HEWLETT PACKARD
Expanding Possibilities

Keys to the Calculator

NCTM-based Middle School Activities for the HP 6S


HYP
INV
R→P
→DEG
1/x
tan
DRG
cos
x²
A b/c
DATA
log
M+
X→M
RM
C/CE
Exp
9
8
7
6
5
4
3
2

Welcome to "Keys to the Calculator"

Hewlett-Packard Calculators is pleased to introduce you to "Keys to the Calculator", an interactive real life learning resource that will help your students learn how the calculator can investigate math concepts.

We have taken great care to ensure this resource works within the NCTM standards and feel we have accomplished this in a way that will make learning realistic and fun for the students.

This material again demonstrates Hewlett-Packard's dedication to the calculator and its educational base. The HP 6S calculator and "Keys to the Calculator" simply take this dedication to a new level, the Middle School. We know this is just the beginning of the support we hope to provide to Middle School educators and students and hope this will grow to the levels our other product categories have achieved.

On that note we would like to invite you, if you have not already, to join more educators as a part of the HP educator program. This is the conduit by which information and support such as workshops and support materials reach the educator.

To find out more about this and the many other products and services we have to offer please visit our web site at www.hp.com/calculators. If you do not have access to the web but are interested in these programs please send your name and address to The HP Educator Program P.O. Box 3226 Salem, Oregon 97302-0226, and we will be happy to get information out to you.

Again thank you for your support and use of this resource in your classroom. If you have any questions or comments on how to improve or change this or any of our products or materials, please contact us at the address above or through the web site where there is an area dedicated to educator feedback.

We look forward to hearing from you.

Hewlett-Packard Calculators

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How To Use This Resource

The activities in this resource are based on an inquiry approach to mathematics education. They are designed to relate to a student's world, with investigations that answer the oft-asked student question – when am I ever going to use this?

By synthesizing concepts and providing a real-world context for activities, "Keys to the Calculator" also demonstrates the interconnectedness between NCTM standards.

All of the five NCTM processes – problem solving, reasoning and proof, communication, connections and representation – are integrated into activities that are easy-to-follow and hands-on.

The Lessons

Teachers may choose to use a lesson as an adjunct to concepts being explored through a text book. Alternately, they may choose to use these lessons to introduce concepts.

Each lesson should be used in its entirety. However, the individual activities are, for the most part, self-contained.

Teacher's Notes

Teachers notes at the beginning of each lesson provide:

- A clear curriculum matrix that outlines the NCTM Standards upon which each lesson is based.
- A *Getting Started* section that shows the skills and concepts taught in the activities.
- A vocabulary list that offers the teacher an opportunity to teach or review key words before the lesson.
- A list of materials that allows the teacher and/or students time to accumulate what is needed for the lesson.

Each lesson also comes complete with an *Activity Overview* matrix; outlining the activity, the teaching method and the assessment criteria.

Student Worksheets

Each activity includes a worksheet to be photocopied for the student.

At the side of the worksheet, a clear step-by-step description of the use of the HP 6S helps the student use the different calculator keys to be mastered in each activity.

Related *Did You Knows?* and *Web Sites* offer both students and teachers opportunities to extend the learning of particular areas of interest.

Numbers Around Us!

Teachers' Notes

This three-part lesson explores which form of a rational number – an equivalent fraction or decimal – best helps to solve specific problems.

NCTM Standards

Number and Operation

Understand numbers and ways of representing numbers, relationships among numbers and number systems

- work flexibly with equivalent fractions, decimals, percents; and choose appropriate and convenient forms of these numbers for solving problems
- develop meaning for integers and be able to represent, compare, and order them
- develop an understanding of large numbers, including the use of benchmarks to comprehend their magnitude; and recognize, understand and appropriately use various representations for large numbers (e.g., exponential, scientific, and calculator notation)
- use number theory concepts (e.g., factors, multiples) to solve problems and to understand ideas about rational numbers

Getting Started

Use this table to prepare yourself and your students for this lesson.

Sheet No.	Materials	Vocabulary	Skills & Concepts
1	• pencils and paper • newspapers • flyers • HP 6S calculators	• rational numbers • fraction • decimal • percents	• equivalent fractions, decimals and percents • operations with fractions • terminating and repeating decimals
2	• newspapers	• integers • scientific notation • calculator display	• positive and negative numbers • exponents • base 10
3	• newspaper business pages • computer with Internet access	• currency • currency cross rates • truncating	• fixed decimal places • rounding • truncating • mean

Numbers Around Us!

Activity Overview

This table shows what students are doing in each activity, how they should do it and how their success can be measured.

Sheet No.	Activity	Method	Assessment
1	• investigating when it is appropriate to keep a fraction as a fraction or change a fraction to decimal form	• find fraction and percent equivalents of reductions in sales ads • find the percent and fraction reduction of sale prices	• correctly describe, using proper math terminology, how fractions, decimals and percents can be equivalents • correctly describe the appropriate use of fractions, decimals and percents to solve specific types of problems
2	• looking for decimal, fractions, percents, integers and large numbers in newspapers and deciding how and why they are used	• find equivalents of decimals, fractions and percents • re-write sentences with equivalent values • order integers (weather temperatures) on thermometer • look for analogies to large numbers	• justify, with proper math terminology, the replacement of one number form with an equivalent • describing how positive and negative numbers can be placed on a number line • support, with proper math terminology, the conversion of large numbers from words to standard form to scientific notation and vice versa.
3	• describing the importance of decimal place values in currency exchange	• examine effect of the four decimal places in exchange rates by fixing the decimal place on the calculator • track the changes in exchange rates over a period of time • assess the effect of current events on the exchange rates	• clearly and concisely describe the effect of current events on currency rates • correctly calculating mean currency rate • clearly and concisely describing the appropriate use of mean and median • describe, with proper math terminology, how decimal places change the value of converted currency

Numbers Around Us!

Sales Math!

When is it appropriate to convert a common fraction to its decimal form or keep a fraction as a fraction?

Clip Your Coupons!

1. Find 3 to 5 sales ads from newspapers or flyers that use different fractions and/or percents to show a reduction in price.
2. Clip out the ads and glue each one to a sheet of paper.
3. Next to the ads that use percents, write the equivalent fractions. Next to ads that use fractions, write the equivalent percents.
4. Compare the equivalent values. In each case, which is easier to understand. Why?
5. Why do you think some ads use fractions and some use percents? Why are sales reductions not shown as decimals?

Keys to the Calculator

To display a fraction on the HP 6S;

Choose a fraction less than 1.

Enter the numerator, press $A^B/_C$. Enter the denominator

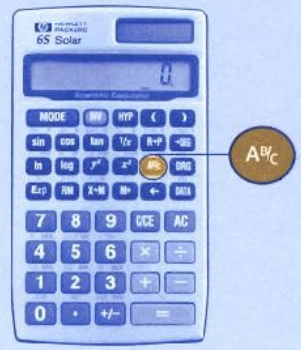
1. Describe what you see. One way to convert the fraction to a decimal is as follows,

Press = and then the $A^B/_C$ key.

2. How does this decimal value show you the percent equivalent of the fraction?

3. What is the greatest denominator you can enter? What is the least?

4. What is the greatest numerator you can enter? What is the least?



Numbers Around Us!

Oops!

How do you make corrections on the HP 6S?

Enter **57**. Oops! You really meant to press **56**. Press **AC**. What happens? (On the battery-powered HP 6S, press **C/CE**.)

Enter **57** again. What key might clear only the last digit? Replace it with a 6.

Enter **56 x 3**. Oops! You really meant **2**. What key might clear your last entry?

Write about how you've learned to make corrections and when to use each method.

Compare your methods with a partner.

Let's practice.
a) Enter **125 x 624**. Clear the last entry and replace it with **534**.
b) Enter **1784**. Clear the last digit and replace it with a **9**.
c) Enter the present year. Clear it to enter the number for next year.

Calculate Your Savings

1. The following table shows the original price of a number of items purchased on sale. The left-hand column indicates the fraction or percent of the reduction.
2. Use a method of your choice to mentally calculate a) the amount you saved and b) the sale price of each item.
3. Record which question was easy (E) to do mentally and which was difficult (D).

Reduction	$36.00		$29.50		$18.95		E or D
	Amount Saved	Sale Price	Amount Saved	Sale Price	Amount Saved	Sale Price	
1/4 off							
10% off							
1/3 off							
50% off							

4. Write about how you performed each of the mental calculations. Explain your choices.
5. When is using a decimal an appropriate way to perform mental calculations?

Sale Prices

At one store, a CD listed at $18.00 is reduced to $12.00.
At another store, the same CD is 30% off.

1. Estimate the fraction and percent equivalents of the reductions at both stores.
2. Calculate the fraction and percent equivalents of the reductions.
3. Which store offers the best buy? Explain why.

Numbers Around Us!

Newspaper Numbers

Numbers are represented in different ways depending on the type of information the writer wishes to communicate. How can you best understand the meaning of these numbers?

Be a News Number Hound!

With a partner, look through newspapers for articles, maps and tables that use:

- fractions
- decimals
- percents
- integers (positive and negative numbers)
- large numbers (numbers in the millions or greater)

1. Cut out your examples and paste them on large sheets of paper.
2. Label each clipping with the section and page number of the newspaper.

Articling Numbers

1. Underline the sentences in your articles that use fractions, decimals and percents.
2. Convert the number to as many equivalent forms as you can. Record each form.
3. Re-write the sentence using the equivalent form you found easiest to understand. Explain why you chose this form.

