL	23	x	71	D	14
C	13	+	61	RCL 3	34 03
STO 3	33 03	RCL 7	34 07	RCL 6	34 06
g R↓	35 08	x	71	x	71
STO 5	33 05	RCL 6	34 06	RCL 2	34 02
RTN	24	RCL 3	34 03	÷	81
LBL	23	x	71	RCL 7	34 07
D	14	6	06	x	71
RCL 2	34 02	÷	81	RTN	24
RCL 5	34 05	RCL 1	34 01	LBL	23
—	51	÷	81	D	14
STO 6	33 06	RCL 2	34 02	RCL 6	34 06
RCL 4	34 04	÷	81	RCL 3	34 03
STO 7	33 07	x	71	x	71
RCL 5	34 05	RTN	24	RCL 2	34 02
g x>y	35 24	LBL	23	÷	81
GTO	22	D	14	RTN	24
1	01	D	14	g NOP	35 01
CHS	42	g LST X	35 00	g NOP	35 01
STO 6	33 06	3	03	g NOP	35 01
RCL 4	34 04	RCL 7	34 07	g NOP	35 01
RCL 2	34 02	f <sup>-1</sup>	32		
—	51	√x	09		
STO 7	33 07	x	71		
LBL	23	RCL 6	34 06		
1	01	↑	41		

<b>R<sub>1</sub></b>	EI	<b>R<sub>4</sub></b>	x	<b>R<sub>7</sub></b>	x or x - ℓ
<b>R<sub>2</sub></b>	ℓ	<b>R<sub>5</sub></b>	a	<b>R<sub>8</sub></b>	
<b>R<sub>3</sub></b>	P	<b>R<sub>6</sub></b>	ℓ - a or a	<b>R<sub>9</sub></b>	Used

## SIMPLE 2

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 2	33 02	LBL	23	D	14
g R↓	35 08	D	14	RCL 2	34 02
x	71	RCL 4	34 04	2	02
STO 1	33 01	4	04	÷	81
RTN	24	x	71	RCL 4	34 04
LBL	23	RCL 2	34 02	–	51
C	13	6	06	RCL 6	34 06
STO 6	33 06	x	71	RTN	24
g R↓	35 08	–	51	LBL	23
STO 5	33 05	E	15	E	15
RTN	24	2	02	RCL 4	34 04
LBL	23	÷	81	↑	41
B	12	RCL 5	34 05	x	71
STO 4	33 04	x	71	x	71
RTN	24	g x↔y	35 07	RCL 2	34 02
LBL	23	–	51	↑	41
D	14	RCL 1	34 01	x	71
RCL 4	34 04	÷	81	g LST X	35 00
RCL 2	34 02	RTN	24	x	71
2	02	LBL	23	+	61
x	71	D	14	RCL 6	34 06
–	51	RCL 5	34 05	x	71
E	15	RCL 4	34 04	2	02
–	51	RCL 2	34 02	4	04
RCL 5	34 05	–	51	÷	81
x	71	RCL 4	34 04	RCL 4	34 04
2	02	x	71	RCL 2	34 02
÷	81	RCL 6	34 06	RTN	24
g x↔y	35 07	x	71	g NOP	35 01
–	51	2	02		
RCL 4	34 04	÷	81		
x	71	–	51		
RCL 1	34 01	RTN	24		
÷	81	LBL	23		
RTN	24				

<b>R<sub>1</sub></b>	EI	<b>R<sub>4</sub></b>	x	<b>R<sub>7</sub></b>
<b>R<sub>2</sub></b>	ℓ	<b>R<sub>5</sub></b>	M	<b>R<sub>8</sub></b>
<b>R<sub>3</sub></b>		<b>R<sub>6</sub></b>	W	<b>R<sub>9</sub></b>

