

HP-70 your key to better real estate decisions



solutions to more than forty types of real estate problems





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INTRODUCTION

The HP70: Your Key to Better Real Estate Decisions is written to present the problem solving capabilities of the HP-70 specifically in real estate terms. Step by step keystroke procedures and examples for over 40 problem types are explained. These can be altered and combined to provide solutions to many more real estate problems.

Whether you're a real estate professional or investor, you'll find the HP-70 and this book invaluable for reducing your calculation time and effort in analyzing real estate transactions. No longer are cumbersome financial tables and texts needed—the HP-70 allows you to merely press the appropriate keys and obtain your answer in seconds.

To help you find the solution applicable to your situation, the table of contents lists each problem type contained in the book. For example, if you want to determine the periodic payment amount of a fully amortized mortgage loan, refer to **Periodic Payment Amount** under **Simple Mortgages**. Or if you need to know the interest paid on a loan during the year, see **Accumulated Interest and Remaining Principal Balance** under **Simple Mortgages**.

A special word of thanks goes to the many real estate professionals who have been most helpful in advising us, without whom this book could not have been completed. In particular, we would like to thank Mr. LeR Burton of Salt Lake City, Utah.

HOW TO READ THIS HANDBOOK

HP-70 problem solutions in this book are presented in step by step keystroke form. The general procedure is shown first, followed by an example. For ease of understanding, the examples show solution step numbers corresponding to the numbers shown in the general procedure. To further clarify the examples, intermediate results are shown and comments explaining the displayed answer are given where needed.

In both the procedure and examples, numbers to be keyed in are shown without boxes while function keys are shown with boxes around them \Box . Problems and solutions appear as follows:

Example:

What is the monthly payment amount for a 30 year, \$35,000 mortgage at 8.75%?

Solution

	Enter	See Displayed	
	CLR	0.00	
1)	35000 PV	35000.00	Mortgage amount.
	30 K 🗙 n	360.00	Total periods in mortgage life.
	8.75 K 🗧 i	.73	Monthly interest rate.
2)	РМТ	275.35	Monthly payment.

The data entry order indicated in the general procedure must be followed in sequence. Where "Calculate and Key in (any order)" is indicated, as in step 1) of the above problem, the data within that step can be entered in any order.

NOTE:

Throughout this book where monthly calculations are shown, the K register is assumed to have a 12 in it. If this is not true, adjust the keystrokes by either putting 12 into K (12 STO K) and proceed as shown or replace K in the solution keystrokes by ENTER+ 12.

Additional information regarding the operation of your HP-70 calculator is contained in your HP-70 Owner's Handbook.

REMEMBER-

The following assumptions are made within the preprogrammed financial functions contained in the top row keys.

- -Payments occur at the end of the time period.
- -The initial amount (present value) occurs at the beginning of the first period.
- -The final amount (future value) is the amount at the end of the last period.
- -Interest is compounded at the same time periodic payments are due.
- -The interest rate per period is entered as a percent, not a decimal.
- -If a rounded payment amount is used, the exact number of payment periods must be calculated (see page 29) to yield exact answers when payment amount and number of payments are both used as inputs.
- -K contains a 12 in storage when machine is turned on.

If your problem differs from any of these assumptions, you'll need to vary the keystrokes. For example, if your payments occur at the beginning of the period, the keystroke solution must be adjusted (see Chapter 7 entitled Annuity Due Calculations.)

Now you're ready to start saving hours of calculation time!

1. PERCENTAGE CALCULATIONS

Percentage calculations are frequently used in real estate, for example, in calculating commissions or down payment amount. In the following sections, many problem solutions involving percentages are explained.

PERCENTAGE RATE

To find what percent one number is of a base amount the following keystrokes are used:

- 1) Key in the specified number, press **ENTER**.
- 2) Key in the base amount, press 🗄 100 🔀.

Example 1:

If the required down payment on a \$44,500 house is \$10,000, what percentage rate does this represent?

Solution

Enter		See Displayed	
1)	10000 ENTER+	10000.00	
2)	44500 🚼 100 🔀	22.47	Percent down.

Example 2:

Mr. Anderson has assets totaling \$65,000. What percent of his assets are in real estate if he owns a home worth \$35,000 and a small lot worth \$6000?

Solution

Enter		See Displayed	
1)	35000 ENTER+ 6000 +	41000.00	
2)	65000 芒 100 🔀	63.08	Percent of assets in real estate.

PERCENTAGE OF BASE

A base amount multiplied by a percentage rate yields a percentage of base, performed as follows:

- 1) Key in the base amount, press **ENTER**.
- 2) Key in the percentage rate, press %.

10 Percentage Calculations

NOTE:

Use the percentage rate, not the decimal equivalent, when entering.

Example 1:

Mortgage requirements for the Hidden Oaks Townhouses include a 10% down payment. What is the down payment on a \$39,950 unit?

Solution

	Enter	See Displayed	
1)	39950 ENTER+	39950.00	
2)	10 %	3995.00	Down payment amount.

Example 2:

If a real estate broker receives 6% commission on a sale, what is the commission on a \$37,300 house?

Solution

Enter		See Displayed	
1)	37300 ENTER+	37300.00	
2)	6 %	2238.00	Broker's commission.

NET AMOUNT

A percentage of the base amount can be added to or subtracted from the base amount to give the net amount as follows:

- 1) Key in the base amount, press **ENTER**
- 2) Key in the percentage rate, press %.
- 3) Press 🛨 or 🚍 to obtain net amount.

Example 1:

A home valued two years ago at \$51,500 has appreciated 4%. What is its current value?

Solution

	Enter	See Displayed	
1)	51500 ENTER+	51500.00	
2)	4 %	2060.00	
3)	+	53560.00	Current value.

Example 2:

A salesman sold his broker's listing for \$39,500 and is entitled to 40% of the 6% commission. What are the two commissions?

Solution

	Enter	See Displayed	
1)	39500 ENTER+	39500.00	
2)	6 %	2370.00	Total commission.
	40 %	948.00	Salesman's commission
3)	•	1422.00	Broker's commission.

PERCENT DIFFERENCE

Percent difference (percent of increase or decrease) is defined as the difference between a base amount and another amount, divided by the base amount. Information is entered as follows:

- 1) Enter the base amount, press **ENTER**.
- 2) Enter the other amount, press Δ %.

Example:

One year ago new houses in Sunshine Acres sold for \$39,500. The current price of new units is \$45,000. What percent increase does this represent?

Solution

	Enter	See Displayed	
1)	39500 ENTER+	39500.00	
2)	45000 🛆%	13.92	Percent increase.

NOTE:

The answer displayed is a percent of the base amount, \$39,500.

2. APPRECIATION CALCULATIONS

One of the most important factors in many real estate transactions is appreciation. Usually an appreciation rate is compounded over time in the same way interest is compounded on savings accounts. You may hear about property increasing 5% per year for the last 6 years. This means that the overall appreciation rate has been 34% rather than 30% due to the effect of compounding.

FUTURE VALUE OF A COMPOUNDED AMOUNT

This calculation finds the future value of an initial amount appreciated/ depreciated at a given rate, compounded over a specified number of periods.

Information is entered as follows:

CLR

- 1) Calculate and key in (any order):
 - Total number of time periods, press
 - Rate per period (expressed as a %), press
 - Initial principal (present value), press **PV**.
- 2) Press **FV** to obtain the future value.

NOTE:

Under some circumstances, property values may actually decline rather than increase. The above keystrokes are still valid; the only change is that when entering a rate of decline, the CHS key must be pressed before pressing in to indicate the negative rate.

Example 1:

Land in Forest Acres is predicted to appreciate at the rate of 9% per year for at least the next four years. At this rate, what will land be worth at the end of four years, if it is currently selling for \$8000 per acre?

	Enter	See Displayed	
	CLR	0.00	
1)	9	9.00	Appreciation rate per year.
	4 n	4.00	Number of years.
	8000 PV	8000.00	Current price.

2) F۷ 11292.65 Future price.

What would the future price be if appreciation is 14% per year?

14	F۷	13511.68	Future price.
			i atare pricer

Example 2:

Property values in the older part of the city have been experiencing a decline of 4% per year. If this rate continues, what will a \$32,000 house be worth in three years?

Solution

	Enter	See Displayed	
	CLR	0.00	
1)	3 n	3.00	Number of years.
	4 CHS	-4.00	Rate of decline.
	32000 PV	32000.00	Current value.
2)	FV	28311.55	Future value in 3 years.

NOTE:

Often it is useful to have an appreciation schedule, showing yearly values based on a given appreciation rate. To do this the following keystrokes can be used.

- Press 1 ENTERA . Key in assumed annual appreciation rate, press %. 1)
- Press 🚹 STO M 2)
- Enter the base value, press \mathbf{M} \mathbf{X} to see the value after one year. 3)
- Continue pressing M 🔀 to obtain each successive year's value. 4)

Example:

If the current rate of appreciation is 8% annually and this rate is expected to continue for the next four years, how much will a house currently worth \$38,000 be valued at in each of the next four years?



14 Appreciation Calculations

3)	38000 M 🔀	41040.00	Value after first year.
4)	M	44323.20	Value after second year.
	M	47869.06	Value after third year.
	M	51698.58	Value after fourth year.

PRESENT VALUE OF A COMPOUNDED AMOUNT

This calculation finds the present value (initial principal amount) when the future value, number of periods, and appreciation/depreciation rate are known. Keystrokes are:

CLR

- 1) Calculate and key in (any order);
 - Total number of periods, press
 - Appreciation/depreciation rate per period, press
 - Future value, press **FV**.
- 2) Press **PV** to obtain the present value.

Example 1:

A piece of land in a surburban community is within walking distance of a recently completed rapid transit system, and it can be purchased for 10,000. Neighbors claim that their property has been appreciating at 1% per month since the construction of the commuter system began four and a half years ago. If this is true, what would have been the value of the property then?

	Enter	See Displayed	
	CLR	0.00	
1)	10000 FV	10000.00	Current property value.
	4.5 K x n	54.00	Total number of months.
	1	1.00	Appreciation rate per month.
2)	PV	5843.13	Value of the property 4½ years ago.

PERIODIC APPRECIATION/DEPRECIATION RATE

Given the number of periods, the value of the property at the beginning of the first period and end of the last period, the periodic appreciation/depreciation rate can be found as follows:

CLR

- 1) Calculate and key in (any order):
 - Total periods, press
 - Initial value of the property, press **PV**.
 - Final value of the property, press **FV**.
- 2) Press to obtain the periodic appreciation/depreciation rate.

Example 1:

A Realtor has just listed a house in Happy Valley which was bought 3 years ago for \$29,000. The current asking price is \$36,750. What yearly appreciation rate does this represent?

Solution

	Enter	See Displayed	
	CLR	0.00	
1)	3 ⁿ	3.00	Total periods.
	29000 PV	29000.00	Value 3 years ago.
	36750 FV	36750.00	Today's value.
2)		8.21	Annual appreciation rate.

Example 2:

Mr. Brown purchased his house 4 years ago for \$45,000. Since that time, the planning commission has proposed a freeway that will adjoin his backyard. His house is for sale and the best offer he has received is \$42,000. What annual rate of decline does this represent?

	Enter	See Displayed	
	CLR	0.00	
1)	4 n	4.00	Number of years.
	45000 PV	45000.00	Original price.
	42000 FV	42000.00	Current price.
2)	ì	-1.71	Annual rate of decline.

