

Civil Engineering Pac I


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## Introduction

The 18 programs of CE Pac I have been drawn from the fields of statics and stress analysis.
Each program in this pac is represented by one or more magnetic cards and a section in this manual. The manual provides a description of the program with relevant equations, a set of instructions for using the program, and one or more example problems, each of which includes a list of the actual keystrokes required for its solution. Program listings for all the programs in the pac appear at the back of this manual. Explanatory comments have been incorporated in the listings to facilitate your understanding of the actual working of each program. Thorough study of a commented listing can help you to expand your programming repertoire since interesting techniques can often be found in this way.
On the face of each magnetic card are various mnemonic symbols which provide shorthand instructions to the use of the program. You should first familiarize yourself with a program by running it once or twice while following the complete User Instructions in the manual. Thereafter, the mnemonics on the cards themselves should provide the necessary instructions, including what variables are to be input, which user-definable keys are to be pressed, and what values will be output. A full explanation of the mnemonic symbols for magnetic cards may be found in appendix A.
If you have already worked through a few programs in Standard Pac, you will understand how to load a program and how to interpret the User Instructions form. If these procedures are not clear to you, take a few minutes to review the sections, Loading a Program and Format of User Instructions, in your Standard Pac.

We hope that CE Pac I will assist you in the solution of numerous problems in your discipline. We would very much appreciate knowing your reactions to the programs in this pac, and to this end we have provided a questionnaire inside the front cover of this manual. Would you please take a few minutes to give us your comments on these programs? It is in the comments we receive from you that we learn how best to increase the usefulness of programs like these.

## CONTENTS

1. Vector Statics ..... 01-01
Performs basic vector operations of addition, cross product, and dot product, and finds the angle between vectors.
2. Section Properties (2 cards) ..... 02-01
The area, centroid, and moments of an arbitrarily complex polygon may be calculated using this program.
3. Properties of Special Sections ..... 03-01
Section properties for rectangles, triangles, ellipses circles and concentric circles are provided by this program.
4. Stress on an Element ..... 04-01
Reduces data from rosette strain gage measurement and performs Mohr circle analysis.
5. Bending or Torsional Stress ..... 05-01
Solves either the bending stress equation ( $\mathrm{s}=\mathrm{Mv} / \mathrm{I}$ ) or the analogous torsional shear stress equation ( $s=T R / J$ ) interchangeably for all variables.
6. Linear or Angular Deformation ..... 06-01
This program solves for linear deflection under tensile load or the analogous angular deflection under torque. The solution is inter- changeable between the five variables.
7. Cantilever Beams ..... 07-01
Calculates deflection, slope, moment and shear for point, distributed, and moment loads applied to cantilever beams.
8. Cantilever Beams-Trapezoidal Load ..... 08-01
Calculates deflection, slope, moment and shear for cantilever beams with distributed trapezoidal loads.
9. Simply Supported Beams ..... 09-01
Calculates deflection, slope, moment, and shear for point, distributed, and moment loads applied to simply supported beams.
10. Simply Supported Beams-Trapezoidal Load ..... 10-01
Calculates deflection, slope, moment and shear for simply supported beams with distributed trapezoidal loads.
11. Beams Fixed at Both Ends ..... 11-01
Calculates deflection, slope, moment, and shear for point, distributed, and moment loads applied to beams with rigidly fixed ends.
12. Beams Fixed at Both Ends-Trapezoidal Loads ..... 12-01
Calculates deflection, slope, moment, and shear for distributed trape- zoidal loads applied to beams with rigidly fixed ends.
13. Propped Cantilever Beams ..... 13-01
Calculates deflection, slope, moment, and shear for point, moment and distributed loads applied to propped cantilever beams.
14. Propped Cantilever Beams-Trapezoidal Load ..... 14-01
Calculates deflection, slope, moment and shear for distributed trapezoidal loads applied to propped cantilever beams.
15. Six-span Continuous Beams ..... 15-01
Solves for the intermediate couples present at the supports of continuous beams. Two to six spans are allowed.
16. Steel Column Formula ..... 16-01
Computes allowable loads for steel columns. Column ends must be constrained by welds, rivets or in some other means which prevents deflection and rotation.
17. Reinforced Concrete Beams ..... 17-01
Solves interchangeably between steel area, width, depth, concrete strength, steel strength and internal moment for reinforced concrete beams. Based on the American Concrete Institute code-ACI 318-71.
18. Bolt Torque ..... 18-01
Calculates the torque that will yield a specified bolt load or the load resulting from a specified torque. The shear stress in the bolt may be calculated as an option.

## A WORD ABOUT PROGRAM USAGE

This application pac has been designed for both the HP-97 Programmable Printing Calculator and the HP-67 Programmable Pocket Calculator. The most significant difference between the HP-67 and the HP-97 calculators is the printing capability of the HP-97. The two calculators also differ in a few minor ways. The purpose of this section is to discuss the ways that the programs in this pac are affected by the difference in the two machines and to suggest how you can make optimal use of your machine, be it an HP-67 or an HP-97.

Many of the computed results in this pac are output by PRINT statements; on the HP-97 these results will be output on the printer. On the HP-67 each PRINT command will be interpreted as a PAUSE: the program will halt, display the result for about five seconds, then continue execution. The term "PRINT/ PAUSE' ${ }^{\prime}$ is used to describe this output condition.

If you own an HP-67, you may want more time to copy down the number displayed by a PRINT/PAUSE. All you need to do is press any key on the keyboard. If the command being executed is PRINTx (eight rapid blinks of the decimal point), pressing a key will cause the program to halt. Execution of the halted program may be re-initiated by pressing R/S .
HP-97 users may also want to keep a permanent record of the values input to a certain program. A convenient way to do this is to set the Print Mode switch to NORMAL before running the program. In this mode all input values and their corresponding user-definable keys will be listed on the printer, thus providing a record of the entire operation of the program.

Another area that could reflect differences between the HP-67 and the HP-97 is in the keystroke solutions to example problems. It is sometimes necessary in these solutions to include operations that involve prefix keys, namely, f on the HP-97 and $\mathbf{f}$, $\mathbf{g}$, and $\boldsymbol{h}$ on the HP-67. For example, the operation $10^{x}$ is performed on the HP-97 as $f 10^{x}$ and on the HP-67 as $98.10^{x}$. In such cases, the keystroke solution omits the prefix key and indicates only the operation (as here, $1 \mathbf{1 0}^{x}$ ). As you work through the example problems, take care to press the appropriate prefix keys (if any) for your calculator.
Also in keystroke solutions, those values that are output by the PRINT command will be followed by three asterisks ( ${ }^{* * *)}$.

Notes

## VECTOR STATICS

|  | VECTOR |  |  | CE1-01A |
| :---: | :---: | :---: | :---: | :---: |
| $x_{1}+y_{1}$ | $\mathrm{x}_{2}+y_{2}$ |  | F* $\dagger$ | - $\mathrm{R}_{1}, \mathrm{R}_{2}$ |
| $\mathrm{r}_{1}+\theta_{1}$ | $\mathrm{r}_{2} \cdot \theta_{2}$ | $-\dot{\mathbf{v}}_{1}+\dot{\mathrm{v}}_{2}$ | $-\stackrel{\rightharpoonup}{v}_{1} \times \dot{v}_{2}$ | - $\dot{v}_{1} \cdot \dot{v}_{2}: \mathrm{V}^{\prime}$ |

Part I of this program performs the basic two dimensional vector operations of addition, cross product and dot, scalar, or inner product. In addition, the angle between vectors may be found. Vectors may be input in polar form $(\mathrm{r}, \theta)$ or rectangular form $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$.

## Equations:

for addition: $\vec{V}_{1}+\vec{V}_{2}=\left(x_{1}+x_{2}\right) \vec{i}+\left(y_{1}+y_{2}\right) \vec{j}$
for cross products: $\vec{V}_{1} \times \vec{V}_{2}=\left(\mathrm{x}_{1} \mathrm{y}_{2}-\mathrm{x}_{2} \mathrm{y}_{1}\right) \overrightarrow{\mathrm{k}}$
for dot, scalar, or inner product: $\overrightarrow{\mathrm{V}}_{1} \cdot \overrightarrow{\mathrm{~V}}_{2}=\mathrm{x}_{1} \mathrm{x}_{2}+\mathrm{y}_{1} \mathrm{y}_{2}$
for the angle between vectors: $\gamma=\cos ^{-1} \frac{\overrightarrow{\mathrm{~V}}_{1} \cdot \overrightarrow{\mathrm{~V}}_{2}}{\left|\overrightarrow{\mathrm{~V}}_{1}\right|\left|\overrightarrow{\mathrm{V}}_{2}\right|}$
where:
$x_{1}$ is the $x$ component of $\vec{V}_{1}\left(x_{1}=r_{1} \cos \theta_{1}\right)$;
$x_{2}$ is the $x$ component of $\vec{V}_{2}\left(x_{2}=r_{2} \cos \theta_{2}\right)$;
$y_{1}$ is the $y$ component of $\vec{V}_{1}\left(y_{1}=r_{1} \sin \theta_{1}\right)$;
$y_{2}$ is the $y$ component of $\vec{V}_{2}\left(y_{2}=r_{2} \sin \theta_{2}\right)$;
Part II of this program calculates the two reaction forces necessary to balance a given two-dimensional force vector. The direction of the reaction forces may be specified as a vector of arbitrary length or by Cartesian coordinates using the point of force application as the origin.


## Equations:

$$
\begin{aligned}
\mathrm{R}_{1} \cos \theta_{1}+\mathrm{R}_{2} \cos \theta_{2} & =\mathrm{F} \cos \phi \\
\mathrm{R}_{1} \sin \theta_{1}+\mathrm{R}_{2} \sin \theta_{2} & =\mathrm{F} \sin \phi
\end{aligned}
$$

where:
F is the known force;
$\phi$ is the direction of the known force;
$\mathrm{R}_{1}$ is one reaction force;
$\theta_{1}$ is the direction of $\mathrm{R}_{1}$;
$R_{2}$ is the second reaction force;
$\theta_{2}$ is the direction of $\mathrm{R}_{2}$.
The coordinates $x_{1}$ and $y_{1}$ are referenced from the point where $F$ is applied to the end of the member along which $\mathrm{R}_{1}$ acts; $\mathrm{x}_{2}$ and $\mathrm{y}_{2}$ are the coordinates referenced from the point where F is applied to the end of the member along which $\mathrm{R}_{2}$ acts.

## Remarks:

Registers $\mathrm{R}_{0}-\mathrm{R}_{3} ; \mathrm{R}_{\mathrm{S} 0}-\mathrm{R}_{\mathrm{S9}}$ and I are available for user storage.

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 and side 2. |  |  |  |
| 2 | To resolve a force in two |  |  |  |
|  | known directions, go to step 6. |  |  |  |
|  | For vector addition, cross |  |  |  |
|  | product, or dot product con- |  |  |  |
|  | tinue with step 3. |  |  |  |
| 3 | Input $\vec{V}_{1}$ and $\vec{V}_{2}$ : |  |  |  |
|  | $\vec{V}_{1}$ in polar form | $\mathrm{r}_{1}$ | ENTERA | $\mathrm{r}_{1}$ |
|  |  | $\theta_{1}$ | A | $\mathrm{y}_{1}$ |
|  | or |  |  |  |
|  | $\vec{V}_{1}$ in rectangular form | ${ }_{1}$ | ENTERA | $\mathrm{x}_{1}$ |
|  |  | $y_{1}$ | 1 $\square^{\text {a }}$ | $y_{1}$ |
|  | and |  |  |  |
|  | $\vec{V}_{2}$ in polar form | $\mathrm{r}_{2}$ | ENTER | $\mathrm{r}_{2}$ |
|  |  | $\theta_{2}$ | B | $y_{2}$ |
|  | or |  |  |  |


| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
|  | $\vec{V}_{2}$ in rectangular form. | $\mathrm{X}_{2}$ | ENTER | $\mathrm{x}_{2}$ |
|  |  | $y_{2}$ | 1 B | $\mathrm{y}_{2}$ |
| 4 | Perform vector operation: |  |  |  |
|  | add vectors |  | c | r, $\theta$ |
|  | or |  |  |  |
|  | take cross product |  | D | $\vec{V}_{1} \times \vec{V}_{2}$ |
|  | or |  |  |  |
|  | take dot (or scalar) product. |  | E | $\vec{V}_{1} \cdot \vec{V}_{2}$ |
|  | (Optionally, calculate angle |  |  |  |
|  | between vectors after dot |  |  |  |
|  | product.) |  | R/S | $\gamma$ |
| 5 | For a new case, go to step 3 |  |  |  |
|  | and change $\vec{V}_{1}$ and/or $\vec{V}_{2}$. |  |  |  |
| 6 | Define reaction directions as |  |  |  |
|  | Cartesian coordinates or as |  |  |  |
|  | vectors of arbitrary magnitude. |  |  |  |
|  | (Use the point of force appli- |  |  |  |
|  | cations as the origin): |  |  |  |
|  | define direction one in polar |  |  |  |
|  | form | 1 | ENTERA | 1.00 |
|  |  | $\theta_{1}$ | $\triangle$ | $\sin \theta_{1}$ |
|  | or |  |  |  |
|  | in rectangular form | $\mathrm{x}_{1}$ | ENTER | $\mathrm{x}_{1}$ |
|  |  | $y_{1}$ | - A | $y_{1}$ |
|  | and |  |  |  |
|  | define direction two in polar |  |  |  |
|  | form | 1 | ENTER 4 | 1.00 |
|  |  | $\theta_{2}$ | B | $\sin \theta_{2}$ |
|  | or |  |  |  |
|  | in rectangular form. | $\mathrm{x}_{2}$ | ENTER | $\mathrm{x}_{2}$ |
|  |  | $\mathrm{y}_{2}$ | 1 B | $y_{2}$ |


| STEP | INSTRUCTIONS | INPUT <br> DATA/UNITS | KEYS | OUTPUT <br> DATA/UNITS |
| :---: | :--- | :---: | :---: | :---: |
| 7 | Input known force: |  |  |  |
|  | magnitude | F | ENTER | F |
|  | then direction. | $\phi$ | D | $\mathrm{F} \sin \phi$ |
| 8 | Compute reactions |  | E E | $\mathrm{R}_{1}, \mathrm{R}_{2}$ |
| 9 | To change force, go to step 7. |  |  |  |
|  | To change either or both |  |  |  |
|  | directions, go to step 6. |  |  |  |

## Example 1:

Forces A and B are shown below. If static equilibrium exists, what is force C.


## Keystrokes:

Outputs:
To obtain $\vec{C}$, add $\vec{A}$ and $\vec{B}$ using negative magnitudes for both.
45 CHS ENTERA 110 A 100 CHS

## ENTERt 30

BC

116.57 ***
-127.66 ***

$$
\overrightarrow{\mathrm{C}}=116.57 \angle-127.66^{\circ}
$$

## 01-05

## Example 2:

Resolve the following three loads along a 175 degree line.


## Keystrokes:

First add $\overrightarrow{\mathrm{L}}_{1}$ and $\overrightarrow{\mathrm{L}}_{2}$.
185 ENTER4 62 A 170 ENTER4
143 BCC

Define the result as $\overrightarrow{\mathrm{V}}_{1}$ and add $\overrightarrow{\mathrm{L}}_{3}$.
A 100 ENTER4 261 BC $\longrightarrow$

To resolve the vector, just calculated along the $175^{\circ}$ line.
A 1 ENTER4 175 BE
$78.86^{* * *}$ (lb)

What is the angle between the vector and the line?
R/S $\qquad$ $63.85^{* * *}$ (deg)

## Example 3:

What is the moment at the shaft of the crank pictured below? What is the reaction force transmitted along the member?


## Keystrokes:

Moment by cross product ( $\overrightarrow{\mathrm{V}}_{1} \times \overrightarrow{\mathrm{F}}$ ).
30 ENTERA 50 A 300 ENTER4

Resolution along crank
1 ENTERA 50 AE $\longrightarrow \quad-271.89 \mathrm{lb}$

## Example 4:

Find the reaction forces in the pin-jointed structure shown below.


Keystrokes:


## Outputs:

$$
\begin{aligned}
&-8.00 \\
& 0.00 \\
&-500.00 \\
&-664.38 \\
& 437.50 * *\left(\mathrm{R}_{1}\right) \\
& * * * \\
&\left(\mathrm{R}_{2}\right)
\end{aligned}
$$

Notes

## SECTION PROPERTIES



The properties of polygonal sections (see figure 1) may be calculated using this program. The ( $\mathrm{x}, \mathrm{y}$ ) coordinates of the vertices of the polygon (which must be located entirely within the first quadrant) are input sequentially for a complete, clockwise path around the polygon. Holes in the cross section, which do not intersect the boundary, may be deleted by following a counter-clockwise path.


Figure 1 - Polygonal Sections

A special feature allows addition or deletion of circular areas. After the point by point traverse of the section has been completed, circular deletions or additions are specified by the ( $x, y$ ) coordinates of the circle centers and by the circle diameters. If the diameter is specified as a positive number, the circular areas are added. A negative diameter causes circular areas to be deleted. Example 4 shows an application of this feature.

After all values have been input, the coordinates of the centroid ( $\bar{x}, \bar{y}$ ) and the area (A) of the section may be output using card 2, key $\boldsymbol{A}$. The moment of inertia about the $x$ axis $\left(I_{x}\right)$, about the $y$ axis $\left(I_{y}\right)$ and the product of inertia ( $\mathrm{I}_{\mathrm{x}, \mathrm{y}}$ ) are output using B. Similar moments, $\mathrm{I}_{\overline{\mathrm{x}}}$, $\mathrm{I}_{\overline{\mathrm{y}}}$ and $\mathrm{I}_{\overline{\mathrm{xy}}}$, about an axis translated to the centroid of the section are calculated when $\mathbf{C}$ is pressed.

