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HEWLETT PACKARD

HP-80

REAL ESTATE APPLICATIONS

A Guide to Profitable Real Estate Decisions

HEWLETT hp PACKARD

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Real Estate Applications

PRINTED IN U.S.A.

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INTRODUCTION

The HP-80 REAL ESTATE APPLICATIONS HANDBOOK has been designed to supplement the *HP-80 Owner's Handbook* by providing a comprehensive collection of key sequences for solving problems specifically associated with real estate transactions. Hopefully, it will show you how to redesign our examples to fit your specific needs and provide a quick and easy reference guide to the majority of your problems.

Real estate professionals are now able to simply press the appropriate keys to rapidly see results on the HP-80 without remembering the exact formulas or having a large library of theory texts and data volumes. This pocket-sized calculator and handbook eliminate the need for cumbersome and, sometimes, less precise financial tables. The time and trouble formerly required to obtain answers to calculations are greatly reduced, thus freeing the professional to do what he is best at—making real property and other financial decisions.

In view of the fact that there are tremendous self-education activities occurring within the real estate profession, the HP-80 and this handbook will prove to be valuable to brokers and salesmen, investors and investment specialists, appraisers, assessors, escrow officials, developers, real property managers, mortgage bankers and brokers, bank, insurance and savings and loan officers, institutional fund managers, syndicators, trust fund analysts, and other top decision-makers.

A complete table of contents is provided in this handbook to help you find the specific key sequence necessary to solve any problem. For example, if you want to determine the periodic payment amount of a fully-amortized mortgage loan, just refer to Periodic Payment Amount under Simple Mortgages. Or, if your job is to construct a depreciation schedule, you will find several methods located under Depreciation Calculations.

Before proceeding to actual calculations, it is essential that you read the chapter entitled Time and the Top Row Keys starting on page 8 .

This handbook could not have been completed without the knowledge and advice of some of the top real estate professionals in the United States and Canada. Although space limitations prevent us from mentioning all of them we would like to offer our appreciation to a few enthusiastic supporters. Our special thanks go to Mr. LeR Burton, Salt Lake City; Mr. K. W. Dees, Fairfield, California; Mr. Dick Robinson, Walnut Creek, California; Mr. John Nance, Santa Cruz County, California; Mr. Russ Ellwood, Ridgewood, New Jersey; and Mr. William Fitzpatrick, Dayton, Ohio.

HOW TO READ THIS HANDBOOK

HP-80 key sequence routines are arranged in left to right order. Examples appear as follows:

> A builder paid $400 to have a concrete patio installed and the current cost is $500. What is the percentage increase in cost?

The correct procedure to solve this calculation on your HP-80 is shown as:

Solution

Enter:	See Displayed:
400 [SAVE ↑]	400.00
500 [ ] [%] (Δ%)	25.00

Therefore there has been a 25% increase in cost.

This illustration means you would press in order the "4" key, the "0" key, the "0" key, the [SAVE ↑] key, then the 5,0,0, keys followed by the gold key [ ] and finally the percent key [%]. The gold key performs an operation similar to a shift key on a typewriter. Functions appearing in gold above the function keys are accessed by pressing the gold key prior to the function key.

To clarify examples, numbers to be keyed in are shown without boxes while function keys are shown with boxes around them [ ].

NOTE:

Pressing the gold key followed by a number from 0-6, will cause the display to be rounded to that number of decimal places. The HP-80 however, retains and uses a full ten digits internally.

Pressing the gold key followed by [CLx] (CLEAR) ***clears all the registers except the*** [STO] ***register.***

Additional information regarding the operation of your HP-80 calculator is contained in your *HP-80 Owner's Handbook*.

TIME AND THE TOP ROW KEYS

Real estate calculations on the HP-80 most frequently use the top row keys— n , i , PMT , PV , FV . These keys access internal programming which is based upon calculations involving the interest compounding process. The following key descriptions and timing assumptions within the compounding calculations should be noted by the HP-80 user in order to make problem solving easier.

Throughout this book, compounding periods, payment periods, time periods, and periods are assumed to be synonymous and are used interchangeably. For any given problem, the total number of these periods is associated with the n key and can be determined by multiplying the number of time periods per year by the number of years and/or fraction of a year. For example, 30 years of monthly payments is equal to 360 total payments. One additional point to remember is that payment and compounding periods are not only equal but coincident, that is, interest is compounded at the same time that periodic payments are due.

The rate per period (interest, appreciation, yield, etc.) is associated with i , and this rate must be entered into the HP-80 as a percent. When the rate per period is given it may be entered as is. Otherwise it must be calculated by dividing the annual rate by the number of compounding periods per year. For example, 12% per year is equal to 1% per month if compounded monthly. One percent is then entered for i .

NOTE:

As explained above, n and i may or may not require a calculation. That is, on a 30 year mortgage with monthly payments, you can enter 360 directly into n or enter 30 SAVE↑ 12 × and then press n .

The periodic payment amount (annuity, loan amortization payment, etc.) is associated with the PMT key. *Payments* in the HP-80 are assumed to be equal, equally spaced, and *occur at the end of the payment periods*. Mortgage and loan repayment schedules, ordinary annuities, payments in arrears as well as most business applications fall into this category. Problems involving payments at the beginning of each period can be handled as explained in Chapter 9, Annuity Due Calculations. This ability to adjust data and accommodate a wide variety of applications beyond the original design is one of the reasons the HP-80 is such a versatile business tool.

The PV key is used to enter or calculate the initial base amount (present value, principal, investment, price, beginning balance, etc.). *This is the amount present at the beginning of the first period.*

The FV key is used to enter or calculate the final amount (future value, compounded amount, balance, etc.) and *is the amount present at the end of the last time period.*

An expansion of the relationships between the periodic payment amount and total time periods is required to finalize this discussion on time and the top row keys. Most people who have been associated with a loan scheduled to be fully amortized by a series of equal periodic payments, may have noted that the last payment is slightly more or less than the rest. This last payment differs because it adjusts for the gain or loss caused by rounding the periodic payment from its exact calculated value. In problem solutions that require values for both the total number of payment periods and the periodic payment amount, some error will occur in the answer if the payment amount has been rounded. When a precise answer is needed, the exact number of payment periods required to amortize the loan using the rounded payment amount must first be determined. This exact number of periods, which usually contains a fraction, is then entered in the solution where total periods are required. (The procedure for finding n is shown in Chapter 5 under Number of Periodic Payments for Full Amortization.) For some problems the rounded numbers for payment and number of periods give sufficient approximations, but it should be noted that a precise answer can be obtained easily. Wherever total number of payments and payment amounts are required for solutions in this text, methods for calculating precise answers are shown.

With these facts, you now are ready to fully explore the power of the HP-80.

PERCENTAGE CALCULATIONS

Percentage calculations play an important role in many real estate calculations. The following explanations and examples point out the wide number of applications on the HP-80.

PERCENTAGE OF BASE

A base amount multiplied by a percentage rate yields a percentage of base, performed as follows:

1) Enter the base amount, press [SAVE ↑].
2) Enter the percentage rate, press [%] .

NOTE:

Use the percentage rate, not the decimal equivalent, when entering.

Example 1:

A real estate broker wants to use his HP-80 to calculate the commission on his listing. If the commission is 6% of the $39,950 sales price, how much will the commission be?

Solution

	Enter:	See Displayed:	
1)	38950 [SAVE ↑]	38950.00	
2)	6 [%]	2337.00	commission on the sale

Example 2:

A house valued one year ago at $51,500 has appreciated 4%. How much money does this represent?

Solution

	Enter:	See Displayed:	
1)	51500 [SAVE ↑]	51500.00	initial amount
2)	4 [%]	2060.00	appreciation

NET AMOUNT

A percentage of the base amount can be added to or subtracted from the base amount to give a net figure as follows:

1) Enter the base amount, press SAVE ↑ .
2) Enter the percentage rate, press % .
3) Press + or − to obtain net amount.

Example 1:

A home valued two years ago at $51,500 has appreciated 4%. What is its current value?

Solution

	Enter:	See Displayed:	
1)	51500 SAVE ↑	51500.00	
2)	4 %	2060.00	
3)	+	53560.00	current value

Example 2:

A salesman sold his broker's listing for $38,950 and is entitled to 45% of the 6% commission. What are the two commissions?

Solution

	Enter:	See Displayed:	
1)	38950 SAVE ↑	38950.00	
2)	6 %	2337.00	total commission
	45 %	1051.65	salesman's commission
3)	−	1285.35	broker's commission

PERCENT DIFFERENCE

Percent difference is defined as the difference between a base amount and another amount, divided by the base amount. Information is entered as follows:

1) Enter the base amount, press SAVE ↑.
2) Enter the other amount, press [shift] % (Δ%) .

Example:

A building valued 3 years ago at $51,500 is now worth $53,560. What percent increase does this represent?

Solution

	Enter:	See Displayed:	
1)	51500 SAVE ↑	51500.00	
2)	53560 [ ] % (Δ%)	4.00	percent increase

NOTE:

The answer displayed is a percent of the base amount, $51,500.

APPRECIATION CALCULATIONS

The valuation of property may involve the calculation of some appreciation amount for a given period of time using a periodic appreciation rate. This periodic rate is compounded over the time frame being considered in the same way as interest is compounded on savings accounts. This should not be confused with a linear overall appreciation rate as shown in the examples on pages 11 and 12 and used in "Linear Growth Trend." Under some circumstances, property values may actually decline rather than increase. The keystrokes shown below are still valid; the only change is that when entering a rate of decline, the CHS key must be used to change the rate to a negative value.

FUTURE VALUE OF A COMPOUNDED AMOUNT

This calculation finds the future value of an initial amount appreciated/depreciated at a given rate compounded over a specified number of periods.

Information is entered as follows:

1) Calculate and enter the total number of time periods, press n .
2) Calculate and enter the rate per period (expressed as a %), press i .
3) Enter the initial principal (present value), press PV .
4) Press FV to obtain the future value.

Example 1:
A home purchased five years ago for $23,850 is located in an area where similar models have been appreciating at about 4% per year. What is the current approximate value of the house?

Solution

	Enter:	See Displayed:	
1)	5 n	5.00	number of periods (years)
2)	4 i	4.00	appreciation rate per year
3)	23850 PV	23850.00	initial amount
4)	FV	29017.17	the approximate current value

Example 2:

Recent relaxation of pollution control laws (brought on by the energy crisis) and the announcement of the intention to build an oil refinery are causing property values in the immediate vicinity of the intended site to decline. Estimates are that property in the area will decline at the rate of 2% per year until the plant is completed six years from now. What will property presently valued at $32,000 be worth at the end of six years if this estimate is correct?

Solution

	Enter:	See Displayed:	
1)	6 [n]	6.00	number of periods (years)
2)	2 [CHS] [i]	−2.00	yearly rate of decline
3)	32000 [PV]	32000.00	present value
4)	[FV]	28346.96	value six years from now

PRESENT VALUE OF A COMPOUNDED AMOUNT

This calculation finds the present value (initial principal amount) when the future value, number of periods, and appreciation rate are known. Keystrokes are:

1) Calculate and enter the total number of periods, press [n] .
2) Calculate and enter the appreciation rate per period, press [i] .
3) Enter the future value, press [FV] .
4) Press [PV] to obtain the present value.

Example 1:

A piece of land in a surburban community is within walking distance of a recently completed rapid transit system, and it can be purchased for $10,000. Neighbors claim that their property has been appreciating at 1% per month since the construction of the commuter system began four and a half years ago. If this is true, what would have been the value of the property then?

Solution

	Enter:	See Displayed:	
1)	12 SAVE ↑ 4.5 × n	54.00	total number of months
2)	1 i	1.00	appreciation rate per month
3)	10000 FV	10000.00	current property value
4)	PV	5843.13	value of the property 4½ years ago

PERIODIC APPRECIATION RATE

Given the number of periods, the value of the investment at the beginning of the first period and end of the last period, the periodic appreciation rate can be found as follows:

1) Calculate and enter total periods, press n .
2) Enter initial value of investment, press PV .
3) Enter final value of investment, press FV .
4) Press i to obtain periodic appreciation rate.

Example 1:

A realtor has just listed a house which was bought 3 years ago for $29,000. The current asking price is $36,750. What yearly appreciation rate does this represent?

Solution

	Enter:	See Displayed:	
1)	3 n	3.00	total periods
2)	29000 PV	29000.00	value 3 years ago
3)	36750 FV	36750.00	today's value
4)	i	8.21	annual appreciation rate

Example 2:

Mr. Brown purchased his house 4 years ago for $45,000. Since that time, the planning commission has proposed a freeway that will adjoin his backyard. His house is for sale and the best offer he has received is $42,000. What annual rate of decline does this represent?

Solution

	Enter:	See Displayed:	
1)	4 [n]	4.00	number of years
2)	45000 [PV]	45000.00	original price
3)	42000 [FV]	42000.00	current price
4)	[i]	−1.71	annual rate of decline

NUMBER OF PERIODS IN A COMPOUNDED AMOUNT

This calculation finds the number of compounding periods when the periodic appreciation or depreciation rate, initial principal (present value) and compounded amount (future value) are given.

Information is entered as follows:

1) Enter the rate per period as a percent, press [i] .
2) Enter the initial principal, press [PV] .
3) Enter the compounded amount (future value), press [FV] .
4) Press [n] to obtain the number of time periods.

Example 1:

Property currently worth $42,000 is in an area that has been appreciating at 4% annually. If this rate continues, how many years until the property will be worth $55,000?

Solution

	Enter:	See Displayed:	
1)	4 i	4.00	annual growth rate
2)	42000 PV	42000.00	present value
3)	55000 FV	55000.00	desired future value
4)	n	6.88	years

LINEAR GROWTH TREND

When it is desired to linearly project future values based on past data, a trend line calculation can be used. This type of calculation requires input data that is evenly distributed in time and in chronological order. (If certain data points are missing, see Chapter 11, Linear Regression.)

NOTE:

Calculations in other parts of this Appreciation section assume a compounded periodic rate whereas the following calculation assumes a linear, not compounded, rate.

Information is entered as follows:

1) Press CLEAR CLx to clear calculator of existing data.
2) Enter successive historical values, press TL after each value. The entry sequence number is displayed after each entry.
3) After all data are entered, press COMPUTE TL to obtain the value at time period 0—i.e., the point at which the trend line, traveling the horizontal axis (time line), intersects the vertical axis (quantity line). This is the y-intercept.
4) Enter number of time period for which a future value is desired, press n .
5) Press TL to obtain trend line value for that time period.
6) Repeat step 5) to obtain each successive trend value per time unit, or go back to step 4) to find values for a unique time position.