Keys to the Calculator

To display a number in scientific notation;

Enter 623, press **Exp**, then enter 5.

1. Describe what happens when you press the = key.

2. What direction does the decimal point move? How many places?

Re-enter 623, **Exp** and 5. Now, press the +/- key and =.

3. What direction does the decimal point move? How many places does it move?

4. Press **INV, SCI,** then repeat all the above steps and questions. How does pressing **INV, SCI** first affect your results?

Numbers Around Us!

Keys to the Calculator

To change a number to scientific notation:

Enter the greatest number you can display on your HP 6S.

Locate **SCI** on your calculator.

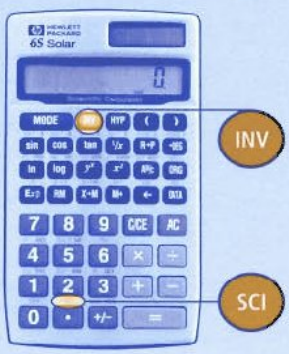
To use this key, Press = first, then **INV** and **SCI.**

1. Describe how your original number now looks on the HP 6S.

Now press plus 1. Describe what happens.

2. What do the small numbers on the right of your display signify?

Try this with the least number you can display.



Awesome Numbers

1. In which sections of newspapers do you find large numbers expressed as words?
2. Write out three examples of these numbers in standard form showing all their place values. Read these numbers to your partner using place value names.
3. On your HP 6S express these large numbers in scientific notation. Record these next to the numbers from #2.

Example:

Words	Standard Form	Scientific Notation
three trillion dollars	= $3,000,000,000,000.00	= $3.00 x 10^{12}

Positive and Negative

1. In what sections of the newspaper did you find positive and negative numbers?
2. Explain what the positive and negative signs indicate in each case.
3. Enter some of the integers into your HP 6S. What key did you use to change the sign of the number? What did your display read each time?

Weathering Integers

1. Select 10 cities with different temperatures from the weather page. Choose 5 positive and 5 negative temperatures.
2. Why are integers used in this section?
3. Draw a thermometer with a range of degrees that covers your selected temperatures.
4. Place the temperatures on the thermometer. Label each one with the city's name. Indicate the range of temperatures from low to high.
5. How does placing them on the thermometer help you understand the temperatures?
6. Find these cities on a map of the world. Name three things that help to explain the temperatures of the cities at this time of year.

Numbers Around Us!

Digits Matter

You are considering a vacation outside your country. Choose three possible countries as destinations. How might the value of their currencies, compared to yours, affect your travel plans?

Example:

1 U.S.$ = $1.50 Canadian dollars = 10 Mexican pesos

Track The Currencies

1. Over the period of at least a week, use the Internet *(see Web Links)* or newspapers to track the value of the three currencies versus your own.
2. Record the changes in the currencies and the date and time of the posted value.
3. Keep a clipping file of news events that effect the countries you plan to visit.

Date	Country 1:	Country 2:	Country 3:

The Difference in a Digit

1. What does it mean if the currency of one of your countries is a number greater than 1, *e.g. $1.00 U.S.$ = $1.50 Canadian?* Is your currency or the other countries' currency worth more? Explain.
2. What does it mean if the currency of one of your countries is a number less than 1, *e.g. $1.00 U.S. = 0.6117 British pounds?* Is your currency or the other country's currency worth more? Explain.

Keys to the Calculator

To fix the decimal places on the HP 6S;

Press **INV ← (FIX)**, then enter the number of decimal places e.g. 2 for hundredths.

What happens to your display? Try fixing decimals to the thousandths, ten-thousandths.

To calculate the mean of your data on the HP 6S;

Press **MODE**, then press **5 (SD)** (Statistical Data)

Press **DATA** after entering each value. Press **INV** 4 **($\bar{x}$)** for the mean.

Numbers Around Us!

Did You Know?

Currencies are traded in units called "basis points." One basis point equals one-hundredth of a cent. Write this as a decimal.

What would the following be as a decimal:
a) 12 basis points
b) 6 basis points
c) 112 basis points

Web Links

http://cnnfn.com/markets/currencies
tables of cross currency rates for most of the world's countries are up-dated throughout the day.

3. How many decimal places are used in calculating currency rates?
4. Using the highest currency rate that you found for each of your countries, calculate the value of a $100.00 of your money. Use the FIX key on your HP 6S to see the effect of decimal places on the values. *(See Keys to the Calculator.)*

Country	Currency Rate	Value of $100 with Rate Fixed to 1 Decimals	Value of $100 with Rate Fixed to 2 Decimals	Value of $100 with Rate Fixed to 3 Decimals

5. How does the digit on the far right affect the conversion of your money into other currencies?
6. When would the effect be greater? When would it be less?

The Mean Exchange

1. Calculate the mean of each country's currency using the instructions in *Keys to the Calculator.*

Example:

1 U.S.$ = a mean of 10.070 Mexican pesos

2. Find the median rate of each country's currency. Compare the median with the mean. Which would you use to calculate exchange rates? Why?
3. Convert a thousand dollars of your currency into the currencies of each country.
4. Which country offers the best vacation value? Why?

Food Fractions

This two-part lesson explores the effect of operations on fractions associated with portions of food.

NCTM Standards

Number and Operation

Understand the meaning of operations and how they relate to each other

- extend understanding of operations to include operations on fractions, decimals, per-cents
- understand the effects of operating among fractions, decimals, and percents
- recognize and use the properties of operations on integers and other rationals, such as closure, associative, commutative, and distributive properties
- understand and use the inverse relationships of addition and subtraction, multiplication and division to solve problems
- extend understanding of counting to include elementary combinatorics

Use computational tools and strategies fluently and estimate appropriately

- develop, analyze, and compare algorithms for computing with fractions, decimals, and percents and become efficient and accurate in computing with them
- develop, analyze, and explain methods for solving problems involving proportions (e.g. scaling, finding equivalent ratios)
- develop and refine strategies for estimating (including fractional quantities)
- use estimation as a means to check the reasonableness of results
- select and use appropriate methods for computing from among mental arithmetic, estimation, paper-and pencil, and calculator

Getting Started

Use this table to prepare yourself and your students for this lesson.

Sheet No.	Materials	Vocabulary	Skills & Concepts
1	• pizza take-out menus • paper, pencils • HP 6S calculators	• fraction • estimation	• checking reasonableness of answers • inverse relationships of operations of adding and subtracting fractions
2	• HP 6S calculators	• automatic constant • ratio • proportion • algorithm	• scaling • ratio table • inverse relationships of operations of multiplying and dividing fractions

Food Fractions

Activity Overview

This table shows what students are doing in each activity, how they should do it, and how their success can be measured.

Sheet No.	Activity	Method	Assessment
1	• determining the most economical way to have a class pizza party	• find how many slices of different types of pizza the class will eat • use operations on fractions to find the best way to feed everyone	• accurately estimate a quantity within a reasonable range • correctly calculate an amount using operations with fractions • provide appropriate reasons for the choice of method of calculation • correctly and concisely describe the effect of operations on fractions • describe, using appropriate mathematical terminology, the inverse relationships of operations with fractions • correctly and concisely describe algorithms for solving ratio problems
2	• changing recipes to feed larger and smaller numbers of students	• use ratio tables to find the proportions of ingredients in changed recipes	• describe, using appropriate mathematical terminology, a method chosen to solve ratio problems • correctly and concisely describe the properties of the results when operating on fractions • describe, using appropriate mathematical terminology, the inverse relationships of operations with fractions • correctly and concisely describe algorithms for solving ratio problems

Food Fractions

Operation Pizza !

How can we use operations with fractions to organize the most economical pizza party for your class?

Pizza Party

1. Determine your class' three favorite types of pizzas.
2. Tally the number of slices of each pizza that students plan to eat.
3. As a group, plan how to purchase pizzas that will satisfy your class' appetite by:
   a) Estimating the amount of each type of pizza you will need.
   b) Finding the total in problem 2 to determine how many slices of each type of pizza are needed.
   c) Making a model of each type of pizza using fraction circle pieces or drawings. Write the fraction to represent each model.

Example:

$3\frac{7}{8}$ cheese pizzas

   d) Writing math sentences using fractions and mixed numbers to show the sum of the amount of all the different types of pizzas.

Example:

cheese		vegetarian		pepperoni	
$3\frac{7}{8}$	+	$2\frac{1}{3}$	+	$7\frac{11}{12}$	=

Keys to the Calculator

To display a mixed number on the HP 6S,

Enter $3\frac{7}{12}$ by pressing **3**, **$A^B/_C$**, **7**, **$A^B/_C$**, **12**

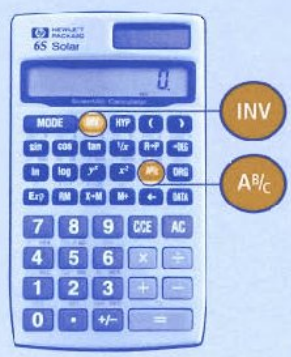
1. What does the display read? Press **INV**, **$A^B/_C$**

2. What do these keystrokes do? Do not clear. Add another $\frac{7}{12}$.

3. What happens when you press = ?

4. What did the calculator do to the answer?

5. Answer questions 2 and 3 again after a) subtracting $\frac{4}{5}$ cups from $7\frac{3}{12}$ cups and b) taking $3\frac{3}{4}$ in. away from $13\frac{1}{8}$ in.



Food Fractions

Did You Know?

