HP-80 APPLICATION NOTES

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PRICE AND YIELD CALCULATIONS FOR MORTGAGES TRADED AT A DISCOUNT/PREMIUM

GENERAL

This note will be useful to HP-80 users who buy and/or sell mortgages at prices lower (discounted) or higher (at a premium) than the remaining balance of the loan at the time of purchase.

Keystrokes solving for ***annual yield*** (given a price) and ***price*** (given a desired yield) are shown for mortgages both ***with and without balloon payments.***

Most mortgage loans are not paid to maturity, but are prepaid, and values for yield or price change depending on the time of prepayment. While there are regional assumptions available for estimating the time of prepayment is often desirable to determine values for price or yield for several prepayment assumptions. This necessitates generating a remaining balance (balloon payment) for each new assumption, and for this reason a simplified method for ***calculating the remaining balance*** of a direct reduction loan (mortgage loan) has been included.

The following symbolic values will be used in the keystroke sequences shown below:

A = Number of payment periods in a year

B = Number of years (and/or fraction of a year as appropriate).

C = Annual yield (for price and yield calculations) or annual percentage rate (for remaining balance calculations).

D = Periodic payment amount

E = Purchase price (for yield and price calculations), or mortgage amount (for remaining balance calculations).

G = Balloon payment amount, remaining balance.

MORTGAGES WITH A BALLOON PAYMENT

The balloon payment of a mortgage may occur in one of two ways. It may be part of the original loan. That is, it is agreed that the mortgage is to be repaid by a series of equal payments plus a balloon payment. Alternatively, the schedule of payments is arranged such that the loan is to be fully amortized (without a balloon) by a series of equal payments, but the mortgage is paid in full prior to maturity. The balloon payment in this case would at least be equal to the remaining balance of the mortgage at the time of prepayment. (The balloon payment could also include prepayment penalties, etc.). The keystroke solutions shown are valid for either situation. However, the examples refer to balloon payments on prepaid mortgages, since this situation occurs more frequently. It should be noted that the balloon payment amount, for the purposes of these keystrokes, occurs coincident with, and does not include the last periodic payment amount.

HEWLETT PACKARD 19310 Pruneridge Avenue, Cupertino, California 95014, Telephone 408-996-0100

5952-6104 10/73

Keystrokes (yield):

1. Divide the balloon payment that will occur at the time of prepayment by 100 and store the result.

   G [SAVE↑] 100 [÷] [STO]

2. Enter the number of payments from the time of purchase to the time of prepayment, divide by 2, multiply by 365 and press [n].

   A [SAVE↑] B [×] 2 [÷] 365 [×] [n]

3. Enter the periodic payment amount, multiply by 2, recall and divide by the value stored in step 1., and press [PMT].

   D [SAVE↑] 2 [×] [RCL] [÷] [PMT]

4. Enter purchase amount, recall and divide by the value stored in step 1., and press [PV].

   E [RCL] [÷] [PV]

5. Calculate the annual yield.

   [gold key] (gold key) [i] A [×] 2 [÷] ——→ C

Example:

A mortgage with monthly (A=12) payments of $265.07 (D) can be purchased for $35,000 (E). It is assumed that this mortgage will be prepaid in 7 years (B), and there will be a remaining balance (balloon payment) of $34,099.87 (G) at that time. Calculate the annual yield (C) if these conditions prevail.

Procedure:	*See Displayed*
1. 34099.87 [SAVE↑] 100 [÷] [STO] ——→	341.00
2. 12 [SAVE↑] 7 [×] 2 [÷] 365 [×] [n] ——→	15330.00
3. 265.07 [SAVE↑] 2 [×] [RCL] [÷] [PMT] ——→	1.55
4. 35000 [RCL] [÷] [PV] ——→	102.64
5. [gold key] (gold key) [i] 12 [×] 2 [÷] ——→	8.82 (8.82% annual yield)

Notes:

1. The HP-80 yield to maturity programming for bonds can be used for finding the yield because the cash flows of bond and mortgage transactions are analogous. The price of a mortgage corresponds to the price of a bond, the periodic mortgage payments correspond to bond coupons, and the balloon payment of a prepaid mortgage compares to the redemption (face) value of a bond.

   The HP-80 bond yield programming has built-in assumptions, however, which are specifically tied to bond calculations.

   Some of these are:

   - Bond coupons are paid semiannually
   - Time is entered in days
   - Bond price is expressed as a percent of redemption value

   To change these assumptions and use another set of conditions, the data must be adjusted. This is the reason for all the numerical data entries (i.e., 365 [×], 2 [÷]) and the [RCL] [÷] sequence in the general symbolic keystroke solutions shown.

2. Since the bond yield algorithm is being used, the same operating limits as expressed in Appendix D of the *HP-80 Owner's Handbook* apply. For these applications the limits can be expressed as follows:

   The absolute value of the number entered for PMT must be greater than .125 and less than the value entered for PV. The absolute value entered for PV must be greater than 20 and less than 5000.