Pressing D calculates the moments of inertia, $\mathrm{I}_{\overline{\mathrm{x}} \phi}$ and $\mathrm{I}_{\overline{\mathrm{y}} \phi}$, about the principal axis. The rotation angle $(\phi)$ between the principal axis and the axis which was translated to the centroid is also calculated. The moments of inertia $\mathrm{I}_{\mathrm{x}}{ }^{\prime}, \mathrm{I}_{\mathrm{y}}{ }^{\prime}$, the polar moment of inertia J and the product of inertia $\mathrm{I}_{\mathrm{xy}}{ }^{\prime}$ may be calculated about any arbitrary axis by specifying its location and rotation with respect to the original axis and pressing $\boldsymbol{D}$

## Equations:

$$
\begin{gathered}
A=-\sum_{i=0}^{n}\left(y_{i+1}-y_{i}\right)\left(x_{i+1}+x_{i}\right) / 2 \\
\bar{x}=\frac{-1}{A} \sum_{i=0}^{n}\left[\left(y_{i+1}-y_{i}\right) / 8\right]\left[\left(x_{i+1}+x_{i}\right)^{2}+\left(x_{i+1}-x_{i}\right)^{2} / 3\right] \\
\bar{y}= \\
\frac{1}{A} \sum_{i=0}^{n}\left[\left(x_{i+1}-x_{i}\right) / 8\right]\left[\left(y_{i+1}+y_{i}\right)^{2}+\left(y_{i+1}-y_{i}\right)^{2} / 3\right] \\
I_{x}=\sum_{i=0}^{n}\left[\left(x_{i+1}-x_{i}\right)\left(y_{i+1}+y_{i}\right) / 24\right]\left[\left(y_{i+1}+y_{i}\right)^{2}+\left(y_{i+1}-y_{i}\right)^{2}\right] \\
I_{y}=- \\
\sum_{i=0}^{n}\left[\left(y_{i+1}-y_{i}\right)\left(x_{i+1}+x_{i}\right) / 24\right]\left[\left(x_{i+1}+x_{i}\right)^{2}+\left(x_{i+1}-x_{i}\right)^{2}\right] \\
I_{i=0}^{n} \frac{1}{\left(x_{i+1}-x_{i}\right)}\left[\frac{1}{8}\left(y_{i+1}-y_{i}\right)^{2}\left(x_{i+1}+x_{i}\right)\left(x_{i+1}{ }^{2}+x_{i}^{2}\right)\right. \\
\\
+\frac{1}{3}\left(y_{i+1}-y_{i}\right)\left(x_{i+1} y_{i}-x_{i} y_{i+1}\right)\left(x_{i+1}^{2}+x_{i+1} x_{i}+x_{i}^{2}\right) \\
\\
\left.+\frac{1}{4}\left(x_{i+1} y_{i}-x_{i} y_{i+1}\right)^{2}\left(x_{i+1}+x_{i}\right)\right] \\
I_{\bar{x}}=I_{x}-A \bar{y}^{2} \\
I_{\bar{y}}=I_{y}-A \bar{x}^{2} \\
I_{\bar{x} \bar{y}}=I_{x y}-A \bar{x} \bar{y}
\end{gathered}
$$

$$
\begin{gathered}
\phi=\frac{1}{2} \tan ^{-1}\left(\frac{-2 \mathrm{I}_{\overline{\mathrm{x}} \overline{\mathrm{y}}}}{\mathrm{I}_{\overline{\mathrm{x}}}-\mathrm{I}_{\overline{\mathrm{y}}}}\right) \\
\mathrm{I}_{\mathrm{x}}^{\prime}=\mathrm{I}_{\overline{\mathrm{x}}} \cos ^{2} \theta+\mathrm{I}_{\overline{\mathrm{y}}} \sin ^{2} \theta-\mathrm{I}_{\overline{\mathrm{x}} \overline{\mathrm{y}}} \sin 2 \theta \\
\mathrm{I}_{\mathrm{y}}^{\prime}=\mathrm{I}_{\overline{\mathrm{y}}} \cos ^{2} \theta+\mathrm{I}_{\overline{\mathrm{x}}} \sin ^{2} \theta+\mathrm{I}_{\overline{\mathrm{x}} \overline{\mathrm{y}}} \sin 2 \theta \\
\mathrm{~J}=\mathrm{I}_{\mathrm{x}}^{\prime}+\mathrm{I}_{\mathrm{y}}^{\prime} \\
\mathrm{I}_{\mathrm{xy}}^{\prime} \\
\mathrm{I}_{\mathrm{circle}}=\frac{\left(\mathrm{I}_{\overline{\mathrm{x}}}-\mathrm{I}_{\overline{\mathrm{y}}}\right)}{2} \sin 2 \theta+\mathrm{I}_{\overline{\mathrm{x}} \overline{\mathrm{y}}} \cos 2 \theta \\
\mathrm{~A}_{\text {circle }}=\frac{\pi \mathrm{d}^{2}}{4} \\
\mathrm{I}^{4}
\end{gathered}
$$

where:
$\mathrm{X}_{\mathrm{i}+1}$ is the x coordinate of the current vertex point;
$y_{i+1}$ is the $y$ coordinate of the current vertex point;
$x_{i}$ is the $x$ coordinate of the previous vertex point;
$y_{i}$ is the $y$ coordinate of the previous vertex point;
A is the area;
$\overline{\mathrm{x}}$ is the x coordinate of the centroid;
$\bar{y}$ is the $y$ coordinate of the centroid;
$\mathrm{I}_{\mathrm{x}}$ is the moment of inertia about the x -axis;
$I_{y}$ is the moment of inertia about the $y$-axis;
$\mathrm{I}_{\mathrm{xy}}$ is the product of inertia;
$\mathrm{I}_{\overline{\mathrm{x}}}$ is the moment of inertia about the x -axis translated to the centroid; $\mathrm{I}_{\overline{\mathrm{y}}}$ is the moment of inertia about the y -axis translated to the centroid;
$\mathrm{I}_{\bar{x} \bar{y}}$ is the product of inertia about the translated axis;
$\phi$ is the angle between the translated axis and the principal axis;
$\mathrm{I}_{\overline{\mathrm{x}} \phi}$ is the moment of inertia about the translated, rotated, principal x -axis;
$\mathrm{I}_{\overline{\mathrm{y}} \phi}$ is the moment of inertia about the translated, rotated, principal $y$-axis;
$\theta$ is the angle between the original axis and an arbitrary axis.
$\mathrm{I}_{\mathrm{x}}{ }^{\prime}$ is the x moment of inertia about the arbitrary axis;
$I_{y}{ }^{\prime}$ is the $y$ moment of inertia about the arbitrary axis;

J is the polar moment of inertia about the arbitrary axis;
$\mathrm{I}_{\mathrm{xy}}{ }^{\prime}$ is the product of inertia about the arbitrary axis;
$d$ is the diameter of a circular area.

## Reference:

Wojciechowski, Felix; Properties of Plane Cross Sections; Machine Design; p. 105, Jan. 22, 1976.

## Remarks:

Registers $\mathrm{R}_{\mathrm{S} 0}-\mathrm{R}_{\mathrm{S} 9}$ are available for user storage.
The polygon must be entirely contained in the first quadrant.
Rounding errors will accumulate if the centroid of the section is a large distance from the origin of the coordinate system.

Curved boundaries may be approximated by straight line segments.

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 and side 2 of |  |  |  |
|  | card 1. |  |  |  |
| 2 | Initialize. |  | (1) |  |
| 3 | Key in ( $\mathrm{x}, \mathrm{y}$ ) coordinates of |  |  |  |
|  | first vertex. | $\mathrm{x}_{\mathrm{i}}$ | ENTER | $y_{i}$ |
|  |  | $y_{i}$ | ENTER4 | $y_{i}$ |
| 4 | Key in (x, y) coordinates of |  |  |  |
|  | next clockwise vertex. | $\mathrm{x}_{\text {i }}$ | ENTER | $\mathrm{x}_{\text {i }}$ |
|  |  | $y_{i+1}$ | A | $y_{i+1}$ |
| 5 | Wait for execution to end, then |  |  |  |
|  | repeat step 4 for next point. |  |  |  |
|  | Go to step 6 after you have |  |  |  |
|  | reinput the starting point. |  |  |  |
| 6 | To delete subsections within |  |  |  |
|  | the section just traversed, |  |  |  |
|  | return to step 3, but traverse in |  |  |  |
|  | a counter-clockwise direction. |  |  |  |


| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 7 | Optional: Add circular areas, | x | ENTER | x |
|  |  | $y$ | ENTER | y |
|  |  | d | C | 0.00 |
|  | or delete circular areas. | x | ENTER | x |
|  |  | y | ENTER | $y$ |
|  |  | d | CHS C | 0.00 |
| 8 | Load side 1 and side 2 of |  |  |  |
|  | card 2. |  |  |  |
| 9 | Calculate any or all of the |  |  |  |
|  | following: |  |  |  |
|  | Centroid and area; |  | A | $\overline{\mathrm{x}}, \overline{\mathrm{y}}, \mathrm{A}$ |
|  | Properties about original |  |  |  |
|  | axis; |  | B | $\mathrm{I}_{\mathrm{x}}, \mathrm{I}_{\mathrm{y}}, \mathrm{I}_{\mathrm{xy}}$ |
|  | Properties about axis trans- |  |  |  |
|  | lated to centroid; |  | c | $\mathrm{I}_{\overline{\mathrm{x}}}, \mathrm{I}_{\overline{\mathrm{y}}}, \mathrm{I}_{\overline{\mathrm{x}} \mathrm{l}}$ |
|  | Angular orientation of |  |  |  |
|  | principal axis and properties |  |  |  |
|  | about principal axis; |  | D | $\phi, \mathrm{I}_{\overline{\mathrm{x}} \boldsymbol{\prime}}, \mathrm{I}_{\overline{\mathrm{y}} \phi}$ |
|  | or |  |  |  |
|  | Specify arbitrary axis and |  |  |  |
|  | rotation and calculate |  |  |  |
|  | properties. | $\mathrm{x}^{\prime}$ | ENTER |  |
|  |  | $\mathrm{y}^{\prime}$ | ENTER |  |
|  |  | $\theta$ | 1 D | $\mathrm{I}_{\mathrm{x}}{ }^{\prime}, \mathrm{I}_{\mathrm{y}}{ }^{\prime}, \mathrm{J}, \mathrm{I}_{\mathrm{xy}}{ }^{\prime}$ |
| 10 | To modify the section, go to |  |  |  |
|  | step 1, but skip step 2. For a |  |  |  |
|  | new case, go to step 1. |  |  |  |

## Example 1:

What is the moment of inertia about the x -axis $\left(\mathrm{I}_{\mathrm{x}}\right)$ for the rectangular section shown? What is the moment of inertia about the neutral axis through the centroid of the section $\left(\mathrm{I}_{\overline{\mathrm{x}} \phi}\right)$ ?


Keystrokes:

## Outputs:

Load side 1 and side 2 of card 1.


Load side 1 and side 2 of card 2 .

B

$$
\begin{array}{rl}
125.00 & * * *\left(\mathrm{I}_{\mathrm{x}}\right) \\
45.00 & * * *\left(\mathrm{I}_{\mathrm{y}}\right) \\
56.25 & * * *\left(\mathrm{I}_{\mathrm{xy}}\right) \\
0.00^{* * *}(\phi) \\
31.25 & \text { *** }\left(\mathrm{I}_{\overline{\mathrm{x} \phi}}\right) \\
11.25 & \text { })
\end{array}
$$

Since $\phi=0$ we would expect $\mathrm{I}_{\overline{\mathrm{x}} \phi}$ to equal $\mathrm{I}_{\overline{\mathrm{x}}}$. Press $\mathbf{C}$ to calculate $\mathrm{I}_{\overline{\mathrm{x}}}, \mathrm{I}_{\bar{y}}$ and $\mathrm{I}_{\overline{\mathrm{x}}}$ and you will see that this prediction is correct. Also, $\mathrm{I}_{\overline{\mathrm{x}}}$ is zero about the principal axis.

C


$$
\begin{array}{rl}
31.25 & * * *\left(\mathrm{I}_{\mathrm{x}}\right) \\
11.25 & * * *\left(\mathrm{I}_{\overline{\mathrm{y}}}\right) \\
0.00^{* * *}\left(\mathrm{I}_{\overline{\mathrm{x}}}\right)
\end{array}
$$

## Example 2:

Calculate the section properties for the beam shown below.


Load side 1 and side 2 of card 1 .

0 ENTERA 0 A
0.00

Load side 1 and side 2 of card 2 .
A

|  | ( $\overline{\mathrm{x}}$ ) |
| :---: | :---: |
|  | ( |
| 49 | (A) |
| 676 | ( |
| 2256 | , |
| 890 | *** |
| 580 | *** |
| 934 | ( $\mathrm{I}_{\overline{\mathrm{y}}}$ ) |
| 22 | *** ( $\mathrm{I}_{\overline{\mathrm{x}} \overline{\mathrm{y}}}$ |
| -17 | *** ( $\phi$ ) |
| 165 | *** ( $\mathrm{I}_{\overline{\mathrm{x}} \boldsymbol{\phi}}$ ) |
| 863.4 | * ( $\mathrm{I}_{\overline{\mathrm{y}} \phi}$ |

Below is a figure showing the translated axis and the rotated, principal axis of example 2.


## Example 3:

What is the centroid of the section below? The inner triangular boundary denotes an area to be deleted.


Keystrokes:
Outputs:
Load side 1 and side 2 of card 1.
f A 3 ENTER4 1 ENTER4

| 3 ENTERA $7 \boldsymbol{A} \longrightarrow$ | 7.00 <br> 14 ENTERA $7 \boldsymbol{A} \longrightarrow$ <br> 3 ENTERA 1 A $\longrightarrow$ |
| :--- | :--- |
| 7.00 |  |
| 1.00 |  |

Delete inner triangle:
4 ENTERA 4 ENTERA 9 ENTERA
6 A
6.00

4 ENTERA 6 A $\longrightarrow \quad 6.00$
4 ENTERA $4 \boldsymbol{A} \longrightarrow \quad 4.00$
Load side 1 and side 2 of card 2 .
Compute Centroid
A
$6.85^{* * *}(\overline{\mathrm{x}})$
$4.94^{* * *}(\overline{\mathrm{y}})$
$28.00^{* * *}(\mathrm{~A})$

## Example 4:

For the part below, compute the polar moment of inertia about point A . Point A denotes the center of a hole about which the part rotates. The area of the hole must be deleted from the cross section.


Keystrokes:
Outputs:
Load side 1 and side 2 of card 1.
f A 0 ENTER4 0 ENTER4 0 ENTER
2 A 5 ENTER4 2 A 5 ENTER4
1.4 A . 8 ENTERA 1.4 A . 8 ENTER4
$0 \boldsymbol{A} 0$ ENTER $0 \boldsymbol{A} \longrightarrow 0.00$
Delete the hole.
. 2 ENTER4 6 ENTER
.5 CHS C $\longrightarrow 0.00$
Load side 1 and side 2 of card 2 .
Compute J about point (.2, .6) with $\theta$ of zero.
. 2 ENTER4 6 ENTER4


$$
\begin{array}{rl}
3.91 & * * *\left(\mathrm{I}_{\mathrm{x}^{\prime}}\right) \\
22.22 & * * *\left(\mathrm{I}_{\mathrm{y}^{\prime}}\right) \\
26.13 & * * *\left(\mathrm{~J}^{2}\right) \\
7.61 & * * *\left(\mathrm{I}_{\mathrm{xy}}{ }^{\prime}\right)
\end{array}
$$

## PROPERTIES OF SPECIAL SECTIONS



For rectangles, triangles, ellipses, circles, and concentric circles, this program performs an interchangeable solution between the section dimensions and the principle moment of inertia about the x axis. The section area and the principle moment of inertia about the $y$ axis may also be calculated.

## Sections and Equations:



$$
\begin{gathered}
\mathrm{I}_{\mathrm{x}}=\mathrm{a}^{3} \mathrm{~b} / 12 \\
\mathrm{I}_{\mathrm{y}}=\mathrm{ab} b^{3} / 12 \\
\mathrm{~A}=\mathrm{ab}
\end{gathered}
$$

$$
\begin{gathered}
I_{x}=a^{3} b / 36 \\
I_{y}=a b^{3} / 36 \\
A=a b / 2
\end{gathered}
$$



$$
\mathrm{I}_{\mathrm{x}}=\pi \mathrm{a}^{3} \mathrm{~b} / 64
$$



$$
\mathrm{I}_{\mathrm{y}}=\pi \mathrm{ab}^{3} / 64
$$

$$
\begin{gathered}
\mathrm{I}_{\mathrm{x}}=\pi \mathrm{a}^{4} / 4=\mathrm{I}_{\mathrm{y}} \\
\mathrm{~A}=\pi \mathrm{a}^{2} / 4
\end{gathered}
$$

$$
\mathrm{I}_{\mathrm{x}}=\frac{\pi\left(\mathrm{a}^{4}-\mathrm{b}^{4}\right)}{64}=\mathrm{I}_{\mathrm{y}}
$$

$$
\mathrm{A}=\pi \mathrm{ab} / 4
$$

$$
\mathrm{A}=\frac{\pi\left(\mathrm{a}^{2}-\mathrm{b}^{2}\right)}{4}
$$

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 and side 2. |  |  |  |
| 2 | Select cross section shape. |  |  |  |
|  | Rectangle |  | 18 | 1.00000 |
|  | Triangle |  | B | 2.00000 |
|  | Ellipse |  | C | 3.00000 |
|  | Circle |  | 1 D | 4.00000 |
|  | Concentric Circles |  | 1 E | 5.00000 |
| 3 | Input two of the following:* | a | A | a |
|  |  | b | B | b |
|  |  | $\mathrm{I}_{\mathrm{x}}$ | c | $\mathrm{I}_{\mathrm{x}}$ |
| 4 | Compute unknown value:* |  | A | a |
|  |  |  | B | b |
|  |  |  | c | $\mathrm{I}_{\mathrm{x}}$ |
| 5 | Optional: Compute area |  | D | A |
| 6 | Optional: Compute Iy |  | $E$ | I, |
| 7 | For a new case, go to step 3 |  |  |  |
|  | and change inputs |  |  |  |
|  | *For circles, only one input or |  |  |  |
|  | output is allowed. |  |  |  |
|  | Input $\mathrm{I}_{\mathrm{x}}$ or a only. |  |  |  |

## Example 1:

For the rectangular section below, what is the moment of inertia about the x axis? What is the moment of inertia about the y axis?


