HP-80
APPLICATION NOTES
PUBLISHED AS A SERVICE FOR USERS OF THE HP-80 FINANCIAL POCKET CALCULATOR

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DIRECT REDUCTION LOAN AMORTIZATION CALCULATIONS

General

The topics within this NOTE should be of assistance in using the HP-80 to solve problems in the category of *payment-to-principal/payment-to-interest* loans (e.g. mortgages).

First, the precise method of determining *accumulated interest/remaining balance* is shown. It is a result of, and a response to questions arising when an HP-80 solution has not compared exactly with that generated by other methods.

The remainder of the material concerns procedures for finding the *time to reach a specified balance,* accumulated interest when the current payment number is unknown, and generating an *amortization schedule.* Finally, the accumulated interest/remaining balance *equations* used by the HP-80 are included.

Using the HP-80 Accumulated Interest/Remaining Balance Calculation

The HP-80 assumes that the periodic payment amount entered is the *exact* payment to fully amortize the loan over its life. In many loans the payment amount has been rounded up to the nearest cent or dollar. To get the correct answer when the payment has been rounded, solve first for the exact number of time periods to fully amortize the loan and use this value in the accumulated interest/remaining principal calculation.

EXAMPLE:

What is the accumulated interest and remaining balance on a 25 year (300 month) 7% mortgage on $20,000 where the monthly payments are $141.40?

(The exact payment to amortize this loan over 300 months is $141.355842)

PROCEDURE:

1. Solve for number of time periods to exactly pay off the loan and write this number down (we recommend writing it down to 6 decimal places).

   Use these keystrokes:

   7 [SAVE↑] 12 [÷] [i] 141.40 [PMT] 20000 [PV] [n] ⟶ 299.746473

   (This is the exact number of months . . . write it down)

2. Now solve for accumulated interest and remaining balance. Use these keystrokes:

   0 [STO] 12 [n] 299.746473 [n] 7 [SAVE↑] 12 [÷] [i] 141.40 [PMT] [Σ+] ⟶ 1390.29

   (accumulated interest of $1390.29)

   [x≷y] ⟶ 19693.49

   (remaining balance of $19693.49)

HEWLETT PACKARD 19310 Pruneridge Avenue, Cupertino, California 95014, Telephone 408-996-0100

5952-6100D 7/73

NOTE:

To get the accumulated interest for the first year (months 1–12) we begin the calculation by entering:

0 [STO] 12 [n] . . .

To get the answers for the second year (months 13–24) we enter:

12 [STO] 24 [n] . . .

The first time period that we enter is always *one less* than the actual first month number in the time span.

Finding Accumulated Interest for a Prior Period

We would like to thank Mr. Jacob Heskes of New York, New York, for the following HP-80 application:

Occasionally the payment number may not be known, but the remaining balance, interest rate and payments are known. It is still possible to find the accumulated interest for some prior period (i.e., the previous 12 months). Now, however, the month of the remaining balance is considered the first or reference period, and prior months are counted backwards from this point and have negative values.

EXAMPLE:

Assume a 7.5% loan that now has a remaining balance of $1,367.04. Payments are $118.71 per month. The payment number is unknown. How much interest has been paid over the past 12 months?

PROCEDURE:

1. First find (again to 6 decimal places) how many months it will take to pay off the remaining balance:

   7.5 [SAVE↑] 12 [÷] [i] 118.71 [PMT] 1367.04 [PV] [n] ⟶ 11.988545

   (This answer should be written down)

2. Now find accumulated interest for the last 12 months (time periods –12 to 0):

   12 [CHS] [STO] 0 [n] 11.988545 [n] 7.5 [SAVE↑] 12 [÷] [i] 118.71 [PMT] [Σ+] ⟶ 154.70

   (accumulated interest of $154.70)

   [x≷y] ⟶ 1367.04

   (remaining balance of 1367.04 previously used)

Finding the Time to Reach a Specified Balance

The number of payment periods to reach a specified balance can be determined by first calculating the number of periods to pay off the remaining balance and subtracting that answer from the total payment periods.

EXAMPLE:

Assume a 30 year mortgage at 6.9% on $45,854 with payments of $301.99. How long will it take to reach a remaining balance of $45,000?

PROCEDURE:

1. Calculate the time to pay off $45,000

   6.9 [SAVE↑] 12 [÷] [i] 301.99 [PMT] 45000 [PV] [n] ⟶ 338.99

2. Subtract from 360 months.

   360 [x≷y] [−] ⟶ 21.01

   (answer rounded is 21 months)

NOTE:

This is also a solution to the question, "How long will it take to build an equity of $854?".

Direct Reduction Loan Amortization Schedule

This calculation generates the interest paid per period, the payment toward principal each period, and the remaining balance each period over the life of a direct reduction loan.

EXAMPLE:

Generate an amortization schedule for a loan with a principal amount of $30,000, monthly payments of $200, and an interest rate of 7%.

PROCEDURE:

1. Store payment amount and load monthly interest rate and beginning principal in the operational stack locations.

   200 [CHS] [STO] 7 [SAVE↑] 12 [÷] [SAVE↑] [SAVE↑] 30000

2. Calculate monthly payment on interest

   [x≷y] [%] ⟶ 175.00
   (payment to interest on first payment)

3. Calculate payment toward principal

   [RCL] [+] [CHS] ⟶ 25.00
   (payment toward principal on first payment)

4. Calculate remaining balance

   [−] ⟶ 29975.00
   (remaining balance after first payment)

5. Return to step number 2 to calculate values for next time period.

HP-80 Accumulated Interest/Remaining Balance Equations

The equations used by the HP-80 are:

$$I_{j-k} = PMT\left[k - j - \frac{(1+i)^{k-n}}{i} + \frac{(1+i)^{j-n}}{i}\right]$$

$$PV_k = \frac{PMT}{i}\left[1 - (1+i)^{k-n}\right]$$

WHERE:

I_{j-k} = accumulated interest from payment j to payment k

PV_k = remaining balance after payment k

PMT = period payment amount

i = periodic interest rate

n = total number of payment periods