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ADA 071 609

Author M. A. Johnson	Subject Category Calculator Programs	No. R79EMH5 Date May 1979
Title RADAR DETECTION CALCULATIONS WITH THE HP-65 AND HP-67		
Copies Available at HMED TIS Distribution Center Box 4840 (CSP 4-18) Syracuse, New York 13221	GE Class 1 Govt Class Unclassified	No. of Pages 77
Summary <p>Card programmable calculators together with suitable programs can easily provide numerical answers which formerly took large computers and volumes of tabular or plotted outputs for everyday reference. Presented here are derivations and program listings for the HP-65 and HP-67 calculators to provide the user with commonly used radar detection performance data.</p> <p>Both fixed threshold and adaptive threshold CFAR detection with noncoherent integration are covered. Recursive programs for the general chi-squared target fluctuation distribution are treated as well as faster running recursive programs for the Swerling target Cases I - IV. Also included are fast programs for the required detection signal-to-noise ratio based on the simplified algorithms of Barton.</p>		

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SECTION I  
INTRODUCTION

The problem treated here is the classical detection of RF target signals in a Gaussian noise background. This was initially analyzed and presented by Marcum and Swerling<sup>1, 2</sup> and extended, since that time by a number of writers. Since tabulated lists and curves are often somewhat awkward to use, the writers were interested in hand-calculator programs which could give numerical results over a wide variety of detection parameters. In particular, programs for HP-65 and HP-67 calculators are presented, but the algorithmic approaches could easily be programmed on other calculators.

The writer started this work by following Barton's<sup>3</sup> approach, an empirical approximation which directly provides required signal-to-noise ratio (SNR) for given probability of detection (PD) for various target models. The accuracy of the approximation is within a dB for normal parameter values, but the approach does not lend itself to finding probability of detection, given the SNR, without an iterative approach which is beyond the capability of the HP-65.

Later, the writer found an excellent report by Shnidman<sup>4</sup> who had found finite recursive series solutions for probability of detection for the basic four Swerling target models for fixed-threshold detection, and who also presented infinite series algorithms for non-fluctuating and the generalized chi-squared target models. These algorithms were found to be directly programmable for the HP-65. S. P. Applebaum has translated these programs for the HP-67 and these are included here. For the special case of a Swerling Case II target, these programs can solve for either PD or for SNR.

In spite of the importance of both CFAR detection and noncoherent integration, there are few papers in the literature which combine the two. One excellent paper, however, is that of Mitchell and Walker<sup>5</sup>. This paper treats the background estimation type of CFAR, and it has provided algorithms used here to cover these cases. For the special case of Swerling Case II, a finite series solution and programmed iterative inverse can provide either PD or SNR as with the fixed-threshold case. For other target models, a truncated infinite series solution provides PD given the SNR.

The implementations which these analyses treat are illustrated in Figures 1-1 and 1-2. In Figure 1-1, the noise background is considered to be constant and known.  $N$  samples of signal-plus-noise are summed and compared to a fixed threshold. If the threshold is exceeded, a target detection is declared. The threshold is set according to a specified false-alarm probability in the absence of signal. In the recursive solutions programmed here, the first step is the calculation of the threshold value given the false-alarm probability.

Figure 1-2 considers the case where the background noise is unknown or slowly varying so that a noise estimate must be made in order to establish a detection threshold value. The noise estimate here considered is the sum of  $R$  independent detected noise samples, and corresponds to the most common type of constant false-alarm rate (CFAR) detector.

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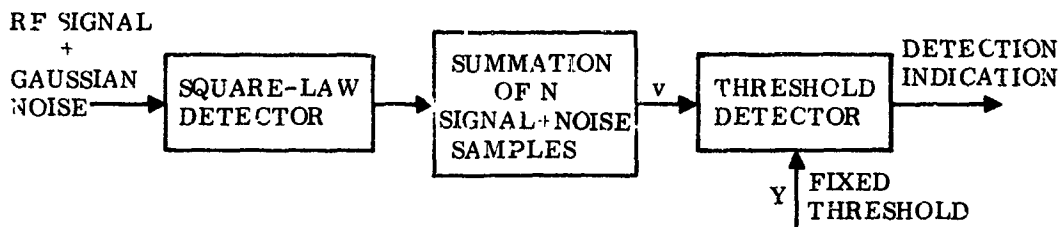


Figure 1-1. Fixed-Threshold Detection

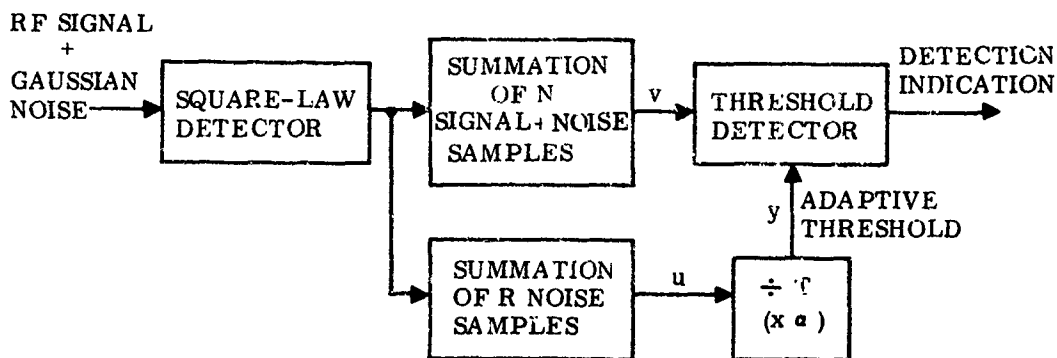


Figure 1-2. CFAR Detection (Background Normalized)



The target models used here follow that of Swerling and the integrated target SNR distributions all may be considered special cases of the chi-squared (or gamma) distribution given by

$$w(z, Z) = \frac{1}{(K-1)!} \left(\frac{K}{Z}\right)^K z^{K-1} e^{-\frac{Kz}{Z}} \quad (1-1)$$

where  $z$  is the integrated target power SNR for any trial and  $Z = \bar{z}$ .  $K$  is a distribution parameter which can have any value greater than zero and certain values of  $K$  correspond to standard target fluctuation cases\* as follows:

$0 < K < 1$  : Weinstock case

$K = 1$  : Swerling case I

$$w(z, Z) = \frac{1}{Z} e^{-\frac{z}{Z}} \quad (1-2)$$

Exponential power or Rayleigh voltage distribution. Target constant over  $N$  integrated samples. Results from radar target composed of many separate scatterers. Often satisfied by aircraft.

$K = 2$  : Swerling case III

$$w(z, z) = \frac{4}{Z^2} e^{-\frac{2z}{Z}}$$

Approximate distribution of target with Rayleigh and fixed components of equal average power. Target constant over  $N$  integrated samples.

$K = N$  : Swerling case II

Same basic target distribution as for  $K = 1$  but with target amplitude independent over  $N$  integrated samples. Often satisfied by aircraft targets with pulse-to-pulse radar frequency agility.

$K = 2N$  : Swerling case IV

Same basic target distribution as for  $K = 2$  but with target amplitude independent over  $N$  integrated samples.

---

\* Nathanson<sup>6</sup> provides a good discussion of these target models.

$K = \infty$  : Nonfluctuating target or case 0

$z = Z$

Note that other values of  $K$  may be useful for cases with different target fluctuation rates or with block correlation within the  $N$  samples integrated.<sup>7</sup> If the diversity order within the  $N$  samples is  $N_e$  then with a Rayleigh target  $K$  should be taken equal to  $N_e$ . For a Case III target distribution  $K$  should be taken as  $2N_e$ .

## SECTION II

### FIXED THRESHOLD DETECTION - RECURSIVE SOLUTION

#### 1. SWERLING CASE II

Although, as shown later, generalized programs can be written to cover chi-squared distributions of any  $K$ , it is worthwhile to consider some special cases since they can provide both simpler and faster running calculator programs. We shall start with the simplest of these cases - fixed threshold detection with a Swerling Case II target model.

The probability density function of  $v$ , the integrated signal-plus-noise variate, is given by, \*

$$f(v) = \frac{v^{N-1}}{(1+X)^N (N-1)!} e^{-\frac{v}{1+X}} \quad (2-1)$$

where  $X$  is the average SNR of each sample. The probability of detection is then given by

$$P_2 = \int_Y^{\infty} \frac{v^{N-1}}{(1+X)^N (N-1)!} e^{-\frac{v}{1+X}} dv \quad (2-2)$$

This may be integrated by parts to give

$$P_2 = \sum_{m=0}^{N-1} \frac{Y^m}{m! (1+X)^m} e^{-\frac{Y}{1+X}} \quad (2-3)$$

Notice that for  $X = 0$  this reduces to the false alarm probability (PF)

$$PF = \sum_{m=0}^{N-1} \frac{Y^m}{m!} e^{-Y} \quad (2-4)$$

\* Eqn. III. 10 of Swerling or Eqn. (39) for  $f_N(V|X)$  of Mitchell and Walker on page 675 noting that our  $X = Z/N$  corresponds to their  $X/N$ .

which provides an implicit solution for Y given PF. Notice also that by substituting  $Y = Y/1+X$  in Equation (2-4), we obtain Equation (2-3) so that one program routine can be used for both. This common equation shall be written as

$$P = \sum_{m=0}^{N-1} \frac{Y^m}{m!} e^{-Y} \quad (2-5)$$

with each definition of Y giving the appropriate corresponding definition of P.

Given a desired PF, the first step in finding either P2 given X or X given P2 must be to find Y using Equation (2-5) with  $P = PF$ . Perhaps one's first thought might be to use Newton's method to find the root of  $P - PIN$  where PIN is the specified value and P is obtained from the equation for a given Y. However, since P versus Y has an inflection point, convergence is not assured and it is better to use  $\ln(P/PIN)$ . This leads to incrementing Y for successive trials by

$$\Delta Y = - \frac{\ln P - \ln PIN}{\frac{d}{dY} (\ln P)} = - \frac{P}{P'} \ln (P/PIN)$$

We find by differentiating Equation (2-5) that

$$P' = - \frac{Y^{N-1}}{(N-1)!} e^{-Y}$$

= - last term of P series.

In the algorithmic expressions, the  $m^{\text{th}}$  term of this series is used and shall be designated YM. Each term is determined recursively from the previous term. After completing the series, we will have in storage the last term, YM, so we can use this for  $-P'$ . Therefore, in applying Newton's method, the Y-increment for successive trials is given by

$$\Delta Y = \frac{P}{YM} \ln \frac{P}{PIN} \quad (2-6)$$

To begin the iteration of Newton's method a starting value for Y is also needed which shall be designated Y0. The writer found empirically that the following expression approximated Y quite closely for small values of PIN as may customarily be desired for PF, and was programmable with very few program steps on the HP-65.

$$\begin{aligned}
 Y_0 &= N - \sqrt{N} + 2.3 \sqrt{L} (\sqrt{L} + \sqrt{N} - 1) \\
 L &= -\log \text{PIN}
 \end{aligned}
 \tag{2-7}$$

Using this start only three or four iterations are needed to calculate Y to 10 significant figures for any value of PIN of interest.

This solution of Equation (2-5) for Y can more completely be specified by using the algorithmic notation of Iverson (following Shnidman's practice) as given in Figure 2-1.

A brief explanation of this notation is first in order. The arrow notation implies a specification, that is, the statement,  $L \leftarrow -\log \text{PIN}$ , is translated to mean that the quantity L is specified by  $-\log \text{PIN}$ . The normal execution of the statements is line by line starting at the top, but a branch may be designated by an arrow between two statement lines. A conditional branch is denoted by a colon statement, and the branch is executed if the comparison condition specified on the arrow is satisfied. Otherwise the next statement in the sequence is executed.

The brackets labeled D and E on Figure 2-1 correspond to subroutines in the HP-65 program which follows and are shown here for convenience. Notice that the iteration is terminated when  $|\Delta Y/Y|$  is less than  $10^{-6}$ . Since the stored value of Y has already been corrected by the indicated  $\Delta Y$  and the convergence is quite rapid, Y is usually accurate to 9 or 10 significant figures.

Having obtained Y for a given PF using this algorithm, we can calculate P2 directly from Equation (2-5) [i.e., subroutine E] by the substitution  $Y = Y/(1+X)$ . Alternatively, if P2 is given and it is desired to find X, we can substitute  $\text{PIN} = P2$ , find a corresponding Y2 using the program of Figure 2-1 and then find  $X = Y/Y2 - 1$ .

These features are all contained in the Program HP-65 Y-P2 given here. Most of the program comes directly from the algorithmic program of Figure 2-1, but a few comments may help in its understanding. First of all, the writer has often recorded in the comment space on each line of the HP-65 programs the stack contents in the order, x, y, z, t. This may be useful to understanding since the stack is often used in these programs for temporary storage. This practice saves on use of the storage registers which is sometimes necessary and also often leads to shorter, faster running programs.

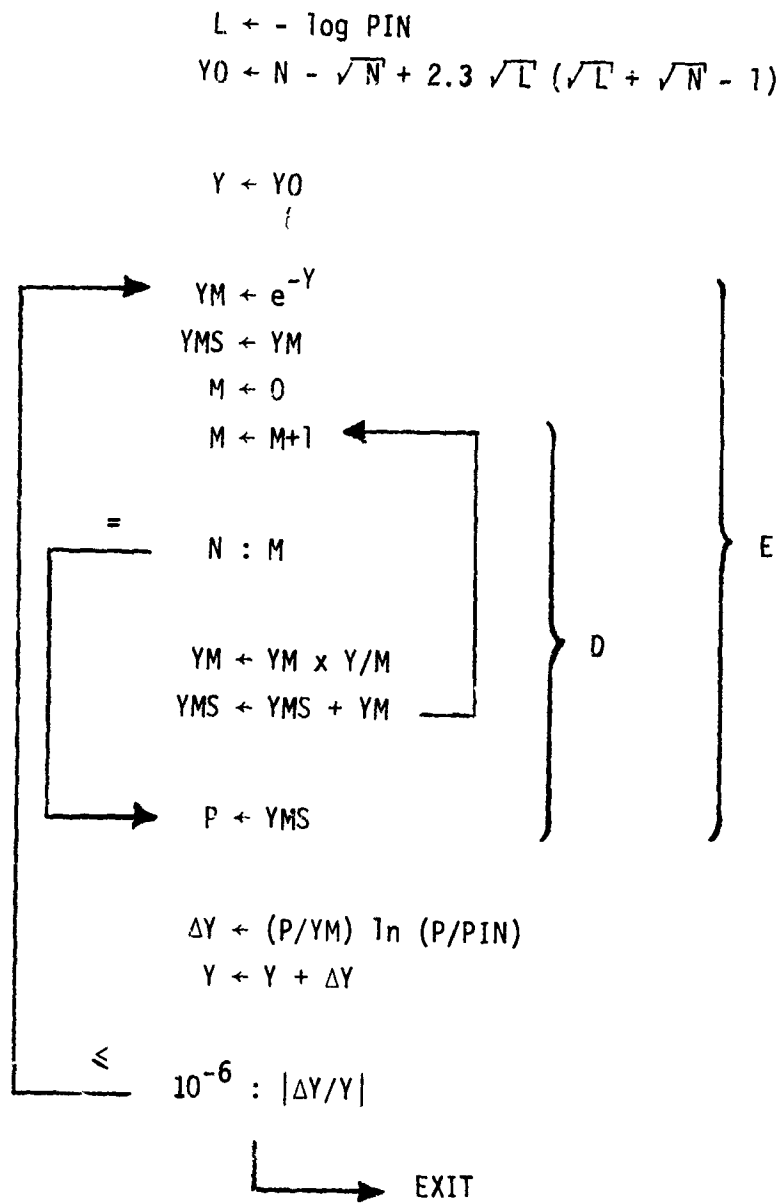


Figure 2-1. Algorithmic Program for Y Given P

Next, the LBLE subroutine incorporates LBLD as a loop to save a program step. When the D statement is reached on the last step, it jumps to LBLD. When the test  $N \leq M$  is satisfied, the RTN jumps back to the E-subroutine call, since the HP-65 has only one program step register, and that holds only the initial subroutine call step number no matter how many successive subroutines are called before the RTN statement is executed.

Since for finding X given PF and P2, the Y-algorithm programmed here is used twice, it is preceded by storing the previously calculated Y from R2 in R6. The first time through nothing exists in R2 anyway, so these steps can be ignored in trying to understand the program. After running the second time with  $PIN \leftarrow P2$  the program stops with Y2 displayed. Depressing the R/S key then restores Y in R2 and calculates X from the Y2 and Y previously found.

Finding P2 given X uses the LBLA function and is straightforward, requiring only that Y has been previously calculated or otherwise stored in R2.

It is interesting to note that Shnidman was concerned about accuracy of the calculation and underflow for certain cases such as  $e^{-Y}$  for large values of Y. His computer was equivalent to about 7 digit words and he went to double-precision arithmetic and logarithmic calculation in underflow cases. With the 10 digit words and  $10^{\pm 99}$  range of the HP calculators, together with direct monitoring by the operator, such measures are really unnecessary. The programs have been written such that if input parameters which would lead to underflow are entered, the underflow condition results almost immediately. This is indicated on the HP-65 by interruption of the program sequence with the display reading zero.

## 2. SWERLING CASES I, III, IV

In this section, the basic expressions to be programmed will not be derived but will be taken directly from Shnidman<sup>2</sup> to which the reader is referred for more detail.

We shall refer to the function represented by Equation (2-5) as  $P(N, Y)$ . The probability of detection of a Swerling Case I target can then be found as

$$P1 = e^{-Y/(Z+1)} \quad \text{for } N = 1,$$

or

$$P1 = P(N-1, Y) + \left(\frac{Z+1}{Z}\right)^{N-1} e^{-Y/(Z+1)} \left[1 - P\left(N-1, \frac{YZ}{Z+1}\right)\right] \quad \text{for } N \geq 2 \quad (2-8)$$

Here the integrated power signal-to-noise ratio,  $Z = NX$  was used.

The HP-65 P1 program directly implements this expression. Prior to running this, the Y-P2 program must be run to store Y in the R2 register. The same basic LBL E-subroutine is used here as in the prior program, modified slightly to give P(N-1, Y1) where Y1 is in the X-register prior to calling the subroutine. The coefficient of [1 - P] is also tested so that if too small, the LBLE subroutine is not run a second time.

