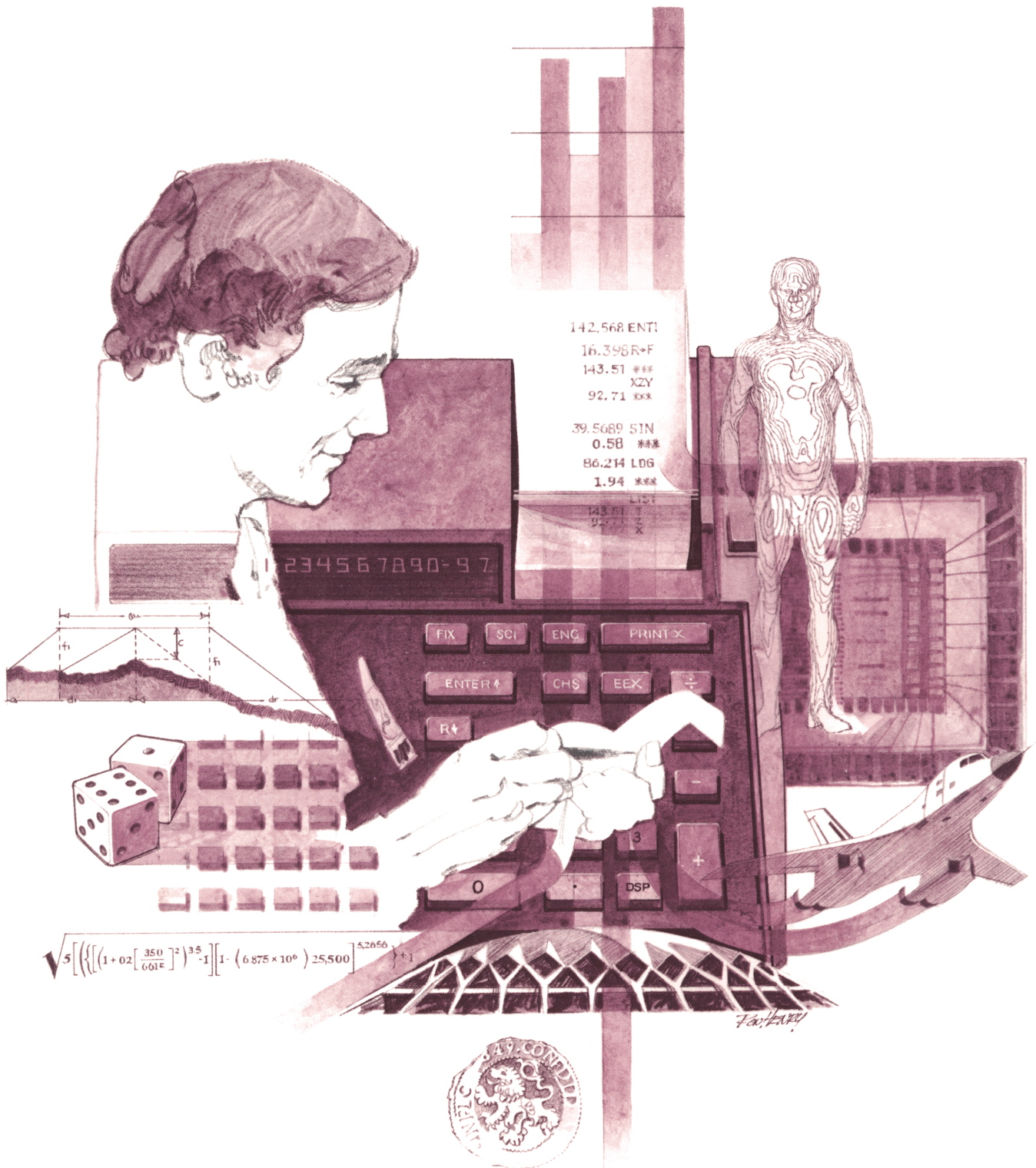


HEWLETT-PACKARD

HP-67/HP-97

Users' Library Solutions

Test Statistics



INTRODUCTION

In an effort to provide continued value to its customers, Hewlett-Packard is introducing a unique service for the HP fully programmable calculator user. This service is designed to save you time and programming effort. As users are aware, Programmable Calculators are capable of delivering tremendous problem solving potential in terms of power and flexibility, but the real genie in the bottle is program solutions. HP's introduction of the first handheld programmable calculator in 1974 immediately led to a request for program **solutions** — hence the beginning of the HP-65 Users' Library. In order to save HP calculator customers time, users wrote their own programs and sent them to the Library for the benefit of other program users. In a short period of time over 5,000 programs were accepted and made available. This overwhelming response indicated the value of the program library and a Users' Library was then established for the HP-67/97 users.

To extend the value of the Users' Library, Hewlett-Packard is introducing a unique service—a service designed to save you time and money. The Users' Library has collected the best programs in the most popular categories from the HP-67/97 and HP-65 Libraries. These programs have been packaged into a series of low-cost books, resulting in substantial savings for our valued HP-67/97 users.

We feel this new software service will extend the capabilities of our programmable calculators and provide a great benefit to our HP-67/97 users.

A WORD ABOUT PROGRAM USAGE

Each program contained herein is reproduced on the standard forms used by the Users' Library. Magnetic cards are not included. The Program Description I page gives a basic description of the program. The Program Description II page provides a sample problem and the keystrokes used to solve it. The User Instructions page contains a description of the keystrokes used to solve problems in general and the options which are available to the user. The Program Listing I and Program Listing II pages list the program steps necessary to operate the calculator. The comments, listed next to the steps, describe the reason for a step or group of steps. Other pertinent information about data register contents, uses of labels and flags and the initial calculator status mode is also found on these pages. Following the directions in your HP-67 or HP-97 **Owners' Handbook and Programming Guide**, "Loading a Program" (page 134, HP-67; page 119, HP-97), key in the program from the Program Listing I and Program Listing II pages. A number at the top of the Program Listing indicates on which calculator the program was written (HP-67 or HP-97). If the calculator indicated differs from the calculator you will be using, consult Appendix E of your **Owner's Handbook** for the corresponding keycodes and keystrokes converting HP-67 to HP-97 keycodes and vice versa. No program conversion is necessary. The HP-67 and HP-97 are totally compatible, but some differences do occur in the keycodes used to represent some of the functions.

A program loaded into the HP-67 or HP-97 is not permanent—once the calculator is turned off, the program will not be retained. You can, however, permanently save any program by recording it on a blank magnetic card, several of which were provided in the Standard Pac that was shipped with your calculator. Consult your **Owner's Handbook** for full instructions. A few points to remember:

The Set Status section indicates the status of flags, angular mode, and display setting. After keying in your program, review the status section and set the conditions as indicated before using or permanently recording the program.

REMEMBER! To save the program permanently, **clip** the corners of the magnetic card once you have recorded the program. This simple step will protect the magnetic card and keep the program from being inadvertently erased.

As a part of HP's continuing effort to provide value to our customers, we hope you will enjoy our newest concept.

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Program Description I

Program Title **ONE SAMPLE TEST STATISTICS
FOR THE MEAN**

Contributor's Name **Hewlett-Packard**

Address **1000 N.E. Circle Blvd.**

City **Corvallis** State **Oregon** Zip Code **97330**

Program Description, Equations, Variables

Suppose $\{x_1, x_2, \dots, x_n\}$ is a sample from a normal population with a known variance σ^2 and unknown mean μ . A test of the null hypothesis

$$H_0: \mu = \mu_0$$

is based on the z statistic which has a standard normal distribution.

If the variance σ^2 is unknown then the t statistic, which has the t distribution with $n - 1$ degrees of freedom, is used instead.

Equations:

$$z = \frac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma}$$

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s}$$

where \bar{x} and s are sample mean and sample standard deviation.

Operating Limits and Warnings

Remark:

$$n > 1.$$

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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Program Description II

Sketch(es)

Sample Problem(s)

Example:

Compute the z and the t statistics for the following set of data if $\mu_0 = 2$ and $\sigma = 1$.

$\{2.73, 0.45, 2.52, 1.19, 3.51, 2.75, 1.79, 1.83, 1, 0.87, 1.9, 1.62, 1.74, 1.92, 1.24, 2.68\}$

Keystrokes:

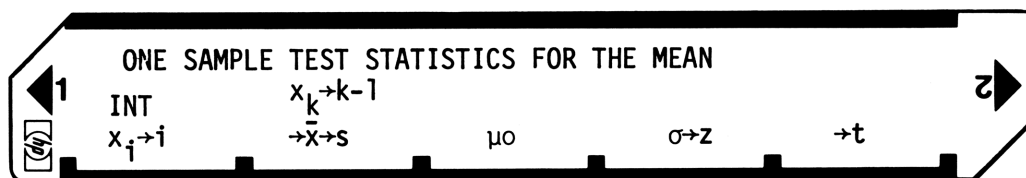
[f]	[A]	→	0.00
2.73	[A]	.45	[A] ... 2.68 [A]
[B]		→	1.86 (\bar{x})
[B]		→	0.82 (s)
2	[C]	→	2.00
1	[D]	→	-0.57 (z)
[E]		→	-0.69 (t)

Solution(s)

Reference(s) This program is a translation of the HP-65 Stat Pac 2 program.