Keystrokes:

42.5 B 90 AC

E

Outputs:

| 1.00000 | (Select) <br> (rectangles) |
| :--- | :--- |
| 2.58206 | $\mathrm{cm}^{4}\left(\mathrm{I}_{\mathrm{x}}\right)$ <br> 575.7 $\mathrm{~cm}^{4}\left(\mathrm{I}_{\mathrm{y}}\right)$ |

## Example 2:

For the elliptical section below, what is the required value of b to make $\mathrm{I}_{\mathrm{x}}=1000$ ? What is the area of the section?


Keystrokes:


Outputs:
$6.03600 \quad$ in (b)
$71.1100 \quad$ in $^{2}$ (A)

## STRESS ON AN ELEMENT



This program reduces data from rosette strain gage measurements and/or performs Mohr circle stress analysis calculations.

Correlations for rectangular and equiangular rosette configurations are included.

## Strain Gage Equations:

| CONFIGURATION CODE | 1 | 2 |
| :---: | :---: | :---: |
| TYPE OF ROSETTE | RECTANGULAR | DELTA (EQUIANGULAR) |
|  |  |  |
| PRINCIPAL STRAINS: $\epsilon_{1}, \epsilon_{2}$ | $\frac{1}{2}\left[\epsilon_{\mathrm{a}}+\epsilon_{\mathrm{c}} \pm \sqrt{2\left(\epsilon_{\mathrm{a}}-\epsilon_{\mathrm{h}}\right)^{2}+2\left(\epsilon_{\mathrm{b}}-\epsilon_{\mathrm{C}}\right)^{2}}\right]$ | $\begin{aligned} & \frac{1}{3}\left[\epsilon_{\mathrm{a}}+\epsilon_{\mathrm{b}}+\epsilon_{\mathrm{c}}\right. \\ & \left. \pm \sqrt{2\left(\epsilon_{\mathrm{a}}-\epsilon_{\mathrm{b}}\right)^{2}+2\left(\epsilon_{\mathrm{b}}-\epsilon_{\mathrm{c}}\right)^{2}+2\left(\epsilon_{\mathrm{c}}-\epsilon_{\mathrm{a}}\right)^{2}}\right] \end{aligned}$ |
| CENTER OF MOHR CIRCLE: $\frac{\mathrm{s}_{1}+\mathrm{s}_{2}}{2}$ | $\frac{\mathrm{E}\left(\boldsymbol{\epsilon}_{\mathrm{a}}+\boldsymbol{\epsilon}_{\mathrm{c}}\right)}{2(1-\nu)}$ | $\frac{E\left(\boldsymbol{\epsilon}_{\mathrm{a}}+\boldsymbol{\epsilon}_{\mathrm{b}}+\boldsymbol{\epsilon}_{\mathrm{c}}\right)}{3(1-\boldsymbol{\nu})}$ |
| NAXIMUM SHEAR STRESS: $\tau_{\text {max }}$ | $\frac{\mathrm{E}}{2(1+\nu)} \sqrt{2\left(\epsilon_{\mathrm{a}}-\epsilon_{\mathrm{b}}\right)^{2}+2\left(\epsilon_{\mathrm{b}}-\epsilon_{\mathrm{c}}\right)^{2}}$ | $\frac{\mathrm{E}}{3(1+\nu)} \sqrt{2\left(\epsilon_{\mathrm{a}}-\epsilon_{\mathrm{b}}\right)^{2}+2\left(\epsilon_{\mathrm{b}}-\epsilon_{\mathrm{c}}\right)^{2}+2\left(\epsilon_{\mathrm{c}}-\epsilon_{\mathrm{a}}\right)^{2}}$ |
| ORIENTATION OF PRINCIPAL STRESSES | $\tan ^{-1}\left[\frac{2 \epsilon_{\mathrm{b}}-\epsilon_{\mathrm{a}}-\epsilon_{\mathrm{c}}}{\epsilon_{\mathrm{a}}-\epsilon_{\mathrm{c}}}\right]$ | $\tan ^{-1}\left[\frac{\sqrt{3}\left(\epsilon_{\mathrm{c}}-\epsilon_{\mathrm{b}}\right)}{\left(2 \epsilon_{\mathrm{a}}-\epsilon_{\mathrm{b}}-\epsilon_{\mathrm{c}}\right)}\right]$ |

The Mohr circle portion of the program converts an arbitrary stress configuration to principal stresses, maximum shear stress and rotation angle. It is then possible to calculate the state of stress for an arbitrary orientation $\boldsymbol{\theta}^{\prime}$.


## Mohr Circle Equations:

$$
\begin{gathered}
\tau_{\max }=\sqrt{\left(\frac{\mathrm{s}_{\mathrm{x}}-\mathrm{s}_{\mathrm{y}}}{2}\right)^{2}+\tau_{\mathrm{xy}}^{2}} \\
\mathrm{~s}_{1}=\frac{\mathrm{s}_{\mathrm{x}}+\mathrm{s}_{\mathrm{y}}}{2}+\tau_{\max } \\
\mathrm{s}_{2}=\frac{\mathrm{s}_{\mathrm{x}}+\mathrm{s}_{\mathrm{y}}}{2}-\tau_{\max } \\
\theta=1 / 2 \tan ^{-1}\left(\frac{2 \tau_{\mathrm{xy}}}{\mathrm{~s}_{\mathrm{x}}-\mathrm{s}_{\mathrm{y}}}\right) \\
\mathrm{s}=\frac{\mathrm{s}_{1}+\mathrm{s}_{2}}{2}+\tau_{\max } \cos 2 \theta^{\prime} \\
\tau=\tau_{\max } \sin 2 \theta^{\prime}
\end{gathered}
$$

where:
$s$ is the normal stress, and $\tau$ is the shear stress.
$\epsilon_{\mathrm{a}}, \epsilon_{\mathrm{b}}$, and $\epsilon_{\mathrm{c}}$ are the strains measured using rosette gages;
$\mathrm{s}_{\mathrm{x}}$ is the stress in the x direction for Mohr circle input;
$s_{y}$ is the stress in the $y$ direction for Mohr circle input;
$\tau_{\mathrm{xy}}$ is the shear stress on the element for Mohr circle input;
$\epsilon_{1}$ and $\epsilon_{2}$ are the principal strains;
$\mathrm{s}_{1}$ and $\mathrm{s}_{2}$ are the principal normal stresses;
$\tau_{\text {max }}$ is the maximum shear stress;
$\nu$ is Poisson's ratio;
$\theta$ is the counterclockwise angle of rotation from the specified axis to the principal axis. Note that this is opposite to the normal Mohr circle convention.
$\boldsymbol{\theta}^{\prime}$ is an arbitrary rotation angle from the original ( $\mathrm{x}, \mathrm{y}$ ) axis;
$E$ is modulus of elasticity.

## Reference:

Spotts, M.F., Design of Machine Elements, Prentice-Hall, 1971.
Beckwith, T. G., Buck, N. L., Mechanical Measurements, Addison-Wesley, 1969

## Remarks:

$\mathrm{R}_{0}, \mathrm{R}_{1}, \mathrm{R}_{7}, \mathrm{R}_{8}, \mathrm{R}_{\mathrm{D}}$ and $\mathrm{R}_{\mathrm{S} 0}-\mathrm{R}_{\mathrm{S} 9}$ are available for user storage.
Negative stresses and strains indicate compression. Positive and negative shear are represented below:


| STEP | INSTRUCTIONS | INPUT <br> DATA/UNITS | KEYS | OUTPUT <br> DATA/UNITS |
| ---: | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | Load side 1 and side 2. |  |  |  |
| 2 | If a stress configuration is |  |  |  |
|  | known, go to step 8 for Mohr |  |  |  |
|  | circle evaluation. Continue |  |  |  |
|  | with step 3 for strain gage |  |  |  |
|  | data reduction. |  |  |  |
| 3 | Select strain gage |  |  |  |
|  | configuration: |  |  |  |
|  | Rectangular |  | B | 1.00000 |
|  | or Delta. |  |  |  |


| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 4 | Input modulus of elasticity, | E | Entert | E |
|  | then Poisson's ratio. | $\nu$ | 18 | E |
| 5 | Input strains: |  |  |  |
|  |  | $\epsilon_{\text {a }}$ | ENTER | $\epsilon_{\text {a }}$ |
|  |  | $\epsilon_{\mathrm{b}}$ | ENTER | $\epsilon_{\text {b }}$ |
|  |  | $\epsilon_{\text {c }}$ | A | $\epsilon_{\text {a }}$ |
| 6 | Calculate principal strains |  |  |  |
|  | and rotation angle. |  | B | $\epsilon_{1}, \epsilon_{2}, \theta$ |
| 7 | Skip to step 9 for Mohr circle |  |  |  |
|  | applications of calculations |  |  |  |
|  | just completed. |  |  |  |
| 8 | Input stress on element in x |  |  |  |
|  | direction | $S_{x}$ | ENTER | $S_{x}$ |
|  | then stress in y direction | $\mathrm{s}_{\mathrm{y}}$ | ENTER | $S_{y}$ |
|  | then shear stress. | $\tau_{\text {xy }}$ | c | 0.00000 |
| 9 | Calculate principal stresses. |  | D | $\mathrm{s}_{1}, \mathrm{~s}_{2}, \tau_{\text {max }}$, |
|  |  |  |  | $\theta$ |
| 10 | Optional: Calculate stress |  |  |  |
|  | configuration at a specified |  |  |  |
|  | angle. | $\theta^{\prime}$ | E | s, $\tau$ |
| 11 | To specify another angle go |  |  |  |
|  | to step 10. For a new case go |  |  |  |
|  | to step 2. |  |  |  |

## Example 1:

If $\mathrm{s}_{\mathrm{x}}=25000 \mathrm{psi}, \mathrm{s}_{\mathrm{y}}=-5000 \mathrm{psi}$, and $\tau_{\mathrm{xy}}=4000$ psi, compute the principal stresses and the maximum shear stress. Compute the normal stresses, where shear stress is maximum $\left(\theta+45^{\circ}\right)$.


Keystrokes:


## Outputs:

$$
\begin{array}{rlll}
25.52 & 03 & * * *\left(\mathrm{~s}_{1}\right) \\
-5.524 & 03 & * * * & \left(\mathrm{~s}_{2}\right) \\
15.52 & 03 & * * *\left(\tau_{\max }\right) \\
-7.466 & 00^{* * *}(\theta) \\
37.53 & 00 & \\
10.00 & 03 & \text { *** }(\mathrm{s}) \\
15.52 & 03 & * * *\left(\tau_{1}\right)
\end{array}
$$

## Example 2:

A rectangular rosette measures the strains below. What are the principal strains and principal stresses?

$$
\begin{array}{lll}
\epsilon_{\mathrm{a}}=90 \times 10^{-6} & \epsilon_{\mathrm{b}}=137 \times 10^{-6} & \epsilon_{\mathrm{c}}=305 \times 10^{-6} \\
\nu=0.3 & \mathrm{E}=30 \times 10^{6} \mathrm{psi} &
\end{array}
$$

Keystrokes:


Outputs:

$$
1.00000
$$

30.0006

$$
\begin{aligned}
& 90.00-06 \\
& 320.9-06
\end{aligned}{ }^{* * *}\left(\epsilon_{1}\right)
$$

## Example 3:

An equiangular rosette measures the strains below. What are the principal strains and stresses?

$$
\epsilon_{\mathrm{b}}=-20 \times 10^{-6}
$$

$60^{\circ}$

$$
\epsilon_{\mathrm{a}}=400 \times 10^{-6}
$$

Keystrokes:


Outputs:

400 EEX CHS 6 ENTER4 20
CHS EEX CHS 6 ENTERA 200


D $\qquad$


## BENDING OR TORSIONAL STRESS



This card solves either the bending stress equation or the analogous torsional shear stress equation, using an interchangeable solution. Given three known values, the remaining unknown value is calculated.
Variables involved in torsional shear stress calculations are shown in parentheses on the magnetic card.

## Equations:


where:
$\mathbf{s}$ is the normal stress at $\mathbf{v}$;
$M$ is the moment applied to the beam;
$v$ is the distance from the neutral axis of the beam;
I is the moment of inertia of the beam;
$s_{s}$ is the shear stress at $r$;
T is the applied torque;
$r$ is the distance from the shaft center to the point of interest;
$J$ is the polar moment of inertia.

## Remarks:

This program is not applicable for non-elastic media or elastic media where stresses exceed the elastic limit. Materials must be isotropic.

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 or side 2. |  |  |  |
| 2 | Input 3 of the following: |  |  |  |
|  | Bending stress (or shear |  |  |  |
|  | stress) | $\mathrm{s}\left(\mathrm{s}_{\mathrm{s}}\right)$ | A | $\mathrm{s}\left(\mathrm{s}_{\mathrm{s}}\right)$ |
|  | Bending moment (or applied |  |  |  |
|  | torque) | M (T) | B | M (T) |
|  | Distance from neutral axis |  |  |  |
|  | (or radius) | $v$ (r) | c | $v(r)$ |
|  | Moment of inertia (or polar |  |  |  |
|  | moment) | I (J) | D | I (J) |
| 3 | Calculate the remaining value: |  |  |  |
|  | Bending stress (or shear |  |  |  |
|  | stress) |  | A | $\mathrm{s}\left(\mathrm{s}_{\mathrm{s}}\right)$ |
|  | Bending moment (or torque) |  | B | M (T) |
|  | Distance from neutral axis |  |  |  |
|  | (or radius) |  | c | $v(r)$ |
|  | Moment of inertia (or polar |  |  |  |
|  | moment) |  | D | I (J) |
| 4 | For a new case, go to step 2 |  |  |  |
|  | and change appropriate inputs. |  |  |  |

## Example 1:

If the maximum stress allowed in a beam is 10,000 pounds per square inch, the moment of inertia is $4.80 \mathrm{in}^{4}$, and the maximum distance from the neutral axis to the surface is 2 inches, what is the maximum applied moment?

Keystrokes:

## Outputs:

10000 A 4.8 D 2 CB
24.0003 in-lb (M)

## Example 2:

What torque will result in a stress of 12000 pounds per square inch at a radius of 1 inch for a 2 inch diameter shaft?

## Keystrokes:



D 1 C 12000 AB

Outputs:
$1.57100 \quad \mathrm{in}^{4}$ (J)
18.8503 in-lb (T)

## Example 3:

A moment of $30,000 \mathrm{in}-\mathrm{lb}$ is applied to a beam with a moment of inertia of $3.8 \mathrm{in}^{4}$. If the neutral axis is 1 inch from the surface, what is the stress at the surface?

## Keystrokes:

30000 B 3.8 D 1 CA $\longrightarrow$

## Outputs:

7.89503 psi (x)

## LINEAR OR ANGULAR DEFORMATION



This card solves for linear deflection under tensile load or the analogous angular deflection under torque using an interchangeable solution. Given four of the five variables, the unknown is calculated.
Variables for circular shafts in torsion are shown in parentheses on the magnetic cards.

## Equations:


where:
$\Delta l$ is the change in length;
P is the applied load;
$\ell$ is the length;
A is the cross sectional area;
E is the modulus of elasticity;
$\theta$ is the deflection angle in radians;
T is the applied torque;
J is the polar moment of the section;
G is the modulus of elasticity in shear.

## Remarks:

This program is not applicable for non-elastic media or elastic media where stress exceeds the elastic limit. Materials must be isotropic. The equation for angular deflection is not valid in the neighborhood of the applied torque.

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 or side 2. |  |  |  |
| 2 | Input four of the following: |  |  |  |
|  | Area (or polar moment of |  |  |  |
|  | inertia) | A (J) | A | A (J) |
|  | Linear deflection (or torsional |  |  |  |
|  | deflection) | $\Delta(\theta)$ | B | $\Delta(\theta)$ |
|  | Length of member | $\ell$ | c | $\ell$ |
|  | Applied load (or torque) | P (T) | D | P (T) |
|  | Modulus of elasticity (in shear) | E (G) | E | E (G) |
| 3 | Calculate remaining value: |  |  |  |
|  | Area (or polar moment of |  |  |  |
|  | inertia) |  | A | a (J) |
|  | Linear deflection (or torsional |  |  |  |
|  | deflection |  | B | $\Delta(\theta)$ |
|  | Length of member |  | c | $\ell$ |
|  | Applied load (or torque) |  | D | P (T) |
|  | Modulus of elasticity (in |  |  |  |
|  | shear) |  | E | E (G) |
| 4 | For a new case, go to step 2 |  |  |  |
|  | and change appropriate inputs. |  |  |  |

## Example 1:

Steel bars, affixed to the roof are to be used to support the end of a cantilever balcony. The load on each bar will be 50,000 newtons. If the maximum allowable deflection is 0.001 meters, what should the area of the bars be? $\ell=10$ meters $\quad \mathrm{E}=2.068 \times 10^{11} \mathrm{~N} / \mathrm{m}^{2}$

## Keystrokes:

50000 D . 001 B 10 C
2.068 EEX 11 EA

## Outputs:

For square bars, .05 meters on a side, what will the deflection be?

## Example 2:

A 6 inch outside/5.5 inch inside diameter steel pipe $\left(G=11.5 \times 10^{6} \mathrm{psi}\right)$ is 15 feet long. How much torque will it resist with an angular deflection of 1.00 degree?

Keystrokes:
First compute $\mathrm{J}=\pi\left(\mathrm{D}_{0}{ }^{4}-\mathrm{D}_{\mathrm{i}}{ }^{4}\right) / 32$.
$6 x^{2} x^{2} 5.5 x^{2} x^{2}-\pi x$
$32 \div$
A 15 ENTER4 $12 \times$ C 11.5 EEX
6 E 1 D $\rightarrow$ B

Outputs:
$37.4000 \quad \mathrm{in}^{4}(\mathrm{~J})$
$41.7003 \quad$ in-lb (T)

## CANTILEVER BEAMS



This program calculates deflection, slope, moment and shear at any specified point along a rigidly fixed, cantilever beam of uniform cross section. Distributed loads, point loads, applied moments or combinations of all three may be modeled. By using the principle of superposition, complicated beams with multiple point loads, applied moments and combined distributed loads may be analyzed.