NUMBER OF PERIODS IN A COMPOUNDED AMOUNT

This calculation finds the number of compounding periods when the appreciation or depreciation rate per period, initial amount (present value) and compounded amount (future value) are known.

Information is entered as follows:

CLR

- 1) Calculate and key in (any order):
 - Rate per period as a percent, press
 - Initial amount, press PV .
 - Compounded amount (future value), press **FV**.
- 2) Press **n** to obtain the number of time periods.

Example 1:

Property currently worth \$42,000 is in an area that has been appreciating at 4% annually. If this rate continues, how many years until the property will be worth \$55,000?

Solution	
----------	--

	Enter	See Displayed	
	CLR	0.00	
1)	42000 PV	42000.00	Present value.
	4 i	4.00	Annual growth rate.
	55000 FV	55000.00	Desired future value.
2)	n	6.88	Number of years.

3. DEPRECIATION CALCULATIONS

Depreciation is a means of allocating the cost of an asset over its useful life. The three common accounting methods of calculating depreciation are straight line, declining balance, and sum-of-the years' digits (SOYD) depreciation. Declining balance and SOYD are accelerated methods which provide higher depreciation amounts in the early years of an asset's life than straight line. Figure 1 shows a comparison of the three amounts.

One important fact to remember—since land does not wear out, its cost may not be depreciated. Therefore, the cost of improvements (building, fixtures) must be separated from the cost of the land before calculating a property's depreciation. Also, if there is an expected salvage value at the end of the asset's life, this must be subtracted from the cost of the property when using the straight line or sum-of-the-years' digits method. If the declining balance method is used, the salvage value is not subtracted initially, but the asset may not be depreciated below this salvage value.

Another point regarding accelerated depreciation—at the time of resale, the excess depreciation must be calculated for tax purposes. Excess depreciation is the difference between the depreciation amount taken and the amount that would have been taken using the straight line method. (See example p. 22.)

The calculations below show you how to easily solve for depreciation amounts and remaining depreciable values.

NOTE:

Salvage value added to depreciable value equals book value. Since salvage value is not subtracted before applying the declining balance method, the remaining value found with that method is the book value. The remaining value found with SOYD or straight line is the remaining depreciable value.



Figure 1: Annual depreciation amounts for the first 5 years of useful life using each depreciation method. Figures based on a \$10,000 asset with a 10 year useful life.

STRAIGHT-LINE METHOD

The annual depreciation allowance using this method is determined by dividing the cost of the property (excluding land costs) less its estimated salvage value by its useful life expectancy. Information is entered as follows:

- 1) Calculate and key in depreciable amount (improvements cost less salvage value) press ENTER+ ENTER+.
- 2) Key in asset's useful life (number of years), press 🗄 to obtain each year's depreciation.
- 3) Press STO M lto obtain depreciable value after the first year.
- 4) Continue pressing M = to obtain the remaining depreciable value for each subsequent year. If book value is needed, add salvage value to depreciable value.

Example 1:

A duplex costing \$41,500 (exclusive of land) is depreciated over 25 years, using the straight line method. What is the annual depreciation amount and remaining depreciable value for years 1 and 2, if it has no salvage value?

	Enter	See Displayed	
1)	41500 ENTER+ ENTER+	41500.00	Depreciable value.

Depreciation 19

2)	25 💼	1660.00	Annual depreciation amount.
3)	STO M -	39840.00	Remaining depreciable value year 1.
4	M	38180.00	Remaining depreciable value vear 2.

NOTE:

The remaining balance at the end of a particular year can be found directly without calculating data for each preceding year. The following keystrokes are used (also see Example 2).

- 1) Calculate and enter depreciable amount, press ENTER+ ENTER+.
- 2) Enter estimate of useful life; press 🗧 STO M ENTER↑.
- 3) Enter number of year for which data is desired, press 🔀 to obtain total depreciation to date.
- 4) Press = to obtain remaining depreciable value to date.
- 5) Press M = to obtain remaining depreciable value for each subsequent year.

Example 2:

Using the duplex in Example 1, what is the remaining depreciable value at the end of year 7? At the end of year 8?

	Enter	See Displayed	
1)	41500 ENTER+ ENTER+	41500.00	Depreciable value.
2)	25 🛨 STO M ENTER+	1660.00	Annual depreciation.
3)	7 🗙	11620.00	Total depreciation first 7 years.
4)		29880.00	Remaining depreciable value year 7.
5)	M 😑	28220.00	Remaining depreciable value, year 8.

DECLINING BALANCE METHOD

The declining balance method is one form of accelerated depreciation; as such it provides for more depreciation in the earlier years of ownership and less depreciation in the later years than the straight line method. A constant percentage of the remaining balance is applied each year to find the depreciation amount.

The following calculations find the depreciation and remaining book value for each year of an asset's depreciable life when the declining factor, cost, salvage value, and life expectancy are known. Calculations under the section entitled Full Year are valid when an asset is held for a full twelve months in the first year of depreciation, while the calculation under the section entitled Partial Year is used in cases where the asset is held less than twelve months in its first year of depreciable life.

Full Year

To find the depreciation and remaining balance for each year, information is entered as follows:

- Key in declining factor (1.25 for 125% declining balance, 2.00 for double declining etc.), press ENTER↑ 100 × .
- 2) Key in number of years of useful life, press 🚼 STO M .
- 3) Key in cost (do not deduct salvage value).
- 4) Press M % to obtain first year's depreciation.
- 5) Press 🔤 to obtain remaining book value after first year.
- 6) Repeat steps 4 and 5 to obtain each succeeding year's depreciation and remaining book value until book value is equal to or less than the salvage value. In the period when the remaining book value is less than the salvage value, the previous book value should be reduced by the salvage value to obtain the final year's depreciation.

Example 1:

The Drifter Apartments have a cost basis of \$86,000. The owner wishes to use 125% declining balance depreciation over 20 years. What is the annual depreciation amount and remaining book value in years 1 and 2?



Depreciation 21

3)	86000	86000	Depreciable value.
4)	M %	5375.00	First year depreciation.
5)		80625.00	Remaining depreciable value, year 1.
6)	M %	5039.06	Second year depreciation.
		75585.94	Remaining depreciable value, year 2.

NOTE:

To find the depreciation allowance and remaining depreciable value for specific years without starting with year one and sequentially proceeding, use the following keystrokes:

CLR

- 1) Key in the year for which the depreciation and book value are desired, press ENTERA 1 n.
- 2) Key in the declining factor (1.25, 2.00 etc.), press ENTER↑ 100 ×.
- 3) Key in the useful life expectancy, press 🚼 STO M .
- 4) Press CHS i
- 5) Key in cost basis, press **PV EV** to obtain the remaining book value at the beginning of the specified year.
- 6) Press M % to obtain depreciation in specified year.
- 7) Press is to obtain remaining value at end of the specified year.
- 8) Repeat steps 6) and 7) to obtain each succeeding year's depreciation and remaining book value.
- 9) To skip to another year.
 - a. Enter new specified year number, press ENTER I –
 - b. Repeat steps 6) and 7) to obtain depreciation and remaining book value.

Example 2:

Considering the apartments in Example 1, what is the depreciation amount and remaining book value in years 6, 7 and 11?



Example 3: Excess Depreciation Calculation

Suppose the Drifter Apartments (Example 1 and 2 above) are sold after year 7. What would the excess depreciation amount be?

Solution

Excess depreciation equals the difference between the depreciation taken and what would have been taken had straight line been used.

If straight line had been used, total depreciation for the first seven years would be:

Enter	See Displayed	
86000 ENTER+ ENTER+	86000.00	Depreciable value.
20 🕂 STO M ENTER+	4300.00	Annual depreciation.
7 🗙 STO M	30100.00	Total depreciation first 7 years.

With 125% declining balance, total depreciation for the first seven years is:

86000 ENTER+		
54739.07 🧧	31260.93	Total depreciation
excess depreciation is:		inst 7 yours.
M	1160.93	Excess depreciation.

Partial Year

The

If the asset is held for less than twelve months in the first year, the depreciation using the declining balance method can be found as follows:

- Key in declining factor (1.25 for 125% declining balance, 1.50 for 150% declining balance etc.), press ENTER▲ 100 × .
- 2) Key in depreciable life, press 🗧 STO M .
- 3) Key in initial book value, press M %.
- 4) Key in number of months held in first year, press K 🗄 🔁 to obtain first year's depreciation.
- 5) Press 🔤 to see remaining book value.
- 6) Press M % to obtain second year's depreciation.
- 7) Press = for remaining book value.
- 8) Repeat steps 6) and 7) for successive years' depreciation and remaining book value.

Example:

Fixtures in the Blue Boar Restaurant are valued at \$50,000 with an expected life of 16 years. They are held for 6 months the first year and double declining balance depreciation is used. What are the depreciation and remaining book value for the first three years?





SUM-OF-THE-YEARS'-DIGITS METHOD

Like the declining balance method, the sum-of-the-years-digits method (SOYD) is an accelerated form of depreciation, allowing more depreciation in the early years of an asset's life than allowed under the straight line method. Each year a fraction of the original cost is used for depreciation. The numerator of the fraction is the remaining years of useful life (as of the beginning of the year) and the denominator is the sum of the years of useful life. For example, a \$1000 asset with a 5 year life would have $\frac{5}{5+4+3+2+1=15} \times 1000 depreciation the first year, $\frac{4}{15} \times 1000 the second year, $\frac{3}{15} \times 1000 the third year and so on. The calculations below find the depreciation and remaining depreciable value using the SOYD method for each year of an asset's depreciable life when its useful life expectancy and cost (less salvage value) are known.

Procedure

- 1) Key in the beginning depreciable value, press **ENTER**.
- 2) Key in asset's depreciable life, press ENTERA.
- 3) Press ENTER 1 + CHS STO M × 2 ÷ ÷ STO K .
- 4) Press 1 M+ \mathbf{K} x to obtain the depreciation amount.
- 5) Press = to obtain the remaining depreciable value at the end of the year.
- 6) Repeat steps 4) and 5) for subsequent year's depreciation and remaining depreciable value.

Example:

Solution

Apartments valued at \$88,000 are depreciated over 25 years using SOYD depreciation. What is the depreciation amount and remaining depreciable value for the first 2 years?

Solution			
	Enter	See Displayed	
1)	88000 ENTER+	88000.00	Depreciable value.
2)	25 ENTER+	25.00	Depreciable life.
3)	ENTER+ 1 + CHS STO M		
	🔀 2 🚼 🚼 STO 🛛 К	-270.77	
4)	1 м+ к 🗙	6769.23	Depreciation amount year 1.
5)		81230.77	Remaining depreciable value, year 1.
6)	1 М+ К 🗙	6498.46	Depreciation amount year 2.
		74732.31	Remaining depreciable value, year 2.

NOTE:

To obtain the depreciation amount and remaining depreciable value for a particular year, the following procedure can be used:

- 1) Key in number of years of useful life, press STO M ENTER*
- 2) Key in the desired year, press = 1 + 2 × M 1 + M × ÷.
- 3) Key in the depreciable value, press 🔀 to obtain depreciation amount.
- 4) Press , key in desired year, press 2 : X to obtain remaining depreciable value.