In a similar way, the PD for a Swerling Case III target can be found as

$$P3 = e^{-\frac{2Y}{Z+2}} \left[ 1 + \frac{2YZ}{(Z+2)^2} \right] \quad \text{for } N = 1$$

or

$$P3 = \frac{Y^{N-2} e^{-Y}}{(N-2)!} \cdot \frac{2Y}{Z+2} + P(N-1, Y) \\ + \left( \frac{Z+2}{Z} \right)^{N-2} e^{-\frac{2Y}{Z+2}} \left[ 1 - \frac{2(N-2)}{Z} + \frac{2Y}{Z+2} \right] \cdot \left[ 1 - P(N-1, \frac{YZ}{Z+2}) \right] \quad \text{for } N \geq 2$$

(2-9)

The HP-65 P3 program directly implements this expression. It is run following the Y-P2 program to find Y as was the P1 program.

Shnidman shows that the PD for a Swerling Case IV target can be written as

$$P4 = \sum_{M=0}^{N-1} \frac{V^M}{M!} e^{-V} + \sum_{M=N}^{2N-1} \frac{V^M}{M!} e^{-V} \left[ 1 - \sum_{K=0}^{M-N} \frac{N!}{K!(N-K)!} \left( \frac{X}{X+2} \right)^K \left( \frac{2}{X+2} \right)^{N-K} \right]$$

(2-10)

where

$$V = 2Y/(X+2).$$

Although the first term is clearly equal to P(N, V), the second term is an extended summation of the same form as the first with a more complex term-by-term multiplier. The programming involves a doubly recursive approach which is best illustrated in algorithmic form on Figure 2-2.



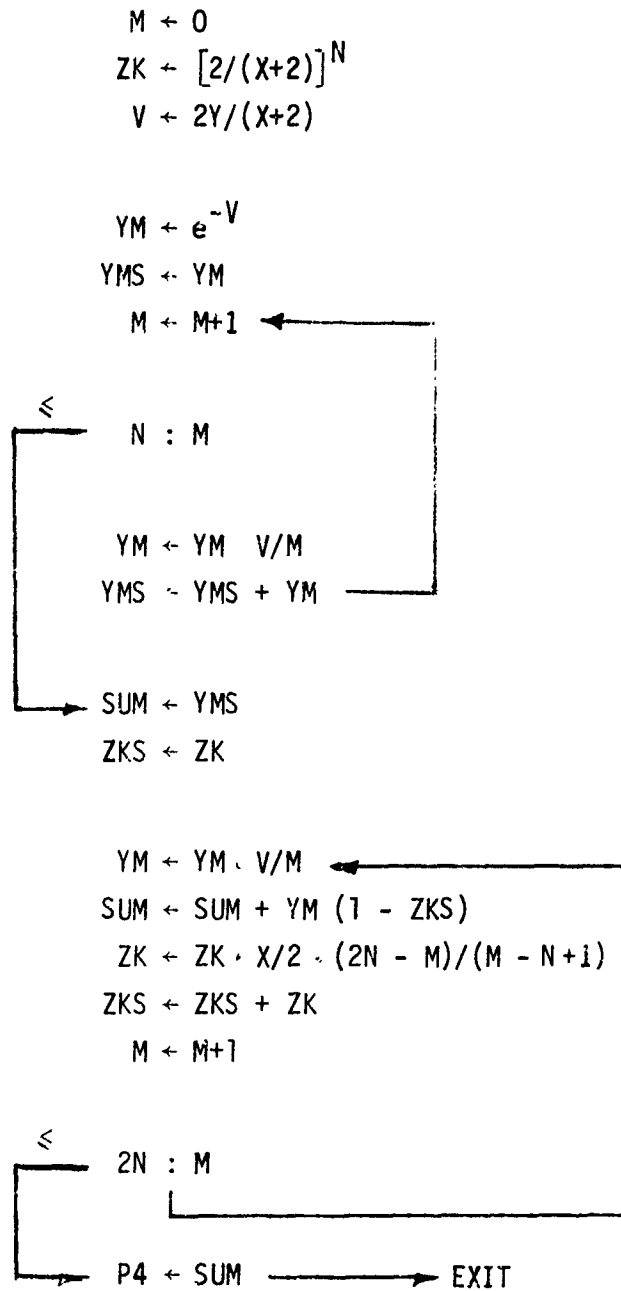


Figure 2-2. Algorithmic Program for P4

For relating Figure 2-2 to Equation (2-10), note that  $K = M - N$  so that no separate index is needed. The nomenclature YM corresponds to  $(V^M / M!) e^{-V}$  while ZK corresponds to each term of the K summation and YMS and ZKS have corresponding relationships to YM and ZK, respectively.

The order of some of the steps listed here is arbitrary and are written to correspond to the program HP-65 P4 for consistency. This program is used, as for P1 and P3 after running Y-P2 to find Y.

### 3. GENERALIZED AND NONFLUCTUATING TARGET MODEL

For the nonfluctuating target, as well as the general case, the summation of an infinite series is required, and the nonfluctuating target can be considered a special case of the general formulation. Mitchell and Walker give a straightforward derivation which in our nomenclature can be written as follows.

The distribution of  $v$  for a given integrated signal-to-noise ratio,  $z$ , is given by

$$\begin{aligned}
 f_N(v|z) &= \left(\frac{v}{z}\right)^{\frac{N-1}{2}} e^{-(v+z)} I_{N-1}(2\sqrt{vz}) \\
 &= \sum_{b=0}^{\infty} \frac{z^b v^{N+b-1} e^{-(v+z)}}{b! (N+b-1)!}
 \end{aligned} \tag{2-11}$$

as given by Marcum and Swerling.

The probability that  $v$  will exceed a fixed threshold,  $Y$ , is then

$$\begin{aligned}
 P(v > Y | z) &= \int_Y^{\infty} f_N(v|z) dv \\
 &= \sum_{b=0}^{\infty} \frac{z^b}{b!} e^{-z} \sum_{m=0}^{N+b-1} \frac{Y^m}{m!} e^{-Y}
 \end{aligned} \tag{2-12}$$

For a nonfluctuating target, this gives the desired PD by letting  $z = Z$ . For a fluctuating target, we must integrate over the distribution of  $z$  as follows

$$P = \int_0^{\infty} w(z, Z) P(v > Y | z) dz \quad (2-13)$$

Using  $w(z, Z)$  for the generalized chi-squared distribution of Equation (1-1), this yields the generalized PD,

$$PG = \sum_{b=0}^{\infty} \frac{(K+b-1)!}{b!(K-1)!} \left(\frac{K}{K+Z}\right)^K \left(\frac{Z}{K+Z}\right)^b \sum_{m=0}^{N+b-1} \frac{Y^m}{m!} e^{-Y} \quad (2-14)$$

Shnidman changes the order of summation of these expressions so as to get a more direct measure of error which can be used to truncate the summation to a finite number of terms. This expression then becomes

$$PG = \sum_{m=0}^{N-1} Y^m + \sum_{M=N}^{\infty} Y^M \left(1 - \sum_{b=0}^{M-N} X_B\right)$$

where

$$Y^M = \frac{Y^M}{M!} e^{-Y}$$

and

$$X_B = \frac{(K+b-1)!}{b!(K-1)!} (1-V)^K V^b, \quad V = \frac{Z}{K+Z} \quad (2-15)$$

Note that a nonfluctuating target corresponds to the limit as  $K \rightarrow \infty$  for which

$$X_B \rightarrow \frac{Z^b}{b!} e^{-Z} \quad (2-16)$$

The error in Equation (2-15) for a truncated summation is shown by Shnidman to be given by the product,

$$\epsilon_M = \left(1 - \sum_{m=0}^M Y^m\right) \left(1 - \sum_{b=0}^{M-N} X_B\right) \quad (2-17)$$

The programs for PG given here test this product after each term of the PG summation and when it becomes less than  $10^{-8}$ , which seems a suitably small number, the summation is stopped.

An algorithmic program for PG is given in Figure 2-3, the program corresponding directly to the HP-65 PG program. As with the other programs, it is necessary to run Y-P2 first to find Y. The initiation of the PG program requires entry of both X, and the target distribution parameter, K.

Suitable values of K were discussed in the introduction, but the special case of a non-fluctuating target provides some difficulty since infinity is not an allowable entry value. The best large number to substitute for infinity in this case was found to be about  $10^5$  (entered with only two keystrokes as EEX 5). Larger values for K give difficulties for some values of  $Z = NX$  in calculating the initial value of XB, while smaller values are less accurate approximations of infinity. This compromise, however, apparently gives an accuracy for the calculated P0 of at least three places for any value of Z. To avoid the required entry of K in this case, as well as to provide greater accuracy if wanted, a modified form of the PG program is given here as the P0 program which calculates the detection probability for a nonfluctuating target. This is based on using Equation (2-16) for XB in place of the more general Equation (2-15). This eliminates the computation difficulty for a large value of K, since it is not used.

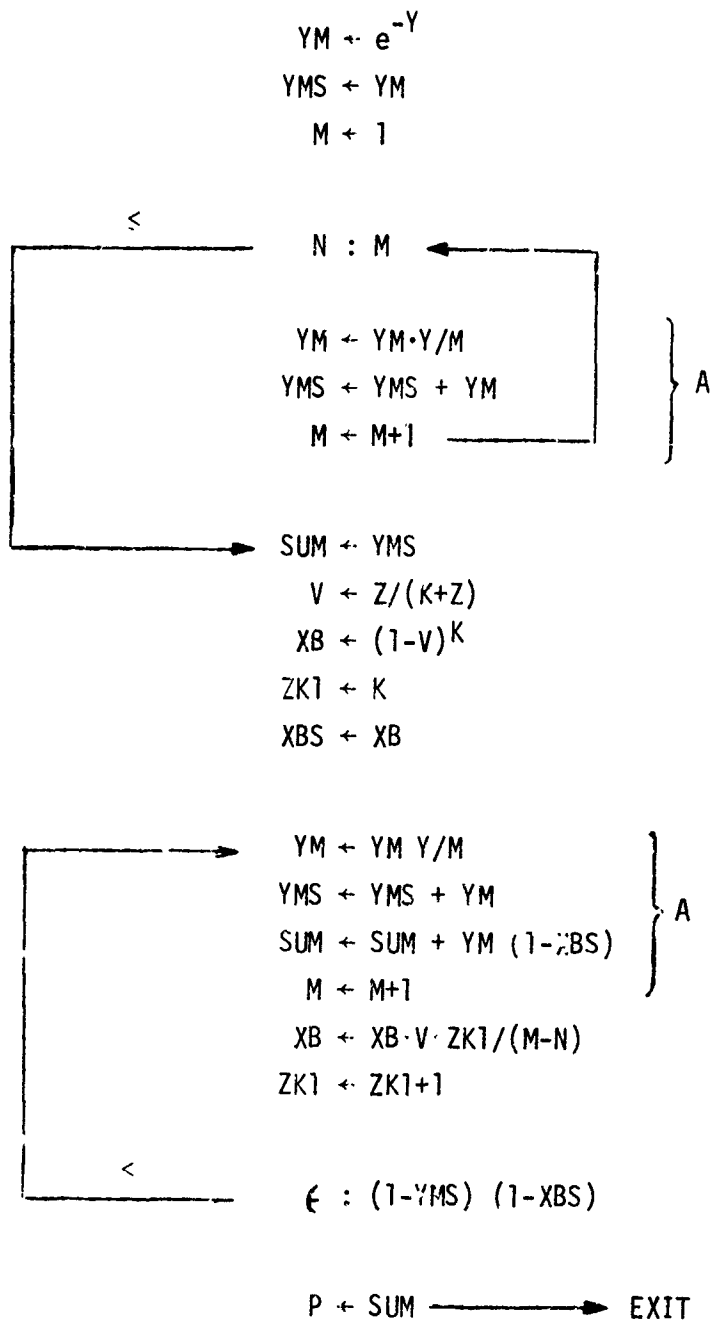


Figure 2-3. Algorithmic Program for PG



### SECTION III

#### FIXED THRESHOLD DETECTION - BARTON ALGORITHM

Barton<sup>3</sup> and Cann<sup>8</sup> were interested in a somewhat universal set of curves which could be used simply to find radar detection performance over various target and radar parameters. They found that an ideal detector curve plus a set of relatively simple loss factors, i. e., detector loss, integration loss, collapsing loss, and fluctuation loss, gave very reasonable accuracy for normal values of PD, PF, and N, and for the target distributions we have been considering. Barton's algorithms have also been programmed for the HP-65 and HP-67 and are included here. These programs are complementary to the PD programs previously given in that they calculate a required signal-to-noise ratio for a given PD rather than the other way around.

For the nonfluctuating target, and no collapsing ratio, the Barton/Cann algorithm can be written

$$\text{SNR (dB)} = 10 \log \left\{ \frac{1}{2} \left[ X + \sqrt{X(X+9.2)} \right] \right\}$$

$$X = \frac{1}{2N} \left[ Q^{-1}(\text{PF}) + Q^{-1}(\text{PD}) \right]^2$$

$$Q(y) = \frac{1}{2\sqrt{\pi}} \int_Y^{\infty} e^{-\frac{t^2}{2}} dt \quad (3-1)$$

(To correlate this with Barton's nomenclature  $X_0 = 2X$ )

The inverse Q function is calculated by the approximation.<sup>9</sup> For

$$P \leq \frac{1}{2}, \quad Q^{-1}(P) = t - \frac{a_0 + a_1 t}{1 + b_1 t + b_2 t^2} \quad (3-2)$$

where

$$a_0 = 2.31,$$

$$a_1 = 0.271,$$

$$b_1 = 0.992,$$

$$b_2 = 0.0448,$$

$$t = \sqrt{\ln(1/P^2)},$$

and for  $P > 1/2$ ,  $Q^{-1}(P) = -Q^{-1}(1 - P)$ .

Also, integration gain is given by:

$$G_i = \text{SNRN} - \text{SNR}_1$$

These are directly programmed in the HP-65 SNRN and HP-67 SNR programs.

For a nonfluctuating target with collapsing loss, or for a fluctuating target, the HP-65 SNRF and HP-67 SNR programs make the following calculation:

$$\text{SNRF (dB)} = \text{SNRN} + \log \frac{N}{N_s} + L_f, \quad (3-3)$$

where  $L_f$ , the fluctuation loss, is given by

$$L_f(\text{dB}) = \frac{10 \log D - \text{SNR}_1}{N_e} \quad (3-4)$$

SNRN is the value calculated by program SNRN for  $N$  and  $\text{SNR}_1$  is the value calculated by program SNRN for  $N = 1$ .  $D$  is the single pulse average SNR for fluctuating target detection and depends on the target model used. For the Rayleigh fluctuation model of Swerling's Cases 1 and 2

$$D_{12} = \frac{\ln PF}{\ln PD} - 1 \quad (3-5)$$



For the one dominant plus Rayleigh fluctuation model of Swerling's Cases III and IV,  $D_{34}$  is given implicitly by<sup>10</sup>

$$PD = \left( 1 - \frac{2 D_{34} \ln PF}{(2 + D_{34})^2} \right) PF^{\frac{2}{2 + D_{34}}} \quad (3-6)$$

The writer found that the solution to this equation is well approximated by

$$D_{34} = (0.361 - \log PD) \left( \frac{3.27}{\sqrt{1 - PD}} - 1.29 - 0.96 \sqrt{1 - PD} \right) - 2 \quad (3-7)$$

and this expression is used in HP-55 SNRF and HP-67 SNR to avoid the need for reiteration. The greatest error in this approximation occurs for low values of PD but it is accurate to better than 0.5 dB for PD equal 50% and within 1 dB for PD equal 30%. For PD greater than 90%, it is accurate to within 0.2 dB. An extremely bad choice of PD too low, or PF too large may cause  $D_{34}$  to be negative and flashing zeros will indicate this error when running the program.

Finally, the programs calculate the diversity gain as

$$G_d(\text{dB}) = (N_e - 1) L_f$$

and the range ratio, re Swerling

$$R/R_o = 10^{\text{SNRF}/40}$$



## SECTION IV

### CFAR DETECTION - RECURSIVE SOLUTION

#### 1. SWERLING CASE II

As with the fixed threshold, the Case II target model leads to a simple analysis and finite summation for finding the probability-of-detection. Starting with Equation (2-3), the constant  $Y$  can be replaced by the variable  $y$  to have

$$P(v > y) = \sum_{m=0}^{N-1} \left( \frac{y}{X+1} \right)^m \frac{1}{m!} e^{-\frac{y}{X+1}} \quad (4-1)$$

for each specific value of  $y$ .

Note on Figure 1-2 that  $y$  is derived from  $u$  and that  $u$  is the sum of  $R$  independent Rayleigh noise samples of unit average power - unity since we also normalized the magnitude of  $v$  to the average noise power. Therefore,  $u$  has the distribution

$$p(u) = \frac{u^{R-1}}{(R-1)!} e^{-u} \quad (4-2)$$

Then the overall probability of  $v$  exceeding  $y$  is given by

$$P = \int_0^{\infty} p(u) P\left(v > \frac{u}{T}\right) du \quad (4-3)$$

where  $T$  is a calibrating factor which must be set to achieve the desired false-alarm probability and is analogous in our further derivation here to  $Y$  which determined the false-alarm probability in the fixed threshold case. Substituting Equations (4-1) and (4-2) into Equation (4-3), interchanging the order of summation and integration, and integrating, one gets

$$P = \sum_{m=0}^{N-1} P_m = \sum_{m=0}^{N-1} \frac{(R+m-1)!}{m! (R-1)!} \frac{(T2)^R}{(T2+1)^{R+m}} \quad (4-4)$$

where  $T2 = T(X+1)$ .

In a similar manner to that for finding Y previously, let X = 0 so that T2 = T and find the value of T for which P equals the false-alarm probability.

This process is best done by using Newton's method on ln(P/PIN) as before so that

$$\Delta T = \frac{P \ln(P/PIN)}{\frac{dP}{dT}} \quad (4-5)$$

and we find from Equation (4-4) that

$$\frac{dP}{dT} = \frac{R}{T} \sum_{m=0}^{N-1} P_m - \sum_{m=1}^N m P_m \quad (4-6)$$

Denoting the last summation as Q, Equations (4-4), (4-5) and (4-6) yield

$$\Delta T = \frac{\ln(P/PIN)}{Q/P - R/T} \quad (4-7)$$

Since the terms of Q are closely related to those of P, both sums can be formed at the same time.

We are left with the problem of the initial value to use for T. Extending the curve fitting approach of before the writer found a reasonable initial value to be given by

$$\frac{1}{T_0} = \frac{1 - (PIN)^{1/R}}{(PIN)^{1/R}} \left[ (1-B)N+B \left( \frac{N - \sqrt{N}}{2.3L} + \frac{\sqrt{L} + \sqrt{N-1}}{\sqrt{L}} \right) \right] \quad (4-8)$$

$$B = \frac{R-1}{R+0.922}, \quad L = -\log PIN$$

The value of T0 from Equation (4-8) was found to provide a sufficiently good start for iterative convergence over the range of  $10^{-10} < P < 1$  and  $1 < R < 1000$ .