NOTES:

1. *The time position (or entry number) can be seen at any time by pressing the* [x⇄y] *key. Be sure to press the* [x⇄y] *key again before resuming with step 4 or 5.*

2. *The slope of the trend line (the change in quantity per time period) may be found after step 5 by pressing* [R↓] [R↓] *(this value is the periodic appreciation amount when used in examples similar to those below). Be sure to press* [R↓] [R↓] *before resuming with steps 4 or 5.*

Example 1:

A record of sale prices for a particular model home has been:

Time Period	Year	Sale Price
1)	1969	$26,500
2)	1970	$27,750
3)	1971	$31,500
4)	1972	$34,500

Using the HP-80 trend line calculation, what are the projected sale prices for the years 1973, 1974, 1975, 1980 (i.e., years 5, 6, 7, 12).

Solution

	Enter:	See Displayed:	
1)	[■] [CLx] (CLEAR)	0.00	
2)	Enter data as described above		
	26500 [TL]	1.00	entry sequence number
	27750 [TL]	2.00	entry sequence number
	31500 [TL]	3.00	entry sequence number
	34500 [TL]	4.00	entry sequence number
3)	[■] [TL] (COMPUTE)	23125.00	"Y" intercept (value projected back to 1968)

	Enter:	**See Displayed:**	
4)	5 n	5.00	starting at the 5th time period (year)
5)	Obtaining successive values:		
	TL	37000.00	1973 (5th year)
	TL	39775.00	1974 (6th year)
	TL	42550.00	1975 (7th year)
	12 n	12.00	jumping ahead to the 12th year
	TL	56425.00	1980
	R↓ R↓	2775.00	yearly appreciation amount

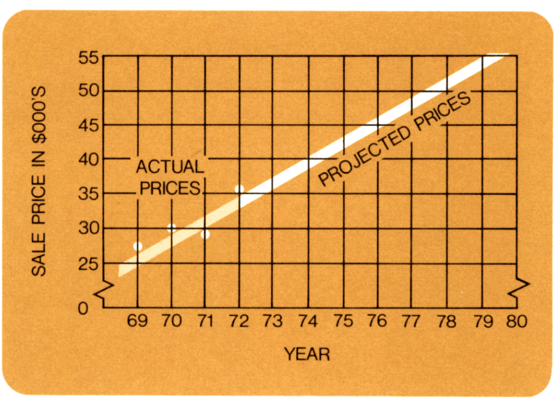

Figure 1: *Trend Line as Projected in Example 1.*

DEPRECIATION CALCULATIONS

The three common accounting methods of providing for the return of a capital investment over its useful life expectancy are—straight-line, declining balance, and sum-of-the-years' digits depreciation (SOD). Declining balance and SOD are accelerated methods, providing higher depreciation amounts initially than the straight line method as Fig. 2 shows.

NOTES:

1. ***Since land does not wear out, its cost may not be depreciated. Thus, the real estate investor must separate the cost of his improvements from the cost of the land.***
2. ***If the improvements are expected to have a salvage value at the end of the useful life of the property, this expected salvage value must be deducted from the cost or other valuation basis of the property to determine the depreciable amount when using the straight-line and sum-of-the-years' digits methods only. The improvement may not be depreciated below its salvage value when using the declining balance method.***
3. ***If accelerated depreciation is used in the early years of depreciation, at the time of resale, the excess depreciation must be calculated for tax purposes. Excess depreciation is the difference between the depreciation amount taken and the amount that would have been taken using the straight line method.***

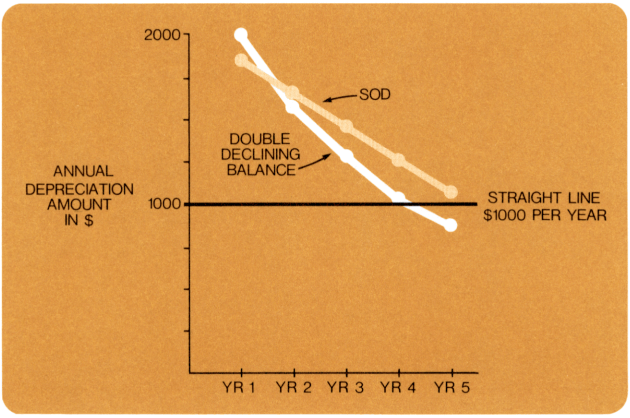

Figure 2: *Annual depreciation amounts for the first 5 years of useful life using each depreciation method. Figures based on a $10,000 asset with a 10 year useful life.*

STRAIGHT-LINE METHOD

The annual depreciation allowance using this method is determined by dividing the cost or other basis of the property valuation (excluding land costs) less its estimated salvage value by its useful life expectancy. Information is entered as follows:

1) Enter depreciable amount (improvements cost less salvage value), press [SAVE ↑] [SAVE ↑] .
2) Enter estimate of asset's useful life (number of years), press [÷] to obtain each year's depreciation.
3) Press [STO] [−] to obtain the depreciable value after the first year.
4) Press [RCL] [−] to obtain the remaining depreciable value for each subsequent year. If book value is needed, add salvage value to depreciable value.

Example 1:

An investor purchased a building for $200,000, excluding the cost of the land, which has an estimated useful life of 40 years and an estimated salvage value of $30,000. Using the straight-line method of depreciation, what are the building's annual depreciation allowance and remaining depreciable value for the first three years of its useful life?

Solution

	Enter:	See Displayed:	
1)	170000 [SAVE ↑] [SAVE ↑]	170000.00	depreciable amount
2)	40 [÷]	4250.00	annual depreciation allowance
3)	[STO] [−]	165750.00	remaining depreciable value, year 1
4)	[RCL] [−]	161500.00	remaining depreciable value, year 2
	[RCL] [−]	157250.00	remaining depreciable value, year 3

SPECIAL NOTE:

The remaining depreciable value for a particular year can be found directly without calculating the initial period data by using the following keystrokes: (as shown in Example 2).

1) Calculate and enter depreciable amount, press SAVE ↑ SAVE ↑ .
2) Enter estimate of useful life, press ÷ STO SAVE ↑ .
3) Enter number of the year for which data is desired, press × to obtain total straight line depreciation to date.
4) Press − to obtain remaining depreciable value to date.
5) Press RCL − to obtain remaining depreciable value for each subsequent year.

Example 2:

An apartment building which cost $1,565,000 49 years ago, has 4 years of useful life remaining. The value of the land at the time of purchase was $208,000 and the buildings estimated salvage value is $85,000. Using straight line depreciation, what are the annual depreciation allowance, remaining depreciable value for the current year, and remaining depreciable value for each year of remaining useful life?

Solution

	Enter:	See Displayed:	
1)	1565000 SAVE ↑ 208000 −		
	85000 − SAVE ↑ SAVE ↑	1272000.00	depreciable amount
2)	53 ÷ STO SAVE ↑	24000.00	annual depreciation amount
3)	49 ×	1176000.00	total depreciation first 49 years
4)	−	96000.00	remaining depreciable value year 49

	Enter:	See Displayed:	
5)	RCL −	72000.00	remaining depreciable value year 50
	RCL −	48000.00	remaining depreciable value year 51
	RCL −	24000.00	remaining depreciable value year 52
	RCL −	0.00	remaining depreciable value year 53

DECLINING BALANCE METHOD

The declining balance method is one form of accelerated depreciation;as such it provides for more depreciation in the earlier years of ownership and less depreciation in the later years than the straight line method. The following calculations find the depreciation and remaining book value for each year of an asset's depreciable life when the cost or other valuation basis, salvage value, and life expectancy are known. Calculations under the section entitled Full Year are valid when an asset is held for a full twelve months in the first year of depreciation, while the calculation under the section entitled Partial Year is used in cases where the asset is held less than twelve months in its first year of depreciable life.

Full Year

To find the depreciation and remaining balance for each year, information is entered as follows:

1) Enter declining factor (1.25 for 125% declining balance, 2.00 for double declining, etc.), press [SAVE ↑].
2) Enter 100, press [×] .
3) Enter number of years of useful life expectancy, press [÷] to obtain multiplier. Press [STO].
4) Enter cost or other basis (do not deduct salvage value).
5) Press [RCL] , press [%] to obtain first year's depreciation.
6) Press [−] to obtain remaining book value after first year.
7) Repeat steps 5) and 6) to obtain each succeeding year's depreciation and remaining book value until the book value is equal to or less than the salvage value. In the latter case, the previous book value is reduced by the salvage value to obtain the final year's depreciation.

Example 1:

A new office building has a cost basis of $250,000.00 (exclusive of land) and a 35 year useful life expectancy. Using the 150% declining balance method, calculate the building's annual depreciation allowance and remaining book value for each of its first two years.

Solution

	Enter:	See Displayed:	
1-2)	1.50 [SAVE ↑] 100 [×]	150.00	
3)	35 [÷] [STO]	4.29	multiplier
4)	250000	250000	
5)	[RCL] [%]	10714.29	first year depreciation
6)	[−]	239285.71	remaining depreciable value
7)	[RCL] [%]	10255.10	second year depreciation
	[−]	229030.61	remaining depreciable value

SPECIAL NOTE:

The following modifications to the solution steps (preceding Example 1) eliminate the necessity to begin declining balance depreciation calculations with the initial years of an asset's useful life; that is, depreciation allowances and remaining depreciable book value amounts for specific years can be determined immediately without having to begin at year one and sequentially proceed to the desired years. Information is entered as follows:

1) Enter the year the depreciation and book value are desired, press [SAVE ↑] 1 [−] [n] .
2) Enter the declining factor, (1.25, 2.00 etc.), press [SAVE ↑].
3) Enter 100, press [×] .
4) Enter useful life expectancy (number of years), press [÷] to obtain multiplier. Press [STO].
5) Press [CHS] [i] to obtain declining rate factor (the negative of the multiplier).
6) Enter the cost or other basis, press [PV] [FV] to obtain the remaining value at the beginning of the specified year.
7) Press [RCL] [%] to obtain depreciation in specified year.
8) Press [−] to obtain remaining value at the end of the specified year.
9) Repeat steps 7) and 8) to obtain each succeeding year's depreciation and remaining book value.
10) To skip a successive year
    a) Enter the new specified year (number), press [SAVE ↑] 1 [−] [n] .
    b) Press [RCL] [CHS] [i] to obtain declining rate factor.
    c) Enter cost or other basis, press [PV] [FV] .
    d) Repeat steps 7) and 8) to obtain depreciation and remaining book value.

Example 2:

Using the same information as in Example 1 above, calculate the depreciation and remaining book value for years 3, 4, 6, and 7.

Solution

	Enter:	See Displayed	
1)	3 [SAVE ↑] 1 [−] [n]	2.00	
2-3)	1.50 [SAVE ↑] 100 [×]	150.00	
4)	35 [÷] [STO]	4.29	
5)	[CHS] [i]	−4.29	
6)	250000 [PV] [FV]	229030.61	remaining book value at beginning of 3rd year
7)	[RCL] [%]	9815.60	depreciation in 3rd year
8)	[−]	219215.02	remaining book value at end of 3rd year
9)	[RCL] [%]	9394.93	depreciation in 4th year
	[−]	209820.09	remaining book value at end of 4th year
10)	a) 6 [SAVE ↑] 1 [−] [n]	5.00	
	b) [RCL] [CHS] [i]	−4.29	
	c) 250000 [PV] [FV]	200827.80	
	d) [RCL] [%]	8606.91	depreciation in 6th year
	[−]	192220.89	remaining book value at end of 6th year
	[RCL] [%]	8238.04	depreciation in 7th year
	[−]	183982.85	remaining book value at end of 7th year

Partial Year

If the asset is held for less than twelve months in the first year, the depreciation using the declining balance method can be found as follows:

1) Enter declining factor (1.25 for 125% declining balance, 1.50 for 150% declining balance etc.), press [SAVE ↑] 100 [×] .
2) Enter depreciable life, press [÷] [STO] .
3) Enter initial book value, press [RCL] [%] .
4) Enter number of months held in first year, press [SAVE ↑] 12 [÷] [×] to obtain first year's depreciation.
5) Press [−] to see remaining book value.
6) Press [RCL] [%] to obtain second year's depreciation.
7) Press [−] for remaining book value.
8) Repeat steps 6 and 7 for successive years depreciation and remaining book value.

Example:

An asset is valued at $50,000 with an expected life of 16 years. It is held for 6 months the first year and double declining balance depreciation is used. What are the depreciation and remaining balance for the first three years?

Solution

	Enter:	See Displayed:	
1)	2.00 [SAVE ↑] 100 [×]	200.00	
2)	16 [÷] [STO]	12.50	
3)	50000 [RCL] [%]	6250.00	
4)	6 [SAVE ↑] 12 [÷] [×]	3125.00	first year's depreciation
5)	[−]	46875.00	remaining book value
6)	[RCL] [%]	5859.38	second year's depreciation
7)	[−]	41015.63	remaining book value
8)	[RCL] [%]	5126.95	third year's depreciation
	[−]	35888.67	remaining book value

SUM-OF-THE-YEARS'-DIGITS METHOD

Like the declining balance method, the sum-of-the-years-digits method (SOD) is an accelerated form of depreciation, allowing more depreciation in the early years of an asset's life than allowed under the straight line method. The calculations below find the depreciation and remaining depreciable value using the SOD method for each year of an asset's depreciable life when its useful life expectancy and cost or other basis (less salvage value) are known. The section entitled Full Year is used if the asset is held the full twelve months of the first year and the section entitled Partial Year is used if the asset is held for less than twelve months the first year of depreciation.

Full Year

To find the depreciation and remaining depreciable value, information is entered as follows:

1) Enter beginning year number (e.g., year 1), press n .
2) Enter asset's useful life expectancy (number of years), press n .
3) Enter cost or other depreciable basis of asset, press PV .
4) Press [ ] COMPUTE SOD to obtain beginning year's depreciation.
5) Press x⇄y to obtain remaining depreciable value.
6) Press SOD to obtain next year's depreciation.
7) Press x⇄y to obtain remaining depreciable value.
8) Repeat steps 6) and 7) to obtain each subsequent year's depreciation and remaining depreciable value until the asset is completely depreciated.

NOTES:

1. *You can bypass the remaining depreciable value calculation by skipping steps 5)–8), and can obtain each subsequent year's depreciation by pressing* SOD *repeatedly after completing step 4).*
2. *You can start calculating depreciation at any year within the depreciable life span of the asset. Furthermore, you can skip to any new starting point by simply entering the year number (e.g., year 4), pressing* n *and then pressing* SOD *for each year the depreciation amount is desired.*

Example 1:

An office building has a cost basis (excluding land cost and salvage value) of $210,000.00 and a useful life expectancy of 25 years. Using the sum-of-the-years'-digits method, what are the depreciation allowances and remaining depreciable values for each of the first two years?