User Instructions

3



STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program		<input type="text"/> <input type="text"/>	
2	If \bar{x} and s are known, go to 8		<input type="text"/> <input type="text"/>	
3	Initialize		RTN <input type="text"/> R/S <input type="text"/>	0.00
4	Perform 4 for $i = 1, 2, \dots, n$	x_i	A <input type="text"/>	i
5	Optional—delete erroneous		<input type="text"/> <input type="text"/>	
	data x_k ($k \neq 1$)	x_k	f <input type="text"/> A <input type="text"/>	
			<input type="text"/> <input type="text"/>	
6	Compute \bar{x} and s		B <input type="text"/>	\bar{x}
			B <input type="text"/>	s
7	Go to 9		<input type="text"/> <input type="text"/>	
8	Store \bar{x} and s	\bar{x}	STO <input type="text"/> 2 <input type="text"/>	
		s	STO <input type="text"/> 5 <input type="text"/>	
9	Input μ_0	μ_0	C <input type="text"/>	
10	Input σ and compute z	σ	D <input type="text"/>	z
	or		<input type="text"/> <input type="text"/>	
	Compute t		E <input type="text"/>	t
11	For a new case, go to 2		<input type="text"/> <input type="text"/>	

97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLa	21 16 11	Initialize	057	-	-45	$\bar{x}-\mu_0$
002	CLRG	16-53		058	X \div Y	-41	
003	RTN	24		059	\div	-24	
004	*LBLA	21 11		060	RCL1	36 01	
005	RCL2	36 02		061	JX	54	Display z
006	-	-45		062	x	-35	
007	RCL4	36 04		063	RTN	24	
008	-	-45		064	*LBLB	21 15	
009	RCL1	36 01		065	RCL2	36 02	$\bar{x}-\mu_0$
010	1	01		066	RCL6	36 06	
011	+	-55		067	-	-45	
012	\div	-24		068	RCL5	36 05	
013	ENT \uparrow	-21		069	\div	-24	Display t
014	ENT \uparrow	-21		070	RCL1	36 01	
015	RCL4	36 04		071	JX	54	
016	+	-55		072	x	-35	
017	ENT \uparrow	-21		073	RTN	24	Delete data
018	ENT \uparrow	-21		074	*LBLb	21 16 12	
019	RCL2	36 02		075	RCL1	36 01	
020	+	-55		076	CHS	-22	
021	STO2	35 02	077	STO1	35 01		
022	LSTX	16-63	078	R \downarrow	-31		
023	-	-45	079	GSBA	23 11		
024	-	-45	080	R/S	51		
025	STO4	35 04					
026	R \downarrow	-31					
027	x	-35					
028	RCL1	36 01					
029	x	-35					
030	1	01					
031	LSTX	16-63					
032	+	-55					
033	ABS	16 31	090				
034	STO1	35 01					
035	x	-35					
036	ST+3	35-55 03					
037	RCL1	36 01					
038	RTN	24					
039	*LBLB	21 12	Display the mean				
040	RCL2	36 02					
041	R/S	51					
042	*LBLB	21 12					
043	RCL3	36 03					
044	RCL1	36 01		100			
045	1	01					
046	-	-45					
047	\div	-24					
048	JX	54					
049	STO5	35 05	Store s				
050	RTN	24					
051	*LBLC	21 13					
052	STO6	35 06					
053	RTN	24					
054	*LBLC	21 14					
055	RCL2	36 02					
056	RCL6	36 06					
					</		

SET STATUS

FLAGS		TRIG	DISP
0	<input type="checkbox"/> ON <input checked="" type="checkbox"/> OFF	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
1	<input type="checkbox"/> <input checked="" type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
2	<input type="checkbox"/> <input checked="" type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
3	<input type="checkbox"/> <input checked="" type="checkbox"/>		n <u>2</u>

Program Description I

Program Title	TEST STATISTICS FOR THE CORRELATION COEFFICIENT		
Contributor's Name	Hewlett-Packard		
Address	1000 N.E. Circle Blvd.		
City	Corvallis	State	Oregon
		Zip Code	97330

Program Description, Equations, Variables

Under the assumptions of normal correlation analysis, the t statistic, which has the t distribution with $n - 2$ degrees of freedom, can be used to test the null hypothesis that the true correlation coefficient $\rho = 0$.

To test the null hypothesis $\rho = \rho_0$, where ρ_0 is a given number, the z statistic is used. z has approximately the standard normal distribution.

Equations:

$$t = \frac{r \sqrt{n-2}}{\sqrt{1-r^2}}$$

$$z = \frac{\sqrt{n-3}}{2} \ln \left[\frac{(1+r)(1-\rho_0)}{(1-r)(1+\rho_0)} \right]$$

where r is an estimate (based on a sample of size n) of the correlation coefficient ρ .

Operating Limits and Warnings

Remarks:

1. This program requires that $n > 3$, $|r| < 1$ and $|\rho_0| < 1$; otherwise, flashing zeros will result.
2. Usually, the z statistic is used when the sample size is large.

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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Program Description II

[illegible]

Sample Problem(s)	
<p>Example:</p> <p>Given $r = 0.12$, $n = 31$, and $\rho_0 = 0$, find t and z.</p> <p>Keystrokes:</p> <p>.12 A 31 B C → 0.65 (t)</p> <p>0 D E → 0.64 (z)</p>	
<p>Solution(s)</p>	

Reference(s) 1. Hogg and Craig, Introduction to Mathematical Statistics,
Macmillan Co., 1970.

2. J. Freund, Mathematical Statistics, Prentice-Hall, 1971.

3. This program is a translation of the HP-65 Stat Pac 2 program.

User Instructions

7



TEST STATISTICS FOR THE
CORRELATION COEFFICIENT

r n ►t ρ_0 ►z

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program		<input type="text"/> <input type="text"/>	
2	Input r and n in any order	r	<input type="text"/> A <input type="text"/>	
		n	<input type="text"/> B <input type="text"/>	
3	Compute t		<input type="text"/> C <input type="text"/>	t
	or		<input type="text"/> <input type="text"/>	
	Input ρ_0 and compute z	ρ_0	<input type="text"/> D <input type="text"/>	
			<input type="text"/> E <input type="text"/>	z
4	For a new case, go to 2		<input type="text"/> <input type="text"/>	

97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Store r	057	=	-24	
002	STO1	35 01		058	RTN	24	
003	*LBLB	21 00		060			
004	ABS	16 31					
005	1	01					
006	X<Y	-41					
007	X>Y?	16-34					
008	GT09	22 09					
009	LSTX	16-63					
010	RTN	24					
011	*LBLB	21 12	Store n				
012	STO2	35 02					
013	3	03		070			
014	X<Y	-41					
015	X<Y?	16-35					
016	GT09	22 09					
017	RTN	24					
018	*LBLC	21 13					
019	RCL2	36 02					
020	2	02					
021	-	-45	n-2				
022	1	01					
023	RCL1	36 01		080			
024	X ²	53					
025	-	-45					
026	=	-24					
027	JX	54					
028	RCL1	36 01					
029	X	-35					
030	RTN	24					
031	*LBLD	21 14	Store ρ_0				
032	STO3	35 03					
033	GT00	22 00		090			
034	*LBLE	21 15					
035	RCL1	36 01					
036	1	01					
037	+	-55					
038	1	01					
039	RCL1	36 01					
040	-	-45					
041	=	-24	(1+4)/(1-r)				
042	1	01					
043	RCL3	36 03		100			
044	-	-45					
045	X	-35					
046	1	01					
047	RCL3	36 03					
048	+	-55					
049	=	-24					
050	LN	32					
051	RCL2	36 02	n-3				
052	3	03					
053	-	-45		110			
054	JX	54					
055	X	-35					
056	2	02					

SET STATUS

FLAGS

TRIG

DISP

ON OFF

0 ☐ ☒1 ☐ ☒2 ☐ ☒3 ☐ ☒DEG ☒GRAD ☐RAD ☐FIX ☒SCI ☐ENG ☐n 2

REGISTERS

0	1	2	3	4	5	6	7	8	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

Program Description I

Program Title	DIFFERENCES AMONG PROPORTIONS		
Contributor's Name	Hewlett-Packard		
Address	1000 N.E. Circle Blvd.		
City	Corvallis	State	Oregon
		Zip Code	97330

Program Description, Equations, Variables

Suppose x_1, x_2, \dots, x_k are observed values of a set of independent random variables having binomial distributions with parameters n_i and θ_i ($i = 1, 2, \dots, k$).

A chi-square statistic χ^2 can be used to test the null hypothesis $\theta_1 = \theta_2 = \dots = \theta_k$. The χ^2 statistic has the chi-square distribution with $k - 1$ degrees of freedom.

Equation:

$$\chi^2 = \sum_{i=1}^k \frac{(x_i - n_i \hat{\theta})^2}{n_i \hat{\theta} (1 - \hat{\theta})} = \sum_{i=1}^k n_i \left[\frac{1}{\sum_{i=1}^k x_i} \sum_{i=1}^k \frac{x_i^2}{n_i} + \frac{1}{\sum_{i=1}^k (n_i - x_i)} \sum_{i=1}^k \frac{(n_i - x_i)^2}{n_i} - 1 \right]$$

where

$$\hat{\theta} = \frac{\sum_{i=1}^k x_i}{\sum_{i=1}^k n_i}$$

Operating Limits and Warnings

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Program Description II

[illegible]

Sample Problem(s)		
Example:		
	n_i	x_i
Sample 1	400	232
Sample 2	500	260
Sample 3	400	197

Keystrokes:


A 400 ↓ 232 B 500 ↓ 260 B 400 ↓ 197 B →	3.00 (k)
C →	6.47 (χ^2)
D →	2.00 (df)
E →	0.53 ($\hat{\theta}$)

[illegible]

Reference(s) 1. J. Freund, Mathematical Statistics, Prentice-Hall, 1971.