Products and quotients aren't always what you might think they should be.

Use the fraction keys on the HP 6S to find
a) $\frac{1}{2}$ of $\frac{3}{4}$ of a pizza
b) $\frac{1}{5}$ of $\frac{2}{3}$ and
c) $\frac{1}{3}$ of $\frac{7}{8}$.

1. Are your products larger or smaller than the original pairs of fractions? Why might this be?

Find:
a) the number of $\frac{1}{3}$'s in a remaining $\frac{3}{4}$ of a pizza ($\frac{3}{4} \div \frac{1}{3}$)
b) the number of $\frac{1}{4}$'s in $\frac{7}{10}$ and
c) the number of $\frac{3}{4}$'s in $\frac{1}{4}$.

2. Are your quotients larger or smaller than the original pairs of fractions? Why might this be?

3. What conclusion can you make about the products and quotients of two fractions less than 1? Make a sketch to show why your conclusions are true.

Best Buy

1. Choose a pizza take-out menu to determine the number of slices there are in each size of pizza.

Size of Pizza	Small	Medium	Large	Extra Large
Slices of Pizza				
Cost of Pizza				

2. Estimate what size(s) of pizza are most appropriate to purchase so you have the least number of slices leftover.
3. Calculate the number of whole pizzas you will need to purchase, knowing you will have some slices leftover,
   a) How does this compare with your estimate?
   b) Calculate the cost of the pizza party based on your findings.
4. Compare your costs by using different sizes of pizzas. What if you bought only large? Only medium or small? How do these different sizes reduce or raise your costs?
5. What is the best combination of sizes? Explain why.

Leftovers

Determine number of slices of each pizza that will be leftover from the best combination of sizes. Show your work. Express these amounts as fractions.

Drinks

Use the above approach to determine the most economical way to provide drinks for your class. Compare using cases of canned drinks with large bottles and cups.

Prices

Determine the cost per student for the pizza party. Be sure to show and explain your work.

Food Fractions

Automatic Menu

How can we change a recipe to accommodate different numbers of students?

Doctor Pizza!

For Italian night, the class decided to customize their basic three-topping pizzas. Use the automatic constant on the HP 6S *(see Keys to the Calculator)* to determine the amount of each ingredient they need. Show your keystrokes.

Number of Pizzas	1	4	8	16	24
Teaspoons of Oregano	1/2				
Cups of Mushrooms	1/3				
Slices of Tomato	5				
Cups of Sliced Olives	1/4				

How could you use proportions to calculate these amounts?

Keys to the Calculator

Let's find out how the automatic constant works in the HP 6S.

A pizza recipe for 4 people calls for 4/5 cup of olives. How many cups are needed for 12 people?

1. Use the multiplication key to find the answer.

2. Use the **4, A$^{B}/_{C}$, 5,** + and = keys to find the same answer.

3. Use the **4, A$^{B}/_{C}$, 5,** + and = keys to find number of cups of olives for
a) 20 people and
b) 32 people.

4. What keystrokes did you use in 2 and 3? How many times did you press = to get the answer in each case?

5. How does this compare to the information given in the problem?

6. What pattern do you see?

Food Fractions

Did You Know?

The memory in the HP 6S automatically stores parts of your math sentences so you can re-use them. The numbers and the operations stored are called "automatic constants."

Chocolate Chip Pancakes

Your class is sponsoring a breakfast fund-raiser. Use the following recipe and table to find the quantities of ingredients you will need to feed the different numbers of students.

Follow this process:

- write your estimated answer in the top corner of each box
- use a different method to complete each column of the table
- record your method *(see Keys to the Calculator)*
- place your answer in the bottom corner of each box
- check the reasonableness of your answers

Ingredients	Servings	Students in Your Class: _____	Students in Your Grade: _____	Students in Your School: _____
Cups of Flour	$1\frac{1}{2}$			
Teaspoons of Salt	$\frac{1}{4}$			
Tablespoons of Sugar	1			
Teaspoons of Baking Powder	$1\frac{1}{2}$			
Teaspoons Baking Soda	$\frac{1}{2}$			
Tablespoons of Oil	3			
Number of Eggs	2			
Cups of Buttermilk	$1\frac{3}{4}$			
Cups of Chocolate Chips	$\frac{1}{3}$			

1. Write about your methods indicating which you prefer and why.
2. Use your preferred method to find out how much of each ingredient would be needed to make pancakes for you and a friend?

Money Patterns

This three-part lesson explores patterns in earning, spending, saving and investing money.

NCTM Standards

Patterns, Functions and Algebra

Understand various types of patterns and functional relationships

- analyze, create, and generalize numeric and visual patterns paying particular attention to patterns that have a recursive nature
- use patterns to solve mathematical and applied problems
- represent a variety of relations and functions with tables, graphs, verbal rules, and, when possible, symbolic rules

Use symbolic forms to represent and analyze mathematical situations and structures

- develop a sound conceptual understanding of equation and of variable
- become fluent in generating equivalent expressions for simple algebraic expressions and in solving linear equations and inequalities;
- use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships

Use mathematical models and analyze change in both real and abstract contexts

- model and solve contextualized problems using various representations, such as graphs and tables, and to understand the purpose and utility of each representation
- explore different types of change occurring in discrete patterns, such as proportional and linear change

Number and Operation

- understand the meaning of operations and how they relate to each other
- extend understanding of counting to include elementary combinatorics

Money Patterns

Getting Started

Use this table to prepare yourself and your students for this lesson.

Sheet No.	Materials	Vocabulary	Skills & Concepts
1	• pencils, paper • HP 6S calculators	• combinatorics	• elementary combinatorics
2	• HP 6S calculators • Internet access	• recursive • federal minimum wage • percent of salary • savings • take-home pay	• problem solving with patterns • recursive patterns
3	• HP 6S calculators • newspapers • Internet access	• principal • compound interest • investment • term	• use of equation and variables • order of operation

Activity Overview

This table shows what students are doing in each activity, how they should do it, and how their success can be measured.

Sheet No.	Activity	Method	Assessment
1	• developing plans to buy school snacks and still have money left over	• using an illustration of a school vending machine on the sheet, students select two different school snacks a day for a week	• concisely and correctly describe recursive patterns • correctly represent the recursive patterns with a table • confidently solve problems using the recursive patterns • describe elementary combinatorics, using proper mathematical terminology
2	• analyzing the benefits of different approaches to saving and investing	• students set a savings goal, calculate how much they need to save a month to reach their goal in a year, then explore the impact of interest on their savings	• concisely and correctly describe recursive patterns • correctly represent the patterns with a table • confidently solve problems using the recursive patterns
3	• investigating the affects of the different variables on compound interest	• students use a compound interest formula to see how different values affect the amount of money earned at the end of a term of investment	• correctly use a formula to calculate the results of different values of variables • confidently solve problems using a compound interest formula • concisely and correctly describe effects of changing values for the variable in a compound interest formula

Money Patterns

A Memory for Money

How you can use the money you earn to buy school snacks and have some money leftover?

Weekly Allowance

You receive $10.00 a week for doing chores. Plan how you would use this money to buy two snacks a day at the school vending machine shown below.

Keys To The Calculator

To explore how the memory on the HP 6S works:

1. With a partner, find all the keys labeled with an **"M"**.

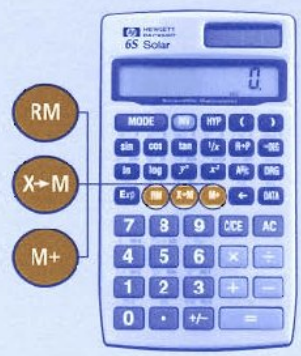
2. Use the **X→M** key to put $5.19 into memory. What does your display read?

3. Enter the difference between $7.00 and $5.19. Press **M+**. What does your display read?

4. Press the **RM** key. What does your display read? What do you think this key does?

5. Use these keys to change the value in your memory to $20.00?

(continued on reverse)



Money Patterns

Keys To The Calculator

6. Subtract $5.19 from the $20.00 in memory?

7. With $7.00 on your display, press **INV X→M**. What does this key do?

8. Clear your memory. How do you know your memory is clear?

Web Links:

The Kid's Consumer Corner
tqjunior.advanced.org/3643
This site has great ideas for saving, investing and earning money, with lots of tips on how to spend your money wisely.

Snack Plan

Use this table and the memory on your HP 6S to plan your snack choices. *(See Keys to the Calculator.)* Buy 2 snacks a day for 5 days. Try different ways of getting snacks and having money left over.

Day	Snack #1	Cost	Snack #2	Cost	Total Snack Cost Per Day
Monday					
Tuesday					
Wednesday					
Thursday					
Friday					
				Week's Total Snack Cost	

Snack Questions

1. a) How could you change your snack choices to make them healthier?
   b) How might this affect your weekly snack cost?
2. How many different pairs of snacks did you make?
3. What's the maximum number of different pairs of snacks you could have with two snacks a day for five days?

Leftovers

1. How much of your $10.00 could you save each week, based on your original snack plan?
2. Saving at this rate, how long would it take you to save for:
   i) a $150.00 pair of in-line skates
   ii) a $350.00 portable stereo
   iii) $1,000.00 for your education after high school
3. Change your snack plan to try to reach the above goals more quickly. How long would it take you to save for each item using your new snack plan?

Earning and Saving!

How do different types of saving plans help your money grow?

Time to Earn!