Keystrokes (price):

1. Determine the present value of the periodic payments from the time of purchase to the time of prepayment using the desired yield. Store the result.

   A STO SAVE↑ B × n C RCL ÷ i D PMT PV STO

2. Using the desired yield, the balloon payment, and the same number of payment periods, find the present value of that balloon payment amount.

   A SAVE↑ B × n C SAVE↑ A ÷ i G FV PV

3. Find the price to pay by adding these two results.

   RCL + ——————————→ E

Example:

A mortgage with monthly (A=12) payments of $271.49 (D) can be purchased. It is assumed that the mortgage will be prepaid in 12 years (B), and the remaining balance (balloon payment amount) at that time will be $31,029.08 (G). Determine the price to pay for this mortgage if the desired annual yield (C) is 13%.

Procedure:	***See Displayed***
1. 12 STO SAVE↑ 12 × n 13 RCL ÷ i 271.49 PMT PV STO ——→	19750.11 (present value of series of payments)
2. 12 SAVE↑ 12 × n 13 SAVE↑ 12 ÷ i 31029.08 FV PV ——→	6575.26 (present value of the remaining balance)
3. RCL + ——————————→	26325.37 ($26,325.37, total present value)

CALCULATING THE REMAINING BALANCE OF A DIRECT REDUCTION LOAN (MORTGAGE)

The HP-80 is programmed to calculate the remaining balance and accumulated interest at a particular point in time and this procedure is outlined in the "HP-80 Owner's Handbook". The keystrokes below are presented as an alternative if only the remaining balance is desired.

1. Calculate the exact number of periods required to pay off the mortgage.

   C SAVE↑ 12 ÷ i D PMT E PV n

2. Subtract from the value just calculated the number of the payment period in which the remaining balance will occur and press n.

   A SAVE↑ B × − n

3. The remaining keystrokes are the same as those that calculate the present value of a series of equal payments.

   C SAVE↑ 12 ÷ i D PMT PV ——————————→ G

Note:

Step 1 is required only if the exact remaining balance is desired, because payment amounts are often rounded to the nearest cent, ten cents, or dollar, with the last payment adjusting for this rounding. A close approximation of the remaining balance can be achieved by replacing the calculation shown for step 1 with a simple entry of the actual total of number of payments. This ignores the fact that the last payment is not equal to the rest, but the amount of error in the remaining balance calculated in this manner would not generally affect the calculated values for price or yield significantly.

Example:

A 30 year (360 payments) loan of $20,000 (E), with an 8% (C) annual interest rate, is being paid back with 359 monthly (A=12) payments of $146.75 (D) and a final payment (360) of $151.10. What will be the outstanding balance (G) at the end of 7 (B) years.

Procedure:	*See Displayed*
1. 8 [SAVE↑] 12 [÷] [i] 146.75 [PMT] 20000 [PV] [n] ⟶	360.03
(This is the exact number of $146.75 payments required)	
2. 12 [SAVE↑] 7 [×] [−] [n] ⟶	276.03
3. 8 [SAVE↑] 12 [÷] [i] 146.75 [PMT] [PV] ⟶	18495.81 ($18,495.81–remaining balance)

Example:

Calculate the remaining balance for the preceding example, replacing step 1 by simply entering the total number of payments (360).

Procedure:	*See Displayed*
1. 360 [SAVE↑] ⟶	360.00
2. 12 [SAVE↑] 7 [×] [−] [n] ⟶	276.00
3. 8 [SAVE↑] 12 [÷] [i] 146.75 [PMT] [PV] ⟶	18495.12 ($18,495.12–remaining balance)

MORTGAGES WITHOUT BALLOON PAYMENTS

If the schedule of payments will fully amortize a loan and it is not prepaid there will of course be no balloon payment. The keystrokes for yield and price in this case are simple top row calculations.

Keystrokes (yield):

A [SAVE↑] B [×] [n] D [PMT] E [PV] [i] A [×] ⟶ C

Example:

A mortgage with monthly (A=12) payments of $311.12 (D) has a life of 7 years (B). It is assumed the mortgage will be held for the full 7 years and the mortgage can be purchased for $17,000 (E). Determine the yield.

Procedure: | *See Displayed*

12 [SAVE↑] 7 [×] [n] 311.12 [PMT] 17000 [PV] [i] 12 [×] ——→ 13.20
(13.20% – annual yield)

Keystrokes (price):

A [STO] [SAVE↑] B [×] [n] C [RCL] [÷] [i] D [PMT] [PV] ——→ E

Example:

Assume that the payments (D=$311.12) and the term (A=12 payments/year, B=7 years) of the preceding example remain the same, but a 14% (C) yield is desired. Determine the price to pay for this mortgage to achieve this yield.

Procedure: | *See Displayed*

12 [STO] [SAVE↑] 7 [×] [n] 14 [RCL] [÷] [i] 311.12 [PMT] [PV] ——→ 16601.91
($16,601.91)