HP-80 APPLICATION NOTES

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ANNUITY DUE CALCULATIONS FOR SAVINGS PLANS WHEN COMPOUNDING PERIODS DIFFER FROM PAYMENT PERIODS

GENERAL

In financial calculations involving a series of payments equally spaced in time with periodic compounding, both periods of time are normally equal and coincident. This is an assumption built into the HP-80 pre-programming.

In savings plans however, money may become available for deposit or investment at a frequency different from the compounding frequencies offered. The HP-80 can easily be used in these calculations. However, because of the assumptions mentioned, adjustments must be made to the data so that the two periods can be considered to occur at the same time. When the compounding periods occur more frequently than payment periods, additional keystrokes adjust the compounding period interest rate to an equivalent rate for the payment period. When payments occur more frequently than compounding, the payment amount is adjusted to reflect the fact that payments accrue simple interest between compounding periods.

This issue presents keystroke solutions for *future value, payment amount, and number of payments* for both situations of compounding periods differing from payment periods. In addition it should be noted that only annuity due (payments at the beginning of payment periods) calculations are shown since this is most common in savings plan calculations.

The following symbolic values will be used in the keystroke sequences shown below:

A = number of payment periods in a year

U = number of compounding periods in a year

B = number of years (and/or fraction of a year as appropriate)

C = annual interest rate expressed as a percent

D = payment amount

F = future value of the series of payments at the end of the last payment period

COMPOUNDING PERIODS MORE FREQUENT

Keystrokes (solving for future value):

1. Calculate an equivalent payment period interest rate and store it.

   U [STO] [SAVE↑] A [÷] [n] C [RCL] [÷] [i] 100 [STO] [PV] [FV] [RCL] [−] [STO]

2. Assuming payments are made at the beginning of the payment periods, calculate the future value.

   A [SAVE↑] B [×] [n] [RCL] [i] D [RCL] [%] [+] [PMT] [FV] ⟶ F

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Example:

Quarterly deposits (A = 4) of $95 (D) are to be made into a savings account paying 5% (C), compounded monthly (U = 12). What amount (F) will be in the account at the end of 7 (B) years?

Procedure:

1. Calculate the equivalent interest rate

   12 [STO] [SAVE ↑] 4 [÷] [n] 5 [RCL] [÷] [i] 100 [STO] [PV] [FV] [RCL] [−] [STO] ⟶ 1.255215

2. Calculate the future value

   4 [SAVE ↑] 7 [×] [n] [RCL] [i] 95 [RCL] [%] [+] [PMT] [FV] ⟶ 3203.586230 ($3203.59)

Keystrokes (solving for payment amount):

1. Calculate an equivalent interest rate

   U [STO] [SAVE ↑] A [÷] [n] C [RCL] [÷] [i] 100 [STO] [PV] [FV] [RCL] [−] [STO]

2. Calculate the payment amount assuming again that payments occur at the beginning of payment periods.

   A [SAVE ↑] B [×] [n] [RCL] [i] 1 [RCL] [%] [+] F [x⇄y] [÷] [FV] [PMT] ⟶ D

Example:

A sum of money will become available annually (A = 1) for 20 years (B) starting tomorrow. It is desired to place this money in a savings institution paying 7% (C) compounded daily (U = 365, see NOTE below) so that at the end of 20 years $19,000 (F) will have been accumulated. How large must this annual deposit be?

Procedure:

1. Calculate the equivalent interest

   365 [STO] [SAVE ↑] 1 [÷] [n] 7 [RCL] [÷] [i] 100 [STO] [PV] [FV] [RCL] [−] [STO] ⟶ 7.250066

2. Calculate the annual payment

   1 [SAVE ↑] 20 [×] [n] [RCL] [i] 1 [RCL] [%] [+] 19000 [x⇄y] [÷] [FV] [PMT] ⟶ 420.474071 ($420.47)

Note: Using 365 or 360 (days) as the number of compounding periods in a year when daily compounding is available would depend on the practice of the particular institution being considered.

Keystrokes (solving for number of payment periods)

1. Calculate the equivalent interest rate

   U [STO] [SAVE ↑] A [÷] [n] C [RCL] [÷] [i] 100 [STO] [PV] [FV] [RCL] [−] [STO]

2. Calculate number of payments

   [i] D [RCL] [%] [+] [PMT] F [FV] [n] ⟶ A · B (total number of payments)

Example:

Semi-annual payments (A = 2) of $1200 (D) are to be made into an account paying 8% (C) compounded monthly (U = 12). How many payment periods will it take to accrue $25,000?