Solution

	Enter:	See Displayed:	
1-4)	1 [n] 25 [n] 210000 [PV]		
	[ ] COMPUTE [SOD]	16153.85	year 1 depreciation
5)	[x⇄y]	193846.15	remaining depreciable value, year 1
6)	[SOD]	15507.69	year 2 depreciation
7)	[x⇄y]	178338.46	remaining depreciable value, year 2

Example 2:

Using the values from Example 1 above, what are the depreciation allowances and remaining depreciable values for years 3 and 4, and the depreciation allowance only for years 5 through 7?

Solution

	Enter:	See Displayed:	
1-4)	3 [n] 25 [n] 210000 [PV]		
	[ ] COMPUTE [SOD]	14861.54	year 3 depreciation
5)	[x⇄y]	163476.92	remaining depreciable value, year 3
6)	[SOD]	14215.38	year 4 depreciation
7)	[x⇄y]	149261.54	remaining depreciable value, year 4
8)	[SOD]	13569.23	year 5 depreciation
	[SOD]	12923.08	year 6 depreciation
	[SOD]	12276.92	year 7 depreciation

Partial Year

When the total depreciable life is an integer but the asset is held less than 12 months the first year of depreciation, the keystrokes for calculating depreciation using SOD are as follows:

1) Enter depreciable life of asset, press [SAVE ↑] [SAVE ↑] [SAVE ↑].
2) Press 1 [+] [x⇄y].
3) Enter number of months asset is held in the first year, press [×] 12 [÷] [−] [n].
4) Press [x⇄y] [n].
5) Enter initial depreciable value, press [PV].
6) Press [ ] COMPUTE [SOD] to obtain first year's depreciation value.
7) Press [R↓], then enter number of months asset is held in the first year, press [SAVE ↑] 12 [÷] [R↓] [SOD].
8) Continue pressing [SOD] for successive year's depreciation.

Example:

Fixtures in an office building have a depreciable value of $15,000 with an estimated total life of 5 years. The owner wishes to take six months depreciation in the first year and the remaining sixth months in the sixth year. Using the sum-of-the-years' digits method, what would each year's depreciation be?

Solution

	Enter:	See Displayed:	
1)	5 [SAVE ↑] [SAVE ↑] [SAVE ↑]	5.00	depreciable life
2)	1 [+] [x⇄y]	5.00	
3)	6 [×] 12 [÷] [−] [n]	3.50	
4)	[x⇄y] [n]	5.00	
5)	15000 [PV]	15000.00	depreciable amount
6)	[ ] COMPUTE [SOD]	2500.00	first year's depreciation
7)	[R↓] 6 [SAVE ↑] 12 [÷] [R↓]		
	[SOD]	4500.00	second year's depreciation

(cont'd)

	Enter:	See Displayed:	
8)	SOD	3500.00	third year's depreciation
	SOD	2500.00	fourth year's depreciation
	SOD	1500.00	fifth year's depreciation
	SOD	500.00	sixth year's depreciation

SIMPLE MORTGAGES
(Fully Amortized Mortgages or Direct Reduction Loans)

Simple mortgages provide for the complete repayment of debt through equal periodic installments which include varying amounts of principal and interest. Since interest is accrued on the remaining balance, the early payments are applied mostly to interest with a small reduction in principal. As the payment number increases, an increasing portion of the payment is applied to principal as Figure 2 shows.

Most simple mortgage calculations can be solved on the HP-80 simply by using the top row keys. No real estate or other financial tables are required.

PERIODIC PAYMENT AMOUNT

This calculation solves for the periodic payment amount (monthly, quarterly, yearly, etc.) to fully amortize a mortgage, given the life of the mortgage, the number of payment periods per year, the annual interest rate, and the amount of the mortgage.

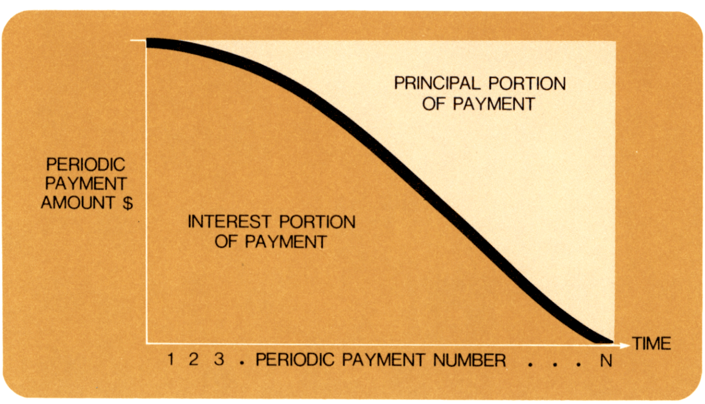

Figure 3: *Interest and Principal Portion of Periodic Payment.*

Information is entered as follows:

1) Calculate and enter the total number of payment periods, press n .
2) Calculate and enter periodic interest rate, press i .
3) Enter the mortgage amount, press PV .
4) To obtain the periodic payment amount, press PMT .

Example 1:

What is the monthly payment required to fully amortize a 30 year, $30,000 mortgage if the interest rate is 9%?

Solution

	Enter:	See Displayed:	
1)	30 SAVE↑ 12 × n	360.00	total monthly periods in mortgage life
2)	9 SAVE↑ 12 ÷ i	.75	monthly interest rate
3)	30000 PV	30000.00	mortgage amount
4)	PMT	241.39	monthly principal and interest payment

Example 2:

What quarterly payments are required on a home if the sales price is $51,950, the down payment is 20% of the sales price, and the buyer can obtain a 25 year mortgage at 8% interest?

Solution

	Enter:	See Displayed:	
1)	25 SAVE↑ 4 × n	100.00	total quarterly periods in mortgage life
2)	2 i	2.00	quarterly interest rate = 8 ÷ 4
3)	51950 SAVE↑ 20 % − PV	41560.00	mortgage amount
4)	PMT	964.31	quarterly payment

NUMBER OF PERIODIC PAYMENTS FOR FULL AMORTIZATION

This calculation solves for the total number of equal periodic payments required to fully amortize a mortgage, given the interest rate, periodic payment, and mortgage amount. It also can determine the remaining number of payment periods when the periodic payment amount, interest rate, and present remaining balance are known.

As indicated in Chapter 1, "Time and the Top Row Keys" these same keystrokes are used to determine the exact number of payment periods required to amortize a specified amount when the payment amount has been rounded. Keystrokes are as follows:

1) Calculate and enter the periodic interest rate; press [i].
2) Enter the periodic payment amount; press [PMT].
3) Enter the mortgage amount, press [PV].
4) To obtain the total number of periodic payment installments (total number of periods), press [n].

   The number of years corresponding to the total number of periods can be obtained by entering the number of periods per year, then pressing [÷].

Example 1:

An investor can afford to pay $380 per month (principal and interest) on a $56,000 mortgage. If the annual interest rate is 7 ¾%, how long will it take to completely amortize this mortgage?

Solution

	Enter:	See Displayed:	
1)	7.75 [SAVE ↑] 12 [÷] [i]	0.65	monthly interest rate
2)	380 [PMT]	380.00	monthly payment
3)	56000 [PV]	56000.00	mortgage amount
4)	[n]	470.90	total monthly periods
	12 [÷]	39.24	total years

Example 2:

An 8% mortgage with annual payments of $25,000 has a remaining balance of $167,752.04. How many payments remain to be paid?

Solution

	Enter:	See Displayed:	
1)	8 [i]	8.00	annual interest rate
2)	25000 [PMT]	25000.00	annual payment
3)	167752.04 [PV]	167752.04	mortgage amount
4)	[n]	10.00	annual payments remaining

NUMBER OF PERIODIC PAYMENTS TO REACH A SPECIFIED PRINCIPAL BALANCE

While a mortgage may be scheduled to be fully amortized, it is often intended to pay off or refinance the loan prior to maturity, at some specified remaining balance or equity position. Given the periodic interest rate, total number of periods in the mortgage, the periodic payment amount, and the specified remaining balance, the number of periods to reach this balance can be found as follows:

1) Calculate and enter the periodic interest rate, press [i] .
2) Enter the periodic payment amount, press [PMT] .
3) Enter the remaining balance, press [PV] [n] to obtain the number of periods required to amortize the remaining balance.
4) Calculate and enter the total number of payment periods during the life of the original loan.
5) Press [x⇄y] [−] to obtain the number of periods required to reach the remaining balance.

Enter the number of payment periods per year, press [÷] to obtain the number of years this answer represents.

Example 1:

An investor intends to refinance his $750,000, 8.9%, 20 year mortgage when the principal balance declines to $500,000. Given that his monthly principal and interest payment is $6,699.79, how many monthly installments will he have to make in order to reach this remaining balance? How many years does this represent?

Solution

	Enter:	See Displayed:	
1)	8.9 [SAVE ↑] 12 [÷] [i]	0.74	monthly interest rate
2)	6699.79 [PMT]	6699.79	monthly payment
3)	500000 [PV] [n]	109.12	total number of monthly payments to amortize $500,000
4)	20 [SAVE ↑] 12 [×]	240.00	total number of monthly periods in mortgage life
5)	[x⇄y] [−]	130.88	total monthly payments or months to reach $500,000 remaining balance
	12 [÷]	10.91	number of years to reach $500,000 remaining balance

NOTE:

Finding the number of periods to reach a specified equity position simply requires determining the remaining balance associated with this equity and then proceeding with the keystrokes shown above. (Purchase price minus equity position equals remaining balance).

Example 2:

Financing of a $750,000 acquisition consists of a $150,000 down payment and an 8 2/3%, 30 year mortgage with quarterly payments of $14,074.82. Business conditions suggest to the purchaser that he should divest himself of this property on or before achieving an equity equal to 30% of the purchase price. When will this amount of equity be realized?

Solution

	Enter:	See Displayed:	
1)	8 [SAVE ↑] 2 [SAVE ↑] 3 [÷] [+]		
	4 [÷] [i]	2.17	quarterly interest rate
2)	14074.82 [PMT]	14074.82	quarterly payment
3)	525000 [PV] [n]	77.03	number of payments required to amortize the remaining balance of $525,000.00 which is 70% of original price
4)	30 [SAVE ↑] 4 [×]	120.00	total number of quarterly payments
5)	[x⇄y] [−]	42.97	total quarterly payments to reach $225,000 equity
6)	4 [÷]	10.74	number of years to reach $225,000 equity

ANNUAL PERCENTAGE RATE CALCULATIONS WITHOUT FEES

This calculation finds the annual percentage rate (APR) associated with a fully amortized mortgage not involving mortgage issuance related fees, given the life of the mortgage, the periodic payment amount, and the mortgage amount. Information is entered as follows:

1) Calculate and enter the total number of payment periods; press [n].
2) Enter the periodic payment amount; press [PMT].
3) Enter the mortgage amount; press [PV] and [i] to obtain the percentage rate per period.
4) To obtain the annual percentage rate, enter the number of periods per year, then press [×].

Example 1:

A 30 year, $50,000 mortgage has monthly payments of $320, including principal and interest. What is the annual percentage rate?

Solution

	Enter:	See Displayed:	
1)	30 [SAVE↑] 12 [×] [n]	360.00	total monthly periods in mortgage life
2)	320 [PMT]	320.00	monthly payment
3)	50000 [PV] [i]	0.55	monthly percentage rate
4)	12 [×]	6.62	annual percentage rate

Example 2:

A $56,554.50 semi-annual principal and interest installment on a 35 year, $1,200,465.98 mortgage corresponds to what annual percentage rate?

Solution

	Enter:	See Displayed:	
1)	70 n	70.00	total semi-annual periods in mortgage life
2)	56554.50 PMT	56554.50	semi-annual payment
3)	1200465.98 PV i	4.49	semi-annual interest rate
4)	2 ×	8.99	annual percentage rate

ANNUAL PERCENTAGE RATE CALCULATIONS WITH FEES

Borrowers are sometimes charged fees related to the issuance of a mortgage, which effectively raises the APR. Given the life of the mortgage, the interest rate, the mortgage amount, and the basis of the fee charge (how the fee is calculated), the true Annual Percentage Rate can be calculated. Information is entered as follows:

1) Calculate the periodic payment amount and store it.
   a) Calculate and enter the total number of payment periods; press n .
   b) Calculate and enter the periodic interest rate; press i .
   c) Enter the mortgage amount; press PV , PMT to obtain the periodic payment amount and press STO .
2) Calculate and enter the total number of payment periods; press n .
3) Calculate the mortgage amount less fees:
   a) *If fees are stated as a percentage of the mortgage amount (points)*, enter the mortgage amount, press SAVE↑; enter the fee (percentage) rate, press % − ;
   b) *If fees are stated as a flat charge*, enter the mortgage amount, press SAVE↑ ; enter the fee amount (flat charge) press − ;
   c) *If fees are stated as a percentage of the mortgage amount plus a flat charge*, enter the mortgage amount, press SAVE↑ ; enter the fee (percentage) rate, press % − ;enter the fee amount (flat charge), press − ;

4) Enter the periodic payment amount stored in step (1) (c) above, by pressing RCL and PMT.

5) Position the mortgage amount less fees, calculated in step (3) above by pressing x⇄y. Press PV i to obtain the percentage rate per compounding period.

6) To obtain the annual percentage rate, enter the number of periods per year, then press ×.

Example 1:

A borrower is charged 2 points for the issuance of his mortgage and note. If the mortgage amount is $50,000 for 30 years, and the interest rate is 9% per year, with monthly payments, what annual percentage rate is the borrower paying? (1 point is equal to 1% of the mortgage amount.)

Solution

	Enter:	See Displayed:	
1)	a) 30 SAVE↑ 12 × n	360.00	total monthly periods in mortgage life
	b) 9 SAVE↑ 12 ÷ i	0.75	monthly interest rate
	c) 50000 PV PMT STO	402.31	monthly payment
2)	30 SAVE↑ 12 × n	360.00	number of payment periods
3)	a) 50000 SAVE↑ 2 % −	49000.00	effective borrowed amount
4)	RCL PMT	402.31	
5)	x⇄y PV i	0.77	monthly percentage rate
6)	12 ×	9.23	annual percentage rate

Example 2:

Using the same information as given in Example 1, calculate the APR if the mortgage fee is $150 instead of a percentage.

Solution

	Enter:	See Displayed:	
1)	a)-c) 360 n .75 i		
	50000 PV PMT STO	402.31	monthly payment amount
2)	360 n	360.00	number of monthly payments
3)	b) 50000 SAVE↑ 150 −	49850.00	effective mortgage amount
4)-5)	RCL PMT x⇄y PV i	.75	monthly interest rate
6)	12 ×	9.03	annual percentage rate

Example 3:

Again using the information given in Example 1, what is the APR if the mortgage fee is stated as 2 points plus $150?