2. This program is a translation of the HP-65 Stat Pac 2 program.

User Instructions



DIFFERENCES
AMONG PROPORTIONS

INIT $n_i \nabla x_i \triangleright i$ $\triangleright \chi^2$ $\triangleright df$ $\triangleright \hat{\theta}$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program		<div></div> <div></div>	
2	Initialize		<div>A</div> <div></div>	0.00
3	Perform 3 for $i = 1, 2, \dots, k$	n_i	<div>↑</div> <div></div>	
		x_i	<div>B</div> <div></div>	i
4	Compute χ^2 statistic		<div>C</div> <div></div>	χ^2
5	Compute df		<div>D</div> <div></div>	df
6	Compute $\hat{\theta}$		<div>E</div> <div></div>	$\hat{\theta}$
7	For a new case, go to 2		<div></div> <div></div>	

97 Program Listing I

12 STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Initialize				
002	CLRG	16-53					
003	0	00					
004	RTN	24		060			
005	*LBLB	21 12	Accumulate sums				
006	ST+1	35-55 01					
007	-	-45					
008	ST04	35 04					
009	ST+2	35-55 02	$n_i - x_i$				
010	LSTX	16-63					
011	+	-55					
012	LSTX	16-63	x_i				
013	ENT↑	-21	n_i	070			
014	x	-35					
015	XZY	-41	x_i^2				
016	÷	-24	x_i^2/n_i				
017	ST+5	35-55 05					
018	LSTX	16-63	n_i				
019	RCL4	36 04	$(n_i - x_i)^2$				
020	ENT↑	-21					
021	x	-35					
022	XZY	-41					
023	÷	-24		080			
024	ST+6	35-55 06	Compute χ^2				
025	1	01					
026	RCL3	36 03					
027	+	-55					
028	ST03	35 03					
029	RTN	24					
030	*LBLC	21 13					
031	RCL5	36 05					
032	RCL1	36 01					
033	÷	-24		090			
034	RCL6	36 06					
035	RCL2	36 02					
036	÷	-24					
037	+	-55					
038	1	01					
039	-	-45					
040	RCL1	36 01					
041	RCL2	36 02					
042	+	-55					
043	x	-35					
044	RTN	24		100			
045	*LBLD	21 14	Compute df				
046	RCL3	36 03					
047	1	01					
048	-	-45					
049	RTN	24					
050	*LBL E	21 15	Compute θ				
051	RCL1	36 01					
052	RCL1	36 01					
053	RCL2	36 02					
054	+	-55		110			
055	÷	-24					
056	RTN	24					

REGISTERS									
0	1	2	3	4	5	6	7	8	9
	Σx_i	$\Sigma (n_i - x_i)$	k	$n_i - x_i$	$\Sigma (x_i^2/n_i)$	$\Sigma (n_i - x_i)^2/n_i$			
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

SET STATUS		
FLAGS	TRIG	DISP
ON OFF		
0 <input type="checkbox"/> <input checked="" type="checkbox"/>	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
1 <input type="checkbox"/> <input checked="" type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
2 <input type="checkbox"/> <input checked="" type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
3 <input type="checkbox"/> <input checked="" type="checkbox"/>		n <u>2</u>

Program Description I

Program Title	BEHRENS-FISHER STATISTIC		
Contributor's Name	Hewlett-Packard		
Address	1000 N.E. Circle Blvd.		
City	Corvallis	State	Oregon
		Zip Code	97330

Program Description, Equations, Variables

Suppose $\{x_1, x_2, \dots, x_{n_1}\}$ and $\{y_1, y_2, \dots, y_{n_2}\}$ are independent random samples from two normal populations having means μ_1, μ_2 (unknown). If the variances σ_1^2, σ_2^2 cannot be assumed equal, then the Behrens-Fisher statistic d is used instead of the t statistic to test the null hypothesis

$$H_0: \mu_1 - \mu_2 = D.$$

Equation:

$$d = \frac{\bar{x} - \bar{y} - D}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

where \bar{x}, \bar{y} and s_1^2, s_2^2 are sample means and variances.

Critical values of this test are tabulated in the Fisher-Yates Tables for various values of n_1, n_2, α and θ , where α is the level of significance and

$$\theta = \tan^{-1} \left(\frac{s_1}{s_2} \sqrt{\frac{n_2}{n_1}} \right).$$

Operating Limits and Warnings

Remark:

$$n_1 > 1, n_2 > 1.$$

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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[illegible]

Sample Problem(s)

Example:

Compute the Behrens-Fisher statistic for $D = 0$.

x: 79, 84, 108, 114, 120, 103, 122, 120

y: 91, 103, 90, 113, 108, 87, 100, 80, 99, 54

Keystrokes:

[f] [A] 79 [A] 84 [A] ... 120 [A] → 8.00 (n_1)

[B] → 34.60 (s_1^2/n_1)

[f] [A] 91 [A] 103 [A] ... 54 [A] → 10.00 (n_2)

0 [C] [D] → 1.73 (d)

[E] → 47.88° (θ)

or 0.84 radians

or 53.20 grads

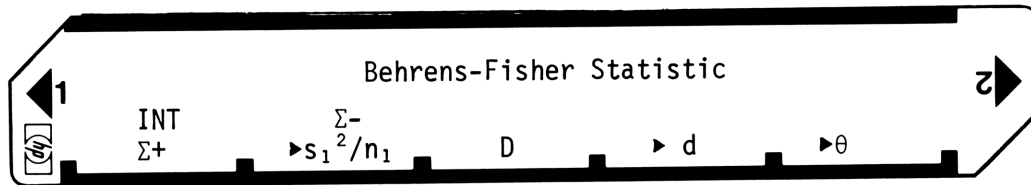
Solution(s)

Reference(s) 1. Fisher and Yates, Statistical Tables for Biological, Agricultural and Medical Research, Hafner, Publishing Co., 1970.

2. This program is a translation of the HP-65 Stat Pac 2 program.

User Instructions

15



STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program		<input type="text"/> <input type="text"/>	
2	If \bar{x} , \bar{y} and s_1^2 , s_2^2 are known, go		<input type="text"/> <input type="text"/>	
	to 11		<input type="text"/> <input type="text"/>	
3	Initialize		<input type="text"/> f <input type="text"/> A	0.00
4	Perform 4 for $i = 1, 2, \dots, n_1$	x_i	<input type="text"/> A <input type="text"/>	i
5	Optional—delete erroneous x_k	x_k	<input type="text"/> f <input type="text"/> B	
	($k \neq 1$)		<input type="text"/> <input type="text"/>	
6	Compute and store \bar{x} , s_1^2/n_1		<input type="text"/> B <input type="text"/>	s_1^2/n_1
7	Initialize		<input type="text"/> f <input type="text"/> A	0.00
8	Perform 8 for $i = 1, 2, \dots, n_2$	y_i	<input type="text"/> A <input type="text"/>	i
9	Optional—delete erroneous y_h	y_h	<input type="text"/> f <input type="text"/> B	
	($h \neq 1$)		<input type="text"/> <input type="text"/>	
10	Go to 12		<input type="text"/> <input type="text"/>	
11	Store \bar{x} , \bar{y} and s_1^2/n_1 , s_2^2/n_2		<input type="text"/> <input type="text"/>	
	in any order	\bar{x}	<input type="text"/> STO <input type="text"/> 5	
		s_1^2/n_1	<input type="text"/> STO <input type="text"/> 6	
		\bar{y}	<input type="text"/> STO <input type="text"/> 2	
		s_2^2/n_2	<input type="text"/> STO <input type="text"/> 3	
12	Input D	D	<input type="text"/> C <input type="text"/>	
13	Compute d and θ		<input type="text"/> D <input type="text"/>	d
			<input type="text"/> E <input type="text"/>	θ
14	Optional—recall means		<input type="text"/> RCL <input type="text"/> 5	\bar{x}
			<input type="text"/> RCL <input type="text"/> 2	\bar{y}
15	For a different D, go to 12		<input type="text"/> <input type="text"/>	
16	For a new case, go to 2		<input type="text"/> <input type="text"/>	

97 Program Listing I

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STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 16 11	Initialize	057	RTN	24	Compute d
002	0	00		058	*LBLD	21 14	
003	ST01	35 01		059	RCL5	36 05	
004	ST02	35 02		060	RCL2	36 02	
005	ST03	35 03		061	-	-45	
006	ST04	35 04	Accumulate sums	062	RCL7	36 07	Store s_2^2/n_2
007	RTN	24		063	-	-45	
008	*LBLA	21 11		064	RCL3	36 03	
009	RCL2	36 02		065	RCL1	36 01	
010	-	-45		066	1	01	
011	RCL4	36 04		067	-	-45	
012	-	-45		068	÷	-24	
013	RCL1	36 01		069	RCL1	36 01	
014	1	01		070	÷	-24	
015	+	-55		071	ST08	35 08	Compute θ
016	÷	-24	Store s_2^2/n_2	072	RCL6	36 06	
017	ENT↑	-21		073	+	-55	
018	ENT↑	-21		074	JX	54	
019	RCL4	36 04		075	÷	-24	
020	+	-55		076	RTN	24	Delete erroneous data
021	ENT↑	-21		077	*LBLA	21 15	
022	ENT↑	-21		078	RCL6	36 06	
023	RCL2	36 02		079	RCL8	36 08	
024	+	-55		080	÷	-24	
025	ST02	35 02	Store \bar{x}	081	JX	54	Delete erroneous data
026	LSTX	16-63		082	TAN↑	16 43	
027	-	-45		083	RTN	24	
028	-	-45		084	*LBLA	21 16 12	
029	ST04	35 04		085	RCL1	36 01	
030	R↓	-31		086	CHS	-22	
031	x	-35		087	ST01	35 01	
032	RCL1	36 01		088	R↓	-31	
033	x	-35		089	GTOA	22 11	
034	1	01	$n_i - 1$	090			
035	LSTX	16-63					
036	+	-55					
037	ABS	16 31					
038	ST01	35 01					
039	x	-35					
040	ST+3	35-55 03					
041	RCL1	36 01					
042	RTN	24					
043	*LBLB	21 12					
044	RCL2	36 02	Store s_1^2/n_1	100			
045	ST05	35 05					
046	RCL3	36 03					
047	RCL1	36 01					
048	1	01					
049	-	-45					
050	÷	-24					
051	RCL1	36 01					
052	÷	-24					
053	ST06	35 06					
054	RTN	24	Store D	110			
055	*LBLC	21 13					
056	ST07	35 07					