## Equations:



$$
\mathrm{y}=\mathrm{y}_{1}+\mathrm{y}_{2}+\mathrm{y}_{3} \quad \text { (total deflection) }
$$

$$
y_{1}=\frac{P X_{1}^{2}}{6 E I}\left(X_{1}-3 a\right)-\frac{P a^{2}}{2 E I}(x-a)(x>a)^{*} \quad(\text { deflection due to point load })
$$

$$
\mathrm{y}_{2}=\frac{-\mathrm{WX}_{2}^{2}}{6 \mathrm{EI}}\left[\mathrm{X}_{2}\left(\frac{\mathrm{X}_{2}}{4}-\mathrm{b}\right)+1.5 \mathrm{~b}^{2}\right]
$$

$$
-\frac{\mathrm{Wb}^{3}}{6 E I}(x-b)(x>b) \quad(\text { distributed load })
$$

$y_{3}=\frac{\mathrm{MX}_{3}{ }^{2}}{2 \mathrm{EI}}+\frac{\mathrm{Mc}}{\mathrm{EI}}(\mathrm{x}-\mathrm{c})(\mathrm{x}>\mathrm{c}) \quad$ (applied moment)
$\theta=\theta_{1}+\theta_{2}+\theta_{3} \quad$ (total slope)
$\theta_{1}=\frac{\mathrm{PX}_{1}}{2 \mathrm{EI}}\left(\mathrm{X}_{1}-2 \mathrm{a}\right) \quad$ (slope due to point load)
$\theta_{2}=\frac{\mathrm{WX}_{2}}{\mathrm{EI}}\left[\mathrm{X}_{2}\left(\frac{\mathrm{X}_{2}}{6}-\frac{\mathrm{b}}{2}\right)+\frac{\mathrm{b}^{2}}{2}\right] \quad$ (distributed load)
$\theta_{3}=\frac{\mathrm{MX}_{3}}{\mathrm{EI}} \quad$ (applied moment)
$\mathrm{M}_{\mathrm{x}}=\mathrm{M}_{\mathrm{x} 1}+\mathrm{M}_{\mathrm{x} 2}+\mathrm{M}_{\mathrm{x} 3} \quad$ (total moment)
$\mathrm{M}_{\mathrm{x} 1}=\mathrm{P}\left(\mathrm{X}_{1}-\mathrm{a}\right) \quad$ (moment due to point load)
$\mathrm{M}_{\mathrm{x} 2}=-\mathrm{W}\left(\mathrm{X}_{2}\left(\mathrm{X}_{2} / 2-\mathrm{b}\right)+\mathrm{b}^{2} / 2\right) \quad$ (distributed load)
$\mathrm{M}_{\mathrm{x} 3}=\mathrm{M}(\mathrm{x} \leqslant \mathrm{c}) \quad$ (applied moment)
$\mathrm{V}=\mathrm{V}_{1}+\mathrm{V}_{2}+\mathrm{V}_{3} \quad$ (total shear)
$\mathrm{V}_{1}=\mathrm{P}(\mathrm{x} \leqslant \mathrm{a}) \quad$ (shear due to point load)
$\mathrm{V}_{2}=\mathrm{W}\left(\mathrm{b}-\mathrm{X}_{2}\right) \quad$ (distributed load)
$\mathrm{V}_{3}=0 \quad$ (applied moment)
where:
y is the deflection at a distance x from the wall;
$\theta$ is the slope (change in $y$ per change in x ) at x ;
$\mathrm{M}_{\mathrm{x}}$ is the moment at x ;
V is the shear at x ;
I is the moment of inertia of the beam;
E is the modulus of elasticity of the beam;
$\ell$ is the length of the beam;
P is a concentrated load;
W is a uniformly distributed load with dimensions of force per unit length.

M is an applied moment;
a is the distance from the foundation to the point load;
b is the distance to the end of the distributed load;
c is the distance to the applied moment;

$$
\begin{aligned}
& X_{1}=x \text { if } x \leqslant a \text { or } a \text { if } x>a ; \\
& X_{2}=x \text { if } x \leqslant b \text { or } b \text { if } x>b \\
& X_{3}=x \text { if } x \leqslant c \text { or } c \text { if } x>c .
\end{aligned}
$$

*The notation $(x>a)$ is interpreted as 1.00 if $x$ is greater than a and as 0.00 if $x$ is less than or equal to $a$.

## Remarks:

Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.

Registers $\mathrm{R}_{\mathrm{S} 0}-\mathrm{R}_{\mathrm{S} 9}$ are available for user storage.
SIGN CONVENTIONS FOR BEAMS

| NAME | VARIABLE | SENSE | SIGN |
| :--- | :---: | :---: | :---: |
| DEFLECTION | y | $\uparrow$ | + |
| SLOPE | $\boldsymbol{\theta}$ | $\uparrow$ | + |
| INTERNAL MOMENT | $\mathrm{M}_{\mathrm{x}}$ | $\uparrow$ | + |
| SHEAR | V | $\uparrow$ |  |
|  | $\downarrow$ | + |  |
| EXTERNAL FORCE OR LOAD | P or W | $\downarrow$ | + |
| EXTERNAL MOMENT | M | C | + |

Sums of $\mathrm{y}, \theta, \mathrm{M}_{\mathrm{x}}$ and V may be stored in $\mathrm{R}_{6}, \mathrm{R}_{7}, \mathrm{R}_{8}$, and $\mathrm{R}_{9}$, respectively. Note that these registers are indicated on the magnetic card.

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 and side 2. |  |  |  |
| 2 | Initialize. |  | 18 | 0.00000 |
| 3 | Input moment of inertia | I | ENTER4 | I |
|  | then modulus of elasticity | E | ENTERA | E |
|  | then beam length. | $\ell$ | 1 B | EI |
| 4 | Input load(s): |  |  |  |
|  | Location of point load | a | ENTERA | a |
|  | Point load | P | 1 C | a |
|  | Length of distributed load | b | ENTER* | b |
|  | Distributed load (force/length) | W | 1 D | b |
|  | Location of applied moment | c | ENTER ${ }^{\text {d }}$ | c |
|  | Applied moment | M | $\square$ E | c |
| 5 | Key in x to specify the point |  |  |  |
|  | of interest and calculate |  |  |  |
|  | deflection | $x$ | A | $y$ |
|  | or slope | x | B | $\theta$ |


| STEP | INSTRUCTIONS | INPUT <br> DATA/UNITS | KEYS | OUTPUT <br> DATA/UNITS |
| ---: | :--- | :---: | :---: | :---: |
|  | or moment | x | C | $\mathbf{M}_{\mathbf{x}}$ |
|  | or shear. | x | D | V |
| 6 | For a new calculation with the |  |  |  |
|  | same loading, go to step 5. |  |  |  |
|  | For new loads, go to step 4. |  |  |  |
|  | Be sure to set obsolete |  |  |  |
|  | loadings to zero. For new |  |  |  |
|  | beam properties, go to step 3. |  |  |  |
|  | To restart, go to step 2. |  |  |  |

## Example 1:

What is the deflection at $\mathrm{x}=12$ ? Neglect the weight of the beam.


Keystrokes:

## Outputs:

A A 4.7 ENTER4 30 EEX
6
ENTERA 15 f B $\longrightarrow \quad 141.0 \quad 06$
Compute deflection at 12 inches due to 100 lb weight:
8 ENTER 100 f C $12 \boldsymbol{A} \longrightarrow \quad-211.8-06$
Store deflection due to 100 lb load for addition to deflection due to 200 lb load:

## STO 9

$\qquad$ -211.8-06
Compute deflection at 12 inches due to 200 lb load:
15 ENTER4 200 © $12 \boldsymbol{A} \longrightarrow \quad-1.123-03$
Compute total deflection:
RCL $9+$

## 07-05

## Example 2:

For the beam below, compute deflection, slope, moment and shear at 0,50 , and 90 inches. Neglect the weight of the beam.


$$
\begin{gathered}
I=23 \mathrm{in}^{4} \\
E=30 \times 10^{6} \mathrm{psi}
\end{gathered}
$$

Keystrokes:
Outputs:

## A 23 ENTER4 30 EEX

6 ENTER 110 B 40 ENTER 4
300 f C 60 ENTER4 10 D
80 ENTER4 20000 fe

| 0 A | 0.000 | 00 (y) |
| :---: | :---: | :---: |
| 0 B $\longrightarrow$ | 0.000 | 00 ( $\theta$ ) |
| 0 C | -10.00 | $03\left(\mathrm{M}_{\mathrm{x}}\right)$ |
| 0 D $\longrightarrow$ | 900.0 | 00 (V) |
| 50 A | 5.211 |  |
| 50 B | 582.1 |  |
| 50 C | 19.50 | 03 |
| 50 D $\longrightarrow$ | 100.0 | 00 |
| 90 A | 50.14 | -03 |
| 90 B $\longrightarrow$ | 1.449 | -03 |
| 90 C $\longrightarrow$ | 0.000 | 00 |
| 90 D $\longrightarrow$ | 0.000 | 00 |

Notes

## CANTILEVER BEAMS—TRAPEZOIDAL LOADING



This program calculates deflection, slope, moment, and shear at any specified point along a cantilever beam of uniform cross section with a distributed trapezoidal load. By using the principle of superposition, complicated distribted loads may be analyzed.

## Equations:



$$
\begin{gathered}
y=y_{d}-y_{e} \\
y_{d}=\theta_{0} x+y_{0}-\langle x-d\rangle^{4} *\left[\frac{w_{d}}{24 E I}+\frac{\left(w_{\ell}-w_{d}\right)}{120 E I(\ell-d)}\langle x-d\rangle\right]
\end{gathered}
$$

$$
\theta_{0}=\frac{(\ell-d)^{3}}{6 E I}\left[w_{d}+\frac{\left(w_{\ell}-w_{d}\right)}{4}\right]
$$

$$
\mathrm{y}_{0}=-\frac{(\ell-\mathrm{d})^{3}}{24 \mathrm{EI}}\left[\mathrm{w}_{\mathrm{d}}(3 \ell+\mathrm{d})+\frac{\left(\mathrm{w}_{\ell}-\mathrm{w}_{\mathrm{d}}\right)}{5}(4 \ell+\mathrm{d})\right]
$$

$$
w_{l}=w_{e}+\frac{\left(w_{e}-w_{d}\right)}{(e-d)}(\ell-d)
$$

$y_{e}$ is analogous to $y_{d}$ except $w_{d}$ is replaced by $w_{e}$ and $d$ is replaced by $e$. Equations for slope, moment, and shear are the first, second, and third x derivitives of the equations above.
${ }^{*}$ If $x-d<0,\langle x-d\rangle=0$.

## Definitions:

I is the moment of inertia of the section;
$E$ is the modulus of elasticity of the material;
$\ell$ is the length of the beam;
d is the distance to the beginning of the load;
$w_{d}$ is the initial value of the load with units of force per unit length;
$e$ is the distance to the end of the load;
$w_{e}$ is the final value of the load;
$x$ is the point of interest along the beam;
y is the deflection at x ;
$\theta$ is the slope at x ;
$\mathrm{M}_{\mathrm{x}}$ is the internal bending moment at x ;
V is the shear at x .

## Reference:

Roark, Raymond J., Young, Warren C., Formulas for Stress and Strain, McGraw-Hill Book Company, 1975.

## Remarks:

Deflections must not significantly alter the geometry of the problem.
Beams must be of constant cross section for deflection and slope equations to be valid

Stresses must be in the elastic region.
Registers $\mathbf{R}_{\mathbf{6}}-\mathbf{R}_{\mathbf{9}}$ are available for problems involving superposition.

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 and side 2. |  |  |  |
| 2 | Input the moment of inertia | I | ENTER4 | I |
|  | then the modulus of elasticity | E | ENTER4 | E |
|  | then the length of the beam. | $\ell$ | 18 | IE |
| 3 | Input distance to load | d | ENTER* | d |
|  | then initial value of load | $w_{\text {d }}$ | ENTER ${ }^{\text {a }}$ | $w_{\text {d }}$ |
|  | then distance to end of load | e | ENTER4 | e |
|  | then final value of loading. | $\mathrm{w}_{\text {e }}$ | 1 C | $\mathrm{w}_{\text {e }}$ |
| 4 | Key in x to specify points of |  |  |  |
|  | interest and calculate |  |  |  |
|  | deflection | x | A | $y$ |
|  | or slope | x | B | $\boldsymbol{\theta}$ |
|  | or moment | x | c | M ${ }_{\text {x }}$ |
|  | or shear. | x | D | V |
| 5 | For a new calculation with the |  |  |  |
|  | same loading, go to step 4. For |  |  |  |
|  | new loads, go to step 3. |  |  |  |

## Example:

Calculate deflection, slope, moment and shear for the beam above using the following values:

| $\mathrm{d}=23$ inches | $\mathrm{w}_{\mathrm{d}}=35 \mathrm{lb} / \mathrm{in}$ | $\mathrm{e}=47 \mathrm{inches}$ | $\mathrm{w}_{\mathrm{e}}=27 \mathrm{lb} / \mathrm{in}$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{I}=5 \mathrm{in}^{4}$ | $\mathrm{E}=30 \times 10^{6} \mathrm{psi}$ | $\ell=75 \mathrm{in}$ | $\mathrm{x}=40 \mathrm{in}$ |

What is the deflection at $\mathrm{x}=55$ ?
Keystrokes:
23 ENTERA 35 ENTERA 47 ENTERA
27 C 5 ENTERA 30 EEX

| 6 ENTER 75 B $\longrightarrow$ | 150.006 |
| :---: | :---: |
| 40 A | -84.71-03 |
| $40 \mathrm{~B} \longrightarrow$ | -3.057-03 |
| $40 \mathrm{C} \longrightarrow$ | -680.6 00 |
| $40 \mathrm{D} \longrightarrow$ | 197.200 |
| $55 \mathrm{~A} \longrightarrow$ | -130.7-03 |

## SIMPLY SUPPORTED BEAMS



This program calculates deflection, slope, moment and shear at any specified point along a simply supported beam of uniform cross section. Distributed loads, point loads, applied moments or combinations of all three may be modeled. By using the principle of superposition, complicated beams with multiple point loads, and multiple applied moments can be analyzed.

## Equations:


$\mathrm{y}=\mathrm{y}_{1}+\mathrm{y}_{2}+\mathrm{y}_{3} \quad$ (total deflection)
$y_{1}=\frac{P(\ell-a) x}{6 E I}\left[x^{2}+(\ell-a)^{2}-\ell^{2}\right]^{*} \quad$ (deflection due to point load)
$y_{2}=\frac{-W x}{24 E I}\left[\ell^{3}+x^{2}(x-2 \ell)\right] \quad$ (distributed load)
$\mathrm{y}_{3}=\frac{-\mathrm{Mx}}{\mathrm{EI}}\left[\mathrm{c}-\frac{\mathrm{x}^{2}}{6 \ell}-\frac{\ell}{3}-\frac{\mathrm{c}^{2}}{2 \ell}\right]^{* *} \quad$ (applied moment)
$\theta=\theta_{1}+\theta_{2}+\theta_{3} \quad$ (total moment)
$\theta_{1}=\frac{P(\ell-a)}{6 E I}\left[3 x^{2}+(\ell-a)^{2}-\ell^{2}\right]^{*} \quad$ (slope due to point load)
$\theta_{2}=-\frac{\mathrm{W}}{24 \mathrm{EI}}\left[\ell^{3}+\mathrm{x}^{2}(4 \mathrm{x}-6 \ell)\right] \quad$ (distributed load)
$\theta_{3}=\frac{-\mathrm{M}}{\mathrm{EI}}\left[\mathrm{c}-\frac{\mathrm{x}^{2}}{2 \ell}-\frac{\ell}{3}-\frac{\mathrm{c}^{2}}{2 \ell}\right]^{* *} \quad$ (applied moment)
$\mathrm{M}_{\mathrm{x}}=\mathrm{M}_{\mathrm{x} 1}+\mathrm{M}_{\mathrm{x} 2}+\mathrm{M}_{\mathrm{x} 3} \quad$ (total moment)
$\mathrm{M}_{\mathrm{x} 1}=\frac{\mathrm{P}(\ell-\mathrm{a}) \mathrm{x}}{\ell} \quad$ (moment due to point load)
$\mathrm{M}_{\mathrm{x} 2}=-\frac{\mathrm{Wx}}{2}[\mathrm{x}-\ell] \quad$ (distributed load)
$\mathrm{M}_{\mathrm{x} 3}=\frac{\mathrm{Mx}^{* *}}{\ell} \quad$ (applied moment)
$\mathrm{V}=\mathrm{V}_{1}+\mathrm{V}_{2}+\mathrm{V}_{3} \quad$ (total shear)
$\mathrm{V}_{1}=\frac{\mathrm{P}(\ell-\mathrm{a})^{*}}{\ell} \quad$ (shear due to point load)
$\mathrm{V}_{2}=\mathrm{W}\left(\frac{\ell}{2}-\mathrm{x}\right) \quad$ (distributed load)
$\mathrm{V}_{3}=\frac{\mathrm{M}}{\ell} \quad$ (applied moment)
where:
y is the deflection at a distance x from the left support;
$\theta$ is the slope (change in $y$ per change in $x$ ) at x ;
$\mathrm{M}_{\mathrm{x}}$ is the moment at x ;
V is the shear at x ;
I is the moment of intertia of the beam;
E is the modulus of elasticity of the beam;
$\ell$ is the length of the beam;
P is a concentrated load;
W is a uniformly distributed load with dimensions of force per unit length;
M is an applied moment;
a is the distance from the left support to the point load;
c is the distance to the applied moment.

[^0]
## Remarks:

Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.