1. You have found a part-time job that earns the federal minimum wage ($5.15). You work 12 hours a week. How much money do you earn in a) a week and b) a month?
2. If you pay 12% of your earnings to taxes, what is your take-home pay a) for the week and b) for the month?

Time to Save!

1. Choose something you would like to own – one that would take a year to save up for. Determine its cost.
2. Choose three different percentages of your take home pay that you can use to save for this item. Enter these in the left column of the table on page 20.
3. Complete the table to show the total amount of savings you would accumulate each month. Find the total saved after one year. Use the memory on your calculator to help you.

Keys To The Calculator

To explore how much interest you can earn on your money, let's use the simple interest formula:
i = PRT

i - interest earned
P – principal: money invested
R – rate of interest: shown as a percentage per year; converted to a decimal equivalent for calculations
T – term or time invested: expressed as a decimal

Predict ahead of time, then find the answers to the following two questions if:
P = $500.00
R = 5%
T = 1 year.

1. What is the effect on the interest of:
a) doubling
b) tripling and
c) halving the principal?

(continued on reverse)

Money Patterns

Keys To The Calculator

2. What is the effect on the interest of:
a) doubling the principal and doubling the rate and
b) tripling the principal and quadrupling the rate?

3. Show how you find the missing values:
a) If i = $80.00, R = 4% and T = 2 years, what is P?
b) If i = $81.25, P = $250.00 and T = 5 years, what is R?
c) If i = $75.00, P = $5000.00, and R = 3%, what is T?

% Savings	Jan.	Feb.	March	April	May	June

July	Aug.	Sept.	Oct.	Nov.	Dec.	Total

Reaching Your Goal

1. Which percentage of savings gets you closest to your goal?
2. How could you change the percentage of savings to get you closer to your goal in exactly one year?
3. What do you find most surprising when you look at the pattern of money saved over a year?

Predict Your Salary

1. Your $60.00 weekly earnings were increased by 4%. After four months, you decide to work less. This reduces your salary by 4%. Predict how your new salary compares to your original. Calculate your new salary. What did you discover?
2. Repeat question 1 for a) if you get a 5% increase and then a 3% reduction and b) if you get a 6% decrease and then a 9% increase.
3. Suppose you have a 2% reduction on your $60.00 earnings, predict, then calculate the percentage increase you would need to return to $60.00 per week.

Money Patterns

Time to Invest!

How might interest on your savings help you reach your investment goal faster?

Compounding Interest

When you put your money into a savings account, your money earns interest. Most savings accounts give you "compound interest." Compound interest is interest that is added to interest. It is added to your account annually, semi-annually, monthly or daily.

A formula for compound interest is:

$A = P\ (1 + i)^n$

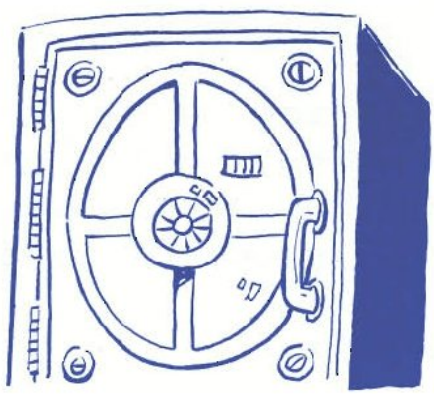
- A - total amount of money at the end of a period of time
- P - principal initially deposited
- i - rate of interest (usually shown as an annual amount and as a decimal)
- n - number of times interest is compounded

This formula is based on annual compound interest but it can be adapted for shorter terms.

Example:

compounding semi-annually would result in:

$A = P\ (1 + {}^{i}/_{2})^{2n}$ – *the amount of interest is halved because it is compounded semi-annually; but the times it is compounded is doubled because you get interest 2 times a year.*



Keys To The Calculator

To explore the keystrokes needed to solve problems using a compound interest formula - **$A = P\ (1 + i)^n$**

P = $150.00
i = 3%
n = 1 year

Solve for A by entering the data in order:
150 multiplied by
$(1 + .03)\ y^x\ 1 =$

1. What keystrokes would you use to find A for 5 years? Write them out.

2. Write out the keystrokes if
a) your interest is 6.5% and
b) your principal is $375.00?

3. Describe the way you enter data into the HP 6S when you use a formula.

Money Patterns

Web Links:

Moneyopolis
www.moneyopolis.com
This is a kid-friendly interactive Internet game about investing money.

On2 Money
www.pbs.org/newshour/on2/money.html
Here lies good background information about the history of money, the stock market, paying for college, getting a job and running your own business.

Growing Money

How do these different variables affect the way your money grows?

1. Use newspapers, the Internet or your local bank to find out the current interest rates on different kinds of savings plans.
2. Use an interest rate for one of the savings plans and the compound interest formula to investigate:
   a) The effect on the amount (A) as the principle (P) changes in the following four questions, but i = annual rate you chose and n = 5 years

      i) P = $1,000.00
      ii) P = $2,000.00
      iii) P = $10,000
      iv) P = your choice
      - How does (A) grow as the principal increases by a factor of 2, 10 and your choice?

   b) The effect on the amount (A) as the interest rate (i) is calculated for a shorter term in the following 3 questions, with P = $1,000.00 and n being counted over 5 years

      i) i = your semi-annual rate
      ii) i = your monthly rate
      iii) i = your daily rate
      - How does (A) grow as interest is calculated over a shorter term?
      - How much more would you need to add to $1,000.00 compounded annually to equal the amount (A) of $1,000.00 compounded daily after 1 year?

   c) Which causes (A) to grow more quickly – changing the principle or the term of interest? Explain your answer
3. How much would you need to invest now to pay for your first year of post-secondary education?

Power Sports

This two-part lesson explores irrational numbers and exponents by investigating the geometry of sports balls and athletic shoes.

NCTM Standards

Geometry and Spatial Sense

Analyze characteristics and properties of two- and three-dimensional geometric objects

- precisely describe, classify, and compare types of plane and solid figures (e.g. triangles, quadrilaterals, cylinders, cones, etc.) according to their main features
- analyze and understand geometric relationships among two-dimensional and three-dimensional figures
- use proportions to examine relationships between similar plane figures
- create and critique inductive and deductive arguments concerning geometric ideas and relationships
- recognize and apply geometric ideas and relationships outside the mathematics class-room, in areas such as art, science, and everyday life

Use visualization and spatial reasoning to solve problems both within and outside of mathematics

- develop fluency with two-dimensional representations of three-dimensional objects
- compose and decompose two- and three-dimensional figures in order to solve problems

Measurement

Understand attributes, units, and systems of measurement

- select appropriate units and scale to estimate and measure perimeter, area, surface area, and volume

Apply a variety of techniques, tools, and formulas for determining measurements

- develop and use formulas for the perimeter and area of parallelograms, trapezoids, circles, and simple composite figure
- develop and use formulas for the surface area and volume of prisms, pyramids, and cylinders
- select techniques and tools to measure accurately with levels of precision appropriate to the situation
- use ratios and proportions to solve problems involving scale factors

Power Sports

Getting Started

Use this table to prepare yourself and your students for this lesson.

Sheet No.	Materials	Vocabulary	Skills & Concepts
1	• pencils, paper • ping pong balls, tennis balls • squash balls/handballs,baseballs • baseballs, basketballs • string, scissors • tape measures • Bristol board, cardboard • HP 6S calculators	• circle • sphere • diameter • radius • pi • volume • circumference • net	• ratio of circumference to diameter • surface area of cylinders • and cubes
2	• student's own athletic shoes • chart paper with gridlines • rulers, protractors, compasses • HP 6S calculators	• mean • volume • cylinders • pyramids • rectangular prism • triangular prism	• bottom, side and back views of an object • volume of spheres, cylinders, pyramids, rectangular and triangular prisms • mean of data

Activity Overview

This table shows what students are doing in each activity, how they should do it and how their success can be measured.

Sheet No.	Activity	Method	Assessment
1	• measuring the circumference and diameter of spheres to calculate surface area of alternate packaging	• measure the diameter and circumference of sports balls to approximate the value of pi • calculate the surface area of circumscribed cylinders and cubes	• correctly and concisely measuring the diameter and circumference of the sports balls • correctly calculating pi to 3 decimal places • describing, with proper mathematical terminology, how to use surface area to calculate the amount of material needed to package an object
2	• measuring back, side and bottom areas of athletic shoes to calculate volume	• trace personal athletic shoe on large grid paper • re-write sentence with equivalent values • order integers (weather temperatures) on thermometer • looking for analogies to large numbers	• correctly and concisely partitioning a whole into familiar geometric shapes • correctly calculating volume of geometric solids • correctly representing equivalencies between fractions and percents

Power Sports

Sports Spheres!

How can you determine the most efficient way to package sports balls by using the constants found in circles?

Wrap it Up!

1. Gather together five sports balls of different sizes.
2. Find the circumference of each sports ball. Use a method that will give you the most accurate measurement. Explain your method.
3. Find the diameter of each sports ball. Use the most accurate method and explain it.
4. Complete the chart below.

Sports Ball	Circumference (C)	Diameter (D)	C/D as a Fraction	C/D as a Decimal
			Mean	

Verify Your Ratios

1. What is the name given to the ratio of the $\frac{circumference}{diameter}$?
2. How close did your mean come to the calculator's built-in value for this ratio? *(See Keys to the Calculator.)* Explain why there may be a difference.

Keys To The Calculator

Locate **pi** on your HP 6S.