REGISTERS

0	1	2	3	4	5	6	7	8	9
	n	running mean	sum of squares	Used	\bar{x}	s_1^2/n_1	D	s_2^2/n_2	
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

SET STATUS					
FLAGS		TRIG		DISP	
0	<input type="checkbox"/> ON <input checked="" type="checkbox"/> OFF	DEG	<input checked="" type="checkbox"/>	FIX	<input checked="" type="checkbox"/>
1	<input type="checkbox"/> <input checked="" type="checkbox"/>	GRAD	<input type="checkbox"/>	SCI	<input type="checkbox"/>
2	<input type="checkbox"/> <input checked="" type="checkbox"/>	RAD	<input type="checkbox"/>	ENG	<input type="checkbox"/>
3	<input type="checkbox"/> <input checked="" type="checkbox"/>			n_{-2}	

Program Description I

Program Title **KRUSKAL-WALLIS STATISTIC**

Contributor's Name **Hewlett-Packard**

Address **1000 N.E. Circle Blvd.**

City **Corvallis** State **Oregon** Zip Code **97330**

Program Description, Equations, Variables

Suppose we want to test the null hypothesis that k independent random samples of sizes n_1, n_2, \dots, n_k come from identical continuous populations.

Arrange all values from k samples jointly (as if they were one sample) in an increasing order of magnitude. Let R_{ij} ($i = 1, 2, \dots, k, j = 1, 2, \dots, n_i$) be the rank of the j^{th} value in the i^{th} sample.

The Kruskal-Wallis statistic H can be used to test the null hypothesis.

When all sample sizes are large (> 5), H is distributed approximately as the chi-square with $k - 1$ degrees of freedom. For small samples, the test is based on special tables.

Equation:

$$H = \frac{12}{N(N+1)} \sum_{i=1}^k \frac{\left(\sum_{j=1}^{n_i} R_{ij} \right)^2}{n_i} - 3(N+1)$$

where

$$N = \sum_{i=1}^k n_i$$

Operating Limits and Warnings

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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Program Description I

Program Title	KRUSKAL-WALLIS STATISTIC		
Contributor's Name	Hewlett-Packard		
Address	1000 N.E. Circle Blvd.		
City	Corvallis	State	Oregon Zip Code 97330

Program Description, Equations, Variables	
<p>Table for small samples ($k = 3$):</p> <p>Alexander and Quade, <i>On the Kruskal-Wallis Three Sample H-statistic</i>, University of North Carolina, Department of Biostatistics, Inst. Statistics Mimeo Ser. 602, 1968.</p>	
<p>Operating Limits and Warnings</p>	

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A full-page sheet of white graph paper featuring a uniform grid of small squares. The grid consists of 20 columns and 15 rows of squares, creating a total of 300 square units. The lines are thin and black, forming a continuous pattern across the entire page.

1. The first step in the process of creating a new product is to identify a market need. This involves conducting market research to understand the preferences and behaviors of potential customers. Once a need is identified, the next step is to develop a concept that addresses this need. This concept should be unique and offer a clear value proposition to the target market.

2. After developing a concept, the next step is to create a prototype. A prototype is a preliminary model of the product that allows the company to test its functionality and gather feedback from potential users. This step is crucial for identifying any design flaws or usability issues before moving forward with full-scale production.

3. Once a prototype is developed, the company must conduct a feasibility study. This study evaluates the technical, financial, and operational aspects of the product. It helps the company determine if the product is viable and if the resources are available to bring it to market. If the study is successful, the company can proceed to the next step: securing funding.

4. Securing funding is a critical step in the product development process. This can be done through various means, including venture capital, angel investors, crowdfunding, or bank loans. Each option has its own requirements and risks, so the company must carefully evaluate its options and choose the most appropriate funding source for its needs.

5. After securing funding, the company can move forward with the development and production of the product. This involves hiring a team of engineers and designers to create the final product, as well as establishing a manufacturing process. The company must also consider distribution channels and marketing strategies to ensure the product reaches its target audience.

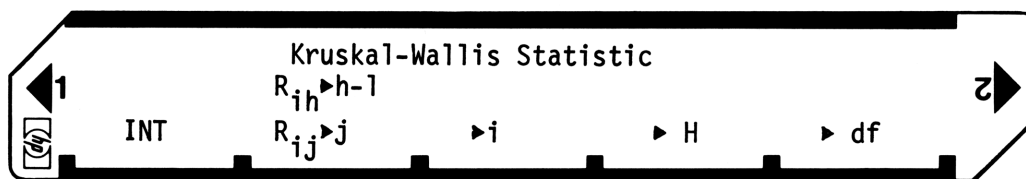
6. Finally, the product is launched into the market. The company must monitor sales and customer feedback closely to ensure the product is meeting expectations and making a positive impact. If necessary, the company may need to make adjustments to the product or its marketing strategy to improve its performance in the market.

		Ranks R_{ij}									
		1	2	3	4	5	6	7	8	9	10
i \ j	1	29	5	26	10	33	30				
	2	11	12	9	7	20	18	19	21		
	3	14	28	8	25	17	15	32	4	2	
	4	6	27	3	16	24	13	1	31	22	23

A 29 **B** 5 **B** ... 30 **B** → 6.00
C → 1.00
 11 **B** 12 **B** ... 21 **B** **C** → 2.00
 14 **B** 28 **B** ... 2 **B** **C** → 3.00
 6 **B** 27 **B** ... 23 **B** **C** → 4.00
D → 2.29 (H)
E → 3.00 (df)

1. W.J. Conover, Practical Nonparametric Statistics, John Wiley and Sons, 1971.
2. This program is a translation of the HP-65 Stat Pac 2 program.

User Instructions

[illegible]

User Instructions

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•KRUSKAL-WALLIS STATISTIC
 INIT $R_{ij} \triangleright j$ $\triangleright i$ $\triangleright H$ $\triangleright df$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program		<input type="text"/> <input type="text"/>	
2	Initialize		<input type="text"/> A <input type="text"/>	0.00
3	Perform 3-6 for $i = 1, 2, \dots, k$		<input type="text"/> <input type="text"/>	
4	Perform 4 for $j = 1, 2, \dots, n_i$	R_{ij}	<input type="text"/> B <input type="text"/>	j
5	Optional—delete erroneous R_{ih}	R_{ih}	<input type="text"/> GTO <input type="text"/> 1	
			<input type="text"/> R/S <input type="text"/>	
6	End of the i th sample		<input type="text"/> C <input type="text"/>	i
7	Compute H statistic		<input type="text"/> D <input type="text"/>	H
8	Compute df		<input type="text"/> E <input type="text"/>	df
9	Optional—recall N		<input type="text"/> RCL <input type="text"/> 5	N
10	For a new case, go to 2		<input type="text"/> <input type="text"/>	

97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS		
001	*LBLA	21 11	Initialize						
002	CLRG	16-53							
003	0	00		060					
004	RTN	24							
005	*LBLB	21 12							
006	ST+2	35-55 02	Accumulate sums						
007	RCL1	36 01							
008	1	01							
009	+	-55							
010	ST01	35 01							
011	RTN	24	Prepare for a new sample						
012	*LBLC	21 13							
013	RCL1	36 01		070					
014	ST+5	35-55 05							
015	RCL2	36 02							
016	X ²	53	$(\sum R_{ij})^2$						
017	X*Y	-41							
018	=	-24							
019	ST+3	35-55 03							
020	RCL4	36 04							
021	1	01	Reinitialize registers R ₁ , R ₂						
022	+	-55							
023	ST04	35 04		080					
024	0	00							
025	ST01	35 01							
026	ST02	35 02	Display sample number						
027	RCL4	36 04							
028	RTN	24							
029	*LBLD	21 14							
030	RCL3	36 03							
031	4	04	Compute H						
032	x	-35							
033	RCL5	36 05		090					
034	=	-24							
035	RCL5	36 05							
036	1	01							
037	+	-55							
038	=	-24							
039	LSTX	16-63							
040	-	-45							
041	3	03	N + 1						
042	x	-35							
043	RTN	24							
044	*LBLE	21 15		100					
045	RCL4	36 04							
046	1	01	Compute df						
047	-	-45							
048	RTN	24							
049	*LBL6	21 16 12							
050	ST-2	35-45 02							
051	RCL1	36 01	Delete erroneous data						
052	1	01							
053	-	-45		110					
054	ST01	35 01							
055	RTN	24							
<div>SET STATUS</div> <div><div>FLAGS</div><div>ON OFF</div><div>0 <input type="checkbox"/> <input checked="" type="checkbox"/> 1 <input type="checkbox"/> <input checked="" type="checkbox"/> 2 <input type="checkbox"/> <input checked="" type="checkbox"/> 3 <input type="checkbox"/> <input checked="" type="checkbox"/></div></div> <div><div>TRIG</div><div>DEG <input checked="" type="checkbox"/> GRAD <input type="checkbox"/> RAD <input type="checkbox"/></div></div> <div><div>DISP</div><div>FIX <input checked="" type="checkbox"/> SCI <input type="checkbox"/> ENG₂ <input type="checkbox"/> n _____</div></div>									
REGISTERS									
0	1	2	3	4	5	6	7	8	9
n_i	$\sum R_{ij}$	$\sum [(\sum R_{ij})^2]$	k	N					
S0	S1	S2	n_i	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

Program Description I

Program Title

MEAN-SQUARE SUCCESSIVE DIFFERENCE

Contributor's Name

Hewlett-Packard

Address

1000 N.E. Circle Blvd.

City

Corvallis

State

Oregon

Zip Code

97330

Program Description, Equations, Variables

When test and estimation techniques are used, the method of drawing the sample from the population is specified to be random in most cases. If observations are chosen in sequence x_1, x_2, \dots, x_n , the mean-square successive difference η can be used to test for randomness.