Registers $\mathrm{R}_{\mathrm{S} 0}-\mathrm{R}_{\mathrm{S} 9}$ are available for user storage.
Sums of $y, \theta, M_{x}$ and $V$ may be stored in $R_{6}, R_{7}, R_{8}$, and $R_{9}$, respectively. Note that these registers are indicated on the magnetic card.

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 and side 2. |  |  |  |
| 2 | Initialize. |  | 18 | 0.00000 |
| 3 | Input moment of inertia | I | ENTERA | I |
|  | then modulus of elasticity | E | ENTERA | E |
|  | then beam length. | $l$ | 18 | EI |
| 4 | Input load(s): |  |  |  |
|  | Location of point load | a | ENTERA | a |
|  | Point load | P | $\square \mathrm{C}$ | a |
|  | Distributed load (force/length) | W | 18 | W |
|  | Location of applied moment | c | ENTERA | c |
|  | Applied moment | M | 1 E | C |
| 5 | Key in x to specify the point of |  |  |  |
|  | interest and calculate |  |  |  |
|  | deflection | x | A | $y$ |
|  | or slope | x | B | $\theta$ |
|  | or moment | x | c | $\mathrm{M}_{\mathrm{x}}$ |
|  | or shear. | x | D | V |
| 6 | For a new calculation with the |  |  |  |
|  | same loading, go to step 5. |  |  |  |
|  | For new loads, go to step 4. Be |  |  |  |
|  | sure to set obsolete loadings |  |  |  |
|  | to zero. For new beam |  |  |  |
|  | properties, go to step 3. To |  |  |  |
|  | restart, go to step 2. |  |  |  |

## Example 1:

Find the deflection, slope, internal moment and shear at distances of 0,24 and 60 inches for the beam below. Neglect the weight of the beam.


Keystrokes:
Outputs:

| A . 92 ENTER 30 EEX |  |  |
| :---: | :---: | :---: |
| 6 ENTER4 $72 \rightarrow \mathrm{~B} \longrightarrow$ | 27.6006 |  |
| 40 ENTER4 10000 E ${ }^{\text {E }} \longrightarrow$ | $40.00 \quad 00$ |  |
| 0 A | $0.000 \quad 00$ | $\left(y_{0}\right)$ |
| 0 B | -1.771-03 | $\left(\theta_{0}\right)$ |
| 0 C | 0.00000 | $\left(\mathrm{M}_{0}\right)$ |
| 0 D $\longrightarrow$ | 138.900 | ( $\mathrm{V}_{0}$ ) |
| 24 A | -30.92-03 | $\left(\mathrm{y}_{24}\right)$ |
| 24 B | -322.1-06 | ( $\theta_{24}$ ) |
| $24 \mathrm{C} \longrightarrow$ | 3.33303 | $\left(\mathrm{M}_{24}\right)$ |
| 24 D | 138.900 | $\left(\mathrm{V}_{24}\right)$ |
| 60 A $\longrightarrow$ | $2.415-03$ | ( $\mathrm{y}_{60}$ ) |
| 60 B $\longrightarrow$ | 40.26 -06 | $\left(\theta_{60}\right)$ |
| 60 C $\longrightarrow$ | -1.667 03 | $\left(\mathrm{M}_{60}\right)$ |
| $60 \mathrm{D} \longrightarrow$ | 138.900 | $\left(\mathrm{V}_{60}\right)$ |

## Example 2:

What is the slope of the beam below at $\mathrm{x}=38$ inches?


## Keystrokes:

f A 1.30 ENTERA 30 EEX

| 6 ENTER4 50 - B | $39.00 \quad 06$ |  |
| :---: | :---: | :---: |
| 44 ENTERA 1000 ¢ $\mathbf{C}$ | $44.00 \quad 00$ |  |
| 25 f D $\longrightarrow$ | $25.00 \quad 00$ |  |
| 38 B $\longrightarrow$ | 3.327 -03 | (in/in) |

## Example 3:

What is the total moment at the center of the beam below? (It is not necessary to know the values of E or I to solve the problem. Simply key in 70 and press fB.)


First solve for the effect of the distributed load, $\mathrm{P}_{1}$, and M .

Keystrokes:
f A 70 f B 20 ENTER
400 C C $\longrightarrow \quad 20.00 \quad 00$
37 f 70 ENTER
10000 CHS $f$ E
70 ENTER4 2 - C $\qquad$
$70.00 \quad 00$

Store values in $\mathrm{R}_{6}$.
STO 6

$21.66 \quad 03$

## Outputs:

$$
20.00 \quad 00
$$

Now solve for the effect of $P_{2}$ and add it to the content of $R_{6}$. This is the final answer assuming superposition is valid.

| (A) 50 ENTER4 1000 ¢ $\mathbf{C} \rightarrow$ | 50.00 | 00 |  |
| :---: | :---: | :---: | :---: |
| 35 C | 10.00 | 03 | (in-lb) |
| RCL $6+\longrightarrow$ | 31.66 | 03 | (in-lb) |

Notes

## SIMPLY SUPPORTED BEAMS-TRAPEZOIDAL LOADING



This program calculates deflection, slope, moment, and shear at any specified point along a simply supported beam of uniform cross section with a distributed trapezoidal load. By using the principle of superposition, complicated distributed loads may be analyzed.


Equations:

$$
\begin{gathered}
y=y_{d}-y_{e} \\
y_{d}=\theta_{0} x+\frac{R_{0} x^{3}}{6 E I}-\langle x-d\rangle^{4}\left[\frac{w_{d}}{24 E I}+\frac{w_{\ell}-w_{d}\langle x-d\rangle}{120 E I(\ell-d)}\right] \\
\theta_{0}=\frac{(\ell-d)^{2}}{24 \ell E I}\left[-w_{d}\left(\ell^{2}+2 d \ell-d^{2}\right)-\frac{w_{l}-w_{d}}{15}\left(7 \ell^{2}+6 d \ell-3 d^{2}\right)\right] \\
R_{0}=\frac{(\ell-d)^{2}}{2 l}\left[w_{d}+\frac{w_{l}-w_{d}}{3 \ell}\right] \\
w_{\ell}=w_{e}+\frac{\left(w_{e}-w_{d}\right)}{(e-d)}(\ell-e)
\end{gathered}
$$

$y_{e}$ is analogous to $y_{d}$ except $w_{d}$ is replaced by $w_{e}$ and $d$ is replaced by e. Equations for slope, moment, and shear are the first, second and third $x$ derivitives of the equations above.

## Definitions:

I is the moment of inertia of the section;
E is the modulous of elasticity of the material;
$\ell$ is the length of the beam;
d is the distance to the beginning of the load;
$w_{d}$ is the initial value of the load with units of force per unit length;
$e$ is the distance to the end of the load;
$w_{e}$ is the final value of the load;
x is the point of interest along the beam;
y is the deflection at x ;
$\boldsymbol{\theta}$ is the slope at x ;
$\mathrm{M}_{\mathrm{x}}$ is the internal bending moment at x ;
V is the shear at x .

## Reference:

Roark, Raymond J., Young, Warren C., Formulas for Stress and Strain, McGraw-Hill Book Company, 1975.

## Remarks:

Deflections must not significantly alter the geometry of the problem.
Beams must be of constant cross section for deflection and slope equations to be valid.
Stresses must be in the elastic region.
Registers $\mathrm{R}_{6}, \mathrm{R}_{7}, \mathrm{R}_{8}$, and $\mathrm{R}_{9}$ are available for problems involving superposition.

10-03

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 and side 2. |  |  |  |
| 2 | Input the moment of inertia | I | ENTER4 | I |
|  | then the modulus of elasticity | E | ENTER | E |
|  | then the length of the beam. | $\ell$ | B | IE |
| 3 | Input distance to load | d | ENTER4 | d |
|  | then initial value of load | $w_{\text {d }}$ | ENTER4 | $\mathrm{w}_{\text {d }}$ |
|  | then distance to end of load | e | ENTER4 | e |
|  | then final value of loading. | $\mathrm{w}_{\text {e }}$ | C $C$ | $\mathrm{w}_{\text {e }}$ |
| 4 | Key in x to specify point of |  |  |  |
|  | interest and calculate |  |  |  |
|  | deflection | x | A | $y$ |
|  | or slope | x | B | $\theta$ |
|  | or moment | X | C | $\mathrm{M}_{\mathrm{x}}$ |
|  | or shear. | x | D | V |
| 5 | For a new calculation with the |  |  |  |
|  | same loading, go to step 4. For |  |  |  |
|  | new loads, go to step 3. |  |  |  |

## Example:

Calculate deflection, slope, moment and shear for the beam above using the following values:

| $d=23$ inches | $\mathrm{w}_{\mathrm{d}}=35 \mathrm{lb} / \mathrm{in}$ | $\mathrm{e}=47 \mathrm{inches}$ | $\mathrm{w}_{\mathrm{e}}=27 \mathrm{lb} / \mathrm{in}$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{I}=5 \mathrm{in}^{4}$ | $\mathrm{E}=30 \times 10^{6} \mathrm{psi}$ | $\ell=75 \mathrm{in}$ | $\mathrm{x}=55 \mathrm{in}$ |

What is the deflection at $\mathrm{x}=40$ ?

## Keystrokes:

## Outputs:

| 23 ENTER4 35 ENTER4 47 ENTER4 |  |
| :---: | :---: |
| 27 f C 5 ENTER4 30 EEX |  |
| 6 ENTERA 75 ¢ B | 150.006 |
| 55 A | -29.58-03 |
| 55 B | 1.175-03 |
| 55 C | 6.84203 |
| $55 \mathrm{D} \longrightarrow$ | -342.100 |
| $40 \triangle \longrightarrow$ | -40.82-03 |

Notes

## BEAMS FIXED AT BOTH ENDS



This program calculates deflection, slope, moment and shear at any specified point along a beam of uniform cross section, fixed at both ends. Distributed loads, point loads, applied moments or combinations of all three may be modeled. By using the principle of superposition, complicated beams with multiple point loads, and multiple applied moments can be analyzed.

## Equations:


$y=y_{1}+y_{2}+y_{3} \quad$ (total deflection)
$y_{1}=\frac{P(\ell-a)^{2} x^{2}}{6 E I^{3}}[x(\ell+2 a)-3 a \ell)^{*} \quad$ (deflection due to point load)
$y_{2}=\frac{W x^{2}}{24 E I}\left[x(2 \ell-x)-\ell^{2}\right] \quad$ (distributed load)
$\mathrm{y}_{3}=\frac{\mathrm{M}(\ell-\mathrm{c}) \mathrm{x}^{2}}{\ell^{2} \mathrm{EI}}\left[\frac{\mathrm{cx}}{\ell}+\frac{\ell-3 \mathrm{c}}{2}\right]^{* *} \quad$ (applied moment)
$\theta=\theta_{1}+\theta_{2}+\theta_{3} \quad$ (total slope)
$\theta_{1}=\frac{\mathrm{P}(\ell-\mathrm{a})^{2} \mathrm{x}}{2 E I^{3}}[\mathrm{x}(\ell+2 \mathrm{a})-2 \mathrm{a} \ell]^{*} \quad$ (slope due to point load)
$\theta_{2}=\frac{\mathrm{Wx}}{12 E I}\left[x(3 \ell-2 x)-\ell^{2}\right] \quad$ (distributed load)
$\theta_{3}=\frac{\mathrm{M}(\ell-\mathrm{c}) \mathrm{x}}{\ell^{2} \mathrm{EI}}\left[\frac{3 \mathrm{cx}}{\ell}+\ell-3 \mathrm{c}\right]^{* *} \quad$ (applied moment)
$\mathrm{M}_{\mathrm{x}}=\mathrm{M}_{\mathrm{x} 1}+\mathrm{M}_{\mathrm{x} 2}+\mathrm{M}_{\mathrm{x} 3} \quad$ (total moment)
$M_{x 1}=\frac{P(\ell-a)^{2}}{\ell^{3}}[x(\ell+2 a)-a \ell] * \quad$ (moment due to point load)
$M_{x 2}=\frac{W}{12}\left[6 x(\ell-x)-\ell^{2}\right] \quad$ (distributed load)
$\mathbf{M}_{\mathrm{x} 3}=\frac{\mathbf{M}(\ell-\mathrm{c})}{\ell^{2}}\left[\frac{6 \mathrm{cx}}{\ell}+\ell-3 \mathrm{c}\right]^{* *} \quad$ (applied moment)
$\mathrm{V}=\mathrm{V}_{1}+\mathrm{V}_{2}+\mathrm{V}_{3} \quad$ (total shear)
$V_{1}=\frac{P(\ell-a)^{2}}{\ell^{3}}(\ell+2 a) \quad$ (shear due to point load)
$\mathrm{V}_{2}=\frac{-\mathrm{W}}{2}(2 \mathrm{x}-\ell) \quad$ (distributed load)
$\mathrm{V}_{3}=\frac{-6 \mathrm{M}(\ell-\mathrm{c}) \mathrm{c}^{* *}}{\ell^{3}} \quad$ (applied moment)
where:
y is the deflection at a distance x from the left support;
$\theta$ is the slope (change in $y$ per change in $x$ ) at $x$;
$M_{x}$ is the moment at $x$;
$V$ is the shear at $x$;
I is the moment of inertia of the beam;
$E$ is the modulus of elasticity of the beam;
$\ell$ is the length of the beam;
P is a concentrated load;
W is a uniformly distributed load with dimensions of force per unit length;
M is an applied moment;
a is the distance from the left support to the point load;
c is the distance to the applied moment.

[^1]
## Remarks:

This card differs from other beam cards. The "start'" function is not included on LBL $f$ A. You must manually perform the 'start' function by storing zero when $\mathrm{P}, \mathrm{W}$ or M are not included in the problem.
Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.

Registers $\mathrm{R}_{\mathrm{S} 0}-\mathrm{R}_{\mathrm{S} 9}$ are available for user storage.
Sums of $\mathrm{y}, \theta, \mathrm{M}_{\mathrm{x}}$ and V may be stored in $\mathrm{R}_{6}, \mathrm{R}_{7}, \mathrm{R}_{8}, \mathrm{R}_{9}$, respectively. Note that these registers are indicated on the magnetic card.

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 and side 2. |  |  |  |
| 2 | Input moment of inertia | I | ENTER | I |
|  | then modulus of elasticity | E | ENTERA | E |
|  | then beam length. | $\ell$ | 1 B | EI |
| 3 | Input load(s):* |  |  |  |
|  | Location of point load | a | ENTER | a |
|  | Point load | P | 1 C | a |
|  | Distributed load (force/length) | W | 10 | W |
|  | Location of applied moment | c | ENTERA | c |
|  | Applied moment | M | 15 | c |
| 4 | Key in x to specify the point |  |  |  |
|  | of interest and calculate |  |  |  |
|  | deflection | x | A | y |
|  | or slope | x | B | $\theta$ |
|  | or moment | x | c | $\mathrm{M}_{\mathrm{x}}$ |
|  | or shear. | x | D | V |


| STEP | INSTRUCTIONS | INPUT <br> DATA/UNITS | KEYS | OUTPUT <br> DATA/UNITS |
| :---: | :--- | :--- | :--- | :--- |
| 5 | For a new calculation with the |  |  |  |
|  | same loading, go to step 4. For |  |  |  |
|  | new loads, go to step 3. Be |  |  |  |
|  | sure to set obsolete loadings to |  |  |  |
|  | zero. For new beam properties, |  |  |  |
|  | go to step 2. |  |  |  |
|  | *Loads must be input, even if |  |  |  |
|  | zero. |  |  |  |

## Example 1:

For the beam below, what are the values of deflection, slope, moment, and shear at an x of 114 inches?


## 11-05

## Example 2:

Find the internal moment at $\mathrm{x}=0$ for the configuration below.


Keystrokes:
9.75 ENTER4 10 EEX 6 ENTER4 $75 \mathrm{~B} \longrightarrow \quad 97.50 \quad 06$
0 EE 100 f(D 50 ENTER4
$1000-\mathbf{C}$ c

0 C
Also, find the deflection at $\mathrm{x}=40$.
40 A
$-101.0-03\left(\mathrm{Y}_{40}\right)$

Notes

## BEAMS FIXED AT BOTH ENDS—TRAPEZOIDAL LOADING



This program calculates deflection, slope, moment, and shear at any specified point along a beam fixed at both ends, of uniform cross section, supporting a distributed trapezoidal load. By using the principle of superposition, complicated distributed loads may be analyzed.

## Equations:



$$
\begin{gathered}
y=y_{d}-y_{e} \\
y_{d}=\frac{M_{0} x^{2}}{2 E I}+\frac{R_{0} x^{3}}{6 E I}-\langle x-d\rangle\left[\frac{w_{d}}{24 E I}+\frac{\left(w_{l}-w_{d}\right)\langle x-d\rangle}{120 E I(l-d)}\right] \\
M_{0}=-\frac{(\ell-d)}{12 \ell^{2}}\left[w_{d}(\ell+3 d)-\frac{\left(w_{\ell}-w_{d}\right)}{5}(2 l+3 d)\right] \\
R_{0}=\frac{(\ell-d)}{2 \ell^{3}}\left[w_{d}(l+d)+\frac{\left(w_{l}-w_{d}\right)}{10}(3 \ell+2 d)\right] \\
w_{\ell}=w_{e}+\frac{\left(w_{e}-w_{d}\right)}{(e-d)}(\ell-e)
\end{gathered}
$$

$y_{e}$ is analogous to $y_{d}$ except $w_{e}$ replaces $w_{d}$ and e replaces $d$.
Equations for slope, moment and shear are the first, second, and third $x$ derivitives of the equations above.

## Definitions:

I is the moment of inertia of the section;
E is the modulus of elasticity of the material;
$\ell$ is the length of the beam;
d is the distance to the beginning of the load;
$w_{d}$ is the initial value of the load with units of force per unit length;
$e$ is the distance to the end of the load;
$w_{e}$ is the final value of the load;
$x$ is the point of interest along the beam;
y is the deflection at x ;
$\boldsymbol{\theta}$ is the slope at x ;
$\mathrm{M}_{\mathrm{x}}$ is the internal bending moment at x ;
V is the shear at x .