1. Press **INV Exp**. What does your display read?

2. Is this all of **pi**? Explain.

3. Do not clear your display. Press **X** (times), 2 x^2 =. What does your display read? Explain what have you calculated. What happened when you pressed x^2 ?

4. Do not clear your display. List the keystrokes you use to retrace your steps and find the radius in #3.

5. To find the mean of C/D, press **INV. (SCI)**, enter the decimal values, press **DATA** after each entry then press **INV 4 ($\bar{x}$)** for the mean.

Power Sports

Did You Know?

Archimedes called the ratio of the circumference to the diameter "pi". He found that $223/71 < \text{pi} < 22/7$. What are the decimal equivalents of this range?

Today, pi is being calculated to millions of decimal places using computers. Why do you think the value of pi will never be exactly determined?

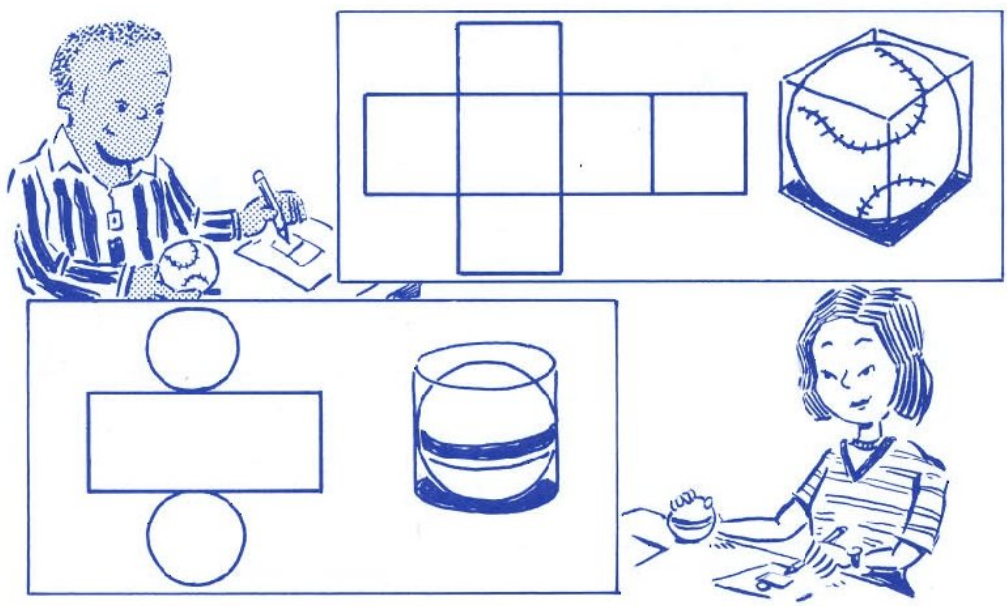
Web Links:

On-Line Math Applications
http://tqjunior.advanced.org/4116/History/history.html
Here, you can click on pages for Archimedes's exploration of spheres and cylinders and Pythagoras' exploration of triangles.

Search for 10 Million Digits of Pi Page
http://gryphon.ccs.brandeis.edu:80/~grath/attractions/gpi/index.html
This site explores the fact that pi isn't terminating or repeating.

Squaring the Circle

1. Pick one ball.
   a) Design a cube so that the ball fits inside the cube, touches all six faces of the cube and uses the minimum amount of material.
   b) Sketch a net for the cube you designed. Label the dimensions.
   c) Design a cylinder so that the ball fits inside the cylinder, touches the sides, top and bottom and uses the minimum amount of material.
   d) Sketch a net for the cylinder you designed. Label the dimensions.



2. Calculate the surface area (the amount of packaging material) required by the cube and the cylinder. Show your work.
3. a) Which method of packaging requires less material? Explain why.
   b) Why do you think this analysis of packaging is essential for manufacturers of sporting equipment?

Power Sports

Really Big Shoes!

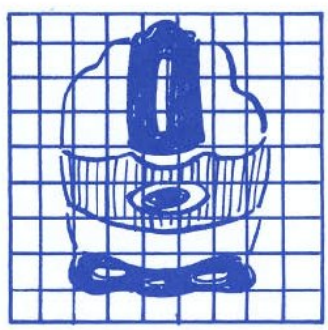
With the size of athletic shoes these days, it's getting hard to find space to store them. How much space would it take to store the class's athletic shoes?

Take Your Footprint

1. Choose an athletic shoe.
   a) How does your shoe compare to its original shoe box? How is it similar? How is it different?
   b) How would you find the volume of your shoe compared to its box?
2. The shape of your shoe makes it hard to accurately calculate its volume. Why does the shape of your shoe make it hard to calculate its volume?
   a) Find familiar three-dimensional geometric shapes that make up the shape of your shoe.
   b) Use your understanding of the volume of these shapes to estimate the volume of your shoe. Record your thinking.
   c) Trace three views of your shoe on grid paper. Label the views 1) bottom (sole), 2) side and 3) back (heel).
3. Draw and measure the familiar three-dimensional geometric shapes that make up the views of your shoe on the grid paper.

BOTTOM VIEW OF SHOE

SIDE VIEW OF SHOE



BACK VIEW OF SHOE

Keys To The Calculator

You can use the y^x key on the HP 6S to find the volume of a solid.

Example:
if a cube has 1.5 ft. sides, its volume is calculated by entering **1.5 y^x 3 =**.

1. What happened after you entered your value for x? Why did you have to press =?

2. Find the key for cube root. How would you use this key to retrace your steps to 1.5?

3. Did you need to press = to retrace your steps? Why or why not?

Power Sports

Web Links

Mathletics
http://www.richmond.edu/~ed344/96/math/sports.html
This excellent site has questions and facts about finding the math in sports.

SportLink
http://www.sportlink.com/research/1996~research/footwearmarket96/afmt96~toc.html
The Sporting Good Manufacturers Association operates this site. It has the latest market research in sports footwear.

Shoes in 3D

1. Calculate the volume of each familiar geometric shape.
2. Find the total volume for each shoe.
3. Compare your calculated volume to your estimate.
   a) Does the calculation seem reasonable? Why or why not?
   b) How accurate is your calculation? Explain.

Shoes of Class

1. As a class, determine the volume required to store all your running shoes.
2. How many shoes would be needed to fill the classroom?
3. How many students would belong to those shoes?

Athletic World!

In marketing, the word "volume" refers to the number of units sold. Let's look at the volume of the "volume" of athletic shoes sold in the United States.

1. Classify the categories of athletic shoes in your entire class. Use the following chart.

Type of Shoe	Number of Shoes in the Class	Fraction of Total	Percent of Total
Basketball			
Running/Jogging			
Cross Training			
Just Sneakers			
Tennis/Court			
Other Styles			

2. In 1995, there were 350.8 million pairs of athletic shoes sold in the U.S. Use your class' percentages to calculate the total number of shoes sold in each of the above categories.

Animating Geometry

This three-part lesson explores how geometry and measurement concepts and skills are applied to create backgrounds for animation and video games.

NCTM Standards

Geometry and Spatial Sense

Analyze characteristics and properties of two-dimensional geometric objects

- use proportions to examine relationships between similar plane figures
- create and critique inductive and deductive arguments concerning geometric ideas and relationships
- recognize and apply geometric ideas and relationships outside the mathematics classroom, in areas such as art, science, and everyday life

Measurement

Understand attributes, units and systems of measurement

- select appropriate units and scale to estimate and measure angles, perimeter
- understand both metric and customary systems of measurement, including relationships among units of the same system

Apply a variety of techniques, tools and formulas for determining measurement

- be proficient in measuring angles in plane figures
- select techniques and tools to measure accurately with levels of precision appropriate to the situation
- use ratios and proportions to solve problems involving scale factors
- determine an appropriate scale and use scale drawings or models in applications

Getting Started

Use this table to prepare yourself and your students for this lesson.

Sheet No.	Materials	Vocabulary	Skills & Concepts
1	•rulers •pencils and paper •protractors •HP 6S calculators	•angles •opposite, adjacent •hypotenuse •similar •tangent • vertex	• angles of triangles • ratios of sides as fractions and decimals • similar triangles • measurement of angles and sides
2	•rectangular piece of cardboard, scissors, string, jumbo paper clips, straws, measuring tape •HP 6S calculators	•clinometer (incline) •unreachable object	• relationships inside triangles • use of TAN key on the calculator
3	•graph paper, pencil crayons, ruler •HP 6S calculators	• rendering • background	• two-dimensional objects • converting conventional to metric units • choosing scale • measuring and drawing to scale

Animating Geometry

Activity Overview

This table shows what students are doing in each activity, how they should do it and how their success can be measured.