## SIMPLE 3

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 2	33 02	RCL 6	34 06	-	51
g R↓	35 08	RCL 1	34 01	x	71
x	71	÷	81	RTN	24
STO 1	33 01	RCL 7	34 07	LBL	23
RTN	24	RCL 7	34 07	D	14
LBL	23	↑	41	RCL 3	34 03
C	13	x	71	RCL 6	34 06
CHS	42	2	02	x	71
STO 6	33 06	÷	81	RCL 2	34 02
g R↓	35 08	RCL 2	34 02	÷	81
STO 5	33 05	÷	81	CHS	42
RTN	24	-	51	RTN	24
LBL	23	RCL 2	34 02	LBL	23
B	12	3	03	D	14
STO 4	33 04	÷	81	RCL 6	34 06
RTN	24	-	51	RCL 2	34 02
LBL	23	RCL 3	34 03	÷	81
D	14	↑	41	CHS	42
A	11	x	71	RTN	24
RCL 5	34 05	6	06	g NOP	35 01
STO 7	33 07	÷	81	g NOP	35 01
RCL 4	34 04	RCL 2	34 02	g NOP	35 01
STO 3	33 03	÷	81	g NOP	35 01
g x≤y	35 22	-	51	g NOP	35 01
GTO	22	RTN	24	g NOP	35 01
E	15	x	71	g NOP	35 01
RCL 2	34 02	RCL 3	34 03	g NOP	35 01
-	51	x	71	g NOP	35 01
STO 3	33 03	RTN	24	g NOP	35 01
RCL 2	34 02	LBL	23	g NOP	35 01
RCL 5	34 05	D	14	g NOP	35 01
-	51	E	15	g LST X	35 00
STO 7	33 07	g LST X	35 00	2	02
LBL	23	x	71		
E	15				

<b>R<sub>1</sub></b>	EI	<b>R<sub>4</sub></b>	x	<b>R<sub>7</sub></b>	a, ℓ - a
<b>R<sub>2</sub></b>	ℓ	<b>R<sub>5</sub></b>	a	<b>R<sub>8</sub></b>	
<b>R<sub>3</sub></b>	x, x - ℓ	<b>R<sub>6</sub></b>	-M	<b>R<sub>9</sub></b>	Used

## FIXED-FIXED 1

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 2	33 02	6	06	RCL 1	34 01
g R↓	35 08	÷	81	÷	81
x	71	RCL 1	34 01	RCL 7	34 07
STO 1	33 01	÷	81	x	71
RTN	24	RCL 2	34 02	RTN	24
LBL	23	RCL 5	34 05	LBL	23
C	13	—	51	D	14
STO 3	33 03	↑	41	RCL 6	34 06
g R↓	35 08	x	71	RCL 4	34 04
STO 5	33 05	RCL 3	34 03	x	71
RTN	24	x	71	RCL 5	34 05
LBL	23	RCL 2	34 02	RCL 2	34 02
B	12	↑	41	x	71
STO 4	33 04	x	71	—	51
RTN	24	g LST X	35 00	RCL 7	34 07
LBL	23	x	71	x	71
D	14	÷	81	RTN	24
RCL 4	34 04	STO 7	33 07	LBL	23
RCL 2	34 02	x	71	D	14
RCL 5	34 05	RTN	24	RCL 6	34 06
2	02	LBL	23	RCL 7	34 07
x	71	D	14	x	71
+	61	RCL 6	34 06	RTN	24
STO 6	33 06	RCL 4	34 04	g NOP	35 01
x	71	x	71	g NOP	35 01
RCL 5	34 05	RCL 2	34 02	g NOP	35 01
RCL 2	34 02	RCL 5	34 05	g NOP	35 01
x	71	x	71	g NOP	35 01
3	03	2	02	g NOP	35 01
x	71	x	71	g NOP	35 01
—	51	—	51	g NOP	35 01
RCL 4	34 04	RCL 4	34 04		
↑	41	x	71		
x	71	2	02		
x	71	÷	81		