Solution

	Enter:	See Displayed:	
1)	a)-c) 360 n .75 i		
	50000 PV PMT STO	402.31	monthly payment
2)	360 n	360.00	total periods
3)	c) 50000 SAVE↑ 2 % −	49000.00	
	150 −	48850.00	effective mortgage amount
4)	RCL PMT	402.31	
5)	x⇄y PV i	0.77	monthly interest rate
6)	12 ×	9.26	annual percentage rate

MORTGAGE AMOUNT

When the mortgage life, interest rate, and the periodic payment amount are known, the full mortgage amount can be found as follows:

1) Calculate and enter the total payment periods, press [n].
2) Calculate and enter the periodic interest rate; press [i].
3) Enter the periodic payment amount; press [PMT] and [PV] to obtain the mortgage amount.

Example 1:

A borrower can afford a $368.21 monthly principal and interest payment on a 30 year, 9¼% mortgage. What is the largest such mortgage he can obtain?

Solution

	Enter:	See Displayed:	
1)	30 [SAVE ↑] 12 [×] [n]	360.00	total monthly periods in mortgage life
2)	9.25 [SAVE ↑] 12 [÷] [i]	0.77	monthly interest rate
3)	368.21 [PMT] [PV]	44757.63	mortgage amount

Example 2:

A home-buyer's monthly gross pay is $1131.60 and he has no current debt. He wants to acquire a $40,000 mortgage for 30 years at 8% annual interest. If the buyer must qualify at 4 to 1 (i.e. his gross pay minus long term debt must be 4 times his monthly principal and interest payment installment), can he afford this mortgage?

Solution

	Enter:	See Displayed:	
1)	30 SAVE↑ 12 × n	360.00	total monthly periods in mortgage life
2)	8 SAVE↑ 12 ÷ i	.67	monthly interest rate
3)	1131.60 SAVE↑ 4 ÷ PMT PV	38554.60	mortgage amount

No. The buyer can only afford a $38554.60 mortgage.

ACCUMULATED INTEREST AND REMAINING PRINCIPAL BALANCE

This calculation finds both the total interest paid during a specified number of time periods and the remaining balance at the end of the last of the specified time periods, given the periodic interest rate, periodic payment amount, total number of periods, mortgage amount, and the beginning and ending payment numbers for the time span being considered. Since most periodic payment amounts are rounded, it is necessary to first find the exact number of payments required to amortize the loan and then use this number for *n* in the calculation of interest and remaining balance. This method is shown in the keystrokes below and in Examples 1 and 2.

If the payment amount is exact or a good approximation of remaining balance is all that is required, a shorter solution is possible by simply entering the number of payment periods as shown in Example 3. ''Time and the Top Row Keys'' explains more fully the relationship between exact payment amounts and exact payment periods.

To find the accumulated interest and remaining balance, information is entered as follows:

1) Calculate or enter the periodic interest rate; press i .
2) Enter the periodic payment amount; press PMT .
3) Enter the mortgage amount; press PV and n to obtain the exact number of payment periods required at this payment amount.

4) Enter one less than the first payment number of the time span being considered; press STO.

5) Enter the last payment number of time span; press x⇄y.

6) Calculate and enter the periodic interest rate; press i.

7) Enter the periodic payment amount; press PMT and Σ+ to obtain the accumulated interest over the desired time span.

8) Press x⇄y to obtain remaining principal balance (at end of the last time period entered in step (4)).

Example 1:

What are the accumulated interest and remaining principal balance on a 25 year (300 month), 7%, $20,000 mortgage after the first year when the monthly principal and interest payment is $141.40?

Solution

	Enter:	See Displayed:	
1)	7 SAVE↑ 12 ÷ i	0.58	monthly interest rate
2)	141.40 PMT	141.40	monthly payment
3)	20000 PV n	299.75	exact number of periods to pay off mortgage
4)	0 STO	0.00	
5)	12 x⇄y	299.75	
6)	7 SAVE↑ 12 ÷ i	0.58	
7)	141.40 PMT Σ+	1390.29	accumulated interest for periods 1 through 12
8)	x⇄y	19693.49	remaining balance after payment 12

NOTE:
To get the accumulated interest for the first 12 periods (months 1-12) we begin the calculation by entering: 0 [STO] 12 [x⇄y] . . . . in steps 4 and 5.
To get the answers for the second 12 payment periods (months 13-24) we enter: 12 [STO] 24 [x⇄y] . . . . in steps 4 and 5.

Example 2:

Assume that the loan in example 1 is arranged such that the first payment occurs at the end of October 1973 (i.e., October = payment number 1). How much interest may be applied to taxes for 1973 and 1974?

Solution

	Enter:	**See Displayed:**	
1)-2)	7 [SAVE↑] 12 [÷] [i]		
	141.40 [PMT]	141.40	
3)	20000 [PV] [n]	299.75	exact periods to amortize mortgage
4)	0 [STO]	0.00	
5)	3 [x⇄y]	299.75	
6-7)	7 [SAVE↑] 12 [÷] [i]		
	141.40 [PMT] [Σ+]	349.57	total interest paid in 1973

1-3)	7 [SAVE↑] 12 [÷] [i] 141.40		
	[PMT] 20000 [PV] [n]	299.75	
4)	3 [STO]	3.00	
5)	15 [x⇄y]	299.75	

January is 4th payment, minus one is 3, December is 15th payment.

6-7)	7 [SAVE↑] 12 [÷] [i]		
	141.40 [PMT] [Σ+]	1384.89	total interest paid in 1974

NOTE:

In the preceding two examples the exact number of time periods was calculated three times (steps 1 through 3.) If this value is going to be used repeatedly it may be desirable to write it down, to be manually re-entered. To preserve accuracy, however, it is necessary to copy the answer to at least six decimal places. Pressing 6 will display the answer to six decimal places. Had this been done for these examples, 299.746473 would have been displayed after step 3.

Example 3:

Again using the same information as given in Example 1, estimate the accumulated interest and remaining principal balance the first year, by entering the actual number of payments (300) instead of calculating the exact number (299.75).

(Keystrokes in *HP-80 Owner's Handbook* under Accumulated Interest between Two Points, Remaining Principal):

Solution

	Enter:	See Displayed:	
1)	0 STO 12 n 300 n	300.00	number of payment periods
2)	7 SAVE↑ 12 ÷ i	0.58	monthly interest rate
3)	141.40 PMT Σ+	1390.74	accumulated interest for the first year
4)	x⇄y	19700.19	remaining balance

ACCUMULATED INTEREST FOR A PRIOR PERIOD[1]

Occasionally, the periodic payment number associated with a remaining principal balance may not be known. It is still possible to find the accumulated interest for any period beginning prior to and ending with such an unknown payment number given the annual interest rate, the number of periods per year, the periodic payment amount, the remaining principal balance, and the number of prior periods desired in the calculation. The unknown payment number associated with the remaining balance is considered the first or reference period, period 0, and the prior periods are counted backwards from this point and have negative values.

[1]We would like to thank Mr. Jacob Heskes of New York, New York for this HP-80 calculation.

Information is entered as follows:

1) Calculate and enter the periodic interest rate; press **i** .
2) Enter the periodic payment amount, press **PMT** .
3) Enter the remaining principal balance, press **PV** to obtain the number of periods required to pay off this remaining balance, press **n** .
4) Enter the number of prior periods for which information is desired, press [CHS] [STO]; enter 0 (reference period); position result of step (3) by pressing [x⇄y] **n** .
5) Calculate and enter the periodic interest rate, press **i** .
6) To obtain accumulated interest for a prior period (entered in step (4)), enter the periodic payment amount, then press **PMT** [Σ+] .
7) Press [x⇄y] to obtain remaining principal balance used in step (3).
8) For other prior periods, repeat steps 4)-6).

Example 1:

A 7½% mortgage has a remaining principal balance of $1,367.04. Payments are $118.71 per month and the current payment number (the one associated with the remaining balance) is unknown. How much interest has accumulated over the past 12 months?

Solution

	Enter:	See Displayed:	
1)	7.5 SAVE↑ 12 ÷ i	0.63	monthly interest rate
2)	118.71 PMT	118.71	monthly payment
3)	1367.04 PV n	11.99	required months to pay off $1367.04
4)	12 CHS STO 0 x⇄y n	11.99	
5)	7.5 SAVE↑ 12 ÷ i	0.63	
6)	118.71 PMT Σ+	154.70	prior 12 months accumulated interest
7)	x⇄y	1367.04	remaining balance

REMAINING BALANCE ONLY

Given the interest rate, the periodic payment amount, and the payment number, the remaining loan balance can be calculated as follows:

1) Calculate and enter the periodic interest rate; press i .
2) Enter the periodic payment amount; press STO PMT .
3) Enter the original loan amount; press PV n , to obtain the exact number of payment periods required to amortize the loan with this payment amount.
4) Calculate and enter the number of the payment period associated with the remaining balance; press − n .
5) Calculate and enter the periodic interest rate; press i .
6) Press RCL PMT PV to obtain the remaining balance.

Example 1:

An individual will receive, as extraordinary income, approximately $50,000. He hopes this is sufficient to pay the outstanding balance of his $80,000, 7½%, 30 year mortgage. Monthly payments are $559 and he will receive this unexpected income when the mortgage has matured 20 years and 8 months. Will the sum be sufficient to pay the remaining balance at this point in time?

Solution

	Enter:	See Displayed:	
1)	7.5 [SAVE↑] 12 [÷] [i]	0.63	periodic interest rate
2)	559 [STO] [PMT]	559.00	monthly payment
3)	80000 [PV] [n]	360.90	exact number of payments required to amortize $80,000
4)	20 [SAVE↑] 12 [×]		
	8 [+] [−] [n]	112.90	
5)	7.5 [SAVE↑] 12 [÷] [i]	0.63	periodic interest rate
6)	[RCL] [PMT] [PV]	45177.74	the remaining balance after payment 248

The remaining balance can certainly be paid with money to spare.

LAST PAYMENT AMOUNT

As indicated previously, if the periodic mortgage payment amount has been rounded (up or down) then the last payment will have to be adjusted in order to account for the gain or loss induced by the rounding.

To solve for this last payment amount, information is entered as follows:

1) Calculate and enter the periodic interest rate; press [i].
2) Enter the periodic payment amount; press [STO] [PMT].
3) Enter the original loan amount; press [PV] [n], to obtain the exact number of payment periods (at this payment amount) required to amortize the loan amount.
4) Calculate and enter the actual number of total payment periods; press [−] [n].
5) Calculate and enter the periodic interest rate; press [i].
6) Enter the payment amount stored in step (2) by pressing [RCL] [PMT] and press [PV] to obtain the remaining balance.
7) To obtain the amount of the last payment add the value of step (6) to the payment stored in step (2) by pressing [RCL] [+].

Example 1:

The exact monthly payment (as calculated to six decimal places on the HP-80) to fully amortize a 30 year, $30,000, 9% mortgage is $241.386785. What would the last payment (number 360) be if the monthly payment were rounded to $241.39?

Solution

	Enter:	See Displayed:	
1)	9 [SAVE ↑] 12 [÷] [i]	0.75	monthly interest
2)	241.39 [STO] [PMT]	241.39	
3)	30000 [PV] [n]	359.98	the exact number of payments required with $241.39
4)	360 [−] [n]	−0.02	
5)	9 [SAVE ↑] 12 [÷] [i]	0.75	
6)	[RCL] [PMT] [PV]	−5.89	remaining balance if all payments were $241.39
7)	[RCL] [+]	235.50	the last payment amount

Example 2:

If the payment amount of the previous example had been rounded to $241 what would the last payment be?

Solution

	Enter:	See Displayed:	
1)-3)	9 [SAVE ↑] 12 [÷] [i] 241 [STO]		
	[PMT] 30000 [PV] [n]	362.98	the exact number of payments at $241
4)-6)	360 [−] [n] 9 [SAVE ↑] 12 [÷]		
	[i] [RCL] [PMT] [PV]	708.10	the remaining balance if all payments were $241
7)	[RCL] [+]	949.10	the last payment amount

This is an unusually large last payment when compared to the periodic payments of $241, and for this reason the rounding would probably not be to the nearest dollar.

MORTGAGE AMORTIZATION SCHEDULE

This calculation generates the interest paid per period, the payment toward principal each period, and the remaining principal balance each period over the life of a fully amortized mortgage loan, given the periodic payment amount, the annual interest rate, the number of payment periods per year, and the mortgage amount. Information is entered as follows:

1) Enter the periodic payment amount, press [STO] .
2) Calculate and enter the periodic interest rate; press [SAVE ↑] [SAVE ↑] .
3) Enter mortgage amount.
4) To obtain interest portion of payment, press [x⇄y] [%] .
5) Press [RCL] [x⇄y] [−] to obtain principal portion of payment.
6) Calculate the remaining principal balance by pressing [−] .
7) Return to step (4) to calculate values for subsequent payments.

Example 1:

Generate an amortization schedule for the first two periods (first two monthly payments) of a $30,000, 7% mortgage loan, when the monthly are $200.

Solution

	Enter:	See Displayed:	
1)	200 [STO]	200.00	monthly payment
2)	7 [SAVE ↑] 12 [÷] [SAVE ↑] [SAVE ↑]	0.58	monthly interest rate
3)-4)	30000 [x⇄y] [%]	175.00	interest portion of first monthly payment
5)	[RCL] [x⇄y] [−]	25.00	principal portion of first payment
6)	[−]	29975.00	remaining principal balance
7)	[x⇄y] [%]	174.85	interest portion of second payment
	[RCL] [x⇄y] [−]	25.15	principal portion of second payment
	[−]	29949.85	remaining balance after second payment

MORTGAGES WITH BALLOON PAYMENTS

If the final payment on a mortage or trust deed is sufficiently greater than the equal periodic payments, it is referred to as a balloon payment. The following sections cover calculations pertaining to balloon payments.

BALLOON PAYMENT AMOUNT

This calculation determines the balloon payment amount (occurring coincident with last periodic payment), given the total number of periods in the mortgage life, the annual interest rate, the periodic payment amount, and the mortgage amount. Information is entered as follows:

1) Calculate or enter the periodic interest rate, press [i].
2) Enter the periodic payment amount; press [STO] [PMT].
3) Enter the original loan amount; press [PV] [n] to obtain exact number of payment periods required to amortize the loan with this payment amount.
4) Calculate or enter the number of the payment period associated with the balloon payment; press [−] [n].
5) Calculate or enter periodic interest rate; press [i].
6) Press [RCL] [PMT] [PV] to obtain the balloon payment amount.

(These keystrokes are identical to those in Chapter 5, ''Remaining Balance Only.'')

Example 1:

A buyer wishes to obtain a $10,000 ten-year mortgage at 8% annual interest requiring monthly principal and interest installments of $100 and a balloon payment. Calculate the balloon payment amount.