## Reference:

Roark, Raymond J., Young, Warren C., Formulas for Stress and Strain, McGraw-Hill Book Company, 1975.

## Remarks:

Deflections must not significantly alter the geometry of the problem.
Beams must be of constant cross section for deflection and slope equations to be valid.
Stresses must be in the elastic region.
Registers $\mathrm{R}_{\mathbf{6}}-\mathrm{R}_{9}$ are available for problem involving superposition.

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT <br> DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 and side 2. |  |  |  |
| 2 | Input the moment of inertia | I | ENTER | I |
|  | then the modulus of elasticity | E | ENTER4 | E |
|  | then the length of the beam. | $\ell$ | [B | IE |
| 3 | Input distance to load | d | ENTER | d |
|  | then initial value of load | $w_{\text {d }}$ | ENTER4 | $w_{\text {d }}$ |
|  | then distance to end of load | e | ENTER4 | e |
|  | then final value of loading. | $\mathrm{w}_{\text {e }}$ | C | $\mathrm{w}_{\text {e }}$ |
| 4 | Key in x to specify point of |  |  |  |
|  | interest and calculate |  |  |  |
|  | deflection | $x$ | A | y |
|  | or slope | x | B | $\theta$ |
|  | or moment | x | c | $\mathrm{M}_{\mathrm{x}}$ |
|  | or shear. | x | D | V |
| 5 | For a new calculation with the |  |  |  |
|  | same loading, go to step 4. For |  |  |  |
|  | new loads, go to step 3. |  |  |  |

## Example:

Calculate deflection, slope, moment and shear for the beam above using the following values:

| $\mathrm{d}=23$ inches | $\mathrm{w}_{\mathrm{d}}=35 \mathrm{lb} / \mathrm{in}$ | $\mathrm{e}=47 \mathrm{inches}$ | $\mathrm{w}_{\mathrm{e}}=27 \mathrm{lb} / \mathrm{in}$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{I}=5 \mathrm{in}^{4}$ | $\mathrm{E}=30 \times 10^{6} \mathrm{psi}$ | $\ell=75 \mathrm{in}$ | $\mathrm{x}=55 \mathrm{in}$ |

What is the deflection at $\mathrm{x}=40$ ?

## Keystrokes:

## Outputs:

## 23 ENTER4 35 ENTERA 47 ENTER4

27 © C 5 ENTERA 30 EEX

| 6 ENTER4 75-B | 150.006 |
| :---: | :---: |
| 55 A | -5.331-03 |
| 55 B | 387.0-06 |
| 55 c | 383.700 |
| 55 D | -328.6 00 |
| 40 A | -9.634-03 |

Notes

## PROPPED CANTILEVER BEAMS



This program calculates deflection, slope, moment and shear at any specified point along a propped cantilever beam of uniform cross section. Distributed loads, point loads, applied moments or combinations of all three may be modeled. By using the principle of superposition, complicated beams with multiple point loads, and multiple applied moments can be analyzed.

## Equations:


$\mathrm{y}=\mathrm{y}_{1}+\mathrm{y}_{2}+\mathrm{y}_{3} \quad$ (total deflection)
$y_{1}=\frac{P}{6 E I}\left[F\left(x^{3}-3 \ell^{2} x\right)+3 b^{2} x\right] ; x \leqslant a \quad$ (deflection due to point load)
$y_{2}=\frac{W}{48 E I}\left(3 \ell x^{3}-2 x^{4}-\ell^{3} x\right) \quad$ (distributed load)
$y_{3}=\frac{M}{E I} G\left(x^{3}-3 \ell^{2} x\right)+\ell x-c x ; x \leqslant c \quad$ (applied moment)
$y_{3}=\frac{M}{E I} G\left(x^{3}-3 \ell^{2} x\right)+\ell x-1 / 2\left(x^{2}+c^{2}\right) ; x>c$
$\theta=\theta_{1}+\theta_{2}+\theta_{3} \quad$ (total slope)
$\theta_{1}=\frac{P}{6 E I}\left[F\left(3 x^{2}-3 \ell^{2}\right)+3 b^{2}\right] ; \quad x \leqslant a \quad$ (slope due to point load)

$$
\begin{aligned}
& \theta_{1}=\frac{P}{6 E I}\left[F\left(3 x^{2}-3 l^{2}\right)-3(x-a)^{2}\right] ; x>a \\
& \theta_{2}=\frac{W}{48 E I}\left(9 x^{2}-8 x^{3}-\ell^{3}\right) \quad \text { (distributed load) } \\
& \theta_{3}=\frac{\mathrm{M}}{\mathrm{EI}}\left[\mathrm{G}\left(3 \mathrm{x}^{2}-3 \ell^{2}\right)+\ell-\mathrm{c}\right] ; \mathrm{x} \leqslant \mathrm{c} \quad \text { (applied moment) } \\
& \theta_{3}=\frac{M}{E I}\left[G\left(3 x^{2}-3 l^{2}\right)+\ell-x\right] ; x>c \\
& \mathrm{M}_{\mathrm{x}}=\mathrm{M}_{\mathrm{x} 1}+\mathrm{M}_{\mathrm{x} 2}+\mathrm{M}_{\mathrm{x} 3} \quad \text { (total moment) } \\
& \mathrm{M}_{\mathrm{x} 1}=\mathrm{PFx} ; \mathrm{x} \leqslant \mathrm{a} \quad \text { (moment due to point load) } \\
& \mathrm{M}_{\mathrm{x} 1}=\mathrm{PFx}-\mathrm{P}(\mathrm{x}-\mathrm{b}) ; \mathrm{x}>\mathrm{a} \\
& \mathrm{M}_{\mathrm{x} 2}=\mathrm{W}\left(3 / 8 \mathrm{x} \ell-\mathrm{x}^{2} / 2\right) \quad \text { (distributed load) } \\
& \mathrm{M}_{\mathrm{x} 3}=6 \mathrm{MGx} ; \mathrm{x} \leqslant \mathrm{c} \quad \text { (applied moment) } \\
& \mathrm{M}_{\mathrm{x} 3}=6 \mathrm{MGx}-\mathrm{M} ; \mathrm{x}>\mathrm{c} \\
& \mathrm{~V}=\mathrm{V}_{1}+\mathrm{V}_{2}+\mathrm{V}_{3} \quad \text { (total shear) } \\
& \mathrm{V}_{1}=\mathrm{PF} ; \mathrm{x} \leqslant \mathrm{a} \quad \text { (shear due to point load) } \\
& \mathrm{V}_{1}=\mathrm{PF}-\mathrm{P} ; \mathrm{x}>\mathrm{a} \\
& \mathrm{~V}_{2}=\mathrm{W}\left(\frac{3}{8} \ell-\mathrm{x}\right) \quad \text { (distributed load) } \\
& \mathrm{V}_{3}=6 \mathrm{MG} \quad \text { (applied moment) } \\
& \mathrm{F}=\left[\frac{3 \mathrm{~b}^{2} \ell-\mathrm{b}^{3}}{2 \ell^{3}}\right] \\
& \mathrm{b}=(\ell-\mathrm{a}) \\
& G=\frac{\ell^{2}-c^{2}}{4 \ell^{3}}
\end{aligned}
$$

where:
y is the deflection at a distance x from the left support;
$\theta$ is the slope (change in $y$ per change in $x$ ) at $x$;
$M_{x}$ is the moment at $x$;
$V$ is the shear at $x$;
I is the moment of inertia of the beam;
$E$ is the modulus of elasticity of the beam;
$\ell$ is the length of the beam;
P is a concentrated load;
W is a uniformly distributed load with dimensions of force per unit length;
$\mathbf{M}$ is an applied moment;
a is the distance from the left support to the point load;
c is the distance to the applied moment.

## Remarks;

Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.
Registers $\mathrm{R}_{\mathrm{S} 0}-\mathrm{R}_{\mathrm{S} 9}$ and $\mathrm{R}_{\mathrm{B}}$ are available for user storage.
Sums of $y, \theta, M_{X}$ and $V$ may be stored in $R_{6}, R_{7}, R_{8}$ and $R_{9}$, respectively. Note that those registers are indicated on the magnetic card.

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 and side 2. |  |  |  |
| 2 | Initialize. |  | 1 A | 0.00000 |
| 3 | Input moment of inertia | 1 | ENTERA | I |
|  | then modulus of elasticity | E | ENTERA | E |
|  | then beam length. | $\ell$ | 1 B | EI |
| 4 | Input load(s): |  |  |  |
|  | Location of point load | a | ENTERA | a |
|  | Point load | P | 1 C | a |
|  | Distributed load (force/length) | W | 1 D | W |
|  | Location of applied moment | c | ENTERA | c |
|  | Applied moment. | M | 1 E | c |


| STEP | INSTRUCTIONS | INPUT <br> DATA/UNITS | KEYS | OUTPUT <br> DATA/UNITS |
| ---: | :--- | :--- | :--- | :--- |
| 5 | Key in x to specify the point of |  |  |  |
|  | interest and calculate |  |  |  |
|  | deflection | x | $\mathbf{A}$ | y |
|  | or slope | x | $\mathbf{B}$ | $\boldsymbol{\theta}$ |
|  | or moment | x | C | $\mathrm{M}_{\mathbf{x}}$ |
|  | or shear. | x | $\mathbf{D}$ | V |
| 6 | For a new calculation with the |  |  |  |
|  | same loading, go to step 5. |  |  |  |
|  | For new loads, go to step 4. |  |  |  |
|  | Be sure to set obsolete |  |  |  |
|  | loadings to zero. For new |  |  |  |
|  | beam properties, go to step 3. |  |  |  |
|  | To restart, go to step 2. |  |  |  |

## Example 1:

What are the values of moment and shear at both ends of the beam below? (It is not necessary to know the values of E or I since deflection and slope are not required.)


Keystrokes:

## Outputs:

| 1 A 120 B 30 ENTER |  |  |  |
| :---: | :---: | :---: | :---: |
| 1000 C | 30.00 | 00 |  |
| 80 ENTERA 35000 CHS |  |  |  |
| 15 ¢D $\longrightarrow$ | 15.00 | 00 |  |
| 0 C | 0.000 | 00 | (in-lb) |
| 0 D $\longrightarrow$ | 1.065 | 03 | (lb) |
| 120 C $\longrightarrow$ | -35.23 | 03 | (in-lb) |
| 120 D | -1.735 | 03 | (lb) |

## 13-05

## Example 2:

Calculate the deflection, slope, moment and shear at $\mathrm{x}=90$ for the beam below.


Keystrokes:
Outputs:
IA 23 ENTERA 30 EEX 6 ENTERA
170 В В $\longrightarrow \quad 690.0 \quad 06$


| 90 A | -75.73-03 | (in) |
| :---: | :---: | :---: |
| 90 B | 920.8-06 | (in/in) |
| 90 c | 11.8903 | (in-lb) |
| 90 D | -229.0 00 | (lb) |

Notes

## PROPPED CANTILEVER BEAMS—TRAPEZOIDAL LOADING



This program calculates deflection, slope, moment and shear at any specified point along a propped cantilever beam of uniform cross section with a distributed trapezoidal load. By using the principle of superposition, complicated distributed loads may be analyzed.

## Equations:



$$
\begin{gathered}
y=y_{d}+y_{e} \\
y_{d}=\theta_{0} x+R_{0} x^{3} / 6 E I-\langle x-d\rangle^{4}\left[\frac{w_{d}}{24 E I}+\frac{\left(w_{\ell}-w_{d}\right)\langle x-d\rangle}{120 E I}\right] \\
R_{0}=\frac{(\ell-d)^{3}}{8 \ell^{3}}\left[w_{d}(3 \ell+d)+\frac{\left(w_{\ell}-w_{d}\right)}{5}(4 \ell+d)\right] \\
\theta_{0}=-\frac{(\ell-d)^{3}}{48 E I \ell}\left[w_{d}(\ell+3 d)-\frac{\left(w_{\ell}-w_{d}\right)}{5}(2 \ell+3 d)\right] \\
w_{\ell}=w_{e}+\frac{\left(w_{e}-w_{d}\right)}{(e-d)}(\ell-e)
\end{gathered}
$$

$y_{e}$ is analogous to $y_{d}$ except $w_{d}$ is replaced by $w_{e}$ and $d$ is replaced by $e$.
Equations for slope moment and shear are the first, second and third $\mathbf{x}$ derivatives of the equations above.

## Definitions:

I is the moment of inertia of the section;
E is the modulus of elasticity of the material;
$\ell$ is the length of the beam;
d is the distance to the beginning of the load;
$w_{d}$ is the initial value of the load with units of force per unit length;
$e$ is the distance to the end of the load;
$\mathrm{w}_{\mathrm{e}}$ is the final value of the load;
x is the point of interest along the beam;
y is the deflection at x ;
$\theta$ is the slope at x ;
$\mathrm{M}_{\mathrm{x}}$ is the internal bending moment at x ;
V is the shear at x .

## Reference:

Roark, Raymond J., Young, Warren C., Formulas for Stress and Strain, McGraw-Hill Book Company, 1975.

## Remarks:

Deflections must not significantly alter the geometry of the problem.
Beams must be of constant cross section for deflection and slope equations to be valid.

Stresses must be in the elastic region.
Registers $R_{6}-R_{9}$ are available for problems involving superposition.

| STEP | INSTRUCTIONS | INPUT DATAUNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 and side 2. |  |  |  |
| 2 | Input the moment of inertia | 1 | ENTER | I |
|  | then the modulus of elasticity | E | ENTER4 | E |
|  | then the length of the beam. | $\ell$ | B | IE |
| 3 | Input distance to load | d | ENTER | d |
|  | then initial value of load | $w_{\text {d }}$ | ENTERA | $w_{\text {d }}$ |
|  | then distance to end of load | e | ENTER | e |
|  | then final value of loading. | $\mathrm{w}_{\text {e }}$ | $\square$ | $\mathrm{w}_{\text {e }}$ |
| 4 | Key in x to specify point of in- |  |  |  |
|  | terest and calculate deflection | x | A | $y$ |
|  | or slope | x | B | $\theta$ |
|  | or moment | x | C | M ${ }_{\text {x }}$ |
|  | or shear. | x | D | V |
| 5 | For a new calculation with the |  |  |  |
|  | same loading, go to step 4. For |  |  |  |
|  | new loads, go to step 3. |  |  |  |

## Example:

Calculate deflection, slope, moment and shear for the beam above using the following values:
$\mathrm{d}=23$ inches
$\mathrm{w}_{\mathrm{d}}=35 \mathrm{lb} / \mathrm{in}$
$\mathrm{e}=47$ inches
$\mathrm{w}_{\mathrm{e}}=27 \mathrm{lb} / \mathrm{in}$
$\mathrm{I}=5 \mathrm{in}^{4}$
$\mathrm{E}=30 \times 10^{6} \mathrm{psi}$
$\ell=75$ in
$x=55$ in

What is the deflection at $\mathrm{x}=40$ ?
Keystrokes:
23
3 ENTERA 35 ENTER4 47 ENTER4
27 CC 5 ENTERA 30 EEX

| 6 ENTER 75 - ${ }^{\text {B }}$ | 150.006 |
| :---: | :---: |
| 55 A | -8.849-03 |
| 55 B | 674.9-06 |
| 55 c | -336.0 00 |
| 55 D | -472.6 00 |
| 40 回 | -17.47-03 |

Notes

## SIX-SPAN CONTINUOUS BEAMS



This program solves for the intermediate couples present at the support points of a continuous beam. From two to six span beams may be analyzed.


Each span of the beam may have a unique length, cross section, and/or modulus of elasticity but properties may not change within a span.
The first step in using this program is computation of the slope factors at each support of the span. This is best accomplished with programs designed for this purpose such as CE-09 and CE-10. Simply break the continuous beam at each support and calculate the slope at each end assuming no moment is transmitted across supports (it is not necessary to calculate the slope at the left end of the first section or the right end of the last section).
After all slope factors have been calculated for the beam sections, you are ready to use Six-Span Continuous Beam to solve for the unknown moments which develop across the intermediate supports of the continuous beam. After loading the program and specifying the number of spans ( N ), the moment acting at the left end of the beam is specified $\left(\mathrm{M}_{0}\right)$, even if zero, then the slope factors from the left side and the right side of the first intermediate support are input. The moment of inertia, modulus of elasticity, and length of the first span are input next.

For subsequent spans (except the last span) input the slope factors and beam properties only. In cases where sections repeat (same load and same properties) the B keys cause automatic span replication. This saves the effort involved in keying in five pieces of repeated data. If the loadings on successive spans change but beam properties remain constant, input the slope factors but use the automatic property duplication function on the © $\mathbf{C}$ keys.
The last span requires input of only the beam properties and the applied moment at the end of the beam $\left(\mathrm{M}_{\mathrm{N}}\right)$, even if zero. After input of the end moment, calculation begins. About one minute later, the values of the moments acting at each end of each segment of the beam are output. The first output is the left end applied moment $\mathbf{M}_{0}$, the last output is the right end applied moment $\mathbf{M}_{\mathrm{N}}$. All moments, inputs and outputs, follow the right hand rule sign convention. If you have a HP-67 and you miss the output of the moments it is not necessary to start over. Simply leave $\mathrm{M}_{\mathrm{N}}$ in the display and press $\mathbf{D}$, the output routine will be repeated after a few seconds of calculation.

## Algorithm:

The program starts by assumming that all internal moments are zero. Based on this assumption it calculates the moment across the first intermediate support using:
$\left.M_{1}=\left\{\left(\theta_{1}-\theta_{1}^{\prime}\right)-\frac{M_{1} \ell_{1}}{6 \mathrm{E}_{1} \mathrm{I}_{1}}-\frac{\mathrm{M}_{2} \ell_{2}}{6 \mathrm{E}_{2} \mathrm{I}_{2}}\right\} /\left(\frac{\ell_{1}}{3 \mathrm{E}_{1} \mathrm{I}_{1}}+\frac{\ell_{2}}{3 \mathrm{E}_{2} \mathrm{I}_{2}}\right)\right\}$
It then uses $\mathrm{M}_{1}$ in an analogous equation for the next support and the next until the end of the beam is reached. The program repeats this procedure until all calculated moments remain unchanged within the specified display setting for one complete cycle of moment calculations.