Example:

Solution

Ellen's Elegant Eats is located in a new building which cost \$90,000. The estimated useful life of the building is 22 years with no salvage value. Assuming SOYD depreciation is used, what is the depreciation amount and remaining depreciable value in year 3? In year 7?

	Enter	See Displayed	
1)	22 STO M ENTER+	22.00	Number of years useful life.
2)	3 🚍 1 🛨 2 🔀 📶 1 🛨		
	M × ÷	.08	
3)	90000 🔀	7114.62	Depreciation amount, year 3.
4)	M 3 - 2 ÷ 🗙	67588.93	Remaining depreciable value, year 3.
2)	M 7 🚍 1 🛨 2 🔀 M 1		
	+ M × ÷	.06	
3)	90000 🔀	5691.70	Depreciation amount, year 7.
4)	M 7 <mark>-</mark> 2 🕂 🗙	42687.75	Remaining depreciable value, year 7.

4. SIMPLE MORTGAGES

(Fully Amortized Mortgages or Direct Reduction Loans)

Simple mortgages provide for the complete repayment of debt through equal periodic installments, which include varying amounts of principal and interest. Since interest is accrued on the remaining balance, the early payments are applied mostly to interest with a small reduction in principal. As the payment number increases, an increasing portion of the payment is applied to principal as Figure 2 shows.

Most simple mortgage calculations can be solved on the HP-70 simply by using the top row keys. No real estate or other financial tables are required.

NOTE:

The keystroke procedures and examples in this chapter which have monthly payments assume that the K register contains a 12. If this is not the case, press 12 STO \mathbf{K} to restore K for use in the calculations.



Figure 2: Interest and Principal Portion of Periodic Payment.

PERIODIC PAYMENT AMOUNT

This calculation solves for the periodic payment amount (monthly, quarterly, yearly, etc.) to fully amortize a mortgage, given the life of the mortgage, the number of payment periods per year, the periodic interest rate, and the amount of the mortgage.

Procedure:

CLR

- 1) Calculate and key in (any order).
 - Total periods in mortgage life, press
 - Periodic interest rate, press
 - Mortgage amount, press **PV**.
- 2) Press **PMT** to obtain periodic payment amount.

Example 1:

Solution

What is the required monthly payment to fully amortize a 30 year, 30,000 home mortgage if the interest rate is 83/4%?

0010					
	Enter	See Displayed			
	CLR	0.00			
1)	30 K 🗙 n	360.00	Total payments.		
	30000 PV	30000.00	Mortgage amount.		
	8.75 🔣 🖶 i	.73	Monthly interest rate.		
2)	РМТ	236.01	Monthly payment.		
Wha	What if the interest rate is 9%?				
	9 K 🕂 i	.75	Monthly interest rate.		
	РМТ	241.39	Monthly payment.		

If the mortgage amount is \$33,000 at 9%, what is the payment?

33000 PV	33000.00	Mortgage amount.
РМТ	265.53	Monthly payment.

Example 2:

What quarterly payments are required on a \$10,000, 10 year loan at 8 % % interest?



NUMBER OF PERIODIC PAYMENTS FOR FULL AMORTIZATION

This calculation solves for the total number of equal periodic payments required to fully amortize a mortgage, if the interest rate, periodic payment amount, and mortgage amount are known. It can also determine the remaining number of payment periods when the periodic payment amount, interest rate, and present remaining balance are known.

These same keystrokes are used to determine the exact number of payment periods required to amortize a specified amount when the payment amount has been rounded.

Procedure:

CLR

- 1) Calculate and key in (any order):
 - Periodic interest rate, press
 - Loan amount, press **PV**.
 - Periodic payment amount, press PMT .
- 2) Press **n** to obtain the exact total number of periodic payment installments to amortize the loan.

Example 1:

A 7¾% mortgage with monthly payments of \$216.01 has a remaining balance of \$24,994. How many payments remain to be paid?

Sol	luti	on

	Enter	See Displayed	
	CLR	0.00	
1)	7.75 🔣 🕂 🖬	.65	Monthly interest rate.
	216.01 PMT	216.01	Monthly payment.
	24994 PV	24994.00	Remaining balance.
2)	n	213.66	Number of payments remaining.

Example 2:

A home buyer can afford \$300 monthly for loan payments. He needs a \$40,000 mortgage. If the annual interest rate is 8¼%, how long would it take to completely amortize this mortgage?

Solution

	Enter	See Displayed	
	CLR	0.00	
1)	300 PMT 40000 PV	40000.00	Mortgage amount.
	8.25 K 🗧 i	.69	Monthly interest rate.
2)	n	362.68	Number of payments to amortize loan.

NUMBER OF PERIODIC PAYMENTS TO REACH A SPECIFIED PRINCI-PAL BALANCE

While a mortgage may be scheduled to be fully amortized, it is often intended to pay off or refinance the loan prior to maturity, at some specified remaining balance or equity position. Given the periodic interest rate, total number of periods in the mortgage, the periodic payment amount, and the specified remaining balance, the number of periods to reach this balance can be found as follows:

CLR

- 1) Calculate and key in (any order):
 - Periodic interest rate, press
 - Periodic payment amount, press **PMT**.
 - Specified remaining balance, press **PV**.
- 2) Press **n** to obtain the number of periods required to amortize the remaining balance.
- 3) Calculate and key in the total number of payments during the life of the original loan.
- 4) Press **XXY** is to obtain the number of periods required to reach the specified remaining balance.

Example 1:

1) Mr. Astute intends to pay off his \$35,000, 30 year mortgage when the remaining balance declines to \$25,000. If the annual interest is 8½% and his monthly payment is \$262.94, when will he reach this balance?

	Enter	See Displayed	
	CLR	0.00	
1)	8.25 K 🖶 i	.69	Monthly interest rate.
	262.94 PMT	262.94	Monthly payment.
	25000 PV	25000.00	Specified remaining balance.
2)	n	154.76	Number of periods to amortize balance.
3)	30 K 🗙	360.00	Total periods in mortgage life.
4)	x2y -	205.24	Number of periods to reach specified balance.
	ĸ	17.10	Number of years to reach \$25,000 remaining balance.

NOTE:

Finding the number of periods to reach a specified equity position simply requires determining the remaining balance associated with this equity and then proceeding with the keystrokes shown above. (Purchase price minus equity position equals remaining balance.)

Example 2:

Colution

An investor has just purchased a small apartment complex for \$95,000 with an \$18,000 down payment. The remaining \$77,000 is financed with an 8%, 25 year mortgage having monthly payments of \$594.30. He would like to dispose of the property when his equity position reaches 30% of the purchase price. When will this amount of equity be reached?

Solu	Solution			
	Enter	See Displayed		
	CLR	0.00		
1)	95000 ENTER+ 30 %	66500.00	Specified remaining balance.	
	8 K 📑 🖬	.67	Monthly interest rate.	
	594.30 PMT	594.30	Monthly payment.	
2)	n	206.23	Number of periods to amortize remaining balance.	
3)	25 K ×	300.00	Number of periods in mortgage life.	
4)	x2y 😑	93.77	Number of payments to reach specified balance.	

ANNUAL PERCENTAGE RATE (APR)

Knowing the actual annual percentage rate (APR) on mortgages is extremely important in real estate both for the borrower and the lender. For the borrower, it is important to know the APR so loan comparisons can be made. The lender needs to know the APR because he is required by law (Regulation Z, Truth in Lending) to state the APR.

The sections which follow show how to compute the APR, both when there is no fee and when a loan origination fee is charged.
ANNUAL PERCENTAGE RATE CALCULATIONS WITHOUT FEES

This calculation finds the annual percentage rate (APR) associated with a fully amortized mortgage not involving fees related to the issue of the mortgage, given the life of the mortgage, the periodic payment amount, and the mortgage amount. Information is entered as follows:

CLR

- 1) Calculate and key in (any order):
 - Total number of payment periods, press
 - Periodic payment amount, press **PMT**.
 - Mortgage amount, press **PV**.
- 2) Press i to obtain the percentage rate per period. Enter the number of periods per year, press x to obtain the annual percentage rate (APR).

Example 1:

A 30 year, \$45,000 mortgage has monthly payments of \$362.08. What is the annual percentage rate?

Solution

	Enter	See Displayed	
	CLR	0.00	
1)	30 K 🗙 n	360.00	Total periods in mortgage life.
	45000 PV	45000.00	Mortgage amount.
	362.08 PMT	362.08	Monthly payment.
2)		.75	Monthly interest rate.
	К	9.00	Annual percentage rate.

Example 2:

Mr. Anderson has a 20 year, \$37,000 mortgage with quarterly principal and interest installments of \$1019.31. What annual percentage rate is charged on the mortgage?

Enter	See Displayed	
CLR	0.00	
4 STO K	4.00	Periods per year.



1)	37000 PV 1019.31 PMT	1019.31	Quarterly payment.
	20 K 🗙 n	80.00	Total periods in mortgage life.
2)		2.31	Quarterly interest rate.
	К	9.25	Annual interest rate.

ANNUAL PERCENTAGE RATE CALCULATIONS WITH FEES

Borrowers are sometimes charged fees related to the issuance of a mortgage, which effectively raise the APR because the actual amount received by the borrower is less yet the payments are still based on the full amount. Given the life of the mortgage, the interest rate, the mortgage amount, and the basis of the fee charge (how the fee is calculated), the true Annual Percentage Rate can be calculated.

Procedure

CLR

- 1) Calculate and key in (any order):
 - Total number of payment periods, press
 - Periodic interest rate, press
 - Mortgage amount, press **PV**.
- 2) Press PMT STO M.
- 3) Press CLR.
- 4) Calculate the mortgage amount less fees:
 - a. If fees are stated as a percentage of the mortgage amount (often stated as points; one point = 1%)
 - 1) Key in mortgage amount, press **ENTER**
 - 2) Key in the fee (expressed as a percentage rate) press %
 PV .
 - b. If fees are stated as a flat charge:
 - 1) Key in the mortgage amount, press **ENTER**.
 - 2) Key in the fee amount (flat charge), press **PV**.
 - c. If fees are stated as a percentage of the mortgage amount plus a flat charge:
 - 1) Key in the mortgage amount, press **ENTER**
 - 2) Key in the fee (percentage rate), press 🥦 📒
 - 3) Key in the fee amount (flat charge), press **PV**.

- 5) Calculate and key in the total number of payment periods; press
- 6) Press M PMT i to obtain the percentage rate per period.
- 7) To obtain the annual percentage rate, enter the number of periods per year and press \times .

Example 1:

A borrower is charged 3 points for the issuance of his mortgage. If it is a 42,000 mortgage for 30 years at $8\frac{1}{2}\%$ interest, requiring monthly payments, what annual percentage rate is the borrower paying? (1 point equals 1% of the mortgage amount).

	Enter	See Displayed	
	CLA	0.00	
1)	42000 PV	42000.00	Mortgage amount.
	30 K 🗙 n	360.00	Total number of payments in mortgage life.
	8.5 K 🗧 i	.71	Monthly interest rate.
2)	PMT STO M	322.94	Monthly payment amount.
3)	CLR	0.00	
4)	42000 ENTER+ 3 % - PV	40740.00	Effective borrowed amount.
5)	30 K 🗙 n	360.00	Total periods in mortgage life.
6)	M PMT i	.74	Monthly interest rate.
7)	KX	8.83	Annual percentage rate (APR).

Solution

Example 2:

Using the same information as given in Example 1, calculate the APR if the mortgage fee is \$175 instead of a percentage.