Solution

	Enter:	See Displayed:	
1)	8 SAVE↑ 12 ÷ i	.67	periodic interest rate
2)	100 STO PMT	100.00	
3)	10000 PV n	165.34	exact number of periods to amortize loan
4)	120 − n	45.34	
5)	8 SAVE↑ 12 ÷ i	.67	
6)	RCL PMT PV	3901.80	balloon payment amount

Example 2:

A property sold for $43,950 with the seller carrying 10% of the sales price in a second mortgage at 10% annual interest. The scheduled maturity of this note is in 5 years, with a monthly payment of 1% of the original second mortgage amount. What is the amount of the balloon payment received by the holder of the second at maturity?

Solution

	Enter:	See Displayed:	
1)	10 SAVE↑ 12 ÷ i	.83	monthly yield
2)	43.95 STO PMT	43.95	
	(The seller lends 10% of the 43,950 sales price or $4395—the monthly payment is 1% of this amount, $43.95)		
3)	4395 PV n	215.91	exact number of periods to amortize the mortgage
4)	60 − n	155.91	
5)	10 SAVE↑ 12 ÷ i	.83	
6)	RCL PMT PV	3827.77	balloon payment amount

PERIODIC PAYMENT AMOUNT FOR A SPECIFIED BALLOON PAYMENT

Given the total periods in the mortgage life, the periodic interest rate, the balloon payment amount (coincident with last periodic payment), and the mortgage amount, the HP-80 can solve for the minimum periodic payment amount.

Information is entered as follows:

1) Calculate and enter total number of periods in mortgage life, press n .
2) Calculate and enter the periodic interest rate, press i .
3) Enter the balloon payment amount; press FV PV to calculate the discounted value of the balloon.
4) Enter the mortgage amount, press x⇄y − STO .
5) Enter total number of periods in mortgage life press n ; enter periodic interest, press i .
6) Press RCL PV PMT to obtain the periodic payment amount.

Example 1:

A borrower specifies that the balloon payment due on his $12,000, 10%, 10 year mortgage should not exceed $5000. What minimum monthly payment is required on this loan?

Solution

	Enter:	See Displayed:	
1)	10 SAVE↑ 12 × n	120.00	total monthly payments in mortgage life
2)	10 SAVE↑ 12 ÷ i	0.83	monthly interest rate
3)	5000 FV PV	1847.03	discounted value of balloon payment
4)	12000 x⇄y − STO	10152.97	
5)	120 n 10 SAVE↑		
	12 ÷ i	0.83	
6)	RCL PV PMT	134.17	monthly payment amount

Example 2:

A $49,950 building with a 20% second mortgage is expected to appreciate at 4% per year. The second has a maturity of 3 years, requiring equal semi-annual installments and a balloon payment. The investor stipulates that the balloon payment be no larger than the anticipated building appreciation, hoping to refinance the property when the note matures in order to pay off the remaining principal balance (i.e., the balloon). If the interest rate is 10% on this second mortgage, what is the amount of each semi-annual payment?

Solution

	Enter:	See Displayed:	
	3 [n] 4 [i]		
	49950 [STO] [PV] [FV]	56186.96	value of building at note maturity
	[RCL] [−] [STO]	6236.96	appreciation amount
1)	3 [SAVE ↑] 2 [×] [n]	6.00	total payment periods
2)	10 [SAVE ↑] 2 [÷] [i]	5.00	semi-annual interest rate
3)	[RCL] [FV] [PV]	4654.11	discounted value of balloon payment
4)	49950 [SAVE ↑] 80 [%] [−]		
	[x⇄y] [−] [STO]	5335.89	amount of second mortgage
5-6)	6 [n] 5 [i] [RCL] [PV] [PMT]	1051.26	semi-annual payment amount

APR WHEN BALLOON OCCURS COINCIDENT WITH LAST PAYMENT

This calculation finds the annual percentage rate (APR) of a mortgage loan with equal periodic payments and a balloon payment occurring coincident with the last periodic payment, given the balloon payment amount, the life of the mortgage, the number of payment periods per year, the periodic payment amount, and the mortgage amount. Information is entered as follows:

1) Enter the balloon payment amount, press [SAVE ↑]; enter 100, press [÷] [STO].
2) Calculate and enter number of periodic payments in mortgage life, press [SAVE ↑] ; enter 365, press [×] ; enter 2, press [÷] [n] .
3) Enter periodic payment amount, press [SAVE ↑] ; enter 2, press [×] [RCL] [÷] [PMT] .
4) Enter the mortgage amount, press [RCL] [÷] [PV] .
5) Press [ ] [i] (YTM) ; enter number of payment periods per year, press [×] ; Enter 2 then press [÷] to obtain the annual percentage rate.

NOTE:

Solving for the Annual Percentage or Yield Rate uses the HP-80 yield to maturity bond calculation. The loan amount corresponds to the bond price, payments correspond to bond coupons,and the balloon payment compares to the redemption value of a bond. Within the HP-80 bond calculations, there are certain assumptions (i.e., bond coupons paid semiannually, time entered in days, bond price as a percent of redemption value) which require data adjustments when the APR is desired. This explains the keystrokes using 365 [×] , 2 [÷] , and [RCL] [÷] .

For these calculations, the operating limits can be expressed as follows: The absolute value of the number entered for [PMT] must be greater than .125 and less than the value entered for [PV] . The value entered for [PV] must be greater than 20 and less than 5000.

Example:

Find the APR on a $2,100 second mortgage requiring equal monthly payments of $42.52 for 2 years and a $1,500 balloon payment in addition to the last periodic payment.

Solution

	Enter:	See Displayed:	
1)	1500 [SAVE ↑] 100 [÷] [STO]	15.00	
2)	2 [SAVE ↑] 12 [×] 365 [×]		
	2 [÷] [n]	4380.00	
3)	42.52 [SAVE ↑] 2 [×] [RCL]		
	[÷] [PMT]	5.67	
4)	2100 [RCL] [÷] [PV]	140.00	
5)	[ ] [i] (YTM) 12 [×]		
	2 [÷]	11.53	annual percentage rate

APR WHEN BALLOON OCCURS ONE PERIOD AFTER LAST PAYMENT

Given the balloon payment amount, the periodic payment amount, the total number of periods in the mortgage life, and the mortgage amount, this calculation finds the annual percentage rate (APR) of a loan with a balloon payment occurring one period after the last periodic payment.

Keystrokes are:

1) Enter balloon payment amount, press [SAVE ↑]; enter periodic payment, press [−] 100 [÷] [STO] .
2) Enter total number of periods in mortgage, press [SAVE ↑] 1 [+] 365 [×] 2 [÷] [n] .
3) Enter periodic payment amount, press [SAVE ↑] 2 [×] [RCL] [÷] [PMT] .
4) Enter the loan amount, press [RCL] [÷] [PV] .
5) Press [ ] [i] (YTM); enter number of periods per year, press [×] 2 [÷] to obtain the APR.

NOTE:
This calculation uses the HP-80 bond calculations as explained in the note under the last section.

Example:

What is the annual interest rate on a $21,000 loan requiring monthly payments of $425.20 for 2 years and a balloon payment of $15,000 one month after the last payment (i.e. at month 25)?

Solution

	Enter:	See Displayed:	
1)	15000 [SAVE ↑] 425.20 [−] 100 [÷] [STO]	145.75	
2)	24 [SAVE ↑] 1 [+] 365 [×] 2 [÷] [n]	4562.50	
3)	425.20 [SAVE ↑] 2 [×] [RCL] [÷] [PMT]	5.83	
4)	21000 [RCL] [÷] [PV]	144.08	
5)	[ ] [i] (YTM) 12 [×] 2 [÷]	11.19	annual percentage rate

LOANS WITH A CONSTANT AMOUNT PAID TOWARDS PRINCIPAL

This type of loan is structured such that the principal is repaid in equal installments with the interest paid in addition. Therefore each periodic payment has a constant amount applied toward the principal and a varying amount of interest.

PAYMENT SCHEDULE

Given the number of payments per year, the number of years in the mortgage life, the constant periodic payment to principal, the annual interest rate, and loan amount, the schedule of payments can be found as follows:

1) Enter constant periodic payment to principal, press [SAVE ↑] [SAVE ↑]
2) Enter annual interest rate, press [SAVE ↑] 100 [÷].
3) Enter number of payments per year, press [÷] [STO] [×] [CHS] [R↓] [R↓] .
4) Enter initial loan amount, press [+] [RCL] [×] [+] [STO] 1 [n] .
5) Press [TL] to obtain first total payment amount.

Continue pressing [TL] for each successive total payment.

NOTE:

At any point after step 5, the total payment for any particular period can be found by entering the payment number and pressing [n] [TL] .

Example:

A ranch loan of $100,000 has a 20 year payoff with 8% annual interest and annual payments. What would the payments be for years 1, 2, 3, and 8? (The constant payment to principal is $5000 per year.)

Solution

	Enter:	See Displayed:	
1)	5000 [SAVE ↑] [SAVE ↑]	5000.00	
2)	8 [SAVE ↑] 100 [÷]	.08	
3)	1 [÷] [STO] [×] [CHS]		
	[R↓] [R↓]	5000.00	
4)	100000 [+] [RCL] [×] [+]		
	[STO] 1 [n]	1.00	
5)	[TL]	13000.00	payment 1
	[TL]	12600.00	payment 2
	[TL]	12200.00	payment 3
	8 [n] [TL]	10200.00	payment 8

LOAN REDUCTION CHART

If the constant periodic payment to principal, annual interest rate, and loan amount are known, the total payment, interest portion of each payment, and remaining balance can be calculated as follows:

1) Enter constant periodic payment to principal, press [STO] ; enter annual interest rate, press [SAVE ↑] [SAVE ↑] [SAVE ↑] ; enter loan amount.
2) Press [x⇄y] [%] to obtain interest portion of payment.
3) Press [RCL] [+] to obtain total payment.
4) Press [R↓] [RCL] [−] to see remaining balance.
5) Return to step 2 for each successive payment.

Example:

Assuming an $80,000, 20 year, 9% farm loan with annual payments, construct a loan reduction chart for the first two years. (Constant payment to principal is $4000 per year.)

Solution

	Enter:	See Displayed:	
1)	4000 [STO] 9 [SAVE ↑] [SAVE ↑] [SAVE ↑] 80000	80000	
2)	[x⇄y] [%]	7200.00	first payment interest
3)	[RCL] [+]	11200.00	total first payment
4)	[R↓] [RCL] [−]	76000.00	remaining balance
5)	[x⇄y] [%]	6840.00	second payment interest
	[RCL] [+]	10840.00	total second payment
	[R↓] [RCL] [−]	72000.00	remaining balance

THE PRICE AND YIELD OF A MORTGAGE

A mortgage bought at a discount is purchased for an amount less than the principal balance of the note—one bought for more than this balance is purchased at a premium. Calculations of the price for and yield of discounted and premium mortgages have similar HP-80 solutions. Other sections of this book, especially those concerning balloon payments, periodic payment amount, and annual percentage rate, are useful in developing data for many of the following problem examples.

PRICE OF FULLY AMORTIZED MORTGAGES

Given the life of the mortgage, number of periods per year, the desired annual yield, and the periodic payment, the price of a mortgage is calculated as follows:

1) Calculate and enter exact* total number of payment periods in mortgage life, press **n** .
2) Calculate and enter the yield per compounding period, press **i** .
3) Enter the periodic payment amount, press **PMT** ; press **PV** to obtain the price of the mortgage.

*If an exact answer is required, the total periods entered here must be exact. Using the actual number of periods gives an approximate answer.

Example 1:

The monthly payment is $183.44 on a fully amortized 30 year mortgage of $25,000 at 8% annual interest. Our investor wants to discount this mortgage to yield 12% annually. How much should he pay?

Solution

	Enter:	See Displayed:	
1)	8 SAVE ↑ 12 ÷ i		
	183.44 PMT 25000 PV n	360.01	exact number of periods to amortize mortgage
2)	1 i	1.00	monthly yield rate
3)	183.44 PMT PV	17833.78	price of mortgage bought at discount

Example 2:

A 5 year second mortgage of $7800 carries an annual interest rate of 10% with quarterly payments of $500. What is the price of this mortgage if bought to yield 8%?

Solution

	Enter:	See Displayed:	
1)	10 SAVE ↑ 4 ÷ i		
	500 PMT 7800 PV n	20.02	exact number of periods to amortize mortgage
2)	8 SAVE ↑ 4 ÷ i	2.00	quarterly yield rate
3)	500 PMT PV	8181.71	price of mortgage bought at a premium

YIELD OF FULLY AMORTIZED MORTGAGES

The annual yield of a mortgage bought at a discount or premium can be calculated, given the life of the mortgage, the number of payment periods per year, the periodic payment amount, and the price paid for the mortgage.

Information is entered as follows:

1) Calculate exact total number of periods in mortgage life, press **n**.

2) Enter the periodic payment amount, press **PMT**.

3) Enter the mortgage price, press **PV** **i** to obtain the yield per period.

   Enter the number of periods per year, then press [×] to obtain the annual yield.

Example 1:

A $25,000, 8%, fully-amortized 30 year mortgage requires monthly principal and interest payments of $183.40. What is the annual yield of this mortgage if it was purchased for $17,833.73?

Solution

	Enter:	See Displayed:	
1)	8 [SAVE ↑] 12 [÷] **i**		
	183.40 **PMT** 25000 **PV** **n**	360.34	exact number of payments
2)	183.40 **PMT**	183.40	
3)	17833.73 **PV** **i**	1.00	monthly yield
	12 [×]	12.00	annual yield

Example 2:

If the mortgage described in Example 1 above were purchased for $30,000 what would be its annual yield?

Solution

	Enter:	**Displayed:**	
1)	8 SAVE↑ 12 ÷ i		
	183.40 PMT 25000 PV n	360.34	exact number of payments
2)	183.40 PMT	183.40	
3)	30000 PV i	.52	monthly yield
	12 ×	6.19	annual yield

PRICE OF PREPAID MORTGAGES OR MORTGAGES WITH A BALLOON PAYMENT

Since both a prepayment and a balloon payment pay off the remaining balance of the loan, calculations for these situations are identical, as the keystrokes and examples below illustrate.

Given the life and amount of the mortgage, the periodic interest rate and payment amount, the timing and amount of the balloon or prepayment, and the desired yield rate, the price of the mortgage can be found.

Information is entered as follows:

1) Calculate or enter Remaining Balance or Balloon (as shown in Chapters 5 and 6), press STO .

2) Enter total number of periods until prepayment or balloon payment occurs, press n .

   CAUTION:
   The balloon payment may occur one period after the last periodic payment instead of being coincident with it. In this case n will be one larger than the n used in step 5. Example 2 shows this type.

3) Calculate and enter the yield per compounding period, press i .

4) Press RCL FV PV STO to obtain the present value of the prepayment amount or balloon discounted at the yield rate.