$R_1$	EI	$R_4$	x	$R_7$	$P(\ell - a)^2 / \ell^3$
$R_2$	$\ell$	$R_5$	a	$R_8$	
$R_3$	P	$R_6$	$(\ell + 2a)$	$R_9$	

## FIXED-FIXED 2

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 2	33 02	RCL 1	34 01	RCL 4	34 04
g R↓	35 08	÷	81	x	71
x	71	RTN	24	RCL 2	34 02
STO 1	33 01	LBL	23	↑	41
RTN	24	D	14	x	71
LBL	23	RCL 2	34 02	—	51
C	13	3	03	RCL 5	34 05
STO 5	33 05	x	71	x	71
RTN	24	RCL 4	34 04	1	01
LBL	23	2	02	2	02
B	12	x	71	÷	81
STO 4	33 04	—	51	RTN	24
RTN	24	RCL 4	34 04	LBL	23
LBL	23	x	71	D	14
D	14	RCL 2	34 02	RCL 4	34 04
RCL 2	34 02	↑	41	2	02
2	02	x	71	x	71
x	71	—	51	RCL 2	34 02
RCL 4	34 04	RCL 5	34 05	—	51
—	51	x	71	2	02
RCL 4	34 04	RCL 4	34 04	÷	81
x	71	x	71	RCL 5	34 05
RCL 2	34 02	1	01	x	71
↑	41	2	02	CHS	42
x	71	÷	81	RTN	24
—	51	RCL 1	34 01	g NOP	35 01
RCL 4	34 04	÷	81	g NOP	35 01
↑	41	RTN	24	g NOP	35 01
x	71	LBL	23	g NOP	35 01
x	71	D	14	g NOP	35 01
RCL 5	34 05	RCL 2	34 02		
x	71	RCL 4	34 04		
2	02	—	51		
4	04	6	06		
÷	81	x	71		

<b>R<sub>1</sub></b>	EI	<b>R<sub>4</sub></b>	x	<b>R<sub>7</sub></b>
<b>R<sub>2</sub></b>	ℓ	<b>R<sub>5</sub></b>	W	<b>R<sub>8</sub></b>
<b>R<sub>3</sub></b>		<b>R<sub>6</sub></b>		<b>R<sub>9</sub></b>

## FIXED-FIXED 3

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 2	33 02	RCL 2	34 02	RCL 5	34 05
g R↓	35 08	RCL 5	34 05	x	71
x	71	—	51	6	06
STO 1	33 01	RCL 6	34 06	x	71
RTN	24	RCL 2	34 02	RCL 2	34 02
LBL	23	↑	41	÷	81
C	13	x	71	RCL 3	34 03
STO 6	33 06	÷	81	+	61
g R↓	35 08	STO 7	33 07	RCL 7	34 07
STO 5	33 05	x	71	x	71
RTN	24	RCL 1	34 01	RTN	24
LBL	23	÷	81	LBL	23
B	12	RTN	24	D	14
STO 4	33 04	LBL	23	RCL 7	34 07
RTN	24	D	14	6	06
LBL	23	RCL 4	34 04	x	71
D	14	RCL 5	34 05	RCL 5	34 05
RCL 5	34 05	x	71	÷	81
RCL 4	34 04	3	03	RTN	24
x	71	x	71	g NOP	35 01
RCL 2	34 02	RCL 2	34 02	g NOP	35 01
÷	81	÷	81	g NOP	35 01
RCL 2	34 02	RCL 3	34 03	g NOP	35 01
RCL 5	34 05	+	61	g NOP	35 01
3	03	RCL 4	34 04	g NOP	35 01
x	71	x	71	g NOP	35 01
—	51	RCL 7	34 07	g NOP	35 01
STO 3	33 03	x	71	g NOP	35 01
2	02	RCL 1	34 01	g NOP	35 01
÷	81	÷	81	g NOP	35 01
+	61	RTN	24	g NOP	35 01
RCL 4	34 04	LBL	23	RCL 2	34 02
↑	41	D	14	÷	81
x	71	RCL 4	34 04	RTN	24
x	71			g NOP	35 01