If the sample size n is large (say, greater than 20) and the population is normal, then a z statistic has approximately the standard normal distribution. Long trends are associated with large positive values of z and short oscillations with large negative values.

Equations:

$$\eta = \sum_{i=2}^n (x_i - x_{i-1})^2 \bigg/ \sum_{i=1}^n (x_i - \bar{x})^2$$

$$= \sum_{i=2}^n (x_i - x_{i-1})^2 \bigg/ \left[\sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i \right)^2}{n} \right]$$

$$z = \frac{1 - \eta/2}{\sqrt{\frac{n-2}{n^2 - 1}}}$$

Operating Limits and Warnings

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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[illegible]

Example:

Find the mean-square successive difference for the following set of data:

{0.53, 0.52, 0.39, 0.49, 0.97, 0.29, 0.65, 0.30, 0.40, 0.06, 0.14, 0.16, 0.68, 0.22, 0.68, 0.08, 0.52, 0.50, 0.63, 0.20, 0.67, 0.44, 0.64, 0.40, 0.97, 0.03, 0.73, 0.24, 0.57, 0.35 }

Keystrokes:

A .53 **B** → 1.00

.52 **C** .39 **C**35 **C** → 30.00

D → 2.81 (η)

E → -2.29 (z)

-E \longrightarrow **-2.29 (z)**

Reference(s) 1. This program is a translation of the HP-65 Stat Pac 2 program.
2. Dixon and Massey, Introduction to Statistical Analysis, McGraw-Hill, 1969.

User Instructions

MEAN-SQUARE
SUCCESSIVE DIFFERENCE

INIT x_1 x_i ▶ i ▶ η ▶ z

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program		<div></div> <div></div>	
2	Initialize		<div>A</div> <div></div>	0.00
3	Input x_1	x_1	<div>B</div> <div></div>	1.00
4	Perform 4 for $i = 2, 3, \dots, n$	x_i	<div>C</div> <div></div>	i
5	Compute η		<div>D</div> <div></div>	η
6	Compute z		<div>E</div> <div></div>	z
7	For a new case, go to 2		<div></div> <div></div>	

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS		
001	*LBLA	21 11	Initialize						
002	CLRG	16-53							
003	0	00							
004	RTN	24		060					
005	*LBLB	21 12							
006	ST05	35 05		Store x_1 and accumulate sums x_1^2					
007	ST+2	35-55 02							
008	ENT↑	-21							
009	x	-35							
010	ST+3	35-55 03							
011	RCL1	36 01							
012	1	01							
013	+	-55							
014	ST01	35 01	070						
015	RTN	24							
016	*LBLC	21 13	Store x_i						
017	RCL5	36 05							
018	XZY	-41							
019	ST05	35 05							
020	-	-45		$x_i - x_{i-1}$					
021	ENT↑	-21							
022	x	-35							
023	ST+4	35-55 04							
024	RCL5	36 05			080				
025	GT0B	22 12							
026	*LBLD	21 14							
027	RCL4	36 04							
028	RCL3	36 03							
029	RCL2	36 02							
030	ENT↑	-21	Compute η $(\sum x_i)^2$						
031	x	-35							
032	RCL1	36 01							
033	÷	-24							
034	-	-45		090					
035	÷	-24		η Compute z					
036	ST05	35 05							
037	RTN	24							
038	*LBLE	21 15							
039	1	01							
040	RCL5	36 05							
041	2	02							
042	÷	-24							
043	-	-45							
044	RCL1	36 01							
045	2	02	Z						
046	-	-45							
047	RCL1	36 01							
048	ENT↑	-21							
049	x	-35							
050	1	01							
051	-	-45							
052	÷	-24							
053	JX	54							
054	÷	-24							
055	RTN	24							

LABELS									
A	INT	B	x_1	C	x_i	D	η	E	z
a		b		c		d		e	
0		1		2		3		4	
5		6		7		8		9	

		FLAGS		SET STATUS		
				FLAGS	TRIG	DISP
	0					
	1		ON OFF			
	2	0	<input type="checkbox"/>	DEG	<input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
	3	1	<input type="checkbox"/>	GRAD	<input type="checkbox"/>	SCI <input type="checkbox"/>
		2	<input type="checkbox"/>	RAD	<input type="checkbox"/>	ENG <input type="checkbox"/>
		3	<input type="checkbox"/>			n <u>2</u>

REGISTERS									
0	1	2	3	4	5	6	7	8	9
	n	$\sum x_i$	$\sum x_i^2$	$\sum (x_i - x_{i-1})^2$	$x_{i,\eta}$				
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

Program Description I

Program Title	THE RUN TEST FOR RANDOMNESS		
Contributor's Name	Hewlett-Packard		
Address	1000 N.E. Circle Blvd.		
City	Corvallis	State	Oregon
		Zip Code	97330

Program Description, Equations, Variables

Consider a sequence of symbols such that the symbols are of two types only. A run is a continuous string of identical symbols preceded and followed by a different symbol or no symbol at all. For example, the sequence 1110100011 has five runs.

Let the total number of runs in a given sequence be u , and let n_1 and n_2 represent the number of symbols of type 1 and type 2 respectively. If the sample sizes are large (say, n_1 and n_2 are both greater than 10), then the randomness of the sequence may be tested using a z statistic which has the standard normal distribution.

Equations:

The sample distribution of the run has the mean μ and the standard deviation σ .

$$\mu = \frac{2 n_1 n_2}{n_1 + n_2} + 1$$

$$\sigma = \sqrt{\frac{2 n_1 n_2 (2 n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}}$$

The test is based on the statistic

$$z = \frac{u - \mu}{\sigma}$$

Operating Limits and Warnings

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Program Description I

Program Title **THE RUN TEST FOR RANDOMNESS**

Contributor's Name **Hewlett-Packard**

Address **1000 N.E. Circle Blvd.**

City **Corvallis** State **Oregon** Zip Code **97330**

Program Description, Equations, Variables

Remarks:

1. For small samples, the test is based on special tables.
2. This program can also be used for other tests involving runs. For example, one might want to test runs of scores above and below the median based on the order in which the scores were obtained. In this case, a sequence could be constructed in which each score would be replaced by a 1 if it was above the median or a 0, if below the median.
The run test for randomness can then be applied to the sequence of 0's and 1's.

Another use might be for Wald-Wolfowitz run test, which tests the null hypothesis that two random samples have been drawn from identical populations. The data from both groups are combined into one sequence according to magnitude. Each value may be assigned a 0 or 1 depending on which population it came from, and the run test for randomness then performed on the resulting sequence.

Operating Limits and Warnings

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Program Description II

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Sketch(es)

Sample Problem(s)

Example:

A statistician sits by the roulette table one night in a Las Vegas casino, suspiciously watching the house rake in stake upon stake. To test the null hypothesis that the sequence of numbers is random, the statistician observes the following sequence of red (R) and black (B) numbers (ignoring 0 and 00):

RRRR B RRR BBBB RR BBB RR BB RRR

In the sequence are 14 R's, 11B's, and a total of 9 runs. Find the mean and standard deviation of the sampling distribution and the z statistic.

Keystrokes:

14 \uparrow 11 **A** 9 **B** **C** \longrightarrow 13.32 (μ)

D \longrightarrow 2.41 (σ)

E \longrightarrow -1.79 (z)


(His suspicion is not entirely unjustified.)

Solution(s)

Reference(s) 1. Freund and Williams, Dictionary/Outline of Basic Statistics, McGraw-Hill, 1966.

