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INTERNAL RATE OF RETURN (IRR) FOR UNEVEN CASH FLOWS

GENERAL

The *internal rate of return* (sometimes called the *discount rate*, *yield*, or *cost of capital*) is that rate which equates the present value of all future net cash flows to the original investment amount. This is easily found with a simple top row key calculation (n i PMT PV) if the cash flows are equal amounts and equally spaced. An iterative method is required, however, to find the internal rate of return if the cash flows are unequally spaced or are unequal amounts or both. This is accomplished with the HP-80 discounted cash flow analysis procedure shown below.

ITERATIVE METHOD FOR UNEVEN CASH FLOWS

The procedure which follows begins by choosing a “best guess” periodic internal rate of return, and performing the keystrokes for discounted cash flow analysis (see page 37 of the HP-80 Owner’s Handbook). If the final net present value is positive, the actual periodic internal rate of return is greater than the rate chosen; if the amount displayed is negative, the actual rate is lower than the rate chosen. Based on the final net present value, different rates can be tried until the final net present value is zero or as close to zero as desired. The internal rate of return will be the last rate used multiplied by the number of cash flow periods per year.

Investments (sometimes called cash outlays) are considered to be negative cash flows, periods with no activity have cash flows of zero, and periods with return or income have positive cash flows. Remember that the HP-80 assumes cash flows occur at the end of the time period.

Keystrokes:

1. \square (gold key) CLx
2. Enter a “best guess” periodic internal rate of return; press i .
3. Enter the amount of the original investment, press CHS to make it negative; press PV .
4. Enter the first period’s net cash flow (if it is a cash outlay press CHS); press PV $\Sigma+$ to obtain the current net present value.

Continue step 4 for all subsequent periods.

Zero should be entered for periods with no cash flow.

5. After the last cash flow has been entered, choose a new rate of return based on the net present value shown in the display, and repeat steps 1 through 4. Continue this process until the final net present value is zero or sufficiently close to zero.
6. When the final net present value is zero, the value last entered in step 2 is the periodic internal rate of return. To obtain the annual internal rate of return, multiply this value by the number of cash flow periods per year.

Example:

An investor purchases an apartment complex with a down payment of \$80,000.00. The property is held for 5 years and then sold, resulting in the following net annual cash flows:

-\$80,000.00	original investment
-\$13,250.00	end of year 1
\$14,710.00	end of year 2
\$16,590.00	end of year 3
\$22,500.00	end of year 4
\$109,000.00	end of year 5

What was his internal rate of return for this investment?

Procedure:

See Displayed:

1. (gold key) CLx → 0.00

2. 15 i → 15.00
(a "best guess" of 15% is tried first)

3. 80000 CHS PV → -80000.00

4. 13250 CHS PV Σ+ → -91521.74

14710 PV Σ+ → -80398.87

16590 PV Σ+ → -69490.67

22500 PV Σ+ → -56626.22

109000 PV Σ+ → -2433.96
(-\$2,433.96 final net present value)

Since the final net present value is less than zero, a lower rate, 14%, is tried next.

1. (gold key) CLx → 0.00

2. 14 i → 14.00

3. 80000 CHS PV → -80000.00

4. 13250 CHS PV Σ+ → -91622.81

14710 PV Σ+ → -80303.94

16590 PV Σ+ → -69106.16

22500 PV Σ+ → -55784.36

109000 PV Σ+ → 826.83
(\$826.83 final net present value)

Since the final net present value is greater than zero, a higher rate, 14.25%, is tried next.

1. (gold key) CLx → 0.00

2. 14.25 i → 14.25

3. 80000 CHS PV → -80000.00

4. 13250 CHS PV Σ+ → -91597.37

14710 PV Σ+ → -80327.99

16590 PV Σ+ → -69203.56

22500 PV Σ+ → -55997.97

109000 PV Σ+ → -3.46
(-\$3.46 final net present value)

The internal rate of return for this investment is slightly lower than 14.25%. (It is actually 14.249%)

“BEST GUESS” TECHNIQUES

The preceding example begins with a “best guess” of 15% as the internal rate of return. Any rate could have been chosen to begin with; however, when the “best guess” is close to the actual internal rate of return, fewer iterations are necessary to arrive at a final answer.

There are numerous ways to decide upon a first “best guess”. Two methods which accommodate most situations are shown below.

Keystrokes: (One cash flow significantly different from the rest)

If one of the cash flows is significantly different (larger or smaller) from the others, add all the cash flows together and assume that this amount occurs at the time period of this “different” cash flow. A top row key calculation can then be used to solve for a “best guess” periodic internal rate of return.

1. Add together all of the cash flows; press **STO**.
2. Enter the number of periods from the initial investment to the period of the “different” cash flow; press **n**.
3. Enter the amount of the original investment; press **PV**.
4. Press **RCL** **FV** to recall and enter the total of all the cash flows.
5. Press **i** to obtain the value to use for a “best guess” periodic internal rate of return.

Example:

Using the values of the previous example, apply the method shown to calculate a first “best guess” internal rate of return.

Procedure:

- | | | <i>See Displayed:</i> |
|----|--|---|
| 1. | 13250 CHS SAVE + 14710 + 16590 + 22500 + 109000 + STO | 149550.00
(\$149,550.00, total of cash flow amounts) |
| 2. | 5 n | 5.00
(period number of the “different” cash flow) |
| 3. | 80000 PV | 80000.00
(original investment amount) |
| 4. | RCL FV | 149550.00 |
| 5. | i | 13.33
(13.33% first “best guess”) |

(The actual IRR for this example is 14.249%)

Keystrokes: (No cash flow significantly different from the rest)

If the cash flows are not significantly different from each other, begin by finding their average (arithmetic mean). Then a top row key calculation, which assumes this average cash flow occurs each period, can be used to solve for a first “best guess” periodic internal rate of return.

1. **□** (gold key) **CLx**
2. Key in each periodic cash flow amount followed by the **Σ+** key.
3. Press **\bar{x}** to obtain the average cash flow amount; press **STO**.

4. Enter the total number of cash flow periods; press **n**.
5. Press **RCL** **PMT** to recall and enter the average cash flow amount.
6. Enter the amount of the original investment; press **PV**.
7. Press **i** to obtain the value to use for a first “best guess” periodic internal rate of return.

Example:

An original investment of \$100,000.00 resulted in the following annual cash flows:

\$18,000.00	end of year 1
\$24,000.00	end of year 2
\$36,000.00	end of year 3
\$10,000.00	end of year 4
\$41,000.00	end of year 5

Calculate a first “best guess” internal rate of return of this investment.

Procedure:

See Displayed:

- | | | |
|----|--|--|
| 1 | [] (gold key) [CLx] | 0.00 |
| 2. | 18000 [Σ+] 24000 [Σ+] 36000 [Σ+] 10000 [Σ+] 41000 [Σ+] → | 129000.00
(sum of cash flows) |
| 3. | [x̄] [STO] → | 25800.00
(average periodic cash flow) |
| 4. | 5 [n] → | 5.00
(number of cash flow periods) |
| 5. | [RCL] [PMT] → | 25800.00
(average periodic cash flow) |
| 6. | 100000 [PV] → | 100000.00
(original investment) |
| 7. | [i] → | 9.14
(9.14% first “best guess”) |

(The actual IRR for this example is 8.3746%)