5) Calculate and enter total number of payments from the beginning of the mortgage to the period of the balloon or prepayment, press n .

6) Calculate and enter yield per compounding period, press i .

7) Enter the periodic payment amount, press PMT PV to obtain present value of the payments.

8) Press RCL + to obtain mortgage price.

Example 1:

A ten year mortgage calls for monthly installments of $100 and a balloon payment of $3901.80 to be paid at the same time as the last periodic payment. The mortgage amount is $10,000 and the annual interest rate is 8% (see Mortgages With Balloon Payments, Chapter 6). If the desired annual yield is 12% what is the price of this mortgage?

Solution

	Enter:	**See Displayed:**	
1)	3901.80 STO	3901.80	balloon payment
2)	120 n	120.00	total payments until balloon payment
3)	12 SAVE↑ 12 ÷ i	1.00	monthly yield
4)	RCL FV PV STO	1182.23	present value of balloon payment
5)	120 n	120.00	total payments in mortgage life
6)	12 SAVE↑ 12 ÷ i	1.00	periodic yield
7)	100 PMT PV	6970.05	present value of payments
8)	RCL +	8152.28	mortgage price

Example 2:

The profit sharing trust of a local real estate office requires an annual 20% yield on mortgages purchased for speculation. One of its salesmen has assured a property seller that if he will carry a 3 year 8½% second mortgage for $11,500, the trust will buy it from him so that he can receive all cash in the transaction. Given that the payment is 1% per month ($115) and the balloon payment amount is $10,130.07, occurring one period after the last periodic payment, how much will the trust pay for this mortgage?

Solution

	Enter:	See Displayed:	
1)	10130.07 [STO]	10130.07	balloon payment
2)	37 [n]	37.00	number of periods until balloon payment
3)	20 [SAVE ↑] 12 [÷] [i]	1.67	monthly yield rate
4)	[RCL] [FV] [PV] [STO]	5495.47	present value of balloon payment
5)	36 [n]	36.00	total periods in mortgage life
6)	20 [SAVE ↑] 12 [÷] [i]	1.67	periodic yield rate
7)	115 [PMT] [PV]	3094.43	present value of payments
8)	[RCL] [+]	8589.90	mortgage price

Example 3:

Find the price of an 8%, 30 year mortgage prepaid in full after 5 years, if the mortgage amount is $45,000, monthly payments of $330 are required and the investor desires an annual yield of 12%.

Solution

	Enter:	See Displayed:	
1)	8 [SAVE ↑] 12 [÷] [i] 330		
	[STO] [PMT] 45000 [PV] [n]	360.88	exact number of payment periods
	60 [−] [n]	300.88	remaining periods when prepayment occurs
	8 [SAVE ↑] 12 [÷] [i]	.67	monthly interest rate
	[RCL] [PMT] [PV] [STO]	42795.69	remaining balance at end of 5th year
2)	60 [n]	60.00	total periods until prepayment
3)	12 [SAVE ↑] 12 [÷] [i]	1.00	monthly yield
4)	[RCL] [FV] [PV] [STO]	23556.87	present value of remaining balance
5)	60 [n]	60.00	total periods in mortgage life
6)	12 [SAVE ↑] 12 [÷] [i]	1.00	monthly yield
7)	330 [PMT] [PV]	14835.16	present value of payments
8)	[RCL] [+]	38392.04	mortgage price

YIELD OF PREPAID MORTGAGES OR MORTGAGES WITH A BALLOON PAYMENT

If the mortgage is prepaid or the balloon payment is coincident with the last payment, the annual yield can be calculated as follows:

1) Enter the balloon payment amount, press [SAVE ↑] ; enter 100, press [÷] [STO] .
2) Calculate and enter the total number of payment periods from the beginning of the mortgage life to the period of the balloon or prepayment; enter 365, press [×] ; enter 2, press [÷] [n] .
3) Enter periodic payment amount, press [SAVE ↑] ; enter 2, press [×] [RCL] [÷] [PMT] .
4) Enter the price of the mortgage, press [RCL] [÷] [PV] .
5) Press [ ] [i] (YTM) ; enter number of payment periods per year, press [×] ; to obtain the annual yield, enter 2, then press [÷] .

NOTE:
This calculation uses the HP-80 bond yield calculation. See Note under Annual Percentage Rate for Mortgages with Balloon Payments, Chapter 6.

Example 1:

Find the annual yield of a mortgage purchased for $1878.58 requiring monthly payments of $42.52 for 2 years and a $1500 balloon payment coincident with the last periodic payment.

Solution

	Enter:	See Displayed	
1)	1500 [SAVE ↑] 100 [÷] [STO]	15.00	
2)	2 [SAVE ↑] 12 [×] 365 [×]		
	2 [÷] [n]	4380.00	
3)	42.52 [SAVE ↑] 2 [×] [RCL]		
	[÷] [PMT]	5.67	
4)	1878.58 [RCL] [÷] [PV]	125.24	
5)	[ ] [i] (YTM)	3.13	
	12 [×] 2 [÷]	18.78	annual yield

Example 2:

Find the annual yield of a 7%, 21 year mortgage prepaid in full at the end of the 12th year, if the mortgage amount is $100,000, the purchase price is $86,000, and equal monthly payments of $758.45 are required. (The remaining balance at the end of the 12th year is $60,652.17 as calculated using the keystrokes in Chapter 5 for Remaining Balance Only.)

Solution

	Enter:	Displayed:	
1)	60652.17 [SAVE ↑] 100 [÷] [STO]	606.52	
2)	144 [SAVE ↑] 365 [×]		
	2 [÷] [n]	26280.00	
3)	758.45 [SAVE ↑] 2 [×] [RCL]		
	[÷] [PMT]	2.50	
4)	86000 [RCL] [÷] [PV]	141.79	
5)	[ ] [i] (YTM)	1.54	
	12 [×] 2 [÷]	9.23	annual yield

YIELD OF MORTGAGES WITH BALLOON ONE PERIOD AFTER LAST PAYMENT

Given the periodic payment amount, total number of periods in mortgage life, mortgage price, and the balloon payment amount which occurs one period after the last payment, the yield is calculated as follows:

1) Enter balloon payment amount, press [SAVE ↑] ; enter periodic payment, press [−] 100 [÷] [STO] .

2) Calculate and enter the total number of periods in mortgage life, press [SAVE ↑] .

   Press 1 [+] 365 [×] 2 [÷] [n] .

3) Enter the periodic payment amount, press [SAVE ↑] 2 [×] [RCL] [÷] [PMT] .

4) Enter the price of the mortgage, press [RCL] [÷] [PV] .

5) Press [ ] [i] (YTM) ; enter number of periods per year, press [×] 2 [÷] to obtain the annual yield.

Example:

What is the annual yield of a mortgage purchased for $7900 which has monthly payments of $80 for 5 years and a balloon payment of $7000 occurring one period after the last periodic payment?

Solution

	Enter:	See Displayed:	
1)	7000 [SAVE ↑] 80 [−]		
	100 [÷] [STO]	69.20	
2)	60 [SAVE ↑] 1 [+] 365 [×]		
	2 [÷] [n]	11132.50	
3)	80 [SAVE ↑] 2 [×] [RCL]		
	[÷] [PMT]	2.31	
4)	7900 [RCL] [÷] [PV]	114.16	
5)	[ ] [i] (YTM)	1.71	
	12 [×] 2 [÷]	10.28	annual yield

ANNUITY DUE CALCULATIONS

As mentioned in Time and the Top Row Keys, the HP-80 assumes payments to occur at the end of each period (ordinary annuity or payment in arrears). However, by slightly modifying the standard keystrokes, the HP-80 can easily solve annuity due problems where payments are made at the beginning of each period (payments in advance), e.g. some rental or lease payments. The following sections explain some of the standard calculations covered under Simple Mortgages, Chapter 5, and Mortgages With Balloon Payments, Chapter 6, with the assumption that *payments are made in advance* instead of in arrears.

PRESENT VALUE OF ANNUITY DUE

This calculation solves for the present value of a series of payments (made at the beginning of each period) given the number of payments, periodic interest rate, and payment amount.

Keystrokes:

1) Enter or calculate total number of periods, press [n] .
2) Enter or calculate periodic interest rate press [STO] [i] .
3) Enter payment, press [RCL] [%] [+] [PMT] [PV] .

Example:

The owner of a downtown parking lot has been able to achieve full occupancy and a 7% annual yield by renting parking spaces for $40 per month payable in advance. Some regular customers have expressed interest in renting their spaces on an annual basis. What annual rent, also payable in advance, will maintain a 7% annual yield rate?

Solution

	Enter:	See Displayed:	
1)	12 [n]	12.00	
2)	7 [SAVE ↑] 12 [÷] [STO] [i]	.58	periodic interest rate
3)	40 [RCL] [%] [+] [PMT] [PV]	464.98	present value of payments or minimum annual charge

PERIODIC PAYMENT AMOUNT WITHOUT BALLOON OR RESIDUAL VALUE

Given the number of periods, periodic interest rate, and initial value, the periodic payment occurring at the beginning of each period can be found as follows:

1) Calculate and enter total number of periods, press [n] .
2) Calculate and enter periodic interest, press [STO] [i] .
3) Press 1 [RCL] [%] [+] .
4) Enter initial value, press [x⇄y] [÷] [PV] [PMT] to obtain periodic payment amount.

Example:

The owner of a building which is presently worth $70,000 intends to lease it for 20 years. He estimates the building will be worthless at the end of the lease and he desires a 10% annual yield. What quarterly payments in advance must he receive to achieve his yield?

Solution

	Enter:	See Displayed:	
1)	20 [SAVE ↑] 4 [×] [n]	80.00	total quarterly payments
2)	10 [SAVE ↑] 4 [÷] [STO] [i]	2.50	quarterly interest
3)	1 [RCL] [%] [+]	1.03	
4)	70000 [x⇄y] [÷] [PV] [PMT]	1982.27	quarterly payment

PERIODIC PAYMENT AMOUNT WITH BALLOON OR RESIDUAL AT END OF LAST PERIOD

Often a building or piece of equipment still has a residual value after several years use. This in effect reduces the value the yield is based on, thus lowering the payments to achieve the same yield. If the initial value, residual value, periodic rate, and number of periods are known, the periodic payment is calculated as follows:

1) Calculate or enter total periods, press [n] .
2) Calculate or enter periodic interest rate, press [STO] [i] .
3) Enter residual value, press [FV] [PV] to obtain present value of residual.
4) Enter initial value, press [x⇄y] [−] to obtain the net present value to be amortized.
5) Press 1 [RCL] [%] [+] [÷] .
6) Enter total periods, press [n] [x⇄y] .
7) Press [RCL] [i] [x⇄y] [PV] [PMT] to obtain periodic payment amount.

NOTE:

The balloon or residual is assumed to occur at the end of the last period.

Example:

Suppose in the last example, the owner decides the building will be worth $20,000 at the end of the lease. What must quarterly payments be to achieve a 10% yield?

Solution

	Enter:	See Displayed:	
1)	20 [SAVE ↑] 4 [×] [n]	80.00	total periods
2)	10 [SAVE ↑] 4 [÷] [STO] [i]	2.50	periodic interest rate

(cont'd)

	Enter:	See Displayed:	
3)	20000 FV PV	2774.09	present value of residual
4)	70000 x⇄y −	67225.91	
5)	1 RCL % + ÷	65586.25	
6)	80 n x⇄y	65586.25	
7)	RCL i x⇄y PV PMT	1903.71	quarterly payment

NUMBER OF PERIODS TO FULLY AMORTIZE INITIAL AMOUNT

If the periodic payment amount, initial amount, and periodic rate are given, the number of periods required to pay off the initial amount is calculated as follows:

1) Enter or calculate periodic interest rate, press STO i .
2) Enter payment amount, press RCL % + PMT .
3) Enter initial amount, press PV n .

Example:

The buyer of 3 acres of land can afford to pay $375.00 per month toward interest and principal. If the asking price is $35,000.00 and the seller wants 8% annual interest with payments in advance, how long will it take to pay off the mortgage?

Solution

	Enter:	See Displayed:	
1)	8 SAVE↑ 12 ÷ STO i	.67	monthly interest
2)	375 RCL % + PMT	377.50	
3)	35000 PV n	144.87	number of months
	12 ÷	12.07	years

ANNUAL PERCENTAGE OR YIELD RATE WITHOUT BALLOON OR RESIDUAL VALUE

Given the initial amount, periodic payment, and number of payments, the annual interest or yield rate is calculated as follows:

1) Calculate or enter one less than the total number of periods, press [n].
2) Enter payment amount press [SAVE ↑] [PMT].
3) Enter initial amount, press [x⇄y] [−] [PV].
4) Press [i] to obtain periodic rate.
5) Enter number of payment periods per year; press [×] to obtain annual rate.

Example:

Equipment worth $12,000.00 is leased for 8 years with monthly payments in advance of $200.00. The equipment is assumed to have no salvage value at the end of the lease. What yield rate does this represent?

Solution

	Enter:	See Displayed:	
1)	8 [SAVE ↑] 12 [×]		
	1 [−]	95.00	
2)	200 [SAVE ↑] [PMT]	200.00	
3)	12000 [x⇄y] [−] [PV]	11800.00	
4)	[i]	1.09	periodic rate
5)	12 [×]	13.07	annual rate

ANNUAL PERCENTAGE OR YIELD RATE WITH BALLOON OR RESIDUAL VALUE

When the investment (equipment) is expected to have some value at the end of the considered time period, this effectively raises the yield. If the total periods, periodic payment amount, initial and residual values are known, the yield rate can be found.

Information is entered as follows:

1) Enter balloon or residual amount, press [SAVE ↑] ; enter periodic payment amount, press [−] 100 [÷] [STO] .
2) Calculate and enter total number of periods, press [SAVE ↑] 365 [×] 2 [÷] [n] .
3) Enter periodic payment amount, press [SAVE ↑] [SAVE ↑] 2 [×] [RCL] [÷] [PMT] [x⇄y] .
4) Enter initial amount, press [x⇄y] [−] [RCL] [÷] [PV] .
5) Press [ ] [i] (YTM) ; enter periods per year, press [×] 2 [÷] to obtain the annual interest or yield rate.

NOTE:
This calculation uses the HP-80 bond yield algorithm. See note in Chapter 6, Mortgages With Balloon Payments under Annual Percentage Rate.

Example:

An office building worth $160,000.00 is leased for 15 years with monthly payments in advance of $1685.00. The tenant has a purchase option at the end of the 15 years enabling him to buy the building for $15,000.00. If he exercises this option, what will the lessor's yield be?