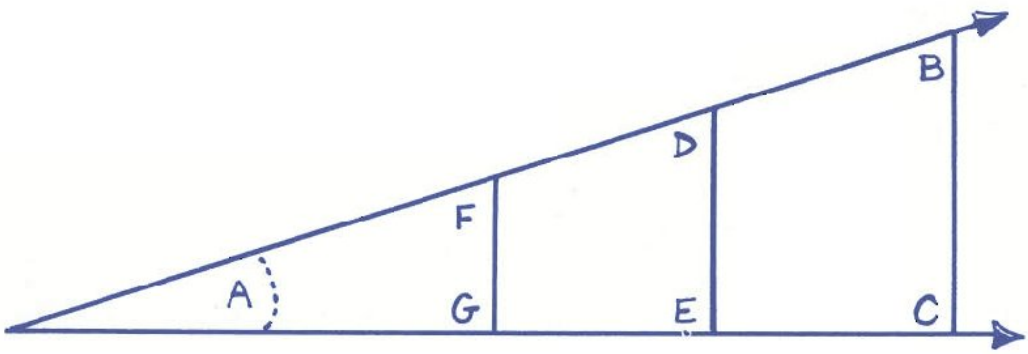
Sheet No.	Activity	Method	Assessment
1	• investigating ratios of sides of an angle in similar triangles	• measure angles • draw different sized triangles • measure the sides • calculate ratios • verify ratios with a calculator	• describing, using mathematical terminology, the relationships between the sides of angles for similar right triangles • accurately measuring the sides and angles of right triangles • accurately calculating ratios as equivalent fractions and decimals
2	• making a clinometer and using it measure the heights of unreachable objects	• work in pairs to create clinometer • measure distance to base of object and height to viewer's eyepiece • calculate unreachable heights using TAN key on calculator	• correctly creating a measuring instrument • correctly and concisely measuring with appropriate units • correctly calculating height of unreachable objects
3	• creating a detailed 2D scale drawing of a skyscape or landscape using measurements gathered with a clinometer	• plan an interesting background • choose objects to measure • conduct measurements of heighth and width of objects • measure placement of significant details • calculate unreachable heights	• choosing an appropriate scale and measurement units for rendering • accurately drawing scale and proportions • describing the development of the rendering using proper mathematical terminology

Starting Triangles!

How does the size of an angle affect the ratios of the lengths of its sides in triangles.

Draw Your Triangles

1. In your groups, choose any angle between 0 and 90 degrees.
2. Draw the angle with a protractor and ruler so that the lengths of its rays cover your page. Label the vertex of this angle "A".
3. Choose a point near the end of your bottom ray to draw a vertical perpendicular line that connects the two rays.
4. Measure the two new angles with your protractor. Record your measurements on your triangle. Label the vertices of the angles "B" and "C".
5. Draw two perpendicular lines – parallel to the first one – inside your triangle.
6. Label the four newly created vertices D, E, F and G.
7. How would you describe triangles ABC, ADE and AFG?



Remember!

Triangles are "similar" when they have the same shape and their angles have the same measurements.

Keys to the Calculator

The ratio

$$\frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

of an angle in a triangle is called its *tangent*.

What happens to the tangent when your angle "A" in a right triangle increases in degrees?

1. Find the tangent key on your HP 6S. How is it labeled?

2. Enter the degrees of your angle A. Press the tangent key. Describe what happens.

(continued on reverse)

Animating Geometry

Keys to the Calculator

3. Find the tangents for the angles measuring 0 to 90 degrees for every ten degrees. Record these on a table.
a) What happened to the value of the tangents as the angle increases?
b) For what angle does the tangent equal 1?
c) What type of right angle triangle gives you a tangent of 1?

4. Find the tangents for the angles measuring 80 to 90 degrees going up by one degree at a time. Record these on a table.
a) What happens to the values of the tangent as you get closer and closer to 90 degrees?
b) What does the calculator read at the tangent for 90 degrees? Explain the result on the display. Use ratios of sides as part of your discussion. Draw the triangles to show their shape as angle "A" approaches 90 degrees.

Investigate Your Triangles

1. Complete the following table for angle "A". Use your HP 6S to display the ratios as a fraction and then as a decimal.

Triangle	Length of Opposite Side	Length of Adjacent Side	Ratio $\frac{\text{Opposite}}{\text{Adjacent}}$	Decimal Equivalent
ABC				
ADE				
AFG				

2. What do you notice about the decimal values?
3. Use your HP 6S to verify the decimal value for each triangle. *(See Keys to the Calculator.)* Does the calculator give you the same result? Explain any difference.
4. Share your findings.
   a. Describe the patterns you have discovered.
   b. From what you have discovered, what conjecture can you make about the relationship between the angles and the ratios of the lengths of sides of these kinds of triangles? Why do you think this is true?
   c. Test your conjecture by investigating other ratios for these kinds of triangles:

$$\frac{\text{length of the opposite side}}{\text{length the hypotenuse}} \quad \textit{and} \quad \frac{\text{length of the adjacent side}}{\text{length of the hypotenuse}}$$

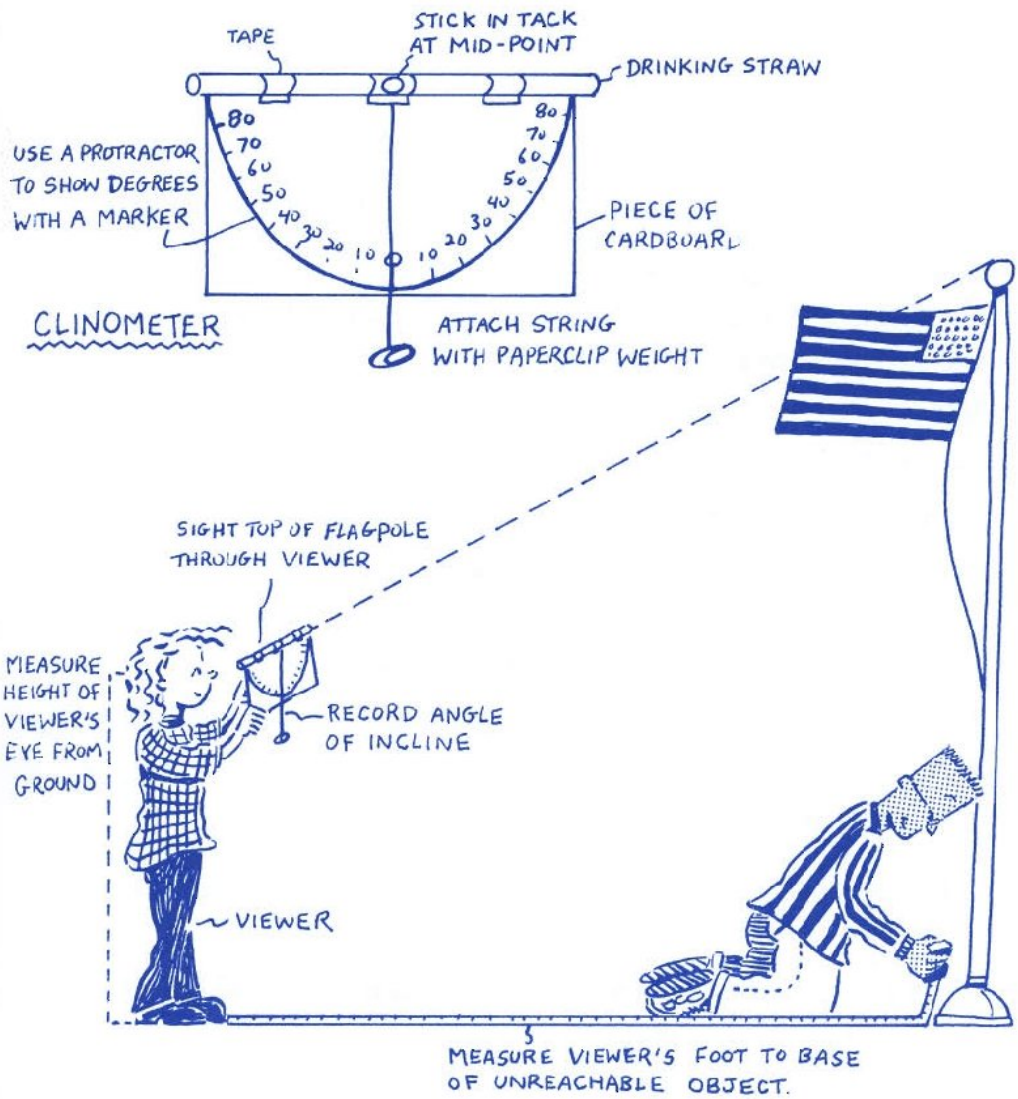
5. Does your conjecture fit all cases? If it does, restate it as a mathematical truth. If it does not, modify it.

Reaching the Unreachable!

How can you accurately find the heights of objects you cannot easily measure?

Making Your Clinometer:

A device that allows you to measure unreachable objects is called a "clinometer." This name comes from its use of inclines. Use this diagram to make your clinometer.



Did You Know?

Ancient mariners used an instrument called a sextant to calculate their ship's latitude. A sextant works on the same principle as a clinometer. However, it uses one sixth of a circle rather than half a circle as its graduated arc for measuring angles. Find out more about sextants and other similar measurement devices.

Animating Geometry

Web Links

Cool Math
www.coolmath.com
This site offers a wealth of math material. Enter "triangles" in the search engine to find problems using non-congruent triangles on different-sized grids.

Geometry Center
www.ScienceU.com/geometry
Shapes, patterns and symmetry are explored at this interactive site.

Native American Geometry
www.earthmeasure.com
This site offers an exploration of a physical, proportional geometry that originates from the simple circle.

Using Your Clinometer:

1. Work with a partner. Take turns being the viewer and the measurer.
2. In your schoolyard, choose one object that is too high to measure – your school building, a flagpole, playground equipment, tall trees, etc.
3. Use this table to record the following measurements.

Object Viewed	Angle of Clinometer (A)	Distance from Viewer to Base of Object (D)	Height of Viewer's Eye from the Ground (H)

4. Why do we need to measure the height of the viewer's eye from the ground?
5. Describe the type of triangle you are creating when you view the unreachable object?
6. What do you know about the ratios of sides in this type of triangle?

