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ESTIMATION OF A CONTACT'S COURSE, SPEED AND POSITION BASED ON BEARINGS-ONLY INFORMATION FROM TWO MOVING SENSORS WITH A PROGRAM FOR AN HP-67/97 CALCULATOR

by

R. H. Shudde

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This report provides a procedure for estimating a contact's course, speed and position based on bearings-only data from two moving sensors. This report also contains a program for the HP-67/97 calculator to implement the procedure.

KEYWORDS:

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The programs in this report are for use within the Department of the Navy, and they are presented without representation or warranty of any kind.

TABLE OF CONTENTS

A. Problem Statement

Bearings-only data for a single target from two sensors which may be moving or stationary are available at two distinct times. The following quantities are required: an estimate of course, speed and position of the target at the latest time; an estimate of a future position of the target and/or an estimate of a point on the track of the target with a specified lead distance at a future time. The relative positions of the two sensors are assumed to be known at the time of each target bearing determination.

B. Operational Analysis

Two simultaneous bearings from two sensors at two distinct times and with known relative positions are used to estimate the course and speed of a target. The HP-67/97 program presented here was designed so that the data corresponding to the earliest time point is purged if data corresponding to a third time point is introduced. The relative position of the sensors may be updated when required. Thus the estimated target position, course and speed are continually updated as new information becomes available. No course smoothing is performed.

C. Computational Algorithm

1. Enter the course ψ_s and speed V_s of the primary sensor S_1.

2. Enter the bearing ϕ and range ρ of the secondary sensor S_2 from the primary sensor S_1 at the time of the latest bearing observation.

3. Enter the time t_1, the bearing of the target from S_1, and the bearing of the target from S_2. Output the target range from S_1.

4. Repeat Step 3 or Steps 2 and 3 for a second time $t_2 > t_1$.

5. Compute and output:
   a. The estimated course and speed of the target.
   b. The bearing and range (n.mi.) of the target from S_1 at time t_2.
   c. The bearing and range (n.mi.) of the target from S_2 at time t_2.

6. If required, enter a time $t_\ell > t_2$ at which a lead distance ℓ (n.mi.) is required. Then compute and output the target's predicted bearing and range from both S_1 and S_2.

7. Repeat from Steps 1, 2, 3 or 4 as required.

D. HP-67/97 Calculator Program

1. User Instructions

Step	Instruction	Input	Key(s)	Output
1	Enter program card (both sides).			
2a	Set for HP-67 output (default) or		fA	none
b	set for HP-97 output.		fB	none
3	Enter course and speed of the primary sensor S_1.	course ψ_s speed V_s	↑ A	 course
4	Enter bearing and range of S_2 from S_1 at datum.	bearing ϕ range ρ	↑ B	 bearing
5	Enter time t_1 of datum, bearing from S_1, and bearing from S_2.	t_1 (HH.MM) θ_{11} (degrees) θ_{21} (degrees)	↑ ↑ C	 R_1 (n.mi.)
	Optional: Display range from S_2.		R/S	r_1 (n.mi.)
6	Repeat Step 4 or 5 or Step 5 only for a second (subsequent) time, then proceed to Step 7.			
7a	Compute: Target course		D	ψ_T (degrees)
b	Target speed		(R/S)*	V_T (knots)
c	Display Sensor S_1 prompt.		(R/S)	1.
d	Target bearing from S_1 at latest time.		(R/S)	θ_{12} (degrees)
e	Target range from S_1 at latest time.		(R/S)	R_2 (n.mi.)
f	Display Sensor S_2 prompt.		(R/S)	2.
g	Target bearing from S_2 at latest time.		(R/S)	θ_{22} (degrees)
h	Target range from S_2 at latest time.		(R/S)	r_2 (n.mi.)

Step	Instruction	Input	Key(s)	Output
8	Compute bearing and range at time t_ℓ with lead distance ℓ (n.mi.):	t_ℓ(HH.MM) ℓ (n.mi.)	↑ E	
a	Display Sensor S_1 prompt.			1.
b	Target bearing from S_1 at t_ℓ.		(R/S)*	$\theta_{1\ell}$ (degrees)
c	Target range from S_1 at t_ℓ.		(R/S)	R_ℓ (n.mi.)
d	Display Sensor S_2 prompt.		(R/S)	2.
e	Target bearing from S_2 at t_ℓ.		(R/S)	$\theta_{2\ell}$ (degrees)
f	Target range from S_2 at t_ℓ.		(R/S)	r_ℓ (n.mi.)
9	Repeat from Step 3 or from Step 6 as required.			