Unfortunately, this takes more than 50 program steps so that a separate program card is necessary for data entry and calculation of T0 with the HP-65. After running this program, HP-65 P2C can be used to perform the iterative calculation of T for a given false-alarm probability and P2 for various input values of SNR = 10 log X. If SNR is to be found for a given P2 program, HP-65 T0 must be rerun with P2 input, followed by HP-65 P2C again, the process being directly analogous to that of the HP-65 Y-P2 program for a fixed threshold. Since the HP-67 has more program storage, the T0 calculation is included in the HP-67 P2C program.

## 2. GENERALIZED TARGET MODEL

The relationship of the CFAR process to be fixed threshold process in general is the same as it was for Case II. Starting with Equation (2-14), let  $Y = u/T$  and integrate over the distribution of  $u$  from Equation (4-2) to find the overall PD. By this process, one obtains

$$PGC = \sum_{b=0}^{\infty} XB \sum_{m=0}^{N+b-1} PM$$

or

$$PGC = \sum_{m=0}^{N-1} PM + \sum_{m=N}^{\infty} PM \left[ 1 - \sum_{b=0}^{m-N} XB \right]$$

where

$$XB = \frac{(k+b-1)!}{b! (K-1)!} (1-V)^K V^b, \quad V = \frac{Z}{K+Z}$$

and

$$PM = \frac{(R+M-1)!}{m! (R-1)!} (1-A)^R A^m, \quad A = \frac{1}{T+1}.$$

(4-9)

Note that  $XB$  is the same as used for the fixed threshold case and for the nonfluctuating case

$$XB \rightarrow \frac{Z^b}{b!} e^{-Z}$$

Similarly, the fixed threshold case is approached by letting  $R \rightarrow \infty$  so that  $y = \frac{u}{T} \rightarrow \frac{R}{T} = Y$  so that

$$A \rightarrow \frac{Y}{Y+R}$$

and

$$PM \rightarrow \frac{Y^m}{m!} e^{-Y}$$

An algorithmic program for PGC is given in Figure 4-1 which follows closely that for PG. Unfortunately, this could not be fitted into 100 HP-65 program steps so it had to be programmed on two cards, HP-65 PGC(1) and PGC(2). PGC(1) incorporates the iteration for T given T0 and finds  $A = 1/(T+1)$ . Therefore, PGC is found by running in sequence HP-65 T0 to find T0, PGC(1) to find A and to enter SNR and K, and finally by running PGC(2) as noted on the CFAR Detection instruction sheet. For the HP-67, the entire calculation is included on the single program card, HP-67 PGC.

### 3. CFAR LOSS

The increase in SNR required with a CFAR detector, as compared to a fixed threshold detector, has been termed CFAR loss. This concept is a convenient one because the CFAR loss is essentially independent of the target fluctuation model, at least as far as the five Marcum and Swerling models are concerned. Although many papers in the literature deal with CFAR loss for various CFAR detector schemes, the paper by Mitchell and Walker<sup>5</sup> is the only one found by the writer to cover the combination of noncoherent signal integration with a background normalizer threshold. Using the HP-65 Programs of this paper for Case II targets, the data of Figures 4-2 through 4-5 were calculated and are presented here for convenience. The loss values from these curves may be used as a correction to fixed threshold SNRs for the fixed threshold programs. This may be handier than running the CFAR programs for many cases because of the long running time of the generalized CFAR program. Although the writer has verified in a few sample cases that other target models and other detection probabilities give essentially the same CFAR loss values, it will be left as an exercise for the reader to be convinced that this is true for the cases of concern.

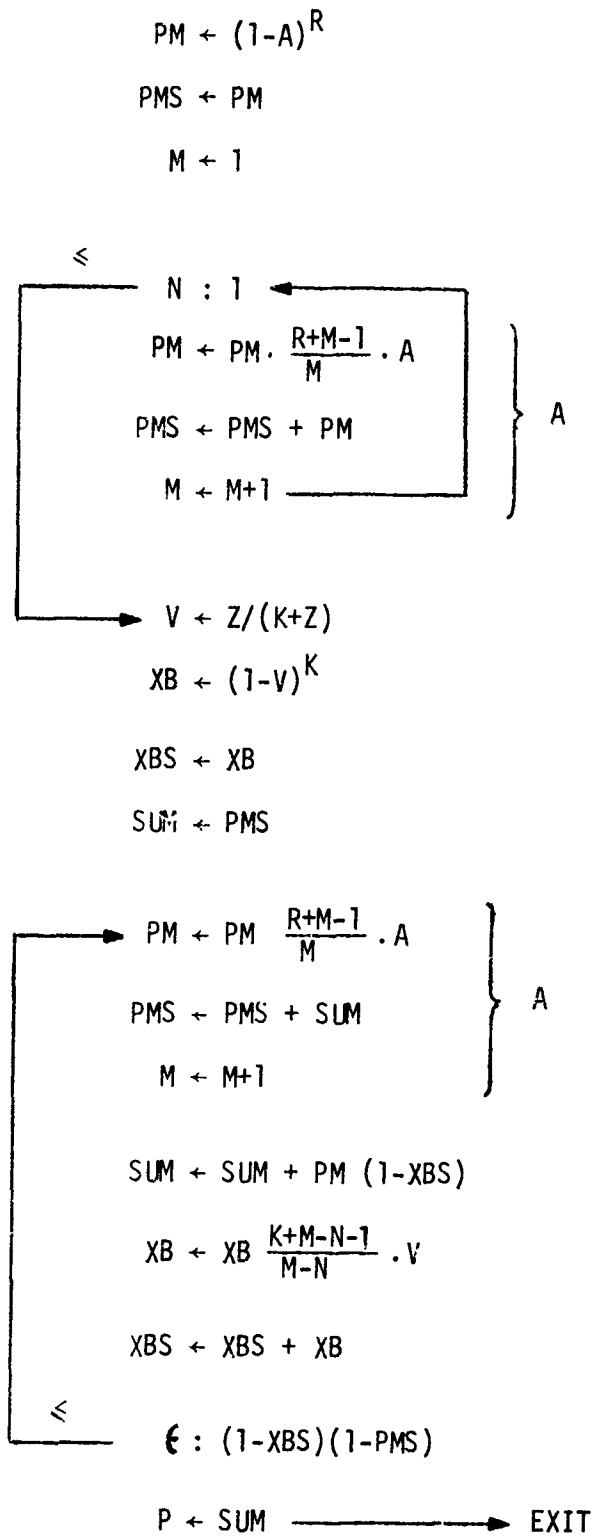


Figure 4-1. Algorithmic Program for PGC

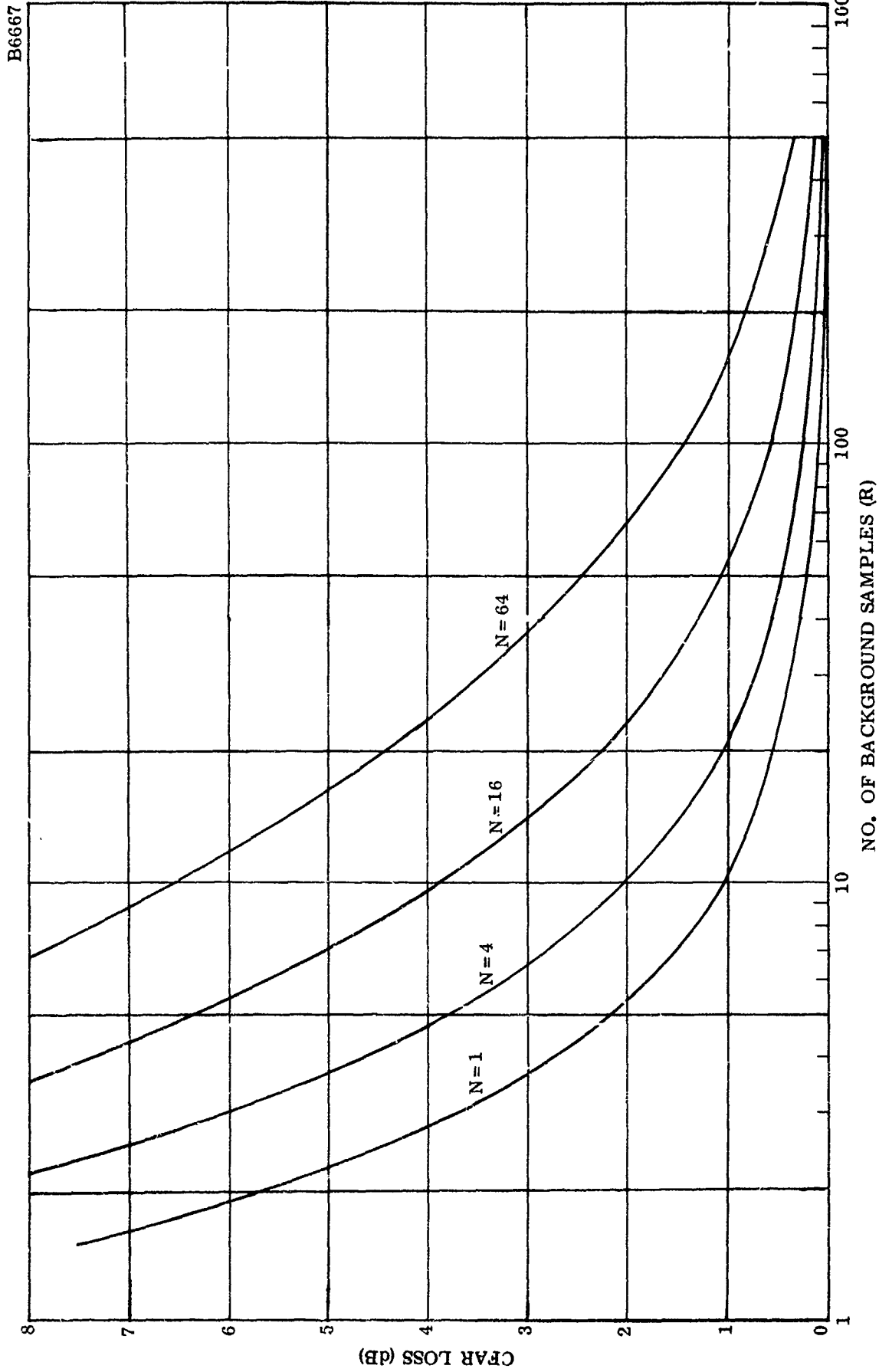


Figure 4-2. Square-Law CFAR Loss Swerling Case II Target,  $PF = 10^{-2}$ ,  $PD = 0.5$



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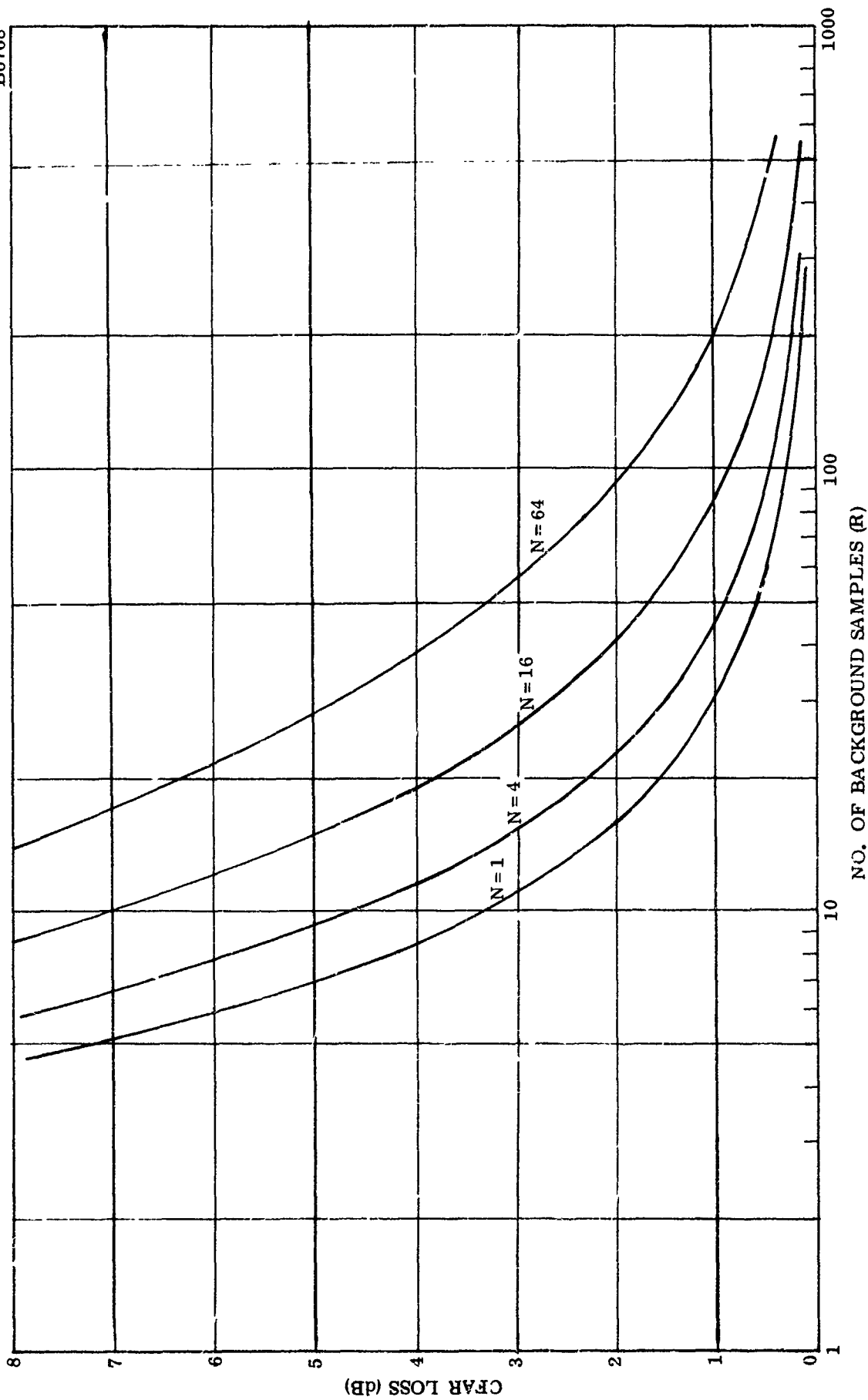


Figure 4-3. Square-Law CFAR Loss Swerling Case II Target,  $PF = 10^{-6}$ ,  $PD = 0.5$

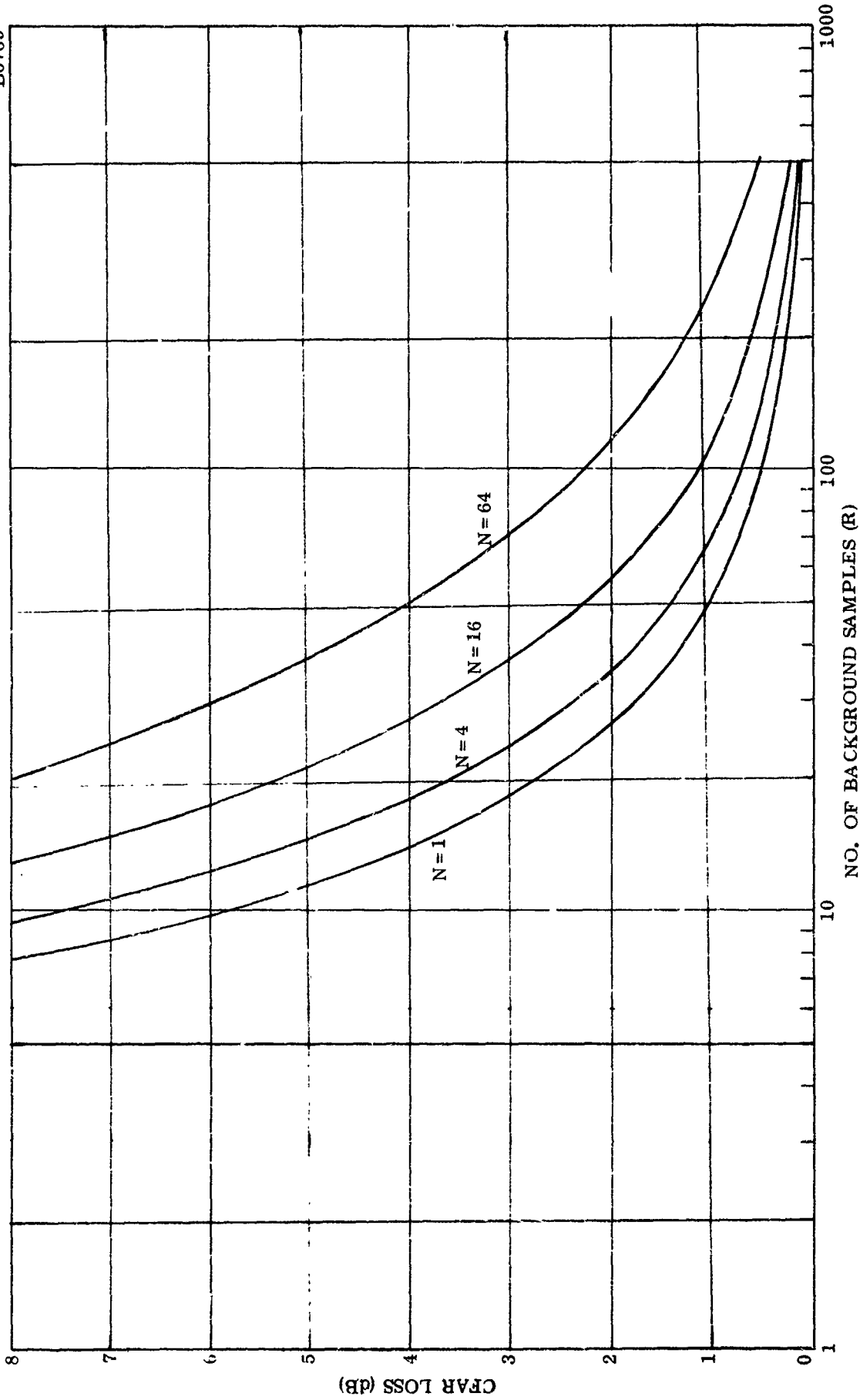


Figure 4-4. Square-Law CFAR Loss Swerling Case II Target,  $PF = 10^{-10}$ ,  $PD = 0.5$

SECTION V  
HP-65 PROGRAMS

1. FIXED-THRESHOLD, RECURSIVE SOLUTIONS

These HP-65 programs calculate the probability of detection, given the number of samples noncoherently integrated, the false-alarm probability, and the average sample signal-to-noise ratio, for the various Swerling target models. The PG program does this for the generalized chi-squared target model. The Y-P2 program must be used to calculate the threshold value, Y, before using any of the other programs, and in addition it can calculate for a Case II target, either probability of detection given average signal-to-noise ratio or average signal-to-noise ratio given probability of detection.