Solution

	Enter:	See Displayed:	
1)	15000 [SAVE↑] 1685 [−]		
	100 [÷] [STO]	133.15	
2)	15 [SAVE↑] 12 [×] 365		
	[×] 2 [÷] [n]	32850.00	
3)	1685 [SAVE↑] [SAVE↑] 2 [×]		
	[RCL] [÷] [PMT] [x⇄y]	1685.00	
4)	160000 [x⇄y] [−] [RCL]		
	[÷] [PV]	1189.00	
5)	[ ] [i] (YTM) 12 [×] 2 [÷]	10.16	annual yield rate

DISCOUNTED CASH FLOW ANALYSIS

Two methods of evaluating investments that consider the time value of money are the net present value approach, which assumes a yield rate, and the discounted or internal rate of return approach, which finds a yield rate.

NET PRESENT VALUE (NPV)

Assuming a minimum desired yield (this could also be a cost of capital or discount rate), the net present value method finds the present value of the future cash flows generated by the investment and compares this value to the initial investment. If this present value is greater than or equal to the investment, the investment meets the profit objectives assumed under minimum yield. If the present value is less than the investment, it is not profitable to the extent of the desired yield.

NOTE:
The yield rate is dependent upon the cash flows. That is, if the cash flows are pretax, the yield rate will be pretax. If the cash flows are after tax, the yield rate will be after tax.

PRESENT VALUE OF CASH FLOWS

Given the number of periods, cash flows per period, investment amount, and the assumed yield per period, the HP-80 can solve for NPV as follows:

1) Clear the HP-80 by pressing [ ] [CLEAR CLx] .
2) Calculate and enter the desired yield rate per period, press [i] .
3) Enter original investment amount; press [CHS] to change it to a negative number (indicating it is a cash outlay), press [PV]
4) Enter first period's cash flow (if the flow is an outlay, press [CHS]); press [PV] .
5) Press [Σ+] to obtain current net present value (first cash flow less original investment).
6) Continue steps 4) and 5) for all periods. If the flow is an outlay, press [CHS] before pressing [PV] . For periods with no cash flow, enter 0, and press [PV] [Σ+] . After pressing [Σ+] for every cash flow you will see the NPV. If the final NPV is a positive number or zero, the investment meets the profit criteria.

NOTES:

1) As soon as the display shows a positive number after pressing [Σ+] *, the investment is recovered on a discounted cash flow basis.*

2) To find the present value of uneven cash flows without an initial investment, enter zero for the initial investment in the above keystrokes.

Example 1

An apartment building costing $100,000.00 is expected to return 10% per year. Based on the anticipated cash flows below, will the investment meet the profit objectives?

Year	Cash Flow
1	$7000
2	$8500
3	$9000
4	$120000

(property is sold in the fourth year)

Solution

	Enter:	See Displayed:	
1)	[ ] CLEAR [CLx]	0.00	clear the HP-80
2)	10 [i]	10.00	desired yield
3)	100000 [CHS] [PV]	−100000.00	original investment
4-5)	7000 [PV] [Σ+]	−93636.36	net present value, first year
4-5)	8500 [PV] [Σ+]	−86611.57	net present value, second year
4-5)	9000 [PV] [Σ+]	−79849.74	net present value, third year
4-5)	120000 [PV]	2111.88	net present value, fourth year

The value at the end of the fourth year is positive; therefore the investment returns greater than 10% per year.

Example 2:

A shopping complex which costs $260,000 has annual cash flows as follows:

Year	Cash Flow
1	−1000
2	15000
3	23,000
4	310,000

The desired annual minimum yield is 9%. Will this rate be achieved by the above cash flows?

Solution

	Enter:	See Displayed	
1)	[ ] CLEAR [CLx]	0.00	clear the HP-80
2)	9 [i]	9.00	desired yield
3)	260000 [CHS] [PV]	−260000.00	original cash outlay
4-5)	1000 [CHS] [PV] [Σ+]	−260917.43	NPV first year
4-5)	15000 [PV] [Σ+]	−248292.23	NPV second year
4-5)	23000 [PV] [Σ+]	−230532.01	NPV third year
4-6)	310000 [PV] [Σ+]	−10920.20	NPV fourth year

The value at the end of the fourth year is negative; therefore, investment does not meet the profit objective.

PRESENT VALUE OF DEFERRED ANNUITIES

If the annuity does not start until some point in the future, the following keystrokes can be used to determine the present value of the payment stream.

1) Calculate and enter the number of payment periods in payment stream; press [n] .
2) Calculate and enter the periodic interest rate; press [i] .
3) Enter the periodic payment amount; press [PMT] [PV] .
4) Enter the number of payment periods from the present until the beginning of the annuity; press [n] [x⇄y] .

5) Calculate and enter the periodic interest rate; press [i] [x⇄y] [FV].

6) Press [PV] to obtain the present value.

Example:

An annuity of $100 per month will begin in 2 years and continue for 2 years. What is its present value if the interest rate is 12% annually?

Solution

	Enter:	See Displayed:	
1)	24 [n]	24.00	total payments
2)	1 [i]	1.00	periodic interest
3)	100 [PMT] [PV]	2124.34	
4)	24 [n] [x⇄y]	2124.34	
5)	1 [i] [x⇄y] [FV]	2124.34	
6)	[PV]	1673.06	present value of annuity

PRESENT VALUE OF UNEVEN PAYMENT STREAMS

In many situations there is an even stream of payments followed by another even stream at a different value. To find the net present value of these cash flows, each payment need not be entered. Instead the consecutive even flows can be grouped in order to shorten the solution steps.

Information is entered as follows:

1) Enter the initial investment; press [CHS] [STO].

2) For each stream of payments (in sequence)

   a) Calculate or enter the number of payment periods from the *initial investment* until the end of this stream of payments; press [n].

   b) Calculate or enter the periodic interest rate; press [i].

   c) Enter the periodic payment amount press [SAVE ↑].

   d) Enter the periodic payment amount of the next stream of payments; press [−] [PMT].

   e) Press [PV] [RCL] [+] [STO].

3) Repeat steps 2a—2e for each payment stream.

After the last stream of payments the net present value will be in the display and storage register.

Example:

An office building costing $250,000 is expected to yield 13% per year. The monthly income streams are shown below:

Mos. 1—12	$2000/mo.
Mos. 13—24	$1900/mo.
Mos. 25—36	$2100/mo.

At the end of the third year the building is sold for $260,000. Based on this information alone, does this meet the profit objective?

Solution

	Enter:	See Displayed:
1)	250000 [CHS] [STO]	−250000.00
2) a.	12 [n]	12.00
b.	13 [SAVE ↑] 12 [÷] [i]	1.08
c.	2000 [SAVE ↑]	2000.00
d.	1900 [−] [PMT]	100.00
e.	[PV] [RCL] [+] [STO]	−248880.40
3) a.	24 [n]	
b.	13 [SAVE ↑] 12 [÷] [i]	1.08
c.	1900 [SAVE ↑]	1900.00
d.	2100 [−] [PMT]	−200.00
e.	[PV] [RCL] [+] [STO]	−253087.22
3) a.	36 [n]	36.00
b.	13 [SAVE ↑] 12 [÷] [i]	1.08
c.	2100 [SAVE ↑]	
d.	0 [−] [PMT]	2100.00
e.	[PV] [RCL] [+] [STO]	−190761.49

Find present value of sales price.

36 [n] 13 [SAVE ↑] 12 [÷]		
[i] 260000 [FV] [PV]	176404.39	present value of sales price
[RCL] [+]	−14357.11	net present value

Present value is negative; profit objective is not met.

DISCOUNTED OR INTERNAL RATE OF RETURN (IRR)

The interest rate that equates the present value of all future cash flows with the original investment is known as the internal rate of return (also called discounted rate of return). Given the initial investment and uneven periodic cash flows, the IRR can be calculated as follows:

Iterative Method

1) Clear the HP-80 by pressing [ ] CLEAR [CLx].

2) Enter a best guess (or desired) rate of return (yield) per period; press [i].

3) Enter original investment amount; press [CHS] to change it to a negative number (indicating cash outlay); press [PV].

4) Enter first period's cash flow (if it is an outflow press [CHS]); press [PV]. Press 0 [PV] for periods with no cash flow.

   Press [Σ+] to obtain the current net present value.

5) Continue step 4) for subsequent periods for all cash flows. If this final NPV is positive, the actual rate of return is greater than the value entered in step 2). Repeat steps 1) — 5) using a higher rate in step 2). If this final NPV is negative, the actual rate of return is less than the value entered in step 2). Repeat steps 1) — 5) using a lower rate in step 2).

6) Continue iterating steps 1) — 5) until the NPV is zero or as close to zero as desired. The IRR will be the rate used to get this NPV.

NOTE:

If the cash flows are pretax, the IRR will be pretax. If the cash flows are after tax, the IRR will be after tax.

Example:

What is the internal rate of return (yield on investment) for a shopping center costing $200,000.00 if the cash flows over the next 3 years are as follows:

Year	Cash Flow
1	$18,000
2	$21,000
3	$225,000

Solution

	Enter:	See Displayed:	
1)	CLEAR CLx	0.00	clear the HP-80
2-3)	10 i 200000 CHS PV	−200000.00	10% as a best first guess of IRR, 200000 investment
4)	18000 PV Σ+	−183636.36	net present value after first year
5)	21000 PV Σ+	−166280.99	net present value after second year
	225000 PV Σ+	2764.84	net present value after third year

Since this NPV is positive, the actual IRR is higher than 10%; therefore, try 11% and iterate steps 1)—5).

1)	CLEAR CLx	0.00	
2-3)	11 i 200000 CHS PV	−200000.00	11% IRR assumed
4)	18000 PV Σ+	−183783.78	NPV after first year
5)	21000 PV Σ+	−166739.71	NPV after second year
	225000 PV Σ+	−2221.65	NPV after third year

This time the NPV is negative so 11% is higher than the actual IRR. As a result of these 2 iterations, the IRR must be between 10% and 11%. Since the NPV for 11% is closer to zero than the NPV for 10%, the actual IRR must be closer to 11%. One last iteration of steps 1)—5) at a rate of 10.55% yields a NPV = .63 which is close to 0 and, therefore, 10.55% is a good approximation of the IRR or yield.

LINEAR REGRESSION CALCULATIONS

Linear regression is a statistical method for finding a straight line that best fits a set of data points, thus providing a relationship between two variables. For example, an appraiser might want to know how much the value of an office building may change if its square footage is increased. If the appraiser has data on several similar buildings with different square footages and values, he can compute the regression line which gives an approximate relationship between the two variables, square footage and value, (e.g. he may find that the building value increases by $200 for each additional square foot of space).

Given the observations (comparables) of two variables each, the HP-80 can solve for the slope, b, and y-intercept, a, of the standard regression line equation, $y = a+bx$. In addition the procedure calculates the correlation coefficient, r, which indicates goodness of fit of the line to the points $(-1 \leq r \leq 1)$, the coefficient of determination, r^2, and the standard error, s, of the estimate of y on x, which is a measure of the scatter about the regression line of y on x. (Refer to any basic statistics text for detailed discussions of these terms.)

NOTE:
Linear regression calculations differ from the HP-80 Trend Line solutions listed under **Appreciation Calculations** ***because unequally spaced and missing data points are allowed on observations of the independent variable (x).***

The following input data, notation, and equations are used:

Input Data:

$$(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$$

where n = number of observations
x = independent variable
y = dependent variable

Notation:

$S_x = x_1 + x_2 \ldots + x_n$	sum of x
$S_y = y_1 + y_2 \ldots + y_n$	sum of y
$SS_x = x_1^2 + x_2^2 + \ldots + x^2{}_n$	sum of squares of x
$SS_y = y_1^2 + y_2^2 + \ldots + y^2{}_n$	sum of squares of y
$S_{xy} = x_1y_1 + x_2y_2 + \ldots + x_ny_n$	sum of x times y
σ_x = standard deviation of x	
σ_y = standard deviation of y	

Equations:

$$b = \frac{S_{xy} - \dfrac{S_xS_y}{n}}{SS_x - \dfrac{(S_x)^2}{n}}$$

$$r = b\left(\frac{\sigma_x}{\sigma_y}\right)$$

$$a = \frac{1}{n}\,(S_y - bS_x)$$

$$s = \left(\frac{SS_y - aS_y - bS_{xy}}{n-2}\right)^{1/2}$$

Solution

1) Solve for S_x (and store it), SS_x, and σ_x (write down the answers to these and the following calculations as you generate them):

[ ] [CLEAR CLx] x_1 [Σ+] x_2 [Σ+] . . . x_n [Σ+] [STO]	gives S_x
[R↓] [R↓]	gives SS_x
[R↓] [R↓] [$\bar{x}$] [x⇄y]	gives σ_x

2) Solve for S_y, SS_y, and σ_y (remember to write the answers down):

[ ] [CLEAR CLx] y_1 [Σ+] . . . y_n [Σ+]	gives S_y
[R↓] [R↓]	gives SS_y
[R↓] [R↓] [$\bar{x}$] [x⇄y]	gives σ_y

3) Solve for S_{xy}:

x_1 [SAVE↑] y_1 [×] x_2 [SAVE↑] y_2 [×] [+] . . .

x_n [SAVE↑] y_n [×] [+] gives S_{xy}

4) To get b:

Press RCL S_y × n ÷ −

SS_x RCL SAVE↑ × n ÷ − ÷ gives b

5) To get a:

RCL × S_y x⇄y − n ÷ gives a

6) To get r:

b SAVE↑ σ_x × σ_y ÷ gives r

7) To get s (the standard error of y on x):

SS_y SAVE↑ a SAVE↑ S_y × − b SAVE↑

S_{xy} × − n SAVE↑ 2 − ÷ [gold] y^x ($\sqrt{x}$) gives s

(n is the number of observations)

Example 1:

A commercial land appraiser has examined 6 vacant lots in the downtown section of a local community, all of which have the same depths but different frontages and values. Based on the following input data, what is the relationship between frontage and lot value?