$R_1$	EI	$R_4$	x	$R_7$	$M(\ell-a)/\ell^2$
$R_2$	$\ell$	$R_5$	a	$R_8$	
$R_3$	$(\ell-3a)$	$R_6$	M	$R_9$	

FIXED-SIMPLE 1,  $x \leq a$ 

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 2	33 02	-	51	RCL 4	34 04
g R↓	35 08	RCL 5	34 05	x	71
x	71	x	71	RCL 7	34 07
STO 1	33 01	STO 7	33 07	-	51
RTN	24	3	03	6	06
LBL	23	x	71	x	71
C	13	-	51	E	15
STO 3	33 03	RCL 4	34 04	RTN	24
g R↓	35 08	↑	41	LBL	23
STO 5	33 05	x	71	D	14
RTN	24	x	71	RCL 6	34 06
LBL	23	RCL 1	34 01	6	06
B	12	÷	81	x	71
STO 4	33 04	E	15	LBL	23
RTN	24	RTN	24	E	15
LBL	23	LBL	23	RCL 3	34 03
D	14	D	14	x	71
RCL 2	34 02	RCL 6	34 06	RCL 2	34 02
RCL 5	34 05	RCL 4	34 04	RCL 5	34 05
+	61	x	71	-	51
2	02	3	03	x	71
x	71	x	71	1	01
RCL 5	34 05	RCL 7	34 07	2	02
↑	41	6	06	÷	81
x	71	x	71	RCL 2	34 02
RCL 2	34 02	-	51	↑	41
÷	81	RCL 4	34 04	x	71
-	51	x	71	÷	81
STO 6	33 06	RCL 1	34 01	RTN	24
RCL 4	34 04	÷	81	g NOP	35 01
x	71	E	15		
RCL 2	34 02	RTN	24		
2	02	LBL	23		
x	71	D	14		
RCL 5	34 05	RCL 6	34 06		

$R_1$	EI	$R_4$	x	$R_7$	$a(2\ell-a)$
$R_2$	$\ell$	$R_5$	a	$R_8$	
$R_3$	P	$R_6$	Used	$R_9$	

FIXED—SIMPLE 2,  $x \geq a$ 

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 2	33 02	x	71	6	06
g R↓	35 08	STO 7	33 07	x	71
x	71	—	51	E	15
STO 1	33 01	RCL 2	34 02	RTN	24
RTN	24	RCL 4	34 04	LBL	23
LBL	23	—	51	D	14
C	13	x	71	RCL 6	34 06
STO 3	33 03	RCL 1	34 01	6	06
g R↓	35 08	÷	81	x	71
STO 5	33 05	E	15	CHS	42
RTN	24	RTN	24	LBL	23
LBL	23	LBL	23	E	15
B	12	D	14	RCL 3	34 03
STO 4	33 04	RCL 7	34 07	x	71
RTN	24	RCL 2	34 02	RCL 5	34 05
LBL	23	RCL 4	34 04	↑	41
D	14	—	51	x	71
3	03	↑	41	x	71
RCL 5	34 05	x	71	1	01
RCL 2	34 02	RCL 6	34 06	2	02
÷	81	x	71	÷	81
—	51	3	03	RCL 2	34 02
STO 6	33 06	x	71	↑	41
RCL 2	34 02	—	51	x	71
RCL 4	34 04	RCL 1	34 01	÷	81
—	51	÷	81	RTN	24
↑	41	E	15	g NOP	35 01
x	71	RTN	24	g NOP	35 01
x	71	LBL	23	g NOP	35 01
RCL 2	34 02	D	14	g NOP	35 01
RCL 5	34 05	RCL 2	34 02		
—	51	RCL 4	34 04		
RCL 2	34 02	—	51		
x	71	RCL 6	34 06		
3	03	x	71		