Specific user instructions are as follows:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program Y-P2	N	STO   1	
		PF	A	Y
	Go to step 2, 3, 4 or 6			
2	For P2 (Repeat or go to step 3 as desired)	X	B	P2
3	For X given P2	P2	A	Y2
			R/S	X
	Repeat or go to step 2, 4, or 6 as desired			
4	For P0, P1, P3, or P4 enter that program			
5		X	B	D
	Repeat or go to step 4 or enter Y-P2 and go to step 2 as desired			
6	For general target model enter program PG			
7		K		
		X	B	P
	Repeat or go to step 4 or enter Y-P2 and go to step 2 as desired			

K : Chi-squared distribution parameter  
 N : Signal and/or noise samples integrated  
 Ne : Target diversity within N samples  
 PF : Probability of false alarm  
 P : Probability of detection for chi-squared target  
 P0 : Probability of detection for nonfluctuating target  
 P1-P4 : Probability of detection for Cases I-IV  
 X : Avg sample power S/N within N samples  
 Y : Fixed detection threshold  
 Y2 : Y/(1+X)

Case 0 :  $K=10^5$  (~3 place acc)  
 Case I :  $K=1$   
 Case II :  $K=N$   
 (Genl Rayleigh target :  $K=Ne$ )  
 Case III :  $K=2$   
 Case IV :  $K=2N$   
 (Genl Rayleigh + equal constant target :  $K=Ne$ )  
 Weinstock :  $0 < K < 1$

# HP-65 Program Form

Title HP-65 Y-P2 Program Listing

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SWITCH TO W PRGM PRESS [ ] PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS	REGISTERS
STO4	3304	PIN	1	01		R <sub>1</sub> N
RCL2	3402	SAVE Y IN R6	RCL2	3402	Y	
STO6	3306		R/S	84		
RCL4	3404	PIN	RCL6	3406	RESTORE Y IN R2	R <sub>2</sub> Y
f	31		STO2	3302		
LOG	08		gx ← y	3507	Y2, Y	
CHS	42	L	÷	81		R <sub>3</sub>
RCL1	3401	N	1	01		
	41		-	51		
f	31		R/S	84	X ← Y/Y2 - 1	R <sub>4</sub> PIN
√	09	√N, N, L	LBL	23		
-	51	N - √N, L	B	12	X	
g STX	3500	√N	1	01		R <sub>5</sub>
1	01		+	61		
-	51	√N-1, N - √N, L →	RCL2	3402	SAVE Y IN R6	
gR↓	3509	L	STO6	3306		R <sub>6</sub> Y
f	31		gx ← y	3507		
√	09	√L	÷	81	Y2 ← Y/(1+X)	
+	61	√L + √N - 1, N - √N	STO2	3302		R <sub>7</sub>
g STX	3500	√L	E	15	P2	
x	71		RCL6	3406	RESTORE Y IN R2	
2	02		STO2	3302		R <sub>8</sub> YMS
.	83		gR↓	3508		
3	03		R/S	84	P2	
X	71	2, 3 √L(√L+√N-1, N-√N)	LBL	23		R <sub>9</sub>
+	61		E	15		
STO2	3302	Y0	RCL2	3402	Y	
LBL	23		CHS	42		LABELS
1	01		f-1	32		A
E	15	P	LN	07	YM ← e <sup>-Y</sup>	B X → P2
gR↓	3509	YM	STO8	3308	YMS ← YM	C
÷	81	P/YM	0	00	M ← 0	D YM Loop
RCL8	3408	P	LBL	23		E Y → P
RCL4	3404	PIN	D	14	M, YM	0
÷	81	P/PIN, P/YM	1	01		1
f	31		+	61	M ← M+1	2
LN	07		RCL1	3401	N	3
x	71	ΔY ← (P/YM) LN(P/PIN)	gx ≤ y	3522	N ≤ M	4
STO	33		RCL8	3408	P ← YMS	5
+	61		RTN	24		6
2	02	(Y ← Y+ΔY)	RCL2	3402	Y, N, M, YM	7
RCL2	3402	Y	gR↓	3509	YM, Y, N, M	8
÷	81		x	71	YM, Y, N, M →	9
g	35		gR↓	3509	M, YM · Y, N, M	
ABS	06	ΔY/Y	÷	81	YM ← YM · Y/M	FLAGS
EEX	43		STO	33		1
CHS	42		+	61		
6	06	-6	8	08	(YMS ← YMS + YM)	2
gx ≤ y	3522	10 ≤  ΔY/Y	gR↓	3509		
GTO	22		D	14	M, YM	

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# HP-65 Program Form

Title HP-65 P0 Program Listing

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SWITCH TO W/PRGM PRESS  PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS	REGISTERS
RCL1	3401	N	X	71		R <sub>1</sub> N
x	71		RCL6	3406		
STO3	3302	Z	1	01		
RCL2	3402		+	61		R <sub>2</sub> Y
CHS	42		STO6	3306	M ← M+1	
f-1	32		RCL1	3401		
LN	07		-	51	K ← M-N	R <sub>3</sub> Z
STO7	3307	YM ← e <sup>-Y</sup>	÷	81		
STO8	3308	YMS ← YM	STO4	3304	XB ← XB · Z/K	
1	01		+	61	XBS ← XBS + XB	R <sub>4</sub> XB
STO6	3306	M ← 1	↑	41		
LBL	23		CHS	42		
1	01		↑	41		R <sub>5</sub> SUM
RCL1	3401	N, M, YM	1	01		
EX<Y	3522		+	61	1 - XBS	
GTO	22		RCL8	3408	YMS	R <sub>6</sub> M
2	02	YM ← YM · Y/M	CHS	42		
A	11	YMS ← YMS + YM	1	01		
1	01		+	61		R <sub>7</sub> YM
RCL6	3406		x	71	(1 - YMS)(1 - XBS), XBS →	
+	61		EEX	43		
STO6	3306	M ← M+1	CHS	42		R <sub>8</sub> YMS
GTO	22		9	09		
1	01		EX ≤ Y	3522	10 <sup>-9</sup> ≤ (1 - YMS)(1 - XBS)	
LBL	23		GTO	22		R <sub>9</sub> Used
2	02		3	03		
RCL8	3408		RCL5	3405		
STO5	3305	SUM ← YMS	R/S	84	P0 ← SUM	LABELS
RCL3	3403		LBL	23		A YM, YMS
CHS	42		A	11	R	B
f-1	32		RCL7	3407	YM	C
LN	07		RCL2	3402	Y	D
STO4	3304	XB ← e <sup>-Z</sup>	RCL2	3406	M, Y, YM, R	E
gR ↓	3508	XBS ← XB	÷	81		0
LBL	23		x	71		1
3	03		STO7	3307	YM ← YM · Y/M, R →	2
gR ↓	3509	XBS { YM ← YM · Y/M	STO	33		3
A	11	{ YMS ← YMS + YM	+	61		4
gR ↓	3508	XBS	8	08	(YMS ← YMS + YM)	5
CHS	42		RTN	24		6
1	01					7
+	61					8
RCL7	3407					9
X	71	YM(1-XBS), XBS →				FLAGS
STO	33					1
+	61					2
5	05	(SUM ← SUM + YM(1-XBS))				
gR ↓	3509	XBS				
RCL4	3404	XB				
RCL3	3403	Z				

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# HP-65 Program Form

Title HP-65 P1 Program Listing Page        of       

SWITCH TO W PRGM    PRESS  $\uparrow$  [ PRGM ] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS	REGISTERS
RCL1	3401	N	E	15		R <sub>1</sub> N
X	71		STO7	3307	Y1	
STO3	3303	Z	CHS	42		
RCL2	3403	Y	f-1	32		R <sub>2</sub> Y
gx=y	3507	Z, Y	LN	07	YM ← e-YI	
1	01		STO8	3308	YMS ← YM	
+	61	Z+1, Y	1	01	M←1	R <sub>3</sub> Z
STO4	3304		LBL	23		
÷	81		D	14		
CHS	42		RCL1	3401	N	R <sub>4</sub> Z+1
f-1	32		1	01		Z/(Z+1)
LN	07		-	51	N-1	
STO5	3305	e-Y/(Z+1)	gx≤y	3522	N-1 ≤ M, YM	R <sub>5</sub> e-Y/(Z+1)
RCL1	3401	N	RCL8	3408	P←YMS, N-1, M, YM	S
1	01		RTN	24		
gx=y	3523	1=N	RCL7	3407	Y1, N-1, M, YM	R <sub>6</sub> P(N-1, Y)
RCL5	3405		gR↑	3509	YM	
R/S	84	P1←e-Y/(Z+1)	X	71	YM·Y1	
RCL2	3302	Y	gR↑	3509	M	R <sub>7</sub> Y1
E	15	P(N-1, Y), N-1	÷	f1	YM←YM·Y1/M	
STO6	3306		STO	33		
RCL3	3403	Z	+	61		R <sub>8</sub> YMS
RCL4	3404	Z+1, Z, P, N-1	8	08	(YMS←YMS+YM)	
÷	81		gR↑	3509	M	
STO4	3304	Z/(Z+1)	1	01		R <sub>9</sub> Used
gR↑	3509	N-1	+	61	M←M+1	
g	35		D	14		
y <sup>x</sup>	05	[Z/(Z+1)] <sup>N-1</sup>				LABELS
RCL5	3405					A
÷	81					B
STO5	3305	S←[Z/(Z+1)] <sup>N-1</sup> e-Y/(Z+1)				C
EEX	43					D YM Loop
7	07					E V→P
gx≤y	3522	10 <sup>7</sup> ≤ S				0
RCL6	3406					1
R/S	84	P1←P(N-1, Y)				2
CLX	44					3
RCL4	3404	Z/(Z+1)				4
RCL2	3402	Y				5
X	71	YZ/(Z+1)				6
E	15	P(N-1, YZ/(Z+1))				7
CHS	42					8
1	01					9
+	61	1-P				FLAGS
RCL5	3405	S				1
÷	81	(1-P)/S				2
RCL6	3406	P(N-1, Y)				
+	61					
R/S	84	P				
LBL	23					

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# HP-65 Program Form

Title HP-65 P3 Program Listing

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SWITCH TO W PRGM PRESS **F** **PRGM** TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS	REGISTERS
RCL1	3401	N	RCL5	3405	2Y/(Z+2)	R1 N
X	71		RCL3	3403	Z/2	
2	02		X	71	YZ/(Z+2)	
÷	81		E	15	P(N-1, YZ/(Z+2))	R2 Y
STO3	3303	Z/2	CHS	42		
RCL3	3403		1	01		
1	01		+	61	1-P	R3 Z/2
+	61	1+Z/2, Z/2	RCL5	3404	C	
÷	81	Z/(Z+2)	X	71	C(1-P)	
RCL2	3402	Y	STO4	3304		R4 C
gLSTX	3500	1+Z/2, Y	RCL2	3402	P	C(1-P)
÷	81	2Y/(Z+2), Z/(Z+2)	E	15	P(N-1, Y)	
STO5	3305		gR↑	3509	YM = Y <sup>N-2</sup> e <sup>-Y</sup> /(N-2)!	R5 2Y/(Z+2)
RCL1	3401	N	RCL5	3405	2Y/(Z+2)	
2	02		X	71		
gx ≤ y	3522	2 ≤ N	+	61	P(Y) + YM · 2Y/(Z+2)	R6 N-2
GTO	22		RCL4	3404	C(1-P)	
1	01		+	61		
gR↑	3509		R/S	84	P3	R7 Y3
gR↑	3509		LBL	23		
X	71	2YZ/(Z+2) <sup>2</sup>	E	15		R8 YMS
1	01		STO7	3307	Y3	
+	61		CHS	42		
RCL5	3405	2Y(Z+2)	f <sup>-1</sup>	32		
CHS	42		LN	07	YM ← e <sup>-Y3</sup>	R9
f <sup>-1</sup>	32		STO8	3308	YMS ← YM	
LN	07	-2Y/(Z+2)	1	01	M-1	
X	71	e <sup>[1+2YZ/(Z+2)]<sup>2</sup></sup>	LBL	23		
R/S	84		D	14		LABELS
LBL	23		RCL1	3401	N	A
1	01		1	01		B
-	51	N-2	-	51	N-1	C
STO6	3306		gx ≤ y	3522	N-1 ≤ M, YM	D YM Loop
gR↑	3509	Z/(Z+2), N-2, 2Y/(Z+2)	RCL8	3408	P-YMS, N-1, M, YM	E Y→P
f	31		RTN	24		0
LN	07		RCL7	3407	Y3, N-1, M, YM	1
X	71	-(N-2) ln[(Z+2)/Z]	gR↑	3509	YM	2
+	61	EXP ← 2Y/(Z+2) - (N-2)ln[ ]	X	71	YM · Y3	3
CHS	42		gR↑	3509	M	4
f <sup>-1</sup>	32		÷	81	YM ← YM · Y3/M	5
LN	07	e <sup>-EXP</sup>	STO	33		6
1	01		+	61		7
RCL6	3406		8	08	(YMS ← YMS+YM)	8
RCL3	3403	Z/2, N-2, 1, e <sup>-EXP</sup>	gR↑	3509	M	9
÷	81	2(N-2)/Z	1	01		FLACS
-	51	1-2(N-2)/Z	+	61	M ← M+1	1
RCL5	3405	2Y/(Z+2)	D	14		2
+	61		-EXP			
X	71	C ← [1-2(N-2)/Z+2Y/(Z+2)] e <sup>-EXP</sup>	100			
STO4	3304					

HEWLETT-PACKARD

# HP-65 Program Form

Title HP-65 P4 Program Listing Page      of     

SWITCH TO W/PRGM PRESS  PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS	REGISTERS
0	00		1	01		R <sub>1</sub> N
STO6	3306	M ← 0	+	61	M ← N+1, 2N-M, ZKS	
gR	3508		÷	81	[(2N-M)/(M-N+1)], ZKS	
2	02		RCL14	3404	ZK	R <sub>2</sub> Y
÷	81		RCL3	3403	S, ZK, [ ], ZKS	
STO3	3303	X/2	X	71		
1	01		X	71		R <sub>3</sub> X/2 = S
+	61		STO4	3304	ZK ← ZK · S [ ], ZKS	
STO7	3307	1+X/2	+	61	ZKS ← ZKS+ZK	
RCL1	3401	N	RCL6	3406		R <sub>4</sub> ZK
CHS	42		1	01		
g	35		+	61		
Y <sup>x</sup>	05		STO6	3306	M ← M+1	R <sub>5</sub> YM
STO4	3304	ZK ← [2/(X+2)] <sup>N</sup>	RCL1	3401	N	
RCL2	3402	Y	2	02		
RCL7	3407	1+X/2	X	71		R <sub>6</sub> M
÷	81		gx ≤ y	3522	2N ≤ M	
STO7	3307	V ← 2Y/(X+2)	RCL8	3408		
CHS	42		R/S	84	P4 ← SUM	R <sub>7</sub> 1+X/2
f-I	32		gR↑	3509	ZKS	V
LN	07	YM ← e <sup>-V</sup>	RCL5	3405	YM, ZKS	
STO8	3308	YMS ← YM	GTO	22		R <sub>8</sub> YMS
E	15	SUM ← P(N, V)	1	01		SUM
gR↓	3508		LBL	23		
STO5	3305	YM	E	15	YM	
RCL4	3404	ZK	RCL6	3406		R <sub>9</sub>
gx ← y	3522	YM, ZKS ← ZK	1	01		
LBL	23		+	61		LABELS
1	01	YM, ZKS	STO6	3306	M ← M+1	A
RCL7	3407		RCL1	3401		B
RCL6	3406	M, V, YM, ZKS	gx ≤ y	3522	N ≤ M	C
÷	81		gR↓	3508	M, YM	D
X	71		RTN	24		E YM Loop
STO5	3305	YM ← YM · V/M	gR↓	3508	M, YM	0
1	01		÷	81	YM/M	1
gR↑	3509		RCL7	3407	V	2
-	51	1-ZKS, YM, ZKS	X	71	YM ← YM · V/M	3
X	71		STO	33		4
STO	33		+	61		5
+	61		8	08	(YMS ← YMS + YM)	6
8	08	(SUM ← SUM + YM(1-ZKS))	E	15		7
CLX	44					8
RCL1	3401	N, ZKS				9
2	02					FLAGS
X	71					1
RCL6	3406	M, 2N, ZKS				2
-	51	2N-M				
gLSTX	3500	M				
RCL1	3401	N				
7	51	M-N				

HP-65 P4



# HP-65 Program Form

Title HP-65 PG Program Listing

Page        of       

SWITCH TO W PRGM PRESS    PRGM    TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS	REGISTERS
RCL1	3401	N, X, K	RCL3	3403	V	R <sub>1</sub> N
X	71	$\left. \begin{array}{l} \text{---} \\ \text{---} \end{array} \right\} \begin{array}{l} \text{K} \\ \text{X} \end{array}$	X	71	XB · V/B	
STO3	3303		Z	gx ← y	3507	ZK1
gR↓	3508		X	71		R <sub>2</sub> Y
STO4	3303	K	STO4	3304	XB ← XB · ZK1 · V/(M-N)	
RCL2	3402	Y	gLSTX	3500	ZK1	
CHS	42		gR↓	3508	XB, (1-XBS), (1-XBS), ZK1	R <sub>3</sub> Z
f <sup>-1</sup>	32		-	51	(1-XBS) ← (1-XBS) - XB	V
LN	07		gR↑	3509	ZK1, (1-XBS)	
STO7	3307	YM ← e <sup>-Y</sup>	1	01		R <sub>4</sub> K
STO8	3308	YMS ← YM	+	61	ZK1 ← ZK1+1	XB
1	01		1	01		
STO6	3306	M ← 1	RCL8	3408	YMS, 1, ZK1, (1-XBS)	R <sub>5</sub> SUM
LBL	23		-	51		
1	01		gR↑	3509		
RCL1	3401	N, M	X	71	(1-XBS)(1-YMS), ZK1, (1-XBS)	M
gx ≤ y	3522	N ≤ M	EEX	43		
GTO	22		CHS	42		
2	02	$\left. \begin{array}{l} \text{YM} \leftarrow \text{YM} \cdot \text{Y/M} \\ \text{YMS} \leftarrow \text{YMS} + \text{YM} \\ \text{M} \leftarrow \text{M} + 1 \end{array} \right\}$	8	08	10 <sup>-8</sup> , ( ) ( )	R <sub>7</sub> YM
A	11			gx > y	3424	
GTO	22		RCL5	3405	P ← SUM	R <sub>8</sub> YMS
1	01		R/S	84		
LBL	23		gR↑	3509		
2	02		gR↑	3509	ZK1, (1-XBS)	
RCL8	3408	YMS	GTO	22		R <sub>9</sub> Logic
STO5	3305	SUM ← YMS	3	03		
1	01		LBL	23		
RCL3	3403	Z	A	11	ZK1, (1-XBS)	
RCL3	3403	Z	RCL7	3407	YM	LABELS
RCL4	3404	K, Z, Z, 1	RCL2	3402	Y, YM, ZK1, (1-XBS)	A Recursion
+	61		X	71	YM · Y	B
÷	81		RCL6	3406	M, YM · Y, ZK1, (1-XBS)	C
STO3	3303	V ← Z/(K+Z)	÷	81		D
-	51	1-V	STO7	3307	YM ← YM · Y/M	E
RCL4	3404	K	STO	33		0 Used
g	35		+	61		1 Used
y <sup>x</sup>	05		8	08	(YMS ← YMS + YM)	2 Used
STO4	3304	XB ← (1-V) <sup>K</sup>	gR↑	3509	(1-XBS), YM, ZK1(1-XBS)	3 Used
gLSTx	3500	ZK1 ← K, XB, 1 ←	X	71	YM(1-XBS), ZK1, ( ) ←	4
gR↓	3508	XBS ← XB, 1, 1, ZK1	STO	33		5
-	51	(1-XBS), 1, ZK1 ←	+	61		6
gR↑	3509	ZK1, (1-XBS)	5	05	(SUM ← SUM + YM(1-XBS))	7
LBL	23	YM ← YMS · Y/M	gR↓	3508	ZK1, ( )	8
3	03	YMS ← YMS + YM	RCL6	3406	M	9
A	11	M SUM ← SUM + YM(1-XBS)	1	01	1, M, ZK1, (1-XBS)	FLAGS
RCL1	3401	N M ← M+1	+	61		1
-	51	M-N	STO6	3306	M ← M+1	2
RCL4	3404	XB	RTN	24		
gx → y	3507	m-N, XB, ZK1, (1-XBS)				
÷	81	XB/(M-N)				

HP-65 PROGRAM PACKING



## 2. FIXED-THRESHOLD, BARTON ALGORITHM

These HP-65 programs calculate the required average SNR in dB for a given detection probability and target model. The target model is specified by its probability density function, i. e., nonfluctuating, Rayleigh (ala Swerling Case I) or Rayleigh plus an constant component of equal power (ala Swerling Case III), and by its diversity order,  $N_e$ , defined as the number of independent target values within the N samples noncoherently integrated. It is always necessary to run SNRN, the calculation for a nonfluctuating target, after entering N, PF, and PD and before running SNRF, the calculation for any fluctuating target.