Calculating the Height

1. What is the definition of the tangent of an angle?
2. How do you think knowing the tangent will help you calculate the height of your unreachable object?
3. Use the following formulas to calculate the height of your object.

a) tangent of A = $\frac{\text{height of object from viewer's eye level}}{D}$

b) height of object from viewer's eye level + H

3. Find the height of three different unreachable objects in or near your schoolyard.

Videoscapes!

Use five actual buildings, local landmarks or elements from landscapes in your area to create the background of a brand new video game.

Planning

Decide what objects you would like to use in your videoscape.

Measuring and Calculating

1. Work with a partner. Each of you will create your own videoscape.
2. Use your clinometer to measure the height of your objects.
3. How do you measure the width of buildings?
4. Record your measurements on the following table.
5. Use your HP 6S to calculate the actual height of all five objects.

Object Viewed	Angle of Clinometer	Distance from Viewer to Base of Object	Height of Viewer's Eye from the Ground	Width of Building	Calculated Height of Building

Did You Know?

Reproducing or representing a real-life person, place or thing is called "rendering." To render an object means to reproduce it in as much detail as possible.

Animating Geometry

Software Connection

Use a computer illustration program to draw your videoscape. What do you notice about the use of geometry in such a program?

Did You Know?

The first four function mechanical calculator was made by Gottfried Leibniz in 1694. It was sold commercially in the mid-1800s.

Two-dimensional Rendering

What do you think would be the most useful scale for the objects you measured?

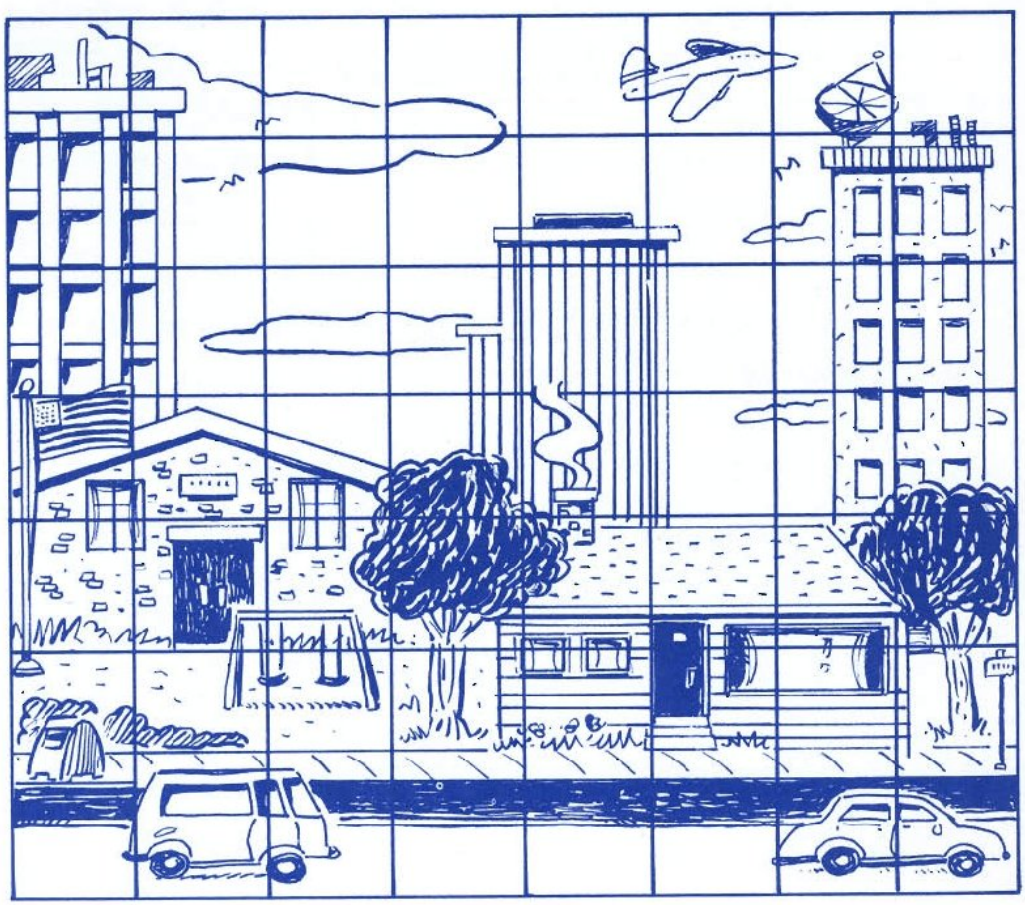
If conventional units are too large, consider converting your measurements into metric

Example:

1 yard = 1 inch or 36:1 vs. 1 meter = 1 millimeter or 1000:1

Use graph paper to draw your videoscape background to scale. Show as much detail as possible so your background is true to life.

If you have a digital camera, take shots of the objects in your videoscape. They could help fill in the details on your rendering.



Imagine The Image

This three-part lesson investigates relationships in rectangles as they are used in magazines and art.

NCTM Standards

Data Analysis, Statistics and Probability

Pose questions and collect, organize, and represent data to answer those questions

- recognize types of data (e.g. categorical, count, continuous or measurement, and organize collections of data
- choose, create and utilize various graphical representations of data (line plots, bar graphs, and circle graphs) appropriately and effectively

Interpret data using methods of exploratory data analysis

- find, describe, and interpret mean as a measure of the center of a data set
- interpret graphical representations of data, including description and discussion of the meaning of the shape and features of the graph

Develop and evaluate inferences, predictions, and arguments that are based on data

- develop conclusions about a characteristic in the population from a well-constructed sample
- use data to answer the questions that were posed, understand the limitations of those answers, and pose new questions that arise from the data

Getting Started

Use this table to prepare yourself and your students for this lesson.

Sheet No.	Materials	Vocabulary	Skills & Concepts
1	• magazines • HP 6S calculators	• ad formats • gutter • margin	• percents • graphing
2	• pencils, rulers, paper • magazines • artbooks • HP 6S calculators	• Golden Mean	• mean • measurement
3	• photocopier • magazines • HP 6S calculators	• magnification • enlargement • reduction	• measurement • dimensions • areas • percents

Imagine The Image

Activity Overview

This table shows what students are doing in each activity, how they should do it, and how their success can be measured.

Sheet No.	Activity	Method	Assessment
1	• finding percentage of advertising versus information in popular magazines	• categorize ads and graphing the results to find the types of ads associated with specific magazines	• correctly and accurately use operations with fractions • choose appropriate graphs to solve a problem • use proper techniques to graph data • correctly and accurately find percentages
2	• conducting statistical analysis of pleasing rectangles to determine common elements	• measure pleasing and non-pleasing rectangles to find patterns in the ratio of length to width	• accurately finding ratios of prescribed measurements • correctly determining the mean of data
3	• using a photocopier to investigate the nature of magnification	• enlarge and reduce a rectangle by various percentages and measuring dimensions	• accurately measuring rectangles • correctly calculating area • describe, using correct mathematical terminology, what percent means to the size of a photocopied reproduction

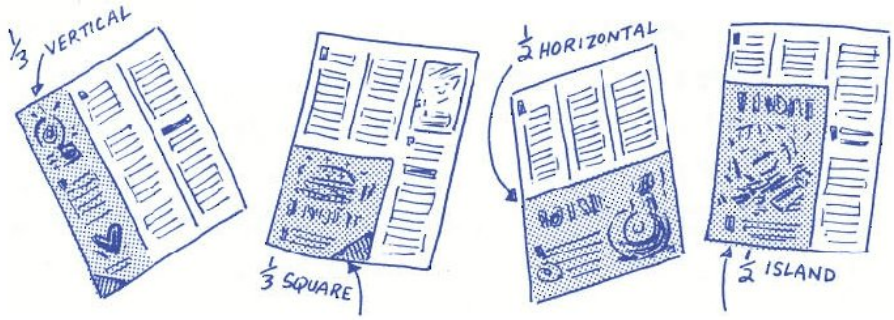
Imagine The Image

Magazine Percents

When you purchase a magazine, how much are you spending on advertising and how much on information?

Grab a Mag!

1. Choose an issue of your favorite magazine. Use a magazine that has a three-column format.
2. Use the illustrations of the ad formats below to help you tally all the advertising in your magazine. Consider a page full of different ads as a one-page ad.
3. Calculate the total number of ads of each format in your issue.
4. Calculate how many pages in total each ad format amounts to.
5. Find the sum of all the pages taken by advertising.



Ad Format	Tally	Number of Ads	Number of Pages
1/3 vertical			
1/3 square			
1/2 horizontal			
1/2 island			
1 page			
2 page			
		Total Number of Pages	

Keys to the Calculator

Find the percent sign on the HP 6S keypad: Enter 33, **INV** =.

1. What does the display read? Explain why.

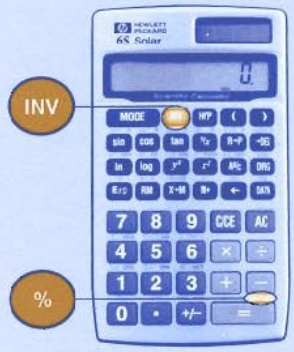
To find what percent is 15 ads out of 65 ads, enter **15 ÷ 65 INV % =**

2. What happened when you pressed the percent key?

3. How would you find the same answer without using the percent key?

4. Find the percentage of:
a) 220 half page ads out of 1650 ads
b) 72 out of 95 marks on a math test

(continued on reverse)



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Keys to the Calculator

5. How would you use the percent key to find 12% of the ads out of 250 ads?

6. How could you use the automatic constant to find 12% of 450 ads? Explain your method to your partner.

7. Use the automatic constant to find 6% of:
a) 250 pages
b) 216 pages
c) 104 pages

Did You Know?