Input Data:

Lot frontage (feet)	Lot value ($)
70.8	10100
60.0	8219
85.0	15000
75.2	11120
69.5	9995
84.0	13500

Solution

	Enter:	See Displayed:	
1)	[gold] CLx (CLEAR)		
	70.8 Σ+ 60 Σ+ 85 Σ+		
	75.2 Σ+ 69.5 Σ+ 84 Σ+ STO	444.50 S_x	*write down*
	R↓ R↓	33378.93 SS_x	*write down*
	R↓ R↓ $\bar{x}$ x⇄y	9.48 σ_x	*write down*

2) [ ] CLEAR [CLx]

10100 [Σ+] 8219 [Σ+]

15000 [Σ+] 11120 [Σ+]

9995 [Σ+] 13500 [Σ+] 67934.00 S_y *write down*

[R↓] [R↓] 800366386.0 SS_y *write down*

[R↓] [R↓] [$\bar{x}$] [x⇄y] 2497.80 σ_y *write down*

3) 70.8 [SAVE ↑] 10100 [×]

60 [SAVE ↑] 8219 [×] [+]

85 [SAVE ↑] 15000 [×] [+]

75.2 [SAVE ↑] 11120 [×] [+]

69.5 [SAVE ↑] 9995 [×] [+]

84 [SAVE ↑] 13500 [×] [+] 5148096.50 S_{xy} *write down*

4) [RCL] 67934 [×] 6 [÷] [−]

33378.93 [RCL] [SAVE ↑] [×]

6 [÷] [−] [÷] 256.90 b *write down*

5) [RCL] [×] 67934 [x⇄y] [−] 6 [÷] −7709.66 a *write down*

Thus the equation of the regression line is:

$y = -7709.66 + 256.9\,x$

Without considering the other variables, the appraiser could now predict (based on past sales) that a lot with 65 feet frontage would have an approximate value of:

$$\begin{aligned} y &= -7709.66 + 256.9\,x \\ &= -7709.66 + 256.9\,(65) \\ &= \$8988.84 \end{aligned}$$

Continuing the keystrokes from 5), r, r^2, and s can be found;

6) 256.9 [SAVE ↑] 9.48 [×] 2497.8 [÷] .98 r

2 [y^x] .95 r^2

7) 800366386 [SAVE ↑] 7709.66 [CHS] [SAVE ↑]

67934 [×] [−] 256.9 [SAVE ↑] 5148096.5

[×] [−] 6 [SAVE ↑] 2 [−] [÷] [ ] [y^x] ($\sqrt{x}$) 626.19 s

CANADIAN MORTGAGE CALCULATIONS

All the monthly payment mortgage calculations explained thus far in this text have based their solutions on the United States convention of compounding interest monthly. In Canada, interest is compounded semi-annually with payments occurring monthly, resulting in a different monthly mortgage factor than programmed in the HP-80. This difference can be handled easily on the HP-80 by the addition of a few keystrokes. For any problem requiring an input for [i], the Canadian mortgage factor is calculated first and then this value is entered for [i] in the calculation to give the answer for Canada.

To find this factor, information is entered as follows:

1) Enter 6 [n] 1 [PV].
2) Enter the annual interest rate [SAVE ↑].
3) Enter 200 [÷] 1 [+] [FV] [i] to obtain the Canadian monthly mortgage factor.

The examples below show how this factor is used for [i] in Canadian mortgage problems.

Example 1:
Periodic Payment Amount

What is the monthly payment required to fully amortize a 30 year, $30,000.00 mortgage if the interest rate is 9%?

Solution

	Enter:	See Displayed:	
1)-2)	6 [n] 1 [PV] 9 [SAVE ↑]		
3)	200 [÷] 1 [+] [FV] [i] [STO]	.74	Canadian mortgage factor
	30 [SAVE ↑] 12 [×] [n]	360.00	total monthly periods in mortgage life
	[RCL] [i]	.74	monthly interest factor
	30000 [PV] [PMT]	237.85	monthly payment

Example 2:

Number of Periodic Payments to Fully Amortize a Mortgage

An investor can afford to pay $380 per month on a $56,000 mortgage. If the annual interest rate is 7¾%, how long will it take to completely amortize this mortgage?

Solution

	Enter:	See Displayed:	
1)-2)	6 [n] 1 [PV] 7.75 [SAVE ↑]		
3)	200 [÷] 1 [+] [FV] [i]	.64	Canadian monthly mortgage factor
	380 [PMT]	380.00	monthly payment
	56000 [PV] [n]	435.67	total monthly payments
	12 [÷]	36.31	total years

EQUITY INVESTMENT ANALYSIS

Equity Investment Analysis is a method of evaluating income producing real estate investment alternatives on a *pretax* basis. Two key factors in this type of analysis are the anticipated income stream that the property will provide and the property's projected resale value at the end of the investment horizon. Based on this and the current price of the property, an equity yield rate can be found giving an indication of the profitability of the investment.

One of the basic equations used in real estate equity analysis relates the income stream, sales price, projected appreciation or depreciation, and amount of mortgage as follows:

$$R = Y - MC \begin{matrix} -\text{Apprec.} \\ +\text{Deprec.} \end{matrix} \frac{1}{S_n} = \frac{\text{NOI}}{\text{Price}} = \text{Overall Capitalization Rate}$$

Where

R = reversion
Y = equity yield rate
M = mortgage
C = mortgage coefficient (imbedded in calculation)
$1/S_n$ = sinking fund factor for depreciation or appreciation
NOI = net operating income

Without using any tables the HP-80 enables the user to evaluate his potential investment to determine if it meets his objectives. Solutions for equity yield rate, equity investment value, and future sales price are all included in the sections which follow.

A brief explanation of terms frequently used in real estate analysis is given here in order to aid in understanding the problems and results more fully.

Annual Net Cash Flow is the annual net operating income minus the annual debt service (i.e., annual mortgage payments).

Reversion is the future sales price minus the mortgage balance at the end of the projection period.

Equity yield rate is that annual rate at which the present value of the net annual cash flows plus the present value of the equity reversion equals the equity investment value.

Equity investment value is the equity in the property at the beginning of the projection period.

Overall Capitalization Rate is the net operating income divided by the selling price.

NOTE:

Keystroke explanations for calculations explained earlier in the text will not be repeated here (e.g. periodic payments, appreciation or depreciation, remaining balance).

EQUITY YIELD RATE

Given the projection period in years, reversion amount, annual net cash flow, and equity investment value, the equity yield rate can be calculated as follows:

1) Calculate and enter reversion; press [SAVE ↑] 100 [÷] [STO] .

2) Enter number of years projection; press [SAVE ↑] 2 [÷] 365 [×] [n] .

3) Enter net annual cash flow; press [RCL] [÷] 2 [×] [PMT] .

4) Enter equity investment value, press [RCL] [÷] [PV] .

5) Press [ ] [i] (YTM) 2 [÷] to obtain equity yield rate.

Example:

An apartment complex is listed for $1,960,500 and has an annual net operating income of $166,315.40. The prospective buyer is considering a down payment of $572,500 and will finance the remaining $1,388,000 for 29 years at 8%. If the property appreciates a total of 20% over the next 10 years, what would the equity yield rate be?

Solution

Using calculations from other sections it is found that the monthly mortgage payments are $10,270.45 and therefore the annual net cash flow is $43,070.

(NOI − debt service = net cash flow).

The remaining mortgage balance at the end of 10 years will be $1,201,922.57.

To calculate the reversion at the end of the tenth year, find the future sales price and subtract the remaining balance.

	Enter:	See Displayed:	
	1960500 SAVE ↑ 20 % +	2352600.00	future sales price
	1201922.57 −	1150677.43	reversion
	To find equity yield rate:		
1)	1150677.43 SAVE ↑		
	100 ÷ STO	11506.77	
2)	10 SAVE ↑ 2 ÷ 365		
	× n	1825.00	
3)	43070 RCL ÷ 2 × PMT	7.49	
4)	572500 RCL ÷ PV	49.75	
5)	[orange key] YTM i 2 ÷	13.00	equity yield rate

EQUITY INVESTMENT VALUE AND PRESENT VALUE

Given the desired equity yield rate, projection period, annual net cash flow, and the reversion, the HP-80 can solve for the equity investment value and present value of the investment (curent sales price).

Information is entered as follows:

1) Enter projection period in years; press n . Enter equity yield rate in percent; press i . Enter the reversion, press FV PV STO to find the present value of the reversion.

2) Enter the projection period in years; press n . Enter the equity yield rate in percent; press i .
Enter the annual net cash flow press PMT PV to obtain the present value of the net cash flows.

3) Press RCL + for the equity investment value.

4) Enter mortgage amount, press + to obtain current sales price or present value.

Example:

An investor has some money he wants to invest in real estate. One of his alternatives is a warehouse, currently leased for 10 years, which generates $26,460 annually before debt service (NOI). Because the warehouse is located in a growth area, he estimates the property should sell for $420,000 at the end of 10 years. He can obtain an 8½%, 20 year mortgage for $240,000 which would have monthly payments of $2,082.78. If his desired yield is 11% over 10 years, what would his equity investment value be and how much could he pay for the property (what is the current sales price)?

Solution

	Enter:	See Displayed:	
	Calculate Reversion		
	8.5 SAVE↑ 12 ÷ i	0.71	periodic interest rate
	2082.78 STO PMT	2082.78	
	240000 PV n	240.00	exact number of payments to amortize loan
	120 − n	120.00	
	8.5 SAVE↑ 12 ÷ i	0.71	
	RCL PMT PV	167984.38	remaining loan balance after 10 years
	CHS 420000 + STO	252015.62	reversion value
1)	10 n 11 i RCL FV		
	PV STO	88755.99	present value of reversion
2)	10 n 11 i 2082.78		
	SAVE↑ 12 × CHS	−24993.36	annual debt service
	26460 + PMT PV	8637.38	present value of net cash flows
3)	RCL +	97393.37	equity investment value
4)	240000 +	337393.37	current sales price

FUTURE SALES PRICE AND OVERALL DEPRECIATION/APPRECIATION RATE

This calculation solves for the sales price at the end of the projection period given the desired equity yield rate, annual net cash flow, equity investment value, projection period, and the mortgage balance at the end of the projection period.

Information is entered as follows:

1) Enter the projection period in years press [n]; enter equity yield rate press [i]; enter annual net cash flow press [PMT] [FV] [STO] to get the future value of the annual net cash flows.

2) Enter the projection period in years press [n]; enter equity yield rate press [i]; enter equity investment value press [PV] [FV] to find the future value of the equity investment; press [RCL] [−] to get the reversion amount.

3) Enter mortgage balance at the end of projection period, press [+] to obtain the required future sales price.

4) Enter the purchase price, press [x⇄y] [ ] [Δ% / %] to obtain the overall appreciation (if the answer is positive) or depreciation (if the answer is negative).

NOTE:
These same keystrokes could be used for investment property where there is a net cash outflow instead of income (e.g. Land with no improvements would have monthly debt service and tax payments). The only modification to the above keystrokes would be in step one. After entering the net cash flow, press [CHS] and then press [PMT] [FV] [STO].

Example:

A shopping center has an annual net cash flow of $14211.24. The desired equity yield rate is 14% over a 9 year period. If the current asking price is $616,000 what must the sales price at the end of year 9 be in order to achieve the desired 14% return? What overall appreciation does this represent?

(Assume 25% equity ($154,000), 25 year mortgage at 8%, monthly payment = $3,565.79, leaving a remaining balance of $385,522.04 at the end of year 9).

Solution

	Enter:	See Displayed:	
1)	9 [n] 14 [i] 14211.24		
	[PMT] [FV] [STO]	228592.72	future value of cash flows
2)	9 [n] 14 [i]		
	154000 [PV] [%FV]	500800.07	future value of equity investment
	[RCL] [−]	272207.35	reversion
3)	385522.04 [+]	657729.39	future sales price
4)	616000 [x⇄y] [ ] [Δ%]	6.77	overall appreciation

ADDITIONAL CONSIDERATIONS

In the preceding sections of this chapter, all investment analyses were calculated on a pretax basis. However, income taxes play an important part in the profitability of an investment, possibly making the difference between a very profitable undertaking and a marginal one. Although it is beyond the scope of this book to cover all tax considerations, it should be noted that by combining keystroke solutions listed in previous chapters, many after tax problems can be handled on the HP-80.

To do an after tax analysis, the yearly cash flows must be after tax. These cash flows can change depending on the depreciation method used, mortgage interest rate, and other factors.

Using an accelerated method (declining balance or SOD) causes high depreciation in the early years of ownership and thus lower taxable income and higher after tax cash flows. Based on IRS guidelines and investor's objectives, the right depreciation method can be chosen and the amount calculated (see Chapter 4, Depreciation Calculations). Yearly accumulated interest can be calculated using Chapter 5, Simple Mortgages.

The after tax cash flow in the year of resale requires considerations in addition to the yearly depreciation, expenses, interest, and income taxes. Transaction costs, excess depreciation, recapture, capital gains tax, and ordinary income tax must all be factored into the calculation to arrive at a net after tax figure. Once all of the yearly tax flows have been calculated, undoubtedly they will be uneven so Chapter 10, Discounted Cash Flow Analysis, can be used to find an after tax net present value or yield rate.

Hopefully the above comments have generated ideas and additional considerations to be used in analysis, enabling you to match your needs and the HP-80's capabilities in order to perform after tax calculations.

The examples below,[1] are just two possible applications of an after tax approach. In both cases, the annual cash flows are net of expenses, depreciation, interest, and income tax with the final year amount including the net proceeds from resale.

[1]*We wish to thank Mr. LeR Burton of Salt Lake City, Utah, for these two examples.*

Example 1:

What initial investment amount will yield 9% on the following projected after tax cash flows (net spendable incomes)?

Year	Cash Flow
1	$12,000
2	$11,000
3	$10,000
4	$ 9,000
5	$28,000 (includes reversion)

Solution

The required initial investment equals the present value of the cash flows discounted at the required 9% yield rate.

Enter:	See Displayed:	
[orange key] CLEAR CLx		
9 i 0 PV	0.00	

(To find present value of uneven cash flows, enter 0 as initial investment.)

12000 PV Σ+	11009.17	
11000 PV Σ+	20267.65	
10000 PV Σ+	27989.49	
9000 PV Σ+	34365.32	
28000 PV Σ+	52563.39	required investment

Example 2:

An investor purchases an investment property for $572,500 down payment with the following after tax cash flows (net spendable incomes) over a six year period.

Year	Cash Flow
1	$65,786
2	$63,575
3	$61,321
4	$59,017
5	$56,465
6	$54,255

What is the amount of the reversion necessary to produce an internal rate of return of 10.6%?

(Reversion for after tax analysis equals the sales price minus the mortgage balance, income tax, and transaction cost at the end of the projection period.)

Solution

Enter:	See Displayed:
1) Find Net Present Value of Cash Flows	
[gold] CLEAR [CLx]	
10.6 [i] 572500 [CHS] [PV]	−572500.00
65786 [PV] [Σ+]	−513018.99
63575 [PV] [Σ+]	−461046.19
61321 [PV] [Σ+]	−415720.56
59017 [PV] [Σ+]	−376278.77
56465 [PV] [Σ+]	−342159.19
54255 [PV] [Σ+] [STO]	−312517.08
2) Solve for the future value of the present value of the cash flows	
6 [n] 10.6 [i] [RCL]	
[PV] [FV] [CHS]	572011.19 reversion required to achieve a 10.6% yield rate

CONCLUSION

In this book we have endeavored to provide basic calculations for Real Estate and other business purposes. Obviously, many more solutions are possible on the HP-80. It is hoped that from the basic examples given here, the reader will be able to recognize similar problem types and combine solutions applicable to his individual needs, thus shortening his calculation time and expanding the usefulness of the HP-80.

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