<b>R<sub>1</sub></b>	EI	<b>R<sub>4</sub></b>	x	<b>R<sub>7</sub></b>	$3\ell(\ell-a)$
<b>R<sub>2</sub></b>	$\ell$	<b>R<sub>5</sub></b>	a	<b>R<sub>8</sub></b>	
<b>R<sub>3</sub></b>	P	<b>R<sub>6</sub></b>	$3 - a/\ell$	<b>R<sub>9</sub></b>	

## FIXED-SIMPLE 3

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 2	33 02	x	71	RCL 6	34 06
g R↓	35 08	RCL 6	34 06	x	71
x	71	↑	41	RCL 2	34 02
STO 1	33 01	x	71	3	03
RTN	24	x	71	x	71
LBL	23	8	08	RCL 6	34 06
C	13	LBL	23	4	04
STO 5	33 05	E	15	x	71
RTN	24	RCL 6	34 06	—	51
LBL	23	↑	41	x	71
B	12	x	71	RTN	24
STO 4	33 04	g LST X	35 00	LBL	23
RTN	24	x	71	D	14
LBL	23	x	71	RCL 5	34 05
D	14	—	51	8	08
3	03	RCL 2	34 02	÷	81
RCL 2	34 02	↑	41	RCL 2	34 02
x	71	x	71	3	03
g LST X	35 00	g LST X	35 00	x	71
RCL 4	34 04	x	71	RCL 6	34 06
—	51	—	51	8	08
STO 6	33 06	4	04	x	71
↑	41	8	08	—	51
x	71	÷	81	x	71
x	71	RCL 5	34 05	CHS	42
2	02	x	71	RTN	24
E	15	RCL 1	34 01	g NOP	35 01
RCL 6	34 06	÷	81	g NOP	35 01
x	71	CHS	42	g NOP	35 01
CHS	42	RTN	24	g NOP	35 01
RTN	24	LBL	23		
LBL	23	D	14		
D	14	RCL 5	34 05		
9	09	8	08		
RCL 2	34 02	÷	81		

<b>R<sub>1</sub></b>	EI	<b>R<sub>4</sub></b>	x	<b>R<sub>7</sub></b>
<b>R<sub>2</sub></b>	ℓ	<b>R<sub>5</sub></b>	w	<b>R<sub>8</sub></b>
<b>R<sub>3</sub></b>		<b>R<sub>6</sub></b>	ℓ - x	<b>R<sub>9</sub></b>

FIXED-SIMPLE 4,  $x \leq a$ 

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 2	33 02	x	71	RCL 4	34 04
g R↓	35 08	2	02	x	71
x	71	x	71	RCL 3	34 03
STO 1	33 01	STO 7	33 07	x	71
RTN	24	÷	81	RCL 4	34 04
LBL	23	STO 3	33 03	+	61
C	13	3	03	RCL 7	34 07
STO 6	33 06	RCL 2	34 02	x	71
g R↓	35 08	x	71	RTN	24
STO 5	33 05	RCL 4	34 04	LBL	23
RTN	24	—	51	D	14
LBL	23	g LST X	35 00	E	15
B	12	f <sup>-1</sup>	32	RCL 4	34 04
STO 4	33 04	√x	09	RCL 2	34 02
RTN	24	x	71	—	51
LBL	23	CHS	42	x	71
D	14	RCL 7	34 07	RCL 6	34 06
RCL 4	34 04	+	61	+	61
↑	41	x	71	RTN	24
x	71	+	61	LBL	23
2	02	RCL 6	34 06	E	15
÷	81	RCL 1	34 01	LBL	23
RCL 2	34 02	÷	81	D	14
RCL 5	34 05	STO 7	33 07	RCL 3	34 03
2	02	x	71	6	06
÷	81	RTN	24	x	71
—	51	LBL	23	RCL 6	34 06
RCL 5	34 05	D	14	x	71
x	71	RCL 4	34 04	RTN	24
—	51	RCL 2	34 02	g NOP	35 01
g LST X	35 00	↑	41		
RCL 2	34 02	+	61		
↑	41	—	51		
x	71	3	03		
g LST X	35 00	x	71		