Example 3:

Again using the information given in Example 1, what is the APR if the mortgage fee is stated as 2 points plus \$100?

	Enter	See Displayed	
	CLR	0.00	
1)	360 ⁿ 42000 PV	42000.00	
	8.5 K 😫 i	.71	
2)	PMT STO M	322.94	Monthly payment.
3)	CLR	0.00	
4)	42000 ENTER+ 2 % -		
	100 🚍 🛛 PV	41060.00	Effective mortgage amount.
5)	360 n	360.00	Total periods in mortgage life.
6)	М РМТ і	.73	Monthly interest rate.
7)	К 🗙	8.75	Annual percentage rate.

MORTGAGE AMOUNT

When the mortgage life, the interest rate, and the periodic payment amount are known, the full mortgage amount can be found as follows:

- 1) Calculate and key in (any order):
 - Total payment periods, press .
 - Periodic interest rate, press
 - Periodic payment amount, press **PMT**.
- 2) Press **PV** to obtain the mortgage amount.

Example 1:

Mr. Theodore feels he can comfortably afford a \$325 principal and interest payment monthly. If the current mortgage terms are 30 years at $8\frac{3}{3}\%$ interest, what is the largest mortgage he can obtain?

Solu	tion		
	Enter	See Displayed	
	CLR	0.00	
1)	30 K 🔀 n	360.00	Total number of monthly payments.
	325 PMT	325.00	Monthly payment.
	8.75 K 🗧 🖬	.73	Monthly interest rate.
2)	PV	41311.79	Mortgage amount.
What	if the interest rate is 9¼%?		
	9.25 🔣 🚼 i	.77	
	PV	39505.25	Mortgage amount.

Example 2:

A home-buyer's monthly gross pay is \$1131.60 and he has no current debt. He wants to acquire a \$40,000 mortgage for 30 years at 8% annual interest. If the buyer must qualify at 4 to 1 (i.e. his gross pay minus long term debt must be 4 times his monthly principal and interest payment installment), can he afford this mortgage?



No. The buyer can only afford a \$38554.60 mortgage.

Example 3:

A seller decides he will carry a \$25,000 mortgage at 8% for 25 years resulting in a monthly payment of \$192.95. If he decides he would rather require 6% interest but wishes to retain the same monthly payment, what would the mortgage amount have to be?



ACCUMULATED INTEREST AND REMAINING PRINCIPAL BALANCE

The amount of interest paid during a specified period and the remaining balance associated with the end of the specified period are very often used in real estate. For example, because interest is a tax deductible expense, you may need to know how much interest is paid during the year on a mortgage. This is just one of the many uses for the procedure below.

The following calculation finds both the total interest paid during a specified number of time periods and the remaining balance at the beginning and end of the specified time, given the periodic interest rate, periodic payment amount, total number of periods, mortgage amount, and the beginning and ending payment numbers for the time span being considered. Since most periodic payment amounts are rounded, it is necessary to first find the exact number of payments required to amortize the loan.

Procedure:

CLR

- 1) Calculate and key in (any order):
 - Periodic interest rate, press
 - Mortgage amount, press **PV**.
 - Periodic payment amount, press **PMT**.
- 2) Press n STO M.
- 3) Calculate and key in (any order):
 - Periodic interest rate, press
 - Periodic payment amount, press **PMT**.
 - Press M, key in the first payment number of the time frame being considered, press ENTER 1 - - n.
- 4) Press **PV** to obtain the balance at the beginning of the time frame.
- 5) Press M, key in the last payment number of the specified time frame, press n PV to obtain the remaining balance at the end of the time frame.
- 6) Press = to obtain the total principal paid (equity buildup) during the time period.
- 7) Recall the periodic payment amount by pressing **DSP PMT**, key in the number of payments made during the time frame, press **X** to obtain the total paid during the time frame.
- 8) Press **XXY** to obtain the accumulated interest paid during the time frame.

Example:

Mr. Newcomer needs to know, for income tax purposes, how much interest he paid on his \$40,000 mortgage last year. His monthly payments are \$279.69, the mortgage was written for a 30 year life, and the stated interest rate is 7.5%. Last year contained payments numbered 17-28.



	Enter CLR	See Displayed 0.00	
1)	40000 PV	40000.00	Mortgage amount.
	279.69 PMT	279.69	Monthly payment.
	7.5 🔣 🚼 🚺	0.63	Monthly interest.
2)	n STO M	359.98	Exact number of payments in
	CLR	0.00	mortgage life.
3)	M 17 ENTER+ 1 - n	343.98	
	7.5 K 🕂 I	.63	Periodic interest rate.
	279.69 PMT	279.69	Monthly payment.
4)	PV	39502.03	Balance at beginning of time frame (after payment 16).
5)	M 28 <mark>–</mark> n PV	39094.59	Balance after payment 28.
6)	8	407.44	Principal paid during year.
7)	DSP PMT 12 X	3356.28	Total paid during year.
8)	x2y -	2948.84	Accumulated interest paid during year.

REMAINING BALANCE ONLY

If you only need to know the remaining balance at a specified point in time and you know the interest rate, periodic payment amount, and the last payment number occurring prior to the desired balance, the following keystrokes can be used:

CLR

- 1) Calculate and key in (any order):
 - Periodic interest rate, press
 - Periodic payment amount, press **PMT**.
 - Original loan amount, press **PV**.

- 2) Press n STO M
- 3) Press CLR .
- 4) Calculate and key in (any order):
 - Periodic interest rate, press
 - Periodic payment amount, press **PMT**.
 - Press M, enter the payment number associated with the desired remaining balance, press = 1.
- 5) Press **PV** to obtain the remaining balance.

Example 1:

What is the remaining balance after 5 years on a 30 year, 8% mortgage for \$35,000 with monthly payments of \$256.82?

	Enter	See Displayed	
	CLR	0.00	
1)	8 K 🕂 i	.67	Monthly interest rate.
	35000 PV	35000.00	Mortgage amount.
	256.82 PMT	256.82	Monthly payment.
2)	n STO M	359.99	Total exact payments in mortgage.
3)	CLR	0.00	
4)	M 5 K 🗙 드 n	299.99	
	8 K 🕂 i	.67	Monthly interest rate.
	256.82 PMT	256.82	Monthly payment.
5)	PV	33274.27	Remaining balance.

Solution

Example 2:

Mr. Stewart has just inherited \$25,000 cash. He would like to pay off his \$32,000 home mortgage. The mortgage was written exactly 8 years ago for 25 years at $8\frac{1}{4}\%$ interest with monthly payments of \$252.30. Will his inheritance be sufficient to pay off the mortgage?



No. Mr. Stewart does not have quite enough to fully pay off his mortgage.

LAST PAYMENT AMOUNT

If the periodic mortgage payment amount has been rounded (up or down) then the last payment will have to be adjusted in order to account for the gain or loss induced by the rounding.

To solve for this last payment amount, information is entered as follows:

CLR

- 1) Calculate and key in (any order):
 - Periodic interest rate, press
 - Periodic payment amount, press **PMT**.
 - Original loan amount, press **PV**.
- Press n to obtain the exact number of payment periods (at the given payment amount) required to amortize the loan amount. Press STO
 M .
- 3) Press CLR.

- Calculate and key in (any order): 4)
 - Periodic interest rate, press
 - Periodic payment amount, press **PMT**. •
 - Press M, enter the *actual* number of payment periods, press • n. -
- Press **PV DSP PMT +** to obtain the amount of the last payment. 5)

Example 1:

The exact monthly payment (as calculated to five decimal places on the HP-70) to fully amortize a 30 year, \$35,000, 9¼% mortgage is \$287.93640. What would the last payment be if the monthly payment were rounded to \$287.95?

3010			
	Enter	See Displayed	
	CLR	0.00	
1)	9.25 🔣 🚼 i	.77	Monthly interest rate.
	35000 PV	35000.00	Mortgage amount.
	287.95 PMT	287.95	Monthly payment amount.
2)	n STO M	359.91	Exact number of payments required with \$287.95.
3)	CLR	0.00	
4)	9.25 🔣 븑 🖬	.77	
	287.95 PMT	287.95	
	M 360 🧮 n	09	
5)	PV DSP PMT 🕂	261.72	Last payment amount.

Example 2:

If the payment amount of the previous example had been rounded to 287.90?



MORTGAGE AMORTIZATION SCHEDULE

This calculation generates the interest paid per period, the payment toward principal each period, and the remaining principal balance each period over the life of a fully amortized mortgage loan, given the periodic payment amount, the annual interest rate, the number of payment periods per year, and the mortgage amount.

Procedure:

- 1) Key in the periodic payment amount, press **STO** M.
- Calculate and key in the periodic interest rate, press ENTER↑ ENTER↑
 ENTER↑
- 3) Key in the loan amount.
- 4) Press $x \ge y$ % to obtain the interest portion of the first payment.
- 5) Press M XXY = to obtain the principal portion of the first payment.
- 6) Press to obtain the remaining principal balance.
- 7) Repeat steps 4–6 for subsequent periods.

Example:

Generate an amortization schedule for the first two months of a \$40,000 loan at 9% with monthly payments of \$321.85.



See Displayed	
321.85	Monthly payment.
.75	Monthly interest rate.
40000	Mortgage amount.
300.00	Interest portion of first payment.
21.85	Principal portion of first payment.
39978.15	Remaining balance after first payment.
299.84	Interest portion of second payment.
22.01	Principal portion of second payment.
39956.14	Remaining balance after second payment.

If the final payment on a mortgage or trust deed is sufficiently greater than the equal periodic payments, it is referred to as a balloon payment. The following sections cover calculations pertaining to this type of mortgage.

BALLOON PAYMENT AMOUNT

This calculation determines the balloon payment amount (occurring coincident with the last periodic payment), given the total number of periods in the mortgage life, the annual interest rate, the periodic payment amount, and the mortgage amount. Information is entered as follows:

CLR

- 1) Calculate and key in (any order):
 - Periodic interest rate, press
 - Periodic payment amount, press **PMT**.
 - Original loan amount, press **PV**.
- 2) Press n STO M
- 3) Press CLR.
- 4) Calculate and key in (any order):
 - Periodic interest rate, press
 - Periodic payment amount, press **PMT**.
 - Press M, enter the payment number associated with the balloon payment, press .

5) Press PV.

Example 1:

A property sold for \$43,950 with the seller carrying 10% of the sales price in a second mortgage at 10% annual interest. The scheduled maturity of this note is in 6 years, with a monthly payment of 1% of the original second mortgage amount. What is the amount of the balloon payment to be received by the holder of the second at maturity?





Example 2:

a i ...

A buyer needs to obtain a \$6500 second mortgage. The terms he is given are 7 years at 10% interest and a quarterly payment of \$300 with a balloon payment at the end of the 7 years. What is the balloon payment?

Solu	Solution				
	Enter	See Displayed			
	CLR	0.00			
1)	4 STO K	4.00	Number of periods per year.		
	6500 PV	6500.00	Mortgage amount.		
	300 PMT	300.00	Quarterly payment.		
	10 K 🚼 i	2.50	Quarterly interest rate.		
2)	n STO M	31.59	Exact number of payments to amortize mortgage.		
3)	CLR	0.00			
4)	M 7 K 🔀 🚍 n	3.59			
	300 PMT	300.00			
	10 🔣 🚼 🔲	2.50			
5)	PV	1019.28	Balloon payment amount.		