## Reference:

Roark, Raymond J.; Young, Warren C.; Formulas for Stress and Strain, McGraw-Hill, 1975.

## Remarks:

This program uses a trial and error procedure. It is possible that no answer would ever be found for some loadings.
The display setting is used to determine when answers are of satisfactory accuracy. Display of Engineering 3 is recommended for best operation. Larger numbers for display setting will take longer to converge.

| STEP | INSTRUCTIONS | INPUT DATAUNITS | KEYS | OUTPUT DATAUNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Calculate all intermediate |  |  |  |
|  | slope factors using simply sup- |  |  |  |
|  | ported beam programs. |  |  |  |
| 2 | Load side 1 and side 2. |  |  |  |
| 3 | Input number of spans in beam |  |  |  |
|  | $(2 \leqslant n \leqslant 6)$ | N | (1) | 0.00000 |
| 4 | Input moment applied at left |  |  |  |
|  | support (even if zero). | M | A | $M_{0}$ |
| 5 | Input slope factor from left side |  |  |  |
|  | of next intermediate support | $\theta_{\text {n }}$ | ENTER | $\theta_{\mathrm{n}}$ |
|  | and from right side of support.* | $\theta_{\mathrm{n}}{ }^{\prime}$ | B | $\theta_{\mathrm{n}}-\theta_{\mathrm{n}}{ }^{\prime}$ |


| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 6 | Input properties of the span:** |  |  |  |
|  | moment of inertia | I |  | I |
|  | modulus of elasticity | E |  | E |
|  | and length of span. | $\ell$ | C | n |
| 7 | For next span, go to step 5. For |  |  |  |
|  | last span, go to step 6 and then |  |  |  |
|  | skip to step 8. |  |  |  |
| 8 | Input moment at end of last |  |  |  |
|  | span. | $M_{N}$ | D | $M_{0}, M_{1}, M_{1}{ }^{\prime}$, |
|  |  |  |  | $\mathrm{M}_{2}, \ldots, \mathrm{M}_{\mathrm{N}}$ |
| 9 | To change any span, key in |  |  |  |
|  | number of span | $n$ | 10 | n |
|  | Go to step 5 for intermediate |  |  |  |
|  | spans, step 4 if first span, or |  |  |  |
|  | step 6 and skip to step 8 if last |  |  |  |
|  | span. |  |  |  |
| 10 | For a new case, go to step 1. |  |  |  |
|  | * To duplicate $\theta_{\mathrm{i}}, \theta_{\mathrm{i}}{ }^{\prime}, \mathrm{I}, \mathrm{E}$, and l |  |  |  |
|  | from previous span to next |  |  |  |
|  | span press |  | 18 | $n$ |
|  | and go to step 7. |  |  |  |
|  | **To duplicate I, E, and l from |  |  |  |
|  | previous span to next span, |  |  |  |
|  | press |  | $1{ }^{\text {c }}$ | n |
|  | and go to step 7. |  |  |  |

Example:


For the three span beam above, calculate the internal moments transmitted across the two intermediate supports.
Separate the beam into three independent sections and use the Simply Supported Beam-Trapezoidal Load program to solve for the slope factors at the points of support.

What is $\theta_{1}$, the slope factor at the end of the beam?


Keystrokes using CE-10:

## Outputs:

109 ENTERA 30 EEX 6 ENTER4
480 B $\longrightarrow \quad 3.27009$
0 ENTERA 16.67 ENTERA
480 ENTERA $16.67 \boldsymbol{C} \longrightarrow 0.00000$
480 B $\longrightarrow \quad 23.49-03 \quad\left(\theta_{1}\right)$

Section 2 is loaded the same as section 1 . No values change, so compute the slope factors at the two ends of section 2 .
$0 \mathrm{~B} \longrightarrow$

$480 \mathrm{~B} \longrightarrow$ | $-23.49-03$ |
| ---: | :--- |
| $23.49-03$ | | $\left(\theta_{1}{ }^{\prime}\right)$ |
| :--- |
| $\left(\theta_{2}\right)$ |

Section 3 requires solution by superposition of the continuous load and the trapezoidal load. First solve for the continuous load and store the result in $\mathrm{R}_{7}$, then add the result of the trapezoidal load.

Keystrokes using CE-15:
272 ENTERA 30 EEX 6 ENTERA
480 B 0 ENTER4
20.83 ENTERA 480 ENTER4
20.83 C 0 B $\longrightarrow$-11.76-03

STO 7120 ENTERA 58.3 ENTER4
340 ENTER4 66.7 © $\mathbf{C} 0$ B $\rightarrow$-22.74-03
$\boldsymbol{R C L} 7+\longrightarrow \quad-34.50-03$
Now we have the slopes at the supports. Using the continuous span program, we can compute the internal moments at the intermediate supports.

## Summary of Knowns

$\quad$ Span 1
$M_{0}=6000 \mathrm{Ib} \times 120 \mathrm{in}$
$\ell=+720,000 \mathrm{in-lb}$
$\theta_{1}=23.49 \times 10^{-3}$
$\mathrm{I}=109$
$\mathrm{E}=30 \times 10^{6}$
$\ell=480$

Span 2
Span 3
$\theta_{2}^{\prime}=-34.50 \times 10^{-3}$
$\mathrm{I}=272$
$E=30 \times 10^{6}$
$\ell=480$
$M_{3}=6000 \times 120$
$=-720,000 \mathrm{in}-\mathrm{lb}$
( $\mathrm{M}_{3}$ is negative by right hand rule)

Keystrokes:
Outputs:
Span 1
3 A A 720 EEX 3 A 23.49 EEX CHS
3ENTERt 23.49 CHS EEX CHS 3 B
109 ENTERA 30 EEX 6 ENTER4
$480 \mathrm{C} \longrightarrow \quad 1.00000$

Input for span 1 complete.

Span 2
23.49 EEX CHS 3 ENTERA 34.50 CHS EEX CHS 3 B

Since I, E, and $\ell$ remain the same between span 1 and span 2 , use the automatic section property duplicate function instead of keying the values in again.

Span 3
272 ENTERA 30 EEX 6 ENTER4
480 C $\longrightarrow \quad 3.00000$
720 CHSEEX 3 D $\longrightarrow \quad 720.003$

| -125.5 | 03 | $\mathrm{M}_{1}{ }^{\prime}$ |
| ---: | :--- | :--- |
| 125.5 | 03 | $\mathrm{M}_{1}{ }^{\prime}$ |
| -698.3 | 03 | $\mathrm{M}_{2}$ |
| 698.3 | 03 | $\mathrm{M}_{2}{ }^{\prime}$ |
| -720.0 | 03 | $\mathrm{M}_{3}$ |

Since we now know all loads and the moments at the ends of each span, we could calculate deflection, moment and shear for any point along the span using program CE-09 and program CE-10.

## STEEL COLUMN FORMULA



This program computes the allowable load and the maximum load for structural steel columns using the American Institute of Steel Construction formula (1961). The column ends must be welded, riveted, or otherwise constrained against deflection and rotation.

## Equations:



$$
\begin{array}{cc}
\mathrm{P}_{\text {allow }}=\mathrm{A} \sigma_{\mathrm{y}}\left[1-(\ell / \mathrm{k})^{2} / 2 \mathrm{C}^{2}\right] / \mathrm{m} & \text { for } \ell / \mathrm{k}<\mathrm{C} \\
\mathrm{P}_{\text {allow }}=\mathrm{A}\left(1.0273 \times 10^{12} \mathrm{~N} / \mathrm{m}^{2}\right) /(\ell / \mathrm{k})^{2} & \text { for } \mathrm{C}<\ell / \mathrm{k} \leqslant 200
\end{array}
$$

$$
\begin{gathered}
\mathrm{C}^{2}=2 \pi^{2} \mathrm{E} / \sigma_{\mathrm{y}} \\
\mathrm{~m}=5 / 3 \times 3(\ell / \mathrm{k}) / 8 \mathrm{C}-[(\ell / \mathrm{k}) / 2 \mathrm{C}]^{3}
\end{gathered}
$$

$$
P_{\max }=P_{\text {allow }} m
$$

## Definitions:

$P_{\text {allow }}$ is the allowable load;
$P_{\text {max }}$ is the maximum load the column could carry;
A is the area of the section;
$\ell$ is the length of the column;
$\mathbf{k}$ is the minimum radius of gyration of the column cross section;
$I$ is the minimum moment of inertia of the cross section;
$\sigma_{y}$ is the yield point of the steel.
$E$ is the modulous of elasticity of steel.

## Remarks:

Either SI (metric) or English units may be used. For SI units, input the yield point stress of the material using the $\boldsymbol{A}$ key and use meters as the unit of length for all other inputs. For English units, input the yield point stress in pounds per square inch using the B key and use inches as the unit of length in all other inputs.
You may input the minimum moment of inertia I, instead of the minimum radius of gyration k . If I is input it will automatically be converted to k using the relation:

$$
\mathrm{k}^{2}=\mathrm{I} / \mathrm{A}
$$

## Reference:

Roark, Raymond J.; Young, Warren C.; Formulas for Stress and Strain, McGraw-Hill, 1975.

## Remarks:

Columns must be nominally straight, homogeneous, and of uniform cross section.

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATAUUNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 and side 2. |  |  |  |
| 2 | Input the following values: |  |  |  |
|  | Input yield point stress of the |  |  |  |
|  | material in newtons per square |  |  |  |
|  | meter | $\sigma \mathrm{y}\left(\mathrm{N} / \mathrm{m}^{2}\right)$ | 1 A | 0.0000 |
|  | or pounds per square inch | $\sigma y$ (psi) | 1 B | 0.0000 |
|  | and section area | A | A | A |
|  | and column length | $\ell$ | B | $\ell$ |
|  | and minimum radius of gyration | k | C | k |
|  | or minimum moment of inertia | I | $\square$ | I |
| 3 | Calculate allowable load |  | D | $\mathrm{P}_{\text {allow }}$ |
|  | and/or maximum load |  | E | $\mathrm{P}_{\text {max }}$ |
| 4 | For a new case, go to step 2 |  |  |  |
|  | and change any or all of the |  |  |  |
|  | inputs. |  |  |  |

## Example 1:

Two steel channels are lased together to form the cross section below:


Calculate the allowable and maximum loads using the following specifications:
$\mathrm{k}=81.0 \times 10^{-3} \mathrm{~m} \quad \mathrm{~A}=9.46 \times 10^{-3} \mathrm{~m}^{2} \quad \sigma_{\mathrm{y}}=248 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}$
$\ell=7.5 \mathrm{~m}$ and 12 m

Keystrokes:

## 248 EEX 6 f A 9.46 EEX CHS

3 A 7.5 B 81 EEX CHS


E


Outputs:

| 918.2 | 03 | $\mathrm{P}_{\text {allow }}(\mathrm{N})$ |
| :--- | :--- | :--- |
| 1.736 | 06 | $\mathrm{P}_{\max }(\mathrm{N})$ |
| 442.8 | 03 | $\mathrm{P}_{\text {allow }}(\mathrm{N})$ |
| 844.5 | 03 | $\mathrm{P}_{\max }(\mathrm{N})$ |

## Example 2:

For a column with the properties below, what is the allowable load?

$$
\sigma_{y}=33,000 \mathrm{psi} \quad \mathrm{~A}=20 \mathrm{in}^{2} \quad \mathrm{I}=223 \mathrm{in}^{4} \quad \ell=350 \mathrm{in}
$$

Keystrokes:
33000 f 20 A 223 f $C$
350 B D

## Outputs:

$241.003 \quad P_{\text {allow }}$ (Pounds)

## REINFORCED CONCRETE BEAMS

| MET? | +a | -NA |  | $\Phi f_{y}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\triangle A_{s}$ | $\Phi$ b | ¢ M | $\triangle \mathrm{d}$ | $\pm \mathrm{f}_{\mathrm{c}}$ |

This program can be used in the design and analysis of rectangular reinforced concrete beams in accordance with the strength design method of the American Concrete Institute Code (ACI 318-71). The program solves interchangeably between the following six variables:
$\mathrm{A}_{\mathrm{s}}$-The area of nonprestressed tension reinforcement ( psi or $\mathrm{kg} / \mathrm{cm}^{2}$ );
b -The width of the member (in or cm );
$\mathrm{M}-$ The maximum internal bending moment ( $\mathrm{lb}-\mathrm{in}$ or $\mathrm{kg}-\mathrm{cm}$ );
d-The depth to the centroid of the reinforcing steel (in or cm );
$\mathrm{f}_{\mathrm{c}}$-The compressive strength of the concrete ( psi or $\mathrm{kg} / \mathrm{cm}^{2}$ );
$f_{y}$ - The yield strength of the steel ( psi or $\mathrm{kg} / \mathrm{cm}^{2}$ ).


During calculation of the parameters listed above, the calculator checks to be sure that enough reinforcement has been specified to meet the minimum allowable value:

$$
\frac{A_{s}}{b d}>\frac{200}{f_{y}}
$$

If this condition is not met the display will flash 10.50 which signifies that the design does not meet section 10.5 of the ACI code. Stop the flashing by pressing R/S. Press Rt to see the current value of $\mathrm{A}_{\mathbf{s}}$. Press Rt again to see the minimum allowable value of $\mathrm{A}_{\mathrm{s}}$. Pressing © at this point stores the minimum value of $\mathrm{A}_{\mathrm{s}}$ and readys the calculator for calculation of the desired variable.

The program also checks for too much steel. Code section 10.32 specifies the maximum steel area as:

$$
\frac{A_{\text {smax }}}{b d}=(0.6375) \beta_{1} \frac{f_{c}}{f_{y}} \frac{87000}{87000+f_{y}}
$$

where

$$
\beta_{1}=\left\{\begin{array}{l}
0.85 \text { for } \mathrm{f}_{\mathrm{c}} \leqslant 4000 \\
0.85-\left(\mathrm{f}_{\mathrm{c}}-4000\right) / 20000 \text { for } \mathrm{f}_{\mathrm{c}}>4000
\end{array}\right.
$$

If too much steel has been specified, the calculator flashes 10.32. Stop the flashing by pressing $\mathbf{R / S}$, then press $\mathbf{R T}$ to see the current steel area. Press RT again to see the maximum allowable tension steel area. Press $\boldsymbol{A}$ if you wish to use the maximum amount of steel in subsequent calculations.
If the program halts displaying "Error," the input values are mathematically impossible to satisfy. This may be due to an entry error (you may review the values by recalling $R_{1}$ for $A_{s}, R_{2}$ for $b, R_{3}$ for $M$ etc....) or the configuration may be mathematically undefined. If this is the case, increase the beam size and/or decrease the moment.
Optionally, the depth of the compression zone (a) may be calculated using the
B keys and the depth of the neutral axis (NA) may be calculated using
C. The depth of the neutral axis is important since T-beams may be modeled as rectangular beams if the slab or flange equals or exceedes the depth of the neutral axis.

## Equations:

$$
\begin{gathered}
\mathrm{M}=\mathrm{d} \phi \mathrm{~A}_{\mathrm{s}} \mathrm{f}_{\mathrm{y}}-\left(0.59 \phi \mathrm{~A}_{\mathrm{s}}^{2} \mathrm{f}_{\mathrm{y}}^{2}\right) /\left(\mathrm{b} \mathrm{f}_{\mathrm{c}}\right) \\
\phi=\text { factor of safety }=0.9
\end{gathered}
$$

## Reference:

ACI Standard Building Code Requirements for Reinforced Concrete (ACI 318-71), American Concrete Institute, May 1976 printing.

## Remarks:

This program is intended as an aid to computation and cannot replace an understanding of ACI 318-71.

This program does not check for deflection of shear stress modes of failure. Refer to ACI 318-71 for specifics on deflection and shear stress.