Specific user instructions follow:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program SNRN		<input type="text"/>	<input type="text"/>	
2	(Repeat as desired)	N	STO	1	
		PD $\geq$ 0.3	STO	2	
		PF $\leq$ 0.5	STO	3	
	If desired		A	<input type="text"/>	SNRN dB
	To include collapsing loss or tgt fluct go to step 3		-	<input type="text"/>	Gi dB
3	Enter program SNRF		<input type="text"/>	<input type="text"/>	
	Samples containing signal	$1 \leq N_s \leq N$	STO	6	
	Order of target diversity	$1 \leq N_e \leq N_s$	STO	7	
	Go to step 4, 5, or 6		<input type="text"/>	<input type="text"/>	
4	For nonfluctuating target		A	<input type="text"/>	SNRC dB
5	For Rayleigh const tgt (Inc Cases III & IV)		B	<input type="text"/>	SNRF dB
6	For Rayleigh & const tgt (Inc Cases III & IV)		C	<input type="text"/>	SNRF dB
	If desired		R/S	<input type="text"/>	Lf dB
			R/S	<input type="text"/>	Gd dB
	(Steps 4, 5, and 6 may be repeated in any order)		E	<input type="text"/>	R/R <sub>0</sub>

- Gd : Diversity gain (dB)
- Gi : Integration gain (dB)
- Lf : Fluctuation loss (dB)
- N : Signal and/or noise samples integrated
  - For Cases I and III :  $N_e=1$
  - For Cases II & IV :  $N_e=N_s$
- Ns : Samples within N containing signal
- PD : Probability of detection
- PF : Probability of false alarm
- R/R<sub>0</sub> : Ratio of detection range to that for which SNRf = 0 dB
- SNRC : SNR per sample for nonfluctuating target w/collapsing loss (dB)
- SNRF : Avg SNR per sample for fluctuating target (dB)
- SNRN : SNR per sample for nonfluctuating target (dB)

# HP-65 Program Form

Title HP-65 SNRN Program Listing Page        of       

SWITCH TO W. PRGM    PRESS   PRGM    TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS	REGISTERS
0	00		4	04		R <sub>1</sub> N
STO8	3308		4	04		
RCL2	3402	PD	8	08	$b_2 = 0.0448$	
2	02		X	71	$b_2t$	R <sub>2</sub> PD
X	71		.	83		
1	01		9	09		
gx>y	3524	$1 > 2PD$	9	09		R <sub>3</sub> PF
STO8	3308		2	C2	$b_1 = 0.992$	
gR↓	3508	2PD	+	61	$b_1+b_2t$	
RCL2	3402	PD	X	71	$b_1t+b_2t^2$	R <sub>4</sub> $-Q^{-1}(PD)$
-	51	{ PD for PD < 0.5	1	01		X1
B	12	{ 1-PD for PD > 0.5	+	61	$1+b_1t+b_2t^2=D$	R <sub>5</sub> SNR1
g	35		gR↑	3509	t	SNRN
DSZ	83		.	83		
CHS	42		2	02		
gNOP	3501		7	07		R <sub>6</sub>
STO4	3304	$-Q^{-1}(PD)$	1	01	$a_1 = 0.271$	
RCL3	3403	PF	X	71		
B	12	$Q^{-1}(PF)$	2	02		R <sub>7</sub>
RCL4	3404		.	83		
-	51	$[Q^{-1}(PF)+Q^{-1}(PD)]$	3	03		
↑	41		1	01	$a_0 = 2.31$	R <sub>8</sub> SNRN
X	71	$[ ]^2$	+	61	$a_0 = a_1t = N$	
2	02		gx←y	3507	D	
÷	81		÷	81	N/D	R <sub>9</sub> Used
STO4	3304	X1	-	51	$Q^{-1}(P)$	
RCL1	3401	N	STO	33		
÷	81	XN	9	09		
E	15		RTN	24	$Q^{-1}(P)$	LABELS
STO5	3305	SNRN	LBL	23		A B $Q^{-1}(P)$
STO8	3308		E	15	X	C
RCL4	3404	X1	↑	41		D
E	15		↑	41		E SNRX
STO4	3304	SNR1	↑	41		
RCL5	3405		9	09		
R/S	84	SNRN	.	83		
LBL	23		2	02		
E	12	P	+	61	$X+0.92$	
↑	41		X	71	$X(X+0.92)$	
X	71	$P^2$	f	31		
f	31		√	09	$\sqrt{X(X+0.92)}$	
LN	07	$\ln P^2$	+	61	$x + \sqrt{\quad}$	
CHS	42		2	02		
f	31		÷	81		
√	09	t	f	31		
↑	41		LOG	08		FLAGS
↑	41		1	01		1
↑	41		0	00		
.	83		X	71		2
0	00		RTN	24	SNRX	

# HP-65 Program Form

Title HP-65 SNRF Program Listing

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SWITCH TO W/PRGM PRESS [ ] [PRGM] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS	REGISTERS
LBL	23		X	71		R1 N
A	11		2	02		
0	00		-	51	D34	
GTO	22		LBL	23		R2 PD
1	01		2	02	D	
LBL	23		f	31		
B	12		LOG	08		R3 PF
RCL3	3403	PF	1	01		
f	31		0	00		
LN	07		X	71	10 log D	R4 SNR1
RCL2	3402	PD	RCL4	3404	SNR1	
f	31		-	51		
LN	07		RCL7	3407	Ne	R5 SNRN
÷	81	ln PF/ln PD	÷	81	Lf	SNRF
1	01		LBL	23		
-	51	D12	1	01	lf	R6 Ns
GTO	22		↑	41		
2	02		↑	41		
LBL	23		RCL1	3401	N	R7 Ne
C	13		RCL6	3406	Ns	
.	83		÷	81		
3	03		f	31		R8 SNRN
6	06		LOG	08		
1	01		1	01		
RCL3	3403	PF	0	00		R9
f	31		X	71	10 log (N/Ns)	
LOG	08		+	61		
-	51	(.361 - log PF)	RCL8	3408	SNRN	LABELS
3	03		+	61		A Nonfluct
.	83		STO5	3305		B Sw 1&2
2	02		R/S	84	SNRF	C Sw 3&4
7	07		gR↓	3508		D
RCL2	3402	PD	R/S	84	Lf	E
CHS	42		RCL7	3407	Ne	0
1	01		1	01		1 Used
+	61		-	51	Ne-1	2 Used
.	31		X	71		3
√	09		R/S	84	Gd	4
÷	81	3.27/√1-PD	LBL	23		5
GLSTX	3500	√1-PD	E	15		6
.	83		RCL5	3405	SNRF	7
9	09		CHS	42		8
6	06		4	04		9
X	71		0	00		
-	51	(3.27/√1-PD) - 0.98√1-PD	÷	81		FLAGS
1	01		f-1	32		1
.	83		LOG	08		
2	02		R/S	84	R/Ro	2
9	09					
5	51	( )				

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### 3. CFAR DETECTION, RECURSIVE SOLUTIONS

These HP-65 programs calculate the detection probability, given the average sample SNR and target model for an adaptive detector threshold which is set proportional to the noncoherent integration of R noise samples. These programs require initial calculation of the threshold proportionality constant, T, or equivalently  $A = 1/(T+1)$ . Two cards must be entered for the P2 case or three cards must be entered for the general case. The partitioning is such that the first program calculates T0, a starting value of an iterative solution for T. The HP-65 P2C program does the iteration for T and also calculates P2. By rerunning HP-65 T0 with a given P2, the required average SNR can also be calculated by HP-65 P2C.

The programs, PGC(1) and PGC(2), are used with T0 to calculate detection probability for the general chi-squared target model.

Specific user instructions follow:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program T0	N	<input type="checkbox"/> ↑ <input type="checkbox"/>	T0
		R	<input type="checkbox"/> ↑ <input type="checkbox"/>	
		PF	<input type="checkbox"/> A <input type="checkbox"/>	
		Go to step 2 or 6	<input type="checkbox"/> <input type="checkbox"/>	
2	Enter program P2C		<input type="checkbox"/> R/S <input type="checkbox"/>	T
		Go to step 3 or 4	<input type="checkbox"/> <input type="checkbox"/>	
3	For P2	SNR dB	<input type="checkbox"/> B <input type="checkbox"/>	P2
		Repeat for new SNR or go to step 4 or 6 as desired	<input type="checkbox"/> <input type="checkbox"/>	
4	For SNR, given P2, enter program T0	P2	<input type="checkbox"/> C <input type="checkbox"/>	T20
		Enter program P2C	<input type="checkbox"/> R/S <input type="checkbox"/>	
5	Repeat steps 4 & 5 for new P2 or to to step 3 or 6 as desired		<input type="checkbox"/> <input type="checkbox"/>	SNR dB
			<input type="checkbox"/> <input type="checkbox"/>	
6	For general target model enter program PGC(1)		<input type="checkbox"/> R/S <input type="checkbox"/>	A
		SNR dB	<input type="checkbox"/> ↑ <input type="checkbox"/>	
7		K	<input type="checkbox"/> A <input type="checkbox"/>	1
			<input type="checkbox"/> <input type="checkbox"/>	
8	Enter program PGC(2)		<input type="checkbox"/> R/S <input type="checkbox"/>	P
		For new SNR or K enter PGC(1) and go to step 7	<input type="checkbox"/> <input type="checkbox"/>	

- |       |  |  |
|-------|--|--|
| A     | : $1/(T+1)$  | { Case 0 : $K=10^5$ ( 3 place acc)<br>Case I : $K=1$<br>Case II : $K=N$<br>(Genl Rayleigh target : $K=Ne$ )<br>Case III : $K=2$<br>Case IV : $K=2N$<br>(Genl Rayleigh + equal constant target : $K=2Ne$ )<br>Weinstock : $0 < K < 1$ |
| K     | : Chi-squared distribution parameter                 |  |
| N     | : Signal and/or noise samples integrated             |  |
| PF    | : Probability of false alarm                         |  |
| P     | : Probability of detection for chi-squared target    |  |
| P0    | : Probability of detection for nonfluctuating target |  |
| P1-P4 | : Probability of detection for Swerling Cases I-IV   |  |
| R     | : Noise samples integrated to set threshold          |  |
| SNR   | : $10 \log X$  |  |
| T     | : Threshold setting divisor                          |  |
| T0    | : Iterative solution start value for T               |  |
| T20   | : Iterative solution start value for $T2=T/(1+X)$    |  |
| X     | : Average sample S/N within N samples                |  |

# HP-65 Program Form

Title HP-65 T0 Program Listing

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SWITCH TO W/PRGM PRESS [ ] PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS	REGISTERS	
LBL		← P2 C	2			R1 N	
C			2				
f			+		R+, 922, R-1		
SF1			÷		B	R2 R	
STO4	P2	} SAVE TF IN R5	X		B( )		
RCL3	TF			1			
STO8				gLSTx		B, I, B( )	R3 T0
GTO			-				
1			RCL1		N		
LBL		N	X		(1-B) N, B( )	R4 PIN	
A				+		[ ]	
STO4	PF, R, N	R	1				
g↓				RCL4		PIN	R5
STO2	R	PF	RCL2		R		
g↓				g			
STO1	N	A	1/X			R6	
LBL			g				
1			y <sup>x</sup>		PIN <sup>1/R</sup> , 1, [ ]		
RCL4	PIN		-			R7	
f			gLSTx				
LOG			÷		(1-PIN <sup>1/R</sup> /R)/PIN <sup>1/R</sup> , [ ]		
CHS	L		X		1/T0	R8 TF	
RCL1	N		g				
↑			1/X				
f			STO3		T0	R9	
√	√N		R/S				
-	N-√N, L						
gLSTx	√N						
1							
-	√N-1, N-√N, L→						
gR↑	L					LABELS	
f						A PIN ← PF	
√	√L					B	
+	√L+√N-1					C PIN ← P2	
gLSTx	√L					D	
÷	(√L+√N-1)/√L, N-√N, L→					E	
gX→Y						0	
gR↑	L, N-√N					1	
÷						2	
2						3	
.						4	
3						5	
÷	(N-√N)/2.3L, (√L+√N-1)/√L					6	
+						7	
RCL2	R					8	
1						9	
-	R-1					FLAGS	
RCL2	R					1 PIN ← P2	
.							
0						2	



# HP-65 Program Form

Title HP-65 P2C Program Listing

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SWITCH TO W/PRGM PRESS  $f^{-1}$  PRCM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS	REGISTERS
RCL3		T0	$f^{-1}$			R <sub>1</sub> N
D		P	LOG			
RCL4		PIN	1			
$\div$		P/PIN	+			R <sub>2</sub> R
f			RCL3		T	
LN		ln(P/PIN)	X		T2 ← T(1+X)	
RCL6		Q	D			R <sub>3</sub> T
RCL5		P	R/S		P2	
$\div$		Q/P	LBL			
RCL2		R	D		T	R <sub>4</sub> PIN
RCL3		T	$\uparrow$			
$\div$		R/T, Q/P, ln(P/PIN)	$\uparrow$			
-			1			R <sub>5</sub> PMS
$\div$		$\Delta T$	+			
STO			STO7		T+1	
+			$\div$			R <sub>6</sub> QMS
3		(T ← T+ $\Delta T$ )	RCL2		R	
RCL3		T	g			
$\div$			y <sup>x</sup>		PM ← [T/(T+1)] <sup>R</sup>	R <sub>7</sub> T+1
g			STO5		PMS ← PM	
ABS			0		M ← 0	
EEX			STO6		QMS ← 0	R <sub>8</sub> T
CHS			LBL			
6		10 <sup>-6</sup> ,   $\Delta T/T$	J		M, PM	
$EX \leq Y$			RCL2		R	R <sub>9</sub> logic
GTO			$EX \leftarrow Y$		M, R, PM	
0		RETURN TO PRGM START	+		R+M	
RCL3		T	gLSTx		M	LABELS
$f^{-1}$			g↓		R+M, PM, ~ M	A
TF1			X			B SNR → P2
R/S		T	RCL7		T+1	C
gNOP			$\div$		M · PM ← PM(R+M)/(T+1)	D T → P
$f^{-1}$			STO			E
SF1			+			PRGMST
RCL8		T, T2 } RESTORE T	6		(QMS ← QMS + M · PM)	1
STO3		} IN R3	gR↑		M	2
$\div$			1			3
i			+		M ← M+1	4
-		X	RCL1		N, M, M · PM	5
f			$EX \leq Y$			6
LOG			RCL5			7
1			RTN		P ← PMS	8
0			gR↓		M, M · PM	9
Y			$\div$		PM	
R/S		SNR dB	STO			FLAGS
LBL			+			1 P2 ← PIN
B		SNR dB	5		(PMS ← PMS + PM)	
1			gLSTx		M, PM	2
0			GTO			
$\div$			1			