<b>R<sub>1</sub></b>	EI	<b>R<sub>4</sub></b>	x	<b>R<sub>7</sub></b>	M/EI
<b>R<sub>2</sub></b>	ℓ	<b>R<sub>5</sub></b>	a	<b>R<sub>8</sub></b>	
<b>R<sub>3</sub></b>	Used	<b>R<sub>6</sub></b>	M	<b>R<sub>9</sub></b>	

FIXED-SIMPLE 5,  $x \geq a$ 

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 2	33 02	g LST X	35 00	RCL 4	34 04
g R↓	35 08	↑	41	x	71
x	71	+	61	RCL 3	34 03
STO 1	33 01	RCL 2	34 02	x	71
RTN	24	3	03	RCL 5	34 05
LBL	23	x	71	+	61
C	13	RCL 4	34 04	RCL 7	34 07
STO 6	33 06	—	51	RTN	24
g R↓	35 08	RCL 4	34 04	LBL	23
STO 5	33 05	f <sup>-1</sup>	32	D	14
RTN	24	√x	09	RCL 4	34 04
LBL	23	x	71	RCL 2	34 02
B	12	—	51	—	51
STO 4	33 04	x	71	RCL 3	34 03
RTN	24	RCL 2	34 02	x	71
LBL	23	RCL 4	34 04	6	06
D	14	—	51	RCL 5	34 05
RCL 5	34 05	RCL 5	34 05	x	71
RCL 2	34 02	x	71	RCL 6	34 06
x	71	—	51	x	71
↑	41	RCL 6	34 06	RTN	24
+	61	RCL 1	34 01	LBL	23
RCL 5	34 05	÷	81	D	14
↑	41	STO 7	33 07	RCL 6	34 06
x	71	x	71	RCL 3	34 03
—	51	RTN	24	x	71
4	04	LBL	23	6	06
÷	81	D	14	x	71
RCL 2	34 02	3	03	RTN	24
↑	41	RCL 4	34 04	g NOP	35 01
↑	41	x	71		
x	71	RCL 2	34 02		
x	71	6	06		
÷	81	x	71		
STO 3	33 03	—	51		

<b>R<sub>1</sub></b>	EI	<b>R<sub>4</sub></b>	x	<b>R<sub>7</sub></b>	M/EI
<b>R<sub>2</sub></b>	ℓ	<b>R<sub>5</sub></b>	a	<b>R<sub>8</sub></b>	
<b>R<sub>3</sub></b>	Used	<b>R<sub>6</sub></b>	M	<b>R<sub>9</sub></b>	

## COMPRESSIVE BUCKLING

KEYS	CODE	KEYS	CODE	KEYS	CODE
g	35	RCL 1	34 01	RCL 3	34 03
$\pi$	02	RCL 3	34 03	$\div$	81
$\uparrow$	41	x	71	RCL 5	34 05
x	71	RCL 4	34 04	x	71
STO 1	33 01	x	71	STO 4	33 04
g R↓	35 08	RCL 5	34 05	RTN	24
0	00	$\div$	81	LBL	23
g $x=y$	35 23	STO 2	33 02	E	15
RTN	24	RTN	24	$\uparrow$	41
RTN	24	LBL	23	x	71
CLX	44	C	13	STO 5	33 05
4	04	STO 3	33 03	0	00
STO	33	0	00	g $x \neq y$	35 21
$\div$	81	g $x \neq y$	35 21	0	00
1	01	0	00	RTN	24
CLX	44	RTN	24	RCL 1	34 01
1	01	RCL 2	34 02	RCL 2	34 02
g $x=y$	35 23	RCL 1	34 01	$\div$	81
RTN	24	$\div$	81	RCL 3	34 03
RTN	24	RCL 4	34 04	x	71
CLX	44	$\div$	81	RCL 4	34 04
1	01	RCL 5	34 05	x	71
6	06	x	71	STO 5	33 05
STO	33	STO 3	33 03	f	31
x	71	RTN	24	$\sqrt{x}$	09
1	01	LBL	23	RTN	24
2	02	D	14	g NOP	35 01
RTN	24	STO 4	33 04	g NOP	35 01
LBL	23	0	00	g NOP	35 01
B	12	g $x \neq y$	35 21	g NOP	35 01
STO 2	33 02	0	00		
0	00	RTN	24		
g $x \neq y$	35 21	RCL 2	34 02		
0	00	RCL 1	34 01		
RTN	24	$\div$	81		