2. This program is a translation of the HP-65 Stat Pac 2 program.

User Instructions



THE RUN TEST FOR RANDOMNESS

$n_1 \uparrow n_2$ u $\triangleright \mu$ $\triangleright \sigma$ $\triangleright z$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program		<input type="text"/>	<input type="text"/>	
2	Input		<input type="text"/>	<input type="text"/>	
	number of symbols of type 1	n_1	<input type="text" value="↑"/>	<input type="text"/>	
	number of symbols of type 2	n_2	<input type="text" value="A"/>	<input type="text"/>	n_1
3	Input number of runs	u	<input type="text" value="B"/>	<input type="text"/>	u
4	Compute the mean		<input type="text" value="C"/>	<input type="text"/>	μ
5	Compute the standard de-		<input type="text"/>	<input type="text"/>	
	viation		<input type="text" value="D"/>	<input type="text"/>	σ
6	Compute the z statistic		<input type="text" value="E"/>	<input type="text"/>	z
7	For a new case, go to 2		<input type="text"/>	<input type="text"/>	

97 Program Listing I

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STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11					
002	ST02	35 02					
003	R↓	-31	Store n_2				
004	ST01	35 01	Store n_1	060			
005	RTN	24					
006	*LBLB	21 12					
007	ST03	35 03	Store u				
008	RTN	24					
009	*LBLC	21 13					
010	RCL1	36 01	Compute the mean				
011	RCL2	36 02					
012	X	-35					
013	2	02					
014	X	-35		070			
015	ST07	35 07					
016	RCL1	36 01					
017	RCL2	36 02					
018	+	-55					
019	ST08	35 08					
020	÷	-24					
021	1	01					
022	+	-55					
023	ST04	35 04					
024	RTN	24		080			
025	*LBLD	21 14					
026	RCL7	36 07	Compute the				
027	RCL8	36 08	standard deviation				
028	-	-45					
029	RCL7	36 07					
030	X	-35	$(n_1+n_2)^2$				
031	RCL8	36 08					
032	ENT↑	-21					
033	X	-35					
034	RCL8	36 08		090			
035	1	01	n_1+n_2-1				
036	-	-45					
037	X	-35					
038	÷	-24					
039	JX	54					
040	ST05	35 05					
041	RTN	24					
042	*LBLE	21 15					
043	RCL3	36 03	Compute the z				
044	RCL4	36 04	statistic	100			
045	-	-45					
046	RCL5	36 05					
047	÷	-24					
048	ST06	35 06					
049	RTN	24					

LABELS					FLAGS		SET STATUS		
A	B	C	D	E	0	1	2	3	4
n_1	n_2	u	μ	σ	z				
a	b	c	d	e					
0	1	2	3	4					
5	6	7	8	9					
REGISTERS									
0	1	2	3	4	5	6	7	8	9
n_1	n_2	u	μ	σ	z	zn_1n_2	n_1+n_2		
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

Program Description I

Program Title **INTRACCLASS CORRELATION COEFFICIENT**

Contributor's Name Hewlett-Packard

Address 1000 N.E. Circle Blvd.

City Corvallis

State Oregon

Zip Code 97330

Program Description, Equations, Variables

The intraclass correlation coefficient r_I measures the degree of association among individuals within classes or groups.

		Observations			
Groups	1	x_{11}	x_{12}	...	x_{1n}
	2	x_{21}	x_{22}	...	x_{2n}

	k	x_{k1}	x_{k2}	...	x_{kn}

The coefficient is most easily calculated using the analysis of variance techniques. r_I is the sample estimate of the population intraclass correlation coefficient ρ_I . If we can assume that the individuals within groups are random samples from normal populations with the same variance, then the hypothesis $\rho_I = 0$ can be tested using the F statistic.

Equations:

1. Sums
Group $T_i = \sum_{j=1}^n x_{ij} \quad i = 1, 2, \dots, k$

Total $T = \sum_{i=1}^k T_i$

2. Sums of squares
Mean

$$MSS = T^2 / k n$$

Operating Li

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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Program Description I

Program Title INTRAClass CORRELATION COEFFICIENT

Contributor's Name Hewlett-Packard

Address 1000 N.E. Circle Blvd.

City Corvallis

State Oregon

Zip Code 97330

Program Description, Equations, Variables

Among groups

$$ASS = \sum_{i=1}^k T_i^2 / n - MSS$$

Within groups

$$WSS = \sum_{i=1}^k \sum_{j=1}^n x_{ij}^2 - MSS - ASS$$

3. Intraclass correlation coefficient

$$r_1 = \left(\frac{ASS}{k-1} - \frac{WSS}{k(n-1)} \right) / \left(\frac{ASS}{k-1} + \frac{WSS}{k} \right)$$

4. F statistic

$$F = \frac{ASS}{k-1} / \frac{WSS}{k(n-1)}$$

with $df_1 = k - 1$ and $df_2 = k(n - 1)$ degrees of freedom.

Operating Limits and Warnings

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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User Instructions

•INTRACLASS
CORRELATION COEFFICIENT

x_{ij}

$\triangleright T_i$

$\triangleright r_1$

$\triangleright F$

$\triangleright df_1, \triangleright df_2$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program		<div></div> <div></div>	
2	Initialize		<div>f</div> <div>REG</div>	
3	Perform 3-5 for $i = 1, 2, \dots, k$		<div></div> <div></div>	
4	Perform 4 for $j = 1, 2, \dots, n$	x_{ij}	<div>A</div> <div></div>	j
5	Compute the group mean		<div>B</div> <div></div>	T_i
6	Compute the coefficient		<div>C</div> <div></div>	r_1
7	Compute the F statistic		<div>D</div> <div></div>	F
8	Compute the degrees of freedom		<div>E</div> <div></div>	df_1
			<div>E</div> <div></div>	df_2
9	For a new case, go to 2		<div></div> <div></div>	

97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	x_{ij}	057	RCL8	36 08	Display r_1
002	ST+6	35-55 06		058	+	-55	
003	x^2	53		059	=	-24	
004	ST+5	35-55 05	Increase counter	060	RTN	24	Compute F
005	1	01		061	*LBLD	21 14	
006	RCL1	36 01		062	RCL7	36 07	
007	+	-55	Compute group sum	063	RCL8	36 08	Compute degrees of freedom
008	ST01	35 01		064	RCL1	36 01	
009	RTN	24		065	=	-24	
010	*LBLB	21 12	Reinitialize	066	=	-24	
011	RCL6	36 06		067	RTN	24	
012	ST08	35 08		068	*LBLB	21 15	
013	ST+3	35-55 03	Display sum	069	RCL2	36 02	
014	x^2	53		070	1	01	
015	ST+4	35-55 04	Compute coefficient	071	-	-45	
016	RCL1	36 01		072	R/8	51	
017	ST07	35 07		073	*LBLB	21 15	
018	0	00	ASS/k-1	074	RCL1	36 01	
019	ST01	35 01		075	RCL2	36 02	
020	ST06	35 06		076	x	-35	
021	1	01		077	RTN	24	
022	RCL2	36 02					
023	+	-55		080			
024	ST02	35 02					
025	RCL8	36 08					
026	RTN	24					
027	*LBLC	21 13					
028	RCL4	36 04					
029	RCL3	36 03					
030	x^2	53					
031	RCL2	36 02					
032	=	-24					
033	-	-45					
034	RCL7	36 07		090			
035	ST01	35 01					
036	=	-24					
037	RCL2	36 02					
038	1	01					
039	-	-45					
040	=	-24					
041	ST07	35 07					
042	RCL5	36 05					
043	RCL4	36 04					
044	RCL1	36 01					
045	=	-24					
046	-	-45					
047	RCL2	36 02					
048	=	-24					
049	ST08	35 08					
050	RCL1	36 01					
051	1	01					
052	-	-45					
053	ST01	35 01					
054	=	-24					
055	-	-45					
056	RCL7	36 07					

FLAGS				SET STATUS		
0	FLAGS			TRIG	DISP	
1	ON	OFF		DEG	<input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
2	<input type="checkbox"/>	<input checked="" type="checkbox"/>		GRAD	<input type="checkbox"/>	SCI <input type="checkbox"/>
3	<input type="checkbox"/>	<input checked="" type="checkbox"/>		RAD	<input type="checkbox"/>	ENG <input type="checkbox"/>
	3	<input checked="" type="checkbox"/>				n <u>2</u>

LABELS				
A	$\Sigma +$	B	T_i	C
a		b		c
0		1		2
5		6		7

REGISTERS									
0	1	2	3	4	5	6	7	8	9
	n-1	k	ΣT_i	ΣT_i^2	Σx_{ij}^2	T_i	ASS/k-1	T_i WSS/k	0
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

Program Description I

Program Title

FISHER'S EXACT TEST FOR A 2 x 2 CONTINGENCY TABLE

Contributor's Name Hewlett-Packard

Address 1000 N.E. Circle Blvd.

City Corvallis State Oregon Zip Code 97330

Program Description, Equations, Variables

Fisher's exact probability test is used for analyzing a 2 x 2 contingency table when the two independent samples are small in size.

a	b
c	d

Suppose a, b, c, d are the frequencies and a is the smallest frequency, this program computes the following:

1. The exact probability p_0 of observing the given frequencies in a 2 x 2 table, when the marginal totals are regarded as fixed.
2. The exact probability p_i ($i = 1, 2, \dots, a$) of each more extreme table having the same marginal totals.
3. The sum S_i of the probabilities of the first $i + 1$ tables.
4. The sum S of the probabilities of all tables with the same margins (i.e., $S = S_a$).

Equations:

$$1. \quad p_0 = \frac{(a+b)! (c+d)! (a+c)! (b+d)!}{N! a! b! c! d!}$$

where

$$N = a + b + c + d.$$

Operating Limits and Warnings

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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Program Description I

Program Title _____	FISHER'S EXACT TEST FOR A 2 x 2 CONTINGENCY TABLE		
Contributor's Name _____	Hewlett-Packard		
Address _____	1000 N.E. Circle Blvd.		
City _____	State _____	Zip Code _____	
Corvallis	Oregon	97330	

Program Description, Equations, Variables _____					
	<p>2. For the more extreme table (with the same margins)</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">$a - i$</td> <td style="padding: 5px;">$b + i$</td> </tr> <tr> <td style="padding: 5px;">$c + i$</td> <td style="padding: 5px;">$d - i$</td> </tr> </table> $p_i = \frac{(a+b)! (c+d)! (a+c)! (b+d)!}{N! (a-i)! (b+i)! (c+i)! (d-i)!}$ <p>where</p> <p style="margin-left: 100px;">i can be 1, 2, ... or a.</p> <p>3.</p> $S_n = \sum_{i=0}^n p_i$ <p>where</p> <p style="margin-left: 100px;">n can be 1, 2, ..., a.</p> <p>4.</p> $S = \sum_{i=0}^a p_i$	$a - i$	$b + i$	$c + i$	$d - i$
$a - i$	$b + i$				
$c + i$	$d - i$				
Operating Limits and Warnings _____					
<p>Remarks:</p> <ol style="list-style-type: none"> 1. a must be the smallest among the frequencies. Rearrange the table if necessary. 2. This program requires $N \leq 69$. However, Fisher's exact test is normally used for $N \leq 30$. 					