PERIODIC PAYMENT AMOUNT FOR A SPECIFIED BALLOON PAYMENT

If you know the total number of payments in the mortgage, the periodic interest rate, the balloon payment amount, and the mortgage amount, you can find the periodic payment amount by using the keystrokes which follow:

CLR

- 1) Calculate and key in (any order):
 - Periodic interest rate, press
 - Total number of payment periods until balloon occurs, press
 - <u>Balloon payment amount, press</u> **FV**.
- 2) Press PV.
- 3) Key in the mortgage amount, press XXY = STO M.
- 4) Press CLR.
- 5) Calculate and key in (any order):
 - Total number of payment periods in mortgage life, press
 - Periodic interest rate, press
 - Press M, PV.
- 6) Press **PMT** to obtain the periodic payment amount.

Example 1:

A borrower specifies that the balloon payment (occurring simultaneously with the last periodic payment) due on his \$12,000, 10%, 10 year mortgage should not exceed \$5000. What minimum monthly payment is required on this loan?

Dona	tion		
	Enter	See Displayed	
	CLR	0.00	
1)	10 K 🗧 i	.83	Monthly interest rate.
	10 K 🗙 n	120.00	Total periods in mortgage.
	5000 FV	5000.00	Balloon payment amount.
2)	PV	1847.03	Discounted value of balloon.
3)	12000 🔀 📑 STO М	10152.97	
4)	CLR	0.00	
5)	M PV	10152.97	
	10 🔣 🚼 🚺	.83	
	10 K 🗙 n	120.00	
6)	РМТ	134.17	Monthly payment amount.

Example 2:

Mr. Crawford needs to finance 20% of his \$58,000 building with a second mortgage at 10% interest, maturing in 5 years. He expects the building will appreciate about 3% per year and hopes to refinance the building at that time. What monthly payment must he make for the balloon payment to be no greater than the building's appreciation?

Sol	ution		
	Enter	See Displayed	
	CLR	0.00	
	5 n 3 i		
	58000 PV FV	67237.90	Value of building at note maturity (5 years).
	DSP PV 📑 STO M	9237.90	Appreciation amount (balloon amount).
	CLR	0.00	
1)	10 K 🖶 i	.83	Monthly interest rate.
	5 K 🗙 n	60.00	Total periods in mortgage.
	MFV	9237.90	Balloon amount.
2)	PV	5614.69	Discounted value of balloon payment.
3)	11600	11600	Amount of second mortgage.
	х2у 😑 STO М	5985.31	Amount to be amortized.
4)	CLR	0.00	
5)	5 K 🗙 n	60.00	
	10 K 🚼 i	.83	
	MPV	5985.31	
6)	РМТ	127.17	Monthly payment.

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6. PRICE AND YIELD OF MORTGAGES

Mortgages are marketable much like bonds—the price paid for a mortgage determines the yield an investor will receive. A mortgage bought at a discount is purchased for an amount less than the principal balance of the note—one bought for more than this balance is purchased at a premium. The less you pay for the mortgage, the higher the yield and vice versa.

Using the calculations below will enable you to determine what price can be paid for a mortgage in order to meet your yield objectives. Other sections of this book, especially those concerning balloon payment and periodic payment amounts are useful as background for many of the following problems.

PRICE OF FULLY AMORTIZED MORTGAGES

Given the desired periodic yield, periodic payment amount, and number of payments in mortgage, the price of a mortgage can be determined as follows:

- 1) Calculate and key in (any order):
 - -Periodic payment amount, press PMT.
 - -Desired periodic yield, press
 - -*Number of payments in mortgage, press
- 2) Press **PV** to obtain the price of the mortgage.

*If an exact answer is required the total number of payments entered here must be exact. To determine the exact answer use the keystrokes shown below (also see Example 2).

CLR

- 1) Calculate and key in (any order):
 - Periodic payment amount, press **PMT**.
 - Mortgage amount, press **PV**.
 - Periodic interest rate, press
- 2) Press n STO M
- 3) Press CLR.
- 4) Calculate and key in (any order):
 - Desired yield per compounding period, press
 - Periodic payment amount, press **PMT**.
 - Press M n.
- 5) Press **PV** to obtain the price of the mortgage.

Example 1:

Ms. Carey wishes an 11% yield on first mortgages she buys. What price should she pay for a \$25,000 fully amortized mortgage for 30 years at 8% interest and monthly payments of \$183.44?

Solution



Example 2:

A 5 year second mortgage of \$7800 carries an annual interest rate of 10% with quarterly payments of \$500. What is the price of this mortgage if bought to yield 12%?

	Enter	See Displayed	
	CLR	0.00	
1)	4 STO K	4.00	Number of periods per year.
	7800 PV	7800.00	Mortgage amount.
	500 PMT	500.00	Quarterly payment.
	10 K 😫 i	2.50	Quarterly interest rate.
2)	n STO M	20.02	Exact number of payments in mortgage.
3)	CLR	0.00	
4)	12 K 😝 i	3.00	Quarterly yield rate.
	M n	20.02	
	500 PMT	500.00	
5)	PV	7443.64	Price of mortgage bought at discount.

PRICE OF PREPAID MORTGAGES OR MORTGAGES WITH A BALLOON PAYMENT

Since both a prepayment and balloon payment pay off the remaining balance of the loan, calculations for these situations are identical, as the keystrokes and examples below illustrate.

Given the life and amount of the mortgage, the periodic interest rate and payment amount, the timing and amount of the balloon or prepayment, and the desired yield rate, the price of the mortgage can be found.

Procedure:



- 1) Calculate and key in (any order):
 - Remaining balance or balloon, press **FV**.
 - Total number of periods until prepayment or balloon occurs, press
 - Desired periodic yield rate, press
- 2) Press PV STO M.

CAUTION:

The balloon payment may occur one period after the last periodic payment instead of being coincident with it. In this case the n in step 1) will be one larger than the n used in step 4). Example 2 shows this type.

- 3) Press CLR .
- 4) Calculate and key in (any order):
 - Total number of payments from beginning of mortgage to period of balloon or prepayment, press
 - Periodic yield rate, press
 - Periodic payment amount, press **PMT**.
- 5) Press PV.
- 6) Press M+ to obtain mortgage price.

Example 1:

A seven year second mortgage calls for monthly installments of \$70 and a balloon payment of \$5588.91 to be paid at the same time as the last periodic payment. The mortgage amount is \$7000 and the interest rate is 10%. If the desired yield is 13%, what is the price of this mortgage?



Example 2:

Home Real Estate sells second mortgages to investors as a means of providing funds to help sell the houses it lists. A recently sold listing required using a second mortgage for \$9,000 at 10%. The monthly payment is \$90 for 6 years with a balloon of \$7501.07 due one period after the last monthly payment. If the investors desire a 13% yield, what price should the mortgage be sold for?



PV STO M	3415.98	Present value of balloon payment.
CLR	0.00	
6 K 🔀 n 90 PMT		
13 K 🗧 i	1.08	Desired monthly yield.
PV	4483.39	
M+	7899.37	Price of mortgage.
	PV STO M CLR 6 K × n 90 PMT 13 K ÷ i PV M+	PV STO M 3415.98 CLR 0.00 6 K N 90 PMT 13 K E I 1.08 PV 4483.39 4483.39 M+ 7899.37

Example 3:

Find the price of an 8%, 30 year mortgage to be prepaid in full after 5 years if the mortgage amount is 37,000, monthly payments of 271.49 are required and the investor desires an annual yield of 12% (The remaining balance after 5 years is 35176.06).

	Enter	See Displayed	
	CLR	0.00	
1)	35176.06 FV	35176.06	
	5 K 🔀 n	60.00	
	12 🔣 🚼 🚺	1.00	
2)	PV STO M	19362.65	Present value of remaining balance.
3)	CLR	0.00	
4)	60 n 12 K 🕂 i	1.00	Desired periodic yield.
	271.49 PMT	271.49	Monthly payment.
5)	PV	12204.84	Present value of payment.
6)	M+	31567.49	Price of mortgage.

YIELD OF FULLY AMORTIZED MORTGAGES

Given the price of a fully amortized mortgage, periodic payment amount, and the number of payments in the mortgage, the yield can be found as follows:

- 1) Calculate and key in (any order):
 - -Periodic payment amount, press **PMT**.
 - -Mortgage price, press **PV**.
 - -*Number of payments in mortgage life, press
- Press i to obtain the period yield. Key in number of periods per year, press to obtain annual yield.

*If an exact answer is required, the number of payments entered here must be exact. If this is the case, use the following keystrokes to determine the yield (also see Example 2 below).

- 1) Calculate and key in (any order):
 - Mortgage amount, press **PV**.
 - Periodic interest rate, press
 - Periodic payment amount, press **PMT**.
- 2) Press **STO M** to obtain exact number of periods in mortgage life.
- 3) Press CLR.
- 4) Calculate and key in (any order):
 - Mortgage price, press **PV**.
 - Periodic payment amount, press **PMT**.
 - Press M n .
- 5) Press i to obtain the yield per period. Enter the number of periods per year, press X to obtain the annual yield.

Example 1:

What is the yield on a mortgage purchased for \$32000 if the monthly payments are \$318.89 and there are 300 payments remaining?

	Enter	See Displayed	
	CLR	0.00	
1)	318.89 PMT	318.89	Monthly payment.
	32000 PV	32000.00	Mortgage price.
	300 n	300.00	Number of payments.

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2)	i	.94	Monthly yield.
	12 🔀	11.23	Annual yield.

Example 2:

The price of a \$28000, 30 year mortgage at $8\frac{4}{3}$ with monthly payments of \$210.35 is \$20,260. What is the annual yield?

Enter	See Displayed	
CLR	0.00	
28000 PV	28000.00	Mortgage amount.
8.25 K 🖶 i	.69	Monthly interest.
210.35 PMT	210.35	Monthly payment.
n STO M	360.03	Exact number of periods in mortgage life.
CLR	0.00	
20260 PV	20260.00	Mortgage price.
210.35 PMT	210.35	Monthly payment.
Mn	360.03	Number of payments in mortgage.
i	1.01	Periodic yield.
К	12.13	Annual yield.
	Enter CLR 28000 PV 8.25 K ÷ i 210.35 PMT n STO M CLR 20260 PV 210.35 PMT M n i K ×	Enter See Displayed CLR 0.00 28000 PV 28000.00 8.25 K ÷ i .69 210.35 PMT 210.35 n STO M 360.03 CLR 0.00 20260 PV 20260.00 210.35 PMT 210.35 m n 360.03 i 1.01 K K 12.13

7. ANNUITY DUE CALCULATIONS

Annuity due payments are payments which occur at the beginning of each period (also called payments in advance). The HP-70 assumes that payments occur at the end of each period (also called ordinary annuity or payment in arrears). However, by slightly changing the keystrokes, problems involving annuity due payments can easily be solved. (e.g. some rental or lease situations).

In the sections which follow, many of the problems covered earlier under Simple Mortgages and Mortgages with Balloon Payments, are solved with the assumption that payments are made in advance instead of in arrears.

PRESENT VALUE OF ANNUITY DUE

This calculation solves for the present value of a series of annuity due payments (i.e. payments at the beginning of each period) given the number of payments, periodic interest rate, and payment amount.