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 and side 2. |  |  |  |
| 2 | Optional: toggle metric units |  |  |  |
|  | ( 1 = kilograms and centi- |  |  |  |
|  | meters) or English units |  |  |  |
|  | ( $0=$ pounds and inches). |  | 14 | 1 or 0 |
| 3 | Input 5 of the following |  |  |  |
|  | variables: |  |  |  |
|  | Area of tension reinforcement | $\mathrm{A}_{\text {s }}$ | A | $\mathrm{A}_{\text {s }}$ |
|  | Width of beam | b | B | b |
|  | Bending moment | M | c | M |
|  | Depth of section to centroid of |  |  |  |
|  | steel | d | D | d |
|  | Compressive strength of |  |  |  |
|  | concrete | $\mathrm{f}_{\mathrm{c}}$ | E | $\mathrm{f}_{\mathrm{c}}$ |
|  | Yield strength of tension |  |  |  |
|  | reinforcement | $f_{y}$ | [E | $\mathrm{f}_{\mathrm{y}}$ |
| 4 | Calculate remaining unknown |  |  |  |
|  | value: |  |  |  |
|  | Area of tension reinforcement |  | A | $\mathrm{A}_{5}$ |
|  | Width of beam |  | B | b |
|  | Bending moment |  | C | M |
|  | Depth of section to centroid |  |  |  |
|  | of steel |  | D | d |
|  | Compressive strength of |  |  |  |
|  | concrete |  | E | $\mathrm{f}_{\mathrm{c}}$ |
|  | Yield strength of tension |  |  |  |
|  | reinforcement |  | 1 E | $\mathrm{f}_{\mathrm{y}}$ |
| 5 | If step 4 resulted in an "Error" |  |  |  |
|  | or a flashing display, refer to |  |  |  |
|  | description for explanation. |  |  |  |


| STEP | INSTRUCTIONS | INPUT <br> DATA/UNITS | KEYS | OUTPUT <br> DATA/UNITS |
| :---: | :--- | :---: | :---: | :---: |
| 6 | Optional: Calculate depth of |  |  |  |
|  | compressive stress block |  | B | a |
|  | and/or depth of neutral axis |  | C | NA |
| 7 | For a new case, go to step 3 |  |  |  |
|  | and change any or all of the |  |  |  |
|  | input values. |  |  |  |

## Example 1:

For the specifications below, calculate the amount of reinforcing steel required.
$\mathrm{M}=1.2 \times 10^{6} \mathrm{in}-\mathrm{lb} \quad \mathrm{b}=18 \mathrm{in} \quad \mathrm{d}=26$ in $\quad \mathrm{f}_{\mathrm{c}}=3500 \mathrm{psi}$
$\mathrm{f}_{\mathrm{y}}=50000 \mathrm{psi}$

## Keystrokes:

A A $\qquad$

## Outputs:

0.00000 (Set for English units.)
1.2 EEX 6 C 18 B 26 D

3500 E 50000 EEA
10.5000
(Flashing display indicates that calculated steel area is too small to meet ACI minimum as specified in ACI 10.5. Press R/S to halt the flashing display. Press $\boldsymbol{R t}$ to see the calculated value, then press $\boldsymbol{R t}$ again to see the minimum value, then use the minimum value to recalculate M.)

| R/S $\mathrm{Rt} \longrightarrow$ | 1.04500 | $\mathrm{in}^{2}$ (calc) |
| :---: | :---: | :---: |
| Rt $\longrightarrow$ | 1.87200 | $\mathrm{in}^{2}$ (min) |
| AC $\longrightarrow$ | 2.11606 | in-lb (M) |

## Example 2:

For the beam specifications below, calculate the area of steel required.
$\mathrm{b}=25 \mathrm{~cm} \quad \mathrm{~d}=30 \mathrm{~cm} \quad \mathrm{M}=1.6 \times 10^{6} \mathrm{~kg}-\mathrm{cm} \quad \mathrm{f}_{\mathrm{c}}=281 \mathrm{~kg} / \mathrm{cm}^{2}$
$\mathrm{f}_{\mathrm{y}}=4219 \mathrm{~kg} / \mathrm{cm}^{2}$

Keystrokes:


## Outputs:

$$
1.00000 \quad \text { (metric units) }
$$

(Flashing display indicates that calculated steel area is too large to meet ACI
specification 10.32. Press R/S to halt flashing display. Press Rt to see calculated value, then Rt again to see maximum value.)

| $\mathrm{R} / \mathrm{S} / \mathrm{Rt} \longrightarrow$ | 17.7800 <br> $\mathrm{Rt} \longrightarrow$ | 16.0200 | $\mathrm{~cm}^{2}$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{~cm}^{2}$ |  |  |  |

Using $16 \mathrm{~cm}^{2}$ for $A_{s}$, what is the minimum value for d ?
16 A D
32.0100
cm

## Example 3:

Calculate the area of the steel and the depth of the slab or flange for the T-beam data below. Use the depth of the neutral axis as the minimum depth of the flange so that the T-beam can be modeled as a rectangular beam.
$\mathrm{M}=2 \times 10^{6} \mathrm{in}-\mathrm{lb}$
$b=20$ in
$\mathrm{d}=20$ in
$\mathrm{f}_{\mathrm{c}}=4000 \mathrm{psi}$
$f_{y}=60,000 \mathrm{psi}$


## Keystrokes:

(A) A

Outputs:

2 EEX 6 C 20 B 20 D 4000 E 60000 EA $\longrightarrow$

C
0.00000
(English units)

## BOLT TORQUE



This program may be used to calculate either the torque that will yield a specified bolt load or the load resulting from a specified torque. The maximum shear stress in the body of the screw may also be calculated.

## Equations:

$$
\begin{gathered}
\mathrm{T}=\mathrm{W} \frac{\mathrm{D}_{\mathrm{m}}}{2}\left[\frac{\tan \alpha+\mathrm{f}_{\mathrm{t}} / \cos \theta}{1-\mathrm{f}_{\mathrm{t}} \tan \alpha / \cos \theta}\right]+\mathrm{W} \mathrm{f}_{\mathrm{c}} \frac{\mathrm{D}_{\mathrm{c}}}{2} \\
\tau_{\max }=\sqrt{\left(\mathrm{W} / 2 \mathrm{~A}_{\mathrm{r}}\right)^{2}+\left(16 \mathrm{~T}_{\mathrm{t}} / \pi \mathrm{D}_{\mathrm{r}}^{3}\right)^{2}} \\
\mathrm{~T}_{\mathrm{t}}=\mathrm{T}-\mathrm{Wf}_{\mathrm{c}} \frac{\mathrm{D}_{\mathrm{c}}}{2}
\end{gathered}
$$

where:
T is the applied torque;
W is the bolt load;
$\mathrm{D}_{\mathrm{m}}$ is the mean thread diameter;
$\alpha$ is the helix angle of the thread;
$\mathrm{f}_{\mathrm{t}}$ is the coefficient of thread friction;
$\theta$ is one-half of the thread angle;
$\mathrm{f}_{\mathrm{c}}$ is the collar coefficient of friction;
$\mathrm{D}_{\mathrm{c}}$ is the collar diameter;
$\tau_{\text {max }}$ is the maximum shear stress in the body of the screw;
$\mathrm{A}_{\mathrm{r}}$ is the root area;
$\mathrm{D}_{\mathrm{r}}$ is the diameter at the root of the thread.

## Remarks:

The accuracy with which $f_{t}$ and $f_{c}$ are approximated has a significant effect on the applicability of the resulting computations.

## Reference;

Hall, Holowenko, Laughlin Machine Design, Schaum's Outline Series, McGraw-Hill Co., 1961.

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | OUTPUT DATA/UNITS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Load side 1 or side 2. |  |  |  |
| 2 | Input helix angle of thread | $\alpha$ | ENTER | $\alpha$ |
|  | then one-half of thread angle | $\theta$ | ENTER4 | $\theta$ |
|  | then coefficient of thread |  |  |  |
|  | friction | $\mathrm{f}_{\mathrm{t}}$ | A | 0.00 |
|  | Input mean thread diameter | $\mathrm{D}_{\mathrm{m}}$ | ENTERA | $\mathrm{D}_{\mathrm{m}}$ |
|  | then collar diameter | $\mathrm{D}_{\mathrm{c}}$ | ENTER ${ }^{\text {d }}$ | $\mathrm{D}_{\mathrm{c}}$ |
|  | then collar coefficient of |  |  |  |
|  | friction | $\mathrm{f}_{\mathrm{c}}$ | B | 0.00 |
| 3 | Input one of the following |  |  |  |
|  | bolt load | W | c | W |
|  | bolt torque | T | D | T |
| 4 | Calculate one of the following |  |  |  |
|  | bolt load |  | C | W |
|  | bolt torque |  | D | T |
| 5 | Optional: Input diameter of the |  |  |  |
|  | root of the thread and compute |  |  |  |
|  | shear stress | $\mathrm{D}_{\text {r }}$ | $E$ | $\tau_{\text {max }}$ |
| 6 | For a new load or torque go to |  |  |  |
|  | step 3. For a new case go to |  |  |  |
|  | step 2. |  |  |  |

## Example:

Some bolts must exert a force of 11,000 pounds each. What torque is necessary to achieve this load assuming the following specifications? What is the shear stress in the bolt?

$$
\begin{array}{rlr}
\mathrm{D}_{\mathrm{m}} & =0.3344 \text { in } & \mathrm{f}_{\mathrm{c}}=0.30 \\
\alpha & =3.40^{\circ} & \mathrm{D}_{\mathrm{c}}=0.8750 \\
\mathrm{f}_{\mathrm{t}} & =0.15 & \mathrm{D}_{\mathrm{r}}=0.2983 \\
\theta & =30^{\circ} &
\end{array}
$$

Keystrokes:
3.40 ENTER 430 ENTER 4.15 A
.3344 ENTER 4.8750 ENTER $\uparrow$
.3 B 11000 C D
$12 \div$ .2983 E B $\longrightarrow$

If the torque were set at 140 foot-pounds ( 1680 inch-pounds), what would be the bolt load?

1680 D C
9850.61 lbs

## PROGRAM LISTINGS

The following listings are included for your reference. A table of keycodes and keystrokes corresponding to the symbols used in the listings can be found in Appendix E of your Owners Handbook.
Program Page

1. Vector Statistics ..... L01-01
2. Section Properties (2 Cards) ..... L02-01
3. Properties of Special Sections ..... L03-01
4. Stress on an Element ..... L04-01
5. Bending or Torsional Stress ..... L05-01
6. Linear or Angular Deformation ..... L06-01
7. Cantilever Beams ..... L07-01
8. Cantilever Beams-Trapezoidal Load ..... L08-01
9. Simply Supported Beams ..... L09-01
10. Simply Supported Beams-Trapezoidal Load ..... L10-01
11. Beams Fixed at Both Ends ..... L11-01
12. Beams Fixed at Both Ends-Trapezoidal Load ..... L12-01
13. Propped Cantilever Beams ..... L13-01
14. Propped Cantilever Beams-Trapezoidal Load ..... L14-01
15. Six-span Continuous Beams ..... L15-01
16. Steel Column Formula ..... L16-01
17. Reinforced Concrete Beams ..... L17-01
18. Bolt Torque ..... L18-01

VECTOR STATICS



SECTION PROPERTIES

|  | 061 <br> 082 <br> 093 <br> 004 <br> 165 <br> 096 <br> $667^{7}$ <br> 008 <br> 089 <br> 010 <br> 011 <br> 012 <br> 013 <br> 014 <br> 015 <br> 016 <br> 017 <br> 018 <br> 819 <br> 020 <br> 021 <br> 922 <br> Q2? <br> 024 <br> 025 <br> 626 <br> 827 <br> 028 <br> 829 <br> Q30 <br> 031 <br> Q32 <br> 033 <br> 034 <br> 035 <br> 036 <br> 037 <br> 038 <br> 039 <br> 449 <br> 841 <br> 042 <br> 24? <br> 044 <br> 845 <br> 046 <br> 047 <br> 848 <br> 849 <br> 850 <br> 85! <br> 852 <br> 853 <br> 854 <br> 855 <br> 856 |  |  | Clear re | tes. |  | 857 <br> 858 <br> 859 <br> 868 <br> 861 <br> 862 <br> 863 <br> 664 <br> 865 <br> 066 <br> 867 <br> 968 <br> 869 <br> A70 <br> 071 <br> 072 <br> 873 <br> 874 <br> 875 <br> 076 <br> 677 <br> 678 <br> 879 <br> 680 <br> e8: <br> 082 <br> 083 <br> 084 <br> 085 <br> 086 <br> 087 <br> 088 <br> 089 <br> 098 <br> 091 <br> 092 <br> 093 <br> 094 <br> 095 <br> 896 <br> 097 <br> 898 <br> 899 <br> 180 <br> 101 <br> 182 <br> :03 <br> 104 <br> 185 <br> 106 <br> 10 ? <br> 108 <br> 189 <br> 118 <br> 111 <br> 112 | ST-1 RCLC RCLB $x$ $R C L A$ RCLD $x$ - $E N T 4$ $E N T \uparrow$ 4 $\vdots$ $R C L 8$ $x$ $R C L A$ $R C L C$ $x$ $R C L C$ $x 2$ $S T O 9$ + $R C L A$ $x$ $S T+9$ + $R C L 7$ $x$ 3 |  | Sum $\Delta I_{\mathbf{x y}}$. <br> Recall $x_{i}$ and $y_{i}$ for next segment. <br> Calculate $\Delta M_{\mathbf{x}}$ and $\Delta M_{\mathbf{r}}$. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| REGISTERS |  |  |  |  |  |  |  |  |  |  |  |
| ${ }^{0} \mathrm{EA}$ |  |  |  | $]^{3} \Sigma \mathrm{I}_{\mathbf{x}} \quad{ }^{4} \Sigma \mathrm{I}_{\mathbf{y}}$ |  | ${ }^{5} \quad \Sigma I_{x y}$ | ${ }^{6}\left(x_{i+1}-x_{i}\right)$ |  | $\sqrt{7 \quad\left(y_{i+1}-y_{i}\right)}$ |  |  |
| So | S1 |  | S2 | S3 | S4 | S5 | S6 |  |  |  |  |
| ${ }^{\text {A }}{ }^{\text {a }}$ | ${ }^{B} y_{i}$ |  |  | ${ }^{C_{x_{i+1}}}$ |  | $\begin{array}{\|l\|l\|} \hline D^{2} & \\ y_{i+1} & \\ \hline \end{array}$ |  |  | $\mathrm{E}$ |  |  |


| 11 11 11 11 11 11 11 12 12 12 12 12 12 12 12 12 12 13 13 13 13 13 13 13 13 139 149 14 14 143 14 14 14 14 14 14 15 15 15 153 | 113 114 115 116 117 118 119 128 121 122 123 124 125 126 127 128 129 138 131 132 133 134 135 136 137 138 139 149 141 142 143 144 145 146 148 148 149 159 159 151 |  | Calcula <br> $---$ <br> Add to regions. | ion subroutine. <br> sums for circular |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Labels |  |  |  |  |  | FLAGS | SET STATUS |  |  |
| ${ }^{4} x_{i+1} \uparrow y_{i+1}$ | ${ }^{8}$ |  | ${ }^{c} \times{ }^{\text {¢ }}$ y ${ }^{\text {d }}$ |  |  |  |  |  |  | flags | trig | Disp |
| a | $\bigcirc$ |  | ${ }^{\circ}$ | $\bigcirc$ |  |  |  | $\bigcirc{ }^{\text {ONOFF }}$ |  |  |
| $\bigcirc$ |  | Calculate | 2 | ${ }^{3}$ |  | Calculate | ${ }^{2}$ |  | ${ }_{\text {GRAD }}^{\text {GRA }}$ |  |
| 5 |  |  | - |  |  |  | ${ }^{3}$ | $3{ }^{1}$ |  |  |

(CARD 2 )



## PROPERTIES OF SPECIAL SECTIONS




## STRESS ON AN ELEMENT




L05-01
BENDING OR TORSIONAL STRESS



LINEAR OR ANGULAR DEFORMATION



CANTILEVER BEAMS



CANTILEVER BEAMS—TRAPEZIODAL LOAD



## SIMPLY SUPPORTED BEAMS




SIMPLY SUPPORTED BEAMS—TRAPEZIODAL LOAD



BEAMS FIXED AT BOTH ENDS



BEAMS FIXED AT BOTH ENDS—TRAPEZOIDAL LOAD



PROPPED CANTILEVER BEAMS



PROPPED CANTILEVER BEAMS—TRAPEZIODAL LOAD



## SIX-SPAN CONTINUOUS BEAMS




STEEL COLUMN FORMULA



## REINFORCED CONCRETE BEAMS




BOLT TORQUE


Notes

## Appendix A <br> MAGNETIC CARD SYMBOLS AND CONVENTIONS

| SYMBOL OR CONVENTION | INDICATED MEANING |
| :---: | :---: |
| White mnemonic: <br> x <br> A | White mnemonics are associated with the userdefinable key they are above when the card is inserted in the calculator's window slot. In this case the value of $x$ could be input by keying it in and pressing A. |
| Gold mnemonic: <br> y <br> x E | Gold mnemonics are similar to white mnemonics except that the gold $f$ key must be pressed before the user-definable key. In this case y could be input by pressing $f$ E. |
| $x \uparrow y$ | 4 is the symbol for ENTERA. In this case ENTERA is used to separate the input variables x and y . To input both $x$ and $y$ you would key in $x$, pressENTERA, key in y and press A. |
| $\begin{aligned} & \mathrm{X} \\ & \hline \mathrm{~A} \end{aligned}$ | The box around the variable x indicates input by pressing STO A. |
| (x) A | Parentheses indicate an option. In this case, x is not a required input but could be input in special cases. |
| $\begin{array}{r} \rightarrow X \\ \boldsymbol{A} \end{array}$ | is the symbol for calculate. This indicates that you may calculate x by pressing key $\boldsymbol{A}$. |
| $\rightarrow x, y, z$ <br> A | This indicates that $x, y$, and $z$ are calculated by pressing $A$ once. The values would be printed in $\mathrm{x}, \mathrm{y}, \mathrm{z}$ order. |
| $\rightarrow x ; y ; z$ <br> A | The semi-colons indicate that after $x$ has been calculated using A, y and z may be calculated by pressing $\mathbf{R} / \mathbf{S}$. |
| $\rightarrow \underset{A}{\prime \prime} x,{ }_{A}^{\prime} y$ | The quote marks indicate that the x value will be "paused" or held in the display for one second. The pause will be followed by the display of $y$. |
| $\stackrel{\Delta x}{\Delta}$ | The two-way arrow $\triangleleft$ indicates that x may be either output or input when the associated userdefinable key is pressed. If numeric keys have been pressed between user-definable keys, x is stored. If numeric keys have not been pressed, the program will calculate x . |


| SYMBOL OR <br> CONVENTION | INDICATED MEANING |
| :---: | :--- |
| P? | The question mark indicates that this is a mode <br> setting, while the mnemonic indicates the type of <br> mode being set. In this case a print mode is con- <br> trolled. Mode settings typically have a 1.00 or 0.00 <br> indicator displayed after they are executed. If 1.00 <br> is displayed, the mode is on. If 0.00 is displayed, <br> it is off. <br> The word START is an example of a command. The <br> start function should be performed to begin or start <br> a program. It is included when initialization is <br> necessary. |
| ATART | This special command indicates that the last value <br> or set of values input may be deleted by pressing <br> A. |
| DEL |  |

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[^0]:    *If $x$ is greater than $a,(\ell-a)$ is replaced by $-a$ and $x$ is replaced by $(x-\ell)$.
    ${ }^{* *}$ If $x$ is greater than $c, x$ is replaced by $(x-\ell)$ and $c$ is replaced by $(\ell-c)$.

[^1]:    *If x is greater than a , a is replaced by $(\ell-\mathrm{a})$ and x is replaced by ( $\ell-\mathrm{x})$. The signs of $\theta_{1}$ and $\mathrm{V}_{1}$ are also changed.
    ${ }^{* *}$ If x is greater than $\mathrm{c}, \mathrm{x}$ is replaced by $(\ell-\mathrm{x})$ and c is replaced by $(\ell-\mathrm{c})$. The signs of $\mathrm{y}_{3}$ and $M_{x 3}$ are also changed.