*Note: The (R/S) function is required when using the HP-67 mode. This output is automatically printed on the HP-97.

2. <u>Sample Problem</u>

a. The Primary Sensor S_1 is traveling on a course of 210° at 10 knots.

b. At the time of the first contact sensor, S_2 is 115° and 3.5 n.mi. from S_1.

c. At 1200 hours the first contact is at 245° from S_1 and 260° from S_2. How far is the contact from S_1 and S_2? (Ans.: 8 n.mi. from S_1 and 10 n.mi. from S_2.)

d. At the next time mark sensor S_2 is 100° and 5.0 n.mi. from S_1.

e. This next time mark is at 1230 hours with the contact at 160° from S_1 and 239° from S_2.

f. Estimate the course and speed of the contact.
(Ans.: 126° and 14 knots.)

g. What is the bearing and range of the contact from S_1 at 1230 hours? (Ans.: 160° and 3 n.mi.)
From S_2? (Ans.: 239° and 4 n.mi.)

h. Estimate the bearing and range of the contact from S_1 and S_2 at 1245 hours with a lead distance of 3.5 n.mi.
(S_1 Ans.: 137° and 10 n.mi.)
(S_2 Ans.: 164° and 7 n.mi.)

a.	210. ENT↑	ψ_s S_1 course
	10. GSBA	V_s S_1 speed
b.	115. ENT↑	ϕ_1 Bearing of S_2
	3.5 GSBB	ρ_1 Range of S_2
c.	12.00 ENT↑	t_1 First contact time
	245. ENT↑	θ_{11} Target bearing from S_1
	260. GSBC	θ_{21} Target bearing from S_2
	8. ***	R_1 Est. range from S_1
	R/S	
	10. ***	r_1 Est. range from S_2
d.	100. ENT↑	ϕ_2 New bearing of S_2
	5. GSBB	ρ_2 New range of S_2
e.	12.30 ENT↑	t_2 Second time mark
	160. ENT↑	θ_{12} Target bearing from S_1
	239. GSBC	θ_{22} Target bearing from S_2
f.	GSBD	Compute
	125. ***	Est. target course
	14. ***	Est. target speed (kts)
g.	1. ***	S_1:
	160. ***	Target bearing
	3. ***	Target range
	2. ***	S_2:
	239. ***	Target bearing
	4. ***	Target range
h.	12.45 ENT↑	Lead time (HH.MM)
	3.5 GSBE	Lead distance (n.mi.)
	1. ***	S_1:
	137. ***	Bearing
	10. ***	Range
	2. ***	S_2:
	164. ***	Bearing
	7. ***	Range

3. Program Storage Allocation and Program Listing

Registers:

R0:	X_2^T	S0:	X_1^T	A:	ρ_i
R1:	Y_2^T	S1:	Y_1^T	B:	ϕ_i
R2:	t_2	S2:	t_1	C:	V_s
R3:	Δt	S3:		D:	ψ_s
R4:	θ_{12}	S4:	Σx	E:	V_T
R5:	θ_{22}	S5:	Used	I:	ψ_T
R6:	R_2	S6:	Σy		
R7:	r_2	S7:	Used		
R8:	t_ℓ	S8:	Used		
R9:	ℓ, R_ℓ and r_ℓ	S9:	Used		

Initial Flag Status and Use:

0: OFF, HP67 or HP97 output

1: OFF, Unused

2: OFF, Used for t_ℓ option

3: OFF, Unused

Display Status: DSP0, FIX, DEG

User Control Keys:

A: $\psi_s \uparrow V_s$

B: $\phi_i \uparrow \rho_i$

C: $t_i \uparrow \theta_{1i} \uparrow \theta_{2i}$

D: Compute ψ_T, V_T and position

E: $t_\ell \uparrow \ell$

a: HP-67 output mode

b: HP-97 output mode

c. Unused

d: Unused

e. Unused

Step	Key	Code	Comment
001	*LBLA	21 11	Primary Sensor S_1
002	STOC	35 13	Store speed V_s
003	R↓	-31	
004	STOD	35 14	Store course ψ_s
005	RTN	24	
006	*LBLB	21 12	S_2 position
007	STOA	35 11	Store range ρ_i
008	R↓	-31	
009	STOB	35 12	Store bearing ϕ_i
010	RTN	24	
011	*LBLC	21 13	Time and Bearing Input
012	P⇄S	16-51	
013	STO5	35 05	Store θ_{2i}
014	R↓	-31	
015	STO4	35 04	Store θ_{1i}
016	R↓	-31	
017	HMS→	16 36	Store t_i
018	STO2	35 02	
019	RCL4	36 04	
020	GSB9	23 09	Compute and store r_i
021	STO7	35 07	
022	RCL5	36 05	
023	GSB9	23 09	Compute and store R_i
024	STO6	35 06	
025	RCL4	36 04	
026	X⇄Y	-41	
027	→R	44	Compute and store
028	STO0	35 00	X_i^T
029	R↓	-31	
030	STO1	35 01	and
031	RCL6	36 06	Y_i^T
032	R/S	51	Display R_i
033	RCL7	36 07	
034	R/S	51	Display r_i
035	GTO0	22 00	
036	*LBL9	21 09	Subroutine
037	RCLB	36 12	
038	-	-45	

Step	Key	Code	Comment
039	SIN	41	Compute range
040	RCL5	36 05	from S_1 or S_2
041	RCL4	36 04	using bearing
042	-	-45	
043	SIN	41	data.
044	÷	-24	
045	RCLA	36 11	
046	×	-35	
047	RTN	24	
048	*LBLD	21 14	
049	RCLΣ	36 56	Computation of
050	Σ-	16 56	target course
051	P⇄S	16-51	
052	RCL1	36 01	and speed
053	RCL0	36 00	
054	RCL2	36 02	
055	P⇄S	16-51	
056	RCL2	36 02	$t_2 - t_1 = \Delta t$
057	-	-45	
058	CHS	-22	Error if $\Delta t < 0$
059	X<0?	16-45	
060	GTO0	22 00	
061	STO3	35 03	
062	R↓	-31	
063	Σ-	16 56	Store $-\vec{R}_1$
064	RCL1	36 01	
065	RCL0	36 00	
066	Σ+	56	Compute $\vec{R}_2 - \vec{R}_1$
067	RCLD	36 14	
068	RCLC	36 13	
069	RCL3	36 03	Compute
070	×	-35	$\vec{V}_s\ \Delta t$
071	→R	44	
072	Σ+	56	
073	RCLΣ	36 56	Compute
074	→P	34	$\vec{V}_T\ \Delta t$ and $\vec{V}_T$
075	RCL3	36 03	
076	÷	-24	

077	STOE	35 15	Store V_T
078	X⇄Y	-41	Store ψ_T
079	GSB7	23 07	
080	STOI	35 46	
081	SPC	16-11	Double space HP-97
082	SPC	16-11	
083	GSB6	23 06	Display ψ_T
084	RCLE	36 15	
085	GSB6	23 06	Display V_T
086	SPC	16-11	
087	1	01	Display 1. (S_1)
088	GSB6	23 06	
089	RCL4	36 04	Display bearing from S_1
090	GSB6	23 06	
091	RCL6	36 06	Display range from S_1
092	GSB6	23 06	
093	SPC	16-11	
094	2	02	Display 2. (S_2)
095	GSB6	23 06	
096	RCL5	36 05	Display bearing from S_2
097	GSB6	23 06	
098	RCL7	36 07	Display bearing from S_1
099	GSB6	23 06	
100	R/S	51	
101	GTO0	22 00	Error
102	*LBLE	21 15	Lead time and lead
103	SF2	16 21 02	distance
104	STO9	35 09	
105	R↓	-31	Store ℓ
106	HMS→	16 36	
107	STO8	35 08	Store t_ℓ
108	SPC	16-11	
109	1	01	Set for S_1 and display
110	GSB6	23 06	
111	RCLΣ	36 56	
112	Σ-	16 56	Clear Σx and Σy
113	RCL1	36 01	
114	RCL0	36 00	Store $\vec{R}_2$