Procedure:



- 1) Calculate and key in the number of payment periods, press
- 2) Calculate and key in the periodic interest rate, press
- 3) Key in periodic payment amount, press DSP i % + PMT
 PV to obtain the present value of the payments.

Example:

Ms. Paris rents out half of her duplex for \$225 per month, payable in advance. If she assumes a $7 \frac{1}{2}\%$ annual interest rate, what would the present value of these payments be for one year?

	Enter	See Displayed	
	CLR	0.00	
1)	12 n	12.00	Number of months in year.
2)	7.5 K 😫 i	.63	Monthly interest rate.
3)	225 DSP i % + PMT PV	2609.65	Present value of payments.

ANNUAL PERCENTAGE OR YIELD RATE WITHOUT BALLOON OR RESIDUAL VALUE

To find the annual interest or yield rate when the initial amount, periodic payment, and number of payments are known, use the following keystrokes:

CLR

- 1) Calculate and key in *one less* than the total number of periods, press
- 2) Key in payment amount, press **PMT**.
- 3) Key in initial amount, press DSP PMT = PV .
- 4) Press **t** to obtain periodic rate.
- Key in number of payment periods per year; press × to obtain annual rate.

Example:

Solution

Equipment worth \$12,000.00 is leased for 8 years with monthly payments in advance of \$200.00. The equipment is assumed to have no salvage value at the end of the lease. What yield rate does this represent?

	Enter	See Displayed	
	CLR	0.00	
1)	8 🔣 1 🔚 n	95.00	
2)	200 PMT	200.00	Payment amount.
3)	12000 DSP PMT 📃 PV	11800.00	
4)	i	1.09	Periodic rate.
5)	K	13.07	Annual rate

NUMBER OF PERIODS TO FULLY AMORTIZE INITIAL AMOUNT

If the periodic payment amount, initial amount, and periodic rate are known, the number of periods required to pay off the initial amount is calculated as follows:

CLR

- 1) Calculate and key in periodic interest rate, press
- 2) Key in payment amount, press DSP i % 🕂 PMT
- 3) Key in initial amount, press **PV n**

Example:

The buyer of 2 acres of land can afford to pay \$225 per month toward interest and principal. If the asking price is \$23,000 and the seller wants $8\frac{14}{\%}$ annual interest with monthly payments in advance, how long will it take to pay off the mortgage?



	Enter	See Displayed	
	CLR	0.00	
1)	8.25 🔣 🚼 🚺	.69	Periodic interest.
2)	225 DSP i % 🕂 PMT	226.55	
3)	23000 PV n	174.74	Number of months.
	К 😫	14.56	Number of years.

PERIODIC PAYMENT AMOUNT WITHOUT BALLOON OR RESIDUAL VALUE

If the number of periods, periodic interest rate, and initial value are known, the payment occurring at the beginning of each period can be found as follows:

CLR

- 1) Calculate and key in the number of payment periods, press
- 2) Calculate and key in the periodic interest rate, press **i**.
- 3) Press 1 DSP i % 🛨.
- 4) Enter initial value, press **XXY FV PMT** to obtain the periodic payment amount.

Example 1:

Mr. Porter obtains an \$8300 loan for 5 years at $8\frac{1}{2}\%$ interest with monthly payments due in advance. What is the payment amount?

	Enter	See Displayed	
	CLR	0.00	
1)	5 K 🗙 n	60.00	Total number of payments.
2)	8.5 🔣 🚼 🚺	.71	Monthly interest.
3)	1 DSP i % 🕂	1.01	
4)	8300 XXY 🕂 PV PMT	169.09	Monthly payment amount.

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Example 2:

The owner of a warehouse which is currently worth \$35,000 wishes to lease it for 20 years. He doubts it will be worth anything at the end of that time. If his desired yield is 9% annually, what quarterly payments in advance must be received?



PERIODIC PAYMENT AMOUNT WITH BALLOON OR RESIDUAL AT END OF LAST PERIOD

The following calculation is very useful in determining the payment amount on a lease since most leases have payments in advance. Often a building or piece of equipment still has a residual value after several years of use. Because this residual is returned to the investor, it effectively reduces the value the yield is based on, thus lowering the payments required to achieve the same yield.

The procedure below shows how to find the periodic payment amount for an annuity due loan with a balloon payment, a lease with a residual, or any other situation with annuity due payments followed by a larger amount at the end of the last period. Known values are the initial value, residual (balloon) value, periodic rate, and number of periods.

CLR

- 1) Calculate and key in (any order):
 - Periodic interest rate, press
 - Number of time periods until balloon (residual) occurs, press
 .
 - Balloon payment amount, press **FV**.

- 2) Press **PV** to obtain the present value of the balloon.
- 3) Key in the initial value, press X = 1 DSP i.
- 4) Press 50 for store we adjusted initial value to be amortized.
- 5) Press CLR.
- 6) Calculate and key in (any order):
 - Total payment periods, press
 - Periodic interest rate, press
 - M PV .
- 7) Press **PMT** to obtain periodic payment amount.

Example 1:

Suppose the warehouse in Example 2 above is estimated to be worth 5000 at the end of the lease. What quarterly payments would be required to achieve the 9% yield?

	Enter	See Displayed	
	CLR	0.00	
	4 STO K	4.00	Periods per year.
1)	9 K 😫 i	2.25	Quarterly yield rate.
	20 K 🗙 n	80.00	Total periods in lease.
	5000 FV	5000.00	Residual value.
2)	PV	843.15	Present value of residual.
3)	35000 🔀 🔄 1 DSP 🗾 i	2.25	
4)	% 🕂 🕂 STO М	33405.23	Value yield is based on.
5)	CLR	0.00	
6)	80 n 2.25 i	2.25	
	Μ	33405.23	
7)	РМТ	904.07	Quarterly payment amount.

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Example 2:

A loan for \$23,000 with a \$3200 balloon payment due at the end of the last period, has a 5 year term and a 10% interest rate. What is the monthly payment amount?



8. LOANS WITH A CONSTANT AMOUNT PAID TOWARDS PRINCIPAL

This type of loan is structured such that the principal is repaid in equal installments with the interest paid in addition. Therefore each periodic payment has a constant amount applied toward the principal and a varying amount of interest. For example, on a \$20,000 loan for 5 years with annual payments, the constant payment to principal is \$4000 per year. Interest is in addition to this amount and obviously declines with each payment as the balance declines.

An amortization schedule for this type of loan can be constructed when the constant payment to principal, periodic interest rate, and loan amount are known by using the following keystrokes.

- 1) Key in constant payment to principal, press **STO** .
- Calculate and key in periodic interest rate, press ENTER↑ ENTER↑
 ENTER↑. Key in loan amount.
- 3) Press x y % to obtain interest portion of payment.
- 4) Press M 🕂 to obtain total payment.
- 5) Press **R** M **G** to see remaining balance.
- 6) Return to step 3 for each successive payment.

Example:

Assuming an \$80,000, 20 year, 9% farm loan with annual payments, construct a loan reduction chart for the first two years. (Constant payment to principal is \$4000 per year.)



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9. CANADIAN MORTGAGE CALCULATIONS

The mortgage calculations explained in the preceding chapters have been based on the United States convention of compounding interest monthly. However, Canadian mortgages use a different monthly mortgage factor than programmed in the HP-70. This factor results from semi-annual compounding with payments occurring monthly.

To do Canadian mortgage calculations, the Canadian mortgage factor must be calculated first; this factor is then entered for in problems where is required as an input.

Keystrokes for finding the Canadian mortgage factor are:

- 1) Key in (any order):
 - 6 <u>n</u>.
 - 1 PV .
 - Annual interest rate, press ENTER 200 🕂 1 🕂 FV.
- 2) Press to obtain the Canadian mortgage factor.

The examples below show how this factor is used for **t** in Canadian mortgage problems.

Example 1:

Periodic Payment Amount

What is the monthly payment required to fully amortize a 30 year, \$30,000.00 mortgage if the interest rate is 9%?



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M	.74	Monthly interest factor
30000 PV	30000.00	Mortgage amount.
РМТ	237.85	Monthly payment.

Example 2:

Number of Periodic Payments to Fully Amortize a Mortgage

An investor can afford to pay \$380 per month on a \$56,000 mortgage. If the annual interest rate is $7\frac{3}{7}$, how long will it take to completely amortize this mortgage?

Solu	tion		
	Enter	See Displayed	
	CLR	0.00	
1)	6 n 1 PV		
	7.75 ENTER+ 200 ÷ 1 + FV	1.04	
2)	i STO M	.64	Canadian monthly mortgage factor.
	CLR	0.00	
	380 PMT	380.00	Monthly payment.
	M	.64	Monthly mortgage factor
	56000 PV n	435.67	Total monthly payments.
	К	36.31	Total years.

10. DISCOUNTED CASH FLOW ANALYSIS

Discounted Cash Flow Analysis is a way of evaluating investment alternatives which considers the time value of money i.e. a dollar received today is not worth the same as a dollar received 5 years from now. This type of analysis puts all future cash flows in terms of their present value. Therefore investments can be compared on the same basis—their present dollar worth.

Two forms of discounted cash flow analysis are the net present value approach, which assumes a yield rate to discount the cash flows, and the discounted or internal rate of return approach, which finds a yield rate such that the present value of the cash flows equals the initial investment amount.

NET PRESENT VALUE (NPV)

Assuming a minimum desired yield (this could also be a cost of capital or discount rate), the Net Present Value method finds the present value of the future cash flows generated by the investment and compares this value to the initial investment. If this present value is greater than or equal to the investment, the investment meets the profit objectives assumed under minimum yield. If the present value is less than the investment, it is not profitable to the extent of the desired yield.

NOTE:

The yield rate is dependent upon the cash flows. That is, if the cash flows are pretax, the yield rate will be pretax. If the cash flows are after tax, the yield rate will be after tax.

The following procedure is used to find the net present value of an investment, when the assumed yield rate (interest or cost of capital), periodic cash flows and time of occurrence are known. The same procedure is also used if you need to find the present value of a series of cash flows with no initial investment; simply key in **O STO M** in step 1 below.

CLR

- 1) Key in the initial investment amount, press CHS STO M.
- 2) Key in the number of the time period of the first cash flow, press **n**.
- 3) Key in the periodic interest rate, press i. Key in the amount of the first cash flow, press FV PV M+. (For periods with negative cash flows (cash outlays) press CHS before pressing FV.)
- 4) Key in the number of the next period, press **n**.
- 5) Key in the cash flow amount, press **FV PV M+** to obtain the current net present value.
- 6) Repeat steps 4 and 5 for each period.

As soon as the display shows a positive number after pressing M^+ , the investment is recovered on a discounted cash flow basis. If the final NPV (after M^+ is pressed following last cash flow) is negative, the investment is not recovered on a discounted cash flow basis assuming the yield used for

To help in visualizing the timing of the cash flows, drawings appear with the examples in this section.

Example 1:

An investor pays \$65,000 for a duplex which he intends to keep five years and then sell. The first year he knows he will have to spend a considerable amount for repairs. If he desires a 9% after tax yield rate and the after tax cash flows are as follows, will he achieve this yield?

Year	Cash Flows (\$)
1	-100
2	4900
3	5300
4	4800
5	74500




Since the final NPV is negative, the investment does not achieve the desired yield.