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# HP-65 Program Form

Title HP-65 PGC(1) Program Listing

Page      of     

SWITCH TO W/PRGM PRESS **[f]** **[PRGM]** TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS	REGISTERS
LBL			RCL5		P ← PMS	R <sub>1</sub> N
0			÷		Q/P	
RCL3			RCL2		R	R <sub>2</sub> R
RCL3			RCL3		T	
1		1, T, T	÷		R/T, Q/P, ln (P/PIN)	
+			-			
STO7		T+1	÷		ΔT	R <sub>3</sub> T
÷			STO			A
RCL2		R	+			
<del>R</del>			3		(T ← T+ΔT)	R <sub>4</sub> PIN
<del>y<sup>x</sup></del>		PM ← [T/(T+1)] <sup>R</sup>	RCL3		T	Z
STO5		PMS ← PM	÷			
0		M ← 0	g			R <sub>5</sub> PMS
STO6		QMS ← 0	ABS		ΔT/T	K
LBL			EEx			
1		M, PM	CHS			R <sub>6</sub> QMS
RCL2		R	6		10 <sup>-6</sup> ,  ΔT/T	M
gx → y		M, R, PM	gx ≤ y			
+		R+M	GTO			R <sub>7</sub> T+I
gLSTx		M	0			PM
gR↓		R+M, PM, ~, M	RCL3		T	
X			1			R <sub>8</sub> PMS
RCL7		T+1	+			
÷		M · PM	g			
STO			1/X			R <sub>9</sub> logic
+			STO3		A ← 1/(T+1)	
6		(QMS ← QMS+M · PM)	R/S			
gR↓		M	LBL			
1			A			LABELS
+		M ← M+1	STO5		K	SNR, K entry
RCL1		N, M	gR↓		SNR	A
gx ≤ y			1			B
GTO			0			C
2			÷			D
gR↓		M, M · PM	f-1			E
÷		PM	LOG		X	0
STO			RCL1		N	1
+			X			2
5		(PMS ← PMS+PM)	STO4		Z	3
gLSTx		M, PM	1			4
GTO			RCL3		A	5
1			-			6
LBL			RCL2		R	7
2			g			8
RCL5		P ← PMS	y <sup>x</sup>			9
RCL4		PIN	STO7		PM ← (1-A) <sup>R</sup>	FLAGS
÷			STO8		PMS ← PM	1
f			1			2
LN		ln (P/PIN)	STO6		M ← 1	
RCL6		Q ← QMS	R/S			

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# HP-65 Program Form

Title HP-65 PGC(2) Program Listing Page        of       

SWITCH TO W/PRGM PRESS [ ] [ PRGM ] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS	REGISTERS
LBL			gLSTx			R <sub>1</sub> N
1		1	÷		(K+M-N-1)/(M-N), SUM, K-1	→
RCL1		N	RCL4		V	
gx < y			X			R <sub>2</sub> R
GTO			RCL5		XB	
2			X			
A			STO5		XB ← XB · V(K+M-N-1)/(M-N)	R <sub>3</sub> A
GTO			STO			
1			-			
LBL			9		{(1-XBS) ← (1-XBS)-XB}	R <sub>4</sub> Z
2			CLX			V
1			1			
RCL4			RCL8		PMS, 1, SUM, K-1	R <sub>5</sub> K
RCL4			-		1-PMS	XB
RCL5		K, Z, Z, 1	EEX			
+			8			R <sub>6</sub> M
÷			X			
STO4		V ← Z/(K+Z), 1 →	g			
-			1/X			R <sub>7</sub> PM
RCL5		K	RCL			
g			9		1-XBS, 1/10 <sup>8</sup> (1-PMS), SUM, K-1	K-1
y <sup>x</sup>			gx > y			R <sub>8</sub> PMS
STO5		XB ← [K/(K+Z)] <sup>K</sup>	GTO			
gLSTx		K	3			
1			gR ↓			R <sub>9</sub> logic
-		K-1	gR ↓			1-XBS
RCL8		SUM, K-1, XB, 1	R/S		P ← SUM	
gR ↓			LBL			
gR ↓		XB, 1, SUM, K-1	A		SUM, K-1	LABELS
-			RCL2		R	PM PMS,
STO			RCL6		M, R, SUM, K-1	AM Recurs.
9		(1-XBS) ← 1-XB	+			B
gR ↑			1			C
LBL			-			D
3		~, ~, SUM, K-1	RCL6		M	E
+			÷		(R+M-1)/M, SUM, K-1	0
CLX		0, SUM, K-1	RCL3		A	1 Initial
A		M, SUM, K-1	X			2 PM loop
CLX			RCL7		PM	3 Sum loop
RCL			X			4
9		1-XBS, SUM, K-1	STO7		PM ← PM · A(R+M-1)/M	5
RCL7		PM, 1-XBS, SUM, K-1	STO			6
X			+			7
+		SUM ← SUM + PM(1-XBS)	8		(PMS ← PMS + PM)	8
RCL6		M	gR ↓		SUM, K-1	9
RCL1		N, M, SUM, K-1	RCL6		M	FLAGS
-		M-N	1			1
gR ↑		K-1	+			2
gx → y		M-N, K-1	STO6		M ← M+1	
st			RTN		M, SUM, K-1	

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SECTION VI  
HP-67 PROGRAMS

The HP-67 is very similar to its predecessor, the HP-65. As a result, programs written for the HP-65, such as the programs of this report, can almost be transcribed one-to-one for the HP-67. The major difference is the greater memory and programming capacity of the 67. This was used to combine and store multiple, related, HP-65 programs on single HP-67 program cards.

The programming differences between the two calculators that prevent exact one-to-one transcription are noted here for future reference:

- MERGED INSTRUCTIONS

The HP-67 has more merged instructions; e. g., "STO + 8" on the HP-67 requires 3 program lines on the 65.

- CONDITIONAL BRANCHING

The HP-65 skips over two program steps if the conditional test is false. The HP-67 only skips one program step when the test is false.

- INDEX REGISTER

The HP-65 uses register R8 as an index register. The index register in the HP-67 is denoted "I". It can be used for real number storage as well. The register R8 in the HP-67 is for data storage only.

- PROGRAM STORAGE

The HP-65 has a capacity of 100 program steps. The HP-67 has 224.

- DATA STORAGE

The HP-65 has 9 storage registers including register 9 which is not fully available because it is used for internal subroutines. The HP-67 has 26 data storage registers including the "I" register. All are fully available.

- LABELS

The HP-65 has 15 labels for program entry points, subroutines, and branch points. The HP-67 has 20.

The HP-67 detection programs are not as completely annotated as the HP-65 programs. However, the HP-67 programs can be readily related to the corresponding HP-65 programs. Program steps and labels that are different are noted.

# 1. FIXED-THRESHOLD, RECURSIVE PROGRAMS

These HP-67 programs calculate the detection probability, given the number of samples noncoherently integrated, the false-alarm probability, and the average sample signal-to-noise ratio (SNR) (in dB) for the various target models. Unlike the HP-65 programs, each program includes the threshold determination. The P1 and P2 calculations are also included on a single program. In addition, the P1-P2 program can also calculate by iteration the value of SNR required for a Case II target with a given P2.

Specific user instructions follow:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1.	Enter P0, P1-P2, P3, P4 or PG program	N	[ ] [ ]	
		PF	A [ ]	PF (calc)
	Go to step 2, 3, 4 or 5 as appropriate		[ ] [ ]	
2.	For P0, P2, P3 or P4 do either:	SNR dB	B [ ]	P
	or:	X	f [ ] b [ ]	P
	Repeat as desired		[ ] [ ]	
	For P2 go to step 3 if desired		[ ] [ ]	
3.	For SNR or X given P2 (P2 prgm)	P2	C [ ]	SNR dB
			RCL A [ ]	X
	Repeat or go to step 2 as desired		[ ] [ ]	
4.	For P1 do either:	SNR dB	D [ ]	P1
	or:	X	f [ ] d [ ]	P1
	Repeat as desired		[ ] [ ]	
5.	For P6 do either:	SNR dB	B [ ]	K
	or:	X	f [ ] b [ ]	K
6.	Enter desired K if different from display	K	C [ ]	P
	Repeat or go to step 5 as desired		[ ] [ ]	
	After steps 2, 3, 4 or 6 do any of the following if desired:		[ ] [ ]	
			RCL 0 [ ]	PF (calc)
			RCL 1 [ ]	N
	(May be done after step 1)		RCL 2 [ ]	Y
			RCL 9 [ ]	P
			RCL A [ ]	X
			RCL B [ ]	SNR dB
	(PG program only)		RCL E [ ]	K

K : Chi-squared distribution parameter  
 N : Signal and/or noise samples integrated  
 Ne : Target diversity within N samples  
 PF : Probability of false alarm  
 P : Probability of detection for chi-squared target  
 P0 : Probability of detection for nonfluctuating target  
 P1-P4 : Probability of detection for Cases I-IV  
 SNR :  $10 \log X$   
 X : Average sample power S/N within N  
 Y : Fixed detection threshold  
 Y2 :  $Y/(1+X)$

Case 0 :  $K=10^5$  (~3 place acc)  
 Case I :  $K=1$   
 Case II :  $K=N$   
 (Genl Rayleigh target :  $K=Ne$ )  
 Case III :  $K=2$   
 Case IV :  $K=2N$   
 (Genl Rayleigh + equal constant target :  $K=Ne$ )  
 Weinstock :  $0 < K < 1$

HP-67 P0 Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBL A	312511			+	61	
	STO4	3304			RCL1	3401	
	LOG	3153			X ≤ Y?	3271	
	CHS	42		060	RTN	3522	
	X ↔ Y	3552			RCL2	3402	
	STO1	3301			R ^	3554	
	↑	41			X	71	
	√	3154			R ^	3554	
	-	51			÷	81	
010	LSTX	3582			STO+8	336108	
	1	01			R ^	3554	
	-	51			GTO(i)	2224	
	R ^	3554			R/S	84	
	√	3154		070	LBLB	312512	
	+	61			1	01	
	LSTX	3582			0	00	
	X	71			÷	81	
	2	02			10 <sup>x</sup>	3253	
	.	83			LBLb	322512	
020	3	03			STOA	3311	
	X	71			LOG	3153	
	+	61			1	01	
	STO2	3302			0	00	
	LBL1	312501		080	X	71	
	GSBE	312215			STOB	3312	
	RCL8	3408			RCLA	3411	
	R ^	3554			RCL1	3401	
	÷	81			X	71	
	RCL8	3408			STO3	3303	
030	RCL4	3404			RCL2	3402	
	÷	81			CHS	42	
	Ln	3152			e <sup>x</sup>	3252	
	X	71			STO7	3307	
	STO+2	336102		090	STO8	3308	
	LSTX	3582			1	01	
	ABS	3564			STO6	3306	
	EEX	43			LBL0	312500	
	CHS	42			RCL1	3401	
	6	06			X ≤ Y?	3271	
040	X ≤ Y?	3271			GTO2	2202	
	GTO1	2201			D	312214	
	RCL8	3408			1	01	
	STO0	3300			RCL6	3406	
	RTN	3522		100	+	61	
	LBL E	312515			STC6	3306	
	1	01			GTO0	2200	
	2	02			LBL2	312502	
	CHS	42			RCL8	3408	
	STI	3533			STO5	3305	
050	RCL2	3402			RCL3	3403	
	CHS	42			CHS	42	
	e <sup>x</sup>	3252			e <sup>x</sup>	3252	
	STO8	3308			STO4	3304	
	0	00		110	Rv	3553	
	↑	41			LBL3	312503	
	1	01			R ^	3554	

REGISTERS

0 PFC	1 N	2 Y	3 Z	4 PF/XK	5 SUM	6 M	7 YM	8 YMS	9 P0
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A X	B SNR dB	C	D	E	I				





# HP-67 P1-P2 Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBLA	312511			e <sup>x</sup>	3252	
	STO4	3304			STO8	3308	
	LOG	3153			0	00	
	CHS	42		060	↑	41	
	X↔Y	3552			1	01	
	STO1	3301			+	61	
	↑	41			RCL1	3401	
	√	3154			X ≤ Y ?	3271	
	-	51			RTN	3522	
010	LSTX	3582			RCL2	3402	
	1	01			R <sup>^</sup>	3554	
	-	51			X	71	
	R <sup>^</sup>	3554			R <sup>^</sup>	3554	
	√	3154		070	÷	81	
	+	61			STO+8	336108	
	LSTX	3582			R <sup>^</sup>	3554	
	X	71			GTO(6)	2224	
	2	02			R/S	84	
	.	.			LBLB	312512	
020	3	03			1	01	
	X	71			0	00	
	+	61			÷	81	
	STO2	3302			10 <sup>x</sup>	3253	
	LBL1	312501		080	LBLb	322512	
	E	312215			STOA	3311	
	RCL8	3408			LOG	3153	
	R <sup>^</sup>	3554			1	01	
	÷	81			0	00	
	RCL8	3408			X	71	
030	RCL4	3404			STOB	3312	
	÷	81			RCLA	3411	
	Ln	3152			1	01	
	X	71			X	61	
	STO+2	336102		090	RCL2	3402	
	LSTX	3582			STO6	3306	
	ABS	3564			X↔Y	3552	
	EEX	43			÷	81	
	CHS	42			STO2	3302	
	6	06			E	312215	
040	X ≤ Y ?	3271			RCL6	3406	
	GTO1	2201			STO2	3302	
	RCL8	3408			RCL8	3408	
	F ? 2	357102			STO9	3309	
	GTO0	2200		100	RTN	3522	
	STO0	3300			LBLC	312513	
	RTN	3522			SF2	355102	
	LBL0	312500			RCL2	3402	
	STO9	3309			STO6	3306	
	RTN	3522			R <sup>v</sup>	3553	
050	LBL E	312515			RCL1	3401	
	1	01			X↔Y	3552	
	2	02			A	312211	
	CHS	42			RCL2	3402	
	STI	3533		110	RCL6	3406	
	RCL2	3402			STO2	3302	
	CHS	42			X↔Y	3552	

### REGISTERS

0 PFC	1 N	2 Y	3 Z	4 PF	5	6 YTEMP	7 Y1	8 YMS	9 P
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A X	B SNR dB	C	D	E	F	G	H	I	J

HP-67 P1-P2 Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
	STO6	3306			Y <sup>x</sup>	3563	
	÷	81		170	RCL5	3405	
	1	01			÷	81	
	-	51			STO5	3305	
	STOA	3311			EEX	43	
	LOG	3153			7	07	
	1	01			X ≤ Y ?	3271	
120	0	00			GTO7	2207	
	X	71			CLX	44	
	STOB	3312			RCL4	3404	
	RTN	3522			RCL2	3402	
	LBLD	312514		180	E	312215	
	1	01			X	71	
	0	00			GSB8	312208	
	÷	81			CHS	42	
	10 <sup>x</sup>	3253			1	01	
	LBLd	322514			+	61	
130	STOA	3311			RCL5	3405	
	LOG	3153			÷	81	
	1	01			RCL6	3406	
	0	00			+	61	
	X	71		190	STO9	3309	
	STOB	3312			RTN	3522	
	1	01			LBL7	312507	
	4	04			RCL6	3406	
	CHS	42			RTN	3522	
	STI	3533			LBL8	312508	
140	FCLA	3411			STO7	3307	
	FCL1	3401			CHS	42	
	X	71			e <sup>x</sup>	3252	
	STO3	3303			STO8	3308	
	RCL2	3402		200	1	01	
	X → Y	3552			RCL1	3401	
	1	01			1	01	
	+	61			-	51	
	STO4	3304			X ≤ Y ?	3271	
	÷	81			GTO6	2206	
150	CHS	42			RCL7	3407	
	e <sup>x</sup>	3252			R ^	3554	
	STO5	3305			X	71	
	1	01			R ^	3554	
	RCL1	3401		210	÷	81	
	X > Y ?	3281			STO+8	336108	
	GTO9	2209			R ^	3554	
	RCL5	3405			1	01	
	STO9	3309			+	61	
	RTN	3522			GTO (i)	2224	
160	LBL9	312509			LBL6	312506	
	RCL2	3402			FCL8	3408	
	GSB8	312208			RTN	3522	
	STO6	3306					
	RCL3	3403		220			
	RCL4	3404					
	÷	81					
	STO4	3304					
	R ^	3554					

LABELS					FLAGS	SET STATUS									
A	N   PF →	B	SNR dB → P2	C	P2 → X	D	SNR dB →	E	Y SUB	0	FLAGS			TRG	DISP
a		b	X → P2	c		d	X →	e		1	ON OFF			DEG	FIX
0		1	Y LOOP	2		3		4		2	0	<input type="checkbox"/>	<input type="checkbox"/>	GRAD	SCI
5		6	BRANCH	7	BRANCH	8	SUB	9	BRANCH	3	2	<input type="checkbox"/>	<input type="checkbox"/>	RAD	ENG
											3	<input type="checkbox"/>	<input type="checkbox"/>		n

HP-67 P3 Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBLA	312511			+	61	
	STO4	3304			RCL1	3401	
	LOG	3153			X ≤ Y	3271	
	CHS	42		060	RTN	3522	
	X ← Y	3552			RCL2	3402	
	STO1	3301			R ^	3554	
	↑	41			X	71	
	√	3154			R ^	3554	
	-	51			÷	81	
010	STX	3582			STO+8	336108	
	1	01			R ^	3554	
	-	51			GTO(i)	2224	
	R ^	3554			LBLB	312512	
	√	3154		070	1	01	
	+	61			0	00	
	LSTX	3582			÷	81	
	X	71			10*	3253	
	2	02			LBLb	322512	
	.	83			STOA	3311	
020	3	03			LOG	3153	
	X	71			1	01	
	+	61			0	00	
	STO2	3302			X	71	
	LBL1	312501		080	STOB	3312	
	GSBE	312215			RCLA	3411	
	RCL8	3408			RCL1	3401	
	R ^	3554			X	71	
	÷	81			2	02	
	RCL8	3408			÷	81	
030	RCL4	3404			STO3	3303	
	÷	81			RCL3	3403	
	Ln	3152			1	01	
	X	71			+	61	
	STO+2	336102		090	÷	81	
	LSTX	3582			RCL2	3402	
	ABS	3564			LSTX	3582	
	EEX	43			÷	81	
	CHS	42			STO5	3305	
	6	06			RCL1	3401	
040	X ≤ Y	3271			2	02	
	GTO1	2201			X ≤ Y ?	3271	
	RCL8	3408			GTO0	2200	
	STO0	3300			R ^	3554	
	RTN	3522		100	R ^	3554	
	LBLB	312515			X	71	
	1	01			1	01	
	2	02			+	61	
	CHS	42			RCL5	3405	
	STI	3533			CHS	42	
050	RCL2	3402			e <sup>x</sup>	3252	
	CHS	42			X	71	
	e <sup>x</sup>	3252			R/S	84	
	STO8	3308			LBL0	312500	
	0	00		110	-	51	
	↑	41			STO6	3306	
	1	01			R ^	3554	

REGISTERS

0	PF C	1	N	2	Y	3	Z/2	4	PF	5	Z+2	6	N-2	7	Y3	8	YMS	9	P
S0		S1		S2		S3		S4		S5		S6		S7		S8		S9	
A		B		C		D		E		F		G		H		I		J	