115	Σ+	56	
116	RCLI	36 46	
117	RCLE	36 15	
118	RCL8	36 08	$t_\ell - t_2$
119	RCL2	36 02	
120	-	-45	Error if $t_\ell - t_2 < 0$
121	X<0?	16-45	
122	GTO0	22 00	
123	×	-35	$\vec{V}_T(t_\ell - t_2)$
124	RCL9	36 09	
125	+	-55	ℓ added
126	→R	44	
127	Σ+	56	Add to $\vec{R}_2$
128	*LBL1	21 01	
129	RCLΣ	36 56	Convert position to
130	→P	34	polar
131	STO9	35 09	
132	X⇄Y	-41	Store range
133	GSB7	23 07	Display bearing
134	GSB6	23 06	
135	RCL9	36 09	Display range
136	GSB6	23 06	
137	F2?	16 23 02	S_2 position to be
138	GTO2	22 02	computed?
139	R/S	51	
140	GTO0	22 00	Error
141	*LBL2	21 02	
142	SPC	16-11	Set for S_2
143	2	02	
144	GSB6	23 06	and display
145	RCLB	36 12	Subtract $\vec{\rho}$ to
146	RCLA	36 11	
147	→R	44	obtain $\vec{r}_2$.
148	Σ-	16 56	
149	GTO1	22 01	
150	*LBL7	21 07	Subroutine.
151	X<0?	16-45	
152	X=0?	16-43	

153	RTN	24	Add 360° to negative bearings
154	3	03	
155	6	06	
156	0	00	
157	+	-55	
158	RTN	24	
159	*LBL6	21 06	HP-67 display and HP-97 print routine
160	F0?	16 23 00	
161	PRTX	-14	
162	F0?	16 23 00	
163	RTN	24	
164	R/S	51	
165	RTN	24	
166	*LBLa	21 16 11	Set for HP-67 display mode
167	CF0	16 22 00	
168	RTN	24	
169	*LBLb	21 16 12	Set for Hp-97 print mode.
170	SF0	16 21 00	
171	RTN	24	
172	R/S	51	

E. Geometric Analysis

1. Static Geometry

Let $\vec{R}_i = (\theta_{1i}, R_i)$ denote the bearing and range of the target from the reference (primary) sensor S_1 at time t_i, and let $\vec{r}_i = (\theta_{2i}, r_i)$ denote the bearing and range of the target from the secondary sensor S_2 at time t_i, $i = 1,2$, where $t_1 < t_2$. Let $\vec{\rho}_i = (\phi_i, \rho_i)$ denote the bearing and range of S_2 from S_1 at time t_i. The static geometry for some fixed time t_i is depicted in Figure 1.

From Figure 1 we see that

$$\vec{R}_i = \vec{\rho}_i + \vec{r}_i \ . \tag{1}$$

By equating the rectangular components of Equation (1) we have

$$R_i \cos \theta_{1i} = \rho \cos \phi + r_i \cos \theta_{2i} \tag{2a}$$

and

$$R_i \sin \theta_{1i} = \rho \sin \phi + r_i \sin \theta_{2i} \ . \tag{2b}$$

Equations (2) are two equations in the two unknown ranges R_i and r_i. Solving this system of equations we obtain

$$R_i = \rho_i \frac{\sin(\theta_{2i} - \phi_i)}{\sin(\theta_{21} - \theta_{i1})} \qquad \text{for any } i, \tag{3}$$