<b>R<sub>1</sub></b> #Fixed Ends, C	<b>R<sub>4</sub></b> I	<b>R<sub>7</sub></b>
<b>R<sub>2</sub></b> P	<b>R<sub>5</sub></b> $\ell^2$	<b>R<sub>8</sub></b>
<b>R<sub>3</sub></b> E	<b>R<sub>6</sub></b>	<b>R<sub>9</sub></b> Used

## ECCENTRICALLY LOADED COLUMNS

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 3	33 03	RCL 4	34 04	RCL 4	34 04
g R↓	35 08	x	71	x	71
STO 2	33 02	RTN	24	RCL 6	34 06
g x↔y	35 07	LBL	23	x	71
STO 1	33 01	D	14	RCL 1	34 01
RTN	24	C	13	÷	81
LBL	23	RCL 4	34 04	RCL 7	34 07
B	12	+	61	x	71
STO 5	33 05	RCL 5	34 05	1	01
g x↔y	35 07	x	71	+	61
STO 4	33 04	RTN	24	RCL 5	34 05
RTN	24	LBL	23	x	71
LBL	23	E	15	RCL 7	34 07
C	13	STO 7	33 07	÷	81
RCL 5	34 05	g x↔y	35 07	RTN	24
RCL 2	34 02	STO 6	33 06	g NOP	35 01
÷	81	RCL 5	34 05	g NOP	35 01
RCL 1	34 01	RCL 2	34 02	g NOP	35 01
÷	81	÷	81	g NOP	35 01
f	31	4	04	g NOP	35 01
√x	09	÷	81	g NOP	35 01
RCL 3	34 03	RCL 1	34 01	g NOP	35 01
x	71	÷	81	g NOP	35 01
2	02	f	31	g NOP	35 01
÷	81	√x	09	g NOP	35 01
g	35	RCL 3	34 03	g NOP	35 01
RAD	42	x	71	g NOP	35 01
f	31	g	35	g NOP	35 01
COS	05	RAD	42	g NOP	35 01
g	35	f	31	g NOP	35 01
DEG	41	COS	05	g NOP	35 01
g	35	g	35	g NOP	35 01
1/x	04	DEG	41	g NOP	35 01
1	01	g	35	g NOP	35 01
-	51	1/x	04	g NOP	35 01

R <sub>1</sub>	I	R <sub>4</sub>	e	R <sub>7</sub>	A
R <sub>2</sub>	E	R <sub>5</sub>	P	R <sub>8</sub>	
R <sub>3</sub>	l	R <sub>6</sub>	c	R <sub>9</sub>	Used