JP-67 P3 Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
	LN	3152					
	X	71			1	01	
	+	61		170	L	61	
	CHS	42			GTCD	312214	
	e <sup>x</sup>	3252			R/S	84	
	1	01					
	RCL6	3406					
120	RCL3	3403					
	÷	81					
	-	51					
	RCL5	3405					
	+	61					
	X	71		180			
	STO4	3304					
	RCL5	3405					
	RCL3	3403					
	X	71					
130	GSB2	312202					
	RCL8	3408					
	CHS	42					
	1	01					
	+	61					
	RC	3404		190			
	X	71					
	STO4	3304					
	RCL2	3402					
	GSB2	312202					
140	RCL8	3408					
	R <sup>^</sup>	3554					
	RCL5	3405					
	X	71					
	+	61					
	RCL4	3404		200			
	+	61					
	STO9	3309					
	R/S	84					
	LBL2	312502					
150	STO7	3307					
	CHS	42					
	e <sup>x</sup>	3252					
	STC8	3303					
	1	01					
	LBLD	312514		210			
	RCL1	3401					
	1	01					
	-	51					
	X ≤ Y?	3271					
160	RCL8	3408					
	RTN	3522					
	RCL7	3107					
	R <sup>^</sup>	3554					
	X	71					
	R <sup>^</sup>	3554		220			
	÷	81					
	STO+8	336108					
	R <sup>^</sup>	3554					

LABELS					FLAGS	SET STATUS				
A N   PF →	B SNR dB →	C	D P3 LOOP	E Y SUB	0	FLAGS		TRIG	DISP	
a	b	c	d	e	1	ON OFF				
						0	<input type="checkbox"/>	<input type="checkbox"/>	DEG <input type="checkbox"/>	FIX <input type="checkbox"/>
						1	<input type="checkbox"/>	<input type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
						2	<input type="checkbox"/>	<input type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
						3	<input type="checkbox"/>	<input type="checkbox"/>		n _____

# HP-67 P4 Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBLA	312511			+	61	
	STD4	3304			RCL1	3401	
	LOG	3153			$X \leq Y ?$	3271	
	CHS	42		060	RTN	3522	
	X ↔ Y	3552			RCL2	3402	
	STO1	3301			R ^	3554	
	↑	41			X	71	
	√	3154			R ^	3554	
	-	51			÷	81	
010	LSTX	3582			STO+8	336108	
	1	01			R ^	3554	
	-	51			GTO (i)	2224	
	√	3154			R/S	84	
	+	61		070	LBLB	312512	
	LSTX	3582			1	01	
	X	71			0	00	
	2	02			÷	81	
	.	83			10 <sup>x</sup>	3253	
020	3	03			LBLb	322512	
	X	71			STOA	3311	
	+	61			LOG	3153	
	STO2	3302			1	01	
	LBL1	312501			0	00	
	E	312215		080	X	71	
	RCL8	3408			STOB	3312	
	R ^	3554			RCLA	3411	
	÷	81			0	00	
	RCL8	3408			STO6	3306	
030	RCL4	3404			R√	3553	
	÷	81			2	02	
	LN	3152			÷	81	
	X	71			STO3	3303	
	STO+2	336102			1	01	
	LSTX	3582		090	+	61	
	ABS	3564			STO7	3307	
	EEX	43			RCL1	3401	
	CHS	42			CHS	42	
	6	06			Y <sup>x</sup>	3563	
040	$X \leq Y ?$	3271			STO4	3304	
	GTO1	2201			RCL2	3402	
	RCL8	3408			RCL7	3407	
	STO0	3300			÷	81	
	RTN	3522			STO7	3307	
	LBL E	312515		100	CHS	42	
	1	01			e <sup>x</sup>	3252	
	2	02			STO8	3308	
	CHS	42			GSB 2	312202	
	STI	3553			R√	3553	
050	RCL	3402			R√	3553	
	CHS	42			STO5	3305	
	e <sup>x</sup>	3252			RCL4	3404	
	STO8	3308			X ↔ Y	3552	
	0	00			LBL3	312503	
	↑	41		110	RCL7	3407	
	1	01			RCL6	3406	
					÷	81	

## REGISTERS

0 PFC	1 N	2 Y	3 X/2	4 PF/ZK	5 YM	6 M	7 1+X/2	8 YMS	9 P
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	X	B	SNR dB	C	D	E	I		

HP-67 P4 Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
	X	71					
	STO5	3305					
	1	01					
	R ^	3554					
	-	51					
	X	71					
	STO+8	336108					
120	CLX	44					
	RCL1	3401					
	2	02					
	X	71					
	RCL	3406					
	-	51					
	LSTX	3582					
	RCL1	3401					
	-	51					
	1	01					
130	+	61					
	÷	81					
	RCL4	3404					
	RCL3	3403					
	X	71					
	X	71					
	STO4	3304					
	+	61					
	RCL6	3406					
	1	01					
140	+	61					
	STO6	3306					
	RCL1	3401					
	2	02					
	X	71					
	X ≤ Y ?	3271					
	GTO4	2204					
	R ^	3554					
	RCL5	3405					
	GTO3	2203					
150	LBL4	312504					
	RCL8	3408					
	STO9	3309					
	RTN	3522					
	LBL2	312502					
	RCL6	3406					
	1	01					
	+	61					
	STO6	3306					
	RCL1	3401					
160	X ≤ Y ?	3271					
	RTN	3522					
	R v	3553					
	÷	81					
	RCL7	3407					
	X	71					
	STO+8	336108					
	GTO2	2202					
	R/S	84					

LABELS					FLAGS	SET STATUS					
A N   PF →	B SNR dB → P	C	D	E Y SUB	0	FLAGS		TRIG	DISP		
a	b X → P	c	d	e	1	ON	OFF				
0	1	2 YM LOOP	3 LOOP	4 BRANCH	2	0	<input type="checkbox"/>	DEG	<input type="checkbox"/>	FIX	<input type="checkbox"/>
5	6	7	8	9	3	1	<input type="checkbox"/>	GRAD	<input type="checkbox"/>	SCI	<input type="checkbox"/>
						2	<input type="checkbox"/>	RAD	<input type="checkbox"/>	ENG	<input type="checkbox"/>
						3	<input type="checkbox"/>			n	_____

# HP-67 PG Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBLA	312511			+	61	
	STO4	3304			RCL1	3401	
	LOG	3153			X ≤ Y ?	3271	
	CHS	42		060	RTN	3522	
	X ≤ Y	3552			RCL2	3402	
	STO1	3301			R ^	3554	
		41			X	71	
	√	3154			R ^	3554	
	-	51			÷	81	
010	LSTX	3582			STO+8	336108	
	1	01			R ^	3554	
	-	51			GTO(i)	2224	
	R ^	3554			R/S	84	
	√	3154		070	LBLB	312512	
	+	61			1	01	
	LSTX	3582			0	00	
	X	71			÷	81	
	2	02			10 <sup>x</sup>	3253	
	.	83			LBLb	322512	
020	3	03			STOA	3311	
	X	71			LOG	3153	
	+	61			1	01	
	STO2	3302			0	00	
	LBL1	312501		080	X	71	
	E	312215			STOB	3312	
	RCL8	3408			RCLA	3411	
	R ^	3554			RCL1	3401	
	÷	81			X	71	
	RCL8	3408			STC13	3313	
030	RCL4	3404			RCLE	3415	
	÷	81			R/S	84	
	LN	3152			LBLC	312513	
	X	71			STOE	3315	
	STO+2	336102		090	RCL2	3402	
	LSTX	3582			CHS	42	
	ABS	3564			e <sup>x</sup>	3252	
	EEX	43			STO7	3307	
	CHS	42			STO8	3308	
	6	06			1	01	
040	X ≤ Y ?	3271			STO6	3306	
	GTO1	2201			LBL0	312500	
	RCL8	3408			RCL1	3401	
	STO0	3300			X ≤ Y ?	3271	
	RTN	3522		100	GTO2	2202	
	LBLB	312515			GSB a	322211	
	1	01			GTC0	2200	
	2	02			LBL2	312502	
	CHS	42			RCL8	3408	
	STI	5533			STO5	3305	
050	RCL2	3402			1	01	
	CHS	42			RCL13	3413	
	e <sup>x</sup>	3252			RCL13	3413	
	STO8	3308			RCLE	3415	
	0	00		110	+	61	
		41			÷	81	
	1	01			STO3	3303	

REGISTERS									
0	1	2	3	4	5	6	7	8	9
PF C	N	Y	V	PF XB	SUM	M	YM	YMS	P
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	X	B	SNR dB	C	Z	D	E	K	I

HP-67 PG Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
	-	51			1	01	
	RCLE	3415		170	+	61	
	YX	3563			STO6	3306	
	STO4	3304			RTN	3522	
	LSTX	3582					
	RV	3553					
	-	51					
120	R^	3554					
	LBL3	312503					
	GSB a	322211					
	RCL1	3401					
	-	51		180			
	RCL4	3404					
	X ↔ Y	3552					
	÷	81					
	RCL3	3403					
	X	71					
130	X ↔ Y	3552					
	X	71					
	STO4	3304					
	LSTX	3582					
	RV	3553		190			
	-	51					
	R^	3554					
	1	01					
	+	61					
	1	01					
140	RCL8	3408					
	-	51					
	R^	3554					
	X	71					
	EEX	43		200			
	CHS	42					
	g	08					
	X > Y ?	3281					
	GTO4	2204					
	R^	3554					
150	R^	3554					
	GTO3	2203					
	LBL4	312504					
	RCL5	3405					
	STO9	3309					
	RTN	3522		210			
	LBL a	322511					
	RCL7	3407					
	RCL2	3402					
	X	71					
160	RCL6	3406					
	÷	81					
	STO7	3307					
	STO+8	336108					
	R^	3554					
	X	71		220			
	STO+5	336105					
	RV	3553					
	RCL6	3406					

LABELS					FLAGS	SET STATUS				
A N PF →	B SNR →	C K → P	D	E Y SUB	0	FLAGS		TRIG	DISP	
a SUB	b X →	c	d	e	1	ON	OFF	DEG	FIX	
1" LOOP	Y LOOP	2 BRANCH	3 LOOP	4 BRANCH	2	0	<input type="checkbox"/>	<input type="checkbox"/>	DEG <input type="checkbox"/>	FIX <input type="checkbox"/>
5	6	7	8	9	3	1	<input type="checkbox"/>	<input type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
						2	<input type="checkbox"/>	<input type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
						3	<input type="checkbox"/>	<input type="checkbox"/>		n _____



## 2. FIXED-THRESHOLD, BARTON ALGORITHM

The program, HP-67 SNR, calculates the required average SNR in dB for a given detection probability and target model. The target model is specified by its probability density function, i. e., nonfluctuating, Rayleigh (i. e., Swerling Case I) or Rayleigh plus an equal power constant component (i. e., Swerling Case II) and by its diversity order,  $N_e$ , defined as the number of independent target values within the  $N$  samples noncoherently integrated.

Specific user instructions follow:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter SNR program		[ ] [ ]	
2	(Repeat as desired)	N	[↑] [ ]	
		PD $\geq$ 0.3	[↑] [ ]	
		PF $\leq$ 0.5	[D] [ ]	SNRN dB
	For integration gain if desired:		[RC] [0]	Gi dB
3	To include collapsing loss or target fluctuation	$1 \leq N_s \leq N$	[↑] [ ]	
		$1 \leq N_e \leq N_s$	[f] [d]	
	Go to step 4, 5 or 6		[ ] [ ]	
4	For nonfluctuating target		[A] [ ]	SNRC dB
5	For Rayleigh target (inc. Cases I and II)		[B] [ ]	SNRF dB
6	For Rayleigh constant target (inc. Cases III & IV)		[C] [ ]	SNRF dB
	(Steps 4, 5 and 6 may be repeated in any desired order)		[ ] [ ]	
	After running each, if desired:		[R/S] [ ]	Ls dB
			[R/S] [ ]	Gd dB
			[E] [ ]	R/R <sub>0</sub>

- Gd : Diversity gain (dB)
- Gi : Integration gain (dB)
- Lf : Fluctuation loss (dB)
- N : Signal and/or noise samples integrated
- Ne : Target diversity within  $N_s$  samples
  - For Cases I and III :  $N_e=1$
  - For Cases II and IV :  $N_e=N_s$
- Ns : Samples within  $N$  containing signal
- PD : Probability of detection
- PF : Probability of false alarm
- R/R<sub>0</sub> : Ratio of detected range to that for which SNRF = 0 dB
- SNRC : SNR sample for nonfluctuating target w/collapsing loss (dB)
- SNRF : Average SNR per sample for fluctuating target (dB)
- SNRN : SNR per sample for nonfluctuating target (dB)

HP-67 SNR Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBLD	312514			0	00	
	STO3	3303			4	04	
	Rv	3553			4	04	
	STO2	3302		060	8	08	
	Rv	3553			X	71	
	STO1	3301			.	83	
	0	00			9	09	
	STI	3533			9	09	
	RCL2	3402			2	02	
010	2	02			+	61	
	X	71			X	71	
	1	01			1	01	
	X ≤ Y ?	3271			+	61	
	GTO9	2209		070	R ^	3554	
	STI	3533			.	83	
	Rv	3553			2	02	
	LBL9	312509			7	07	
	RCL2	3402			1	01	
	-	51			X	71	
020	fb	322212			2	02	
	DSZ	3133			.	83	
	CHS	42			3	03	
	SPACE	3584			1	01	
	STO4	3304		080	+	61	
	RCL3	3403			X ↔ Y	3552	
	fb	322212			÷	81	
	RCL4	3404			-	51	
	-	51			RTN	3522	
	↑	41			LBL e	322515	
030	X	71			↑	41	
	2	02			↑	41	
	÷	81			↑	41	
	STO4	3304			9	09	
	RCL1	3401		090	.	83	
	÷	81			2	02	
	fe	322215			+	61	
	STO5	3305			X	71	
	STO8	3308			√	3154	
	RCL4	3404			+	61	
040	fe	322215			2	02	
	STO4	3304			÷	81	
	RCL5	3405			LOG	3153	
	-	51			1	01	
	STO0	3300		100	0	00	
	RCL5	3405			X	71	
	RTN	3522			RTN	3522	
	LBL b	322512			R/S	84	
	↑	41			LBL d	322514	
	X	71			STO7	3307	
050	LN	3152			Rv	3553	
	CHS	42			STO6	3306	
	√	3154			RTN	3522	
	↑	41			LBL A	312511	
	↑	41		110	0	00	
	↑	41			GTO1	2201	
	.	83			LBL B	312512	

REGISTERS

0	1	2	3	4 - Q <sup>-1</sup> , X, SNR	5	6	7	8	9
Gi dB	N	PD	PF	SNRN	Ns	Ne	SNRN dB		
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

HP-67 SNR Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
	RCL3	3403			1	01	
	LN	3152		170	0	00	
	RCL2	3402			X	71	
	LN	3152			+	61	
	÷	81			RCL8	3408	
	1	01			+	61	
	-	51			STO5	3305	
120	GTO2	2202			R/S	84	
	LBLC	312513			Rv	3553	
	.	83			R/S	84	
	3	03			RCL7	3407	
	6	06		180	1	01	
	1	01			-	51	
	RCL3	3403			X	71	
	LOG	3153			R/S	84	
	-	51			LBLC	312513	
	3	03			RCL5	3405	
130	.	83			CHS	42	
	2	02			4	04	
	7	07			0	00	
	RCL2	3402			÷	81	
	CHS	42		190	10*	3253	
	1	01			RTN	3522	
	+	61					
	√	3154					
	÷	81					
	LSTX	3582					
140	.	83					
	9	09					
	6	06					
	X	71					
	-	51					
	1	01					
	.	83					
	2	02					
	9	09					
	-	51					
150	X	71					
	2	02					
	-	51					
	LBL2	312502					
	LOG	3153					
	1	01					
	0	00					
	X	71					
	RCL4	3404					
	-	51					
160	RCL7	3407					
	÷	81					
	LBL1	312501					
	↑	41					
	↑	41					
	RCL1	3401					
	RCL6	3406					
	÷	81					
	LOG	3153					

LABELS					FLAGS	SET STATUS			
A → SNRC dB	B → SNRF dB	C → SNRF dB	D → N, PD, PF	E → R/Ro	0	FLAGS		TRIG	DISP
a	b Subroutine	c	d Ns ↑ Ne	e Subroutine	1	ON	OFF		
0	1 Branch	2 Branch	3	4	2	0	<input type="checkbox"/>	DEG <input type="checkbox"/>	FIX <input type="checkbox"/>
5	6	7	8	9 Branch	3	1	<input type="checkbox"/>	GRAP <input type="checkbox"/>	SCI <input type="checkbox"/>
						2	<input type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
						3	<input type="checkbox"/>		n _____



### 3. CFAR DETECTION, RECURSIVE SOLUTIONS

These HP-67 programs calculate the detection probability, given the average sample SNR and target model for an adaptive detector threshold which is set proportional to the non-coherent integration of R noise samples. They require an initial calculation of the threshold proportionality constant, T, or equivalently  $A = 1/(T+1)$ . Each program includes the required T or A iterative calculation. The HP-67 P2C program can also calculate iteratively the required average SNR for a Case II target and a given detection probability. The HP-67 PGC program calculates detection probability for the general chi-squared target model.