Procedure:

1. Calculate the equivalent interest rate

   12 [STO] [SAVE ↑] 2 [÷] [n] 8 [RCL] [÷] [i] 100 [STO] [PV] [FV] [RCL] [−] [STO] ⟶ 4.067262

2. Now calculate the number of payments

   [i] 1200 [RCL] [%] [+] [PMT] 25000 [FV] [n] ⟶ 14.941105 (approximately 15 semi annual payments)

PAYMENT PERIODS MORE FREQUENT

Keystrokes (solving for future value):

1. Assuming payments occur at the beginning of payment periods, calculate an equivalent *payment amount* that can be considered to occur with the same frequency as compounding.

   A [SAVE↑] [SAVE↑] U [STO] [÷] 1 [+] 2 [÷] C [%] [x⇄y] [R↓] [+] [RCL] [÷] D [×] [STO]

2. Using this equivalent payment amount, future value can be calculated.

   U [SAVE↑] B [×] [n] C [SAVE↑] U [÷] [i] [RCL] [PMT] [FV] ——→ F

Example:

Deposits of $100 (D) per month (A = 12) will be made into a savings account paying 6% (C) compounded quarterly (U = 4).

Find the value of the series of payments at the end of 14 (B) years.

Procedure:

1. Calculate an equivalent quarterly payment.

   12 [SAVE↑] [SAVE↑] 4 [STO] [÷] 1 [+] 2 [÷] 6 [%] [x⇄y] [R↓]

   [+] [RCL] [÷] 100 [×] [STO] ——→ 303.00

2. Now calculate the future value.

   4 [SAVE↑] 14 [×] [n] 6 [SAVE↑] 4 [÷] [i] [RCL] [PMT] [FV] ——→ 26299.65551 ($26,299.66)

Keystrokes (solving for payment):

Unlike the other situations presented, in solving for payment amount when payments occur more frequently than compounding, the adjustments are made to an HP-80 answer rather than to input data.

1. Calculate and store a payment factor to be recalled for use in step 3.

   A [SAVE↑] [SAVE↑] U [STO] [÷] 1 [+] 2 [÷] C [%] [x⇄y] [R↓] [+] [RCL] [÷] [STO]

2. Using the ordinary annuity keystrokes solve for the intermediate payment amount if payments were to occur with the same frequency as compounding.

   U [SAVE↑] B [×] [n] C [SAVE↑] U [÷] [i] F [FV] [PMT]

3. Now divide by the payment factor calculated in step 1.

   [RCL] [÷] ——→ D

Example:

A mutual fund has consistently paid annual dividends (U = 1) of 6% (C) on money invested, and offers investors the opportunity to automatically reinvest this money. In addition, investors can make monthly (A = 12) investments in the fund. Assuming the dividend rate stays constant and without considering stock appreciation, what should the monthly payments be if an investor wishes to accrue an investment of $25,000 (F) at the end of 15 years (B)?

Procedure:

1. Calculate and store the monthly payment factor

   12 [SAVE↑] [SAVE↑] 1 [STO] [÷] 1 [+] 2 [÷] 6 [%] [x⇄y] [R↓] [+] [RCL] [÷] [STO] ——→ 12.39

2. Calculate the equivalent annual payment to reach $25,000 at the end of 15 years at 6%.

   1 [SAVE↑] 15 [×] [n] 6 [SAVE↑] 1 [÷] [i] 25000 [FV] [PMT] ——→ 1074.06910

3. Divide by the payment factor to find the monthly payment required.

RCL ÷ ——→ 86.688386
($86.69)

Keystrokes (solving for number of payment periods):

1. Calculate and store the equivalent payment amount.

A SAVE↑ SAVE↑ U STO ÷ 1 + 2 ÷ C % x⇄y R↓ + RCL ÷ D × STO

2. The number of payment periods is arrived at by first calculating the number of compounding periods.

C SAVE↑ U ÷ i RCL PMT F FV n ——→ U · B
(Total compounding periods)

3. Dividing by the compounding periods per year results in total years. . .

U ÷ ——→ B

4. and multiplying by payment periods per year gives the number of payments required.

A × ——→ A · B
(Total payment periods)

Example:

Deposits of $100 (D) per month (A = 12) will be made into a savings account paying 6% (C) compounded quarterly (U = 4). How long (B), and how many payments will it take to accrue $26,299.66?

Procedure:

1. Calculate and store an equivalent payment amount.

12 SAVE↑ SAVE↑ 4 STO ÷ 1 + 2 ÷ 6 % x⇄y R↓ +

RCL ÷ 100 × STO ——→ 303.00

2. Find the total compounding periods.

6 SAVE↑ 4 ÷ i RCL PMT 26299.66 FV n ——→ 56.000006

3. Find the number of years.

4 ÷ ——→ 14.000002
(14 years)

4. Calculate total number of payments.

12 × ——→ 168.000019
(168 payments)