Magazines commonly use a three-column design. The space between each column is called a gutter. The gutter is also the term used to describe the fold in the center where two pages are joined together. Most people flip through a magazine like a deck of cards. Why does this explain the placement of ads on the outside left or right side of a page?

Analyze Your Ads

1. Choose an appropriate graph to show each ad size as a percentage of all the ads. Use your HP 6S to calculate the percentages. *(See Keys to the Calculator.)*
2. Which ad size is used most often? Which ad size is used least? Look at the type of ads in each of these formats.
3. Compare your graph to the graph of another classmate who has used:
   a) A magazine similar to yours
   b) A magazine different to yours
4. How are these graphs similar? Different?
5. Why do you think this is so? Discuss.

Ads or Information?

1. Choose an appropriate graph to show the number of pages of ads as a percentage of all the pages in the magazine. Use your HP 6S to calculate the percentages. *(See Keys to the Calculator.)*
2. What is the ratio of advertising to non-advertising in this magazine?
3. Use this ratio and the price of the magazine to determine how much the reader pays for advertising and how much for content.
4. Do you believe the newsstand price covers the cost of producing the magazine? Why or why not? How would you find this out?
5. In purchasing the magazine, do you value the advertising and the content in the same relationship to the price that you pay for each? Why or why not?

The Golden Mean

What is the relationship between the sides of a rectangle that is pleasing to look at compared to one that is not?

A Mean Rectangle

1. Use the measurements on the table to construct the six rectangles.
2. Complete the table based on your assessment of each rectangle.

Rectangle No.	Length	Width	Pleasing or Non-pleasing?	Fraction Ratio Length/Width	Decimal
1	4 in.	2 in.			
2	$4\frac{7}{8}$ in.	3 in.			
3	3 in.	2 in.			
4	$6\frac{1}{4}$ in.	4 in.			
5	$3\frac{1}{2}$ in.	$2\frac{3}{4}$ in.			
6	$1\frac{1}{4}$ in.	1 in.			

3. How did you decide whether or not a rectangle was pleasing or non-pleasing?

Keys to the Calculator

To find the mean on the HP 6S:
Press **MODE. (SCI)**

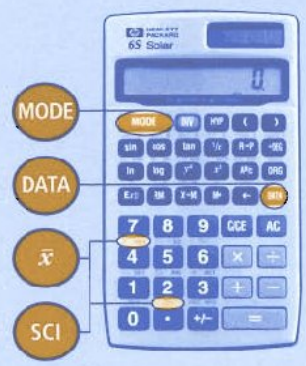
Enter the decimal values for your pleasing rectangles. Press **DATA** after each entry.

Press **INV** 4 ($\bar{x}$) to find the mean.

1. What is another way to find the mean of the data?

2. Find the mean of the decimal ratio of length to width for:
a) a 3 by 5 inch file card
b) a 5 by 7 inch photo
c) a sheet of notebook paper
d) the HP 6S
e) a blackboard or chart paper

(continued on reverse)



Imagine The Image

Keys to the Calculator

3) Enter 1.618033988 or the golden mean on your HP 6S.
a) Square it. What do you notice about the square?
b) Enter $^1/_{1.618033988}$ What do you notice about the quotient?

4) Find the class mean of student heights over the height of their navels from the ground. What does this mean approximate?

Did You Know?

The Golden Mean or phi is a mysterious non-repeating, non-terminating decimal like pi. It was first used by the Egyptians to build the Great Pyramid in 2600 B.C., for the ratio of the length of one side to the altitude is about 8 to 5 or 1.625.

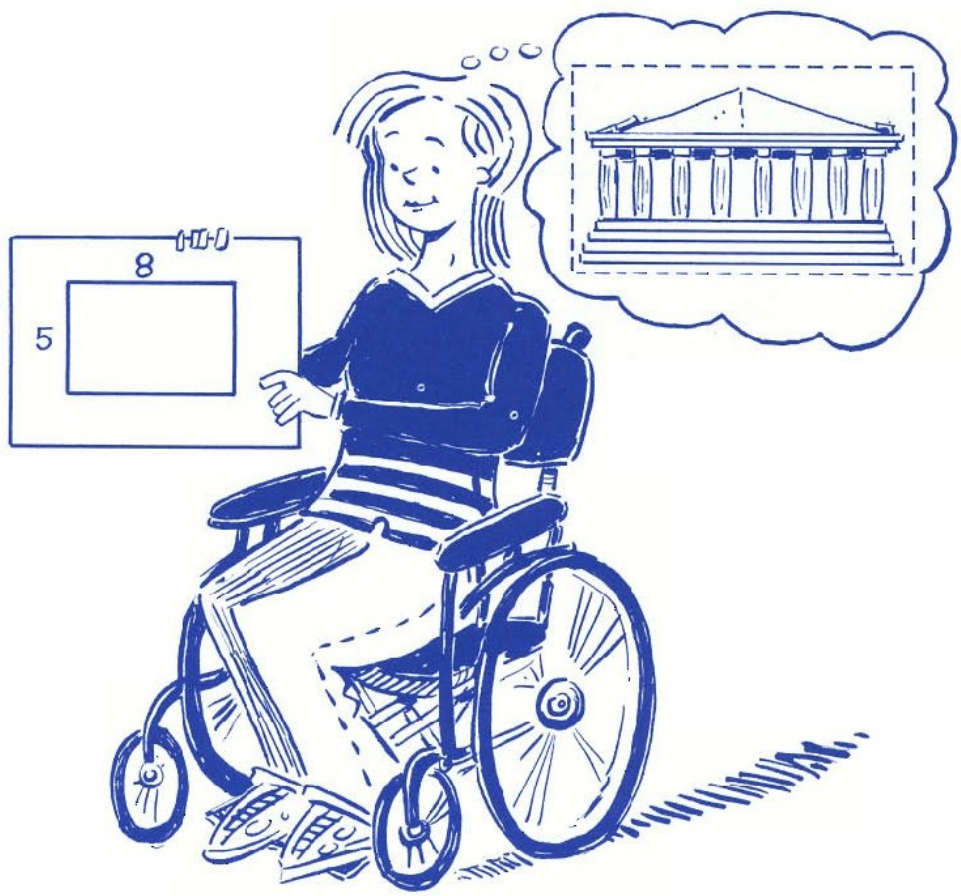
The Greeks used this ratio in their art and architecture. This pleasing proportion was felt to be the embodiment of "beauty."

Where's the Gold?

1. a) Describe the decimal values for the non-pleasing rectangles.
   b) Describe the decimal values for the pleasing rectangles.
   c) What pattern do you find in the decimal values of the pleasing rectangles compared to the non-pleasing?

It is said that the ratio of length to width for pleasing rectangles approaches 1.618033988... This decimal is called the "Golden Mean." It was named "phi" after the greatest Greek sculptor Phidias, who used this proportion in his work.

2. What is the simplest fraction equivalent that approximates phi?
3. Find five examples of pleasing rectangles in magazines, art books or elsewhere. Determine how close their proportions approximate the Golden Mean.



Imagine The Image

Magnificent Magnifiers!

When you use a photocopier to enlarge or reduce an image, do you increase its area, its length, its width, its length and width, or all of them?

Your Favorite Rectangle!

1. When a photocopier reproduces an image at less than 100%, what does that mean for the size of the reproduced image? What does it mean if it reproduces at greater than 100%?
2. Draw a Golden Mean rectangle so that it covers no more than half the area of the sheet of paper. Record its length, width and area in the table.
3. Use a photocopier to reproduce your rectangle according to the percentages on the following table.
4. Measure the copied rectangles and calculate their areas.

Percentage	Length	Width	Area
50%			
75%			
100% (original)			
125%			
150%			
200%			

Keys to the Calculator

To use the percent key on the HP 6S to find enlargements and reductions:
15% enlargement of a 250 sq. in. photograph – enter
250 + 15 INV % =

1. How would you change these keystrokes to find a 15% reduction?

2. Find the following enlargements and reductions for a magazine lay-out:
a) 25% added onto $2\frac{1}{2}$ inches
b) 65% taken away from $5\frac{3}{4}$ inches
c) 45% large than 18 sq. in.

Imagine The Image

Web Links

The Golden Ratio
www.kutztown.edu/~fithian/Geometry/Activity-02.html
This site contains some more activities that explore the occurrence of the Golden Mean.

The Most Pleasing Rectangle Web Poll
homepage.esoterica.pt/~madureir/
Place your vote for the most pleasing rectangle.

Copy or Original?

1. Look at the information in the 200% row.
   a) By what factor is the length of the enlargement greater than the length of the original?
   b) Repeat 1. a) for the width and area.
   c) How do these factors compare to the percentage of enlargement?
2. Make a conjecture about the percentage of enlargement (200%) compared to the length, width and area of the original.
3. Test your conjecture using the other enlargements.
4. Is your conjecture still true? Why or why not?
5. Test your conjecture on the results of the reductions. Is it still true?
6. How would you modify your conjecture to hold for enlargements and reductions?

Fix a Design

1. Find a page of a magazine that you think could be better designed. Make sure it has several different elements such as text, photographs or illustrations.
2. Photocopy the page in its original form.
3. Cut out all the elements.
4. Use your knowledge of the Golden Mean to enlarge or reduce the size of each element to rearrange them into a more pleasing design on the same sized page.

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