Example 2:

A small shopping complex, which costs \$137,000, is estimated to have annual cash flows as follows:

Year	Cash Flow (\$)
1	10,000
2	13,000
3	19,000
4	152,000
	property sold in 4th year

The desired minimum yield is 10%. Will this rate be achieved by the above cash flows?



Solution



Because the final NPV is positive, the investment more than achieves the desired yield.

PRESENT VALUE OF DEFERRED ANNUITIES

For annuities where payments do not begin until some point in the future, the following keystrokes can be used to determine the present value of the payment stream:

CLR

- 1) Calculate and key in (any order):
 - Total number of payments in the payment stream press
 - Periodic interest rate, press
 - Periodic payment amount, press PMT.
- 2) Press PV STO M
- 3) CLR
- 4) Calculate and key in (any order):
 - Number of payment periods from the present until the beginning of the annuity, press
 - Periodic interest rate, press
 - Press M FV.
- 5) Press **PV** to obtain the present value of the annuity.

NOTE:

If there is an initial investment, simply subtract its value from the present value of the payment streams to obtain a net present value of the investment.

Example:

An annuity of \$150 per month will start in 3 years and continue for 5 years. What is the present value of this payment stream if the interest rate is 10% annually?







PRESENT VALUE OF UNEVEN PAYMENT STREAMS

In many situations there is an even stream of payments followed by another even stream at a different value, e.g. first year rental payments at \$150 per month, second year payments at \$175 per month. To find the present value of these cash flows, each payment need not be entered. Instead the consecutive even payments can be grouped in order to shorten the solution steps.

Procedure:

- 1) Press 0 STO M \cdot
- 2) For each stream of payments

CLR

- a. Calculate and key in (any order):
 - Number of equal consecutive payment periods, press .
 - Periodic interest rate, press
 - Payment amount, press **PMT**.
- b. Press PV STO K CLR.
- c. Calculate and key in (any order):
 - Number of periods from present to start of payment stream, press
 - Periodic interest rate, press

- d. Press **K FV PV** to obtain present value of payment stream.
- e. Press **M+** to see the present value of total payment streams thus far.
- 3) Repeat steps a-e for each payment stream.

NOTE:

If there is an initial investment, simply subtract this value from the present value of the payment streams to obtain a net present value of the investment.

Example:

Mrs. Holmes is considering purchase of a 6 unit apartment for \$80,000. The estimated monthly cash flows are as follows:

Year	Cash Flow (\$)
1	375/month
2	350/month
3	365/month

If she intends to sell the building after 3 years and expects to receive \$88,000 for it, will an annual yield of 8% be achieved?



Solution

	Enter		See Displayed	
		CLR	0.00	
1)		0 STO M	0.00	
2)	a)	12 8 ENTER+ 12		
		÷ i 375 PMT	.67 375.00	Monthly yield rate.
	b)	PV STO K	4310.92	Present value of first year's cash flows.
		CLR	0.00	
	c)	Since the start of this pasame, simply press K cash flows.	ayment stream and M+ CLR and	d the present are the proceed to the next
2)	a)	12 n 8 ENTER+ 12		
		🕂 👘	.67	Monthly yield rate.
		350 PMT	350.00	
	b)	PV STO K	4023.52	
		CLR	0.00	
	c)	12 n 8 ENTER+ 12		
		÷	.67	
	d)	K FV PV	3715.17	Present value of second year's cash flows.
	e)	M+	8026.09	Present value of first 2 years' cash flows.
		CLR	0.00	
2)	a)	12 n 8 ENTER+ 12		
			.67	
			365.00	
	0)		4195.96	
			0.00	
	C)	24 1 8 ENTER+ 12		
			.67	

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d) K FV PV	3577.46	Present value of third year's cash flows.
e) M+	11603.55	Present value of 3 years of cash flows.
CLR	0.00	
What is now in the M streams. To find the prese 36 n 8 ENTER+ 12	register is the present ent value of the sale pr	value of all the payment rice in 3 years:
88000 FV PV	69278.41	Present value of future sales price.
M+ ENTER+	80881.95	Present value of sales price plus income streams.
Finally, subtract out the value.	initial investment to	o obtain the net present
80000 🧮	881.95	Net present value.

Since the NPV is positive, the investment meets the desired yield.

DISCOUNTED OR INTERNAL RATE OF RETURN (IRR)

The interest rate that equates the present value of all future cash flows with the original investment is known as the internal rate of return (also called discounted rate of return). The method here is to use the net present value approach, trying various rates until a rate is found which causes the NPV to be zero or close to it. If there is no initial investment simply key in **O STO M** in step 1 below.

Procedure:

- 1) Key in initial investment amount, press CHS STO M.
- 2) Key in a best guess (or desired) rate of return (yield) per period, pressi
- 3) Key in the time period number of the first cash flow, press .
- 4) Key in amount of first cash flow, press **FV PV M+**.
- 5) Key in number of next cash flow, press n. Key in next cash flow amount, press FV PV M+ to obtain current net present value. (For periods with cash outlays, press CHS before pressing FV.)

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- 6) Repeat step 5 for each cash flow. If after doing step 5 for all cash flows the final NPV is positive, the actual rate of return is greater than the value entered in step 3. Repeat the procedure using a higher rate of return. If the final NPV is negative, the actual rate of return is less than the value entered in step 3. Repeat the procedure using a lower rate in step 3.
- 7) Continue the procedure until the NPV is zero or as close to zero as desired. The IRR will be the rate used to obtain this NPV.

NOTE:

If the cash flows are pretax, the IRR will be pretax. If the cash flows are after tax, the IRR will be after tax.

Example:

What is the internal rate of return (yield on investment) for an office building costing \$115,000 if the cash flows over the next 4 years are as follows:

		Year	Cash Flow	
		1	\$10,000	
		2	\$9,500	
		3	\$9,000	
		4	\$135,000	
Solı	ition			
	Enter		See Displayed	
	CLR		0.00	
1)	115000 CH	STO M	-115000.00	Initial investment.
2)	10 ⁱ		10.00	Best guess of IRR.
3)	1 n		1.00	
4)	10000 FV	PV M+	-105909.09	NPV after first year.
5)	2 n 950	0 FV P	V	
	M+		-98057.85	NPV after second year.
6)	3 n 9000	FV PV	M+ -91296.02	NPV after third year.
	4 n 1350	000 FV	PV	
	M+		910.80	Final NPV with 10%

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Since the NPV is positive, the actual IRR is higher than 10%, therefore, try 10.5% in step 3 and repeat the procedure.



This time the NPV is negative, so the IRR is less than 10.5%. As a result of these 2 trials, we know that the IRR is between 10% and 10.5%. One last trial of the procedure using 10.25% gives an NPV of -24.78, so the actual IRR is slightly less than 10.25%.

11. EQUITY INVESTMENT ANALYSIS

Equity Investment Analysis is a method of evaluating income producing real estate investment alternatives on a pretax basis. Two key factors in this type of analysis are the anticipated income stream that the property will provide and the property's projected resale value at the end of the investment horizon. The profitability of the investment is indicated by the equity yield rate which is based on the relationship of the current price of the property, the future cash flows, and the future sales price.

One of the basic equations used in real estate equity analysis relates the income stream, sales price, projected appreciation or depreciation, and amount of mortgage as follows:

R=Y-MC
$$\frac{-\text{Apprec.}}{+\text{Deprec.}} \frac{1}{S_n} = \frac{\text{NOI}}{\text{Price}}$$

Where

R = Overall capitalization rate

Y = Equity yield rate

M = Mortgage to value ratio

C = Mortgage coefficient (imbedded in calculation)

 $1/S_n$ = Sinking fund factor for depreciation or appreciation

NOI = Net operating income

However, the HP-70 does not solve for the basic unknowns by applying the equation directly. Without using any tables the HP-70 enables the user to evaluate his potential investment to determine if it meets his objectives. Solutions for equity investment value, present value (purchase price), future sales price, and overall appreciation rate are included in the sections which follow. The section entitled Additional Considerations includes some ideas for after tax analysis as well as two applications.

A brief explanation of terms frequently used in real estate analysis is given here in order to aid in understanding the problems and results more fully.

Gross Operating Income is the total income from all sources.

Net Operating Income (NOI) is the gross income less all operating costs excluding debt service and depreciation.

Annual Net Cash Flow is the annual net operating income minus the annual debt service (i.e., annual mortgage payments).

Reversion is the future sales price minus the mortgage balance at the end of the projection period. (Sometimes called net proceeds from sale.)

Equity Yield Rate is that annual rate at which the present value of the net annual cash flows plus the present value of the equity reversion equals the equity investment value.

Equity Investment Value is the equity in the property at the beginning of the projection period.

Overall Capitalization Rate is the net operating income divided by the purchase price.

Projection Period is the length of time asset is held or holding period.

NOTE:

Keystroke explanations for calculations explained earlier in the text will not be repeated here (e.g., periodic payments, appreciation or depreciation, remaining balance).

EQUITY INVESTMENT VALUE AND PRESENT VALUE

Given the desired equity yield rate, projection period, annual net cash flow, and the reversion, the HP-70 can solve for the equity investment value and present value of the investment (current sales price).

Procedure

CLR

- 1) Calculate and key in (any order):
 - Projection period in years, press
 - Annual equity yield rate, press
 - Reversion amount, press **FV**.
- 2) Press **PV STO M** to find present value of the reversion.
- 3) Press CLR.
- 4) Calculate and key in (any order):
 - Projection period in years, press
 - Annual equity yield rate, press
 - Annual net cash flow, press **PMT**.
- 5) Press **PV** to obtain the present value of the net cash flows.
- 6) Press **M+** to see the equity investment value.
- 7) Key in mortgage amount, press **M**⁺ to obtain the current sales price (present value of investment).

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Example:

Mr. Baumann would like to invest in real estate. One of his alternatives is a small office building, currently leased for 12 years, which generates \$10,100 annually before debt service. Based on growth projections for the area, he estimates the property should be worth \$145,000 after 9 years. He can obtain an $8-\frac{1}{3}$, 20 year mortgage for \$90,000 which would have monthly payments of \$766.86.

If his desired pretax yield is 10% over 9 years, what should the equity investment value and current sales price be?

Solution

First calculate the reversion by computing the remaining loan balance after 9 years and subtracting it from the future sales price \$145,000. (See Remaining Balance only page 40.)

Reversion value is \$78,608.20. Store this in **M** by pressing **STO M**.

	Enter	See Displayed	
	CLR	0.00	
	78608.20 STO M	78608.20	
1-2)	9 n 10 i M FV		
	PV STO M	33337.55	Present value of
3)	CLR	0.00	reversion.
4-5)	9 n 10 i 766.86		
	ENTER 12 × CHS	-9202.32	Annual debt service.
	10100 🛨 РМТ РV	5169.76	Present value of net cash flows.
6)	M+	38507.31	Equity investment value.
7)	90000 M+	128507.31	Current sales price.

FUTURE SALES PRICE AND OVERALL DEPRECIATION/ APPRECIATION RATE

This calculation solves for the future sales price at the end of the projection period required to achieve a desired equity yield rate, given this rate, annual net cash flow, equity investment value, projection period, and the mortgage balance at the end of the projection period.