and

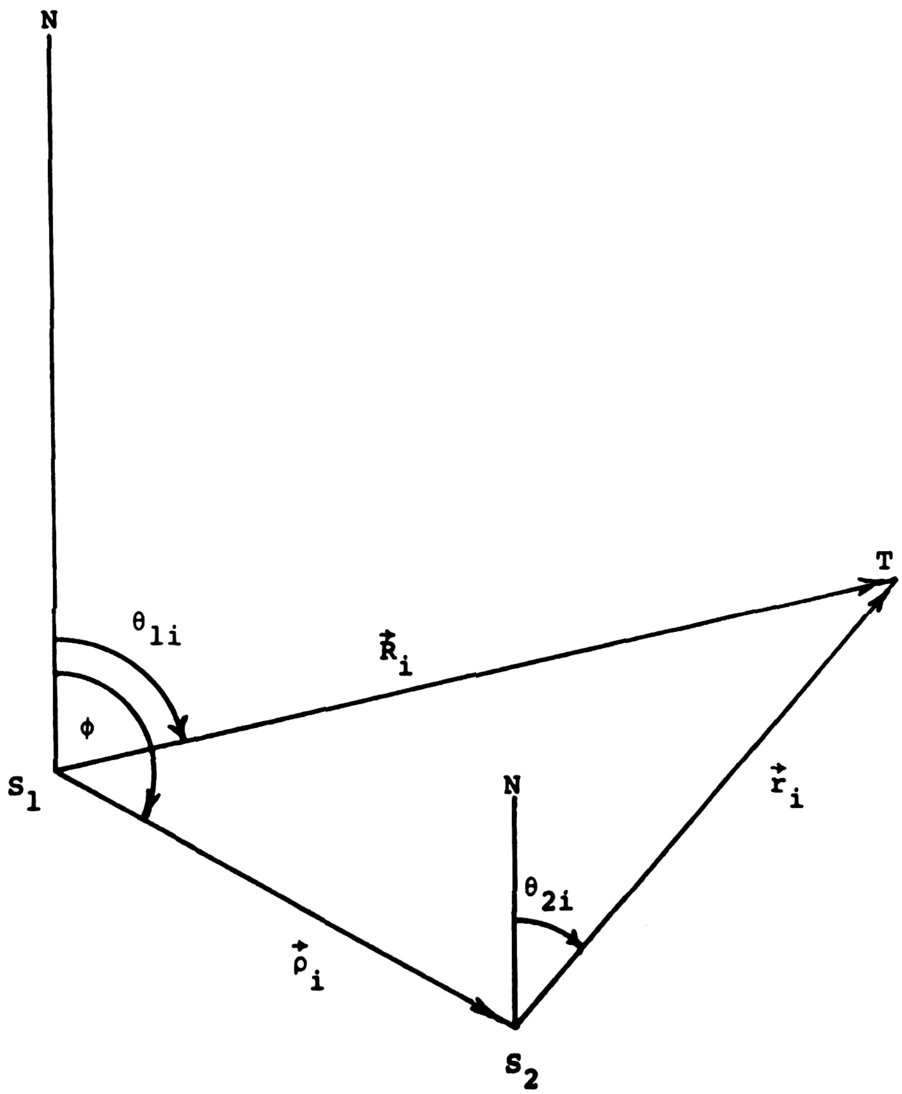

FIGURE 1. The Relative Sensor and Target Geometry at Time t_i.

$$r_i = \rho_i \frac{\sin(\theta_{1i} - \phi_i)}{\sin(\theta_{2i} - \theta_{1i})} \quad \text{for any } i \ . \tag{4}$$

At any time t_i the target range R_i from sensor S_1 and the target range r_i from sensor S_2 may be computed from Equations (3) and (4), respectively. Thus $\vec{R}_i$ and $\vec{r}_i$ are determined at any time t_i.

2. <u>Dynamic Geometry</u>

Let $\vec{V}_s = (\psi_s, V_s)$ denote the course and speed of the primary sensor S_1, and let $\vec{V}_T = (\psi_T, V_T)$ denote the unknown course and speed of the target. Let $\Delta t = t_2 - t_1 > 0$ be the time between first and second observations of the target. The absolute motion of sensors and the target is depicted in Figure 2. From Figure 2 it is evident that one of the many vectorial relationships is

$$\vec{R}_1 + \vec{V}_T \, \Delta t = \vec{V}_s \, \Delta t + \vec{R}_2 \ . \tag{5}$$

The target course and speed vector $\vec{V}_T$ is then found to be

$$\vec{V}_T = \vec{V}_s + \frac{1}{\Delta t} (\vec{R}_2 - \vec{R}_1) \ . \tag{6}$$