## RECTANGULAR, REINFORCED CONCRETE SECTION

KEYS	CODE	KEYS	CODE	KEYS	CODE
LBL	23	RCL 1	34 01	÷	81
A	11	RCL 5	34 05	RTN	24
STO 3	33 03	x	71	LBL	23
÷	81	—	51	E	15
STO 4	33 04	RTN	24	f	31
g R↓	35 08	LBL	23	$\sqrt{x}$	09
STO 1	33 01	C	13	D	14
RTN	24	STO 2	33 02	f	31
LBL	23	RTN	24	$\sqrt{x}$	09
B	12	LBL	23	RTN	24
.	83	D	14	g NOP	35 01
5	05	RCL 2	34 02	g NOP	35 01
RCL 4	34 04	g x $\geq$ y	35 07	g NOP	35 01
RCL 1	34 01	÷	81	g NOP	35 01
÷	81	g LST X	35 00	g NOP	35 01
1	01	÷	81	g NOP	35 01
+	61	6	06	g NOP	35 01
RCL 4	34 04	x	71	g NOP	35 01
x	71	RCL 4	34 04	g NOP	35 01
÷	81	RCL 1	34 01	g NOP	35 01
STO 5	33 05	+	61	g NOP	35 01
RTN	24	↑	41	g NOP	35 01
LBL	23	x	71	g NOP	35 01
B	12	x	71	g NOP	35 01
RCL 1	34 01	RCL 1	34 01	g NOP	35 01
RCL 5	34 05	÷	81	g NOP	35 01
x	71	RCL 3	34 03	g NOP	35 01
↑	41	÷	81	g NOP	35 01
+	61	3	03	g NOP	35 01
g LST X	35 00	RCL 4	34 04	g NOP	35 01
↑	41	x	71	g NOP	35 01
x	71	RCL 1	34 01	g NOP	35 01
+	61	+	61	g LST X	35 00
f	31	g LST X	35 00	+	61
$\sqrt{x}$	09				

<b>R<sub>1</sub></b>	n	<b>R<sub>4</sub></b>	$f_s/f_c$	<b>R<sub>7</sub></b>
<b>R<sub>2</sub></b>	M	<b>R<sub>5</sub></b>	P	<b>R<sub>8</sub></b>
<b>R<sub>3</sub></b>	$f_c$	<b>R<sub>6</sub></b>		<b>R<sub>9</sub></b>

## BOLT TORQUE

KEYS	CODE	KEYS	CODE	KEYS	CODE
STO 1	33 01	0	00	x	71
g R↓	35 08	RTN	24	STO 8	33 08
f	31	LBL	23	RTN	24
COS	05	C	13	LBL	23
STO 2	33 02	STO 7	33 07	E	15
g R↓	35 08	0	00	2	02
f	31	g x≠y	35 21	g	35
TAN	06	RTN	24	π	02
STO 3	33 03	RTN	24	÷	81
RCL 1	34 01	RCL 8	34 08	g x↔y	35 07
RCL 2	34 02	RCL 1	34 01	÷	81
÷	81	RCL 6	34 06	g LST X	35 00
+	61	RCL 4	34 04	÷	81
1	01	RCL 5	34 05	8	08
RCL 1	34 01	x	71	g LST X	35 00
RCL 3	34 03	+	61	÷	81
x	71	÷	81	RCL 1	34 01
RCL 2	34 02	STO 7	33 07	RCL 6	34 06
÷	81	RTN	24	x	71
—	51	LBL	23	RCL 7	34 07
÷	81	D	14	x	71
STO 1	33 01	STO 8	33 08	RCL 7	34 07
0	00	0	00	f	31
RTN	24	g x≠y	35 21	R→P	01
LBL	23	RTN	24	g x↔y	35 07
B	12	RTN	24	g R↓	35 08
STO 4	33 04	RCL 1	34 01	x	71
g R↓	35 08	RCL 6	34 06	RTN	24
2	02	x	71	g NOP	35 01
÷	81	RCL 4	34 04		
STO 5	33 05	RCL 5	34 05		
g R↓	35 08	x	71		
2	02	+	61		
÷	81	RCL 7	34 07		
STO 6	33 06				

$R_1$	f, used	$R_4$	$f_c$	$R_7$	W
$R_2$	$\cos \theta$	$R_5$	$D_c/2$	$R_8$	T
$R_3$	$\tan \alpha$	$R_6$	$D_m/2$	$R_9$	Used









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