81

Procedure CLR

- 1) Calculate and key in (any order):
 - Projection period in years, press
 - Annual equity yield rate, press
 - Annual net cash flow, press **PMT**.
- 2) Press **FV STO M**, to get the future value of the annual net cash flows.
- 3) Press CLR.
- 4) Calculate and key in (any order):
 - Projection period in years, press n.
 - Annual equity yield rate, press
 - Equity investment value, press **PV**.
- 5) Press **FV** to obtain the future value of the equity investment. Press **M** = to find the reversion amount.
- 6) Key in mortgage balance at the end of the projection period, press + to obtain the future sales price required to meet desired yield rate.
- 7) Key in the purchase price, press XXY \triangle % to obtain the overall appreciation (if the answer is positive) or depreciation (if the answer is negative).

Example:

A warehouse has an Annual Net Cash Flow of \$1670. (The mortgage payment of \$616 has been subtracted from the NOI). The desired equity yield rate is 12% over an 11 year period. If the current asking price is \$102,000 what must the sales price be at the end of year 11 in order to achieve the desired 12% return? What overall percent appreciation does this represent?

(Assume 25% equity of \$25,500, 25 year mortgage at 8-½% with a monthly payment of \$616 leaving a remaining balance of \$60396.58 at the end of year 11)

Solution

	Enter	See Displayed	
	CLR	0.00	
1)	11 ⁿ	11.00	Projection period.
	12 i	12.00	Desired annual yield rate.
	1670 PMT	1670.00	Annual net cash flow.



ADDITIONAL CONSIDERATIONS

In the preceding sections of this chapter, the investment analyses were calculated on a pretax basis. However, income taxes play an important role in determining the correct property for investment, often making the difference between a very attractive investment and an unprofitable one. Because of the wide range of tax implications and investor's objectives, this book cannot possibly cover all tax considerations. However, it is hoped that using various sections of this book will enable you to combine calculations to fit your tax situation and objectives.

The basic procedure for doing an after tax analysis is to first compute the yearly after tax cash flows and then using a discounted cash flow analysis, find the present value of the cash flows to see if the profit objective (yield) has been met.

Computing the yearly cash flows involves determining gross income, expenses, depreciation, debt service, and income tax. The after tax cash flow in the year of resale involves even further considerations—transaction costs, excess depreciation, recapture, capital gains tax, and ordinary income tax. All the calculations necessary to do an after tax analysis have been covered in other chapters of this book. A basic income property analysis outline has been included in Appendix 3 of this book to help generate ideas for your individual analysis.

The examples below, are just two possible applications of an after tax approach. In both cases, the annual cash flows are net of expenses, depreciation, interest, and income tax with the final year amount including the net proceeds from resale.

We wish to thank Mr. LeR Burton of Salt Lake City, Utah, for these two examples.

Example 1:

What initial investment amount will yield 9% on the following projected after tax cash flows (net spendable incomes)?

Year	Cash Flow
1	\$12,000
2	11,000
3	10,000
4	9,000
5	\$28,000 (includes reversion

Solution

The required initial investment equals the present value of the cash flows discounted at the required 9% yield rate.

	Enter				See Displayed	
	CLR				0.00	
	0 ST	0 M			0.00	
1	n PV	9 i M+	12000 FV		11009.17	Present value of first cash flow.
2	n	11000	FV PV	M+	20267.65	PV of two years' cash flow.
3	n	10000	FV PV	M+	27989.49	PV of three years' cash flows.
4	n	9000	FV PV	M+	34365.32	PV of four years' cash flows.
5	n	28000	FV PV	M+	52563.39	Required investment (PV of five years' cash flows).

Example 2:

An investor purchases an investment property for \$572,500 down payment with the following after tax cash flows (net spendable incomes) over a six year period.

Year	Cash Flow
1	\$65,786
2	63,575
3	61,321
4	59,017
5	56,465
6	54,255

What is the amount of the reversion necessary to produce an internal rate of return of 10.6%?

Reversion for after tax analysis equals the sales price minus the mortgage balance, income tax, and transaction costs at the time of sale. The reversion amount necessary to achieve a certain yield rate can be easily found because the reversion equals the future value of the equity investment value minus the future value of the annual net cash flows.

Solution

1) Find the future value of the cash flows.

Ε	nter	See Displayed	
0	STO M	0.00	
5	n 10.6 i 65786		
	PV FV M+	108870.22	
4	n 63575		
	PV FV M+	203997.90	
3	n 61321		
	PV FV M+	286959.02	
2	n 59017		
	PV FV M+	359150.74	
1	n 56465		
	PV FV M+	421601.03	
54	4255 M+	475856.03	Future value of cash flows.

2) Find the future value of the equity investment value and subtract this from the answer found in step 1.



CONCLUSION

This book has presented a wide variety of basic calculations used in real estate. Many more variations of these calculations are possible. It is hoped that from the basic problem types presented here, you will be able to modify and combine these procedures to solve your specific problems, thus saving you hours of calculation time and broadening your HP-70's usage.

APPENDIX I – FINANCING ALTERNATIVES

Financing plays an important part in real estate decision making. Usually there are alternate sources of financing available which differ in percent down, interest rates, prepayment penalties, etc. The HP-70 makes a comparison of the alternatives easy.

For Example

Suppose you have just purchased a house, subject to financing, for \$45,000 and have a maximum of \$5,400 to apply towards purchase.

You have the following financing alternatives available

- 1. A 90% loan at 8-1/2% interest for 30 years with two points due now.
- 2. An 80% loan at 8% interest for 30 years with no points and a 10% second mortgage at 10% interest for 7 years.

How will each plan affect you now and over the life of the mortgage?

1. This option requires \$4500 plus 2% of \$40,500 or \$5310 now. Its monthly payments would be \$311.41.

The true annual interest rate is 8.72% because of the points due now. Total interest over the loan life would be \$71,607.54.

2. With this option only \$4500 is required now but monthly cash payments would be

\$264.16 1st mortgage
<u>74.71</u> 2nd mortgage (for 7 years)
\$338.87 Total

Total interest over each loan's life would be:

Of these two choices, alternative 2 appears to be the best. This is just one set of alternatives; you may have many others. Whatever your options, they can be easily analyzed with the HP-70 so that you can make the proper decision.

APPENDIX II - RENT OR BUY ANALYSIS

The rent or buy decision is one that is encountered frequently. Many factors enter into the analysis of this decision-rental payments, interest rate, appreciation rate, tax bracket, and property taxes, to name a few. Of course, in the end, the decision may be strictly based on emotional factors but at least you can first evaluate the financial implications with ease using the HP-70.

The example below shows one possible analysis of a rent-buy situation.

Ed Anderson is currently renting a 2 bedroom, 2 bath apartment. Ed is tired of paying out \$285 per month in rent with nothing to show for it and wonders if he really would be better off buying a house. After looking at some houses, he realizes he will have to spend about \$39,000 to meet his needs. The property taxes for this house would run about \$1050 per year.

All this money is beginning to overwhelm Ed and he questions whether buying is really cheaper. Using his HP-70, he decides to actually see what buying would cost and compare this to renting as follows:

If he buys:

1. Financing

Checking with the banks, the financing available is a 90% loan for 30 years at $8\frac{1}{2}$ %.

Sales price	\$39,000
Down payment	3,900
Mortgage	35,100
Closing costs	350
Monthly payment =	\$269.89

2. Income Tax Effect

Annual Property Taxes (estimated)	\$1,050
Annual Interest on Mortgage	2,973
Total tax deductions (property tax plus interest)	\$4,023
Tax savings (Ed's bracket is approximately 36%)	\$1,448

3. Appreciation

Property values in the area he is considering have appreciated historically by 4% per year. Based on this the annual appreciation will be about \$1560 this year.

4. Maintenance

Ed assumes he will be spending at least an additional \$50 per month for maintenance or \$600 annually.

5. Interest on Down Payment

If Ed does not buy, he could be earning interest on the down payment and closing costs money. He figures he could earn 7% annually, compounded quarterly, on the \$4250 or \$305.40. Applying his 36% tax bracket to this, he would realize only \$195.46 per year.

Combining the above information, he constructs a table for comparison, looking at annual costs*.

	Rent	Buy
Gross payments	\$3420	\$3238.68
	(12 x \$285)	(12 x \$269.89 principal and interest)
Property tax	0	\$1050
Insurance	50	\$175
Equity buildup	0	<265.36>
Maintenance	0	600
Income tax savings	0	<1448>
Appreciation	0	<1560>
Interest on down payment	<195.46>	0
Total annual cost	\$3274.54	\$1790.32
Difference	\$1484.22	

Ed decides buying really does make the most sense for his situation.

*Figures presented here are for the first year. Figures for later years will differ slightly due to less interest paid, the compounding effect on appreciation, changes in property and income taxes, etc.

APPENDIX III – WORKSHEET IDEAS

Many of our users, who frequently do the same types of calculations, have found it convenient to draw up forms containing the actual keystrokes necessary to obtain the required answers. In this way, consistency is guaranteed no matter who does the actual computation.

To give you an idea of how this would work, a sample Realtors' worksheet is shown below.

Purchaser		_ Seller		
Address		. Address		
Phone: Office	_ Res	Phone:	Office	Res
Property	Selling O	ffice		. Phone
Address	Salesmar			Phone
A.P.N	Previous Taxes		_ Assessme	nts
Date of Sale	Date of Application	า	_ Estimated	d Closing Date
Loan type	Loan pla	ced with		
Appraised at \$	Originati	on Fee		. Loan Points
Interest Rate % ENTER+ ENTER+ Perc	Number of Year	rs	_ Selling Pr	ice \$
Down payment \$	Rt	= Sale	price	
	FIXED CLOSIN		S	
Standard Title Policy & Es	crow Fees			
- ENTER+ EN	NTER+ X		+	s x≿y
(base)	(rate) (I	base amou	unt.)	Ψ
ATA Title Policy & Inspec	ction			
×	+			\$ 🛃
(rate) (bas	e amount)			¢ 🚹
California Tax Service				\$
Drawing & Recording				\$
Appraisal Report				\$
Termite Report				\$
Photos, Inspections, Milea	qe			\$ 🚺
Drawing Loan Documents				\$ 🛃
Septic Tank & Water Inspe	ection			\$ 🛃
M Origination Fee,	Pts % X≿y	R+		\$ 🛃
Other				\$ 🚹
EST. TOTAL FIXED COS	STS			\$



MONTHLY PAYMENT			
Mortgage Payment (prin + int)			
n í M PV PMT	\$		
(periods) (int/pd)			
FHA/MGIC Insurance/month	\$₽		
Home & Fire Insurance/month	\$		
Taxes/month	\$		
EST. TOTAL MONTHLY PAYMENT	\$		

Another area where you might find a worksheet helpful is in income property analysis. The following outline covers most basic considerations used in such analyses. It is not intended to be the definitive analysis worksheet but rather to be used as a guide in making your own form for analyzing properties.

Property Description			
Location			
Type of Property			
Market Value	Land		
	Improvements		
	Personal Property		
	Total		



Income	
Gross Scheduled Income less Vacancy and Credit Losses	
equals Gross Operating Income less Operating Expenses less Taxes less Insurance, etc.	
equals Net Operating Income less Interest (See "Accumulated Interest" page 38) less Depreciation (See "Depreciation Calculations" page equals Taxable Income (Loss) Income Tax (Savings)	= 17) =
Net Operating Income less Annual Debt Service (See ''Periodic Payment Amour	nt" page 27)
equals Gross Spendable Income less Income Tax less Capital Improvements	8
equals Net Spendable Income plus principal payment (See ''Remaining Balance'' page equals Net Equity Income	40)+



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