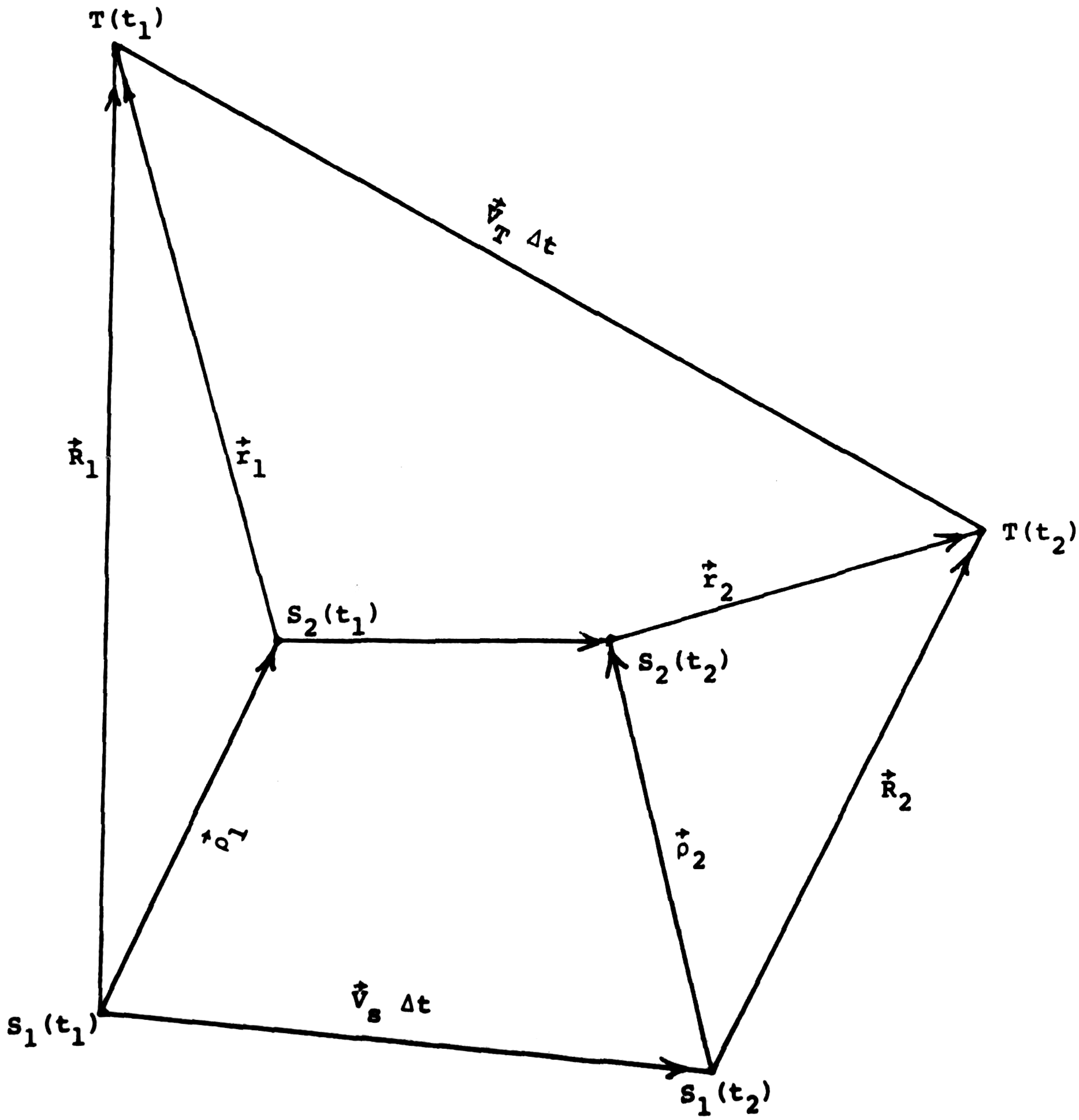

FIGURE 2. Motion of Sensors S_1 and S_2 and of the Target T from Time t_1 to Time t_2.