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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Program Description II

Sketch(es)

Sample Problem(s)

Example:

Compute p_0 , p_1 , p_2 , S_4 and S for the following table

7	10
8	5

Note:

The table must be rearranged as

5	8
10	7

Keystrokes:

5 $\boxed{+}$ 8 $\boxed{+}$ 10 $\boxed{+}$ 7 \boxed{A} \longrightarrow 0.16 (p_0)

\boxed{B} \longrightarrow 0.06 (p_1)

\boxed{B} \longrightarrow 0.01 (p_2)

\boxed{B} \boxed{B} \boxed{C} \longrightarrow 0.23 (S_4)

\boxed{D} \longrightarrow 0.23 (S)

Sol

Reference(s)

1. S. Siegel, *Nonparametric Statistics*, McGraw-Hill, 1956.
2. Sir R. A. Fisher, *Statistical Methods for Research Workers*, Oliver and Boyd, 1950.

This program is a translation of the HP-65 Stat Pac 2 program.

User Instructions



FISHER'S EXACT TEST FOR A
2 × 2 CONTINGENCY TABLE

a↕b↕
c↕d↕p₀

↗p_i

↗S_i

↗S

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter program		<div></div> <div></div>	
2	Enter frequencies and compute		<div></div> <div></div>	
	p ₀	a	<div>↑</div> <div></div>	
		b	<div>↑</div> <div></div>	
		c	<div>↑</div> <div></div>	
		d	<div>A</div> <div></div>	p ₀
3*	Optional—perform 3 or 3-4 for		<div></div> <div></div>	
	i = 1, 2,..., a		<div>B</div> <div></div>	p _i
4	Optional—recall current S _i		<div>C</div> <div></div>	S _i
5	Compute the sum of all		<div></div> <div></div>	
	probabilities		<div>D</div> <div></div>	S
6	For a new case, go to 2		<div></div> <div></div>	
			<div></div> <div></div>	
	* It is not necessary to com-		<div></div> <div></div>	
	plete the loop of 3 and 4. Go to		<div></div> <div></div>	
	5 for S when desired.		<div></div> <div></div>	

97 Program Listing I

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STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11		057	ST+3	35-55 03	
002	ST04	35 04		058	ST-4	35-45 04	
003	R4	-31	Stored, c,b,a	059	ST-8	35-45 08	
004	ST03	35 03		060	RCL7	36 07	
005	R4	-31		061	GT00	22 00	
006	ST02	35 02		062	*LBLC	21 13	Recall the sum S_n
007	XZY	-41		063	RCL5	36 05	
008	ST01	35 01		064	R/S	51	
009	ST08	35 08		065	*LBLD	21 14	
010	+	-55		066	RCL8	36 06	Compute the sum S
011	ST05	35 05		067	0	00	
012	R4	-31		068	X=Y?	16-33	
013	+	-55		069	GSB1	23 01	
014	ST06	35 06		070	GSBB	23 12	
015	N!	16 52	(c+d)!	071	GT00	22 14	
016	RCL5	36 05		072	*LBL1	21 01	
017	N!	16 52	(a+b)!	073	RCL5	36 05	
018	X	-35		074	R/S	51	
019	RCL5	36 05		075	RTN	24	
020	RCL6	36 06					
021	+	-55					
022	N!	16 52					
023	=	-24					
024	RCL1	36 01		080			
025	RCL3	36 03					
026	+	-55	(a+c)!				
027	N!	16 52					
028	X	-35					
029	RCL2	36 02					
030	RCL4	36 04					
031	+	-55					
032	N!	16 52	(b+d)!				
033	X	-35					
034	ST07	35 07		090			
035	0	00					
036	ST05	35 05					
037	R4	-31					
038	*LBL0	21 00					
039	RCL1	36 01	Loop for computing				
040	N!	16 52	probability				
041	=	-24					
042	RCL2	36 02					
043	N!	16 52					
044	=	-24		100			
045	RCL3	36 03					
046	N!	16 52					
047	=	-24					
048	RCL4	36 04					
049	N!	16 52					
050	=	-24					
051	ST+5	35-55 05	Accumulate the sum				
052	RTN	24	Display p_0				
053	*LBLB	21 12	Compute p_i for				
054	1	01	more extreme				
055	ST-1	35-45 01	tables				
056	ST+2	35-55 02					

Recall the sum S_n

Compute the sum S

(c+d)!

(a+b)!

(a+c)!

(b+d)!

Loop for computing probability

Accumulate the sum

Display p_0

Compute p_i for more extreme tables

FLAGS	SET STATUS		
	FLAGS	TRIG	DISP
0	ON OFF	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
1	0 <input type="checkbox"/> 1 <input checked="" type="checkbox"/> 2 <input type="checkbox"/> 3 <input checked="" type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
2		RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
3			n <u>2</u>

REGISTERS									
0	1	2	3	4	5	6	7	8	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

Program Description I

Program Title BARTLETT'S CHI-SQUARE STATISTIC

Contributor's Name Hewlett-Packard

Address 1000 N.E. Circle Blvd.

City Corvallis

State Oregon

Zip Code 97330

Program Description, Equations, Variables, etc.

$$\chi^2 = \frac{f \ln s^2 - \sum_{i=1}^k f_i \ln s_i^2}{1 + \frac{1}{3(k-1)} \left[\left(\sum_{i=1}^k \frac{1}{f_i} \right) - \frac{1}{f} \right]}$$

where s_i^2 = sample variance of the i^{th} sample

f_i = degrees of freedom associated s_i^2

$i = 1, 2, \dots, k$

k = number of samples

$$s^2 = \frac{\sum_{i=1}^k f_i s_i^2}{f}$$

$$f = \sum_{i=1}^k f_i$$

This χ^2 has a chi-square distribution (approximately) with $k - 1$ degrees of freedom which can be used to test the null hypothesis that $s_1^2, s_2^2, \dots, s_k^2$ are all estimates of the same population variance σ^2 ; i.e. H_0 : Each of $s_1^2, s_2^2, \dots, s_k^2$ is an estimate of σ^2 .

Note: Erroneous data can be corrected by using the **[D]** key.

Operating Limits and Warnings

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[illegible]

Apply the program to the following data:

i	1	2	3	4	5	6
s_i^2	5.5	5.1	5.2	4.7	4.8	4.3
f_i	10	20	17	18	8	15

[A]

5.5[ENT↑] 10[B], 5.1[ENT↑] 20[B],

4.3[ENT+] 15[B] -----> 6.00

$[C] \xrightarrow{\quad\quad\quad} 0.25 (x^2)$

[R/S] -----> 5.00 (df)

1. **Chapter 1: The Engineering Applications of Fluid Mechanics** A. Held, John Wiley

1. Statistical Theory with Engineering Applications, A. Hald, John Wiley and Sons, 1960.
2. This program is a translation of the HP-65 Stat Pac 1 program.

[illegible]

97 Program Listing I

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STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Initialize Clear storage registers Accumulate sums	057	R↓	-31	
002	CLRG	16-53		058	ENT↑	-21	
003	0	00		059	ENT↑	-21	
004	RTN	24		060	RCL1	36 01	
005	*LBLB	21 12		061	X	-35	
006	ST01	35 01		062	ST-8	35-45 08	
007	ST+3	35-55 03		063	X*Y	-41	
008	1/X	52		064	LN	32	
009	ST+4	35-55 04		065	RCL1	36 01	
010	R↓	-31		066	X	-35	
011	ENT↑	-21		067	ST-7	35-45 07	
012	ENT↑	-21		068	RCL5	36 05	
013	RCL1	36 01		069	1	01	
014	X	-35		070	-	-45	
015	ST+8	35-55 08		071	ST05	35 05	
016	X*Y	-41		072	RTN	24	
017	LN	32					
018	RCL1	36 01					
019	X	-35					
020	ST+7	35-55 07					
021	RCL5	36 05					
022	1	01					
023	+	-55					
024	ST05	35 05		080			
025	RTN	24					
026	*LBLC	21 13					
027	RCL8	36 08					
028	RCL3	36 03					
029	=	-24					
030	LN	32	Compute chi-square				
031	RCL3	36 03					
032	X	-35					
033	RCL7	36 07					
034	-	-45		090			
035	RCL4	36 04					
036	RCL3	36 03					
037	1/X	52					
038	-	-45					
039	RCL5	36 05					
040	1	01					
041	-	-45					
042	ST02	35 02					
043	3	03					
044	X	-35		100			
045	=	-24					
046	1	01					
047	+	-55					
048	=	-24					
049	P/S	51					
050	RCL2	36 02	Display chi-square Error corrector				
051	RTN	24					
052	*LBLD	21 14					
053	ST01	35 01					
054	ST-3	35-45 03		110			
055	1/X	52					
056	ST-4	35-45 04					

SET STATUS			
FLAGS		TRIG	DISP
ON	OFF		
0	<input type="checkbox"/>	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
1	<input type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
2	<input type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
3	<input type="checkbox"/>		n <u>2</u>

REGISTERS									
0	1 f_i	2 df	3 Σf_i	4 $\Sigma 1/f_i$	5 k	6 0	7 $\Sigma f_i \ln s_i$	8	9 0
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

Program Description I

Program Title MANN-WHITNEY STATISTIC

Contributor's Name Hewlett-Packard

Address 1000 N.E. Circle Blvd.