Specific user instructions follow:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter P2C or PGC program	N	[↑] [ ]	
		R	[↑] [ ]	
		PF	[A] [ ]	PF (calc)
		For P2C go to step 2 or 3. For PGC go to step 4.	[ ] [ ]	
2	For P2 do either:	SNR dB	[B] [ ]	P2
	or:	X	[f] [b]	P2
	Repeat for new SNR or X or go to step 3 as desired.		[ ] [ ]	
3	For SNR or X given P2	P2	[C] [ ]	SNR dB
			[RCL] [A]	X
	Repeat or go to step 2 as desired		[ ] [ ]	
4	For PG	SNR dB	[B] [ ]	K
	Enter desired K if different from display	K	[R/S] [ ]	P
	Repeat 4 as desired.		[ ] [ ]	
	After step 2, 3 or 4 do any of the following desired:		[RCL] [0]	PF (calc)
			[RCL] [1]	N
			[RCL] [2]	R
			[RCL] [3]	T
			[RCL] [9]	P
			[RCL] [A]	X
			[RCL] [B]	SNR (dB)
			[RCL] [3]	A
			[RCL] [4]	Z
			[RCL] [E]	K
	Note: PGC does not store P, X, or SNR to completion. They may be recalculated after running as $X=Z/N$ , $SNR=10 \log X$	PGC only		

A :  $1/(T+1)$   
 K : Chi-squared distribution parameter  
 N : Signal and/or noise samples integrated  
 PF : Probability of false alarm  
 P : Probability of detection for chi-squared target  
 P0 : Probability of detection for nonfluctuating target  
 P1-P4 : Probability of detection for Swerling Cases I-IV  
 R : Noise samples integrated to set threshold  
 SNR :  $10 \log X$   
 T : Threshold setting divisor  
 T0 : Iterative solution start value for T  
 T20 : Iterative solution start value for  $T2=T/(1+X)$   
 X : Average sample S/N within N samples

Case 0 :  $K=10^5$  (~ 3 place acc)  
 Case I :  $K=1$   
 Case II :  $K=N$   
 (Genl Rayleigh target :  $K=Ne^*$ )  
 Case III :  $K=2$   
 Case IV :  $K=2N$   
 (Genl Rayleigh + equal constant target :  $K=2Ne^*$ )  
 Weinstock :  $0 < K < 1$

\*Ne: Target diversity within N samples

HP-67 P2C Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LE LA	312511			X	71	
	SE2	355102			1/X	3562	
	STO4	3304			STO3	3303	
	Rv	3553		060	1	01	
	STO2	3302			9	09	
	Rv	3553			CHS	42	
	STO1	3301			STI	3533	
	LBL1	312501			LBL0	312500	
	RCL4	3404			RCL3	3403	
010	LOG	3153			D	312214	
	CHS	42			RCL5	3405	
	RCL1	3401			RCL4	3404	
	↑	41			÷	81	
	√	3154		070	LN	3152	
	-	51			STOC	3313	
	LSTX	3582			RCL6	3406	
	1	01			RCL5	3405	
	-	51			÷	81	
	R^	3554			RCL2	3402	
020	√	3154			RCL3	3403	
	+	61			÷	81	
	LSTX	3582			-	51	
	÷	81			÷	81	
	X ↔ V	3552		080	STO+3	336103	
	R^	3554			RCLC	3413	
	÷	81			ABS	3564	
	2	02			EEX	43	
	.	83			CHS	42	
	3	03			6	06	
030	÷	81			X ≤ Y ?	3271	
	+	61			GTO 0	2200	
	RCL2	3402			RCL5	3405	
	1	01			TF2	357102	
	-	51		090	STO 0	3300	
	RCL2	3402			TF3	357103	
	.	83			STO 9	3309	
	9	09			RTN	3522	
	2	02			LBLB	312512	
	2	02			1	01	
040	+	61			0	00	
	÷	81			÷	81	
	X	71			10 <sup>x</sup>	3253	
	1	01			LBLb	312512	
	LSTX	3582		100	STOA	3311	
	-	51			LOG	3153	
	RCL1	3401			I	01	
	X	71			0	00	
	+	61			X	71	
	1	01			STOB	3312	
050	RCL4	3404			RCLA	3411	
	RCL2	3402			I	01	
	1/X	3562			+	61	
	Y <sup>x</sup>	3563			RCL3	3403	
	-	51		110	X	71	
	LSTX	3582			D	312214	
	÷	81			RCL5	3405	

REGISTERS

0 PF	1 N	2 R	3 T	4 PIN	5 PC	6 Q	7 T+1	8 <sup>T</sup> TEMP	9 P2
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A X	B SNR dB	C	D	E	F	G	H	I 19	

HP-67 P2C Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
	STO9	3309					
	RTN	3522					
	LBLC	312513		170			
	SF3	355103					
	STO4	3304					
	RCL3	3403					
	STO8	3308					
120	GSB1	312201					
	RCL3	3403					
	RCL8	3408					
	STO3	3303					
	÷	81		180			
	1	01					
	-	51					
	STOA	3311					
	LOG	3153					
	1	01					
130	0	00					
	X	71					
	STOB	3312					
	RTN	3522					
	LBLD	312514		190			
	↑	41					
	↑	41					
	1	01					
	+	61					
	STO7	3307					
140	÷	81					
	RCL2	3402					
	Y <sup>x</sup>	3563					
	STO5	3305					
	0	00		200			
	STO6	3306					
	RCL2	3402					
	X ↔ Y	3552					
	+	61					
	LSTX	3582					
150	R v	3553					
	X	71					
	RCL7	3407					
	÷	81					
	STO+6	336106		210			
	R^	3554					
	1	01					
	+	61					
	RCL1	3401					
	X ≤ Y?	3271					
160	RTN	3522					
	R v	3553					
	÷	81					
	STO+5	336105		220			
	LSTX	3582					
	GTO(i)	2224					

LABELS					FLAGS		SET STATUS		
A, R, PF →	B SNR db → P	C P → SNR dB	D SUB	E	0	SET STATUS			
a	b X → P	c	d	e	1	FLAGS	TRIG	DISP	
0	LOOP	1	LOOP	2	3	0 <input type="checkbox"/> ON <input type="checkbox"/> OFF	DEG <input type="checkbox"/>	FIX <input checked="" type="checkbox"/>	
5		6		7	8	1 <input type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>	
		7		8	9	2 <input type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>	
						3 <input type="checkbox"/>		n <u>6</u>	

# HP-67 PGC Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBLA	312511			STO3	3303	
	STO4	3304			LBL0	312500	
	RV	3553			RCL3	3403	
	STO2	3302		060	↑	41	
	RV	3553			↑	41	
	STO1	3301			1	01	
	RCL4	3404			+	61	
	LOG	3153			STO7	3307	
	CHS	42			÷	81	
010	RCL1	3401			RCL2	3402	
	↑	41			Y <sup>x</sup>	3563	
	√	3154			STO0	3300	
	-	51			0	00	
	LSTX	3582		070	STO6	3306	
	1	01			LBL3	312503	
	-	51			RCL2	3402	
	R <sup>^</sup>	3554			X ↔ Y	3552	
	√	3154			+	61	
	+	61			LSTX	3582	
020	LSTX	3582			RV	3553	
	÷	81			X	71	
	X ↔ Y	3552			RCL7	3407	
	R <sup>^</sup>	3554			÷	81	
	÷	81		080	STO+6	336106	
	2	02			R <sup>^</sup>	3554	
	.	83			1	01	
	3	03			+	61	
	÷	81			RCL1	3401	
	+	61			X ≤ Y?	3271	
030	RCL2	3402			GTO7	2207	
	1	01			RV	3553	
	-	51			÷	81	
	RCL2	3402			STO+0	336100	
	.	83		090	LSTX	3582	
	9	09			GTO3	2203	
	2	02			LBL7	312507	
	2	02			RCL0	3400	
	+	61			RCL4	3404	
	÷	81			÷	81	
040	X	71			LN	3152	
	1	01			STOC	3313	
	LSTX	3582			RCL6	3406	
	-	51			RCL0	3400	
	RCL1	3401		100	÷	81	
	X	71			RCL2	3402	
	+	61			RCL3	3403	
	1	01			÷	81	
	RCL4	3404			-	51	
	RCL2	3402			÷	81	
050	1/X	3562			STO+3	336103	
	Y <sup>x</sup>	3563			RCLC	3413	
	-	51			ABS	3564	
	LSTX	3582			EEX	43	
	÷	81		110	CHS	42	
	X	71			6	06	
	1/X	3562			X ≤ Y?	3271	

### REGISTERS

PFC(PMS)	N	R	T, A	PF, Z	XB	QMS, M	T+1	PMS	1-XBS
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A		B		C LN(PFC/PF)		D V		E K	



HP-67 PGC Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
	GTO0	2200			RV	3553	
	RCL3	3403		170	RV	3553	
	1	01			a	322311	
	+	61			RV	3553	
	1/X	3562			RCL9	3409	
	STO3	3303			RCL7	3407	
	RCL0	3400			X	71	
120	RTN	3522			+	61	
	LBLB	312512			RCL6	3406	
	1	01			RCL1	3401	
	0	00			-	51	
	:	81		180	R^	3554	
	10^x	3253			X ↔ Y	3552	
	RCL1	3401			+	61	
	X	71			LSTX	3582	
	STO4	3304			÷	81	
	RCLE	3415			RCLD	3414	
130	R/S	84			X	71	
	STOE	3315			RCL5	3405	
	1	01			X	71	
	RCL3	3403		190	STO5	3305	
	-	51			STO-9	335109	
	RCL2	3402			CLX	44	
	y^x	3563			1	01	
	STO7	3307			RCL8	3408	
	STO8	3308			-	51	
	1	01			EEX	43	
140	STO6	3306			8	08	
	LBL1	312501			X	71	
	RCL1	3401			1/X	3562	
	X ≤ Y	3271			RCL9	3409	
	GTO2	2202		200	X > Y ?	3281	
	a	322211			GTO6	2206	
	GTO1	2201			RV	3553	
	LBL2	312502			RV	3553	
	1	01			RTN	3522	
	RCL4	3404			LBLa	322511	
150	RCL4	3404			RCL2	3402	
	RCLE	3415			RCL6	3406	
	+	61			+	61	
	÷	81			1	01	
	STOD	3314		210	-	51	
	-	51			RCL6	3406	
	RCLE	3415			÷	81	
	y^x	3563			RCL3	3403	
	STO5	3305			X	71	
	LSTX	3582			RCL7	3407	
160	1	01			X	71	
	-	51			STO7	3307	
	RCL8	3408			STO+8	335108	
	RV	3553			RV	3553	
	RV	3553		220	RCL6	3406	
	-	51			1	01	
	STO9	3309			+	61	
	R^	3554			STO6	3306	
	LBL6	312506			RTN	3522	

LABELS					FLAGS	SET STATUS		
A	B	C	D	E	0			
N ↓ R →	SNR dB + K →		SUB			FLAGS	TRIG	DISP
a	b	c	d	e	1	ON OFF		
0	1	2	3	4	2	0 <input type="checkbox"/> <input type="checkbox"/>	DEG <input type="checkbox"/>	FIX <input type="checkbox"/>
			LOOP	LOOP		1 <input type="checkbox"/> <input type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
						2 <input type="checkbox"/> <input type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
5	6	7	8	9	3	3 <input type="checkbox"/> <input type="checkbox"/>		n _____
BRANCH	LOOP	BRANCH						



SECTION VII  
ILLUSTRATIVE EXAMPLES

1. HP-65 FIXED-THRESHOLD, RECURSIVE

<u>Program</u>	<u>Step</u>	<u>Input</u>	<u>Key</u>	<u>Output</u>	<u>Time (m:s)</u>
<u>Y-P2</u>	1	N=10 PF=10 <sup>-6</sup>	↑ A	Y=32.710341	0:28
	2	X=√10	B	P2=0.733987	0:08
	3	P2=0.9	A R/S	Y2=6.221305 X=4.257794	0:45 0:01
<u>P0</u>	4	X=√10	B	P0=0.853317	2:23
<u>P1</u>	4	X=√10	B	P1=0.485543	0:18
<u>P3</u>	4	X=√10	B	P3=0.569375	0:20
<u>P4</u>	4	X=√10	B	P4=0.781789	0:33
<u>P6</u>	7	K=10 <sup>5</sup> X=√10	↑ B	P0≈0.853297	2:19
	7	K=1 X=√10	↑ B	P1=0.485543	2:35
	7	K=10 X=√10	↑ B	P2=0.733987	2:31
	7	K=2 X=√10	↑ B	P3=0.569375	2:34
	7	K=20 X=√10	↑ B	P4=0.781789	2:27

2. HP-65 FIXED-THRESHOLD, BARTON

<u>Program</u>	<u>Step</u>	<u>Input</u>	<u>Key</u>	<u>Output</u>	<u>Time (m:s)</u>
<u>SNRN</u>	2	N=10	STO 1		
		PD=0.75	STO 2		
		PF=10 <sup>-6</sup>	STO 3		
			A	SNRN=4.34	0:09
			-	G <sub>i</sub> =7.89	
	3	N <sub>s</sub> =10	STO 6		
		N <sub>e</sub> =10	STO 7		
	4		A	SNRC=4.34	0:01
	5		B	SNRF=4.79	0:03
	6		C	SNRF=4.57	0:04
		R/S	L <sub>f</sub> =0.22	0:01	
		R/S	G <sub>d</sub> =2.05	0:01	
		E	R/R <sub>o</sub> =0.769	0:01	

3. HP-65 CFAR DETECTION

<u>Program</u>	<u>Step</u>	<u>Input</u>	<u>Key</u>	<u>Output</u>	<u>Time (m:s)</u>
<u>T0</u>	1	N=10	↑		
		R=16	↑		
		PF=10 <sup>-6</sup>	A	T0=0.224789	0:04
<u>P2C</u>	2		R/S	T=0.234428	0:41
	3	SNR=10	B	P2=0.867652	0:13
<u>T0</u>	4	P2=0.9	C	T20=2.207242	0:04
<u>P2C</u>	5		R/S	SNR=10.539693	1:10
<u>PGC(1)</u>	6		R/S	A=0.810092	0:41*
	7	SNR=10	↑		
		K=10	A	1	0:01
<u>PGC(2)</u>	8		R/S	P2=0.867652	9:57
<u>PGC(1)</u>	7	SNR=10	↑		
		K=1	A	1	0:01
<u>PGC(2)</u>	8		R/S	P1=0.564214	10:42

\*0:14 if step 2 has been run

4. HP-67 FIXED-THRESHOLD, RECURSIVE

<u>Program</u>	<u>Step</u>	<u>Input</u>	<u>Key</u>	<u>Output</u>	<u>Time(m:s)</u>
<u>Any below</u>	1	N=10	↑		
		PF=10 <sup>-6</sup>	A	PF=0.000001	0:32
<u>P1-P2</u>	2	SNR=5	B	P2=0.733987	0:11
	3	P2=0.9	C	SNR=6.291847	0:56
	4	SNR=5	D	P1=0.485543	0:24
<u>P0</u>	2	SNR=5	B	P0=0.853317	2:46
<u>P3</u>	2	SNR=5	B	P3=0.569375	0:08
<u>P4</u>	2	SNR=5	B	P4=0.781789	0:39
<u>PG</u>	5	SNR=5	B		
	6	K=10 <sup>5</sup>	C	P0≈0.853298	2:40
	6	K=1	C	P1=0.485543	2:59
	6	K=10	C	P2=0.733987	2:55
	6	K=2	C	P3=0.569375	2:58
	6	K=20	C	P4=0.781789	2:51

5. HP-67 FIXED-THRESHOLD, MARTON

<u>Program</u>	<u>Step</u>	<u>Input</u>	<u>Key</u>	<u>Output</u>	<u>Time(m:s)</u>
<u>SNR</u>	2	N=10	↑		
		PD=0.75 PF=10 <sup>-6</sup>	↑ D	SNRN=4.34	0:13
	3	N <sub>g</sub> =10 N <sub>e</sub> =10	↑ fd		
	4		A	SNRC=4.34	0:02
	5		B	SNRF=4.79	0:05
	6		C	SNRF=4.57	0:06
			R/S	L <sub>f</sub> =0.22	0:01
			R/S	G <sub>d</sub> =2.05	0:01
			E	R/R <sub>o</sub> =0.769	0:01

6. HP-67 CFAR DETECTION

<u>Program</u>	<u>Step</u>	<u>Input</u>	<u>Key</u>	<u>Output</u>	<u>Time (m:s)</u>
<u>P2C or PGC</u>	1	N=10 R=16 PF=10 <sup>-6</sup>	A	PF=0.000001	1:01
<u>P2C</u>	2	SNR=10	B	P2=0.867652	0:14
	3	P2=0.9	C	SNR=10.339693	1:15
<u>PGC</u>	4	SNR=10 K=10	B R/S	P2=0.867552	10:24
	4	SNR=10 K=1	B R/S	P1=0.564214	11:11

## SECTION VIII

### REFERENCES

1. Marcum, J.I., "A Statistical Theory of Target Detection by Pulsed Radar", RAND Corp. Res. Memo RM-754, Dec. 1, 1947 (reprinted in IEEE Transactions IT-6, No. 2, April 1960, pp 59 - 144).
2. Swerling, P., "Probability of Detection for Fluctuating Targets", RAND Corp. Res. Memo RM-1217, March 17, 1954 (reprinted in IEEE Transactions IT-6, No. 2, April 1960, pp 269 - 308).
3. Barton, D.K., "Simple Procedures for Radar Detection Calculations", IEEE Transactions AES-5, No. 5, September 1969, pp 837 - 46.
4. Shnidman, D.A., "Evaluation of Probability of Detection for Several Target Fluctuation Models", Lincoln Lab Tech Note 1975 - 35, July 9, 1975, ADA 013733.
5. Mitchell, R.L. and Walker, J.F., "Recursive Methods for Computing Detection Probabilities", IEEE Transactions AES-7, No. 4, July 1971, pp 671 - 676.
6. Nathanson, F.E., Radar Design Principles, McGraw-Hill, New York, 1969, pp 148 - 154.
7. Cann, A.J., "Simple Radar Detection Calculation," IEEE Transactions AES-8, No. 1, January 1972, pp 73 - 74.
8. Mayer, H.A. and Meyer, D.P., "Chi-Square Target Models of Low Degrees of Freedom", IEEE Transactions AES-11, No. 5, September 1975, p 694 - 707.
9. Abramowitz, M. and Stegun, I.A., Handbook of Mathematical Functions, Nat. Bureau of Stds, Applied Math Series 55, June 1964, p 933, 26.2.22.
10. DiFranco, J.V. and Rubin, W.L., Radar Detection, Prentice Hall, Englewood Cliffs, N.J., 1968, p 315, Eqn 9.5-8b, with  $R=2D$ .





## ERRATA SHEET

September 12, 1979

Please make the following changes to TIS R79EMH5:

- p. 2-5 Change reference number for Shnidman from 2 to 4.
- p. 3-1 Equation (3-1) center should read  $X - \frac{1}{2N} \left[ Q^{-1}(PF) - Q^{-1}(PD) \right]^2$
- p. 3-1 Change reference number for Cann from 8 to 7.
- p. 5-1 Step 5 output: Change "D" to "P".
- p. 5-1 In instruction table - step 7, first key block after K should contain "↑".
- p. 5-2 Step 25, Comment should read:  $2.3\sqrt{L}(\sqrt{L} + \sqrt{N} - 1), N - \sqrt{N}$
- p. 5-3 Step 3, change "3302" to "3303".
- p. 5-4 Step 4, change "3403" to "3402".
- p. 5-4 Step 19, change "3302" to "3402".
- p. 5-7 On program line 52, change comment from "XB · V/B" to "XB · V/(M-N)".
- p. 5-7/  
5-8 Step 5, change "3303" to "3304".
- p. 5-9 Instr. step 5 should read: "For Rayleigh tgt. (Inc Cases I & II)
- p. 5-10 Line 21 comment should read  $\left[ Q^{-1}(PF) - Q^{-1}(PD) \right]$ ; Comment column  
line 17, remove minus sign.
- p. 5-13 In nomenclature table: Case 0:  $k = 10^5$
- p. 6-2 Step 5, "P6" should be "PG".
- p. 6-2 Add to step 1, 1st block, insert ↑.
- p. 6-5 On program line 089, change "X" to "+".
- p. 7-1 Change last item under Program from P6 to PG.
- p. 7-1 Line 1, step 1, change "↑" to "STO1".
- p. 7-2 Par. 2, step 3, add SNRF in Program Column.
- p. 7-4 Step 1 in Key column, add ↑ in Key Column after N = 10 and R = 16.