3. Lead Distance Geometry

If, at some time t_ℓ $(t_\ell > t_2)$, it is desired to lead the target on its track by a distance ℓ, then the bearing $\theta_{1\ell}$ and range R_ℓ to this position from the primary sensor S_1 is obtained by converting the vector $[\psi_T, V_T(t_\ell - t_2) + \ell]$ to rectangular coordinates and adding it to the rectangular form of the position vector $\vec{R}_2$ (see Figure 3). The resulting vector is then converted to polar coordinates to obtain the vector $(\theta_{1\ell}, R_\ell)$. The predicted bearing and range $\vec{r}_\ell$ of the target from the secondary sensor S_2 is computed from

$$\vec{r}_\ell = \vec{R}_\ell - \vec{\rho} \quad . \tag{7}$$

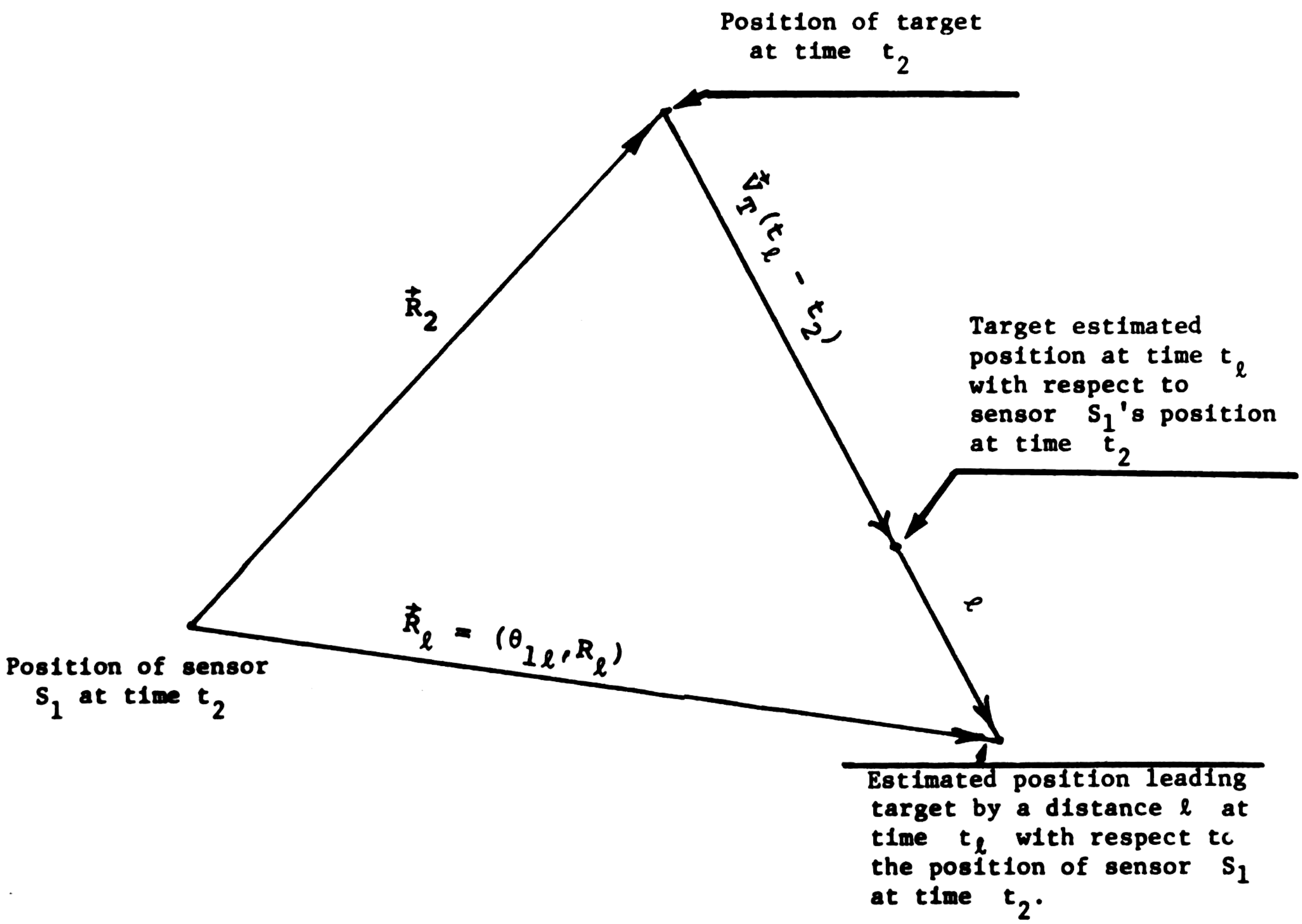

FIGURE 3. Target Lead Distance Geometry.