City Corvalli

State Oregon

Zip Code 97330

Program Description, Equations, Variables, etc.

This program computes the Mann-Whitney test statistic on two independent samples of equal or unequal sizes. This test is designed for testing the null hypothesis of no difference between two populations.

Mann-Whitney test statistic is defined as

$$U = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - \sum_{i=1}^{n_1} R_i$$

where n_1 and n_2 are the sizes of the two samples. Arrange all values from both samples jointly (as if they were one sample) in an increasing order of magnitude, let R_i ($i = 1, 2, \dots, n_1$) be the ranks assigned to the values of the first sample (it is immaterial which sample is referred to as the "first").

When n_1 and n_2 are small, the Mann-Whitney test bases on the exact distribution of U and specially constructed tables. When n_1 and n_2 are both large (say, greater than 8) then

$$z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{n_1 n_2 (n_1 + n_2 + 1)/12}}$$

is approximately a random variable having the standard normal distribution.

Operating Limits and Warnings

For small samples (say, less than or equal to 8) the specially constructed tables should be used.

For example:

Handbook of Statistical Tables, D. B. Owen, Addison-Wesley, 1962

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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Program Description II

Sketch(es)

Sample Problem(s)

Find U and Z for the following data:

Sample 1	14.9	11.3	13.2	16.6	17	14.1	15.4	13	16.9	
Rank R_i	7	1	4	12	14	5	10	3	13	
Sample 2	15.2	19.8	14.7	18.3	16.2	21.1	18.9	12.2	15.3	19.4
Rank	8	18	6	15	11	19	16	2	9	17

Note: 1. $n_1 = 9, n_2 = 10$

2. The ranks have already been assigned in the example.

Solution(s)

Keystrokes:

10[A] 7[B] 1[B] 4[B] -----

3[B] 13[B] -----> 9.00

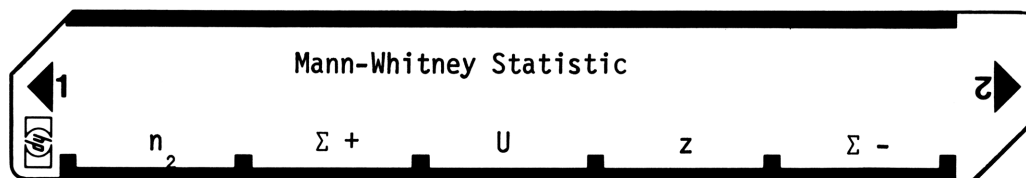
[C] -----> 66.00 (U)

[D] -----> 1.71 (Z)

Reference(s)

1. Mathematical Statistics, J.E. Freund, Prentic Hall, 1962.
2. This program is a translation of the HP-65 Stat Pac 1 program.

User Instructions

[illegible]

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STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	$n_2 \rightarrow R_2$ Store 0 in R_1, R_3				
002	ST02	35 02					
003	0	00					
004	ST01	35 01		060			
005	ST03	35 03	Accumulate sums				
006	RTN	24					
007	*LBLB	21 12					
008	ST+3	35-55 03					
009	RCL1	36 01	Compute U				
010	1	01					
011	+	-55					
012	ST01	35 01					
013	RTN	24	Display U				
014	*LBLC	21 13		070			
015	RCL2	36 02					
016	RCL1	36 01					
017	1	01	Compute z				
018	+	-55					
019	2	02					
020	=	-24					
021	+	-55	Display z				
022	x	-35					
023	RCL3	36 03					
024	-	-45					
025	RTN	24	Error corrector				
026	*LBLD	21 14					
027	RCL1	36 01					
028	RCL2	36 02					
029	x	-35	Error corrector				
030	2	02					
031	=	-24					
032	-	-45					
033	RCL1	36 01	Error corrector				
034	RCL2	36 02		090			
035	+	-55					
036	1	01					
037	+	-55	Error corrector				
038	RCL1	36 01					
039	x	-35					
040	RCL2	36 02					
041	x	-35	Error corrector				
042	1	01					
043	2	02					
044	=	-24					
045	IX	54	Error corrector				
046	=	-24					
047	RTN	24					
048	*LBLE	21 15					
049	ST-3	35-45 03	Error corrector				
050	RCL1	36 01					
051	1	01					
052	-	-45					
053	ST01	35 01	Error corrector				
054	RTN	24					

REGISTERS									
0	1	2	3	4	5	6	7	8	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	F	G	H	I	J

SET STATUS		
FLAGS	TRIG	DISP
0 <input type="checkbox"/> ON <input checked="" type="checkbox"/> OFF	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
1 <input type="checkbox"/> <input checked="" type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
2 <input type="checkbox"/> <input checked="" type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
3 <input type="checkbox"/> <input checked="" type="checkbox"/>		n <u>2</u>

Program Description I

Program Title	KENDALL'S COEFFICIENT OF CONCORDANCE		
Contributor's Name	Hewlett-Packard		
Address	1000 N.E. Circle Blvd.		
City	Corvallis	State Oregon	Zip Code 97330

Program Description, Equations, Variables, etc.

Suppose n individuals are ranked from 1 to n according to some specified characteristic by k observers, the coefficient of concordance W measures the agreement between observers (or concordance between rankings).

$$W = \frac{12 \sum_{i=1}^n \left(\sum_{j=1}^k R_{ij} \right)^2}{k^2 n(n^2 - 1)} - \frac{3(n+1)}{n-1}$$

Where R_{ij} is the rank assigned to the i^{th} individual by the j^{th} observer.

W varies from 0 (no community of preference) to 1 (perfect agreement). The null hypothesis that the observers have no community of preference may be tested using special tables, or if $n > 7$, by computing

$$\chi^2 = k(n-1)W$$

which has approximately the chi-square distribution with $n-1$ degrees of freedom (df).

Operating Limits and Warnings

For small samples (say, less than or equal to 7) the specially constructed tables should be used. For example:

Rank Correlation Methods, M.G. Kendall, Hafner Publishing Co., 1962

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Program Description II

Sketch(es)

Sample Problem(s)

1. Find W , χ^2 , and df for the following data:

Table for R_{ij} ($n = 10, k = 3$)			
$i \backslash j$	1	2	3
1	6	7	3
2	1	4	2
3	9	3	5
4	2	6	1
5	10	8	9
6	3	2	6
7	5	9	8
8	4	1	4
9	8	10	10
10	7	5	7

Solution(s) Keystrokes:

[f] [CL REG]

6[A] 7[A] 3[A] [B],

1[A] 4[A] 2[A] [B]

.

7[A] 5[A] 7[A] [B]

[C] -----> 0.69 (W)

[D] -----> 18.64 (χ^2)

[R/S] -----> 9.00 (df)

Reference(s)

1. Nonparametric Statistical Inference, J.D. Gibbons, McGraw-Hill, 1971.
2. This program is a translation of the HP-65 Stat Pac 1 program.

User Instructions

1
Kendall's Coefficient of Concordance
2

$\Sigma +$
 $\Sigma \Sigma +$
 W
 χ^2, df
 $\Sigma -$

[illegible]

97 Program Listing I

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STEP	KEY ENTRY ~	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11		057	RCL4	36 04	
002	ST+2	35-55 02		058	1	01	
003	RCL1	36 01	Accumulate sums (i is fixed)	059	-	-45	
004	1	01		060	RTN	24	
005	+	-55		061	*LBLB	21 15	
006	ST01	35 01	Display current j	062	ST-2	35-45 02	Error corrector (i is fixed)
007	RTN	24		063	RCL1	36 01	
008	*LBLB	21 12		064	1	01	
009	RCL1	36 01	Accumulate sum over i	065	-	-45	
010	ST05	35 05		066	ST01	35 01	
011	RCL2	36 02		067	RTN	24	
012	X²	53					
013	ST+3	35-55 03		070			
014	RCL4	36 04					
015	1	01					
016	+	-55					
017	ST04	35 04					
018	0	00					
019	ST01	35 01	Reinitialize R ₁ , R ₂				
020	ST02	35 02					
021	RCL4	36 04	Display current i				
022	RTN	24					
023	*LBLC	21 13					
024	RCL3	36 03	Compute W	080			
025	1	01					
026	2	02					
027	X	-35					
028	RCL5	36 05					
029	X²	53					
030	=	-24					
031	RCL4	36 04					
032	=	-24					
033	RCL4	36 04					
034	X²	53		090			
035	1	01					
036	-	-45					
037	=	-24					
038	RCL4	36 04					
039	1	01					
040	+	-55					
041	3	03					
042	X	-35					
043	RCL4	36 04					
044	1	01		100			
045	-	-45					
046	=	-24					
047	-	-45					
048	RTN	24	Display answer W				
049	*LBLD	21 14					
050	RCL5	36 05	Compute X² and df k				
051	X	-35					
052	RCL4	36 04					
053	1	01					
054	-	-45		110			
055	X	-35					
056	R/S	51	Display X²				

REGISTERS									
0	1 j ΣR_{ij}	2	3 $(\Sigma R_{ij})^2$	4 n	5 k	6	7	8	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

SET STATUS									
FLAGS		TRIG		DISP					
0	<input type="checkbox"/> ON <input checked="" type="checkbox"/> OFF	DEG	<input checked="" type="checkbox"/>	FIX	<input checked="" type="checkbox"/>				
1	<input type="checkbox"/> <input checked="" type="checkbox"/>	GRAD	<input type="checkbox"/>	SCI	<input type="checkbox"/>				
2	<input type="checkbox"/> <input checked="" type="checkbox"/>	RAD	<input type="checkbox"/>	ENG	<input type="checkbox"/>				
3	<input type="checkbox"/> <input checked="" type="checkbox"/>			n	2				

NOTES

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